AE 371 Propulsion

Tuesdays		Thursdays
January 25		January 27
February 1		
February 8		February 10
February 15		
February 22		February 24
February 29		
March 7		March 9
March 14		
March 21		March 23
April ~~8~~ 11		APRIL 13 - EXAM
April 18		~~May~~ APRIL 20
April 25		APRIL 27
May 2	2R	May 4 (FINAL EXAM)
~~May 9~~ (must show up for return of exams)		

Section A	Room 505	Tuesday and Thursdays
Section A2	Room 600	Tuesdays
	Room 215	Thursdays

POLYTECHNIC INSTITUTE OF BROOKLYN

Department of Aerospace Engineering
and Applied Mechanics

Course AE 301 - Propulsion

Professor Marian Visich, Jr.

Text: Hesse and Mumford: Jet Propulsion for Aerospace Applications. Pitman Publishing Co. , New York, 1964.

References:

Corliss: Propulsion Systems for Space Flight. McGraw-Hill Book Co. , New York, 1960.

Stuhlinger: Ion Propulsion for Space Flight. McGraw-Hill Book Co. , New York, 1964.

Godsey and Young: Gas Turbines for Aircraft. McGraw-Hill, 1949.

Durham: Aircraft Jet Powerplants. Prentice Hall, 1951.

Kuchemann and Weber: Aerodynamics of Propulsion. McGraw-Hill, 1953.

Shepard: Principles of Turbomachinery. MacMillan, 1956.

Zucrow, M. J. : Aircraft and Missile Propulsion.
Vol. I - Thermodynamics of Fluid Flow and Application to Propulsion Engines

Vol. II - The Gas Turbine Power Plant, the Turboprop, Turbojet, Ramjet and Rocket Engines

John Wiley and Sons, Inc. , 1958.

Buzzard and DeLauer: Nuclear Rocket Propulsion. McGraw-Hill, 1958.

Hill and Peterson: Mechanics and Thermodynamics of Propulsion. Addison-Wesley Publishing Co., 1965.

Topics to be covered:

Airbreathing Engines

1. Principles of Jet Propulsion and Engine Classification - Chapter 2.
2. Energy Transfer in Turbo-Machinery - Chapter 6.
3. Axial-Flow Compressors - Chapter 8.

4. Gas Turbines - Chapter 10.
5. Combustion, Combustion Chambers, Fuels and Controls - Chapter 4, Section 1-2. Chapter 9.
6. Exhaust Nozzles - Chapter 4, Sections 3-8. Chapter 5, Sections 10-13.
7. Inlets - Chapter 5, Sections 1-9.
8. Turbojet and Turbofan Engines - Chapter 11 .
9. Thrust Augmentation of the Turbojet and Turbofan Engines - Chapter 12.
10. High Flight Mach Number Air-Breathing Engines - Chapter 14.
11. Nuclear Turbojet and Ramjet Engines - Chapter 16, Sections 1-8.

Rocket Propulsion Systems

12. Chemical Rocket Engines - Chapter 15.
13. Nuclear Rocket Propulsion - Chapter 16, Section 9.
14. Electric Rocket Propulsion - Chapter 17.

AE 371 A Prof. VISICH 1-25-72

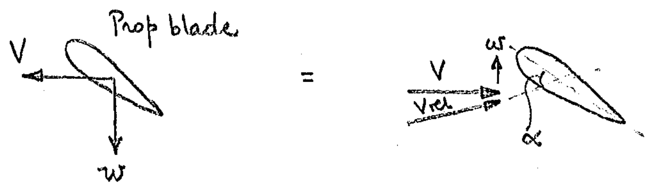
3 EXAMS 2 HR FINAL MAY 9

ROCKET System – any system whereby all the fuel is carried on board

Prop. System

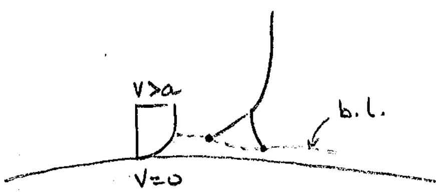
$$\frac{BHP}{BHP_{SL}} = (1+a)\sigma - a \qquad \sigma = \frac{\rho}{\rho_0}$$

$a \sim .1324$ BHP $\to 0$ at $h = 56{,}000$ ft.



at ~ 3000 rpm compressibility

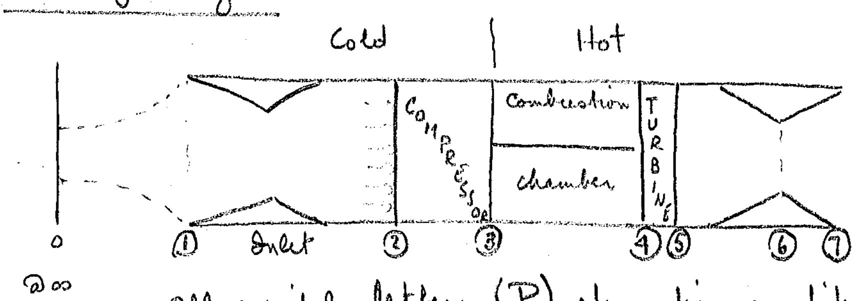
separation caused by shock
due to shock static pressure increases
causing adverse pressure gradient
which gives separation



Propeller System advantage – const thrust horsepower engine
maneuverability
specific fuel consumption

Turbo Jet Engine:

Notation



All capital letters (P) stagnation conditions
" lower case " (p) static conditions

Assume front of engine is cold $\gamma = 1.4$
Past Compressor $\gamma = 1.33$ Hot Section
Ramjet Engine Hot Section $\gamma = 1.2 - 1.4$

Inlet captures right amount of air & bring it to compressor face at the right conditions. if $V > a$ Inlet reduces speed to .3 a

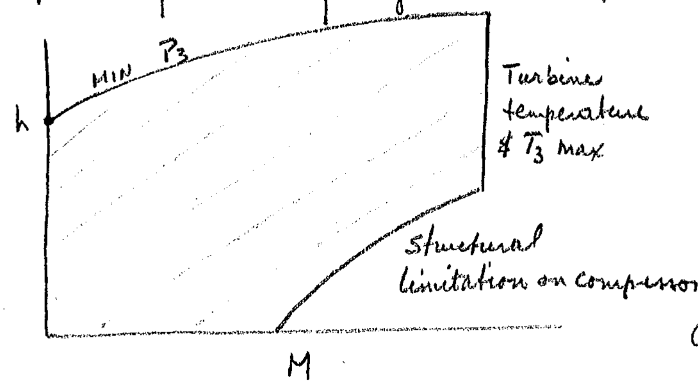
Compressor increases pressure still operate subsonic, temperature increases

Combustion Chamber ignite air by mixing fuel & increases temp of flame to ~ 5000°F

turbine in front T = 2000 – 2500°F sect 4 if higher get creep

turbine takes out energy to run compressor, pressure ↓ temp ↓

expand to high velocity to get thrust V_7



defined STP
normal hot day
normal cold day

Cannot fly at high M @ SL since structurally impossible

at M = 0 minimum pressure in combustion chamber (complete energy release

@ $M = M_4$ due to temp at front of turbine the amount of fuel is fixed by $T_4 - T_3$.

@ S.L.

M_0	P_0/p_0
.3	1.064
1	1.893
2	7.825
3	36.733
4	151.81

Assume adiabatic $T_0 = T_2$

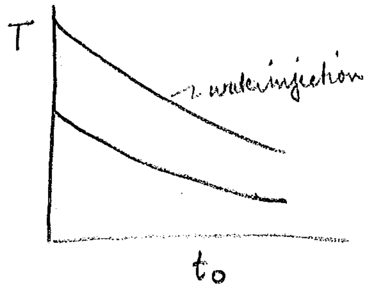
$\eta_c = 100\%$ $P_3/P_2 = 10$

@ S.L $t_0 = 520°R$

M	T_0	T_3
1	624	1205
2	936	1801
3	1456	2812
4	2184	4217

@ 35'000 $t_0 = 393°R$

M	T_0	T_3
1	472	910
2	707	1366
3	1100	2125
4	1650	3187



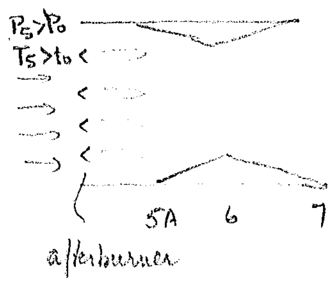
water injection decreases static pressure in front of compressor. It boosts curve up.

$$\omega_{fuel} \sim T_4 - T_3$$

Advantages - smaller diameter
relatively lightweight
noise level
τ higher but you use kerosene less fuel economic

Bad things - low speed characteristics
short haul usage

Afterburners



to increase thrust

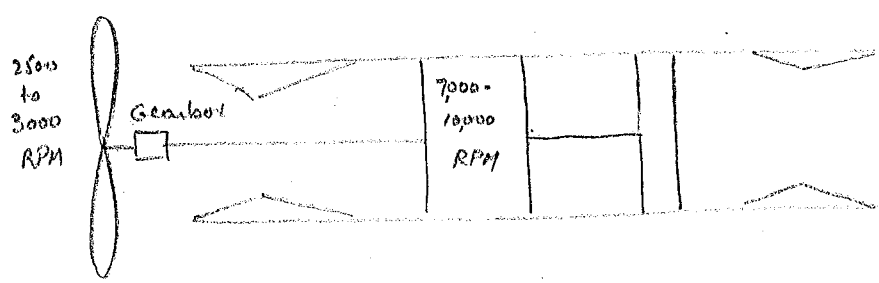
GE4/J5P SST

57500 Thrust Continuous SFC = 1.0
After 68600 " SFC = 1.77

50% prop system & fuel
< 10% payload.

Turbo Prop System

Used for short flights

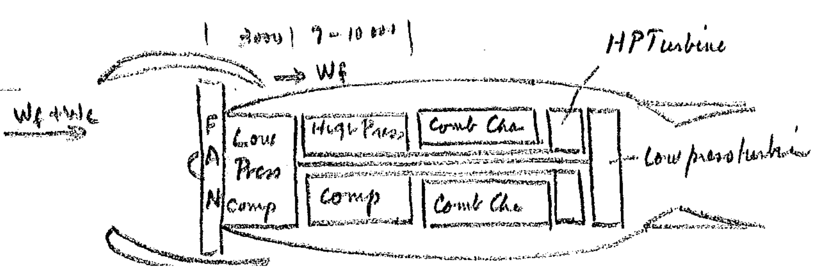


60-70% thrust

$$T \sim m(V_7 - V_0) \sim m\Delta V$$

$m >$ turbo
$\Delta V <$

Turbo Fan



SFC = .6
low speed char of fan
high speed char of engine

β = By pass-ratio = $\dfrac{w_f}{w_{core}}$

__P & W__ JT9D7 __747__

@ SL. $w_f + w_c = 1600$ lb/sec

$\beta = 5$ $w_f = 1333$

$w_c = 267$

Thrust 45,500 lb

Fan + 3 stag LPC pressure ratio 2.4:1 total press ratio

11 stage HPC pressure ratio 10:1 24:1

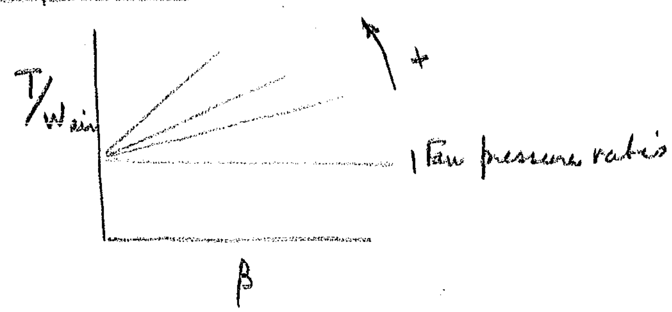
Fan driven by 4 stage turbine

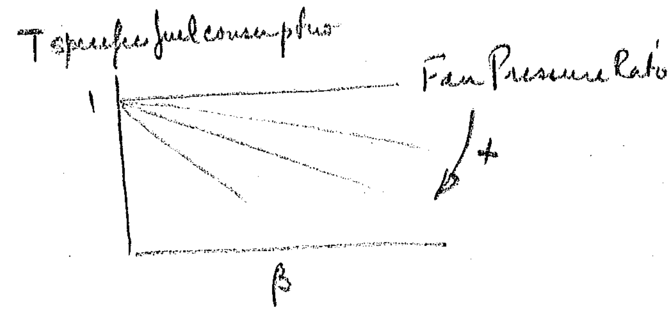
HPC - driven by 2 stage turbine

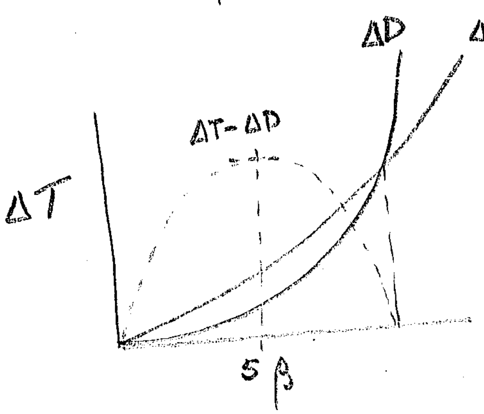
residual thrust from core is used to keep core from becoming a drag producer

$T \sim m(V_7 - V_0)$ if $V_7 < V_0$ T is low $\therefore$ core increases drag of engine

__Bypass Ratio__ __Opt__



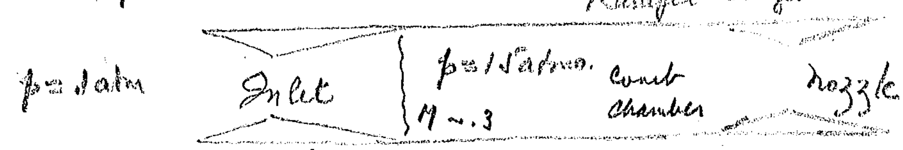




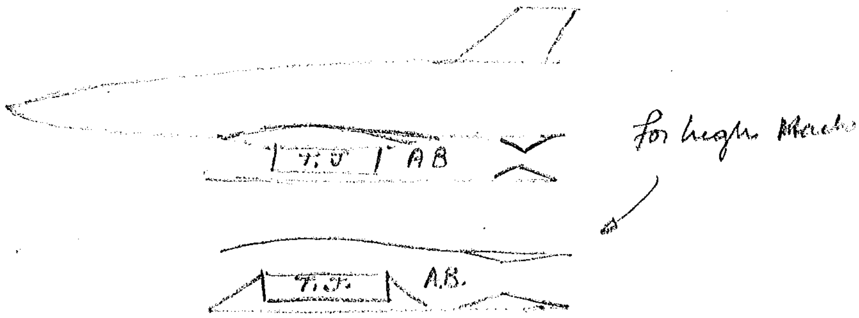
optimum pressure ratio for 1 stage fan 1.6-1.5 to reduce adverse pressure grad. __increase size of fan blade__ or decrease pressure grad.

$M_0 = 4$ $(P/p)_0 = 151$

Ramjet Engine



limitation - it can't go slow.



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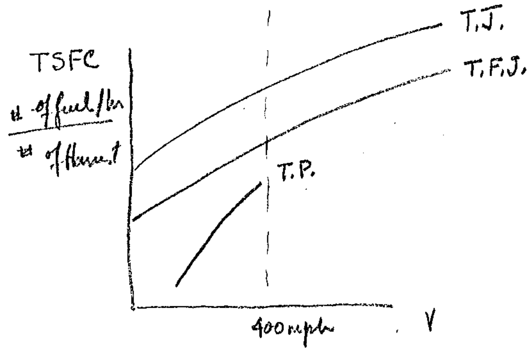


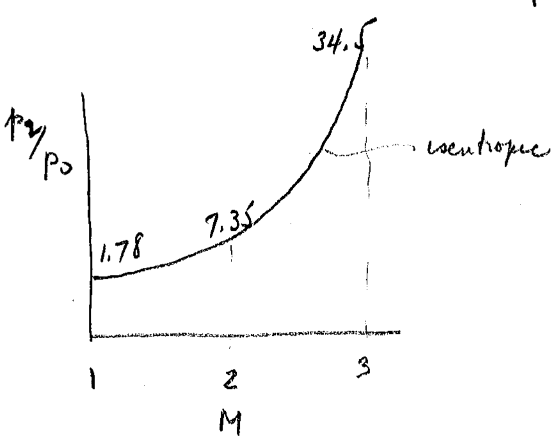
Turbo jet

$$T = m(V_7 - V_0) + A_7(p_7 - p_0)$$

variable exhaust nozzle can bring $A_7 \Delta p \to 0$
corresponds to max thrust.

Turbo fan

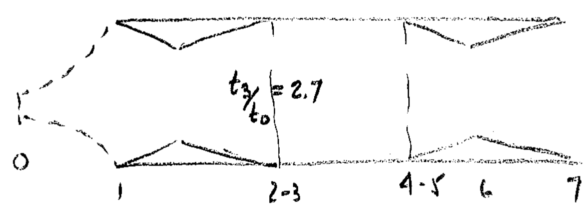
$$T = m(V_7 - V_0) + A_7(p_7 - p_0) + m_f(V_7' - V_0) + A_7'(p_7 - p_0)$$



$M_2 = .3$

Ramjet at high mach compressor is not needed because slowing down the air will cause p_2/p_0 ↑.

$M_0 = 3$



for $M_0 = 6$
$h > 35000$

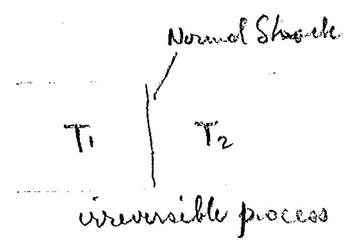
$M_3 = .3$

$\dot{Q}$ in inlet thru walls is small with respect to flow

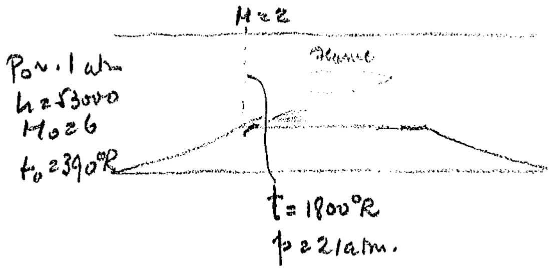
stag temp is temp that flow will achieve if fluid is brought adiabatically to rest. $(\Delta s \neq 0)$

$$c_p t + \frac{1}{2} V^2 = c_p T \qquad T/t = 1 + \frac{\gamma - 1}{2} M^2$$

if $t = 400$ $\qquad T_0 = 3280°R$



Supersonic Combustion Ramjet SCRAMJET

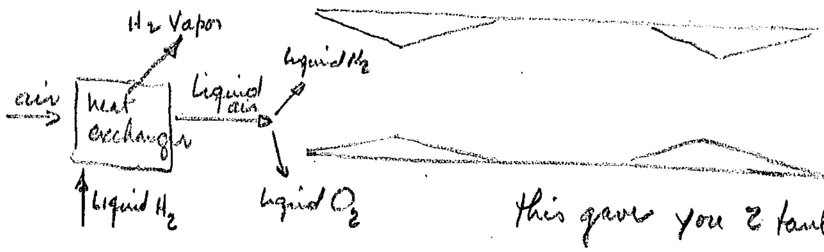


if at $M_0 = 10$
$h = 53000$

$M = 4.2$

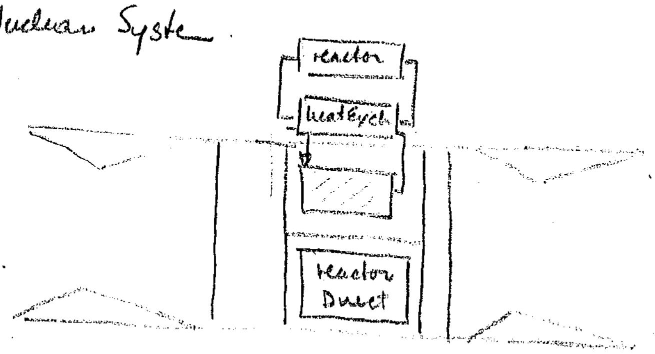
$t = 1800°R$
$p = 21$ atm.
$T = 8200°R$
$P = 4200$ atm.

<u>Liquid Air Cycle Engine</u> <u>LACE</u> H_2

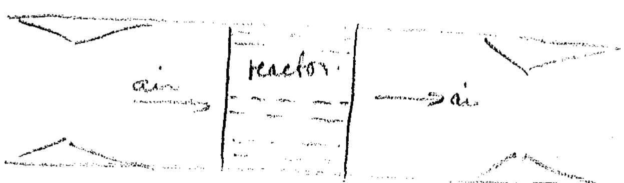


this gave you 2 tanks of H_2 & O_2 which was rocket system

Nuclear System



Standard Turbojet
heat exchanger



fly for long periods of time
$M_0 \sim 3$ Project PLUTO.
limited by struct consideration

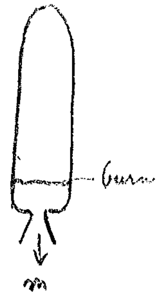
<u>Rocket Engine</u> (no air taken in)

$$T = \dot{M}(V_7) + A_7(p_7 - p_0) \sim \dot{m} V_E$$

define $$I_s = \frac{\text{thrust}}{\text{weight flow}} = \frac{T}{\dot{W}} = \frac{V_e}{g} \text{ (sec)}$$

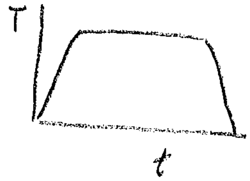
all masses & weights are $\dot{m}$ & $\dot{W}$

I_s	V_e	
250 sec	8050 fps.	Chemical
500 sec	16100 fps.	Chemical
1500 sec	48300 fps.	— nuclear
10,000 sec	322000 fps	— elect

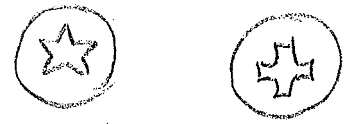


depends on surface area   if surface area is const burning is const & so is thrust

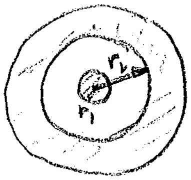
get T vs time



Cross Sect

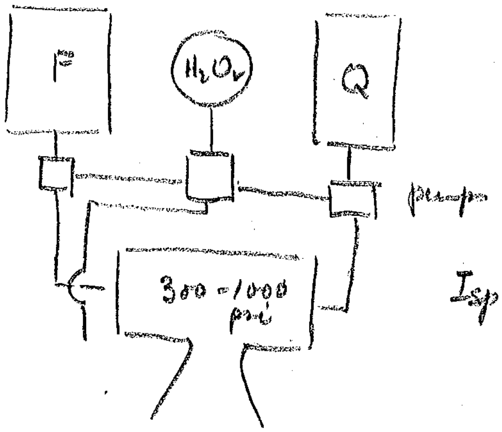


Advantage - simple, reliable, storage is possible



originally $2\pi(R_1+R_2)$

after change $2\pi(R_1 - \Delta R + R_2 + \Delta R) = 2\pi(R_1+R_2)$ const.



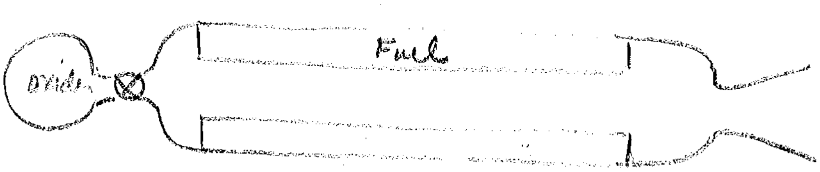
H_2O_2 drives turbine which drives pumps which pressurizes tanks

$I_{sp} = 500$

$2H_2O_2 \rightarrow 2H_2O + O_2$

liquid prop - half size of solid prop
- quick shutdown.
- gimbal easier
- cannot package.

LIQUID-SOLID PROP



hypergollic - fuel & oxider ignite when in contact with each other

- can be stored
- can be shut down.
- $I_{sp} > 250$

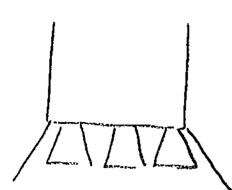
ATLAS

$T_{TO} = 367000$ lbs.

$I_S \sim 300$ $W = 1230$ lbs/sec

$W_{TO} = 260\,000$ lbs $T/W_{TO} = 1.4$

TO 11LE ORBIT   PAYLOAD = 2700 $W_0/W_{PE} = 96$



Saturn V

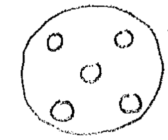
$T_{TO} = 7.5 \times 10^6$

$I_s \sim 300$ $\quad W \sim 25000$ lb/sec

$W_{TO} \approx 6 \times 10^6$ $\quad T/W_{TO} \approx 1.25$

300 MILE ORBIT $\quad W_{PL} \approx 220{,}000$ lbs $\quad W_0/W_{PL} \approx 27$

LUNAR $\quad W_{PL} \approx 102400$ $\quad W_0/W_{PL} \approx 62$



ICBM

$R = 5000$ N.MI.

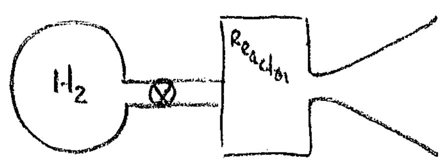
↑ 1% change I_s → 7½% increase in range

300 → 303

↑ 5% change I_s → 45% increase in range.

$\frac{W_0}{W_f} = 10$

I_s	Mission
200	1300 miles (Drag free)
300	4100 miles "
400	Earth satellite
500	Escape.



$V_e \sim \sqrt{T/m}$

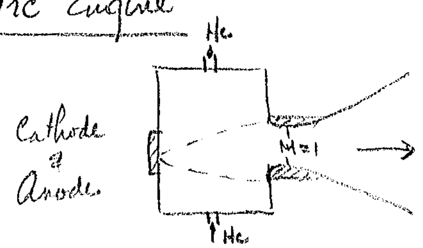
Electric Rocket

Energy Source	Power Converter	Thrust Device
Solar	Mechanical	ARC, MHD (Plasma)
nuclear	Direct	ion

Electrical Power.

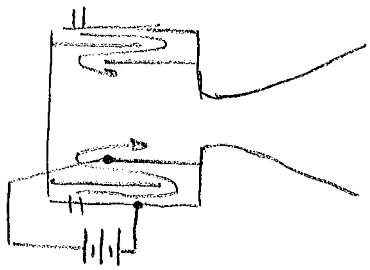
Reliability
Usage

Arc Engine



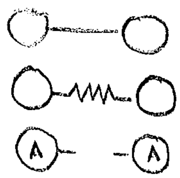
He goes through arc $\quad V_e \sim \sqrt{T/m}$ $\quad I_s \sim 1000$
& comes out back end
Get Erosion of arc components etc.

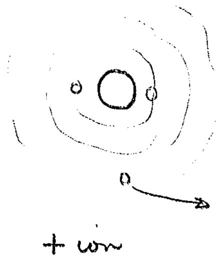
Resisto Jet



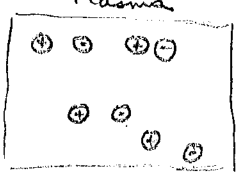
Magneto Hydrodynamic — Plasma

Increasing Energy





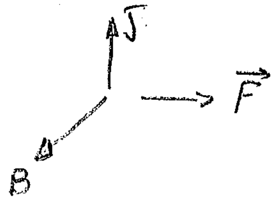
Plasma

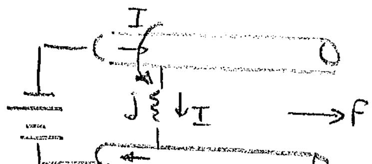


if ⊕ = ⊖ neutral gas

Can conduct electrical energy through gas.

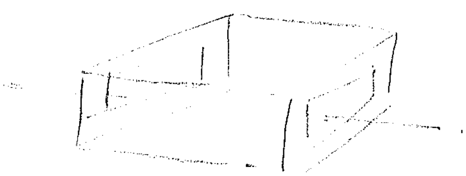
$\vec{F} \sim \vec{J} \times \vec{B}$ apply electric field



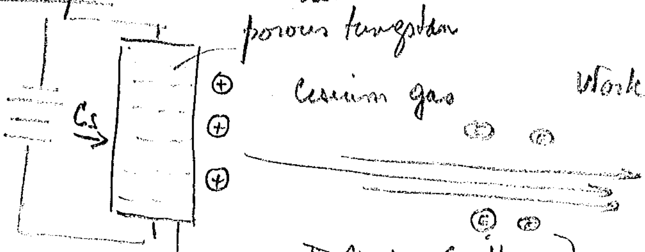


wire will be plasma.

$I_s = 5000$ sec



Ion Prop

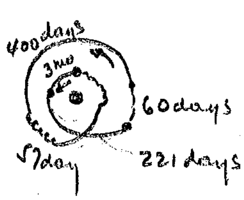


Cesium gas will give up Electrons & leave cesium ions + which will be attracted by negative electrodes.

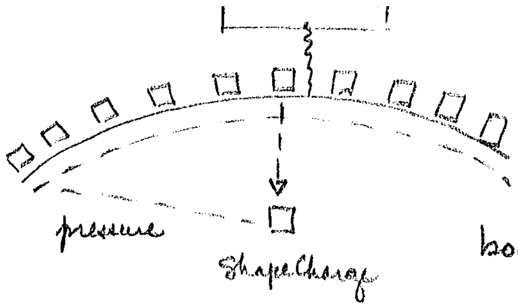
If a space charge builds at rear of engine shooting + ions will be repelled ∴ Electrons emitted must be placed ∋. this space charge becomes neutral.

$I_s = 10\,000$

$T/W = 10^{-4}$

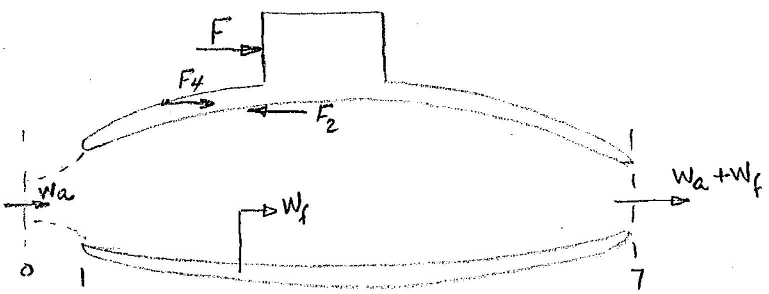


powerplant on 3/2 days



boosts it

2-1-72



weight flow rate

F_2 force exerted on walls due to prop system

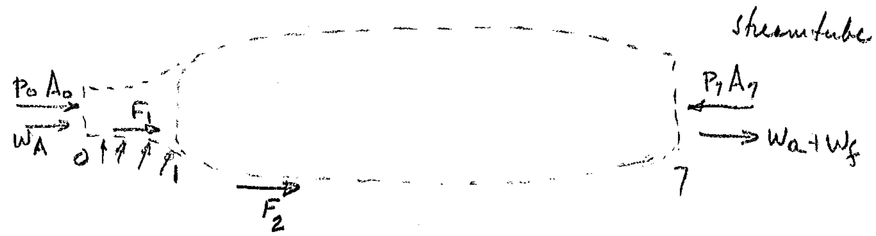
F_4 due to press distrib & viscous effect acting on nacelle

F force exerted on nacelle by rest of airplane

$$F + F_4 - F_2 = 0$$

$$F = F_2 - F_4 \quad \text{(force exerted on nacelle acted on by rest of airplane)}$$

Momentum principle



$$F_1 = \int_0^1 p\,dA$$

$$\frac{w_a + w_f}{g} V_7 - \frac{w_a}{g} V_0 = p_0 A_0 + F_1 + F_2 - P_7 A_7$$

$$F_4 = \int_1^7 p\,dA + D_F \quad \text{skin friction drag}$$

$$F = F_2 - F_4 = \frac{w_a + w_f}{g} V_7 - \frac{w_a}{g} V_0 + P_7 A_7 - P_0 A_0 - F_1 - F_4 + p_0 A_7 - p_0 A_7$$

$$F = \underbrace{\frac{w_a + w_f}{g} V_7 - \frac{w_a}{g} V_0 + A_7 (p_7 - p_0)}_{F_N \text{ net thrust}} - \underbrace{\left[p_0 (A_0 - A_7) + F_1 + F_4 \right]}_{D_N \text{ nacelle drag}}$$

$$= F_N - D_N$$

net thrust nacelle drag

$$F_N = \frac{w_a + w_f}{g} V_7 + A_7 (p_7 - p_0) - \frac{w_a}{g} V_0 = \underset{\text{gross thrust}}{F_G} - \underset{\text{ram drag}}{F_R}$$

$$D_N = F_1 + F_4 + p_0 (A_0 - A_7) = \int_0^7 (p - p_0)\,dA + D_F = \underset{\text{pressure drag}}{D_p} + D_F$$

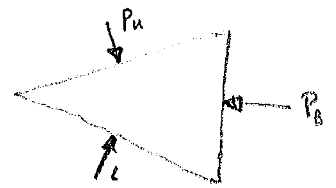
define $D_p = \int (p - p_0)\,dA$

D_{P_4} = press drag of the force F_4 $\quad D_{P_4} = \int_1^7 (p - p_0)\,dA = F_4 + p_0 (A_1 - A_7)$

D_{P_1} = " " " " " F_1 $\quad D_{P_1} = \int_0^1 (p - p_0)\,dA = F_1 + p_0 (A_0 - A_1)$

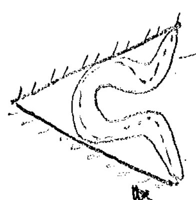
$$D_N = p_0(A_0 - A_7) + F_1 + F_4$$
$$= p_0 A_0 + D_{p_4} + p_0(A_7 - A_1) + D_F + D_{p_1} + p_0(A_1 - A_0) - p_0 A_7$$
$$= D_{p_4} + D_{p_1} + D_F = D_p + D_F$$

additive drag.



$\bar{D}_p = \int p\,dA$

consistent for closed body: $D_p = \int p\,dA = \int (p - p_0)\,dA$ since $\int p_0\,dA \equiv 0$

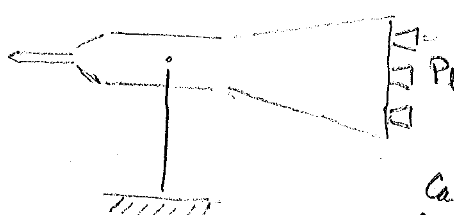


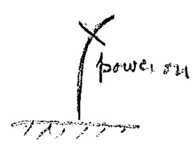
changed to prop susle.

$$F_N = \frac{\omega_a + \omega_f}{g} V_7 + (p_7 - p_0) A_7$$

is $\bar{D}_p \overset{?}{=} D_p$ no since $\int_{\text{body}} p_0\,dA \neq 0$

V_0





Can be used with Power off

for power on measure base pressure

$$D_{P_{ON}} = D_{P_{OFF}} + (p_0 - p_{base}) A_7$$
$$D_{P_{ON}} < D_{P_{OFF}}$$

HW Pg 42 : 2, 3, 5

$$F_N = \frac{W_a + W_f}{g} V_7 - \frac{W_a}{g} V_0 + (p_7 - p_0) A_7 \approx \frac{W_a}{g}(V_7 - V_0) + A_7 (p_7 - p_0) = \dot{m}\Delta V + A_7(\Delta p)$$

$\frac{W_f}{W_a} << 1$ turbojet ramjet

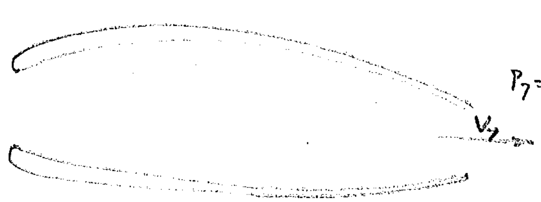
V_{eff} exhaust veloc. $= V_{eff} = V_7 + \frac{g}{w} A_7 (p_7 - p_0)$

$$F_N = \frac{w}{g}(V_{eff} - V_0)$$

if compressor failure $A_0 = 0$ (pitot tube)

if $V_0 = 0$ $A_0 = \infty$ $(m = \rho_0 A_0 V_0)$

p_0



$p_7 = p_0$ V_7 is subsonic

$$F_N = \frac{w_a}{g}(V_7 - V_0)$$

$$\frac{w_a V}{g} = mV = \rho A V^2 \qquad M = \frac{V}{a} \qquad a = \sqrt{\gamma R T} = \sqrt{\gamma p/\rho}$$

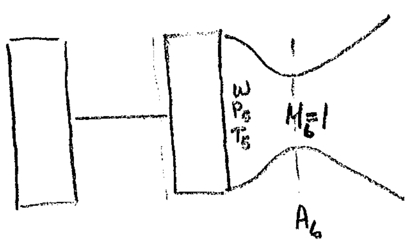
$$= \rho A M^2 a^2$$

$$= \rho A M^2 \frac{\gamma p}{\rho} = \gamma A p M^2$$

$$F_g = \frac{w}{g} V_7 + A_7(p_7 - p_0) = \gamma_7 A_7 p_7 M_7^2 + A_7(p_7 - p_0)$$

$$= A_7\left[p_7(1 + \gamma_7 M_7^2) - p_0\right] \qquad \gamma_7\ (1.15 \to 1.4)$$

$$F_R = A_0 \gamma_0 p_0 M_0^2 = 1.4\,(A_0 p_0 M_0^2)$$



adiabatic

if A_7 changes
p_7 & V_7 are affected

$$T = m(V_7 - V_0) + A_7(p_7 - p_0)$$
$$\uparrow \qquad\qquad \uparrow \quad \downarrow$$

$$c_{p_7} t_7 + \frac{1}{2g} V_7^2 = c_{p_7} T_7$$

$$T_7 = T_5$$

$$V_7 = \sqrt{2g c_{p_7}(T_7 - t_7)} = \sqrt{2 c_{p_7} T_7 g\left(1 - \frac{t_7}{T_7}\right)}$$

$$V_7 = \sqrt{2g \frac{\gamma_7}{\gamma_7 - 1} \frac{\mathcal{R}}{m} T_7 \left(1 - \left(p_7/P_7\right)^{\frac{\gamma_7-1}{\gamma_7}}\right)}$$

$$\frac{t_7}{T_7} = \left(\frac{p_7}{P_7}\right)^{\frac{\gamma-1}{\gamma}}$$

for: $V_7 \uparrow \quad T_7 \uparrow \quad m \downarrow \quad \gamma_7 \to 1$

$$\gamma = \frac{c_p}{c_v} \qquad R = c_p - c_v \qquad c_p = \frac{\gamma}{\gamma - 1} R$$

$$R = \mathcal{R}/m$$

53.3 air   1545   molecular weight

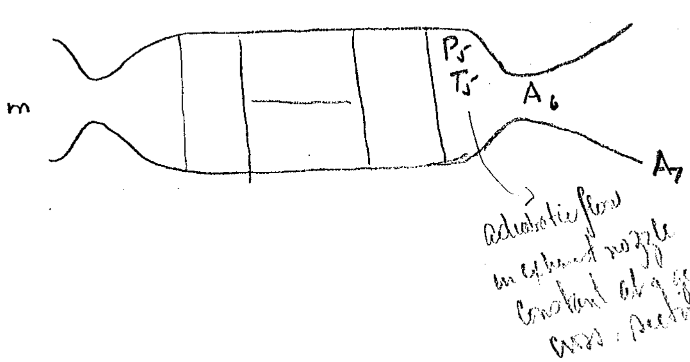
$$p_7/P_7 = \frac{p_7}{p_0}\,\frac{p_0}{P_0}\,\frac{P_0}{P_7}$$

for ideal nozzle $p_7 = p_0$ for max thrust

$p_0/P_0 \to$ flight alt & velocity fixed

$P_0/P_7 \to$ stag press losses in system. occur in inlet

2-8-72



$$F_N = m(V_7 - V_0) + A_7(p_7 - p_0)$$

$$\frac{dF}{dA_7} = 0 \qquad \frac{d^2F}{dA_7^2} < 0 \qquad \text{possibilities}$$

$$\frac{dF}{dV_7} = 0 \qquad \frac{d^2F}{dV_7^2} < 0$$

$$\frac{dF}{dp_7} = 0 \qquad \frac{d^2F}{dp_7^2} < 0$$

$$\frac{dF}{dp_7} = m\left(\frac{dV_7}{dp_7}\right) + A_7 + \frac{dA_7}{dp_7}(p_7 - p_0)$$

subsonic $P = p + \frac{1}{2}\rho V^2$

Bernoulli $VdV + \frac{dp}{\rho} = 0$ $\qquad \frac{dV}{dp} = -\frac{1}{\rho V}$

Supersonic $c_0 = \frac{\gamma}{(\gamma - 1)\rho} + \frac{1}{2}V^2$?

$$\frac{dF}{dp_7} = \rho_7 A_7 V_7\left(-\frac{1}{\rho_7 V_7}\right) + A_7 + \frac{dA_7}{dp_7}(p_7 - p_0) = 0$$

either $\frac{dA_7}{dp_7} = 0$ or $(p_7 - p_0) = 0$

occurs at throat

$M_7 = 1$

$$\frac{d^2F}{dp_7^2} = \frac{d^2A_7}{dp_7^2}(p_7 - p_0) + \frac{dA_7}{dp_7}$$

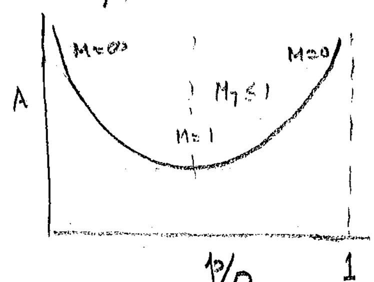
$\frac{d^2A_7}{dp_7^2}(p_7 - p_0) < 0$

at $M_7 = 1$ $p_7 = p_0$

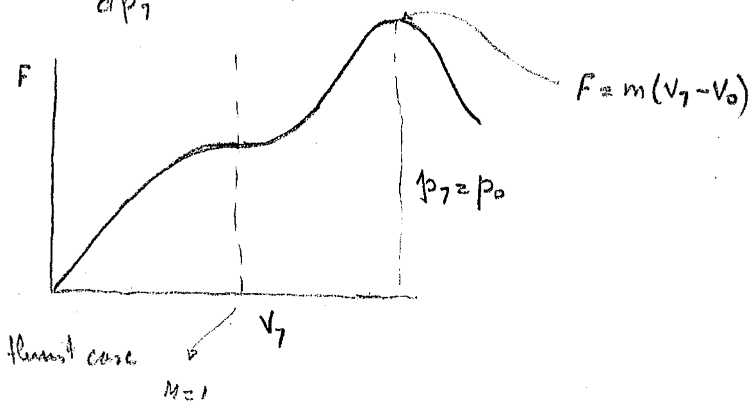
$\frac{d^2F}{dp_7^2} = 0$ inflect.

$\frac{dA_7}{dp_7} < 0$ for supersonic exhaust since $p_7 - p_0 \equiv 0$

for subs $\therefore \frac{dA_7}{dp_7} \neq 0$

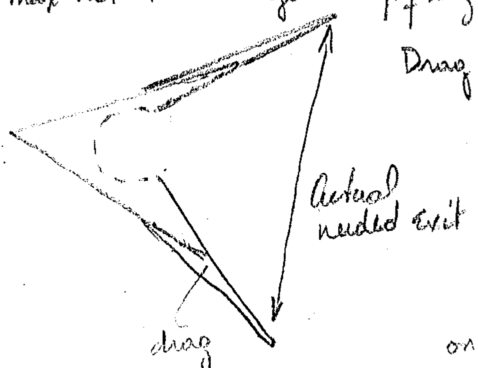


Max. net thrust

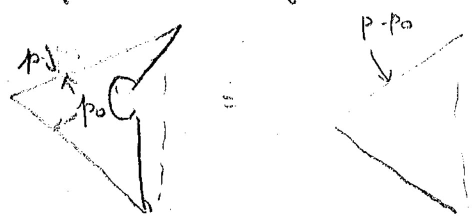


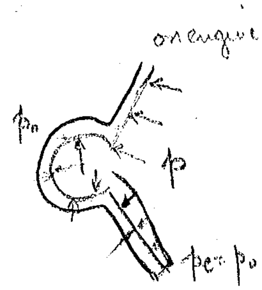
For max net thrust geometry of nozzle

Drag ↑ for max net thrust case



forebody pressure drag

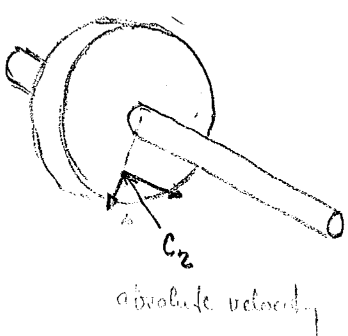




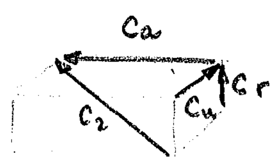
as more area is increasing p on outside < p_0

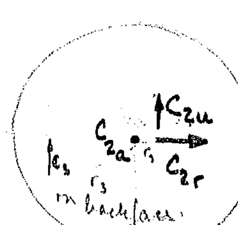
$\therefore$ net force on nozzle will decrease

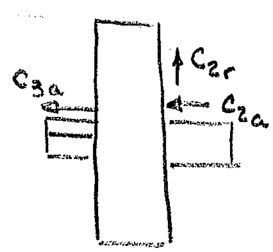
reduce area & reduce force acting on nozzle in thrust direction.



resolve C_2







$F_a = m(C_{3a} - C_{2a})$

need thrust bearings so that it doesn't move in axial direct

Change in angular momentum, or moment of momentum of the fluid is equal to the summation of all the applied torques acting on the fluid.

$$\text{Torque} = \frac{W}{g}\left(\underbrace{r_3 C_{3u}}_{\text{whirl}} - r_2 C_{2u}\right) \quad \text{ft-lb}$$

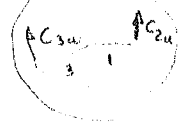
$$\text{Power} = \frac{\text{ft-lb}}{\text{sec}} = \omega\,\text{Torque} = \frac{W}{g}\,\omega\left[r_3 C_{3u} - r_2 C_{2u}\right] \quad \frac{\text{ft-lbs}}{\text{sec}}$$

u is velocity of wheel & ω is revol of wheel then

$$\text{Power} = \frac{W}{g}\left[u_3 C_{3u} - u_2 C_{2u}\right] \quad \text{if } \begin{array}{l} \geq 0 \text{ compressor. rotating body doing work on fluid} \\ \leq 0 \text{ turbine fluid doing work on rotating body} \end{array}$$

$$\mathcal{W} = \text{work done/lbs of fluid} = \frac{P}{W} = \frac{1}{g}\left[u_3 C_{3u} - u_2 C_{2u}\right]$$

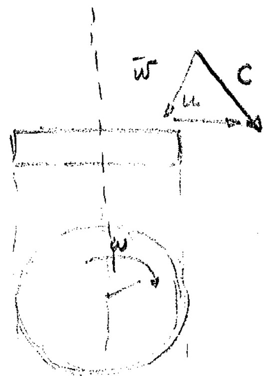
if $\xrightarrow{u}$ $\longleftarrow C_u(-)$ $\longrightarrow C_u(+)$



turbine

u_3 & u_2 are positive

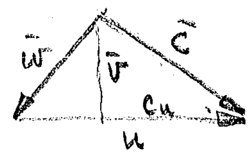
wheel axis



$\bar{w}$ = relative velocity to blade (stationary)

fix axis to blade

C_a, C_u, C_r



$$v^2 = C_a^2 + C_r^2$$

$$c^2 = v^2 + C_u^2$$

$$\bar{w}^2 = v^2 + (u - C_u)^2 = v^2 + u^2 - 2uC_u + C_u^2 = c^2 + u^2 - 2uC_u$$

$$c^2 - w^2 = -u^2 + 2C_u u$$

$$C_u u = \frac{c^2 + u^2 - w^2}{2}$$

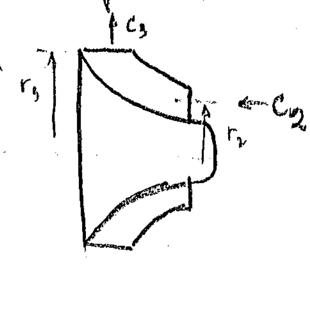
positive quantities

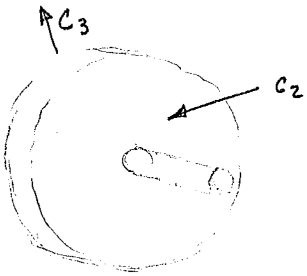
$$\mathcal{W}_c = \frac{1}{g}\left[C_{3u} u_3 - C_{2u} u_2\right] = \frac{1}{2g}\left[(C_3^2 - C_2^2) + (u_3^2 - u_2^2) - (w_3^2 - w_2^2)\right]$$

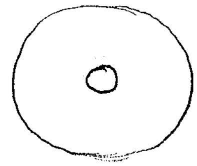
$$\mathcal{W}_T = \frac{1}{g}\left[C_{4u} u_4 - C_{5u} u_5\right]$$

Axial Flow compressor $C_r \cong 0$ $\qquad \mathcal{W} = \frac{u}{g}\left[C_{3u} - C_{2u}\right]$

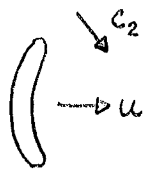
Centrifugal



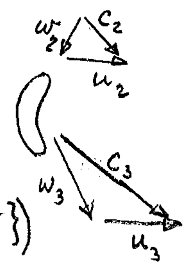




$$\mathcal{W}_{\text{on fluid}} = \frac{1}{g}\left(u_3\, C_{3_u} - u_2\, C_{2_u}\right) \quad \begin{matrix} \geq 0 & \text{compressor} \\ < 0 & \text{turbine} \end{matrix}$$



Consider a stationary blade

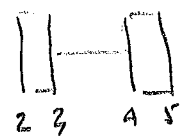


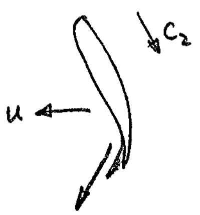
$\bar{w}$ relative to blade

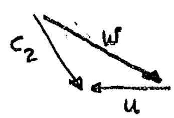
$$\mathcal{W}_{\text{on fluid}} = \frac{1}{2g}\left(\{C_3^2 - C_2^2\} + \{u_3^2 - u_2^2\} - \{w_3^2 - w_2^2\}\right)$$

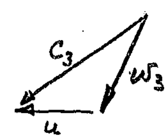
$$W_c = \frac{1}{g}\left(u_3 C_{3u} - u_2 C_{2u}\right)$$

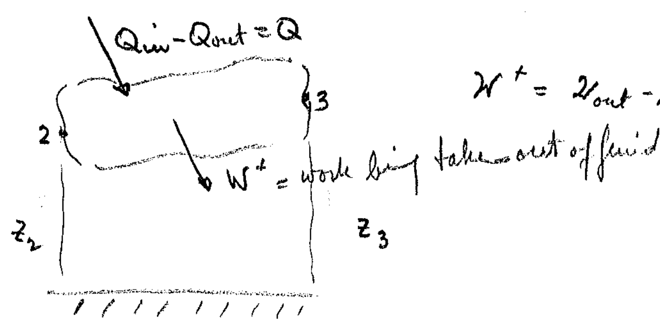
$$W_T = \frac{1}{g}\left(u_4 C_{4u} - u_5 C_{5u}\right)$$











$\mathcal{W}^+ = \mathcal{W}_{out} - \mathcal{W}_{in}$

At station 2 Energy / lb of fluid

flow work

$$\frac{1}{2}\frac{V_2^2}{g} + Z_2 + u_2 + Q + p_2 v_2 = \frac{1}{2}\frac{V_3^2}{g} + Z_3 + u_3 + \mathcal{W} + p_3 v_3$$

for our problems $Z_2 \cong Z_3$ assumed process is adiabatic $Q \cong 0$ (due to heat transfer)

$$\frac{1}{2}\frac{V_2^2}{g} + u_2 + p_2 v_2 = \frac{1}{2}\frac{V_3^2}{g} + \underbrace{u_3 + \mathcal{W} + p_3 v_3}_{\text{static enthalpy } h_3 = U_3 + p_3/\rho_3}$$

$$H_2 = \frac{1}{2}\frac{V_2^2}{g} + h_2 = \frac{1}{2}\frac{V_3^2}{g} + h_3 + W = H_3 + W_T$$ work being taken out of fluid

stag enthalpy

$$\mathcal{W}_c = H_3 - H_2 \qquad \text{BTU/lb}$$
$$\mathcal{W}_T = H_4 - H_5$$

$H = \int C_p\,dT$ for our temp range $\quad C_p = \bar{C_p} \quad H = C_p T$

$\mathcal{W}_c = C_p(T_3 - T_2)$

$\mathcal{W}_T = C_p(T_4 - T_5)$

$C_{p_{cold}} = .24$ BTU/lb °R $= 186.7$ ft/°R

conversion factor 778.16 $\frac{ft\text{-}lb}{BTU}$

$C_{p_{hot}} = .276$ BTU/lb °R

<u>Efficiency of compressor</u>

For a given P_3/P_2 define $\eta_{actual_c} = \dfrac{\text{Ideal work done on fluid to achieve } P_3/P_2}{\text{to actual work done on fluid to achieve } P_3/P_2}$

$$= \frac{C_p\,(T_3' - T_2)}{C_p\,(T_3 - T_2)}$$

' - ideal case

ideal is isentropic

$p \sim \rho^{\gamma}$

$\rho \sim p/t \qquad p \sim (p/t)^{\gamma} \qquad p^{1/\gamma} \sim p/t$

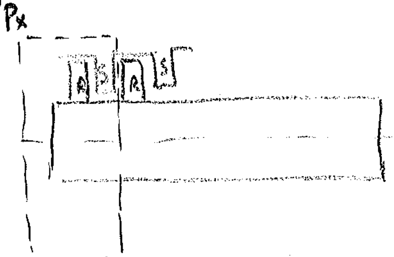
$t \sim p^{1 - 1/\gamma}$

$$\therefore \quad \frac{T_3'/T_2 - 1}{T_3/T_2 - 1} = \frac{(p_3/p_2)^{\frac{\gamma-1}{\gamma}} - 1}{T_3/T_2 - 1}$$

$$\mathcal{W}_{on\ fluid_c} = C_p\,(T_3 - T_2) = \frac{C_p T_2}{\eta_{ad_c}}\left\{\left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\} = \frac{1}{g}\left[u_3 C_{3u} - u_2 C_{2u}\right]$$

multistage effect.

P_Y/P_X



R = rotor

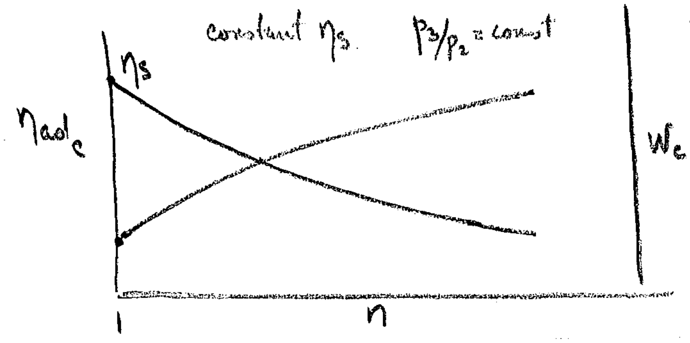
S - stator turns air to correct angle $\Rightarrow$ more work can be done on the fluid

stage rotor & stator $\quad \eta_s$ - efficiency of stage

n-stage compressor $(P_Y/P_X) = (P_Y/P_X)_2 \qquad P_3/P_2 = (P_Y/P_X)^n$

$$\eta_s = \frac{(P_Y/P_X)^{\frac{\gamma-1}{\gamma}} - 1}{T_Y/T_X - 1} \qquad \eta_{ad_c} = \frac{(P_3/P_2)^{\frac{\gamma-1}{\gamma}} - 1}{\left\{1 + \frac{1}{\eta_s}\left[\left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{n\gamma}} - 1\right]\right\}^n - 1}$$

constant $\eta_s \quad P_3/P_2 = $ const



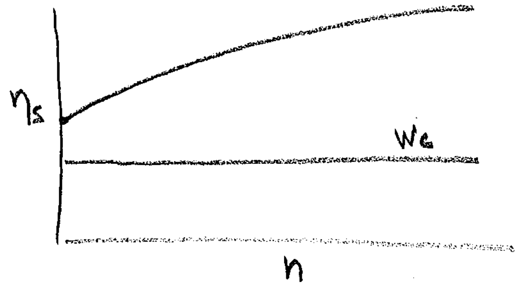
a) Centrifugal- compressor advantages
   1. High pressure rise per stage (6)
   2. Good efficiencies over wide rotational speed range
   3. Simplicity of manufacture and low cost
   4. Low weight
   5. Starting power requirements are low.

b) Centrifugal-compressor disadvantages
   1. Large frontal area for given air flow
   2. Two or more stages not practical because of losses in turns between stages.

c) Axial-flow compressor advantages
   1. High peak efficiencies
   2. Low frontal area for given air flow
   3. Straight-through flow allows high ram efficiency
   4. Increased pressure rise by increasing number of stages with negligible losses.

d) Axial-flow compressor disadvantages
   1. Good efficiencies over narrow rotational-speed range
   2. Difficulty of manufacture and high cost
   3. Relatively high weight
   4. High starting power requirements
   5. Low pressure rise per stage.

Manufacturer	Name	Pressure Ratio	Frontal Area	Static Thrust
16 stage axial flow compressors				
Allison	J71-A-13	8:01:1 (1.14)	8.5	10,000
Pratt-Whitney	J75	12:1 (1.17)	10.1	16,000
Westinghouse	J54-WG-2	9:1 (1.15)	6.7	6,500
GE	J79-GE11A	13:1 (1.16) (17 stage)	5.6	15,800
1 stage centrifugal compressors				
DeHavilland	Ghost	4.7:1	15.2	5,300
DeHavilland	Goblin	3.67:1	13.6	3,500
Rolls-Royce	Nine	4.5:1	13.4	5,400



P_3/P_2 = const

η_{ad_c} = const

given amount of turbine work

HW 163/2,8 164/9

Adiabatic Efficiency of Expansion

For a given $P_5/P_4 < 1$

$$\eta_{ad_{Turbine}} = \frac{\text{actual work removed from the fluid for a given } P_5/P_4}{\text{the ideal work removed from fluid for a given } P_5/P_4}$$

$$\eta_{ad_T} = \frac{C_p(T_4 - T_5)}{C_p(T_4 - T_5')} = \frac{1 - T_5/T_4}{1 - T_5'/T_4} \qquad \frac{T_5'}{T_4} = \left(P_5/P_4\right)^{\frac{\gamma-1}{\gamma}}$$

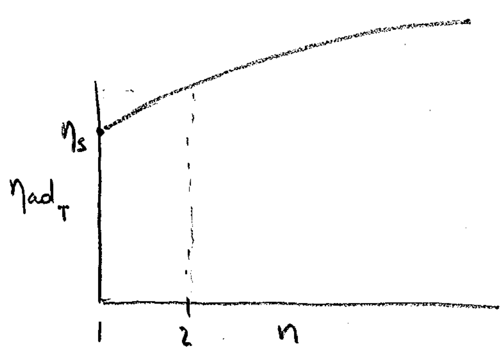
$$= \frac{1 - T_5/T_4}{1 - \left(P_5/P_4\right)^{\frac{\gamma-1}{\gamma}}}$$

$$W_T = C_p T_4 \eta_{ad_T}\left[1 - \left(P_5/P_4\right)^{\frac{\gamma-1}{\gamma}}\right] = C_p T_4\left[1 - T_5/T_4\right]$$

let P_y/P_x = pressure ratio/stage of turbine

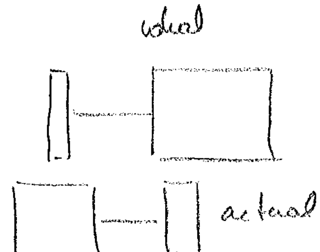
$$P_5/P_4 = \left(P_y/P_x\right)^n$$

$$\eta_{ad_T} = (\eta_s, \dots) = \frac{1 - \left[1 - \eta_s\left\{1 - \left(P_5/P_4\right)^{\frac{\gamma-1}{\gamma n}}\right\}\right]^n}{1 - \left(P_5/P_4\right)^{\frac{\gamma-1}{\gamma}}}$$



P_5/P_4 = const

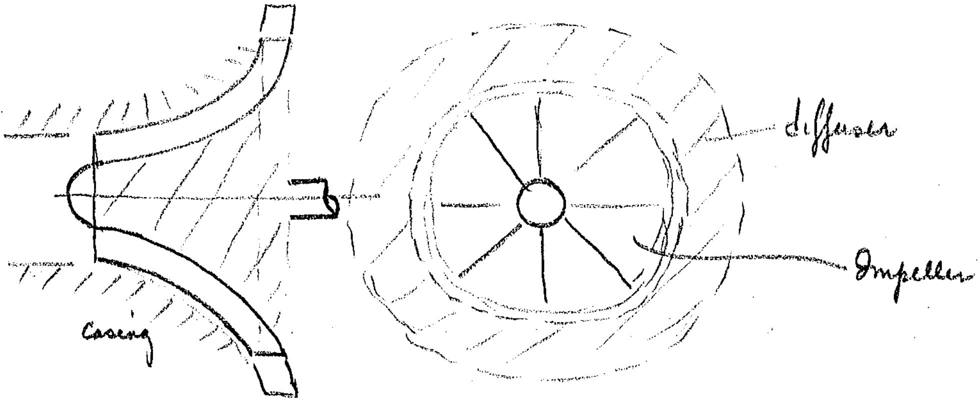
η_s = const

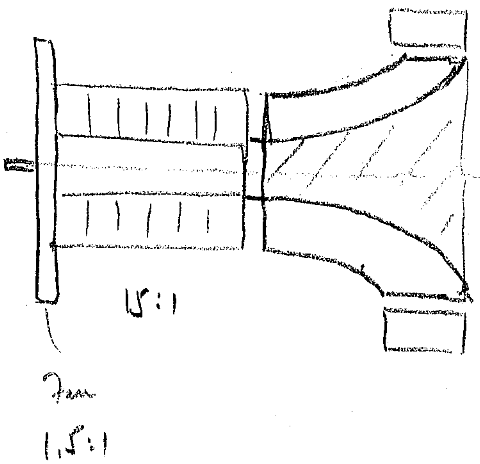


to keep separation from occuring I must have large number of stages in compressor to achieve P_3/P_2 large. If only 1 stage $P_3/P_2 \gg$ will cause large adverse pressure gradient → produce separation.

no. of turbine stages is low since P_5/P_4 is low so no worries about separation

increase in number of turbine stage : will weight ↑ be offset by high $\Delta\eta_{ad_T}$

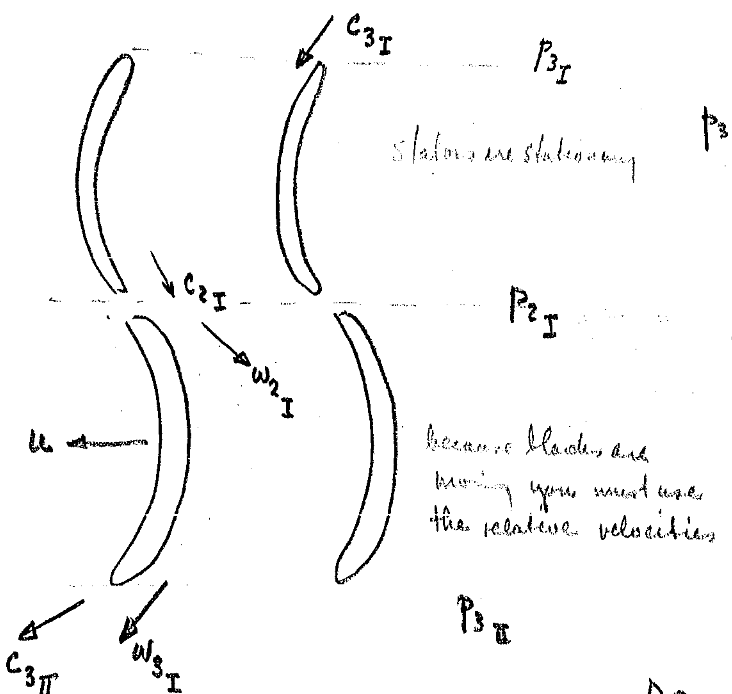




$$\mathcal{H}_c = \frac{\omega}{g}\left[r_3 C_{3u} - r_2 C_{2u}\right]$$

Centr $r_3 >> r_2$

Axial $r_3 \cong r_2$ $\quad \mathcal{H}_c = \frac{\omega r}{g}\left[C_{3u} - C_{2u}\right]$



$$p_{3_I} = p_{2_I} + \frac{1}{2}\rho c_{2_I}^2 - \frac{1}{2}\rho c_{3_I}^2$$

$$p_{2_I} + \frac{1}{2}\rho w_{2_I}^2 = p_{3_{II}} + \frac{1}{2}\rho w_{3_I}^2$$

$$\underset{\text{stator}}{\Delta p_s} = p_{2_I} - p_{3_I} = \frac{1}{2}\rho\left(c_{3_I}^2 - c_{2_I}^2\right)$$

$$\underset{\text{rotor}}{\Delta p_r} = p_{3_{II}} - p_{2_I} = \frac{1}{2}\rho\left(w_{2_I}^2 - w_{3_I}^2\right)$$

$$\Delta p = \Delta p_s + \Delta p_r = \frac{1}{2}\rho\left[\left(c_{3_I}^2 - c_{2_I}^2\right) + \left(w_{2_I}^2 - w_{3_I}^2\right)\right]$$

$$P_{3_I} = p_{3_I} + \frac{1}{2}\rho c_{3_I}^2$$

$$P_{3_{II}} = p_{3_{II}} + \frac{1}{2}\rho c_{3_{II}}^2$$

$$P_{3_{II}} - P_{3_I} = \left(p_{3_{II}} - p_{3_I}\right) + \frac{1}{2}\rho\left(c_{3_{II}}^2 - c_{3_I}^2\right)$$

$$\Delta P = \Delta p + \frac{1}{2}\rho\left(c_{3_{II}}^2 - c_{3_I}^2\right)$$

usually $c_{3_{II}} \cong c_{3_I} \cong c_3$

$$\Delta P \cong \Delta p$$

a) Centrifugal-compressor advantages

1. High pressure rise per state (6)
2. Good efficiencies over wide rotational speed range
3. Simplicity of manufacture and low cost
4. Low weight
5. Starting power requirements are low.

b) Centrifugal-compressor disadvantages

1. Large frontal area for given air flow
2. Two or more stages not practical because of losses in turns between stages.

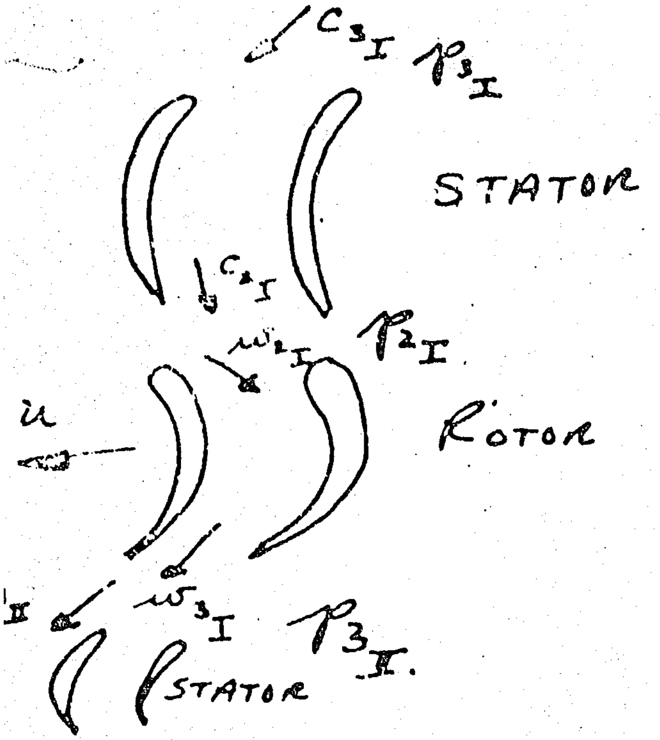
c) Axial-flow compressor advantages

1. High peak efficiencies
2. Low frontal area for given air flow
3. Straight-through flow allows high ram efficiency
4. Increased pressure rise by increasing number of stages with negligible losses.

d) Axial-flow compressor disadvantages

1. Good efficiencies over narrow rotational-speed range
2. Difficulty of manufacture and high cost
3. Relatively high weight
4. High starting power requirements
5. Low pressure rise per stage.

Manufacturer	Name	Pressure ratio at Max RPM	Stages	Max. Envelope Diameter (in.)	Thrust
Curtiss Wright	J65-W-18	---	13	37.5	10500
General Electric	CJ 805-3B	12.5	17	48.	11650
	J79-G19	13.5	17	39.1	17900
Pratt Whitney	J75-P-19W	12.0	15	43	26500
	JT4A-9	12.0	15	43	16800
De Havilland	Ghost	47.1:1	1	~53	5300
	Goblin	3.67:1	1	~50	3500
Rolls Royce	Nine	4.5:1	1	~50	5400



Static pressure change in stator:

$$p_{2_I} + \frac{1}{2}\rho C_{2_I}^2 = p_{3_I} + \frac{1}{2}\rho C_{3_I}^2$$

$$\Delta p_s = p_{2_I} - p_{3_I} = \frac{1}{2}\rho\left[C_{3_I}^2 - C_{2_I}^2\right]$$

Static pressure change ~~in~~ rotor:

$$p_{3_{II}} + \frac{1}{2}\rho w_{3_I}^2 = p_{2_I} + \frac{1}{2}\rho w_{2_I}^2$$

$$\Delta p_R = p_{3_{II}} - p_{2_I} = \frac{1}{2}\rho\left[w_{2_I}^2 - w_{3_I}^2\right]$$

Total change in static pressure

$$\Delta p = \Delta p_s + \Delta p_R = \frac{1}{2}\rho\left[w_{2_I}^2 - w_{3_I}^2 + C_{3_I}^2 - C_{2_I}^2\right]$$

also $P_{3_{II}} = p_{3_{II}} + \frac{1}{2}\rho C_{3_{II}}^2$

$P_{3_I} = p_{3_I} + \frac{1}{2}\rho C_{3_I}^2$

For $C_{3_I} = C_{3_{II}} --- = C_{3_n}$

$\Delta P = \Delta p$

r the change in static pressure across the

change in stagnation pres

DIMENSIONAL ANALYSIS AND COMPRESSOR PERFORMANCE

The general requirements of a gas-turbine compressor are a high pressure ratio coupled with a high efficiency and flow rate. Dimensional analysis may be applied to compressor performance in order to determine the significant dimensional groups ~~w~~ upon which the performance depends. The results of many tests have indicated that for the normal operating range the performance of both the centrifugal and axial flow compressor may be described by the following measurable quantities:

P_2 - inlet total pressure
T_2 - inlet total temperature
gR - gas constants
μ - coefficient of viscosity
P_3 - outlet total pressure
m - mass flow rate
n - rotational speed
D - characteristic diameter

It is possible to write the following functional equation:

$$P_3 = f(P_2, T_2, m, gR, n, \mu, D)$$

π Theorem

A complete physical equation $Q_1 = f(Q_2, Q_3 \cdots Q_N)$ may be expressed in the form of a number of π terms, each π term representing a product of powers of some of the Q's which in terms of primary dimensions form a dimensionless group.

$$\pi_1 = \phi(\pi_2, \pi_3 \cdots \pi_{N-K})$$

where each π is given by

$$\pi = Q_1^a \, Q_2^b \, Q_3^c \, --- \, Q_N^x$$

K is the number of primary dimensions

M - mass; L - length, T - time, β - Temp.

In the compressor analysis there are 8 variables and 4 primary dimensions, therefore $N-K = 4$.

$$P \to \frac{M}{LT^2} \; ; \; T \to \beta \; , \; m \to \frac{M}{T} \; ; \; n \to \frac{1}{T}$$

$$\mu \to \frac{M}{LT} \; ; \; D \to L \; ; \; gR \to L^2/T^2\beta$$

Therefore

$$\frac{M}{LT^2} = \left(\frac{M}{LT^2}\right)^a \beta^b \left(\frac{M}{T}\right)^c \left(\frac{L^2}{T^2\beta}\right)^d \left(\frac{1}{T}\right)^e \left(\frac{M}{LT}\right)^f (L)^g$$

$$1 = a + c + f$$

$$-1 = -a + 2d + g - f$$

$$2 = 2a + c + 2d + e + f$$

$$b = d$$

Solving for a, b, e, g in terms of c, d, f

$$a = 1 - c - f$$

$$b = d$$

$$e = 2 - 2a - c - 2d = 2 - 2 + 2c + 2f - c - 2d - f$$

$$= c - 2d + f$$

$$g = a - 1 - 2d + f = 1 - c - f - 1 - 2d + f = -c - 2d$$

Therefore

$$P_3 = P_2^{\,1-c-f}\; T_2^{\,d}\; m^{c}\,(gR)^{d}\; n^{\,c-2d+f}\; \mu^{f}\; D^{-c-2d}$$

$$= P_2 \left[\frac{m\,n}{D\,P_2}\right]^{c} \left[\frac{T_2\, g R}{n^2 D^2}\right]^{d} \left[\frac{n\mu}{P_2}\right]^{f}$$

$$\pi_1 = P_3/P_2 \quad ; \quad \pi_3 = \frac{n D}{\sqrt{gRT_2}} \;;\; \pi_A = \frac{n m}{D P_2} \;;\; \pi_B = \frac{n\mu}{P_2}$$

$$\pi_2 = \frac{\pi_A}{\pi_3} = \frac{m\, n}{D\, P_2}\cdot\frac{\sqrt{gRT_2}}{n D} = \frac{m\sqrt{gRT_2}}{P_2\, D^2}$$

$$\pi_4 = \frac{\pi_A}{\pi_B} = \frac{m\, n}{D\, P_2}\cdot\frac{P_2}{n\mu} = \frac{m}{D\mu} = \frac{\rho A V}{D\mu} \sim \frac{\rho D V}{\mu} \left\{ \text{Reynolds number} \right.$$

Therefore

$$\frac{P_3}{P_2} = g\left(\frac{m\sqrt{gRT_2}}{D^2\, P_2}\;;\; \frac{n D}{\sqrt{gRT_2}}\;,\; \frac{m}{D\mu}\right)$$

For a given compressor there is no loss in generality ~~by~~ if m is replaced by w, omitting D and gR where they occur; and if T_2 and P_2 are replaced by θ_2 and δ_2, respectively. $\theta_2 = \frac{T_2}{T_{ref}} = \frac{T_2}{520°R}$, $\delta_2 = \frac{P_2}{P_{ref}} = \frac{P_2}{14.7\,psi}$

Therefore

$$\pi_2 \longrightarrow \frac{w\sqrt{\theta_2}}{\delta_2} \quad ; \quad \pi_3 \longrightarrow \frac{n}{\sqrt{\theta_2}}$$

Assuming a constant Mach number before the compressor $V_2 = Ma \sim \sqrt{T_2}$

$\rho = P_2/RT_2 \sim P_2/T_2$

$$\Pi_4 = \frac{m}{D\mu} = K_1 \frac{P_2 V_2}{\mu_2} \sim \frac{P_2}{T_2} \frac{\sqrt{T_2}}{\mu_2} \sim \frac{\delta}{\sqrt{\theta}\,\phi}$$

where

ϕ = viscosity at t_2 / viscosity at 520°R

Reynolds number index = RNI = $\delta_2 / \sqrt{\theta_2}\,\phi_2$

w = weight flow #/sec;

n = revolutions per second.

$w\sqrt{\theta_2}/\delta_2$ = corrected weight flow

$n/\sqrt{\theta_2}$ = corrected R.P.S.

Therefore

$$\frac{P_3}{P_2} = F\left(\frac{w\sqrt{\theta_2}}{\delta_2} ; \frac{n}{\sqrt{\theta_2}} ; \frac{\delta_2}{\sqrt{\theta_2}\,\phi_2} \right)$$

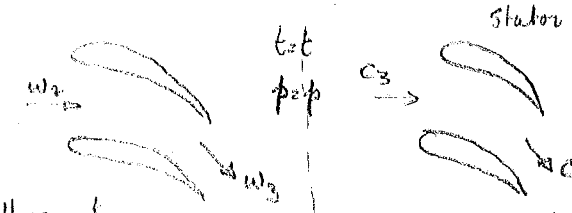
From compressor experiments it is found that P_3/P_2 is almost independent of RNI.

M. Visich

Oct 4, 1961

- %-reaction $= \dfrac{\Delta h_{rotor}}{\Delta h_{rotor} + \Delta h_{stator}}$

$\Delta h_r = C_p (t_3 - t_2)$



$\Delta h_s = \frac{1}{2}(c_3^2 - c_2^2)$

H = const

$\Delta H_{rotor} = \Delta H_{stator} = 0$

$\Delta H_{TOT} \neq 0$

H = const

adiabatic

$\Delta h_r = \frac{1}{2}(w_2^2 - w_3^2)$ $\Delta H = \frac{1}{2}(c_3^2 - w_3^2)$ at interface

P = const P = const

$\Delta P = \frac{1}{2}\rho(c_3^2 - w_3^2)$

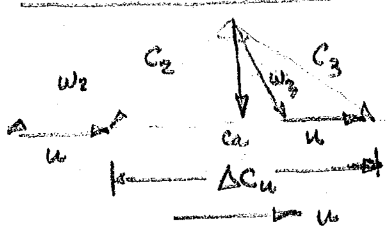
ΔH & ΔP exist because of the fact we are going from a rotating frame to a stationary frame

$$\% \text{Reaction} = \frac{\frac{1}{2}(w_2^2 - w_3^2)}{\frac{1}{2}(w_2^2 - w_3^2 + c_3^2 - c_2^2)} = \frac{(w_2^2 - w_3^2)}{(w_2^2 - w_3^2) + (c_3^2 - c_2^2)}$$

$\Delta p_{rotor} = \rho/_2 (w_2^2 - w_3^2)$

$\Delta p_{stator} = \rho/_2 (c_3^2 - c_2^2)$ $\rightarrow$ % reaction $\dfrac{\Delta p_{rotor}}{\Delta p_{rotor} + \Delta p_{stator}}$

Consider 100 percent reaction stage

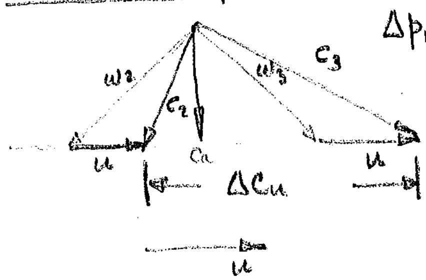


$\Delta p_s = 0$ $c_3 = c_2$ $\mathcal{H} \sim C_{3u} - C_{2u}$

$\Delta p_r \neq 0$ $c_r = 0$

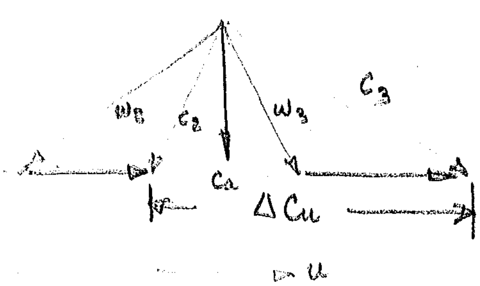
Consider a 0 percent reaction stage (impulse stage)

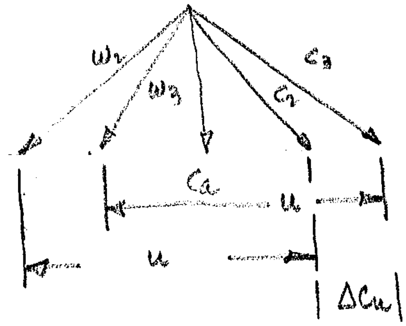
$\Delta p_r = 0$ $W_2 = W_3$



Consider a 50% reaction

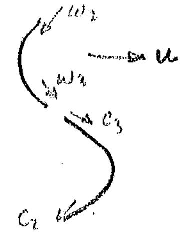
$\Delta p_r = \Delta p_s$ $w_2 = c_3$, $w_3 = c_2$

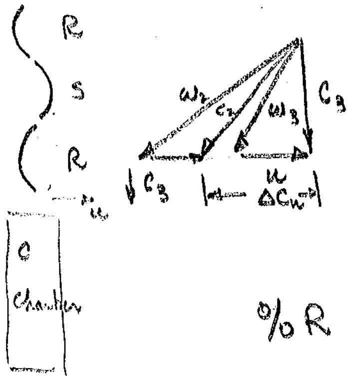




since $p_3/p_2 \cong 1.17$

$\therefore$ $w_3 \sim w_2$



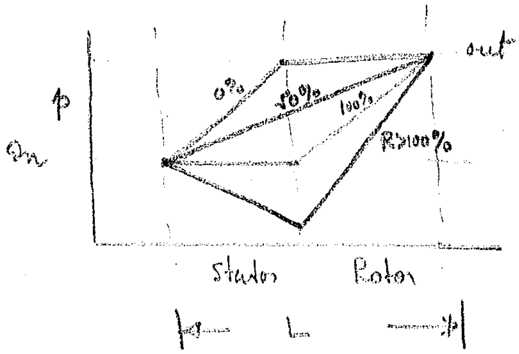


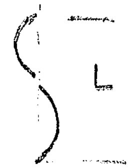
$\mathcal{H}_c \sim C_{3u} - C_{2u}$ $\quad C_{3u} = 0 \quad C_{2u} < 0$

Δp_r (+)

Δp_s (−)

%R > 100%



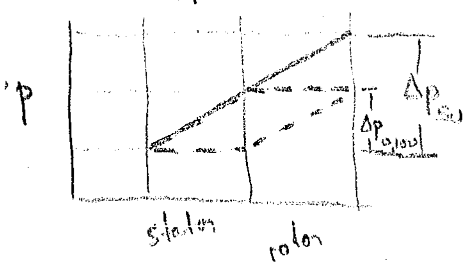


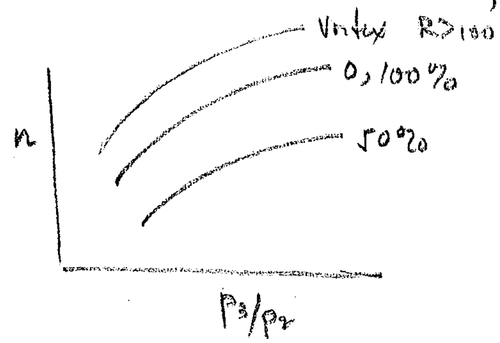
100% react

To min length of compressor

$\frac{\Delta p}{\Delta x} \to$ 50% reaction · $\left(\frac{\Delta p}{\Delta x}\right)_{S,R}$ is smallest ∴ separation will be least likely

for same p_3/p_2

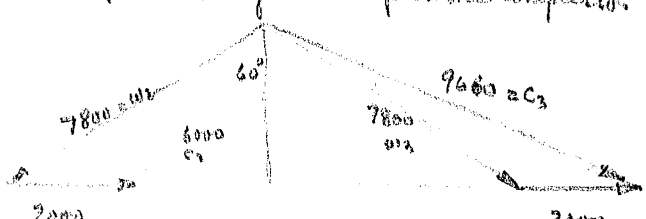


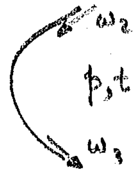


Impulse stage – supersonic compressor $\quad t_2 = 418°R \quad a_2 = 1000$ ft/sec

$M_2 = 6.0 \quad C_2 = 6000 \quad u = 2000$ ft/sec

$w_2 = w_3$



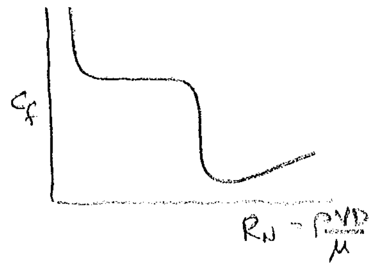


p, t const

$M_3 = 9.66$ since $p_2 = p_3$

$$\left(\frac{p}{P}\right)_{M=6} \Big/ \left(\frac{p}{P}\right)_{M=9.66} = 21.4$$

$$\left(\frac{t}{T}\right)_{M=6} \Big/ \left(\frac{t}{T}\right)_{M=9.66} = 2.4$$



force on body $= f(\rho, V, d, \mu, t, gR)$

$N = 7$ no of variables $\quad K = 4$ primary dimension $\quad L, T, M, \beta$ β = beta

$$\frac{F}{\rho V^2 d^2} = \varphi\left[\frac{\rho V d}{\mu}, \frac{V}{\sqrt{Rgt}}\right] \qquad \pi_1 = C_f \sim \frac{F}{\frac{1}{2}\rho V^2\left(\frac{\pi d^2}{4}\right)}$$

$$\pi_2 = R_N \qquad \pi_3 = \frac{V}{\sqrt{\gamma g R T}} = M$$

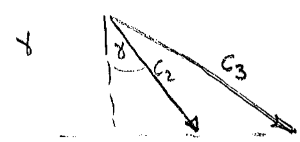
$P_3 = f(P_2, T_2, \text{Massflow}, gR, n = \text{rotational speed}, \mu, \text{Diameter})$

$N = 8 \quad K = 4$

2-22-72

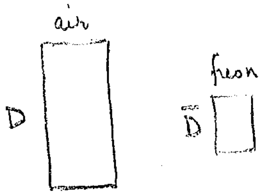
No Class 2-24-72 ; Exam 3/2/72 up to compressor

Assume $C_2 = 400$ ft/sec
$u = 1000$ ft/sec $\quad P_y/P_x = 1.15$
$\eta_c = .9$ plot % react as a function of γ



Sketch accurately the blades for $\gamma = 30°$ & $\gamma = 60°$

$$P_3/P_2 = g\left[\frac{m\sqrt{gRT_2}}{D^2 P_2}, \frac{nD}{\sqrt{gRT_2}}, \frac{m}{D\mu}\right] \qquad \pi_1 = g\left[\pi_2, \pi_3, \pi_4\right]$$



$$\frac{m_{air}}{D\mu_{air}} = \frac{m_{freon}}{D\mu_{freon}} \qquad \left[\frac{m\sqrt{gRT_2}}{D^2 p_2}\right]_F = \text{const} \rightarrow \left[\sqrt{T_2}/P_2\right]_F = K_1$$

$$\left[\frac{nD}{\sqrt{gRT_2}}\right]_F = \text{const} \rightarrow \left[\frac{n}{\sqrt{T_2}}\right]_F = K_2$$

n is to be limited due to centrifugal effect on element.

neglect D, R

define $\Theta = T/T_{ref} = 519,520°R$

$$\delta = \frac{P}{P_{ref}} = \frac{P}{14.7\,psi} = \frac{P}{2116\,psf}$$

$$\pi_1 = P_3/P_2 \qquad \pi_2 = \frac{m\sqrt{gRT_2}}{D^2 P_2} \sim \frac{\omega\sqrt{\theta_2}}{\delta_2} \text{ corrected weight flow} \qquad \pi_3 = \frac{nD}{\sqrt{gRT_2}} \sim \frac{N}{\sqrt{\theta_2}}$$

N rpm of system

$\frac{2\pi N}{60} = n$ corrected rpm.

$$\pi_4 = \frac{m}{D\mu} \quad (\text{let } M_2 \sim \text{constant}) \quad V_2 = M_2\sqrt{\gamma R t_2} \sim \sqrt{t_2} \sim \sqrt{T_2}$$

$$\rho_2 = p_2/_R T_2 \sim p_2/T_2 \sim P_2/T_2$$

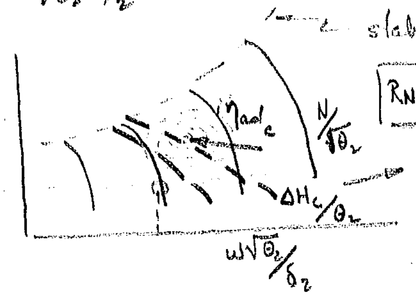
define $\phi = \frac{\mu}{\mu_{ref}} = \frac{\mu}{\mu_{T=520°F}}$ $\qquad\qquad$ $\phi, \delta, \theta = 1$ for SSL

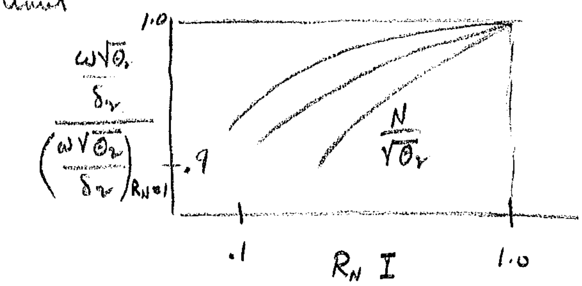
$$\Pi_4 = \frac{\rho A V}{D\mu} \sim \frac{\rho D V}{\mu} \sim \frac{P_2}{T_2}\frac{\sqrt{T_2}}{\mu} \sim \frac{\delta_2}{\sqrt{\theta_2}\,\phi_2} \quad R_N \text{ Index}$$

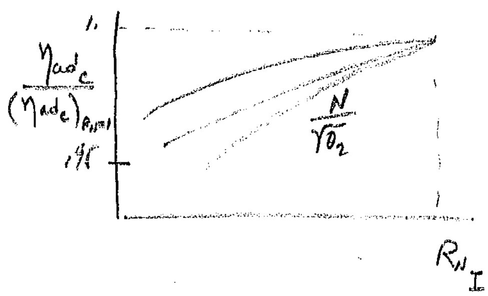
P_3/P_2 : Corrected weight flow $- \frac{w\sqrt{\theta_2}}{\delta_2}$ $\qquad$ if compressor has these 3 equal, 2 compressors are same

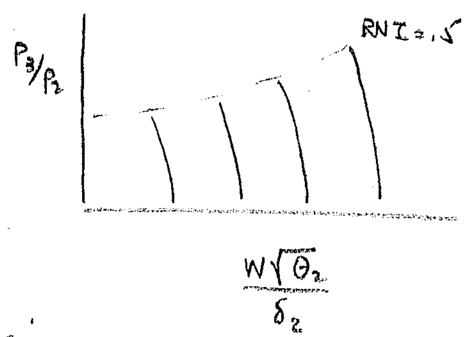
corrected RPM $- \frac{N}{\sqrt{\theta_2}}$

R_N Index $- \frac{\delta_2}{\sqrt{\theta_2}\,\phi_2}$ $\qquad$ least effect on performance







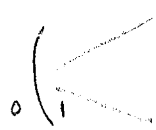


get P_3/P_2 & $\left(\frac{w\sqrt{\theta_2}}{\delta_2}\right)_{R_N=}$ go to 2 charts & get $\left(\frac{w\sqrt{\theta_2}}{\delta_2}\right)_{R_N=.1}$

$\frac{w\sqrt{\theta}}{\delta}$ will change throughout engine

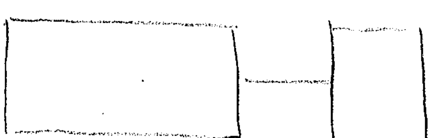
in nozzle flow is adiabatic T = const, no shock ∴ p = const

with normal shock



$\frac{\text{corrected}_0}{\text{corrected}_1} = \frac{P_0}{P_1}$ since T = const.

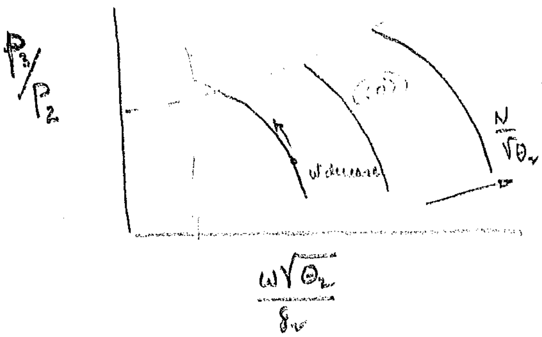
$$\frac{w\sqrt{\theta_2}}{\delta_2} = \frac{w\sqrt{\theta_4}}{\delta_4} \cdot \sqrt{\frac{T_2}{T_4}}\,\frac{P_4}{P_2}$$

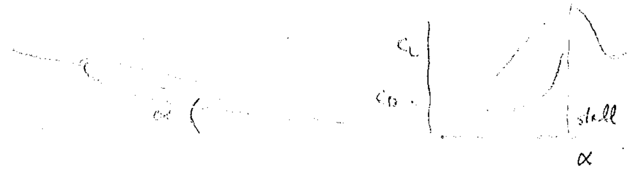


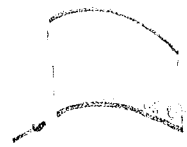
$\mathcal{W}_T = \mathcal{W}_c + \Delta\mathcal{W}$ $\quad$ as first approx $\mathcal{W}_T = \mathcal{W}_c$

$$\frac{\mathcal{W}_c}{\theta_2} = \frac{\Delta H_c}{\theta_2} = \frac{c_p(T_3 - T_2)}{\theta_2} = c_p T_{ref}\left(T_3/T_2 - 1\right)$$

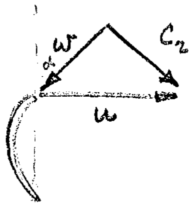
$\eta_{ad_c} = f\left(P_3/P_2, T_3/P_2\right)$ $\qquad$ $\dfrac{\Delta H_c}{\theta_2} = \dfrac{c_p T_{ref}}{\eta_{ad_c}}\left[\left(P_3/P_2\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$



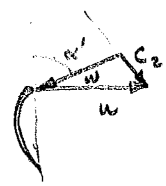




once blade stalls out get compressor surge.

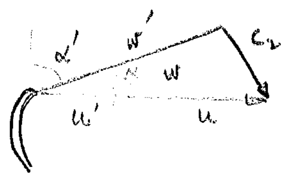


assume C_2 decreases if w decreases



if $\alpha' > \alpha_{stall}$

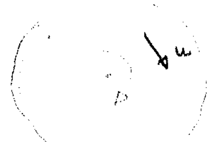
if w remains & u increases



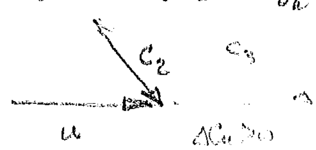
rotating stall, high altitudes, α_{stall} decreases

exam up to here:

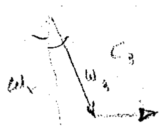
in axial flow no comp in r direct



$W_c > 0 \quad \Delta C_u = C_{3_u} - C_{2_u} > 0$



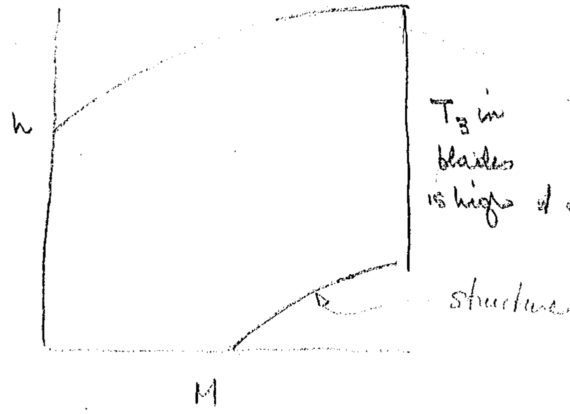
100% R. $\quad w_3 \neq w_2$
$c_3 = c_1 \quad \Delta p_s = 0$



$W_c = \dfrac{u \Delta C_u}{g}$ $\qquad$ if velocity diag is const $\quad W_c = \text{const}$
η_{ad_c} changes

2-28-72

types of equations – definitions $p \sim \rho^\gamma$
derivations
know engines



T_3 in blades is high at turbine blades.

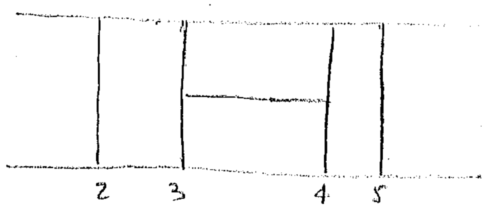
compressor purpose make sure pressure in combustor is high enough

$p/P_{M=3} = .025$ $P = 40$ atm.

at entrance of compressor

if $P_3/P_2 = 10$ $P_3 = 400$

$\therefore$ wall thickness is very large to sustain



$$W_T = C_p(T_4 - T_5) = \frac{u}{g}(C_{4u} - C_{5u}) = C_p T_4 \eta_{ad_T}\left[1 - (P_5/P_4\right.$$

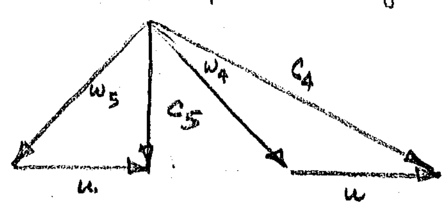
$C_p = .276$ $\gamma = 1.33$

$P_5/P_4 < 1$ $T_5/T_4 < 1$

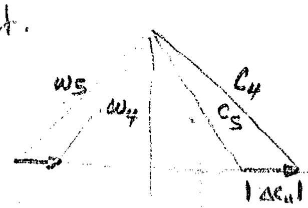
$$\%\text{ reaction} = \frac{(w_5^2 - w_4^2)}{(w_5^2 - w_4^2) + (C_4^2 - C_5^2)}$$

For our compressor Impulse stage $w_5 = w_4$

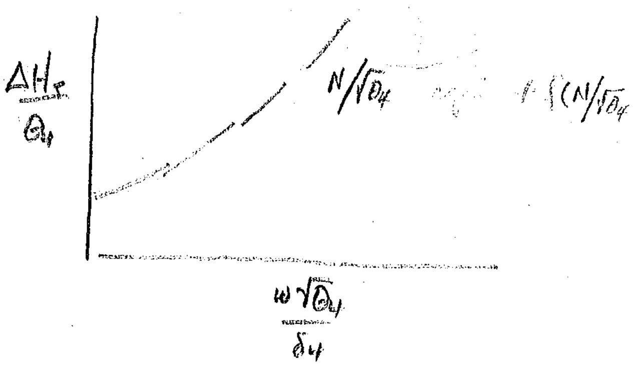
no stators needed

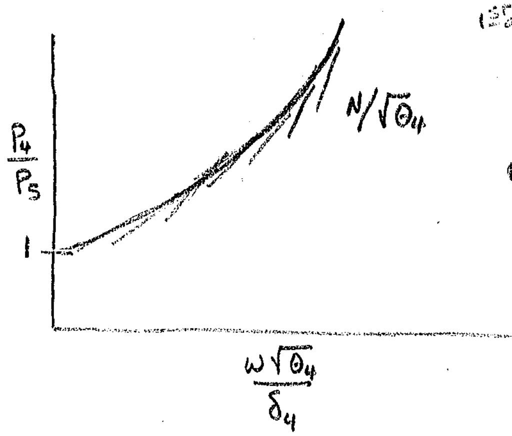


50% react.



$w_5 = c_4$
$c_5 = w_4$





approx

P_4/P_5 is not a fn of corrected rpm.

at take off specify $T_{4\,max}$ $\therefore$ V_7 is high

$V_7 = \sqrt{g\gamma R T_7} \sim \sqrt{T_7} \sim \sqrt{T_5}$ adiab

$W_T = C_p(T_4 - T_5)$ if $T_4 \uparrow$ $T_5 \uparrow$ $\therefore V_7 \uparrow$

P&W   JT 18D $T_4 \sim 2000°F$ $T_{T.O.} = 35{,}000$ lb

$T_4 \sim 2150°F$ $T_{T.O.} = 36{,}600$ lb.

JT 9D-3

$T_4 = 2100°F$

(1) $t_0 = 80°F$ — 43,500 lb

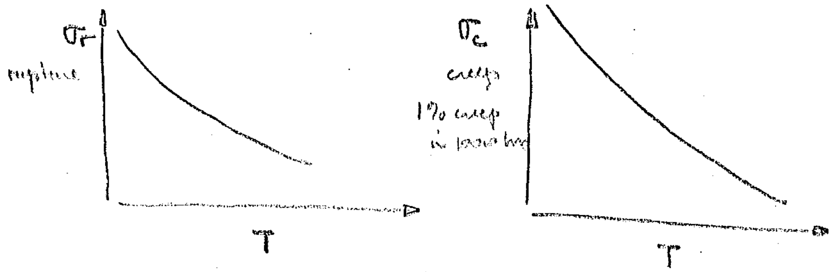
(2) $t_0 = 90°F$ — 42,000 lb

$T_3 \uparrow$

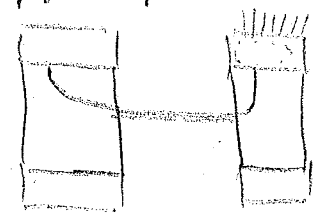
$\Delta H_{\text{combustion chamber}}$ depends of energy liberated in chamber

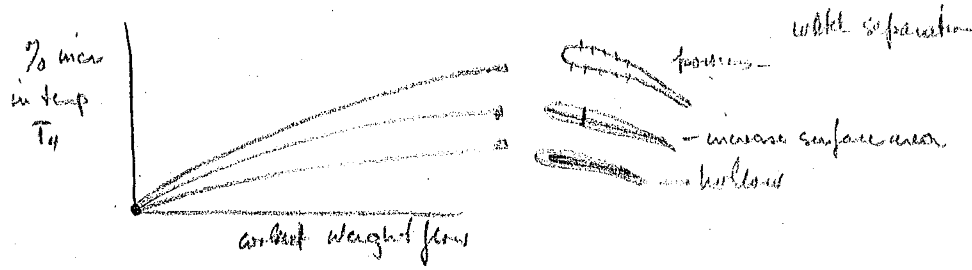
amount of fuel in (1) > amount of fuel in (2)

$T_{3(1)} < T_{3(2)}$

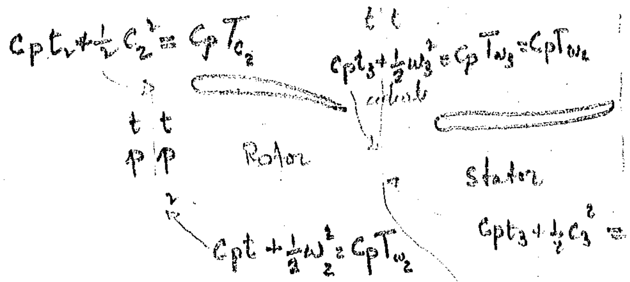


Cooling Scheme for turbine blades. take air out from compressor pump them to turbine blade & dump from top.

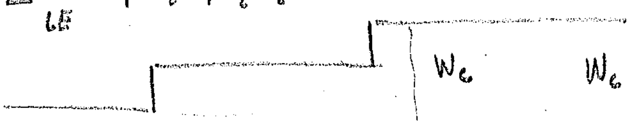




$$W_c = C_p (T_3 - T_2) = \frac{u}{g}\left(c_{3u} - c_{2u}\right) = \frac{1}{2g}\left[\left(c_3^2 - c_2^2\right) + \left(w_2^2 - w_3^2\right)\right]$$

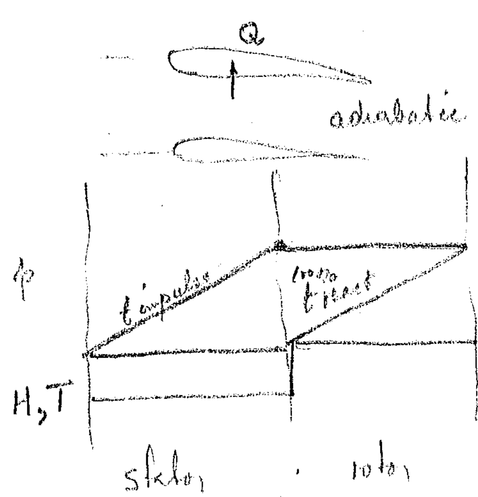


$$\Delta H_{LE} = c_p T_{w_2} - c_p T_{c_2} = \frac{1}{2}\left(w_2^2 - c_2^2\right) \qquad \Delta H_{TE} = c_p\left(T_{c_3} - T_{w_3}\right) = \frac{1}{2}\left(c_3^2 - w_3^2\right)$$



W_c

$$W_c = \Delta H_{LE} + \Delta H_{TE} = \frac{1}{2}\left(c_3^2 - c_2^2\right) + \frac{1}{2}\left(w_2^2 - w_3^2\right)$$



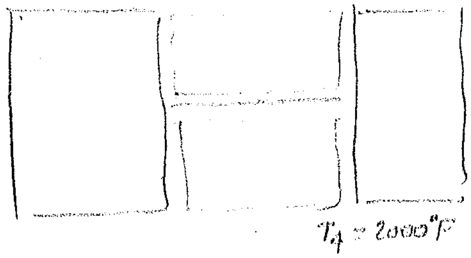
adiabatic in passage

talk always about relative velocity to body

impulse stage $\Delta p_r = 0$ V const t = const rotor

incomp. flow $\rho \approx C$ $p = \rho RT$ due to change in velocity.

Combustion Chambers.



Stoichiometric

$\boxed{\text{fuel}} + \boxed{\text{oxidz}} \longrightarrow \boxed{\text{Product combust}}$ $\neq$ fuel or oxidiz.

air

$\frac{79}{21} N_2 + 2H_2 + O_2 \longrightarrow 2H_2O + \frac{79}{21} N_2$

$(2+6)H_2 + O_2 \longrightarrow 2H_2O + 6H_2$ fuel rich

$2H_2 + (1+6)O_2 \longrightarrow 2H_2O + 6O_2$ fuel lean

$$\left(\frac{w_f}{w_o}\right)_{stoich} = \frac{1}{8} \quad 2\cdot 2/32$$

$$\left(\frac{w_f}{w_o}\right)_{rich} > \frac{1}{8}$$

Equiv Ratio $= \dfrac{\left(\frac{w_f}{w_o}\right)}{\left(w_f/w_o\right)_{stoich}} = \varphi$

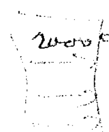
stoich $\varphi = 1$

rich $\varphi > 1$

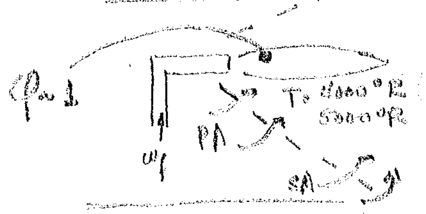
lean $\varphi < 1$

$\boxed{\text{Fuel}} + \boxed{O_2 \mid N_2} \longrightarrow \boxed{\text{Prod of Comb}} + \boxed{N_2}$

stable combustion occurs at roughly stoich. ($\varphi \sim O(1)$)



$\varphi < 1$ fuel lean. $\approx .1 - .4$

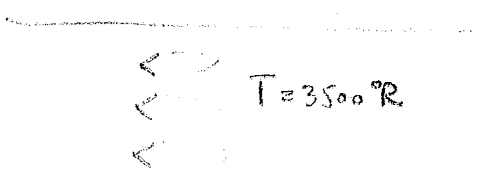


∴ conflict with $T_4 = 2000°F$

in vicinity of jet, primary air combines at stoich with fuel

secondary air reduces temp of product of comb to get to T_4

Ramjet Engine

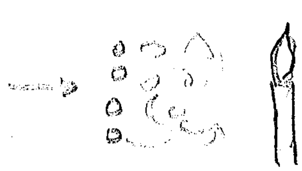


propagation speed of combustion

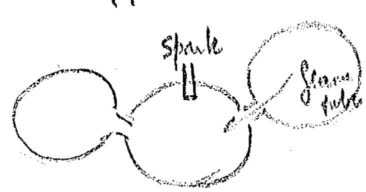
= velocity that will blow out flame

$V_{P_{laminar}} \sim 5$ to 15 ft/sec

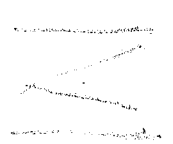
to increase V_{P_t} put screen in front of flame to generate turbulence



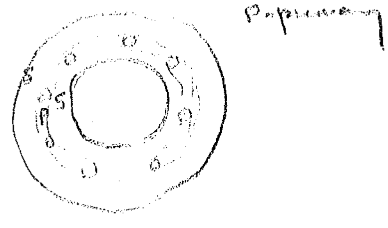
turbulence $\xrightarrow{\text{induces}}$ pressure losses ∴ pressures losses must be kept to min.

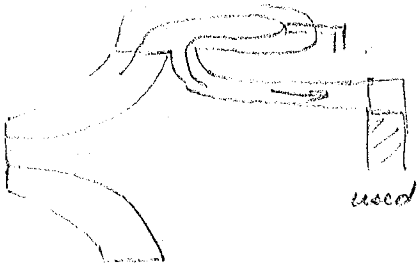


one surface seen



2D - wrap around ⟳.





loss of pressure due to ; counter flow
turn flow blade never feels flame.

used to reduce size of engine

$$\eta_b = \frac{\text{actual heat released}}{\text{Ideal Heat Released}} \qquad \dot{Q} > 0 \quad W \equiv 0 \quad = \frac{(W_a + W_f) H_4 - H_3 W_a - H_f W_f}{W_f\,(HV)}$$

defined by enthalpy

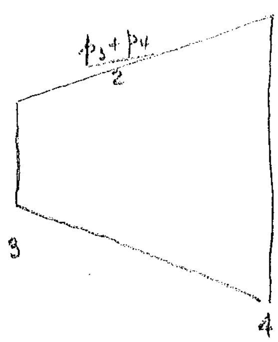
$$\eta_b \approx \frac{W_a (H_4 - H_3)}{W_f\, HV} = \frac{C_p (T_4 - T_3)}{\left(\frac{W_f}{W_a}\right) H.V.}$$

jet gasoline $HV \sim 18-19000$ BTU/lb.

JP4 + Air $\longrightarrow \underbrace{H_2O}_{\text{steam}} + \dots$ heat released corresponds to lower heating value of fuel

$\longrightarrow \underbrace{H_2O}_{\text{liquid}} + \dots$ higher heating value of fuel more heat released
difference is heat of vaporization recovered.

Quasi-1-D flow: 2 dim problem where flow is assumed uniform in one direct do not satisfy b.c.

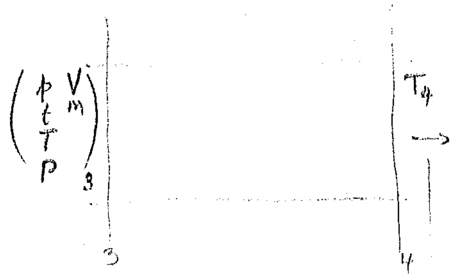


$$\rho_3 A_3 V_3 = \rho_4 A_4 V_4$$

$$p_3 A_3 - p_4 A_4 + \left(\frac{p_3+p_4}{2}\right)(A_4 - A_3) = m(V_4 - V_3)$$

$$\frac{1}{2}(p_3 - p_4)(A_4 + A_3) = m(V_4 - V_3)$$

Assume uniform



$$\rho_3 V_3 = \rho_4 V_4$$

$$p_3 - p_4 = \frac{W}{gA}(V_4 - V_3)$$

$$p_4/p_3 = \frac{1+\gamma M_3^2}{1+\gamma M_4^2} \qquad \frac{t_4}{t_3} = \left[\frac{1+\gamma M_3^2}{1+\gamma M_4^2}\right]^2 \left(\frac{M_4}{M_3}\right)^2$$

p_4
t_4
V_4
M_4
$\vdots$

$$T_4 = t_4\left[1 + \frac{\gamma - 1}{2} M_4^2\right] \quad \text{point funct}$$

$$\frac{T_4}{T_3} = \frac{t_4}{t_3}\left[\frac{1 + \frac{\gamma-1}{2} M_4^2}{1 + \frac{\gamma-1}{2} M_3^2}\right].$$

$$M_4 = f\left(\frac{T_4}{T_3}, M_3\right)$$

if M_3, T_3 known differential form

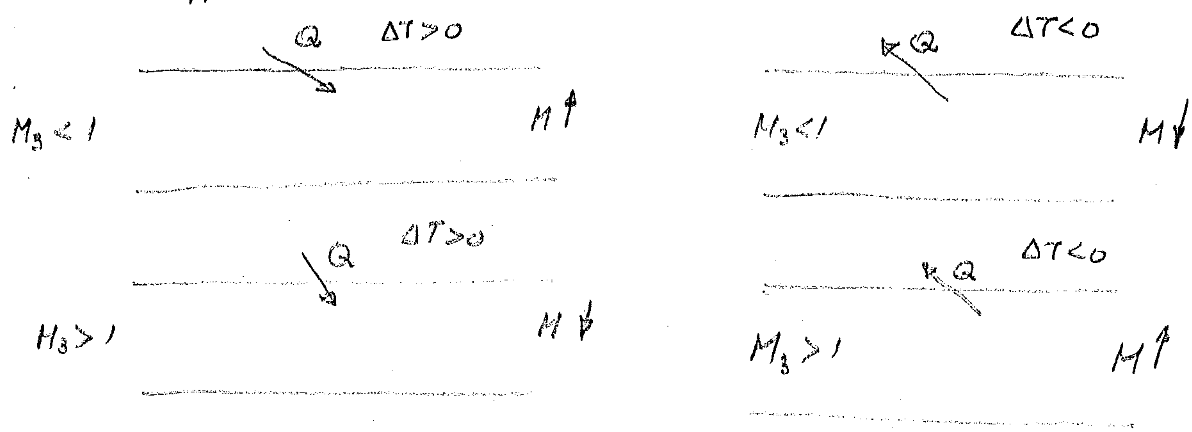
$$\frac{T_4\,(1+\gamma M_4^2)^2}{M_4^2\left(1+\frac{\gamma-1}{2}M_4^2\right)} = \text{constant} = \frac{T_3\,(1+\gamma M_3^2)^2}{M_3^2\left(1+\frac{\gamma-1}{2}M_3^2\right)}$$

$$T_4 = \frac{C\,M_4^2\left(1+\frac{\gamma-1}{2}M_4^2\right)}{(1+\gamma M_4^2)^2}$$

$$\log T_4 = \log C + \log M_4^2 + \log\left(1+\tfrac{\gamma-1}{2}M_4^2\right) - 2\log\left(1+\gamma M_4^2\right)$$

$$\frac{dT_4}{T_4} = \frac{dM_4^2}{M_4^2} + \frac{\frac{\gamma-1}{2}\,dM_4^2}{1+\frac{\gamma-1}{2}M_4^2} - 2\,\frac{\gamma\,dM_4^2}{1+\gamma M_4^2}$$

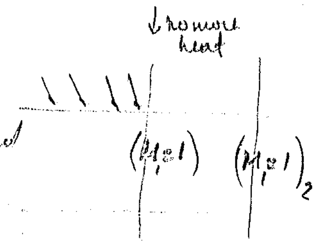
$$\text{Eq}(10) = \frac{dM^2}{M^2} = \frac{(1+\gamma M^2)\left(1+\frac{\gamma-1}{2}M^2\right)}{1-M^2}\,\frac{dT}{T}$$



$$\left(\frac{dT}{dM}\right)_{M=1} = 0 \qquad T \text{ is max} \qquad \left.\frac{d^2T}{d(M^2)^2}\right|_{M=1} = \frac{-2\,T_{M=1}}{(1+\gamma)^2}$$

Thermal Choking
Cannot add more heat to flow at $M=1$

physically



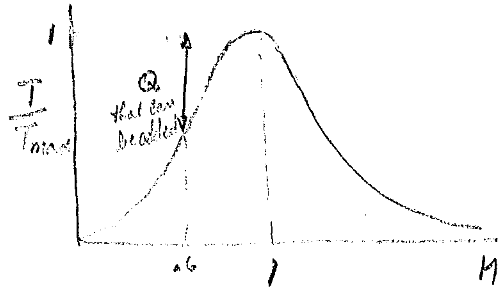
as more heat is added

M_1 mod $\downarrow$, $M_2<M_1$

and $M=1$ pt moves more into duct

readjustment so that more heat can be added.

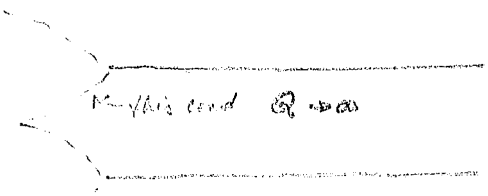
Pg 228 : 1, 3, 4



$$\frac{T}{T_{max}} = \frac{2(\gamma+1)M^2\left(1+\frac{\gamma-1}{2}M^2\right)}{(1+\gamma M^2)^2}$$

constant area tube as $T/T_{max}\downarrow$ $Q\uparrow$

as $M \to \infty$ $\quad \frac{T}{T_{max}} \approx \frac{2(\gamma+1)\cancel{M^2}\left(\frac{\gamma-1}{2}\right)M^2}{\gamma^2 \cancel{M^4}} = \frac{\gamma^2-1}{\gamma^2}$



M_3	$(T_4/T_3)_{max}$
0	∞
.05	82
.1	22
.2	5.7
.3	2.9
.4	1.9
.5	1.5
.75	1.2
1.0	1.0
2	1.26
∞	2.04

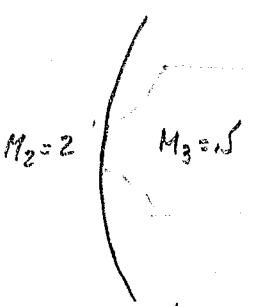
$M_3 = .4$ $\qquad M_4 = 1$ $\qquad \left(\frac{T_4}{T_3}\right)_{max} = 1.9$

$M_3 = .2$ $\qquad \left(\frac{T_4}{T_3}\right)_{max} = 5.7$

Supersonic

$M_3 = 2$ $\qquad \left(T_4/T_3\right) = 1.26$

$M_4 = 1.0$



$(T_4/T_3) = 1.26$

if $(T_4/T_3) = 1.5$

$M = 1$

readj occurs at front of combustion chamber strong shock.

non isentropic

Eg 7 $\quad t = \frac{M^2}{(1+\gamma M^2)^2}$ constant

$\log t = \log M^2 - 2\log(1+\gamma M^2) + \log\text{const}$

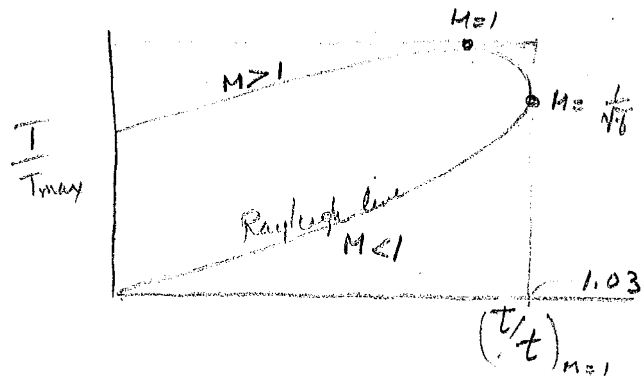
$\frac{dt}{t} = \frac{dM^2}{M^2} - \frac{2\gamma\, dM^2}{1+\gamma M^2}$

$$\boxed{\frac{dt}{t} = \frac{\left(1+\frac{\gamma-1}{2}M^2\right)(1-\gamma M^2)}{1-M^2}\frac{dT}{T}}$$

$M < \frac{1}{\sqrt{\gamma}}$ $\qquad \frac{dt}{t} \approx \frac{dT}{T}$

$\frac{1}{\sqrt{\gamma}} < M < 1$ $\qquad \frac{dt}{t} \sim -\frac{dT}{T}$

$M > 1$ $\qquad \frac{dt}{t} \sim \frac{dT}{T}$



Eq (13) change in stag pressure in combustion chamber coming in with as low a mach no. as possible.

if const pressure $\rho V dV + dp = 0$ leads to $dV = 0$ const veloc.

$$\frac{A_4}{A_3} = \left(\frac{M_3}{M_4}\right)^2 \qquad \frac{T_4}{T_3} = \left(\frac{M_3}{M_4}\right)^2 \left[\frac{1 + \frac{\gamma-1}{2} M_4^2}{1 + \frac{\gamma-1}{2} M_3^2}\right]$$

no difficulty with thermal choking

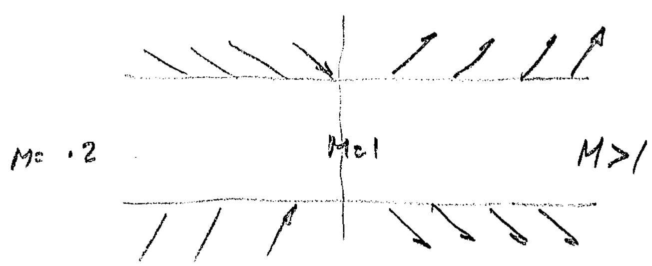
if $\frac{T_4}{T_3} = 2$

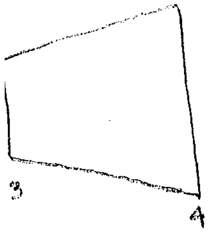
constant area $M_3 = .3$, $M_4 = .5$ $\quad p_4/p_3 = .834 \quad P_4/P_3 = .929$ $\qquad$ turbo jet

$M_3 = 2$ no solution thermal choking

Const pressure $M_3 = .3$, $M_4 = .2$ $\quad A_4/A_3 = 2.25 \quad P_4/P_3 = .966$ $\qquad$ Scramjet type

$M_3 = 2$, $M_4 = 1.2$ $\quad P_4/P_3 = .309 \quad \frac{A_4}{A_3} = 2.78$





p = constant

1-D flow $\quad \rho V dV + dp = 0 \qquad V$ = constant

$$V_4 = V_3$$

$$M_4\sqrt{\gamma R t_4} = M_3\sqrt{\gamma R t_3} \qquad M_4\sqrt{t_4} = M_3\sqrt{t_3} \qquad \frac{t_4}{t_3} = \left(\frac{M_3}{M_4}\right)^2$$

$$\frac{P_4}{P_3} = \left[\frac{1+\frac{\gamma-1}{2}M_4^2}{1+\frac{\gamma-1}{2}M_3^2}\right]^{\gamma/\gamma-1}, \qquad \frac{T_4}{T_3} = \frac{t_4}{t_3}\left[\frac{1+\frac{\gamma-1}{2}M_4^2}{1+\frac{\gamma-1}{2}M_3^2}\right] = \left(\frac{M_3}{M_4}\right)^2\left[\frac{1+\frac{\gamma-1}{2}M_4^2}{1+\frac{\gamma-1}{2}M_3^2}\right]$$

$$\rho_3 A_3 V_3 = \rho_4 A_4 V_4$$

$$\frac{\rho_3 V_3^2}{\gamma p_3} A_3 = \frac{\rho_4 V_4^2}{\gamma p_4} A_4 \qquad \text{since } V \text{ \& } p \text{ are const}$$

$$M_3^2 A_3 = M_4^2 A_4 \qquad \boxed{\frac{A_4}{A_3} = \left(\frac{M_3}{M_4}\right)^2 = \frac{t_4}{t_3}}$$

$$T = K\,\frac{1+\frac{\gamma-1}{2}M^2}{M^2} \qquad \log T = \log K + \log\left[1+\tfrac{\gamma-1}{2}M^2\right] - \log M^2$$

$$\frac{dT}{T} = \frac{\frac{\gamma-1}{2}dM^2}{1+\frac{\gamma-1}{2}M^2} - \frac{dM^2}{M^2}$$

$$= \frac{dM^2}{M^2}\left[\frac{\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M^2} - 1\right] = \frac{dM^2}{M^2}\left[\frac{-1}{1+\frac{\gamma-1}{2}M^2}\right]$$

as $\Delta T > 0 \quad \Delta M^2 < 0$

to balance

$$P = K\left(1+\tfrac{\gamma-1}{2}M^2\right)^{\gamma/\gamma-1} \qquad \frac{dP}{P} = \frac{\frac{\gamma}{\gamma-1}\left(\frac{\gamma-1}{2}\right)dM^2}{1+\frac{\gamma-1}{2}M^2} = \frac{\frac{\gamma}{2}\,dM^2}{1+\frac{\gamma-1}{2}M^2}$$

as $\Delta M^2 < 0 \qquad \Delta P < 0$

$$\frac{dP}{P} = -\gamma/2\, M^2\,\frac{dT}{T}$$

constant velocity & const pressure combustion to get const stat. temp combs use same method of handling equations.

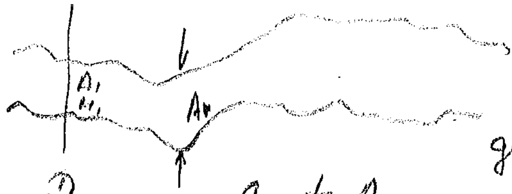
$$m = \rho A V \qquad V = Ma = \frac{Ma^2}{a} = \frac{M\gamma p}{\sqrt{\gamma R t}\,\rho}$$

$$= \frac{\rho A M\sqrt{\frac{\gamma}{R}}\,p}{\rho\sqrt{t}} = \frac{\sqrt{\frac{\gamma}{R}}AMp}{\sqrt{t}} = \frac{\sqrt{\frac{\gamma}{R}}\,AMP\left[1+\frac{\gamma-1}{2}M^2\right]^{-\gamma/\gamma-1}}{\sqrt{T}\left[1+\frac{\gamma-1}{2}M^2\right]^{-1/2}}$$

$$m = \sqrt{\frac{\gamma}{R}}\frac{AP}{\sqrt{T}}M\left[1+\frac{\gamma-1}{2}M^2\right]^{-\frac{(\gamma+1)}{2(\gamma-1)}} = \sqrt{\frac{\gamma}{R}}\frac{A^*P^*}{\sqrt{T^*}}\left[\frac{\gamma+1}{2}\right]^{-\frac{(\gamma+1)}{2(\gamma-1)}}$$

can be used between any 2 pts which are in isentropic sect.

$$\frac{A}{A_*} = \frac{1}{M}\left(\frac{1+\frac{\gamma-1}{2}M^2}{\frac{\gamma+1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

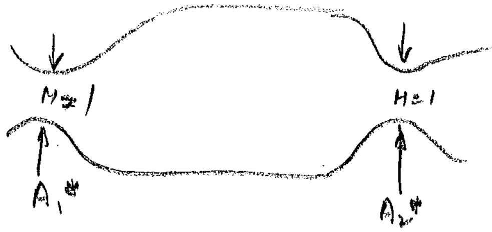


given $A = A(x)$

go to $\frac{A}{A_*}$ get A_* fictitious

check to see if $A \le A_*$ if $A_* \geq A$ then chocked flow. ∴ not isentropic
will adjust somewhere

if $A = A_*$ somewhere from 1 to A_* subsonic
from A_* on depends on p



$A_1^* \neq A_2^*$?

$$\cancel{\sqrt{\frac{\gamma}{R}}}\ \frac{A_1^* P_1^*}{\cancel{\sqrt{T_1^*}}}\ \cancel{\left(\frac{\gamma+1}{2}\right)^n} = \cancel{\sqrt{\frac{\gamma}{R}}}\ \left(\frac{A_2^* P_2^*}{\cancel{\sqrt{T_2^*}}}\right)\cancel{\left(\frac{\gamma+1}{2}\right)^n}$$

$$A_1^* P_1^* = A_2^* P_2^*$$

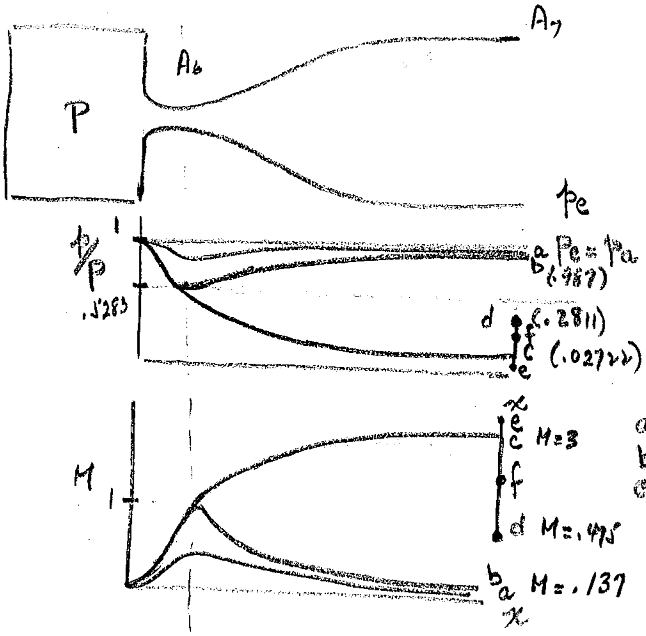
$$\frac{A_2^*}{A_1^*} = \frac{P_1^*}{P_2^*}$$

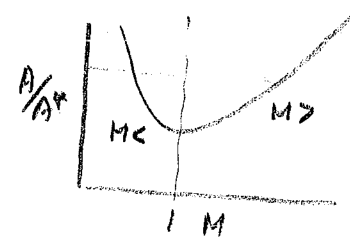
if $A_2^* > A_1^*$ $P_1^* > P_2^*$ ∴ ∃ a shock in syst
non isentropic

$$\frac{dA}{A} = \frac{f(M)}{1-M^2}\ \frac{dM^2}{M^2}$$

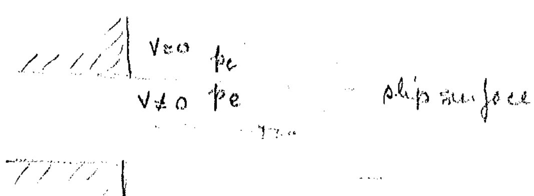
if $\frac{A}{A_*} = f(M)$

if $M < 1$ if $dA > 0$ $dM(-)$
$dA < 0$ $dM(+)$



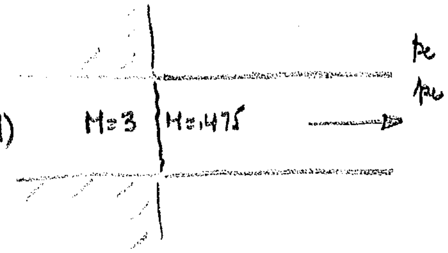


$A_7/A_7 = 4.235$

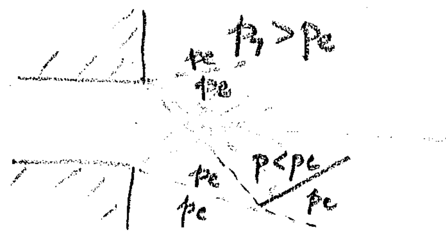


$p/p_b = .987$

d)

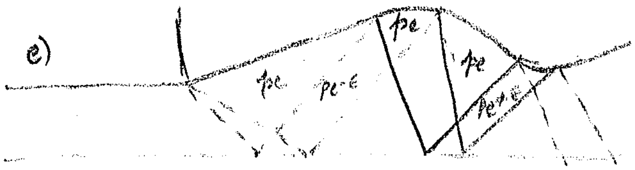


e)

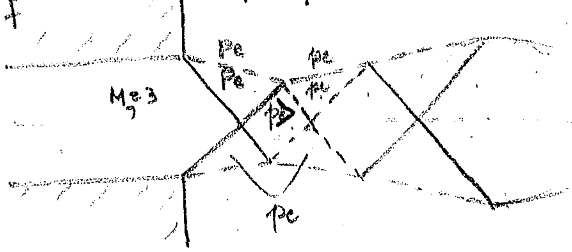


underexpanded nozzle

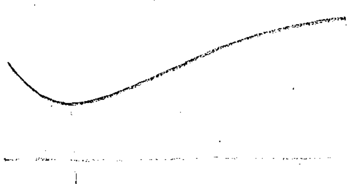
e)

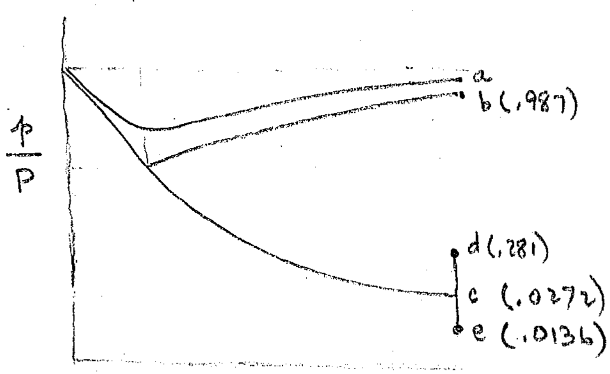


f

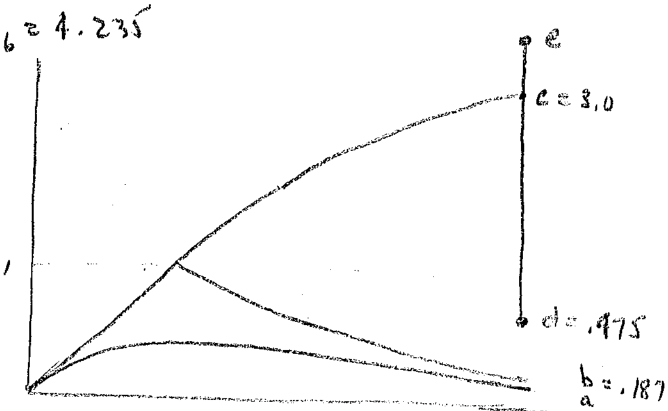


3-14-72

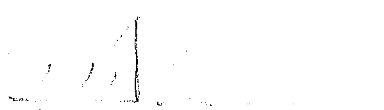




for $A_7/A_6 = 4.235$

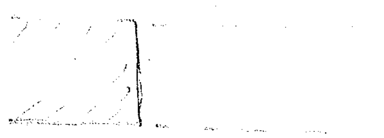


a, b



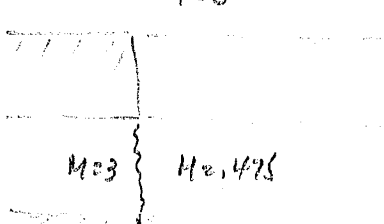
M<1

c

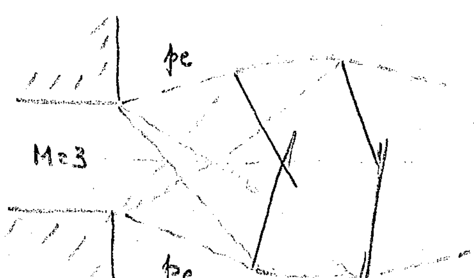


M=3

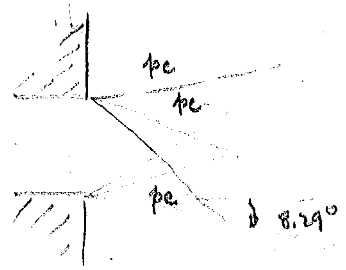
d



a



point e $(.0136 = p/p)$ $\qquad p_e/p = .0136 = p_e/p_7$



P is the same isentropic exp.

$M_7 = 3.47$

$$\nu = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}\left(M^2-1\right)} - \cos^{-1}\frac{1}{M}$$

M=1 θ=0

M	ν
1	0
2	26.38
3	49.76
5	76.92
20	116.20
∞	130.5

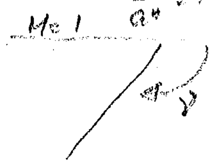
constant γ

t=0 at M=∞

$$c_p t + \frac{1}{2}V^2 = c_p T = \frac{1}{2}V_\ell^2$$

limit velocity

T, P $\quad V = a^*\quad \frac{V}{a^*} = 1$ $\quad$ M=1



$a^*(2.45) = V_\ell$ @ M=100

$t \to 0 \quad a \to 0$

T, P $\quad p = 0$

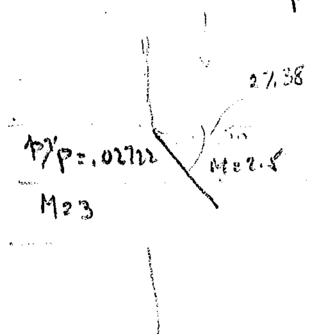
M=3 $\quad \nu_{M=3} =$ ——

M=3.47 $\quad \nu_{M=3.47} =$ ——

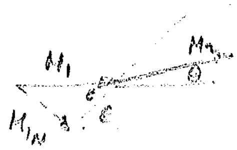
$\Delta\nu = \nu_{M=3.47} - \nu_{M=3}$

$\Delta\nu = 58.05 - 49.76 = 8.2$

point d $\quad \left(.05591 = \frac{p_\infty}{p}\right)$



$\frac{p_\infty}{p_7} = 2.054 \rightarrow M_{\text{after shock}}$

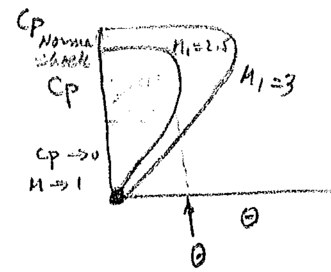


θ=10°

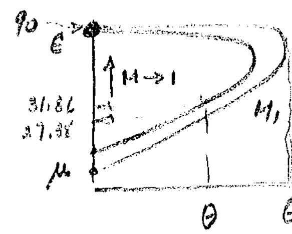
ε=27.38°

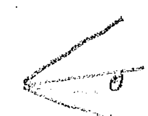
given $\frac{p_7}{p_\infty} \rightarrow M_{1N}$ $\therefore M_1, M_{1N}$ are known

$\therefore \epsilon$ is given



$$C_p = \frac{p_2 - p_1}{\frac{1}{2}\gamma M^2 p_1} = \left(\frac{p_2}{p_1} - 1\right)\frac{2}{\gamma M^2}$$

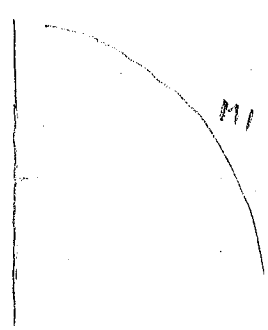




if $\theta < \theta_{max}$ attached

$M_{1N} = M_1 \sin\mu$

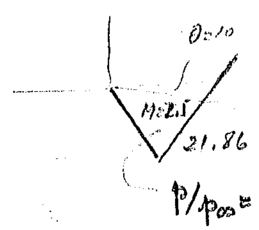
$M_{2N} = M_2 \sin(\mu - \theta)$



θ

if $\theta > \theta_{max}$ detached

get stagnation behind normal shock



get Cp → p_2/p_1

$p/p_\infty = 1.864$

$M = 2.09$

$p/p_\infty = 1$

$$p_2/p_1 = C_p\frac{\gamma M_1^2}{2} + 1$$

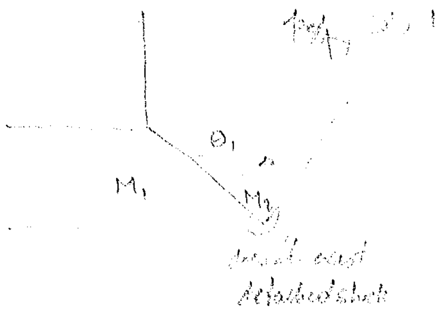
as $M_1 \uparrow \quad \epsilon - \theta \rightarrow 0$

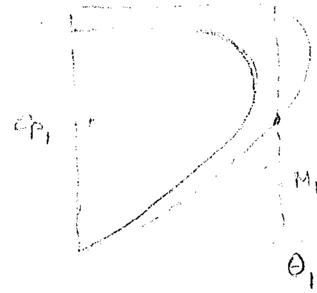
18

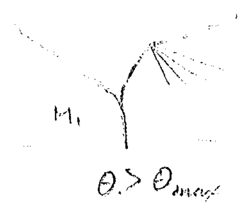
b. $M = 2.09$ $\nu = 28.83$ $p/P_{M=2.09} = .111$ $\left(\frac{p}{P}\right)_{\text{in new reg}} = \frac{.1111}{1.864} = .0596$ $M = 2.49$

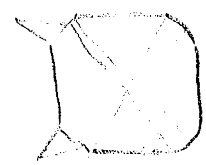
$\nu_{2.49} = 38.89$

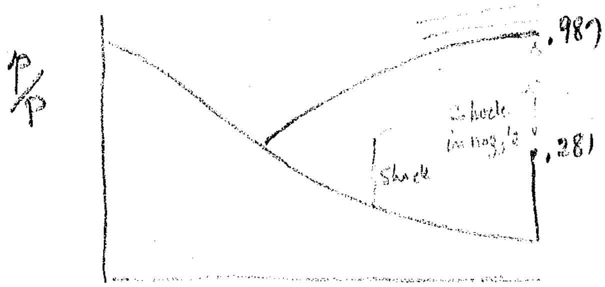
$\Delta\nu = 10.06$

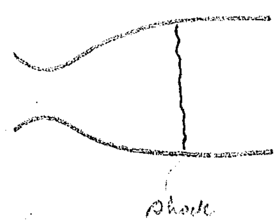


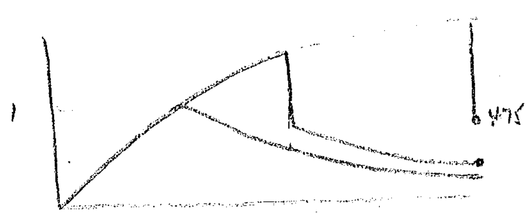












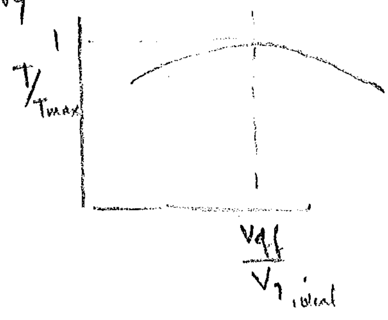
Given $A_e/A_{th} = 4.235$ shock exists at $\frac{A}{A_*} = 2$ $p/P_5 = .5$

assume then M_1, M_2 $p_2/p_1 \rightarrow \bar{A}_*$ depended on M_2 & A_5

then get M_7, p_7/P_7, $p_7/P_5 = \frac{p_7}{P_7} \cdot \frac{P_7}{P_5}$

3-21-71

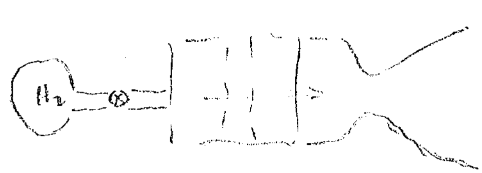
Exam. $V_{eff} = \frac{A_7}{\dot{m}}(p_7 - p_0) + V_7$



Increase T_4 $\mathcal{W}_c$ is const turbine work $\mathcal{W}_T = Cp(T_4 - T_5)$ if $\mathcal{W}_T$ is const $T_4\uparrow$ $T_5\uparrow \sim V_7$

$V_7 \sim \sqrt{T_7} \sim \sqrt{T_5}$

b. Nuclear rocket:

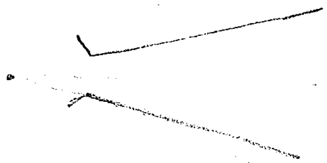


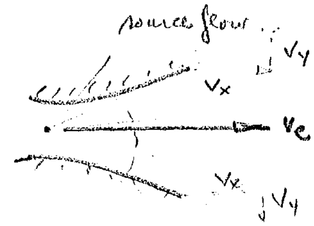
hot wall transfers heat to gases

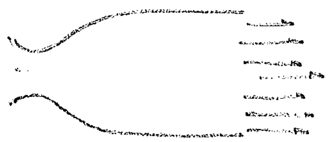
$I_{sp}\uparrow$

helium - heat gas & expand it no dissociation

2D

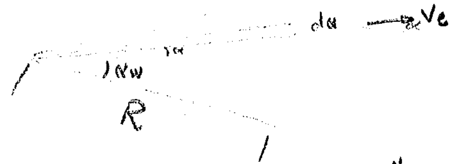






$T = \dot m V_e$

Rocket



$$dA = R\,d\alpha$$

$$dm = \rho_e V_e\, dA = \rho_e V_e R\, d\alpha \qquad m = \int_{-\alpha_w}^{\alpha_w} \rho_e V_e R\, d\alpha = 2\rho_e V_e R \alpha_w$$

$$dT = V_x\, dm = \rho_e V_e^2 R \cos\alpha\, d\alpha$$

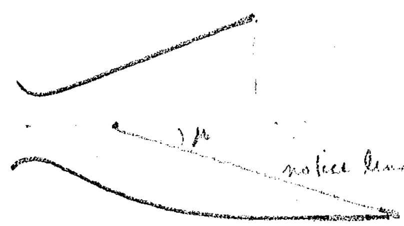
$$T = \int_{-\alpha_w}^{\alpha_w} \rho_e V_e^2 R \cos\alpha\, d\alpha = \rho_e V_e^2 R \sin\alpha \Big|_{-\alpha_w}^{\alpha_w}$$

$$2\rho_e V_e^2 R \sin\alpha_w$$

$$\lambda = \frac{T}{mV_e} = \frac{2\rho_e V_e^2 R \sin\alpha_w}{2\rho_e V_e^2 R \alpha_w}$$

$$\lambda = \frac{\sin\alpha_w}{\alpha_w}$$

α_w	λ
0	1.0
10	.996
20	.980
30	.954



notice length is larger

min length draw mach wave from end of nozzle to get uniform flow, first approx

3-D

$$dA = 2\pi h\, R\, d\alpha$$

$$\sin\alpha = \frac{h}{R}$$

$$dA = 2\pi R^2 \sin\alpha\, d\alpha$$

$$A = 2\pi R^2 [1 - \cos\alpha_w]$$

$$m = 2\pi R^2 \rho_e V_e [1 - \cos\alpha_w]$$

$$dT = V_e \rho_e V_e \cos\alpha\, dA$$

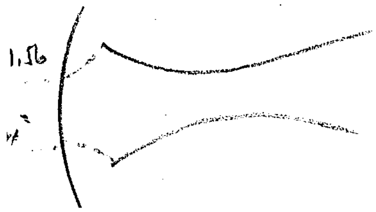
$$= \rho_e V_e^2\, 2\pi R^2 \cos\alpha \sin\alpha\, d\alpha$$

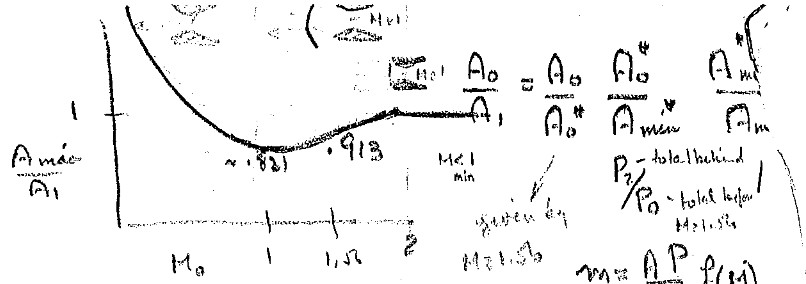
$$T = 2\pi R^2 \rho_e V_e^2 \int_0^{\alpha_w} \cos\alpha \sin\alpha\, d\alpha$$

$$T = \pi R^2 \rho_e V_e^2 [1 - \cos^2\alpha_w]$$

$$\lambda = \frac{T}{mV_e} = \frac{2\pi R^2 \rho_e V_e^2 \frac{\sin^2\alpha_w}{2}}{2\pi R^2 \rho_e V_e^2 [1 - \cos\alpha_w]}$$

$$= \frac{\sin^2\alpha_w}{2[1 - \cos\alpha_w]} \qquad \frac{(1 - \cos\alpha_w)(1 + \cos\alpha_w)}{2\,(1 - \cos\alpha_w)} = \frac{1 + \cos\alpha_w}{2}$$





$$\frac{A_0}{A_1} = \frac{A_0}{A_0^*}\,\frac{A_0^*}{A_{min}^*}\,\frac{A_{min}^*}{A_{min}}$$

P_2/P_0 – total behind / total before, $M=1.56$

$$\dot m = \frac{A\,P}{\sqrt{T}}\,f(M)$$

$$= 1.217\,(.913)\left(1/1.217\right) = .913$$

Starting Mach no for inlet

M at which NS at inlet ⇒ M=1 at throat. $M < M_s$ steady shock outside

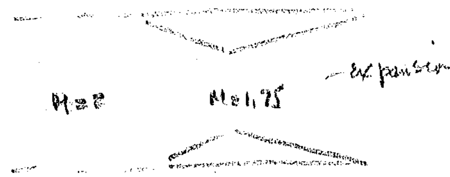
$M > M_s$ shock travels in inlet

get $\frac{A}{A_*} = 1.217 \rightarrow M = .58 < 1$

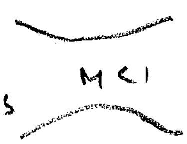
go to supersonic tables using $M_{behind} = .58$ $M_{before} = 2$

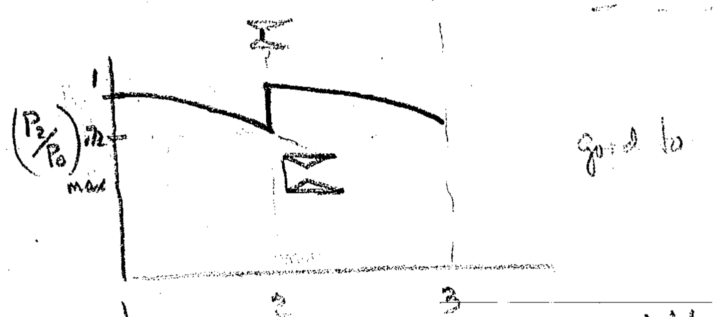
$$A_0/A_1 = \frac{A_0}{A_0^*}\,\frac{A_0^*}{A_{mi}^*}\,\frac{A_{mi}^*}{A_{mi}}\,\frac{A_{mi}}{A_1} \qquad \frac{A_0}{A_1} = 1$$

$$1 = \left(\frac{A}{A_*}\right)_{NS}\left(P_2/P_0\right)_{NS}\frac{A_{mi}}{A_1}$$

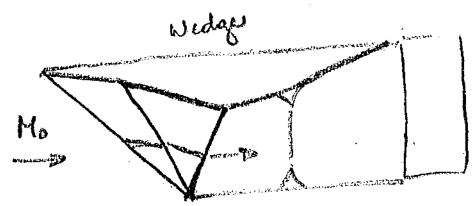


$M_0 = 3$
$> M_s$

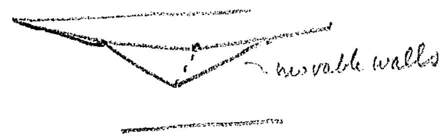




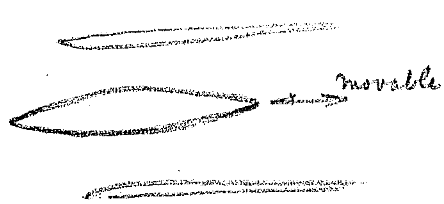
Good to M=1.8 – 1.9



As $M_0\uparrow$ contraction must decrease



B.58



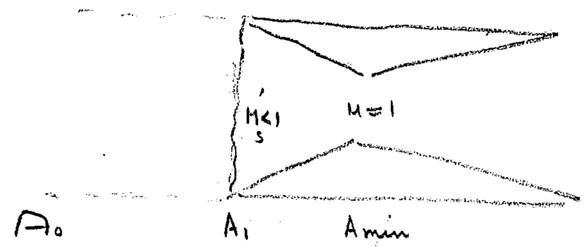
Oswatitsch

book exam – bring slide rule

Combustion Chambers
Exhaust nozzles
inlets.
Turbines – small amount.

box type may have leakage due to bulging of strut due to high pressure

Starting M of inlet – normal shock at LE of inlet



$\frac{A_1}{A_{min}}$

$\left(\frac{A}{A_*}\right)_{M_s} = \frac{A}{A_{min}}$ go to tables & get jump cond.

$$\frac{A_0}{A_{min}} = \frac{A_0}{A_{0*}}\ \frac{A_{0*}}{A_{1*}}\ \frac{A_{1*}}{A_{min}^*}\ \frac{A_{min}^*}{A_{min}}$$

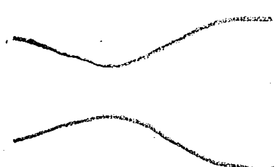
$$\frac{A_0}{A_0^*}\quad P_2/P_0\quad 1\quad 1$$

$$\left(\frac{A}{A_*}\right)_{M_0}\ \left(\frac{P_{t_2}}{P_{t_1}}\right)_{M_0}$$

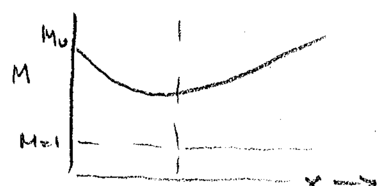
$A_0^* P_0^* = A_1^* P_1^*$

Always want to generate normal shock at lowest mach no.

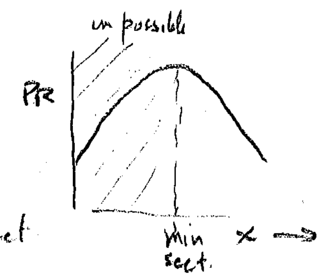
$M_0 > 1$



$P_2/P_0 \sim 1$



shock wave should be closest to min inlet sect



control shock by $T_6 = T_5$ $\qquad M_6 = 1$ at A_6

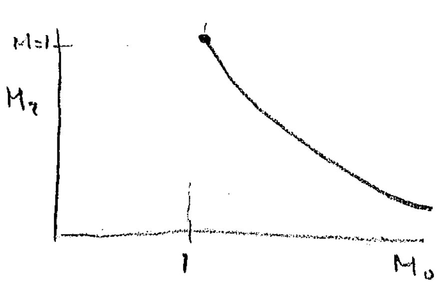
Assume T_6 = const assume shock is not near A_{min} assume A_6 decreases.

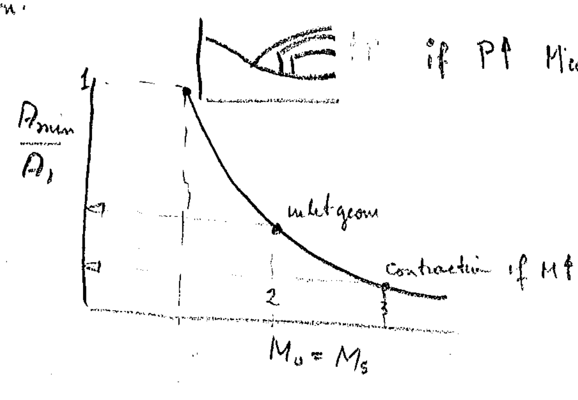
mass flow thru system is const since M_0 is supersonic

$\dot{m} = \frac{PA}{\sqrt{T}} f(M)$ at A_6 T_6 = const $PA_6 = PA_{new}$ = const

shock moves toward min section.

if P↑ M const p↑



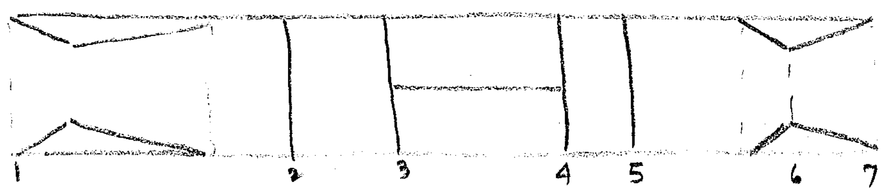


since inlet geometry is fixed & contract can occur ∴ $M_{min} < 1$

A_6 = const   vary fuel ↑ T_6 increases   mass flow eq says P↑   shock moves toward throat

for $M_0 < 1$ $m \sim \frac{PA}{\sqrt{T}}$ P is const   ∴ $m \sim \frac{A}{\sqrt{T}}$

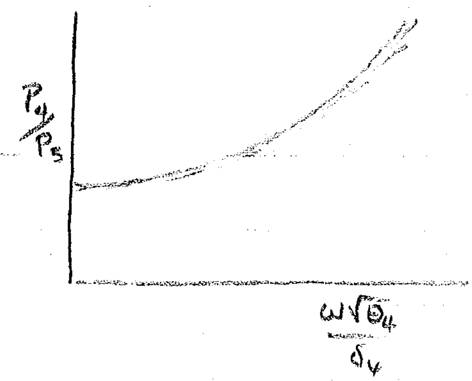
INLET BUZZ – where shock wave is pushed out of inlet (movement of shock back & forth)

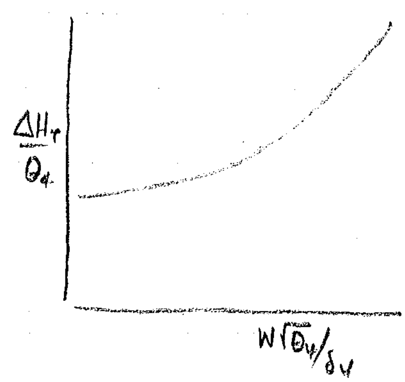


Given M_0, h, P_2/P_0, T_4, A_6 Find: $\dot{N}$, A_0 mass flow   Start from rear of engine & work forward

$m = \frac{PA}{\sqrt{T}} f(M)$ $\frac{\omega\sqrt{\theta_6}}{\delta_6}$ eq 2: $f(\gamma)\, A_6$ if no after burner

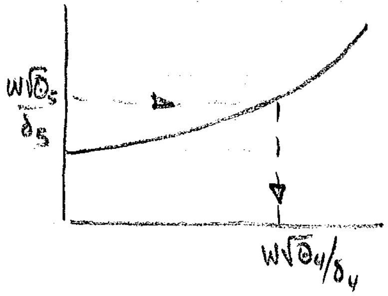
$\frac{\omega\sqrt{\theta_6}}{\delta_6} = \frac{\omega\sqrt{\theta_5}}{\delta_5}$ go to turbine charts





$$\frac{W\sqrt{\theta_5}}{\delta_5} = \frac{W\sqrt{\theta_4}}{\delta_4}\,\frac{P_4}{P_5}\sqrt{\frac{T_5}{T_4}} = \frac{\omega\sqrt{\theta_4}}{\delta_4}\,\frac{P_4}{P_5}\sqrt{1-\frac{\Delta H_T}{\theta_4}\frac{T_{ref}}{c_p}}$$

$$\frac{\Delta H_T}{\theta_4} = \frac{c_p\,(T_4-T_5)}{T_4/T_r}, \qquad \frac{\Delta H_T}{\theta_4}\frac{T_r}{c_p} = \left(1-\frac{T_5}{T_4}\right)$$



$$A_6 \rightarrow \frac{\omega\sqrt{\theta_6}}{\delta_6} \rightarrow \frac{\omega\sqrt{\theta_5}}{\delta_5} \longrightarrow \frac{\omega\sqrt{\theta_4}}{\delta_4},\ P_4/P_5,\ \frac{\Delta H_T}{\theta_4}$$

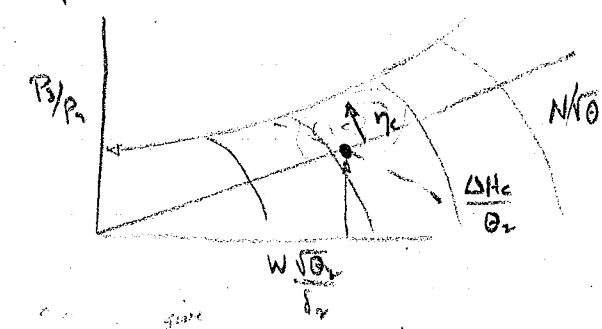
$$\frac{\omega\sqrt{\theta_2}}{\delta_2} = K\, P_3/P_2,\quad \frac{\Delta H_c}{\theta_2} = \frac{\Delta H_T}{\theta_4}\,\frac{T_4}{T_2} \longrightarrow \frac{\omega\sqrt{\theta_2}}{\delta_2},\ P_3/P_2,\ N$$

$$T_2 = T_0$$

$$\frac{\omega\sqrt{\theta_3}}{\delta_2} = \frac{\omega\sqrt{\theta_4}}{\delta_4}\;\frac{P_4}{P_2}\cdot\sqrt{\frac{T_2}{T_4}}\text{ - given}\qquad P_3 = P_4\,,\; T_2 = T_0 \quad \text{adiab inlet;}$$

$$= K\; P_3/P_2$$

Compressor data



Satisfy $\quad \frac{\Delta H_c}{\theta_2} = \frac{\Delta H_T}{\theta_4}\;\frac{T_4}{T_2}$

given Given

$$\frac{\omega\sqrt{\theta_0}}{\delta_0} = \frac{\omega\sqrt{\theta_2}}{\delta_2}\;\frac{P_2}{P_0}\sqrt{\frac{T_0}{T_2}} = f(\gamma, M_0)\cdot A_0 \qquad A_0 \text{ is found}$$

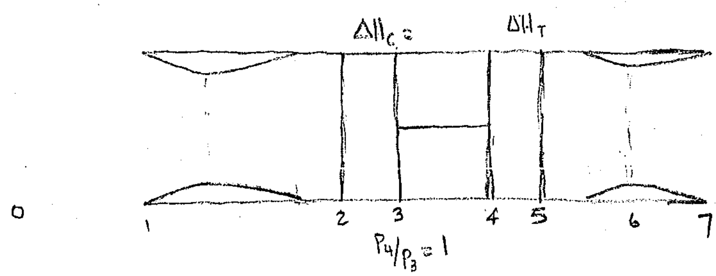
$$m_0 = \rho_0 V_0 A_0$$

at exhaust $\quad P_7/P_5 \cdot P_5/P_4 \cdot P_4/P_3 \cdot P_3/P_2 \cdot P_2/P_0 \cdot P_0 = P_7$

1 turbine 1 comp inlet given

ideal valve $\quad p_7 = p_0 \longrightarrow A_7$

1-18-72



review Given $M=1$ at station six

$\frac{A_7}{A_6} = \text{const} \longrightarrow M_7 \qquad$ know T_4 & $\Delta H_T \longrightarrow T_5 \qquad \Delta H_T = c_p(T_4 - T_5)$

$T_5 = T_7$ adiab $\qquad T_7 = \left(1 + \frac{\gamma-1}{2} M_7^2\right) t_7 \longrightarrow t_7\;;\; a_7 = \sqrt{\gamma R t_7} \;\therefore\; V_7 = M_7 a_7$

from P_7 as above $\qquad p_7 = \left(1 + \frac{\gamma-1}{2} M_7^2\right)^{-\frac{\gamma-1}{\gamma}} P_7$

now $\qquad T = m(V_7 - V_0) + A_7(p_7 - p_0)$

ideal thrust $\quad T = m(V_7 - V_0) \qquad A_7 = ? \qquad P_7$ found as above $\quad p_7/P_7 = \frac{p_0}{P_7} \longrightarrow M_7$

$M_7 \longrightarrow T_5 = T_7 \longrightarrow t_7 \longrightarrow a_7 \longrightarrow V_7 \qquad$ from $M_7 \longrightarrow \frac{A_7}{A_*} = \frac{A_7}{A_6}$

T_4 is a critical value that must be given.

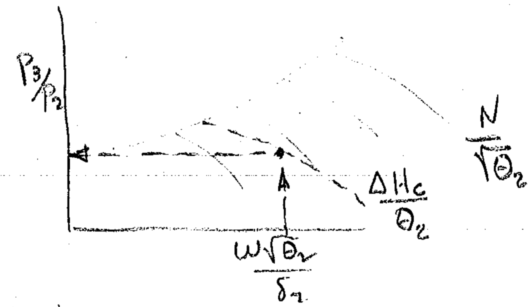
Given: h, M_0, P_2/P_0, A_0, N (m_0)

Unknown: T_4, A_6

Start from front of engine & work to rear.

$M_0, h \rightarrow P_0$ $\qquad \dfrac{w\sqrt{\theta_0}}{\delta_0} = f(M_0)A_0$ $\qquad \dfrac{w\sqrt{\theta_2}}{\delta_2} = \dfrac{w\sqrt{\theta_0}}{\delta_0}\left(\dfrac{P_0}{P_2}\right)$ adiab inlet

from compressor map

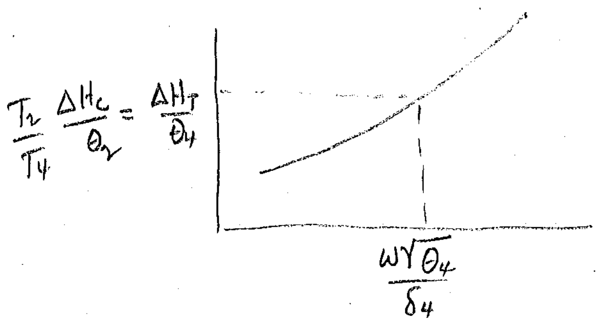


get P_3/P_2 & $\dfrac{\Delta H_c}{\theta_2}$

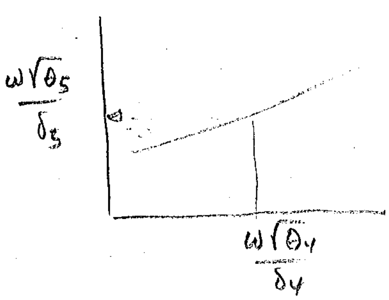
$$\frac{w\sqrt{\theta_4}}{\delta_4} = \underbrace{\frac{w\sqrt{\theta_2}}{\delta_2}}_{\text{known}}\ \underbrace{\frac{P_2}{P_4}}_{\text{known}}\sqrt{\frac{T_4}{T_2}} \quad \leftarrow T_2 \text{ known} = T_0$$

$P_3/P_2 = P_4/P_2$ $\qquad$ pick a $T_4 \rightarrow \dfrac{w\sqrt{\theta_4}}{\delta_4}$

get $\dfrac{\Delta H_T}{\theta_4}$ go to map & get $\dfrac{w\sqrt{\theta_4}}{\delta_4}$ it should match calculated value. If not pick a new T_4 & do it again



$$\frac{w\sqrt{\theta_4}}{\delta_4} \longrightarrow \frac{w\sqrt{\theta_5}}{\delta_5}$$



given $\dfrac{w\sqrt{\theta_5}}{\delta_5} = \dfrac{w\sqrt{\theta_6}}{\delta_6}$ $\qquad M=1$ @ 6 $\quad \therefore$ get A_6

depending on $A_7/A_6 \rightarrow M_7$ & go as before.

$M = 2.7$ $\qquad$ SCAT 15F $\qquad$ 3500 n.m. $\qquad$ 250 People (200 lb)

nozzle $\quad 97.7\% \xrightarrow{\text{increased}} 98.7\%$ $\quad$ increases range to 3620 nm $\quad$ or $\quad$ 25 more people

$\eta_T \quad 86.7\% \rightarrow 87.7\%$ $\quad$ range 3539 nm $\quad$ or 5 to 6 more people

$\eta_c \quad 88\% \rightarrow 89\%$ $\quad$ change in range 34 nm $\quad$ or $\quad$ 6 people

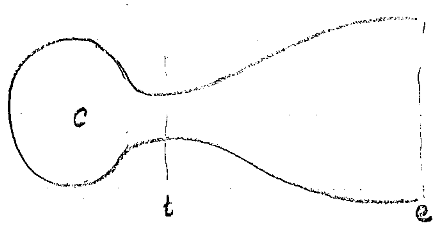
η_B – 98% – 99%   3 more people

GE engine $T = 11100$ lb $M = .85$ $W = 618$ lb/sec $h > 35{,}000$ ft/sec

$V_0 \rightarrow 820$ ft/sec $V_e = V_0 + \frac{Tg}{W} = 820 + \frac{11000\,(32.2)}{618} \cong 1400$ ft/sec   578

½% change ↓ $V_e = 7$ ft/sec   % change in thrust 1.2% $\frac{7}{578}$

$M_0 = 2.7$ at $h \geq 35{,}000$ $V_0 \sim 2500$ ft/sec $V_e \sim 3000 - 3500$ ft/sec



$$T = \dot{m} V_e + A_e (p_e - p_o)$$

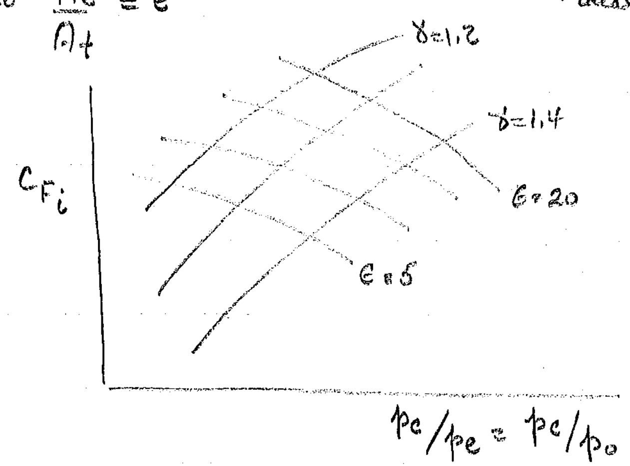
$$T = \dot{m} c = \dot{m}\left\{V_e + \frac{A_e}{\dot{m}}(p_e - p_o)\right\}$$

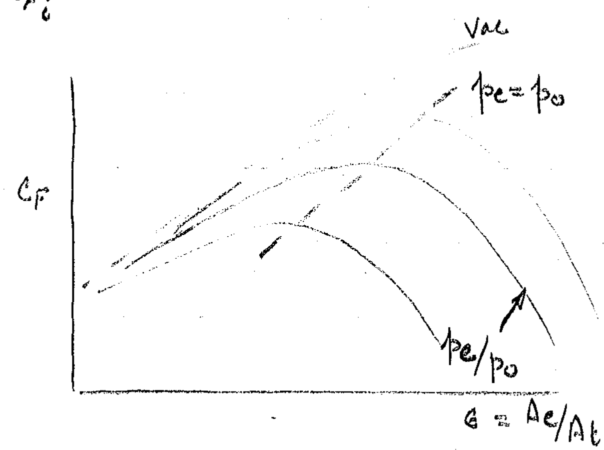
$$T = C_F A_t p_c$$

p_o

$$C_F = \frac{T}{A_t p_c} = \underbrace{\sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1 - \left(p_e/p_c\right)^{\frac{\gamma-1}{\gamma}}\right]}}_{C_{F_{ideal}} = C_{F_i}} + \left(\frac{p_e}{p_c} - \frac{p_o}{p_c}\right)\frac{A_e}{A_t}$$

define $\frac{A_e}{A_t} = \epsilon$





$c^* = c/C_F$ characteristic velocity $= \sqrt{\frac{gRT_c}{\gamma}}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = \frac{a_c}{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma+1)}}$

$I_{sp} = \frac{T}{\dot{w}} = \frac{c}{g_0}$ based on conditions on the earth.

$$V_e = \sqrt{\frac{2g_0\gamma}{\gamma-1}\frac{R'}{\mathfrak{M}}T_c\left[1-\left(\frac{p_e}{p_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

T_c should be high $\mathfrak{M}$ should be low

$$I_{sp} = \frac{T}{\dot{w}} = \frac{\dot{m}V_e + A_e(p_e-p_o)}{\dot{m}g_0} = \frac{V_e}{g_0} + \frac{A_e(p_e-p_o)}{\dot{w}}$$

I_{sp} at high alt is depended on area ratio

$$= \frac{V_e}{g_0}\left[1 + \frac{A_e g_0(p_e-p_o)}{\rho_e V_e A_e g_0 V_e}\right] = \frac{V_e}{g_0}\left[1 + \frac{p_e-p_o}{\rho_e V_e^2}\right] = \frac{V_e}{g_0}\left[1 + \frac{\gamma p_e(1-p_o/p_e)}{\rho_e\,\gamma V_e^2}\right]$$

$$= \frac{V_e}{g_0}\left[1 + \frac{(1-p_o/p_e)}{\gamma M_e^2}\right]$$

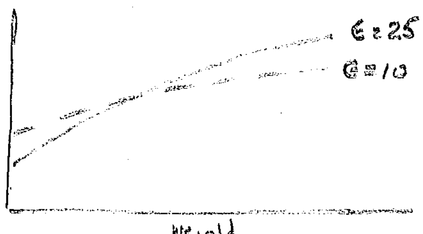
A_e/A_t will influence V_e & M_e

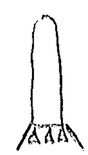
$I_{sp}\uparrow$ as $h\uparrow$ since $p_o/p_e \to 0$

$\gamma_e = 1.4$ $P_c = 1000$ psia $T_c = 5000°R$

$\epsilon = 10$ $I_{sp} = 199$ at s.l. $\epsilon = 25$ $I_{sp} = 177$ @ SL

$\epsilon = 10$ $I_{sp} = 219$ at vac $\epsilon = 25$ $I_{sp} = 226$ @ vac





drop 2 outer engines at high alt.

Oxidizer	Fuel	O/F	T_c °F	γ	C^*	I_{sp} eq	I_{sp} froz
LOX	LH_2	4.0	4915	1.26	7985	391	388
	RP-1	2.45	6145	1.24	5915	301	286
LF_2	LH_2	8.0	6670	1.33	8355	410	398
	RP-1	2.80	7480	1.22	6390	326	304

$p_c = 1000$ psia $p_e = 14.7$ psia

$H_2 + \frac{1}{2}O_2 \to H_2O$ $\mathfrak{M} = 18$ $T_c > 4915$

$2H_2 + \frac{1}{2}O_2 \to \underbrace{H_2}_{2} + \underbrace{H_2O}_{18}$ $\mathfrak{M} = 10$ $\frac{20}{2 \text{ moles}}$

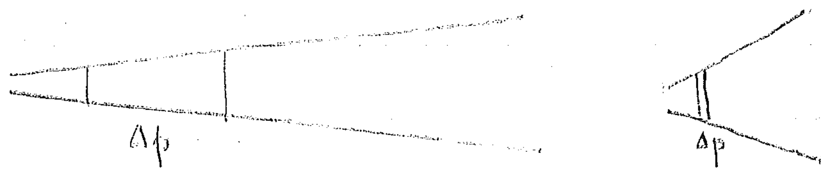
the mixture is rich thus T_c is lower than for that stoich burning

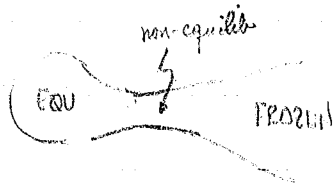
$2H_2 + \frac{1}{2}O_2 \rightarrow a\,H + b\,H_2 + c\,O + d\,O_2 + e\,OH + f\,H_2O$ combust at 4415°F

In expansion $V\uparrow$ $T\downarrow$ if V_i decreases ∋ T is low enough that reaction doesn't occur : frozen process, non equil — state changes.

Composition of products of combustion doesn't change from 1 station to another. because it doesn't remain at any one station long enough to reach equil.

differences in I_{sp} are due to lower energy levels due to dissociation since ions exist more energy must by used by ions to remain dissociated energy for frozen flow. — longer nozzles tend to drive products of combustion to equil: since same Δp is achieved in longer time period in longer nozzle.





4-20-72

$$m\frac{dv}{dt} = F - mg$$

$$\frac{dv}{dt} = \frac{F}{m} - g \qquad F = \dot{m}_p c$$

$$m = m_0 - \frac{\overset{\dot{m}_p}{m_{prop}}\,t}{t_{powered}} = m_0\left(1 - \frac{m_p}{m_0}\frac{t}{t_{powered}}\right) = m_0\left(1 - \varsigma\frac{t}{t_p}\right)$$

$$\frac{dv}{dt} = \frac{m_p/t_p\,c}{m_0(1-\varsigma\,t/t_p)} - g \qquad I = \frac{T}{\dot{W}} = \frac{c}{g}$$

$$\frac{dv}{dt} = \left[\frac{c\varsigma}{t_p}\Big/\left(1-\varsigma\,t/t_p\right)\right] - g$$

$$\Delta V = -c\ln\left(1-\varsigma\,t/t_p\right) - gt$$

$$\Delta V_{t=t_p} = -c\ln(1-\varsigma) - g t_p$$

$$h_p = \int_0^{t_p} v\,dt = c t_p \left[1 + \frac{1-\zeta}{\zeta} \ln(1-\zeta)\right] - \frac{1}{2} g t_p^2$$

$h_{max} = h_p + V_p^2/2g$ neglect $g t_p$ since it is small in comparison to $c \ln(1-\zeta)$

$$\Delta V_p = -c \ln(1-\zeta) = -c \ln\left(\frac{m_f}{m_o}\right) = -c \ln \lambda$$

$W_{PL} = 100$

$\Delta V_{TOT} = 24000$ mph $= 35200$ ft/sec

$\Delta V/_{stage} = \Delta V_{TOT}/N$ stage.

s – structure $\quad \frac{W_s}{W_o} = .092$

$\Delta V = -I_{sp}\, g \ln(1-\zeta)$ $\quad I_{sp} = 300$ $\quad \Delta V = -9660 \ln(1-\zeta)$

$N = 1$ $\quad \ln(1-\zeta) = -\frac{35200}{9660}$ $\quad \zeta = .9734$ $\quad W_s/W_o = .0266$

$N = 2$ $\quad \Delta V/_{ST} = 17600$ $\quad \ln(1-\zeta) = -\frac{17600}{9660}$ $\quad \zeta = .838$ $\quad W_s/W_o = .162$

$\frac{m_p}{m_o} = .838$ $\quad \frac{m_s}{m_o} = .092$ $\quad \frac{m_{PL}}{m_o} = .07$ $\quad m_{PL} = 100/32.2$ $\quad W_o = 1430$

$W_{PL} = 100$ $\quad 1430$

$W_s = 132$ $\quad 1860$

$W_p = 1198$ $\quad 16910$

1430 $\quad 20200$

$W_o = W_{PL}/.07$

$N = 3$ $\quad \Delta V/_{ST} = 11734$ $\quad \zeta = .703$ $\quad W_s/W_o = .092$ $\quad \frac{W_{PL}}{W_o} = .205$

$W_3 = 488$

$W_2 \quad 2380 \quad W_{PL} = 488$

$W_1 \quad 11600 \quad W_{PL} = 2380$

$I_{sp} = 250$

$N = 1$ $\quad W_{T_o} =$ ——

$N = 2$ $\quad = 250,000$

$N = 3$ $\quad = 32,468$

$I_{sp} = 300$

$N = 1$ $\quad W_{T_o} =$ ——

$N = 2$ $\quad = 20200$

$N = 3$ $\quad = 11600$

$\underline{I_{sp} = 400}$

$\omega_{T_0} =$ ——— $N = 1$

$\omega_{T_0} = 3761$ $N = 2$

$\omega_{T_0} = 3340$ $N = 3$

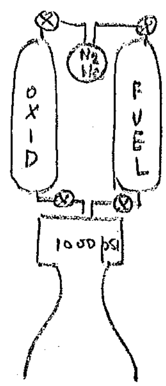
$\underline{I_{sp} = 800}$ nuclear

$N = 1$ $\omega_{T_0} = 613$

$N = 2$ $\omega_{T_0} = 455$

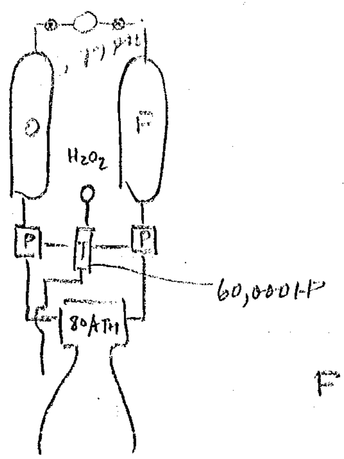
$N = 3$ $= 591$

LIQUID ROCKET



low thrust short duration Gas Press.

open valves - N_2, He at high pressure push OX FUEL into chamber.

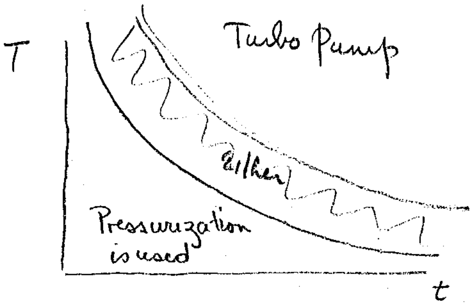


large tanks are at atmospheric so tank do not buckle. hot gas from gas generator drive turbines which drive pumps which pump oxid & fuel to combust chamber. Decomposition of H_2O_2 would drive turbine. Inert gas keeps syste pressurized at 1 atmosphere.

$F = 7.5 \times 10^6$ lb. $I_{sp} \sim 300$ $\dot{\omega} \sim 25000$ lbs/sec

HYPERGOLIC - comb of fuel & oxid that instantaneously ignites when brought together, to start engines have initial F & O in the lines being hypergolic & rest being standard F & O

4-25-72

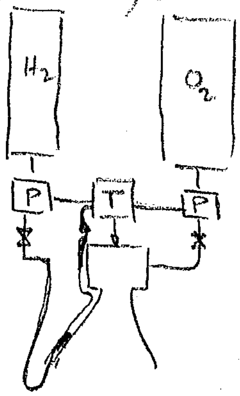


T↑ t↓ or T↓ t↑ use pressurization

N_2 - He must be at 3-5000 psi. Whole syste must be at p_c (1000) when whole fuel-ox is used tanks must therefore be heavy.

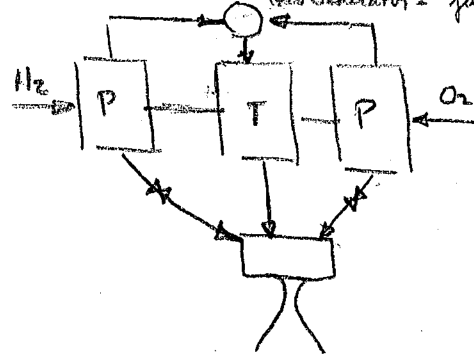
to reduce sloshing use baffles or a pressure bag that is pressurized while fuel is being used

expander - cycle



use H_2 as heat sink pass thru turbine to drive it & out into combust. chamber.

Gas Generator - fuel rich product

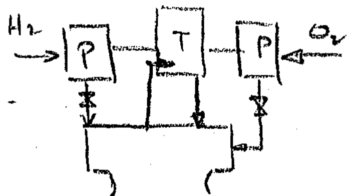


drives turbine & into chamber. if temp. is high piping gets hot ∴ keep more H_2 to cool gases (turbine blades melt)

$\therefore\ W_T = wc_p(T_4 - T_5)$

W_T depends the ox fuel flows.

Tapoff cycle



use comb chamber itself to produce hot gas & remove some to drive turbine then back into chamber.

Must be steadystate comb.

<u>Injectors</u> — L fuel & ox. meet & must become atomized

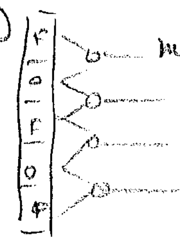
V-2 type

① 

mixing due to diffusion

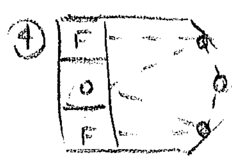
showerhead

for easy to atomize O/F

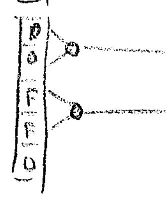
② 

mixing due to impinging

like unlike, Impinging

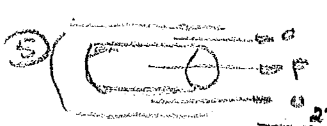
④ 

Double Impinging type

③ 

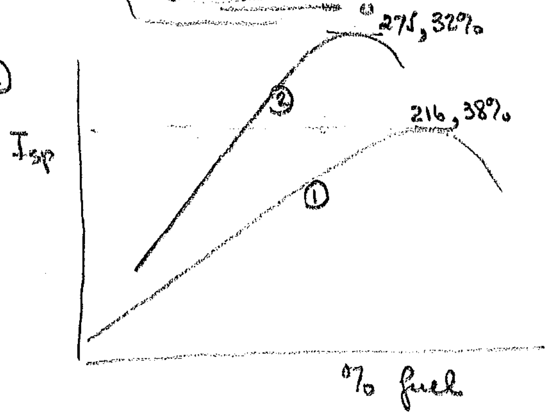
mix { due to impinging
& then due to diff O & F mix

LIKE - LIKE

⑤ 

test must be on total engine cannot be done on small scale.

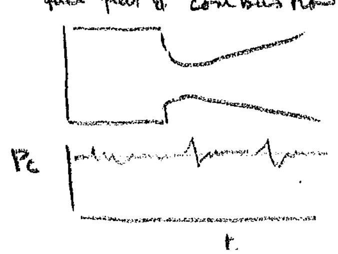
① & ②



NH_3 N_2H_4 F_2

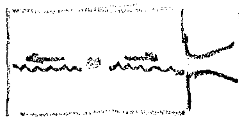
high Δp cause design problems.

to force fuel into comb chamber ΔP must exist across injector. To lower ΔP across injector combust. instability results due to coupling between fuel feed & combustion process. Instability makes heat transfer rates go up

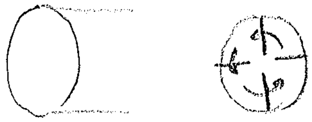


high freq osc. screaming motor
low " osc. chugging motor
chugging motor can be eliminated by modifying pipe lengths to motor

high freq



reverberation of amplification occurs, since pipe out of chamber is small with respect to length & width of chamber acoustic wave will be reverberating



vortex can be cut down by baffles
in liquid prop. $p_c \uparrow$ $\dot{Q} \uparrow$
in solid prop $p_c \uparrow$

$I_{sp_{liquid}} > I_{sp_{solid}}$. Disadvantages — long countdown times

Fuel & Oxidizer

Grain - propellant Homogeneous - oxidizer & fuel in one heavy molecules
Heterogeneous - mixed fuel & oxidizer rubber based fuel used to decrease danger of propellant cracking

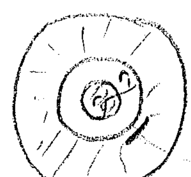
Inhibitor slows down combustion to get desired thrust level.

$T = \dot{m} V_e$ constant thrust ⟹ constant mass

burning rate normal to propellant surface depends on p_c - $r = a\, p_c^n$ r - in/sec

a - depends on curing temperature & temperature prior to combustion $a = \frac{A}{T_1 - T_p}$

disadv
① more of wall is exposed
② A_b is a constant



area of burning is increased

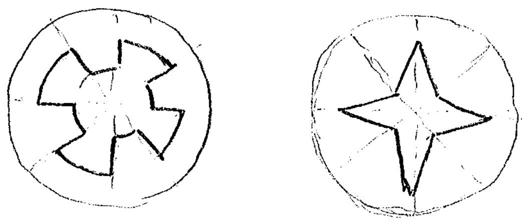
$A_b = 2\pi(R_1 + R_2)L$ $t=0$

$\Delta = rt$

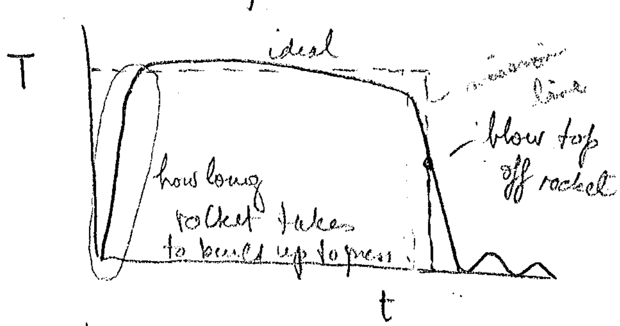
$A_{b_{t=t_1}} = 2\pi(R_1 + \Delta + R_2 - \Delta)L$
$= 2\pi(R_1 + R_2)L$

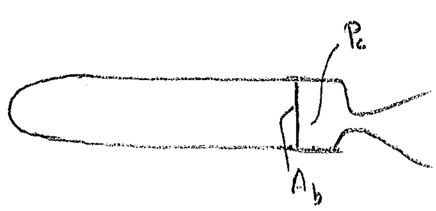
if a crack occurs force occurring due to cracking will be unbalanced

$r = a\, p_c^n$

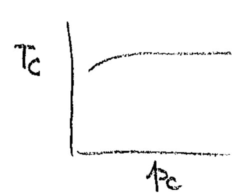


constant burning

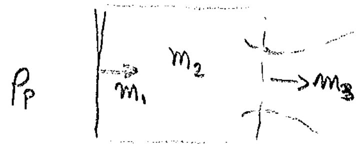




$t = 0$
$V_c = V_0$

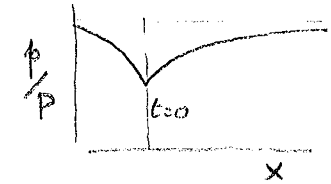


p_c



$m_1 = m_2 + m_3$ $m_1 = \rho_p A_b r$ $m_2 = (\rho_c V_c)_{,t}$

$$\dot m_3 = \frac{p_c A_t}{\sqrt{R T_c}} \Gamma \qquad \Gamma = \left[\gamma \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]^{1/2}$$



$p_e/p = .9994$

$A/A_t = 20$

$$\dot m_2 = V_c \frac{d\rho_c}{dt} + \rho_c \frac{dV_c}{dt} = \frac{V_c}{RT_c}\frac{dp_c}{dt} + \rho_c A_b r$$

$A_b r$

$$\rho_c = \frac{p_c}{RT_c}$$

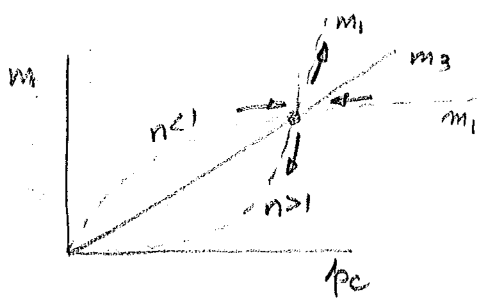
$$V_c = V_0 + \int_0^t A_b r\, dt$$

$$\overset{\dot m_1}{\rho_p A_b r} = \overbrace{\frac{V_c}{RT_c}\frac{dp_c}{dt} + \rho_c A_b r}^{\dot m_2} + \overbrace{\frac{p_c A_t}{\sqrt{RT_c}}\Gamma}^{\dot m_3} = \frac{V_0 + \int_0^t A_b r\, dt}{RT_c}\frac{dp_c}{dt} + \rho_c A_b r + \frac{p_c A_t}{\sqrt{RT_c}}\Gamma$$

since $\rho_c << \rho_p$ drop $\rho_c A_b r$

steadystate-pressure constant $\therefore dp_c/dt = 0$

$$\rho_p A_b r = \frac{p_c A_t}{\sqrt{T_c R}}\Gamma \qquad p_c = \left[\frac{A_b}{A_t}\frac{a\rho_p}{\sqrt{\frac{\gamma}{RT_c}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}}\right]^{\frac{1}{1-n}}$$



$$\dot m_1 = \rho_p A_b a p_c^n$$

$n > 1$

stable $n < 1$

$n > 1$ on disturbance will cause buildup of pressure

$n \sim .4 \sim .85 \qquad \frac{1}{1-n} \sim 1.667 - 6.667$

$$p_c \sim \left(\frac{A_b}{A_t}\right)^{\frac{1}{1-n}}$$

4-27-72

<u>Final Exam</u>

2 Parts
- 1 HR Closed } on last segment
- 1 HR open } problems, slide rules — inlet problem, nozzle problems, problem from notes.

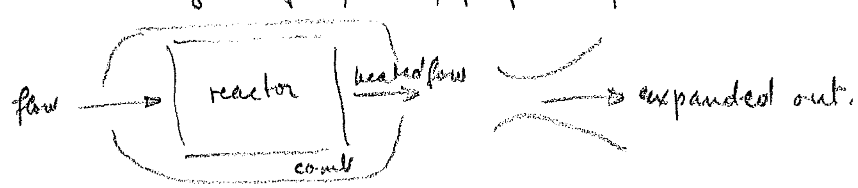
For space mission $T\uparrow\ t\downarrow$, $T\downarrow\ t\uparrow$ shut on-off type.

$I_{sp} \uparrow$ $\frac{\text{lb - thrust}}{\text{lb} \cdot \text{prop/sec}}$ chemical impulse up to 450

nuclear, electric prop & ion prop.

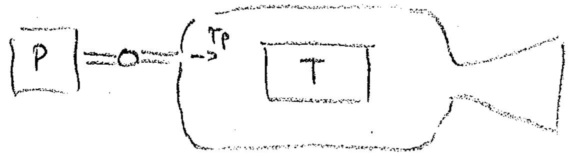
electric, ion } small part. at high veloc. $I_{sp}\uparrow\ T\downarrow$

nuclear $I_{sp}\uparrow\ T\uparrow$ due to large weight flow of prop. Prop is not part of reaction



chemical
solid prop } energy due to chem. react.

plasma — E M



$T > T_p$ Brings $T_p \xrightarrow{\text{up to}} T \approx T_c$

$v_e \sim \sqrt{\frac{T_c}{\mathcal{M}}}$ increase T_c, $v_e \uparrow$
decrease $\mathcal{M}$, $v_e \uparrow$

chemical rocket $H_2 - O_2$ prop.
$T_c \approx 10{,}000°R$
$\mathcal{M} \approx 16$

Nuclear Rocket H_2 prop.
$T_c = 5000°R$
$\mathcal{M} = 2$

$\frac{v_{e_{nuc}}}{v_{e_{chem}}} \cong \sqrt{\frac{T_{c_{nuc}}}{T_{c_{chem}}} \frac{\mathcal{M}_{chem}}{\mathcal{M}_{nuc}}} \cong \sqrt{\frac{1}{2}(8)} \cong 2$

Direct or Indirect Cycle solid core nuclear Rocket
is limited by material properties. Temp react. walls
can withstand without melting.
fuel - radioactive small amount → great heat values
long lasting
storeable

disadv. large weight due to shielding to protect passengers if it were to blow up.
expensive & heaviness makes $v_{e_{nuc}}$ increase negligible wrt its disadv.

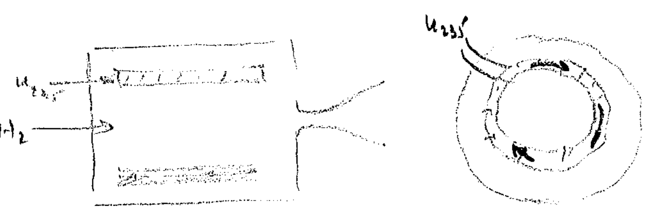
Gaseous Core
Nuclear Rockets.

reflector to keep
neutrons in chamber.
"FIZZLER"

propellant gas includes reactor fuel.
When self sustaining, will get large
energy of reaction. Accelerator H_2 & U_{235}

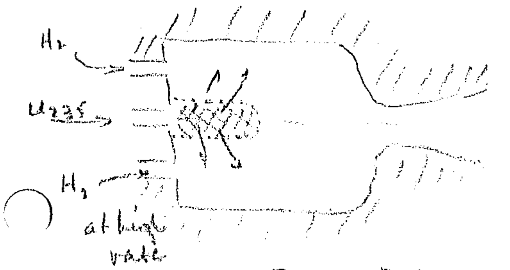
disad.
1. lose fuel \$7000/lb = Nuclear fuel.
2. trail of radioactive debris
3. Twall may be > Tmelting.
4. no shielding
5. no control.

1. Make settling chamber reactor.
2. heat transfer by thermal radiation from gaseous core.



"CENTRIFUGAL DIFFUSION CONCEPT"

propellant heated & expanded.
by whirling U_{235}. Unstable
to hold. This was to shield walls
from high temp. Turbulence tend to
revert this engine to FIZZLER type.



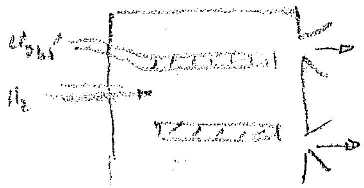
U_{235} at low rate

U_{235} heat radiation will heat large amount of H_2 & since little mixing not too much would be lost. Reduction of mixing will maintain higher temperature. Flow is very unstable.

FLUID DYNAMIC CONTAINMENT coaxial

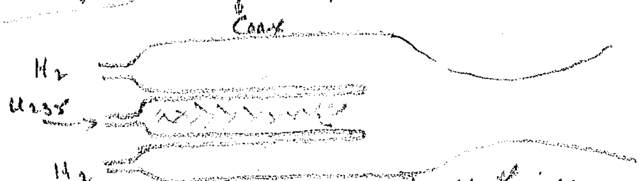
Another system again as centrif introduced U_{235} in vortex type arrangement as above small amount for no mixing

NUCLEAR - LIGHTBULB - combination of FLUID DYNAMIC & VORTEX STAB.



Fluid Dynamic Centrifugal

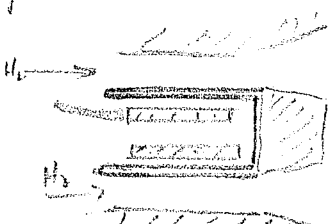
Vortex stabilized | put wall to try & cut down on mixing



1. plates prevent mixing fuel & prop.
2. shouldn't hinder radiation
3. Must not melt

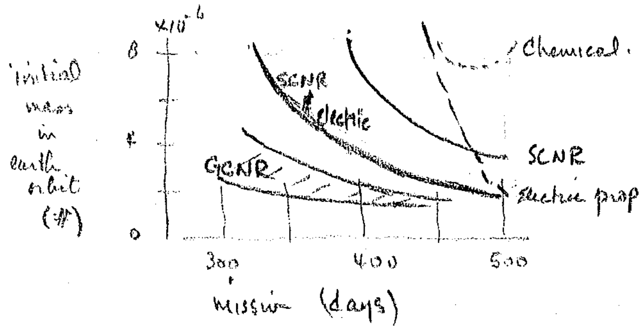
2. Must be as optically thin as possible to let thermal radiation to pass thru without increasing its own temperature. Can be cooled.

centrifugal is the same which has the same function as coaxial



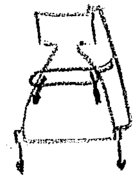
this one doesn't lose fuel.
fused alumina
fused Silicon
beryllium oxide.

Most problems with these have been solved.



SCNR - solid core nuclear rocket

Exhaust from turbine of liquid prop. can be shot out from bottom of engine to prevent flowback



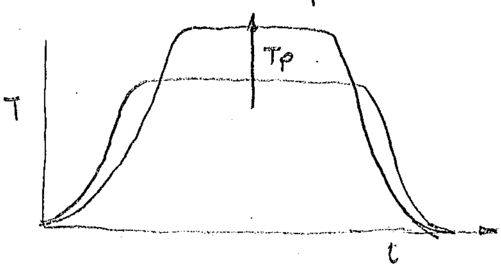
this keeps flame from doubling back on skirt wall. Can also be shot on inside to keep cooling wall temperature. If flow is high enough another motor can be included to get thrust.

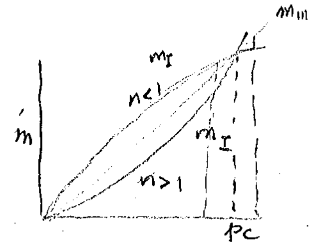
2 HR EXAM

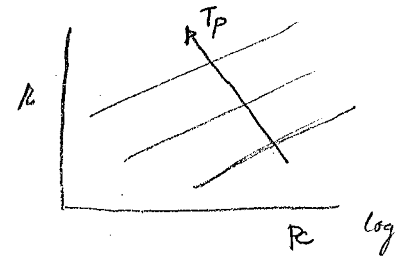
$\sqrt{-2-72}$

1 HR CLOSED up until last thursday's lecture.
2 HR OPEN

$r = a p_c^n = \frac{A}{T_1 - T_p} p_c^n \qquad n<1$







Impulse $= \int F\,dt$ will remain the same even if $T_p \uparrow$
if no drag ΔV will remain the same.

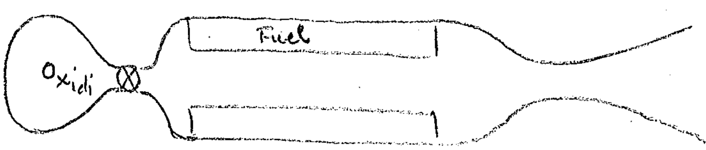
TALOS BOOSTER $\quad \theta_{Launch} = 60^\circ$

T_p	t_p	$V_{separat}$	M_{sep}	T
125°F	3.5 sec	1985	1.803	↑
80°F	4 sec	1933	1.767	
40°F	4.5 sec	1907	1.730	

T–D will be higher at higher T_p
∴ movement through lighter atmosphere will cause V_s to be higher

cannot shut down engine very simply
thrust vector control. – inject fluid cause separation of flow.

Hybrid Rockets



Isp 250
Can be stored.

Nuclear Rockets

Fission heavy element bombard with neutrons split heavy element – Mass loss produces thrust.

$n + \text{fuel} \longrightarrow 2\,ff + 2 \text{ or } 3\,n + \text{energy} \qquad E = mc^2$

neutrons $\left| \begin{matrix} U^{235} \\ Th \end{matrix} \right|$ fission fragments; energy: mass defect; 2 or 3 n: allows continuous reaction.

prot + neutro

$_0n^1 + {}_{92}U^{235} \longrightarrow {}_{42}Mo^{95} + {}_{57}La^{139} + 2\,{}_0n^1 \qquad 1\text{ amu} = 1.66 \times 10^{-24}$ grams.

protons

236.133 amu $\qquad$ 235.918 amu $\qquad \Delta m = .215$ amu

1 gram/day runs 1300 HP motor.

Fusion – light element fuse to form heavier elements

$_1D^2 + {}_1D^2 \longrightarrow {}_2He^3 + {}_0n^1 + \text{energy}$

$_1T^3$

1 atom of $_1D^2$ per 6500 of $_1H^1$

Oceans ⟶ 10^4 tons of $_1D^2$ ⟶ efficient use ⟶ 10^{21} KW-YRS.

World annual rate - 4×10^9 KW last 2.5×10^{11} yrs.

k - multiplication factor

l - average time between succeeding generation of neutrons.

$_0n^1 + \text{fuel} \longrightarrow 2\,ff + \underbrace{2 \text{ or } 3}_{k}\ _0n^1 + \text{energy}$

number of neutrons are produced $n(k-1)$

$$\frac{dn}{dt} = \frac{n(k-1)}{l} \qquad n = n_0 e^{t(k-1)/l}$$

critical reaction - $k = 1$ $\quad _0n^1$ before $= {}_0n^1$ after $\quad$ if $k = 2$ — one will decelerate to very low velo. — one will be left for cont react.

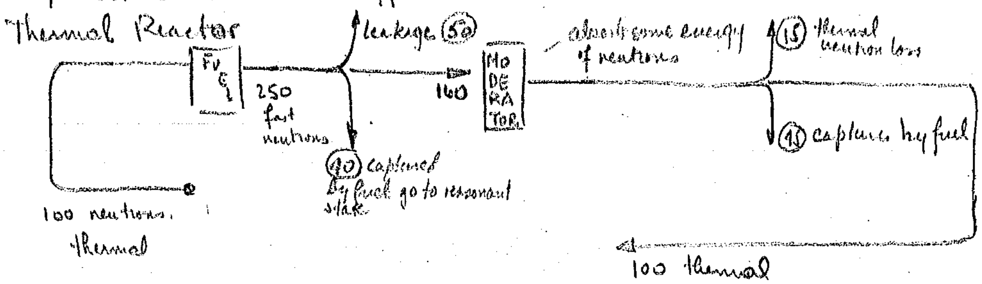
supercritical reaction - $k > 1$

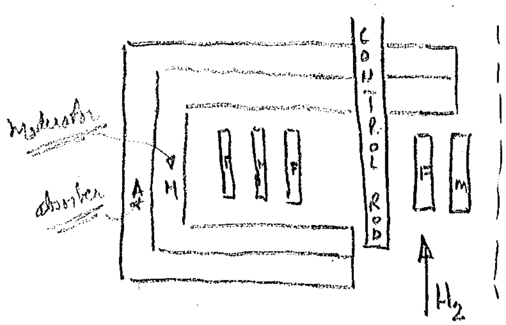
$_{92}U^{235}$ reaction $l = .0002$ sec $\quad n = n_0 e^{10t}$
$k = 1.0002$ sec

t = .1	n/n_0 =	2.71828
.5	=	148
1.0	=	20 000

Start supercritical then tail off

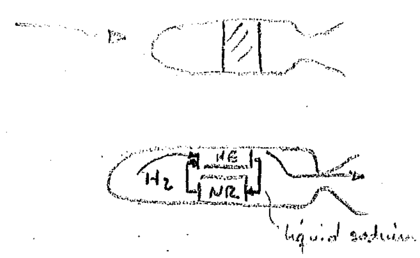
Thermal Reactor



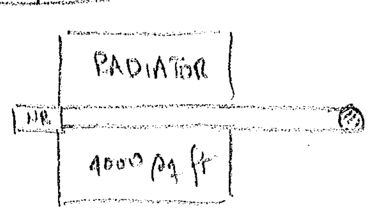


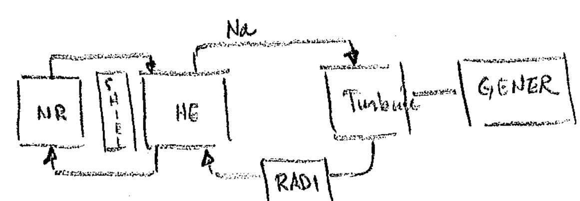
to start reactor pull rod out
to slow reactor push rod in.

rod controls rate of reaction



Electric Prop System





rejects heat to atmosphere
large areas since p is low

in space $\Delta V = -C \ln(1 - \zeta)$
(C ↑ increase, ζ ↑ increase)

Chemical - ζ ↑ since C is limited
nuclear - C ↑ since ζ is limited

Cesar Lewy

AE 371

P 42 2,3,5

2. $w_a = 60$ lb/sec $w_f = .7$ lb/sec $T_{SSL} = 3000$ lb

Since static SL $V_0 = 0$ also because subsonic $p_0 = p_e$

$$\therefore \quad F = \left(\frac{w_a + w_f}{g}\right) V_e \quad \text{or} \quad V_e = \frac{F g}{w_a + w_f}$$

$$V_e = \frac{(3000\text{ lb})(32.2\text{ ft/sec}^2)}{60.7\text{ lb/sec}} = 1591.5\text{ ft/sec}$$

3. @ 20,000 ft $T = 413.6°R$ $p_e = 9$ psia $A_e = 180$ sq in $p_0 = 6.76$ psia

$$F_g = A_e\left[p_e(\gamma M_e^2 + 1) - p_0\right] \qquad \gamma = 1.33 \quad M_e = 1$$

$$F_g = 180\left[9(1.33 + 1) - 6.76\right] = 2563.2\text{ lb}$$

$$A_e(p_e - p_0) = 180(9 - 6.76) = 403.2 \qquad \% = \frac{403.2}{2563.2} \times 100 = 15.73\%$$

$$(w_f + w_a) = p_e A_e M_e \sqrt{\frac{g\gamma}{R T_e}} = 9(180)(1)\sqrt{\frac{(32.2)(1.33)}{(53.35)(1460)}} = 38.1\text{ lb/sec}$$

since $\frac{w_f}{w_a} = .02$ $w_a + w_f = 38.1$ or $w_a(1.02) = 38.1$ $w_a = 37.4$ lb/sec

$$F_R = \frac{37.4(550)}{32.2} = \frac{w_a}{g}(V_0) = 639\text{ lb.}$$

$$F_N = F_g - F_R = 2563.2 - 639 = 1924.2\text{ lb}$$

$$THP = \frac{F V_0}{550} = \frac{1924.2(550)}{550} = 1924.2\text{ HP}$$

$$V_e = \left\{F_G - A_e(p_e - p_0)\right\}\frac{g}{w_f + w_a} = \frac{2160(32.2)}{38.1} = 1824\text{ ft/sec}$$

$$V_{ef} = \frac{F_G\, g}{w_f + w_a} = \frac{(2563.2)(32.2)}{38.1} = 2163\text{ ft/sec}$$

$$\eta_p = \frac{2V_o}{V_{ef}+V_o} = \frac{1100}{2713} = .405 \text{ or } 40.5\%$$

$$\eta_{TH} = \left\{ \frac{F_N V_o + \frac{w_f + w_a}{2g}(V_{ef} - V_o)^2}{w_f\ LHV} \right\}$$

$$= \left\{ \frac{(1924)(550) + \frac{38.1}{64.4}(1613)^2}{(.7)(18500)(778.16\ \frac{ft\text{-}lb}{BTU})} \right\} = 25.76\%$$

$$\eta_o = \eta_p \times \eta_{TH} = (40.5)(25.76) = 10.43\%$$

~~latest~~

$$\frac{.7(3600)}{1924} = sfc = \frac{V_o}{\eta_o(HV)} = \frac{550(3600\ sec/hr)}{.1043(1850)(778.16\ \frac{Ft\text{-}lb}{BTU})} = 1.32\ lb/lb\text{-}hr$$

5. $w_a = 130$ lb/sec $F_{SSL} = 7000$ lb $w_f = 1.77$ lb/sec $d_e = 20$ in

$$V_{ef} = V_e = \frac{Fg}{w_f + w_a} = \frac{7000(32.2)}{131.77} = 1710\ ft/sec$$

$$\dot{w} = w_f + w_a = 131.77\ lb/sec$$

$$sfc = \frac{w_f}{F} = \frac{1.77(3600\ sec/hr)}{7000\ lb} = .911\ lb/lb\text{-}hrs$$

$F_R = 0$ since $V_o = 0$

$$\rho A_g V_1 = \dot{w} \qquad V_1 = \frac{\dot{w}}{\rho A_1 g} = \frac{131.77}{(.002378)(2)(32.2)} = 857\ ft/sec$$

$$\frac{9\times10}{1.2\times10^4}$$

2601769
1,539,00

2. $\omega_a = 60 \quad \omega_f = .7 \quad T_{ssl} = 3000 \quad V_0 = 0$

$$3000 = \frac{\omega_f + \omega_a}{g} V_e = \frac{60.7}{32.2} V_e = 1,885$$

$$V_e = 1591.5 \text{ ft/sec}$$

3. 20,000 ft $\quad T = 413.6°R \quad p_e = 9 \text{ psia} \quad A_e = 180 \text{ sq in} \quad p_o = 973 \text{ psfa}$

$$F_g = \frac{(\omega_f + \omega_a) V_e}{g} + A_e (p_e - p_o) = A_e \left[p_e (\gamma M_e^2 + 1) - p_o \right]$$

$$F_g = 180 \left[9(1.33 + 1) - 6.76 \right] = 180 \left[9(2.33) - 6.76 \right] = 180 \left[21 - 6.76 \right]$$

$$= 180 \left[14.24 \right] = 2563.2 \text{ lb}$$

$$A_e (p_e - p_o) = 180 (2.24) = 403.2 \qquad \% = 15.73\%$$

$$G = pAM\sqrt{\frac{g\gamma}{RT}} = 9(180)(1)\sqrt{\frac{(32.2)(1.33)}{(1460)(53.35)}} = 1620\sqrt{5.51 \times 10^{-4}} = 1620(2.35 \times 10^{-2}) = 38.1$$

$$\omega_a + \omega_f = 38.1 \qquad \omega_f = .02\omega_a \qquad \omega_a = \frac{38.1}{1.02} = 37.4$$

$$\frac{\omega_f}{\omega_a} = .02 \qquad \cancel{\omega_a = \frac{.7}{.02} = \frac{70}{2} = 35} \qquad F_R = \frac{37.4(550)}{32.2} = 639 \text{ lb}$$

$$F_N = F_g - F_R = 2563.2 - 639 = 1924.2 \text{ lb}$$

$$THP = \frac{FV_0}{550} = \frac{1924.2}{550}(550) = 1924.2 \text{ HP}$$

$$V_e = (2563.2 - 403.2)\frac{g}{\omega_f + \omega_a} = \frac{2160 (32.2)}{38.1} = 1824 \text{ ft/sec}$$

$$\cancel{F_g = 2563.2} \quad \cancel{G_e = 38.1} \quad \cancel{g = 32.2} \quad \cancel{V_{e_f} = \frac{F_G g}{G_e}} = 2163 \text{ ft/sec}$$

$$\eta_p = \frac{2(550)}{2163 + 550} = \frac{1100}{2713} = 40.5\%$$

$$\eta_{TH} = \left\{ 1924.2(550) + \frac{38.1}{64.4}(1613)^2 \right\} \Big/ .7(18500)(778.16) = 25.76\%$$

$\eta_0 = \eta_P \times \eta_{TH} = 40.6 \times 25.76 = 10.48\ \%$

$$sfc = \frac{V_0}{\eta_0 (HV)} = \frac{550\,(3600)\ \frac{ft}{hr}}{(.1048)(18500)(778.16)} = 1.32\ lb/lb\text{-}hr$$

1980000

14,395,960

1,501,499

5. $w_a = 130$ $F = 7000$ $w_f = 1.77$ $d_e = 20$ $\frac{\pi d^2}{4} = \frac{314 in^2}{144 in^2/ft^2} = 2.18\ ft^2$

a $\cancel{V_{ef}} = V_e = \frac{Fg}{G_e} = \frac{7000\,(32.2)}{131.77} = 1710\ ft/sec$ OK

$(\cancel{32.2})(.002378)\left(\frac{314}{144}\right)(1710) = 2.855$

b $\cancel{\dot{W} = \rho g A V = \overset{V}{1710}\,(\overset{\rho}{.002378})(\overset{g}{32.2})(\overset{A}{2.18})} =$ $\dot{W} = 130 + 1.77 = 131.77$

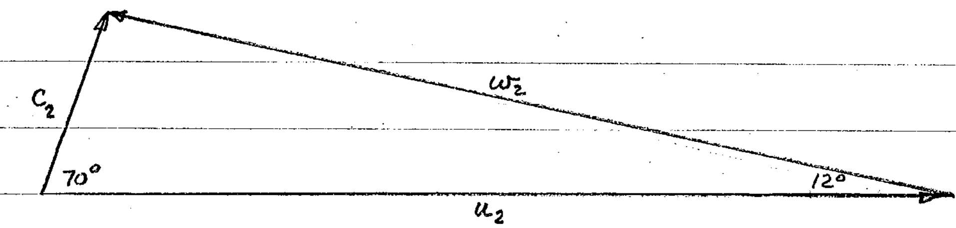
c $sfc = \frac{1.77\,(3600)}{7000}\ \frac{lb/hr}{lb} = .911\ lb/lb\text{-}hrs.$

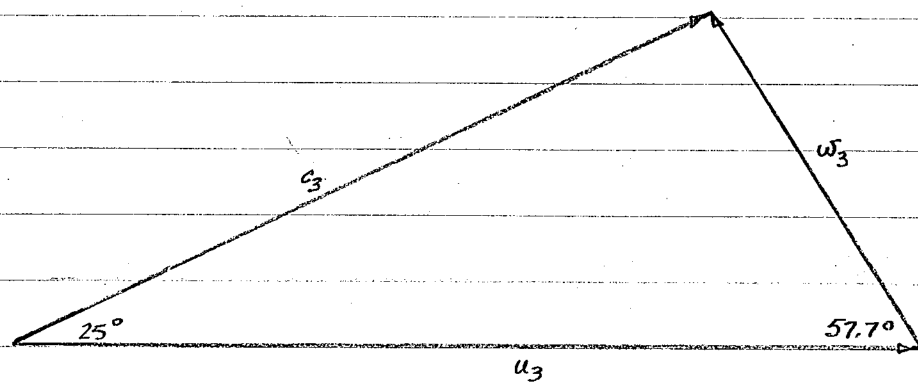
d 0 since $V_0 = 0$

e $\rho g A V = \dot{W}$ $V = \frac{\dot{W}}{\rho g A} = \frac{131.77}{(.002378)(32.2)(2)} = 851\ ft/sec$

Cesar Levy
AE 371

2. let $1'' = 300$ ft/sec





$9000 \frac{rev}{min} \longrightarrow u_3 = u_2 = \frac{9000}{60} 2\pi \frac{18}{12} = 1415$ ft/sec

$$w_2^2 = u_2^2 + c_2^2 - 2u_2 c_2 \cos\alpha_2 = (1415)^2 + (300)^2 - 2(1415)(300)\cos 70° = (1340)^2$$

$$w_3^2 = u_3^2 + c_3^2 - 2u_3 c_3 \cos\alpha_3 = (1415)^2 + (1200)^2 - 2(1415)(1200)\cos 25° = (600)^2$$

$\frac{1340}{\sin 70°} = \frac{300}{\sin\beta_2}$ $\beta_2 = 12°$

$\frac{600}{\sin 25°} = \frac{1200}{\sin\beta_3}$ $\beta_3 = 57.7°$

(d) $C_{3u} = C_3 \cos\alpha_3 = 1200 \cos 25° = 1089.5$ ft/sec

$C_{2u} = C_2 \cos\alpha_2 = 300 \cos 70° = 102.5$ ft/sec

$$Tq = \frac{G}{g}\left(r_3 C_{3u} - r_2 C_{2u}\right) = \frac{32.2}{32.2}(1.5)\left[1089.5 - 102.5\right] = 1481 \text{ lb-ft}$$

(e) no since η_c depends on p_3/p_2, T_3/T_2 not Tq

(f) $HP = Tq \frac{\omega}{550} = \frac{9000(2\pi)}{550(60)} 1481 = 2540$ HP

(g) $H_t' = \frac{1}{g}\left[u_3 C_{3u} - u_2 C_{2u}\right] = \frac{1}{32.2}(1415)\left[1089.5 - 102.5\right] = 43400 \text{ ft}$

$= 55.8$ BTU/lb.

(h) none since $u_3 = u_2$

(i) $\frac{w_2^2 - w_3^2}{2g} = \frac{(180 - 36)\times 10^4}{64.4} = 22400 \text{ ft}$

(j) $\frac{C_3^2 - C_2^2}{2g} = \frac{(1200)^2 - (300)^2}{64.4} = 21000 \text{ ft}$

(k) entrance $r_2 C_{2u} = 1.5(102.5) = 154 \text{ ft}^2/\text{sec}$

exit $r_3 C_{3u} = 1.5(1089.5) = 1634 \text{ ft}^2/\text{sec}$

(l) $\Delta C_u = C_{3u} - C_{2u} = 1089.5 - 102.5 = 987 \text{ ft/sec}$

(m) $c_p T_2 = c_p t_2 + \frac{1}{2g} C_2^2$ $C_2 = 300 \text{ ft/sec}$ $t_2 = 519$

$T_2 = 519 + \frac{\frac{1}{2}(300)^2}{(32.2)(186.7)} = 526.5°R$

$W_c = \frac{c_p T_2}{\eta_c} X = 55.8 = \frac{.24(526.5)}{1} X$ $X = .442$ $p_{t3}/p_{t2} = 3.62$ ✓

(8) (a) $W_c = c_p(T_3 - T_2) = .24(880 - 519) = 86.5$ BTU/lb

(b) $HP = \frac{W_c}{.707}(60) = 7350$ HP

(c) $\eta_c = \frac{c_p T_2}{W_c} X = \frac{.24(519)}{86.5} X$ $p_{t3}/p_{t2} = \frac{70}{14.7} = 4.73$, $X = .5589$

$\eta_c = 80.4\%$

(d) $\Delta r c_u = \frac{W_c g}{\omega} = \frac{86.5(32.2)60}{12000(2\pi)} = 2.22 \frac{\text{ft}}{\text{sec}}\frac{\text{BTU}}{\text{lb}}$, $1727 \text{ ft}^2/\text{sec}$

(e) $T_q = \frac{G}{g}\Delta r c_u$ $\frac{60}{32.2}(2.22) = 4.14 \frac{\text{lb-ft}}{\text{ft-lb}}$ BTU or 3220 lb-ft

(9) $p_3/p_2 = 4$ $T_3 = 800°R$ $T_2 = 500°R$

$$\eta_c = \frac{(p_3/p_2)^{\frac{\gamma-1}{\gamma}} - 1}{T_3/T_2 - 1} = \frac{.486}{1.6 - 1} = .81$$

81% eff.

(1) $P_3/P_2 = 4$ $\quad T_3 = 800°R$

$T_2 = 500°R$

$$\frac{.4860}{1.6-1} = \frac{.486}{.6} = .81$$

987
444

(2) $C_{2u} = 300 \cos 70°$ $\quad C_{3u} = 1200 \cos 25°$

$1089.5 \quad 102.5 = 987$

(d) $$T_q = \frac{G}{g}\left(r_3 C_{3u} - r_2 C_{2u}\right) = \frac{32.2}{32.2}\left(\frac{18}{12}\right)\left(.906 - .342\right) = \frac{18}{12}\left(\cancel{.564}\right) = .845$$

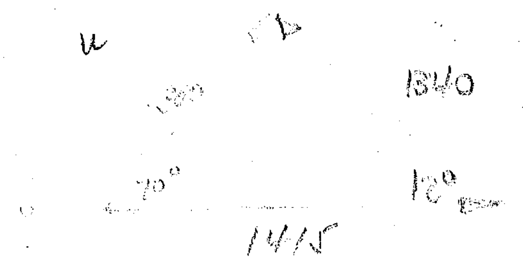
1481 lb-ft

$9000 \frac{rev}{min}$ $\quad V = \omega r$

$$u_3 = \frac{9000}{60} 2\pi r = 943 \text{ ft/sec}$$

943
472
1415

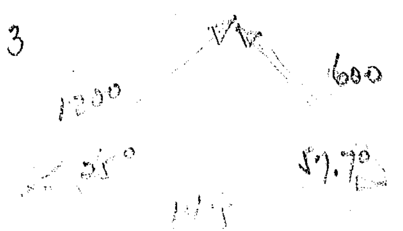
(a)



$$w_2^2 = (1415)^2 + (300)^2 - 2(1415)(300)\cos 70°$$

$$= 2\times10^6 + 9\times10^4 - 2.9\times10^5$$

(c)



209×10^4
-29
$180\times10^4 = 13.40$

$$w_3 = 144\times10^4 + 200\times10^4 - 308\times10^4$$

344×10^4 $\quad 36\times10^4 \quad 600$
308×10^4

$$\frac{\sin 70°}{1340} = \frac{\sin x}{300} \quad (b)$$

$$\frac{300}{1340}($$

$\cancel{\frac{600}{\sin 25°}}$ $\quad \sin x = 2\sin 25°$

(e) no since η_c depends on P_3/P_2, T_3/T_2 not T_q

(f) $$HP = \frac{1481(9000)(2\pi)}{550\times60} = \frac{1481(3)(2\pi)}{11} = 2540 \text{ HP}$$

(g) $$H_i' = \frac{1}{32.2}\left[(1415)(1089) - 1415(102.5)\right] = \frac{1415(987)}{32.2} = \frac{43400 \text{ ft}}{778.16 \frac{ft\text{-}lb}{BTU}}$$

$= 56.8$ BTU/lb

(h) 0 since $u_3 = u_2$

(i) $$\frac{-36\times10^4 + 180\times10^4}{64.4} = \frac{144\times10^4}{64.4} = 22400 \text{ ft}$$

(j) $\frac{C_3^2 - C_2^2}{2g} = \frac{144 \times 10^4 - 9 \times 10^4}{2g} = \frac{135 \times 10^4}{64.4} = 2.1 \times 10^4 = 21000$ ft

(k) C_u $1\frac{1}{2}(1089.5) = 1089.5 + 545 = 1634$ ft/sec

C_u $1\frac{1}{2}(102.5) = 102.5 + 52 = 154$ ft/sec

$\Delta C_u = 987$ ft/sec

$c_p T_2 = c_p t + \frac{1}{2} c_2^2$

$(186.7) T_2 = 186.7(519)$ $\frac{1}{2}(90{,}000)$ 45000

97000

$\frac{P_{t_3}}{P_{t_2}} = \left[1 + \frac{9000(2\pi)(1.5)(987)}{.24(519)(32.2)(60)} \right]$

$.761 = T_2$ $\frac{200(500)}{10 \times 10^4}$ $\frac{142000}{186.7} = T_2$

$T_3 - T_2$ $= [\ 349\ ]$

$c_p T_3 = c_p(t_3) +$

$43{,}500 / 186.7(761)$

58.8 BTU/lb $= W_c = .24(761)$

$\frac{56.8}{180 \cdot .24(519)} = 397$

778.16

176 $\frac{2}{3}(.437)$

2.55

(8) $T_t = 880°R$ $p_t = 70$ $G = 60$ $P_{t_3}/P_t = \frac{70}{14.7} = 4.75$

$(880 - 519).24 = W_c = 361(.24) = 86.5$ BTU/lb

$HP = \frac{86.5(60)}{.707} = 7350$ HP

ok

$\eta_c = \frac{c_p T_{t_2} X_c}{W_c} = \frac{(.24)(519)(.5589)}{86.5} =$ 80.4%

$\frac{(60)86.5(32.2)}{12000(2\pi)} = (\Delta r C_u) = 2.22$ ft²/sec (778.16) BTU/lb

1727.61

$T_q = \frac{G}{g}(\Delta r C_u) = \frac{60}{32.2}(2.22) = 4.14(778.16) = 3220$ lbf

$T_{t_3} = 519 + \frac{86.5}{.24} \frac{BTU}{lb} =$ $1 =$ $86.5 = .24(T_{t_3}' - T_{t_2})$ $T_{t_3}'/T_{t_2} = ($

$= \frac{361}{519} = \frac{T_{t_3}' - T_{t_2}}{T_{t_2}}$ .695

Cesar Levy AE371

Given: $C_2 = 400$ ft/sec $\quad u = 1000$ ft/sec $\quad p_y/p_x = 1.15 \quad \eta_c = .9$

$$\mathcal{W}_c = \frac{c_p T_2}{\eta_c}\left\{\left(p_y/p_x\right)^{\frac{\gamma-1}{\gamma}} - 1\right\} = \frac{u}{g}\left[C_{3u} - C_{2u}\right]$$

using $\gamma = 1.4 \quad c_p = 186.7$ ft/°R $\quad T_2 = 520°R$

$$\left(p_y/p_x\right)^{\frac{\gamma-1}{\gamma}} - 1 = .0407$$

$$C_{3u} - C_{2u} = \frac{g}{u}\frac{c_p T_2}{\eta_c}\left\{\left(p_y/p_x\right)^{\frac{\gamma-1}{\gamma}} - 1\right\} = 141.4 \text{ ft/sec}$$

γ	C_{2u}	C_{3u}	C_{3a}	C_3	ω_2	ω_3	%
0°	0	141.4	400	424	1078	947	93
15°	103.5	244.9	386	457	976	847	82.6
30°	200	341.4	346.5	486	872	744	73.0
45°	282.8	424.2	282.8	509	772	641	64.7
60°	346.5	487.9	200	527	683	549	58.2
75°	386	527.4	103.5	537	622	482	54.3
90°	400	541.4	0	541.4	600	459	53.
-15°	-103.5	37.9	386	388	1169	1037	103.4
-30°	-200	-58.6	346.5	357	1249	1115	112.8
-45°	-282.8	-141.4	282.8	316	1313	1176	121.1
-60°	-346.5	-205.1	200	286.5	1361	1222	127.5
-75°	-386	-244.6	103.5	265.5	1390	1248	131.7
-90°	-400	-258.6	0	258.6	1400	1260	132.9

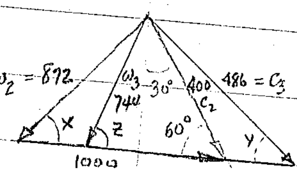
Equations used

$C_{2u} = C_2 \sin\gamma$ $\qquad C_{3u} = 141.4 + C_{2u}$ $\qquad C_{3a} = C_2 \cos\gamma$

$C_3 = (C_{3a}^2 + C_{3u}^2)^{1/2}$ $\qquad w_2^2 = C_2^2 + u_2^2 - 2u_2 C_2 \cos(90-\gamma)$

$w_3^2 = (C_3^2 - C_2^2) + w_2^2 - 2u(C_{3u} - C_{2u})$

at 30°



$\frac{C_2}{\sin x} = \frac{w_2}{\sin 60°}$

$\sin x = \frac{C_2}{w_2} \sin 60° = \frac{(.866)(400)}{872} = .397$

$X = 23\tfrac{1}{3}°$ angle of entry into rotor

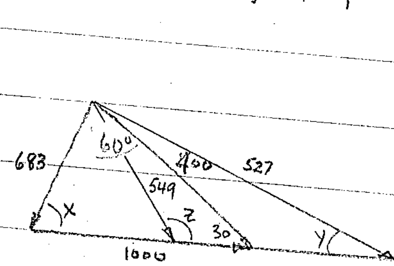
$C_{3u} = C_3 \cos y$ $\qquad \cos y = \frac{C_{3u}}{C_3} = \frac{341.4}{486} = .702$ $\qquad y = 45.5°$ angle of entry into stator

60° is angle of departure from stator

$\frac{C_3}{\sin Z} = \frac{w_3}{\sin Y}$ $\qquad \sin Z = \frac{C_3}{w_3} \sin Y = \frac{(.714)(486)}{744} = .466$

$Z = 27.7°$ angle of departure from rotor.

at 60°



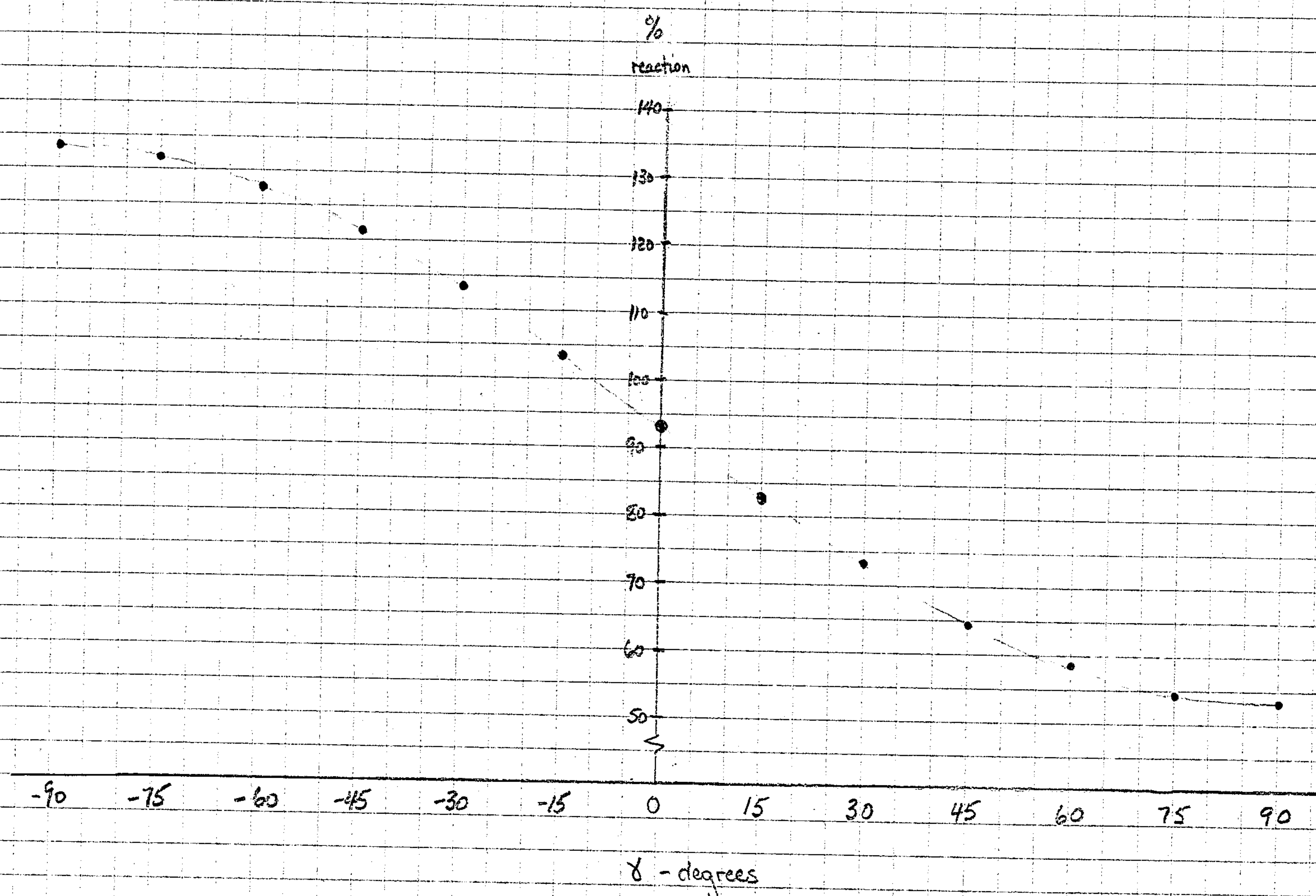
$\sin X = \frac{C_2}{w_2} \sin 30° = \frac{400}{683}(.5) = .293$

$X = 17.1°$ angle of entry into rotor.

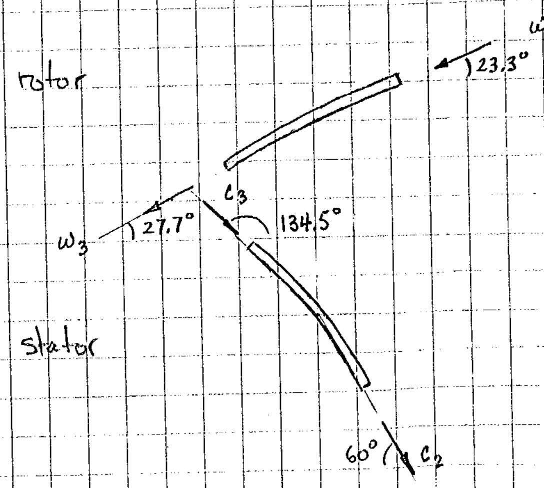
$\cos y = \frac{C_{3u}}{C_3} = \frac{487.9}{527} = .926$ $\qquad y = 22.2°$ angle of entry into stator

30° angle of departure from stator

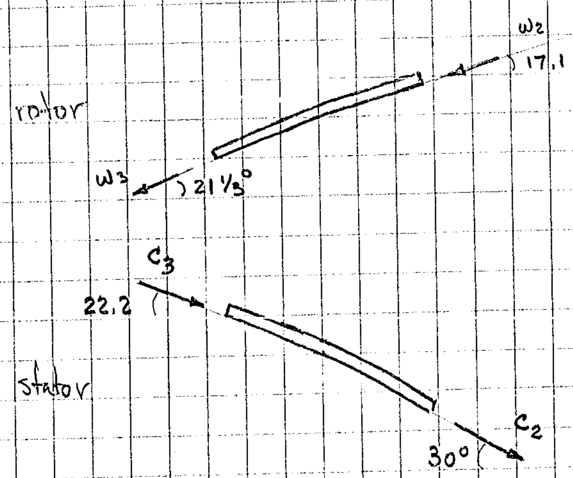
$\sin Z = \frac{C_3}{w_3} \sin Y = \frac{527}{549}(.377) = .364$ $\qquad Z = 21\tfrac{1}{3}°$ angle of departure for rotor


%
reaction
140
130
120
110
100
90
80
70
60
50
-90
-75
-60
-45
-30
-15
0
15
30
45
60
75
90
γ - degrees

$\gamma = 30^\circ$



$\gamma = 60^\circ$



$C_2 = 400$ ft/sec
$u = 1000$ ft/sec $\qquad p_3/p_2 = 1.15 \qquad \eta_c = .9$

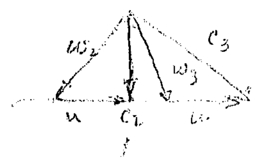
C_{3u}, C_{3a}, C_{2u}
w_2, w_3, C_3, η_0

$$W_c = \frac{c_p T_2}{\eta_{ad_c}} \left\{ (p_3/p_2)^{\frac{\gamma-1}{\gamma}} - 1 \right\} = \frac{1}{g} u \left[C_{3u} - C_{2u} \right] \qquad \gamma = 1.4$$

$$(p_3/p_2)^{\frac{\gamma-1}{\gamma}} - 1 = .0407$$

$$W_c = \frac{.24(520)}{.9}(.0407)(778.16) = 4390 \text{ ft} = \frac{1000}{32.2}\left[C_{3u} - C_{2u}\right] \quad C_{3u} - C_{2u} = 141.4$$

$\gamma = 0°$ $\qquad C_{2a} \sin\delta = C_{2u} = 0 \qquad C_{3u} = 141.4$



~~$V = C_2 = 400$ ft/sec~~

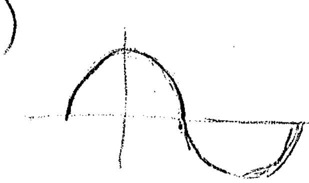
$C_2^2 + u^2 = w_2^2 \qquad (160000) + (100,0000) = 116,0000 \qquad w_2 = 1078$ ft/sec

$C_{3a} = C_2 = 400 \qquad C_{3u} = 141.4 \qquad C_3^2 = \sqrt{C_{3a}^2 + C_{3u}^2} = \sqrt{160000 + 20000} = 180000 = 424$ ft/sec

$(180000 - 160000) + (116,0000 - w_3^2) = 2(141400) = 28,2800$

$2,0000 + 116,0000 - w_3^2 = 28,2800$

$118,0000 - 28,2800 = 89,7200 = w_3^2 \qquad w_3^2 = (947 \text{ ft/sec})^2$



$$\% \text{ reaction} = \frac{116,0000 - 89,7200}{28,2800} = \frac{26,2800}{28,2800} = 93\%$$

$\gamma = -15°$ $\qquad w_2^2 = c_2^2 + u_2^2 - 2u_2c_2\cos 75° = 160000 + 100,0000 - 2(400)(1000)(.259) = 922500$
$80\,0000 \qquad 20.75$

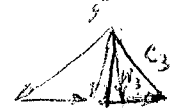
$C_{2u} = C_2 \sin 15° = 400(.259) = 103.5$ ft/sec $\qquad C_{3u} = 37.9$ (116)

$C_{3a} = C_2 \cos 15° = 400(.9659) = 386 \qquad C_3^2 = C_{3a}^2 + C_{3u}^2 = (386)^2 + (37.9)^2 = 149000 + 1437 = 150,437 = (388)$

$(150,437 - 160,000) + (922,500 - w_3^2) = 282800$
$-9,563$

$282800 + 9563 = 292,363$

$\frac{292363}{282800} \qquad \%_0\text{react} = 103.4$

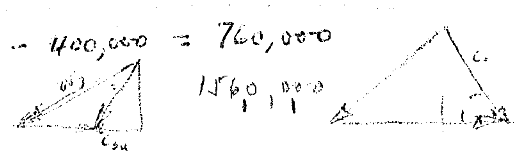


$\delta = -30° \qquad w_2^2 = c_2^2 + u_2^2 - 2u_2c_2\cos 60° = 1160,000 - 400,000 = 760,000$
$1560,000$

$C_{2u} = C_2 \sin 30° = 200 \qquad C_{3u} = -58.6$ ft/sec

turbine

$C_{3a} = C_2 \cos 30° = 346.5 \qquad C_3^2 = (346.5)^2 + (58.6)^2 = 120,000 + 3430 = 123430$



$(123,430 - 160,000) + \qquad = 282,800$
$-36,570$

$$\frac{319,370}{282,800} = 112.8\%$$

$\gamma = 45^\circ$ $\quad w_2^2 = c_2^2 + u_2^2 - 2uc\cos 45^\circ = 1{,}160{,}000 - 800{,}000(.707) = 1160{,}000 - 565{,}000 = 595{,}000$

$c_{2u} = c_2 \sin 45^\circ = 400(.707) = 282.8$ $\quad c_{3u} = -141.4$ $\quad \frac{565}{1725000}$

turbine $\quad c_{3a} = c_2 \cos 45^\circ = 282.8$ $\quad (80{,}000) + (20{,}000) = (100{,}000) = c_3^2$

$(100{,}000 - 160{,}000) + (\quad) = 282{,}800$ $\quad 332800$

$-60{,}000$

$\%\text{react} = \frac{\cancel{332800}\ 342800}{282800} = 117.6\%$ $\quad 541.4$ $\quad 258.6$ $\quad 800.0$

$\gamma = 60^\circ$ $\quad w_2^2 = c_2^2 + u_2^2 - 2uc\cos 30^\circ = 1{,}160{,}000 - 800{,}000(.866) = 467{,}000$

$1{,}160{,}000 \; 693{,}000$ $\quad 1160{,}000$ $\quad 693{,}000$ $\quad 1853{,}000$

$c_{2u} = c_2 \sin 60^\circ = 400(.866) = 346.4$ $\quad c_{3u} = -205.1$

$c_{3a} = c_2 \cos 60^\circ = 200$ $\quad (200)^2 + (205.1)^2 = 40000 + 42100 = 82100 = c_3^2$

turbine $\quad (82100 - 160{,}000) + (\quad) = 282{,}800 + 77900$ $\quad \frac{360700}{282{,}800} = 127.5\%$

-77900 $\quad 360700$

$\gamma = 75^\circ$ $\quad w_2^2 = c_2^2 + u_2^2 - 2uc\cos 15^\circ = 1{,}160{,}000 - 800{,}000(.9659) = 387{,}000$

1160 $\quad 773{,}000$

$c_{2u} = c_2 \sin 75^\circ = 400(.9659) = 386$ $\quad c_{3u} = -244.6$ $\quad 1933000$

turbine $\quad c_{3a} = c_2 \cos 75^\circ = 400(.259) = 103.5$ $\quad (103.5)^2 + (244.6)^2 = 10700 + 59800 = 70500$

$282{,}800$

$(70500 - 160{,}000) = -89500$ $\quad \frac{372300}{282800} = 131.7\%$

372300

$\gamma = 90^\circ$ $\quad w_2^2 = c_2^2 + u_2^2 - 2uc = 1{,}160{,}000 - 800{,}000 = 360{,}000$

16 $\quad 1960$

$c_{2u} = c_2 = 400$ $\quad c_{3u} = -258.6$

turbine $\quad c_{3a} = 0$ $\quad c_3^2 = (258.6)^2 = 66900$

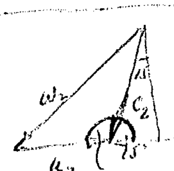
282800

$(66900 - 160{,}000) = 93100$ $\quad \frac{375900}{282800} = 132.9\%$

375900

$\gamma = -15^\circ$ $\quad w_2^2 = c_2^2 + u_2^2 + 2u_2c_2 \cos 75^\circ \; (.259)$ $\quad \cos(90+15^\circ) = \cos 90 \cos 15 - \sin 90 \sin 15$

$= 1{,}160{,}000 + 207500 = 1{,}367{,}500 \; 1{,}384{,}100$



375900 $\quad 282800$ $\quad 1931$ $\quad -400$ $\quad +141.4$ $\quad 258.6$

$c_{3u} - c_{2u} = 141.4$

$c_{2u} = -c_2 \sin 15^\circ = -103.5$ $\quad c_{3u} = 37.9$

$c_{3a} = c_2 \cos 15^\circ = 386$ $\quad c_3^2 = 150{,}437$ $\quad w_3^2 = \frac{1960{,}000 \; 375{,}900 \; 292363}{282{,}800} = 103.4$ $\quad 99300$

282800 $\quad -93100$

$c_3^2 - c_2^2 = 150{,}437 - 160{,}000 = -9563$ $\quad (1960{,}000 - w_3^2) + (66900 - 160{,}000) = 3759$

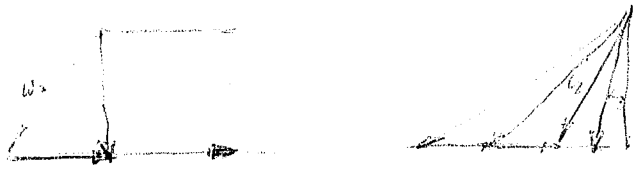
273263

$\gamma = -30^\circ$ $\quad w_2^2 = c_2^2 + u_2^2 + 2u_2c_2(\;.5) = 1360{,}000$

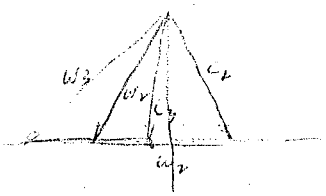
$C_{2u} = -C_2 \sin 30^\circ = -200$ $\quad C_{3u} = -\;381$

$C_{3a} = 346.5$

40000



$$\begin{array}{r} 149000 \\ 60000 \\ \hline \sqrt{209{,}000} \end{array}$$



$c_3^2 = 209{,}000$ $\quad c_2^2 = 160{,}000$

$w_2^2 = 952{,}500$ $\quad \Delta C^2 = 49{,}000$

$w_3^2 = 718{,}700$

$$\begin{array}{r} 952{,}500 \\ \hline 1{,}001{,}500 \\ -282{,}800 \\ \hline 718{,}700 \end{array}$$

$$\frac{w_2^2 - w_3^2}{282{,}800} = \frac{233{,}800}{282{,}800}$$

$$\begin{array}{r} -9563 + 1{,}367{,}500 \\ -292{,}400 \\ \hline 1075{,}100 \end{array}$$

$$\begin{array}{r} 120{,}000 \\ 116{,}500 \\ \hline 236{,}500 \leftarrow c_3^2 \\ 160{,}000 \\ \hline 76{,}500 \end{array}$$

$$\begin{array}{r} 282800 \\ -76500 \\ \hline 206{,}300 \end{array}$$

$$\begin{array}{r} 760{,}000 \\ \cancel{76{,}500} \\ 206{,}300 \\ \hline \sqrt{553{,}700} \end{array}$$

150000

$$\begin{array}{r} 18\;000 \\ 8\;000 \\ 260{,}000 \quad c_3^2 \\ -160\;000 \\ \hline 100{,}000 = c_3^2 - c_2^2 \end{array}$$

$$\begin{array}{r} 595{,}000 \\ 182{,}800 \\ \hline 412{,}200 \end{array}$$

$$\frac{182{,}800}{282{,}800}$$

$$\begin{array}{r} 40000 \\ 238500 \\ \hline 278500 \leftarrow c_3^2 \\ 160 \\ \hline 118500 \leftarrow c_3^2 - c_2^2 \end{array}$$

$$\frac{\quad}{282{,}800}$$

$$\begin{array}{r} 282{,}800 \\ -118\;500 \\ \hline \rightarrow 164{,}300 \end{array}$$

$$\begin{array}{r} 467000 \\ -164300 \\ \hline 302700 \end{array}$$

$$\cancel{\begin{array}{r} 595{,}000 \\ -182{,}800 \\ \hline 312{,}200 \end{array}}$$

37.

75°

$$\begin{array}{r} 10{,}700 \\ 278{,}000 \\ \hline 288700 \leftarrow c_3^2 - c_2 + \\ 160 \\ \hline 1\cancel{28}\,700 \leftarrow c_3^2 - c_2^2 \end{array}$$

$$\begin{array}{r} 282800 \\ -128700 \\ \hline 154{,}100 \end{array} \leftarrow w_2^2 - w_3^2$$

$$\begin{array}{r} 294000 \\ -160 \\ \hline 134000 \end{array} + (w_2^2 - w_3^2) = \begin{array}{r} 282800 \\ 134 \\ \hline =148800 \end{array}$$

1

250000

360000

$386000 = w_2^2$

$160,000 = c_2^2$

$289,000 = c_3^2$

$129,000 + 386000 - w_3^2 = 282800$

-129000

153800

386000

153800

$232,200$

$360000 = w_2^2$

$160,000 = c_2^2$

$293,000 = c_3^2$

$133,000 + 360,000 - w_3^2 = 282800$

-133

149800

$1,560,000 = w_2^2$

$\left.\begin{array}{l}160,000 = c_2^2 \\ 123,000 = c_3^2\end{array}\right\} -37000$

$360,000$

$-149,800$

$210,200$

$-37000 + 1,560,000 = 282800$

319800

1560000

319400

$1,240,600$

$w_2^2 = 1,725,000$

$c_3^2 = 100,000$

$c_2^2 = 160,000$

$-60,000 + 1,725,000 - w_3^2$

$= 282800$

60

342800

$1,725,000$

$342,800$

$1,382,200$

$w_2^2 = 1,853,000$

$\left.\begin{array}{l}c_3^2 = 82100 \\ c_2^2 = 160,000\end{array}\right\} -77900$

282800

779

$360,700$

$1,853,000$

$360,700$

$1492,300$

$1,960,000 = w_2^2$

$66900 = c_3^2$

931

$160,000 = c_2^2$

282800

931

375900

1933000

375900

1557100

$1,960,000$

375900

1584100

Cushing AE 371

① $1060°R = T_{t_3}$ $T_{t_4} = 2000°R$ $\Delta T_t = 940°R$

$\eta_b = .9$ $HV = 18900$

$$f/a = \frac{1}{\frac{\eta_b \, H.V.}{c_p(T_{t_4} - T_{t_3})}} - 1 = \frac{1}{\frac{.9(18900)}{.276(940)}} - 1 = \frac{1}{64.9 - 1} = .01542$$

From graph $\eta_b (f/a) = .014$ $f/a = \frac{.014}{.9} = .01555$

③ $\eta_b = .8$ $HV = 18,900$ $T_{t_3} = 1660°R$ $T_{t_4} = 3200°R$ $\Delta T_t = 1540°R$

$c_p = .276$

$$f/a = \frac{1}{\frac{\eta_b \, H.V.}{c_p \, \Delta T_t}} = \frac{1}{\frac{.8(18900)}{.276(1540)}} - 1 = \frac{1}{35.6 - 1} = .0289$$

from the graph $\eta_b (f/a) = .026$ $f/a = \frac{.026}{.8} = .0325$

④ $M_0 = .8$ $\gamma = 1.4$ $\eta_c = 1.00$ $P_3/P_2 = 7$ $\eta_b = 1.00$ $HV = 18900$

$T_{t_4} = 2100°R$

altitude	a_0 (ft/sec)	t_2 (°R)	T_{t_2} (°R)
SL	1116	518.69	585
10,000	1077.4	483.04	544.7
20,000	1036.9	447.43	504.5
30,000	994.85	411.86	464.4
40,000	968.08	389.99	439.8

$$T_{t_2} = t_2 + \frac{M_0^2}{2 g c_p} a_0^2 = t_2 + .532 \times 10^{-4} a_0^2$$

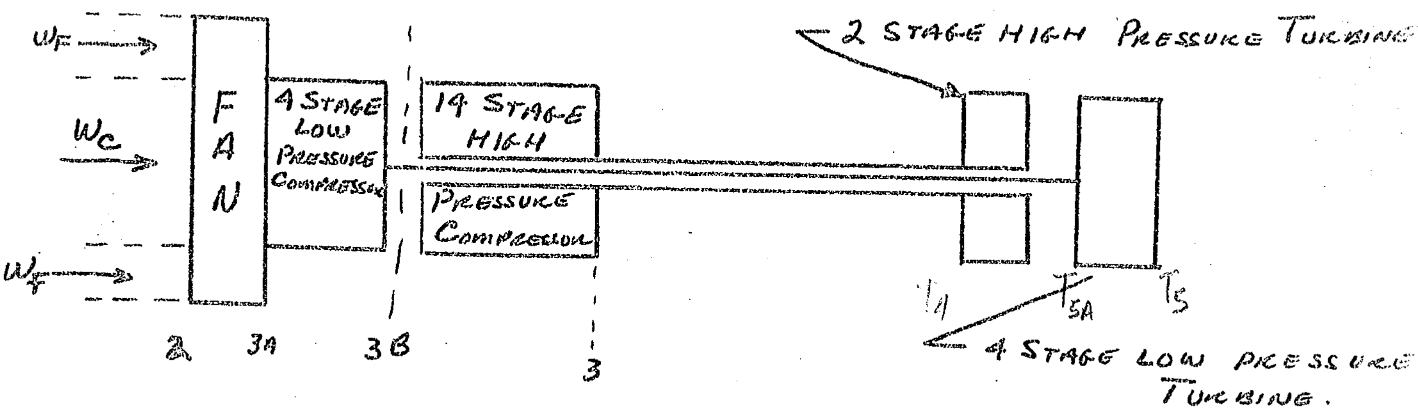
$$\eta_c = \frac{\left(P_3/P_2\right)^{\frac{\gamma - 1}{\gamma}} - 1}{\frac{T_{t_3}}{T_{t_2}} - 1}$$

$T_{t_3}/T_{t_2} - 1 = .7436$

T_{t_2} (°R)	$T_{t_3} - T_{t_2}$	T_{t_3}	$\frac{\eta_b HV}{c_p}$	$T_{t_4} - T_{t_3}$	$\frac{\eta_b HV}{c_p(T_{t_4} - T_{t_3})} - 1$
585	435	1020	68500	1080	62.4
544.7	404.5	949.2		1150.8	58.5
504.5	375	879.5		1220.5	56.1
464.4	345	809.4		1290.6	52.1
439.8	326.5	766.3		1333.7	50.4

(f/a)	from Graph $\eta_b(f/a) = (f/a)$
.01603	.016
.0171	.017
.01785	.018
.01921	.019
.01986	.0195

The characteristics of the General Electric CF 6-50 D turbofan engine are as follows



Fan pressure ratio $P_{3A}/P_2 = 1.72$

Low pressure compressor pressure ratio $\frac{P_{3B}}{P_{3A}} = 1.69$

Overall pressure ratio $P_3/P_2 = 29.4$

$\eta_c = \eta_T = .9$ $\eta_B = .95$

$\gamma_c = 1.4$ $\gamma_T = 1.33$

$T_4 = 2480°F$ $HV = 19000$ BTU/#

At sea level static conditions $W_c = 274$ lbs/s

$W_F = 1192$ lbs/sec ($\beta = 4.35$)

Determine: a) Horsepower of high pressure and low pressure turbines

b) Thrust developed by engine for the case of ideal expansion of the fan flow and the core flow

c) Specific fuel consumption.

T ~ 50,000 lb ± 10%

Cesar Levy AE 371

Since static sea level $V_2 = 0$ $t_2 = T_2 = 518.7°R$ $P_2 = 2116$ psf

for $\eta_F = .9$ $P_{3A}/P_2 = 1.72$ $\gamma = 1.4$

$$T_{3A} - T_2 = \frac{T_2}{\eta_F}\left[\left(P_{3A}/P_2\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = .1863\, T_2 = 96.8$$

$$\mathcal{W}_F = c_p\,(T_{3A} - T_2) = .24\,(96.8) = 23.24 \text{ BTU/lb}$$

$$HP_F = \frac{(w_F + w_c)\,\mathcal{W}_F}{.707} = \frac{1466\,(23.24)}{.707} = 48200 \text{ HP}$$

Now $T_{3A} = 96.8 + 518.7 = 615.5°R$, $P_{3A} = 1.72 P_2 = 3637$ psf

for $\eta_{c_{LPC}} = .9$ $P_{3B}/P_{3A} = 1.69$ $\gamma = 1.4$

$$T_{3B} - T_{3A} = \frac{T_{3A}}{\eta_{c_{LPC}}}\left[\left(P_{3B}/P_{3A}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = .1796\, T_{3A} = 110.5$$

$$\mathcal{W}_{LPC} = c_p\,(T_{3B} - T_{3A}) = .24\,(110.5) = 25.5 \text{ BTU/lb}$$

$$HP_{LPC} = \frac{w_c\,\mathcal{W}_{LPC}}{.707} = \frac{274\,(25.5)}{.707} = 9890 \text{ HP}$$

Now $T_{3B} = 110.5 + 615.5 = 726°R$, $P_{3B} = 1.69 P_{3A} = 6145$ psf

$$P_3/P_{3B} = \frac{P_3/P_2}{(P_{3B}/P_{3A})(P_{3A}/P_2)} = \frac{29.4}{(1.69)(1.72)} = 10.12$$

for $\eta_{c_{HPC}} = .9$ $P_3/P_{3B} = 10.12$ $\gamma = 1.4$

$$T_3 - T_{3B} = \frac{T_{3B}}{\eta_{c_{HPC}}}\left[\left(P_3/P_{3B}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = 1.042\, T_{3B} = 756$$

$$\mathcal{W}_{HPC} = c_p\,(T_3 - T_{3B}) = .24\,(756) = 181.4 \text{ BTU/lb}$$

$$HP_{HPC} = \frac{w_c\,\mathcal{W}_{HPC}}{.707} = \frac{274\,(181.4)}{.707} = 70350 \text{ HP}$$

Now $T_3 = 756 + 726 = 1482°R$, $P_3 = 10.12 P_{3B} = 62300$ psf

1. HP of HPT = HP of HPC = 70350 HP

HP of LPT = HP of FAN + HP of LPC = 58090 HP

TOTAL HP = 128440 HP

$$T_4 - T_{5A} = \frac{W_{HPT}}{c_{p_{HOT}}} = \frac{W_{HPC}}{c_{p_{HOT}}} = \frac{181.4}{.276} = 658$$

Given $T_4 = 2480°F = 2939.7°R$

$T_{5A} = 2939.7 - 658 = 2281.7°R$ (note: T_{5A} is temp at interface of high & low pressure turbines)

$$T_5 - T_{5A} = \frac{W_{LPT}}{c_{p_{HOT}}} = \frac{HP_{LPT}(.707)}{w_c \; c_{p_{HOT}}} = \frac{58090(.707)}{.276(274)} = 544$$

$T_5 = 2281.7 - 544 = 1737.7°R$ ✓

$$P_{5A}/P_4 = \left(1 - \frac{W_{T_{HPT}}}{c_p T_4 \eta_T}\right)^{\gamma/\gamma-1} \quad \text{for } \eta_T = .9 \quad \gamma = 1.33$$

$P_{5A}/P_4 = .316$, $P_{5A} = .316\,P_4 = .316\,P_3 = 19670$ psf

$W_{LPT} = .276\,(T_5 - T_{5A}) = .276\,(544) = 150.1$ BTU/lb

$$(P_5/P_{5A}) = \left(1 - \frac{W_{LPT}}{c_p T_{5A} \eta_T}\right)^{\gamma/\gamma-1} \quad \text{for } \eta_T = .9 \quad \gamma = 1.33$$

$P_5/P_{5A} = .289$, $P_5 = .289\,P_{5A} = 5685$ psf

now $P_5/P_2 = \frac{P_5}{P_{5A}} \frac{P_{5A}}{P_4} \frac{P_4}{P_3} \frac{P_3}{P_2} = (.289)(.316)(1)(29.4) = 2.685$

Since isentropic expansion $P_5 = P_7$ $T_5 = T_7$; $p_7 = p_2 = P_2$

For core flow:

$P_7/p_7 = P_3/P_2 = 2.685$, $\gamma = 1.33$

$$\frac{P_7}{p_7} = \left[1 + \frac{\gamma - 1}{2} M_c^2\right]^{\gamma/\gamma-1} = \left[1 + \frac{1}{6} M_c^2\right]^4 \qquad M_{core} = 1.246$$

$$T_7 = t_7\left[1 + \frac{1}{6} M_c^2\right] = 1.259\, t_7 \qquad t_7 = \frac{(T_7 = T_3)}{1.259} = \frac{1,739.7}{1.259} = 1380\,°R$$

$a_c = \sqrt{\gamma g R t_7} = 1772$ ft/sec

$V_{core} = M_{core}\, a_{core} = 1772(1.246) = 2207$ ft/sec

$$F_{core} = \frac{w_{core} V_{core}}{g} = \frac{274(2207)}{32.2} = 18750 \text{ lb}$$

For bypass flow

$P_7/p_7 = \dfrac{P_{3A}}{P_2} = 1.72$, $\gamma = 1.4$ from tables $M = .9155$

$$T_{3A} = T_7 = t_7\left[1 + M_F^2\right] = 1.168\, t_7 \qquad t_7 = \frac{T_{3A}}{1.168} = \frac{615.5}{1.168} = 527\,°R$$

$a_c = \sqrt{\gamma g R t_7} = 1124$ ft/sec

$V_{BP} = M_F\, a_F = .9155(1124) = 1030$ ft/sec

$$F_{BP} = \frac{w_F V_{BP}}{g} = \frac{1192(1030)}{32.2} = 38100 \text{ lb}$$

2. $F_{TOT} = F_{core} + F_{BP} = 56850$ lb ✓

$T_4 - T_3 = 2939.7 - 1482 = 1457.7\,°R$

$$f/a = \frac{1}{\dfrac{\eta_b\, HV}{Cp_{Hot}(T_4 - T_3)} - 1} = \frac{1}{\dfrac{.95(19000)}{.276(1457.7)} - 1} = \frac{1}{43.55} = .02297$$

$w_f = w_c(f/a) = 274(.02297) = 6.285$ lb/sec

8. $sfc = \frac{W_f\,(3600)}{F_{TOT}} = \frac{6.285\,(3600)}{56850} = .3985\ \frac{lb}{lb\text{-}hr.}$

	T_{t_2}	$T_{t_3} - T_{t_2}$	T_{t_3}	$\frac{\eta_b HV}{c_p}$	$T_{t_4} - T_{t_3}$	$\frac{\eta_b HV}{c_p(T_{t_4}-T_{t_3})} - 1$
SL	585	435	1020	68500	1080	62.4
10	544.7	404.5	949.2		1150.8	58.5
20	504.5	375	879.5		1220.5	56.1
30	464.4	345	809.4		1290.6	52.1
40	439.8	326.5	766.3		1333.7	50.4

$T_{t_4} = 2100$

f/a	from graph η_b f/a = f/a
.01603	.016
.0171	.017
.01785	.018
.01921	.019
.01986	.0195

(3)

$$\frac{f}{a} = \frac{1}{\dfrac{\eta_b (H.V.)}{c_p (T_{t_4} - T_{t_3})} - 1}$$

$\eta_b = .8$ $HV = 18,900$

$T_{t_3} = 1660°R$ $T_{t_4} = 3200°R$

$C_p = .276$

$T_{t_4} - T_{t_3} = 1540°R$

$$\frac{1}{\dfrac{(.8)(18900)}{(.276)(1540)} - 1} = \frac{1}{35.6 - 1} = \frac{1}{34.6} = .0289$$

from graph $.026 = \eta_b (f/a) = \dfrac{.026}{.8} = .0325$

(1) $1060°R = T_{t_3}$ $T_{t_4} = 2000°R$ $\Delta T_t = 540°R$

$\eta_b = .9$ $HV = 18900$

$$\frac{1}{\dfrac{(.9)(18900)}{.276\ (940)} - 1} = \frac{1}{64.9 - 1} = \frac{1}{63.9} = .01542$$

$\eta_b (f/a) = .014$ $f/a = \dfrac{.014}{.9} = .01555$

(4) $M_0 = .8$ $t_2 + \dfrac{1}{2g}\dfrac{M_0^2 a_0^2}{c_p} = T_{t_2}$

at SL $a_0 = 1116$ ft/sec. $t_2 = 518.69°R$ $M_0^2 = .64$ $\gamma = 1.4$

10,000 $a_0 = 1077.4$ $t_2 = 483.04$ $c_p = 186.7$

20,000 $a_0 = 1036.9$ $t_2 = 447.43$ $\dfrac{M_0^2}{2gC_p} = \dfrac{.64}{(64.4)(186.7)} = .532 \times 10^{-4}$

30,000 $a_0 = 994.85$ $t_2 = 411.86$

40,000 $a_0 = 968.08$ $t_2 = 389.99$

$$\left.\begin{array}{ll}
SL = & 518.69 + .532\times10^{-4}(1.247\times10^6) = 518.69 + 66.31 = 585°R \\
10{,}000 = & 483.04 + .532\times10^{-4}(1.159\times10^6) = 483.04 + 61.66 = 544.7°R \\
20{,}000 = & 447.43 + .532\times10^{-4}(1.071\times10^6) = 447.43 + 57.07 = 504.5°R \\
30{,}000 = & 411.86 + .532\times10^{-4}(.988\times10^6) = 411.86 + 52.54 = 464.4°R
\end{array}\right\} T_{t_2}$$

$$40{,}000 = 389.99 + .532\times10^{-4}(.938\times10^6) = 389.99 + 49.81 = 439.8°R$$

$$\eta_c = \frac{(p_3/p_2)^{\frac{\gamma-1}{\gamma}} - 1}{\dfrac{T_{t_3}}{T_{t_2}} - 1}$$

$T_{t_3}/T_{t_2} - 1 = \dfrac{X}{1} = .7436$

$T_{t_3} - T_{t_2} = .7436\, T_{t_2}$

since static sea level $V_2 = 0$ $t_2 = T_2 = 520°R$ $P_2 = 2116$

Assume $\eta_F = .9$

$$\eta_F = \frac{\left(\frac{P_{3A}}{P_2}\right)^{\frac{\gamma-1}{\gamma}} - 1}{T_{3A}/T_2 - 1} \qquad T_{3A}/T_2 - 1 = \frac{\left(\frac{P_{3A}}{P_2}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_F} = \frac{(1.72)^{2/7} - 1}{.9}$$

$$\frac{T_{3A} - T_2}{T_2} = \frac{.1676}{.9} = .1863$$

$T_{3A} - T_2 = 96.8$ $\underline{T_{3A} = 616.8\,°R}$ $P_{3A} = 3640$ psf

$$\eta_{c_{LPC}} = \frac{\left(\frac{P_{3B}}{P_{3A}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{T_{3B}/T_{3A} - 1} \qquad \frac{T_{3B} - T_{3A}}{T_{3A}} = \frac{\left(\frac{P_{3B}}{P_{3A}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{c_{LPC}}} = \frac{(1.69)^{2/7} - 1}{.9} = \frac{.1617}{.9}$$

$= .1796$

$T_{3B} - T_{3A} = 110.8$ $\underline{T_{3B} = 727.6\,°R}$ $P_{3B} = 6145$ psf

$$P_3/P_2 = \frac{P_3}{P_{3B}}\,\frac{P_{3B}}{P_{3A}}\,\frac{P_{3A}}{P_2} = \frac{P_3}{P_{3B}}(1.69)(1.72) \qquad P_3/P_{3B} = \frac{29.4}{(1.69)(1.72)} = 10.12$$

$$\eta_{c_{HPC}} = \frac{\left(P_3/P_{3B}\right)^{\frac{\gamma-1}{\gamma}} - 1}{T_3/T_{3B} - 1} \qquad \frac{T_3 - T_{3B}}{T_{3B}} = \frac{\left(P_3/P_{3B}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{c_{HPC}}} = \frac{(10.12)^{2/7} - 1}{.9} = \frac{.9373}{.9}$$

$= \cancel{.942}\ 1.042$

$T_3 - T_{3B} = \cancel{685}\ 758$ $\underline{T_3 = \cancel{1412.6}\ 1485.6\,°R}$ $P_4 = P_3 = 62200$ psf

$$\eta_B = \frac{\left(P_4/P_3\right)^{\frac{\gamma-1}{\gamma}} - 1}{T_4/T_3 - 1}\ ; \qquad \eta_B\left(T_4/T_3 - 1\right) = X \ (\gamma = 1.33)$$

$$.95\left(\frac{2940}{1412.6} - 1\right) = X = 1.08(.95) = 1.025 \qquad \left(P_4/P_3\right)^{1/4} = 2.025$$

$\frac{1}{4}\log_{10}(P_4/P_3) = \log 2.025$ $\log_{10}(P_4/P_3) = 4 \log 2.025 = 4(.307) = 1.228 = 16.9$

$\cancel{P_4/P_3 = 16.9}$

$T_4 - T_3 = 2940 - 1485.6 = 1454.4$

$\mathcal{W}_F = .24(96.8) = 23.24$ BTU/lb

$\mathcal{W}_{c_{LPC}} = .24(110.8) = 25.6$ BTU/lb

$\} \ \mathcal{W}_{T_{LPT}}$ 18100 ft 19900 ft

$HP_{LPT} = \frac{48.84(274)}{.707} = 18900$ HP

$HP_{HPT} = \frac{182(274)}{.707} = 70500$ HP

$\mathcal{W}_{c_{HPC}} = .24(758) = 182$ BTU/lb $= \mathcal{W}_{T_{HPT}}$ 141700 ft

$\mathcal{W}_{T_{HPT}} = C_p(T_4 - T_{4A}) = .276(T_4 - T_{4A})\ ;\ \frac{182}{.276} = 659$

$T_4 = 2940\,°R$ $T_{4A} = 2281\,°R$

$\mathcal{W}_{T_{LPT}} = .276\,(T_{5A} - T_5)\ ; \quad \frac{48.84}{.276} = 176.9$

$25.6 + 4.35(23.24) = 135.7$

$T_5 = 2104.1\,°R$

$491°$ $\quad T_5 \quad \frac{2281}{491} \quad 1790°R$

$\cancel{\eta_T = .9 =}$

$\frac{182}{(.276)(2940)(.9)} = .249 = \frac{a}{1+a}$

$.249 = .751a$

$a = .3315$

$$1 - \left(\frac{P_{5A}}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = \frac{\mathcal{W}_{T_{HPT}}}{C_p T_4 \eta_T} = \frac{182}{(.276)(2940)(.9)} = .249$$

$$1 - .249 = \left(\frac{P_{5A}}{P_4}\right)^{\frac{\gamma-1}{\gamma}} \qquad .751 = \left(\frac{P_{5A}}{P_4}\right)^{\frac{1}{5}}$$

$\frac{.2485}{.7515}$

$.875 - 1 = -.125 \quad \frac{-.125}{4} \quad = \quad P_{5A}/P_4 \quad = .316$

$.5 \quad .5-1$

$3.18 = \frac{P_4}{P_{5A}}$

$.316 = \frac{P_{5A}}{P_4} \qquad P_{5A} = 19650\ psf$

$$1 - \left(\frac{P_5}{P_{5A}}\right)^{\frac{\gamma-1}{\gamma}} = \frac{\mathcal{W}_{T_{LPT}}}{C_p T_{5A}\eta_T} = \frac{\overset{150.1}{\cancel{48.84}}}{.276\,(2280)(.9)} = \overset{.265}{\cancel{.0863}}$$

$\frac{a}{1+a}$

$.0863 = .9137a$

$.0945 \quad .9137$

$.9137 = \left(P_5/P_{5A}\right)^{\frac{\gamma-1}{\gamma}}$

$P_5/P_{5A} = .698$

$1.37 \quad -.059 \quad .118 \quad .844 - 1$

$P_5/P_{5A} = .698 \qquad P_5 = 13700\ psf$

$$f/a = \frac{1}{\frac{\eta_b\,HV}{C_p\,(T_4 - T_3)} - 1} = \frac{1}{\frac{.95\,(19000)}{.276\,(1484.4)} - 1} = \frac{1}{44.9 - 1} = \frac{1}{43.9}$$

$.0228$

$w_f = 274(.0228) = 6.24\ lb/sec$

$P_5/P_0 = 6.48$

Primary

$P^* = P_{egg} = .5 \times P_5 = 6850\ psf$

$T^* = T_{egg} = \frac{2103.1\,°R}{1.165} = 1805\,°R$

$.265 = \frac{a}{1+a}$

$.265 = .735a$

$.3605$

$V_{eq} = V^* = \sqrt{g \times \gamma \times R \times T} = \sqrt{4{,}110{,}000} = 2027\ fps$

$A^* = \frac{\dot m}{\rho V} = \frac{\dot m \times R \times T}{P \times V} = \frac{274\,(53.3)(1805)}{6850 \times 2027} = 1.895\ ft^2$

Fan nozzle

$\gamma = 1.4$ pressure det for Mach 1.0

$P^* = P_{ef} = .495 \times (3640) = 1800\ psf$

$T^* = T_{eff} = 616.8/1.2 = 514.5\,°R$

$V_{ef} = \sqrt{g \times R \times T \times \gamma} = \sqrt{1235000} = 1111 \text{ ft/sec}$

$A_{ef} = \dfrac{w_f \times R \times T}{P \times V} = \dfrac{1192 \times 53.3 \times 514.5}{1800 \times 1111} = 1.635 \text{ ft}^2$

$F_G = \dfrac{w_c V_{egg}}{g} + (p_{egg} - p_0) A_{egg} + \dfrac{w_f V_f}{g} + (p_{ef} - p_0) A_{ef}$

$= \dfrac{274(2027)}{32.2} + (6850 - 2116)(1.895) + \dfrac{1192(1111)}{32.2} + (1800 - 2116) 1.635$

4734   -316

$= 17250 + 8960 + 41150 - 517$

17250
41150
58400
8960
67360

67360 - 517 = 66,843 lb.

$s f c = \dfrac{w_f (3600)}{F} = \dfrac{6.24(3600)}{66,843} = .336 \text{ lb/lb hr.}$

$P_3/P_2 = 29.4$

$P_4/P_2 = 29.4$

$P_{5A}/P_2 = 29.4(.316)$
$9.29(.698)$
$= 6.5$

$W_F = .24(96.8) = 23.24 \text{ BTU/lb}$ $HP_F = \dfrac{(23.24)(1466)}{.707} = 48,200 \text{ HP}$

$W_{c_{LPC}} = .24(110.8) = 25.5 \text{ BTU/lb}$ $HP = \dfrac{(25.5)(274)}{.707} = 9,890 \text{ HP}$

$W_{c_{HPC}} = .24(758) = 182 \text{ BTU/lb}$ $HP = \dfrac{182(274)}{.707} = 70500 \text{ HP}$

$\dfrac{W_{T_{HPT}}}{c_p} = \dfrac{182}{.276} = 659 = T_4 - T_{5A}$ $T_{5A} = 2281°R$

$T_4 = 2940°R$

$\dfrac{W_{T_{LPT}}}{c_p} = \dfrac{48.84}{.276} = 176.9 = T_{5A} - T_5$ $T_5 = 2104.1°R$

$1 - \left(\dfrac{P_{5A}}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = \dfrac{W_{T_{HPT}}}{c_p T_4 \eta_T} = \dfrac{182}{.276(2940)(.9)} = .249$ $P_{5A}/P_4 = .316$

$1 - \left(\dfrac{P_5}{P_{5A}}\right)^{\frac{\gamma-1}{\gamma}} = \dfrac{W_{T_{LPT}}}{c_p T_{5A} \eta_T} = \dfrac{48.84}{.276(2280)(.9)} = .0863$ $P_5/P_{5A} = .698$

$f/a = \dfrac{1}{\dfrac{\eta_b HV}{c_p(T_4 - T_3)} - 1} = \dfrac{1}{43.9} = .0228$ $w_f = 278(.0228) = 6.24$ lb/sec

$\cancel{P_5 = \frac{1}{2}\rho V_7^2 + p_7}$

$\cancel{p_7/P_5 = \frac{2116}{13700} = .1547}$

find

$W_{T_{LPT}} \rightarrow \dfrac{\Delta P}{w_c}(.707)$

isentropic expansion $T_5 = T_7$ $P_5 = P_7$ $p_7 = 2116$

$p_7/P_7 = \dfrac{1}{6.48} = .154$

$V_{ex} = \sqrt{\dfrac{2gRT_t\gamma}{\gamma - 1}\dfrac{X}{1+X}} = \sqrt{\dfrac{2gRT_t\gamma}{\gamma-1}\dfrac{.7071}{1.7071}}$

$V_x' = \sqrt{\dfrac{64.4(1.4)(53.3)(2103.1)(.7071)}{.3\ (1.7071)}} =$

$.107$

$\dfrac{2.685}{\cancel{6.44}} = \left[1 + \frac{1}{6}M_e^2\right]^4$ $\frac{1}{4}(\cancel{.812})\ .429 = 1 + \frac{1}{6}M^2$

antil(.203) = $\cancel{1.59}$ 1.259 $6(.59) = 3.56$

1.554

$M_e = 1.885$

$T_{7_e} = t_{7_e}\left[1 + \frac{1}{6}M^2\right] = \cancel{1.59}\ 1.259\ t_7$

1770

$\dfrac{T_{7_e}}{1.595} = t_{7_e} = \dfrac{2104.1}{1.595} = \cancel{1323}\ 1380°R$ $a_e = \sqrt{g\gamma R t_7} = \sqrt{3000000}$ $\cancel{1735}$

2207

$V_{ex_e} = M_e a_e = \cancel{3263}$ ft/sec

$\dfrac{P_{3a}}{P_2} = 1.72 = \dfrac{P_7}{p_7}$ $2(\frac{.236}{7}) = \cancel{.059}\ .068$ $\cancel{1.145}\ 1.168 = 1 + \frac{1}{5}M^2$ $\cancel{.725}\ .84 = M^2$ $M = \cancel{.852}\ .916$

.840

$T_{3A} = 616.8°R$ $\dfrac{T_{7_F}}{\cancel{1.145}\ 1.168} = t_{7_F} = \cancel{534}\ 527°R$ $a_F = \sqrt{g\gamma R t_{7_F}} = \sqrt{1.64 \times 10^6}$

$a_F = \cancel{1137}\ 1124$

$V_{ex_F} = M_F a_F = \cancel{.852}\ .916(\cancel{1137}\ 1124) = \cancel{967}\ 1030$ ft/sec

$F_N = \dfrac{w_c V_c}{g} + \dfrac{w_F V_F}{g} = \dfrac{274(3275)}{32.2} + \dfrac{1192(967)}{32.2}$

381
18750
56850

26650 + 35800 = 65850 64750
38 -11

$\dfrac{1790°R}{1.59} = 1125$ $a_c = \sqrt{2.565} = 1600$

$V_c = 3020$ 26650

EXAMINATION BOOK

Name Cesar Levy

Subject AE 371

Instructor Prof Visich

Exam Seat No. Section A

Date 3/2/72 Grade

C. P. LESH PAPER CO., INDIANAPOLIS, IND.

1 12

2 33

3 19

$$\begin{array}{r} 19 \\ \hline 83 \end{array}$$

4

1. a) Adiabatic Eff of Compression: the ideal work done to achieve a pressure ratio P_3/P_2, divided by the actual work done to achieve that pressure ratio $\eta_{ad_c} = \dfrac{c_p(T_3' - T_2)}{c_p(T_3 - T_2)}$

It is a function of the temperatures before and after the compressor and the ideal P_3/P_2 and γ.

b) Corrected RPM $N/\sqrt{\theta_2}$ where $\theta_2 = T_2/T_{ref} = 520°R$
it is the rpm needed at altitude to produce the same pressure ratio P_3/P_2 (for a compressor) ~~at altitude~~ as at S.L. It is a function of the rpm at S.L. and the reference temp & T_2 at altitude.

c) ~~d)~~ Ram Drag $= \dfrac{W_f V_0}{g}$ the drag needed to push the correct amount of air through the inlet of the nozzle it depends on the velocity of the air relative to the vehicle & the amount of air passing station 1 per second

d) X

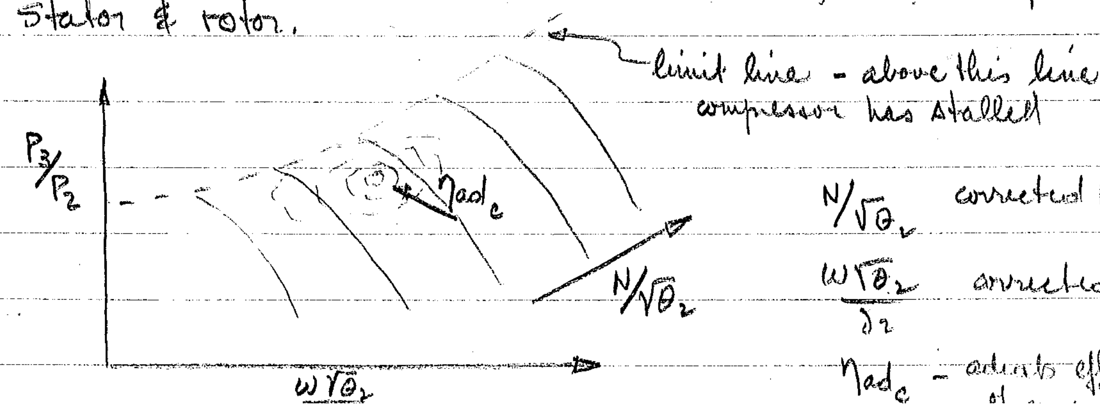
e) ~~d)~~ % react $= \dfrac{\Delta h_r}{\Delta h_r + \Delta h_s} = \dfrac{\Delta p_r}{\Delta p_r + \Delta p_s} = \dfrac{(w_2^2 - w_3^2)}{(c_3^2 - c_2^2) + (w_2^2 - w_3^2)}$

defines the (enthalpy) pressure drop across the rotor / (enthalpy) pressure drops across the rotor and stator. Also defines the velocity diagrams of the flow at the face & rear of compressor. It is a fn of the relative & absolute velocities entering & leaving the stator & rotor.

RNJ

2.

10



$N/\sqrt{\theta_2}$ corrected RPM

$\dfrac{w\sqrt{\theta_2}}{\delta_2}$ corrected weight f[low]

η_{ad_c} - adiab. efficiency of ...

b. The axial flow compressor is relatively lightweight. Its specific fuel consumption is high yet it uses a ~~kerosene base fuel which~~ is relatively cheap. Its frontal diameter is much smaller than the centrifugal flow compressor. It has high peak efficiency. It can use several stages whereas ~~on~~ a cent. flow comp loses turns when more than one stage is used

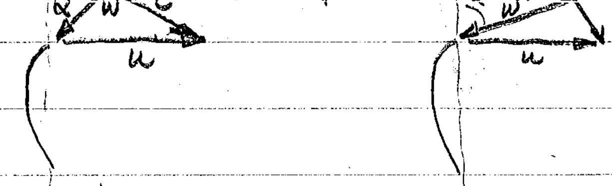
7 ?

8

The centrifugal flow compressor is relatively easy to manufacture its operating efficiency is high for large rotational speeds. It has high pressure changes per stage whereas the axial flow comp. must use several stages (flow separation being the reason)

c. If the absolute velocity of the flow coming into the wheel drops drastically while the rotational speed is constant, The relative velocity into the rotor will be coming in at a much steeper angle to the blade. If this angle is greater than the stall angle of the blade compressor will stall

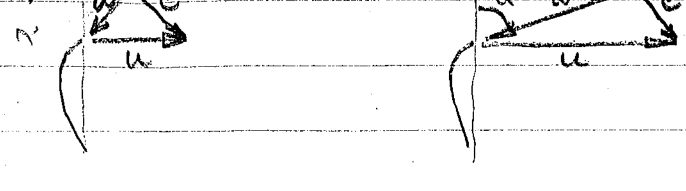
weight flow decreases



if $\alpha' > \alpha_{stall}$

fuel flow input increases

If the rotational speed of the wheel is such that it increases drastically ~~again~~ & if the absolute speed & direction of flow is constant, again the relative speed to the rotor will be such that the angle of entrance may be larger than the stall angle.

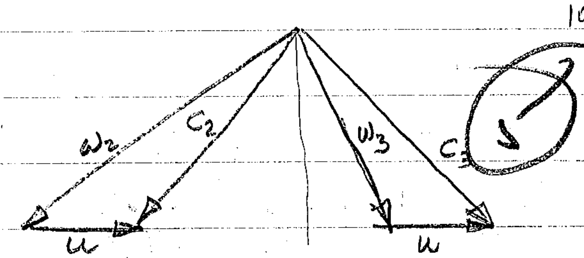


if $\alpha' > \alpha_{stall}$

5 a turbine tends to drop the pressure ratio P_5/P_4 therefore no adverse pressure gradients exists, thus ~~can need be retain for~~ separation contour therefore no stall. Thus a turbo jet engine is not limited by turbine stall.

4 3. By maximizing T_4 one maximizes the velocity V_7 exiting the nozzle therefore increasing the thrust. ~~If one stage~~ ~~exists~~

100% react $\Delta p_s = 0 \quad C_3 = C_2$



$c_p t_2 + \frac{1}{2g} c_2^2 = c_p T_2' \qquad c_p t_2 + \frac{1}{2g} w_2^2 = c_p \overline{T}_2$

$$c_p(\overline{T}_2 - T_2) = \frac{1}{2g}(w_2^2 - c_2^2)$$

$\begin{matrix} 2 & \overline{2} \\ t & t \\ P & P \end{matrix}$ $\qquad \begin{matrix} & 3 \\ t & t \\ P & P \end{matrix}$

$C_p(\overline{T}_2 - T_2)$ is the energy imparted from the rotor to the fluid at the leading edge of the rotor

10

$c_p t_3 + \frac{1}{2g} w_3^2 = c_p \overline{T}_2 \qquad c_p t_3 + \frac{1}{2g} c_3^2 = c_p T_3 \qquad c_p(T_3 - \overline{T}_2) = \frac{1}{2g}(c_3^2 - w_3^2)$

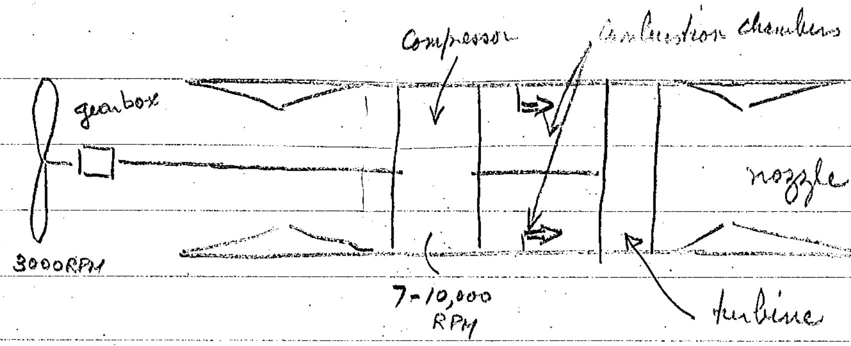
~~$C_p(\overline{T}_2 - T_3)$ is the energy~~ $C_p(T_3 - \overline{T}_2)$ is the energy imparted from the rotor to the fluid at the trailing edge

$$\therefore \Delta H_{tot\ on\ fluid} = C_p(T_3 - \overline{T}_2) + C_p(\overline{T}_2 - T_2) = C_p(T_3 - T_2)$$
$$= \frac{1}{2g}\left[(c_3^2 - c_2^2) + (w_2^2 - w_3^2)\right]$$

if no centrifugal effect exists $\therefore \Delta H_{tot\ on\ fluid} = -\Delta H_{TOT\ on\ rotor}$

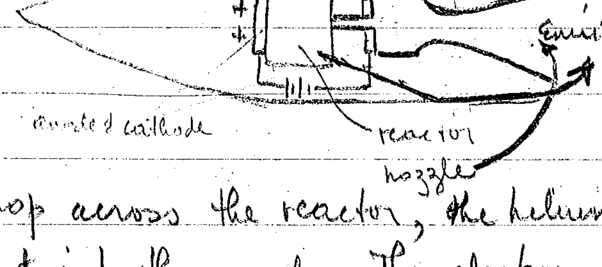
$$\therefore \underset{\text{to rotor}}{W} = -\Delta H_{on\ fluid} = -\frac{1}{2g}\left[\underbrace{(c_3^2 - c_2^2)}_{\text{external effect}} + \underbrace{(w_2^2 - w_3^2)}_{\text{internal diffusion}}\right]$$

4. a

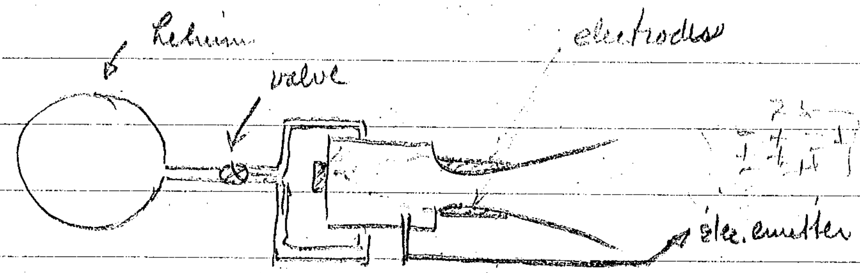


a gear box is introduced to reduce the rotational speed obtained from the turbine system to a speed which will not induce separation or cavitation of the propeller. the inlet takes the air in & slows it down to a workable speed & introduces it to the face of the compressor. The compressor increases the pressure & temperature ratios of the fluid. The fluid & the fuel are ignited in the combustion chambers & the temperature at the face of the turbine is increased to 2000°F ~ 2500°F. The turbine takes work out of the fluid to run compressor & propeller, and drops temperature of combustion products. The products are exhausted after expansion, through the nozzle to a higher speed than the vehicle speed (this type ~~vehicle~~ Engine has low speed charact of a prop.)

b. The reason is that the nuclear rocket can have its engines gimballed & can be shut down by slowing down the nuclear reaction rate of the reactor whereas this is not possible in a solid propellant rocket. It can also function for much longer period.



By manipulating the graphite rods, reaction occurs so that the helium nucleus is created. Because of potential drop across the reactor, the helium nucleus is accelerated out into the nozzle. The electrons also produced as expelled by an electron emitter into the He nucleus to neutralize them & also to make sure that the ship builds up no space charge c.



Pressurized helium thru valve is introduces into chamber where the electrodes form an arc which ionizes the helium thereby releasing He^+ ions through the nozzle & propelling rocket. In order for the rocket not to build up a space charge & in order for the [illegible] not to be repelled an electron emitter emits electrons into the discharging He^+ ions so that the charge behind the nozzle is neutral.

EXAMINATION BOOK

Name Cesar Levy

Subject AE 371

Instructor Prof Visich

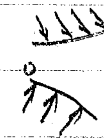
Exam Seat No. ______ Section ______

Date 4/13/72 Grade ______

C. F. LESH PAPER CO., INDIANAPOLIS, IND.

1	15
2	24
3	30
4	22
	91

a) Stoichiometric fuel air ratio - stoichometric means that the product of combustion do not contain the oxidizer & fuel. Stoichiometric fuel air ratio ratio that will make φ - equivalence ratio (ratio of weight of fuel/weight of air to weight of fuel to weight of air for total combustion) go to 1. depends on the mass of the oxidizer & fuel & the no. of molecules of each

b) Aditive drag is defined as the drag ($\int_0^1 p dA$) on the stream tube it is a function of the pressure on the stream tube in the direction of motion & the area change from station 0 to station 1. It is also a fn. , of the amount of mass inlet must capture.

c) Overexpanded nozzle flow - flow for which $p_7 < p_{amb}$ the flow expands too much in the nozzle and exits isentropic supersonic for the pressure to increase to ambient oblique shock waves stand at the ~~exist~~ area of the nozzle. The flow must go through a series of shock waves that increase p_7 to pamb to $p > p_{amb}$. Then a series of expansion waves that decrease p to pamb to $p < p_{amb}$ & pattern repeats itself. At the same time the flow must satisfy the restrictions on the flow direction. This type of nozzle flow depends on the $A/A_*)_{exit}$, M_{exit}, $p_{ambient}$.

d) combustion efficiency

$$\frac{w}{a} = \frac{1}{\frac{\eta_b HV}{c_p(T_4 - T_3)} - 1} \longrightarrow \frac{w_f}{a} = \frac{c_p(T_4 - T_3)}{\eta_b HV - c_p(T_4 - T_3)}$$

$$\eta_b = \left[\frac{c_p(T_4 - T_3)}{w/a} + c_p(T_4 - T_3)\right]\frac{1}{HV}$$

efficiency at which fuel releases its heat in combustion chamber. It is dependent on the temperature difference between the back face of the compressor & the front face of turbine & the heating value of the fuel, as well as the C_p

c. Stagnation pressure is the pressure that a flow would have if it were brought isentropically to rest. It is a function of the static pressure, mach number & the ratio of specific heats γ.

2. $dm = \rho V_e \, dA$

$$A = \int dA = \int_0^{\alpha_w} 2\pi r \, d\alpha \, R$$

$\sin\alpha = r/R$ $\quad \therefore \quad dA = 2\pi R \sin\alpha \, d\alpha$

$$m = \rho V_e \, 2\pi R \int_0^{\alpha_w} \sin\alpha \, d\alpha$$

$$= \rho V_e \, 2\pi R \left(-\cos\alpha\right)\Big|_0^{\alpha_w} = \rho V_e \, 2\pi R \left(1-\cos\alpha_w\right)$$

$$T = \int V_x \, dm = \int_0^{\alpha_w} \rho V_e^2 \cos\alpha \, 2\pi R \sin\alpha \, d\alpha = \rho V_e^2 R 2\pi \int_0^{\alpha_w} \cos\alpha \sin\alpha \, d\alpha$$

$$T = \rho V_e^2 R \pi \sin^2\alpha \Big|_0^{\alpha_w} = \rho V_e^2 R \pi \sin^2\alpha_w$$

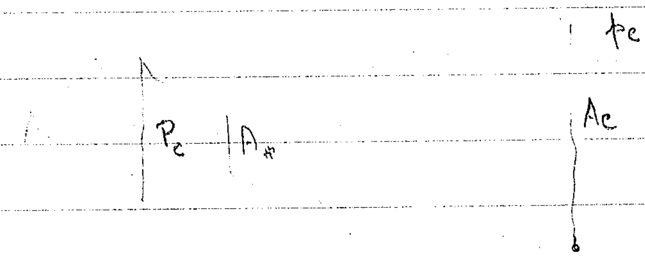
18

$\sin^2\alpha$

$(1-\cos^2\alpha_w)$

$$= \frac{T}{\rho V_e^2 \cdot 2\pi R \left(1-\cos\alpha_w\right)} = \frac{\rho V_e^2 R \, \pi \left(1-\cos^2\alpha_w\right)}{2\rho V_e^2 \pi R \left(1-\cos\alpha_w\right)} = \frac{1+\cos\alpha_w}{2}$$

.1278



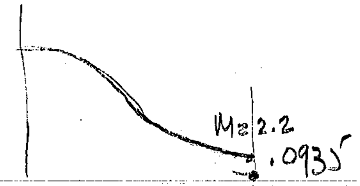
$A_e/A_* = 2.00$

$P/p_e = 30$

$p_e/p = .0333$

if shock at exist $p_2/p_1 = 4.48$ $\qquad p/p_{0_1} = .1278 \qquad p_1 = .1278 \, p_{01}$

$p_2 = 4.5(.1278)P$



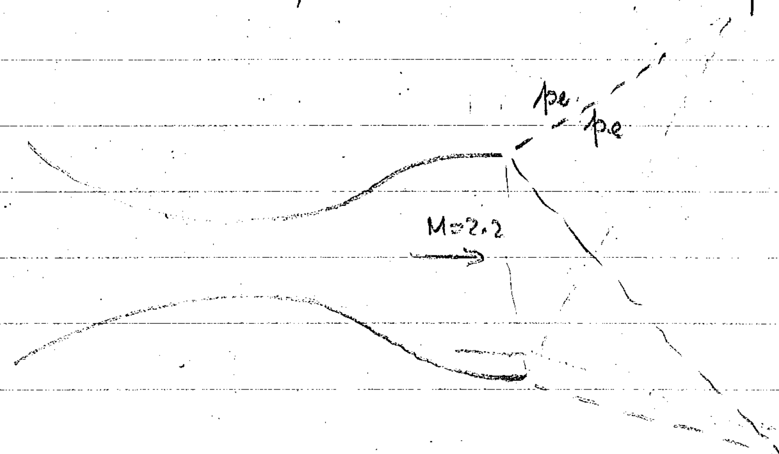
$p_2 = .575\,P$

~~$\frac{A_e}{A_*} = 4$~~ Since $p_e/p = .0333 \rightarrow A/A_* = 3.98$

∴ look at $A/A_* = 2 \longrightarrow M = 2.2 \longrightarrow (p/P)_{\text{isentr exit}} = .0935$

∴ $(p_e/p) < (p_7/p)_{\text{isentrop}}$ ∴ flow is underexpanded

6 for flow to decrease in static pressure expansion waves sit at exit ∴ flow must move away from centerline



$\nu_{M=2.2} = 31.732$ $\nu_{M=2.87} = 47.185$ $\theta = 15.453^\circ$

3. $\cancel{A_1/A_{min}} = 1.23$

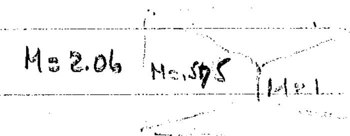
$$\cancel{A_0/A_1 = 1 = \left.\frac{A_0}{A_0^*}\right)_{M_S} \frac{A_0^*}{A_{min}^*} \frac{A_{min}^*}{A_{min}} \frac{A_{min}}{A_1}}$$

$1.775 \qquad 1 \qquad \frac{1}{1.23} \qquad .264$

$A_1/A_{min} = 1.23$ mach behind shock $M = .5674$

M in front of shock $= 2.06$

$(P_2/P_0)_{max}$ occurs when shock at throat



$$\frac{A_1/A_*}{A_1/A_{min}} = \frac{1.775}{1.23} = 1.442 = \frac{A_{min}}{A_*} \rightarrow M_{at\ thr} = 1.80$$

$M_{behind} = .6165 \qquad (P_{t_2}/P_{t_1})_{max} = .8127$ S

$(A_0/A_1)_{max} = 1$ S ; A_0/A_1 for shock outside inlet < 1

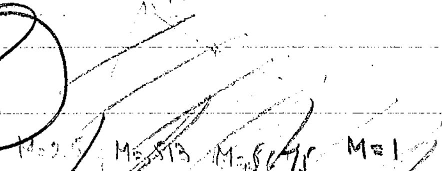
$M_0 = 2.5 \qquad M_0 > M_S$

$(P_{t_2}/P_{t_1})_{max}$

$$\frac{A_1/A_*}{A_1/A_{min}} = \frac{2.637}{1.23} = 2.14 \qquad M = 2.275$$

$M_{behind} = .5374$

$P_2/P_0\ _{max} = .5944$ S

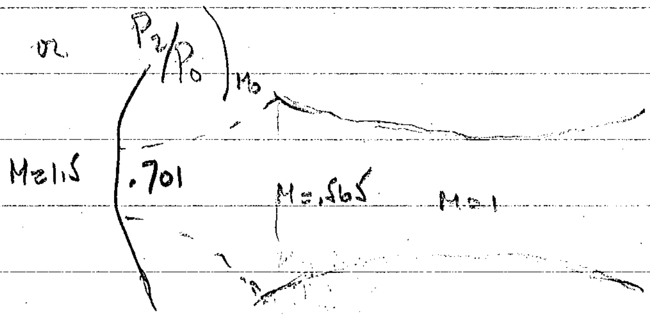


$(A_0/A_1)_{max} = 1$; if S shock stands outside inlet $A_0/A_1 < 1$

$\cancel{A_0/A_* = 1.315} \qquad \cancel{\frac{A_1}{A_*} = 1.23}$

$\cancel{(A_0/A_1)_{max} = \frac{1.315}{1.23} = 1.07}$

$M = 1.5$ $\quad$ $M_0 < M_s$ $\quad$ $P_2/P_{0_{max}} = .9298$



$A_0/A_* = 1.0940$

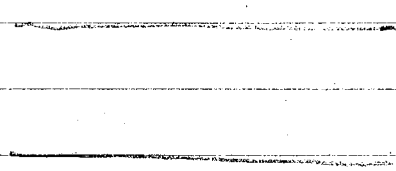
$A_0/A_{1_{max}} = \frac{1.094}{1.23} = .89$

10

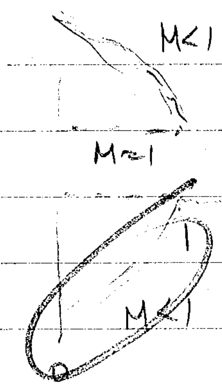
$A_1/A_* = 1.23$

4a. primary air is used so that the fuel is burned at stoichiometric proportion $\varphi = 1$. Secondary air is used to decrease the temp of hot gases down to the prescribed T_4 or else blades of turbine will creep & turbine blades will fail

b.

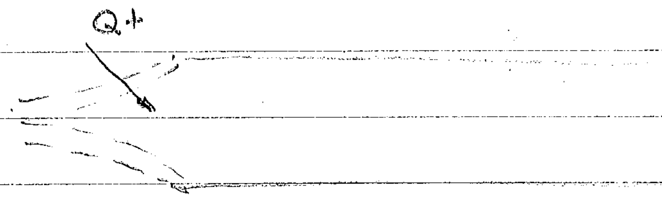


as the flow moves through the tube & heat is added the velocity in the tube will increase until mach 1 is reached whereupon chocking will occur. For more heat to be added the flow must readjust so that the velocity of the fluid at the entrance decreases. When this occurs more heat can be added. The $M = 1$ point in the tube moves further from the entrance, More heat is added until the flow chokes again.



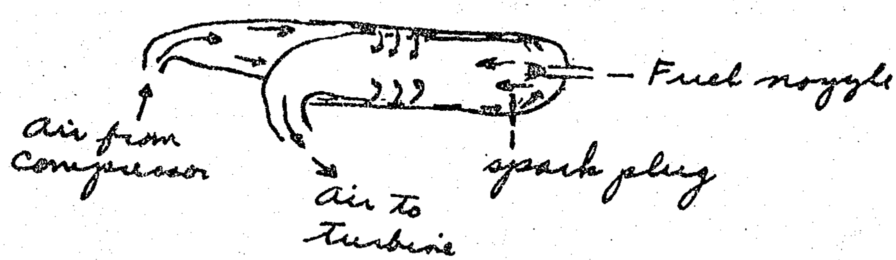
the stream tube will decrease
until it is straight or $M = 1$.
at that pt no more heat can be added
as readj occurs stream tube increases in size
$\frac{A_0}{A_1} > 1$ $M<1$ & more heat is added until
stream tube becomes straight again etc.

12



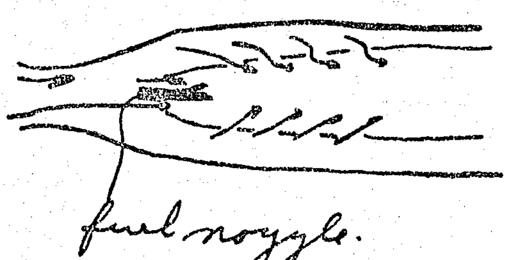
Types of Combustion Chambers

a) Counterflow can-type burner.

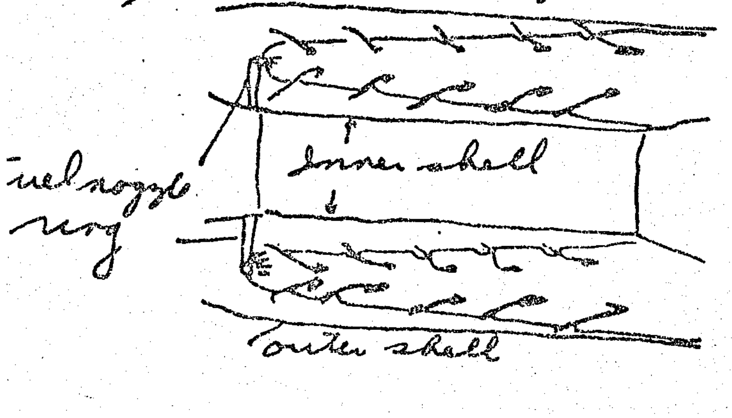


The counterflow can-type burner requires the primary air to travel the full length of the chamber before mixing with the fuel spray whereas the secondary air flow reverses its direction as it enters the inner chamber.

b) Straight-flow can-type burner.



c) Annular burner.



single burner, completely surrounding the engine shaft between the compressor and the turbine, and contains several fuel nozzles spaced evenly around the burner inlet

COMPARISON OF COMBUSTION CHAMBERS

Counter Flow Can Type

+ 1. High turbulence which is conductive to high combustion efficiency.
- 2. High turbulence and long flow path have disadvantage of large pressure losses.
- 3. Burner must completely surround the turbine tending to a large overall engine diameter.
- 4. Losses associated with two 180° bends.
+ 5. Is such that the turbine stator and rotor blades are shielded from direct high temperature radiation of the flame.

Straight Flow Can Type

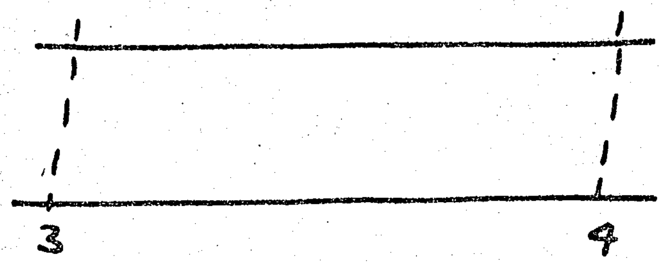
+ 1. Low friction losses.
+ 2. Provides even distribution of air flow.
+ 3. Relatively easy to manufacture.
+ 4. Several of these burners are used on an engine so that only a small fraction of the entire flow passes through each burner in comparison to the annular burner where the entire flow passes through one burner. For this reason, testing and development costs of the combustion chamber are much less.
+ 5. Arrangment of these burners around the shaft of the engine offers the possibility of small diameter engines.
- 6. Subjects turbine rotor blades to direct high temperature radiation from the flame.

Annular

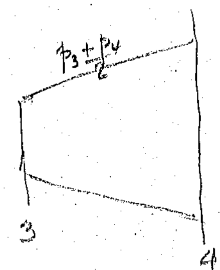
+ 1. Offers low friction and high flow rates.
+ 2. For a given overall diameter the annular burner offers the greatest cross-sectional area normal to the flow and should lead to the lowest flow velocity in the combustion chamber.
- 3. Inaccessibility of the rotor shaft.
- 4. Disassembly and assembly difficulties for service replacement.

CONSTANT AREA BURNING

SIMPLE T CHANGE



General Eq



Conservation of Mass

$$\frac{W}{g} = \rho A V = \text{constant} \qquad \rho_3 A_3 V_3 = \rho_4 A_4 V_4$$

$$\frac{\rho_3}{\rho_4} = \frac{V_4}{V_3} \qquad (1)$$

Momentum Balance

$$p_3 A_3 - p_4 A_4 + \left(\frac{p_3+p_4}{2}\right)\left(A_4 - A_3\right) = m\left(V_4 - V_3\right)$$

$$p_3 - p_4 = \frac{W}{gA}\left(V_4 - V_3\right) \qquad (2)$$

$$\frac{W}{gA} = \rho V, \qquad \rho V^2 = \gamma p M^2$$

$$p_3 - p_4 = \rho_4 V_4^2 - \rho_3 V_3^2 = \gamma p_4 M_4^2 - \gamma p_3 M_3^2$$

$$\frac{p_4}{p_3} = \frac{1+\gamma M_3^2}{1+\gamma M_4^2} \qquad (3)$$

Perfect Gas Law

$$\frac{p_4}{p_3} = \frac{\rho_4 \, t_4}{\rho_3 \, t_3} \qquad (4)$$

Speed of Sound

$$a^2 = \gamma p / \rho = \gamma R t \qquad (5)$$

$$\frac{V_3^2}{V_4^2} = \frac{M_3^2}{M_4^2}\,\frac{\gamma R t_3}{\gamma R t_4}, \qquad \frac{V_3}{V_4} = \frac{M_3}{M_4}\sqrt{\frac{t_3}{t_4}} \qquad (6)$$

Combining (1), (3), (4) and (6):

$$\frac{t_4}{t_3} = \frac{\rho_4}{\rho_3}\cdot\frac{P_3}{P_4} = \frac{1+\gamma M_3^2}{1+\gamma M_4^2}\,\frac{V_4}{V_3}$$

$$= \frac{1+\gamma M_3^2}{1+\gamma M_4^2}\cdot\frac{M_4}{M_3}\sqrt{\frac{t_4}{t_3}}$$

$$\frac{t_4}{t_3} = \left[\frac{1+\gamma M_3^2}{1+\gamma M_4^2}\right]^2\left(\frac{M_4}{M_3}\right)^2 \qquad (7)$$

From the definition of the stagnation pressure and temperature

$$\frac{T}{t} = 1+\frac{\gamma-1}{2}M^2, \qquad \frac{T_4}{T_3} = \frac{M_4^2}{M_3^2}\,\frac{(1+\gamma M_3^2)^2}{(1+\gamma M_4^2)^2}\,\frac{\left(1+\frac{\gamma-1}{2}M_4^2\right)}{\left(1+\frac{\gamma-1}{2}M_3^2\right)} \qquad (8)$$

$$\frac{P}{p} = \left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)}; \qquad \frac{P_4}{P_3} = \left[\frac{1+\gamma M_3^2}{1+\gamma M_4^2}\right]\left[\frac{1+\frac{\gamma-1}{2}M_4^2}{1+\frac{\gamma-1}{2}M_3^2}\right]^{\gamma/\gamma-1} \qquad (9)$$

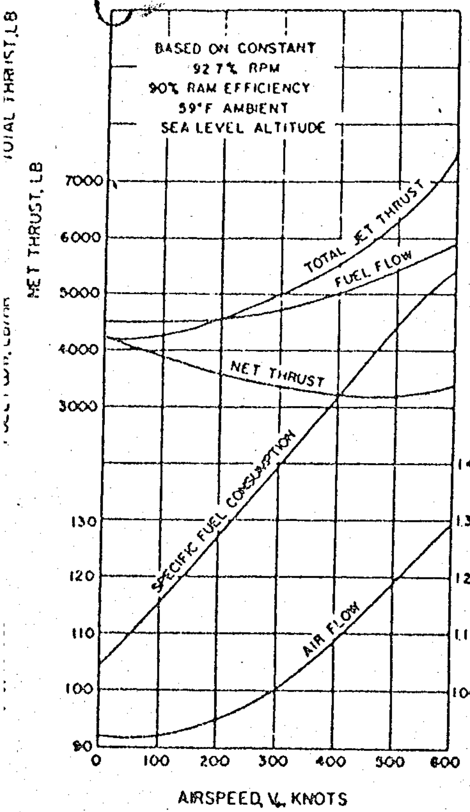
also.
$$\frac{dM^2}{M^2} = \frac{[1+\gamma M^2]\left[1+\frac{\gamma-1}{2}M^2\right]}{1-M^2}\,\frac{dT}{T} \qquad (10)$$

$$\frac{dV}{V} = \frac{1+\frac{\gamma-1}{2}M^2}{1-M^2}\,\frac{dT}{T} \qquad (11)$$

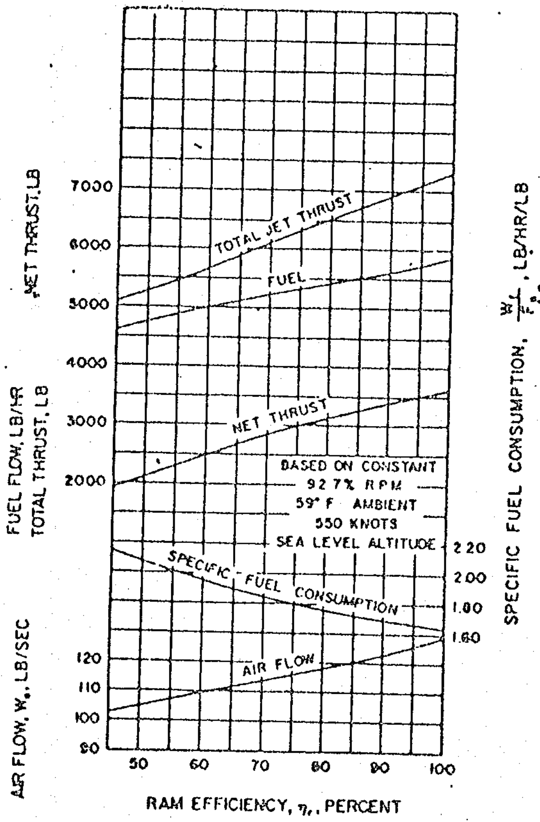
$$\frac{dt}{t} = \frac{\left(1+\frac{\gamma-1}{2}M^2\right)(1-\gamma M^2)}{1-M^2}\,\frac{dT}{T} \qquad (12)$$

$$\frac{dP}{P} = -\frac{\gamma M^2}{2}\,\frac{dT}{T} \qquad (13)$$

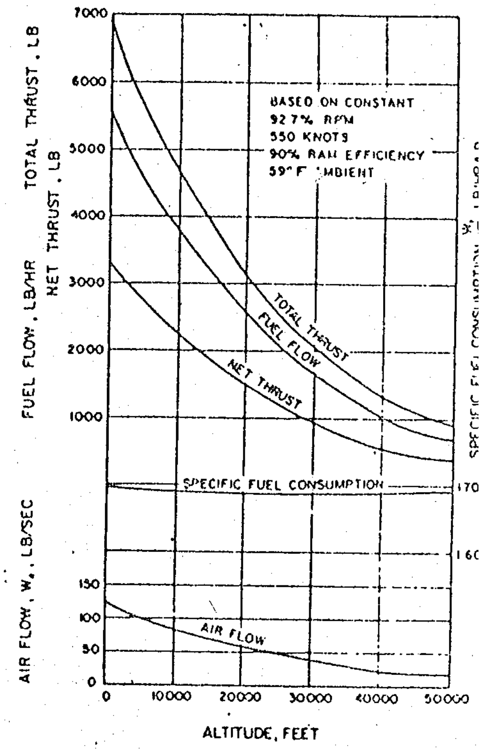
EFFECT OF AIRSPEED ON TURBOJET ENGINE PERFORMANCE



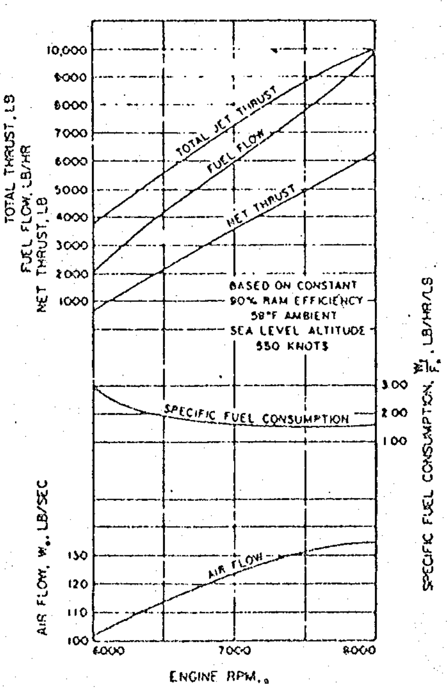
EFFECT OF RAM EFFICIENCY ON TURBOJET ENGINE PERFORMANCE



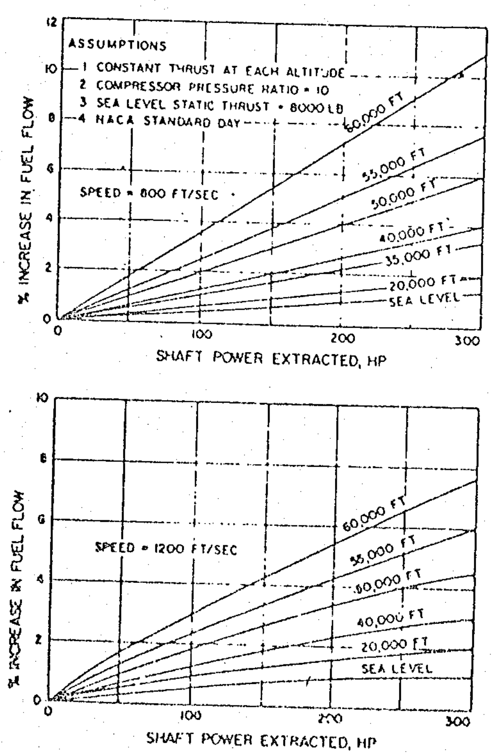
EFFECT OF ALTITUDE ON TURBOJET ENGINE PERFORMANCE



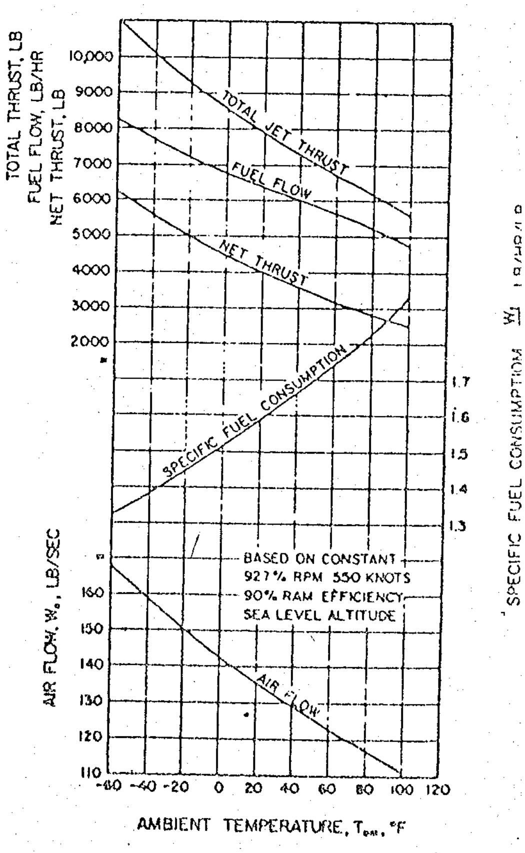
EFFECT OF ENGINE RPM ON TURBOJET ENGINE PERFORMANCE



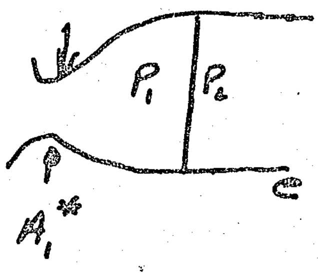
EFFECT OF SHAFT HORSEPOWER EXTRACTION ON FUEL FLOW FOR A HIGH PRESSURE RATIO TURBOJET ENGINE



EFFECT OF AMBIENT TEMPERATURE ON TURBOJET ENGINE PERFORMANCE



Find the position of the normal shock in an expansion nozzle with p_e/P_1 given



$$A_1^* P_1 = A_2^* P_2$$

$$\frac{A^*}{A} = K_1\left[1-\left(\frac{p}{P}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2}\left(\frac{p}{P}\right)^{1/\gamma}$$

$$K_1 = \left(\frac{2}{\gamma-1}\right)^{1/2}\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{A_2^*}{A_e} = K_1\left[1-\left(\frac{p_e}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2}\left(\frac{p_e}{P_2}\right)^{1/\gamma} = \frac{A_2^*}{A_1^*}\cdot\frac{A_1^*}{A_e} = \frac{P_1}{P_2}\,\frac{A_1^*}{A_e}$$

$$\frac{p_e}{P_2} = \frac{p_e}{P_1}\cdot\frac{P_1}{P_2} = K_2\,\frac{P_1}{P_2}\ , \quad K_3 = \frac{A_1^*}{A_e}$$

Therefore

$$K_3'\,\frac{P_1}{P_2} = K_1 K_2^{\frac{1}{\gamma}}\left(\frac{P_1}{P_2}\right)^{1/\gamma}\left[1-K_2^{\frac{\gamma-1}{\gamma}}\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2}$$

Let $K_4 = \dfrac{K_1 K_2^{1/\gamma}}{K_3}$ $\qquad K_5 = K_2^{\frac{\gamma-1}{\gamma}}$

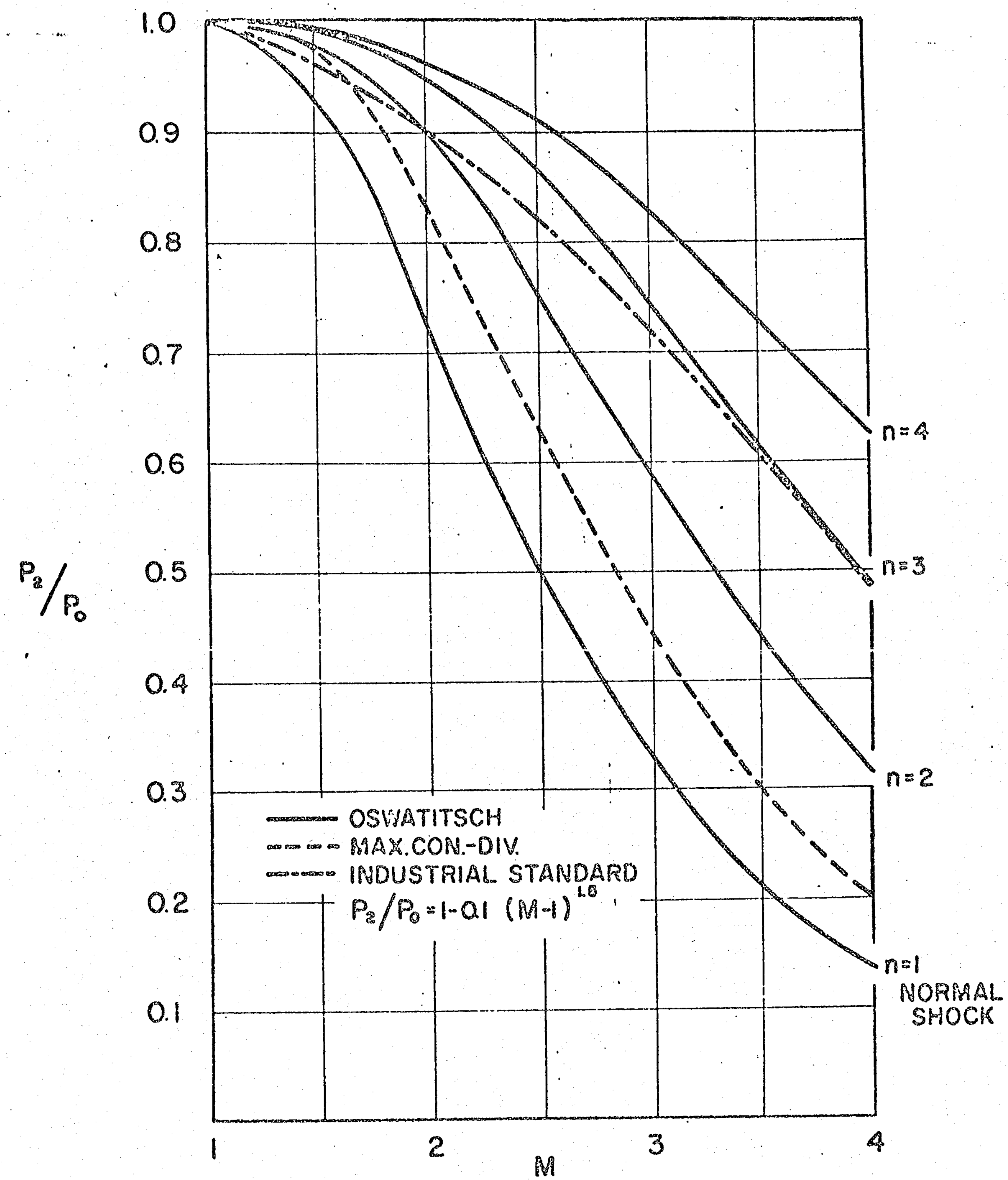
$$1 = K_4^2\left(\frac{P_1}{P_2}\right)^{\frac{2(1-\gamma)}{\gamma}}\left[1-K_5\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$-K_4^2\,K_5\left[\frac{P_2}{P_1}\right]^{\frac{\gamma-1}{\gamma}} + K_4\left(\frac{P_2}{P_1}\right)^{\frac{2(\gamma-1)}{\gamma}} - 1 = 0$$

Let $X = \left(\dfrac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$, $\quad K_4^2 x^2 - K_4^2 K_5 X - 1 = 0$

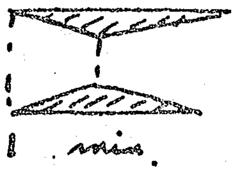
$$X = \left\{K_4^2 K_5 \pm \sqrt{K_4^4 K_5^2 + 4K_4^2}\right\}\Big/ 2K_4^2$$

Negative sign can be neglected



INLET PRESSURE RECOVERY

Consider a convergent - divergent inlet with the following geometry:



Let A_0 equal the free-stream tube capture area

A_1 equal the entrance area of the inlet

A_{min} equal the minimum area of the inlet

The starting Mach number is defined as the free-stream Mach number at which the Mach number at the minimum section is unity when a normal shock stands at the entrance to the inlet $(A_0 = A_1)$

For a given starting Mach number the area ratio A_1/A_{min} is given by

$$\frac{A_1}{A_{min}} = \left(\frac{A}{A^*}\right)_{M_s} \frac{A_0^*}{A_{min}} = \left(\frac{A}{A^*}\right)_{M_s} \frac{P_Y}{P_0}$$

where $\left(\frac{A}{A^*}\right)_{M_s}$ is the isentropic area ratio evaluated at $M = M_s$

P_Y/P_0 is the stagnation pressure ratio across a normal shock at $M = M_s$.

M_s	$(A/A^*)_{M_s}$	P_Y/P_0	A_1/A_{min}
1.0	1.0	1.0	1.0
1.5	1.176	.9298	1.093
2.0	1.688	.7209	1.217
2.5	2.637	.4990	1.316
3.0	4.235	.3283	1.390

Consider the inlet designed for a given M_s and flying at M_s. By regulating the exhaust nozzle minimum area and/or burner temperature of a ram jet it is possible to swallow the normal shock into the inlet and then position it at the minimum section. The maximum attainable pressure recovery with the inlet at $M_0 = M_s$ corresponds to the case where the normal shock is located at the minimum section

Consider flight Mach numbers greater than M_s. Since the minimum area of the inlet is larger than that required to obtain $M_{min} = 1$ when the normal shock stands at the entrance to the inlet, the normal shock can be swallowed if the pressure downstream of the inlet is low enough. The maximum pressure recovery obtainable with this inlet is obtained when the normal shock is located at the minimum section of the inlet. The maximum mass flow for $M_0 > M_s$ corresponds to $A_0 = A_1$

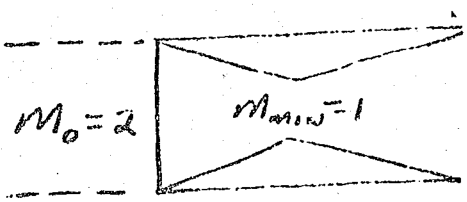
Consider flight Mach numbers less than M_s. Since the minimum area of the inlet is smaller than that required to obtain $M_{min} = 1$ when the normal shock stands at the entrance to the inlet, the normal shock must stand ahead of the inlet such that $A_0 < A_1$. Therefore, for $M_0 < M_s$ the maximum attainable pressure recovery is equal to the normal shock pressure recovery at the flight Mach number. The maximum mass flow that will pass through the inlet occurs when the Mach number at the minimum section is equal to unity and a normal shock stands ahead of the inlet.

For $M_0 < 1$ the pressure recovery of the inlet is equal to unity and the maximum mass flow that can pass through the inlet occurs when $M_{min} = 1$. The ratio $\frac{A_0}{A_1}$ is therefore given by

$$\frac{A_0}{A_1} = \left(\frac{A}{A_0^*}\right)_{M_0} \frac{A_0^*}{A_1} = \left(\frac{A}{A^*}\right)_{M_0} \frac{A_{min}}{A_1}$$

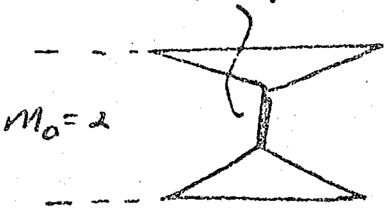
Consider an inlet designed for $M_s = 2.0$
$(A_1/A_{min} = 1.217)$

<u>at $M_0 = M_s = 2$.</u>



$P_2/P_0 = 0.7209$
$A_0/A_1 = 1$

max Mass



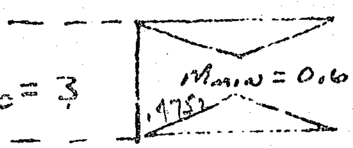
$\frac{P_2}{P_0} = .854$
$\frac{A_0}{A_1} = 1.0$

max P_2/P_0

For $A_0/A_1 < 1$ a normal shock stands ahead of the inlet.

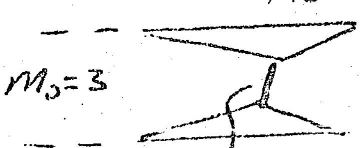
<u>at $M_0 = 3$</u>

max Mass



$P_2/P_0 = 0.3283$
$A_0/A_1 = 1.0$

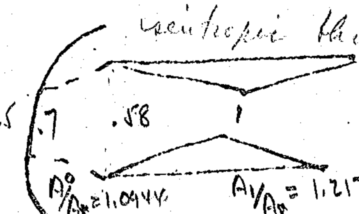
max P_2/P_0



$P_2/P_0 = .3926$
$A_0/A_1 = 1.00$

For $A_0/A_1 < 1$ a normal shock stands ahead of the inlet

<u>at $M_0 = 1.5$</u>



$\frac{A_0}{A_1} = \frac{1.0944}{1.217} = .898$

$\left(\frac{A_0}{A_1}\right)_{max}$ occurs when

$M_{min} = 1.0$, $\left(\frac{A_0}{A_1}\right)_{max} = 0.895$

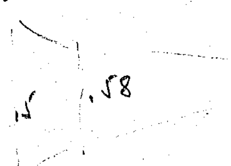
$P_2/P_0 = .4248$ - max P_2/P_0

<u>at $M_0 = 0.5$</u> $\frac{A_0}{A_1} = \frac{A_0}{A^*} \cdot \frac{A^*}{A_1} = \frac{1.3398}{1.217} = 1.10$

$.4752 \quad A_1/A_* = \frac{1.39}{1.217}$ $\quad M_{min} = .64$ ✓

$\frac{A_{min}}{A_1}$

$\frac{1.235}{1.217} = \quad M = 2.79$



$\frac{A_0}{A_0/A_1/A_*} = \frac{1.3398}{1.217} = 1.1$ ✓✓

1. Stoichiometric combustion : where the products of combustion do not contain either the fuel or the oxidizers.

2. Equiv Ratio = $\dfrac{(w_f/w_o)}{(w_f/w_o)_{stoich}} = \varphi$

$\varphi = 1$ stoic
$\varphi > 1$ rich
$\varphi < 1$ lean

3. propagation speed of combustion = velocity that will blowout flame ; to increase the speed generate turbulence ; this however induces pressure losses

a) Counter flow can

+ blades are not directly exposed to flame
- because of the double 180° turns pressure losses are incurred
+ high turbulence conductive to high combustion efficiency.
- burner must completely surround turbine tending to increase overall engine diameter.

b) Straight Flow can

+ low friction losses , even flow distribution
+ relatively easy to manufacture , testing & development costs are low.
- turbine blades are directly exposed to flame

c) Annular

+ low friction & high flow rates
+ greatest cross-sectional area normal to flow $\rightarrow$ lowest flow velocity
- Inaccessibility of the rotor shaft, servicing is difficult

4. $w_f/w_a = \dfrac{1}{\dfrac{\eta_b \, H.V.}{c_p(T_4 - T_3)} - 1}$

5. Continuity $\rho A V = const$

Momentum $\quad p_3 A_3 - p_4 A_4 + \left(\frac{p_3+p_4}{2}\right)\left(A_3 - A_3\right) = m(V_4 - V_3)$

since $\quad C_p T = C_p t + \frac{1}{2} V^2 \longrightarrow \quad T = t + \frac{1}{2}\frac{V^2}{C_p} = t + M^2 \frac{\gamma R t}{2 C_p} = t\left[1 + \frac{\gamma-1}{2} M^2\right]$

since $\quad P/p = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\gamma/\gamma-1}$

Constant Area $\quad \frac{V_3}{V_4} = \frac{M_3}{M_4}\sqrt{t_3/t_4} \qquad \rho_3/\rho_4 = V_4/V_3 \qquad p_4/p_3 = \frac{1+\gamma M_3^2}{1+\gamma M_4^2}$

$$\frac{t_4}{t_3} = \left[\frac{1+\gamma M_3^2}{1+\gamma M_4^2}\right]^2 \left(\frac{M_4}{M_3}\right)^2 \qquad \frac{T_4}{T_3} = \frac{M_4^2}{M_3^2}\left[\frac{1+\gamma M_3^2}{1+\gamma M_4^2}\right]^2 \left[\frac{1+\frac{\gamma-1}{2} M_4^2}{1+\frac{\gamma-1}{2} M_3^2}\right]$$

$$P_4/P_3 = \left[\frac{1+\gamma M_3^2}{1+\gamma M_4^2}\right]\left[\frac{1+\frac{\gamma-1}{2} M_4^2}{1+\frac{\gamma-1}{2} M_3^2}\right]^{\gamma/\gamma-1}$$

Thermal choking occurs.

for $\quad M_3 < 1 \quad \Delta T > 0 \quad M\uparrow \qquad M_3 < 1 \quad \Delta T < 0 \quad M\downarrow$

$M_3 > 1 \quad \Delta T > 0 \quad M\downarrow \qquad M_3 > 1 \quad \Delta T < 0 \quad M\uparrow$

$M < \frac{1}{\sqrt{\gamma}} \quad \Delta T > 0 \quad \Delta t > 0, \quad \frac{1}{\sqrt{\gamma}} < M < 1 \quad \Delta T > 0 \quad \Delta t < 0$

$M > 1 \quad \Delta T > 0 \quad \Delta t > 0.$

Constant pressure, Const velocity

$$A_4/A_3 = \left(M_3/M_4\right)^2 \qquad T_4/T_3 = \left(\frac{M_3}{M_4}\right)^2 \left[\frac{1+\frac{\gamma-1}{2} M_4^2}{1+\frac{\gamma-1}{2} M_3^2}\right]$$

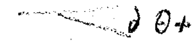
no thermal choke

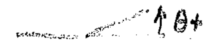
$\Delta T > 0 \qquad \Delta M^2 < 0$

$\Delta P < 0 \qquad \Delta M^2 < 0$

Using $\quad m = \rho A V = \frac{\rho A M \gamma P/\rho}{\sqrt{\gamma R T}} \longrightarrow \quad m = \sqrt{\frac{\gamma}{R}}\frac{AP}{\sqrt{T}} M\left[1 + \frac{\gamma-1}{2} M^2\right]^{-\frac{(\gamma+1)}{2(\gamma-1)}}$

$$\frac{A}{A_*} = \frac{1}{M}\left[\frac{1+\frac{\gamma-1}{2} M^2}{\frac{\gamma+1}{2}}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \qquad A_1^* P_1^* = A_2^* P_2^*$$

1. underexpanded flow $p_7 > p_e$ exp, comp $\nu - \theta = \text{const}$ 

2. overexpanded flow $p_7 < p_e$ comp, exp. $\nu + \theta = \text{const.}$ 

$M_n = M \sin\theta$ read p_2/p_1 for M_n

$$\rho A V = m \qquad \rho \frac{AM}{\sqrt{\gamma R t}} \frac{\gamma p}{\rho} = AM\sqrt{\frac{\gamma}{R}} \frac{p}{\sqrt{t}} = A \frac{\sqrt{\gamma/R}}{\sqrt{T}} M P \frac{\left(1 + \frac{\gamma-1}{2}M^2\right)^{-\gamma/\gamma-1}}{\left(1 + \frac{\gamma-1}{2}M^2\right)^{-1/2}}$$

$$\frac{\sqrt{\gamma/R}}{\sqrt{T}} PAM \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma-1}{2(\gamma-1)} - \frac{2\gamma}{2(\gamma-1)} = -\frac{(\gamma+1)}{2(\gamma-1)}}$$

$$m = \sqrt{\gamma/R} \frac{PA}{\sqrt{T}} M \left(1 + \frac{\gamma-1}{2}M^2\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}}$$

$$= \sqrt{\gamma/R} \frac{P^* A^*}{\sqrt{T^*}} \left(\frac{\gamma+1}{2}\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}} \qquad \frac{A}{A^*} = \frac{1}{M}\left(\frac{1 + \frac{\gamma-1}{2}M^2}{\frac{\gamma+1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$c_p T = c_p t + \frac{1}{2g} V_7^2 \qquad \sqrt{2g\, c_p T_7 \left(1 - \frac{t_7}{T_7}\right)}$$

$$c_p (T - t) = \frac{1}{2g} V_7^2$$

$$\frac{p_0}{p} = \frac{\rho_0}{\rho} \frac{t_0}{t}$$

$$\frac{T}{t} = \left(\frac{P}{p}\right)^{\gamma-1/\gamma}$$

$$V_7 = \sqrt{2g c_p T_7 \left(1 - [p/P]^{\frac{\gamma-1}{\gamma}}\right)} \qquad p_0/p = \left(\frac{\rho_0}{\rho}\right)^{\gamma}$$

$$p_0/p = \left(\frac{t_0}{t}\right)^{\frac{\gamma}{\gamma-1}} \qquad \frac{t_0}{t} = \frac{\rho}{\rho_0} p_0/p = \left(\frac{\rho_0}{\rho}\right)^{\gamma-1}$$

if $M_0 = M_S$ $\qquad A_1/A_{min} = A/A_0\Big)_{MS} \quad \frac{A_0^*}{A_{min}} = A/A^*_{M_0} \quad P_2/P_0\Big)_{M_0}$ max mass flow

for max P_2/P_0 when normal shock is located at min section

if $M_0 > M_S$ $\qquad A_{min} \geq A_*$

for max P_2/P_0 normal shock at min section

for max mass flow $\qquad A_0 = A_1$

if $M_0 < M_S$ $\qquad A_{min} \leq A_*$ $\qquad$ normal shock ahead of inlet

P_2/P_0 max $\qquad \left(P_2/P_0\right)_{M_0}$

max mass $\qquad M_{inlet} = 1$ shock ahead of inlet

$M_0 < 1$ $\qquad P_2/P_0 = 1$ $\qquad m = m$ at A_* $\qquad A_0/A_1 = \frac{A}{A_*}\Big)_{M_0} \quad \frac{A_{min}}{A_1}$

$$\frac{dT}{T} = \frac{dM^2}{M^2} + \frac{\frac{\gamma-1}{2}dM^2}{1+\frac{\gamma-1}{2}M^2} - \frac{2\gamma\, dM^2}{1+\gamma M^2}$$

$$\frac{dM^2}{M^2}\left[1 + \frac{\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M^2} - \frac{2\gamma M^2}{1+\gamma M^2}\right]$$

$$\left(1+\frac{\gamma-1}{2}M^2\right)\left(1+\gamma M^2\right) + \frac{\gamma-1}{2}M^2\left(1+\gamma M^2\right) - 2\gamma M^2\left(1+\frac{\gamma-1}{2}M^2\right)$$

$$1+\gamma M^2 + \frac{\gamma-1}{2}M^2 + \cancel{\frac{\gamma-1}{2}\gamma M^4} + \frac{\gamma-1}{2}M^2 + \cancel{\frac{\gamma-1}{2}\gamma M^4} - 2\gamma M^2 - \cancel{\gamma\frac{\gamma-1}{2}M^4}$$

$$+\cancel{\gamma M^2} \quad (\gamma-1)M^2 - \cancel{2\gamma M^2}$$

$$\cancel{\gamma M^2}$$

$$1-M^2$$

$$\frac{dT}{T}\left[\frac{\left(1+\frac{\gamma-1}{2}M^2\right)\left(1+\gamma M^2\right)}{1-M^2}\right] = \frac{dM^2}{M^2}$$

$$\frac{dt}{t} = \frac{dM^2}{M^2} - \frac{2\gamma\, dM^2}{1+\gamma M^2} = \frac{dM^2}{M^2}\left[1+\gamma M^2 - 2\gamma M^2\right]$$

$$\frac{1-\gamma M^2}{1+\gamma M^2}$$

$$\frac{dT}{T}\left[\frac{\left(1+\frac{\gamma-1}{2}M^2\right)\left(1-\gamma M^2\right)}{1-M^2}\right] = \frac{dt}{t}\left[\cancel{\frac{1+\gamma M^2}{1-\gamma M^2}}\right]$$

to control shock at inlet vary A_6 decrease A_6 shock moves toward throat if $M_0>1$ $P_6\uparrow$
" " " " " " fuel increase fuel $T_6\uparrow$ $P_6\uparrow$ shock toward throat.

Matching

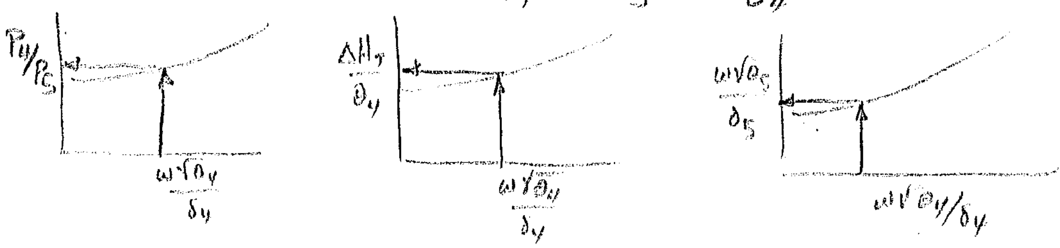
M_0, h, P_2/P_0, T_4, A_6 find N, A_0

$$\frac{w\sqrt{\theta_5}}{\delta_5} = \frac{w\sqrt{\theta_6}}{\delta_6} = A_6\, f(\gamma) \quad \text{if no afterburner.}$$

$$\frac{w\sqrt{\theta_5}}{\delta_5} = \frac{w\sqrt{\theta_4}}{\delta_4}\,\frac{P_4}{P_5}\sqrt{\frac{T_5}{T_4}}$$

$$\frac{\Delta H_T}{\theta_4} = c_p T_{ref}\left(1 - T_5/T_4\right)$$

from turbine charts get for $\frac{w\sqrt{\theta_4}}{\delta_4}$ $\frac{P_4}{P_5}$ & $\frac{\Delta H_T}{\theta_4}$ this gives $\frac{w\sqrt{\theta_5}}{\delta_5}$ vs $\frac{w\sqrt{\theta_4}}{\delta_4}$



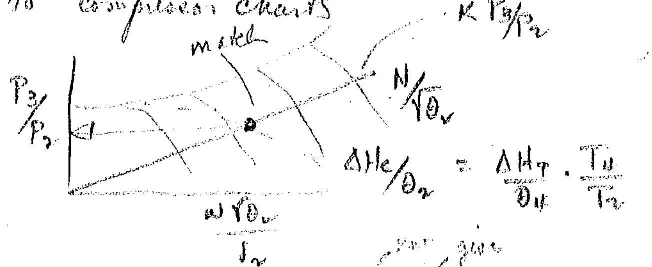
this determines $\frac{w\sqrt{\theta_4}}{\delta_4}$

now

$$\frac{w\sqrt{\theta_2}}{\delta_2} = \frac{w\sqrt{\theta_4}}{\delta_4}\left(\frac{P_4}{P_2} = \frac{P_3}{P_2}\right)\sqrt{\frac{T_2}{T_4}} = K\,\frac{P_3}{P_2}$$

T_2 obtained from T_0; T_4 given

go to compressor charts



get P_3/P_2 & $N/\sqrt{\theta_2}$

$$\Delta H_c/\theta_2 = \frac{\Delta H_T}{\theta_4}\cdot\frac{T_4}{T_2}$$

$$\frac{w\sqrt{\theta_0}}{\delta_0} = \frac{w\sqrt{\theta_2}}{\delta_2}\,\frac{P_2}{P_0}\sqrt{\frac{T_0}{T_2}}$$

P_2/P_0 given; $\sqrt{T_0/T_2}$ = 1 adiab inlet

$$\frac{w\sqrt{\theta_0}}{\delta_0} = f(M_0)\,A_0\,\frac{P_r}{\sqrt{T_r}}$$

knowing $P_7/P_5 \cdot P_5/P_4 \cdot P_4/P_3 \cdot P_3/P_2 \cdot P_2/P_0 \cdot P_0 = P_7$
1, found, 1, found, given, given
T, Comb, Comp I

assume $p_7 = p_0$ $\therefore$ $p_7/P_7 \rightarrow M_7$, $\frac{A_7}{A_6}$

$$T = m(V_7 - V_0) + A_7\underbrace{(p_7 - p_0)}_{0} = m(V_7 - V_0)$$

(V_7 from M_7, t_7; V_0 from M_0, h)

from A_7/A_6 = const $\rightarrow M_7$ $\quad \Delta H_T \rightarrow T_5 = T_7 \rightarrow t_7 \rightarrow a_7$, $M_7 \rightarrow V_7$, from M_7, $P_7 \rightarrow p_7$

$$\therefore\ T = m(V_7 - V_0) + A_7(p_7 - p_0)$$

h, M_0, P_2/P_0, A_0, N find T_4, A_6

$M_0, h \rightarrow P_0, T_0$ $\quad \frac{w\sqrt{\theta_0}}{\delta_0} = f(M_0)\,A_0\,\frac{P_r}{\sqrt{T_r}} \rightarrow A_0$

$$\frac{w\sqrt{\theta_0}}{\delta_0} = \frac{w\sqrt{\theta_2}}{\delta_2}\,\frac{P_2}{P_0}\sqrt{\frac{T_0}{T_2}} = 1$$

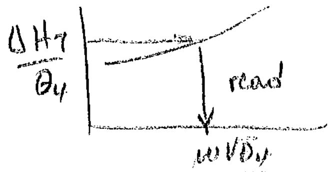
(known, get, given)

$\frac{w\sqrt{\theta_2}}{\delta_2}$ go to compressor charts

get P_3/P_2, $\frac{\Delta H_c}{\theta_2}$

$$\frac{w\sqrt{\theta_4}}{\delta_4} = \underbrace{\frac{w\sqrt{\theta_2}}{\delta_2}}_{\text{Know}} \; \underbrace{\frac{P_2}{P_4}}_{\text{Know}} \sqrt{T_4/T_2}$$

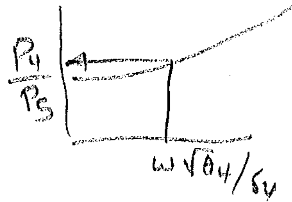
pick $T_4 \rightarrow \frac{\Delta H_7}{\theta_4} = \frac{\Delta H_c}{\theta_2}\,\frac{T_2}{T_4} \rightarrow \frac{\Delta H_7}{\theta_4}$



if not ⇒ pick T_4 & repeat

T_4 found, ΔH_7 found $\therefore$ T_5 found.

$$\frac{w\sqrt{\theta_5}}{\delta_5} = \underbrace{\frac{w\sqrt{\theta_4}}{\delta_4}}_{\text{Known}} \left(\frac{P_4}{P_5}\right)\left(\sqrt{\overbrace{T_5}^{\text{Known}}/T_4}\right)$$



$\frac{w\sqrt{\theta_5}}{\delta_5} = \frac{w\sqrt{\theta_6}}{\delta_6} = f(\gamma)\,A_6 \longrightarrow A_6$ depending on $A_7/A_6 \rightarrow M_7$ etc

---

$T = \dot{m} V_E + A_E(p_E - p_0)$

$T = \dot{m} c = A_t\, p_c\, C_F$ $\qquad c^* = c/C_F = f(\gamma, a_c)$ $\qquad V_E = f\left(\frac{T_c}{\mathcal{M}}, p_e/p_c\right)$

$I_{sp} = \frac{c}{g_0} = \frac{T}{\dot{w}} = \frac{V_e}{g_0}\left[1 + \frac{(1 - p_0/p_e)}{\gamma M_e^2}\right]$

mixtures are fuel rich to decrease the chamber pressure for structural purposes but weight/mole decreases

frozen flow – as flow expands $V\uparrow$ $T\downarrow$ such that reaction cannot occur composition from station to another remains same since flow doesn't remain at any one station long enough to reach equil.

$I_{sp_{frozen}}$ is lower since energy of flow must be removed to keep ions dissociated.

longer nozzles tend to drive products of combustion to equil. since same Δp takes place in longer distance; $\therefore$ Δt is longer ⇒ flow equilibrize

---

$m = m_0\left(1 - \varsigma \frac{t}{t_p}\right) \qquad \varsigma = M_p/M_0$

$\Delta V = -c \ln(1 - \varsigma t/t_p) - gt \quad ; \quad h_p = c t_p\left[1 + \frac{1-\varsigma}{\varsigma}\ln(1-\varsigma)\right] - \frac{1}{2} g t_p^2$

$\Delta V_{p_{ideal}} = -I_s g_0 \ln(1-\varsigma) = -c \ln(M_f/M_0) = -c \ln\lambda$ $\qquad$ assume $\Delta V_{TOT} = N_{STA} \cdot \Delta V/_{STAGE}$

Centrifugal — propellant heated & expanded. Heated by rotating U^{235} but instability causes turbulence → FIZZLE

Fluid Dynamic Contain. Case of Rotating. — U^{235} small amount comes out very slowly so no mixing; heating by radiation of large H_2 around it. Flow unstable mix.

Nuclear Lightbulb

wall extension around U^{235} — 1. walls are optically thin so as not to interfere with radiation

same principal of FLUID DYNAMIC CONTAIN — 2. plates to prevent mixing

3. must not melt

Centrif same however fuel is not lost.

Gaseous are best for low mission duration; all about same for large duration

use Gas Press for T↑ t↓, T↓ t↑

other wise use Turbopump.

low thrust, ~~was mass~~ short duration   high press N_2, He push O/F into chamber ∴ tanks & lines must be heavy $P_c \sim 1000$ 3-5000

Pump Pressuriz

O/F pumped into chamber. These pumps run by turbine which is run by decomposition of H_2O_2 . . . . since system is at one atmosphere. tanks are light but system complex.

Expander - Cycle   run H_2 thru pump use H_2 to cool walls take H_2 pass thru turbine which runs pumps. Put H_2 & O_2 in chamber

Tapoff Cycle   run H_2 & O_2 thru pump & into chamber tap off some of gases to run turbine & back into chamber.

Gas Generator   → Temp will not ruin turbines

run H_2 & O_2 thru generator fuel rich → thru turbine → thru chamber along with O_2.

waste from turbine shot out from rear engine to prevent flare back, cool wall or form another thrust source

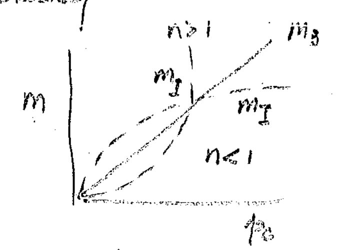
Injectors

showerhead mixing by diffusion if fuel & oxid are easily atomized.
like unlike - mixing by impinging
like like " " impinging then by diffusion.

ΔP must exist across injector   if ΔP↓ combustion instability due to coupling between fuel feed & combustion process. $\dot{Q}$↑ engine experiences oscillation
low freq → chugging motor
high freq → screaming motor.   instability due to vortex can be reduced by installing baffles efficiency decreases.

Solid Prop
Grain - Propellant   Homogeneous - oxid & fuel in one heavy molecules
Heterogeneous - mixed fuel & oxidizer ← rubber based fuel to reduce cracking

$r = \frac{A}{T_1 - T_p} p_c^n$ $n < 1$ for stability $p_c \sim \left(\frac{A_b}{A_t}\right)^{\frac{1}{1-n}}$



nuclear I_{sp}↑ T↑ propellant not part of reaction

Solid Core N.R.   limited by wall material since Treact > Twall melt.   fuel passes around reactor & gets heated & then exploded
fuel radioactive but small amount long lasting
shielding → very heavy   high energy release storable.

Gaseous core   fuel & radioactive material   prop is fuel & U^{235} when self sustaining will get large energy release. No shielding, loss of expensive fuel, no control, Twall↑ radioactive debris
fuel & U^{235} put into chamber where fission occurs. Neutrons released are reflected off walls back to react with U^{235}. Heat released heats H_2 & expands out nozzle

May 1968

POLYTECHNIC INSTITUTE OF BROOKLYN

Department of Aerospace Engineering and Applied Mechanics

Course AE 361 - Propulsion

Prof. Marian Visich, Jr.

not on exam

TURBOJET COMPONENT MATCHING

Let us consider a turbojet engine where the conditions at various stations in the engine are given by the following subscripts:

0	Free stream
1	Entrance to inlet
2	Entrance to compressor
3	Exit of compressor, entrance to combustion chamber
4	Exit of combustion chamber, entrance to turbine
5	Exit of turbine, entrance to exhaust nozzle
6	Exhaust nozzle minimum section
7	Exhaust nozzle exit

Symbols

A	Area, ft^2
C_P	Specific heat at constant pressure, $ft^2/sec^2\ {}^oR$
g	Acceleration of gravity, ft/sec^2

ΔH_c	$C_P\,(T_3 - T_2)$, change in stagnation enthalpy across compressor
ΔH_T	$C_P\,(T_4 - T_5)$, change in stagnation enthalpy across turbine
M	Mach number
N	Rotational speed of turbine or compressor
$N/\sqrt{\Theta}$	Corrected rotational speed of turbine or compressor
p	Static pressure, lbs/ft^2
P	Stagnation pressure, lbs/ft^2
P_r	Reference pressure, lbs/ft^2
R	Gas constant, $ft/^{o}R$
T	Stagnation temperature, ^{o}R
T_r	Reference temperature, ^{o}R
W	Weight flow, lbs/sec
$W\sqrt{\Theta}/\delta$	Corrected weight flow, lbs/sec
γ	Ratio of specific heats
Θ	T/T_r
δ	P/P_r
η_{ad}	Adiabatic efficiency of compressor

Case I Given: Flight Mach number, Altitude, P_2/P_0 A_6 and T_4

Find: N, A_0 and A_7

1. For a given A_6 and A_5 the Mach number at the exit of the turbine can be obtained from equation 1 for sonic velocity at station 6

$$\frac{A_5}{A_6} = \frac{1}{M_5}\left[\frac{2}{\gamma + 1}\left(1+\frac{\gamma-1}{2} M_5{}^2\right)\right]^{(\gamma+1)/2(\gamma - 1)} \tag{1}$$

2. The corrected weight flow at stations 5, 6, and 7 are equal and given by equation 2

$$\frac{W\sqrt{\Theta_5}}{\delta_5} = A_6 \frac{P_r}{\sqrt{T_r}}\sqrt{\frac{\gamma g}{R}}\left(\frac{\gamma+1}{2}\right)^{(\gamma+1)/2(1-\gamma)} \tag{2}$$

3. Turbine data is usually given by a unique plot of $\Delta H_T/\Theta_4$ versus $W\sqrt{\Theta_4}/\delta_4$ for all values of $N/\sqrt{\Theta_4}$ with values of P_4/P_5 specified on the curve. This data can be transformed into a plot of $W\sqrt{\Theta_5}/\delta_5$ versus $\frac{W\sqrt{\Theta_4}}{\delta_4}$ since

$$\frac{W\sqrt{\Theta_5}}{\delta_5} = \frac{W\sqrt{\Theta_4}}{\delta_4}\frac{P_4}{P_5}\sqrt{1-(\Delta H_T/C_P T_r \Theta_4)} \tag{3}$$

4. Knowing $W\sqrt{\Theta_5}/\delta_5$ from equation 2, $W\sqrt{\Theta_4}/\delta_4$ can be determined from the plot of $W\sqrt{\Theta_5}/\delta_5$ versus $W\sqrt{\Theta_4}/\delta_4$. For this value of $W\sqrt{\Theta_4}/\delta_4$, $\frac{P_4}{P_5}$ and $\frac{\Delta H_T}{\Theta_4}$ can be obtained from the turbine data.

5. As a first approximation it may be assumed that $P_3 = P_4$.

6. The equation for the corrected weight flow in front of the compressor (station 2) is given by equation 4

$$\frac{W\sqrt{\Theta_2}}{\delta_2} = \frac{W\sqrt{\Theta_4}}{\delta_4}\,\frac{P_4}{P_2}\sqrt{\frac{T_2}{T_4}} = \text{constant}\,\frac{P_3}{P_2} \qquad (4)$$

In equation 4, T_2/T_4 and $W\sqrt{\Theta_4}/\delta_4$ are known. From step 5, we note that P_4/P_2 and P_3/P_2 are equal. From the compressor map the above equation must be satisfied for the condition that the work absorbed from the fluid by the turbine is equal to the work done on the fluid by the compressor. The RPM which satisfies this condition is the RPM at which the turbine-compressor unit will rotate.

7. The condition on the work is that $\Delta H_T = \Delta H_c$. From step 4, $\Delta H_T/\Theta_4$ is known and by specifying the value of T_4, $\Delta H_c/\Theta_2$ is known from equation 5.

$$\frac{\Delta H_c}{\Theta_2} = \frac{\Delta H_T}{\Theta_4}\,\frac{T_4}{T_2} \qquad (5)$$

8. The basic compressor map consists of a plot of P_3/P_2 versus $W\sqrt{\Theta_2}/\delta_2$ for constant values of $N/\sqrt{\Theta_2}$ and η_{ad}. From this data it is possible to also plot on this map curves of constant $\Delta H_c/\Theta_2$ since

$$\frac{\Delta H_c}{\Theta_2} = \frac{C_p\,T_r\left[(P_3/P_2)^{(\gamma - 1)/\gamma} - 1\right]}{\eta_{ad}} \qquad (6)$$

9. The intersection of the straight line represented by equation 4 and the line $\Delta H_c/\Theta_2$ = constant, corresponding to the turbine requirement, represents the condition for which the turbine and compressor are matched. The RPM of the compressor-turbine unit is now determined.

10. Knowing the pressure recovery of the inlet P_2/P_0 it is possible to find the corrected weight flow based on the free stream conditions

$$\frac{W\sqrt{\Theta_0}}{\delta_0} = \frac{W\sqrt{\Theta_2}}{\delta_2}\ \frac{P_2}{P_0} \tag{7}$$

since $T_2 = T_0$,

11. The free stream tube area A_0 which the inlet must capture can be obtained from equation 8

$$\frac{W\sqrt{\Theta_0}}{\delta_0} = A_0\, M_0 \sqrt{\frac{\gamma g}{R}}\ \frac{P_r}{\sqrt{T_r}}\left(1 + \frac{\gamma - 1}{2} M_0^2\right)^{\frac{\gamma + 1}{2(1-\gamma)}} \tag{8}$$

12. The exhaust area at the exit of the nozzle can be obtained for the condition $p_0 = p_7$ from equation 9

$$\frac{A_7}{A_6} = \sqrt{\frac{\gamma - 1}{2}}\left(\frac{\gamma+1}{2}\right)^{(\gamma + 1)/2(1-\gamma)}\left[\left(\frac{p_7}{P_7}\right)^{2/\gamma} - \left(\frac{p_7}{P_7}\right)^{(\gamma+1)/\gamma}\right]^{-1/2} \tag{9}$$

Case II Given: Flight Mach number, Altitude, W, P_2/P_0, and N

Find: T_4, A_6 and A_7

1. From the given data it is possible to calculate

$$W\sqrt{\Theta_0}/\delta_0\ ,\ A_0,\ W\sqrt{\Theta_2}/\delta_2 \text{ and } N/\sqrt{\Theta_2}$$

2. From the standard compressor map it is possible to obtain for these conditions η_{ad} and P_3/P_2.

3. The work done by the compressor on the fluid, $\Delta H_c/\Theta_2$, can be obtained from equation 6.

4. The temperature ratio across the compressor is obtained from equation 10.

$$\frac{T_3}{T_2} = 1 + \frac{\left[(P_3/P_2)^{(\gamma - 1)/\gamma} - 1\right]}{\eta_{ad}} \qquad (10)$$

5. The Mach number at the exit of the compressor can be calculated from equation 8 where the subscripts 0 are replaced by subscripts 3, and the corrected weight flow at station 3 is given by equation 11.

$$\frac{W\sqrt{\theta_3}}{\delta_3} = \frac{W\sqrt{\theta_2}}{\delta_2}\sqrt{\frac{T_3}{T_2}}\;\frac{P_2}{P_3} \qquad (11)$$

6. Knowing M_3 and assuming a temperature ratio across the burner, T_4/T_3, the Mach number at the exit of the burner, and the pressure ratio across the burner can be obtained as a first approximation from equations 13 and 14.

$$\frac{T_4}{T_3} = \frac{M_4^2}{M_3^2}\;\frac{(1 + \gamma M_3^2)^2}{(1 + \gamma M_4^2)^2}\;\frac{\left(1 + \frac{\gamma - 1}{2} M_4^2\right)}{\left(1 + \frac{\gamma - 1}{2} M_3^2\right)} \qquad (12)$$

$$\frac{P_4}{P_3} = \frac{(1 + \gamma M_3^2)}{(1 + \gamma M_4^2)}\left[\left(\frac{1 + \frac{\gamma - 1}{2} M_4^2}{1 + \frac{\gamma - 1}{2} M_3^2}\right)\right]^{\gamma/(\gamma - 1)} \qquad (13)$$

7. The corrected weight flow in front of the turbine can now be calculated

$$W\sqrt{\theta_4}/\delta_4 = \frac{W\sqrt{\theta_3}}{\delta_3}\sqrt{\frac{T_4}{T_3}}\cdot\frac{P_3}{P_4} \qquad (14)$$

8. The work absorbed by the turbine is

$$\frac{\Delta H_T}{\theta_4} = \frac{\Delta H_c}{\theta_3}\;\frac{T_3}{T_4}$$

9. From the turbine map of $\Delta H_T/\theta_4$ versus $W\sqrt{\theta_4}/\delta_4$ and the value of the corrected weight flow found in step 7, $\Delta H_T/\theta_4$ can be obtained.

9. If this value of $\Delta H_T / \Theta_4$ is equal to the value given in step 7, the turbine and compressor are matched. If they do not agree, steps 6 thru 9 are repeated for a new value of T_4 until they do agree.

10. For the final value of $\Delta H_T / \Theta_4$ obtained in step 9, one can find P_4 / P_5 from the turbine map and T_5 / T_4 from equation 15.

$$\frac{T_5}{T_4} = 1 - \frac{\Delta H_T}{C_P T_r \Theta_4} \tag{15}$$

11. $W\sqrt{\Theta_5} / \delta_5$ can now be calculated and the minimum area of the nozzle, A_6, and the Mach number at the exit of the turbine, M_5 can be obtained from equations 2 and 1.

12. The exit area of the exhaust nozzle, A_7, can be obtained from equation 9.

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C. P. LESH PAPER CO., INDIANAPOLIS, IND.

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1.

a. hypergolic propellants - when fuel and oxidizer are brought together they instantaneously ignite. These propellants must be such that they have an afinity for one another, i.e. they must readily form products of combustion in a short time period.

b. Thrust coefficient - defined as the $T/A_t p_c$. This thrust coefficient is a function of

A_e/A_t where A_e is the exit area, A_t throat area

$\frac{p_e - p_o}{p_c}$, (pressure ate exit - pressure in atmosphere) / chamber pres

γ ratio of specific heats, T_c - chamber temperature.

A_e/A_t is a function of M at throat & M at exit.

c. Stagnation temperature - temperature that a flow would exhibit if it were brought adiabatically to rest

$T = t\left(1 + \frac{\gamma - 1}{2} M^2\right)$ function of mach number & static temperature & ratio of specific heats.

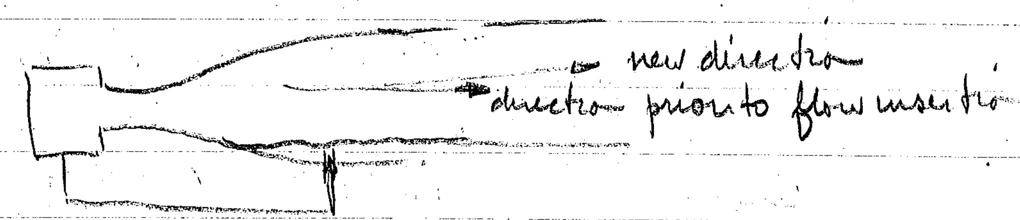
d. Specific impulse - ratio of Thrust of an engine to the weight flow of that engine

$I_{sp} = \frac{T}{\dot{w}} = \frac{c}{g_o}$ where c is effective exhaust velocity

& g_o is acceleration of gravity at surface of earth

$c = f(\text{exhaust veloc } v_e,\ p_e - p_o,\ A_e,\ \dot{m})$

e. thrust vector control - a means by which the flow out of the engine can be changed ("directioned") so that the desired trajectory is followed. One such method is inserting a flow into the nozzle $\perp$ to wall causing flow to separate from that wall



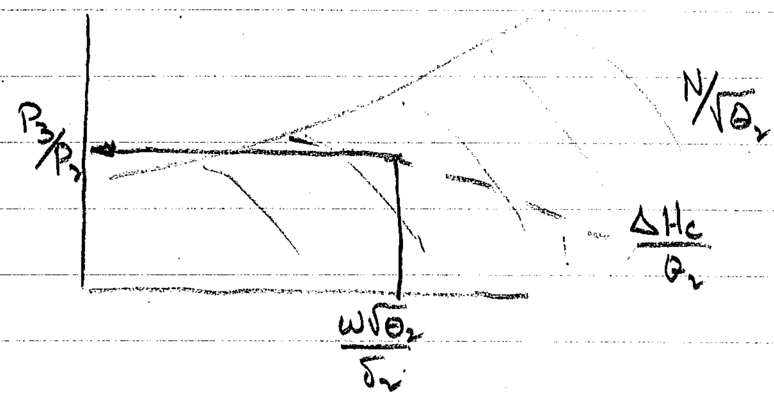
Given: M_0, h, P_2/P_0, N, A_0, ideal nozzle find: T_4 & A_6, A_7
$M=1$ @ A_6
from M_0 & $h \rightarrow P_0$

from $\frac{\omega\sqrt{\theta_0}}{\delta_0} = f(\gamma, M_0) A_0$ one obtains $\frac{\omega\sqrt{\theta_0}}{\delta_0}$ (1)

using $\frac{\omega\sqrt{\theta_2}}{\delta_2} = \frac{\omega\sqrt{\theta_0}}{\delta_0} \frac{P_0}{P_2}\left(\sqrt{\frac{T_2}{T_0}}\right)$ (2) from (1) one knows $\frac{\omega\sqrt{\theta_0}}{\delta_0}$

P_0/P_2 is given & since adiabatic inlet assumed $\sqrt{T_2/T_0} = 1$

$\therefore$ $\frac{\omega\sqrt{\theta_2}}{\delta_2}$ is found, going to compressor data charts

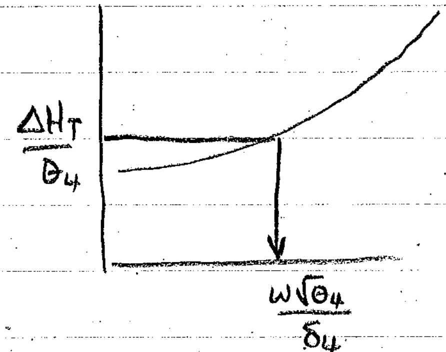


since N is given & $T_0 = T_2$ with $\frac{\omega\sqrt{\theta_2}}{\delta_2}$ found from eq (2) read P_3/P_2 & $\frac{\Delta H_c}{\theta_2}$.

now assume $P_3 = P_4$ & $\Delta H_c = \Delta H_T$
then

$\frac{\omega\sqrt{\theta_4}}{\delta_4} = \frac{\omega\sqrt{\theta_2}}{\delta_2}\left(\frac{P_2}{P_4}\right)\sqrt{T_2/T_4}$ (3) pick a T_4 get $\frac{\omega\sqrt{\theta_4}}{\delta_4}$

go to



$$\frac{\Delta H_T}{\theta_4} = \frac{\Delta H_c}{\theta_2}\,\frac{T_2}{T_4} \qquad (4)$$

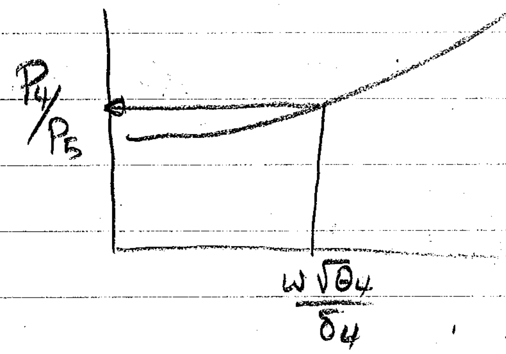
get $\frac{\Delta H_T}{\theta_4}$, read $\frac{w\sqrt{\theta_4}}{\delta_4}$ if

$\frac{w\sqrt{\theta_4}}{\delta_4}$ not same as $\frac{w\sqrt{\theta_4}}{\delta_4}$ found by (3)

pick new T_4 & iterate until T_4 is found.

<u>Caution:</u> T_4 may be > T_4 that turbine blades can withstand.

since T_4 is known & $w\sqrt{\theta_4}/\delta_4$ is found go to



read P_4/P_5

now $$\frac{w\sqrt{\theta_5}}{\delta_5} = \frac{w\sqrt{\theta_4}}{\delta_4}\,\frac{P_4}{P_5}\sqrt{\frac{T_5}{T_4}}$$

but $$\frac{\Delta H_T}{\theta_4} = T_{ref} C_p \left(1 - T_5/T_4\right)$$

$$\therefore \quad \frac{w\sqrt{\theta_5}}{\delta_5} = \frac{w\sqrt{\theta_4}}{\delta_4}\,\frac{P_4}{P_5}\sqrt{1 - \frac{\Delta H_T}{\theta_4 C_p T_{ref}}} \qquad (5)$$

known is $\frac{w\sqrt{\theta_4}}{\delta_4}$ from (3), $\frac{\Delta H_T}{\theta_4}$ from (4), P_4/P_5 from graph

$\therefore$ $\frac{w\sqrt{\theta_5}}{\delta_5}$ is found from (5)

Assume isentropic nozzle $\Rightarrow$ $\frac{w\sqrt{\theta_5}}{\delta_5} = \frac{w\sqrt{\theta_6}}{\delta_6}$

since $M = 1$ at A_6 then $w\sqrt{\theta_6} = f(\gamma)\,A_6$ (6) $\therefore$ A_6 is found since isentropic expansion $P_5 = P_7$, $T_5 = T_7$ T_5 found from ΔH_7

$$\therefore \quad P_7 = P_0 \underbrace{\frac{P_2}{P_0}}_{\text{give inlet}} \underbrace{\frac{P_4}{P_2}}_{1} \underbrace{P_5/P_4}_{\text{turbine}}, \underbrace{P_7/P_5}_{1} \qquad (7)$$

since ideal nozzle $p_7 = p_0$

by $P_7/p_7 = \left(1 + \frac{\gamma-1}{2} M_7^2\right)^{\gamma/\gamma-1}$ M_7 is found

since isentropic exp. $T_5 = T_7$

$$\therefore \quad t_7 = \frac{T_7}{\left(1 + \frac{\gamma-1}{2} M_7^2\right)} \longrightarrow a_7 = \sqrt{\gamma R t_7}$$

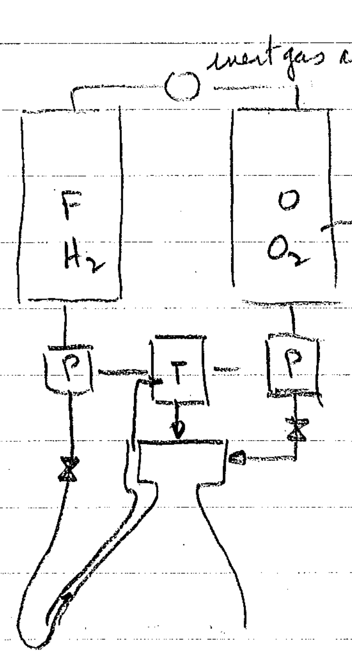
$\therefore$ $V_7 = M_7 a_7$, from M_7 get get $A_7/A_6 = A_7/A_4$

since A_6 is known from (6)

$$\therefore \quad T = m(V_7 - V_0) + A_7 \cancel{(p_7 - p_0)}^{\,0}$$

V_0 found from M_0 & $h \rightarrow a_0$

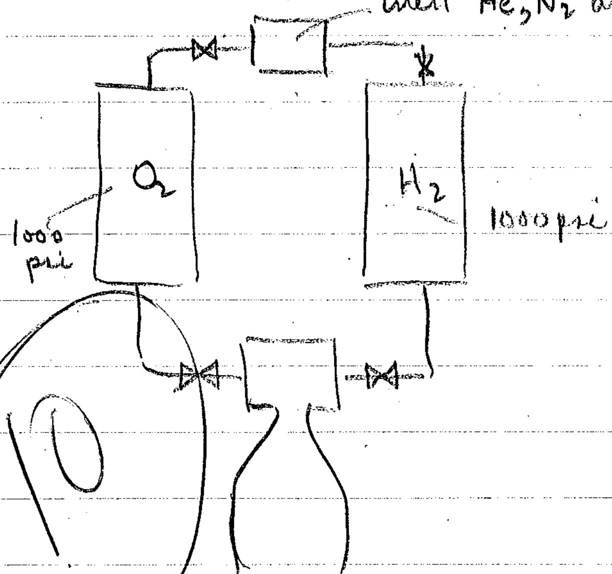
3.



Pump Pressurization

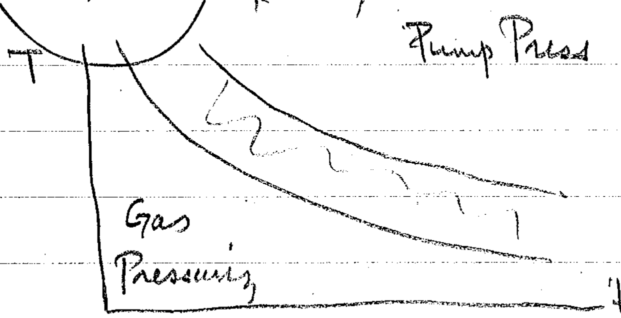
expando cycle
used H_2 as heat sink around engine walls - Take heated H_2 thru turbine which then runs pumps. Take Heated H_2 & O_2, which is pumped in, into combustion chamber to mix & expand thru nozzle. H_2 & O_2 tanks at ~ 1 atm
turbine runs both pumps

Gas Pressurization



He & N_2 since at high pressure force O_2 & H_2 ($p \approx 1000$) into chamber to mix burn & expand out nozzle

10

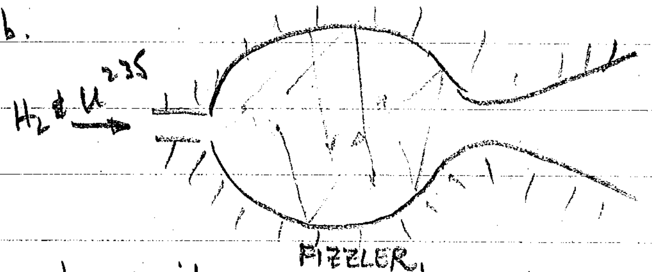


for high T, low time or high time, low T use gas pressuriz system. For higher thrust & longer times use Pump pressuriz.

Since high pressures in gas press system, tubing, tanks & piping must be thick to keep from bursting, whereas since pump pressurization system must be kept at ~ 1 atmosphe tubing & tanks can be made thinner.

Even though pump $_\wedge$press is more complex in terms of tubing & piping it is better because it is lighter.

b.



H_2 & U^{235} is fed into combustion chamber, as soon as U^{235} can sustain reaction the U^{235} fissions to emits more neutrons & energy which heats H_2; walls of chamber are made of reflector material so that neutrons cannot pass thru walls but are reflected back into mainstream so that they keep reaction going. Heated H_2 as well as U^{235} then expand through nozzle & exit.

This engine is not used since 1. U^{235} fuel is lost

2. no shielding
3. no control of fission reaction
4. trail of radioactive debris
5. Twall > melting temperature of material.

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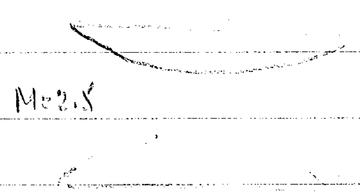
-5

28

if $M_S = 2.18$ since mach after shock = .5498 → $\frac{A_1}{A_{min}} = 1.26$

∴ $M_0 > M_S$ & from max pressure recovery shock should be at the throat ∴



(10)

$\left.\frac{A_1}{A_*}\right)_{M=2.5} = 2.637$ $\qquad \frac{A_{min}}{A_*} = \frac{A_1}{A_*}\cdot\frac{A_{min}}{A_1}$

$= 2.637\left(\frac{1}{1.26}\right) = 2.09$

M at throat before shock = 2.25

M after shock = .5406 $\qquad \boxed{P_2/P_0 = .6055}$ ✓

Table

$T_0 = t_0\left(1+\frac{\gamma-1}{2}M_0^2\right) = 390\left(1+.2(6.25)\right) = 390\left(1+1.25\right) = 2.25(390)$ $\quad +2\frac{1}{2}$

$= 877°R$ ✓

since stag temp are equal after shock $T_0 = T_2 = T_3 = T_4$

$f/a = \frac{1}{\frac{\eta_b HV}{C_p(T_4-T_3)} - 1}$

$\frac{w\sqrt{\theta_0}}{\delta_0} = A_0 M_0 \sqrt{\frac{\gamma g}{R}}\,\frac{P_r}{\sqrt{T_r}}\left(1+\frac{\gamma-1}{2}M_0^2\right)^{\frac{\gamma+1}{2(1-\gamma)}}$

$P_r = 2116$ atm $\quad T_r = 520°R$

$$\cancel{\frac{\omega\sqrt{\theta_5}}{\delta_5} = 1 \; \frac{2116}{\sqrt{520}} \sqrt{\frac{\gamma g}{R}} \left(\frac{\gamma+1}{2}\right)^{(\gamma+1)/2(1-\gamma)}}$$

A_0

$\cancel{m = \sqrt{\;}}$

$$f/a = \frac{1}{\dfrac{\eta_b (HV)}{c_p (2500 - 877)} - 1} = \frac{1}{\dfrac{.95 (18500)}{.24 (1623)} - 1} = \frac{1}{45.2 - 1} = \frac{1}{44.2}$$

$$\boxed{f/a = .0227}$$

✓

$T_4 = T_5 = T_7$

Tables

$$P_0 = p_0 \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/\gamma-1} = 1.69 (2.25)^{2.5} = 1.69 (5.07)(1.5) = 12.85 \text{ psi}$$

$P_2 = 7.7$ psi

isentropic nozzle

$$P_7 = \underbrace{P_7/P_5}_{1} \cdot \underbrace{P_5/P_4}_{1} \cdot \underbrace{P_4/P_3}_{1} \cdot \underbrace{P_3/P_2}_{1} \cdot \underbrace{P_2/P_0}_{.6055} \cdot \underbrace{P_0}_{12.85} = 7.7 \text{ psi}$$

$M_{exit} = 2.2 \qquad p_7/P_7 = .09352 \qquad A_7 = 2\ ft^2$

$p_7 = 103.8$ psf $\qquad p_0 = 243.5$ psf

$$A_7 (p_7 - p_0) = 2 (103.8 - 243.5)$$

since $M_7 = 2.2$ $\quad t/T = .5081$ $\quad t = .5081(2500) = 1270$

$R = 1545$

$$a = \sqrt{\gamma g R t_7} = \sqrt{89.} \quad 10^3$$

~~310~~ $1.\times 10^7$ $\quad R = 53.3$

$a =$ ~~2980~~ ft/sec 1750

$V_7 =$ ~~6500~~ 3850

$a_0 \to V_0 = 967$

m_0

get $\dfrac{\omega\sqrt{\theta_6}}{\delta_6} = A_6 f(\gamma)$

$$\cancel{\frac{\omega\sqrt{\theta_0}}{\delta_0} = \frac{\omega\sqrt{\theta}}{\delta}\,P}$$

$$\frac{\omega\sqrt{\theta_5}}{\delta_5} =$$

$$m_5 = A_6 \frac{P_5}{\sqrt{T_5}} \sqrt{\frac{1.4(32.2)}{53.3}} \left(1.2\right)^{1.75}$$ Units

$$1 \quad \frac{7.7(144)}{\sqrt{2500}}\left(.92\right)\left(1.38\right)$$

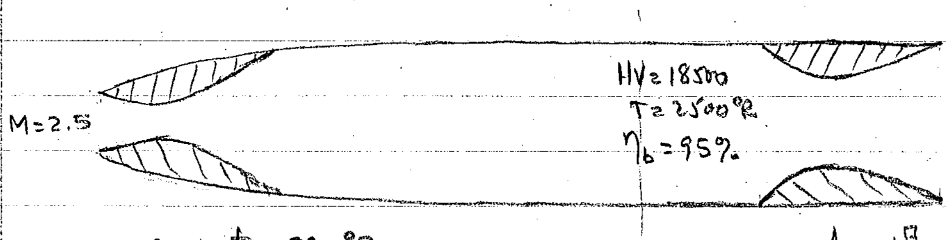
$$m_5 = \frac{7.7(144)}{50}(.92)(1.38)$$ ~~slugs/sec~~

$$= 28 \quad \text{lbm/sec}$$

$m_f/m_a = .0227$

$m_f + m_a = 28$

$$\frac{m_f}{m_a} + 1 = \frac{28}{m_a} \qquad \frac{28}{1.0227} = m_a$$



$p_0 = 1.69$ $\quad t_0 = 390°R$ $\qquad A_6 = 1^{\square}$

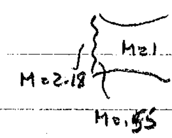
$A_1/A_{min} = 1.26$ $\qquad A_7 = 2^{\square}$

Assume $\gamma = 1.4$

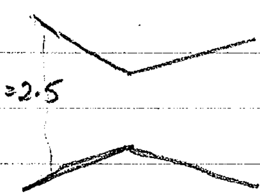
$$T = \underset{.83?}{\dot m}\,(V_7 - \underset{2420}{V_0}) + \underset{2}{A_7}\,(p_7 - \underset{1.69}{p_0})$$

$a_0 = \sqrt{\gamma R t_0} = 968$ ft/sec

$V_0 = M_0 a_0 = 2420$ ft/sec



$M_s < M_0$



$$\frac{A_{min}}{A_{min}^*} = \frac{A_0}{A_0^*}\,\frac{A_0^*}{A_{min}^*}\,\frac{A_1}{A_0}\,\frac{A_{min}}{A_1}$$

$$= 2.637 \cdot \underline{1} \cdot 1 \cdot \frac{1}{1.26}$$

$$= 2.093$$

$M_{min} = 2.25 \qquad P_2/P_0 = .6055$

$P_1 = P_0 = \dfrac{1.69}{.05853} = 28.87$ psi

7.7
2.25
15.4
1.9
17.3

$P_5 = 28.87 \cdot .6055 = 17.48$ psi

$m = \rho A V = \rho^* A^* V^*$

$M^* = 1 \qquad \rho^*/\rho_t = .6339 \qquad \rho_t = \dfrac{P_5}{R T_5} = 5.867 \times 10^{-4}$

$\rho^* = 3.719 \times 10^{-4}$

$V^* = a^* = \sqrt{\gamma R t^*} \qquad t^*/T_t = .8333 \qquad t^* = 2083$

$V^* = 2237 \qquad m = .832$ slugs

.87
32.2 ⟌ 28
25.76
2.24

$$V_7 = \left(\frac{V}{a^*}\right) a^* = 1.718\,(2237) = 3843$$

$M_{exit} = 2.2 \rightarrow A_7/A_6 = 2$ $\quad$ $t/T = .5081 \rightarrow t_7 = 1270°R$

$a = \sqrt{\gamma R T} = 1747$ $\quad$ $V_7 = 3843$

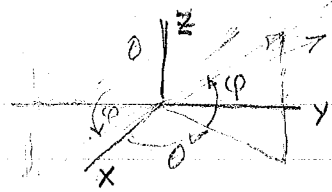
$p/p_t = .09357$ $\quad$ $p_7 = 1.64$

$T = 1184 - 14 = 1170$

$sfc \rightarrow$ $\quad$ $w_f/w_a = \dfrac{c_p(T_4 - T_3)}{\eta_B \; HV} = \dfrac{.24\,(2500 - 897)}{.95\;(18500)}$

$w_f = .581$

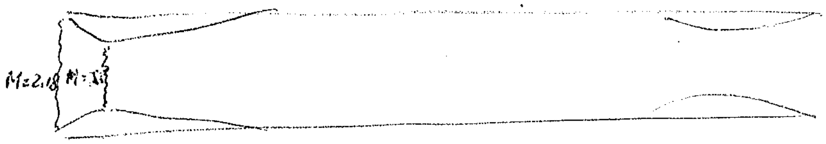
$sfc = \dfrac{w_f}{T} = 1.788$



let $\bar{T} = T_x$

$$\dot{x} - V\cos\varphi\cos\theta = 0$$
$$\dot{y} - V\cos\varphi\sin\theta = 0$$
$$\dot{z} - V\sin\varphi = 0$$
$$\cancel{m\dot{V} = D(z,V,L) - g\sin\varphi} + T\cos e$$
$$m\ddot{x} = D(z,V,L)\cos\varphi\cos(180+\theta)$$
$$m\ddot{y} = D(z,V,L)\cos\varphi\sin(180+\theta)$$
$$m\ddot{z} = -mg - D(z,V,L)\sin\varphi$$

$M_0 = 2.5 \quad p_0 = 1.69$ psi @ 50,000 ft. $\quad t_0 = 390°R \qquad 243.5$ psf



$A_6 = 1$

$A_7/A_6 = 2$

$A_1/A_{min} = 1.26 \qquad$ since $\quad M_s < M_0$

for max press ice. shock at throat

$$\frac{A_1}{A_*} = 2.637 \qquad \frac{A_{min}}{A_*} = \frac{A_1}{A_{star}} \cdot \frac{A_{min}}{A_1} = \frac{2.637}{1.26} = 2.09$$

$M_{\text{before shock}} = 2.25 \qquad P_2/P_0 = .6055$

$$T_0 = t_0\left(1 + .2\,(2.5)^2\right) = t_0\,(1 + 1.25) = 2.25\,t_0 = 2.25\,(390) = 877°R$$

1.25

$\cancel{P_0 = p_0 \ldots}$

$$\frac{P_0}{p_0} = \frac{R_0}{\rho_0}\frac{T_0}{t_0} = \left(\frac{P_0}{p_0}\right)^{1/\gamma}\frac{T_0}{t_0} \qquad \left(\frac{P_0}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_0}{t_0} \qquad \frac{P_0}{p_0} = \left(\frac{T_0}{t_0}\right)^{\gamma/\gamma-1}$$

$$\cancel{\frac{T_0}{t_0} = 2.25} \qquad \cancel{\gamma/\gamma-1 = \frac{1.4}{.4} = 3.5} \qquad \cancel{\frac{P_0}{p_0} = 17.1}$$

$$P_0 = 17.1\,(243.5)\text{ psf} = 4160\text{ psf}$$

$$T_0 = T_2 = T_3 \qquad T_4 = 2500°R$$

$$f/a = \frac{1}{\frac{\eta_b HV}{C_p(T_4 - T_3)} - 1} = \frac{1}{\frac{.95(18500)}{.24(2500 - 877)} - 1} = \frac{1}{45.1 - 1} = \frac{1}{44.1} = .0227$$

$$P_2 = P_3 = P_4 = P_5 = P_7 = .6055 P_0 = 2520\text{ psf}$$

$$A_7/A_6 = 2 \longrightarrow M_7 = 2.20$$

$$T_7 = T_5 = T_4 = 2500°R \qquad t_7 = \frac{2500}{1 + .2(4.84)} = \frac{2500}{1.968} = 1270°R$$

$$p_7 = \frac{P_7}{(T_7/t_7)^{3.5}} = \frac{2520\text{ psf}}{(1.968)^{3.5}} = \frac{2520}{10.68} = 236\text{ psf}$$

$A_7 (p_7 - p_0) = 2(236 - 243.5) = -15$ psf

$a_7 = \sqrt{\gamma g R t_7} = \sqrt{1.4(1715)(1270)} = 1747$ ft/sec

$V_7 = 3840$ ft/sec

$a_0 = \sqrt{\gamma g R t_0} = \sqrt{1.4(1715)(390)} = 967$

$V_0 = 2415$ ft/sec

$$m = A_6 \frac{P_6}{\sqrt{T_6}} \sqrt{\frac{\gamma}{gR}} \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(1-\gamma)}}$$

since isentropic $P_5 = P_6 = P_7$
$T_5 = T_6 = T_7$

Assume $w_f <<< w_a$

$$= 1 \cdot \frac{2520}{50} \sqrt{\frac{1.4}{.1715}} \left[1.2\right]^{\frac{2.4}{-.8}} = -3$$

$$= 50.4 \sqrt{8.17 \times 10^{-4}} \frac{1}{1.728} = \frac{50.4}{1.728} .0286 = .835 \text{ slugs/sec}$$

$w_a = 26.9$

$T = .835 (V_7 - V_0) - 15 = .835 (1425) - 15 = 1190 - 15 = 1175$ lb

$w_f / w_a = .0227$

$w_f = .0227(26.9) = .61$

$$sfc = \frac{.61(3600)}{1175} = .187 \frac{\text{lb/hr}}{\text{lb}}$$