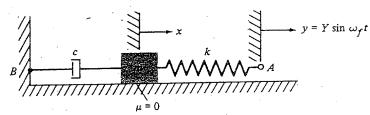
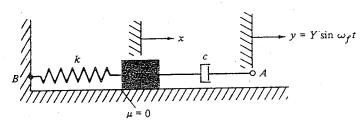
- the forced motion for the following forced frequencies f_f : 1.5, 3, 3.25, and 7.5 Hz.
- 4-2. The mass of an undamped mass-spring system is subjected to a harmonic force having a maximum value of 45 lb and a period of 0.25 sec. The body weighs 2.5 lb and the spring constant is 15 lb/in. Determine the amplitude of the forced motion.
- 4-3. An undamped system is composed of mass m = 1.1 kg and spring k = 4400 N/m. It is acted on by a harmonic force having maximum value of 440 N and a frequency of 180 cycles/min. Determine the amplitude of the forced motion.
- 4-4. A weight of 8 lb is suspended from a spring having a constant of 25 lb/in. The system is undamped but the mass is subjected to a harmonic force with a frequency of 7 Hz, resulting in a forced-motion amplitude of 1.59 in. Determine the maximum value of the impressed force.
- 4-5. The body of an undamped system is driven by a harmonic force having an amplitude of 36 N and a frequency of 450 cycles/min. The body has a mass of 8 kg and exhibits a forced-displacement amplitude of 4.06 mm. Obtain the value of the spring modulus.
- 4-6. An undamped system consists of an 8.75-kg mass suspended from a spring having a constant of 3500 N/m. A harmonic force acting on the mass and having a maximum value of 187 N causes the system to vibrate with a forced amplitude of 7.6 cm. Determine the frequency of the impressed force.
- 4-7. A weight W is suspended from a spring having a constant of 25 lb/in. The system is undamped, but the mass is driven by a harmonic force having a frequency of 2 Hz and a maximum value of 10 lb, causing a forced-motion amplitude of 0.5 in. Determine the value of weight W.
- 4-8. The mass of an undamped system is acted on by force $P = P_0 \cos \omega_f t$, having a maximum value of 10 lb. The spring constant is 3 lb/in. The forced frequency can be any integral multiple of the natural frequency of the system other than 1, thereby avoiding resonance. Only a single forced frequency can exist at a time. Determine the maximum amplitude of the forced motion that could occur.
- 4-9. Express the relation for the complete motion of an undamped system, excited by a force $P = P_0 \sin \omega_f t$, for the initial conditions of x = 0, $\dot{x} = 0$, at t = 0.
- 4-10. Write the relation that defines the complete motion of an undamped system, subjected to the harmonic force $P = P_0 \sin \omega_f t$, for the initial conditions of x = 0, $\dot{x} = \dot{x}_0$, at t = 0. $(r \neq 1)$
- 4-11. Express the equation for the complete motion of an undamped system, subjected to the forcing condition $P = P_0 \sin \omega_f t$, for the initial conditions of $x = x_0, \dot{x} = 0, \text{ at } t = 0. (r \neq 1.)$
- 4-12. Write the complete solution for the motion of an undamped system, driven by the force $P = P_0 \sin \omega_f t$, for the initial conditions of $x = x_0$, $\dot{x} = \dot{x}_0$, at t = 0. $(r \neq 1.)$
- 4-13. An undamped system consists of a weight of 19.3 lb and a spring having a modulus of 10 lb/in. The system mass is driven at resonance by harmonic force $P = P_0 \sin \omega_f t$ having a maximum value of 4 lb. Determine the amplitude of the forced motion at the end of (a) $\frac{1}{2}$ cycle, (b) $2\frac{1}{2}$ cycles, (c) $4\frac{1}{2}$ cycles, and (d) $6\frac{1}{2}$ cycles.

- having a maximum value of 14 N. Determine the amplitude of the forced motion at the end of (a) 1 cycle, (b) 5 cycles, and (c) 10 cycles.
- 4-15. An undamped system is harmonically forced near resonance, resulting in a beating condition. The natural frequency of the system is 1765 cycles/min, and the forced frequency is 1752 cycles/min. Determine the beat period of the motion.
- 4-16. For the arrangement shown, x represents the absolute displacement of the mass m, and y is the absolute displacement of the end A of the spring k. Point Ais moved according to the relation $y = Y \sin \omega_f t$. (a) Construct the free-body diagram for m. (b) Write the differential equation of motion for m. (c) Obtain the solution for the steady-state motion of m. (d) Determine the relation for the impressed force at A. (e) Obtain the expression for the force transmitted to the support at B.



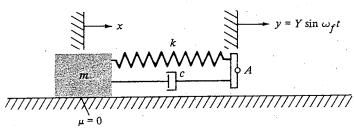
Problem 4-16

4-17. For the arrangement shown, x represents the absolute displacement of the mass m, and y is the absolute displacement of the end A of the dashpot c. The motion of point A is defined by $y = Y \sin \omega_t t$. (a) Construct the free-body diagram for m. (b) Write the differential equation of motion for m. (c) Obtain the solution for the steady-state motion of m. (d) Determine the relation for the impressed force at A. (e) Obtain the expression for the force transmitted to the support at B.



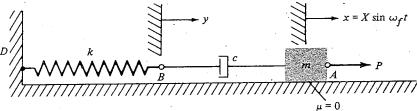
Problem 4-17

4-18. The spring k and the dashpot c of the accompanying diagram are fastened together at A; x represents the absolute displacement of m, and y is the absolute displacement of the point A. The motion of A is defined by $y = Y \sin \omega_f t$. (a) Construct the free-body diagram for m. (b) Write the differential equation of motion for m. (c) Obtain the solution for the steady-state motion of m. (d) Determine the relation for the impressed force at A.



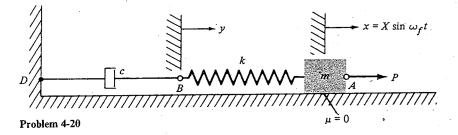
Problem 4-18

4-19. For the arrangement shown, x represents the absolute displacement of the mass m, and y is the absolute displacement of the point B. The impressed force P moves m in accordance with the relation $x = X \sin \omega_f t$. (a) Construct the free-body diagram for m. (b) Write the differential equation for the dynamic condition of m. (c) Construct the free-body diagram for the connection point B. (d) Write the differential equation for the connection point B. (e) Obtain the solution for part (d), representing the relation that governs the motion of point B. (f) Determine the relation for the impressed force P. (g) Obtain the expression for the force transmitted to the support at D.



Problem 4-19

4-20. For the arrangement shown, x represents the absolute displacement of the mass m, and y is the absolute displacement of the point B. The impressed force P moves m in accordance with the relation $x = X \sin \omega_f t$. (a) Construct the free-body diagram for m. (b) Write the differential equation for the dynamic condition of m. (c) Construct the free-body diagram for the connection point B. (d) Write the differential equation for the connection point B. (e) Obtain the solution for part (d), representing the relation that governs the motion of point B. (f) Determine the relation for the impressed force P. (g) Obtain the expression for the force transmitted to the support at D.



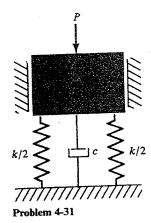
- 4-21. A damped system is composed of a mass of 8.75 kg, a spring having a modulus of 1750 N/m, and a dashpot having a damping constant of 37.13 N·s/m. The mass is acted on by a harmonic force $P = P_0 \sin \omega_f t$ having a maximum value of 220 N and a frequency of 4.50 Hz. Using the symbols of X' and ϕ for the arbitrary constants of the transient, write the complete solution representing the motion of the mass.
- 4-22. A body having a mass of 100 kg moves along a straight line according to the relation

$$x = 0.12 \sin (3t - \beta) + \frac{0.05 \sin (5.96t + \pi/6)}{e^{0.6t}}$$

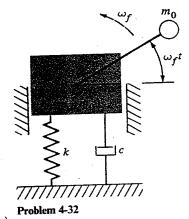
where x is the displacement along the line in meters and t is time in seconds. (a) Obtain the physical constants and parameters of the system, including appropriate frequencies, and so on. (b) Obtain the initial displacement and velocity.

- 4-23. For a damped system excited by the harmonic force $P = P_0 \sin \omega_f t$, plot the magnification-factor curve for $\zeta = 0.3$. Do this from r = 0 to r = 4, carefully determining the peak value.
- 4-24. For a damped system driven by the harmonic force $P = P_0 \sin \omega_f t$, plot the phase-angle curve for $\zeta = 0.3$. Do this from r = 0 to r = 4.
- **4-25.** Develop a relation for the ratio of the maximum amplitude to the resonant amplitude for steady-state motion for the case of a harmonically forced damped system.
- 4-26. For a damped system excited by the harmonic force $P = P_0 \sin \omega_f t$, plot the curve of the steady-state amplitude against the spring constant k for $c = 0.707m\omega_f$. Carry this out from k = 0 to $k = 4m\omega_f^2$.
- 4-27. A damped system has a spring modulus of 24 lb/in. and a damping constant of 0.88 lb sec/in. It is subjected to a harmonic force having a force amplitude of 15 lb. When excited at resonance, the steady-state amplitude is measured as 2.8409 in. Determine (a) the damping factor, (b) the natural undamped frequency of the system, and (c) the damped natural frequency.
- 4-28. A damped system is subjected to a harmonic force for which the frequency can be adjusted. It is determined experimentally that at twice the resonant frequency, the steady amplitude is one-tenth of that which occurs at resonance. Determine the damping factor for the system.
- 4-29. A damped torsional system is composed of a shaft having a torsional spring constant $k_T = 60\,000$ lb in./rad, a disk with a mass moment of inertia I = 24 lb in. \sec^2 , and a torsional damping device having a torsional damping constant $c_T = 840$ lb in. \sec/rad . A harmonic torque with a maximum value of 2700 lb in., acting on the disk, produces a sustained angular oscillation of 3.368-degree amplitude. (a) Determine the frequency of the impressed torque. (b) Obtain the maximum torque transmitted to the support.
- 4-30. A torsional system consists of a shaft having a torsional spring constant $k_T = 12\,800~\mathrm{N}\cdot\mathrm{m/rad}$, a disk with a mass moment of inertia $I = 8~\mathrm{kg}\cdot\mathrm{m}^2$, and a torsional damping device with a torsional damping constant $c_T = 192~\mathrm{N}\cdot\mathrm{m}\cdot\mathrm{s/rad}$. A harmonic torque with a maximum value of 480 N·m produces a steady angular oscillation of 1.5-degree amplitude. (a) Determine the frequency of the impressed torque. (b) Obtain the maximum torque transmitted to the support.

4-31. A machine having a mass of 70 kg is mounted as shown on springs having a total stiffness of 33 880 N/m. The damping factor is $\zeta = 0.20$. A harmonic force $P = 450 \sin 13.2t$ (where P is in newtons and t is in seconds) acts on the mass. For the sustained or steady-state vibration, determine (a) the amplitude of the motion of the machine, (b) its phase with respect to the exciting force, (c) the transmissibility, (d) the maximum dynamic force transmitted to the foundation, and (e) the maximum velocity of the motion.



4-32. A machine having a total weight of 96.5 lb is mounted as shown on a spring having a modulus of 900 lb/in. and is connected to a dashpot with a damping factor of 0.25. The machine contains a rotating unbalance (w_0e) of 5 lb in. If the speed of rotation is 401.1 rpm, obtain (a) the amplitude of the steady-state motion, (b) the maximum dynamic force transmitted to the foundation, and (c) the angular position of the arm when the structure goes upward through its neutral position.



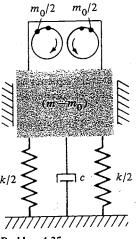
4-33. A machine having a total weight of 128.67 lb contains a part that rotates at 859.44 rpm. The machine is supported equally on four identical springs and is

connected to a dashpot for which the damping constant is 10 lb sec/in. In operation the machine has a steady amplitude of 0.116 in. and transmits a maximum dynamic force of 174 lb to the supporting base. (a) Determine the modulus for the springs. (b) Calculate the unbalanced moment $(w_0 e)$ for the rotating part.

4-34. A machine with a total mass of 50 kg contains a shaft mechanism that rotates at 1800 rpm. The machine is supported by springs for which the equivalent modulus is 20000 N/m but the damping constant is unknown. Due to the rotating unbalance, the steady-state amplitude is 1.32 cm and the maximum dynamic force carried by the base is 278 N. Determine (a) the damping con-

stant and (b) the unbalanced moment $m_0 e$.

4-35. A typical vibration exciter is composed of two eccentric masses that rotate oppositely, as represented in the top of the accompanying figure. Thus an oscillation is produced in the vertical direction only, since the horizontal effect cancels. This device is used to determine the vibrational characteristics of the structure to which it is attached. The unbalance $(ew_0/2)$ of each exciter wheel is 3 lb in. The total arrangement has a weight mg = 200 lb. The exciter speed (of eccentric rotation) was adjusted until a stroboscope showed the structure to be moving upward through its equilibrium position at the instant the eccentric weights were at their top position. The exciter speed then was 840 rpm, and the steady amplitude was 0.75 in. Determine (a) the natural frequency of the structure and the damping factor for the structure. If the speed were changed to 1260 rpm, (b) obtain the steady-state amplitude of the structure and (c) the angular position of the eccentrics as the structure moves upward through its equilibrium position.



Problem 4-35

• 4-36. A small compressor weighs 69 lb and runs at 875 rpm. It is to be supported by four springs (equally) and no damping is provided. Design the springs, thus determining k, so that only 15% of the shaking force is transmitted to the supporting foundation or structure.

- 4-37. Solve Prob. 4-36 considering that damping is also to be included such that the damping factor will be 0.4 for the system. Equal damping at each spring will be provided by a plug or block of energy-absorbing material. Determine the required damping constant as well as the spring constant.
- 4-38. A refrigerator compressor unit weighs 179 lb and operates at 590 rpm. The maximum dynamic force imposed on the compressor is 20 lb. The unit is to be supported equally by three springs and is to be undamped. (a) Determine the required spring constant if 10% of the dynamic force is to be transmitted to the supporting base. (b) Determine the clearance that must be provided for the unit.
- 4-39. Solve Prob. 4-38, considering that each spring contains a viscous damping absorber having a damping constant of 0.8333 lb sec/in.
- 4-40. A small machine is known to contain a rotating unbalance on its main shaft. The machine weighs 65 lb and when placed on elastic vibration isolators (which serve as a viscous damping member as well as the elastic support), the static equilibrium displacement is 2.166 in. Also, when the machine is displaced further and released, the subsequent vibration diminishes from an amplitude of 2.350 in. to 0.135 in. in exactly 3 cycles. Operating the machine at resonance produces a sustained amplitude of 0.0193 in. (a) Determine the damping constant for the system. (b) Considering that the shaft diameter is 3 in. so that the eccentric moment arm is 1.5 in., calculate the unbalanced weight. (c) Determine the steady amplitude at an operation speed of twice the resonant speed.
- 4-41. A damped system with a rotating unbalance is composed of a 96.5-lb body and a spring having a constant of 80 lb/in. but the damping constant is unknown. When operated at resonance, the sustained amplitude is 2.143 in. If the speed of operation is greatly increased, the steady amplitude eventually approaches the value of 0.3572 in. Determine the damping constant for the system.
- 4-42. The support for a damped system (see Fig. 4-26) is oscillated harmonically with a frequency of 75 cycles/min, causing the mass to vibrate with a steady-state amplitude of 0.6 in. and a maximum dynamic force of 48.75 lb to be transmitted to the support. For the system, the spring modulus is 13 lb/in. and the damping constant is 0.454 lb sec/in. Determine (a) the natural frequency of the system, (b) the mass of the system, and (c) the amplitude of oscillation of the support.
- 4-43. The support for a damped system is oscillated harmonically. The body weighs 20 lb and the spring constant is 14 lb/in. When the support is oscillated at a rate of 300 cycles/min, the sustained amplitude of the body is 0.793 in. (a) Determine the maximum dynamic force carried by the support. (b) If the support is oscillated at resonance with an amplitude of 1.121 in., the body has a steady amplitude of 3.069 in. Obtain the damping constant for the system.
- 4-44. A mass-spring system, arranged horizontally, is damped only by Coulomb friction (refer to Fig. 4-27). The system is driven by a harmonic force having a frequency of 54.356 cycles/min and a maximum value of 30 lb, causing the body to oscillate with a steady amplitude of 3.09 in. The spring constant is 12 lb/in. and the body weighs 28.95 lb. Determine the average coefficient of sliding friction for the surfaces involved.
- 4-45. A machine is designed to produce an abrasive action on a horizontal surface in order to wear and smoothen the surface. It consists of a sliding member weigh-

- ing 20 lb fixed to an elastic member having a stiffness coefficient of 8 lb/in. The average coefficient of sliding friction of the body against the surface is 0.3. A harmonic force having a maximum value of 15 lb actuates the body at a frequency of 0.9 of the resonant value. Determine the total excursion or stroke of the member.
- 4-46. A certain mass-spring system exhibits hysteresis damping only. The spring modulus is 310 lb/in. When excited harmonically at resonance, the steady amplitude is 1.540 in. for an energy input of 32 in. lb. When the resonant energy input is increased to 87 in. lb, the sustained amplitude changes to 2.464 in. Determine the hysteresis coefficient β and the exponent γ .
- 4-47. A small built-up structure shows solid damping characteristics with $\gamma=2$. A 1000-lb load on the structure causes a static displacement of 2.500 in., and a supported machine that is subjected to a harmonic force having a constant amplitude of 75 lb produces a resonant amplitude of 6.818 in. (a) Determine the hysteresis damping coefficient β and (b) the energy dissipated per cycle by hysteresis at resonance. (c) Calculate the steady amplitude at twice the resonant frequency and (d) at one-half the resonant frequency.

