Fluid Mechanics - basic study of fluids, forces in fluids and motion of fluids

Continuum - where the characteristic length of problem >> mean free path of molecules (distance that a molecule travels before encountering another molecule.) - we need only look at a conglomerate of molecules

Eulerian Coordinates - look at specific point in space and describe what happens as particles move through that point.

Lagrangian - usually wrt a fixed coordinate system. Describe properties of particles as particles move through space.

Pathline - locus of points of the path of a particle

Streamline - locus of tangents to the instantaneous velocity vector.

Streakline - locus of all particles that pass through particular point

Conservation of mass: $\frac{\partial}{\partial t}\int_{\mho} \rho \, d\mho + \int_A \rho \underset{\sim}{V}_{rel} \cdot \underset{\sim}{n} \, dA = 0$

Momentum: $\frac{\partial}{\partial t}\int_{\mho} \rho \underset{\sim}{V} d\mho + \int_A \underset{\sim}{V} (\rho \underset{\sim}{V}_{rel} \cdot \underset{\sim}{n} \, dA) = -\int_A p \bar{n} \, dA + \int_{\mho} \rho \underset{\sim}{F}_b \, d\mho + \int_A \text{friction forces} \, dA$

Equation of state $p = p(\rho, T)$ gibbs: basic $T ds = dh - v dp = du + p dv$; isentropic $P/\rho^k = \text{const}$

Energy: $\frac{\partial}{\partial t}\int_{\mho} \rho e \, d\mho + \int_A e(\rho \underset{\sim}{V}_{rel} \cdot \underset{\sim}{n}) dA = -\int_A p \underset{\sim}{n} \cdot \underset{\sim}{V} \, dA + \int_{\mho} \rho \underset{\sim}{F}_b \cdot \underset{\sim}{V} \, d\mho + \int_{\mho} \dot{Q} \, d\mho + \int_A \text{frict} \cdot \underset{\sim}{V} \, dA$

($\dot{Q}$: rate at which heat is added)

Note all the above are written for fixed mass

Differential form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \underset{\sim}{V} = 0$$

$$\frac{\partial \rho \underset{\sim}{V}}{\partial t} + \nabla \cdot (\rho \underset{\sim}{V}\,\underset{\sim}{V}) = -\nabla p + \rho \underset{\sim}{F}_b + \nabla \cdot (\text{friction terms} = \mu \nabla \underset{\sim}{V})$$

only if Stokes Hypothesis is correct

$$\nabla \cdot (\rho \underset{\sim}{V}\,\underset{\sim}{V}) = \underset{\sim}{V} \cdot \nabla (\rho \underset{\sim}{V}) + \rho \underset{\sim}{V}\, \nabla \cdot \underset{\sim}{V}$$

frictional effects

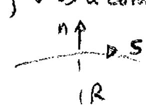
$$\frac{\partial}{\partial t} \rho e + \nabla \cdot (\rho \underset{\sim}{V}_{rel}\, e) = -(\nabla p)\underset{\sim}{V} + \rho \underset{\sim}{F}_b \cdot \underset{\sim}{V} + \dot{Q} + (\mu \underset{\sim}{V} \nabla \underset{\sim}{V})$$

$$e = \underset{IE}{u} + \underset{KE}{V^2/2}$$

for fixed control volume $\frac{\partial}{\partial t} d\mho = 0$ & $q_{rel} = q_{vo}$

Euler Eqs $\underset{\sim}{n}$ is positive outward, s is distance along streamline, R is local radius of curvature, U is a conservative bodyforce; no friction;

$\therefore$ Mom: $\frac{\partial V_s}{\partial t} + V\frac{\partial V}{\partial s} = -\frac{1}{\rho}\frac{\partial p}{\partial s} + \frac{\partial U}{\partial s}$; $\frac{\partial V_n}{\partial t} - \frac{V^2}{R} = -\frac{1}{\rho}\frac{\partial p}{\partial n} + \frac{\partial U}{\partial n}$; $V^2 = V_s^2 + V_n^2$



$U = -gz$

not necessary to assume 2-D flow

for integration along stream line $\int \frac{\partial V_s}{\partial t} + \frac{1}{2}V^2 + \int dp/\rho - U = B(t)$ if ρ=const $\Rightarrow \int \frac{\partial V_s}{\partial t} ds + \frac{V^2}{2} + P/\rho - U = B(t)$

* for steady flow B is constant along a streamline; for constant along all streamlines $\nabla B = 0$

but $-\frac{\partial \underline{v}}{\partial t} + \underline{v} \times (\nabla \times \underline{v}) = \nabla\left(\frac{V^2}{2} + P/\rho - U\right) = 0 \Rightarrow \frac{\partial \underline{v}}{\partial t} = 0$ and $\underline{v} \times \underline{\omega} = 0$ or $\underline{\omega} = 0$

steady state; $\underline{v} \parallel \underline{\omega}$ Beltrami flows; irrotational flow

* Another way for 2-D case to look at it is take $\frac{\partial}{\partial n}(\text{bernoulli}) = \frac{\partial B}{\partial n} = 0$ everywhere; use Euler n eq to get that $\frac{\partial V}{\partial n} + \frac{V}{R} = 0$ is condition necessary for B = const everywhere but $\frac{\partial V}{\partial n} + \frac{V}{R} = \omega_z = -2\Omega_z$ (vorticity) ~ rate of rotation = irrot flow

* Note in flows $\frac{\partial P}{\partial S} < 0$ favorable pressure grads retard separation; p in subsonic flows is really p_{gage}. (ρ=const) (irrot.)

Aside: Potential flow $\nabla \cdot \underline{v} = 0$, $\nabla \times \underline{v} = 0$ give enough equations to get $\underline{v}$ if we relax no slip condition

Vorticity exists where there are non conservative forces, 3D flow, differential heating

$\nabla \times$ eq of motion: $\frac{D\omega}{Dt} = \bar{\omega}(\nabla \cdot \underline{v}) + \nabla p \times \nabla\left(\frac{1}{\rho}\right) + \nabla \times [f_{NC} + f_\mu] + \underbrace{\bar{\omega} \cdot (\nabla \underline{v})}_{0 \text{ in 2D flow}}$ $\bar{\omega} = \omega_z$ in 2d flows $\nabla \underline{v} = f(x,y)$ only $\therefore \bar{\omega} \perp \nabla \underline{v}$

INVISCID STEADY FLOWS $\partial P/\partial t = 0$

Define stream fn. to satisfy continuity $\psi_{,y} = \rho u$ $\psi_{,x} = -\rho v$ no flow across streamlines

* also $Q = VL$ (Area/unit depth) $= \psi \Big|_1^2$. Since must also satisfy irrotational flow therefore ρ=const $\nabla^2\psi = 0$ & ψ must be 2-D plane flow.

* Define potential fn. $\ni$. $\nabla \times (\nabla\phi) = \nabla \times \underline{v} = \underline{\omega} = 0$ ie $\underline{v} = \nabla\phi$ $\phi(\underline{x},t)$ is scalar $\psi(\underline{x},t)$ is scalar

defn ϕ: makes no restriction on ρ or $\frac{\partial}{\partial t}$ of ρ, u, v, ω, p etc. thus $\nabla \times \nabla\phi = 0$ satisfies vorticity but to satisfy continuity $\nabla^2\phi = 0$! ϕ lines are $\perp$ to ψ lines $\nabla\psi \cdot \nabla\phi = (-v,u)\cdot(u,v) = 0$ $\nabla\psi \times \nabla\phi = -v^2 - u^2 = -|V|^2$

B.C. Dirichlet BC. ψ or ϕ is specified on surface soln is unique max/min on bdy.

Neumann $\frac{\partial\psi}{\partial n}$ or $\frac{\partial\phi}{\partial n}$ " " " soln is unique to a constant

Methods of Soln for 2D problems

Analytic - restricted to simple problems

Graphing

Numerical

* Complex fn - conformal mapping $\Phi = \phi + i\psi$ $\frac{d\Phi}{dz} = \bar{\omega} = u - iv$

Potential flow soln $\Phi = U_\infty z$ parallel flow

$\Phi = \frac{Q}{2\pi}\ln z$ source or sink located at origin [$\ln(z - z_0)$ loc at $z = z_0$

* limit of $\frac{Q}{2\pi}\ln(z-a) - \frac{Q}{2\pi}\ln(z+a)$ as $a \to 0 \ni \frac{Qa}{\pi} = S$ = const

$\Phi = \frac{-i\Gamma}{2\pi}\ln z$ point vortex $\Gamma > 0$ ccw :

$\Phi = S/z$ doublet at origin

$\Phi = \frac{A}{n}z^n$ is is pot for corner or wedge flow $n = \pi/\alpha$ α is angle of wedge.

$\Phi = U_\infty z + \frac{Q}{2\pi}\ln z$ defines flow over a half body [ie nose of pitot probe parallel + source + sink $\quad \Phi = U_\infty z + \frac{Q}{2\pi} \ln(z+z_0) - \frac{Q}{2\pi} \ln(z-z_0)$ $\quad$ rankine oval

parallel + doublet $\quad \Phi = U_\infty z + \frac{s}{z}$ $\quad$ flow over cylinder

$\rightarrow$ Drag = 0 $\quad$ D'Alembert Paradox

parallel + doublet + vortex $\quad \Phi = U_\infty z + \frac{s}{z} + \frac{i\Gamma}{2\pi} \ln z$ $\quad$ flow over cylinder w/ lift (define Γ so that rear point is a stag point - Joukowsky law $\quad$ Lift $= -\rho U_\infty \Gamma$)

$$\Gamma = \oint_c \underline{V} \cdot d\underline{s} = \oiint_A (\nabla \times \underline{V}) \cdot d\underline{A} = \iint_A \underline{\omega} \cdot d\underline{A} \qquad \underline{\omega} = 0 \Rightarrow \Gamma = 0$$

* define flow over cambered airfoil by N bound vortices + $U_\infty z + \frac{s}{z}$ $\quad$ this gives better soln than simple vortex

For flat plate : N vortices + $U_\infty z$

Similitude I Buckingham π theorem.

1. List all parameters involved
2. List dimension of all param in terms of M, L, t (primary dimensions)
3. Select from parameters a list of param = to # of primary dimensions and including all primary dimensions
4. Set up n - m dimensional eqs w/ remaining params.

Example : drag over a sphere

$F = f(\rho, V, D, \mu)$

1. $F \quad \rho \quad V \quad D \quad \mu$ $\quad$ span
2. $F = \frac{ML}{t^2} \quad \rho = \frac{M}{L^3} \quad V = \frac{L}{t} \quad D = L \quad \mu = \frac{M}{Lt}$

   3 primary dimensions
3. Select ρ, V, D
4. $\Pi_1 = \rho^a V^b D^c F = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b (L)^c \frac{ML}{t^2} = M^0 L^0 t^0$

   $\Rightarrow a = -1,\ c = -2,\ b = -2$

   $\Pi_1 = \frac{F}{\rho V^2 D^2}$

   $\Pi_2 = \rho^d V^f D^g \mu = \left(\frac{M}{L^3}\right)^d \left(\frac{L}{t}\right)^f (L)^g \frac{M}{Lt} = M^0 L^0 t^0$

   $\Rightarrow d = -1,\ e = -1,\ f = -1 \qquad \Pi_2 = \frac{\mu}{\rho V D} = \frac{1}{Re}$

   $\frac{F}{\rho V^2 D^2} = f\left(\frac{1}{Re}\right)$

II normalizing of eqns let $u^* = u/u_0$, $T = T^*/T_i$, $P_0/\rho u_0^2 = p^*$ $\quad x/l = x^*$ $\quad t^* = \frac{t u_0}{l_0}$

→ Froude no. important in water waves $\frac{\text{inertia}}{\text{gravity}}$ ratio

→ R_N no. important for b.l. = $\frac{\text{inertia}}{\text{viscous}}$ ratio

Navier Stokes Eqns

Includes the viscous effects $\Rightarrow$ $\rho \frac{D\underline{v}}{Dt} = -\nabla\bar{p} + \nabla\cdot\sigma + \rho \underset{\sim}{f}_b$ $\bar{p}$ is mean pressure

assume fluid continuum, $\sigma_{ij} \propto \dot{\epsilon}_{ij}$, fluid isotropic $\Rightarrow$ $\sigma_{ij} = 2\mu\dot{\epsilon}_{ij} + \lambda\delta_{ij}\dot{\epsilon}_{kk}$

mean press $\approx$ thermodynamic pressure p and $\sigma_{ij_{TOT}} = \sigma_{ij_{visc}} - p\delta_{ij}$

Thus without the stokes hypothesis $(\lambda + \frac{2}{3}\mu = 0)$

N.S. Eqn $\quad \rho\frac{D\underline{v}}{Dt} = -\nabla p + \nabla\cdot\left\{\mu\left[\nabla\underline{v} + \underline{v}\nabla\right] + (\lambda\nabla\cdot\underline{v})\right\} + \rho \underset{\sim}{f}_b$ or

$$\rho\frac{Du_i}{Dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left\{\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right\} + \frac{\partial}{\partial x_j}\left(\lambda\frac{\partial u_i}{\partial x_i}\right) + \rho f_i \qquad (1)$$

total change of energy of syste.

Conservation of energy becomes $\frac{\partial(\rho e)}{\partial t} + \frac{\partial(\rho u_j e)}{\partial x_j} = \frac{\partial q_j}{\partial x_j} + \left[\frac{\partial(u_i[\sigma_{ij} - p\delta_{ij}])}{\partial x_j} + \hat{f}_i u_i\right]$

storage + energy flux out = Heat flow in to volume. + inflow of mech power to the fluid

$e = u + V^2/2$, the $g z$ term is the f_i

or simply $\quad \rho\frac{Du}{Dt} = -\nabla\cdot\underset{\sim}{q} - p\nabla\cdot\underline{v} + (\sigma:\nabla)\underline{v}$ or $\rho\frac{Du}{Dt} = -\frac{\partial q_j}{\partial x_j} + \left(-p\frac{\partial v_i}{\partial x_i}\right) + \sigma_{ij}\frac{\partial v_i}{\partial x_j}$

rate of heat transfer into particle; dissipative term

$\rightarrow$ or $\rho\frac{Dh}{Dt} = \frac{Dp}{Dt} - \frac{\partial q_j}{\partial x_j} + \sigma_{ij}\frac{\partial v_i}{\partial x_j}$ Φ dissipative fn. find out about

note $q_j = -k\frac{\partial T}{\partial x_j}$

$\rightarrow$ 2nd law of thermo $ds > 0$ or $\frac{\partial}{\partial t}(\rho s) + \frac{\partial(\rho u_j s)}{\partial x_j} = -\frac{\partial}{\partial x_j}\left(q_j/T\right) + \mathcal{P}$ (irreversible production) ≥ 0

get this

for ρ = const, μ = const $\frac{\partial u_i}{\partial x_i} = 0$ (cont.) $\Rightarrow$ NS $\rho\frac{D\underline{v}}{Dt} = -\nabla p + \mu\nabla^2\underline{v} + \underline{f}_b$ this includes stokes hypothesis

also $\rho\frac{De}{Dt} = -\frac{\partial q_j}{\partial x_j} - \frac{\partial u_j p}{\partial x_j} + \frac{\partial(u_i\sigma_{ij})}{\partial x_j} + \rho f_i u_i$ becomes $\rho\frac{De}{Dt} = k\nabla^2 T - p\nabla\cdot\underline{v} - (\underline{v}\cdot\nabla)p + \rho\underline{f}\cdot\underline{v}$

To solve problem start with (1) + continuity + eqns of state + b.c. [no slip, given T_{wall} or $(q_1)_{wall}$] + IC

Exact Solns for constant properties   cont $\Rightarrow \nabla\cdot\underline{v} = 0$, NS: $\frac{D\underline{v}}{Dt} = \underline{f}_b - \frac{1}{\rho}\nabla p + \nu\nabla^2\underline{v}$

non dimensionalize $\hat{x} = x/L$, $\hat{t} = \frac{tV}{L}$, $\hat{p} = \frac{p}{\rho V^2}$, $\hat{u}_i = \frac{u_i}{V}$ for flow over body (char length L, velocity V)

this gives $\quad \frac{\partial\hat{u}_i}{\partial\hat{t}} + \hat{u}_j\frac{\partial\hat{u}_i}{\partial\hat{x}_j} = -\frac{\partial\hat{p}}{\partial\hat{x}_i} + \frac{1}{R}\frac{\partial^2\hat{u}_i}{\partial\hat{x}_j\partial\hat{x}_j}$ and $\frac{\partial\hat{u}_j}{\partial\hat{x}_j} = 0$

$R \to \infty \Rightarrow$ we need bl eqns; we can't sat. noslip

$R \to 0 \Rightarrow$ creeping flows $\nabla^4\psi = 0$

we need to keep highest order terms & apply no slip bc

Convective Accel Terms = 0 : Couette flows (linear profile), Fully Developed flows (Poiseuille) (Parabolic Profile); Semi infinite flows (stokes problems)

$\partial(\text{veloc.})/\partial x = 0$ for Fully Developed flows + no suction $\quad \frac{\partial u}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x} = \nu\frac{\partial^2 u}{\partial y^2}$, $\partial p/\partial y = 0$ $\quad \frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial y^2}$

$p \neq p(x)$ in all infinite flows

in infinite flows we can use similarity if no char length exist

Creeping flow $\nabla p = \mu \nabla^2 \underline{v}$; $\nabla\cdot\underline{v}=0$ viscous effects >> inertial effects

take $\nabla\cdot(\nabla p) = \mu \nabla\cdot(\nabla^2 \underline{v}) = \mu \nabla^2(\nabla\cdot\underline{v}) = 0$ or $\nabla^2 p = 0$ => if $\nabla\cdot\underline{v} = \nabla^2\psi$ $\nabla^4\psi = 0$ in 2D : non dimensionalize $\hat{p} = \frac{PL}{\mu V}$

Stokes flow, Oseen improvement attempts to treat fact that no wake by adding linearized inertia terms.

LAMINAR BL Steady 2-D, ρ=const, μ=const.

- Prandtl BL Eqn. non dim x, y wrt char L, δ u, v wrt U_∞, p wrt ρU_∞^2 $\delta \ll L$

Keep highest order terms : use continuity to give order of v use this in mom eqns to get $\frac{\partial p}{\partial y} = 0$; use inviscid flow bernoulli eqn to get order of $\frac{\partial p}{\partial x}$: This leads to $\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{1}{R_N}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}$ where $R_N \sim O(\frac{1}{\delta^2})$

since $p_{,y} = 0$ $\frac{\partial p}{\partial x} = \frac{dp}{dx}$ $\frac{dp}{dx} = -\rho u_\infty \frac{du_\infty}{dx}$ or $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$ and $\nabla\cdot\underline{v} = 0$

TURB BL eddy viscosity >> μ replace $\mu\nabla^2\underline{v}$ by $(\nabla\cdot\sigma)$

INTEGRAL eqns - solutions in the mean gives overall effect. Integrate BL eqn over y.

define displacement thickness $\delta^* = \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dy$ mom thickness $\theta = \int_0^\delta \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy$

this gives $\frac{d}{dx}(u_\infty^2 \theta) + \delta^* u_\infty \frac{du_\infty}{dx} = \frac{\tau_w}{\rho} + v_w(u_e - u_w)$ if suction at wall w/ u_w, v_w

δ^* represents the displacement of streamlines due to shear layer existing ; θ represents momentum lost in the shear layer compared to potential flow.

flat plate soln

for $dp/dx = 0$ => $\frac{du_\infty}{dx} = 0$ we can integrate the Karman MI equation. Polhausen solns let $\frac{u}{u_\infty} = f(y/\delta)$

plug into eqns and get soln for τ_w as fn of δ (which is fn of R_N) w/ $\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$

for $dp/dx \neq 0$ Polhausen defines a pressure parameter $\Lambda = \frac{\delta^2}{\nu}\frac{du_\infty}{dx}$ solutions obtained good for $|\Lambda| \leq 12$ for 4th order b.l.

b.c. for the integral eqn $\left.\frac{u}{u_\infty}\right)_w = 0$ $\left.\frac{u}{u_\infty}\right)_\delta = 1$ $\left.\frac{\partial u}{\partial y}\right)_\delta = 0$, $\left.\frac{\partial^2 u}{\partial y^2}\right)_\delta = 0, \ldots$ thus order of $f(y/\delta)$ define how many of these to use

better solutions are obtained $\frac{u}{u_\infty} = f(y/\delta, \Pi_1, \Pi_2, \ldots)$ where Π_i are other paramaters. Assume $u/u_\infty = f(y/\delta, \Pi_1)$

- if $\Pi_1 = \frac{\delta^2}{\nu}\frac{du_\infty}{dx}$ & 4th order polynomial we have pohlhausen, if $\Pi_1 = \frac{\theta^2 u_e'}{\nu} = \lambda$ we have Thwaite soln (give better results)
- if Π_1 = const then we have Faulkner Skan solution ($u_e = cx^m$)

Thwaite results depends on u_e not derivatives as Pohlhausen (u is easier to measure)

Thwaite method; we can write MI eqn for $\frac{d\theta}{dx} = \frac{\tau_w}{\rho u_e^2} - (2+H)\frac{\theta u_e'}{u_e}$ $H = \delta^*/\theta$, do not assume profile but correlated data of $\frac{\tau_w \theta}{\mu u_e} = \ell$ vs. λ and $H = \delta^*/\theta$ vs λ. Mult MI eq by $\frac{\theta u_e}{\nu}$ to get $u_e\frac{d\theta^2}{dx} = 2\left[\ell(\lambda) + (2+H)\lambda\right] = F(\lambda)$

data correlation gave $F = .45 - 6\lambda$. Mult both sides by u_e^5

Thus $\frac{u_e^6}{\nu}\frac{d\theta^2}{dx} = u_e^5\left(.45 - 6\frac{\theta^2}{\nu}\frac{du_e}{dx}\right)$ => $\frac{1}{\nu}\frac{d}{dx}(\theta^2 u_e^6) = .45 u_e^5$ => $\frac{\theta^2 u_e^6}{\nu} = .45\int_0^x u_e^5 dx + \left(\frac{\theta^2 u_e^6}{\nu}\right)_{x=0}$

if we assume $u_e = u_e(x)$ we can get λ as a fn of x.

Separation occurs in all cases when $\tau_w = 0$ => $\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0$

Transition from laminar to turb occurs around $Re \sim 5\times10^5$ to 10^6

Thwaite's method gives better results than Pohlhausen. For adverse pressure gradients we know that

$u/u_\infty = f(y/\delta, \Pi_1)$ is not a good parametriz but must go to $f(y/\delta, \Pi_1, \Pi_2)$ at least.

use empirical result from pipe flow w/ R_n^{PU} for τ_0, the MI eqn & $u/u_e = f(y/\delta)^{1/7}$

Turbulent layer made up of laminar sublayer $\{u/u_e \sim (y/\delta)^{1/7}\}$ & a buffer layer + outer layer

ie $(\,) = \overline{(\,)} + (\,)'$

Stability theory Assume superposition on mean flows $(\bar p, \bar u, \bar v)$ fluctuations (u', v', p'). Put into NS and keep only 1st order terms and use B.L. approximations. Subtract the mean eqns to get

$$(1)\quad u'_{,t} + \bar u \frac{\partial u'}{\partial x} + \bar v \frac{\partial u'}{\partial y} + v' \frac{\partial \bar u}{\partial y} = -\frac{1}{\rho}\frac{\partial p'}{\partial x} + \nu \nabla^2 u' \qquad + \qquad \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0$$

$$(2)\quad v'_{,t} + \bar u \frac{\partial v'}{\partial x} + \bar v \frac{\partial v'}{\partial y} + v' \frac{\partial \bar v}{\partial y} = -\frac{1}{\rho}\frac{\partial p'}{\partial y} + \nu \nabla^2 v'$$

this decouples disturbance eqns & mean eqns.

take $\frac{\partial}{\partial x}(2) - \frac{\partial}{\partial y}(1)$ and define $u' = \frac{\partial \psi}{\partial y}$ $v' = -\frac{\partial \psi}{\partial x}$ to satisfy then

$$(1)\quad \left[\frac{\partial}{\partial t} + \bar u \frac{\partial}{\partial x}\right]\nabla^2\psi - \bar u'' \frac{\partial \psi}{\partial x} = \nu \nabla^4 \psi$$

after using parallel flow approx (when disturbances in x dir are small) $\bar v = 0$, $\bar u(y)$ only, $\frac{\partial \bar p}{\partial y} = 0$

if we assume $\psi = \phi(y) e^{i(\alpha x - \omega t)}$ and look at real part of soln where $\omega = \omega_r + i\omega_i$ $c = \frac{\omega}{\alpha}$ $\phi = \phi_r + i\phi_i$

then stability is dependent only on sign of ω_i $\alpha = 2\pi/\lambda$ λ = wave length

put ψ into DE (1) to get Orr-Sommerfeld eqn.

$$(\bar u - c)(\phi'' - \alpha^2\phi) - \bar u''\phi = -\frac{i\nu}{\alpha}\left[\phi^{IV} - 2\alpha^2\phi'' + \alpha^4\phi\right] \qquad \text{with } \begin{array}{ll} y=0 & \phi=\phi'=0 \\ y=\infty & \phi=\phi'=0 \end{array}$$

for instability (from the EV problem) given $\bar u(y)$, α, ν $\omega_i > 0$ $c_i > 0$ where $c = c_r + i c_i$

basically $\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0$ $\frac{dp}{dx} > 0$ (adverse pres grad) $\frac{\partial^2 u}{\partial y^2} = 0$ @ some point in bl.

GAS DYN

Everything we talk about is wrt a system: a fixed & identifiable quantity of mass

Flux: flow of an extensive property through the control surface.

Extensive property: property proportional to mass or volume

Define internal energy $u = u(p, T)$ if its a simple substance, one reversible mode of work ($p\,dv$)

one reversible mode: doesn't include effects of EM, surface tension, fluid stresses, motion, gravity or reference state

Eqn of state comes from Dfn of Eqn of State, + 1st & 2nd law of thermo + how many indep variables you assume.

basic eqns are $\underbrace{\text{cont., mom, energy}}_{\text{only conservation eqns}}$, eqn of state (6 eqns) u, v, w, p, T, ρ

for infinitely weak wave. 1-D w/o friction ; this is essentially isentropic

unsteady problem

$v = dv \rightarrow$ $\} \rightarrow c$ $v=0$

$p+dp$ p

$\rho+d\rho$ ρ

observer not w/ wave

transform by Galilean transformation

quasi steady

$c-dv \leftarrow$ $\leftarrow V = c$

$p+dp$ p

$\rho+d\rho$ ρ

obser w/wave

Cont: $c\,d\rho - \rho\,dV = 0$

Mom: $dp = \rho c\,dV$

$\Rightarrow c^2 d\rho = c\rho\,dV = dp$

or $c^2 = \left.\frac{dp}{d\rho}\right)_{isentropic}$

when $\dot q = 0$ (or negligible), process is reversible = isentropic = reversible adiabatic

perfect gas: c_v, c_p are constant and $p = \rho RT$ $\quad c_v = \left.\frac{\partial u}{\partial T}\right)_\rho \quad c_p = \left.\frac{\partial h}{\partial T}\right)_p$

if process is isentropic $\left.\frac{\partial p}{\partial \rho}\right)_s = \frac{kp}{\rho} = kRT = c^2$ for a perfect gas (ideal gas), since $p/\rho^k = \text{const}$

for a non isentropic gas $p/\rho^k = f(s)$ ie $p/p_0 = (\rho/\rho_0)^k\, e^{\left(\frac{s-s_0}{c_v}\right)}$ $\quad \left.\frac{\partial p}{\partial \rho}\right)_s = \frac{\gamma p}{\rho} = c^2 = kRT$

Thermo - don't care how long process takes & only types of energy transport are work & heat

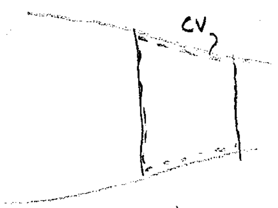
System + eqn of state define energy

define $M^2 = \frac{u^2}{c^2} = \frac{\rho u^2}{\gamma p}\frac{\text{dynamic press}}{\text{Static press}} = \frac{u^2}{\gamma RT} = \frac{KE}{\text{Thermal energy}}$

Simple processes 1) isentropic w/area change w/o heating or friction 2) friction w/o area change heating 3) heating w/o area ch or fricti

For 1-D flows we fix state of substance by 2 thermo variables and 1 motion variables ie (p, T, u)

Isentropic flow: Assume 1-D, steady state, adiab, w/o gravity, EM effects etc. one reversible mode



Cont.: $\rho AV = \text{const}$ $\quad d\rho/\rho + dA/A + dV/V = 0$

Mom: Euler's eqn $\quad dp = -\rho V dV$ $\quad$ use $p/\rho^k = \text{const} \Rightarrow \frac{k}{k-1}\frac{p}{\rho} + \frac{V^2}{2} = \text{const}$

Energy: $h + V^2/2 = \text{const}$ or $dh + V dV = 0$

Eqn of state $h = h(\rho, s)$ $\quad \cancel{Tds} = dh - \frac{dp}{\rho}$ gibbs RELATION

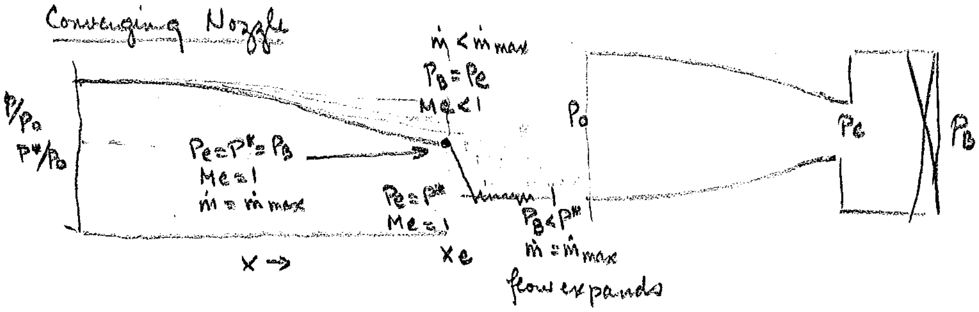
critical mach no = 1

$dA<0$ converg	$M^2<1$	$du>0$	$d\rho<0$	$dp<0$	$dT<0$
	$M^2>1$	$du<0$	$d\rho>0$	$dp>0$	$dT>0$
$dA>0$ diverg	$M^2<1$	$du<0$	$d\rho>0$	$dp>0$	$dT>0$
	$M^2>1$	$du>0$	$d\rho<0$	$dp<0$	$dT<0$

from energy eq $T_0/T = 1 + \frac{k-1}{2}M^2$ using eqn of state & isentropic eqn $p_0/p = \left(\frac{T_0}{T}\right)^{\frac{k}{k-1}}$ $\quad \rho_0/\rho = \left(\frac{T_0}{T}\right)^{\frac{1}{k-1}}$

mass flow rate ↑ as $M \to 1$ at $\underline{M=1}$ $\underline{\dot m = max}$ flow is choked occurs at $\underline{min\ A}$ (denote crit by ( )*)

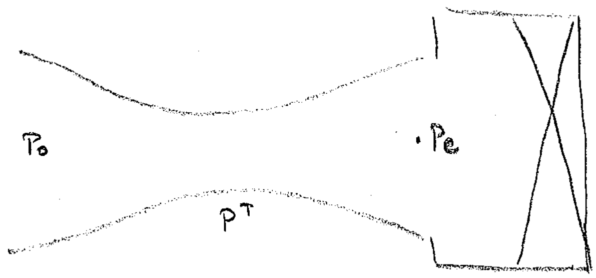
Converging Nozzle

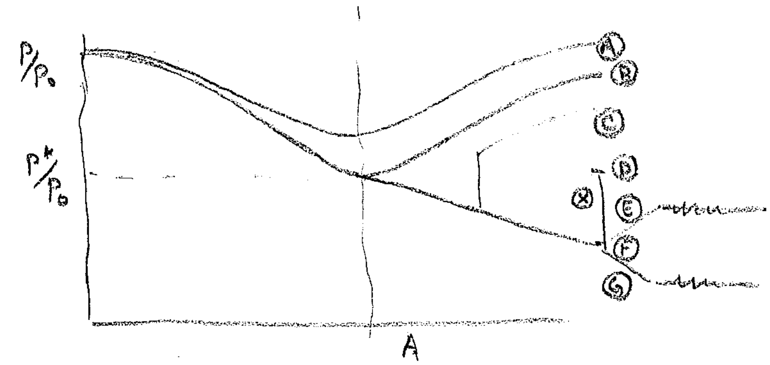


Fliegner formula for air $\frac{w\sqrt{T_0}}{A^* p_0} = .532$

$w = \frac{lbm}{sec}$

p^* is controlling parameter





at A $M_e < 1$ $M_T < 1$ $\dot m < \dot m_{max}$ $p_e = p_B > p^*$

B $M_e < 1$ $M_T = 1$ $\dot m = \dot m_{max}$ $p_e = p_B > p^*$ ↓ flow choked.

C $M_e < 1$ $M_T = 1$ $\dot m = \dot m_{max}$ $p_e = p_B$

D $M_e < 1$ $M_x > 1$, $M_T = 1$ $\dot m = \dot m_{max}$ $p_e = p_B$

E $M_e > 1$ $M_T = 1$ $\dot m = \dot m_{max}$ $p_e = p_{(F)} < p_B$ flow is turned toward centerline underexpands to recover press

F $M_e > 1$ $M_T = 1$ $\dot m = \dot m_{max}$ $p_e = p_B = p_{(F)}$ design conditions

G $M_e > 1$ $M_T = 1$ $\dot m = \dot m_{max}$ $p_e = p_{(F)} > p_B$ flow is turned away from centerline overexpands to recover press contact discont

3 index parameters 4 flow regimes
$p_{shock\ exit}$ $p_{subsonic\ vent}$ $p_{sup\ vent}$

Choked flow when $\dot m$ becomes independent of p_B for given p_0, T_0

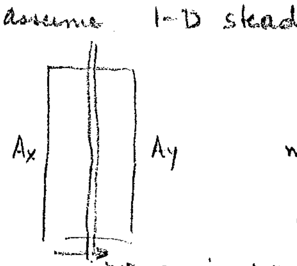
Gas tables link motion of fluid to the state of fluid through governing eqns & process path. { state of substance is fixed by 2 state variables + 1 motion variable (velocity) for 1-D flow

They also provide solutions for similar flows, reduce # of indep variables & generalize results

Methods of irreversibility for pure substance: friction, heat loss over a finite temp. drop.

4th simple process SHOCK - look at control volume. Energy balance & momentum balance tell you nothing about how shock works; 2nd law does. Changes in Δp, ΔT, ΔV etc occur in a very short distance ~ 10^{-8} in - AD/AB_{AT}

We assume 1-D steady state & look at control volume. Assume $A_x = A_y$ and adiabatic ($\dot q = 0$)



cont: $\rho VA = \text{const}$ this is reason that $\dot m$ remains $\dot m_{max}$ in C, D

mom: $p + \rho V^2 = \text{const}$ $p_0 \neq \text{const.}$ | $dp + \dot m dv = 0$

Energy: $h + \frac{V^2}{2} = \text{const.} \Rightarrow T_0 = \text{const}$ $\dot q = 0$, work $= 0$ $dh + VdV = 0$

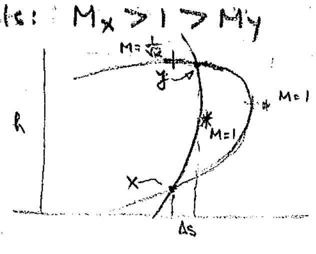
$\tau = \mu \frac{\partial v}{\partial n} = 0$ since 1-D flow

Entropy: $\dot m S_x = \dot m S_y -$ (entropy flow from surroundings) $\therefore S_x < S_y$

Intersection of Fanno / Rayleigh lines give x, y sides of shock

Results: $M_x > 1 > M_y$

$P_{0y} < P_{0x}$, $T_{0y} = T_{0x}$, $P_y > P_x$, $T_y > T_x$



if given

$V=0$ $\xleftarrow{c}$ | p_y, T_y, M_y, T_{oy}, c_y | $V=V_y$

p_x, T_x, M_x, T_{ox}, c_x

cannot use tables unless you transform the problem

$p_x'=p_x$, $V_x'=c$, $T_x'=T_x$, $M_x'=\frac{c}{c_x}$, $T_{ox}'\neq T_{ox}$ | $\longrightarrow$ $p_y'=p_y$, $V_y'=V_y-c$, $T_y'=T_y$, $M_y=V_y/c_y$, $T_{oy}'\neq T_{oy}$

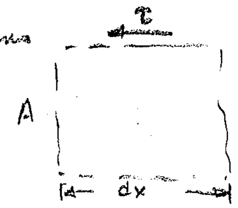
all p, T remain same under transformation since $\neq f(\text{velocity})$

but p_o, T_o change since they depend on V of wave (ie all stagnation conditions change)

__Fanno flows__ flow through constant area duct w/ friction

Look at 1-D flow, friction only at wall, no heating, no body forces. No work done by fluid.

Governing eqns



cont: $\rho VA = \text{const} = \dot m$

Eqn of state: $s=s(T,\rho)$

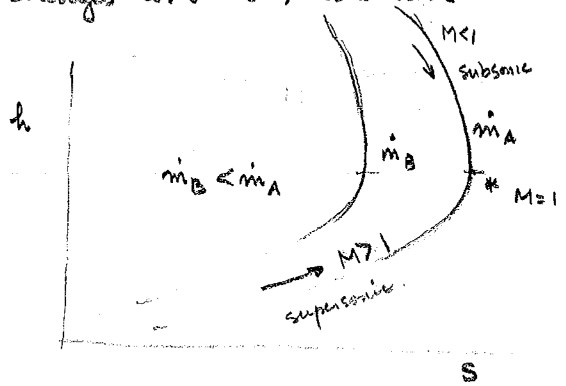
Energy: $h+\frac{V^2}{2}=\text{const}$. $T_o=\text{const}$.

Mom: $-Adp - \tau_w dA_w = \rho VAdV$

define friction factor $4f\frac{dx}{D}$ $D=\frac{4A_{\text{crosssection}}}{\text{perimeter of duct}}$. $f=\frac{\tau_w}{\frac{1}{2}\rho V^2}$

define changes wrt $(\ )^*$ conditions

$\exists$ a length L_{max} .э. when $L=L_{max}$ $M_{L_{max}}=1$. Flow always tends to $M=1$.



if $M_i<1$ & $L<L_{max}$ $\dot m<\dot m_{max_{duct}}$ $M_{exit}<1$ $p_e=p_B$

$L=L_{max}$ $\dot m=\dot m_{max_{duct}}$ $M_{exit}=1$ $p_e=p_B=p^*$

$L>L_{max}$ flow will choke and the flow will adjust .э. $M_{exit}=1$. Adjusts by changing $\overbrace{\ }^{\text{initial conditions}}$ $\Rightarrow$ $\dot m$ will decrease and will jump from one fanno line to another. Flow adjusts outside to reach p_B. $p_e=p^*>p_B$

if $M_i>1$ $L<L_{max}$ $\dot m=\dot m_{max}$ $M_{exit}>1$ $p_{exit}=p_B$

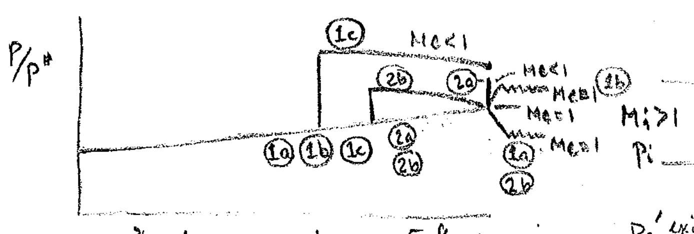
$L=L_{max}$ " $M_{exit}=1$ $p_{exit}=p_B=p^*$

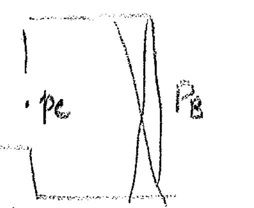
$L>L_{max}$ $\dot m=\dot m_{max}$ $M_{exit}=1$ and shock exists .э. flow adjustment will give $M_{exit}=1$

$p_e=p_B$

we may have conditions p_B .э. a shock stands at exit. Then we get same effect as with deLaval nozzle

Note that for $M_i<1$ $p\downarrow$ to p^*, for $M_i>1$ $p\uparrow$ to p^* : p^* needn't be same as isent flow





3 index parameters 5 flow regimes p_e' exit press for sup flow in nozzle & duct; p_e = pressure behind shock at exit

Construct Fanno line ∴ $\Delta s > 0$ which means we always tend to $M=1$. Also since x condition on shock is lower on h-s diag than y cond then lower branch must be supersonic

For isothermal flow mom, cont same as Fanno (friction flow), energy changes $dQ \neq 0$ critical mach no. $= \frac{1}{\sqrt{k}}$
long ducts are normally isothermal rather than adiabatic. Eqn of state is $p = C\rho$ or $\frac{dp}{p} = \frac{d\rho}{\rho}$

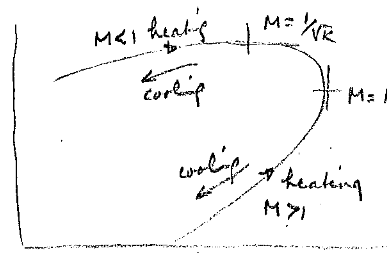
Rayleigh flows - flow with heating/cooling no friction, no area change 1-D, steady state, no work

Energy: $dQ = dh + VdV$

Continuity: $\rho V = \text{const} = G$

Momentum: $p + \rho V^2 = \text{const} = F/A$

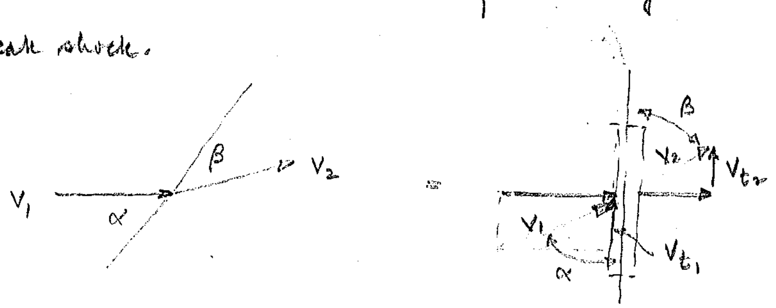
critical mach numbers $M = \frac{1}{\sqrt{k}}$ & $M=1$



if cooling there is no restriction: you move away from crit mach no. - you can never choke flow
Tabulate vs. M^*. if $M_{in} < 1$ & if you add heat you move toward M^* you can thus choke flow and jump to another Rayleigh line. This will cause $\dot{m}$ to decrease to an extent that new $\dot{m} = \dot{m}_{max}$ for given Q and $M_{exit} = 1$.

\- if $M_{in} > 1$ and $L > L_{max}$ ⇒ conditions must change; cannot be through shock ∴ shock must be in supplying nozzle.
Rayleigh flow will ~~not~~ not tell you where shock stands (since $T_{0y} = T_{0x}$ for shock) whereas T_0 changes in Rayleigh flow - friction will.

for oblique shocks break up into components of velocity normal and tangential $V_{t_1} = V_{t_2}$. If M_2 turns out to be subsonic then it's a strong shock. If M_2 turns out to be supersonic still then it's a weak shock.



$V_{t_1} = V_{t_2}$ since no pressure variation in tangential direction ∴ in t direction

$\oint V_t (\rho V \cdot n \, dA) = 0$ since $\rho V \cdot n \, dA = \text{const}$ by continuity ⇒ $V_{t_1} = V_{t_2}$

II-10 Newtonian fluids – linear relationship between shearing stress and rate of strain

II-13 Nondimensional Parameters

Rate of change of momentum $\approx \frac{\rho U^2 \delta V}{L}$ Grav. force $\approx \rho g \delta V$ Pressure force $\approx \frac{\delta p}{L}\delta V$ Viscous force $\approx \frac{\tau}{L}\delta V \approx \frac{\mu U}{L^2}$

Froude Number:

$$\frac{\text{Inertia}}{\text{gravity}} \approx \frac{\rho U^2 \delta V}{L \rho g \delta V} = \frac{U^2}{Lg} = Fr^2$$

$$\boxed{Fr = \frac{U}{\sqrt{Lg}}}$$

Reynold's number:

$$\frac{\text{Inertia}}{\text{Viscous}} \approx \frac{\rho U^2 \delta V L^2}{L \mu U \delta V} = \frac{\rho U L}{\mu}$$

$$\boxed{Re = \frac{\rho U L}{\mu}}$$

$M^2 = \frac{U^2}{c^2} \quad \frac{\rho U^2}{\rho R T k}$

forces due to inertia = dyn. press.

forces due to press. is ~~[illegible]~~ thermo. press.

$$\frac{\text{Inertia}}{\text{pressure}} \approx \frac{\rho U^2}{L}\frac{\delta V L}{\delta p \, \delta V} = \frac{U^2}{\delta p/\rho} = \frac{U^2}{p/\rho}\frac{1}{\delta p/p}$$

For a perfect gas (a = speed of sound)

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_{\text{const entropy}} = \frac{\gamma p}{\rho}$$

$$\Rightarrow \frac{\text{Inertia}}{\text{pressure}} \approx \frac{U^2}{a^2}\frac{1}{\delta p/p} = M^2 \frac{1}{\delta p/p}$$

$$\boxed{M = \frac{U}{a} = \text{Mach number}}$$

General Remarks

Generally neglect compressibility effects for $M <\sim 1/2$

$$\nu = \mu/\rho$$

Water : $\nu \approx .0114\ cm^2/sec$

Air : $\nu \approx .15\ cm^2/sec$

Boats, airplanes, etc $L \approx 10$ meters $= 10^3$ c

Low speed planes $U \approx 100\ \frac{meters}{sec} = \frac{10^4\ cm}{sec}$

Boats $U \approx 10\ \frac{meters}{sec} = \frac{10^3\ cm}{sec}$

$$Re = \frac{UL}{\nu}$$

$$\Rightarrow \quad Re_{water} \simeq \frac{10^3 \cdot 10^3}{.0114} \frac{cm^2/sec}{cm^2/sec} \simeq 10^8$$

$$Re_{air} \simeq \frac{10^4\ 10^3}{.15} \simeq 10^8$$

$\Rightarrow$ viscous terms very small relative to inertial terms

In boundary layer (high Re applications) more appropriate characteristic dimension might be λ, the thickness of the boundary layer

$$g \simeq 980\ \frac{cm}{sec^2}$$

$$Fr = \frac{U}{\sqrt{980 L}}$$

for parameters listed above:

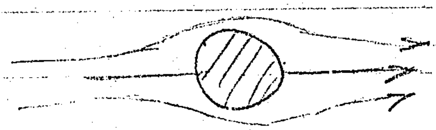
$Fr_{air} \simeq 10$ gravitational forces normally neglected (except sonic boom problems)

$Fr_{water} \sim 1$ Wave resistance of ships, shallow submerged objects usually depend on Froude number

Gravitational effects due to free surface of the fluid

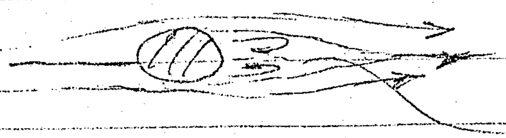
Variation of Flow Field with Re

$M \ll 1$ Highly viscous fluid around a right circular cylinder



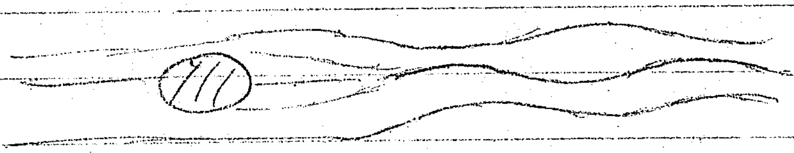
$Re \sim 1$

Behaves much like inviscid fluid



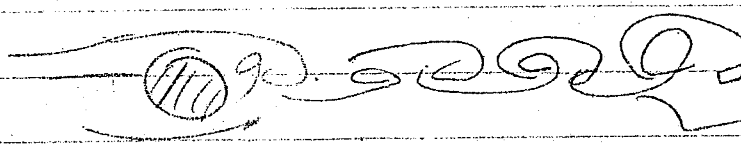
$Re \sim 25$

Stable separation bubble



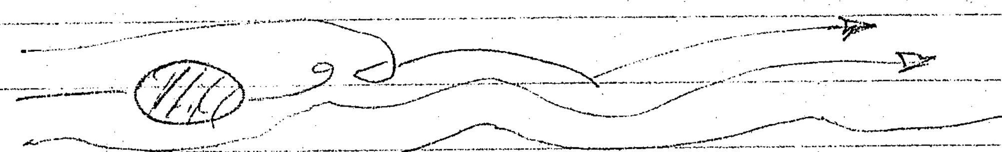
$Re \sim 50$

Instability begins to set in at $Re \sim 40$

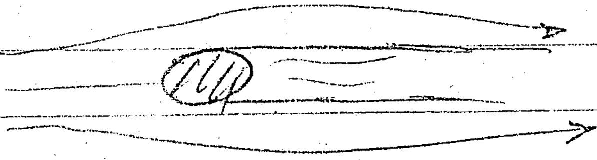


$Re \sim 100$

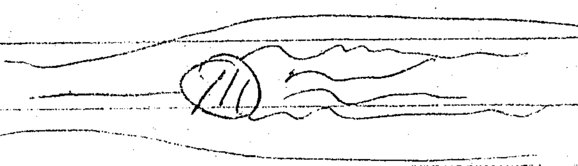
Karman Vortex Street



$Re \sim 250$



Laminar Separation of Boundary Layer

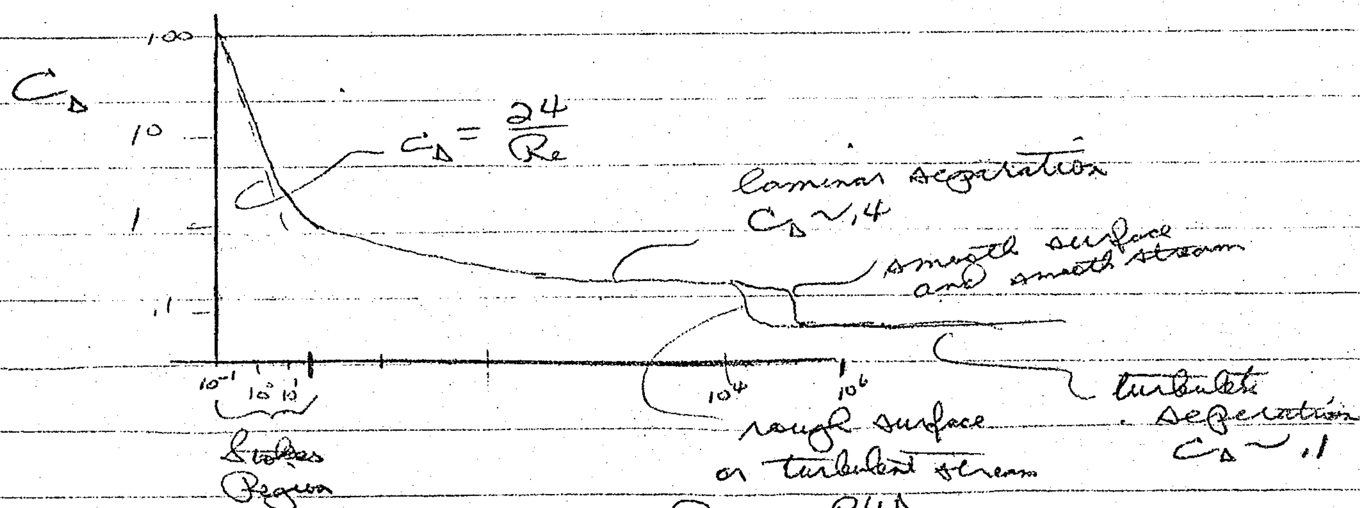


Turbulent Separation of Boundary Layer

Very High Re

Ideal fluid theory still useful, especially for streamlined shapes

Drag Coefficients of a Sphere



$$Re = \frac{\rho U D}{\mu}$$

$$C_D = \frac{\text{Drag force}}{\frac{\rho}{2} U^2 \pi R^2_{\text{radius}}} = \begin{cases} \frac{24}{Re} & 0 < Re \leq 60 \\ .4 & 60 < Re \leq 2 \times 10^5 \\ .1 & 2\times 10^5 \leq Re \end{cases}$$

For cylinder, curve for drag coefficients has much the same character and

$$C_D = \frac{\text{Drag}}{\frac{\rho}{2} U^2 D L}$$

Navier - Stokes Equ.

Incompressible, Newtonian fluid

II-37 Simplification of Navier Stokes Equations Based on relative sizes of ~~the~~ terms