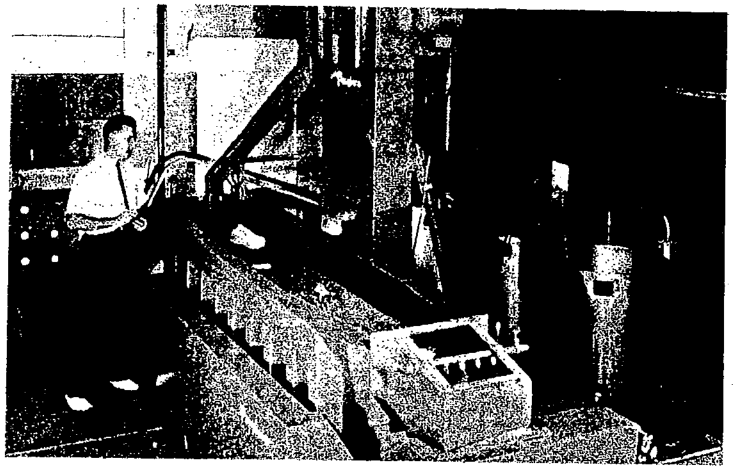


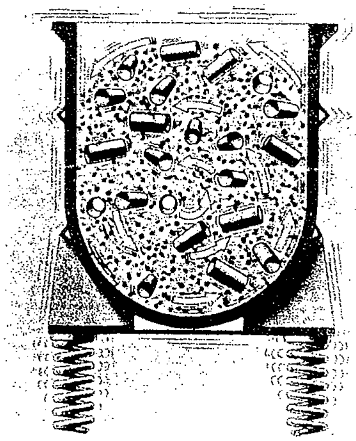


FIGURE 1.10 Vibratory finishing process. (Reprinted courtesy of the Society of Manufacturing Engineers, © 1964 The Tool and Manufacturing Engineer.)

have increased considerably in recent years [1.21]. For example, vibration is put to work in vibratory conveyors, hoppers, sieves, compactors, washing machines, electric toothbrushes, dentist's drills, clocks, and electric massaging units. Vibration is also used in pile driving, vibratory testing of materials, vibratory finishing processes, and electronic circuits to filter out the unwanted frequencies (see Fig. 1.10). Vibration has been found to improve the efficiency of certain machining, casting, forging, and welding processes. It is employed to simulate earthquakes for geological research and also to conduct studies in the design of nuclear reactors.

1.4 Basic Concepts of Vibration

1.4.1 Vibration

Any motion that repeats itself after an interval of time is called *vibration* or *oscillation*. The swinging of a pendulum and the motion of a plucked string are typical examples of vibration. The theory of vibration deals with the study of oscillatory motions of bodies and the forces associated with them.

1.4.2 Elementary Parts of Vibrating Systems

A vibratory system, in general, includes a means for storing potential energy (spring or elasticity), a means for storing kinetic energy (mass or inertia), and a means by which energy is gradually lost (damper).

The vibration of a system involves the transfer of its potential energy to kinetic energy and kinetic energy to potential energy, alternately. If the system is damped, some energy is dissipated in each cycle of vibration and must be replaced by an external source if a state of steady vibration is to be maintained.

As an example, consider the vibration of the simple pendulum shown in Fig. 1.11. Let the bob of mass m be released after giving it an angular displacement θ.

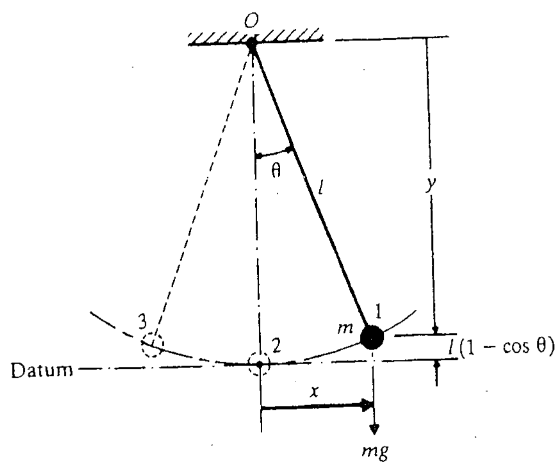


FIGURE 1.11 A simple pendulum.

At position 1 the velocity of the bob and hence its kinetic energy is zero. But it has a potential energy of magnitude $mgl(1 - \cos\theta)$ with respect to the datum position 2. Since the gravitational force mg induces a torque $mgl \sin\theta$ about the point O, the bob starts swinging to the left from position 1. This gives the bob certain angular acceleration in the clockwise direction, and by the time it reaches position 2, all of its potential energy will be converted into kinetic energy. Hence the bob will not stop in position 2, but will continue to swing to position 3. However, as it passes the mean position 2, a counterclockwise torque starts acting on the bob due to gravity and causes the bob to decelerate. The velocity of the bob reduces to zero at the left extreme position. By this time, all the kinetic energy of the bob will be converted to potential energy. Again due to the gravity torque, the bob continues to attain a counterclockwise velocity. Hence the bob starts swinging back with progressively increasing velocity and passes the mean position again. This process keeps repeating, and the pendulum will have oscillatory motion. However, in practice, the magnitude of oscillation (θ) gradually decreases and the pendulum ultimately stops due to the resistance (damping) offered by the surrounding medium (air). This means that some energy is dissipated in each cycle of vibration due to damping by the air.

1.4.3 Degree of Freedom

The minimum number of independent coordinates required to determine completely the positions of all parts of a system at any instant of time defines the degree of freedom of the system. The simple pendulum shown in Fig. 1.11, as well as each of the systems shown in Fig. 1.12, represents a single degree of freedom system. For example, the motion of the simple pendulum (Fig. 1.11) can be stated either in terms of the angle θ or in terms of the Cartesian coordinates x and y. If the coordinates x and y are used to describe the motion, it must be recognized that these coordinates are not independent. They are related to each other through the relation $x^2 + y^2 =$

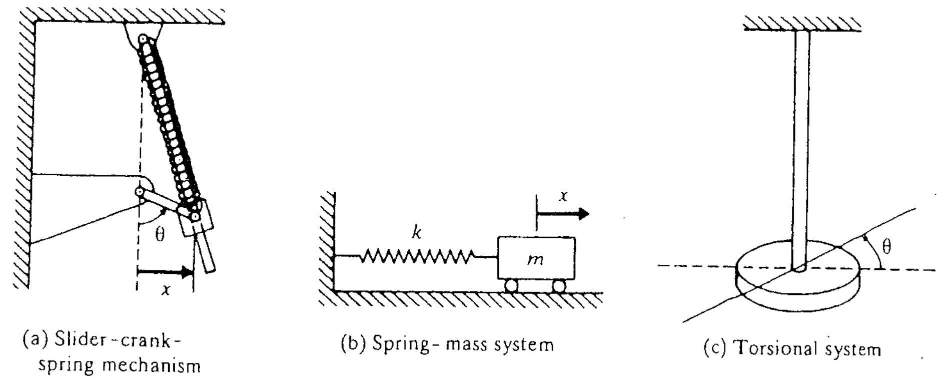


FIGURE 1.12 Single degree of freedom systems.

l^2, where l is the constant length of the pendulum. Thus any one coordinate can describe the motion of the pendulum. In this example, we find that the choice of θ as the independent coordinate will be more convenient than the choice of x or y. For the slider shown in Fig. 1.12(a), either the angular coordinate θ or the coordinate x can be used to describe the motion. In Fig. 1.12(b), the linear coordinate x can be used to specify the motion. For the torsional system (long bar with a heavy disk at the end) shown in Fig. 1.12(c), the angular coordinate θ can be used to describe the motion.

Some examples of two and three degree of freedom systems are shown in Figs. 1.13 and 1.14, respectively. Figure 1.13(a) shows a two mass–two spring system that is described by the two linear coordinates x_1 and x_2. Figure 1.13(b) denotes a two rotor system whose motion can be specified in terms of θ_1 and θ_2. The motion of the system shown in Fig. 1.13(c) can be described completely either by X and θ or by x, y, and X. In the latter case, x and y are constrained as $x^2 + y^2 = l^2$ where l is a constant.

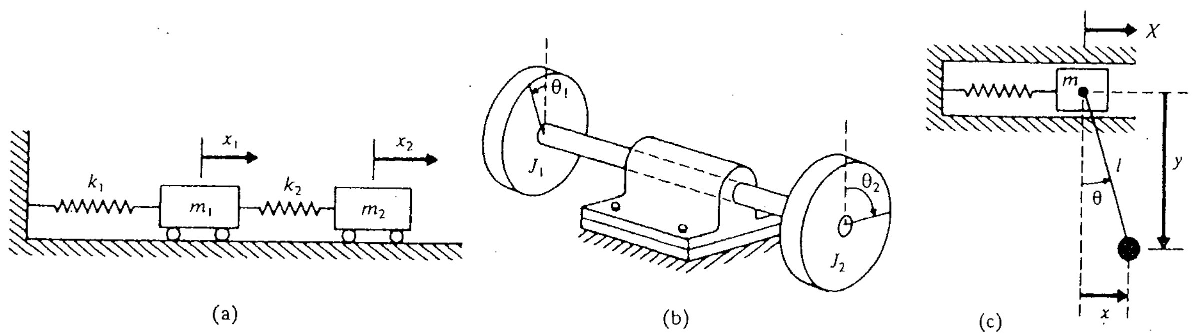


FIGURE 1.13 Two degree of freedom systems.

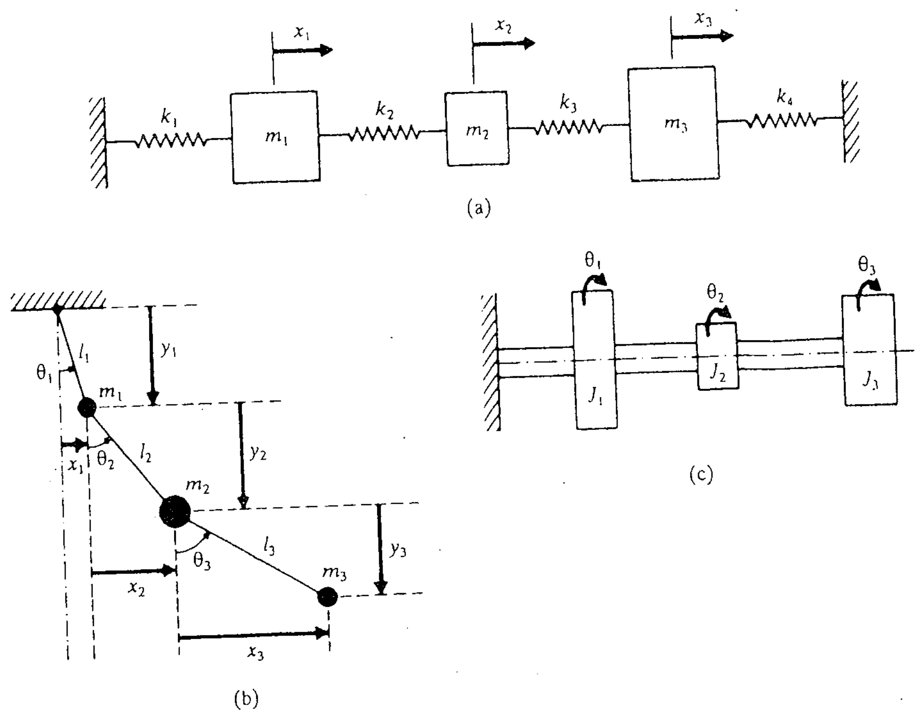


FIGURE 1.14 Three degree of freedom systems.

For the systems shown in Figs. 1.14(a) and 1.14(c), the coordinates x_i (i = 1, 2, 3) and θ_i (i = 1, 2, 3) can be used, respectively, to describe the motion. In the case of the system shown in Fig. 1.14(b), θ_i (i = 1, 2, 3) specifies the positions of the masses m_i (i = 1, 2, 3). An alternate method of describing this system is in terms of x_i and y_i (i = 1, 2, 3); but in this case the constraints $x_i^2 + y_i^2 = l_i^2$ (i = 1, 2, 3) have to be considered.

The coordinates necessary to describe the motion of a system constitute a set of *generalized coordinates*. The generalized coordinates are usually denoted as q_1, q_2, . . . and may represent Cartesian and/or non-Cartesian coordinates.

1.4.4 Discrete and Continuous Systems

A large number of practical systems can be described using a finite number of degrees of freedom, such as the simple systems shown in Figs. 1.11 to 1.14. Some systems, especially those involving continuous elastic members, have an infinite number of degrees of freedom. As a simple example, consider the cantilever beam shown in Fig. 1.15. Since the beam has an infinite number of mass points, we need an infinite number of coordinates to specify its deflected configuration. The infinite number of coordinates defines its elastic deflection curve. Thus the cantilever beam has an infinite number of degrees of freedom. Most structural and machine systems have deformable (elastic) members and therefore have an infinite number of degrees of freedom.

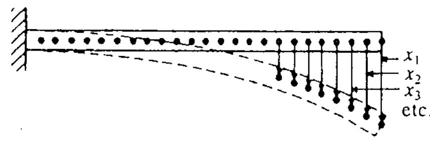


FIGURE 1.15 A cantilever beam (an infinite number of degrees of freedom system).

Systems with a finite number of degrees of freedom are called *discrete* or *lumped parameter* systems, and those with an infinite number of degrees of freedom are called *continuous* or *distributed* systems.

Most of the time, continuous systems are approximated as discrete systems, and solutions are obtained in a simpler manner. Although treatment of a system as continuous gives exact results, the analysis methods available for dealing with continuous systems are limited to a narrow selection of problems, such as uniform beams, slender rods, and thin plates. Hence most of the practical systems are studied by treating them as finite lumped masses, springs, and dampers. In general, more accurate results are obtained by increasing the number of masses, springs, and dampers—that is, by increasing the number of degrees of freedom.

1.5 Classification of Vibration

Vibration can be classified in several ways. Some of the important classifications are as follows.

1.5.1 Free and Forced Vibration

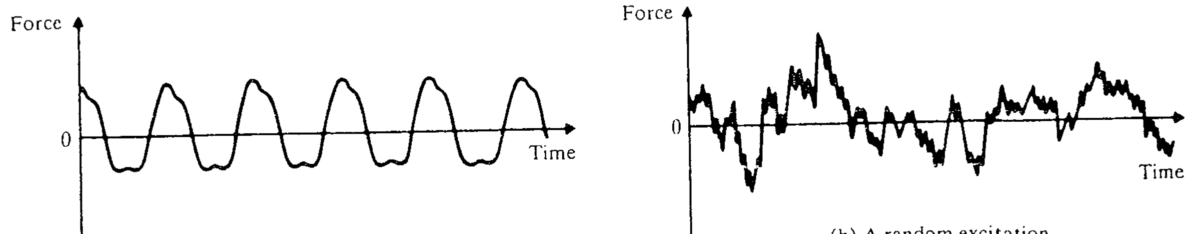
Free Vibration. If a system, after an initial disturbance, is left to vibrate on its own, the ensuing vibration is known as free vibration. No external force acts on the system. The oscillation of a simple pendulum is an example of free vibration.

Forced Vibration. If a system is subjected to an external force (often, a repeating type of force), the resulting vibration is known as *forced vibration.* The oscillation that arises in machines such as diesel engines is an example of forced vibration.

If the frequency of the external force coincides with one of the natural frequencies of the system, a condition known as *resonance* occurs, and the system undergoes dangerously large oscillations. Failures of such structures as buildings, bridges, turbines, and airplane wings have been associated with the occurrence of resonance.

1.5.2 Undamped and Damped Vibration

If no energy is lost or dissipated in friction or other resistance during oscillation, the vibration is known as *undamped vibration.* If any energy is lost in this way, on the other hand, it is called *damped vibration.* In many physical systems, the amount of damping is so small that it can be disregarded for most engineering purposes. However, consideration of damping becomes extremely important in analyzing vibratory systems near resonance.



(a) A deterministic (periodic) excitation

(b) A random excitation

FIGURE 1.16

1.5.3 Linear and Nonlinear Vibration

If all the basic components of a vibratory system—the spring, the mass, and the damper—behave linearly, the resulting vibration is known as *linear vibration*. On the other hand, if any of the basic components behave nonlinearly, the vibration is called *nonlinear vibration*. The differential equations that govern the behavior of linear and nonlinear vibratory systems are linear and nonlinear, respectively. If the vibration is linear, the principle of superposition holds, and the mathematical techniques of analysis are well developed. For nonlinear vibration, the superposition principle is not valid, and techniques of analysis are less well known. Since all vibratory systems tend to behave nonlinearly with increasing amplitude of oscillation, a knowledge of nonlinear vibration is desirable in dealing with practical vibratory systems.

1.5.4 Deterministic and Random Vibration

If the value or magnitude of the excitation (force or motion) acting on a vibratory system is known at any given time, the excitation is called *deterministic*. The resulting vibration is known as *deterministic vibration*.

In some cases, the excitation is *nondeterministic* or *random*; the value of the excitation at a given time cannot be predicted. In these cases, a large collection of records of the excitation may exhibit some statistical regularity. It is possible to estimate averages such as the mean and mean square values of the excitation. Examples of random excitations are wind velocity, road roughness, and ground motion during earthquakes. If the excitation is random, the resulting vibration is called *random vibration*. In the case of random vibration, the vibratory response of the system is also random; it can be described only in terms of statistical quantities. Figure 1.16 shows examples of deterministic and random excitations.

1.6 Vibration Analysis Procedure

A vibratory system is a dynamic system for which the variables such as the excitations (inputs) and responses (outputs) are time-dependent. The response of a vibrating system generally depends on the initial conditions as well as the external excitations. Most practical vibrating systems are very complex, and it is impossible to consider all the details for a mathematical analysis. Only the most important features are considered in the analysis to predict the behavior of the system under specified input conditions. Often, the overall behavior of the system can be determined by considering even a simple model of the complex physical system. Thus the analysis of a vibrating system usually involves mathematical modeling, derivation of the governing equations, solution of the equations, and interpretation of the results.

Step 1: Mathematical Modeling. The purpose of mathematical modeling is to represent all the important features of the system for the purpose of deriving the mathematical (or analytical) equations governing the behavior of the system. The mathematical model should include enough details to be able to describe the system in terms of equations without making it too complex. The mathematical model may be linear or nonlinear, depending on the behavior of the components of the system. Linear models permit quick solutions and are simple to handle; however, nonlinear models sometimes reveal certain characteristics of the system that cannot be predicted using linear models. Thus a great deal of engineering judgment is needed to come up with a suitable mathematical model of a vibrating system.

Sometimes the mathematical model is gradually improved to obtain more accurate results. In this approach, first a very crude or elementary model is used to get a quick insight into the overall behavior of the system. Subsequently, the model is refined by including more components and/or details so that the behavior of the system can be observed more closely. To illustrate the procedure of refinement used in mathematical modeling, consider the forging hammer shown in Fig. 1.17(a). The forging hammer consists of a frame, a falling weight known as the tup, an anvil, and a foundation block. The anvil is a massive steel block on which material is forged into desired shape by the repeated blows of the tup. The anvil is usually mounted on an elastic pad to reduce the transmission of vibration to the foundation block and the frame [1.22]. For a first approximation, the frame, anvil, elastic pad, foundation block, and soil are modeled as a single degree of freedom system as shown in Fig. 1.17(b). For a refined approximation, the weights of the frame and anvil and the foundation block are represented separately with a two degree of freedom model as shown in Fig. 1.17(c). Further refinement of the model can be made by considering eccentric impacts of the tup, which cause each of the masses shown in Fig. 1.17(c) to have both vertical and rocking (rotation) motions in the plane of the paper.

Step 2: Derivation of Governing Equations. Once the mathematical model is available, we use the principles of dynamics and derive the equations that describe the vibration of the system. The equations of motion can be derived conveniently by drawing the free-body diagrams of all the masses involved. The free-body diagram of a mass can be obtained by isolating the mass and indicating all externally applied forces, the reactive forces, and the inertia forces. The equations of motion of a vibrating system are usually in the form of a set of ordinary differential equations for a discrete system and partial differential equations for a continuous system. The equations may be linear or nonlinear depending on the behavior of the components of the system. Several approaches are commonly used to derive the governing equations. Among them are Newton's second law of motion, d'Alembert's principle, and the principle of conservation of energy.

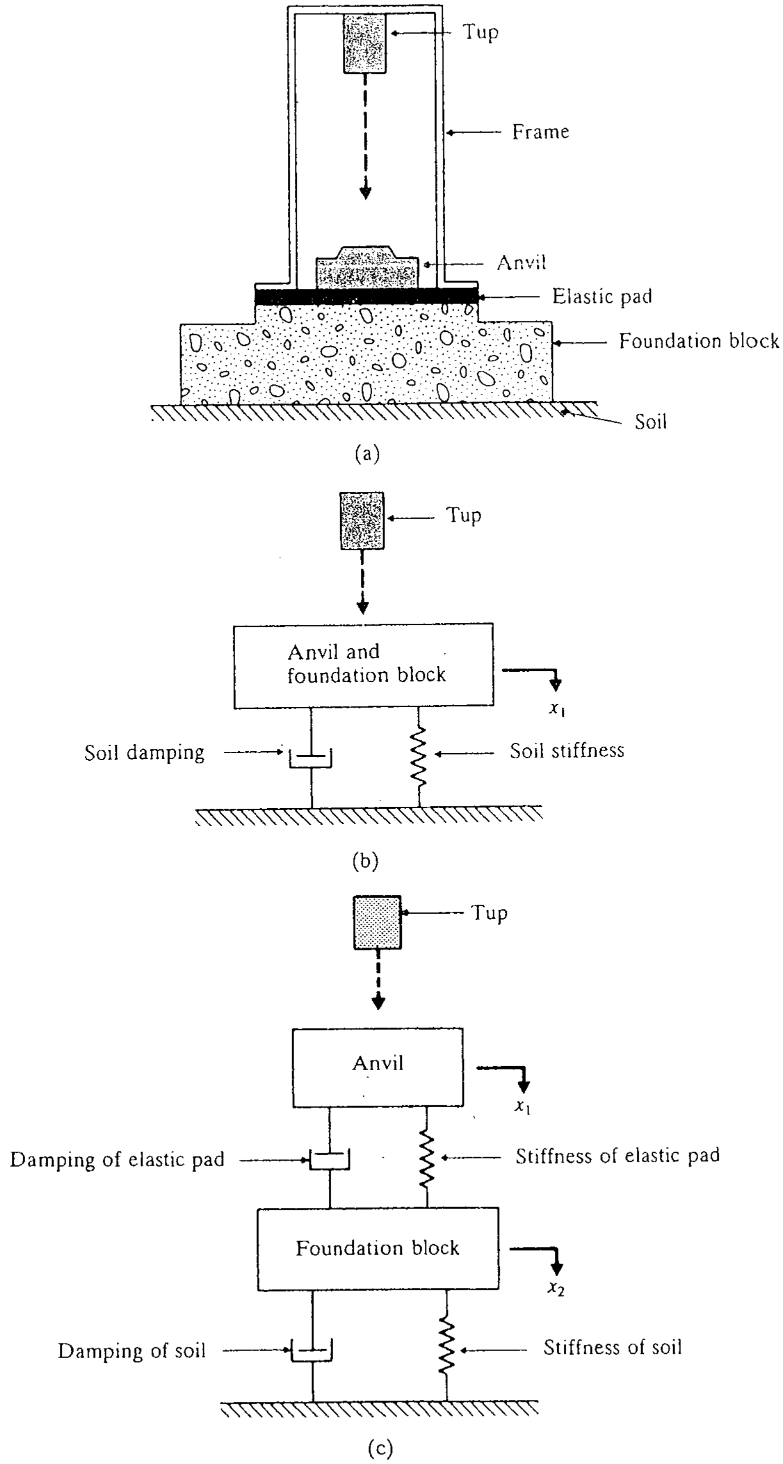


FIGURE 1.17 Modeling of a forging hammer.

Step 3: Solution of the Governing Equations. The equations of motion must be solved to find the response of the vibrating system. Depending on the nature of the problem, we can use one of the following techniques for finding the solution: standard methods of solving differential equations, Laplace transformation methods, matrix methods,[1] and numerical methods. If the governing equations are nonlinear, they can seldom be solved in closed form. Furthermore, the solution of partial differential equations is far more involved than that of ordinary differential equations. Numerical methods involving computers can be used to solve the equations. However, it will be difficult to draw general conclusions about the behavior of the system using computer results.

Step 4: Interpretation of the Results. The solution of the governing equations gives the displacements, velocities, and accelerations of the various masses of the system. These results must be interpreted with a clear view of the purpose of the analysis and the possible design implications of the results.

EXAMPLE 1.1 Mathematical Model of a Motorcycle

Figure 1.18(a) shows a motorcycle with a rider. Develop a sequence of three mathematical models of the system for investigating vibration in the vertical direction. Consider the elasticity of the tires, elasticity and damping of the struts (in the vertical direction), masses of the wheels, and elasticity, damping, and mass of the rider.

Given: Spring constants, damping constants, and masses of the various parts of the motorcycle and the rider.

Find: A sequence of three mathematical models.

Approach: Start with the simplest model and refine it gradually.

Solution: By using the equivalent values of the mass, stiffness, and damping of the system, a single degree of freedom model of the motorcycle with the rider can be obtained as indicated in Fig. 1.18(b). In this model, the equivalent stiffness (k_{eq}) includes the stiffnesses of the tires, struts, and rider. The equivalent damping constant (c_{eq}) includes the damping of the struts and the rider. The equivalent mass includes the masses of the wheels, vehicle body, and the rider. This model can be refined by representing the masses of wheels, elasticity of the tires, and elasticity and damping of the struts separately, as shown in Fig. 1.18(c). In this model, the mass of the vehicle body (m_v) and the mass of the rider (m_r) are shown as a single mass, $m_v + m_r$. When the elasticity (as spring constant, k_r) and damping (as damping constant, c_r) of the rider are considered, the refined model shown in Fig. 1.18(d) can be obtained.

Note that the models shown in Figs. 1.18(b) to (d) are not unique. For example, by combining the spring constants of both tires, the masses of both wheels, and the spring and damping constants of both struts as single quantities, the model shown in Fig. 1.18(e) can be obtained instead of Fig. 1.18(c).

[1]The basic definitions and operations of matrix theory are given in Appendix A.

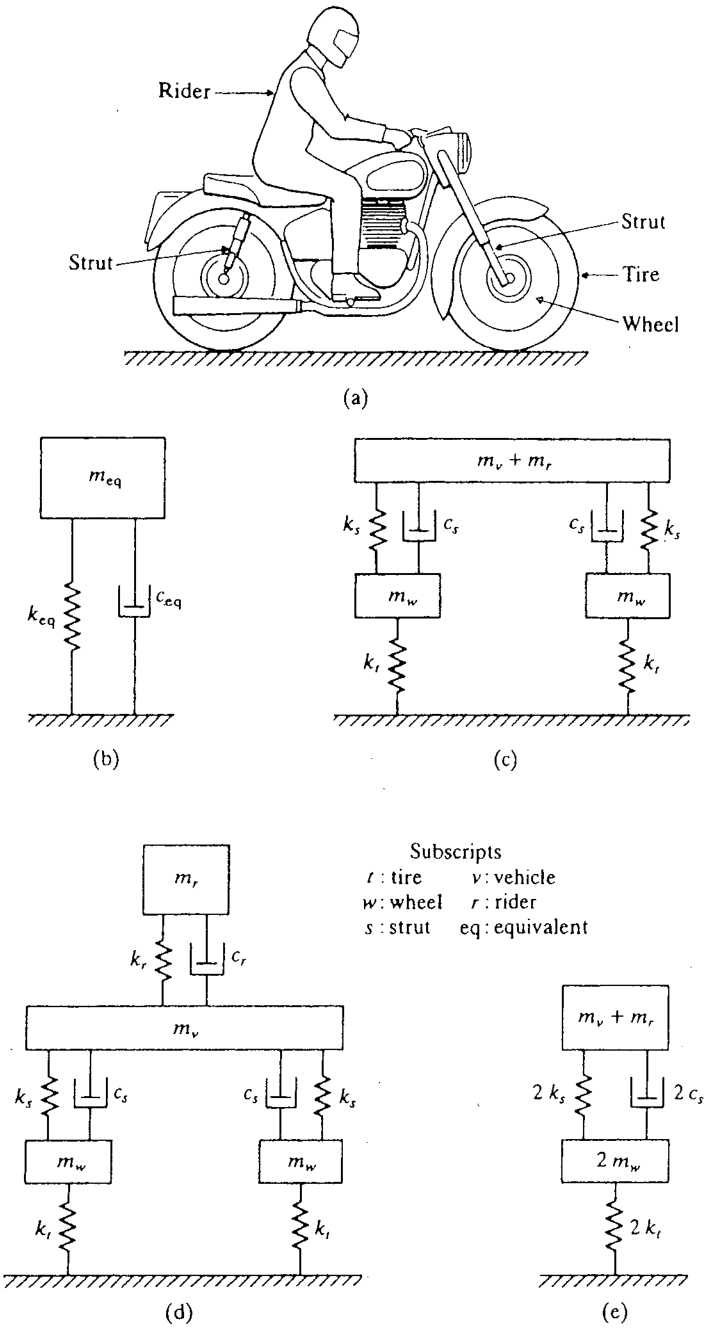


FIGURE 1.18 Motorcycle with a rider—a physical system and mathematical model.

1.7 Spring Elements

A linear spring is a type of mechanical link that is generally assumed to have negligible mass and damping. A force is developed in the spring whenever there is relative motion between the two ends of the spring. The spring force is proportional to the amount of deformation and is given by

$$F = kx \tag{1.1}$$

where F is the spring force, x is the deformation (displacement of one end with respect to the other), and k is the *spring stiffness* or *spring constant.* If we plot a graph between F and x, the result is a straight line according to Eq. (1.1). The work done (U) in deforming a spring is stored as strain or potential energy in the spring, and it is given by

$$U = \frac{1}{2}kx^2 \tag{1.2}$$

Actual springs are nonlinear and follow Eq. (1.1) only up to a certain deformation. Beyond a certain value of deformation (after point A in Fig. 1.19), the stress exceeds the yield point of the material and the force-deformation relation becomes nonlinear [1.23, 1.24]. In many practical applications we assume that the deflections are small and make use of the linear relation in Eq. (1.1). Even if the force-deflection relation of a spring is nonlinear, as shown in Fig. 1.20, we often approximate it as a linear one by using a linearization process [1.24, 1.25]. To illustrate the linearization process, let the static equilibrium load F acting on the spring cause a deflection of x^*. If an incremental force ΔF is added to F, the spring deflects by an additional quantity Δx. The new spring force $F + \Delta F$ can be expressed using Taylor's series expansion about the static equilibrium position x^* as

$$\begin{aligned} F + \Delta F &= F(x^* + \Delta x) \\ &= F(x^*) + \left.\frac{dF}{dx}\right|_{x^*}(\Delta x) + \frac{1}{2!}\left.\frac{d^2F}{dx^2}\right|_{x^*}(\Delta x)^2 + \dots \end{aligned} \tag{1.3}$$

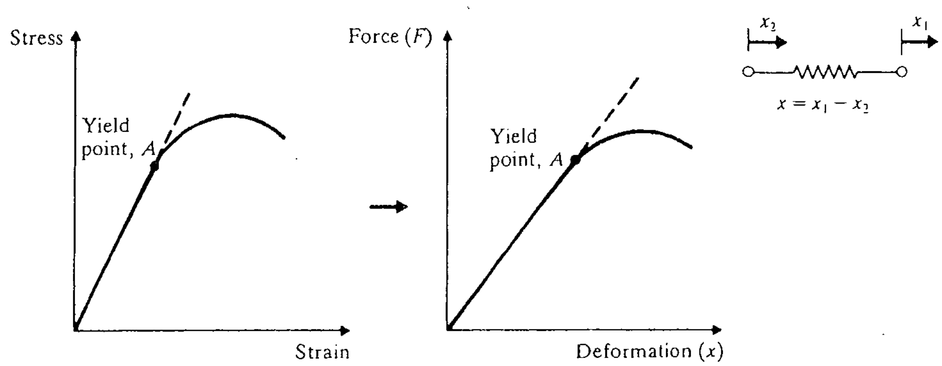


FIGURE 1.19 Nonlinearity beyond proportionality limit.

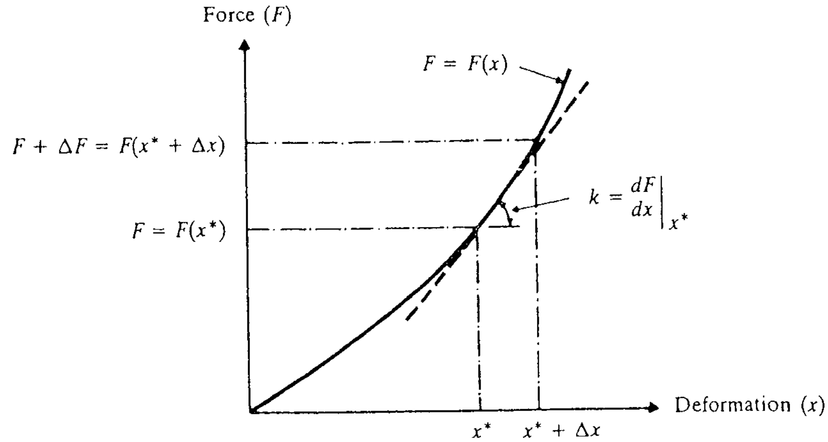


FIGURE 1.20 Linearization process.

For small values of Δx, the higher order derivative terms can be neglected to obtain

$$F + \Delta F = F(x^*) + \left.\frac{dF}{dx}\right|_{x^*} (\Delta x) \tag{1.4}$$

Since $F = F(x^*)$, we can express ΔF as

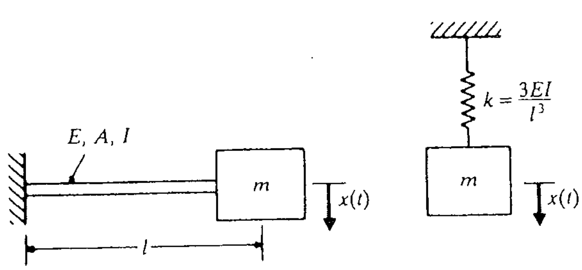
$$\Delta F = k \Delta x \tag{1.5}$$

where k is the linearized spring constant at x^* given by

$$k = \left.\frac{dF}{dx}\right|_{x^*}$$

We may use Eq. (1.5) for simplicity, but sometimes the error involved in the approximation may be very large.

Elastic elements like beams also behave as springs. For example, consider a cantilever beam with an end mass m, as shown in Fig. 1.21. We assume, for simplicity,



(a) Actual system (b) Single degree of freedom model

FIGURE 1.21 Cantilever with end mass.

that the mass of the beam is negligible in comparison with the mass m. From strength of materials [1.26], we know that the static deflection of the beam at the free end is given by

$$\delta_{st} = \frac{Wl^3}{3EI} \tag{1.6}$$

where $W = mg$ is the weight of the mass m, E is Young's modulus, and I is the moment of inertia of the cross section of the beam. Hence the spring constant is

$$k = \frac{W}{\delta_{st}} = \frac{3EI}{l^3} \tag{1.7}$$

Similar results can be obtained for beams with different end conditions.

The formulas given in Appendix B can be used to find the spring constants of beams and plates.

1.7.1 Combination of Springs

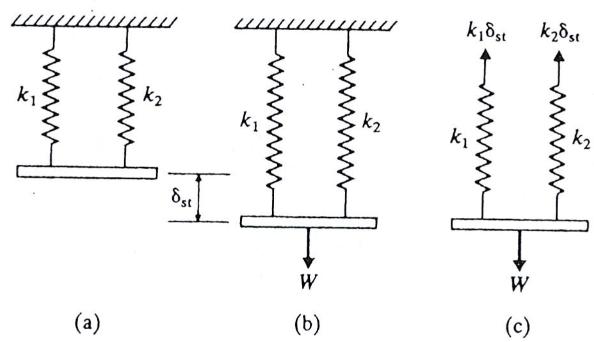
In many practical applications, several linear springs are used in combination. These springs can be combined into a single equivalent spring as indicated below.

Case 1: Springs in Parallel. To derive an expression for the equivalent spring constant of springs connected in parallel, consider the two springs shown in Fig. 1.22(a). When a load W is applied, the system undergoes a static deflection δ_{st} as shown in Fig. 1.22(b). Then the free body diagram, shown in Fig. 1.22(c), gives the equilibrium equation

$$W = k_1\,\delta_{st} + k_2\,\delta_{st} \tag{1.8}$$

If k_{eq} denotes the equivalent spring constant of the combination of the two springs, then for the same static deflection δ_{st}, we have

$$W = k_{eq}\delta_{st} \tag{1.9}$$



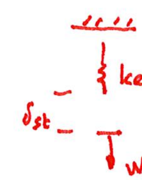


FIGURE 1.22 Springs in parallel.

Equations (1.8) and (1.9) give

$$k_{eq} = k_1 + k_2 \tag{1.10}$$

In general, if we have n springs with spring constants $k_1, k_2, \ldots, k_n$ in parallel, then the equivalent spring constant k_{eq} can be obtained:

$$k_{eq} = k_1 + k_2 + \cdots + k_n \tag{1.11}$$

Case 2: Springs in Series. Next we derive an expression for the equivalent spring constant of springs connected in series by considering the two springs shown in Fig. 1.23(a). Under the action of a load W, springs 1 and 2 undergo elongations δ_1 and δ_2, respectively, as shown in Fig. 1.23(b). The total elongation (or static deflection) of the system, δ_{st}, is given by

$$\delta_{st} = \delta_1 + \delta_2 \tag{1.12}$$

Since both springs are subjected to the same force W, we have the equilibrium shown in Fig. 1.23(c):

$$W = k_1 \delta_1$$
$$W = k_2 \delta_2 \tag{1.13}$$

If k_{eq} denotes the equivalent spring constant, then for the same static deflection,

$$W = k_{eq}\delta_{st} \tag{1.14}$$

Equations (1.13) and (1.14) give

$$k_1\delta_1 = k_2\delta_2 = k_{eq}\delta_{st}$$

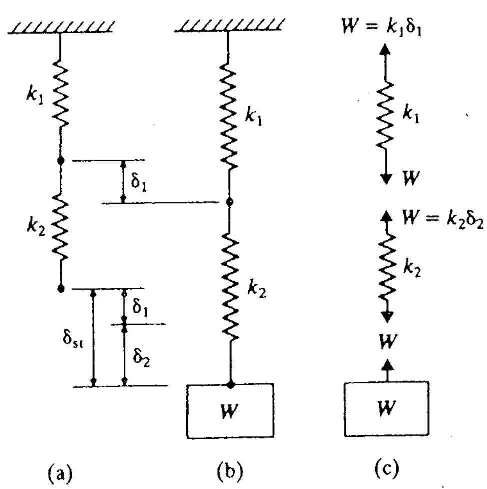


FIGURE 1.23 Springs in series.

or

$$\delta_1 = \frac{k_{eq}\delta_{st}}{k_1} \quad \text{and} \quad \delta_2 = \frac{k_{eq}\delta_{st}}{k_2} \tag{1.15}$$

Substituting these values of δ_1 and δ_2 into Eq. (1.12), we obtain

$$\frac{k_{eq}\delta_{st}}{k_1} + \frac{k_{eq}\delta_{st}}{k_2} = \delta_{st}$$

that is,

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} \tag{1.16}$$

Equation (1.16) can be generalized to the case of n springs in series:

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_n} \tag{1.17}$$

In certain applications, springs are connected to rigid components such as pulleys, levers, and gears. In such cases, an equivalent spring constant can be found using energy equivalence, as illustrated in Example 1.5.

EXAMPLE 1.2 Equivalent k of a Suspension System

Figure 1.24 shows the suspension system of a freight truck with a parallel-spring arrangement. Find the equivalent spring constant of the suspension if each of the three helical springs is

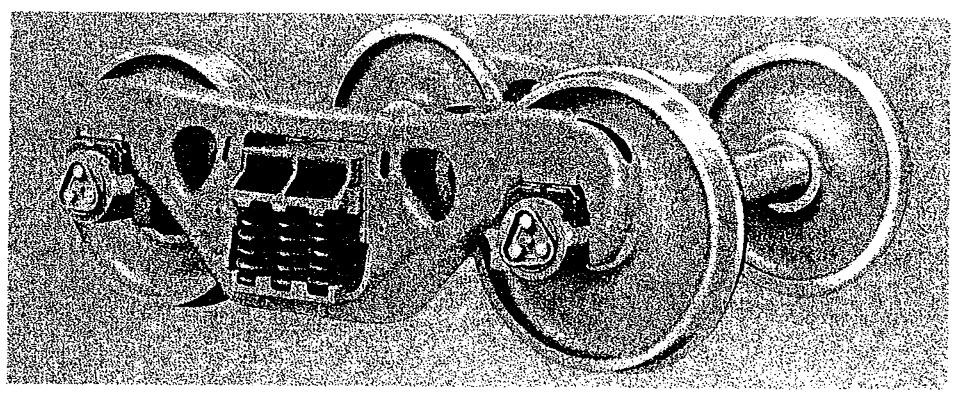


FIGURE 1.24 Parallel arrangement of springs in a freight truck. (Courtesy of Buckeye Steel Castings Company).

made of steel with a shear modulus $G = 80 \times 10^9$ N/m^2, and has five effective turns, mean coil diameter $D = 20$ cm, and wire diameter $d = 2$ cm.

Given: Suspension system with helical springs.

Find: Equivalent spring constant, k_{eq}.

Approach: Use the formula corresponding to springs in parallel.

Solution: The stiffness of each helical spring is given by

$$k = \frac{d^4 G}{8 D^3 n} = \frac{(0.02)^4 (80 \times 10^9)}{8 (0.2)^3 (5)} = 40{,}000.0 \text{ N/m}$$

(See inside front cover for the formula.) Since the springs are identical, the equivalent spring constant of the suspension system is given by

$$k_{eq} = 3k = 3\ (40{,}000.0) = 120{,}000.0 \text{ N/m}$$

■

EXAMPLE 1.3 Torsional Spring Constant of a Propeller Shaft

Determine the torsional spring constant of the steel propeller shaft shown in Fig. 1.25.

Given: Geometry and material of a stepped shaft.

Find: Torsional spring constant, k_{eq}.

Approach: Consider the segments 12 and 23 of the shaft as springs in combination.

Solution: From Fig. 1.25, the torque induced at any cross section of the shaft (such as *AA* or *BB*) can be seen to be equal to the torque applied at the propeller, *T*. Hence the elasticities

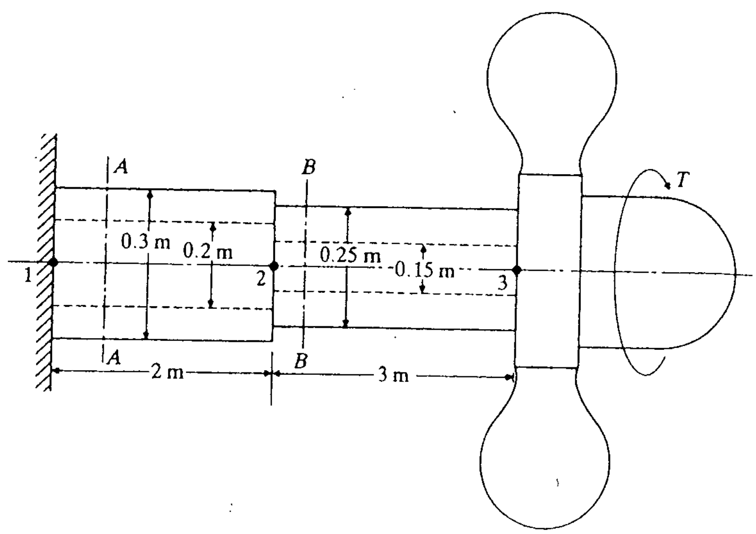


FIGURE 1.25

(springs) corresponding to the two segments 12 and 23 are to be considered as series springs. The spring constants of segments 12 and 23 of the shaft ($k_{t_{12}}$ and $k_{t_{23}}$) are given by

$$k_{t_{12}} = \frac{GJ_{12}}{l_{12}} = \frac{G\pi(D_{12}^4 - d_{12}^4)}{32\, l_{12}} = \frac{(80 \times 10^9)\pi(0.3^4 - 0.2^4)}{32\,(2)}$$
$$= 25.5255 \times 10^6 \text{ N-m/rad}$$

$$k_{t_{23}} = \frac{GJ_{23}}{l_{23}} = \frac{G\pi(D_{23}^4 - d_{23}^4)}{32\, l_{23}} = \frac{(80 \times 10^9)\pi(0.25^4 - 0.15^4)}{32\,(3)}$$
$$= 8.9012 \times 10^6 \text{ N-m/rad}$$

Since the springs are in series, Eq. (1.16) gives

$$k_{t_{eq}} = \frac{k_{t_{12}}\, k_{t_{23}}}{k_{t_{12}} + k_{t_{23}}} = \frac{(25.5255\ 10^6)\ (8.9012 \times 10^6)}{(25.5255 \times 10^6 + 8.9012 \times 10^6)} = 6.5997 \times 10^6 \text{ N-m/rad}$$ ■

EXAMPLE 1.4 Equivalent k of Hoisting Drum

A hoisting drum, carrying a steel wire rope, is mounted at the end of a cantilever beam as shown in Fig. 1.26(a). Determine the equivalent spring constant of the system when the suspended length of the wire rope is l. Assume that the net cross-sectional diameter of the wire rope is d and the Young's modulus of the beam and the wire rope is E.

Given: Dimensions of the cantilever beam: length $= b$, width $= a$, and thickness $= t$. Young's modulus of the beam $= E$. Wire rope: length $= l$, diameter $= d$, and Young's modulus $= E$.

Find: Equivalent spring constant of the system.

Approach: Series springs.

Solution: The spring constant of the cantilever beam is given by

$$k_b = \frac{3EI}{b^3} = \frac{3E}{b^3}\left(\frac{1}{12}at^3\right) = \frac{Eat^3}{4b^3} \tag{E.1}$$

The stiffness of the wire rope subjected to axial loading is

$$k_r = \frac{AE}{l} = \frac{\pi d^2 E}{4l} \tag{E.2}$$

Since both the wire rope and the cantilever beam experience the same load W, as shown in Fig. 1.26(b), they can be modeled as springs in series, as shown in Fig. 1.26(c). The equivalent spring constant k_{eq} is given by

$$\frac{1}{k_{eq}} = \frac{1}{k_b} + \frac{1}{k_r} = \frac{4b^3}{Eat^3} + \frac{4l}{\pi d^2 E}$$

or

$$k_{eq} = \frac{E}{4}\left(\frac{\pi a t^3 d^2}{\pi d^2 b^3 + l a t^3}\right) \tag{E.3}$$

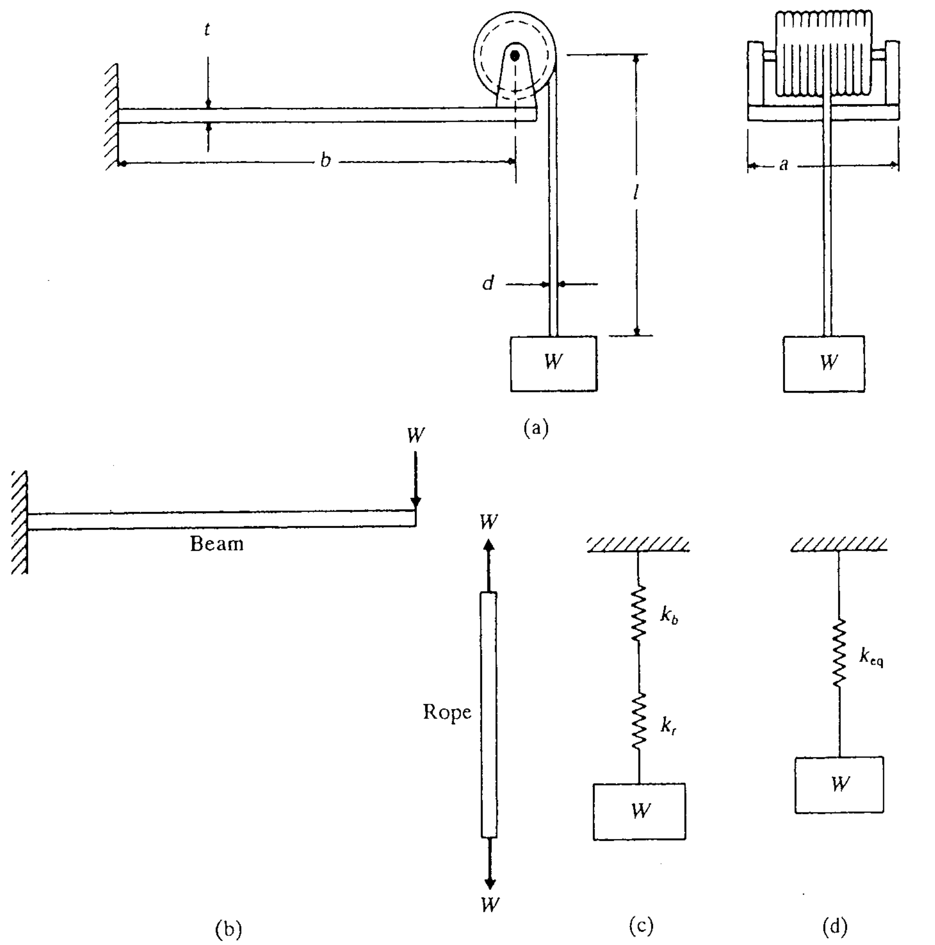


FIGURE 1.26 Hoisting drum. ■

EXAMPLE 1.5 Equivalent *k* of a Crane

The boom *AB* of the crane shown in Fig. 1.27(a) is a uniform steel bar of length 10 m and area of cross section 2500 mm^2. A weight *W* is suspended while the crane is stationary. The cable *CDEBF* is made of steel and has a cross-sectional area of 100 mm^2. Neglecting the effect of the cable *CDEB*, find the equivalent spring constant of the system in the vertical direction.

Given: Steel boom: length = 10 m, cross-sectional area = 2500 mm^2, and material = steel. Cable *FB:* material = steel and cross-sectional area = 100 mm^2. Base: *FA* = 3 m.

A vertical displacement x of point B will cause the spring k_2 (boom) to deform by an amount $x_2 = x \cos 45°$ and the spring k_1 (cable) to deform by an amount $x_1 = x \cos (90° - \theta)$. The length of the cable FB, l_1, is given by Fig. 1.27(b):

$$l_1^2 = 3^2 + 10^2 - 2(3)(10)\cos 135° = 151.426, \qquad l_1 = 12.3055 \text{ m}$$

The angle θ satisfies the relation

$$l_1^2 + 3^2 - 2(l_1)(3)\cos \theta = 10^2, \quad \cos \theta = 0.8184, \quad \theta = 35.0736°$$

The total potential energy (U) stored in the springs k_1 and k_2 can be expressed, using Eq. (1.2), as

$$U = \tfrac{1}{2}k_1(x \cos 45°)^2 + \tfrac{1}{2}k_2[x \cos(90° - \theta)]^2 \tag{E.1}$$

where

$$k_1 = \frac{A_1E_1}{l_1} = \frac{(100 \times 10^{-6})(207 \times 10^9)}{12.3055} = 1.6822 \times 10^6 \text{ N/m}$$

and

$$k_2 = \frac{A_2E_2}{l_2} = \frac{(2500 \times 10^{-6})(207 \times 10^9)}{10} = 5.1750 \times 10^7 \text{ N/m}$$

Since the equivalent spring in the vertical direction undergoes a deformation x, the potential energy of the equivalent spring (U_{eq}) is given by

$$U_{eq} = \tfrac{1}{2}k_{eq}x^2 \tag{E.2}$$

By setting $U = U_{eq}$, we obtain the equivalent spring constant of the system as

$$k_{eq} = 26.4304 \times 10^6 \text{ N/m}$$ ■

1.8 Mass or Inertia Elements

The mass or inertia element is assumed to be a rigid body; it can gain or lose kinetic energy whenever the velocity of the body changes. From Newton's second law of motion, the product of the mass and its acceleration is equal to the force applied to the mass. Work is equal to the force multiplied by the displacement in the direction of the force and the work done on a mass is stored in the form of kinetic energy of the mass.

In most cases, we must use a mathematical model to represent the actual vibrating system, and there are often several possible models. The purpose of the analysis often determines which mathematical model is appropriate. Once the model is chosen, the mass or inertia elements of the system can be easily identified. For example, consider again the cantilever beam with an end mass shown in Fig. 1.21(a). For a quick and reasonably accurate analysis, the mass and damping of the beam can be disregarded; the system can be modeled as a spring-mass system, as shown in Fig. 1.21(b). The tip mass m represents the mass element, and the elasticity of the beam denotes the stiffness of the spring. Next, consider a multistory building subjected to an earthquake. Assuming that the mass of the frame is negligible compared to the masses of the floors, the building can be modeled as a multidegree of freedom system, as shown in Fig. 1.28. The masses at the various floor levels represent the mass elements, and the elasticities of the vertical members denote the spring elements.

1.8.1 Combination of Masses

In many practical applications, several masses appear in combination. For a simple analysis, we can replace these masses by a single equivalent mass, as indicated below [1.27].

Case 1: Translational Masses Connected by a Rigid Bar. Let the masses be attached to a rigid bar that is pivoted at one end, as shown in Fig. 1.29(a). The equivalent mass can be assumed to be located at any point along the bar. To be specific, we assume the location of the equivalent mass to be that of mass m_1. The velocities of masses m_2 ($\dot{x}_2$) and m_3 ($\dot{x}_3$) can be expressed in terms of the velocity of mass m_1 ($\dot{x}_1$), by assuming small angular displacements for the bar, as

$$\dot{x}_2 = \frac{l_2}{l_1}\dot{x}_1, \quad \dot{x}_3 = \frac{l_3}{l_1}\dot{x}_1 \tag{1.18}$$

and

$$\dot{x}_{eq} = \dot{x}_1 \tag{1.19}$$

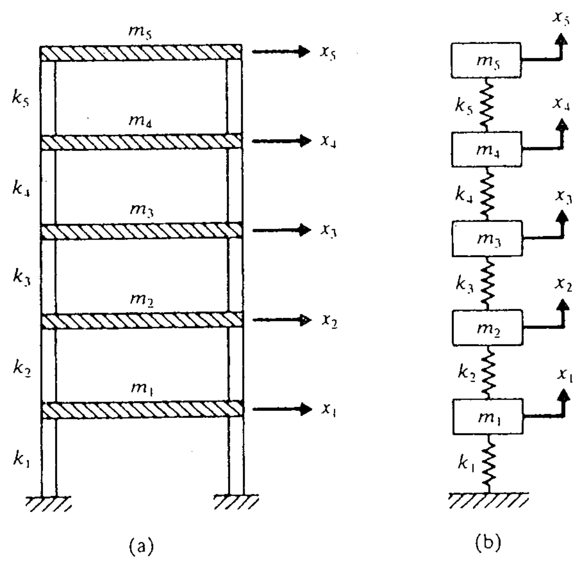


FIGURE 1.28 Idealization of a multistory building as a multidegree of freedom system.

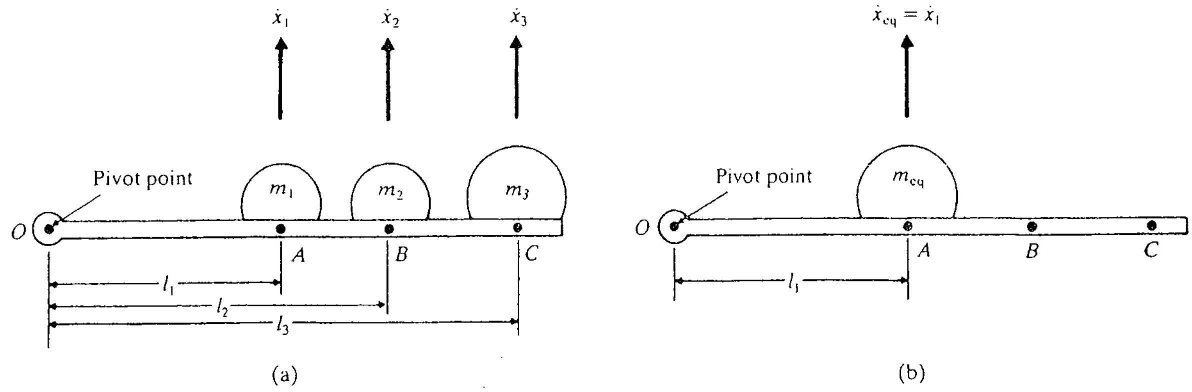


FIGURE 1.29 Translational masses connected by a rigid bar.

By equating the kinetic energy of the three mass system to that of the equivalent mass system, we obtain

$$\frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{2}m_3\dot{x}_3^2 = \frac{1}{2}m_{eq}\dot{x}_{eq}^2 \tag{1.20}$$

This equation gives, in view of Eqs. (1.18) and (1.19).

$$m_{eq} = m_1 + \left(\frac{l_2}{l_1}\right)^2 m_2 + \left(\frac{l_3}{l_1}\right)^2 m_3 \tag{1.21}$$

Case 2: Translational and Rotational Masses Coupled Together. Let a mass m having a translational velocity $\dot{x}$ be coupled to another mass (of mass moment of inertia J_0) having a rotational velocity $\dot{\theta}$, as in the rack and pinion arrangement shown in Fig. 1.30. These two masses can be combined to obtain either (1) a single equivalent translational mass m_{eq} or (2) a single equivalent rotational mass J_{eq}, as shown below.

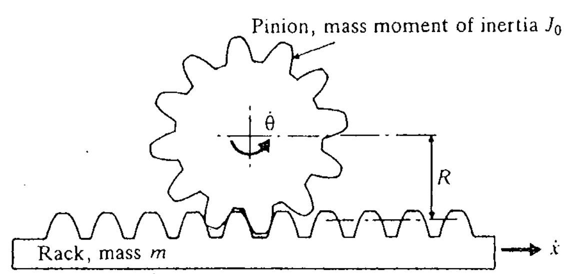


FIGURE 1.30 Translational and rotational masses in a rack and pinion arrangement.

1. *Equivalent translational mass.* The kinetic energy of the two masses is given by

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_0\dot{\theta}^2 \tag{1.22}$$

and the kinetic energy of the equivalent mass can be expressed as

$$T_{eq} = \frac{1}{2}m_{eq}\dot{x}_{eq}^2 \tag{1.23}$$

Since $\dot{x}_{eq} = \dot{x}$ and $\dot{\theta} = \dot{x}/R$, the equivalence of T and T_{eq} gives

$$\frac{1}{2}m_{eq}\dot{x}^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_0\left(\frac{\dot{x}}{R}\right)^2$$

that is,

$$m_{eq} = m + \frac{J_0}{R^2} \tag{1.24}$$

2. *Equivalent rotational mass.* Here $\dot{\theta}_{eq} = \dot{\theta}$ and $\dot{x} = \dot{\theta}R$, and the equivalence of T and T_{eq} leads to

$$\frac{1}{2}J_{eq}\dot{\theta}^2 = \frac{1}{2}m(\dot{\theta}R)^2 + \frac{1}{2}J_0\dot{\theta}^2$$

or

$$J_{eq} = J_0 + mR^2 \tag{1.25}$$

EXAMPLE 1.6 Equivalent Mass of a System

Find the equivalent mass of the system shown in Fig. 1.31, where the rigid link 1 is attached to the pulley and rotates with it.

Given: System composed of a mass, pulley, rigid links, and a cylinder, Fig. 1.31.

Find: Equivalent mass, m_{eq}.

Approach: Equivalence of kinetic energy (assuming small displacements).

Solution: When the mass m is displaced by a distance x, the pulley and the rigid link 1 rotate by an angle $\theta_p = \theta_1 = \frac{x}{r_p}$. This causes the rigid link 2 and the cylinder to be displaced by a distance $x_2 = \theta_p l_1 = \frac{x l_1}{r_p}$. Since the cylinder rolls without slippage, it rotates by an

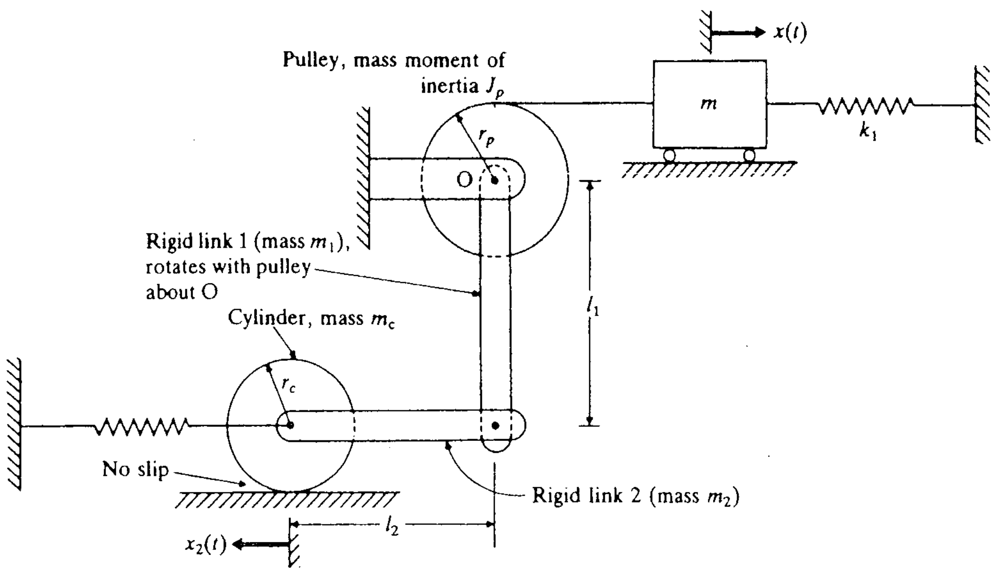


FIGURE 1.31

angle $\theta_c = \dfrac{x_2}{r_c} = \dfrac{x l_1}{r_p\, r_c}$. The kinetic energy of the system (T) can be expressed (for small displacements) as:

$$T = \frac{1}{2} m\, \dot{x}^2 + \frac{1}{2} J_p\, \dot{\theta}_p^2 + \frac{1}{2} J_1\, \dot{\theta}_1^2 + \frac{1}{2} m_2\, \dot{x}_2^2 + \frac{1}{2} J_c\, \dot{\theta}_c^2 \tag{E.1}$$

where J_p, J_1, and J_c denote the mass moments of inertia of the pulley, link 1 (about O), and cylinder, respectively, $\dot{\theta}_p$, $\dot{\theta}_1$, and $\dot{\theta}_c$ indicate the angular velocities of the pulley, link 1 (about O), and cylinder, respectively, and $\dot{x}$ and $\dot{x}_2$ represent the linear velocities of the mass m and link 2, respectively. Noting that $J_c = \dfrac{m_c\, r_c^2}{2}$ and $J_1 = \dfrac{m_1\, l_1^2}{3}$, Eq. (E.1) can be rewritten as

$$T = \frac{1}{2} m\, \dot{x}^2 + \frac{1}{2} J_p \left(\frac{\dot{x}}{r_p}\right)^2 + \frac{1}{2}\left(\frac{m_1\, l_1^2}{3}\right)\left(\frac{\dot{x}}{r_p}\right)^2 + \frac{1}{2} m_2 \left(\frac{\dot{x}\, l_1}{r_p}\right)^2 + \frac{1}{2}\left(\frac{m_c\, r_c^2}{2}\right)\left(\frac{\dot{x}\, l_1}{r_p\, r_c}\right)^2 \tag{E.2}$$

By equating Eq. (E.2) to the kinetic energy of the equivalent system,

$$T = \frac{1}{2} m_{\text{eq}}\, \dot{x}^2 \tag{E.3}$$

we obtain the equivalent mass of the system as

$$m_{\text{eq}} = m + \frac{J_p}{r_p^2} + \frac{1}{3}\frac{m_1\, l_1^2}{r_p^2} + \frac{m_2\, l_1^2}{r_p^2} + \frac{1}{2}\frac{m_c\, l_1^2}{r_p^2} \tag{E.4}$$

■

EXAMPLE 1.7 Cam-Follower Mechanism

A cam-follower mechanism (Fig. 1.32) is used to convert the rotary motion of a shaft into the oscillating or reciprocating motion of a valve. The follower system consists of a pushrod of mass m_p, a rocker arm of mass m_r, and mass moment of inertia J_r about its C.G., a valve of mass m_v, and a valve spring of negligible mass [1.28–1.30]. Find the equivalent mass (m_{eq}) of this cam-follower system by assuming the location of m_{eq} as (i) point A and (ii) point C.

Given: Mass of pushrod $= m_p$, mass of rocker arm $= m_r$, mass moment of inertia of rocker arm $= J_r$, and mass of valve $= m_v$. Linear displacement of pushrod $= x_p$.

Find: Equivalent mass of the cam-follower system (i) at point A, (ii) at point C.

Approach: Equivalence of kinetic energy.

Solution: Due to a vertical displacement x of the pushrod, the rocker arm rotates by an angle $\theta_r = x/l_1$ about the pivot point, the valve moves downward by $x_v = \theta_r l_2 = x l_2/l_1$,

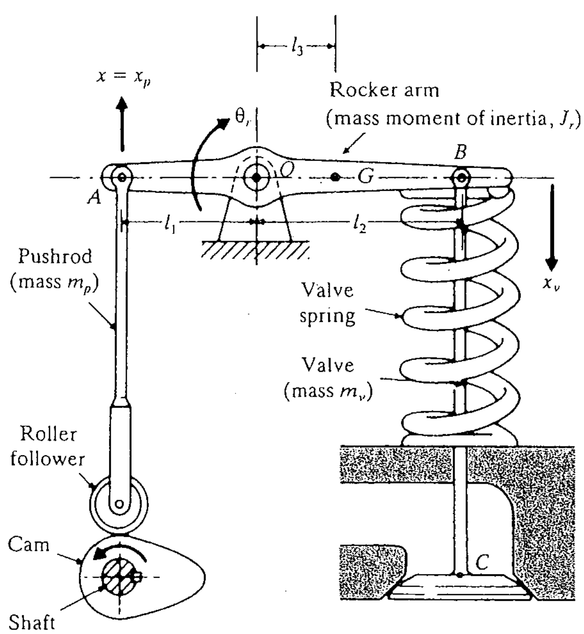


FIGURE 1.32 Cam-follower system.

and the C.G. of the rocker arm moves downward by $x_r = \theta_r l_3 = x l_3 / l_1$. The kinetic energy of the system (T) can be expressed as[2]

$$T = \frac{1}{2} m_p \dot{x}_p^2 + \frac{1}{2} m_v \dot{x}_v^2 + \frac{1}{2} J_r \dot{\theta}_r^2 + \frac{1}{2} m_r \dot{x}_r^2 \tag{E.1}$$

where $\dot{x}_p$, $\dot{x}_r$, and $\dot{x}_v$ are the linear velocities of the pushrod, C.G. of the rocker arm and the valve, respectively, and $\dot{\theta}_r$ is the angular velocity of the rocker arm.

(i) If m_{eq} denotes the equivalent mass placed at point A, with $\dot{x}_{eq} = \dot{x}$, the kinetic energy of the equivalent mass system T_{eq} is given by

$$T_{eq} = \frac{1}{2} m_{eq} \dot{x}_{eq}^2 \tag{E.2}$$

By equating T and T_{eq}, and noting that

$$\dot{x}_p = \dot{x}, \qquad \dot{x}_v = \frac{\dot{x} l_2}{l_1}, \qquad \dot{x}_r = \frac{\dot{x} l_3}{l_1}, \qquad \text{and} \qquad \dot{\theta}_r = \frac{\dot{x}}{l_1}$$

we obtain

$$m_{eq} = m_p + \frac{J_r}{l_1^2} + m_v \frac{l_2^2}{l_1^2} + m_r \frac{l_3^2}{l_1^2} \tag{E.3}$$

(ii) Similarly, if the equivalent mass is located at point C, $\dot{x}_{eq} = \dot{x}_v$ and

$$T_{eq} = \frac{1}{2} m_{eq} \dot{x}_{eq}^2 = \frac{1}{2} m_{eq} \dot{x}_v^2 \tag{E.4}$$

Equating (E.4) and (E.1) gives

$$m_{eq} = m_v + \frac{J_r}{l_2^2} + m_p \left(\frac{l_1}{l_2}\right)^2 + m_r \left(\frac{l_3}{l_2}\right)^2 \tag{E.5}$$

■

1.9 Damping Elements

In many practical systems, the vibrational energy is gradually converted to heat or sound. Due to the reduction in the energy, the response, such as the displacement of the system, gradually decreases. The mechanism by which the vibrational energy is gradually converted into heat or sound is known as damping. Although the amount of energy converted into heat or sound is relatively small, the consideration of damping becomes important for an accurate prediction of the vibration response of a system. A damper is assumed to have neither mass nor elasticity, and damping force exists only if there is relative velocity between the two ends of the damper. It is difficult to determine the causes of damping in practical systems. Hence damping is modeled as one or more of the following types.

[2]If the valve spring has a mass m_s, then its equivalent mass will be $\frac{1}{3} m_s$ (see Example 2.7). Thus the kinetic energy of the valve spring will be $\frac{1}{2}(\frac{1}{3} m_s)\dot{x}_v^2$.

Viscous Damping. Viscous damping is the most commonly used damping mechanism in vibration analysis. When mechanical systems vibrate in a fluid medium such as air, gas, water, and oil, the resistance offered by the fluid to the moving body causes energy to be dissipated. In this case, the amount of dissipated energy depends on many factors, such as the size and shape of the vibrating body, the viscosity of the fluid, the frequency of vibration, and the velocity of the vibrating body. In viscous damping, the damping force is proportional to the velocity of the vibrating body. Typical examples of viscous damping include (1) fluid film between sliding surfaces, (2) fluid flow around a piston in a cylinder, (3) fluid flow through an orifice, and (4) fluid film around a journal in a bearing.

Coulomb or Dry Friction Damping. Here the damping force is constant in magnitude but opposite in direction to that of the motion of the vibrating body. It is caused by friction between rubbing surfaces that are either dry or have insufficient lubrication.

Material or Solid or Hysteretic Damping. When materials are deformed, energy is absorbed and dissipated by the material [1.31]. The effect is due to friction between the internal planes, which slip or slide as the deformations take place. When a body having material damping is subjected to vibration, the stress-strain diagram shows a hysteresis loop as indicated in Fig. 1.33(a). The area of this loop denotes the energy lost per unit volume of the body per cycle due to damping.[3]

1.9.1 Construction of Viscous Dampers

A viscous damper can be constructed using two parallel plates separated by a distance h, with a fluid of viscosity μ between the plates (see Fig. 1.34). Let one plate be fixed and let the other plate be moved with a velocity v in its own plane. The fluid layers in contact with the moving plate move with a velocity v, while those in contact with the fixed plate do not move. The velocities of intermediate fluid layers are assumed to vary linearly between 0 and v, as shown in Fig. 1.34. According to Newton's law of viscous flow, the shear stress (τ) developed in the fluid layer at a distance y from the fixed plate is given by

$$\tau = \mu \frac{du}{dy} \tag{1.26}$$

[3] When the load applied to an elastic body is increased, the stress (σ) and the strain (ε) in the body also increase. The area under the $\sigma - \varepsilon$ curve, given by

$$u = \int \sigma \, d\varepsilon$$

denotes the energy expended (work done) per unit volume of the body. When the load on the body is decreased, energy will be recovered. When the unloading path is different from the loading path, the area ABC in Fig. 1.33(b)—the area of the hysteresis loop in Fig. 1.33(a)—denotes the energy lost per unit volume of the body.

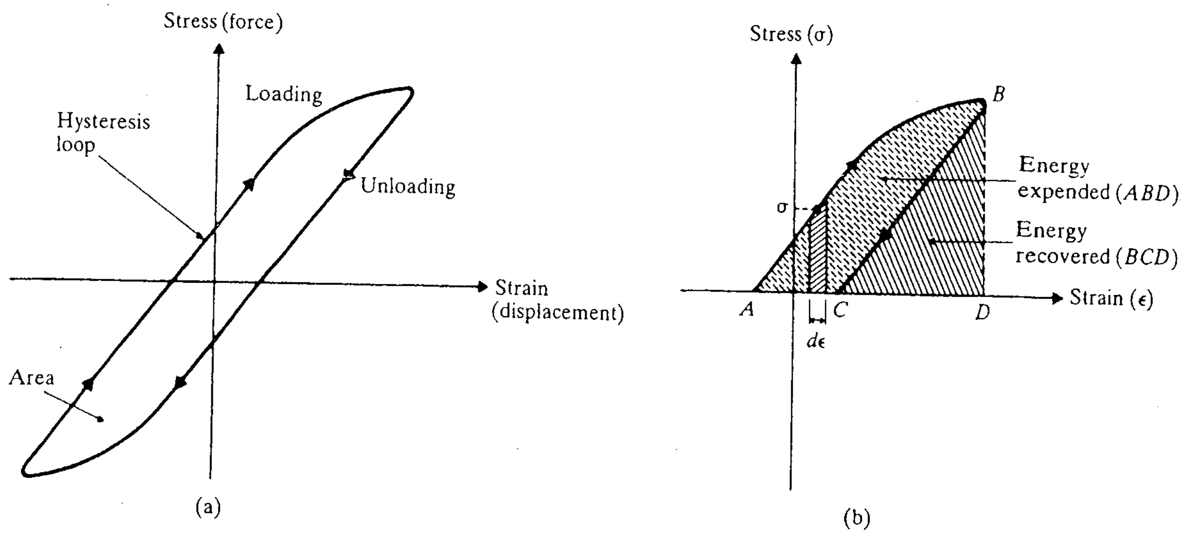


FIGURE 1.33 Hysteresis loop for elastic materials.

where $du/dy = v/h$ is the velocity gradient. The shear or resisting force (F) developed at the bottom surface of the moving plate is

$$F = \tau A = \frac{\mu A v}{h} = cv \tag{1.27}$$

where A is the surface area of the moving plate and

$$c = \frac{\mu A}{h} \tag{1.28}$$

is called the damping constant.

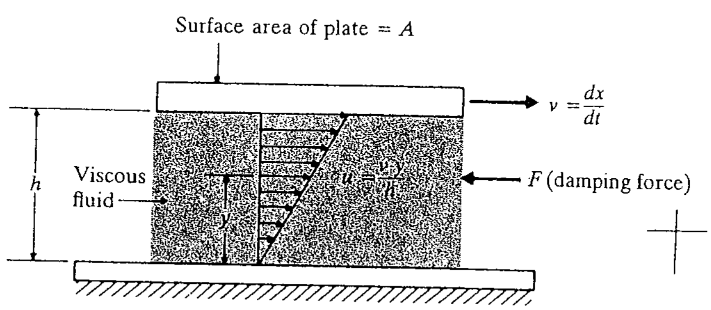


FIGURE 1.34 Parallel plates with a viscous fluid in between.

If a damper is nonlinear, a linearization procedure is generally used about the operating velocity (v^*), as in the case of a nonlinear spring. The linearization process gives the equivalent damping constant as

$$c = \left. \frac{dF}{dv} \right|_{v^*} \tag{1.29}$$

1.9.2 Combination of Dampers

When dampers appear in combination, they can be replaced by an equivalent damper by adopting a procedure similar to the one described in Sections 1.7 and 1.8 (see Problem 1.32).

EXAMPLE 1.8 Clearance in a Bearing

A bearing, which can be approximated as two flat plates separated by a thin film of lubricant (Fig. 1.35), is found to offer a resistance of 400 N when SAE30 oil is used as the lubricant and the relative velocity between the plates is 10 m/s. If the area of the plates (A) is 0.1 m^2, determine the clearance between the plates. Assume the absolute viscosity of SAE30 oil as 50 μ reyn or 0.3445 Pa-s.

Given: Characteristics of a bearing and the lubricant.

Find: Distance between the plates of the bearing.

Approach: Use the definition of damping constant.

Solution: Since the resisting force (F) can be expressed as $F = c\,v$, where c is the damping constant and v is the velocity, we have

$$c = \frac{F}{v} = \frac{400}{10} = 40 \text{ N-s/m} \tag{E.1}$$

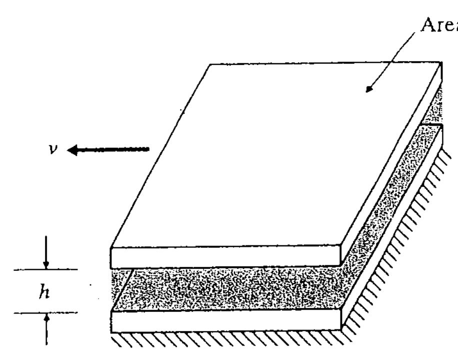
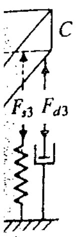


FIGURE 1.35



Let the total forces acting on all the springs and all the dampers be F_s and F_d, respectively (see Fig. 1.37(d)). The force equilibrium equations can thus be expressed as

$$F_s = F_{s1} + F_{s2} + F_{s3} + F_{s4}$$

$$F_d = F_{d1} + F_{d2} + F_{d3} + F_{d4} \tag{E.2}$$

where $F_s + F_d = W$, with W denoting the total vertical force (including the inertia force) acting on the milling machine. From Fig. 1.37(d), we have

$$F_s = k_{eq}\, x$$

$$F_d = c_{eq}\, \dot{x} \tag{E.3}$$

Equations (E.2) along with Eqs. (E.1) and (E.3), yield

$$k_{eq} = k_1 + k_2 + k_3 + k_4 = 4k$$

$$c_{eq} = c_1 + c_2 + c_3 + c_4 = 4c \tag{E.4}$$

when $k_i = k$ and $c_i = c$ for $i = 1, 2, 3, 4$.

Note: If the center of mass, G, is not located symmetrically with respect to the four springs and dampers, the i^{th} spring experiences a displacement of x_i and the i^{th} damper experiences a velocity of $\dot{x}_i$ where x_i and $\dot{x}_i$ can be related to the displacement x and velocity $\dot{x}$ of the center of mass of the milling machine, G. In such a case, Eqs. (E.1) and (E.4) need to be modified suitably. ■

1.10 Harmonic Motion

Oscillatory motion may repeat itself regularly, as in the case of a simple pendulum, or it may display considerable irregularity, as in the case of ground motion during an earthquake. If the motion is repeated after equal intervals of time, it is called *periodic motion*. The simplest type of periodic motion is *harmonic motion*. The motion imparted to the mass m due to the Scotch yoke mechanism shown in Fig. 1.38 is an example of simple harmonic motion [1.24, 1.34, 1.35]. In this system, a crank of radius A rotates about the point O. The other end of the crank P slides in a slotted rod, which reciprocates in the vertical guide R. When the crank rotates at an angular velocity ω, the end point S of the slotted link and hence the mass m of the spring-mass system are displaced from their middle positions by an amount x (in time t) given by

$$x = A \sin \theta = A \sin \omega t \tag{1.30}$$

This motion is shown by the sinusoidal curve in Fig. 1.38. The velocity of the mass m at time t is given by

$$\frac{dx}{dt} = \omega A \cos \omega t \tag{1.31}$$

and the acceleration by

$$\frac{d^2x}{dt^2} = -\omega^2 A \sin \omega t = -\omega^2 x \tag{1.32}$$

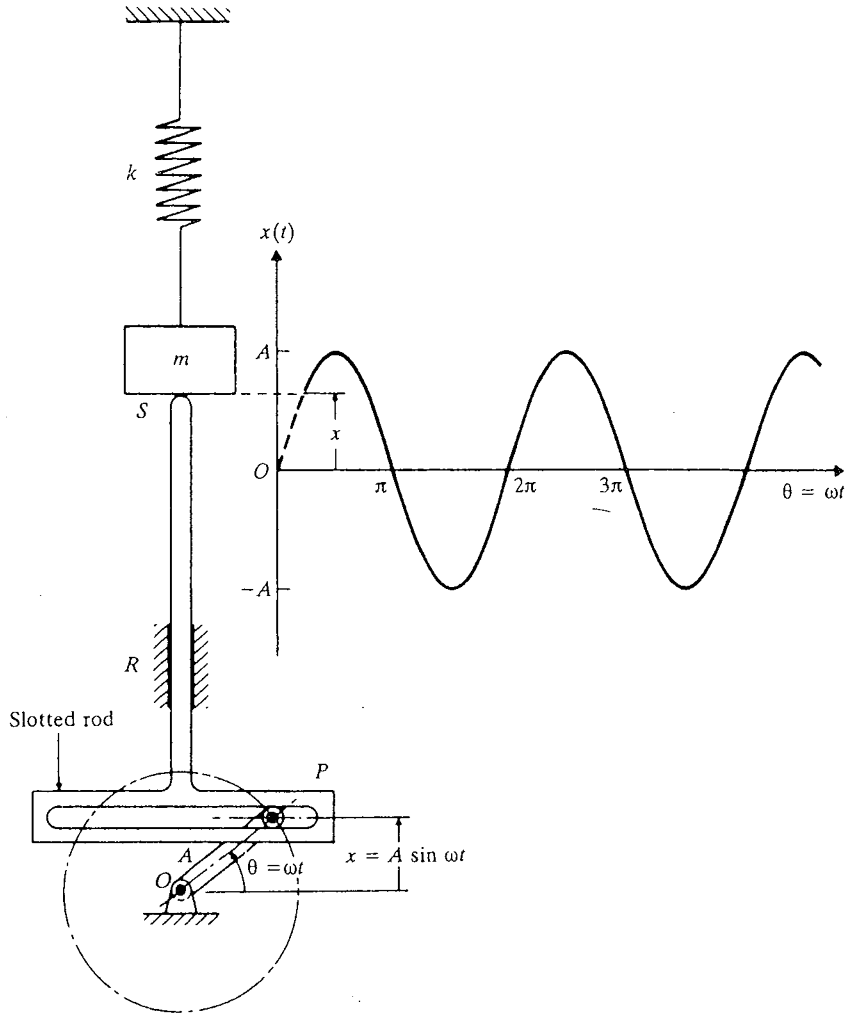


FIGURE 1.38 Scotch yoke mechanism.

It can be seen that the acceleration is directly proportional to the displacement. Such a vibration, with the acceleration proportional to the displacement and directed toward the mean position, is known as *simple harmonic motion*. The motion given by $x = A \cos \omega t$ is another example of a simple harmonic motion. Figure 1.38 clearly shows the similarity between cyclic (harmonic) motion and sinusoidal motion.

1.10.1 Vectorial Representation of Harmonic Motion

Harmonic motion can be represented conveniently by means of a vector $\overrightarrow{OP}$ of magnitude A rotating at a constant angular velocity ω. In Fig. 1.39, the projection of the tip of the vector $\vec{X} = \overrightarrow{OP}$ on the vertical axis is given by

$$y = A \sin \omega t \tag{1.33}$$

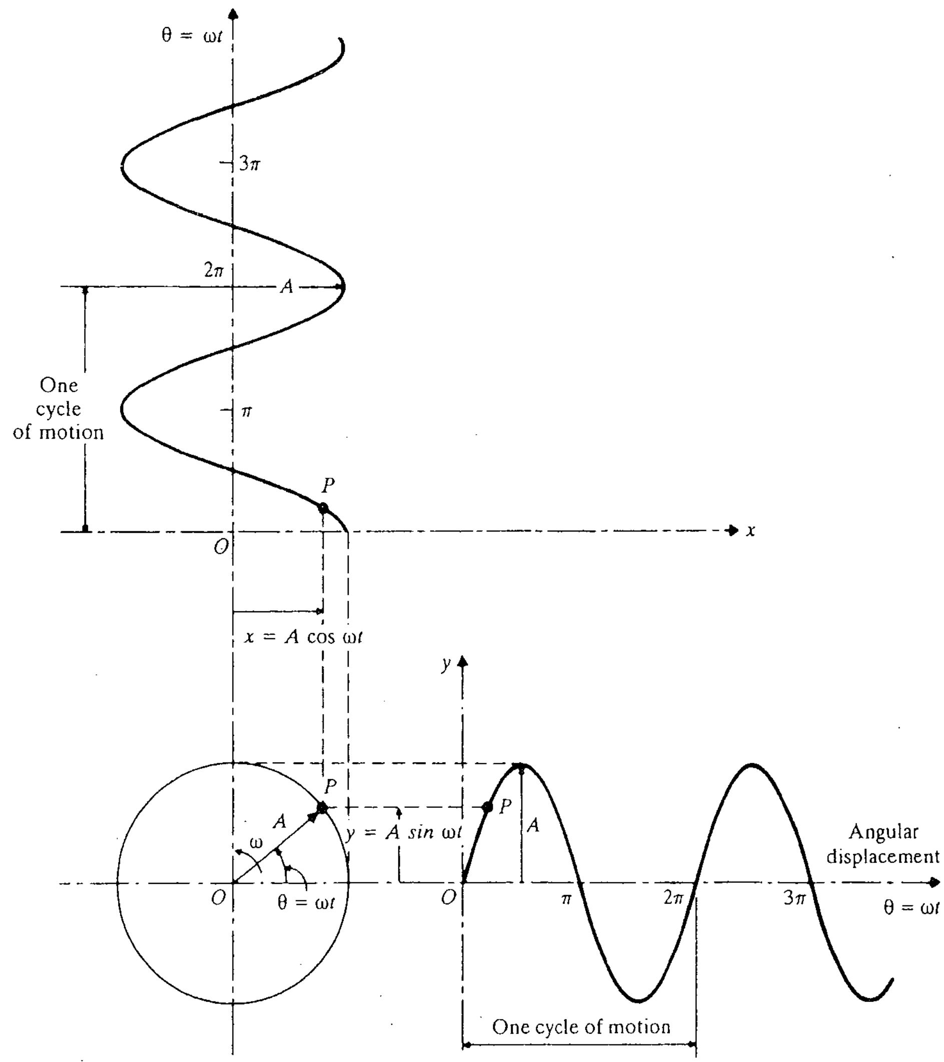


FIGURE 1.39 Harmonic motion as the projection of the end of a rotating vector.

and its projection on the horizontal axis by

$$x = A \cos \omega t \tag{1.34}$$

1.10.2 Complex Number Representation of Harmonic Motion

As seen above, the vectorial method of representing harmonic motion requires the description of both the horizontal and vertical components. It is more convenient to represent harmonic motion using a complex number representation. Any vector $\vec{X}$ in the xy plane can be represented as a complex number:

$$\vec{X} = a + ib \tag{1.35}$$

Problems

The problem assignments are organized as follows:

Problems	Section Covered	Topic Covered
1.1–1.6	1.6	Vibration analysis procedure
1.7–1.26	1.7	Spring elements
1.13, 1.26–1.31	1.8	Mass elements
1.32–1.38	1.9	Damping elements
1.39–1.59	1.10	Harmonic motion
1.60–1.70	1.11	Harmonic analysis
1.71–1.74	1.13	Computer program
1.75–1.80	—	Design projects

1.1* A study of the response of a human body subjected to vibration/shock is important in many applications. In a standing posture, the masses of head, upper torso, hips, and legs, and the elasticity/damping of neck, spinal column, abdomen, and legs influence the response characteristics. Develop a sequence of three improved approximations for modeling the human body.

1.2* Figure 1.54 shows a human body and a restraint system at the time of an automobile collision [1.47]. Suggest a simple mathematical model by considering the elasticity,

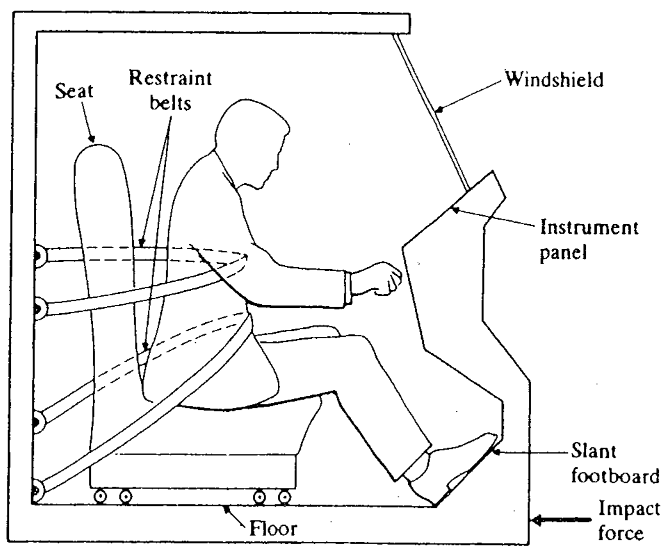


FIGURE 1.54 A human body and a restraint system.

*The asterisk denotes a design type problem or a problem with no unique answer.

mass, and damping of the seat, human body, and the restraints for a vibration analysis of the system.

1.3* A reciprocating engine is mounted on a foundation as shown in Fig. 1.55. The unbalanced forces and moments developed in the engine are transmitted to the frame and the foundation. An elastic pad is placed between the engine and the foundation block to reduce the transmission of vibration. Develop two mathematical models of the system using a gradual refinement of the modeling process.

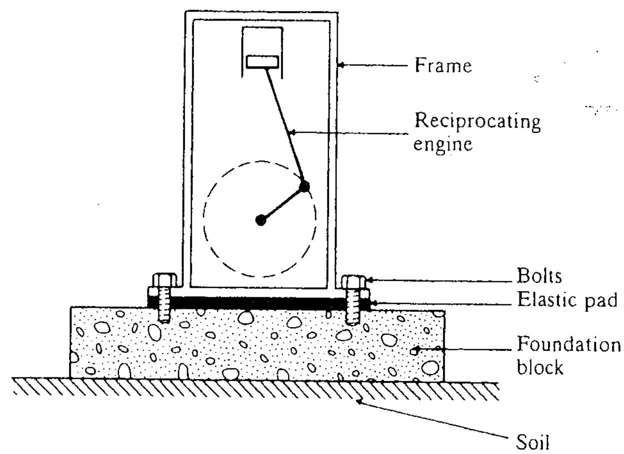


FIGURE 1.55 A reciprocating engine on a foundation.

1.4* An automobile moving over a rough road (Fig. 1.56) can be modeled considering (a) weight of the car body, passengers, seats, front wheels, and rear wheels; (b) elasticity of tires (suspension), main springs, and seats; and (c) damping of the seats, shock absorbers, and tires. Develop three mathematical models of the system using a gradual refinement in the modeling process.

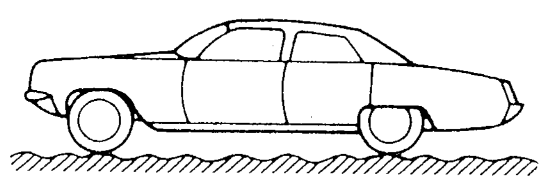


FIGURE 1.56 An automobile moving on a rough road.

1.5* The consequences of a head-on collision of two automobiles can be studied by considering the impact of the automobile on a barrier, as shown in Fig. 1.57. Construct a mathematical model by considering the masses of the automobile body, engine, transmission, and suspension, the elasticity of the bumpers, radiator, sheet metal body, driveline, and engine mounts.

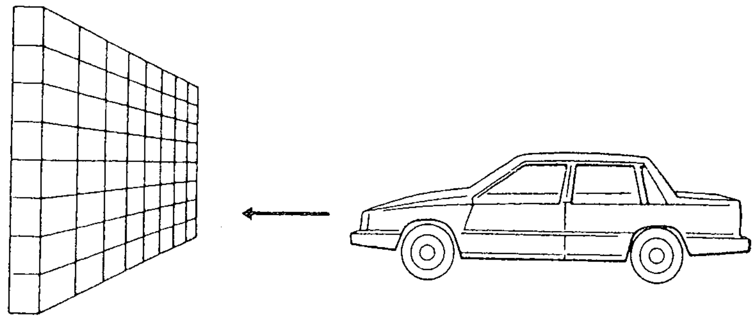


FIGURE 1.57 An automobile colliding on a barrier.

1.6* Develop a mathematical model for the tractor and plow shown in Fig. 1.58 by considering the mass, elasticity, and damping of the tires, shock absorbers, and the plows (blades).

1.7 Determine the equivalent spring constant of the system shown in Fig. 1.59.

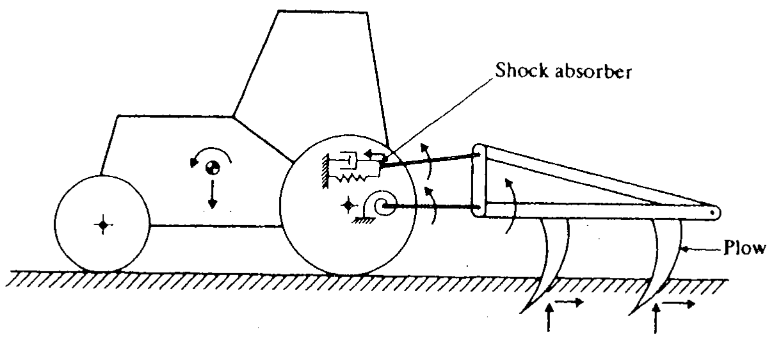


FIGURE 1.58 A tractor and plow.

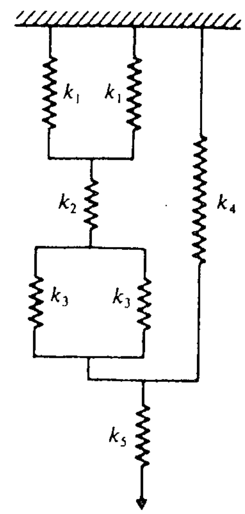


FIGURE 1.59

1.8 In Fig. 1.60, find the equivalent spring constant of the system in the direction of θ.

1.9 Find the equivalent torsional spring constant of the system shown in Fig. 1.61. Assume that k_1, k_2, k_3, and k_4 are torsional and k_5 and k_6 are linear spring constants.

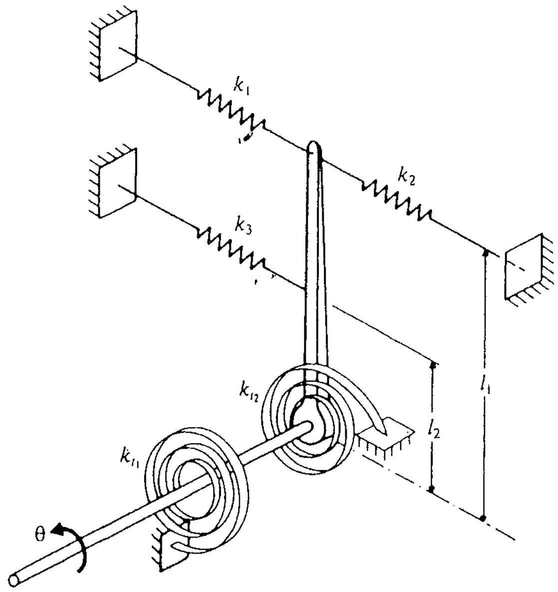


FIGURE 1.60

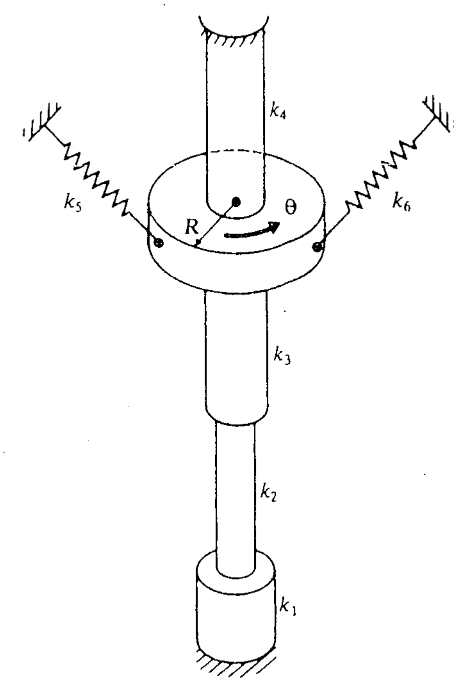


FIGURE 1.61

1.10 A machine of mass $m = 500$ kg is mounted on a simply supported steel beam of length $l = 2$ m having a rectangular cross section (depth = 0.1, m, width = 1.2 m) and Young's modulus $E = 2.06 \times 10^{11}$ N/m^2. To reduce the vertical deflection of the beam, a spring of stiffness k is attached at the mid-span, as shown in Fig. 1.62. Determine the value of k needed to reduce the deflection of the beam to one-third of its original value. Assume that the mass of the beam is negligible.

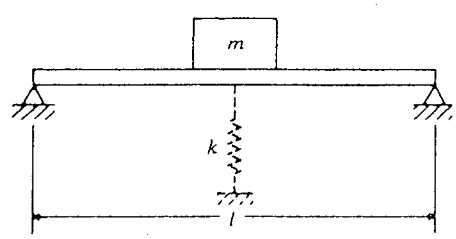


FIGURE 1.62

1.11 Four identical rigid bars—each of length a—are connected to a spring of stiffness k to form a structure for carrying a vertical load P, as shown in Figs. 1.63(a) and (b). Find the equivalent spring constant of the system k_{eq}, for each case, disregarding the masses of the bars and the friction in the joints.

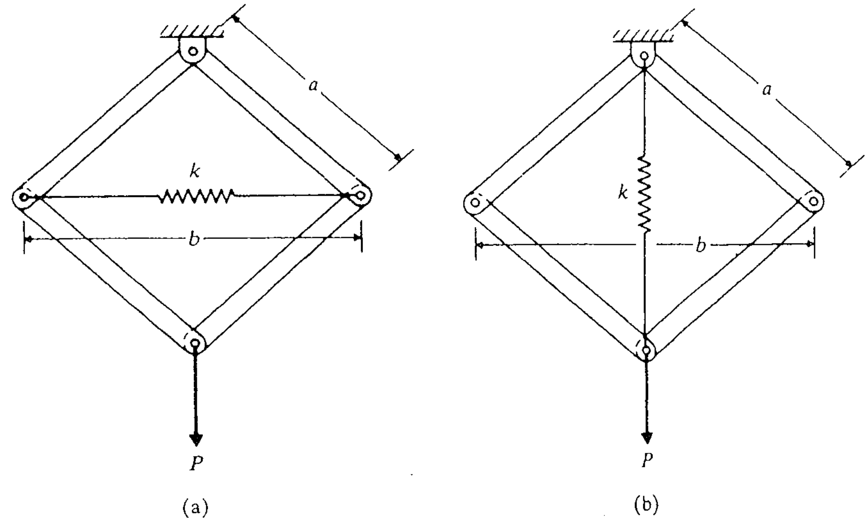


FIGURE 1.63

1.12 The tripod shown in Fig. 1.64 is used for mounting an electronic instrument that finds the distance between two points in space. The legs of the tripod are located symmetrically about the mid-vertical axis, each leg making an angle α with the vertical. If each leg has a length of l and axial stiffness of k, find the equivalent spring stiffness of the tripod in the vertical direction.

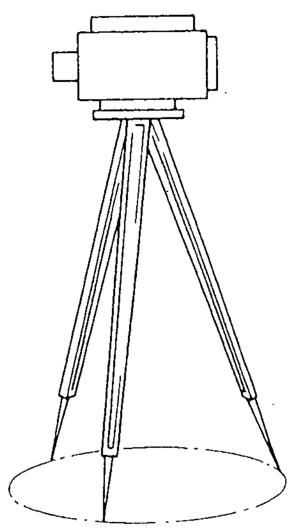


FIGURE 1.64

1.13 Find the equivalent spring constant and equivalent mass of the system shown in Fig. 1.65 with reference to θ. Assume that the bars AOB and CD are rigid with negligible mass.

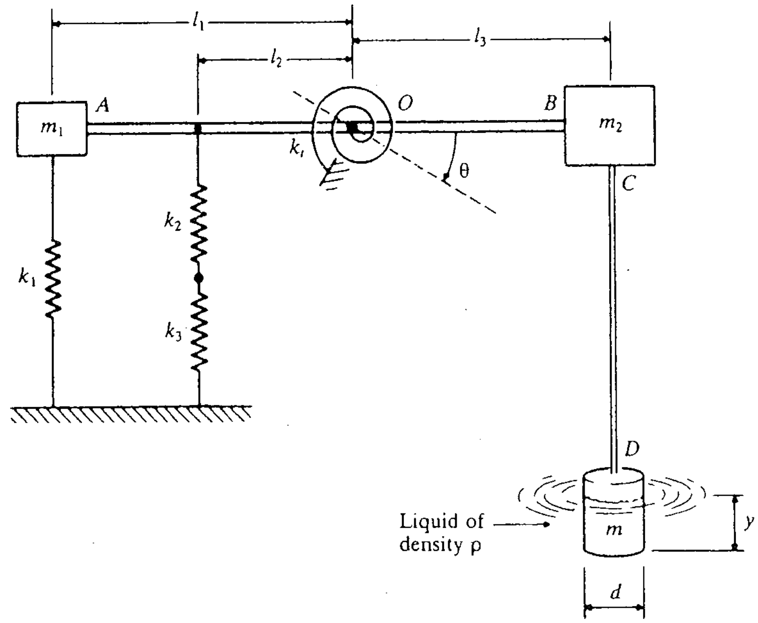


FIGURE 1.65

1.14 Find the length of the equivalent uniform hollow shaft of inner diameter d and thickness t that has the same axial spring constant as that of the solid conical shaft shown in Fig. 1.66.

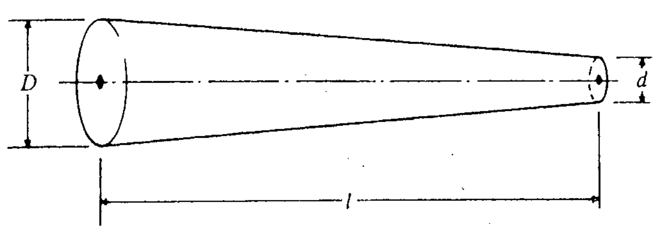


FIGURE 1.66

1.15 The force-deflection characteristic of a spring is described by $F = 500x + 2x^3$ where the force (F) is in Newtons and the deflection (x) is in millimeters. Find (a) the linearized spring constant at $x = 10$ mm, and (b) the spring forces at $x = 9$ mm and $x = 11$ mm using the linearized spring constant. Also find the error in the spring forces found in (b).

1.16 Figure 1.67 shows an air spring. This type of spring is generally used for obtaining very low natural frequencies while maintaining zero deflection under static loads. Find the spring constant of this air spring by assuming that the pressure p and volume v change adiabatically when the mass m moves.
Hint: $pv^\gamma =$ constant for an adiabatic process, where γ is the ratio of specific heats. For air, $\gamma = 1.4$.

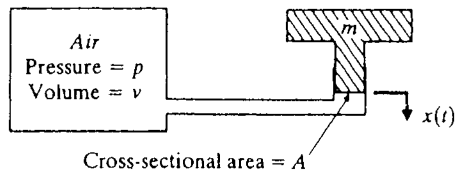


FIGURE 1.67

1.17 Find the equivalent spring constant of the system shown in Fig. 1.68 in the direction of the load P.

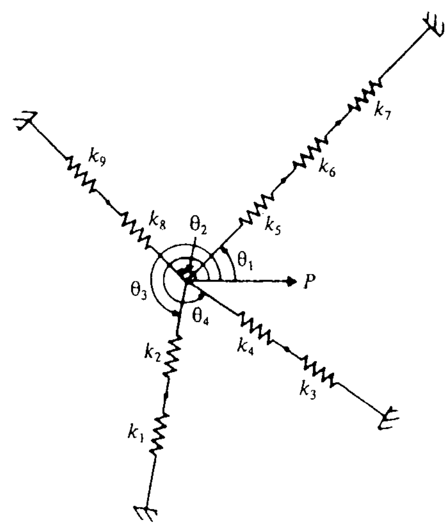


FIGURE 1.68

1.18* Design an air spring using a cylindrical container and a piston to achieve a spring constant of 75 lb/in. Assume that the maximum air pressure available is 200 psi.

1.19 The force (F)-deflection (x) relationship of a nonlinear spring is given by

$$F = ax + bx^3$$

where a and b are constants. Find the equivalent linear spring constant when the deflection is 0.01 m with $a = 20{,}000$ N/m and $b = 40 \times 10^6$ N/m^3.

1.20 Two nonlinear springs, S_1 and S_2, are connected in two different ways as indicated in Fig. 1.69. The force, F_i, in spring S_i is related to its deflection (x_i) as

$$F_i = a_i\, x_i + b_i\, x_i^3;\ i = 1, 2$$

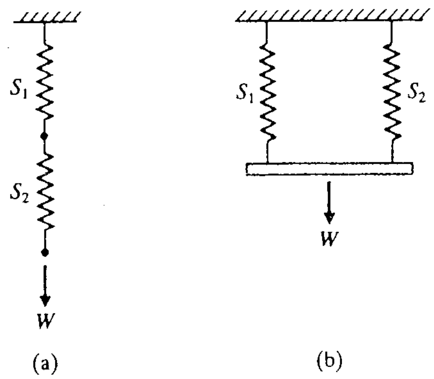


FIGURE 1.69

where a_i and b_i are constants. If an equivalent linear spring constant, k_{eq}, is defined by $W = k_{eq}\,x$ where x is the total deflection of the system, find an expression for k_{eq} in each case.

1.21* Design a steel helical compression spring to satisfy the following requirements:

Spring stiffness $(k) \geqslant 8000$ N/mm

Fundamental natural frequency of vibration $(f_1) \geqslant 0.4$ Hz

Spring index $(D/d) \geqslant 6$

Number of active turns $(N) \geqslant 10$.

The stiffness and fundamental natural frequency of the spring are given by [1.43]:

$$k = \frac{Gd^4}{8D^3N} \quad \text{and} \quad f_1 = \frac{1}{2}\sqrt{\frac{kg}{W}}$$

where G = shear modulus, d = wire diameter, D = coil diameter, W = weight of the spring, and g = acceleration due to gravity.

1.22 Find the spring constant of the bimetallic bar shown in Fig. 1.70 in axial motion.

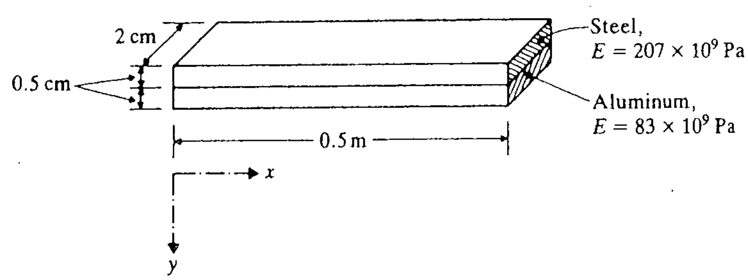


FIGURE 1.70

1.23 A tapered solid steel propeller shaft is shown in Fig. 1.71. Determine the torsional spring constant of the shaft.

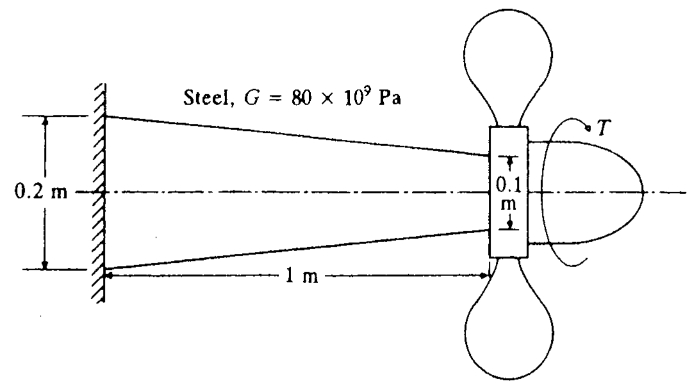


FIGURE 1.71

1.24 A composite propeller shaft, made of steel and aluminum, is shown in Fig. 1.72. Determine the torsional spring constant of the shaft.

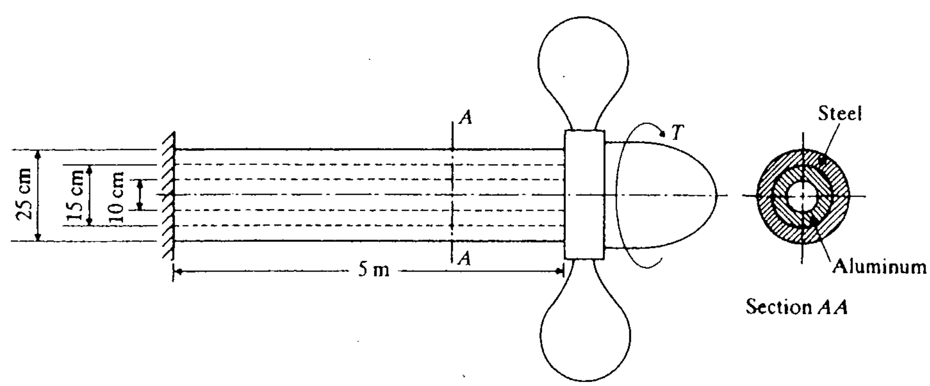


FIGURE 1.72

1.25 Consider two helical springs with the following characteristics:

Spring 1: material–steel; number of turns–10; mean coil diameter–12 in; wire diameter–2 in; free length–15 in; shear modulus–12 $\times$ 10^6 psi.

Spring 2: material–aluminum; number of turns–10; mean coil diameter–10 in; wire diameter–1 in; free length–15 in; shear modulus–4 $\times$ 10^6 psi.

Determine the equivalent spring constant when (a) spring 2 is placed inside spring 1, and (b) spring 2 is placed on top of spring 1.

1.26 Two sector gears, located at the ends of links 1 and 2, are engaged together and rotate about O_1 and O_2, as shown in Fig. 1.73. If links 1 and 2 are connected to springs k_1 to k_4 and k_{t1} and k_{t2} as shown, find the equivalent torsional spring stiffness and equivalent mass moment of inertia of the system with reference to θ_1. Assume (a) the mass moment of inertia of link 1 (including the sector gear) about O_1 as J_1 and that of link 2 (including the sector gear) about O_2 as J_2, and (b) the angles θ_1 and θ_2 to be small.

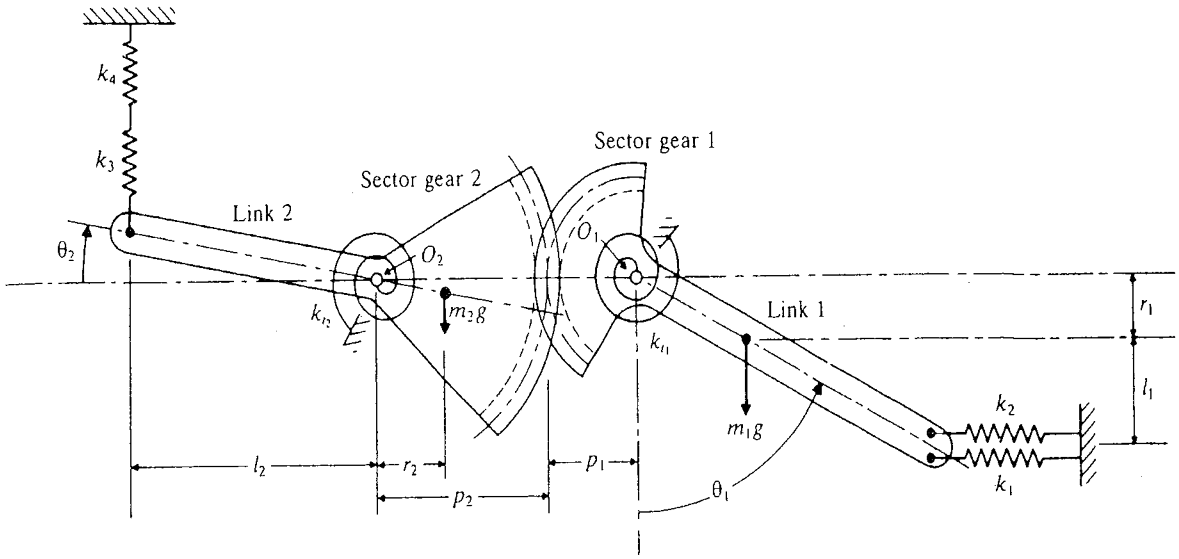


FIGURE 1.73

1.27 In Fig. 1.74 find the equivalent mass of the rocker arm assembly, referred to the x coordinate.

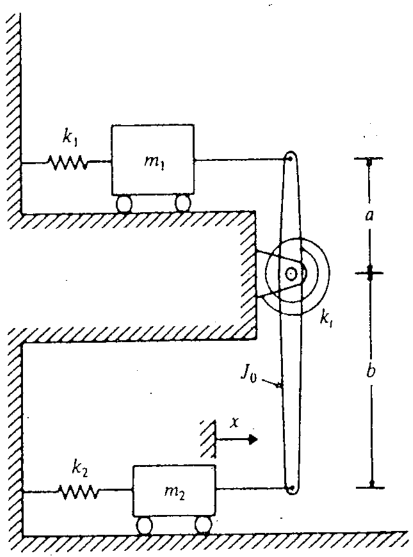


FIGURE 1.74

1.28 Find the equivalent mass moment of inertia of the gear train shown in Fig. 1.75 with reference to the driving shaft. In Fig. 1.75, J_i and n_i denote the mass moment of inertia and the number of teeth, respectively, of gear i, $i = 1,2, \ldots, 2N$.

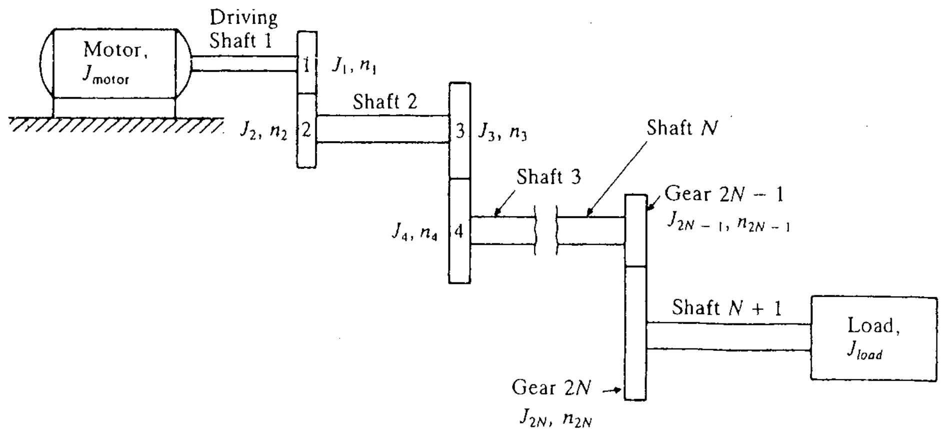


FIGURE 1.75

1.29 Two masses, having mass moments of inertia J_1 and J_2, are placed on rotating rigid shafts that are connected by gears, as shown in Fig. 1.76. If the number of teeth on gears 1 and 2 are n_1 and n_2, respectively, find the equivalent mass moment of inertia corresponding to θ_1.

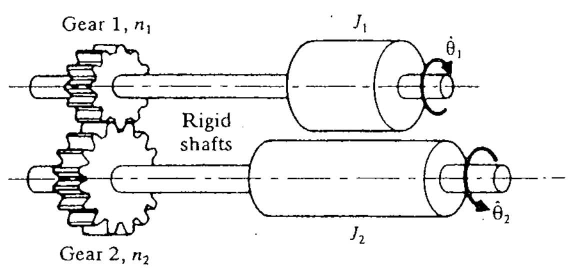


FIGURE 1.76 Rotational masses on geared shafts.

1.30 A simplified model of a petroleum pump is shown in Fig. 1.77, where the rotary motion of the crank is converted to the reciprocating motion of the piston. Find the equivalent mass, m_{eq}, of the system at location A.

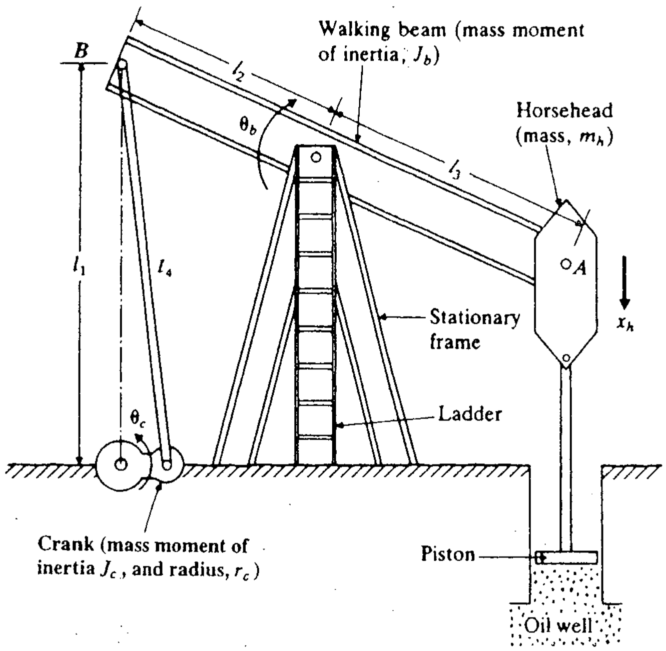


FIGURE 1.77

1.31 Find the equivalent mass of the system shown in Fig. 1.78.

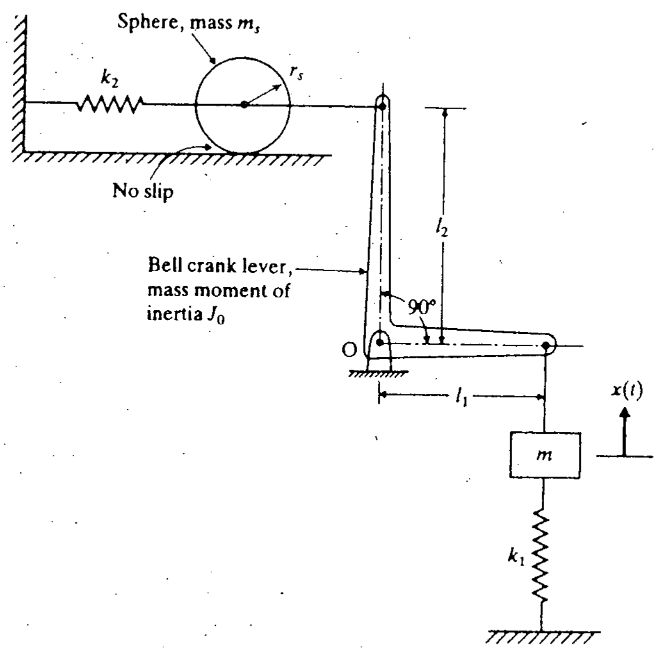


FIGURE 1.78

1.32 Find a single equivalent damping constant for the following cases:

a. When three dampers are parallel.
b. When three dampers are in series.
c. When three dampers are connected to a rigid bar (Fig. 1.79) and the equivalent damper is at site c_1.

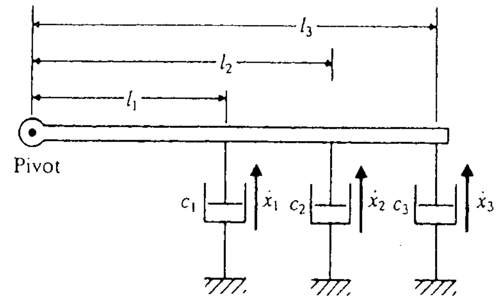


FIGURE 1.79 Dampers connected to a rigid bar.

d. When three torsional dampers are located on geared shafts (Fig. 1.80) and the equivalent damper is at location c_{t1}.

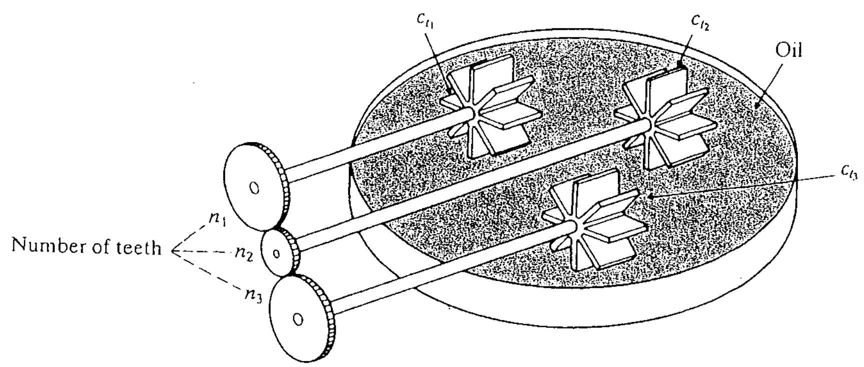


FIGURE 1.80 Dampers located on geared shafts.

Hint: The energy dissipated by a viscous damper in a cycle during harmonic motion is given by $\pi c \omega X^2$, where c is the damping constant, ω is the frequency, and X is the amplitude of oscillation.

1.33* Design a piston-cylinder type viscous damper to achieve a damping constant of 1 lbf-sec/in using a fluid of viscosity 4 μreyn (1 reyn = 1 lbf-sec/in^2).

1.34* Design a shock absorber (piston-cylinder type dashpot) to obtain a damping constant of 10^5 lb-sec/in using SAE 30 oil at 70° F. The diameter of the piston has to be less than 2.5 inches.

1.35 Develop an expression for the damping constant of the rotational damper shown in Fig. 1.81 in terms of D, d, l, h, ω, and μ, where ω denotes the constant angular velocity of the inner cylinder, and d and h represent the radial and axial clearances between the inner and outer cylinders.

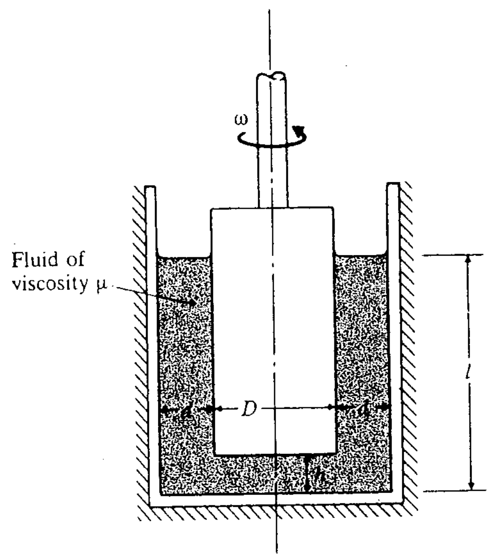


FIGURE 1.81

1.36 The force (F)-velocity $(\dot{x})$ relationship of a nonlinear damper is given by

$$F = a\,\dot{x} + b\,\dot{x}^2$$

where a and b are constants. Find the equivalent linear damping constant when the relative velocity is 5 m/s with a = 5N-s/m and b = 0.2N-s^2/m^2.

1.37 The damping constant (c) due to skin friction drag of a rectangular plate moving in a fluid of viscosity μ is given by (see Fig. 1.82):

$c = 100\;\mu\; l^2\; d$

Design a plate-type damper (shown in Fig. 1.35) that provides an identical damping constant for the same fluid.

FIGURE 1.82