

3-23 $\Delta_{st} = .375 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}}$, $W = 12 \text{ lb} \Rightarrow m = \frac{W}{g} = \frac{W}{32.2}$, $C = 0.15 \frac{\text{lb-s}}{\text{in}} \times \frac{12 \text{ in}}{\text{ft}}$
 ζ ? ω_n ? ω_d ?

$$\omega_n = \sqrt{\frac{g}{\Delta_{st}}} \Rightarrow C_{cr} = 2m\omega_n \Rightarrow \zeta = \frac{C}{C_{cr}} < 1, \omega_d = \omega_n \sqrt{1 - \zeta^2}$$

3-24 δ ? X_0/X_1 ?

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \Rightarrow \delta = \frac{1}{n} \ln\left(\frac{X_0}{X_1}\right) \Rightarrow \delta = \ln\left(\frac{X_0}{X_1}\right) \Rightarrow e^\delta = \frac{X_0}{X_1} = \frac{X_1}{X_2} = \dots = \frac{X_n}{X_{n+1}}$$

3-28 15 cycles 80% reduction $\Rightarrow X_{15} = 0.2X_0$, $C_{cr} = 70 \frac{\text{N-s}}{\text{m}}$

C ? $\delta = \frac{1}{n} \ln\left(\frac{X_0}{X_n}\right) = \frac{1}{15} \ln\left(\frac{X_0}{0.2X_0}\right) = \frac{1}{15} \ln 5$; $\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \Rightarrow \zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$
 $\Rightarrow C = C_{cr} \cdot \zeta$

3-29 $W = 7.72 \text{ lb} \Rightarrow m = \frac{W}{g}$, $k = \frac{5 \text{ lb}}{\text{in}} = \frac{60 \text{ lb}}{\text{ft}}$, $C = 0.125 \frac{\text{lb-s}}{\text{in}} \times \frac{12 \text{ in}}{\text{ft}} = 1.5 \frac{\text{lb-s}}{\text{ft}}$

$$\omega_n = \sqrt{\frac{k}{m}} \Rightarrow C_{cr} = 2m\omega_n \Rightarrow \zeta = \frac{C}{C_{cr}} \begin{cases} \text{O.D. } \zeta > 1 \\ \text{C.D. } \zeta = 1 \\ \text{U.D. } 0 < \zeta < 1 \end{cases}$$

$X_0, V_0 = 0$ $\zeta > 1$ $x(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$ $s_1, s_2 = -\zeta\omega_n \pm \sqrt{\zeta^2 - 1} \cdot \omega_n$

$x(t^*) = X_0 \cdot \text{number}$

$\zeta = 1$ $x(t) = (C_1 + C_2 t) e^{-\omega_n t}$ $t^* = \frac{1}{\omega_n} - \frac{C_1}{C_2}$

$x(t') = X_0 \cdot \text{number}$