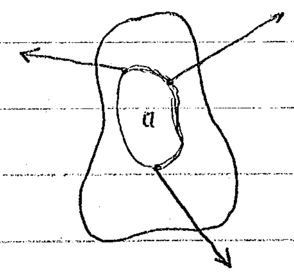
EGOR P. POPOV - Introduction to Mechanics of Solids

References
E.P. POPOV - Mechanics of Materials
BYARS & SNYDER - Engineering Mechanics of Deformable Bodies
TIMOSHENKO & YOUNG - Elements of Strength of Materials
SINGER - Strength of Materials
GRINTER - Engineering Mechanics
BEER & ~~SMITH~~ JOHNSTON - AE 100 Book
SEELY & SMITH - Advanced Mechanics of Materials.

H.W. Read Pages 1-53 Prob 2 1, 2, 3, 5, 7, 9, 11

BASIC CONCEPTS - STATICS



Full Knowledge (Practical) must be had in:

a) Resolution & Vectoral Manipulation of Forces
b) Equilibrium Problems, equilibrium conditions
c) Free-Body diagram.

Look at structure through strength, stiffness, stability

Strength - ability to withstand load without breaking
stiffness - the ability to withstand change in dimensions (elasticity)
stability - depends on structure & dimensions of body
support given load w/o experiencing sudden change in its configuration
We will use Method of Sections - If body in equilibrium, any portion of body is equilibrium.

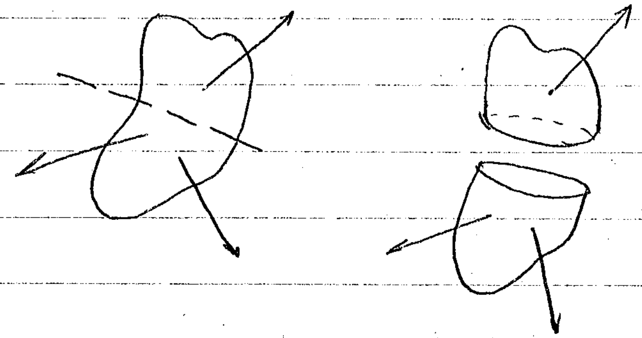
Method of Sections

Approach No.

1. Free-Body diagram showing all Forces - body must be in

equilibrium.

2. Pass arbitrary plane through body separating two pieces.



MAIN Problem: Approach

1) Isolate Member to be analyzed or designed.

2) Find reactions by

   a) Statics $\Sigma F_x = \Sigma F_y = \Sigma F_z = \Sigma M_x = \Sigma M_y = \Sigma M_z = 0$

   b) Differential Equations & Boundary Conditions.

   c) If Indeterminate, By Kinematic Considerations – displacements are defined, restraints put on member.

3) Apply Method of Sections

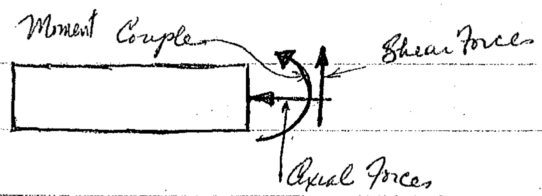
4) At Section find internal forces (INTERNAL REACTIONS)

5) Use appropriate formulas to find stresses

6) In Design, select appropriate material & member.

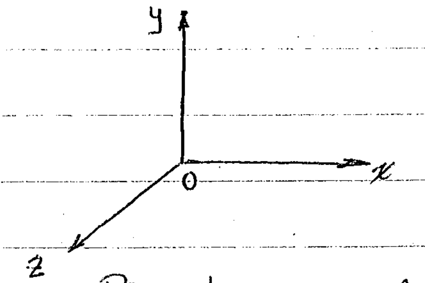
7) Predict deformations of the body

Internal Forces



Equilibrium

Solid body – $\Sigma F_x = \Sigma F_y = \Sigma F_z = \Sigma M_x = \Sigma M_y = \Sigma M_z = 0$



Plane body – neglect all distances & dimensions in z direction

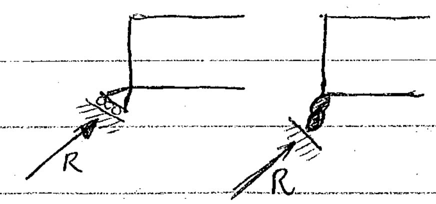
Forces in plane of body.

$\Sigma F_x = \Sigma F_y = \Sigma M_z = 0$

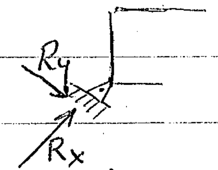
Dimensions will be taken as if before deformation

Supports

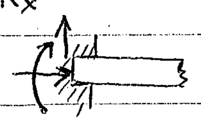
a) Roller or Link

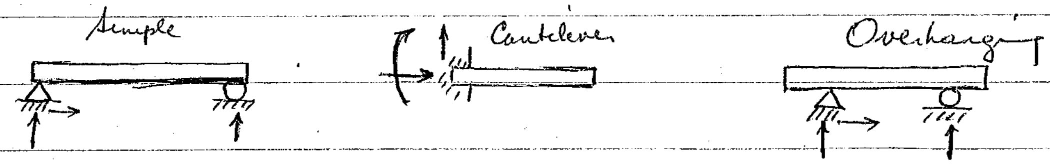


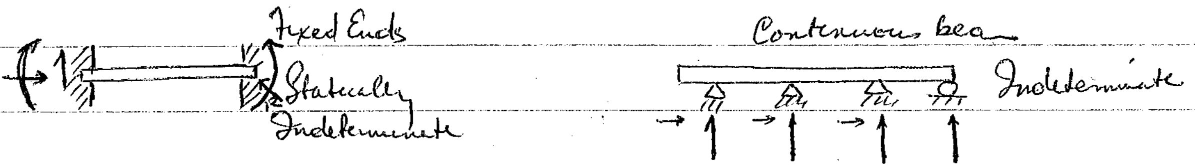
b) Pin



c) Fixed







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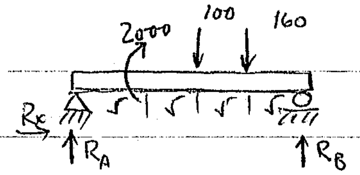


$R_x = 0$

$\Sigma F_y = R_A + R_B - 260 = 0$

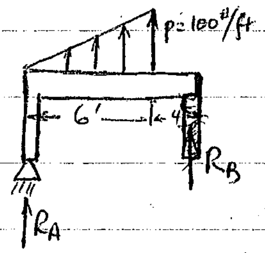
$R_A + R_B = 260$

$\Sigma M_B = 0 \quad R_A \times 20 + 2000 - 100 \times 10 - 160 \times 5 = 0$

$R_A \times 1 + 100 - 50 - 40 = 0 \quad R_A = -10^\# = 10^\# \downarrow$

$R_B = 270^\# \uparrow$

②



$\Sigma F_y = 0$

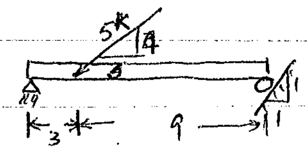
$\Sigma F_y = R_A + R_B + \frac{1}{2} \times 6 \times 100 = 0$

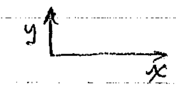
$R_A + R_B = -300$

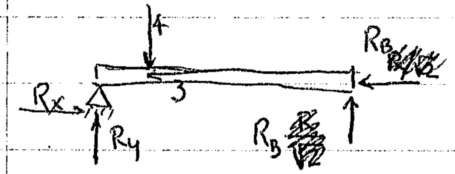
$\Sigma M_B = 0 \quad R_A \times 10 + 300 \times 6 = 0; \; R_A = -180^\# \quad 180^\# \downarrow$

$R_B = 120^\# \downarrow$

③







$R_x - 3^K - R_B = 0$

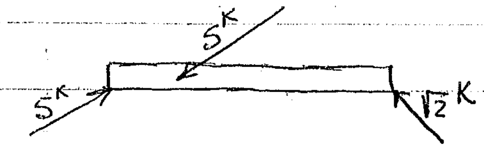
$\boxed{R_x - R_B = 3^K}$

$R_y - 4 + R_B = 0$

$\boxed{R_y + R_B = 4^K}$

$\Sigma M_B = 0 \quad R_y(12) - 4(9) = 0 \quad R_y = 3^K \quad R_B = 1^K$

$R_x = 4^K$



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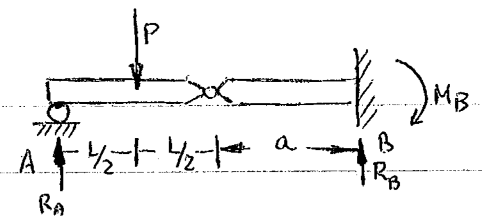
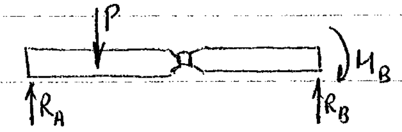
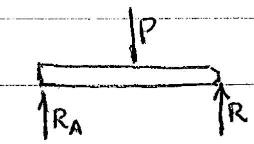


Find R_A, R_B, M_B



$\sum F_y = R_A + R_B - P = 0$ $\quad \boxed{R_A + R_B = P}$

$\sum M_B = R_A(L+a) - P(L/2 + a) + M_B = 0$ $\quad \boxed{R_A(L+a) + M_B = P(L/2 + a)}$



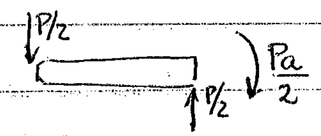
$R_A - P + R = 0$ $\quad \boxed{R = P/2}$

$\sum M_e = 0$ $\quad R_A(L) - PL/2 = 0$ $\quad \boxed{R_A = P/2}$

$\boxed{R_B = P/2}$ $\quad \frac{P}{2}(L+a) + M_B = \frac{P}{2}(L+2a)$

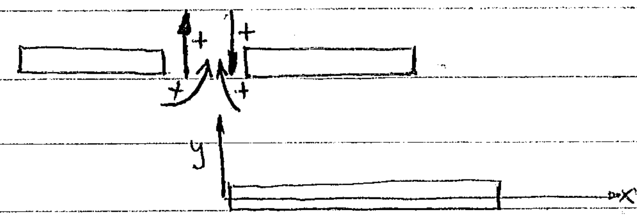
$M_B = P/2\,(L + 2a - L - a) = \frac{Pa}{2}$

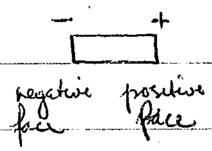
$\boxed{M_B = \frac{Pa}{2}}$



$\sum F_y = 0$ ✓ $\quad \sum M_B = 0$ ✓

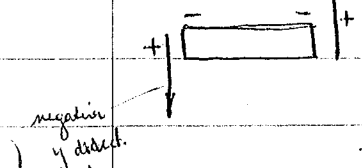
Sign Convention for problems



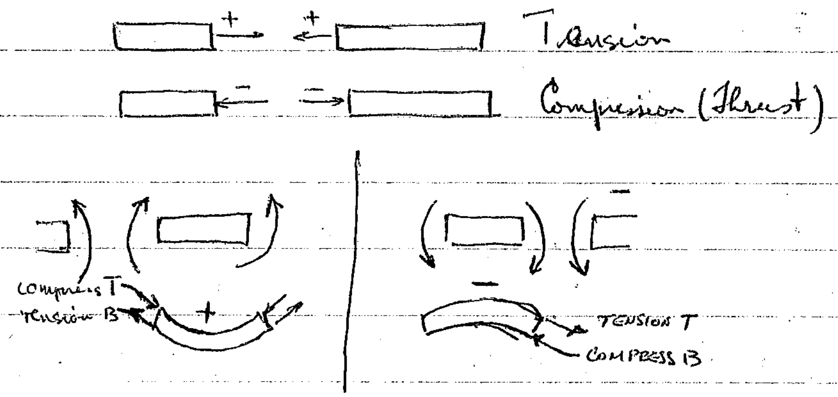


relative to the other end

depends on direction & face.



Shear



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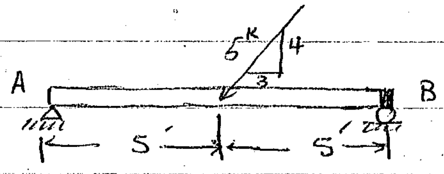
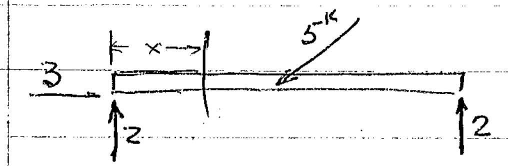
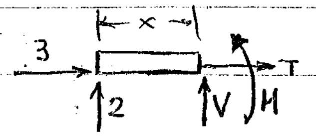


$\boxed{R_{A_x} = 3^K \longrightarrow}$

$R_{Ay} - 4 + R_B = 0$

$\curvearrowright \sum M_A = 4(5') - 10' R_B = 0$ $\boxed{R_B = 2^K \uparrow}$ $\boxed{R_{Ay} = 2^K \uparrow}$



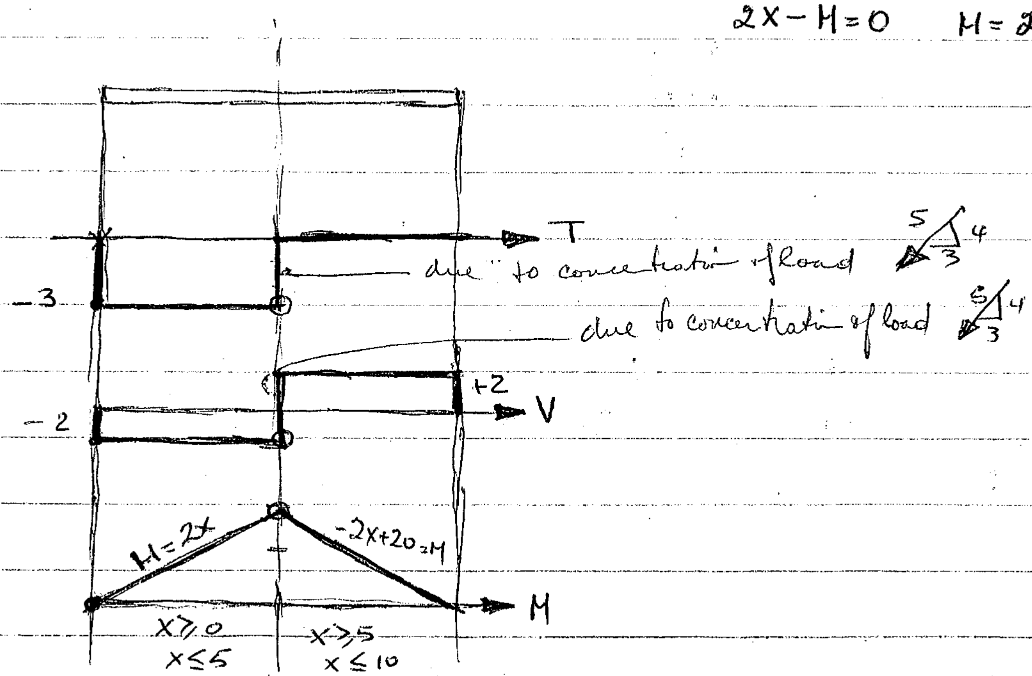


$0 \leq x < 5'$

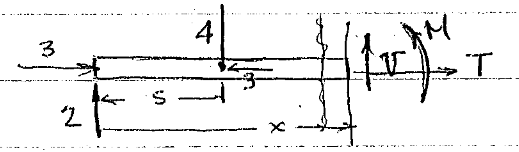
$T = 3^K \longleftarrow$ $V = 2^K \downarrow$

Always take M about cut

$2x - M = 0$ $M = 2x$



$5 \leq x \leq 10$



$\Sigma F_x = 0$

$3 - 3 + T = 0$

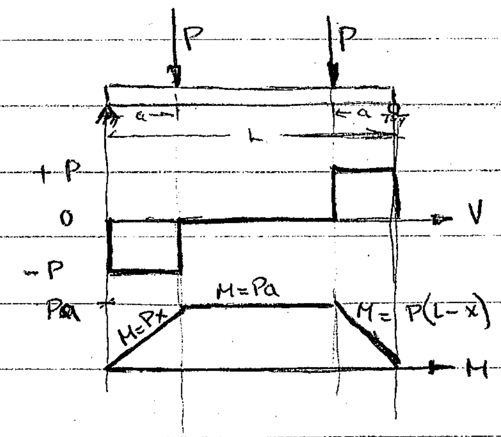
$T = 0$

$\Sigma F_y = 0 \quad 2 - 4 + V = 0 \qquad V = +2$ kips

$\Sigma M_C = 2x - 4(x-5) - M = 0 \qquad -2x + 20 = M$

Shear curve displaces the curve an equal amount in the opposite direction to the direction of the concentrated load

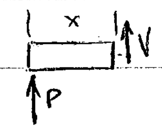
Axial curve is displaced an equal amount in the same direction as the concentrated load.



$R_A + R_B = 2P \; ; \; \Sigma F_y = 0$

$\Sigma M_B = 0 \; ; \; R_A(L) - P(L-a) - P(a) = 0$

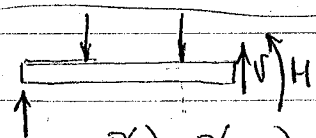
$R_A(L) - PL = 0 \qquad \boxed{R_A = P \quad R_B = P}$



$0 \leq x \leq a$

$V = -P$

$M = P(x) \quad x \geq 0, \; x \leq a$



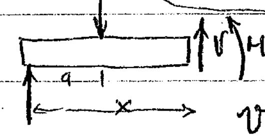
$V = P$

$P(x) - P(x-a) - P(x-(L-a)) - M = 0$

$\cancel{P(x)} - \cancel{P(x)} + P(a) - Px + P(L) - Pa - M = 0$

$\boxed{M = P(L) - P(x)} \quad x \geq L-a, \; x \leq L$

when $x = (L-a)$



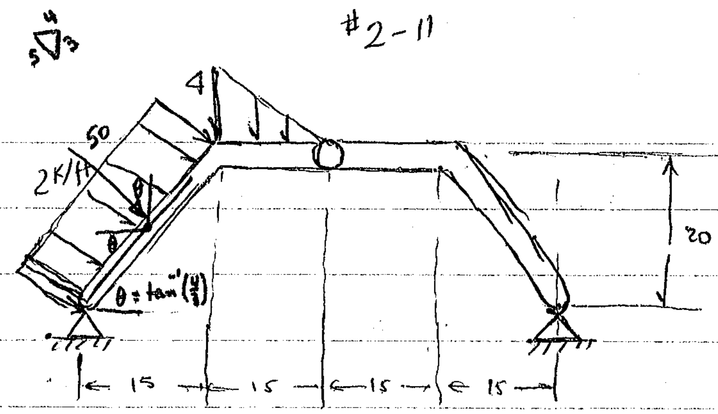
$a \leq x \leq L-a$

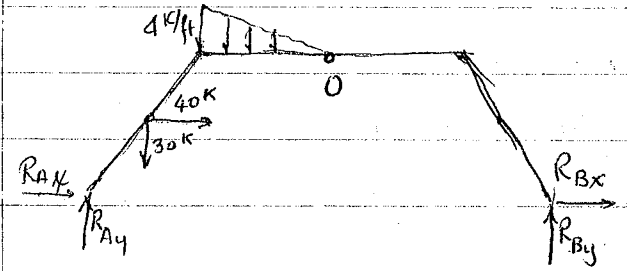
$V = 0$

$P(x) - P(x-a) - M = 0$

$+Pa = M \quad x \geq a, \; x \leq L-a$

#2-11





$\Sigma F_x = 0$

$R_{Ax} + R_{Bx} + 40 = 0$

$R_{Ax} + R_{Bx} = -40$

$\Sigma F_y = 0$

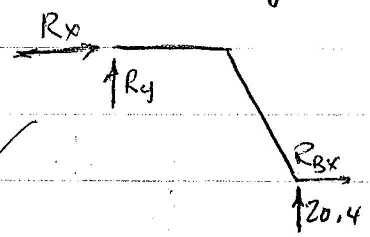
$R_{Ay} - 30 - \frac{1}{2} \times 15 \times 4 + R_{By} = 0$

$R_{Ay} + R_{By} = 60^K$

$\Sigma M_A = 0$: $50 \times 12.5 + \frac{1}{2} \times 15 \times 4 \times 20 - R_{By} \times 60 = 0$

$R_{By} = \frac{5}{6} \times 12.5 + 10 = 20.4^K$

$R_{Ay} = 39.6^K$



$\Sigma M_0 = 0$: $-R_{Ax} \times 20 + 39.6 \times 30$

$- 30 \times 22.5 - 40 \times 10 - \frac{1}{2} \times 4 \times 15 \times 10$

$+ R_{Bx} \times 20 + 20.4 \times 30 = 0$

$R_{Ax} + R_{Bx} = (39.6 - 20.4) \times \frac{30}{20} - \frac{30}{20} \times 22.5 - 35$

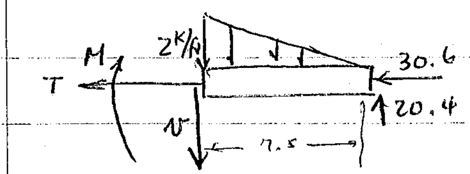
$(-3.3) \times \frac{30}{20} - 35$

$-35 - 5 = -40^K$

$-20.4(30) - R_{Bx}(20) = 0$ $\quad R_{Bx} = +30.6 \leftarrow$

$R_{Ax} = +9.4 \leftarrow$

Since we know everything about right member we can isolate the smaller member betwe pin & cut therefore



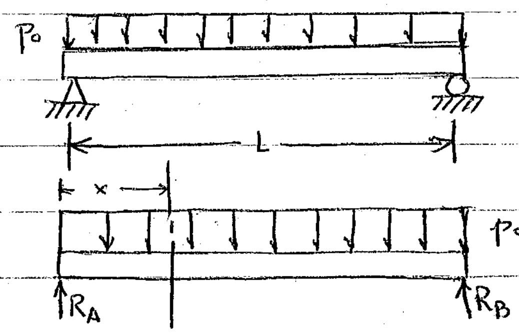
$\sum F_x = 0 \qquad -T - 30.6 = 0 \qquad T = -30.6^K$

$\sum F_y = 0 \qquad -V + 20.4 - \frac{1}{2}(2)(7.5) = 0 \qquad V = 12.9^K$

$\sum M_{\text{at cut}} \circlearrowright = 0 \qquad M + \frac{1}{2}(2)(7.5)(2.5) - 20.4(7.5) = 0$

$M = (17.9)(7.5) \cong 135 \text{ K-ft.}$

Consider

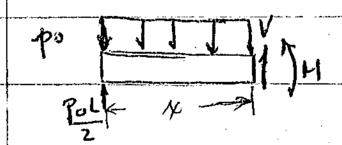


$\sum F_y = R_A - p_o L + R_B = 0$

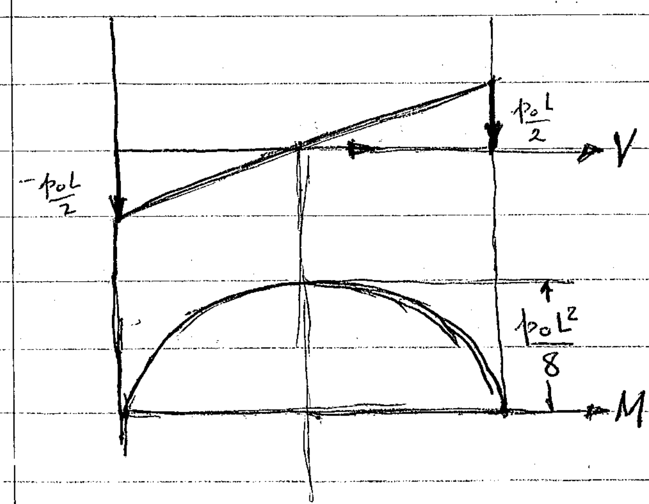
$\boxed{R_A + R_B = p_o L}$

$\sum M_B = R_A L - p_o L \frac{L}{2} = 0$

$\boxed{R_A = \frac{p_o L}{2} \quad R_B = \frac{p_o L}{2}}$



$\sum F_y = 0 \qquad \frac{p_o L}{2} - p_o x + V = 0 \qquad V = p_o\left(x - L/2\right)$

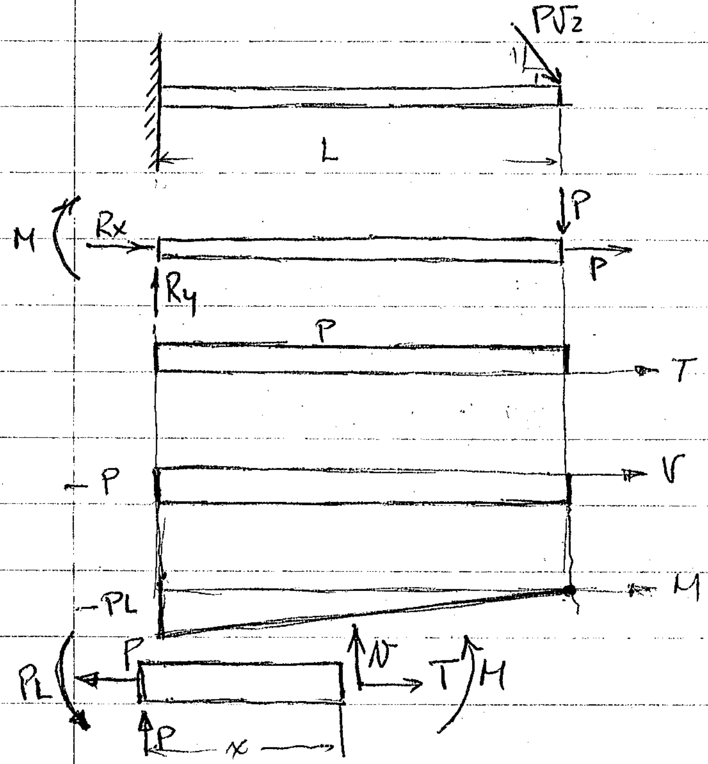


$\sum M_c = 0 \circlearrowright \qquad \frac{p_o L}{2} x - \frac{p_o x^2}{2} - M = 0$

$M = \frac{p_o x}{2}(L - x)$

$\frac{dM}{dx} = \frac{p_o L}{2} - p_o x = p_o\left(\frac{L}{2} - x\right) = 0$

$x = L/2$



$\Sigma F_x = 0$

$R_x + P = 0 \quad \boxed{R_x = -P}$

$\Sigma F_y = 0$

$R_y - P = 0 \quad \boxed{R_y = P}$

$\Sigma M_A = 0 \quad M + P(L) = 0$

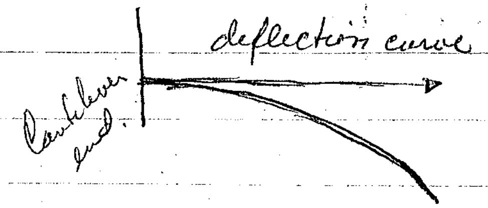
$\boxed{M = -PL}$

$T = P$

$V = -P$

$\Sigma M_C = 0 \quad -PL + Px - M = 0$

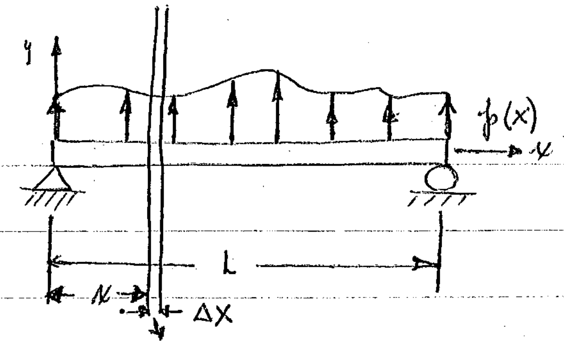
$M = P(x - L)$

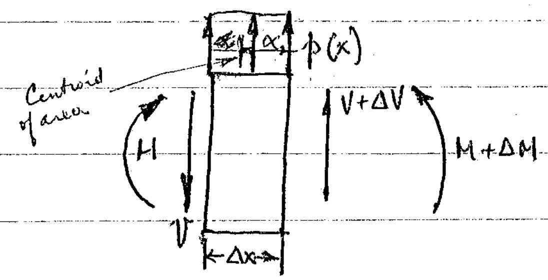


Rd Page 30-31 Problem 2-8 2-15, 17, 19, 21, 29

Steps to solve

1) Draw sketch of Member - which shows all forces & dimensions
2) Unknown reactions are indicated
3) Resolve inclined forces into components
4) Find reactions
5) Cut arbitrary sections (depending on range that is good for that section)
6) Isolate segment to either side of cut. Show forces acting
7) Indicate internal reactions.
8) Find internal unknown reactions.





$$\Sigma F_y = 0$$

$$-V + p(x)\Delta x + V + \Delta V = 0$$

$$\frac{\Delta V}{\Delta x} = -p(x)$$

$$\Sigma M_R = 0 \qquad M + p(x)\Delta x \cdot \alpha \Delta x - V\Delta x - M - \Delta M = 0$$

$$\frac{\Delta M}{\Delta M} = -V + p(x)\alpha\Delta x$$

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = \boxed{\frac{dV}{dx} = -p(x)} \qquad \lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = \boxed{\frac{dM}{dx} = -V}$$

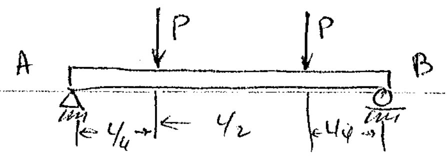
$$\boxed{-\frac{dV}{dx} = p = \frac{d^2M}{dx^2}}$$

$$dV = -p\,dx \qquad \boxed{V = -\int p(x)\,dx + C_1} \quad (1)$$

$$\boxed{M = -\int V(x)\,dx + C_2} \quad (2)$$

$$V_b - V_a = -\int_a^b p(x)\,dx$$

$$M_b - M_a = -\int_a^b V(x)\,dx$$

① 

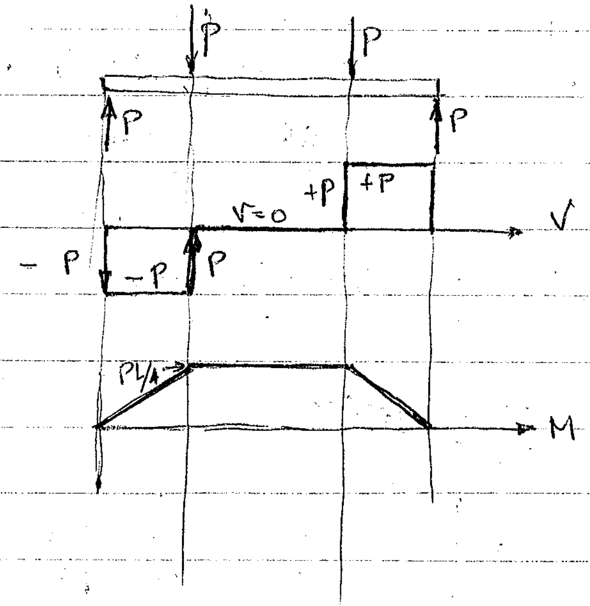
$\sum F_y = 0 \uparrow$ Inspectionally

$R_A + R_B - 2P = 0$

$\boxed{R_A + R_B - 2P}$

$\sum M_B = 0 \circlearrowright$

$R_A L - PL = 0 \qquad R_A = P = R_B$ ✓



Analytically

$p = 0 \qquad 0 \le x \le L/4 \qquad V_1 = C_{11} \qquad M_1 = -C_{11}x + C_{21}$

$C_{21} = 0$; when $x = 0, M = 0$.

$M_1 = -C_{11}x \qquad$ at $x = L/4 \qquad V_1 = C_{11} \qquad M_1 = -C_{11}L/4$

$L/4 \le x \le 3L/4 \qquad V_2 = C_{12} \qquad M_2 = -C_{12}x + C_{22}$

at $x = L/4 \qquad C_{11} + p = C_{12} \Rightarrow C_{11} = C_{12} - p$

at $x = L/4 \qquad -C_{11}\frac{L}{4} = -C_{12}L/4 + C_{22} \Rightarrow (-C_{12} + p)L/4 = -C_{12}L/4 + C_{22}$

$\boxed{C_{22} = pL/4}$

at $x = 3L/4 \qquad V_2 = C_{12} \qquad M_2 = -C_{12}\left(\frac{3L}{4}\right) + p\frac{L}{4}$

$3L/4 \le x \le L \qquad V_3 = C_{13} \qquad M_3 = -C_{13}x + C_{23}$

$C_{12} + p = C_{13} \Rightarrow C_{12} = C_{13} - p$

$-C_{12}\frac{3L}{4} + \frac{PL}{4} = -C_{13}\frac{3L}{4} + C_{23}$

$$-\cancel{C_{13}\frac{3L}{4}} + \frac{P\cdot 3L}{4} + \frac{PL}{4} = -\cancel{C_{13}\frac{3L}{4}} + C_{23}$$

$$\boxed{PL = C_{23}}$$

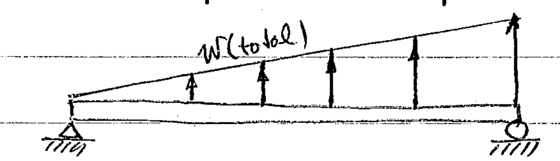
$M_3 = -C_{13}X + PL$ at $X = L$ $M_3 = 0$ $\therefore$ $\boxed{+C_{13} = P}$

$\boxed{C_{12} = 0}$ $\boxed{C_{11} = -p}$

$V_1 = -p$ $V_2 = 0$ $V_3 = P$

$M_1 = +px$ $M_2 = pL/4$ $M_3 = P(L-x)$

(3)



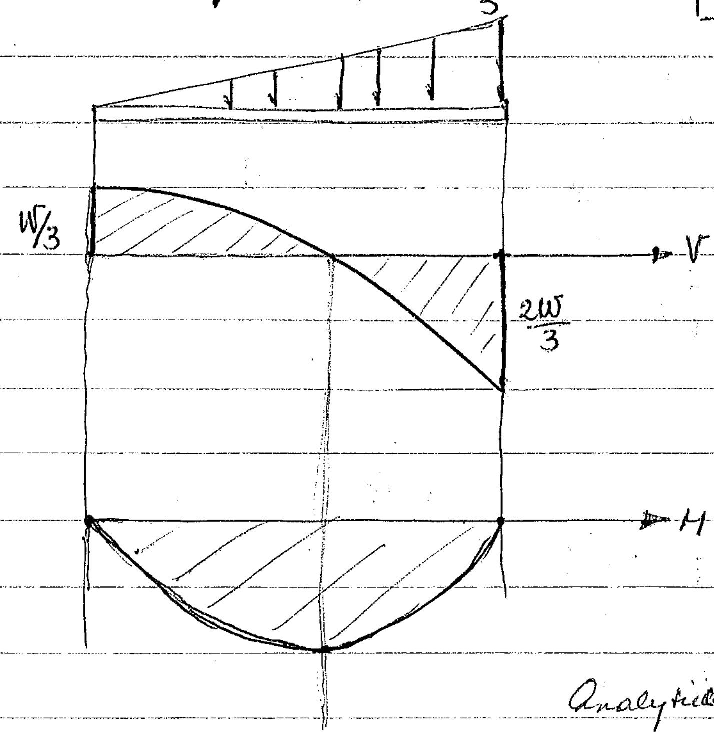
Inspectively

$\Sigma F_y = 0 \uparrow$

$R_A + R_B + W = 0$

$\boxed{R_A + R_B = -W}$

$\Sigma M_B = 0$ $R_A L + \frac{WL}{3} = 0$ $\boxed{R_A = -\frac{W}{3}}$ $\boxed{R_B = -\frac{2W}{3}}$



Analytically

$p = kx$ at L $p = kL$ $W = \frac{1}{2}KL\cdot L$ $k = \frac{2W}{L^2}$

$p = \frac{2W}{L^2}x$

$V = -\frac{\cancel{2}w}{L^2} x^2 + C_1$

$M = \frac{+w}{L^2}\frac{x^3}{3} - C_1 x + C_2$ at $x=0$ $M=0$ $\therefore \boxed{C_2 = 0}$

at $x = L$ $M = 0$

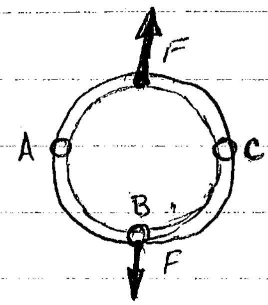
$M = \frac{wL}{3} - C_1 L = 0$ $\boxed{C_1 = \frac{w}{3}}$

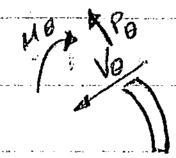
$\boxed{V = -\frac{w}{L^2}x^2 + \frac{w}{3}}$ $M = \frac{w}{3}\left[\frac{x^3}{L^2} - \cancel{x}\right]$

at $V=0$ $x = L/\sqrt{3}$ $M_{max} =$

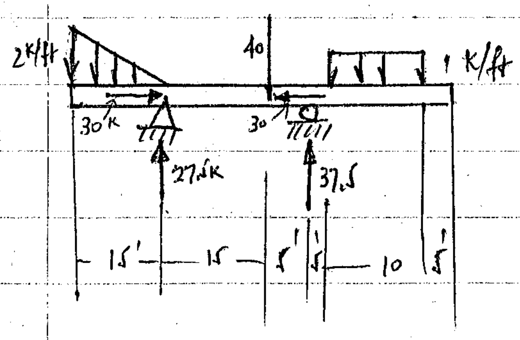
problems: 2-38, 42, 47, 50, 54, 56, 61, 63 Study examples 2-10
P. 37-39 Rd. Chapter 3.

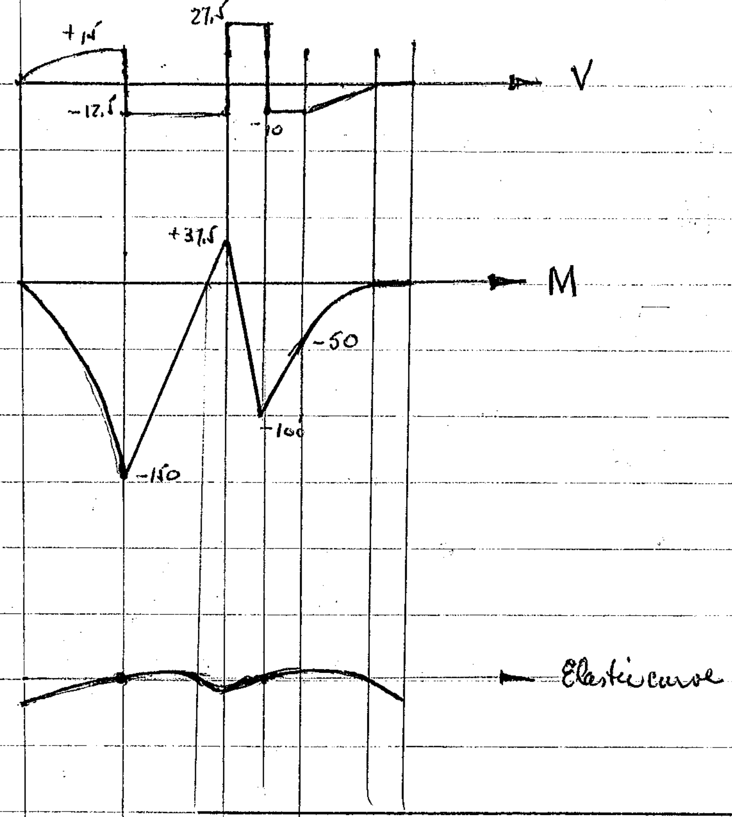
2-29

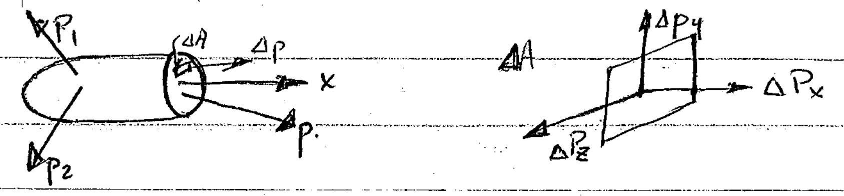




P. 38-9 Example 2-10







ΔP_x - axial (norma[l]

ΔP_y & ΔP_z - shear,

Stress - force intensity per unit area. $\#/in^2 = psi$

$K/in^2 =$ ksi

$$\tau_{xx} = \lim_{\Delta A \to 0} \frac{\Delta P_x}{\Delta A} \rightarrow \begin{matrix}\text{compression} \\ \text{tension}\end{matrix}$$

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta P_y}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta P_y}{\Delta A}$$

$\tau_{a,b}$ a - tells the normal to the plane lies in the a directio[n]

b - direction of force.

when $a = b$ we have an axial type force

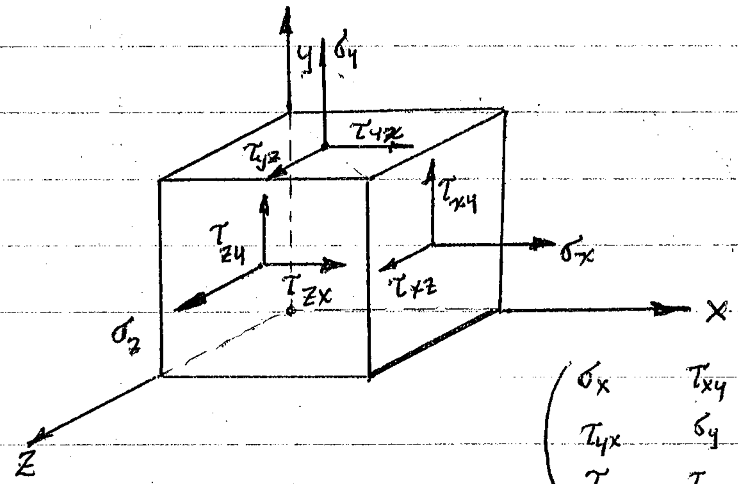
$a \neq b$ we have shear-type forces

$\tau_{xx} \equiv \sigma_x$, $\tau_{yy} \equiv \sigma_y$, $\tau_{zz} \equiv \sigma_z$

Force = Stress by area

if $\sigma_x A_2 = \sigma_x A_1$ we have equilib.

if $\sigma_x A_2 \neq \sigma_x A_1$ we have motion



$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} \Rightarrow \text{Stress Tensor.}$$

Scalar 0-Rank

Vector 1st-Rank Stress-Tensor 2nd rank

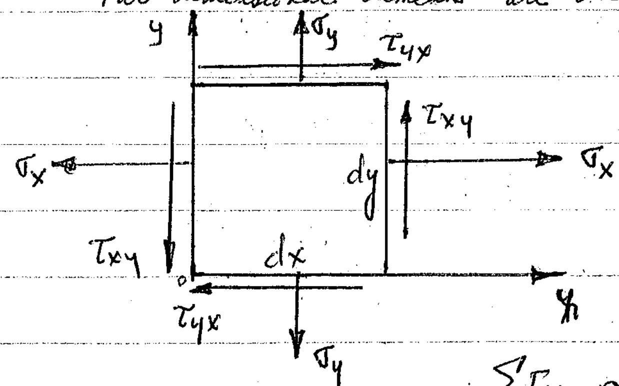
$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$$

tensors $\begin{cases} \text{scalar} - 0 \text{ rank} - t & - 2^0 - 3^0 \\ \text{vector} - 1 \text{ rank} - V_x, V_y, V_z & - 2^1 - 3^1 \\ \text{tensor} - 2 \text{ rank} & - 2^2 - 3^2 \end{cases}$

# of components to describe it in

Plane Free Space

Two dimensional elements are used for the derivation.



Uniform Thickness dz

$\sum F_x = 0$

$\sigma_x \, dy\,dz - \sigma_x \, dy\,dz +$

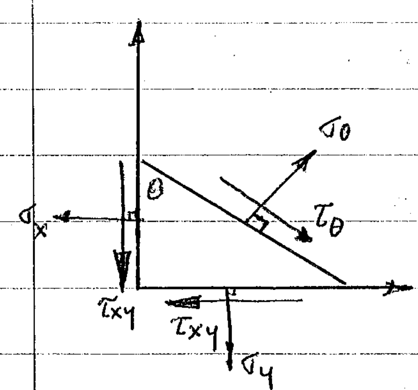
$\tau_{yx}\, dx\,dz - \tau_{yx}\, dx\,dz = 0$

$\sum F_y = 0 \uparrow \quad \sigma_y \, dx\,dz - \sigma_y \, dx\,dz$

$+ \tau_{xy}\, dy\,dz - \tau_{xy}\, dy\,dz = 0$

$\Sigma M_o = 0 \qquad \tau_{yx}\,(dx\,dz)\,dy - \tau_{xy}\,(dy\,dz)\,dx = 0$

$$\boxed{\tau_{yx} = \tau_{xy}} \iff \tau_{xz} = \tau_{zx} \iff \tau_{yz} = \tau_{zy}$$



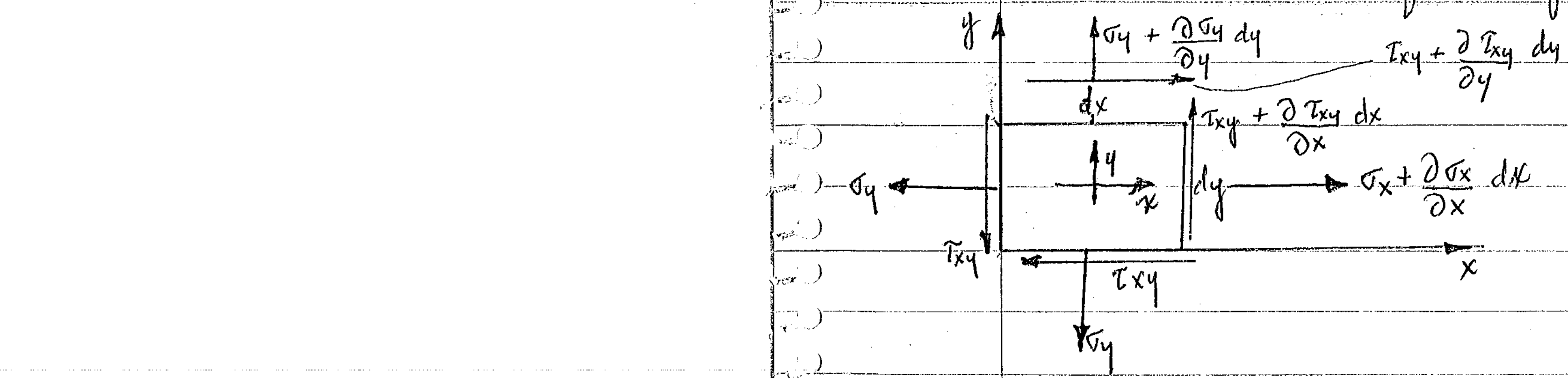
$\sigma_\theta = f(\theta)$
$\tau_\theta = f(\theta)$

since σ_θ depends on $\sigma_x, \sigma_y, \tau_{xy}, \theta$; <u>but</u> $\sigma_x, \sigma_y, \tau_{xy}$ have been already calculated & they are constant; <u>thus</u> σ_θ, and similarly τ_θ, are functions of θ.

we can find some $\theta \Rightarrow \tau_\theta, \sigma_\theta$ are 0 in the xy, yz, xz planes so that we can reduce

$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} \Longrightarrow \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$$

We will derive the equations of unequal stresses



$\Sigma F_x = 0$;
$$\left(\sigma_x + \frac{\partial \sigma_x}{\partial x} dx\right) dy\,dz - \cancel{\sigma_x\, dy\,dz} + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y} dy\right) dx\,dz - \cancel{\tau_{xy}\, dx\,dz} + X\,dy\,dz\,dx = 0$$

$$\Sigma F_y = 0;\quad \left(\sigma_y + \frac{\partial \sigma_y}{\partial y} dy\right) dx\,dz - \cancel{\sigma_y\, dx\,dz} + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x} dx\right) dy\,dz - \cancel{\tau_{xy}\, dy\,dz} + Y\,dx\,dy\,dz = 0$$

$$\boxed{\begin{aligned} &① \quad \frac{\partial}{\partial x}(\sigma_x) + \frac{\partial}{\partial y}(\tau_{xy}) + X = 0 \\ &② \quad \frac{\partial}{\partial y}(\sigma_y) + \frac{\partial}{\partial x}(\tau_{xy}) + Y = 0 \end{aligned}}$$

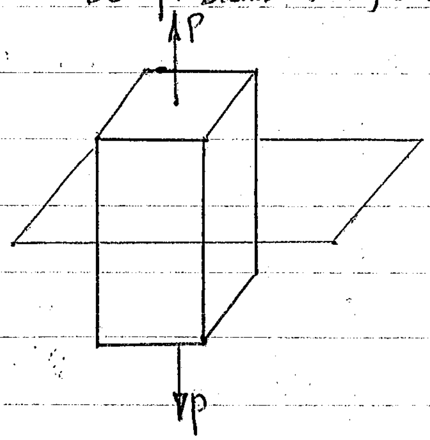
Three dimensionalization

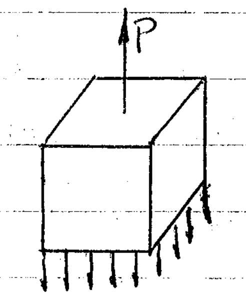
$$\frac{\partial}{\partial x}(\sigma_x) + \frac{\partial}{\partial y}(\tau_{xy}) + \frac{\partial}{\partial z}(\tau_{xz}) + X = 0$$

$$\frac{\partial}{\partial y}(\sigma_y) + \frac{\partial}{\partial x}(\tau_{xy}) + \frac{\partial}{\partial z}(\tau_{yz}) + Y = 0$$

$$\frac{\partial}{\partial z}(\sigma_z) + \frac{\partial}{\partial x}(\tau_{xz}) + \frac{\partial}{\partial y}(\tau_{yz}) + Z = 0$$

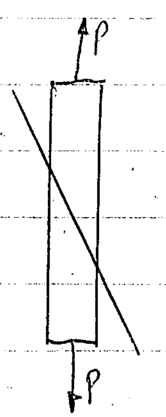
Do problems 3-2, 3-3

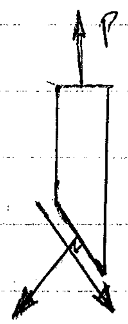


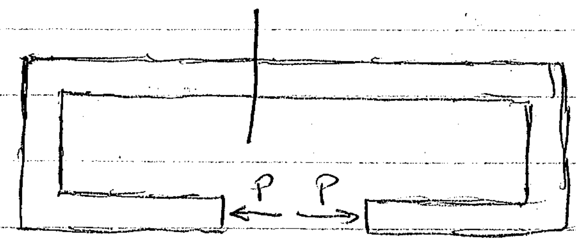


$\sigma = \frac{P}{A}$

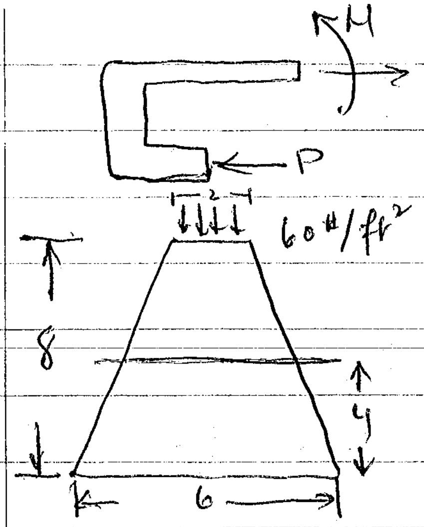
centric load







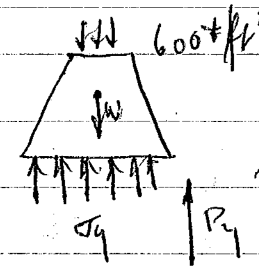
eccentric load



distribution is not uniform

wt of concrete = 120 #/ft³

2 ft wide



Since we have resultant passing thru centroid we can say σ_y must be uniform.

$$b_y = 6 - \tfrac{y}{8}(6-2) = (12-y)/2$$

$$W_y = \frac{\left(2 + \cancel{6}\ \frac{+12-y}{2}\right)}{2}(8-y)\,2 \cdot 150 = 75(8-y)(16-y)$$

$$P_y = W_y + 600(4) = 2400 + 75(8-y)(16-y)$$

$$\sigma_y = \frac{2400 + 75(8-y)(16-y)}{2\left(\frac{12-y}{2}\right)} \qquad \text{if } P_y = \sigma_y \cdot A$$

2nd way

There's a mistake

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + Y = 0 \qquad \text{since } F_x\text{'s are } 0 \text{ we can}$$

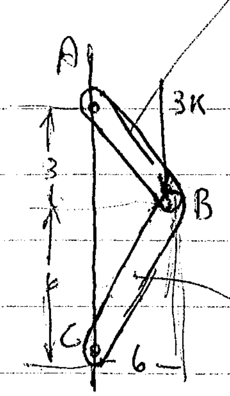
then say $\frac{\partial \sigma_y}{\partial y} + Y = 0$ #/ft³ $\frac{\partial \sigma_y}{\partial y} = -150$ $\int d\sigma_y = \int -150\,dy$

$$\boxed{\sigma_y = -150y + f(x)}$$

at $x=8$ $\sigma = 600$ #/ft²

$600 = -1200 + f(x)$ $f(x) = 1800$ $(\sigma_y = -150y + 1800)$

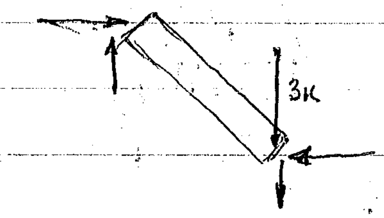
.25"X.50" - cross section area.

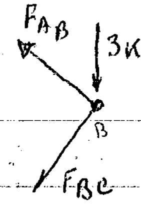


.375" ϕ diameter

Neglect Weight of Members

Consider AB these members must have result in direction of axis of the members.





$\Sigma F_x = 0 \qquad -\frac{2}{\sqrt{5}} F_{AB} - \frac{1}{\sqrt{2}} F_{BC} = 0$

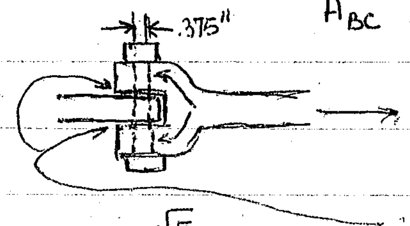
$\Sigma F_y = 0 \qquad \frac{1}{\sqrt{5}} F_{AB} - \frac{1}{\sqrt{2}} F_{BC} = 3$

$\frac{3}{\sqrt{5}} F_{AB} = 3 \qquad F_{AB} = \sqrt{5}\,k\,T \qquad F_{BC} = +2\sqrt{2}\,k\,C$

2.828 KN

Stresses $\qquad \sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{\sqrt{5}}{.025 \times .50} \qquad T$

$\sigma_{BC} = \frac{F_{CB}}{A_{BC}} = \frac{2\sqrt{2}}{.25 \times .875} \qquad C$



2 surface shear

$\tau_{BOLT\,A} = \frac{\sqrt{5}}{\frac{\pi}{4} \times .375^2 \times 2}$

Ultimate Strength.

strength at which member "failed". failure can be total rupture or Excessive deflection.

Factor of Safety - Factor of Ignorance

those factors that can't calculated and those factors which we have doubt.

Allowable Stress -

Maximum stress allowed to occur on job and that aren't exceeded

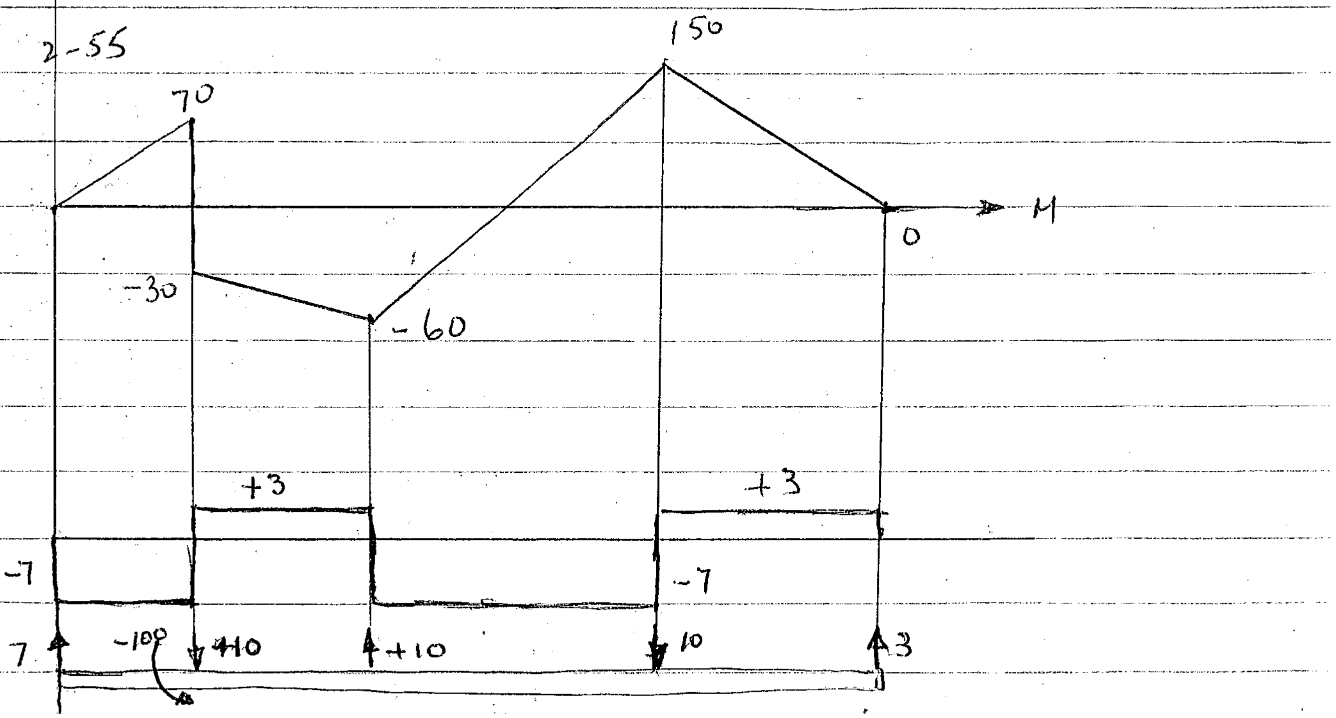
$$\text{Allowable Stress} = \frac{\text{Ult. Strength}}{\text{Factor of Safety}}$$

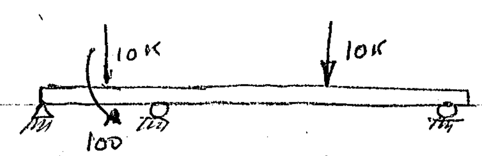
נטית החומר לזרימה פלסטית תחת עומס קבוע לאורך זמן רב.

Creep - tendency of material to flow plastically under sustained load (load should produce stress almost = ultimate stress)

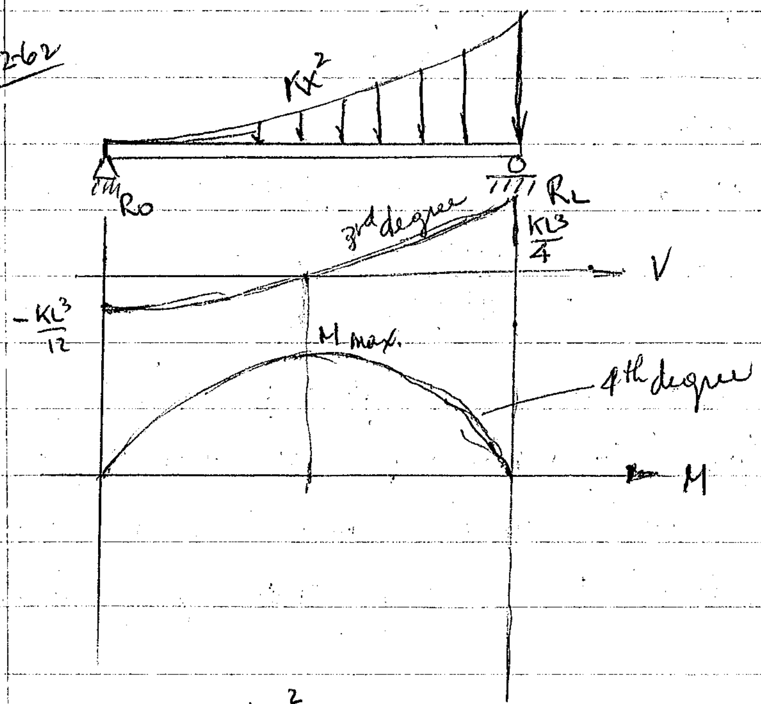
Fatigue - if a material carries a high stress & these stresses are constantly loading & unloading or change of state (T to C to T to C)

Residual stresses -





2-62



$p = -Kx^2$

$V = -\int -Kx^2 dx = \frac{Kx^3}{3} + C_1$

$M = -\int\left(\frac{Kx^3}{3} + C_1\right) = -\frac{Kx^4}{12} - C_1 x + C_2$

at $x = 0$ $M = 0$ $C_2 = 0$

at $x = L$ $M = 0$ $C_1 = -\frac{KL^3}{12}$

$\therefore$ $V = \frac{Kx^3}{3} - \frac{KL^3}{12}$ $M = -\frac{Kx^4}{12} + \frac{KL^3}{12}x$

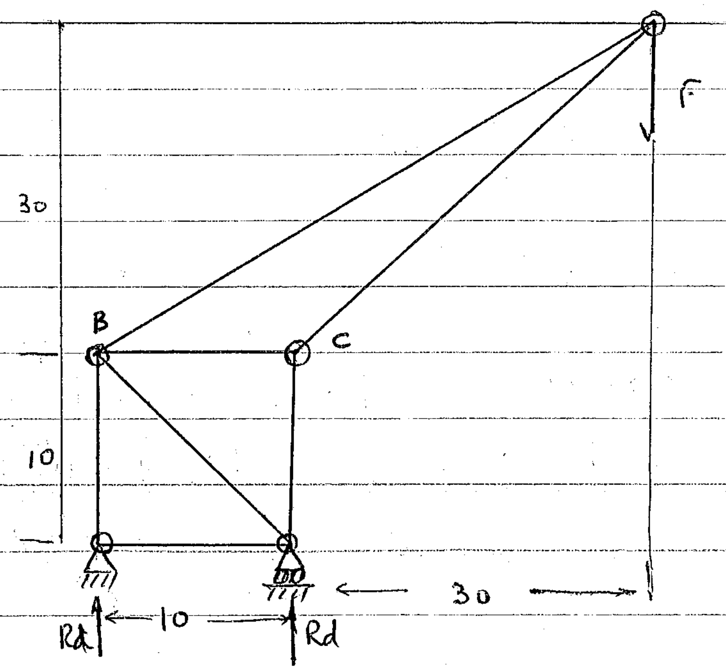
at $V = 0$ $x = \frac{L}{\sqrt[3]{4}}$ $\therefore$ at $V = 0$ $M = max.$

thus $M_{max} = \frac{KL}{12\sqrt[3]{4}}\left[\frac{3L^3}{4}\right] = \frac{KL^4}{16\sqrt[3]{4}}$

$R_0 = -V_0 = +\frac{KL^3}{12}$

$R_L = V_L = \frac{KL^3}{4}$

3-12



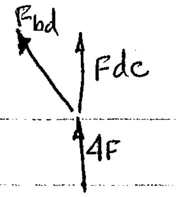
$R_a + R_d = F$

$\Sigma M_d = R_a(10) + F(30) = 0 \qquad R_a = -3F \qquad R_d = 4F$

Member	Force	Stress
AD	0	0
AB	3F (T)	$\frac{3}{8}$F (T)
DC	4F (C)	$\frac{1}{2}$F (C)
BD	0	0
BE	5F (T)	$\frac{5}{8}$F (T)

Member	Force	Stress
BC	4F (C)	$\frac{1}{2}$F (C)
CE	$\frac{2\cancel{\sqrt{2}}F}{\sqrt{2}}$ (C)	$\frac{\cancel{1}\sqrt{2}}{\cancel{2}}$ F (

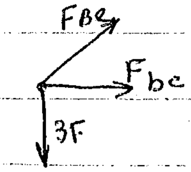
Start at a $\quad$ Fab $\uparrow$, $\rightarrow$ Fad, 3F $\downarrow$ $\qquad$ Fad = 0 $\qquad$ Fab = 3F at d



$F_{bd} = 0$

$F_{dc}\ 4F \downarrow$

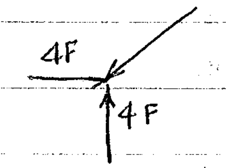
at B



$\frac{3}{5} F_{BE} = 3F \qquad F_{BE} = 5F$

$F_{bc} = -4F\ C$

at c



$F_{CE} = \frac{2}{\sqrt{2}}\ C$

Since $a = 8''$ squared

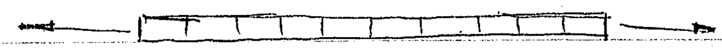
$\frac{5}{8} F \leq 20{,}000 \qquad F \leq 32{,}000 = 16$ TONS

$\frac{4\sqrt{2}}{8} F \leq 15{,}000 \qquad F \leq \frac{30{,}000}{\sqrt{2}} = 15{,}000\sqrt{2}$

This is maximum to be reached first.

$$\begin{array}{r} 15000 \\ \times\ 1.414 \\ \hline 15000 \\ 6000 \\ 150 \\ 60 \\ \hline 21210 \end{array}$$

Deformation - change in dimensions or shape due to stress, temperature



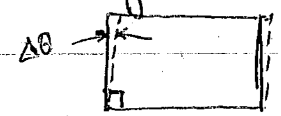
$\Delta l = l - l_0$. $\frac{\Delta l}{l_0} = \frac{l}{l_0} - 1 = \varepsilon$ strain

if uniform strain

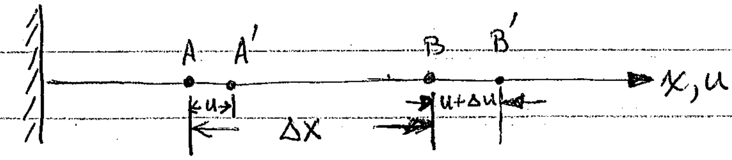
if non-uniform strain $\epsilon = \frac{\Delta l}{l_0} = \int_0^{l_0} \frac{d(\Delta l)}{l_0}$

Order of Magnitude of ϵ: we will take $\epsilon \leq .001$ $\varepsilon \leq .1\%$

Shearing strain - change in $\Delta\theta$ of a right angle of a rectangle

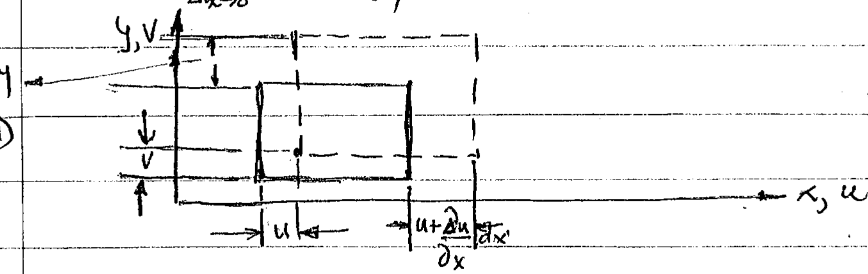


1-D

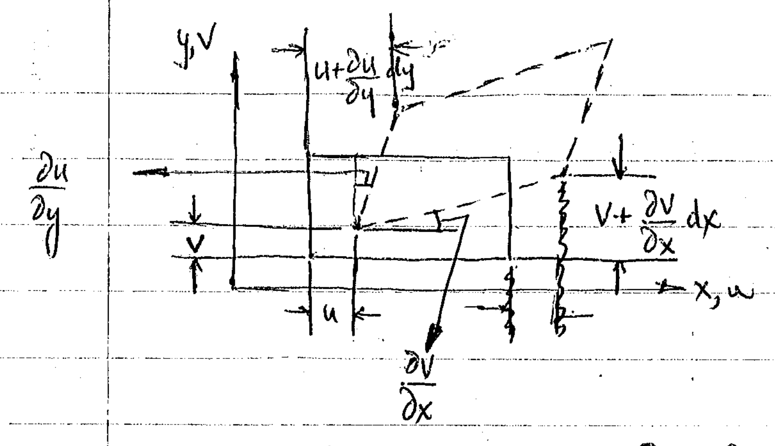


$$\epsilon = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = \frac{du}{dx}$$

2-D



$\epsilon_x = \frac{\partial u}{\partial x}$ $\epsilon_y = \frac{\partial v}{\partial y}$ $\epsilon_z = \frac{\partial w}{\partial z}$

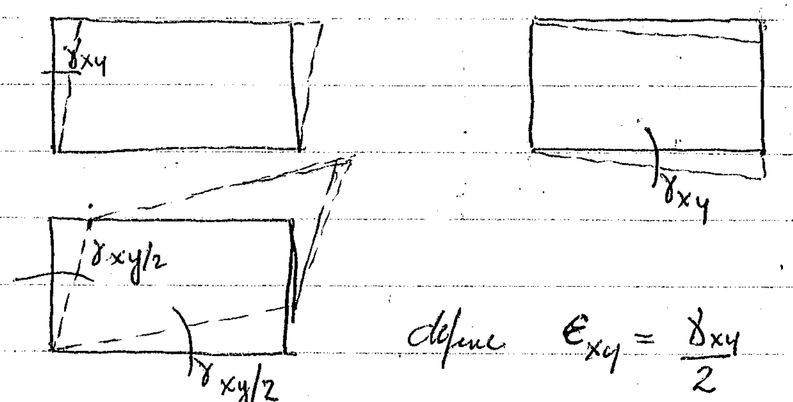


$$\gamma_{xy} \text{ (shearing strain)} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \gamma_{yx}$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \gamma_{zx}$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \gamma_{zy}$$

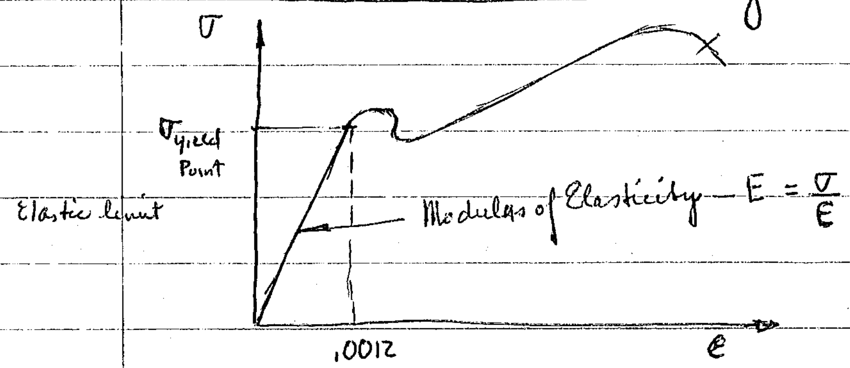
deformation must be such that compatability occurs between each deformation of a body with dimensions $\partial x, \partial y$



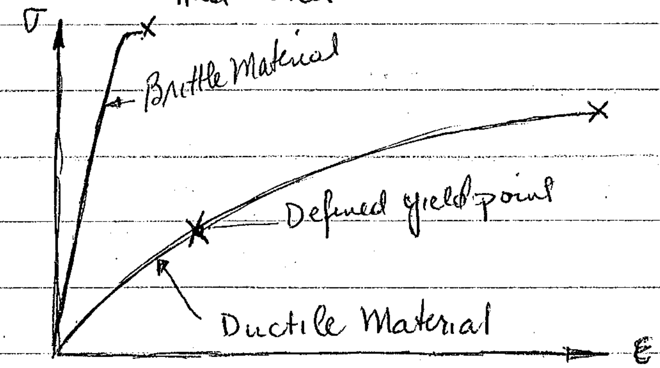
define $\epsilon_{xy} = \frac{\gamma_{xy}}{2}$

$$\begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix} \equiv \begin{pmatrix} \epsilon_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{yx}}{2} & \epsilon_y & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{zx}}{2} & \frac{\gamma_{zy}}{2} & \epsilon_z \end{pmatrix}$$

<u>Constitutive Laws</u> - mostly emperical laws



Mild Steel



Hooke's Law

$\tau = C\,\epsilon$ $\epsilon = A\tau$ depends on material

stress.

<u>Principle of Superposition</u>

1) each strain must be directly and linearly related to the stress causing the strain (Hooke's Law)

2) Strains due to one stress have negligible effect on another stress.

$\epsilon_{xx} = A_{11}\tau_{xx} + A_{12}\tau_{yy} + A_{13}\tau_{zz} + A_{14}\tau_{xy} + A_{15}\tau_{yz} + A_{16}\tau_{zx}$

$\epsilon_{yy} = A_{21}\tau_{xx} + A_{22}\tau_{yy} + A_{23}\tau_{zz} + A_{24}\tau_{xy} + A_{25}\tau_{yz} + A_{26}\tau_{zx}$

$\epsilon_{zz} = \ldots\ldots$

$\epsilon_{xy} = A_{41}\tau_{xx} + A_{42}\tau_{yy} + A_{43}\tau_{zz} + A_{44}\tau_{xy} + A_{45}\tau_{yz} + A_{46}\tau_{zx}$

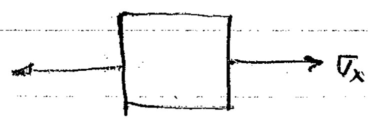
$\epsilon_{yz} = \ldots\ldots$

$\epsilon_{zx} = \ldots\ldots$

A homogeneous ~~element~~ material - every infinitesimal element of the material has the mechanical properties of the material

Isotropic - microscopic - has same mechanical properties in all directions

Homogeneous Isotropic Material



$$\frac{\epsilon_{xx}}{\tau_{xx}} = \frac{\epsilon_{yy}}{\tau_{yy}} = \frac{\epsilon_{zz}}{\tau_{zz}}$$

$$A_{11} = A_{22} = A_{33} = 1/E$$

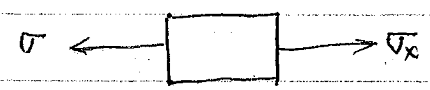
$$\frac{\epsilon_{yy}}{\tau_{xx}} = \frac{\epsilon_{xx}}{\tau_{yy}} = \qquad A_{12} = A_{21} = A_{31} = A_{13} = A_{23} = A_{32} = \frac{-\nu}{E}$$

$$\frac{\epsilon_{xy}}{\tau_{xy}} = \frac{\epsilon_{yz}}{\tau_{yz}} = \frac{\epsilon_{xz}}{\epsilon_{xz}} = A_{44} = A_{55} = A_{66} = \frac{1}{2G}$$

All others are not non-zero.

$\epsilon_x = \sigma_x \frac{\nu}{E} - \sigma_y \frac{\nu}{E} - \sigma_z \frac{\nu}{E}$

$\epsilon_x = 1/E\,\tau_x \quad \tau_x = \nu\sigma_x$



$$\epsilon_x = 1/E\,\sigma_x$$

$$\epsilon_y = -\nu\,\epsilon_x = -\frac{\nu}{E}\sigma_x$$

$$\boxed{\begin{aligned} \epsilon_x &= \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y - \frac{\nu}{E}\sigma_z \\ \epsilon_x &= -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_z \end{aligned}} \qquad \boxed{\epsilon_z = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \frac{\sigma_z}{E}}$$

$$\boxed{\epsilon_{xy} = \frac{1}{2G}\tau_{xy}} \qquad \boxed{\gamma_{xy} = \frac{1}{G}\tau_{xy}} \qquad \boxed{\gamma_{xz} = \frac{\tau_{xz}}{G}, \quad \gamma_{yz} = \frac{\tau_{yz}}{G}}$$

E – modulus of elasticity – Young's modulus.

Mild steel $E \sim 29 \times 10^6$ psi $= 29 \times 10^3$ ksi

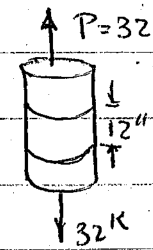
G – shearing modulus of elasticity

Mild steel $G \sim 10\text{-}12 \times 10^6$ psi $= 10\text{-}12 \times 10^3$ ksi

ν – Poisson's ratio

0.1 – 0.5, 0.25 – 0.33 more practical

Problem:



$2\frac{1}{4}''\ \phi$

$\Delta l = .00938$ Find E & ν

$\Delta d = -.000585$

$$E = \frac{\sigma_x}{\epsilon_x} \qquad \boxed{\sigma_x = \frac{32}{\frac{\pi}{4}(2.25)^2}} \qquad \boxed{\epsilon_x = \frac{.00938}{12''}}$$

$$\nu = -\epsilon_y/\epsilon_x; \qquad \boxed{\epsilon_y = \frac{-.000585}{2.25}}$$

$$\boxed{\nu = + \frac{\frac{.000585}{2.25}}{\frac{.00938}{12}}} =$$

2^{in} cube $\quad \sigma_x = \sigma_y = \sigma_z = 30{,}000$ psi $\quad E = 30 \times 10^6$ psi

$\nu = 1/4$

$\epsilon = \frac{\sigma}{E} - \nu\frac{\sigma}{E} - \nu\frac{\sigma}{E}$ $\qquad \epsilon = \frac{\sigma}{E}(1-2\nu) = \frac{30000}{30\times10^6}\cdot\frac{1}{2} = 1/2(1000)$

$l = l_0 + \epsilon l_0 \qquad 2'' + (.0005)2 = 1.999''$

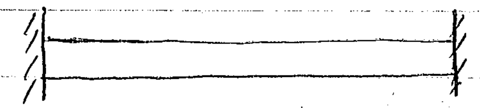
Thermal Strains

$\epsilon_x = \epsilon_y = \epsilon_z$

α – coefficient of thermal expansion

$\alpha = \frac{\epsilon}{\Delta T} \qquad \alpha_{steel} = 6.5\times10^{-6}/{}^\circ F$

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} + \alpha\Delta T$$



$A = 4\,in^2$

$L = 20\,ft$

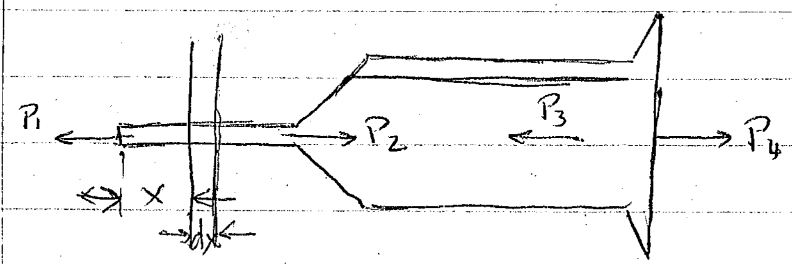
$\Delta T = 100^\circ F \qquad \nu = \qquad E =$

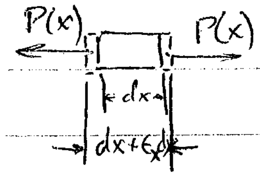
$\epsilon_x = 0 = \frac{\sigma_x}{E} + \alpha\Delta T$ since it is not restrained in y, z direction $\sigma_y, \sigma_z = 0$

$\sigma_x = -E\alpha\Delta T$ - - - - -

$$\epsilon_y = -\frac{\nu\sigma_x}{E} + \alpha\Delta T$$

$$\epsilon_z = -\frac{\nu\sigma_x}{E} + \alpha\Delta T$$





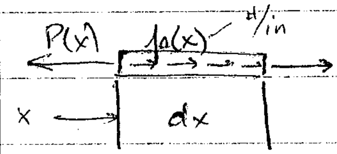
displacement or measure of elongations.

$$du = \epsilon_x dx$$

$$u = \int du = \int \epsilon_x dx = \frac{1}{E}\int \sigma_x dx = \frac{1}{E}\int \frac{P(x)}{A(x)} dx$$

$$\boxed{\frac{du}{dx} = \frac{1}{E}\frac{P(x)}{A(x)}}$$

distributed load



$$\sum F_x = P(x) + dP - P(x) + p_x dx = 0$$

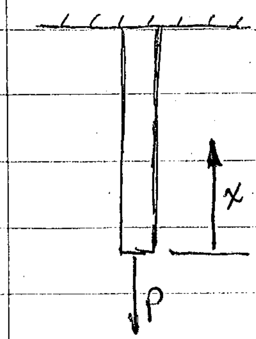
$$\frac{dP}{dx} = -p_x$$

$$\frac{d}{dx}\left(\frac{du}{dx}\right) = \frac{1}{E}\,\frac{d\left(\frac{P(x)}{A(x)}\right)}{dx}$$

if A is a constant

$$\frac{d^2u}{dx^2} = \frac{1}{EA}\frac{dP}{dx} = -\frac{p_x}{EA}$$

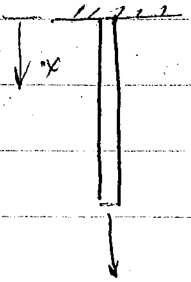
$$\boxed{\frac{d^2u}{dx^2} = \frac{-p_x}{EA}}$$



$$\epsilon_x = \frac{\Delta l}{l} \qquad \sigma_x = P/A \qquad \frac{\sigma_x}{E} = P/AE \qquad \Delta l = \frac{Pl}{AE}$$

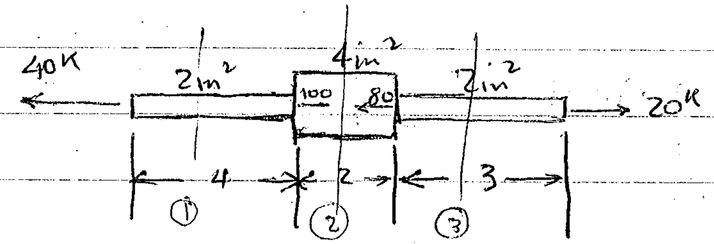
$$u = 1/E\int \frac{P}{A}dx = \frac{Px}{AE} + C_1 \qquad u=0 \quad x=0 \Rightarrow C_1 = 0$$

$$u = \frac{Px}{AE} \qquad u_l = \frac{PL}{AE}$$



$$u(L) = \frac{PL}{AE}$$

Problem 2



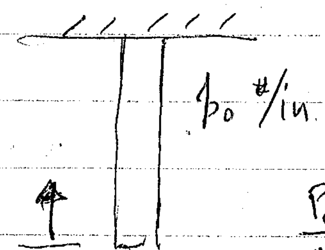
$$u = \sum \frac{PL}{AE}$$

$$u_1 = \frac{40^K \times 48 in}{2 in^2 \times 30 \times 10^3 K} \qquad u_2 = -\frac{60^K \times 24 in}{4 in^2 \times 30 \times 10^3}$$

$$u_3 = \frac{20 \times 36}{2 in^2 \times 30 \times 10^3 K}$$

$$u_T = 32 \times 10^{-3} - 12 \times 10^{-3} + 12 \times 10^{-3} = 32 \times 10^{-3}.$$

Problem 3



$$p_x = -P_0 \qquad \frac{du}{dx} = -\int \frac{p_x dx}{AE} dx = \left\{ \right.$$

$$\frac{P_0 x}{AE} + C_1$$

$$\frac{du}{dx} = \frac{P(x)}{AE} \qquad \frac{du}{dx} = \frac{P_0 x}{AE} \text{ since } C_1 = 0$$

$$u = \frac{p_0 x^2}{2AE} + C_2 \qquad u=0 \quad x=0 \qquad C_2 = 0$$

$$u = \frac{p_0 x^2}{2AE} \qquad u(L) = \frac{p_0 L^2}{2AE}$$

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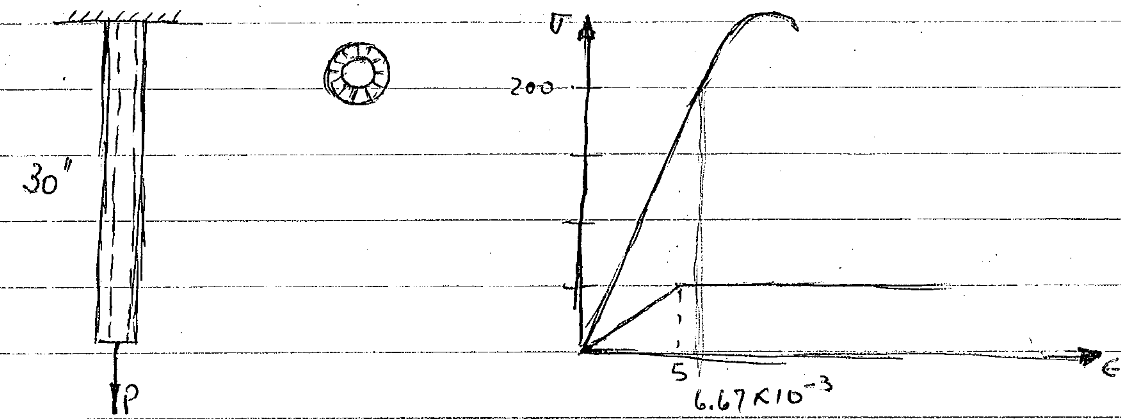


$$E_s = \frac{\sigma}{\epsilon} = \frac{200}{6.67\times10^{-3}} = 30\times10^3 \text{ KSI}$$

$$E_a = \frac{\sigma}{\epsilon} = \frac{50}{5\times10^{-3}} = 10\times10^3 \text{ KSI}$$

$P_a + P_s = P$ ← equil. $\qquad \epsilon_s = \epsilon_a$ ← compatability $\qquad$ since $u_s = u_a$

$\epsilon_s = \frac{\sigma_s}{E_s}$; $\epsilon_a = \frac{\sigma_a}{E_a}$ ← elastic

$\epsilon_s = \frac{\sigma_s}{30\times10^3} \qquad \epsilon_a = \frac{\sigma_a}{10\times10^3} \qquad \sigma_s = 9/3\,\sigma_a \qquad \sigma_s = 3\sigma_a$

$P_a = \sigma_a A_a \qquad P_s = \sigma_s A_s \qquad P_a = .5\sigma_a \qquad P_s = .5(3\sigma_a)$

$.5\sigma_A + 1.5\sigma_A = P \qquad \sigma_a = P/2.0$

$\sigma_s = \frac{3P}{2.0}$

$$\Delta l = \epsilon_s l = \epsilon_a l = \frac{\sigma_a}{10\times10^3} 30 = 3\sigma_a\times10^3 = \sigma_s\times10^3$$

for $P_1 = 80K$

$\sigma_a = \frac{80}{2.0} = 40$ KSI $\qquad \sigma_s = \frac{3 \cdot 80}{2.0} = 120$ KSI

$\Delta l = 120 \times 10^{-3} = .12$ in

suppose $P_z = 125$ K $\qquad \sigma_a = \frac{125}{2} = 62.5 > 50$

$\sigma_s = \frac{3(125)}{2} = 187.5 < 200$

$\sigma_a \cong 50$ ksi

$P_a \cong 25$ K $\qquad P_a + P_s = P$

$P_s \cong 100$ K

$\therefore \quad \sigma_s = \frac{100}{.5} = 200$

$\epsilon_s = \frac{200}{30 \times 10^3} = 6.67 \times 10^{-3}$

$\Delta l = \epsilon_s \times 30 = 200 \times 10^{-3} = .200$ in

4-3 $\quad \epsilon_x = \frac{\sigma_x}{E} - \frac{\nu \sigma_y}{E} - \frac{\nu \sigma_z}{E}$

$\epsilon_y = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_z$

$\epsilon_z = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \frac{\sigma_z}{E}$

$\sigma_x = \lambda e + 2\mu \epsilon_x$

$$\sigma_x = \frac{(1-\nu^2)E\,\epsilon_x + \nu E(1+\nu)[\epsilon_y + \epsilon_z]}{1 - 3\nu^2 - 2\nu^3}$$

$$= \frac{(1-\nu)E\,\epsilon_x + \nu E[\epsilon_z + \epsilon_y]}{1 - \nu - 2\nu^2}$$

4-8 $\quad \epsilon_x = \frac{\sigma_x}{E} - \frac{\nu \sigma_y}{E} - \frac{\nu \sigma_z}{E} + \alpha \Delta T$

$\epsilon_y = -\frac{\nu \sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_z + \alpha \Delta T$

$\therefore \quad 0 = \frac{(1-\nu)}{E}\sigma + \alpha \Delta T$

$\sigma = \frac{\alpha E \Delta T}{\nu - 1}$

Can't contract in 3 direction

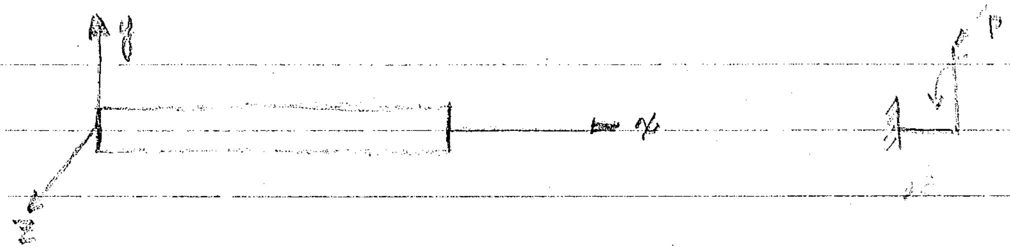
~~they are constrained~~

$\sigma_z = \epsilon_x = \epsilon_y = 0$

$\sigma_x = \sigma_y$

Chapter 5 and Chapter 6 - Deal with internal reaction

Torsion



1 hp. $\rightarrow$ 550 ft-lb/sec = 550×12×60 in-lb/min

if crankshaft rotates at N rpm Torque = T

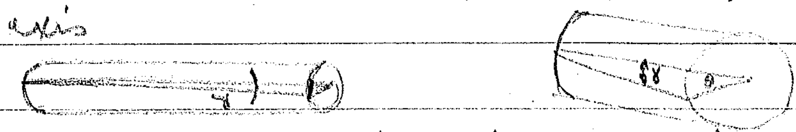
power = $\underbrace{2\pi N}_{rad} T$ = 550×12×60 × # hp developed

$$\boxed{T = 63000(hp)/N} \qquad T(\text{in lb-in})$$

Assumptions

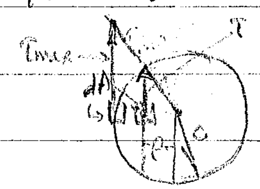
1) Plane sections normal to axis remain plane after deformation no warping or distortion

2) In circular members, shearing strains, vary linearly from central axis



$r\,d\theta = \gamma L$

3) Materials remain elastic, hooke's law applies, then stress proportional to strain.



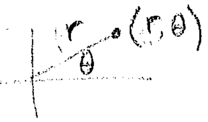
$$\frac{\tau}{\rho} = \frac{\tau_{max}}{c}$$

$$\tau = \frac{\tau_{max}}{c}\rho$$

$\underbrace{dT}_{moment} = \underbrace{\tau dA}_{force}\rho$ over

$$dT = \frac{\tau_{max}}{c}\rho\, dA\, \rho$$

$$T = \int_A \frac{\tau_{max}}{c}\rho^2 dA = \frac{\tau_{max}}{c}\int_A \rho^2 dA$$

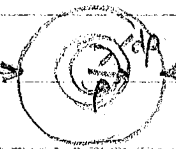


J (polar moment of inertia) $= \int_A \rho^2 dA$

$T = \frac{\tau_{max}}{c} J$ $\qquad \boxed{\tau_{max} = Tc/J}$ torsion formula for circular shaft.

max stress

$J = \int_A \rho^2 dA$ $\quad d \rightarrow$



$\leftarrow dA = 2\pi\rho\, dr$

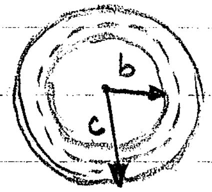
for solid circle

$$J = \int_0^c \rho^2 \times 2\pi\rho\, d\rho = 2\pi \left.\frac{\rho^4}{4}\right|_0^c = 2\pi \frac{c^4}{4} = \frac{\pi c^4}{2} = \frac{\pi d^4}{32}$$

$$\tau = \frac{\tau_{max}}{c}\rho \qquad \frac{\tau_{max}}{c} = \frac{Tc^2}{\rho c} = \frac{Tc}{J} \qquad \boxed{\tau = T\rho/J}$$

stress at any pt.

For circular tube :—



$$J = \int_b^c 2\pi\rho^3 d\rho = \frac{2\pi\rho^4}{4} = \frac{\pi c^4}{2} - \frac{\pi b^4}{2}$$

for a thin tube $b \cong c$ $\qquad c - b = t$ is assumed

$$J = \frac{\pi}{2}(c^4 - b^4) = \frac{\pi}{2}(c^2 - b^2)(c^2 + b^2) = \frac{\pi}{2}(c^2 + b^2)(c - b)(c + b)$$

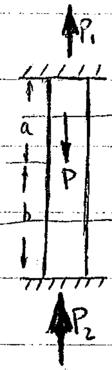
$$\cong \frac{\pi}{2} \cdot 2c^2 t \cdot 2c \approx 2c^3 t\pi$$

$$J \cong 2\pi c^3 t$$

$c = 10''$ $\quad t = 1''$ $\qquad J = \frac{\pi}{2}(10^4 - 9^4) = \frac{\pi}{2}(10000 - 6561)$

$J \cong \pi\left(\frac{3439}{2}\right) = 1719.5\pi \qquad J \cong 2000\pi$

4-35



$P = P_1 + P_2$ - equil $\Sigma F_y = 0$

$u = \sum \frac{PL}{AE} = \frac{P_1 a}{AE} - \frac{P_2 b}{AE} = 0$

$P_1 = P - P_2 \quad a = L - b \qquad P_1 = \frac{bP}{L} \quad P_2 = \frac{aP}{L}$

4-31

$\tau_{max} = \frac{Tc}{J}$ solid shaft $J = \frac{\pi d^4}{32}$

hollow shaft $J = \frac{\pi d_o^4}{32} - \frac{\pi d_i^4}{32}$

thin tube $J = 2\pi c^3 t = \frac{\pi d_o^3 t}{4}$

solid shaft $\underset{\text{unit length}}{\frac{J}{Vol.}} = \frac{\pi c^4/2}{\pi c^2} = \frac{c^2}{2}$

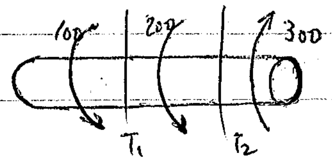
tube: $\frac{J}{V} = \frac{\pi}{2}\left(c_o^4 - c_i^4\right) / \pi\left(c_o^2 - c_i^2\right) = \frac{c_o^2 + c_i^2}{2}$

thin tube: $\frac{J}{V} = \frac{2\pi c^3 t}{2\pi c t} = c^2$

1. Equilibrium - Method of Sections
   Internal Torques
2. Assume shearing strain linear from axis

3. Apply the Material properties & perhaps Hooke's Law.

Problem



$\phi = 1/2''$

$T_1 = 100$

$T_2 = 300$ $\quad \tau_{max} = J T c^{-1}$ $\quad J = \frac{\pi}{516}$

$\tau_{max} = \frac{512(300)(.25)}{\pi} = 128\frac{(300)}{\pi} \approx 12000$ psi

Pblm $1''$ outside dia $\quad T = 400 \quad$ find $\tau_p \; \tau_c$

$.9''$ inside dia $\quad J = \frac{\pi}{32}(1^4 - .9^4) = \frac{\pi}{32}(.6561) = \frac{\pi}{32}(.3439)$

$J \geq .01\pi$ $\quad$ approx $\rightarrow J_{\text{thin tube}} = \frac{\pi}{4} 1^3 (.05) = .0125\pi$

$\tau_{max} = \frac{400(0.5)}{.01\pi} \sim 5930$ psi $\quad \tau_p = \frac{400(0.45)}{.01\pi} \sim 5330$ psi

Design

$\tau_{all} = 8000$ psi Steel (anneal)

$\frac{J}{c} = \frac{T}{\tau_{max}}$ $\quad$ for solid shaft $\frac{\pi c^3}{2} = \frac{J}{c}$

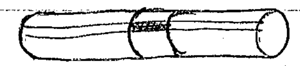
Select a shaft for a ten hp motor at 1800 rpm.

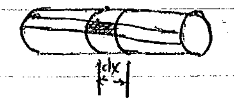
$\tau_{all} = 8000$ $\quad T = \frac{63000 \text{ Hp}}{N} = \frac{630000}{1800} = 350$ in-lb

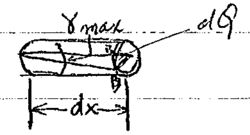
$J/c = \frac{350}{8000} = \frac{\pi c^3}{2}$ $\quad c^3 = \frac{700}{8000\pi} = (.303)^3$

$c = .303$

Angle of twist







$\widehat{BD} = c\,d\theta$

$\gamma_{max}\,dx = \widehat{BD}$

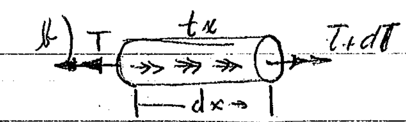
$\dfrac{d\varphi}{dx} = \dfrac{\gamma_{max}}{c}$

if hookes' law applies $\quad \gamma_{max} = \dfrac{\tau_{max}}{G} \qquad \tau_{max} = \dfrac{Tc}{J}\ ; \qquad \gamma_{max} = \dfrac{Tc}{JG}$

or $\dfrac{d\theta}{dx} = \dfrac{T}{JG}$ $\qquad \boxed{\varphi = \int_0^L \dfrac{T}{JG}\,dx}$

1) Suppose $\dfrac{T}{GJ}$ = constant $\qquad \varphi = \dfrac{TL}{GJ} \qquad \varphi_T = \sum \dfrac{TL}{GJ}$

2) Suppose $\dfrac{T}{JG}$ not constant $\qquad \varphi = \int_0^L \dfrac{T(x)}{J(x)G}\,dx$



$dT = -t_x\,dx$

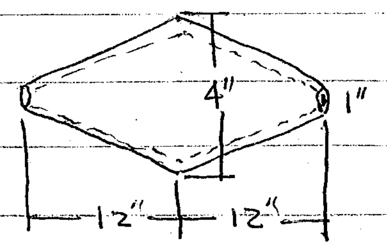
$\dfrac{dT}{dx} = -t_x \qquad \dfrac{d\theta}{dx} = \dfrac{T}{JG}$

$\dfrac{d^2\theta}{dx^2} = \dfrac{1}{JG}\dfrac{dT}{dx} + T\dfrac{d}{dx}\left(\dfrac{1}{JG}\right)$

if $1/JG$ = const $\quad T\dfrac{d}{dx}\left(1/JG\right) = 0 \qquad \boxed{\dfrac{d^2\theta}{dx^2} = \dfrac{-t_x}{JG}}$

$\dfrac{d\theta}{dx} = -\int \dfrac{t_x}{JG}\,dx = f(x) + C_1 = \dfrac{T}{JG}$ $\qquad$ JG - torsional stiffness of rod.

5-14



$\dfrac{d\theta}{dx} = \dfrac{T}{JG}$

$t = C_o - C_i = \dfrac{1}{2\pi}$

$C_i = C_o - t$

$d_o = ax + b \qquad 0 \le x \le 12'' \qquad @\ x=0 \quad d_o = 1$

$1 = b \qquad @\ x = 12 \quad d_o = 4$

$4 = 12a + 1 \qquad a = 1/4 \qquad d_o = \dfrac{x}{4} + 4$

$x = 12 \quad d_o = 4 \qquad 4 = 12a' + b' \qquad 3 = -12a' \qquad a' = -1/4$

$x = 24 \quad d_o = 1 \qquad 1 = 24a' + b' \qquad b' = 28/4$

$d_o = \dfrac{-x + 28}{4}$

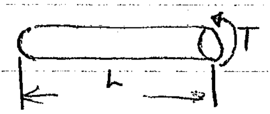
$0 \le x \le 12'' \qquad d_o = \dfrac{x+4}{4} \qquad c_i = c_o - t$

$12''+0 \le x \le 12 + 12'' \qquad d_o = \dfrac{-x+28}{4} \qquad J_x = \dfrac{\pi c_o^4}{2} - \dfrac{\pi c_i^4}{2}$

$$\frac{\pi c_o^4}{2} - \frac{\pi}{2}(c_o - t)^4 = -\frac{\pi}{2}\left(-4c_o^3 t + 6c_o^2 t^2 - 4c_o t^3 + t^4\right).$$

$$J_x = \frac{1}{4}\left(4c_o^3 + 6c_o^2 t + 4c_o t^2 - t^3\right) = c_o^3 - \frac{3}{2}c_o^2 t + c_o t^2 - \frac{1}{4}t^3$$

5-6 $\quad .1\text{in} = c$



$\tau_{max} = 6000$

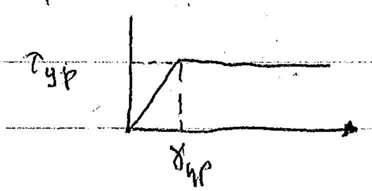
$G = 3.84 \times 10^6$

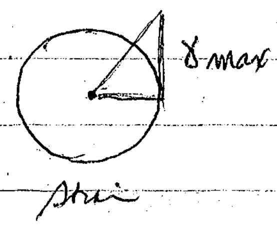
$\phi = \dfrac{TL}{JG} \qquad \tau_{max} = \dfrac{Tc}{J} \qquad \dfrac{\tau_{max} L}{c\, G} = \phi = 2\pi$

$L = \dfrac{2\pi c G}{\tau_{max}} = \dfrac{2\pi(.1)(3.84 \times 10^6\ \text{psi})}{6000\ \text{psi}}$

Relax some of the assumptions

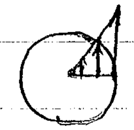
1. relax hookes law





Case I $\quad \gamma_{max} \le \gamma_{yp} \quad \tau_{max} \le \tau_{yp}$

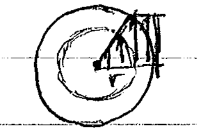
already discussed.



$\tau_{max} \to \tau_{yp}$

$T_{elastic}$; max. stress reaches yield pt $\qquad T_{el} = \frac{\tau_{yp} J}{c}$

Case II $\quad \gamma_{max} > \tau_{yp} \qquad \tau_{max} = \tau_{yp}$

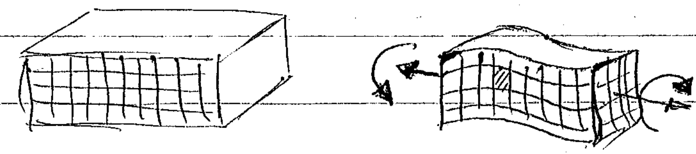


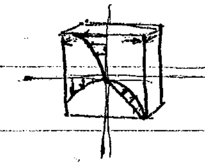
$$T = \int_A \tau\, dA\, \rho = \int_r^c \tau_{yp}\, 2\pi\rho^2 d\rho \; \int_0^r \frac{\tau_{yp}\, \rho^2}{r}\, 2\pi \rho\, d\rho$$

limiting case $r = 0$ $\qquad T_{plastic} = \int_0^c \tau_{yp}\, 2\pi\rho^2 d\rho = \tau_{yp}\frac{2\pi c^3}{3} = \tau_{yp}\frac{2}{3}\frac{\pi c^4}{2\, c}$

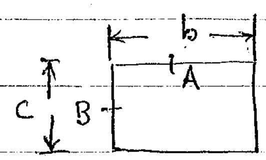
$$T_{plastic} = \tau_{yp}\,\frac{4}{3}\,\frac{J}{c}$$

Solid non-circular member.





$\tau_{max} = \frac{T}{\alpha b c^2} \qquad \varphi = \frac{TL}{\beta b c^3 G}$



$\alpha = f(b/c)$

$\tau_A = \frac{T}{\alpha b c^2} \qquad \tau_B = \frac{T}{\alpha b^2 c}$

$$\int \frac{du}{u^4 - a^4}$$ a contains x & constant term.

$$\int \frac{dx}{(ax+b)^4 - c^4} = \frac{1}{a}\int \frac{a\,dx}{(ax+b)^4 - c^4} = \frac{1}{a}\int \frac{du}{u^4 - c^4}$$

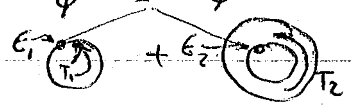
let $u = ax - b$

$du = a\,dx$

19 

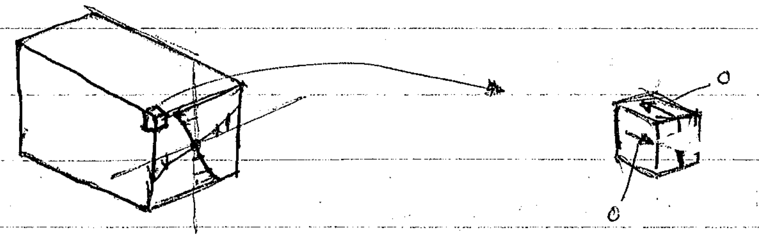
$T_1 + T_2 = T$

$\varphi = \varphi$



$\epsilon_1 = \epsilon_2$

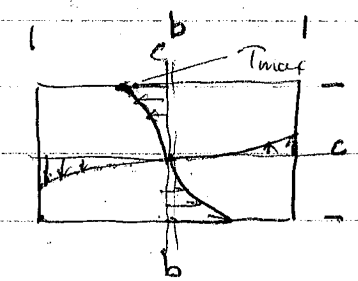
Solid Noncircular Members



Stress at corner = 0

Strain at corner = 0

All assumptions are not valid.

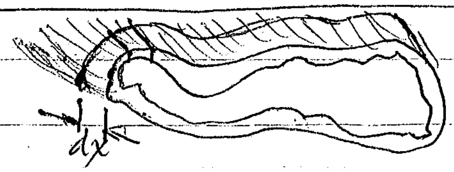


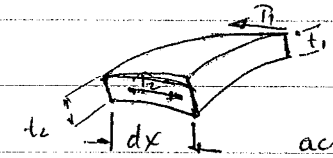
$$\tau_{max} = \frac{T}{\alpha b c^2} \qquad \alpha = f(b/c)$$

$$\varphi = \frac{TL}{G(\beta b c^3)} \qquad \beta = f(b/c)$$

Table on page 167.

Thin-walled hollow members

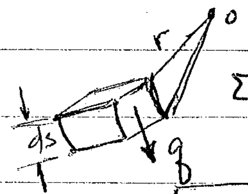




Since t is small assume that τ is constant across thickness

$$\Sigma F_x = 0 \Rightarrow \tau_2(t_2\,dx) - \tau_1(t_1\,dx) = 0$$

$\tau_1 t_1 = \tau_2 t_2 = q$ constant about perimeter. (shear flow) lb/in

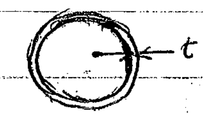


$$\Sigma M_o = \oint q\,ds\,r = T \qquad q\oint r\,ds = T = q\,2A_{enclosed}$$

$$\boxed{\tau = \frac{T}{2A_{enc}t}}$$

$$\boxed{J = 2\pi c^3 t \qquad \tau = Tc/J \qquad T/2\pi c^2 t = \tau \quad ✓}$$

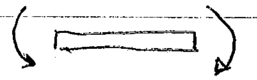
$$\tau = \frac{T}{2A_{enc}t} = \frac{T}{2(\pi c^2)t} \quad ✓✓$$

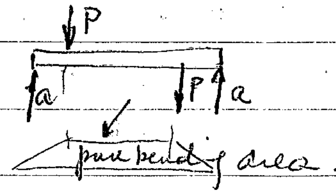


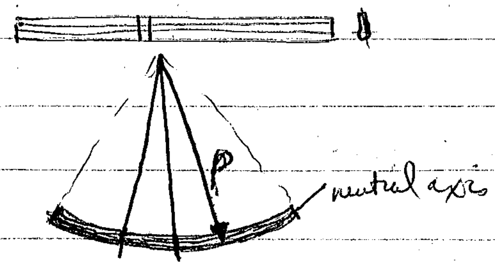
Chapt 6 Flexural (Bending) Stresses in Beams

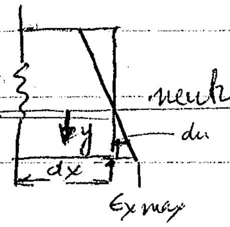
Straight beams (homogeneous)

Pure bending = $V = 0 \Rightarrow M =$ constant







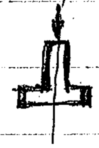


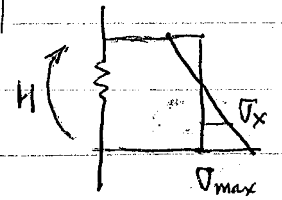
$\epsilon_x = \frac{du}{dx}$ $\quad \epsilon_{max} = \frac{du_{max}}{dx}$

$\epsilon_x = by$ $\quad$ b depends on geometry, moment

Suppose

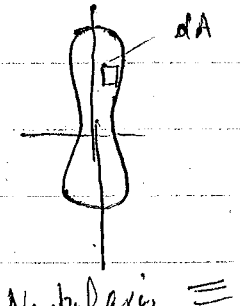
1) Member has axis of symmetry
2) loading along axis of symmetry.





$\sigma_x = Eby$

$\Sigma F_x = 0$



$\int_A \sigma_x dA = 0$ $\qquad Eb\int_A y\,dA$

$\int_A y\,dA = 0 \implies$ Neutral axis $\equiv$ centroid

$$\frac{\int_A y\,dA}{\int_A dA} = \bar{y}$$

but since $\int_A y\,dA = 0$ $\qquad \bar{y} = 0$

$\therefore$ neutral axis $\equiv$ centroid

$\Sigma M_{\phi} = 0$

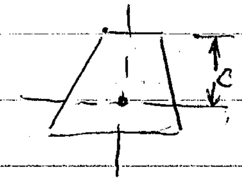
$$M + \int_A \sigma_x \, dA \, y = 0$$

$$M + Eb \int_A y^2 dA = 0 \qquad M = -Eb \int_A y^2 dA$$

$$I_z = \int_A y^2 dA \quad \text{moment of inertia} \qquad I_y = \int_A z^2 dA$$

$$\boxed{M = -EbI} \qquad \text{since } \sigma_x = Eby \qquad \boxed{\frac{\sigma_x I}{y} = -M} \qquad \boxed{\sigma_x = \frac{-My}{I}}$$

call c largest distance from one of the free surfaces to NA.

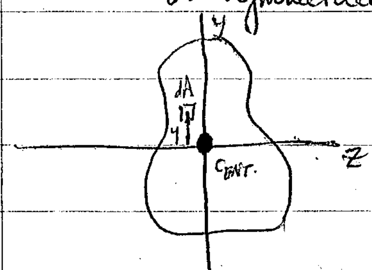


$$\sigma_{max} = Mc/I \qquad \boxed{\sigma_{max} = -Mc/I}$$

Basic Assumption

1) Deformation bases on linear distribution of strain.
2) Material properties relate stress-strain. (Hooke's law)
3) Equilibrium

Unsymmetrical Bending { either member doesn't have axis of symmetry / Load is not along axis of symmetry.

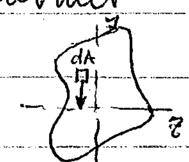


M_z

$$\int dA\,\sigma_x = \int Eby \, dA = F$$

$$M_y = 0$$

Submit on Tuesday we come back → Exam



M_z $\qquad \sigma_x dA \longrightarrow$ force

$$\int_{M_z} dM_z = 0 \Rightarrow M_z = -\int_A \sigma_x \, dA \, y$$

$\sum M_y = 0 \qquad 0 = \int_A \sigma_x \, dA\, z$

Suppose $\epsilon_x = by \qquad \sigma_x = Eby \qquad$ Define: $\int_A yz\,dA = I_{yz}$ product of inertia

$0 \overset{?}{=} Eb \int_A yz\,dA \qquad E \neq b \neq 0$ if $\int_A yz\,dA \neq 0$ the assumption is false

Define If $I_{yz} = 0$ y & z are principle axes.

if $I_{yz} \neq 0$ assumptions are false

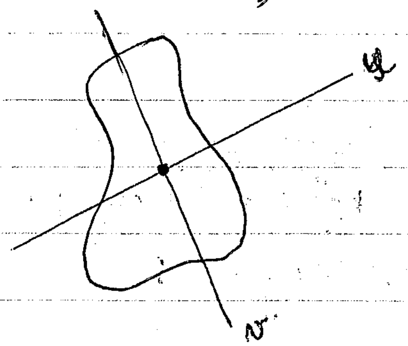
Cor: Axis of symmetry is a principle axis.

Centroidal Axis normal to a principle axis is itself principle

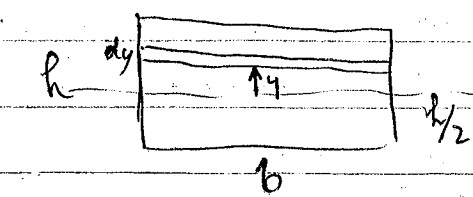
Suppose: $I_{yz} \neq 0 \qquad \sigma_x = E(by + dz)$

$\sum M_y = 0 = \int_A \sigma_x \, dA\, z = E\int (by + dz) z\,dA = E\left[b\int yz\,dA + d\int z^2 dA\right]$

$E\left[b I_{yz} + d I_{yy}\right] = 0 \qquad E \neq 0$



principle axes for unsymmetric ~~bending~~ beams

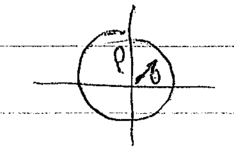


$dA = b\,dy$

$I_{zz} = b\int_A y^2 dy$

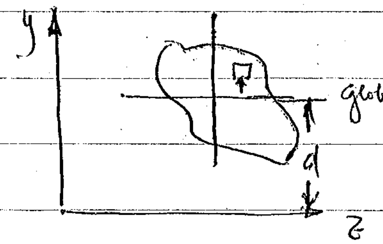
$I_{zz} = b/_{12} h^3$

$I_{yy} = b^3h/12$



$I_{zz} = \int_A y^2 dA \quad ; \quad I_{yy} = \int_A z^2 dA$

$2 I_{zz} = \int_A (y^2+z^2)\, dA = \int_A \rho^2 dA = J \qquad I_{zz} = I_{yy} = J/2$



$I_{zz} = \int_A (y+d)^2 dA$

$\int_A y^2 dA + \int 2yd\,dA + \int d^2 dA$

$\int_A y^2 dA + 2d \int_A y\,dA + d^2 \int_A dA$

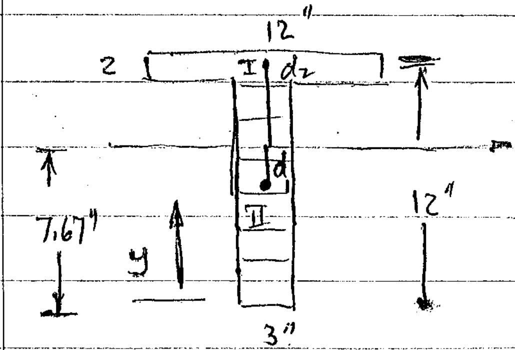
if y is measured from centroid $y=0$ ∴ $\int y\,dA = 0$

$I_{zz} = I_0 + 2d(0) + d^2A$

The Parallel Axis Theorem:

$I_{zz} = I_0 + Ad^2$ ⟶ the take Area x d^2 from global axis

I_0 ⟶ moment about its own centroid



$I_{zz_{total}} = \sum I_{o_i} + \sum A_i d_i^2$

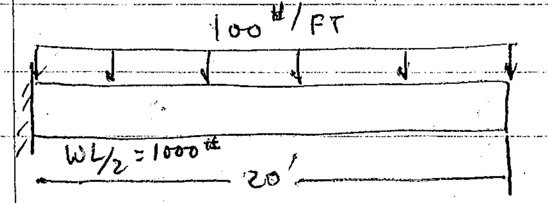
$\bar{y} = \dfrac{\sum Ay}{\sum A}$

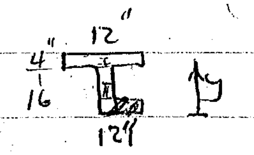
	A	y	Ay	d	Ad^2
I	24	11	264	3.33	
2	30	5	150	-2.67	
	54		414		

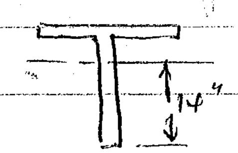
$\dfrac{69}{9} = \dfrac{414}{54} = \dfrac{23}{3}$

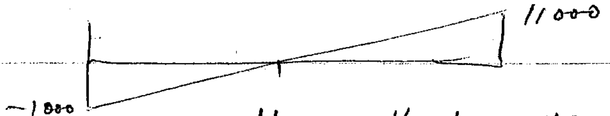
$= 7.67''$

	Ad^2	I_0
I	24×3.3^2	$12 \times 2^3/12$
II	30×2.67^2	$\dfrac{3 \times 10^3}{12}$









$M_{max} = \frac{1}{2} \times 1000 \times 10 = 5000$

$= \frac{1}{2} \times \frac{wL}{2} \times \frac{L}{2}$

$\frac{bh^3}{12}$

	A	Y	Ay	d	Ad^2	I_o
I	48	18	864	4"	768	64
II	32	8	256	−6"	1152	$682\frac{2}{3}$
	80		1120		1920	$746\frac{2}{3}$

$\bar{y} = \frac{\Sigma Ay}{\Sigma A}$

$I_{zz} = \Sigma(I_o + Ad^2)$

$\bar{y} = \frac{112}{8} = 14$

$I_{zz}\ 2666\frac{2}{3}$

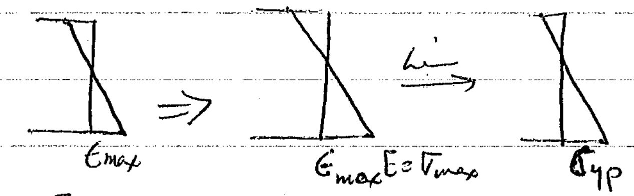
$\sigma^T_{max} = \frac{5000(12)14}{2666.67}$

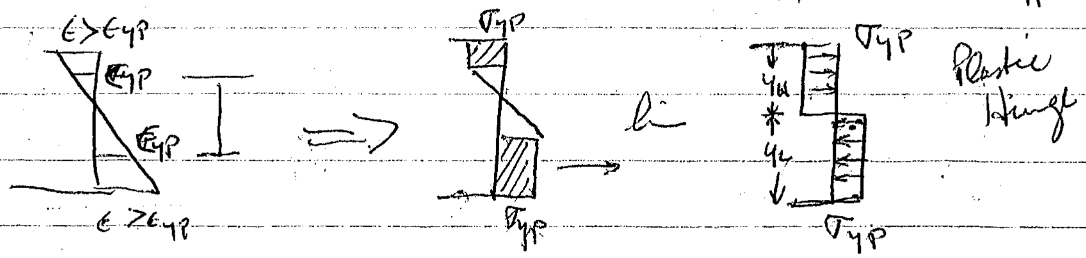
$\sigma^C_{max} = \frac{5000(12)(6)}{2666.67}$

$\sigma_{max} = \frac{Mc}{I}$ let $\frac{I}{c} = S$ - Section modulus

$\sigma_{max} = \frac{M}{S}$

$M_{yp} = \sigma_{yp} S$





We must now find a new neutral axis since Hooke's law doesn't hold any more

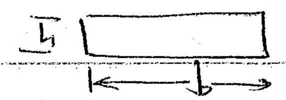
$$\sum F_x = 0 \qquad \int_A \sigma_x \, dA = 0$$

$$\sum M_{\text{Neutral axis}} = 0 \qquad M_{\text{plast.}} + \int_A \sigma_x \, dA \, y = 0$$

$$\sum F_x = 0 = \int_A \sigma_x \, dA = \sigma_{yp} \int_{\text{lower area}} dA - \sigma_{yp} \int_{\text{upper area}} dA = \sigma_{yp} \left[\int_{y_L}^{0} dA - \int_{0}^{y_u} dA \right]$$

$$\int_{y_L}^{0} dA = \int_{0}^{y_u} dA$$ areas above & below the neutral axis are equal.

For rectangle

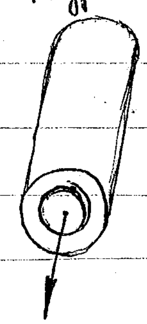


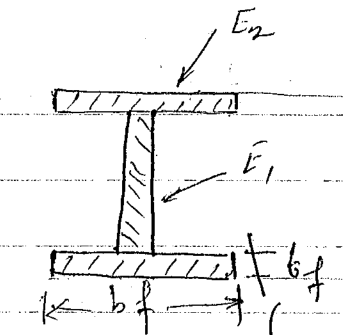
$$M_{plastic} = -\sigma_{yp} b \int_{-h/2}^{0} y \, dy + \sigma_{yp} b \int_{0}^{h/2} y \, dy$$

$$M_{plastic} = \frac{bh^2}{4} (\sigma_{yp})$$

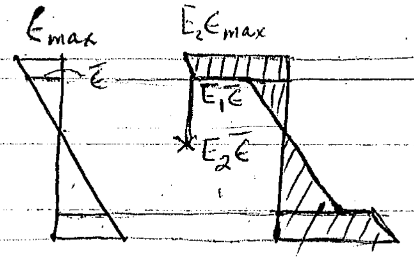
$M_{plastic} = \sigma_{yp} Z$ $\quad$ Z is the plastic modulus.

$$\frac{M_{plastic}}{M_{yp}} = \frac{Z}{S} = \text{Shape Factor.}$$





$E_2 > E_1$



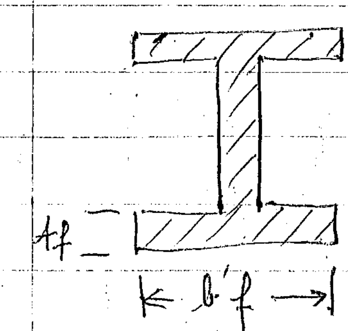
$\eta = \frac{E_2}{E_1}$

$M = -\int_A \sigma_x \, dA (y)$ $\sigma = \frac{My}{I}$

Equivalent beam.

web same.

$F_{in\ flange} = \int_{A_{flange}} \sigma_x \, dA = \frac{E_2(\epsilon_{max} + \bar{\epsilon})}{2} t_f b_f$



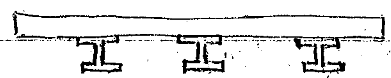
$F'_{flange} = t_f b'_f \frac{E_1(\epsilon_{max} + \bar{\epsilon})}{2}$

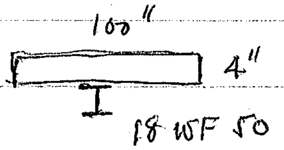
$b'_f E_1 = b_f E_2$

$\boxed{b'_f = \eta b_f}$

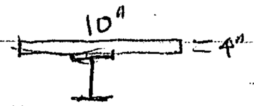
σ_{max} in original flange = $\eta \sigma_{max}$ of equiv material

1)





$\frac{E_s}{E_c} = 10 = \eta$



	A	y	Ay	d	Ad^2	I_o
R	40	20	800	3	360	53.33
WF	14.71	9	132.39	-8	942	800.6
	54.71		932.39		1302	853.93

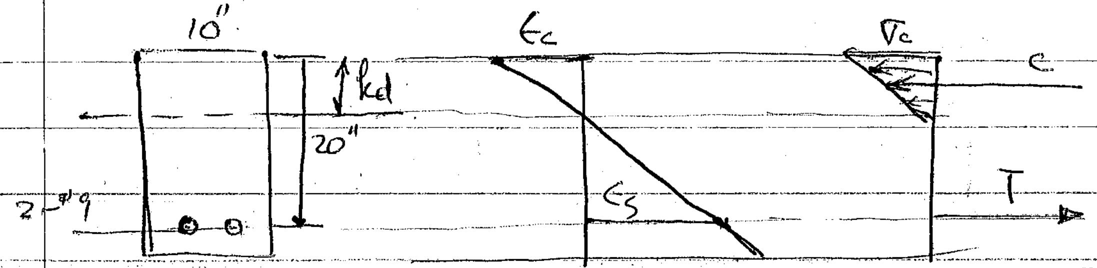
$$\bar{y} = \frac{932.39}{54.71} = 17$$

1) $\sigma^T_{max} = \frac{M}{I} 17''$

2) $\sigma^c_{max} = \frac{M}{I} 1''$

$\Big\}$ WF

$$\sigma^c_{max\,conc} = \frac{M}{I}\frac{5''}{n} = \frac{M}{2I} \quad \text{Concrete}$$



A = 1" sq.

$$n = \frac{E_s}{E_c} = 15$$

$$T = C \qquad \epsilon_s = \epsilon_c \frac{(20-kd)}{kd}$$

$$\epsilon_s = \frac{\sigma_s}{E_s} = \frac{T}{A_s E_s} \;; \qquad \epsilon_c = \frac{\sigma_c}{E_c} \;; \qquad \frac{T}{A_s E_s} = \frac{\sigma_c}{E_c}\frac{(20-kd)}{kd}$$

$$T = \sigma_c\, n\, A_s \frac{(20-kd)}{kd} \qquad T = \tfrac{1}{2}\sigma_c\, kd \cdot 10$$

$$T = 5\sigma_c kd$$

$\boxed{nA_s = 30}$

$$I = \sum (Ad^2 + I_o) \qquad \frac{bh^3}{12} = \frac{Ah^2}{12}$$

$$= \frac{10\,kd^3}{12} + \frac{10kd^3}{4} + \frac{30\cdot(1\tfrac{1}{8})^2}{12} \leftarrow \text{not needed.}$$

$$+ 30(20-kd)^2 \qquad \sigma_c = \frac{M}{I} kd$$

$$5kd = \frac{30(20-kd)}{kd}$$

$$\sigma_s = \frac{M}{I}(20-kd)\eta = \frac{15M}{I}(20-kd)$$

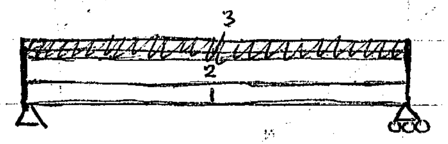
$$T = A_s \sigma_s$$

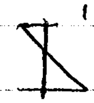
18.28
-2.94
-11.5
18.28
14.44
3.84

Shearing Stresses in Beams.

Shear tensor $\tau = \begin{pmatrix}\sigma_x & \tau_{xy} \\ \tau_{yx} & 0\end{pmatrix}$ $\qquad -\frac{dM}{dx} = V$

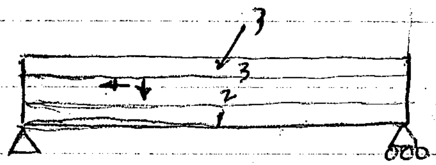
$$dM = -V\,dx$$



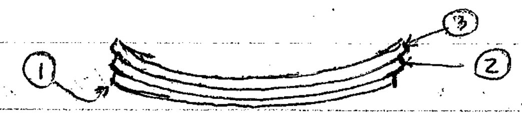


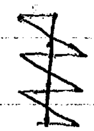
if bonded adequately

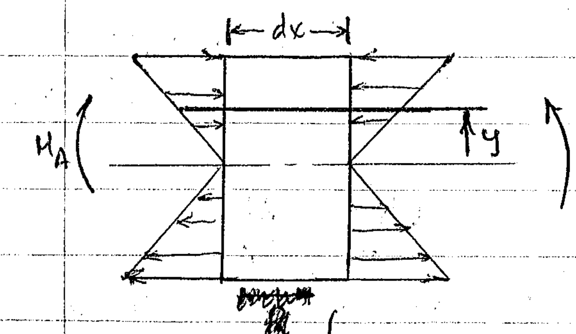
3 same laminar layer

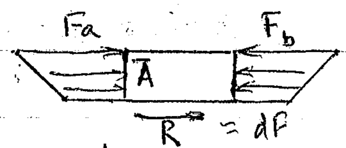


if not bonded









$$F_a = \int_{\bar{A}} \sigma_A \, dA \qquad \sigma_a = -\frac{M_A y}{I}$$

$$F_a = \int_{\bar{A}} -\frac{M_A}{I}(y)\,dA = -\frac{M_A}{I}\int_{\bar{A}} y\,dA = -\frac{M_A}{I} Q_y$$

$Q_y = \int_{\bar{A}} y\,dA$ - statical moment of $\bar{A}$ about the neutral axis

$F_b = \dfrac{-M_B Q_y}{I}$ if $\bar{A}$ is constant across beam & $M_a = M_B$

$F_a \equiv F_b \qquad R = 0$

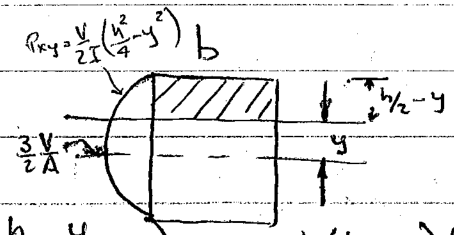
if $\bar{A}$ is constant across beam & $M_a \neq M_b$

$F_a \not\equiv F_b \qquad R \neq 0 \qquad R = F_b - F_a \neq 0$

$$dF = F_b - F_a = -\frac{(M_A + dM) Q_y}{I} + \frac{M_A Q_y}{I} = -\frac{dM\, Q_y}{I}$$

$$\frac{dF}{dx} = -\frac{dM}{dx}\frac{Q_y}{I} \; ; \qquad \boxed{q = \frac{V Q_y}{I}} \qquad q = \tau t \quad \text{Shear Flow}$$

$$\tau_{xy} = \frac{V Q_y}{I t}$$



$$Q = \int y\,dA$$

$$Q = b\left(\frac{h}{2} - y\right)\left(\frac{h}{4} - \frac{y}{2} + y\right) = b\left(\frac{h}{2} - y\right)\left(\frac{2y + h}{4}\right) = \frac{b}{2}\left(\frac{h - 2y}{2}\right)\left(\frac{h + 2y}{2}\right)$$

$$Q = \frac{b}{2}\left(\frac{h^2 - 4y^2}{4}\right) \qquad \tau_{xy} = \frac{VQ}{Ib} = \frac{V}{2I}\left(\frac{h^2}{4} - y^2\right)$$

$$\tau_{xy} = 0 \Rightarrow y = \pm h/2 \qquad \tau_{max} \text{ @ } y = 0$$

$$\tau_{max} = \frac{V}{2I}\frac{h^2}{4} = \frac{12 V h^2}{8 b h^3} = \frac{3}{2}\frac{V}{A}$$

$$\tau = \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & 0 \end{pmatrix} = \begin{pmatrix} My & \frac{V}{2I}\left(\frac{h^2}{4} - y^2\right) \\ \frac{V}{2I}\left(\frac{h^2}{4} - y^2\right) & 0 \end{pmatrix}$$

Prove $\int_A \tau_{xy}\, dA = V$

$$= b\int_{-h/2}^{h/2} \frac{V}{2I}\left(\frac{h^2}{4} - y^2\right) dy = \frac{Vb}{2I}\left[\frac{h^2}{4}y - \frac{y^3}{3}\right]_{-h/2}^{h/2}$$

$$\frac{Vb}{2I}\left[\frac{h^3}{4} - \frac{h^3}{12}\right]$$

$$\frac{Vb}{24I}\left[2h^3\right] = \frac{2V}{2I}\cdot\frac{bh^3}{12} = V$$

$$\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} = 0 \qquad \sigma_x = -\frac{My}{I}$$

$$\frac{\partial\sigma_x}{\partial x} = -\frac{y}{I}\frac{\partial M}{\partial x} = \frac{Vy}{I}$$

$$\frac{\partial\tau_{xy}}{\partial y} = -\frac{Vy}{I}$$

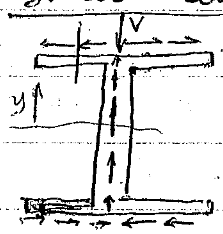
$$\tau_{xy} = -\frac{V}{I}\int y\,\partial y$$ if τ_{xy} is $f(y)$ only

$$\tau_{xy} = -\frac{V}{I}\int y\,dy = -\frac{V}{I}\frac{y^2}{2} + C_1$$ @ $y = \pm\frac{h}{2}$ if symmetry about neutral axis; since τ_{xy} at free surfaces are $= 0$

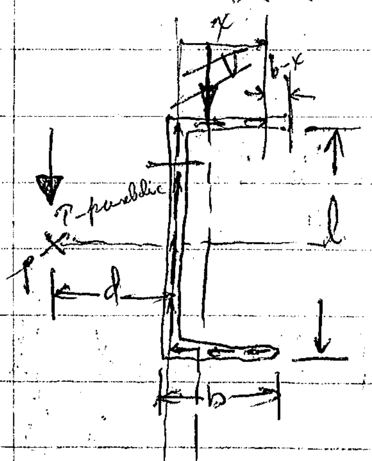
$C_1 = \frac{Vh^2}{8I}$ if body is rectangular in nature

$$\tau_{xy} = \frac{V}{2I}\left(\frac{h^2}{4} - y^2\right) \qquad V = f(x)$$

Shear Center



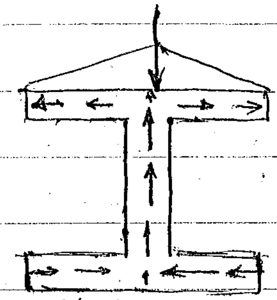
Since y is constant throughout the flange the only variant in Q is dA. ∴ Q is linear. τ_{xy} is the linear. Vertical, Horizontal & rotation equil are satisfy.



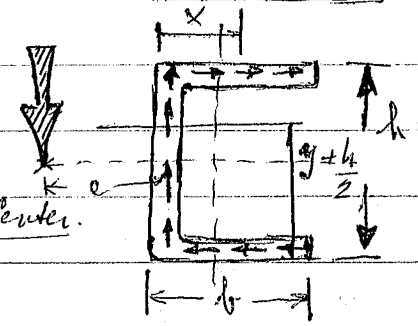
Torsion will occur by unbalanced shears in flange & shear & V of web. To remove torsion there exists a pt ∋

$Vd = T\,l$ P is a shear center.

Assume thin walled-member



P is linear in flanges, P parabolic in web



Thin member. $t \ll h$, $t \ll b$

for flange: $Q = (b-x)\,t\left(\frac{h}{2}\right)$

$$q = \frac{V}{I}\,\frac{th}{2}(b-x)$$

$Q_a = bht/2$

$$q_a = \frac{V}{I}\,\frac{thb}{2}$$

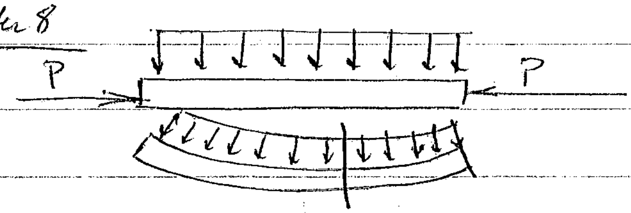
for web: $Q = Q_a + \left(\frac{h}{2}-y\right)t\left(\frac{y+\frac{h}{2}-y}{2}\right)$

$$q = q_a + \frac{V}{I}\left(\frac{t}{2}\right)\left(\frac{h}{2}-y\right)\left(\frac{h}{2}+y\right) = q_a + \frac{V}{I}\left(\frac{t}{2}\right)\left[\frac{h^2}{4}-y^2\right]$$

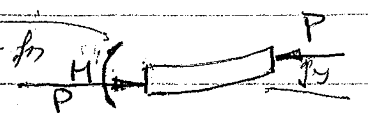
$V = \int_A q\,dy$ $\qquad F_{flanges} = \frac{1}{2}q_a b$ $\qquad T = \frac{1}{2}q_a b\,h = Ve$

$$e = \frac{q_a b h}{2V} = \frac{th^2b^2}{4I}$$

Chapter 8

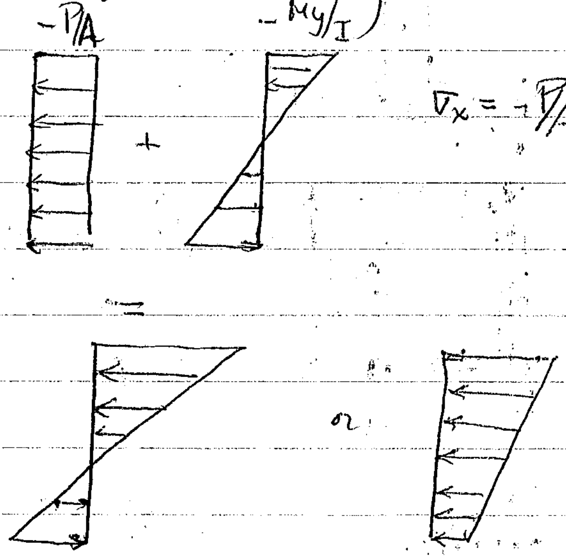


this must change to $Py+M$ to account for superposition of P to bending



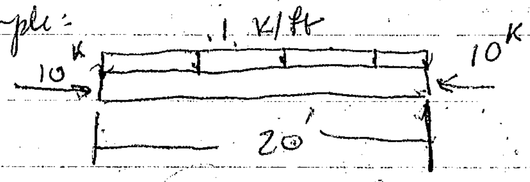
If $\bar{y}$ is small ∋ $P\bar{y} \ll M$ then we can superimpose

If beam is allowed to bend elastically $\bar{y}$ will be small, P must be small enough ∋ buckling doesn't occur.



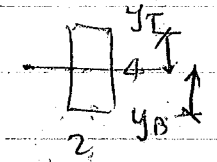
$$\sigma_x = -P/A - \frac{My}{I}$$

Example:



$$M_{max} = \frac{wL^2}{8} = \frac{.1 \times 400}{8} = 5^{k\cdot ft}$$

$$\sigma_x = -\frac{10}{8} - \frac{60}{10.67}y$$



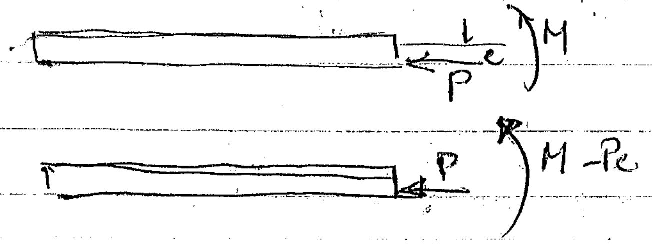
$$I = \frac{2\cdot 4^3}{12} = 10.67$$

M =

$$\sigma_T = \frac{-10}{8} \; \frac{-120}{10.67} =$$

$$\boxed{\sigma_x = -F/A - \frac{Mc}{I}}$$

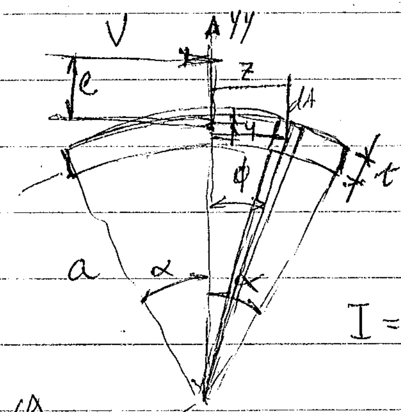
$$\sigma_B = -\frac{10}{8} + \frac{120}{10.67} =$$



$\sigma = -\frac{P}{A} \pm \frac{[M^* c]}{I}$ where $M^* = M - Pe$

In concrete $\sigma \begin{cases} \leq 0 \\ \geq f_c \end{cases}$

7-28



$dA = a\,t\,d\phi$, $z = a \sin\phi$

$q = \frac{VQ}{I}$ $\quad y = a(1-\cos\phi)$

$I = \int_A z^2 dA \qquad Q = \int_\phi^\alpha z\, dA$

$$Q = \int_\phi^\alpha a \sin\phi \; a t\, d\phi = a^2 t \int \sin\phi \, d\phi = -a^2 t \cos(\alpha - \phi)$$

$$I = \int_{-\alpha}^{\alpha} a^2 \sin^2\phi \, a t \, d\phi = a^3 t \int_{-\alpha}^{\alpha} \left(\frac{1 - \cos 2\phi}{2}\right) d\phi$$

$$\frac{a^3 t}{2}\left(\phi - \frac{1}{2}\sin 2\phi\right)_{-\alpha}^{\alpha} = \frac{a^3 t}{2}(2\alpha - \sin 2\alpha) = a^3 t(\alpha - \sin\alpha\cos\alpha$$

$$q = \frac{V(-a^2 t \cos(\alpha-\phi))}{\frac{a^3 t}{2}(2\alpha - \sin 2\alpha)} = -\frac{2V}{a}\frac{(\cos(\alpha-\phi))}{2\alpha - \sin 2\alpha}$$

$$T = \int q\, a\, d\phi\, y = \int_{-\alpha}^{\alpha} \frac{-2V \cos(\alpha-\phi)}{2\alpha - \sin 2\alpha} a(1-\cos\phi)\, d\phi$$

$$\frac{-2Va}{2\alpha - \sin 2\alpha}\int_{-\alpha}^{\alpha} \cos(\alpha-\phi)(1-\cos\phi)\, d\phi =$$

$$\int_{-\alpha}^{\alpha}(\cos\alpha\cos\phi + \sin\alpha\sin\phi)(1-\cos\phi)\, d\phi$$

$\cos\alpha\cos\phi + \sin\alpha\sin\phi - \cos\alpha\left(\frac{1+\cos 2\phi}{2}\right) - \sin\alpha\,\frac{\sin 2\phi}{2}$

$T + Ve = 0 \qquad e = -\frac{T}{V}$

$\left(+\cos\alpha\sin\phi + \sin\alpha\cos\phi - \cos\alpha\left[\frac{\phi}{2} + \frac{\sin 2\phi}{4}\right] + \sin\alpha\left[\frac{\cos 2\phi}{4}\right]\right)\Big|_{-\alpha}^{\alpha}$

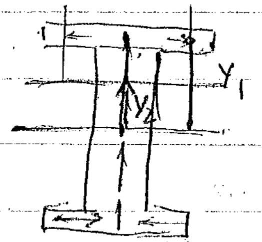
7-21

Capacity - that load at which the max allowable stress is reached.

$q_y \neq q_z$ but $\tau_y = \tau_z$

$I_z = 166$

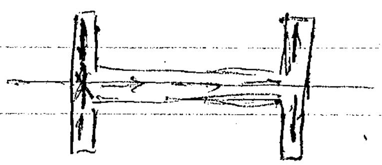
$I_y = 36.5$



$\tau_{max_1} = \frac{V_z}{I_z}\frac{Q_z}{t_z}$ $\qquad Q_z = 6\cdot 1\cdot(4-\tfrac{1}{2}) + 3\cdot 1\cdot 1.5 = 25.5$

$t_z = 1$

$\frac{V_z}{166}\,\frac{25.5}{1}$



$\tau_{max} = \frac{V_y}{I_y}\frac{Q_y}{t_y}$

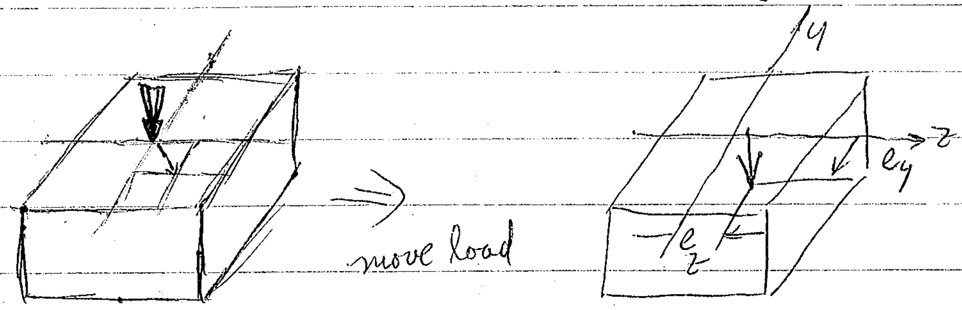
$Q_y = 3\cdot 1\cdot 1.5 + 3\cdot 1\cdot 1.5 + 6\cdot .5\cdot .25$

9.75

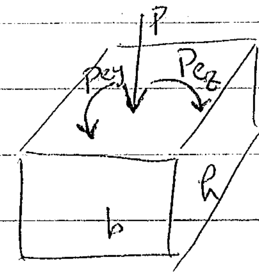
$\tau_{max_2} = \frac{V_y}{I_y}\,\frac{9.75}{2}$ $\qquad t_z$ = thickness of each flange

~~$\tau_{max_1} = \tau_{max_2}$~~

$\frac{V_y}{I_y}\frac{9.75}{2} = \frac{V_z}{I_z}\frac{25.1}{1}$ $\qquad V_y = \frac{I_y}{I_z}\frac{Q_z}{Q_y}\frac{t_z}{t_y}V_z$



$=$



$\sigma_x = -\frac{P}{A} - \frac{\overset{M_y}{Pe_z}}{I_y}z + \frac{\overset{M_z}{Pe_y}}{I_z}y \le 0$

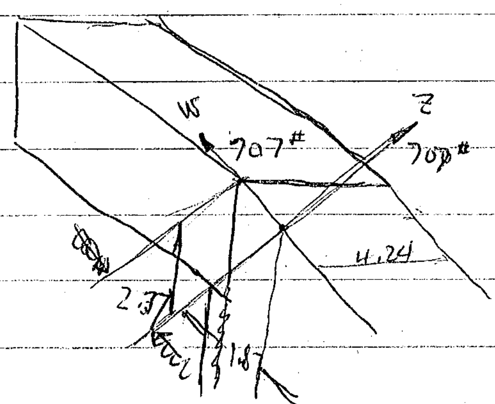
$I_z = bh^3/12$

$I_y = \frac{hb^3}{12} \qquad A = bh$

if $\sigma_x = 0$ then $-1 - \frac{e_z z}{b^2/12} + \frac{e_y y}{h^2/12} = 0$

$\pm\frac{e_z\, b/2}{b^2/12} \pm \frac{e_y\, h/2}{h^2/12} = 1 \qquad \pm\frac{e_z}{b/6} \pm \frac{e_y}{h/6} = 1$ defines delineated area called kern

8-21



$I_{zz} = 8.0 = Ar_z^2$

$I_{ww} = 31.8$

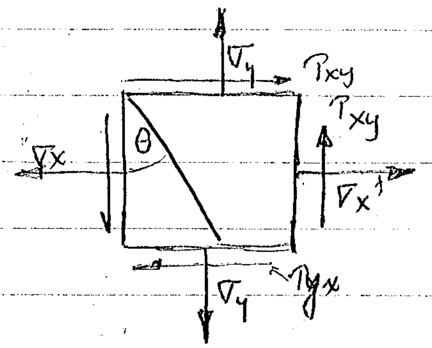
$M_z = M_w = .707 \times 70.7 \approx 50^k$

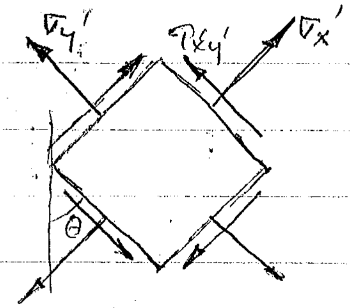
$\sigma_A = \frac{50\times 4.24}{31.8} + \frac{50\times 1.87}{8.0}$

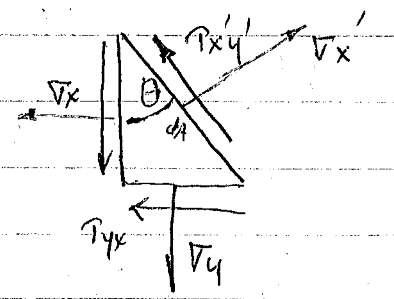
$-\sigma_B = \frac{50\times 2.37}{8.0} =$

$\sigma_C = \frac{50\times 1.87}{8.0} - \frac{50\times 4.24}{31.8}$

Transformation of Plane Stress







$\Sigma F_{x'} = 0$

$$\sigma_x' dA - \sigma_x dA \cos^2\theta - \sigma_y dA \sin^2\theta + \tau_{xy}(-2dA \sin\theta \cos\theta) = 0$$

$$\sigma_x' = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_x' = \sigma_x \left(\frac{1+\cos 2\theta}{2}\right) + \sigma_y\left(\frac{1-\cos 2\theta}{2}\right) + \tau_{xy} \sin 2\theta$$

$$\boxed{\sigma_x' = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy} \sin 2\theta}$$

$$\frac{\partial \sigma_x'}{\partial \theta} = -2\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + 2\tau_{xy} \cos 2\theta$$

$$(\sigma_y - \sigma_x) \sin 2\theta + 2\tau_{xy} \cos 2\theta = 0$$

$$\tan 2\theta_{1,2} = 2\tau_{xy}/(\sigma_x - \sigma_y) \quad \therefore \quad (A)$$

$\Sigma F_{y'} = 0$
$$\tau_{x'y'} dA + \sigma_x dA \cos\theta \sin\theta - \sigma_y dA \sin\theta \cos\theta + \tau_{xy}(dA \cos^2\theta - dA \sin^2\theta) = 0$$

$$\tau_{x'y'} = -\sigma_x \frac{\sin 2\theta}{2} + \sigma_y \frac{\sin 2\theta}{2} + \tau_{xy} \cos 2\theta$$

$$\boxed{\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta}$$

$$\frac{\partial \tau_{x'y'}}{\partial \theta} = -(\sigma_x - \sigma_y) \cos 2\theta - 2\tau_{xy} \sin 2\theta = 0$$

$$\therefore \left(\frac{2\tau_{xy}}{\sigma_y - \sigma_x}\right)^{-1} = \tan 2\theta_3 \quad (B)$$

$$\sigma_{y'} = \frac{\sigma_x+\sigma_y}{2} - \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$\sigma_x' + \sigma_y' = \sigma_x + \sigma_y$ if $\sigma_x' = max$ $\sigma_y' = min$

$\sigma'_{x\,max} \perp \sigma'_{y\,min}$

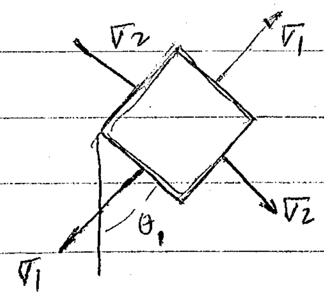
if $\sigma_{x\,\substack{max\\min}}$ $\tau_{xy} = 0$

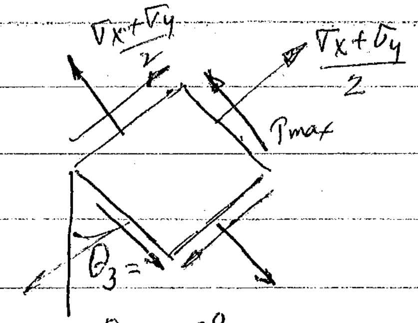
(A) $\sigma_{1,2}$ (principal stresses) $= \dfrac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$

(B) $\tau_{\substack{max\\min}} = \pm\sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \pm\dfrac{\sigma_1-\sigma_2}{2}$

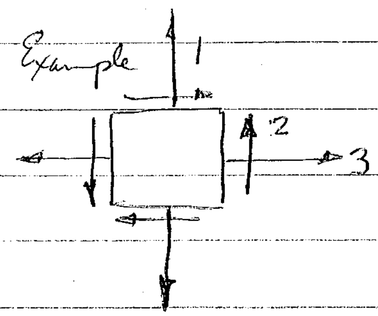
when $\tau_{max} = \dfrac{\sigma_x+\sigma_y}{2}$



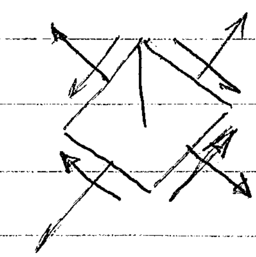


$\theta_1 + 45°$

Example



Suppose $\theta = -22.5°$



$\sigma_x' = 2 + .707 - 1.414$

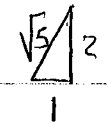
$= 1.293$

$\sigma_y' = 2.707$

$\tau_{x'y'} = +.707 + 3 \times .707 = 2.12$

at $-22.5°$

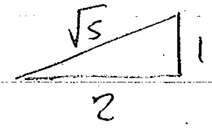
Principal stresses $\tan 2\theta_1 = \frac{2}{1} = 2$



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 2 \pm \sqrt{1+4} = 2 \pm \sqrt{5}$$

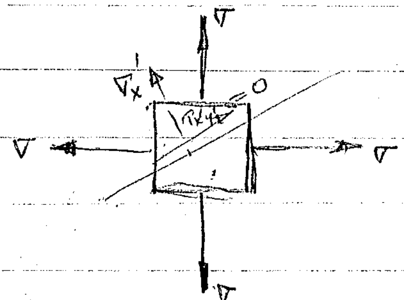
$$\tau_{1,2} = -1 \cdot \frac{2}{\sqrt{5}} + 2 \times \frac{1}{\sqrt{5}} = 0$$

$$\tan 2\theta_3 = -\frac{1}{2}$$



$$\sigma_3 = 2 + 1 \cdot \frac{2}{\sqrt{5}} - 2 \cdot \frac{1}{\sqrt{5}} = 2$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \sqrt{5}$$



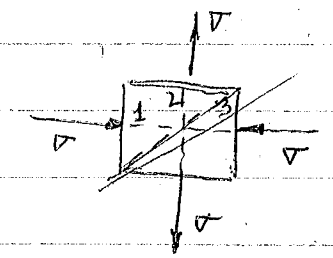
principal stresses no shear

$$\sigma_x' = \frac{\sigma + \sigma}{2} + \frac{\cancel{\sigma - \sigma}}{2}(\quad) + \overset{0}{\tau_{xy}}(\quad)$$

$\sigma_x' = \sigma$ irregardless of angle

$$\tau_{x'y'} = 0$$

degenerate case σ stays constant at any theta, fluid cannot sustain shear $\therefore$ must be in a state $\ni$ $\sigma_x' = \sigma_x = \sigma_y$



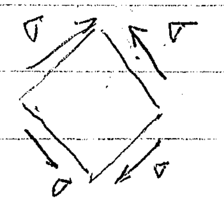
pure shear $\sigma_x' = \overset{0}{\frac{\sigma_x + \sigma_z}{2}} + \frac{\overset{-\sigma}{\sigma_x - \sigma_y}}{2} \cos 2\theta$

$\sigma_y' = -\sigma_x'$

$+ \tau_{x''y''} \sin 2\theta$

$$\tau_{x'y'} = \sigma \sin 2\theta + \overset{0}{\tau_{xy} \cancel{\cos 2\theta}}$$

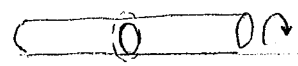
$$\tau_{max} = \sigma$$

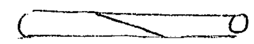


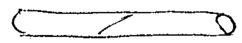
if weak in tension breaks at face 1

if weak in compression breaks at plane 2

if " " shear breaks at plane 3

 if weak in ~~shear~~ torque 2 planes shear

 if weak in tension buckling at 45° angle

 if weak in compression buckling at 45° angle

Mohr's circle of Stress

$$\sigma_x' - \left(\frac{\sigma_x+\sigma_y}{2}\right) = \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

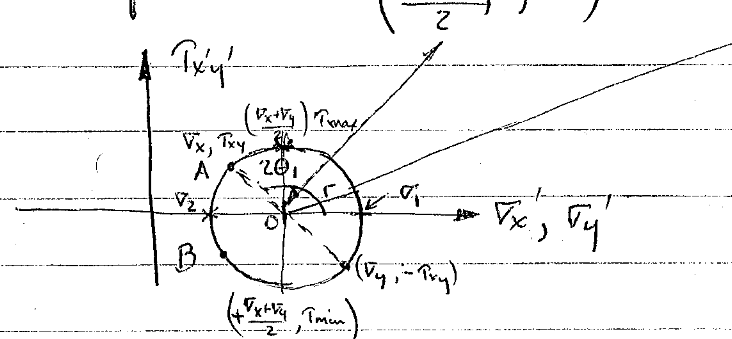
$$\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\left(\sigma_x' - \frac{\sigma_x+\sigma_y}{2}\right)^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2\cos^2 2\theta + 2\tau_{xy}\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta \sin 2\theta + \tau_{xy}^2\sin^2 2\theta$$

$$\tau_{x'y'}^2 = \left(\frac{\sigma_y-\sigma_x}{2}\right)^2\sin^2 2\theta + 2\tau_{xy}\cos 2\theta\left(\frac{\sigma_y-\sigma_x}{2}\right)\sin 2\theta + \tau_{xy}^2\cos^2 2\theta$$

$$\left(\sigma_x' - \left(\frac{\sigma_x+\sigma_y}{2}\right)\right)^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_y-\sigma_x}{2}\right)^2 + \tau_{xy}^2$$

center of circle is at $\left(\frac{\sigma_x+\sigma_y}{2}, 0\right)$ radius $= \sqrt{\left(\frac{\sigma_y-\sigma_x}{2}\right)^2+\tau_{xy}^2} = \tau_{max}$



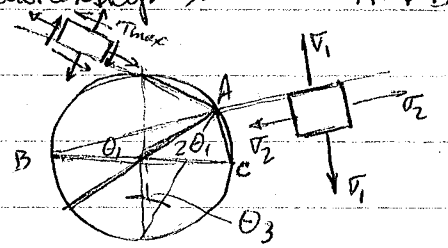
$$\sigma_{1,2} = \left(\frac{\sigma_x+\sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

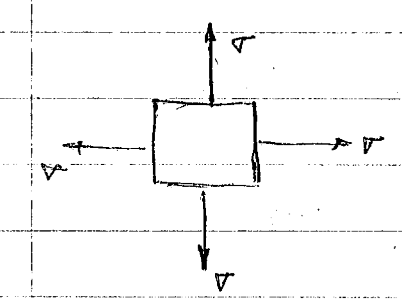
$$\tau_{max} = \sqrt{\left(\frac{\sigma_y-\sigma_x}{2}\right)^2+\tau_{xy}^2}$$

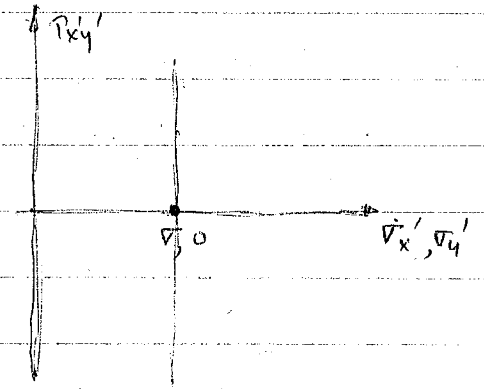
knowing σ_x, σ_y & τ_{xy}: locate center of circle, O @ $\left(\frac{\sigma_x+\sigma_y}{2}, 0\right)$; locate A @ (σ_x, τ_{xy})

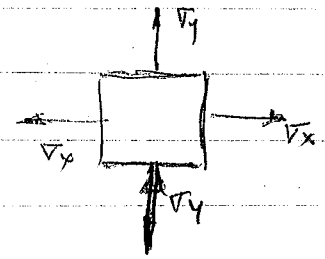
Construct Mohr's circle, with center @ O & radius $\overline{OA}$

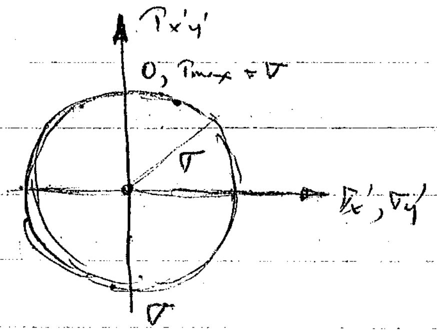
Find a relationship between A & B



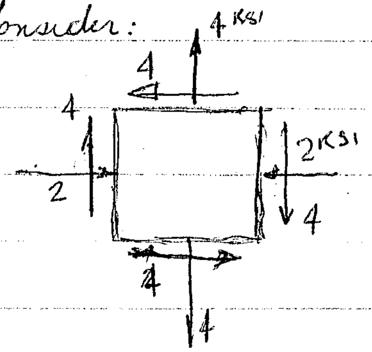




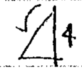




Consider:

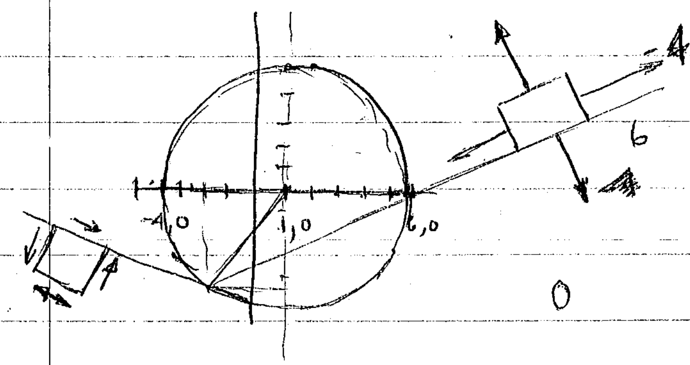


$$\tan 2\theta_{1,2} = \frac{-4(2)}{-2-4} = \frac{4}{3}$$



$$\sigma_{1,2} = \frac{4-2}{2} \pm \sqrt{\left(\frac{4-(-2)}{2}\right)^2 + (-4)^2} = \begin{cases} 6 \\ -4 \end{cases}$$

$$\tau_{max} = \pm 5 \qquad \sigma_{\tau_{max}} = \frac{\sigma_1 + \sigma_2}{2} = 1$$



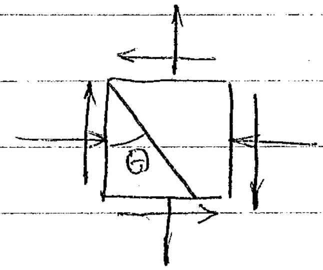
$\sigma_1 = 1+5 = 6$

$\sigma_2 = 1-5 = -4$

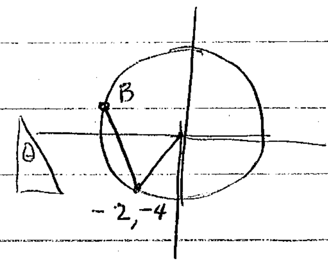
$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = 5$

θ

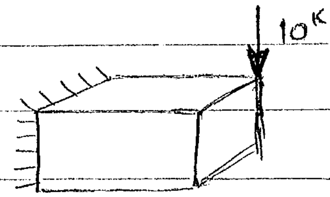
$\tau_{min} = -5$



to find stresses at angle Θ from vertical face draw a line ‖ to cut and where it intersects circle is it.



8-29



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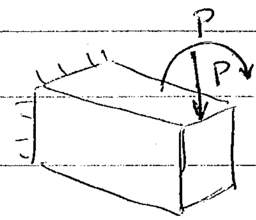
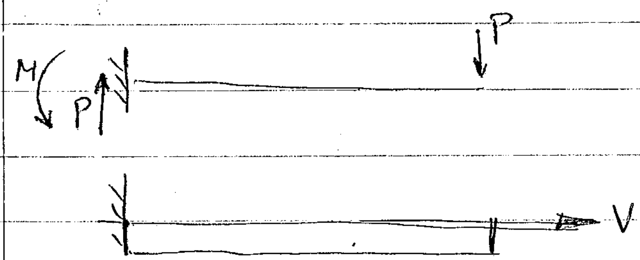
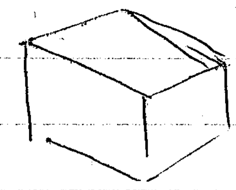
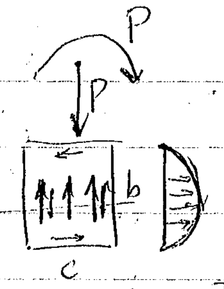




$P = 10^K$ $\quad$ $T = 10^K\text{-in}$



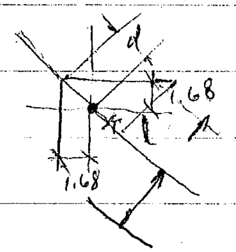


$\tau_v = \frac{3}{2}\frac{V}{A}$

$\tau_T = \frac{T}{\alpha b c^2}$

$\tau_{max} = \frac{3}{2}\frac{V}{A} + \frac{T}{\alpha b c^2} = \frac{3}{2}\frac{10}{4\cdot 2} + \frac{10}{(.246)(4)(2)^2}$

8-21.



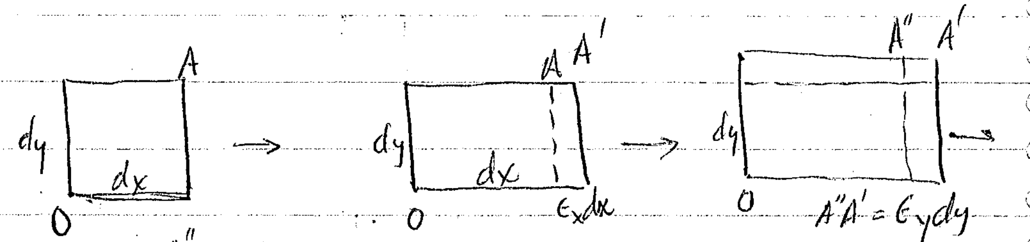
$d = \sqrt{2 \times 1.68^2} = 2.37$

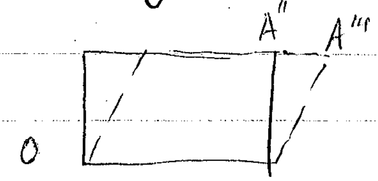
$\cancel{18 = 2b^3}$ $\quad K = \cancel{1.37}\ \sqrt{18} - 2.37 = 1.87$

$l = 4.24$

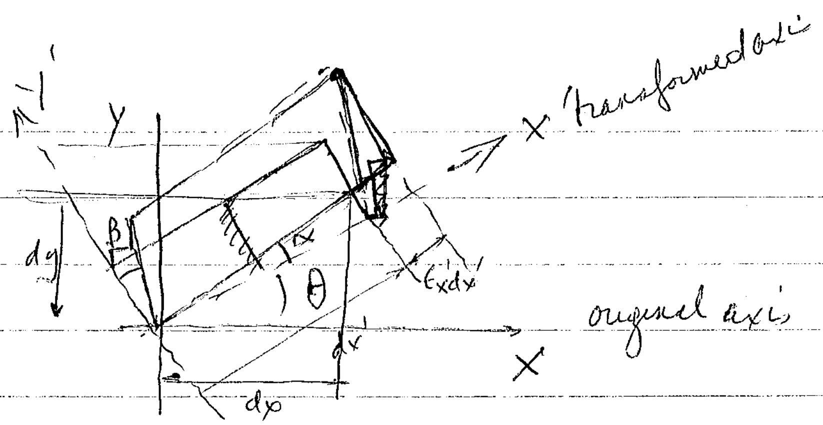
Transformation of Strain

$$\begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} \\ \epsilon_{xy} & \epsilon_{yy} \end{pmatrix}$$





$A''A''' = \gamma_{xy}\, dy$



$$\epsilon_x' dx' = AA' \cos\theta + A'A'' \sin\theta + A''A''' \cos\theta$$

$$\epsilon_x' = \epsilon_x \frac{dx}{dx'} \cos\theta + \epsilon_y \frac{dy}{dx'} \sin\theta + \gamma_{xy} \frac{dy}{dx'} \cos\theta$$

$$= \epsilon_x \underbrace{\cos^2\theta}_{\left(\frac{1+\cos 2\theta}{2}\right)} + \epsilon_y \underbrace{\sin^2\theta}_{\left(\frac{1-\cos 2\theta}{2}\right)} + \gamma_{xy} \frac{\sin 2\theta}{2}$$

$$\epsilon_x' = \frac{\epsilon_x + \epsilon_y}{2} + \left(\frac{\epsilon_x - \epsilon_y}{2}\right)\cos 2\theta + \gamma_{xy}\frac{\sin 2\theta}{2} \text{ ; but } \epsilon_{xy} = \frac{\gamma_{xy}}{2}$$

therefore $\boxed{\epsilon_x' = \frac{\epsilon_x + \epsilon_y}{2} + (\epsilon_x - \epsilon_y)\frac{\cos 2\theta}{2} + \epsilon_{xy} \sin 2\theta}$

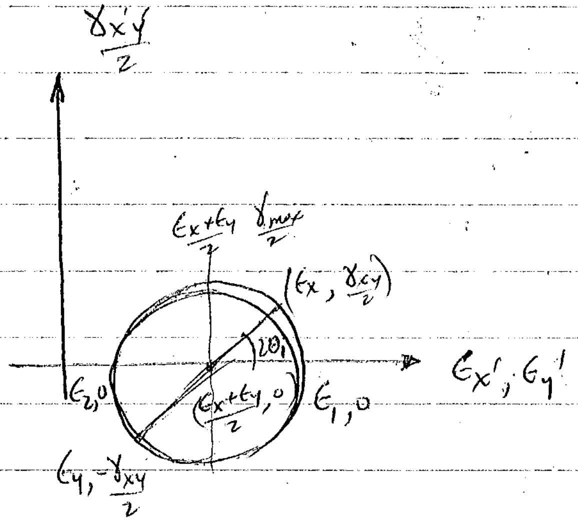
$$\gamma_{x'y'} = \alpha + \beta \qquad \alpha \approx \tan\alpha = \frac{-AA'\sin\theta + A'A''\cos\theta - A''A'''\sin\theta}{dx'}$$

$$\alpha = -\epsilon_x \frac{\sin 2\theta}{2} + \epsilon_y \frac{\cancel{\cos 2\theta}}{2} - \gamma_{xy} \sin^2\theta$$

$$\beta = \cancel{\quad} -\epsilon_x \frac{\sin 2\theta}{2} + \epsilon_y \frac{\cancel{\sin 2\theta}}{2} + \gamma_{xy} \cos^2\theta$$

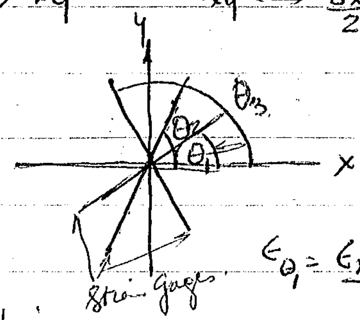
$$\gamma_{x'y'} = \alpha + \beta \quad -(\epsilon_x - \epsilon_y) \sin 2\theta + \gamma_{xy} \cos 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = \epsilon_{x'y'} = -\frac{(\epsilon_x - \epsilon_y)}{2} \sin 2\theta + \frac{\gamma_{xy}}{2} \cos 2\theta$$



$\sigma_x \Longleftrightarrow \epsilon_x \qquad \sigma_y \Longleftrightarrow \epsilon_y \qquad \tau_{xy} \Longleftrightarrow \frac{\gamma_{xy}}{2}$

$$\begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{pmatrix}$$



$$\epsilon_{\theta_1} = \frac{\epsilon_x+\epsilon_y}{2} + \left(\frac{\epsilon_x-\epsilon_y}{2}\right)\cos 2\theta_1 + \frac{\gamma_{xy}}{2}\sin 2\theta_1$$

Starting with

$\sigma_z = 0$

$$\epsilon_{\theta_2} = \ldots + \ldots \cos 2\theta_2 + \ldots \sin 2\theta_2$$

$$\epsilon_{\theta_3} = \ldots + \ldots \cos 2\theta_3 + \ldots \sin 2\theta_3$$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y - \frac{\nu}{E}\sigma_z^{\swarrow 0}$$

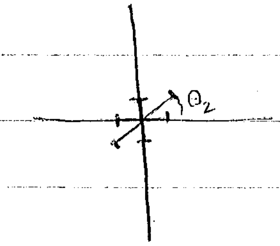
$$\epsilon_x + \nu\epsilon_y = \frac{(1-\nu^2)}{E}\sigma_x$$

$$\epsilon_y = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E}$$

$$\boxed{\sigma_x = \frac{(\epsilon_x+\nu\epsilon_y)E}{1-\nu^2}}$$

$$\boxed{\tau_{xy} = G\gamma_{xy}}$$

if E, G are known

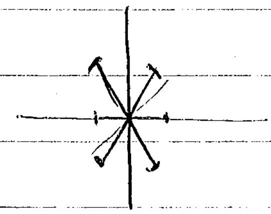


$\theta_1 = 0^\circ \quad \theta_2 = 45^\circ \quad \theta_3 = 90^\circ$

$\epsilon_{\theta_1} = \epsilon_x$

$\epsilon_{\theta_3} = \epsilon_y \qquad \epsilon_{45^\circ} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\gamma_{xy}}{2}$

$\gamma_{xy} = 2\epsilon_{45^\circ} - (\epsilon_{0^\circ} + \epsilon_{90^\circ})$



$\theta_1 = 0^\circ \quad \theta_2 = 60^\circ \quad \theta_3 = 120^\circ$

$\epsilon_{0^\circ} = \epsilon_x \qquad \epsilon_{60^\circ} = \frac{\epsilon_x + \epsilon_y}{2} - \left(\frac{\epsilon_x - \epsilon_y}{4}\right) + \frac{\gamma_{xy}\sqrt{3}}{4}$

$\epsilon_{120^\circ} = \frac{\epsilon_x + \epsilon_y}{2} - \left(\frac{\epsilon_x - \epsilon_y}{4}\right) - \frac{\gamma_{xy}\sqrt{3}}{4}$

$\epsilon_0 = -.0005 \qquad \nu = .3$

$\epsilon_{45^\circ} = .0002 \qquad E = 30 \times 10^6 \text{ psi}$

$\epsilon_{90^\circ} = .0003$

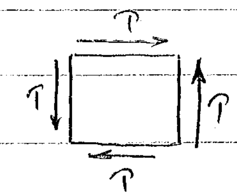
$\epsilon_x = -.0005 \qquad \sigma_x = \frac{-.0005 + (.3)(.0003)}{.91} \, 30 \times 10^6$

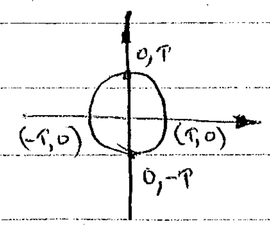
$\epsilon_y = .0003$

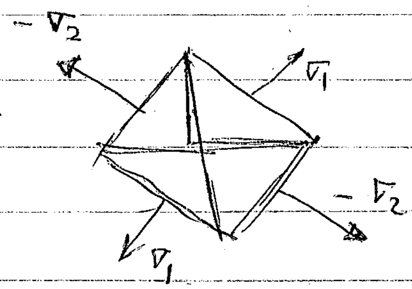
$\sigma_y = \frac{.0003 - .0015}{1.91} \, 30 \times 10^6$

$\gamma_{xy} = 2(.0002) - (-.0005 + .0003)$

$\tau_{xy} = G(\gamma_{xy})$







$\sigma_1 = \tau \qquad \theta_1 = 45^\circ$

$\sigma_2 = -\tau$

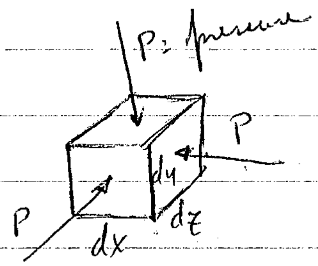
$\epsilon_x = \epsilon_y = 0$

$\gamma_{xy} \neq 0$

$$\epsilon_{45^\circ} = \overset{0}{\frac{\epsilon_x+\epsilon_y}{2}} + \overset{0}{\frac{\epsilon_x-\epsilon_y}{2}}\cos 2\theta_1 + \frac{\gamma_{xy}}{2}\overset{1}{\sin 2\theta_1}$$

$$\epsilon_{45^\circ} = \frac{\gamma_{xy}}{2} = \frac{\tau_{xy}}{2G} = \frac{\sigma_1}{E} - \nu\frac{\sigma_2}{E} = \left(\frac{1+\nu}{E}\right)\tau_{xy}$$

$$\frac{\tau_{xy}}{2G} = \frac{1+\nu}{E}\tau_{xy} \qquad G = \frac{E}{2(1+\nu)}$$



bulk Modulus = K = $\dfrac{\text{compressive stress}}{\text{decrease in volume}}$

$$E = \frac{\sigma}{\epsilon}$$

Change in volume $= (1+\epsilon_x)dx\,(1+\epsilon_y)dy\,(1-\epsilon_z)dz$

$- \, dx\,dy\,dz = \left[(1+\epsilon_x)(1+\epsilon_y)(1+\epsilon_z)-1\right]dx\,dy\,dz$

Assume: second order changes are 0.

$\approx (\epsilon_x + \epsilon_y + \epsilon_z)\,dx\,dy\,dz$

However. dilatation $= e = \dfrac{\text{change in volume}}{\text{original vol}} = \epsilon_x + \epsilon_y + \epsilon_z$

dilatation is an invariant since ΔV, V_{orig} do not change with change in coordinate system!

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$e = \epsilon_x + \epsilon_y + \epsilon_z = \frac{(1-2\nu)}{E}\sigma_x + \left(\frac{1-2\nu}{E}\right)\sigma_y + \left(\frac{1-2\nu}{E}\right)\sigma_z$$

$\sigma_x + \sigma_y + \sigma_z$ is also an invariant.

Suppose $\sigma_x = \sigma_y = \sigma_y = -p$

$$e = \epsilon_x + \epsilon_y + \epsilon_z = \left(\frac{1-2\nu}{E}\right)(-3p)$$

Compressive stress $= -p$

decrease in volume $= e$ $\qquad K = \dfrac{-p}{e} = \dfrac{E}{3(1-2\nu)}$ $\qquad \boxed{K = \dfrac{E}{3(1-2\nu)}}$

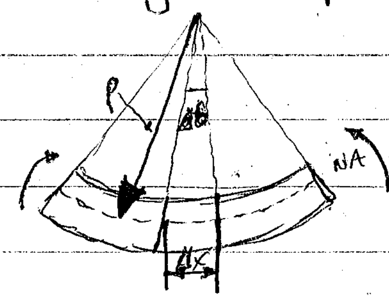
$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} = \frac{1-2\nu}{E}(-p)$$

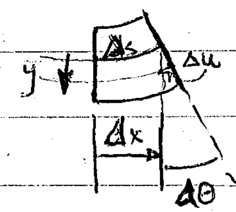
If $\epsilon_x = 0 \implies \dfrac{1-2\nu}{E} = 0 \qquad \nu = .5$

$K = \infty$ you need an infinite compressive stress to change volume

Deflections of Beams

1) Plane Sections Remain Plane
2) Shear deformation are neglected
3) bending about principal axis





$$\frac{\Delta u}{\Delta s} = -y\frac{\Delta\theta}{\Delta s}$$

$$\frac{d\theta}{ds} = \frac{1}{\rho}$$

$$\lim_{\Delta s \to 0} \frac{\Delta u}{\Delta s} = \frac{du}{ds} \qquad \lim_{\Delta s \to 0} -y\frac{\Delta\theta}{\Delta s} = -y\frac{d\theta}{ds}$$

$$\epsilon_x = \lim_{\Delta s \to 0}\frac{\Delta u}{\Delta s} = -y \lim_{\Delta s \to 0}\frac{\Delta\theta}{\Delta s}$$

$$\epsilon_x = -y\frac{1}{\rho} = -yK$$

$\boxed{K = \dfrac{1}{\rho} = -\dfrac{\epsilon_x}{y}}$ Universal (General Equations)

Assume that the problem satisfies three assumptions made & it occurs in elastic range.

if elastic $\quad \epsilon_x = \frac{\sigma_x}{E} = \frac{-My}{IE} \qquad \frac{1}{\rho} = \frac{-\sigma_x}{Ey} = \frac{M}{IE}$

$v = f(x)$

deflection of beam

$$\frac{1}{\rho} = \frac{\frac{d^2v}{dx^2}}{\left[1+\left(\frac{dv}{dx}\right)^2\right]^{3/2}} \qquad \frac{dv}{dx} \ll 1$$

$\left[1+\left(\frac{dv}{dx}\right)^2\right]^{3/2} \approx 1$

$\frac{d^2v}{dx^2} \approx \frac{M}{EI}$ $\qquad$ EI - flexural stiffness

$v(x)$ elastic curve

$\theta(x) = \frac{dv}{dx}$ = slope of elastic curve

$M(x) = EI\frac{d^2v}{dx^2}$ = moment curve.

$V(x) = -\frac{dM}{dx} = \frac{d}{dx}\left(-EI\frac{d^2v}{dx^2}\right)$ = shear curve

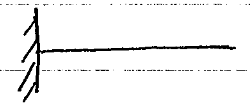
$p(x) = \frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right)$ = loading curve.

if EI is constant

$p(x) = EI\frac{d^4v}{dx^4}$

$V(x) = -EI\frac{d^3v}{dx^3}$

Boundary Conditions

Fixed Support  $v = 0$ $\theta = 0$

$\downarrow$ $\downarrow$

$V \neq 0$ $M \neq 0$

at free end
$V = 0$
$M = 0$

Simply Support (pin or roller) $v = 0 \rightarrow V \neq 0$

$\theta \neq 0 \rightarrow M = 0$

Free End $v \neq 0 \rightarrow V = 0$

$\theta = 0 \rightarrow M = 0$

at e

	A	y	Q	q	T
1.	.5	2.0	1.0	(7.38)(196)	2900
2.	1.5	2.75	2.75	1450	
3.	1.5	1.5	2.25		

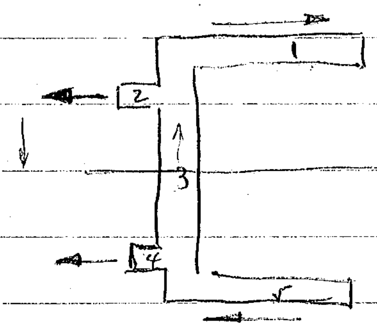
at e $T = 2900$ psi

8-21, 22, 24, 29   9-1, 2, 6, 7

On May 26   Exam 6-9
9-14, 16, 17, 23

[redacted]

26

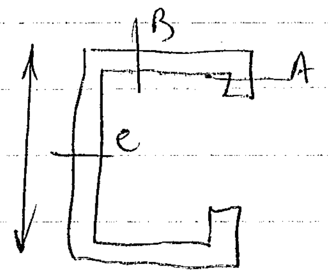


	A	d	Ad^2	I_0
1	.075	2.0	.3	
2	.025	1.5	.056	small
3	.2	0	0	
4	.025	-1.5	.056	
5	.075	-2.0	.3	

$Ve = F_2 h$

$$e = \frac{b^2 h^2 t}{4I} = \frac{(2)^2 (4)^2 (.05)}{4(.712\ in)}$$

$$e = 1.12\ in - .05\ in = 1.07\ in$$



$V = 7000\ \#$

$I = 35.7\ in$

$t = .5\ in$

$V/I = 196$

at A

A	y	Q
.5	2.0	1.0

$q = VQ/I = 196 \qquad \tau = q/t = 392$

$\tau = 392$

at B

A	y	Q	q	τ
.5	2.0	1.0	(5.13)(196)	2000
1.5	2.75	4.13	1000	

at B $\tau = 2000\ psi$

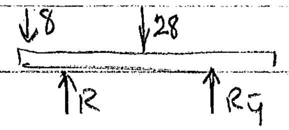
$\cancel{36}\sec^2\theta\, d\theta \cdot \frac{\cos^2\theta}{9}$

$d\theta$

$\cancel{\sec}\ \arccos \frac{3}{\sqrt{9+(x/12)^2}}$

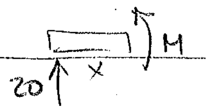
$\frac{4}{18} \arccos \frac{3}{\sqrt{9+(x/12)^2}} \Big|_0^{24}$

$-\frac{4}{18} \ln \left| \frac{36-x}{\sqrt{9(144)-x^2}} \right|$



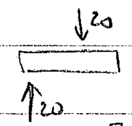
$R + R_g = 36$

$32(\ ) - 28(14)\ + R_y(20) = 0$

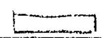


$R_y(20) = 392 - 32 = 360$

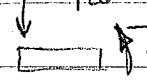
$R_y = 18$



$R_g = 18$

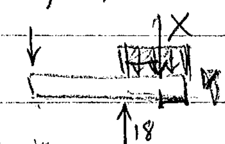


$\cancel{20}$



$-20(x) + 20(x-4) + M =$

$M =$ x



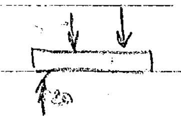
$+10x + 32 - \frac{x^2}{2} + \cancel{8}x + 16$

$M = -8x$

$20x - 240 + M$

$14x - \cancel{48}\,90 - x^2/2$

$56 - 40 - 8$



$-20(x) + 20(x-4)$

$x - \cancel{4}\ \ \frac{x-4}{2}$

$+20(x-8) + M$ $\ 8x - 18(x-4) + \ldots + M = 0$

$-10x + 36 + x^2/2 - 2x + \frac{2}{2} + M = 0$

$M = -8x + 18x - 36 - \frac{x^2}{2} + x$ $\quad 8x - 18(x-2) + \frac{1}{2}(x-2) \ldots - M = 0$

$M = 240 - 20x$ $\quad 11x - 36 - \frac{x^2}{2}$ $\quad M = -36 + 12x - x^2/2$

$x = 4$ $\quad 44 - 36 - 8$ $\quad -36 + 48 - \cancel{16}$

Feb 26 - Exam   Ch 1 & 2 and all ~~part~~ of chapter 3

Read Chapt 4   Prob 4-1

Prob. 4-3, 5, 6, 7, 8

4-20, 23, 24, 26, 31, 34, 35

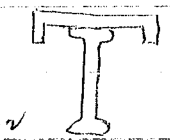
Prob 5-1, 2, 3, 4

Proble 6-10, 6-9, 6-11, 6-17

Proble 6-1, 3, 7, 12

TA 405   P68

TA 405   T485   1962

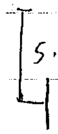


5.98

2.

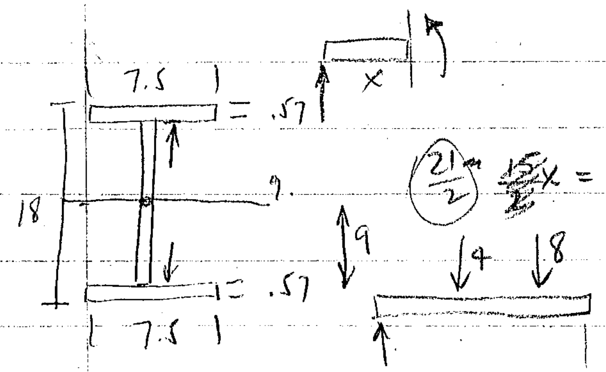
$\frac{I}{\frac{I}{c}} = c$

$\frac{128.1}{21.4}$ .598



2.29

2.29



$-\frac{15}{2}x + (x-2)4 + M = 0$

$\frac{21}{2}$ $\frac{15}{2}x = M$

$M = +\frac{21}{2}x + (2-x)4$

$\frac{21}{2}x + 8 - 4x$

$M = \frac{13}{2}x + 8$

39 + 8

$1.14(7.5)$ $-\frac{21}{2}x + 4(x-2) + 8(x-6) + M = 0$ (10½)

$16.86(.358)$ $M = \frac{21}{2}x - 4x + 8 - 8x + 48$

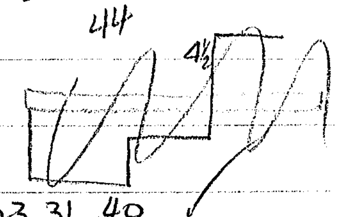
$8.5 + 6.4$ $-\frac{3}{2}x + 56 = M$ $\frac{21}{2}x + 56 = M$

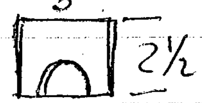
14.9   x = 6   -27 + 56 = 29

x = 8

-12 + 56   x = 8   -(36) + 56 = 20

44



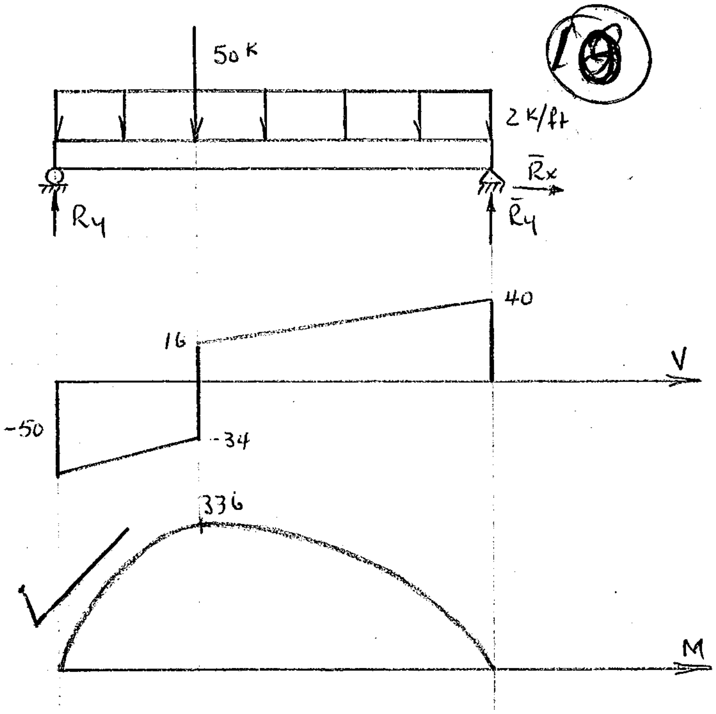


$\frac{3 \cdot \left(\frac{5}{2}\right)^3}{12} - .11(1)$

HW   6-19, 22, 23, 31, 40

7-2, 7-4, 7-9, 7-10, 7-16   7-21, 22, 25, 28

10

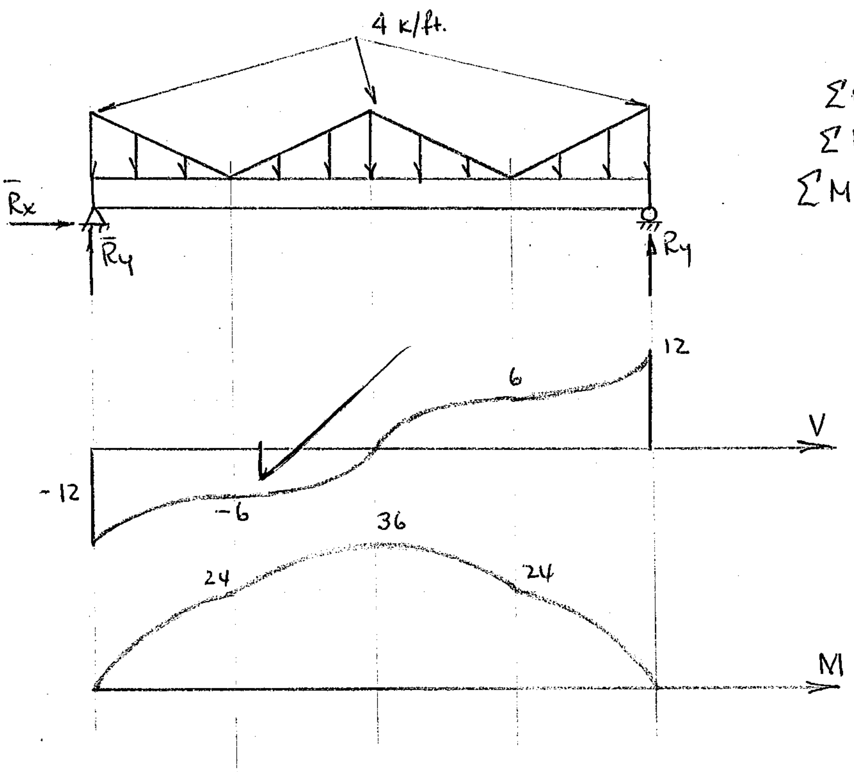


$\Sigma F_x = 0 \qquad \bar{R}_x = 0$

$\Sigma F_y = 0 \qquad 2(20) + 50 = R_y + \bar{R}_y$

$\Sigma M_y = 0 \qquad \bar{R}_y(20) - 50(8) - 40(10) = 0$

$\bar{R}_y = 40\text{k} \qquad R_y = 50\text{k}$

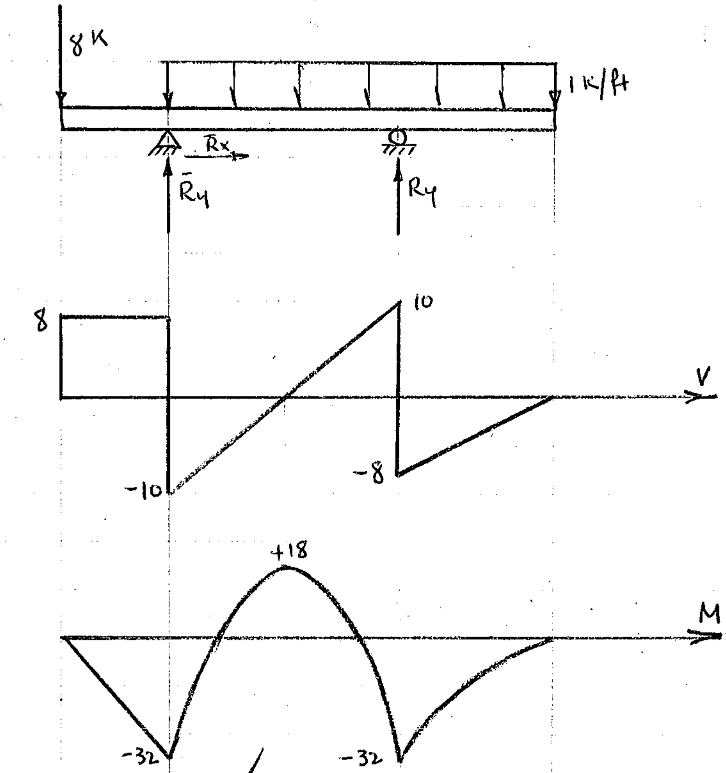


$\Sigma F_x = 0 \qquad \bar{R}_x = 0$

$\Sigma F_y = 0 \qquad R_y + \bar{R}_y = 4[4 \cdot 3 \cdot \tfrac{1}{2}]$

$\Sigma M_{\bar{y}} = 0 \qquad 6 \cdot 1 + 6 \cdot 5 + 6 \cdot 7 + 6 \cdot 11 = R_y(12)$

$R_y = 12\text{k} \qquad \bar{R}_y = 12\text{k}$

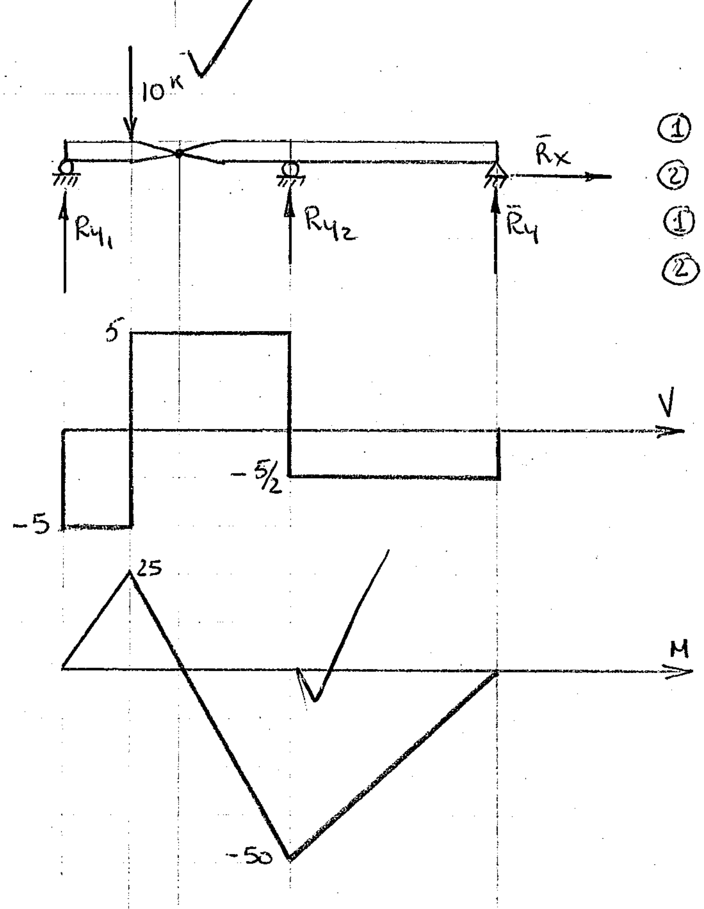


$\Sigma F_x = 0 \qquad \bar{R}_x = 0$

$\Sigma F_y = 0 \qquad R_y + \bar{R}_y = 36$

$\Sigma M_{\bar{y}} = 0 \qquad 32 + R_y(20) - 28(14) = 0$

$R_y = 18\,K \qquad \bar{R}_y = 18\,K$



$\Sigma F_x = 0 \qquad \bar{R}_x = 0$

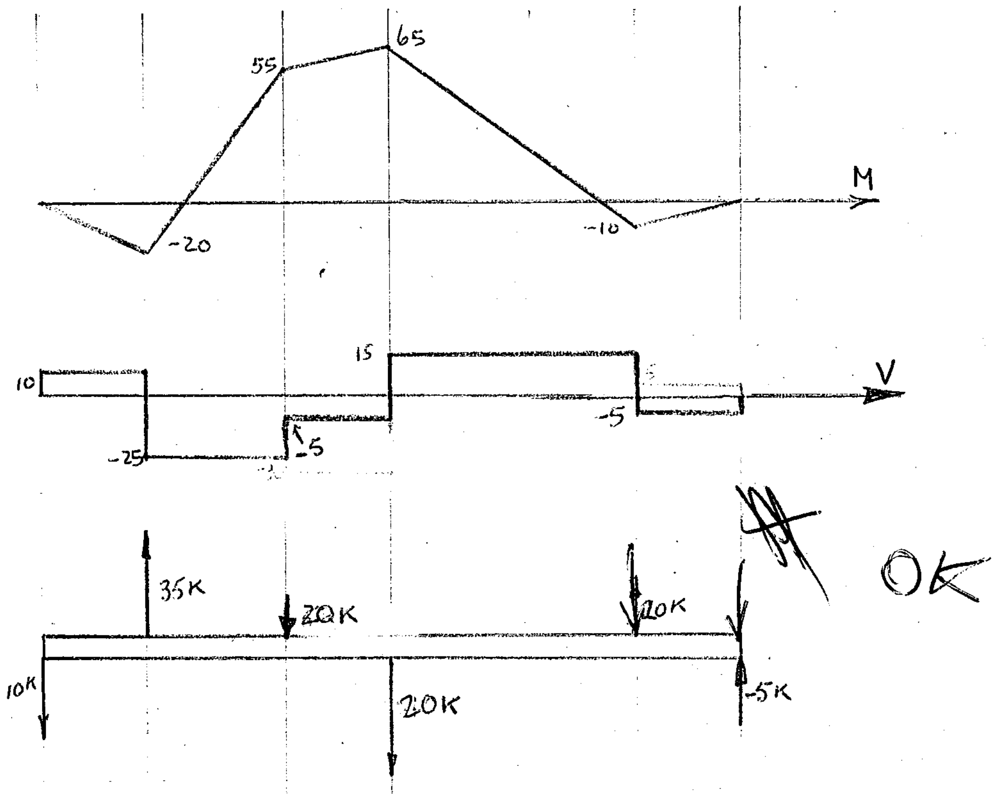
① $\Sigma F_y = 0 \qquad R_{y_1} + V = 10$

② $\Sigma F_y = 0 \qquad R_{y_2} + \bar{R}_y - V = 0$

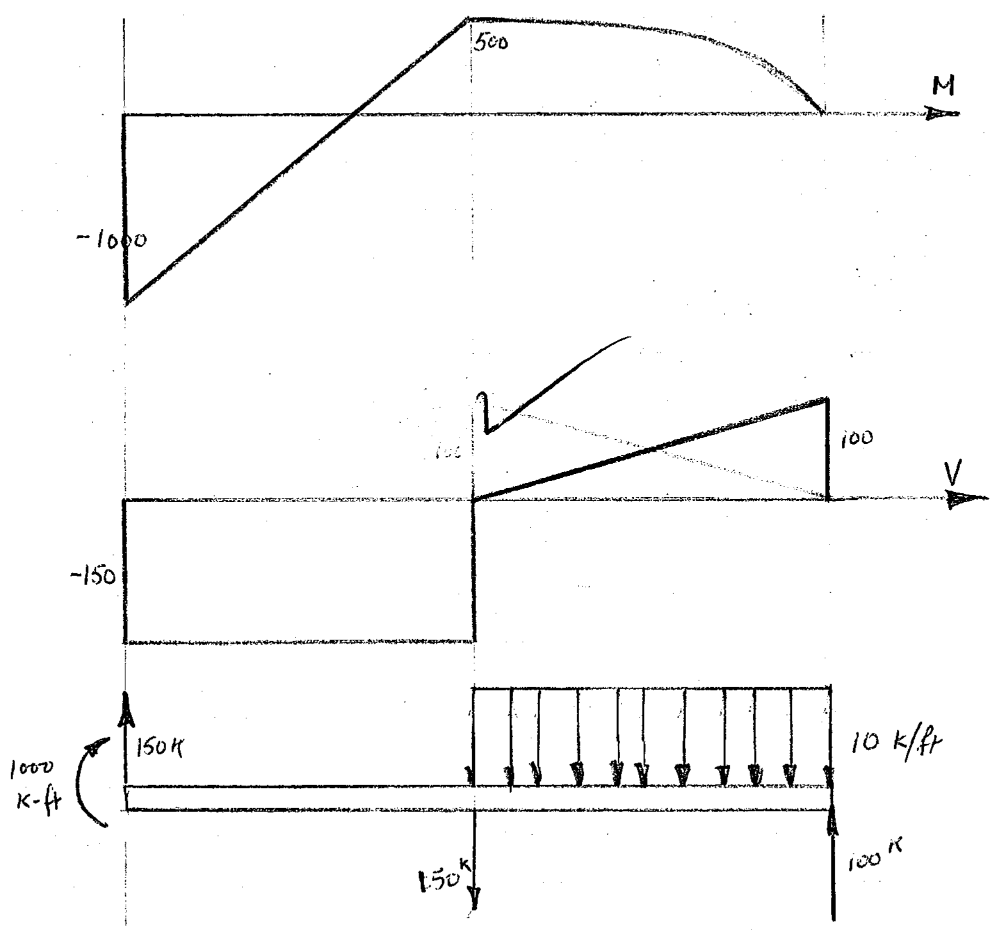
① $\Sigma M_{y_1} = 0 \qquad V(10) = 5(10) \qquad V = 5\,K \qquad \boxed{R_{y_1} = 5\,K}$

② $\Sigma M_{y_2} = 0 \qquad V(10) + \bar{R}_y(20) = 0 \qquad \boxed{\bar{R}_y = -2\tfrac{1}{2}\,K}$

$\boxed{R_{y_2} = 7\tfrac{1}{2}\,K}$



65
55
M
-20
-10
15
10
V
-5
-25
-5
35K
20K
20K
10K
20K
-5K
OK



500
M
-1000
100
V
-150
150K
10 k/ft
1000 k-ft
150K
100K

10

2-61 $\quad p(x) = -k(x) \quad$ at $L \quad p(L) = -KL \qquad \bar{L} = -\frac{KL^2}{2} \qquad K = -\frac{2\bar{L}}{L^2}$

$\therefore \quad p = +\frac{2\bar{L}}{L^2}x \implies V = \frac{2\bar{L}x^2}{2L^2} + c_1 \implies M = +\frac{\bar{L}}{3L^2}x^3 + c_1 x + c_2$

at $x = L \quad M = 0 \quad \therefore \quad M = +\frac{L}{3}\bar{L} - \bar{L}L + C_2 = 0 \qquad C_2 = +\frac{2}{3}L(\bar{L})$

at $x = L \quad V = 0 \quad \therefore \quad c_1 = -\bar{L}$

$$V = -\bar{L}\left(x^2/L^2 - 1\right) \qquad M = +\bar{L}\left(\frac{x^3}{3L^2} - x + \tfrac{2}{3}L\right)$$

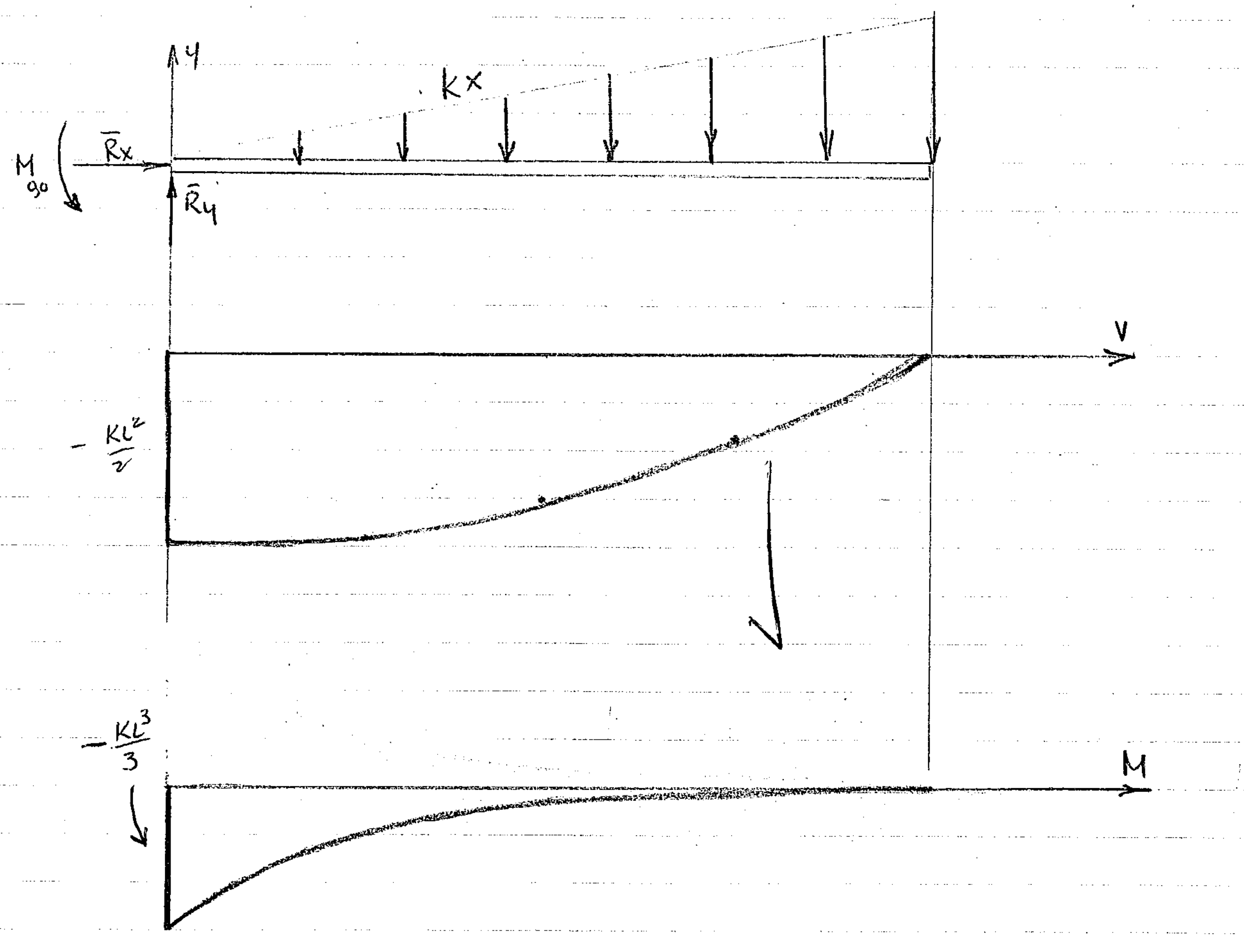
$\sum F_x = 0 \qquad \bar{R}_x = 0 \qquad$ at $x = 0$

$\sum F_y = 0 \qquad \bar{R}_y = \frac{KL^2}{2} \qquad \sum M_{0,0} = 0 \qquad M = \frac{KL^3}{3}$

formulas give at $x=0$ $\quad V = \bar{L} = -\frac{KL^2}{2}$ but the graph value is the negative of the reaction ∴ it checks

for the moment it is in same direction as if we were to take M +

the formulas give at $x = 0$ $\quad M = \frac{KL^3}{3}$ which checks



2-63 $\quad p(x) = K\sin(2\pi x/L)$

$$-V = -\frac{KL}{2\pi}\cos\frac{2\pi x}{L} + C_1$$

$$M = -\frac{KL^2}{4\pi^2}\sin\frac{2\pi x}{L} + C_1 x + C_2$$

at $x = 0; L \quad M = 0 \quad \therefore \quad C_2 = C_1 = 0$

$$V = \frac{KL}{2\pi}\cos\frac{2\pi x}{L} \qquad M = -\frac{KL^2}{4\pi^2}\sin\frac{2\pi x}{L}$$

at $x=0 \quad V = \frac{KL}{2\pi}$ but $V_{graph} = -V_{reaction}$ $\therefore$ at $x=0$

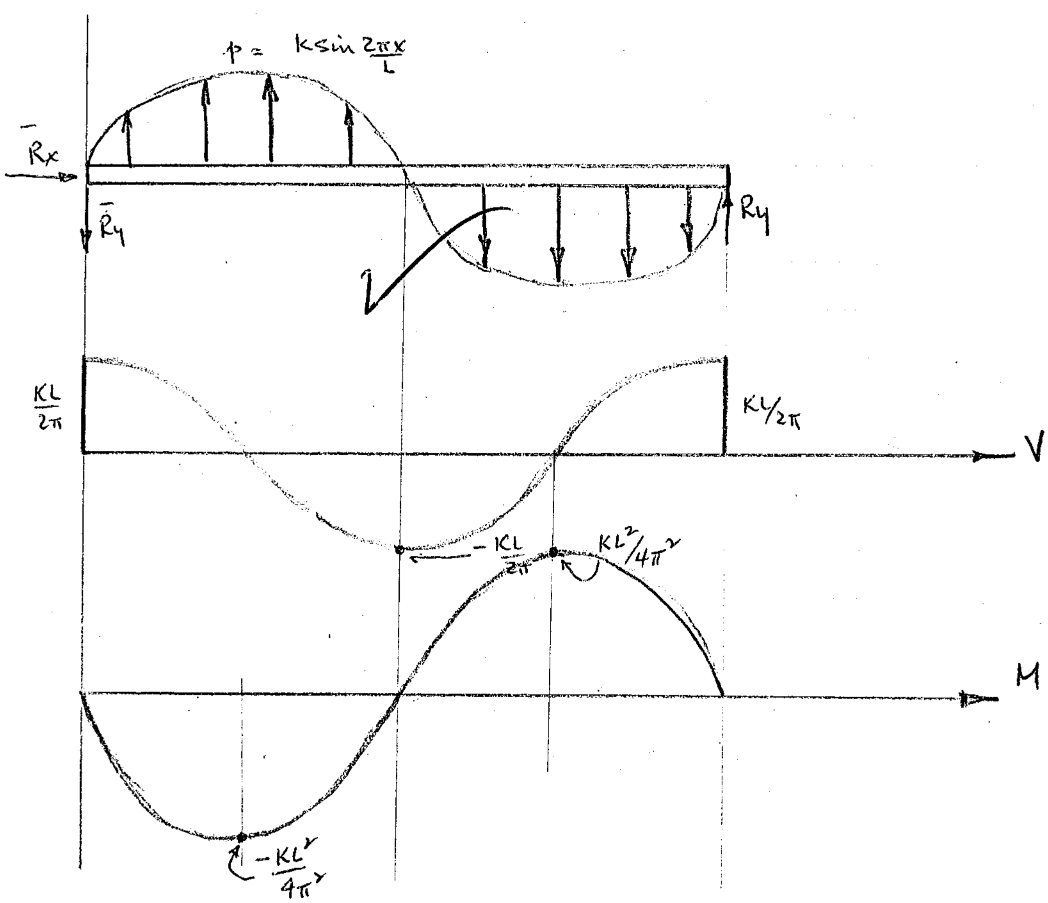
there must be a downward reaction $= \frac{KL}{2\pi}$; at $x = L \quad V$

① $\displaystyle\int_0^{L/2} K\sin\frac{2\pi x}{L}\,dx = \frac{KL}{\pi}$

$\therefore \quad \Sigma F_x = 0 \qquad \bar{R}_x = 0 \qquad \Sigma F_y = 0 \quad -\bar{R}_y + R_y = \frac{KL}{\pi} - \frac{KL}{\pi} = 0$

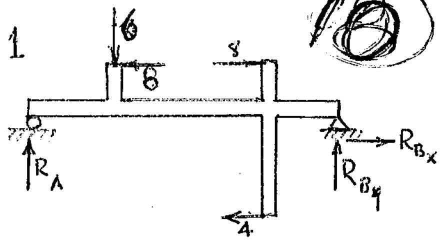
$$\Sigma M_{0,0} = \bar{R}_y\cdot 0 + \frac{KL}{\pi}(L/4) - \frac{KL}{\pi}(3L/4) + R_y L$$

$R_y = \frac{KL}{2\pi} \qquad \bar{R}_y = \frac{KL}{2\pi}$ which checks results.



18

1



$\Sigma F_x = 0 \quad -4+8-8+R_{B_x} = 0 \qquad R_{B_x} = 4 \text{ kips} \rightarrow$

$\Sigma F_y = 0 \quad R_A - 6 + R_{B_y} = 0$

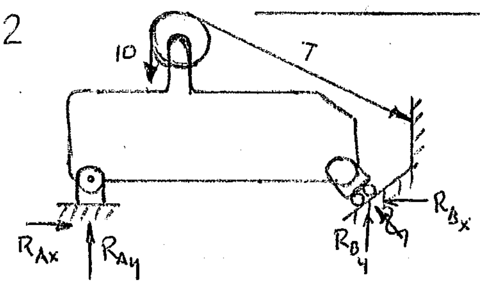
$\Sigma M_B = 0 \circlearrowleft + \quad -8\times3 - 4\times11 - R_A\times20 + 6\times14 + 8\times3 = 0$

$-24 - 44 - 20R_A + 108 = 0$

$R_A = 2 \text{ kip} \uparrow$

$R_{B_y} = 4 \text{ kip} \uparrow$

2



$\Sigma F_x = 0 \quad R_{A_x} - R_{B_x} + T_x = 0 \qquad R_{B_x} = R_B/\sqrt{2}$

$T_x = 2T/\sqrt{5}$

$\Sigma F_y = 0 \quad R_{A_y} + R_{B_y} - 10 - T_y = 0 \qquad R_{B_y} = R_B/\sqrt{2}$

$T_y = T/\sqrt{5}$

$T = 10 \text{ kips}$

$R_{A_x} \cancel{- R_B/\sqrt{2}} + 20/\sqrt{5} = 0$

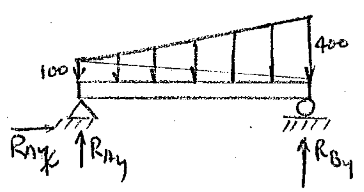
$R_{A_y} + R_B/\sqrt{2} - 10 - 10/\sqrt{5} = 0$

$\Sigma M_A = 0 \circlearrowleft + \quad -10(3) - 20/\sqrt{5}(5) - \frac{20}{\sqrt{5}}(4) + \frac{R_B}{\sqrt{2}}(10) = 0 \qquad R_B = 13.1 \text{ kips}$

$R_{Ax} = 35.5 \text{ kips} \leftarrow$

$R_{A_y} = 23.125 \text{ kips} \rightarrow$

3

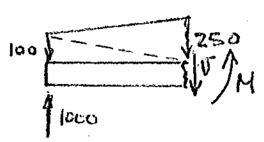


$\Sigma F_x = 0 \quad R_{Ax} = 0$

$\Sigma F_y = 0 \quad R_{A_y} + R_{B_y} - \frac{1}{2}(100)(10) - \frac{1}{2}(400)(10) = 0$

$\Sigma M_A = 0 \circlearrowleft \quad R_{B_y}(10) - 500\left(\frac{10}{3}\right) - 2000\left(\frac{20}{3}\right) = 0$

$R_{B_y} = 4500(1/3) = 1500^{\#} \uparrow \qquad R_{A_y} = 1000^{\#} \uparrow$

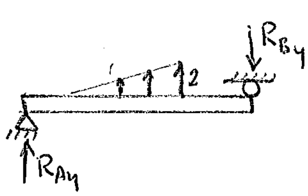


Axial Force = 0

$\Sigma F_y = 1000 - V - \frac{1}{2}(100)(5) - \frac{1}{2}(250)(5) = 0 \qquad V = +125^{\#}\downarrow$

$\Sigma M_A = \frac{5}{3}(-250) - 125(5) - \frac{10}{3}(625) + M = 0 \qquad M = 3125 \text{ kp-ft} \circlearrowleft$

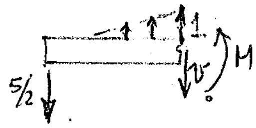
5



$\Sigma F_y = 0 \quad R_{A_y} + 6 - R_{B_y} = 0$

$\Sigma M_A = 0 \quad +6(7) - R_{B_y}(12) = 0 \qquad R_{B_y} = 7/2 \text{ kip} \downarrow$

$R_{Ay} = 5/2 \text{ kip} \downarrow$



$\Sigma F_y = 0 \quad -5/2 - V + 3/2 = 0 \quad V = +1 \text{ kip} \uparrow$

$\Sigma M_A = 0 \quad 5(3/2) + 6 + M = 0 \qquad M = 13.5 \text{ kp ft}$

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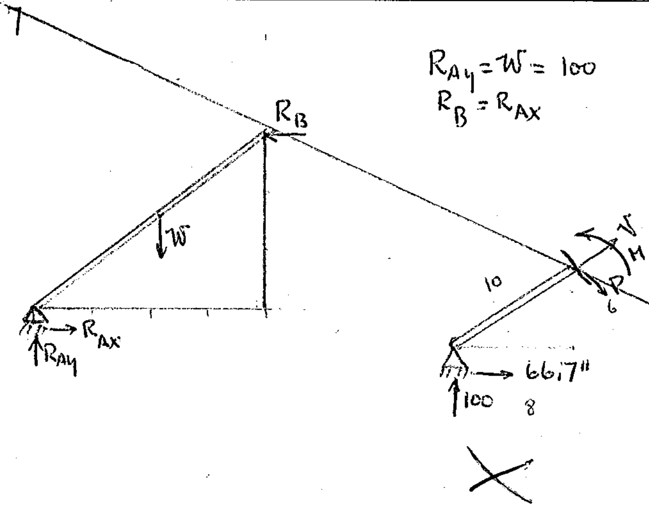


$R_{Ay} = W = 100$
$R_B = R_{Ax}$

$\Sigma M_A = 0 \quad -100(8) + R_B(12) = 0$

$R_B = \frac{100(2)}{3} = 66.7^{\#} = R_{Ax}$

$100 + V\frac{3}{5} - \frac{4P}{5} = 0 \quad ; \quad 3V - 4P = -500$

$66.7^{\#} + \frac{3}{5}P + \frac{4}{5}V = 0 \quad ; \quad 3P + 4V = -\frac{1000}{3}$

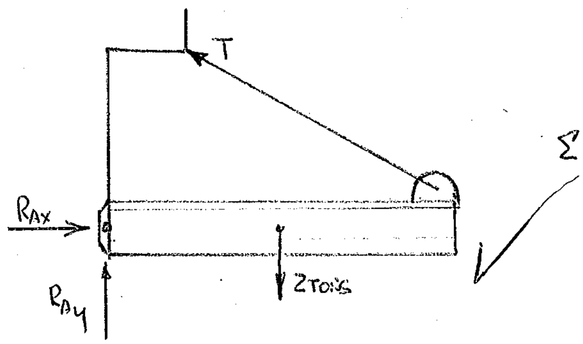
$\Sigma M_A = 0 ; \quad -P(10) + M = 0$

$V = +113\frac{1}{3}$ (3, 4) $\quad P = 210^{\#}$ (4, 3)

$M = 2100^{\#}\text{-ft}$

Problem 7 is found as last problem done

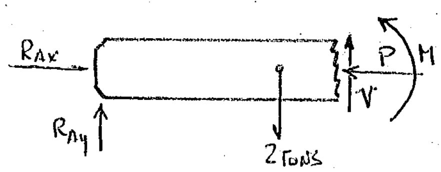
9



$R_{Ax} = \frac{2}{\sqrt{5}}T \quad ; \quad R_{Ay} + \frac{T}{\sqrt{5}} = 2\left(\frac{2000}{1000}\right) = 4k$

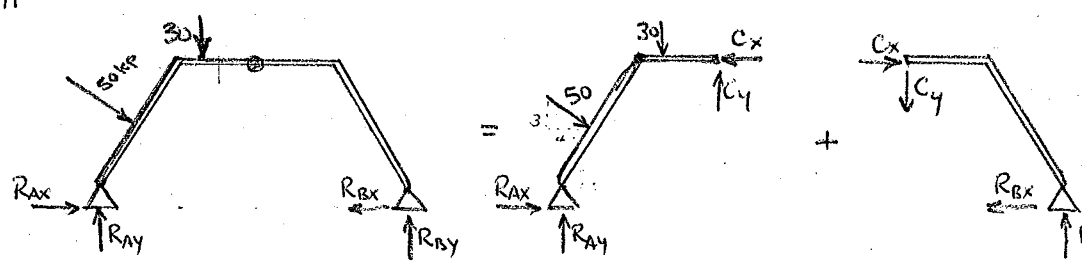
$\Sigma M_A = 0 \quad -4000(3) + \frac{T}{\sqrt{5}}\left(\frac{35}{6}\right) + \frac{2}{\sqrt{5}}T\left(\frac{1}{2}\right) = 0$

$T = 3.93$ kips
$R_{Ax} = 3.5$ kips
$R_{Ay} = 2.25$ kips



$R_{Ax} = P \quad ; \quad P = 3.5$ kips $\leftarrow$
$R_{Ay} - 2000(2) + V = 0 \quad ; \quad V = 1.75$ kips $\uparrow$
$\Sigma M_A = -4000(3) + V(4) + M = 0 \quad ; \quad M = 5$ kft $= 60$ kip-in.

11



$+R_{Ax} - C_x + 40 = 0$
$C_y + R_{Ay} - 30 - 30 = 0$
$\Sigma M_A = -30(15/2) - 40(10) - 30(20) + Cx(20) + Cy(30) = 0$
$-225 - 400 - 600 + 40Cx = 0$

$Cx = R_{Bx} \quad ; \quad C_y = R_{By}$

$\Sigma M_B = 0; \quad C_y(30) - Cx(20) = 0$
$C_y = \frac{2}{3}Cx$

$Cx = \frac{1225}{40} = 30.63$ kp ; $C_y = 20.42$ kp ;

$R_{Ax} = 9.37$ kp $\leftarrow \quad R_{Ay} = 39.58$ kp $\uparrow$

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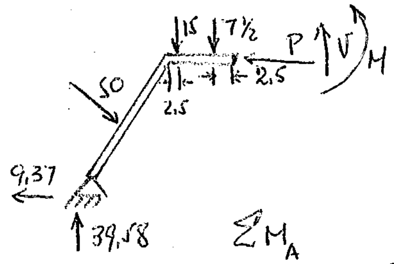
11 CONT.



$-9.37 + 40 - P = 0$

$P = 30.63 \text{ kp} \leftarrow$

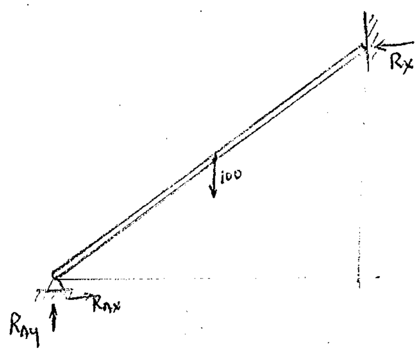
$39.58 - 30 - 22.5 + V = 0$

$V = 12.92 \text{ kp} \uparrow$

$\sum M_A \quad 0 = -30(15/2) - 40(10) - 15(17.5) - 15/2(20) + P(20) + V(22.5) + M$

$-225 - 400 - 262.5 - 150 + 612.6 + 290.7 + M = 0$

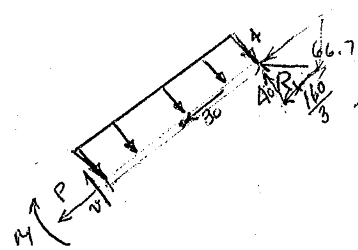
$M = 134.2 \text{ kp-ft}$



$R_{Ay} = 100^{\#}$

$R_{Ax} = R_x$

$\sum M = 0; \quad 100(8) = R_x(12); \quad R_x = R_{Ax} = \frac{100(2)}{3}$



~~$P = 30^{\#}$~~

~~$V = 40^{\#}$~~

$M = 200^{\#}\text{-inches}$

$P + 30 + 53\tfrac{1}{3} = 0 \qquad P = 83\tfrac{1}{3}^{\#}$

$V - 40 + 40 = 0 \qquad V = 0$

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Exam

(100)

1. $\tau_{max} = \frac{T}{\alpha b c^2}$ $\quad \varphi = \frac{TL}{\beta c^3 b G} = \frac{L}{\beta c G}\left(T/bc^2\right)$

$\tau_{max}\,\alpha = T/bc^2$ $\qquad \varphi = \frac{L}{\beta c G}\,\tau_{max}\,\alpha$

a) $\tau_{max} = \frac{\varphi \beta c G}{\alpha L}$

$\tau_{max} = 8000\,psi \quad \varphi = .1 \quad b/c = 2 \Rightarrow \alpha = .246,\ \beta = .229$

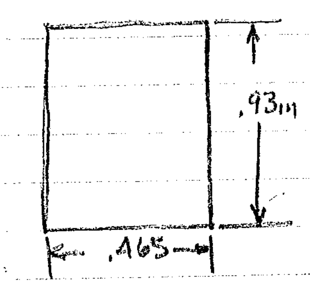
if $\tau_{max} = 8000 \quad \varphi = .1 \quad \alpha = .246,\ \beta = .229$ then $\frac{\tau_{max}\,\alpha}{\beta \varphi} = \frac{cG}{L} = \frac{8000(.246)}{.229(.1)} =$

$85900 = cG/L_{min}$

$\varphi = \frac{TL}{\beta c^3 b G} = \frac{400}{(.229)(c^2 b)} \frac{L}{cG} = \frac{400}{.229(2c^3)} \frac{1}{85900} = .1$

$\frac{400}{2(2.29)(859)} = c^3 = .1018 \qquad c = .465 \qquad b = .93$

$T/\alpha b c^2 = \tau_{max} \qquad \frac{400}{2(.246)c^3} = \tau_{max} \qquad \tau_{max} = \frac{400}{2(.246)(.1018)} = 8000$ ✓

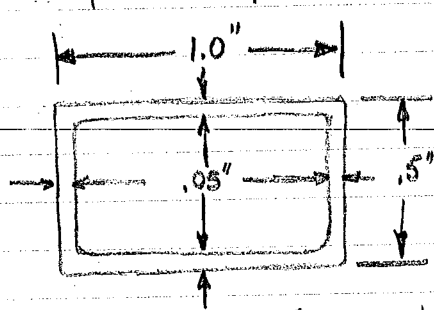


b) $b = .93 in \quad c = .465 in$

$L \le \frac{\varphi \beta c G}{T \alpha} \le .1(.229)(.465)(G)/(8000)(.246)$

$L \le 5.45 \times 10^{-6} G$

Part 2. c)



$\tau_{max} = \frac{T}{2 A_{enc} t} \qquad A_{encl} = \frac{T}{2\tau_{max} t}$

$A_{encl} = \frac{400}{800} = .5 = bc = 2c^2 \qquad c^2 = .5/2 = .25$

$c = .5'' \quad b = 1''$

Ans. $b = 1.0'' \quad c = .5'' \quad t = .05'' \qquad L \le 4.16 \times 10^{-6} G$ from eq in Part I

Part 1 $\quad \tau_{max} = \frac{T}{2 A_m t} \qquad \theta = \frac{d\varphi}{dx} = \frac{T}{4A^2 G} \oint \frac{ds}{t} = \frac{T}{4A^2 G t} \oint ds$ t is uniform

$\oint ds = 2(b+c)$

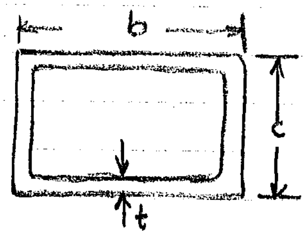
$$\frac{\tau_{max}^2 t^2}{T^2} = \frac{1}{4A_m^2} \qquad \theta = \frac{T}{4A^2 G t}(2b+2c) = \frac{d\varphi}{dx}$$

$$\theta = \frac{\tau_{max}^2 t^2}{T^2} \cdot \frac{T}{Gt}(2b+2c) = \frac{(\tau_{max})^2 t}{TG}(2b+2c)$$

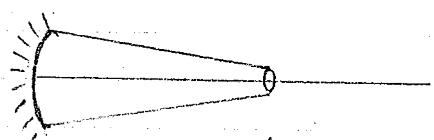
$$\varphi = \int_0^L \frac{\tau_{max}^2 t}{TG}\left[2b+2c\right] dx = \frac{\tau_{max}^2 t}{TG}\left[2b+2c\right]\int_0^L dx$$

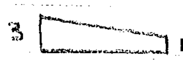
if t is uniform and material is homogeneous

$$\boxed{\varphi = \frac{\tau_{max}^2 t}{TG}\left[2b+2c\right]L \quad \text{or} \quad \varphi = \frac{\tau_{max} L\left[b+c\right]}{AG} \quad A = bc}$$



2) Example 5-12





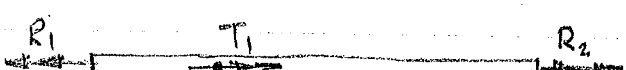
at $x=0$ $y=3$
$x=20$ $y=1$

if $y = ax+b$ $\quad y = 3 - x/10 = c_o$

$$\varphi = \int_0^{20} \frac{T\,dx}{\frac{\pi}{2} G (3 - x/10)^4} = \frac{2T}{\pi G}\int_0^{20} \frac{dx}{(3-x/10)^4} = \frac{2T(10)}{3\pi G}\left[3 - x/10\right]^{-3}\Big|_0^{20}$$

$$\varphi_{\text{in degrees}} = \frac{2T(10)(26)(180)}{27(3\pi G)} = \frac{2(27000)(10)(26)(180)}{27(3\pi^2)(12\times10^6)} = \frac{260}{987} = .263^\circ$$

3)

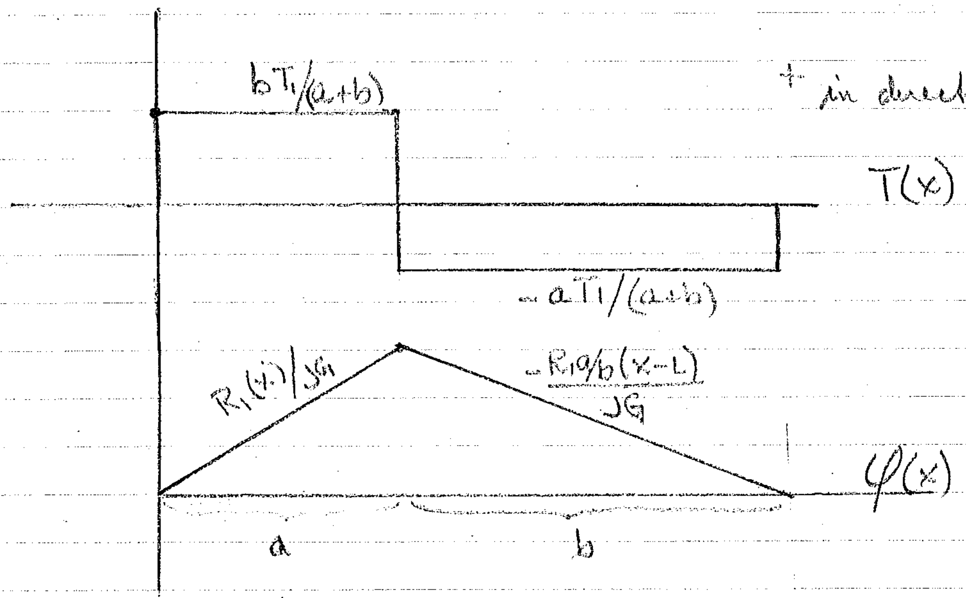


$T_1 + R_2 = R_1$

$$\varphi = 0 = \int_0^L \frac{T\,dx}{JG} = \int_0^a \frac{R_1\,dx}{JG} + \int_a^L \frac{R_2\,dx}{JG}$$

$$\frac{R_1 a}{JG} + \frac{R_2 b}{JG} = 0 \qquad R_2 = -\frac{R_1 a}{b}$$

$$T_1 - \frac{R_1 a}{b} - R_1 = 0 \quad \boxed{\frac{bT_1}{a+b} = R_1} \quad \text{but} \quad \boxed{R_2 = -\frac{R_1 a}{b} = \frac{-aT_1}{a+b}}$$



+ in direction as shown in diagram.
+ Torque

i) $\tau_{max} = Tc/J \qquad J = \frac{\pi}{2}(2)^4 \quad c = 2 \quad T = \frac{bT_1}{a+b} = \frac{2}{3}(10000)$

$$\tau_{max} = \frac{\frac{2}{3}(10000)\,2}{\frac{\pi}{2}\,2^4} = \frac{10000}{6\pi} = 531 \text{ lb/in}^2$$

ii) $$\varphi_{max} = \int_0^a \frac{R_1\,dx}{JG} = \frac{2/3\,(10000)\,(5(12))}{\pi/2\,(2^4)\,(10\times10^6)} = \frac{5}{\pi}\,10^{-3} = 1.593\times10^{-3}\text{ rad.}$$

iii) at larger torque material is fully plastic – $\tau_{max} = \tau_{yp}$

25

$$\cancel{T =} \int_A \tau_{max}\, dA\rho = \tau_{max}\int \rho\, dA = \tau_{max}\int_0^c 2\pi\rho\cdot\rho\, d\rho = \tau_{max}(2\pi)\left.\frac{\rho^3}{3}\right|_0^c$$

$$T = \frac{2}{3}\pi c^3\,\tau_{max}\ ; \quad J = \frac{\pi}{2}c^4 \quad \Rightarrow \quad \frac{2J}{c} = \pi c^3$$

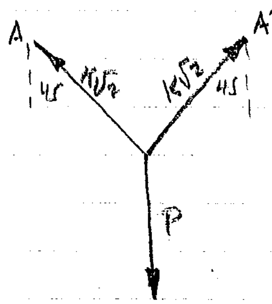
$\therefore\ T_{plastic} = \frac{4}{3}\tau_{max} J/c \qquad$ but $\quad T_{plastic} = R_1 = \frac{2}{3}(10000)$

$$\tau_{max} = \frac{3T_p c}{4J} = \frac{3}{4}\cdot\frac{2}{3}(10000)(2)\cdot\frac{2}{\pi\, 2^4} = \frac{10000}{8\pi} = 398 \text{ lb/in}^2$$

iff $T_{plastic} = R_1$ otherwise not solvable since τ_{yp} wasn't given.

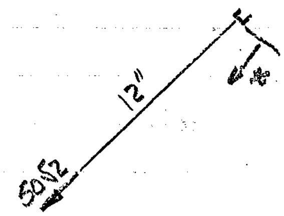
iv) Part IV is at the end of the exam

4.



$\Sigma F_y\,(+\uparrow) = 0 \quad 2K - P = 0 \quad K = P/2 = 50$

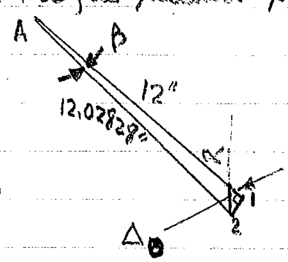
$\Sigma F_x\,(\xrightarrow{+}) = 0 \quad K - K = 0 \checkmark$



$$u = \int_0^L \frac{P(x)\,dx}{E\,A(x)} = \frac{K\sqrt{2}}{EA}L + C_1 \qquad \text{at } x=0 \quad u=0 \quad C_1 = 0$$

$$\therefore \underline{u} = \frac{50\sqrt{2}\,(12)}{(30\times10^6)(1\times10^{-3})} = \frac{600\sqrt{2}}{30\times10^3} = 20\sqrt{2}\times10^{-3} = 2\sqrt{2}\times10^{-2} = 2.828\times10^{-2}\ \text{in.}$$

using $(l+u)$ as radius strike an arc from A' such that it intersects perpendicular

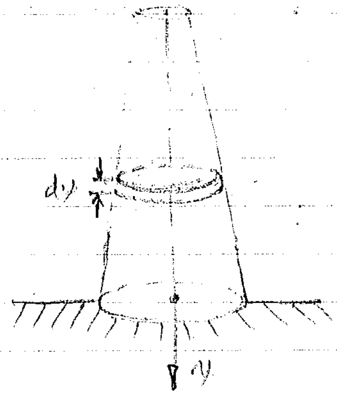


Since deflection is small β will be small therefore arc 1-2 will be small and can be approximated as a line. Therefore from trigonometry $\quad \cos\alpha = \dfrac{u}{\Delta_0} \qquad \Delta_0 = \dfrac{u}{\cos\alpha}$; $\alpha = 45^\circ$

$$\Delta_0 = \frac{2.828\times10^{-2}\,\text{in}}{.707} = 4\times10^{-2}\,\text{in}$$

pt$_0$ deflection $\approx 4\times10^{-2}$ in.

5.



@ $x=0$ $r=1$

@ $x=24$ $r=3$ $\qquad r = x/12 + 1$

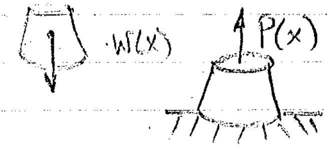
$$dV = A\,dx$$

$$dW = \gamma\,dV = \gamma A\,dx = \gamma \pi r^2 dx$$

$$W = \int \gamma \pi r^2 dx = \gamma\pi \int \left(1 + x/12\right)^2 dx$$

$$W(x) = 4\gamma\pi \left(1 + x/12\right)^3 + C \qquad @\ x=0 \quad W(0)=0$$

$$\therefore \quad C = -4\gamma\pi$$



$$W(x) = 4\gamma\pi\left[\left(1 + x/12\right)^3 - 1\right]$$

$$W(x) = P(x)$$

$$A(x) = \pi r^2 = \pi\left[1 + x/12\right]^2$$

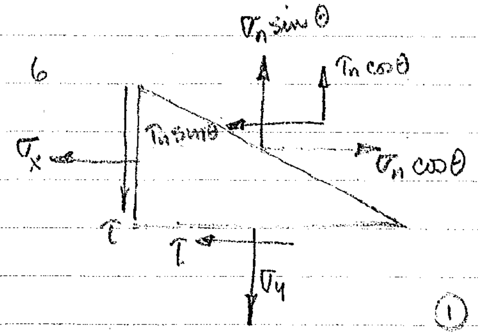
$$\therefore \quad du/dx = \frac{P}{AE} = \frac{4\gamma\pi\left[\left(1+x/12\right)^3 - 1\right]}{\pi\left[1 + x/12\right]^2 E}$$

$$u = \int_0^{24} \frac{4\gamma\left[\left(1+x/12\right)^3 - 1\right]dx}{\left[1+x/12\right]^2 E} = \frac{4\gamma}{E}\int_0^{24}\left[\left(1+x/12\right) - \frac{1}{\left(1+x/12\right)^2}\right]dx$$

$$= \frac{4\gamma}{E}\left[6\left(1+x/12\right)^2 + 12\left(1+x/12\right)^{-1}\right]_0^{24}$$

$$= \frac{4\gamma}{E}\left(54 + 4 - 6 - 12\right) = \frac{160\gamma}{E}$$

6



$\sum F_y (\uparrow +) = 0$

(1) $-\tau\cos\theta\, ds + \sigma_n \sin\theta\, ds + \tau_n \cos\theta\, ds - \sigma_y \sin\theta\, ds = 0$

$\sum F_x (\rightarrow +) = 0$

(2) $-\sigma_x \cos\theta\, ds - \tau_n \sin\theta\, ds + \sigma_n \cos\theta\, ds - \tau \sin\theta\, ds = 0$

(1) $-\tau\cos\theta + \sigma_n \sin\theta + \tau_n \cos\theta - \sigma_y \sin\theta = 0$

(2) $-\sigma_x\cos\theta - \tau_n \sin\theta + \sigma_n\cos\theta - \tau\sin\theta = 0$

$-\tau\cos\theta\sin\theta + \sigma_n \sin^2\theta + \tau_n\cos\theta\sin\theta - \sigma_y \sin^2\theta = 0$

$-\sigma_x\cos^2\theta - \tau_n \sin\theta\cos\theta + \sigma_n\cos^2\theta - \tau\sin\theta\cos\theta = 0$

(i) $-2\tau\cos\theta\sin\theta + \sigma_n - \sigma_y \sin^2\theta - \sigma_x\cos^2\theta = 0$

$\longrightarrow \sigma_n = \sigma_y \sin^2\theta + \sigma_x \cos^2\theta + \tau \sin 2\theta$

(1) $-\tau\cos\theta + \sigma_n\sin\theta + \tau_n\cos\theta - \sigma_y\sin\theta = 0$

(2) $-\sigma_x\cos\theta - \tau_n\sin\theta + \sigma_n\cos\theta - \tau\sin\theta = 0$

$-\tau\cos^2\theta + \sigma_n\sin\theta\cos\theta + \tau_n\cos^2\theta - \sigma_y\sin\theta\cos\theta = 0$

$+\sigma_x\cos\theta\sin\theta + \tau_n\sin^2\theta - \sigma_n\cos\theta\sin\theta + \tau\sin^2\theta = 0$

(ii) $\tau_n + (\sigma_x - \sigma_y)\dfrac{\sin 2\theta}{2} + \tau(\sin^2\theta - \cos^2\theta) = 0$

$\longrightarrow \tau_n = (\sigma_y - \sigma_x)\dfrac{\sin 2\theta}{2} + \tau\cos 2\theta$

$\dfrac{d\sigma_n}{d\theta} = 0$

$0 = -(\sigma_x - \sigma_y)\sin 2\theta + 2\tau\cos 2\theta \qquad \tan 2\theta_p = \dfrac{2\tau}{(\sigma_x - \sigma_y)}$ (iii)

Substituting this equation (iii) into (i) we get

$\sigma_{n_{max}} = \dfrac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$ (work is done on last sheet)

$\dfrac{d\tau_n}{d\theta} = 0$

$0 = (\sigma_y - \sigma_x)\cos 2\theta - 2\tau\sin 2\theta \qquad \tan 2\theta_s = \dfrac{(\sigma_y - \sigma_x)}{2\tau}$ (iv)

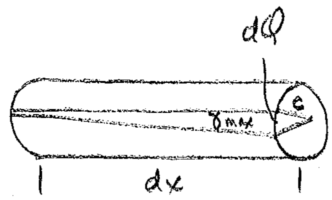
Substituting (iv) into (ii) yields

(work is done on last sheet)

$\tau_{n_{max}} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$

3b.

iv)



$\gamma_{max}\, dx = d\phi\, c$

also $\gamma\, dx = d\phi\, \rho$

since $\tau^n = K\gamma$

since distribution of γ_{max} is still linear

$$\frac{\gamma}{\gamma_{max}} = \rho/c = \frac{\tau^n}{\tau_{max}^n}$$

$$\therefore \quad \tau = (\rho/c)^{1/n}\, \tau_{max}$$

$$T = \int_A \tau\, dA\, \rho = \tau_{max} \int_A (\rho/c)^{1/n}\, dA(\rho) \quad ; \quad dA = 2\pi\rho\, d\rho$$

$$\therefore \quad 2\pi\tau_{max}\left(\frac{1}{c}\right)^{1/n} \int_A \rho^{\frac{2n+1}{n}}\, d\rho = T$$

$$\therefore \quad T = 2\pi\rho^3\tau_{max}\, (\rho/c)^{1/n}\, \frac{n}{3n+1}$$

and $$\tau_{max} = \frac{3n+1}{n}\left(\rho/c\right)^n \frac{T}{2\pi\rho^3}$$ $$T = \frac{10000}{2.585}$$

$T_a = T_1/2.585$ $\quad T_b = T_1/3.828$ $\quad T_a > T_b$

$$\tau_{max} = \frac{10000 \cdot 3.667}{2.585 \cdot 16\pi} = 282 \text{ lb/in}^2$$

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7-21

$\tau_y = \tau_z$

$I_z = 166 \quad I_y = 36.5$

$\tau_{max_z} = \frac{VQ}{It} = \frac{V_z Q_z}{I_z t_z}$ $\quad t_z = 1 in$; $Q_z = (6.1)3.5 + 3 \cdot 1 \cdot 1.5 = 25.5$

$I_z = 166$

$\tau_{max_z} = V_z(.1536)$

$\tau_{max\,y} = \frac{V_y Q_y}{I_y t_y}$ $\quad t_y = 2 in \quad I_y = 36.5 \quad Q_y = 3 \cdot 1 \cdot 1.5 + 3 \cdot 1 \cdot 1.5 + 6 \cdot .5 \cdot .2$
$= 9.75$

$\tau_{max\,y} \cancel{=} V_y(.1335)$ $\quad V_y \frac{(.1335)}{(.1536)} = V_z$; $V_z = .869 V_y$

why was this marked wrong answer:

7-22 $\quad V = 7000^{\#} \quad I = 35.7 in^4 \quad t = .5$

at A $\quad A = (1 \cdot .5) = .5 \quad \bar{y} = 2.0 \quad Q = A\bar{y} = 1.0 in^3$

$\tau = \frac{q}{t} = \frac{VQ}{It} = \frac{7000(1.0)}{35.7(.5)} = 393 psi$

at B $\quad A_T = (2.5)(.5) + (1.5)(.5) = 2 in^4 \quad \bar{y}_1 = 2.25 \quad \bar{y}_2 = 2.75$

$A_1\bar{y}_1 + A_2\bar{y}_2 = (2.5)(.5)(2.75) + (1.5)(.5)(2.25) = Q$

$\tau = q/t = \frac{VQ}{It} = \frac{7000(5.12)}{35.7 \; .5} = 2005 psi$

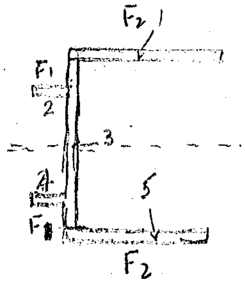
at C $\quad A_1 = (2.5)(.5) = 1.25 \quad A_2 = (3.5)(.5) = 1.75 \quad A_3 = (1.0)(.5) = .5$

$\bar{y}_1 = 1.25 \quad \bar{y}_2 = 2.75 \quad \bar{y}_3 = 2.00$

$A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 = (1.563 + 4.820 + 1.000) = 7.383$

$\tau = \frac{VQ}{It} = \frac{7000(7.383)}{35.7(.5)} = 2900 psi$

7-25

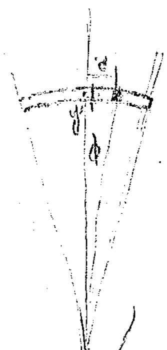


	A	d	Ad^2	I_0
1.	.075	2.0	.3	$\frac{(1.5)(.05)^3}{12} =$
2.	.025	1.5	.056	small $\frac{(.5)(.05)^3}{12}$
3.	.2	0	0	$\frac{.05(4)^3}{12} = .267$
4.	.025	-1.5	.056	$\frac{(.5)(.05)^3}{12}$
5	.075	-2.0	.3	$\frac{(1.5)(.05)^3}{12}$

$Ad^2 + I_0 \approx .712 + .267 \approx .990$

$$e = \frac{b^2h^2t}{4I} = \frac{(2)^2(4)^2(.05)}{4(.99)} = .81 \text{ inches from center}$$

7-28 $dA = at\,d\phi$



$z = a\sin\phi \quad y = a(1-\cos\phi)$

$$I = \int_A z^2\,dA$$

$$I = \int_{-\alpha}^{\alpha} a^2\sin^2\phi\, t\,d\phi = a^3t(\alpha - \sin\alpha\cos\alpha)$$

$$Q = \int_\phi^\alpha z\,dA = \int_\phi^\alpha a^2\sin\phi\, t\,d\phi = -a^2t\cos(\alpha-\phi)$$

$$q = \frac{VQ}{I} = \frac{V\left(-a^2t\cos(\alpha-\phi)\right)}{a^3t(\alpha - \sin\alpha\cos\alpha)}$$

$$T = \int \tau\,dA\,y = \int_{-\alpha}^{\alpha} \frac{q}{t}\,at\,d\phi\;a(1-\cos\phi) = -\int_{-\alpha}^{\alpha}\frac{V\cos(\alpha-\phi)\,a(1-\cos\phi)\,d\phi}{\alpha-\sin\alpha\cos\alpha}$$

$$\frac{-Va}{\alpha - \sin\alpha\cos\alpha}\int_{-\alpha}^{\alpha}\cos(\alpha-\phi)(1-\cos\phi)\,d\phi = \frac{-Va}{\alpha-\sin\alpha\cos\alpha}\int_{-\alpha}^{\alpha}\left(\cos\alpha\cos\phi + \sin\alpha\sin\phi - \cos\alpha\cos^2\phi - \frac{\sin\alpha\sin 2\phi}{2}\right)d\phi$$

$$\frac{-Va}{\alpha-\sin\alpha\cos\alpha}\left[\cos\alpha\sin\phi\Big|_{-\alpha}^{\alpha} + \cancelto{0}{\sin\alpha\cos\phi\Big|_{-\alpha}^{\alpha}} - \cos\alpha\left(\frac{\phi}{2}+\frac{\sin 2\phi}{4}\right)\Big|_{-\alpha}^{\alpha} + \cancelto{0}{\frac{\sin\alpha\cos 2\phi}{4}\Big|}\right]$$

$$\frac{-Va}{\alpha-\sin\alpha\cos\alpha}\left(2\cos\alpha\sin\alpha - \cos\alpha\left(\alpha+\frac{\sin 2\alpha}{2}\right)\right) = \frac{-Va\,2(\sin\alpha-\alpha\cos\alpha)}{\alpha-\sin\alpha\cos\alpha} = T$$

$$e = -\frac{T}{V} = \frac{2a(\sin\alpha-\alpha\cos\alpha)}{\alpha-\sin\alpha\cos\alpha} =$$

9-1 -

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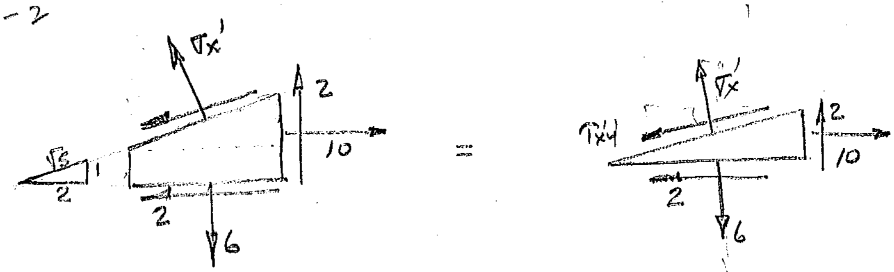
9-2



$$\sigma_{x'}\frac{2}{\sqrt{5}}dA + 2\cdot\frac{1}{\sqrt{5}}dA + 6\cdot\frac{2}{\sqrt{5}}dA - \tau_{x'y'}\,dA\frac{1}{\sqrt{5}} = 0$$

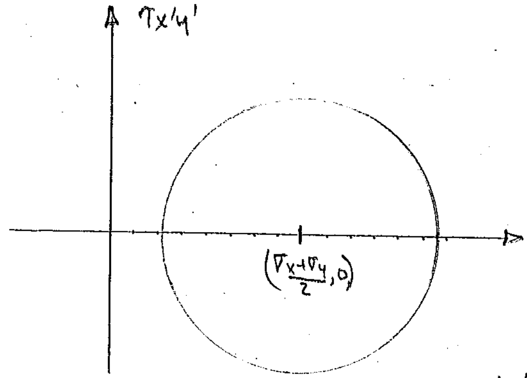
$$10\cdot\frac{1}{\sqrt{5}} - 2\cdot\frac{2}{\sqrt{5}} - \tau_{x'y'}\frac{2}{\sqrt{5}} - \sigma_{x'}\frac{1}{\sqrt{5}} = 0$$

$$\frac{2\sigma_{x'}}{\sqrt{5}} - \frac{\tau_{x'y'}}{\sqrt{5}} = \frac{10}{\sqrt{5}} \longrightarrow 2\sigma_{x'} - \tau_{x'y'} = 10 \qquad 5\sigma_{x'} = 26$$

$$-\frac{\sigma_{x'}}{\sqrt{5}} - \frac{2\tau_{x'y'}}{\sqrt{5}} = -\frac{6}{\sqrt{5}} \longrightarrow +\sigma_{x'} + 2\tau_{x'y'} = 6$$

$$\boxed{\begin{aligned}\sigma_{x'} &= 5.2\ \text{KSI}\\ \tau_{x'y'} &= .4\ \text{KSI}\end{aligned}}$$

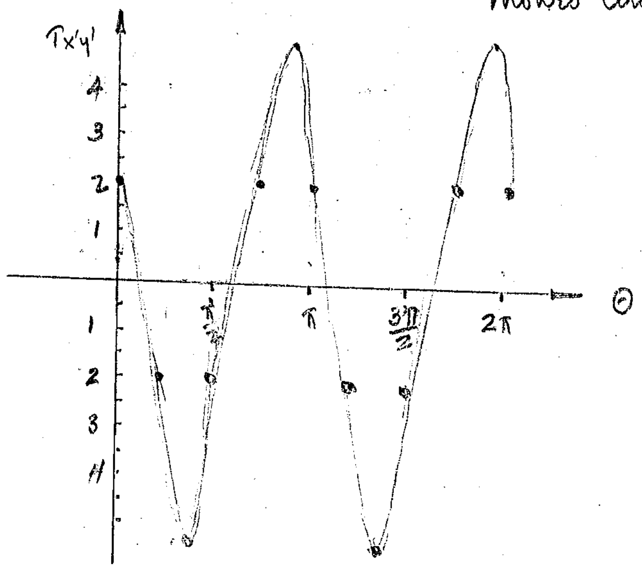
9-6



Mohr's Circle

$$\sigma_{1\text{ or }2} = \left(\frac{\sigma_x+\sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$8 \pm \sqrt{4+4} = 8 \pm 4\sqrt{2}$$



$$\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$\theta = 0 \quad \tau_{x'y'} = 2$

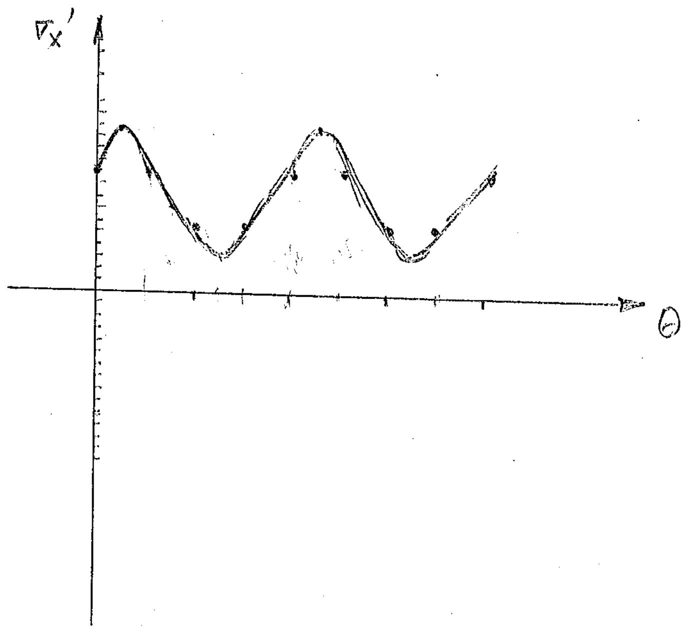
$\theta = \frac{\pi}{4} \quad = -2$

$\theta = \frac{\pi}{2} \quad = -2$

$\theta = \pi \quad = 2$

$\theta = \frac{3\pi}{2} \quad = -2$

$\theta = 2\pi \quad = 2$



$$\sigma_x' = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$\theta = 0 \quad \sigma_x' = 8+2 = 10$

$\theta = \frac{\pi}{2} \quad \sigma_x' = 8-2 = 6$

$\theta = \pi \quad \sigma_x' = 8+2 = 10$

$\theta = \frac{3\pi}{2} \quad \sigma_x' = 8-2 = 6$

$\theta = 2\pi \quad \sigma_x' = 10$

$\theta = \frac{\pi}{4} \quad \sigma_x' = 8+2 = 10$

$\theta = \frac{3\pi}{4} \quad \sigma_x' = 8-2 = 6$

$\theta = \frac{\pi}{8} \quad \sigma_x' = 8+4\sqrt{2}$

$\theta = \frac{5\pi}{8} \quad \sigma_x' = 8+2\sqrt{2}-2\sqrt{2}$

9-7

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$\theta = \arctan .5$

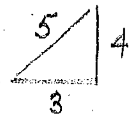
$2\theta = (\arctan .5)2$

$$\tau_{x'y'} = +\frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$\theta = 26°34'$

$2\theta = 53°8'$

$$\sigma_x' = 8 - 2\left(\frac{3}{5}\right) - 2\left(\frac{4}{5}\right) = 8 - 2.8 = 5.2 \text{ KSI} \checkmark$$



$$\tau_{x'y'} = \frac{6-10}{2}\left(-\frac{4}{5}\right) + 2\left(\frac{3}{5}\right) = \frac{2}{5} \text{ KSI} = .4 \text{ KSI} \checkmark$$

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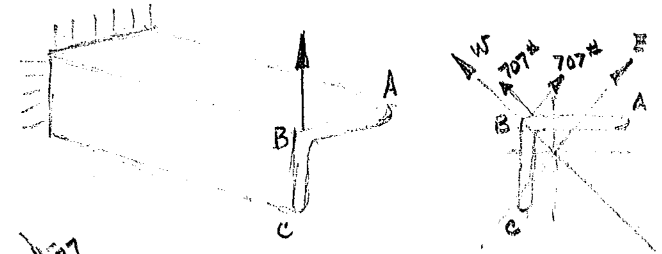
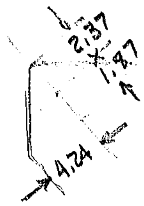
8-21





$I_x + I_y = 39.8$

$I_{min} = A r_{min}^2 = 8.0 = I_z$

$I_{max} = A r_{max}^2 = 31.8 = I_w$

$M_z = M_w = .707^k \times 70.7 = 50^{k\text{-}in}$

$$\sigma = \frac{M_z w}{I_z} + \frac{M_w z}{I_w}$$

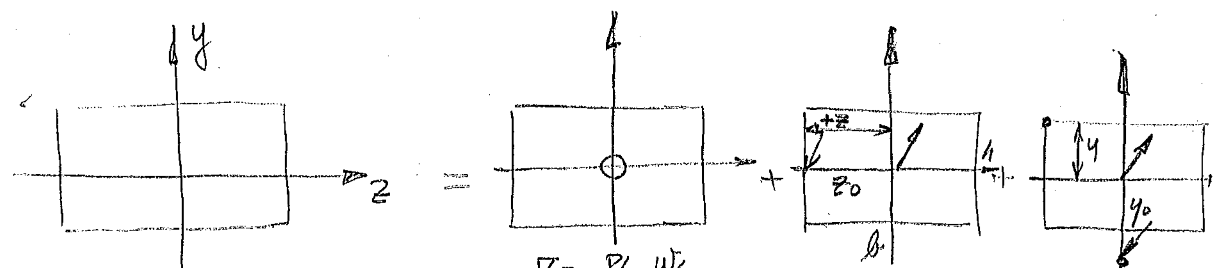
$$\sigma_A = -\frac{50(-1.87)}{8.0} - \frac{50(4.24)}{31.8} = 11.67 - 6.67 = 5.0 \text{ KSI}$$

$$\sigma_B = -\frac{50(2.37)}{8.0} - \frac{50(0)}{31.8} = -14.84 \text{ KSI}$$

$$\sigma_C = -\frac{50(-1.87)}{8.0} - \frac{50(-4.24)}{31.8} = 18.34 \text{ KSI}$$

σ_A at wall $= +5.0$ KSI σ_B at wall $= -14.84$ KSI σ_C at wall $= +18.34$ KSI

8-22



$\sigma = -P/A - W/A$

$M_{yy} = -P z_0$ $I_{yy} = \frac{hb^3}{12}$ $z = -z_0$ when $z > 0, \sigma \leq 0$

$M_{zz} = P y_0$ $I_{zz} = \frac{bh^3}{12}$ $y = y_0/2$ when $y > 0, \sigma > 0$

$W = .283 \times V = 6 \times 12 \times 20 \times .283 = 407.5$

$$\sigma = -\frac{(P+W)}{A} - \frac{M_{yy} z}{I_{yy}} + \frac{M_{zz} y}{I_{zz}}$$

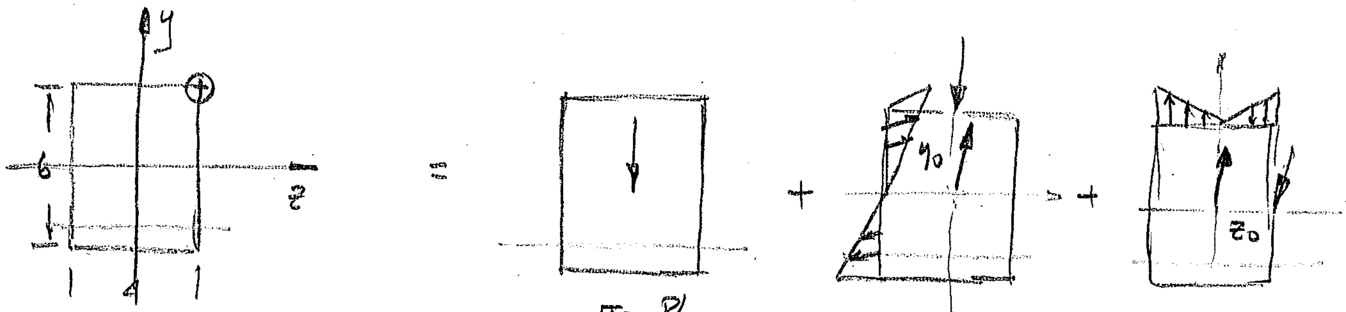
$$0 = -\frac{(P+407.5)}{72} - \frac{P \cdot 6 \cdot 6}{\frac{6 \cdot 12 \cdot 12 \cdot 12}{12}} + \frac{P \cdot 6 \cdot 3}{\frac{12 \cdot 6 \cdot 6 \cdot 6}{12}}$$

$$0 = -\frac{(P+407.5)}{72} + \frac{P}{24} = \frac{2P - 407.5}{72}$$

$P = 203.75^{\#}$

8-24

$$\sigma = E\epsilon = (500\times10^{-6})(10\times10^{6}) = 5000 \text{ psi}$$



$\sigma = -P/A$

$M_{zz} = -P y_0$

$I_{zz} = \dfrac{4\cdot6^3}{12}$

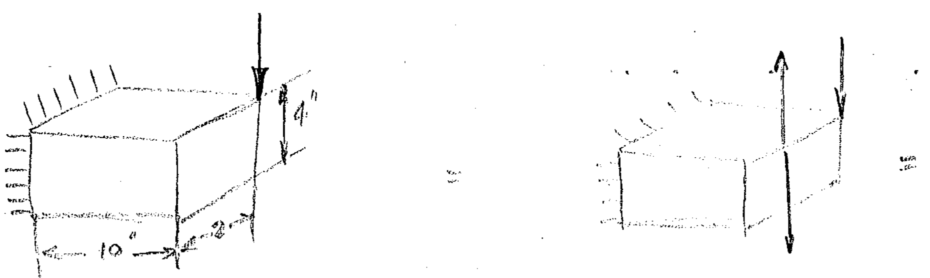
$M_{yy} = P z_0$

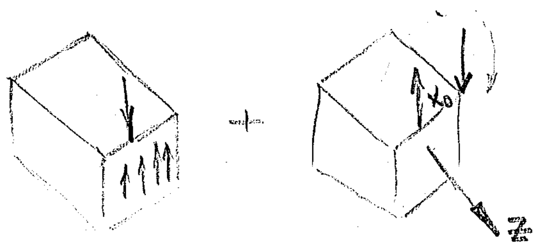
$I_{yy} = \dfrac{6\cdot4^3}{12}$

$$\sigma = -\frac{P}{A} - \frac{M_{yy}z}{I_{yy}} + \frac{M_{zz}y}{I_{zz}}$$

$$= \frac{-P}{24} + \frac{3P}{24} + \frac{2P}{24} \implies 6\sigma = P \qquad P = 30{,}000\,\#$$

8-29

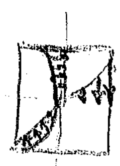




$M_z = -P x_0$

at $x \geq 0$ $\sigma \geq 0$

$\tau_{max} = \frac{3}{2}\frac{V}{A}$ +



$\tau_{max} = \dfrac{T}{\alpha b c^2}$

$\cancel{T + M_z = 0}$ $\quad -T = M_z$

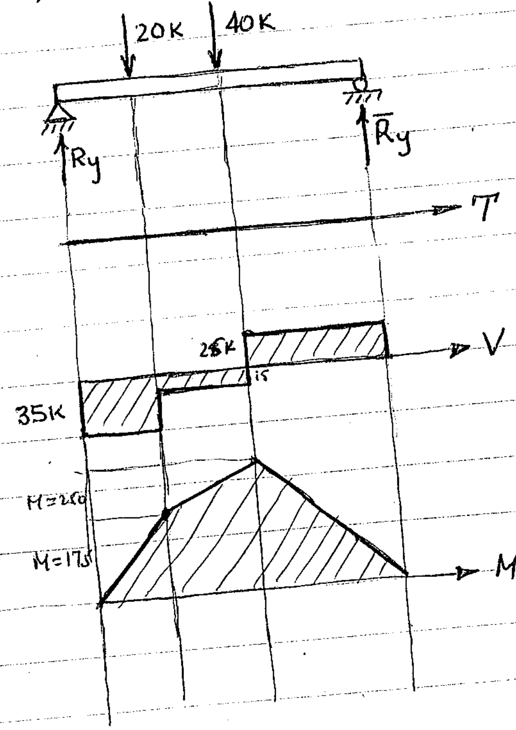
$$\tau_{max} = \frac{1.5V}{A} + \frac{P x_0}{\alpha b c^2} = \frac{1.5(10)}{4.2} + \frac{10(1)}{(.246)4.4} = \frac{15}{8} + 2.54 = 4.415 \text{ KSI}$$

CESAR
LEVY

TO

CESAR LEVY

2-15, 17, 19, 21, 29

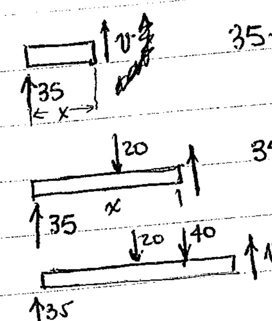


$R_y + \bar{R}_y - 60 = 0$

$R_y \cdot 0 - 100 - 400 + \bar{R}_y(20) = 0$

$\bar{R}_y = 25K$

$R_y = 35K$



$35 + V = 0$

$V = -35K$

$35 + V - 20 = 0$

$V = -15K$

$V = 25K$

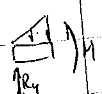
$\Sigma M_c = 0 \quad M = 35x \quad x \geq 0, \; x \leq 5$

$35x - 20(x-5) = M$

$15x + 100 = M$

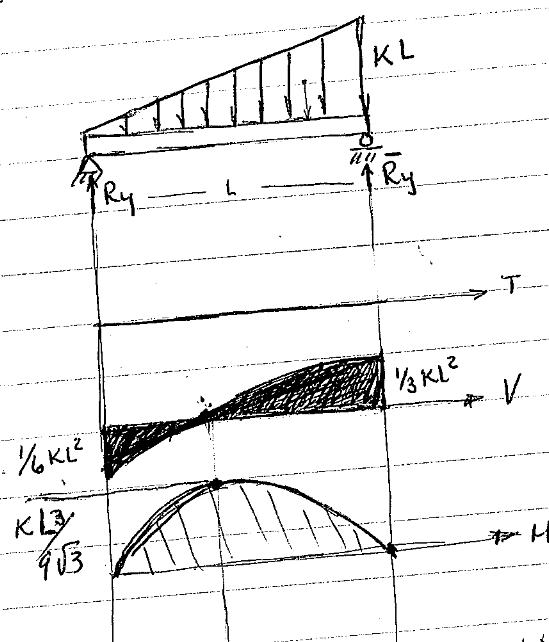
$35x - 20(x-5) - 40(x-10) = M$

$M = -25x + 500$



$-R_y(x) + \frac{1}{2}Kx^2 \cdot \frac{x}{3} + M = 0$

$M = R_y x - \frac{Kx^3}{6}$



$R_y + \bar{R}_y - \frac{1}{2}KL^2 = 0$

$-\frac{1}{2}KL^2 \cdot \frac{2}{3}L + \bar{R}_y L = 0$

$\bar{R}_y = \frac{1}{3}KL^2$

$R_y = \frac{1}{6}KL^2$

$R_y - \frac{1}{2}K(x)^2 = V$

$\frac{1}{6}KL^2 - \frac{1}{2}K(x)^2 = V$

$-\frac{K}{6}\left[L^2 - 3x^2\right] = V$

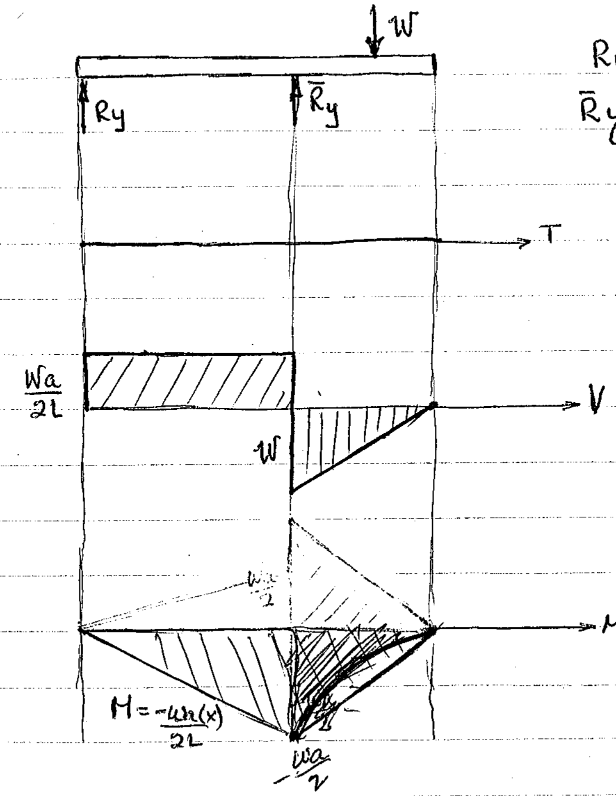
$\frac{K}{6}\left[3x^2 - L^2\right] = V$

$M = \frac{1}{6}K\left[L^2 x - x^3\right]$

$\frac{1}{6}KL^2(x) - \frac{1}{6}K(x)^3 = M$

$R_y x - \frac{1}{6}K(x)^3 = M$

$\frac{dM}{dx} = 0 \; ; \; x = \frac{L\sqrt{3}}{3}$

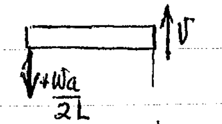


$R_y + \bar{R}_y = W$

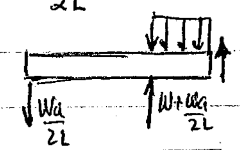
$\bar{R}_y L - W(L + a/2) = 0$

$\bar{R}_y = \frac{W}{L}(L + a/2) = W + \frac{Wa}{2L}$

$R_y = W\left(1 - \frac{L - a/2}{L}\right) = -\frac{Wa}{2L}$



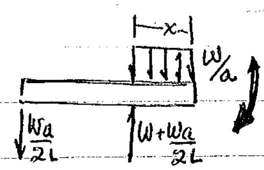
$V = +\frac{Wa}{2L}$



$-\cancel{\frac{Wa}{2L}} + W + \cancel{\frac{Wa}{2L}} - \frac{Wx}{a} + V = 0$

$V = \frac{Wx}{a} - W$

$M = -\frac{Wa}{2L}(x)$



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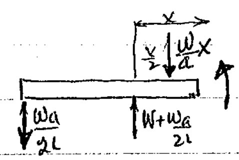


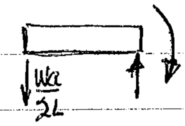
$-\frac{Wa}{2L}(L+x) + \left(\frac{W + Wa}{2L}\right)x - \frac{Wx^2}{a} + M = 0$

$M = \frac{Wa}{2} + \frac{Wax}{2L} - Wx + \frac{Wax}{2L} + \frac{Wx^2}{a} =$

$\frac{W}{2}(a - 2x) + \frac{W}{a}x^2 = M$

$-\frac{Wa}{2L}(L+x) + \left(W + \frac{Wa}{2L}\right)x - \frac{W}{2a}x^2 + M = 0$

$M = \frac{Wa}{2} + \cancel{\frac{Wax}{2L}} - Wx - \cancel{\frac{Wax}{2L}} + \frac{Wx^2}{2a}$



$\frac{Wa}{2} = M$

$M = \frac{W}{2}(a - 2x) + \frac{Wx^2}{2a}$ $-\frac{Wa}{2} + \frac{Wa}{2}$

$+\frac{Wa}{2} - \cancel{\frac{Wax}{2L}} + Wx + \cancel{\frac{Wax}{2L}} + \frac{Wx^2}{2a} = +M$

$+\frac{Wa}{2L}(L+x) - \left(W + \frac{Wa}{2L}\right)x + \frac{Wx^2}{2a} + M = 0$ $M = -\frac{Wa}{2L}(x)$

$-M = \frac{Wa}{2} + \cancel{\frac{Wax}{2L}} - Wx - \cancel{\frac{Wax}{2L}} + \frac{Wx^2}{2a}$

$M = \frac{W}{2}(-a + 2x) - \frac{Wx^2}{2a} = \frac{W}{2}\left[2x - a - \frac{x^2}{a}\right]$

$a - a - \frac{a}{4}$

$-\frac{W}{8}$

$$\begin{array}{r} 3.04 \\ 3.04 \\ \hline 1216 \\ 912 \\ \hline 9.2416 \end{array}$$

$18.33 - 2x^2 = 0$

$9.17 = x^2 \quad = 3.09$

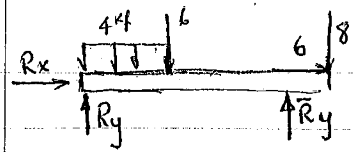
$x = 3.$

12.

$4x - 18.33 = 0$

$x = 4.6$

Problem 2-21



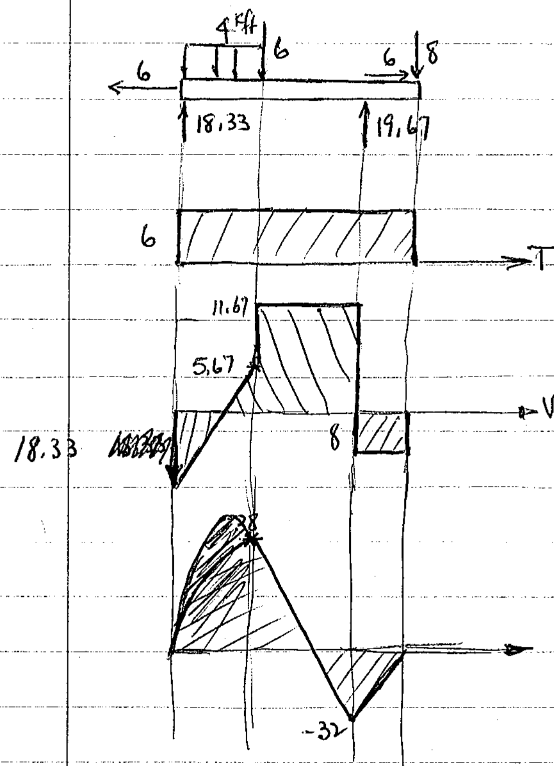
$R_x + 6 = 0 \qquad R_x = -6$

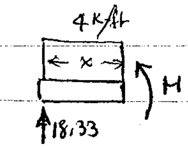
$R_y + \bar{R}_y - 8 - 6 - 24 = 0$

$-72 - 36 + \bar{R}_y(12) - 8(16) = 0$

$+\frac{108 + 128}{12} = \bar{R}_y \qquad \bar{R}_y = 19.67$

$R_y = 18.33$

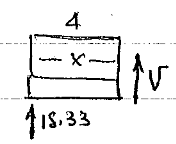




$x \geq 0 \quad x \leq 6$

$-18.33x + \frac{4x^2}{2} + M = 0$

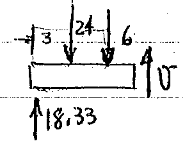
$M = 18.33x - 2x^2$



$18.33 - 4x + V = 0$

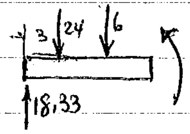
$4x - 18.33 = V$

$5.67 \cdot \quad x \geq 0 \quad x \leq 6$



$V = 30 - 18.33$

$= 11.67 \qquad x \geq 6, \; x \leq 12$



$-18.33x + 24(x-3) + 6(x-6)$

$+ M = 0$

$M = -11.67x + 108 \quad x \geq 6, \; x \leq 12$

$-\frac{35}{3}(9) + 108 = -32$

$55 - 18 = 37$

60
48
2
$110 - 72 = 38$
$18.33(5) - 50$
91.67
-50

$-18.33x + 24x - 72 + 6x - 36$

$-19.67x + 236 + M = 0$

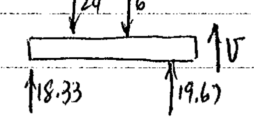
$-38x + 30x + 128 + M = 0$

$+8x - 128 = M$

80
48

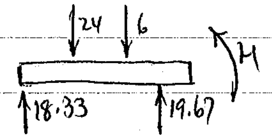
$18.33 - 4x = 0$

$x = \frac{18.33}{4} = 4.58$



$V \geq 12 \quad V \leq 16$

$V + 38 - 30 = 0 \quad V = -8$



$-18.33(x) + 24(x-3)$

$+6(x-6) - 19.67(x-12)$

$+M = 0$

$$V(x) = 4x - 18.33k \qquad 0 \le x \le 6$$
$$= 11.67\ k \qquad 6 \le x \le 12$$
$$= -8\ k \qquad 12 \le x \le 16$$
$$= 0\ k \qquad x = 16$$

$$M(x) = 18.33x - 2x^2 \qquad 0 \le x \le 6$$
$$= -11.67x + 108 \qquad 6 \le x \le 12$$
$$= 8x - 128 \qquad 12 \le x \le 16$$
$$= 0 \qquad x = 16$$

$$\frac{\partial M}{\partial x} = -V_x.$$
$$\frac{\partial^2 M}{\partial x^2} = -\frac{\partial V}{\partial x} = +P$$

$$18.33x - 2x^2 = M$$

$$\frac{\partial M}{\partial x} = 18.33 - 4x = 0$$
$$4.58 = x$$

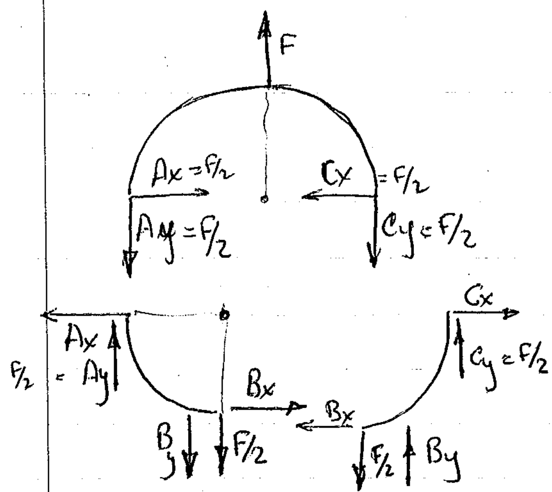
$$\frac{55}{3}(4.58) - 2(4.58)^2 =$$

4.6

$$\begin{array}{r} 230 \\ 23 \\ \hline 253 \end{array}$$
$$\frac{253}{3} = 84$$

$$\begin{array}{r} 4.6 \\ 4.6 \\ \hline 276 \\ 184 \\ \hline 21.16 \end{array}$$

$$84 - 42.32$$
$$41.$$



$Ax - Cx = 0$

$F - Ay - Cy = 0$

$Ay\,R - Cy\,R = 0$

$Ay = Cy$

$F - 2Cy = 0 \quad F/2 = Cy = Ay$

$-Ax + Bx = 0$

$Ay - F/2 - By = 0 \qquad By = 0$

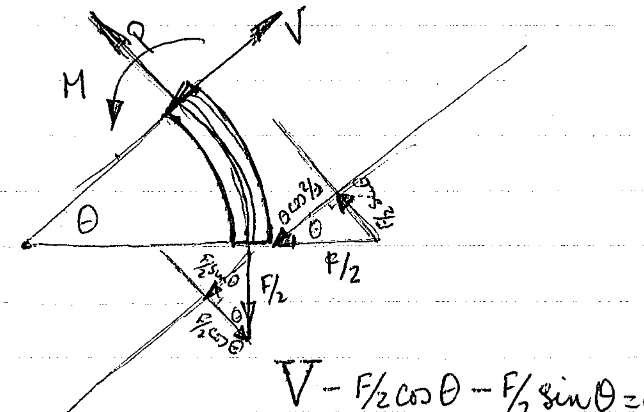
$Bx(R) - Ay(R) = 0 \qquad Bx = Ay = F/2 = Ax$

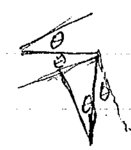
$Cx = F/2$

$Cx - Bx = 0 \qquad F/2 - F/2 = 0$

$Cy + By - F/2 = 0 \qquad F/2 + 0 - F/2 = 0$

$-Bx(R) + Cy\,R = 0 \qquad Bx = Cy = F/2$ ✓





$V - F/2 \cos\theta - F/2 \sin\theta = 0$

$P + F/2 \sin\theta - F/2 \cos\theta = 0$

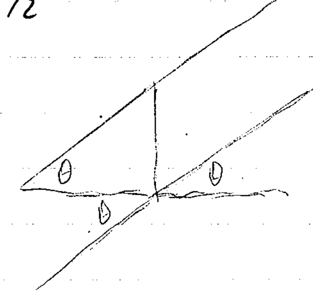
$P = F/2 \left(\cos\theta - \sin\theta\right)$

$V = F/2 \left(\cos\theta + \sin\theta\right)$

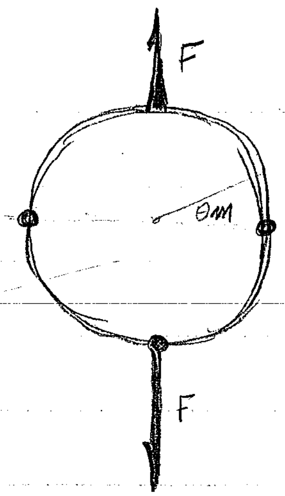
$PR - F/2\,R + M = 0$

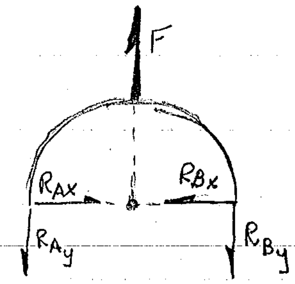
$M = F/2\,R - R\left[F/2 \sin\theta - F/2 \cos\theta\right]$

$M = \frac{FR}{2}\left[1 + \cos\theta - \sin\theta\right]$



$0 < \theta < \pi/2$





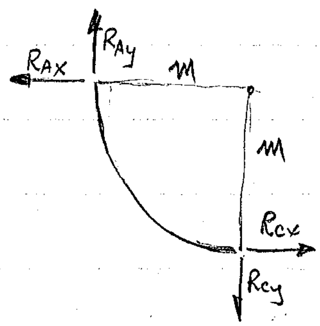
$R_{Ax} = R_{Bx}$

$R_{Ay} + R_{By} = F$

$R_{Ay}[M] = R_{By}[M]$

$R_{Ay} = R_{By}$

$$\boxed{\therefore\ R_{Ay} = R_{By} = F/2}$$



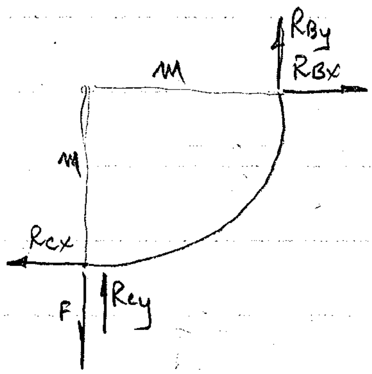
$$\boxed{R_{Ay} = R_{cy} = F/2}$$

$R_{Ax} = R_{cx}$

$R_{cx}[M] = R_{Ay}[M]$

$$\boxed{R_{cx} = R_{Ay} = R_{Ax} = F/2}$$

$$\boxed{\text{since } R_{Ax} = F/2;\ R_{Bx} = F/2}$$

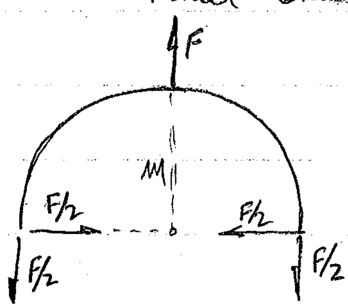


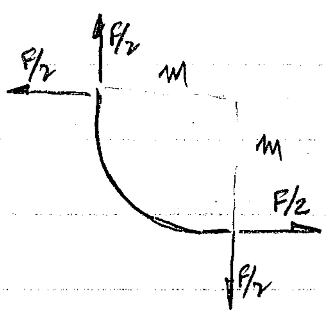
$R_{By} + R_{cy} = F$

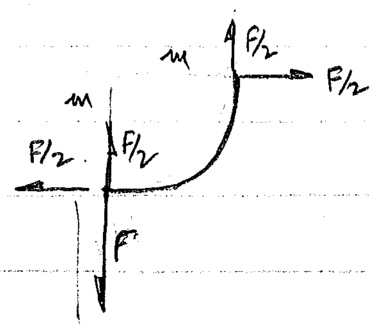
$\therefore\ R_{By} = F/2$ ✓

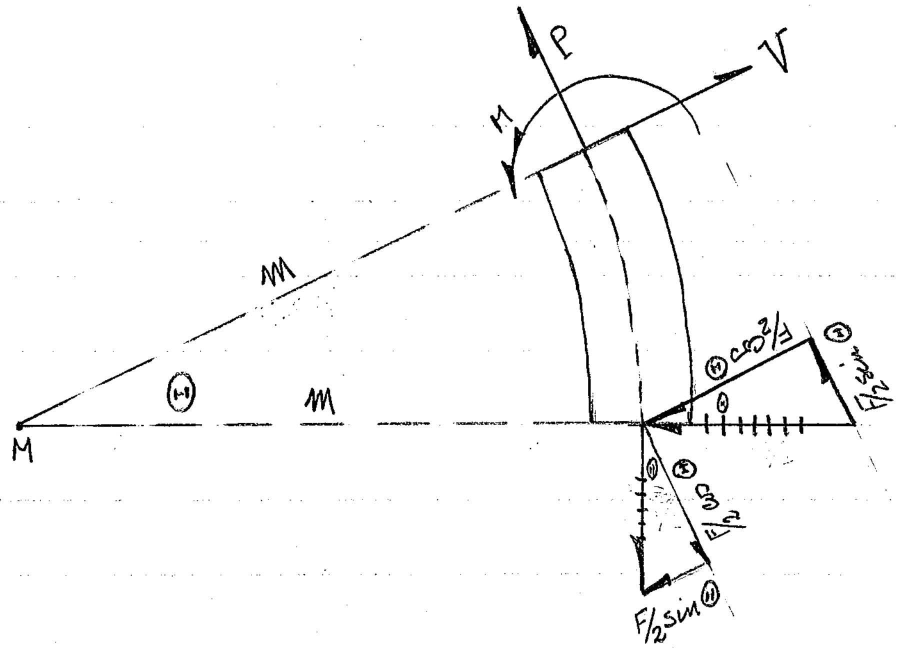
$$\boxed{R_{Bx} = R_{cx} = F/2}$$

Final Shear-Stress Force position









$$\sum F_p = 0 \qquad P + \tfrac{F}{2}\sin\Theta - \tfrac{F}{2}\cos\Theta = 0$$

$$\sum F_V = 0 \qquad V - \tfrac{F}{2}\cos\Theta - \tfrac{F}{2}\sin\Theta = 0$$

$$\sum M_M = 0 \qquad P[m] + M - \tfrac{F}{2}[m] = 0$$

$$\boxed{\begin{aligned} P(\Theta) &= \tfrac{F}{2}\left[\cos\Theta - \sin\Theta\right] \\ V(\Theta) &= \tfrac{F}{2}\left[\sin\Theta + \cos\Theta\right] \\ M(\Theta) &= \frac{Fm}{2}\left[1 + \sin\Theta - \cos\Theta\right] \end{aligned}}$$

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4-1

$$\varepsilon_x = \frac{\partial u}{\partial x}, \qquad \varepsilon_y = \frac{\partial v}{\partial y}, \qquad \varepsilon_z = \frac{\partial w}{\partial z}$$

① $$\gamma_{xy} = \gamma_{yx} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

$$\frac{\partial(\gamma_{xy})}{\partial y} = \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial^2 \gamma_{xy}}{\partial y \partial x} = \frac{\partial^3 v}{\partial x \partial y \partial x} + \frac{\partial^3 u}{\partial y^2 \partial x}$$

but $\dfrac{\partial^2 F}{\partial y \partial x} = \dfrac{\partial^2 F}{\partial x \partial y}$ therefore

② $$\frac{\partial^2 \gamma_{xy}}{\partial y \partial x} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

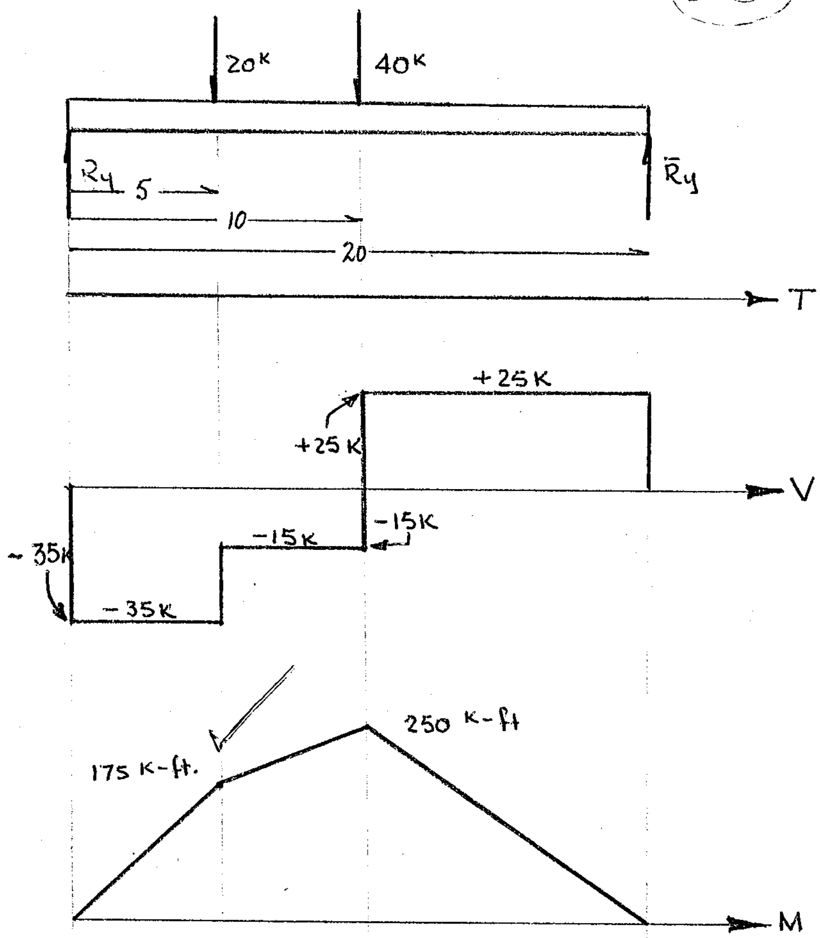
③ $$\frac{\partial^3 v}{\partial x \partial y \partial x} = \frac{\partial^3 v}{\partial x^2 \partial y} = \frac{\partial^2}{\partial x^2}\left[\frac{\partial v}{\partial y}\right] = \frac{\partial^2 \epsilon_y}{\partial x^2}$$

④ $$\frac{\partial^3 u}{\partial y^2 \partial x} = \frac{\partial^2}{\partial y^2}\left[\frac{\partial u}{\partial x}\right] = \frac{\partial^2 \epsilon_x}{\partial y^2}$$

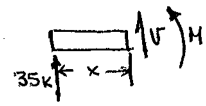
$$\therefore \quad \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2}$$

QED

C. LEVY
CE201D

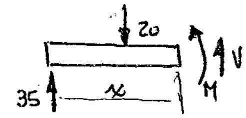


$$R_y + \bar{R}_y - 60 = 0$$
$$-100 - 400 + \bar{R}_y(20) = 0$$
$$\bar{R}_y = 25K$$
$$R_y = 35K$$

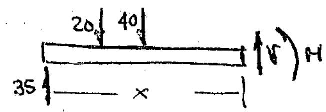


$$35 + V = 0$$
$$V = -35K$$

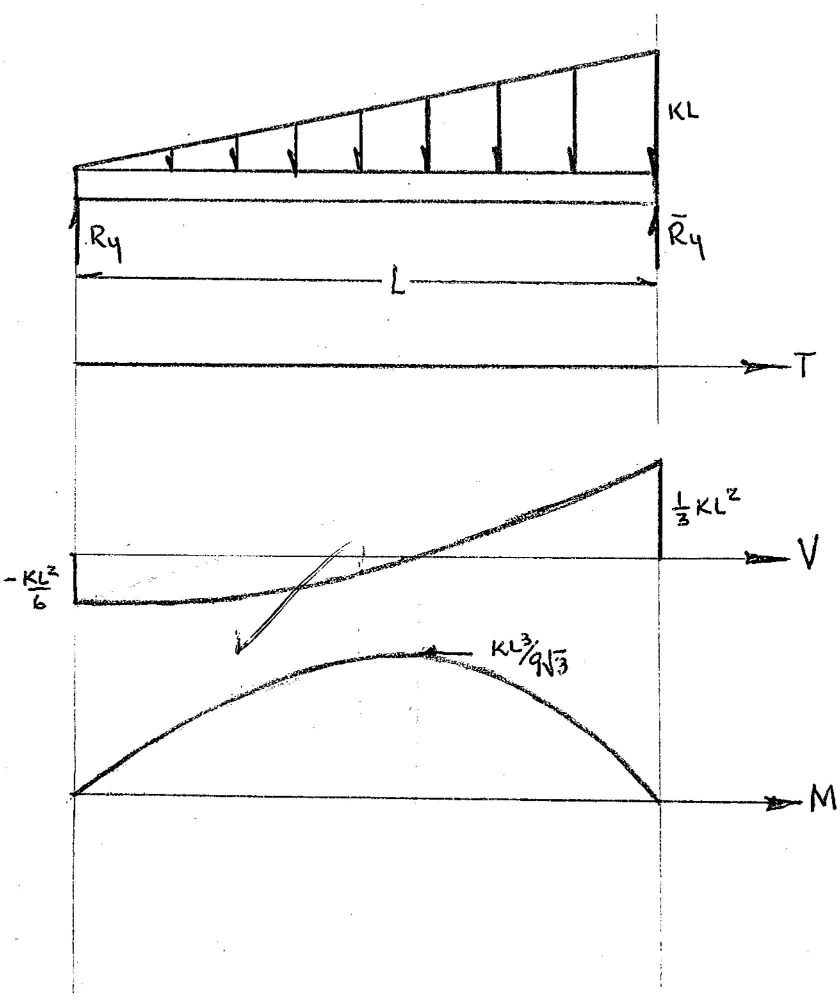
$$35x = M \; ; \; V = -35K \qquad 0 \le x \le 5$$



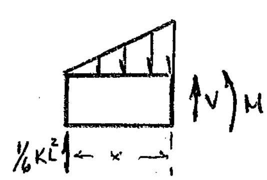
$$V = -15K$$
$$M = 15x + 100 \qquad 5 \le x \le 10$$



$$V = 25K \quad M = -25x + 500 \qquad 10 \le x \le 20$$

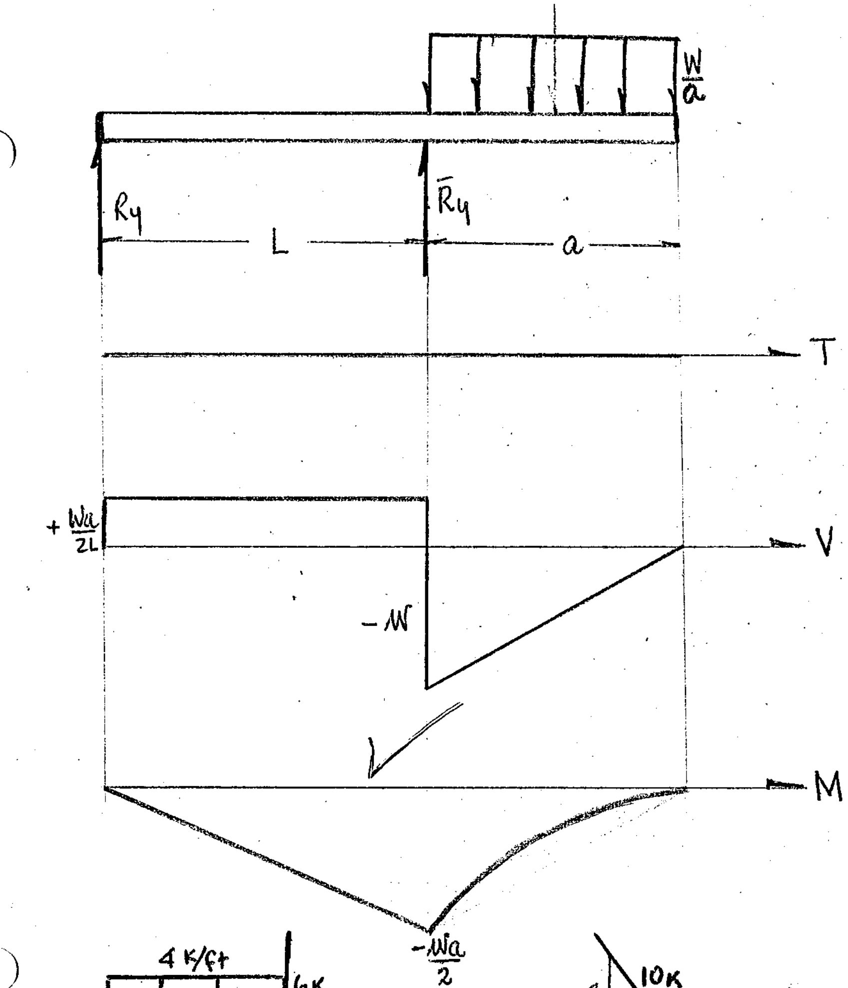


$$R_y + \bar{R}_y - \tfrac{1}{2}KL^2 = 0$$
$$-\frac{KL^3}{3} + \bar{R}_y L = 0$$
$$\bar{R}_y = \tfrac{1}{3}KL^2$$
$$R_y = \tfrac{1}{6}KL^2$$



$$R_y - \tfrac{1}{2}Kx^2 + V = 0$$
$$V = \left[3x^2 - L^2\right]K/6$$

$$-\tfrac{1}{6}KL^2x + \tfrac{1}{6}Kx^3 + M = 0$$
$$M = \tfrac{1}{6}K\left[L^2x - x^3\right]$$

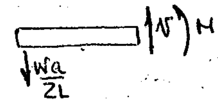


$R_y + \bar{R}_y = W$

$\bar{R}_y L - W\left(L + \frac{a}{2}\right) = 0$

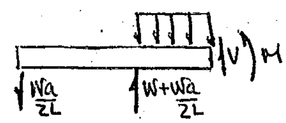
$\bar{R}_y = W + \frac{Wa}{2L}$

$R_y = -\frac{Wa}{2L}$



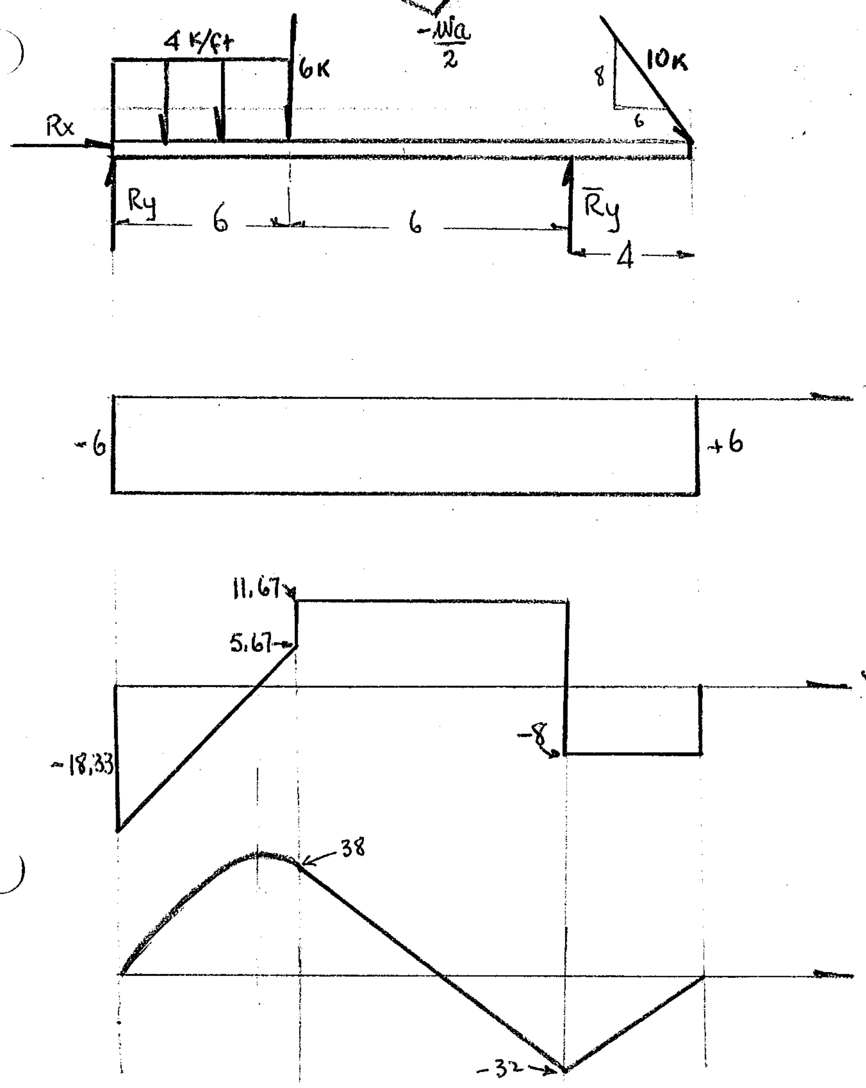
$V = \frac{Wa}{2L}$; $M = -\frac{Wa}{2L}(x)$

$0 \le x \le L$



$V = \frac{Wx}{a} - W$ $\quad L \le x \le L+a$

$M = \frac{W}{2}\left[2x - a - \frac{x^2}{a}\right]$



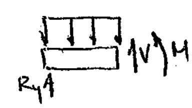
$R_x + 6 = 0$

$R_y + \bar{R}_y - 8 - 6 - 24 = 0$

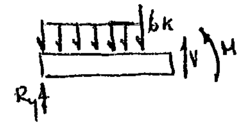
$-72 - 36 + \bar{R}_y(12) - 8(16) = 0$

$\bar{R}_y = 19.67$

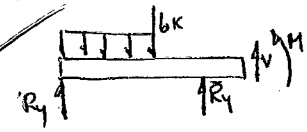
$R_y = 18.33$



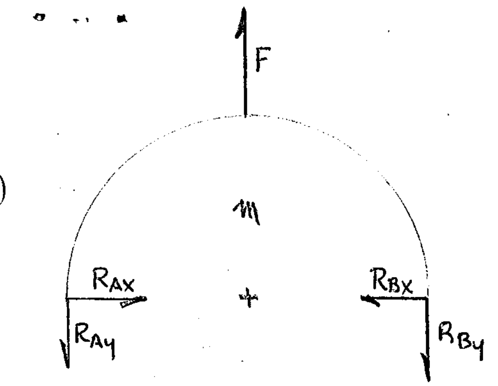
$T = -6$; $V = 4x - 18.33$; $M = 18.33x - 2x^2$; $0 < x \le 6$



$T = -6$; $V = 11.67$; $M = -11.67x + 108$; $6 \le x \le 12$



$T = -6$; $V = -8$; $M = 8x - 128$; $12 \le x \le 16$



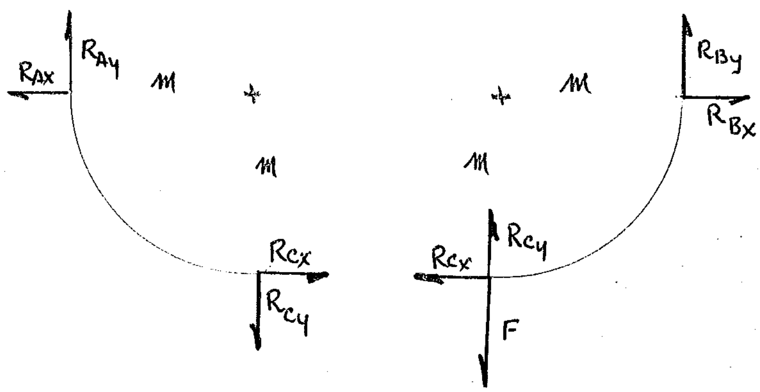
$R_{Ax} = R_{Bx}$

$R_{Ay} + R_{By} = F$

$R_{Ay}[\mathcal{M}] = R_{By}[\mathcal{M}]$

$R_{Ay} = R_{By}$

$\therefore\ R_{Ay} = R_{By} = F/2$



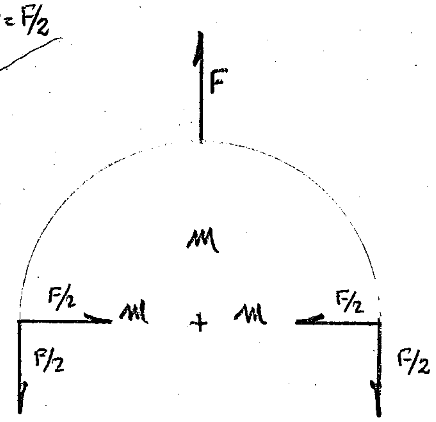
$R_{Ay} = R_{cy} = F/2$

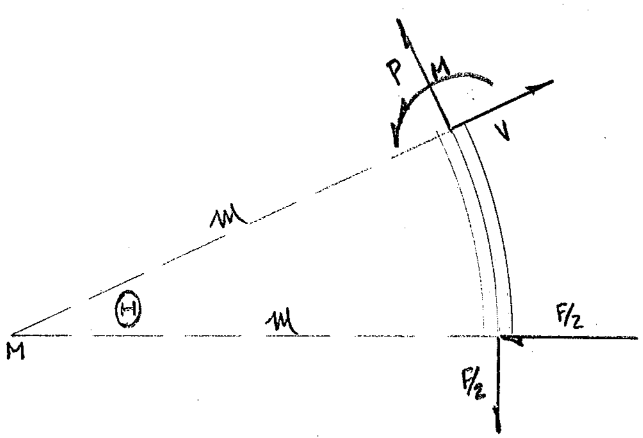
$R_{Ax} = R_{cx}$

$R_{cx}[\mathcal{M}] = R_{Ay}[\mathcal{M}]$

$\therefore\ R_{cx} = R_{Ay} = R_{Ax} = F/2$

Ainsi $R_{Ax} = F/2$; $R_{Bx} = F/2$





$\sum F_p = 0 \qquad P + F/2 \sin\Theta - F/2 \cos\Theta = 0$

$\sum F_V = 0 \qquad V - F/2 \cos\Theta - F/2 \sin\Theta = 0$

$\sum M_M = 0 \qquad P[\mathcal{M}] + M - F/2[\mathcal{M}] = 0$

$P(\Theta) = F/2\left[\cos\Theta - \sin\Theta\right]$

$V(\Theta) = F/2\left[\sin\Theta + \cos\Theta\right]$

$M(\Theta) = \frac{F\mathcal{M}}{2}\left[1 + \sin\Theta - \cos\Theta\right]$

$$d\varphi = \frac{T\,dx}{JG} \qquad \varphi = \int_0^x \frac{T(x)}{J(x)G}\,dx + C_1$$

$$J_{AB} = \frac{\pi c^4}{2} = \frac{\pi}{2} = J_{BK} \qquad J_{KC} = J_{CD} = \frac{\pi}{2}(c_o^4 - c_i^4) = \frac{\pi}{2}\left(\frac{15}{16}\right)$$

$L_{AB} = 10$ in $\quad L_{BK} = 6$ in $\quad L_{KC} = 6$ in $\quad L_{CD} = 10$ in

$T_{AB} = 450\pi \quad T_{BK} = 240\pi \quad T_{KC} = 240\pi \quad T_{CD} = 375\pi$

$$\varphi = \int_A^D \frac{T(x)}{G(J(x))}\,dx = \int_A^B \frac{T_{AB}}{GJ_{AB}}\,dx_{AB} + \int_B^K \frac{T_{BK}}{GJ_{BK}}\,dx_{BK} + \int_K^C \frac{T_{KC}}{GJ_{KC}}\,dx_{KC} + \int_C^D \frac{T_{CD}}{GJ_{CD}}\,dx_{CD}$$

$$= \frac{T_{AB}L_{AB}}{GJ_{AB}} + \frac{T_{BK}L_{BK}}{GJ_{BK}} + \frac{T_{KC}L_{KC}}{GJ_{KC}} + \frac{T_{CD}L_{CD}}{GJ_{CD}}$$

$$= \frac{(450\pi)(10)}{(12\times10^6)(\pi/2)} + \frac{240\pi(6)}{(12\times10^6)(\pi/2)} + \frac{240\pi(6)}{(12\times10^6)\left(\frac{\pi(15)}{32}\right)} + \frac{375\pi(10)}{(12\times10^6)\left(\frac{\pi}{32}(15)\right)}$$

$$= \frac{9000}{12\times10^6} + \frac{480}{2\times10^6} + \frac{240(32)}{(2\times10^6)(15)} + \frac{375(16)}{(6\times10^5)(15)}$$

$$= .75\times10^{-3} + .24\times10^{-3} + .128\times10^{-3} + .667\times10^{-3} = 1.785\times10^{-3}\ \text{rad.}$$

$$\varphi \text{ in degrees} = .001785\left(\frac{180}{\pi}\right) = .123^\circ$$

10

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5-1

$\tau_{max} = \frac{Tc}{J}$ $J = \frac{\pi c^4}{2}$ for solid shaft ; $J = \frac{\pi c^4}{2} - \frac{\pi b^4}{2}$ for hollow shaft

$\tau_{max_1} = \tau_{max_2}$; $T_1 = T_2$; $\therefore$ $c_1 J_2 = c_2 J_1$

$c_1 = 1$ in , $c_2 = 1.5$ in , $J_1 = \frac{\pi}{2}$ $J_2 = \frac{\pi}{2}\left(\frac{81}{16} - b^4\right)$

$1.5\left(\frac{\pi}{2}\right) = 1\left(\frac{\pi}{2}\right)\left(\frac{81}{16} - b^4\right)$; $1.5 = \left(\frac{81}{16} - b^4\right)$

$b = \sqrt[4]{5.0625 - 1.5} = \sqrt[4]{3.5625} = 1.375$ in

thickness must be $1.5 - 1.375 = .125$ in.

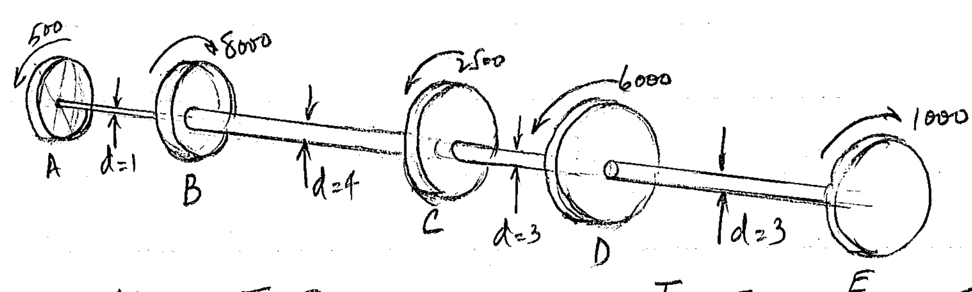
b)

weight of solid / weight of hollow = ρ Volume solid / ρ Volume hollow.

Volume of solid = $\pi r^2 h$ volume of hollow = $\pi R^2 h - \pi r^2 h$

$\frac{W_s}{W_h} = \frac{r^2}{R^2 - r^2}$ $r = 1.375$; $R = 1.5$; $r = 1$; $\frac{W_s}{W_h} = \frac{1}{.365} = \underline{\underline{2.74}}$

5-2



between AB $T = 500$ $c = 1/2$ $J = \frac{\pi}{32}$ $\tau_{max} \stackrel{?}{=} 2545$ psi

between BC $T = 7500$ $c = 2$ $J = 8\pi$ $\tau_{max} \stackrel{?}{=} 596$ psi

between CD $T = 5000$ $c = 1.5$ $J = \frac{\pi}{2}\left(\frac{81}{16}\right)$ $\tau_{max} \stackrel{?}{=} 944$ psi

between DE $T = 1000$ $c = 1.5$ $J = \frac{\pi}{2}\left(\frac{81}{16}\right)$ $\tau_{max} \stackrel{?}{=} 188.5$ psi

$\tau_{max} = 2545$ psi between A & B

5-3 $T_l = \dfrac{63000(90)}{63000} = 9000$ in-lb $\qquad T_r = \dfrac{63000}{63000}(30) = 3000$ in-lb

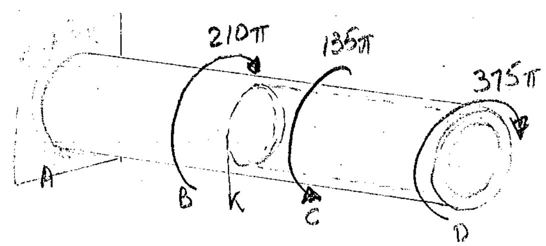
since $kT = \tau_{max}$, we must find a rod that will withstand T_{max}. We can therefore say that we must overcome the torque produced by the left machine since it has the highest torque.

$$\therefore \quad \tau_{max} = \frac{Tc}{J} \; ; \quad \frac{c}{J} = \frac{c}{\frac{\pi}{2}c^4} = \frac{2}{\pi}\frac{1}{c^3} \; ; \quad \frac{\tau_{max}}{T} = \frac{2}{\pi}\frac{1}{c^3}$$

$$\frac{\pi}{2}c^3 = \frac{T}{\tau_{max}} \qquad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}} = \sqrt[3]{\frac{18000}{18090}} \approx \sqrt[3]{1.0} = 1.0$$

$$\therefore \quad c = 1.0 \qquad \underline{d = 2 \text{ in}}$$

5-4



1. belw AB $\quad T = 450\pi \quad c = 1'' \quad J = \frac{\pi c^4}{2}$

$$\tau_{max_1} = \frac{Tc}{J} = \frac{2(450)\pi}{\pi c^3} = \frac{900\pi}{\pi} = 900 \text{ psi}$$

2. between BK $\quad T = 240\pi \quad c = 1 \quad J = \frac{\pi c^4}{2}$

$$\tau_{max_2} = \frac{Tc}{J} = \frac{2(240)\pi}{\pi c^3} = 480 \text{ psi}$$

3. between KC $\quad T = 240\pi \quad c = 1 \quad b = 1/2 \quad J = \frac{\pi b^4}{2} + \frac{\pi c^4}{2}$

$$J = \frac{\pi}{2}\left(1 - \frac{1}{16}\right) = \frac{\pi}{2}\left(\frac{15}{16}\right) \; ; \quad \tau_{max_3} = \frac{Tc}{J} = \frac{240\pi(1)2(16)}{\pi(15)} = 480\left(\frac{16}{15}\right) = 512 \text{ psi}$$

4 between CD $\quad T = 375\pi \quad c = 1 \quad b = 1/2 \quad J = \frac{\pi}{2}\left(\frac{15}{16}\right)$

$$\tau_{max_4} = \frac{Tc}{J} = \frac{375\pi(1)2(16)}{\pi(15)} = \frac{750(16)}{15} = 800 \text{ psi}$$

$$\boxed{\tau_{max} = 900 \text{ psi} \quad \text{bet AB}}$$

Mechanics of Solids

9

CE201 D

Problems:
4-3, 5, 6, 7, 8

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4/9/70

C. LEVY CE201 D
Mr. Blumert

4-3, 5, 6, 7, 8

1)

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\epsilon_y = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_z \qquad \therefore \qquad \begin{bmatrix} 1/E & -\frac{\nu}{E} & -\frac{\nu}{E} \\ -\nu/E & +1/E & -\frac{\nu}{E} \\ -\nu/E & -\nu/E & 1/E \end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y \\ \sigma_z\end{bmatrix} = \begin{bmatrix}\epsilon_x \\ \epsilon_y \\ \epsilon_z\end{bmatrix}$$

$$\epsilon_z = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \frac{\sigma_z}{E}$$

TAKE THE AUGMENTED MATRIX AND REDUCE IT

$$\begin{bmatrix} 1/E & -\nu/E & -\nu/E & \epsilon_x \\ -\nu/E & +1/E & -\nu/E & \epsilon_y \\ -\nu/E & -\nu/E & 1/E & \epsilon_z \end{bmatrix} \longrightarrow \begin{bmatrix} 1/E + \nu/E & 0 & -\nu/E - 1/E & \epsilon_x - \epsilon_z \\ 0 & 1/E + \nu/E & -\nu/E - 1/E & \epsilon_y - \epsilon_z \\ -\nu/E & -\nu/E & 1/E & \epsilon_z \end{bmatrix} \longrightarrow$$

$$\begin{bmatrix} 1 & 0 & -1 & (\epsilon_x - \epsilon_z)/(1/E + \nu/E) \\ 0 & 1 & -1 & (\epsilon_y - \epsilon_z)/(1/E + \nu/E) \\ -\nu/E & -\nu/E & 1/E & \epsilon_z \end{bmatrix} \longrightarrow$$

$$\begin{bmatrix} 1 & 0 & -1 & (\epsilon_x - \epsilon_z)/(1/E + \nu/E) \\ 0 & 1 & -1 & (\epsilon_y - \epsilon_z)/(1/E + \nu/E) \\ 0 & -\nu/E & 1/E - \nu/E & \epsilon_z + \nu/E\left[(\epsilon_x - \epsilon_z)/(1/E + \nu/E)\right] \end{bmatrix} \longrightarrow$$

$$\begin{bmatrix} 1 & 0 & -1 & (\epsilon_x - \epsilon_z)/(1/E + \nu/E) \\ 0 & 1 & -1 & (\epsilon_y - \epsilon_z)/(1/E + \nu/E) \\ 0 & 0 & -2\nu/E + 1/E & \epsilon_z + \nu/E\left[(\epsilon_x - \epsilon_z)/(1/E + \nu/E)\right] + \frac{\nu}{E}\left[(\epsilon_y - \epsilon_z)/(1/E + \nu/E)\right] \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & (\epsilon_x - \epsilon_y)/(1 + \nu/E) \\ 0 & 1 & -1 & (\epsilon_y - \epsilon_z)/(1 + \nu/E) \\ 0 & 0 & 1 & \underbrace{\frac{E}{1-2\nu}\left[\epsilon_z + \nu/E\left[(\epsilon_x - \epsilon_z)/\left(\frac{1+\nu}{E}\right)\right] + \nu/E\left[(\epsilon_y - \epsilon_z)/\left(\frac{1+\nu}{E}\right)\right]\right]}_{K} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & (\epsilon_x - \epsilon_z)/\left(\frac{1+\nu}{E}\right) + K \\ 0 & 1 & 0 & (\epsilon_y - \epsilon_z)/\left(\frac{1+\nu}{E}\right) + K \\ 0 & 0 & 1 & K \end{bmatrix} \Rightarrow \begin{aligned} \sigma_x &= (\epsilon_x - \epsilon_z)/\left(\tfrac{1+\nu}{E}\right) + K \\ \sigma_y &= (\epsilon_y - \epsilon_z)/\left(\tfrac{1+\nu}{E}\right) + K \\ \sigma_z &= K \end{aligned}$$

$$\sigma_x = (\epsilon_x - \epsilon_z)/\left(\frac{1+\nu}{E}\right) + E/(1-2\nu)\left[\epsilon_z + \nu/E\left[(\epsilon_x + \epsilon_y - 2\epsilon_z)/\left(\frac{1+\nu}{E}\right)\right]\right]$$

$$\sigma_y = (\epsilon_y - \epsilon_z)/\left(\frac{1+\nu}{E}\right) + E/(1-2\nu)\left[\epsilon_z + \nu/E\left[(\epsilon_x + \epsilon_y - 2\epsilon_z)/\left(\frac{1+\nu}{E}\right)\right]\right]$$

$$\sigma_z = E/(1-2\nu)\left[\epsilon_z + \nu/E\left[(\epsilon_x + \epsilon_y - 2\epsilon_z)/\left(\frac{1+\nu}{E}\right)\right]\right]$$

from $\sigma_z = \frac{E\nu}{(1+\nu)(1-2\nu)}\left[\frac{\epsilon_z}{\nu} + \epsilon_x + \epsilon_y - \epsilon_z\right]$ add to both sides an $\lambda\epsilon_z$

so that $\sigma_z = \lambda\left[\frac{\epsilon_z}{\nu} + \epsilon_x + \epsilon_y + \epsilon_z - \epsilon_z\right] - \lambda\epsilon_z$

and that $\sigma_z = \lambda e + \lambda\epsilon_z\left[\frac{1}{\nu} - 2\right] = \lambda e + \lambda\epsilon_z\left[\frac{1-2\nu}{\nu}\right]$

$$\lambda\left[\frac{1-2\nu}{\nu}\right] = \frac{E}{(1+\nu)} = 2G$$

$\therefore \quad \sigma_z = \lambda e + 2G\epsilon_z$ if $G = \mu$ then $\sigma_z = \lambda e + 2\mu\epsilon_z$ (same as book)

therefore $\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$ $\quad \mu = G = \frac{E}{2(1+\nu)}$

for σ_y proving that $\sigma_y = \sigma_z - 2\mu\epsilon_z + 2\mu\epsilon_y$ is simple
since $\sigma_y = E(\epsilon_y - \epsilon_z)/(1+\nu) + \sigma_z$

$\therefore \quad E(\epsilon_y - \epsilon_z)/(1+\nu) = 2\mu(\epsilon_y - \epsilon_z) = 2G(\epsilon_y - \epsilon_z) = E/(1+\nu)[\epsilon_y - \epsilon_z]$

since σ_x just is $E(\epsilon_x - \epsilon_z)/(1+\nu) + \sigma_z$ by the same way $2\mu\epsilon_x - 2\mu\epsilon_z = \frac{E}{(1+\nu)}(\epsilon_x - \epsilon_z)$

5. $P_x = 20$ KIPS $\quad P_y = 40$ KIPS $\quad \sigma_z = 0$

$\epsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y)$ $\quad \sigma_x = \frac{20}{\text{in}^2} = 20\ \text{K/in}^2$ $\quad \sigma_y = \frac{40}{5} = 8\ \text{K/in}^2$

$= -\frac{1}{120\times10^3\ \text{KSI}}(28\ \text{KSI})$ $\quad \nu = 1/4 \quad E = 30\times10^3\ \text{KSI}$

$= -\frac{7}{30\times10^3\ \text{KSI}} = -.2335\times10^{-3}$

$\epsilon_z = \frac{\Delta}{z} = \frac{\Delta}{.5\text{in}} = -.2335\times10^{-3};$ $\quad \underline{-.1168\times10^{-3}\text{in} = \Delta}$ $\quad$ depth decreases by $.1168\times10^{-3}$ in.

6. $\sigma_x = \frac{P_x}{\text{in}^2} = P_x/\text{in}^2$ $\quad \sigma_z = \sigma_y = 0$

$\epsilon_z = -.2335\times10^{-3}$ $\quad \epsilon_z = -\frac{\nu}{E}\sigma_x$

$-.2335\times10^{-3} = -\frac{7}{30\times10^3} = -\frac{1}{4(30\times10^{+3})}\sigma_x$ $\quad \therefore \quad \sigma_x = 28\ \text{KSI}$

$\boxed{P_x = 28\ \text{K}}$

6. $\sigma_x = \frac{P_x}{A_x}$ $\sigma_y = \frac{P_y}{A_y}$;

$\epsilon_x = \frac{\Delta x}{L_x} = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y$ $E = 30\times10^3$ KSI $\nu = \frac{E}{2G} - 1$

$G = 12\times10^3$ KSI

$\epsilon_y = \frac{\Delta y}{L_y} = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E}$ $\nu = \frac{30}{24} - 1 = 1/4$

$\epsilon_x = \frac{\Delta x}{L_x} = \frac{.0768}{12(8)} = .0008$; $\epsilon_y = \frac{\Delta y}{L_y} = \frac{.0864}{12(12)} = .0006$

$.0008 = \frac{P_x}{EA_x} - \frac{1}{4E}\frac{P_y}{A_y}$ $A_x = (12\times12)(1/4) = 36\,in^2$

$A_y = (8\times12)(1/4) = 24\,in^2$

$.0006 = -\frac{1}{4E}\frac{P_x}{A_x} + \frac{P_y}{EA_y}$

$.0008 = \frac{P_x}{30\times10^3(36)} - \frac{P_y}{(120\times10^3)(24)}$; $.0006 = -\frac{P_x}{120\times10^3(36)} + \frac{P_y}{(30\times10^3)(24)}$

$P_x = \frac{28.5(576)}{15}$ and P_x in KPF $= \frac{(28.5)(576)}{15(12)} = \underline{91.2 \text{ KPF}}$

P_y by substitution $= \underline{76.8 \text{ KPF}}$

7. $\epsilon_x = \frac{\nu\sigma_x}{\nu E} - \frac{\nu}{E}\sigma_y - \frac{\nu}{E}\sigma_z$; $\sigma_x = \frac{40 \text{ KIPS}}{8\,in^2}$; $\sigma_y = \frac{48 \text{ KIPS}}{6\,in^2}$; $\sigma_z = \frac{36 \text{ KIPS}}{12\,in^2}$

$\epsilon_x = \frac{20}{40\times10^{+3}} - \frac{8}{40\times10^{+3}} - \frac{3}{40\times10^3}$ $\sigma_x = 5$ KSI ; $\sigma_y = 8$ KSI ; $\sigma_z = 3$ KSI

$E = 10\times10^3$ KSI $\nu = 1/4$

$= \frac{9}{40\times10^3} = .225\times10^{-3}$

$\Delta l_x = \epsilon_x l_{o_x}$ $\Delta l_x = (.225\times10^{-3})(3) = .675\times10^{-3}$ in

find $P_x \Rightarrow \Delta l$ is same $\therefore \epsilon_x$ must be same

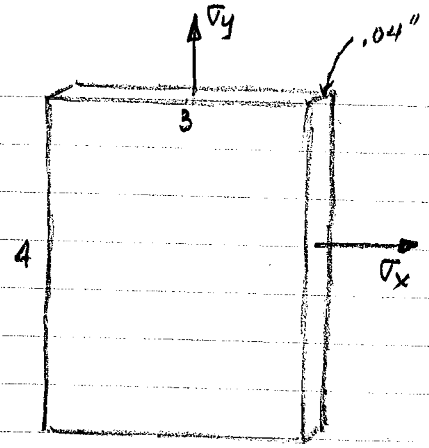
$.225\times10^{-3} = \frac{\sigma_x}{E}$ since $\sigma_y = \sigma_z = 0$

$\frac{9}{40\times10^3}(10\times10^3) = \sigma_x$; $\sigma_x = 2.25$ KSI $P_x = \sigma_x A_x$ $A_x = 8\,in^2$

$\boxed{P_x = 8(2.25) = 18 \text{ KIPS}}$

8. $\Delta T = -200°F$ $E_B = 16\times10^6$ psi , $G_B = 6\times10^6$ psi , $\alpha_B = 11\times10^{-6}\,/°F$

$\alpha_{INVAR} = 0$



$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y + \alpha\Delta T$$

$$\epsilon_y = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E} + \alpha\Delta T$$

since it is constrained to a rigid invar frame then

$$\epsilon_x = \epsilon_y = 0$$

$$-\alpha\Delta T\,E = \sigma_x - \nu\sigma_y$$
$$-\alpha\Delta T\,E = -\nu\sigma_x + \sigma_y$$

then
$$-\alpha\Delta T E\nu = \nu\sigma_x - \nu^2\sigma_y$$
$$-\alpha\Delta T E = -\nu\sigma_x + \sigma_y$$

$$-\alpha\Delta T E(1+\nu) = (1-\nu^2)\sigma_y = (1-\nu)(1+\nu)\sigma_y$$

$$\sigma_y = \frac{-\alpha\Delta T E}{1-\nu} = -\left(11\times10^{-6}\tfrac{in}{in/deg\,F}\right)(-200\,°F)(16\times10^6 psi)/(2/3)$$

$$\sigma_y = 52.8 \text{ KSI}$$

Substituting back we get $\sigma_x = 26.4$ KSI   I made a mistake of computation

$\times$

$-\alpha\Delta T E = +35.2$ $35.2 - 52.8 = -17.6 = -\tfrac{1}{3}\sigma_x$

$\sigma_x = 52.8$ KSI

9

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CE 201 D
Mr. Blument.

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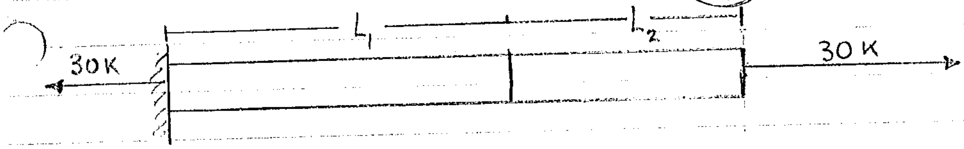
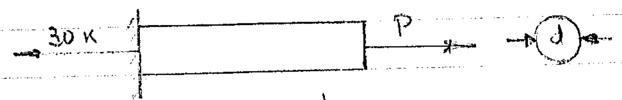




$L_1 + L_2 = 36 in$
$u_s = u_A$

$$u_s = \frac{1}{E_s}\int_0^L \frac{P_s}{A_s}dx = \frac{1}{E_s}\frac{P}{A}L_1 \qquad u_A = \frac{1}{E_A}\int \frac{P_A}{A_A}dx = \frac{1}{E_A}\frac{P}{A}L_2$$

since $u_s = u_A$ $\quad \frac{L_1}{L_2} = \frac{E_s}{E_A} = .333$

$L_1 + L_2 = 36 in \quad . \; L_2 + 3L_2 = 36 in \quad L_2 = 9 in \quad L_1 = 27 in$

$$u_A = \frac{30{,}000}{10\times 10^6(\pi)}(9) = \frac{27}{\pi \times 10^{-3}} = .859\times 10^{-2} \qquad u_T = 2u_A = 1.718\times 10^{-2} in$$

23 $\quad u_T = \frac{1}{E_A}\int \frac{P}{A_A}dx_1 + \frac{1}{E_s}\int \frac{P}{A_s}dx_2$

$$u_T = \frac{P}{E_A A_A}(15) + \frac{P}{E_s A_s}(12) \qquad E_A = 10\times 10^6 psi \quad E_s = 30\times 10^6 psi \quad A_A = 16 in^2 \quad A_s = 4 in^2$$

$$u_T = \cancel{.01} = \frac{3P + 3.2P}{32\times 10^6} \; ; \quad P = 51.6 \text{ kips}$$

21. f is given in units of force/length ∴ $F = \int f dy = \frac{Ky^3}{3} + C_1 \quad C_1 = 0 \quad y = $

$$u = \int \frac{F dy}{EA} = \frac{1}{EA}\int \frac{Ky^3}{3}dy = \frac{1}{EA}\left[\frac{Ky^4}{12}\right]_0^L = \frac{KL^4}{12EA}$$

∴ but $K = \frac{3F}{L^3} \longrightarrow \quad u = \frac{3F}{L^3}\cdot\frac{L^4}{12EA} = \frac{FL}{4EA}$

$P = 96000^{\#} \quad L = 40\times 12 \; in \quad A = 100 \; in^2 \quad E = 1.5\times 10^6 psi$

$$\frac{(96\times 10^3 \#)(40\times 12 \; in)}{4(1.5\times 10^6 \frac{\#}{in^2})(100 in^2)} = \frac{16(40\times 12)}{10^5} = 7.682\times 10^{-2} in$$

26. $u_A = u_B$

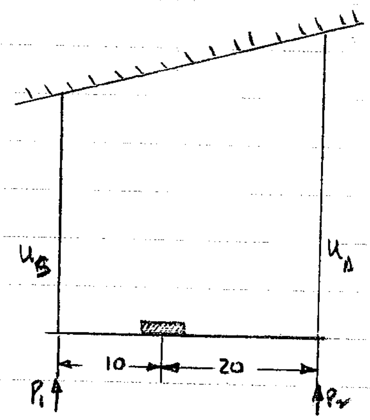
$$u_A = \int_0^{L_A} \frac{1}{E}\frac{P}{A}\,dx = \frac{P}{E\times 2}L_A$$

$$u_B = \int_0^{L_B} \frac{1}{E}\frac{P}{A}\,dx = \frac{P}{E}\int \frac{dx}{A_T} = \frac{PL_B}{E}\int_0^{L_B} \frac{dx}{2x+L_B} = \frac{PL_B}{E}\left\{\frac{1}{2}\ln\left(3L_B/L_B\right)\right\} = \frac{PL_B}{E}[.55]$$

$$u_A = u_B \quad \therefore \quad \frac{P}{E\times 2}L_A = \frac{PL_B}{E}(.55)$$

$$L_A/L_B = 1.1 \quad ✓$$

31.



$P_1 + P_2 = 2000$ $\qquad 3P_2 = 2000$

$P_1(10) = P_2(20)$ $\qquad P_2 = \frac{2000}{3}$ $\qquad P_1 = \frac{4000}{3}$

$$u_B = \int_0^{60''} \frac{1}{E_s}\frac{P_1}{A_s}\,dx = \frac{4000\times 60}{3\times 30\times 10^6\times .1} = \frac{12(2)}{9\times 10^2} = .6667\times 10^{-2}\,\text{in}\downarrow$$
$$= 2.667$$

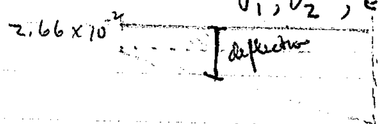
$$u_A = \int_0^{100''} \frac{1}{E_A}\frac{P_2}{A_A}\cancel{dx} = \frac{2000}{3\times 10\times 10^6\times .2}\times 100 = 3.33\times 10^{-2}$$

find P_1, P_2
$\sigma_1, \sigma_2, \epsilon_1, \epsilon_2$

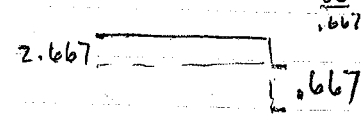
$u_1 = \epsilon_1 \Delta L_1$

$u_2 = \epsilon_2 \Delta L_2$ ~~deflection = .667×10⁻² in + 1/3 4(.667×10⁻² in)~~

~~1.555×10⁻² in~~



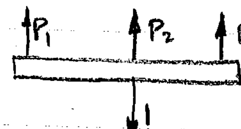
$\frac{30}{.667} = \frac{10}{x}$ $\quad .22$



34. $u_1 = u_2 = u_3$

$P_1 + P_2 + P_3 = 1\ \text{K}$

$P_1 = P_3$



$$u_1 = \int_0^{25\times 12} \frac{P_1}{E_1A_1}\,dx_1 = \int_0^{50\times 12} \frac{P_2}{E_2A_2}\,dx_2 = \int_0^{25\times 12} \frac{P_3}{E_3A_3}\,dx_3$$

$$u_1 = \frac{P_1}{E_1A_1}(25\times 12) = \frac{P_2}{E_2A_2}(50\times 12)$$

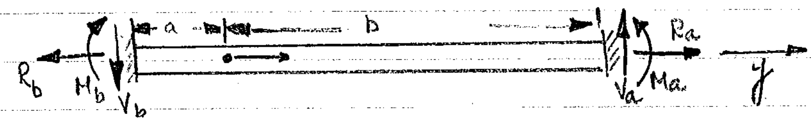
$$\frac{P_1}{E_1A_1} = \frac{2P_2}{E_2A_2}$$

$E_1A_1 = (10\times 10^6)(.3) = 3\times 10^6$

$E_2A_2 = (30\times 10^6)(.2) = 6\times 10^6$ $\qquad \therefore E_1A_1 = \frac{1}{2}E_2A_2 \qquad \therefore 2P_1 = 2P_2$

$\therefore P_1 = P_2 = P_3 \quad \Longrightarrow \quad P_1 = P_2 = P_3 = 1/3\ \text{K}$

$$AE\frac{d^2u}{dx^2} = -p_x$$



$V_a = V_b \qquad R_a + P = R_b$

$u(0)=0 \quad u(L)=0$

$$AE\frac{d^2u}{dx^2} = -p_x = -P\langle x-a\rangle_*^{-1}$$

$$AE\frac{du}{dx} = -P\langle x-a\rangle^0 + C_1$$

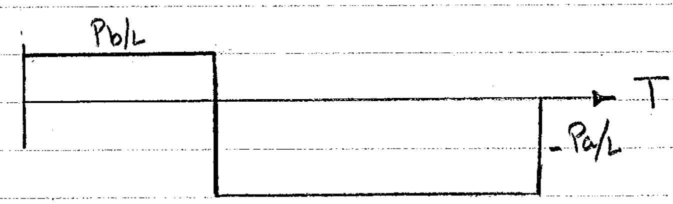
$$AEu = -P\langle x-a\rangle^1 + C_1x + C_2$$

$$AEu(0) = 0 = C_2$$

$$AEu(L) = 0 = -Pb\cancel{+C_1L} \qquad C_1 = Pb/L$$

$$R_b = P(0) = AE\left(\frac{du}{dx}\right)_{x=0} = Pb/L$$

$$R_a = P(L) = AE\left(du/dx\right)_{x=L} = -P + C_1 = -Pa/L$$



for $x<a$ axial force distrib $= Pb/AL$

for $x=a$ " " " $= P/A$

for $x>a$ axial force dist $= Pa/AL$

EXAMINATION BOOK

Name Cesar Levy

Subject CE 201

Instructor Mr Blanchard

Exam Seat No. ______ Section D

Date 2/26/70 Grade ______

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1 - 30/30
2 - 30/35
3 - 25/35

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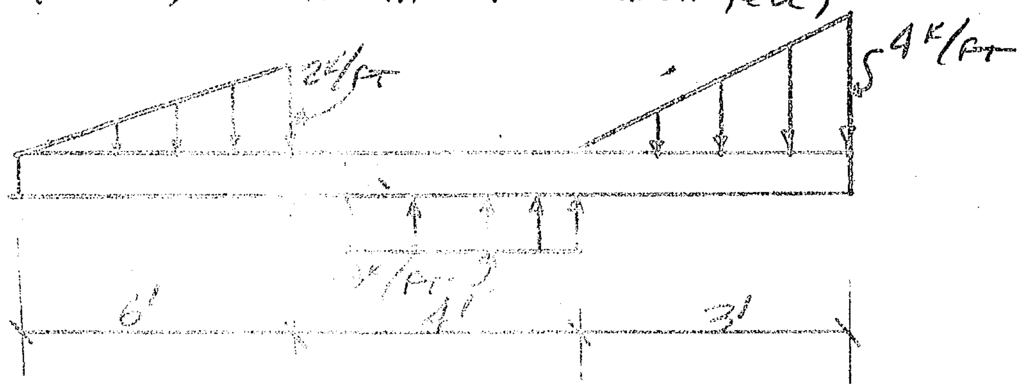
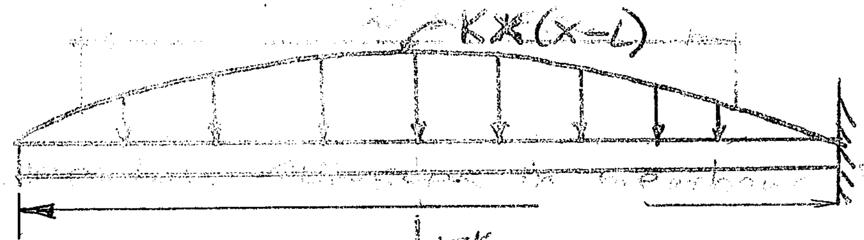
50-12 PAGE

Exam I — February 26, 1970

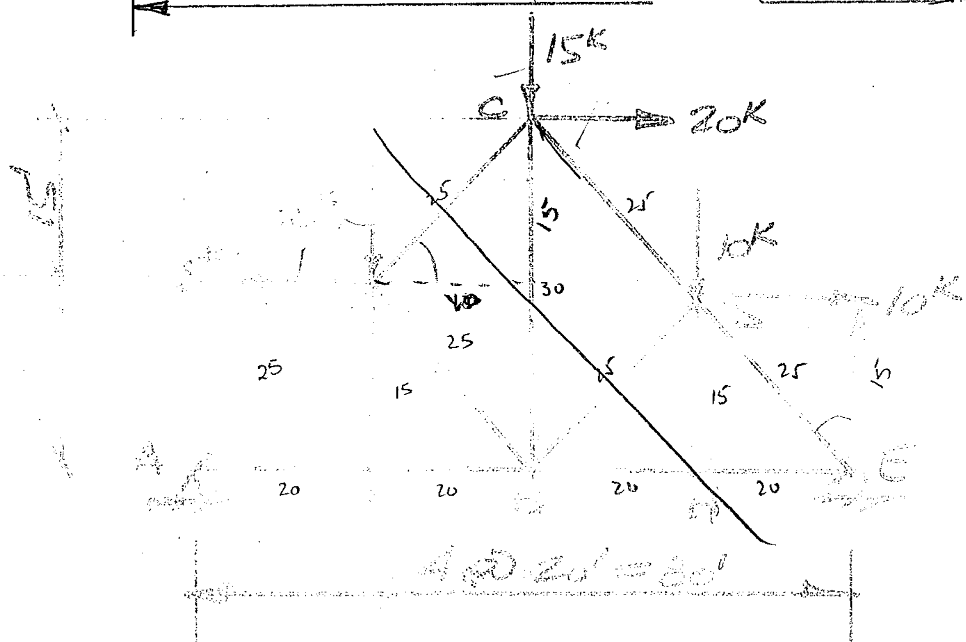
1) Draw a shear & bending moment diagram, using any method. Show values at critical points. (transition points, end points, maximum or minimum, etc)



2) Find expression for shear & bending moment and draw diagrams.



3)



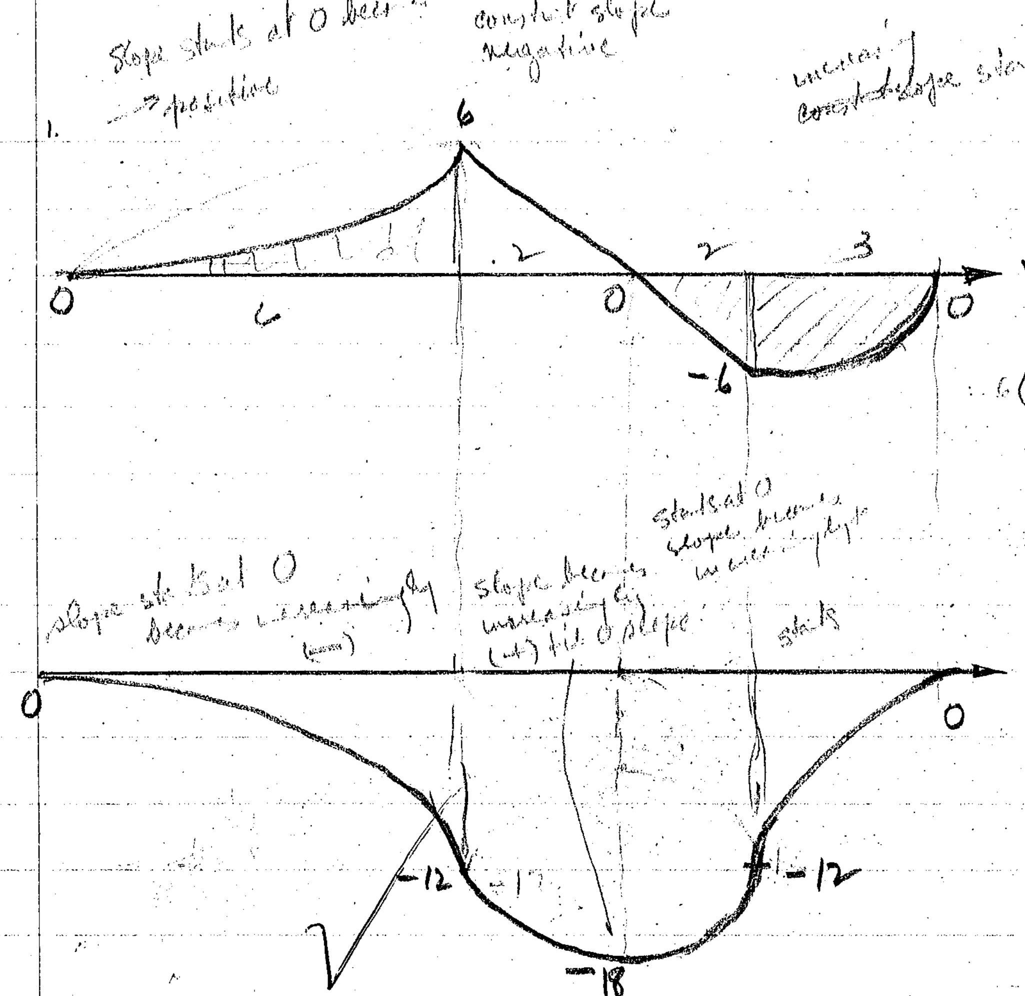
σ_{GH} = 13.34 KSI

σ_{BC} = 8.45 KSI

σ_{CG} = 1 KSI

Find member stresses in members BC, GC, & GH. All members pin-connected & 2.5 in² area.

1.



$6(3)$

$\frac{6(3)}{3}$ $\frac{2}{3}(12)(3)$

$6\frac{(2)}{2}$

$6(3) =$

$\downarrow 6$

$-6 + 3(x-6) + V = 0$

$3(x-6) =$

$V = -3x + 24$

$x = 6$

$M = \frac{3x^2}{2} - 24x + C$

$54 - 144 + C$

$\frac{1}{3}6(6) = 12$

$M =$

$\downarrow 6(4)$

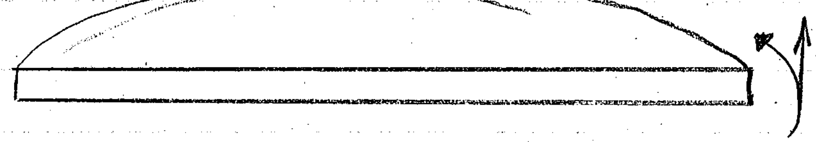
$\downarrow 6(12)$

$\uparrow 8(12)$

$24 + 72 = 96$

All done by inspection starting from right end

2.



$p = +Kx(x-L) = +Kx^2 - KLx$

$-V = -\frac{KLx^2}{2} + \frac{Kx^3}{3} + C_1$

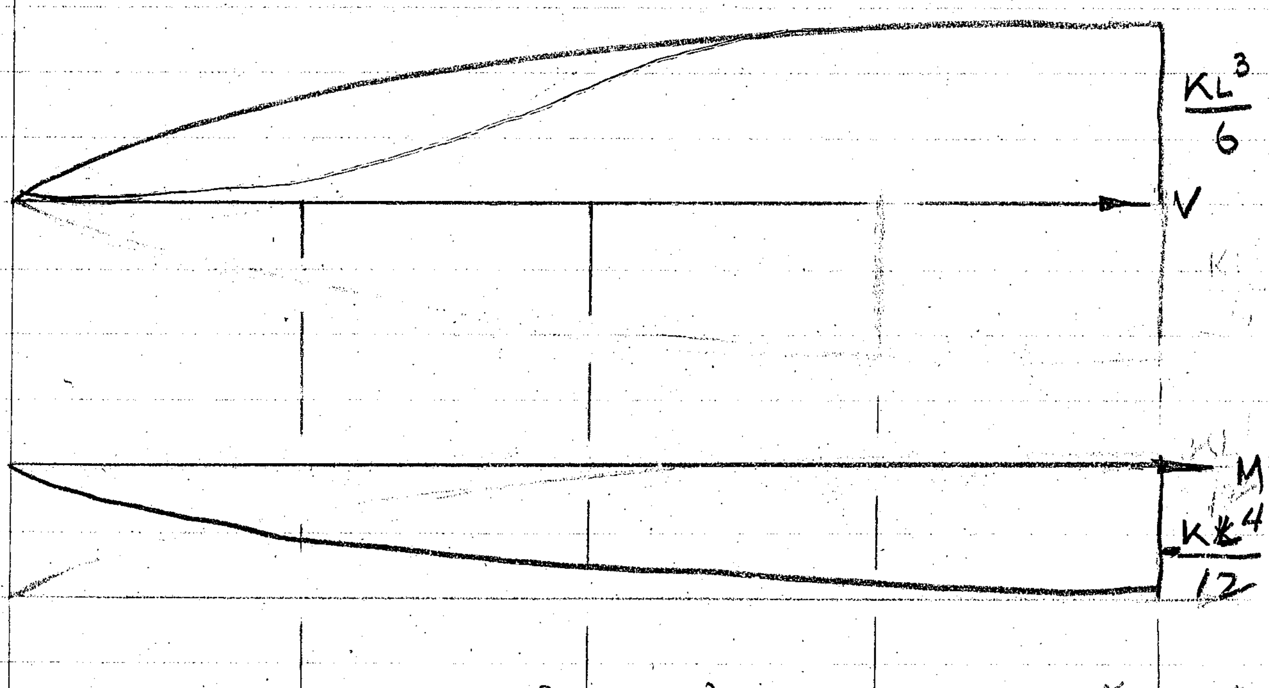
$M = -\frac{Kx^3 L}{6} + \frac{Kx^4}{12} + C_1 x + C_2$ at $x=0$ $M=0$

at $x=0$ $V=0$

$M = 0$ $0 - 0 + 0 + C_2$ $C_2 = 0$

$V=0$ $0 - 0 + C_1 = 0$ $C_1 = 0$

$\checkmark\ V = -\frac{Kx^3}{3} + \frac{KLx^2}{2}$ $\checkmark\ M = -\frac{Kx^3}{6}L + \frac{Kx^4}{12}$



at $x = L/4$ $V = -\frac{KL^3}{192} + \frac{6KL^3}{6(32)} = \frac{15KL^3}{192}$ $\cancel{\frac{Kx^3L}{6} = \frac{Kx^4}{12}}\ x$

at $x = L$ $-\frac{KL^3}{3} + \frac{KL^3}{2} = +\frac{KL^3}{6} = V$ $\frac{2L}{12} = \frac{x}{12}$

at $X = L$ $M = -\frac{KL^4}{6} + \frac{KL^4}{12} = -\frac{KL^4}{12} = M$ $2L = X$

3.

$\overrightarrow{R\bar{x}}$

$\uparrow R\bar{y}$ $\uparrow R_y$

$\Sigma F_x = 0 \qquad R\bar{x} - 10^K + 20^K + 10^K = 0$

$R\bar{x} = +20^K \leftarrow$

$\Sigma F_y = 0 \qquad R\bar{y} - 10^K - 15^K - 10^K + R_y = 0$

$R\bar{y} + R_y = 35^K$

$\Sigma M_A = 0 \; \circlearrowright + \quad 10(15) - 10(20) + 15(40) - 20(30) - 10(60) - 10(15) + R_y(80) = 0$

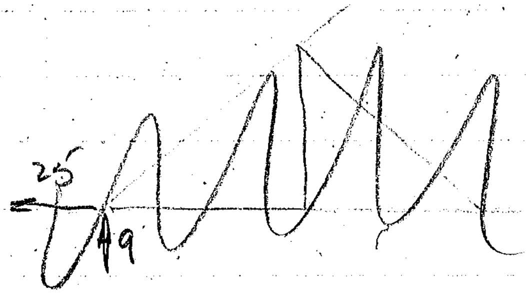
$\dfrac{-5(15) - 10(20) - 15(40) - 20(30) - 10(60)}{80} = -R_y$

$\dfrac{15(+45) + 10(+80) + 20(+30)}{80} = R_y$

$\begin{array}{r} 450 \\ 225 \\ \hline 675 \end{array}$

$\dfrac{675 + 800 + 600}{80} = R_y \qquad \dfrac{2075}{8}$

$\begin{array}{r} 450 \\ 225 \\ \hline 675 \end{array}$



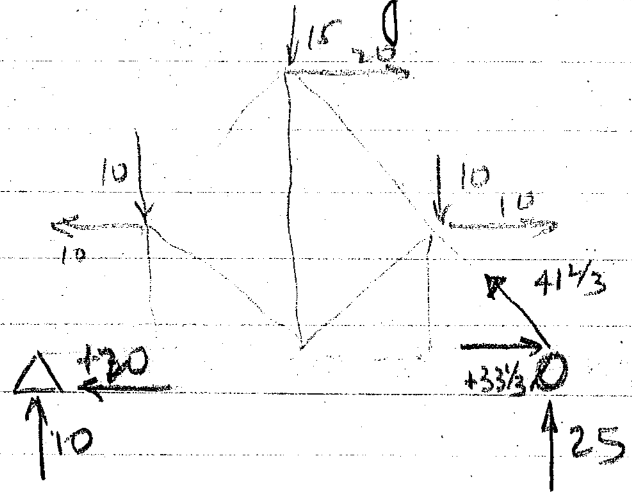
$\Sigma M_A \circlearrowleft = 0 \qquad \cancel{10(15)} - 10(20) - 15(40) - 20(30)$

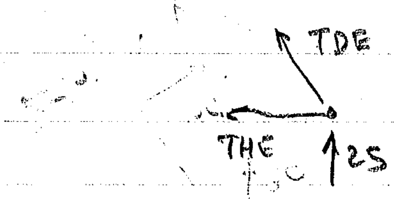
$- 10(60) - \cancel{10(15)} + Ry(80) = 0$

$$\frac{+200 + 600 + 600 + 600}{80} = Ry$$

$$\frac{2000}{80} = 25^K = Ry$$

$$10^K = \bar{Ry}$$



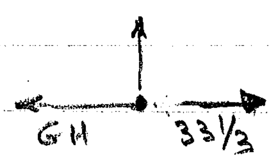


$25 + \frac{3}{5} T_{DE} = 0$

$T_{DE} = \frac{-5(25)}{3} = 41\frac{2}{3}$

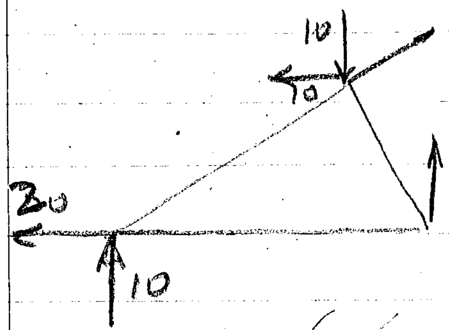
$- HE - \left(-\frac{4}{5}\left(\frac{5(25)}{3}\right)\right) = 0$

$+ 33\frac{1}{3} K$



$GH = 33\frac{1}{3}$ in ~~compression~~ tension

$$\boxed{\sigma_{GH} = \frac{33.3^K}{2.5 \text{ in}^2} = 13.34 \text{ KSI} \quad (T)}$$



$$\cancel{10} - \cancel{10} + GC + \tfrac{3}{5} BC = 0$$

$$-30 + \tfrac{4}{5} GC = 0 \qquad GC = \tfrac{5}{4}(30) = \tfrac{150}{4} = 37\tfrac{1}{2}$$

$$GC = 37.5\,K$$

$$BC = -37.5\left(\frac{5}{3}\right) = -(12.5)5 = -62.5\,K$$

$$\boxed{\begin{array}{l} \sigma_{GC} = \dfrac{37\frac{1}{2}}{2.5\,in^2} = 15\ KSI \quad (T) \\[2em] \sigma_{BC} = \dfrac{62.5}{2.5} = 25\ KSI \quad (C) \end{array}}$$

$$\overset{12.5}{37.5}\left(\frac{5}{\cancel{3}}\right) =$$

$\frac{1}{4}\left(\frac{1}{12}\right) = \frac{1}{48}$

$\sigma_x = \frac{P_x}{12 \times 1/48} \qquad \sigma_y = \frac{P_y}{8 \times 1/48}$

$E = \frac{\sigma}{\epsilon} = \frac{4P_x}{\epsilon} \; K/ft^2$

$144 \text{ KSI} = \text{KSF}$

$\frac{30 \times 10^3 \text{ KSI}}{144 \text{ KSI}/\text{KSF}} = \frac{300}{144} \times 10^5 = 2.083 \times 10^5 \text{ KSF}$

$2.083 \times 10^2 \text{ KSF} = \frac{4P_x}{\epsilon_x}$

$\frac{2.083 \times 10^2}{4} \epsilon_x$

$\epsilon_x = \frac{\Delta x}{L_x} = \frac{.0768}{12.8^F} = .0008 = 8 \times 10^{-4}$ (.0096)

$\frac{2.083 \times 10^2 (8 \times 10^{-4})}{4} = 4.163 \times 10^{-2}$

$24 - 2.535$

$\frac{24}{4} \frac{\cancel{4}}{\cancel{24}} (21.465)$

$\frac{\cancel{24}(21.5)}{\cancel{4.8}}$

$24 - \frac{P_x}{36} = \frac{P_y}{4(24)}$

$24 - \frac{28.5(576)}{15(36)}$

-30.4

$24 - 30.4$

$\frac{24(-6.4)}{4}$

$-\frac{6.4(24)}{4}$

$\frac{1}{4}\left[.0006(30 \times 10^3)\right] = \frac{(-P_x)}{4(144)} + \frac{P_y}{4(24)}$

$.0008(30 \times 10^3) = \frac{P_x}{36} - \frac{P_y}{4(24)}$

$(.0008 + .00015) = \frac{-P_x}{576} + \frac{P_x}{16(36)}$

$(.00095)(30 \times 10^3)$

$28.5 = \frac{-P_x}{576} + \frac{16P_x}{576}$

36
16
360
216

$P_x = \frac{28.5(576)}{15(12)}$

$$\sigma_y = \sigma_z - 2\mu\epsilon_z + 2\mu\epsilon_y$$

$$\sigma_y = (\epsilon_y - \epsilon_z)\left[(1+\nu)/E\right) + \sigma_z$$

$$\frac{E}{(1+\nu)}(\epsilon_y - \epsilon_z) = 2\mu(\epsilon_y - \epsilon_z) \quad QED$$

$$G = \frac{E}{2+2\nu}$$

$$\cancel{2}+\cancel{2}\nu = \frac{E}{2G}$$

$$\nu = \frac{E}{2G} - 1$$

$$\frac{30}{24} - 1 = \nu = 1/4$$

$$\epsilon_x = \frac{\Delta x}{L_x} \qquad \sigma_x = \frac{P_x}{12 \times 1/48} = \cancel{4}\sigma_x \cancel{=} P_x$$

$$\epsilon_y = \frac{\Delta y}{L_y} \qquad \frac{\cancel{.0768}}{\cancel{8(12)}}\ .0096 = .0008 \qquad \sigma_y = \frac{P_y}{8 \times 1/48}$$

$$8 \times 10^{-4} = \epsilon_x$$

$$\frac{.0864}{12(12)} = \frac{.0072}{12} = .0006 = 6\times10^{-4} = \epsilon_y$$

$$6\times10^{-4} = \epsilon_y = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E} = \frac{-\nu 4P_x}{E\ \cancel{4}} + \frac{6P_y}{E}$$

$$8\times10^{-4} = \epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y = \frac{4P_x}{E} - \frac{\nu}{E}6P_y$$

$$E(6\times10^{-4}) = -P_x + 6P_y$$

$$E(8\times10^{-4}) = 4P_x - 6/4 P_y$$

$$180 \times 10^2 \quad -P_x + 6P_y$$

$$4(240\times10^2) = 16P_x - 6P_y$$

$$15 \overline{)1140}^{\,76}$$
$$1050$$
$$90$$
$$90$$

$$\frac{1140}{15}$$

$$180\times10^2 = -90 + 6P_y$$

$$6P_y = 270$$

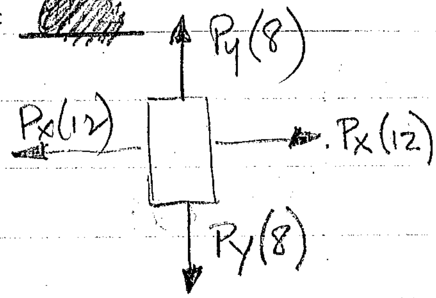
$$P_y = 45$$

$$15P_x = 1140\times10^2$$

$$P_x =$$

$$p_x = 91.2\,(12) = \frac{\frac{30\times10^3\ \text{KSI}}{144\ \frac{\text{KSI}}{\text{KSF}}}}{}$$

$\sigma_x =$



$$\frac{P_x(12)\ \text{FT.}}{12 \times 1/48} \qquad 48\,P_x = \sigma_{x\,\text{KSF}} \qquad \frac{P_y(8)}{8\times 1/48} = 48P_y = \sigma_y$$

K/FT.

48 K/F²

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y \qquad \frac{30\times10^3(.0008)}{144(48)} \qquad E = \frac{\sigma_x}{\epsilon_x} = \qquad \frac{E\epsilon_x}{48} = \sigma_x$$

$$E\epsilon_x = \sigma_x - \nu\sigma_y$$

$$\frac{30\times10^3}{144}\epsilon_y = -12P_x + 48P_y$$

$$\frac{30\times10^3\ \text{KSI}}{144\ \frac{\text{KSI}}{\text{KSF}}}\left[\epsilon_x\right] = 48P_x - \frac{1}{4}(48)P_y$$

$$\frac{30\times10^3}{144}(.0008) = 48P_x - 12P_y \qquad \frac{(18)}{144} = -48P_x + 192P_y$$

$$\frac{24}{144} = 48P_x - 12P_y \qquad \frac{24}{144} = 48P_x - 12P_y$$

$$\frac{4(18)+24}{144} = 180P_y$$

$$E = 30\times10^6\ \#/\text{in}^2$$

$$91.2(12) = P_x \qquad \sigma_x = \frac{P_x}{A_x} = \frac{91.2(12)}{12\times1/48}$$

$$76.8\,(8) = P_y \qquad \sigma_y = \frac{P_y}{A_y} = \frac{76.8(8)}{8\times 1/48}$$

$$\epsilon_x = \frac{192(91.2)\,144}{4(30\times10^3)} - \frac{48(76.8)\,144}{120\times10^3}$$

$$\sigma_{x\,\text{KSF}} = 48(91.2) \qquad \sigma_{y\,\text{KSF}} = 48(76.8)$$

$$\frac{144(48)}{120\times10^3}\left(4(91.2) - (76.8)\right)$$

$$364.8 - 76.8 = 288.$$

$$\frac{144\,(2)(288)}{5\times10^3} \qquad \frac{288(288)}{5000}$$

$$\frac{\epsilon_z(\nu/E)\left(\frac{1+\nu}{E}\right)}{(\nu/E)\left(\frac{1+\nu}{E}\right)} + \frac{\nu/E}{\left(\frac{1+\nu}{E}\right)}$$

$\frac{\nu}{1+\nu}$ $\left[\epsilon_z\left(\frac{1+\nu}{\nu}\right) + \epsilon_x + \epsilon_y - \epsilon_z\right]$ $(\epsilon_y - \epsilon_z)\left(\frac{1+\nu}{E}\right)$

$$\epsilon_z\left(\frac{1+\nu}{\cancel{\nu}}\right) \frac{- \epsilon_z\nu + \epsilon_x\nu + \epsilon_y\nu}{\nu}$$

$\epsilon_z +$

$$\sigma_z = \frac{\nu}{1+\nu}\left[\frac{\epsilon_z}{\nu} + \epsilon_x + \epsilon_y\right]\frac{E}{(1-2\nu)}$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

$$\sigma_y = (\epsilon_y - \epsilon_z)\left(\frac{1+\nu}{E}\right)$$

$$\sigma_z = \lambda\left[\frac{\epsilon_z}{\nu} + \epsilon_x + \epsilon_y\right]$$

$$\lambda\epsilon_z + \sigma_z = \lambda e + \frac{\lambda\epsilon_z}{\nu} = \lambda(\epsilon_x + \epsilon_y + \epsilon_z)$$

$$\sigma_z = \lambda e + \lambda\epsilon_z\left(\frac{1-\nu}{\nu}\right) = \lambda e + \frac{\lambda\epsilon_z}{\nu} - \frac{\lambda\epsilon_z\nu}{\nu}$$

$$\frac{E}{1+\nu} = 2G$$

$$\frac{E\cancel{\nu}}{(1+\nu)(1-2\nu)}\frac{1-\nu}{\cancel{\nu}}$$

$$\lambda\epsilon_z\left(\frac{1-\nu}{\nu}\right)$$

$$\frac{E\cancel{\nu}}{(1+\nu)(1-2\nu)}\left(\frac{1-2\nu}{\cancel{\nu}}\right)\epsilon_z$$

$$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

$$2G\frac{(1-\nu)}{(1-2\nu)}$$