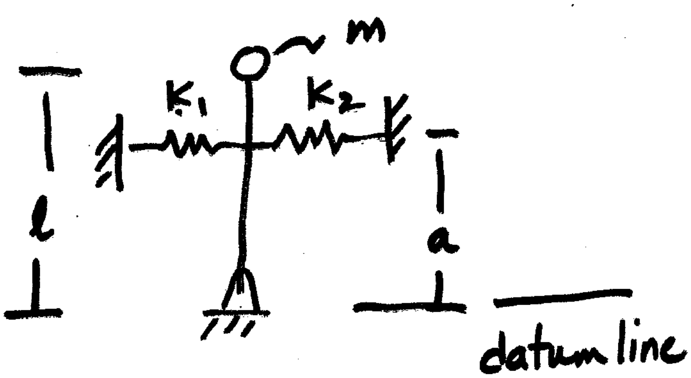
Energy Method

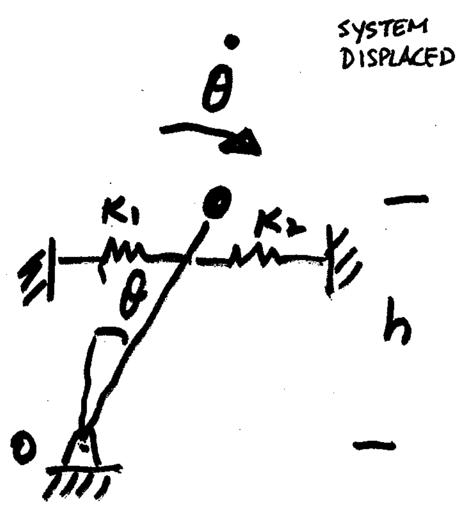
Conservative systems (no damping)

$$(KE + PE) = \text{constant}$$

$$\frac{d}{dt}(KE + PE) = 0 \implies \text{Eqn. of motion}$$

Inverted Pendulum





$$KE_i = 0$$

$$PE_i = \underbrace{\cancel{PE_{springs}}}_{\substack{0 \\ \text{unstretched}}} + \underbrace{PE_{mass}}_{mg\ell}$$

PE of mass $= mgh$

PE of springs $= \frac{1}{2}kx^2$

$$KE_f = \frac{1}{2}m(\ell\dot{\theta})^2 = \frac{1}{2}I_0\dot{\theta}^2$$

$$PE_f = PE_{springs} + PE_{mass}$$

$$= \frac{1}{2}k_1(a\sin\theta)^2 + \frac{1}{2}k_2(-a\sin\theta)^2 + mg\cdot\ell\cos\theta$$

$$(KE_f - KE_i) + (PE_f - PE_i) = 0$$

$$KE_f + PE_f = KE_i + PE_i = \text{const}$$

$$\left(\frac{1}{2}m\ell^2\dot{\theta}^2 - 0\right) + \frac{1}{2}(k_1+k_2)a^2\sin^2\theta + mg\ell\cos\theta - mg\ell = 0$$

$$0 = \frac{1}{\cancel{2}} m l^2 \left[\cancel{2}\dot{\theta}\ddot{\theta}\right] + \frac{1}{\cancel{2}}(k_1+k_2)a^2\left[\cancel{2}\sin\theta\cos\theta\cdot\dot{\theta}\right] + mgl\left[\sin\theta\cdot\dot{\theta}\right]$$

$$0 = \dot{\theta}\left[ml^2\ddot{\theta} + (k_1+k_2)a^2\sin\theta\cos\theta - mgl\sin\theta\right]$$

$$\cancel{\dot{\theta} = 0} \text{ or } [\qquad\downarrow\qquad] = 0$$

after $\frac{d}{dt}$, make small angle assumption, e.g.,

$$\cos\theta \sim 1 \quad \sin\theta \sim \theta$$

$$\cos\theta - 1 \sim \theta^2/2$$

RAYLEIGH METHOD – from energy you can get the natural frequency. For **Previous Prob.**

$\frac{1}{2} m l^2 \dot{\theta}^2 =$ KE of the system

$\frac{1}{2}(k_1+k_2)a^2\sin^2\theta - mgl(1-\cos\theta) =$ PE of ~~stem~~ system

$\cos\theta \sim 1 - \theta^2/2$ **ONLY FOR COS θ - KEEP 2 TERMS**

$$KE_{max} = PE_{max} \Longrightarrow \omega_n^2$$

$\frac{1}{2}(k_1+k_2)a^2\theta^2 - mgl\cdot\frac{\theta^2}{2} =$ PE of the sys. for small θ

if $\theta = A\sin(\omega_n t + \phi)$ for an undamped harmonic system

$\dot{\theta} = A\omega_n\cos(\omega_n t + \phi)$

$$KE = \frac{1}{2} m l^2 \dot{\theta}^2 = \frac{1}{2} m l^2 \left[A \omega_n \cos(\omega_n t + \phi)\right]^2$$

$$= \frac{1}{2} m l^2 A^2 \omega_n^2 \cos^2(\quad)$$

$$\underline{KE_{max}} = \frac{1}{2} m l^2 A^2 \omega_n^2$$

$$PE = \frac{1}{2}\left[(k_1+k_2)a^2 - mgl\right]\theta^2 = \frac{1}{2}\left[(k_1+k_2)a^2 - mgl\right]\left[A\sin(\omega_n t+\phi)\right]$$

$$\underline{PE_{max}} = \frac{1}{2}\left[(k_1+k_2)a^2 - mgl\right]A^2$$

$$KE_{max} = \frac{1}{2} m l^2 \cancel{A^2} \omega_n^2 = \frac{1}{2}\left[(k_1+k_2)a^2 - mgl\right]\cancel{A^2} = PE_{max}$$

solve for $$\omega_n^2 = \frac{\left[(k_1+k_2)a^2 - mgl\right]}{m l^2}$$