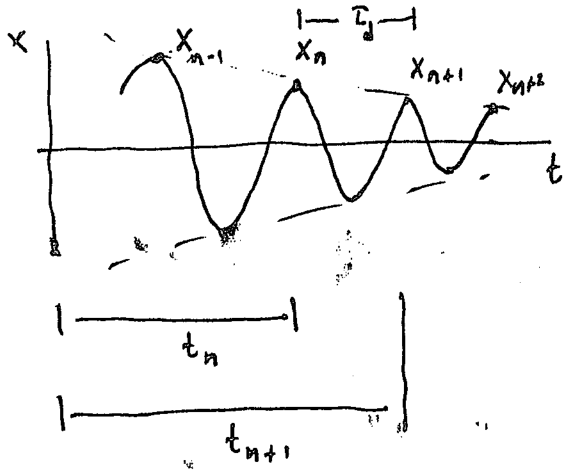
if we compare case III to friction damping

1. Viscous damping takes an ∞ amount of time to come to a stop



$$t_{n+1} = t_n + \tau_d$$

$$x_n = X e^{-\zeta \omega_n t_n} \sin(\omega_d t_n + \phi)$$

$$x_{n+1} = X e^{-\zeta \omega_n (t_n + \tau_d)} \sin(\omega_d [t_n + \tau_d] + \phi)$$

$$\frac{x_n}{x_{n+1}} = \frac{1}{e^{-\zeta \omega_n \tau_d}}$$ and the two sine function are equal

$$\sin(\omega_d (t_n + \tau_d) + \phi) = \sin(\omega_d t_n + 2\pi + \phi) = \sin(\omega_d t_n + \phi)$$

$$\therefore \ \ln\left(\frac{x_n}{x_{n+1}}\right) = \ln\left(e^{\zeta \omega_n \tau_d}\right) = \zeta \omega_n \tau_d = \zeta \omega_n \cdot \frac{2\pi}{\omega_d} = \frac{\zeta \omega_n \cdot 2\pi}{\omega_n \sqrt{1-\zeta^2}}$$

LOGARITHMIC DECREMENT $= \delta = \ln\left(\frac{x_n}{x_{n+1}}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$ CAN GET DAMPING RATIO FROM GRAPH

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

IF $\zeta < 0.3$ $\zeta \cong \frac{\delta}{2\pi}$

2. Viscous damping is not linear but logarithmic $\delta = \ln\left(\frac{x_n}{x_{n+1}}\right)$ x_n, x_{n+1} 2 adjacent points in phase

3. $x_n - x_{n+1} \neq x_{n+1} - x_{n+2}$

Lastly $m\ddot{x} + c\dot{x} + kx = m\left[\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x\right] = m\left[\ddot{x} + 2\cdot\underbrace{\frac{c}{2m}}_{\zeta\omega_n}\dot{x} + \underbrace{\frac{k}{m}}_{\omega_n^2}x\right] = m\left[\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x\right] = 0$

$\Downarrow$

$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$ is also the governing equation for viscous systems.