3-15. A damped mass-spring system is subjected to the initial conditions $x = 0$ and $\dot{x} = \dot{x}_0$ (at $t = 0$). Express the equation of motion for each of the following cases: (a) $\zeta = 2$, (b) $\zeta = 1$, and (c) $\zeta = 0.1$.

3-16. A damped mass-spring system is subjected to the initial conditions $x = x_0$ and $\dot{x} = 0$ (at $t = 0$). Express the equation of motion for each of the following cases: (a) $\zeta = 2$, (b) $\zeta = 1$, and (c) $\zeta = 0.1$.

3-17. A weight of 9.65 lb is suspended from a spring having a constant of 30 lb/in., and the system is critically damped. The mass is held at rest 2 in. from equilibrium and then released. (a) Determine the damping constant. (b) Obtain the displacement at the end of 0.01, 0.1, and 1 sec.

3-18. A mass of 9 kg is suspended from a spring having a modulus of 8100 N/m. The system is also critically damped. The initial displacement is zero and the initial velocity is 60 cm/s. Determine the maximum displacement for the system.

3-19. For a damped system, $k = 7875$ N/m, $m = 8.75$ kg, and the damping factor $\zeta = 0.1$. The initial displacement is 10 cm and the initial velocity is zero. (a) Determine the damping constant. (b) Obtain the amplitude one-half and one cycle after the initial condition.

3-20. A damped system is composed of a body for which $W = 38.6$ lb, spring constant $k = 90$ lb/in., and damping constant $c = 13.416$ lb sec/in. The system is given an initial displacement of 2 in. and released from rest. (a) Determine the damping factor. (b) Obtain the displacement at $t = 0.01$, 0.1, and 1 sec.

3-21. A suspended mass of 4.5 kg causes a static displacement of 1.00 cm for a spring. The system is also connected to a dashpot having a viscous damping constant of 35 N · s/m. Determine (a) the damping factor, (b) the undamped frequency, and (c) the damped frequency.

3-22. Determine (a) the logarithmic decrement and (b) the ratio of successive amplitudes for the data of Prob. 3-21.

3-23. A static displacement of 0.375 in. of a spring is caused by a suspended weight of 12 lb. The system also includes a dashpot having a viscous damping constant of 0.15 lb sec/in. Determine (a) the damping factor, (b) the undamped frequency, and (c) the damped frequency.

3-24. For the data of Prob. 3-23, determine (a) the logarithmic decrement and (b) the ratio of successive amplitudes of free vibration.

3-25. For a viscously damped system, a certain amplitude is measured as 74.8% of the immediately preceding amplitude. Determine the damping factor for the system.

3-26. The physical constants for a damped system are $k = 45$ lb/in., $W = 19.3$ lb, and $c = 0.057$ lb sec/in. If the initial amplitude is 3 in., determine the amplitude 12 cycles later.

3-27. For Prob. 3-26 determine (a) the half-life and (b) the number of cycles occurring in the half-life.

3-28. For a certain viscously damped system, measurements show that the amplitude reduction is 80% in 15 cycles. The critical damping constant is known to be 70 N · s/m. Determine the damping constant for the system.

3-29. A system is composed of a body for which $W = 7.72$ lb, spring constant $k = 5$ lb/in., and damping constant $c = 0.125$ lb sec/in. If the system is given an initial displacement and released from rest, determine the percent overshoot (the displacement attained past the equilibrium position).

3-30. A system composed of a mass of 5 kg and an elastic member having a modulus of 45 N/m is less than critically damped. When the mass is given an initial displacement and released from rest, the overshoot (the displacement attained past the equilibrium position) is 25%. Determine the damping factor and the damping constant.

3-31. A mass-spring system is critically damped. The spring constant is 3 lb/in. and the body weighs 11.58 lb. (a) The system is given an initial displacement $x_0 = 2$ in. and initial velocity $\dot{x}_0 = -30$ in./sec. Determine (a) the overshoot. (b) If $x_0 = 2$ in. and $\dot{x}_0 = -20$ in./sec, obtain the overshoot.

3-32. A large gun with supporting base weighs 2500 lb, and has a recoil system composed of spring $k = 32\,000$ lb/ft and a viscous shock absorber (dashpot) for which damping is critical. The recoil distance is 3 ft. Determine (a) the initial recoil velocity, (b) the time to return to a position 0.1 in. from the initial position, and (c) displacement at $t = 0.5$ sec.

3-33. An artillery piece and integral supporting base weigh 1800 lb and have a recoil spring for which $k = 24\,000$ lb/ft. When fired, the system recoils 40 in. A dashpot having a critical damping constant is then engaged on the return stroke. Determine (a) the initial velocity of recoil, (b) the damping constant, and (c) the position of the gun and base at 0.5 sec of the return stroke.

3-34. For the case of Coulomb damping, by equating the loss of potential energy for a half cycle to the work done by the constant friction force, show that the loss of amplitude is $2F_d/k$. Designate the initial amplitude as x_0 and a half cycle later as $x_{0.5}$.

3-35. A mass-spring system is subjected to an undetermined damping condition. The vibrating body weighs 47 lb and the spring modulus is 30 lb/in. The amplitudes of successive cycles were measured as 1.64, 1.59, 1.54, 1.49, 1.44, ..., in. (a) Define the type and magnitude of the damping force and (b) determine the frequency of the damped oscillation.

3-36. A system composed of mass of 57.61 kg and a spring having a modulus of 9100 N/m is subjected to a Coulomb damping force of 6.825 N. If the initial amplitude is 4 cm, determine (a) the amplitude at the end of 8 cycles and (b) the frequency of the damped oscillation.

3-37. A simple structure exhibits hysteresis damping. The period of the vibration is 0.30 sec and the amplitude at the ninth cycle is 0.9309 times that at the first cycle. Determine the hysteresis damping constant b and write the equation of motion for this case.