AE 332

Dr. Cresci

Lab Experiments

1. Supersonic Tunnel
2. Shock Tube

} Compressible flow held in Farmingdale

3. Airfoil Characteristics
4. Circular Cylinder

First Lab - Feb 10.

Books Used: Fundamentals of Aerodynamics Kuethe & Schetzer

Fund. of Hydro & Aero Prandtl & Tietjens

	Incompressible Flow	Transonic Flow	Supersonic Flow	Hypersonic Flow.
Inviscid	$M \sim 0$		$M > 1$	$M \gg 1$
	$R_N \to \infty$ $\mu = 0$		covered last term	

$R_N \sim \frac{1}{\mu}$

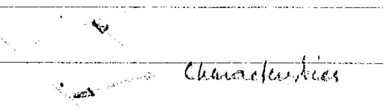
$R_N \to 0$ $\mu \gg 0$

Inviscid Supersonic Flow : Hyperbolic equat $\varphi_{xx} - \varphi_{yy} = 0$

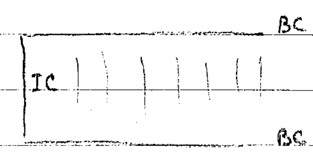
Incomp Inviscid Flow : Laplace Equation (Elliptic) $\varphi_{xx} + \varphi_{yy} = 0$

Incomp Viscous Flow : Parabolic equation $\varphi_x - \varphi_{yy} = 0$

Inviscid Sup. Flow



Incompress Viscous Flow



Schedule

Monday	6-7		
Wed	1-2	once a month	no thursday 7
		Feb. 24, March 24, Apr. 21, May 5	

Lab. F 10/11 M 10/11 A 21/22 M 5/6

2/1/70

Topics

I Hydrostatics - study of a fluid in static equilibrium

P & T - 1,2 K & S - 1

II Surface tension - 4

III Dynamics -

Bernoulli, Laplace, Euler's, Momentum, Mass, Energy 8 & 9 3

IV Kinematics -

Vortex Motion 6 & 12 2

V Stream, Potential Functions

1. Source - a pt. in fluid flow where mass is injected in system. 10 4

VI Thin airfoil Theory 10 5

VII Momentum Theorem

Knowing incoming values & outgoing values of P, ρ, T, mv etc we can relate the change in these values with what happens in the control volume.

$$\frac{\partial}{\partial t}\iiint_R \rho\vec{V}\,dR + \iint_S \rho\vec{V}\,(\vec{V}\cdot\hat{n})\,dS = -\iint_S p\hat{n}\,dS + \vec{F}$$

Compressible flow.

$p = \rho R T \qquad M = V/a \qquad p, \rho, T \to f(M)$

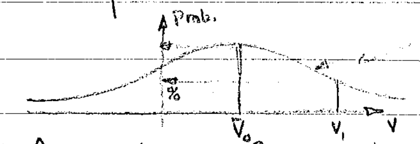
shockless isentropic flow $\quad p = c\rho^{\gamma}$

I Incompressible flow

1. pressure changes exist but are small.
2. density changes are caused mainly by changes in T or composition
3. ρ, T may change and are indep. of V or pressure
4. $M \ll 1$ Compressibility effects are small & neglected (due to velocity) ρ may change.

II Nature of fluid

1. Composed of large number of small particles.
2. Kinetic Theory - assume a distribution fn. plot it against velocity

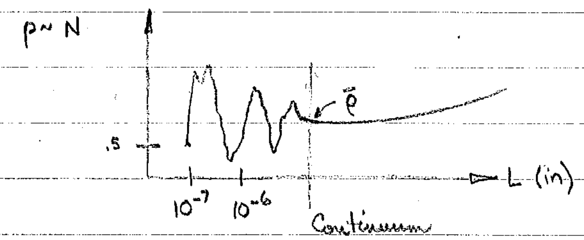


$\bar{V}_0$ average velocity of largest no. of molecules.

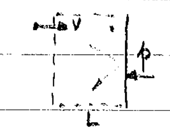
required for low density flows. high altitude flight $h > 200{,}000'$

3. Continuum - 5×10^{20} mol/in^3 for $p = 1$ atm.

a. $\rho = \dfrac{N\omega}{L^3} \qquad \omega$ - weight/molecule.



b. $p = F/A = \dfrac{\frac{d}{dt}(mv)}{A} = \dfrac{\frac{d}{dt}(N\omega V)}{L^2} \qquad \Delta t = L/V$



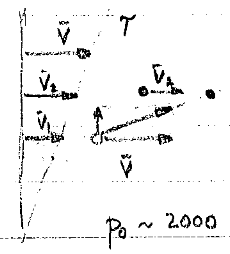
$= \dfrac{N\omega V^2}{L^3} \sim \rho V^2$

1 atm $\quad \rho = .00238 \dfrac{\#\,sec^2}{ft^4} \qquad V^2 = \dfrac{2116\ \#/ft^2}{.00238} \sim 10^6\ ft^2/sec^2$

$V \sim 1000$ ft/sec must be reached is a $\Delta p = 1$ atm is to be obtained.

for $V \sim 100$ ft/sec $\quad \Delta p \sim 10^{-2}$ atm. (assumption 1 checks)

c. pressure always acts normal to surface.

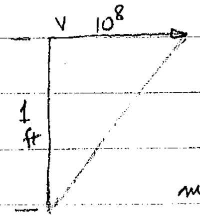


$$\frac{\omega(\bar V_2 - \bar V_1)}{L^2} = F/A = \tau \sim \underbrace{(\quad)}_{\mu} \frac{d\bar v}{dy}$$

$$\mu = .35 \times 10^{-6} \ \frac{\# \ sec}{ft^2}$$

$$p_0 \sim 2000 \ \#/ft^2$$

$$\tau = \Delta p \sim 20 \ \#/ft^2 = (\mu)\frac{d\bar v}{dy} \rightarrow \frac{d\bar v}{dy} \sim 10^8 \ \frac{ft/sec}{ft}$$



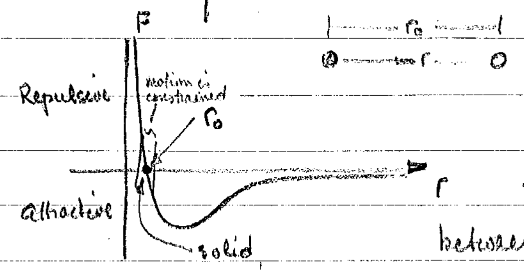
however to work with $V = 100$ ft/sec the change in y must be 10^{-6} ft, $\therefore$ $\frac{d\bar v}{dy} \sim 10^8 \ \frac{ft/sec}{ft}$

d. flow therefore is taken as inviscid since velocity gradient is very small as height increases.

Sections Omitted.

Ch. 1	§ 9
Ch. 2	§ 16–18
Ch. 3	Eliminate

1. viscosity leads to shear stress by virtue of a net exchange of momentum of fluid.
   $\tau_N \sim \frac{\partial v}{\partial n}$ $n \uparrow \longrightarrow v$ $\mu = 0$ in most cases.
2. pressure → normal to surface
3. density → defined whether body can be treated as a continuum
4. compressibility → defines $E = \frac{\text{stress (normal, pressure)}}{\text{strain. } (\Delta \text{ in linear dimensions})} = -\frac{p}{\Delta V / V}$ V is volume.
   (bulk modulus).



in a gas the attractive force ~ 0 and the force is mainly generated by collisions between particles or container walls.

1., 2., 3., 4. deal with macroscopic properties

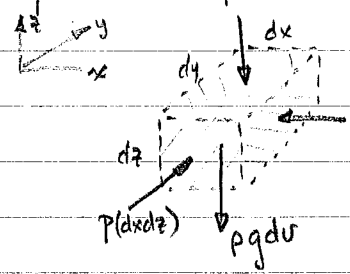
$E_{steel} \sim 10^8$ $E_{water} \sim 10^5$ if $\rho \sim \frac{1}{v}$ $\Delta\rho \sim -\frac{\Delta v}{v^2}$

$$-\frac{\Delta V}{V} = v \Delta \rho \quad \text{or} \quad E = \frac{P}{v\Delta\rho} = \frac{\rho R T}{\Delta\rho/\rho}$$

$E_{gas} \sim 15$ psia

I   Equilibrium of Fluid Element

① zero shear
② Normal forces ⊥ to surface
③ body forces.
④ Continuous change in pressure. $p = p(x,y,$



$dV = dx\,dy\,dz \neq 0$

$$\sum F_y = p\,dx\,dz - \left(p + \frac{\partial p}{\partial y} dy\right) dx\,dz = 0$$

$$\frac{\partial p}{\partial y} dV = 0 \implies \frac{\partial p}{\partial y} = 0$$

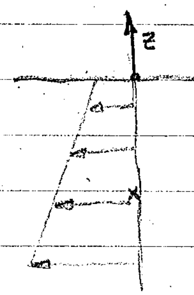
$\Sigma F_x = 0 \Rightarrow \frac{\partial p}{\partial x} = 0$

$\Sigma F_z = 0 \qquad p(dy\,dx) - (p + \frac{\partial p}{\partial z}dz)dy\,dx - \rho g\,dV = 0$

$\frac{\partial p}{\partial z} = -\rho g \frac{dV}{dV} = -\rho g$ since $p \neq p(x,y)$ then we can write $\frac{dp}{dz}$

$\frac{dp}{dz} = -\rho g \rightarrow p = \text{const} - \rho g z$ {liquid or any incomp. fluid}

at a free surface

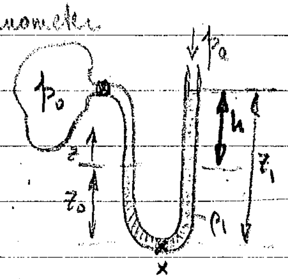


at $z=0$ $p = p_a$

$p = p_a - \rho g z$

we have assumed $\rho = \text{const}$, $g = \text{const}$.

II Manometer



$p_x = p_0 - \rho_1 g z_0$ left hand side of tube

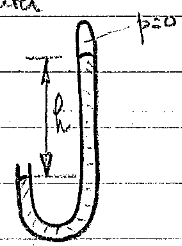
$p_x = p_a - \rho g z_1$ right hand side

$p_0 = p_a - \rho g(z_1 - z_0)$ if $h = z_0 - z_1$

then

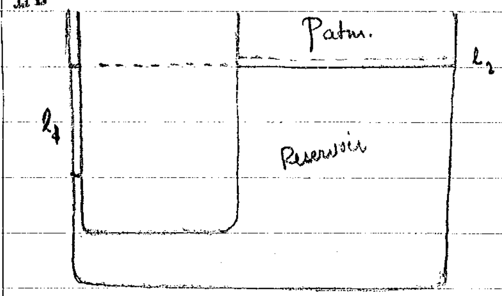
$p_0 = p_a + \rho g h$

III Barometer



$p_{ATM} = \rho g h$ Mercury $h \approx 30''$ Hg.

IIb

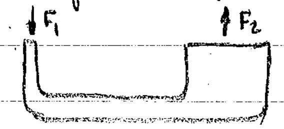


$A_1 l_1 = A_2 l_2$

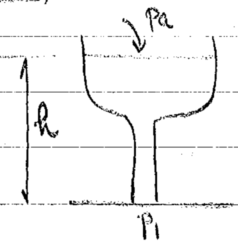
$\frac{l_2}{l_1} = \frac{A_1}{A_2}$ $\quad l_2 \ll 1$ assume $l_2 = 0$

Measure $h = l_1$ not $(l_2 - l_1)$

IIc Hydraulic Jack.



depending on $\frac{A_2}{A_1}$ $\quad F_2 \gg F_1$

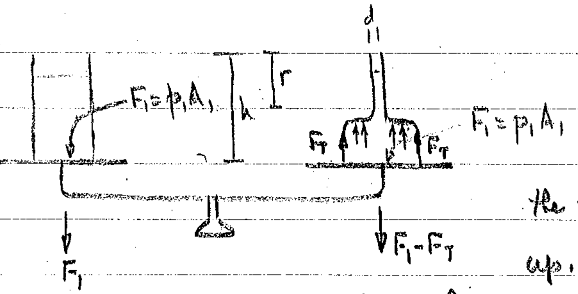


$p_1 = p_2 \qquad p_1 = p_{atm} + \rho g h$

Hydrostatic paradox.

IV

Consider



balance will not equilibriumize itself when water fills up and reaches the neck. A pressure force will lift glass up. If glass is held down there still exists a tension force acting on the walls.

$F_1 - F_T$ = weight of fluid

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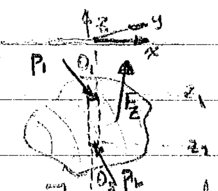
<u>Archimedes Principle</u> for fluid in hydrostatic equil & $\rho = C$.

$p = p_a - \rho g z$



proven: $\frac{dp}{dx} = \frac{dp}{dy} = 0$

Since $\frac{dp}{dx} = \frac{dp}{dy} = 0$ $\quad \nexists$ no net force on the body acting in either direction x, or y. $\quad F_x, F_y = 0$

but we do get a force in the z direction.

$dF_z = p_2\, dS_2 \cos\theta_2 - p_1\, dS_1 \cos\theta_1$

$p_1 = p_a - \rho g z_1$ $\qquad$ $p_2 = p_a - \rho g z_2$

$ds_1 \cos\theta_1 = dA \rightarrow ds_2 \cos\theta_2 = dA$

$\therefore \; dF_z = p_2\, dA - p_1\, dA = dA\left[\rho g (z_1 - z_2)\right]$

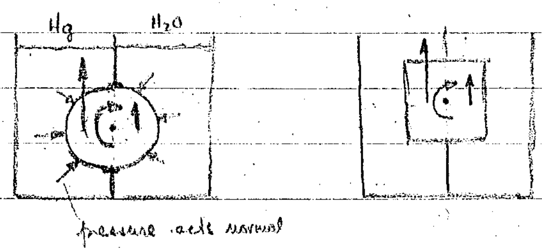
but $z_1 - z_2 = h$

$\therefore \; dF_z = \rho g\, dV$ where $dV = h\, dA$.

$\sum dF_z = F_z = \rho g V$ = weight of fluid displaced.

1. independent of surface pressure.
2. independent of the depth. $F_z \sim$ pressure diff. not absolute pressure.

Example:

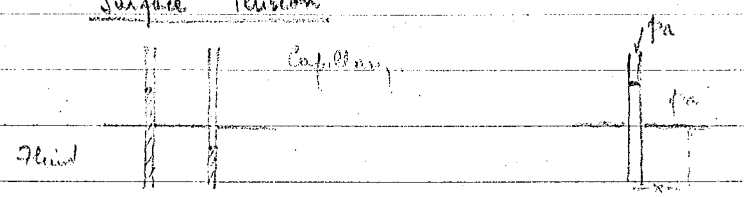


HW #2.

H.W. using arch. principle & hydro. equil. equat. prove there is no net torque.

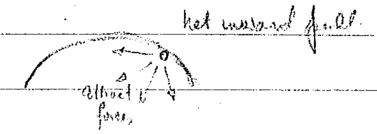
$T = (\rho_{Hg} - \rho_{H_2O}) \frac{s^2 h}{2}$ $\qquad$ $f \sim 2h$.

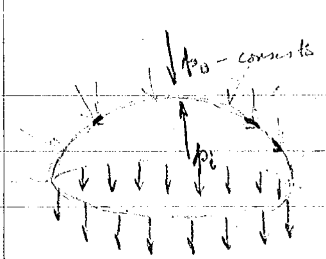
Surface Tension



hydrostatic equat doesn't hold.

Examine Curved Surface.



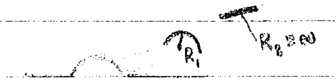


$(p_i - p_0)\pi r^2$ = total force upward $= \sigma(2\pi r)$

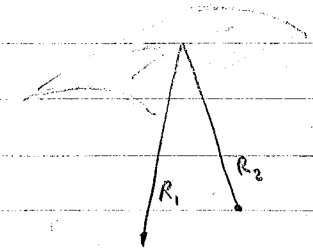
σ - force/length - surface tension

$p_i = \frac{2\sigma}{r} + p_0$ $\qquad$ H_2O - air interface $\sigma \approx .005$ #/ft

for 2-D contours $\quad p_i = p_0 + \frac{\sigma}{r}$



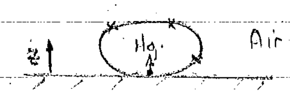
for general case.



$p_i = p_0 + \sigma\left[\frac{R_2 + R_1}{R_1 R_2}\right]$ $\qquad$ if $R_1 = R_2$ $\quad p_i = p_0 + \sigma\left(\frac{2}{r}\right)$

if R_1 or $R_2 \to \infty$ $\quad p_i = p_0 + \frac{\sigma}{r}$

Assume a Hg drop.

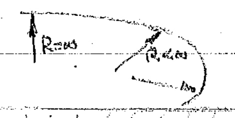


$p_0 = p_a - \rho_a g z$ $\qquad$ $p_i = p_H - \rho_H g z$

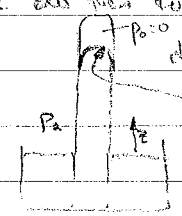
at any z $\quad p_i = p_0 + \sigma\left\{\frac{1}{r} - \frac{d^2z/dr^2}{\left(1 + \left(\frac{dz}{dr}\right)^2\right)^{3/2}}\right\}$

$(p_H - p_a) - gz(\rho_H - \rho_a) = C_1 - C_2 z$ $\quad$ due to the relatively small change in p_0 since ρ_a is small & a relatively large change in p_i since ρ_i is large, the radius of curvature must decrease.

Thus the bubble will look like this:



we do this to find the error in measurement of barometer due to meniscus formation



diameter of tube = d $\quad$ radius of curvature = d/2

$p_i = p_0 + \frac{2\sigma}{r} = \frac{4\sigma}{d}$

$p = p_a - \rho g z$ $\qquad$ @ $z = h$ $\qquad$ $p = \frac{4\sigma}{d}$

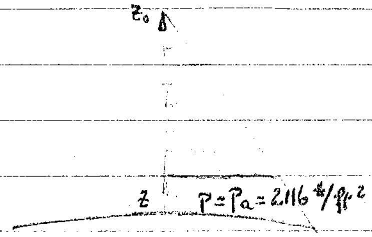
$p_a = \rho g h + \frac{4\sigma}{d}$

$\frac{4\sigma}{d}$ for air-H_2O

d	$\frac{4\sigma}{d}$	d	$\frac{4\sigma}{d}$
1	.02 #/ft^2	.01	2 #/ft^2
.1	.2 #/ft^2	.001	20 #/ft^2

Statics of Gases (Atmosphere)

$$\frac{dp}{dz} = -\rho g$$



(A) density is a constant

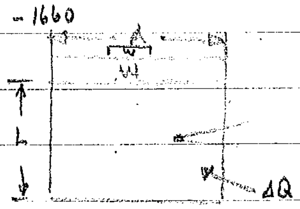
$\rho = \rho_0 = .002378 \ \frac{\# sec^2}{ft^4}$

$$p = p_a - \rho g z = p_a - .07z = 2116 - .07z$$

@ $p = 0$ $\quad z_0 = \frac{2116}{.07} \sim 30{,}000$ ft (height of atmosphere).

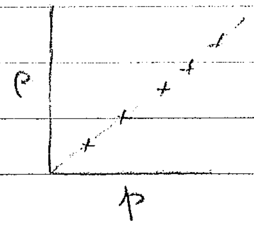
(B) Boyle's law. - 1660

Constant temp.



$p_1 = w/A \qquad p_2 = \frac{w_1 + w_2}{A}$

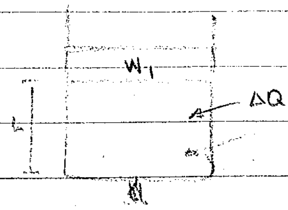
$V_1 = L_1 A = 1/\rho_1 \qquad V_2 = L_2 A = 1/\rho_2$

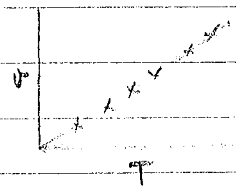


$p = C_1 \rho$

Charles' law

p = constant





$C_2 = \rho T$

Equation of State

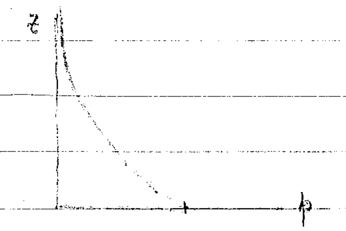
$p = \rho R T$ $\qquad R = 1716 \; \frac{ft^2}{sec^2 - {}^\circ R}$ $\qquad \rho = \frac{\# - sec^2}{ft^4}$ $\qquad T = {}^\circ R.$

(B) Temp = Const (Isothermal.)

$p = \rho R T_a$ $\qquad T_a \sim 520\,{}^\circ R$

$$\frac{dp}{dz} = -\frac{p}{RT_a} g$$

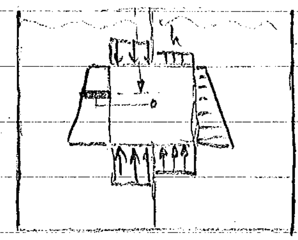
$$\ln p = -\frac{g}{RT} z + \eta \qquad p/p_a = e^{-gz/RT_a}$$



$\rho/\rho_a = e^{-gz/RT_a}$ since $p/p_a = \rho/\rho_a \; T/T_A$ but $T/T_A = 1$

2/22/70

H.W.



$$M = \int_h^{h+s} (p_a + \rho g z)(h + s/2 - z)\, dz$$

2 hr class

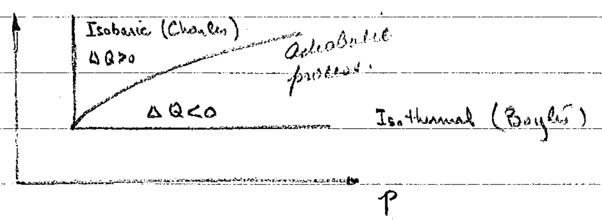
=

(A) density = const.

p, T, ρ $\qquad \frac{dp}{dz} = -\rho g$ $\qquad$ led to results NG.

(B) isothermal.

$\rho = p/RT \quad \rightarrow \quad \rho/\rho_a \rightarrow e^{-gz/RT_a}$



$$dQ = \underset{\substack{\downarrow \\ C_v dT}}{dU} + p\, d(1/\rho)$$

$$dQ = 0 = C_v dT + \rho R T \frac{d\rho}{-\rho^2}$$

$$0 = C_v dT - RT \frac{d\rho}{\rho}$$

$$\ln T = \ln \rho^{\gamma-1}$$
$$T = C\rho^{\gamma-1}$$
$$p = C\rho^{\gamma}$$

(c) polytropic process

$$p = C\rho^{n} \qquad p = C \rightarrow n = \infty$$
$$T = C \qquad n = 1$$

$$p/p_a = (\rho/\rho_a)^{n}$$

$$\frac{dp}{dz} = -g\rho_a (p/p_a)^{1/n}$$

$$-g\rho_a dz = (p/p_a)^{-1/n} dp$$

$$-g\frac{\rho_a}{p_a} z\Big|_0^z = (p/p_a)^{\frac{n-1}{n}} \cdot \frac{n}{n-1}\Big|_{p_a}^{p}$$

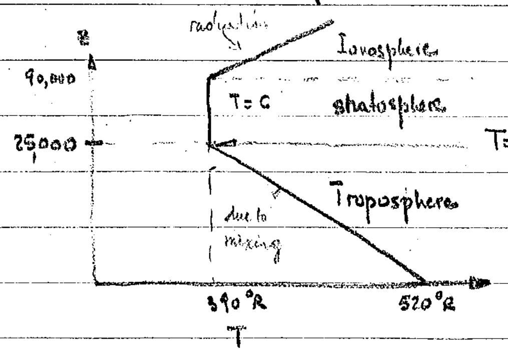
$$(p/p_a)^{\frac{n-1}{n}} = 1 - \frac{n-1}{n}\frac{gz}{RT_a}$$

$$p/p_a = \left[1 - \frac{n-1}{n}\frac{gz}{RT_a}\right]^{n/n-1}$$

$$\rho/\rho_a = (p/p_a)^{1/n} = \left[1 - \frac{n-1}{n}\frac{gz}{RT_a}\right]^{1/n-1}$$

$$\text{if } z = \frac{n}{n-1}\frac{RT_a}{g} \qquad p/p_a,\ \rho/\rho_a,\ T/T_a = 0$$

$$T/T_a = \frac{p}{\rho R}\frac{\rho_a R}{p_a} = \frac{p/p_a}{\rho/\rho_a} = \left[1 - \frac{n-1}{n}\frac{gz}{RT_a}\right]$$

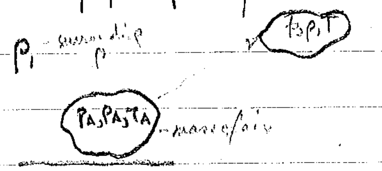


$T = T_T$ @ $z = z_T$ in isothermal equation

Non-perfect, no continuum in Ionosphere.

H.W. #3 Part A

using polytropic equat: calculate n $\qquad n \approx 1.4$. this leads $p = c\rho^{\gamma}$



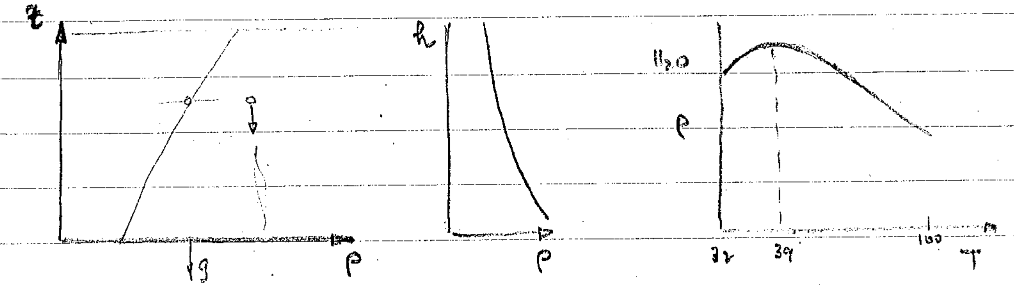
if $\rho > \rho_1$ the fluid will drop to sea level &
if $\rho < \rho_1$ fluid mass continues to rise
} unstable

after large scale mixing $\rho = \rho_1$ stable system

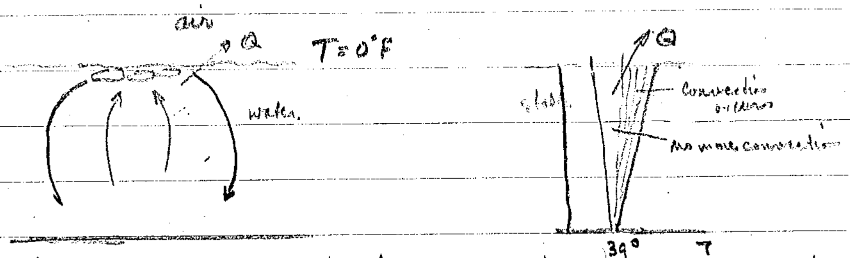
if ΔT existed Q would be inputed or outputed but $Q \sim \underbrace{k}_{\text{thermal conductivity}} \frac{\partial T}{\partial n}$ $\therefore$ since system is unstable not enough heat would be absorbed or exchanged to change T $\therefore$ we take the system to be adiabatic $\Delta Q = 0$. $\therefore$ all equations to be used must be adiabatic or $p = c\rho^{\gamma}$ & $n = \gamma$

10^{-6}

Stability of Liquids



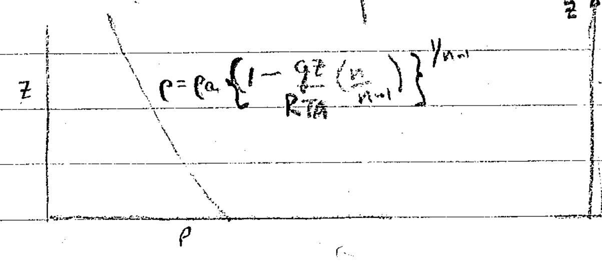
molecules displaced $\therefore$ ρ increases $\therefore$ if $\rho > \rho$ at z $\therefore$ it will fall.



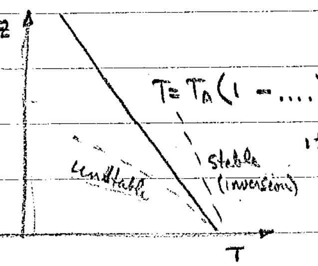
conduction in surface causes temp to drop at surface but at bottom temp remains at 39°

Stability of Atmosphere

Neutral Stability $n = \gamma$



$$\rho = \rho_A \left\{1 - \frac{gz}{RT_A}\left(\frac{n}{n-1}\right)\right\}^{1/n-1}$$



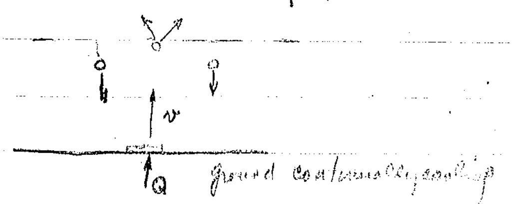
$$\frac{dT}{dz} = -\frac{n-1}{n}\frac{g}{R}$$

$T = T_A(1 - \dots)$

if $\frac{dT}{dz} = -\frac{\gamma-1}{\gamma}\frac{g}{R}$ adiabatic lapse rate

$-.5°F/100\,ft$

HW. 3 (b),(c) Determine stability (stable) of isothermal & constant density (unst.)



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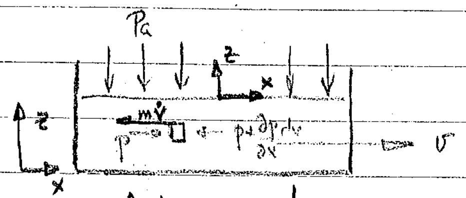


if $v = c$; $\quad \frac{dp}{dz} = -\rho g$

if $v = v(t) \qquad \frac{\partial p}{\partial x} = -\rho \frac{dv}{dt}$

since $p = f(x,z)$ we must write $\frac{\partial p}{\partial z} = -\rho g$

$p = -\rho g z + f(x) \qquad \frac{\partial p}{\partial x} = f'(x) = -\rho \frac{dv}{dt}$

$$f = f_0 - \rho \frac{dv}{dt} x$$

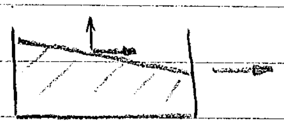
$p = p_a - \rho\left(\frac{dv}{dt} x + gz\right)$ at surface $p = p_a$ for horizontal free surface $z = 0$ but $x \neq 0$ $\therefore$ $\frac{dv}{dt} = 0$

if $\frac{dv}{dt} \neq 0$ determine shape of surface

$p = p_a$ at surface $\qquad \frac{dv}{dt} x + gz = 0$ for $p = p_a$

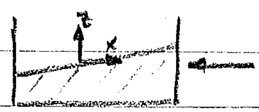
$$z = -\frac{x}{g}\frac{dv}{dt}$$

$$\frac{dz}{dx} = -\frac{1}{g}\frac{dv}{dt}$$

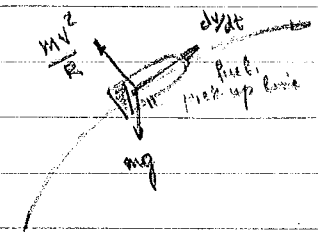


$\frac{dv}{dt} > 0 = \text{const.}$

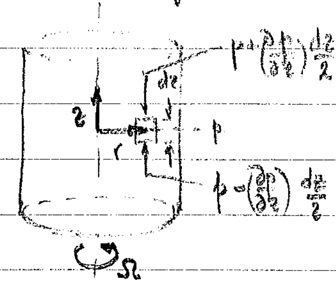
for $\frac{dv}{dt} < 0 \qquad \frac{dz}{dx} > 0$

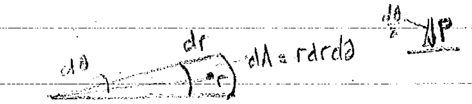


$\frac{dv}{dt} < 0 = \text{const.}$



System Rotating about Given axis.





$$\Sigma F_z = \left[p-\left(\frac{\partial p}{\partial z}\right)\frac{dz}{2}\right] r\,dr\,d\theta - \left[p+\left(\frac{\partial p}{\partial z}\right)\frac{dz}{2}\right] r\,dr\,d\theta$$

$$-\rho g\left[r\,dr\,d\theta\,dz\right] = -\left(\frac{\partial p}{\partial z}\right) - \rho g = 0$$

$$\boxed{\frac{\partial p}{\partial z} = -\rho g}$$

$$\Sigma F_r = -\left[p+\left(\frac{\partial p}{\partial r}\right)\frac{dr}{2}\right]\left(dz\left(r+\frac{dr}{2}\right)d\theta\right) + \left[p-\left(\frac{\partial p}{\partial r}\right)\frac{dr}{2}\right]\left(dz\left[r-\frac{dr}{2}\right]d\theta\right) + \rho\Omega^2 r\left(dz\,dr\,r\,d\theta\right)$$

$$+2p\,dr\frac{d\theta}{2}dz = 0$$

$$\boxed{\underbrace{\frac{mV^2}{R} = \text{c.f.}}_{\text{centrifugal}} = \frac{m\Omega^2R^2}{R} = m\Omega^2 R}$$

$$-\left(\frac{\partial p}{\partial r}\right)r\,dr + \rho\Omega^2 r^2 dr = 0$$

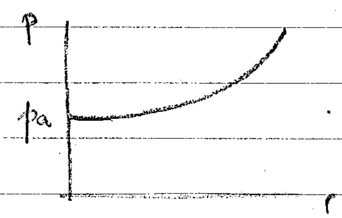
$$\boxed{\frac{\partial p}{\partial r} = \rho\Omega^2 r}$$

$$p = F(r) - \rho g z$$

$$\frac{\partial p}{\partial r} = F'(r) = \rho\Omega^2 r \qquad F = \text{const} + \frac{\rho\Omega^2 r^2}{2}$$

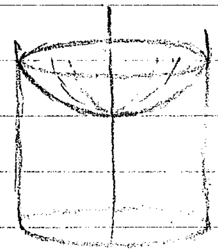
$$\boxed{p = p_a + \rho\left(\frac{\Omega^2 r^2}{2} - gz\right)} \qquad \text{when } r=0,\ g=0 \quad p=p_a$$

if surface is to be horizontal. $\frac{\Omega^2 r^2}{2} = 0$ either $r=0$ or $\Omega = 0$

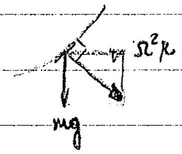


surface is not horizontal.

at free surface $\frac{\Omega^2 r^2}{2} = +gz \quad \therefore \; z = \frac{\Omega^2 r^2}{2g}$

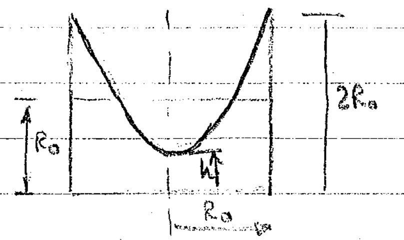


as Ω increases the paraboloid steepens.



when ρ = const p = linear variation.

H.W. # 4



spin it around at Ω (to be found)

$\ni$. at $R = R_0$ z = top of container

total mass of fluid is constant.

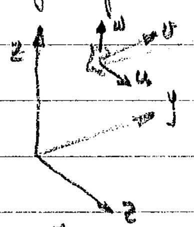
$m = \rho \pi R_0^2 \cdot R_0$

find minimum pt h.

Dynamics of fluids.

In equil motion (relative) is 0.

I. Lagrangian Approach. - rigid body motion.



$u = \frac{dx}{dt} \quad v = \frac{dy}{dt} \quad w = \frac{dz}{dt}$

$p = p(x, z, y)$

$\sum F = ma = \rho \frac{dV}{dt} \Delta v$

$\hookrightarrow f(t)$

II Eulers Approach. let $\Delta V = 1$

$V = V(x,y,z,t)$ $\quad u = u(x,y,z,t)$

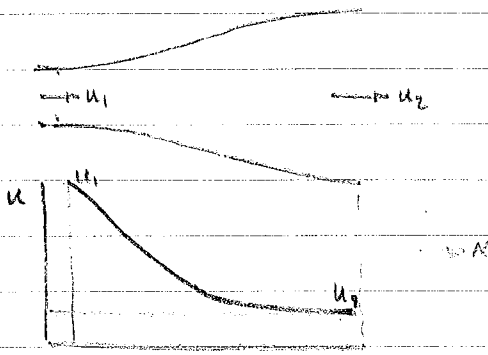
$v = v(x,y,z,t)$

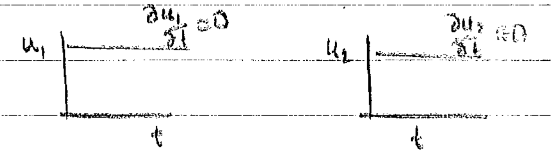
$w = w(x,y,z,t)$

$$\sum F_x = ma_x = \rho a_x = \rho\left(\frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt} + \frac{\partial u}{\partial t}\right) = \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}\right)$$

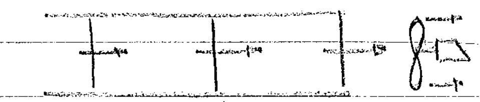
$$\frac{Du}{Dt} = \underbrace{\frac{\partial u}{\partial t}}_{\text{local accel.}} + \underbrace{\bar{V}\cdot\nabla u}_{\text{convective accel.}} \qquad \bar{V} = ui + vj + wk$$

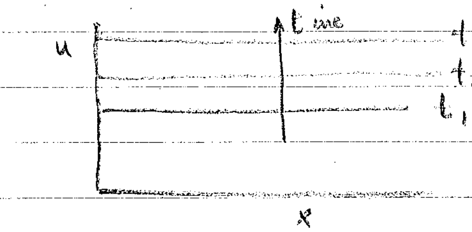
Steady flow $\partial/\partial t = 0$



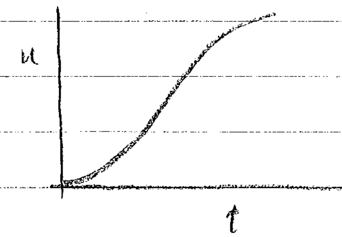


Constant area – unsteady flow





$u_x = 0$



$u_t \neq 0$

Streamline – tangent to velocity vector of fluid. (Eulerian system)

Pathline – trajectory of fluid particle $x,y,z = f(t)$ (Lagrangian system)

streamline & Pathline are coincident in steady flow

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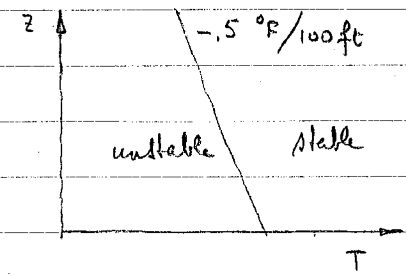


$n > \gamma$ unstable

$n < \gamma$ stable

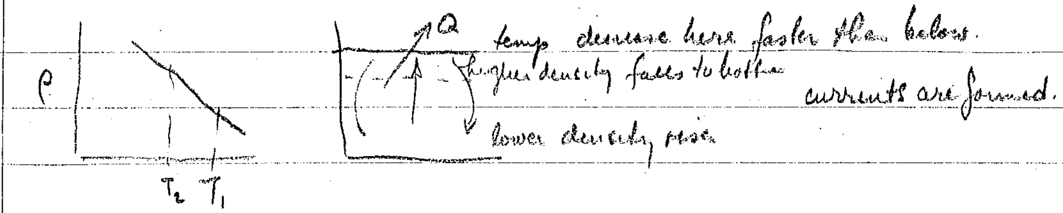
$p = p_a - \rho g z \qquad T = p/\rho R = f(z)$

$$\frac{p_a}{\rho_a R} - \frac{\rho_a g z}{\rho_a R} = T \qquad \frac{dT}{dz} = -g/R = -\frac{32.2}{1716} \approx -.02 \ {}^\circ R/ft$$

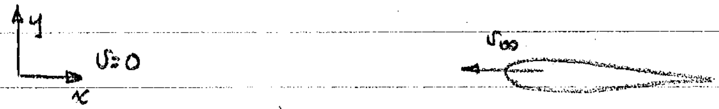
$\therefore \frac{dT}{dz}$ for $\rho = C$ is $\approx -.2 \ {}^\circ R/100ft$

next week 1 hr exam covering everything upto week previous

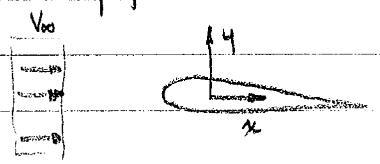
Consider H_2O



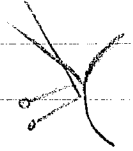
Wind fixed Coord. System,



body fixed Coord. System.

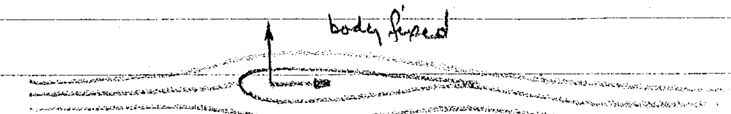


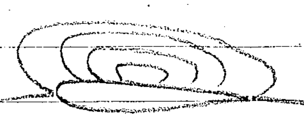
pressure bouncing off surface transferring momentum to surface



Static pressure, if v is same, is measured the same way

Streamlines





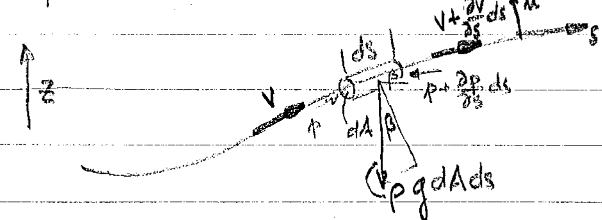
$$a_x = \frac{Du}{Dt} = u_t + u u_x + v u_y + w u_z$$

$$\sum F_x = 0 \qquad \rho \Delta V \frac{Du}{Dt} = -\frac{\partial p}{\partial x}\, dx\, dy\, dz$$

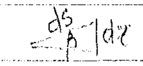
$$\frac{Du}{Dt} = -\frac{p_x}{\rho}\ ; \quad \frac{Dw}{Dt} = -\frac{p_z}{\rho} - g\ ; \quad \frac{Dv}{Dt} = -\frac{p_y}{\rho}\ ;$$

Assume Coord. fixed to body. Euler Equation.

Streamline Coordinates – every pt in fluid field streamline is tangent to velocity



$\sin\beta = \frac{dz}{ds}$



$z(s,t)$

$$\rho \Delta V \frac{DV}{Dt} = \sum F_s \rightarrow \rho \frac{DV}{Dt} = -\frac{\partial p}{\partial s} - \rho g \frac{dz}{ds} \Rightarrow \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s} + \frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{dz}{ds}$$

$$\frac{DV}{Dt} = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s}$$

Assume steady flow $\left(\frac{\partial}{\partial t} = 0\right)$ $\therefore$ we can write $V\frac{dV}{ds} + \frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} = 0$

Constant density

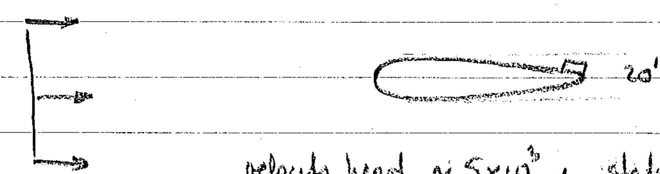
$$\int \left(V\frac{dV}{ds} + \frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} \right) ds = \frac{V^2}{2} + p/\rho + gz = C = \text{total head.}$$

Bernoulli's Equation: velocity head, static pressure head, gravity head

We derived it using total momentum balance but it also represents cons. of energy.

Assume body fixed; unif velocity V $\qquad V_\infty \sim 100$ ft/sec.

$\rho_0 = .002378$



velocity head $\sim 5\times10^3$; static pressure head $\sim 10^6$; gravity head is $\sim .6\times10^3$

since gravity head is small wrt pressure & velocity head, we say that we can neglect gz in Bernoulli's equation.

for infinite distance from wing we hydrostatic equil $\therefore p = p(z)$

but for our variation of height we can take

$p = p_\infty$

$\therefore$ Bernoulli's Equation becomes.

$\frac{\rho V^2}{2} + p = p_s \rightarrow$ stagnation pressure. (if flow decelerates to 0 isentropically)

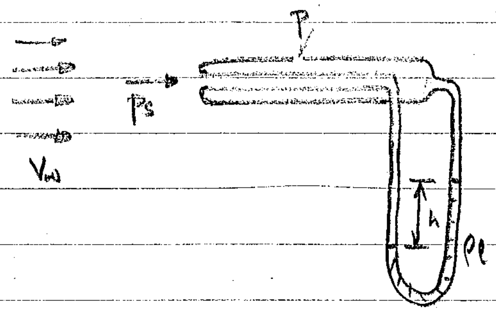
p → static pressure

$\frac{\rho V^2}{2}$ → dynamic pressure

in free stream $p_s = p_\infty + \frac{\rho V_\infty^2}{2}$

Pitot Static Tube

$$V_\infty = \sqrt{\frac{2}{\rho_\infty}(p_s - p_\infty)}$$

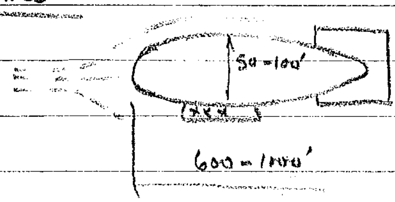


$$p_s - p_\infty = \rho_l g h$$

$$\therefore \quad V_\infty = \sqrt{\frac{2gh}{\rho_\infty}\rho_l}$$

(1) if Δz is large we can't drop out gz term.

Dirigible



(2) or if v is low. we must use the whole Bernoulli term.

(3) liquids $\rho_{H_2O} \sim 1000\, \rho_{air}$ or if $\rho >> \rho_{air}$

Submarines

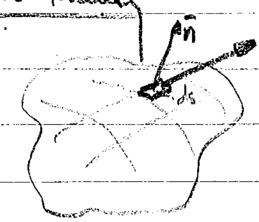
Euler's Equations.

$$u u_x + v u_y = -\frac{p_x}{\rho} \qquad \bar{q}\cdot\nabla\bar{q} = -\frac{1}{\rho}\nabla p$$

$$u v_x + v v_y = -\frac{p_y}{\rho}$$

Continuity

①



$dm = \rho \bar{V}\cdot\hat{n}\, ds.$ & $dm = -\frac{\partial}{\partial t}\iiint_{\hat{R}} \rho V d\hat{R}$

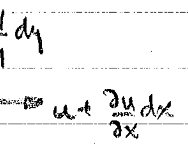
$$\iint_S dm \equiv 0 \quad \therefore \quad \frac{\partial}{\partial t}\iiint \rho\bar{q}\, dv + \rho \iiint \nabla\cdot\rho\bar{q}\, dv = 0$$

$$\iint_S \rho \bar{V}\cdot\hat{n}\, ds = \rho\iint_S \bar{V}\cdot\hat{n} ds = \rho\iiint_v \nabla\cdot\bar{V}\, dv \equiv 0$$

or. $\nabla\cdot\bar{V} = \operatorname{div}\bar{V} \equiv 0 \qquad \bar{V} = \bar{i}u + v\bar{j}$

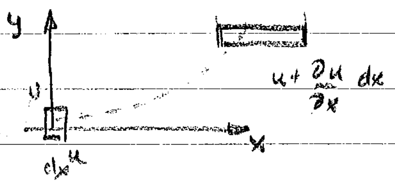
$$\boxed{u_x + v_y = 0} \qquad \boxed{\omega = \tfrac{1}{2}(v_x - u_y)}$$

②

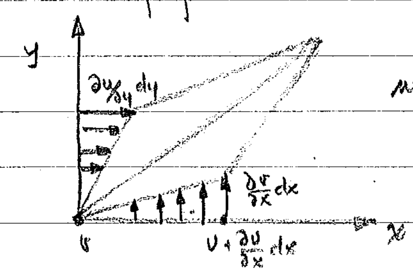


mass in $\equiv$ mass out

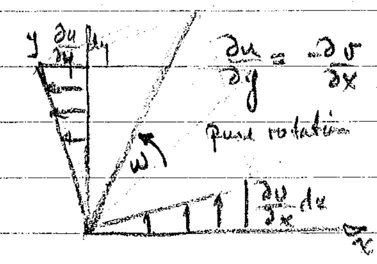
③



Rotation of fluid elements.



no rotation



$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

pure rotation

$\omega = \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

define ω = average $\frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$ average of sum.

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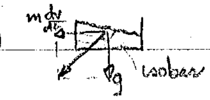
Hydrostatic Equil.

ZERO VELOCITY

$\frac{dp}{dz} = -\rho g$

NON-ZERO VELOCITY

(Rigid Body)



liquid - ρ = const

$p = p_a - \rho g z$

$\downarrow$

Archimedes' Principle

$F_z = \rho_L g \, Vol.$

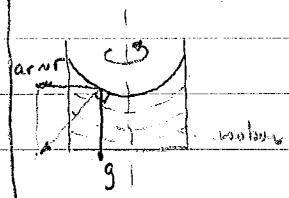
gas - $\rho \neq$ const.

atmos. $p = c\rho^n$

$n = \infty$ const ρ unstable.

$n = 1$ const T

$n = \gamma$ isentropic < neutrally stable



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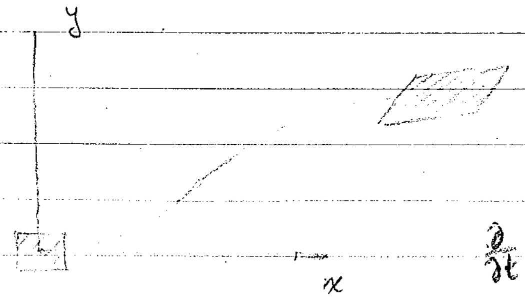


$\frac{\partial}{\partial t} = 0$ steady flow

① Translation $\longrightarrow u, v$

② Rotation $\longrightarrow \omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

③ Extension or compression $\frac{\partial v}{\partial y}, \frac{\partial u}{\partial x} \neq 0$ linear distortion

if ρ = constant $\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0$

④ Shear $\sim \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$ angular distortion

Euler's Equation based on conserv. of mv.

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

Grav field is negligible.

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$\frac{\partial \varphi}{\partial x}\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial \varphi}{\partial y}\frac{\partial^2 \varphi}{\partial x \partial y}$$

Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

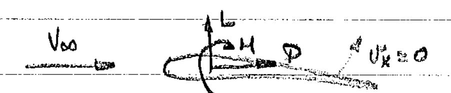
Rotation

$$\omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$\rho = C$ under all conditions we discuss.

unknown u, v, p, ω

equations 4 eq



V as $r \to \infty$ is V_∞ where r is the radius vector.

if Rotational equation is left out

3 equation 3 unknowns $- u, v, p$.

take $\frac{\partial}{\partial y}$ of first eq $\frac{\partial}{\partial x}$ of second equation

① $u\, u_{,xy} + v\, u_{,yy} + u_{,y} u_{,x} + v_{,y} u_{,y} = -\frac{1}{\rho}\, p_{,xy}$

② $u_{,} v_{,xx} + u_{,x} v_{,x} + v\, v_{,yx} + v_{,x} v_{,y} = -\frac{1}{\rho}\, p_{,yx}$

③ $u\left(u_{,xy} - v_{,xx}\right) + u_{,x}\left(u_{,y} - v_{,x}\right) + v\left(u_{,yy} - v_{,yx}\right) + v_{,y}\left(u_{,y} - v_{,x}\right) = 0$

$$u\left(u_{,xy} - v_{,xx}\right) + \left(u_{,x} + v_{,y}\right)\left(u_{,y} - v_{,x}\right) + v\left(u_{,yy} - v_{,yx}\right) = 0$$

$$u\left(u_{,y}-v_{,x}\right)_{,x} + \left(u_{,x}+v_{,y}\right)\left(u_{,y}-v_{,x}\right) + v\left(u_{,y}-v_{,x}\right)_{,y} = 0$$

if cont is satisfied & $\omega = \tfrac{1}{2}\left(v_{,x}-u_{,y}\right)$

$$-u(2\omega)_{,x} + \cancel{\left(u_{,x}+v_{,y}\right)}^{\,0}(-2\omega) - v\,(2\omega)_{,y} = 0$$

$\frac{D\omega}{Dt} \equiv 0$ since $u\,\omega_{,x} + v\,\omega_{,y} = 0$

where $\frac{D\omega}{Dt} = \cancel{\frac{\partial\omega}{\partial t}}^{\,0} + u\frac{\partial\omega}{\partial x} + v\frac{\partial\omega}{\partial y}$ when $u = \frac{dx}{dt}$ $v = \frac{dy}{dt}$

this is equivalent to Euler's Equation. $\therefore$ $\omega = \text{const.} = \omega_0$

Special case: $\omega_0 = 0$ (Irrotational)

$v_{,x} = u_{,y}$

∃ a potential fn. $\varphi(x,y)$ ∋ it satisfies the flow identically

let $u = \frac{\partial\varphi}{\partial x}$ $v = \frac{\partial\varphi}{\partial y}$ then $\omega = \tfrac{1}{2}\left(\frac{\partial^2\varphi}{\partial y\,\partial x} - \frac{\partial^2\varphi}{\partial x\,\partial y}\right) \equiv 0$

From continuity

$\nabla^2\varphi = 0$ since $\frac{\partial}{\partial x}\Big(\underbrace{\frac{\partial\varphi}{\partial x}}_{u}\Big) + \frac{\partial}{\partial y}\Big(\underbrace{\frac{\partial\varphi}{\partial y}}_{v}\Big) = 0$

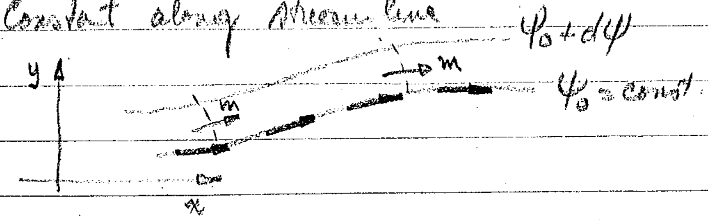
take u & v and put them into Euler's equation & integrate

or

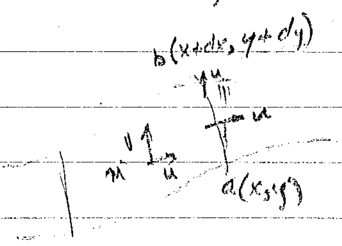
$$\rho\frac{V^2}{2} + p = p_{stag} \quad \text{where } V^2 = u^2 + v^2$$

Stream Function $\psi(x,y)$

Constant along stream line



Mass flow in = mass flow out



$$m = u\rho\, dy - \rho v\, dx = \rho\, d\psi$$

$\psi \sim VA$

$\rho\psi \sim m$

$\rho(\psi + d\psi) - \rho(\psi)$

no mass transferred accross adjacent streamlines

$$d\psi = u\, dy - v\, dx$$

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy \quad \therefore \quad \frac{\partial \psi}{\partial x} = -v \quad \frac{\partial \psi}{\partial y} = u$$

ψ must satisfy continuity $\therefore$

$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right) = 0$$

Potential Fn. $\phi(x,y)$

satisfies irrotationality $u = \frac{\partial \phi}{\partial x}$ $v = \frac{\partial \phi}{\partial y}$

Stream Fn. $\psi(x,y)$

satisfies continuity $u = \frac{\partial \psi}{\partial y}$ $v = -\frac{\partial \psi}{\partial x}$

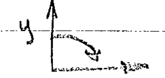
We have defined 2 fns. from all other equations

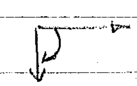
if we take ϕ & subst. in continuity $\nabla^2 \phi = 0$

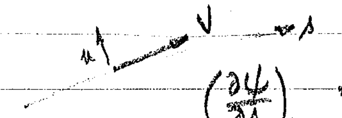
if we take ψ & subs in irrotationality $\nabla^2 \psi = 0$

Differentiate ψ in given direction = velocity component in a direction $\frac{\pi}{2}$ clockwise to the original direction.

$\frac{\partial \psi}{\partial y}$ gives u component. $\frac{\partial \psi}{\partial x}$ gives $-v$ component.



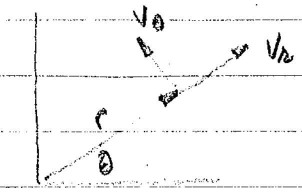




$\left(\frac{\partial\psi}{\partial s}\right)_{n\,const} = 0 \qquad \frac{\partial\psi}{\partial s} = V_n$ but $V_n = 0$

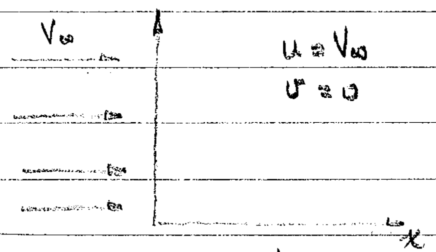
$\frac{\partial\psi}{\partial n} = V \qquad \therefore \psi = \psi(n)$ since $\psi_{,s} = V \quad \psi_{,n} = 0$

Polar



$V_r = \frac{\partial\psi}{r\partial\theta} \qquad -V_\theta = \frac{\partial\psi}{\partial r}$

① Uniform flow // to x axis



$u = V_\infty$
$v = 0$

$u = \frac{\partial\psi}{\partial y} = V_\infty$

$\psi = V_\infty y + f(x)$

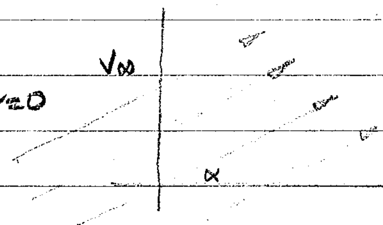
$f(x) = $ constant.

$\psi = \psi_0 + V_\infty y.$

if $\psi_0 = 0$ when $y = 0 \qquad \psi = V_\infty y$

② Airfoil on axis inclined

$\nabla^2\psi = 0$

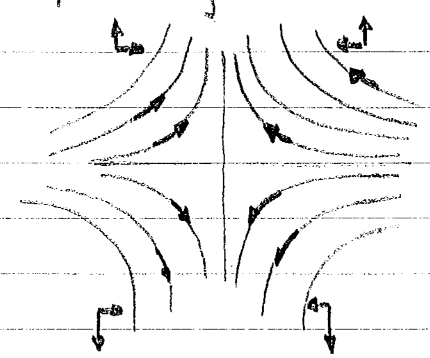


$u = V_\infty \cos\alpha = \frac{\partial\psi}{\partial y}$

$v = V_\infty \sin\alpha = -\frac{\partial\psi}{\partial x}$

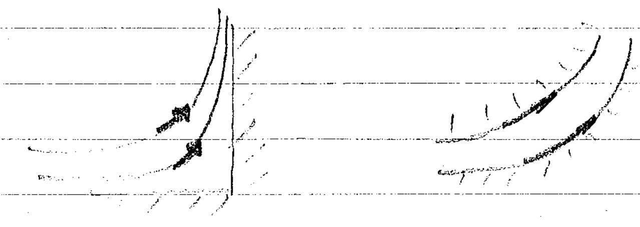
$\psi = V_\infty y \cos\alpha - V_\infty x \sin\alpha + \psi_0 \quad$ let $\psi_0 = 0$ when x & $y = 0$

(3) $\psi = -axy$

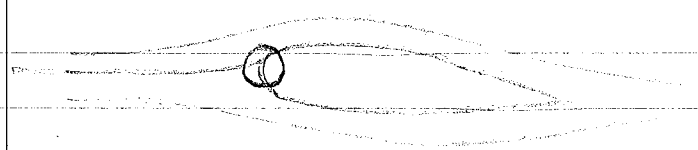


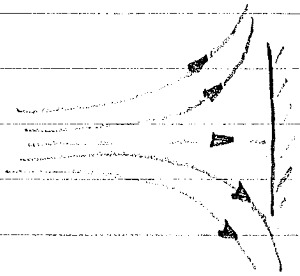
$u = -ax$
$v = ay$

Flow in a corner.



V_∞

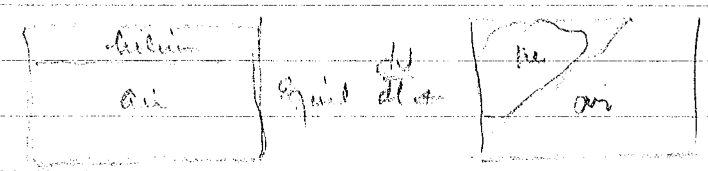




but when $x \to -\infty$ $y \to 0$
$\therefore$ $v = 0$ but $u \to -\infty$
$\therefore$ this is not a realistic problem.

$\nabla^2\psi = 0$ BC on body & at ∞ must be satisfied for a realistic problem to have a valid meaning

Exam.



$\therefore$ balloon moves backwards

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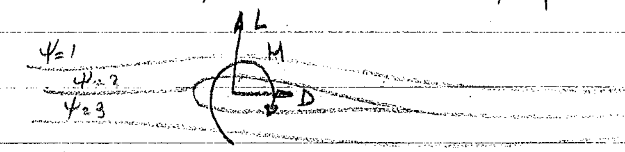
$\psi(x,y) \qquad u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$

Substitute into continuity $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ satisfies continuity

$\varphi(x,y) \qquad u = \frac{\partial \varphi}{\partial x} \qquad v = \frac{\partial \varphi}{\partial y}$

Substitute into irrotationality $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$ satisfies irrotationality

Cross subst. of ψ into irrot. eq $\rightarrow \nabla^2 \psi = 0$
Cross subst. of φ " continuity eq $\rightarrow \nabla^2 \varphi = 0$

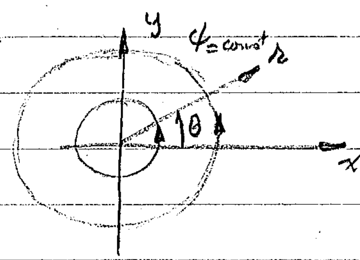


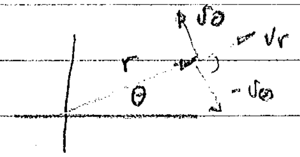
Boundary conditions — streamlines represent body $\therefore v_n = 0$

at ∞

Uniform Parallel Flow $(\alpha = 0 \neq 0)$
Flow in Corners (stagnation pt)
Vortex

Vortex

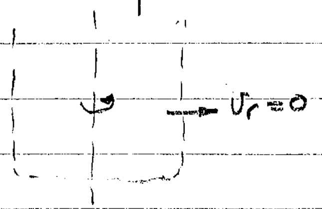




$-v_\theta = \frac{\partial \psi}{\partial r}$ given velocity 90° clockwise

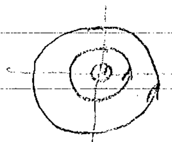
$\frac{\partial \psi}{r \partial \theta} = v_r$

Rotating Fluid



$v_\theta = \Omega r$

Vortex (Potential vortex)



stream lines of Rotating Fluid

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{r^2\partial\theta^2} = 0$$

$v_\theta = \Omega r = -\frac{\partial\psi}{\partial r}$ $\qquad \psi = f(\theta) - \frac{\Omega r^2}{2}$

no mass flow across streamline $\quad v_r = 0 = \frac{\partial\psi}{r\partial\theta} = \frac{f'(\theta)}{r} = 0 \qquad f(\theta) = c$

$\psi = c - \frac{\Omega r^2}{2}$ $\quad$ when $\psi = 0$ when $r = 0$ then $\psi = -\frac{\Omega r^2}{2}$

$\psi_{,\theta\theta} = 0 \qquad \psi_{,r} = -\Omega r \qquad \psi_{,rr} = -\Omega$

$\nabla^2\psi = -\Omega - \Omega = -2\Omega$

this shows that the body is rotational

$\frac{D\omega}{Dt} = 0$ either $\omega = \omega_0$; $\omega = 0$

Irrotational motion

$\omega = \omega_0$ or $\omega = 0$ came from euler's equation $\nabla^2\psi = 0$

which was gotten from Conservation of Momentum $\nabla^2\psi = 0$

$\omega = \omega_0$ leads to rotational flow.

if $\omega_0 = \frac{1}{2}(v_{,x} - u_{,y})$ & $u_{,x} + v_{,y} = 0 \rightarrow \psi(x,y) \rightarrow \nabla^2\psi = -2\omega_0$

$\nabla^2\psi = -2\omega_0$ is poisson's equation.

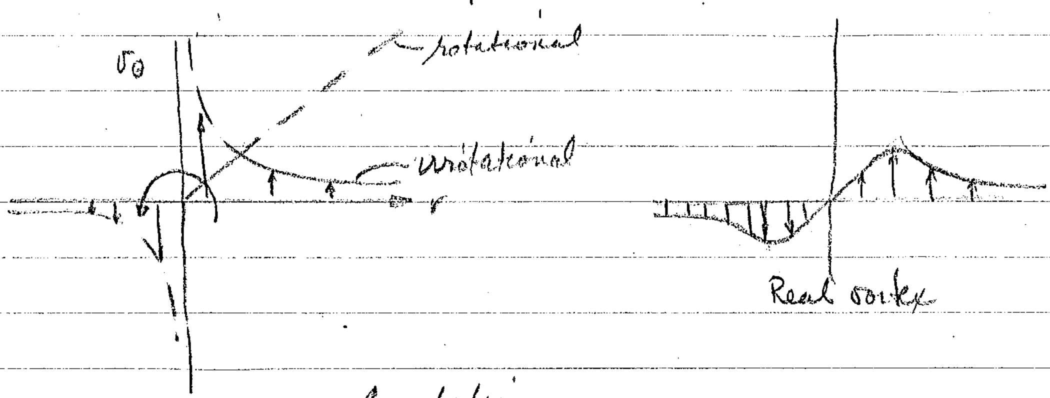
$v_r = 0 \qquad v_\theta =$

$v_r = 0 \qquad \frac{\partial\psi}{\partial\theta} = 0 \qquad \psi = \psi(r)$ only

$$\therefore \nabla^2\psi \rightarrow \frac{d^2\psi}{dr^2} + \frac{1}{r}\frac{d\psi}{dr} = \frac{1}{r}\frac{d}{dr}\left[r\frac{d\psi}{dr}\right] = 0$$

$\psi = c \ln r$

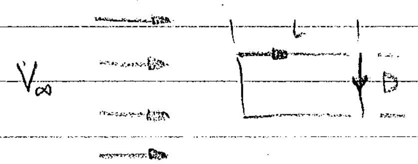
$$v_\theta = -\frac{c}{r}$$



Circulation

$\Gamma = \oint V\, ds$ clockwise +

(a) Uniform flow



$$V_\infty \oint ds = V_\infty \cdot L$$

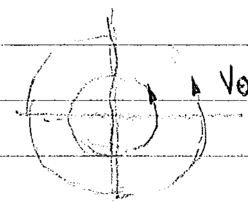
$$\oint V ds = 0 \cdot \oint ds = 0$$

$$\oint V_\infty ds = -V_\infty \oint ds = -V_\infty L$$

$$\oint V ds = 0 \cdot \oint ds = 0$$

$$\therefore \quad \Gamma = V_\infty L + 0 - V_\infty L + 0 = 0$$

(b) Potential Vortex



$$V_\theta = -\frac{c}{r} \qquad \Gamma = \oint_0^{2\pi} +\frac{c}{r} \cdot r\, d\theta$$

$$\Gamma = \int_0^{2\pi} \frac{c}{r} \cdot r\, d\theta = 2\pi c$$

$$\boxed{c = \frac{\Gamma}{2\pi}}$$

$$\therefore \quad V_\theta = -\frac{\Gamma}{2\pi r}$$

(c) rotating fluid

$$v_r = 0 \qquad v_\theta = \Omega r$$

$$\Gamma = \oint (\Omega r)\, r\, d\theta = 2\pi\Omega r^2$$

Irrotational fluid Γ = const (independent of path)

Rotational fluid Γ = fn of path

<u>Potential Vortex</u>

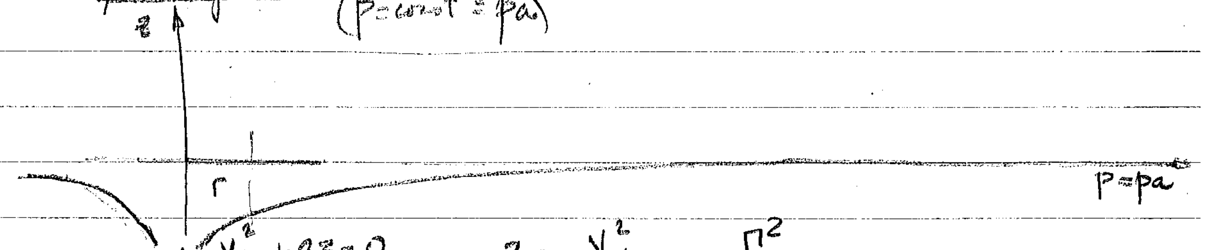
$$V_r = 0 \qquad V_\theta = -\frac{\Gamma}{2\pi r}$$

Along a streamline $\frac{V^2}{2} + p/\rho + gz = \text{const}$ — along all stream lines for potential flow; 1 streamline for rot. flow.

$$V^2 = (v_r^2 + v_\theta^2) = \frac{\Gamma^2}{4\pi^2 r^2} \qquad \text{as } r \to \infty \quad V \to 0$$

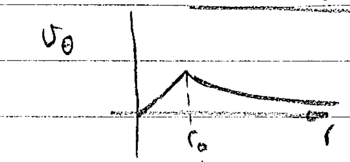
if at $z=0$ $p = p_a$ const of bernoullis eq is $\frac{p_a}{\rho}$

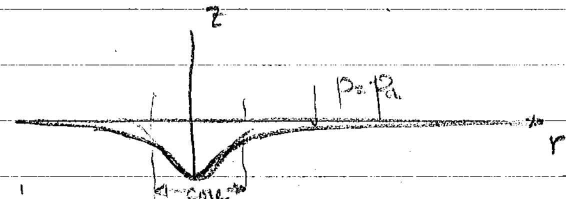
at <u>free surface</u>. (p = const = p_a)



$$\therefore\ V^2/2 + gz = 0 . \qquad z = -V^2/2g = \frac{-\Gamma^2}{8\pi^2 g r^2}$$

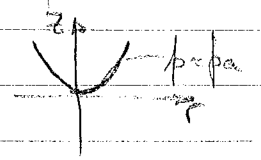
③ <u>Real Vortex</u>





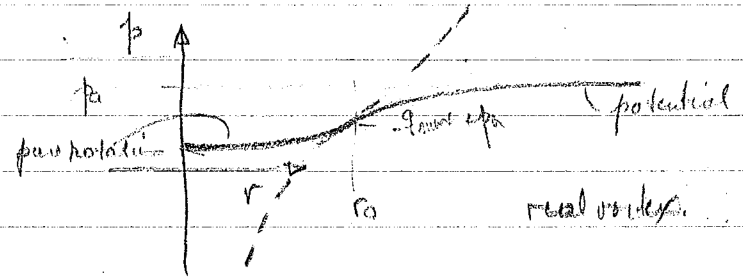
Pure Rotation

$$p/\rho = p_a/\rho - gz' + \frac{\Omega^2 r^2}{2}$$

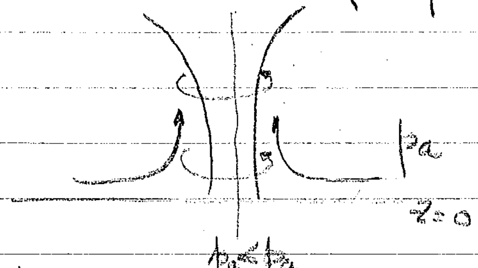


Constant z

$$p/\rho = p_a/\rho - \frac{V^2}{2} = p_a/\rho - \frac{\Gamma^2}{8\pi^2 r^2}$$

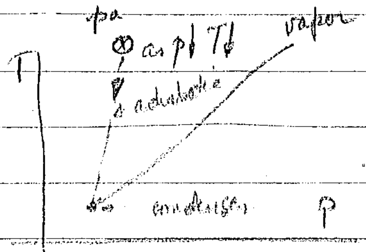


pure rotation let z = const. $\quad p/\rho = \frac{p_a}{\rho} + \frac{\Omega^2 r^2}{2}$



any fluid in p_a region will be subjected to an inward acceleration and rotating as it reaches center.

if warm, moist air



temp of surrounding air increases causing ρ to decrease and system increases in velocity in axial direction

droplets are pulled up causing more to be pulled in at bottom

Bernoulli's Equation is used only because it is a potential flow.

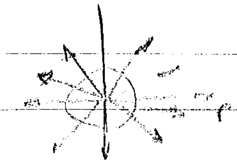
if B.E. is used for pure rotation $v_\theta = \pm\Omega r$

therefore $\frac{\Omega^2 r^2}{2} + p/\rho + gz = c(z)$ only valid along 1 streamline

$c(z) = \text{const} + 2gz$

H.W. 3/24

Source flow $\psi(r,\theta)$



$v_r \neq 0$

$v_\theta \equiv 0$

Mass flow = const = M

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$$\psi = \frac{ar^n}{n} \sin n\theta \qquad (n \neq 0)$$

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} \stackrel{?}{=} 0$$

$\psi_{,r} = -a r^{n-1} \sin n\theta \qquad \psi_{,\theta} = -ar^n \cos n\theta$

$\psi_{,rr} = -a(n-1) r^{n-2} \sin n\theta \qquad \psi_{,\theta\theta} = -nar^n \sin n\theta$

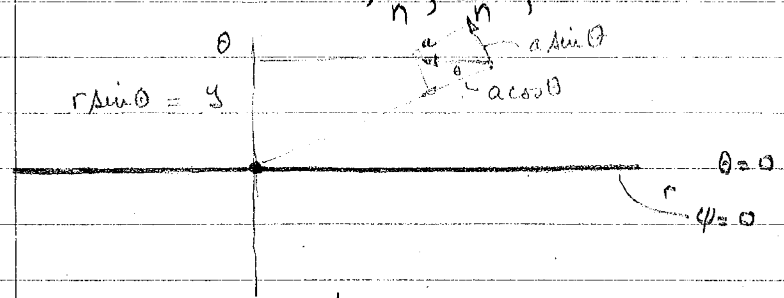
$$\nabla^2\psi = \left[-a(n-1)r^{n-2} - ar^{n-2} + nar^{n-2}\right]\sin n\theta$$

$$= -an + a - a + na = 0 \qquad \therefore \quad \nabla^2\psi \equiv 0$$

determine $\psi = 0 \rightarrow r^n = 0$ or $\sin n\theta = 0$

$r^n = 0 \rightarrow r = 0 \qquad \sin n\theta = 0 \rightarrow n\theta = 0, 2\pi, \pi, 3\pi \ldots$

$\theta = 0, \frac{\pi}{n}, \frac{2\pi}{n},$



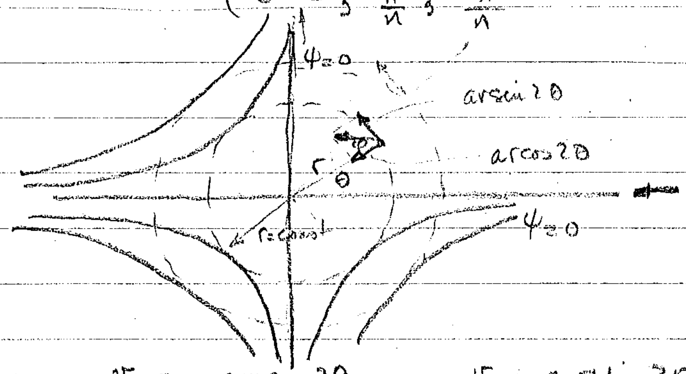
$n = 1 \qquad \psi = 0$ at $\theta = 0 \quad \theta = \pi \quad \theta = 2\pi$

$$\frac{\partial\psi}{r\partial\theta} = -ar^{n-1}\cos n\theta = v_r \qquad \frac{\partial\psi}{\partial r} = +ar^{n-1}\sin n\theta = v_\theta$$

for $n=1$ $\quad v_r = -a\cos\theta \quad v_\theta = a\sin\theta \quad V^2 = a^2$

for $n=1$ uniform parallel flow $\quad \psi = -ar\sin\theta$

$n=2$ $\left(\theta = 0, \frac{\pi}{n}, \frac{2\pi}{n}\right.$



$\tan\varphi = \frac{v_\theta}{-v_r} = \frac{ar\sin 2\theta}{ar\cos 2\theta} = \tan 2\theta$

$\varphi = 2\theta$

$v_r = -ar\cos 2\theta$ $\qquad v_\theta = ar\sin 2\theta$

$V^2 = a^2r^2$ $\qquad V = ar$

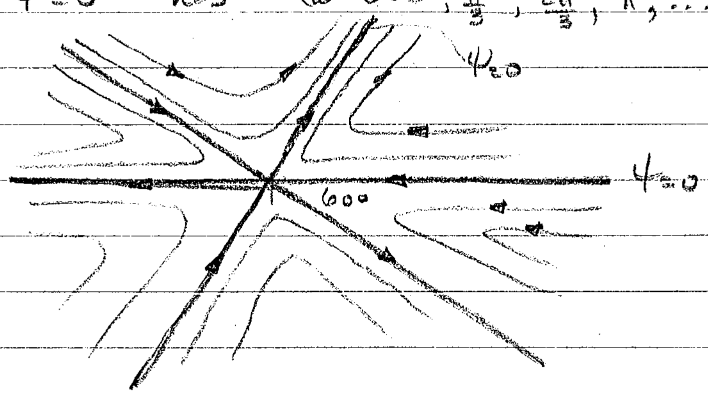
corner flow.

$$\psi = -ar^2\frac{\sin 2\theta}{2} = -ar^2\sin\theta\cos\theta$$

if $x = r\cos\theta$ $\quad y = r\sin\theta$ $\quad \psi = -axy$

for $n=3$

$\psi = 0$ $\quad n=3$ $\quad$ @ $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \ldots$



Uniform flow - $\psi = V_\infty y = V_\infty r\sin\theta$ $\rightleftarrows$

Potential Vortex $\quad \psi = \Gamma/_{2\pi}\ln r = \frac{\Gamma}{2\pi}\ln(x^2+y^2)^{1/2}$ $\quad (\Gamma$

Source Flow $\quad v_\theta = 0 \quad v_r \neq 0$

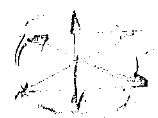
$-\frac{\partial\psi}{\partial r} \rightarrow \psi = f(\theta)$

$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$

$\nabla^2\psi = 0$ must be satisfied

$$\nabla^2\psi = \frac{1}{r^2}\frac{d^2\psi}{d\theta^2} = 0$$

$$\frac{d\psi}{d\theta} = C \quad \text{or} \quad \psi = C\theta + C_2 \qquad \theta = 0 \quad \psi = 0 \quad C_2 = 0$$

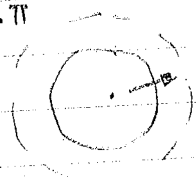
for source  mass = lb/sec   mass = ft³/sec

volumetric mass flow = $v \cdot A = V \cdot 2\pi r(1)$

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{C}{r}$$

$$m = \frac{C}{r}2\pi r = 2C\pi \qquad C = \frac{m}{2\pi}$$

$$\psi = \frac{m\theta}{2\pi} \quad \text{Source}$$



$$m = (2\pi r v_r)_1 \qquad r_1 v_{r_1} = r_2 v_{r_2} = C$$

$$m = (2\pi r v_r)_2$$

$$\frac{C}{r} = v_r$$

$$\frac{\dot{m}}{2\pi r} = v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \qquad \psi = \frac{m\theta}{2\pi} + C$$

<u>Source</u>

$$\psi = \frac{m\theta}{2\pi} \qquad v_\theta = 0 \qquad v_r = m/2\pi r$$

All flows must satisfy $\nabla^2\psi = 0$ @ body
@ ∞ — velocity is finite

Source & Uniform Flow

$$\psi = \psi_1 + \psi_2$$

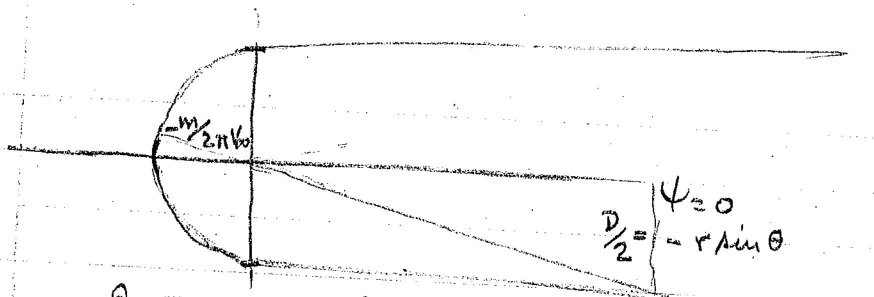
$$\nabla^2\psi_1 = 0 \quad \nabla^2\psi_2 = 0 \qquad \nabla^2(\psi) = 0 = \nabla^2\psi_1 + \nabla^2\psi_2$$

since Laplace's Equation is linear we can superimpose

$$\psi = V_\infty r\sin\theta + \frac{m\theta}{2\pi}$$

$\psi = 0$ streamline $r_0 = -\dfrac{m\theta_0}{2\pi V_\infty \sin\theta_0}$

when $\theta = 0$ $r_0 = -\dfrac{m}{2\pi V_\infty}$



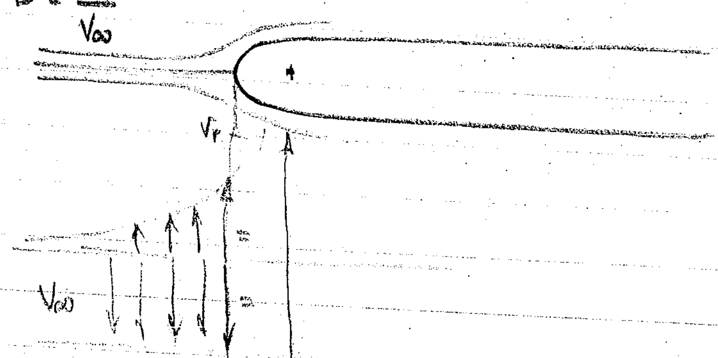
$\theta_0 = \pi/2 \qquad r_0 = \frac{-m\pi}{4\pi V_\infty} = \frac{-m}{4V_\infty}$

$\theta_0 = \pi \qquad r_0 = \infty$

when $\theta < \pi$ diameter of body

$$\frac{D}{2} = -r_0 \sin\theta_0 = \frac{m\theta_0}{2\pi V_\infty} \qquad \lim_{r_0 \to \infty} = \frac{m}{2V_\infty}$$

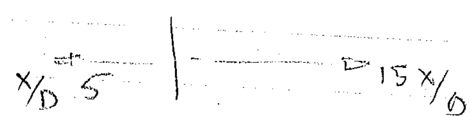
$$D = \frac{m}{V_\infty}$$



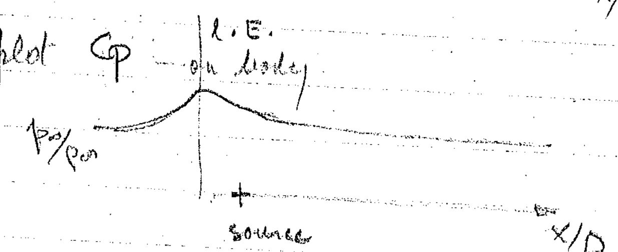
(a) for HW. plot $\psi = 0$, $\psi = 1$, $\psi = 2$ for $\frac{m}{V_\infty} = 1$

$\theta_0 = 0, \; , 60, 90, 120, 155, 165, 175, 177.5^\circ$

for X/D 15 ⟶ ⟵ 5



(b) plot C_p



$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = V_\infty\cos\theta + \frac{m}{2\pi r}$$

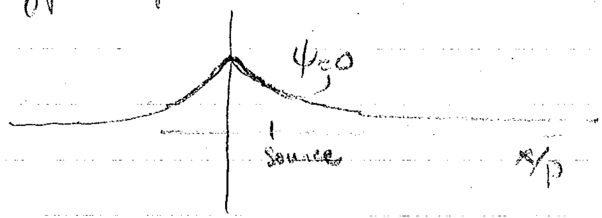
$$v_\theta = -\frac{\partial\psi}{\partial r} = -V_\infty\sin\theta$$

$$p + \rho_\infty \frac{V^2}{2} = p_\infty + \rho_\infty\frac{V_\infty^2}{2}$$

$$= \frac{\Delta p}{\rho_\infty} = \frac{p - p_\infty}{\rho_\infty} = \frac{V_\infty^2 - V^2}{2} \qquad C_p = \frac{\Delta p}{q_\infty}$$

@ ∞ $C_p = 0$

@ stag pt $C_p = 1.0$



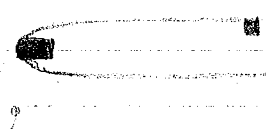
$\theta = 0, \pi \Rightarrow v_\theta = 0$

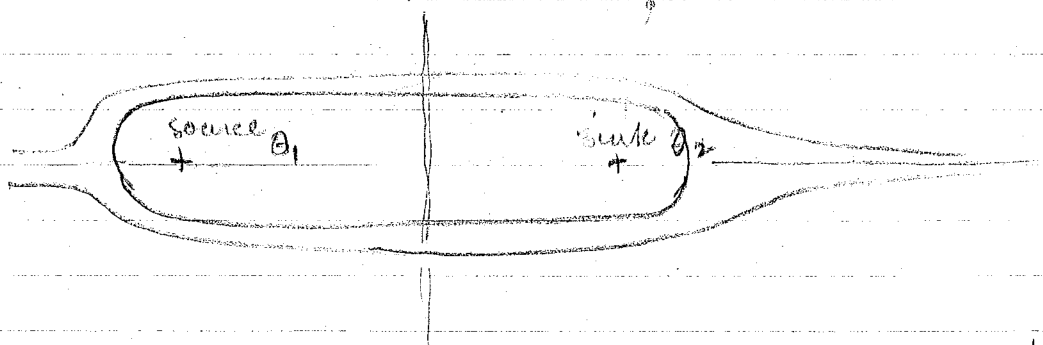
$\theta = \pi \quad v_r = -V_\infty + \frac{m}{2\pi r}$ if $v_r = 0$

$r_{st} = \frac{m}{2\pi V_\infty}$

Pitot-static tube. if a pressure tap on body at $x/D \to \infty$ we measure p_∞ at stag pt pressure tap p_s is measured.

$$V_\infty = \sqrt{\frac{2(p_s - p_\infty)}{\rho_\infty}}$$





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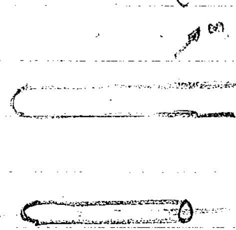
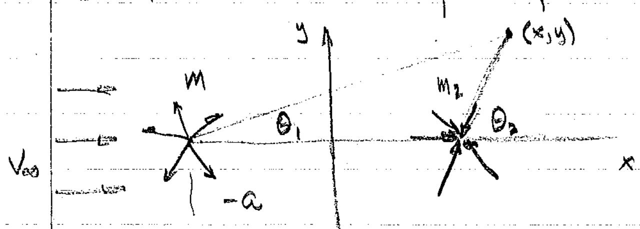
① Source - Uniform Flow (2-D)



2-D Unrealistic goes down to infinity

3-D axisymmetric counterpart.

② Source - sink uniform flow (Rankine Oval)



$\psi_1 = V_\infty y$

$\psi(r,\theta) = \frac{m\theta}{2\pi}$ for source $-\frac{m\theta}{2\pi}$ for sink

if origin at source is $\frac{m\theta_1}{2\pi}$

if origin at sink is $-m\theta_2/_{2\pi}$

$\therefore \quad \psi_2 = -\frac{m}{2\pi}(\theta_2 - \theta_1)$

$\tan\theta_1 = \frac{y}{x+a} \qquad \tan\theta_2 = \frac{y}{x-a}$

$$\tan(\theta_2-\theta_1) = \frac{\tan\theta_2 - \tan\theta_1}{1+\tan\theta_1\tan\theta_2} = \frac{\frac{y}{x-a}-\frac{y}{x+a}}{1+\frac{y^2}{x^2-a^2}} = \frac{y(x+a)-y(x-a)}{x^2-a^2+y^2}$$

$$= \frac{2ay}{x^2+y^2-a^2}$$

$\psi = \psi_1 + \psi_2$

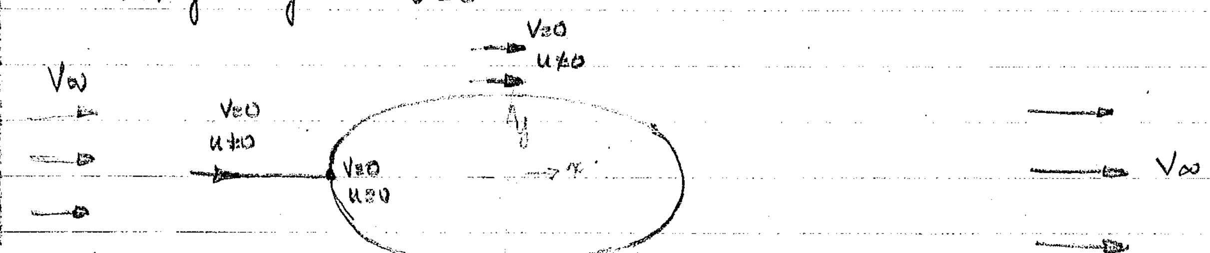
$$= V_\infty y - \frac{m}{2\pi}\arctan\frac{2ay}{x^2+y^2-a^2}$$

at $\psi=0 \qquad \tan\left[\frac{2\pi V_\infty y}{m}\right] = \frac{+2ay}{x^2+y^2-a^2}$

$$u = \psi_{,y} = V_\infty - \frac{ma}{\pi}\frac{(x^2-y^2-a^2)}{(x^2+y^2-a^2)^2+4a^2y^2}$$

$$v = -\psi_{,x} = -\frac{ma}{\pi}\frac{2xy}{(x^2+y^2-a^2)^2+4a^2y^2}$$

along $x=y=0 \quad v=0$



stag pt

at $y=0 \qquad u=0 = V_\infty - \frac{ma}{\pi}\frac{x^2-a^2}{(x^2-a^2)^2} = V_\infty - \frac{ma}{\pi}(x^2-a^2)^{-1}$

as $x\to\infty \quad u\to V_\infty$ realistic body

$\therefore$ at stag pt $\quad V_\infty = \frac{ma}{\pi(x^2-a^2)}$

or $\quad a^2 + \frac{ma}{V_\infty\pi} = x^2 \longrightarrow \quad x = \pm a\left[1+\frac{m}{V_\infty \pi a}\right]^{1/2}$

as $\frac{m}{V_\infty}\uparrow \quad x$ increases away from center of oval.

for the semi minor axis

let $\psi = 0$ and $x = 0$

$$0 = V_\infty y - \frac{m}{2\pi} \arctan\left[\frac{2ay}{y^2 - a^2}\right]$$

$$\tan \frac{2\pi V_\infty y}{m} = \frac{2ay}{y^2 - a^2}$$

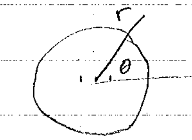
$$\frac{y_{u,l}}{x_{stag\,pt}} = \text{slenderness ratio}$$

upper, lower

③ Circular Cylinder.

let sources & sinks approach each other ∴ Slender Ratio → 1

if a is small $\tan\alpha \approx \alpha$



$$\psi = V_\infty y - \frac{may}{\pi(x^2+y^2-a^2)}$$

since $x^2+y^2 \gg a$ ∴

$$\psi = V_\infty y - \frac{may}{\pi(x^2+y^2)} \qquad \text{let } a \to 0 \quad m \to \infty$$

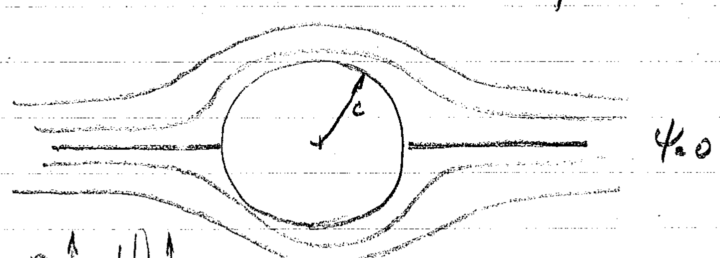
let $\mu = ma = \text{const.}$

$$\psi = V_\infty y - \frac{\mu y}{\pi(x^2+y^2)} = V_\infty r \sin\theta \left[1 - \frac{\mu}{\pi V r^2}\right]$$

define $c^2 = \frac{\mu}{\pi V_\infty}$

$$\therefore \quad \psi = V_\infty r \sin\theta \left[1 - \frac{c^2}{r^2}\right]$$

$\psi = 0$ $c = r$ $\forall\theta$ if $\theta = 0, \pi, \ldots n\pi$ n an integer.

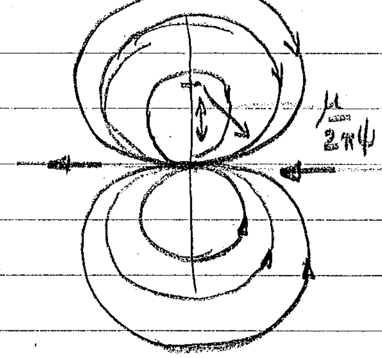


when $r \uparrow$ $\psi \uparrow$

④ (Source sink ma = μ) Doublet

$$\psi = \frac{-\mu y}{\pi (x^2+y^2)} \longrightarrow x^2+y^2 + \frac{\mu y}{\pi \psi}$$

$$x^2 + \left(y + \frac{\mu}{2\pi\psi}\right)^2 = \left(\frac{\mu}{2\pi\psi}\right)^2$$



centre moves with respect to ψ

for: $\psi = V_\infty \, r \sin\theta \left[1 - \left(\frac{c}{r}\right)^2\right]$

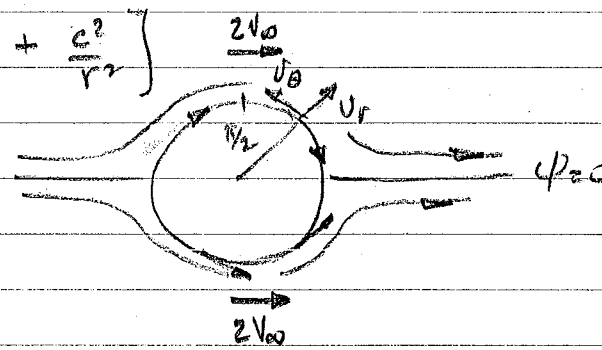
$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = V_\infty \cos\theta \left[1 - \frac{c^2}{r^2}\right]$$

$$v_\theta = \frac{-\partial \psi}{\partial r} = -V_\infty \sin\theta \left[1 + \frac{c^2}{r^2}\right]$$

$v_r \equiv 0$ at $r = c$

$v_\theta = -2V_\infty \sin\theta$ at $r = c$

@ $\theta = 0, \pi$ $v_\theta = 0$



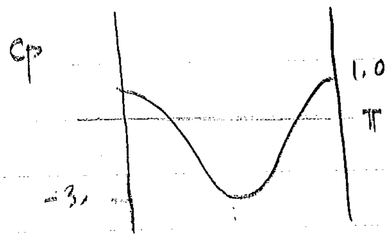
as $r \to \infty$ $V \to V_\infty$

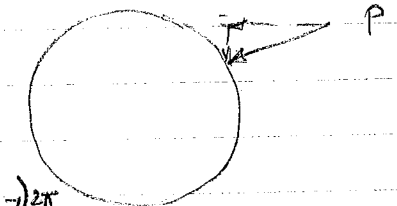
Since potential flow use Bernoulli's eq.

$$p + \frac{\rho_\infty V^2}{2} = p_s = p_\infty + \frac{\rho_\infty V_\infty^2}{2}$$

$$p = p_\infty + \frac{\rho_\infty V_\infty^2}{2} - \frac{\rho_\infty 4 V_\infty^2 \sin^2\theta}{2} \quad \text{at } r = c$$

$$\frac{p - p_\infty}{q_\infty} = \left[1 - 4\sin^2\theta\right]$$





$$D = -\int_0^{2\pi} p \cos\theta \, c\,d\theta$$

$$p = \left\{ p_\infty + q_\infty \left[1 - 4\sin^2\theta \right] \right\}_0^{2\pi} = 0$$

$$L = -\int_0^{2\pi} p \sin\theta \, c\,d\theta = 0$$

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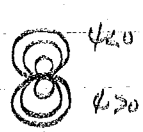
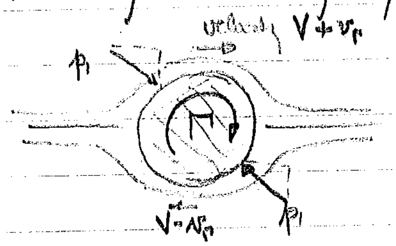
1. Uniform Flow $\qquad \psi = V_\infty r \sin\theta \qquad \varphi$
2. Source $\qquad \psi = m\theta/2\pi$
3. Vortex $\qquad \psi = \frac{\Gamma}{2\pi} \ln r$
4. Doublet ⟶  $\psi < 0$ $\psi > 0$ $\quad x^2 + \left(y + \frac{\mu}{2\pi\psi}\right)^2 = \left(\frac{\mu}{2\pi\psi}\right)^2$
5. Doublet + Uniform Flow (cyl)

Test - after rotating cylinder to today's work.



L =

D =

Adding a vortex

6. Cylinder + Vortex

$$\psi = V_\infty r \sin\theta \left[1 - \frac{a^2}{r^2}\right] + \frac{\Gamma}{2\pi} \ln r$$

when $r = a$ $\quad \psi = C$ $\quad$ to bring $\psi = 0$ at body

$\therefore$ add a const $\therefore$ $\quad \psi = V_\infty r \sin\theta \left[1 - a^2/r^2\right] + \frac{\Gamma}{2\pi} \ln(r/a)$

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = V_\infty \cos\theta \left[1 - a^2/r^2\right]$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -V_\infty \sin\theta \left[1 + a^2/r^2\right] - \frac{\Gamma}{2\pi r}$$

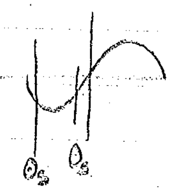
when $r = a$ $\quad \psi = 0 \quad v_r = 0 \quad v_\theta = -2V_\infty \sin\theta - \Gamma/2\pi a$

stag pts of fluid

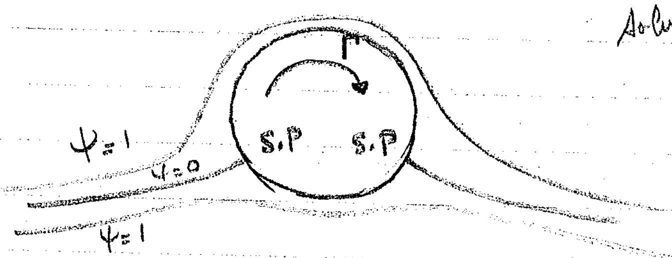
$v_r = v_\theta = 0$

$$v_\theta = -2V_\infty \sin\theta_s - \frac{\Gamma}{2\pi a} = 0$$

$$\sin\theta_s = -\frac{\Gamma}{4\pi a V_\infty}$$



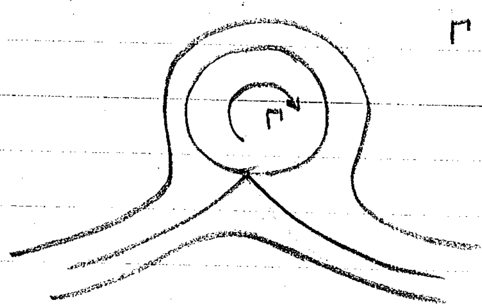
when $\Gamma \neq 0$



Solve from $v_\theta = 0$

$$\sin\theta_s = -\frac{\Gamma}{2\pi r V_\infty\left(1+\left(\frac{a}{r}\right)^2\right)}$$

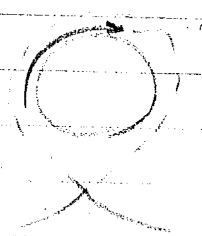
as $\Gamma \uparrow$ θ_s moves towards bottom of cylinder.



$\Gamma >> 0$

$\Gamma = 4\pi V_\infty a$

if $\Gamma > 4\pi V_\infty a$ stag pt moves off body



pure rotation

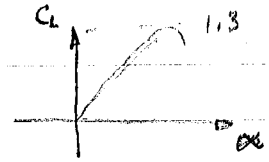
H.W. L & D to be evaluated.

$D \equiv 0$ 2-D potential flow any shape.

$L = \rho_\infty V_\infty \Gamma$ Kutta-Joukowski law. independent of body shape.

for max lift $L = \rho_\infty V_\infty \cdot 4\pi V_\infty a$

$$C_L = \frac{L}{q_\infty \cdot 2a \cdot 1}$$ - unit span



$$C_L = \frac{4\pi V_\infty^2 \rho_\infty a}{\frac{\rho_\infty V_\infty^2}{2}\cdot 2a} = 4\pi \sim 12.57$$

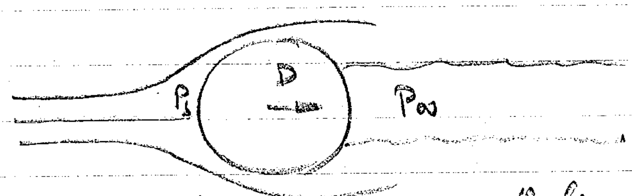
$\underbrace{\quad\Gamma = 0\quad}$ $\qquad \Gamma \neq 0$

$C_D \qquad .3 \to 1.0 \qquad \to \quad 2.5$

Typical Airfoil $\approx .003$

$HP \sim DV \sim C_D V^3$

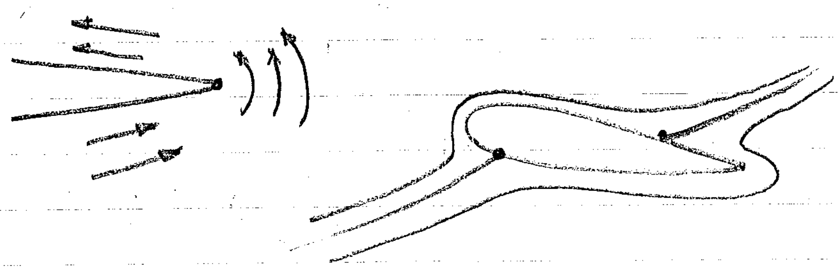
These rotating cylinders are being used in VSTOL aircraft & ships



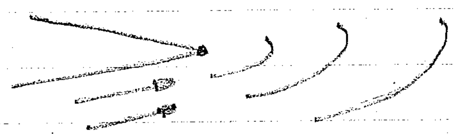
due to shear stresses, the flow separates & flow behind is p_∞ with flow never closing. $P_\infty \ll P_s$ gives high drag.

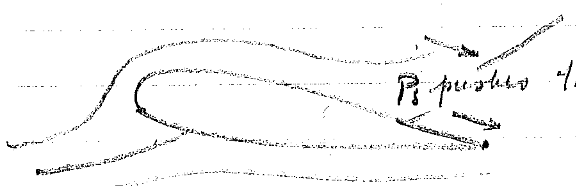
This can't be predicted by pot flow theory since shear stresses neglected

Rankine oval.



if a vortex is introduced at t.e. we get flow detached.





P_s pushes t.e. to trail edge

Kutta Condition - flow leaves trailing edge smoothly.

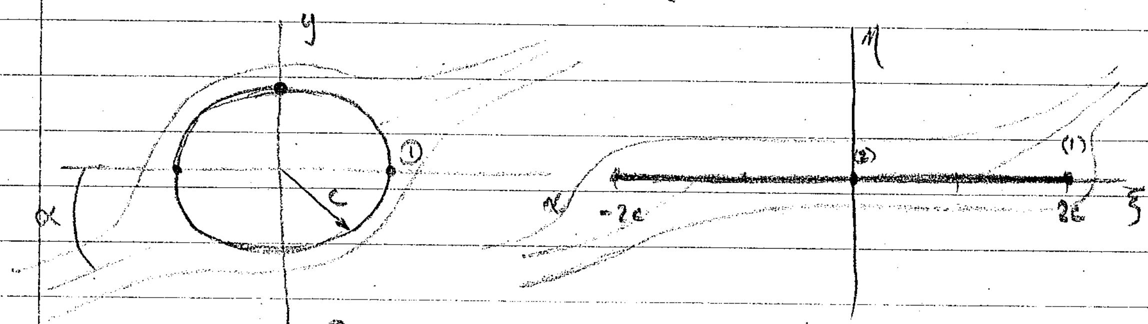
This allows us to fix Γ

Joukowski Airfoil

Transformation $\zeta = z + \frac{c^2}{z}$

$z = x + iy$

$\zeta = \xi + i\eta$



① $x = c \quad y = 0 \quad z = c \quad \zeta = 2c$

② $x = 0 \quad y = c \quad z = ic \quad \zeta = 0$

③ $x = -c$

add a Γ $\ni$ $\theta_s \to 0$ $\quad \alpha = -\theta_s \quad \sin\theta_s = \frac{-\Gamma}{4\pi V_\infty c}$

if $\sin\alpha \sim \alpha \quad \alpha < 10^\circ$

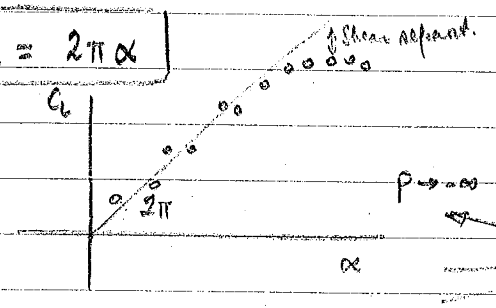
$\therefore \quad \Gamma = 4\pi V_\infty c\alpha$ ← this satisfies Kutta Cond.

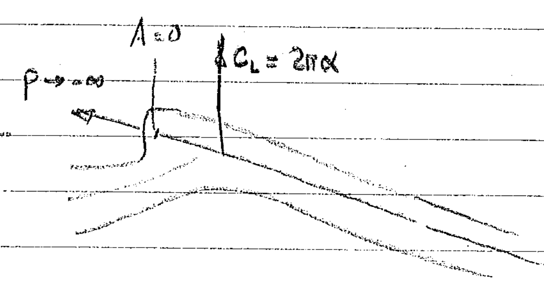
$L = \rho_\infty V_\infty \Gamma$

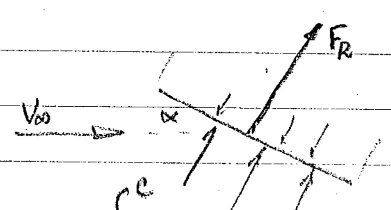
$= \rho_\infty V_\infty^2 4\pi c\alpha \qquad C_L = \frac{L}{S_W\, q_\infty} = \frac{4\pi c\alpha \;\; \rho_\infty V_\infty^2}{4c \cdot \rho_\infty \frac{V_\infty^2}{2}}$

flat plate area

$\boxed{C_L = 2\pi\alpha}$







$F_R = \int_0^c (P_l - P_u)\, dl \qquad L = \rho_\infty V_\infty \Gamma$

$D = \rho_\infty V_\infty \Gamma \alpha$

theory predicts zero drag $\qquad V_\infty \to \infty$ at leading edge

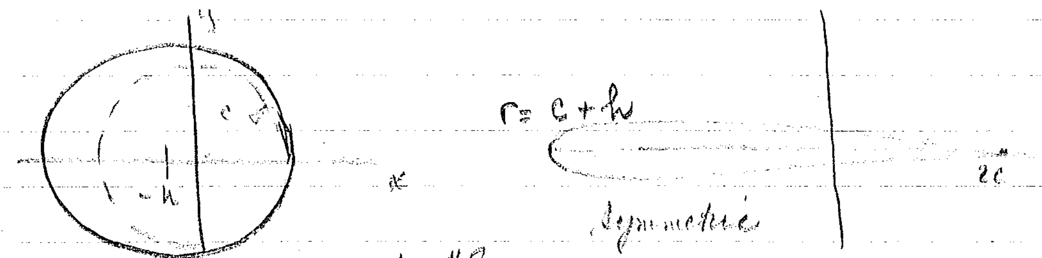
$P_\infty \to -\infty \quad \ni \quad P_\infty + \rho_\infty \frac{V_\infty^2}{2} = C$

p at edge $\to -\infty \quad A = 0 \qquad \therefore \quad 0(-\infty) = \rho_\infty V_\infty \Gamma \alpha$

this is the leading edge suction.

to decrease V around le we must blunt l.e.

$\zeta = z + \frac{c^2}{z}$ $\qquad \zeta = \xi + i\eta$



HW. #8

H.W. draw Joukowski shape for $\theta = 0$ to $180°$ $\quad h/c = 1/4$

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Dr. Ackerberg's Class.

$u = x(y-z)$

$v = y(z-x)$ $\qquad$ assume $\quad x = x(t)$

$w = z(x-y)$ $\qquad\qquad\qquad\quad y = y(t)$

$\qquad\qquad\qquad\qquad\qquad\qquad\quad z = z(t)$

streamlines defined

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \qquad \frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)} = \lambda(t)\,dt$$

$z = f(x,y) \qquad z = g(x,y)$

$$dx + dy + dz = \lambda(t)\,dt\left\{x(y-z) + y(z-x) + z(x-y)\right\} = 0$$

$$x+y+z = C_1$$

$$\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = \lambda(t)\,dt\left\{(y-z) + (z-x) + (x-y)\right\} = 0$$

$\ln x + \ln y + \ln z = C_2 \qquad (x)(y)(z) = C_2'$ gives streamlines.

particle path $\quad \frac{dx}{dt} = x(y-z) \qquad x(0) = a$

$\qquad\qquad\qquad \frac{dy}{dt} = y(z-x) \qquad y(0) = b$

$\qquad\qquad\qquad \frac{dz}{dt} = z(x-y) \qquad z(0) = c$

Streaklines 2 dimension

$$\frac{dx}{dt} = 3xt^2 \qquad \frac{dy}{dt} = 2yt$$

$\ln x = t^3 + c$ when $t = 0$ $x = a$

$\frac{x}{a} = e^{t^3}$ particle path.

$\ln y = t^2 + c'$ when $t = 0$ $y = b$

$y/b = e^{t^2}$ particle path.

streakline point in space through which particles have passed. (x_0, y_0)

if $x = x_0$ $y = y_0$ $a = x_0 e^{-t^3}$

$b = y_0 e^{-t^2}$ let $0 \le t \le t_1$

to const streakline must find coord. of particles now.

$\therefore$ $y/b = e^{t_1^2}$; $\frac{x}{a} = e^{t_1^2}$ let s vary from 0 to t_1

$x = x_0 e^{-s^3} e^{t_1^3}$

$y = y_0 e^{-s^2 + t_1^2}$ $0 \le s \le t_1$ gives pts x,y in space needed to construct streakline.

Bernoulli's Equation:

if $V = \nabla\varphi$ $\oint V \cdot dr = \oint \nabla\varphi \cdot dr = \oint d\phi = \varphi\Big]_a^a = 0$

doublet has an axis correspond'g to direction which source & sink approach each other.

$\oint d\psi = 0$ if no sink or source is present within streamline.

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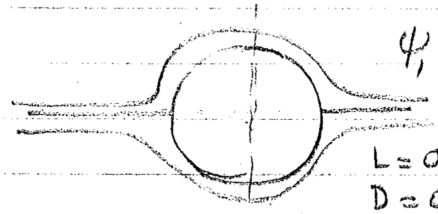
$\oint V dl = 0$ if origin is not included

$\oint V dl = \Gamma$ " " "



$\psi_1 = V_\infty r \sin\theta - \frac{\mu}{\pi r} \sin\theta$

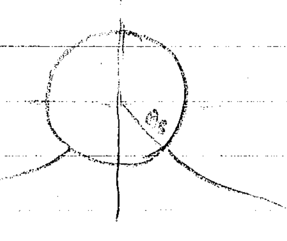
doublet uniform stream

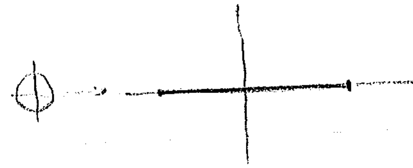
$L = 0$

$D = 0$

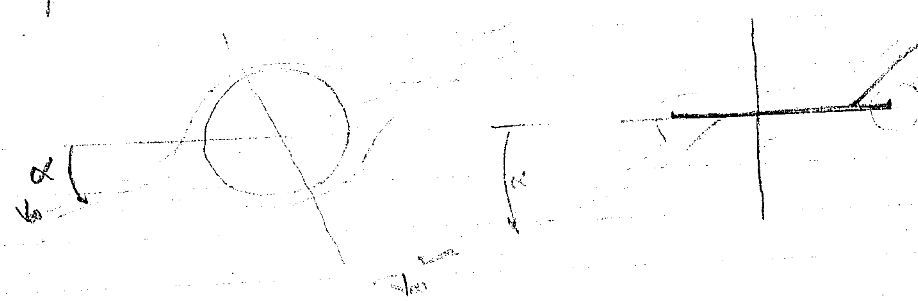
if a vortex is added

$\psi = \psi_1 + \frac{\Gamma}{2\pi r}$



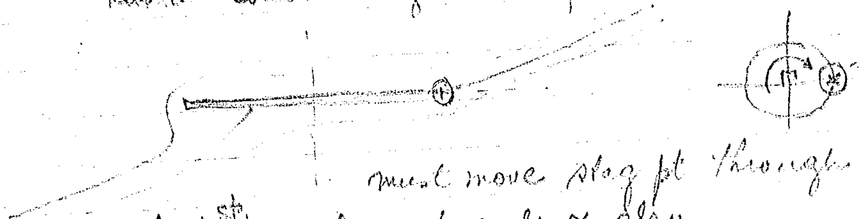
Using transformed plane 

if inclined.



impossible
separation occurs.

Kutta Condition gives real flow.



stagnat.

must move stag pt through angle of attack.

front $\overset{\text{st.}}{}$moves forward angle α also.

$\psi = 0 \quad -\theta_s = \alpha \longrightarrow \Gamma_{\text{necessary}}$

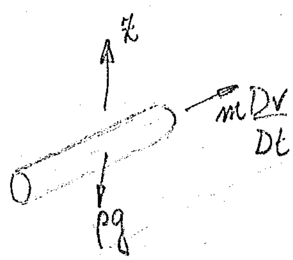
$L = \rho V \Gamma \qquad \Gamma = \Gamma(\alpha) \longrightarrow C_L = 2\pi\alpha$

$\psi = V_\infty r \sin\theta - \frac{\mu}{\pi r}\sin\theta + \frac{\Gamma}{2\pi r}$ at body & stag pt.

$\psi = 0 \quad v_r = 0 \quad v_\theta = 0 \quad \therefore$
$v_\theta = -V_\infty \sin\theta\,(1 + a^2/r^2) - \Gamma/2\pi r \to 0 \qquad r = a$

$\therefore \quad \sin\theta = -\dfrac{\Gamma}{4\pi a V_\infty}$

<u>Bernoulli Equation</u>

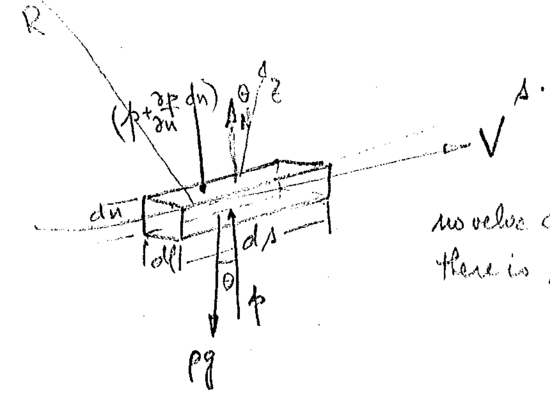


$s \qquad p_{s_2}$

p_{s_1}

$V\dfrac{\partial V}{\partial s} + \dfrac{1}{\rho}\dfrac{\partial p}{\partial s} + g\dfrac{\partial z}{\partial s} = 0$

$\rho\dfrac{V^2}{2} + p + \rho g z = C = p_s$ along a particular streamline



no veloc change in normal direct since it is streamline.
there is centrifugal accel. $= \rho \frac{V^2}{R}$

$$\Sigma F_n \uparrow = p\,ds\,dl - \left(p + \frac{\partial p}{\partial n}dn\right)ds\,dl - \rho\frac{V^2}{R}(ds\,dl\,dn) - \rho g\cos\theta\,(ds\,dl\,dn) = 0 \qquad \cos\theta = \frac{\partial z}{\partial n}$$

$$= \left\{-\frac{\partial p}{\partial n} - \rho\frac{V^2}{R} - \rho g\frac{\partial z}{\partial n}\right\}(ds\,dl\,dn) = 0$$ must be satisfied.

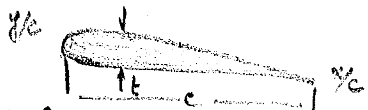
$$\frac{\partial}{\partial n}(\text{Bernoulli}) = \rho V\frac{\partial V}{\partial n} + \frac{\partial p}{\partial n} + \rho g\frac{\partial z}{\partial n} = \frac{\partial p_s}{\partial n}$$ add above.

$$-2\omega\rho V = \frac{\partial p_s}{\partial n} = \rho V\frac{\partial V}{\partial n} - \rho\frac{V^2}{R} \qquad \text{if } \frac{\partial p_s}{\partial n} = 0 \quad \boxed{\frac{\partial V}{\partial n} = \frac{V}{R}}$$ for not fam.

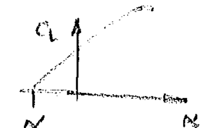
this is just $\omega = \frac{1}{2}\left(\frac{\partial V}{\partial n} - \frac{V}{R}\right)$ for irrotational flow $\omega = 0$ we can use Bernoulli's everywhere.

Thin Airfoil Theory.

Joukowski (Indirect)



α_{0_L}, C_M
a.c.



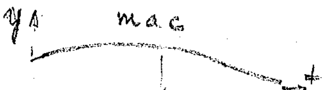
Governing Eq. $\nabla^2\varphi = 0$ with b.c.

Lift Γ must be included

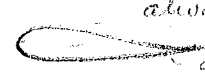
3 general cond. Charact. shape

1. Thickness distrib t/c
2. mean Camber line
3. α.

$y/c = f(x/c)$

solution = 

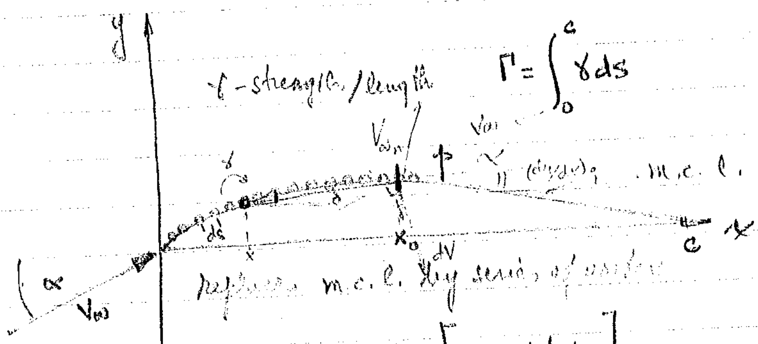
generated by vortices



always symmetric
generated by sources & sinks.

y/c m.a.c $= f(x/c)$

usually not done
only to determine drag



$$\Gamma = \int_0^c \gamma \, ds$$

(1) Assume t of airfoil was taken completely $t/c \ll 1$
Streamlines are ≈ parallel to m.c.l.

(2) ∴ m.c.l. is a streamline.

V ⊥ to m.c.l. ≡ 0

pick some pt at x_0 find total velocity induced at pt P. Take normal veloc at P set equal to 0

$$V_{\infty_n} = V_\infty \sin\left[\alpha - \left(\frac{dy}{dx}\right)_0\right]$$

$$V_p = \int_0^c dV_p \qquad dV_p = \frac{\gamma \, ds}{2\pi r} \cos\left(\delta - \left(\frac{dy}{dx}\right)_0\right)$$ r is distance bet. vortex & pt. P.

$$\cos\delta = \frac{x_0 - x}{r} \qquad \frac{dx}{ds} = \cos\left(\frac{dy}{dx}\right)$$

$$V_p = \int_0^c \frac{\gamma}{2\pi} \frac{dx}{\cos\left(\frac{dy}{dx}\right)} \cdot \frac{\cos\delta}{x_0 - x} \cdot \cos\left(\delta - \left(\frac{dy}{dx}\right)_0\right) dx$$

(3) Assume $\left(\frac{dy}{dx}\right)_{mcl} \ll 1$ $\qquad \cos\left(\frac{dy}{dx}\right), \cos\delta, \cos\left(\delta - \left(\frac{dy}{dx}\right)_0\right) = 1$

(4) Assume α is small.

$$V_p = \int_0^c \frac{\gamma \, dx}{2\pi (x_0 - x)}$$

$$\int_0^c \frac{\gamma \, dx}{x_0 - x} = 2\pi V_\infty \left[\alpha - \left(\frac{dy}{dx}\right)_0\right] \Rightarrow f(x)_0$$

$$\bar{x} = x/c \qquad \int_0^1 \frac{\gamma \, d\bar{x}}{\bar{x}_0 - \bar{x}} = 2\pi V_\infty \left[\alpha - \left(\frac{d\bar{y}}{d\bar{x}}\right)_0\right]$$

$$\bar{y} = y/c$$

$$\bar{x} = x/c = \frac{1 - \cos\theta}{2} \qquad d\bar{x} = \frac{\sin\theta}{2} d\theta$$

$$\int_0^\pi \frac{\gamma \sin\theta \, d\theta}{(1 - \cos\theta_0) - (1 - \cos\theta)} = \int_0^\pi \frac{\gamma \sin\theta \, d\theta}{\cos\theta - \cos\theta_0}$$

$$\int_0^\pi \frac{\cos n\theta \, d\theta}{\cos\theta - \cos\theta_0} \rightarrow \frac{\pi \sin n\theta_0}{\sin\theta_0}$$

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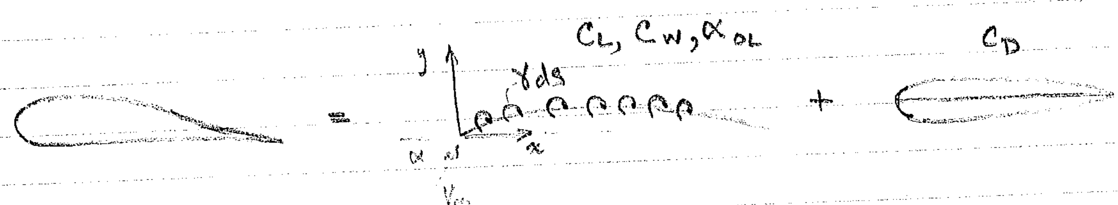


Potential flow theory : $C_D = 0$

Governing Equation

$$2\pi V_\infty\left[\alpha - \left(\frac{dy}{dx}\right)_0\right] = \int_0^\pi \frac{\gamma \sin\theta \, d\theta}{\cos\theta - \cos\theta_0}$$

$\gamma = \gamma(x)$ $\quad \frac{x}{c} = \frac{1-\cos\theta}{2}$

I. Symmetric airfoil. $\quad \int_0^\pi \frac{\cos n\theta \, d\theta}{\cos\theta - \cos\theta_0} = \frac{\pi \sin n\theta_0}{\sin\theta_0}$

$\left(\frac{dy}{dx}\right)_0 = 0$ m.c.l. has no slope.

assume $\gamma = \frac{C_1 + C_2\cos\theta}{\sin\theta}$

$$2\pi V_\infty \alpha = \int_0^\pi \frac{C_1 + C_2\cos\theta}{\cos\theta - \cos\theta_0} d\theta = 0 + C_2\pi$$

$$\boxed{C_2 = 2V_\infty\alpha}$$

γ at trailing edge $= 0$ from Kutta Condition

@ $\frac{x}{c} = 1$ $\quad \theta = \pi$ $\quad \gamma = 0$ $\quad \gamma = \frac{C_1 - 2V_\infty\alpha}{\sin\pi}$

if $C_1 - 2V_\infty\alpha = 0$ $\quad \therefore C_1 = 2V_\infty\alpha$ This satisfies l'Hôpital's rule.

$$\therefore \gamma = 2V_\infty\alpha\left[\frac{1+\cos\theta}{\sin\theta}\right]$$

$$L = \rho_\infty V_\infty \Gamma = \rho_\infty V_\infty \int_0^c \gamma\, dx = \rho_\infty V_\infty c \int_0^1 \gamma\, d(x/c)$$

$$= \rho_\infty V_\infty c \int_0^\pi 2V_\infty\alpha\left[\frac{1+\cos\theta}{\sin\theta}\right]\frac{\sin\theta}{2} d\theta$$

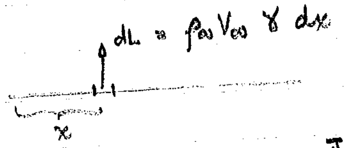
$$= \rho_\infty V_\infty^2 \alpha c\left[\pi + \sin\theta\Big|_0^\pi\right] = \rho_\infty V_\infty^2 \alpha c \pi = q_\infty 2\pi\alpha c$$

let $C_L = \frac{L}{q_\infty \cdot 1 \cdot c}$ (length) $= 2\pi\alpha$ — c_L for flat plate. $\therefore$ lift is determined by m.c.l. V_∞ & α only & ≠ fn (thickness of airfoil)

C_M

$\curvearrowleft +$

$M_{l.e}$



$$M_{l.e} = -\int_0^c x \cdot dL = -c^2\int_0^{\pi}\left[\frac{1-\cos\theta}{2}\right] 2\rho_\infty V_\infty^2 \alpha\left[\frac{1+\cos\theta}{\sin\theta}\right]\frac{\sin\theta}{2}\,d\theta$$

$$= -\frac{\rho_\infty V_\infty^2}{2} c^2 \cdot \alpha \int_0^{\pi}\left[1-\cos^2\theta\right] d\theta = -\frac{\rho_\infty}{2} V_\infty^2 c^2 \alpha\left[\frac{\pi}{2} + \frac{\sin 2\theta}{4}\Big|_0^{\pi}\right]$$

$$= -\frac{\rho_\infty}{4} V_\infty^2 c^2 \alpha \pi = -q_\infty \frac{c^2 \alpha \pi}{2}$$ nose down.

$C_{M_{ac}} = 0 \qquad \therefore \quad C_{M_{l.e}} = C_{M_{ac}} - C_l\,(ac)$

Aerodynamic center - pt on airfoil where moment coefficient will be invariant with change in angle of attack.

$$a.c. = -\frac{C_{M_{l.e}}}{C_L} \qquad C_{M_{l.e.}} = \frac{M_{l.e}}{q \cdot \underbrace{1 \cdot c}_{\text{area}} \cdot \underset{\text{chord}}{c}} = -\frac{\pi\alpha}{2}$$

$$= +\frac{\pi\alpha}{2}\Big/ 2\pi\alpha = .25c$$

for $M > 1.0$ $\quad ac \approx .5c$
$\quad\;\; M < 1.0$ $\quad a.c. = .25c$

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$$2V_\infty \pi\left[\alpha - \left(\frac{dy}{dx}\right)_0\right] = \int_0^{\pi}\frac{\gamma \sin\theta\, d\theta}{\cos\theta - \cos\theta_0} \qquad \text{where } \Gamma = \int_0^c \gamma\, dx$$

if $x/c = \dfrac{1-\cos\theta}{2}$

For symmetric

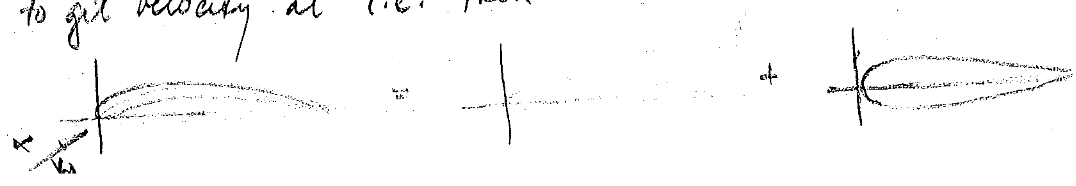
$C_L = 2\pi\alpha$

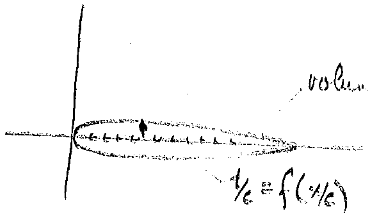
$\alpha_{0_L} = 0$ $\qquad\qquad \gamma = 2V_\infty\left[\dfrac{1+\cos\theta}{\sin\theta}\right]\alpha$

$C_{M_{ac}} = 0 \qquad a.c. = .25c$

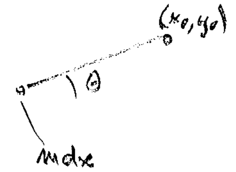
as $\frac{x}{c} = 0 \quad \theta = 0 \quad \gamma_{LE} \to \infty$

this integrable for given finite lift (This theory is not good near l.e.) not good near l.e. since V is ⊥ m.c.
approximation m.c.l. is streamline ($V_\infty \parallel$ m.c.l.)
to get velocity at l.e. then





volume obtained by putting source or sinks on 0 mcl.



$$\psi_i = \frac{m_i \theta}{2\pi}$$

$\theta = \arctan \dfrac{y_0}{x_0 - x}$

$$\psi(x_0, y_0) = \int_0^c \frac{m\theta}{2\pi}\,dx = \int_0^c \frac{m}{2\pi} \arctan \frac{y_0}{x_0 - x}\,dx$$

$\psi_0 = \text{const.}$ is condition

equiv condition to Kutta Condition $\displaystyle\int_0^c m\,dx = 0$

non Symmetric Airfoil

use $\sin n\theta \sin\theta = \frac{1}{2}\left[\cos(n-1)\theta - \cos(n+1)\theta\right]$

$$\gamma = \left[A_0\left[\frac{1+\cos\theta}{\sin\theta}\right] + \sum_{n=1}^{\infty} A_n \sin n\theta\right] 2V_\infty$$

$$\pi\left(\alpha - \left(\frac{dy}{dx}\right)_0\right) = A_0 \int_0^{\pi} \left(\frac{1+\cos\theta}{\cos\theta - \cos\theta_0}\right) d\theta + \sum_{n=1}^{\infty} A_n \int_0^{\pi} \frac{\sin n\theta \sin\theta\, d\theta}{\cos\theta - \cos\theta_0}$$

$$= A_0 \pi + \sum_{n=1}^{\infty} \frac{A_n}{2}\left[\pi \frac{\sin(n-1)\theta_0}{\sin\theta_0} - \pi \frac{\sin(n+1)\theta_0}{\sin\theta_0}\right]$$

$$\pi\left(\alpha - \left(\frac{dy}{dx}\right)_0\right) = A_0 \pi - \sum_{n=1}^{\infty} A_n \pi \left[\cos n\theta_0\right]$$

$$\alpha - A_0 + \sum_{n=1}^{\infty} A_n \cos n\theta_0 = \left(\frac{dy}{dx}\right)_0$$

$$A_n = \frac{2}{\pi}\int_0^{\pi}\left(\frac{dy}{dx}\right)\cos n\theta\, d\theta$$

$$A_0 = \alpha - \frac{1}{\pi}\int_0^{\pi}\left(\frac{dy}{dx}\right) d\theta$$

Evaluate lift coefficient for h.w.

$$C_L = \pi\left(2A_0 + A_1\right)$$

$$C_{m_{le}} = -\frac{\pi}{2}\left(A_0 + A_1 - \frac{A_2}{2}\right)$$

$$A_0 = \alpha - \frac{1}{\pi}\int \frac{dy}{dx}\, d\theta$$

$A_n = f(dy/dx)$ $\qquad C_L = $ fn of α & geometry

$$\alpha_{0L} \neq 0 \implies C_L = 0 = 2\alpha_{0L} - \frac{2}{\pi}\int_0^{\pi} \frac{dy}{dx}\left(1 - \cos\theta\right) d\theta$$

$$\alpha_{0L} = \frac{1}{\pi}\int_0^{\pi}\left(\frac{dy}{dx}\right)\left[1 - \cos\theta\right] d\theta$$

$C_{m\,le} = C_{m\,ap} - C_L\,(ac)$
evaluate at $\alpha = \alpha_{0L}$ $C_L = 0$

$\therefore A_0 = -\dfrac{A_1}{2}$ $\quad C_{m_{ac}} = C_{m_{l.e}} = -\pi/4\left[A_1 - A_2\right]$ $\qquad C_{m_{ac}} = \dfrac{\pi}{4}\left[A_2 - A_1\right]$

$$ac = -\frac{C_{m\,le}}{C_L} + \frac{C_{m_{ac}}}{C_L} = \frac{+\pi/2\left(A_0 + A_1 - \frac{A_2}{2}\right) + \pi/4\left[A_2 - A_1\right]}{\pi\left(2A_0 + A_1\right)} = \frac{1}{4}$$

Conservation of Mass $\qquad (\rho u)_{,x} + (\rho v)_{,y} = 0$ compressible flow.

$$\int_{\psi_1}^{\psi_2} V\,dn = \text{const}$$

Momentum

Euler's Eq.
inviscid flow, steady flow.

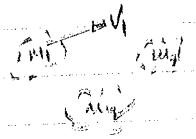
$$\left.\begin{aligned} u\,u_{,x} + v\,u_{,y} &= -\frac{p_{,x}}{\rho} \\ u\,v_{,x} + v\,v_{,y} &= -\frac{p_{,y}}{\rho} + g \end{aligned}\right\}\text{ diff eqs.}$$

Bernoulli's Eq.
$\partial/\partial t = 0$ $\qquad p + \rho\dfrac{V^2}{2} + \rho g y = p_s\,, \quad \dfrac{dp_s}{dn} = -2\omega\rho V$

$$\omega = \frac{V}{R} - \frac{dv}{dn}$$

Assume Continuum, look at Lagrangian pt of view

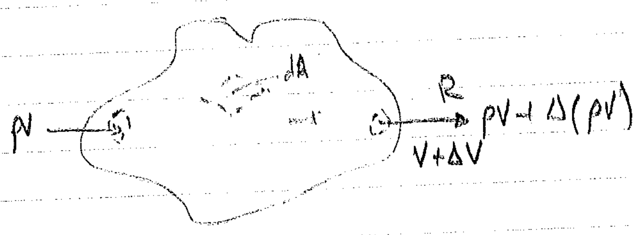
$$F_{ext_1} + f_{int_1} = \frac{d}{dt}\left[m_1 V_1\right]$$



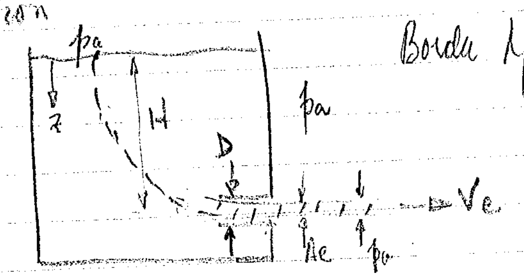
$$F_{ext_2} + f_{int_2} = \frac{d}{dt}\left[m_2 V_2\right] \implies \sum F_{ext_i} = \sum_i \frac{d}{dt}\left[m_i v_i\right]$$

$\vdots$

$$F_{Ext} = \frac{\partial}{\partial t}\iiint_R \rho V\,dR + \iint_A \rho V(V\cdot$$



Application $\qquad$ Borda Mouthpiece

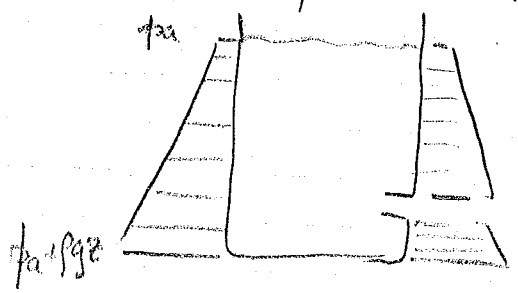


since no vorticity p_s = const on streamline , $g_z = 0$

$$p_a = p_s = p_e + \frac{\rho V_e^2}{2} - \rho g H$$

$$V_e^2 = 2gH$$

Continuity $\rho_e V_e A_e = \rho V_s A_s$ $\qquad V_s \simeq 0 \quad A_s \gg A_e$



since V at top = 0
as h ↑ V must decrease

$$\sum F_x = \int_0^L (p_a + \rho g z)\,dz - \int_0^{H-D/2} (p_a + \rho g z)\,dz - \int_{H-D/2}^{H+D/2} p_a\,dz$$

$$- \int_{H+D/2}^{L} (p_a + \rho g z)\,dz$$

$$= \rho g \left[\frac{L^2}{2} - \frac{(H-D/2)^2}{2} - \frac{L^2}{2} + \frac{(H+D/2)^2}{2} \right] = \rho g H D$$

$$\sum F_x = \rho g H D = \iint_A \rho V (V \cdot \hat{n})\,dA = \rho V_e^2 A_e$$

$$\frac{A_e}{D} = \frac{gH}{V_e^2} = \frac{gH}{2gH} = \frac{1}{2}$$

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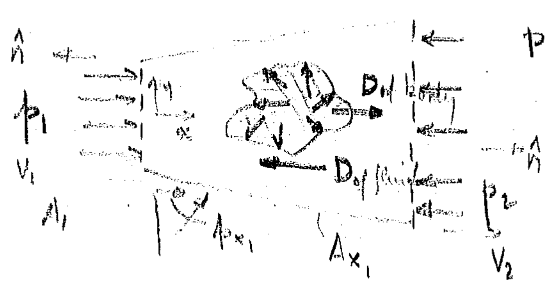
$$\sum F = \frac{\partial}{\partial t} \iiint_R \rho \bar{V}\,dR + \iint_A \rho \bar{V} (\bar{V} \cdot \hat{n})\,dA$$

(forces acting on fluid) $\quad F_x = \frac{\partial}{\partial t} \iiint_R \rho u\,dR + \iint_A \rho u (\bar{V} \cdot \hat{n})\,dA$; $F_y = \frac{\partial}{\partial t} \iiint \rho v\,dR + \iint_A \rho v (\bar{V} \cdot \hat{n})\,dA$

mass flux



pick a particular region of fluid

$$F_x = \int_{A_1} p_1\,dA_1 - \int_{A_2} p_2\,dA_2 + \int p_{x_1}\,dA_{x_1} + \int p_{x_2}\,dA_{x_2}$$

+ Force due to body (forces on fluid)

$$\iint \rho \bar{V} (\bar{V} \cdot \hat{n})\,dA = -\rho V_1^2 A_1 + \rho V_2^2 A_2$$

since body is there $V \cdot \hat{n} = 0$

Mom out − mom in

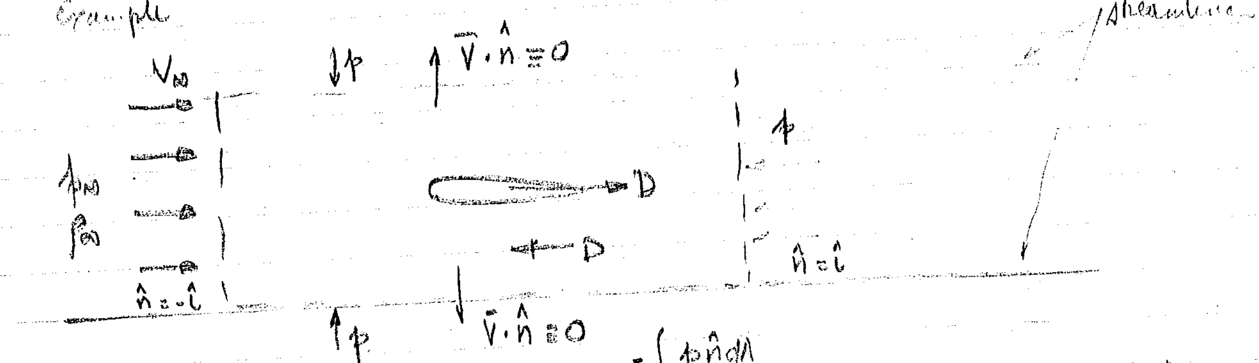
$\bar{V} \cdot \hat{n}$ along wall = 0

When mass injected then we have $\frac{\partial}{\partial t}\int_R \rho u \, dR$

have the same $\rho V_1 \, dR + \rho_e V_e A = \rho V_2 A_2$

and $\iint \rho \bar{V}(\bar{V}\cdot\hat{n})\,dA$ would give $\rho V_e^2 A_e$ for inner body

Example



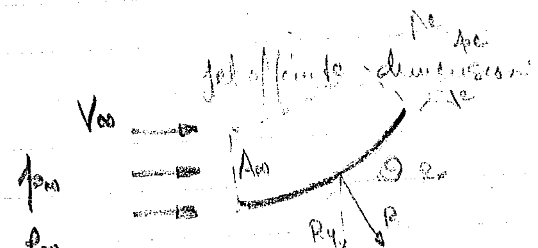
$-\int_A p\hat{n}\,dA$

$$F_x = p_\infty A_1 - p A_1 - D = \rho_\infty V_\infty(-V_\infty)A_1 + \rho_\infty \int V(V)\,dA$$

for $D = \rho_\infty \int dA_1 (V_\infty^2 - V^2) + (p_\infty - p)A_1 = \left[\rho_\infty V_\infty^2 \int \left(1 - \left(\frac{V}{V_\infty}\right)^2\right) + \int (p_\infty - p)\right] dA_1$

if $p = p_\infty$ $\quad D = \rho_\infty V_\infty^2 \int \left(1 - \left(\frac{V}{V_\infty}\right)^2\right) dA_1$

Vane



R_x acts on blades; $-R_x$ acts on fluid

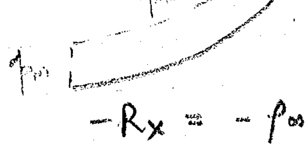
for upper and lower $\qquad (V_e \cos\theta) V_e$

$$F_x = p_\infty A_\infty - p_e A_e \cos\theta - R_x = -\rho_\infty V_\infty^2 A_\infty + \rho_\infty A_e V_e^2 \cos\theta$$

on walls $V\cdot\hat{n} = 0$

$\rho_\infty V_\infty A_\infty = \rho_\infty V_e A_e$ $\quad$ p_e is unknown; if $p_e = p_\infty$

for water wheel



$$F_x = -R_x = -\rho_\infty V_\infty^2 A_\infty + \rho_\infty (V_e \cos\theta) V_e A_e$$

$$-R_x = -\rho_\infty V_\infty^2 A_\infty + V_e\cos\theta\,(\rho_\infty V_\infty A_e)$$

$$R_x = \rho_\infty V_\infty A_\infty \left[V_\infty - V_e \cos\theta \right]$$

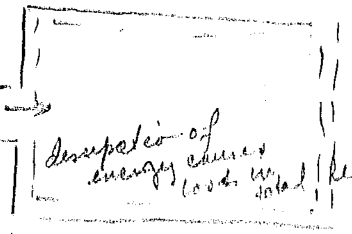
$F_y = R_y = 0$ from first $+ \rho_\infty (V_e \sin\theta) V_e A_e = \rho_\infty V_\infty A_\infty (V_e \sin\theta)$

$$R^2 = R_x^2 + R_y^2 = \rho_\infty^2 V_\infty^2 A_\infty^2 \left[V_\infty^2 - 2 V_e V_\infty \cos\theta + V_e^2 \right]$$

$$R^2_{max} \quad (\theta \rightarrow \pi) =$$

$V_1 P_1 \rho$

A_1



V_2, p_2, ρ, A_2

$\frac{\partial}{\partial t}\iiint \rho u \, dR = 0$ since no mass sources or sinks

$$F_x = p_1 A_2 - p_2 A_2 = -\rho V_1^2 A_1 + \rho V_2^2 A_2$$

$$p_2 = p_1 + \rho V_1^2 \frac{A_1}{A_2} - \rho \frac{V_2^2 A_2}{A_2} = p_1 + \rho \frac{V_1 A_1}{A_2}\left[V_1 - V_2 \right] = p_1 + \rho V_2 (V_1 - V_2$$

continuity $\rho V_1 A_1 = \rho V_2 A_2$

Using Bernoulli's Eq $\quad p_1 + \rho \frac{V_1^2}{2} = p_s = p_2 + \rho \frac{V_2^2}{2}$

$$p_2 = p_1 + \rho\left(\frac{V_1^2}{2} - \frac{V_2^2}{2}\right) = p_1 + \frac{\rho}{2}(V_1 + V_2)(V_1 - V_2)$$

Momentum is more general since it takes into account viscous forces.

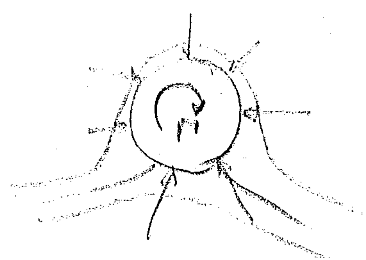
$$\Delta p_s = p_{s_1} - p_{s_2} = p_1 + \rho \frac{V_1^2}{2} - \left(p_2 + \rho \frac{V_2^2}{2}\right)$$

$$\rho V_2 (V_2 - V_1) + \frac{\rho}{2}(V_1^2 - V_2^2)$$

$$\frac{\Delta p_s}{\rho} = \frac{(V_2 - V_1)^2}{2}$$

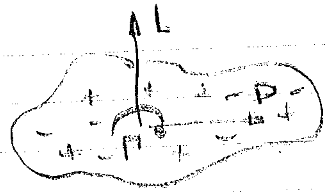
Examples:

Cylinder in Flow with Circulation



$$\left.\begin{array}{l} L = \rho_\infty V_\infty \Gamma \\ D = 0 \end{array}\right\}$$

airfoil



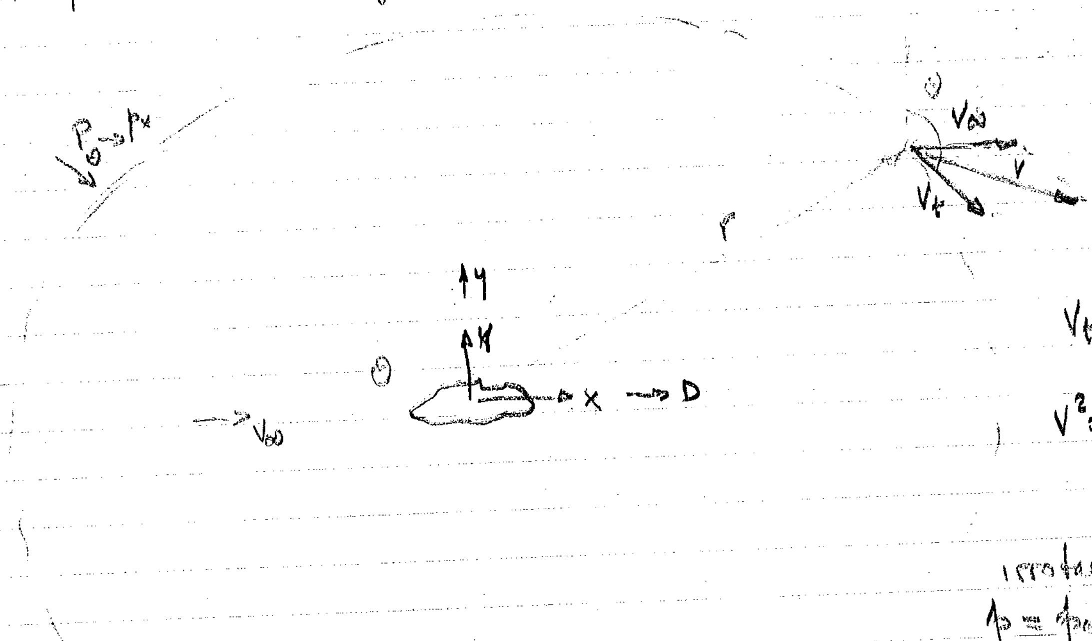
+ − source or sink

$\int_0^c m\,dx = 0$ for closed body

m, Γ $\quad V \sim \frac{1}{r}$

source & sink $\quad \pm m \quad V \sim \frac{1}{r^2}$

if $R \gg 1$ effect of so & si die out much faster than V, Γ combination



$V_t = \frac{\Gamma}{2\pi r}$

$V^2 = V_\infty^2 + V_t^2 + 2V_t V_\infty \times \sin\theta$

irrotational flow

$p = p_\infty + \rho_\infty \frac{V_\infty^2}{2} - \rho_\infty \frac{V^2}{2}$

$p = p_\infty + q_\infty \left[1 - \frac{V^2}{V_\infty^2}\right]$

$= p_\infty + q_\infty \left[1 - \left(1 + \frac{V_t^2}{V_\infty^2} + \frac{2V_t}{V_\infty}\sin\theta\right)\right]$

no contrib for $\int_0^{2\pi} p_\infty \cos\theta = 0$

$p = p_\infty - q_\infty \left[\frac{V_t^2}{V_\infty^2} + \frac{2V_t}{V_\infty}\sin\theta\right]$

V_t never contributes

Drag $\quad F_x = -D + \int_0^{2\pi} p\cos\theta\, r\,d\theta = 0$ steady flow $+ \int_0^{2\pi} \rho_\infty \left(V_\infty + \frac{\Gamma}{2\pi r}\sin\theta\right)\left(-V_\infty\cos\theta\right) r\,d\theta$

$D = -r q_\infty \int_0^{2\pi} \left(\frac{2V_t}{V_\infty}\sin\theta\cos\theta\,d\theta\right) + \frac{\rho_\infty V_\infty \Gamma}{2\pi}\int_0^{2\pi}\sin\theta\cos\theta\,d\theta$

$= 0$

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Continuity

$\frac{D\rho}{Dt} + \rho\nabla\cdot\bar{q} = 0$

$$\bar{q} = u\bar{i} + v\bar{j} + w\bar{k}$$

$$\bar{F} = F_x\bar{i} + F_y\bar{j} + F_z\bar{k}$$

Momentum (Euler's)

$$\rho \frac{D\bar{q}}{Dt} = -\nabla p + \bar{F}$$

$$\bar{q}\frac{D\rho}{Dt} + \rho\bar{q}\,\nabla\cdot\bar{q} = 0 \qquad + \text{mom.}$$

$$\frac{D}{Dt}(\rho\bar{q}) + \rho\bar{q}\,\nabla\cdot\bar{q} = -\nabla p + \bar{F}$$

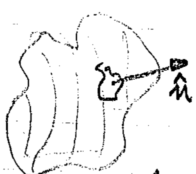
For $\bar{i}$ component

$$\frac{D}{Dt}(\rho u) + \rho u\,\nabla\cdot\bar{q} = -p_{,x} + F_x$$

$$\frac{\partial}{\partial t}(\rho u) + \bar{q}\cdot\nabla\rho u + \rho u\,\nabla\cdot\bar{q} = -p_{,x} + F_x$$

$$\frac{\partial}{\partial t}(\rho u) + \nabla\cdot(\rho u\bar{q}) = -p_{,x} + F_x$$ ← units are force/unit volume.

$$\frac{\partial}{\partial t}\iiint_R \rho u\, dR + \iiint_R \nabla\cdot(\rho u\bar{q})\, dR = -\iiint_R p_{,x}\, dR + \iiint_R F_x\, dR$$



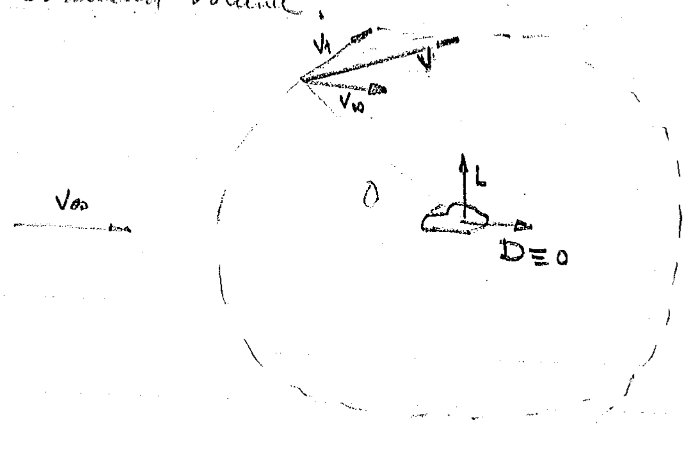
$$\frac{\partial}{\partial t}\iiint_R \rho u\, dR + \iint_A \rho u(\bar{q}\cdot\hat{n})\, dA = -\iint_A p\,\bar{i}\cdot\hat{n}\, dA + F_x$$

For all 3 directions

$$\boxed{\frac{\partial}{\partial t}\iiint_R \rho\bar{q}\, dR + \iint_A \rho\bar{q}(\bar{q}\cdot\hat{n})\, dA = -\iint p\,\hat{n}\, dA + \bar{F}}$$

More general than Bernoulli's or Euler's since it allows for viscous effects in the bounded volume.

$$V^2 = V_\infty^2 + V_t^2 + 2V_tV_\infty \sin\theta$$



body acting on fluid

$$\sum F_y = -L - \int_0^{2\pi} p \sin\theta \, r\,d\theta = -\int_0^{2\pi} \rho_\infty V_t \cos\theta \; V_\infty \cos\theta \,(r\,d\theta)$$

$$p = p_\infty + \frac{\rho_\infty V_\infty^2}{2}\left[1 - \left(\frac{V}{V_\infty}\right)^2\right] = p_\infty - \frac{\rho_\infty V_\infty^2}{2}\left[\frac{V_t^2}{V_\infty^2} + \frac{2V_t}{V_\infty}\sin\theta\right]$$

$$p = p_\infty + \frac{\rho_\infty V_t^2}{2} - \frac{\rho_\infty V_\infty \Gamma}{2\pi r}\sin\theta$$

$$-L + \int_0^{2\pi} \frac{\rho_\infty V_\infty \Gamma}{2\pi} \sin^2\theta\, d\theta = -\int_0^{2\pi} \rho_\infty \frac{\Gamma V_\infty}{2\pi} \cos^2\theta\, d\theta$$

$$L = \rho_\infty V_\infty \Gamma$$

Review

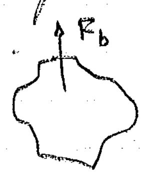
I. Hydrostatics

Static Equilibrium $\quad V \equiv 0 \qquad p_{,z} = -\rho g$

liquid ($\rho \neq f(p)$): stable $\rho_{,z} < 0$; unstable $\rho_{,z} > 0$; neutral $\rho_{,z} = 0$

gas ($p = C\rho^\gamma$) ← adiabatic process: stable $T_{,z} >$; unstable $T_{,z} < -g/R\left(\frac{\gamma-1}{\gamma}\right)$; neutral $T_{,z} = -\frac{g}{R}\left(\frac{\gamma-1}{\gamma}\right)$

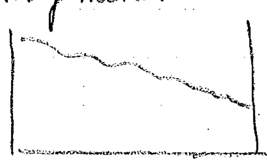
Archimedes Principle



$F_b = \rho_{fl}(Vol)$

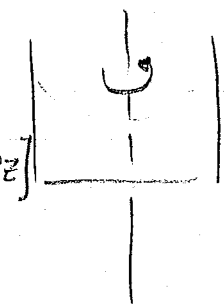
Dynamic Equil $\quad V \neq 0 \qquad$ acceleration $\neq f(t)$

rigid body motion



$\dot{V} = \text{const}$

$$p = p_a + \rho\left[x\frac{dV}{dt} + gz\right]$$



acc $= f(r)$

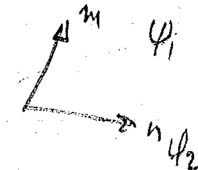
$$p = p_a + \rho\left[\frac{\Omega^2 r^2}{2} - gz\right]$$

Hydrodynamics

(A) Cons. of mass (continuity)

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \bar{V}) = 0$$

$$\int_{\psi_1}^{\psi_2} \rho \bar{V}\cdot\hat{n}\, dA = \text{const} \quad \text{integral independent of path}$$



$$\frac{\partial \psi}{\partial m} = V_n \qquad \frac{\partial \psi}{\partial n} = -V_m$$

Conservation of Momentum

diff eq — Euler's $\rho \frac{D\bar{q}}{Dt} = -\nabla p + \bar{F}$ ← can be valid for viscous & compressible fluid.

if $\bar{F} = \nabla T$ (potential) then only valid for inviscid fluid.

Momentum integral.

$$\frac{\partial}{\partial t}\iint \rho \bar{q}\, dR + \ldots$$

Potential flow $\omega \equiv 0$ $\bar{q} = \nabla \varphi$

continuity = (ρ = const) $\nabla \cdot \bar{V} = 0$ = $\nabla^2 \varphi = 0$

rotational flow $\omega \neq 0$

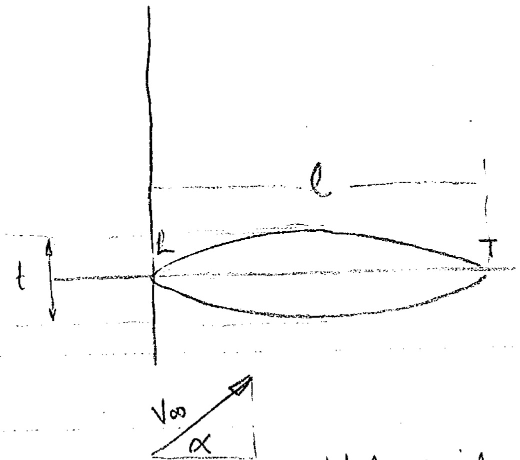
$\nabla^2 \phi = -2\omega_0$

Bernoulli's Eq

$$p + \frac{\rho V^2}{2} = p_s \quad \text{along streamline}$$

$$p_{s,n} = -2\omega V \rho \qquad \omega = \frac{V}{R} - \frac{dV}{dn}$$

Thin Airfoil Theory.



$y = \epsilon\, \eta(x) \qquad |\epsilon| << 1$

$= \epsilon\, \eta_+(x) \qquad \epsilon\eta_+$ can be $\epsilon \frac{x}{\ell}\left(1 - \frac{x}{\ell}\right)$

$= \epsilon\, \eta_-(x) \qquad \epsilon\eta_-$ can be $-\epsilon\left(\frac{x}{\ell}\right)^2\left[1 - \left(\frac{x}{\ell}\right)^2\right]$

total velocity potent Φ

total perturbation to free stream φ

$$\Phi = \varphi + x\, U_\infty \cos\alpha + y\, U_\infty \sin\alpha$$

$$\nabla^2 \varphi = 0$$

$$\frac{\partial \varphi}{\partial y} = \begin{cases} U_\infty \epsilon \eta_+'(x) - U_\infty \alpha & y = 0+ \\ U_\infty \epsilon \eta_-'(x) - U_\infty \alpha & y = 0- \end{cases}$$

$\nabla \varphi \to 0$ for $(x^2 + y^2)^{1/2} \to \infty$

for $\alpha = 0$ $\quad \left.\begin{array}{l} U_\infty \epsilon \eta_+'(x) \\ U_\infty \epsilon \eta_-'(x) \end{array}\right\} = \frac{\partial \varphi}{\partial y} \quad \begin{array}{l} y = 0+ \\ y = 0- \end{array}$

Kutta Cond

flat plate

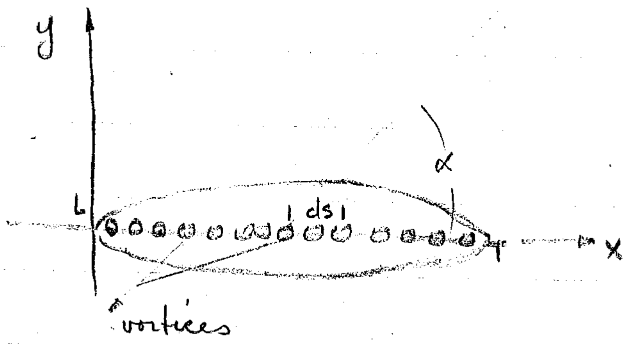
$$\frac{\partial \varphi}{\partial y} = \begin{array}{l} -U_\infty \alpha \\ -U_\infty \alpha \end{array}$$

Replace $\frac{\partial \varphi}{\partial y}$ by $\frac{\partial \Phi}{\partial y} = U_\infty \underbrace{y_c'(x)}_{\text{camber line}} \quad y = \pm 0$

gives no lift $\longrightarrow \quad \frac{\partial \Phi}{\partial y} = \pm U_\infty y_t'(x) \quad y = \pm 0$

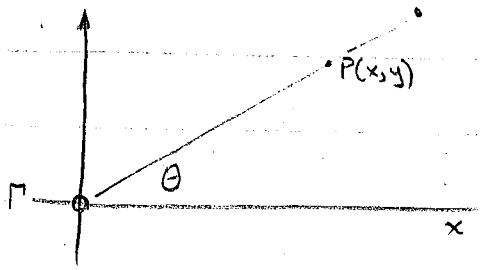
flat plate is same as $\frac{\partial \Phi}{\partial y} = U_\infty y_c'(x)$ where $y_c'(x) = -\alpha$

solve now $\frac{\partial \Phi}{\partial y} = U_\infty y_c'(x) \quad y = \pm 0$



let $\gamma(s)$ = vortex strength/unit length.

$\gamma(s)\,ds$



$$\phi = \frac{\Gamma}{2\pi}\,\theta$$

$$d\phi = \frac{\gamma(s)\,ds}{2\pi}\,\alpha(x,y,s)$$

$$\varphi = \int_0^l \frac{\gamma(s)\,ds}{2\pi}\,\alpha(x,y,s) \qquad \tan\alpha = \frac{y}{x-s}$$

$$\varphi = \frac{-1}{2\pi}\int_0^l \gamma(s)\,ds\,\tan^{-1}\left(\frac{y}{x-s}\right)ds \qquad \alpha = \tan^{-1}\frac{y}{x-s}$$

check this satisfies $\nabla^2\varphi = 0$

$$u(x,y) = \frac{\partial\varphi}{\partial x} = -\frac{1}{2\pi}\int_0^l \gamma(s)\left\{\frac{1}{1+\frac{y^2}{(x-s)^2}}\right\}\left\{-\frac{y}{(x-s)^2}\right\}ds$$

$$u(x,y) = \frac{1}{2\pi}\int_0^l \gamma(s)\,\frac{y}{(x-s)^2+y^2}\,ds$$

$$v(x,y) = -\frac{1}{2\pi}\int_0^l \gamma(s)\,\frac{x-s}{(x-s)^2+y^2}\,ds$$

satisfies $\nabla\varphi \to 0$ for $(x^2+y^2)^{1/2} \to \infty$; take $v(x,y)$ let $y=0$ then

$$-\frac{1}{2\pi}\int_0^l \frac{\gamma(s)}{(x-s)}\,ds = U_\infty\,y_c'(x) \qquad 0 \le x \le l$$

let $$-I = \frac{1}{2\pi}\int_0^l \gamma(s)\,\frac{x-s}{(x-s)^2+y^2}\,ds = \frac{1}{2\pi}\int_0^l \frac{[\gamma(s)-\gamma(x)](x-s)}{(x-s)^2+y^2}\,ds \; +$$

$$\frac{\gamma(x)}{2\pi}\int_0^l \frac{x-s}{(x-s)^2+y^2}\,ds = \gamma'(x) + \frac{\gamma(x)}{4\pi}\ln\left[(x-s)^2+y^2\right]_0^l$$

$$-I(x,0) = \frac{1}{2\pi}\int_0^l \frac{\gamma(s)-\gamma(x)}{x-s}\,ds + \frac{1}{4\pi}\left[\gamma(l)\ln(x-l)^2 - \gamma(0)\ln(x)^2\right]$$

to keep $-I(x,0)$ finite $\gamma(l)$, $\gamma(0)=0$ Kutta Condition $\therefore$ accept $V_\infty=\infty$ at leading edge.

Principal Value of an Integral.

$$-\frac{P}{2\pi}\int_0^l \frac{\gamma(s)}{(x-s)}\,ds = U_\infty\, y_c'(x) \qquad \gamma(l)=0$$

Solution of Integral Eq.

trailing, leading

$$x = \frac{l}{2}(1+\cos\theta) \qquad 0\le\theta\le\pi$$

$$s = \frac{l}{2}(1+\cos\varphi) \qquad 0\le\varphi\le\pi$$

$$ds = -\frac{l}{2}\sin\varphi\, d\varphi$$

$$\frac{P}{2\pi}\int_0^\pi \frac{\gamma(\varphi)\sin\phi\, d\varphi}{\cos\varphi-\cos\theta} = V_\infty\, y_c'(\theta) \qquad \gamma(\pi)=0$$

$$P\int_0^\pi \frac{\cos n\varphi}{\cos\varphi-\cos\theta}\,d\varphi = \pi\frac{\sin n\theta}{\sin\theta}$$

when $n=0$ $\quad P\int_0^\pi \frac{d\varphi}{\cos\varphi-\cos\theta}=0$ $\quad$ then $\gamma(\varphi) = \frac{k}{\sin\phi} + \gamma_1(\varphi)$

Solution of Potential Problems in 2-d is not unique unless Γ specified

$y_c'(\theta) = B_0 + \sum_{n=1}^{\infty} B_n\cos n\theta$ $\quad$ substitute into $V_\infty\, y_c'(\theta)$

$$\frac{1}{2\pi}P\int_0^\pi \frac{\gamma_1(\varphi)\sin\phi\, d\varphi}{\cos\varphi-\cos\theta} = U_\infty\left[B_0+\sum_{n=1}^{\infty}B_n\cos n\theta\right]$$

let $\gamma_1(\varphi) = C_0 \dfrac{\cos\varphi}{\sin\varphi} + \gamma_2(\varphi)$

Subst.

$$\frac{1}{2\pi} C_0 \, P\int_0^{\pi} \frac{\cos\phi}{\cos\varphi - \cos\theta}\, d\varphi + \frac{1}{2\pi} P\int_0^{\pi} \frac{\gamma_2(\varphi)\sin\varphi}{\cos\varphi - \cos\theta}\, d\varphi = U_\infty B_0 + U_\infty \sum_{n=1}^{\infty} B_n \cos n\theta$$

$$\frac{C_0}{2} = U_\infty B_0 \qquad \text{let } C_0 = 2U_\infty B_0$$

now solve

$$\frac{1}{2\pi} P\int_0^{\pi} \frac{\gamma_2(\varphi)\sin\varphi}{\cos\varphi - \cos\theta}\, d\varphi = U_\infty \sum_{n=1}^{\infty} B_n \cos n\theta$$

let $\gamma_2(\varphi) = \sum_{n=1}^{\infty} C_n \sin n\varphi$

$$\frac{1}{2\pi} \sum_{n=1}^{\infty} C_n \int_0^{\pi} \frac{\sin n\varphi \sin\varphi}{\cos\varphi - \cos\theta}\, d\varphi = U_\infty \sum_{n=1}^{\infty} B_n \cos n\theta$$

$$P\int_0^{\pi} \frac{\sin n\varphi \sin\varphi}{\cos\varphi - \cos\theta}\, d\varphi = -\pi \cos n\theta$$

$$\frac{1}{2\pi} \sum_{n=1}^{\infty} C_n \left(-\pi \cos n\theta\right) = U_\infty \sum_{n=1}^{\infty} B_n \cos n\theta$$

$$C_n = -2U_\infty B_n$$

$$\gamma_c(\varphi) = \frac{k}{\sin\phi} + 2U_\infty B_0 \frac{\cos\varphi}{\sin\varphi} - 2U_\infty \sum_{n=1}^{\infty} B_n \sin n\varphi$$

$$y_c'(x) = B_0 + \sum_{n=1}^{\infty} B_n \cos n\theta$$

$$B_0 = \frac{1}{\pi}\int_0^{\pi} y_c'(\varphi)\, d\phi \; ; \quad B_n = \frac{2}{\pi}\int_0^{\pi} y_c'(\varphi) \cos n\varphi \, d\varphi \quad \text{for } n \geq 1$$

at $\varphi = 0$ $\quad \frac{1}{\sin\varphi} \to \infty$ $\quad \therefore$ choose k .э. numerator goes to zero when $\varphi = 0$

Angle of Attack Prob. $\quad \frac{dy_c}{dx} = -\alpha$

$B_0 = -\alpha \quad B_n = 0 \quad n > 0$

$$\gamma_\alpha(\varphi) = \frac{1}{\sin\varphi}\left[k' - 2U_\infty \alpha \cos\varphi\right]$$

$$\gamma_{total} = \gamma_\alpha + \gamma_c = \frac{1}{\sin\varphi}\left[k + k' + \cos\varphi\left(2U_\infty B_0 - 2U_\infty \alpha\right)\right] - 2U_\infty \sum_{n=1}^{\infty} B_n \sin n\varphi$$

$$k + k' + \left(2U_\infty B_0 - 2U_\infty \alpha\right) = 0$$

$$\gamma_{tot}(x = l) = \gamma(\varphi = 0) = 0 \qquad k + k' = 2U_\infty(\alpha - B_0)$$

$$\gamma_{TOT} = \left[2U_\infty(\alpha - B_0)\left(\frac{1-\cos\varphi}{\sin\varphi}\right) - 2U_\infty \sum_{n=1}^{\infty} B_n \sin n\varphi\right]$$

$$\Gamma = \int_0^l \gamma_{TOT}(s)\, ds = -\frac{l}{2}\int_\pi^0 \gamma_T(\varphi)\sin\varphi\, d\varphi$$

$$= lU_\infty(\alpha - B_0)\int_0^\pi (1 - \cos\varphi)\, d\varphi - lU_\infty \sum_{n=1}^{\infty} B_n \int_0^\pi \sin\varphi \sin n\varphi\, d\varphi$$

$$= lU_\infty(\alpha - B_0)\left[\phi - \sin\phi\right]_0^\pi - lU_\infty B_1 \cdot \frac{\pi}{2}$$

$$= \pi l U_\infty(\alpha - B_0) - \frac{\pi l U_\infty B_1}{2}$$

$$= \pi l U_\infty\left[\alpha - B_0 - \frac{B_1}{2}\right]$$

$$L = \rho U_\infty \Gamma = \rho U_\infty^2 \pi l\left[\alpha - B_0 - \frac{B_1}{2}\right] \; ; \quad C_L = 2\pi\left[\alpha - B_0 - \frac{B_1}{2}\right]$$

$$B_0 + \frac{B_1}{2} = \frac{1}{\pi}\int_0^\pi y_c'(\varphi)\left[1 + \cos\varphi\right] d\varphi$$

$$x = \frac{\ell}{2}(1+\cos\varphi)$$

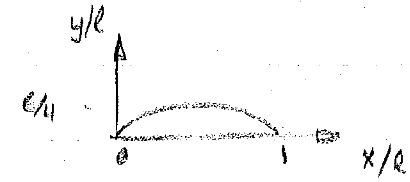
$$M_J = \int_0^\ell \left[p(x,0+) - p(x,0-) \right] x\,dx = \rho U_\infty \int_0^\ell x \left[\varphi_x(x,0-) - \varphi_x(x,0+) \right] dx$$

$$\varphi_x(x,0+) - \varphi_x(x,0-) = u(x,0+) - u(x,0-) = \gamma(x)$$

$$M_J = -\rho U_\infty \int_0^\ell x\, \gamma(x)\, dx = -\frac{\rho U_\infty \ell^2}{4} \int_0^\pi (1+\cos\varphi)\sin\varphi\, \gamma_{tot}(\varphi)\, d\varphi$$

$$C_M = \frac{M_J}{\frac{1}{2}\rho U_\infty^2 \ell^2} = -\frac{1}{4}\left[2\pi\alpha - (2B_0 + B_1)\pi \right] + \frac{\pi}{4}(B_1 + B_2)$$

Example



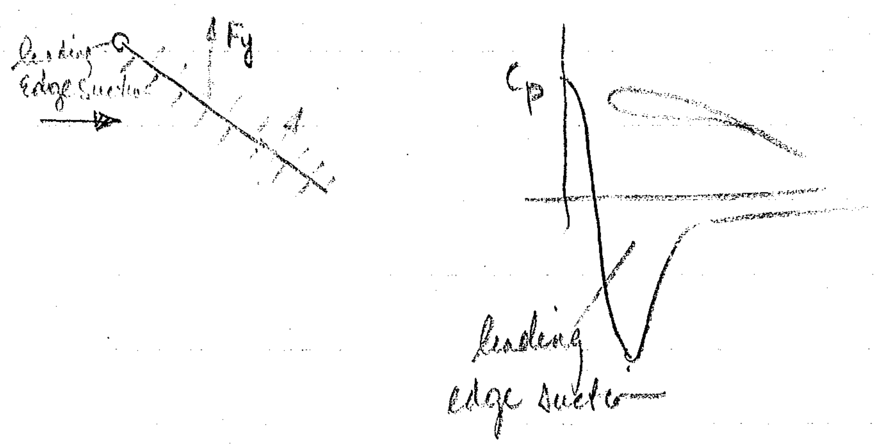
$$y_c(x) = \epsilon\,\ell\left(\frac{x}{\ell}\right)\left(1-\frac{x}{\ell}\right)$$

$$y_c'(x) = \epsilon\frac{\ell}{\ell}\left(1-\frac{x}{\ell}\right) + \epsilon\ell\left(\frac{x}{\ell}\right)\left(-\frac{1}{\ell}\right)$$

$$\epsilon(1 - x/\ell) - \epsilon\frac{x}{\ell} = \epsilon\left(1-\frac{2x}{\ell}\right) = \epsilon\left(1 - \frac{2}{\ell}\cdot\frac{\ell}{2}(1+\cos\varphi)\right) = -\epsilon\cos\varphi$$

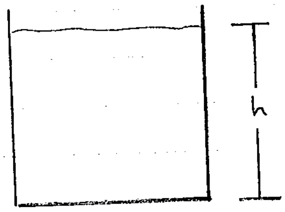
$$B_0 = \frac{1}{\pi}\int_0^\pi -\epsilon\cos\varphi\, d\varphi = 0 \qquad B_n = \frac{2}{\pi}\int_0^\pi -\epsilon\cos\varphi\cos n\varphi\, d\varphi = -\epsilon$$

$$C_\ell = 2\pi\left(\alpha + \frac{\epsilon}{2}\right) \qquad C_M = -\frac{\pi}{2}(\alpha + \epsilon)$$



2/8/71 AE582

Cesar Levy.

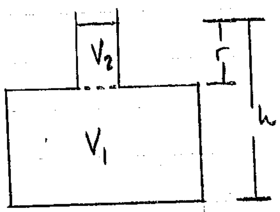


@ $h=0$ $p=p_a$

@ $h=h$ $p=p_h$

$$p_h = p_a + \rho g h$$

$$F_h = p_h A = \frac{\pi}{4} D^2 p_h = \frac{\pi}{4} D^2 (p_a + \rho g h)$$



@ $h=0$ $p=p_a$

@ $h=r$ $p=p_r$

@ $h=h$ $p=p_h$

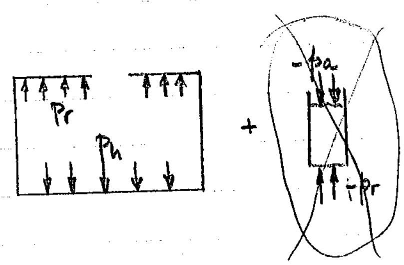
$$p_r = p_a + \rho g r$$
$$p_h = p_a + \rho g h$$

$$F_h = p_h A = \frac{\pi D^2}{4} p_h = \frac{\pi D^2}{4}(p_a + \rho g h)$$

$$F_r = p_r A_r = (p_a + \rho g r)\left[\frac{\pi D^2}{4} - \frac{\pi d^2}{4}\right];$$

$$F_p = (A - A_r) p_r = \frac{\pi d^2}{4}(p_a + \rho g r) \quad ?$$

should be A_r



$$= (p_h A - p_r A) + (p_r[A - A_r] - p_a[A - A_r]) = \rho g(h-r)A + \rho g r(A - A_r)$$

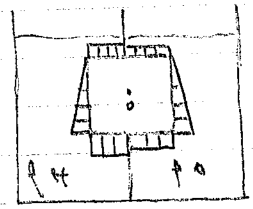
$$= \rho g(h-r)\frac{\pi D^2}{4} + \rho g r \frac{\pi d^2}{4}$$

$$= \rho g V_1 + \rho g V_2 = \text{wt of liquid}$$

HW#2 & #3 Cesar Levy

AE332

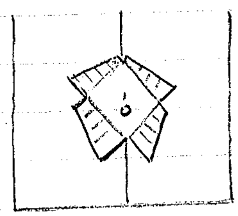
10)

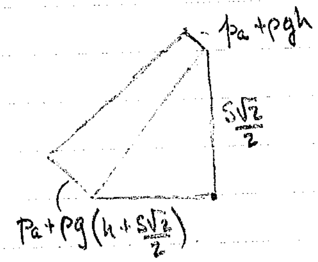


$$M\circlearrowright = \rho_H g\, s\left(\frac{sL}{2}\right)\frac{s}{6} + \rho_o g\, s\left(\frac{sL}{2}\right)\frac{s}{4}$$

$$M\circlearrowleft = \rho_H g\, s\left(\frac{sL}{2}\right)\frac{s}{4} + \rho_o g\, s\left(\frac{sL}{2}\right)\frac{s}{6}$$

$$\Sigma M_o\downarrow = g\frac{s^3L}{24}\left[\rho_H - \rho_o\right]$$ ✓





$$M\circlearrowright = \rho_H g\,\frac{s\sqrt{2}}{2}\left(\frac{sL}{2}\right)\frac{s}{6} + \rho_H g\,\frac{s\sqrt{2}}{2}\left(\frac{sL}{2}\right)\frac{s}{6}$$

$$M\circlearrowleft = \rho_o g\,\frac{s\sqrt{2}}{2}\left(\frac{sL}{2}\right)\frac{s}{6} + \rho_o g\,\frac{s\sqrt{2}}{2}\left(\frac{sL}{2}\right)\frac{s}{6}$$

$$\Sigma M_o\big) = g\frac{s^3L\sqrt{2}}{12}\left[\rho_o - \rho_H\right]$$ ✓ ∴ this produces a counterclockwise moment therefore the equil. position lies between 0° & 45° rotation. The box will rotate until equil is reached & then be opposed by a negative moment if it tries to go past its neutral position.

8) $T/T_A = \left[1 - \frac{n-1}{n}\left(\frac{g z}{R T_A}\right)\right]$ for $g = 32.2$ ft/sec² $z = 25{,}000$ ft

$R = 1716\ \frac{ft^2}{sec^2 \cdot °R}$ $T_A = 520°R$

show numerical work ✓

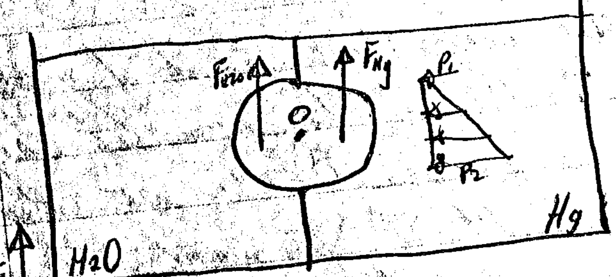
we get $n = 1.3834$

for $T = C$ as h increases $\frac{dT}{dz} = 0$ on the z-T graph $\frac{dT}{dz}$ lies to the right of the neutral stable line $n = \gamma$; therefore it is stable

for $p = C$ as h increases $\frac{dT}{dz} = -\infty$ ✗. If we plot the graphs for n increasing from 1 (T=C) to ∞ the lines will change from vertical to negative sloped to horizontal (p=C). Since the horiz. line lies to the left of neutrally stable curve then p=C is an unstable atmosphere; in fact for $n > \gamma$ we have unstable atmospheres.

Frank Tarulli AE 332C 2/13/71

1. The level of the lake decreases. When the stones are in the boat, the stones and the boat are buoyed up with a force equal to the weight of the displaced water. After the stones are thrown out of the boat, the boat rises and the water level decreases.

2. 

$p_2 = p_1 + \rho g h$

$p_2 = p_1 + 2r\rho g$

$p - p_1 = \rho g(h - y)$

$p - p_1 = \rho g(2r - y)$

Summing the moments due to buoyant forces about O

$\circlearrowleft \Sigma M_1 = \frac{4r}{3\pi}[F_{Hg} - F_{H2O}] = \frac{4r}{3\pi}[\rho_{Hg} g \frac{V}{2} - \rho_{H2O} g \frac{V}{2}]$

$= \frac{2r}{3\pi} V g [\rho_{Hg} - \rho_{H2O}]$

F_H = horizontal force (due to pressure forces)

$dF_H = (p - p_1) dA = (p - p_1) L\,dy$

$F_H = \int dF_H = \int_0^{2r} L(p - p_1)\,dy = L\rho g[hy - \frac{y^2}{2}]_0^{2r}$

$F_H = \int_0^{2r} L\rho g(h - y)\,dy$

$F_H = L\rho g[2ry - \frac{y^2}{2}]_0^{2r}$

$F_H = L\rho g[\frac{4r^2}{2} - \frac{4r^2}{2}]$

$F_H = 2L\rho g r$

Assume F_H is cancelled by membrane restraints

$M = F_H \cdot d_0$ = moment created by F_H about origin O

d bottom of cylinder $= \frac{1}{3} h = \frac{2r}{3}$

Fra...
cont. at r p...

$d_o = d$ from origin $= r - \frac{2r}{3} = \frac{r}{3}$

$M = F_H \cdot d_o = \frac{2}{3} L \rho g r^3$

Summing the moments due to pressure forces about origin O

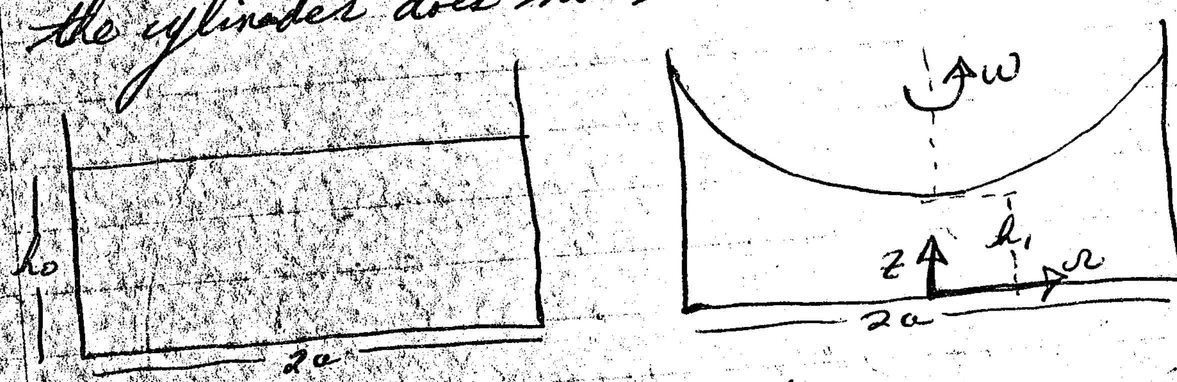
$\Sigma M_2 = \frac{2}{3} L g r^3 (\rho_{Hg} - \rho_{H_2O})$

$\Sigma M_2 = \frac{2r(\pi r^2 L) g}{3\pi} (\rho_{Hg} - \rho_{H_2O})$

$\Sigma M_2 = \frac{2r}{3\pi} V g (\rho_{Hg} - \rho_{H_2O})$

Notice that the moment created by the buoyant force ΣM_1 = the moment created by the pressure forces ΣM_2 so that there is no resultant moment. Therefore the cylinder does not rotate.

3.



Set up polar coordinate system

$\frac{v}{r} = \omega, \quad a_r = \frac{V^2}{r} = \frac{\omega^2 r^2}{r} = \omega^2 r$

$r = a$ = radius of cylinder

$\frac{\partial p}{\partial z} = -\rho g \qquad \frac{\partial p}{\partial r} = \rho \omega^2 r$

$dp = \frac{\partial p}{\partial z} dz + \frac{\partial p}{\partial r} dr$

$dp = -\rho g\, dz + \rho \omega^2 r\, dr$

$p = -\rho g z + \frac{1}{2} \rho \omega^2 r + C$

EXAMINATION BOOK

Name Cesar Levy

Subject AE 332

Instructor Dr. Cresci

Exam Seat No. ________ Section ________

Date 3/8/71 Grade ________

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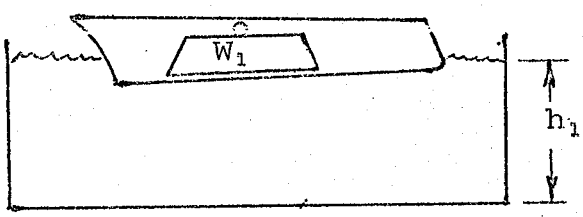
POLYTECHNIC INSTITUTE OF BROOKLYN

Department of Aerospace Engineering
and Applied Mechanics

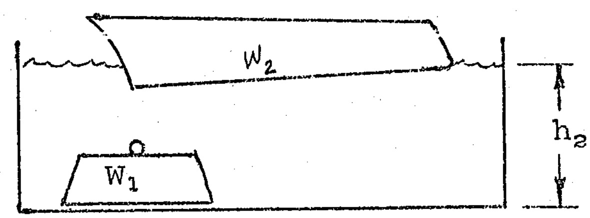
Course AE 332
One Hour Exam - Hydrostatics
March 8, 1971

1. Suppose you are driving a car at constant velocity and <u>inside</u> the car is a helium-filled balloon (resting on the roof). Suddenly the car is decelerated to stop for a red light! Will the balloon move (with respect to the automobile) or not? If it moves, which direction will it go?

2. A naval architect is studying the stability of a ship model in a water tank. He does this by removing lead ballast weights from the ship and dropping them into the water. After the weights are all removed from the model, is the water level in the tank the same as it was when the weights were in the ship? If not, is it higher or lower?



Original Condition



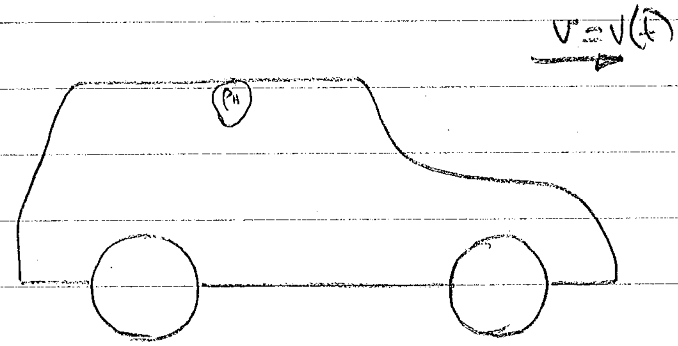
Final Condition

$\rho_{lead} = 10\ \rho_{water}$

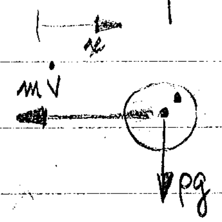
W_1 = weight of lead ballast

W_2 = weight of empty ship

1.



if it is decelerating the following forces act on the balloon.

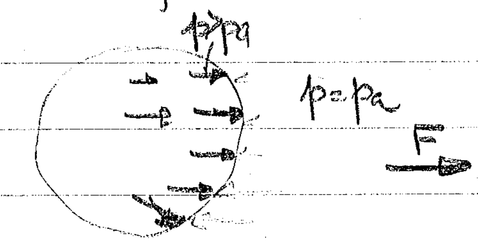


from this we get $p = p_a - \rho\left(\frac{dv}{dt}x + gz\right)$

if $\frac{dv}{dt}$ is negative and the fluid remains at its height z and its position x, y; p at the point will be greater than p_a outside causing the system to move forward.

or if $p = p_a$ at surface $\rightarrow \frac{dv}{dt}x + gz = 0$ causing $\frac{dz}{dx} = -\frac{1}{g}\frac{dv}{dt}$

and $\frac{dz}{dx} > 0$ $\therefore$ the fluid would tend to move forward.



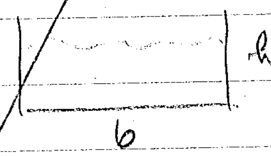
This can be seen when people lurch forward during sudden stops. X

2.

For the original conditions:

the boyant force $= \rho g V$

the weight of the ship $= m_A g =$



$w_1 = m_1 g = \rho_1 V_1 g \qquad \omega_2 = m_2 g = \rho_2 V_2 g \qquad V_2 = \dfrac{\omega_2}{\rho_2 g}$

(4) if the tank is taken as bh = volume without the ship

the boyant force on the total weights $= \rho_{H_2O}\, g \left[\dfrac{W_2}{\rho_2 g} + \dfrac{W_1}{\rho_1 g}\right]$

the boyant force $F = \rho_{water}\left[\dfrac{W_2}{\rho_S}\ \dfrac{W_1}{\rho_1}\right]$

~~the volume displaced must~~ the mass of the water is a constant.

$\therefore M = \rho_H bh$ the volume displaced is $\dfrac{F}{\rho_{H_2O}\, g} = V$

Volume displaced in first case:

$\omega_1 = m_1 g \rightarrow V_1 = \dfrac{\omega_1}{\rho_1 g}$ the ship will displace

the same amount of water in either case.

not true — V of weight is the same causing same displacement as before yet when it was put into the hollow ship the total effect causes the ship to lower in the water or the height of the tank to be higher than if the weights were thrown over the side.

Therefore the height $h_1 > h_2$ $\qquad \rho g V$

Proof: volume displaced in case one is $\dfrac{(W_1 + W_2)}{\rho_H g}$.

$V = \frac{W_1}{\rho_1 g}$

in case 2 $\quad V_{dis} = \cancel{\frac{W_1}{\rho_H g}} + \cancel{\frac{W_2}{\rho_H g}}$ ~~under~~ $\frac{W_1}{\rho_1 g} + \frac{W_2}{\rho_2 g}$

should be ρ_H

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AE 332

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HW #4

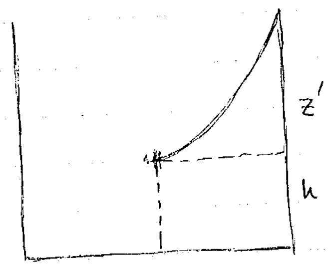
$Z = kr^2$ is the equation of the parabola the surface makes.

@ $z=0$ $r=0$

@ $z=z'$ $r=R_0$ $\qquad k = \frac{z'}{R_0^2}$

$$z = z'\left(\frac{r}{R_0}\right)^2$$

if the configuration is:



$$\rho\frac{\pi R_0^2}{2}h + \rho\int_0^{R_0}\pi r z\,dr = \rho\frac{\pi R_0^2 h}{2} + \rho\frac{\pi z'}{R_0^2}\int_0^{R_0} r^3 dr$$

$$= \rho\frac{\pi R_0^2 h}{2} + \rho\frac{\pi R_0^2 z'}{4} = \rho\frac{\pi R_0^3}{2}$$

$$\rho\frac{\pi R_0^2}{4}\left[2h+z'\right] = \rho\frac{\pi R_0^2}{4}\left[2R_0\right] \quad \text{therefore}$$

$$2h+z' = h+(h+z') = 2R_0 \qquad \text{where } h+z' = 2R_0$$

$$\therefore\ h \equiv 0 \qquad \therefore\ z' = 2R_0$$

when $r=R_0$ $z=z'$ and $p = p_a + \rho\left(\frac{\Omega^2 r^2}{2} - gz\right)$

therefore $\frac{\Omega^2 r^2}{2} = gz$ and $\Omega = \sqrt{\frac{2gz'}{r^2}} = \sqrt{\frac{4gR_0}{R_0^2}} = 2\sqrt{g/R_0}$

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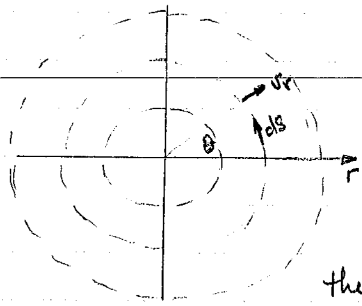

$v_\theta = 0$

$v_r \neq 0$

Mass flow = constant

the mass flow crossing any circle whose center is at the source must be equal to the quantity of fluid coming from the source.

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \quad \text{since} \quad \psi \neq \psi(r) \quad \text{we can write} \quad v_r = \frac{1}{r}\frac{d\psi}{d\theta}$$

the strength of the source is defined as the rate of mass fluid flow with unit density across a closed curve enclosing the source and thus has a value of $2\pi K$, where K is a constant.

If the quantity of fluid streaming from the source is m ; the total fluid crossing any circle is $\rho(2\pi r)\, v_r = \rho\, ds\, v_r$

since the mass flow in = mass flow out

$$v_r = \frac{m}{2\pi r \cdot \rho} \qquad K = \frac{m}{2\pi} \text{ when } \rho = 1$$

$$\text{from } v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{1}{r}\frac{m}{2\pi \cdot \rho} \qquad \frac{\partial \psi}{\partial \theta} = \frac{d\psi}{d\theta} = \frac{m}{2\pi \cdot \rho}$$

$$\psi = \frac{m}{2\pi\rho}\theta + \text{constant}$$

$$\text{if } \psi = 0 \text{ when } \theta = 0 \qquad \psi = \frac{m\theta}{2\pi\rho}$$

when m is the mass flow ρ = density and θ is measured from the streamline $\psi = 0$.

To check dimensions $\psi \sim v A$ or L^3/T , $m = M/T$ $\rho = M/L^3$

$\therefore$ the right hand side $\sim \dfrac{M/T}{M/L^3} = L^3/T$ which are the units of ψ.

1D

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AE 332

$V_\infty = 10$ fps $m/V_\infty = 1$

$\psi = 0$

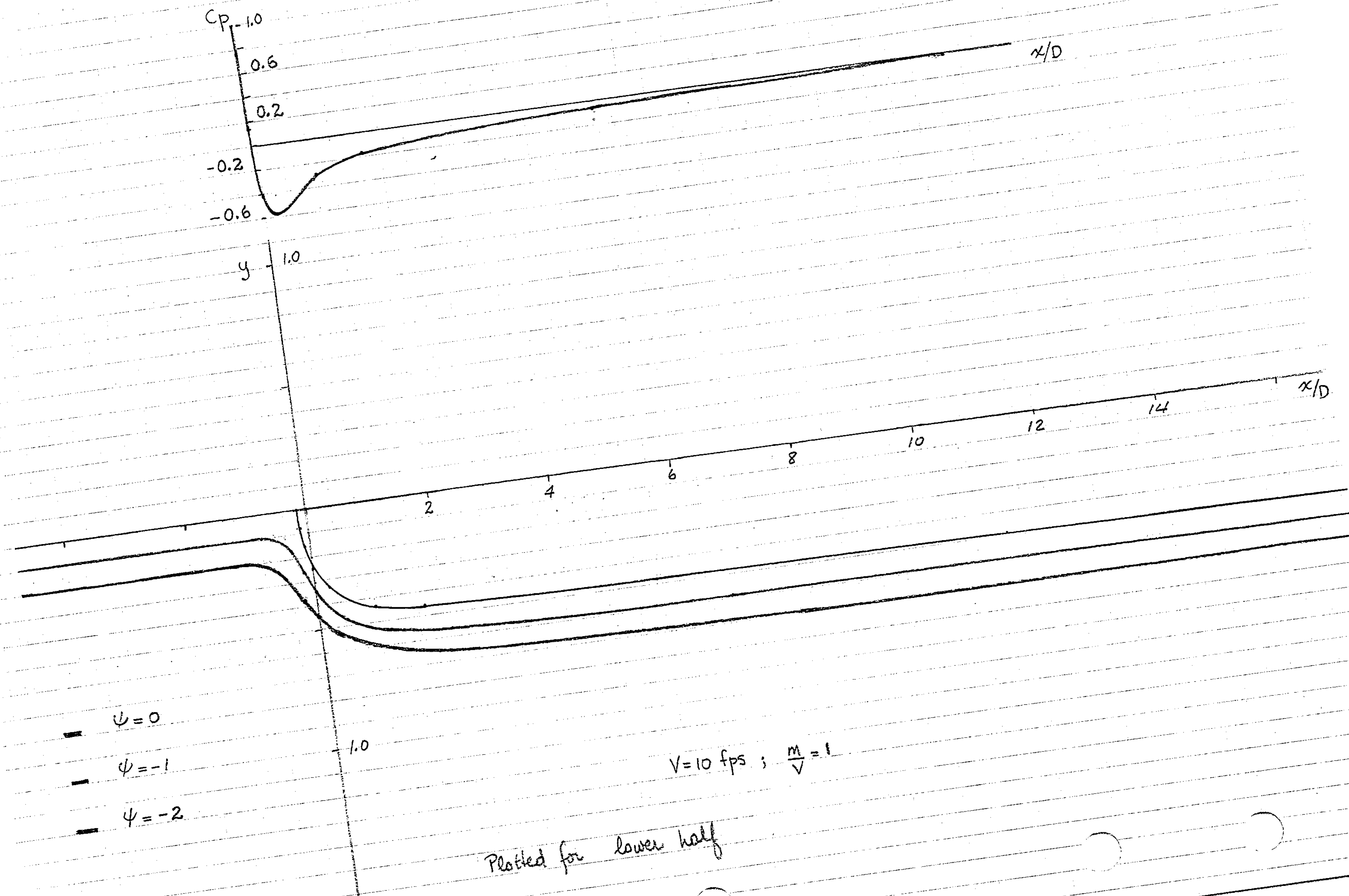
θ	X	Y	Cp	
0	0.0	0.0	0.0	← mass source
30°	-.144	-.083	.742	
60°	-.096	-.167	.143	
90°	.000	-.250	-.410	
120°	.192	-.333	-.584	
155°	.923	-.431	-.308	
165°	1.710	-.458	-.182	
175°	5.556	-.486	-.058	
177.5°	11.291	-.493	-.028	

$\psi = -1$

X	Y	
0.0	0.0	← mass source
-.318	-.183	
-.154	-.267	as $x/D \to -5$ $y \to -.1$
.000	-.350	
.250	-.433	
1.138	-.531	
2.084	-.558	
6.699	-.586	
13.581	-.593	

$\psi = -2$

X	Y	
0.0	0.0	← mass source
-.491	-.283	
-.212	-.367	as $x/D \to -5$ $y \to -.2$
0.00	-.450	
.308	-.533	
1.352	-.631	
2.457	-.658	
7.842	-.686	
15.872	-.693	



Cp
1.0
0.6
0.2
-0.2
-0.6
x/D
y
1.0
2
4
6
8
10
12
14
x/D
1.0
ψ = 0
ψ = -1
ψ = -2
V=10 fps ; m/V = 1
Plotted for lower half

Cevarhery
AE332

from previous eq.

$$V_r = 0 \qquad V_\theta = -\left[2V_\infty \sin\theta + \frac{\Gamma}{2\pi a}\right]$$

$$V = \sqrt{V_r^2 + V_\theta^2} = 2V_\infty \sin\theta + \frac{\Gamma}{2\pi a}$$

using bernoulli since we have potential flow.

$$P + \frac{\rho_\infty V^2}{2} = P_\infty + \frac{\rho_\infty V_\infty^2}{2}$$

$$P = P_\infty + \frac{\rho_\infty V_\infty^2}{2} - \frac{\rho_\infty V^2}{2}$$

$$= P_\infty + \frac{\rho_\infty V_\infty^2}{2}\left[1 - 4\sin^2\theta\right] - \frac{\rho_\infty V_\infty \sin\theta\, \Gamma}{\pi a} - \frac{\rho_\infty}{8}\frac{\Gamma^2}{\pi^2 a^2}$$

$$D = -\int_0^{2\pi} P\cos\theta\, a\, d\theta = -a\left[P_\infty \sin\theta + q_\infty\left[\sin\theta - \frac{4}{3}\sin^3\theta\right] - \frac{\rho_\infty V_\infty}{2\pi a}\sin 2\theta - \frac{\rho_\infty \Gamma^2}{8\pi^2 a^2}\sin\theta\right]$$

evaluated from 0 to 2π all sine fns are 0 $\therefore$ $D = 0$.

$$L = -\int_0^{2\pi} P\sin\theta\, a\, d\theta = -a\left[-P_\infty\cos\theta + q_\infty\left[\cos\theta - 4\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)\sin\theta\, d\theta\right] - \frac{\rho_\infty V_\infty \Gamma}{\pi a}\left(\frac{\theta}{2} - \frac{\cos 2\theta}{4}\right)\right.$$

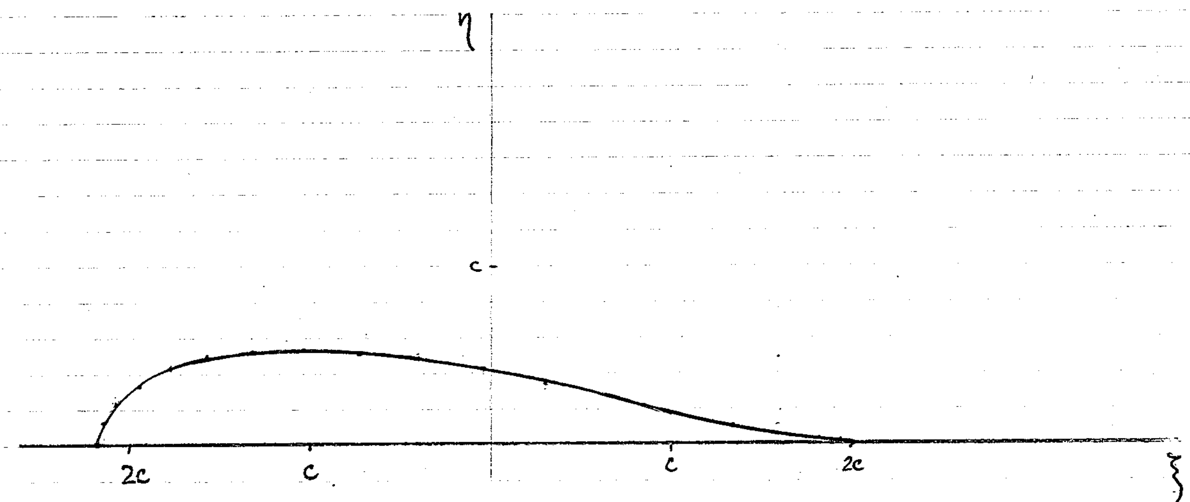
$$\left. + \frac{\rho_\infty \Gamma^2}{8\pi^2 a^2}\cos\theta\right]$$

evaluated from 0 to 2π

$$L = +\frac{\rho_\infty V_\infty \Gamma}{\pi a}\left[\frac{2\pi}{2} a\right] = \rho_\infty V_\infty \Gamma$$

Cesar Levy
AE 332

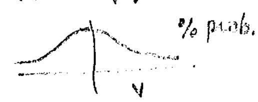
ZETA PLANE



Incompressible flow

1. Δp are small and exist
2. $\Delta \rho$ exist due to ΔT or composition
3. ρ, T change $\neq f(V, p)$, ρ defined if body can be treated as a continuum
4. $M \ll 1$ compressibility effects neglected , $E = \dfrac{F/A}{\Delta l / l} = -p/_{\Delta V/V}$

Liquid

1. Considered a continuum – when properties of fluid particles are described as the average of all the molecules in the particles. $\bar{\lambda} \ll$ Rad of molec.
2. Kinetic Theory assume a Maxwellian fn for velocity 

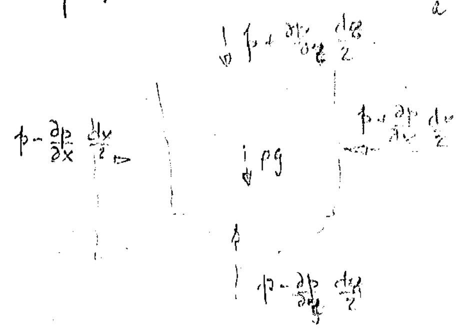
since $\dfrac{dv}{dy} \sim 10^{8}$ and for $V = 100$ ft/sec $\Delta y \to 10^{-6}$ ft $\therefore$ fluid is taken as inviscid
3. $\tau \sim {dv}/{dn}$ due to exchange of momentum of fluid. (pressure – change of momentum will a surface)

Statics Equilib.
zero shear
forces $\perp$ to surface
$p \in C^1$
body forces ...

$$\partial p/\partial x = \frac{\partial p}{\partial y} = 0$$

$$\frac{\partial p}{\partial z} = -\rho g$$

$p = f(z)$ only



$p = p_a - \rho g z$ ... $\uparrow z$ $\quad \rightarrow \quad p - p_a = \rho g h$

Archimedes Principle

for fluid in hydrostatic equil and $\rho = c$; if $\dfrac{dp}{dx} = \dfrac{dp}{dy} = 0$

$F = \rho g$ Vol of liq displaced. if $\rho = f(z)$ then $F \int f(z)\, g\, dA \cdot z$

1. independent of depth , independent of surface pressure

Surface tension

$$p_i = p_0 + \sigma \left[\frac{R_2 + R_1}{R_2 R_1} \right] \quad \text{\#/ft}$$

for 2-D $R_1 \to \infty$

also $p = p_a - \rho g z$ when $R_1 = \infty$ & $p = p_i$ at $z = h$

Statics of Gases

$$\frac{dp}{dz} = -\rho g \qquad \text{if } \rho \neq f(z) \text{ then } p = p_a - \rho g z$$

interface condition for p if $\rho = f(z)$

if $T = T_a \neq f(z)$ then using $p = \rho R T_a$ $\quad \frac{dp}{dz} = -\frac{p}{RT_a} g \quad p/p_a = e^{-gz/RT_a}$

also $p/p_a = \rho/\rho_a$ since $T/T_A = 1$

using an adiabatic process

$$p = C\rho^n \rightarrow \frac{dp}{dz} = -\rho g \text{ gives } p/p_a = \left[1 - \frac{n-1}{n}\, \frac{gz}{RT_a}\right]^{n/n-1}$$

$$\rho/\rho_a = \left[1 - \frac{n-1}{n}\, \frac{gz}{RT_a}\right]^{1/n-1} \qquad T/T_A = \left[1 - \frac{n-1}{n}\, \frac{gz}{RT_a}\right]$$

for a stable system. $\frac{dT}{dz} > -\frac{n-1}{n}\, g/R$ $\qquad n < \gamma$ stable

neutral system $\frac{dT}{dz} = -\frac{n-1}{n}\, g/R$ $\qquad n = \gamma$ stable

unstable system $\frac{dT}{dz} < -\frac{n-1}{n}\, g/R$ $\qquad n > \gamma$ unstable

isothermal atmosphere is stable, isometric atmosphere is unstable

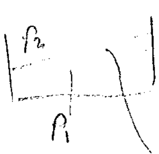
Consider now

$$V = c, \quad \frac{dp}{dz} = -\rho g \qquad \text{if } V = V(t) \quad \frac{\partial p}{\partial x} = -\rho \frac{dV}{dt}$$

$$\therefore \quad p = p(x, z) \quad \therefore \quad p = -\rho g z + f(x); \quad \frac{\partial p}{\partial x} = +f'(x) = -\rho \frac{dV}{dt}$$

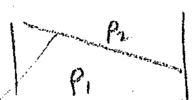
$$f(x) = f_0 - \rho \frac{dV}{dt} x \quad \therefore \quad p = f_0 - \rho\left(gz + x\frac{dV}{dt}\right) \qquad \text{when } z = 0 \text{ and } \frac{dV}{dt} = 0 \quad p = p_a$$

$$\therefore \quad \boxed{p = p_a - \rho\left(gz + x\frac{dV}{dt}\right)}$$



$p_1 > p_2$ deceleration

$\frac{dV}{dt} < 0$



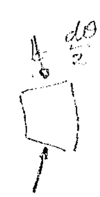
$\frac{dV}{dt} > 0 \quad p_1 > p_2$

acceleration

$$\frac{dz}{dx} = -\frac{1}{g}\frac{dV}{dt}$$

For rotating system Ω

$$\frac{\partial p}{\partial z} = -\rho g$$



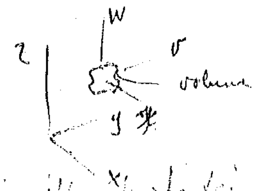
$$\frac{\partial p}{\partial r} = \rho \Omega^2 r$$

$$\boxed{p = p_a + \rho\left(\frac{\Omega^2 r^2}{2} - gz\right)}$$

surface is not horizontal since $z = f(r^2)$

Dynamics

Lagrange $u = \frac{dx}{dt} \quad v = \frac{dy}{dt} \quad w = \frac{dz}{dt}$



$$\sum F = ma = \rho \frac{d\bar{q}}{dt} \qquad p = p(x, y, z)$$

look at particle examine its properties as it moves in space

Euler $V = V(x, y, z, t)$

Euler's

$$\sum F_x = ma_x = \rho \Delta V a_x = \rho \Delta V \left(\bar{q} \cdot \nabla u \right) + \frac{\partial u}{\partial t} \Big) = \rho \Delta V \frac{Du}{Dt}$$

look at specific point and examine properties of particle as it moves past point.

steady flow $\partial/\partial t = 0$

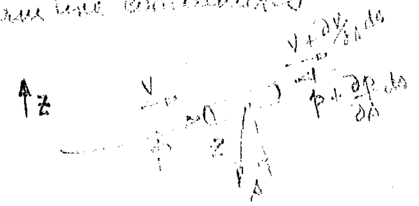
Streamline - tangent to velocity vector of fluid part. $\frac{dy}{dx} = \frac{v}{u}$

Pathline - trajectory of fluid particle $x, y, z = f(t)$

steady flows - Pathline, Streamline are coincident

Streakline - trajectory of all particles passing through a given pt.

using streamline coordinates



if $\sum F = 0 \quad \rho \Delta V \frac{D}{Dt} \bar{q} = -\Delta V [(\nabla p) + \rho \bar{g}]$

Euler's $\bar{q} = \bar{q}(x, y, z)$

also $\frac{\partial \bar{q}}{\partial n} = 0$ streamline

$$\rho \Delta V \frac{D}{Dt} \bar{q} = \sum F_s \longrightarrow \frac{\partial \bar{q}}{\partial t} + \bar{q} \frac{\partial \bar{q}}{\partial s} + \frac{1}{\rho} \frac{\partial p}{\partial s} + g \frac{dz}{ds} = 0$$

if steady flow $\int \left(\bar{q} \cdot \frac{d\bar{q}}{ds} + \frac{1}{\rho} \frac{dp}{ds} + g \frac{dz}{ds} \right) ds = \text{const on streamline}$

derived by use of momentum but it also represents conservation of energy.

if gravity head is << veloc, press head neglect. gravity head term

for infinite distance from wing $p = p(z)$

stagnation conditions - those conditions in flow of flow where brought to rest isentropically.

if Δz is large or V is low, the whole bernoulli must be used.
or $\rho >> \rho_{air}$

Euler's Eqn:

$$\left. \begin{aligned} \bar{q} \cdot \nabla u &= -\frac{1}{\rho} \frac{\partial p}{\partial x} \\ \bar{q} \cdot \nabla v &= -\frac{1}{\rho} \frac{\partial p}{\partial y} \end{aligned} \right\} \quad \bar{q} \cdot \nabla \bar{q} = -\frac{1}{\rho} \nabla p$$

Continuity

$$dm = \rho \bar{q}\cdot\bar{n}\,dA \quad ; \quad \iint_A \rho\bar{q}\cdot\bar{n}\,dA = \iiint_R \rho\nabla\cdot\bar{q}\,dR = -\frac{\partial}{\partial t}\iiint_R \rho\bar{q}\,dR$$

$$\iint_A dm \equiv 0 \qquad \nabla\cdot\bar{q} = 0 \rightarrow u_{,x} + v_{,y} = 0$$

also $\omega = \frac{1}{2}(v_x - u_y) = 0 \quad \nabla\times\bar{q} = 0$

mass in − mass out = 0

if $\nabla\times\bar{q} = 0 \quad \bar{q} = \nabla\varphi$ where $\varphi = \varphi(x,y)$ & $u = \frac{\partial\varphi}{\partial x} \quad v = \frac{\partial\varphi}{\partial y}$

$\therefore \nabla^2\varphi = 0$ Laplace

also $m = \rho u\,dy - \rho v\,dx = \rho\,d\psi$

$\therefore \frac{\partial\psi}{\partial x} = -v \quad \frac{\partial\psi}{\partial y} = u \quad \therefore \nabla^2\psi = 0$ for irrotationality

$\nabla^2\psi = -2\omega_0$ Poisson for rotationality.

$0 = \nabla^2\psi$ must satisfy BC on body & at ∞ $\qquad v_n$ on body $\equiv 0$

Circulation $\quad \Gamma = \oint \bar{q}\cdot d\bar{s} = \oint (u\,dx + v\,dy + w\,dz)$

2-D flows are not unique ; when Γ is defined and BC at ∞ & body satisfied uniqueness is established.

Γ for $\nabla\times\bar{q} = 0$ is not fn of path

Γ for $\nabla\times\bar{q} \neq 0$ is fn. of path.

Bernoulli's equation used only for potential flow
- if $\nabla\times\bar{q} = 0$ const. $\forall\,\psi$
- if $\nabla\times\bar{q} \neq 0$ const for ψ_i

for source & sink uniform flow (Rankine Oval)

for stag pt. − $u_\theta, u_r, v_\theta, v_r, u, v = 0$

for source-sink doublet $ma = \mu$, slenderness ratio $\frac{y_{oL}}{x_{stag}}$

$$D = -\int_0^{2\pi} p\cos\theta\,d\theta = 0$$

$$L = -\int_0^{2\pi} p\sin\theta\,d\theta = \rho V\Gamma$$

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Kutta Condition - flow leaves trailing edge smoothly.

$C_{L_{theory}} = 2\pi\alpha$ if Kutta condition satisfied.

Bernoulli use $\sum F_n \uparrow$ gives $\frac{\partial p_s}{\partial n} = \rho V \frac{\partial V}{\partial n} - \rho \frac{V^2}{R} = -2\omega\rho V$

this leads to $\frac{\partial p_s}{\partial n} = 0 \Rightarrow \frac{\partial V}{\partial n} = \frac{V}{R}$ or $\omega = \frac{1}{2}\left(\frac{\partial V}{\partial n} - \frac{V}{R}\right)$

Thin airfoil theory $\nabla^2 \varphi = 0$ with B.C. Governing Eq. for $\bar{V}$

3 general condition

1. Thickness distribution - generated by sources and sinks $\int_0^c m\,dx = 0$ $\psi = \frac{m\theta}{2\pi}$
2. mean camber line . generated by vortices $\gamma(c) = 0$ Kutta cond. $\psi = \frac{\Gamma}{2\pi}\ln r$
3. angle of attack

replace mcl by vortex lines whose Total strength is $\Gamma = \int_0^c \gamma\,ds$; $L = \rho V_\infty \Gamma$

use $x = f(\theta) = \frac{c}{2}(1-\cos\theta)$

assumptions for airfoil ① $t/c \ll 1$ ② m.c.l. is a streamline $\therefore V\perp$ is $\equiv 0$

③ assume $\frac{d}{dx} y_{mcl} \ll 1$ ④ assume α is small.

$\boxed{C_{m_{l.e.}} = C_{m_{a.c.}} - (a.c.)C_l}$ a.c. pt where $C_{m_{ac}} =$ const. $= 0$.
with increasing angle of attack

$V_{L.e.}$ is not finite.

for a cambered airfoil where $\frac{dy}{dx}$ of mcl is given

$$\gamma = \left[A_0 \frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta\right] 2V_\infty$$

where $A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \left(\frac{dy}{dx}\right) d\theta$ $\frac{dy}{dx} = f(\theta)$ where $x = \frac{c}{2}(1-\cos\theta)$

$$A_n = \frac{2}{\pi}\int_0^\pi \left(\frac{dy}{dx}\right)\cos n\theta\, d\theta$$

$C_L = 2\pi\left(A_0 + \frac{A_1}{2}\right)$ fn of α & geometry. $C_{m_{ac}} = \frac{\pi}{4}(A_2 - A_1)$

$C_{m_{le}} = -\frac{\pi}{2}\left(A_0 + A_1 - \frac{A_2}{2}\right)$ $\alpha_{0L} = \alpha \ni C_L = 0 \therefore C_{m_{ac}} = C_{m_{le}}$

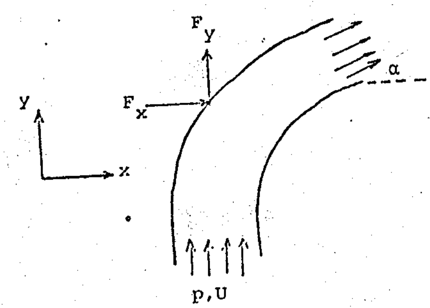
use of momentum theorem.

$$\partial/\partial t \int_R \rho \bar{q} \, d\bar{R} + \int_S \rho \bar{q} (\bar{q} \cdot n) \, dS = -\int_S p \bar{n} \, dS + \bar{F}$$ forces acting on fluid

AE 332 - Problem Set No. 6

1. (a) Applying the principle of conservation of momentum to the fluid contained in the elbow shown below, derive expressions for the components of force necessary to hold the elbow in a stationary position when the system is in a steady state. Assume the fluid is of constant density ρ, the velocity U does not vary across the cross section of area A, and neglect any frictional losses. Be careful to get your signs correct.

   (b) In what direction must a fireman push to hold a fire hose steady assuming the hose bends up from the ground as shown?



(a) Prove : $\int \nabla \phi \, dV = \int \phi \hat{n} \, dS$

Since F_y acts on the elbow $-F_y$ acts on the fluid

$$-F_y + \rho U(-U)A + \rho U(U \sin\alpha)A = pA - pA\sin\alpha$$

$$F_y = pA(1-\sin\alpha) + \rho U^2 A(1-\sin\alpha)$$

$$-F_x + \rho U(U\cos\alpha)A = -pA\cos\alpha$$

$$F_x = (\rho U^2 A + pA)\cos\alpha$$

EXAMINATION BOOK

Name [illegible]

Subject AF 382

Instructor Dr. [illegible]

Exam Seat No. ______ Section ______

Date 4/24/7? Grade ______

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~~17~~ 19

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1.

a. A stream function is any fn which satisfies laplace's equation $\nabla^2\psi = 0$ for an irrotational flow and poisson's equation $\nabla^2\psi = -2\omega_0$ for rotational flow; it also satisfies the equation of continuity $u_{,x} + v_{,y} = 0$

b. A Potential function is any fn which satisfies laplaces equation $\nabla^2\varphi = 0$ and irrotationality $\nabla \times \bar{q} = 0$

c. Irrotational - any flow which can be represented by $\nabla \times \bar{q} = 0$ where $\bar{q} = u\bar{i} + v\bar{j} + w\bar{k}$. In an irrotational flow the velocity vector $\bar{q}$ can be replaced by $\nabla\varphi$.

1/2

d. The substantial derivative consists of local derivative and a convective derivative, it describes the change in property of a particle in the Eulerian sense in that

if $a = a(x,y,z,t)$

$$\frac{Da}{Dt} = \frac{\partial a}{\partial t} + \frac{\partial a}{\partial x}\frac{dx}{dt} + \frac{\partial a}{\partial y}\frac{dy}{dt} + \frac{\partial a}{\partial z}\frac{dz}{dt} = \frac{\partial a}{\partial t} + \bar{q}\cdot\nabla a$$

e. A real vortex is any vortex whose velocity varies directly with the radius in the core and inversely as the radius outside

f. A doublet is any potential flow such that it is made up of a sink and a source of same strength, and properties of such configuration are investigated as the sink and source approach each other & become infinitesimally separated & $m \to \infty$

1/2

$\mu = ma = \text{const.}$

g. the Kutta - Joukowski Law states that any body which has circulation around it produces the following results

~~Drag = 0~~

Lift $= \rho_\infty V_\infty \Gamma$ and it is $\perp$ to direction of flow.

2.

a The flow is assumed inviscid so that shear stresses are neglected the flow was assumed to be steady $\Rightarrow \bar{q} = \bar{q}(x,y,z)$ only. The fluid was in equilibrium so that there was no relative motion. Gravity was neglected and only pressure forces are accounted for. $\rho \Delta V \frac{D\bar{q}}{Dt} = -\nabla p \ \Delta V$ also x, y, z are indeped of each other and only a fn. of time $\therefore \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}$ exist

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b the density is considered constant and the flow is considered irrotational. The flow must also obey the equation of continuity

c. yes, except it must satisfy $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -2\omega_0$

~~Since a potential fn is defined as $\nabla \times \bar{q} = a$~~

~~There exists no potential fn.~~

since a potential fn is defined as $\nabla \times \bar{q} = 0$

since $v_{,x} - u_{,y} = 2\omega_0$ and $\frac{\partial \varphi}{\partial x} = u$ $\frac{\partial \varphi}{\partial y} = v$

$\left[\frac{\partial \varphi}{\partial x \partial y}\right](1 - 1) \stackrel{?}{=} 2\omega_0$ Not true.

there exists no potential fn.

3. $\psi = k(x^2+y^2)$ let $x^2+y^2=r^2$

$$\frac{\partial \psi}{\partial x} = -v = 2kx \qquad \frac{\partial \psi}{\partial y} = u = 2ky$$

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$$v = -2kx \qquad u = 2ky$$

since $v = \frac{\partial \varphi}{\partial y}$ then

$$\varphi = -2kxy + f(x)$$

$$\frac{\partial \varphi}{\partial x} = -2ky + f'(x) = 2ky \quad \text{impossible}$$

try polar

$$\psi = kr^2$$

$$-v_\theta = \frac{\partial \psi}{\partial r} = 2kr \qquad v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = 0$$

$$v_\theta = -2kr = \frac{1}{r}\frac{\partial \varphi}{\partial \theta}$$

$$\varphi = -2kr^2\theta + f(r)$$

$$\frac{\partial \varphi}{\partial r} = -4kr\theta + f'(r) = 0$$

therefore there exists no potential fn, unless $\psi = 0$

2a. continuum.

ρ = const.

inviscid $\mu = 0$

Conservation of Momentum

Euler's

b) Continuity $\nabla^2\psi = -2\omega_0$

irrotational $\nabla^2\psi = 0 \Rightarrow \nabla^2\varphi = 0$

(c) ψ exist φ cannot if $\omega \neq 0$

3 rotational fluid

EXAMINATION BOOK

Name Anne Levy

Subject H 332

Instructor Dr. Amaro

Exam Seat No. ______ Section ______

Date [illegible] Grade ______

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1. assume $p_1 \neq f(y)$ $p_2 \neq f(y)$
do not take into account gravity

$u_1 = \frac{V_\infty y}{h}$ $v_1 = 0$ $\nabla^2\psi = 0$ for stream funct.
$\nabla \times \bar{q} = 0$ irrot.

to satisfy irrotationality $v_{3x} - u_{3y} = 0$
$0 - \frac{V_\infty}{h} \neq 0$

a) Flow is rotational.

b.) The total mass $\bar{n} = \bar{i}$ $\bar{q} = ui + vj + wk$
$$m = \rho \int \nabla \cdot \bar{q}\, dV = \rho \int_S \bar{q} \cdot \bar{n}\, dS = \rho \int u_1 dS = \rho \int_0^h \frac{V_\infty y}{h}\, dy \cdot 1$$
$$= \frac{\rho}{2} \frac{V_\infty}{h} y^2 \Big|_0^h = \rho \frac{V_\infty h}{2}$$

c) ~~Since flow is rotational and~~
~~since ρ is a constant~~ or created
since mass cannot be destroyed then
$$m_2 = m_1 = \cancel{\rho} \int u_2\, dS = \cancel{\rho} \frac{V_\infty h}{2}$$

u_2 can be obtained as $2 \frac{V_\infty y}{h}$
$$\int \frac{2 V_\infty}{h} y\, dy = \frac{2 V_\infty}{h} \frac{h^2}{4} = \frac{V_\infty h}{2}$$

$\therefore$ $u_2 = \frac{2 V_\infty y}{h}$

d. for irrotationality $v_{3x} - u_{3y} = 0$
but $v_{2,x} = 0$ $u_{2,y} = \frac{2 V_\infty}{h}$ $\therefore$ flow is rotational

$\sqrt{x^2+(y-a)^2}$

2.

Γ

$$\left[\ \Psi = \frac{\Gamma}{2\pi}\ln(r-a) - \frac{\Gamma}{2\pi}\ln(r+a)\ \right]\ ?$$

since it is additive V_θ

velocity at center must be $-\dfrac{\Gamma}{\pi a}$

$-\Gamma$

$$V_\theta = \frac{\Gamma}{2\pi(r-a)} - \frac{\Gamma}{2\pi(r+a)}$$

as $r \to 0$

$$V_\theta = -\frac{\Gamma}{\pi a} \text{ which is true}$$

b. from the equation

$$\Psi = \frac{\Gamma}{2\pi}\ln\left(\frac{r-a}{r+a}\right) \to \ln(1+\epsilon) = \epsilon$$

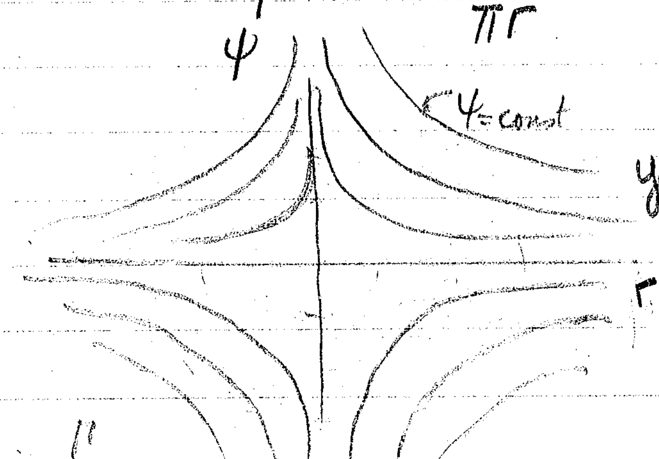
$$\frac{r+a}{r+a}\quad \frac{-2a}{r+a}$$

$$\psi = \frac{\Gamma}{2\pi}\ln\left(1 + \frac{-2a}{r+a}\right)$$

$$\psi = \frac{\Gamma}{2\pi}\left(\frac{-2a}{r+a}\right) \quad \text{for } a \text{ small then}$$

$$\psi = -\frac{\Gamma a}{\pi r} = \frac{\text{Const}}{r}$$

c.



yet it looks like the corner flow

$\dfrac{\partial \psi}{\partial r}$, Γ

1(c) Euler's equation

Conservation of Mass

Conservation of Momentum.

(i) since $\bar{V}$ increases then $\bar{q}_{\infty}$ increases & $\bar{p}$ decreases

$\therefore$ at bottom edge $V \neq 0$

(ii) $\frac{\partial \omega}{\partial s} \cdot V + \frac{\partial \omega}{\partial t} = 0 \qquad \frac{D\omega}{Dt} = 0$

$\therefore \frac{\partial \omega}{\partial s} \equiv 0$

d $\therefore$ ω is const since $\omega = \frac{1}{2}(v_x - u_y) = -\frac{V_\infty}{2h}$

$$\omega_2 = -\frac{V_\infty}{2h} = \frac{1}{2}(v_{2x} - u_{2y}) = -\frac{1}{2}\frac{\partial u}{\partial y}$$

$$\frac{\partial u_2}{\partial y} = \frac{V_\infty}{h} \qquad u_2 = \frac{V_\infty}{h} y + C_0$$

$$m = \rho \int \bar{q} \cdot \bar{n} \, dS = \rho \frac{V_\infty h}{2} = \rho \int_0^{h/2} \left(\frac{V_\infty}{h} y + C_0\right) dy$$

$$= \rho_\infty \left(\frac{V_\infty y^2}{2h} + C_0 y\right)_0^{h/2} = \rho_\infty \left(\frac{V_\infty h}{8} + C_0 \frac{h}{2}\right) =$$

$$\therefore \frac{3 \rho_\infty V h}{8} = \frac{C_0 h}{2} \qquad C_0 = \frac{3}{4} V_\infty$$

$$\therefore u_2 = \frac{V_\infty}{h} y + \tfrac{3}{4} V_\infty$$

(ii) Bernoulli's Equat.

$$p + \frac{\rho_\infty u_1^2}{2} + \cancel{\rho g z} = p_s \quad \text{along 1 stream line}$$

$$p_{10} + \frac{\rho_\infty u_{10}^2}{2} = p_{s_0} = p_2 + \frac{\rho_\infty u_{20}^2}{2}$$

$$p_{1_t} + \frac{\rho_\infty u_{1_t}^2}{2} = p_{s_1} = p_{2_t} + \frac{\rho_\infty}{2} u_{2_t}^2$$

$$\frac{1}{\rho}\frac{dp_s}{dn} = V\frac{\partial V}{\partial n} - \frac{V^2}{R} = -2V\omega$$

$$\Delta p_{s_1} = p_{s_t} - p_{s_0} = \Delta p_{s_2}$$

$$\Delta p_s = \int_0^h dp_s = -2\rho \int_0^h -\frac{V_\infty}{2h} \cdot \frac{V_\infty y}{h}\, dy$$

$$= \frac{\rho_\infty V_\infty^2}{2}$$

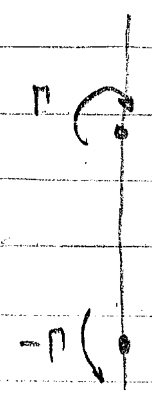
Assume $u_2 = u_0 + ky$

$$\Delta p_{s_2} = -\rho_\infty \int_0^{h/2} 2(u_0 + ky) - \frac{k}{2}\, dy$$

$$= f(u_0, k) = \frac{\rho_\infty V_\infty^2}{2}$$

continuity $\dot{m} = \frac{\rho_\infty V_\infty h}{2} = \rho_\infty \int_0^{h/2} (u_0 + ky)\, dy$

(3)



$$\psi = \frac{\Gamma}{2\pi} \ln r$$

$$r_1 = \sqrt{x^2 + (y-a)^2}$$

$$r_2 = \sqrt{x^2 + (y+a)^2}$$

$$\psi_T = \frac{\Gamma}{4\pi} \ln\left[\frac{x^2 + (y-a)^2}{x^2 + (y+a)^2}\right]$$

$$\psi = \frac{\Gamma}{4\pi} \ln\left[\frac{r^2 - 2ay}{r^2 + 2ay}\right] = \frac{\Gamma}{4\pi} \ln\left[\frac{1 - 2ay/r^2}{1 + 2ay/r^2}\right]$$

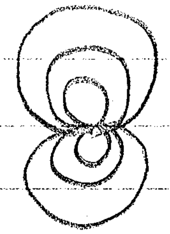
$$= \Gamma/_{4\pi} \ln\left[1 - \frac{4ay}{r^2} + \cancel{O(a^2)} \right]$$

$$= \Gamma/_{4\pi} \left[- \frac{4ay}{r^2} \right]$$

$$\psi = - \frac{\Gamma a y}{\pi r^2} = \frac{\mu y}{\pi r^2}$$

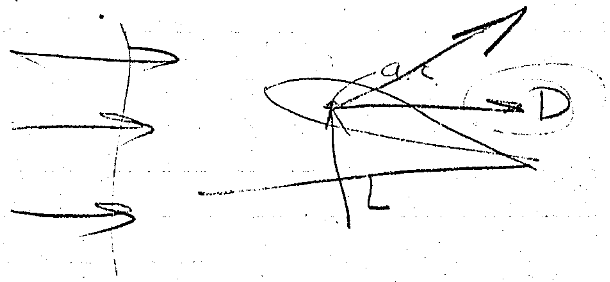
$$x^2 + \left(y + \frac{\lambda}{\pi\psi}\right)^2 = \left(\frac{\lambda}{\pi\psi}\right)^2$$

$\psi < 0$



dipole.

$\psi > 0$



$$\tan\varphi = \frac{-F_y}{-F_x} = \frac{A[p + \rho V^2](1 - \sin\alpha)}{-A[p+\rho V^2]\cos\alpha}$$

$$\tan\varphi = \frac{-(1-\sin\alpha)}{\cos\alpha} = \frac{\sin\alpha - 1}{\cos\alpha} = \tan\alpha - \sec\alpha$$

$$* \ \varphi = \tan^{-1}[\tan\alpha - \sec\alpha]$$

$A = \text{const}$, $\rho V_1 A_1 = \rho V_2 A_2$

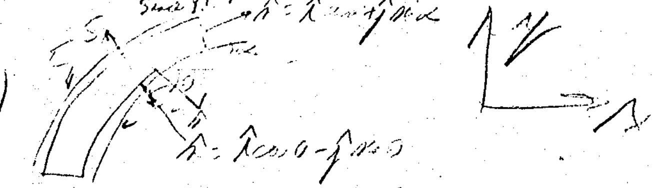
Frompcts: $\rho V A = \text{const}$

1) From Ber.: $p_1 + \rho \frac{V_1^2}{2} = p_2 + \rho \frac{V_2^2}{2} \rightarrow p_1 = p_2$

Since $V_1 = V_2 \rightarrow \hat{n} = \hat{i}\cos\alpha + \hat{j}\sin\alpha$

H 6

(a)



$\hat{n} = \hat{i}\cos\theta - \hat{j}\sin\theta$

x: $p A \cos\alpha (\hat{i}) + \int_{S_2} p(\hat{i}\cos\theta) dS_2 + \int_{S_1} p(-\hat{i}\cos\theta) dS_1$

$+ A\rho U^2 \cos\alpha\, \hat{i} = 0$

$F_x = \int_{S_2} p\, \hat{i}\cos\theta\, dS_2 - \int_{S_1} p\, \hat{i}\cos\theta\, dS_1$

$\therefore F_x = -pA\cos\alpha - \rho A U^2 \cos\alpha$

$R_x = -F_x$

y: $pA\sin\alpha\,\hat{j} - pA\hat{j} - \int_{S_2} p\sin\theta\,\hat{j}\, dS_2 + \int_{S_1} p\sin\theta\,\hat{j}\, dS_1$

$+ \rho A U^2 \sin\alpha\,\hat{j} - \rho A U^2 \hat{j} = 0$

$F_y = \int_{S_1} p\sin\theta\,\hat{j}\, dS_1 - \int_{S_2} p\sin\theta\,\hat{j}\, dS_2$

$F_y = pA(1-\sin\alpha) + \rho A U^2 (1-\sin\alpha)$

$\therefore F_y = A(1-\sin\alpha)[p + \rho U^2]$ $R_y = -F_y$

2) $\int_V \nabla\cdot\vec{Q}\,dV = \int_S \vec{Q}\cdot\hat{n}\,dS$ (Div. Thm.)

$\vec{Q} = \nabla\varphi$

$\int_V \nabla\cdot\nabla\varphi\,dV = \int_S \nabla\varphi\cdot\hat{n}\,dS$

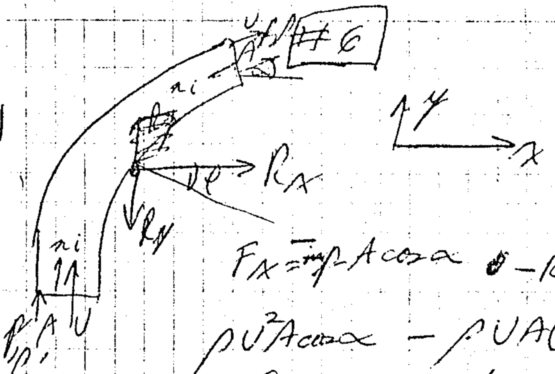
$\int_V \nabla^2\varphi\,dV = \int_S \nabla\varphi\cdot\hat{n}\,dS$

$\therefore \int_V \nabla\varphi\,dV = \int_S \varphi\hat{n}\,dS$

'S

S

1) (a)



$F_x = pA\cos\alpha - R_x$

$\rho U^2 A\cos\alpha - \rho U A(U\cos 90^\circ)$

$= \int \rho \bar{q}(\bar{q}\cdot\hat{n})\,dS$

$\therefore pA\cos\alpha - R_x = +\rho U^2 A\cos\alpha$

$R_x = -\rho U^2 A\cos\alpha - pA\cos\alpha$

* $R_x = A\cos\alpha[-\rho U^2 - p]$

$F_y = -pA\sin\alpha + R_y + pA$

$\int \rho\bar{q}(\bar{q}\cdot\hat{n})\,dS = \rho U^2 A\sin\alpha - \rho U^2 A$

$pA - pA\sin\alpha + R_y = \rho U^2 A[\sin\alpha - 1]$

* $R_y = \cancel{-pA}\, A\sin\alpha[\rho U^2 + p] - \rho U^2 A$

(b) $\tan\varphi = \frac{R_y}{R_x} = \frac{A\sin\alpha(\rho U^2+p) - \rho U^2 A}{-A\cos\alpha(\rho U^2+p)}$

* $\tan\varphi = -\tan\alpha + \frac{\rho U^2}{\cos\alpha(\rho U^2+p)}$

AE-332 - Homework

May 1970

Consider an airfoil of length ℓ with maximum camber/chord ratio $\varepsilon<<1$ placed in a uniform stream with velocity U_∞. In dimensional form

$$\bar{y}_c(\bar{x}) = \varepsilon\ell\left(\frac{\bar{x}}{\ell}\right)\left[1-\left(\frac{\bar{x}}{\ell}\right)^2\right] = \epsilon c (x/c)\left[1-\left(\frac{x}{c}\right)^2\right]$$

Assuming the airfoil is at a small angle of attack α, use the formulas developed in class to determine

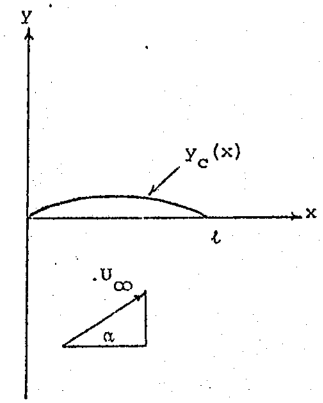
a. The total vortex strength per unit length along the airfoil. $\gamma =$

b. The total circulation about the airfoil. $\Gamma = \int_0^c \gamma\, ds$

c. The coefficient of lift C_L. $2\pi\left[A_0 + A_1/2\right]$

d. The coefficient of the moment C_M about the leading edge. $C_{M_{le}} = -\frac{\pi}{2}\left[A_0 + A_1 - \frac{A_2}{2}\right]$

a useful formula is $\cos^2\varphi = \frac{1}{2}(1+\cos 2\varphi)$



(2)

$n\varphi\, d\varphi$

$\varphi \cos n\varphi)\, d\varphi$

$\sum_{n=1}^{\infty} b_n \sin n\varphi$

$\frac{3\epsilon}{2}) \sin\varphi$

$\sin\varphi + \frac{3\epsilon b_0}{4}\sin 2\varphi$

$^{\pi}(+3\epsilon b_0)\sin\varphi \sin\varphi\, d\varphi$

$\frac{7}{8}\epsilon + 8\alpha)$

(1)

$\cancel{b_n = \frac{2}{\pi}\int_0^{\pi} -\epsilon \cos n\varphi\, d\varphi - \frac{3}{\pi}\epsilon \int_0^{\pi} (\cos\varphi \cos n\varphi)\, d\varphi}$

$$\bar{y}_c(\bar{x}) = \epsilon l \left(\frac{x}{l}\right)\left[1-\left(\frac{x}{l}\right)^2\right]$$

$$\frac{dy_c}{dx} = \epsilon l \left[\frac{x}{l}\right]\left[-2\left(\frac{x}{l}\right)\left(\frac{1}{l}\right)\right] + \frac{\epsilon l}{l}\left[1-\left(\frac{x}{l}\right)^2\right]$$

$$= -\epsilon 2\left(\frac{x}{l}\right)^2 + \epsilon - \epsilon\left(\frac{x}{l}\right)^2$$

$$= \epsilon - 3\epsilon\left(\frac{x}{l}\right)^2$$

$$s = \frac{l}{2}(1+\cos\varphi) = x$$

$$\frac{dy_c}{dx} = \epsilon - \frac{3\epsilon}{l^2}\frac{l^2}{4}(1+\cos\varphi)^2 = \epsilon - \frac{3}{4}\epsilon(1+2\cos\varphi+\cos^2\varphi)$$

$$\frac{dy_c}{dx} = \frac{\epsilon}{4} - \frac{3}{2}\epsilon\cos\varphi - \frac{3}{8}\epsilon(1+\cos 2\varphi)$$

$$\frac{dy_c}{dx} = -\frac{\epsilon}{8} - \frac{3}{2}\epsilon\cos\varphi - \frac{3}{8}\epsilon\cos 2\varphi$$

$$b_0 = \frac{1}{\pi}\int_0^{\pi}\left[-\frac{\epsilon}{8} - \frac{3}{2}\epsilon\cos\varphi - \frac{3}{8}\epsilon\cos 2\varphi\right] d\varphi$$

$$b_0 = \frac{1}{\pi}\left[-\frac{\epsilon\varphi}{8} - \frac{3}{2}\epsilon\sin\varphi - \frac{3}{8}\epsilon\frac{\sin 2\varphi}{2}\right]_0^{\pi}$$

$$b_0 = \frac{1}{\pi}\left[-\frac{\pi\epsilon}{8} - \frac{3}{2}\epsilon\cancel{\sin\pi} - \frac{3}{8}\epsilon\frac{\cancel{\sin 2\pi}}{2}\right]$$

$$b_0 = -\frac{\epsilon}{8}$$

or $\Gamma = \pi l U_\infty \left[\alpha - B_0 - \frac{B_1}{2}\right]$

* $\Gamma = \pi l U_\infty \left[\alpha + \frac{\epsilon}{8} + \frac{3\epsilon}{4}\right] = \pi l U_\infty \left[\alpha + \frac{7}{8}\epsilon\right] = \frac{\pi l U_\infty}{8}\left[8\alpha + 7\epsilon\right]$

(c) $C_L = 2\pi \left[\alpha - B_0 - \frac{B_1}{2}\right]$

* $C_L = 2\pi \left[\alpha + \frac{\epsilon}{8} + \frac{3\epsilon}{4}\right] = 2\pi \left[\alpha + \frac{7\epsilon}{8}\right]$

(d)

$C_M = -\frac{1}{4}\left[2\pi\alpha - (2B_0 + B_1)\pi\right] + \frac{\pi}{4}(B_1 + B_2)$

$C_M = -\frac{1}{4}\left[2\pi\alpha - \left(-\frac{2\epsilon}{8} - \frac{3\epsilon}{2}\right)\pi\right] + \frac{\pi}{4}\left(-\frac{3\epsilon}{2} - \frac{3\epsilon}{8}\right)$

$C_M = -\frac{1}{4}\left[2\pi\alpha + \frac{14}{8}\epsilon\pi\right] + \frac{\pi}{4}\left(-\frac{15\epsilon}{8}\right)$

* $C_M = \frac{\pi}{4}\left[-2\alpha + \frac{14\epsilon}{8} - \frac{15\epsilon}{8}\right] = \frac{\pi}{4}\left[-\frac{\epsilon}{8} - 2\alpha\right]$

$C_M = -\frac{\pi}{4}\left[2\alpha + \frac{\epsilon}{8}\right]$