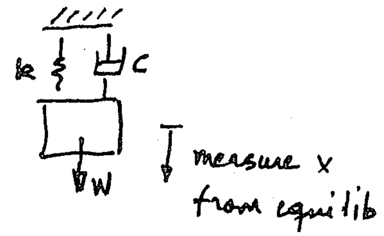
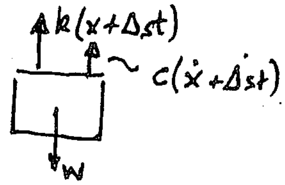


$\Delta_{st} = \frac{W}{k}$

$F = c\dot{x}$ in direction opposite to motion $\quad [c] = \frac{\text{lb-sec}}{\text{ft}} = \frac{\text{N-s}}{\text{m}}$



$$\Sigma F = W - k(x+\Delta_{st}) - c(\dot{x}+\dot{\Delta}_{st}) = -kx - c\dot{x} = m\ddot{x}$$

$$m\ddot{x} + c\dot{x} + kx = 0$$

let $x(t) = Ce^{st} \implies Ce^{st}(ms^2 + cs + k) = 0 \implies s = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$

<u>Case I</u> $\quad c^2 - 4mk > 0 \qquad s_1 = \frac{-c + \sqrt{c^2-4mk}}{2m} < 0 \qquad s_2 = \frac{-c - \sqrt{c^2-4mk}}{2m} < 0$

$$x(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

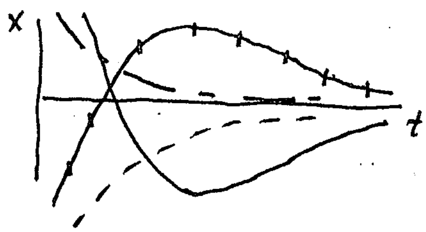
dying exponentials

solution is APERIODIC (no oscillatory behavior)

$x(t=0) = X_0 = C_1 + C_2$

$\dot{x}(t=0) = V_0 = C_1 s_1 + C_2 s_2$

get C_1 & C_2 from here



can be any one of these depending on Initial Conditions (IC)

This is called the <u>overdamped case</u>

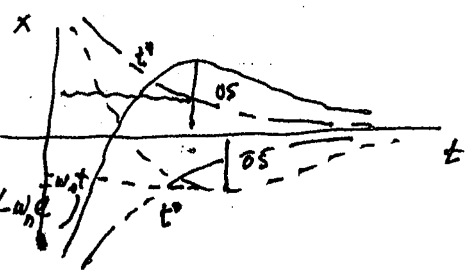
Case II $\quad c^2 - 4mk = 0$ or $c = \sqrt{4mk} = 2\sqrt{mk} = 2\sqrt{m^2 \cdot \frac{k}{m}} = 2m\omega_n$ $\quad$ this value of $c = c_{crit}$

c_{crit} = critical damping constant ; $\quad s_1 = s_2 = -\frac{c}{2m} = -\frac{c_{crit}}{2m} = -\frac{2m\omega_n}{2m} = -\omega_n$

$$\therefore \quad x(t) = C_1 e^{-\omega_n t} + C_2 t e^{-\omega_n t} = (C_1 + C_2 t)e^{-\omega_n t}$$

dying exponentials

solution is APERIODIC



can be any one of these 4 depending on ICs.

$x(t=0) = X_0 = C_1$

$\dot{x}(t) = C_2 e^{-\omega_n t} + (C_1 + C_2 t)(-\omega_n e^{-\omega_n t})$

$\dot{x}(t=0) = C_2 - \omega_n C_1 = V_0$

$\therefore C_2 = V_0 + \omega_n X_0$

$C_2 - \omega_n(C_1 + C_2 t^*) = 0$

or $\quad C_2 - \omega_n C_1 = \omega_n C_2 t^* \qquad t^* = \frac{1}{\omega_n} - \frac{C_1}{C_2}$

$x(t^*) = \text{O.S.} = (C_1 + C_2 t^*)e^{-\omega_n t^*}$

This is called the <u>critically damped case</u>