Given an airplane with the following characteristics

$W = 250,000$ lbs. $e = 0.85$

$W/S = 100$ lbs/ft^2 $T_{SL} = 34,000$ lbs.

$AR = 7$ $T_\sigma = \sigma T_{SL}$

$C_{D_0} = 0.015$

The thrust at any altitude is considered to be independent of velocity.

a) Draw the T_R Vs. V_e curve.

b) Calculate $V_{e_{max}}$ and $V_{e_{min}}$ at sea level.

c) Draw the T_a(thrust available) curves at diff. altitudes on the curve of T_R Vs. V_e. Read off values of $V_{e_{min}}$ and $V_{e_{max}}$ at different altitudes (at 5,000 ft. intervals) and plot a curve of h vs. V_{max}. (This can also be accomplished by equating $T_A = T_R$ and solving the equation for $V_{e_{max}}$ and hence V_{max}.)

d) Find the absolute ceiling either graphically or analytically

e) Assuming small angles of climb, calculate the rate of climb at SL at different velocities between $V_{e_{min}}$ and $V_{e_{max}}$. Plot variation of R/C with V_e at SL. Calculate γ_{max} and $(R/C)_{max}$ at SL

f) Calculate the $V_{(R/C)_{max}}$ and $(R/C)_{max}$ at different altitudes. Plot against altitude V_{min}, V_{max}, $V_{R/C_{max}}$ and $(R/C)_{max}$.

SOLUTIONS (H. W. PROBLEMS)

Given $w = 250{,}000$ lbs. $T_{SL} = 34\ 000$ lbs.

$W/S = 100$ lbs./ft^2 $T_\sigma = \sigma T_{SL}$

$AR = 7$ $\rho_o = 0.002378$ Slugs/ft^3

$C_{D_0} = 0.015$

$e = 0.85$

$$A = \frac{1}{2}\rho_o S C_{D_0}, \quad S = \frac{W}{W/S} = 2{,}500 \text{ ft}^2, \quad B = \frac{2W^2}{\pi b^2 e \rho_o}$$

$A = 0.0446$ $\frac{b^2}{S} = 7.,\quad b^2 = 17{,}500$

$B = 11.24 \times 10^8$

b) At sea level

$$T_A = T_R = AV_e^2 + \frac{B}{V_e^2} = 34{,}000 \text{ lbs.}$$

$$V_e^2 = \frac{T_A \pm \sqrt{T_A^2 - 4AB}}{2A} = \frac{34{,}000 \pm \sqrt{34{,}000^2 - 4 \times .0446 \times 11.24 \times 10^8}}{2 \times 0.0446}$$

$V_{e_{min}} = \underline{186}$ ft/sec (127 m.p.h.)

$V_{e_{max}} = \underline{853}$ ft/sec (581 m.p.h.)

a) T_R vs. V_e $T_R = AV_e^2 + \frac{B}{V_e^2}$, $A = 0.0446$, $B = 11.24 \times 10^8$

Choose values of V_e between $V_{e_{min}}$ and $V_{e_{max}}$

CALCULATE T_R FOR THESE VALUES OF V_e

PLOT T_R Vs V_e

SEE FOLLOWING TABLE.

V_e f.p.s.	$0.0446\ V_e^2$ ①	$11.24 \times 10^8 / V_e^2$ ②	①+② = $AV_e^2 + \frac{B}{V_e^2}$ lbs. T_R	
200	1784	28100	29884	
300	4014	12489	16503	
$V_{e(L/D)max}$ From (d) → 398.5			14165	
400	7136	7025	14161	
500	11150	4496	15646	
600	16056	3122	19178	T_R vs. V_e - p
700	21854	2294	24148	
800	28544	1756	30300	
186			34000	From (b)
853			34000	

c) T_σ (available) = $T_{SL}\,\sigma$, T_{SL} = 34000 lbs

	ALT (FT)	σ	√σ	T_σ (AVAILABLE) LBS.
	5000'	0.862	.928	29308
	10000'	0.738	.859	25092
	15000'	0.629	.793	21386
	20000'	0.533	.730	18122
	25000'	0.448	.669	15232
ABS CEILING *	27050	0.4166	.646	14165

*From (d)

Plot these values on T_R vs. V_e graph I. $V_{e_{min}}$ and $V_{e_{max}}$ values at these altitudes may be read off at the intersection points when $T_A = T_R$.

SEE TABLE ON PAGE 3

ALT	σ	$V_{e_{min}}$ FPS	$V_{e_{max}}$ FPS	V_{min} FPS	V_{max} FPS
SL	1	186	853	186	853
5000	0.862	203	785	219	846
10000	0.738	220	715	256	832
15000	0.629	245	645	309	813
20000	0.533	277	572	379	783
25000	0.448	335	481	501	719
27050	0.4166	≈400	≈400	619	619

$V_{e_{min}}$ and $V_{e_{max}}$ are read off graph I.

V_{min} $V_{e_{min}} / \sqrt{\sigma}$

V_{max} $V_{e_{max}} / \sqrt{\sigma}$

Plot h vs. V_{max} and $V_{e_{max}}$ graph II.

d) Absolute Ceiling: Since T_A is independent of velocity at any altitude, at the absolute ceiling $T_A = T_R = D_{min}$.

$$V_{e_{D_{min}}} = \left(\frac{B}{A}\right)^{1/4} = \left(\frac{11.24 \times 10^8}{0.0446}\right)^{1/4} = 398.5 \text{ ft/sec.} \approx 400 \text{ ft/sec}$$

$$T_A = T_R = AV_{e_{D_{min}}}^2 + \frac{B}{V_{e_{D_{min}}}^2} = 2AV_{e_{(L/D)_{max}}}^2 = 14165 \text{ lbs.} = \sigma \, T_{A_{SL}}$$

$$\sigma_{abs\ ceiling} = \frac{14165}{34000} = 0.4166$$

h_{abs} corresponding to $\sigma = 0.4166$ is 27050 ft.

8) S.Level

$$V_{e(R/C)_{max}} = \left[\frac{T_A \pm \sqrt{T_A^2 + 12AB}}{6A}\right]^{1/2}$$

$$V^2_{e(R/C)_{max}} = \frac{34000 \pm \sqrt{34000^2 + 6.02X\ 10^8}}{0.2676}, \quad V_{e(R/C)_{max}} = 533 \text{ FPS}$$

$$= 284000$$

$$\therefore \quad SIN\ \gamma_{max} = \frac{T_A - D_{min}}{W} = \frac{34000 - 14160}{250000} = \frac{19840}{250000} = 0.08 \text{ rad.}$$

$$\gamma_{max} \simeq 5^{o}$$

$$(R/C)_{max} = V_{e(R/C)_{max}} \left(T_A - AV^2_{e_{R/C_{max}}} - \frac{B}{V^2_{e_{R/C_{max}}}}\right) \frac{1}{W}$$

$$= \frac{533}{250,000} \quad (34000 - 16626) = 37 \text{ FPS} = 2220 \text{ FT/MIN}$$

$R/C = \frac{V}{W}\ (T_A - D)$ at SL $(T_A - D)$ values at velocities V may be read off graph I and (R/C) at SL calculated. $V = V_e$ at SL and T_A = 34000 lbs. $D = T_R$.

SEA LEVEL R/C Vs V

V(F.P.S)	$D=T_R$	$T_A - T_R$	R/C(FPS)	R/C(FT/MIN)	
186	34000	0	0	0	
200	29884	4116	3.3	198	
300	16503	17497	21.0	1260	
400	14161	19839	31.6	1896	
500	15646	18354	36.5	2190	
533	16626	17374	37	2220	$V_{R/C_{MAX}}$ and R/C_{MAX} at S.L.
600	19178	14822	35.4	2124	Note: $V=V_e$ at S.L.
700	24148	9852	27.3	1638	
800	30300	3700	12.0	720	
853	34000	0	0	0	

A plot of R/C Vs V at sea level is found in Fig IIa .

f) At any altitude $V_{e(R/C)_{max}} = \left[\dfrac{T_{A_\sigma} + \sqrt{T_{A_\sigma}^2 + 12AB}}{6A}\right]^{\frac{1}{2}}$

$12AB = 6.02 \times 10^8$

$6A = 0.2676$

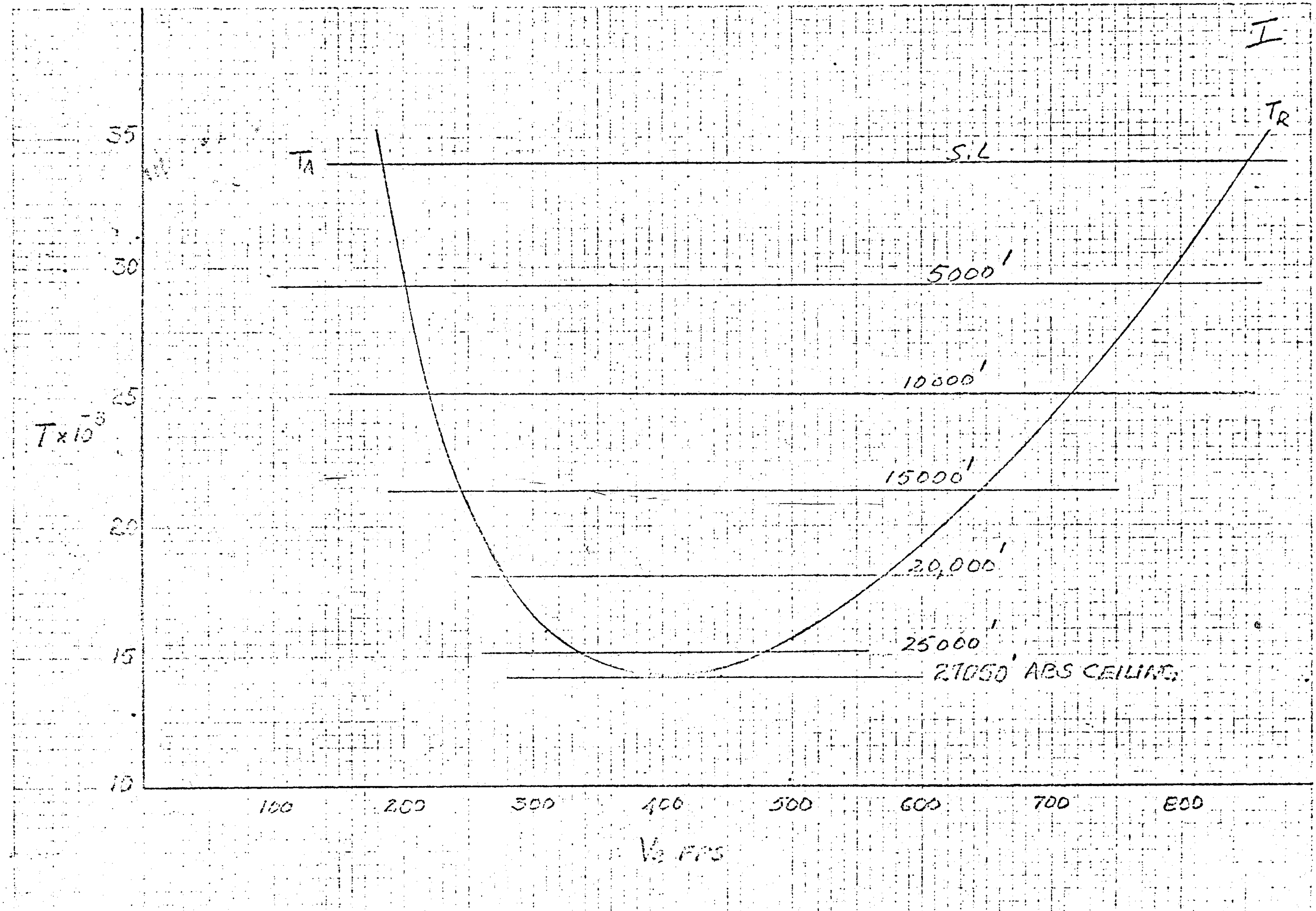
ALT	① T_A	② T_A^2	③ $T_A^2 + 12AB$	④ $\sqrt{T_A^2 + 12AB}$	⑤ $T_A + \sqrt{T^2 + 12AB}$	⑥ ⑤/6A	⑦ $\sqrt{⑥} = V_{e_{R/C_{max}}}$ FPS
SL	34000						533 from (e)
5000	29308	8.59×10^8	14.61×10^8	3.82×10^4	6.751×10^4	25.23×10^4	503
10000	25092	6.30×10^8	12.32×10^8	3.51×10^4	6.02×10^4	22.5×10^4	475
15000	21386	4.57×10^{88}	10.59×10^8	3.254×10^4	5.39×10^4	20.14×10^4	449
20000	18122	3.28×10^8	9.3×10^8	3.05×10^4	4.86×10^4	18.16×10^4	426
25000	15232	2.32×10^8	8.34×10^8	2.888×10^4	4.41×10^4	16.48×10^4	406
27050	14165	2.01×10^8	ABSOLUTE CEILING		$T_A = T_R$		399

ALT	$V_{e(R/C)_{max}}$	$AV_e^2 + \frac{B}{V_e^2} = T_R$	$T_A - T_R$	$\frac{V_e}{W}(T_A - T_R) = (R/C)_{Emax}$	$(R/C)_{max}$ FPS	$= \frac{(R/C)_{Emax}}{\sqrt{\sigma}}$	$V_{(R/C)_{max}} = \frac{V_{e_{R/C}}}{\sqrt{\sigma}}$
SL	533				37	→ From (e)	533 (FPS)
5000	503	15707	13601	27.36	29.5		542
10000	475	15030	10062	19.1	22.24		553
15000	449	14563	6823	12.26	15.46		566
20000	426	14288	3834	6.52	8.93		583
25000	406	14170	1062	1.73	2.586		607
27050	399		0	0	0	ABSOLUTE CEILING	417

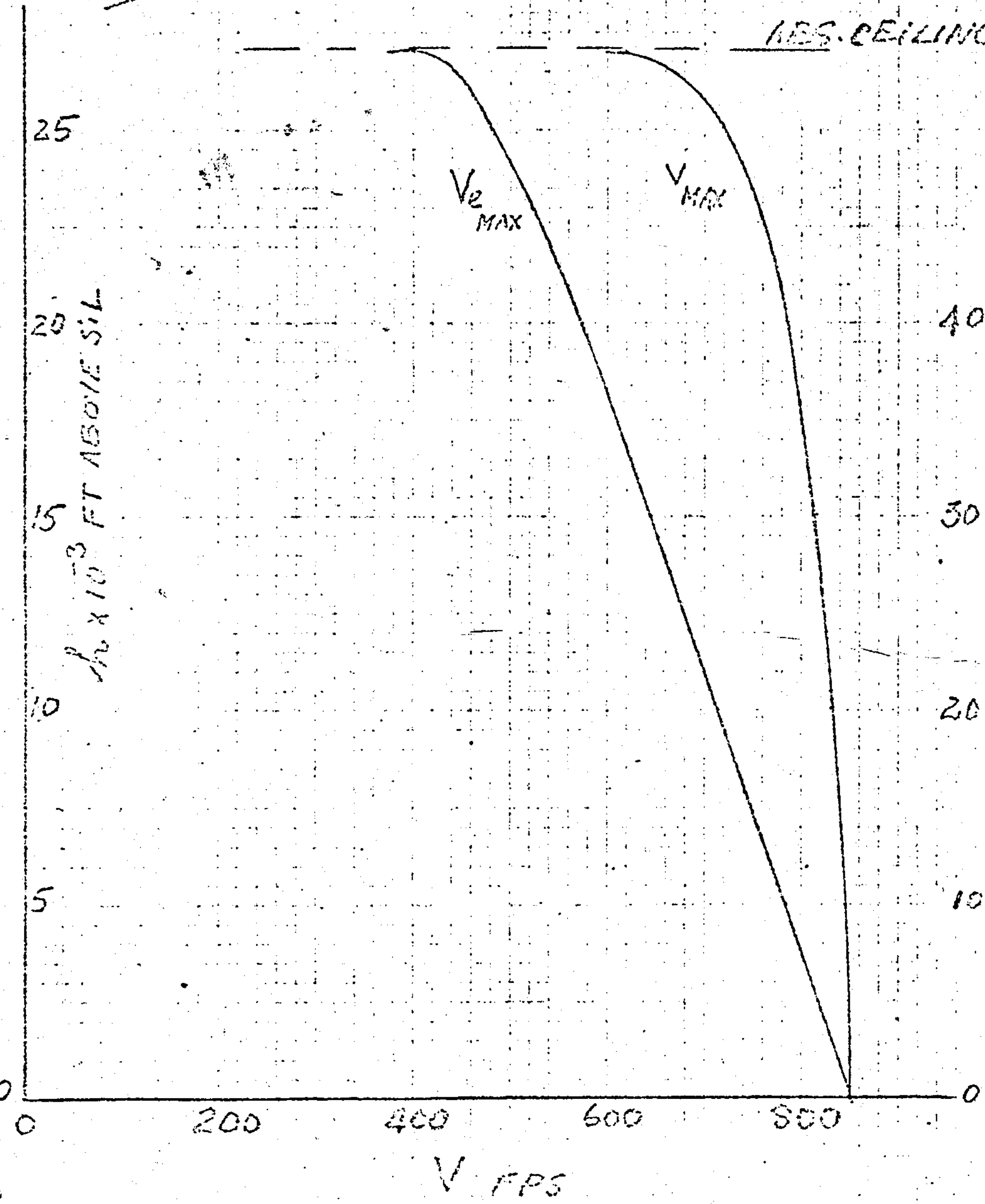
$$(R/C)_{max\ eq.} = V_{e_{(R/C)_{max}}} \left[T_{A_\sigma} - AV_{e_{R/C_{max}}}^{2} - \frac{B}{V_{e_{R/C_{max}}}^{2}} \right] \frac{1}{W}$$

$$(R/C)_{max} = (R/C)_{max\ equiv} \times \frac{1}{\sqrt{\sigma}}$$

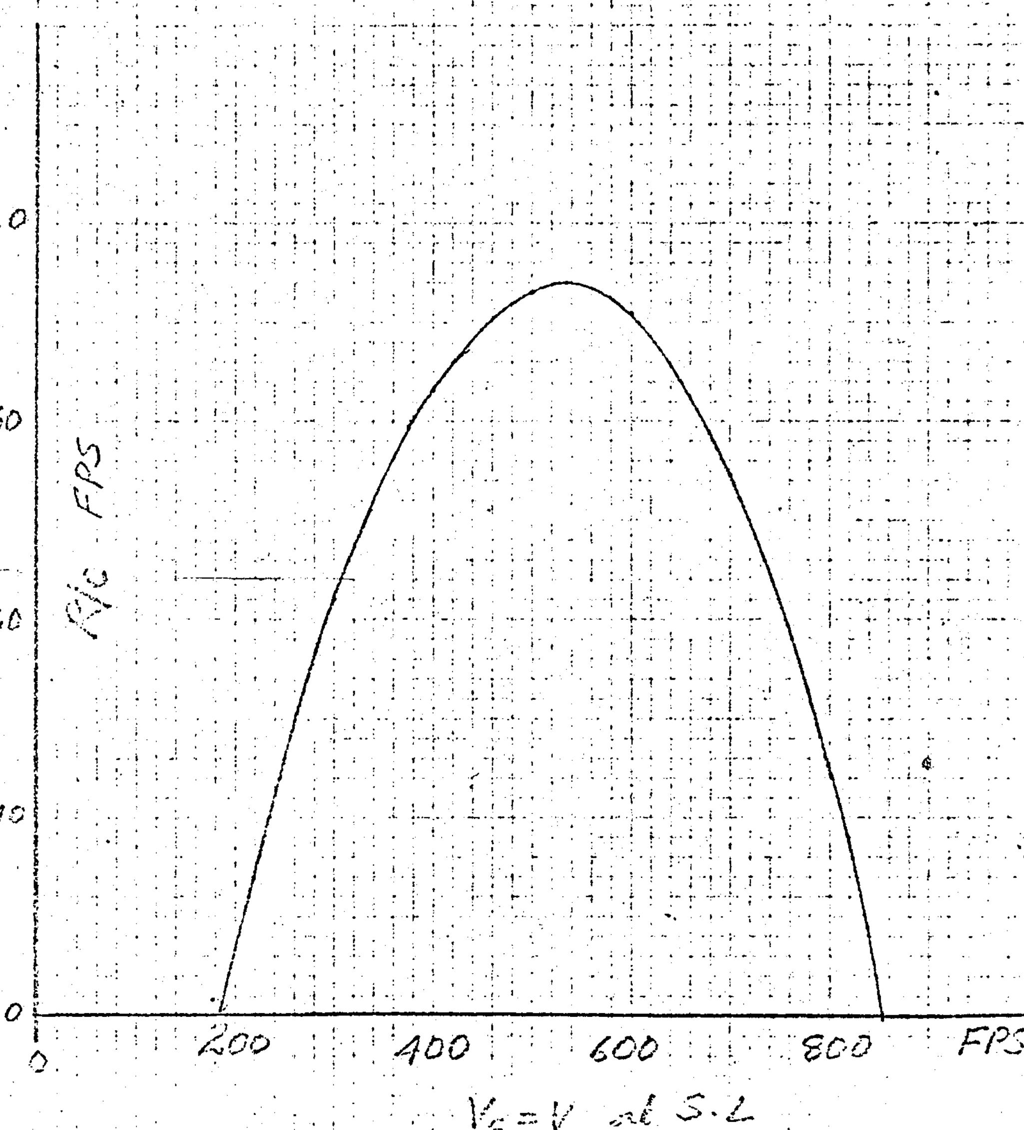
ALT Vs. V_{min}, V_{max}, $V_{R/C_{max}}$ and $(R/C)_{max}$ are plotted in Graph III.

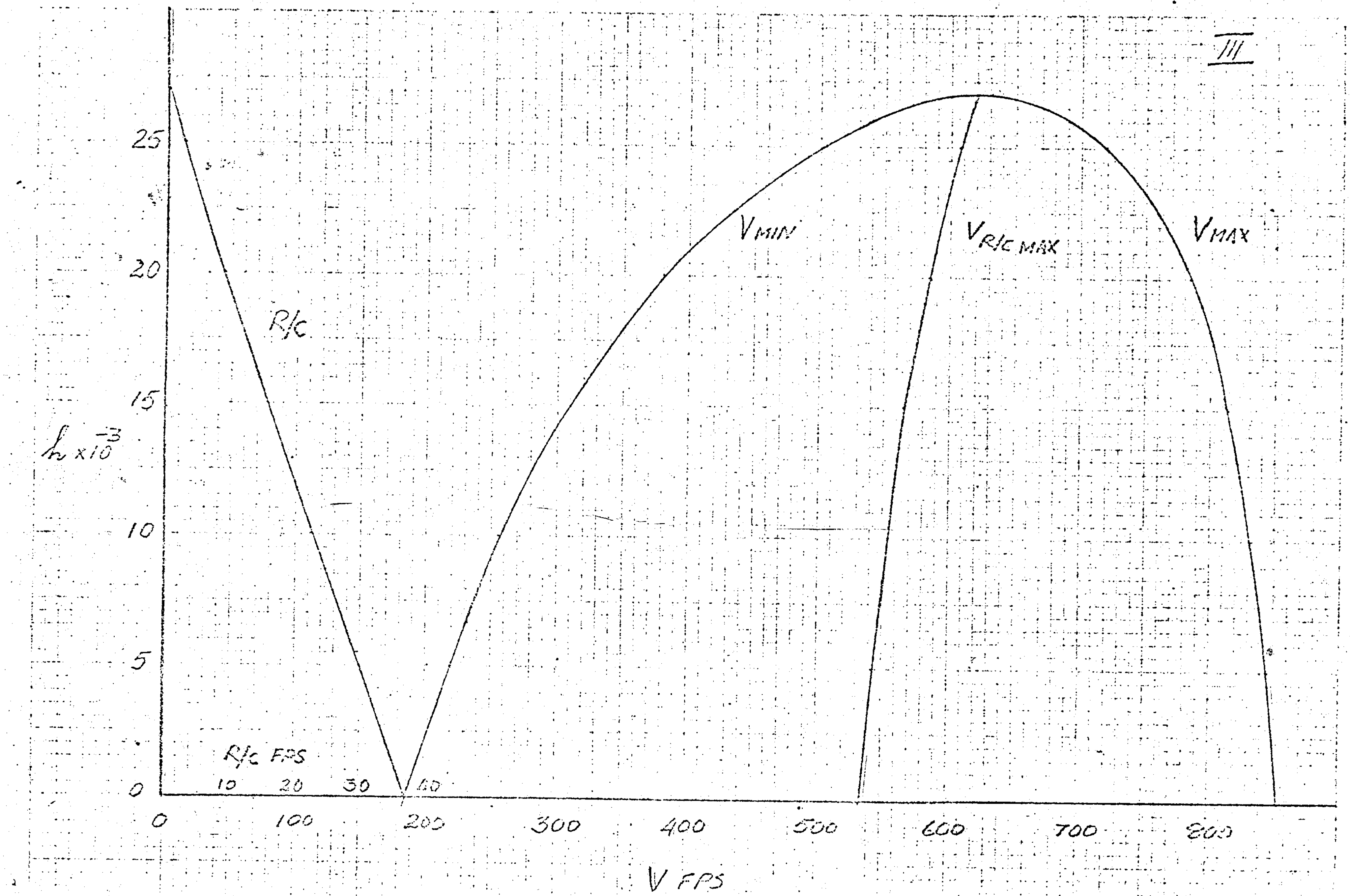

I
T x 10^-3
35
30
25
20
15
10
100
200
300
400
500
600
700
800
V_2 FPS
T_A
T_R
S.L
5000'
10000'
15000'
20,000'
25000'
27050' ABS CEILING

II


ABS. CEILING
Ve MAX
V MAX
h x 10^-3 FT ABOVE S.L.
0
5
10
15
20
25
0
200
400
600
800
V FPS

II A


R/C FPS
0
10
20
30
40
0
200
400
600
800
FPS
Ve = V at S.L.



III
25
20
15
10
5
0
h x10^-3
R/c
V MIN
V R/C MAX
V MAX
R/c FPS
10
20
30
40
0
100
200
300
400
500
600
700
800
V FPS

AE 271

Homework Assignment

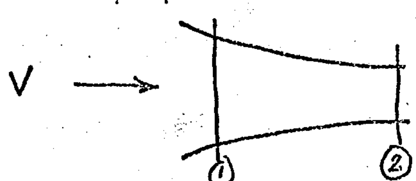
1. You are given that the pressure at the center of a ~~tornado~~ hurricane is 27" of mercury. The Barometer reads 29.92 far from the storm center. Calculate the maximum velocity existing in the Tornado. Assume that the core region rotates as a rigid body, and the transition region does not exist, i.e., $V = \omega r$ in the core, and $V = \Gamma/2\pi r$ outside the core.

2. Calculate the value of Γ as a function of radius. Assume that the radius of the core is one mile. The paths of interest are circles concentric with the origin. (Use data of prob. 1)

3. Determine equations for calculating the pressure as a function of radial distance in the core, and outside of the core, assuming V_{MAX} is known, i.e. determine
   - a.) $p = f_1(r)$ in the core
   - b.) $p = f_2(r)$ outside the core.

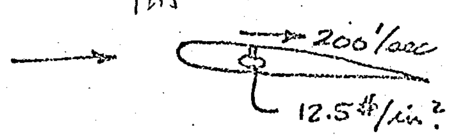
4. The velocity at the top of a circular cylinder of radius r_0 placed in a uniform flow is equal to 2.75 V_∞, and at the bottom is equal to 1.250 V_∞. Take $\rho = \rho_{S.L.}$, determine the lift per unit span L', and the circulation Γ, assuming the flow is perfect.

5. The area at stations 1 and 2 are 15 and 10 sq. ft., respectively. The mass flow rate is 30 lbs./sec, and the density and temperature at station 1 is .00200 slugs/ft.^{3} and $T = 10°F$, respectively. Determine all flow properties at station 2.



6. Assume that A_2 of Problem 5 is the minimum area in the channel. Calculate the maximum mass flow that can be passed, given that $P_1 = 10\ \#/in.^2$ and $T_1 = 10°F$. The flow is isentropic, flying at sea level.

7. The flow velocity over an airplane wing is known to be 200 ft./sec. where the measured pressure is 12.5 #/in.^{2}. Calculate the airplane velocity.



8. The same airplane as used in Problem #7 is now found to have a velocity of 800 ft./sec. (on the wing) However the temperature is measured at the position of the pressure orifice and found to be $T = 10°F$.

$p = 11.5\#/in^2$

9. Calculate the factors $\tau + \delta$ for an unswept untwisted trapezoidal wing of $AR = 6$ and taper ratio $\lambda = 1.0$. Use $n = 1, 3, 5,$ and $a_0 = 2\pi$ 1/deg.

10. Calculate the lift curve slope and induced drag coefficient of an elliptical wing of aspect Ratio $AR = 6$. (Wing uses symmetric airfoil and is untwisted.) If this wing is used on an aircraft whose cruise speed is $V_E = 150$ mph @ $h = 10,000'$, determine C_L, α, and C_D. The airplane has a wing loading of $W/S = 40$ and has a profile drag coefficient of .017.

11. The airplane of problem 1 is modified by using an airfoil section whose angle of zero lift is $\alpha_{L_0} = -1.5°$. Reestimate C_L, C_D, and α.