3-20 $\quad K = 90\,\frac{lb}{in} \times \frac{12\,in}{ft} = 1080\,\frac{lb}{ft} \qquad m = \frac{38.6\,lb}{32.2\,ft/sec^2} = 1.199\text{ slugs}$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{1080}{1.199}} = 30\,\frac{rad}{s}$$

$$C_{cr} = 2m\omega_n = 2(1.2)(30) = 72\,lb_f\text{-sec}/ft \qquad C = 13.416\,\frac{lb_f\text{-sec}}{in} \times \frac{12\,in}{ft} = 161\,\frac{lb_f\text{-sec}}{ft} \qquad \zeta = \frac{C}{C_{cr}} = 2.236$$

$$s_1 = \left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n = -7.08 \qquad s_2 = \left(-\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n = -127.08$$

$$x(t) = C_1 e^{-7.08t} + C_2 e^{-127.08t}$$

@t=0 $\quad x(t) = 2in$ & $\dot{x}(t) = 0$

$\therefore$

$$\left.\begin{aligned} x(t) &= C_1 + C_2 = 2 \\ \dot{x}(t) &= -7.08C_1 - 127.08C_2 = 0 \end{aligned}\right] \text{ solve } \quad C_1 = 2.118, \quad C_2 = -0.118$$

$\therefore \quad x(t) = 2.118e^{-7.08t} - .118e^{-127.08t}$

to find overshoot $\quad \dot{x}(t^*) = 0 \Rightarrow C_1(-7.08)e^{-7.08t^*} + C_2(-127.08)e^{-127.08t^*} = 0$

$$\Rightarrow \frac{C_2(-127.08)}{-C_1(-7.08)} = \frac{e^{-7.08t^*}}{e^{-127.08t^*}} = e^{120t^*} \quad \text{or} \quad \frac{C_2 s_2}{-C_1 s_1} = e^{(s_1 - s_2)t^*}$$

So $\quad \ln\left(\frac{C_2 s_2}{-C_1 s_1}\right) = (s_1 - s_2)t^*$ & $\frac{1}{s_1 - s_2}\ln\left(\frac{C_2 s_2}{-C_1 s_1}\right) = t^*$

$$= \frac{1}{120}\ln\left[\frac{-.118(-127.08)}{-(2.118)(-7.08)}\right] = \frac{1}{120}\ln(1) = 0 = t^*$$

not part of problem statement

so overshoot is $x(t^*) = x(0) = 2in$

321. $\quad m = 4.5\,kg \qquad \Delta_{st} = 1cm = .01m \qquad c = 35\,\frac{N\text{-}s}{m}$

find ζ, ω_n, ω_d $\qquad \omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{g}{\Delta_{st}}} = 31.32\,rad/s \qquad C_{cr} = 2m\omega_n = 2(4.5)(31.32) = 281.89\,\frac{N\text{-}s}{m}$

$\zeta = \frac{c}{C_{cr}} = \frac{35}{281.89} = .1242 \qquad$ now $\omega_d = \omega_n\sqrt{1 - \zeta^2} = 31.08\,rad/s$

Suppose: system starts @ $X_o = 0.1m$ & $V_o = 0\,m/s$ then $x(t) = Xe^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$

$$X = \sqrt{X_o^2 + \left(\frac{V_o + \zeta\omega_n X_o}{\omega_d}\right)^2} = .1008\,m \qquad \tan\phi = \frac{X_o\omega_d}{V_o + \zeta\omega_n X_o} = 7.9898 \Rightarrow \phi = 1.446\,rad$$

what is amplitude 5 cycles later. Since $V_o = 0 \Rightarrow X$ is the max displacement @ $t=0$

$$\delta = \frac{1}{n}\ln\left(\frac{X_o}{X_n}\right) = \frac{2\pi\zeta}{\sqrt{1 - \zeta^2}} = .7865 = \frac{1}{5}\ln\left(\frac{.1008}{X_5}\right) \Rightarrow X_5 = .001976\,m$$

another way $\tau_d = \frac{2\pi}{\omega_d} = .2022\,sec \qquad t_{5\,cycles} = 5\tau_d = 1.0108\,sec$ and put into $Xe^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$

not part of problem statement

3-32 $W = 2500\,lb$ $k = 32000\,lb/ft$ $c = c_{cr}$ $x(t^*) = 3\,ft$ find V_0,

$x(t=0) = 0$ $m = \frac{W}{g} = 77.64$ slugs $\omega_n = \sqrt{\frac{k}{m}} = 20.30\,rad/s$ $c_{cr} = 2m\omega_n = 3152.5\ \frac{N\text{-}s}{m}$

since $c = c_r$ $\zeta = 1 \Rightarrow x(t) = (c_1 + c_2 t)e^{-\omega_n t}$

$x(t) = 0 = (c_1 + c_2 \cdot 0)e^{0} \Rightarrow c_1 = 0$ & $x(t) = c_2 t e^{-\omega_n t} \Rightarrow \dot{x}(t) = c_2[1 - \omega_n t]e^{-\omega_n t}$. when $t = t^*$ $\dot{x} = 0$

$\Rightarrow 1 - \omega_n t^* = 0$ $\therefore t^* = \frac{1}{\omega_n} = .04926$ sec

$\therefore x(t^*) = 3\,ft = c_2 t^* e^{-\omega_n t^*} = c_2(.04926)e^{-0} \Rightarrow c_2 = 165.47\,ft/s$

and $\dot{x}(t=0) = c_2 e^{-\omega_n(0)} = 165.47\,ft/s = V_0$