TABLE 1.2: Unit Conversion Factors for Common SI and English Engineering Units

Quantity	From	Multiply by	To produce
Length	m	3.2808	ft
	ft	0.3048	m
Force	N	0.2248	lb
	lb	4.4482	N
Mass	kg	0.06852	slug
	slug	14.594	kg
Energy	J	0.7376	ft-lb
	ft-lb	1.3558	J
Pressure	Pa	0.02088	lb/ft^2
	lb/ft^2	47.8806	Pa
Temperature	K	$\frac{9}{5}$	°R
	°R	$\frac{5}{9}$	K

Other factors:	
Temperature	°F = °R − 459.67
	°C = K − 273.15
	°C = $\frac{5}{9}$(°F − 32)
Energy	1.0 Btu = 778.16 ft-lb
Power	1.0 hp = 550 ft-lb/s
Pressure	1.0 lb/in.^{2} = 144 lb/ft^2
	1.0 Pa = 1 N/m^2
Gravity	g = 9.81 m/s^2 = 32.17 ft/s^2

Solution From Table 1.2, the 150 hp may be converted to the fundamental English units of ft-lb/s by using the factor 550 ft-lb/s/hp:

$$(150 \text{ hp})\ (550 \text{ ft-lb/s/hp}) = 82{,}500 \text{ ft-lb/s}$$

This power level may be converted to SI units (W or J/s) by the factor of 1.3558 J/ft-lb in Table 1.2:

$$(82{,}500 \text{ ft-lb/s})\ (1.3558 \text{ J/ft-lb}) = 111{,}854 \text{ J/s}$$

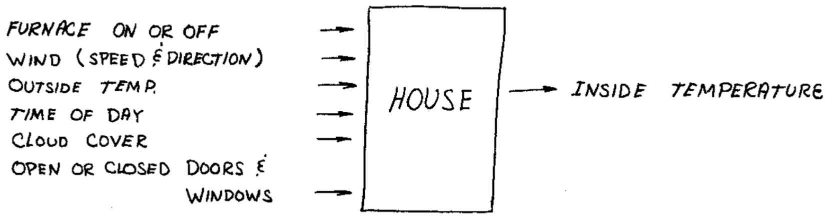
Noting that 1 W is equal to 1 J/s, approximately 111.854 kW is available from a 150-hp generator.

PROBLEMS

1.1. Consider a home heating system. The outside temperature and radiant heat from the sun influence the internal room temperature. The furnace and its thermostat control are used to maintain the house temperature at a desired level as the external weather conditions change.

(a) Use an engineering sketch to describe the system of interest and its environment, identifying the system inputs and outputs. Identify the sources of heat flow between the system and the environment (i) when the room temperature is above the temperature set on the thermostat, and (ii) when the room temperature is below the set point.

(b) Discuss how an increase in the effectiveness of wall insulation in the house walls influences the system behavior.



SYSTEM PARAMETERS

HOUSE SIZE
AMOUNT & TYPE OF INSULATION
TIME DELAYS — BOILER, PIPING, No. & EFFICIENCY OF HEATING ELEMENTS (PUMPS, FANS, ETC.)
NUMBER OF WINDOWS AND DOORS
TYPE OF SYSTEM (HOT WATER, STEAM, FORCED AIR, ETC.)

This is what I wanted from you

1.4. Many urban areas receive their water supply from reservoirs located in nearby mountains. Rain and melting snow in the catchment area flows through streams into the reservoir. Water is drawn from the reservoir by pumps for supply to the urban area. In addition, water evaporates into the atmosphere from the surface of the reservoir, and water seeps into the ground around the reservoir. We are interested in the variation of the total volume of water stored in the reservoir.

(a) Describe the system of interest using a sketch and identify the system inputs and the output of interest.

(b) The net volume flow rate of water into the reservoir varies with the time of year, as does the net flow leaving the reservoir. The net input and output volume flow rates are shown in Fig. 1.8. If the net flow into the reservoir is Q_{in} and the net flow out is Q_{out}, the resultant total reservoir net flow $Q_{net} = Q_{in} - Q_{out}$ determines the change in the total volume of water in the reservoir at any given time. Determine and plot the total reservoir net flow as a function of time.

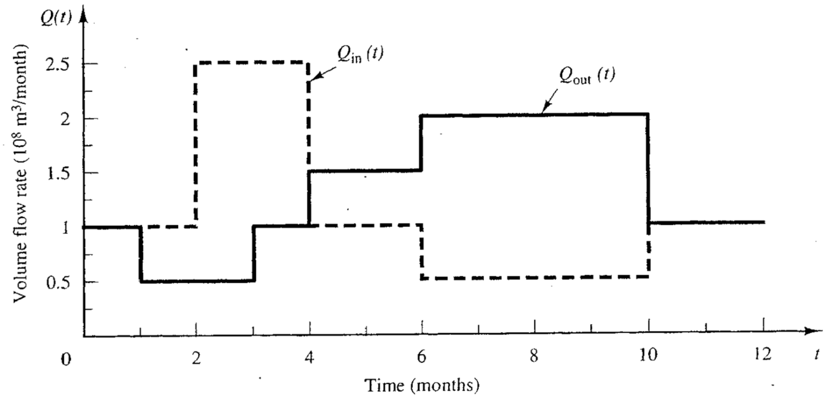


Figure 1.8: Reservoir volume flow rates.

(c) The volume of water $V(t)$ in the reservoir at any time, t, is equal to the initial volume V_0 at time $t = 0$ plus the integral of the total reservoir net flow over time

$$V(t) = V_0 + \int_0^t Q_{net}dt$$

Determine the total volume of the water in the reservoir as a function of time if at time $t = 0$, $V_0 = 4 \times 10^8$ m^3. Plot the volume as a function of time. When is the volume a minimum? In many urban areas, a water emergency is declared if a reservoir reaches a sufficiently low level, which for the example is 2.5×10^8 m^3. Does the volume of water in the reservoir ever decrease to an emergency level during the year?

1.5. A large truck pulls into a highway weighing station to ensure that it meets state weight limits. The truck's weight is 24,000 lb.

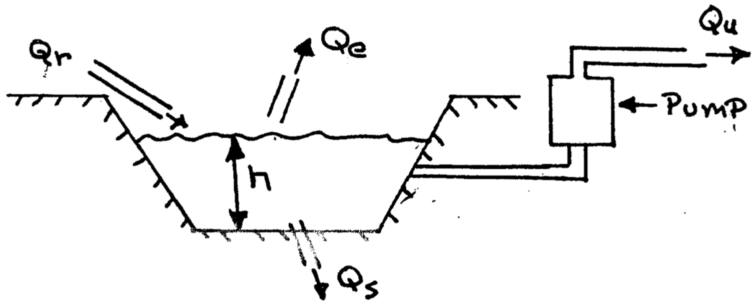
(a) What is the mass of the truck (i) in SI units and (ii) in English units.

(b) Use Newton's laws of motion to determine the propulsive force that the truck's engine must develop to accelerate the truck at 2.5 ft/s^2 in (i) SI units and (ii) in English units.

(c) If the truck accelerates from rest with a constant force of 10,000 lb, how far has it traveled after 10 seconds? What is its speed at this time?

PROBLEM 1.4

(a)



Q_r: INFLOW FROM RAIN AND MELTING SNOW

Q_u: PUMPED OUT TO URBAN REGION

Q_s: SEEPAGE INTO GROUND

Q_e: EVAPORATION TO ATMOSPHERE

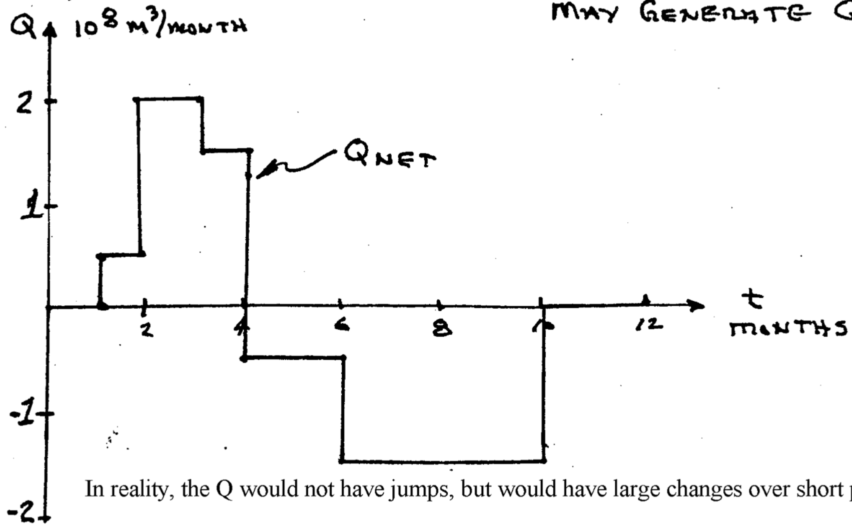
NET FLOW; $Q_N = Q_r - Q_s - Q_e - Q_u$

INPUTS: Q_r, Q_u, Q_e

OUTPUTS: h (height of water in reservoir)

Q_s (assuming depends on h)

(b) $Q_{NET} = Q_{IN} - Q_{OUT}$: DIRECTLY FROM GRAPH MAY GENERATE Q_{NET}

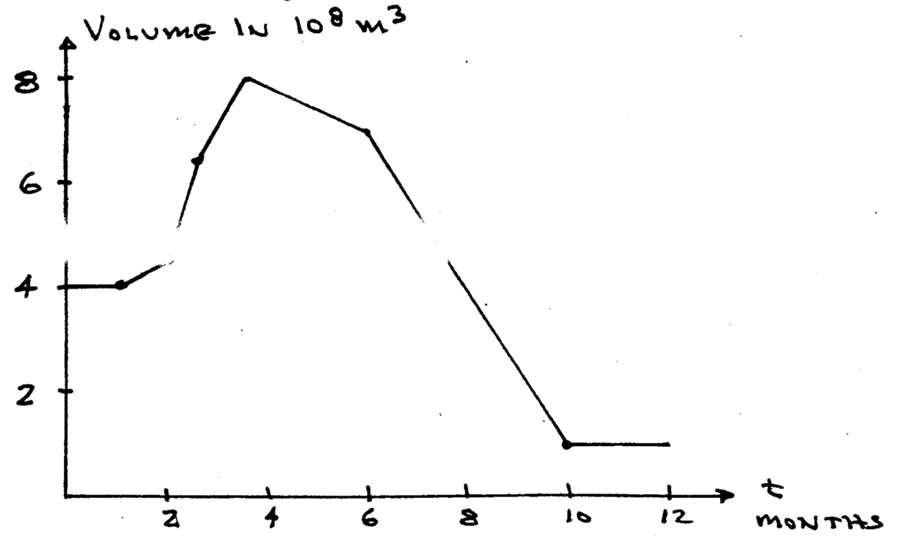


In reality, the Q would not have jumps, but would have large changes over short periods of time

1- 8

PROBLEM 1.4 CONTINUED

(c) $V = V_0 + \int_0^t Q_{NET}\, dt$ $\qquad V_0\,(t=0) = 4\cdot 10^8\ m^3$



4

Q_{NET} IN: 10^8 m^3/MONTH $\qquad$ V IN: 10^8 m^3

$0<t<1$	$Q_{NET}=0$ $\therefore$	$V=V_0$	$V(1)=V_0$	$V(1)=$
$1<t<2$	$Q_{NET}=0.5$ $\therefore$	$V=V(1)+0.5t'$	$t'=t-1$	$V(2)=$
$2<t<3$	$Q_{NET}=2$ $\therefore$	$V=V(2)+2t'$	$t'=t-2$	$V(3)=$
$3<t<4$	$Q_{NET}=1.5$ $\therefore$	$V=V(3)+1.5t'$	$t'=t-3$	$V(4)=$
$4<t<6$	$Q_{NET}=-.5$ $\therefore$	$V=V(4)-.5t'$	$t'=t-4$	$V(6)$
$6<t<10$	$Q_{NET}=-1.5$ $\therefore$	$V=V(6)-1.5t'$	$t'=t-6$	$V(10)$
$10<t<12$	$Q_{NET}=0$ $\therefore$	$V=V(10)$		$V(12)$:

THE VOLUME IS A MINIMUM FOR MONTHS 10-12 AT $1\cdot 10^8$ m^3

2

THE VOLUME DECREASES BELOW $2.5\cdot 10^8\ m^3$ AT MONTH 9 AND REMAINS BELOW FOR MONTHS 9 → 12, THUS FOR 9→12 MONTHS AN EMERGENCY

Note that even though Q has discontinuities, V is continuous.
Integration smooths out discontinuities

Note that each line segment must start where the last one left off.
That is because the volume is a continuous function of time

PROBLEMS

2.1. Consider a mechanical system in which a force F acts through an infinitesimal distance dx, where the force and the displacement are in the same direction. The infinitesimal amount of work done is $dW = Fdx$. The power flow $\mathcal{P}$ into the system is $\mathcal{P} = dW/dt = Fdx/dt = Fv$, where v is the velocity of the point at which the force is applied.

(a) Check the units of $\mathcal{P} = Fv$ to show that they are consistent.

(b) For an accelerating mass m moving at velocity v, show that the time rate of increase of stored energy is equal to Fv.

(c) For a spring with one end free, the force required to displace the spring an amount x from its rest length is $F = Kx$ (Hooke's Law), where K is the spring constant. Determine the amount of energy required to stretch the spring a distance x.

2.2. An external force applied to a point in a system, and the velocity of that point, are shown in Fig. 2.41. Assume that the velocity reference direction is the same as the force reference direction. Plot a graph of the power input to the system at this point, and a graph of the energy input to the system from time $t = 0$ to $3T$.

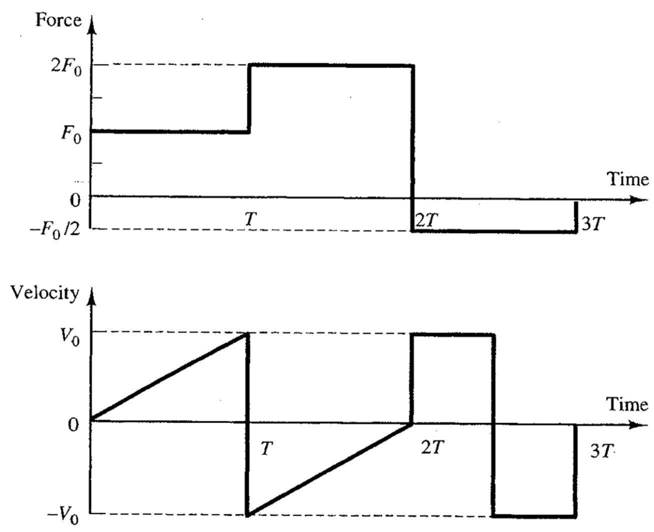


Figure 2.41: The force and velocity at a point in a system.

2.3. The velocity of a machine element over a 10 second period is:

$$v = 2.5t \text{ m/s} \qquad 0 < t < 4 \text{ s}$$

$$v = 10 \text{ m/s} \qquad 4 \leq t < 10 \text{ s}$$

(a) If the element is an ideal damper with $B = 10$ N-s/m, determine the power absorbed as a function of time, and the total energy dissipated over the 10 s period.

(b) If the element represents aerodynamic drag, approximated by the characteristic $F = Cv^2$, determine the power absorbed as a function of time, and the total energy dissipated if $C = 1.0$ N-s^2/m^2.

CHAPTER 2

PROBLEM 2.1

(a) $P = F \cdot V$ UNITS $F: N$ $V: m/s$ $P: w$

$w = N \cdot m/s = \text{Joule}/s$ $\text{JOULE} = N \cdot m$

$\therefore$ UNITS CONSISTENT

(b) E of MOVING MASS $= E = \frac{1}{2} m v^2$

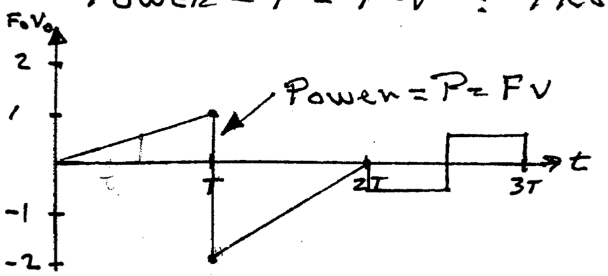
$P = \frac{dE}{dt} = \frac{1}{2} m\, 2v \frac{dv}{dt} = \left(m \frac{dv}{dt}\right) v = F \cdot V$

SINCE $F = m \frac{dv}{dt}$

(c) $F = Kx$, $E = \int_0^{x_0} F\, dx = \int_0^{x_0} Kx\, dx = \frac{1}{2} K x_0^2$

PROBLEM 2.2

POWER $= P = F \cdot V$: FROM GRAPH YIELD:



Note that because F and v have jumps so must the power. The jumps occur where F and v have jumps

$$\text{NET ENERGY} = \int P\, dt = \int_{T_1}^{T_2} P\, dt + E_{T_1}$$

CONSIDER THE FOLLOWING TIME PERIODS

$0 < t < T$ $E_0 = 0$ $E = \int_0^T \left(\frac{F_0 V_0}{T}\right) t\, dt = \frac{F_0 V_0}{T} \frac{t^2}{2}$

$E(t) = \frac{F_0 V_0}{T} \frac{t^2}{2}$ $E_T = F_0 V_0 T/2$

$T < t < 2T$ $E = \int_0^T 2 F_0 V_0 \left(-1 + \frac{t'}{T}\right) dt' + E_T$

$t' = t - T$

$E(t') = 2 F_0 V_0 \left(-t' + \frac{t'^2}{2T}\right) + E_T$

PROBLEM 2.2 CONTINUED

$$E_{2T} = 2F_0V_0\left(-T + \frac{T^2}{2T}\right) + \frac{F_0V_0T}{2}$$

$$E_{2T} = -\frac{F_0V_0T}{2}$$

$2T < t < 2.5T$

$t' = t - 2T$

$$E = \int_0^{.5T} -0.5F_0V_0\,dt' + E_{2T}$$

$$E(t') = -0.5F_0V_0t' + E_{2T}$$

$$E_{2.5T} = -.25F_0V_0T - \frac{F_0V_0T}{2} = -.75F_0$$

$2.5T < t < 3T$

$t' = t - 2.5T$

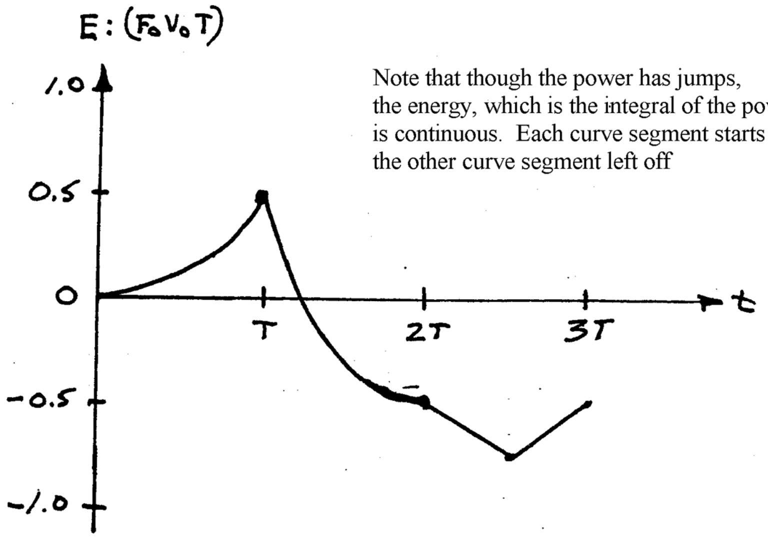
$$E = \int_0^{.5T} 0.5F_0V_0\,dt' + E_{2.5T}$$

$$E(t') = 0.5F_0V_0t' + E_{2.5T}$$

$$E_{3T} = 0.25F_0V_0T - 0.75F_0V_0T$$

$$E_{3T} = -0.5F_0V_0T$$

PLOT OF ENERGY CHANGE



Note that though the power has jumps, the energy, which is the integral of the power, is continuous. Each curve segment starts where the other curve segment left off

5

(c) In the time period $t < 4$ s, how does the power absorbed by the two dampers compare? Which damper dissipates the most energy in the 10 s period?

(d) At a velocity of 20 m/s, which damper absorbs the most power?

2.4. Automobiles must be able to sustain a frontal impact. The automobile design must allow low speed impacts with little sustained damage, while allowing the vehicle front end structure to deform and absorb impact energy at higher speeds. Consider a frontal impact test of a 1000 kg mass vehicle.

(a) For a low speed test at 2.5 m/s, compute the energy in the vehicle just prior to impact. If the bumper is a pure elastic element, what is the effective design stiffness required to limit the bumper maximum deflection during impact to 4 cm?

(b) At a higher speed impact of 25 m/s, considerable deformation occurs. To absorb the energy the front end of a vehicle is designed to deform while providing a nearly constant force. For this condition, what is the amount of energy that must be absorbed by the deformation [neglecting the energy stored in the elastic deformation in (a)]? If it is desired to limit the deformation to 10 cm, what level of resistance force is required? What is the deacceleration of the vehicle in this condition?

2.5. Consider two mechanical springs: spring A is a simple linear spring with a characteristic $F = K_1 x$, while spring B is a nonlinear hardening spring which becomes stiffer as the deflection increases $F_B = K_2 x^2$. (Hardening springs are often designed in this manner to prevent bottoming of the load.)

(a) In a laboratory test a 100 N force was found to deflect both springs by 5 cm. Find the values (and units) of K_1 and K_2.

(b) Find the energy stored in each spring when a force of 100 N is applied to each.

(c) If the force is doubled to 200 N, find the deflection and energy stored in each spring. For what range of values of applied force is the energy stored in the nonlinear spring greater than that stored in the linear spring?

2.6. A computer hard disk stores data on a rotating cylindrical disk. Consider such a disk with a radius of 6 cm and a mass of 0.02 kg.

(a) What is the moment of inertia of the disk?

(b) If the disk drive motor provides a torque of 0.1 N-m during spin-up, what is the rotational speed of the disk after 5 seconds?

(c) A new disk design uses composite materials to make a disk of the same mass but with a radius 1.5 times that of the original disk. What is the inertia of the new disk? What motor torque would be required to spin the new disk to the same speed as the original in 5 seconds?

2.7. The torsional stiffness of a cylindrical shaft of diameter D and length l is

$$K_r = G\frac{\pi}{32}\frac{D^4}{l}$$

where G is the shear modulus of the shaft material. Consider the problem in Example 2.2 for the case in which a steel shaft, 5 m long and 5 cm in diameter, drives a cylindrical flyweel with a 30 cm diameter and a thickness of 5 cm at a rotational speed of 90 r/s. Steel has a density of $\rho = 7.8$ gm/cm^3 and a shear modulus of $G = 83$ GPa.

(a) What are the values of the shaft stiffness and the flywheel moment of inertia?

(b) What is the stored energy when the system is spinning?

(c) If, as in Example 2.2, the velocity input to the shaft is suddenly stopped, what will be (i) the maximum energy stored in the shaft and (ii) the maximum angular deflection of the shaft?

PROBLEM 2.4

(a) $E = \frac{1}{2} mv^2 = \frac{1}{2} 1000 (2.5)^2 = 3125$ N-m

ELASTIC BUMPER $F = KX$

ENERGY ABSORBED $E = \frac{1}{2} K X^2$ $\quad X = 4 \cdot 10^{-2}$ m.

$E = \frac{mv^2}{2} = \frac{kx^2}{2} \Rightarrow k = \frac{mv^2}{x^2}$ $\quad K = \frac{2E}{X^2} = \frac{2(3125)}{16 \cdot 10^{-4}} = 390.6 \cdot 10^4 \, N/m$

(b) AT $V = 25 \, m/s$ $\quad E = \frac{1}{2} 1000 (25)^2 = 3125 \cdot 10^2$ N-m

NOW DURING DEFORMATION

$F = F_0$

ENERGY $= \int_0^X F \, dx = F_0 X$ $\quad$ since F = constant

$\therefore \frac{1}{2} mv^2 = Fx$ or $F = \frac{mv^2}{2x} = E/x$

FOR $X = 10$ cm $= 0.1$ m

$F_0 = E/x = 3125 \cdot 10^2 / 0.1$

but for constant F, a = const $\therefore a = \frac{F}{m}$ $\quad F_0 = 3.125 \cdot 10^6 \, N$

MASS DEACCELERATION: F_0/M

$F_0/M = \frac{3.125 \cdot 10^6}{10^3} = 3125 \, m/s^2$