field, a fluid may have a shear-rate-shear-stress relation which includes magnetohydrodynamic effects. For low density gases the fluid may have to be considered as made up of discrete particles, and not be treated as a continuum. These are typical of fluids in this fourth category.

Most of the work on non-Newtonian fluids during the past decade has been done by chemical engineers, but interest is being shown by people in other engineering areas. An informative discussion of the civil engineering aspects of non-Newtonian fluids is given by Bugliarello *et al* [4]. Engineers in many areas are becoming increasingly confronted with the flow of non-Newtonian fluids in pipes as well as in the design and selection of pumps for these fluids [5]. Experiments have shown that frictional effects for some non-Newtonian fluids are less than for a Newtonian fluid under equivalent turbulent flow conditions, resulting in a lower pressure drop in pipes and reduced drag on bodies submerged in liquids. This latter situation is of interest in naval hydrodynamics.

The study of non-Newtonian fluids is a part of the general science of rheology [6].

2–3. SURFACE TENSION AND CAPILLARITY

If a spoon is held under a dripping faucet, the water may rise nearly ⅛ in. above the upper edges of the spoon before spilling. Similarly, water may be poured into a glass to a level nearly ⅛ in. above the lip of the glass. If a small-bore tube is placed vertically in a free water surface, the water will rise in the tube; if the liquid is mercury, the mercury will be depressed within the tube. These are examples of the effects of surface tension and capillarity of liquids.

Surface tension is a characteristic of a liquid surface or skin, and its effects occur at liquid-gas or liquid-liquid interfaces. It is expressed in terms of energy per unit area, or its equivalent, force per unit length. The surface tension of water, for example, varies slightly with temperature, but it is about 0.005 lb_f/ft at ordinary temperatures. It may be reduced to about one-half that value by the addition of wetting agents.

Typical values of surface tension for some liquids are given in Table 2–8. It is seen that the surface tension is essentially the same for a liquid in contact with air or its own vapor. The higher the surface tension, the better is the interface in a manometer (see Sec. 4–3).

Attempts are usually made to avoid the effects of surface tension. Manometers (see Sec. 4–3) contain fluids which are chosen so as to have a readable meniscus (interface), that of mercury under water being exceptionally good, and that of carbon tetrachloride under water being poor. Manometer tubing, especially when an air-liquid interface is used in plastic tubing, should be selected carefully in order to avoid erroneous results.

TABLE 2–8

SURFACE TENSION OF LIQUIDS AT 77 F IN CONTACT WITH AIR, WATER, OR THEIR OWN VAPOR*

Substance	Surface Tension (lb/ft)
Carbon tetrachloride-air	0.0018
Water-air	0.0050
Mercury-air	0.032
Carbon tetrachloride-water	0.0031
Mercury-water	0.026
Carbon tetrachloride-vapor	0.0018
Water-vapor	0.0050

* *Handbook of Chemistry and Physics* (38th ed.; Cleveland: Chemical Rubber Publishing Company, 1956–1957).

Some plastic tubes also exhibit hysteresis effects when used in this way—a falling liquid column comes to rest at a different position than a rising liquid column for the same external pressure condition. Hydraulic models are made large enough so that shallow depths, which otherwise might be affected by surface tension, are avoided. Wetting agents may be used to reduce these effects if the size of the model is made as large as possible, yet smaller than would be desirable. Surface tension plays a role in determining the growth of small gas nuclei in liquids when they pass through low-pressure regions, yet the precise role is not understood. This rapid growth and collapse of bubbles in liquids is one form of cavitation.

Forces due to surface tension are, with the preceding exceptions, generally small compared with the forces due to gravity, viscosity, and pressure in engineering practice.

2–4. COMPRESSIBILITY OR ELASTICITY

Fluids may be deformed by viscous shear or compressed by an external pressure applied to a volume of fluid. All fluids are compressible by this method, liquids to a much smaller degree, however, than gases.

The compressibility is defined in terms of an average bulk modulus of elasticity

What we call β →

$$\overline{K} = -\frac{p_2 - p_1}{(V_2 - V_1)/V_1} \tag{2–11}$$

where V_2 and V_1 are the volumes of the substance at pressure p_2 and p_1, respectively. The bulk modulus varies with the pressure for gases, and with both pressure and temperature (though but slightly) for liquids. Thus, the true bulk modulus of elasticity is the limiting value of Eq. 2–11 when the pressure and volume changes become infinitesimal.

$$K = -\frac{dp}{dV/V} \qquad (2\text{–}12a)$$

If a unit mass of substance is considered,

$$K = -\frac{dp}{dv/v} \qquad (2\text{–}12b)$$

and

$$K = +\frac{dp}{d\rho/\rho} \qquad (2\text{–}12c)$$

The denominators of Eqs. 2–11 and 2–12 are dimensionless, so that K has the dimensions of a pressure, or force per unit area.

The value of K for water at 68 F is about 320,000 $lb_f/in.^2$ at atmospheric pressure, increasing essentially linearly to about 410,000 $lb_f/in.^2$ at a pressure of 15,000 $lb_f/in.^2$ Thus, in this range,

$$K = 320{,}000 + 6\,p \quad lb_f/in.^2$$

where p is the gage pressure in $lb_f/in.^2$

The change in volume of a gas for a change in pressure depends on the compression process. If *isothermal* (constant temperature), the gas equation may be expressed in logarithmic form as $\ln p = \ln \rho + \ln (RT)$ and differentiated to obtain $dp/p = d\rho/\rho$. Thus

$$K_{\text{isothermal}} = \frac{dp}{d\rho/\rho} = \frac{dp}{dp/p} = p \qquad (2\text{–}13)$$

and the elastic modulus equals the absolute pressure during an isothermal compression.

If the compression is adiabatic and is carried out slowly so that equilibrium conditions exist, the compression may be considered reversible and adiabatic, or *isentropic* (see Chapter 3). For this process, p/ρ^k = constant (k is the ratio of specific heat capacities). The logarithmic form, $\ln p - k \ln \rho = \ln C$, may be differentiated to obtain $dp/p = k\, d\rho/\rho$. Thus

$$K_{\text{isentropic}} = \frac{dp}{d\rho/\rho} = \frac{dp}{dp/kp} = kp \qquad (2\text{–}14)$$

and the elastic modulus equals the absolute pressure times the ratio of specific heat capacities ($k = c_p/c_v = 1.4$ for air) during an isentropic compression.

The bulk modulus of elasticity K is of interest in acoustics as well as in fluid mechanics. The velocity of sound in any medium is

$$c = \sqrt{\frac{K}{\rho}} \qquad (2\text{–}15)$$

and for a gas, sound waves are transmitted essentially isentropically (see Sec. 9–1), so that the velocity of sound in a perfect gas is

$$c = \sqrt{\frac{kp}{\rho}} = \sqrt{kRT} \qquad (2\text{–}16)$$

EXAMPLE 2–5. What is the speed of sound in water at 68 F and atmospheric pressure?

Solution:

$$c = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{(320{,}000\ lb_f/in.^2)\ (144\ in.^2/ft^2)}{1.94\ slugs/ft^3}} = 4870\ ft/sec$$

EXAMPLE 2–6. What is the speed of sound in air at 68 F at sea level and at an altitude where the pressure is 10 $lb_f/in.^2$ absolute?

Solution: At either pressure,

$$c = \sqrt{kRT} = \sqrt{(1.4)\left(1715\ \frac{ft\ lb_f}{slug\ R}\right)(528\ R)} = 1126\ ft/sec$$

2–5. VAPOR PRESSURE

If a liquid and its vapor coexist in equilibrium, the vapor is called a saturated vapor, and the pressure exerted by this saturated vapor is called the *vapor pressure*. The vapor pressure is a function of temperature for a given substance.

The vapor pressure of liquids is of practical importance in barometers (see Sec. 4–3), pump-piping systems, and from an elementary point of view, in the formation of cavities in low-pressure regions within a liquid.

Values of vapor pressure for some liquids at various temperatures are shown in Table 2–9.

TABLE 2–9

VAPOR PRESSURE OF SOME LIQUIDS

Temperature (deg F)	Liquid Vapor Pressure (psia)				
	Water	Mercury	Kerosene	Propane	Methyl Alcohol
40	0.122		0.32	78	0.71
60	0.26	0.000025	0.44	107	1.42
100	0.95		0.86	187	4.42
160	4.74		1.23		

REFERENCES

1. W. L. Wilkinson, *Non-Newtonian Fluids* (New York: Pergamon Press, 1960).

2. A. B. Metzner, "Flow of Non-Newtonian Fluids," Section 7 of *Handbook*

PROBLEMS

3–1. A simple hydraulic pump (a source of fluid power) consists of two meshing gears contained in a close-fitting housing (Fig. 3–25). As the gears rotate, each tooth space captures a nearly constant amount of fluid at the inlet and carries it around to the outlet. At the meshing region the gear teeth contact and seal the outlet. Ideally, Q = constant through the device and the pressure rises from 0 to P_s.

a) Qualitatively discuss how Q is related to the gear-tooth size, the speed Ω, and the width w of the gears.

b) If there are no energy losses, how are Q, P_s, T, and Ω related?

c) The above pump is connected to a fluid system. During operation, Q = const = 10 gal/min, and the outlet pressure P_s is found to vary sinusoidally about a mean pressure of 500 psi, $P_s = 500 + 100 \sin 20\pi t$. Determine the average power and the maximum instantaneous power delivered to the system.

3–2. In a chemical-processing plant, liquid flows into a long cylinder with a stainless-steel wall 1 in. thick (Fig. 3–26). The pressure in the tank may be assumed uniform and varies from 100 to 500 psi during operation of the process. In this problem we wish to study the capacitive properties of this tank. The fluid has a bulk modulus of 150,000 psi and a density of 60 lb/ft^3; the steel has a Young's modulus of 30 $\times$ 10^6 psi and a Poisson's ratio of 0.3.

a) Assuming that the tank is rigid, compute the fluid capacitance of the tank, $C_f = (\int(Q_1 - Q_2)\,dt)/P = V/P$.

b) Compute the energy stored when P increases from 0 to 500 psi.

c) Assuming that the fluid is incompressible, compute C_f and compute the energy stored when P = 500 psi.

d) Combine the above to obtain the actual tank capacitance and the total energy stored when P = 500 psi.

e) Discuss the linearity of the V/P relations in (a), (c), and (d).

3–3. Consider an open reservoir being filled with water. The pressure in the tank bottom varies as shown in Fig. 3–27. Sketch curves of the flow rate Q into the tank and the height H of the water in the tank vs. time.

3–4. The flow of water into the tank of Problem 3–3 varies as shown in Fig. 3–28. Given that A = 1 ft^2, determine, as a function of time, the pressure P required at the inlet to produce this flow rate. $P = P_0$ at t = 0.

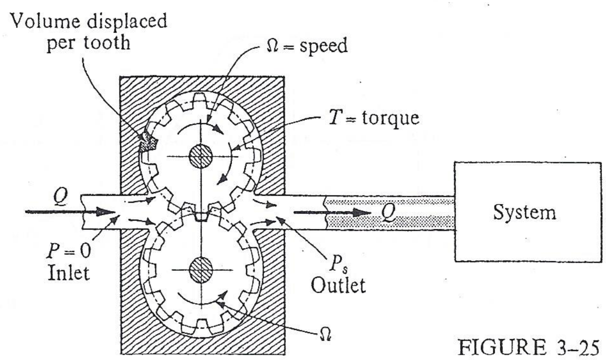

FIGURE 3–25

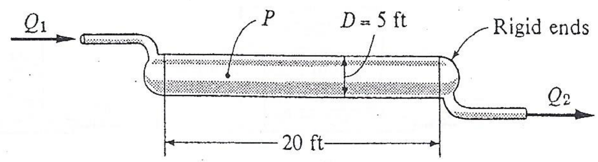

FIGURE 3–26

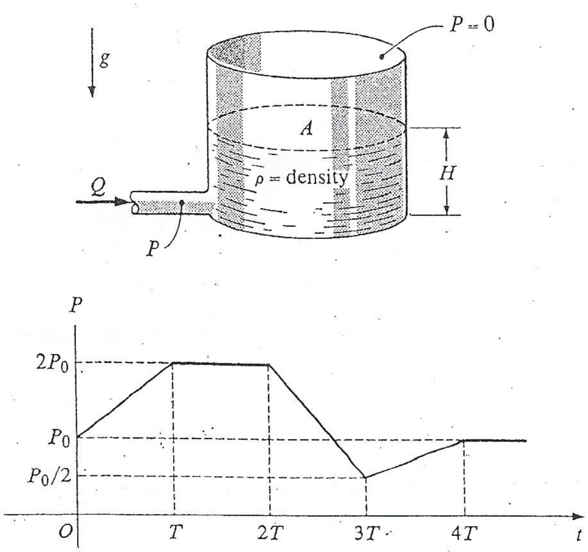

FIGURE 3–27

3–5. A cylindrical steel tank of the dimensions shown in Fig. 3–29 has a single inlet for oil flow. The tank is rigid, and the oil has a bulk modulus β = 200,000 psi and a density ρ_0 = 62 lb/ft^3. The average pressure in the tank

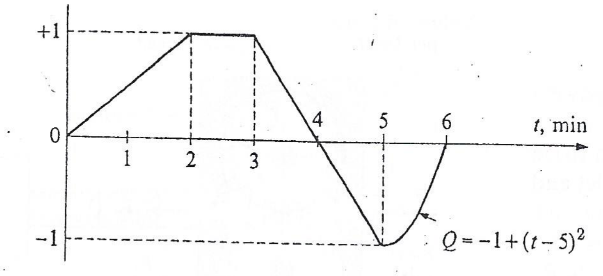

FIGURE 3-28

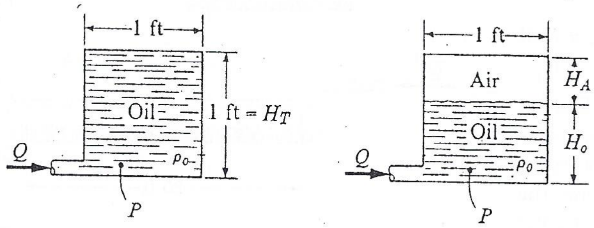

FIGURE 3-29

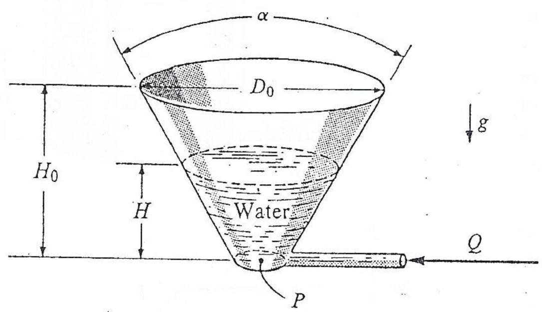

FIGURE 3-30

is P_O, and P varies slightly from this value. Gravity forces are negligible.

a) Assuming that the tank is entirely full of oil as in Fig. 3-29(i), compute the fluid capacitance.

b) In Fig. 3-29(ii) a quantity of air was accidentally left in the tank when it was filled with oil. Derive an expression for the capacitance when $P = P_O + \Delta P$, where ΔP is small in terms of H_O, H_A, P_O, etc., assuming that the air is isothermal and acts like a perfect gas.

c) If $H_A = 0.1\ H_O$ and $P_O = 200$ psi, compute the capacitances of the tanks in Fig. 3-29(i) and (ii). Comment on the effect of entrained gas bubbles on the capacitance of liquid-filled tanks.

3-6. A water storage tank has a conical shape (Fig. 3—30). Assuming that $\alpha = 30°$, $H_O = D_O$, and the tank is rigid, compute and sketch a curve of stored volume V vs. pressure P for $0 < H < H_O$. Sketch a curve of stored energy vs. P. For small changes of P about a mean value P_0 the tank is approximated by an ideal capacitance. Sketch a curve of capacitance vs. P_0.

3-7. A fluid accumulator (Fig. 3-9) is required to have a capacitance of 20 lb/in^5 and a volume of 50 in^3. Design the device; i.e., specify piston area, spring stiffness, length and wall thickness of cylinder, and any other important parameters. Discuss qualitatively the conditions on Q, P_2, and their respective integrals and derivatives which must be met for the device to behave approximately like an ideal capacitance.

3-8. A conduit, called a *penstock*, which leads water from a dam to the inlet of a hydraulic turbine driving an electric generator has a diameter of 5 ft and a length of 700 ft. Assuming that the conduit is rigid, compute the inertance of the fluid. During full-load operation the average flow velocity in the pipe is 50 ft/sec. For this condition, find the energy stored in the fluid inertance and determine how long a 100-watt light bulb could operate from this amount of energy.

3-9. In Problem 3-8, an attempt is made to reduce the flow in the conduit to zero as a linear function of time in 30 sec, starting with full-load conditions. This is done by closing a gate valve near the turbine inlet. Estimate the pressure rise at the valve if the upstream pressure at the inlet to the conduit remains constant.

3-10. A circular tube has a gradual uniform taper, so that its internal diameter decreases linearly from D_2 at the left end to D_1 at the right. The tube is rigid and has length L.

a) Assuming that an incompressible fluid of density ρ flows from left to right in the tube, compute the fluid inertance.

b) If the fluid flows from right to left, how does the inertance change?

c) If the taper is nonuniform, how will this affect the inertance, assuming that D_2 and D_1 do not change?