AE 313 - AE 333 Friday Sessions

Sept.	17	Fluids Lab: 2 hrs. lab. lecture-Experiment 1
	24	Structure Lab: 2 hrs. lab. lecture + 2 hrs. lab.
Oct.	1	Fluids Lecture: 2 hrs. Fluids Lab: 2 hrs.-Exp. 1
	8	Structures Lecture: 2 hrs. Structures Lab: 2 hrs.
	15	Fluids Lab: 2 hrs. lab. lecture + 2 hrs. lab.-Exp. 2
	22	Structures Lab: 2 hrs. lab. lecture + 2 hrs. lab.
	29	Fluids Lecture: 2 hrs.
Nov.	5	Structures Lecture: 2 hrs. Structures Lab: 2 hrs.
	12	Fluids Lab: 2 hrs. lab. lecture + 2 hrs. lab.-Exp. 3
	19	Structures Lab: 2 hrs. lab. lecture + 2 hrs. lab.
Dec.	3	Fluids Lab: 2 hrs. lab lecture + 2 hrs. Lab.-Exp. 4
	10	Structures Lecture: 2 hrs. Structures Lab.:2 hrs.
	17	Fluids Lecture: 2 hrs.

AE 333 Fluids III : Dr Schmidt X 118 Farmingdale
School. Fl. Lab

Sept 17	Rm. 207 Lect	Ex 1	2-3 hr.
Oct 1	Rm 516 Perf	"	4-9 hr.
* Oct 15	Farmingdale	Ex 2	2-9 hrs.
Nov 12.	Rm 207 Lect	Ex 3	2-3 hrs.
	Rm 516 Perf	" "	4-9 hrs.
* Dec 3	Farmingdale	Ex 4	2-9 hr.

* Conflict w. EE 378 B is resolvable

① Airfoil Chain
Measure L, M, D on sym airfoil W & W. O flap @ V_∞ = const
α = var.
η = "

② hypersonic wind tunnel
Measure M_∞ & run times.

③ Stokes flow & Demo. of Hele-Shaw Apparatus

④ Pipe flow Utilize hot wire anemometer to measure prop prof in a pipe

AE 333 Profs Sforza, Visich

HW due 2 lectures after assigned 3 exams Lab exam counts as 1/2 credit

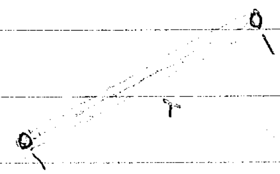
previous courses have not allowed fluids to support shear stresses.

σ = diameter of typical molecule $= 10^{-8}$ cm

$n = 10^{19}$ particles/cc $\dfrac{\text{total volume of particles}}{\text{Total vol of cont.}} \approx \dfrac{d^3 \times n}{1cc} = 10^{-5}$.

volume occupied by molecular matter << volumes in which it is contained.

instead of microscopic view take macroscopic view.

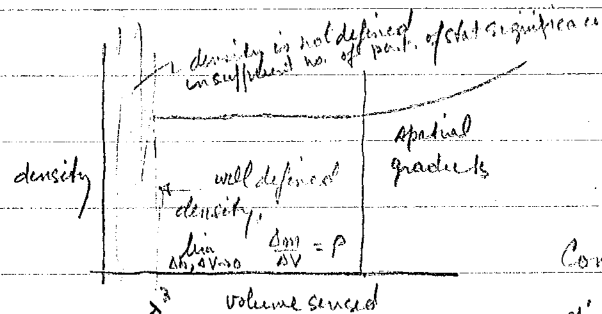


vol swept out $= \pi \sigma^2 \lambda$

no of particles in the vol = no. density of part × volume $= n \times \pi \sigma^2 \lambda$

$\lambda = (n \pi \sigma^2)^{-1}$ for collision path mean free path-gas; 10^{-5} cm

$K_N = \lambda / L$ $\dfrac{\text{local mean free path}}{\text{characteristic dimension of problem}}$ $K \ll 1$ matter is continuum; $K \simeq O(1)$ non continuum.

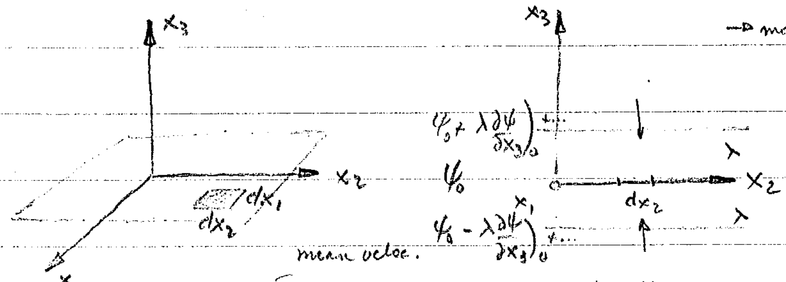


$\rho = \dfrac{mass}{vol}$

Continuum - matter investigated is continuously distributed throughout space

Continuity equation is continuity eq since derivatives must Exist.

All fluids will be considered as continuous, ρ does exist & so do the limit



→ movement of molecules

Transport properties

six possible ways molecules can move. $\frac{1}{6}$ of molecules move in any direction

find flux of property being transp

$\frac{1}{6} nc$ – no. of molecules passing through $dx_1 dx_2$/unit time in $+x_3$ direct

n: no. part / vol.

the net flux of $\psi = \frac{1}{6} nc\left[\psi_0 - \lambda \frac{\partial \psi}{\partial x_3}\Big|_0\right] - \frac{1}{6} nc\left[\psi_0 + \lambda \frac{\partial \psi}{\partial x_3}\Big|_0\right]$

flux in $+x_3$ dir flux in $-x_3$ direct

net flux $= -\frac{1}{3} nc\lambda \frac{\partial \psi}{\partial x_3}\Big)_0$

if $\psi = mu_1$ m is mass of 1 particle ψ is momentum/part in x_1 direc

u_1 is ordered velocity in x_1-dir

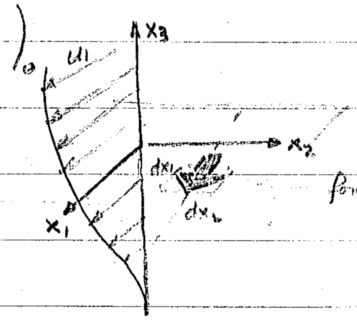
net flux in ψ = net flux in x_1-momentum / unit area ($dx_1 dx_2$)

= force in x_1-direc / unit area ($dx_1 dx_2$)

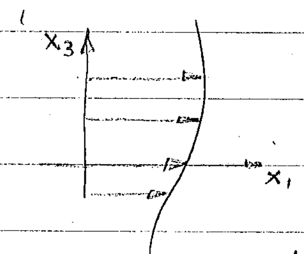
= stress in x_1-direct acting on $dx_1 dx_2$

Stress $= -\frac{1}{3} m n c \lambda \frac{\partial u_1}{\partial x_3}\Big)_0$

acts in x_1 direct, acts on surface $dx_1 dx_2$.



force acting on x_1 direct (shear force)



$\sigma \sim \frac{\partial u_1}{\partial x_3}$ the coeff $\frac{1}{3} mnc\lambda$ is the viscosity coeff, dependent on fluid properties.

$mn = (\text{mass of 1 part})\left(\frac{\text{no. of part}}{\text{vol}}\right) = \frac{\text{mass}}{\text{vol}} = \rho$

$\mu = \frac{1}{3}\rho c \lambda$

$= \frac{2}{3} \frac{\sqrt{mkT/\pi}}{\pi \sigma^2}$ = fn of T only & not fn of pressure.

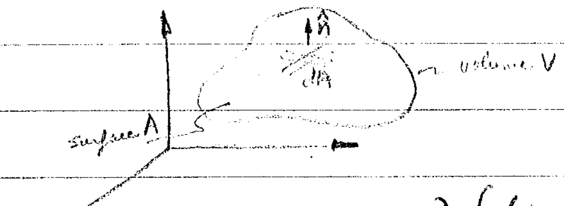
This has been experimentally verified for dilute gases (low pressures)

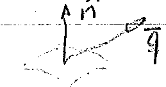
transport is on a microscopic level.

skin friction drag is due to viscosity (boundary layer). Gives you the skin friction drag & lift due to destruction of symmetrical flow.

9-22-70

Assumption of Conservation of Mass, inertial frame of reference, continuum





rate of change of mass in volume V $= \frac{\partial}{\partial t}\left\{\int_V \rho\, dV\right\}$, rate of efflux of mass across A $= -\int_A \rho(\bar{q}\cdot\hat{n})\,dA$

$$\therefore \quad \frac{\partial}{\partial t}\left\{\int_V \rho\, dV\right\} = -\int_A \rho(\bar{q}\cdot\hat{n})\,dA$$

Conservation of Momentum (Newton's second law)

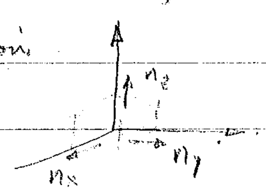
$\frac{\partial}{\partial t}\int_V \rho u_i\, dV$ time rate of change in moment; $\int_A \rho u_i(u_j n_j)\,dA$ momentum flow across the area; $\int_V \rho X_i\, dV$ body forces acting on fluid in V; $\int_A P_{ji}\, n_j\, dA$ the surface forces acting on the fluid over A.

$$\frac{\partial}{\partial t}\int_V \rho u_i\, dV + \int_A \rho u_i(u_j n_j)\,dA = \int_V \rho X_i\, dV + \int_A P_{ji}\, n_j\, dA$$

ρu_i momentum / unit vol., $u_j n_j = \bar{q}\cdot\hat{n} \Rightarrow u_j n_j\, dA = \bar{q}\cdot\hat{n}\, dA$

X_i = force / unit mass, P_{ji} gives stress $\quad P_{ji} n_j = \bar{\bar{P}}\cdot\hat{n}$

Tensor Notation



let i = x-direction

$F_x = p_{xx} + t_{zx} + t_{yx}$

$F_y = t_{xy} + t_{zy} + p_{yy}$ etc. $\therefore$ Surface forces require 9 components for definition

to make notation simpler use ij to denote Σ of directions

Subscripts i, j, k, l, etc. refer to any of x, y, z

$u_i = u\,(i=1),\ v\,(i=2),\ w\,(i=3)$

$a_{ij} b_i = \sum_i a_{ij} b_i$

Momentum Eq. will not give detailed information ∴ must look at differential form.

Differential forms of Cons. of Mass

$$\frac{\partial}{\partial t}\left\{\int_V \rho\, dV\right\} + \int_A \rho u_i n_i\, dA = 0$$

Using divergence theorem $\int_V \nabla\cdot \bar{F}\, dV = \int_A \bar{F}\cdot\bar{n}\, dA$ when $F \in C^n$ in V.

$$\frac{\partial}{\partial t}\int_V \rho\, dV + \int_V \frac{\partial}{\partial x_i}(\rho u_i)\, dV = 0$$

since Volume is fixed $\frac{\partial}{\partial t}\int_V \rho\, dV = \int_V \rho_{,t}\, dV$

$$\therefore \int_V (\rho_{,t} + (\rho u_i)_{,x_i})\, dV = 0$$

since control volume is independent ∴

$$\rho_{,t} + (\rho u_i)_{,x_i} = 0 \quad \text{Continuity Equation}$$

$$= \frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

Differential form of Cons of Mom.

$$\frac{\partial}{\partial t}\int_V \rho u_i\, dV + \int_V \frac{\partial}{\partial x_j}(\rho u_i u_j)\, dV = \int_V \rho X_i\, dV + \int_V \frac{\partial P_{ji}}{\partial x_j}\, dV$$

$$\int_V \left\{(\rho u_i)_{,t} + (\rho u_i u_j)_{,x_j} - \rho X_i - P_{ji,x_j}\right\} dV = 0$$

again since $dV \neq 0$

then $(\rho u_i)_{,t} + (\rho u_i u_j)_{,x_j} - \rho X_i - P_{ji,x_j} = 0$

now $\frac{\partial}{\partial x_j}\rho u_i u_j = u_i \frac{\partial \rho u_j}{\partial x_j} + \rho u_j \frac{\partial u_i}{\partial x_j}$, from continuity we

can write $0 = \frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j}$ mult by u_i then $-u_i \rho_{,t} + \rho u_j u_{i,x_j}$

then $\rho u_{i,t} + u_i \rho_{,t} - u_i \rho_{,t} + \rho u_j u_{i,x_j} = \rho X_i + P_{ji,x_j}$

$$\rho u_{i,t} + \rho u_j u_{i,x_j} = \rho X_i + P_{ji,x_j}$$

in the x direction $\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho X + P_{xx,x} + P_{yx,y} + P_{zx,z}$

$$\rho\left(\frac{\partial u}{\partial t} + (\bar{q}\cdot\nabla)u\right) = \rho X + \nabla\cdot\tilde{P}_x \qquad \tilde{P}_x = P_{xx}\bar{i} + P_{yx}\bar{j} + P_{zx}\bar{k}$$

$$\bar{q} = u\bar{i} + v\bar{j} + w\bar{k}$$

$P_{ji} = P_{ii} = p\delta_{ij}$

10-1-71

Prof. Visich Ext 97 (182) Rm 404A. Our room 303

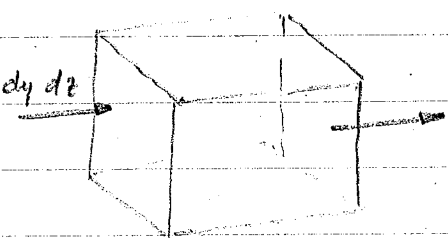
Problems must be solved for u, v, w, p, ρ, T. Gas remains a diatomic gas.

$$\rho_{,t} + (\rho u_i)_{,x_i} = 0 \qquad \text{Continuity}$$

$$\rho_{,t} + (\rho u)_{,x} + (\rho v)_{,y} + (\rho w)_{,z} = 0 \quad \text{for compressible fluid}$$

Can be found by considering mass flow coming into control volume.

$\rho u\, dy\, dz$



$\left(\rho u + \frac{\partial(\rho u)}{\partial x}dx\right)dy\, dz$

Mass flow out − mass flow in in x direct $\frac{\partial(\rho u)}{\partial x}dx\,dy\,dz$

$+$ in y direct $\frac{\partial(\rho v)}{\partial y}dx\,dy\,dz$

$+$ in z direct $\frac{\partial(\rho w)}{\partial z}dx\,dy\,dz$

equals rate of change of mass in volume

$$= -\frac{\partial\rho}{\partial t}dx\,dy\,dz$$

if expanded

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y} + w\frac{\partial\rho}{\partial z} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

$$\frac{D\rho}{Dt} + \rho\,\nabla\cdot\bar{q} = 0 \qquad \text{from } \rho = \text{const} \quad \nabla\cdot\bar{q} = 0$$

if $\rho \neq$ const but flow steady $\frac{\partial \rho}{\partial t} = 0$ $\qquad \nabla \cdot (\rho \bar{q}) = 0$

Euler's Equation 1755

$$\rho \frac{D\bar{q}}{Dt}\, dx\,dy\,dz = \bar{F}$$

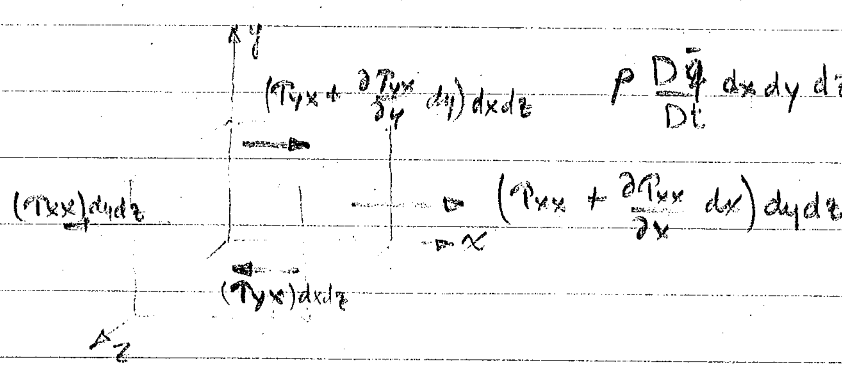
in x direct $\qquad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + X$

$$u_{i,t} + u_j\, u_{i,x_j} = X_i + \frac{1}{\rho} P_{ji,x_j}$$

x dir $\qquad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = X + \frac{1}{\rho}\left[\tau_{xx,x} + \tau_{yx,y} + \tau_{zx,z}\right] = \frac{Du}{Dt}$

y dir $\qquad \frac{Dv}{Dt} = Y + \frac{1}{\rho}\left[\tau_{xy,x} + \tau_{yy,y} + \tau_{zy,z}\right]$

z direct $\qquad \frac{Dw}{Dt} = Z + \frac{1}{\rho}\left[\tau_{xz,x} + \tau_{yz,y} + \tau_{zz,z}\right]$



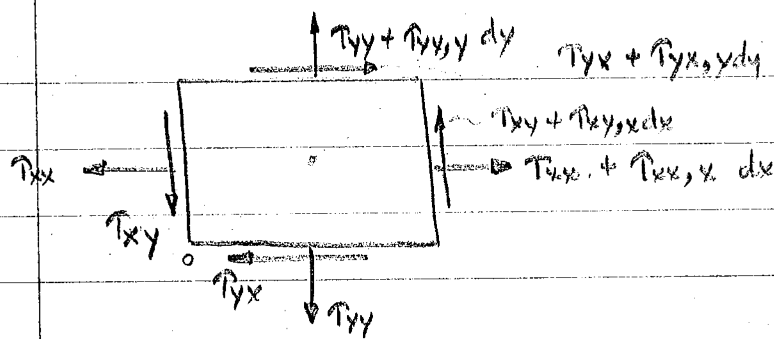
$\rho \frac{D\bar{q}}{Dt}\, dx\,dy\,dz = \bar{F}$ find $\bar{F}$

$$\therefore\ \bar{F} = (\tau_{xx,x} + \tau_{yx,y} + \tau_{zx,z})\,dx\,dy\,dz$$

$$\therefore\ \rho \frac{Du}{Dt}\, dx\,dy\,dz = dy\,dz\,dx\,(\tau_{xx,x} + \tau_{yx,y} + \tau_{zx,z})$$

+ body force/unit mass $X \cdot \rho\, dx\,dy\,dz$

to relate shear stresses one must look at fluid properties



2 subs gives direction act'g in
1 subs gives direct. ⊥ acting to it

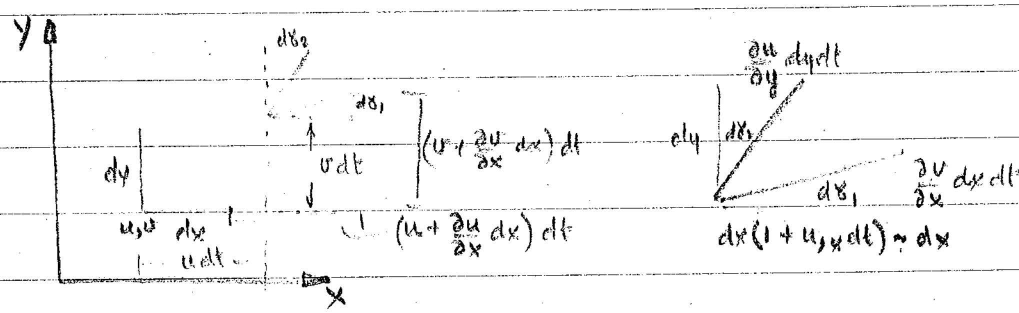
Fluid in Equil $\therefore \sum M_o)^{+} = 0$ $\quad \tau_{xx,x}\,dx\,\frac{dy^2}{2} - \tau_{yy,y}\,dy\,\frac{dx^2}{2}$

$$(\tau_{yx} + \tau_{yx,y}\,dy)\,dx\,dy - (\tau_{xy} + \tau_{xy,x}\,dx)\,dy\,dx = \rho\,dx\,dy \cdot \left[\frac{(dx)^2+(dy)^2}{12}\right]\omega$$

$$dx\,dy\left\{\tau_{yx} - \tau_{xy} + dy\left[\tfrac{1}{2}\tau_{xx,x} + \tau_{yx,y}\right] + dx\left[\tau_{xy,y} - \tfrac{1}{2}\tau_{yy,y}\right]\right\} = 0$$

if $\omega = 0$ $\qquad -\rho J_o\,\partial x\,dy$

as $dx, dy \to 0$ $\quad \tau_{yx} = \tau_{xy} \iff \boxed{\tau_{ij} = \tau_{ji}}$



$$\tan d\gamma_1 = \frac{\partial v}{\partial x}dt \sim d\gamma_1 \qquad \frac{d\gamma}{dt} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

$$\tan d\gamma_2 \approx d\gamma_2 = \frac{\partial u}{\partial y}dt$$

Newtonian fluid $\quad \tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$

$$\boxed{\tau_{ij} = \tau_{ji} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}$$

From Solid Mechanics

$$\tau_i = \lambda^*(e_1 + e_2 + e_3) + 2\mu^* e_i \qquad \mu^* = G \text{ modulus of rigidity}$$

$$\lambda^* = \frac{\nu E}{(1+\nu)(1-2\nu)} = \text{bulk mod}$$

e extensions along principal axis

ν poisson's ratio

for fluid case

$$\tau_{xx} = -p + \lambda \nabla\cdot\bar{q} + 2\mu \frac{\partial u}{\partial x} \qquad \text{prop dilation, viscosity}$$
$$\tau_{yy} = -p + \lambda \nabla\cdot\bar{q} + 2\mu \frac{\partial v}{\partial y}$$
$$\tau_{zz} = -p + \lambda \nabla\cdot\bar{q} + 2\mu \frac{\partial w}{\partial z}$$

$$\tau_{xx} + \tau_{yy} + \tau_{zz} = -3p + 3\lambda\nabla\cdot\bar{q} + 2\mu\nabla\cdot\bar{q}$$
$$\tau_{xx} + \tau_{yy} + \tau_{zz} = -3p + (3\lambda + 2\mu)\nabla\cdot\bar{q}$$

Stokes said that $3\lambda + 2\mu = 0$

$\therefore \tau_{xx} + \tau_{yy} + \tau_{zz} = -3p$ also $\tau_{xx} \neq -p$.

from conti. $\operatorname{div}\bar{q} = -\frac{1}{\rho}\frac{D\rho}{Dt}$

$$\therefore \tau_{xx} + \tau_{yy} + \tau_{zz} = -3p - \frac{\overbrace{(3\lambda + 2\mu)}^{\mu^*}}{\rho}\frac{D\rho}{Dt}$$

Equation of state $p = f(\rho) \neq f(\dot{\rho})$ $\therefore$ coeff of $\frac{D\rho}{Dt} \equiv 0$

which justifies $\tau_{xx} + \tau_{yy} + \tau_{zz} = -3p$

$\therefore \lambda = -\frac{2}{3}\mu$

$$\boxed{\tau_{ii} = -p - 2\mu\left[\frac{1}{3}\nabla\cdot\bar{q} - u_{i,x_i}\right]}$$

if body forces are neglected

$$\rho\frac{Du}{Dt} = \frac{\partial}{\partial x}\left(-p - \frac{2}{3}\mu\nabla\cdot\bar{q} + 2\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]\right)$$
$$+ \frac{\partial}{\partial z}\left(\mu\left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right]\right)$$

$\nabla \cdot \bar{q} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$

$$\rho \frac{Du}{Dt} = -p_{,x} + [\mu u_{,x}]_{,x} + [\mu u_{,y}]_{,y} + [\mu u_{,z}]_{,z}$$
$$+ \frac{1}{3}\left[\mu(u_{,x} + v_{,y} + w_{,z})\right]_{,x}$$

Navier Stokes Equation in x direction

$$\rho \frac{Du}{Dt} = -p_{,x} + [\mu u_{,x}]_{,x} + [\mu u_{,y}]_{,y} + [\mu u_{,z}]_{,z}$$

for ρ = const since $\nabla \cdot \bar{q} = 0$ $\qquad \mu = f(T)$

if ρ, μ const $\quad \rho \frac{Du}{Dt} = -p_{,x} + \mu \nabla^2 u$

u — Conserv Mass

$\left.\begin{matrix} u \\ w \\ p \end{matrix}\right\}$ mom equat 3

if compress flow look at energy equa.

ρ = const

T Equation of state

$p = \rho R T$ thermally perfect
C_p = const calorically " gas

if thermally & calorically

u, v, w, ρ, p, t

for static condition use lower case letters, for stag condition use upper case letter.

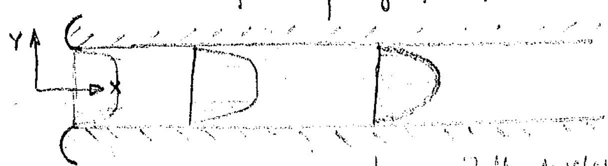
$\rho\left[u_{,t} + (\bar{q}\cdot\nabla)u\right] = -p_{,x} + \mu\nabla^2 u$ ②, ③, ④

2,3,4,5 give p, u, v, w

$\rho = \text{const}$ ① $\qquad \nabla\cdot\bar{q} = 0$ ⑤ $\qquad p = \rho R t$ ⑥

for compressible ① becomes energy equation ② has term containing $\nabla\cdot\bar{q}$ ⑤ $\nabla\cdot\rho\bar{q} = 0$

Flow between two infinitely long flat plates. (POISEUILLE Flow - 1840)



no slip condition at the walls. $u=0$ at surface due to viscosity relative to body.

boundary layer | ... fully developed flow ...

We will use potential flow problems to get initial input into viscous flow equation, potential flow will be used outside boundary layer.
Solution to Entrance Region is complex since potential & viscous flow mix. Fully developed flow $\frac{\partial}{\partial x}$(velocity components) $\equiv 0$. Flow is affected by upper & lower walls due to velocity.

Will find u, v, p, t

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -p_{,x} + \mu\left[\nabla^2 u\right]$$

$$\rho\left[u v_{,x} + v v_{,y}\right] = -p_{,y} + \mu\left[\nabla^2 v\right]$$

$$\nabla\cdot\bar{q} = 0 \qquad t = p/R\rho$$

from $\nabla\cdot q = 0$ & $\frac{\partial u}{\partial x} = 0 \Rightarrow \frac{\partial v}{\partial y} = 0$

For fully developed flow $\therefore$ since $v_{,y} = 0$, $p_{,y} = 0$ (from 2nd mom Eq) $\quad p = p(x) \quad \frac{\partial p}{\partial x} = \frac{dp}{dx}$

$v_{,yy} = 0;\ v_{,xx} = 0 \Rightarrow p_{,y} = 0$ from 2nd mom eqn.

$u_{,x};\ v_{,x};\ u_{,xx};\ v_{,xx} = 0$

since $v_{,y} = 0$
$v = v(x)$ or constant cannot be $f(x)$ since $\frac{\partial v}{\partial x} = 0$ due to partially developed flow.

but $v = 0$ since $v = 0$ at boundary - no slip condition. Using 1st mom eq $\Rightarrow$ w/ $u_{,x} = 0$, $v = 0 \Rightarrow p_{,x} = \mu\nabla^2 u$

but $u_{,xx} = 0$ $\therefore$ $\frac{\partial p}{\partial x} = \mu\frac{\partial^2 u}{\partial y^2}$ since $p_{,y} = 0$ $\frac{\partial p}{\partial x} = \frac{dp}{dx}$ since $u \neq u(x)$ Fully developed flow $\frac{\partial^2 u}{\partial y^2} = \frac{d^2 u}{dy^2}$

$$\therefore \quad \frac{dp}{dx} = \mu\frac{d^2 u}{dy^2} = \text{const}$$

since $p = p(x)$ only $u = u(y)$ only & total differentials.

B.C. $\quad u(\pm a) = 0 \qquad \left.\frac{du}{dy}\right|_{y=0} = 0$

$$\frac{du}{dy} = \frac{y}{\mu}\frac{dp}{dx} + A \qquad A = 0 \text{ due to } y=0,\ \frac{du}{dy} = 0$$

$$u = \frac{y^2}{2\mu}\frac{dp}{dx} + B \qquad B = -\frac{a^2}{2\mu}\frac{dp}{dx}$$

$$\therefore \quad u = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - a^2\right) \qquad \frac{dp}{dx} < 0$$

Prove if $y = \pm a$ $u = 0$ we still get the same equation

$$u_{max} = u_{\mathbb{C}} = u_{\frac{du}{dy}=0} = -\frac{a^2}{2\mu}\frac{dp}{dx} = -\frac{h^2}{8\mu}\frac{dp}{dx} \quad \text{where } 2a = h$$

$$\bar{u}_{mean} = \frac{\int_{-a}^{a} u\,dy}{\int_{-a}^{a} dy} = \frac{\int_0^a u\,dy}{a} = -\frac{1}{2\mu a}\frac{dp}{dx}\int_0^a (a^2-y^2)\,dy = -\frac{1}{2\mu a}\frac{dp}{dx}\left[a^3 - \frac{a^3}{3}\right]$$

$$= -\frac{a^2}{3\mu}\frac{dp}{dx} = \frac{2}{3}u_{\mathbb{C}}$$

Skin friction Coeff

$$C_{f_{\mathbb{C}}} = \frac{\tau_w}{\frac{1}{2}\rho u_{\mathbb{C}}^2} \qquad \tau_w = \mu\left(du/dy\right)_{y=-a} \qquad \therefore\ C_{f_{\mathbb{C}}} = \frac{\mu\left(du/dy\right)_{y=-a}}{\frac{1}{2}\rho u_{\mathbb{C}}^2}$$

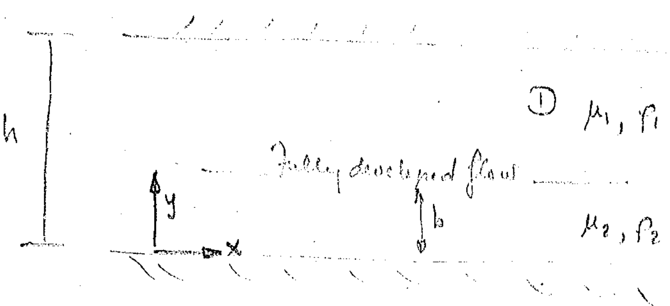
$$\frac{du}{dy} = \frac{y}{\mu}\frac{dp}{dx} \quad \text{at } y=-a \quad -\frac{a}{\mu}\frac{dp}{dx} = \left.\frac{du}{dy}\right|_{y=-a}$$

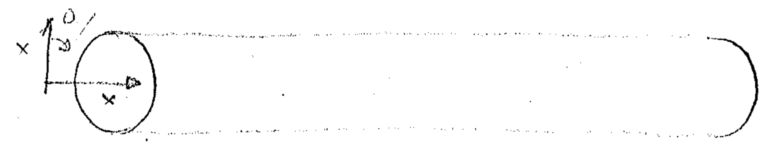
$$C_{f_{\mathbb{C}}} = \frac{-a\ dp/dx}{\frac{1}{2}\rho u_{\mathbb{C}}^2} = \frac{-a\frac{dp}{dx}}{\frac{1}{2}\rho u_{\mathbb{C}}\left(-\frac{a^2}{2\mu}\frac{dp}{dx}\right)} = \frac{4\mu}{\rho u_{\mathbb{C}}\,a} = \frac{4}{R_{N_{a,u_{\mathbb{C}}}}} = \frac{8}{R_{N_{h,u_{\mathbb{C}}}}}$$

$$= \frac{6}{R_{N_{a,\mathbb{C}}}}$$

$u_{\mathbb{C}} = 30$ fps. $\qquad C_{f_{\mathbb{C}}} = 1 \qquad \tau_w = \left(\frac{1}{2}\rho u^2\right)C_f$

$u_{mean} = 20$ fps $\qquad C_{f_{mean}} = 9/4$





Fully developed flow.

$$\frac{\partial}{\partial x}\left(u_x, u_\theta, u_r\right) = 0$$

Due to symmetry $u_\theta = 0 \quad \frac{\partial}{\partial \theta}(\ \ ) = 0$

Cont. $\quad \frac{\partial u}{\partial x} + \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} = 0 \Rightarrow \frac{\partial u_r}{\partial r} + \frac{u_r}{r} = 0 \qquad \frac{du_r}{dr} = -\frac{u_r}{r}$ since $u \neq u(\theta)$

$u_r = \frac{A}{r} = 0$ since no velocity at radius of tube.

①

Momentum reduces to $\frac{\partial p}{\partial \theta} = 0 \quad \frac{\partial p}{\partial r} = 0 \quad \Rightarrow p = p(x)$.

$$\frac{dp}{dx} = -\mu\left[u_{,rr} + \frac{1}{r}u_{,r}\right] = \frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)$$

$f(x) = f(r) = \text{const}$ at $r = R \quad u = 0$

$r = 0 \quad \frac{du}{dr} = 0$.

$$dr\,\frac{r}{\mu}\frac{dp}{dx} = d\left(r\frac{du}{dr}\right)$$

$\frac{r^2}{2\mu}\frac{dp}{dx} + A = r\frac{du}{dr} \qquad A = 0$ at center of tube $\tau = 0$. but $\tau = \mu\frac{du}{dr}$,

$\frac{r^2 dp}{2\mu dx} = r\frac{du}{dr} \qquad \frac{r^2}{4\mu}\frac{dp}{dx} + B = u \; ; \; \left(\frac{r^2 - R^2}{4\mu}\right)\frac{dp}{dx} = u \qquad \frac{dp}{dx} < 0$

$u_{\mathbb{C}} = \frac{-R^2}{4\mu}\frac{dp}{dx} \qquad u_{mean} = \dfrac{\int_0^R u\,(2\pi r\,dr)}{\int_0^R 2\pi r\,dr}$

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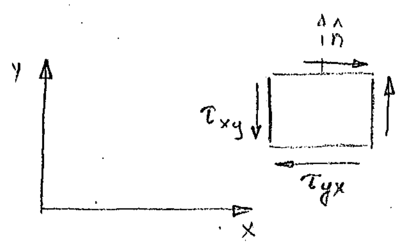
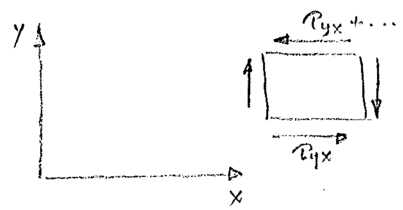


positive as indicated

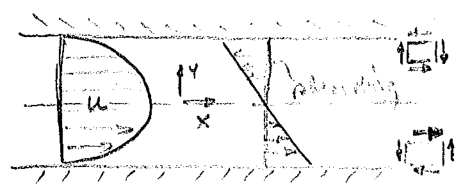
$$\tau = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

τ_{ij} — j denotes direction it acts in; i denotes direction normal to face [?]



negative wrt above

$$\tau = -\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$



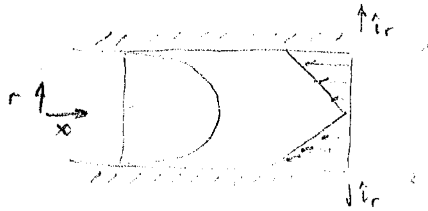
$$u = -\frac{dp}{dx}\frac{1}{2\mu}\left(a^2 - y^2\right)$$

in above $\frac{\partial v}{\partial x} = 0$ & $u \neq u(x) \quad \therefore \quad \tau = \mu\frac{du}{dy}$

$\tau = y\frac{dp}{dx} = \mu\frac{du}{dy}$ shear acts in same direction $\quad \tau \sim y$

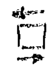
$$\tau_w = C_f\, q = C_f\,\frac{\rho u^2}{2} \qquad C_f\, u^2 = C_{f_{\mathbb{C}}}\, u_{\mathbb{C}}^2 = C_{f_m}\, u_m^2$$

$$u_{\mathbb{C}} = \frac{a^2}{2\mu}\frac{dp}{dx} \qquad C_{f_{\mathbb{C}}} = \frac{4\mu}{\rho u_{\mathbb{C}} a} \qquad \bar{u}_{mean} = \frac{2}{3}u_{\mathbb{C}}$$



$$u = -\frac{R^2}{4\mu}\frac{dp}{dx}\left[1-\left(\frac{r}{R}\right)^2\right]$$

$$\frac{du}{dr} = \frac{r}{2\mu}\frac{dp}{dx} \qquad \tau = \frac{r}{2}\frac{dp}{dx}$$



$$\bar{u}_{mean} = \frac{\int_0^R u\,(2\pi r)\,dr}{\int_0^R 2\pi r\,dr} = \frac{\int_0^{2\pi}\int_0^R r u\,dr\,d\theta}{\int_0^{2\pi}\int_0^R r\,dr\,d\theta} = \frac{\int_0^{2\pi} u\,d\theta \int_0^R r\,dr}{\int_0^{2\pi} d\theta \int_0^R r\,dr}$$

$$= \frac{\int_0^R u r\,dr}{R^2/2} = \frac{1}{R^2/2}\int_0^R -\frac{R^2}{4\mu}\frac{dp}{dx}\left[1-\left(\frac{r}{R}\right)^2\right] r\,dr$$

$$= -\frac{R^2}{2\mu}\frac{dp}{dx}\left[\frac{1}{2}-\frac{1}{4}\right] = -\frac{R^2}{8\mu}\frac{dp}{dx} = \frac{1}{2}u_{℄}$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho u^2} = \frac{\mu\, du/dr}{\frac{1}{2}\rho u^2}$$

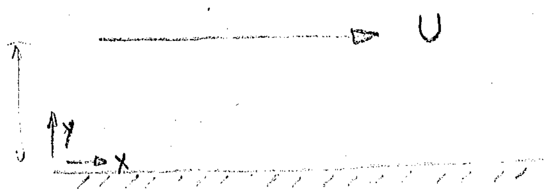
$$C_{f_{℄}} = \frac{8}{R_{N\,R,℄}} \qquad C_{f_{mean}} = \frac{16}{R_{N\,R,\bar{u}_m}}$$

valid until $R_{N_{crit}} = \dfrac{\bar{u}_{mean}\cdot R}{\nu} = 1150$

if $R_N > R_{N_{crit}}$ go to turbulent flow.

Example:

Couette Flow



neglect end effects $\frac{\partial}{\partial x} u_i \to 0$

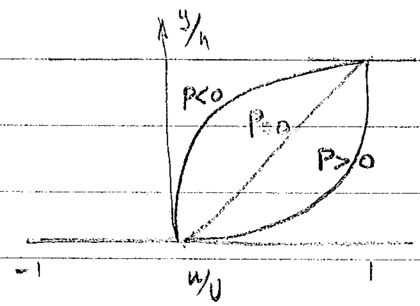
$$\mu\frac{d^2u}{dy^2} = \frac{dp}{dx}$$

$$\frac{1}{\mu}\frac{dp}{dx} = \frac{d^2u}{dy^2}$$

$$\frac{du}{dy} = \frac{y}{\mu}\frac{dp}{dx} + A \qquad \begin{matrix} y=0 & u=0 \\ y=h & u=U \end{matrix}$$

$$u = \frac{y^2}{2\mu}\frac{dp}{dx} + Ay + B \qquad B \to 0$$

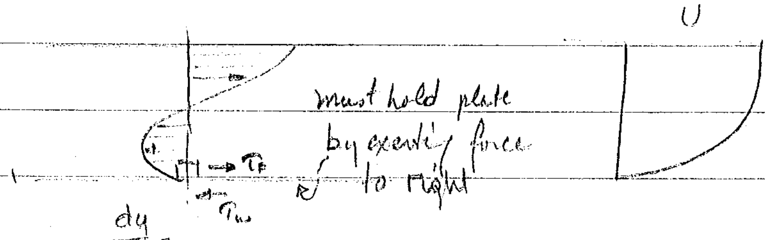
$$U = \frac{h^2}{2\mu}\frac{dp}{dx} + Ah \qquad A = \frac{U}{h} - \frac{h}{2\mu}\frac{dp}{dx} \qquad u = \frac{Uy}{h} + \frac{y}{2\mu}\frac{dp}{dx}(y-h)$$



$$u/U = y/h + P\frac{y}{h}\left(1 - y/h\right)$$

$$P = -\frac{dp}{dx}\frac{h}{U \cdot 2\mu}$$

if favorable pressure gradient p (+)



$\left(\frac{du}{dy}\right)_{y=h} = 0$ no force exerted on upper plate

$$\frac{dy}{dy} = \quad \frac{dU^*}{dy^*} = \frac{d(u/U)}{d(y/h)} = 1 + P\left(1 - 2\,y/h\right) \quad \text{stress}$$

$0 = 1 + P(1-2)$ $\quad P = 1$ at upper plate

$0 = 1 + P$ $\quad P = -1$ at lower plate

} where no force need be exerted on plate

Rayleigh Problem 1851 $\quad$ Continuity equation $\rightarrow \rho VA = \rho VA$ $\therefore$ since V = cons

mom says $u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$

$\frac{\partial u}{\partial x} = 0$ since u = const $\Rightarrow \frac{\partial p}{\partial x} = 0$ p = const

$t \le 0 \quad y = 0 \quad u = 0$

$t > 0 \quad y = 0 \quad u = U$

$y = \infty \quad u = 0$

pressure = const since no mechanism exists to form a Δp in x direction ~~since infinite plate~~ $\cancel{\frac{\partial}{\partial x} = 0}$

Also assume $u = f(y, t)$ we can show $v \equiv 0$ everywhere using continuity

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad \text{fully developed flow}$$

$$\frac{\partial u}{\partial t} = \nu\frac{\partial^2 U}{\partial y^2} \qquad u = U\,T(t)\,Y(y)$$

$$U\,Y(y)\,T'(t) = \nu\,U\,T(t)\,Y''(y) = \text{const.}$$

$$\frac{T'}{\nu t} = \frac{Y''(y)}{Y} = \text{const}$$

$$\frac{u}{U} = e^{\nu k t}\left(C_1 e^{\sqrt{k}\, y} + C_2 e^{-\sqrt{k}\, y}\right) \qquad k > 0$$

$$= e^{\nu k t}\left(C_3 \sin\sqrt{k}\, t + C_4 \cos\sqrt{k}\, t\right) \qquad k < 0$$

$$= C_5\, y + C_6 \qquad k = 0$$

none of solution, satisfy the BC

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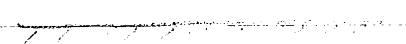
Nov. 10 Exam. (2 hr closed book exam)



$u_{y=0} = 0 \quad t = 0$

$u_{y=0} = U \quad t > 0 \qquad u(y,t)$

$u_{y=\infty} = 0$

$v = 0$ in flow field $\qquad u_{,t} = \nu\, u_{,yy}$. Second moment eq is satisfied identically

$u(y,t) = U\, T(t)\, Y(y)$ assumed as separation of variable.

use $u = U f(\eta) \qquad \eta = k y t^m$

$$\frac{\partial u}{\partial t} = \frac{\partial \eta}{\partial t}\frac{\partial u}{\partial \eta} = U\frac{df}{d\eta} = U k y m t^{m-1}\, \frac{\partial f}{\partial \eta} \qquad m \,\&\, k \text{ are arbitrary}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y} = U\frac{df}{d\eta} = U k t^m \frac{\partial f}{\partial \eta}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y}\right) = \frac{\partial}{\partial \eta}\left(\frac{\partial u}{\partial y}\right)\frac{\partial \eta}{\partial y} = U\frac{d^2 f}{d\eta^2}\, k^2 t^{2m}$$

$$\frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial y^2} \qquad U\frac{df}{d\eta}\underbrace{k y}_{\eta / t^m}\, m t^{m-1} = \nu\, U\frac{d^2 f}{d\eta^2}\, k^2 t^{2m}$$

$$\frac{d^2 f}{d\eta^2} - \frac{\eta m t^{m-1}}{\nu k^2 t^{3m}}\frac{df}{d\eta} = 0$$

$$\frac{d^2 f}{d\eta^2} - \frac{\eta m}{\nu k^2 t^{2m+1}}\frac{df}{d\eta} = 0 \qquad \text{let } 2m+1 = 0 \qquad m = -1/2$$

$$\frac{d^2 f}{d\eta^2} + \frac{\eta}{2\nu k^2}\frac{df}{d\eta} = 0$$

Let $4\nu k^2 = 1$ $\quad k^2 = 1/4\nu \quad k = \frac{1}{2\sqrt{\nu}}$

$$\frac{d^2f}{d\eta^2} + 2\eta\frac{df}{d\eta} = 0 \qquad \eta = \frac{y t^{-1/2}}{2\sqrt{\nu}} = \frac{y}{2\sqrt{\nu t}} \qquad u = U f(\eta)$$

$$f'' + 2\eta f' = 0 \qquad \ln f' = -\eta^2 + \ln A$$

$$f' = Ae^{-\eta^2}$$

$$\boxed{f = A\int_0^\eta e^{-\eta^2}\,d\eta + B} \qquad \begin{matrix} \eta = 0 & f = 1 & t > 0 & u_{y=0} = U \\ \eta = \infty & f = 0 & & y = \infty \quad U = 0 \end{matrix}$$

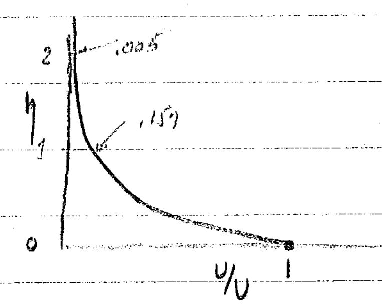
$$0 = A\int_0^\infty e^{-\eta^2}\,d\eta + 1 \qquad A = \frac{-1}{\int_0^\infty e^{-\eta^2}\,d\eta} \qquad \frac{2}{\sqrt{\pi}}\int_0^\eta e^{-\eta^2}\,d\eta = \mathrm{erf}(\eta)$$

$$A = -\frac{2}{\sqrt{\pi}}$$

$$f = 1 - \frac{2}{\sqrt{\pi}}\int_0^\eta e^{-\eta^2}\,d\eta$$

$$\frac{u}{U} = 1 - \frac{2}{\sqrt{\pi}}\int_0^\eta e^{-\eta^2}\,d\eta = 1 - \mathrm{erf}\,\eta = \mathrm{erfc}\,\eta$$ complementary

η	$\mathrm{erf}\,\eta$	u/U
0	0	1
.5	.521	.479
1.0	.843	.157
1.5	.966	.034
2.0	.995	.005
2.5	.9996	.0004



Consider flow over plate of oil $\quad \nu = \frac{1}{6380} \approx \frac{1}{6400} \quad \eta = \frac{40y}{\sqrt{t}}$

6" $\quad$ find t $\quad \frac{u}{U} = \frac{1}{2}\%$ $\quad \eta = 2 \quad \sqrt{t} = \frac{40y}{\eta} = 10$

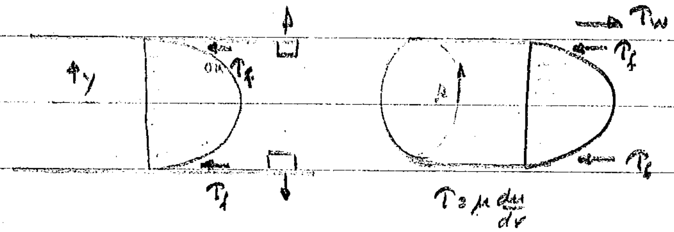
$t = 100$

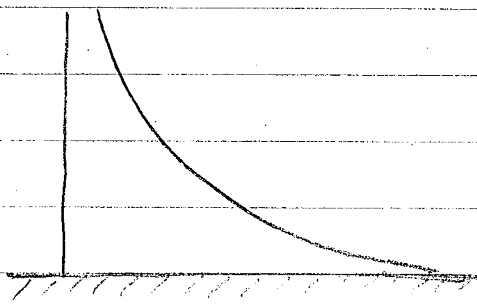
$$\nu_{H_2O}\, t_{H_2O} = \nu_{Air}\, t_{Air} \qquad \frac{t_{H_2O}}{t_{air}} = \frac{\nu_{air}}{\nu_{H_2O}} = 14.65$$

$t_{H_2O} = 1465$ sec. $t_{oil} = 3.72$ sec.

2-D $\tau = \mu \frac{du}{dy}$

<u>Sidelight</u>





Boundary Layer thickness δ for plate moving & fluid stationary | $\eta = \frac{y}{2\sqrt{\nu t}}$ $2\eta\sqrt{\nu t} = \delta$

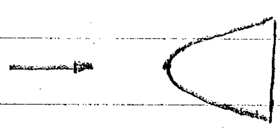
$y = \delta$ $\frac{u}{U} = .01$ $\eta = 1.8$

$\delta = 2\eta_\delta\sqrt{\nu t} = 3.6\sqrt{\nu t}$

$$\tau_w = -\mu \frac{du}{dy} = -\mu \frac{\partial f}{\partial \eta}\left(\frac{\partial \eta}{\partial y}\right) = -\mu U\left(\frac{-2}{\sqrt{\pi}}\right)\left(\frac{1}{2\sqrt{\nu t}}\right)$$

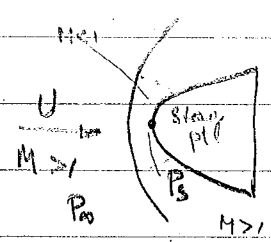
$\tau_w = U\sqrt{\frac{\rho\mu}{\pi t}}$ as $t \to$ $\tau_w \downarrow$

<u>Stagnation Point</u> (2-D) - Hiemenz Flow.



relative velocity = 0

$P = p + \frac{1}{2}\rho V^2$ $M \approx 0$

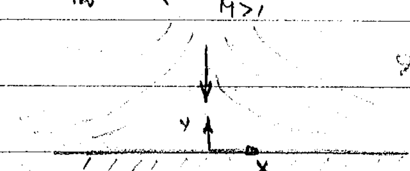


Similar flow about sp except U and diff

$P_s < P_\infty$

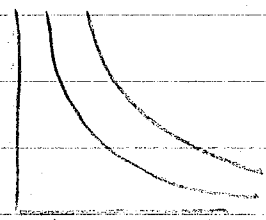
Shock Equation $\Delta P < 0$ $\Delta S > 0$

Impossible Expansion $\Delta P > 0$ $\Delta S < 0$



Inviscid flow sln. $U = ax$ $V = -ay$ at $y = 0$ $U = U(x)$

Stag point press $p_0 - p = \frac{1}{2}\rho(U^2 + V^2) = \frac{1}{2}a^2\rho(x^2 + y^2)$



Same as mirror corner flow.

With viscosity $u, v = 0$ at wall (no slip

$u = x f(y)$ for $y > 0$ $\quad v = -f(y)$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ must be satisfied

$p_0 - p = \frac{1}{2} a^2 \rho (x^2 + F(y))$ must satisfy potential flow outside region of wall.

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \nu [\nabla^2 u]$$

$$x f'(y) \cdot f(y) - f(y) f''(y) x = -\frac{1}{\rho}[-a^2 \rho x] + \nu [0 + x f'']$$

$$f'^2 - f f'' = a^2 + \nu f'''$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{dp}{dy} + \nu [\nabla^2 v]$$

$$x f'(0) + f f' = +\frac{1}{\rho}(\tfrac{1}{2} a^2 \rho) F' - \nu f'' \qquad \text{since } -\frac{\partial p}{\partial y} = \tfrac{1}{2} a^2 \rho F'$$

$$f f' = \tfrac{1}{2} a^2 F' - \nu f''$$

Potential flow $\nabla^2 \varphi = 0$ where $u = \frac{\partial \varphi}{\partial x}$, $v = \frac{\partial \varphi}{\partial y}$

Consider $\varphi = a x^n + b y^n$ to satisfy Laplace $\Rightarrow \varphi = a(x^n - y^n)$

$\therefore \quad u = a n x^{n-1} \qquad v = -a n y^{n-1}$

when $n = 1 \qquad u = -v = \text{const}$

$n = 2 \qquad u = 2ax = \psi_{,y} \quad (1)$

$v = -2ay = -\psi_{,x} \quad (2)$

(1) $\psi = 2axy + f_1(x)$

(2) $\psi = 2axy + f_2(y)$ thus $f_1(x) = f_2(y) = \text{const.}$

$\therefore \quad \psi = 2axy + \text{const}$

Body located on $\psi = 0 \quad \therefore \psi_b = 0 = 2axy + \text{const}$

$\psi - \text{const} = \bar{\psi} = 2axy$ or body is on $\bar{\psi} = 0 = 2axy$

satisfied $\forall_y \; x = 0 \;;\; \forall_x \; y = 0$

Streamlines are $\bar{\psi} = 2axy = \text{const.}$

$\therefore$ y streamline $= \dfrac{\text{const}}{2ax\rho}$

Get pressure by: $p_0 = p + \frac{1}{2}\rho(u^2+v^2)$

But potential theory says that $u = 2ax$ - so that no slip condition at $y=0$ is not satisfied:

Potential flow soln
$$p = p_0 - \tfrac{1}{2}\rho(u^2+v^2)$$
$$u = 2ax$$
$$v = -2ay$$
$$\psi = 2axy$$

In viscous problem - assume $\psi = x f(y)$ as $f \to 2a$, $y \to \infty$

continuity $u_{,x} + v_{,y} = 0 = (\psi_{,y})_{,x} + (\psi_{,x})_{,y} = 0$
$$= (xf')_{,x} - f'$$
$$= f' - f' = 0$$

Thus $u = \psi_{,y} = xf'$
$v = -f$

x mom eq $\quad u\, u_{,x} + v u_{,y} = -\frac{1}{\rho} p_{,x} + \nu(u_{,xx} + u_{,yy})$
$$x f'^2 + (-f)\, x f'' = -\frac{1}{\rho} \frac{p_{,x}}{x} + \nu(0) + \nu x f'''$$
$$f'^2 - ff'' = -\frac{1}{\rho}\frac{p_{,x}}{x} + \nu f'''$$

y mom eq:
$$u\, v_{,x} + v\, v_{,y} = -\frac{1}{\rho} p_{,y} + \nu(v_{,xx} + v_{,yy})$$
$$x f'(0) + (-f)(-f') = -\frac{1}{\rho} p_{,y} + \nu(0) + \nu(-f'')$$
$$ff' = -\frac{1}{\rho} p_{,y} - \nu f''$$

$$\nu f''' + ff'' - f'^2 = \frac{1}{\rho} p_{,x}/x$$

$$\nu f'' + ff' = -\frac{1}{\rho} p_{,y}$$

$$p_{,x} = p_{o,x} - \tfrac{1}{2}\rho\,(2u u_{,x} + 2v v_{,x})$$
$$= -\tfrac{1}{2}\rho\,(2x f'^2 + 2v \cancel{v_{,x}}^{\,0}) = -\rho x f'^2$$
$$p_{,y} = -\tfrac{1}{2}\rho(2u u_{,y} + 2v v_{,y})$$
$$= -\tfrac{1}{2}\rho\,(2xf'(xf'') + 2ff') = -\rho(x^2 f'f'' + ff')$$

Combine to get equat. i terms of f.

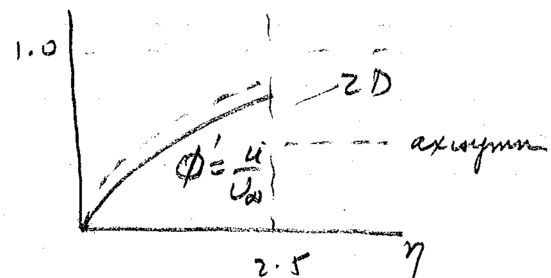
Introduce a stretching coordinate: $\eta = \sqrt{\frac{1}{\nu}}\, y$

$\therefore\ f(y) = \sqrt{\nu}\ \phi(\eta)$

$\therefore\ \phi''' + \phi\phi'' - \phi'^2 + 1 = 0$

$\phi = \phi' = 0$ at $y = 0$ ($u = v = 0$ at wall)

$\phi = 1$ at $\eta \to \infty$



$\delta = 2.5\sqrt{\nu}$ (small

In viscinity of stagnation pt. b.l. thickness is const.

x - mom.

$$\frac{Du}{Dt} = -\frac{1}{\rho}\,p_{,x} + \nu\nabla^2 u$$

Non-dimensionalize x, y, z wrt to l $\quad p$ wrt p_∞

u, v, w wrt U

t wrt l/U

Thus non dimensionalizing:

$$\frac{D\bar{u}}{Dt} = -\frac{p_\infty}{\rho U^2}\,\frac{d\bar{p}}{d\bar{x}} + \frac{\nu}{Ul}\,\nabla^2\bar{u} \qquad 1/R = \frac{\text{viscous}}{\text{inertia}}$$

Eckert

Observation

$R_N \to \infty$ $(\nu = 0)$ Inviscid flow (Euler Eq)

$R_N \to 0$ (ν large) Creeping flow (viscous eff >> inertia effects)
Stokes' flow

For Stokes' Flow – very slow motion:

$$0 = -p_{,x} + \mu \nabla^2 u \quad (1)$$

$$0 = -p_{,y} + \mu \nabla^2 v \quad (2)$$

$$0 = -p_{,z} + \mu \nabla^2 w \quad (3)$$

Diff $\frac{\partial(1)}{\partial y} - \frac{\partial(2)}{\partial x} \Rightarrow \nabla^4 \psi = 0$ for planar creeping flow

10-29-71

Creep Flow - inertia terms << viscous terms   in 3-D:

① $p_{,x} = \mu \nabla^2 u$ ② $p_{,y} = \mu \nabla^2 v$ ③ $p_{,z} = \mu \nabla^2 w$ $\Rightarrow R_N << 1$

$$\frac{\partial}{\partial x}① + \frac{\partial}{\partial y}② + \frac{\partial ③}{\partial z} = \nabla^2 p = 0 \qquad \text{\& ④ } u_{,x} + v_{,y} + w_{,z} = 0$$

2-D case $\frac{\partial p}{\partial x} = \mu \nabla^2 u$ ⑤ $w = 0$

$p_{,y} = \mu \nabla^2 v$ ⑥ $w = 0$

define $\psi = \psi(x,y)$ $u = \frac{\partial \psi}{\partial y}$ $v = -\frac{\partial \psi}{\partial x}$

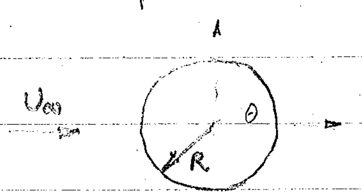
⑦ $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ is then satisfied

$$\text{diff ⑤ \& ⑥ } p_{,xy} - p_{,yx} = 0 = \left[\frac{\partial^3 u}{\partial x^2 \partial y} + \frac{\partial^3 u}{\partial y^3}\right] - \left[\frac{\partial^3 v}{\partial x^3} + \frac{\partial^3 v}{\partial y^2 \partial x}\right] = 0$$

$$\psi_{,xxyy} + \psi_{,yyyy} + \psi_{,xxxx} + \psi_{,xxyy} = 0 \quad \text{or} \quad \nabla^2(\nabla^2 \psi) = \nabla^4 \psi = 0$$

in polar coordinates or cartesian coordinates.

Quasi 2-D problem



$$v_r = \frac{1}{r^2 \sin\theta}\frac{\partial \psi}{\partial \theta}$$

$$v_\theta = -\frac{1}{r\sin\theta}\frac{\partial \psi}{\partial r} \qquad v_\varphi = 0$$

$$\frac{\partial}{\partial \theta} M_r - \frac{\partial}{\partial r} M_\theta = 0 \qquad p_{,r\theta} - p_{,\theta r} = 0 \qquad E^4 \psi = 0$$

$$E^2 = \left[\,{}_{,rr} + \frac{1}{r^2}\,{}_{,\theta\theta} - \frac{\cot\theta}{r^2}\,{}_{,\theta}\right]$$

$$E^4 \psi = [\quad]^2 \psi = \psi_{,rrrr} + \frac{1}{r^4}\psi_{,\theta\theta\theta\theta} + \frac{\cot^2\theta}{r^4}\psi_{,\theta\theta}$$

$$+ \frac{\partial^2}{\partial r^2}\left(\frac{1}{r^2}\psi_{,\theta\theta}\right) + \frac{1}{r^2}\psi_{,rr\theta\theta} + \dots$$

at $r = R$ $v_r = v_\theta = 0$ $\frac{\partial \psi}{\partial \theta} = 0 = \frac{\partial \psi}{\partial r}$ no slip conditions

at $r \to \infty$ $v_r = U_\infty \cos\theta$ $v_\theta = -U_\infty \sin\theta$ $\left\{ \begin{array}{l} u = U_\infty \\ v = 0 \\ w = 0 \end{array} \right.$

ψ is to be used for inviscid case at ∞. $\psi_{INV} = \frac{1}{2} U_\infty r^2 \sin^2\theta$

to solve no slip cond'n viscous flow $\psi_{VIS} = \frac{1}{2} U_\infty f(r) \sin^2\theta$

at any theta ψ varies with r since conditions at diff r change.

put ψ into $E^4\psi = 0$ to get $f(r)$. velocity profile

This gives 4th order total diff eq:

$$f^{(4)} - \frac{2}{r} f^{(3)} - 2\left(f/r^2\right)'' + \frac{4}{r^4} f = 0$$

$$\left(\frac{d^2}{dr^2} - \frac{2}{r^2}\right)^2 f(r) = 0 \qquad \text{Assume } Cr^n$$

$A\cos nr$

let $Cr^n = f$ $\quad C\left[n(n-1)(n-2)(n-3)r^{n-4} - 2n(n-1)r^{n-4} - 2(n-2)(n-3)r^{n-4} + 4r^{n-4}\right] = 0$

for non triv sol C & $r^{n-4} \neq 0$

$$n(n-1)(n-2)(n-3) - 2n(n-1) - 2(n-2)(n-3) + 4 = 0$$

$$\left[n(n-1) - 2\right]\left[(n-2)(n-3) - 2\right] = 0$$

$n = 2 \quad n = 4$

$n = -1 \quad n = 1$

$$\psi = \frac{U_\infty}{2}\left[\frac{A}{r} + Br + Cr^2 + Dr^4\right]\sin^2\theta$$

$$v_r = U_\infty\cos\theta = \frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta} = \frac{1}{r^2\sin\theta}\frac{U_\infty}{2}\left[\frac{A}{r} + Br + Cr^2 + Dr^4\right]2\sin\theta\cos\theta$$

$$v_r = U_\infty\left[\frac{A}{r^3} + \frac{B}{r} + C + Dr^2\right]\cos\theta \qquad D = 0$$

$C = 1$

$$v_\theta = -U_\infty\sin\theta = -\frac{1}{r\sin\theta}\frac{U_\infty}{2}\left[-\frac{A}{r^2} + B + 2Cr + 4Dr^3\right]\sin^2\theta$$

1 0

$$= -U_\infty\left[-\frac{A}{2r^3} + \frac{B}{2r} + C + \frac{Dr^2}{2}\right]\sin\theta \Rightarrow C = 1$$

$$\left(\frac{\partial\psi}{\partial r}\right)_{r=R} = \left[-\frac{A}{R^2} + B + 2R\right]\frac{U_\infty}{2}\sin^2\theta = 0$$

$$\left(\frac{\partial\psi}{\partial\theta}\right)_{r=R} = 0 \quad \left[\frac{U_\infty}{2}2\sin\theta\cos\theta\right]\left[\frac{A}{R} + BR + R^2\right] = 0$$

$A = \frac{R^3}{2} \qquad B = -\frac{3R}{2}$

$$v_r = U_\infty \cos\theta \left[\frac{1}{2}\left(\frac{R}{r}\right)^3 - \frac{3}{2}\left(\frac{R}{r}\right) + 1 \right]$$

$$v_\theta = -U_\infty \sin\theta \left[-\frac{1}{4}\left(\frac{R}{r}\right)^3 - \frac{3}{4}\left(\frac{R}{2}\right) + 1 \right]$$

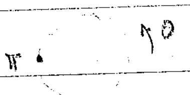
From momentum

$$\frac{\partial p}{\partial r} = \frac{3\mu R U_\infty}{r^3} \cos\theta$$

$$\frac{\partial p}{\partial \theta} = \frac{3\mu R U_\infty}{2r^2} \sin\theta$$

$$p = p_\infty - \frac{3\mu R U_\infty \cos\theta}{2r^2} \qquad \text{at stag pt } \theta = \pi$$

$$p_s - p_\infty = \frac{3\mu U_\infty}{2R}$$



$$(p_s - p_\infty)_{INV} = \frac{1}{2}\rho U_\infty^2 \qquad \frac{(p_s - p_\infty)}{(p_s - p_\infty)_{INV}} = \frac{\frac{3\mu U_\infty}{2R}}{\frac{\rho U_\infty^2}{2}} = \frac{3\mu}{\rho U_\infty R}$$

$$= \frac{6}{R_{N_{U_\infty, D}}} = \frac{3}{R_{N_{U_\infty, R}}}; \qquad R_N < 1$$

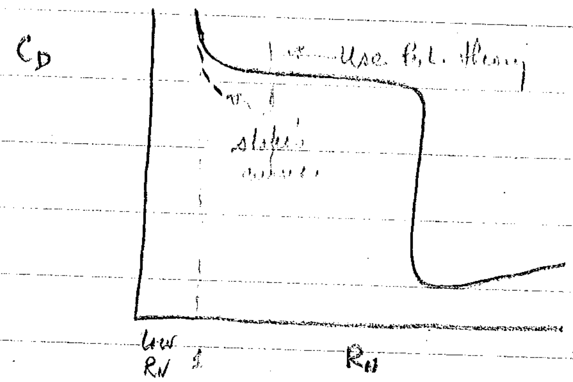
get fairly high pressures acting on body at stag pt.

• $$D = \int (p - p_\infty) \cos\theta \, dA + \int \tau_{r\theta} \sin\theta \, dA$$

$\tau_{\theta\varphi}$ & $\tau_{\varphi r} = 0$ since $w = 0$ $\qquad dA = 2\pi R^2 \sin\theta \, d\theta$

$$D = \underbrace{2\pi R \mu U_\infty}_{\text{Pressure Drag}} + \underbrace{4\pi R \mu U_\infty}_{\text{Viscous drag}} = 6\pi R \mu U_\infty$$

$$C_D = \frac{D}{\frac{1}{2}\rho U_\infty^2 \pi R^2} = \frac{6\pi R \mu U_\infty}{\frac{1}{2}\rho U_\infty^2 \pi R^2} = \frac{12\mu}{\rho U_\infty R} = \frac{12}{R_{N_{U_\infty, R}}} = \frac{24}{R_{N_{U_\infty, D}}}$$



Sphere

difficulties of Stokes soln in vicinity of body $\frac{\text{Inertia}}{\text{Viscous}} < 1$

Oseen Flow 1911 keep inertia forces in

U_∞

$u = U_\infty + u'$
$v = v'$ ' perturbation veloc.
$w = w'$ $u' < U_\infty$

solution breaks down near body but satisfied outside.

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(u_{,xx} + u_{,yy} + u_{,zz}\right)$$

neglect second order terms.

$$U_\infty \frac{\partial u'}{\partial x} + O(u')^2 + O(u')^2 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu[\nabla^2 u']$$

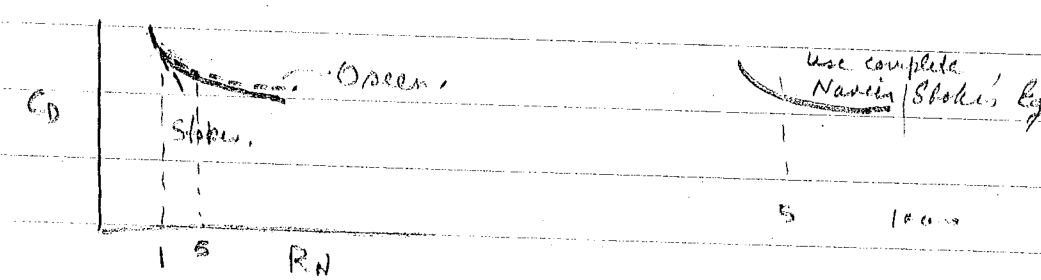
$$U_\infty \frac{\partial v'}{\partial x} = \cdots + \text{pert veloc.} = -\frac{1}{\rho}p_{,y} + \nu[\nabla^2 v']$$

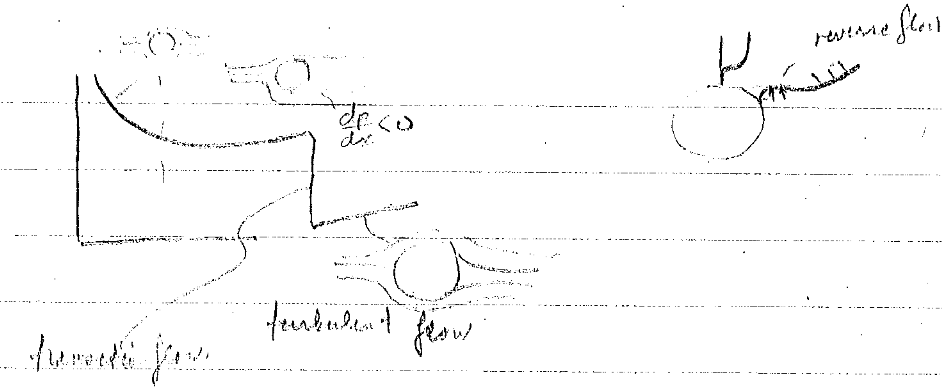
$$U_\infty \frac{\partial w'}{\partial x} = -\frac{1}{\rho}p_{,z} + \nu\nabla^2 w'$$

$$u'_{,x} + v'_{,y} + w'_{,z} = 0 \quad \text{Continuity.}$$

Solns of this give

$$C_D = \frac{24}{R_N}\left(1 + \frac{3}{16}R_N\right)$$





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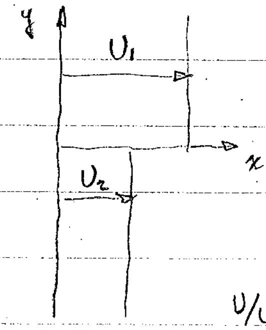


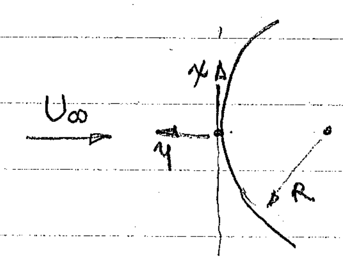
$p = \text{const}$

$\frac{\partial v}{\partial y} = 0$ $\qquad v = 0$ at ∞

$u = u(y,t)$

$\operatorname{erf}\eta = -\operatorname{erf}-\eta$

$$U/U_1 = \tfrac{1}{2}\left[1+\operatorname{erf}\eta\right]\left(1-U_1/U_2\right) + U_1/U_2$$



$$\psi = -U_\infty\, r \sin\theta\left(1-\frac{R^2}{r^2}\right)$$

$$v_r = -U_\infty \cos\theta\left(1-\frac{R^2}{r^2}\right)$$

$$v_\theta = U_\infty \sin\theta\left(1+\frac{R^2}{r^2}\right)$$

in vicinity of stag pt $\qquad y = r - R$

$\sin\theta \sim \theta$ $\qquad V = v_r$

$\cos\theta \sim 1$

$$-U_\infty\left(1-R^2/(R+y)^2\right)$$

$$= -U_\infty\left[\frac{R^2+2yR+y^2-R^2}{(R+y)^2}\right]$$

since $y \ll 1$ $\quad = -U_\infty\left(\frac{2yR}{R^2}\right) = -\frac{U_\infty 2y}{R}$

$\delta = 2.4\sqrt{\frac{\nu}{a}}$

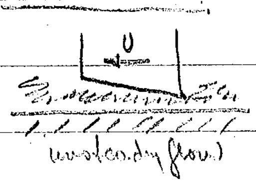
$U_\infty = 100$ ft/sec

$R = 1 \qquad \delta = .00215$ ft

2 Hr Closed Book

for $\eta = 2.4$ $\quad u/U_\infty = .9905$ which defines δ

Lubrication: Reynolds 1886



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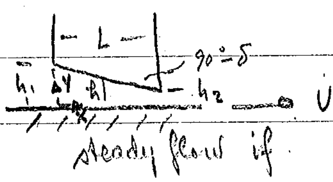
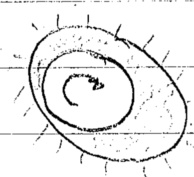


for creep flow forces on sphere $\dfrac{(p_s - p_\infty)_{\psi}}{(p_s - p_\infty)_{\nu}} = \dfrac{3}{R_N}$



$\dfrac{h}{L} << 1$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \frac{U}{L}\frac{\partial u/U}{\partial x/L} + \frac{U}{h}\frac{\partial v/U}{\partial y/h} = 0$$

since δ is small $h_1, h_2 \sim h$

$$\frac{\partial(u/U)}{\partial(x/L)} + \frac{L}{h}\frac{\partial(v/U)}{\partial(y/h)} = 0$$ from order of magnitude analysis $v/U = O(h/L)$

$\therefore u >> v$

$$u\frac{\partial u}{\partial x} + \cancel{v\frac{\partial u}{\partial y}} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(u_{,xx} + u_{,yy}\right)$$

Second momentum eq will lead to $p_{,y} = 0$ since $u >> v$

$$\frac{\dfrac{\partial^2 u}{\partial x^2}}{\dfrac{\partial^2 u}{\partial y^2}} = \frac{\dfrac{U}{L^2}\dfrac{\partial^2 u/U}{\partial(x/L)^2}}{\dfrac{U}{h^2}\dfrac{\partial^2 u/U}{\partial(y/h)^2}} \sim \left(\frac{h}{L}\right)^2 \quad \therefore \frac{\partial^2 u}{\partial x^2} << \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\rho u \dfrac{\partial u}{\partial x}}{\mu \dfrac{\partial^2 u}{\partial y^2}} = \frac{\text{Inertia}}{\text{Viscous}} = \frac{\rho \dfrac{uU}{UL}\dfrac{\partial u/U}{\partial x/L}}{\mu \dfrac{U}{h^2}\dfrac{\partial^2 u/U}{\partial(y/h)^2}} \sim \frac{\rho U}{\mu L}h^2 = \left(\frac{\rho U L}{\mu}\right)\frac{h^2}{L^2} = R^* << 1$$

since $u/U \sim O(1)$

$$R^{*} = \left(\frac{UL}{\nu}\,\frac{h^2}{L^2}\right) < 1$$

$p_{,y} = 0$ since $u \gg v$ $\quad L \gg h$

$$u = U(1 - y/h) - \frac{h^2}{2\mu}\,\frac{dp}{dx}\,(y/h)\left[1 - y/h\right]$$

$$Q = \int_0^h u\,dy = \text{const} \qquad \text{at low } R_N$$

$$Q = \frac{Uh}{2} - \frac{h^3}{12\mu}\,\frac{dp}{dx} \qquad\qquad \frac{dp}{dx} = \left(\frac{Uh}{2} - Q\right)\frac{12\mu}{h^3}$$

$$h = h_1 - \delta x \qquad dh = -\delta dx \qquad Q = U\,\frac{h_1 h_2}{h_1 + h_2}$$

$$p = p_0 + \frac{6\mu UL}{h^2}\,\frac{(h_1 - h)(h - h_2)}{h_1^2 - h_2^2}$$

$$P = \int_0^L (p - p_0)\,dx = \frac{6\mu UL^2}{h_2^2 (K-1)^2}\left[\ln K - \frac{2(K-1)}{K+1}\right] \qquad K = h_1/h_2$$

$$L^2/h_2^2 \gg 1 \qquad D = \int_0^L -\mu \left.\frac{\partial u}{\partial y}\right|_0 dx = \frac{\mu UL}{(K-1)h_2}\left[4\ln K - 6\,\frac{(K-1)}{K+1}\right]$$

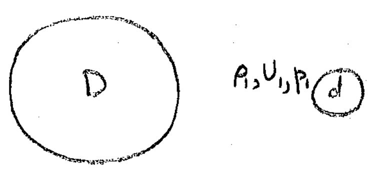
11-24-71

$h_1/h_2 = 2.2$ $\quad P = P_{max}$

Final 2 HR exam during finals week 3 HR exam if possible on lubrication,

Consider

ρ_∞, U_∞
p_∞



similarity flow

Ass: bodies are geometrically similar

flow is steady $\partial/\partial t = 0$, incompressible flow $\rho = c$

$$\rho\left[(\bar{q}\cdot\nabla)u\right] = -p_{,x} + \mu\nabla^2 u$$

Nondimensionalize: $\bar{x} = \frac{x}{D}$ $\quad \bar{y} = \frac{y}{D}$ $\quad \bar{z} = z/D$ $\qquad \bar{u} = \frac{u}{U_\infty}$ $\quad \bar{v} = \frac{v}{U_\infty}$ $\quad \bar{w} = \frac{w}{U_\infty}$

$$\bar{p} = \frac{p}{2q_\infty} = \frac{p}{\rho_\infty U_\infty^2}$$

for large ball

$$\frac{\partial}{\partial x} = \frac{1}{D}\frac{\partial}{\partial \bar{x}} \qquad \frac{\partial^2}{\partial x^2} = \frac{\partial}{\partial x}\left[\frac{1}{D}\frac{\partial}{\partial \bar{x}}\right] = \frac{1}{D^2}\frac{\partial^2}{\partial \bar{x}^2}$$

$\therefore\ u\frac{\partial u}{\partial x}$ can be neglected $\therefore$ if $R^* << 1$

$$\boxed{p_{,x} = \mu\, u_{,yy}}$$

$p = p(x)$ only since $\partial p/\partial y = 0$ $\qquad \frac{1}{\mu}\frac{dp}{dx} = \frac{\partial^2 u}{\partial y^2}$

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{dp}{dx}y + f(x)$$

$$u = \frac{1}{2\mu}\frac{dp}{dx}y^2 + f_1(x)y + f_2(x)$$

at $y=0$ $u=U$

at $y=h$ $u=0$

$$u = U\left[1 - y/h\right] - h^2/2\mu \frac{dp}{dx}(y/h)\left[1 - y/h\right]$$

$$Q = \int_0^h u\,dy = U\frac{h}{2} - \frac{h^3}{12\mu}\frac{dp}{dx}$$ constant due to steady state

$$\frac{-12\mu}{h^3}\left[Q - \frac{Uh}{2}\right] = \frac{dp}{dx} = 12\mu\left[\frac{U}{2h^2} - \frac{Q}{h^3}\right]$$ $\qquad h = h_1 - \delta x$

$$p = \int_0^x 12\mu\left[\frac{U}{2[h_1-\delta x]^2} - \frac{Q}{[h_1-\delta x]^3}\right]dx$$ $\qquad$ at $h = h_1$ $p = p_0$

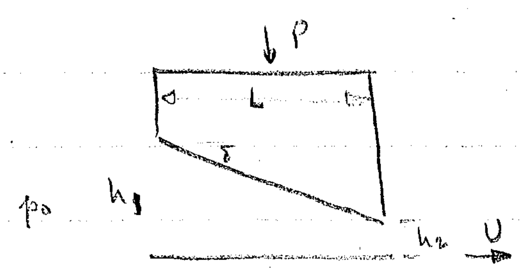
$$p = p_0 + \frac{6\mu U}{\delta}\left[\frac{1}{h_1 - \delta x} - \frac{1}{h_1}\right] - \frac{6\mu Q}{\delta}\left[\frac{1}{(h_1-\delta x)^2} - \frac{1}{h_1^2}\right]$$

at $x = L$ $p = p_0$ $\therefore$ $h_1 - \delta x \to h_1 - \delta L = h_2$

$$U\left(\frac{1}{h_2} - \frac{1}{h_1}\right) = \frac{Q}{1}\left[\frac{1}{h_2^2} - \frac{1}{h_1^2}\right]$$

$$\boxed{Q = \frac{h_1 h_2}{h_1 + h_2}U} \qquad \left[U\frac{h}{2} - Q\right]\frac{12\mu}{h^3} = \frac{dp}{dx}$$

11-19-71



$\frac{dp}{dx} = \mu\frac{\partial^2 u}{\partial y^2}$ $\quad p = p_0$ at X_0 $h = h_1$ $x = L$ $h = h_2$

$u = U$ $y = 0$ $\quad u = 0$ $y = h$

$$\rho u \frac{\partial u}{\partial x} = \rho_\infty \frac{U_\infty^2}{D} \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} = \frac{\rho_\infty U_\infty^2}{D} \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}}$$

$$\rho v \frac{\partial u}{\partial y} = \rho_\infty \frac{U_\infty^2}{D} \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{\rho_\infty U_\infty^2}{D} \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}}$$

$$\frac{\partial p}{\partial x} = \frac{\rho_\infty U_\infty^2}{D} \frac{\partial \bar{p}}{\partial \bar{x}} \qquad \frac{\partial^2 u}{\partial x^2} = \frac{U_\infty}{D^2} \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} \quad \text{then}$$

$$\frac{\rho_\infty U_\infty^2}{D} \left[(\bar{\bar{q}} \cdot \bar{\nabla}) \bar{u} \right] = -\frac{\rho_\infty U_\infty^2}{D} \bar{p}_{,\bar{x}} + \mu_\infty \frac{U_\infty}{D^2} \left[\bar{\nabla}^2 \bar{u} \right]$$

$$\left[\bar{\bar{q}} \cdot \bar{\nabla} \bar{u} \right] = -\bar{p}_{,\bar{x}} + \frac{1}{R_{N_{\infty,D}}} \left[\bar{\nabla}^2 \bar{u} \right]$$

Continuity $\frac{U_\infty}{D}(\bar{\nabla} \cdot \bar{\bar{q}}) = 0$ $\qquad$ $\bar{u} = \bar{v} = \bar{w} = 0$ on surface no slip

$\bar{u} = 1 \quad \bar{v} = \bar{w} = 0$ at infinity

for smaller body

$$(\bar{\bar{q}} \cdot \bar{\nabla}) \bar{u} = -\bar{p}_{,\bar{x}} + \frac{1}{R_{N_{1,d}}} \left[\bar{\nabla}^2 \bar{u} \right] \qquad \bar{u} = \frac{u}{u_1}, \ \bar{x} = \frac{x}{d}, \text{ etc}$$

Complete similarity in flow field $\Rightarrow R_{N_{\infty,D}} \equiv R_{N_{1,d}}$ $\qquad$ Completely similar flow. Flow patterns & streamline patterns are similar.

$$C_p = \frac{p - p_\infty}{q_\infty} = 2 \left[\bar{p} - \bar{p}_\infty \right]$$

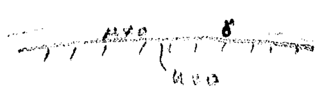
As $R_N \to \infty$ $\quad \mu \to 0$ or $V \to \infty$ $\quad$ Navier Stokes $\to$ Euler's Equation

as $R_N \to 0$ $\quad \mu \to \infty$ or $V \to 0$ $\quad$ Navier Stokes $\to$ Creep Flow.

<u>Boundary Layer Equations</u> 1904 Prandtl

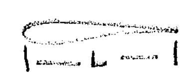
Effective viscosity is felt in a region very close to body

outside this neglect $\mu \to 0$, potential flow

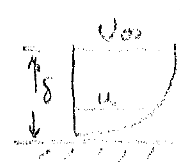


include viscous effects, viscous flow

Characteristic lengths $L \gg\gg \delta$ or $\delta/L \ll 1$ — Assume this



2-D $\quad \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = -\bar{p}_{,\bar{x}} + \frac{1}{R_N} \left[\bar{u}_{,\bar{x}\bar{x}} + \bar{u}_{,\bar{y}\bar{y}} \right]$



$\frac{\partial u}{\partial y} = 0$ $\qquad$ Steady flow

let $\bar{u} = \frac{u}{U_\infty} \sim O(1)$ on body

Assumed
$$\begin{cases} \bar{x} = \frac{x}{L} \sim O(1) \\ \bar{y} = \frac{y}{L} \sim O(\bar{\delta} = \delta/L) \end{cases}$$

$$\partial/\partial\bar{x} \sim O(1), \quad \frac{\partial}{\partial \bar{y}} \sim O(1/\bar{\delta}), \quad \frac{\partial^2}{\partial \bar{x}^2} \sim O(1)$$

$\partial^2/\partial \bar{y}^2 \sim O\left(\frac{1}{\bar\delta^2}\right)$

$$\frac{\partial \bar u}{\partial \bar x} + \frac{\partial \bar v}{\partial \bar y} = 0$$

$$\underbrace{O(1)}_{\bar u} \cdot \underbrace{O\left(\frac{1}{1}\right)}_{\partial/\partial \bar x} + \underbrace{O\left(\frac{1}{\bar\delta}\right)}_{\partial/\partial \bar y} \cdot \underbrace{O(\bar\delta)}_{\bar v} = 0$$

From continuity $\quad v \sim O(\bar\delta)$

$$\bar u \frac{\partial \bar u}{\partial \bar x} + \bar v \frac{\partial \bar u}{\partial \bar y} = -\partial \bar p/\partial \bar x + \frac{1}{R_N}\left[\frac{\partial^2 \bar u}{\partial \bar x^2} + \frac{\partial^2 \bar u}{\partial \bar y^2}\right]$$

$$O(1)\,O(1) + O(\bar\delta)\,O\left(\frac{1}{\bar\delta}\right) = \;?\qquad ?\left[\underbrace{O(1)O(1)}_{\text{neglect}} + \underbrace{O(1)\cdot O(1/\bar\delta^2)}_{\text{much larger than}}\right]$$

On outer edge of b.l.

at $y=\delta$ $\quad \bar u \dfrac{\partial \bar u}{\partial \bar x} = -\bar p_{,\bar x} \qquad \bar p_{,\bar x} \sim O(1)$

therefore $\quad 1/R_N \sim O(\bar\delta^2) \quad$ since $\dfrac{\partial^2 \bar u}{\partial \bar y^2} \gg \dfrac{\partial^2 \bar u}{\partial \bar x^2}$ shown in same manner as lubrication theory.

$$\boxed{R_N \sim O(1/\bar\delta^2)}$$

if $R_N \sim (1/\bar\delta^3)$ then we can neglect viscous terms

if $R_N \sim O(1)$ then we can throw out inertia & pressure terms

$\sim O(1/\delta)$ then

It's not creep flow because one can distinguish 2 regions < viscous, inviscid

in creep flow flow is wholly viscid $\therefore O(\bar p) \ll O(\bar u, \bar y, \bar y)$

Creep Flow - inertia ≪ viscous, valid body to ∞

Boundary layer - valid between body & δ

$$\bar u \frac{\partial \bar v}{\partial \bar x} + \bar v \frac{\partial \bar v}{\partial \bar y} = -\bar p_{,\bar y} + \frac{1}{R_N}\left[\frac{\partial^2 \bar v}{\partial \bar x^2} + \frac{\partial^2 \bar v}{\partial \bar y^2}\right]$$

$$\underbrace{O(1)(O)(\delta)O(1)}_{O(\bar\delta)} + \underbrace{O(\bar\delta)\,O(1)}_{O(\bar\delta)} = \;?\; + O(\bar\delta^2)\underbrace{\left[\underbrace{O(\bar\delta)}_{\text{neglect}} + O\left(\frac{1}{\bar\delta}\right)\right]}_{O(\bar\delta)}$$

$$\boxed{\frac{\partial^2 \bar v}{\partial \bar x^2} \ll \frac{\partial^2 \bar v}{\partial \bar y^2}}$$

therefore $\boxed{\bar p_{,\bar y} \sim O(\bar\delta)}$ $\qquad \bar p_{,\bar x} = \dfrac{d\bar p}{d\bar x}$

- look at $\bar p_{,\bar x}$, $\bar p_{,\bar y}$

$\bar p_{,\bar x} \gg \bar p_{,\bar y}$

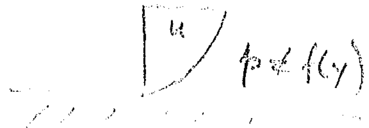
therefore $\bar p = f(\bar x)$ only

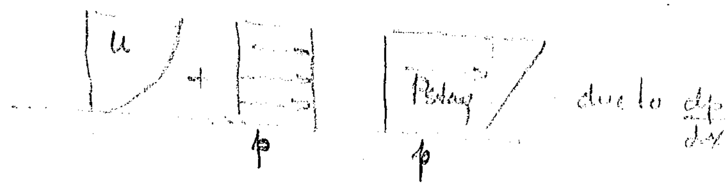
$\longrightarrow \bar p_{,\bar y} = 0$

① or $\boxed{p_{,y} = 0}$

Front

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{1}{R_N}\left[\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right] \quad \text{or} \quad (2)\ \boxed{\rho\left[(\bar{q}\cdot\nabla)u\right] = -p_{,x} + \mu u_{,yy}}$$





$0 = u = v \quad y = 0$

$U_\infty = u \quad y = \delta$

$p(x)$ is given by inviscid flow solution

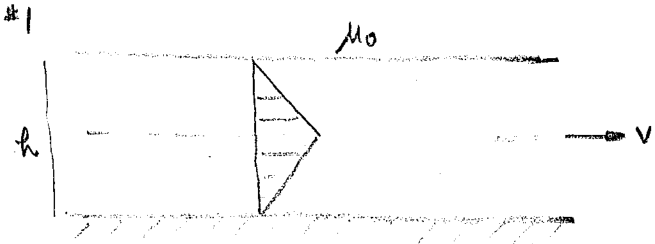
@ δ $\quad u = u_e,\ v = 0$

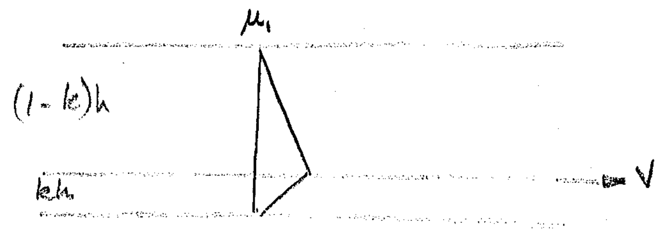
$u = v = 0$

at $y = \delta \quad v = 0$

$$\boxed{u_e \rho_e \frac{\partial u_e}{\partial x} = -\frac{\partial p}{\partial x}}$$

HW #1 $\qquad\qquad$ 12-1-71



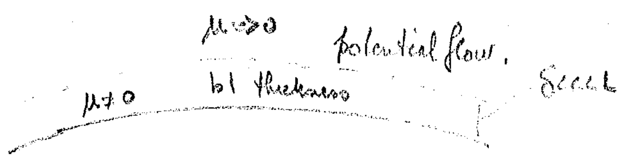


$$\frac{F}{A} = 2\mu_0 \frac{V}{h/2}$$

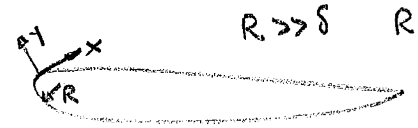
$$\frac{F}{A} = \mu_1 \frac{V}{kh} + \mu_1 \frac{V}{(1-k)h}$$

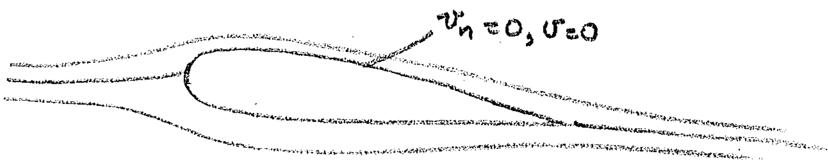
$$= \mu_1 \frac{V}{h}\left(\frac{1}{k(1-k)}\right)$$

$$4\mu_0 = \frac{\mu_1}{k(1-k)} \longrightarrow 4k(1-k) = \frac{\mu_1}{\mu_0}$$



$R \gg \delta \quad R$ rad of curvature



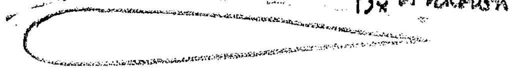


$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -p_{,x} + \mu u_{,yy}$$

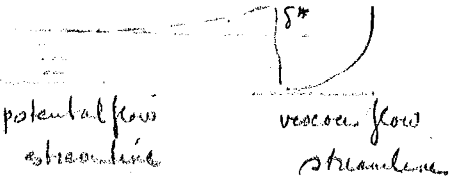
$$p_{,y} = 0$$

$$u_{,x} + v_{,y} = 0$$

take potential flow solution & use these conditions at edge of bl



$u_e = u$ on surface of body in potential flow between any 2 streamlines same mass ∴ if the same mass must pass through 2 streamlines then the streamlines are displaced upward.



δ^* displacement thickness

$y = 0 \quad v = u = 0$

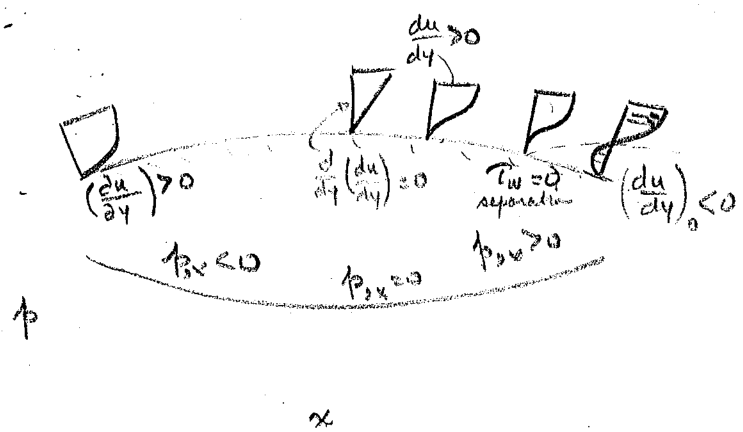
$y = \delta \quad u = u_e \qquad$ get $\delta = \delta(x)$

Is boundary layer rotational or irrotational?

$$\omega_z = \frac{1}{2}(u_{,y} - v_{,x}) = \frac{1}{2}\frac{U}{L}\left[\frac{\partial \bar{u}}{\partial \bar{y}} - \frac{\partial \bar{v}}{\partial \bar{x}}\right]$$

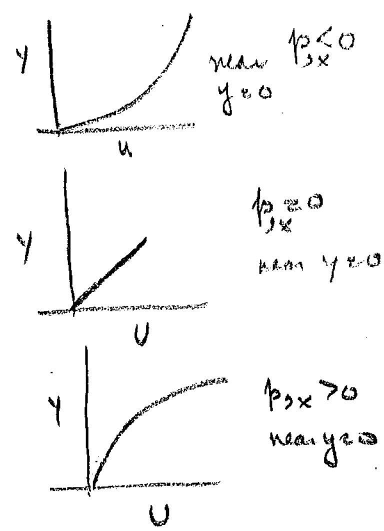
$$O(1)\cdot O(\tfrac{1}{\delta}) - O(\delta_1)(O(1))$$

∴ flow is rotational $\quad \omega_z = \frac{1}{2}\frac{\partial u}{\partial y} \quad$ zero at outer edge.



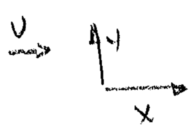
at $y = 0$

$u_{,yy} = \frac{1}{\mu} p_x \qquad$ from Navier Stokes



Separation must occur in $p_{,x} > 0$ region $\quad \frac{du}{dy}$ at $y = \delta = 0$

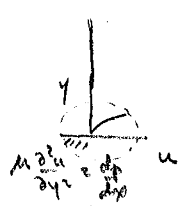
Blasius 1908 $\qquad$ Flat Plate



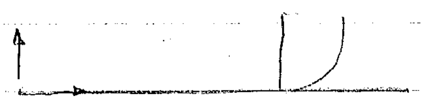
Equations in text for curvature on same order as boundary layer.

invisicity of bl

In vasinity of body about a point



$\frac{dp}{dx} = 0$ for blasius u, v



$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

at $y = 0$ $u = 0$ $v = 0$

at $y = \infty$ $u = u_1$

Introduce $u = \frac{\partial \psi}{\partial y}$ $v = -\frac{\partial \psi}{\partial x}$

Assume, $\eta = C y x^{\alpha}$ $u = u_1 f'(\eta) = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{\partial \psi}{\partial \eta} c x^{\alpha}$

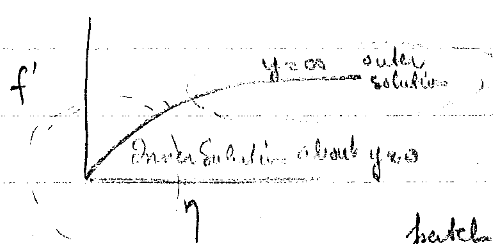
$$\frac{\partial \psi}{\partial \eta} = \frac{u_1 f'(\eta)}{c x^{\alpha}} \qquad \psi = \frac{u_1}{c x^{\alpha}} f(\eta)$$

HW #1 - Show that the diff eq can be reduced to $2f''' + f f'' = 0$

$\eta = y\sqrt{\frac{u_1}{x\nu}}$ at $y = 0$ $\eta = 0$ $u = 0$ $f'(\eta) = 0$

also → $v = 0$ $f(\eta) = 0$

$y = \infty$ $\eta = \infty$ $u = u_1$ $f'(\eta) = 1$



1 constant unknown from Inner Soln

2 const " " outer Soln

patch both solutions ∴ f, f', f'' are the same for inner & outer solutions

HW #2 In vacinity of $\eta = 0$ assume $f = \sum_{n=0}^{\infty} A_n \eta^n = A_0 + A_1\eta + A_2\eta^2 + \dots$ ($A_0 = 0$)

from BC & diff eq find $A_0 \to A_5$

In vacinity of $\eta = \infty$ $\frac{df}{d\eta} = 1$ as $\eta \to \infty$

Assume $f = f_1 + f_2$ where $f_1 \ggg f_2$, at $\eta = \infty$ $f_2 = f_2' = f_2'' = 0$

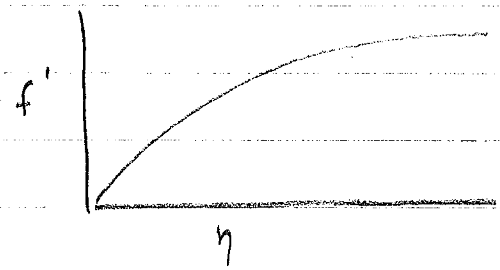
at $\eta = \infty$ $f_2 = 0$ $\frac{df}{d\eta} = \frac{df_1}{d\eta} = 1$ $f_1 = \eta - \beta$

$f = \eta - \beta + f_2$ put into diff equation $f'' = f_2''$, $f''' = f_2'''$ $f \approx \eta - \beta$ since $f_1 \gg f_2$ $2f_2''' + (\eta - \beta) f_2'' = 0$

$f_2'' = \gamma e^{-1/4(\eta-\beta)^2}$

$f_2 = \gamma \int_\infty^\eta d\eta \int_\infty^\eta e^{-\frac{1}{4}(\eta-\beta)^2} d\eta$ $\qquad$ $f = (\eta-\beta) + \gamma \int_\infty^\eta d\eta \int_\infty^\eta e^{-1/4(\eta-\beta)^2} d\eta$

Pick $f''(0) \neq 0$ $\qquad$ $f(0) = f'(0) = 0$



for $f'(\infty) = 1$

Search on $f''(0) \neq 0$

12-8-71

$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$ $\qquad$ $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ $\qquad$ from second eq $\frac{\partial p}{\partial y} = 0$

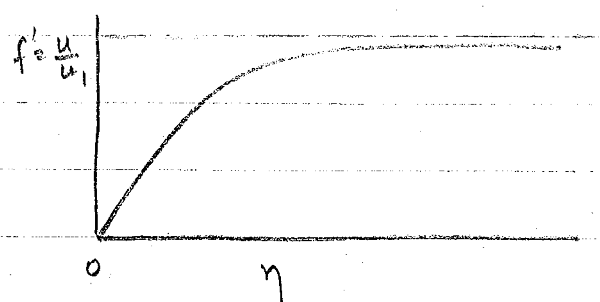
$u = \frac{\partial \psi}{\partial y}$ $\quad$ $v = -\frac{\partial \psi}{\partial x}$ satisfy continuity exactly.

define $\eta = cyx^\alpha$ $\qquad$ $u/u_1 = f'(\eta)$ $\qquad$ $\psi = \frac{u_1 f}{cx^\alpha}$

leads to $2f''' + f''f = 0$ $\qquad$ $\eta = y\sqrt{u_1/x\nu}$

at $y = 0$ $\qquad$ $u = 0$ $f' = 0$ $\qquad$ at $y = \infty$ $\quad$ $u = u_1$

$\eta = 0$ $\qquad$ $v = 0$ $f = 0$ $\qquad$ $\eta = \infty$ $\quad$ $f' = 1$



assume $f_i = \sum_{n=0}^{\infty} A_n \eta^n$ in the inner solution

and 2 bc leaves 1 unknown const A

assume for $\eta \to \infty$ $\quad$ $f = f_1 + f_2$ where $f_2 <<< f_1$

$f' = f_1' + f_2'$ $\quad$ $f_2' <<< f_1'$ $\Rightarrow$ $f' = f_1' = 1$

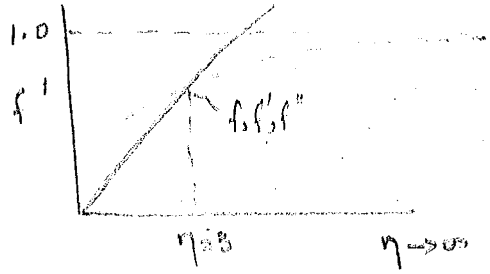
$f_1 = \eta - \beta$

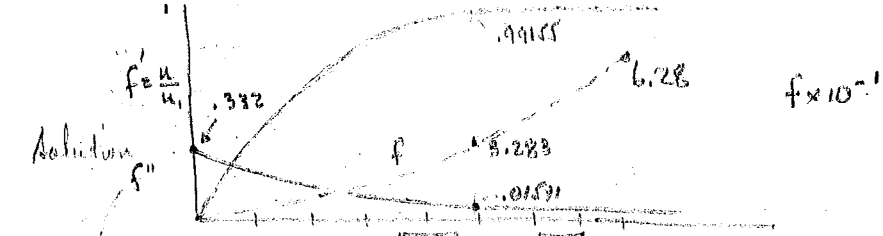
$f = \eta - \beta + f_2 \rightarrow 2f''' + ff'' = 0 \longrightarrow f \sim f_1 \qquad f'' = f_2'' \qquad f''' = f_2'''$

$2f_2''' + (\eta-\beta) f_2'' = 0$

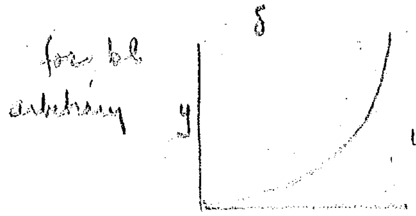
$f_o = (\eta-\beta) + \gamma \int_\infty^\eta d\eta \int_\infty^\eta e^{-1/4(\eta-\beta)^2} d\eta$ $\qquad$ 2 unknowns from outer solutions

3 unknown in problem A, β, γ use patchwork to get them. use it $\ni f, f', f''$ match





$$\eta = y\sqrt{\frac{u_1}{\nu x}} = \frac{y}{x}\sqrt{\frac{u_1 x}{\nu}} = \frac{y}{x}\sqrt{R_{N_x}}$$



$u/u_1 = .99$ or $u/u_1 = .995$

define $\delta \ni$ $\quad u/u_1 = .9915 \qquad \eta = 5 = \frac{\delta}{x}\sqrt{R_{N_x}} \qquad \frac{\delta}{x} = \frac{5.0}{\sqrt{R_{N_x}}}$

$u/u_1 = .99425 \qquad \eta = 5.2 \qquad \delta/x = \frac{5.2}{\sqrt{R_{N_x}}}$

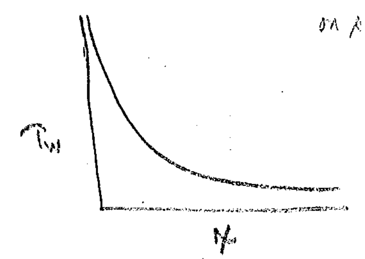
$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_0 = \mu\left(\frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y}\right)_0 = \mu\left(u_1 f''(0)\ \frac{1}{x}\sqrt{R_{N_x}}\right) = \frac{.332\,\mu u_1}{x}\sqrt{R_{N_x}}$$

$$C_f = \frac{\tau_w}{q} = .332\mu\frac{u_1}{x}\sqrt{R_{N_x}}\cdot\frac{1}{\frac{1}{2}\rho u_1^2} = \frac{.664}{\sqrt{R_{N_x}}}$$

$$D/\text{unit width} = \int_0^L \tau_w\,dx = \bar{k}\int_0^L \frac{dx}{x^{1/2}} = \bar{k}\,2x^{1/2}\Big|_0^L = 2L^{1/2}\,\bar{k} \quad \text{where } \bar{k} = .332\mu u_1\sqrt{\frac{u_1}{\nu}}$$

$$D = .664\,\mu u_1\sqrt{R_{N_L}} \qquad C_D = \frac{D}{qL} = \frac{.664\,\mu u_1\sqrt{R_{N_L}}}{\frac{1}{2}\rho u_1^2 L} = \frac{1.328}{\sqrt{R_{N_L}}}$$

on single side



The displacement thickness is defined as that distance by which the external flow is displaced outward as a consequence of the decrease in velocity in the boundary layer. (adjacent to wall

$$\rho_1 u_1 \beta = \int_0^\delta \rho u\,dy$$

$$\rho_1 u_1 (\delta - \delta^*) = \int_0^\delta \rho u\,dy$$

but $\delta = \int_0^\delta dy$

$$\rho u_1\left(\int_0^\delta dy\right) - \int_0^\delta \rho_1 u_1\,dy = \rho_1 u_1 \delta^*$$

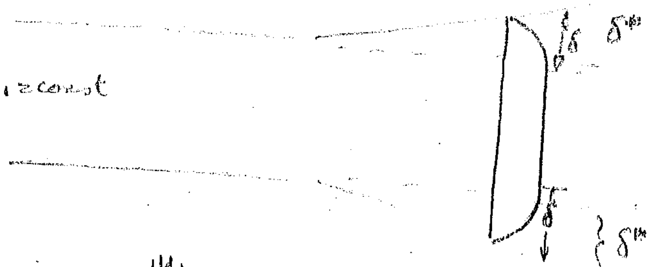
$$\int_0^\delta \left(1 - \frac{\rho u}{\rho_1 u_1}\right)dy = \delta^*$$

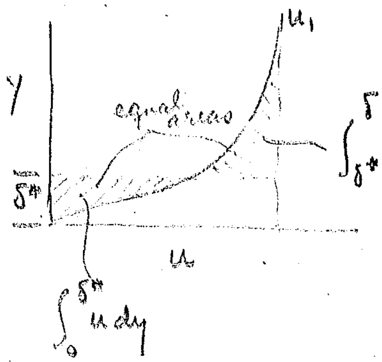
$$\boxed{\delta^* = \int_0^\delta \left(1 - \frac{\rho u}{\rho_1 u_1}\right)dy}$$

in incompressible flow $\quad \delta^* = \int_0^\delta (1 - u/u_1)\,dy$

Another definition

potential flow $\quad u_1 = \text{const}$





$$\int_0^{\delta^*} u\,dy = \int_{\delta^*}^{\delta} (u_1 - u)\,dy$$

$$\int_0^{\delta} u\,dy = \int_{\delta^*}^{\delta} u_1\,dy = \int_0^{\delta} u_1\,dy - \int_0^{\delta^*} u_1\,dy$$

$$\int_0^{\delta} (u_1 - u)\,dy = \int_0^{\delta^*} u_1\,dy = u_1 \delta^*$$

$$\therefore \quad \delta^* = \int_0^\delta (1 - u/u_1)\,dy = \int_0^\infty (1 - u/u_1)\,dy$$

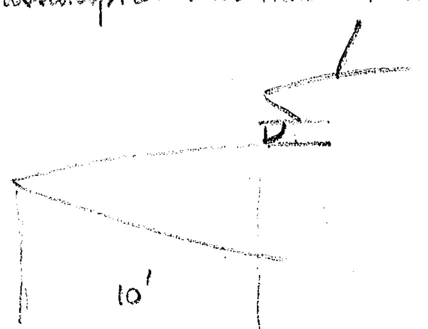
For flat plate $\quad u/u_1 = f'(\eta) \qquad \eta = y\sqrt{\nu x/u_1}$

$$\delta^* = \int_0^\infty \sqrt{\frac{\nu x}{u_1}}\,(1 - f'(\eta))\,d\eta = \sqrt{\frac{\nu x}{u_1}}\,\big(\eta - f(\eta)\big)_0^\infty$$

$$\eta - \beta$$

$$= \sqrt{\frac{\nu x}{u_1}} \cdot \beta = 1.73\sqrt{\frac{\nu x}{u_1}} = \frac{1.73\,x}{\sqrt{RN_x}}$$

an assumption was made that v_1 was small $\qquad$ at δ, $\dfrac{v}{u_1} = \dfrac{.865}{\sqrt{RN_x}} \qquad RN > 10^4$

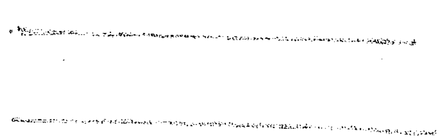


Assume

$u_1 = 200$ ft/sec

$h \sim 20{,}000$ ft. $\quad$ find $\delta, \delta^*, D_{\text{fuselage to inlet}}$

compute mass flow in b.l.



same condition $\quad$ give approximation for fully developed flow.

12-15-71

$\delta^* = \int_0^\delta (1 - u/u_1)\,dy$

Blasius: $\delta^* = \int_0^\infty (1 - u/u_1)\,dy = \sqrt{\frac{\nu x}{U_1}} \int_0^\infty (1-f')\,d\eta = \sqrt{\frac{\nu x}{u_1}}\left[\eta - f\right]_0^\infty = \beta\sqrt{\frac{\nu x}{u_1}}$

$= 1.73 \frac{x}{\sqrt{R_{Nx}}}$ at ∞ $f = \eta - \beta$ $\therefore \eta - f = \beta$

The loss of momentum in the boundary layer as compared with potential flow is $\rho\int_0^\infty u(u_1 - u)\,dy$

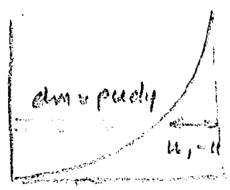
The momentum thickness is defined as:

$\rho u_1^2 \theta = \rho\int_0^\infty u(u_1 - u)\,dy$

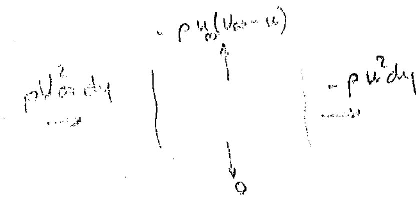
Blasius: $\theta = \int_0^\infty \frac{u}{u_1}(1 - u/u_1)\,dy = \sqrt{\frac{\nu x}{u_1}} \int_0^\infty f'(1-f')\,d\eta = \frac{.664x}{\sqrt{R_{Nx}}}$

if δ^* is defined at $\eta = 5.0$ $\delta^* = 1.717 \frac{x}{\sqrt{R_{Nx}}}$

$1 - f' = dV$ $\eta - f = V$ $u = f'$ $du = f''$ $(\eta - f) f' d\eta$



$d(mV) = d(mV)_{pot} - d(mV)_{visc}$ $dm = \rho u\,dy$ $dV = u_1 - u$



$\frac{\theta}{\delta^*} = .384$ if $u = .99u_1$ $\frac{\delta^*}{\delta} = .346$

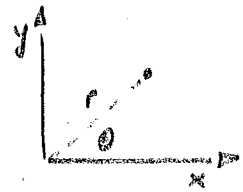
$2f''' + ff'' = 0$ $\eta = y\sqrt{u_1/\nu x}$

Potential Function

$w = \varphi + i\psi$ where w is the potential fn $= uz^n$ $z = x + iy$

write in polar coord

$= ur^n(\cos n\theta + i\sin n\theta)$



$\psi = ur^n \sin n\theta$

to get body shape set $\psi = 0$

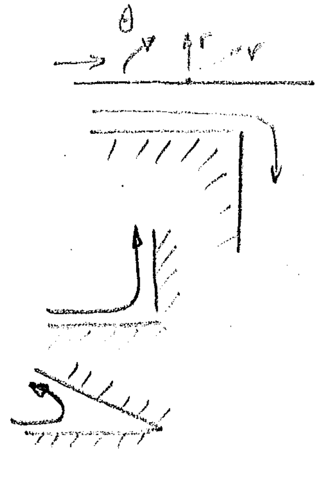
$\rightarrow \theta = 0$ n $n\theta = \pi$

$n = 1$

$n = 2/3$

$n = 2$

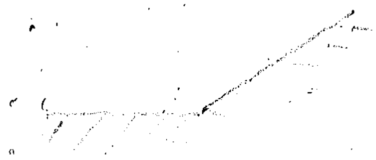
$n = 4$

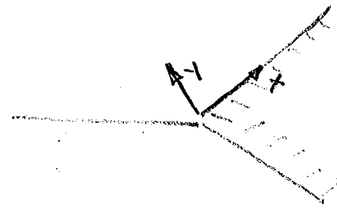


$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = U r^{n-1} n \cos n\theta$ $\qquad v_r = -U r^{n-1} n$ at body

$v_\theta = -\frac{\partial \psi}{\partial r} = -U r^{n-1} n \sin n\theta$ $\qquad v_\theta = 0$ at body $\qquad$ for $\theta = \pi/n$

Consider

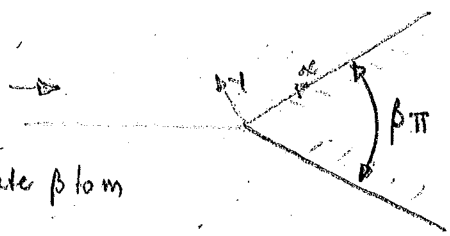




v_{r_b} at body $= u_e$
$= -U r^{n-1} n$
let $r = x$, $m = n-1$
$u_1 = -nU$

then $u_e = u_1 x^m$ $\qquad$ Faulkner – Skan, Solutions

Consider a flow over a wedge



$\pi(1-\beta/2) = \pi/n$ $\qquad \beta = \frac{2m}{m+1}$

relate β to m

Momentum $\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2}$

$\frac{\partial p}{\partial y} = 0$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ $\qquad$ at $y = \delta$ $\quad u = u_e = u_1 x^m = U$

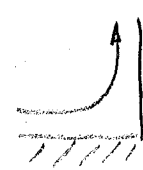
$\qquad y = 0 \quad u = 0$

this gives rise to $f''' + ff'' - \frac{2m}{m+1}\left[f'^2 - 1\right] = 0$

$\eta = \sqrt{\frac{u_e}{\nu x}\frac{(m+1)}{2}}$ $\qquad \alpha = \frac{m-1}{2}$

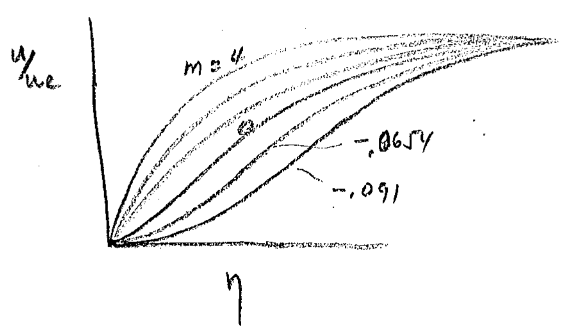
for $m = 0$ $\quad f''' + ff'' = 0 \quad \rightarrow \eta = \sqrt{\frac{u_e}{\nu x}\frac{1}{2}}$

for $m = 1$ $\quad f''' + ff'' - [f'^2 - 1] = 0$ $\qquad n = 2$

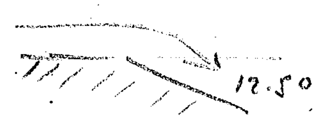


Stagnation Point flow Soln.

2 point boundary problem

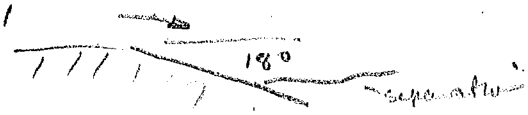


for $m < 0$



for $m = -.0654$

for $m = -.091$



$m > 0$ favorable pressure gradient

$$\int_0^\delta u_s\,dy - u_s\delta^* = \int_0^\delta u\,dy \quad \text{or} \quad u_s\,\delta^* = \int_0^\delta (u_s - u)\,dy$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_s}\right) dy.$$

8. δ_i – that distance from the actual boundary such that the momentum flux through δ_i is the same as the loss of momentum flux through the actual boundary layer.

Momentum flux $= \int_A \rho \underset{\sim}{q} (\underset{\sim}{q} \cdot \hat{\underset{\sim}{n}})\, dA$

Mom flux thru δ_i $\quad \rho\, u_s^2\, \delta_i = \int_0^\delta \rho\,(u_s - u)\, u\, dy$

$$\delta_i = \int_0^\delta \left(1 - u/u_s\right) \frac{u}{u_s}\, dy$$

E. SHEAR ON FLAT PLATE $\frac{\partial p}{\partial x} = 0$

1. Euler $\quad u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \quad \Rightarrow \quad \frac{\partial u}{\partial x} = 0$

2. $\frac{d\delta_i}{dx} = \frac{\tau_w}{\rho u_s^2} \quad$ or $\quad \tau_w = \rho u_s^2 \frac{d}{dx}\int_0^\delta \frac{u}{u_s}\left(1 - u/u_s\right) dy$

3. Let $u/u_s = U$ when $u = 0$ $U = 0$; $u = u_s$ $U = 1$
   Let $y/\delta = Y$ when $y = 0$ $Y = 0$; $y = \delta$ $Y = 1$

$$\tau_w = \rho u_s^2 \frac{d}{dx}\int_0^1 U(1 - U)\,\delta\, dY = \mu \left.\frac{du}{dy}\right|_{y=0} = \mu \frac{u_s}{\delta} \left.\frac{dU}{dY}\right|_{Y=0}$$

If $\frac{du}{dy}=0$ no shear at wall separation profile   this happens at $m=-.091$

<u>Von Karman Integral Techniques</u>

$$\rho\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

to get average values in b.l.

$$\int_0^y\frac{\partial u}{\partial x}dy+\int_0^y\frac{\partial v}{\partial y}dy=0$$

$$\int_0^y\frac{\partial u}{\partial x}dy+\left.v\right|_0^y=0$$

$$\int_0^y\frac{\partial u}{\partial x}dy+v=0 \qquad v=-\int_0^y\frac{\partial u}{\partial x}dy$$

$$\rho\left[\int_0^\delta u\frac{\partial u}{\partial x}dy-\int_0^\delta\left(\int_0^y\frac{\partial u}{\partial x}dy\right)\frac{\partial u}{\partial y}dy\right]=-\int_0^\delta\frac{\partial p}{\partial x}dy+\int_0^\delta\mu\frac{\partial^2 u}{\partial y^2}dy$$

$$\int_0^\delta\mu\frac{\partial^2 u}{\partial y^2}dy=\left.\mu\frac{\partial u}{\partial y}\right|_0^\delta=0-\tau_w=-\tau_w$$

$$-\int_0^\delta\frac{\partial p}{\partial x}dy=+\int_0^\delta\rho u_e\frac{\partial u_e}{\partial x}dy \qquad \text{at outer boundary } -\frac{\partial p}{\partial x}=\rho u_e\frac{\partial u_e}{\partial x}$$

integration by parts using Leibnitz rule.

$$\frac{dI}{dx}=\frac{d}{dx}\int_0^{\delta(x)}f(x,y)\,dy=\int_0^\delta\frac{\partial F}{\partial x}dy+f(x,\delta)\frac{d\delta}{dx}$$

Combining terms & using Leibnitz Rule

$$\frac{d\theta}{dx}+\theta\left[2+\frac{\delta^*}{\theta}\right]\frac{1}{u_e}\frac{du_e}{dx}=\frac{C_f}{2} \qquad \text{where } \frac{C_f}{2}=\frac{\tau_w}{\rho u_e^2}$$

$$\theta=\int_0^\delta\frac{u}{u_e}\left(1-u/u_e\right)dy$$

$$\delta^*=\int_0^\delta\left(1-u/u_e\right)dy$$

for a flat plate $u_e=$ const

$$\frac{d\theta}{dx}=\frac{C_f}{2}$$ this determines variation of boundary layer with position x.

<u>Pohlhausen soln</u>

Assume $u/u_e=A\left(\frac{y}{\delta}\right)+B\left(y/\delta\right)^2+C\left(y/\delta\right)^3+D\left(y/\delta\right)^4+\ldots$

$$\delta^*=\delta\int_0^1\left(1-u/u_e\right)d\left(y/\delta\right)$$ velocity must satisfy no slip

at $y=0$ $u/u_e=0$ $v=0$ no slip

$y=\delta$ $u/u_e=1$

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2}$$

at $y=0$ $\quad u=v=0 \qquad \mu\frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial x}$ $\quad$ 1

at $y=\delta$ $\quad u=u_e \qquad \rho u_e\frac{\partial u_e}{\partial x} = -\frac{\partial p}{\partial x}$

$y=\delta$ $\quad \tau=0 \qquad \left(\frac{\partial u}{\partial y}\right)_\delta = 0$ $\quad$ 3

$y=\delta \qquad \left(\frac{\partial^2 u}{\partial y^2}\right)_\delta = 0$ $\quad$ 4 $\qquad$ diff mom wrt y

for higher order diff mom wrt y at surface.

12-17-71

$$\left.\begin{aligned}\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] &= -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2}\\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0\end{aligned}\right\} u, v$$

δ 

$\frac{\partial p}{\partial y} = 0$ in b.l. $\qquad$ to get an average of the properties $\int_0^\delta \begin{matrix}\text{cont}\\ \text{mom}\end{matrix}\, dy$

$$\int_0^Y \frac{\partial u}{\partial x}dy + \int_0^Y \frac{\partial v}{\partial y}dy = 0 \qquad v = -\int_0^Y \frac{\partial u}{\partial x}dy$$

using Liebnitz's rule to integrate gives

$$\frac{d\theta}{dx} + \theta\left[2 + \frac{\delta^*}{\theta}\right]\frac{1}{u_e}\frac{du_e}{dx} = \frac{C_f}{2}$$ Von Karman Integral eq

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_e}\right)dy \qquad \theta = \int_0^\delta \frac{u}{u_e}\left(1 - \frac{u}{u_e}\right)dy$$

Solved by Polhausen

$$\frac{u}{u_e} = A\left(y/\delta\right) + B\left(y/\delta\right)^2 + C\left(y/\delta\right)^3 + D\left(y/\delta\right)^4$$

at $y=0$ $\quad u=0$

at $y=\delta$ $\quad u/u_e = 1 \quad \partial u/\partial y = 0$ $\quad$ no shear at boundary due to potential flow

$v=0$ $\quad$ momentum reduces to Bernoulli $\therefore \frac{\partial^2 u}{\partial y^2} = 0$

at $y=0$ $\quad u=0, v=0 \qquad \partial p/\partial x = \mu\frac{\partial^2 u}{\partial y^2}$

look at bc at outer edge

$$1 = A + B + C + D$$
$$0 = A + 2B + 3C + 4D$$
$$0 = 2B + 6C + 12D$$
$$\left.\frac{\partial^2 u}{\partial y^2}\right|_0 = \frac{1}{\mu}\frac{\partial p}{\partial x}$$

$$\frac{\partial u}{\partial y} = u_e\left[\frac{A}{\delta} + \frac{2B}{\delta^2}y + \frac{3C}{\delta^3}y^2 + \frac{4Dy^3}{\delta^4}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = u_e\left[\frac{2B}{\delta^2} + \frac{6C}{\delta^3}y + \frac{12Dy^2}{\delta^4}\right] = \mu\frac{2Bu_e}{\delta^2} \quad \text{at } y=0$$

$$\frac{\partial p}{\partial x} = \mu\frac{2Bu_e}{\delta^2} = -\rho u_e\frac{\partial u_e}{\partial x} \qquad B = -\frac{\rho u_e}{2\mu u_e}\frac{\partial u_e}{\partial x}\delta^2 = -\frac{\delta^2}{2\nu}\frac{\partial u_e}{\partial x}$$

define $\lambda = +\frac{\delta^2}{\nu}\frac{du_e}{dx}$ $\qquad \boxed{B = -\frac{\lambda}{2}}$ $\qquad \lambda$ is the Pohlhausen pressure parameter

$$A = 2 + \frac{\lambda}{6}$$
$$C = -2 + \frac{\lambda}{2}$$
$$D = +1 - \lambda/6$$

if 6 order eq take bc as $\left.\frac{\partial^3 u}{\partial y^3}\right|_0 = 0$, $\left.\frac{\partial^3 u}{\partial y^3}\right|_\delta = 0$

$$\frac{u}{u_e} = 2\frac{y}{\delta} - 2\left(y/\delta\right)^3 + \left(y/\delta\right)^4 + \frac{\lambda}{6}\left[(y/\delta) - 3\left(y/\delta\right)^2 + 3\left(y/\delta\right)^3 - \left(y/\delta\right)^4\right]$$

put into δ^* & Θ

$$\therefore \delta^* = \int_0^\delta \left(1 - u/u_e\right)dy = \delta\left[1 - \frac{A}{2} - \frac{B}{3} - \frac{C}{4} - D/5\right]$$

for flat plate $\delta^* = \delta\left[1 - 1 - 0 + \frac{1}{2} - \frac{1}{5}\right] = \frac{3\delta}{10}$

$$\Theta = \int_0^\delta \left(u/u_e - u^2/u_e^2\right)dy = \delta\left[-\frac{\delta^*}{\delta} + 1 - A^2/3 - \frac{AB}{2} - \frac{(2AC + B^2)}{5} - \frac{(AD + BC)}{3} - \frac{(2BD + C^2)}{7} - \frac{CD}{4} - \frac{D^2}{9}\right]$$

for flat plate
$$\delta\left[-3/10 + 1 - \frac{4}{3} + \frac{8}{5} - \frac{2}{3} - \frac{4}{7} + \frac{1}{2} - \frac{1}{9}\right]$$
$-1/2$

put into $\frac{d\Theta}{dx} + \Theta\left[2 + \frac{\delta^*}{\Theta}\right]\frac{1}{u_e}\frac{du_e}{dx} = c_f/2 \longrightarrow \frac{d\delta}{dx} = f(\delta)$

pressure in bl is given by p from potential flow imposed at edge of bl
u_e of bl = u_p from potential flow

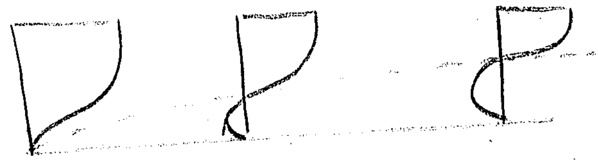
we assume $u/u_e = \sum_{i=0}^{4} A_i\left(y/\delta\right)^i$ when $A_0 = 0$

purpose:
to obtain validity of Pohlhausen λ $\qquad \lambda = \frac{\delta^2}{\nu}\frac{du_e}{dx}$

if separation occurs all the preceding is invalid

λ must be such that no reverse flow or separated flow exists

One extreme



$\frac{\partial u}{\partial y}\Big|_0 = 0 \qquad A = 0 \qquad$ if $\; u/u_e = A\left(y/\delta\right) + B\left(y/\delta\right)^2 + C\left(y/\delta\right)^3 + D\left(y/\delta\right)^4$

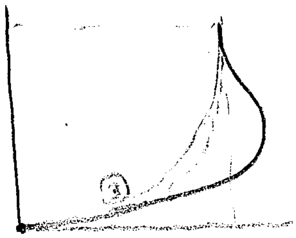
if so. $\quad A = 0 = 2 + \lambda/6 \quad \therefore \quad \lambda \geq -12$ adverse pressure gradient

at $\lambda = -40$ max reverse flow $u/u_e = -.32$

height of reverse flow is $.35\delta$

the other extreme

y/δ



u/u_e $\quad 1$

Excludes values of λ producing this type of flow.

at $\lambda = 40$ $\; u/u_e = 1.274$ at $y/\delta = .4$

Let $\eta = y/\delta$ $\qquad \frac{u}{u_e} = 2\eta - 2\eta^3 + \eta^4 + \frac{\lambda}{6}\left[\eta - 3\eta^2 + 3\eta^3 - \eta^4\right]$

$$\frac{d\,u/u_e}{d\eta} = 2 - 6\eta^2 + 4\eta^3 + \frac{\lambda}{6}\left[1 - 6\eta + 9\eta^2 - 4\eta^3\right]$$

$$\frac{d^2 u/u_e}{d\eta^2} = -12\eta + 12\eta^2 + \frac{\lambda}{6}\left[-6 + 18\eta - 12\eta^2\right]$$

$$= (1-\eta)\left[12\eta\left(\frac{\lambda}{6} - 1\right) - \lambda\right]$$

for a curve to exist as above an inflection pt must occur.

$$\therefore \frac{d^2 u/u_e}{d\eta^2} = 0 = (1-\eta)\left(12\eta\left(\lambda/6 - 1\right) - \lambda\right)$$

$\eta = 1 \qquad\qquad 12\eta\left(\lambda/6 - 1\right) - \lambda = 0$

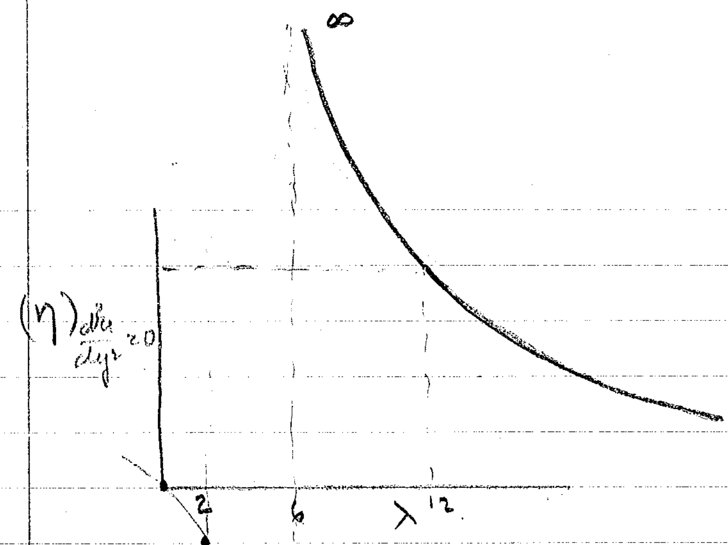
b.c.

$\eta = \lambda/12 \,/\, \lambda/6 - 1$

to impose inflection point $\eta = 1 \qquad \lambda = 12$

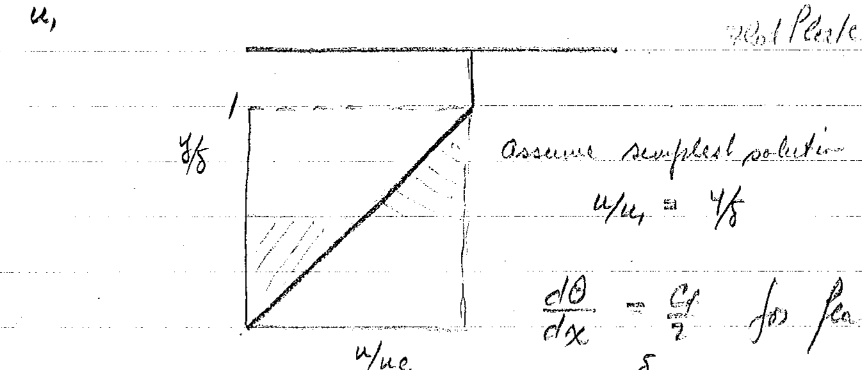
For $\lambda > 12 \quad 0 < \eta < 1$

$\therefore \quad |\lambda| \leq 12$



inflection occurs inside body

Flow over flat plate



assume simplest solution

$$u/u_1 = y/\delta$$

$$\frac{d\theta}{dx} = \frac{C_f}{2} \quad \text{for flat plate, since } u_e = u_e \text{ const}$$

$$\theta = \int_0^\delta u/u_1\,(1 - u/u_1)\,dy$$

$$= \int_0^\delta y/\delta\,(1 - y/\delta)\,dy = \left(\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right)_0^\delta$$

$\boxed{\theta = \delta/6}$

$$\delta^* = \int_0^\delta (1 - u/u_1)\,dy = \left(y - \frac{y^2}{2\delta}\right)_0^\delta = \delta/2$$

$\boxed{\delta^* = \delta/2}$ equal areas

$$\frac{d\theta}{dx} = \frac{1}{6}\frac{d\delta}{dx} = \frac{C_f}{2} = \frac{\tau_w}{\rho u_1^2} = \frac{\mu u_1}{\rho \delta u_1^2}$$

$$\frac{1}{6}\frac{d\delta}{dx} = \frac{\nu}{\delta u_1} \qquad \frac{\delta\, d\delta}{\delta x} = \frac{6\nu}{u_1}$$

$$\frac{\delta^2}{2} = \frac{6\nu}{u_1}x + \text{const} \qquad \text{at } x=0\ \delta=0$$

$$\therefore \quad \delta/x = \frac{3.46}{\sqrt{RN_x}} \qquad \text{Blasius: } \frac{5.0}{\sqrt{RN_x}}$$

$$C_f = \frac{2\mu}{\rho u_1 \delta} = \frac{2\mu\sqrt{R_{N_x}}}{\rho u_1 \cdot 3.46 x} = \frac{2}{3.46}\,\frac{1}{\sqrt{R_{N_x}}} = \frac{.580}{\sqrt{R_{N_x}}}$$

c_f Blasius $\frac{.664}{\sqrt{R_{N_x}}}$

$\delta^* = \frac{\delta}{2} = \frac{1.73x}{\sqrt{R_{N_x}}}$ exact with Blasius

$\frac{d\theta}{dx} = \frac{C_f}{2}$ assume 4th order profile assume $\lambda = 0$

$$\frac{u}{u_1} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4 \qquad \begin{matrix} A=2 & B=0 \\ C=-2 & D=1 \end{matrix}$$

gives $\frac{\theta}{\delta} = \frac{74}{360}$ $\qquad \frac{\delta^*}{\delta} = .3$

$$\frac{d\theta}{dx} = \frac{74}{360}\frac{d\delta}{dx} = \frac{C_f}{2} = \frac{\tau_w}{\rho u_1^2} = \frac{\mu(2/\delta)u_1}{\rho u_1^2} = \frac{2\nu}{\delta u_1}$$

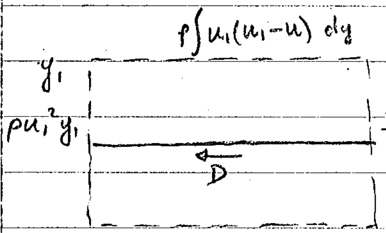
$$\frac{74}{360}\,\delta\, d\delta = \frac{2\nu\, dx}{u_1}$$

$$\frac{\delta}{x} = \frac{5.83}{\sqrt{R_{N_x}}} \text{ vs } \frac{5.0}{\sqrt{R_{N_x}}} \qquad C_f = \frac{4\nu}{\delta u_1} = \frac{4\nu\sqrt{R_{N_x}}}{u_1 \cdot 5.83x}$$

$$= \frac{.686}{\sqrt{R_{N_x}}} \text{ vs } \frac{.664}{\sqrt{R_{N_x}}}$$

$$\frac{\delta^*}{x} = \frac{1.749}{\sqrt{R_{N_x}}} \text{ vs. } \frac{1.73}{\sqrt{R_{N_x}}}$$

12-22-71

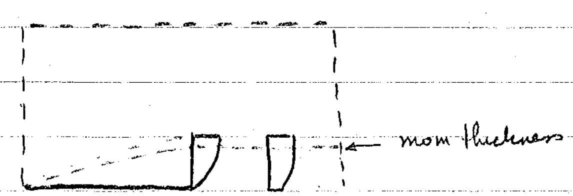


$$D = +\int_0^{y_1}\rho u^2 dy - \rho u_1^2 y_1 + \int_0^{y_1}\rho u_1(u_1-u)\,dy$$

$$= \int_0^{\delta}\rho u^2 dy + \int_\delta^{y_1}\rho u^2 dy - \rho u_1^2 y_1$$

$$= \int_0^{\delta}\rho u^2 dy - \rho u_1^2 \delta$$

$D_{TOT} = 2\rho u_1^2\,\theta(L)$



→ Final: Lubrication, Flow similarity, Prandtl B.L. equations, Flat Plate
→ Faulkner - Skan (General soln); Flat Plate & Stag Pt (special cases
→ of the Faulkner - Skan); Karman - Pohlhausen Technique

$$\frac{d\theta}{dx} + \frac{\theta}{u_e}\left(2 + \frac{\delta^*}{\theta}\right)\frac{du_e}{dx} = \frac{c_f}{2}$$

$$u/u_e = 2\eta - 2\eta^2 + \eta^4 + \frac{\lambda}{6}\eta(1-\eta)^3$$

$$\lambda = \frac{\delta^2}{\nu}\frac{du_e}{dx}$$

$$\frac{\delta^*}{\delta} = .3 - \frac{\lambda}{120}, \quad \frac{\theta}{\delta} = \frac{37}{315} - \frac{\lambda}{945} - \frac{\lambda^2}{9072}$$

$$\tau_0 = (2 + \lambda/6)\frac{\mu u_e}{\delta} \qquad \lambda \geq -12$$

Problem

$$u/u_e = A\eta + B\eta^2 + C\eta^3$$

Prove for this profile, $|\lambda| \leq 6$

$$\frac{d^2(u/u_e)}{d\eta^2} = (1-\eta)\left[12\left(\frac{\lambda}{6} - 1\right)\eta - \lambda\right] = 0$$

$$\eta = \frac{\lambda/12}{\lambda/6 - 1}$$

For no overshoot of velocity profile: $\eta \geq 1$ in above eq

$\eta = 1 \qquad \lambda = 12$

$|\lambda| \leq 12$

Integral eq. satisfies (equations) conditions on the average across the B.L. in comparison with a pt. by pt. solution

Stagnation Pt

$u_e = ax \qquad \lambda = \frac{a\delta^2}{\nu} = A\delta^2 \qquad \underbrace{m=1 \quad n=2 \quad \beta=1}_{\text{faulkner skan}}$

$$\frac{\delta^*}{\delta} = \left[.3 - \frac{A\delta^2}{120}\right]$$

$$\frac{\theta}{\delta} = \frac{1}{63}\left[\frac{37}{5}\delta - \frac{A\delta^3}{15} - \frac{A^2\delta^5}{144}\right]$$

$$\frac{d\theta}{dx} + \frac{\theta}{u_e}\left[2+\frac{\delta^*}{\theta}\right]\frac{du_e}{dx} = \frac{C_f}{2}$$

$$\frac{\theta}{u_e}\left[2+\frac{\delta^*}{\theta}\right]\frac{du_e}{dx} \approx \frac{1}{ax}\,a \approx \frac{1}{x}$$

$$\frac{C_f}{2} = \frac{\nu}{\delta a x}\left[2+\lambda/6\right] = \frac{\delta}{x}\left[\frac{12+\lambda}{\lambda}\right]$$

$$f_1(\delta)\frac{d\delta}{dx} + \frac{1}{x}\left[f_2(\delta)\right] = \frac{1}{x}f_3(\delta)$$

$$\frac{d\delta}{dx} = \frac{1}{x}\frac{[f_3 - f_2]}{f_1}$$

for $x=0$ $\quad \frac{d\delta}{dx} \to \infty$ (not possible)

Then $f_3 - f_2 = 0 \longrightarrow$ indeterminate form
This leads to

$$\delta = 2.655\sqrt{\frac{\nu}{a}} \longrightarrow 2.4\sqrt{\frac{\nu}{a}} \text{ (exact}$$

Problem:

(Impulse Problems)

Solve using Karman - Pohlhausen Technique
3rd order Profile

$$u_t - u_{yy}(\nu) = 0$$

Integrate to satisfy on average
use $u_{,yy} = 0$ @ $y=0$

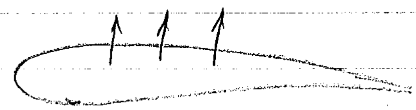
should be $\tau = .539\sqrt{\frac{\rho u_0^2\mu}{t}}$

$\delta = 2.82\sqrt{\nu t}$

Show $\delta = 2.68\sqrt{\nu t}$ vs $\delta = 4\sqrt{\nu t}$

$\tau = .559\sqrt{\frac{\rho u_0^2\mu}{t}}$ vs $\tau = .564\sqrt{\frac{\rho u_0^2\mu}{t}}$

now use $\frac{\partial^2 u}{\partial y^2} = 0$ @ $y = \delta$ $\quad \delta = 4.9\sqrt{\nu t} \quad \tau_w = .612\sqrt{\frac{\rho u_0^2\mu}{t}}$



$u_{,x} + v_{,y} = 0$

with suction or injection

$$v = v_0 - \int_0^y u_{,x}\, dy$$

$$\theta_{,x} + \frac{\theta}{u_e}\left[2 + \frac{\delta^*}{\theta}\right] u_{e,x} + \frac{v_0}{u_e} = \frac{C_f}{2}$$

$$u/u_e = A\eta + B\eta^2 + C\eta^3 + D\eta^4$$

at $\eta = 1$ $\quad u/u_e = 1 \quad \frac{d}{d\eta}(u/u_e) = 0 \quad \frac{d^2}{d\eta^2}(u/u_e) = 0$

at $\eta = 0$ $\quad \rho v \frac{\partial u}{\partial y} = -p_{,x} + \mu u_{,yy} \quad \frac{d\delta}{dx} = f(\zeta, \lambda, \delta)$

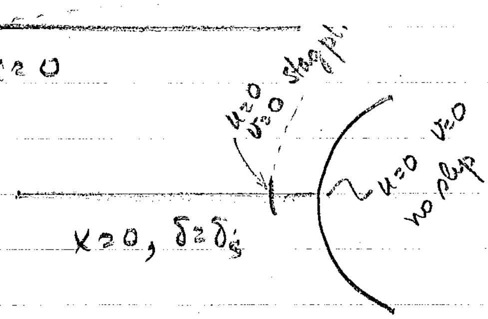
$$u/u_e = \left[f_1(\eta) + \lambda f_2(\eta) + \zeta f_3(\eta)\right] \frac{1}{1+\zeta}$$

$$\zeta = \frac{\delta v_0}{6\nu}$$

$$f_3(\eta) = 6\eta^2 - 8\eta^3 + 3\eta^4$$

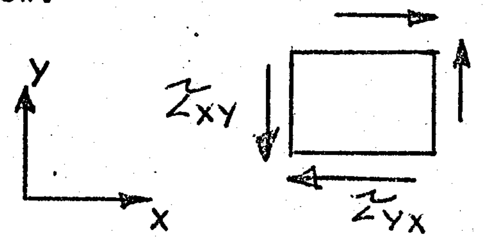
Flat Plate

$\delta = 0 \quad @ x = 0$



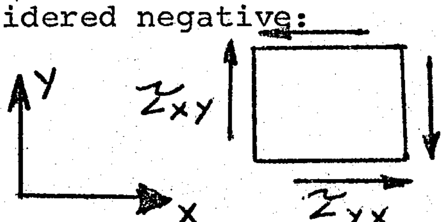
SHEAR STRESS SIGN CONVENTION

Consider a fluid element subjected to shear stresses as shown below:

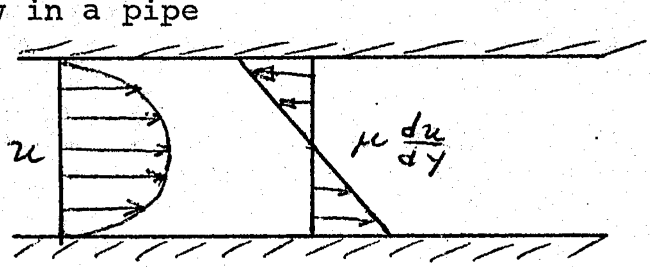


The shear stresses are all considered positive in the direction shown. The convention used is that if the outward normal to the fluid surface points in the positive direction of the corresponding coordinate axis, then the (positive) shear stress acts in the positive direction of its corresponding axis. If the outward normal points in the negative direction of its coordinate axes, then the (positive) shear stress acts in the negative direction of the other coordinate axis.

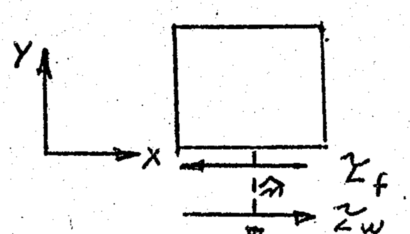
The shear stresses indicated in the figure below are all considered negative:



Consider the flow in a pipe

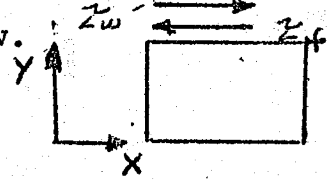


where $\tau = \mu \frac{du}{dy}$ and the subscripts f and w denote conditions in the fluid and at the wall, respectively. Consider the fluid element close to the lower wall. The shear stress (i.e., $\mu \frac{du}{dy}$) is positive and therefore the direction of τ_f is as shown below:

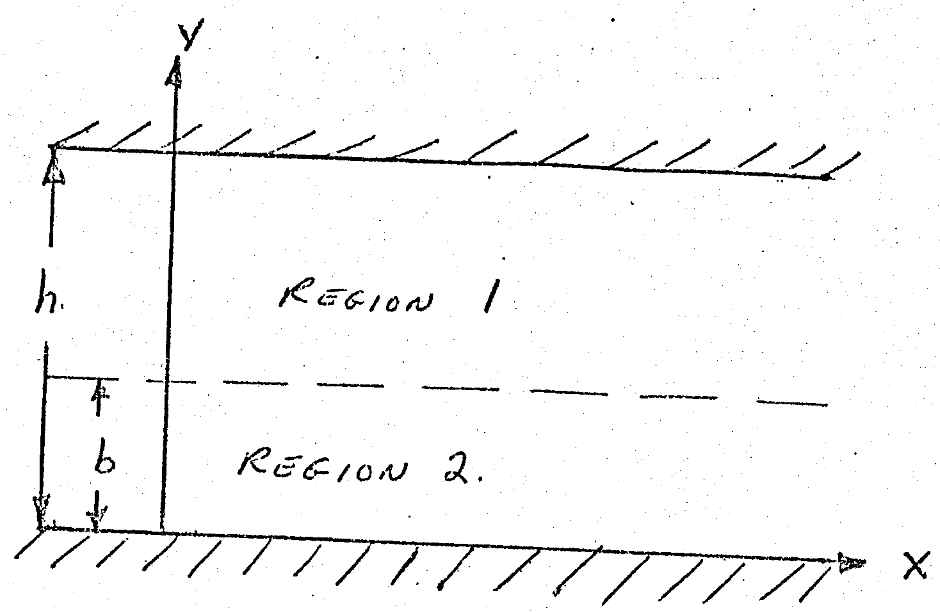


The shear stress acting on the wall is equal in magnitude but opposite in sense to that acting on the fluid.

For the fluid element adjacent to the upper wall the shear stress (i.e., $\mu \frac{du}{dy}$) is negative, which means that the shear that the shear stresses act as shown below.



Consider the steady, uniform flow of two immisicible fluids in a two-dimensional channel. (see figure below). The fluids have different viscosity coefficients μ_1 and μ_2 and different densities ρ_1 and ρ_2. Find (a) an expression for the velocity distribution across the channel (b) the average velocity in regions 1 and 2 (c) the shear stress at each wall and (d) is there any restriction on the relative magnitude of the density and/or viscosity ratios of the two streams?



7

Cesar Levy

10-7-71

Since steady flow $\partial u/\partial t = \frac{\partial v}{\partial t} = 0$

In each region we have

$$\rho_i \left[(\bar{q}_i \cdot \nabla) u_i \right] = -p_{i,x} + \mu_i \nabla^2 u_i$$

$$\rho_i \left[(\bar{q}_i \cdot \nabla) v_i \right] = -p_{i,y_i} + \mu_i \nabla^2 v_i$$

$$\rho_i (\nabla \cdot \bar{q}_i) = 0 \qquad \bar{q}_i = u_i \hat{i} + v_i \hat{j}$$

Since we have fully developed flow $u_{i,x} \quad v_{i,x} \quad u_{i,xx} \quad v_{i,xx} \equiv 0$

Therefore we reduce the above equations to

$$\rho_i v_i u_{i,y_i} = -p_{i,x} + \mu_i u_{i,y_i y_i}$$

$$\rho_i v_i v_{i,y_i} = -p_{i,y_i} + \mu_i v_{i,y_i y_i}$$

$$\rho_i v_{i,y_i} = 0$$

Since $\rho_i \neq 0 \quad v_{i,y_i} = 0$ also $v_{i,y_i y_i} = 0$

Therefore $-p_{i,y_i} = 0$ or $p_i = f_{1i}(x)$ or $p_i = (\text{const})_{1i}$

since $v_{i,y_i} = 0 \quad v_i = f_{2i}(x)$ or $v_i = (\text{const})_{2i}$

but since flow is fully developed $v_i \neq f_{2i}(x)$

y_1 ? due to the no slip condition at $y_1 = h$ and $y_2 = 0$ & $v_{i,y_i} = 0 \Rightarrow v_i \equiv 0$

y_2 ? Thus the above equations reduce to

$$p_{i,x} = \mu_i u_{i,y_i y_i}$$

$$p_{i,y_i} = 0 \qquad \text{take most general case } p_i = f_i(x)$$

since $p_i = p_i(x)$ only and $u_i = u_i(y)$ only then

$$\frac{dp_i}{dx} = \mu_i \frac{d^2 u_i}{dy_i^2} = \text{const} \qquad \text{due to boundary}$$

condition that $y_1(\cancel{u_1}) = h \quad y_2(\cancel{u_2}) = 0$ obtain

$$u_1 = A_1(y_1 - h) + \frac{y_1^2 - h^2}{2\mu_1} \frac{dp_1}{dx}$$

$$u_2 = A_2 y_2 + \frac{y_2^2}{2\mu_2} dp_2/dx$$

Since discontinuities are not allowed in viscous flows (only in shock waves) and if they did it would at most be a contact dis'continuity, at any given position x the pressure at the boundary must be the same. But since $p_i \neq f(y_i)$ then pressure is constant at any given x along cross-section. Since interface must remain at $y=b$ a pressure gradient in the y direction would tend to move boundary in direction of least pressure implying existence of v_{i,y_i} which contradicts results. Therefore p must be constant along cross-section.

At interface $u_1(b) = u_2(b)$ and $\tau_1(b) = \tau_2(b)$

From the above b.c.

$$A_1(b-h) + \frac{b^2-h^2}{2\mu_1}\frac{dp}{dx} = A_2\, b + \frac{b^2}{2\mu_2}\frac{dp}{dx}$$

$$A_1\mu_1 + b\frac{dp}{dx} = A_2\mu_2 + b\frac{dp}{dx}$$

Thus

$$A_1 = \frac{dp}{dx}\left[\frac{\mu_1 b^2 + \mu_2(h^2-b^2)}{2\mu_1(\mu_2(b-h)-\mu_1 b)}\right]$$

$$A_2 = \frac{dp}{dx}\left[\frac{\mu_1 b^2 + \mu_2(h^2-b^2)}{2\mu_2\{\mu_2(b-h)-\mu_1 b\}}\right]$$

$$B_1 = \frac{dp}{dx}\left[\frac{bh(b-h)[\mu_2-\mu_1]}{2\mu_1\{\mu_2(b-h)-\mu_1 b\}}\right]$$

Substitution into equations for u_1 and u_2 gives

$$u_1 = \frac{dp}{dx}(y_1-h)\left[\frac{y_1\{\mu_2(b-h)-\mu_1 b\} + b(b-h)\{\mu_1-\mu_2\}}{2\mu_1\{\mu_2(b-h)-\mu_1 b\}}\right]$$

$$u_2 = \frac{dp}{dx}\, y_2\left[\frac{\mu_1 b^2 + \mu_2(h^2-b^2) + \mu_2(b-h)y_2 - \mu_1 b y_2}{2\mu_2\{\mu_2(b-h)-\mu_1 b\}}\right]$$

$$\bar{u}_1 = \frac{\int_b^h u_1\,dy_1}{\int_b^h dy_1} = \frac{(h-b)}{12\mu_1}\frac{dp}{dx}\left[\frac{\mu_2(h-b)^2 - \mu_1 b(b-4h)}{\mu_2(b-h)-\mu_1 b}\right]$$

$$\bar{u}_2 = \frac{\int_0^b u_2\,dy_2}{\int_0^b dy_2} = \frac{1}{12\mu_2}\frac{dp}{dx}\left[\frac{\mu_1 b^2 - \mu_2(b+3h)(b-h)}{\mu_2(b-h)-\mu_1 b}\right]$$

$$\tau_i = \mu_i \frac{du_i}{dy_i} = A_i\mu_i + y_i\frac{dp_i}{dx}$$

$$\tau_{y=h} = A_1 \overset{\mu_1}{\cancel{y_i}} + h\frac{dp}{dx} = \frac{1}{2}\frac{dp}{dx}\left[(h-b) - \frac{\mu_1 bh}{\mu_2(b-h)-\mu_1 b}\right]$$

$$\tau_{y=0} = A_2\mu_2 = \frac{1}{2}\frac{dp}{dx}\left[\frac{\mu_2 h(h-b)}{\mu_2(b-h)-\mu_1 b} - b\right]$$

Overall Restrictions from math Solution:

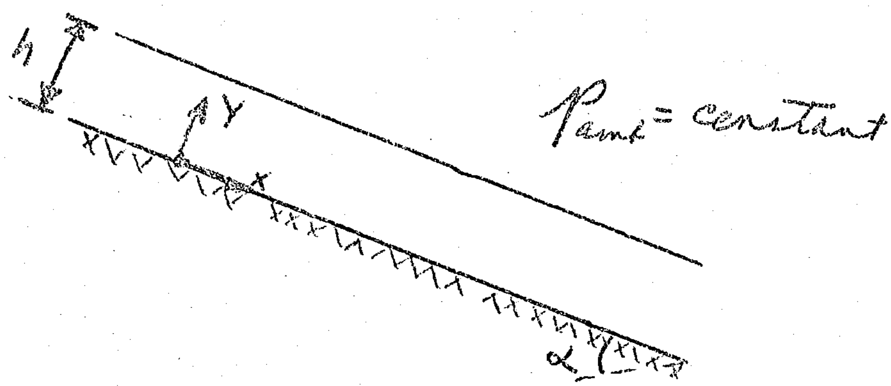
Since $\frac{dp_1}{dx} = \frac{dp_2}{dx}$ $\qquad \mu_1/\mu_2 = \frac{d^2u_2/dy_2^2}{d^2u_1/dy_1^2}$?

Also $\rho_1 < \rho_2$ otherwise Region 1 would displace Region 2

There exists no real life restriction on μ_1 and μ_2 since I can find two liquids having densities less than a third, yet one of the liquids will have viscosity less than the third, while the other liquid will have a viscosity greater than the third.

AE 343 - Homework #2

1. Consider a sheet of fluid of height h flowing down a two-dimensional flat plate inclined at an angle α to the horizontal as shown below. Find (a) the velocity profile for steady fully developed flow; (b) the shear stress at the wall and (c) the mean velocity.



2. Air at standard sea level conditions is flowing through a 4-inch diameter pipe at a rate of 1 pound per second. For the case of fully developed flow, determine the pressure gradient in the pipe and the skin friction drag acting on the pipe per unit length.

$$R_N(y) = \rho \frac{Ua}{\mu} = \rho \frac{U(y)a}{\mu}$$

$$\frac{d R_N(y)}{dy} = \frac{\rho}{\mu} a \, u_{,y}$$

$$\bar{u}_{mean} = \frac{-R^2}{8\mu} \frac{dp}{dx}$$

$$\frac{d^2 R_N(y)_1}{dy_1^2} = \frac{\rho_1}{\mu_1} a \, u_{,y_1 y_1}$$

$$\frac{ft^2}{\frac{lb\text{-}sec}{ft^2}} \cdot \frac{lb}{ft^3 \cdot ft}$$

$$\frac{R_1}{R_2} = \frac{\frac{\rho_1}{\mu_1} u_{,y_1 y_1}}{\frac{\rho_2}{\mu_2} u_{,y_1 y_1}}$$

$$\frac{\mu_1}{\mu_2} = \frac{u_{,y_2 y_2}}{u_{,y_1 y_1}}$$

ν $\quad$ ft/sec

$$= \frac{\rho_1}{\rho_2}$$

$$\frac{\frac{lb\,sec\,ft^2}{ft^4}}{\frac{\#\text{-}sec^2}{ft^4}} \qquad \frac{ft^2}{sec} \qquad \frac{ft^2/sec}{ft^2 \cdot ft/sec^2}$$

$$\frac{lb}{ft^2} = \mu \frac{ft/sec}{ft} \qquad \frac{lb\,sec}{ft^2}$$

$$\frac{M}{L^3} = \frac{\#\text{-}sec^2}{ft^4} = \rho \qquad \frac{\mu}{\rho}$$

$$\frac{\frac{lb\text{-}sec\;ft^2}{\cancel{ft^4}}}{\frac{lb\text{-}sec^2}{\cancel{ft^4}}} = \frac{ft^2}{sec}$$

$$\frac{\Delta u}{\Delta y} = \frac{ft/sec}{ft} = \frac{1}{sec}$$

$\mu \qquad \frac{1}{sec}$

$$\tau = \frac{lb}{ft^2} \qquad \mu = \frac{lb\text{-}sec}{ft^2}$$

$$\frac{lb-sec^2}{ft^4} \cdot \frac{ft}{sec} \cdot ft^2 \cdot g \qquad \rho g \bar{u} A = 1$$

$$\frac{lb-sec}{ft} \cdot \frac{ft}{sec^2}$$

$$72 \overline{)32.2} = .447$$

28.8

3.40

2.88

.52

$.447(207)$

$$\frac{-R^2}{8\mu}\frac{dp}{dx} = \bar{u}_{mean} \qquad ft/sec \cdot ft^2 \cdot \frac{\#}{ft^3}$$

207

.447

1449

8280

82800

92529

$$\pi R^2 \quad A = ft^2 \qquad \frac{\frac{ft^2}{sec} \cdot lb}{-ft^2 \cdot \cancel{\frac{ft}{sec^2}}}$$

$$\rho g = \frac{\# - sec^2}{ft^4} \cdot \frac{ft}{sec^2} = \frac{\#}{ft^3}$$

$$\bar{u}_{mean} = \frac{1}{A\rho g} = \frac{1}{\pi R^2 \rho g}$$

$$\frac{-R^2}{8\mu}\frac{dp}{dx} = \frac{1}{\pi R^2 \rho g} \qquad \frac{dp}{dx} = \frac{-8\mu}{\pi R^4 \rho g} = \frac{-8\nu}{\pi R^4 \cancel{\rho} g}$$

$$\frac{\frac{ft^2}{sec}}{ft^4 \cdot \#/ft^3} = \frac{ft^2/sec}{\# ft} = \frac{ft}{\# sec}$$

D=

$$\frac{-8\nu}{R^4 g} = \frac{ft^2/sec^2\ sec}{ft^4 \cdot ft/sec^2} = \frac{ft^2 - sec}{ft^5} = \frac{sec}{ft^3}$$

$$\frac{\cancel{ft^2}}{\frac{\cancel{lb}-sec}{\cancel{ft^2}}} \cdot \frac{\cancel{\#}}{\cancel{ft^3}} ~~ft~~$$

$$\frac{dp}{dx} = \frac{1}{\frac{-\pi R^4 \rho g}{8\mu}} \quad lb/sec \quad = \frac{8\nu \cdot 1}{-\pi R^4 \cancel{\rho} g}$$

$$= \frac{1}{ft^4.}$$

$$\pi \frac{8\nu}{-\pi R^4 g} \; \pi R^2 = \frac{8\nu \cdot 1}{-R^2 g}$$

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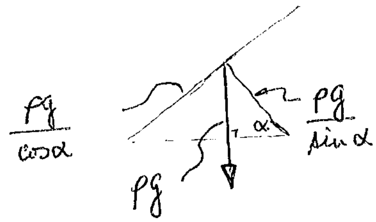
AE333

8, 7

since body force cannot be neglected

$$\rho \frac{Du}{Dt} = -p_{,x} + \mu \nabla^2 u + X\rho$$

$$\rho \frac{Dv}{Dt} = -p_{,y} + \mu \nabla^2 v + Y\rho$$



$$\therefore \quad X = \frac{\rho g}{\sin\alpha}$$

$$Y = \frac{-\rho g}{\cos\alpha}$$

at $y = 0 \quad u = 0$
$y = h \quad \tau = 0$ } since no force is exerted on upper surface

since fully developed steady flow $\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = \frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial x^2} = 0$

$$\rho \left[v \frac{\partial u}{\partial y} \right] = -p_{,x} + \mu \frac{\partial^2 u}{\partial y^2} + \frac{\rho g}{\sin\alpha}$$

$$\rho \left[v \frac{\partial v}{\partial y} \right] = -p_{,y} + \mu \frac{\partial^2 v}{\partial y^2} - \frac{\rho g}{\cos\alpha}$$

since continuity depends only on the change of mass in a control volume

$$\frac{D\rho}{Dt} + \rho(\nabla \cdot \bar{q}) = 0 \quad \text{or} \quad \frac{\partial v}{\partial y} = 0 \qquad \text{since } \rho \neq 0 \quad \rho = \text{const}$$

$\therefore \quad v = f(x)$ or const cannot be $f(x)$ since $\frac{\partial v}{\partial x} = 0$ ✓ $\therefore \; v = \text{const}$

since at wall $v = 0$ $v \equiv 0$ everywhere

$\therefore \quad \frac{\partial v}{\partial y} = \frac{\partial^2 v}{\partial y^2} = 0$ then

$$\rho(0) = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} + \frac{\rho g}{\sin\alpha}$$

$$0 = -\frac{\partial p}{\partial y} + \mu(0) - \frac{\rho g}{\cos\alpha}$$

$$\therefore \quad p = -\frac{\rho g}{\cos\alpha} y + f(x) \quad \Rightarrow \quad p_{,x} = f'(x) = \mu \frac{\partial^2 u}{\partial y^2} + \frac{\rho g}{\sin\alpha}$$

since $f = f(x)$ only and since $u \neq u(x)$

$$f'(x) = \mu \frac{d^2 u}{dy^2} + \frac{\rho g}{\sin\alpha} = C_a$$

$\therefore \quad p = -\frac{\rho g}{\cos\alpha} y + \left[\mu \frac{d^2u}{dy^2} + \frac{\rho g}{\sin\alpha}\right] x + C_2$

at $y = h \quad p = p_{amb} \quad \forall x$

$$p_{amb} = -\frac{\rho g}{\cos\alpha} h + \left[\mu \frac{d^2u}{dy^2} + \frac{\rho g}{\sin\alpha}\right] x + C_2$$

$\therefore \quad C_2 = p_{amb} + \frac{\rho g}{\cos\alpha} h - \left[\mu \frac{d^2u}{dy^2} + \frac{\rho g}{\sin\alpha}\right] x$ then

$p = +\frac{\rho g}{\cos\alpha}\left[h - y\right] + p_{amb} \quad \therefore \quad p = p(y)$ only $\quad \therefore \quad p_{,x} \equiv 0$

or $C_a = 0$

$\therefore \quad \mu \frac{d^2u}{dy^2} + \frac{\rho g}{\sin\alpha} = 0 \qquad u = \frac{-\rho g}{2\mu \sin\alpha} y^2 + C_1 y + C_3$

since $u = 0$ at $y = 0$ (no slip condition) $C_3 = 0$

since $\tau = 0$ at $y = h$ (free surface no shear exists)

$\therefore \quad \frac{\partial u}{\partial y} = 0 = \frac{-\rho g}{\mu \sin\alpha} h + C_1 \qquad C_1 = \frac{\rho g}{\mu \sin\alpha} h$

$$u = \frac{\rho g}{\mu \sin\alpha}\left[h y - \frac{y^2}{2}\right]$$

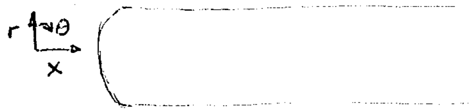
$$\bar{u} = \frac{\int_0^h \frac{\rho g}{\mu \sin\alpha}\left[h y - \frac{y^2}{2}\right] dy}{h} = \frac{\frac{\rho g}{\mu \sin\alpha}\left[\frac{h y^2}{2} - \frac{y^3}{6}\right]}{h}$$

$$= \frac{\rho g}{\mu \sin\alpha}\left[\frac{h^3}{3h}\right] = \frac{\rho g h^2}{3\mu \cancel{\sin\alpha}} \sin\alpha$$

$$\tau_w = \mu \frac{du}{dy} = \mu\left[\frac{\rho g}{\mu \sin\alpha}(h - y)\right]_{y=0} = \frac{\rho g h}{\cancel{\sin\alpha}} \sin\alpha.$$

for fully developed flow $u_r, u_\theta = 0$ since $u_{\theta,x} = u_{r,x} = 0$ due to symmetry $u_\theta = 0$

and since no flow outward $u_r = 0$ also $p = p(x)$ only



$$\frac{dp}{dx} = -\mu \left[u_{,rr} + \frac{1}{r} u_{,r} \right] = \frac{\mu}{r} \frac{d}{dr}\left(r \frac{du}{dr} \right)$$

at $r = R$ $u = 0$; $r = 0$ $\frac{du}{dr} = 0$ using the above

$$u = \frac{r^2 - R^2}{4\mu} \frac{dp}{dx}$$

$$D_{sf} = \tau_w S = \tau_w (2\pi R L) = \mu \left(\frac{du}{dr}\right)_{r=R} \cdot 2\pi R \cdot 1$$

this is drag acting on fluid particles about the wall.

$$\frac{du}{dr} = \frac{r}{2\mu} \frac{dp}{dx} ; \qquad D_{sf} = \frac{R}{2} \frac{dp}{dx} \cdot 2\pi R = \pi R^2 \frac{dp}{dx}$$

$$\bar{u}_{mean} = \frac{\int_0^R u \, (2\pi r) dr}{\int_0^R 2\pi r dr} = \frac{-R^2}{8\mu} \frac{dp}{dx}$$

$$\rho g \bar{u}_{mean} A = 1 \text{ lb/sec}$$

$$\rho g \frac{-R^2}{8\mu} \frac{dp}{dx} (\pi R^2) = 1 \quad \Rightarrow \quad \frac{dp}{dx} = \frac{1 \cdot 8\mu}{-\rho g \pi R^4}$$

$$D_{sf} = \pi R^2 \left[\frac{8\mu}{-\rho g \pi R^4} \right] = \frac{8\mu}{-\rho g R^2}$$

$$= \frac{-8(3.719 \times 10^{-7})}{(.002378)(32.2)(.166)^2} = -1.411 \times 10^{-3} \quad \#$$

this is drag acting on particles of fluid

since the shear on the walls is $-\mu \frac{du}{dr}\Big|_{r=R}$ then

$$D_{sf \text{ on wall}} = +1.411 \times 10^{-3} \quad ✓$$

4

7, 8 C. Levy

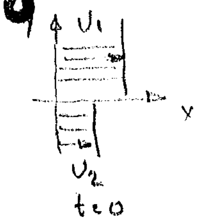
1. Using Rayleigh Problem similarity

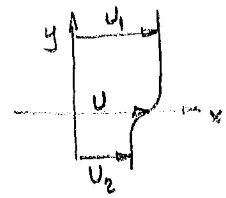
$$u_{,t} = \nu\, u_{,yy} \qquad p = \text{const} \quad v \equiv 0 \quad \forall (y, x)$$

$u = U_1$ @ $y = \infty$ $t \geq 0$

$u = U_2$ @ $y = -\infty$ $t \geq 0$

$u = U$ @ $y = 0$ $t > 0$





if $u = U_1 f(\eta)$ where $\eta = k y t^m$

it can be shown that the governing equation is

$$f'' + 2\eta f' = 0 \qquad \text{where} \quad \eta = \frac{y}{2\sqrt{\nu t}}$$

if $f' = A e^{-\eta^2}$ then

$$f = A \int_0^\eta e^{-\xi^2} d\xi + B \; ;$$

since $y = \infty$ $\eta = \infty$ $t \geq 0$

$$U_1/U_1 = f(\eta = \infty) = 1$$

since $y = -\infty$ $\eta = -\infty$ $t \geq 0$

$$U_2/U_1 = f(\eta = -\infty) = -1$$

since $y = 0$ $\eta = 0$ $t > 0$

$$U/U_1 = f(0)$$

$$\therefore \quad \frac{U_1}{U_1} = A \frac{\sqrt{\pi}}{2}\, \text{erf}(\eta) + B = 1$$

erf(∞)

$$\therefore \quad \frac{U_2}{U_1} = A \frac{\sqrt{\pi}}{2}\, \text{erf}(\eta) + B \; ;$$

erf(−∞)

?

$$\therefore \quad U/U_1 = A \frac{\sqrt{\pi}}{2}\, \text{erf}(0) + B = B \; ;$$

since $1 = A \frac{\sqrt{\pi}}{2} \cdot 1 + U/U_1$ then $A = \dfrac{1 - U/U_1}{\sqrt{\pi}/2}$ then

$$f = (1 - U/U_1)\, \text{erf}(\eta) + U/U_1$$

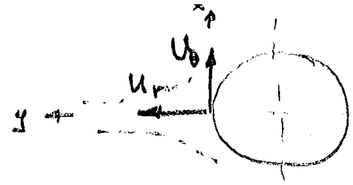
since $U_2/U_1 = (1 - U/U_1)(-1) + U/U_1$ $\qquad U = \dfrac{U_2 + U_1}{2}$

$$\therefore \quad f = \left(1 - \frac{U_2 + U_1}{2U_1}\right) \text{erf}(\eta) + \frac{U_2 + U_1}{2U_1}$$

or

$$u/U_1 = \left[\frac{1 - U_2/U_1}{2}\right] \text{erf}(\eta) + \left(\frac{1 + U_2/U_1}{2}\right)$$

2. since $\psi = -U_\infty r \sin\theta \left(1 - \frac{R^2}{r^2}\right)$



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$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = -U_\infty \cos\theta \left(1 - R^2/r^2\right)$$

$$u_\theta = -\frac{\partial\psi}{\partial r} = +U_\infty \sin\theta \left(1 + R^2/r^2\right)$$

since u_θ at stag pt $= U_\infty \sin\theta\,(2) = 2U_\infty \frac{x}{R}$ since $\sin\theta \sim \theta = \frac{x}{R}$

$$\therefore \quad U = 2U_\infty \frac{x}{R} = x f'(y) \qquad f'(y) = \frac{2U_\infty}{R} \quad ? \quad f(y) = \frac{2U_\infty}{R} y$$

$$\therefore \quad V = -f(y) = -\frac{2U_\infty y}{R}$$

$$\delta = 2.4\sqrt{\frac{\nu}{a}} \qquad \text{where } a = \frac{2U_\infty}{R} = \frac{2(100)}{1} = 200\ \frac{1}{\text{sec}}$$

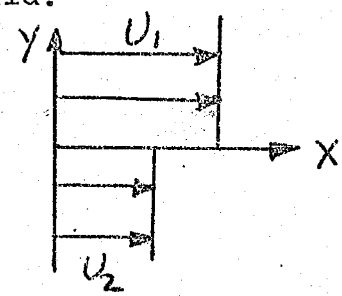
$$\delta = 2.4\sqrt{\frac{1.6\times10^{-4}}{200}} = 2.4\sqrt{.8\times10^{-6}} = 2.4\,(.895\times10^{-3})\ ft$$

$$\delta = 2.145\times10^{-3}\ ft$$

AE-333 Fluid Dynamics III

Homework No. 3

1. Consider at time t=0 the following velocity distribution in an infinite fluid:



Deterime the velocity distribution $(\frac{u}{U_1})$ for the developing viscous flow.

2. The stream function for a circular cylinder in potential flow is

$$\psi = - U_{\infty} r \sin\theta (1 - \frac{R^2}{r^2})$$

solution of stag pt is only valid in region about stag pt.

a) Show that the u and v velocity components (i.e., tangent and normal to the cylinder, respectively) in the vicinity of the stagnation point are

$$U = 2U_{\infty}\frac{x}{R} \quad , \quad V = - 2U_{\infty}\frac{y}{R}$$

b) Determine the boundary layer thickness on the cylinder at the stagnation point for R=1 ft., V_{∞}=100 ft/sec., and ν=1.6x10^{-4} ft^2/sec.

$$\frac{u}{U_1} = \frac{U_2/U_1\,(V+1) + (V-1)}{2V}$$

$$\therefore \quad \tau_{xx} = -3p + 3\lambda\, \nabla\cdot\bar{q} + 2\mu\, \nabla\cdot\bar{q}$$

$$= -3p + \frac{(3\lambda+2\mu)}{\rho}\frac{D\rho}{Dt} \qquad \text{since } p = f(\rho) \neq f(\dot{\rho})$$

$$\therefore \text{ coeff of } \dot{\rho} = 0 \qquad \therefore \quad 3\lambda + 2\mu = 0$$

$$\lambda = -\frac{2}{3}\mu$$

$$\therefore \quad \tau_{ii} = -3p - 2\mu\left[\frac{1}{3}\nabla\cdot\bar{q} - \frac{\partial u_i}{\partial x_i}\right]$$

also proven $\tau_{ij} = \mu\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right]$

$$\therefore \quad \frac{D\bar{q}}{Dt} = X_i + \frac{1}{\rho}\left[\tau_{ii,i} + \tau_{ji,j} + \tau_{ki,k}\right] \longrightarrow$$

in x dir $\quad \frac{\partial u}{\partial t} + \bar{q}\cdot\nabla u = X + \nu\nabla^2 u - p_{,x} \qquad$ if μ & ρ are const.

$$\frac{\partial v}{\partial t} + \bar{q}\cdot\nabla v = Y + \nu\nabla^2 v - p_{,y}$$

$$\frac{\partial w}{\partial t} + \bar{q}\cdot\nabla w = Z + \nu\nabla^2 w - p_{,z}$$

For fully developed flow all derivatives with respect to x are zero

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho u^2}$$

$+ \quad \tau_{ij}$ $\quad j$ denotes direct it acts in
i denotes $\perp$ direction to face it acts on

When flow is radial due to symmetry $u = u(r)$ only
$u_\theta = 0 \quad u_r = 0$ since no velocity at walls of tube

Rayleigh Problems since flow is started suddenly $p_{,x} = 0$, $v = 0$
for boundary layer at stag pt $\delta = 2.4\sqrt{\nu/a}$ where a solves $U = +ax$

$$\frac{\partial}{\partial t}\int_V \rho\, dV = -\int_A \rho \bar{q}\cdot \bar{n}\, dA$$

$$\int_A \rho\bar{q}\cdot\bar{n}\, dA = \int_V \nabla\cdot(\rho\bar{q})\, dV$$

$$\therefore \quad \frac{\partial}{\partial t}\int_V \rho\, dV + \int_V \nabla\cdot(\rho\bar{q})\, dV = 0$$

since dV is arbitrary $\quad \frac{\partial}{\partial t}\int_V \rho\, dV = \int_V \frac{\partial\rho}{\partial t} dV$

$$\int_V \left\{\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\bar{q})\right\} dV = 0$$

note that $\nabla\cdot(\rho\bar{q}) = \rho\nabla\cdot\bar{q} + \bar{q}\cdot\nabla\rho$

$$\frac{\partial\rho}{\partial t} + \bar{q}\cdot\nabla\rho \equiv \frac{D\rho}{Dt} \quad \therefore \quad \frac{D\rho}{Dt} + \rho\nabla\cdot\bar{q} = 0$$

if $\rho = \text{const}$ $\quad \nabla\cdot\bar{q} = 0$

$$\frac{\partial}{\partial t}\int_V \rho\bar{q}\, dV + \int_A \rho\bar{q}\,(\bar{q}\cdot\bar{n})\, dA = \int_V \rho X_i\, dV + \int_A P_{ij}\,\bar{n}\, dA$$

$$\frac{\partial}{\partial t}\int_V \rho\bar{q}\, dV + \int_V \rho\bar{q}\,(\nabla\cdot\bar{q})\, dV = \int_V \rho X_i\, dV + \int_V \nabla P\, dV$$

$$\left[\frac{\partial}{\partial t}\rho\bar{q}\right] + \rho\bar{q}\left(\nabla\cdot\bar{q}\right) = \rho X_i + \nabla P$$

$$\frac{D\rho\bar{q}}{Dt} = \rho X_i + \nabla P$$ if ρ is const then

$$\rho\frac{D\bar{q}}{Dt} = \rho X_i + \nabla P \quad \therefore \quad \frac{D\bar{q}}{Dt} = X_i + \frac{\nabla\tilde{P}}{\rho}$$

note $\tilde{P} = \tau_{xx} + \tau_{yx} + \tau_{zx}$

note $\tau_{ij} = \tau_{ji}$ and also $\tau_{xx} = -p + \lambda\nabla\cdot\bar{q} + 2\mu\frac{\partial u}{\partial x}$

also note that $\nabla\cdot\bar{q} = -\frac{1}{\rho}\frac{D\rho}{Dt}$ from continuity

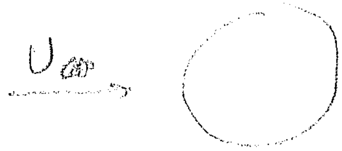
for creep flow all terms on left hand side of Navier-Stokes Equation → 0
for plane creep flow

$$① \; p_{,x_i} = \mu \nabla^2 u_i$$ from these exact equa. $\nabla^4 \psi = 0$ for $R_N \ll 1$

also $\sum_i \frac{\partial}{\partial x_i} ① = \nabla^2 p$ if $u = \frac{\partial \psi}{\partial y}$ $v = -\frac{\partial \psi}{\partial x}$ then $\nabla^4 \psi = 0$

for flow past a sphere $v_\varphi = 0$ $v_r = \frac{1}{r^2 \sin\theta} \frac{\partial \psi}{\partial \theta}$

$$v_\theta = -\frac{1}{r \sin\theta} \frac{\partial \psi}{\partial r}$$



at $r = R$ $v_r = v_\theta = 0$ ∴ $\psi_{,\theta} = \psi_{,r} = 0$ $E^4 \psi = \nabla^4 \psi = 0$ for radial, circum. ∮

$r = \infty$ $v_r = U_\infty \cos\theta$ $v_\theta = -U_\infty \sin\theta$

ψ viscid leads to $\frac{1}{2} U_\infty f(r) \sin^2\theta$ since ψ varies with r
since conditions at diff r change. $\xrightarrow{\text{put into } E^4\psi = 0}$ $\left(\frac{d^2}{dr^2} - \frac{2}{r^2}\right)^2 f(r) = 0$

Assume $f(r) = C r^n \rightarrow n = -1, 2, 1, 4$

or $\psi = \frac{U_\infty}{2}\left[A/r + Br + Cr^2 + Dr^4\right] \frac{1}{r^2 \sin\theta} \sin 2\theta$

from BC $C = 1, D = 0, A = \frac{R^3}{3}$ & $B = -\frac{3R}{2}$.

this gives $\frac{(p_s - p_\infty)\, vis}{(p_s - p_\infty)\, inv} = \frac{3}{R_{N\, U_\infty, R}}$, $R_N < 1$

Oseen flow $u = U_\infty + u'$ neglect higher order terms. not good near body

2nd Highest

1st 78 MLP

Med was 45

EXAMINATION BOOK

Name Cesar Levy

Subject AE 333

Instructor Dr. Visich

Exam Seat No. ______ Section A

Date Nov. 10, 1971 Grade ______

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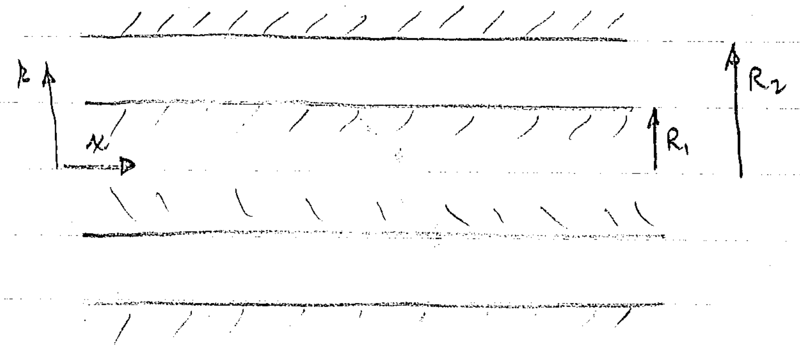


first due to symmetry $\frac{\partial}{\partial\theta}(u, v_r, v_\theta) = 0$, & $v_\theta = 0$ everywhere due to symmetry since flow is fully developed the derivatives of the velocity components with respect to x, are 0

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial u}{\partial x} = 0 \qquad \text{Continuity}$$

$\therefore$ since $v_\theta = 0$ & $u_{,x} = 0$ then

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} = 0 \qquad v_r \neq v_r(x) \ \& \ v_r \neq v_r(\theta)$$

$$\therefore \quad \frac{dv_r}{dr} + \frac{v_r}{r} = 0$$

$$\ln v_r = -\ln r + \ln A \ \therefore$$

$$v_r = \frac{A}{r}$$

since $v_r\Big|_{r=R_1} = 0$ then $A = 0$ $\therefore$

$$v_r, v_\theta = 0$$

thus from the above & Navier Stokes for Cylindrical Coordinates

$$\cancel{\frac{\partial p}{\partial r}} \qquad + \frac{\partial p}{\partial x} = \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{du}{dr}\right)\right]$$

also $\frac{\partial p}{\partial \theta} = 0$ & $\frac{\partial p}{\partial r} = 0$ $\therefore$ $\frac{\partial p}{\partial x} = \frac{dp}{dx}$

$$dr\,\frac{r}{\mu}\frac{dp}{dx} = d\left(r\frac{du}{dr}\right) \quad \therefore$$

$$\frac{r^2}{2\mu}\frac{dp}{dx} + A = r\frac{du}{dr} \ ; \qquad \frac{r}{2\mu}\frac{dp}{dx} + \frac{A}{r} = \frac{du}{dr}$$

$$B + \frac{r^2}{4\mu}\frac{dp}{dx} + A\ln r = u$$

$$\frac{r^2}{4\mu}\frac{dp}{dx} + A\ln r + B = u$$

at $r = R_1$ then $u = 0$ ✓

at $r = R_2$ then $u = 0$ ✓ $\therefore$

$$\frac{R_1^2}{4\mu}\frac{dp}{dx} + A\ln R_1 + B = 0$$

$$\frac{R_2^2}{4\mu}\frac{dp}{dx} + A\ln R_2 + B = 0$$

$$A = \frac{\begin{vmatrix} -\frac{R_1^2}{4\mu}\frac{dp}{dx} & 1 \\ -R_2^2/4\mu\, dp/dx & 1 \end{vmatrix}}{\begin{vmatrix} \ln R_1 & 1 \\ \ln R_2 & 1 \end{vmatrix}} = \frac{\left(R_2^2 - R_1^2\right)\left[\frac{dp}{dx}\cdot\frac{1}{4\mu}\right]}{\ln\left(R_1/R_2\right)}$$

$$A = \frac{-\left(K^2 - 1\right)\frac{dp}{dx}\left(\frac{1}{4\mu}\right)R_1^2}{\ln K} \quad ✓$$

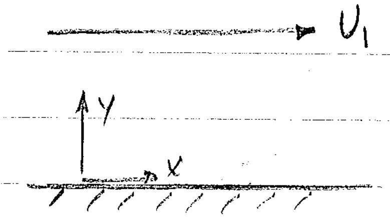
$K = R_2/R_1$

$$B = \frac{\begin{vmatrix} \ln R_1 & -R_1^2/4\mu\,\frac{dp}{dx} \\ \ln R_2 & -R_2^2/4\mu\,\frac{dp}{dx} \end{vmatrix}}{\left| -\ln K \right|} = \frac{\frac{1}{4\mu}\frac{dp}{dx}\left[(\ln R_2)\cdot R_1^2 - (\ln R_1)R_2^2\right]}{-\ln K}$$

$$u = \frac{1}{4\mu}\frac{dp}{dx}\left[r^2 - R_1^2\frac{(K^2-1)}{\ln K}\ln r + \frac{R_2^2\ln R_1 - R_1^2\ln R_2}{\ln K}\right] ✓$$

$$\cancel{K^2\left[\frac{R_1^2\ln R_1 - R_2^2\ln R_2}{\ln K}\right]}$$

2.



at $y=0$ $u=0$ (2)

$y=h$ $u=U_1$

Since fully developed flow all derivatives with respect to x are 0. & $\frac{\partial u}{\partial t}=0$

Then Continuity gives $v_{,y}=0$ $\therefore$ $v=v(y)$ only since $v \neq v(x)$
but since $v\big|_{y=0}=0$ then $v\equiv 0$ $\forall (x,y)$
therefore

$$+\frac{\partial p}{\partial x}=\mu\frac{\partial^2 u}{\partial y^2} \qquad \frac{\partial p}{\partial y}=0 \quad \therefore \quad p\neq p(y) \Rightarrow p=p(x) \text{ only}$$

$$\therefore \quad \frac{dp}{dx}=\mu\frac{d^2u}{dy^2} \quad (3) \quad \text{since} \quad u=u(y) \text{ only } \& \ p=p(x) \text{ only}$$

then $$\frac{du}{dy}=\frac{y}{\mu}\frac{dp}{dx}+A \qquad u=\frac{y^2}{2\mu}\frac{dp}{dx}+Ay+B$$

at $y=0$ $u=0$ $\Rightarrow B=0$

at $y=h$ $u=U_1$ $\therefore$ $$U_1=\frac{h^2}{2\mu}\frac{dp}{dx}+Ah$$

$$yA=\frac{U_1 y}{h}-\frac{hy}{2\mu}\frac{dp}{dx}$$

$$\therefore \quad u=\left(\frac{y^2-yh}{2\mu}\right)\frac{dp}{dx}+\frac{U_1}{h}y$$

now $$u/U_1=\left(\frac{y^2-yh}{2\mu}\right)\frac{1}{U_1}\frac{dp}{dx}+\frac{y}{h}$$

$$u/U_1=\left(\left(\frac{y}{h}\right)^2-\frac{y}{h}\right)\frac{h^2}{2\mu U_1}\frac{dp}{dx}+\frac{y}{h} \quad (5)$$

1. The force required to hold lower plate stationary $=0$, find $\frac{dp}{dx}$

$$\mu\frac{du}{dy}=y\frac{dp}{dx}+\mu\frac{U_1}{h}-\frac{h}{2}\frac{dp}{dx}=\frac{dp}{dx}\left[y-\frac{h}{2}\right]+\mu\frac{U_1}{h}$$

Since the force is 0 at $y=0$ $\quad T\big|_{y=0}=-\frac{dp}{dx}\frac{h}{2}+\mu U_1/h$

$\tau \cdot (\text{area of plate}) = 0$ $\therefore$

$$\text{area}\left[\frac{dp}{dx}\frac{h}{2}\right] = \left[\mu \frac{U_1}{h}\right] \text{area of plate}$$

$$\therefore \quad \frac{dp}{dx} = \frac{2\mu U_1}{h^2} \quad (\text{is } +)$$

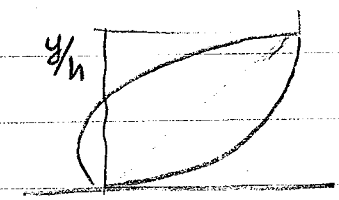
5

2. The direction of force is in + x direction if gradient is favorable.

$$\text{Force} = \tau(\text{area of plate}) = \left[y\Big|_{y=h}\frac{dp}{dx} + \frac{U_1\mu}{h} - \frac{h}{2}\frac{dp}{dx}\right]\text{area of plate}$$

$$= \left[\frac{h}{2}\frac{dp}{dx} + \frac{U_1\mu}{h}\right](\text{area of plate})$$

4



Check for 1:
$$\frac{d(u/U_1)}{d(y/h)} = \left(\frac{2y}{h} - 1\right)\frac{h^2}{2\mu U}\frac{dp}{dx} + 1$$

at $y = 0$
$$-\frac{h^2}{2\mu U}\frac{dp}{dx} = -1$$

$$\frac{dp}{dx} = \frac{2\mu U}{h^2}$$

3. for a suddenly accelerated plane wall at $t \leq 0$ $u = 0$ $y = 0$
at $t > 0$ $u = U$ $y = 0$

Since no mechanism exists for a Δp to exist in x direction $\frac{\partial p}{\partial x} = 0$ and ~~since no mechanism exists for Δp to exist in y direction~~ it is assumed $u = u(y, t)$, it can be shown that $v \equiv 0$ everywhere from continuity thus resulting in $p \neq f(y)$ either

$\therefore$ the navier-stokes relation becomes

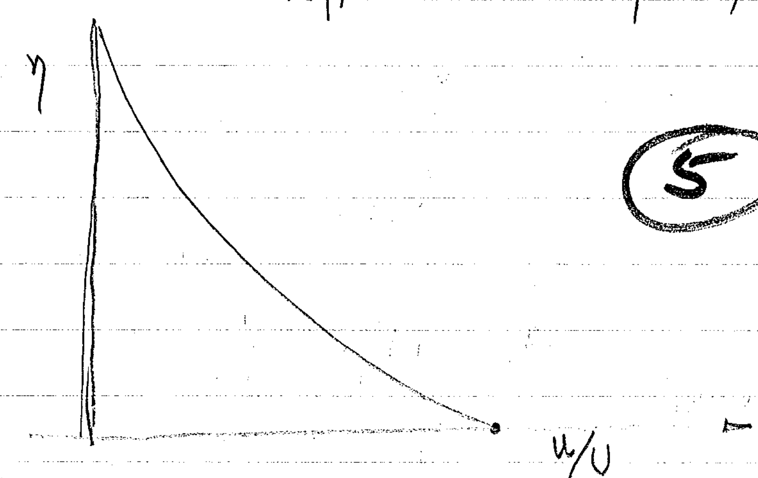
$$u_{,t} = \nu u_{,yy}$$

8

b. assume $u = U f(\eta)$ when $\eta = k y t^m$ when $m = -\frac{1}{2}$, $k = \frac{1}{2\sqrt{\nu}}$

5

c.



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knowing $u = U f(\eta)$ & $\eta = \frac{y}{2\sqrt{\nu t}}$

we can find $f(\eta) = -\frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\xi^2} d\xi + 1$

$$\therefore \quad u = -\frac{2U}{\sqrt{\pi}} \int_0^\eta e^{-\xi^2} d\xi + U$$

d. to find the speed of propagation of the edge ~~of the fluid~~ of disturbance layer into undisturbed fluid take $\left.\frac{dy}{dt}\right|_{\text{for } \eta = 1.8}$ (when $u/U = .01$)

7

Prof. Visich I didn't quite understand part d: My interpretation of it was finding the change in y with respect to t evaluated at the defined limit of the boundary layer

$\eta = \frac{y}{2\sqrt{\nu t}}$ at $y = \delta$ $\eta = 1.8$ $u/U_1 = .01$

$\therefore \quad y = 3.6\sqrt{\nu t}$ $\qquad \frac{dy}{dt} = \frac{3.6}{2}\sqrt{\nu/t} = 1.8\sqrt{\nu/t}$

$\therefore \quad \frac{dy}{dt} = 1.8\sqrt{\nu/t}$

$\frac{dy}{dt} = \frac{dy}{d\eta}\frac{d\eta}{dt} + \frac{dy}{dt}$ $\qquad \eta = f(y,t)$

$y = f(\eta, t)$

$y = 2\eta\sqrt{\nu t}$ $\qquad \frac{\partial y}{\partial \eta} = 2\sqrt{\nu t}\cdot\frac{-\frac{1}{2}y}{2\sqrt{\nu t^3}}$

$= -\frac{1}{2}\frac{y}{t} + \eta\sqrt{\nu/t}$

$$\frac{dy}{dt} = \frac{\partial y}{\partial \eta}\frac{\partial \eta}{\partial t} + \frac{\partial y}{\partial t} = 2\sqrt{\nu t}\cdot\frac{-\frac{1}{2}y}{2\sqrt{\nu t^3}} + \frac{2}{2}\eta\sqrt{\nu/t}$$

$$\frac{dy}{dt} = -\frac{1}{2}\,y/t + \eta\sqrt{\nu/t}$$ $\qquad -\frac{1}{2}\; -1.8\sqrt{\nu/t} + \eta\sqrt{\nu/t}$

$\eta = y/2\sqrt{\nu t}$ at $y = \delta$ $\eta = 1.8$ $u/U = .01$

$\therefore \quad 3.6\sqrt{\nu t} = y \quad \therefore \quad 1.8\sqrt{\nu/t} = dy/dt\Big|_{\text{of b.l.}}$

4. We make the assumption that it is creep flow & that $R_N \ll 1$ and viscous terms are $\gg$ inertia terms $\ni \nabla^2 p = 0$

we assume that the velocity

V are the same μ is the same C_D are the same

$$\therefore \quad C_D = \frac{24}{R_{N,D_1}} = \frac{24}{R_{N,D_2}}$$

$$\therefore \quad \frac{\rho_1 V D_1}{\mu} = \frac{\rho_2 V D_2}{\mu} \qquad \therefore \quad \rho_1 D_1 = \rho_2 D_2$$

$$\frac{\rho_1}{\rho_2} = \frac{D_2}{D_1} \qquad \therefore \quad \frac{\rho_1}{\rho_2} = \frac{1}{2}$$

$D_2 = 2D_1$

$\therefore$ the body with diameter twice as large must have a density one half as large

the drag acting on the body

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 \pi R^2} = \frac{6\mu \pi U R}{\frac{1}{2}\rho U^2 \pi R^2} = \frac{\mu\, 12}{\rho V R}$$

$$\therefore \quad C_D = \frac{24}{R_{N,V,D_1}} = \frac{24}{R_{N,V,D_2}}$$

① body 1
② body 2

?

$$\therefore \quad \frac{\rho_1 V D_1}{\mu} = \frac{\rho_2 V D_2}{\mu} \qquad \therefore \quad \rho_1 D_1 = \rho_2 D_2$$

since $D_1 = 2D_2$ then $\rho_1 = \frac{1}{2}\rho_2$ $\therefore \frac{\rho_1}{\rho_2} = \frac{1}{2}$

Imaginative!

$6\mu\pi V R_1$

Exam

1. $\frac{\partial^n}{\partial x^n}(\text{veloc}) = 0$ symmetry $\frac{\partial^n}{\partial \theta^n} = 0$

at $r = R_1, R_2$ $u = 0$ (no slip)

$$u = \beta\left[1 - \left(\frac{r}{R_1}\right)^2 - \frac{(K^2-1)}{\ln K} \ln\frac{r}{R_1}\right]$$

$$\beta = -\frac{R_1^2}{4\mu}\frac{dp}{dx}$$

2. $p = \text{const}$

$\frac{\partial}{\partial x} = 0$

$u = 0$ $y = 0$ $t = 0$

$u = U$ $y = 0$ $t > 0$

$u = 0$ $y = \infty$

$\cancel{\eta = \frac{\delta}{2\sqrt{\nu t}}}$ $\cancel{\delta = 2\eta\sqrt{\nu t}}$

$\cancel{\frac{d\delta}{dt} = \frac{2\eta}{2}\sqrt{\nu/t}}$

d $\eta = A y t^n$

$\bar{\eta}_{u/U = .01} = A\delta t^n$ $\delta = 3.6\sqrt{\nu t}$

$v_p = \frac{d\delta}{dt} = 1.8\sqrt{\nu/t}$

$$\delta = \frac{\bar{\eta}}{A} t^{-n} \qquad \frac{d\delta}{dt} = \frac{\bar{\eta}}{A}(-n)t^{-(n+1)}$$

4. terminal velocity $D = W$

$W_1 = K\rho_1 d_1^3$ $D_1 = \frac{1}{2}\rho_\infty V_T^2 C_D K_1 d_1^2$

$W_1/W_2 = D_1/D_2$

$$\frac{\rho_1 d_1^3}{\rho_2 d_2^3} = \frac{C_{D_1} d_1^2}{C_{D_2} d_2^2} \qquad \frac{\rho_1}{\rho_2} = \frac{C_{D_1}}{C_{D_2}} \frac{d_2}{d_1}$$

if C_D = const $\qquad \frac{\rho_1}{\rho_2} = \frac{d_2}{d_1}$

$$C_{D_1} \sim \frac{1}{R_N} \approx \frac{1}{d} \qquad \frac{\rho_1}{\rho_2} = \left(\frac{d_2}{d_1}\right)^2$$

$$W = D + B = D + K\rho_\infty d_1^3 =$$

$$D = K d_1^3 (\rho_1 - \rho_\infty)$$

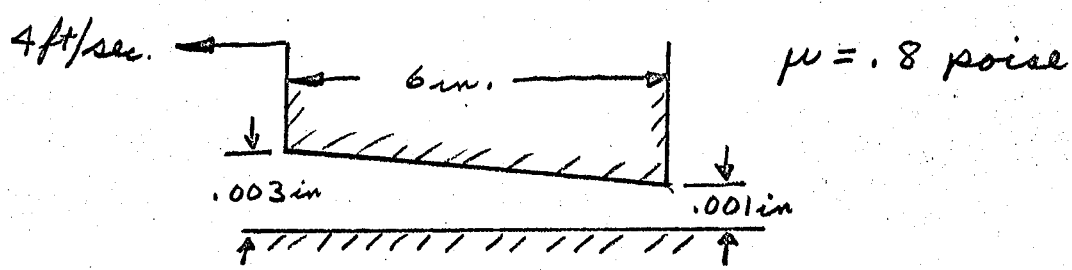
$$\therefore \left(\frac{\rho_1 - \rho_\infty}{\rho_2 - \rho_\infty}\right) \cdot \frac{d_1}{d_2} = \frac{C_{D_1}}{C_{D_2}}$$

AE 333 - Fluid Mechanics III

Due: December 1, 1971

1. A thin plate of large area is placed midway in a gap of height h filled with oil of viscosity μ_0 and the plate is pulled at a constant velocity V. If a lighter oil of visocity μ_1 is then substituted in the gap, it is found that for the same velocity V the drag will be the dame as before if the plate is located unsymmetrically in the gap but parallel to the walls. Find μ_1 in terms of μ_0 and the distance from the nearer wall to the plate.

2. For the slipper bearing shown below, calculate (a) the load the bearing will sustain and (b) the drag of the bearing. Assume that the width of the bearing is 1 ft. and that there is no flow normal to the paper.



3. An infinite flat plate is impulsively accelerated to a velocity of 100 ft./sec.

a) Derive the dimensional equation of motion stating all the assumptions used in the analysis.

b) Reduce the equation to an ordinary differential equation and solve for the velocity distribution.

c) Plot the velocity distribution for t=5 sec. for the cases where the fluid is (i) air and (ii) glycerine.

d) Determine the skin friction coefficient for the conditions specified in part c.

$$\frac{1}{\mu_1}\frac{dp}{dx}\frac{y^2}{2} + Ay + B = u \qquad + \qquad V = \frac{1}{2\mu_0}\frac{dp}{dx}\left(\frac{h^2}{4}\right) + A\,h/2$$

$$\left(V - \frac{1}{2\mu_0}\frac{dp}{dx}\frac{h^2}{4}\right)\frac{2}{h} = A$$

u at $y = h'$ $\quad u = U$

$y = h \quad u = 0$

$$\frac{1}{\mu_1}\frac{dp}{dx}\frac{d^2}{2} + Ad + B = U \qquad \frac{2V}{h} - \frac{1}{\mu_0}\frac{dp}{dx}\left(\frac{h}{4}\right)$$

$$\frac{1}{\mu_1}\frac{dp}{dx}\frac{h^2}{2} + Ah + B = 0$$

$$\frac{1}{\mu_1}\frac{dp}{dx}\left[\frac{d^2}{2} - \frac{h^2}{2}\right] + A\left[d - h\right] = U$$

$(h-d)(h+d$

$$\left\{U + \frac{1}{\mu_1}\frac{dp}{dx}\left[\frac{h^2}{2} - \frac{d^2}{2}\right]\right\}\frac{1}{d-h}$$

$$\frac{U}{d-h} - \frac{1}{2\mu_1}\frac{dp}{dx}\left[h+d\right] = A$$

$$\frac{1}{\mu_1}\frac{dp}{dx}\frac{y^2}{2} + \frac{Uy}{d-h} - \frac{1}{2\mu_1}\frac{dp}{dx}\left[h+d\right]y + B = u$$

$$\cancel{\frac{1}{\mu_1}\frac{dp}{dx}\frac{h^2}{2}} + \frac{Uh}{d-h} - \frac{1}{2\mu_1}\frac{dp}{dx}\left[\cancel{h^2} + dh\right] + B = 0$$

$$B = -\frac{Uh}{d-h} + \frac{dh}{2\mu_1}\frac{dp}{dx}$$

$$\frac{1}{2\mu_1}\frac{dp}{dx}\left[y^2 - y(h+d) + dh\right] + \frac{U}{d-h}\left[y-h\right] = u$$

$$\frac{du}{dy} = \frac{1}{2\mu_1}\frac{dp}{dx}\left[2y - (h+d)\right] + \frac{U}{d-h}$$

at top $\quad \tau_{\text{at } y=d} = \frac{1}{\cancel{2\mu_1}}\frac{dp}{dx}\left[d-h\right] + \frac{U}{d-h}\mu_1$

$$= \frac{1}{2}\frac{dp}{dx}\left[-d+h\right] + \frac{U}{-d+h}\mu_1$$

$$\frac{1}{2}\frac{dp}{dx}\left\{\left[h-d\right] + \left[d\right]\right\} + \left\{\frac{U\mu_1}{d}\right.$$

$$h-d \quad + d$$

$$U\mu_1\left[\frac{1}{d} + \frac{1}{h-d}\right]$$

$$d(h-d)$$

$$\frac{du}{dy} = U\left(-\tfrac{1}{h}\right) - \frac{h^2}{2\mu}\frac{dp}{dx}\left[\tfrac{1}{h} - \frac{2y}{h^2}\right]$$

$$\tau = \mu\frac{du}{dy} = -\frac{\mu U}{h} - \frac{h^2}{2\mu}\frac{dp}{dy}\left[\tfrac{1}{h} - \frac{2y}{h^2}\right] = -\frac{\mu U}{h} - \frac{1}{2}\frac{dp}{dx}\left[h - 2y\right]$$

$$\int_0^L \tau\, dx = +\int_0^L \left(\frac{\mu U}{h} + \frac{1}{2}\frac{dp}{dx}\left[h-2y\right]\right)dx \qquad h = h_1 - \delta x$$

h_1 h_2

$$= \frac{\mu U L}{(k-1)h_2}\left[4\ln k - \frac{6(k-1)}{k+1}\right] \qquad k = h_1/h_2$$

$h_1 = .003$ $\qquad h_1/h_2 = 3$

$h_2 = .001$ $\qquad R^* = 4\left(\tfrac{1}{2}\right)$

$L = 6\text{ in} = \tfrac{1}{2}$

$U = 4$

$$\int_0^L (p - p_0)\,dx = \frac{6\mu U L^2}{h_2^2 (k-1)^2}\left[\ln k - \frac{2(k-1)}{k+1}\right]. \qquad k = 3$$

12
4 8
168.00

$$D = \frac{\mu(4)(\cancel{\tfrac{1}{2}})^{6}}{(2)(.001)}\left[4(1.1) - \frac{6(2)}{4}\right] = \frac{\mu(12)}{.001}\left[4.4 - 3\right] = \frac{\mu}{.001}\overset{12\,(1.4)}{\left[\cancel{1.4}\right]} = \mu\left[\cancel{1400}\right] = \mu[168$$

1.68×10^3

$$P = \frac{\mu\, 24\,(\cancel{\tfrac{1}{4}})^{36}}{(1\times10^{-6})(4)}\left[1.1 - \frac{4}{4}\right] = \frac{\mu\, 24(36)}{4\times10^{-6}}\left[.1\right] = 216\times10^{\cancel{5}}\mu$$

$$2.16\times10^{7} = 3.61\times10^{4}$$

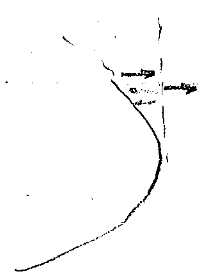
$$D = \left(1.68\times10^{+4}\right)\left(1.675\times10^{-3}\right) = 10(2.815) = 28.15 \text{ lb}$$

$$P = \left(2.16\times10^{-3}\right)\left(1.675\times10^{-3}\right) = 3.62\times10^{-6}\text{ lb} = 3.62\times10^{-\cancel{6}\,4}\text{ lb}$$

$$v = \frac{1}{2\mu_0}\frac{dp}{dx}\left(\frac{h^2}{4}\right) + A\,h/2$$

$$\left[\frac{2V}{h}y - \frac{1}{\cancel{2}\mu_0}\frac{dp}{dx}\left(\frac{h}{2}\right)y\right]$$

$$\frac{2Vy}{h} - \frac{1}{2\mu_0}\frac{dp}{dx}\left(\frac{h}{2}y\right)$$



$h' = h/2$

$$u = \frac{1}{2\mu}\frac{dp}{dx}y^2 + f_1(x)y + \underbrace{f_2(x)} \qquad \text{at } y=0 \quad u=U$$
$$y = h/2 \quad u = 0$$

$$u = \frac{1}{2\mu}\frac{dp}{dx}y^2 + f_1(x)y + U$$

$$0 = \frac{1}{2\mu}\frac{dp}{dx}\left(\frac{h^2}{4}\right) + f_1(x)\frac{h}{2} + U$$

$$-\left[U + \frac{1}{2\mu}\frac{dp}{dx}\cancel{\frac{h^2}{4}}^{\Delta^2}\right]\frac{y}{\Delta} = f_1(x)y \qquad -\frac{Uy}{\Delta} = f_1(x)y$$

$$U\left[1 - y/\Delta\right]$$

$$u = \frac{1}{2\mu}\frac{dp}{dx}\left[y^2 - \Delta y\right] - \frac{Uy}{\Delta} + U$$

$$Q = \int_0^\Delta u\,dy = \int_0^\Delta \frac{1}{2\mu}\frac{dp}{dx}\left[y^2 - \Delta y\right]dy - \int_0^\Delta \frac{Uy}{\Delta}dy + \int_0^\Delta U\,dy$$

$$Q = \frac{1}{2\mu}\frac{dp}{dx}\left[\frac{y^3}{3} - \frac{\Delta y^2}{2}\right]_0^\Delta - \frac{Uy^2}{2\Delta}\Big|_0^\Delta + Uy\Big|_0^\Delta$$

$$= \frac{1}{2\mu}\frac{dp}{dx}\left[\frac{\Delta^3}{3} - \frac{\Delta^3}{2}\right] - \frac{U\Delta^2}{2\Delta} + U\Delta$$

$$= \frac{1}{2\mu}\frac{dp}{dx}\left[-\frac{\Delta^3}{6}\right] + \frac{U\Delta}{2}$$

$$-\left[Q - \frac{U\Delta}{2}\right]\frac{12\mu}{\Delta^3} = \frac{dp}{dx}$$

$$p = \int_0^L \left[\frac{U\Delta}{2} - Q\right]\frac{12\mu}{\Delta^3}dx$$

$$p = \left[\frac{U\Delta}{2} - Q\right]\frac{12\mu}{\Delta^3}x + B$$

when $x = 0 \quad p = p_0$ $\qquad p = \left[\frac{U\Delta}{2} - Q\right]\frac{12\mu}{\Delta^3}x + p_0$

when $x = L \quad p = p_0$ $\qquad \cancel{p_0} = \left[\frac{U\Delta}{2} - Q\right]\frac{12\mu}{\Delta^3}L + \cancel{p_0}$

$$0 = \left[\frac{U\Delta}{2} - Q\right]$$

if so $\frac{dp}{dx} = 0$ $\qquad p = \text{Constant} = p_0$ $\qquad \boxed{Q = \frac{U\Delta}{2}}$

$$u = U\left[1 - y/\Delta\right]$$

$$\frac{du}{dy} = U\left[0 - \frac{1}{\Delta}\right] \qquad \frac{du}{dy} = U\left[-\frac{1}{\Delta}\right] \qquad \tau = +\frac{U\mu}{\Delta} \qquad 2\tau = \frac{2U\mu}{\Delta}$$

$$\frac{4U\mu_o}{h} = 2\tau$$

$$u = U\left[1 - y/d\right]$$

$$u = U\left[1 - y/x\right]$$

U

$$\frac{2U\mu_1}{x} + \frac{2U\mu_1}{d} = \frac{4U\mu_o}{h}$$

$$2U\mu_1\left[\frac{d+x}{xd}\right] = \frac{4U\mu_o}{h}$$

$$\frac{\cancel{2U\mu_1}\, h}{xd} = \frac{\overset{4}{4U\mu_o}}{h} \quad \leftarrow 1$$

$$\longrightarrow \mu_1 = \frac{4\mu_o}{h^2}\left[hd - d^2\right]$$

$$u = U\left[1 - y/d\right]$$

$$u = U\left[1 - y/x\right]$$

$$\cancel{\left[1 - y/d\right] = \left[1 - y/x\right]}$$

poise = dynes - sec/cm^2

lb - sec/ft^2

$$30.48 \frac{cm}{ft}$$

$$444,820 \frac{dynes}{lb}$$

$$\text{poise} \cdot \left(30.48 \frac{cm}{ft}\right)^2 \cdot \frac{lb}{444,820 \text{ dynes}} = lb\text{-}sec/ft^2$$

$$.8 \text{ poise} \cdot \left(\underset{2.09}{2.095 \times 10^{-3}}\right) = 1.675 \times 10^{-3} \frac{lb\text{-}sec}{ft^2} \cdot \frac{\cancel{1000}}{\cancel{400,000}}$$

1.672

$$u_{,t} = \nu u_{,yy}$$

assume $u = U f(\eta)$ $\quad \eta = k y t^m$

$$\Rightarrow \quad f'' + 2\eta f' = 0 \quad \underline{\underline{or}}$$

$$f = A\int_0^\eta e^{-\eta^2} d\eta + B \quad or.$$

$$u = 100 - \frac{200}{\sqrt{\pi}}\int_0^\eta e^{-\eta^2} d\eta \qquad \eta = \frac{y}{2\sqrt{\nu t}}$$

for air $\nu = 160 \times 10^{-6}$ ft²/sec @ 20°C

for glycerine $\nu = 7319 \times 10^{-6}$ ft²/sec @ 20°C

$$\eta = k y \qquad \text{when} \quad k = \frac{1}{2\sqrt{\nu t}} = \frac{1}{2\sqrt{(160\times 10^{-6})(5)}} = \frac{1}{2\sqrt{8\times 10^{-4}}}$$

$$\frac{1}{2(2.54\times 10^{-4})}$$

2.83

$$k_{air} = \frac{1}{\frac{5.09\times 10^4}{\sqrt{.166}\times 10^{-2}}} = .1965 \times 10^{+2} = 1.965 \times 10^{-1}$$

$$\boxed{\eta_{air} = 1.965 \times 10^{+1} y} \qquad 765 \qquad \boxed{\eta_{air} = 17.65 \times 1 \; y}$$

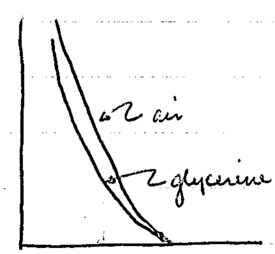
$$\text{when} \quad k = \frac{1}{2\sqrt{\nu t}} = \frac{1}{2\sqrt{(7319\times 10^{-6})(5)}} = \frac{1}{2\,(1.915\times 10^{-2})}$$

$$\frac{1}{3.83\times 10^5} \qquad .261 \times 10^{-5}$$

36595 $\times 10^{-6}$

3.66 $\times 10$ $\qquad$ 3.83 10^{-1}

$$\boxed{\eta_{gly} = 2.61 \times 10 \; y}$$



$$\tau = U\sqrt{\frac{\rho\mu}{\pi t}} = c_f \frac{1}{2}\rho U^2$$

$$c_f = U\sqrt{\frac{\rho\mu}{\pi t}}\,\frac{2}{\rho U^2} = \frac{2}{U}\sqrt{\frac{\mu}{\rho\pi t}} = \frac{2}{U}\sqrt{\frac{\nu}{\pi t}}$$

$$c_{f_{gly}} = \frac{2}{100}\sqrt{\frac{7319\times 10^{-6}}{3.14(5)}} = \frac{1}{50}\sqrt{466\times 10^{-6}} = \frac{1}{50}\, 21.6\times 10^{-3} = \boxed{432 \times 10^{-4}}$$

$$c_{f_{air}} = \frac{1}{50}\sqrt{\frac{160\times 10^{-6}}{3.14(5)}} = \frac{1}{50}\sqrt{10.02\times 10^{-6}} = \frac{3.16+\times 10^{-3}}{50} = \cancel{63.25} \quad .06325\times 10^{-3} \quad \boxed{.06325 \times 10^{-5}}$$

$$\frac{1}{2(17.65)} = \frac{1}{34}$$

Cesar Levy
11-23-71

1. Since large area one can assume the flow is Couette in nature (fully developed) $\checkmark, \checkmark, \checkmark$
thus the Navier Stokes Equation reduces to

$$p_{,x} = \mu\, u_{,yy}$$
$$p_{,y} = 0$$
$$v_{,y} = v \equiv 0$$

From these

$$u = \frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2} + Ay + B$$

for $\mu = \mu_0$ $\quad u=0 \quad y=0$
$\quad u = V \quad y = h/2$

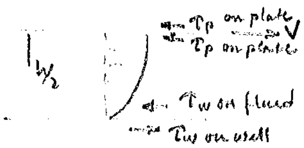
$B = 0$

$$A = \frac{2V}{h} - \frac{1}{2\mu_0}\frac{dp}{dx}\left(\frac{h}{2}\right)$$

or

$$u = \frac{1}{2\mu_0}\frac{dp}{dx}\left[y^2 - y\frac{h}{2}\right] + \frac{2Vy}{h}$$

$$\frac{du}{dy} = \frac{1}{2\mu_0}\frac{dp}{dx}\left[2y - \frac{h}{2}\right] + \frac{2V}{h}$$



$$\tau = \mu_0\frac{du}{dy} = \frac{dp}{dx}\left[y - \frac{h}{4}\right] + \frac{2V\mu_0}{h}$$

$$|\tau|_{\substack{\text{on plate}\\ \text{bottom}}} = \left|\frac{dp}{dx}\left(\frac{h}{4}\right) + \frac{2V\mu_0}{h}\right|$$ acting opposite to shear on fluid

Since problem is symmetric

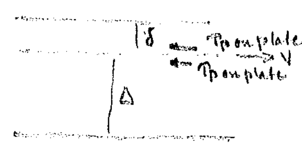
$$|\tau|_{\substack{\text{on plate}\\ \text{top}}} = \left|\frac{dp}{dx}\left(\frac{h}{4}\right) + \frac{2V\mu_0}{h}\right|$$ acting in same direction as shear on bottom surface

thus the total drag is $2\tau A = \left[\frac{h}{2}\frac{dp}{dx} + 4\frac{V\mu_0}{h}\right]A$ where A is the area of the plate

In the second case

$$u = \frac{1}{2\mu_1}\frac{dp}{dx}\left[y^2 - y\Delta\right] + \frac{Vy}{\Delta}$$

$$\frac{du}{dy} = \frac{1}{2\mu_1}\frac{dp}{dx}\left[2y - \Delta\right] + \frac{V}{\Delta}$$



$$|\tau|_{\substack{\text{on plate}\\ \text{bottom}}} = \left|\frac{dp}{dx}\left[\Delta - \frac{\Delta}{2}\right] + \frac{V\mu_1}{\Delta}\right|$$ acting opposite to shear on fluid

$$|\tau|_{\substack{\text{on plate}\\ \text{bot}}} = \left|\frac{dp}{dx}\left[\frac{\delta}{2}\right] + \frac{V\mu_1}{\delta}\right|$$

total drag on the plate is $\left(\frac{dp}{dx}\left[\frac{\Delta+\delta}{2}\right] + V\mu_1\left[\frac{\delta+\Delta}{\delta\Delta}\right]\right)A$

but $\Delta + \delta = h$ or

$$\left(\frac{h}{2}\frac{dp}{dx} + V\mu_1\frac{h}{\delta\Delta}\right)A$$

Equating the drags leads to

$$\frac{4V\mu_0}{h} = V\mu_1 \frac{h}{\delta\Delta} \qquad \text{or} \qquad \mu_1 = \frac{4}{h^2}\mu_0 (\delta\Delta)$$

using the nearer distance to the plate δ

$$\mu_1 = \frac{4}{h^2}\mu_0(\delta h - \delta^2) \quad \checkmark$$

Note: Solution by lubrication theory shows that $p_{3x} = 0$

2.

$$P = \int_0^L (p - p_0)\,dx = \frac{6\mu U L^2}{h_2^2 (K-1)^2}\left[\ln K - \frac{2(K-1)}{K+1}\right] \qquad K = h_1/h_2$$

$$D = \int_0^L -\tau\,dx_{\text{in fluid}} = \int_0^L \left\{\frac{\mu U}{h} + \frac{1}{2}\frac{dp}{dx}\left[h - 2y\right]\right\}dx = \frac{\mu U L}{(K-1)h_2}\left[4\ln K - \frac{6(K-1)}{K+1}\right]$$

$h_1 = .003$

$h_2 = .001$ $\qquad k = 3 \qquad \ln K = 1.1$

$L = 6\,in$

$U = 4\,ft/sec$

$$P = \frac{6\mu\,(4\,ft/sec)(6\,in)^2}{(.001\,in)^2(4)}\left[1.1 - \frac{4}{4}\right] = 2.16\times10^7\mu$$

$$\mu = 2.090\times10^{-3}\ \frac{lb\text{-}sec}{ft^2} \qquad \left(\text{poise units} = \frac{dynes\text{-}sec}{cm^2}\right)$$

$$P = (2.16\times10^7)(1.672\times10^{-3}) = 3.61\times10^4\ lb$$

$$D = \frac{\mu\,(4\,ft/sec)(6\,in)}{(.001\,in)\,(2)}\left[4(1.1) - \frac{6(2)}{4}\right] = 1.68\times10^4\mu$$

$$= (1.68\times10^4)(1.675\times10^{-3}) = 28.15\ lb. \quad \checkmark$$

3. Because the plate is infinite there cannot exist any changes in the parameters in the x direction, Because of this development and continuity $v \equiv 0$ everywhere so that the stokes' equations reduces to

$$\rho u_{3t} = \mu u_{3yy}$$

$$0 \equiv 0$$

Also $\quad t \le 0 \quad u = 0, y = 0$

$\qquad\ \ t > 0 \quad u = U, y = 0$

If $u = u(y,t)$ separation of variables leads to solutions which do not satisfy boundary conditions then assume $u = Uf(\eta)$ where $\eta = k y t^{m}$

then
$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial t} = Uf'\left[kmyt^{m-1}\right]$$
$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y} = Uf'\left[kt^{m}\right]$$
$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial \eta}\left(\frac{\partial u}{\partial y}\right)\frac{\partial \eta}{\partial y} = Uf'' k^2 t^{2m}$$

$$u_{,t} = \nu\, u_{,yy}$$
$$Uf'\left[kmyt^{m-1}\right] = \nu\, U f'' k^2 t^{2m} \qquad \text{since} \quad y = \eta / k t^{m-1}$$

thus
$$f'' - \frac{\eta m}{\nu k^2 t^{2m+1}} f' = 0 \qquad \text{let } 2m+1 = 0 \quad \text{therefore} \quad m = -1/2$$
$$f'' + \frac{\eta}{2\nu k^2} f' = 0 \qquad \text{let } 4\nu k^2 = 1 \quad \text{therefore} \quad k = \frac{1}{2\sqrt{\nu}}$$
$$f'' + 2\eta f' = 0 \qquad \text{if } f' = Ae^{-\eta^2} \text{ then}$$
$$\rightarrow \quad f = A\int_0^{\eta} e^{-\varsigma^2} d\varsigma + B \qquad \begin{array}{llll} \text{when } y=0 & \eta = 0 & u = U & f = 1 \\ y = \infty & \eta = \infty & u = 0 & f = 0 \end{array}$$

therefore
$$A = \frac{-1}{\int_0^{\infty} e^{-\varsigma^2} d\varsigma} = -\frac{2}{\sqrt{\pi}} \quad ; \quad B = 1$$
$$\therefore \quad f = 1 - \frac{2}{\sqrt{\pi}}\int_0^{\eta} e^{-\varsigma^2} d\varsigma \qquad \text{or} \qquad u = U - \frac{2U}{\sqrt{\pi}}\int_0^{\eta} e^{-\varsigma^2} d\varsigma$$
$$\boxed{u = 100 - \frac{200}{\sqrt{\pi}}\int_0^{\eta} e^{-\varsigma^2} d\varsigma} = 100\left[1 - \operatorname{erf}(\eta)\right]$$

since $\eta = \lambda y$ where $\lambda = \frac{1}{2\sqrt{\nu t}}$ @ $t = 5$ and $t = 20°C$

for air $\nu = 160 \times 10^{-6}\ ft^2/sec \longrightarrow \eta = 17.65\, y$

glycerine $\nu = 7319 \times 10^{-6}\ ft^2/sec \longrightarrow \eta = 2.61\, y$

for air

η	y	erf η	$u/100$
0	0	0	1
.5	.0283	.521	.479
1.0	.0567	.843	.157
1.5	.085	.966	.034
2.0	.1133	.995	.005
2.5	.1415	.9996	.0004

for glycerine

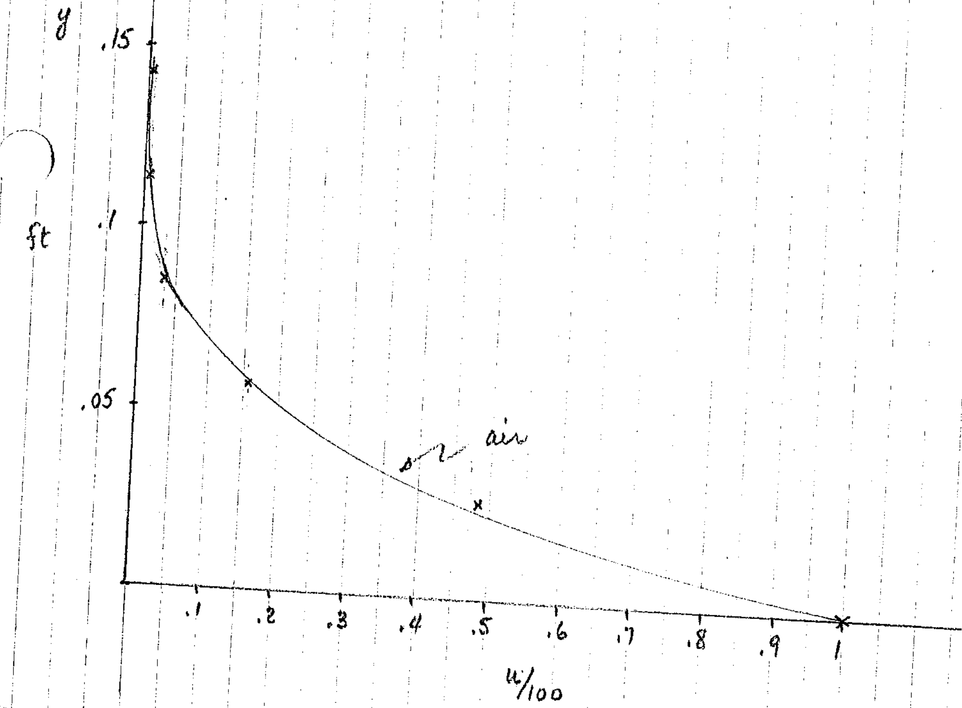
η	y	erf η	$u/100$
0	0	0	1
.5	.1915	.521	.479
1.0	.383	.843	.157
1.5	.575	.966	.034
2.0	.766	.995	.005
2.5	.96	.9996	.0004

$$\tau = U\sqrt{\frac{\rho\mu}{\pi t}} = c_f \, \frac{1}{2}\rho U^2$$

$$c_f = U\sqrt{\frac{\rho\mu}{\pi t}}\left(\frac{2}{\rho U^2}\right) = \frac{2}{U}\sqrt{\frac{\nu}{\pi t}}$$

$$c_{f_{gly}} = \frac{1}{50}\sqrt{\frac{7319\times10^{-6}}{3.14(5)}} = 4.32\times10^{-4} \checkmark$$

$$c_{f_{air}} = \frac{1}{50}\sqrt{\frac{160\times10^{-6}}{3.14(5)}} = 6.325\times10^{-5} \checkmark$$


y
ft
.15
.1
.05
air
.1
.2
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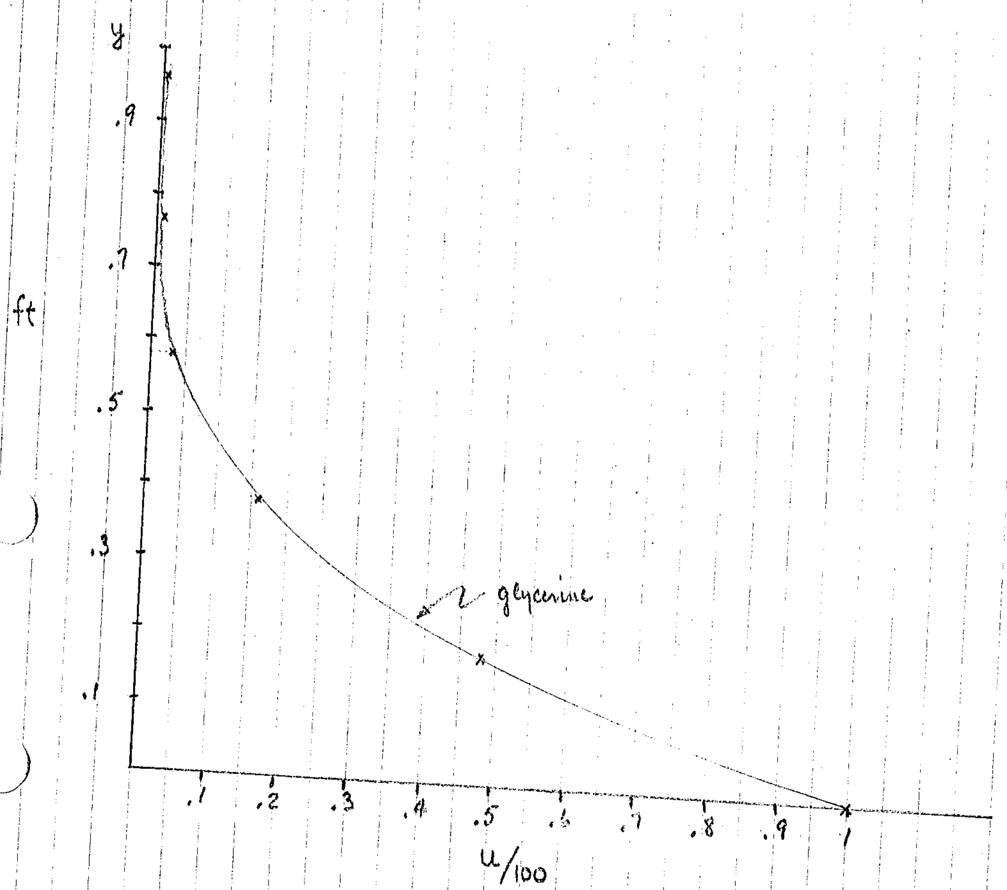
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$$120A_5 + 2A_2^2 = 0$$

$$8\cdot7\cdot6\cdot2A_8 + 20A_2A_5 + 2A_5A_2 = 0$$

$$11\cdot10\cdot9\cdot2A_{11} + 56A_2A_8 + 20A_5A_5 + 2A_8A_2 = 0$$

$$14\cdot13\cdot12\cdot2A_{14} + 110A_2A_{11} + 56A_5A_8 + 20A_8A_5 + 2A_{11}A_2 = 0$$

$$17\cdot16\cdot15\cdot2A_{17} + 14\cdot13A_2A_{14} + 110A_5A_{11} + 56A_8A_8 + 20A_{11}A_5 + 2A_{14}A_2 = 0$$

$$20\cdot19\cdot18\cdot2A_{20}\ 17\cdot16A_{20} + 14\cdot13A_2A_{17} + 110A_5A_{14} + 56A_8A_{11}\ \ 20A_{11}A_8\ \ 2A_{17}A_2 = 0$$

2. $(5+3K)(4+3K)(3+3K)A_{5+3K} + (3K+2)(3K+1)A_2A_{3K+2} + (3K-1)(3K-2)A_5A_{3K-1}$

$+ (3K-4)(3K-5)A_8A_{3K-4} + (3K-7)(3K-8)A_{11}A_{3K-7} + (3K-10)(3K-11)A_{14}A_{3K-10}$

$+ \ldots\ldots 2A_2A_{3K+2} = 0$

for any coeff whose index is negative

$$A_5 = -\frac{2A_2^2}{5!}$$

$$A_8 = +\frac{22A_2^3}{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2} \qquad A_8 = \frac{-22A_2A_5}{8\cdot7\cdot6\cdot2}$$

$$-\frac{(29A_2A_8 + 10A_5^2)}{11\cdot10\cdot9}$$

$$A_{11} = -\frac{526A_2^4}{11!} \qquad \frac{29\cdot11A_2^4}{11!/2} - \frac{10A_2^4}{11\cdot10\cdot9\cdot5\cdot4\cdot3\cdot5\cdot4\cdot3}$$

$$-\frac{638A_2^4}{11!} - \frac{10A_2^4\cdot8\cdot7\cdot6\cdot2}{11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2}$$

$$14\cdot13\cdot12\cdot2A_{14} + 112A_2\left(-\frac{526}{11!}\right) + \frac{76A_2^5\cdot24}{8!\ 5!} = 0 \qquad \frac{526A_2^4}{11!}$$

$$A_{14} = \frac{112\cdot526A_2^5}{11!\cdot14\cdot13\cdot12\cdot2} + \frac{76\cdot44A_2^5}{8!\ 5!\cdot14\cdot13\cdot12\cdot2}$$

$$\frac{56\cdot526A_2^5}{14!} + \frac{76\cdot22A_2^5\cdot11\cdot10\cdot9}{14!\ 5\cdot4\cdot3\cdot2} = 19\cdot22\cdot33$$

$2A_2A_{14}\eta^{14} + 20A_{14}A_5\eta^{11}$

$A_{17}\cdot 2A_2\eta^{17}$

$$f = \sum_{n=0} A_n \eta^n$$

$$f' = \sum_{n=0} A_n n \eta^{n-1}$$

$$f'' = \sum_{n=0} A_n (n)(n-1)\eta^{n-2}$$

$$f''' = \sum_{n=0} A_n (n)(n-1)(n-2)\eta^{n-3}$$

$$2A_n\, n(n-1)(n-2)\eta^{n-3} + A_n\eta^n \sum_{n=0} A_n (n)(n-1)\eta^{n-2} = 0$$

$$2A_8(8\cdot7\cdot6)\ \eta \quad + A_8\eta^8$$

$$f = A_2\eta^2 + A_3\eta^3 + A_4\eta^4 + A_5\eta^5 + A_6\eta^6 + A_7\eta^7 + A_8\eta^8 + A_9\eta^9 + A_{10}\eta^{10} + A_{11}\eta^{11} + A_{12}\eta^{12}$$
$$+ A_{13}\eta^{13} + A_{14}\eta^{14} + A_{15}\eta^{15} + A_{16}\eta^{16} + A_{17}\eta^{17}$$

$$f' = 2A_2\eta + 5A_5\eta^4 + 8A_8\eta^7 + 11A_{11}\eta^{10} + 14A_{14}\eta^{13} + 17A_{17}\eta^{16}$$

$$f'' = 2A_2 + 20A_5\eta^3 + 8\cdot7A_8\eta^6 + 11\cdot10A_{11}\eta^9 + 14\cdot13A_{14}\eta^{12} + 17\cdot16A_{17}\eta^{15}$$

$$f''' = 5\cdot4\cdot3A_5\eta^2 + 8\cdot7\cdot6A_8\eta^5 + 11\cdot10\cdot9A_{11}\eta^8 + 14\cdot13\cdot12A_{14}\eta^{11} + 17\cdot16\cdot15A_{17}\eta^{14}$$

$$f = A_2\eta^2 + A_5\eta^5 + A_8\eta^8 + A_{11}\eta^{11} + A_{14}\eta^{14} + A_{17}\eta^{17}$$

$$(5\cdot4\cdot3\cdot2A_5 + 2A_2\cdot A_2)\eta^2 + (2\cdot8\cdot7\cdot6A_8 + 20A_5A_2 + 2A_2A_5)\eta^5 + (2\cdot11\cdot10\cdot9A_{11} + A_5^2\cdot20 +$$
$$A_2A_8\cdot8\cdot7)\eta^8 + (2\cdot14\cdot13\cdot12A_{14} + A_5A_8\,8\cdot7 + 20A_5A_8)\eta^{11}$$

$$(5\cdot4\cdot3\cdot2A_5)\eta^2 + (2\cdot8\cdot7\cdot6A_8)\eta^5 + (2\cdot11\cdot10\cdot9A_{11})\eta^8 + (2\cdot14\cdot13\cdot12A_{14})\eta^{11} + (17\cdot16\cdot15\cdot2A_{17})\eta^{14}$$

$$(A_2\eta^2 + A_5\eta^5 + A_8\eta^8 + A_{11}\eta^{11} + A_{14}\eta^{14} + A_{17}\eta^{17})(2A_2 + 20A_5\eta^3 + 56A_8\eta^6 + 110A_{11}\eta^9 + 14\cdot13A_{14}\eta^{12} + 17\cdot16\eta$$

$$(5\cdot4\cdot3\cdot2A_5 + 2A_2^2)\eta^2 + (2\cdot8\cdot7\cdot6A_8 + 20A_2A_5 + 2A_2A_5)\eta^5 + (2\cdot11\cdot10\cdot9A_{11} + A_2A_8 56 + 20A_5A_5 + 2A_2A_8\cdot$$

$$2A_2^2\eta^2 + 20A_2A_5\eta^5 + 56A_2A_8\eta^8 + 110A_{11}A_2\eta^{11} + 14\cdot13A_{14}A_2\eta^{14} + 17\cdot16A_{17}A_2\eta^{17}$$
$$2A_2A_5\eta^5 + 20A_5^2\eta^8 + 56A_5A_8\eta^{11} + 110A_{11}A_5\eta^{14} + 14\cdot13A_5A_{14}\eta^{17} + A_5A_{17}\cdot17\cdot16\eta^{20}$$
$$2A_8A_2\eta^8 + 20A_5A_8\eta^{11} + 56A_8^2\eta^{14} + 110A_{11}A_8\eta^{17} + 14\cdot13A_8A_{14}\eta^{20} + 17\cdot16\eta$$
$$2A_2A_{11}\eta^{11} + 20A_{11}A_5\eta^{14} + 56A_8A_{11}\eta^{17} +$$

$2f''' + ff'' = 0$

$A_9\eta^9 + A_{10}\eta^{10} + A_{11}$

let $f = A_0 + A_1\eta + A_2\eta^2 + A_3\eta^3 + A_4\eta^4 + A_5\eta^5 + A_6\eta^6 + A_7\eta^7 + A_8\eta^8$

$f' = A_1 + 2A_2\eta + 3A_3\eta^2 + 4A_4\eta^3 + 5A_5\eta^4 + 6A_6\eta^5 + 7A_7\eta^6 + 8A_8\eta^7$

$$f = \sum_{n=0}^{8} A_n \eta^n \qquad f' = \sum_{n=1}^{8} nA_n\eta^{n-1} \qquad f'' = \sum_{n=2}^{8} n(n-1)A_n\eta^{n-2}$$

$$f''' = \sum_{n=3}^{8} n(n-1)(n-2)A_n\eta^{n-3}$$

$$2\sum_{n=3}^{8} n(n-1)(n-2)A_n\eta^{n-3} + \sum_{n=0}^{8} A_n\eta^n \sum_{n=2}^{8} n(n-1)A_n\eta^{n-2} = 0$$

$$f'' = 2A_2 + 6A_3\eta + 12A_4\eta^2 + 20A_5\eta^3 + 30A_6\eta^4 + 42A_7\eta^5 + 56A_8\eta^6$$

$$f''' = 6A_3 + 24A_4\eta + 60A_5\eta^2 + 120A_6\eta^3 + 210A_7\eta^4 + 336A_8\eta^5$$

from BC A_0 & $A_1 = 0$ $\therefore$

$$2\sum_{n=3}^{8} n(n-1)(n-2)A_n\eta^{n-3} + \sum_{n=2}^{8} A_n\eta^n \sum_{n=2}^{8} n(n-1)A_n\eta^{n-2} = 0$$

$$2\sum_{n=3}^{8} n(n-1)(n-2)A_n\eta^{n-3} + \sum_{n=3}^{8} A_n\eta^n \sum_{n=2}^{8} n(n-1)(A_n\eta^{n-2}) + A_2\eta^2\sum_{n=2}^{8} n(n-1)A_n\eta^{n-2} =$$

$(n+2)(n+1)$

$$\sum_{n=3}^{8}\left[2n(n-1)(n-2)A_n\eta^{n-3} + A_n\eta^n\sum_{n=2}^{8} n(n-1)(A_n\eta^{n-2})\right] + A_2\eta^2\sum_{n=2}^{8} n(n-1)A_n\eta^{n-2} = 0$$

A A A

A_5 is needed highest power to be used is η^5, since 3rd order non lin eq η^5 occurs when A_8 is taken into account in f''' $\therefore$ f used must have $\sum_0^8 A_n\eta^n$. Take all terms for $\eta \le 5$

~~Since [illegible] only goes to η [illegible] have only the 2 terms up to & included [illegible]~~

set coeff of $\eta^n = 0$ $\therefore$

$$12A_3 + 48A_4\eta + 120A_5\eta^2 + 240A_6\eta^3 + 420A_7\eta^4 + 672A_8\eta^5 + 2A_2^2\eta^2 + 2A_2A_3\eta^3$$
$$+ 2A_2A_4\eta^4 + 2A_2A_5\eta^5 + 6A_3A_2\eta^3 + 6A_3A_4\eta^4 + 6A_3A_5\eta^5 + 12A_4A_2\eta^4 + 12A_4A_3\eta^5$$
$$+ 20A_2A_5\eta^5 + \text{higher order terms in } \eta^6 \longrightarrow$$

η^0 $12A_3 = 0$ ✓ η^2 $120A_5 + 2A_2^2 = 0$ η^4 $420A_7 + 14A_2A_4 + 6A_3A_4 = 0$ ✓

η^1 $48A_4 = 0$ ✓ η^3 $240A_6 + 8A_2A_3 = 0$ ✓ η^5 $672A_8 + 22A_2A_5 + 6A_3A_5 + 12A_4A_3 = 0$

$A_3 = 0 \qquad A_4 = 0 \qquad 120 A_5 + 2A_2 = 0 \quad 4 \cdot 0 \qquad A_6 = 0 \qquad A_7 = 0$

$672 A_8 + 22 A_2 A_5 = 0 \leftarrow$

$8 \cdot 7 \; A_n \cdot 6 \qquad A_5 A_5 \qquad 8 \cdot 7 \cdot 6$

$2(n$

$$2\left[n(n-1)(n-2)A_n + \cancel{A_{n-3}} + \; A_n \sum n(n-1) A_n\right]$$

$$2n(n-1)(n-2)A_n + \cancel{A_2\left[(n(n-1))A_{n-3}\right] + \left[\right.}$$

$8 \cdot 7 \cdot 6$

$\cancel{n-3}$

$11(10)(9)$

$2 \quad 990$

1980

$\cancel{A_{n-3}}\left[\right.$

$5 \cdot 4$

$$A_2\left[(n-3)(n-4)A_{n-3}\right]$$

$n = 5, 8,$

$11 \cdot 10 \cdot 9$

$$+ A_{n-3}\left[\cancel{(n-4)(n-3)}A_2\right]$$

$$A_{n-3}\left[\cancel{(n-7)(n-8)}A_2\right]$$

$2(n+6)(n+\cancel{8})(n+4) \ldots \quad 2A_2A_8\eta^8 + 56A_8\eta^6 + A_2\eta^2$

$8 \cdot 7$

$$\cancel{2(n+} \quad A_{n+9} \quad \cancel{A_n} \quad nA_n A_{n+6}\eta^{n+6} + (n+6)(n+5)A_{n+6}\eta^{n+4} \; A_n\eta^n \leftarrow$$

$\delta = 3$

$$2(n+9)(n+8)(n+7)A_{n+6} + nA_nA_{n+3}\eta^{n+3} + (n+3)(n+2)A_{n+3}\eta^{n+1}A_n\eta^n$$

$2 \cdot A_2 \cdot A_5\eta^5 \qquad 5 \cdot 4 \; A_5\eta^3 \; A_2\eta^2$

$$2(n+3)(n+2)(n+1)A_{n+3}\eta^{\cancel{n+1}\,n} + nA_nA_{n}\eta^{n} + (n)(n-1)A_n\eta^{n-2}A_n\eta^n$$

$$\cancel{2(3+\delta)(2+\delta)(1+\delta)A_{\delta+2}\eta^2 \quad 2A_2 A_{2+\delta}\eta^{2+\delta} + (\delta+2)(\delta+1)A_{2+\delta}\eta \quad A_2\eta^2}$$

$$(2+(\delta-1))A_{2+\delta}\eta^\delta A_2\eta^2$$

let δ = increment $\qquad 3K$ where $k = 0, 1, 2, 3$

$$0 = 2(2+\delta)(2+[\delta-1])(2+[\delta-2])A_{2+\delta}\eta^{2+\delta} + 2A_2A_{2+\delta}\eta^{2+\delta} + (2+\delta)(2+[\delta-1])A_{2+\delta}\eta^{2+[\delta-2]}A$$

$3K$

$(2+3K)(1+3K)$

$$0 = 2(\cancel{2}5+3K)(4+3K)(3+3K)A_{5+3K}\eta^{2+3K} + 2A_2A_{5+3K}\eta^{2+3K} + \cancel{(5+3K)(3K)}A_{2+3K}\eta^{3K}A_2\eta^2$$

for $K=1 \longrightarrow$

$$2(5+3K)(4+3K)(3+3K)A_{5+3K}\eta^{2+3K} + 2A_2A_{2+3K}\eta^{3K+2} + (2+3K)(1+3K)A_{2+3K}\eta^{3K}A_2\eta^2 = 0 \quad \checkmark$$

for $K=0$

$$2(5+3K)(4+3K)(3+3K)A_{5+3K}\eta^{2+3K} + 2A_2A_{2+3K}\eta^{3K+2} = 0 \quad \checkmark$$

$$u = u_1 f'(\eta) = \frac{\partial \psi}{\partial \eta} c x^\alpha$$

$$\psi = \frac{u_1}{c x^\alpha} f(\eta) \qquad \cancel{\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial x} = \frac{u_1}{c x^\alpha} f'(\eta)\alpha c \frac{y}{x} x^{\alpha-1} = -u_1 f'(\eta)\alpha y/x}$$

$$\cancel{\frac{\partial u}{\partial x}} \qquad v = -\frac{\partial \psi}{\partial x} = -\frac{\partial \psi}{\partial \eta} c y \alpha \frac{x^\alpha}{x} = -u_1 f'(\eta) y \frac{\alpha}{x}$$

$$u = \frac{\partial \psi}{\partial \eta} c x^\alpha \qquad -\frac{u_1}{c x^\alpha} f' c y \alpha \frac{x^\alpha}{x} + \frac{u_1 f}{c}(-\alpha x^{-\alpha-1})$$

$$-u_1 f' y \frac{\alpha}{x} + \frac{u_1 f \alpha}{c x^{\alpha+1}}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial x} = u_1 f''(\eta) \alpha c y \frac{x^\alpha}{x} \qquad u_1 f' = \partial \psi / \partial y$$

$$u = u_1 f'(\eta)$$

$$\cancel{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0} \qquad u_1 f'' c x^\alpha =$$

$$\cancel{u_1 f''(\eta) c y \alpha x^{\alpha-1} - u_1} \qquad u_1 c^2 x^{2\alpha} f'''$$

$$v = -u_1 f'(\eta) \alpha y / x$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y} = u_1 f''(\eta) c x^\alpha$$

$$\frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{u_1}{c x^\alpha} f'(\eta) c x^\alpha = u_1 f'$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = u_1 c^2 x^{2\alpha} f'''(\eta)$$

$$\frac{\partial}{\partial x}(u_1 f') = u_1 f'' c y \alpha x^{\alpha-1}$$

$$\cancel{u_1^2 f'(\eta) f''(\eta) \alpha c y \frac{x^\alpha}{x} - u_1^2 f'(\eta) \frac{\alpha y}{x} f''(\eta) c x^\alpha}$$

$$\frac{u_1}{c x^\alpha}\left[f' c y \alpha x^{\alpha-1}\right] - \frac{\alpha f(\eta) u_1}{c x^{\alpha+1}}$$

$$= \frac{\partial \psi}{\partial y}\left(\frac{\partial \psi}{\partial x \partial y}\right) - \frac{\partial \psi}{\partial x}\left(\frac{\partial^2 \psi}{\partial y^2}\right) = \nu \frac{\partial^3 \psi}{\partial y^3}$$

$$u_1^2 f'(\eta)\left[f'' c y \alpha x^{\alpha-1}\right] - \frac{u_1^2}{c x^\alpha}\left[f' c y \alpha x^{\alpha-1} - \frac{\alpha f}{x}\right]\left[c x^\alpha f''\right] = \nu u_1 c^2 x^{2\alpha} f'''$$

$$u_1^2 f' f'' \cancel{c y} \alpha x^{\alpha-1} - u_1^2 f' y \frac{\alpha}{x} c x^\alpha f'' + \frac{u_1^2}{x} \alpha f'' f = \nu u_1 c^2 x^{2\alpha} f'''$$

$$\nu u_1 c^2 x^{2\alpha} f''' - \frac{u_1^2}{x} \alpha f'' f = 0$$

$$f''' - \frac{u_1 \alpha}{x^{2\alpha+1} \nu c^2} f'' f = 0 \qquad \text{now let } x^{2\alpha+1} = 1 \quad \therefore 2\alpha + 1 = 0$$

$$\alpha = -1/2$$

$$f''' + \frac{u_1}{2 \nu c^2} f'' f = 0 \quad \text{or} \quad 2 f''' + \frac{u_1}{\nu c^2} f'' f \qquad \therefore c = \sqrt{u_1/\nu}$$

or $\eta = y\sqrt{\dfrac{u_1}{\nu x}}$

$$\psi_{,x} = \frac{u_1}{c x^{\alpha}} f' \, c y \alpha x^{\alpha-1} + \frac{u_1 f}{c}\left(-\alpha x^{-\alpha-1}\right)$$

$$v = \frac{u_1 f}{c} \alpha x^{-\alpha-1} - u_1 f' y \alpha/x$$

$c = \sqrt{\dfrac{u_1}{\nu}}$

$\alpha = -1/2$

since $f' = 0$ for $u = 0$ at $y = 0$

and $v = 0$ then if $u_1, c, \alpha, x \neq 0$ $\quad f(\eta) = 0$

$$0 = \frac{-u_1 f}{2\sqrt{u_1/\nu}}$$

since $\quad f = A_0 + A_1\eta + A_2\eta^2 + A_3\eta^3 + A_4\eta^4 + A_5\eta^5$

and since $v = 0$, $f(\eta) = 0$ $\quad \eta = 0 \rightarrow A_0 = 0$

$$f' = A_1 + 2A_2\eta + 3A_3\eta^2 + 4A_4\eta^3 + 5A_5\eta^4$$

and since $u = 0$ when $\eta = 0$ $\quad f'(\eta) = 0 \quad \therefore \quad A_1 = 0$

$$f'' = 2A_2 + 6A_3\eta + 12A_4\eta^2 + 20A_5\eta^3$$

$$f''' = 6A_3 + 24A_4\eta + 60A_5\eta^2$$

$$0 = 12A_3 + 48A_4\eta + 120A_5\eta^2 + \left[A_2\eta^2 + A_3\eta^3 + A_4\eta^4 + A_5\eta^5\right]\left[2A_2 + 6A_3\eta + 12A_4\eta^2 + 20A_5\eta^3\right]$$

$$12A_3 + 48A_4\eta + \left[120A_5 + 2A_2^2\right]\eta^2 + \left[6A_2A_3 + 2A_2A_3\right]\eta^3$$

$$+ \left[12A_2A_4 + 6A_3^3 + 2A_2A_4\right]\eta^4 + \left[20A_5A_2 + 12A_4A_3 + 6A_4A_3 + 2A_2A_5\right]\eta^5$$

$$+ \left[12A_4^2 + 20A_3A_5 + 6A_3A_5\right]\eta^6 + \left[20A_4A_5 + 12A_4A_5\right]\eta^7 + 20A_5^2\eta^8$$

$$22A_5A_2 = 0$$

$$= 0\cdot\eta + 0 + 0\cdot\eta^2 + \dots + 0\cdot\eta^7 \quad \therefore$$

$A_3 = 0 \quad A_4 = 0 \qquad \underline{120A_5 + 2A_2^2 = 0} \qquad \therefore \quad A_5 = -\frac{1}{60}A_2^2$

$$\therefore \quad f = A_2\left[\eta^2 - \frac{A_2}{60}\eta^5\right]$$

$$f' = A_2\left[2\eta - \frac{A_2}{12}\eta^4\right] \qquad f''' = A_2\left[-A_2\eta\right]$$

$$f'' = A_2\left[2 - \frac{A_2}{3}\eta^3\right]$$

$$12 \dots + \left[2A_2 + 20A_5\eta^3\right]\left[A_2\eta^2 + A_5\eta^5\right] = 0$$

$$120A_5\eta^2 + 2A_2^2\eta^2 + 22A_2A_5\eta^5 + 20A_5^2\eta^8 = 0$$

$$-2\eta A_2^2 + A_2^2\left[2\eta^2 - \frac{11A_2}{30}\eta^5 + \frac{A_2^2}{180}\right]$$

$$A_{5+3K} = -A_2 \frac{A_{2+3K}\{(2+3K)(1+3K)+2\}}{(5+3K)(4+3K)(3+3K)}$$

$$A_{2+3(K+1)} =$$

$$\cancel{A_2 = -\frac{1}{60}\sqrt{A_r}}$$

$$2(5\cdot4\cdot3)A_5 - \frac{[2+2]}{60}A_2^3 = 0$$

$$A_5 = -\frac{1}{60}A_2^2$$

$$2(5+3K)(4+3K)(3+3K)A_{5+3K}\eta^{2+3K} - \eta^{3K+2}A_2\left[2+(3K+2)(3K+1)\right]\frac{A_2^2}{60} = 0$$

$$2(5+3K)(4+3K)(3+3K)A_{5+3K} - \left[2+(3K+2)(3K+1)\right]\frac{A_2^3}{60} = 0$$

1 + 92

3

$K=1 \longrightarrow$

$$2(5+3K)(4+3K)(3+3K)A_{5+3K} + 2A_2A_{2+3K} + (2+3K)(1+3K)A_{2+3K}A_2 = 0$$

$$14\cdot13\cdot12\ A_{14} + 2A_2A_{11} + 11\cdot10\ A_{11}A_2$$

$K=0 \longrightarrow$

92

$$2(5\cdot4\cdot3)A_5 + 2A_2A_2 = 0$$

$f = A_0 + A_1\eta + A_2\eta^2 + A_3\eta^3 + A_4\eta^4 + A_5\eta^5$

$f' = A_1 + 2A_2\eta + 3A_3\eta^2 + 4A_4\eta^3 + 5A_5\eta^4$

$f'' = 2A_2 + 6A_3\eta + 12A_4\eta^2 + 20A_5\eta^3$

$f''' = 6A_3 + 24A_4\eta + 60A_5\eta^2$

$$12A_3 + 48A_4\eta + 120A_5\eta^2 + 2A_0A_2 + 6A_3A_0\eta + 12A_0A_4\eta^2 + 20A_0A_5\eta^3$$
$$+ 2A_1A_2\eta + 6A_1A_3\eta^2 + 12A_1A_4\eta^3 + 20A_1A_5\eta^4 + 2A_2^2\eta^2 + 6A_2A_3\eta^3$$
$$+ 12A_2A_4\eta^4 + 20A_2A_5\eta^5 + 2A_2A_3\eta^3 + 6A_3^2\eta^4 + 12A_3A_4\eta^5 + 20A_3A_5\eta^6$$
$$+ 2A_4A_2\eta^4 + 6A_3A_4\eta^5 + 12A_4^2\eta^6 + 20A_4A_5\eta^7 + 2A_5A_2\eta^5 + 6A_3A_5\eta^6$$
$$+ 12A_4A_5\eta^7 + 20A_5^2\eta^8 = 0 = 0\cdot 1 + 0\cdot\eta + 0\cdot\eta^2 + 0\cdot\eta^3 + 0\cdot\eta^4 + 0\cdot\eta^5 + 0\cdot\eta^6 + \dots$$

0 $12A_3 + 2A_0A_2 = 0 \leftarrow$ IS BC at $\eta = 0$ $f = 0$ A_0 $A_3 = 0$

1 $48A_4 + 6A_3A_0 + 2A_1A_2 = 0 \leftarrow$ IS BC at $\eta = 0$ $f'(0) = 0$ $A_1 = 0$ $A_4 = 0$

2 $120A_5 + 12A_0A_4 + 6A_1A_3 + 2A_2^2 = 0 \leftarrow A_2 = 0$ $120A_5 + 2A_2^2 = 0$

3 $20A_0A_5 + 12A_1A_4 + 6A_2A_3 + 2A_2A_3 = 0 \leftarrow$ IS

4 $20A_1A_5 + 12A_2A_4 + 6A_3^2 + 2A_2A_4 = 0 \leftarrow A_3 = 0$

5 $20A_2A_5 + 12A_3A_4 + 6A_3A_4 + 2A_2A_5 = 0 \leftarrow$ IS $A_2 = A_5 = 0$

6 $20A_3A_5 + 12A_4^2 + 6A_3A_5 = 0 \leftarrow A_4 = 0$ IS

7 $20A_4A_5 + 12A_4A_5 = 0 \leftarrow$ IS IS

8 $20A_5^2 = 0$ $A_5 = 0$ A_5

$$\underset{\downarrow \atop 0}{2(5\cdot 4\cdot 3)} + 2A_2\cdot A_2$$

$$672 A_8 - \frac{22}{60} A_2^3 = 0$$

$5 \cdot 0 + 1 \quad (5 \cdot 2 + 1)$

$7 \cdot 4 + 1$

$1/2$

$$A_8 - \frac{22}{672 \cdot 60} A_2^3 = 0$$

29 $\quad 9 \cdot 6 + 1$

$2 \cdot 14 \cdot 13 \cdot 12 \quad A_{14} + 2 A_2 A_{11} + 11 \cdot 10$

58 $\quad$ 110

$$\frac{11}{6 \cdot 7 \cdot 8 \cdot 5 \cdot 4 \cdot 3}$$

$$2 \cancel{3} (11)(\cdot 10)(\cdot 9) A_{11} + 2A_2 A_8 + 8 \cdot 7 A_2 A_8$$

$(17 \cdot 16 \cdot 15) \; A_{17} + A_2 A_{14} (\cancel{26} + 14 \cdot 13)$

$$A_{11} = -\frac{57 A_2 A_8}{11 \cdot 10 \cdot 9}$$

$2 (14 \cdot 13 \cdot 12) \; A_{14} + A_2 A_{11} (2 + 11 \cdot 10)$

990 $\quad$ 108 $\quad$ 58

$2 (11 \cdot 10 \cdot 9) A_{11} + A_2 A_8 (2 + 8 \cdot 7)$

27 $\quad$ 29

$$A_{11} = \frac{\cancel{56} \; \cancel{57} \cdot 11 \cdot 1}{\underbrace{11 \cdot 10 \cdot 9}\underbrace{\cdot 8 \cdot 7 \cdot 6}\underbrace{\cdot 5 \cdot 4 \cdot 3}} A_2^4$$

$2 (8 \cdot 7 \cdot 6) A_8 + A_2 A_5 (2 + 5 \cdot 4)$

336

$\frac{2}{120} \cdot \frac{22}{672} \cdot \frac{114}{1980}$

$(16 \cdot 3 - 1) \quad (10 \cdot 3 - 1)(4 \cdot 3 - 1)$

$k = 1 \longrightarrow$

$11 \cdot 10 + 1 \, [8 \cdot 7 + 1] \, [5 \cdot 4 + 2]$

$\cancel{1 \cdot 11 \cdot 57}$

$14 \cdot \quad 60 \cdot 336 \cdot 990$

$$A_{3k-1} = (-1)^{3k-1} \frac{2[}{(3k-1)!} A_2^k$$

56

$3k+2$

$\cancel{(k(k-1)+1)}(\quad 4 \cdot 3 - 1$

$+1$

$5 \cdot 2 + 1$

$$\delta = \frac{5.0\,x}{\sqrt{\frac{u_1 x}{\nu}}} \qquad \delta^* = \frac{1.73x}{\sqrt{\frac{u_1 x}{\nu}}}$$

$$R_{N\,at\,20{,}000} = \frac{2.10^7}{2.6235} = 7.63\times10^6 \qquad \sqrt{R_{N_x}} = 2.76\times10^3$$

$$\delta = \frac{50}{2760} = \frac{5}{276} = 1.812\times10^{-2}\,ft$$

$$\delta^* = \frac{17.3}{2760} = .627\times10^{-2}\,ft$$

$$\beta = 1.185\times10^{-2}\,ft$$

$$\frac{\dot m}{ft} = \rho_1 u_1 \beta = (.0012664)(200)(1.185\times10^{-2}) = (.0012664)(2.37) = 3\times10^{-3}\,\frac{slugs}{sec\cdot ft}$$

$$\frac{lb\cdot sec^2}{ft^4}\cdot\frac{ft^2}{sec} = \frac{lb\cdot sec}{ft^2} = \frac{slug}{sec\cdot ft}$$

$$D = .664\,\mu u_1\sqrt{RN_L} = .664\,(3.323\times10^{-7})(2\times10^2) = 1.328\,(3.323\times10^{-5})\sqrt{R_{N_L}}$$

$$= 4.41\times10^{-5}\,lb/ft\cdot(2.76\times10^3) = .1217\ lb/ft$$

for fully developed flow the boundary layer change wrt x is 0

$$\therefore \quad \cancel{\delta = 5.0\sqrt{\frac{\nu x}{u_1}}} \qquad \cancel{\frac{d\delta}{dx} = \frac{5.0}{2}\sqrt{\frac{\nu}{x u_1}}}$$

$$a = 5.0\sqrt{\frac{\nu}{u_1}x} \qquad \frac{a^2}{25}\frac{u_1}{\nu} = x \qquad x = a^2\left(\frac{200}{25}\right)\frac{10^4}{2.6235} = \frac{8\times10^4}{2.6235} =$$

$$3.05\times10^4\,ft$$

$$\nu x/a^2 U_1 = .16 \qquad x = \frac{.16a^2 U_1}{\nu} = \frac{.16a^2(200)}{2.6235\times10^{-4}} =$$

$$\frac{32}{2.6235}$$

$dx = r\,d\theta$

$dy = dr$

$\delta y = \frac{\partial y}{\partial r}\,\delta r$

$x = r\sin\theta$

$x = x(r,\theta)$

$dx = \frac{\partial x}{\partial r}\,dr + \frac{\partial x}{\partial \theta}\,d\theta$

$dx = r\cos\theta\,d\theta + \sin\theta\,dr$

$-r\cos\theta \quad\quad \sin\theta = \frac{\partial x}{\partial r}$

$\frac{\partial x}{\partial \theta}$

$y = r\cos\theta$

$dy = r\sin\theta\,d\theta + \cos\theta\,dr \quad\quad \frac{\partial y}{\partial \theta} = r\sin\theta \quad\quad \frac{\partial y}{\partial r} = \cos\theta$

$\frac{\partial x}{\partial r}\,dr = \sin\theta\,dr \quad\quad \frac{\partial y}{\partial r}\,dr = \cos\theta\,dr$

$\left[-k(r-r_0)\sin\theta\right]\sin\theta\,dr + \left[mg - k(r-r_0)\cos\theta\right]\cos\theta\,dr$

$mg\cos\theta\,dr \quad - k(r-r_0)\,dr \quad\quad \sin\theta$

$mg\cos\theta - k(r-r_0) = Q_r$

$-r\cos\theta = \frac{\partial x}{\partial \theta}$

$-\cos\theta = \frac{1}{r}\frac{\partial x}{\partial \theta}$

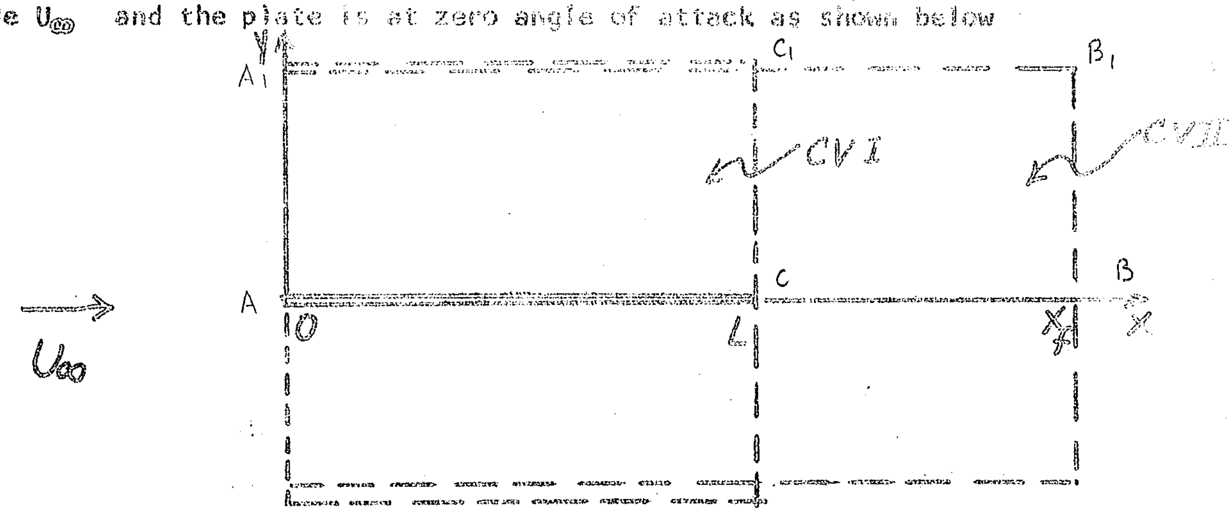
$\sin\theta = \frac{1}{r}\frac{\partial y}{\partial \theta}$

POLYTECHNIC INSTITUTE OF BROOKLYN

DEPARTMENT OF AEROSPACE ENGINEERING AND APPLIED MECHANICS

AE 333
Homework — Assigned Dec. 15 — Due Dec. 22

Consider the flow over a flat plate of length L. The velocity is uniform and of magnitude U_∞ and the plate is at zero angle of attack as shown below



A) Sketch the flow field in CV I. Determine the drag on the flat plate by means of the momentum theorem applied to CV I for $U_\infty = 200$ ft/sec, $L = 20$ ft.; consider the fluid to be air at STP conditions. Plot the skin friction as a function of X for $0 \leq x \leq L$ for the case mentioned. Integrate the skin friction numerically (or graphically) to determine the drag.

B) Sketch the flow field in CV II. Determine the drag on the flat plate by means of the momentum theorem applied to CV II. Plot the skin friction coefficient as a function of X for $0 \leq x \leq x_f$. How does this result compare with that in part (a).

C) Using the results of parts (a) and (b) plot
   (i) the momentum thickness θ as a function of X for $0 \leq x \leq x_f$
   (ii) the v-component of velocity along the x-axis for $0 \leq x \leq x_f$
   (iii) the shear stress t_{xy} along the x-axis for $0 \leq x_f$

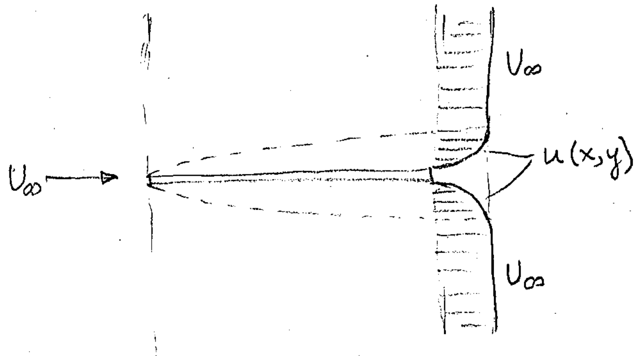
at tail & lead edge

D) Are there any areas in theflow field where the analyses you have used are invalid? Enumerate these and indicate their effects on your results.

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho u_\infty^2} \quad ; \quad \frac{C_f}{2} = \frac{\tau_w}{\rho u_\infty^2} = \frac{d\theta}{dy} = \frac{\tau_w}{\rho u_\infty^2}$$

$$\rho U_\infty^2 \frac{d\theta}{dx} = \tau_w$$

$$\rho u_\infty^2 \theta = \int \tau_w \, dx$$



$$\frac{\sqrt{\nu x^2}}{\sqrt{ux}} = \sqrt{\frac{\nu x}{u}} = \sqrt{\frac{\mu x}{\rho u}}$$

$U_\infty = 200$ ft/sec $\qquad L = 20$ ft $\qquad \rho = .002378 \qquad \mu = 3.719 \times 10^{-7}$

$$R_{N_d} = \frac{200(.002378)}{3.719 \times 10^{-7}} d = .1278 \times 10^{7} d = 1.278 \times 10^{6} d$$

$$\sqrt{R_{N_d}} = 1131 \sqrt{d}$$

$d \qquad \sqrt{d}$

$$\frac{d\theta}{d(d)} = \frac{.664}{2 \cdot 1131 \sqrt{d}} = \frac{.664}{2262 \sqrt{d}}$$

since for flat plat $C_f = \frac{\tau_w}{\frac{1}{2}\rho u_\infty^2} \qquad \frac{C_f}{2} = \frac{\tau_w}{\rho u_\infty^2} = \frac{d\theta}{dx} \qquad \therefore \quad \tau_w = \rho u_\infty^2 \frac{d\theta}{dx}$ on one side

$$\rho u_\infty^2 \frac{d\theta}{dx} = 95.2 \frac{d\theta}{dx} = \frac{95.2\,(.664)}{2262 \sqrt{d}} = \frac{27.94}{\sqrt{d}} \times 10^{-3} =$$

$2.276 \times 10^{-3} (4 \times 10^{4})$

d	$\sqrt{d}$	$\tau_w \times 10^3$
0	0	0 ✓
2	1.414	19.74 ✓
4	2	13.96 ✓
6	2.45	11.41 ✓
8	2.83	9.88 ✓
10	3.16	8.83 ✓
12	3.46	8.07 ✓
14	3.74	7.47 ✓
16	4	6.98 ✓
18	4.24	6.59 ✓
20	4.47	6.24 ✓

$$D/_{ft} = \sum \tau_w \, dx$$
$$= 2\,[3.12 + 92.93] = 2\,[96.05] = 192.1$$

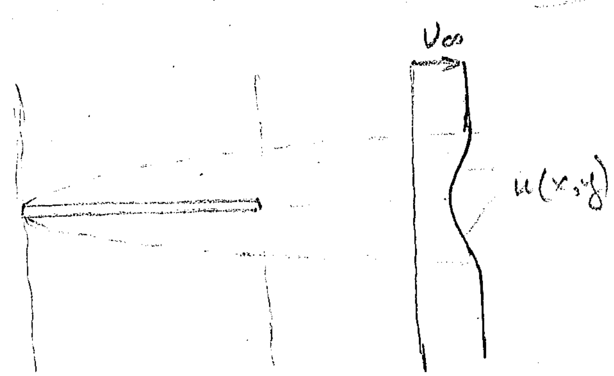
19.74
13.96
33.70
11.41
45.11
9.88
54.99
8.83
63.82
8.07

71.89
7.47
79.36
6.98
86.34
6.59
92.93

D_{TOT} on both sides

384.2 lb/ft $\times 10^{-3}$

$.3842$

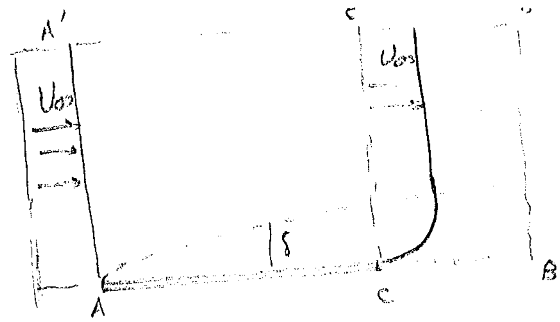


Momentum flux across AA' $\int_0^h \rho b U_\infty^2 \, dy$

AC 0

A_1C_1 $\int_0^h -\rho b u^2 \, dy$

CC_1 $\int_0^h -\rho b U_\infty (U_\infty - u) \, dy$



$$\rho b \int_0^h (U_\infty^2 - u^2 - U_\infty^2 + U_\infty u) \, dy = \rho b \int_0^h u (U_\infty - u) \, dy$$ where b is the plate width

$\sum$ momentum flux = drag on one side

$\therefore \; 2\rho b \int_0^h u (U_\infty - u) \, dy = D_{TOT}$ on plate

however $\int_0^h u (U_\infty - u) \, dy = \Theta u_\infty^2$ or $2\rho b u_\infty^2 \Theta = D_{TOT}$

but $\Theta = \dfrac{.664 x}{\sqrt{R_{Nx}}} = \dfrac{.664 \sqrt{l}}{1131} = \dfrac{.664 (4.47)}{1131} = 2.625 \times 10^{-3}$

using $\rho = .002378$ $\quad U_\infty = 200$

$\mu = 3.719 \times 10^{-7}$

$\therefore \quad \dfrac{D_{TOT}}{ft} = 2(2.625)(.002378)(200)^2 \times 10^{-3} = .499 \;\; lb/ft$

$\tau_0 = .332 \dfrac{\mu U_\infty}{x} \sqrt{R_{Nx}} = .332 \, u_\infty \sqrt{\rho u_\infty \mu} \cdot \dfrac{1}{\sqrt{x}} = .0293/\sqrt{x}$

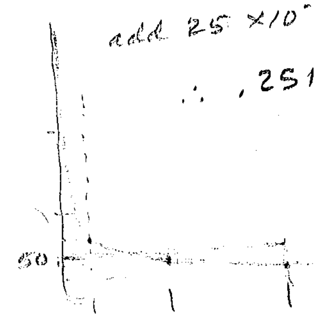
x	$\sqrt{x}$	$\tau_w \times 10^3$	
0	0	∞	
2	1.414	20.7	
4	2	14.65	26.6
6	2.45	11.95	36.95
8	2.83	10.35	46.21
10	3.16	9.26	54.66
12	3.46	8.45	62.49
14	3.74	7.83	69.82
16	4	7.33	76.73
18	4.24	6.91	
20	4.47	6.55	

$D = 2[3.28 + 97.43] \times 10^{-3} = 2[.10071] = .2014$

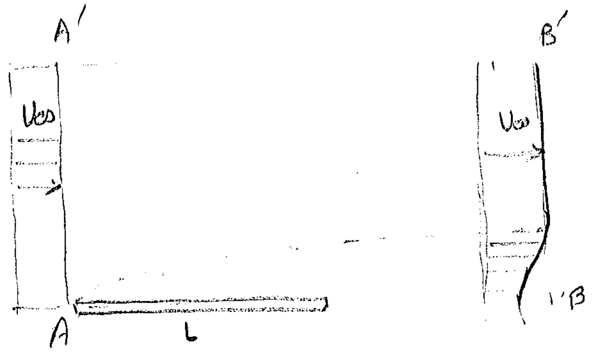
last should be 50

add $25 \times 10^{-3} \cdot 2 = 50 \times 10^{-3}$

$\therefore \; .2514$



$\therefore \; D_{TOT} = .5028$



$2D = 2\rho V_\infty^2 \Theta$ where $\Theta = \dfrac{.664 L}{\sqrt{R_{N_L}}}$

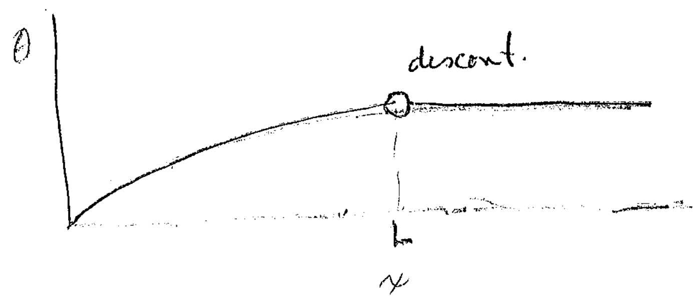
Since no plate exists between $L \le x \le x_f$
$\therefore$ the drag $= .498$ lb/ft as before

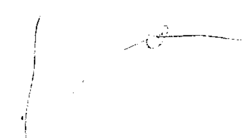
$$C_{f_{\text{both sides of plate}}} = \frac{1.328}{\sqrt{R_{N_x}}} = \frac{1.328}{1131\sqrt{x}} = \frac{1.174 \times 10^{-3}}{\sqrt{x}}$$

$\sqrt{x}$	$C_f \times 10^{+3}$
0	∞
1.414	.831
2	.587
2.45	.479
2.83	.415
3.16	.3715
3.46	.339
3.74	.314
4	.294
4.24	.277
4.47	.266
~~6~~ ~~4.47~~ ↓ x_f	~~6~~ ~~.266~~ 0

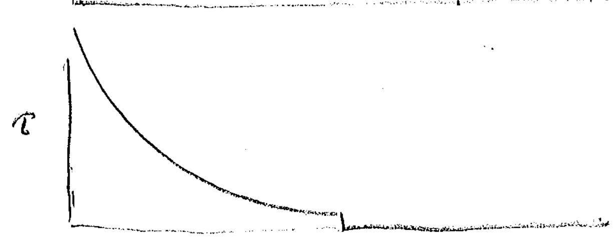
$\dfrac{.664\sqrt{x}}{1131}$

$.587 \times 10^{-3}\sqrt{x}$





over plate it was assumed that θ; also small
because of symmetric flow v≈0 in this area



$$\frac{\partial u}{\partial t} - \nu \frac{\partial^2 u}{\partial y^2} = 0$$

$$\int_0^Y \frac{\partial u}{\partial t} dy - \nu \int_0^Y \frac{\partial^2 u}{\partial y^2} dy = 0$$

$$\int_0^Y \frac{\partial u}{\partial t} dy - \nu \left.\frac{\partial u}{\partial y}\right|_0^Y = 0 \qquad \int_0^Y \frac{\partial u}{\partial t} dy = -\frac{\tau_0}{\rho}$$

$$\frac{d}{dt}\int_0^\delta u\,dy = \int_0^\delta \frac{\partial u}{\partial t} dy + u(\delta,t)\frac{d\delta}{dt}$$

$$\int_0^\delta \frac{\partial u}{\partial t} dy = \frac{d}{dt}\int_0^\delta u\,dy - u(\delta,t)\frac{d\delta}{dt} = -\frac{\tau_0}{\rho}$$

$$\frac{d}{dt}\int_0^\delta u\,dy - u_e \frac{d\delta}{dt} = -\frac{\tau_0}{\rho}$$

$$-\frac{\tau_0}{\rho} - \frac{d}{dt}\int_0^\delta u\,dy = -u_e\frac{d\delta}{dt}$$

now

$u = u_e$ at $\eta = 1$

$\frac{\partial u}{\partial \eta} = 0$ at $\eta = 1$ $\qquad \frac{\partial^2 u}{\partial \eta^2} = 0$ at $\eta = 1$ ← from Euler's Equation $\frac{dp}{dx} = 0$

$A + B + C = 1$ $\qquad \frac{u}{u_e} = A\eta + B\eta^2 + C\eta^3$

$A + 2B + 3C = 0$

$2B + 6C = 0$ $\qquad \frac{\partial u}{\partial \eta} = A + 2B\eta + 3C\eta^2$

$\qquad \frac{\partial^2 u}{\partial \eta^2} = 2B + 6C\eta$

$$A = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 2 & 6 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 2 & 6 \end{vmatrix}} = \frac{[12-6]}{[(12-6)-(6-2)]} = \frac{6}{(6-4)} = 3$$

$$B = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 0 & 0 & 6 \end{vmatrix}}{2} = \frac{6[-1]}{2} = -3 \qquad C = 1$$

$$\boxed{\frac{u}{u_e} = 3\eta - 3\eta^2 + \eta^3} \qquad \frac{1}{u_e}\frac{\partial u}{\partial y} = \frac{3}{\delta} - \frac{6}{\delta}\left(\frac{y}{\delta}\right) + \frac{3}{\delta}\left(\frac{y}{\delta}\right)^2$$

$$\frac{\partial u}{\partial y} = \frac{3}{\delta}u_e \cdot - \frac{6}{\delta}(0) + \frac{3}{\delta}(0) \qquad \frac{3\mu}{\delta}u_e = \mu\left.\frac{\partial u}{\partial y}\right|_0 = \tau_0$$

$$+\frac{\tau_0}{\rho} + \frac{d}{dt}\int_0^\delta u\,dy = u_e \frac{d\delta}{dt}$$

$$+3\frac{\mu u_e}{\delta\rho} + \frac{d}{dt}\int_0^\delta \left[3(y/\delta) - 3(y/\delta)^2 + (y/\delta)^3\right] u_e\,dy = +u_e\frac{d\delta}{dt}$$

$$+\frac{3\mu}{\delta\rho} + \frac{d}{dt}\int_0^\delta \left[3(y/\delta) - 3(y/\delta)^2 + (y/\delta)^3\right] dy = \frac{d\delta}{dt}$$

$$+\frac{3\mu}{\rho\delta} + \frac{d}{dt}\left|\left[\frac{3}{2}\frac{y^2}{\delta} - \frac{3}{3}\,y^3/\delta^2 + \frac{1}{4}(y^4/\delta^3)\right]_0^\delta\right. = \frac{d\delta}{dt}$$

$$+\frac{3\mu}{\rho} + \frac{d}{dt}\left[\frac{3}{2}\delta \overset{1/2}{-} \delta + \frac{1}{4}\delta\right] = \frac{d\delta}{dt}$$

$$+\frac{3\nu}{\delta} + \frac{3\,d\delta}{4\,dt} = \frac{d\delta}{dt}$$

$$+\frac{3\nu}{\delta} = \frac{1}{4}\frac{d\delta}{dt} \qquad +3\nu\cdot 4\,dt = \delta\,d\delta$$

$$+12\nu t = \frac{\delta^2}{2}$$

$$+24\nu t = \delta^2 \qquad \delta = \sqrt{24\nu t}$$

$$= \sqrt{24\nu t} = 4.9\sqrt{\nu t}$$

$$\frac{3\mu}{\rho\delta}u_e = \tau_0$$

$$\frac{3\mu\,u_e}{.4.9\sqrt{\nu t}} = \tau_0 \qquad \frac{3\mu u_e}{4.9\sqrt{\nu t}} = \tau_0 = \frac{3}{4.9}\sqrt{\frac{\mu\rho}{t}u_e^2} = .612\sqrt{\frac{\mu\rho u_e^2}{t}}$$

at $u = u_e$ $\quad \eta = 1$ $\qquad \frac{\partial^2 u}{\partial\eta^2} = 0$ $\quad \eta = 0$ $\quad$ since $\frac{dp}{dx} = 0$ from momentum

$\frac{\partial u}{\partial\eta} = 0 \qquad \eta = 1$

$$u/u_e = A\eta + B\eta^2 + C\eta^3$$

$$1 = A + B + C \qquad B = 0 \qquad A + C = 1 \qquad C = -1/2$$
$$0 = A + 2B + 3C \qquad A + 3C = 0 \qquad A = 3/2$$
$$0 = 2B \qquad A = -3C \quad -3C + C = 1 \quad C = -1/2$$

$$u/u_e = \frac{3}{2}\eta - 1/2\,\eta^3 \qquad A =$$

$$\frac{1}{u_e}\frac{\partial u}{\partial y} = \frac{3}{2\delta} - \frac{1}{2\delta}(y/\delta)^2 \qquad \mu\left.\frac{\partial u}{\partial y}\right|_0 = \mu u_e\left[3/2\delta - 0\right] = \frac{3\mu u_e}{2\delta}$$

$$\frac{3\nu u_e}{2\delta} + \frac{d}{dt}\int_0^{\delta}\left[\frac{3}{2}\eta - \frac{1}{2}\eta^3\right]u_e\,dy = u_e\frac{d\delta}{dt}$$

$$\frac{3\nu}{2\delta} + \frac{d}{dt}\left(\left[\frac{3}{2}\frac{y^2}{2\delta} - \frac{1}{2}\frac{y^4}{4\delta^3}\right)\right|_0^{\delta} = \frac{d\delta}{dt}$$

$$\frac{3\nu}{2\delta} + \frac{d}{dt}\left(\frac{3}{4}\delta - \frac{1}{8}\delta\right) = \frac{d\delta}{dt} \quad ; \quad \frac{3\nu}{2\delta} + \frac{d}{dt}\left(\frac{5}{8}\delta\right) = \frac{d\delta}{dt} \; ; \; \frac{3\nu}{2\delta} = \frac{3}{48}\frac{d\delta}{dt}$$

$$\frac{3\nu}{2\delta} + \frac{5}{8}\cdot\frac{d\delta}{dt} = \frac{d\delta}{dt} \qquad 4\nu t = \frac{\delta^2}{2} \qquad \delta = \sqrt{8\nu t} = 2.83\sqrt{\nu t}$$

$$\frac{3\mu u_e}{2(2.83)\sqrt{\nu t}} = \tau_0 \qquad \frac{3}{5.66}\sqrt{\frac{\rho u_e^2}{t}\mu}$$

$$\frac{3\nu}{2\delta}\cdot\frac{8}{3} = \frac{d\delta}{dt}$$

$$4\nu\,dt = \delta\,d\delta$$

$$4\nu t = \frac{\delta^2}{2}$$

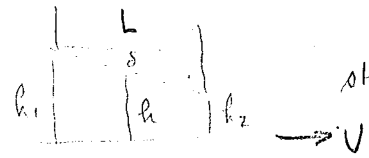
$$8\nu t = \delta^2$$

$$\delta = \sqrt{8\nu t} = 2.83\sqrt{\nu t}$$

$$\tau_0 = .539\sqrt{\frac{\rho\mu u_e^2}{t}}$$

for lubrication

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$



steady flow

non dimensionalize $u \to u/U \quad x \to x/L$

$v \to v/U \quad y \to y/h$ if δ is small high...

$$\frac{U}{L}\frac{\partial(u/U)}{\partial(x/L)} + \frac{U}{h}\frac{\partial(v/U)}{\partial(y/h)} = 0 \qquad \therefore \quad \frac{\partial(u/U)}{\partial(x/L)} + \frac{L}{h}\frac{\partial(v/U)}{\partial(y/h)} = 0$$

since $u/U \sim O(1) \quad y/h \sim O(1)$

$x/L \sim O(1) \quad \therefore \quad v/U \sim O(h/L) \qquad \therefore \quad u \gg v$

go to first equation of momentum

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$$

since $u \gg v$ $\quad v\frac{\partial u}{\partial y}$ can be neglected. also since steady flow $\frac{\partial u}{\partial t}$

go to second equation

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2}\right]$$ since $u \gg v$ all terms involving v are neglected

$$\therefore \quad \boxed{\frac{\partial p}{\partial y} = 0}$$

let's look at $\quad \frac{\partial^2 u}{\partial x^2} \Big/ \frac{\partial^2 u}{\partial y^2} \longrightarrow \frac{U}{L^2}\frac{\partial^2(u/U)}{\partial(x/L)^2} \Big/ \frac{U}{h^2}\frac{\partial^2(u/U)}{\partial(y/h)^2} \sim O\left(\frac{h^2}{L^2}\right)$

$\therefore \quad \frac{\partial^2 u}{\partial x^2} \ll \frac{\partial^2 u}{\partial y^2} \quad \therefore$ neglected $\frac{\partial^2 u}{\partial x^2}$ wrt $\frac{\partial^2 u}{\partial y^2}$ then

$$\rho\left[u\frac{\partial u}{\partial x}\right] = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial y^2}\right]$$

let's look at inertia term to viscous term.

$$\rho u\frac{\partial u}{\partial x} \Big/ \mu\frac{\partial^2 u}{\partial y^2} = \rho\frac{U^2}{L}(u/U)\left(\frac{\partial(u/U)}{\partial(x/L)}\right) \Big/ \mu\frac{U}{h^2}\frac{\partial^2 u/U}{\partial(y/h)^2} \sim \frac{\rho U^2 h^2}{L\mu U} = \frac{\rho UL}{\mu}\left(\frac{h}{L}\right.$$

$R^* \ll 1 \qquad \therefore \quad \rho u\left(\frac{\partial u}{\partial x}\right) \ll \mu\frac{\partial^2 u}{\partial y^2}$ <u>or</u>

$$\frac{\partial p}{\partial x} = \mu\frac{\partial^2 u}{\partial y^2}$$ provided $u \gg v$ & $R^* = \frac{\rho UL}{\mu}\left(\frac{h}{L}\right)^2 \ll 1$

since $\frac{\partial p}{\partial y} = 0 \longrightarrow p = p(x) \quad \therefore$

$$\frac{1}{\mu}\frac{dp}{dx}y + f_1(x) = \frac{\partial u}{\partial y} \qquad u = \frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2} + f_1(x)y + f_2(x)$$

$u = 0$ at $y = h$
$u = U$ at $y = 0$

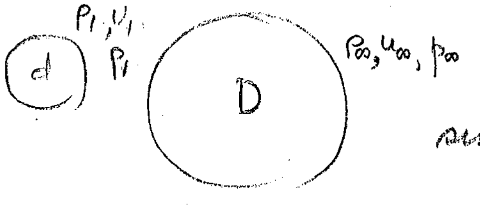
since $u = U$ at $y = 0$ & at $y = h$ u=0 the

$$u = \frac{1}{\mu}\frac{dp}{dx}\left[\frac{y^2}{2} - \frac{yh}{2}\right] + U\left[1 - y/h\right] \qquad D = \int_0^L -\mu \left.\frac{\partial u}{\partial y}\right|_0 dx = \frac{\mu U L}{(k-1)h_2}\left[4\ln k - \frac{6(k-1)}{k+1}\right]$$

constant due to steady state $$Q = \int_0^h u\,dy = \frac{Uh}{2} - \frac{h^3}{12\mu}\frac{dp}{dx} \rightarrow \frac{h_1 h_2}{h_1 + h_2} U \qquad k = h_1/h_2$$

$$p = p_0 + \frac{6\mu U L}{h^2}\frac{(h_1 - h)(h - h_2)}{h_1^2 - h_2^2} \qquad P = \int_0^L (p - p_0)\,dx = \frac{6\mu U L^2}{h_2^2 (k-1)^2}\left[\ln k - \frac{2(k-1)}{k+1}\right]$$

For flow similarity non dimensionalize wrt body dimension (distances $\bar{x} = \frac{x}{D}$) $\bar{u} = \frac{u}{U_\infty}$



$\bar{p} = \frac{p}{2q_\infty}$ $\bar{u} = u/u_1$

since $\bar{u} = \bar{v} = \bar{w} = 0$ @ body
$\bar{u} = 1$ $\bar{v} = \bar{w} = 0$ @ ∞ then $\bar{x} = \frac{x}{d}$

$$\bar{q}(\bar{\nabla}\cdot\bar{u}) = -\bar{p}_{,\bar{x}} + \frac{1}{R_{N_{\infty,D}}}\left[\bar{\nabla}^2\bar{u}\right]$$

Assume geometrically similar bodies flow steady & incompressible

$$\bar{q}(\bar{\nabla}\cdot\bar{u}) = -\bar{p}_{,\bar{x}} + \frac{1}{R_{N_{1,d}}}\left[\bar{\nabla}^2\bar{u}\right]$$

for similar flows $R_{N_{\infty,D}} = R_{N_{1,d}}$

Boundary Layer Equation 1. assume effective viscosity in a region close to body $\ni$ $\exists$ 2 different regions

2. Assume $L >> \delta$

3. Assume $\bar{u} = \frac{u}{U_\infty} \sim O(1)$ on body 4. steady flow

$\bar{x} = \frac{x}{L} \sim O(1)$

$\bar{y} = y/L \sim O(\bar{\delta} = \delta/L)$

in 2-D $$\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}} = -\frac{\partial\bar{p}}{\partial\bar{x}} + \frac{1}{R_N}\left[\frac{\partial^2\bar{u}}{\partial\bar{x}^2} + \frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right] \quad \text{all } O(1)$$

$$\frac{U_\infty}{L}\left[\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}}\right] = 0 \rightarrow \bar{v} \sim O(\bar{\delta})$$

order of magnitude check shows $\frac{\partial^2\bar{u}}{\partial\bar{x}^2} << \frac{\partial^2\bar{u}}{\partial\bar{y}^2}$

at the bl, $\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} = -\frac{\partial\bar{p}}{\partial\bar{x}} \rightarrow \frac{\partial\bar{p}}{\partial\bar{x}} \sim O(1) \rightarrow \frac{1}{R_N} \sim O(\bar{\delta}^2)$ or $R_N \sim O(1/\bar{\delta}^2)$

$$\bar{u}\frac{\partial\bar{v}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{v}}{\partial\bar{y}} = -\frac{\partial\bar{p}}{\partial\bar{y}} + \frac{1}{R_N}\left[\frac{\partial^2\bar{v}}{\partial\bar{x}^2} + \frac{\partial^2\bar{v}}{\partial\bar{y}^2}\right] \qquad \frac{\partial^2\bar{v}}{\partial\bar{x}^2} << \frac{\partial^2\bar{v}}{\partial\bar{y}^2} \rightarrow \frac{\partial\bar{p}}{\partial\bar{y}} \sim O(\bar{\delta})$$

comparison of $\frac{\partial\bar{p}}{\partial\bar{y}}$ to $\frac{\partial\bar{p}}{\partial\bar{x}}$ shows that $\bar{p}_{,\bar{x}} >> \bar{p}_{,\bar{y}}$ or $\boxed{p_{,y} = 0}$

2

$\therefore$ the governing equation leads to

$$\rho\left[(\bar{q}\cdot\nabla)u\right] = -p_{,x} + \mu\, u_{,yy} \qquad \text{at b.l.} \quad \rho u_e \frac{\partial u_e}{\partial x} = -\frac{\partial p}{\partial x}$$

$$p_{,y} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

the boundary layer was found rotational $\omega = \frac{1}{2}\left(u_{,y} - v_{,x}\right) = \frac{1}{2}\frac{U}{L}\left(\frac{\partial \bar{u}}{\partial \bar{y}} - \frac{\partial \bar{v}}{\partial \bar{x}}\right)$

order of magnitude check shows $\omega_z \approx \frac{1}{2} u_{,y}$

Blasius solution shows $\frac{dp}{dx} = 0$ In vicinity of body about a pt $\mu \frac{\partial^2 u}{\partial y^2} = \frac{dp}{dx}$

since Navier Stokes is not a fn of p then define $u = \frac{\partial \psi}{\partial y}$ $v = -\frac{\partial \psi}{\partial x}$ u_1

define $\eta = C y x^{\alpha}$ $\Rightarrow$ $\psi = \frac{u_1}{C x^{\alpha}} f(\eta)$ if $u = u_1 f'(\eta)$

if $\alpha = -\frac{1}{2}$ and $u_1/\nu C^2 = 1$ $\Longrightarrow$ $C = \sqrt{u_1/\nu}$ $\quad 2f''' + f f'' = 0$

Assume inner soln $f = \sum A_n \eta^n$ use bc $y=0,\ u=0,\ v=0 \rightarrow f'(\eta) = f(\eta) = 0$

$y = \infty,\ u = u_1 \rightarrow f'(\eta) = 1$

Assume outer soln $f = f_1 + f_2$

$\Rightarrow$ $f_1 \ggg f_2$, at $\eta = \infty$ $f_2 = f_2' = f_2'' = 0$ $\Longrightarrow$ $f = \underbrace{\eta - \beta}_{\approx \eta - \beta} + f_2$ since $\frac{df}{d\eta} = f_1'(\eta) = 1$

$$f_2 = \gamma \int_{\infty}^{\eta} d\varsigma \int_{\infty}^{\eta} e^{-\frac{1}{4}(\varsigma-\beta)^2} d\varsigma \quad \Longrightarrow \quad f = (\eta - \beta) + \gamma \int_{\infty}^{\eta} d\varsigma \int_{\infty}^{\eta} e^{-\frac{1}{4}(\varsigma-\beta)^2} d\varsigma$$

obtained from diff eq $2f_2''' + (\eta - \beta) f_2'' = 0$. Patch soln.

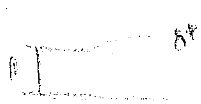
$u/u_1 = .9915$ gives $\eta = 5 = \frac{\delta}{x}\sqrt{R_{N_x}}$ $\quad \delta = \frac{5x}{\sqrt{R_{N_x}}}$

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_0 = \frac{.332\,\mu u_1}{x}\sqrt{R_{N_x}}$$

$$c_f = \frac{\tau_w}{q} = \frac{.664}{\sqrt{R_{N_x}}}$$

$$D/\text{width}_{\text{ on one plate}} = \int_0^L \tau_w\, dx = .664\,\mu\, u_1 \sqrt{R_{N_L}} \qquad C_D = \frac{D}{qL} = \frac{1.328}{\sqrt{R_{N_L}}}$$

define a displacement thickness whereby the external flow is displaced outward as a consequence of the decrease in velocity in the b.l. Mass flow remains the same between 2 streamlines.



$$\rho_1 u_1 \beta = \int_0^{\delta} \rho u\, dy \Longrightarrow \delta^* = \int_0^{\delta} (1 - u/u_1)\, dy \quad \text{if } \rho = \rho_1$$

$$\delta^* = \frac{1.73x}{\sqrt{R_{N_x}}} \quad \text{since } u/u_1 = f'(\eta)$$

$v = \frac{u_1 (.865)}{\sqrt{R_{N_x}}}$ at edge of bl.

Momentum thickness defined by momentum loss $\rho u_1^2 \theta = \int_0^\infty \rho u (u_1 - u) dy$

$$\theta = \int_0^\infty \frac{u}{u_1}(1 - u/u_1)\, dy = \frac{.664x}{\sqrt{R_{N_x}}}$$

Falkner Skan Solution

$\omega = \varphi + \psi i$ where $\omega = Ur^n(\cos n\theta + i \sin n\theta)$

$\psi = Ur^n \sin n\theta$ $\quad \frac{\partial \psi}{\partial r} = -v_\theta = Ur^{n-1} n \sin n\theta = 0$

$\psi = 0$ at body $n\theta = 0, \pi$ $\quad \frac{1}{r}\frac{\partial \psi}{\partial \theta} = v_r = -Ur^{n-1} n \cos n\theta = -Ur^{n-1} n$ $\Big\}$ at body

set $v_{r_{body}} = u_e = -Ur^{n-1} n$ $\quad$ let $r = x$ $\quad m = n-1$ $\quad u_1 = -nU$

$\therefore \quad u_e = +u_1 x^m$

$\pi(1 - \beta/2) = \pi/n$ $\quad \beta = \frac{2m}{m+1}$

using steady flow Prandtl bl with $\frac{dp}{dx} = -\rho U \frac{dU}{dx}$

and let $U = cx^m$ $\quad$ & $u = U f'(\eta)$ $\quad$ where $\eta = Ayx^\alpha$ $\quad$ & $u = u_1 x^m \quad y = \delta$; $\quad u = 0 \quad y = 0$

$\psi = \frac{c}{A} x^{m-\alpha} f$ $\quad$ leads to $f''' + ff'' - \frac{2m}{m+1}[f'^2 - 1] = 0$

& $\eta = \sqrt{\frac{c}{\nu}\frac{m+1}{2}}\, y x^{\frac{m-1}{2}}$, $\alpha = \frac{m-1}{2}$, $\quad A = \sqrt{\frac{c}{\nu}\frac{m+1}{2}}$ $\quad \rightarrow \quad \eta = \sqrt{\frac{U}{\nu x}\frac{m+1}{2}}\, y$

let $\frac{\nu}{c}\frac{A^2}{m} = \frac{m+1}{2m}$ $\quad$ if $m = 1$ $\quad n = 2$ stagnation flow / flow in corner; $\quad m = 0$ $\quad n = 1$ flat plate

Von Karman Integral technique $\quad$ integrate on average of boundary

$$\int_0^y \frac{\partial u}{\partial x} dy + \int_0^y \frac{\partial v}{\partial y} dy = 0 \quad \rightarrow \quad v = -\int_0^y \frac{\partial u}{\partial x} dy$$

place into prandtl bl. using $\int_0^\delta \mu \frac{\partial^2 u}{\partial y^2} dy = \mu \frac{\partial u}{\partial y}\Big|_0^\delta = -\tau_w$

& $\int_0^\delta -\frac{\partial p}{\partial x} dy = \int_0^\delta \rho u_e \frac{\partial u_e}{\partial x} dy$

use of Leibnitz' Rule $\quad \frac{d}{dx}\int_0^\delta f(x,y)\, dy = \int_0^\delta \frac{\partial f}{\partial x} dy + f(x,\delta)\frac{d\delta}{dx}$

leads to $\frac{d\theta}{dx} + \theta\left[2 + \frac{\delta^*}{\theta}\right]\frac{1}{u_e}\frac{du_e}{dx} = \frac{C_f}{2}$ $\quad$ where $\theta = \int_0^\delta \frac{u}{u_e}(1 - u/u_e)\, dy$

$\delta^* = \int_0^\delta (1 - u/u_e)\, dy$

Pohlhausen technique assume $u/u_e = \sum_{i=1}^{n} A_i \, (y/\delta)^i$

define Pohlhausen Pressure Parameter $\lambda = \frac{\delta^2}{\nu} \frac{du_e}{dx}$

separation if $\left.\frac{\partial u}{\partial y}\right|_0 = 0$ inflection $\left.\frac{\partial^2 u}{\partial y^2}\right|_\delta = 0$ useful to get A_i's

with injection $v = v_0 - \int_0^y u_{,x} \, dy$

$\frac{d\theta}{dx} + \theta\left[2 + \frac{\delta^*}{\theta}\right] \frac{1}{u_e} \frac{du_e}{dx} + \frac{v_0}{u_e} = \frac{C_f}{2}$ at $\eta = 0$ $\rho v \frac{\partial u}{\partial y} = -p_{,x} + \mu u_{,yy}$

the navier stokes reduce to

$$\rho\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{dp}{dx}+\mu\frac{\partial^2 u}{\partial y^2}$$

$$p_{,y}=0$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

For blasius solution in potential flow u = constant over a flat plate @ $0°=\alpha$

$\therefore$ $\frac{\partial u}{\partial x}=0 \implies \frac{\partial p}{\partial x}=0$ at body $\therefore$

The blasius equations are

$$\rho\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=\mu\frac{\partial^2 u}{\partial y^2}$$

$$p_{,y}=0$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

from continuity assume $u=\frac{\partial \psi}{\partial y}$ $v=-\frac{\partial \psi}{\partial x}$ and $u=u_1 f'(\eta)$ where $\eta=cyx^{\alpha}$

& using b.c. that $u=v=0$ at $y=0$ $\rightarrow$ $f'(\eta)=0$, $f(\eta)=0$ $\eta=0$

$u=u_1$ at $y=\delta$ $\quad f'(\eta)=1$ $\eta=\infty$

$$\frac{\partial \psi}{\partial y}=\frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y}=u_1 f'(\eta)=\frac{\partial \psi}{\partial \eta}(cx^{\alpha}) \qquad \boxed{f\left[\frac{u_1}{cx^{\alpha}}\right]=\psi}$$

taking this and putting it into (4) using definitions of u & v and making the equation independent of x leads to $\alpha=-\frac{1}{2}$ $c=\sqrt{\frac{u_1}{\nu}}$ & $2f'''+ff''=0$

$\therefore$ $\eta=y\sqrt{\frac{u_1}{\nu x}}$ solution shows that $\frac{u}{u_1}=.9915$ for $\eta=5$ $(y=\delta)$

$\therefore \rightarrow \delta=\frac{5x}{\sqrt{R_{N_x}}}$ $\qquad \frac{\partial u}{\partial y}=\frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y}=u_1 f''(\eta)cx^{\alpha}=f''(\eta)u_1\sqrt{\frac{u_1}{\nu x}}$

$$\tau_0=\mu\frac{\partial u}{\partial y}\Big|_0=\mu f''(\eta)u_1\sqrt{\frac{u_1}{\nu x}}=.332\frac{\mu u_1}{x}\sqrt{R_{N_x}}$$

$$D_{\text{on one side}}=\int_0^L \mu\frac{\partial u}{\partial y}\Big|_0 dx=.332\mu u_1\sqrt{\frac{u_1}{\nu}}\int_0^L\frac{1}{\sqrt{x}}dx$$

$$=.332\mu u_1\sqrt{\frac{u_1}{\nu}}\,2x^{1/2}\Big|_0^L=.664\mu u_1\sqrt{R_{N_L}}$$

$$C_D=\frac{D}{\frac{1}{2}\rho u_1^2 L}=\frac{.664\mu u_1\sqrt{R_{N_L}}}{\frac{1}{2}\rho u_1^2 L}=\frac{1.328}{\sqrt{R_{N_L}}}$$ for either one or both sides

$$C_f=\frac{\tau_0}{\frac{1}{2}\rho u_1^2}=\frac{.332\mu u_1\sqrt{R_{N_x}}}{\frac{x}{2}\rho u_1^2}=\frac{.664}{\sqrt{R_{N_x}}}$$

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -p_{,x} + \mu\left[u_{,xx} + u_{,yy}\right] \quad (1)$$

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -p_{,y} + \mu\left[v_{,xx} + v_{,yy}\right] \quad (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

If $\exists$ a layer $\delta << L$ (characteristic length) $\ni$. $\mu \to 0$ for $y \geq \delta$, $\mu \neq 0$ for $y < \delta$

and that
1. $\bar{u} = \frac{u}{U_\infty} \sim O(1)$
2. $\bar{x} = \frac{x}{L} \sim O(1)$
3. $\bar{y} = y/L \sim O(\bar{\delta} = \delta/L)$
4. steady flow
5. radius of curvature $>> \delta$ on the body then nondimensional analysis of Continuity Eq leads to

$$\frac{\partial}{\partial \bar{x}} \sim O(1) \quad \frac{\partial^2}{\partial \bar{x}^2} \sim O(1)$$
$$\frac{\partial}{\partial \bar{y}} \sim O(\tfrac{1}{\bar{\delta}}) \quad \frac{\partial^2}{\partial \bar{y}^2} \sim O(\tfrac{1}{\bar{\delta}^2})$$

$$\frac{U_\infty}{L}\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}}\frac{U_\infty}{L} = 0$$
$$O(1)\cdot O(1) + ?\ O(1/\bar{\delta})$$

$\therefore$ $\underline{\bar{v} \sim O(\bar{\delta})}$

$$\left[\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right] = -\bar{p}_{,\bar{x}} + \frac{1}{R_N}\left[\bar{u}_{,\bar{y}\bar{y}} + \bar{u}_{,\bar{x}\bar{x}}\right]$$
$$O(1)\,O(1)\,O(1) + O(\bar{\delta})\,O(1)\,O(1/\bar{\delta}) = ? + ?\ \left[O(1)\,O(\tfrac{1}{\bar{\delta}^2}) + O(1)\,O(1)\right]$$

$\therefore$ $\bar{u}_{,\bar{y}\bar{y}} >> \bar{u}_{,\bar{x}\bar{x}}$ neglect then $\bar{u}_{,\bar{x}\bar{x}}$ wrt $\bar{u}_{,\bar{y}\bar{y}}$

On boundary edge
Navier Stokes reduces to Euler Equation $\ni$. $\rho U_e \frac{\partial U_e}{\partial x} = -\frac{\partial p}{\partial x}$ or $\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} = -\frac{\partial \bar{p}}{\partial \bar{x}}$
$O(1)\ O(1)\ O(1) \to O($

$\therefore$ $\frac{\partial \bar{p}}{\partial \bar{x}} \sim O(1)$ then $\frac{1}{R_N} \sim O(\bar{\delta}^2)$ or $R_N \sim O(1/\bar{\delta}^2)$

$$\left[\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}\right] = -\bar{p}_{,\bar{y}} + \frac{1}{R_N}\left[\bar{v}_{,\bar{y}\bar{y}} + \bar{v}_{,\bar{x}\bar{x}}\right]$$
$$O(1)\,O(\bar{\delta})\,O(1) + O(\bar{\delta})\,O(\bar{\delta})\,O(1/\bar{\delta}) = ? + O(\bar{\delta}^2)\left[O(\bar{\delta})\,O(1/\bar{\delta}^2) + O(\bar{\delta})\,O(1)\right]$$

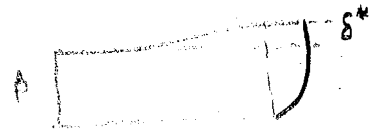
$\therefore$ $\bar{v}_{,\bar{y}\bar{y}} >> \bar{v}_{,\bar{x}\bar{x}}$ neglect then $\bar{v}_{,\bar{x}\bar{x}}$ wrt $\bar{v}_{,\bar{y}\bar{y}}$

$\therefore$ $\bar{p}_{,\bar{y}} \sim O(\bar{\delta})$ look at $\bar{p}_{,\bar{y}}$ wrt $\bar{p}_{,\bar{x}}$ it is seen $\bar{p}_{,\bar{x}} >> \bar{p}_{,\bar{y}}$

Therefore we can neglect $\bar{p}_{,\bar{y}}$ wrt $\bar{p}_{,\bar{x}}$ or $p_{,y} = 0$

define a displacement thickness as the increase in width between two contiguous streamlines, caused by the decrease in velocity in boundary layer. (δ^* - mass flow is the same)

$$\therefore \quad \rho u_1 \beta = \int_0^\delta \rho u \, dy$$



$$\delta^* = \delta - \beta \qquad \rho u_1 (\delta - \delta^*) = \int_0^\delta \rho u \, dy \qquad \therefore \quad \delta^* = \int_0^\delta (1 - u/u_1) \, dy$$

$$\delta^* = \int_0^\delta (1 - f'(\eta)) \, d\eta \sqrt{\frac{\nu x}{u_1}} \qquad \delta^* = \sqrt{\frac{\nu x}{u_1}} \int_0^\delta (d\eta - f' d\eta)$$

$$\delta^* = \sqrt{\frac{\nu x}{u_1}} \quad \eta - [\eta - \beta] = \frac{\beta x}{\sqrt{R_{Nx}}} \qquad \beta = 1.73 \quad \therefore$$

$$\delta^* = \frac{1.73 \, x}{\sqrt{R_{Nx}}}$$

Momentum thickness - increase in width between two streamlines caused by momentum loss in boundary layer is defined as

$$\rho u_1^2 \theta = \int_0^\delta \rho u (u_1 - u) \, dy \qquad \therefore \quad \theta = \int_0^\delta u/u_1 (1 - u/u_1) \, dy = \int_0^\delta f'(1 - f') \, dy$$

solution to this leads to $\theta = \dfrac{.664 \, x}{\sqrt{R_{Nx}}}$

at the outer edge it is found that $v/u_1 = \dfrac{.865}{\sqrt{R_{Nx}}}$

Blasius solution was made as follows assume $f = \sum A_i \eta^i$ about $\eta = 0$ using $f' = f = 0$ at $\eta = 0$. The inner solution resulted in a solution with one unknown. The outer solution was assumed as $f = f_1 + f_2$ where $f_1 \gg f_2$ and at $\eta = \infty$ $f_2 = f_2' = f_2'' = 0$. Using $\frac{df}{d\eta} = 1$ at $\eta = \infty$ a solution was obtained with 2 unknowns. The outer & inner solutions were patched at some $\eta \Rightarrow f, f', f''$ were equal. since $\frac{df}{d\eta} = 1$ at $\eta = \infty$ $\frac{df}{d\eta} = \frac{df_1}{d\eta} = 1$ or $f_1 = \eta - \beta$

going to original diff equation $f_1' = 1$ $f_1'' = 0$ $f_1''' = 0$ $\therefore$ $2f''' + ff'' = 2f_2''' + f f_2'' = 0$

where $f = \eta - \beta + f_2$ but since $f_1 = \eta - \beta \gg f_2$ then

$$2 f_2''' + (\eta - \beta) f_2'' = 0 \longrightarrow f_2 = \gamma \int_0^\eta d\xi \int_0^\eta e^{-1/4(\xi - \beta)^2} d\xi$$

$$\Rightarrow \quad f = \eta - \beta + \gamma \int_0^\eta d\xi \int_0^\eta e^{-1/4(\xi - \beta)^2} d\xi .$$

$$p = p_0 + \frac{6\mu U L}{h_1^2 - h_2^2} \qquad \frac{(h_1 - h)(h - h_2)}{h^2}$$

$$D = \frac{\mu U L}{(k-1)h_2}\left[4 \ln K - \frac{6(K-1)}{K+1}\right]$$

$$L = \frac{6\mu U L^2}{(k-1)^2 h_2^2}\left[\ln K - 2\,\frac{(K-1)}{K+1}\right]$$

NO

EXAMINATION BOOK

Name Cesar Levy

Subject AE 333

Instructor Prof Vlsich

Exam Seat No. ______ Section ______

Date 1/6/72 Grade ______

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1	30
2	30
3	21
4	13
	94

1.

$$\int_0^\delta \frac{\partial u}{\partial t} dy - \nu \int_0^\delta \frac{\partial^2 u}{\partial y^2}\, dy = 0$$

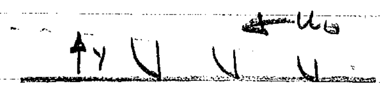
$$\int_0^\delta \frac{\partial u}{\partial t} dy - \nu \left.\frac{\partial u}{\partial y}\right|_0^\delta = 0$$

$$\int_0^\delta \frac{\partial u}{\partial t} dy + \frac{\tau_0}{\rho} = 0 \qquad \text{where} \qquad \tau_0 = \mu \left.\frac{\partial u}{\partial y}\right|_0$$ 10

$$\int_0^\delta \frac{\partial u}{\partial t}\, dy = \frac{d}{dt}\int_0^\delta u(t,y)\,dy - u(t,\delta)\frac{d\delta}{dt}$$

$$\therefore \quad \frac{d}{dt}\int_0^\delta u(t,y)\,dy + \frac{\tau_0}{\rho} = u_0 \frac{d\delta}{dt} \qquad \text{where} \quad u(t,\delta) = u_0$$

$$\frac{u}{u_0} = A\eta + B\eta^2 + C\eta^3 \qquad \text{where} \quad \eta = y/\delta$$



change of coord system ⇒ plate sees air moving over it with velocity u_0

at $\eta = 1$ $\quad u = u_0$

$\eta = 0 \quad u = 0$

$\frac{\partial u}{\partial \eta} = 0$ at $\eta = 1$ ← $\tau_\delta = 0$ $\qquad u_0(A + 2B\eta + 3C\eta^2) = u'$

$u_0(2B + 6C\eta) = u'$

$\frac{\partial^2 u}{\partial \eta^2} = 0$ at $\eta = 0$ since $\frac{dp}{dx} = 0$ no mech. exists for $p = p(x)$

$1 = A + B + C \qquad B = 0$

$0 = A + 2B + 3C \qquad A + C = 1$

$0 = 2B \qquad A + 3C = 0 \qquad 2C = -1$

$C = -1/2$

$A = 3/2$

$$\frac{u}{u_0} = \frac{3}{2}\eta - \frac{1}{2}\eta^3$$

O.K.

$$\tau_0 = \mu \left.\frac{\partial u}{\partial y}\right|_0 = \mu \frac{3}{2} u_0$$

$$\frac{d}{dt}\int_0^\delta u\,dy + \frac{3}{2}\frac{\nu u_0}{\delta} = u_0\frac{d\delta}{dt}$$

$$\frac{d}{dt}u_0\int_0^\delta\left(\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right)dy + \frac{3}{2}\frac{\nu u_0}{\delta} = u_0\frac{d\delta}{dt}$$

$$\frac{d}{dt}\left[\frac{3}{2}\frac{y^2}{2\delta} - \frac{1}{8}\frac{y^4}{\delta^3}\right]_0^\delta + \frac{3\nu}{2\delta} = \frac{d\delta}{dt}$$

$$\frac{d}{dt}\left(\frac{3}{4}\delta - \frac{1}{8}\delta\right) + \frac{3\nu}{2\delta} = \frac{d\delta}{dt}$$

$$\frac{5}{8}\frac{d\delta}{dt} + \frac{3\nu}{2\delta} = \frac{d\delta}{dt}$$

$$\frac{3\nu}{2\delta} = \frac{3}{8}\frac{d\delta}{dt}$$

$$4\nu\,dt = \delta\,d\delta$$

$$8\nu t = \delta^2$$

$$2.83\sqrt{\nu t} = \delta$$

$$\tau_0 = \mu\frac{3}{2}\frac{u_0}{2.83\sqrt{\nu t}} = \frac{\mu u_0}{\sqrt{\nu t}}\;\frac{3}{5.66}$$

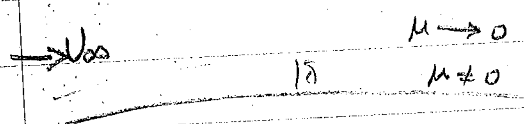
$$c_f = \frac{\tau_0}{\frac{1}{2}\rho u_0^2} = \frac{6}{5.66}\;\frac{\mu u_0}{\sqrt{\nu t}\;\rho u_0^2} = \frac{1.16\,\nu}{u_0\sqrt{\nu t}} = \frac{1.16}{u_0}\sqrt{\frac{\nu}{t}}$$

OK.

2. $\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = F_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = F_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

In this analysis it is assumed that the flow is steady and that the body forces are small compared with the other terms and therefore negligible. It is assumed that viscous effects are felt only within a region close to the body and outside it the solution to the problem is given by potential flow solution. close to the body the following is assumed



$L >> \delta$ where L is the characteristic length of body

$\bar{u} = u/U_\infty \sim O(1)$ $\quad \bar{x} = x/L \sim O(1)$ $\quad \bar{y} = y/L \sim O(\bar{\delta} = \delta/L)$

The continuity equation can be nondimensionalized so that

$$\underset{O(1)\cdot O(1)}{\frac{U_\infty}{L}\frac{\partial(u/U_\infty)}{\partial(x/L)}} + \underset{? \quad O(1/\bar{\delta})}{\frac{U_\infty}{L}\frac{\partial(v/U_\infty)}{\partial(y/L)}} = 0$$

by order of magnitude it is found that $\bar{v} = v/U_\infty \sim O(\bar{\delta})$

the first mom equation is reduced by our assumption to

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\bar{p}_{,\bar{x}} + \frac{1}{R_N}\left[\bar{u}_{,\bar{x}\bar{x}} + \bar{u}_{,\bar{y}\bar{y}}\right]$$

$O(1)\ O(1)\ O(1) \quad O(\bar{\delta})\ O(1)\ O(\bar{\delta}) = ? \qquad ? \quad [O(1)\cdot O(1) + O(1)\cdot O(\frac{1}{\bar{\delta}^2})]$

where $\partial/\partial\bar{x} \sim O(1)$ $\quad \partial/\partial\bar{y} \sim O(1/\bar{\delta})$ $\quad \partial^2/\partial\bar{x}^2 \sim O(1)$ $\quad \partial^2/\partial\bar{y}^2 \sim O(1/\bar{\delta}^2)$

at the edge of the bl. the Navier Stokes reduce to euler's equation $\Rightarrow$

$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} = -\bar{p}_{,\bar{x}}$ order of magnitude analysis shows

$\bar{p}_{,\bar{x}} \sim O(1)$

taking a look at $\bar{u}_{,\bar{x}\bar{x}}$ & $\bar{u}_{,\bar{y}\bar{y}}$ we see $\bar{u}_{,\bar{y}\bar{y}} >> \bar{u}_{,\bar{x}\bar{x}}$ since $O(1/\bar{\delta}) >> O(1)$ $\therefore$ neglect $\bar{u}_{,\bar{x}\bar{x}}$ wrt $\bar{u}_{,\bar{y}\bar{y}}$

because of this ~~ord~~ mag of $\frac{1}{R_N} \sim O(\bar{\delta}^2)$ $\therefore$ $R_N \sim O\left(\frac{1}{\bar{\delta}^2}\right)$

2nd momentum eq shows

$$\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} = -\bar{p}_{,\bar{y}} + \frac{1}{R_N}\left[\bar{v}_{,\bar{x}\bar{x}} + \bar{v}_{,\bar{y}\bar{y}}\right]$$

$$O(1)\,O(\bar{\delta})\,O(1) \quad O(\bar{\delta})\,O(\bar{\delta})\,O(1/\bar{\delta}) = ? + O(\bar{\delta}^2)\left[O(\bar{\delta})\,O(1) \quad O(\bar{\delta})\,O(1/\bar{\delta}^2)\right]$$

taking a look at $\bar{v}_{,\bar{x}\bar{x}}$ & $\bar{v}_{,\bar{y}\bar{y}}$ we see $\bar{v}_{,\bar{y}\bar{y}} >> \bar{v}_{,\bar{x}\bar{x}}$ since $O(\bar{\delta})\,O(1) << O(\bar{\delta})\,O(1/\bar{\delta}^2)$ therefore neglect $\bar{v}_{,\bar{x}\bar{x}}$ wrt $\bar{v}_{,\bar{y}\bar{y}}$

$\therefore$ the order of mag of second mom eq is $O(\bar{\delta})$

$\therefore$ $O(\bar{p}_{,\bar{y}}) \sim O(\bar{\delta})$

take a look at $\bar{p}_{,\bar{y}}$ wrt $\bar{p}_{,\bar{x}}$ $\quad \bar{p}_{,\bar{x}} >> \bar{p}_{,\bar{y}}$ since $O(1) > O(\bar{\delta})$

$\therefore$ we can take $\bar{p}_{,\bar{y}} = 0$ wrt $\bar{p}_{,\bar{x}}$

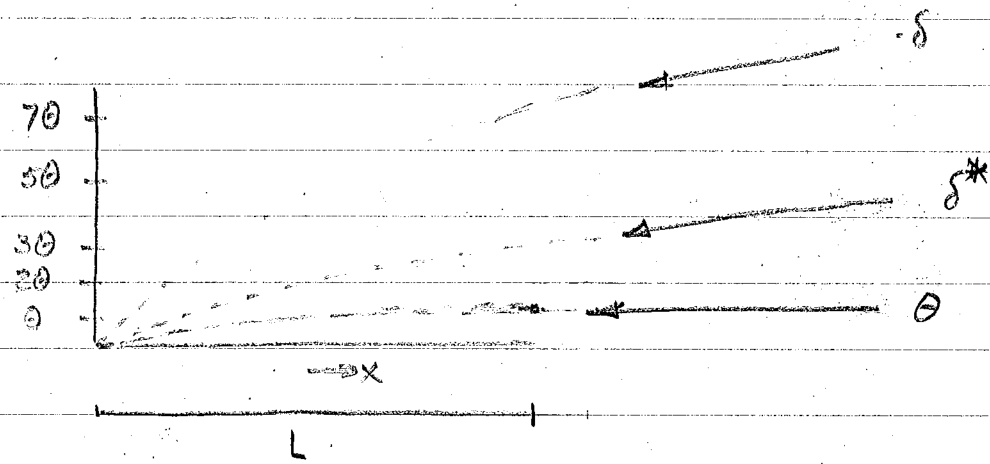
thus the full navier Stokes equat reduce to

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -p_{,x} + \mu\frac{\partial^2 u}{\partial y^2}$$

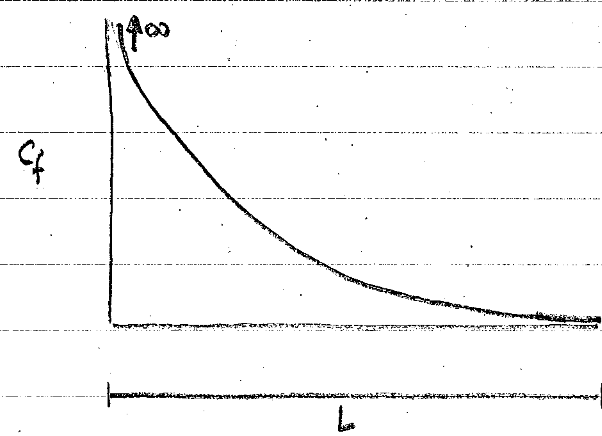
$$p_{,y} = 0$$

since Order of mag of 1st mom eq is larger than Order of mag of 2nd eq

3.



$$\theta = \frac{.664x}{\sqrt{R_{N_x}}} \qquad \delta = \frac{5x}{\sqrt{R_{N_x}}} \qquad \delta^* = \frac{1.73x}{\sqrt{R_{N_x}}}$$



$$c_f = \frac{.664}{\sqrt{R_{N_x}}}$$

if $u \rightarrow 2V_\infty$ $\qquad c_f = \dfrac{.664}{1.414\sqrt{R_{N_x}}} = \dfrac{.47}{\sqrt{R_{N_x}}} = .707\, C_{f_{original}}$

where $R_{N_x} = \rho \dfrac{V_\infty x}{\mu}$

$$D = C_D\, q\, 2L = \frac{1.328}{1.414\sqrt{R_{N_L}}}\, \frac{1}{2}\rho\left[4V_\infty^2\right]2L$$

on both sides of plate

$$= 4 \cdot \frac{1.328}{1.414}\, \rho \frac{V_\infty^2}{2} \frac{2L}{\sqrt{R_{N_L}}} = \frac{4}{1.414} D_{TOT\ original}$$

$$= 2.83\, D_{TOT\ original}$$

b. Flow over bodies having corners, turns, as part of their geometry. $\psi = u r^n \sin n\theta$, $\varphi = u r^n \cos n\theta$

Faulkner Skan produced the equation which described the flow for those bodies having potential flow s.t. at bl, $u = ucx^m$

$$f''' + ff'' - \frac{2m}{m+1}\left[f'^2 - 1\right] = 0 \qquad u = \frac{\partial\psi}{\partial y} \quad v = -\frac{\partial\psi}{\partial x}$$

where $f = f(\eta)$ $\qquad \eta = A y x^{\alpha}$ where $\alpha = \frac{m-1}{2}$ &

$$\frac{A^2 \nu}{c\,m} = \frac{m+1}{2m} \qquad A = \sqrt{\frac{c}{\nu}\,\frac{m+1}{2}} \quad \& \quad u = u_e f'(\eta)$$

$$\eta = y\sqrt{\frac{u_e}{\nu x}\,\frac{m+1}{2}} \qquad \psi = \frac{A}{c} x^{m-\alpha} f$$

now $\left.\frac{\partial u}{\partial y}\right|_0 = 0$ for separation

$\alpha = \frac{m-1}{2}$, for the equat to be independent of x

$$\left.\frac{\partial u}{\partial y}\right|_0 = \left.\frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y}\right|_0 = \left. u_e f''(\eta)\sqrt{\frac{u_e}{\nu x}\,\frac{m+1}{2}}\right|_0$$

$$3 = c x^m f''(0)\sqrt{\frac{u_e}{\nu x}\,\frac{m+1}{2}} = c f''(0)\sqrt{\frac{u_e^3}{\nu x}\,\frac{m+1}{2}}$$

either $c = 0$ or $m = -1$

$c = 0 \rightarrow u_e = 0$ implies $\delta = 0$ since $u = 0$ at body

$m = -1 \rightarrow u_e x = C$

4. a The point at which the flow separates from the body and the shear on the body at that point is zero

b. displacement thickness is that increase in width in b.l. between 2 adjacent streamlines due to the decrease in velocity in b.l. Mass flow is constant. Good way of judging effect of b.l. and extent of bl effect on streamline.

c. Pohlhausen parameter can define region of validity of flow $\lambda = \frac{\delta^2}{\nu}\frac{du_e}{dx}$. Region of validity of solution must satisfy $\left.\frac{\partial u}{\partial y}\right|_{wall} = 0$ & $\left.\frac{\partial^2 u}{\partial y^2}\right|_{\delta} = 0$

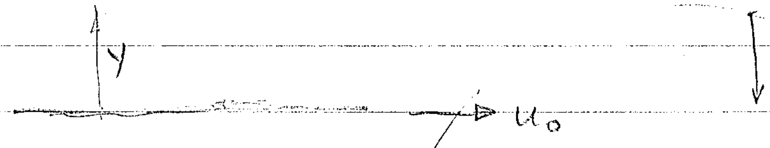
d. stagnation press that pressure that would exist if flow is brought isentropically to rest. Measures to total pressure in the flow. $p_s = p_0 + q$

2

e. Reynolds number $R_{N_{\infty,x}} = \frac{\rho V_\infty x}{\mu}$ use to produce similar flows over a body so that wind tunnel tests results can be applied & modified to estimate actual flow over true size body.

2

(1)



$$\frac{u}{u_0} = 1 + A\eta + B\eta^2 + C\eta^3$$

$\eta = 0$ $\quad u/u_0 = 1$

$\frac{u}{u_0} = 0$ at $\eta = 1$ $\quad$ BC

$\frac{\partial u}{\partial u_0} = 0$ at $\eta = 1$

$\frac{\partial^2 u}{u_0} = 0$ at $\eta = 0$

$0 = 1 + A + B + C$ $\qquad -1 = A + B + C$

$0 = A + 2B + 3C$ $\qquad A + 3C = 0$

$0 = 2B = 0$

$C = \frac{1}{2} \quad A = -3/2$

$$1 - \frac{3}{2}\eta + \frac{1}{2}\eta^3 = u/u_0 \qquad -\frac{3u_0}{2\delta}$$

$\tau_0 = \mu \frac{\partial u}{\partial y}\Big|_0 = -\frac{3\mu}{2\delta}u_0$ using the integral

$$-\frac{3}{2}\frac{\nu}{\delta}u_0 + \frac{\partial u_0}{dt}\left[y - \frac{3}{2}\frac{y^2}{2\delta} + \frac{1}{8}\frac{y^4}{\delta^3}\right]_0^\delta = u_0\frac{d\delta}{dt}$$

$$\delta - 3/4\delta + \frac{1}{8}\delta$$

$$\delta - \frac{5}{8}\delta = \frac{3}{8}\frac{d\delta}{dt}u_0$$

$$-\frac{3}{2}\frac{\nu}{\delta}u_0 + 3/8\frac{d\delta}{dt}u_0 = u_0\frac{d\delta}{dt}$$

$$\frac{3}{2}\nu\, dt = \frac{5}{8}\delta\, d\delta$$

$$3\nu t = \frac{5}{8}\delta^2$$

$$\delta^2 = \frac{24}{5}\nu t \qquad \delta = 2.19\sqrt{\nu t}$$

$$\tau = \frac{3 u_0 \mu}{2\delta}$$

AE 533

Cesar Levy

1. Since $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$

$$u = u_1 f'(\eta) = \frac{\partial \psi}{\partial \eta} c x^{\alpha} \quad , \quad -v = \frac{u_1 \alpha}{x}\left[y f' - \frac{f}{c x^{\alpha}}\right] \quad , \quad \psi = \frac{u_1 f}{c x^{\alpha}}$$

Since $u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$; since $\psi = \psi(x,y)$ then

$$\frac{\partial \psi}{\partial y}\left(\frac{\partial^2 \psi}{\partial x \partial y}\right) - \frac{\partial \psi}{\partial x}\left(\frac{\partial^2 \psi}{\partial y^2}\right) = \nu \frac{\partial^3 \psi}{\partial y^3}$$

$$u_1 f'\left[u_1 f'' c y \alpha x^{\alpha-1}\right] - \frac{u_1}{c x^{\alpha}}\left[f' c y \alpha x^{\alpha-1} - \frac{\alpha f}{x}\right] u_1 c x^{\alpha} f''$$

$$= \nu u_1 c^2 x^{2\alpha} f'''$$

$$u_1^2 f' f'' c y \alpha x^{\alpha-1} - u_1^2 f' y \frac{\alpha}{x} \cdot c x^{\alpha} f'' + \frac{u_1^2 \alpha f'' f}{x} = \nu u_1 c^2 x^{2\alpha} f'''$$

then $\nu u_1 c^2 x^{2\alpha} f''' - \frac{u_1^2}{x} \alpha f'' f = 0$

or $f''' - \frac{u_1 \alpha}{x^{2\alpha+1} \nu c^2} f'' f = 0$

let $x^{2\alpha+1} = 1$ or $2\alpha + 1 = 0$ $\alpha = -1/2$

then $2 f''' + \frac{u_1}{\nu c^2} f'' f = 0$

let $\frac{u_1}{\nu c^2} = 1$ or $c = \sqrt{\frac{u_1}{\nu}}$

If so then $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$ becomes

$$2f''' + f''f = 0$$

when $f'' = \frac{d^2f}{d\eta^2}$ & $f''' = \frac{d^3f}{d\eta^3}$ and $\eta = cyx^{\alpha} = y\sqrt{\frac{u_1}{\nu x}}$

2. since @ $y=0$ $u=v=0$ then $\eta=0$ and $f'(0)=0$ $(u=0)$

but $v = \frac{u_1 f}{c}\alpha x^{-\alpha-1} - u_1 f' y\alpha/x$. From above for $v=0$ at $y=0$ and since $f'(0)=0$ then

$$0 = \frac{u_1 f}{c}\alpha x^{-\alpha-1} - 0.$$ Since $u_1, c, \alpha, \neq 0$ $\forall x$ then

$f(0) = 0$.

3. Assume $f = \sum_{n=0} A_n\eta^n$. Since A_0 to A_5 is to be obtained the highest power of the series needed is η^5. Since the D.E. is a 3^{rd} O.N.L.D.E., to obtain η^5 in the solution for f''' it is necessary for f to go no lower than η^8 $\therefore$ $f = \sum_{n=0}^{8} A_n\eta^n$ for $A_0 \to A_5$ to be obtained.

Since $f=0$ for $\eta=0$ ($v=0$ @ $y=0$ B.C.) $A_0 \equiv 0$

Since $f'=0$ for $\eta=0$ ($u=0$ @ $y=0$ B.C.) $A_1 \equiv 0$

$$\therefore\ f = A_2\eta^2 + A_3\eta^3 + A_4\eta^4 + A_5\eta^5 + A_6\eta^6 + A_7\eta^7 + A_8\eta^8$$

$$f' = 2A_2\eta + 3A_3\eta^2 + 4A_4\eta^3 + 5A_5\eta^4 + 6A_6\eta^5 + 7A_7\eta^6 + 8A_8\eta^7$$

$$f'' = 2A_2 + 6A_3\eta + 12A_4\eta^2 + 20A_5\eta^3 + 30A_6\eta^4 + 42A_7\eta^5 + 56A_8\eta^6$$

$$f''' = 6A_3 + 24A_4\eta + 60A_5\eta^2 + 120A_6\eta^3 + 210A_7\eta^4 + 336A_8\eta^5$$

$2f''' + f''f = 0$ leads to

$$12A_3 + 48A_4\eta + (120A_5 + 2A_2^2)\eta^2 + (240A_6 + 8A_2A_3)\eta^3$$
$$+ (420A_7 + 14A_2A_4 + 6A_3A_4)\eta^4 + (672A_8 + 22A_2A_5 +$$
$$6A_3A_5 + 12A_4A_3)\eta^5 + \text{H.O.T. in } \eta \ldots = 0$$

But $\quad 0 = 0\cdot\eta^0 + 0\cdot\eta^1 + 0\cdot\eta^2 + 0\cdot\eta^3 + 0\cdot\eta^4 + 0\cdot\eta^5 + \ldots\ldots$

Matching Coefficients leads to

$$12A_3 = 0, \quad 48A_4 = 0, \quad 120A_5 + 2A_2^2 = 0,$$
$$240A_6 + 8A_2A_3 = 0, \quad 420A_7 + 14A_2A_4 + 6A_3A_4 = 0,$$
$$672A_8 + 22A_5A_2 + 6A_3A_5 + 12A_4A_3 = 0$$

This gives

$$A_3 = 0, \quad A_4 = 0, \quad A_6 = 0, \quad A_7 = 0, \quad A_2 = A_2$$
$$A_5 = -\frac{1}{60}A_2^2, \quad A_8 = +\frac{11}{336\cdot 60}A_2^3, \quad A_{2+3K} = (-1)^K\cdot \text{const}\ A_2^{K+1}$$

where $K \geq 0$

A recursion formula can be obtained to express A_5 in terms of A_2, A_8 in terms of A_5 and A_2, A_{11} in terms of A_8 & A_2 etc.

Using a new index $K = 0, 1, 2, \ldots$

for $K = 0$

$$2(5+3K)(4+3K)(3+3K)A_{5+3K} + 2A_2A_{2+3K} = 0$$

for $K \geq 1$

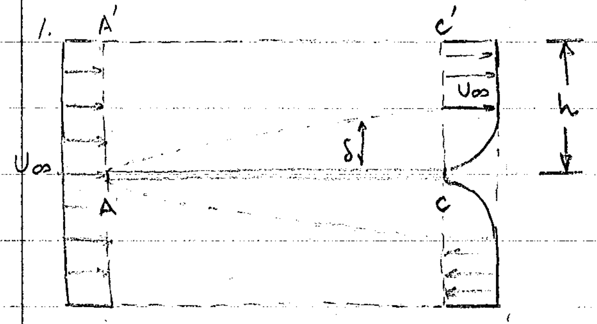
$$2(5+3K)(4+3K)(3+3K)A_{5+3K} + (3K+2)(3K+1)A_2A_{3K+2} + (3K-1)(3K-2)A_5A_{3K-}$$
$$+ (3K-4)(3K-5)A_8A_{3K-4} + (3K-7)(3K-8)A_{11}A_{3K-7} + (3K-10)(3K-11)A_{14}A_{3K-1}$$
$$+ \ldots + 2A_2A_{3K+2} = 0$$

where any coeff of A with a negative index is zero.

8

Cesar Levy
AE 332

1.



if b is the plate width
and h is the height on either side of the plate

the momentum flux across

AA' $\quad \int_0^h \rho b U_\infty^2 \, dy \qquad AC \quad 0$

$A'C'$ $\quad \int_0^h -\rho b u^2 dy \qquad CC' \quad \int_0^h -\rho b U_\infty (U_\infty - u) \, dy$

the sum of the momentum flux = drag on the plate (on one side)

$$\therefore \rho b \int_0^b \{ U_\infty^2 - u^2 - U_\infty^2 + U_\infty u \} dy = \rho b \int_0^b u(U_\infty - u) \, dy$$

but

$$\int_0^b u(U_\infty - u) \, dy = U_\infty^2 \Theta \quad \text{or} \quad D = \rho b U_\infty^2 \Theta$$

total drag is $2D = 2\rho b U_\infty^2 \Theta$ or drag / unit ft wide $= 2\rho U_\infty^2 \Theta$

now $\Theta = \dfrac{.644 x}{\sqrt{R_{Nx}}}$ using STP conditions & $\sqrt{L} = \sqrt{20} = 4.47$

also $U_\infty = 200$ ft/sec

$$\Theta = \frac{.644 \ (4.47)}{1131} = 2.625 \times 10^{-3} \text{ ft}$$

$$\therefore D_{TOT} = 2(2.625 \times 10^{-3})(.002378)(200)^2 = .499 \text{ lb/ft}$$

$$\tau_0 = .332 \frac{\mu U_\infty}{x} \sqrt{R_{Nx}} = .332 \, U_\infty \sqrt{\rho U_\infty \mu} \cdot \frac{1}{\sqrt{x}} = .0293/\sqrt{x}$$

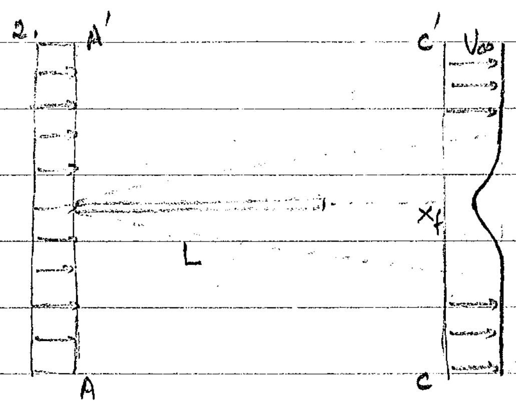
x	$\sqrt{x}$	$\tau_w \times 10^3$	(on one side)
0	0	∞	
2	1.414	20.7	
4	2	14.65	
6	2.45	11.95	
8	2.83	10.35	
10	3.16	9.26	
12	3.46	8.45	
14	3.74	7.83	
16	4	7.33	
18	4.24	6.91	
20	4.47	6.55	
@ .01	.1	293	

100% error

integration between ~~$.01 \le x \le 20$ is made~~ using trapezoidal rule

$D_{TOT} = .986$ lb/ft $\quad .01 \le x \le 20$

using $\quad D_{TOT} = 2 \sum_{i=1}^{10} h\left(\frac{\tau_{wi} + \tau_{wi+1}}{2}\right)$



You need a ~~better~~ smaller step size in the region 0 to 4

Since no plate exists between $L \le x \le x_f$ $\exists$ no shear in that region $\therefore$

$D_{TOT} = 2\rho V_\infty^2 \Theta$ where $\Theta = \frac{.664 L}{\sqrt{R_{N_L}}}$ or $D_{TOT} = .499$ lb/ft

$C_{f\text{ both sides of plate}} = \frac{1.328}{\sqrt{R_{N_x}}} = \frac{1.328}{1131\sqrt{x}} = \frac{1.174 \times 10^{-3}}{\sqrt{x}}$

$\sqrt{x}$	$C_f \times 10^{+3}$	$\Theta \times 10^3$
0	∞	0
1.414	.831	.83
2	.587	1.175
2.45	.479	1.44
2.83	.415	1.66
3.16	.3715	1.855
3.46	.339	2.03
3.74	.314	2.195
4	.294	2.35
4.24	.277	2.49
4.47	.266	2.62
$\downarrow$ x_f	$\downarrow$ 0	2.62

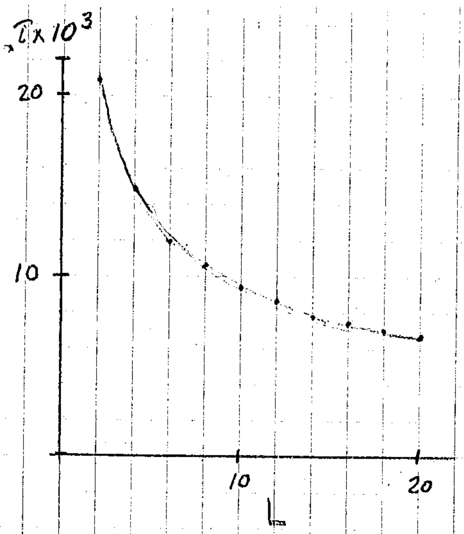
3. Plot of Θ $0 \le x \le x_f$ is made

Plot of v along x-axis $0 \le x \le L$ is 0 on the plate due to no slip condition; for $L \le x \le x_f$ v is 0 because of symmetry of flow

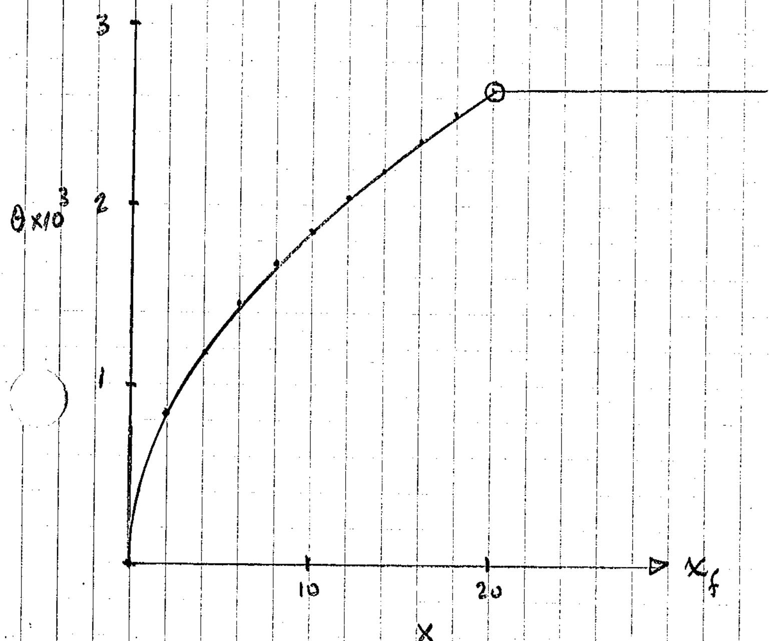
Plot of τ $0 \le x \le x_f$ is made.

4. The flow field is invalid at leading & trailing edge
At T.E. a discontinuity in $\frac{d\Theta}{dx}$ and τ_w exists; this is impossible since discontinuities cannot exist in subsonic flow.
At L.E. the shear is infinite, but the drag is zero since the shear acts on an infinitesimal area. The shear found by integration was much larger than it should have been.

As far as the drag goes (drag found by momentum theorem) it is the same in parts. 1 & 2



for one side of plate



for one side

$D_{TOT} \cong 2\rho U_\infty^2 \theta$