Example: given $y' = f(t,y) = 5y^2t + 3t$ with $y=1$ at $t=0$, use Euler & Mod Euler

euler

$$\frac{dy}{dt} = 5y^2t + 3t = f(t,y)$$

Given: $y_0 = 1$ when $t_0 = 0$

$f(t_0,y_0) = 0$ let $\Delta t = 0.1$ sec (our choice of step)

$y_1 = y_0 + f(t_0,y_0)\Delta t = 1 + 0(.1) = \underline{1}$

$t_1 = t_0 + \Delta t = \underline{0.1}$

$f(t_1,y_1) = 5(1)^2(0.1) + 3(.1) = .8$

$y_2 = y_1 + f(t_1,y_1)\Delta t = 1 + 0.8(.1) = \underline{1.08}$

$t_2 = t_1 + \Delta t = .1 + .1 = \underline{.2}$

$f(t_2,y_2) = 5(1.08)^2(.2) + 3(.2) = 1.7664$

$y_3 = y_2 + f(t_2,y_2)\Delta t = 1.08 + 1.7664(.1) = \underline{1.25664}$

$t_3 = t_2 + \Delta t = 0.2 + 0.1 = \underline{.3}$

t	y
0	1
.1	1
.2	1.08
.3	1.2566

---

mod euler

$$\frac{dy}{dt} = 5y^2t + 3t = f(t,y)$$

let $y_0 = 1$ when $t_0 = 0$

$f(t_0,y_0) = 5(1)^2(0) + 3(0) = 0$ let $\Delta t = 0.1$ sec

predicted $\tilde{y}_1 = y_0 + f(t_0,y_0)\Delta t = 1 + 0(.1) = 1$

$t_1 = t_0 + \Delta t = 0 + 0.1 = 0.1$

$f(t_1,\tilde{y}_1) = 5\tilde{y}_1^2 t_1 + 3t_1 = 5(1)^2(.1) + 3(.1) = 0.8$

corrected $y_1 = y_0 + \left[f(t_0,y_0) + f(t_1,\tilde{y}_1)\right]\frac{\Delta t}{2} = 1 + \left[0 + 0.8\right]\frac{0.1}{2} = \underline{1.04}$

$t_1 = t_0 + \Delta t = \underline{0.1}$

$f(t_1,y_1) = 5(1.04)^2(.1) + 3(.1) = .8408$

predicted $\tilde{y}_2 = y_1 + f(t_1,y_1)\Delta t = 1.04 + .8408(.1) = 1.1241$

$t_2 = t_1 + \Delta t = 0.1 + 0.1 = 0.2$

$f(t_2,\tilde{y}_2) = 5\tilde{y}_2^2 t_2 + 3t_2 = 5(1.1241)^2(.2) + 3(.2) = 1.8636$

corrected $y_2 = y_1 + \left[f(t_1,y_1) + f(t_2,\tilde{y}_2)\right]\frac{\Delta t}{2} = 1.04 + \left[.8408 + 1.8636\right]\frac{.1}{2} = \underline{1.1752}$

$t_2 = t_1 + \Delta t = \underline{0.2}$

t	y
0	1
.1	1.04
.2	1.1752