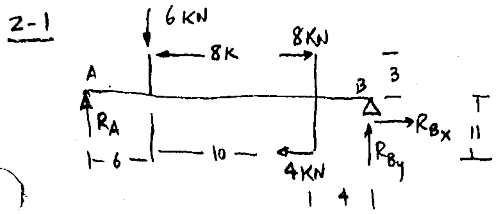
[redacted]

2-1



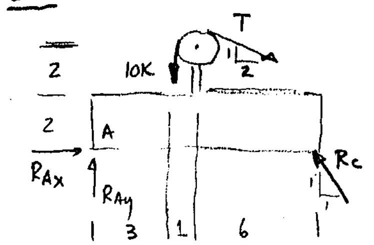
$\Sigma F_x = 0 \Rightarrow -4+8-8+R_{Bx} = 0 \qquad R_{Bx} = 4\text{ KN} \rightarrow$

$\Sigma F_y = 0 \Rightarrow R_A - 6 + R_{By} = 0$

$\Sigma M_B \curvearrowleft^{+} = 0 \Rightarrow -8\times3 - 4\times11 - R_A\times20 + 6\times14 + 8\times3 = 0 \qquad R_A = 2\text{ KN}\uparrow$

$R_{By} = 4\text{ KN}\uparrow$

2-2.



$\Sigma F_x = 0 \Rightarrow R_{Ax} - R_{cx} + T_x = 0$

$\Sigma F_y = 0 \Rightarrow R_{Ay} + R_{By} - 10 - T_y = 0$

$+\curvearrowleft \Sigma M_A = 0 = -10(3) - T\cdot1 - T_x\cdot4 - T_y\cdot4 + \frac{R_c}{\sqrt2}(10)$

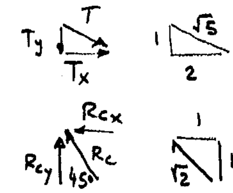
$R_{cx} = R_c/\sqrt2$

$T_x = 2T/\sqrt5$

$R_{cy} = R_c/\sqrt2$

$T_y = T/\sqrt5$

$T = 10\text{ KN}$



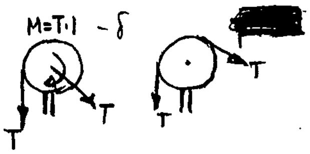
$R_{Ax} = 0.42\text{ kN} \rightarrow$

$R_{Ay} = 5.11\text{ KN}\uparrow$

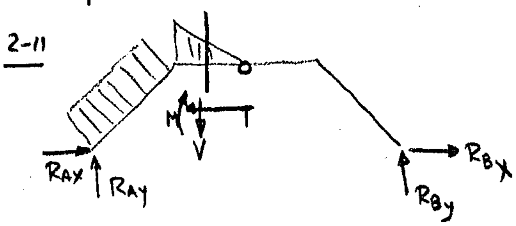
$R_c = 13.25\text{ KN}$

$M = T\cdot1 - \delta$

[redacted]



2-11



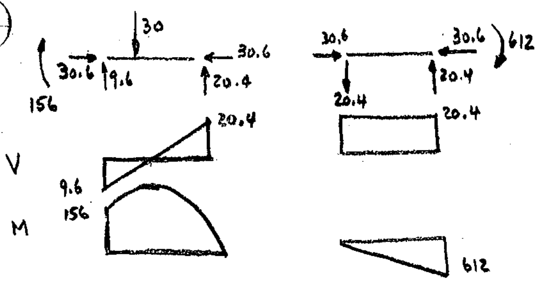
[redacted]

$R_{Bx} = 30.6\text{ KN}\leftarrow \qquad R_{Ay} = 39.6\text{ KN}$

$R_{Ax} = 9.4\text{ kN}\leftarrow \qquad R_{By} = 20.4\text{ KN}$

$M = 135\text{ KN-m}\ \curvearrowright$

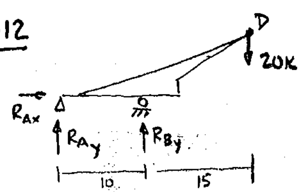
$T = 30.6\text{ KN}\rightarrow \qquad V = 12.9\text{ KN}\downarrow$



M = 156

V & M for horizontal section to left & right of hinge

2-12

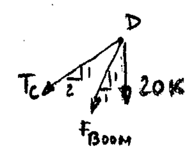


$\Sigma F_x = R_{Ax} = 0$

$\Sigma F_y = R_{Ay} + R_{By} - 20\text{ KN} = 0$

$+\curvearrowleft \Sigma M_A = 0 = -R_{By}\cdot10 + 20\cdot25 = 0 \qquad R_{By} = 50\text{ KN} \qquad R_{Ay} = -30\text{ KN}$

[redacted]

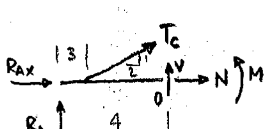


$\Sigma F_x = 0 = -T_c\cdot\frac{2}{\sqrt5} - F_{Boom}\cdot\frac{1}{\sqrt2} = 0$

$\Sigma F_y = 0 = -T_c\cdot\frac{1}{\sqrt5} - F_{Boom}\frac{1}{\sqrt2} - 20\text{ KN}$

$T_c = 20\sqrt5\text{ KN} = 44.72\text{ KN}$

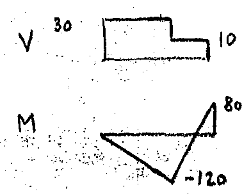
$F_{Boom} = -56.57\text{ KN}$



$\Sigma F_x = 0 \Rightarrow N + R_{Ax} + T_c\cdot\frac{2}{\sqrt5} = 0 \qquad N = -40\text{ KN}$

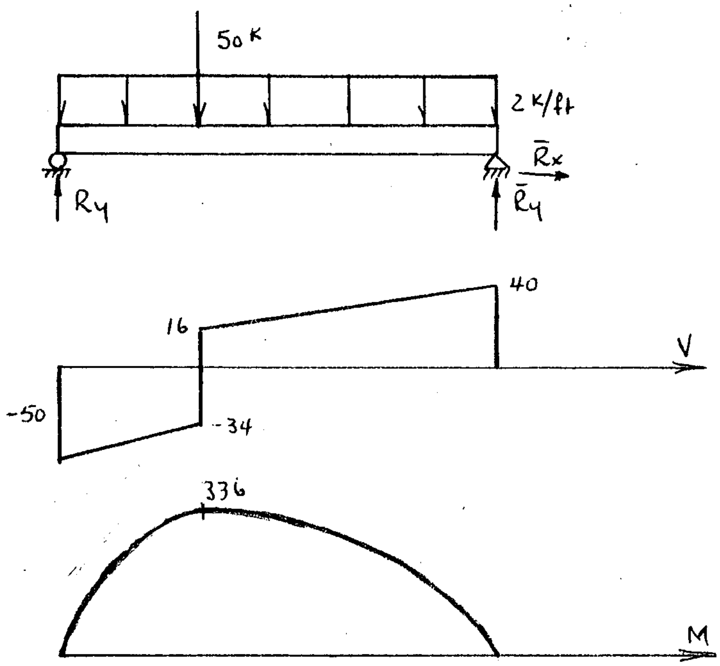
$\Sigma F_y = 0 \Rightarrow V + T_c\cdot\frac{1}{\sqrt5} + R_{Ay} = 0 \qquad V = 10\text{ KN}$

$+\curvearrowleft \Sigma M_0 = 0 \Rightarrow -M + R_{Ay}\cdot4 + T_c\cdot\frac{1}{\sqrt5}\cdot1 + T_c\cdot\frac{2}{\sqrt5}\cdot0.5 = 0 \qquad M = -80\text{ KN}\cdot\text{m or } 80\text{ KN}\cdot\text{m}\curvearrowright$



V & M from A to a-a

2-38



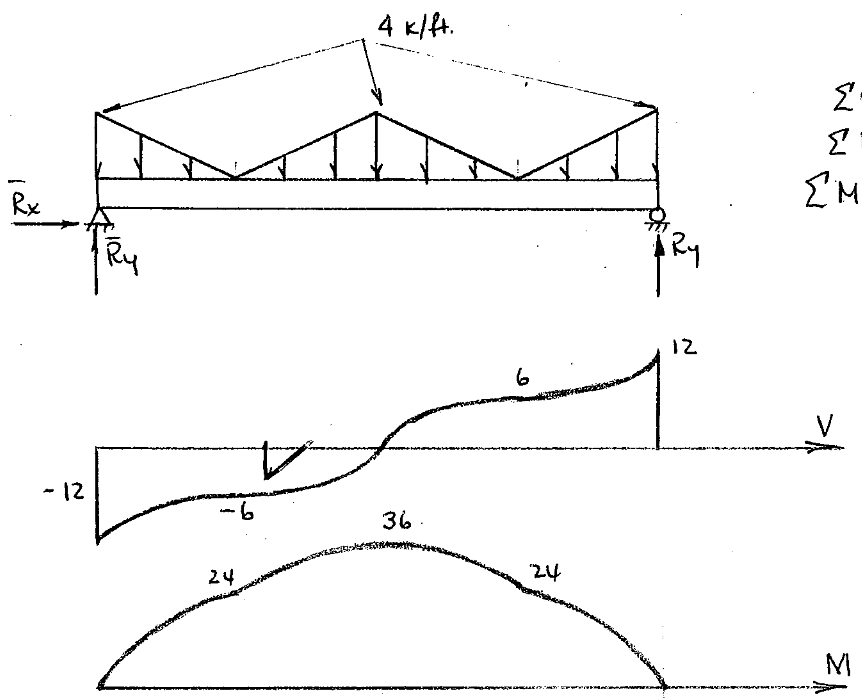
$\Sigma F_x = 0 \qquad \bar{R}_x = 0$

$\Sigma F_y = 0 \qquad 2(20) + 50 = R_y + \bar{R}_y$

$\Sigma M_y \curvearrowleft = 0 \qquad \bar{R}_y(20) - 50(8) - 40(10) = 0$

$\bar{R}_y = 40\text{K} \quad R_y = 50\text{K}$

2-42



$\Sigma F_x = 0 \qquad \bar{R}_x = 0$

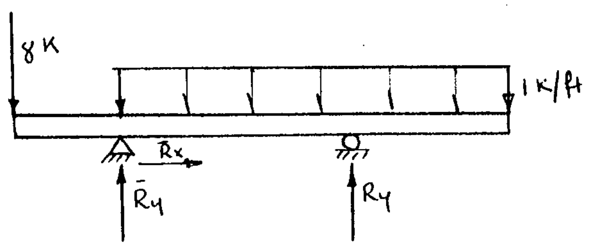
$\Sigma F_y = 0 \qquad R_y + \bar{R}_y = 4[4 \cdot 3 \cdot \tfrac{1}{2}]$

$\Sigma M_{\bar{y}} \curvearrowright = 0 \qquad 6 \cdot 1 + 6 \cdot 5 + 6 \cdot 7 + 6 \cdot 11 = R_y(12)$

$R_y = 12\text{K} \quad \bar{R}_y = 12\text{K}$

C. LEVY

2-47

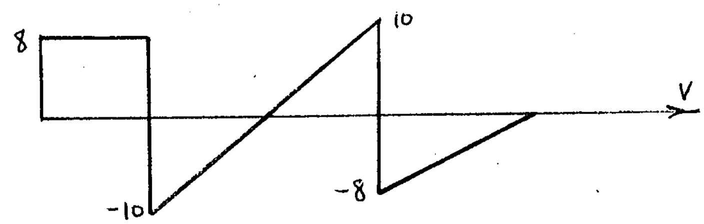


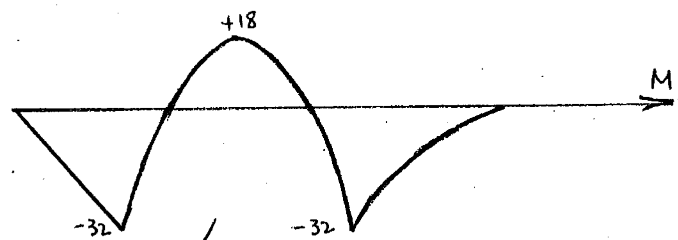
$\Sigma F_x = 0 \quad \bar{R}_x = 0$

$\Sigma F_y = 0 \quad R_y + \bar{R}_y = 36$

$\Sigma M_{\bar{y}} = 0 \quad 32 + R_y(20) - 28(14) = 0$

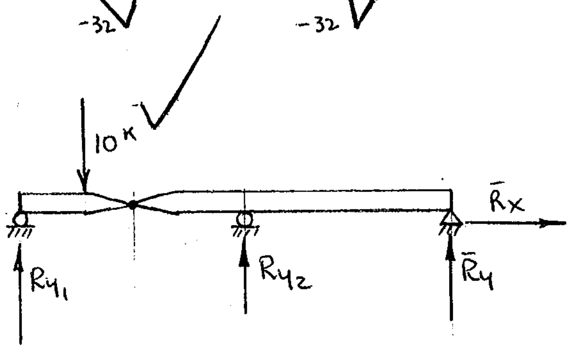
$R_y = 18\,K \quad \bar{R}_y = 18\,K$





Note M max when V = 0

2-50



$\Sigma F_x = 0 \quad \bar{R}_x = 0$

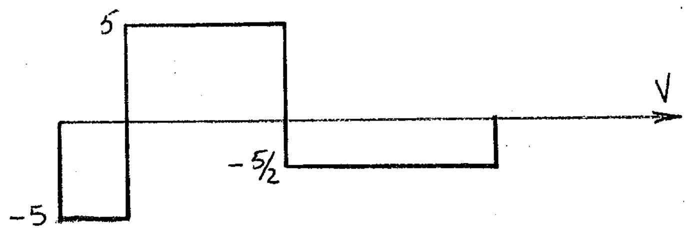
① $\Sigma F_y = 0 \quad R_{y_1} + V = 10$

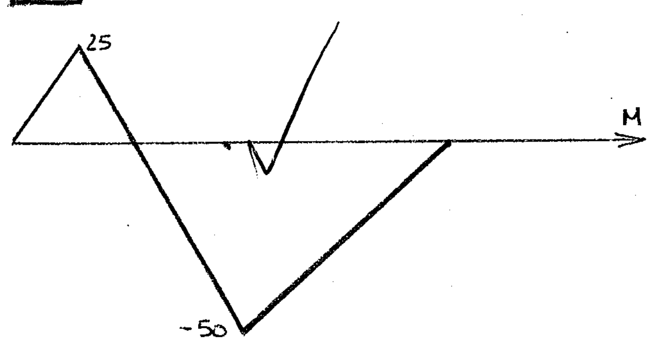
② $\Sigma F_y = 0 \quad R_{y_2} + \bar{R}_y - V = 0$

① $\Sigma M_{y_1} = 0 \quad V(10) = 5(10) \quad V = 5\,K \quad \boxed{R_{y_1} = 5\,K}$

② $\Sigma M_{y_2} = 0 \quad V(10) + \bar{R}_y(20) = 0 \quad \boxed{\bar{R}_y = -2\tfrac{1}{2}\,K}$

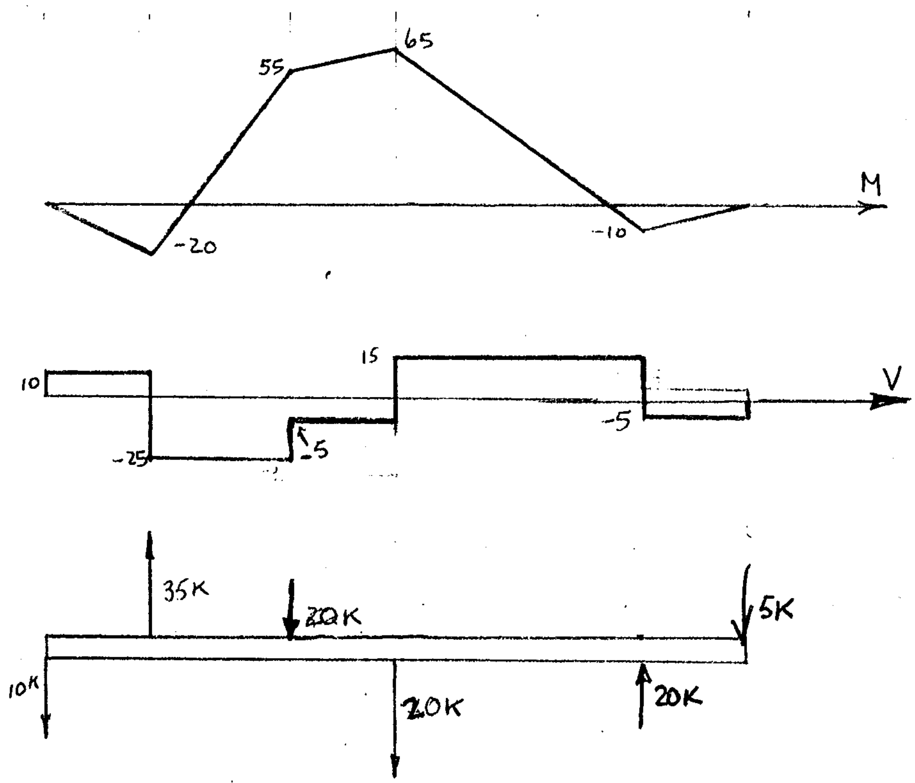
$\boxed{R_{y_2} = 7\tfrac{1}{2}\,K}$



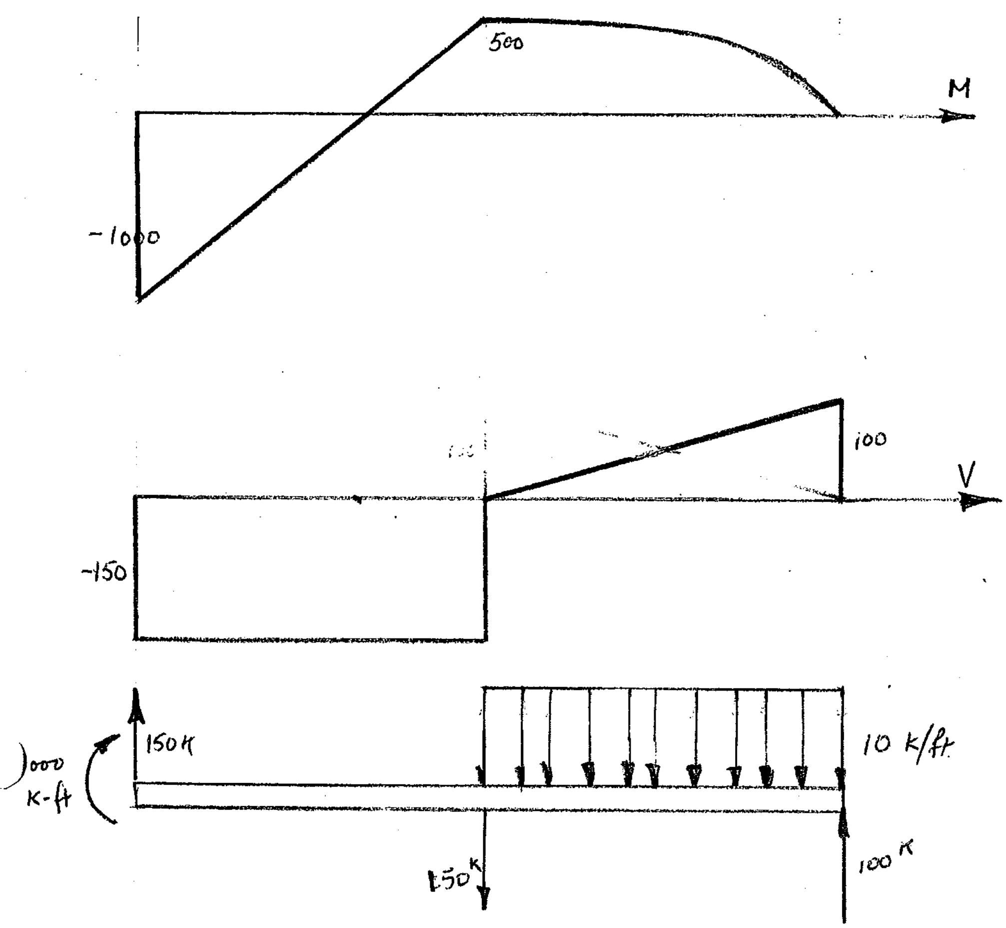


Note M=0 at hinge

2-54

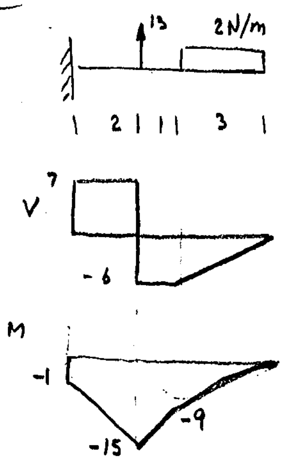

65
55
M
-20
-10
15
V
10
-25
-5
-5
35K
20K
5K
10K
20K
20K

2-56

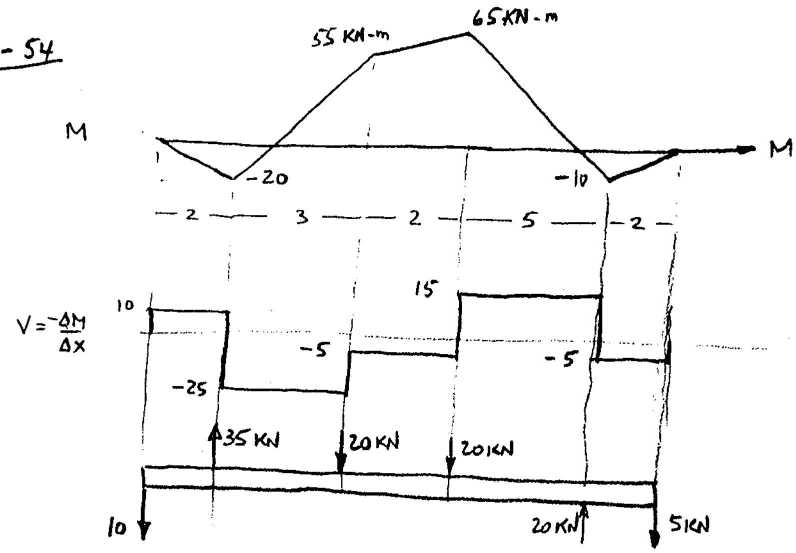

500
M
-1000
100
V
-150
150K
10 k/ft
1000 k-ft
150K
100K

[redacted]

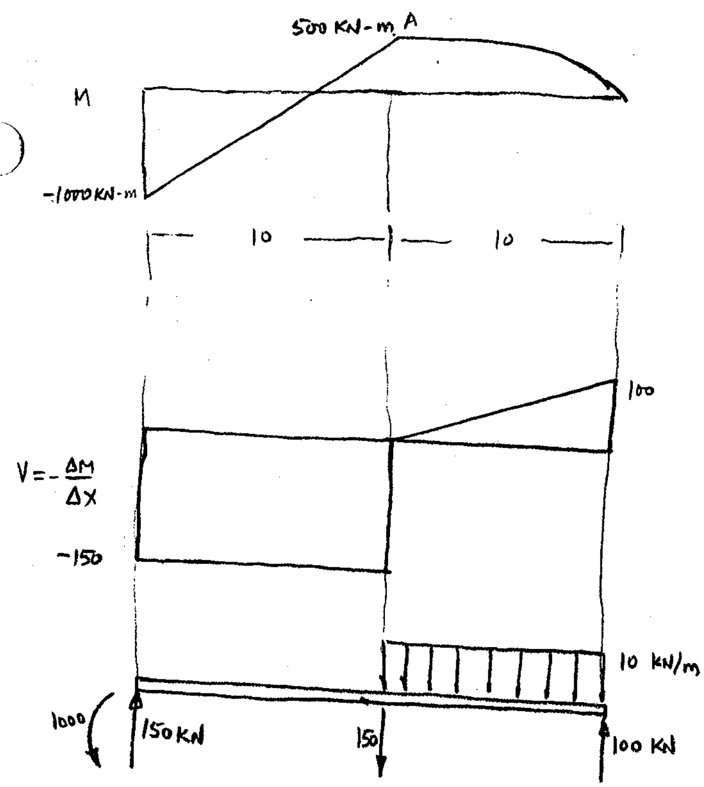
2-39



2-54



2-56

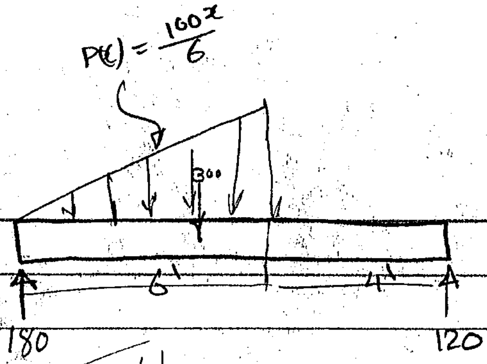


[redacted]

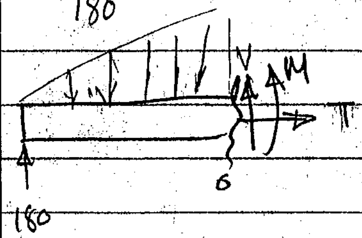
AT A: $\frac{dM}{dx}=0$. But since $-V=\frac{dM}{dx} \Rightarrow V=0$ there

Since $\frac{dM}{dx}$ is <0 for pts. beyond A then $V>0$.

If M is quadratic ie $\sim x^2$ then V is linear & p(x) is constant



$0 \le x \le 6$



$\Sigma F_x = 0 \qquad T = 0$

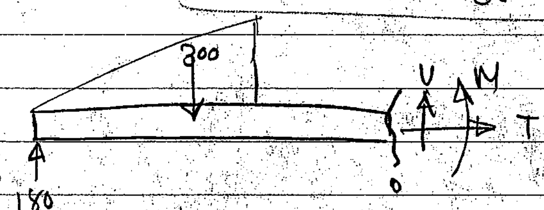
$\Sigma F_y = 0 \qquad V + 180 - \frac{100x}{6} = 0 \qquad V = \int_0^x \frac{100x}{6} - 180$

$$\boxed{V = \frac{100x^2}{12} - 180}$$

$\Sigma M_0 = 0 \qquad M = -\int_0^x \frac{100x^2}{12} - 180$

$$\boxed{M = -\left(\frac{100x^3}{36} - 180x\right)}$$

$6 \le x \le 10$

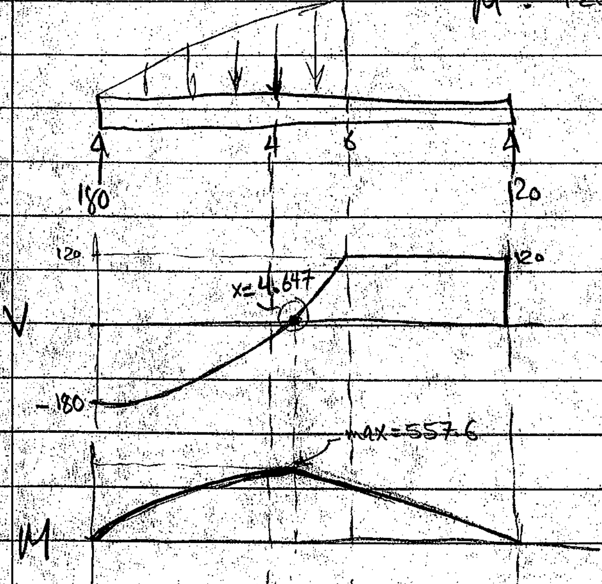


$\Sigma F_x = 0 \qquad T = 0$

$\Sigma F_y = 0 \qquad V + 180 - 300 = 0 \qquad \boxed{V = 120 \text{ lb}}$

$\Sigma M_0 = 0 \qquad M + 300(x-4) - 180x = 0$

$M = 1200 - 120x$



$V = 0 \Rightarrow \frac{100x^2}{12} - 180 = 0$

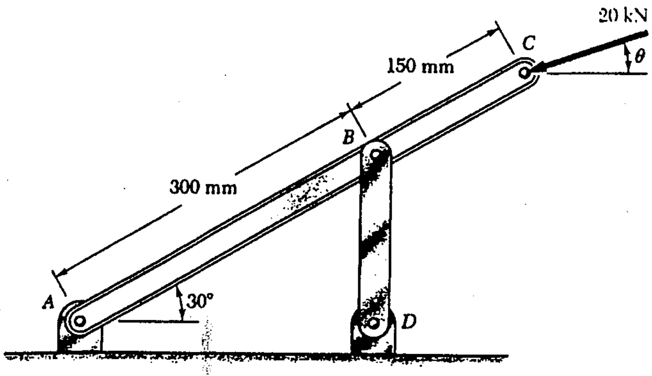
$x^2 = 21.6 \qquad x = 4.65 \text{ ft}$

$M\big|_{x=4.65} = \left(180x - \frac{100x^3}{36}\right) = 557.6 \text{ lb-ft}$

HW #2

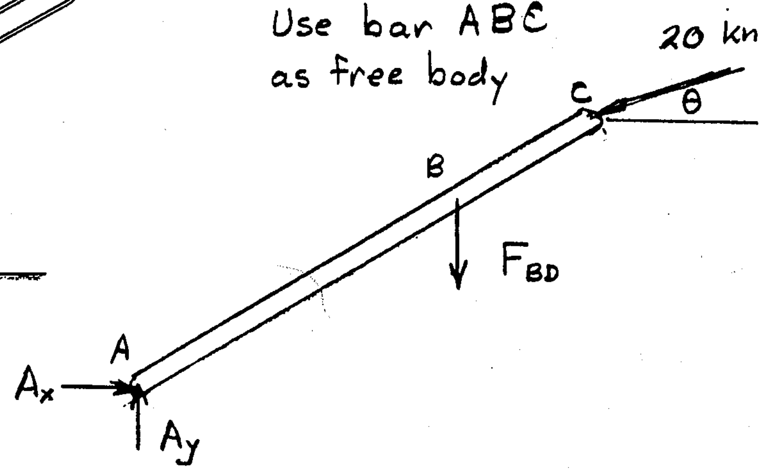
PROBLEM 1.7

1.7 Link BD consists of a single bar 30 mm wide and 12 mm thick. Knowing that each pin has a 10-mm diameter, determine the maximum value of the average normal stress in link BD if (*a*) $\theta = 0$, (*b*) $\theta = 90$



SOLUTION

Use bar ABC as free body



$\Sigma M_A = 0$

(a) $\theta = 0°$ $\quad (0.450 \sin 30°)(20\times10^3) - (0.300 \cos 30°) F_{BD} = 0$

$F_{BD} = 17.32 \times 10^3$ N

(b) $\theta = 90°$ $\quad (0.450 \cos 30°)(20\times10^3) - (0.300 \cos 30°) F_B = 0$

$F_{BD} = -30 \times 10^3$ N

Areas

(a) tension loading $\quad A = (0.030 - 0.010)(0.012) = 240 \times 10^{-6}$ m^2

(b) compression $\quad A = (0.030)(0.012) = 360 \times 10^{-6}$ m^2

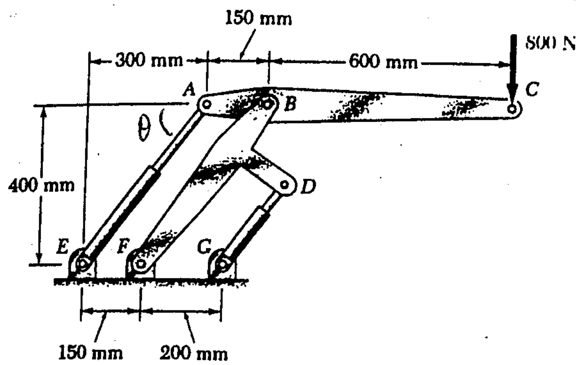
Stresses

(a) $\sigma = \dfrac{F_{BC}}{A} = \dfrac{17.32\times10^3}{240\times10^{-6}} = 72.2\times10^6$ $\quad$ 72.2 MPa

(b) $\sigma = \dfrac{F_{BC}}{A} = \dfrac{-30\times10^3}{360\times10^{-6}} = -83.3\times10^6$ $\quad$ - 83.3 MPa

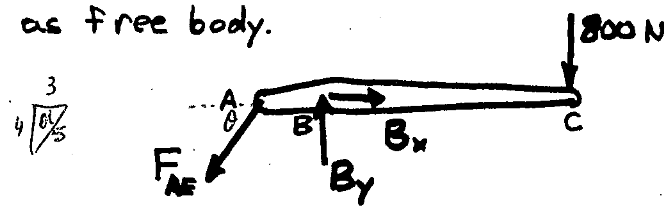
PROBLEM 1.14

1.14 Two hydraulic cylinders are used to control the position of the robotic arm ABC. Knowing that the control rods attached at A and D each have a 20-mm diameter and happen to be parallel in the position shown, determine the average normal stress in (a) member AE, (b) member DG.



SOLUTION

Use member ABC as free body.

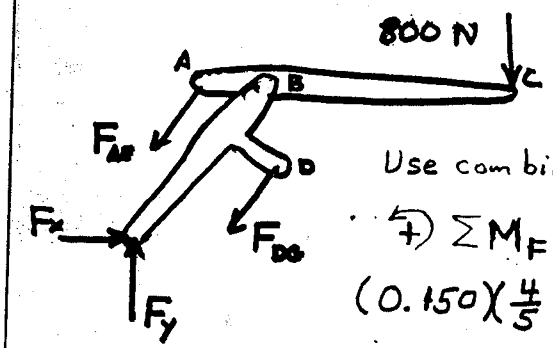


$\circlearrowleft + \; \Sigma M_B = 0 \qquad (0.150)\frac{4}{5}F_{AE} - (0.600)(800) = 0 \qquad F_{AE} = 4\times10^3$ N

Area of rod in member AE is $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20\times10^{-3})^2 = 314.16\times10^{-6}$ m^2

Stress in rod AE: $\sigma_{AE} = \frac{F_{AE}}{A} = \frac{4\times10^3}{314.16\times10^{-6}} = 12.73\times10^6$ Pa

(a) $\sigma_{AE} = 12.73$ MPa ◄



Use combined members ABC and BFD as free body.

$\circlearrowleft + \; \Sigma M_F = 0$

$(0.150)(\frac{4}{5}F_{AE}) - (0.200)(\frac{4}{5}F_{DG})$

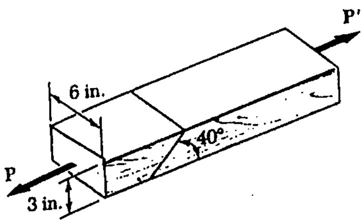
$- (1.050 - 0.350)(800) = 0 \qquad F_{DG} = -1500$ N

Area in rod DG is $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20\times10^{-3})^2 = 314.16\times10^{-6}$ m^2

Stress in rod DG: $\sigma_{DG} = \frac{F_{DG}}{A} = \frac{-1500}{3.1416\times10^{-6}} = -4.77\times10^6$ Pa

(b) $\sigma_{DG} = -4.77$ MPa ◄

PROBLEM 1.31



1.31 Two wooden members of 3 × 6- in. uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 90 psi, determine (*a*) the largest load **P** which can be safely applied, (*b*) the corresponding tensile stress in the splice.

$\tau_{max} = 90$

SOLUTION

$\theta = 90° - 40° = 50°$

$A_o = (3)(6) = 18 \text{ in}^2$

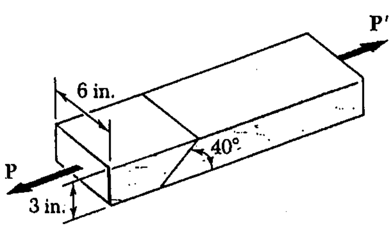
$\tau = \frac{P}{2A} \sin 2\theta$

$P = \frac{2A\tau}{\sin 2\theta} = \frac{(2)(18)(90)}{\sin 100°} = 3290$

(a) $P = 3290$ lb.

(b) $\sigma = \frac{P \cos^2\theta}{A_o} = \frac{3290 \cos^2 50°}{18} = 75.5$ $\quad \sigma = 75.5$ psi

PROBLEM 1.32



1.32 Two wooden members of 3 × 6- in. uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 2400$ lb, determine the normal and shearing stresses in the glued splice.

SOLUTION

$\theta = 90° - 40° = 50°$ $\quad P = 2400$ lb.

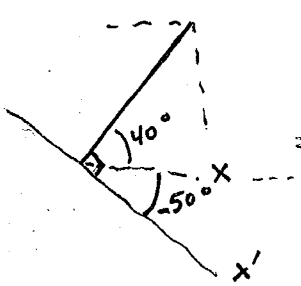
$A_o = (3)(6) = 18 \text{ in}^2$

$\sigma = \frac{P}{A_o}\cos^2\theta = \frac{(2400)\cos^2 50°}{18} = 55.1$

$\sigma = 55.1$ psi

$\tau = \frac{P}{2A}\sin 2\theta = \frac{(2400)\sin 100°}{(2)(18)} = 65.7$

$\tau = 65.7$ psi



draw x' ⊥ to surface

or

$\theta = -50°$ $\quad \sigma_x' = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$

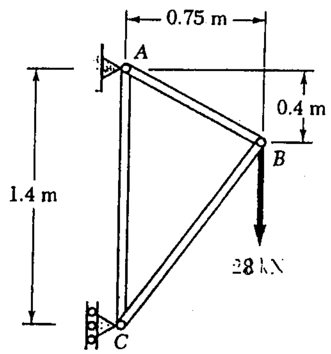
here $\sigma_y = 0 \quad \tau_{xy} = 0$

$\sigma_x' = \frac{\sigma_x}{2} + \frac{\sigma_x}{2}\cos(-110) = \sigma_x \cos^2(-50) = 55.1$ psi

$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$

$= -\frac{P}{2A}\sin(-110) = 65.7$ psi

PROBLEM 1.41



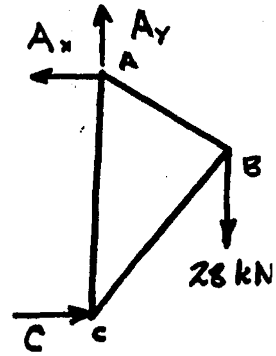
1.41 Members *AB* and *AC* of the truss shown consist of bars of square cross sectic made of the same alloy. It is known that a 20- mm- square bar of the same alloy w tested to failure and that an ultimate load of 120 kN was recorded. If bar *AB* has a 1 mm- square cross section, determine (*a*) the factor of safety for bar *AB*, (*b*) tl dimensions of the cross section of bar *AC* if it is to have the same factor of safety as b *AB*.

SOLUTION

Length of member AB

$$l_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

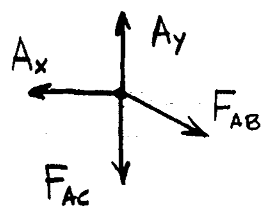
Use entire truss as a free body



$\circlearrowleft \Sigma M_c = 0 \quad 1.4 A_x - (0.75)(28) = 0 \quad A_x = 15 \text{ kN}$

$+\uparrow \Sigma F_y = 0 \quad A_y - 28 = 0 \quad A_y = 28 \text{ kN}$

Use joint A as free body



$\xrightarrow{+} \Sigma F_x = 0 \quad \frac{0.75}{0.85} F_{AB} - A_x = 0$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$+\uparrow \Sigma F_y = 0 \quad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

Need to find stress at failure, σ_u

For the test bar $\quad A = (0.020)^2 = 400 \times 10^{-6} \text{ m}^2 \quad P_U = 120 \times 10^3 \text{ N}$

For the material $\quad \sigma_U = \frac{P_U}{A} = \frac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

(a) For bar AB $\quad F.S. = \frac{P_U}{F_{AB}} = \frac{\sigma_U A}{F_{AB}} = \frac{(300 \times 10^6)(0.015)^2}{17 \times 10^3}$

$$= 3.97$$

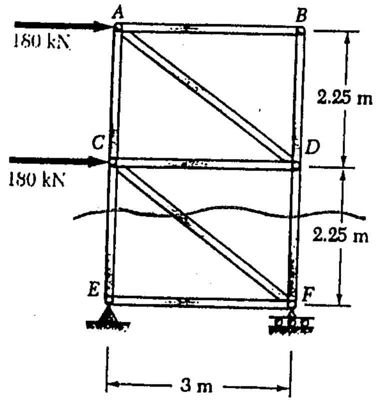
(b) For bar AC $\quad F.S. = \frac{P_U}{F_{AC}} = \frac{\sigma_U A}{F_{AC}} = \frac{\sigma_U a^2}{F_{AC}}$

$$a^2 = \frac{(F.S.) F_{AC}}{\sigma_U} = \frac{(3.97)(20 \times 10^3)}{300 \times 10^6} = 264.7 \times 10^{-6} \text{ m}^2$$

$$a = 16.27 \times 10^{-3} \text{ m} \qquad 16.27 \text{ mm}$$

PROBLEM 1.59

1.59 For the truss and loading shown, determine the average normal stress member DF, knowing that the cross-sectional area of that member is 2500 mm².

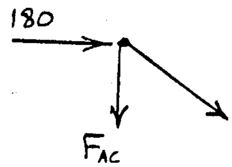


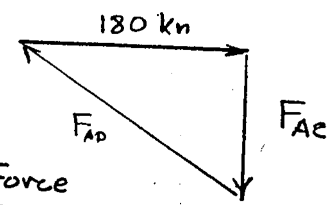
SOLUTION

Using method of joints to find member forces

Joint B : AB and BD are zero force member.

Joint A : $\ell_{AD} = \sqrt{3^2 + 2.25^2} = 3.75$ m





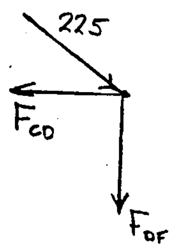
Force Trangle

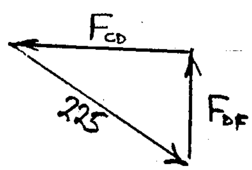
By similar triangles

$$\frac{F_{AD}}{3.75} = \frac{180}{3} \quad \therefore \quad F_{AD} = 225 \text{ lb.}$$

(compression)

Joint D





By similar triangles

$$\frac{F_{DE}}{2.25} = \frac{225}{3.75}$$

$F_{DF} = 135$ kN (comp)

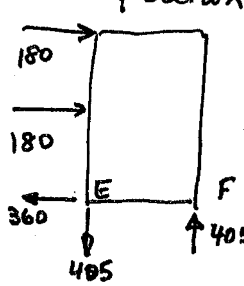
$= 135 \times 10^3$ N

Area : $A_{DE} = 2500 \text{ mm}^2 = 2500 \times 10^{-6} \text{ m}^2$

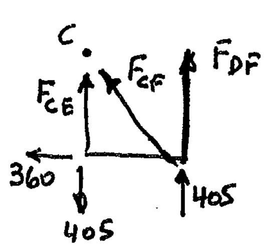
Stress : $\sigma_{DF} = -\dfrac{135 \times 10^3}{2500 \times 10^{-6}} = -54 \times 10^6 \text{ Pa} = -54.0 \text{ MPa}$

or

Method of sections



① Find support reactions



② Cut along red line & take moments about C

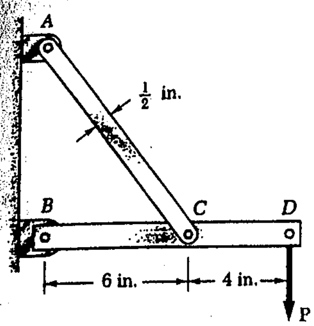
$+\circlearrowleft \Sigma M_C = 0 \quad -360(2.25) + (405 + F_{DF}) \cdot 3 = 0$

$F_{DF} = -135$ kN

rest is same as above

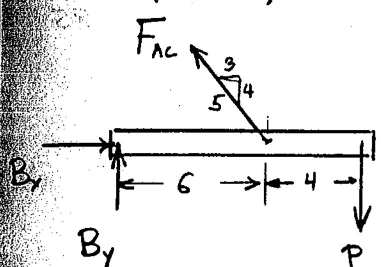
$\sigma_{DF} = \dfrac{F_{DF}}{A} = -54$ MPa

PROBLEM 1.68



1.68 Link AC has a uniform $\frac{1}{4} \times \frac{1}{2}$- in. uniform rectangular cross section and is made of a steel with a 60-ksi ultimate normal stress. It is connected to a support at A and to member BCD at C by $\frac{3}{8}$-in.-diameter pins, while member BCD is connected to a support at B by a $\frac{5}{16}$-in.-diameter pin. All of the pins are in single shear and are made of a steel with a 25-ksi ultimate shearing stress. Knowing that an overall factor of safety of 3.25 is desired, determine the largest load **P** which can be safely applied at D. Note that link AC is not reinforced around the pin holes.

SOLUTION



$\circlearrowleft \Sigma M_B = 0 \qquad (6)(\frac{3}{5} F_{AC}) - 10\ P = 0$

$F_{AC} = 2.0833\ P \qquad P = 0.480\ F_{AC}$

$\xrightarrow{+} \Sigma F_x = 0 \qquad B_x - \frac{3}{5} F_{AC} = 0$

$B_x = \frac{3}{5} F_{AC} = (\frac{3}{5})(2.0833 P) = 1.25\ P$

$+\uparrow \Sigma F_y = 0 \qquad B_y + \frac{4}{5} F_{AC} - P = 0$

$B_y = P - \frac{4}{5}(2.0833 P) = -0.66667\ P$

$B = \sqrt{B_x^2 + B_y^2} = 1.41667\ P, \qquad P = 0.70588\ B$

Based on strength of link AC: $\sigma_U = 60$ ksi

$A_{net} = (\frac{1}{4})(\frac{1}{2} - \frac{3}{8}) = 0.03125\ in^2; \quad F_{AC,U} = \sigma_U A_{net} = (60)(0.03125) = 1.875$ kip.

$P_U = (0.480)(1.875) = 0.900$ kip.

Based on strength of pin at C: $A_{pin} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(\frac{3}{8})^2 = 0.11045\ in^2$

$\tau_U = 25$ ksi $\qquad F_{AC,U} = \tau_U A_{pin} = (25)(0.11045) = 2.761$ kip.

$P_U = (0.480)(2.761) = 1.325$ kip.

Based on strength of pin at B: $A_{pin} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(\frac{5}{16})^2 = 0.07670\ in^2$

$B_U = \tau_U A_{pin} = (25)(0.07670) = 1.9175$ kip.

$P_U = (0.70588)(1.9175) = 1.3535$ kip

Actual P_U is the smallest $\therefore \quad P_U = 0.900$ kip.

Allowable value for P: $\quad P = \frac{P_U}{FS} = \frac{0.900}{3.25} = 0.277$ kip $= 277$ lb. ◄

3, 2, 2, 2, 1

PROBLEM 2.1

load is applied to it. Knowing that $E = 200$ GPa, determine (*a*) the smallest diameter rod which should be used, (*b*) the corresponding normal stress caused by the load.

HW #4

SOLUTION

(a) $\delta = \dfrac{PL}{AE} \quad \therefore \quad A = \dfrac{PL}{E\delta} = \dfrac{(8.5\times10^3)(2.2)}{(200\times10^9)(1.2\times10^{-3})} = 77.92\times10^{-6}\ m^2$

$A = \frac{\pi}{4}d^2 \quad \therefore \quad d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(77.92\times10^{-6})}{\pi}} = 9.96\times10^{-3}\ m$

$= 9.96\ mm$ ◄

(b) $\sigma = \dfrac{P}{A} = \dfrac{8.5\times10^3}{77.92\times10^{-6}} = 109.1\times10^6\ Pa = 109.1\ MPa$ ◄

PROBLEM 2.8

2.8 A cast-iron tube is used to support a compressive load. Knowing that $E = 10 \times 10^6$ psi and that the maximum allowable change in length is 0.025 percent, determine (*a*) the maximum normal stress in the tube, (*b*) the minimum wall thickness for a load of 1600 lb if the outside diameter of the tube is 2.0 in.

SOLUTION

(a) $\dfrac{\delta}{L} = \dfrac{0.025}{100} = 0.00025$

$\sigma = \dfrac{E\delta}{L} = (10\times10^6)(0.00025) = 2.5\times10^3\ psi = 2.5\ ksi$ ◄

(b) $\sigma = \dfrac{P}{A} \quad \therefore \quad A = \dfrac{P}{\sigma} = \dfrac{1600}{2.5\times10^3} = 0.640\ in^2$

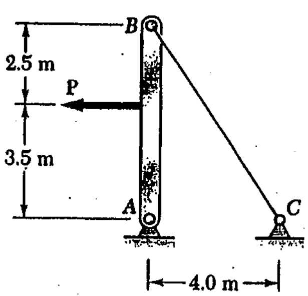
$A = \frac{\pi}{4}(d_o^2 - d_i^2)$

$d_i^2 = d_o^2 - \dfrac{4A}{\pi} = 2.0^2 - \dfrac{(4)(0.64)}{\pi} = 3.1851\ in^2 \quad \therefore \quad d_i = 1.7847\ in.$

$t = \frac{1}{2}(d_o - d_i) = \frac{1}{2}(2.0 - 1.7847) = 0.1077\ in.$ ◄

PROBLEM 2.11

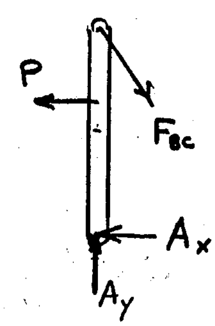
2.11 The 4-mm-diameter cable *BC* is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown.



SOLUTION

$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111\ m$

Use bar AB as a free body



$\circlearrowleft \Sigma M_A = 0 \qquad 3.5P - (6)\left(\frac{4}{7.2111}F_{BC}\right) = 0$

$P = 0.9509\ F_{BC}$

Considering allowable stress $\sigma = 190\times10^6\ Pa$

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.004)^2 = 12.566\times10^{-6}\ m^2$

$\sigma = \dfrac{F_{BC}}{A} \quad \therefore \quad F_{BC} = \sigma A = (190\times10^6)(12.566\times10^{-6}) = 2.388\times10^3\ N$

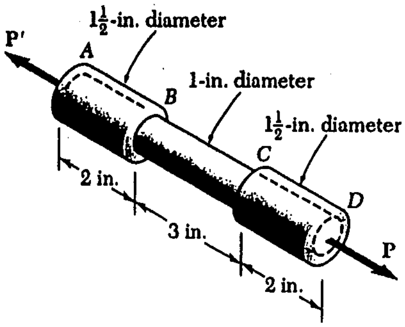
Considering allowable elongation $\delta = 6\times10^{-3}\ m$

$\delta = \dfrac{F_{BC}L_{BC}}{AE} \quad \therefore \quad F_{BC} = \dfrac{AE\delta}{L_{BC}} = \dfrac{(12.566\times10^{-6})(200\times10^9)(6\times10^{-3})}{7.2111} = 2.091\times10^3\ N$

Smaller value governs, i.e., $F_{BC} = 2.091\times10^3\ N$

$P = 0.9509\ F_{BC} = (0.9509)(2.091\times10^3) = 1.988\times10^3\ N = 1.988\ kN$ ◄

PROBLEM 2.15



2.15 The specimen shown is made from a 1-in.-diameter cylindrical steel rod with two 1.5-in.-outer-diameter sleeves bonded to the rod as shown. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the load **P** so that the total deformation is 0.002 in., (*b*) the corresponding deformation of the central portion *BC*.

SOLUTION

(a) $\delta = \sum \frac{P_i L_i}{A_i E_i} = \frac{P}{E} \sum \frac{L_i}{A_i}$

$P = E\delta \left(\sum \frac{L_i}{A_i} \right)^{-1}$ $\qquad A_i = \frac{\pi}{4} d_i^2$

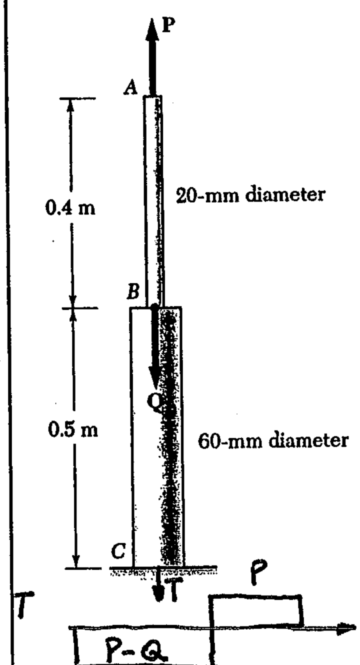
	L, in.	d, in	A_i, in^2	L/A, in^{-1}
AB	2	1.5	1.7671	1.1318
BC	3	1.0	0.7854	3.8197
CD	2	1.5	1.7671	1.1318
				6.083 ← sum

$P = (29\times10^6)(0.002)(6.083)^{-1} = 9.535 \times 10^3$ lb. $= 9.53$ kips

(b) $\delta_{BC} = \frac{P L_{BC}}{A_{BC} E} = \frac{P}{E}\frac{L_{BC}}{A_{BC}} = \frac{9.535\times10^3}{29\times10^6}(3.8197) = 1.254\times10^{-3}$ in

PROBLEM 2.16

2.16 Both portions of the rod *ABC* are made of an aluminum for which $E = 70$ GPa. Knowing that the magnitude of **P** is 4 kN, determine (*a*) the value of **Q** so that the deflection at *A* is zero, (*b*) the corresponding deflection of *B*.



SOLUTION

(a) $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16\times10^{-6}$ m^2

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274\times10^{-3}$ m^2

Force in member AB is P tension

Elongation $\delta_{AB} = \frac{P L_{AB}}{E A_{AB}} = \frac{(4\times10^3)(0.4)}{(70\times10^9)(314.16\times10^{-6})}$

$= 72.756\times10^{-6}$ m

Force in member BC is Q − P compression

Shortening $\delta_{BC} = \frac{(Q-P)L_{BC}}{E A_{BC}} = \frac{(Q-P)(0.5)}{(70\times10^9)(2.8274\times10^{-3})}$

$= 2.5263\times10^{-9}(Q-P)$

For zero deflection at A $\quad \delta_{BC} = \delta_{AB}$

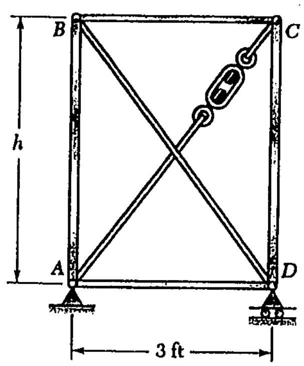
$2.5263\times10^{-9}(Q-P) = 72.756\times10^{-6} \quad \therefore \quad Q - P = 28.8\times10^3$ N

$Q = 28.3\times10^3 + 4\times10^3 = 32.8\times10^3$ N $= 32.8$ kN

(b) $\delta_{AB} = \delta_{BC} = \delta_B = 72.756\times10^{-6}$ m $= 0.0728$ mm

or $\begin{cases} (a) \quad \sum \delta_i = 0 \quad \frac{(P-Q)l_1}{E_1 A_1} + \frac{P l_2}{E_2 A_2} = 0 \\ (b) \quad \delta_B = \frac{(P-Q)l_1}{E_1 A_1} \end{cases}$ $\qquad A_1 = A_{BC} \quad l_1 = 0.5$; $\quad A_2 = A_{BA} \quad l_2 = 0.4$

PROBLEM 2.25

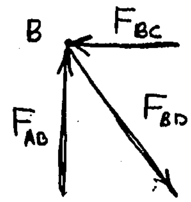


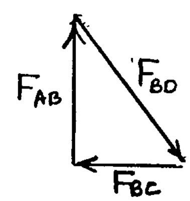
2.24 Members *AB* and *CD* are $1\frac{1}{8}$-in.-diameter steel rods, and members *BC* and *AD* are $\frac{7}{8}$-in.-diameter steel rods. When the turnbuckle is tightened, the diagonal member *AC* is put in tension. Knowing that $E = 29 \times 10^6$ psi and $h = 4$ ft, determine the largest allowable tension in *AC* so that the deformations in members *AB* and *CD* do not exceed 0.04 in.

2.25 For the structure in Prob. of 2.24, determine (*a*) the distance *h* so that the deformations in members *AB, BC, CD* and *AD* are all equal to 0.04 in., (*b*) the corresponding tension in member *AC*.

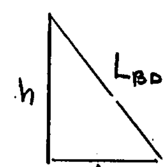
SOLUTION

(a) Statics: Use joint B as a free body





Force Triangle



Geometry

From similar triangles

$$\frac{F_{AB}}{h} = \frac{F_{BC}}{b} = \frac{F_{BD}}{L_{BC}}$$

$$F_{AB} = \frac{h}{b} F_{BC}$$

For equal deformations

$$\delta_{AB} = \delta_{BC} \therefore \quad \frac{F_{AB}\, h}{E A_{AB}} = \frac{F_{BC}\, b}{E A_{BC}} \qquad F_{AB} = \frac{b}{h} \cdot \frac{A_{AB}}{A_{BC}} F_{BC}$$

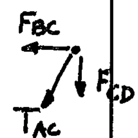
Equating expressions for F_{AB}

$$\frac{h}{b} F_{BC} = \frac{b}{h} \frac{A_{AB}}{A_{BC}} F_{BC} \qquad \frac{h^2}{b^2} = \frac{A_{AB}}{A_{BC}} = \frac{\frac{\pi}{4} d_{AB}^2}{\frac{\pi}{4} d_{BC}^2} = \frac{d_{AB}^2}{d_{BC}^2}$$

$$\frac{h}{b} = \frac{d_{AB}}{d_{BC}} = \frac{9/8}{7/8} = \frac{9}{7} \qquad b = 3\text{ ft} = 36 \text{ in.}$$

$$h = \frac{9}{7} b = \frac{9}{7}(3) = 3.86 \text{ ft} = 46.3 \text{ in.}$$

or



$F_{BC} = T_{AC} \cdot \frac{3}{\sqrt{9+h^2}}$

$F_{CD} = T_{AC} \cdot \frac{h}{\sqrt{9+h^2}}$

but $\delta_{BC} = \delta_{CD} = \frac{F_{BC} \cdot 3}{A_{BC} \cdot E} = \frac{F_{CD} \cdot h}{A_{CD} \cdot E}$

$\therefore \frac{F_{BC} \cdot 3}{A_{BC}} \cdot \frac{A_{CD}}{h} = F_{CD}$ ①

$F_{CD} = F_{BC} \cdot \frac{h}{3}$ ②

these 2 eq'ns lead to

(b) Setting $\delta_{AB} = \delta_{BC} = 0.04$ in.

$$\delta_{BC} = \frac{F_{BC}\, b}{E A_{BC}} \therefore \qquad F_{BC} = \frac{E A_{BC} \delta_{BC}}{b} = \frac{(29 \times 10^6) \frac{\pi}{4} \left(\frac{7}{8}\right)^2 (0.04)}{36}$$

$$= 19.376 \times 10^3 \text{ lb.}$$

$$F_{AB} = \frac{h}{b} F_{BC} = \frac{9}{7}(19.376 \times 10^3) = 24.912 \times 10^3 \text{ lb}$$

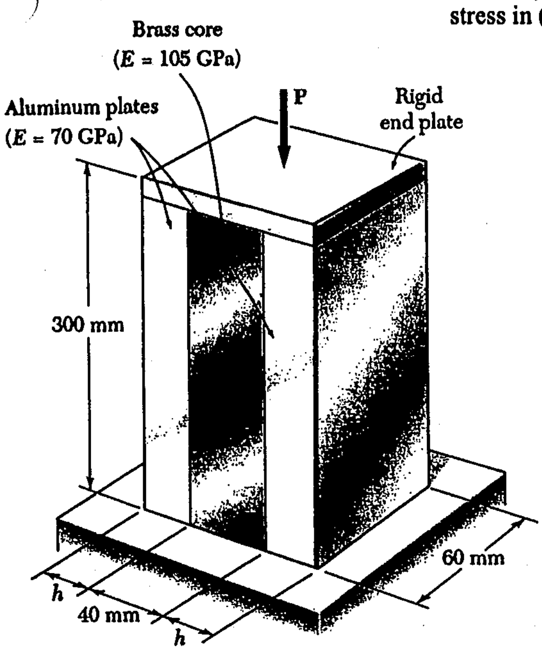
From the force triangle

$$F_{BD} = F_{AC} = \sqrt{F_{BC}^2 + F_{AB}^2} = 31.6 \times 10^3 \text{ lb.}$$

EMA 3702

HW #5

PROBLEM 2.36



2.36 An axial centric force of magnitude $P = 450$ kN is applied to the composite block shown by means of a rigid end plate. Knowing that $h = 10$ mm, determine the normal stress in (*a*) the brass core, (*b*) the aluminum plates.

SOLUTION

Let P_b = portion of axial force carried by brass core

P_a = portion carried by ~~two~~ each aluminum plates

$$\delta = \frac{P_b L}{E_b A_b} \qquad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_a L}{E_a A_a} \qquad P_a = \frac{E_a A_a \delta}{L}$$

all sections see same displacement since rigid plate moves downward same distance everywhere

$$P = P_b + 2P_a = (E_b A_b + 2E_a A_a)\frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{P}{E_b A_b + 2E_a A_a}$$

$$A_b = (60)(40) = 2400 \text{ mm}^2 = 2400\times10^{-6} \text{ m}^2$$

$$A_a = \cancel{(2)}(60)(10) = \cancel{1200}\ 600 \text{ mm}^2 = \cancel{12}\,600\times10^{-6} \text{ m}^2$$

$$\varepsilon = \frac{450\times10^3}{(105\times10^9)(2400\times10^{-6}) + (70\times10^9)(1200\times10^{-6})} = 1.3393\times10^{-3}$$

(a) $\sigma_b = E_b\varepsilon = (105\times10^9)(1.3393\times10^{-3}) = 140.6\times10^6 \text{ Pa} = 140.6 \text{ MPa}$ ◄

(b) $\sigma_a = E_a\varepsilon = (70\times10^9)(1.3393\times10^{-3}) = 93.75\times10^6 \text{ Pa} = 93.75 \text{ MPa}$ ◄

Recommended Method

or

Equilib: $P = P_{br} + 2P_{al}$

& $\delta_{al} = \delta_{brass} \Rightarrow \dfrac{P_{br} L}{E_{br} A_{br}} = \dfrac{P_{al} L}{E_{al} A_{al}} \quad \therefore \quad P_{br} = \dfrac{E_{br} A_{br}}{E_{Al} A_{Al}} P_{al} = \dfrac{105}{70}\cdot\dfrac{24}{6} P_{al} = \dfrac{3}{2}\cdot 4\, P_{al} = 6P_{al}$

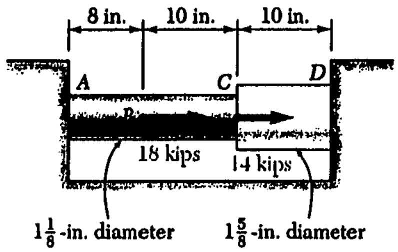
$\therefore\ P = 6P_{al} + 2P_{al} = 8P_{al} \qquad P_{al} = P/8 \qquad P_{br} = \dfrac{3}{4}P$

$$\sigma_{br} = \frac{P_{br}}{A_{br}} = \frac{0.75\,(4.50\times10^5)}{2400\times10^{-6}} = 140.6\times10^6 \ \frac{N}{m^2} \qquad \epsilon_{br} = \frac{\sigma_{br}}{E_{br}} = 1.3393\times10^{-3}$$

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{0.125\,(4.50\times10^5 N)}{600\times10^{-6}} = 93.75\times10^6 \ \frac{N}{m^2} \qquad \epsilon_{al} = \frac{\sigma_{al}}{E_{al}} = 1.3393\times10^{-3}$$

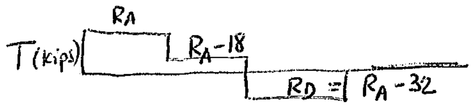
$\epsilon_{br} \equiv \epsilon_{al}$ a must!

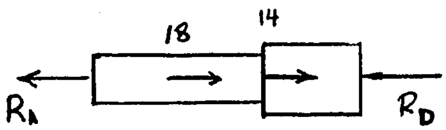
PROBLEM 2.41



2.41 Two cylindrical rods, *CD* made of steel ($E = 29 \times 10^6$ psi) and *AC* made of aluminum ($E = 10.4 \times 10^6$ psi), are joined at *B* and restrained by rigid supports at *A* and *D*. Determine (*a*) the reactions at *A* and *D*, (*b*) the deflection of point *C*.

SOLUTION





AB: $P = R_A$, $L_{AB} = 8$ in

$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(1.125)^2 = 0.99402 \text{ in}^2$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A (8)}{(10.4\times10^6)(0.99402)} = 0.77386 \times 10^{-6} R_A$$

BC: $P = R_A - 18 \times 10^3$, $L = 10$ in, $A = 0.99402 \text{ in}^2$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 18\times10^3)(10)}{(10.4\times10^6)(0.99402)} = 0.96732 \times 10^{-6} R_A - 17.412 \times 10^{-3}$$

CD: $P = R_A - 18\times10^3 - 14\times10^3 = R_A - 32\times10^3$

$L = 10$ in $\quad A = \frac{\pi}{4} d_{CD}^2 = \frac{\pi}{4}(1.625)^2 = 2.0739 \text{ in}^2$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A - 32\times10^3)(10)}{(29\times10^6)(2.0739)} = 0.16627\times10^{-6} R_A - 5.321\times10^{-3}$$

$$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 1.9075\times10^{-6} R_A - 22.733\times10^{-3}$$

Since point D cannot move relative to A $\quad \delta_{AD} = 0$

(a) $\quad 1.9075\times10^{-6} R_A - 22.733\times10^{-3} = 0 \qquad R_A = 11.92\times10^3$ lb. $\leftarrow$

$\quad R_D = 32\times10^3 - R_A = 20.08\times10^3$ lb $\leftarrow$

(b) $\quad \delta_C = \delta_{AB} + \delta_{CD}$

$= 1.7412\times10^{-6} R_A - 17.412\times10^{-3}$

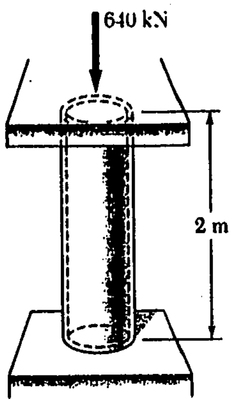
$= (1.7412\times10^{-6})(11.92\times10^3) - 17.412\times10^{-3} = 3.34\times10^{-3}$ in

or $\quad \delta_C = \frac{R_D L_{CD}}{E_{CD} A_{CD}} = \frac{(20.08\times10^3)(10)}{(29\times10^6)(2.0739)} = 3.34\times10^{-3}$ in.

or $\quad \frac{R_A \cdot 8_{in}}{10.4\times10^6 \cdot A_{AB}} + \frac{(R_A - 18\times10^3)\cdot 10_{in}}{10.4\times10^6 \cdot A_{AB}} + \frac{(R_A - 32\times10^3)\times 10_{in}}{29\times10^6 \cdot A_{AC}} = 0 = \frac{\Sigma P_i L_i}{A_i E_i}$

$R_A = 11.92\times10^3$ lb $\qquad R_D = 32\times10^3 - R_A = 20.08\times10^3$ lb

PROBLEM 2.65



2.65 A 2-m length of an aluminum pipe of 240-mm outer diameter and 10-mm wall thickness is used as a short column and carries a centric axial load of 640 kN. Knowing that $E = 73$ GPa and $\nu = 0.33$, determine (*a*) the change in length of the pipe, (*b*) the change in its outer diameter, (*c*) the change in its wall thickness.

SOLUTION

$d_o = 240$ mm $t = 10$ mm $d_i = d_o - 2t = 220$ mm

$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(240^2 - 220^2) = 7.2257 \times 10^3 \text{ mm}^2$

$= 7.2257 \times 10^{-3} \text{ m}^2$

$P = 640 \times 10^3$ N

(a) $\delta = -\dfrac{PL}{AE} = -\dfrac{(640 \times 10^3)(2.00)}{(7.2257 \times 10^{-3})(73 \times 10^9)} = -2.427 \times 10^{-3}$ m

$= -2.43$ mm

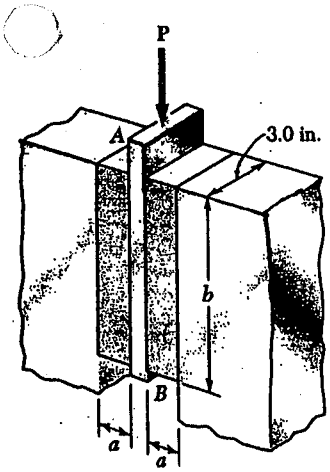
$\varepsilon = \dfrac{\delta}{L} = -\dfrac{2.427 \times 10^{-3}}{2.00} = -1.2133 \times 10^{-3}$

$\varepsilon_{lat} = -\nu\varepsilon = -(0.33)(-1.2133 \times 10^{-3}) = 400.4 \times 10^{-6}$

(b) $\Delta d_o = d_o \varepsilon_{lat} = (240)(400.4 \times 10^{-6}) = 0.0961$ mm

(c) $\Delta t = t \varepsilon_{lat} = (10)(400.4 \times 10^{-6}) = 0.00400$ mm

PROBLEM 2.80



2.80 A vibration isolation unit consists of two blocks of hard rubber bonded to plate *AB* and to rigid supports as shown. For the type and grade of rubber used $\tau_{all} = 220$ psi and $G = 1800$ psi. Knowing that a centric vertical force of magnitude $P = 3.2$ kips must cause a 0.1 in. vertical deflection of the plate *AB*, determine the smallest allowable dimensions *a* and *b* of the block.

SOLUTION

Consider the rubber block on the right. It carries a shearing force equal to $\frac{1}{2}P$.

The shearing stress is $\tau = \dfrac{\frac{1}{2}P}{A}$

or required $A = \dfrac{P}{2\tau} = \dfrac{3.2 \times 10^3}{(2)(220)} = 7.2727 \text{ in}^2$

But $A = (3.0)\,b$

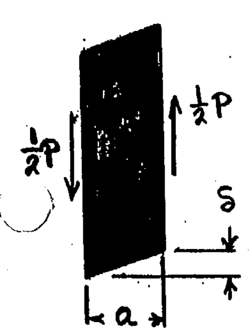
Hence $b = \dfrac{A}{3.0} = 2.42$ in.

Use $b = 2.42$ in and $\tau = 220$ psi

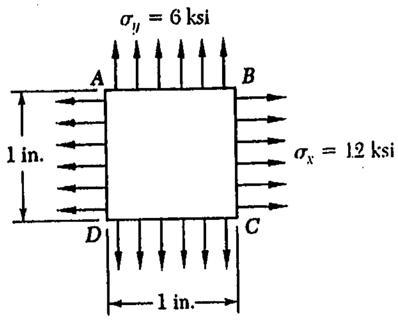
Shearing strain $\gamma = \dfrac{\tau}{G} = \dfrac{220}{1800} = 0.12222$

But $\gamma = \dfrac{\delta}{a}$

Hence $a = \dfrac{\delta}{\gamma} = \dfrac{0.1}{0.12222} = 0.818$ in.



PROBLEM 2.69



2.69 A 1-in. square is scribed on the side of a large steel pressure vessel. After pressurization the biaxial stress condition at the square is as shown. Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

SOLUTION

since $\sigma_z = 0$, then

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{29\times10^6}\left[12\times10^3 - (0.30)(6\times10^3)\right]$$
$$= 351.72\times10^{-6}$$
$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{29\times10^6}\left[6\times10^3 - (0.30)(12\times10^3)\right]$$
$$= 82.76\times10^{-8}$$

using $\Delta l = l_0 \cdot \varepsilon$

(a) $\delta_{AB} = (\overline{AB})_0\varepsilon_x = (1.00)(351.72\times10^{-6}) = 351.7\times10^{-6}$ in.

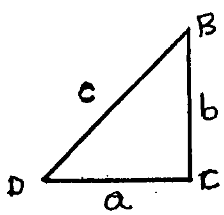
(b) $\delta_{BC} = (\overline{BC})_0\varepsilon_y = (1.00)(82.76\times10^{-6}) = 82.8\times10^{-6}$ in.

(c) $(\overline{AC}) = \sqrt{(\overline{AB})^2 + (\overline{BC})^2} = \sqrt{(\overline{AB}_0 + \delta_x)^2 + (\overline{BC}_0 + \delta_y)^2}$

$$= \sqrt{(1 + 352\times10^{-6})^2 + (1 + 82.8\times10^{-6})^2}$$
$$= 1.41452 \text{ new diagonal length}$$

original $(\overline{AC})_0 = \sqrt{2}$ length of diag $\qquad \overline{AC} - (\overline{AC})_0 = 307\times10^{-6}$ in.

or use calculus as follows:



Label sides using a, b, and c as shown

$$c^2 = a^2 + b^2$$

Obtain differentials $\quad 2c\,dc = 2a\,da + 2b\,dc$

from which $\quad dc = \frac{a}{c}\,da + \frac{b}{c}\,dc$

But $\quad a = 1.00$ in, $b = 1.00$ in, $c = \sqrt{2}$ in.

$$da = \delta_{AB} = 351.72\times10^{-6} \text{ in}, \quad db = \delta_{BC} = 82.8\times10^{-6} \text{ in}$$
$$\delta_{AC} = dc = \frac{1.00}{\sqrt{2}}(351.7\times10^{-6}) + \frac{1.00}{\sqrt{2}}(82.8\times10^{6})$$
$$= 307\times10^{-6} \text{ in.}$$

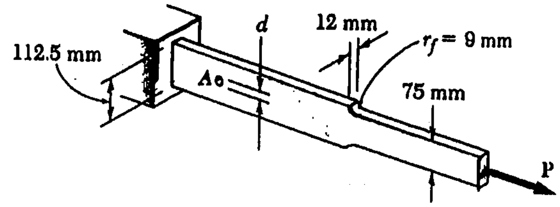
to find ϵ in AC dir

$$\epsilon_{AC} = \frac{\overline{AC} - \overline{AC}_0}{AC_0} = \frac{307\times10^{-6} \text{ in}}{1.41428 \text{ in}} \cong 215\times10^{-6} \text{ or } 215\,\frac{\mu\text{in}}{\text{in}}$$

shown. (b) Solve part a, assuming that the hole at A is not drilled.

2.134

SOLUTION



Maximum stress at hole

Use Fig. 2.64 a for values of K

$$\frac{r}{d} = \frac{6}{112.5 - 12} = 0.0597, \qquad K = 2.80 \text{ from pg 108 Fig 2.64a}$$

$$A_{net} = (12)(112.5 - 12) = 1206 \text{ mm}^2 = 1206 \times 10^{-6} \text{ m}^2$$

$$\sigma_{max} = K\frac{P}{A_{net}} = \frac{(2.80)(58 \times 10^3)}{1206 \times 10^{-6}} = 134.7 \times 10^6 \text{ Pa}$$

Maximum stress at fillets: if no hole, fillets cause stress concentration

Use Fig. 2.64 b

$$\frac{r}{d} = \frac{9}{75} = 0.12, \qquad \frac{D}{d} = \frac{112.5}{75} = 1.50, \qquad K = 2.10 \text{ from pg 108 Fig 2.64b}$$

$$A_{min} = (12)(75) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

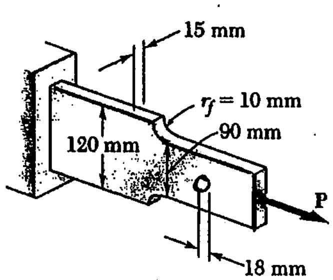
$$\sigma_{max} = K\frac{P}{A_{min}} = \frac{(2.10)(58 \times 10^3)}{900 \times 10^{-6}} = 135.3 \times 10^6 \text{ Pa}$$

(a) With hole and fillets $\qquad \sigma_{max} = 134.7$ MPa

(b) Without hole but with fillets $\qquad \sigma_{max} = 135.3$ MPa

PROBLEM 2.100

2.100 A centric axial force is applied to the steel bar shown. Knowing that σ_{all} is 135 MPa, determine the maximum allowable load **P**.



SOLUTION

At the hole: $\quad r = 9$ mm $\quad d = 90 - 18 = 72$ mm

$\frac{r}{d} = 0.125 \quad$ From Fig 2.64 a $\quad K = 2.65$

$$A_{net} = t\,d = (15)(72) = 1.08 \times 10^3 \text{ mm}^2 = 1.08 \times 10^{-3} \text{ m}^2$$

$$\sigma_{max} = \frac{KP}{A_{net}}$$

$$P = \frac{A_{net}\,\sigma_{max}}{K} = \frac{(1.08 \times 10^{-3})(135 \times 10^6)}{2.65} = 55 \times 10^3 \text{ N} = 55 \text{ kN}$$

At the fillet: $\quad D = 120$ mm, $\quad d = 90$ mm, $\quad \frac{D}{d} = \frac{120}{90} = 1.333$

$r = 10$ mm $\quad \frac{r}{d} = \frac{10}{90} = 0.1111 \quad$ From Fig 2.64 b $\quad K = 2.02$

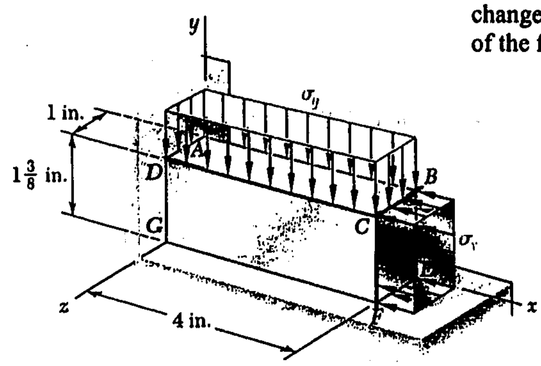
$$A_{min} = t\,d = (15)(90) = 1.35 \times 10^3 \text{ mm}^2 = 1.35 \times 10^{-3} \text{ m}^2$$

$$\sigma_{max} = \frac{KP}{A_{min}}$$

$$P = \frac{A_{min}\,\sigma_{max}}{K} = \frac{(1.35 \times 10^{-3})(135 \times 10^6)}{2.02} = 90 \times 10^3 \text{ N} = 90 \text{ kN}$$

Smaller value for P controls $\qquad P = 55$ kN

PROBLEM 2.127



2.127 The block shown is made of a magnesium alloy for which $E = 6.5 \times 10^6$ psi and $\nu = 0.35$. Knowing that $\sigma_x = -20$ ksi, determine (*a*) the magnitude of σ_y for which the change in the height of the block will be zero, (*b*) the corresponding change in the area of the face *ABCD*, (*c*) the corresponding change in the volume of the block.

SOLUTION

$\delta_y = 0 \qquad \varepsilon_y = 0$ if no change in height

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = 0$$

(a) $\sigma_y = \nu\sigma_x = (0.35)(-20\times10^3)$

$= -7\times10^3$ psi $= -7$ ksi

to find change in area find change in length in z & x dir

(b) $\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y) = -\dfrac{\nu(\sigma_x+\sigma_y)}{E}$

$$= \frac{(0.35)(-20\times10^3 - 7\times10^3)}{6.5\times10^6} = 1.4538\times10^{-3}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{-20\times10^3 - (0.35)(-7\times10^3)}{6.5\times10^6}$$

$= -2.7\times10^{-3}$

$$A_o + \Delta A = \overbrace{L_x(1+\varepsilon_x)}^{\text{new } L_x}\overbrace{L_z(1+\varepsilon_z)}^{\text{new } L_z} = L_xL_z(1+\varepsilon_x+\varepsilon_z+\varepsilon_x\varepsilon_z)$$

But $A_o = L_xL_z$ original area

$$\Delta A = L_xL_z(\varepsilon_x + \varepsilon_z + \varepsilon_x\varepsilon_z)$$

$= (4.0)(1.0)(1.4538\times10^{-3} - 2.7\times10^{-3} + \text{small term})$

$= -4.98\times10^{-3}\ \text{in}^2 = -0.00498\ \text{in}^2$

(c) Since L_y is constant

$\Delta V = L_y(\Delta A) = (1.375)(-4.98\times10^{-3}) = -6.85\times10^{-3}\ \text{in}^3$

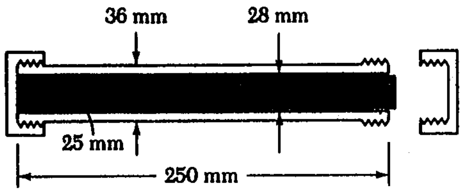
$= -0.00685\ \text{in}^3$

or $e = \epsilon_x + \epsilon_y + \epsilon_z = (1.4538\times10^{-3} + 0 - 2.7\times10^{-3}) = -1.2472\times10^{-3}$

$e = \dfrac{\Delta V}{V} \quad \therefore \quad \Delta V = e\cdot V = e\,(L_x\cdot L_y\cdot L_z) = -1.2472\times10^{-3}(4\cdot1.375\cdot1)$

$= -1.2472\times10^{-3}(5.5) = -6.85\times10^{-3}\ \text{in}^3$

PROBLEM 2.126



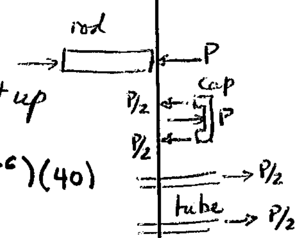
2.125 A 250-mm-long aluminum tube ($E = 70$ GPa) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod ($E = 105$ GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (*a*) the average normal stress in the tube and in the rod, (*b*) the deformations of the tube and of the rod.

2.126 In Prob. 2.125, determine the average normal stress in the tube and the rod, assuming that the temperature was 15° C when the nuts were snugly fitted and that the final temperature is 55° C. (For aluminum, $\alpha = 23.6 \times 10^{6}/°C$; for brass, $\alpha = 20.9 \times 10^{6}/°C$.)

SOLUTION

$$A_{tube} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12\ mm^2 = 402.12 \times 10^{-6}\ m^2$$

$$A_{rod} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(25)^2 = 490.87\ mm^2 = 490.87 \times 10^{-6}\ m^2$$



$$\Delta T = 55 - 15 = 40\ °C.$$ Due to temp change & closing tube a stress is set up

$$\delta_{tube} = \frac{PL}{E_{tube}A_{tube}} + L\alpha_{tube}(\Delta T) = \frac{P(0.250)}{(70\times10^9)(402.12\times10^{-6})} + (0.250)(23.6\times10^{-6})(40)$$
$$= 8.8815\times10^{-9}P + 236\times10^{-6}$$

$$\delta_{rod} = -\frac{PL}{E_{rod}A_{rod}} + L\alpha_{rod}(\Delta T) = -\frac{P(0.250)}{(105\times10^9)(490.87\times10^{-6})} + (0.250)(20.9\times10^{-6})(40)$$
$$= -4.8505\times10^{-9}P + 209\times10^{-6}$$

$$\delta^* = \frac{1}{4}\text{turn} \times 1.5\ mm = 0.375\ mm = 375\times10^{-6}\ m$$

$$\delta_{tube} = \delta_{rod} + \delta^*$$ change in tube = change in rod + ¼ turn of cap

$$8.8815\times10^{-9}P + 236\times10^{-6} = -4.8505\times10^{-9}P + 209\times10^{-6} + 375\times10^{-6}$$

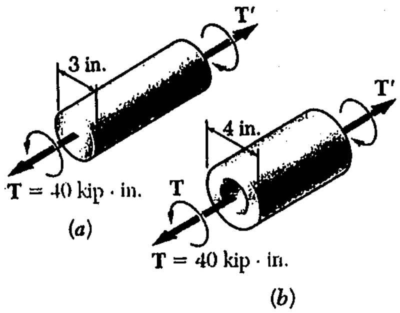
$$13.732\times10^{-9}P = 348\times10^{-6} \qquad P = 25.342\times10^3\ N$$

once you have P now find σ using P/A with appropriate A

$$\sigma_{tube} = \frac{P}{A_{tube}} = \frac{25.342\times10^3}{402.12\times10^{-6}} = 63.0\times10^6\ Pa = 63.0\ MPa$$

$$\sigma_{rod} = -\frac{P}{A_{rod}} = -\frac{25.342\times10^3}{490.87\times10^{-6}} = -51.6\times10^6\ Pa = -51.6\ MPa$$

PROBLEM 3.7



3.7 (*a*) For the 3-in.-diameter solid cylinder and loading shown, determine the maximum shearing stress. (*b*) Determine the inner diameter of the hollow cylinder, of 4-in. outer diameter, for which the maximum stress is the same as in part *a*.

SOLUTION

(a) Solid shaft $\quad c = \frac{1}{2}d = \frac{1}{2}(3.0) = 1.5 \text{ in.}$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(40)}{\pi(1.5)^3} = 7.545 \text{ ksi}$$

(b) Hollow shaft $\quad c_2 = \frac{1}{2}d = \frac{1}{2}(4.0) = 2.0 \text{ in.}$

$$\frac{J}{c_2} = \frac{\frac{\pi}{2}(c_2^4 - c_1^4)}{c_2} = \frac{T}{\tau_{max}}$$

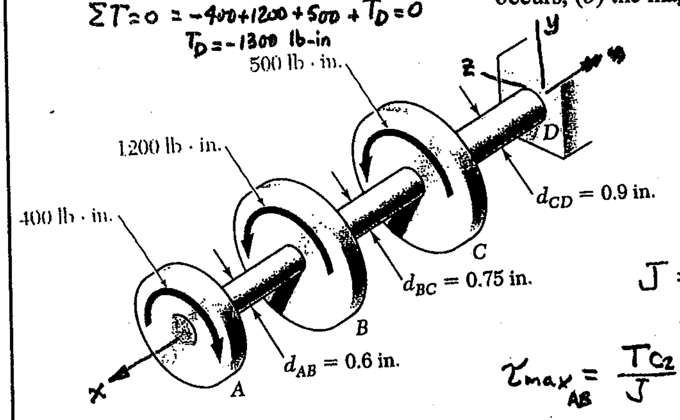
$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi \tau_{max}} = 2.0^4 - \frac{(2)(40)(2.0)}{\pi(7.545)} = 9.25 \text{ in}^4$$

$$c_1 = 1.74395 \text{ in} \qquad d_1 = 2c_1 = 3.49 \text{ in}$$

PROBLEM 3.12

$\Sigma T = 0 = -400 + 1200 + 500 + T_D = 0$

$T_D = -1300$ lb-in



3.12 Knowing that a 0.30-in.-diameter hole has been drilled through each portion of shaft *AD*, determine (*a*) the portion of the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.

SOLUTION

Hole: $c_1 = \frac{1}{2}d_1 = 0.15$ in

Shaft AB: $T_{AB} = \underline{400}$ lb·in

$c_2 = \frac{1}{2}d_2 = 0.30$ in

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.30^4 - 0.15^4) = 0.011928 \text{ in}^4$$

$$\tau_{max_{AB}} = \frac{Tc_2}{J} = \frac{(400)(0.30)}{0.011928} = \underline{10600 \text{ psi}}$$

Shaft BC: $T_{BC} = -400 + 1200 = \underline{800 \text{ lb·in}} \qquad c_2 = \frac{1}{2}d_2 = 0.375$ in.

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.375^4 - 0.15^4) = 0.030268 \text{ in}^4$$

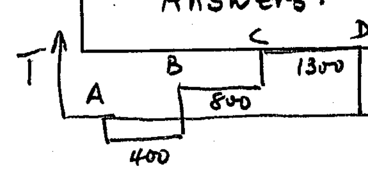
$$\tau_{max_{BC}} = \frac{Tc_2}{J} = \frac{(800)(0.375)}{0.030268} = \underline{9911 \text{ psi}}$$

Shaft CD: $T_{CD} = -400 + 1200 + 500 = \underline{1300 \text{ lb·in}} \qquad c_2 = \frac{1}{2}d_2 = 0.45$ in

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.45^4 - 0.15^4) = 0.063617 \text{ in}^4$$

$$\tau_{max_{CD}} = \frac{Tc_2}{J} = \frac{(1300)(0.45)}{0.063617} = \underline{9196 \text{ psi}}$$

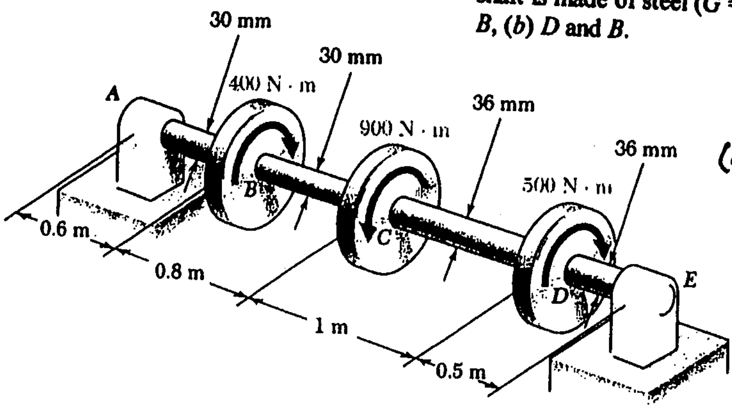
Answers: (a) shaft AB $\qquad$ (b) 10.06 ksi



torque diagram

$T_{AB} = 400$
$T_{BC} = 800$
$T_{CD} = 1300$

PROBLEM 3.36



3.36 The torques shown are exerted on pulleys *B*, *C* and *D*. Knowing that the entire shaft is made of steel (G = 27 GPa), determine the angle of twist between (*a*) *C* and *B*, (*b*) *D* and *B*.

SOLUTION

(a) Shaft BC: $c = \frac{1}{2}d = 0.015\ m$

$J_{BC} = \frac{\pi}{4}c^4 = 79.522 \times 10^{-9}\ m^4$

$L_{BC} = 0.8\ m, \quad G = 27 \times 10^9\ Pa$

$\varphi_{BC} = \frac{TL}{GJ} = \frac{(400)(0.8)}{(27 \times 10^9)(79.522 \times 10^{-9})}$

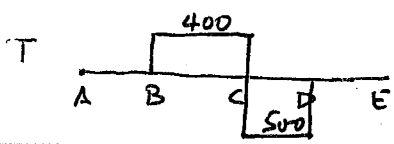
$= 0.149904\ rad = 8.54°$

(b) Shaft CD: $c = \frac{1}{2}d = 0.018\ m \qquad J_{CD} = \frac{\pi}{4}c^4 = 164.896 \times 10^{-9}\ m^4$

$L_{CD} = 1.0\ m \qquad T_{CD} \doteq 400 - 900 = -500\ N \cdot m$

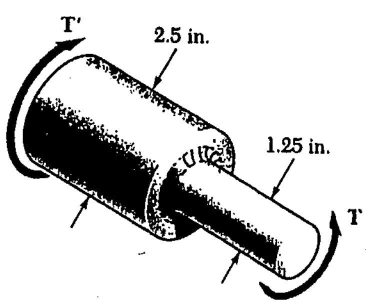
$\varphi_{CD} = \frac{TL}{GJ} = \frac{(-500)(1.0)}{(27 \times 10^9)(164.896 \times 10^{-9})} = -0.11230\ rad$

$\varphi_{BD} = \varphi_{BC} + \varphi_{CD} = 0.14904 - 0.11230 = 0.03674\ rad = 2.11°$



$\Sigma T = 0 \Rightarrow T_{AB} = T_{DE} = 0$

PROBLEM 3.89



3.89 Knowing that the stepped shaft shown must transmit 60 hp at a speed of 2100 rpm, determine the minimum radius *r* of the fillet if an allowable stress of 6000 psi is not to be exceeded.

SOLUTION

$f = \frac{2100}{60} = 35\ Hz$

$P = 60\ hp = (60)(6600) = 396 \times 10^3\ lb \cdot in/s$

$T = \frac{P}{2\pi f} = \frac{396 \times 10^3}{2\pi\,(35)} = 1.8007 \times 10^3\ lb \cdot in$

For smaller shaft $\quad c = \frac{1}{2}d = 0.625\ in \qquad \tau = K\frac{Tc}{J} = \frac{2KT}{\pi c^3}$

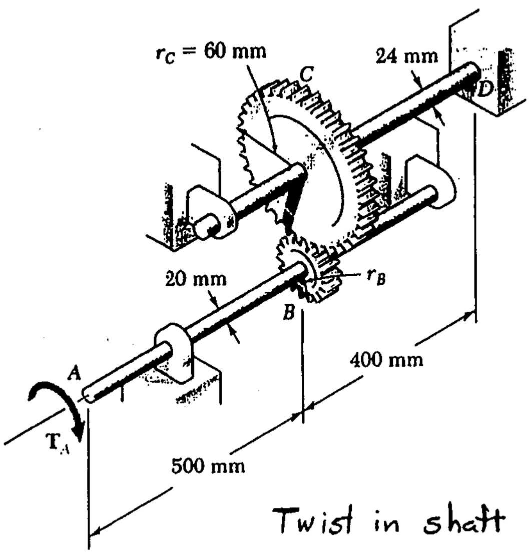
$K = \frac{\pi c^3 \tau}{2T} = \frac{\pi(0.625)^3(6000)}{(2)(1.8007 \times 10^3)} = 1.28$

$\frac{D}{d} = \frac{2.5}{1.25} = 2 \qquad$ From Fig 3.32 $\qquad \frac{r}{d} = 0.18$

$r = 0.18\,d = (0.050)(1.25\ in) = 0.225\ in.$

PROBLEM 3.39

3.39 Two solid steel shafts (G = 77 GPa) are connected by the gears shown. Knowing that the radius of gear B is r_B = 20 mm, determine the angle through which end A rotates when $\mathbf{T}_A$ = 75 N·m.



SOLUTION

Calculation of torques.

Circumferential contact force between gears B and C

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \quad \therefore \quad T_{CD} = \frac{r_C}{r_B} T_{AB}$$

$$T_{AB} = T_A = 75 \text{ Nm}$$

$$T_{CD} = \frac{0.060}{0.020}(75) = 225 \text{ N-m}$$

Twist in shaft CD

$$J_{CD} = \frac{\pi}{2} c_{CD}^4 = \frac{\pi}{2}(0.012)^4 = 32.572 \times 10^{-9} \text{ m}^4, \qquad L_{CD} = 0.400 \text{ m}$$

$$G = 77 \times 10^9 \text{ Pa} \qquad \varphi_{CD} = \frac{TL}{GJ} = \frac{(225)(0.400)}{(77\times10^9)(32.572\times10^{-9})} = 35.885 \times 10^{-3} \text{ rad.}$$

Rotation angle at C $\qquad \varphi_C = \varphi_{CD} = 35.885 \times 10^{-3}$ rad

Circumferential displacement at contact points of gears B and C

$$\delta = r_C \varphi_C = r_B \varphi_B$$

Rotation angle at B: $\quad \varphi_B = \frac{r_C}{r_B}\varphi_C = \frac{0.060}{0.020}(35.885\times10^{-3}) = 107.654\times10^{-3}$ rad

Twist in shaft AB:

$$J_{AB} = \frac{\pi}{2} c_{AB}^4 = \frac{\pi}{2}(0.010)^4 = 15.708 \times 10^{-9} \text{ m}^4, \qquad L_{AB} = 0.500 \text{ m}$$

$$G = 77 \times 10^9 \text{ Pa}, \quad \varphi_{AB} = \frac{TL}{GJ} = \frac{(75)(0.500)}{(77\times10^9)(15.708\times10^{-9})} = 31.004 \times 10^{-3} \text{ rad}$$

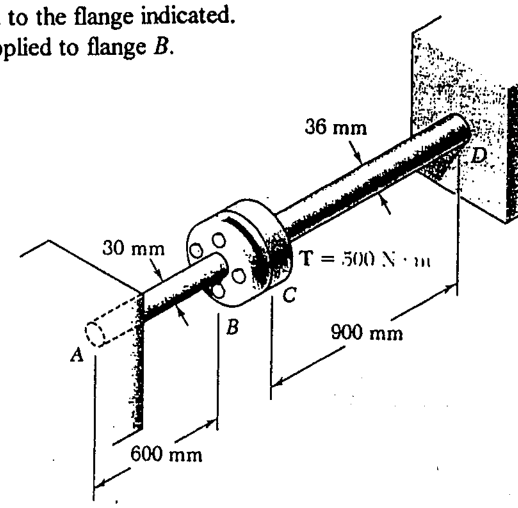
Rotation at A $\qquad \varphi_A = \varphi_B + \varphi_{AB} = 138.7 \times 10^{-3} \text{ rad} = 7.94°$ ◄

PROBLEM 3.57

3.57 and 3.58 Two solid steel shafts are fitted with flanges which are then connected by bolts as shown. The bolts are slightly undersized and permit a 1.5° rotation of one flange with respect to the other before the flanges begin to rotate as a single unit. Knowing that $G = 77$ GPa, determine the maximum shearing stress in each shaft when a 500 N·m torque **T** is applied to the flange indicated.

3.57 The torque **T** is applied to flange B.

SOLUTION



Shaft AB

$T = T_{AB}, \quad L = 0.6\text{ m}, \quad c = \frac{1}{2}d = 0.015\text{ m}$

$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4 = 79.52\times10^{-9}\text{ m}^4$

$$\varphi_B = \frac{T_{AB}L_{AB}}{G_{AB}J_{AB}}$$

$$T_{AB} = \frac{G_{AB}J_{AB}}{L}\varphi_B = \frac{(77\times10^9)(79.52\times10^{-9})}{0.6}\varphi_B$$

$$= 10.205\times10^3\ \varphi_B$$

Shaft CD

$T = T_{CD}, \quad L_{CD} = 0.9\text{ m}, \quad c = \frac{1}{2}d = 0.018\text{ m}, \quad J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4$

$J_{CD} = 164.896\times10^{-9}\text{ m}^4$

$$T_{CD} = \frac{G_{CD}J_{CD}}{L_{CD}}\varphi_C = \frac{(77\times10^9)(164.896\times10^{-9})}{0.9}\varphi_C = 14.108\times10^3\ \varphi_C$$

Clearance rotation for flange B $\quad \varphi_B' = 1.5° = 26.18\times10^{-3}$ rad

Torque to remove clearance: $T_{AB}' = (10.205\times10^{+3})(26.18\times10^{-3}) = 267.17$ N·m

Torque T'' to cause additional rotation φ'': $T'' = 500 - 267.17 = 232.83$ N·m

$$T'' = T_{AB}'' + T_{CD}''$$

$$232.83 = (10.205\times10^3 + 14.108\times10^3)\varphi'' \quad \therefore \quad \varphi'' = 9.5765\times10^{-3}\text{ rad}$$

$$T_{AB}'' = (10.205\times10^3)(9.5765\times10^{-3}) = 97.73\text{ N·m}$$

$$T_{CD}'' = (14.108\times10^3)(9.5765\times10^{-3}) = 135.10\text{ N·m}$$

Maximum shearing stress in AB

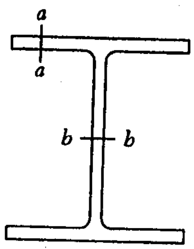
$$\tau_{AB} = \frac{T_{AB}c}{J_{AB}} = \frac{(267.17 + 97.73)(0.015)}{79.52\times10^{-9}} = 68.8\times10^6\text{ Pa} \qquad 68.8\text{ MPa}$$

Maximum shearing stress in CD

$$\tau_{CD} = \frac{T_{CD}c}{J_{CD}} = \frac{(135.10)(0.018)}{164.896\times10^{-9}} = 14.75\times10^6\text{ Pa} \qquad 14.75\text{ MPa}$$

PROBLEM 3.135

3.135 An 8-ft-long steel member with a W 8 × 31 cross section is subjected to a 5 kip·in. torque. From Appendix C we find that the thickness of the section is $\frac{3}{8}$ in. and that its area is 2.86 in^2. Knowing that $G = 11.2 \times 10^6$ psi, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the maximum shearing stress along line *b-b*, (*c*) the angle of twist. (*Hint:* Consider the web and flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)



W8 × 31

SOLUTION

Flange: $a = 7.995$ in, $b = 0.435$, $\frac{a}{b} = \frac{7.995}{0.435} = 18.38$

$c_1 = c_2 = \frac{1}{3}(1 - 0.630\frac{b}{a}) = 0.3219 \qquad \varphi_F = \frac{T_F L}{c_2 a b^3 G}$

$T_F = c_2 a b^3 \frac{G\varphi_F}{L} = K_F \frac{G\varphi}{L} \qquad$ where $K_F = c_2 a b^3$

$K_F = (0.3219)(7.995)(0.435)^3 = 0.2188 \text{ in}^3$

Web: $a = 8.0 - (2)(0.435) = 7.13$ in, $b = 0.285$ in, $\frac{a}{b} = \frac{7.13}{0.285} = 25.02$

$c_1 = c_2 = \frac{1}{3}(1 - 0.630\frac{b}{a}) = 0.3249 \qquad \varphi_W = \frac{T_W L}{c_2 a b^3 G}$

$T_W = c_2 a b^3 \frac{G\varphi_W}{L} = K_W \frac{G\varphi}{L} \qquad$ where $K_W = c_2 a b^3$

$K_W = (0.3249)(7.13)(0.285)^3 = 0.0563 \text{ in}^4$

For matching twist angles $\qquad \varphi_F = \varphi_W = \varphi$

Total torque $\qquad T = 2T_F + T_W = (2K_F + K_W)\frac{G\varphi}{L}$

$\frac{G\varphi}{L} = \frac{T}{2K_F + K_W}, \quad T_F = \frac{K_F T}{2K_F + K_W}, \quad T_W = \frac{K_W T}{2K_W + K_W}$

$T_F = \frac{(0.2138)(5000)}{(2)(0.2138) + 0.0563} = 2221 \text{ lb}\cdot\text{in}; \quad T_W = \frac{(0.0563)(5000)}{(2)(0.2138) + 0.0563} = 557 \text{ lb}\cdot\text{in}$

(a) $\tau_F = \frac{T_F}{c_1 a b^2} = \frac{2221}{(0.3219)(7.995)(0.435)^2} = 4570 \text{ psi} = 4.57 \text{ ksi}$

(b) $\tau_W = \frac{T_W}{c_1 a b^2} = \frac{557}{(0.3249)(7.13)(0.285)^2} = 2960 \text{ psi} = 2.96 \text{ ksi}$

(c) $\frac{G\varphi}{L} = \frac{T}{2K_F + K_W} \quad \therefore \varphi = \frac{TL}{G(2K_F + K_W)} \quad$ where $L = 8 \text{ ft} = 96$ in.

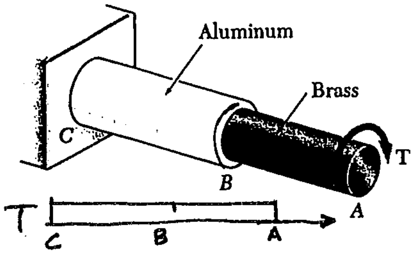
$\varphi = \frac{(5000)(96)}{(11.2 \times 10^6)[(2)(0.2138) + 0.563]} = 88.6 \times 10^{-3} \text{ rad} = 5.08°$

also $\quad \tau = \frac{Tt}{J} \quad$ where $J = K_w + 2K_f$

for τ_{ab} take $t = b_{flange} = .435$

τ_{bb} take $t = b_{web} = .285$

PROBLEM 3.20



the allowable shearing stress is 25 MPa. Rod *AB* is hollow and has an outer diameter of 25 mm; it is made a brass for which the allowable shearing stress is 50 MPa. Determine (*a*) the largest inner diameter of rod *AB* for which the factor of safety is the same for each rod, (*b*) the largest torque that may be applied at *A*.

SOLUTION

Solid rod BC: $\tau = \frac{Tc}{J}$ $\quad J = \frac{\pi}{2}c^4$

$\tau_{all} = 25 \times 10^6$ Pa $\quad c = \frac{1}{2}d = 0.015$ m

$T_{all} = \frac{\pi}{2}c^3\tau_{all} = \frac{\pi}{2}(0.015)^3(25\times10^6) = 132.536$ N·m $= \frac{\tau J}{c}$

Hollow rod AB: $\tau_{all} = 50\times10^6$ Pa $\quad T_{all} = 132.536$ N·m

$c_2 = \frac{1}{2}d_2 = \frac{1}{2}(0.025) = 0.0125$ m $\quad c_1 = ?$

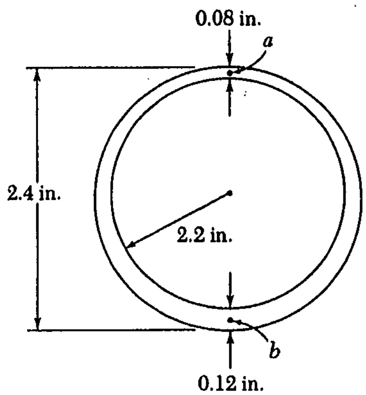
$T_{all} = \frac{J\tau_{all}}{c_2} = \frac{\pi}{2}(c_2^4 - c_1^4)\frac{\tau_{all}}{c_2}$

$c_1^4 = c_2^4 - \frac{2T_{all}c_2}{\pi\tau_{all}} = 0.0125^4 - \frac{(2)(132.536)(0.0125)}{\pi(50\times10^6)} = 3.3203\times10^{-9}$ m^4

$c_1 = 7.59\times10^{-3}$ m $= 7.59$ mm $\quad d_1 = 2c_1 = 15.18$ mm ◄

(b) Allowable torque $\quad T_{all} = 132.5$ N·m ◄

PROBLEM 3.145



3.145 A hollow cylindrical shaft was designed to have a uniform wall thickness of 0.1 in. Defective fabrication, however, resulted in the shaft having the cross section shown. Knowing that a 15-kip·in. torque **T** is applied to the shaft, determine the shearing stress at points *a* and *b*.

SOLUTION

Radius of outer circle = 1.2 in

Radius of inner circle = 1.1 in

Mean radius = 1.15 in

Area bounded by centerline

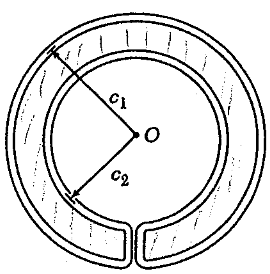
$$a = \pi r_m^2 = \pi (1.15)^2 = 4.155 \text{ in}^2$$

or $a_{bounded} = \dfrac{\pi (R_o^2 + R_i^2)}{2}$ $= \pi \dfrac{1.44 + 1.21}{2}$ $= \pi \cdot 1.325$

At point a $\quad t = 0.08$ in $\quad \tau = \dfrac{T}{2ta} = \dfrac{15}{(2)(0.08)(4.155)} = 22.6$ ksi ◄

At point b $\quad t = 0.12$ in $\quad \tau = \dfrac{T}{2ta} = \dfrac{15}{(2)(0.12)(4.155)} = 15.04$ ksi ◄

PROBLEM 3.146



3.146 A cooling tube having the cross section shown is formed from a sheet of stainless steel of 3 mm thickness. The radii $c_1 = 150$ mm and $c_2 = 100$ mm are measured to the centerline of the sheet metal. Knowing that a torque of magnitude $T = 3$ kN·m is applied to the tube, determine (*a*) the maximum shearing stress in the tube, (*b*) the magnitude of the torque carried by the outer circular shell. Neglect the dimension of the small opening where the outer and inner shells are connected.

SOLUTION

Area bounded by centerline even though this appears to be an open figure, it is closed

$$a = \pi (c_1^2 - c_2^2) = \pi (150^2 - 100^2) = 39.27 \times 10^3 \text{ mm}^2$$

$$= 39.27 \times 10^{-3} \text{ m}^2$$

$$t = 0.003 \text{ m}$$

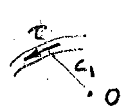
(a) $\tau = \dfrac{T}{2ta} = \dfrac{3 \times 10^3}{(2)(0.003)(39.27 \times 10^{-3})} = 12.73 \times 10^6 \text{ Pa} = 12.73 \text{ MPa}$ ◄

(b) $T_1 = (2\pi c_1 t \, \tau \, c_1) = 2\pi c_1^2 t \tau$

$= 2\pi (0.150)^2 (0.003)(12.73 \times 10^6) = 5.40 \times 10^3$ N·m

$= 5.40$ kN·m ◄

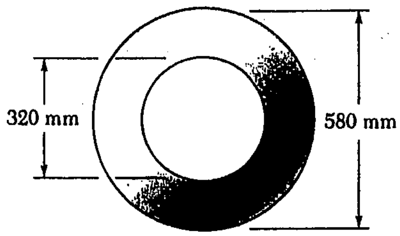
or $T_1 = \oint \underbrace{\tau \cdot dA}_{\text{elemental force}} \cdot \underbrace{c_1}_{\text{moment arm}}$



$dA = t\,ds$

$$= \oint_0^{2\pi} \tau \cdot c_1 \cdot t \cdot c_1 \, d\theta = \tau \cdot t c_1^2 \cdot 2\pi$$

PROBLEM 3.158



3.158 One of the two hollow steel drive shafts of an ocean liner is 75 m long and has the cross section shown. knowing that $G = 77$ GPa and that the shaft transmits 44 MW to its propeller when rotating at 144 rpm, determine (*a*) the maximum shearing stress in the shaft, (*b*) the angle of twist of the shaft.

SOLUTION

$L = 75 \text{ m}, \qquad f = 144 \text{ rpm} = \frac{144}{60} = 2.4 \text{ Hz}$

$P = 44 \text{ MW} = 44 \times 10^6 \text{ W}$

$P = 2\pi f T \quad \therefore \quad T = \frac{P}{2\pi f} = \frac{44 \times 10^6}{2\pi (2.4)} = 2.9178 \times 10^6 \text{ N}\cdot\text{m}$

$c_1 = \frac{d_1}{2} = \frac{320}{2} = 160 \text{ mm} = 0.160 \text{ m}$

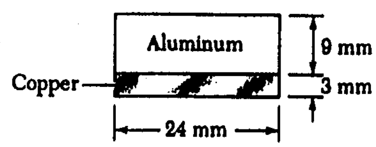
$c_2 = \frac{d_2}{2} = \frac{580}{2} = 290 \text{ mm} = 0.290 \text{ m}$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.290^4 - 0.160^4) = 10.08 \times 10^{-3} \text{ m}$

(a) $\tau = \frac{T c_2}{J} = \frac{(2.9178 \times 10^6)(0.290)}{10.08 \times 10^{-3}} = 83.9 \times 10^6 \text{ Pa} = 83.9 \text{ MPa}$ ◀

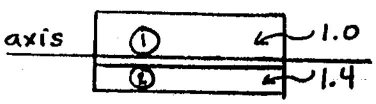
(b) $\varphi = \frac{TL}{GJ} = \frac{(2.9178 \times 10^6)(75)}{(77 \times 10^9)(10.08 \times 10^{-3})} = 281.9 \times 10^{-3} \text{ rad} = 16.15°$ ◀

PROBLEM 4.46



4.45 and 4.46 A copper strip (E_c = 105 GPa) and an aluminum strip (E_a = 75 GPa) are bonded together to form the composite bar shown. Knowing that the bar is bent about a horizontal axis by a couple of moment 35 N·m, determine the maximum stress in (*a*) the aluminum strip, (*b*) the copper strip.

SOLUTION



Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = E_c/E_a = 105/75 = 1.4$ in copper

Transformed section

	A, mm²	nA, mm²	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm³
①	216	216	7.5	1620
②	72	100.8	1.5	151.8
Σ		316.8		1771.2

$$\bar{Y}_o = \frac{1771.2}{316.8} = 5.5909 \text{ mm}$$

The neutral axis lies 5.5909 mm above the bottom

$$I_1 = \frac{n_1}{12}b_1h_1^3 + n_1A_1d_1^2 = \frac{1.0}{12}(24)(9)^3 + (1.0)(24)(9)(1.9091)^2 = 2245.2 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12}b_2h_2^3 + n_2A_2d_2^2 = \frac{1.4}{12}(24)(3)^3 + (1.4)(24)(3)(4.0909)^2 = 1762.5 \text{ mm}^4$$

$$I = I_1 + I_2 = 4008 \text{ mm}^4 = 4.008 \times 10^{-9} \text{ m}^4$$

(a) Aluminum: $n = 1.0$ $y = 12 - 5.5909 = 6.4091 \text{ mm} = 0.0064091$

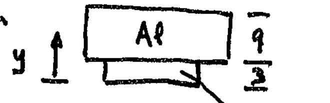
$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(35)(0.0064091)}{4.008 \times 10^{-9}} = -56.0 \times 10^{6} \text{ Pa} = -56.0 \text{ MPa}$$

(b) Copper: $n = 1.4$, $y = -5.5909 \text{ mm} = -0.0055909$ m

$$\sigma = -\frac{nMy}{I} = -\frac{(1.4)(35)(-0.0055909)}{4.008 \times 10^{-9}} = 68.4 \times 10^{6} \text{ Pa} = 68.4 \text{ MPa}$$

or

convert copper to aluminum

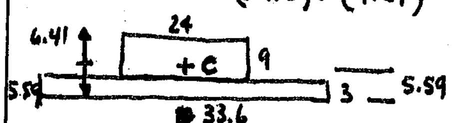


$E_c b_c = E_{al} b_{al}$

$\frac{E_c}{E_{al}} b_c =$ $b_{al} = 1.4 \cdot 24$

$\cong 33.6$ mm

$$\bar{y} = \frac{[33.6 \cdot 3]1.5 + [9 \cdot 24][7.5]}{(33.6 \cdot 3) + (9 \cdot 24)} = 5.59 \text{ mm}$$



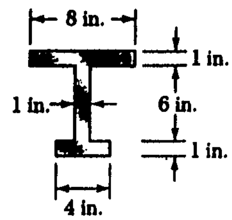
I_{zo}	A	d	I
$24 \cdot 9^3/12$	24.9	(7.5-5.59)	
$33.6 \cdot 3^3/12$	33.6·3	(5.59-1.5)	
			4008 mm⁴

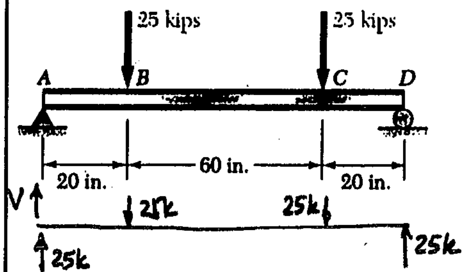
$= 4.008 \times 10^{-9}$ m⁴

since converted to alum $\sigma = \frac{-My}{I}$ as usual & $y = 6.41$

for copper $\sigma = -\frac{E_c}{E_{al}} \cdot \frac{My}{I}$ & $y = -5.59$

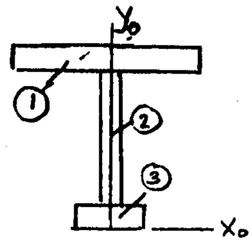
PROBLEM 4.11



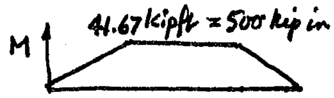


4.9 through 4.11 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

SOLUTION

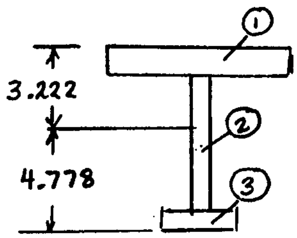


	A	$\bar{y}_0$	$A\bar{y}_0$
①	8	7.5	60
②	6	4	24
③	4	0.5	2
Σ	18		86



$$\bar{Y}_0 = \frac{86}{18} = 4.778 \text{ in}$$

Neutral axis lies 4.778 in above the base.



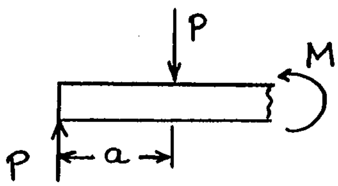
$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(8)(1)^3 + (8)(2.722)^2$$
$$= 59.94 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(1)(6)^3 + (6)(0.778)^2$$
$$= 21.63 \text{ in}^4$$

$$I_3 = \frac{1}{12} b_3 h_3^3 + A_3 d_3^2 = \frac{1}{12}(4)(1)^3 + (4)(4.278)^2 = 73.54 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 59.94 + 21.63 + 73.54 = 155.16 \text{ in}^4$$

$y_{top} = 3.222$ in $\qquad y_{bot} = -4.778$ in

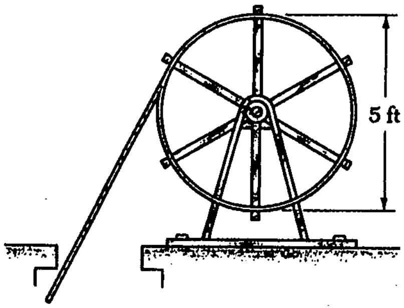


$$M - Pa = 0$$
$$M = Pa = (25)(20) = 500 \text{ kip·in.}$$

$$\sigma_{top} = -\frac{M y_{top}}{I} = -\frac{(500)(3.222)}{155.16} = -10.38 \text{ ksi}$$

$$\sigma_{bot} = -\frac{M y_{bot}}{I} = -\frac{(500)(-4.778)}{155.16} = 15.40 \text{ ksi}$$

PROBLEM 4.26



4.26 Straight rods of 0.30-in. diameter and 200-ft length are sometimes used to clear underground conduits of obstructions or to thread wires through a new conduit. The rods are made of high-strength steel and, for storage and transportation, are wrapped on spools of 5-ft diameter. Assuming that the yield strength is not exceeded, determine (*a*) the maximum stress in a rod, when the rod, which was initially straight, is wrapped on a spool, (*b*) the corresponding bending moment in the rod. Use $E = 29 \times 10^6$ psi.

SOLUTION

$$r = \tfrac{1}{2}d = \tfrac{1}{2}(0.30) = 0.15 \text{ in}$$

$$I = \tfrac{\pi}{4}r^4 = \tfrac{\pi}{4}(0.15)^4 = 397.61 \times 10^{-6} \text{ in}^4$$

$$D = 5 \text{ ft} = 60 \text{ in} \qquad \rho = \tfrac{1}{2}D = 30 \text{ in} = 2.5 \text{ ft} \times \frac{12 \text{ in}}{\text{ft}} = 30 \text{ in}$$

$$c_1 = r = 0.15 \text{ in.}$$

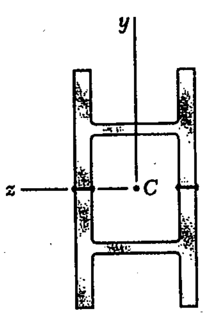
(a) $$\sigma_{max} = \frac{Ec}{\rho} = \frac{(29\times10^6)(0.15)}{30} = 145 \times 10^3 \text{ psi} = 145 \text{ ksi}$$ ◄

(b) $$M = \frac{EI}{\rho} = \frac{(29\times10^6)(397.61\times10^{-6})}{30} = 384 \text{ lb}\cdot\text{in.}$$ ◄

or $$\frac{1}{R} = \frac{M}{EI} \rightarrow M = \frac{EI}{R} \text{ as in (b)}$$

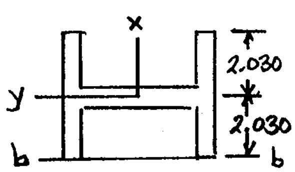
since $$\sigma_{max} = \frac{My}{I}$$ with $y_{max} = \frac{1}{2}$ diam of rod $= 0.15$ in

PROBLEM 4.8



4.7 and 4.8 Two W 4 × 13 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_Y = 36$ ksi and $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the *z* axis.

SOLUTION



Properties of W 4×13 rolled section
See Appendix B

Area = 3.83 in^2 Width = 4.060 in
I_y = 3.86 in^4

For one rolled section, moment of inertia about axis b-b is

$$I_b = I_y + Ad^2 = 3.86 + (3.83)(2.030)^2 = 19.643 \text{ in}^4$$

For both sections $I_z = 2I_b = 39.286$ in^4

c = width = 4.060 in

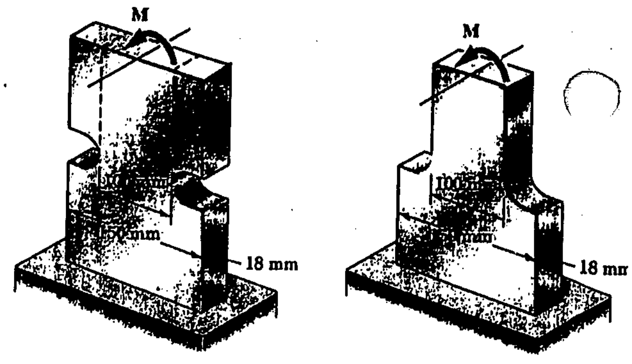
$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \qquad \sigma = \frac{Mc}{I}$$

$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(19.333)(39.286)}{4.060} = 187.1 \text{ kip}\cdot\text{in}$$ ◄

PROBLEM 4.74

4.74 A couple of moment $M = 2$ kN·m is to be applied to the end of a steel ba
Determine the maximum stress in the bar (*a*) if the bar is designed with groove having semicircular portions of radius $r = 10$ mm, as shown in Fig. *a*, (*b*) if the ba is redesigned by removing the material above the grooves as shown in Fig. *b*.

SOLUTION



(*a*) (*b*)

For both configurations

$D = 150$ mm, $d = 100$ mm

$r = 10$ mm.

$$\frac{D}{d} = \frac{150}{100} = 1.50$$

$$\frac{r}{d} = \frac{10}{100} = 0.10$$

For configuration (a),
Fig 4.32 give $K_a = 2.21$

For configuration (b), Fig. 4.31 gives $K_b = 1.79$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(18)(100)^3 = 1.5\times10^6 \text{ mm}^4 = 1.5\times10^{-6} \text{ m}^4$$

$$c = \tfrac{1}{2}d = 50 \text{ mm} = 0.05 \text{ m}$$

(a) $$\sigma = \frac{KMc}{I} = \frac{(2.21)(2\times10^3)(0.05)}{1.5\times10^{-6}} = 147\times10^6 \text{ Pa} = 147 \text{ MPa}$$ ◄

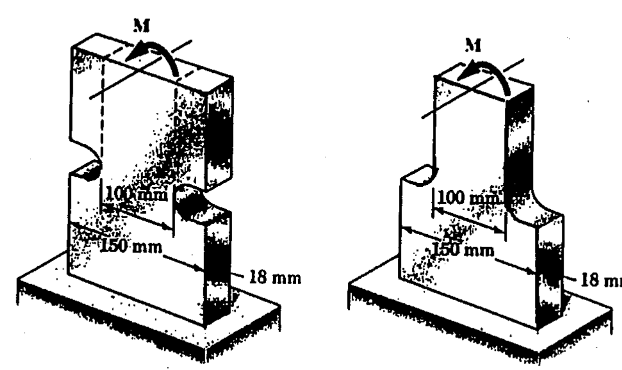
(b) $$\sigma = \frac{KMc}{I} = \frac{(1.79)(2\times10^3)(0.05)}{1.5\times10^{-6}} = 119\times10^6 \text{ Pa} = 119 \text{ MPa}$$ ◄

better (b) than (a)

PROBLEM 4.73

4.73 The allowable stress used in the design of a steel bar is 80 MPa. Determi the largest couple M that can be applied to the bar (*a*) if the bar is designed wi grooves having semicircular portions of radius $r = 15$ mm, as shown in Fig. *a*, (*b* if the bar is redesigned by removing the material above the grooves as shown in Fi *b*.

SOLUTION



(*a*) (*b*)

For both configuations

$D = 150$ mm, $d = 100$ mm,

$r = 15$ mm.

$$\frac{D}{d} = \frac{150}{100} = 1.50$$

$$\frac{r}{d} = \frac{15}{100} = 0.15$$

For configuration (a), Fig 4.32 gives $K_a = 1.92$.

For configuration (b) Fig 4.31 gives $K_b = 1.57$.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(18)(100)^3 = 1.5\times10^6 \text{ mm}^4 = 1.5\times10^{-6} \text{ m}^4$$

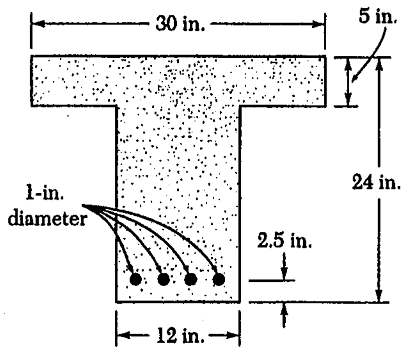
$$c = \tfrac{1}{2}d = 50 \text{ mm} = 0.050 \text{ m}$$

(a) $$\sigma = \frac{KMc}{I} \quad \therefore \quad M = \frac{\sigma I}{Kc} = \frac{(80\times10^6)(1.5\times10^{-6})}{(1.92)(0.05)} = 1.25\times10^3 \text{ N·m} = 1.25 \text{ kN·m}$$

(b) $$M = \frac{\sigma I}{Kc} = \frac{(80\times10^6)(1.5\times10^{-6})}{(1.57)(0.050)} = 1.53\times10^3 \text{ N·m} = 1.53 \text{ kN·m}$$ ◄

better (a) than (b)

PROBLEM 4.57



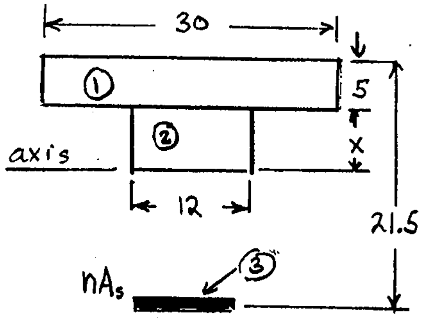
4.57 Knowing that the bending moment in the reinforced concrete beam shown is +150 kip·ft and that the modulus of elasticity is 3.75×10^6 psi for the concrete and 30×10^6 psi for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{30\times10^6}{3.75\times10^6} = 8.0$$

$$A_s = 4\,\frac{\pi}{4}d^2 = 4\left(\frac{\pi}{4}\right)(1)^2 = 3.1416 \text{ in}^2$$

$$nA_s = 25.133 \text{ in}^2$$



Locate the neutral axis

$$(30)(5)(x+2.5) + 12\,x\,\frac{x}{2} - (25.133)(16.5 - x) = 0$$

$$150x + 375 + 6x^2 - 414.69 + 25.133x = 0$$

$$6x^2 + 175.133x - 39.69 = 0$$

Solve for x $\quad x = \dfrac{-175.133 + \sqrt{(175.133)^2 + (4)(6)(39.69)}}{(2)(6)} = 0.225$ in.

$$16.5 - x = 16.275 \text{ in.}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(30)(5)^3 + (30)(5)(2.725)^2 = 1426.3 \text{ in}^4$$

$$I_2 = \frac{1}{3}b_2x^3 = \frac{1}{3}(12)(0.225)^3 = 0.1 \text{ in}^4$$

$$I_3 = nA_sd_3^2 = (25.133)(16.275)^2 = 6657.1 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 8083.5 \text{ in}^4$$

$\sigma = -\dfrac{nMy}{I}$ where $M = 150$ kip·ft $= 1800$ kip·in.

(a) Steel $\quad n = 8.0,\ y = -16.275$ in

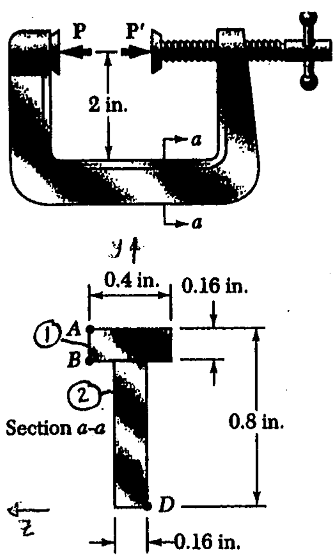
$$\sigma = -\frac{(8.0)(1800)(-16.275)}{8083.5} = 29.0 \text{ ksi}$$

(b) Concrete $\quad n = 1.0,\ y = 5.225$ in

$$\sigma = -\frac{(1.0)(1800)(5.225)}{8083.5} = -1.163 \text{ ksi}$$

PROBLEM 4.133

4.133 Knowing that the clamp shown has been tightened until $P = 75$ lb, determine in section *a-a* (*a*) the stress at point *A*, (*b*) the stress at point , (*c*) the location of the neutral axis.



SOLUTION

Locate centroid

Part	A, in²	$\bar{y}_o$, in	$A\bar{y}_o$, in³
①	0.064	0.72	0.04608
②	0.1024	0.32	0.03277
Σ	0.1664		0.07885

$$\bar{Y}_o = \frac{\Sigma A\bar{y}_o}{\Sigma A} = \frac{0.07885}{0.1664} = 0.4739 \text{ in.}$$

The centroid lies 0.4739 in. above point D.

Bending couple $M = Pe$

$$e = -(2 + 0.8 - 0.4739) = -2.3261 \text{ in}$$

$$I_1 = \tfrac{1}{12}(0.4)(0.16)^3 + (0.064)(0.72 - 0.4739)^2 = 4.013 \times 10^{-3} \text{ in}^4$$

$$I_2 = \tfrac{1}{12}(0.16)(0.64)^3 + (0.1024)(0.4739 - 0.32)^2 = 5.921 \times 10^{-3} \text{ in}^4$$

$$I = I_1 + I_2 = 9.934 \times 10^{-3} \text{ in}^4$$

(a) Stress at point A: $y = 0.8 - 0.4739 = 0.3261$ in

$$\sigma_A = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pey}{I} = \frac{75}{0.1664} - \frac{(75)(-2.3261)(0.3261)}{9.934 \times 10^{-3}}$$

$$= 6.18 \times 10^3 \text{ psi} = 6.18 \text{ ksi}$$

(b) Stress at point D: $y = -0.4739$ in. $= 0.1661$ in

$$\sigma_D = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pey}{I} = \frac{75}{0.1664} - \frac{(75)(-2.3261)(-0.4739)}{9.934 \times 10^{-3}}$$

$$= -7.87 \times 10^3 \text{ psi} = -7.87 \text{ ksi}$$

(c) Location of neutral axis $\sigma = 0$

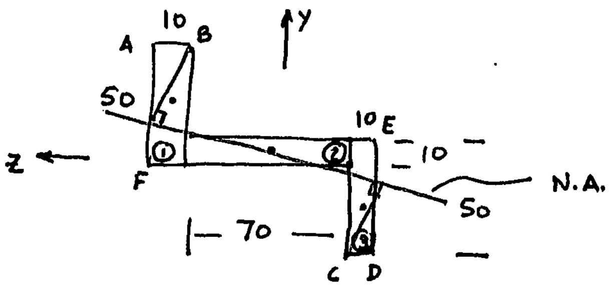
$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pey}{I} = 0 \qquad \frac{ey}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{9.934 \times 10^{-3}}{(0.1664)(-2.3261)} = -0.0257 \text{ in}$$

The neutral axis lies $0.4739 - 0.0257 = 0.448$ in. above point D.

NA

4.170



$$I_{zz} = 2\underbrace{\left(\frac{1}{12}\cdot 10\cdot 50^3 + 50\cdot 10\cdot 20^2\right)}_{1+3} + \underbrace{\frac{1}{12}\cdot 70\cdot 10^3}_{2} = .6142\times 10^6\ mm^4 \quad \text{or} \quad 0.6142\times 10^{-6}\ m^4$$

$$I_{yy} = 2\underbrace{\left(\frac{1}{12}\cdot 50\cdot 10^3 + 50\cdot 10\cdot 40^2\right)}_{1+3} + \underbrace{\frac{1}{12}\cdot 10\cdot 70^3}_{2} = 1.8942\times 10^6\ mm^4 \quad \text{or} \quad 1.8942\times 10^{-6}\ m^4$$

$$I_{yz} = \underbrace{0 + 50\cdot 10\cdot 40\cdot 20}_{1} + \underbrace{0}_{2} + 0 + 50\cdot 10\cdot 40\cdot 20 = 8\times 10^5\ mm^4 \quad \text{or} \quad 8\times 10^{-7}\ m^4$$

$$\sigma = -\frac{(+I_y M_z + M_y I_{yz})\,y}{I_y I_z - I_{yz}^2} + \frac{(M_y I_z + I_{yz} M_z)\,z}{I_y I_z - I_{yz}^2}$$

here $M_z = M_o$ so no M_y

$$\sigma_x = -\frac{(I_y M_o)\,y}{I_y I_z - I_{yz}^2} + \frac{I_{yz} M_o\, z}{I_y I_z - I_{yz}^2}$$

Neutral axis $\sigma_x = 0$ means

$-I_y \cdot y + I_{yz} z = 0$

$-1.8942y + 0.8z = 0$

since B & C furthest away from neutral axis, then max. σ should occur there

@ A	$y = .045$	$z = .045$	E =	$y = .005$	$z = -.045$
B	$y = .045$	$z = .035$	C =	$y = -.045$	$z = -.035$
F	$y = -.005$	$z = .045$	D	$y = -.045$	$z = -.045$

$$\sigma_x = \frac{M_o\left[-I_y\, y + I_{yz}\, z\right]}{I_y I_z - I_{yz}^2}$$

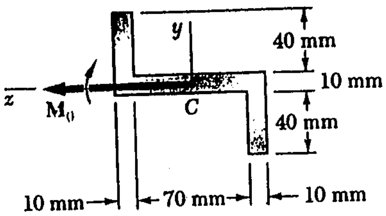
Iy	Iz	Iyz		IyIz-Iyz^2 = denom
1.89E-06	6.14E-07	8.00E-07		5.23E-13

Mo	Moly/deno	Molyz/denom		y	z	sigma
732	2.65E+09	1.12E+09	a	0.045	0.045	-68860782
			b	0.045	0.035	-80048789
			c	-0.045	-0.035	80048788.6
			d	-0.045	-0.045	68860782
			e	0.005	-0.045	-63591231
			f	-0.005	0.045	63591230.9

solution 732 N-m

2nd method.

PROBLEM 4.170

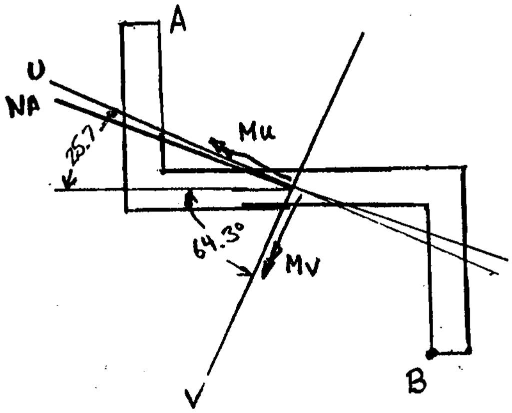


4.170 The Z section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given:* $I_{max} = 2.28 \times 10^6\ mm^4$, $I_{min} = 0.23 \times 10^6\ mm^4$, principal axes 25.7° ↘ and 64.3° ↗.

SOLUTION

$$I_v = I_{max} = 2.28 \times 10^6\ mm^4 = 2.28 \times 10^{-6}\ m^4$$

$$I_u = I_{min} = 0.23 \times 10^6\ mm^4 = 0.23 \times 10^{-6}\ m^4$$



$$M_v = M_o \cos 64.3°$$

$$M_u = M_o \sin 64.3°$$

$$\theta = 64.3°$$

$$\tan \varphi = \frac{I_v}{I_u} \tan \theta$$

$$= \frac{2.28 \times 10^{-6}}{0.23 \times 10^{-6}} \tan 64.3° = 20.597$$

$$\varphi = 87.22°$$

Points A and B are farthest from the neutral axis.

$$U_B = y_B \cos 64.3° + z_B \sin 64.3° = (-45)\cos 64.3° + (-35)\sin 64.3°$$

$$= -51.05\ mm$$

$$V_B = z_B \cos 64.3° - y_B \sin 64.3° = (-35)\cos 64.3° - (-45)\sin 64.3°$$

$$= +25.37\ mm$$

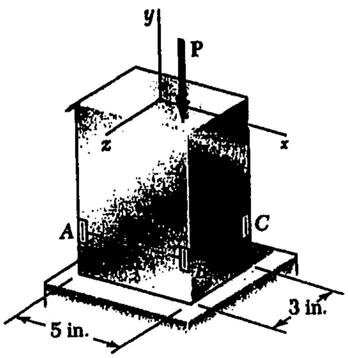
$$\sigma_B = -\frac{M_v U_B}{I_v} + \frac{M_u V_B}{I_u}$$

$$80 \times 10^6 = -\frac{(M_o \cos 64.3°)(-51.05 \times 10^{-3})}{2.28 \times 10^{-6}} + \frac{(M_o \sin 64.3°)(25.37 \times 10^{-3})}{0.23 \times 10^{-6}}$$

$$= 93.81 \times 10^3\ M_o$$

$$M_o = \frac{80 \times 10^6}{109.1 \times 10^3} = 733\ N \cdot m$$

PROBLEM 4.211



4.211 A single vertical force **P** is applied to a short steel post as shown. Gages located at *A*, *B*, and *C* indicate the following strains:

$\epsilon_A = -500\ \mu \qquad \epsilon_B = -1000\ \mu \qquad \epsilon_C = -200\ \mu$

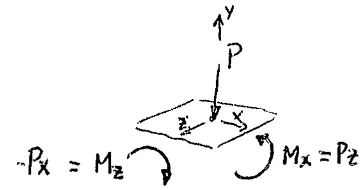
Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the magnitude of **P**, (*b*) the line of action of **P**, (*c*) the corresponding strain at the hidden edge of the post, where $x = -2.5$ in. and $z = -1.5$ in.

SOLUTION

$$I_x = \frac{1}{12}(5)(3)^3 = 11.25 \text{ in}^4$$

$$I_z = \frac{1}{12}(3)(5)^3 = 31.25 \text{ in.}^4$$

$$A = (5)(3) = 15 \text{ in}^2$$



$M_x = Pz \qquad M_z = -Px \qquad$ here we assume P is located at $(x, 0, z)$

$x_A = -2.5$ in, $x_B = 2.5$ in, $x_C = 2.5$ in, $x_D = -2.5$ in.

$z_A = 1.5$ in, $z_B = 1.5$ in, $z_C = -1.5$ in, $z_D = -1.5$ in

$$\sigma_A = E\varepsilon_A = (29\times10^6)(-500\times10^{-6}) = -14500 \text{ psi} = -14.5 \text{ ksi}$$

$$\sigma_B = E\varepsilon_B = (29\times10^6)(-1000\times10^{-6}) = -29000 \text{ psi} = -29 \text{ ksi}$$

$$\sigma_C = E\varepsilon_C = (29\times10^6)(-200\times10^{-6}) = -5800 \text{ psi} = -5.8 \text{ ksi}$$

$$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -0.06667P - 0.13333M_x - 0.08M_z \qquad (1)$$

$$\sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -0.06667P - 0.13333M_x + 0.08M_z \qquad (2)$$

$$\sigma_C = -\frac{P}{A} - \frac{M_x z_C}{I_x} + \frac{M_z x_C}{I_z} = -0.06667P + 0.13333M_x + 0.08M_z \qquad (3)$$

Substituting the values for σ_A, σ_B, and σ_C into (1), (2), and (3) and solving the simultaneous equations gives

$M_x = 87$ kip·in, $M_z = -90.625$ kip·in, $P = 152.25$ kips

$$x = -\frac{M_z}{P} = -\frac{-90.625}{152.25} = 0.595 \text{ in.}$$

$$z = \frac{M_x}{P} = \frac{87}{152.25} = 0.571 \text{ in.}$$

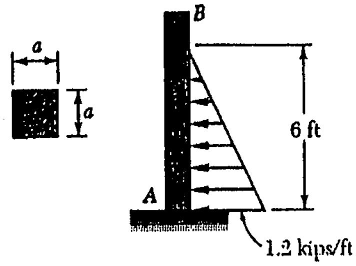
$$\sigma_D = -\frac{P}{A} - \frac{M_x z_D}{I_x} + \frac{M_z x_D}{I_z} = -0.06667P + 0.13333M_x - 0.08M_z$$

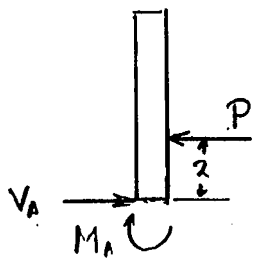
$$= -(0.06667)(152.25) + (0.13333)(87) - (0.08)(-90.625)$$

$$= 8.70 \text{ ksi}$$

PROBLEM 5.77

5.77 and 5.78 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1.75 ksi.





SOLUTION

Equivalent concentrated load

$$P = (\tfrac{1}{2})(6)(1.2) = 3.6 \text{ kips}$$

Bending moment at A

$$M_A = (2)(3.6) = 7.2 \text{ kip}\cdot\text{ft} = 86.4 \text{ kip}\cdot\text{in}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{86.4}{1.75} = 49.37 \text{ in}^3$$

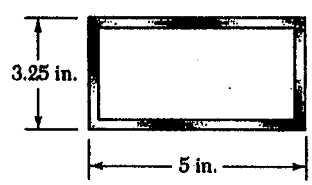
For a square section $\quad S = \frac{1}{6}a^3$

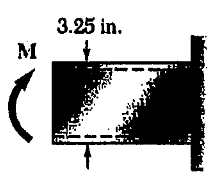
$$a = \sqrt[3]{6\,S}$$

$$a_{min} = \sqrt[3]{(6)(49.37)} = 6.67 \text{ in.}$$

PROBLEM 4.220

4.220 Knowing that the hollow beam shown has a uniform wall thickness of 0.25 in. determine (*a*) the largest couple that can be applied without exceeding the allowable stress of 20 ksi, (*b*) the corresponding radius of curvature of the beam.





SOLUTION $\qquad E = 10.6 \times 10^6$ psi

$$I = \frac{1}{12} b_o h_o^3 - \frac{1}{12} b_i h_i^3 = \frac{1}{12}(5)(3.25)^3 - \frac{1}{12}(4.5)(2.75)^3 = 6.5046 \text{ in}^4$$

$$c = \frac{3.25}{2} = 1.625 \text{ in.}$$

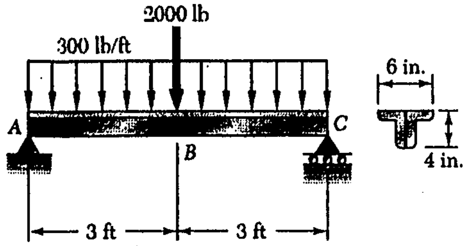
(a) $$\sigma_{max} = \frac{Mc}{I} \quad \therefore \quad M = \frac{\sigma_{max} I}{c} = \frac{(20)(6.5046)}{1.625} = 80.1 \text{ kip}\cdot\text{in.}$$

(b) $$\varepsilon_{max} = \frac{c}{\rho} = \frac{\sigma_{max}}{E} \quad \therefore \quad \rho = \frac{Ec}{\sigma_{max}} = \frac{(10.6 \times 10^6)(1.625)}{20 \times 10^3}$$

$$= 861 \text{ in} = 71.8 \text{ ft.}$$

PROBLEM 5.91

5.91 Two L 4 × 3 rolled-steel angles are bolted together to support the loading shown. Knowing that the allowable normal stress for the steel used is 24 ksi, determine the minimum angle thickness that can be used.



SOLUTION

By symmetry $A = C$

$+\sum F_y = 0 \qquad A + C - 2000 - (6)(300) = 0$

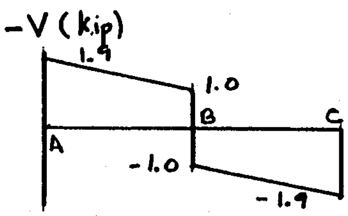
$A = C = 1900 \text{ lb.}$

Shear: $V_A = 1900 \text{ lb.} = 1.9 \text{ kips}$

$V_{B^-} = 1900 - (3)(300) = 1000 \text{ lb} = 1 \text{ kip}$

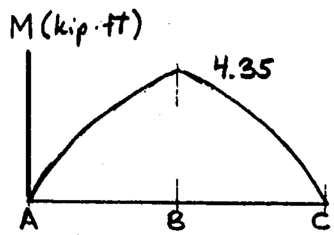
$V_{B^+} = 1000 - 2000 = -1000 \text{ lb} = -1 \text{ kip}$

$V_C = -1000 - (3)(300) = -1900 \text{ lb} = -1.9 \text{ kip}$



Areas: A to B $(\frac{1}{2})(3)(1.9+1) = 4.35$ kip·ft trapezoidal area

B to C $(\frac{1}{2})(3)(-1-1.9) = -4.35$ kip ft.



Bending moments: $M_A = 0$

$M_B = 0 + 4.35 = 4.35$ kip·ft

$M_C = 4.35 - 4.35 = 0$

Maximum $|M| = 4.35$ kip·ft $= 52.2$ kip-in

$\sigma_{all} = 24$ ksi

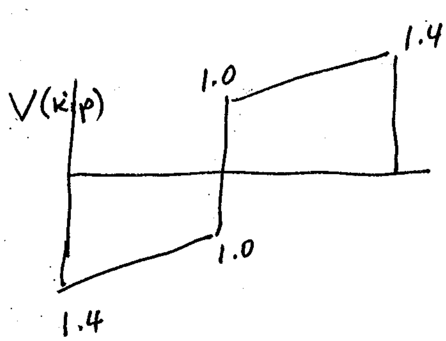
For section consisting of two angles $\quad S_{min} = \dfrac{|M|}{\sigma_{all}} = \dfrac{52.2}{24} = 2.175 \text{ in}^3$

For each angle $\quad S_{min} = (\frac{1}{2})(2.175) = 1.0875 \text{ in}^3$

Angle section	S (in³)
L 4×3×½	1.89
L 4×3×⅜	1.46
L 4×3×¼	1.00

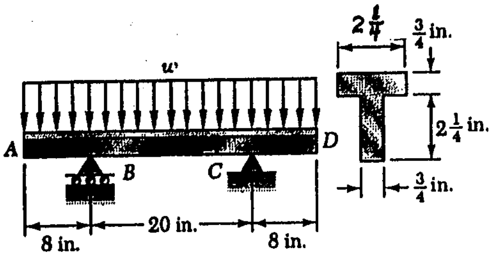
Smallest allowable thickness

$t = \frac{3}{8}$ in.



PROBLEM 5.97

5.97 Determine the largest permissible uniformly distributed load w for the beam shown, knowing that the allowable normal stress is +12 ksi in tension and − 19,5 ksi in compression.



SOLUTION

Reactions: $B + C - 36w = 0 \qquad B = C = 18 w$

Shear: $V_A = 0$

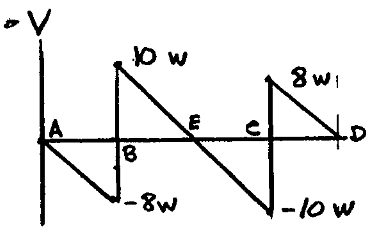
$V_{B^-} = 0 - 8w = -8w$

$V_{B^+} = -8w + 18w = 10w$

$V_{C^-} = 10w - 20w = -10w$

$V_{C^+} = -10w + 18w = 8w$

$V_D = 8w - 8w = 0$



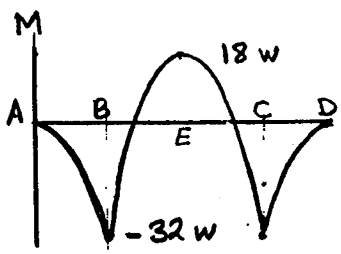
Areas: A to B $(\tfrac{1}{2})(8)(-8w) = -32w$

B to E $(\tfrac{1}{2})(10)(10w) = 50w$

Bending moments: $M_A = 0$

$M_B = 0 - 32w = -32w$

$M_E = -32w + 50w = 18w$

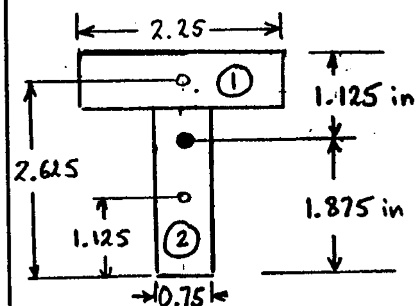


Centroid and moment of inertia

Part	A (in²)	$\bar{y}$ (in)	$A\bar{y}$ (in³)	d in	Ad^2 (in⁴)	$\bar{I}$ (in⁴)
①	1.6875	2.625	4.4297	0.75	0.9492	0.0791
②	1.6875	1.125	1.8984	0.75	0.9492	0.7119
Σ	3.375		6.3281		1.8984	0.7910

$$\bar{Y} = \frac{6.3281}{3.375} = 1.875 \text{ in}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 2.6894 \text{ in}^4$$



Top: $y = 1.125$

Bottom: $y = -1.875$

$I/y = 2.3906 \text{ in}^3$

$I/y = -1.4343 \text{ in}^3$

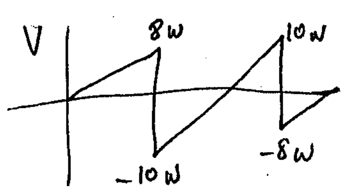
section modulus

Bending moment limits $\qquad M = -\sigma I/y = -\sigma S$

Tension at B and C	$-(12)(2.3906) =$	-28.687 kip·in
Comp. at B and C	$-(-19.5)(-1.4343) =$	-27.969 kip·in.
Tension at E	$-(12)(-1.4343) =$	17.212 kip·in
Compression at E	$-(-19.5)(2.3906) =$	46.6 kip·in

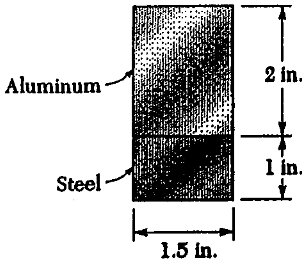
Allowable load w

B & C	$-32w = -27.969$	$w = 0.874$ kip/in
E	$18w = 17.212$	$w = 0.956$ kip/in



Smallest $\quad w = 0.874$ kip/in $= 10.49$ kip/ft.

PROBLEM 6.59

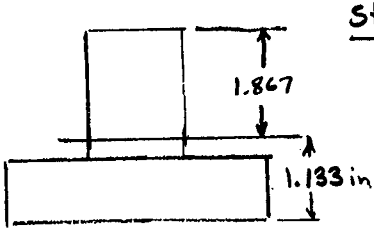


6.59 and 6.60 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum stress in the beam. (*Hint.* Use the method indicated in Prob. 6.56.)

SOLUTION

$n = 1$ in aluminum $\qquad n = \dfrac{29 \times 10^6 \text{ psi}}{10.6 \times 10^6 \text{ psi}} = 2.7358$ in steel

Part	nA (in^2)	$\bar{y}$ (in)	$nA\bar{y}$ (in^3)	d (in)	nAd^2 (in^4)	$n\bar{I}$ (in^4)
Alum.	3.0	2.0	6.0	0.8665	2.2525	1.0
Steel	4.1038	0.5	2.0519	0.6335	1.6469	0.3420
Σ	7.1038		8.0519		3.8994	1.3420



$$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma nA} = \frac{8.0519}{7.1038} = 1.1335$$

$$I = \Sigma nAd^2 + \Sigma n\bar{I} = 5.2414 \text{ in}^4$$

(a) At the bonded surface $\qquad Q = (1.5)(2)(0.8665) = 2.5995 \text{ in}^3$

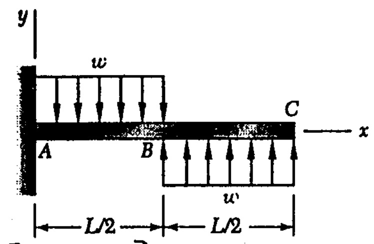
$$\tau = \frac{VQ}{It} = \frac{(4)(2.5995)}{(5.2414)(1.5)} = 1.323 \text{ ksi}$$

(b) At the neutral axis $\qquad Q = (1.5)(1.8665)\left(\frac{1.8665}{2}\right) = 2.6129 \text{ in}^2$

$$\tau_{max} = \frac{VQ}{It} = \frac{(4)(2.6129)}{(5.2414)(1.5)} = 1.329 \text{ ksi}$$

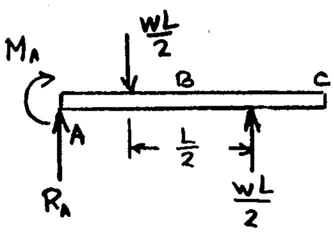
PROBLEM 9.7

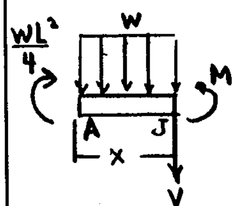
9.7 For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.



$[x = 0,\ y = 0]$

$[x = 0,\ \frac{dy}{dx} = 0]$





SOLUTION

Using ABC as a free body

$+\uparrow \Sigma F_y = 0 \qquad R_A - \frac{wL}{2} + \frac{wL}{2} = 0 \qquad R_A = 0$

$+\circlearrowleft M_A = 0 \qquad -M_A + \left(\frac{wL}{2}\right)\left(\frac{L}{2}\right) = 0 \qquad M_A = \frac{wL^2}{4}$

Using AJ as a free body (Portion AB only)

$\circlearrowleft \Sigma M_J = 0 \qquad -\frac{wL^2}{4} + (wx)\frac{x}{2} + M = 0$

$M = \frac{1}{4}wL^2 - \frac{1}{2}wx^2$

$EI\frac{d^2y}{dx^2} = \frac{1}{4}wL^2 - \frac{1}{2}wx^2$

$EI\frac{dy}{dx} = \frac{1}{4}wL^2x - \frac{1}{6}wx^3 + C_1$

$[x = 0,\ \frac{dy}{dx} = 0] \qquad 0 = 0 - 0 + C_1 \qquad C_1 = 0$

$EIy = \frac{1}{8}wL^2x^2 - \frac{1}{24}wx^4 + C_1x + C_2$

$[x = 0,\ y = 0] \qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

(a) Elastic curve

$y = \frac{w}{EI}\left(\frac{1}{8}L^2x^2 - \frac{1}{24}x^4\right)$ ◄

$\frac{dy}{dx} = \frac{w}{EI}\left(\frac{1}{4}L^2x - \frac{1}{6}x^3\right)$

(b) y at $x = \frac{L}{2}$

$y_B = \frac{w}{EI}\left\{\frac{1}{8}L^2\left(\frac{L}{2}\right)^2 - \frac{1}{24}\left(\frac{L}{2}\right)^4\right\} = \frac{wL^4}{EI}\left\{\frac{1}{32} - \frac{1}{384}\right\}$

$= \frac{11\,wL^4}{384\,EI} \qquad y_B = \frac{11\,wL^4}{384\,EI}\ \uparrow$ ◄

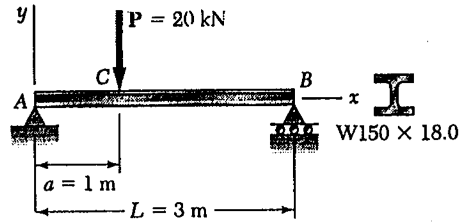
(c) $\frac{dy}{dx}$ at $x = \frac{L}{2}$

$\theta_B = \frac{w}{EI}\left\{\frac{1}{4}L^2\left(\frac{L}{2}\right) - \frac{1}{6}\left(\frac{L}{2}\right)^3\right\} = \frac{wL^3}{EI}\left\{\frac{1}{8} - \frac{1}{48}\right\}$

$= \frac{5\,wL^3}{48\,EI} \qquad \theta_B = \frac{5\,wL^3}{48\,EI}$ ◄

PROBLEM 9.14

9.13 and 9.14 For the beam and loading shown, determine the deflection at point C. Use $E = 200$ GPa.



$[x=0, y=0]$ $\qquad [x=L, y=0]$

$[x=a, y=y]$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$

SOLUTION

Let $b = L - a$

Reactions: $R_A = \frac{Pb}{L}\uparrow$, $R_B = \frac{Pa}{L}\uparrow$

Bending moments

$0 < x < a \qquad M = \frac{Pb}{L}x$

$a < x < L \qquad M = \frac{P}{L}[bx - L(x-a)]$

$0 < x < a$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}(bx)$$

$$EI\frac{dy}{dx} = \frac{P}{L}\left(\tfrac{1}{2}bx^2\right) + C_1 \qquad (1)$$

$$EI\,y = \frac{P}{L}\left(\tfrac{1}{6}bx^3\right) + C_1x + C_2 \qquad (2)$$

$a < x < L$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}[bx - L(x-a)]$$

$$EI\frac{dy}{dx} = \frac{P}{L}\left[\tfrac{1}{2}bx^2 - \tfrac{1}{2}L(x-a)^2\right] + C_3 \qquad (2)$$

$$EI\,y = \frac{P}{L}\left[\tfrac{1}{6}bx^3 - \tfrac{1}{6}L(x-a)^3\right] + C_3x + C_4 \qquad (4)$$

$[x=0, y=0]$ Eq (2) $\qquad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$ Eqs. (1) and (3) $\qquad \frac{P}{L}\left(\cancel{\tfrac{1}{2}ba^2}\right) + C_1 = \frac{P}{L}\left[\cancel{\tfrac{1}{2}ba^2} + 0\right] + C_3 \quad \therefore \quad C_3 = C_1$

$[x=a, y=y]$ Eqs (2) and (4) $\qquad \frac{P}{L}\left(\cancel{\tfrac{1}{6}ba^3}\right) + C_1a + C_2$

$$= \frac{P}{L}\left[\cancel{\tfrac{1}{6}ba^3} + 0\right] + \cancel{C_1a} + C_4 \qquad C_4 = C_2 = 0$$

$[x=L, y=0]$ Eq. (4) $\qquad \frac{P}{L}\left[\tfrac{1}{6}bL^3 - \tfrac{1}{6}L(L-a)^3\right] + C_3L = 0$

$$C_1 = C_3 = \frac{P}{L}\left[\tfrac{1}{6}(L-a)^3 - \tfrac{1}{6}bL^2\right] = \frac{P}{L}\left(\tfrac{1}{6}b^3 - \tfrac{1}{6}bL^2\right)$$

Make $x = a$ in Eq (2)

$$y_C = \frac{P}{EIL}\left[\tfrac{1}{6}ba^3 + \tfrac{1}{6}b^3a - \tfrac{1}{6}bL^2a\right] = \frac{P(ba^3 + b^3a - L^2ab)}{6EIL}$$

Data: $P = 20\times10^3$ N, $E = 200\times10^9$ Pa, $I = 9.17\times10^6\ \text{mm}^4 = 9.17\times10^{-6}\ \text{m}^4$

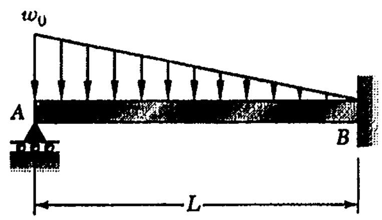
$L = 3$ m, $a = 1$ m, $b = 2$ m

$$y_C = \frac{(20\times10^3)[(2)(1)^3 + (2)^3(1) - (3)^2(1)(2)]}{(6)(200\times10^9)(9.17\times10^{-6})(3)} = -4.85\times10^{-3}\ \text{m}$$

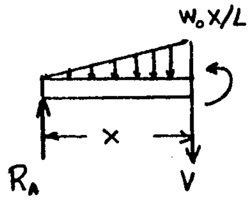
i.e. 4.85 mm $\downarrow$

PROBLEM 9.20

9.17 through 9.20 For the beam and loading shown, determine the reaction at the roller support.



$[x=0, y=0]$ $\quad$ $[x=L, y=0]$ $\quad$ $[x=L, \frac{dy}{dx}=0]$



SOLUTION

Reactions are statically indeterminate.

Boundary conditions are shown at left.

$$w = \frac{w_0}{L}(L-x)$$

$$\frac{dV}{dx} = +w = +\frac{w_0}{L}(L-x)$$

$$\frac{dM}{dx} = -V = -\frac{w_0}{L}\left(Lx - \tfrac{1}{2}x^2\right) + R_A$$

$$M = -\frac{w_0}{L}\left(\tfrac{1}{2}Lx^2 - \tfrac{1}{6}x^3\right) + R_A x$$

$$EI\frac{d^2y}{dx^2} = -\frac{w_0}{L}\left(\tfrac{1}{2}Lx^2 - \tfrac{1}{6}x^3\right) + R_A x$$

$$EI\frac{dy}{dx} = -\frac{w_0}{L}\left(\tfrac{1}{6}Lx^3 - \tfrac{1}{24}x^4\right) + \tfrac{1}{2}R_A x^2 + C_1$$

$$EI\,y = -\frac{w_0}{L}\left(\tfrac{1}{24}Lx^3 - \tfrac{1}{120}x^5\right) + \tfrac{1}{6}R_A x^3 + C_1 x + C_2$$

$[x=0, y=0]$ $\qquad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L, \frac{dy}{dx}=0]$ $\qquad -\frac{w_0}{L}\left(\tfrac{1}{6}L^4 - \tfrac{1}{24}L^4\right) + \tfrac{1}{2}R_A L^2 + C_1 = 0$

$$C_1 = \tfrac{1}{8}w_0 L^3 - \tfrac{1}{2}R_A L^2$$

$[x=L, y=0]$ $\qquad -\frac{w_0}{L}\left(\tfrac{1}{24}L^4 - \tfrac{1}{120}L^4\right) + \tfrac{1}{6}R_A L^3 + \left(\tfrac{1}{8}w_0 L^3 - \tfrac{1}{2}R_A L^2\right)L = 0$

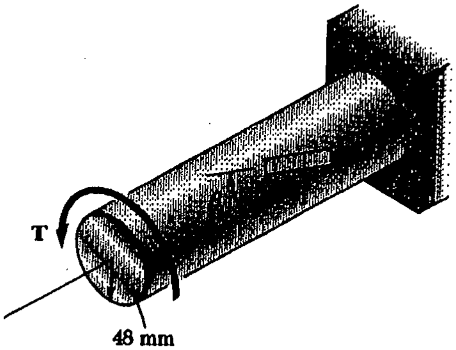
$$\left(\tfrac{1}{2} - \tfrac{1}{6}\right)R_A = \left(\tfrac{1}{8} - \tfrac{1}{24} + \tfrac{1}{120}\right)w_0 L$$

$$\tfrac{1}{3}R_A = \tfrac{11}{120}w_0 L \qquad R_A = \tfrac{11}{40}w_0 L \uparrow$$

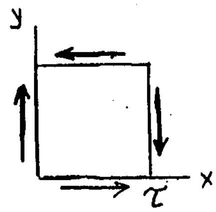
PLEASE ADD TO EMA3702
FOLDER

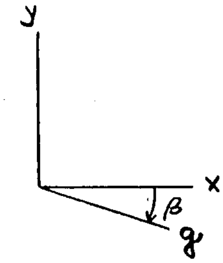
PROBLEM 7.152

7.152 A single strain gage is cemented to a solid 96-mm-diameter aluminum shaft at an angle $\beta = 20°$ with a line parallel to the axis of the shaft. Knowing that $G = 27$ GPa, determine the torque **T** corresponding to a gage reading of 400 μ.



SOLUTION





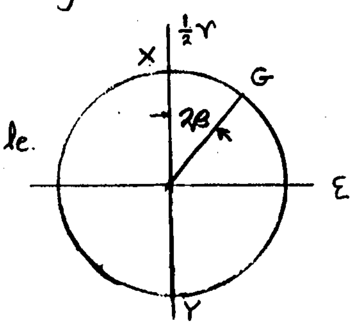
$$\tau = \frac{Tc}{J} \qquad J = \frac{\pi}{2}c^4 \qquad \gamma = \frac{\tau}{G}$$

$$\sigma_x = \sigma_y = 0 \qquad \varepsilon_x = \varepsilon_y = 0$$

Sketch Mohr's circle for strain.

Gage direction is β clockwise from x

Point G is 2β clockwise from X on Mohr's circle.



$$\varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) = 0$$

$$R = \tfrac{1}{2}\gamma_{xy}$$

$$\varepsilon_g = \varepsilon_{ave} + R\sin 2\beta = \tfrac{1}{2}\gamma_{xy}\sin 2\beta = \frac{\tau_{xy}}{2G}\sin 2\beta$$

$$= \frac{Tc}{2GJ}\sin 2\beta =$$

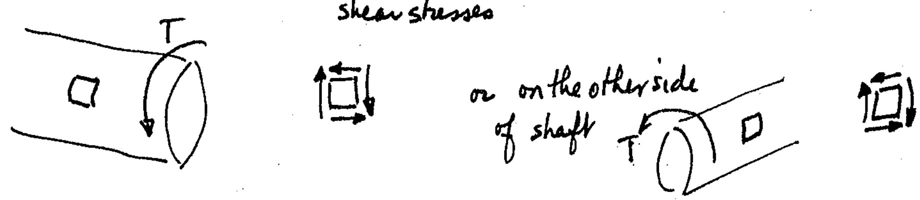
Solving for T

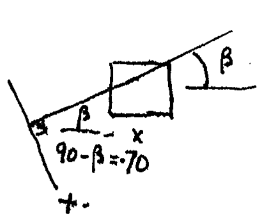
$$T = \frac{2GJ\varepsilon_g}{c\sin 2\beta} = \frac{\pi G c^3 \varepsilon_g}{\sin 2\beta}$$

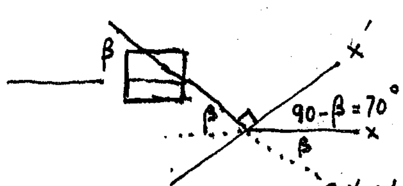
$$T = \frac{\pi(27\times10^9)(48\times10^{-3})^3(400\times10^{-6})}{\sin 40°} = 5.84\times10^3\ \text{N}\cdot\text{m}$$

$$= 5.84\ \text{kN}\cdot\text{m}$$

OR since torsion only produces shear stress then an element on the side only shows shear stresses







$$\tau_{x'y'} = \tau_{xy}\cos 2\theta - \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta$$

$$G\gamma_{x'y'} = \tau_{x'y'} = \tau_{xy}\cos 140°$$

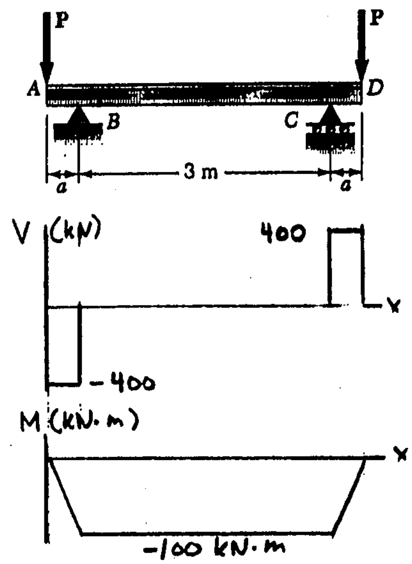
$$\therefore\ \gamma_{x'y'} = \frac{\tau_{xy}}{G}\cos 140° = \frac{Tc}{JG}\cos 140°$$

$\cos(140) = \cos(180-40) = \cos 180 \cos 40 = -\cos 40°$

$$T = \frac{\gamma_{x'y'}}{-\cos 40°}\cdot\frac{JG}{c}$$

PROBLEM 8.1

8.1 An overhanging W250 × 58 rolled-steel beam supports two loads as shown. Knowing that P = 400 kN, a = 0.25 m, and σ_{all} = 250 MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of aflange and the web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned..



$|V|_{max} = 400 \text{ kN} = 400 \times 10^3 \text{ N}$

$|M|_{max} = (400 \times 10^3)(0.25) = 100 \times 10^3 \text{ N·m}$

For W 250 × 58 rolled steel section

$d = 252$ mm. $\quad b_f = 203$ mm $\quad t_f = 13.5$ mm

$t_w = 8.0$ mm $\quad I_z = 87.3 \times 10^6 \text{ mm}^4 \quad S_z = 693 \times 10^3 \text{ mm}^3$

$c = \frac{1}{2}d = 126$ mm $\quad y_b = c - t_f = 112.5$ mm.

(a) $\sigma_m = \dfrac{|M|_{max}}{S_z} = \dfrac{100 \times 10^3}{693 \times 10^{-6}} = 144.3 \times 10^6 \text{ Pa} = 144.3 \text{ MPa}$

$\sigma_b = \dfrac{y_b}{c}\sigma_m = \dfrac{112.5}{126}(144.3) = 128.84 \text{ MPa}$

$A_f = b_f t_f = (203)(13.5) = 2740.5 \text{ mm}^2$

$\bar{y}_f = \frac{1}{2}(c + y_b) = 119.25 \text{ mm}^3$

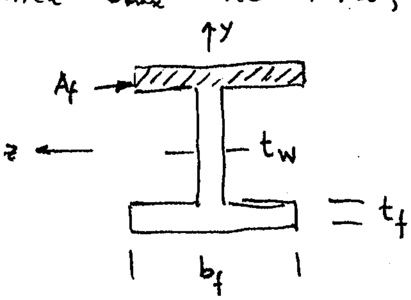
$Q_b = A_f \bar{y}_f = 326.80 \times 10^3 \text{ mm}^3 = 326.80 \times 10^{-6} \text{ m}^3$

$\tau_{xy} = \dfrac{|V|_{max} Q_b}{I_z t_w} = \dfrac{(400 \times 10^3)(326.80 \times 10^{-6})}{(87.3 \times 10^{-6})(8 \times 10^{-3})} = 187.2 \times 10^6 \text{ Pa} = 187.2 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 197.97 \text{ MPa}$

(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 262 \text{ MPa}$

(c) Since $\sigma_{max} > 250$ MPa, W250 × 58 is <u>not acceptable</u>.



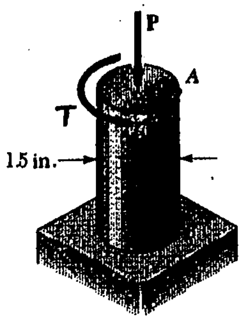
$S_z = \dfrac{I_{zz}}{c} = \dfrac{I_{zz}}{d/2}$

when σ_x is max $\tau = 0$

$\therefore \sigma_{max} = \dfrac{\sigma_x + \cancel{\sigma_y}}{2} + \sqrt{\left(\dfrac{\sigma_x - \cancel{\sigma_y}}{2}\right)^2 + \cancel{\tau_{xy}^2}}$

$= \sigma_x$

PROBLEM 7.87



7.87 The 1.5-in-diameter shaft AB is made of a grade of steel for which the yield strength is $\sigma_Y = 42$ ksi.. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 60$ kips.

SOLUTION

$P = 60$ kips $\qquad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.5)^2 = 1.7671 \text{ in}^2$

$$\sigma_x = -\frac{P}{A} = -\frac{60}{1.7671} = -33.953 \text{ ksi}$$

$$\sigma_y = 0 \qquad \sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = \tfrac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\tfrac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$2\tau_{max} = 2R = \sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_Y$$

$$4\tau_{xy}^2 = \sigma_Y^2 - \sigma_x^2 \qquad \tau_{xy} = \tfrac{1}{2}\sqrt{\sigma_Y^2 - \sigma_x^2} = \tfrac{1}{2}\sqrt{42^2 - 33.953^2}$$

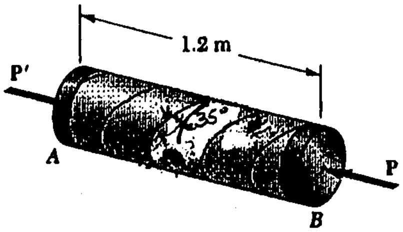
$$= 12.361 \text{ ksi}$$

From torsion $\quad \tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c}$

$$c = \tfrac{1}{2}d = 0.75 \text{ in} \qquad J = \tfrac{\pi}{2}c^4 = 0.49701 \text{ in}^4$$

$$T = \frac{(0.49701)(12.361)}{0.75} = 8.19 \text{ kip}\cdot\text{in}$$

PROBLEM 7.120



7.120 A pressure vessel of 250-mm inside diameter and 6-mm wall thickness is fabricated from a 1.2-m section of spirally welded pipe *AB* and is with two rigid end plates. The gage pressure inside the vessel is 2 MPa and 45-kN centric axial forces **P** and **P'** are applied to the end plates.. Determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$r = \frac{1}{2}d = 125 \text{ mm} \qquad t = 6 \text{ mm}$

$\sigma_1 = \frac{pr}{t} = \frac{(2)(125)}{6} = 41.67 \text{ MPa}$ hoop

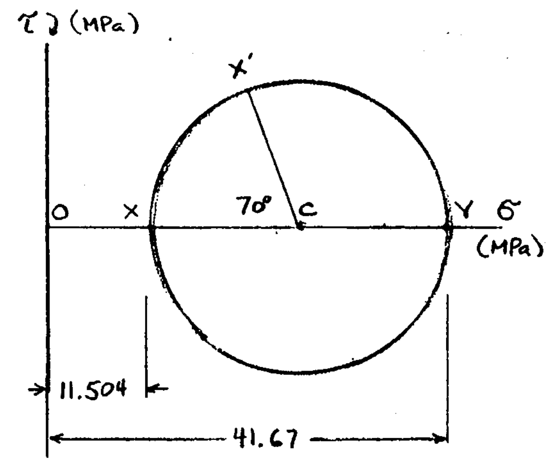
$\sigma_2 = \frac{pr}{2t} = \frac{(2)(125)}{(2)(6)} = 20.83 \text{ MPa}$ axial

$r_o = r + t = 125 + 6 = 131 \text{ mm} \qquad A = \pi(r_o^2 - r^2) = 4.825 \times 10^3 \text{ mm}^2 = 4.825 \times 10^{-3} \text{ m}^2$

$\sigma = -\frac{P}{A} = -\frac{45 \times 10^3}{4.825 \times 10^{-3}} = -9.326 \times 10^6 \text{ Pa} = -9.326 \text{ MPa}$

Total stresses: Longitudinal $\sigma_x = 20.83 - 9.326 = 11.504 \text{ MPa}$

Circumferential $\sigma_y = 41.67 \text{ MPa}$



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 26.585 \text{ MPa}$

$R = \frac{\sigma_x - \sigma_y}{2} = 15.081$

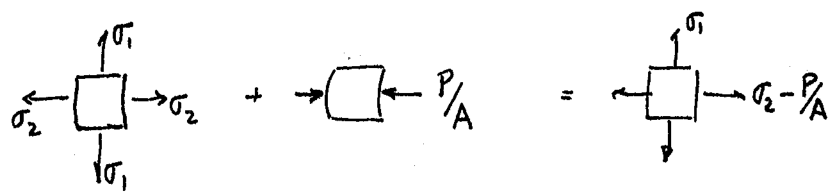
(a) $\sigma_{x'} = \sigma_{ave} + R\cos 70°$

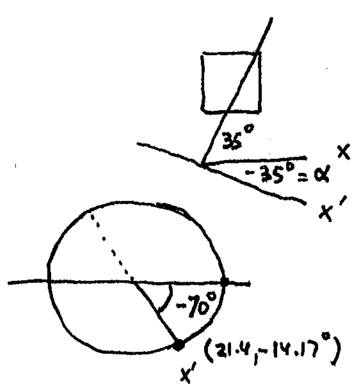
$= 26.585 - 15.081 \cos 70°$

$= 21.4 \text{ MPa}$ ◄

(b) $\tau_{x'y'} = R \sin 70° = 15.081 \sin 70°$

$= 14.17 \text{ MPa}$ ◄





$\sigma_x' = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\alpha + \cancel{\tau_{xy}} \sin 2\alpha$ is stress ⊥ to weld

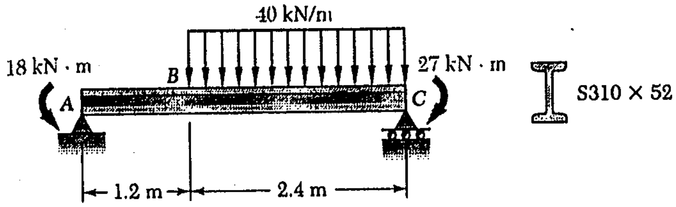
radius of mohr circle $= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \cancel{\tau_{xy}^2}^{\,0}} = \frac{\sigma_x - \sigma_y}{2}$ when $\tau_{xy} = 0$

so $\sigma_x' = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos(-70°) = 21.4 \text{ MPa}$

$\tau_{x'y'} = \tau_{xy}\cos 2\alpha - \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\alpha = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin(-70°) = 14.17$

PROBLEM 5.124

5.123 and 5.124 (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.



SOLUTION

$\circlearrowleft \Sigma M_C = 0$

$18 - 3.6 R_A + (1.2)(2.4)(40) - 27 = 0$

$R_A = 29.5 \text{ kN}$

$V = 29.5 - 40\langle x - 1.2\rangle^1 \text{ kN}$

Point D $\quad V = 0 \qquad 29.5 - 40(x_D - 1.2) = 0$

$x_D = 1.9375 \text{ m}$

since $V = \frac{dM}{dx}$ when $V = 0$ M is max or min

$M = -18 + 29.5x - 20\langle x - 1.2\rangle^2 \text{ kN·m}$

$M_A = -18 \text{ kN·m}$ check end point

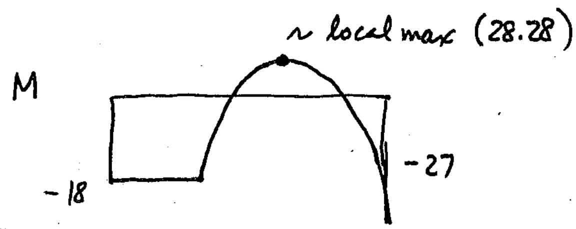
$M_D = -18 + (29.5)(1.9375) - (20)(0.7375)^2 = 28.278 \text{ kN·m}$

$M_C = -18 + (29.5)(3.6) - (20)(2.4)^2 = -27 \text{ kN·m}$ check other end point

Maximum $|M| = 28.278 \text{ kN·m}$ at $x = 1.9375 \text{ m}$ ◄

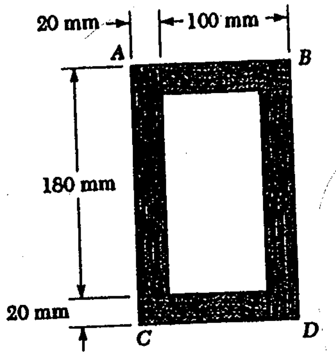
For S310 × 52 rolled steel section $\quad S = 625 \times 10^3 \text{ mm}^3 = 625 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{28.278 \times 10^3}{625 \times 10^{-6}} = 45.2 \times 10^6 \text{ Pa} = 45.2 \text{ MPa}$ ◄



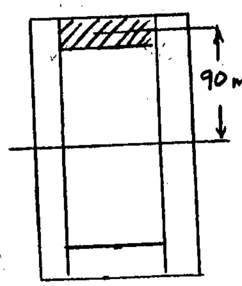
PROBLEM 6.33

6.33 Two 20 × 100-mm and two 20 × 180-mm boards are glued together as shown to form a 120 × 200-mm box beam. Knowing that the beam is subjected to a vertical shear of 3.5 kN, determine the average shearing stress in the glued joint (*a*) at *A*, (*b*) at *B*.



SOLUTION

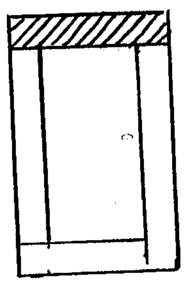
$$I = \frac{1}{12}(120)(200)^3 - \frac{1}{12}(80)(160)^3 = 52.693 \times 10^6 \ mm^4 = 52.693 \times 10^{-6} \ m^4$$



(a) $Q_A = (80)(20)(90) = 144 \times 10^3 \ mm^3 = 144 \times 10^{-6} \ m^3$

$t_A = (2)(20) = 40 \ mm = 0.040 \ m$

$$\tau_A = \frac{VQ_A}{I t_A} = \frac{(3.5 \times 10^3)(144 \times 10^{-6})}{(52.693 \times 10^{-6})(0.040)} = 239 \times 10^3 \ Pa = 239 \ kPa$$

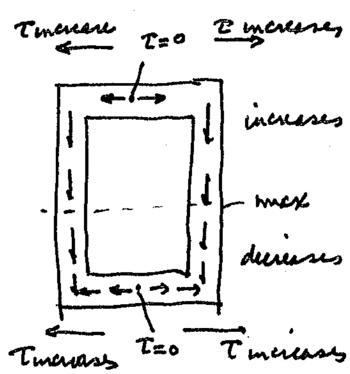


(b) $Q_B = (120)(20)(90) = 216 \times 10^3 \ mm^3 = 216 \times 10^{-6} \ m^3$

$t_B = (2)(20) = 40 \ mm = 0.040 \ m$

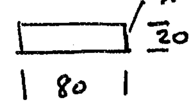
$$\tau_B = \frac{VQ_B}{I t_B} = \frac{(3.5 \times 10^3)(216 \times 10^{-6})}{(52.693 \times 10^{-6})(0.040)} = 359 \times 10^3 \ Pa = 359 \ kPa$$

since τ for box beam



to find τ_A

isolate

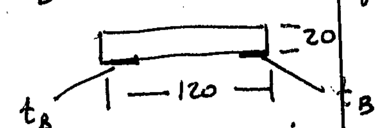


since symmetric use t_A = 2 thickness

$Q_A = 80 \cdot 20 \cdot 90$

to find τ_B

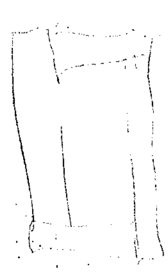
isolate section to find we want



since symmetric

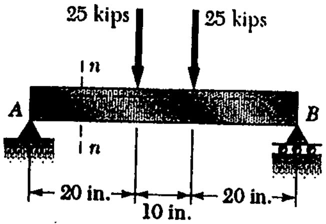
$\tau_B = \dfrac{VQ_B}{It_B}$ $\quad t_B$ = 2. thickness

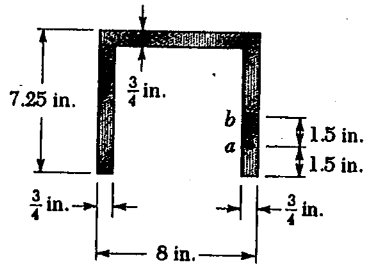
$Q_B = 120 \cdot 20 \cdot 90$



PROBLEM 6.24

6.23 and 6.24 For the beam and loading shown, determine the largest shearing stress in section *n-n*.



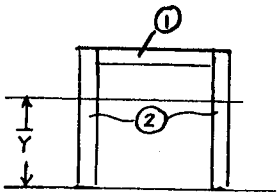


SOLUTION

$R_A = R_B = 25$ kips

At section n-n $V = 25$ kips.

Locate centroid and calculate moment of inertia

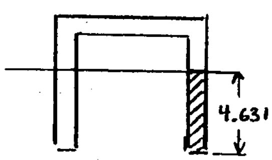


Part	$A\,(in^2)$	$\bar{y}$ (in)	$A\bar{y}\,(in^3)$	d (in)	$Ad^2\,(in^4)$	$\bar{I}\,(in^4)$
1	4.875	6.875	33.52	2.244	24.55	0.23
2	10.875	3.625	39.42	1.006	11.01	47.68
Σ	15.75		72.94		35.56	47.86

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{72.94}{15.75} = 4.631 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 35.56 + 47.86 = 83.42 \text{ in}^4$$

Largest shearing stress occurs on section through centroid of entire cross section.



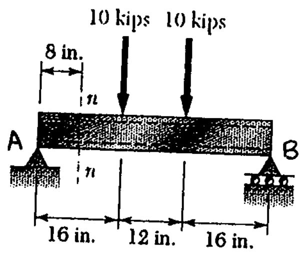
$$Q = A\bar{y} = \left(\tfrac{3}{4}\right)(4.631)\left(\frac{4.631}{2}\right) = 8.042 \text{ in}^3$$

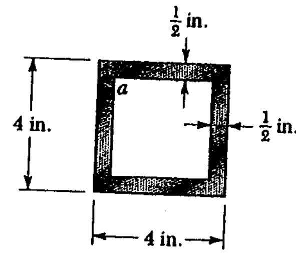
$$t = \tfrac{3}{4} = 0.75 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(25)(8.042)}{(83.42)(0.75)} = 3.21 \text{ ksi}$$

PROBLEM 6.10

6.9 through 6.12 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.



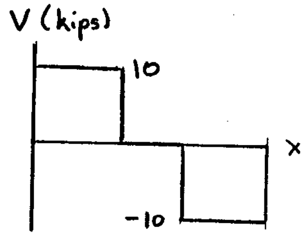


SOLUTION

By symmetry $R_A = R_B$

$+\uparrow \Sigma F_y = 0 \quad R_A + R_B - 10 - 10 = 0$

$R_A = R_B = 10$ kips

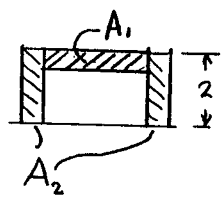


From the shear diagram $V = 10$ kips at n-n.

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(4)(4)^3 - \frac{1}{12}(3)(3)^3 = 14.583 \text{ in}^4$$

(a)



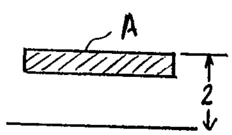
$$Q = A_1 \bar{y}_1 + A_2 \bar{y}_2 = (3)(\tfrac{1}{2})(1.75) + (2)(\tfrac{1}{2})(2)(1)$$

$$= 4.625 \text{ in}^3$$

$$t = \tfrac{1}{2} + \tfrac{1}{2} = 1 \text{ in.}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(10)(4.625)}{(14.583)(1)} = 3.17 \text{ ksi}$$

(b)

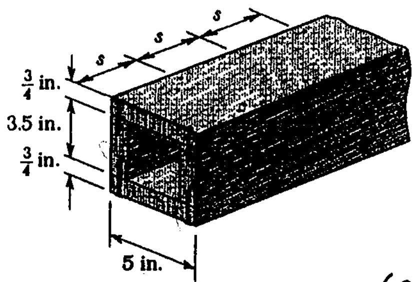


$$Q = A\bar{y} = (4)(\tfrac{1}{2})(1.75) = 3.5 \text{ in}^3$$

$$t = \tfrac{1}{2} + \tfrac{1}{2} = 1 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(10)(3.5)}{(14.583)(1)} = 2.40 \text{ ksi}$$

PROBLEM 6.3

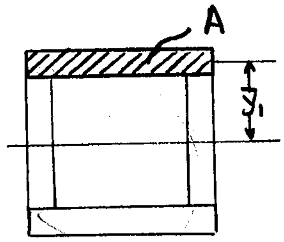


6.3 A square box beam is made of two $\frac{3}{4} \times 3.5$-in. planks and two $\frac{3}{4} \times 5$-in. planks nailed together as shown. Knowing that the spacing between nails is s = 1.25 in. and that the vertical shear in the beam is V = 250 lb, determine (*a*) the shearing force in each nail, (*b*) the maximum shearing stress in the beam.

SOLUTION

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(5)(5)^3 - \frac{1}{12}(3.5)(3.5)^3 = 39.578 \text{ in}^4$$



(a) $A = (5)(\frac{3}{4}) = 3.75 \text{ in}^2$

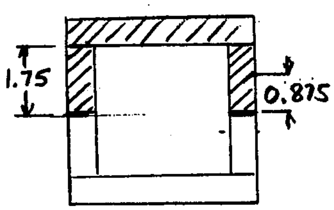
$$\bar{y}_1 = 2.5 - \frac{3}{8} = 2.125 \text{ in}$$

$$Q_1 = A\bar{y}_1 = (3.75)(2.125) = 7.969 \text{ in}^3$$

$$q = \frac{VQ_1}{I} = \frac{(250)(7.969)}{39.578} = 50.34 \text{ lb/in}$$

$q s = 2 F_{nail}$

$$F_{nail} = \frac{qs}{2} = \frac{(50.34)(1.25)}{2} = 31.5 \text{ lb.}$$



(b) $Q_2 = Q_1 + (2)(1.75)(\frac{3}{4})(0.875)$

$$= 7.969 + 2.297 = 10.266 \text{ in}^3$$

$$t = (2)(\frac{3}{4}) = 1.5 \text{ in.}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(250)(10.266)}{(39.578)(1.5)} = 43.2 \text{ psi}$$