Case III $c^2 - 4mk < 0$ $\quad s_1 = \dfrac{-c + i\sqrt{4mk - c^2}}{2m} = +p + iq \qquad s_2 = \dfrac{-c - i\sqrt{4mk-c^2}}{2m} = +p - iq \qquad p = \dfrac{c}{2m} \quad q = \dfrac{\sqrt{4mk - c^2}}{2m}$

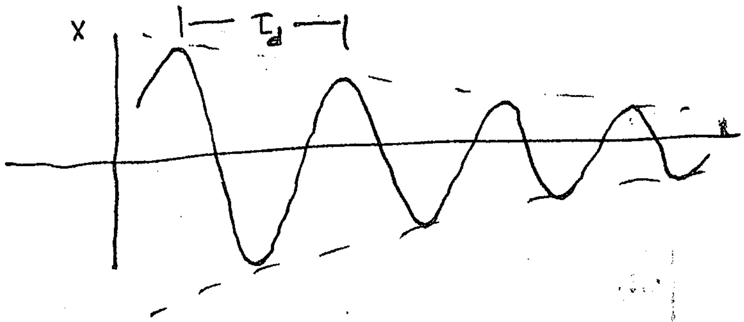
$$x(t) = e^{+pt}\left[C_1 \sin qt + C_2 \cos qt\right] \quad (1)$$

q = damped frequency of oscillation = ω_d

$$= X e^{+pt}\left[\sin(qt + \phi)\right] \quad (2)$$

where X is the amplitude, ϕ is the lead or lag angle

note that unlike the undamped case, where $c = 0$, here Xe^{-pt} acts like the amplitude for the sine function and decreases over time



$\omega_d = q < \omega_n$

$\omega_d \tau_d = 2\pi$

motion is periodic with period $\tau_d > \tau_n$

$$x(t=0) = C_2 = X \sin\phi = x_0$$

$$\dot{x}(t) = +pe^{+pt}\left[C_1 \sin qt + C_2 \cos qt\right] + e^{+pt}\left[qC_1 \cos qt - qC_2 \sin qt\right] \quad \text{from (1)}$$

$$= +pXe^{+pt}\sin(qt+\phi) + Xe^{+pt}\left[q\cos(qt+\phi)\right) \quad \text{from (2)}$$

$$\dot{x}(t=0) = +pC_2 + qC_1 = V_0 \quad \therefore \quad C_1 = \frac{V_0 + px_0}{q}$$

$$= +pX\sin\phi + Xq\cos\phi = V_0 \quad \therefore \quad X\cos\phi = \frac{V_0 + pX\sin\phi}{q} = \frac{V_0 + px_0}{q} = C_1$$

$$\therefore \quad X = \sqrt{(X\sin\phi)^2 + (X\cos\phi)^2} = \sqrt{x_0^2 + \left(\frac{V_0 + px_0}{q}\right)^2} = \sqrt{C_1^2 + C_2^2}$$

$$\tan\phi = \frac{X\sin\phi}{X\cos\phi} = \frac{C_2}{C_1} = \frac{x_0 q}{V_0 + px_0} \qquad p = \frac{c}{2m}, \quad q = \frac{\sqrt{4mk - c^2}}{2m}$$

Case III is the <u>underdamped</u> case