HW#8

4-2, 6, 13

4-2

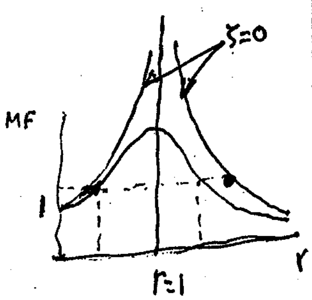
$P = P_o \sin \omega_f t$: $P_o = 45\,lb$ $\quad \tau = .25\,sec = \frac{2\pi}{\omega_f}$; $W = 2.5\,lb$ $\quad k = 15\,lb/in$

find $X = \frac{X_o}{1-r^2}$ $\quad r < 1$

① $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{W}} = \sqrt{\frac{15 \cdot 32.2 \cdot 12}{2.5}} = 48.15\ rad/sec$ $\Big\}$ $r = \frac{\omega_f}{\omega} = .522$

② $\omega_f = \frac{2\pi}{\tau} = 8\pi = 25.133\ rad/sec$

$X = \frac{P_o/k}{1-r^2} = \frac{3\,in}{.7275} = 4.123\,in$



4-6 $\quad m = 8.75\,kg$ $\quad k = 3500\,N/m$ $\quad P_o = 187\,N$ $\quad X = 7.6\,cm$

find ω_f (or f_f)

$\omega_n = \sqrt{\frac{k}{m}} = 20\ rad/sec$; $\quad X_o = P_o/k = .0534\,m = 5.34\,cm$ $\quad MF = 1.423 = \frac{X}{X_o}$

Since $MF > 1$ there are 2 solutions. First assume $r < 1$

$X = \frac{X_o}{1-r^2} \Rightarrow 1 - r^2 = \frac{X_o}{X} \Rightarrow r = \sqrt{1 - \frac{X_o}{X}} = .545$

$r = \frac{\omega_f}{\omega_n} \Rightarrow \omega_f = r\omega_n = 10.9\ rad/sec$ $\quad f_f = \frac{\omega_f}{2\pi} = 1.735\ Hz$

Now Assume $r > 1 \Rightarrow X = \frac{X_o}{r^2 - 1} \Rightarrow r^2 - 1 = \frac{X_o}{X} \Rightarrow r = \sqrt{1 + \frac{X_o}{X}} = 1.305$

$r = \frac{\omega_f}{\omega_n} \Rightarrow \omega_f = r\omega_n = 26.097\ rad/sec$ $\quad f_f = \frac{\omega_f}{2\pi} = 4.153\ Hz$

4-13

$W = 19.3\,lb$, $k = 10\,lb/in$; $P = P_o \sin \omega_f t$ where $P_o = 4\,lb$

system is in resonance $r = 1$. From work in class

$$x_p = -\frac{X_o \omega_f t}{2} \cos \omega_f t \qquad X_o = P_o/k = .4\,in$$

$\omega = \omega_f = \sqrt{\frac{kg}{W}} = 14.149\ rad/sec$

@ $\frac{1}{2}$ cycle $\quad \omega_f t = \pi \quad x_p = -\left(\frac{.4}{2}\right)\pi \cos(\pi) = .2\pi = .628\,in$

@ $2\frac{1}{2}$ cycles $\quad \omega_f t = 5\pi \quad x_p = -\left(\frac{.4}{2}\right)5\pi \cos(5\pi) = \pi = 3.142\,in$

@ $4\frac{1}{2}$ cycles $\quad \omega_f t = 9\pi \quad x_p = -\left(\frac{.4}{2}\right)9\pi \cos(9\pi) = 1.8\pi = 5.655\,in$

@ $6\frac{1}{2}$ cycles $\quad \omega_f t = 13\pi \quad x_p = -\left(\frac{.4}{2}\right)13\pi \cos(13\pi) = 2.6\pi = 8.168\,in$

HW # 9

4-21 $m = 8.75\,kg$; $k = 1750\,N/m$; $c = 37.13\,N\,sec/m$. $P = P_o \sin \omega_f t \Rightarrow P_o = 220 N$

$f_f = 4.50\,Hz$.

$\omega_n = \sqrt{\frac{k}{m}} = 14.142\,rad/sec$ $\quad c_c = 2m\omega_n = 247.49\,\frac{N\text{-}sec}{m}$ $\quad \zeta = \frac{c}{c_c} = .15$

since $\zeta < 1$ system is underdamped

$\omega_d = \sqrt{1-\zeta^2}\,\omega_n = 13.982\,rad/sec$

transient $x_c = X' e^{-\zeta\omega_n t} \sin(\omega_d t + \phi) = X' e^{-2.1617t} \sin(13.982t + \phi)$ ✓

$X_o = P_o/k = .1257\,m$; note X', ϕ obtained from initial conditions

$f_f = 4.50\,Hz = \frac{\omega_f}{2\pi} \Rightarrow \omega_f = 28.274\,rad/sec \Rightarrow r = \frac{\omega_f}{\omega_n} = 2.0$

$X_{ss} = \frac{X_o}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = .0411\,m$ $\quad \tan\psi = \frac{2\zeta r}{1-r^2} = -.2$ $\quad \psi = -11.31° = -.1974\,rad$

steadystate $x_p = \overset{X_{ss}}{.0411} \sin(\overset{\omega_f}{28.274}t + \overset{\psi}{.1974})$ ✓; $\underline{x_{total} = x_c + x_p}$

4-27 $k = 24\,lb/in$; $c = .88\,lb\text{-}sec/in$; $P_o = 15\,lb$ when $r = 1$ $x_p = 2.8409\,in$

find ζ ; ω_n ; ω_d note that $x_p|_{r=1} = X_{res}$

$X_{res} = \frac{X_o}{2\zeta} \Rightarrow \underline{X_o = P_o/k = .625\,in} \Rightarrow \zeta = \frac{1}{2}\frac{X_o}{X_{res}} = \frac{1}{2}\left(\frac{.625}{2.8409}\right) = \underline{.11 = \zeta}$

Since we don't have it

$\zeta = \frac{c}{c_c}$ $\therefore$ $\underline{c_c = \frac{c}{\zeta} = 8\,lb\text{-}sec/in}$; now $\omega_n^2 = k/m \Rightarrow k = m\omega_n^2$ & $c_c = 2m\omega_n$

Thus $\frac{k}{c_c} = \frac{\omega_n}{2}$ or $\underline{\omega_n = \frac{2k}{c_c} = 6\,rad/sec}$ $\quad f_n = \frac{\omega_n}{2\pi} = .955\,Hz$

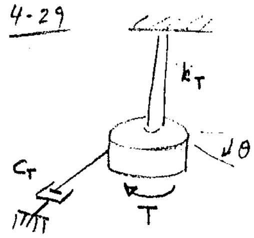
Now $\underline{\omega_d} = \omega_n\sqrt{1-\zeta^2} = \underline{5.964\,rad/sec}$ $\quad f_d = .949\,Hz$

4-28 gives when $r = 2$ $\quad X_{r=2} = \frac{1}{10} X_{res}$

$$X_{r=2} = \frac{X_o}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{X_o}{\sqrt{(-3)^2 + (4\zeta)^2}} = \frac{1}{10} X_{res} = \frac{1}{10}\frac{X_o}{2\zeta}$$

Solving those terms indicated by (~~~) gives $\zeta = .1531$

4-29



$k_T = 60000$ lb in/rad ; $I = 24$ lb in sec²/rad, $C_T = 840$ lb in sec/rad

the ODE is $I\ddot{\theta} + c_T\dot{\theta} + k_T\theta = T = T_0 \sin \omega_f t$

also for given $T_0 = 2700$ lb-in results in $\theta = 3.368^\circ = .0588$ rad

In this case everything we said about the linear case holds here

Thus $MF = \dfrac{\theta}{\theta_0}$ where $\theta_0 = T_0/k_T = .045$ rad $\Rightarrow$ $MF = 1.3067 = \dfrac{0.0588}{0.0450}$

now $\sqrt{k_T/I} = \omega_n = 50$ rad/sec ; $C_{T_{cr}} = 2I\omega_n = 2400$ lb-in sec/rad $\Rightarrow \zeta = \dfrac{c}{c_{cr}} = .35$

$$MF = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = 1.3067; \text{ solve for } r \Longrightarrow r = 1.0722, \; r = .6 .$$

$$r^4 - 1.51 r^2 + .414 = 0$$

For $r = 1.0722 = \omega_f/\omega_n \Rightarrow \omega_f = 1.0722\omega_n = 53.61$ rad/sec $\quad f_f = \dfrac{\omega}{2\pi} = 8.532$ Hz

$r = .6 = \omega_f/\omega_n \Rightarrow \omega_f = .6\omega_n = 30$ rad/sec $\Rightarrow f_f = \dfrac{\omega}{2\pi} = 4.775$ Hz

TO OBTAIN MF_{RES} & MF_{MAX}:

Now $MF_{RES} = \dfrac{\theta_{RES}}{\theta_0} = \dfrac{1}{2\zeta} = 1.429$ @ $r = 1$; $MF_{MAX} = MF_{RES} \cdot \dfrac{1}{\sqrt{1-\zeta^2}} = 1.525$ @ $r = .869 = \sqrt{1-2\zeta^2}$

$= \dfrac{1}{2\zeta} \cdot \dfrac{1}{\sqrt{1-\zeta^2}}$

TO OBTAIN THE BANDWIDTH:

now $MF_{RES} = Q$ for the bandwidth $\dfrac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \dfrac{1}{2\sqrt{2}\zeta} \Rightarrow r^2 = (1-2\zeta^2) \pm 2\zeta\sqrt{1+\zeta^2}$

$\Rightarrow r_2 = 1.2234$ and $r_1 = .1156 \Rightarrow$ bandwidth is $r_2 - r_1 = 1.1078$

NOTE: Must use full formula for r since ζ is not small.

TO FIND $T_{transmitted}$ $\quad TR = \dfrac{T_{trans}}{T_o} = \dfrac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$

when $r = 0.6$ $\quad TR = 1.4169 \quad T_{trans} = TR \cdot TO = 3825.5$ lb-in

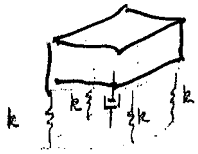
$r = 1.0724 \quad TR = 1.6334 \quad T_{trans} = TR \cdot T_o = 4410.1$ lb-in

HW #10

4-33, 35, 40, 41

4-33 Given $\begin{cases} W = 128.67\,lb \;;\; \omega_f = 859.44 \frac{rev}{min} \cdot 2\pi \frac{rad}{rev} \cdot \frac{1 min}{60 sec} = 90\ rad/sec\ ; \\ c = 10\ lb sec/in \;;\; X_{ss,RU} = 0.116\,in \;;\; F_T = 174\,lb \end{cases}$

Find k, $W_0 e$



$k_{EQ} = 4k$ now $F_T = X_{ss,RU}\sqrt{k_{eq}^2 + (c\omega_f)^2}$

$174\,lb = .0116\,in \sqrt{k_{EQ}^2 + \left(10\frac{lb sec}{in} \cdot 90 \frac{rad}{sec}\right)^2}$

$\sqrt{(F_T/X)^2 - (c\omega_f)^2} = k_{EQ} = 1200.00\ lb/in$ $\quad k = \frac{1}{4}k_{EQ} = 300.0\ lb/in$

$X_{ss} = \frac{P_o}{\sqrt{(k_{EQ} - m\omega_f^2)^2 + (c\omega_f)^2}}$ $\quad m = \frac{W}{g} = .333\ \frac{lb\text{-}sec^2}{in}$ $\quad \boxed{P_o = m_o e \omega_f^2}$

$g = 386.4\ in/s^2$

$P_o = 0.116\,in \underbrace{\sqrt{(k_{EQ} - m\omega_f^2)^2 + (c\omega_f)^2}}_{known} = 202.92\,lb = m_o e\,\omega_f^2 \Rightarrow W_0 e = \frac{(202.92)(32.2)(12)}{(90)^2}\ lb\,in$

$W_0 e = 9.68\ lb\cdot in$

as $r \to \infty$ $\quad X_{ss,RU} \to \frac{m_o e}{m} = \frac{W_o e}{W} = \frac{9.68\ lb\text{-}in}{128.67\ lb} \sim .075\ in$

4-35

Given $\begin{cases} eW_0/2 = 3\ lb in \;;\; W = 200 lb \;;\; \text{when } x = 0 \quad \psi = 90^\circ \Rightarrow r = 1 \text{ from } \tan\psi = \frac{2\zeta r}{1-r^2} \\ \omega_f = 840 \frac{rev}{min} \cdot 2\pi\frac{rad}{rev} \cdot \frac{1 min}{60 sec} = 87.965\ rad/sec \;;\; X = .75\ in \end{cases}$

Find (a) ω_n (or f_n), ζ

(b) Suppose $\omega_f = 1260 \frac{rev}{min} = 131.95\ rad/sec$ find X and ψ

(a) $X_{ss,RU} = \frac{m_o e}{m} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{W_o e}{W} \cdot \frac{1}{2\zeta}$ since $r = 1$

$X_{ss,RU} = .75\,in = \frac{2(3)}{200}\ \frac{1}{2\zeta}$

thus $\zeta = \frac{W_0 e}{2 W X} = \frac{2(3)}{400(.75)} = \boxed{.02 = \zeta}$ ✓

since $r = 1$ $\quad \omega_n = \omega_f = 87.965\ rad/sec$ or $f = 14\ Hz$

(b) For the system $\omega_f = 131.95\ rad/sec$; $\omega_n = 87.965\ rad/sec \Rightarrow r = 1.5$ also $\zeta = .02$

$\tan\psi = \frac{2\zeta r}{1-r^2} = \frac{2(.02)(1.5)}{1-(2.25)} = -.048 \qquad \underline{\psi = -2.748^\circ}$

$$X_{ss,RU} = \frac{m_o e}{m}\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = \frac{W_o e}{W}\frac{(1.5)^2}{\sqrt{(1.25)^2+(.06)^2}} = \underline{.054\ in} = X$$

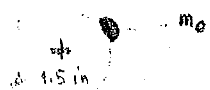
as $r \to \infty \quad X \to \frac{W_o e}{W} = .03\ in$

<u>4-40</u>

Given $W = 65\ lb$; $\Delta_{st} = \frac{W}{k_{eq}} = 2.166\,in$; $n = 3 = \frac{\sqrt{1-\zeta^2}}{2\pi\zeta}\ln\left(\frac{2.35}{.135}\right)$ $\qquad \delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{1}{n}\ln\left(\frac{X_0}{X_n}\right)$

Finally when $r = 1$, $X_{ss,RU} = .0193\ in$; also $e = 1.5\,in$

Find: c, m_o (or W_o), if $r = 2$ find X_{RU} $\qquad m = \frac{W}{g} = \frac{65\ lb}{386.4\ in/s^2}$



From $\Delta_{st} \Rightarrow k_{eq} = \frac{W}{\Delta_{st}} = \frac{65\ lb}{2.166\ in} = 30.01\ lb/in$

but $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{W}} = \underline{13.356\ rad/sec} = \sqrt{g/\Delta_{st}}$

thus $c_{cr} = 2m\omega_n = \underline{4.494\ \frac{lb\text{-}sec}{in}}$; let $\ln\left(\frac{2.35}{.135}\right) = \rho = \ln(17.41) = 2.857$

From $n = \frac{\sqrt{1-\zeta^2}}{2\pi\zeta}\ln\left(\frac{2.35}{.135}\right) \Rightarrow 4\pi^2\zeta^2 n^2 = (1-\zeta^2)\rho^2$ or $\zeta = \frac{\rho}{\sqrt{\rho^2+4\pi^2n^2}}$

thus $\underline{\zeta = .1499}$. $\qquad \underline{c = \zeta c_{cr} = .6734\ \frac{lb\text{-}sec}{in}}$

Now $X_{RU} = \frac{m_o e}{m}\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = \frac{W_o e}{W}\frac{1}{2\zeta}$ at $r=1$ $\Rightarrow \underline{W_o = \frac{X W \cdot 2\zeta}{e} = .2507\ lb}$

$X_{RU}|_{r=1}$ 0.0193 in

Knowing $X_{RU} = \frac{m_o e}{m}\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$ then for $r = 2$, $\zeta = .1499$, $W_o = .2507\ lb$, $e = 1.5\,in$, $W = 65\,lb$

$\underline{X = .00756\ in}$.

<u>4-41</u> Given $W = 96.5\,lb$; $k = 80\ lb/in$; $r = 1 \Rightarrow X = 2.143\,in$; $r \to \infty \Rightarrow X \to .3572\ in$

Find c : $\qquad X = \frac{m_o e}{m}\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$; when $r \to \infty$ $X_{r\to\infty} \to \frac{m_o e}{m} = .3572\ in$

now when $r = 1$ $\quad X = \frac{m_o e}{m}\cdot\frac{1}{2\zeta}$ or $\zeta = \frac{X_{r=\infty}}{2X_{r=1}} = \frac{.3572}{2(2.143)} = \underline{.08334 = \zeta}$

$\underline{c_c} = 2m\omega = 2\sqrt{mk} = 2\sqrt{\frac{Wk}{g}} = \underline{8.94\ \frac{lb\,sec}{in}} \qquad \underline{c = c_c\zeta = 0.74504\ \frac{lb\,sec}{in}}$