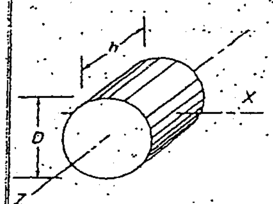
Appendix 2
Modeling of System Elements

COCHIN "ANALYSIS & DESIGN OF DYNAMIC SYSTEMS"

Table 2.1.3-1 MASSES AND MOMENTS OF INERTIA OF RIGID BODIES

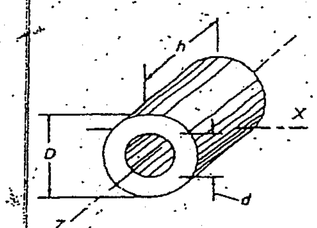


1. Solid disk

$$M = \frac{\rho\pi D^2 h}{4}$$

$$J_z = \frac{\rho\pi D^4 h}{32} = \frac{MD^2}{8}$$

$$J_x = \frac{\rho\pi D^2 h}{16}\left[\frac{h^2}{3} + \frac{D^2}{4}\right] = M\left[\frac{h^2}{12} + \frac{D^2}{16}\right]$$



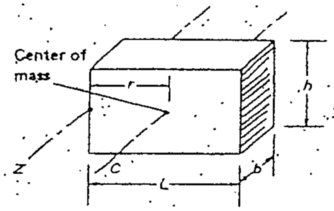
2. Tube

$$M = \frac{\rho\pi h(D^2 - d^2)}{4}$$

$$J_z = \frac{\rho\pi h(D^4 - d^4)}{32} = \frac{M(D^2 + d^2)}{8}$$

$$J_x = \frac{\rho\pi h}{16}(D^2 - d^2)\left[\frac{h^2}{3} + \frac{D^2}{4} + \frac{d^2}{4}\right]$$

$$= M\left[\frac{h^2}{12} + \frac{D^2}{16} + \frac{d^2}{16}\right]$$



3. Rectangular solid

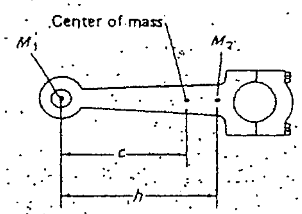
$$M = \rho Lbh$$

$$J_z = \frac{\rho Lbh}{12}(L^2 + h^2 + 12r^2)$$

$$= M\left[\frac{L^2}{12} + \frac{h^2}{12} + r^2\right]$$

$$J_c = \frac{\rho Lbh(L^2 + h^2)}{12}$$

$$= \frac{M[L^2 + h^2]}{12}$$



4. Connecting rod

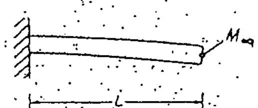
$$M_1 = M\left(1 - \frac{c}{h}\right)$$

$$M_2 = \frac{Mc}{h}$$

$$h = \frac{J_1}{Mc}$$

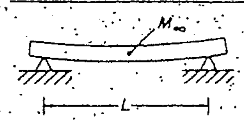
J_1 = moment of inertia about M_1

Table 2.1.4-1 MASSES AND MOMENTS OF INERTIA OF ELASTIC BODIES



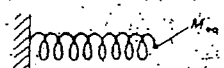
1. Cantilever beam

$$M_{eq} \approx \frac{M}{4}$$



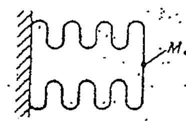
2. Simply supported beam

$$M_{eq} = \tfrac{1}{2}M$$



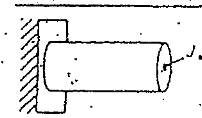
3. Coiled spring

$$M_{eq} = \frac{M}{3}$$



4. Bellows

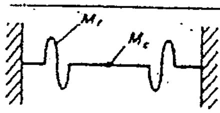
$$M_{eq} = \frac{M}{3}$$



5. Shaft in torsion

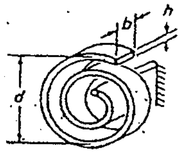
$$J_{eq} = \frac{J}{3}$$

Table 2.1.4-1 (cont.)



6. Diaphragm

$$M_{eq} = M_d + \frac{M_f}{4}$$



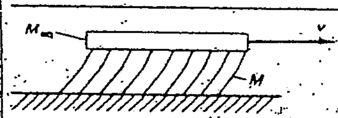
7. Clock spring

$$M = \rho bhL$$

$$J_{eq} \approx \frac{Md^2}{16}$$

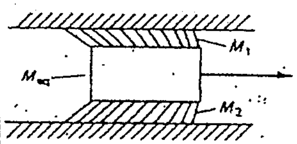
L = total length of spiral

Table 2.1.5-1 FLUID MASSES



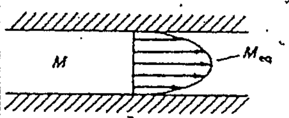
1. Translating surfaces

$$M_{eq} = \frac{M}{3}$$



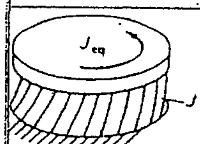
2. Translating body

$$M_{eq} = \frac{M_1 + M_2}{3}$$



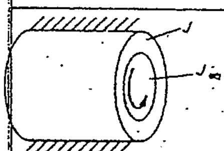
3. Laminar flow in pipe

$$M_{eq} = \frac{4M}{3}$$



4. Rotating parallel surfaces

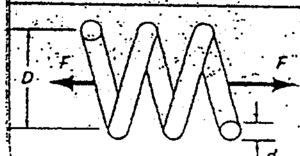
$$J_{eq} = \frac{J}{3}$$



5. Rotating concentric cylinders

$$J_{eq} = \frac{J}{3}$$

Table 2.2.3-1 MECHANICAL SPRINGS (TRANSLATORY)



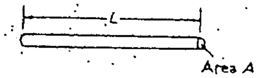
1. Coiled spring (round wire)

$$K_{eq} = \frac{d^4G}{64R^3N}$$

where

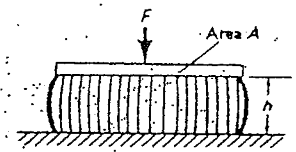
G = modulus in shear

N = number of coils



2. Rod or wire in tension

$$K_{eq} = \frac{EA}{L}$$

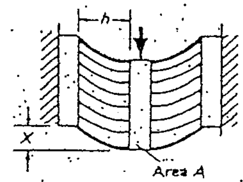


3. Rubber in compression

$$K_{eq} = \frac{1 + F/AE}{h/AE}$$

where

A = area of plate

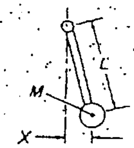


4. Rubber in double shear

$$K_{eq} = \frac{2AG}{h}$$

where

A = area of plate providing shear surface



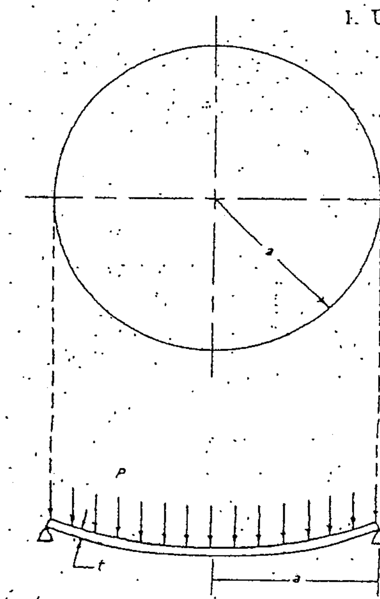
5. Pendulum

$$K_{eq} = \frac{gM}{L}$$

where

g = acceleration due to gravity

Table 2.2.3-2 DISK SPRINGS



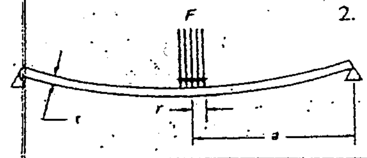
1. Uniform load, edges simply supported

$$K = \frac{64\pi Z(1 + \nu)}{a^2(5 + \nu)}$$

$$\sigma_{max} = \frac{3Pa^2(3 + \nu)}{8t^2}$$

$$Z = \frac{Et^3}{12(1 - \nu^2)}$$

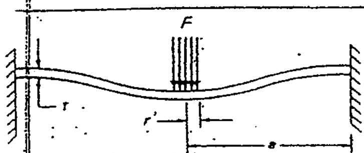
Table 2.2.3-2 (cont.)



2. Concentrated load, edges simply supported

$$K = \frac{16\pi Z(1+\nu)}{a^2(3+\nu)}$$

$$\sigma_{max} = \frac{3F}{2\pi t^2}\left[(1+\nu)\ln\frac{a}{r}+1\right]$$

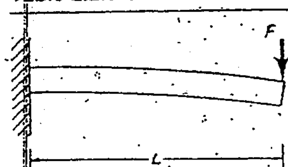


3. Concentrated load, edges clamped

$$K = \frac{16\pi Z}{a^2}$$

$$\sigma_{max} = \frac{3F}{2\pi t^2}(1+\nu)\ln\frac{a}{r}$$

Table 2.2.3-3 EQUIVALENT SPRINGS FOR BEAMS

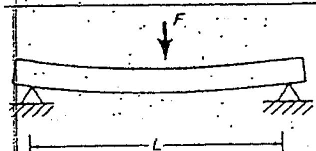


1. Cantilever

$$K_{eq} = \frac{3EI}{L^3}$$

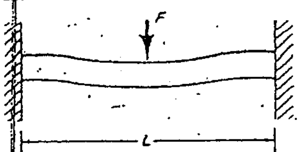
where

$$I = \frac{bh^3}{12}$$



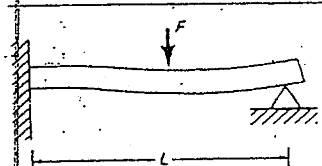
2. Simply supported

$$K_{eq} = \frac{48EI}{L^3}$$



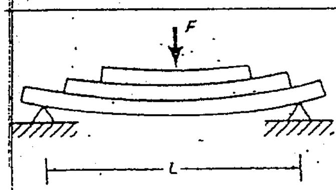
3. Both ends fixed or clamped

$$K_{eq} = \frac{192EI}{L^3}$$



4. Cantilever with pinned end

$$K_{eq} = \frac{110EI}{L^3}$$



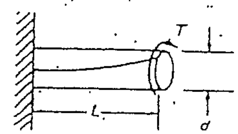
5. Leaf spring

$$K_{eq} = \sum K_i$$

where

K_i = spring constant for each leaf

Table 2.2.3-4 TORSION SPRINGS

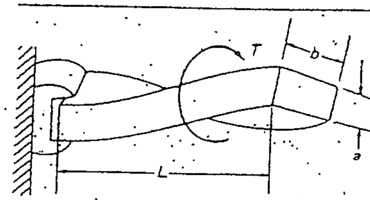


1. Round rod

$$K_{eq} = \frac{\pi d^4 G}{32L}$$

where

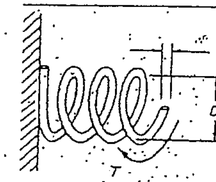
G = modulus in shear



2. Rectangular bar

$$K_{eq} = \frac{C_3 a^3 bG}{L}$$

C_3 = Saint Venant's coefficient



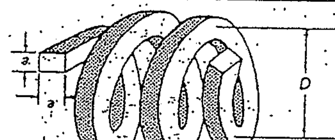
3. Coiled spring in torsion (round wire)

$$K_{eq} = \frac{Ed^4}{64DN}$$

where

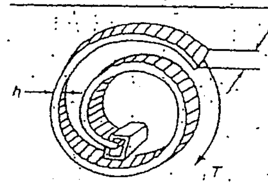
E = Young's modulus

N = number of coils



4. Square wire coiled spring

$$K_{eq} = \frac{Ea^4}{12\pi DN}$$



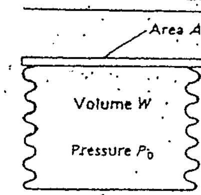
5. Clock spring

$$K_{eq} = \frac{EI}{L}$$

or

$$K_{eq} = \frac{Ebh^3}{12L}$$

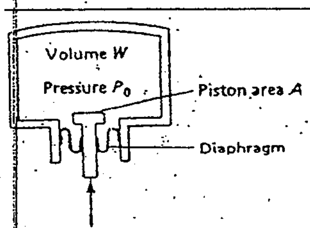
Table 2.2.4-1 FLUID SPRINGS



1. Air spring, bellows

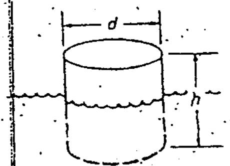
$$K = \frac{A^2 P_0}{W}$$

Table 2.2.4-1 (cont.)



2. Air spring, piston

$$K = \frac{A^2 P_0}{W}$$

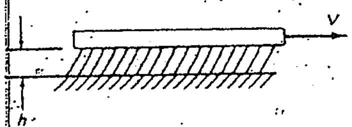


3. Floating cylinder

$$K = \frac{\pi d^2 \rho_f g}{4}$$

ρ_f = mass density of fluid
g = acceleration due to gravity

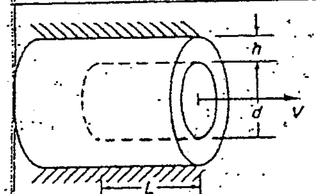
Table 2.3.3-1 FLUID DAMPERS USING VISCOUS SHEAR



1. Translating surfaces

$$D_{eq} = \frac{\mu A}{h}$$

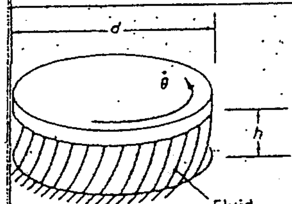
where
A = wetted surface area
μ = coefficient of viscosity (absolute)



2. Translating cylinder

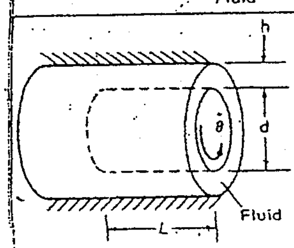
$$D_{eq} = \frac{\mu A}{h}$$

where
A = wetted surface area
$= \pi L d$



3. Rotating disks

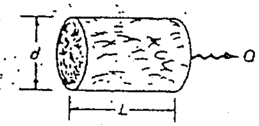
$$D_{eq} = \frac{\pi \mu d^4}{16h}$$



4. Rotating cylinders

$$D_{eq} = \frac{\pi \mu d^3 L}{4h}$$

Table 2.3.4-1 DAMPING DUE TO FLOW THROUGH RESTRICTION

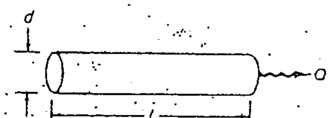


1. Porous material[a]

$$R = \frac{4C_1 L}{\pi d^2 \rho g}$$

C_1 = coefficient of fluid resistance

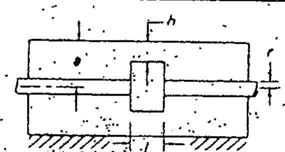
$$D_{eq} = \frac{A^2 4 C_1 L}{\pi d^2}$$



2. Pipe[a]

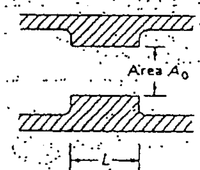
$$R = \frac{128 \mu L}{\pi d^4 \rho g}$$

$$D_{eq} = \frac{A^2 128 \mu L}{\pi d^4}$$



3. Dashpot

$$D_{eq} = \frac{6\pi\mu L}{h^3}\left[\left(a - \frac{h}{2}\right)^2 - r^2\right]\left[\frac{a^2 - r^2}{a - h/2} - h\right]$$



4. Orifice

$$Q = C_d A_0 \sqrt{\frac{2\Delta p}{\rho}}$$

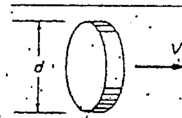
A_0 = area of orifice
C_d = discharge coefficient
ρ = fluid density
Δp = pressure drop through orifice
$N_r = \frac{\rho Q d}{A_0 \mu}$ Reynolds #

[a] A = area of piston in Fig. 2.3.4-1. μ = absolute (dynamic) viscosity. See Appendix 6.

Table 2.3.5-1 DAMPING DUE TO DRAG IN A VISCOUS MEDIUM

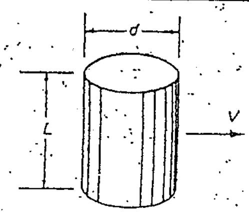
1. Sphere[a]

$$D_{eq} = 3\pi\mu d$$



2. Disk[a]

$$D_{eq} = 30\mu d$$



3. Cylinder[a]

$$D_{eq} = 3\mu L$$