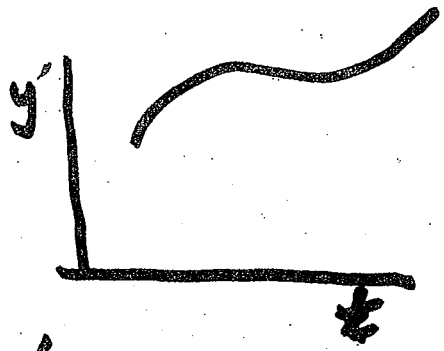
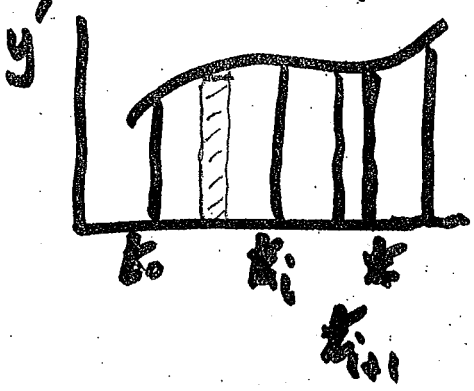
$$y' = \frac{dy}{dx} = f(x,y)$$

$$y = y(x)$$



$$\int dy = \int \frac{dy}{dx} dx = \int f(x,y)\, dx$$



$$y_i' = f(x_i, y_i)$$

$$\frac{y_{i+1} - y_i}{\Delta x} = y_i' = f(x_i, y_i)$$

$$y(x_{i+1}) \cong y_{i+1} = y_i + f(x_i, y_i)\,\Delta x \qquad \text{rect. approx}$$

$$x_{i+1} = x_i + \Delta x \qquad \text{Euler's Method.}$$

EXAMPLE: $y' = 5xy + 3y^2 - 2x = f(x,y)$

$$y(x = x_0) = y_0$$

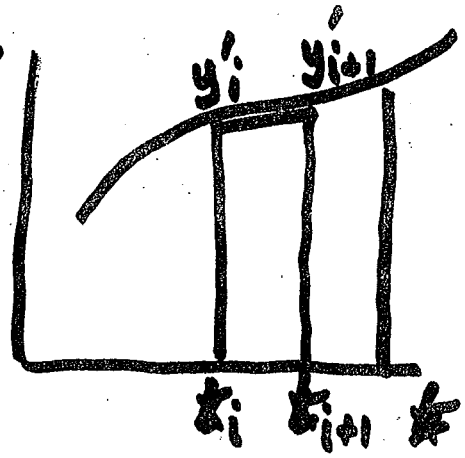
LOCAL ERROR $O(\Delta x^2)$

GLOBAL $O(\Delta x)$

$$y(x_{i+1}) = y_{i+1} = y_i + y_i' \Delta x + y_i'' \frac{\Delta x^2}{2!} + \ldots$$

$$y_{i+1} = y_i + f(x_i, y_i)\Delta x + \underline{\quad\quad}$$

MODIFIED EULER
TRAPEZ. RULE



$$y_{i+1} = y_i + \frac{y_i' + y_{i+1}'}{2}\Delta x$$

$$y_{i+1}' = f(x_{i+1}, y_{i+1})$$

M.E.

$$\tilde{y}_{i+1} = y_i + f(x_i, y_i)\Delta x \qquad \text{EULER}$$

$$\tilde{y}_{i+1}' = f(x_{i+1}, \tilde{y}_{i+1})$$

$$y_{i+1} = y_i + \frac{y_i' + \tilde{y}_{i+1}'}{2}\Delta x$$

LOCAL ERROR $O(\Delta x^3)$

GLOBAL $O(\Delta x^2)$

$$x_{i+1} = x_i + \Delta x$$

$$y_i' = f(x_i, y_i)$$

$$y_{i+1} = y_i + [f(x_i, y_i) + f(x_{i+1}, \tilde{y}_{i+1})]\frac{\Delta x}{2}$$

$$y_{i+1} = y_i + y_i'\Delta x + y_i''\frac{\Delta x^2}{2!} + \dots$$

$$y_i'' = (y_i')' = \frac{d}{dx}y_i' = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{dy}{dx}$$

$$y' = 5xy + 3y^2 - 2x = f(x,y)$$

$$\partial f/\partial x = 5y - 2 \qquad \frac{\partial f}{\partial y} = 5x + 6y$$

$$y_{i+1} = y_i + f(x_i, y_i)\Delta x + \left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} f\right)\frac{\Delta x^2}{2!} +$$

$$y'' = f(x, y', y)$$

$$\underline{y' = p} \qquad y(x_0) = y_0$$

$$y'' = \underline{p' = f(x, y, p)} \qquad p(x_0) = p_0$$

EULER METHOD

$$\frac{y_{i+1} - y_i}{\Delta x} = p_i \Rightarrow y_{i+1} = y_i + p_i \Delta x$$

$$\frac{p_{i+1} - p_i}{\Delta x} = f(x_i, y_i, p_i) \Rightarrow p_{i+1} = p_i + f(x_i, y_i, p_i)\Delta x$$

$$x_{i+1} = x_i + \Delta x$$

MOD. EULER : GIVEN y_i, x_i, p_i

③ $y_{i+1} = y_i + \dfrac{p_i + \tilde{p}_{i+1}}{2}\Delta x$

④ $p_{i+1} = p_i + \dfrac{f(x_i, y_i, p_i) + f(x_{i+1}, \tilde{y}_{i+1}, \tilde{p}_{i+1})}{2}\Delta x$

① $\tilde{y}_{i+1} = y_i + p_i \Delta x$

② $\tilde{p}_{i+1} = p_i + f(x_i, y_i, p_i)\Delta x$