SOLUTION

$$k = \frac{AE}{l} = \frac{(0.5)^2 \times 10^7}{100} = 2.5 \times 10^4 \text{ lb/in.}$$

a. $$\omega = \sqrt{\frac{k}{M}} = \sqrt{\frac{2.5 \times 10^4 \times 386}{5}} = 1389 \text{ rad/sec}$$

$$f = \frac{\omega}{2\pi} = \frac{1389}{2\pi} = 221 \text{ Hz}$$

b. $$\omega' = \sqrt{\frac{k}{M + m/3}} = \sqrt{\frac{2.5 \times 10^4 \times 386}{5 + 2.5/3}} = 1286 \text{ rad/sec}$$

$$f' = \frac{\omega'}{2\pi} = \frac{1286}{2\pi} = 205 \text{ Hz}$$

The exact results† are

$$\omega'' = 1283 \text{ rad/sec}$$

$$f'' = 204 \text{ Hz}$$

PROBLEMS

2-1. A body weighing 12 lb is suspended by a spring having a modulus of 4 lb/in. Determine the natural frequency for the system.

2-2. A 5-lb weight, attached to a spring, causes a static equilibrium elongation of 0.143 in. Determine the natural frequency of oscillation that the system will exhibit if it is displaced from its equilibrium position and then released.

2-3. A body having a mass of 9 kg is suspended by a spring having a modulus of 800 N/m. Determine the natural period of vibration for the system.

2-4. A 1.4-kg mass causes a 4-mm static equilibrium elongation of a certain spring. Determine the natural frequency of vibration for this system.

2-5. A weight of 20 lb, attached to a spring, is found to oscillate at 330 cycles/min. Determine the spring constant k.

2-6. A motor is placed on rubber mountings to isolate it from its foundation. If the dead weight of the motor compresses the mountings 3.1 mm, determine the natural frequency of the system.

2-7. Determine the mass which must be attached to a spring having a modulus of 1750 N/m in order that the resulting system will have a frequency of 21 Hz.

2-8. Determine the length of a simple pendulum so that the half period will be 0.5 sec.

2-9. A mass-spring system has a natural period of 0.152 sec. Determine the period if the spring constant is increased by 60%.

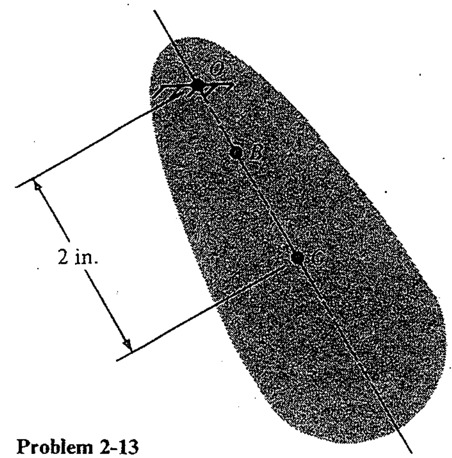
2-10. A weight W is suspended from the end of spring k, and the system has a natural frequency of 90 cycles/min. The weight W is removed and replaced by a weight $W/4$ at the middle of the spring, and the upper and lower ends of the spring are then fixed. Determine the frequency of the new system.

† See Example 10-6.

2-11. A mass-spring system has a frequency of 2 Hz. When the mass is decreased by 0.4 kg, the frequency is changed by 25%. Determine (a) the mass m, and (b) the spring constant k for the original system.

2-12. A mass-spring system has a frequency of 5 Hz. When the spring constant is reduced by 4 lb/in., the frequency is altered by 30%. Determine (a) the spring constant k and (b) the weight W for the original system.

2-13. The cam-shaped member shown in the accompanying figure is free to rotate about the axis at O perpendicular to the view and exhibits a natural frequency of angular oscillation of 54 cycles/min with respect to this axis. The rotation axis is moved to point B, located somewhere between O and the mass center G. The frequency about the axis at B is found to be 36 cycles/min. Determine the distance OB.



Problem 2-13

2-14. A harmonic motion is expressed by the relation $x = 0.5 \sin (10t + 4)$, where x is measured in inches, t in seconds, and the phase angle in radians. Determine (a) the circular and natural frequencies of vibration and (b) the displacement, velocity, and acceleration at $t = 0.3$ sec.

2-15. A vibratory motion has an amplitude of 0.30 in. and a period of 0.4 sec. Determine (a) the frequency of vibration, (b) the maximum velocity, and (c) the maximum acceleration.

2-16. From experimental measurements it has been found that a structure is vibrating harmonically. If the amplitude of motion was measured to be 2 cm and the period was 3 sec, determine (a) the maximum velocity and (b) the maximum acceleration to which the structure is subjected.

2-17. A simple harmonic motion has a maximum amplitude of 50 mm and a period of 10 sec. Determine (a) the displacement, (b) the velocity, and (c) the acceleration, at the time $t = 8$ sec, if the time is zero when the motion passes through the equilibrium position.

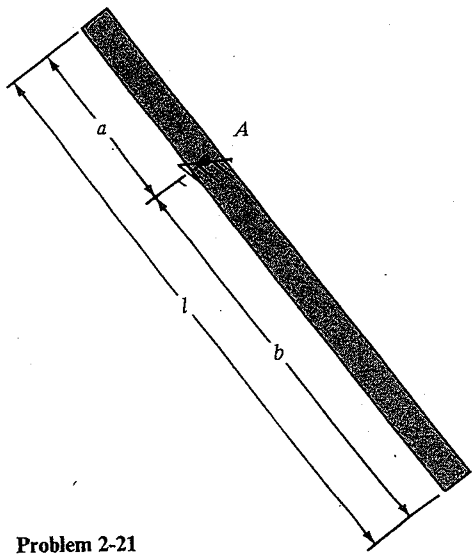
2-18. Vibration-measuring instruments indicate that a structure is vibrating harmonically at a frequency of 4 Hz, with a maximum acceleration of 324 in./sec^2.

Determine (a) the displacement amplitude and (b) the maximum velocity during motion of the structure.

2-19. An accelerometer is used to measure a vibratory motion. If the frequency is found to be 20 Hz and the maximum acceleration is 12 m/s^2, determine (a) the period, (b) the amplitude, and (c) the maximum velocity of the motion.

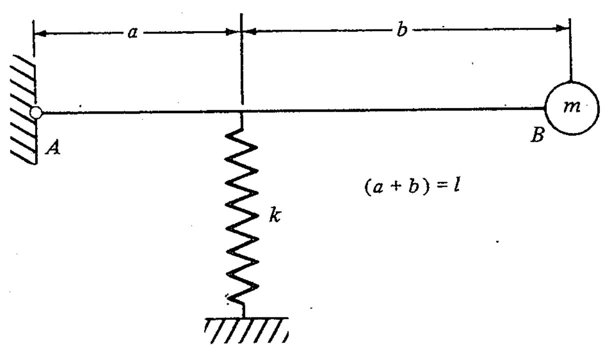
2-20. If the frequency of a vibrating body is 5 Hz and its maximum velocity is 90 in./sec, determine (a) the amplitude, (b) the maximum acceleration, and (c) the period of the motion.

2-21. A slender uniform bar pivots in a vertical plane about axis A located distance a from the upper end. For small angular vibration, obtain the differential equation of motion and express the frequency.



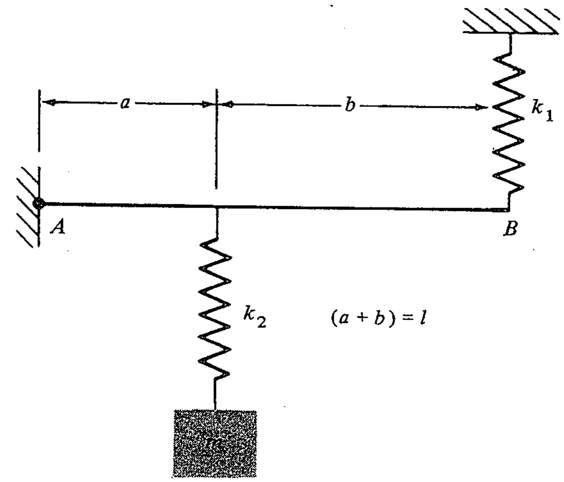
Problem 2-21

2-22. A rigid bar AB is weightless and pivots at A as shown in the accompanying figure. Obtain (a) the differential equation of motion and (b) the expression for the natural frequency of angular vibration of the system.



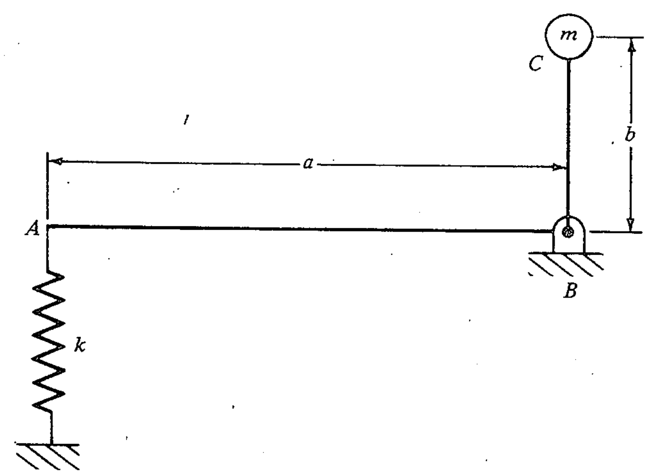
Problem 2-22

2-23. A rigid bar AB is weightless and pivots at A as shown in the accompanying figure. Determine (a) the differential equation for the vertical motion of m and (b) the expression for the natural frequency of oscillation of m.



Problem 2-23

2-24. A rigid member ABC is weightless and pivots at B as shown in the accompanying figure. Obtain (a) the differential equation of motion and (b) the expression for the natural frequency of angular vibration of the system. (c) Indicate the condition for stability of the vibration.

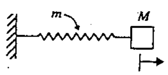
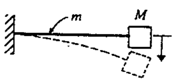
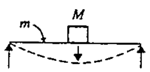
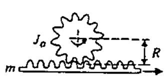
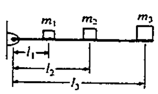


Problem 2-24

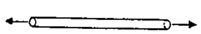
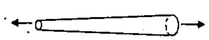
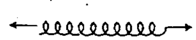
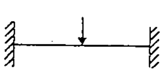
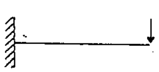
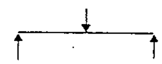
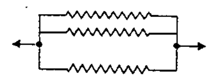
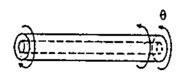
2-25. Determine (a) the differential equation of motion and (b) the expression for the natural frequency of vibration of the system shown herewith.

Equivalent Masses, Springs and Dampers

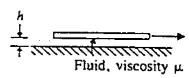
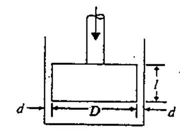
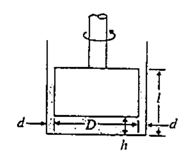
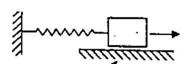
Equivalent masses

	Mass (M) attached at end of spring of mass m	$m_{eq} = M + \frac{m}{3}$
	Cantilever beam of mass m carrying an end mass M	$m_{eq} = M + 0.23\,m$
	Simply supported beam of mass m carrying a mass M at the middle	$m_{eq} = M + 0.5\,m$
	Coupled translational and rotational masses	$m_{eq} = m + \frac{J_0}{R^2}$ $J_{eq} = J_0 + mR^2$
	Masses on a hinged bar	$m_{eq_1} = m_1 + \left(\frac{l_2}{l_1}\right)^2 m_2 + \left(\frac{l_3}{l_1}\right)^2 m_3$

Equivalent springs

	Rod under axial load (l = length, A = cross sectional area)	$k_{eq} = \frac{EA}{l}$
	Tapered rod under axial load (D, d = end diameters)	$k_{eq} = \frac{\pi EDd}{4l}$
	Helical spring under axial load (d = wire diameter, D = mean coil diameter, n = number of active turns)	$k_{eq} = \frac{Gd^4}{8nD^3}$
	Fixed-fixed beam with load at the middle	$k_{eq} = \frac{192\,EI}{l^3}$
	Cantilever beam with end load	$k_{eq} = \frac{3EI}{l^3}$
	Simply supported beam with load at the middle	$k_{eq} = \frac{48EI}{l^3}$
	Springs in series	$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_n}$
	Springs in parallel	$k_{eq} = k_1 + k_2 + \cdots + k_n$
	Hollow shaft under torsion (l = length, D = outer diameter, d = inner diameter)	$k_{eq} = \frac{\pi G}{32l}(D^4 - d^4)$

Equivalent viscous dampers

	Relative motion between parallel surfaces (A = area of smaller plate)	$c_{eq} = \frac{\mu A}{h}$
	Dashpot (axial motion of a piston in a cylinder)	$c_{eq} = \mu \frac{3\pi D^3 l}{4d^3}\left(1 + \frac{2d}{D}\right)$
	Torsional damper	$c_{eq} = \frac{\pi\mu D^2(l-h)}{2d} + \frac{\pi\mu D^3}{32h}$
	Dry friction (Coulomb damping) (fN = friction force, ω = frequency, X = amplitude of vibration)	$c_{eq} = \frac{4fN}{\pi\omega X}$

Quantity	SI Equivalence	English Equivalence
Work or energy	1 in. $-$ lb_f = 0.1129848 J 1 ft $-$ lb_f = 1.355818 J 1 Btu = 1055.056 J 1 kWh = 3.6×10^6 J	1 J = 8.850744 in. $-$ lb_f 1 J = 0.737562 ft $-$ lb_f = 0.947817×10^{-3} Btu = 0.277778 kWh
Power	1 in $-$ lb_f/sec = 0.1129848 W 1 ft $-$ lb_f/sec = 1.355818 W = 0.0018182 hp 1 hp = 745.7 W	1 W = 8.850744 in. $-$ lb_f/sec 1 W = 0.737562 ft $-$ lb_f/sec = 1.34102×10^{-3} hp
Area moment of inertia or second moment of area	1 in^4 = 41.6231×10^{-8} m^4 1 ft^4 = 86.3097×10^{-4} m^4	1 m^4 = 240.251×10^4 in^4 = 115.862 ft^4
Mass moment of inertia	1 in $-$ lb_f $-$ sec^2 = 0.1129848 $m^2 \cdot kg$	1 $m^2 \cdot kg$ = 8.850744 in. $-$ lb_f $-$ sec^2
Spring constant: translatory	1 lb_f/in. = 175.1268 N/m 1 lb_f/ft = 14.5939 N/m	1 N/m = 5.71017×10^{-3} lb_f/in. = 68.5221×10^{-3} lb_f/ft
torsional	1 in. $-$ lb_f/rad = 0.1129848 $m \cdot N$/rad	1 $m \cdot N$/rad = 8.850744 in-lb_f/rad = 0.737562 lb_f $-$ ft/rad
Damping constant: translatory	1 lb_f $-$ sec/in = 175.1268 $N \cdot s/m$	1 $N \cdot s/m$ = 5.71017×10^{-3} lb_f $-$ sec/in.
torsional	1 in $-$ lb_f $-$ sec/rad = 0.1129848 $m \cdot N \cdot s$/rad	1 $m \cdot N \cdot s$/rad = 8.850744 lb_f $-$ in $-$ sec/rad
Angles	1 rad = 57.295754 degrees; 1 degree = 0.0174533 rad; 1 rpm = 0.166667 rev/sec = 0.104720 rad/sec; 1 rad/sec = 9.54909 rpm	