

HW#8

4-2, 6, 13

4-2

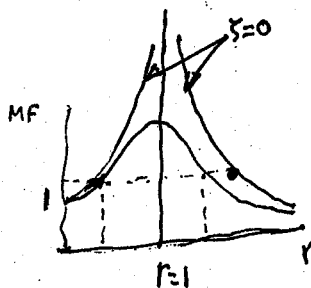
$$P = P_0 \sin \omega_f t : P_0 = 45 \text{ lb} \quad \tau = .25 \text{ sec} = \frac{2\pi}{\omega_f} ; \quad W = 2.5 \text{ lb} \quad k = 15 \text{ lb/in}$$

$$\text{find } \Delta = \frac{\Delta_0}{1-r^2} \quad r < 1$$

$$\textcircled{1} \quad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{W}} = \sqrt{\frac{15 \cdot 32.2 \cdot 12}{2.5}} = 48.15 \text{ rad/sec} \quad \left. \vphantom{\omega = \sqrt{\frac{k}{m}}} \right\} r = \frac{\omega_f}{\omega} = .522$$

$$\textcircled{2} \quad \omega_f = \frac{2\pi}{\tau} = 8\pi = 25.133 \text{ rad/sec}$$

$$\Delta = \frac{P_0/k}{1-r^2} = \frac{3 \text{ in}}{.7275} = 4.123 \text{ in}$$



$$\text{4-6} \quad m = 8.75 \text{ kg} \quad k = 3500 \text{ N/m} \quad P_0 = 187 \text{ N} \quad \Delta = 7.6 \text{ cm}$$

find ω_f (or f_f)

$$\omega_n = \sqrt{\frac{k}{m}} = 20 \text{ rad/sec} ; \quad \Delta_0 = P_0/k = .0534 \text{ m} = 5.34 \text{ cm} \quad MF = 1.423 = \frac{\Delta}{\Delta_0}$$

Since $MF > 1$ there are 2 solutions. First assume $r < 1$

$$\Delta = \frac{\Delta_0}{1-r^2} \Rightarrow 1-r^2 = \frac{\Delta_0}{\Delta} \Rightarrow r = \sqrt{1 - \frac{\Delta_0}{\Delta}} = .545$$

$$r = \frac{\omega_f}{\omega_n} \Rightarrow \omega_f = r\omega_n = 10.9 \text{ rad/sec} \quad f_f = \frac{\omega_f}{2\pi} = 1.735 \text{ Hz}$$

$$\text{Now Assume } r > 1 \Rightarrow \Delta = \frac{\Delta_0}{r^2-1} \Rightarrow r^2-1 = \frac{\Delta_0}{\Delta} \Rightarrow r = \sqrt{1 + \frac{\Delta_0}{\Delta}} = 1.305$$

$$r = \frac{\omega_f}{\omega_n} \Rightarrow \omega_f = r\omega_n = 26.097 \text{ rad/sec} \quad f_f = \frac{\omega_f}{2\pi} = 4.153 \text{ Hz}$$

4-13

$$W = 19.3 \text{ lb} , \quad k = 10 \text{ lb/in} ; \quad P = P_0 \sin \omega_f t \quad \text{when } P_0 = 4 \text{ lb}$$

system is in resonance $r = 1$. From work in class

$$x_p = - \frac{\Delta_0 \omega_f t}{2} \cos \omega_f t \quad \Delta_0 = P_0/k = .4 \text{ in}$$

$$\omega = \omega_f = \sqrt{\frac{kg}{W}} = 14.149 \text{ rad/sec}$$

$$\textcircled{a} \quad \frac{1}{2} \text{ cycle} \quad \omega_f t = \pi \quad x_p = - \left(\frac{.4}{2} \right) \pi \cos(\pi) = .2\pi = .628 \text{ in}$$

$$\textcircled{a} \quad 2\frac{1}{2} \text{ cycles} \quad \omega_f t = 5\pi \quad x_p = - \left(\frac{.4}{2} \right) 5\pi \cos(5\pi) = \pi = 3.142 \text{ in}$$

$$\textcircled{a} \quad 4\frac{1}{2} \text{ cycles} \quad \omega_f t = 9\pi \quad x_p = - \left(\frac{.4}{2} \right) 9\pi \cos(9\pi) = 1.8\pi = 5.655 \text{ in}$$

$$\textcircled{a} \quad 6\frac{1}{2} \text{ cycles} \quad \omega_f t = 13\pi \quad x_p = - \left(\frac{.4}{2} \right) 13\pi \cos(13\pi) = 2.6\pi = 8.168 \text{ in}$$

4-21 $m = 8.75 \text{ kg}$; $k = 1750 \text{ N/m}$; $c = 37.13 \text{ N-sec/m}$. $P = P_0 \sin \omega_f t \Rightarrow P_0 = 220 \text{ N}$

$$f_f = 4.50 \text{ Hz}$$

$$\omega_n = \sqrt{\frac{k}{m}} = 14.142 \text{ rad/sec}$$

$$c_c = 2m\omega_n = 247.49 \text{ N-sec/m}$$

$$\zeta = \frac{c}{c_c} = .15$$

since $\zeta < 1$ system is underdamped

$$\omega_d = \sqrt{1 - \zeta^2} \omega_n = 13.982 \text{ rad/sec}$$

transient $x_c = \Sigma' e^{-\zeta \omega_n t} \sin(\omega_d t + \phi) = \Sigma' e^{-2.1617t} \sin(13.982t + \phi)$ ✓

$$\Sigma_0 = P_0/k = .1257 \text{ m}; \text{ note } \Sigma', \phi \text{ obtained from initial conditions}$$

$$f_f = 4.50 \text{ Hz} = \frac{\omega_f}{2\pi} \Rightarrow \omega_f = 28.274 \text{ rad/sec} \Rightarrow r = \frac{\omega_f}{\omega_n} = 2.0$$

$$\Sigma_{ss} = \frac{\Sigma_0}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = .0411 \text{ m} \quad \tan \psi = \frac{2\zeta r}{1-r^2} = -.2 \quad \psi = -11.31^\circ = -.1974 \text{ rad}$$

steadystate $x_p = .0411 \sin\left(\frac{\omega_f}{28.274t} + \frac{\psi}{.1974}\right)$ ✓; $x_{total} = x_c + x_p$

4-27 $k = 24 \text{ lb/in}$; $c = .88 \text{ lb-sec/in}$; $P_0 = 15 \text{ lb}$ when $r=1$ $x_p = 2.8409 \text{ in}$

find ζ ; ω_n ; ω_d note that $x_p|_{r=1} = \Sigma_{res}$

$$\Sigma_{res} = \frac{\Sigma_0}{2\zeta} \Rightarrow \Sigma_0 = P_0/k = .625 \text{ in} \Rightarrow \zeta = \frac{1}{2} \frac{\Sigma_0}{\Sigma_{res}} = \frac{1}{2} \left(\frac{.625}{2.8409} \right) = .11 = \zeta$$

$\zeta = \frac{c}{c_c} \therefore c_c = \frac{c}{\zeta} = 8 \text{ lb-sec/in}$; now $\omega_n^2 = \frac{k}{m} \Rightarrow k = m\omega_n^2$ & $c_c = 2m\omega_n$ Since we don't have

Thus $\frac{k}{c_c} = \frac{\omega_n}{2}$ or $\omega_n = \frac{2k}{c_c} = 6 \text{ rad/sec}$ $f_n = \frac{\omega_n}{2\pi} = .955 \text{ Hz}$

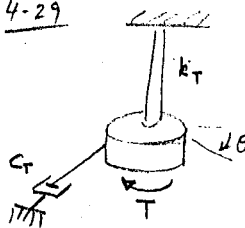
Now $\omega_d = \omega_n \sqrt{1 - \zeta^2} = 5.964 \text{ rad/sec}$ $f_d = .949 \text{ Hz}$

4-28 gives when $r=2$ $\Sigma_{r=2} = \frac{1}{10} \Sigma_{res}$

$$\Sigma_{r=2} = \frac{\Sigma_0}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{\Sigma_0}{\sqrt{(-3)^2 + (4\zeta)^2}} = \frac{1}{10} \Sigma_{res} = \frac{1}{10} \frac{\Sigma_0}{2\zeta}$$

Solving those terms indicated by (~~~~) gives $\zeta = .1531$

4-29



$$k_T = 60000 \text{ lb-in/rad}; \quad I = 24 \text{ lb-in-sec}^2/\text{rad}; \quad c_T = 840 \text{ lb-in-sec/rad}$$

The ODE is $I\ddot{\theta} + c_T\dot{\theta} + k_T\theta = T = T_0 \sin \omega_f t$

also for given $T_0 = 2700 \text{ lb-in}$ results in $\theta = 3.368^\circ = .0588 \text{ rad}$

In this case everything we said about the linear case holds here

Thus $MF = \frac{\theta}{\theta_0}$ where $\theta_0 = T_0/k_T = .045 \text{ rad} \Rightarrow MF = 1.3067 = \frac{0.0588}{0.045}$

now $\sqrt{k_T/I} = \omega_n = 50 \text{ rad/sec}; \quad c_{Tcr} = 2I\omega_n = 2400 \text{ lb-in/rad} \Rightarrow \zeta = \frac{c}{c_{cr}} = .35$

$MF = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = 1.3067$; solve for $r \Rightarrow r = 1.0722, r = .6$

$$r^4 - 1.51r^2 + .414 = 0$$

For $r = 1.0722 = \omega_f/\omega_n \Rightarrow \omega_f = 1.0722\omega_n = 53.61 \text{ rad/sec} \quad f_f = \frac{\omega}{2\pi} = 8.532 \text{ Hz}$

$r = .6 = \omega_f/\omega_n \Rightarrow \omega_f = .6\omega_n = 30 \text{ rad/sec} \Rightarrow f_f = \frac{\omega}{2\pi} = 4.775 \text{ Hz}$

TO OBTAIN MF_{RES} & MF_{MAX} :

Now $MF_{RES} = \frac{\theta_{RES}}{\theta_0} = \frac{1}{2\zeta} = 1.429 @ r=1; MF_{MAX} = MF_{RES} \cdot \frac{1}{\sqrt{1-\zeta^2}} = 1.525 @ r=.869$

TO OBTAIN THE BANDWIDTH:

now $MF_{RES} = Q$ for the bandwidth $\frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{1}{2\zeta} \Rightarrow r^2 = (1-2\zeta^2) \pm 2\zeta\sqrt{1+\zeta^2}$

$\Rightarrow r_2 = 1.2234$ and $r_1 = .1156 \Rightarrow \text{bandwidth is } r_2 - r_1 = 1.1078$

NOTE: Must use full formula for r since ζ is not small.

TO FIND $T_{transmitted}$

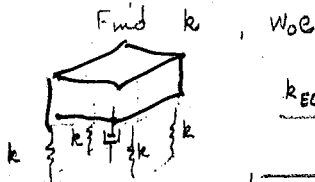
$$TR = \frac{T_{trans}}{T_0} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

when $r = 0.6 \quad TR = 1.4169 \quad T_{trans} = TR \cdot T_0 = 3825.5 \text{ lb-in}$

$r = 1.0724 \quad TR = 1.6334 \quad T_{trans} = TR \cdot T_0 = 4410.1 \text{ lb-in}$

4-33, 35, 40, 41

4-33 Given $W = 128.67 \text{ lb}$; $\omega_f = 859.44 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = 90 \text{ rad/sec}$;
 $c = 10 \text{ lbsec/in}$; $\Sigma = 0.116 \text{ in}$; $F_T = 174 \text{ lb}$



$$k_{EA} = 4k$$

$$\text{now } F_T = \Sigma \sqrt{k_{eq}^2 + (c\omega_f)^2}$$

$$174 \text{ lb} = 0.116 \text{ in} \sqrt{k_{eq}^2 + \left(10 \frac{\text{lbsec}}{\text{in}} \cdot 90 \frac{\text{rad}}{\text{sec}}\right)^2}$$

$$\sqrt{(F_T/\Sigma)^2 - (c\omega_f)^2} = k_{eq} = 1200.00 \text{ lb/in}$$

$$k = \frac{1}{4} k_{eq} = 300.0 \text{ lb/in}$$

$$\Sigma = \frac{P_0}{k_{eq}} = \frac{P_0}{\sqrt{(k_{eq} - m\omega_f^2)^2 + (c\omega_f)^2}}$$

$$P_0 = 0.116 \text{ in} \sqrt{(k_{eq} - m\omega_f^2)^2 + (c\omega_f)^2} = 202.92 \text{ lb} = m\omega_f^2$$

known

$$m = \frac{W}{g} = .333 \frac{\text{lb-sec}^2}{\text{in}}$$

$$g = 386.4 \text{ in/sec}^2$$

$$P_0 = m\omega_f^2$$

$$\omega_e = \frac{(202.92)(32.2)(12)}{(90)^2} \text{ lb/in}$$

$$\omega_e = 9.68 \text{ lb/in}$$

$$\text{as } r \rightarrow \infty \quad \Sigma \rightarrow \frac{m\omega_e}{m} = \frac{\omega_e}{W} = \frac{9.68 \text{ lb-in}}{128.67 \text{ lb}} \sim .075 \text{ in}$$

4-35

Given $e\omega_0/2 = 3 \text{ lb/in}$; $W = 200 \text{ lb}$; when $x=0$ $\psi = 90^\circ \Rightarrow r=1$ from $\tan \psi = \frac{2\zeta r}{1-r^2}$
 $\omega_f = 840 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = 87.965 \text{ rad/sec}$; $\Sigma = .75 \text{ in}$

Find (a) ω_n (or f_n) , ζ (b) Suppose $\omega_f = 1260 \frac{\text{rev}}{\text{min}} = 131.95 \text{ rad/sec}$ find Σ and ψ

$$(a) \quad \Sigma = \frac{m\omega_e}{m} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{\omega_e}{W} \cdot \frac{1}{2\zeta}$$

$$\Sigma = .75 \text{ in} = \frac{2(3)}{200} \frac{1}{2\zeta}$$

since $r=1$

$$\text{then } \zeta = \frac{\omega_e}{2W\Sigma} = \frac{2(3)}{400(.75)} = .02 = \zeta$$

$$\text{since } r=1 \quad \omega_0 = \omega_f = 87.965 \text{ rad/sec} \quad \text{or } f = 14 \text{ Hz}$$

(b) For the system $\omega_f = 131.95 \text{ rad/sec}$; $\omega_0 = 87.965 \text{ rad/sec} \Rightarrow r=1.5$ also $\zeta = .02$

$$\tan \psi = \frac{2\zeta r}{1-r^2} = \frac{2(.02)(1.5)}{1-(2.25)} = -.048 \quad \psi = -2.748^\circ$$

$$\Sigma_{RV} = \frac{m_0 e}{m} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{W_0 e}{W} \frac{(1.5)^2}{\sqrt{(1.25)^2 + (.06)^2}} = .054 \text{ in} = \Sigma$$

$$\text{as } r \rightarrow \infty \quad \Sigma \rightarrow \frac{W_0 e}{W} = .03 \text{ in}$$

4-40

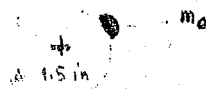
$$\text{Given } W = 65 \text{ lb} ; \Delta_{st} = \frac{W}{k_{eq}} = 2.166 \text{ in} ; n = 3 = \frac{\sqrt{1-\zeta^2}}{2\pi\zeta} \ln\left(\frac{2.35}{.135}\right)$$

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{1}{n} \ln\left(\frac{x}{\Sigma}\right)$$

$$\text{Finally when } r=1, \Sigma_{RV} = .0193 \text{ in}; \text{ also } e=1.5 \text{ in}$$

$$\text{Find: } c, m_0 (\text{or } W_0), \text{ if } r=2 \text{ find } \Sigma_{RV}$$

$$m = \frac{W}{g} = \frac{65 \text{ lb}}{386.4 \text{ in/sec}^2}$$



$$\text{From } \Delta_{st} \Rightarrow k_{eq} = \frac{W}{\Delta_{st}} = \frac{65 \text{ lb}}{2.166 \text{ in}} = 30.01 \text{ lb/in}$$

$$\text{but } \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_{eq}}{W}} = \frac{13.356 \text{ rad/sec}}{\sqrt{g_{bst}}}$$

$$\text{thus } c_{cr} = 2m\omega_n = \frac{4.494 \text{ lb-sec}}{\text{in}} ; \text{ let } \ln\left(\frac{2.35}{.135}\right) = \rho = \ln(17.41) = 2.857$$

$$\text{From } n = \frac{\sqrt{1-\zeta^2}}{2\pi\zeta} \ln\left(\frac{2.35}{.135}\right) \Rightarrow 4\pi^2\zeta^2 n^2 = (1-\zeta^2)\rho^2 \text{ or } \zeta = \frac{\rho}{\sqrt{\rho^2 + 4\pi^2 n^2}}$$

$$\text{thus } \zeta = .1499 \quad c = \zeta c_{cr} = .6734 \frac{\text{lb-sec}}{\text{in}}$$

$$\text{Now } \Sigma_{RV} = \frac{m_0 e}{m} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{W_0 e}{W} \frac{1}{2\zeta} \text{ at } r=1 \Rightarrow W_0 = \frac{\Sigma W \cdot 2\zeta}{e} = .2507 \text{ lb}$$

$$\text{Knowing } \Sigma_{RV} = \frac{m_0 e}{m} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$\text{then for } r=2, \zeta = .1499, W_0 = .2507 \text{ lb}, e=1.5 \text{ in}, W=65 \text{ lb}$$

$$\Sigma = .00756 \text{ in}$$

4-41 Given $W = 96.5 \text{ lb} ; k = 80 \text{ lb/in} ; r=1 \Rightarrow \Sigma = 2.143 \text{ in} ; r \rightarrow \infty \Rightarrow \Sigma \rightarrow .3572 \text{ in}$

$$\text{Find } c : \Sigma = \frac{m_0 e}{m} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} ; \text{ when } r \rightarrow \infty \Sigma \rightarrow \frac{m_0 e}{m} = .3572 \text{ in}$$

$$\text{now when } r=1 \quad \Sigma = \frac{m_0 e}{m} \cdot \frac{1}{2\zeta} \text{ or } \zeta = \frac{\Sigma_{r=\infty}}{2\Sigma_{r=1}} = \frac{.3572}{2(2.143)} = .08334 = \zeta$$

$$c_c = 2m\omega = 2\sqrt{mk} = 2\sqrt{\frac{Wk}{g}} = \frac{8.94 \text{ lb-sec}}{\text{in}}$$

$$c = c_c \zeta = 0.74504 \frac{\text{lb-sec}}{\text{in}}$$