OTHER PROBLEMS
FROM VIERCK
VIBRATION ANALYSIS

5.27

HW # 14 – ~~5.30~~, 5.42 in 2 pages

SOLUTIONS TO

$$\ddot{x} + \omega^2 x = \cos\omega_0 t$$

$$\ddot{x} + \omega_0^2 x = \cos\omega_0 t$$

and

Problems ~~1.1~~ 1.7, ~~1.2~~ 1.8, ~~1.6~~ 1.12, ~~1.10~~ 1.27 from RAO ~~Second~~ 3rd Ed

3 PAGES

$\ddot{x} + \omega^2 x = \cos \omega_0 t$

Solution to the homogeneous equation is

$$x_h = A\cos \omega t + B \sin \omega t$$

Solution to the non-homogeneous equation

$$x_p = \frac{1}{\omega^2 - \omega_0^2} \cos \omega_0 t$$

$x_{TOT} = A\cos\omega t + B\sin\omega t + \frac{1}{\omega^2-\omega_0^2}\cos\omega_0 t$. If A & B and $\frac{1}{\omega^2-\omega_0^2}$ are finite (bounded values), then x_{TOT} is bounded. This is due to the fact that $\sin\omega t$, $\cos\omega t$ and $\cos\omega_0 t$ vary between -1 and $+1$. Thus x_{TOT} is bounded as $t \to \infty$

$\ddot{x} + \omega_0^2 x = \cos\omega_0 t$

Solution to the homogeneous equation is

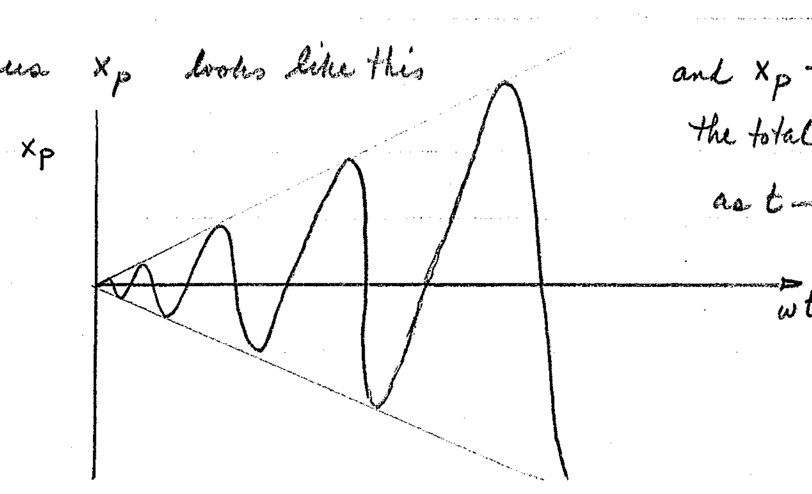
$$x_h = A\cos\omega_0 t + B\sin\omega_0 t$$

Solution to the nonhomogeneous equation is

$$x_p = \frac{t}{2\omega_0} \sin\omega_0 t$$

$x_{TOT} = x_h + x_p = A\cos\omega_0 t + B\cos\omega_0 t + \frac{t}{2\omega_0}\sin\omega_0 t$. If A & B are bounded X_h is bounded as $t \to \infty$. But $\frac{t}{2\omega_0} \to \infty$ as $t \to \infty$

Thus x_p looks like this



and $x_p \to \infty$ as $t \to \infty$. Thus the total solution is unbounded as $t \to \infty$.

HW #2 - 3 PAGES

~~1.15~~, ~~1.24~~, ~~1.25~~, ~~1.29~~

~~1.20~~, ~~1.33~~, ~~1.30~~, ~~1.35~~ in ~~Second Ed.~~

1.32 1.50 1.49 1.55 3rd ed

SOLUTIONS TO HW #3

Problems ~~2.4~~, ~~2.5~~, like ~~2.6~~, like ~~2.12~~ ~~2.13~~ & like ~~2.13~~ ~~2.12~~

like 2.4 2.6 like 2.12, like 2.13, like 2.17 in 3rd ed

in 3 pages

2.26 like 2.59 like 2.70 3rd ed

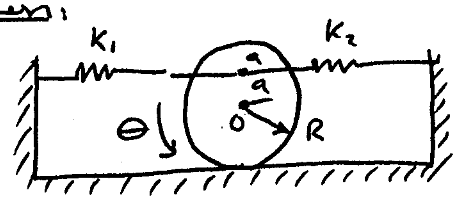
~~2.18~~ 2.38 ~~like 2.31~~ ~~like 2.40~~

HW #4 → ~~2.17~~, ~~2.23~~, ~~2.32~~, ~~2.42~~

in 3 pages

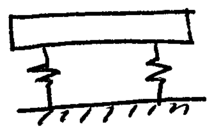
2.69 in 4th

2.60) Given:



Find:
natural frequency of vibration
value of a that maximizes ω_n

Solution:



$K_{eqv} = (k_1 + k_2)$

$$PE_{sys} = \frac{1}{2} k x^2 \Rightarrow \frac{1}{2}(k_1+k_2)(R+a)^2\theta^2$$

$$KE_s = \frac{1}{2} m v^2 \Rightarrow \frac{1}{2} J_r \dot{\theta}^2 \Rightarrow \frac{1}{2}(J_o + mR^2)\dot{\theta}^2$$

$$KE_{sys} = \frac{1}{2}\left(\frac{1}{2}mR^2 + mR^2\right)\dot{\theta}^2$$

$$\frac{1}{2}(k_1+k_2)(R+a)^2\theta^2 + \frac{1}{2}\left(\frac{1}{2}mR^2 + mR^2\right)\dot{\theta}^2 = \text{Const}$$

$$\left[(k_1+k_2)(R+a)^2\dot{\theta}\theta + (1.5mR^2)\ddot{\theta}\dot{\theta}\right] = 0$$

$$\left[(k_1+k_2)(R+a)^2\theta + (1.5mR^2\ddot{\theta})\right]\dot{\theta} = 0$$

$$\omega_n \sqrt{\frac{(k_1+k_2)(R+a)^2}{(1.5mR^2)}}$$

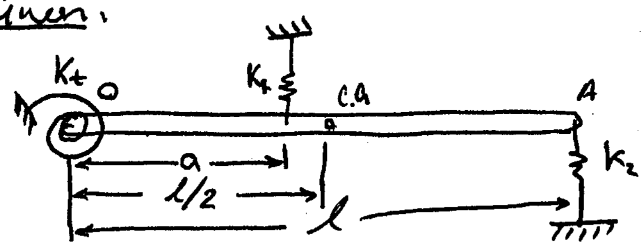
10

when the numerator is maximized, the natural frequency will also be maximized. This is done when

$R = a$

2.71 in 4th

2.62) Given:



Find:

equation of motion
natural frequency

Solution

$PE_{sys} = \cancel{\frac{1}{2}} k_t \cancel{\theta^2}$

$PE_{sys} = \frac{1}{2} k_1 X_1^2 + \frac{1}{2} k_2 X_2^2$

$PE_{equ} = \cancel{\frac{1}{2}} k_1 (a\cancel{\theta})^2 + \cancel{\frac{1}{2}} k_2 (l\cancel{\theta})^2 + \cancel{\frac{1}{2}} \cancel{k_t} \theta^2$

$k_{t\,equ} = k_t + k_1 a^2 + k_2 l^2$

$PE_{sys} = \frac{1}{2}(k_t + k_1 a^2 + k_2 l^2)\theta^2$

where from? show it

$KE_{equ} = \frac{1}{2} J_r \dot{\theta}^2$ $\quad J_r = \frac{ml^2}{3}$

$\frac{1}{2}(ml^2/3)\dot{\theta}^2$ $\quad$ for uniform rigid bar

$\frac{1}{2}(k_t + k_1 a^2 + k_2 l^2)\theta^2 + \frac{1}{2}(ml^2/3)\dot{\theta}^2 = \text{Const.}$

$(k_t + k_1 a^2 + k_2 l^2)\theta\dot{\theta} + \left(\frac{ml^2}{3}\right)\dot{\theta}\ddot{\theta} = 0$

$$\underbrace{\left[(k_t + k_1 a^2 + k_2 l^2)\theta + \left(\frac{ml^2}{3}\right)\ddot{\theta}\right]}_{\text{equation}}\dot{\theta} = 0$$

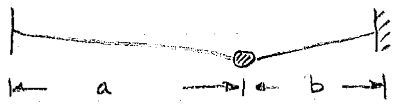
$$\omega_n = \sqrt{\frac{(k_t + k_1 a^2 + k_2 l^2)}{\cancel{3}\frac{m}{3} l^2}}$$

9/10

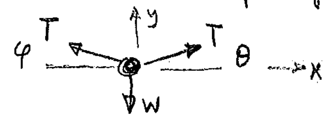
$J_o = J_{CG} + md^2 = J_{CG} + ml^2/4$ $\quad J_{CG} = \frac{1}{12} ml^2$

$= \frac{1}{3} ml^2$

2.26
2.18



look at the Free Body Diagram (FBD) for static equil



$\sum F_y = 0 = T\sin\theta + T\sin\varphi - W$

$W = T\dfrac{\Delta y}{a} + T\dfrac{\Delta y}{b}$

$W = T\left[\frac{1}{a} + \frac{1}{b}\right]\Delta y$

$= k_{eq}\,\Delta y$

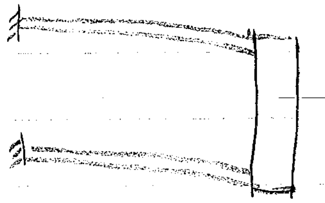
$\sum F_x = T\cos\theta - T\cos\varphi = 0 \Rightarrow \theta \cong \varphi$

From the above $k_{eq} = T\left[\frac{1}{a} + \frac{1}{b}\right]$. If we measure everything from static equilib position, then the DE $\Rightarrow$ $m\ddot{y} + k_{eq}\,y = 0$ or $m\ddot{y} + T\left[\frac{1}{a} + \frac{1}{b}\right]y = 0$

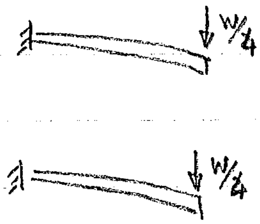
and $\omega_n = \sqrt{\frac{T}{m}\left[\frac{1}{a} + \frac{1}{b}\right]}$ $\quad f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{T}{m}\left[\frac{1}{a} + \frac{1}{b}\right]}$

if measured from the horizontal: $m\ddot{y} + k_{eq}\,y + mg = 0$

2.38
2.22



$\Rightarrow$

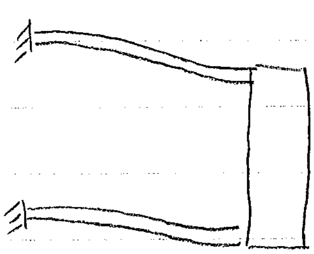


thus $\Delta x = \dfrac{W/4}{k_{eq}} = \dfrac{W/4}{\frac{3EI}{\ell^3}}$

$= \dfrac{W}{12EI/\ell^3}$

Now $\omega_n = \sqrt{\dfrac{k_{eq}}{m}} = \sqrt{\dfrac{k_{eq}\,g}{W}} = \sqrt{\dfrac{g}{\Delta x}} = \sqrt{\dfrac{g \cdot 12EI}{mg\,\ell^3}} = \sqrt{\dfrac{12EI\,g}{W\ell^3}} = \sqrt{\dfrac{g}{\delta_{ST}}}$

and the DE $= m\ddot{x} + \dfrac{12EI}{\ell^3}x = 0$



The displacement for $w(x)$ for a beam in bending is $w(x) = c_1x^3 + c_2x^2 + c_3x + c_4$ with $w(0) = w'(0) = 0$

and $w'(x) = 3c_1x^2 + 2c_2x + c_3$. for $w(0) = w'(0) = 0$

$\Rightarrow c_4 = c_3 = 0$. also for no rotation at $\ell \Rightarrow w'(\ell) = 0$

$w'(\ell) = 3c_1\ell^2 + 2c_2\ell = 0$ or $c_2 = -\dfrac{3c_1\ell}{2}$

thus $w(x) = c_1x^3 - \frac{3}{2}c_1x^2\ell = c_1x^2\left[x - \frac{3}{2}\ell\right]$. Finally the shear must be $=$ to $\frac{W}{4}$ at $x = \ell$ $\Rightarrow$ $V = -EI\,w'''(x) = -EI\left[6c_1\right] = \frac{W}{4} \Rightarrow c_1 = \dfrac{-W}{24EI}$

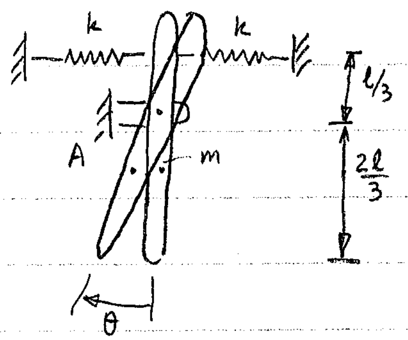
or $w(x) = -\dfrac{W}{24EI}x^2\left[x - \frac{3}{2}\ell\right]$ and $w(\ell) = -\dfrac{W\ell^2}{24EI}\left(-\frac{1}{2}\ell\right) = \dfrac{W\ell^3}{48EI} = \Delta x$

thus if $\Delta x = \dfrac{W}{k_{eq}} \Rightarrow \underline{k_{eq} = 48EI/\ell^3}$

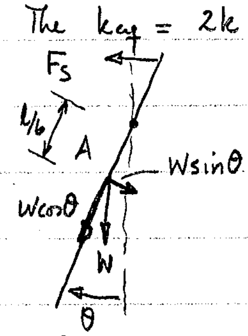
DE $\Rightarrow$ $m\ddot{x} + k_{eq}\,x = 0 \Longrightarrow m\ddot{x} + \left(48EI/\ell^3\right)x = 0$

and $\omega_n = \sqrt{\frac{24EIg}{Wl^3}} = \sqrt{\frac{24EI}{ml^3}}$ · based on $W_n = \sqrt{\frac{g}{\Delta x}} = \sqrt{\frac{g}{\delta_{ST}}}$

like ~~2.49~~ 2.68 in 4th
~~like 2.31~~
~~2.32~~



look at the FBD when system undergoes a rotation θ. The $k_{eq} = 2k$



Both F_s & $W\sin\theta$ provide restoring torques about A

Take $\Sigma T_A = -W\sin\theta\left[\frac{l}{2} - \frac{l}{3}\right] - F_s \cdot \frac{l}{3}\cos\theta$ $\qquad F_s = k_{eq} \cdot \frac{l}{3}\sin\theta$

$= -W\sin\theta\left[\frac{l}{6}\right] - 2k\left(\frac{l}{3}\right)^2 \sin\theta\cos\theta$

inertia term $I_A\ddot{\theta} = \left[I_{CG} + m\left(\frac{l}{2} - \frac{l}{3}\right)^2\right]\ddot{\theta}$ and in class for a slender rod we showed $I_{CG} = \frac{ml^2}{12}$

$= \left[\frac{ml^2}{12} + \frac{ml^2}{36}\right]\ddot{\theta} = \frac{ml^2}{9}\ddot{\theta}$

$\therefore$ DE is $\frac{ml^2}{9}\ddot{\theta} + \frac{Wl}{6}\sin\theta + \frac{2}{9}kl^2\sin\theta\cos\theta = 0$

for small vibs $\sin\theta \to \theta \quad \cos\theta \to 1 \quad \Rightarrow \quad \frac{ml^2}{9}\ddot{\theta} + \left(\frac{Wl}{6} + \frac{2kl^2}{9}\right)\theta = 0$

$$\omega_n = \sqrt{\frac{\frac{Wl}{6} + \frac{2kl^2}{9}}{ml^2/9}} = \sqrt{\frac{6Wl + 8kl^2}{4ml^2}} = \sqrt{\frac{3W + 4kl}{2ml}}$$

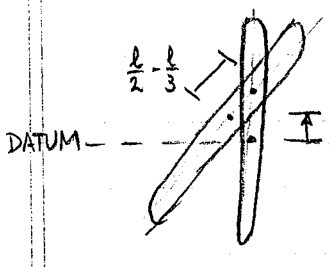
with the given data $\omega_n = \sqrt{\frac{3(10)(9.81) + 4(2000)(5)}{2(10)(5)}} = 20.073$ rad/s

~~2.42~~ ~~like 2.40~~ like ~~2.70~~ 2.79 in 4th

the KE for the system is $\frac{1}{2}I_A\dot{\theta}^2$

the PE for the springs is given by $2\left[\frac{1}{2}k\Delta x^2\right]$ where $\Delta x = \frac{l}{3}\sin\theta$

the PE for the rod is given by $W[\Delta y]$ where datum line is at the CG location when the rod hangs vertically



Δy for the weight is $\left(\frac{l}{2}-\frac{l}{3}\right)(1-\cos\theta)$

Now $PE + KE = \text{const} \Rightarrow \frac{1}{2} I_A \dot{\theta}^2 + 2\left[\frac{1}{2} k \left(\frac{l}{3}\sin\theta\right)^2\right] + W\left(\frac{l}{2}-\frac{l}{3}\right)(1-\cos\theta) = C$

Now take $\frac{d}{dt}(PE+KE) = 0 \Rightarrow \frac{1}{2} I_A \cdot 2\dot{\theta}\ddot{\theta} + 2\cdot\frac{1}{2}k\cdot 2\frac{l^2}{9}\sin\theta\cos\theta\,\dot{\theta} + W\left(\frac{l}{2}-\frac{l}{3}\right)(\sin\theta\,\dot{\theta}) = 0$

or

$$\dot{\theta}\left[I_A\ddot{\theta} + 2k\frac{l^2}{9}\sin\theta\cos\theta + \frac{Wl}{6}\sin\theta\right] = 0 \qquad \text{where } I_A = \frac{ml^2}{9}$$

$\dot{\theta} = 0$ only implies $\theta = \text{const}$, & for this system to be in equilibrium $\theta = 0 \Rightarrow$ rod remains vertical for all time. This is the trivial solution, so

$$I_A\ddot{\theta} + 2k\frac{l^2}{9}\sin\theta\cos\theta + \frac{Wl}{6}\sin\theta = 0$$ as before (in problem $\cancel{2.32}$ 2.68)

To use Rayleigh's Method assume 1) vibrations are small and harmonic

thus $KE = \frac{1}{2} I_A \dot{\theta}^2$

$PE = 2\left[\frac{1}{2}k\left(\frac{l}{3}\sin\theta\right)^2\right] + W\left(\frac{l}{2}-\frac{l}{3}\right)(1-\cos\theta) \Rightarrow k\left(\frac{l}{3}\theta\right)^2 + \frac{Wl}{6}\frac{\theta^2}{2}$

Note: $\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \dots \qquad 1-\cos\theta = \frac{\theta^2}{2!}$ + higher order terms.

Now let $\theta = \Theta\sin(\omega_n t + \phi)$

$\dot{\theta} = \Theta\omega_n\cos(\omega_n t + \phi)$

$KE = \frac{1}{2} I_A \left(\omega_n^2\Theta^2\cos^2(\omega_n t+\phi)\right); \qquad KE_{max} = \frac{1}{2} I_A \Theta^2\omega_n^2$

$PE = \left[k\frac{l^2}{9} + \frac{Wl}{12}\right]\theta^2 = \left(\frac{kl^2}{9} + \frac{Wl}{12}\right)\left(\Theta^2\sin^2(\omega_n t+\phi)\right); \quad PE_{max} = \left(\frac{kl^2}{9} + \frac{Wl}{12}\right)\Theta^2$

$$PE_{max} = KE_{max} = \frac{1}{2}\frac{ml^2}{9}\omega_n^2\Theta^2 = \left(\frac{kl^2}{9} + \frac{Wl}{12}\right)\Theta^2$$

or $$\omega_n = \sqrt{\frac{kl^2/9 + \frac{Wl}{12}}{\frac{ml^2}{18}}} = \sqrt{\frac{4kl + 3W}{2ml}}$$ as before

HW #7

like like
3.1 , 3.2 , 3.4 , 3.6

in 1 page

HW #8

like 3.10 ~~like 3.15~~ ~~like 3.14~~

Problems ~~3.7~~, 3.8, ~~3.14~~, ~~3.16~~

in 2 pages

like 3.21 like 3.25

Y730

HW # 10 in 2 pages

~~3.26~~, like ~~3.29~~ ~~3.27~~, 3.30, like 3.30 ~~3.31~~ ~~other 2 From first edition~~

like 3.45 like 3.49 like 3.44

Solutions to HW# 5

~~like 2.43~~ ~~2.50~~ ~~2.51~~ ~~like 2.73~~

~~2.48~~, ~~2.53~~, ~~2.54~~, ~~2.55~~

like 2.75, 2.83, 2.86, like 2.118 3rd ed

in 2 pages

Solutions to HW # 6

~~like 2.56~~ ~~2.57~~ ~~like 2.60~~

~~2.60~~ , ~~2.62~~ to ~~2.64~~

in 3 pages

like 2.98 2.100 , 2.102 , like 2.83

Undamped system consists of a body weighing 14.475 lb supported by elastic member having $k = 15$ lb/in. Acted on by harmonic force whose max is 24 lb. find $\frac{W}{k}$ P_0/k

static displ. the spring due to the maximum impressed force. (c) Obtain the amplitude of the forced motion for the following forced frequencies f_f: 1.5, 3, 3.25, and 7.5 Hz.

4-2. The mass of an undamped mass-spring system is subjected to a harmonic force having a maximum value of 45 lb and a period of 0.25 sec. The body weighs 2.5 lb and the spring constant is 15 lb/in. Determine the amplitude of the forced motion.

4-3. An undamped system is composed of mass $m = 1.1$ kg and spring $k = 4400$ N/m. It is acted on by a harmonic force having maximum value of 440 N and a frequency of 180 cycles/min. Determine the amplitude of the forced motion.

4-4. A weight of 8 lb is suspended from a spring having a constant of 25 lb/in. The system is undamped but the mass is subjected to a harmonic force with a frequency of 7 Hz, resulting in a forced-motion amplitude of 1.59 in. Determine the maximum value of the impressed force.

4-5. The body of an undamped system is driven by a harmonic force having an amplitude of 36 N and a frequency of 450 cycles/min. The body has a mass of 8 kg and exhibits a forced-displacement amplitude of 4.06 mm. Obtain the value of the spring modulus.

4-6. An undamped system consists of an 8.75-kg mass suspended from a spring having a constant of 3500 N/m. A harmonic force acting on the mass and having a maximum value of 187 N causes the system to vibrate with a forced amplitude of 7.6 cm. Determine the frequency of the impressed force.

4-7. A weight W is suspended from a spring having a constant of 25 lb/in. The system is undamped, but the mass is driven by a harmonic force having a frequency of 2 Hz and a maximum value of 10 lb, causing a forced-motion amplitude of 0.5 in. Determine the value of weight W.

4-8. The mass of an undamped system is acted on by force $P = P_0 \cos \omega_f t$, having a maximum value of 10 lb. The spring constant is 3 lb/in. The forced frequency can be any integral multiple of the natural frequency of the system other than 1, thereby avoiding resonance. Only a single forced frequency can exist at a time. Determine the maximum amplitude of the forced motion that could occur.

4-9. Express the relation for the complete motion of an undamped system, excited by a force $P = P_0 \sin \omega_f t$, for the initial conditions of $x = 0$, $\dot{x} = 0$, at $t = 0$. $(r \neq 1.)$

4-10. Write the relation that defines the complete motion of an undamped system, subjected to the harmonic force $P = P_0 \sin \omega_f t$, for the initial conditions of $x = 0$, $\dot{x} = \dot{x}_0$, at $t = 0$. $(r \neq 1.)$

4-11. Express the equation for the complete motion of an undamped system, subjected to the forcing condition $P = P_0 \sin \omega_f t$, for the initial conditions of $x = x_0$, $\dot{x} = 0$, at $t = 0$. $(r \neq 1.)$

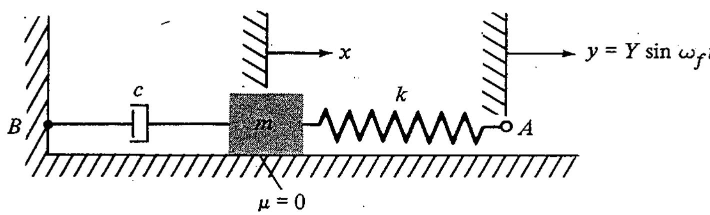
4-12. Write the complete solution for the motion of an undamped system, driven by the force $P = P_0 \sin \omega_f t$, for the initial conditions of $x = x_0$, $\dot{x} = \dot{x}_0$, at $t = 0$. $(r \neq 1.)$

4-13. An undamped system consists of a weight of 19.3 lb and a spring having a modulus of 10 lb/in. The system mass is driven at resonance by harmonic force $P = P_0 \sin \omega_f t$ having a maximum value of 4 lb. Determine the amplitude of the forced motion at the end of (a) $\frac{1}{2}$ cycle, (b) $2\frac{1}{2}$ cycles, (c) $4\frac{1}{2}$ cycles, and (d) $6\frac{1}{2}$ cycles.

4-14. An undamped system is composed of a 4.375-kg mass and a spring having a modulus of 3500 N/m. The mass is driven at resonance by a harmonic force having a maximum value of 14 N. Determine the amplitude of the forced motion at the end of (a) 1 cycle, (b) 5 cycles, and (c) 10 cycles.

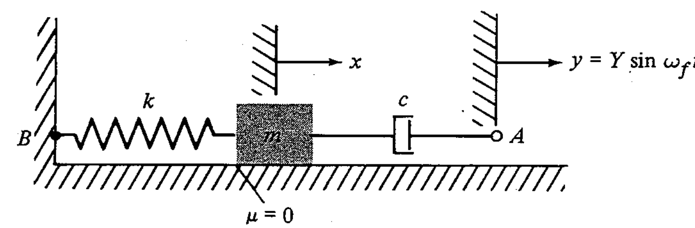
4-15. An undamped system is harmonically forced near resonance, resulting in a beating condition. The natural frequency of the system is 1765 cycles/min, and the forced frequency is 1752 cycles/min. Determine the beat period of the motion.

4-16. For the arrangement shown, x represents the absolute displacement of the mass m, and y is the absolute displacement of the end A of the spring k. Point A is moved according to the relation $y = Y \sin \omega_f t$. (a) Construct the free-body diagram for m. (b) Write the differential equation of motion for m. (c) Obtain the solution for the steady-state motion of m. (d) Determine the relation for the impressed force at A. (e) Obtain the expression for the force transmitted to the support at B.



Problem 4-16

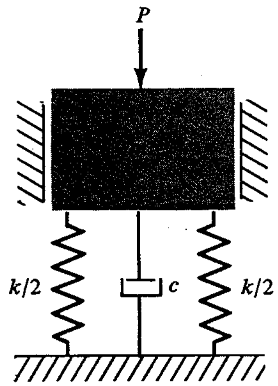
4-17. For the arrangement shown, x represents the absolute displacement of the mass m, and y is the absolute displacement of the end A of the dashpot c. The motion of point A is defined by $y = Y \sin \omega_f t$. (a) Construct the free-body diagram for m. (b) Write the differential equation of motion for m. (c) Obtain the solution for the steady-state motion of m. (d) Determine the relation for the impressed force at A. (e) Obtain the expression for the force transmitted to the support at B.



Problem 4-17

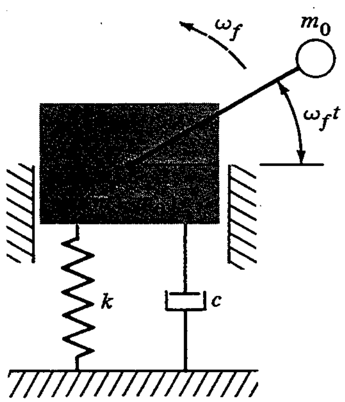
4-18. The spring k and the dashpot c of the accompanying diagram are fastened together at A; x represents the absolute displacement of m, and y is the absolute displacement of the point A. The motion of A is defined by $y = Y \sin \omega_f t$. (a) Construct the free-body diagram for m. (b) Write the differential equation of motion for m. (c) Obtain the solution for the steady-state motion of m. (d) Determine the relation for the impressed force at A.

4-31. A machine having a mass of 70 kg is mounted as shown on springs having a total stiffness of 33 880 N/m. The damping factor is $\zeta = 0.20$. A harmonic force $P = 450 \sin 13.2t$ (where P is in newtons and t is in seconds) acts on the mass. For the sustained or steady-state vibration, determine (a) the amplitude of the motion of the machine, (b) its phase with respect to the exciting force, (c) the transmissibility, (d) the maximum dynamic force transmitted to the foundation, and (e) the maximum velocity of the motion.



Problem 4-31

4-32. A machine having a total weight of 96.5 lb is mounted as shown on a spring having a modulus of 900 lb/in. and is connected to a dashpot with a damping *factor* of 0.25. The machine contains a rotating unbalance ($w_0 e$) of 5 lb in. If the speed of rotation is 401.1 rpm, obtain (a) the amplitude of the steady-state motion, (b) the maximum dynamic force transmitted to the foundation, and (c) the angular position of the arm when the structure goes upward through its neutral position.



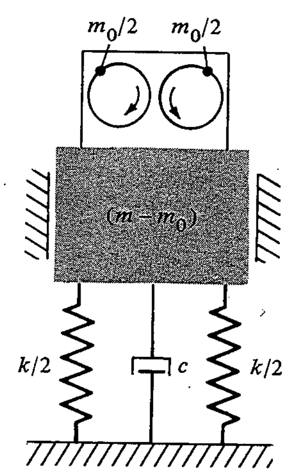
Problem 4-32

4-33. A machine having a total weight of 128.67 lb contains a part that rotates at 859.44 rpm. The machine is supported equally on four identical springs and is

connected to a dashpot for which the damping constant is 10 lb sec/in. In operation the machine has a steady amplitude of 0.116 in. and transmits a maximum dynamic force of 174 lb to the supporting base. (a) Determine the modulus for the springs. (b) Calculate the unbalanced moment ($w_0 e$) for the rotating part.

4-34. A machine with a total mass of 50 kg contains a shaft mechanism that rotates at 1800 rpm. The machine is supported by springs for which the equivalent modulus is 20 000 N/m but the damping constant is unknown. Due to the rotating unbalance, the steady-state amplitude is 1.32 cm and the maximum dynamic force carried by the base is 278 N. Determine (a) the damping constant and (b) the unbalanced moment $m_0 e$.

4-35. A typical vibration exciter is composed of two eccentric masses that rotate oppositely, as represented in the top of the accompanying figure. Thus an oscillation is produced in the vertical direction only, since the horizontal effect cancels. This device is used to determine the vibrational characteristics of the structure to which it is attached. The unbalance ($ew_0/2$) of each exciter wheel is 3 lb in. The total arrangement has a weight $mg = 200$ lb. The exciter speed (of eccentric rotation) was adjusted until a stroboscope showed the structure to be moving upward through its equilibrium position at the instant the eccentric weights were at their top position. The exciter speed then was 840 rpm, and the steady amplitude was 0.75 in. Determine (a) the natural frequency of the structure and the damping factor for the structure. If the speed were changed to 1260 rpm, (b) obtain the steady-state amplitude of the structure and (c) the angular position of the eccentrics as the structure moves upward through its equilibrium position.



Problem 4-35

4-36. A small compressor weighs 69 lb and runs at 875 rpm. It is to be supported by four springs (equally) and no damping is provided. Design the springs, thus determining k, so that only 15% of the shaking force is transmitted to the supporting foundation or structure.

4-37. Solve Prob. 4-36 considering that damping is also to be included such that the damping factor will be 0.4 for the system. Equal damping at each spring will be provided by a plug or block of energy-absorbing material. Determine the required damping constant as well as the spring constant.

4-38. A refrigerator compressor unit weighs 179 lb and operates at 590 rpm. The maximum dynamic force imposed on the compressor is 20 lb. The unit is to be supported equally by three springs and is to be undamped. (a) Determine the required spring constant if 10% of the dynamic force is to be transmitted to the supporting base. (b) Determine the clearance that must be provided for the unit.

4-39. Solve Prob. 4-38, considering that *each* spring contains a viscous damping absorber having a damping constant of 0.8333 lb sec/in.

4-40. A small machine is known to contain a rotating unbalance on its main shaft. • The machine weighs 65 lb and when placed on elastic vibration isolators (which serve as a viscous damping member as well as the elastic support), the static equilibrium displacement is 2.166 in. Also, when the machine is displaced further and released, the subsequent vibration diminishes from an amplitude of 2.350 in. to 0.135 in. in exactly 3 cycles. Operating the machine at resonance produces a sustained amplitude of 0.0193 in. (a) Determine the damping constant for the system. (b) Considering that the shaft diameter is 3 in. so that the eccentric moment arm is 1.5 in., calculate the unbalanced weight. (c) Determine the steady amplitude at an operation speed of twice the resonant speed.

4-41. A damped system with a rotating unbalance is composed of a 96.5-lb body and a spring having a constant of 80 lb/in. but the damping constant is unknown. When operated at resonance, the sustained amplitude is 2.143 in. If the speed of operation is greatly increased, the steady amplitude eventually approaches the value of 0.3572 in. Determine the damping constant for the system.

4-42. The support for a damped system (see Fig. 4-26) is oscillated harmonically with a frequency of 75 cycles/min, causing the mass to vibrate with a steady-state amplitude of 0.6 in. and a maximum dynamic force of 48.75 lb to be transmitted to the support. For the system, the spring modulus is 13 lb/in. and the damping constant is 0.454 lb sec/in. Determine (a) the natural frequency of the system, (b) the mass of the system, and (c) the amplitude of oscillation of the support.

4-43. The support for a damped system is oscillated harmonically. The body weighs 20 lb and the spring constant is 14 lb/in. When the support is oscillated at a rate of 300 cycles/min, the sustained amplitude of the body is 0.793 in. (a) Determine the maximum dynamic force carried by the support. (b) If the support is oscillated at resonance with an amplitude of 1.121 in., the body has a steady amplitude of 3.069 in. Obtain the damping constant for the system.

4-44. A mass-spring system, arranged horizontally, is damped only by Coulomb friction (refer to Fig. 4-27). The system is driven by a harmonic force having a frequency of 54.356 cycles/min and a maximum value of 30 lb, causing the body to oscillate with a steady amplitude of 3.09 in. The spring constant is 12 lb/in. and the body weighs 28.95 lb. Determine the average coefficient of sliding friction for the surfaces involved.

4-45. A machine is designed to produce an abrasive action on a horizontal surface in

ing 20 lb fixed to an elastic member having a stiffness coefficient of 8 lb/in. The average coefficient of sliding friction of the body against the surface is 0.3. A harmonic force having a maximum value of 15 lb actuates the body at a frequency of 0.9 of the resonant value. Determine the total excursion or stroke of the member.

4-46. A certain mass-spring system exhibits hysteresis damping only. The spring modulus is 310 lb/in. When excited harmonically at resonance, the steady amplitude is 1.540 in. for an energy input of 32 in. lb. When the resonant energy input is increased to 87 in. lb, the sustained amplitude changes to 2.464 in. Determine the hysteresis coefficient β and the exponent γ.

4-47. A small built-up structure shows solid damping characteristics with $\gamma = 2$. A 1000-lb load on the structure causes a static displacement of 2.500 in., and a supported machine that is subjected to a harmonic force having a constant amplitude of 75 lb produces a resonant amplitude of 6.818 in. (a) Determine the hysteresis damping coefficient β and (b) the energy dissipated per cycle by hysteresis at resonance. (c) Calculate the steady amplitude at twice the resonant frequency and (d) at one-half the resonant frequency.

4-40 2

$k = \frac{W}{\Delta} = 30\,\text{lb/in}$ $\qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{W}} = 13.347\,\text{rad/s}$ 2

$\delta = \frac{1}{n}\ln\left(\frac{X_0}{X_n}\right) = \frac{1}{3}\ln\left(\frac{2.35}{.135}\right) = .9523$ $\qquad c_c = 2m\omega_n = 4.495\,\text{lb-sec/in}$

$\zeta = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} = .14985$ $\qquad c = \zeta c_c = .6736\,\text{lb-sec/in}$ 2

$X = \frac{W_0 e}{W}\frac{r^2}{\sqrt{\quad}} = .0193\,\text{in} = \frac{W_0 e}{W}\frac{1}{2\zeta} \Rightarrow W_0 \doteq .251\,\text{lb}$ 2

$= W_0 \frac{1.5}{65} \cdot \frac{1}{.2997}$

$X\big|_{r=2} \doteq .00756\,\text{in}$ 2

HW#8

4-2, 6, 13

4-2

$P = P_o \sin \omega_f t$: $P_o = 45\,lb$ $\tau = .25\,sec = \frac{2\pi}{\omega_f}$; $W = 2.5\,lb$ $k = 15\,lb/in$

find $X = \frac{X_o}{1-r^2}$ $r < 1$

① $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{W}} = \sqrt{\frac{15 \cdot 32.2 \cdot 12}{2.5}} = 48.15\ rad/sec$ } $r = \frac{\omega_f}{\omega} = .522$

② $\omega_f = \frac{2\pi}{\tau} = 8\pi = 25.133\ rad/sec$

$X = \frac{P_o/k}{1-r^2} = \frac{3\,in}{.7275} = 4.123\,in$

4-6 $m = 8.75\,kg$ $k = 3500\,N/m$ $P_o = 187\,N$ $X = 7.6\,cm$

find ω_f (or f_f)

$\omega = \sqrt{\frac{k}{m}} = 20\ rad/sec$; $X_o = P_o/k = .0534\,m = 5.34\,cm$ $MF = 1.423$

Since $MF > 1$ there are 2 solutions. First assume $r < 1$

$X = \frac{X_o}{1-r^2}$ => $1 - r^2 = \frac{X_o}{X}$ => $r = \sqrt{1 - \frac{X_o}{X}} = .545$

$r = \frac{\omega_f}{\omega}$ => $\omega_f = r\omega = 10.9\ rad/sec$ $f_f = \frac{\omega_f}{2\pi} = 1.735\,Hz$

Now assume $r > 1$ => $X = \frac{X_o}{r^2-1}$ => $r^2 - 1 = \frac{X_o}{X}$ => $r = \sqrt{1 + \frac{X_o}{X}} = 1.305$

$r = \frac{\omega_f}{\omega}$ => $\omega_f = r\omega = 26.097\ rad/sec$ $f_f = \frac{\omega_f}{2\pi} = 4.153\,Hz$

4-13

$W = 19.3\,lb$, $k = 10\,lb/in$; $P = P_o \sin \omega_f t$ where $P_o = 4\,lb$

system is in resonance ~~r = 1. From work in class~~

$x_p = -\frac{X_o \omega_f t}{2} \cos \omega_f t$ $X_o = P_o/k = .4\,in$

$\omega = \omega_f = \sqrt{\frac{kg}{W}} = 14.149\ rad/sec$

@ $\frac{1}{2}$ cycle $\omega_f t = \pi$ $x_p = -\frac{(.4)}{2}\pi \cos(\pi) = .2\pi = .628\,in$

@ 2½ cycles $\omega_f t = 5\pi$ $x_p = -\frac{(.4)}{2} 5\pi \cos(5\pi) = \pi = 3.142\,in$

@ 4½ cycles $\omega_f t = 9\pi$ $x_p = -\frac{(.4)}{2} 9\pi \cos(9\pi) = 1.8\pi = 5.655\,in$

@ 6½ cycles $\omega_f t = 13\pi$ $x_p = -\frac{(.4)}{2} 13\pi \cos(13\pi) = 2.6\pi = 8.168\,in$

HW # 9

4-21 $m = 8.75\ kg$; $k = 1750\ N/m$; $c = 37.13\ N\,sec/m$. $P = P_0 \sin\omega_f t \Rightarrow P_0 = 220N$

$f_f = 4.50\ Hz$.

$\omega_n = \sqrt{\frac{k}{m}} = 14.142\ rad/sec$ $\quad c_c = 2m\omega_n = 247.49\ \frac{N\text{-}sec}{m}$ $\quad \zeta = \frac{c}{c_c} = .15$

since $\zeta < 1$ system is underdamped

$\omega_d = \sqrt{1-\zeta^2}\,\omega_n = 13.982\ rad/sec$

transient $x_c = X' e^{-\zeta\omega_n t}\sin(\omega_d t + \phi) = X' e^{-2.1617t}\sin(13.982t + \phi)$ ✓

$X_0 = P_0/k = .1257\ m$; note X', ϕ obtained from initial conditions

$f_f = 4.50\ Hz = \frac{\omega_f}{2\pi} \Rightarrow \omega_f = 28.2741\ rad/sec \Rightarrow r = \frac{\omega_f}{\omega_n} = 2.0$

$X = \frac{X_0}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = .0411\ m$ $\quad \tan\psi = \frac{2\zeta r}{1-r^2} = -.2$ $\quad \psi = -11.31° = -.1974\ rad$

steadystate $x_p = .0411 \sin(28.274t + .1974)$ ✓; $x_{total} = x_c + x_p$

4-27 $k = 24\ lb/in$; $c = .88\ lb\text{-}sec/in$; $P_0 = 15\ lb$ when $r = 1$ $x_p = 2.8409\ in$

find ζ ; ω_n ; ω_d note that $x_p|_{r=1} = X_{res}$

$X_{res} = \frac{X_0}{2\zeta} \Rightarrow \underline{X_0 = P_0/k = .625 in} \Rightarrow \zeta = \frac{1}{2}\frac{X_0}{X_{res}} = \frac{1}{2}\left(\frac{.625}{2.8409}\right) = \underline{.11 = \zeta}$

$\zeta = \frac{c}{c_c}$ $\therefore$ $\underline{c_c = \frac{c}{\zeta} = 8\ lb\text{-}sec/in}$; now $\omega_n^2 = k/m \Rightarrow k = m\omega_n^2$ & $c_c = 2m\omega_n$

Thus $\frac{k}{c_c} = \frac{\omega_n}{2}$ or $\underline{\omega_n = \frac{2k}{c_c} = 6\ rad/sec}$ $\quad f_n = \frac{\omega_n}{2\pi} = .955\ Hz$

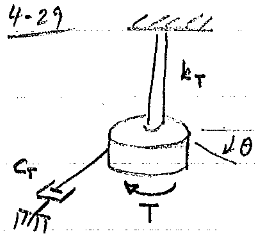
Now $\underline{\omega_d = \omega_n\sqrt{1-\zeta^2} = 5.964\ rad/sec}$ $\quad f_d = .949\ Hz$

4-28 gives when $r = 2$ $\quad X_{r=2} = \frac{1}{10} X_{res}$

$$X_{r=2} = \frac{X_0}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \underbrace{\frac{X_0}{\sqrt{(-3)^2 + (4\zeta)^2}}} = \frac{1}{10} X_{res} = \underbrace{\frac{1}{10}\frac{X_0}{2\zeta}}$$

Solving those terms indicated by ($\sim\sim$) gives $\zeta = .1531$

4-29



$k_T = 60000$ lb in/rad ; $I = 24$ lb in sec^2/rad; $c_T = 840$ lb in sec/rad

the ODE is $I\ddot{\theta} + c_T\dot{\theta} + k_T\theta = T = T_0 \sin \omega_f t$

also for given $T_0 = 2700$ lb-in results in $\theta = 3.368^\circ = .0588$ rad

In this case everything we said about the linear case holds here

Thus $MF = \dfrac{\theta}{\theta_0}$ where $\theta_0 = T_0/k_T = .045$ rad $\Rightarrow$ $MF = 1.3067$

now $\sqrt{k_T/I} = \omega_n = 50$ rad/sec ; $c_{T_c} = 2I\omega_n = 2400$ lb-in sec/rad $\Rightarrow \zeta = \dfrac{c}{c_c} = .35$

$$MF = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = 1.3067; \text{ solve for } r \Rightarrow \quad r = 1.0722,\ r = .6.$$

$$r^4 - 1.51r^2 + .414 = 0$$

For $r = 1.0722 = \omega_f/\omega_n \Rightarrow \omega_f = 1.0722\omega_n = 53.61$ rad/sec $\quad f_f = \dfrac{\omega}{2\pi} = 8.532$ Hz

$r = .6 = \omega_f/\omega_n \Rightarrow \omega_f = .6\omega_n = 30$ rad/sec $\Rightarrow f_f = \dfrac{\omega}{2\pi} = 4.775$ Hz

TO OBTAIN MF_{RES} & MF_{MAX}:

Now $MF_{RES} = \dfrac{\theta_{RES}}{\theta_0} = \dfrac{1}{2\zeta} = 1.429$ @ $r = 1$; $MF_{MAX} = MF_{RES} \cdot \dfrac{1}{\sqrt{1-\zeta^2}} = 1.525$ @ $r = .869$

$= \dfrac{1}{2\zeta} \cdot \dfrac{1}{\sqrt{1-\zeta^2}} \qquad = \dfrac{1}{\sqrt{1-2\zeta^2}}$

TO OBTAIN THE BANDWIDTH:

now $MF_{RES} = Q$ for the bandwidth $\dfrac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \dfrac{1}{2\sqrt{2}\zeta} \Rightarrow r^2 = (1-2\zeta^2) \pm 2\zeta\sqrt{1+\zeta^2}$

$\Rightarrow r_2 = 1.2234$ and $r_1 = .1156 \Rightarrow$ bandwidth is $r_2 - r_1 = 1.1078$

NOTE: must use full formula for r since ζ is not small.

TO FIND $T_{transmitted}$ $\quad TR = \dfrac{T_{trans}}{T_0} = \dfrac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$

when $r = 0.6$ $\quad TR = 1.4169 \quad T_{trans} = TR \cdot T_0 = 3825.5$ lb-in

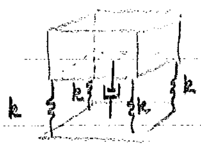
$r = 1.0724 \quad TR = 1.6334 \quad T_{trans} = TR \cdot T_0 = 4410.1$ lb-in

HW #10

4-33, 35, 40, 41

4-33 Given $\begin{cases} W = 128.67\ lb \ ; \ \omega_f = 859.44 \frac{rev}{min} \cdot 2\pi \frac{rad}{rev} \cdot \frac{1 min}{60 sec} = 90\ rad/sec; \\ c = 10\ lb sec/in \ ; \ X = 0.116 in \ ; \ F_T = 174\ lb \end{cases}$

Find k, $W_0 e$



$\underline{k_{EQ} = 4k}$ now $F_T = X\sqrt{k_{eq}^2 + (c\omega_f)^2}$

$174\ lb = .0116\ in \sqrt{k_{EQ}^2 + \left(10 \frac{lb sec}{in} \cdot 90 \frac{rad}{sec}\right)^2}$

$\sqrt{(F_T/X)^2 - (c\omega_f)^2} = k_{EQ} = 1200.00\ lb/in \qquad \underline{k = \frac{1}{4} k_{EQ} = 300.0\ lb/in}$

$X = \dfrac{P_0}{\sqrt{(k_{EQ} - m\omega_f^2)^2 + (c\omega_f)^2}} \qquad m = \frac{W}{g} = .333 \frac{lb\text{-}sec^2}{in} \qquad P_0 = m_0 e \omega_f^2$

$g = 386.4\ in/s^2$

$P_0 = 0.116 in \sqrt{(k_{EQ} - m\omega_f^2)^2 + (c\omega_f)^2} = 202.92\ lb = m_0 e \omega_f^2 \quad \therefore \quad W_0 e = \dfrac{(202.92)(32.2)(12)}{(90)^2}\ lb\ in$

$\underline{W_0 e = 9.68\ lb\cdot in}$

as $r \to \infty \quad X \to \dfrac{m_0 e}{m} = \dfrac{W_0 e}{W} = \dfrac{9.68\ lb\text{-}in}{128.67\ lb} \sim .075\ in$

4-35

Given $\begin{cases} eW_0/2 = 3\ lb in \ ; \ W = 200 lb \ ; \ \text{when } x = 0 \quad \psi = 90° \Rightarrow r = 1 \text{ from } \tan\psi = \frac{2\zeta r}{1-r^2} \\ \omega_f = 840 \frac{rev}{min} \cdot 2\pi \frac{rad}{rev} \cdot \frac{1 min}{60 sec} = 87.965\ rad/sec \ ; \ X = .75\ in \end{cases}$

Find (a) ω_n (or f_n), ζ

(b) Suppose $\omega_f = 1260 \frac{rev}{min} = 131.95\ rad/sec$ find X and ψ

(a) $X = \dfrac{m_0 e}{m} \dfrac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \dfrac{W_0 e}{W} \cdot \dfrac{1}{2\zeta}$ since $r = 1$

$X = .75 in = \dfrac{2(3)}{200} \dfrac{1}{2\zeta}$

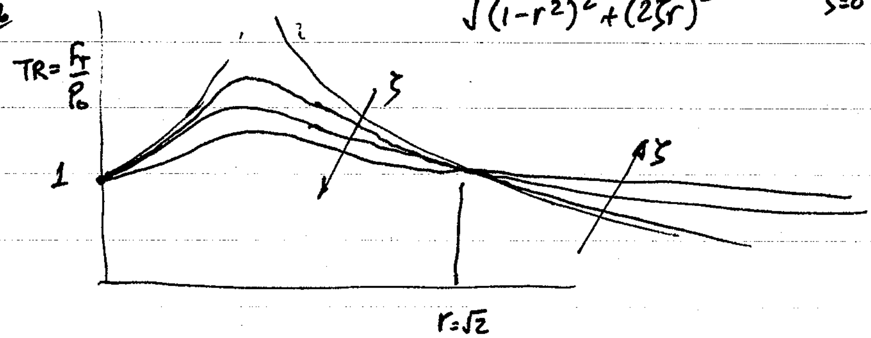
thus $\zeta = \dfrac{W_0 e}{2WX} = \dfrac{2(3)}{400(.75)} = \boxed{.02 = \zeta}$

since $r = 1$ $\quad \underline{\omega_n = \omega_f = 87.965\ rad/sec}$ or $\underline{f = 14\ Hz}$

(b) For the system $\omega_f = 131.95\ rad/sec$; $\omega_n = 87.965\ rad/sec \Rightarrow r = 1.5$ also $\zeta = .02$

$$TR = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \quad ; \quad TR\Big|_{\zeta=0} = \frac{1}{|1-r^2|}$$

4-36



$$TR = 0.15 = \frac{1}{r^2-1} \qquad r^2 = 7.67 \qquad r = 2.769$$

$$\omega_f = 875 \cdot \frac{2\pi}{60} = 91.63 \text{ rad/s}$$

$$\omega_n = \frac{\omega_f}{r} = 33.09 \text{ rad/s}$$

$$\omega_n = \sqrt{\frac{K_{eq}}{m}} \qquad K_{eq} = 4k = \omega_n^2 \cdot m$$

$$k = \frac{\omega_n^2}{4} m = \frac{33.09^2}{4} \cdot \frac{69}{32.2} = 586.59 \frac{lb}{ft}$$

$$\tan\psi = \frac{2\zeta r}{1-r^2} = \frac{2(.02)(1.5)}{1-(2.25)} = -.048 \qquad \underline{\psi = -2.748^\circ}$$

$$X = \frac{m_0 e}{m}\,\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = \frac{W_0 e}{W}\,\frac{(1.5)^2}{\sqrt{(1.25)^2+(.06)^2}} = \underline{.054 \text{ in} = X}$$

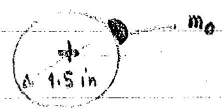
as $r \to \infty \quad X \to \frac{W_0 e}{W} = .03$ in

4-40

Given $W = 65$ lb ; $\Delta_{st} = \frac{W}{k_{eq}} = 2.166$ in ; $n = 3 = \frac{\sqrt{1-\zeta^2}}{2\pi\zeta}\ln\left(\frac{2.35}{.135}\right)$

Finally when $r = 1$, $X = .0193$ in; also $e = 1.5$ in

Find: c, m_0 (or W_0), if $r = 2$ find X



From $\Delta_{st} \Rightarrow k_{eq} = \frac{W}{\Delta_{st}} = \frac{65 \text{ lb}}{2.166 \text{ in}} = 30.01$ lb/in

but $\quad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{W}} = \underline{13.356 \text{ rad}}/\text{sec}$

thus $\quad c_c = 2m\omega = \underline{4.494} \ \frac{\text{lb-sec}}{\text{in}}$; let $\ln\left(\frac{2.35}{.135}\right) = \rho = \ln(17.41) = 2.857$

From $\quad n = \frac{\sqrt{1-\zeta^2}}{2\pi\zeta}\ln\left(\frac{2.35}{.135}\right) \Rightarrow \quad 4\pi^2\zeta^2 n^2 = (1-\zeta^2)\rho^2$ or $\zeta = \frac{\rho}{\sqrt{\rho^2+4\pi^2 n^2}}$

thus $\quad \underline{\zeta = .1499}$. $\quad \underline{c = \zeta c_c = .6734 \ \frac{\text{lb-sec}}{\text{in}}}$.

Now $\quad X = \frac{m_0 e}{m}\,\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = \frac{W_0 e}{W}\,\frac{1}{2\zeta} \Rightarrow \quad W_0 = \frac{X W \cdot 2\zeta}{e} = \underline{.2507 \text{ lb}}$

Knowing $\quad X = \frac{m_0 e}{m}\,\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$ then for $r = 2$, $\zeta = .1499$, $W_0 = .2507$ lb

$$X = \underline{.00756 \text{ in}} \ .$$

4-41 Given $W = 96.5$ lb ; $k = 80$ lb/in ; $r = 1 \Rightarrow X = 2.143$ in ; $r \to \infty \Rightarrow X \to .3572$ in

Find c : $\quad X = \frac{m_0 e}{m}\,\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$; when $r \to \infty \quad X_{r\to\infty} \to \frac{m_0 e}{m} = .3572$ in

now when $r = 1 \quad X = \frac{m_0 e}{m} \cdot \frac{1}{2\zeta}$ or $\zeta = \frac{X_{r=\infty}}{2X_{r=1}} = \frac{.3572}{2(2.143)} = \underline{.08334 = \zeta}$

$\underline{c_c} = 2m\omega = 2\sqrt{mk} = 2\sqrt{\frac{Wk}{g}} = \underline{8.94 \ \frac{\text{lb sec}}{\text{in}}} \qquad \underline{c = c_c\zeta = 0.74504 \ \frac{\text{lb sec}}{\text{in}}}$

9.49 $J_1 = 15 \text{ kg-m}^2$, $k_{t1} = 0.6 \times 10^6$ N-m/rad

$\omega_1 = \sqrt{k_{t1}/J_1} = \sqrt{0.6 \times 10^6/15} = 200$ rad/sec

Absorber:

k_{t2}, J_2, $\omega_2 = \sqrt{k_{t2}/J_2}$

For the combined system with tuned absorber, the natural frequencies Ω_1 and Ω_2 are given by an equation similar to Eq. (9.128) as

$$r_1^2, r_2^2 = \left(1 + \frac{\mu}{2}\right) \mp \sqrt{\left(1 + \frac{\mu}{2}\right)^2 - 1} \qquad (E_1)$$

where $\mu = J_2/J_1$, $r_1 = \Omega_1/\omega_2$ and $r_2 = \Omega_2/\omega_2$.

Let Ω_1 be 25% less than ω_1. Then $r_1 = \frac{150}{200} = 0.75$ (if 20% $r_1 = .8$)

$\therefore$ Eq. (E_1) gives

$$(0.75)^2 = \left(1 + \frac{\mu}{2}\right) - \sqrt{\left(1 + \frac{\mu}{2}\right)^2 - 1} \Rightarrow \mu = 0.3403$$ (if 20% $\mu = .2025$)

$\therefore J_2 = \mu J_1 = 5.1045 \text{ kg-m}^2$ (if 20% $J_2 = 3.0375$)

Since $\omega_2 = \sqrt{k_{t2}/J_2} = 200$, $k_{t2} = 4 \times 10^4 J_2$

$= 0.2042 \times 10^6$ N-m/rad (if 20% $k_{t_2} = .121$

Since Ω_2 has to be at least 20% greater than ω_1,

$r_2 = \frac{\Omega_2}{\omega_2} \geq 1.2$ ($r_2^2 = 2(1 + \mu/2) - r_1^2 = 1.5625$ if 20% $\therefore r_2 = 1.25 > 1.2$)

Eq. (E_1) gives, for $\mu = 0.3403$,

$$r_2^2 = \left(1 + \frac{0.3403}{2}\right) + \sqrt{\left(1 + \frac{0.3403}{2}\right)^2 - 1} = 1.7779 \Rightarrow r_2 = 1.3333$$

Hence $J_2 = 5.1045 \text{ kg-m}^2$ and $k_{t2} = 0.2042$ MN-m/rad are acceptable. If 20% then $J_2 = 3.0375$ & $K_{t_2} = .1215$ MN-m/rad.

9.52 $\omega_1 = \sqrt{\frac{k_1}{m_1}} = \sqrt{\frac{10^5}{40}} = 50$ rad/sec

Assuming that $\omega_2 = \omega_1$, we obtain $\omega_2 = \left\{\frac{k_2}{m_2}\right\}^{\frac{1}{2}} = \left\{\frac{k}{30}\right\}^{\frac{1}{2}} = 50$

$k_2 = 75000$ N/m, and

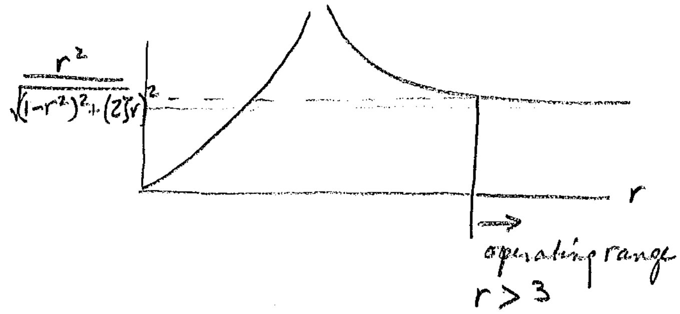
$$X_2 = -\frac{F_0}{m_2\, \omega^2} \approx -\frac{F_0}{m_2\, \omega_1^2} = -\frac{300}{30\,(50^2)} = -0.004 \text{ m}$$

10-10

$f_f = 100\ Hz \Rightarrow 2\pi f_f = \omega_f = 628.3\ rad/s$

$Z = .001\, m \qquad error = 2\%$

$k = 4000\, N/m$



$$\frac{Z}{Y} = 1.02 = \frac{r^2}{\sqrt{(1-r^2)^2 + (\underbrace{2\zeta r}_{0})^2}}$$

$$\therefore\ 1.0404 = \frac{r^4}{1-2r^2+r^4} \Rightarrow .0404r^4 - 2.0808r^2 + 1.0404 = 0 \qquad r^2 = \sqrt{51} \quad r^2 = .5$$

$r = 7.1414 \qquad r = .71$ (this is value which is not acceptable

$$r = \frac{\omega_f}{\omega_n} \Rightarrow \omega_n = \frac{\omega_f}{r} = \frac{628.3}{7.14} = 88\ \frac{rad}{s}$$

$$m = \frac{k}{\omega_n^2} = \frac{4000}{(88)^2} = \underline{.515\ kg}$$

10.15 $\quad m = 0.1\, kg \qquad k = 10{,}000\ N/m \qquad$ since peak to peak = 10 mm then $X = 5\, mm$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{10^5} = 316.3\ \frac{rad}{s}$$

$$f_f = 1000\ rpm \quad \therefore\ \omega_f = 1000 \cdot \frac{2\pi}{60} = 104.72\ rad/s \qquad r = \frac{\omega_f}{\omega_n} = 0.331$$

$$\frac{X}{Y} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(\underbrace{2\zeta r}_{=0})^2}}$$ for an accelerometer ; $\frac{X}{Y} = \frac{1}{1-r^2} = \frac{9}{8} = 1.125$

$$\therefore\ Y = .005/(1.125) = .0044\ m$$

velocity $= Y\omega_f = .0044\ (104.72) = 0.4633\, m/s$

accel $= Y\omega_f^2 = .0044\ (104.72)^2 = 48.30\ m/s^2$

note that here if the peak to peak is a relative displacement, then for

accelerometer $Z\omega_n^2 = \frac{Y\omega_f^2}{\sqrt{(1-r^2)^2+(\underbrace{2\zeta r}_{0})^2}} \Rightarrow \frac{Z}{Y} = \frac{r^2}{1-r^2}$

$Y = \frac{Z(1-r^2)}{r^2} = .0406\ m \qquad Y\omega_f^2 = 445.3\ m/s^2$

$Y\omega_f = 4.25\ m/s$

$Y\omega_f^2 = 445.3\ m/s^2$

$F: k(x-y) + c(\dot{x}-\dot{y}) = -m\ddot{x}$

$F = k(x-y) + c(\dot{x}-\dot{y}) = -m\ddot{x}$
$= +m\omega_f^2 x = \frac{Yk\, r^2\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \sin(\cdot$

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Department of Mechanical Engineering

EML 4220 QUIZ NO. 4 VA November 4, 2003

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1. Your calculator
2. Your seven sheets of paper 8.5" x 11"

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PROBLEM - A **vibrometer** consisting of a mass-spring-dashpot system has a static displacement due to its weight of 0.62 mm. **During its transient motion**, the system is found to have a damped natural frequency of 4 Hz. If the **vibrometer** is used to measure the vibration of an engine, find:

a) The lowest operating frequency, ω_f, of the engine for which the **vibrometer** measurement is within 2.5% of the engine's vibration amplitude. (18)

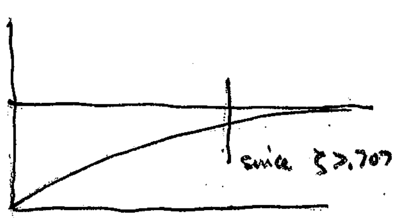
b) For this lowest operating frequency, find the maximum transmitted force to the **vibrometer** base if the base has a maximum displacement of Y = 2 mm. (2)

$\Delta_{st} = .62\text{ mm}$ $\quad \omega_n = \sqrt{g/\Delta} = 125.79\text{ rad/s}$ (2)

$f_d = 4\text{ Hz}$ $\quad \omega_d = 2\pi f_d = 25.13\text{ rad/s}$ (1)

$\zeta = \sqrt{1-\left(\frac{\omega_d}{\omega_n}\right)^2} = .98$ (2)

$\omega_d = \omega_n\sqrt{1-\zeta^2}$



$1-\epsilon = \frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$ (2) or $.975 = \frac{r^2}{\sqrt{(1-r^2)^2+(1.96r)^2}}$

$.975^2\left[1-2r^2+r^4+3.8416r^2\right] = r^4$

$\left[.9506 - 1.90125r^2 + .9506r^4 + 3.6519r^2 - r^4\right] = 0$

$.9506 + 1.75065r^2 - .0494r^4 = 0$

$r^2 = \frac{-1.75065 \pm \sqrt{(1.75065)^2 - 4(.9506)(-.0494)}}{2(-.0494)} = -.5349 \quad 35.9732$ (4)

$\underline{r = 6}$ $\quad \omega_f = r \cdot \omega_n = 754.45\text{ rad/s}$ (2)

$F = Yk\sqrt{1+(2\zeta r)^2}\left(\frac{r^2}{\quad}\right) = Yk\sqrt{1+(2\zeta r)^2}(1-\epsilon) = \underline{23.015k}$ (2)

$m\ddot{x} + c\dot{x} + kx = ky + c\dot{y}$

$= kY\sin + c\omega_f Y \cos = Y\sqrt{1+(2\zeta r)^2}\,\sin(\omega_f t - \beta)$

$x = \dfrac{Y\sqrt{\quad}}{\sqrt{\quad}}\sin(\omega_f t - \beta - \psi)$

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PROBLEM - A **velometer** is bought by your company but the service department loses instructions to the velometer. You observe that the velometer in free vibrations has a damped frequency of 12 Hz. You also find that the mass of the velometer causes a static displacement due to its own weight of 1.64 mm.

Find:

a) The lowest operating frequency that the **velometer** can measure accurately so that the error remain within 3.5% error. 12

b) For this lowest operating frequency, find the equation x(t) that describes the steady state displacement of the **velometer** mass. Write the solution in terms of Y, the amplitude of motion of the base. 6

$\omega_d = 2\pi f_d = 24\pi = 75.4$ rad/s 1

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$\omega_n = \sqrt{g/\Delta_{st}} = \sqrt{\dfrac{9.81}{1.64\times10^{-3}}} = 77.34$ rad/s 1

$\zeta = \sqrt{1-(\omega_d/\omega_n)^2} = .223$ 2

$1+\epsilon = \dfrac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = 1.035 = \dfrac{r^2}{\sqrt{(1-r^2)^2+(.4452r)^2}}$ or $(1.035)^2\left[1-2r^2+r^4+.1982r^2\right] = r^4$

$1.0712 - 2.1425r^2 + 1.0712r^4 + .1982r^2 - r^4 = 1.0712 - 1.9443r^2 + .0712r^4 = 0$ 4

$r^2 = \dfrac{1.9443 \pm \sqrt{(1.9443)^2 - 4(1.0712)(.0712)}}{2(.0712)} = 26.744$ $r = 5.1715$ $\omega_f = r\omega_n = 400$ rad/s 2

$.5625$ $\cancel{.75}$

$z+y = x(t) = Y\sin\omega_f t + \dfrac{Y r^2}{\sqrt{\quad}}\sin(\omega_f t - \psi) = 2\sin 400t + 2(1.035)\sin(400t-\psi)$ $\psi = \tan^{-1}\dfrac{2\zeta r}{1-r^2}$

$= \dfrac{Y\sqrt{1+(2\zeta r)^2}}{\sqrt{\quad}}\sin(\omega_f t - \beta - \psi) = .0973Y\sin 400t$ 6 $= -.0894$

$\beta = -\tan^{-1} 2\zeta r$

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Department of Mechanical Engineering

EML 4220 QUIZ NO. 4 VC November 4, 2003

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PROBLEM - An accelerometer is composed of a suspended **weight** of 0.386 lb force and a spring having a modulus of 10 lb/in., but the damping is not known. When tested on a vibration having a frequency of 382 cycles/min., the acceleration measurement is found to be 1% greater than the correct value.

a) based on the damping RATIO you found, what is the maximum error that the accelerometer should have 16

b) based on the damping RATIO found, what is the acceptable frequency range of the accelerometer 6

$m = \frac{.386}{32.2} = .012$ slug (2)

$k = 10\frac{lb}{in} \cdot \frac{12 in}{ft} = 120 \frac{lb}{ft}$ (2)

$\omega_f = 382 \frac{cyc}{min} \cdot \frac{1 min}{60 sec} \cdot 2\pi \frac{rad}{cy} = 40$ rad/s (2)

$\omega_n = \sqrt{\frac{k}{m}} = 100$ rad/s (2)

$r = \frac{\omega_f}{\omega_n} = .4$ (2)

$$1+\epsilon = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$1.01 = \frac{1}{\sqrt{(1-.4^2)^2 + (2\zeta \cdot .4)^2}}$$

$$1.0201\left((1-.16)^2 + (.8\zeta)^2\right) = 1$$

$$.7056 + .64\zeta^2$$

$$\zeta^2 = \left[\frac{1}{1.0201} - .7056\right]\frac{1}{.64} = .4292$$

$\zeta = .6551$ (4)

max error $1+\epsilon = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 1.0102$ (2) $-1+(1+\epsilon) = \epsilon$

$\underline{1.02\%}$

$1-\epsilon$

$$1-.0102 = \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = .98983$$

$(.98983)^2\left[1-2r^2+r^4+1.7166r^2\right] = 1$ or $.9798 - 1.9595r^2 + .9798r^4 + 1.6819r^2 - 1 = -.0202 - .2776r^2 + .9798r^4$

$r^2 = \left(.2776 + \sqrt{.2776^2 - 4(.9798)(-.0202)}\right)/2(.9798) = .3434$ (4) $r = .5859$ (2) $\omega_1 = r\omega_n = 58.59$ rad/s

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Department of Mechanical and Materials Engineering

EML 4220 QUIZ NO.7B 4 Dec. 2003

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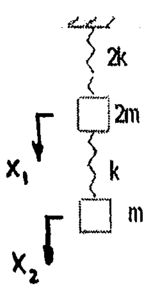
For the system shown, find:

(a) The matrix [K] and the matrix [M]
(b) Use Rayleigh's method to find ω_n, given that $[A_1 \quad A_2] = [1 \quad 5]$
(d) What are the actual values of ω_n.
(e) What are the actual relationships between the amplitudes, $\eta = A_2/A_1$.



$$\begin{bmatrix} 3K & -K \\ -K & K \end{bmatrix} \qquad \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix}$$

$$\frac{[1\ 5]\begin{bmatrix} 3K & -K \\ -K & K \end{bmatrix}\begin{bmatrix} 1 \\ 5 \end{bmatrix}}{[1\ 5]\begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} 1 \\ 5 \end{bmatrix}} \quad \frac{(1\ 5)\begin{pmatrix} -2K \\ 4K \end{pmatrix}}{(1\ 5)\begin{pmatrix} 2m \\ 5m \end{pmatrix}} = \frac{18K}{27m}$$

$$\omega_n^2 = \frac{2}{3}\frac{K}{m} \Rightarrow .817\sqrt{\frac{K}{m}}$$

$$2m^2\omega_n^4 - \omega_n^2\Big[\underbrace{2m(K) + m(3K)}_{5\,Km}\Big] + 2K^2 = 0$$

$$\omega_n^2 = \frac{5Km \pm \sqrt{25k^2m^2 - 4\cdot 2m^2\cdot 2k^2}}{4m^2} = \frac{5Km \pm 3Km}{4m^2} = \frac{2K}{m},\ \frac{1}{2}\frac{K}{m}$$

$$1.414\sqrt{\frac{K}{m}} \quad .71\sqrt{\frac{K}{m}}$$

$$\eta \; \cancel{\omega_n^2} = \frac{-2m\omega_n^2 + 3k}{K} = \begin{array}{ll} -1 & \text{for larger} \\ 2 & \text{for smaller} \end{array}$$

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EML 4220 QUIZ NO. 5B 20 Nov. 2003

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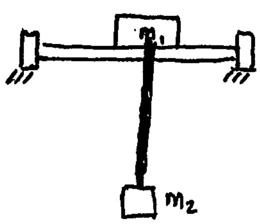
PRINT NAME SIGN NAME

PROBLEM - An overhead traveling crane can be modeled as shown below. The beam has a moment of inertia I of 0.04 m^4 and a modulus of elasticity E of 2.06 x 10^{11} N/m^2, the truck has a mass (m_1) of 2000 kg, the load being lifted has a mass (m_2) of 10000 kg, and the cable through which the mass (m_2) is lifted has a stiffness (k) of 6.0 x 10^5 N/m. Determine the natural frequencies and mode shapes of the system. Assume the span of the beam to be 40 m.



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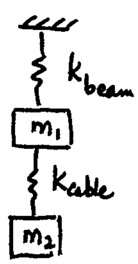

$k_1 = k_{beam} = \frac{48EI}{l^3} = 6.18\times10^6$ N/m

$k_2 = 6\times10^5$ N/m

$m_1 = 2000$

$m_2 = 10000$

$$m_1 m_2 \omega_n^4 - [m_1 k_2 + m_2(k_1+k_2)]\,\omega_n^2 + k_1 k_2$$

$$2\times10^7 \omega_n^4 - 6.9\times10^{10}\,\omega_n^2 + 3.708\times10^{12} = 0$$

$\omega_{n_1}^2 = 54.6$ $\omega_{n_2}^2 = 3395.4$

$\omega_{n_1} = 7.4$ rad/s $\omega_{n_2} = 58.27$ rad/s

$\eta = 11.18$ $\eta = -.018$

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EML 4220 QUIZ NO.7A 4 Dec. 2003

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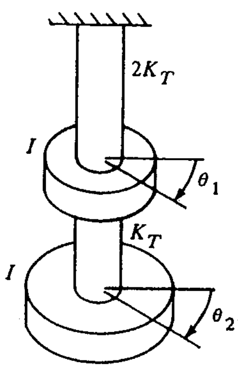

For the system shown, find:

(a) The matrix [K] and the matrix [M]
(b) Use Rayleigh's method to find ω_n, given that $[\Theta_1 \quad \Theta_2] = [1 \quad 3]$
(c) What are the actual values of ω_n.
(d) What are the actual relationships between the amplitudes, $\eta = \Theta_2/\Theta_1$.

$$\begin{bmatrix} 3K_T & -K_T \\ -K_T & K_T \end{bmatrix} \qquad \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

$$\omega_n^2 = \frac{\begin{bmatrix} 1 & 3 \end{bmatrix}\begin{bmatrix} 3K_T & -K_T \\ -K_T & K_T \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix}}{\begin{bmatrix} 1 & 3 \end{bmatrix}\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix}} = \frac{(1 \quad 3)\begin{pmatrix} 0K_T \\ 2K_T \end{pmatrix}}{(1 \cdot 3)\begin{pmatrix} I \\ 3I \end{pmatrix}} = \frac{6K_T}{10I}$$

$$\omega_n^2 \cong .6\frac{K_T}{I} \qquad \omega_n = .775\sqrt{\frac{K_T}{I}}$$

$$I^2\omega_n^4 - \underbrace{[I \cdot 3K_T + I \cdot K_T]}_{4K_TI}\omega_n^2 + 2K_T^2 = 0$$

$$\omega_n^2 = \frac{4K_TI \pm \sqrt{16K_T^2I^2 - 8K_T^2I^2}}{2I^2}$$

$$\omega_n = \sqrt{2\frac{K_T}{I} \pm \sqrt{2}\frac{K_T}{I}} = \sqrt{(2 \pm \sqrt{2})\frac{K_T}{I}}$$

$$(-\omega_n^2+3K_T)A_1 - K_T A_2 = 0$$

$$\frac{A_2}{A_1} = \frac{-\omega_n^2 I + 3K_T}{K_T} = \frac{-(2+\sqrt{2})K_T + 3K_T}{K_T} = 2.414$$

$$\frac{A_2}{A_1} = \frac{-\omega_n^2 I + 3K_T}{K_T} = \frac{-(2+\sqrt{2})K_T + 3K_T}{K_T} = -.414$$

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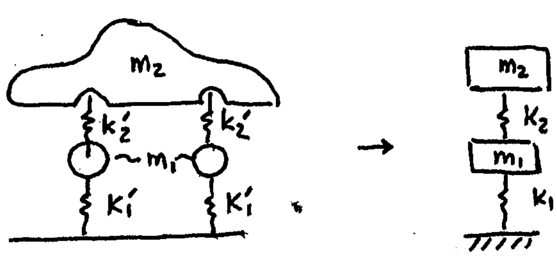
PROBLEM - A large automobile manufacturer analyzed the problem of the automobile by taking an entire automobile apart. By weighing each section, the following values of equivalent masses were found

m_1 axle mass	360 kg
m_2 body mass	1340 kg
k_2 springs	91 N/mm
k_1 tires	1076 N/mm

By assuming vertical motion only, determine the two natural frequencies of motion and the associated mode shapes A_2/A_1.



$$m_1 m_2 \omega_n^4 + \left[m_1(k_2 + \cancel{k_3}) + m_2(k_1 + k_2)\right]\omega_n^2 + k_1 k_2 + k_2 k_3 + k_3 k_1$$

$$1340 \cdot 360\ \omega_n^4 - \left[360(91000) + 1340(1167000)\right]\omega_n^2 + 91 \cdot 1076 \cdot 10^6 = 0$$

$$4.824 \times 10^5 \omega_n^4 - 1.59654 \times 10^9 \omega_n^2 + 9.792 \times 10^{10} = 0$$

$$\frac{A_2}{A_1} = \frac{-m_1\omega_n^2 + k_1 + k_2}{k_2}$$

$\omega_n^2 = 62.5$ $\omega_{n_1} = 7.91$ rad/s $\eta_1 = 12.$

$\omega_n^2 = 3246.99$ $\omega_{n_2} = 56.98$ rad/s $\eta_2 = -0.$

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EML 4220 QUIZ NO.5C 20 Nov. 2003

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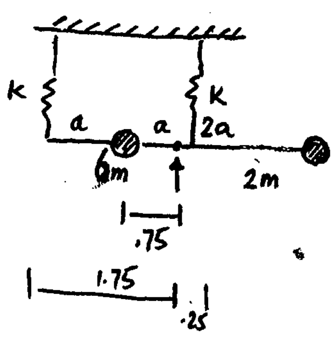
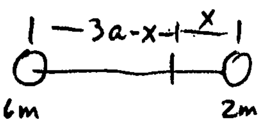
Problem. Two identical springs support a rigid rod and two identical masses. Determine the two natural frequencies of motion for the two-mass system as well as their respective modal relationships (i.e., A/Θ or A/B).

b) If the right spring is replaced by a new spring, what would the new spring value be in order to decouple the equations.



2 CG => 

$6m(3a-x) = 2mx$

$18ma - 6mx = 2mx$

$18ma = 8mx$

$\frac{18}{8}a = x$

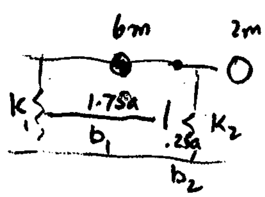
$2.25a$

2 $I = 6m(.75a)^2 + (2.25a)^2 \cdot 2m = 13.5ma^2$

2 $m = 8m$

2 coupling $= k_1 b_1 - k_2 b_2$

$= k \cdot 1.75a - .25a k_2 = 0$ $\underline{k_2 = k \cdot 7}$



$$\begin{bmatrix} -8m\omega_n^2 + 2k & -1.5ka \\ -1.5ka & -13.5ma^2\omega_n^2 + [(1.75a)^2 + (.25a)^2]k \end{bmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$3.125a^2$

~~k(.6b+x)~~

$108m^2a^2\omega_n^4 - (27mka^2 + 25ma^2k)\omega_n^2 + 6.25a^2k^2 - 2.25k^2a^2 = 0$

$108m^2a^2\omega_n^4 - 52mka^2\omega_n^2 + 4a^2k^2 = 0$

$\omega_n^2 = 52mka^2 \pm \sqrt{(52mka^2)^2 - 4(108m^2a^2)(4a^2k^2)} = .385\frac{k}{m}$ $.096 k$ .61

$$\eta_1 = -\frac{8m\omega_n^2 + k_1 + k_2}{1.5ka} = -\frac{.72}{a}$$

$$\eta_2 = \qquad = \frac{.82}{a}$$

Florida International University
Department of Mechanical and Materials Engineering

EML 4220 QUIZ NO.7C 4 Dec. 2003

This examination is 30 minutes long. Put away all papers, note- books, texts, except for:

1. Your calculator
2. Your two sheets of paper 8.5" x 11"

DO YOUR OWN WORK -- USE THE PAPER PROVIDED FOR YOU

Please sign the following:

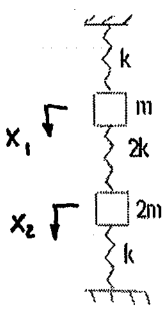
I certify that I will neither receive nor give unpermitted aid on this examination. Violation of this will result in failure of the examination.

____________________ PRINT NAME

____________________ SIGN NAME

For the system shown, find:

(a) The matrix [K] and the matrix [M]
(b) The characteristic equation for this system
(c) Use Rayleigh's method to find ω_n, using the static displacement method to find $[A_1 \;\; A_2]$
(d) What are the actual values of ω_n.



$$\begin{bmatrix} 3K & -2K \\ -2K & 3K \end{bmatrix} \qquad \begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix}$$

$$\delta_{st_1} = \frac{3W}{K} \qquad \delta_{st_2} = \frac{3W}{K} + \frac{W}{K} = \frac{4W}{K} \qquad \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\frac{\begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} 3K & -2K \\ -2K & 3K \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}}{\begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}} = \frac{(3\;4)\begin{pmatrix} K \\ 6K \end{pmatrix}}{(3\;4)\begin{pmatrix} 3m \\ 8m \end{pmatrix}} = \frac{27K}{41m}$$

$$\omega_n \cong .66\sqrt{\frac{k}{m}} \qquad \omega_n = .81\sqrt{k/m}$$

$$2m^2\omega_n^4 - \omega_n^2\left[m(3K) + 2m(3K)\right] + \underbrace{2K^2 + 2K^2 + K^2}_{5K^2}$$

$$9mK$$

$$\omega_n^2 = \frac{9mk \pm \sqrt{81m^2k^2 - 40m^2k^2}}{4m^2} = \frac{9mk \pm 6.40\,mk}{4m^2}$$

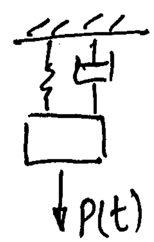
$$\omega_n^2 = 3.85\,\frac{K}{m} \qquad \omega_n = 1.962\sqrt{\frac{K}{m}}$$

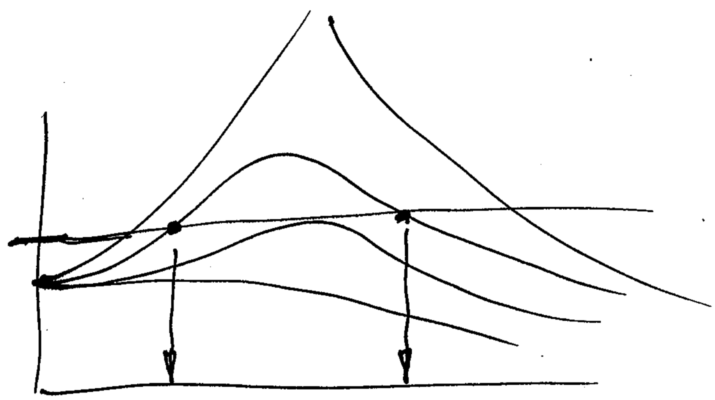
$$= .65\,\frac{K}{m} \qquad \omega_n = .806\sqrt{\frac{K}{m}}$$

EML 4220

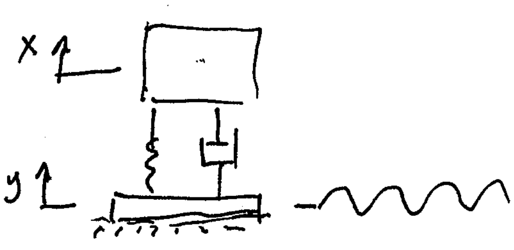
MECH. VIBS - PROBLEM SOLVING

CH. 3, 9, 10 RAO

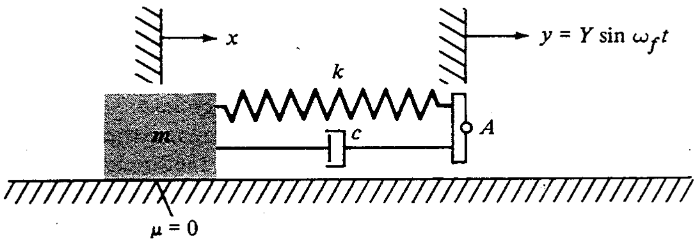




$$F_{trans} = \frac{P_0 \sqrt{k^2 + (c\omega_f)^2}}{\sqrt{(k - m\omega_f^2)^2 + (c\omega_f)^2}} = X \sqrt{k^2 + (c\omega_f)^2}$$

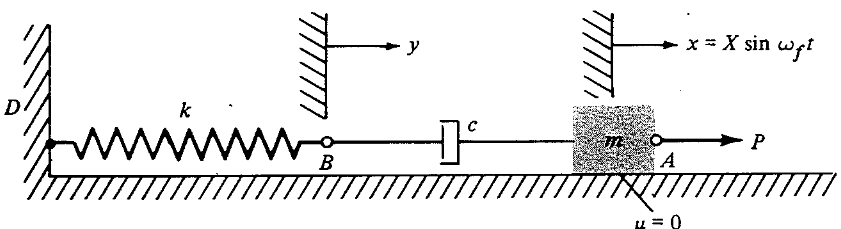


$y = Y \sin \omega_f t$



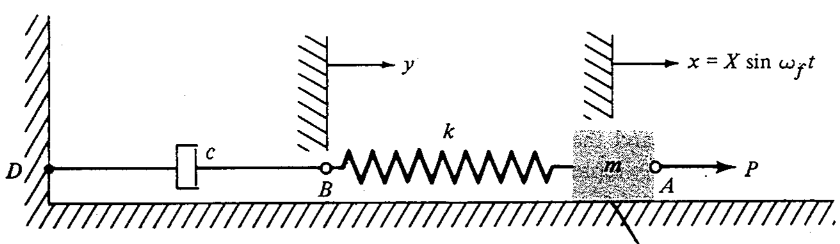
Problem 4-18

4-19. For the arrangement shown, x represents the absolute displacement of the mass m, and y is the absolute displacement of the point B. The impressed force P moves m in accordance with the relation $x = X \sin \omega_f t$. (a) Construct the free-body diagram for m. (b) Write the differential equation for the dynamic condition of m. (c) Construct the free-body diagram for the connection point B. (d) Write the differential equation for the connection point B. (e) Obtain the solution for part (d), representing the relation that governs the motion of point B. (f) Determine the relation for the impressed force P. (g) Obtain the expression for the force transmitted to the support at D.



Problem 4-19

4-20. For the arrangement shown, x represents the absolute displacement of the mass m, and y is the absolute displacement of the point B. The impressed force P moves m in accordance with the relation $x = X \sin \omega_f t$. (a) Construct the free-body diagram for m. (b) Write the differential equation for the dynamic condition of m. (c) Construct the free-body diagram for the connection point B. (d) Write the differential equation for the connection point B. (e) Obtain the solution for part (d), representing the relation that governs the motion of point B. (f) Determine the relation for the impressed force P. (g) Obtain the expression for the force transmitted to the support at D.



Problem 4-20

4-21. A damped system is composed of a mass of 8.75 kg, a spring having a modulus of 1750 N/m, and a dashpot having a damping constant of 37.13 N · s/m. The mass is acted on by a harmonic force $P = P_0 \sin \omega_f t$ having a maximum value of 220 N and a frequency of 4.50 Hz. Using the symbols of X' and ϕ for the arbitrary constants of the transient, write the complete solution representing the motion of the mass.

4-22. A body having a mass of 100 kg moves along a straight line according to the relation

$$x = 0.12 \sin (3t - \beta) + \frac{0.05 \sin (5.96t + \pi/6)}{e^{0.6t}}$$

where x is the displacement along the line in meters and t is time in seconds. (a) Obtain the physical constants and parameters of the system, including appropriate frequencies, and so on. (b) Obtain the initial displacement and velocity.

4-23. For a damped system excited by the harmonic force $P = P_0 \sin \omega_f t$, plot the magnification-factor curve for $\zeta = 0.3$. Do this from $r = 0$ to $r = 4$, carefully determining the peak value.

4-24. For a damped system driven by the harmonic force $P = P_0 \sin \omega_f t$, plot the phase-angle curve for $\zeta = 0.3$. Do this from $r = 0$ to $r = 4$.

4-25. Develop a relation for the ratio of the maximum amplitude to the resonant amplitude for steady-state motion for the case of a harmonically forced damped system.

4-26. For a damped system excited by the harmonic force $P = P_0 \sin \omega_f t$, plot the curve of the steady-state amplitude against the spring constant k for $c = 0.707 m\omega_f$. Carry this out from $k = 0$ to $k = 4m\omega_f^2$.

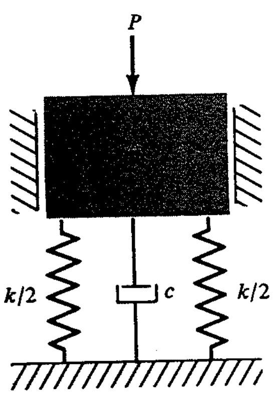
4-27. A damped system has a spring modulus of 24 lb/in. and a damping constant of 0.88 lb sec/in. It is subjected to a harmonic force having a force amplitude of 15 lb. When excited at resonance, the steady-state amplitude is measured as 2.8409 in. Determine (a) the damping factor, (b) the natural undamped frequency of the system, and (c) the damped natural frequency.

4-28. A damped system is subjected to a harmonic force for which the frequency can be adjusted. It is determined experimentally that at twice the resonant frequency, the steady amplitude is one-tenth of that which occurs at resonance. Determine the damping factor for the system.

4-29. A damped torsional system is composed of a shaft having a torsional spring constant $k_T = 60\,000$ lb in./rad, a disk with a mass moment of inertia $I = 24$ lb in. sec^2, and a torsional damping device having a torsional damping constant $c_T = 840$ lb in. sec/rad. A harmonic torque with a maximum value of 2700 lb in., acting on the disk, produces a sustained angular oscillation of 3.368-degree amplitude. (a) Determine the frequency of the impressed torque. (b) Obtain the maximum torque transmitted to the support.

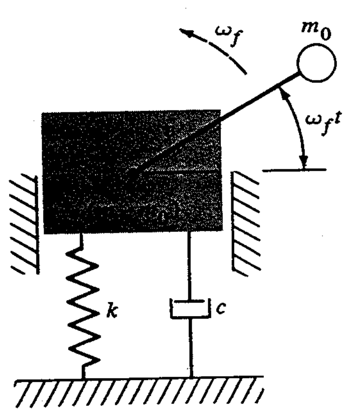
4-30. A torsional system consists of a shaft having a torsional spring constant $k_T = 12\,800$ N · m/rad, a disk with a mass moment of inertia $I = 8$ kg · m^2, and a torsional damping device with a torsional damping constant $c_T = 192$ N · m · s/rad. A harmonic torque with a maximum value of 480 N · m produces a steady angular oscillation of 1.5-degree amplitude. (a) Determine the frequency of the impressed torque. (b) Obtain the maximum torque transmitted to the support.

4-31. A machine having a mass of 70 kg is mounted as shown on springs having a total stiffness of 33 880 N/m. The damping factor is $\zeta = 0.20$. A harmonic force $P = 450 \sin 13.2t$ (where P is in newtons and t is in seconds) acts on the mass. For the sustained or steady-state vibration, determine (a) the amplitude of the motion of the machine, (b) its phase with respect to the exciting force, (c) the transmissibility, (d) the maximum dynamic force transmitted to the foundation, and (e) the maximum velocity of the motion.



Problem 4-31

4-32. A machine having a total weight of 96.5 lb is mounted as shown on a spring having a modulus of 900 lb/in. and is connected to a dashpot with a damping *factor* of 0.25. The machine contains a rotating unbalance ($w_0 e$) of 5 lb in. If the speed of rotation is 401.1 rpm, obtain (a) the amplitude of the steady-state motion, (b) the maximum dynamic force transmitted to the foundation, and (c) the angular position of the arm when the structure goes upward through its neutral position.



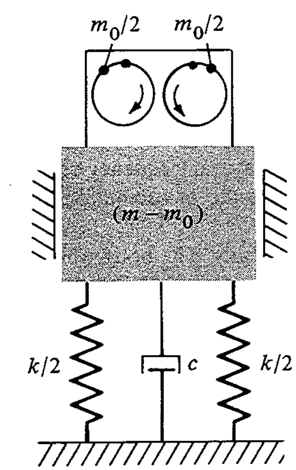
Problem 4-32

4-33. A machine having a total weight of 128.67 lb contains a part that rotates at 859.44 rpm. The machine is supported equally on four identical springs and is

connected to a dashpot for which the damping constant is 10 lb sec/in. In operation the machine has a steady amplitude of 0.116 in. and transmits a maximum dynamic force of 174 lb to the supporting base. (a) Determine the modulus for the springs. (b) Calculate the unbalanced moment ($w_0 e$) for the rotating part.

4-34. A machine with a total mass of 50 kg contains a shaft mechanism that rotates at 1800 rpm. The machine is supported by springs for which the equivalent modulus is 20 000 N/m but the damping constant is unknown. Due to the rotating unbalance, the steady-state amplitude is 1.32 cm and the maximum dynamic force carried by the base is 278 N. Determine (a) the damping constant and (b) the unbalanced moment $m_0 e$.

4-35. A typical vibration exciter is composed of two eccentric masses that rotate oppositely, as represented in the top of the accompanying figure. Thus an oscillation is produced in the vertical direction only, since the horizontal effect cancels. This device is used to determine the vibrational characteristics of the structure to which it is attached. The unbalance ($ew_0/2$) of each exciter wheel is 3 lb in. The total arrangement has a weight $mg = 200$ lb. The exciter speed (of eccentric rotation) was adjusted until a stroboscope showed the structure to be moving upward through its equilibrium position at the instant the eccentric weights were at their top position. The exciter speed then was 840 rpm, and the steady amplitude was 0.75 in. Determine (a) the natural frequency of the structure and the damping factor for the structure. If the speed were changed to 1260 rpm, (b) obtain the steady-state amplitude of the structure and (c) the angular position of the eccentrics as the structure moves upward through its equilibrium position.



Problem 4-35

4-36. A small compressor weighs 69 lb and runs at 875 rpm. It is to be supported by four springs (equally) and no damping is provided. Design the springs, thus determining k, so that only 15% of the shaking force is transmitted to the supporting foundation or structure.

4-37. Solve Prob. 4-36 considering that damping is also to be included such that the damping factor will be 0.4 for the system. Equal damping at each spring will be provided by a plug or block of energy-absorbing material. Determine the required damping constant as well as the spring constant.

4-38. A refrigerator compressor unit weighs 179 lb and operates at 590 rpm. The maximum dynamic force imposed on the compressor is 20 lb. The unit is to be supported equally by three springs and is to be undamped. (a) Determine the required spring constant if 10% of the dynamic force is to be transmitted to the supporting base. (b) Determine the clearance that must be provided for the unit.

4-39. Solve Prob. 4-38, considering that *each* spring contains a viscous damping absorber having a damping constant of 0.8333 lb sec/in.

4-40. A small machine is known to contain a rotating unbalance on its main shaft. The machine weighs 65 lb and when placed on elastic vibration isolators (which serve as a viscous damping member as well as the elastic support), the static equilibrium displacement is 2.166 in. Also, when the machine is displaced further and released, the subsequent vibration diminishes from an amplitude of 2.350 in. to 0.135 in. in exactly 3 cycles. Operating the machine at resonance produces a sustained amplitude of 0.0193 in. (a) Determine the damping constant for the system. (b) Considering that the shaft diameter is 3 in. so that the eccentric moment arm is 1.5 in., calculate the unbalanced weight. (c) Determine the steady amplitude at an operation speed of twice the resonant speed.

4-41. A damped system with a rotating unbalance is composed of a 96.5-lb body and a spring having a constant of 80 lb/in. but the damping constant is unknown. When operated at resonance, the sustained amplitude is 2.143 in. If the speed of operation is greatly increased, the steady amplitude eventually approaches the value of 0.3572 in. Determine the damping constant for the system.

4-42. The support for a damped system (see Fig. 4-26) is oscillated harmonically with a frequency of 75 cycles/min, causing the mass to vibrate with a steady-state amplitude of 0.6 in. and a maximum dynamic force of 48.75 lb to be transmitted to the support. For the system, the spring modulus is 13 lb/in. and the damping constant is 0.454 lb sec/in. Determine (a) the natural frequency of the system, (b) the mass of the system, and (c) the amplitude of oscillation of the support.

4-43. The support for a damped system is oscillated harmonically. The body weighs 20 lb and the spring constant is 14 lb/in. When the support is oscillated at a rate of 300 cycles/min, the sustained amplitude of the body is 0.793 in. (a) Determine the maximum dynamic force carried by the support. (b) If the support is oscillated at resonance with an amplitude of 1.121 in., the body has a steady amplitude of 3.069 in. Obtain the damping constant for the system.

4-44. A mass-spring system, arranged horizontally, is damped only by Coulomb friction (refer to Fig. 4-27). The system is driven by a harmonic force having a frequency of 54.356 cycles/min and a maximum value of 30 lb, causing the body to oscillate with a steady amplitude of 3.09 in. The spring constant is 12 lb/in. and the body weighs 28.95 lb. Determine the average coefficient of sliding friction for the surfaces involved.

4-45. A machine is designed to produce an abrasive action on a horizontal surface in order to wear and smoothen the surface. It consists of a sliding member weighing 20 lb fixed to an elastic member having a stiffness coefficient of 8 lb/in. The average coefficient of sliding friction of the body against the surface is 0.3. A harmonic force having a maximum value of 15 lb actuates the body at a frequency of 0.9 of the resonant value. Determine the total excursion or stroke of the member.

4-46. A certain mass-spring system exhibits hysteresis damping only. The spring modulus is 310 lb/in. When excited harmonically at resonance, the steady amplitude is 1.540 in. for an energy input of 32 in. lb. When the resonant energy input is increased to 87 in. lb, the sustained amplitude changes to 2.464 in. Determine the hysteresis coefficient β and the exponent γ.

4-47. A small built-up structure shows solid damping characteristics with $\gamma = 2$. A 1000-lb load on the structure causes a static displacement of 2.500 in., and a supported machine that is subjected to a harmonic force having a constant amplitude of 75 lb produces a resonant amplitude of 6.818 in. (a) Determine the hysteresis damping coefficient β and (b) the energy dissipated per cycle by hysteresis at resonance. (c) Calculate the steady amplitude at twice the resonant frequency and (d) at one-half the resonant frequency.

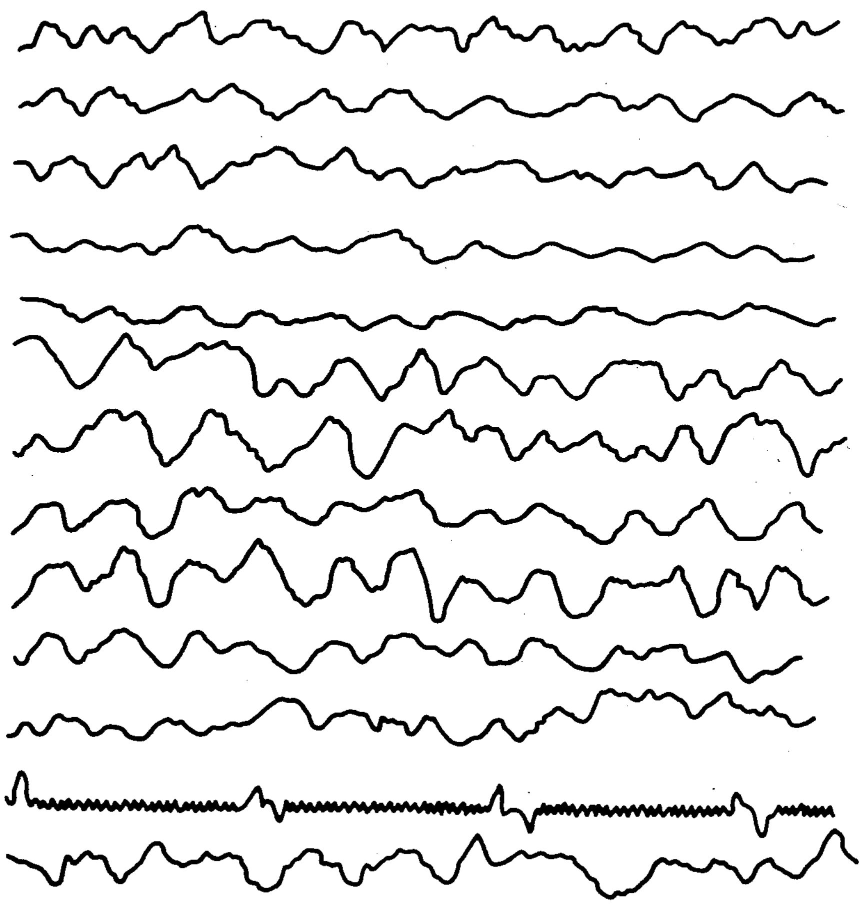


Figure 6-9

traces as well as for displacement records. In such analyses, various methods† are employed, including numerical procedures, superposition, the envelope method, and so forth. Mechanical and other aids are also used to facilitate these determinations.

PROBLEMS

6-1. Determine the percent error of a vibrometer for the frequency ratios of 3, 5, and 10 for each of the following damping factors: (a) $\zeta = 0$, (b) $\zeta = 0.6$, and (c) $\zeta = 0.7$.

6-2. For a vibrometer, determine the greatest percent error that occurs for the range of frequency ratio from $r = 3$ to $r = \infty$. Also obtain the corresponding value of r at which this occurs. Do this for each of the following damping factors: (a) $\zeta = 0$, (b) $\zeta = 0.6$, and (c) $\zeta = 0.7$.

† See R. G. Manley, *Waveform Analysis* (New York: John Wiley, 1945).

6-3. Determine the percent error of an accelerometer for the frequency ratios of 0, 0.2, 0.4, and 0.6 for each of the following damping ratios: (a) $\zeta = 0$, (b) $\zeta = 0.66$, and (c) $\zeta = 0.71$.

6-4. For an accelerometer, determine the greatest percent error in measurement that occurs for the range of the frequency ratio from $r = 0$ to $r = 0.6$. Also obtain the value of r at which this occurs. Carry this out for (a) $\zeta = 0$, (b) $\zeta = 0.66$, and (c) $\zeta = 0.71$.

6-5. An accelerometer is composed of weight $W = 0.042$ lb suspended by a spring for which $k = 5$ lb/in., and the fluid damping factor for the instrument is $\zeta = 0.65$. When used on a machine vibrating at a frequency of 13.647 Hz, the instrument reading for the maximum acceleration is 50 in./sec^2. Determine the correct value of the maximum acceleration of the machine.

• 6-6. A vibrometer is composed of a spring for which $k = 28.8$ lb/in., a suspended weight $W = 19.3$ lb, and a viscous damper for which $c = 0.96$ lb sec/in. The amplitude indicated by the instrument is 0.73 in. when attached to a structure oscillating with a frequency of 14 Hz. Determine the correct value for the amplitude of displacement of the structure.

6-7. An accelerometer is composed of weight $W = 0.0965$ lb suspended by a spring for which $k = 0.355$ lb/in., and the fluid damping factor for the instrument is $\zeta = 0.65$. When used on a machine vibrating at a frequency of 2.4 Hz, the instrument reading for the maximum acceleration is 20 in./sec^2. Determine the correct value for the maximum acceleration of the machine.

6-8. A vibrometer is composed of a spring for which $k = 1.44$ lb/in., a suspended weight of $W = 3.86$ lb, and a viscous damper for which $c = 0.096$ lb sec/in. The amplitude indicated by the instrument is 0.5 in. when attached to a structure oscillating with a frequency of 7 Hz. Determine the correct value for the amplitude of displacement of the structure.

6-9. A vibrometer is to be designed, based on a maximum error of 3% when used for frequencies of 1200 cycles/min and above. The instrument mass is to weigh 1 lb, and the damping factor is to be 0.6. Determine the required spring modulus and damping constant for the instrument.

• 6-10. An accelerometer is composed of a suspended weight of 0.193 lb and a spring having a modulus of 5 lb/in., but the damping is not known. When tested on a vibration having a frequency of 382 cycles/min, the acceleration measurement is found to be 1% greater than the correct value. Determine the damping constant for the instrument.

6-11. It is desired to design an accelerometer that will have an optimum *range* of frequency ratio for a maximum accelerometer error of 1%. (a) Calculate the required damping factor. (b) Determine this optimum range of frequency ratio.

6-12. An accelerometer is to be designed that will have an optimum *range* of frequency ratio for a maximum accelerometer error of 2%. (a) Calculate the required damping factor. (b) Determine this optimum range of frequency ratio.

6-13. Design an accelerometer based on a maximum error of 1% when used for vibration measurements having frequencies in the range from 0 to 1800 cycles/min. The instrument mass is to have a weight of 0.05 lb. Determine the required spring modulus and damping constant for the instrument. (Suggestion: Refer to Prob. 6-11.)

6-14. Design an accelerometer based on a maximum error of 2% when used for vibration measurements having frequencies in the range from 0 to 6000

6-6

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{28.8 \times 12}{19.3/32.2}} = 24 \text{ rad/s}$$

$$r = \frac{\omega_f}{\omega_n} = \frac{2\pi \cdot 14}{24} = 3.665$$

$$c_c = 2m\omega_n = 2 \times \frac{19.3}{32.2 \cdot 12} \cdot 24 = 2.4 \ \frac{\text{lb-s}}{\text{in}}$$

$$\zeta = c/c_c = \frac{0.96}{2.4} = .4$$

for a vibrometer $\frac{X}{Y} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = 1.052 = 1 + \epsilon$

ie 5.2% high

$\therefore Y = \frac{X}{1.052} = \frac{.73}{1.052} = .694 \text{ in}$ β, ψ $x = \frac{r^2 Y}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \sin(\omega_f t - \psi - \beta)$

6.10

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{(5)(12)}{.193/32.2}} = 100 \ \frac{\text{rad}}{\text{s}}$$

$$\omega_f = 382 \ \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi}{60} = 40 \text{ rad/s}$$

$$r = \frac{\omega_f}{\omega_n} = .4$$

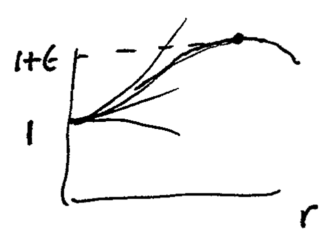
for accel $\frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = 1.01$ $\rightarrow$ $\zeta = .655$

$$\left\{ \frac{1}{4r^2}\left[\left(\frac{1}{1.01}\right)^2 - (1-r^2)^2\right] \right\}^{1/2} = \zeta$$

$$c_c = 2m\omega_n = 2\frac{(.193)}{32.2} \cdot 100 = 1.2 \ \frac{\text{lb-s}}{\text{ft}}$$

$$c = c_c\zeta = .4795 \ \frac{\text{lb-s}}{\text{ft}}$$

To find max error $\frac{Z}{Y}_{max} = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 1 + \epsilon$



solve to determine range $1 - \epsilon = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$ solve for r

6-6

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{28.8 \times 12}{19.3/32.2}} = 24 \text{ rad/s}$$

$$r = \frac{\omega_f}{\omega_n} = \frac{2\pi \cdot 14}{24} = 3.665$$

$$c_c = 2m\omega_n = 2 \times \frac{19.3}{32.2 \cdot 12} \cdot 24 = 2.4 \ \frac{\text{lb-s}}{\text{in}}$$

$$\zeta = c/c_c = \frac{0.96}{2.4} = .4$$

for a vibrometer $\dfrac{X}{Y} = \dfrac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = 1.052 = 1+\epsilon$

ie 5.2% high

$\therefore\ Y = \dfrac{X}{1.052} = \dfrac{.73}{1.052} = .694 \text{ in}$ $\qquad \beta, \psi \qquad x = \dfrac{r^2 Y}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \sin(\omega_f t - \psi - \beta)$

6.10

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{(5)(12)}{.193/32.2}} = 100 \ \frac{\text{rad}}{\text{s}}$$

$$\omega_f = 382 \ \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi}{60} = 40 \text{ rad/s}$$

$$r = \frac{\omega_f}{\omega_n} = .4$$

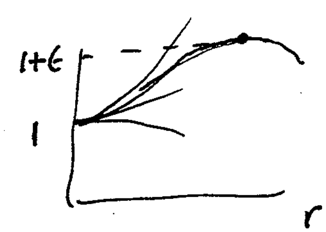
for accel $\dfrac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = 1.01 \rightarrow \zeta = .655$

$$c_c = 2m\omega_n = 2\frac{(.193)}{32.2} \cdot 100 = 1.2 \ \frac{\text{lb-s}}{\text{ft}}$$

$$c = c_c\zeta = .4795 \ \frac{\text{lb-s}}{\text{ft}}$$

$$\left\{ \frac{1}{4r^2}\left[\left(\frac{1}{1.01}\right)^2 - (1-r^2)^2 \right] \right\}^{1/2} = \zeta$$

To find max error $\qquad \dfrac{Z}{Y_{max}} = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}} = 1+\epsilon$



solve to determine range $\qquad 1-\epsilon = \dfrac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \qquad$ solve for r