

Midpoint

$$a_2 = 1 \quad a_1 = 0 \quad p_1 = 1/2 \quad q_{11} = 1/2$$

$$y_{i+1} = y_i + k_2$$

$$k_1 = \Delta t f(t_i, y_i)$$

$$k_2 = \Delta t f\left(t_i + \frac{\Delta t}{2}, y_i + \frac{1}{2}k_1\right)$$

Given $t_0, y_0, f(t, y), \Delta t$

DO $I = 1, 2, \dots, N$

$$k_1 = \Delta t f(t_{I-1}, y_{I-1})$$
~~$$k_2 = \Delta t f(t_{I-1}, y_{I-1})$$~~

$$t_{IM} = t_{I-1} + \Delta t/2$$

$$y_{IP} = y_{I-1} + \frac{1}{2}k_1$$

$$k_2 = \Delta t f(t_{IM}, y_{IP})$$

$$y_I = y_{I-1} + k_2$$

$$t_I = t_{I-1} + \Delta t$$

print t_I, y_I

$$\text{LE } O(\Delta t^3) = C \Delta t^3$$

$$\text{G.E. } O(\Delta t^2) = D \Delta t^2$$

$$\text{L.E. euler } O(\Delta t^2) = C \Delta t_E^2$$

$$\text{G.E. euler } O(\Delta t) = D \Delta t_E$$

$$\text{LE midpoint} = O(\Delta t^3) = C \Delta t_M^3$$

$$\text{G.E. midpoint} = O(\Delta t^2) = D \Delta t_M^2$$

many steps $D_E \Delta t_E = D_M \Delta t_M^2$

$$\Delta t_M = \sqrt{\frac{D_E \Delta t_E}{D_M}} \quad \text{assume } D_E = D_M$$

$$\Delta t_M = \sqrt{\Delta t_E} \quad \Delta t_E = .1 \Rightarrow \Delta t_M = .3163$$

$$5 \text{ min} = 300 \text{ sec} = 3000 \text{ time steps} = 3000 \text{ FLOPS}$$

$$5 \text{ min} = 300 \text{ sec} \quad TS = \frac{300 \text{ sec}}{.3163 \text{ sec/step}} \cong 1100 \text{ TS} \times \frac{2 \text{ FLOPS}}{TS} = 2200 \text{ FLOPS}$$

$$\text{RK4} \quad \text{LE } O(\Delta t^5) = C \Delta t_{\text{RK4}}^5$$

$$\text{GE } O(\Delta t^4) = D \Delta t_{\text{RK4}}^4$$

RK2 2SV

$$t_0, V_0, Q_0, f(t, V, Q), g(t, V, Q), \Delta t \quad \frac{dV}{dt} = f$$

$$\frac{dQ}{dt} = g$$

DO $I = 1, 2, 3, \dots, N$

$$k_1 = \Delta t f(t_{I-1}, V_{I-1}, Q_{I-1})$$

$$l_1 = \Delta t g(t_{I-1}, V_{I-1}, Q_{I-1})$$

$$k_1 = \Delta t f(t_{I-1}, v_{I-1}, Q_{I-1})$$

$$l_1 = \Delta t g(t_{I-1}, v_{I-1}, Q_{I-1})$$

$$v_{IP} = v_{I-1} + k_1$$

$$Q_{IP} = Q_{I-1} + l_1$$

$$t_I = t_{I-1} + \Delta t$$

$$k_2 = \Delta t f(t_I, v_{IP}, Q_{IP})$$

$$l_2 = \Delta t g(t_I, v_{IP}, Q_{IP})$$

$$v_I = v_{I-1} + \frac{1}{2} (k_1 + k_2)$$

$$Q_I = Q_{I-1} + \frac{1}{2} (l_1 + l_2)$$

$$\text{Print } t_I, v_I, Q_I \quad \leftarrow$$