EMA 3702 Mechanics of Materials (3)

Professor:

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Textbook: Beer, Johnston and DeWolf, Mechanics of Materials, 3rd Ed., McGraw Hill

References: There are plenty of books at the FIU Library on the subject. Here is a list of a few books on the subject available at the library.

Engineering mechanics of solids. Egor P. Popov. Englewood Cliffs, N.J. : Prentice Hall, c1990.

Mechanics of materials. James M. Gere, Stephen P. Timoshenko. 3rd ed. Boston : PWS-KENT Pub. Co., c1990.

Problem solver in strength of materials and mechanics of solids staff of Research and Education Association ; M. Fogiel, director. Rev. print. Piscataway, N.J. : The Association, 1988, c1980.

Engineering considerations of stress, strain, and strength, Robert C. Juvinall, New York, McGraw-Hill [1967].

Elements of mechanics of materials Gerner A. Olsen. Englewood Cliffs, N.J. Prentice-Hall, c1982.

Course Objectives

1. Identify mechanical properties and the characteristics of elastic behavior for material types.
2. Calculate the stress and strain configuration at a point for a specific loading arrangement.
3. Transform plane stress and strain configurations and identify principal stress and Principal Axes.
4. Use the appropriate failure criteria for diverse situation and/or materials (elastic behavior only).
5. Design prismatic beams.

Topics By Week

1. Introduction. Shear and Bending Moment Diagrams. Relation among load, shear and bending moment Normal and Shear stresses. Bearing stresses. Stress on an oblique plane. Stresses under general conditions **Quiz 1**
2. Stress and Strain – Axial Loading. Stress vs. strain. Hooke's law. Fatigue and deformations of members under axial load Thermal Stresses, Poisson ratio. Generalized Hooke's law **Quiz 2**
3. Saint Venant's Principle. Stress Concentration. Torsion. Stresses in a shaft Deformations and angle of twist Statically indeterminate shafts. Stress concentration in circular shaft **Quiz 3**
4. Analysis and Design of Beams. Pure Bending. Stresses and deformations in the elastic range Bending on members made of several materials. Stress Concentration. General case of eccentric axial loading **Quiz 4**
5. Prismatic beams for bending. Transformation of Stresses and Strains Principal Stresses. Mohr's Circle: Plane Stress. General State of Stress. **Quiz 5.**

6. Stresses in Thin-Walled pressure vessels. Mohr's Circle: Plane Strain. Strain Rosette. Deflection of Beams. Beam deflection by integration. Statically indeterminate beam. **Quiz 6**
7. Superposition Method. Columns. Euler's formula. Extension Euler's formula. Design of columns: centric and eccentric load. Failure Criteria

Homeworks will be assigned but not collected. These problems should be worked out at home because you might see them again on your quizzes. Use the "Given, Required, Solution" format and completely draw appropriate diagrams and coordinate systems. All numerical answers should have the appropriate units. You must keep up with the homework in order to do well in class

Grading

Quizzes (4 15% each) lowest two grades will be dropped	60%
Final Exam	40%
Total	100%

Grades will be assigned based on your performance on the activities above. Final letter grades will be assigned as follows:

100 – 90	A	77-79.99	B-	60-64.99	D
87-89.99	A-	73-76.99	C+	55-60	D-
84-86.99	B+	70-72.99	C	Below 55	F
80-83.99	B	65-69.99	C-		

Class will meet MTWR at 5:40 - 6:55pm and on Thursday there will be an extra hour from 6:55-8:20 pm

FINAL EXAM (Cumulative): Tuesday 24 June at 540pm.

NOTE: ***This is a preliminary syllabus and it might be changed during the semester. Any change will be announced in class.***

Introduction to Mechanics and Materials Science

In structural engineering, one has to give dimensions to the particular members, such as beams, floors, walls, shafts and the like, so that they can support the forces that act upon them, both the known ones as well as those that could act upon them.

Therefore, the parts of a composite structure, must be able to withstand the forces, i.e., be strong enough in order not to deflect much when the forces act upon them.

Thin wall members can buckle when they are loaded by small loads before they fail in bending.

One must also design bars and beams without waste of materials.

Our subject deals with strength, stiffness and stability of members of our structure that carry the loads.

Galileo, in the 17th Century, first investigated the behavior of materials that were loaded in a rational manner, bars, beams in both tension and compression.

From that time until now, there were many investigators such as Coulomb, Poisson, Navier, St. Venant and Cauchy.

Mechanics and Materials Science crosses all realms-- from Civil Engineering, to Aeronautical Engineering, mechanical Engineering, Aerospace Engineering and any field that requires building structures of any kind.

The behavior of bars and beams depend not only on the basic laws of Newton that require equilibrium, but also upon the mechanical characteristics of materials that make up the bars and beams.

Materials behavior are obtained from many rational experiments done in laboratories. The knowledge has given us the information to choose the material that is proper to design our structures.

We will begin with a review of statics and after that we will begin to learn our subject matter.

and the slope dv/dx must vanish. Hence at the end considered, where $x = a$,

$$v(a) = 0, \qquad v'(a) = 0 \tag{11-18a}$$

(B) ROLLER OR PINNED SUPPORT: At the end considered no deflection v nor moment M can exist. Hence

$$v(a) = 0, \qquad M(a) = EIv''(a) = 0 \tag{11-18b}$$

Here the physically evident condition for M is related to the derivative of v with respect to x from one part of Eq. 11-14.

(C) FREE END: Such an end is free of moment and shear. Hence

$$M(a) = EIv''(a) = 0, \qquad V(a) = -(EIv'')'_{x=a} = 0 \tag{11-18c}$$

(D) GUIDED SUPPORT: In this case free vertical movement is permitted, but the rotation of the end is prevented. The support is not capable of resisting any shear. Therefore

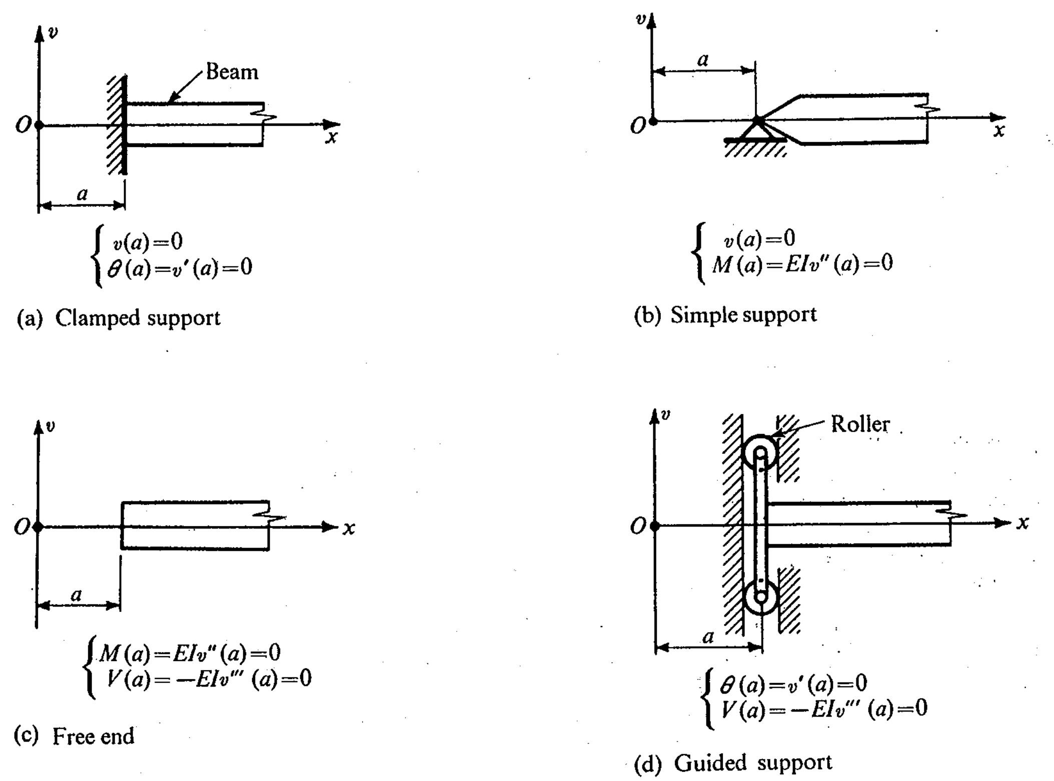

Fig. 11-4. Homogeneous boundary conditions for beams with constant EI. In (a) both conditions are *kinematic*; in (c) both are *static*; in (b) and (d) conditions are mixed.

11-5. ALTERNATIVE DIFFERENTIAL EQUATIONS OF ELASTIC BEAMS

In Chapter 2 a number of differential relations were shown among shear, moment, and the applied load (Eqs. 2-4, 2-5, and 2-6). These can be combined with Eq. 11-10 to yield the following useful sequence of equations:

$$v = \text{deflection of the elastic curve}$$

$$\theta = \frac{dv}{dx} = v' = \text{slope of the elastic curve}$$

$$M = EI\frac{d^2v}{dx^2} = EIv'' \tag{11-14}$$

$$V = -\frac{dM}{dx} = -\frac{d}{dx}\left(EI\frac{d^2v}{dx^2}\right) = -(EIv'')'$$

$$p = -\frac{dV}{dx} = \frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right) = (EIv'')''$$

For beams with constant flexural rigidity *EI*, Eqs. 11-14 simplifies into three alternative equations for determining the deflection of a loaded beam:

$$EI\frac{d^2v}{dx^2} = M(x) \tag{11-15}$$

$$EI\frac{d^3v}{dx^3} = -V(x) \tag{11-16}$$

$$EI\frac{d^4v}{dx^4} = p(x) \tag{11-17*}$$

The choice of equation for a given case depends on the ease with which an expression for load, shear, or moment can be formulated. Fewer constants of integration are needed in the lower-order equations.

11-6. BOUNDARY CONDITIONS

For the solution of beam deflection problems, in addition to the differential equations, boundary conditions must be prescribed. Several types of homogeneous boundary conditions are as follows:

(A) CLAMPED OR FIXED SUPPORT: In this case the displacement v

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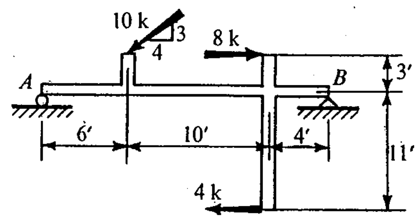
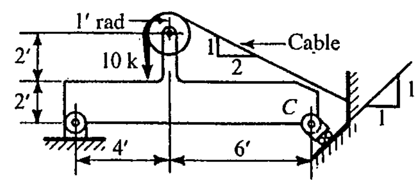
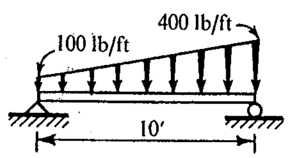
* If in Eq. 11-17 in accordance with the d'Alembert principle one sets $p = -m\ddot{v}$, where m is the mass of the beam per unit length and $\ddot{v} = \partial^2 v/\partial t^2$, the basic equation for the free lateral vibration of a beam is obtained. This equation is $EI\,\partial^4 v/\partial x^4 + m\,\partial^2 v/\partial t^2 = 0$.



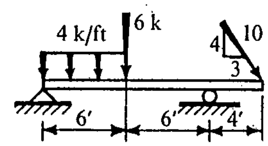
PROB. 2-1



PROB. 2-2



PROB. 2-3



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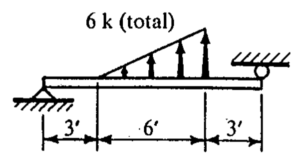
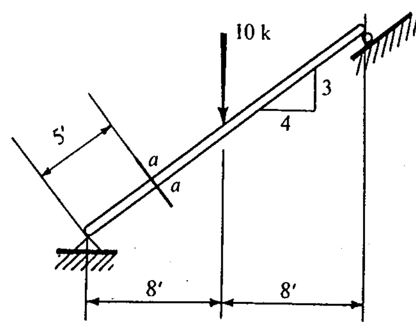
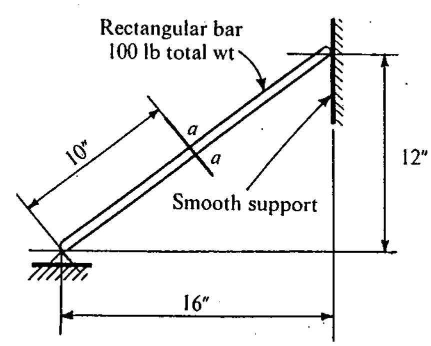


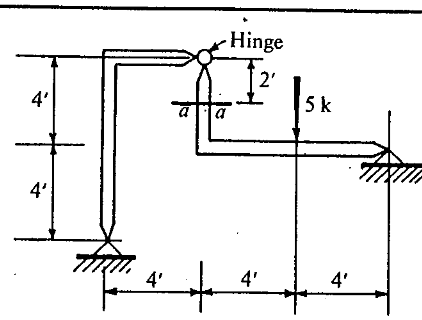
PROB. 2-5



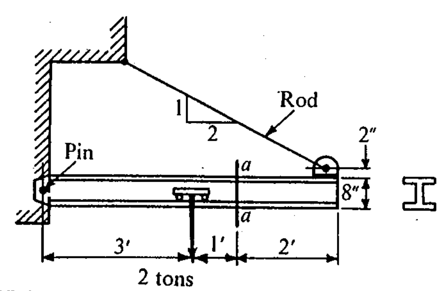
PROB. 2-6



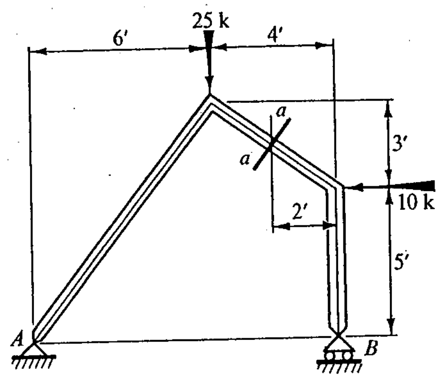
PROB. 2-7



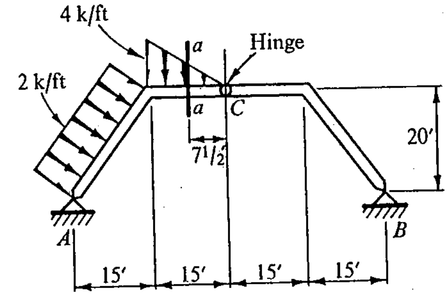
PROB. 2-8



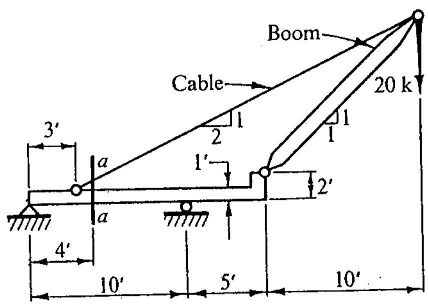
PROB. 2-9



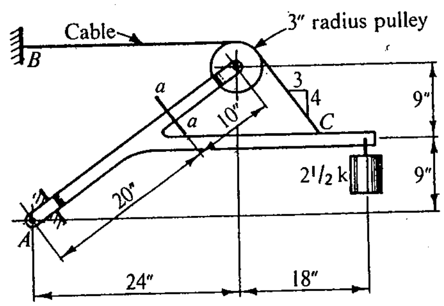
PROB. 2-10



PROB. 2-11



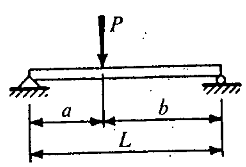
PROB. 2-12



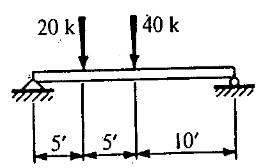
PROB. 2-13

2-14 through 2-19. For the beams loaded as shown in the figures write general expressions for the shear and bending moments for each region over the length of the member. Also plot the corresponding shear and moment diagrams. *Ans.* Maximum moment in parentheses by the figure.

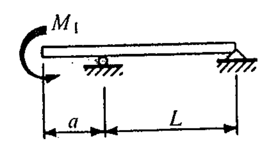
For additional loading conditions see other problems in this chapter.



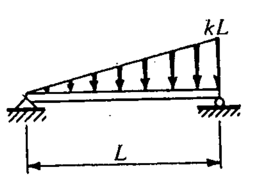
PROB. 2-14 (Pab/L)



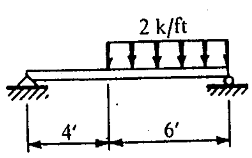
PROB. 2-15 (250)



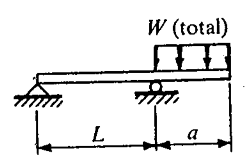
PROB. 2-16 $(-M_1)$



PROB. 2-17 $(kL^3/9\sqrt{3})$



PROB. 2-18 (17.6)



PROB. 2-19 $(-Wa/2)$

tions assumed for
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of p, V, and M are
figure, derive the
. 2-4, 2-5, and 2-6.

e beams loaded in
figures, neglecting
, solve as directed:
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in addition, show
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ent, and, wherever
grams for the main

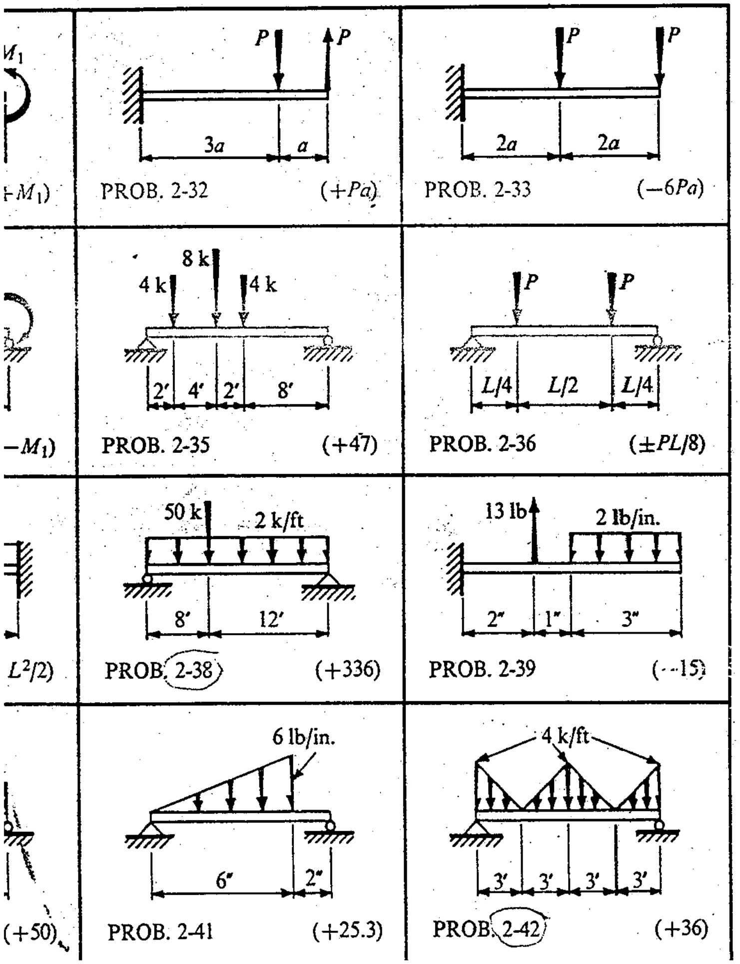
horizontal members. Determine all critical ordinates.

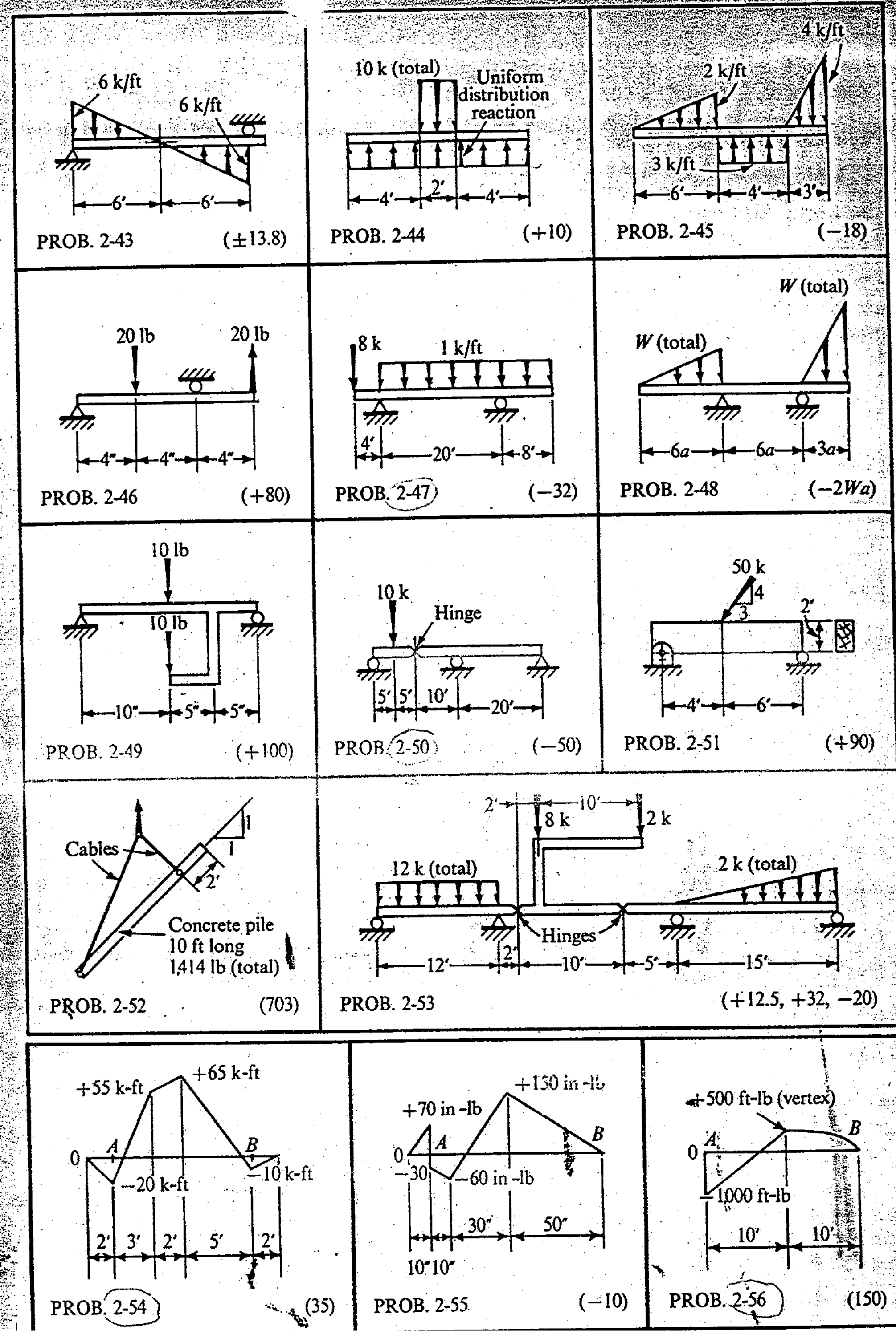
D. Same as **C**, and, in addition, determine the points of inflection and show the shape of the elastic curve.

Ans. All shear and moment diagrams must close. The largest moment is given in parentheses by the figures in the units of the problem.

For additional loading conditions see other problems in this chapter.

2-54 through 2-56. The moment diagrams for beams supported at A and B are as shown in the figures. How are these beams loaded? All curved lines represent parabolas, i.e., plots of equations of the second degree. (*Hint:* construction of shear diagrams aids the





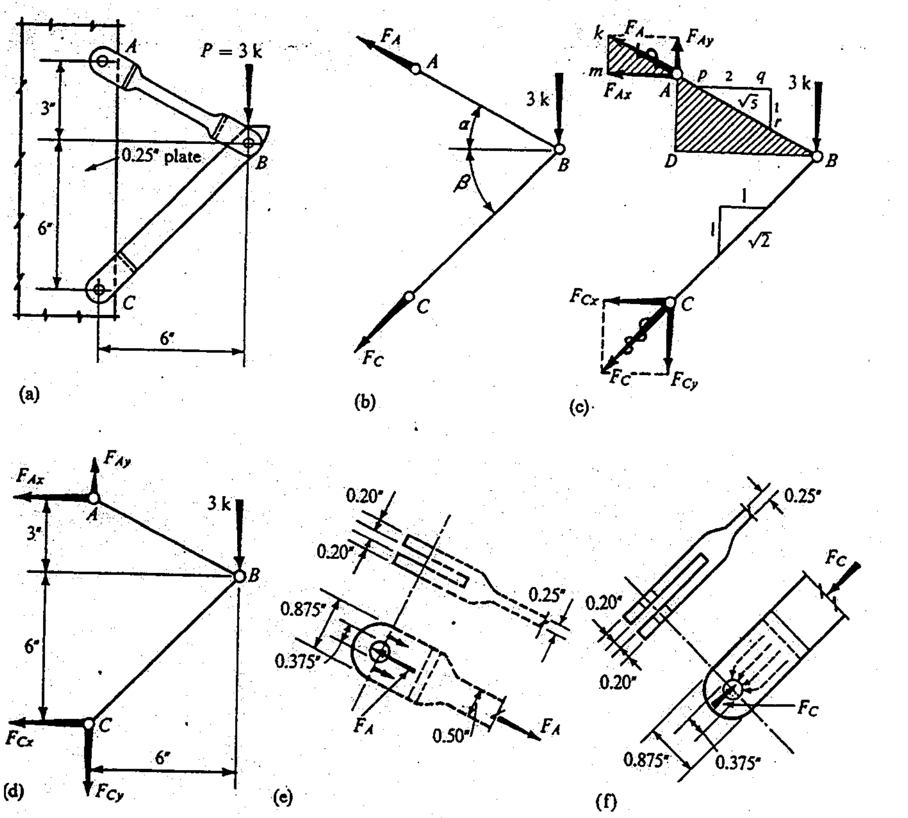


Fig. 3-12

SOLUTION

First an idealized free-body diagram consisting of the two bars pinned at the ends is prepared, Fig. 3-12(b). As there are no intermediate forces acting on the bars and the applied force acts through the joint at B, the forces in the bars are directed along the lines AB and BC, and the bars AB and BC are loaded axially. The magnitudes of the forces are unknown and are labeled F_A and F_C in the diagram.* These forces may be determined graphically by completing a triangle of forces F_A, F_B, and P. These forces may also be found analytically from two simultaneous

* In frameworks it is convenient to assume all unknown forces are tensile. A negative answer in the solution then indicates that the bar is in compression.

equations $\Sigma F_y = 0$ and $\Sigma F_x = 0$, written in terms of the unknowns F_A and F_C, a known force P, and two known angles α and β. Both these procedures are possible. However, in this course usually it will be found advantageous to proceed in a different way. Instead of treating forces F_A and F_C directly, their components are used; and instead of $\Sigma F = 0$, $\Sigma M = 0$ becomes the main tool.

Any force may be resolved into components. For example, F_A may be resolved into F_{Ax} and F_{Ay} as in Fig. 3-12(c). Conversely, if any one of the components of a directed force is known, the force itself may be determined. This follows from similarity of dimension and force triangles. In Fig. 3-12(c) the triangles Akm and BAD are similar triangles (both are shaded in the diagram). Hence, if F_{Ax} is known,

$$F_A = (AB/DB)F_{Ax}$$

Similarly, $F_{Ay} = (AD/DB)F_{Ax}$. However, note further that AB/DB or AD/DB are ratios, hence relative dimensions of members may be used. Such relative dimensions are shown by a little triangle on the member AB and again on BC. In the problem at hand

$$F_A = (\sqrt{5}/2)F_{Ax} \quad \text{and} \quad F_{Ay} = F_{Ax}/2$$

Adopting the above procedure of resolving forces, the revised free-body diagram, Fig. 3-12(d), is prepared. Two components of force are necessary at the pin joints. After the forces are determined by statics, Eq. 3-5 is applied several times, thinking in terms of a free body of an individual member:

$$\sum M_C = 0 \circlearrowleft +, \quad +F_{Ax}(3+6) - 3(6) = 0, \qquad F_{Ax} = +2 \text{ kips}$$

$$F_{Ay} = F_{Ax}/2 = 2/2 = 1 \text{ kip},$$

$$F_A = 2(\sqrt{5}/2) = +2.23 \text{ kips}$$

$$\sum M_A = 0 \circlearrowright +, \quad +3(6) + F_{Cx}(9) = 0,$$

$$F_{Cx} = -2 \text{ kips} \quad \text{(compression)}$$

$$F_{Cy} = F_{Cx} = -2 \text{ kips},$$

$$F_C = \sqrt{2}(-2) = -2.83 \text{ kips}$$

Check: $\sum F_x = 0, \quad F_{Ax} + F_{Cx} = 2 - 2 = 0$

$\sum F_y = 0, \quad F_{Ay} - F_{Cy} - P = 1 - (-2) - 3 = 0$

Stress in main bar AB:

$$\sigma_{AB} = \frac{F_A}{A} = \frac{2.23}{(0.25)(0.50)} = 17.8 \text{ ksi} \qquad \text{(tension)}$$

possible.

EXAMPLE 3-3

A bracket of negligible weight shown in Fig. 3-12(a) is loaded with a force P of 3 kips. For interconnection purposes the bar ends are clevised (forked). Pertinent dimensions are shown in the figure. Find the normal stresses in the members AB and BC and the bearing and shearing stresses for the pin C. All pins are 0.375 in. in diameter.

Stress in clevis of bar AB, Fig. 3-12(e):

$$(\sigma_{AB})_{\text{clevis}} = \frac{F_A}{A_{\text{net}}} = \frac{2.23}{2(0.20)(0.875 - 0.375)} = 11.2 \text{ ksi} \qquad \text{(tension)}$$

Stress in main bar BC:

$$\sigma_{BC} = \frac{F_C}{A} = \frac{2.83}{(0.875)(0.25)} = 12.9 \text{ ksi} \qquad \text{(compression)}$$

In the compression member the net section at the clevis need not be investigated; see Fig. 3-12(f) for the transfer of forces. The bearing stress at the pin is more critical. Bearing between pin C and clevis:

$$\sigma_b = \frac{F_C}{A_{\text{bearing}}} = \frac{2.83}{(0.375)(0.20)2} = 18.8 \text{ ksi}$$

Bearing between the pin C and the main plate:

$$\sigma_b = \frac{F_C}{A} = \frac{2.83}{(0.375)(0.25)} = 30.1 \text{ ksi}$$

Double shear in the pin C:

$$\tau = \frac{F_C}{A} = \frac{2.83}{2\pi(0.375/2)^2} = 12.9 \text{ ksi*}$$

For a complete analysis of this bracket, other pins should be investigated. However, it may be seen by inspection that the other pins in this case are stressed the same amount as computed above, or less.

The advantages of the method used in the above example for finding forces in members should now be apparent. It can also be applied with success in a problem such as the one shown in Fig. 3-13. The force F_A transmitted by the curved member AB acts through points A and B since the forces applied at A and B must be collinear. By resolving this force at A', the same procedure may be followed. Wavy lines through F_A and F_C indicate that these forces are replaced by the two components shown. Alternatively, the force F_A may be resolved at A, and since $F_{Ay} = (x/y)F_{Ax}$, the application of $\Sigma M_C = 0$ yields F_{Ax}.

In frames where the applied forces do not act through a joint, proceed as above as far as possible. Then isolate an individual member, and using its free-body diagram, complete the determination of forces.

* Considering the pin in a two-dimensional state of stress $\tau_{xy} = \tau_{yx}$, the tensor representation of the results becomes $\begin{pmatrix} 0 & 12.9 \\ 12.9 & 0 \end{pmatrix}$ ksi

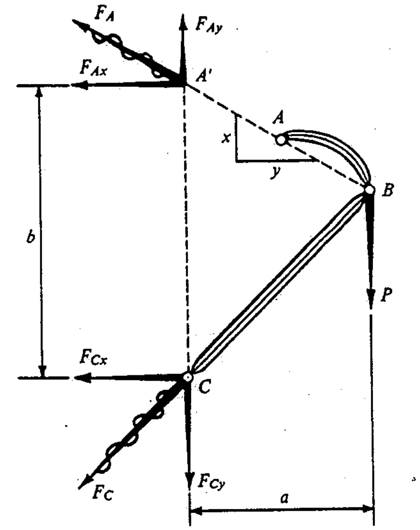


Fig. 3-13

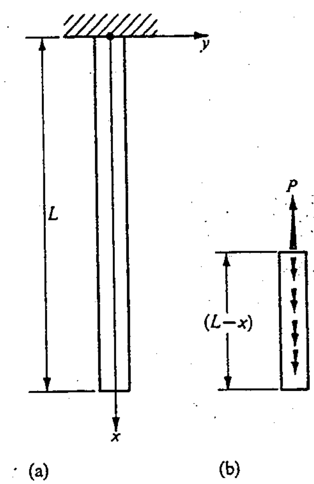


Fig. 3-14

If inclined forces are acting on the structure, resolve them into convenient components.

EXAMPLE 3-4

A 1-in.² rod L in. long is suspended vertically as shown in Fig. 3-14(a). The unit weight of the material is γ. Determine the normal stress in this rod using differential equations of equilibrium.

SOLUTION

With the axes shown in the figure, $\tau_{xy} = 0$, and only the first part of Eq. 3-3 has relevance. The body force $X = \gamma$. By virtue of the boundary condition at the free end of the rod $\sigma_x(L) = 0$. On this basis, setting up a differential equation, intergating it, and determining the constant of integration from the boundary conditions, one has

$$\frac{d\sigma_x}{dx} + \gamma = 0 \qquad \text{and} \qquad \sigma_x = -\gamma x + C_1$$

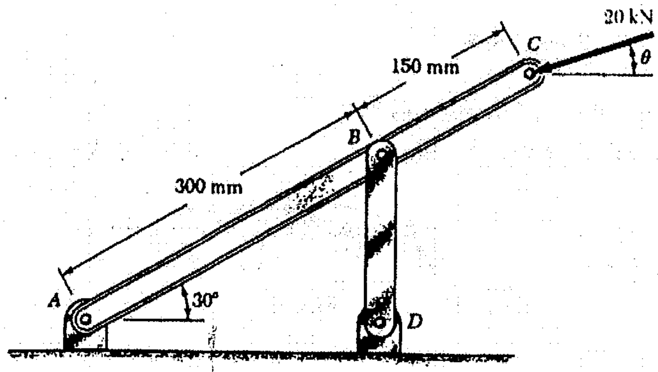
$$\sigma_x(L) = -\gamma L + C_1 = 0 \qquad \text{and} \qquad \sigma_x = (L - x)\gamma$$

This result can be easily checked by cutting the rod $(L - x)$ above the free end, Fig. 3-14(b), and applying Eq. 3-5. Only very few problems can be analyzed using Eq. 3-3 alone. In more general problems deformations must be considered simultaneously in the analysis.

HW #~~2~~ 3

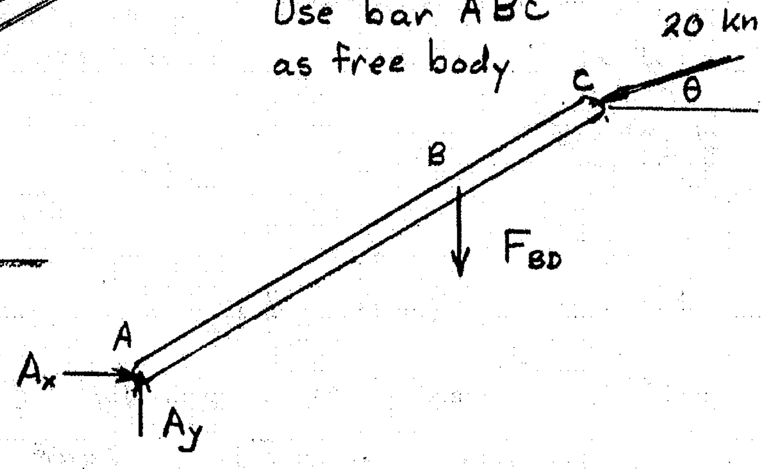
PROBLEM 1.7

1.7 Link *BD* consists of a single bar 30 mm wide and 12 mm thick. Knowing that each pin has a 10-mm diameter, determine the maximum value of the average normal stress in link *BD* if (*a*) $\theta = 0$, (*b*) $\theta = 90$



SOLUTION

Use bar ABC as free body



$\Sigma M_A = 0$

(a) $\theta = 0°$ $\quad (0.450 \sin 30°)(20 \times 10^3) - (0.300 \cos 30°) F_{BD} = 0$

$F_{BD} = 17.32 \times 10^3 \text{ N}$

(b) $\theta = 90°$ $\quad (0.450 \cos 30°)(20 \times 10^3) - (0.300 \cos 30°) F_B = 0$

$F_{BD} = -30 \times 10^3 \text{ N}$

Areas

(a) tension loading $\quad A = (0.030 - 0.010)(0.012) = 240 \times 10^{-6} \text{ m}^2$

(b) compression $\quad A = (0.030)(0.012) = 360 \times 10^{-6} \text{ m}^2$

Stresses

(a) $\sigma = \frac{F_{BC}}{A} = \frac{17.32 \times 10^3}{240 \times 10^{-6}} = 72.2 \times 10^6$ $\quad$ 72.2 MPa ◄

(b) $\sigma = \frac{F_{BC}}{A} = \frac{-30 \times 10^3}{360 \times 10^{-6}} = -83.3 \times 10^6$ $\quad$ −83.3 MPa ◄

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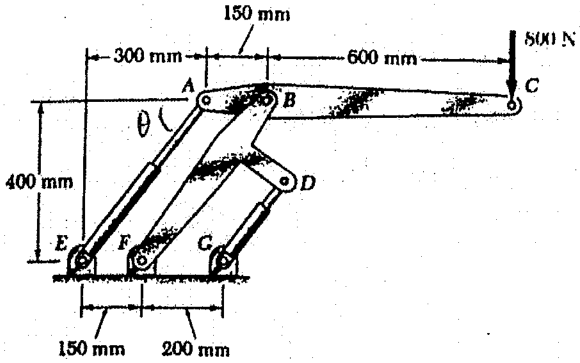
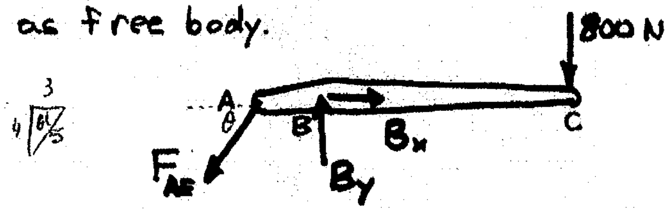
PROBLEM ~~1.14~~

1.14 Two hydraulic cylinders are used to control the position of the robotic arm *ABC*. Knowing that the control rods attached at *A* and *D* each have a 20-mm diameter and happen to be parallel in the position shown, determine the average normal stress in (*a*) member *AE*, (*b*) member *DG*.



SOLUTION

Use member ABC as free body.

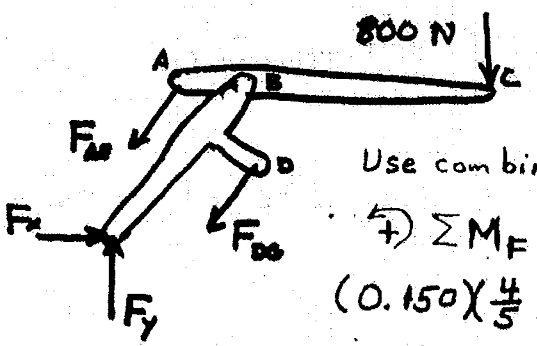


$\circlearrowleft \sum M_B = 0 \qquad (0.150)\frac{4}{5}F_{AE} - (0.600)(800) = 0 \qquad F_{AE} = 4 \times 10^3$ N

Area of rod in member AE is $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20\times10^{-3})^2 = 314.16\times10^{-6}$ m^2

Stress in rod AE: $\sigma_{AE} = \frac{F_{AE}}{A} = \frac{4\times10^3}{314.16\times10^{-6}} = 12.73\times10^6$ Pa

(a) $\sigma_{AE} = 12.73$ MPa ◄



Use combined members ABC and BFD as free body.

$\circlearrowleft \sum M_F = 0$

$(0.150)(\frac{4}{5}F_{AE}) - (0.200)(\frac{4}{5}F_{DG})$

$- (1.050 - 0.350)(800) = 0 \qquad F_{DG} = -1500$ N

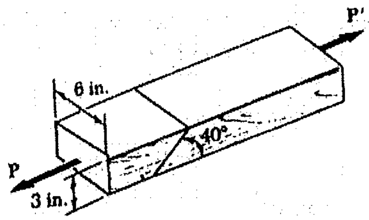
Area in rod DG is $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20\times10^{-3})^2 = 314.16\times10^{-6}$ m^2

Stress in rod DG: $\sigma_{DG} = \frac{F_{DG}}{A} = \frac{-1500}{3.1416\times10^{-6}} = -4.77\times10^6$ Pa

(b) $\sigma_{DG} = -4.77$ MPa ◄

PROBLEM ~~1.31~~ 1.29

1.31 Two wooden members of 3 × 6- in. uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 90 psi, determine (*a*) the largest load **P** which can be safely applied, (*b*) the corresponding tensile stress in the splice.



$P_{max} = 90$

SOLUTION

$\theta = 90° - 40° = 50°$

$A_o = (3)(6) = 18 \text{ in}^2$

$\tau = \frac{P}{2A} \sin 2\theta$

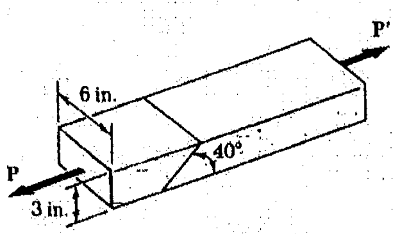
$P = \frac{2A\tau}{\sin 2\theta} = \frac{(2)(18)(90)}{\sin 100°} = 3290$

(a) $P = 3290$ lb.

(b) $\sigma = \frac{P\cos^2\theta}{A_o} = \frac{3290 \cos^2 50°}{18} = 75.5$ $\qquad \sigma = 75.5$ psi

PROBLEM ~~1.32~~ 1.30

1.32 Two wooden members of 3 × 6- in. uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 2400$ lb, determine the normal and shearing stresses in the glued splice.



SOLUTION

$\theta = 90° - 40° = 50°$ $\qquad P = 2400$ lb.

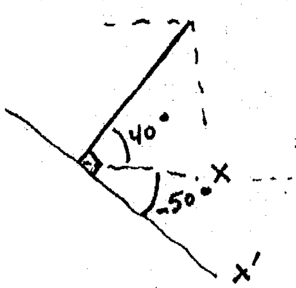
$A_o = (3)(6) = 18 \text{ in}^2$

$\sigma = \frac{P}{A_o}\cos^2\theta = \frac{(2400)\cos^2 50°}{18} = 55.1$

$\sigma = 55.1$ psi

$\tau = \frac{P}{2A}\sin 2\theta = \frac{(2400)\sin 100°}{(2)(18)} = 65.7$

$\tau = 65.7$ psi



draw x' ⊥ to surface

or $\qquad \theta = -50°$ $\qquad \sigma_x' = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$

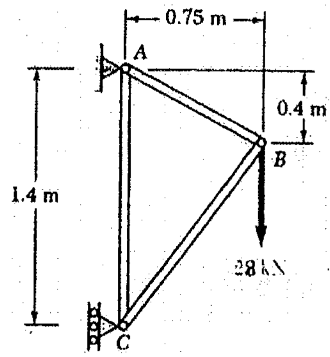
here $\sigma_y = 0 \quad \tau_{xy} = 0$

$\sigma_x' = \frac{\sigma_x}{2} + \frac{\sigma_x}{2}\cos(-110) = \sigma_x \cos^2(-50) = 55.1$ psi

$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$

1.42

PROBLEM ~~1.41~~



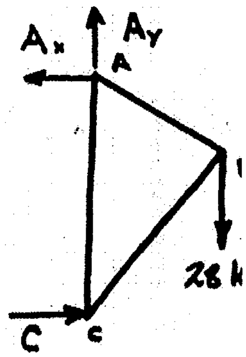
1.41 Members *AB* and *AC* of the truss shown consist of bars of square cross sec made of the same alloy. It is known that a 20- mm- square bar of the same alloy tested to failure and that an ultimate load of 120 kN was recorded. If bar *AB* has a mm- square cross section, determine (*a*) the factor of safety for bar *AB*, (*b*) dimensions of the cross section of bar *AC* if it is to have the same factor of safety a *AB*.

SOLUTION

Length of member AB

$$\ell_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

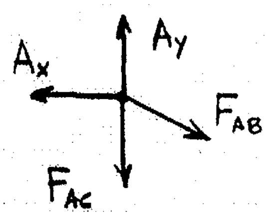
Use entire truss as a free body



$\circlearrowleft \Sigma M_C = 0 \quad 1.4 A_x - (0.75)(28) = 0 \quad A_x = 15 \text{ kN}$

$+\uparrow \Sigma F_y = 0 \quad A_y - 28 = 0 \quad A_y = 28 \text{ kN}$

Use joint A as free body



$\xrightarrow{+} \Sigma F_x = 0 \quad \frac{0.75}{0.85} F_{AB} - A_x = 0$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$+\uparrow \Sigma F_y = 0 \quad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

Need to find stress at failure, σ_u

For the test bar $\quad A = (0.020)^2 = 400 \times 10^{-6} \text{ m}^2 \quad P_U = 120 \times 10^3$

For the material $\quad \sigma_U = \frac{P_U}{A} = \frac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

(a) For bar AB $\quad F.S. = \frac{P_U}{F_{AB}} = \frac{\sigma_U A}{F_{AB}} = \frac{(300 \times 10^6)(0.015)^2}{17 \times 10^3}$

$$= 3.97$$

(b) For bar AC $\quad F.S. = \frac{P_U}{F_{AC}} = \frac{\sigma_U A}{F_{AC}} = \frac{\sigma_U a^2}{F_{AC}}$

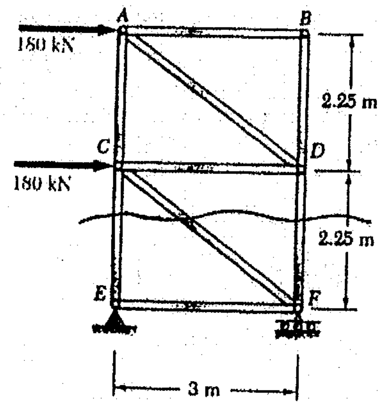
$$a^2 = \frac{(F.S.) F_{AC}}{\sigma_U} = \frac{(3.97)(20 \times 10^3)}{300 \times 10^6} = 264.7 \times 10^{-6} \text{ m}^2$$

$$a = 16.27 \times 10^{-3} \text{ m} \qquad 16.27 \text{ mm}$$

HW #3

PROBLEM 1.59

1.59 For the truss and loading shown, determine the average normal st[ress in] member DF, knowing that the cross-sectional area of that member is 2500 mm[²]

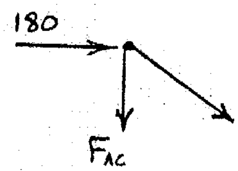


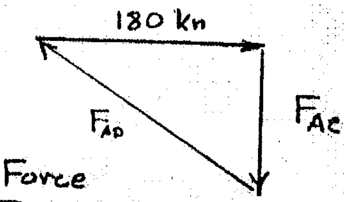
SOLUTION

Using method of joints to find member forces

Joint B: AB and BD are zero force memb[ers]

Joint A: $l_{AD} = \sqrt{3^2 + 2.25^2} = 3.75$ m





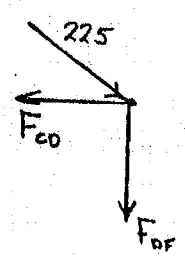
Force Trangle

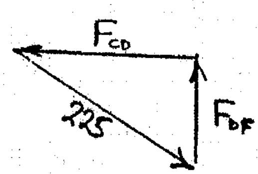
By similar triangles

$$\frac{F_{AD}}{3.75} = \frac{180}{3} \quad \therefore \quad F_{AD} = 225 \text{ lb.}$$

(compression)

Joint D





By similar triangles

$$\frac{F_{DE}}{2.25} = \frac{225}{3.75}$$

$$F_{DF} = 135 \text{ kN (comp)}$$

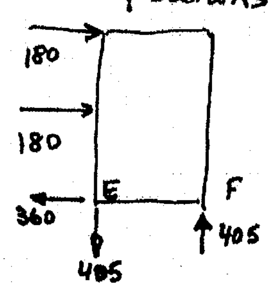
$$= 135 \times 10^3 \text{ N}$$

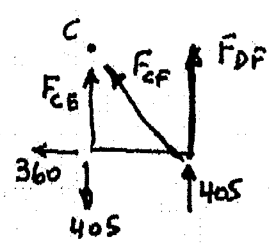
Area: $A_{DE} = 2500 \text{ mm}^2 = 2500 \times 10^{-6} \text{ m}^2$

Stress: $\sigma_{DF} = -\dfrac{135 \times 10^3}{2500 \times 10^{-6}} = -54 \times 10^6 \text{ Pa} = -54.0 \text{ MPa}$ ◄

or

Method of sections





$\circlearrowleft \Sigma M_C = 0 \quad -360(2.25) + (405 + F_{DF}) \cdot 3 =$

$F_{DF} = -135$ kN

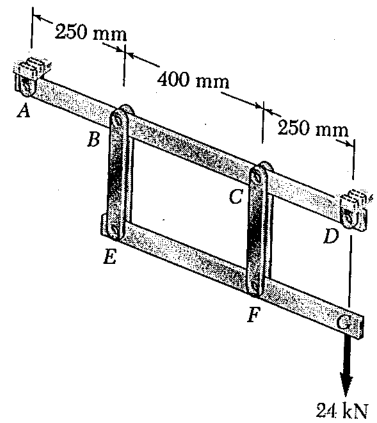
rest is same as above

$\sigma_{DF} = \dfrac{F_{DF}}{A} = -54$ M[Pa]

① Find support reactions

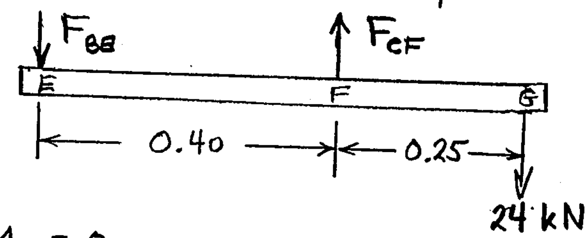
② Cut along red line & take moments about C

Problem 1.67



1.67 Each of the two vertical links *CF* connecting the two horizontal members *AD* and *EG* has a 10 × 40-mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at *C* and *F* has a 20-mm diameter and is made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links *CF* and the pins connecting them to the horizontal members.

Use member EFG as free body.



$\circlearrowleft \Sigma M_E = 0$

$0.40\,F_{CF} - (0.65)(24\times10^3) = 0$

$F_{CF} = 39\times10^3$ N

Based on tension in links CF

$A = (b-d)t = (0.040 - 0.02)(0.010) = 200\times10^{-6}\ \text{m}^2$ (one link)

$F_U = 2\sigma_U A = (2)(400\times10^6)(200\times10^{-6}) = 160.0\times10^3$ N

Based on double shear in pins

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.020)^2 = 314.16\times10^{-6}\ \text{m}^2$

$F_U = 2\tau_U A = (2)(150\times10^6)(314.16\times10^{-6}) = 94.248\times10^3$ N

Actual F_U is smaller value, i.e. $F_U = 94.248\times10^3$ N

Factor of safety $\quad F.S. = \dfrac{F_U}{F_{CF}} = \dfrac{94.248\times10^3}{39\times10^3} = 2.42$ ◄

2.9 The 36-mm-diameter steel rod ABC and a brass rod CD of the same diameter are joined at point C to form the 7.5-m rod $ABCD$. For the loading shown, and neglecting the weight of the rod, determine the deflection (*a*) of point C, (*b*) of point D.

Answer: $\delta_C = 2.95\ mm\downarrow$, $\delta_D = 5.29\ mm\downarrow$.

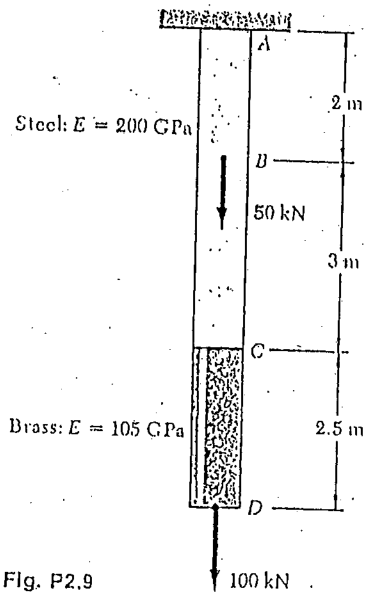


Fig. P2.9

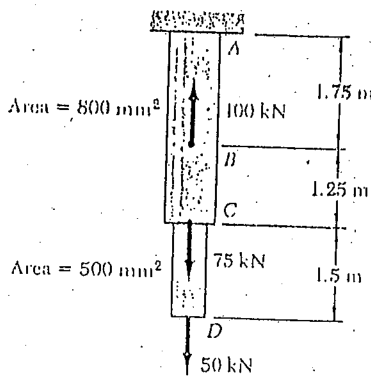


Fig. P2.10

2.10 The rod $ABCD$ is made of an aluminum alloy for which $E = 70$ GPa. For the loading shown, and neglecting the weight of the rod, determine the deflection (*a*) of point B, (*b*) of point D.

Answer: $\delta_B = 0.781\ mm\downarrow$, $\delta_D = 5.71\ mm\downarrow$.

2.26 An axial centric force of magnitude $P = 385$ kN is applied to the composite block shown by means of a rigid end plate. Determine the normal stress in (*a*) the steel core, (*b*) the aluminum plates.

Answer: $\sigma_S = -175\ MPa$, $\sigma_{Al} = -61.3\ MPa$.

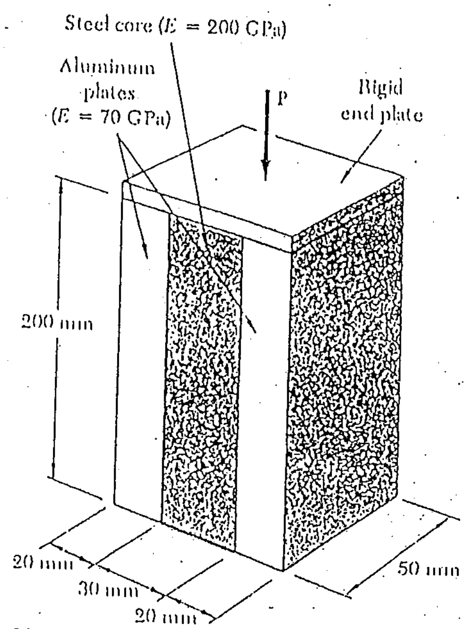


Fig. P2.26

2.37 The rigid rod ABC is suspended by three identical wires. Knowing that $x = \frac{2}{3}L$, determine the tension in each wire due to the force **P**.

Answer: $T_B = P/3$, $T_C = P/6$, $T_A = P/2$.

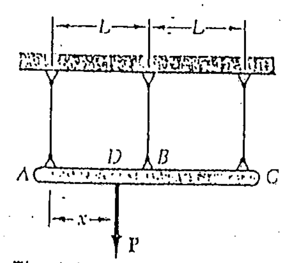


Fig. P2.37 and P2.38

2.39 The rigid bar *ABCD* is suspended from four identical wires. Determine the tension in each wire caused by the load P shown.

Answer: $T_B = P/5$, $T_D = 2P/5$, $T_A = P/10$, $T_C = 3P/10$.

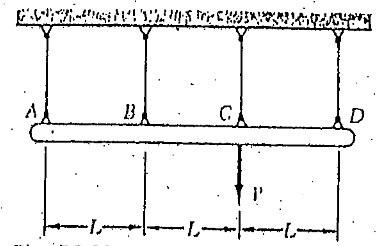
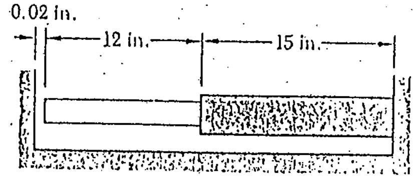


Fig. P2.39

2.45 Determine (*a*) the compressive force in the bars shown after a temperature rise of 200°F, (*b*) the corresponding change in length of the aluminum bar.

Answer: $P = -52\ kips$, $\delta_{Al} = 0.0124\ in.$

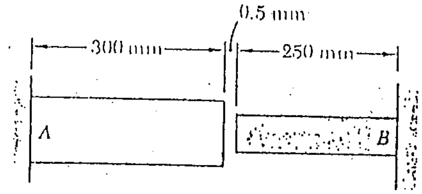


Bronze	Aluminum
$A = 2.5\ in^2$	$A = 3\ in^2$
$E = 15 \times 10^6$ psi	$E = 10 \times 10^6$ psi
$\alpha = 10.1 \times 10^{-6}/°F$	$\alpha = 12.8 \times 10^{-6}/°F$

Fig. P2.45

2.46 At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature reaches 140°C, determine (*a*) the normal stress in the aluminum, (*b*) the exact length of the aluminum rod.

Answer: $\sigma_{Al} = -114.6\ MPa$, $L = 300.34\ mm.$



Aluminum	Stainless steel
$A = 2000\ mm^2$	$A = 800\ mm^2$
$E = 70$ GPa	$E = 190$ GPa
$\alpha = 23 \times 10^{-6}/°C$	$\alpha = 18 \times 10^{-6}/°C$

Fig. P2.46 and P2.47

2.47 Knowing that a 0.5-mm gap exists between the rods shown when the temperature is 20°C, determine (*a*) the temperature at which the normal stress in the stainless steel rod will be $\sigma = -150$ MPa, (*b*) the corresponding exact length of the stainless steel rod.

Answer: $T = 103.7°C$, $L = 250.18\ mm.$

2/5

2.7. A straight piece of wire PQ lies along the line $y = mx$ as shown in Fig. P2.7. The wire is strained and displaced to the line $y = nx$ in such a way that a point originally at x is displaced to $x^2/2$. Show that the extensional strain at any point along the wire is given by

$$\epsilon = \sqrt{\frac{1 + n^2}{1 + m^2}}\, x - 1$$

(where x is the original coordinate of the point).

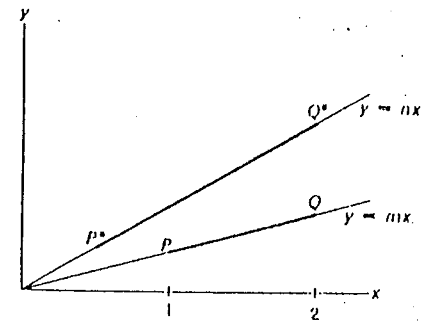


Fig. P2.7

2.16. The point C is displaced to C^* as shown in Fig. P2.16. The horizontal and vertical components of this displacement are u and v. Express the average extensional strains of AC and BC in terms of u, v and L. If u and v are small, what are the approximate expressions for these average extensional strains?

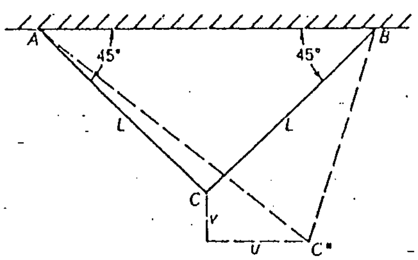


Fig. P2.16

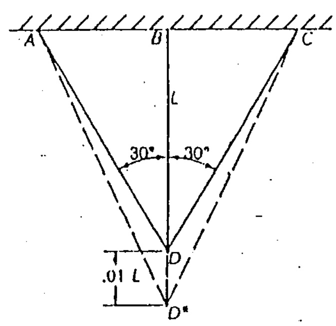


Fig. P2.17

2.17. The point D in Fig. P2.17 is displaced vertically an amount $.01L$. Obtain the approximate (first-order) average extensional strain of AD, BD, and CD.

2.21. The bar AB is rigid and is supported by two wires DB and CB as shown in Fig. P2.21. If the bar rotates through a small angle θ, determine the average extensional strain of DB and CB.

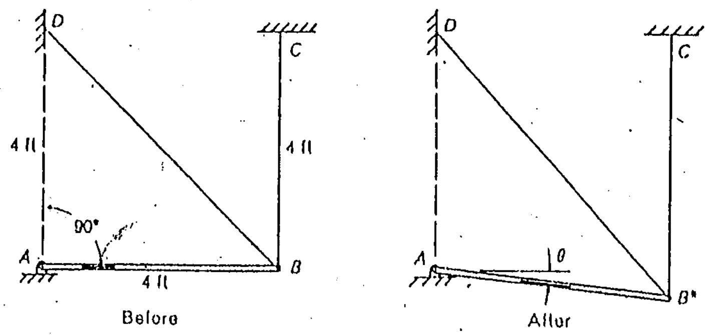


Fig. P2.21

2.32. A thin triangular plate is deformed as indicated in Fig. P2.32. Compute approximately the shear strain $\gamma_{st}(P)$.

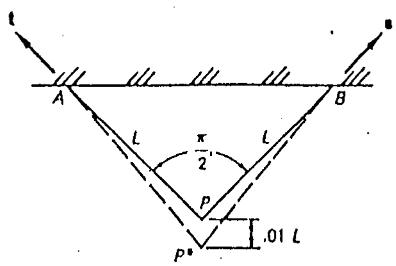


Fig. P2.32

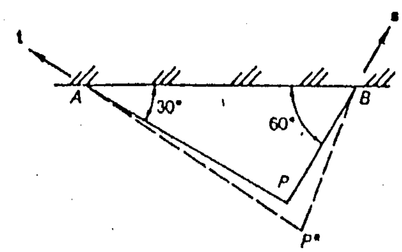


Fig. P2.34

2.34. A thin plate is in the form of a 30°–60°–90° triangle and is supported along its hypotenuse as shown in Fig. P2.34. It is subjected to the uniform strains $\epsilon_s = .004$, $\epsilon_t = .003$. Compute the approximate value of $\gamma_{st}(P)$.

2.37. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.37. It elongates 10 per cent of its original length and contracts 3 per cent of its original width. Compute approximately the shear strain γ_{st}.

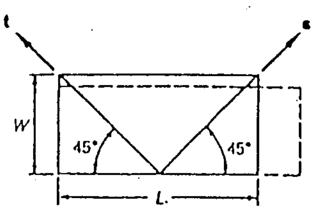


Fig. P2.37

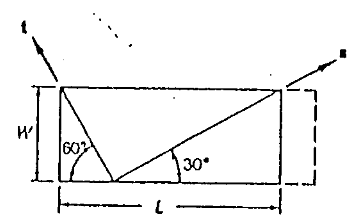


Fig. P2.38

2.38. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.38. The extensional strain along its length is .02 and the width remains unchanged. Compute approximately the shear strain γ_{st}.

Answers:

2.16 $\varepsilon_{AC} = \dfrac{\sqrt{L^2+\sqrt{2}L(v+u)+u^2+v^2}-L}{L}$; $\varepsilon_{AC} \approx \dfrac{\sqrt{2}(u+v)}{2L}$, $\varepsilon_{BC} \approx \dfrac{\sqrt{2}(u-v)}{2L}$

2.17 $\varepsilon_{AD} = \varepsilon_{CD} = 0.00754$; $\varepsilon_{BD} = 0.01$

2.21 $\varepsilon_{DB} = \theta/2$; $\varepsilon_{CB} = \theta$

2.32 $\sqrt{2}(0.01)$

2.34 0.0075

2.37 −0.125

2.38 $-0.01\sqrt{3}$

2.32. A thin triangular plate is deformed as indicated in Fig. P2.32. Compute approximately the shear strain $\gamma_{st}(P)$.

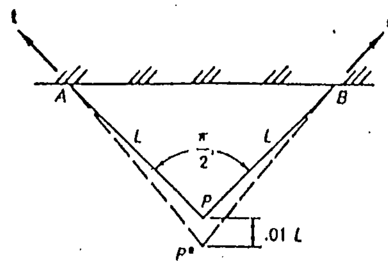


Fig. P2.32

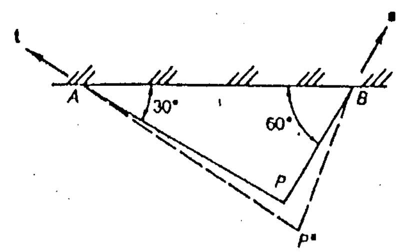


Fig. P2.34

2.34. A thin plate is in the form of a 30°–60°–90° triangle and is supported along its hypotenuse as shown in Fig. P2.34. It is subjected to the uniform strains $\epsilon_s = .004$, $\epsilon_t = .003$. Compute the approximate value of $\gamma_{st}(P)$.

2.37. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.37. It elongates 10 per cent of its original length and contracts 3 per cent of its original width. Compute approximately the shear strain γ_{st}.

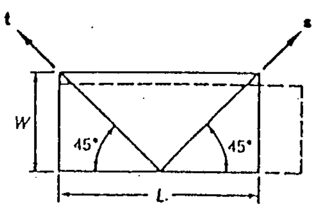


Fig. P2.37

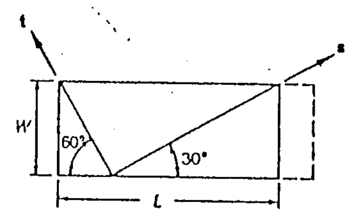


Fig. P2.38

2.38. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.38. The extensional strain along its length is .02 and the width remains unchanged. Compute approximately the shear strain γ_{st}.

Answers:

2.16 $\varepsilon_{AC} = \dfrac{\sqrt{L^2 + \sqrt{2}L(v+u) + u^2 + v^2} - L}{L}$; $\varepsilon_{AC} \approx \dfrac{\sqrt{2}(u+v)}{2L}$, $\varepsilon_{BC} \approx \dfrac{\sqrt{2}(u-v)}{2L}$

2.17 $\varepsilon_{AD} = \varepsilon_{CD} = 0.00754$; $\varepsilon_{BD} = 0.01$

2.21 $\varepsilon_{DB} = \theta/2$; $\varepsilon_{CB} = \theta$

2.32 $\sqrt{2}(0.01)$

2.34 0.0075

2.37 -0.125

2.38 $-0.01\sqrt{3}$

2.7. A straight piece of wire PQ lies along the line $y = mx$ as shown in Fig. P2.7. The wire is strained and displaced to the line $y = nx$ in such a way that a point originally at x is displaced to $x^2/2$. Show that the extensional strain at any point along the wire is given by

$$\epsilon = \sqrt{\frac{1 + n^2}{1 + m^2}}\, x - 1$$

(where x is the original coordinate of the point).

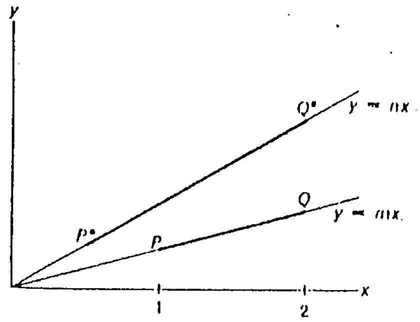


Fig. P2.7

2.16. The point C is displaced to C^* as shown in Fig. P2.16. The horizontal and vertical components of this displacement are u and v. Express the average extensional strains of AC and BC in terms of u, v and L. If u and v are small, what are the approximate expressions for these average extensional strains?

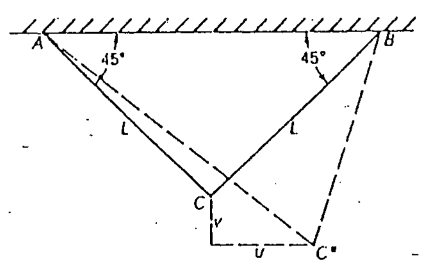


Fig. P2.16

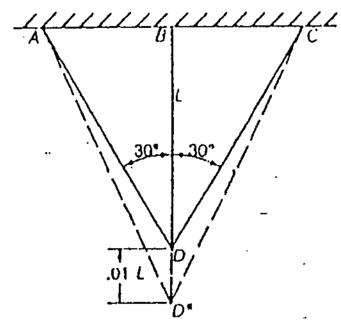


Fig. P2.17

2.17. The point D in Fig. P2.17 is displaced vertically an amount $.01L$. Obtain the approximate (first-order) average extensional strain of AD, BD, and CD.

2.21. The bar AB is rigid and is supported by two wires DB and CB as shown in Fig. P2.21. If the bar rotates through a small angle θ, determine the average extensional strain of DB and CB.

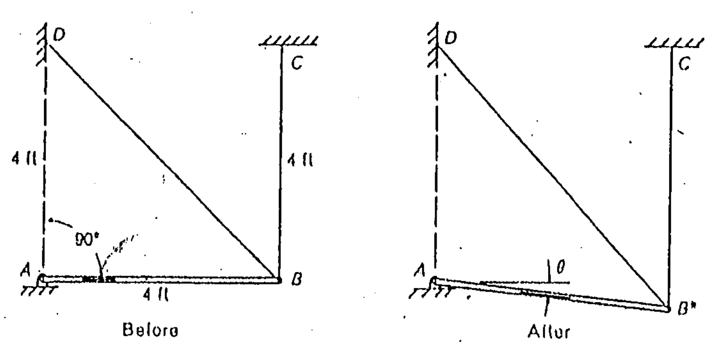
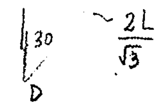
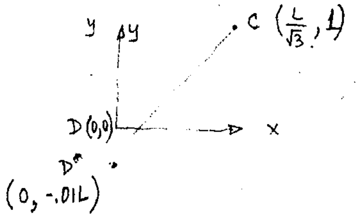


Fig. P2.21



$\sin 30 = \frac{BC}{CD} \Rightarrow BC = CD \sin 30° = \frac{2L}{\sqrt{3}} \cdot \frac{1}{2} = L/\sqrt{3}$

$\epsilon_{BD} = \frac{\overline{BD}^* - BD}{BD} = \frac{1.01L - L}{L} = .01$



$\epsilon_{CD} = \frac{\overline{CD}^* - CD}{CD} = \frac{1.163L - 1.155L}{1.155L} = .0075$

$CD = \frac{L}{\sqrt{3}}\underline{i} + L(1.01)\underline{j} \Rightarrow L\sqrt{\frac{1}{3} + 1.02} = 1.163L$

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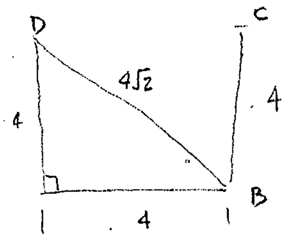
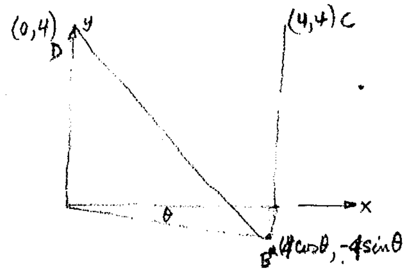
מפני שהמבנה סמטרי

$0.0075 = \epsilon_{AD} = \epsilon_{CD}$

2.21





$\overline{B^*D} = \left[(0 - 4\cos\theta)^2 + (4 + 4\sin\theta)^2\right]^{1/2}$
$= \left[16\cos^2\theta + 16 + 32\sin\theta + 16\sin^2\theta\right]^{1/2}$
$= \left[32 + 32\sin\theta\right]^{1/2} = 4\sqrt{2}\,(1+\sin\theta)^{1/2}$

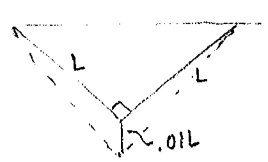
$\epsilon_{BD} = \frac{\overline{B^*D} - BD}{BD} = \frac{4\sqrt{2}(1+\sin\theta)^{1/2} - 4\sqrt{2}}{4\sqrt{2}} = (1+\sin\theta)^{1/2} - 1$

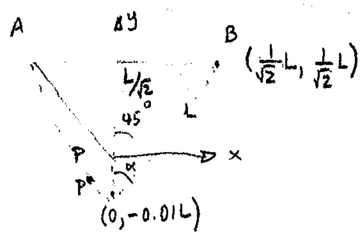
$\epsilon_{BD} \cong \left(1 + \frac{\sin\theta}{2} + \ldots\right) - 1 = \frac{\sin\theta}{2} \approx \theta/2 \text{ rad}$

$\overline{CB}^* = \left[(4\cos\theta - 4)^2 + (-4\sin\theta - 4)^2\right]^{1/2}$
$= \left[16\cos^2\theta - 32\cos\theta + 16 + 16\sin^2\theta + 32\sin\theta + 16\right]^{1/2}$
$= \left[48 + 32\sin\theta - 32\cos\theta\right]^{1/2} \cong \left[16 + 32\sin\theta\right]^{1/2}$
$= 4\left[1 + 2\sin\theta\right]^{1/2}$

$\epsilon_{CB} = \frac{\overline{CB}^* - CB}{CB} = \frac{4(1+2\sin\theta)^{1/2} - 4}{4} = (1+2\sin\theta)^{1/2} - 1$
$= \left(1 + \frac{2\sin\theta}{2} + \ldots\right) - 1 = \sin\theta \cong \theta$

2.32



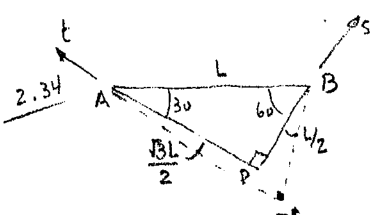


$\tan\alpha = \frac{\frac{1}{\sqrt{2}}L}{\frac{L}{\sqrt{2}} + .01L} = .986 \Rightarrow \alpha = 44.598° = .7784$

$45° = .7854 \text{ rad}$

$\gamma_{st} = 90° - 2(44.598°) = .8046° = \underline{.0140 \text{ rad.}}$

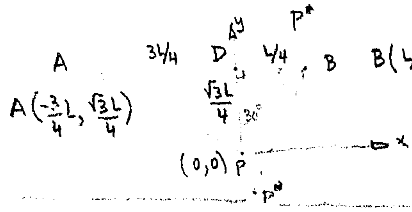
2.34



$\epsilon_s = .004$
$\epsilon_t = .003$

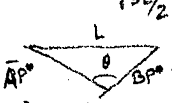
ASSUME AB IS OF LENGTH L
עוד פעם, מניחים שאחד מהדברים שווה ל-L.

$\frac{\sqrt{3}L}{4} = \overline{PD}$, $\frac{\sqrt{3}L}{2} = \overline{AP}$, $L/2 = \overline{BP}$ $\Leftarrow$ L-נניח ש- AB שווה ל



$\epsilon_s = \frac{\overline{BP}^* - BP}{BP} = \frac{\overline{BP}^* - L/2}{L/2} \Rightarrow \overline{BP}^* = (\epsilon_s + 1)L/2 = (1.004)L/2$

$\epsilon_t = \frac{\overline{AP}^* - AP}{AP} = \frac{\overline{AP}^* - \sqrt{3}L/2}{\sqrt{3}L/2} \Rightarrow \overline{AP}^* = (\epsilon_t + 1)\frac{\sqrt{3}L}{2} = (1.003)\frac{\sqrt{3}L}{2}$



$\Leftarrow \theta = \measuredangle AP^*B$ את הזווית

$L^2 = \overline{AP^*}^2 + \overline{BP^*}^2 - 2\overline{AP^*}\,\overline{BP^*}\cos\theta \Rightarrow \cos\theta = \frac{L^2 - \overline{AP^*}^2 - \overline{BP^*}^2}{-2\overline{AP^*}\,\overline{BP^*}}$

$= \frac{(1.003)^2\frac{3}{4}L^2 + (1.004)^2\frac{L^2}{4} - L^2}{\ldots} = .00745$

כאשר θ קרובה ל-90°, 90-θ קרובה ל-0°. לכן,

$\cos\theta = \cos(-\theta) = \cos(90° - \theta - 90°) = \cos(90-\theta)\cos 90° + \sin(90-\theta)\sin 90°$
$= \sin(90-\theta) \cong 90° - \theta$

PROBLEM 2.1

2.1 A steel rod is 2.2 m long and must not stretch more than 1.2 mm when a 8.5 kN load is applied to it. Knowing that $E = 200$ GPa, determine (*a*) the smallest diameter rod which should be used, (*b*) the corresponding normal stress caused by the load.

SOLUTION

(a) $\delta = \frac{PL}{AE} \therefore A = \frac{PL}{E\delta} = \frac{(8.5\times10^3)(2.2)}{(200\times10^9)(1.2\times10^{-3})} = 77.92\times10^{-6}\ m^2$

$A = \frac{\pi}{4}d^2 \therefore d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(77.92\times10^{-6})}{\pi}} = 9.96\times10^{-3}\ m$

$= 9.96\ mm$ ◄

(b) $\sigma = \frac{P}{A} = \frac{8.5\times10^3}{77.92\times10^{-6}} = 109.1\times10^6\ Pa = 109.1\ MPa$ ◄ 2

PROBLEM 2.8 like 2.6 HW Sp07

2.8 A cast-iron tube is used to support a compressive load. Knowing that $E = 10 \times 10^6$ psi and that the maximum allowable change in length is 0.025 percent, determine (*a*) the maximum normal stress in the tube, (*b*) the minimum wall thickness for a load of 1600 lb if the outside diameter of the tube is 2.0 in.

SOLUTION

(a) $\frac{\delta}{L} = \frac{0.025}{100} = 0.00025$ 2

$\sigma = \frac{E\delta}{L} = (10\times10^6)(0.00025) = 2.5\times10^3\ psi = 2.5\ ksi$ ◄ 2

(b) $\sigma = \frac{P}{A} \therefore A = \frac{P}{\sigma} = \frac{1600}{2.5\times10^3} = 0.640\ in^2$

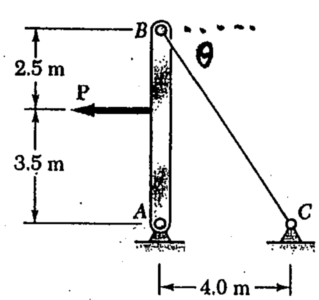
$A = \frac{\pi}{4}(d_o^2 - d_i^2)$

$d_i^2 = d_o^2 - \frac{4A}{\pi} = 2.0^2 - \frac{(4)(0.64)}{\pi} = 3.1851\ in^2 \therefore d_i = 1.7847\ in.$

$t = \frac{1}{2}(d_o - d_i) = \frac{1}{2}(2.0 - 1.7847) = 0.1077\ in.$ ◄ 1

PROBLEM 2.11 2.13

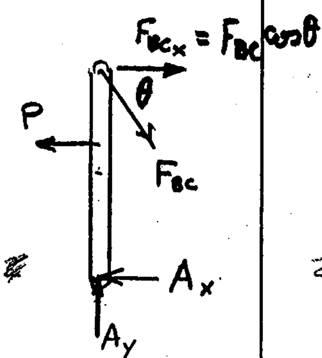
2.11 The 4-mm-diameter cable *BC* is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown.



SOLUTION

$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111\ m$

Use bar AB as a free body



$\circlearrowleft \Sigma M_A = 0 \quad 3.5P - (6)(\frac{4}{7.2111}F_{BC}) = 0$

$P = 0.9509\ F_{BC}$ 2

Considering allowable stress $\sigma = 190\times10^6\ Pa$

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.004)^2 = 12.566\times10^{-6}\ m^2$

$\sigma = \frac{F_{BC}}{A} \therefore F_{BC} = \sigma A = (190\times10^6)(12.566\times10^{-6}) = 2.388\times10^3\ N$ 2

Considering allowable elongation $\delta = 6\times10^{-3}\ m$

$\delta = \frac{F_{BC}L_{BC}}{AE} \therefore F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566\times10^{-6})(200\times10^9)(6\times10^{-3})}{7.2111} = 2.091\times10^3\ N$ 2

Smaller value governs $F_{BC} = 2.091\times10^3\ N$

$P = 0.9509\ F_{BC} = (0.9509)(2.091\times10^3) = 1.988\times10^3\ N = 1.988\ kN$ ◄ 2

$$\sigma_x = \lambda(\epsilon_x + \epsilon_y + \epsilon_z) + 2\mu\epsilon_x$$

$$I_{1_\epsilon}$$

$$\epsilon_x = \lambda(\epsilon_1 + \epsilon_2 + \epsilon_3) + 2\mu\epsilon_x$$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z) \qquad \sigma_z = 0$$

$$\epsilon_x = \frac{\sigma_x}{E} + \frac{\nu}{E}\sigma_y \qquad \epsilon_1 = \frac{\sigma_1}{E} - \frac{\nu}{E}\sigma_2$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z) \qquad \epsilon_2 = \frac{\sigma_2}{E} - \frac{\nu}{E}\sigma_1$$

$$\sigma_2 = \frac{E}{(1-\nu^2)}(\epsilon_2 + \nu\epsilon_1)$$

only for $\sigma_z = 0$

$$\sigma_1 = \frac{E}{(1-\nu^2)}(\epsilon_1 + \nu\epsilon_2)$$

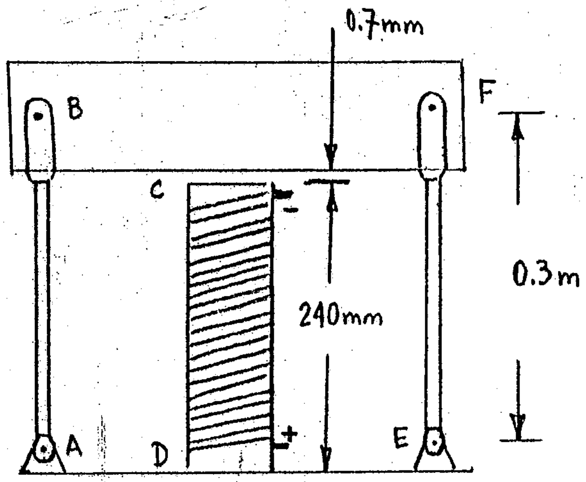
$\sigma_1 = -9725 \qquad \sigma_2 = -32418$

תרגיל מס. 4

המוט המרכזי CD של ההרכבה מחומם מ- T_1=30 C ל- T_2=180 C דרך חמום התנגדות חשמלי.
כאשר הטמפרטורה שווה ל- T_1 הרווח בין C והמוט הקשיח הוא 0.7 mm

א) מיצאו את הכוח במוטות AB ו- EF שבא משינוי הטמפרטורה של מוט CD.
מוטות AB ו- EF עשוים מפלדה, ולכל אחד יש שטח חתך של 125 mm^2, המוט CD נעשה מאלומיניום ולו יש שטח חתך של 375 mm^2.

ב) מה תהיה ההיתארכות/ההיתכווצות הסופית של המוט CD



$_{אל}E = 70\ GPa$ $\quad$ $_{פל}E = 210\ GPa$

$_{אל}\alpha = 23\times10^{-6}/°C$ $\quad$ $_{פל}\alpha = 18\times10^{-6}/°C$

$-2F_{st} = F_{al} \quad -F_{st} = F_{al}/2$ (3)

$\Delta_{טמפרטורה} = \frac{F_{al}\cdot L_{al}}{A_{al}E_{al}} - \frac{F_{st}L_{st}}{A_{st}\cdot E_{st}}$ (3)

$_{st}\Delta = \Delta_{al} - \Delta_{temp}$ after CD touches BF

$_{tep}\Delta = \Delta_{al} - \Delta_{st}$

$= F_{al}\left(\frac{L_{al}}{A_{al}E_{al}} + \frac{L_{st}}{2A_{st}E_{st}}\right)$

$23\times10^{-6}/°C \cdot \Delta T \cdot 0.24 = {_{אל}\alpha}\,\Delta T\,L = 0.7mm$ לסגור הרווח של (3)

$\Delta T = 126.81$

לכן ΔT הנשאר = $150 - 126.81 = 23.19°$

$1.280\times10^{-4}m = 23\times10^{-6}\times 23.19°\times 0.240 = \alpha\Delta T\cdot L' = \Delta_{טמפרטורה}$ (2)

$= F_{al}\left(\frac{0.2400\ m}{375\times10^{-6}\cdot 70\times10^{9}\frac{N}{m^2}} + \frac{0.3}{2\cdot125\times10^{-6}\cdot 210\times10^{9}}\right) = 1.486\times10^{-8}F_{al}$

$F_{al} = 8616\ N$

$F_{st} = -4308\ N = F_{BC} = F_{EF}$ (2)

$\Delta_{al} = \frac{F_{al}\cdot L_{al}}{A_{al}E_{al}} + \alpha\Delta T L_{al} = -\Delta_{st}$ (2)

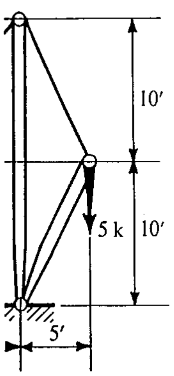
$\Delta al + .7\times10^{-4}$ $= +4.92\times10^{-5}m + .7\times10^{-4} = 1.192\times10^{-4}m.$ = התארכות של האלומיניום

15 נק'

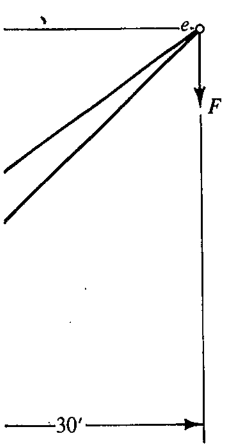
ıre. The total weight he two hangers is 15 k. g stresses in the 1-in.- s A and B due to the glect the weight of the that contact between angers is frictionless.

the mast of the derrick ıll members are in the ıd are joined by pins. ı an 8 in. standard steel ɔer foot. (See Appendix weight of the members.

ıeck the capacity of the wn in the figure. All steel and have the same f 8 in.^{2} Determine the



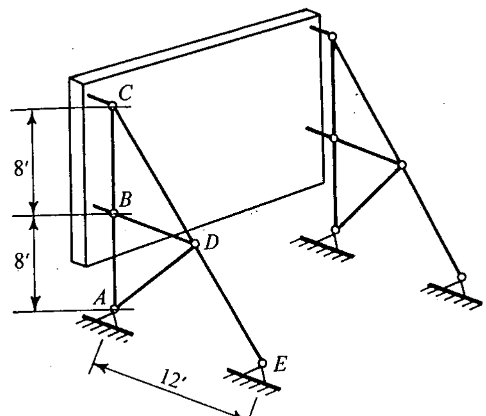
B. 3-11



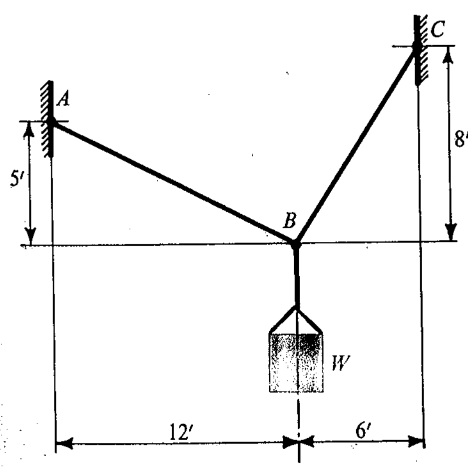
. 3-12

maximum allowable load F if the allowable stresses are 20,000 psi in tension and 15,000 psi in compression. All joints are pinned. *Ans.* 21.2 k.

3-13. A signboard 15 ft by 20 ft in area is supported by two frames as shown in the figure. All members are actually 2 in. by 4 in. in cross section. Calculate the stress in each member due to a horizontal wind load on the sign of 20 lb per square foot. Assume that all joints are connected by pins and that one-quarter of the total wind force acts at B and at C. Neglect the possibility of buckling of the compression members. Neglect the weight of the structure.



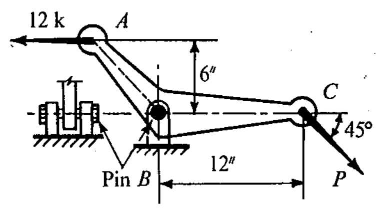
PROB. 3-13



PROB. 3-14

3-14. Two high-strength rods of different sizes are attached at A and C and support a load W at B as shown in the figure. What load W can be supported? The ultimate strength of the rods is 160 ksi, and the factor of safety is to be 4. The rod AB has $A = 0.20$ in.^{2}; the rod BC has $A = 0.10$ in.^{2}

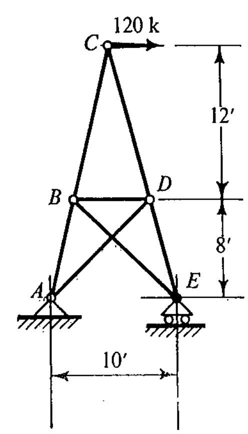
3-15. What is the required diameter of the pin B for the bell-crank mechanism shown in the figure if an applied force of 12 kips at A is resisted by a force P at C? The allowable shearing stress is 15,000 psi. *Ans.* 0.60 in.



PROB. 3-15

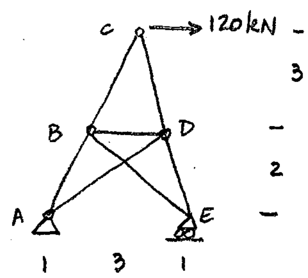
3-16. Find the required cross-sectional areas for all tension members in Example 3-6. The allowable stress is 20 ksi.

3-17. A tower used for a high line is shown in the figure. If it is subjected to a horizontal force of 120 kips and the allowable stresses are 15 ksi in compression and 20 ksi in tension, what is the required cross-sectional area of each member? All members are pin-connected.

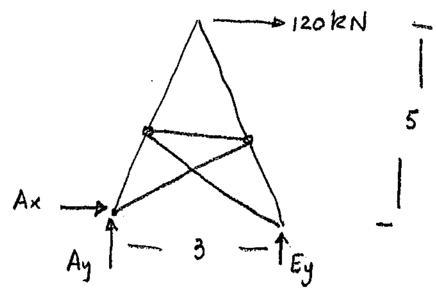


PROB. 3-17

תורת החוזק 1



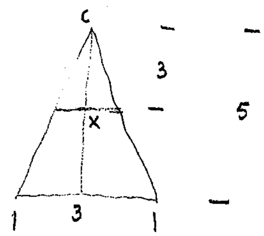
דיאגרמת גוף חופשי



$$\curvearrowleft + \sum M_A = -120(5) + E_y \cdot 3 = 0 \quad E_y = 200\text{ kN}$$

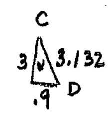
$$\overset{+}{\rightarrow} \sum F_x = 0 \quad 120 + A_x = 0 \Rightarrow A_x = -120\text{kN}$$

$$+\uparrow \sum F_y = 0 \quad E_y + A_y = 0 \Rightarrow A_y = -200\text{kN}$$

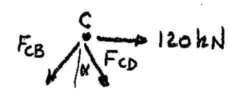


$$\frac{5}{3} = \frac{\text{גובה}}{\text{רוחב}} = \frac{3}{x}$$

$$x = \frac{9}{5} = 1.8m$$



בנקודה C



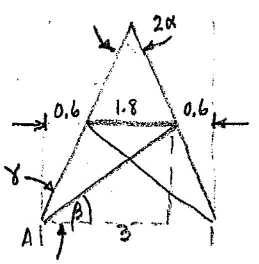
$$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow 120 + F_{CD}\sin\alpha - F_{CB}\sin\alpha = 0$$

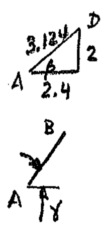
$$+\uparrow \sum F_y = 0 \Rightarrow -(F_{CD}\cos\alpha + F_{CB}\cos\alpha) = 0 \Rightarrow F_{CD} = -F_{CB}$$

$$\Rightarrow 2F_{CD}\sin\alpha = -120$$

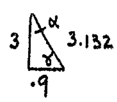
$$2F_{CD}\left(\frac{.9}{3.132}\right) = -120 \qquad \underline{F_{CD} = -208.8\text{ kN}}$$

$$\underline{F_{CB} = 208.8\text{ kN}}$$

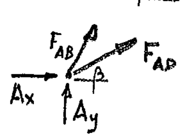




$$\gamma = \frac{180-2\alpha}{2} = 90-\alpha \Rightarrow$$



בנקודה A



$$\sum F_x = 0 \Rightarrow A_x + F_{AD}\cos\beta + F_{AB}\cos\gamma = 0$$

$$\sum F_y = 0 \Rightarrow A_y + F_{AD}\sin\beta + F_{AB}\sin\gamma = 0$$

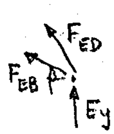
$$-120 + F_{AD}\left(\frac{2.4}{3.124}\right) + F_{AB}\left(\frac{.9}{3.132}\right) = 0$$

$$-200 + F_{AD}\left(\frac{2}{3.124}\right) + F_{AB}\left(\frac{3}{3.132}\right) = 0$$

$$\underline{F_{AD} = 104.132\text{kN}}$$

$$\underline{F_{AB} = 139.201\text{ kN}}$$

בנקודה E



$$\overset{+}{\rightarrow} \sum F_x = -F_{ED}\cos\gamma - F_{EB}\cos\beta = 0$$

$$\sum F_y = F_{ED}\sin\gamma + F_{EB}\sin\beta + E_y = 0$$

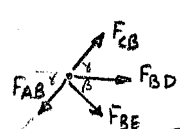
$$-F_{ED}\left(\frac{.9}{3.132}\right) - F_{EB}\left(\frac{2.4}{3.124}\right) = 0$$

$$F_{ED}\left(\frac{3}{3.132}\right) + F_{EB}\left(\frac{2}{3.124}\right) + 200 = 0$$

$$\underline{F_{ED} = -278.403\text{ kN}}$$

$$\underline{F_{EB} = 104.135\text{ kN}}$$

בנקודה B



$$\sum F_x = 0 =$$

$$F_{CB}\cos\gamma + F_{BD} + F_{BE}\cos\beta - F_{AB}\cos\gamma$$

$$(F_{CB} - F_{AB})\cos\gamma + F_{BE}\cos\beta + F_{BD} = 0$$

$$\sum F_y = 0 = (F_{CB} - F_{AB})\sin\gamma - F_{BE}\sin\beta$$

$$69.6\left(\frac{3}{3.132}\right) - 104.135\left(\frac{2}{3.124}\right) \approx 0$$

$$69.6\left(\frac{.9}{3.132}\right) + 104.135\left(\frac{2.4}{3.124}\right) = -F_{BD}$$

$$\underline{F_{BD} = -100\text{ kN}}$$

כוח במוט (kN)	מאמץ המותר (kPa)	שטח המוט $= \frac{F}{\sigma_{\text{מותר}}}$ (m^2)	
$F_{AB} = 139.201$	15 kPa	9.28	1.719
$F_{AD} = 104.132$	15 kPa	6.95	1.487
$F_{BD} = -100$	-12 kPa	8.34	1.629
$F_{CD} = -208.8$	-12 kPa	17.4	2.354
$F_{CB} = 208.8$	15 kPa	13.92	2.105
$F_{ED} = -278.403$	-12 kPa	23.20	2.718

- על פי המספרים, שטח המוטות גדולים ביותר. אבל התהליך לקבלם עדיין עבוד בנכונות. אם המאמץ נתון כ- kN/cm^2, אז המספרים יהיו יותר סבירים. זאת אומרת, שיחידות השטחים

IF $\sigma_{max} = 15MPa$ for AB

then $A = .00927m^2$

$r = .055$ m

$$P_E = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(206\times10^9)\left(\frac{\pi}{4}\right)(.055}{}$$

SOLUTION

In this case $P(x)$ is variable. It is conveniently expressed as $p_o x$ if the origin is taken at A. Here again Eq. 4-29 can be applied:

$$u = \int_0^x \frac{P(x)\,dx}{A(x)E} + C_1 = \frac{1}{AE}\int_0^x p_o x\,dx + C_1 = \frac{p_o x^2}{2AE} + C_1$$

At the boundary B, where $x = L$, the displacement is zero, i.e., $u(L) = 0$. This condition must be used to evaluate the constant of integration: $C_1 = -p_o L^2/2AE$. Thus $u = -p_o(L^2 - x^2)/2AE$ and $u(0) = -p_o L^2/2AE$. The negative sign indicates that the displacement u is in the opposite direction to that of positive x. If W designates the total weight of the rod, the absolute maximum deflection is $WL/2AE$. Compare this expression with Eq. 4-33.

In this problem Eq. 4-32 instead of Eq. 4-29 could be applied. With the gravity load acting downward and with the positive x axis directed upward, the sign of the load in Eq. 4-32 must be negative, i.e., $AE\,d^2u/dx^2 = -(-p_o)$. As in the previous solution, one of the boundary conditions is $u(L) = 0$. The second one is $u'(0) = 0$, where $u' = du/dx$; this follows from the fact that at the free end $P = 0$. (See Eq. 4-30.)

If a concentrated force P, in addition to the bar's own weight, were acting on the bar AB at the end A, the total end deflection due to the two causes by superposition would be

$$|u| = \frac{PL}{AE} + \frac{WL}{2AE} = \frac{[P + (W/2)]L}{AE}$$

EXAMPLE 4-10

A 30-in.-long aluminum rod is enclosed within a steel-alloy tube, Figs. 4-29(a) and (b). The two materials are bonded together. If the stress-strain diagrams for the two materials can be idealized as shown, respectively, in Fig. 4-29(d), what end deflection will occur for $P_1 = 80$ kips and for $P_2 = 125$ kips? The cross-sectional areas of steel A_s and of aluminum A_a are the same and equal to 0.5 in.².

SOLUTION

By applying the method of sections, one can easily determine the axial force at an arbitrary section, Fig. 4-29(c). However, unlike the case in any problem considered so far, the manner in which the resistance to the force P is distributed between the two materials is not known. Thus, the problem is internally statically indeterminate. The requirements of equilibrium (statics) remain valid, but additional conditions are necessary to solve the problem. One of the auxiliary conditions comes from the requirements of the compatibility of deformations. However, since the requirements of statics involve forces and deformations involve displacements, a connecting condition based on the property of materials must be added.

Let subscripts a and s on P, ε, and σ identify these quantities as being for aluminum and steel, respectively. Then, noting that the applied force is supported by a force developed in steel and aluminum and that

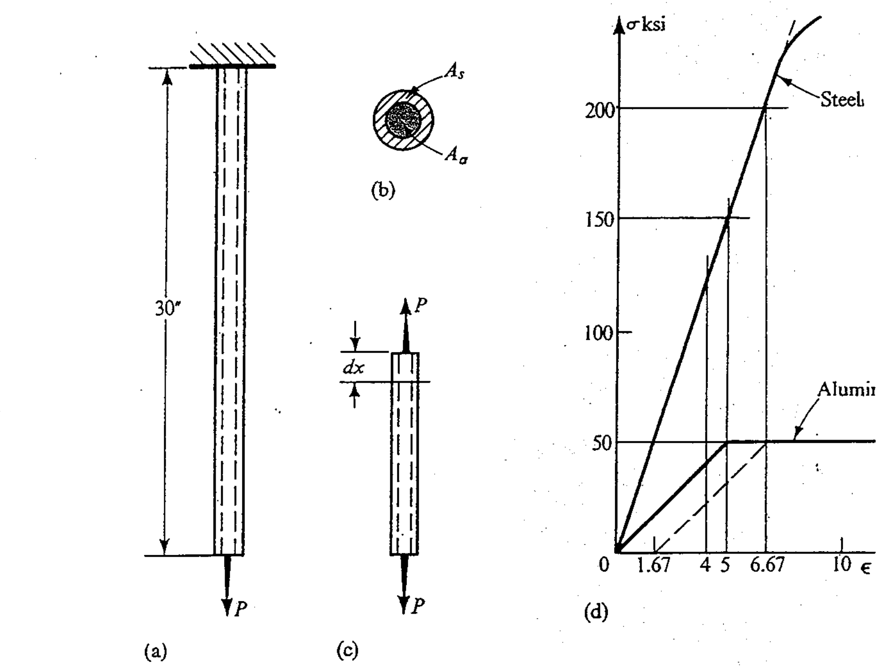


Fig. 4-29

at every section the displacement or the strain of the two materials is the same, and tentatively assuming elastic response of both materials, one has

Equilibrium: $P_a + P_s = P_1$ or P_2

Deformation: $u_a = u_s$ or $\varepsilon_a = \varepsilon_s$

Material properties: $\varepsilon_a = \sigma_a/E_a$ and $\varepsilon_s = \sigma_s/E_s$

By noting that $\sigma_a = P_a/A_a$ and $\sigma_s = P_s/A_s$, one can solve the three equations. From the diagram the elastic moduli are $E_s = 30 \times 10^6$ psi and $E_a = 10 \times 10^6$ psi. Thus

$$\varepsilon_a = \varepsilon_s = \frac{\sigma_a}{E_a} = \frac{\sigma_s}{E_s} = \frac{P_a}{A_a E_a} = \frac{P_s}{A_s E_s}$$

Hence $P_s = [A_s E_s/(A_a E_a)]P_a = 3P_a$, and $P_a + 3P_a = P_1 = 80$ k; therefore, $P_a = 20$ k, and $P_s = 60$ k.

Applying Eq. 4-33 to either material, the tip deflection for 80 kips will be

$$u = \frac{P_s L}{A_s E_s} = \frac{P_a L}{A_a E_a} = \frac{20(10^3)30}{0.5(10)10^6} = 0.120 \text{ in.}$$

This corresponds to a strain of $0.120/30 = 4 \times 10^{-3}$ in. per inch. In this range both materials respond elastically, which satisfies the material-property assumption made at the beginning of this solution. In fact, as may be seen from Fig. 4-29(d), since for the linearly elastic response the strain can reach 5×10^{-3} in. per inch for both materials, by direct proportion the applied force P can be as large as 100 kips.

At $P = 100$ kips the stress in aluminum reaches 50 ksi. According to the idealized stress-strain diagram no higher stress can be resisted by this material, although the strains may continue to increase. Therefore, beyond $P = 100$ kips, the aluminum rod can be counted upon to resist only $P_a = A_a\sigma_{yp} = 0.5 \times 50 = 25$ kips. The remainder of the applied load must be carried by the steel tube. For $P_2 = 125$ kips, 100 kips must be carried by the steel tube. Hence $\sigma_s = 100/0.5 = 200$ ksi. At this stress level $\varepsilon_s = 200/(30 \times 10^3) = 6.67 \times 10^{-3}$ in. per inch. Therefore, the tip deflection

$$u = \varepsilon_s L = 6.67 \times 10^{-3} \times 30 = 0.200 \text{ in.}$$

Note that it is not possible to determine u from the strain in aluminum since no unique strain corresponds to the stress of 50 ksi, which is all that the aluminum rod can carry. However, in this case the elastic steel tube contains the plastic flow. Thus, the strains in both materials are the same, i.e., $\varepsilon_s = \varepsilon_a = 6.67 \times 10^{-3}$ in. per inch, see Fig. 4-29(d).

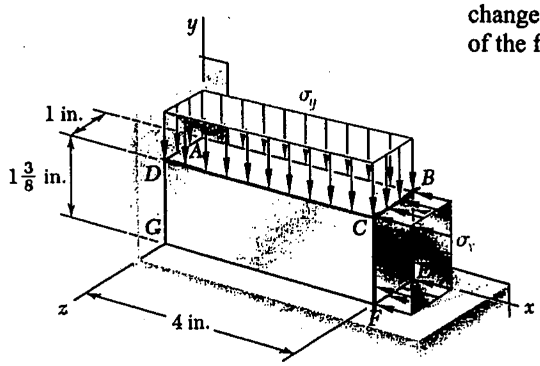
If the applied load $P_2 = 125$ kips were removed, both materials in the rod would rebound elastically. Thus if one imagines the bond between the two materials broken, the steel tube would return to its initial shape. But a permanent set (stretch) of $(6.67 - 5) \times 10^{-3} = 1.67 \times 10^{-3}$ in. per inch would occur in the aluminum rod. This incompatibility of strain cannot develop if the two materials are bonded together. Instead, residual stresses develop, which maintain the same axial deformations in both materials. In this case, the aluminum rod remains slightly compressed, and the steel tube is slightly stretched. The solution of such statically indeterminate problems is considered in greater detail in Chapter 12. The small effect due to Poisson's ratio is neglected in the above discussion.

4-18. STRESS CONCENTRATIONS

From the preceding articles of this chapter it is seen that stresses are accompanied by deformations. If such deformations take place at the same uniform rate in adjoining elements, no additional stresses, other than [illegible] the cross-sectional area of a member is interrupted or if the force is actually applied over a very small area, a perturbation in stresses takes place because the adjoining elements must be physically continuous in a deformed state. They must stretch or contract equal amounts at the adjoining sides of all particles. These deformations result from linear and shearing deformations involving the properties of materials E, G, and ν and the applied forces. Methods of obtaining this disturbed-stress distri-

PROBLEM 2.127

2.127 The block shown is made of a magnesium alloy for which $E = 6.5 \times 10^6$ psi and $\nu = 0.35$. Knowing that $\sigma_x = -20$ ksi, determine (*a*) the magnitude of σ_y for which the change in the height of the block will be zero, (*b*) the corresponding change in the area of the face *ABCD*, (*c*) the corresponding change in the volume of the block.



SOLUTION

$\delta_y = 0 \qquad \varepsilon_y = 0$ 2

$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = 0$

(a) $\sigma_y = \nu\sigma_x = (0.35)(-20\times10^3)$

$= -7\times10^3 \text{psi} = -7 \text{ ksi}$ ◄

2

(b) $\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y) = -\frac{\nu(\sigma_x+\sigma_y)}{E}$

$= \frac{(0.35)(-20\times10^3 - 7\times10^3)}{6.5\times10^6} = 1.4538\times10^{-3}$

$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{-20\times10^3 - (0.35)(-7\times10^3)}{6.5\times10^6}$

$= -2.7\times10^{-3}$

$A_o + \Delta A = L_x(1+\varepsilon_x)L_z(1+\varepsilon_z) = L_xL_z(1+\varepsilon_x+\varepsilon_z+\varepsilon_x\varepsilon_z)$

But $A_o = L_xL_z$

$\Delta A = L_xL_z(\varepsilon_x + \varepsilon_z + \varepsilon_x\varepsilon_z)$

$= (4.0)(1.0)(1.4538\times10^{-3} - 2.7\times10^{-3} + \text{small term})$

$= -4.98\times10^{-3} \text{ in}^2 = -0.00498 \text{ in}^2$ 4 ◄

(c) Since L_y is constant

$\Delta V = L_y(\Delta A) = (1.375)(-4.98\times10^{-3}) = -6.85\times10^{-3} \text{ in}^3$

$= -0.00685 \text{ in}^3$ 2 ◄

$\epsilon_z = 1.454\times10^{-3}$
$\epsilon_x = -2.7\times10^{-3}$

$L_{x_{new}} = L_x(1+\epsilon_x)$
$L_{z_{new}} = L_y(1+\epsilon_y)$

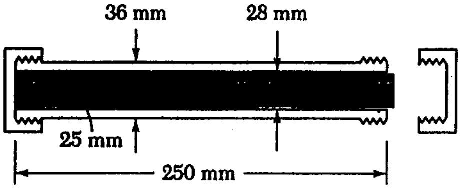
$A_{new} = L_{x_{new}} \cdot L_{z_{new}}$ $\qquad \Delta A = A_{new} - L_x \cdot L_z$

$\frac{\Delta V}{V} = e = \epsilon_x + \epsilon_y + \epsilon_z = -1.2462\times10^{-3}$ $\qquad \Delta V = V(-1.2462\times10^{-3})$

$= (1\cdot1.375\cdot4)(-1.2462\times10^{-3})$

$=$

PROBLEM 2.126



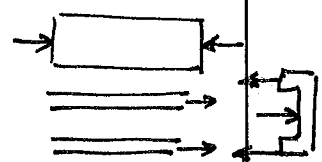
2.125 A 250-mm-long aluminum tube (E = 70 GPa) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod (E = 105 GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (*a*) the average normal stress in the tube and in the rod, (*b*) the deformations of the tube and of the rod.

2.126 In Prob. 2.125, determine the average normal stress in the tube and the rod, assuming that the temperature was 15° C when the nuts were snugly fitted and that the final temperature is 55° C. (For aluminum, $\alpha = 23.6 \times 10^{-6}/°C$; for brass, $\alpha = 20.9 \times 10^{-6}/°C$.)

SOLUTION

$$A_{tube} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12 \times 10^{-6} \text{ m}^2$$

$$A_{rod} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2 = 490.87 \times 10^{-6} \text{ m}^2$$



$$\Delta T = 55 - 15 = 40 \text{ °C}$$

$$\delta_{tube} = \frac{PL}{E_{tube}A_{tube}} + L\alpha_{tube}(\Delta T) = \frac{P(0.250)}{(70\times10^9)(402.12\times10^{-6})} + (0.250)(23.6\times10^{-6})(40)$$

$$= 8.8815\times10^{-9}P + 236\times10^{-6}$$ 2

$$\delta_{rod} = -\frac{PL}{E_{rod}A_{rod}} + L\alpha_{rod}(\Delta T) = -\frac{P(0.250)}{(105\times10^9)(490.87\times10^{-6})} + (0.250)(20.9\times10^{-6})(40)$$

$$= -4.8505\times10^{-9}P + 209\times10^{-6}$$ 2

$$\delta^* = \frac{1}{4}\text{ turn} \times 1.5 \text{ mm} = 0.375 \text{ mm} = 375\times10^{-6} \text{ m}$$

$$\delta_{tube} = \delta_{rod} + \delta^*$$ 1

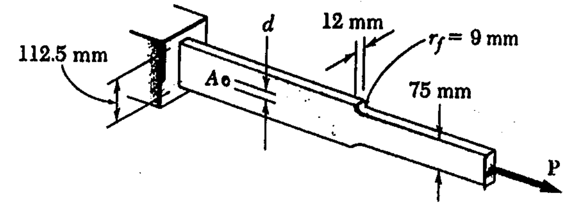
$$8.8815\times10^{-9}P + 236\times10^{-6} = -4.8505\times10^{-9}P + 209\times10^{-6} + 375\times10^{-6}$$

$$13.732\times10^{-9}P = 348\times10^{-6} \qquad P = 25.342\times10^3 \text{ N}$$ 1

$$\sigma_{tube} = \frac{P}{A_{tube}} = \frac{25.342\times10^3}{402.12\times10^{-6}} = 63.0\times10^6 \text{ Pa} = 63.0 \text{ MPa}$$ ◄ 2

$$\sigma_{rod} = -\frac{P}{A_{rod}} = -\frac{25.342\times10^3}{490.87\times10^{-6}} = -51.6\times10^6 \text{ Pa} = -51.6 \text{ MPa}$$ ◄ 2

PROBLEM 2.134 shown. (*b*) Solve part *a*, assuming that the hole at *A* is not drilled.



SOLUTION

Maximum stress at hole

Use Fig. 2.64 a for values of K

$\frac{r}{d} = \frac{6}{112.5 - 12} = 0.0597, \quad K = 2.80$ 2

$A_{net} = (12)(112.5 - 12) = 1206\ mm^2 = 1206 \times 10^{-6}\ m^2$

$\sigma_{max} = K\frac{P}{A_{net}} = \frac{(2.80)(58 \times 10^3)}{1206 \times 10^{-6}} = 134.7 \times 10^6\ Pa$ 2

Maximum stress at fillets

Use Fig. 2.64 b

$\frac{r}{d} = \frac{9}{75} = 0.12, \quad \frac{D}{d} = \frac{112.5}{75} = 1.50, \quad K = 2.10$ 2

$A_{min} = (12)(75) = 900\ mm^2 = 900 \times 10^{-6}\ m^2$

$\sigma_{max} = K\frac{P}{A_{min}} = \frac{(2.10)(58 \times 10^3)}{900 \times 10^{-6}} = 135.3 \times 10^6\ Pa$ 2

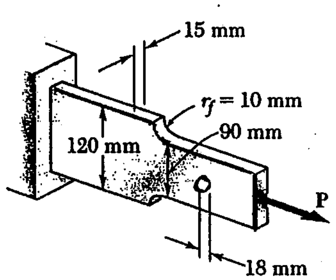
(a) With hole and fillets $\sigma_{max} = 134.7\ MPa$ ◀

2

(b) Without hole $\sigma_{max} = 135.3\ MPa$ ◀

PROBLEM 2.100

2.100 A centric axial force is applied to the steel bar shown. Knowing that σ_{all} is 135 MPa, determine the maximum allowable load **P**.



SOLUTION

At the hole: $r = 9\ mm \quad d = 90 - 18 = 72\ mm$

$\frac{r}{d} = 0.125$ From Fig 2.64 a $K = 2.65$ 2

$A_{net} = t\,d = (15)(72) = 1.08 \times 10^3\ mm^2 = 1.08 \times 10^{-3}\ m^2$

$\sigma_{max} = \frac{KP}{A_{net}}$

$P = \frac{A_{net}\,\sigma_{max}}{K} = \frac{(1.08 \times 10^{-3})(135 \times 10^6)}{2.65} = 55 \times 10^3\ N = 55\ kN$ 2

At the fillet $D = 120\ mm, \quad d = 90\ mm, \quad \frac{D}{d} = \frac{120}{90} = 1.333$

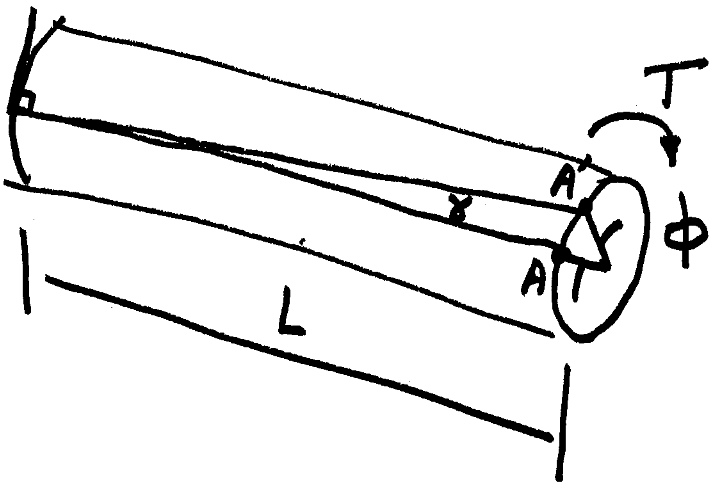
$r = 10\ mm \quad \frac{r}{d} = \frac{10}{90} = 0.1111$ From Fig 2.64 b $K = 2.02$ 2

$A_{min} = t\,d = (15)(90) = 1.35 \times 10^3\ mm^2 = 1.35 \times 10^{-3}\ m^2$

$\sigma_{max} = \frac{KP}{A_{min}}$

$P = \frac{A_{min}\,\sigma_{max}}{K} = \frac{(1.35 \times 10^{-3})(135 \times 10^6)}{2.02} = 90 \times 10^3\ N = 90\ kN$ 2

angle of twist.



$$\overline{AA'} = L \cdot \gamma = c\phi$$

$$\gamma = \frac{\tau_{max}}{G}$$

$$L\frac{\tau_{max}}{G} = c\,\phi$$

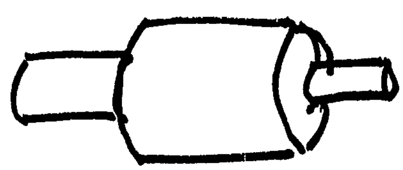
Angle of Twist

$$\phi = \frac{L}{G}\frac{\tau_{max}}{c} = \frac{L}{G}\frac{T}{J}$$

$$\frac{\phi}{L} \text{ - unit angle of twist} = \frac{T}{GJ}$$

$$\phi_{TOT} = \sum \phi_i = \sum \frac{T_i L_i}{G_i J_i}$$

$$= \int_0^L \frac{T\,dx}{GJ}$$



$$u_{TOT} = \sum \frac{P_i L_i}{A_i E_i}$$

$$u = \int_0^L \frac{P\,dx}{AE}$$

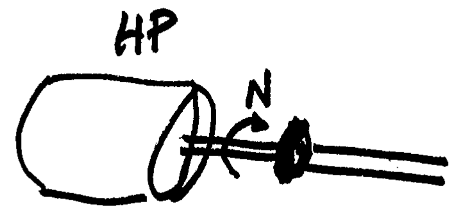
$$\text{Power} = F \cdot V \quad \text{or} \quad T \cdot \omega = T \cdot 2\pi N$$

(N: revs/min)

$$[\text{lb-in}]\ T = 63025 \cdot \frac{HP}{N\ (\text{revs/min})}$$

$$[\text{lb-ft}]\ T = 5252 \cdot \frac{HP}{N\ (\text{revs/min})}$$

$$[\text{N-m}]\ T = 9540 \cdot \frac{P\ (\text{kw})}{N\ (\text{revs/min})}$$



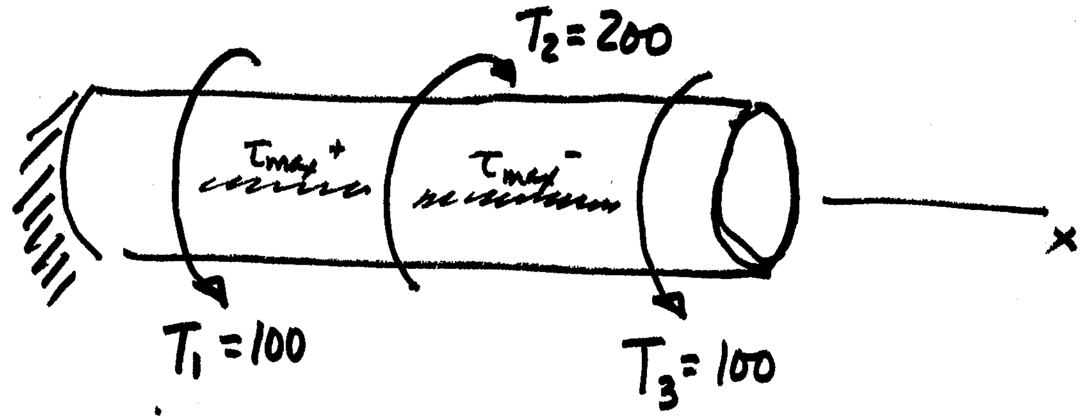
Design a steel shaft for 10 Hp motor operating at 1800 rpm

Max. shear is 8000 psi

$$T = 63025 \frac{HP}{N} = \frac{63025 \cdot 10}{1800} = 350 \text{ lb-in}$$

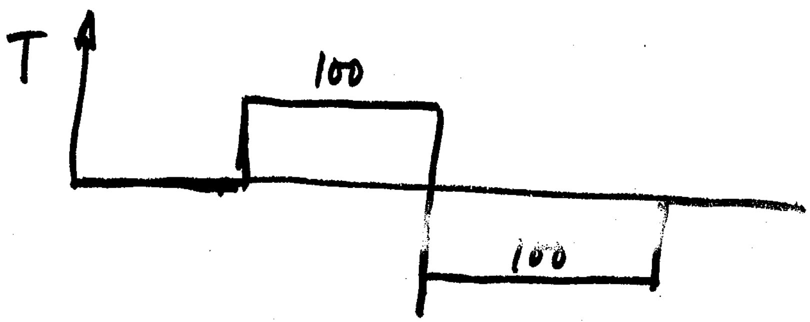
$$\tau_{max} = \frac{Tc}{J} = 350 \frac{c}{\pi c^4/2} = \frac{700}{\pi c^3} = 8000$$

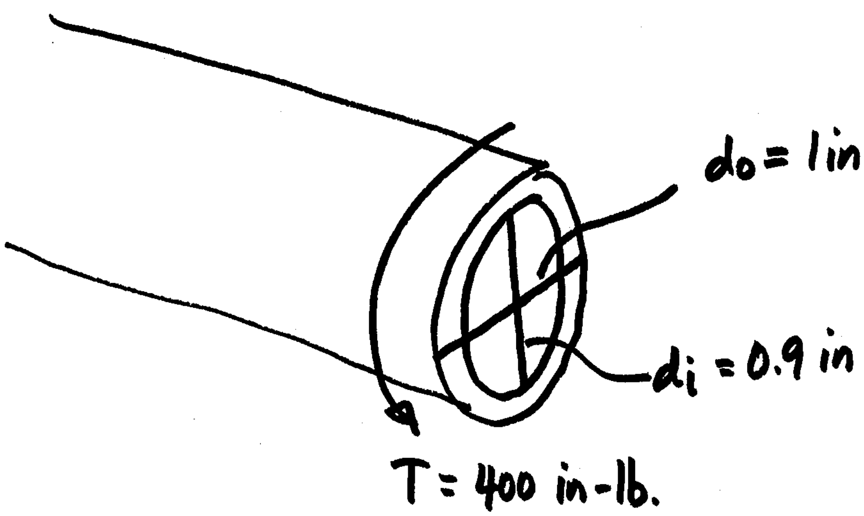
$$c = \sqrt[3]{\frac{700}{\pi(8000)}} = .303 \text{ in}$$



WHERE is τ_{max}

$$\tau_{max} = \frac{Tc}{J}$$



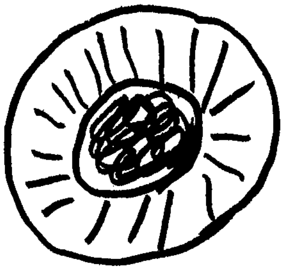


what is τ at the inner surface $c_i = 0.45in$

τ at the outer surface $c_o = 0.5in$

$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right) = 0.0337\ in^4$$

$\tau_{max} = \frac{Tc_i}{} = \frac{400(.45)}{} \cong 5330$ psi ; $\tau_{outer} = \frac{Tc_o}{} = \frac{400(.5)}{} \cong 5930$



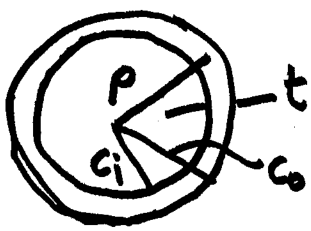
$\rho = .448 \Rightarrow$ carries only 4% of torque

$$T = \frac{\tau_{max}}{c} J = \frac{\tau}{\rho} J = \frac{\tau_{max}}{c} \cdot \frac{\pi c^4}{2}$$

$$.04T = \frac{\tau_{max}}{c} \cdot \frac{2\pi r^4}{4}$$

$$.04 = \left(\frac{r}{c}\right)^4$$

$$.448 = \frac{r}{c}$$



Thin tube

$$c = \frac{c_i + c_o}{2}$$

$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right)$$

$$= \frac{\pi}{2}\left(c_o^2 - c_i^2\right)\left(c_o^2 + c_i^2\right)$$

$$= \frac{\pi}{2}\left(c_o - c_i\right)\left(c_o + c_i\right)\left(c_o^2 + c_i^2\right)$$

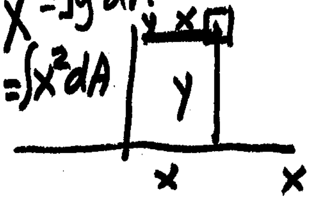
$$= \pi \cdot tc \cdot 2c^2$$

$$\underline{J = 2\pi t c^3}$$

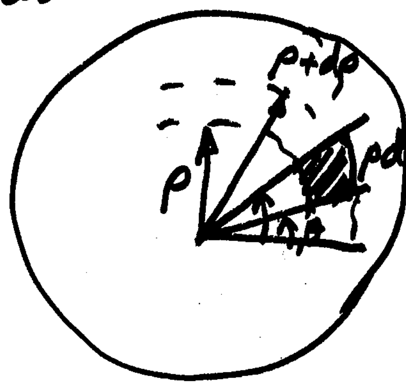
$J = \int \rho^2 dA$ Polar moment of inertia

$I_x = \int y^2 dA$

$I_y = \int x^2 dA$



$dA = \rho\, d\rho\, d\theta$



$J = \iint \rho^2 \cdot \rho\, d\rho \cdot d\theta$

SOLID CYLINDER

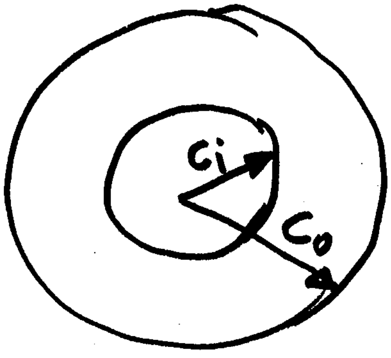
$= \int_0^{2\pi} d\theta \int_0^c \rho^3 d\rho = 2\pi \cdot \frac{\rho^4}{4}\Big|_0^c = \frac{\pi c^4}{2} = J$

$T = \frac{\tau_{max}}{c} \cdot J \Rightarrow \quad T = \frac{\tau}{\rho} J \quad \boxed{\tau = \frac{T\rho}{J}}$

$\downarrow$

$\tau_{max} = \frac{Tc}{J}$

Thick Tube

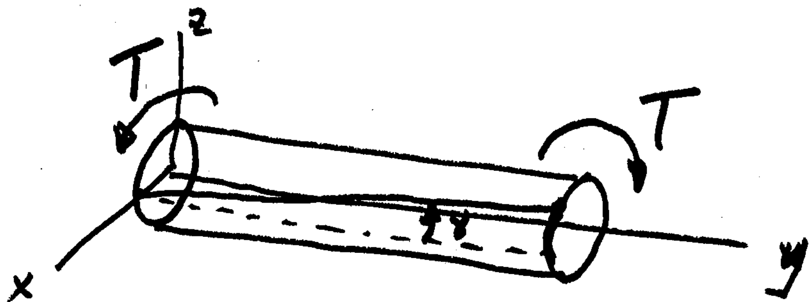


$J = \int \rho^2 dA = \int_0^{2\pi} d\theta \cdot \int_{c_i}^{c_o} \rho^3 d\rho$

$J = 2\pi \left[\frac{c_o^4}{4} - \frac{c_i^4}{4} \right]$

EMA 3702 2nd TAPE

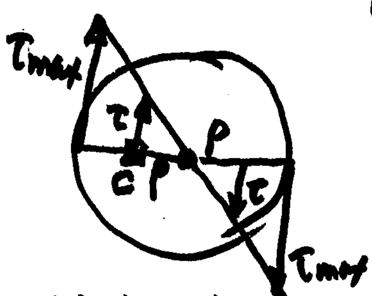
TORSION



Assumptions

(1) Plane sections remain plane after deformation & no warping or distortion of the plane section normal to the long axis

(2) For cylindrical members, shear stresses are not constant but vary linearly from the center of the member to a maximum on the outer surface



$$\frac{\tau_{max}}{c} = \frac{\tau}{\rho}$$

(3) Material remains elastic & Hooke's Law applies

$\tau \sim \gamma$ $\quad\quad$ $\tau = G\gamma$

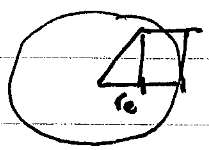
$dF = \tau dA$

$dT = (\tau dA)\rho$

$$T = \int dT = \int \tau \rho \, dA = \int \rho \frac{\tau_{max}}{c} \cdot \rho dA$$

$$= \frac{\tau_{max}}{c} \int \rho^2 dA$$

- Talk about sheet
- Talk about HP & Torque] do proble 3-84 on pg 171
- Talk about K for torsion] combo.
- Talk about ~~...~~ plastic torque



$$T = \int \tau \cdot dA \cdot \rho = \int_0^{r_e} \tau_e \cdot dA \cdot \rho + \int_{r_e}^{R} \tau_y \cdot dA \cdot \rho$$

$$= \int_0^{r_e} \tau_{max} \cdot \frac{\rho}{r_e} \cdot dA \cdot \rho + \int_{r_e}^{R} \tau_y \cdot dA \rho$$

if $\tau_y = \tau_{max}$ when $r_e = R$ $\quad T = \frac{\tau_y \cdot R}{\pi R^4/2} = \frac{\tau_{max} R}{J}$

if $\tau_y = \tau_{max}$ when $r_e = 0$ $\quad T = \tau_y \cdot \frac{\rho^3}{3}\Big|_0^R \cdot 2\pi = \tau_y \cdot R^3$

1

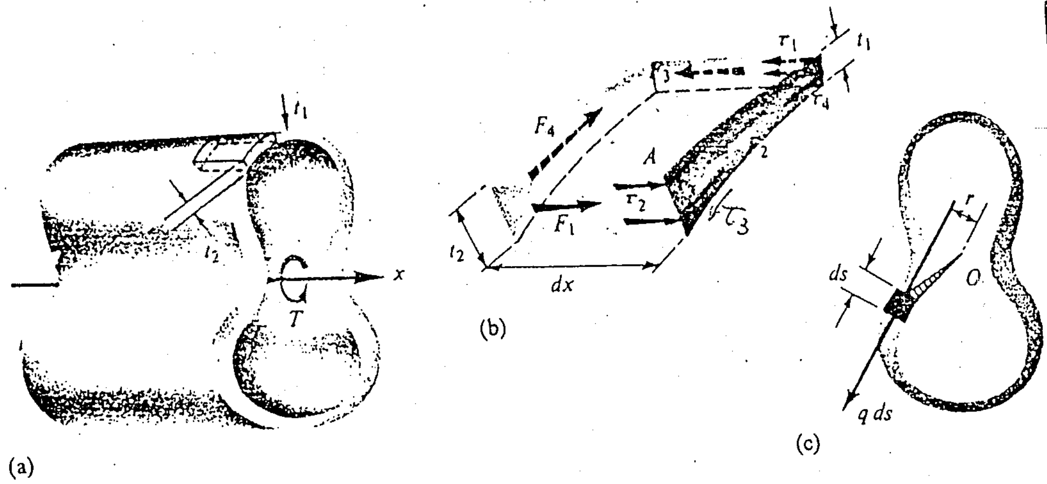


Fig. 3-22. Thin-walled member of variable thickness.

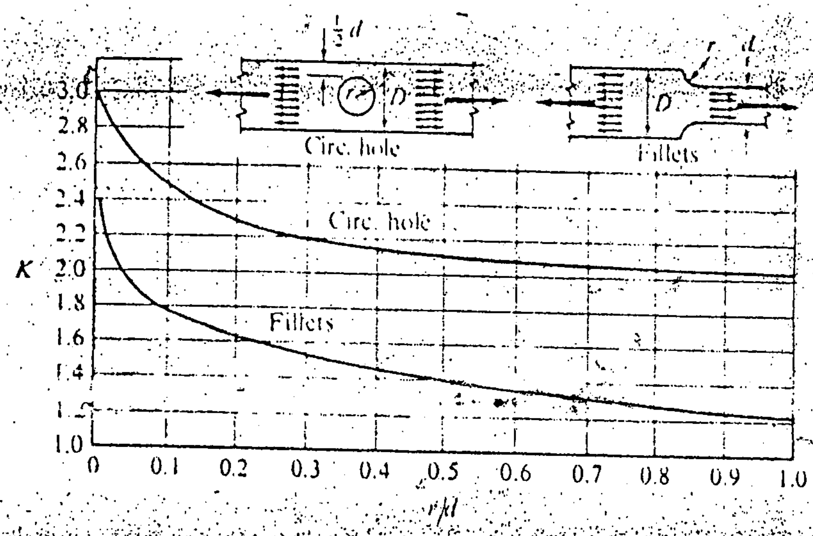


Fig. 2-17. Stress-concentration factors for flat bars in tension

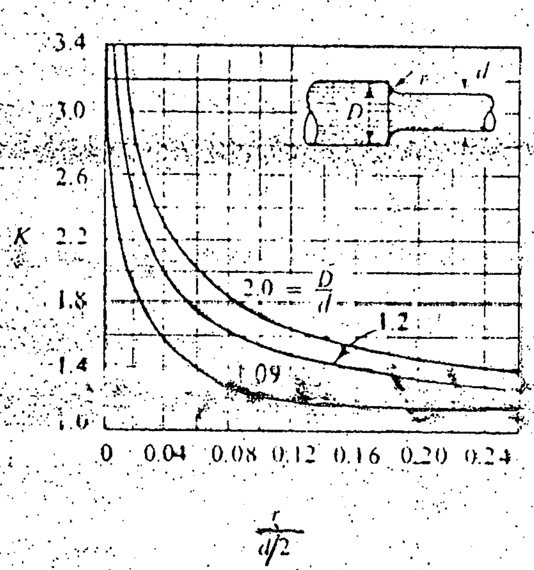
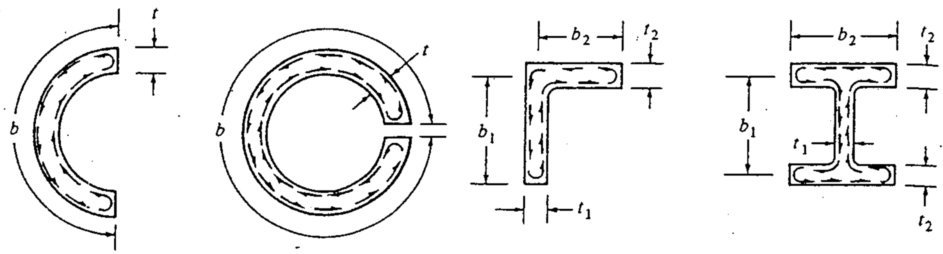


Fig. 3-15. Torsional stress-concentration factors in circular shafts of two diameters.



$J_R = \frac{bt^3}{3}$ (a) $J_R = \frac{bt^3}{3}$ (b) $J_R = \frac{b_1 t_1^3 + b_2 t_2^3}{3}$ (c) $J_R = \frac{b_1 t_1^3 + 2b_2 t_2^3}{3}$ (d)

FIGURE 8.4.3. Thin-walled open sections, showing their shear stresses and torsional constants J_R. In (a) and (b), dimension b is the length of the midline of the cross section.

when a bar is bent to form a helical spring, some corrections to these formulas may be necessary [8.3].

The membrane analogy permits a very useful generalization of the foregoing results to other thin-walled open cross sections. Imagine that the narrow rectangular cross section of Fig. 8.4.2 is distorted into a C or an L shape, or attached to another narrow rectangle to make a T or an I section. By visualizing the inflated membranes for these shapes, we decide that ϕ surfaces of all of them remain parabolic (excepting near ends and near reentrant corners, which we discuss later). The total torque is the sum of torques carried by each part of the cross section. Since $G\beta$ is the same for each part, Eqs. 8.4.5 and 8.4.8 still apply but with the $bt^3/3$ contributions summed to yield

$$J_R = \frac{1}{3}\sum_{i=1}^{n} b_i t_i^3 \tag{8.4.10}$$

where n is the number of parts into which the cross section is divided for purposes of calculation. Examples of this calculation appear in Fig. 8.4.3. In angle and I sections, the part of greater thickness displays greater shear stress. Where there is taper, as in the cross section of a turbine blade or in flanges of a rolled section, we can use

$$J_R = \frac{1}{3}\int t^3\, ds \tag{8.4.11}$$

where ds is an increment of length along the medial line of the cross section. More exact formulas are available [8.3]. These formulas, experimental results [8.5], and coefficient C_β in Table 8.4.1 suggest that, for standard rolled structural shapes, J_R may actually be some 10 percent higher than predicted by Eqs. 8.4.10 and 8.4.11. Omission of this adjustment yields a conservative design under either a stress limit or a deflection limit.

8.5 A HYDRODYNAMIC ANALOGY

There are several hydrodynamic analogies to the torsion problem [8.1]. In section we outline one, without proof, and use it to draw useful conclus about shear stresses in twisted bars.

If a fluid without viscosity executes motion in the yz plane with con vorticity, its motion is described by the equation

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = -2\omega \qquad (8.$$

where ϕ is the stream function and ω is the (constant) vorticity. Fluid veloc v and w in y and z directions, respectively, are

$$v = \frac{\partial \phi}{\partial z} \qquad w = -\frac{\partial \phi}{\partial y} \qquad (8.$$

Clearly, Eqs. 8.5.1 and 8.5.2 are analogous to Eqs. 8.3.5 and 8.3.1, res tively. Thus fluid velocities are proportional to shear stresses. Lines ϕ = stant are streamlines. The fluid boundary must be a streamline, therefore boundary condition $d\phi = 0$ of Eq. 8.3.6 is met. Thus the analogy is compl

Experiments have been done in the following way. Imagine that a sq cross section is to be studied. A shallow square tank is painted black and pla on a turntable. A camera, looking down onto the tank, is also attached to turntable. The tank is filled with water (whose viscosity is low, if not q zero) and aluminum powder is sprinkled on the water. When all is quiet, camera shutter is opened while the turntable is rotated about 10°. From viewpoint of the photograph, the tank is stationary while the fluid rotates nearly constant vorticity. The aluminum particles show as streaks on the f Each streak is in the direction of a shear stress. The length of the strea proportional to the magnitude of the shear stress.

Experiments aside, the hydrodynamic analogy has other uses. One is in aid visualization of torsion problems. Another is that known solutions for fluid f can be applied to the torsion problem. For example, consider an elliptical stacle (Fig. 8.5.1a). Far from the obstacle the flow is uniform and horizon so $w = 0$ and $v = v_o$, a constant. Points A are stagnation points, where v $w = 0$. Fluid theory shows that at points B, where $z = \pm b$, fluid veloci are $v = v_o(1 + b/a)$ and $w = 0$. Applying these results to a twisted bar, F 8.5.1b, we conclude that stress is zero at the sharp external corners C. At root of the small elliptical notch, shear stress is approximately

$$\tau = \tau_o\left(1 + \frac{b}{a}\right) \qquad (8.5$$

where τ_o is the stress that would prevail near the boundary if the notch w not there. The calculation is approximate because the undisturbed stress is the uniform value τ_o unless the notch is very small. The quantity in parenthe

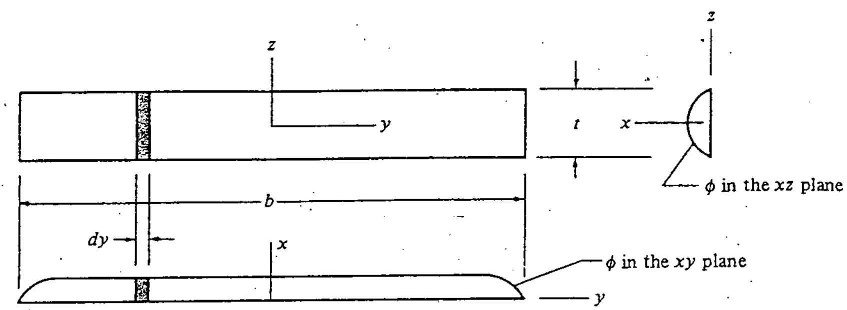


FIGURE 8.4.2. Narrow rectangular cross section ($b >> t$), with views of the stress function (or inflated membrane).

where C is a constant. Substituting into Eq. 8.3.5, we find

$$-2C = -2G\beta \qquad \text{so} \qquad C = G\beta \tag{8.4.4}$$

The torque is

$$T = 2\int_{\text{area}} \phi \, dA = 2\int_{-t/2}^{t/2} \phi b \, dz = G\beta \frac{bt^3}{3} \tag{8.4.5}$$

The maximum shear stress, found along the edges $z = \pm t/2$, is

$$\tau = \left| \frac{\partial \phi}{\partial z} \right|_{z=\pm t/2} = 2C\frac{t}{2} = G\beta t \tag{8.4.6}$$

Substituting for β from Eq. 8.4.5, we find

$$\tau = \frac{Tt}{bt^3/3} = \frac{3T}{bt^2} \qquad \text{at} \quad z = \pm\frac{t}{2} \tag{8.4.7}$$

Equations 8.4.5 and 8.4.7 can be written in the forms

$$\frac{d\theta}{dx} = \beta = \frac{T}{GJ_R} \qquad \tau = \frac{Tt}{J_R} \tag{8.4.8}$$

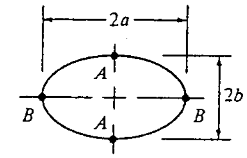
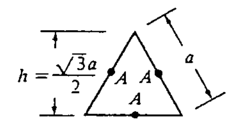
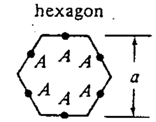
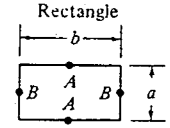
where

$$J_R = \frac{bt^3}{3} \qquad (\text{for } b >> t) \tag{8.4.9}$$

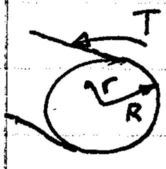
These expressions for β and τ are similar in form to the corresponding expressions for a circular cross section, Eqs. 8.1.2 and 8.1.3. However, J_R is *emphatically NOT* the polar moment of the cross-sectional area about the centroidal x axis.

If (say) $b/t = 10$, Eqs. 8.4.8 give β and τ values that are approximately 6.5 percent low. Accuracy improves as b/t increases. Aspect ratios in the range $1 < b/t < 10$ can be analyzed with tabulated data obtained by other analytical or numerical methods (Table 8.4.1). If the centerline of the bar is curved, as

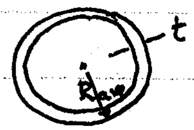
TABLE 8.4.1 Expressions for Maximum Shear Stress and Rate of Twist in Selected Solid Sections [8.1, 8.4]

Cross Section and Area	Maximum Shear Stress	Rate of Twist
Ellipse  Area = πab	$\tau_A = \dfrac{2T}{\pi ab^2}$ $(a > b)$ (τ_{max} at B if $b > a$)	$\beta = \dfrac{a^2 + b^2}{\pi a^3 b^3}\dfrac{T}{G} = \dfrac{d\theta}{dx}$
Equilateral triangle  Area = $0.433\,a^2$	$\tau_A = \dfrac{20T}{a^3}$	$\beta = \dfrac{46.2}{a^4}\dfrac{T}{G} = \dfrac{d\theta}{dx}$
Regular hexagon  Area = $0.866a^2$	$\tau_A = \dfrac{5.7T}{a^3}$	$\beta = \dfrac{8.8}{a^4}\dfrac{T}{G} = \dfrac{d\theta}{dx}$
Rectangle  Area = ab	$\tau_A = \dfrac{T}{C_\tau ba^2}$	$\beta = \dfrac{1}{C_\beta ba^3}\dfrac{T}{G} = \dfrac{d\theta}{dx}$

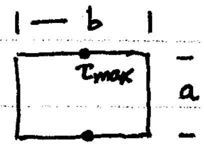
b/a	C_τ	τ_B/τ_A	C_β
1.0	0.208	1.000	0.1406
1.2	0.219	0.935	0.166
1.5	0.231	0.859	0.196
2.0	0.246	0.795	0.229
2.5	0.258	0.766	0.249
3.0	0.267	0.753	0.263
4.0	0.282	0.745	0.281
6.0	0.299	0.743	0.299
10.0	0.312	0.742	0.312
∞	0.333	0.742	0.333



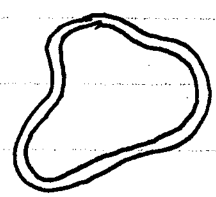
$$\tau = \frac{Tr}{J} \qquad \tau_{max} = \frac{TR}{J} \qquad \varphi = \frac{TL}{JG} \qquad J = \frac{\pi R^4}{2}$$



$$\tau = \frac{Tr}{J} \qquad \tau_{max} = \frac{TR_{ave}}{J} \qquad \varphi = \frac{TL}{JG} \qquad J = 2\pi R_{ave}^2 t$$

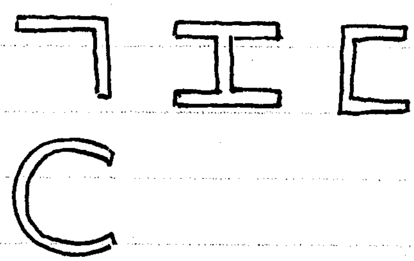


$$\tau_{max} = \frac{T}{C_\tau b a^2} \qquad C_\tau, C_\beta = \frac{1}{3}\left(1 - .63\frac{a}{b}\right) \qquad \varphi = \frac{TL}{C_\beta b a^3 G}$$



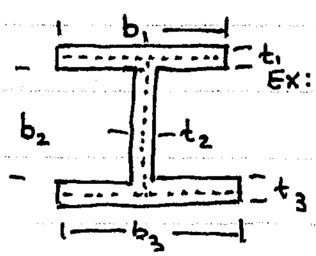
$$\tau_{ave} = \frac{T}{2A_{inc}t} \qquad \varphi = \frac{TL}{4A_{inc}^2 G}\oint \frac{ds}{t}$$

Open sections



$$\tau = \frac{Tt}{J_R} \qquad \varphi = \frac{TL}{J_R G}$$

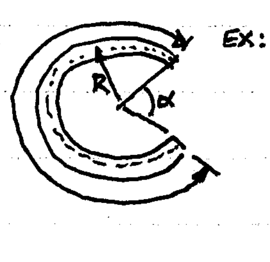
where J_R is the moment of inertia for the section



EX: $J_R = C_{\tau_1} b_1 t_1^3 + C_{\tau_2} b_2 t_2^3 + C_{\tau_3} b_3 t_3^3$

IF $b_1, b_2, b_3 \gg t_1, t_2, t_3$ $\quad C_\tau = \frac{1}{3}$

here $C_\tau = \frac{1}{3}\left(1 - 0.63\frac{t}{b}\right)$



EX:

$$J_R = \underbrace{(2\pi - \alpha)R}_{b} \cdot t^3 \cdot C_\tau$$

where $C_\tau = \frac{1}{3}(1 - .63\frac{t}{b}$

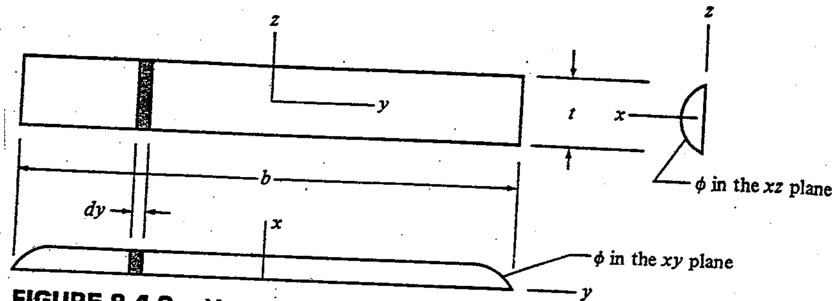


FIGURE 8.4.2. Narrow rectangular cross section ($b >> t$), with views of the stress function (or inflated membrane).

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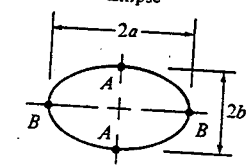
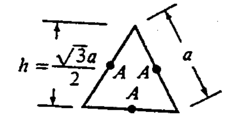
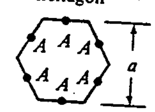
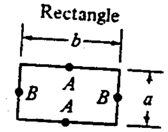
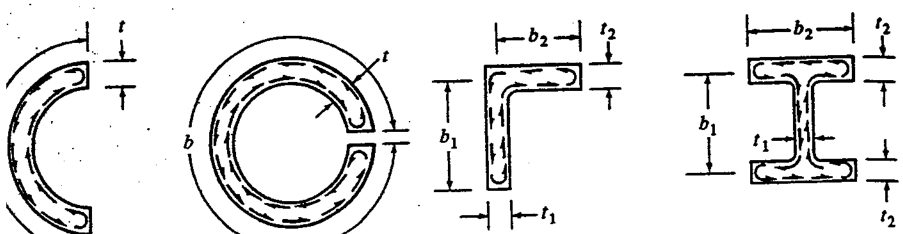
Ellipse  Area = πab	$\tau_A = \frac{2T}{\pi ab^2}$ $(a > b)$ (τ_{max} at B if $b > a$)	$\beta = \frac{a^2 + b^2}{\pi a^3 b^3}\frac{T}{G} = \frac{d\theta}{dx}$
Equilateral triangle  Area = $0.433\,a^2$	$\tau_A = \frac{20T}{a^3}$	$\beta = \frac{46.2}{a^4}\frac{T}{G} = \frac{d\theta}{dx}$
Regular hexagon  Area = $0.866a^2$	$\tau_A = \frac{5.7T}{a^3}$	$\beta = \frac{8.8}{a^4}\frac{T}{G} = \frac{d\theta}{dx}$
Rectangle  Area = ab	$\tau_A = \frac{T}{C_\tau ba^2}$	$\beta = \frac{1}{C_\beta ba^3}\frac{T}{G} = \frac{d\theta}{dx}$

b/a	C_τ	τ_B/τ_A	C_β
1.0	0.208	1.000	0.1406
1.2	0.219	0.935	0.166
1.5	0.231	0.859	0.196
2.0	0.246	0.795	0.229
2.5	0.258	0.766	0.249
3.0	0.267	0.753	0.263
4.0	0.282	0.745	0.281
6.0	0.299	0.743	0.299
10.0	0.312	0.742	0.312
∞	0.333	0.742	0.333



$J_R = \frac{bt^3}{3}$ (a) $\quad J_R = \frac{bt^3}{3}$ (b) $\quad J_R = \frac{b_1 t_1^3 + b_2 t_2^3}{3}$ (c) $\quad J_R = \frac{b_1 t_1^3 + 2b_2 t_2^3}{3}$ (d)

IGURE 8.4.3. Thin-walled open sections, showing their shear stresses and torsional constants J_R. In (a) and (b), dimension b is the length of the midline of the cross section.

when a bar is bent to form a helical spring, some corrections to these formulas may be necessary [8.3].

The membrane analogy permits a very useful generalization of the foregoing results to other thin-walled open cross sections. Imagine that the narrow rectangular cross section of Fig. 8.4.2 is distorted into a C or an L shape, or attached to another narrow rectangle to make a T or an I section. By visualizing the inflated membranes for these shapes, we decide that ϕ surfaces of all of them remain parabolic (excepting near ends and near reentrant corners, which we discuss later). The total torque is the sum of torques carried by each part of the cross section. Since $G\beta$ is the same for each part, Eqs. 8.4.5 and 8.4.8 still apply but with the $bt^3/3$ contributions summed to yield

$$J_R = \frac{1}{3}\sum_{i=1}^{n} b_i t_i^3 \tag{8.4.10}$$

where n is the number of parts into which the cross section is divided for purposes of calculation. Examples of this calculation appear in Fig. 8.4.3. In angle and I sections, the part of greater thickness displays greater shear stress. Where there is taper, as in the cross section of a turbine blade or in flanges of a rolled section, we can use

$$J_R = \frac{1}{3}\int t^3\,ds \tag{8.4.11}$$

where ds is an increment of length along the medial line of the cross section. More exact formulas are available [8.3]. These formulas, experimental results [8.5], and coefficient C_β in Table 8.4.1 suggest that, for standard rolled structural shapes, J_R may actually be some 10 percent higher than predicted by Eqs. 8.4.10 and 8.4.11. Omission of this adjustment yields a conservative design under either a stress limit or a deflection limit.

8.5 A HYDRODYNAMIC ANALOGY

There are several hydrodynamic analogies to the torsion problem [8.1]. In this section we outline one, without proof, and use it to draw useful conclusions about shear stresses in twisted bars.

If a fluid without viscosity executes motion in the yz plane with constant vorticity, its motion is described by the equation

$$\frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = -2\omega \tag{8.5.1}$$

where ϕ is the stream function and ω is the (constant) vorticity. Fluid velocities v and w in y and z directions, respectively, are

$$v = \frac{\partial\phi}{\partial z} \qquad w = -\frac{\partial\phi}{\partial y} \tag{8.5.2}$$

Clearly, Eqs. 8.5.1 and 8.5.2 are analogous to Eqs. 8.3.5 and 8.3.1, respectively. Thus fluid velocities are proportional to shear stresses. Lines ϕ = constant are streamlines. The fluid boundary must be a streamline, therefore the boundary condition $d\phi = 0$ of Eq. 8.3.6 is met. Thus the analogy is complete.

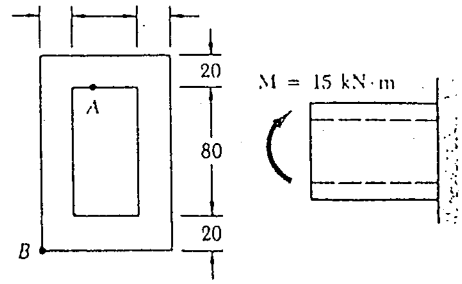
Experiments have been done in the following way. Imagine that a square cross section is to be studied. A shallow square tank is painted black and placed on a turntable. A camera, looking down onto the tank, is also attached to the turntable. The tank is filled with water (whose viscosity is low, if not quite zero) and aluminum powder is sprinkled on the water. When all is quiet, the camera shutter is opened while the turntable is rotated about 10°. From the viewpoint of the photograph, the tank is stationary while the fluid rotates with nearly constant vorticity. The aluminum particles show as streaks on the film. Each streak is in the direction of a shear stress. The length of the streak is proportional to the magnitude of the shear stress.

Experiments aside, the hydrodynamic analogy has other uses. One is in aiding visualization of torsion problems. Another is that known solutions for fluid flow can be applied to the torsion problem. For example, consider an elliptical obstacle (Fig. 8.5.1a). Far from the obstacle the flow is uniform and horizontal, so $w = 0$ and $v = v_o$, a constant. Points A are stagnation points, where $v = w = 0$. Fluid theory shows that at points B, where $z = \pm b$, fluid velocities are $v = v_o(1 + b/a)$ and $w = 0$. Applying these results to a twisted bar, Fig. 8.5.1b, we conclude that stress is zero at the sharp external corners C. At the root of the small elliptical notch, shear stress is approximately

$$\tau = \tau_o\left(1 + \frac{b}{a}\right) \tag{8.5.3}$$

where τ_o is the stress that would prevail near the boundary if the notch were not there. The calculation is approximate because the undisturbed stress is not the uniform value τ_o unless the notch is very small. The quantity in parentheses

4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.



Dimensions in mm
Fig. P4.1

4.9 and 4.10 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in the beam.

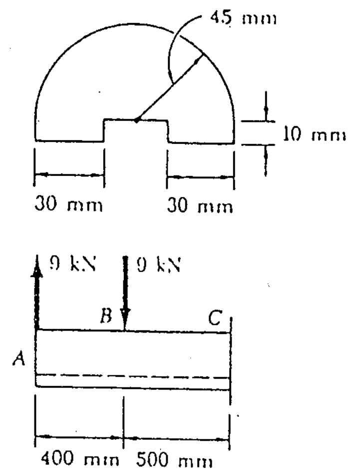


Fig. P4.9

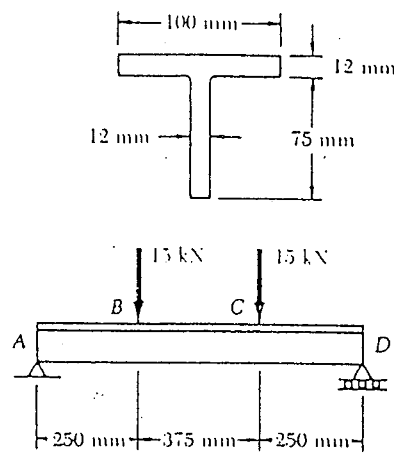


Fig. P4.10

4.26 A couple M will be applied to a beam of rectangular cross section which is to be sawed from a log of circular cross section. Determine the ratio *d*/*b*, for which (*a*) the maximum stress σ_m will be as small as possible, (*b*) the radius of curvature of the beam will be maximum.

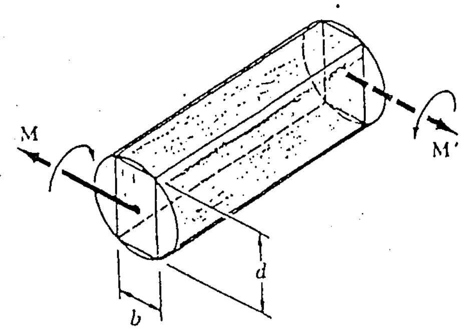
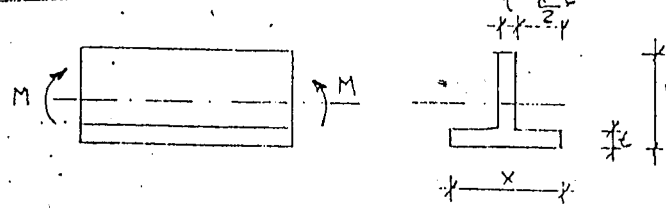


Fig. P4.26

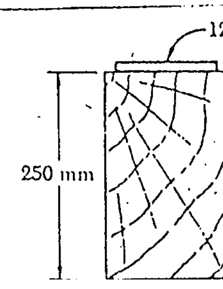
4.

What must be the width x of the beam so that the σ_{max} in tension is 3 times the value of σ_{max} in compression?



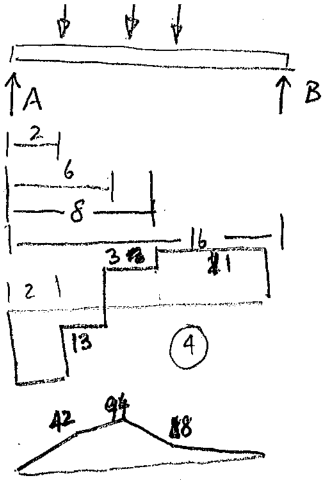
h = 10 cm
t = 2,5 cm.

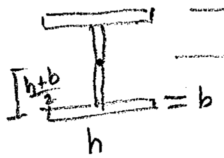
4.38 The 150 × 250 mm timber beam has been strengthened by bolting to it two steel strips to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.



	Wood	Steel
Modulus of elasticity	13 GPa	200 GPa

$B = 14\ Kip \qquad A = 32 - 14 = 28\ Kip$ (5)





$\bar{y} = \frac{h}{2} + b = .6h$ by symmetry (1)

$b = .1h$

$$I = 2\left(\frac{hb^3}{12} + h \cdot b\left[\frac{h+b}{2}\right]^2\right) + \frac{b \cdot h^3}{12}$$ (5)

$M_{max} = 94\ kip\text{-}ft$

$$= h^4\left[\frac{2}{12}\times 10^{-3} + \frac{.1 \cdot 2}{4}(1.1)^2 + \frac{0.1}{12}\right] = h^4(0.069)$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{94000\left(\frac{h+b}{2}\right)}{h^4(0.069)} = \frac{94000\ h\,(0.6)}{h^4(0.069}$$ (1) (1) (1)

$$h^3 = \frac{94000(0.6)\cdot 12}{0.069\sigma_{max}} = \frac{94000(0.6)\cdot 12}{.069(48000)} = 204.35$$

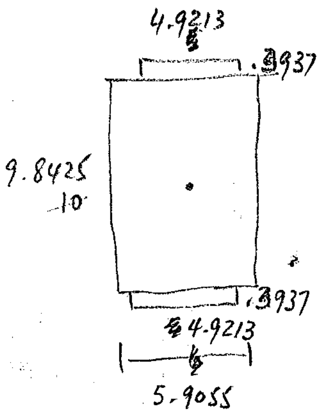
$h = 5.89\ in$

$b = 0.589\ in$

$$= \frac{lb\text{-}ft \cdot \frac{12 in}{ft} = lb\,in}{\frac{lb}{in^2}} = in^3$$ (2)

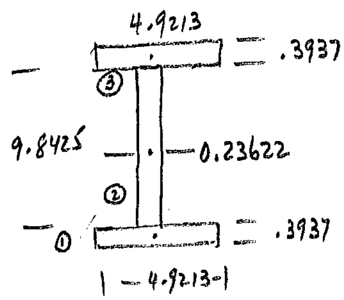
$\bar{x} = 78.55$

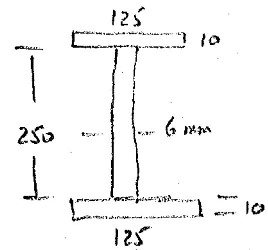
$\sigma = 16.52$



Centroid is at center by symmetry

$E_s b_s = E_w b_w \qquad b_s = \frac{E_w}{E_s} b_w = \frac{1}{25}(5.9055) = .23622$ (2)





	I_{zz_0}	A	d	I_{zz}	
(1)	$\frac{1}{12}(4.9213)(.3937)^3$	4.9213(.3937)	5.1181	$50.7782\ in^4$	(4) $21.136\times10^6\ mm^4$
(2)	$\frac{1}{12}(.23622)(9.8425)^3$	9.8425(.2362)	0	$18.7695\ in^4$	(4) $7.8125\times10^6\ mm^4$
(3)	$\frac{1}{12}(4.9213)(.3937)^3$	4.9213(.3937)	5.1181	$50.7782\ in^4$	(4) $21.136\times10^6\ mm^4$
				$120.3258\ in^4$	$50.084\times10^6\ mm^4$

from wood to steel

$\sigma_{s_{allow}} = \frac{Mc}{I} \qquad c = 5.31495\ in$ (1) $= 135\ mm$

$$M = \frac{\sigma_{s_{allow}} \cdot I}{c} = 20000\frac{lb}{in^2}\frac{(120.3258\ in^4)}{5.31495\ in}$$ (1) (1)

$M = 452782\ lb\,in$ on steel

$$\sigma_{w_{allow}} = \frac{M\bar{c}}{I}\cdot\frac{E_w}{E_s} \Rightarrow M = \sigma_{w_{allow}}\cdot\frac{I}{\bar{c}}\,\frac{E_s}{E_w} = 1200\frac{lb}{in^2}\cdot\frac{120.3258\ in^4}{4.92125\ in}\cdot 25 = 733507\ lb\text{-}in$$ (1) (1) on wood

$\bar{c} = \frac{9.8425}{2} = 4.92125\ in$ (1)

$\frac{\sigma_w}{E_w} \quad \epsilon_w = \epsilon_s = \frac{\sigma_s}{E_s}$

$\sigma_w = \sigma_s \cdot \frac{E_w}{E_s}$

use smaller of two $M = 452782$

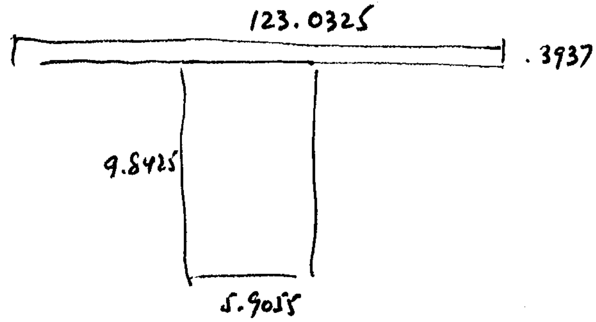
$\bar{x} = 67.5$

$\sigma = 18.87$

$1\ lb = 1\ slugs\ \frac{1 ft}{s^2}\ \frac{.12}{39.37}$ $\frac{32.2\ lb_m}{slug}$ $\frac{in/ft}{in/m}$

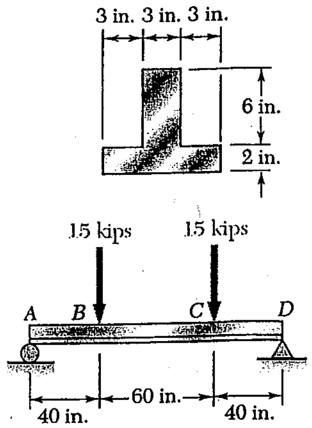
$1N = 1\ kg \times \frac{1m}{s^2}$

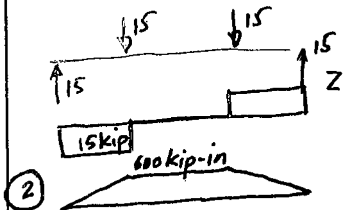
$I_W = 3008.15\ in^4 = .001252\ m^4 = 1.2521\times10^9\ mm^4$


123.0325
.3937
9.8425
5.9055

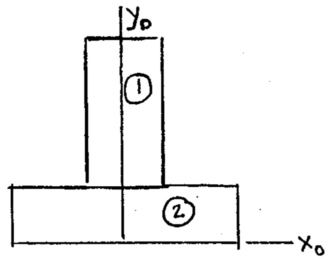
Problem 4.7

4.7 through 4.9 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.





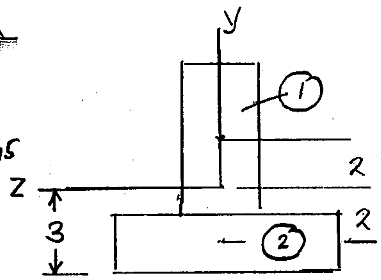
(2)



	A	$\bar{y}_0$	$A\bar{y}_0$
①	18	5	90
②	18	1	18
Σ	36		108

$$\bar{Y}_0 = \frac{108}{36} = 3 \text{ in.}$$ (2)

Neutral axis lies 3 in. above the base.

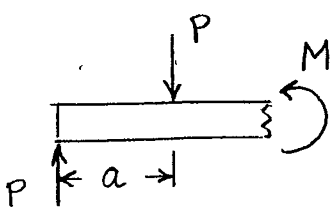


$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(6)^3 + (18)(2)^2 = 126 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(9)(2)^3 + (18)(2)^2 = 78 \text{ in}^4$$

$$I = I_1 + I_2 = 126 + 78 = 204 \text{ in}^4$$ (4)

$y_{top} = 5$ in. $\qquad y_{bot} = -3$ in.



$$M - Pa = 0$$

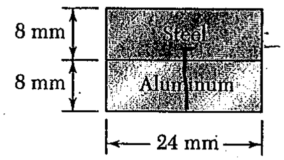
$$M = Pa = (15)(40) = 600 \text{ kip}\cdot\text{in.}$$

$$\sigma_{top} = -\frac{M y_{top}}{I} = -\frac{(600)(5)}{204} = -14.71 \text{ ksi}$$ (compression) (1) ◄

$$\sigma_{bot} = -\frac{M y_{bot}}{I} = -\frac{(600)(-3)}{204} = 8.82 \text{ ksi}$$ (tension) (1) ◄

Problem 4.39

4.39 and 4.40 A steel bar (E_s = 210 GPa) and an aluminum bar (E_a = 70 GPa) are bonded together to form the composite bar shown. Determine the maximum stress in (*a*) the aluminum, (*b*) the steel, when the bar is bent about a horizontal axis with M = 60 N · m.

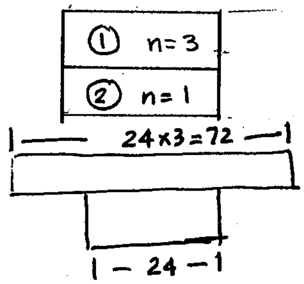


Use aluminum as the reference material.

For aluminum $n = 1$

For steel $n = E_s/E_a = 210/70 = 3$ (1)

Transformed section



(1)

	A, mm²	nA, mm²	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm³
①	192	576	12	6912
②	192	192	4	768
Σ		768		7680

$$\bar{Y}_o = \frac{7680}{768} = 10 \text{ mm}$$ (2)

The neutral axis lies 10 mm above the bottom.

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{3}{12}(24)(8)^3 + (576)(2)^2 = 5.376 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12}(24)(8)^3 + (192)(6)^3 = 7.936 \times 10^3 \text{ mm}^4$$ (4)

$$I = I_1 + I_2 = 13.312 \times 10^3 \text{ mm}^4 = 13.312 \times 10^{-9} \text{ m}^4$$

$M = 60$ N·m

$$\sigma = -\frac{nMy}{I}$$

(a) Aluminum $n = 1$, $y = -10 \text{ mm} = -0.010 \text{ m}$

$$\sigma_a = -\frac{(1)(60)(-0.010)}{13.312 \times 10^{-9}} = 45.072 \times 10^6 \text{ Pa}$$ (1)

$\sigma_a = 45.1$ MPa ◄

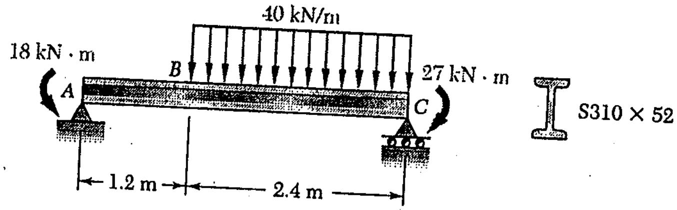
(b) Steel $n = 3$, $y = 6 \text{ mm} = 0.006 \text{ m}$

$$\sigma_s = -\frac{(3)(60)(0.006)}{13.312 \times 10^{-9}} = 81.130 \times 10^6 \text{ Pa}$$ (1)

$\sigma_s = -81.1$ MPa ◄

PROBLEM 5.124

5.123 and 5.124 (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.



SOLUTION

$\circlearrowleft \Sigma M_C = 0$

$18 - 3.6 R_A + (1.2)(2.4)(40) - 27 = 0$

$R_A = 29.5 \text{ kN}$

$V = 29.5 - 40 \langle x - 1.2 \rangle^1 \text{ kN}$

Point D $\quad V = 0$ $\qquad 29.5 - 40(x_D - 1.2) = 0$ $\quad x_D = 1.9375 \text{ m}$

when $V = -\frac{dM}{dx}$

M max when $V = 0$

$M = -18 + 29.5x - 20\langle x - 1.2 \rangle^2 \text{ kN-m}$

$M_A = -18 \text{ kN}\cdot\text{m}$

$M_D = -18 + (29.5)(1.9375) - (20)(0.7375)^2 = 28.278 \text{ kN}\cdot\text{m}$

$M_C = -18 + (29.5)(3.6) - (20)(2.4)^2 = -27 \text{ kN}\cdot\text{m}$

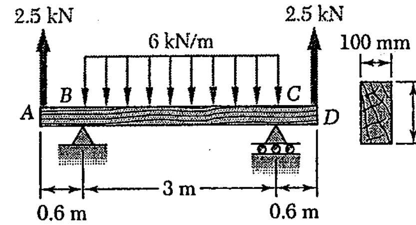
Maximum $|M| = 28.278 \text{ kN}\cdot\text{m}$ at $x = 1.9375 \text{ m}$ ◄ ◄

For S 310 × 52 rolled steel section $\quad S = 625 \times 10^3 \text{ mm}^3 = 625 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{28.278 \times 10^3}{625 \times 10^{-6}} = 45.2 \times 10^6 \text{ Pa} = 45.2 \text{ MPa}$ ◄

Problem 5.70

5.69 and 5.70 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



By symmetry $B = C$

$+\uparrow \Sigma F_y = 0 \qquad B + C + 2.5 + 2.5 - (3)(6) = 0$

$B = C = 6.5 \text{ kN}$ (2)

Shear: A to B $V = 2.5$ kN

$V_{B^+} = 2.5 + 6.5 = 9$ kN

$V_{C^-} = 9 - (3)(6) = -9$ kN

C to D $V = -9 + 6.5 = -2.5$ kN

Areas under shear diagram

A to B $\int V dx = (0.6)(2.5) = 1.5$ kN·m

B to E $\int V dx = (\frac{1}{2})(1.5)(9) = 6.75$ kN·m

E to C $\int V dx = -6.75$ kN·m

C to D $\int V dx = -1.5$ kN·m

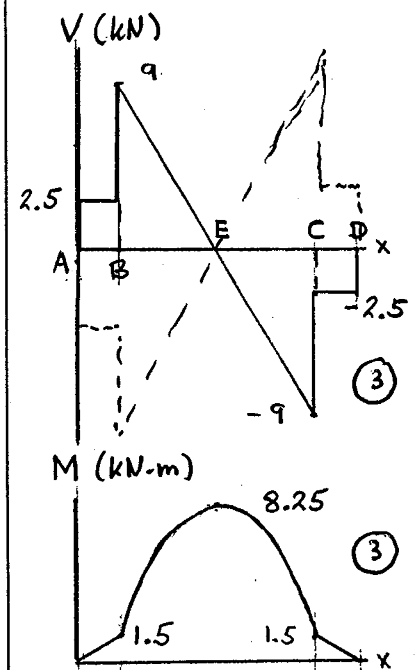
Bending moments $M_A = 0$

$M_B = 0 + 1.5 = 1.5$ kN·m

$M_E = 1.5 + 6.75 = 8.25$ kN·m

$M_C = 8.25 - 6.75 = 1.5$ kN·m

$M_D = 1.5 - 1.5 = 0$



Maximum $|M| = 8.25$ kN·m $= 8.25 \times 10^3$ N·m

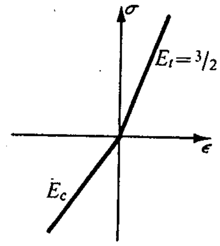
$\sigma_{all} = 12$ MPa $= 12 \times 10^6$ Pa

$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{8.25 \times 10^3}{12 \times 10^6} = 687.5 \times 10^{-6} \text{ m}^3 = 687.5 \times 10^3 \text{ mm}^3$

For a rectangular section $S = \frac{1}{6} b h^2$

$687.5 \times 10^3 = (\frac{1}{6})(100) h^2 \qquad h^2 = \frac{(6)(687.5 \times 10^3)}{100} = 41.25 \times 10^3 \text{ mm}^2$

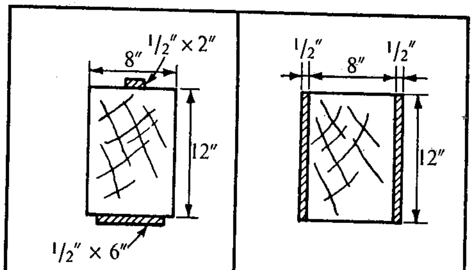
$h = 203$ mm (2) ◄



PROB. 6-37

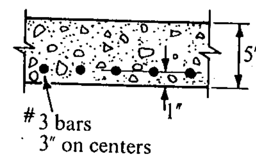
180,000 ft-lb around the "strong" axis. The material of the beam is nonisotropic and is such that the modulus of elasticity in tension is 1½ times as great as in compression, see figure. If the stresses do not exceed the proportional limit, find the maximum tensile and compressive stresses in the beam. *Ans.* 16.7 ksi, −13.6 ksi.

6-38 and 6-39. Determine the allowable bending moment around horizontal neutral axes for the composite beams of wood and steel having the cross-sectional dimensions shown in the figures. Materials are fastened together so that they act as a unit. $E_s = 30 \times 10^6$ psi; $E_w = 1.2 \times 10^6$ psi. The allowable bending stresses are $\sigma_s = 20{,}000$ psi and $\sigma_w = 1{,}200$ psi. *Ans. Prob.* 6-38. 450 k-in.



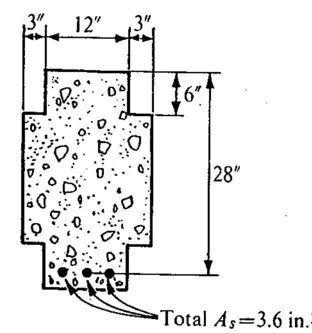
PROB. 6-38 PROB. 6-39

6-40. A 5-in.-thick concrete slab is longitudinally reinforced with steel bars as shown in the figure. (a) Determine the allowable bending moment per 1-ft width of this slab. Assume $n = 12$ and that the allowable stresses for steel and concrete are 18,000 psi and 900 psi, respectively. (b) Find the ultimate moment capacity per foot of width of the slab if for steel $\sigma_{yp} = 40$ ksi and for concrete $f'_c = 3{,}000$ psi. (*Note:* #3 bars are ⅜ in. in diameter having $A = 0.11$ in.^{2} each.)



PROB. 6-40

6-41. (a) A beam has a cross section as shown in the figure, and is subjected to a positive bending moment which causes a tensile stress in the steel of 18,000 psi. If $n = 10$, what is the value of the bending moment? (b) If $\sigma_{yp} = 50$ ksi, and $f'_c = 4{,}000$ psi, what is the ultimate moment capacity of the section? *Ans.* (a) 131.6 k-ft.



PROB. 6-41

6-42. Rework Example 6-12 by changing h to 4 in.

6-43. Derive Eq. 6-23.

6-44. What is the largest bending moment which may be applied to a curved bar, such as shown in Fig. 6-25(a), with $\bar{r} = 3$ in., if it has a circular cross-sectional area of 2-in. diameter and the allowable stress is 12 ksi?

to as the *shear flow*. Since force is usually measured in pounds, shear flow q has units of pounds per inch. Then, recalling that $dM/dx = -V$, one obtains the following expression for the shear flow in beams:

$$q = \frac{dF}{dx} = -\frac{dM}{dx}\frac{1}{I}\int_{\substack{\text{area}\\ fghj}} y\,dA = \frac{V A_{fghj}\bar{y}}{I} = \frac{VQ}{I} \qquad (7\text{-}5)$$

In this equation I stands for the moment of inertia of the entire cross-sectional area around the neutral axis, just as it does in the flexure formula from which it came. The total shearing force at the section investigated is represented by V, and the integral of $y\,dA$ for determining Q extends only over the cross-sectional area of the beam to one side of this area at which q is investigated.

In retrospect, note carefully that Eq. 7-5 was derived on the basis of the elastic flexure formula, but no term for a bending moment appears in the final expressions. This resulted from the fact that only the change in the bending moments at the adjoining sections had to be considered, and the latter quantity is linked with the shear V. The shear V was substituted for $-dM/dx$, and this masks the origin of the established relations. Equation 7-5 is very useful in determining the necessary interconnection between the elements making up a beam. This will be illustrated by examples.

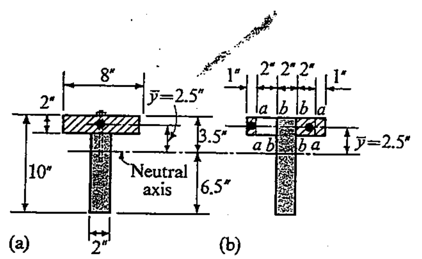


Fig. 7-6

EXAMPLE 7-1

Two long wooden planks form a T section of a beam as shown in Fig. 7-6(a). If this beam transmits a constant vertical shear of 690 lb, find the necessary spacing of the nails between the two planks to make the beam act as a unit. Assume that the allowable shearing force per nail is 150 lb.

SOLUTION

In attacking such problems the analyst must ask: What part of a beam has a tendency to slide longitudinally from the remainder? Here it is the plane of contact of the two planks; Eq. 7-5 must be applied to determine the shear flow in this plane. To do this the neutral axis of the whole section and its moment of inertia around the neutral axis must be found. Then as V is known and Q is defined as the statical moment of the area of the upper plank around the neutral axis, q may be determined. The distance y_c from the top to the neutral axis is

$$y_c = \frac{2(8)1 + 2(8)6}{2(8) + 2(8)} = 3.5 \text{ in.}$$

$$I = \frac{8(2)^3}{12} + (2)8(2.5)^2 + \frac{2(8)^3}{12} + (2)8(2.5)^2 = 291 \text{ in.}^4$$

$$Q = A_{fghj}\bar{y} = (2)8(3.5 - 1) = 40 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{690(40)}{291} = 95 \text{ lb per in.}$$

Thus, a force of 95 lb must be transferred from one plank to the other in every linear inch along the length of the beam. However, from the data given, each nail is capable of resisting a force of 150 lb, hence one nail can take care of 150/95 = 1.59 linear inches along the length of the beam. As shear remains constant at the consecutive sections of the beam, the nails should be spaced throughout at 1.59-in. intervals. In a practical problem a 1.5-in. spacing would probably be used.

SOLUTION FOR AN ALTERNATE ARRANGEMENT OF PLANKS

If, instead of using the two planks as above, a beam of the same cross section were made from five pieces, Fig. 7-6(b), a different nailing schedule would be required.

To begin, the shear flow between one of the 1-in.-by-2-in. pieces and the remainder of the beam is found, and although the contact surface *a-a* is vertical, the procedure is the same as before. The push or pull on an element is built up in the same manner as formerly:

$$Q = A_{fghj}\bar{y} = (1)2(2.5) = 5 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{690(5)}{291} = 11.8 \text{ lb per in.}$$

If the same nails as before are used to join the 1-in.-by-2-in. piece to the 2-in.-by-2-in. piece, they may be 150/11.8 = 12.7 in. apart. This nailing applies to both sections *a-a*.

To determine the shear flow between the 2-in.-by-10-in. vertical piece and either one of the 2-in.-by-2-in. pieces, the whole 3-in.-by-2-in. area must be used to determine Q. It is the difference of pushes (or pulls) on this whole area that causes the unbalanced force which must be transferred at the surface *b-b*:

$$Q = A_{fghj}\bar{y} = (3)2(2.5) = 15 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{690(15)}{291} = 35.6 \text{ lb per in.}$$

Space nails at 150/35.6 = 4.2 in. or, in practice, 4-in. intervals along the length of the beam in both sections *b-b*. These nails could be driven in first and then the 1-in.-by-2-in. pieces put on.

EXAMPLE 7-2

A simple beam on a 20-ft span carries a load of 200 lb per foot including its own weight. The beam cross section is to be made from several full-sized wooden pieces as in Fig. 7-7(a). Specify the spacing of the ½-in.

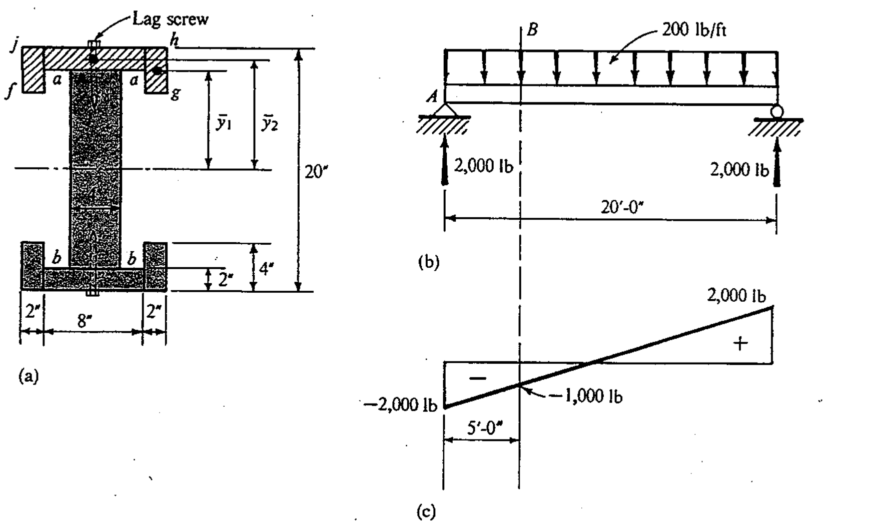


Fig. 7-7

lag screws shown which is necessary to fasten this beam together. Assume that one ½-in. lag screw, as determined by laboratory tests, is good for 500 lb when transmitting a lateral load parallel to the grain of the wood. For the entire section I is equal to 6,060 in.[4]

SOLUTION

To find the spacing of the lag screws, the shear flow at section a-a must be determined. The loading on the given beam is shown in Fig. 7-7(b); to show the variation of the shear along the beam, the shear diagram is constructed in Fig. 7-7(c). Then, to apply the shear flow formula, Q must be determined. This is done by considering the shaded area to one side of the cut a-a in Fig. 7-7(a). The statical moment of this area is most conveniently computed by multiplying the area of the two 2-in.-by-4-in. pieces by the distance from their centroid to the neutral axis of the beam and adding to this product a similar quantity for the 2-in.-by-8-in. piece. The largest shear flow occurs at the supports, as the largest vertical shears V of 2,000 lb act there:

$$Q = A_{fghj}\bar{y} = \sum A_i\bar{y}_i = 2A_1\bar{y}_1 + A_2\bar{y}_2$$

$$= 2(2)4(8) + 2(8)9 = 272 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{2{,}000(272)}{6{,}060} = 90 \text{ lb per in.}$$

At the supports the spacing of the lag screws must be 500/90 = 5.56 in. apart. This spacing of the lag screws applies only at a section where the shear V is equal to 2,000 lb. Similar calculations for a section where V = 1,000 lb gives q = 45 lb per inch, and the spacing of the lag screws becomes 500/45 = 11.1 in. Thus it is proper to specify the use of ½-in. lag screws at 5½ in. on centers for a distance of 5 ft nearest both the supports and at 11 in. for the middle half of the beam. A greater refinement in making the transition from one spacing of fastenings to another may be desirable in some problems. The same spacing of lag screws should be used at the section b-b as at the section a-a.

In a manner analogous to the above, the spacing of rivets or bolts in fabricated beams made from continuous angles and plates, Fig. 7-8, may be determined. Welding requirements are established similarly. The nominal shearing stress in a rivet is determined by dividing the total shearing force transmitted by the rivet (shear flow times spacing of the rivets) by the cross-sectional area of the rivet.

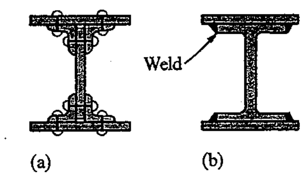


Fig. 7-8. Typical beam sections consisting of several components: (a) plate girder, (b) I beam reinforced with plates.

7-4. THE SHEARING STRESS FORMULA FOR BEAMS

The shearing stress formula for beams may be obtained from the shear flow formula. Analogously to the earlier procedure, an element of a beam may be isolated between two adjoining sections taken perpendicular to the axis of the beam. Then by passing another imaginary section through this element parallel to the axis of the beam, a new element is obtained, which corresponds to the element of one "plank" used in the earlier derivations. A side view of such an element is shown in Fig. 7-9(a), where the imaginary longitudinal cut is made at a distance y_1 from the neutral axis.* The cross-sectional area of the beam is shown in Fig. 7-9(c).

If shearing forces exist at the sections through the beam, a different bending moment acts at section A than at B. Hence more push or pull is developed on one side of the area $fghj$ than on the other, and, as before, the difference in the longitudinal forces in a distance dx is

$$dF = -\frac{dM}{I}\int_{\substack{\text{area}\\ fghj}} y\, dA = -\frac{dM}{I} A_{fghj}\bar{y} = -\frac{dM}{I} Q$$

The force equilibrating dF is developed in the plane of the longi-

* Since $dM/dx = -V$, for a positive V the change in moment $dM = -V\,dx$. For this reason $M_A > M_B$ and the magnitudes of the normal stresses in Fig. 7-9(a) are shown accordingly.

occur in the plastic zones, no unbalance in longitudinal forces occurs and no shearing stresses are developed.

This elementary solution has been refined by using a more carefully formulated criterion of yielding caused by the simultaneous action of normal and shearing stresses.* Some fundamental aspects of the interaction of such stresses will be considered in Chapter 9.

EXAMPLE 7-6

An I beam is loaded as in Fig. 7-14(a). If it has the cross section shown in Fig. 7-14(c), determine the shearing stresses at the levels indicated. Neglect the weight of the beam.

SOLUTION

A free-body diagram of a segment of the beam is in Fig. 7-14(b). It is seen from this diagram that the vertical shear at every section is 50 kips. Bending moments do not enter directly into the present problem. The shear flow at the various levels of the beam is computed in the table below using Eq. 7-5. Since $\tau = q/t$ (Eq. 7-6), the shearing stresses are obtained by dividing the shear flows by the respective widths of the beam.

$$I = \frac{6(12)^3}{12} - \frac{(5.5)(11)^3}{12} = 254 \text{ in.}^4$$

For use in Eq. 7-5 the ratio $V/I = -50{,}000/254 = -197$ lb/in.^{4}

Level	A_{fghj}*	$\bar{y}$**	$Q = A_{fghj}\bar{y}$	$q = VQ/I$	t	τ, psi
1-1	0	6	0	0	6.0	0
2-2	(0.5)6 = 3.00	5.75	17.25	−3,400	6.0 0.5	−570 −6,800
3-3	(0.5)6 = 3.00 (0.5)(0.5) = 0.25	5.75 5.25	17.25 1.31 } 18.56	−3,650	0.5	−7,300
4-4	(0.5)6 = 3.00 (0.5)(5.5) = 2.75	5.75 2.75	17.25 7.56 } 24.81	−4,890	0.5	−9,780

* A_{fghj} is the partial area of the cross section above a given level in square inches.
** $\bar{y}$ is the distance from the neutral axis to the centroid of the partial area in inches.

The negative signs of τ show that, for the section considered, the stresses act downward on the right face of the elements. The sense of the shearing stresses acting on the section coincides with the sense of the shearing force V. For this reason a strict adherence to the sign convention is often unnecessary. It is always true that $\int_A \tau \, dA$ is equal to V and has the same sense.

* D. C. Drucker, "The Effect of Shear on the Plastic Bending of Beams," *Journal of Applied Mechanics*, 23 (1956), pp. 509–14.

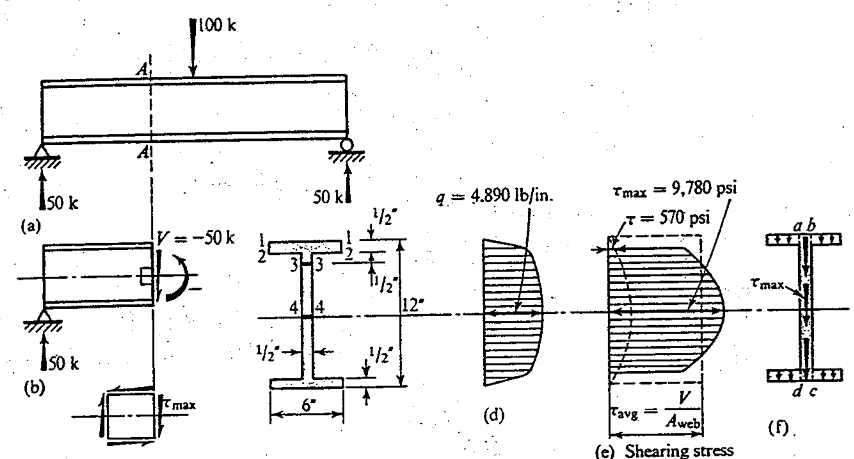


Fig. 7-14

Note that at the level 2-2 two widths are used to determine the shearing stress—one just above the line 2-2, and one just below. A width of 6 in. corresponds to the first case, and 0.5 in. to the second. This transition point will be discussed in the next article. The results obtained, which by virtue of symmetry are also applicable to the lower half of the section, are plotted in Fig. 7-14(d) and (e). By a method similar to the one used in the preceding example, it may be shown that the curves in Fig. 7-14(e) are parts of a second-degree parabola.

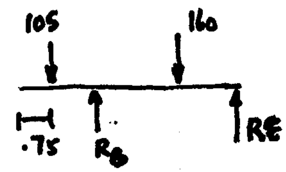
The variation of the shearing stress indicated by Fig. 7-14(e) may be interpreted as is shown in Fig. 7-14(f). The maximum shearing stress occurs at the neutral axis; the vertical shearing stresses throughout the web of the beam are nearly of the same magnitude. The shearing stresses occurring in the flanges are very small. For this reason the maximum shearing stress in an I beam is often approximated by dividing the total shear V by the cross-sectional area of the web (area $abcd$ in Fig. 7-11(f)). Hence

$$(\tau_{max})_{approx} = V/A_{web} \tag{7-9}$$

In the example considered this gives

$$(\tau_{max})_{approx} = \frac{50{,}000}{(0.5)12} = 8{,}330 \text{ psi}$$

This stress differs by about 15 per cent from the one found by the accurate formula. For most cross sections a much closer approximation



5.16 and 5.17 For the wide-flange beam and loading shown, determine in a section located halfway between points D and E, (a) the largest normal stress, (b) the largest shearing stress,

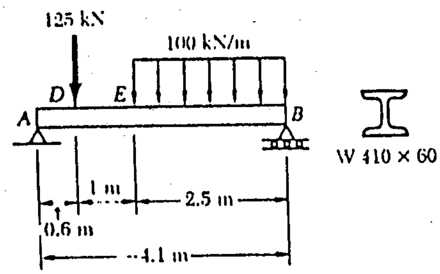


Fig. P5.16

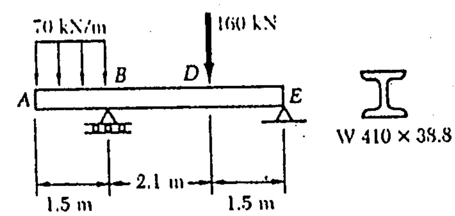


Fig. P5.17

5.18 Two rectangular plates are welded to the 310-mm-wide-flange beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 90 MPa.

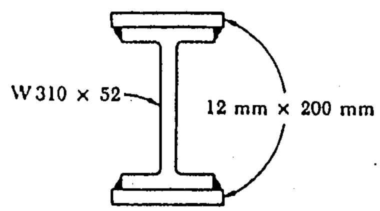
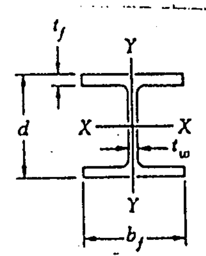


Fig. P5.18

Designation†	Area A, mm²	Depth d, mm	Flange Width b_f, mm	Flange Thickness t_f, mm	Web Thickness t_w, mm	Axis X-X I_x 10^6 mm⁴	Axis X-X S_x 10^3 mm³	Axis X-X r_x ($\sqrt{I/A}$) mm
W310 × 52	6650	317	167	13.2	7.6	118.6		
W410 × 114	14600	420	261	19.3	11.6	462	2200	177.8
85	10800	417	181	18.2	10.9	316	1516	170.7
60	7610	407	178	12.8	7.7	216	1061	168.4
46.1	5880	403	140	11.2	7.0	156.1	775	162.8
38.8	4950	399	140	8.8	6.4	125.3	628	159.0



3. For a timber beam having the cross section shown, determine the dimension w if the maximum allowable vertical shear force is 7.67kN, and the shearing stress is not to exceed 1.MPa.

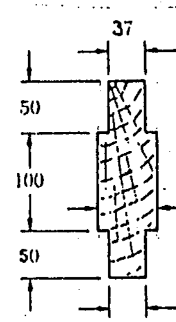
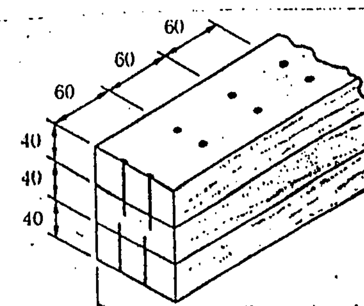


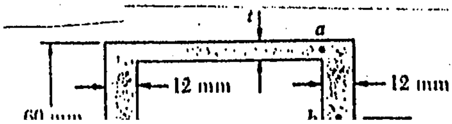
Fig. P5.20 Dimensions in mm

5.1 Three boards, each of 40 mm × 90 mm rectangular cross section, are nailed together to form a beam which is subjected to a vertical shear of 1 kN. Knowing that the spacing between each pair of nails is 60 mm, determine the shearing force in each nail.



Dimensions in mm

5.32 An extruded beam has the cross section shown and is subjected to a vertical shear of 50 kN. For t = 6 mm, determine the shearing stress at (a) point a, (b) point b.



Florida International University
Department of Mechanical and Materials Engineering

EMA 3702 QUIZ 2A March 28, 2007

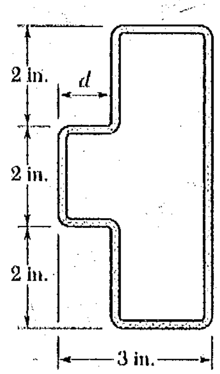
You are allowed three sheets of 8 ½ x 11 inch paper with whatever you wish on the sheet

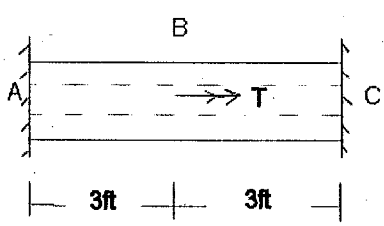
Print your name and sign the following statement:

I will not give nor take any unpermitted aid during this quiz. I understand that violation of this statement will lead to automatic failure of the quiz.

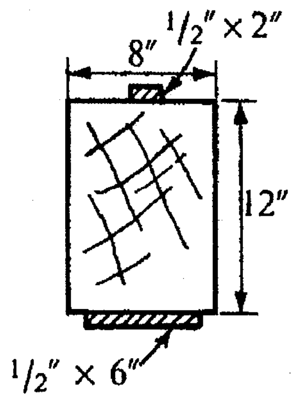
PRINT NAME SIGN NAME

Problem. The member, having the cross-section shown, is to be formed from the sheet metal of 0.06 in. thickness. If the member is fixed between two walls and has an applied torque, T, of 2500 lb-in., determine the smallest dimension *d* that may be used if the shearing stress is not to exceed 750 psi. The Young's Modulus, $E = 30 \times 10^6$ psi and that the Poisson ratio, $\nu = 0.3$.





Problem. Determine the allowable bending moment around the horizontal neutral axes for the composite beams of wood and steel having the cross-sectional dimensions shown in the figures. Materials are fastened together so that they act as a unit.
Given: E_S=30 x 10^6 psi; E_W=1.2 x 10^6 psi. The allowable bending stresses are σ_S=20 ksi and σ_W=1.2 ksi



5.16 and 5.17 For the wide-flange beam and loading shown, determine in a section located halfway between points D and E, (*a*) the largest normal stress, (*b*) the largest shearing stress.

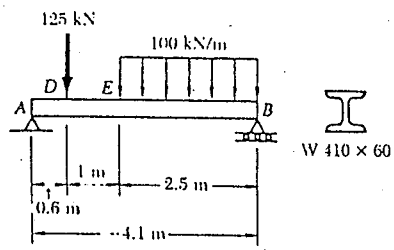


Fig. P5.16

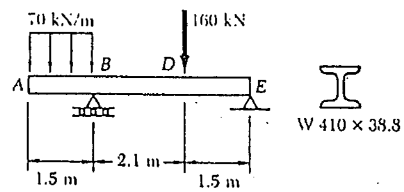


Fig. P5.17

5.18 Two rectangular plates are welded to the 310-mm-wide-flange beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 90 MPa.

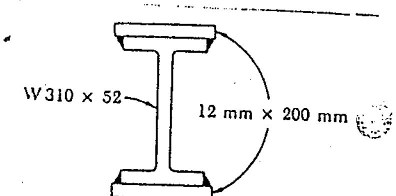
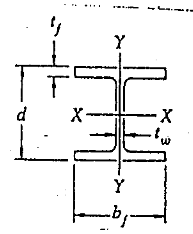


Fig. P5.18

Designation†	Area A, mm²	Depth (גובה) d, mm	Flange: Width (רוחב) b_f, mm	Flange: Thickness (עובי) t_f, mm	Web Thickness t_w, mm	Axis X-X: I_x 10^6 mm⁴	Axis X-X: S_x 10^3 mm³	Axis X-X: r_x mm
W310 × 52	6650	317	167	13.2	7.6	118.6		
W410 × 114	14600	420	261	19.3	11.6	462	2200	177.8
85	10800	417	181	18.2	10.9	316	1516	170.7
60	7610	407	178	12.8	7.7	216	1061	168.4
46.1	5880	403	140	11.2	7.0	156.1	775	162.8
38.8	4950	399	140	8.8	6.4	125.3	628	159.0



3. For a timber beam having the cross section shown, determine the dimension w if the maximum allowable vertical shear force is 7.67kN, and the shearing stress is not to exceed 1.MPa.

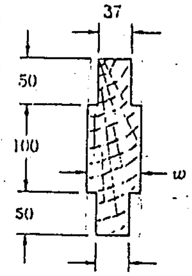


Fig. P5.20 Dimensions in mm

5.1 Three boards, each of 40 mm × 90 mm rectangular cross section, are nailed together to form a beam which is subjected to a vertical shear of 1 kN. Knowing that the spacing between each pair of nails is 60 mm, determine the shearing force in each nail.

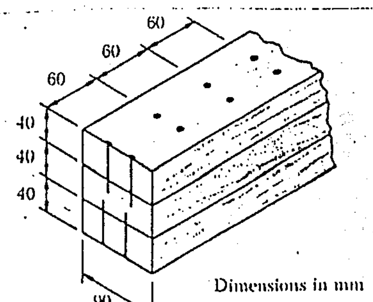
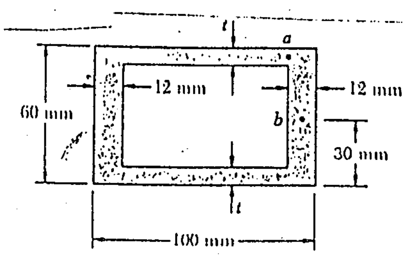
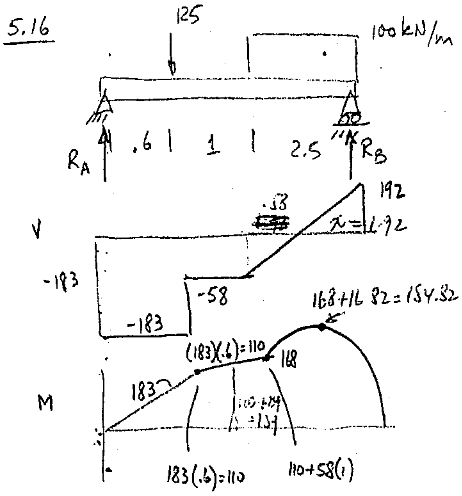


Fig. P5.1

5.32 An extruded beam has the cross section shown and is subjected to a vertical shear of 50 kN. For $t = 6$ mm, determine the shearing stress at (*a*) point a, (*b*) point b.



5.16

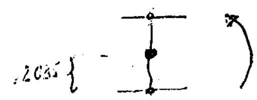


$R_A + R_B = 375$

$\Sigma M_A = 0 \quad 125(.6) + 250(2.85) - R_B(4.1) = 0$

$R_B = 192.1\ kN \quad R_A = 183\ kN$

$\frac{192}{x} = \frac{58}{2.5 - x} \qquad x = 1.92$



$\frac{My}{I} = \sigma$

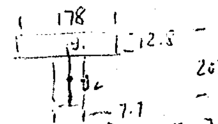
$I_{zz} = 2.16 \times 10^{-4}\ m^4$

$kN\text{-}m \quad \frac{(184.82)(.2035)}{2.16 \times 10^{-4}} = 174.1\ MPa$ max

@A 130.96 MPa

$\tau = \frac{VQ}{Ib}$

$Q = \int_A y\,dA =$



$= [178(12.8)][203.5 - 6.4] + [7.7][203.5 - 12.8][203.5 - 12.8]/2 = 589084\ mm^3$

$= 5.89 \times 10^{-4}\ m^3$

$\tau = \frac{(58\ kN)(5.89 \times 10^{-4})}{2.16 \times 10^{-4}(.0077)} = 20.543\ MPa$

1. The stress distribution on the rectangular cross-section shown in Fig. 1 is given by $\sigma_{xx}=1000y-500z+800$ kPa, $\sigma_{xy}=200z$ kPa, $\sigma_{xz}=0$. What is the net internal force system on this cross-section ?

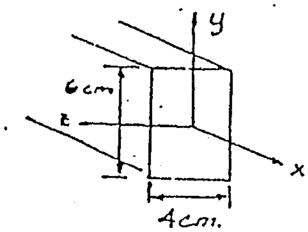


Fig.

Answer : $F=1920$ N, $V_y=V_z=0$
$M_y=-1600$ N-cm., $M_z=-7200$ N-cm., $T=-640$ N-cm.

2. Suppose the stress distribution on a cross-section of the circular cylinder of Fig. 2 is given by $\sigma_{xx}=\sigma_{xr}=0$, $\sigma_{x\theta}=k\sqrt{r}$, where k is unknown. What is the value of k in this case?

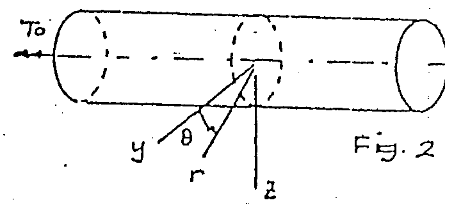


Fig. 2

Answer : $k=7T_0/4\pi R^{7/2}$

6.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium equations of that element, as was done in the derivations presented in Sec. 6.2.

6.6 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

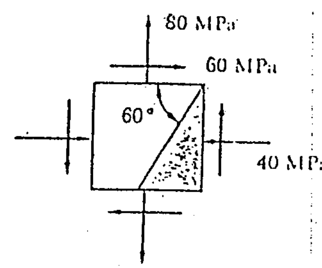


Fig. P6.4

6.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

6.14 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 40° counterclockwise, (*b*) 15° clockwise.

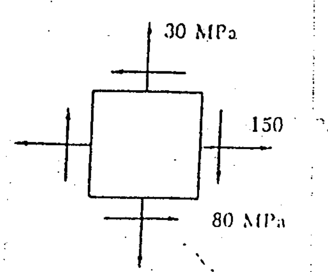
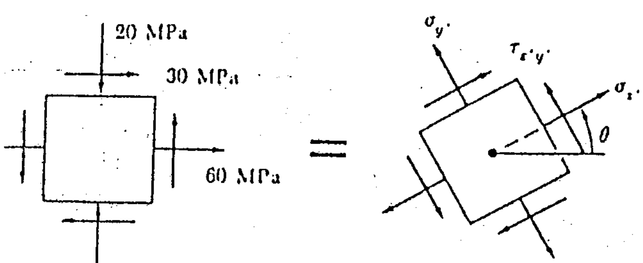


Fig. P6.6 and P6.10

6.56 For the state of plane stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than +65 MPa.



P6.55 and P6.56

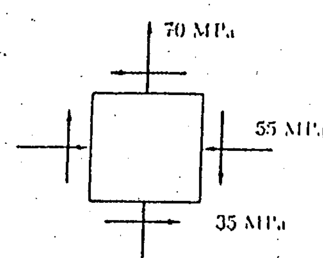


Fig. P6.14

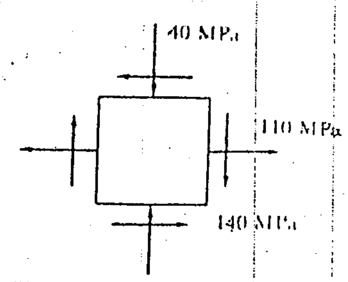
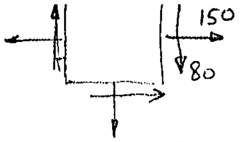


Fig. P6.8 and P6.12

2 5
1/2



$\sigma_x = 150$ MPa
$\sigma_y = 30$ MPa
$\tau_{xy} = -80$ MPa

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

לכן

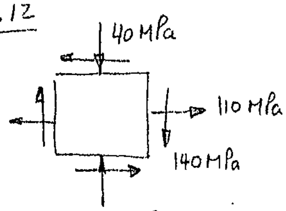
$$\sigma_{1,2} = \left(\frac{150+30}{2}\right) \pm \left[\left(\frac{150-30}{2}\right)^2 + (-80)^2\right]^{1/2}$$

$$= 90 \pm 100 \quad \Rightarrow \quad \sigma_1 = 190.- \text{ MPa} \quad \sigma_2 = -10.- \text{ MPa}$$

$$\tan 2\theta = \frac{2(-80)}{150-30} = -1.333$$

$$2\theta = -53.13 \quad \theta = -26.57^\circ \qquad \theta_2 = \theta + 90^\circ = 63.43^\circ$$

6.12



$\sigma_x = 110$ MPa
$\sigma_y = -40$ MPa
$\tau_{xy} = -140$ MPa

למצוא את הצירים של מאמץ הגזירה המקסימלי והמאמץ המקס.

$$\tau_{\text{מקס}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left[\left(\frac{110-(-40)}{2}\right)^2 + (-140)^2\right]^{1/2} = \pm 158.82 \text{ MPa}$$

$$\tan 2\alpha = \frac{-(\sigma_x - \sigma_y)}{2\tau_{xy}} = \frac{-[110-(-40)]}{2(-140)} = \frac{-150}{-280} = 208.18^\circ$$

מפני שהיחס הוא שלילי מחלקי שלילי.

$$\alpha = 104.09^\circ$$

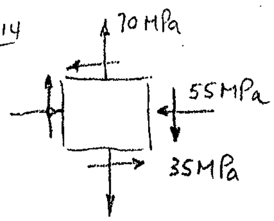
$$\alpha + 90^\circ = 194.09^\circ = 14.09^\circ$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\alpha + \tau_{xy} \sin 2\alpha$$

$$= \frac{110-40}{2} + \left(\frac{110-(-40)}{2}\right)\cos 208.18 + (-140)\sin 208.18^\circ$$

$$= 35 + 75\cos 208.18 - 140 \sin 208.18 = 35 \text{ MPa} = (\sigma_x + \sigma_y)/2$$

6.14



$\sigma_x = -55$ MPa
$\sigma_y = 70$ MPa
$\tau_{xy} = -35$ MPa

בסעיף א) למצוא את המאמצים כאשר החלקיק הולך ומסתובב 40° נגד השעון (מאמצים נורמלי וגזירה).
בסעיף ב) אותו תהליך כאשר החלקיק הולך ומסובב 15° בכיוון השעון.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\alpha + \tau_{xy} \sin 2\alpha \qquad \alpha = 40^\circ$$

$$= \frac{-55+70}{2} + \left(\frac{-55-70}{2}\right)\cos 80 + (-35)\sin 80$$

$$= 7.5 - 62.5\cos 80 - 35 \sin 80 = -37.82 \text{ MPa}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\alpha + \tau_{xy}\cos 2\alpha$$

$$= -\left(\frac{-55-70}{2}\right)\sin 80 + (-35)\cos 80$$

$$= 62.5 \sin 80 - 35 \cos 80 = 55.47 \text{ MPa}$$

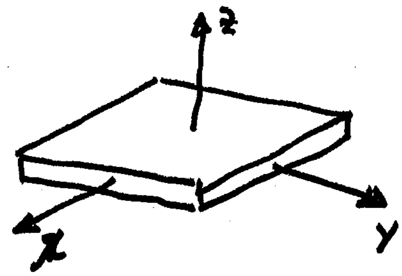
TO FIND σ_1 & σ_2 NEED $\sigma_x, \sigma_y, \tau_{xy}, \lambda, \mu$

$$\sigma_x = \lambda e + 2\mu\epsilon_x = \lambda e + 2G\epsilon_x \qquad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \qquad \mu = G = \frac{E}{2(1+\nu)}$$

$$\sigma_y = \lambda e + 2\mu\epsilon_y$$

$$= 17019231 \frac{N}{m^2} \qquad = 11346154 \frac{N}{m^2}$$

$$e = \epsilon_x + \epsilon_y + \epsilon_z$$



$\sigma_z = 0$

$$\sigma_z = \lambda e + 2\mu\epsilon_z = 0 \qquad \lambda(\epsilon_x+\epsilon_y) + (2\mu+\lambda)\epsilon_z = 0$$

$$\epsilon_z = \frac{-\lambda}{\lambda+2\mu}(\epsilon_x+\epsilon_y) = 428.57\times10^{-6}$$

$$\sigma_x = \lambda e + 2\mu\epsilon_x = 17.02\times10^9(-571.43\times10^{-6}) + 2\cdot11.35\times10^9(-800\times10^{-6})$$
$$= -27879 \text{ psi}$$

$$\sigma_y = \lambda e + 2\mu\epsilon_y = 17.02\times10^9(-571.43\times10^{-6}) + 2\cdot11.35\times10^9(-200\times10^{-6})$$
$$= -14264 \text{ psi}$$

$$\tau_{xy} = G\gamma_{xy} = 11.35\times10^9(800\times10^{-6}) = 9077 \text{ psi}$$

$$\sigma_{max,min} = \left(\frac{\sigma_x+\sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left(\frac{-27879 - 14264}{2}\right) \pm \sqrt{\left(\frac{-27879+14264}{2}\right)^2 + (9077)^2}$$

$$= -21071.5 \pm 11346.1 = \begin{cases} -9725 \text{ psi} - \sigma_1 \\ -32418 \text{ psi} - \sigma_2 \end{cases}$$

$$\tan 2\theta_\sigma = \frac{\tau_{xy}}{(\sigma_x-\sigma_y)/2} \qquad 2\theta_\sigma = 126.86^\circ$$

Suppose $\epsilon_x = -800 \times 10^{-6}$ m/m

$\epsilon_y = -200 \times 10^{-6}$ m/m

$\gamma_{xy} = 800 \times 10^{-6}$

WHAT ARE THE MAX & MIN DIRECT STRAINS (ϵ_1, ϵ_2) ✓

WHAT ARE THE PRINCIPAL DIRECTIONS FOR ϵ_1 & ϵ_2 ✓

SUPPOSE THESE STRAINS OCCUR ON A PLATE OF STEEL W/ $E = 29.5 \times 10^6$ psi & $\nu = 0.3$, WHAT ARE THE MAX STRESSES (σ_1, σ_2) & WHAT ARE PRINCIPAL DIRECTIONS θ_σ

$$\epsilon_{max,min} = \frac{\epsilon_x + \epsilon_y}{2} \pm \frac{1}{2}\sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}$$

$$= \left\{ \frac{-800 + (-200)}{2} \pm \frac{1}{2}\sqrt{(-800 - (-200))^2 + (800)^2} \right\} \times 10^{-6}$$

$$= \left\{ -500 \pm 500 \right\} \times 10^{-6} = \begin{cases} 0 & \text{— } \epsilon_1 \\ -1000 \times 10^{-6} & \text{— } \epsilon_2 \end{cases}$$

$$\tan 2\theta_\epsilon = \frac{2\epsilon_{xy}}{\epsilon_x - \epsilon_y} = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{800}{-800 - (-200)} = \frac{800}{-600}$$

$2\theta_\epsilon' = -53.14^\circ$

$2\theta_\epsilon'' = 2\theta_\epsilon' + 180 = 126.86^\circ$

$\theta_{\epsilon_1} = 63.43^\circ$

$\theta_{\epsilon_2} = 153.43^\circ$

$$\epsilon_x' = \frac{\epsilon_x + \epsilon_y}{2} + \left(\frac{\epsilon_x - \epsilon_y}{2}\right) 2\theta + \epsilon_{xy} \cos 2\theta$$

$\epsilon_{xy} \to \frac{\gamma_{xy}}{2}$

1. The stress distribution on the rectangular cross-section shown in Fig. 1 is given by $\sigma_{xx}=1000y-500z+800$ kPa, $\sigma_{xy}=200z$ kPa, $\sigma_{xz}=0$. What is the net internal force system on this cross-section ?

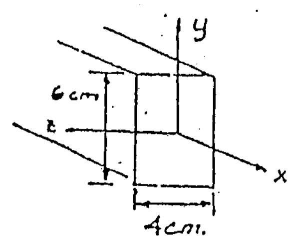


Fig. 1

Answer : $F= 1920$ N, $V_y=V_z=0$
$M_y=-1600$ N-cm., $M_z=-7200$ N-cm., $T=-640$ N-cm.

2. Suppose the stress distribution on a cross-section of the circular cylinder of Fig. 2 is given by $\sigma_{xx}=\sigma_{xr}=0$, $\sigma_{x\theta}=k\sqrt{r}$, where k is unknown. What is the value of k in this case?

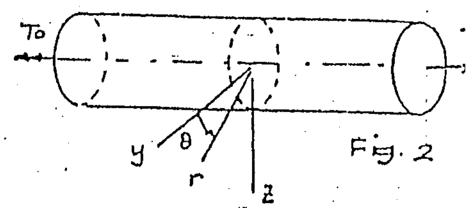


Fig. 2

Answer : $k= 7T_0/4\pi R^{7/2}$

6.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium equations of that element, as was done in the derivations presented in Sec. 6.2.

6.6 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

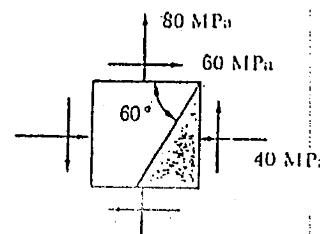


Fig. P6.4

6.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

6.14 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 40° counterclockwise, (*b*) 15° clockwise.

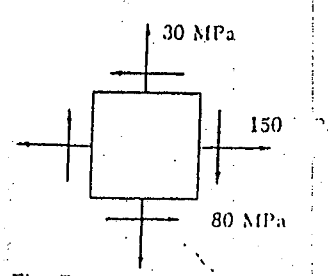
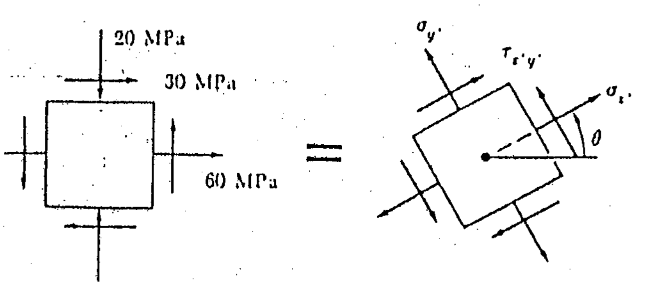


Fig. P6.6 and P6.10

6.56 For the state of plane stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than +65 MPa.



P6.55 and P6.56

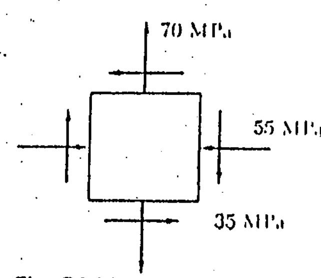


Fig. P6.14

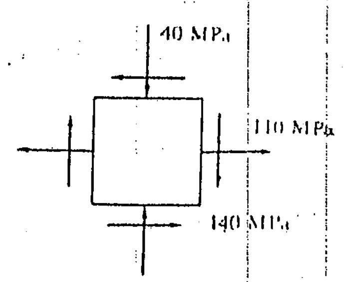


Fig. P6.8 and P6.12

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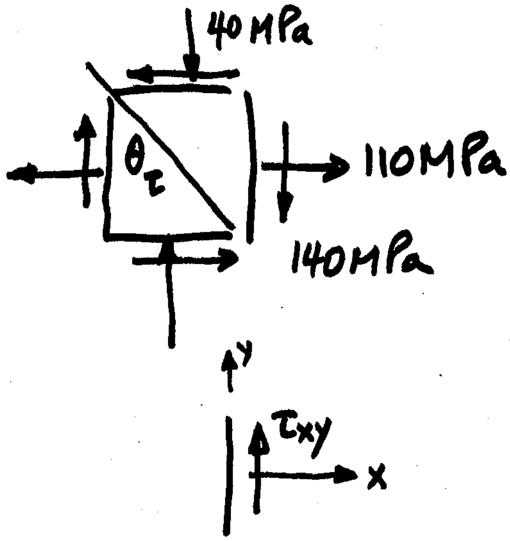
EMA 3702 PART I

θ_τ ?, τ_{max}, σ_x' ?



$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_x = 110\ MPa$

$\sigma_y = -40\ MPa$

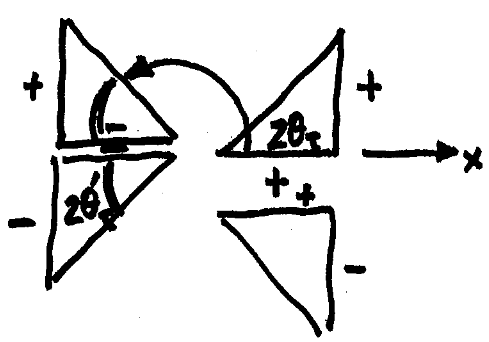
$\tau_{xy} = -140\ MPa$

$$\tau_{max} = \sqrt{\left(\frac{110-(-40)}{2}\right)^2 + (-140)^2}$$

$$= 158.82\ MPa$$

$$\tan 2\theta_\tau = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(110-(-40))/2}{-140}$$

$$= \frac{-150}{-280}$$



$$2\theta_\tau' = 28.18^\circ$$

$$2\theta_\tau = 180 + 28.18^\circ = 208.18^\circ$$

$$\theta_\tau = 104.09^\circ$$

$$\tau_{x'y'} = \tau_{xy} \cos 2\theta - \left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta$$

$$2\theta_\tau = 208.18^\circ + 180^\circ = 388.18^\circ \text{ or } 28.18^\circ$$

$$\theta_\tau = 14.09^\circ$$

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} = \frac{70}{2} = \underline{\underline{35\ MPa}}$$

EMA 3702

Mechanics & Materials Science

4/11/07 - LECTURE 1

element, equal stresses must be exerted on the horizontal faces of the same element. We thus conclude that longitudinal shearing stresses must exist in any member subjected to a transverse loading. This can be verified by considering a cantilever beam made of separate planks clamped together at one end (Fig. 6.3*a*). When a transverse load **P** is applied to the free end of this composite beam, the planks are observed to slide with respect to each other (Fig. 6.3*b*). In contrast, if a couple **M** is applied to the free end of the same composite beam (Fig. 6.3*c*), the various planks will bend into concentric arcs of circle and will not slide with respect to each other, thus verifying the fact that shear does not occur in a beam subjected to pure bending (cf. Sec. 4.3).

While sliding does not actually take place when a transverse load **P** is applied to a beam made of a homogeneous and cohesive material such as steel, the tendency to slide does exist, showing that stresses occur on horizontal longitudinal planes as well as on vertical transverse planes. In the case of timber beams, whose resistance to shear is weaker between fibers, failure due to shear will occur along a longitudinal plane rather than a transverse plane (Fig. 6.4).

In Sec. 6.2, a beam element of length Δx bounded by two transverse planes and a horizontal one will be considered and the shearing force $\Delta \mathbf{H}$ exerted on its horizontal face will be determined, as well as the shear per unit length, q, also known as *shear flow*. A formula for the shearing stress in a beam with a vertical plane of symmetry will be derived in Sec. 6.3 and used in Sec. 6.4 to determine the shearing stresses in common types of beams. The distribution of stresses in a narrow rectangular beam will be further discussed in Sec. 6.5.

The derivation given in Sec. 6.2 will be extended in Sec. 6.6 to cover the case of a beam element bounded by two transverse planes and a curved surface. This will allow us in Sec. 6.7 to determine the shearing stresses at any point of a symmetric thin-walled member, such as the flanges of wide-flange beams and box beams. The effect of plastic deformations on the magnitude and distribution of shearing stresses will be discussed in Sec. 6.8.

In the last section of the chapter (Sec. 6.9), the unsymmetric loading of thin-walled members will be considered and the concept of *shear center* will be introduced. You will then learn to determine the distribution of shearing stresses in such members.

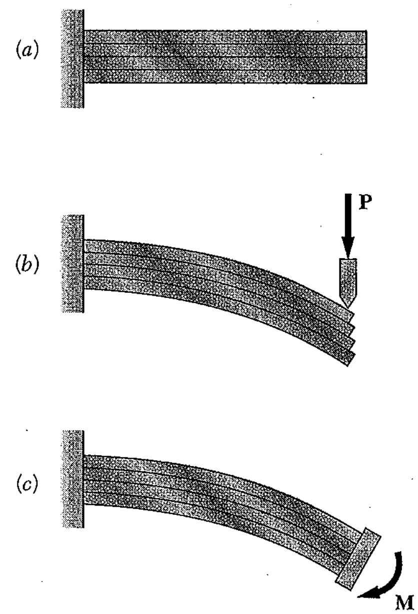


Fig. 6.3

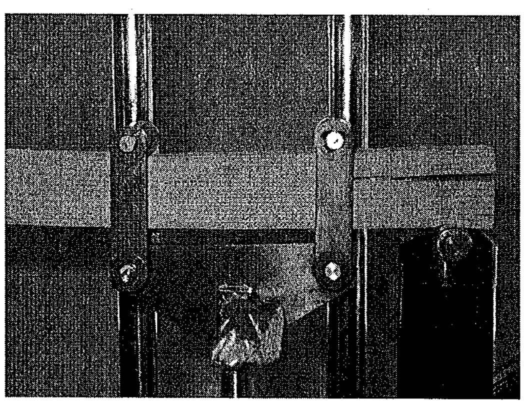
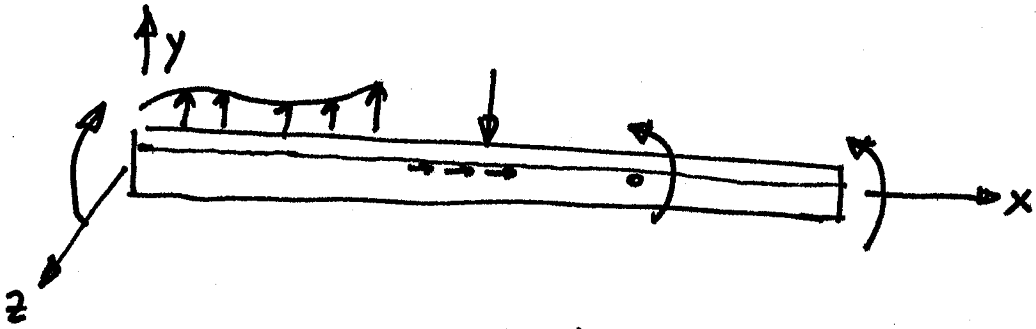
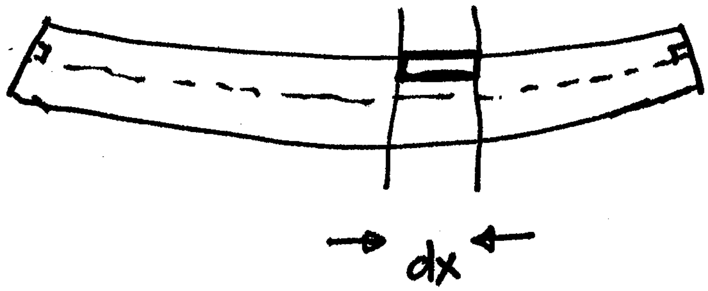
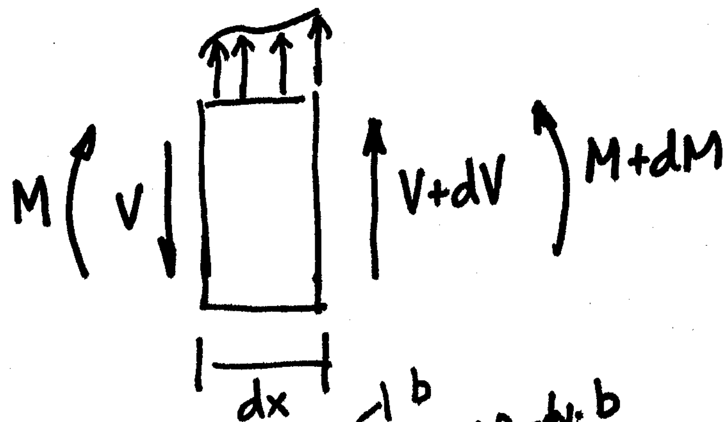


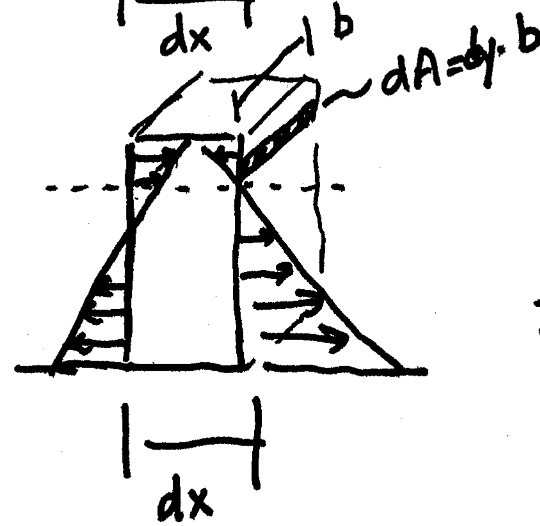
Fig. 6.4

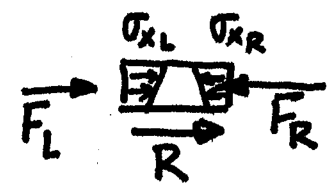




$$\sigma_x = \frac{-M_z y}{I_{zz}}$$







$$R = F_R - F_L$$

$$= \int \sigma_{x_R} dA - \int \sigma_{x_L} dA$$

$$= b \int \sigma_{x_R} dy - b \int \sigma_{x_L} dy$$

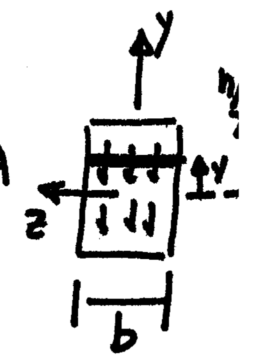
$$= b \int \frac{-(M+dM) y \, dy}{I_{zz}} - b \int \frac{-M y \, dy}{I_{zz}}$$

$$R = -b \int \frac{[dM] y \, dy}{I_{zz}} \Rightarrow \frac{R}{dx \cdot b} = \tau_{xy} = \int \frac{-[dM]}{dx} \frac{y \, dA}{I}$$

$$\tau_{xy} = \tau_{yx} = \int -\frac{[dM]}{dx} \cdot \frac{y\,dA}{b\,I_{zz}}$$

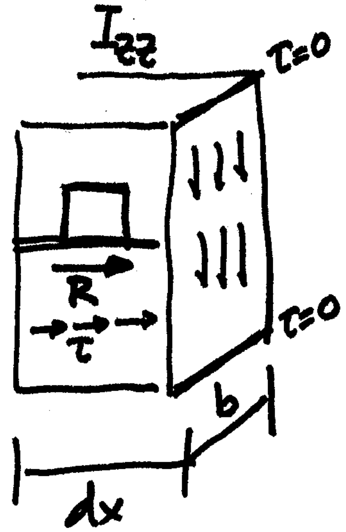
$$\tau_{yx} \cdot b = q = \text{shear flow} = -\int \frac{dM}{dx} \cdot \frac{y\,dA}{I_{zz}}$$

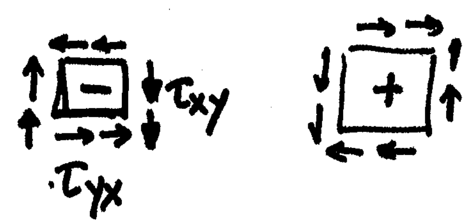
$$\frac{dM}{dx} = -V \quad ; \quad q = \int V \frac{y\,dA}{I_{zz}} = \frac{V}{I_{zz}} \int_y^{h/2} \tilde{y}\,dA$$



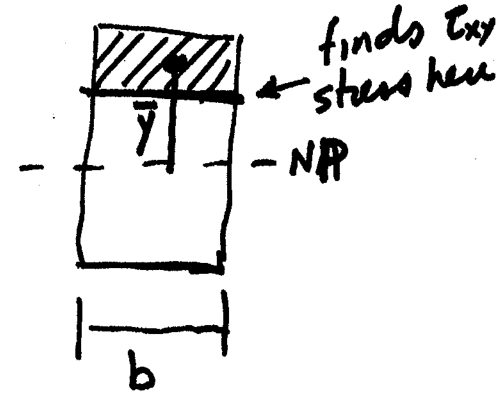
$$\int_y^{h/2} \tilde{y}\,dA = \text{first moment of area} = Q$$

$$q = \frac{VQ}{I_{zz}} \qquad \tau_{yx} = \tau_{xy} = \frac{q}{b} = \frac{VQ}{I_{zz} \cdot b}$$

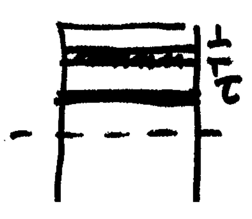




$$\int_y^{h/2} \tilde{y}\,dA = \bar{y} \cdot A$$

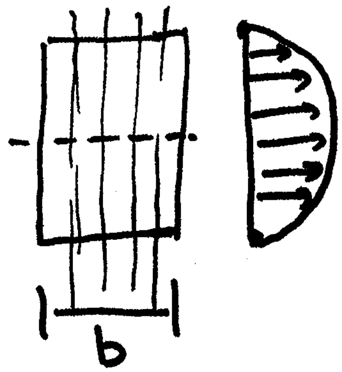


$$\tau_{xy} = \tau_{yx} = \frac{V}{I_{zz} \cdot b} \int_y^{h/2} \tilde{y}\,dA$$



$$= \frac{V}{I_{zz} \cdot b} \int_y^{h/2} \tilde{y} \cdot b\,d\tilde{y} = \frac{V}{I_{zz}} \left[\frac{\tilde{y}^2}{2}\right]_y^{h/2}$$

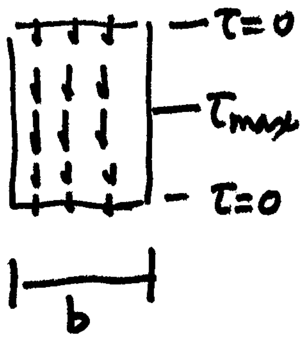
$$\tau = \frac{V}{\ } \left[(h/2)^2 - y^2\right]$$



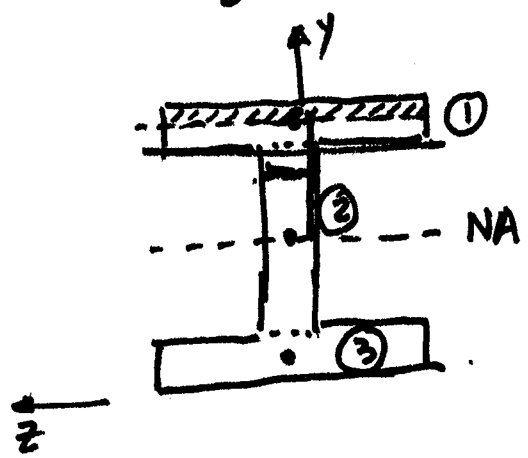
τ_{max} @ $y=0$ $\quad \dfrac{Vh^2}{8 I_{zz}} = \dfrac{Vh^2}{8 \cdot \frac{bh^3}{12}}$

$$= \frac{3}{2}\frac{V}{bh} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\tau_{ave}$$

only true for a rectangular cross-section and occurs at the N.A.



$$\tau = \frac{VQ}{I_{zz} b}$$



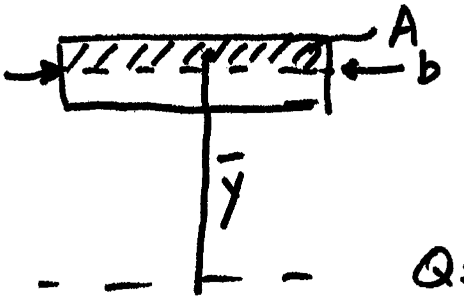
	$\bar{y}$	A	$\bar{y}A$
①			
②			
③			
		ΣA	$\Sigma \bar{y} A$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

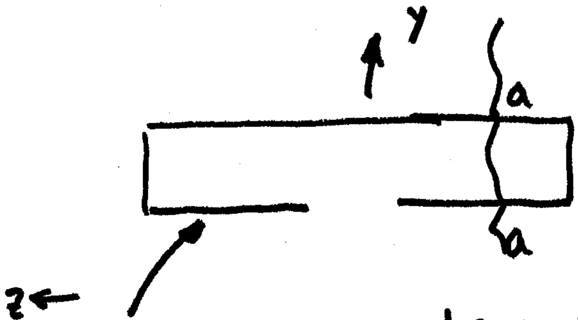
GIVES LOC OF N.A.

	I_{zz_0}	A	d	$I_{zz} = I_{zz_0} + Ad^2$
①				
②				
③				

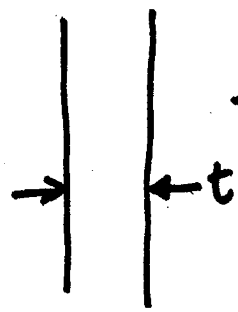
$$\Sigma I_{zz} = I_{zz_{TOT}}$$



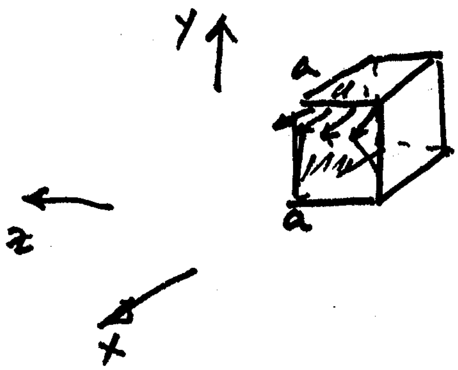
$Q = \bar{y}A$



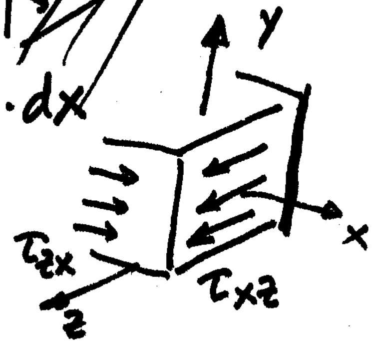
Formula says that $\tau \neq 0$ on this free surface and this isn't true



$$\tau = \frac{VQ}{I_{zz}\, t}$$



$$R = \tau_{zx} \cdot t_f \cdot dx$$

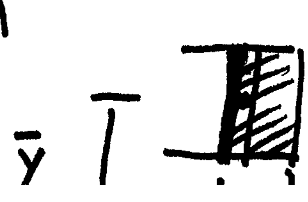


$$\frac{R}{t_f \cdot dx} = \tau_{zx}$$

$$\frac{R}{dx} = \tau_{zx} \cdot t_f = q$$

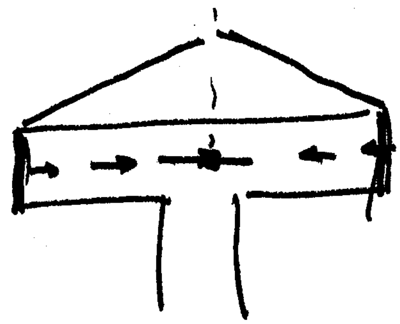
$$\tau_{xz} = \tau_{zx} = \frac{VQ}{I_{zz}\, t_f}$$

$$Q = \int \bar{y}\, dA$$



$t_f \cdot dz$

$$Q = \bar{y} \cdot t_f \int dz = \bar{y} t_z z$$



EMA 3702

Mechanics & Materials Science

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5.16 and 5.17 For the wide-flange beam and loading shown, determine in a section located halfway between points D and E, (*a*) the largest normal stress, (*b*) the largest shearing stress,

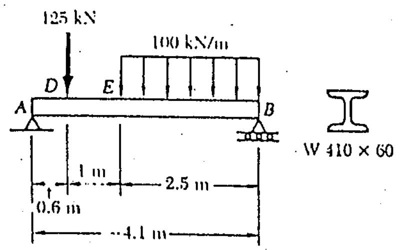


Fig. P5.16

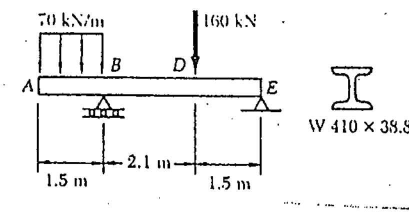


Fig. P5.17

5.18 Two rectangular plates are welded to the 310-mm-wide-flange beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 90 MPa.

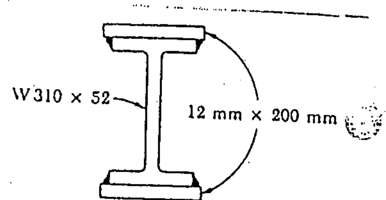
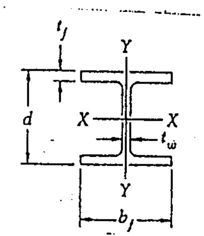


Fig. P5.18

Designation†	Area A, mm²	Depth d, mm	Flange Width b_f, mm	Flange Thickness t_f, mm	Web Thickness t_w, mm	Axis X-X I_x 10^6 mm⁴	Axis X-X S_x 10^3 mm³	Axis X-X r_x mm $\sqrt{I/A}$
W310 × 52	6650	317	167	13.2	7.6	118.6		
W410 × 114	14600	420	261	19.3	11.6	462	2200	177.8
85	10800	417	181	18.2	10.9	316	1516	170.7
60	7610	407	178	12.8	7.7	216	1061	168.4
46.1	5880	403	140	11.2	7.0	156.1	775	162.8
38.8	4950	399	140	8.8	6.4	125.3	628	159.0



3. For a timber beam having the cross section shown, determine the dimension w if the maximum allowable vertical shear force is 7.67kN, and the shearing stress is not to exceed 1.MPa.

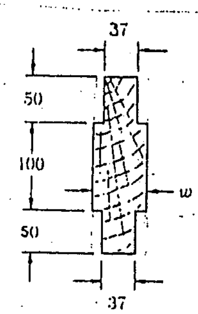


Fig. P5.20 Dimensions in mm

5.1 Three boards, each of 40 mm × 90 mm rectangular cross section, are nailed together to form a beam which is subjected to a vertical shear of 1 kN. Knowing that the spacing between each pair of nails is 60 mm, determine the shearing force in each nail.

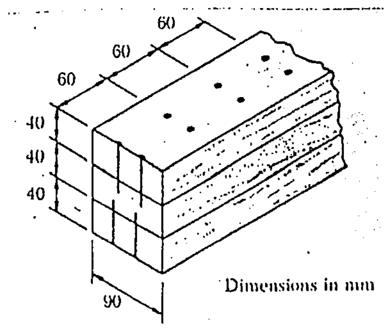
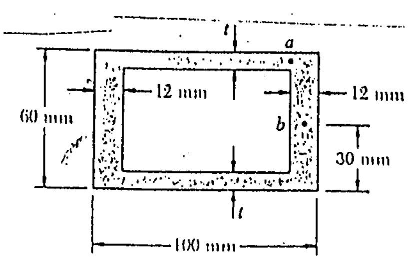


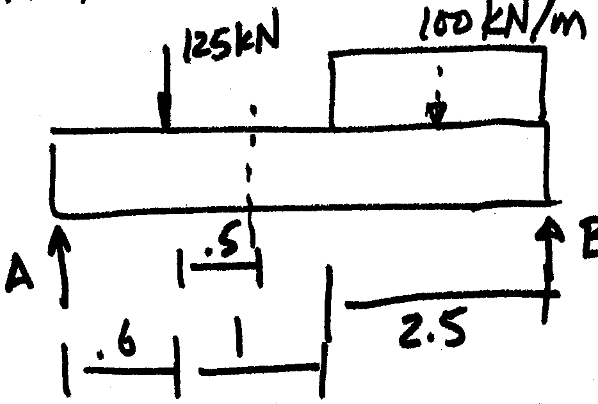
Fig. P5.1

5.32 An extruded beam has the cross section shown and is subjected to a vertical shear of 50 kN. For t = 6 mm, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.



15
17

1. Find the support forces
2. Draw Shear & moment diagrams
3. find shear & moment at section
4. apply formulas.



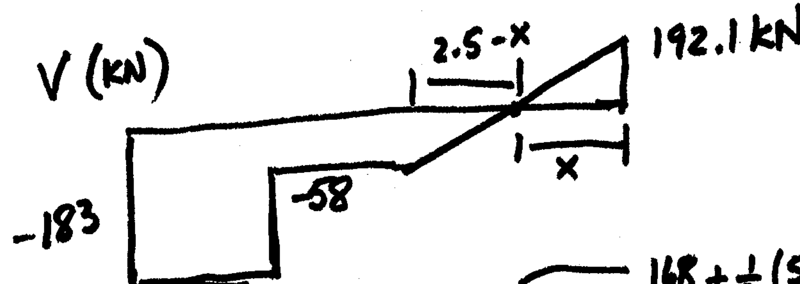
$\Sigma F_y = 0 \quad A + B - 125 - 100(2.5) = 0$

$A + B = 375 \text{ kN}$

$\Sigma M_A = -125(.6) - 100(2.5)(2.85)$

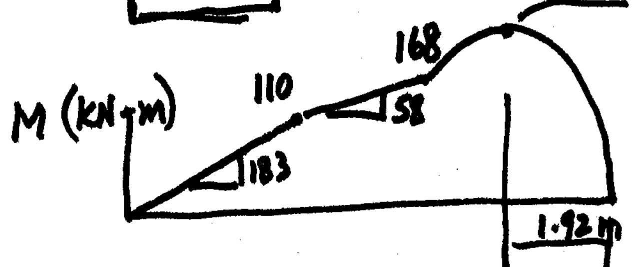
$+ B(4.1) = 0$

$B = 192.1 \text{ kN} \qquad A = 183 \text{kN}$



$\frac{192.1}{x} = \frac{2.5 - x}{58} \qquad x = 1.92$

$168 + \frac{1}{2}(58)(.58) = 184.82$



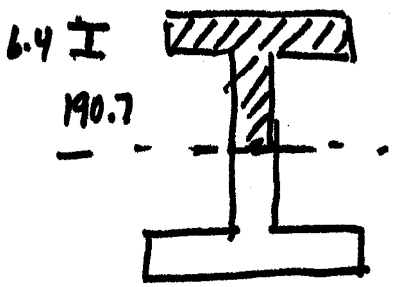
$\frac{dM}{dx} = -V$

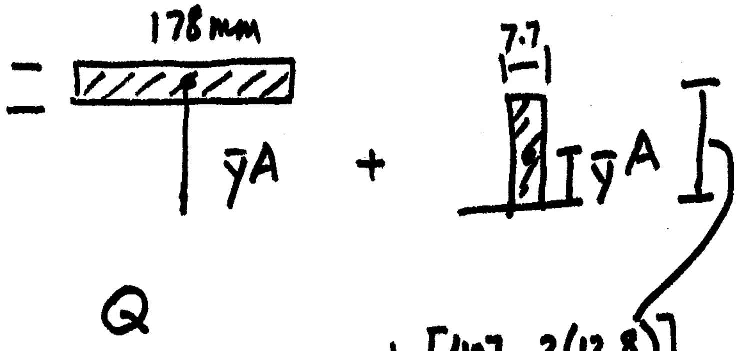
$M = \int -V dx$

$$\sigma_{x_{max}} = \frac{-(184.82 \text{ kN-m})\left(\frac{.407 \text{m}}{2}\right)}{216 \times 10^6 \text{ mm}^4 \cdot \left(1 \times 10^{-3} \frac{\text{m}}{\text{mm}}\right)^4}$$

$= 174.1 \text{ MPa}$

$V = -58\,kN$ halfway between D-E





$\frac{1}{2}\left[407-2(12.8)\right]$

$203.5 - 12.8 = 190.7\,mm$

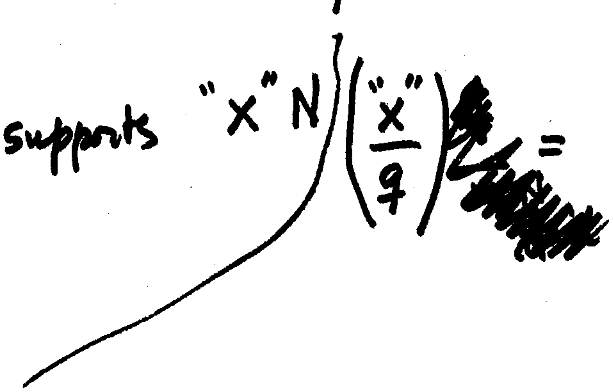
$Q = [12.8][178][190.7+6.4] + [190.7][7.7][190.7/2] = 589084\,mm^3$
$= 5.89 \times 10^{-4}\,m^3$

$I_{zz} = 216 \times 10^{6}\,mm^4 = 216 \times 10^{-6}\,m^4$; $\quad b = t_w = 7.7\,mm = .0077\,m$

$$\tau = \frac{QV}{I_{zz}t_w} = \frac{(58000\,N)(5.89 \times 10^{-4}\,m^3)}{216 \times 10^{-6}\,m^4\,(.0077\,m)} = 20.54\,MPa$$

$q = \tau t_w = \frac{\text{force}}{\text{length}}$

Nail supports "x" N $\left(\frac{\text{"x"}}{q}\right) = \frac{\text{dist}}{\text{nail}}$



occur in the plastic zones, no unbalance in longitudinal forces occurs and no shearing stresses are developed.

This elementary solution has been refined by using a more carefully formulated criterion of yielding caused by the simultaneous action of normal and shearing stresses.* Some fundamental aspects of the interaction of such stresses will be considered in Chapter 9.

EXAMPLE 7-6

An I beam is loaded as in Fig. 7-14(a). If it has the cross section shown in Fig. 7-14(c), determine the shearing stresses at the levels indicated. Neglect the weight of the beam.

SOLUTION

A free-body diagram of a segment of the beam is in Fig. 7-14(b). It is seen from this diagram that the vertical shear at every section is 50 kips. Bending moments do not enter directly into the present problem. The shear flow at the various levels of the beam is computed in the table below using Eq. 7-5. Since $\tau = q/t$ (Eq. 7-6), the shearing stresses are obtained by dividing the shear flows by the respective widths of the beam.

$$I = \frac{6(12)^3}{12} - \frac{(5.5)(11)^3}{12} = 254 \text{ in.}^4$$

For use in Eq. 7-5 the ratio $V/I = -50{,}000/254 = -197$ lb/in.^{4}

Level	A_{fghj}*	$\bar{y}$**	$Q = A_{fghj}\bar{y}$	$q = VQ/I$	t	τ, psi
1-1	0	6	0	0	6.0	0
2-2	(0.5)6 = 3.00	5.75	17.25	−3,400	6.0 0.5	−570 −6,800
3-3	(0.5)6 = 3.00 (0.5)(0.5) = 0.25	5.75 5.25	17.25 1.31 } 18.56	−3,650	0.5	−7,300
4-4	(0.5)6 = 3.00 (0.5)(5.5) = 2.75	5.75 2.75	17.25 7.56 } 24.81	−4,890	0.5	−9,780

* A_{fghj} is the partial area of the cross section above a given level in square inches.
** $\bar{y}$ is the distance from the neutral axis to the centroid of the partial area in inches.

The negative signs of τ show that, for the section considered, the stresses act downward on the right face of the elements. The sense of the shearing stresses acting on the section coincides with the sense of the shearing force V. For this reason a strict adherence to the sign convention is often unnecessary. It is always true that $\int_A \tau\, dA$ is equal to V and has the same sense.

* D. C. Drucker, "The Effect of Shear on the Plastic Bending of Beams," *Journal of Applied Mechanics*, **23** (1956), pp. 509–14.

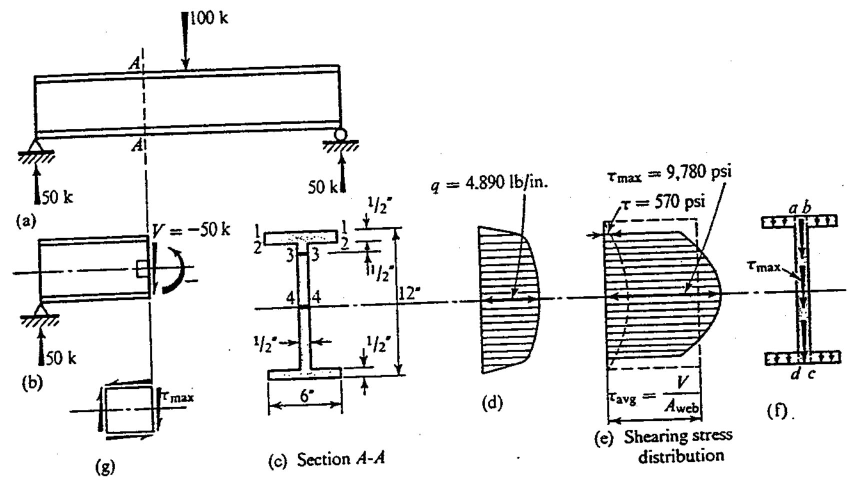


Fig. 7-14

Note that at the level 2-2 two widths are used to determine the shearing stress—one just above the line 2-2, and one just below. A width of 6 in. corresponds to the first case, and 0.5 in. to the second. This transition point will be discussed in the next article. The results obtained, which by virtue of symmetry are also applicable to the lower half of the section, are plotted in Fig. 7-14(d) and (e). By a method similar to the one used in the preceding example, it may be shown that the curves in Fig. 7-14(e) are parts of a second-degree parabola.

The variation of the shearing stress indicated by Fig. 7-14(e) may be interpreted as is shown in Fig. 7-14(f). The maximum shearing stress occurs at the neutral axis; the vertical shearing stresses throughout the web of the beam are nearly of the same magnitude. The shearing stresses occurring in the flanges are very small. For this reason the maximum shearing stress in an I beam is often approximated by dividing the total shear V by the cross-sectional area of the web (area *abcd* in Fig. 7-11(f)). Hence

$$(\tau_{max})_{approx} = V/A_{web} \tag{7-9}$$

In the example considered this gives

$$(\tau_{max})_{approx} = \frac{50{,}000}{(0.5)12} = 8{,}330 \text{ psi}$$

This stress differs by about 15 per cent from the one found by the accurate formula. For most cross sections a much closer approximation

תורת החוזק 1

תרגיל מס 9

5.16 and 5.17 For the wide-flange beam and loading shown, determine in a section located halfway between points D and E, (*a*) the largest normal stress, (*b*) the largest shearing stress.

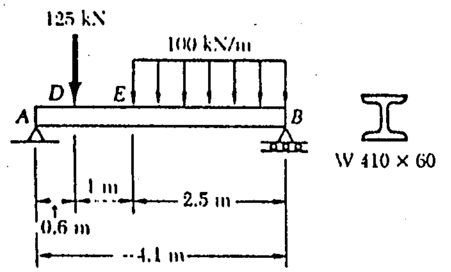


Fig. P5.16

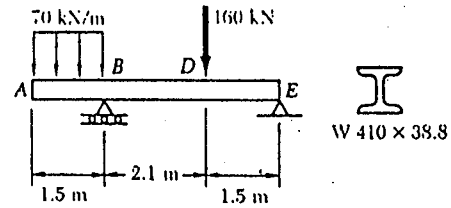


Fig. P5.17

5.18 Two rectangular plates are welded to the 310-mm-wide-flange beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 90 MPa.

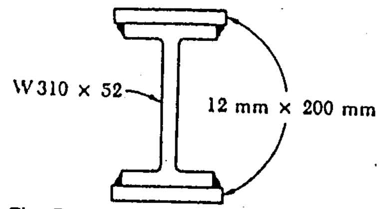
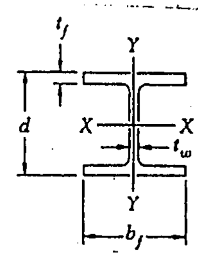


Fig. P5.18

Designation†	Area A, mm²	Depth d, mm	Flange Width b_f, mm	Flange Thickness t_f, mm	Web Thickness t_w, mm	Axis X-X I_x 10^6 mm⁴	Axis X-X S_x 10^3 mm³	Axis X-X r_x mm
W310 × 52	6650	317	167	13.2	7.6	118.6		
W410 × 114	14600	420	261	19.3	11.6	462	2200	177.8
85	10800	417	181	18.2	10.9	316	1516	170.7
60	7610	407	178	12.8	7.7	216	1061	168.4
46.1	5880	403	140	11.2	7.0	156.1	775	162.8
38.8	4950	399	140	8.8	6.4	125.3	628	159.0



3. For a timber beam having the cross section shown, determine the dimension w if the maximum allowable vertical shear force is 7.67kN, and the shearing stress is not to exceed 1.MPa.

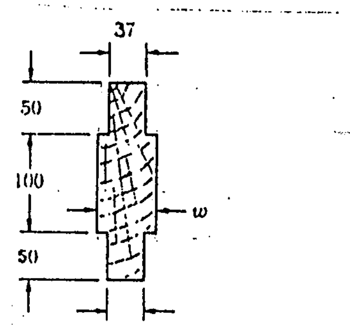
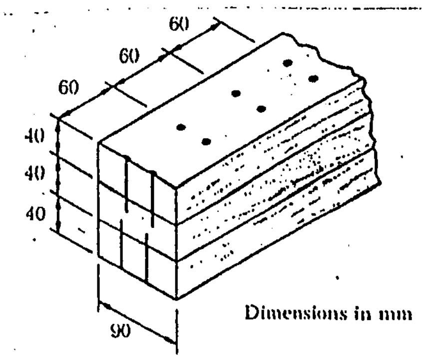
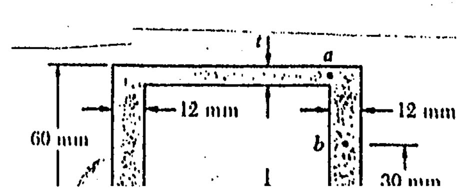


Fig. P5.20

5.1 Three boards, each of 40 mm × 90 mm rectangular cross section, are nailed together to form a beam which is subjected to a vertical shear of 1 kN. Knowing that the spacing between each pair of nails is 60 mm, determine the shearing force in each nail.



5.32 An extruded beam has the cross section shown and is subjected to a vertical shear of 50 kN. For t = 6 mm, determine the shearing stress at (*a*) point a, (*b*) point b.



to as the *shear flow*. Since force is usually measured in pounds, shear flow q has units of pounds per inch. Then, recalling that $dM/dx = -V$, one obtains the following expression for the shear flow in beams:

$$q = \frac{dF}{dx} = -\frac{dM}{dx}\frac{1}{I}\int_{\substack{\text{area}\\ fghj}} y\,dA = \frac{V A_{fghj}\bar{y}}{I} = \frac{VQ}{I} \qquad (7\text{-}5)$$

In this equation I stands for the moment of inertia of the entire cross-sectional area around the neutral axis, just as it does in the flexure formula from which it came. The total shearing force at the section investigated is represented by V, and the integral of $y\,dA$ for determining Q extends only over the cross-sectional area of the beam to one side of this area at which q is investigated.

In retrospect, note carefully that Eq. 7-5 was derived on the basis of the elastic flexure formula, but no term for a bending moment appears in the final expressions. This resulted from the fact that only the change in the bending moments at the adjoining sections had to be considered, and the latter quantity is linked with the shear V. The shear V was substituted for $-dM/dx$, and this masks the origin of the established relations. Equation 7-5 is very useful in determining the necessary interconnection between the elements making up a beam. This will be illustrated by examples.

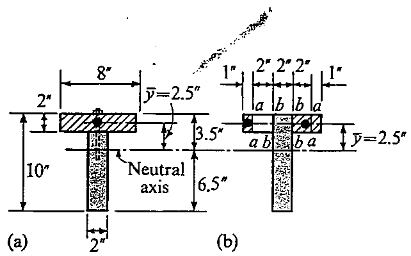


Fig. 7-6

EXAMPLE 7-1

Two long wooden planks form a T section of a beam as shown in Fig. 7-6(a). If this beam transmits a constant vertical shear of 690 lb, find the necessary spacing of the nails between the two planks to make the beam act as a unit. Assume that the allowable shearing force per nail is 150 lb.

SOLUTION

In attacking such problems the analyst must ask: What part of a beam has a tendency to slide longitudinally from the remainder? Here it is the plane of contact of the two planks; Eq. 7-5 must be applied to determine the shear flow in this plane. To do this the neutral axis of the whole section and its moment of inertia around the neutral axis must be found. Then as V is known and Q is defined as the statical moment of the area of the upper plank around the neutral axis, q may be determined. The distance y_c from the top to the neutral axis is

$$y_c = \frac{2(8)1 + 2(8)6}{2(8) + 2(8)} = 3.5 \text{ in.}$$

$$I = \frac{8(2)^3}{12} + (2)8(2.5)^2 + \frac{2(8)^3}{12} + (2)8(2.5)^2 = 291 \text{ in.}^4$$

$$Q = A_{fghj}\bar{y} = (2)8(3.5 - 1) = 40 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{690(40)}{291} = 95 \text{ lb per in.}$$

Thus, a force of 95 lb must be transferred from one plank to the other in every linear inch along the length of the beam. However, from the data given, each nail is capable of resisting a force of 150 lb, hence one nail can take care of 150/95 = 1.59 linear inches along the length of the beam. As shear remains constant at the consecutive sections of the beam, the nails should be spaced throughout at 1.59-in. intervals. In a practical problem a 1.5-in. spacing would probably be used.

SOLUTION FOR AN ALTERNATE ARRANGEMENT OF PLANKS

If, instead of using the two planks as above, a beam of the same cross section were made from five pieces, Fig. 7-6(b), a different nailing schedule would be required.

To begin, the shear flow between one of the 1-in.-by-2-in. pieces and the remainder of the beam is found, and although the contact surface *a-a* is vertical, the procedure is the same as before. The push or pull on an element is built up in the same manner as formerly:

$$Q = A_{fghj}\bar{y} = (1)2(2.5) = 5 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{690(5)}{291} = 11.8 \text{ lb per in.}$$

If the same nails as before are used to join the 1-in.-by-2-in. piece to the 2-in.-by-2-in. piece, they may be 150/11.8 = 12.7 in. apart. This nailing applies to both sections *a-a*.

To determine the shear flow between the 2-in.-by-10-in. vertical piece and either one of the 2-in.-by-2-in. pieces, the whole 3-in.-by-2-in. area must be used to determine Q. It is the difference of pushes (or pulls) on this whole area that causes the unbalanced force which must be transferred at the surface *b-b*:

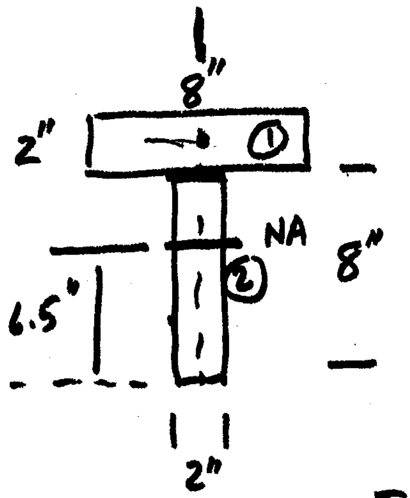
$$Q = A_{fghj}\bar{y} = (3)2(2.5) = 15 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{690(15)}{291} = 35.6 \text{ lb per in.}$$

Space nails at 150/35.6 = 4.2 in. or, in practice, 4-in. intervals along the length of the beam in both sections *b-b*. These nails could be driven in first and then the 1-in.-by-2-in. pieces put on.

EXAMPLE 7-2

A simple beam on a 20-ft span carries a load of 200 lb per foot including its own weight. The beam cross section is to be made from several full-sized wooden pieces as in Fig. 7-7(a). Specify the spacing of the ½-in.

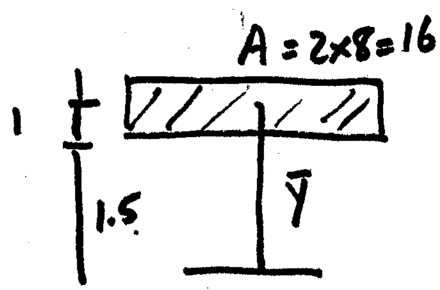


	$\bar{y}$	A	$\bar{y}A=$
①	9	$2\cdot8=16$	144
②	4	$2\cdot8=16$	64
		32	208

$$\bar{y}=\frac{208}{32}=6.5''$$

$$\tau=\frac{VQ}{I_{zz}b}$$

	I_{zz_0}	A	d	$I_{zz}=I_{zz_0}+Ad^2$
①	$\frac{1}{12}8\cdot2^3$	$2\cdot8=16$	-2.5	$5.33+6.25(16)=105.33\,in^4$
②	$\frac{1}{12}\cdot2\cdot8^3$	$8\cdot2=16$	2.5	$=185.67\,in^4$
				$291\,in^4$



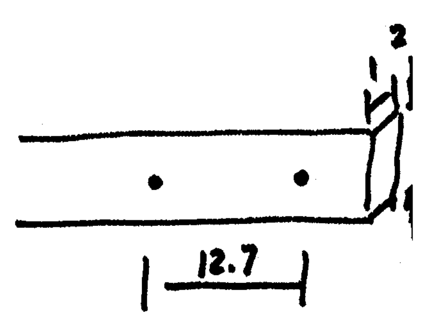
$$Q=A\bar{y}=16\times2.5=40\,in^3$$

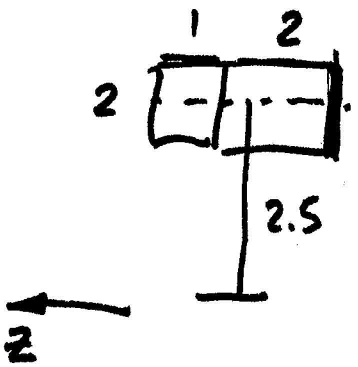
$$q=\frac{VQ}{I}=\frac{690\,lb\cdot40\,in^3}{291\,in^4}=95\,\frac{lb}{in}$$

$$\frac{\text{shear load/nail}}{q}=\frac{150\,lb}{95\,lb/in}=1.59\,in$$

$$Q=\bar{y}t_w\cdot z=2.5\cdot2\cdot1=5\,in^3$$

$$q=\frac{VQ}{I}=\frac{690\cdot5}{291}=11.8\,\frac{lb}{in}$$

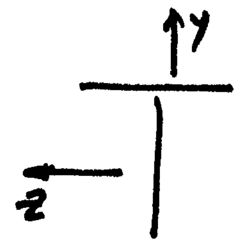


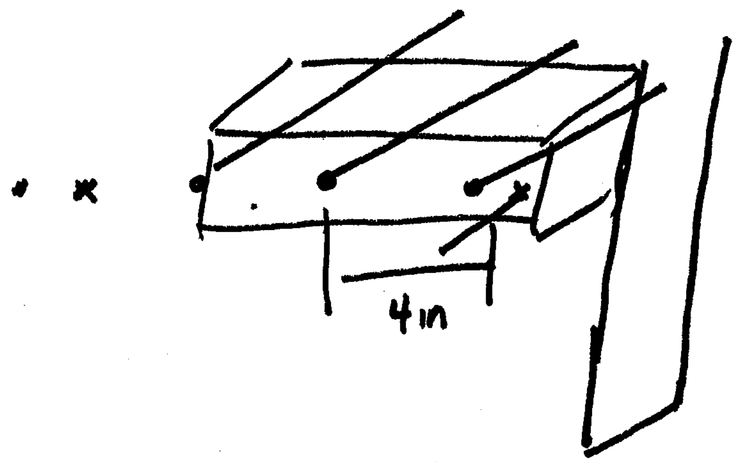


$$Q = A\bar{y} \Rightarrow A = t_w \cdot z$$

$$t_w = 2 \quad z = 3$$

$$Q = 2 \cdot 3 \cdot 2.5 = 15\,in^3$$





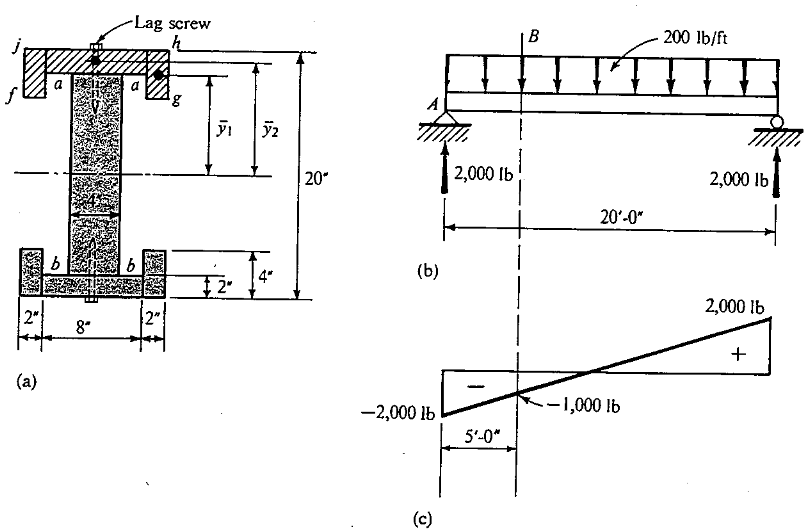


Fig. 7-7

lag screws shown which is necessary to fasten this beam together. Assume that one ½-in. lag screw, as determined by laboratory tests, is good for 500 lb when transmitting a lateral load parallel to the grain of the wood. For the entire section I is equal to 6,060 in.[4]

SOLUTION

To find the spacing of the lag screws, the shear flow at section *a-a* must be determined. The loading on the given beam is shown in Fig. 7-7(b); to show the variation of the shear along the beam, the shear diagram is constructed in Fig. 7-7(c). Then, to apply the shear flow formula, Q must be determined. This is done by considering the shaded area to one side of the cut *a-a* in Fig. 7-7(a). The statical moment of this area is most conveniently computed by multiplying the area of the two 2-in.-by-4-in. pieces by the distance from their centroid to the neutral axis of the beam and adding to this product a similar quantity for the 2-in.-by-8-in. piece. The largest shear flow occurs at the supports, as the largest vertical shears V of 2,000 lb act there:

$$Q = A_{fghj}\bar{y} = \sum A_i\bar{y}_i = 2A_1\bar{y}_1 + A_2\bar{y}_2$$

$$= 2(2)4(8) + 2(8)9 = 272 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{2{,}000(272)}{6{,}060} = 90 \text{ lb per in.}$$

At the supports the spacing of the lag screws must be 500/90 = 5.56 in. apart. This spacing of the lag screws applies only at a section where the shear V is equal to 2,000 lb. Similar calculations for a section where $V = 1{,}000$ lb gives $q = 45$ lb per inch, and the spacing of the lag screws becomes 500/45 = 11.12 in. Thus it is proper to specify the use of ½-in. lag screws at 5½ in. on centers for a distance of 5 ft nearest both the supports and at 11 in. for the middle half of the beam. A greater refinement in making the transition from one spacing of fastenings to another may be desirable in some problems. The same spacing of lag screws should be used at the section *b-b* as at the section *a-a*.

In a manner analogous to the above, the spacing of rivets or bolts in fabricated beams made from continuous angles and plates, Fig. 7-8, may be determined. Welding requirements are established similarly. The nominal shearing stress in a rivet is determined by dividing the total shearing force transmitted by the rivet (shear flow times spacing of the rivets) by the cross-sectional area of the rivet.

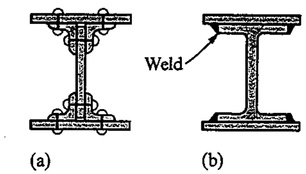


Fig. 7-8. Typical beam sections consisting of several components: (a) plate girder, (b) *I* beam reinforced with plates.

7-4. THE SHEARING STRESS FORMULA FOR BEAMS

The shearing stress formula for beams may be obtained from the shear flow formula. Analogously to the earlier procedure, an element of a beam may be isolated between two adjoining sections taken perpendicular to the axis of the beam. Then by passing another imaginary section through this element parallel to the axis of the beam, a new element is obtained, which corresponds to the element of one "plank" used in the earlier derivations. A side view of such an element is shown in Fig. 7-9(a), where the imaginary longitudinal cut is made at a distance y_1 from the neutral axis.* The cross-sectional area of the beam is shown in Fig. 7-9(c).

If shearing forces exist at the sections through the beam, a different bending moment acts at section A than at B. Hence more push or pull is developed on one side of the area $fghj$ than on the other, and, as before, the difference in the longitudinal forces in a distance dx is

$$dF = -\frac{dM}{I}\int_{\substack{\text{area}\\ fghj}} y\,dA = -\frac{dM}{I}A_{fghj}\bar{y} = -\frac{dM}{I}Q$$

The force equilibrating dF is developed in the plane of the longi-

* Since $dM/dx = -V$, for a positive V the change in moment $dM = -V\,dx$. For this reason $M_A > M_B$ and the magnitudes of the normal stresses in Fig. 7-9(a) are shown accordingly.

2.56 A homogeneous plate $ABCD$ is subjected to a biaxial loading which results in the normal stresses $\sigma_x = 150$ MPa and $\sigma_z = 100$ MPa. Knowing that the plate is made of steel for which $E = 200$ GPa and $\nu = 0.30$, determine the change in length of (*a*) edge AB, (*b*) edge BC, (*c*) diagonal AC.

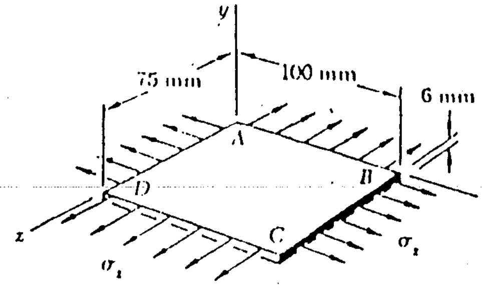


Fig. P2.56 and P2.57

Answer: $(a)\delta_{AB} = 60\mu m$, $(b)\delta_{BC} = 20.6\mu m$, $(c)\delta_{AC} = 60.4\mu m$

$1\,\mu m = 10^{-6} m$

2.57 The homogeneous plate $ABCD$ is subjected to a biaxial loading as shown. It is known that $\sigma_z = \sigma_0$ and that the change in length of the plate in the x direction must be zero, that is, $\varepsilon_x = 0$. Denoting by E the modulus of elasticity and by ν Poisson's ration, determine (*a*) the required magnitude of σ_x (*b*) the ratio σ_0/ε_z.

Answer: $(a)\sigma_x = \nu\sigma_0$, $(b)\frac{\sigma_0}{\varepsilon_z} = \frac{E}{1-\nu^2}$

4.12. A steel member ($E = 30 \times 10^6$ psi, $\nu = .3$) is subjected to the stresses

$$\sigma_x = 15{,}000 \text{ psi}, \quad \sigma_y = -5000 \text{ psi}, \quad \sigma_z = 0,$$
$$\tau_{xy} = -8000 \text{ psi}, \quad \tau_{xz} = 0, \quad \tau_{yz} = 0.$$

Determine the principal strains and the principal directions.

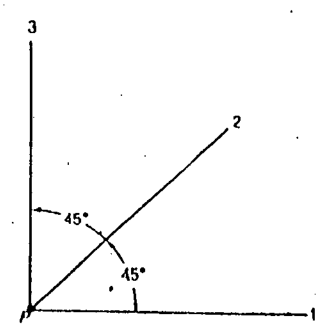
Answer: $\varepsilon_1 = 6.66\ 10^{-4}$, $\varepsilon_2 = -4.34\ 10^{-4}$, $\theta = 70.7°,\ 160.7°$

4.13. For a steel, $E = 30 \times 10^6$ psi, $\nu = \frac{1}{3}$. Determine the state of stress which corresponds to the following state of strain if the material obeys Hooke's law:

$$\epsilon_x = .001, \quad \epsilon_y = -.005, \quad \epsilon_z = 0,$$
$$\gamma_{xy} = -.0025, \quad \gamma_{yz} = -.0025, \quad \gamma_{xz} = 0.$$

Answer: $\sigma_{xy} = \sigma_{yz} = -2.81\ 10^4$, $\sigma_{xx} = -6.75\ 10^4$, $\sigma_{yy} = -20.25\ 10^4$, $\sigma_{zz} = -9\ 10^4$ psi

4.17. A state of plane stress (see Prob. 4.7) exists at the free surface of a body. Strain measurements are taken at a point P on such a free surface by the 45° strain rosette shown in Fig. P4.17. If the material obeys Hooke's law, compute the principal stresses in terms of ϵ_1, ϵ_2, ϵ_3, E, and ν.



P 4.17

Answer: $\sigma_{1,2} = \frac{E}{2(1-\nu)}(\varepsilon_1 + \varepsilon_3) \pm \frac{E}{2(1+\nu)}\left[(\varepsilon_1 - \varepsilon_3)^2 + (2\varepsilon_2 - \varepsilon_1 - \varepsilon_3)^2\right]^{1/2}$

4.18. Repeat Prob. 4.17 for the 60° strain rosette shown in Fig. P4.18.

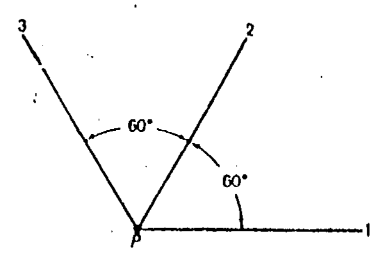


Fig. P4.18

Answer: $\sigma_{1,2} = \frac{E}{3}\left[\frac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{1-\nu} \pm \frac{2}{1+\nu}\sqrt{(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2) - (\varepsilon_1\varepsilon_2 + \varepsilon_1\varepsilon_3 + \varepsilon_2\varepsilon_3)}\right]$

6.96. Shafts AB and BC are made of different materials and unstressed when welded together at B as shown in Fig. P6.96. The twisting couples at A and C are equal in magnitude when the couple T_0 is applied at B. Determine the relation between the elastic moduli of the two materials.

Answer: $G_A = 0.592 G_C$

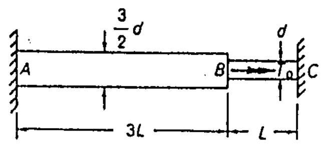


Fig. P6.96

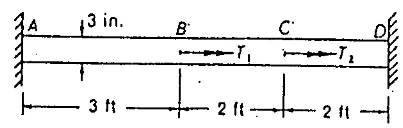


Fig. P6.97

6.97. An aluminum shaft is rigidly restrained at each end and loaded by couples as shown in Fig. P6.97. If $T_1 = 12{,}000$ in.-lb and $T_2 = 10{,}000$ in.-lb, compute the maximum shear stress in each section and the rotations of cross sections at B and C.

Answer: $\tau_{CD} = -2300\ psi$, $\tau_{BC} = -433\ psi$, $\tau_{AB} = 1840\ psi$,

3.108 Each of the two aluminum bars shown is subjected to a torque of magnitude $T = 1800$ N · m. Knowing that $G = 26$ GPa, determine for each bar the maximum shearing stress and the angle of twist at B.

Answer: (a) 40.1 MPa 0.653°
(b) 50.9 MPa 0.917°

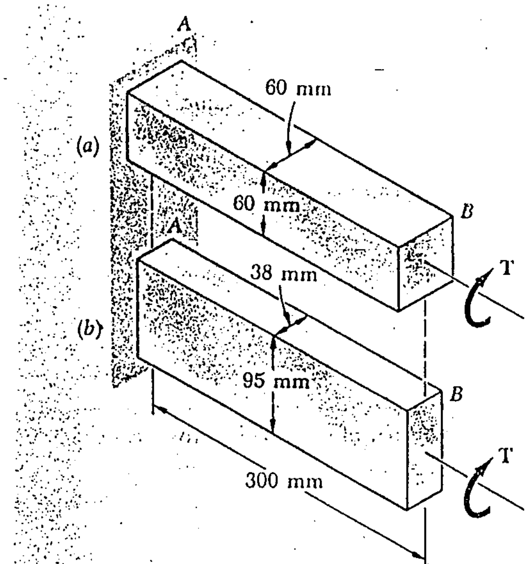


Fig. P3.108 and P3.109

3.132 A thin-walled tube has been fabricated by bending a metal plate of thickness t into a cylinder of radius c and bonding together the edges of the plate. A torque T is then applied to the tube, producing a shearing stress τ_1 and an angle of twist ϕ_1. Denoting by τ_2 and ϕ_2, respectively, the shearing stress and the angle of twist which will develop if the bond suddenly fails, express the ratios τ_2/τ_1 and ϕ_2/ϕ_1 in terms of the ratio c/t.

$\phi_2/\phi_1 = 3\left(\frac{c}{t}\right)^2$

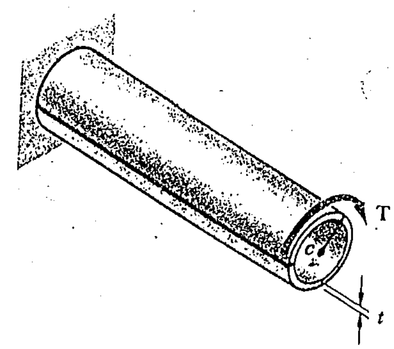
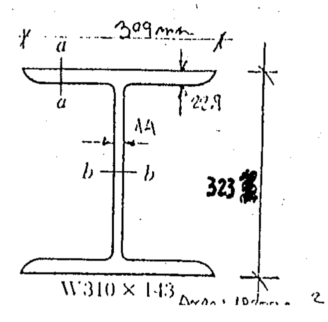


Fig. P3.132

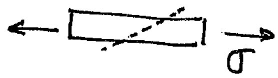
3.122 A 3.5-m-long steel member with a W310 × 143 cross section is subjected to a 4.5 kN · m torque. Knowing that $G = 77$ GPa and referring to Appendix C for the dimensions of the cross section, determine (a) the maximum shearing stress along line a-a, (b) the maximum shearing stress along line b-b, (c) the angle of twist. (*Hint*: Consider the web and the flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

39.7 MPa 24.7 MPa 6.72°



Failure theories

Ductile materials fail in shear when $\tau = \tau_{max}$. How to find τ_{max}

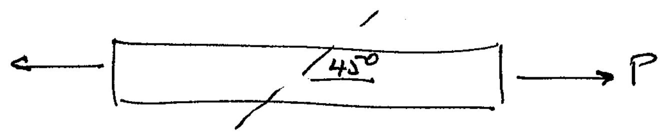
Do a uniaxial test  . When $\sigma = \sigma_1$ then $\tau = \tau_{max}$

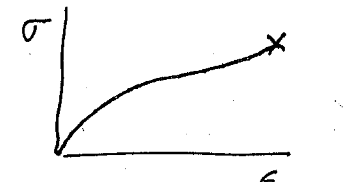
but max $\sigma_1 = \sigma_{yp}$

For a general situation $\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_{yp} - 0}{2}$

$$\therefore \quad \tau_{max} = \frac{\sigma_{yp}}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \, .$$

This is called the max shear theory. Material fails on a 45° angle to loading direction for uniaxial test



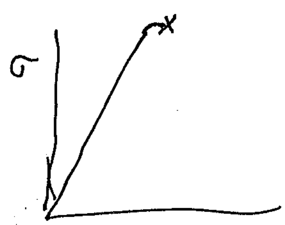


σ-ε looks like this

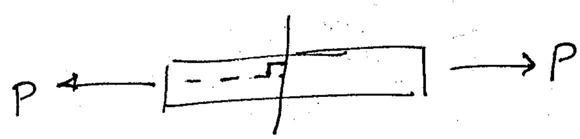
Almost brittle materials or **brittle type materials**

failure occurs when $\sigma_1 \geq \sigma_{yp}$ but $\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \geq \sigma_{yp}$ (for general situation)

failure occurs 90° to load direction in uniaxial test

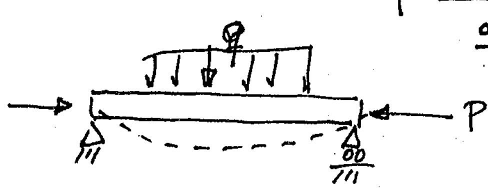


σ-ε usually looks like this



Another Failure method is by **buckling** (Chapter 10: Sections 1-4, 6 read!)

axial loads pushing on beam can cause it to buckle



Governing equation is

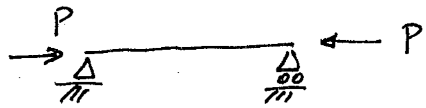
$$EI v^{IV} + P v'' = -q$$

Therefore the solution depends on $\frac{P}{EI} = \lambda^2 \Rightarrow v^{IV} + \lambda^2 v'' = -q/EI$

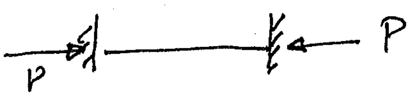
$v = A\cos\lambda x + B\sin\lambda x + Cx + D - \frac{q}{EI}\frac{x^2}{2\lambda^2}$; A, B, C, D are functions of λ

Depending on BC's, v becomes very large when $\sin\lambda L = 0$ or $\cos\lambda L = 0$ or some other function. For simply supported beam $\lambda L = n\pi$ ∴ the lowest value is when $\lambda L = \pi$. Note that buckling depends on P not

since $\lambda L = \pi$ $\qquad \sqrt{\dfrac{P}{EI}} \cdot L = \pi$ or $\qquad P_{cr} = EI \cdot \dfrac{\pi^2}{L^2}$ this is the Euler Buckling load for simply supported beams

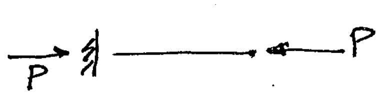


$$P_{cr} = \frac{\pi^2 EI}{L^2}$$



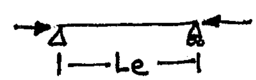
$$P_{cr} = \frac{4\pi^2 EI}{L^2} = \frac{\pi^2 EI}{L_e^2} \Rightarrow L_e = \frac{L}{2}$$

$$P_{cr} = 2.05\,\frac{\pi^2 EI}{L^2} = \frac{\pi^2 EI}{L_e^2} \Rightarrow L_e = 0.7L$$



$$P_{cr} = \frac{\pi^2 EI}{4L^2} = \frac{\pi^2 EI}{L_e^2} \Rightarrow L_e = 2L$$

these are equivalent lengths for simply supported beams to have same P_{cr} as original beam.

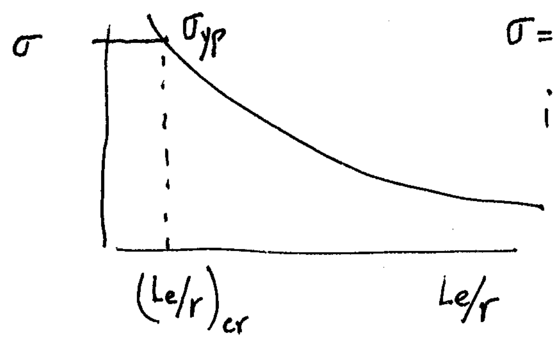


To find the stress in each case due to P_{cr} : $\sigma = \dfrac{P_{cr}}{A}$

or $\quad \sigma = \dfrac{\pi^2 EI}{L_e^2 A} = \pi^2 E\left(\dfrac{1}{L_e/r}\right)^2$; thus stress depends on L_e/r = slenderness ratio

• Here $I = r^2 A$ (r = radius of gyration)

Plot of this curve is $\quad \sigma\left(L_e/r\right)^2 = \pi^2 E$ = constant for each material. Only good until $\sigma = \sigma_{yp}$. When $\sigma = \sigma_{yp}$ $\quad L_e/r = \left(L_e/r\right)_{cr}$.

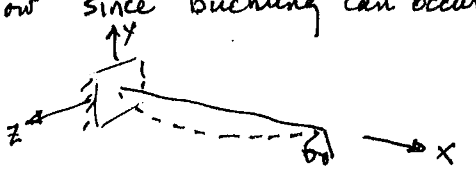


if $\left(L_e/r\right) \geq \left(L_e/r\right)_{cr}$ then the beam is in the elastic range.

For $\left(L_e/r\right) < \left(L_e/r\right)_{cr}$, the beam is considered a short beam an the equations for buckling must be modified.

Note

Now since buckling can occur in x-y plane or x-z plane, you must check P_{cr}



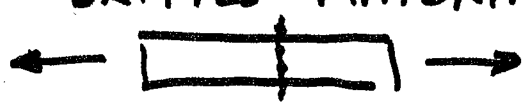
ie. $\quad P_{cr_1} = \dfrac{\pi^2 E I_{zz}}{L^2}$; $\quad P_{cr_2} = \dfrac{\pi^2 E I_{yy}}{L^2}$; the smaller value determines direction of buckling

EMA 3702

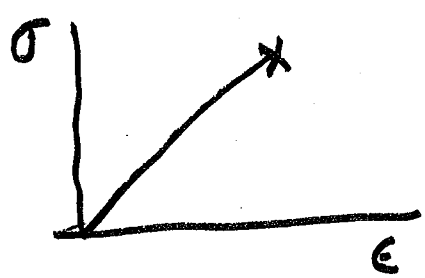
EXTRA SESSIONS

1. FAILURE CRITERION - Chpt 7
2. BUCKLING - Chpt 10

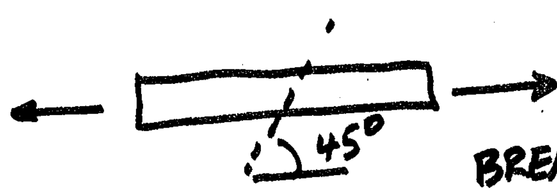
BRITTLE MATERIAL



BREAK ⊥ TO AXIS OF LOAD $\sigma_1 > \sigma_{max}$



MATERIALS w/ PLASTICITY



BREAK SURFACE IS 45° TO APPLIED LOAD.

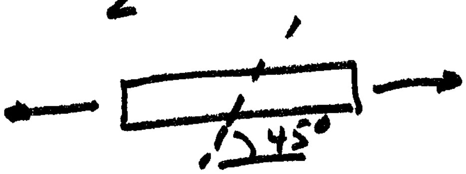
TRESCA – MATERIALS w/ PLASTICITY

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\sigma_{1_{max}} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{2_{min}} = (\;\text{"}\;) - \sqrt{\;\text{"}\;}$$

$$\frac{\sigma_1 - \sigma_2}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \tau_{max}$$



$$\tau_{max} = \frac{\sigma_{yp}}{2} = \frac{(\sigma_1 - \sigma_2)}{2}$$

uniaxial test BIAXIAL

$\sigma_1 > \sigma_2 > \sigma_3$ MOST GEN $\tau_{max} = \frac{\sigma_{yp}}{2} = \frac{(\sigma_1 - \sigma_3)}{2}$

$\sigma_1 > 0 > \sigma_2$ BIAXIAL PROB

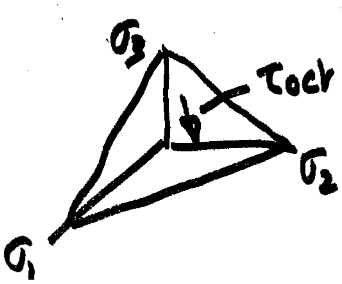
$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$$

VON MISES – ENERGY CRITERION – MATERIALS w/ PLASTICITY

FAILURE: σ_e – effective stress $\geq \sigma_{yp}$

$$\sigma_{yp} \leq \sigma_e = \frac{3}{\sqrt{2}}\tau_{oct} = \frac{1}{\sqrt{2}}\sqrt{\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)\right]}$$

$$= \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

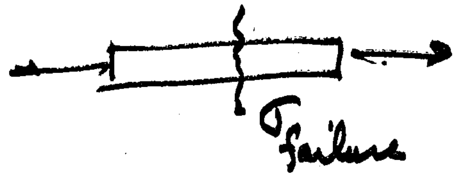


FOR BIAXIAL all $\sigma_z, \tau_{yz}, \tau_{zx} = 0$

or $\sigma_z = 0$

PRINCIPAL STRESS - BRITTLE

$\sigma_1 > \sigma_{failure} \Rightarrow$ FAILED



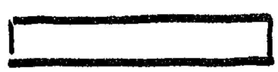
IN 2-D

$$\sigma_{max} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} > \sigma_{fail}$$

FOR EXAMPLE $\sigma_1 \geq \sigma_{yp}$ is failure

$$\sigma_{max} = \quad \geq \sigma_{yp}$$

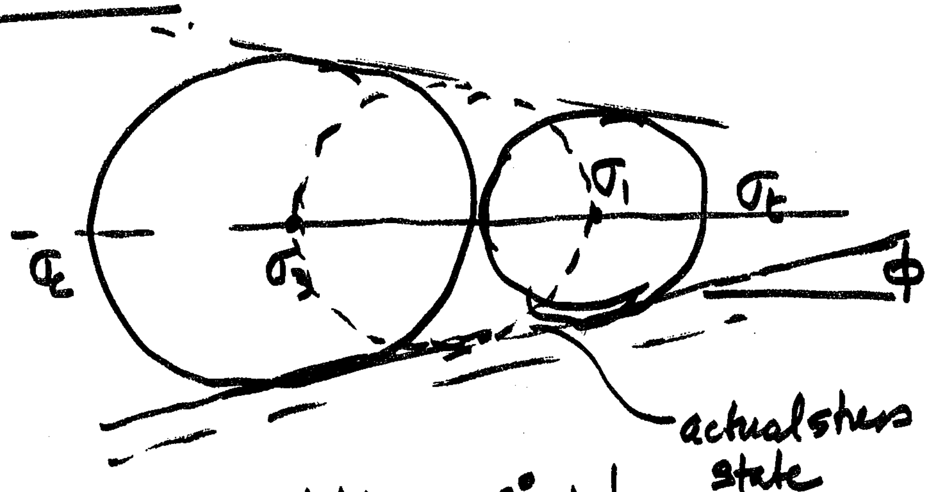
PRINC. STRAIN



$$\epsilon_1 > \epsilon_{failure} \Rightarrow \frac{1}{2}\left[(\epsilon_x + \epsilon_y) + \sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}\right] > \epsilon_{fail}$$

FOR EXAMPE $\epsilon_{fail} = \frac{\sigma_{yp}}{E}$

MOHR CRITERION - BRITTLE MATERIAL



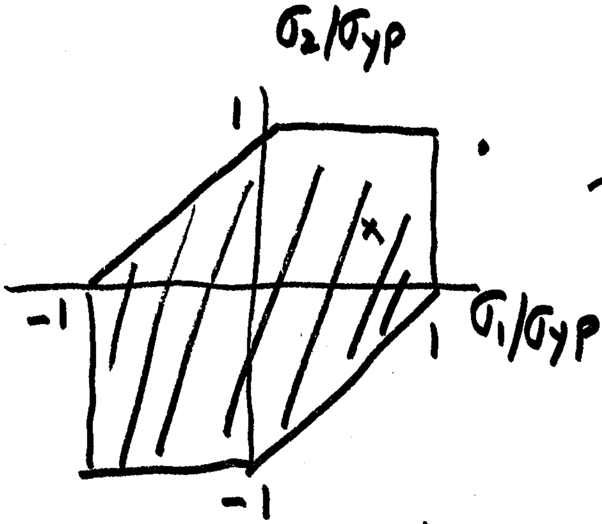
plane of failure $45° + \frac{1}{2}\phi$

FAILURE IF $\frac{\sigma_1}{\sigma_t} - \frac{\sigma_3}{|\sigma_c|} \geq 1$

σ_t - max failure stress in tension

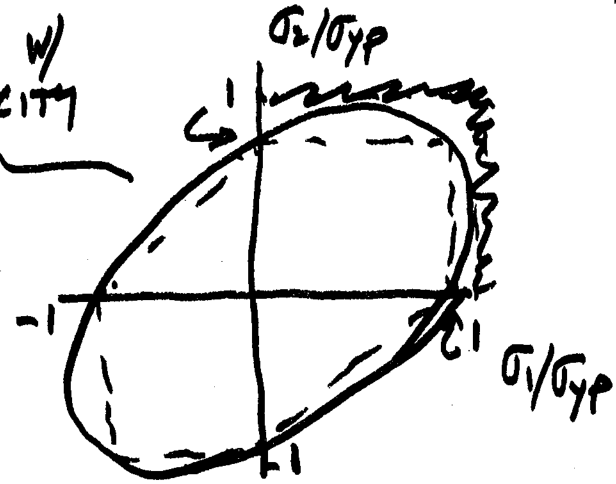
σ_c - max failure stress in compression

MATERIALS W/ PLASTICITY



inside you are safe

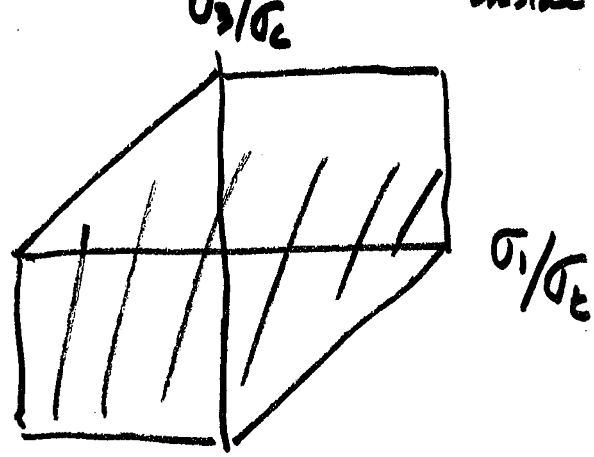
TRESCA



VON MISES

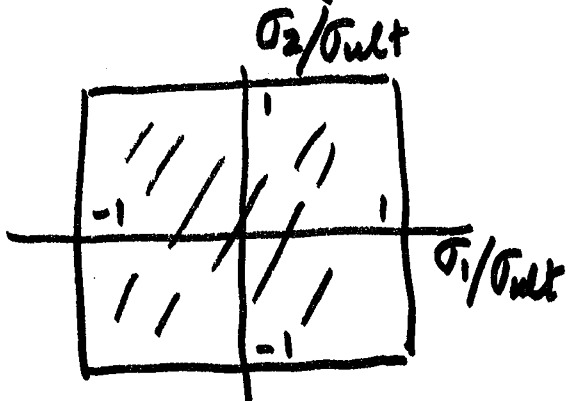
inside you are safe

$$\begin{pmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_y & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



MOHR

inside you are safe



PRINCIPAL STRESS

inside safe

BRITTLE

TENSION & COMP. TESTS FOR BRITTLE MATERIAL

$\sigma_t = 14$ MPa $\qquad \sigma_c = 120$ MPa

UNDER A CERTAIN LOADING : $\sigma_x = 0$, $\sigma_y = -18$ MPa, $\tau_{xy} = 20$ MPa

ARE WE SAFE ? CONSIDER PLANE STRESS $(\sigma_z, \tau_{xz}, \tau_{yz} = 0)$

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 - 18}{2} + \sqrt{\left(\frac{0-(-18)}{2}\right)^2 + (20)^2} = -9 + 21.9 = 12.9 \text{ MPa}$$

$$\sigma_2 = \left(\frac{\sigma_x - \sigma_y}{2}\right) - \sqrt{} = -9 - 21.9 = -30.9 \text{ MPa}$$

$$\sigma_1 > 0 > \sigma_2 \implies 12.9 > 0 > -30.9$$

$$\sigma_e = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)\right]^{1/2}$$

$$= \frac{1}{\sqrt{2}}\left[(0+18)^2 + (-18-0)^2 + (0-0)^2 + 6\left(20^2 + 0^2 + 0^2\right)\right]^{1/2}$$

$$= 39.04 \text{ MPa}$$

PRINC. STRESS CRIT : $\sigma_1 > \sigma_t \implies$ FAIL $\qquad 12.9 \overset{?}{>} 14$ NO ✓

TRESCA : $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{12.9 - (-30.9)}{2} = 21.9$ MPa

$\implies$ 1-D TEST $\quad \sigma_{yp} = \sigma_t = 14$ MPa , $\tau_{max} = \frac{\sigma_{yp}}{2} = 7$ MPa

$\tau_{max} \overset{?}{>} \frac{\sigma_{yp}}{2} \qquad 21.9 > 7$ MPa YES ✗

MISES : $\sigma_e = 39.04$ MPa $\qquad \sigma_e \geq \sigma_{yp} \implies$ FAIL $\qquad \sigma_{yp} = \sigma_t = 14$ MPa

$39.04 \overset{?}{>} 14$ YES ✗

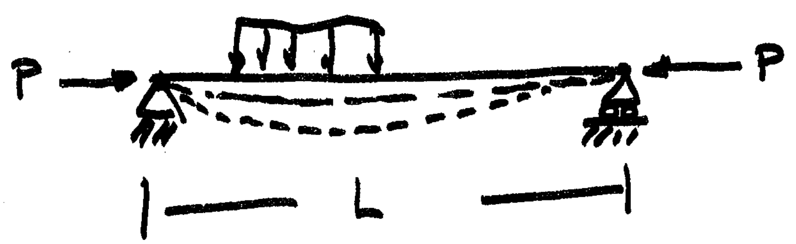
MOHR CRITERIA

$$\frac{\sigma_1}{\sigma_t} - \frac{\sigma_3}{|\sigma_c|} \geqslant 1 \implies \text{FAIL}$$

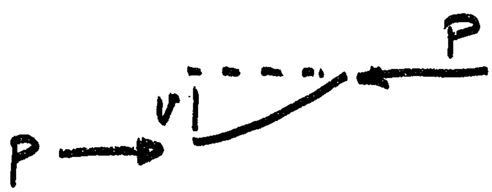
$$\frac{12.9}{14} - \frac{(-30.9)}{120} = 1.18 \overset{?}{\geqslant} 1 \quad \text{YES} \qquad \times$$

FAILURE – DUE TO EXCESSIVE DISPLACEMENT

BUCKLING



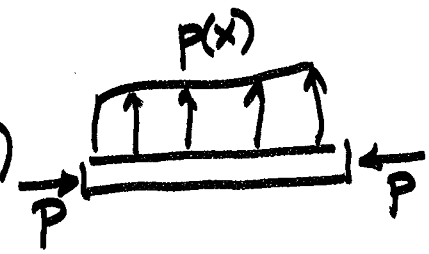
$$\frac{v}{L} \ll 1$$



Normally we have $EIv'' = M$

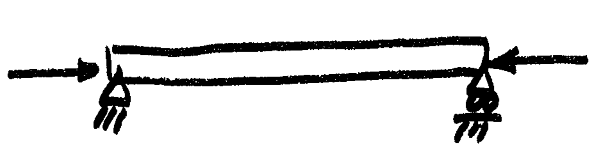
$$EIv'' + Pv = M$$

$$(EIv'')'' + (Pv)'' = +p(x)$$



IF $EI = \text{const.}$ $\quad v^{IV} + \frac{P}{EI} v'' = \frac{p(x)}{EI}$

$$\frac{P}{EI} = \lambda^2$$



$$v^{IV} + \lambda^2 v'' = 0$$

$$v''' + \lambda^2 v' = C_1$$

$$v'' + \lambda^2 v = C_1 x + C_2$$

homog soln $v'' + \lambda^2 v = 0 \Rightarrow A\cos\lambda x + B\sin\lambda x = v_h$

particular $v'' + \lambda^2 v = C_1 x + C_2$

$\longrightarrow v_p = D$

$\longrightarrow v_p = Cx$

$v_p = D: \; v_p'' + \lambda^2 v_p = 0 + \lambda^2 D = C_2 \qquad D = C_2/\lambda^2$

$$v_T = v_h + v_p = A\cos\lambda x + B\sin\lambda x + Cx + D$$

$$v_T'' = -\lambda^2 A\cos\lambda x - \lambda^2 B\sin\lambda x$$

$v(x=0)=0 \qquad v(x=L)=0$

$v''(x=0)=0 \qquad v''(x=L)=0$

$$A + 0B + 0C + D = 0$$
$$-\lambda^2 A + 0\cdot B + 0C + 0\cdot D = 0$$
$$A\cos\lambda L + B\sin\lambda L + CL + D = 0$$
$$-\lambda^2 A\cos\lambda L - \lambda^2 B\sin\lambda L + 0\cdot C + 0\cdot D = 0$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -\lambda^2 & 0 & 0 & 0 \\ \cos\lambda L & \sin\lambda L & L & 1 \\ -\lambda^2\cos\lambda L & -\lambda^2\sin\lambda L & 0 & 0 \end{bmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

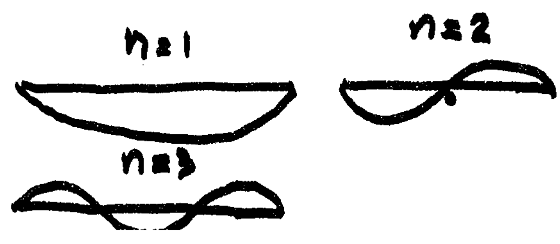
det matrix $= L\lambda^4 \sin\lambda L = 0 \Rightarrow \sin\lambda L = 0$

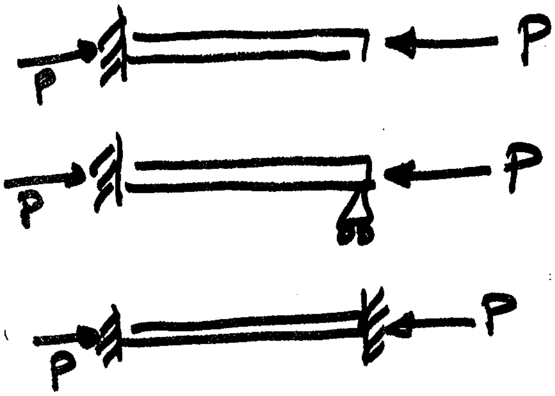
$\Rightarrow \lambda L = n\pi$

$$\sqrt{\frac{P}{EI}} = \lambda = \frac{n\pi}{L}$$

$$P = \frac{n^2\pi^2}{L^2}\cdot EI \qquad n=1 \quad P = \frac{\pi^2 EI}{L^2} \quad \text{Euler Buckling Load.}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

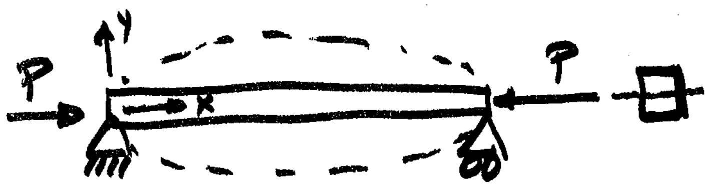




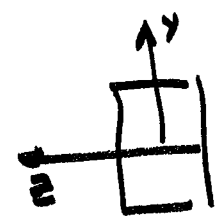
$$P = \frac{1}{4} P_{cr} = \frac{1}{4} \frac{\pi^2 EI}{L^2}$$

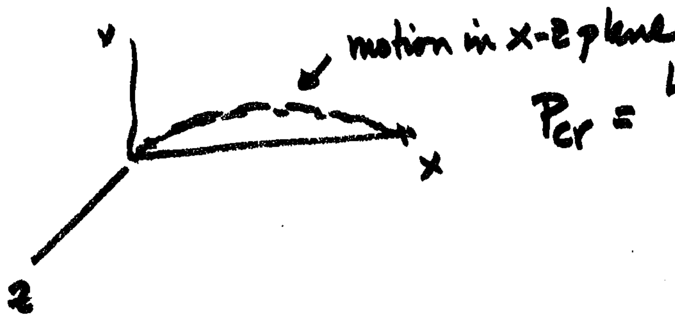
$$P = 2.05\, P_{cr} = 2.05 \frac{\pi^2 EI}{L^2}$$

$$P = 4 P_{cr} = \frac{4\pi^2 EI}{L^2}$$

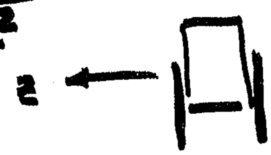


$$P_{cr} = \frac{\pi^2 EI_z}{L^2}$$



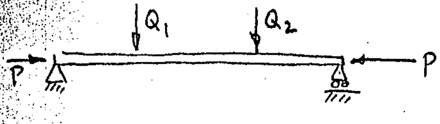


$$P_{cr} = \frac{4\pi^2 EI_y}{L^2} \quad \gg \quad \text{if } I_z = I_y$$

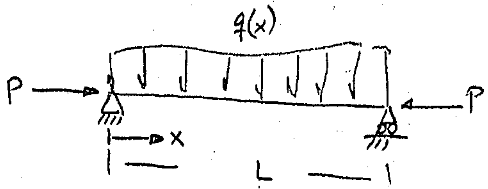


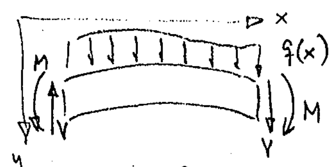
if $4I_y < I_z$ then buckling is out of the plane of the paper

קריסה צירית



בדרך כלל אנחנו יכולים להשתמש בסופרפוזיציה
כדי לקבל את התזוזה של קורה או מוט עם כמה עומסים.
אבל כאשר יש קורה או מוט שבהם יש כוח צירי כמו P
שפועל על המוט או קורה עם הכוחות Q_1, Q_2 אי אפשר לקבל
את התזוזות בסופרפוזיציה. זה בא בגלל שהתזוזות מיוחסים בדרך לינארית
ל Q_2, Q_1 אבל היחס לא לינארי ל-P.





$$M = EI\frac{d^2v}{dx^2} \qquad -V=\frac{dM}{dx} \qquad -q=\frac{dV}{dx}$$

אבל הכוח P מכניס לנוסחה מומנט של Pv. המומנט הזה קורה לגזירה של $P\frac{dv}{dx}$
וגם עומס מפורס של $-P\frac{d^2v}{dx^2}=q$. לכן הנוסחה שפועלת במצב הזה היא

$$\frac{d^2M}{dx^2}+\lambda^2 M = q \qquad \lambda^2=\frac{P}{EI}$$ כפונקציה של מומנט

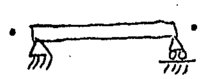
$$\frac{d^4v}{dx^4}+\lambda^2\frac{d^2v}{dx^2}=q/EI$$ כפונקציה של התזוזה

אם EI פונקציה של x

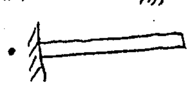
$$\frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right)+P\frac{d^2v}{dx^2}=q$$

פתרון כאשר EI קבוע ו- $q_0=q(x)$

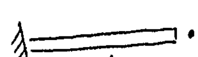
$$v(x)= A\cos\lambda x + B\sin\lambda x + Cx + D + \frac{q_0x^2}{2\lambda^2EI}$$



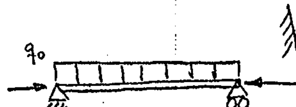
בסמך נייד ונייח: $M=EI\frac{d^2v}{dx^2}=0,\ v=0$



בסמך רתום: $\frac{dv}{dx}=0,\ v=0$



בצד חופשי: $M=0$, $V=\frac{d^3v}{dx^3}+\frac{P}{EI}\frac{dv}{dx}=0$



בעמוד - קורה בקצוות נייח ונייד

$$v=\frac{q_0}{\lambda^4EI\sin\lambda L}\left\{(1-\cos\lambda L)\sin\lambda x-\left[(1-\cos\lambda x)+\frac{\lambda^2x}{2}(L-x)\right]\sin\lambda L\right\}$$

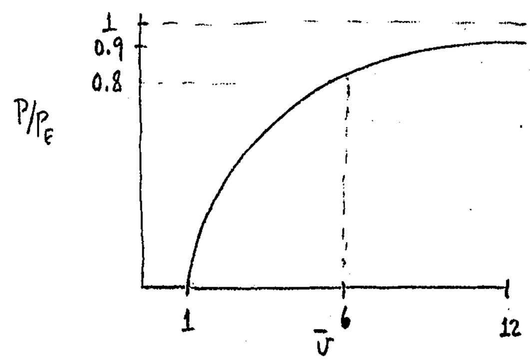
$$v(L/2) = \frac{5 q_0 L^4}{384 EI} \left\{ \frac{(1-\cos\lambda L)\sin(\lambda L/2) - \left(1-\cos\frac{\lambda L}{2} + \lambda^2 L^2/8\right)\sin\lambda L}{\frac{5}{384}(\lambda L)^4 \sin\lambda L} \right\}$$ כאשר $\lambda \neq 0$

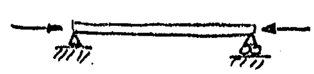
ותגובה הולכת לאין סוף כאשר $(\lambda L)^4 \sin\lambda L$ הולך לאפס, או כאשר $\sin\lambda L = 0$.

$$\sin\lambda L = 0 \Rightarrow \lambda L = n\pi \Rightarrow P = \frac{n^2\pi^2}{L^2} EI$$

לכן התגובה לא תלויה ב- q_0. ה- P הקטן הוא $P_E = \frac{\pi^2 EI}{L^2}$ וזה העומס הקריטי או <u>עומס הקריסה</u>

אם נסתכל במקדם $\left(\frac{\lambda L}{\pi}\right)^2 = \frac{\lambda^2 EI}{\pi^2 EI/L^2} = P/P_E$ כפונקציה של $\bar{v} = \frac{v(L/2,\lambda)}{v(L/2,\lambda=0)}$



אם הסתכלנו רק בקורה  היינו מקבל $\frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right) + P\frac{d^2v}{dx^2} = 0$

והפיתרון למשוואה הנ"ל הוא ש-

$$v = A\cos\lambda x + B\sin\lambda x + Cx + D$$

אם היינו משתמשים בתנאי הקצוות, זאת אומרת

$$v(0) = 0 \longrightarrow A + D = 0$$

$$M(x=0) = EI\frac{d^2v}{dx^2} = 0 \longrightarrow -\lambda^2 A = 0$$

$$v(L) = 0 \qquad A\cos\lambda L + B\sin\lambda L + CL + D = 0$$

$$M(x=L) = EI\frac{d^2v}{dx^2} = 0 \qquad -\lambda^2 A\cos\lambda L - B\lambda^2\sin\lambda L = 0$$

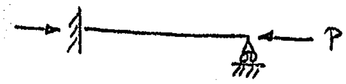
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -\lambda^2 & 0 & 0 & 0 \\ \cos\lambda L & \sin\lambda L & L & 1 \\ -\lambda^2\cos\lambda L & -\lambda^2\sin\lambda L & 0 & 0 \end{bmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

כדי למצוא פיתרון, או $A=B=C=D=0$ (זאת אומרת ש- $v=0$ הפיתרון הפשוט) <u>או</u> ש- ה DETERMINANT שווה לאפס. לכן, לפי העמוד השלישי, ה- DETERMINANT שווה ל-

$$L \begin{vmatrix} 1 & 0 & 1 \\ -\lambda^2 & 0 & 0 \\ -\lambda^2\cos\lambda L & -\lambda^2\sin\lambda L & 0 \end{vmatrix} = L\lambda^4 \sin\lambda L = 0$$

solutions

בתגובה מקסימלית של B. אבל, כאשר שמים עומס אנכי או עומס מפורס כמו q התגובה המקסימלית הולכת וגודלת. ואם העומס P הוא P_E, אז העמוד מתחיל לקרוס.

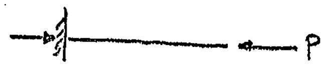


יישום תנאי הקצוות לעמוד-קורה כבצורה נותן המשוואה הקריטית $\tan\lambda L = \lambda L$

$$P_{קריסה} = 2.05\frac{\pi^2 EI}{L^2} = 2.05P_E$$

המשוואה הקריטית $\sin\frac{\lambda L}{2}\left(\sin\frac{\lambda L}{2} - \frac{\lambda L}{2}\cos\frac{\lambda L}{2}\right) = 0$

$$P_{קריסה} = \frac{4\pi^2 EI}{L^2} = 4P_E$$

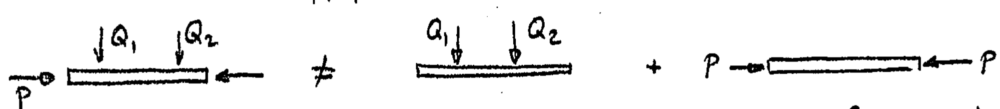


המשוואה הקריטית $\cos kL = 0$

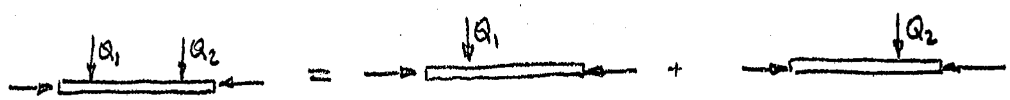
$$P_{קריסה} = \frac{1}{4}\frac{\pi^2 EI}{L^2} = \frac{1}{4}P_E$$

<u>כאן, לא נכלל את עומס הגזירה</u>. אם הגזירה נכללת, העומס הקריטי שגורם לקריסה יהיה יותר נמוך. רואים שתנאי הקצוות משפיעים לעומס הקריטי.

אף על פי שאי אפשר להשתמש בסופרפוזיציה כזאת



<u>אפשר</u> לעשות ככה



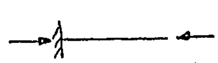
We got results assuming we remain in elastic range

<u>הערות</u> קבלנו את התוצאות האלה על תנאי שנישאר בחלק האלסטי הליניארי של החומר

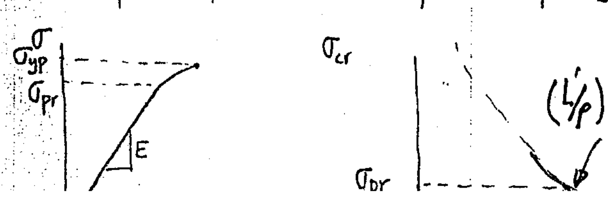
בדרך כלל, $P_{cr} = C\frac{\pi^2 EI}{L^2}$ והמספר C תלוי <u>בתנאי הקצוות</u>. אם $C=(L/L')^2$

relate other BC's to simply supported case

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{AL'^2} = \frac{\pi^2 E(\rho^2 A)}{A(L')^2} = \frac{\pi^2 E}{(L'/\rho)^2}$$

כאן L' = אורך האפקטיבי של הקורה. לדוגמא  $C=1/4$ לכן $L'=2L$

- $I = \rho^2 A$ ו- ρ הוא רדיוס ההסתובבות
- המקדם L/ρ הוא תלוי בגאומטרית הקורה וקוראים לזה מקדם הדקות slenderness ratio
- רואים ש- $\sigma_{cr}(L'/\rho)^2 = \pi^2 E$



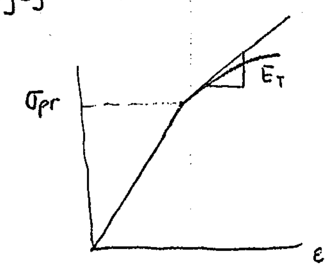
היפרבולה של אוילר

זאת אומרת ש-לפי הגאומטריה אנחנו יכולים לקבל את $(L'/\rho)_{גאום}$ ולפי הנוסחה

$$(L'/\rho)_{cr} = \pi\sqrt{E/\sigma_{pr}}$$

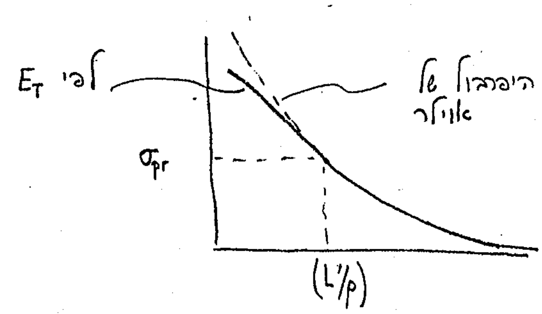
יכולים לקבל את הערך הקריטי. אם $(L'/\rho)_{cr} > (L'/\rho)_{גאום}$ המאמץ הקריטי לא יהיה שוה ל- P_{cr}/A אבל יהיה יותר גדול.

לתקן בזה, כדי לעבוד ב $(L'/\rho)_{cr} > (L'/\rho)_{גאום}$ משתמשים במודול הטנגנטי אחרי נקודת הפרופורציה עד σ_{yp}.

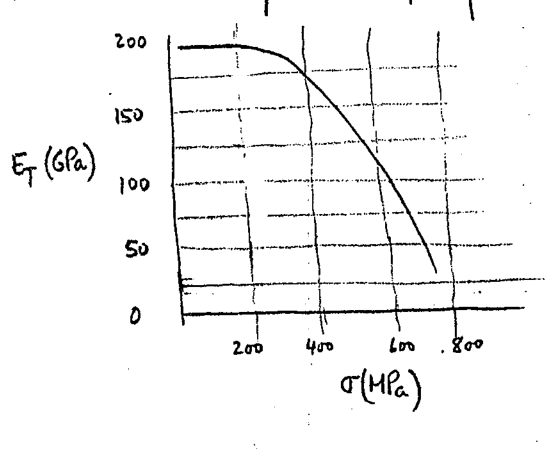


$$\sigma_{t,cr} = \frac{\pi^2 E_T}{(L'/\rho)^2}$$

ו- בדרך ניסיונות מקבלים את E_T.



לפלדה מסוימת קיבלו את הקשר בין E_T למאמץ



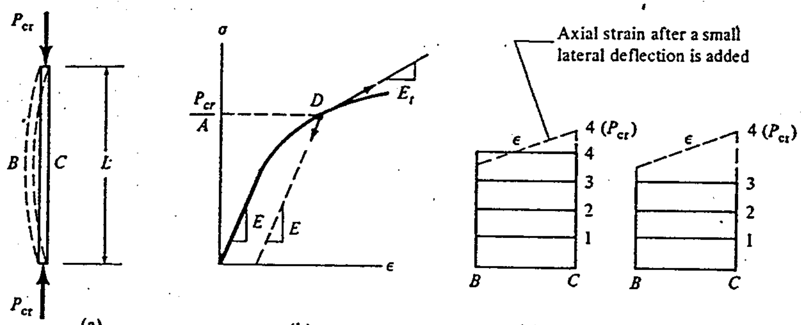


FIGURE 12.4.1. (a) Buckling of a pin-ended column under centroidal axial load. (b) Compressive stress–strain diagram, showing loading and unloading paths from point D, which corresponds to inelastic buckling. (c) Distribution of axial strain across the column at increasing load levels, according to double-modulus theory. (d) Possible distribution of axial strain across the column in tangent modulus theory.

cross section. Therefore, the column must bend before reaching the double-modulus load. But this is in contradiction to a basic assumption in double-modulus theory. The contradiction is resolved by noting that lateral deflection may occur *simultaneously* with application of the last increment of load. There need be no unloading on the convex side, and modulus E_t may prevail all across the section (Fig. 12.4.1d). Under near-perfect test conditions the collapse load slightly exceeds the theoretical tangent-modulus load, but it does not reach the double-modulus load.

In summary, inelastic buckling of a straight, axially loaded column does not occur at a unique value of axial load P. Instead, buckling begins at the tangent-modulus load and is complete (meaning that collapse takes place) before the theoretical double-modulus load is reached. Tests of real columns, which have larger imperfections than laboratory specimens, are in excellent agreement with tangent modulus theory.

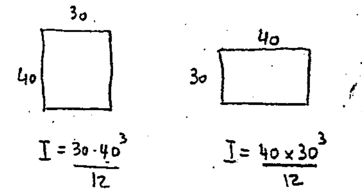
Euler did not realize that bending stiffness EI could be calculated rather than obtained by experiment. However, he anticipated Engesser by remarking in 1757 that EI represents a resistance to bending that need not pertain only to elastic bodies [12.4].

Example 12.4.1. A column has a solid rectangular cross section, 40 mm by 30 mm. It is 200 mm long, free at the top, and fixed at the base. Material properties are shown in Fig. 12.4.2. What centroidal axial compressive load at the top will make the column buckle?

The appropriate equation is $P_{cr} = \pi^2 EI/4L^2$, where

$$I = \frac{bh^3}{12} = \frac{40(30)^3}{12} = 90{,}000 \text{ mm}^4 \qquad (12.4.2)$$

because buckling will take place about the weaker axis of the cross section.



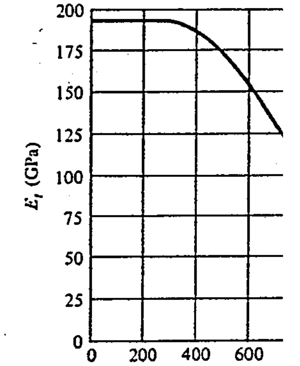


FIGURE 12.4.2. Tangent modulus E_t versus compressive stress σ for a particular steel, obtained from a stress–strain plot.

P_{cr} = 1077 kN, or $\sigma_{cr} = P_{cr}/A$ = 898 MPa. This stress i[s]
higher than the proportional limit stress, which appears to be a
in Fig. 12.4.2. Therefore, buckling is inelastic, the effective
pends on load, and an iterative method of calculation is need[ed]
as follows.

Assume that σ_{cr} will be, say, 600 MPa. At this stress, Fig
E_t = 160 GPa. Hence

$$P_{cr} = \frac{\pi^2 E_t I}{4L^2} = 888 \text{ kN} \qquad \frac{P_{cr}}{A} = 740 \text{ MPa}$$

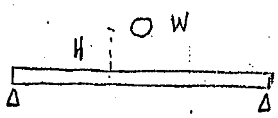
As P_{cr}/A exceeds the assumed σ_{cr} of 600 MPa, another trial i[s]
Assume that σ_{cr} = 660 MPa; then

$$E_t = 142 \text{ GPa} \qquad P_{cr} = \frac{\pi^2 E_t I}{4L^2} = 788 \text{ kN} \qquad \frac{P_{cr}}{A} = 657 \text{ M}$$

Now the assumed value of σ_{cr} agrees well enough with the cal[culated]
and P_{cr} = 788 kN is accepted as the tangent modulus bucklin[g]

Creep Buckling. As the name implies, creep buckling theor[y]
material that creeps, that is, a material whose strain changes with ti[me]
stress. A creeping column may display a small but gradually inc[reasing]
deflection, then fail suddenly by buckling. The phenomenon is
examination of creep curves (Fig. 12.4.3). One may enter the c[urves]
a certain time, say t_1, and read the strain for each of several
Stress–strain data thus obtained is then plotted as a stress–strain c[urve]
the curve labeled t_1 in Fig. 12.4.3. Repetition of this procedure a[t]
produces a set of isochronous stress–strain curves (stress versus str[ain]
time). These curves show that at a given stress level, the tan[gent]
decreases with time. This implies that however light the load, a cre[eping column]
will eventually buckle if we wait long enough for its tangent modul[us]

מאמץ הדינמי עקב נגיפה



משקל W שנופל על קורה גורם למאמצים דינמיים והמאמצים האלה יותר גבוהים ממצב שמשקל W לוחץ על הקורה בלבד

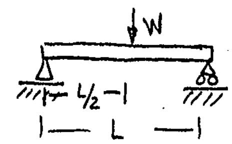
העומס הדינמי נחשב כ

$$P_{dyn} = W\left(1 + \sqrt{1 + 2H/\Delta_{st}}\right)$$

h - זה מרחק האנכי בין המשקל W לקורה.

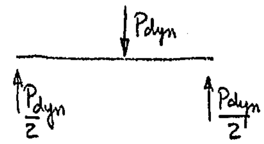
Δ_{st} - זה התזוזה הסטטית של הקורה שבאה מיישום המשקל על הקורה.

דוגמא -

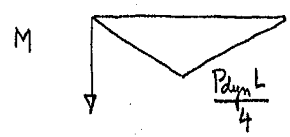


$$\Delta_{st} = \frac{WL^3}{48EI}$$

המאמץ הדינמי כאן -



① למצוא העומס הדינמי
② למצוא המומנט המקס.
③ למצוא המאמץ



$$\sigma_{מקס} = \frac{My}{I} = \frac{P_{dyn}L\cdot(h/2)}{4\cdot bh^3/12} = \frac{3P_{dyn}\cdot L}{2bh^2}$$

הערות ליישום הנוסחה הבאות.

- אין איבוד אנרגיה בנקודת הנגיפה או בסביבה
- אינרציית המערכת נחשבת כאפס (משקל המערכת לא התחשב)
- לתזוזה של המערכת יש יחס לינארי לגובה העומס המיושם ואין אכפת אם זה עומס סטטי או דינמי

1. Write the second-order differential equation for the bending of the column shown in Fig. P1-2 and use it to determine the critical load of the column. At its lower end the column is completely fixed. At the upper end the column is prevented from rotating, but free to translate laterally. (ans: $P_{cr}=\pi^2 EI/L^2$)

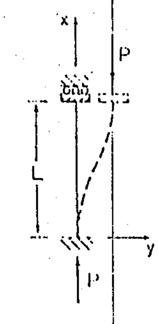
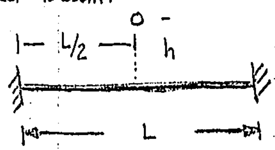


Fig. P1-2

2. Find an expression for the maximum stress when a ball weighing W Newtons is dropped onto a fixed-fixed beam.



3. . A linearly elastic beam-column having a flexural rigidity EI, is subjected to a thrust P and a moment M_0 as shown in Fig. A below.

(a) Determine the lateral displacement v(x).

(b) From part (a), write the solution for the system subjected to a force P acting as shown in Fig. B.

(c) Determine Δ_c, the horizontal displacement of point C, assuming small rotations. (Assume also that the horizontal displacement of point B is negligible).

(d) Determine the bending moment M(x).

(g) Explain why, although the beam is made of a linearly elastic material, the results of part (c) are non-linear.

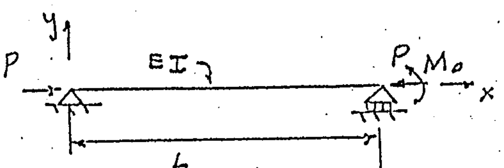


Fig. A

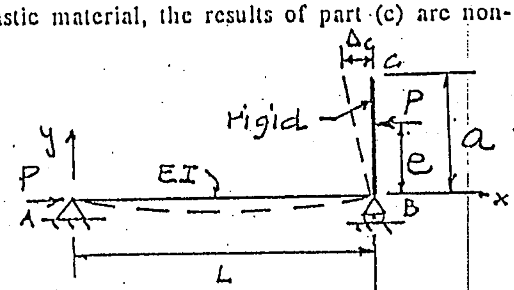


Fig. B

Answers :

(a) $y(x) = -\frac{M_0}{P}\left[\frac{\sin Kx}{\sin KL} - \frac{x}{L}\right], \; K^2 = \frac{P}{EI}$

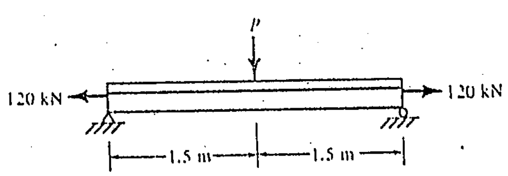
(c) $\Delta_c = \frac{ac}{L}\,[1 - L\sqrt{P/EI}\,\cot(L\sqrt{P/EI})\,] = \frac{ac}{L}\,(1 - KL\cot KL)$

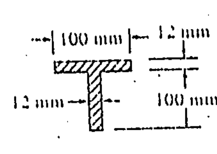
(d) $M(x)=M_0 \sin kl/\sin kL$

*12.10 A T section carries an axial tensile force of 120 kN, applied through the centroid of the cross section. The allowable stress in tension or compression is 130 MPa. Let $E = 200$ GPa. What transverse force P can be applied at midspan if the beam is

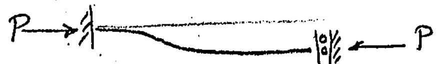
(a) Stem down (as shown)?

(b) Stem up?





1.



מהנוסחה שמצאנו בשעור שעבר והפיתרון הוא

$$EI\frac{d^4}{dx^4}v + P\frac{d^2v}{dx^2} = 0$$

$$v = A\cos\lambda x + B\sin\lambda x + Cx + D$$

$$\lambda^2 = P/EI$$

תנאי הקצוות

① תזוזה $v(x=0)=0$

② זווית $\frac{dv}{dx}(x=0)=0$

③ זווית $\frac{dv}{dx}(x=L)=0$

④ גזירה : $EI\frac{d^3v}{dx^3}(x=L) + P\frac{dv}{dx}(x=L) = 0$

מפני ש $\frac{dv}{dx} = 0$ ב-$x=L$, לכן ④ נותנת $\frac{d^3v}{dx^3}=0$ ב-$x=L$

מקבלים

$$A + D = 0 \quad ①\text{-מ}$$
$$\lambda B + C = 0 \quad ②$$
$$-A(\lambda\sin\lambda L) + B(\lambda\cos\lambda L) + C = 0 \quad ③$$
$$A\lambda^3\sin\lambda L - B\lambda^3\cos\lambda L = 0 \quad ④$$

לכן

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \lambda & 1 & 0 \\ -\lambda\sin\lambda L & \lambda\cos\lambda L & 1 & 0 \\ \lambda^3\sin\lambda L & -\lambda^3\cos\lambda L & 0 & 0 \end{bmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

כדי לקבל תוצאות לא טריוויאליות הדיטרמיננט צריך להיות אפס. לפי העמוד הרביעי

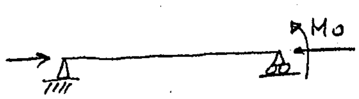
$$-1\begin{pmatrix} 0 & \lambda & 1 \\ -\lambda\sin\lambda L & \lambda\cos\lambda L & 1 \\ \lambda^3\sin\lambda L & -\lambda^3\cos\lambda L & 0 \end{pmatrix} = 0$$

לכן

$$\lambda^4\sin\lambda L = 0$$

$\lambda \neq 0$ (זה הפיתרון $v=0$) ; לכן $\sin\lambda L = 0 \Leftarrow \lambda L = n\pi$ או $P_E = \left(\frac{\pi}{L}\right)^2 EI$

3.



$$v = A\cos\lambda x + B\sin\lambda x + Cx + D \qquad \lambda^2 = P/EI$$

לפי הסכם הסימנים

① $v(x=0)=0$

② $EI\frac{d^2v}{dx^2}(x=0) = M = 0$

③ $v(x=L)=0$

④ $EI\frac{d^2v}{dx^2}(x=L) = -M_0$

$$A + D = 0 \quad ①\text{-מ}$$
$$-A\lambda^2 = 0 \quad ②$$
$$A\cos\lambda L + B\sin\lambda L + CL + D = 0 \quad ③$$
$$EI(-A\lambda^2\cos\lambda L - B\lambda^2\sin\lambda L) = -M_0 \quad ④$$

אם $\lambda \neq 0$ $\Leftarrow$ $A \overset{(2)}{=} 0$ $\Leftarrow$ $D \overset{(1)}{=} 0$ $\Leftarrow$ $B\sin\lambda L + CL \overset{(3)}{=} 0$ $\Leftarrow$ $B(EI\lambda^2\sin\lambda L) \overset{(4)}{=} M_0$ $\Leftarrow$ $B = \frac{M_0}{EI\lambda^2\sin\lambda L}$

ו- $C = -\frac{M_0}{EI\lambda^2 L}$

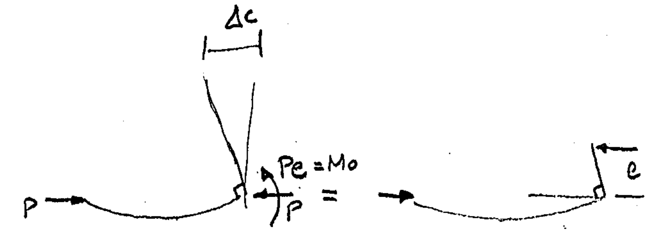
לכן,

$$v = \frac{M_0}{EI\lambda^2}\left(\frac{\sin\lambda x}{\sin\lambda L} - \frac{x}{L}\right)$$

$$\boxed{v = \frac{M_0}{P}\left(\frac{\sin\lambda x}{\sin\lambda L} - \frac{x}{L}\right)}$$

לקבל $M(x)$, גוזר פעמיים לפי x: $EI\frac{d^2v}{dx^2} = M = -EI\,M_0\lambda^2\sin\lambda x$ ...

הסימון וכן לפי ...



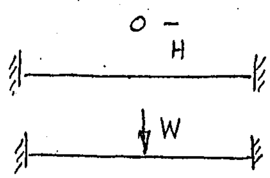
לפי הצורה

לכן $\theta|_{x=L} = \frac{Pe}{P}\left(\lambda\cot\lambda L - \frac{1}{L}\right)$ ו- $a\theta|_{x=L} = \Delta_c$ כי העמוד קשיח וקשור לקורה בזווית אנכית של $90°$. לכן,

$$\Delta_c = \frac{ae}{L}\left(\lambda L\cot\lambda L - 1\right)$$

2.

לפתור את

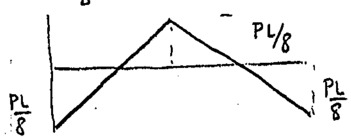


פותרים את והתזוזה היא $\Delta_{st} = v = \frac{WL^3}{192EI}$

לפי הנוסחה $P_{dyn} = W\left(1+\sqrt{1+2H/\Delta_{st}}\right)$, בגלל שמצאנו את Δ_{st}, מצאנו את P_{dyn}.

למצוא את המומנט המקס. לפי הנוסחה שנתונה ב- COOK & YOUNG, $v = \frac{P}{48EI}(3Lx^2 - 4x^3)$

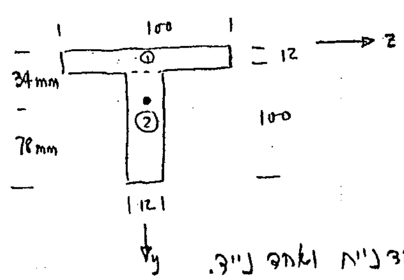
ו- $M = EI\frac{d^2v}{dx^2}$. לכן, $M = \frac{P}{48}(6L - 24x)$. ב- $x=0$ $M = \frac{PL}{8}$; ב- $x=L/2$, $M = -\frac{PL}{8}$, וגרף המומנט



לכן יש שלושה מקומות שבהם המומנט הוא מקסימלי והמאמץ המקסימלי הוא $\sigma = \frac{Mc}{I} = \frac{PL}{8}\frac{6}{bh^2}$

$$\frac{3LW\left(1+\sqrt{1+\frac{2H}{WL^3/192EI}}\right)}{4bh^3} = \frac{3}{4}\frac{PL}{bh^3} = \sigma_{max}$$

12.10



שטח החתך הוא $2(100\times12)\ mm = 2400\ mm^2$

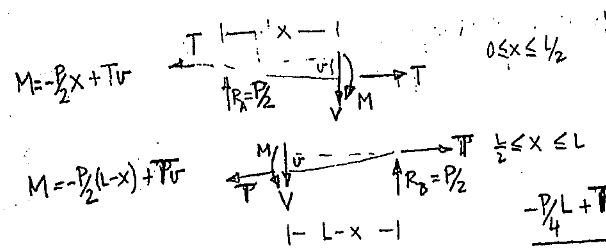
מרכז הכובד הוא

	A	y	Ay
①	100x12	6	7200
②	100x12	62	74400
	2400		81600

$$\bar{y} = \frac{\Sigma Ay}{A} = 34mm$$

הכוח הקריסה הוא $T_E = \frac{\pi^2 EI}{L^2}$ מפני הסמכים, אחד נייח ואחד נייד.

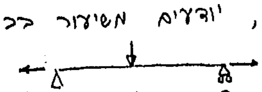
בגלל הכוח הצירי, T, המומנט בקורה הוא המומנט שמצאנו בקורה בלי הכוח הצירי נוסף המומנט שבא מהכוח הצירי, Pv; v הוא התזוזה באותו מקום שמחשבים את המומנט.



לכן, המומנט המקס. נמצא ב- $x=L/2$ והוא $-\frac{P}{4}L + Tv$

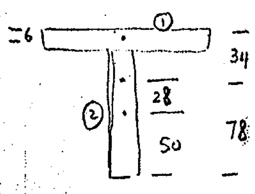
כדי למצוא את $v|_{x=L/2}$, יודעים משיעור הכיתה ש- $v = A\cosh\lambda x + B\sinh\lambda x + Cx + D$ ו- $\lambda^2 = \frac{T}{EI}$ בכל חלק מהקורה.

זאת אומרת שצריך להשתמש ב $v_L = A\cosh\lambda x + B\sinh\lambda x + Cx + D$



בין AC, ו- $v_R = \bar{A}\cosh\lambda(x-L) + \bar{B}\sinh\lambda(x-L) + \bar{C}(x-L) + \bar{D}$ בין CB. תנאי השפה: $v_L(x=0)\overset{(1)}{=}0$, $v_L''(x=0)\overset{(2)}{=}0$, $v_R(x=L)\overset{(3)}{=}0$,

$v_R''(x=L)\overset{(4)}{=}0$. ב- C: $v_L(x=L/2)\overset{(5)}{=}v_R(x=L/2)$, $v_L'(x=L/2)\overset{(6)}{=}v_R'(x=L/2)$ ...



I	A	d^2	I_{tt}
$\frac{1}{12}(100)(12)^3$	1200	$(28)^2$	955200
$\frac{1}{12}(12)(100)^3$	1200	$(28)^2$	1940800
			$2.896\times10^6\ mm^4$

לכן, דין A-C: $v = \frac{P}{2EI}\frac{\sinh\lambda x}{\lambda^3\cosh\lambda L/2} - \frac{Px}{2EI\lambda^2}$. מפני ש- $\lambda^2 = T/EI$ ורוצים $v|_{x=L/2}$

$$v|_{x=L/2} = P\left\{\frac{\sinh\lambda L/2}{2T\lambda\cosh\lambda L/2} - \frac{L}{4T}\right\}$$

צריך לדעת λ לחשב את v. גם צריך לדעת I.

$\lambda = \sqrt{\frac{120,000}{(200\times10^9)(2.896\times10^{-6})}} = .4552$, $T_E = \pi^2\frac{EI}{L^2} = 135164N$. $T < 135164N$.

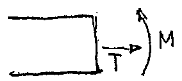
$$v|_{x=L/2} = P\left\{\frac{0.7371}{2(120,000)(1.2423)(.4552)} - \frac{3}{4(120000)}\right\}$$

$$= P(8.19\times10^{-7} m)$$

לכן המומנט יהיה $M = -\frac{PL}{4} + T\cdot P\times 8.19\times10^{-7}$

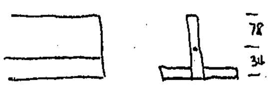
$$M = P[-0.75 + .09828] = -.65172P$$

המאמצים במקום שהמומנט מקס.



$$\sigma = T/A + \frac{Mc}{I}$$

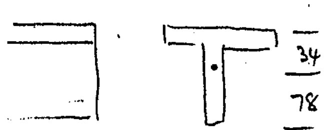
אם צורת החתך כמו מצורלל שמאלה:



$\sigma_{עליון} = \frac{120,000}{2400\times10^{-6}} - \frac{0.65172(.078)P}{2.896\times10^{-6}} = -130\times10^6$ $\quad$ $P = 10,250\ N$

$\sigma_{תחתון} = \frac{120,000}{2400\times10^{-6}} + \frac{0.65172(.034)P}{2.896\times10^{-6}} = 130\times10^6$ $\quad$ $P = 10,451N$

אם צורת החתך כמו מצורלל שמאלה:

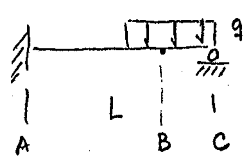


$\sigma_{עליון} = \frac{120,000}{2400\times10^{-6}} - \frac{0.65172(.034)P}{2.896\times10^{-6}} = -130\times10^6$ $\quad$ $P = 23,515N$

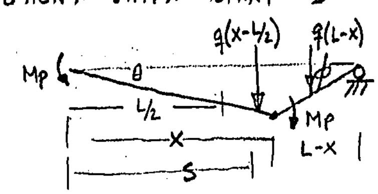
$\sigma_{תחתון} = \frac{120,000}{2400\times10^{-6}} + \frac{0.65172(.078)P}{2.896\times10^{-6}} = 130\times10^6$ $\quad$ $P = 4556N$

לכן, העומס הגבוה ביותר בא כאשר לשטח החתך יש צורה כזה ⊥

11.26



כאשר שהמומנט מגיע ל-M_p, פרק פלסי קורה בסמך הרתום וגם בנקודה B
צריך למצוא את מקום נקודה B ומהוא העומס המפורס שגורם ל-M_p



$$\theta x = \phi(L-x)$$

$$\frac{\theta x}{L-x} = \phi$$

$$s = \frac{1}{2}(x+L/2)$$

לפי עקרון עבודה וירטואלית

$$M_p\theta + M_p(\theta+\phi) = q(x-L/2)\theta s + q(L-x)\left(\frac{L-x}{2}\right)\phi$$

מחליפים ϕ ו-s ביחסים ופותרים בשביל q

$$q = \frac{8M_p(2L-x)}{(5Lx-4x^2-L^2)L}$$

כדי למצוא את ה-q שגורם ל-M_p, גוזרים q לגבי x
ומקבלים ש- $0 = 4x^2-16Lx+9L^2$. מצה יש פשובת אחת

לפי נוסחאות שקבלנו בנושא הקריסה, כל פעם צריך לפתור

$$v = A\cos\lambda x + B\sin\lambda x + Cx + D$$

לפי תנאי השפות של הקורה.

כדי לקצר את הדרך לפתור את הבעיה נזכיר ש-

$$EI\frac{d^2v}{dx^2} + Pv = M_t$$

כאשר $P=0$

$$EI\frac{d^2v_t}{dx^2} = M_t$$

פה M_t הוא המומנט שבא מעומסים אנכיים בלבד, ו- v_t היא התזוזה בכוון y שבאה מהעומסים האנכיים. מניחים שצורת עקום התזוזה יהיה אותו דבר <u>עם או בלי</u> הכוח האופקי P.

לכן, לפי תנאי השפה, כמו קורה על סמך נייד וסמך נייד, מניחים ש- $v = \bar{v}\sin\frac{\pi x}{L}$
ו- $v_t = \bar{v}_t \sin\frac{\pi x}{L}$. $\bar{v}$, ו- $\bar{v}_t$ הם ה-AMPLITUDE של v ו- v_t. לפי הנוסחאות למעלה

$$-EI\frac{\pi^2}{L^2}\bar{v} + P\bar{v} = -EI\frac{\pi^2}{L^2}\bar{v}_t$$

מפני ש- $P_E = \frac{\pi^2 EI}{L^2}$, $\bar{v}(P-P_E) = -P_E\bar{v}_t$ או

$$\bar{v} = \frac{\bar{v}_t}{1 - P/P_E}$$

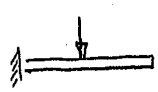
המומנט גם כן עוקב אחרי נוסחה דומה לזה

$$M = \frac{M_t}{1 - P/P_E}$$

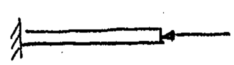
אם P חיובי, $\bar{v}_t < \bar{v}$ ו- $M_t < M$. אם P שלילי, $\bar{v}_t > \bar{v}$ ו- $M_t > M$

<u>TACHLIS</u>

פותר



וקבל את v מתחת העומס או בכל מקום אחר. קורא לזה $\bar{v}_t$.

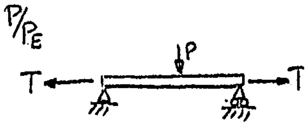


קבל P_E לקורה עמוסה כמו בצורה

v יהיה

$$v = \frac{\bar{v}_t}{1 - P/P_E}$$

בתרגיל בית 12.10



$T_E = \frac{\pi^2 EI}{L^2}$, $\bar{v}_t\Big|_{x=L/2} = \frac{PL^3}{48EI}$, ו- $v = \frac{\bar{v}_t}{1 + T/T_E}$

כי P היא שלילי.

1. Write the second-order differential equation for the bending of the column shown in Fig. P1-2 and use it to determine the critical load of the column. At its lower end the column is completely fixed. At the upper end the column is prevented from rotating, but free to translate laterally. (ans: $P_{cr}=\pi^2 EI/L^2$)

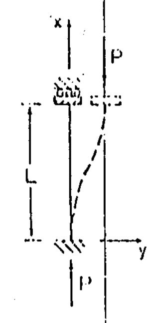
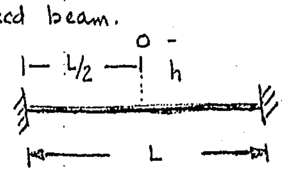


Fig. P1-2

2. Find an expression for the maximum stress when a ball weighing W Newtons is dropped onto a fixed-fixed beam.



3. A linearly elastic beam-column having a flexural rigidity EI, is subjected to a thrust P and a moment M_0 as shown in Fig. A below.

(a) Determine the lateral displacement v(x).

(b) From part (a), write the solution for the system subjected to a force P acting as shown in Fig. B.

(c) Determine Δ_c, the horizontal displacement of point C, assuming small rotations. (Assume also that the horizontal displacement of point B is negligible).

(d) Determine the bending moment M(x).

(g) Explain why, although the beam is made of a linearly elastic material, the results of part (c) are non-linear.

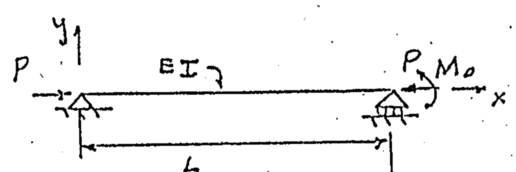


Fig. A

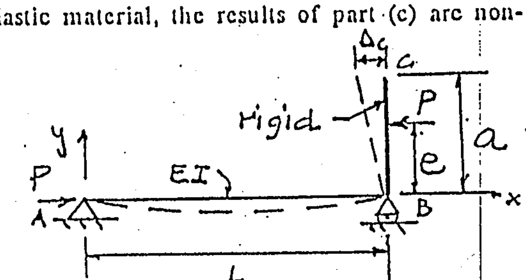


Fig. B

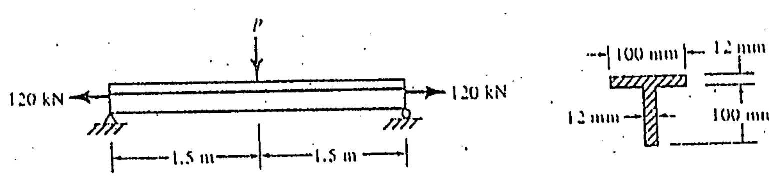
Answers :

(a) $y(x) = -\frac{M_0}{P}\left[\frac{\sin Kx}{\sin KL} - \frac{x}{L}\right], \; K^2 = \frac{P}{EI}$

(c) $\Delta_c = \frac{ac}{L}\left[1 - L\sqrt{P/EI}\cot\left(L\sqrt{P/EI}\right)\right] = \frac{ac}{L}(1 - KL\cot KL)$

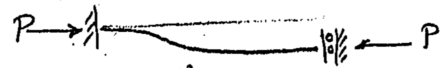
(d) $M(x) = M_0 \sin kl/\sin kL$

*12.10 A T section carries an axial tensile force of 120 kN, applied through the centroid of the cross section. The allowable stress in tension or compression is 130 MPa. Let $E = 200$ GPa. What transverse force P can be applied at midspan if the beam is
(a) Stem down (as shown)?
(b) Stem up?



PROBLEM 12.10

1.



מהנוסחה שמצאנו בשעור שעבר והפיתרון הוא

$$EI\frac{d^4v}{dx^4} + P\frac{d^2v}{dx^2} = 0$$

$$v = A\cos\lambda x + B\sin\lambda x + Cx + D$$

$$\lambda^2 = P/EI$$

תנאי הקצוות

① תזוזה $v(x=0)=0$

② זווית $\frac{dv}{dx}(x=0)=0$

③ זווית $\frac{dv}{dx}(x=L)=0$

④ גזירה: $EI\frac{d^3v}{dx^3}(x=L) + P\frac{dv}{dx}(x=L) = 0$

מפני ש $\frac{dv}{dx}=0$ ב־$x=L$, לכן ④ נותנת $\frac{d^3v}{dx^3}=0$ ב־$x=L$

מקבלים

$$A+D=0 \qquad \text{①-מ}$$

$$\lambda B + C = 0 \qquad \text{②}$$

$$-A(\lambda\sin\lambda L) + B(\lambda\cos\lambda L) + C = 0 \qquad \text{③}$$

$$A\lambda^3\sin\lambda L - B\lambda^3\cos\lambda L = 0 \qquad \text{④}$$

לכן

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \lambda & 1 & 0 \\ -\lambda\sin\lambda L & \lambda\cos\lambda L & 1 & 0 \\ \lambda^3\sin\lambda L & -\lambda^3\cos\lambda L & 0 & 0 \end{bmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

כדי לקבל תוצאות לא טריוויאליות הדיטרמיננט צריך להיות אפס. לפי העמוד הרביעי

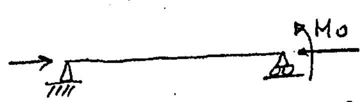
$$-1\begin{pmatrix} 0 & \lambda & 1 \\ -\lambda\sin\lambda L & \lambda\cos\lambda L & 1 \\ \lambda^3\sin\lambda L & -\lambda^3\cos\lambda L & 0 \end{pmatrix} = 0$$

לכן

$$\lambda^4\sin\lambda L = 0$$

$\lambda \neq 0$ (זה הפיתרון $v=0$) ; לכן $\sin\lambda L = 0 \Leftarrow \lambda L = n\pi$ או $P_E = \left(\frac{\pi}{L}\right)^2 EI$

3.



$$v = A\cos\lambda x + B\sin\lambda x + Cx + D \qquad \lambda^2 = P/EI$$

לפי הסכם הסימנים

① $v(x=0)=0$ ③ $v(x=L)=0$

② $EI\frac{d^2v}{dx^2}(x=0) = M = 0$ ④ $EI\frac{d^2v}{dx^2}(x=L) = -M_0$

$$A+D=0 \qquad \text{①-מ}$$

$$-A\lambda^2 = 0 \qquad \text{②}$$

$$A\cos\lambda L + B\sin\lambda L + CL + D = 0 \qquad \text{③}$$

$$EI(-A\lambda^2\cos\lambda L - B\lambda^2\sin\lambda L) = -M_0 \qquad \text{④}$$

אם $\lambda \neq 0$ $\Leftarrow$ $A \overset{(2)}{=} 0$ $\Leftarrow$ $D \overset{(1)}{=} 0$ $\Leftarrow$ $B\sin\lambda L + CL \overset{(3)}{=} 0$ $\Leftarrow$ $B(EI\lambda^2\sin\lambda L) \overset{(4)}{=} M_0$ $\Leftarrow$ $B = \frac{M_0}{EI\lambda^2\sin\lambda L}$

$C = -\frac{M_0}{EI\lambda^2 L}$ לכן,

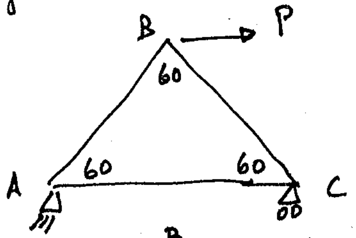
$$v = \frac{M_0}{EI\lambda^2}\left(\frac{\sin\lambda x}{\sin\lambda L} - \frac{x}{L}\right)$$

$$\boxed{v = \frac{M_0}{P}\left(\frac{\sin\lambda x}{\sin\lambda L} - \frac{x}{L}\right)}$$

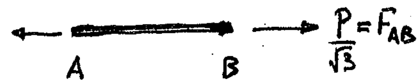
לקבל $M(x)$, גוזר פעמיים לפי x: $EI\frac{d^2v}{dx^2} = M = -EI\,M_0\lambda^2\sin\lambda x$ ...

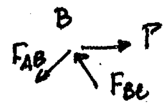
הסימן נכון לפי הסכם ...

For example



find forces in AB, BC, AC





$F_{AB} = F_{BC}$ from $\Sigma F_y = 0$

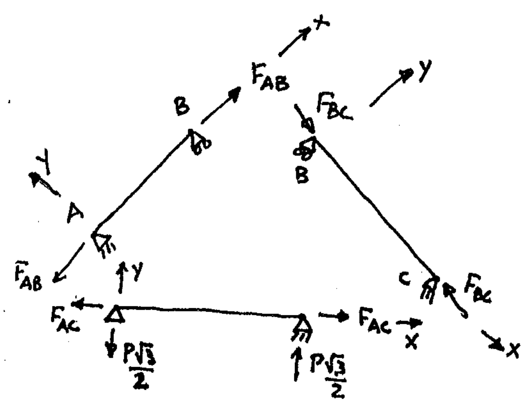
$\Sigma F_x = -2 \cdot \frac{\sqrt{3}}{2} F_{AB} + P = 0$ $\qquad F_{AB} = \frac{P}{\sqrt{3}}$. Now treat each member as if pinned for buckling in x-y plane!

You set up a local coord. system.

what must P be for BC to buckle?

$$F_{BC} = P_{cr} = \frac{P}{\sqrt{3}} = \frac{\pi^2 E I_{zz}}{L_{eBC}^2} \quad \text{or} \quad \frac{P}{\sqrt{3}} = \frac{\pi^2 E I_{yy}}{L_{eBC}^2}$$

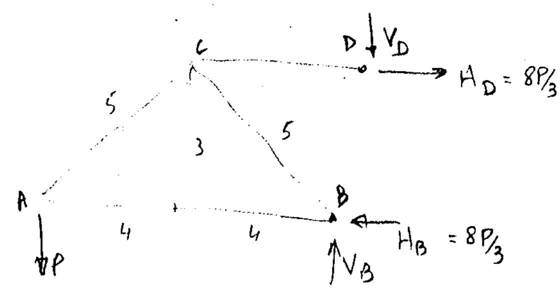
- whichever P is smaller will be the maximum load required to buckle BC
- You must also check to see if AC is in compression and do the same as above to find the P that will cause it to buckle. The minimum value of all the P's you get will be the largest P you can place on the structure.
- Note beams, bars that are pulled do not buckle but fail in maximum direct stress.



here each beam has its local coordinate system

- since AB & AC have positive axial forces they won't buckle
- we normally consider buckling in the x-z plane as fixed-fixed since the sides of the supports act as fixed supports so check

$$\frac{P}{\sqrt{3}} = F_{BC} = P_{cr} = \frac{4\pi^2 E I_{yy}}{L_{BC}^2} \quad \text{for out of plane buckling}$$

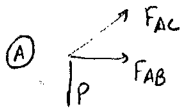


$\Sigma F_x = 0 \quad H_D - H_B = 0 \quad H_D = 8P/3$ (2)

$\Sigma F_y = 0 \quad V_B - V_D - P = 0$

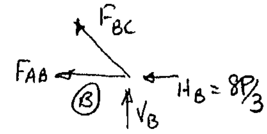
$+\circlearrowleft \Sigma M_D = 0 \quad H_B \cdot 3 - P \cdot 8 = 0 \quad H_B = \frac{8P}{3}$ (2)

(A)



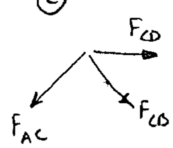
$F_{AC} \cdot 3/5 - P = 0 \quad \frac{5}{3}P = F_{AC}$ (2)

$F_{AC} \cdot \frac{4}{5} + F_{AB} = 0 \quad F_{AB} = -\frac{4}{5}F_{AC} = -\frac{4}{5}\cdot\frac{5}{3}P = -\frac{4}{3}P$ (2)



$V_B = P$ (2)

(C)

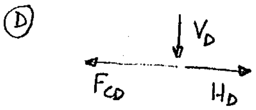


$F_{CD} - F_{AC} \cdot 4/5 + F_{CB} 4/5 = 0$

$F_{AC} \cdot 3/5 + F_{CB} \cdot 3/5 = 0 \quad F_{AC} = -F_{CB} \quad F_{CB} = -5P/3$ (2)

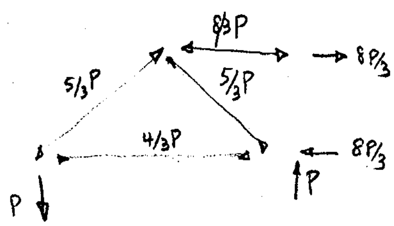
$F_{CD} - \frac{8F_{AC}}{5} = 0 \quad F_{AC} = \frac{5}{8}F_{CD} = \frac{5}{8}\cdot\frac{8}{3}P = \frac{5P}{3}$ (2)

(D)



$-F_{CD} + H_D = 0 \quad F_{CD} = 8P/3 = H_D$ (2)

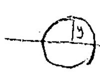
$V_D = 0$ (2)



$F_{crit} = \frac{4}{3}P = \frac{\pi^2 EI}{l^2}$ (2) in AB (2)

$P = \frac{3}{4}\pi^2\frac{EI}{l^2} = \frac{\frac{3}{4}\pi^2\left(29\times10^6 \frac{lb}{in^2}\right)\left(\frac{\pi}{4}(1.5)^4\right)}{(8\cdot12)^2}$

$= 92613$ lb (2)



$I = \int y^2 dA = \int r^2 \sin^2\theta \cdot r\,dr\,d\theta$

$= \int r^3 dr \int \sin^2\theta\, d\theta$

$= r^4/4 \Big|_0^R \; \theta/2 - \frac{\sin 2\theta}{4}\Big|_0^{2\pi} = \frac{R^4}{4}\pi$ (2)

($\sin^2\theta = \frac{1-\cos 2\theta}{2}$)

$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{4/3 P}{\pi r^2} = \frac{4/3(92613)}{\pi(1.5)^2} = 17470 \text{ psi} < \sigma_y$ (2)

since $|F_{BC}| > |F_{AB}|$ F_{AB} buckles first

$\sigma_{CD} = \frac{F_{CD}}{A} = \frac{2F_{AB}}{A} = 34939 \text{ psi} < \sigma_y$

(2)

out of plane buckling $F_{crit} = 4/3P = 4\pi^2 EI/L^2$ or $P_{FF} = 3\pi^2 EI/L^2 = 370452 \text{ lb} > P_{s.s.}$ (2)

For simultaneous buckle & yield

yield at CD first

$\sigma_y = \frac{F_{CD}}{A} = \frac{8/3 P_{crit}}{\pi r^2} = \frac{8/3\left[3/4\, \pi^2 E \frac{\pi r^4}{L_{AB}^2\, 4}\right]}{\pi r^2} = \frac{\pi^2 E r^2}{2L_{AB}^2}$ (2)

$36000 = \frac{\pi^2 E r^2}{2L_{CD}^2} \qquad r = \sqrt{\frac{36000 \cdot 2(8\cdot12)^2}{29\times10^6\cdot\pi^2}} = 1.523$ in

$r = \sqrt{\frac{\sigma_y \cdot 2L_{AB}^2}{\pi^2 E}} \qquad d = 3.045$ in (2)

and $P_{crit} = \frac{3}{4}\frac{\pi^2 EI}{L^2} = \frac{\frac{3}{4}\pi^2\left(29\times10^6\frac{lb}{in^2}\right)\left(\frac{\pi}{4}(1.523)^4\right)}{(8\cdot12)^2} = 98325$ lb (2)

תורת החוזק 2 מועד ב | מבחן סופי | אפריל 3 2000

פרופ עזרא לוי

אורך המבחן הזה יהיה 180 דקות. במבחן יש ארבעה שאלות. עליכם לענות לכל השאלות.
רק להשתמש ברשמות הכיתה, תרגילי בית שלכם, ומחשבון. ואם יש דברים הנתונים לכם עם המבחן בלבד.

לחתום על סעיף הבא בבקשה:
לאורך המבחן אני לא רשאי(ת) לקבל עזרה מאף אחד או לתת עזרה לאף אחד. אם אני אעבור על אלה, אני אכשול את המבחן.

חתימת הסטודנט________________________

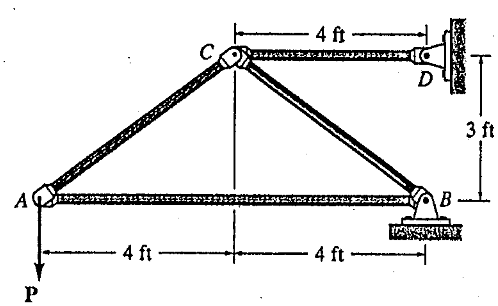
תרגיל מס. 1 (40)

מסבך מוטות, עשוים מפלדה, מעומס בכוח P כמשורטט בציור. אם לכל מוט יש שטח החתך עגול בקוטר של 3 in.

א. מיצאו את הכוח שיגרום לקריסה ובאיזה מוט זה יקרה.
ב. מה צריך להיות הקוטר כדי שיהיה קריסה וכניעה בו-זמנית במסבך.

חשבו את P לפי התנאים הבאים: כל המוטות מחוברות בסיכה בשני הצדדים נגד קריסה בכוון x, ורתומים בשני הצדדים נגד קריסה בכוון z.

הניחו ש- $E = 29 \times 10^6$ lb/in^2 ו- σ_{yp}=36000 lb/in^2. המדדים של הקורות נתונים בצורה, ויש לזכור ש- 12 in.= 1 ft.



Florida International University
Department of Mechanical and Materials Engineering

EMA 3702 QUIZ 1A February 19, 2009

You are allowed two sheets of 8 ½ x 11 inch paper with whatever you wish on the sheet

Print your name and sign the following statement:

I will not give nor take any unpermitted aid during this quiz. I understand that violation of this statement will lead to automatic failure of the quiz.

____________________ PRINT NAME ____________________ SIGN NAME

Problem.

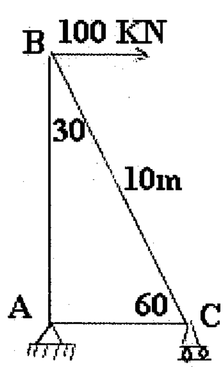
a) For the truss shown, find the maximum elongation that the bar BC will see if the bar is a solid cylinder of 12.5 cm diameter and the Young's Modulus, E = 206 GPa.

b) Link AB is connected to the support at A and to Link BC by 1 cm diameter pins. All the pins are in single shear with ultimate stress in shear of 200 MPa. Determine the shearing stress in the pins and whether the pins will fail.

c) For part b) if the pins don't fail, what is the safety factor?

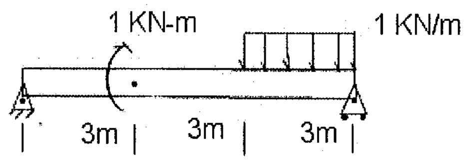
d) Find the strains in the non-loaded directions for AC when, on top of the axial load, the truss sees a temperature **increase** ΔT of 40 °C.

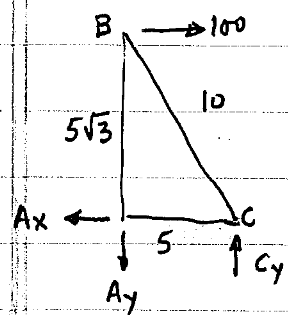
You are given that the shear modulus, G = 78 GPa and the coefficient of thermal expansion, α, is 12.8×10^{-6} in/in-°C

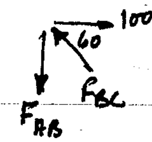


a) For the following beam determine the shear and moment diagram.

b) If the beam is 50 cm high and 50 cm wide, determine the bearing stress on the pin, if the pin has a 5 cm diameter and the Young's modulus is 200 GPa







$-F_{BC}\cos 60 + 100 = 0$

$F_{BC} = 200$ kN ..

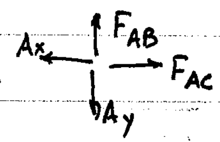
$F_{BC}\cos 30 - F_{AB} = 0$

$F_{AB} = 173.2$ kN ..

$\Sigma F_x \quad A_x = 100$ kN

$\Sigma F_y \quad A_y = C_y$

$\Sigma M_A \circlearrowleft + = 0 \qquad C_y \cdot 5 - 100(5\sqrt{3}) = 0 \qquad C_y = 173.2\text{ kN} = A_y$

@A 

$F_{AC} = A_x = 100$ kN ..

$F_{BA} = A_y = 173.2$ kN ..

L_{AB}, ~~L_{BC}~~, L_{AC} (2)

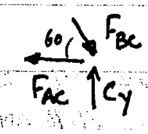
ⓐ F_{BC}(2), A(2), δ(2)

ⓑ R(2), A(2), τ(2), fail (1), same (1)

ⓒ 1 No FS

ⓓ F_{AB}(2), A_x(2), ϵ_{AC}(2), ν(2), ϵ_{TOT}(2)

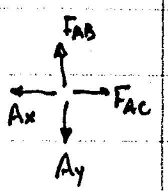
27

@ C 

$F_{BC}\cos 60 = F_{AC} \qquad F_{BC} = 200$ kN

$A_{BC} = A_{AC} = A_{BA} = \pi d^4/4 = \pi(.125)^2/4 = .01227\ m^4$ ..

$\delta_{BC} = \dfrac{F_{BC} L_{BC}}{A_{BC} E} = \dfrac{-200\times 10^3 (10)}{.01227 \times 206\times 10^9} = -7.91\times 10^{-4}\ m = -.791$ mm ..



b) at A Total Force $= \sqrt{F_{AB}^2 + F_{BC}^2} = 200$ kN ..

$\tau = \dfrac{P}{A}$. $\qquad A = \dfrac{\pi d^2}{4} = \dfrac{\pi(.01)^2}{4} = 7.854\times 10^{-5}\ m$ ..

$= \dfrac{200\times 10^3}{7.854\times 10^{-5}} = 2.5465\times 10^9$ Pa

$= 2.5465$ GPa will fail .

@ B Total force 200kN same result

c) no FS .

d) ~~$\epsilon_x = \frac{\delta}{L} = \frac{-.791\times 10^{-3}\ m}{m}$~~ $\quad \epsilon_{AC} = \dfrac{F_{AC}}{A_{AC} E} = 3.956\times 10^{-5}$ ..

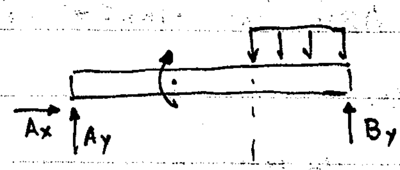
$G = \dfrac{E}{2(1+\nu)}$

$\nu = \dfrac{E}{2G} - 1 =$ .3205 ..

$\epsilon_{unloaded} = -\nu\,\epsilon_{AC} = -1.268\times 10^{-5}$ ..

$\epsilon_{TOT} = \epsilon_{unloaded} + \alpha\Delta T$

$= -1.268\times 10^{-5} + 12.8\times 10^{-6}(40) = 4.99\times 10^{-4}$ ..



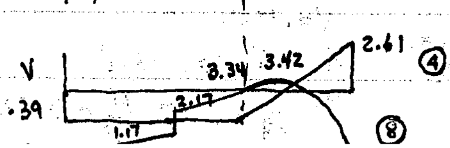
$\Sigma F_x = 0 = A_x$.

$\Sigma F_y = A_y + B_y - 1\times 3 = 0$ ...

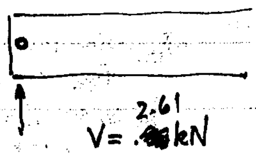
$\circlearrowleft + \Sigma M_A = -1\text{kN·m} + B_y \cdot 9 - 1\times 3\times 7.5 = 0$ ...

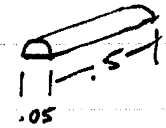
$B_y = \dfrac{23.5}{9} = 2.61$ kN

$A_y = .39$ kN .



$\dfrac{.39\times .39}{2} =$





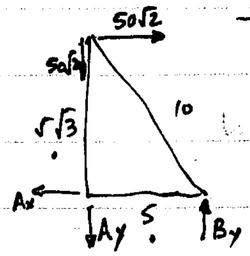
$V = 2.61\ kN$ ..

bearing stress

$A_{pin} = .05(.5) = .025\ m^2$ ..

$\tau = \frac{V}{A} = \frac{2.61\ kN}{.025\ m^2} = 104440 \frac{N}{m^2} = 104.4\ kPa$ ..

27 pts



$\Sigma F_x = 50\sqrt{2} - A_x = 0 \quad A_x = 70.71\ kN$

$\Sigma F_y = B_y - A_y - 70.71 = 0$

$\circlearrowleft \Sigma M_A = B_y \cdot 5 - 50\sqrt{2} \cdot 5\sqrt{3} = 0 \quad B_y = 50\sqrt{6} = 122.47\ kN$

$A_y = 51.76\ kN$

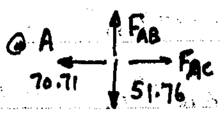
$A_{BC} = \frac{\pi d^2}{4} = \frac{\pi}{4}(.125)^2 = .01227\ m^2$ ..

$\delta_{BC} = \frac{F_{BC} \cdot L_{BC}}{A_{BC} E} = 5.595 \times 10^{-4}\ m$ ..

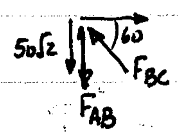
$= .5595 \times 10^{-3}\ m$

$= .56\ mm$

@ A



$A_{pin} = \frac{\pi d^2}{4} = \frac{\pi (.01)^2}{4} = 7.854 \times 10^{-5}\ m^2$ ..



$-F_{BC} \cos 60 + 50\sqrt{2} = 0 \quad F_{BC} = 100\sqrt{2} = 141.42\ kN$ ..

$F_{BC} \sin 60 - F_{AB} - 50\sqrt{2} = 0 \quad F_{AB} = 51.76$

$R = F_{BC}$ .. @ B double shear $\tau = \frac{R}{A} = \frac{141.42\ kN}{2 \times 7.854 \times 10^{-5}} = 900.31\ MPa$ ~~does not fail~~ fails

$R = \sqrt{A_x^2 + A_y^2}$ .. @ A $\tau = R/A = \frac{87.63\ kN}{2 \times 7.854 \times 10^{-5}} = 557.87\ MPa$ ~~does not fail~~ fails

~~$FS_B = \frac{200}{90.031} = 2.22$~~ .. ~~$FS_A = 3.585$~~ .. ~~FS is 2.22~~ . no

$\epsilon_{AC} = \frac{F_{AC}}{A_{AC} E} = \frac{70.71\ kN}{.01227 \cdot 206 \times 10^9} = 2.798 \times 10^{-5}$

$G = \frac{E}{2(1+\nu)}$

$1 + \nu = \frac{E}{2G}$

$\nu = \frac{E}{2G} - 1 = .3205$

$\epsilon_{mech} = -\nu \epsilon_{AC} = -.3205(2.798 \times 10^{-5}) = -8.966 \times 10^{-6}$

$\epsilon_{TOT} = \epsilon_{mech} + \alpha \Delta T = -8.966 \times 10^{-6} + 12.8 \times 10^{-6}(40) = 5.03 \times 10^{-4}$

L_{AB}, L_{AC} (2)

(a) F_{BC}, A, δ (2+2+2)

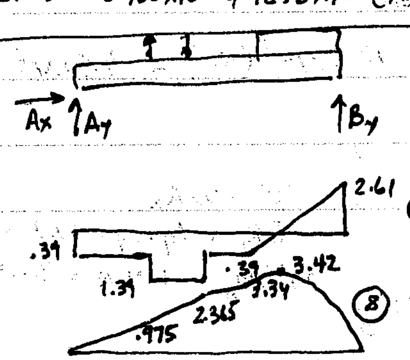
(b) R, A, τ, ~~no fail~~ ~~(2+2+2+1)~~ 2+2+2+1

R, ~~A~~, τ, no fail ~~(2+2+2+1)~~ 1

(c) FS ~~2, 2, 1~~

(d) F_{AC}, ϵ_{AC}, ϵ_{mech}, ϵ_{TOT}, ν (2)×5 /27

27



$\Sigma F_x = A_x = 0$

$\Sigma F_y = B_y + A_y - 1 + 1 - 3 = 0$ ...

$\circlearrowleft \Sigma M_A = 1 \times 2.5 - 1 \times 3.5 + 3 \cdot 7.5 + B_y \cdot 9 = 0$ ....

$B_y = \frac{23.5}{9} = 2.$

$A_y = .39$

$V = .39\ kN$ ..

$A = .05(.5) = .025\ m^2$

$\tau = V/A = 15520 \frac{N}{m^2} = 15.52\ kPa$ ..

28

$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$ $\quad \sigma_x = \frac{30000}{.1 \times .01} = 30\ MPa$ .. $\quad G = \frac{E}{2(1+\nu)}$

$= \frac{30\ MPa}{206\ GPa} - .3205 \frac{24\ MPa}{206\ GPa} - 0$ $\quad \sigma_y = \frac{30000}{.125 \times .01} = 24\ MPa$ .. $\quad \nu = \frac{E}{2G} - 1 = .3205$ ..

$\sigma_z = 0$ ..

$= .14563 \times 10^{-3} - .03734 \times 10^{-3}$

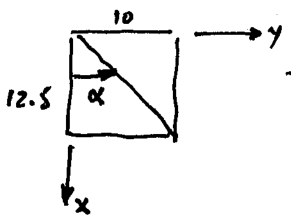
$\epsilon_x = .10829 \times 10^{-3}$ ..

$\delta_{AB} = \epsilon_x \cdot L_{AB} = 1.354 \times 10^{-5}\ m$ .. (12)

$\gamma_{xy} = 1.5^\circ \times \frac{\pi}{180^\circ} = .0262$ .. $\quad G\gamma_{xy} = \tau_{xy} = 2.042\ GPa$ ..

$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha = \frac{30+24}{2} + \left(\frac{30-24}{2}\right)\cos(77.32^\circ) + 204.2 \sin(77.32^\circ)$

$27 + 3 \cdot (.2195) + 2042\ (.9756)$



$\tan\alpha = \frac{10}{12.5}$ $\quad \alpha = 38.66^\circ$ ..

$= 2.02\ GPa$ ..

$\tau_n = \tau_{xy}\cos 2\alpha - \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\alpha = 2042\ (.2195) - 3 \cdot (.9756) = 445.29\ MPa$ .. (10)

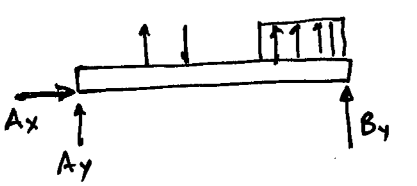
$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 2.042\ GPa$ ..

$\tan 2\alpha_\epsilon = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = \frac{3}{-4084} = -.042^\circ$

$\alpha_\epsilon = -.042^\circ,\ 89.958^\circ$ .. (4)

$\epsilon_{z_{mech}} = \frac{\sigma_z}{E} - \frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} = -\frac{\nu}{E}(54\ MPa) = -\frac{.3205}{206 \times 10^9}(54 \times 10^6) = -8.4 \times 10^{-5}$ ..

$\epsilon_{z_{TOT}} = \epsilon_{z_{mech}} + \alpha\Delta T = -8.4 \times 10^{-5} + 12.8 \times 10^{-6}(-20) = -.00034$ .. (4)



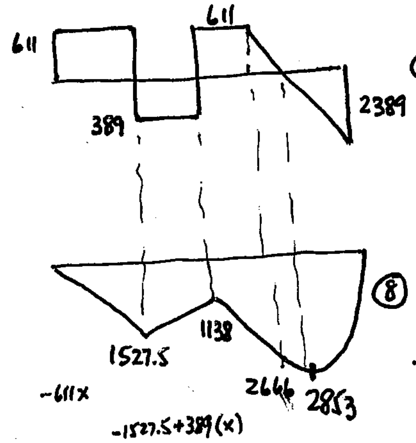
$\Sigma F_x = 0 = A_x$ (2)

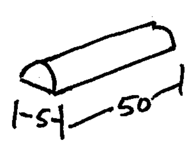
$\Sigma F_y = A_y + B_y + 1 - 1 + 3 = 0$ (4)

$\circlearrowleft \Sigma M_A = 0 = 1 \cdot 2.5 - 1 \cdot 3.5 + 3 \cdot 7.5 + B_y \cdot 9$ (4)

$= 21.5 + B_y \cdot 9 = 0$ $\quad B_y = -\frac{21.5}{9} = -2389\ N$ (1)

$A_y = -611\ N$ (1)





$A = .05(.5) = .025\ m^2$ ..

@A $\sigma_b = \frac{611}{.025} = 24440\ \frac{N}{m^2} = 24.44\ kPa$ ..

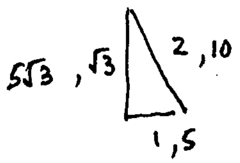
@B $\sigma_b = \frac{2389}{.025} = 95560 = 95.56\ kPa$ ..

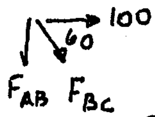
ultimate stress must be

$95.56\ kPa \times 2.5 = 238.9\ kPa$ ..

(2)

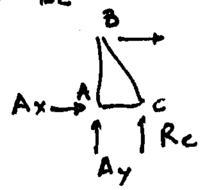
30 TOT





$-F_{BC} \cdot \cos 60 = 100$ $\quad F_{BC} = -200$ kN

$F_{BC} \sin 60 + F_{AB} = 0$ $\quad F_{AB} = 173.2$ kN



$R_c \cdot 5 - 100 \cdot 5\sqrt{3} = 0$

$R_c = 173\text{kN} = -A_y$

$A_x = -100\text{kN}$

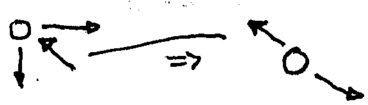
$A_{BC} = \frac{\pi d^2}{4} = \frac{\pi(.025)^2}{4} = \frac{\pi}{256}\ m^2$

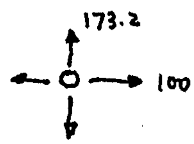
$= .01227$

$\sigma_{BC} = \frac{-200{,}000}{\pi/256} \cong \frac{-200\ MPa}{12.5} \cong -163 MPa$

$\epsilon = \frac{\sigma}{E} \cong \frac{-163 \times 10^{+6}}{206 \times 10^9} \cong -.079 \times 10^{-3}$

$\Delta u = \epsilon \cdot \ell_{BC} \cong -.79 \times 10^{-3}\ m = -.79\ mm$





$\sigma_{AC} = \frac{100\text{kN}}{A_{AC}} = \frac{100{,}000}{\pi/256} \cong 8\ MPa$

$\epsilon_{AC} = \frac{\sigma_{AC}}{E} = +.035 \times 10^{-3}$

$\nu = \frac{E}{2G} - 1$

$= \frac{206}{156} - 1 = .33$

$\epsilon_{NL} = -\nu \epsilon_{load}$

$\epsilon_{TOT} = \epsilon_{NL} + \epsilon_T = -\nu \epsilon_{AC} + \alpha \Delta T$

$= -\frac{1}{3}(+.035 \times 10^{-3}) + 12.8 \times 10^{-6}(40)$

Florida International University
Department of Mechanical and Materials Engineering

EMA 3702 QUIZ 1B February 23, 2007

You are allowed two sheets of 8 ½ x 11 inch paper with whatever you wish on the sheet

Print your name and sign the following statement:

I will not give nor take any unpermitted aid during this quiz. I understand that violation of this statement will lead to automatic failure of the quiz.

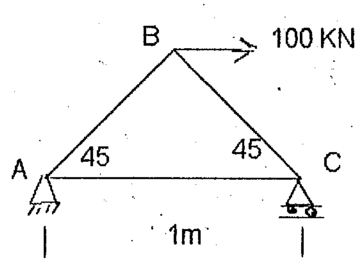
PRINT NAME SIGN NAME

Problem.

a) For the truss shown, find the maximum elongation that the bar AB will see if the bar is a solid cylinder of 10 cm diameter and the Young's Modulus, E = 206 GPa.

b) If the axial direction is called the "x axis", find the strains in the "y- and z-directions" for AB when, on top of the axial load, the truss sees a temperature **increase** ΔT of 50 °C.

You are given that the shear modulus, G = 78 GPa and the coefficient of thermal expansion, α, is 12.8×10^{-6} mm/mm-°C



Faculty Council on Governance

College of Engineering and Computing

Florida International University

AGENDA

Meeting Place & Time: EC-2300 1:00pm
Date: Wednesday, February 14, 2007

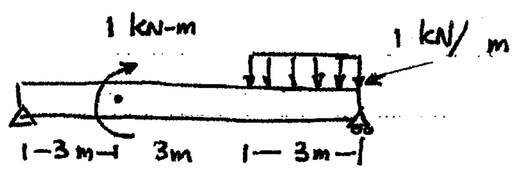
Estimated Start Time	Time allowed	Item
1:00	1	1. Approval of the Agenda
	1	2. Approval of the Minutes of the previous meeting
1.03	15	3. Standing Committee Reports 3a. Curriculum Committee (Chair or representative) 3b. Library Committee (Chair or representative) 3c. Tenure and Promotion Committee (Chair or representative)
1:18	45	4. Items brought forward from last meeting: Update, Discussion and possible Vote 4a. Department vs. College Policies item 4b. Departmental Restructuring 4c. Chair and Administrative Complaints 4d. Course Load Assignment
2:03	57	5. New Business 5a. College Strategic Plan- have dean speak or representative speak (20 minutes) Discussion 25 minutes max 5b. Appointment of Committee on written subcommittee policies item 5c. Appointment of Committee on Research facilities item
3:00	0	6. Adjournment

All are welcome to participate in the proceedings

Problem 3

For the following loaded beam, determine the location and magnitude of the maximum shear stress and maximum direct stress using the following formulas.

The maximum shear stress is given by $\tau = 5V$ where the shear stress is in MPa ($=10^6$ N/m^2) and the shear V is in KN
The maximum direct stress is given by σ=3M where the direct stress is in MPa ($=10^6$ N/m^2) and the moment M is in KN-m



College of Engineering & Computing
Curriculum Committee

BERRIN TANSEL (CHAIR)
JOSE MITRANI(VICE CHAIR)
REBECCA ANDERSON (SECRETARY)
BILL KRAYNEK (SECRETARY)
JOE CHOW
YIDING CAO
HAO ZHU

WWW.ENG.FIU.EDU/CURRICULUM

Report for College Curriculum Committee

February 14, 2007

Prepared by: Dr. Berrin Tansel, Chair, College Curriculum Committee

Meetings:

The Curriculum Committee met 8 times during this academic year, until now.

Minutes of the meeting are distributed to all faculty and chairs.

Approvals

The Committee approved:

19	New course proposals,
10	Course change proposals
1	New degree program proposal
1	New minor program proposal,
3	Proposals for unit specific graduate admission standards,
4	Catalog change proposals

Issues:

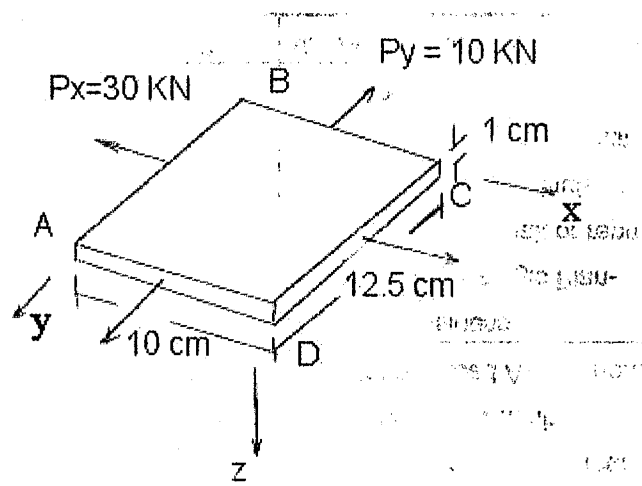
Approved courses are not finalized by faculty in the official forms. This is preventing these courses from being officially forwarded to the University Curriculum Committee for final approval.

Problem 2

a) For the plate shown, find the maximum stresses (σ_{max}, τ_{max}) if the plate is acted upon by the loads shown and has the dimensions shown. The plate has a Young's Modulus, E = 100 GPa.

b) What are the directions that will produce these stresses? Show the directions clearly on a diagram of the plate.

You are given that the shear modulus, G = 38 GPa and the coefficient of thermal expansion, α, is 12.8×10^{-6} in/in-°C



Faculty Council on Governance

College of Engineering and Computing

Florida International University

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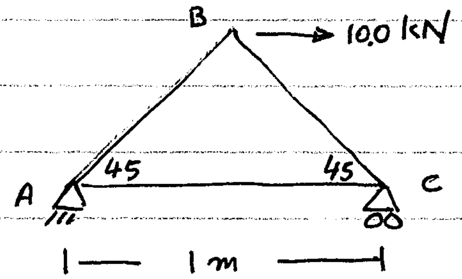


$\frac{F_{AB}}{\sin 45} = \frac{10}{\sin 90} = \frac{F_{BC}}{\sin 45}$

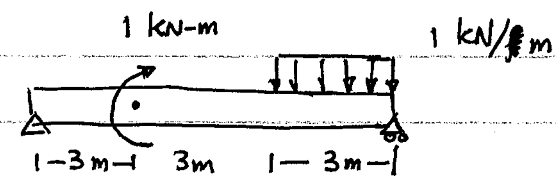
$F_{AB} = 7.07 = -F_{BC}$

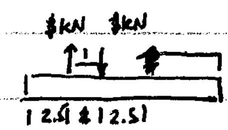
$\tau = \frac{-7.07}{2\pi\frac{(.02)^2}{4}}$

For the truss shown, find the maximum shear stress in pin C assuming double shear conditions. Assume pin is 2 cm in diameter

find the maximum (normal or shear) stress in AB assuming that A & B are double & single shear type pins of diameter 2cm respectively

$\tau_A = \frac{1}{2}\tau_B =$ σ

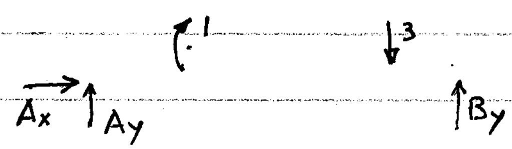


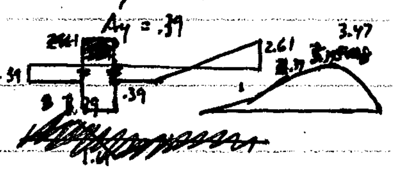


$\Sigma M_A = 0 = \ldots + 3 \cdot 7.5 + B_y \cdot 9$

$B_y = 2.61$ kN

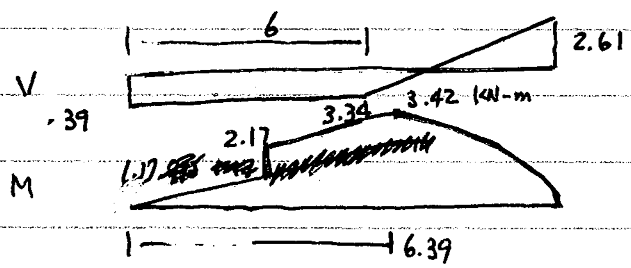
$A_y = .39$

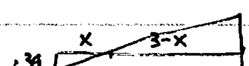




$\Sigma M_A = 0 = -1 + 3(7.5) + B_y \cdot 9$ $\qquad \frac{23.5}{9} = B_y = 2.61$ kN

$A_y + B_y - 3 = 0 \qquad A_y = .39$ kN

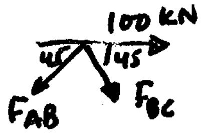




$\frac{2.61}{3-x} = \frac{3}{3}$ $\qquad x = .39$

Given the following beam loaded as shown, find the location of the maximum moment.

what is the shear stress 4m from the left end if the beam cross sectional area is 10 cm^2

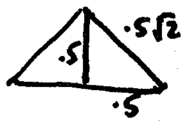


$-F_{BC}\sin 45 - F_{AB}\sin 45 = 0 \Rightarrow F_{BC} = -F_{AB}$ (2)

$100 + F_{BC}\cos 45 - F_{AB}\cos 45 = 0 \Rightarrow 100 - 2F_{AB}\cos 45 = 0$ (3)

$F_{AB} = \frac{100}{2\cdot\sqrt{2}/2} = 70.7\text{ kN}$ (1)

$F_{BC} = -70.7\text{ lb}$



$L = .5\sqrt{2} = .707\text{ m}$ (2)

$A = \frac{(.1\text{m})^2}{4}\pi = .007854\text{ m}^2$ (2)

$\delta = \frac{PL}{AE} = \frac{70.71\times10^3\text{N}(.707\text{ m})}{.007854\text{ m}^2(206\times10^9\frac{N}{m^2})}$

$= 3.09\times10^{-5}\text{ m}$ (2) $= 3.09\times10^{-2}\text{ mm} = .030\text{ mm}$

12

$G = \frac{E}{2(1+\nu)} \Rightarrow \frac{E}{2G} - 1 = \nu = .3205$ (2)

$\epsilon_x = \frac{\delta}{L} = 4.37\times10^{-5}$ (2)

$\epsilon_y = \epsilon_z = -\nu\epsilon_x = -1.4\times10^{-5}$ (2)

there are mechanical strains

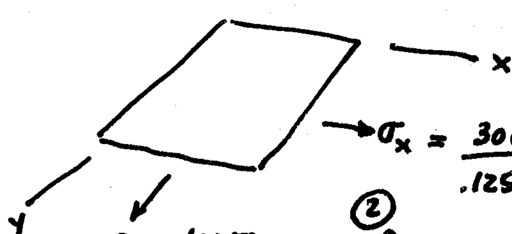
$\epsilon_{y_{TOT}} = \epsilon_{z_{TOT}} = \epsilon_y + \alpha\Delta T = 6.26\times10^{-4}$ (1)

$= -1.4\times10^{-5} + 6.4\times10^{-4}$ (2)

9

$n = 14$
$\bar{x} = 46$
$\sigma = 22.10$

21 pants



$\sigma_x = \frac{30000}{.125\times.01} = 24\text{ MPa}$ (2)

$\sigma_y = \frac{10000}{.1\times.01} = 10\text{ MPa}$ (2)

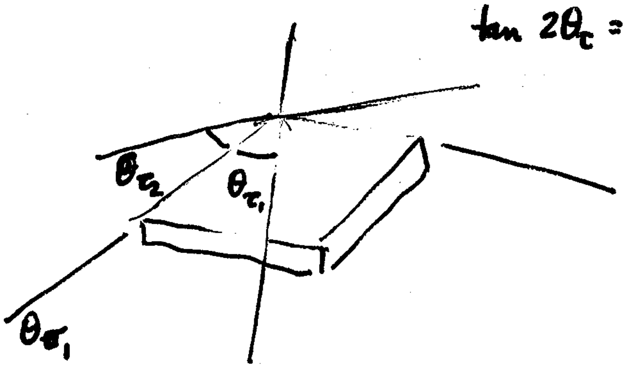
$\sigma_{max} = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= 17\text{ MPa} + \sqrt{\left(\frac{24-10}{2}\right)^2 + 0} = 24\text{ MPa}$ (2)

$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7\text{ MPa}$ (2)

$\tan 2\theta_\sigma = \frac{\tau_{xy}}{\frac{\sigma_x-\sigma_y}{2}} = 0 \Rightarrow \theta_\sigma = 0, 90$ (2)

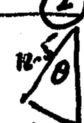
$\tan 2\theta_\tau = -\frac{\left(\frac{\sigma_x-\sigma_y}{2}\right)}{\tau_{xy}} = -\infty \Rightarrow \theta_\tau = -45°, 45$ (2)



(4)

16 pts

$\sigma_x = 10\text{ mpa}$ (2)
$\sigma_y = 24\text{ mpa}$ (2)



(2)

38.6°

$\tan\theta = \frac{4}{5}$ $\sin\theta = \frac{4}{\sqrt{41}}$ $\cos = \frac{5}{\sqrt{41}}$ (2)

$\sigma_x' =$ (2)
$\tau_{xy}' =$ (2)

Parshas Tetzaveh
Parshas Zachor

March 2-3 2007
Adar 13, 5767

Shabbos Schedule

Candle Lighting	6:02 PM
Mincha	6:12 PM
1st Shacharis	7:30 AM
2nd Shacharis	9:00 AM
* No afternoon classes today *	
Mincha	6:00 PM
Shabbos Ends	7:02 PM
Megillah	7:32 PM

Learning Schedule

Daily

Daf Yomi *Rabbi Sapirman*	45 min. before 1st minyan
Gemara *Rabbi Lehrfield*	7:25 AM to 7:55 AM
Halacha *Shimshon Mindick*	10:00 AM to 11:30 AM
Parsha *Rabbi Lehrfield*	between Mincha and Maariv

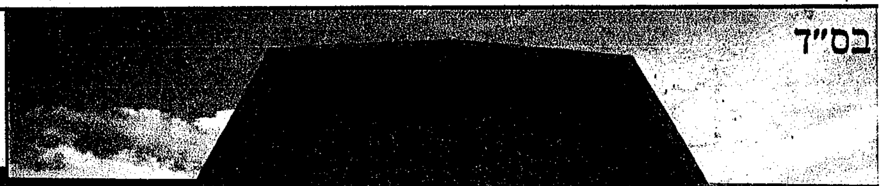
Thursday

Parsha *Rabbi Lehrfield*	8:00 PM to 9:00 PM
Chovos Halvovos *Rabbi Yachnes*	9:00 PM to 10:00 PM

Eruv Hotline
305.249.8939

YOUNG ISRAEL of Greater Miami

Young Israel of Greater Miami
990 NE 171st Street
N. Miami Beach, FL 33162
P: 305.651.3591
F: 305.651.3601



בס"ד

Young Israel Yarn

Sponsors Kiddush

- Wainberg and Benson families in memory of the Yahrzeit of their father Chaim Meyer Wainberg.

Mazel Tov

- Dr. Joseph and Maxine Shuman and Dr. Elliot and Lillian Hahn on the birth of a granddaughter, Daniela Lia, born Sunday, February 25th to Dr. Binny and Shira Hahn of Teaneck, New Jersey. Mazel to the entire Shuman and Hahn Families.

Upcoming Shul Events

Event	Date/Time	Location
Purim Seudah *$20 Adults; $8 Children under 12*	Sunday, March 4 5:00 PM	Young Israel
Oneg Shabbat *Dr. Abe Gittelson*	Friday, March 9 8:30	Young Israel
Purim Carnival *Purim Carnival! Games, Prizes, Costumes and a lot of fun!*	March 3 After the Megilah reading	Young Israel
Comedy Night *An evening with Yisrael Campbell $15 in advance; $18 at the door*	Saturday, March 17 9:00 PM	Young Israel
Shul Dinner *Honoring Dr. & Mrs. Kenneth Israel*	Sunday April 29	Beth Torah

We need drivers on Sunday (Purim Day) to deliver Shalach Manos. To volunteer, for more information, or if you have any questions please call Debra Weitz @ 305-613-2888 or Sarah Bauer @ 305-653-9646.

Look inside for Divrei Torah, kashrus information, community news, youth activities and more...

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NORTH MIAMI BEACH, FL 33162
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03/04/2007 Page: 1
3054931906

	Actual Level	Ideal Level	High/Low
Total Chlorine	1.0 ppm	2 TO 4 ppm	Low
Free Chlorine	1.0 ppm	2 TO 4 ppm	Low
Combined Chlorine	0.0 ppm	0 TO 0 ppm	
pH	7.6	7.4 TO 7.6	
Acid Demand	*	0	
Base Demand	*	0	
Total Alkalinity	125 ppm	80 TO 120 ppm	High
Calcium Hardness	375 ppm	250 TO 400 ppm	
Stabilizer	100 ppm	40 TO 100 ppm	
Total Dissolved Solids	3,300 ppm	300 TO 3,000 ppm	High
Salt	* ppm	0 TO 0 ppm	

** = Not Tested*

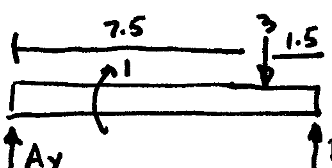
A. Maintain Chlorine Lvl
B. 2.8 gals Suncoast Gold
C. 11.2 ozs All In One

Analysis for: 28080 Gallons Marcite/Gunite Pool
Prescription for: LEVY, CESAR on Mar 4 2007

Analyst: S, L

STABILIZED CHLORINE:
Add Suncoast Stabilized Chlorinating Tablets to your Automatic Chlorinator or Float Feeder.

LIQUID CHLORINE:
Shock your pool by pouring 2.8 gallons of fresh Suncoast Gold Liquid Chlorine around the perimeter of your swimming pool while the circulation system is operating. Do not swim for 24 hours after shocking.



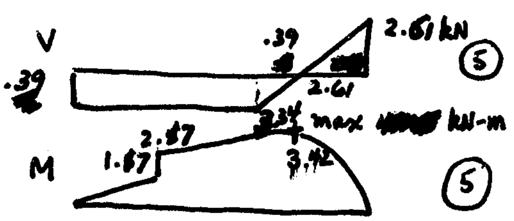
(3)

$\Sigma M = B_y \cdot 9 - 3(7.5) - 1 = 0 \quad B_y = \frac{23.5}{9} = 2.61\ kN$

$\Sigma F = A_y + B_y - 3 = 0 \quad A_y = .39\ kN$ (3)

$\tau = 10^6 \frac{N}{m^2} = 5\ V\ 10^3 N \uparrow \frac{10^3}{m^2}$

$\sigma = 10^6 \frac{N}{m^2}\ 3\ M (= 10^3 N\text{-}m) \uparrow \frac{10^3}{m}$



(5)

(5)

$5 V_{max} = \tau_{max} \quad 5 \times 10^3 \cdot 2610 = 13.01\ MPa$ (2)

$3 M_{max} = \sigma_{max} \quad 3 \times 10^3 \cdot 3420 = 10.28\ MPa$ (2)

20

PLEASE SEE ADDITIONAL INSTRUCTIONS AND PRECAUTIONS ON THE BACK OF THIS PAGE.

WATER COMPONENT SYMPTOMS/INDICATIONS

	LOW	HIGH
CHLORINE *Total, Free & Combined*	Low levels of Chlorine promote the growth of algae and bacteria, which causes the pool to look bad and be unhealthy to swimmers.	High levels of chlorine result in higher sanitation costs and can cause certain types of stains. High Combined Chlorine will result in reduced effectiveness of Free Available Chlorine.
pH *Acid/Base Demand*	Low pH means your water is too acidic, which can cause burning eyes and skin irritation. In addition, Low pH is corrosive on the finish of most pool surfaces and equipment. Your pool will also consume chlorine/bromine at a faster rate.	High pH means your water is not acidic enough, causing burning eyes and skin irritation. High pH also causes chlorine and bromine to be less effective and promotes the buildup of scale.
TOTAL ALKALINITY	Low Total Alkalinity causes rapid deterioration of some types of pool surfaces and fittings, and may result in metal stains. Low Total Alkalinity can also cause the pH to change rapidly when acid is added and prevent the pH from being balanced.	High Total Alkalinity can cause the buildup of scale and stains. This can also cause your equipment to run inefficiently. High Total Alkalinity can also prevent the pH from being balanced.
CALCIUM HARDNESS	Low Calcium Hardness levels cause pool water to be corrosive and may result in staining and etching of your pool finish. Low Calcium Hardness levels are also corrosive on equipment and metal fittings.	High Calcium Hardness levels can lead to the buildup of scale and promote stains. This could also lead to having your pool or spa equipment run inefficiently. The only way to reduce your Calcium Hardness level is through the addition of fresh water.
STABILIZER	Low Stabilizer levels will cause your pool to use more chlorine than is necessary. A correct Stabilizer level makes chlorination simple and more effective by slowing down the release of chlorine and providing continuous chlorine residual.	High Stabilizer levels can lead to stains or spot etching. If you are having problems you believe are associated with High Stabilizer levels, you may need to drain some pool water and replace with fresh water. Always consult with a pool care professional before draining pool water.
TOTAL DISSOLVED SOLIDS (TDS)	There are no chemical drawbacks to having Low TDS. However, a TDS level that reads significantly lower than prior readings can indicate a period of heavy rains, frequent backwashing, splash-out, or a possible leak in the pool due to the frequent addition of new source water.	The higher your TDS levels, the more difficult it is for the chemicals to dissolve in the water and do their job. Elevated TDS levels can result in persistent algae blooms, cloudiness, staining, or salty tasting water. The only way to lower TDS is to dilute the pool with fresh water. Always consult with a pool care professional before draining pool water.
SALT	Salt is only needed for pools sanitized by Salt-Chlorine Generators. For such systems to work properly and produce enough chlorine, the pool must maintain a certain level of salt.	Salt is only needed for pools sanitized by Salt-Chlorine Generators. For such systems, an excessively high level of salt may damage the Salt-Chlorine Generator and can increase scale build-up.

NOTE: The instructions provided by this computer water analysis report are a summary and only for your convenience. For more information on each of the chemicals recommended in your analysis, please read the entire label on each chemical, which includes instructions, warnings, and precautionary statements.

For more information about maintaining proper water balance, ask your pool care professional. To find the store nearest you, call 1-800-234-1616 or visit www.pinchapenny.com.

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EMA 3702 SPRING 2008 DR. C. LEVY

FINAL EXAMINATION-B

April 22, 2008

General Instructions -- This examination is 2 hours long. You are allowed your help aids from previous quizzes, your notes but not your book, and any help aids attached to the examination. SHOW ALL WORK!!!

Please sign the following:

I certify that I will neither receive nor give unpermitted aid on this examination. Violation of this will result in failure of the course and possibly other academic disciplinary actions.

________________________ ________________________

Print your name Sign your name

This examination consists of 3 problems with several parts to each of the problems. You are to answer all the problems!

GOOD LUCK!

Problem #	Breakdown by Problem	Score
1	30%	
2	40%	
3	30%	
	TOTAL	

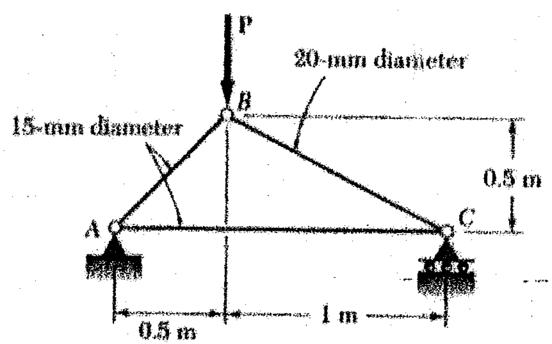
Problem 1B.

Knowing that a factor of safety of 2.6 is required,

a) Determine the largest load P that can be applied to the structure shown.
Use E=200 GPA.

Calculate the load under the following conditions:
The rods are pin connected at both ends for buckling in the plane of the page and are considered fixed at both ends for buckling in the out of page direction.

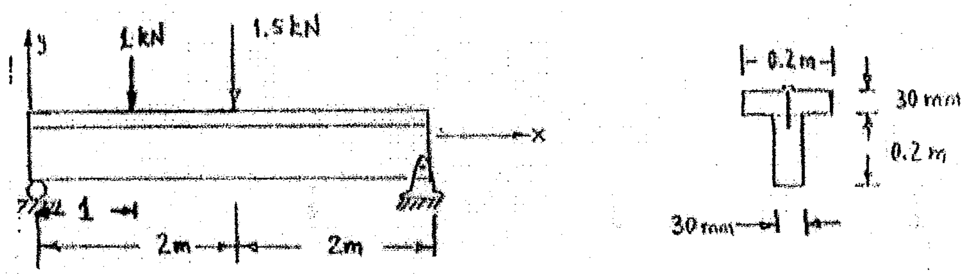
b) For the load found in (a), find the cross-sectional area in the other two members so that their allowable stresses meet the safety factor. Assume that σ_{yp} = 360 MPa



Problem 2B.

The T shaped beam is made of 2 wood planks 200 mm x 30 mm which are joined by nails. If the allowable bending stress is 12 MPa, and the allowable shearing stress is 0.8 MPa, find:

a) what is the equation of the elastic curve
b) if the beam is able to support safely the loads shown in the picture
c) the maximum spacing between the nails if each nail is able to support safely 1500 N of shear force.

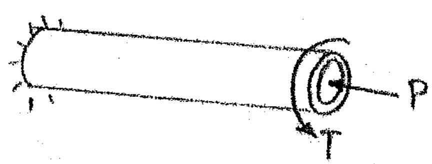


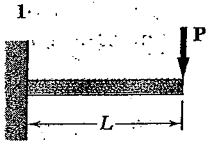
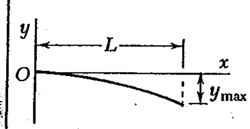
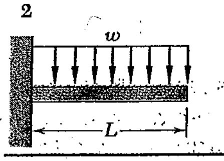
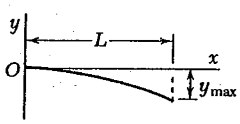
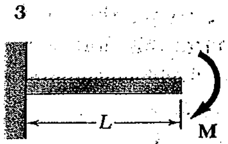
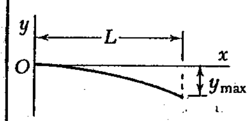
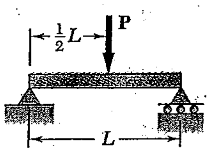
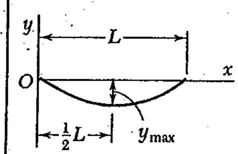
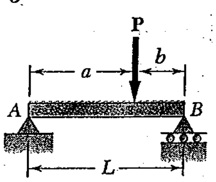
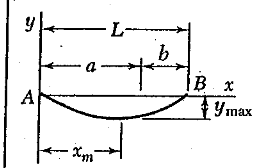
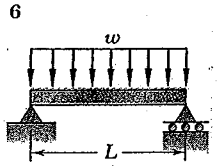
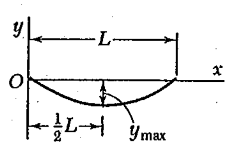
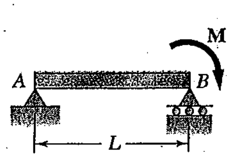
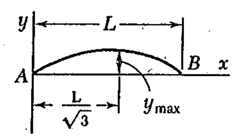
Problem 3B.

Consider a hollow cylindrical tube of outer radius $R_0 = 140$ mm and inner radius $R_i = 125$ mm. The tube has a flat end cap. The tube is fixed at one end and subjected to a torque of 35 kN-m together with an axial compressive force of 68 kN as shown in the diagram. If the tube is also pressurized to a pressure of 2.1 MPa:

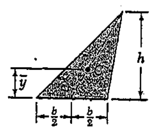
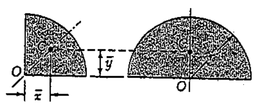
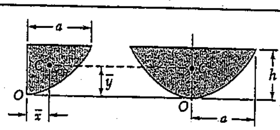
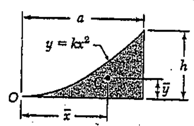
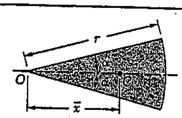
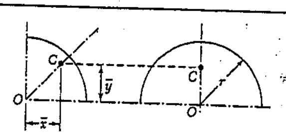
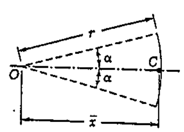
a. Determine the principal stresses and where they occur

b. What would the length of the tube have to be for the tube to buckle?

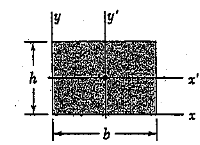
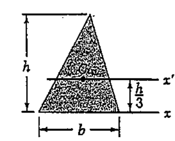
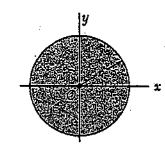
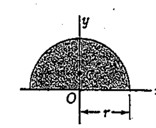
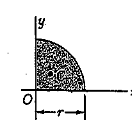
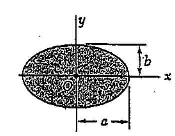


Beam and Loading	Elastic Curve	Maximum Deflection	Slope at End	Equation of Elastic Curve
1 		$-\frac{PL^3}{3EI}$	$-\frac{PL^2}{2EI}$	$y = \frac{P}{6EI}(x^3 - 3Lx^2)$
2 		$-\frac{wL^4}{8EI}$	$-\frac{wL^3}{6EI}$	$y = -\frac{w}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$
3 		$-\frac{ML^2}{2EI}$	$-\frac{ML}{EI}$	$y = -\frac{M}{2EI}x^2$
4 		$-\frac{PL^3}{48EI}$	$\pm\frac{PL^2}{16EI}$	For $x \leq \frac{1}{2}L$: $y = \frac{P}{48EI}(4x^3 - 3L^2x)$
5 		For $a > b$: $-\frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}EIL}$ at $x_m = \sqrt{\frac{L^2 - b^2}{3}}$	$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL}$ $\theta_B = +\frac{Pa(L^2 - a^2)}{6EIL}$	For $x < a$: $y = \frac{Pb}{6EIL}[x^3 - (L^2 - b^2)x]$ For $x = a$: $y = -\frac{Pa^2b^2}{3EIL}$
6 		$-\frac{5wL^4}{384EI}$	$\pm\frac{wL^3}{24EI}$	$y = -\frac{w}{24EI}(x^4 - 2Lx^3 + L^3x)$
7 		$\frac{ML^2}{9\sqrt{3}EI}$	$\theta_A = +\frac{ML}{6EI}$ $\theta_B = -\frac{ML}{3EI}$	$y = -\frac{M}{6EIL}(x^3 - L^2x)$

Centroids of Common Shapes of Areas and Lines

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r\sin\alpha}{\alpha}$	0	$2\alpha r$

Moments of Inertia of Common Geometric Shapes

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

Florida International University
Department of Mechanical and Materials Engineering

EMA 3702 QUIZ 4A April 2, 2009

You are allowed five sheets of 8 ½ x 11 inch paper, with whatever you wish except solutions

Print your name and sign the following statement:

I will not give nor take any unpermitted aid during this quiz. I understand that violation of this statement will lead to automatic failure of the quiz.

PRINT NAME SIGN NAME

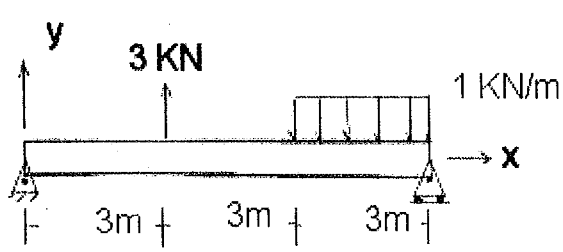
You are given 3 problems: CHOOSE 2 of the 3 problems and indicate which you have chosen. If you decide to do the third problem for extra credit, indicate the extra credit problem by EC. Extra credit problem is worth 25 points.

Problem 1 (50 points).

a) Given the following beam loaded as shown, find the deflection v as a function of x. Leave results in terms of EI

b) What is the moment as a function of x? Evaluate it at x=4.5 m. Leave results in terms of EI

c) What is the shear as a function of x? Evaluate it at x=7.5 m. Leave results in terms of EI

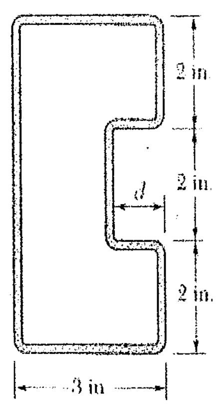


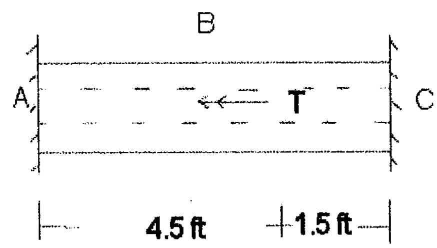
Problem 2 (50 points).

The member, having the cross-section shown, is to be formed from the sheet metal of 0.06 in. thickness. Also d=0.95 in. If the member is fixed between two walls and has an applied torque, T, of 2500 lb-in,

Determine the maximum shearing stress in sections AB and BC and the maximum angle of twist.

The Young's Modulus, $E = 30 \times 10^6$ psi and that the Poisson ratio, $\nu = 0.3$. Remember, this is a closed thin walled section.

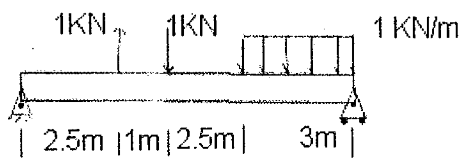


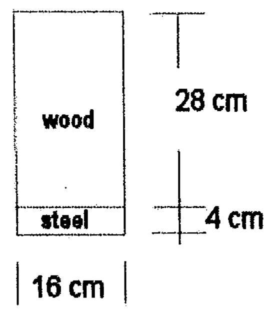


Problem 3 (50 points)

a) Given the following beam loaded as shown, find the location of the maximum moment.

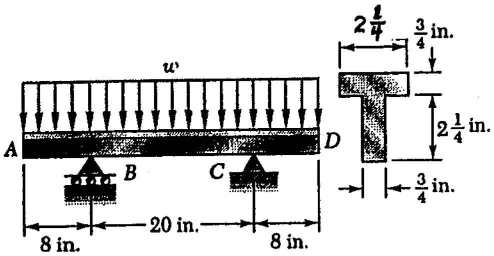
b) What is the maximum stress σ_x and where can it be found, given the following information: $E_{steel} = 206$ GPa, $E_{wood} = 10.3$ GPa, for the cross-section below.





PROBLEM 5.97

5.97 Determine the largest permissible uniformly distributed load w for the beam shown, knowing that the allowable normal stress is +12 ksi in tension and −19,5 ksi in compression.



SOLUTION

Reactions: $B + C - 36w = 0 \qquad B = C = 18w$

Shear: $V_A = 0$

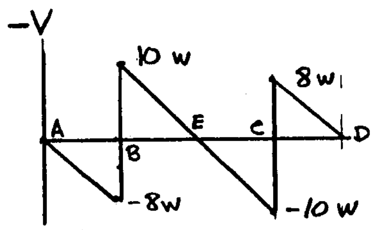
$V_{B^-} = 0 - 8w = -8w$

$V_{B^+} = -8w + 18w = 10w$

$V_{C^-} = 10w - 20w = -10w$

$V_{C^+} = -10w + 18w = 8w$

$V_D = 8w - 8w = 0$



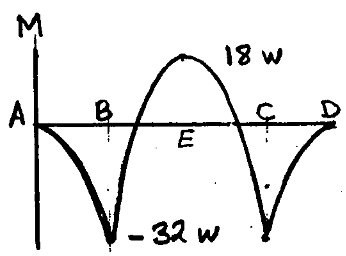
Areas: A to B $(\frac{1}{2})(8)(-8w) = -32w$

B to E $(\frac{1}{2})(10)(10w) = 50w$

Bending moments: $M_A = 0$

$M_B = 0 - 32w = -32w$

$M_E = -32w + 50w = 18w$

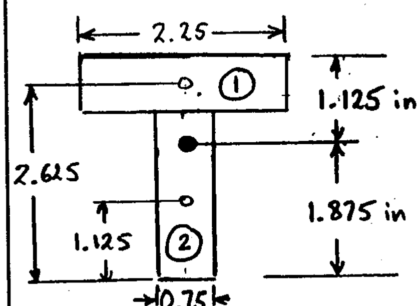


Centroid and moment of inertia

Part	A (in²)	$\bar{y}$ (in)	$A\bar{y}$ (in³)	d in.	Ad² (in⁴)	$\bar{I}$ (in⁴)
①	1.6875	2.625	4.4297	0.75	0.9492	0.0791
②	1.6875	1.125	1.8984	0.75	0.9492	0.7119
Σ	3.375		6.3281		1.8984	0.7910

$\bar{Y} = \frac{6.3281}{3.375} = 1.875$ in

$I = \sum Ad^2 + \sum \bar{I} = 2.6894$ in⁴



Top: $y = 1.125$

Bottom: $y = -1.875$

$I/y = 2.3906$ in³ = Section modulus

$I/y = -1.4343$ in³ = Section modulus } S

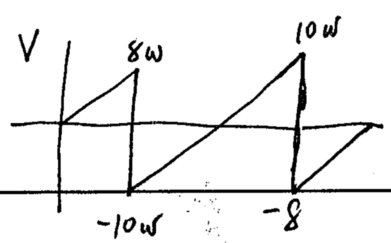
Bending moment limits $\qquad M = -\sigma I/y = -\sigma S$

Tension at B and C	$-(12)(2.3906) =$	-28.687 kip·in
Comp. at B and C	$-(-19.5)(-1.4343) =$	-27.969 kip·in. ◁
Tension at E	$-(12)(-1.4343) =$	17.212 kip·in ◁
Compression at E	$-(-19.5)(2.3906) =$	46.6 kip·in

Allowable load w

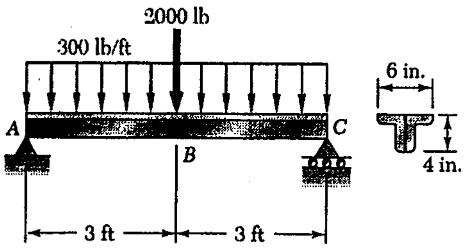
B & C	$-32w = -27.969$	$w = 0.874$ kip/in
E	$18w = 17.212$	$w = 0.956$ kip/in

Smallest $\quad w = 0.874$ kip/in $= 10.49$ kip/ft. ◄



PROBLEM 5.91

5.91 Two L 4 × 3 rolled-steel angles are bolted together to support the loading shown. Knowing that the allowable normal stress for the steel used is 24 ksi, determine the minimum angle thickness that can be used.



SOLUTION

By symmetry $A = C$

$+\Sigma F_y = 0 \quad A + C - 2000 - (6)(300) = 0$

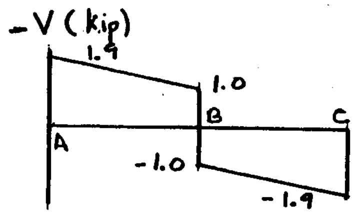
$A = C = 1900$ lb.

Shear: $V_A = 1900$ lb. $= 1.9$ kips

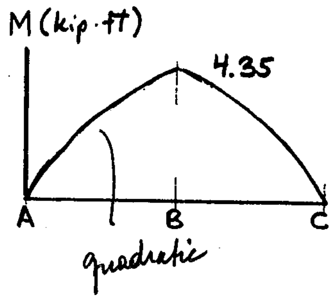
$V_{B^-} = 1900 - (3)(300) = 1000$ lb $= 1$ kip

$V_{B^+} = 1000 - 2000 = -1000$ lb $= -1$ kip

$V_C = -1000 - (3)(300) = -1900$ lb $= -1.9$ kip



Areas: A to B $(\frac{1}{2})(3)(1.9+1) = 4.35$ kip·ft
B to C $(\frac{1}{2})(3)(-1-1.9) = -4.35$ kip·ft } trapezoidal areas



Bending moments: $M_A = 0$

$M_B = 0 + 4.35 = 4.35$ kip·ft

$M_C = 4.35 - 4.35 = 0$

Maximum $|M| = 4.35$ kip·ft $= 52.2$ kip·in

$\sigma_{all} = 24$ ksi

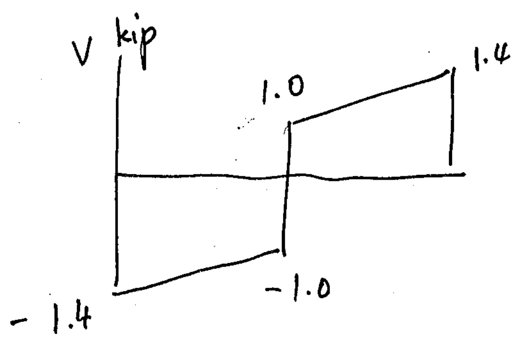
For section consisting of two angles $S_{min} = \frac{|M|}{\sigma_{all}} = \frac{52.2}{24} = 2.175 \text{ in}^3$

For each angle $S_{min} = (\frac{1}{2})(2.175) = 1.0875 \text{ in}^3$

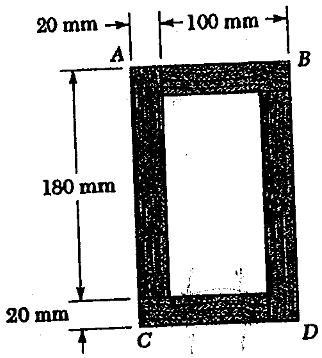
Angle section	S (in³)
L 4×3 × $\frac{1}{2}$	1.89
L 4×3 × $\frac{3}{8}$	1.46
L 4×3 × $\frac{1}{4}$	1.00

Smallest allowable thickness

$t = \frac{3}{8}$ in. ◀



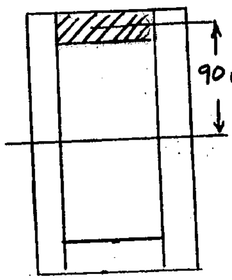
PROBLEM 6.33



6.33 Two 20 × 100-mm and two 20 × 180-mm boards are glued together as shown to form a 120 × 200-mm box beam. Knowing that the beam is subjected to a vertical shear of 3.5 kN, determine the average shearing stress in the glued joint (*a*) at *A*, (*b*) at *B*.

SOLUTION

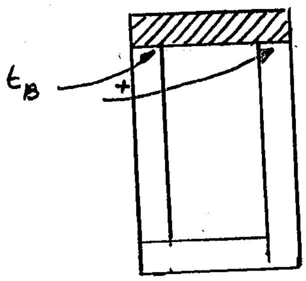
$$I = \frac{1}{12}(120)(200)^3 - \frac{1}{12}(80)(160)^3 = 52.693 \times 10^6 \text{ mm}^4 = 52.693 \times 10^{-6} \text{ m}^4$$



(a) $Q_A = (80)(20)(90) = 144 \times 10^3 \text{ mm}^3 = 144 \times 10^{-6} \text{ m}^3$

$t_A = (2)(20) = 40 \text{ mm} = 0.040 \text{ m}$

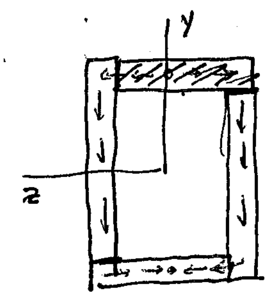
$$\tau_A = \frac{VQ_A}{It_A} = \frac{(3.5\times 10^3)(144\times 10^{-6})}{(52.693\times 10^{-6})(0.040)} = 239 \times 10^3 \text{ Pa} = 239 \text{ kPa}$$



(b) $Q_B = (120)(20)(90) = 216 \times 10^3 \text{ mm}^3 = 216 \times 10^{-6} \text{ m}^3$

$t_B = (2)(20) = 40 \text{ mm} = 0.040 \text{ m}$

$$\tau_B = \frac{VQ_B}{It_B} = \frac{(3.5\times 10^3)(216\times 10^{-6})}{(52.693\times 10^{-6})(0.040)} = 359 \times 10^3 \text{ Pa} = 359 \text{ kPa}$$



$Q_A = 100 \cdot 20 \cdot 90 = \frac{5}{4} Q_A$ above

$t_A = 0.040$ m

$$\tau_A = \frac{VQ_A}{It_A} = \frac{5}{4}\, \tau_A \text{ above} \cong 478 \text{ kPa}$$

$Q = (b/2 - z)t\bar{y}$

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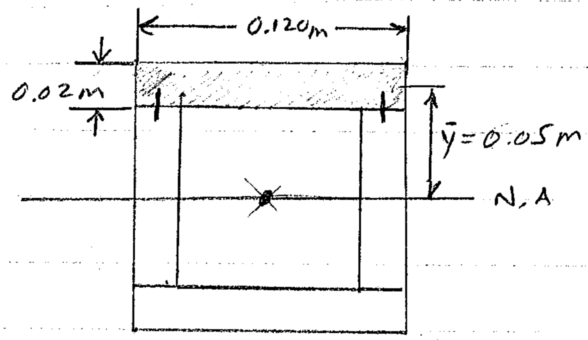
25/30

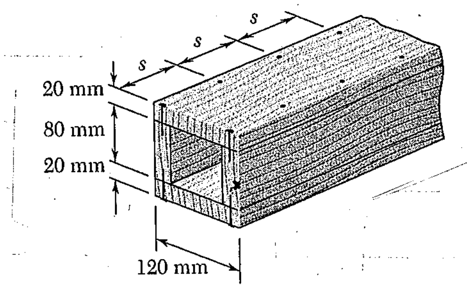
(6.3)

6.3 A square box beam is made of two 20 × 80-mm planks and two 20 × 120-mm planks nailed together as shown. Knowing that the spacing between the nails is $s = 50$ mm and that the allowable shearing force in each nail is 300 N, determine (*a*) the largest allowable vertical shear in the beam, (*b*) the corresponding maximum shearing stress in the beam.

$\tau_{all} = 300$ N

(a)





$$Q = A\bar{y} = (0.02m \times 0.120m)(0.05m) = 0.00012\ m^3$$

$$I = 2\left[\tfrac{1}{12}(.120m)(0.02m)^3\right] \qquad 3.2\times10^{-7}$$
$$+ 2\left[\tfrac{1}{12}(0.02m)(0.08m)^3\right] \qquad 1.7067\times10^{-6}$$
$$+ (0.02m \times 0.120m)(0.05m)^2 \qquad 6.0\times10^{-6}$$

$$\boxed{I = 8.0267\times10^{-6}\ m^4}$$

q

$$q = \frac{VQ}{I}$$

$$2F = (0.05m)\, q$$
$$2(300N) = (0.05m)\, q$$
$$q = 12{,}000\ N/m$$

$$q = \frac{VQ}{I} \Rightarrow V = \frac{Iq}{Q} = \frac{(8.0267\times10^{-6}\ m^4)(12{,}000\ N/m)}{0.00012\ m^3}$$

$$\boxed{V = 802.67N}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(802.67\,N)(0.00012\,m^3)}{(8.0267\times10^{-6}\,m^4)(.04\,m)} = 300\,kPa$$

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6.10

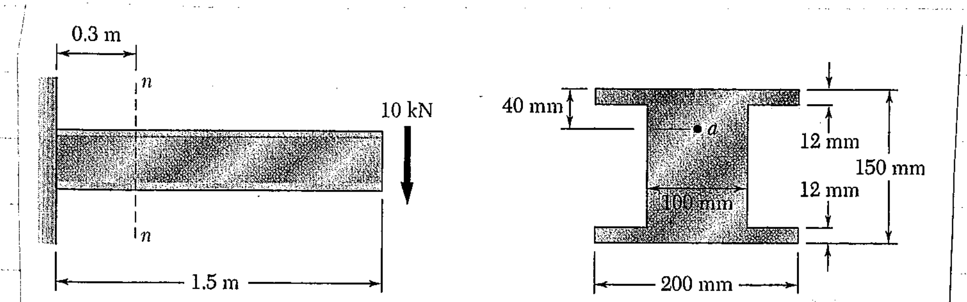
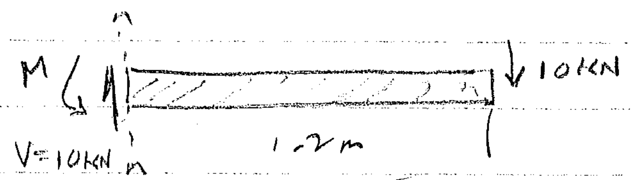


Fig. P6.10

FOR THE BEAM LOADING SHOWN, CONSIDER SECTION n-n AND DETERMINE (a) THE LARGEST SHEARING STRESS IN THAT SECTION (b) THE SHEARING STRESS AT POINT (a)

SOLUTION: VERTICAL SHEAR AT SECTION n-n

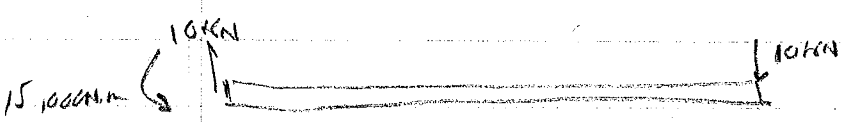


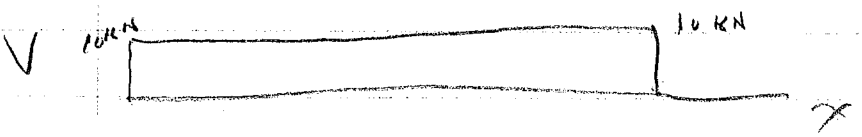
$\Sigma F_y = 0 \qquad V - 10KN = 0$

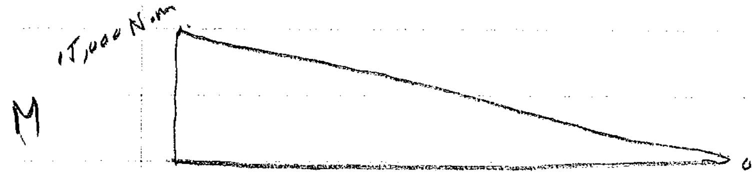
$V = 10KN \uparrow$

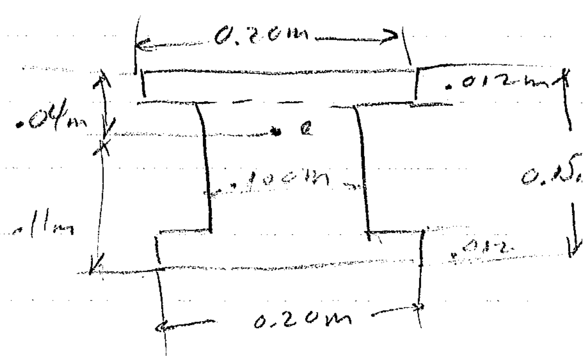
$\Sigma M_{n-n} = 0 \qquad -10\,kN \cdot 1.2\,m + M = 0$

$M = 12{,}000\ N \cdot m$



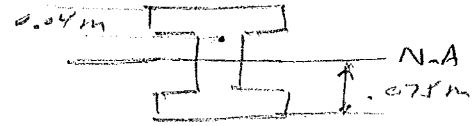






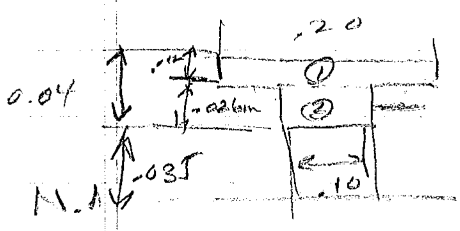
(a) $V_{MAX} = 10\ KN$

(b) $Q = A\bar{y}$



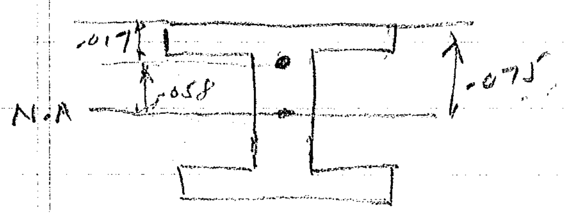
DETERMINE Q BY COMPUTING THE FIRST MOMENT WITH RESPECT TO THE NEUTRAL AXIS OF THE AREA ABOVE a

$Q = A\bar{y} =$



$\Sigma A\bar{y} = \bar{y}\Sigma A$

	A	$\bar{y}$	$A\bar{y}$
①	$(.012)(.20) = .0024\,m^2$	.069	.0001656
②	$(.028m)(.10) = .0028\,m^2$	.049	.0001372
		$\Sigma A\bar{y} =$	.0003028



$$\bar{y} = \frac{.0003028}{.0052} = .058$$

$$Q = A\bar{y} = (.0052\,m^2)(.058\,m) = 3.016 \times 10^{-4}\,m^3$$

$$\tau = \frac{VQ}{It} = \frac{(10{,}000\,N)(3.016 \times 10^{-4}\,m^3)}{(I)\quad(0.10\,m)}$$

$$I = \tfrac{1}{12}(.20)(.012\,m)^3 + \tfrac{1}{12}(.10\,m)(.028)^3 + (.0052\,m^2)(.058)^2$$

$$2.88 \times 10^{-8} + 1.83 \times 10^{-7} + 1.75 \times 10^{-5}$$

$$\boxed{I = 1.77 \times 10^{-5}\,m^4}$$

$$\tau = \frac{(10000\,N)(3.016 \times 10^{-4}\,m^3)}{(1.77 \times 10^{-5}\,m^4)(0.10\,m)} = 1.7 \times 10^{6}\,N/m^2$$

$$\tau = 1.7 \times 10^{6}\,N/m^2$$

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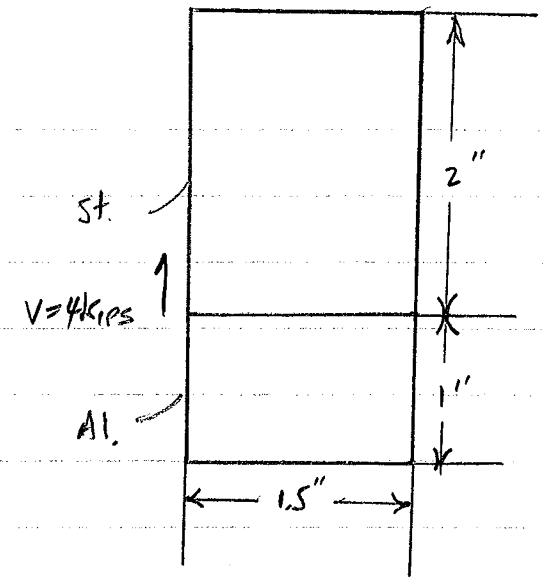


(6.59) $\tau_{ave} = \frac{VQ}{It}$

$E_{st} = 29 \times 10^6$ psi

$E_{al} = 10.6 \times 10^6$ psi

FIND: (a) AVG. STRESS AT BONDING SURFACE.

(b) MAXIMUM SHEARING STRESS IN BEAM

SEE §4.6 $n = \frac{E_{st}}{E_{Al}} = \frac{29 \times 10^6}{10.6 \times 10^6} = 2.736$

WIDTH OF STEEL $= (1.5")(2.736) = 4.104"$

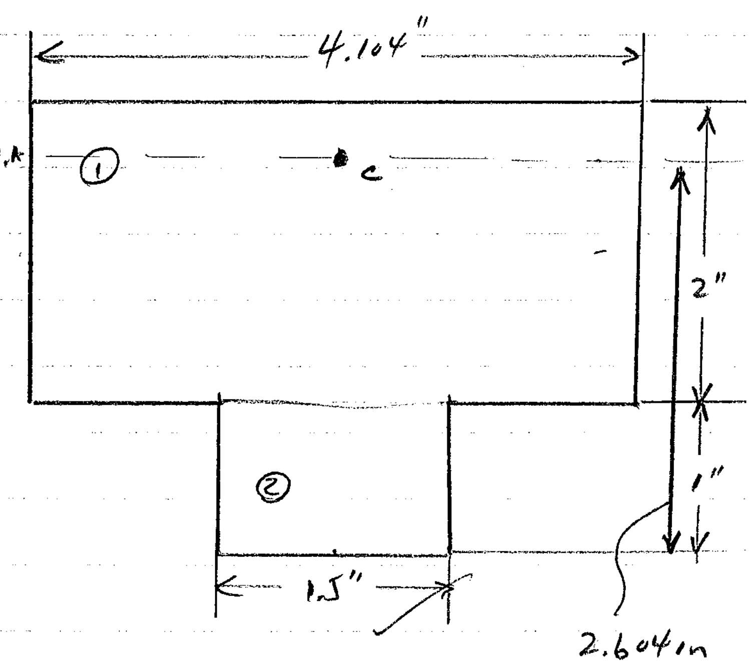
CENTROID

	AREA in^2	$\bar{y}$ in	$\bar{y}A$
①	$8.208\ in^2$	3 in	$24.624\ in^3$
②	$1.5\ in^2$	.5 in	$.75\ in^3$
$\Sigma A =$	$9.708\ in^2$	3.5 in.	25.374

$\bar{Y}\Sigma A = \Sigma \bar{y} A$

$\bar{Y}(9.708\ in^2) = 25.374\ in^3$

$\boxed{\bar{Y} = 2.614\ in}$



8

CENTROIDAL MOMENT OF INERTIA

$$I = \frac{1}{12}(4.104)(2)^3 + (4.104 \times 2)(.604)^2$$
$$+ \frac{1}{12}(1.5)(1)^3 + (1.5 \times 1)(2.104)^2$$

$$I = 2.736 + 2.994 + .125 + 6.640$$

$$I = 12.495 \text{ in}^4$$

(a) AVG. STRESS AT BONDING SURFACE

$$\tau_{AVG.} = \frac{VQ}{It} = \frac{(4 \text{ Kips})(Q)}{(12.495 \text{ in}^4)(1.0'')} = \frac{(4 \text{ Kips})(4.968 \text{ in}^3)}{(12.495 \text{ in}^4)(1.0 \text{ in})}$$

$$\boxed{\tau_{AVG.} = 1590. \text{ lb/in}^2}$$

$$Q = (2)(1.5)(.604) + (1)(1.5)(2.104)$$
$$Q = 1.812 + 3.156 = \boxed{4.968 \text{ in}^3}$$

τ_{MAX} = OCCURS AT NEUTRAL AXIS WHERE Q IS MAX.

$$Q = (4.104)(.604)(.604/2) = .749 \text{ in}^3$$

$$\tau_{MAX} = \frac{(4,000 \text{ lbs})(.749 \text{ in}^3)}{(12.495 \text{ in}^4)(.396)} = 605.49 \text{ p.s.i}$$

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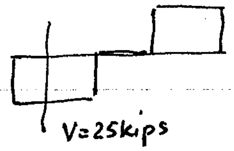
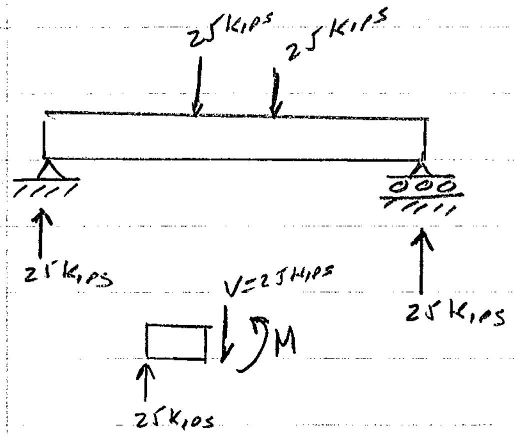
(6.93)





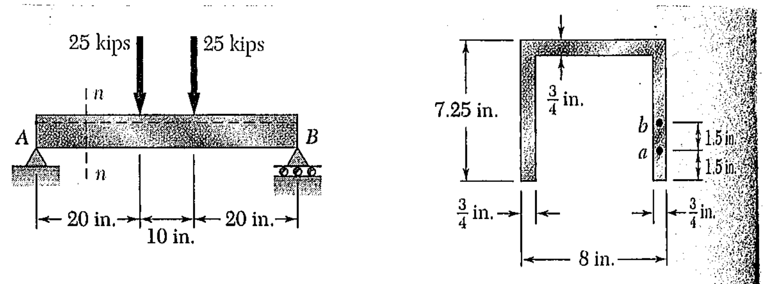


Fig. P6.92

6.93 For the beam and loading shown in Prob. 6.92, determine the largest shearing stress in section *n-n*.

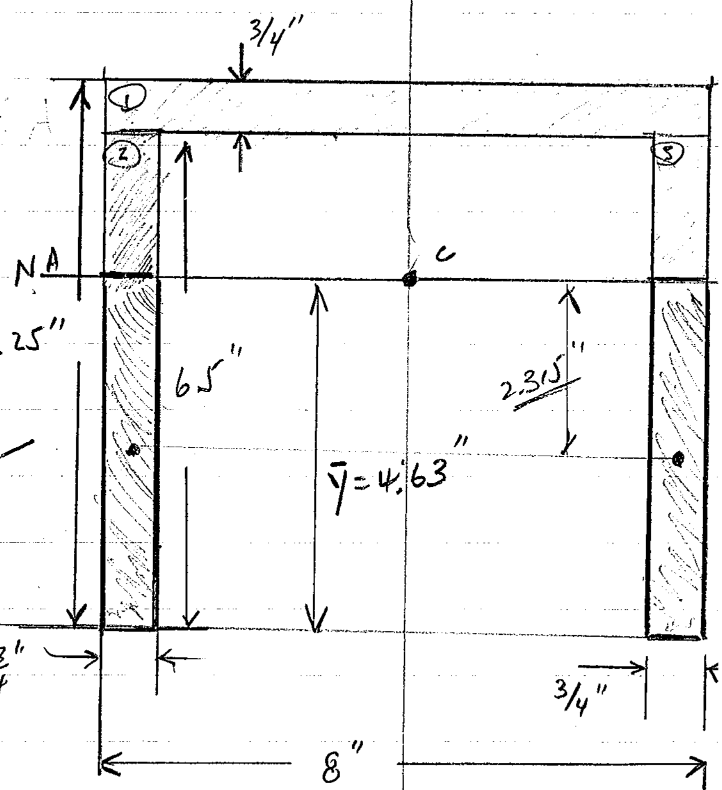
MAX STRESS OCCURS AT CENTROID

SINCE THE BEAM AND LOADING ARE BOTH SYMMETRIC WITH RESPECT TO THE CENTER OF THE BEAM, WE HAVE A=B=25 Kips CONSIDERING

<u>CENTROID</u>

$\Sigma A\bar{y} = \bar{y}\Sigma A$

	A	$\bar{y}$	$A\bar{y}$
①	$(3/4)(8'') = 6 in^2$	6.875	41.25
②	$(3/4)(6\frac{1}{2}) = 4.875 in^2$	3.25	15.844
③	$(3/4'')(6\frac{1}{2}'') = 4.875 in^2$	3.25	15.844
	15.75		72.49



4

$\Sigma A\bar{y} = \bar{y}\,\Sigma A$

$72.94 = \bar{y}(15.75)$

$\bar{y} = 4.63$

$Q = A\bar{y}_1 = 2(.75'')(4.63'')(2.315'') = 16.078 \; in^3$

$Q = 16.078 in^3$

MOMENT OF INERTIA

I

$I_{x_1} = \frac{1}{12} b h^3 = \frac{1}{12}(8'')(.75)^3 = .28125 + Ad^2$

$= .28125 + (8'')(.75'')(2.245'')^2$

$= .28125 + 30.24$

$I_{x_1} = 30.52\ in^4$

$I_{x(2+3)} = \frac{1}{12} b h^3 + A d^2$

$= \frac{1}{12}(1.5)(6.5)^3 + (1.5)(6.5)(2.315)^2$ X

$= 34.33 + 52.25$

$I_{x(2+3)} = 86.58\ in^4$ X

$I_x = 30.52 + 86.58 = 117.1\ in^4$ X

$\tau_{MAX} = \frac{VQ}{It} = \frac{(25\ kips)(16.078\ in^3)}{(86.58\ in^4)(.75)} = 6190\ ksi$

X (86.58) — I of entire beam.

(.75) → $2 \cdot (.75)$

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$q = \frac{VQ}{I}$

6.97 Three plates, each 12 mm thick, are welded together to form the section shown. For a vertical shear of 100 kN, determine the shear flow through the welded surfaces and sketch the shear flow in the cross section.

$q = Q/m$

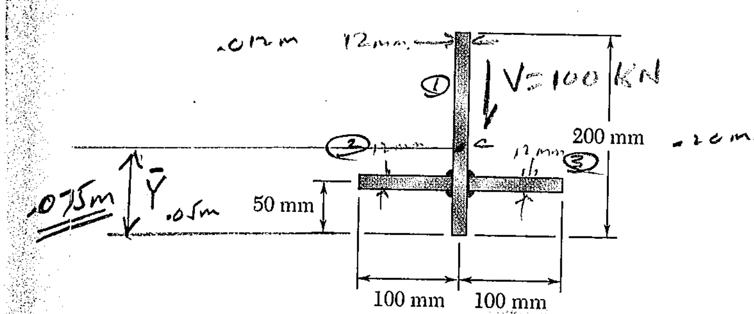


Fig. P6.97 .1m .1m

CENTROID $\Sigma \bar{y} A = \bar{Y} \Sigma A$

Area	$\bar{y}$	$\bar{y} A$
① $(.012m)(.20m) = 0.0024 m^2$	0.10 m	$.00024 m^3$
② $(.012m)(.10m) = 0.0012 m^2$	0.05 m	$.00006 m^3$
③ $(.012m)(.10m) = 0.0012 m^2$	0.05 m	$.00006 m^3$

$\Sigma A = 0.0048 m^2$ $\quad \Sigma \bar{y} A = 0.00036 m^3$

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.00036 m^3}{0.0048 m^2} = .075 m$$

I (MOMENT OF INERTIA)

$$I_{1x} = \frac{1}{12} bh^3 + Ad^2 = \frac{1}{12}(.012m)(.2m)^3 + (.012)(.2)(.025)^2$$
$$= 8\times10^{-6} + 1.5\times10^{-6} = \boxed{9.5\times10^{-6}}$$

$$I_{2x} = \frac{1}{12} bh^3 + Ad^2 = \frac{1}{12}(0.10m)(.012m)^3 + (.012)(.10)(.025)^2$$
$$= 1.44\times10^{-8} + 7.5\times10^{-7} = \boxed{7.644\times10^{-7}}$$

$$I_{3x} = \frac{1}{12} bh^3 + Ad^2 = \frac{1}{12}(0.10m)(.012m)^3 + (.012)(.10)(.025)^2$$
$$= \boxed{7.644\times10^{-7}}$$

$$\boxed{I = 1.103\times10^{-5} m^4}$$

$$Q = A\bar{y} = (.012m)(.1m) \times (.025)$$

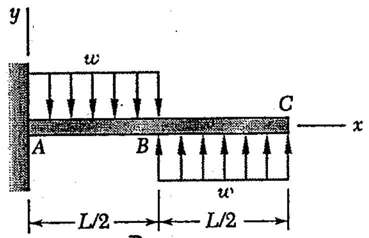
$$Q = .00003$$

$$q = \frac{(100\ KN)(.00003\ m^3)}{1.103 \times 10^{-5}\ m^4} = 271.985\ KN/m$$

$$\boxed{q = 271.985\ kN/m}$$

Problem 9.6

9.5 and 9.6 For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.



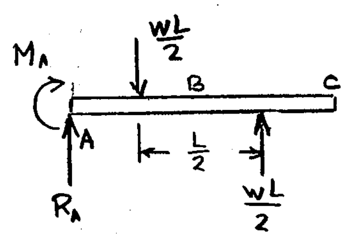
$[x=0,\ y=0]$

$[x=0,\ \frac{dy}{dx}=0]$

Using ABC as a free body.

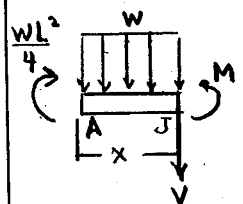
$+\uparrow \Sigma F_y = 0: \quad R_A - \frac{wL}{2} + \frac{wL}{2} = 0 \qquad R_A = 0$ ①

$+\circlearrowleft M_A = 0: \quad -M_A + \left(\frac{wL}{2}\right)\left(\frac{L}{2}\right) = 0 \qquad M_A = \frac{wL^2}{4}$ ①



Using AJ as a free body (Portion AB only)

$\circlearrowleft \Sigma M_J = 0: \quad -\frac{wL^2}{4} + (wx)\frac{x}{2} + M = 0$



$M = \frac{1}{4}wL^2 - \frac{1}{2}wx^2$ ②

$EI\frac{d^2y}{dx^2} = \left(\frac{1}{4}wL^2\right) - \frac{1}{2}wx^2$

$EI\frac{dy}{dx} = \frac{1}{4}wL^2x - \frac{1}{6}wx^3 + C_1$ C_3

$\left[x=0,\ \frac{dy}{dx}=0\right] \qquad 0 = 0 - 0 + C_1 \qquad C_1 = 0$ ①

$EIy = \frac{1}{8}wL^2x^2 - \frac{1}{24}wx^4 + C_1x + C_2$ C_4

$[x=0,\ y=0] \qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$ ①

(a) Elastic curve. $\quad y = \frac{w}{EI}\left(\frac{1}{8}L^2x^2 - \frac{1}{24}x^4\right)$ ①

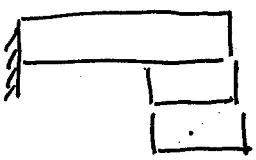
$\frac{dy}{dx} = \frac{w}{EI}\left(\frac{1}{4}L^2x - \frac{1}{6}x^3\right)$

(b) y at $x = \frac{L}{2}$: $\quad y_B = \frac{w}{EI}\left\{\frac{1}{8}L^2\left(\frac{L}{2}\right)^2 - \frac{1}{24}\left(\frac{L}{2}\right)^4\right\} = \frac{wL^4}{EI}\left\{\frac{1}{32} - \frac{1}{384}\right\}$

$= \frac{11wL^4}{384EI} \qquad y_B = \frac{11wL^4}{384EI} \uparrow$ ①

(c) $\frac{dy}{dx}$ at $x = \frac{L}{2}$: $\quad \theta_B = \frac{w}{EI}\left\{\frac{1}{4}L^2\left(\frac{L}{2}\right) - \frac{1}{6}\left(\frac{L}{2}\right)^3\right\} = \frac{wL^3}{EI}\left\{\frac{1}{8} - \frac{1}{48}\right\}$

$= \frac{5wL^3}{48EI} \qquad \theta_B = \frac{5wL^3}{48EI}$ ①



①

$q(x) = -w + 2w\langle x - L/2\rangle^0$ ①

$v(x=0) = 0 \qquad EIv''' = -V_0 = 0$

$v'(x=0) = 0 \qquad EIv'''' = M_0 = 0$ $\}\ x=L$

④ for the coeffs $\quad EIv'' = -wx^2/2 + \frac{2w}{2}\langle x-L/2\rangle^2 + C_1x + C_2$ ④

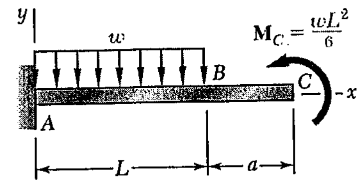
$C_2 = \frac{wL^2}{4}$

$EIv''' = -wx + 2w\langle x - L/2\rangle^1 + C_1x$

$= -wL + 2w \cdot L/2 + C_1 \cdot L = 0 \Rightarrow C_1 = 0$

Problem 9.5

9.5 and 9.6 For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.



Using ABC as a free body

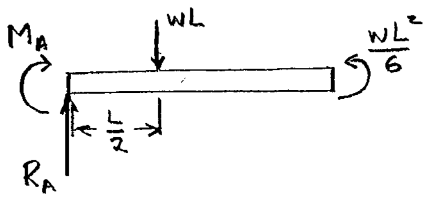
$+\uparrow \Sigma F_y = 0: \quad R_A - wL = 0 \qquad R_A = wL$ ①

$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - (wL)(\frac{L}{2}) + \frac{wL^2}{6} = 0$

$M_A = -\frac{1}{3} wL$ ①

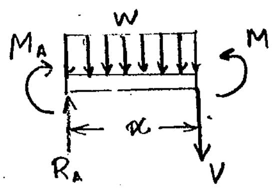
$[x = 0, y = 0]$

$[x = 0, \frac{dy}{dx} = 0]$



Using AJ as a free body (Portion AB only)

$+\circlearrowleft M_J = 0: \quad M + (wx)(\frac{x}{2}) - R_A x - M_A = 0$



$M = -\frac{1}{2}wx^2 + R_A x + M_A$

$= -\frac{1}{2}wx^2 + wLx - \frac{1}{3}wL^2$ ②

$EI\frac{d^2y}{dx^2} = -\frac{1}{2}wx^2 + wLx - \frac{1}{3}wL^2$

$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{2}wLx^2 - \frac{1}{3}wLx + C_1$

$[x = 0, \frac{dy}{dx} = 0]: \quad -0 + 0 - 0 + C_1 = 0 \qquad C_1 = 0$ ①

$EIy = -\frac{1}{24}wx^4 + \frac{1}{6}wLx^3 - \frac{1}{6}wLx^2 + C_2$

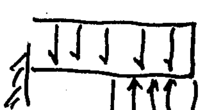
$[x = 0, y = 0] \quad -0 + 0 - 0 + C_2 = 0 \qquad C_2 = 0$ ①

(a) Elastic curve over AB. $\quad y = \frac{w}{24EI}(-x^4 + 4Lx^3 - 4L^2x^2)$ ①

$\frac{dy}{dx} = \frac{w}{6EI}(-x^3 + 2Lx^2 - L^2x)$

(b) y at $x = L$: $\quad v_B = y_B = -\frac{wL^4}{24EI}$ ① $\qquad y_B = \frac{wL^4}{24EI} \downarrow$

(c) $\frac{dy}{dx}$ at $x = L$: $\quad \frac{dy}{dx}\Big|_B = 0 \qquad \frac{dv}{dx_B} = \theta_B = 0$ ①



①

$q(x) = -w + w\langle x - L\rangle^0$

$EIv^{IV} = q(x)$ ①

$w(x=0) = 0$

$v'(x=0) = 0$

$EIv'''(x = L+a) = -V_0 = 0$ ④

$EIv''(x = L+a) = M_0 = \frac{wL^2}{6}$

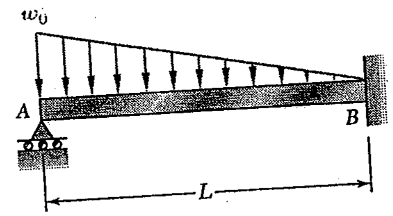
$EIv''' = -wx + w\langle x-L\rangle^1 + C_1$ ④ for coeffs.

$EIv'' = -wx^2/2 + w\langle x-L\rangle^2 + C_1x + C_2$

$EIv' = -wx^3/6 + w\langle x-L\rangle^3/6 + C_1x^2/2 + C_2x + C_3$

Problem 9.21

***9.19* through 9.22** For the beam and loading shown, determine the reaction at the roller support.



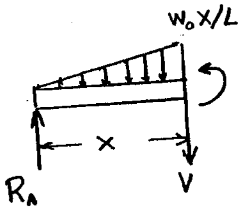
Reactions are statically indeterminate.

Boundary conditions are shown at left.

$[x=0, y=0]$ $[x=L, y=0]$ $[x=L, \frac{dy}{dx}=0]$

$w = \frac{w_0}{L}(L-x)$ (1) $q(x) = \frac{-w_0}{L}(L-x)$

$\frac{dV}{dx} = -w = -\frac{w_0}{L}(L-x)$ $EIv^{IV} = q(x)$



(2)

$\frac{dM}{dx} = V = -\frac{w_0}{L}(Lx - \frac{1}{2}x^2) + R_A$

$M = -\frac{w_0}{L}(\frac{1}{2}Lx^2 - \frac{1}{6}x^3) + R_A x$ (2)

$EI\frac{d^2y}{dx^2} = -\frac{w_0}{L}(\frac{1}{2}Lx^2 - \frac{1}{6}x^3) + R_A x$ (1) $= M$

$EI\frac{dy}{dx} = -\frac{w_0}{L}(\frac{1}{6}Lx^3 - \frac{1}{24}x^4) + \frac{1}{2}R_A x^2 + C_1$

$EI\,y = -\frac{w_0}{L}(\frac{1}{24}Lx^3 - \frac{1}{120}x^5) + \frac{1}{6}R_A x^3 + C_1 x + C_2$ (1)

$[x=0, y=0]$ $0 = 0 + 0 + 0 + C_2$ $C_2 = 0$ (1)

$[x=L, \frac{dy}{dx}=0]$ $-\frac{w_0}{L}(\frac{1}{6}L^4 - \frac{1}{24}L^4) + \frac{1}{2}R_A L^2 + C_1 = 0$

$C_1 = \frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2$ (1)

$[x=L, y=0]$ $-\frac{w_0}{L}(\frac{1}{24}L^4 - \frac{1}{120}L^4) + \frac{1}{6}R_A L^3 + (\frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2)L = 0$

$(\frac{1}{2} - \frac{1}{6})R_A = (\frac{1}{8} - \frac{1}{24} + \frac{1}{120})w_0 L$

$\frac{1}{3}R_A = \frac{11}{120}w_0 L$ $R_A = \frac{11}{40}w_0 L \uparrow$ (1)

$EIv = \iiiint q(x)\,dddd + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4$ (1) (4) for coeffs

$v=0$ @ $x=0$ $v(x=L)=0$

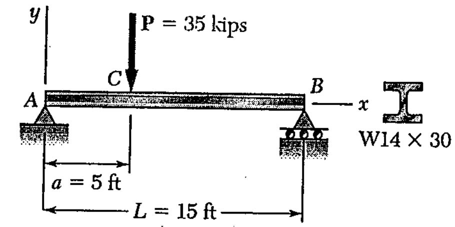
$EIv''=0$ @ $x=0$ $v'(x=L)=0$ (4)

solution is (1)

$-EIv''' = V$ $V = -R_A$

Problem 9.13

9.13 For the beam and loading shown, determine the deflection at point C. Use $E = 29 \times 10^6$ psi.



$[x=0, y=0]$ $[x=L, y=0]$

$[x=a, y=y]$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$

Let $b = L - a$

Reactions: $R_A = \frac{Pb}{L}\uparrow$, $R_B = \frac{Pa}{L}\uparrow$ ②

Bending moments

$0 < x < a \qquad M = \frac{Pb}{L}x$

$a < x < L \qquad M = \frac{P}{L}[bx - L(x-a)]$

$0 < x < a$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}(bx)$$

$$EI\frac{dy}{dx} = \frac{P}{L}(\tfrac{1}{2}bx^2) + C_1 \qquad (1)$$

$$EI\,y = \frac{P}{L}(\tfrac{1}{6}bx^3) + C_1x + C_2 \qquad (2)$$

$a < x < L$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}[bx - L(x-a)]$$

$$EI\frac{dy}{dx} = \frac{P}{L}[\tfrac{1}{2}bx^2 - \tfrac{1}{2}L(x-a)^2] + C_3 \qquad (2)$$

$$EI\,y = \frac{P}{L}[\tfrac{1}{6}bx^3 - \tfrac{1}{6}L(x-a)^3] + C_3x + C_4 \qquad (4)$$

$[x=0, y=0]$ Eq (2) $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$ Eqs. (1) and (3) $\quad \frac{P}{L}(\tfrac{1}{2}ba^2) + C_1 = \frac{P}{L}[\tfrac{1}{2}ba^2 + 0] + C_3 \therefore C_3 = C_1$

$[x=a, y=y]$ Eqs (2) and (4) $\quad \frac{P}{L}(\tfrac{1}{6}ba^3) + C_1a + C_2$

$= \frac{P}{L}[\tfrac{1}{6}ba^3 + 0] + C_1a + C_4 \qquad C_4 = C_2 = 0$

$[x=L, y=0]$ Eq. (4) $\quad \frac{P}{L}[\tfrac{1}{6}bL^3 - \tfrac{1}{6}L(L-a)^3] + C_3L = 0$

$$C_1 = C_3 = \frac{P}{L}[\tfrac{1}{6}(L-a)^3 - \tfrac{1}{6}bL^2] = \frac{P}{L}(\tfrac{1}{6}b^3 - \tfrac{1}{6}bL^2)$$

Make $x = a$ in Eq. (2).

$$y_c = \frac{P}{EIL}[\tfrac{1}{6}ba^3 + \tfrac{1}{6}b^3a - \tfrac{1}{6}bL^2a] = \frac{P(ba^3 + b^3a - L^2ab)}{6EIL}$$

Data: $P = 35$ kips, $E = 29\times10^6$ psi $= 29\times10^3$ kips/in^2

$L = 15$ ft, $a = 5$ ft, $b = 10$ ft.

$I = 291$ in^4, $EI = 8.439\times10^6$ kip·in^2 $= 58.604\times10^3$ kip·ft^2

$$y_c = \frac{35}{(6)(58.604\times10^3)(15)}[(10)(5)^3 + (10^3)(5) - (15)^2(5)(10)]$$

$= -33.179\times10^{-3}$ ft $= -0.398$ in. $\qquad y_c = 0.398$ in. ↓ ◄

$V = -EIv''' = -\frac{Pb}{L} + P\langle x-a\rangle^0$ ①

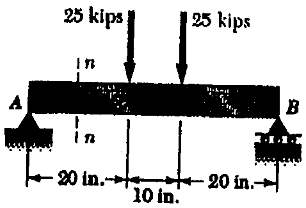
~~$EIv''' = \frac{Pb}{L} - P\langle x-a\rangle^0$~~

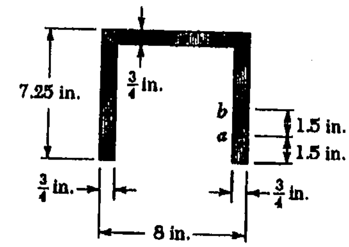
$EIv'' = \frac{Pbx}{L} - P\langle x-a\rangle^1 + C_1$

$EIv = \frac{Pbx^3}{6L} - \frac{P}{6}\langle x-a\rangle^3 + C_1\frac{x^2}{2} + C_2x + C_3$ ③

PROBLEM 6.24

6.23 and 6.24 For the beam and loading shown, determine the largest shearing stress in section *n-n*.



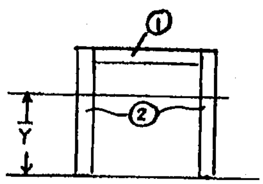


SOLUTION

$R_A = R_B = 25$ kips

At section n-n $V = 25$ kips.

Locate centroid and calculate moment of inertia

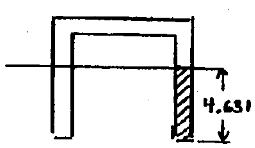


Part	A (in²)	$\bar{y}$ (in)	$A\bar{y}$ (in³)	d (in)	Ad^2 (in⁴)	$\bar{I}$ (in⁴)
①	4.875	6.875	33.52	2.244	24.55	0.23
②	10.875	3.625	39.42	1.006	11.01	47.68
Σ	15.75		72.94		35.56	47.86

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{72.94}{15.75} = 4.631 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 35.56 + 47.86 = 83.42 \text{ in}^4$$

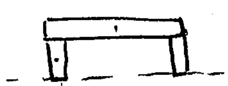
Largest shearing stress occurs on section through centroid of entire cross section.



$$Q = A\bar{y} = \left(\tfrac{3}{4}\right)(4.631)\left(\frac{4.631}{2}\right) = 8.042 \text{ in}^3$$

$$t = \tfrac{3}{4} = 0.75 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(25)(8.042)}{(83.42)(0.75)} = 3.21 \text{ ksi}$$



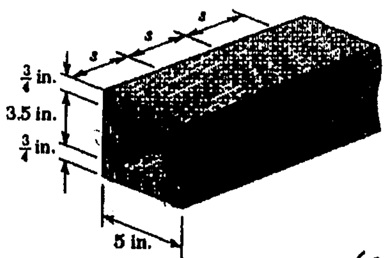
$$Q = (3/4 \times 8)(7.25 - .375 - 4.631) + 2(7.25 - .75 - 4.631)(3/4)(7.25 - .75 - 4.631)/2$$

$=$

$t = 3/4 \times 2$

$$\tau = \frac{VQ}{It}$$

PROBLEM 6.3

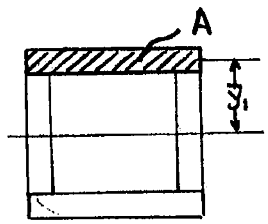


6.3 A square box beam is made of two $\frac{3}{4} \times 3.5$-in. planks and two $\frac{3}{4} \times 5$-in. planks nailed together as shown. Knowing that the spacing between nails is $s = 1.25$ in. and that the vertical shear in the beam is $V = 250$ lb, determine (*a*) the shearing force in each nail, (*b*) the maximum shearing stress in the beam.

SOLUTION

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3 \qquad \frac{1}{12}(120)(120)^3 - \frac{1}{12}(80)(80)^3$$

$$= \frac{1}{12}(5)(5)^3 - \frac{1}{12}(3.5)(3.5)^3 = 39.578 \text{ in}^4 \qquad 13.87 \times 10^{-6} \text{ m}^4$$



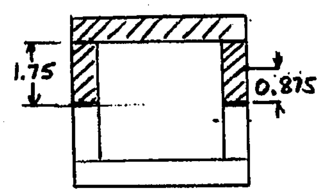
(a) $A = (5)(\frac{3}{4}) = 3.75 \text{ in}^2 \qquad 120 \times 20 = 2400 \text{ mm}^2$

$\bar{y}_1 = 2.5 - \frac{3}{8} = 2.125 \text{ in} \qquad \bar{y} = 50 \text{ mm}$

$Q_1 = A\bar{y}_1 = (3.75)(2.125) = 7.969 \text{ in}^3 \qquad 120 \times 10^{-5} \text{ m}^3$

$q = \frac{VQ}{I} = \frac{(250)(7.969)}{39.578} = 50.34 \text{ lb/in}$

$qs = 2F_{nail} \qquad F_{nail} = \frac{qs}{2} = \frac{(50.34)(1.25)}{2} = 31.5 \text{ lb.}$ ◄



(b) $Q_2 = Q_1 + (2)(1.75)(\frac{3}{4})(0.875)$

$= 7.969 + 2.297 = 10.266 \text{ in}^3$

$t = (2)(\frac{3}{4}) = 1.5 \text{ in.}$

$\tau_{max} = \frac{VQ}{It} = \frac{(250)(10.266)}{(39.578)(1.5)} = 43.2 \text{ psi}$ ◄

$q_{al} = \frac{2F_{nail}}{S\text{ - distance between nail}} = \frac{2(300)}{1.25} = 12 \times 10^3 N$

$q = \frac{VQ}{I} = \frac{V(120 \times 10^{-5})}{13.87 \times 10^{-6}} \Rightarrow V = 1.387 \text{ kN}$

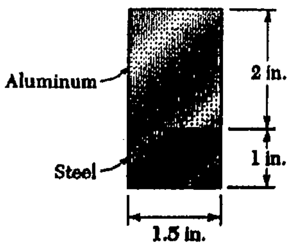
b) $Q = Q_1 + 2A\bar{y}_2 = 2400 \times 50 + 2 \cdot 40 \cdot 20 \cdot 20 = 152 \times 10^{-6} \text{ m}^3$

$\tau_{max} = \frac{VQ}{It} = \qquad 380 \times 10^3 \text{ Pa} = 380 \text{ kPa}$

t: 2×20 mm

PROBLEM 6.59

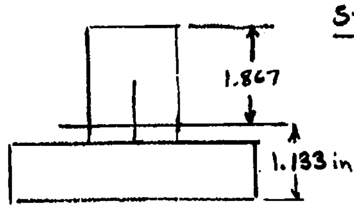
6.59 and 6.60 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum stress in the beam. (*Hint.* Use the method indicated in Prob. 6.56.)



SOLUTION

$n = 1$ in aluminum $\qquad n = \frac{29\times10^6 \text{ psi}}{10.6\times10^6\text{ psi}} = 2.7358$ in steel

Part	nA (in²)	$\bar{y}$ (in)	$nA\bar{y}$ (in³)	d (in)	nAd^2 (in⁴)	$n\bar{I}$ (in⁴)
Alum.	3.0	2.0	6.0	0.8665	2.2525	1.0
Steel	4.1038	0.5	2.0519	0.6335	1.6469	0.3420
Σ	7.1038		8.0519		3.8994	1.3420



$$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma nA} = \frac{8.0519}{7.1038} = 1.1335 \qquad 1.768$$

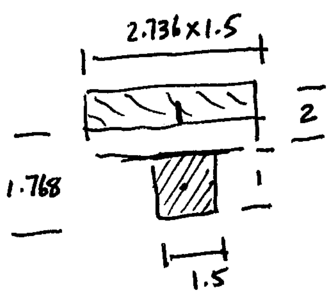
$$I = \Sigma nAd^2 + \Sigma n\bar{I} = 5.2414 \text{ in}^4 \qquad 5.714$$

(a) At the bonded surface $\quad Q = (1.5)(2)(0.8665) = 2.5995 \text{ in}^3$

$$\tau = \frac{VQ}{It} = \frac{(4)(2.5995)}{(5.2414)(1.5)} = 1.323 \text{ ksi}$$

(b) At the neutral axis $\quad Q = (1.5)(1.8665)\left(\frac{1.8665}{2}\right) = 2.6129 \text{ in}^2$

$$\tau_{max} = \frac{VQ}{It} = \frac{(4)(2.6129)}{(5.2414)(1.5)} = 1.329 \text{ ksi}$$



conversion of steel to alum.

$1.768 - .5 = 1.268$

at bonded surface $\tau = \frac{VQ}{It}$

$Q = A_{al}\cdot\bar{y}$
$= (1.5\times1)(1.268)$
$= 1.902 \text{ in}^3$

$\tau = .888$ ksi

at NA in steel $\quad Q = n\cdot A\cdot\bar{y} = 2.736\cdot1.5\cdot1.232\ (1.232/2) = 3.113 \text{ in}^3$

$$\tau_{max} = \frac{VQ}{It} = \frac{4000\,(3.113)}{(5.714)(1.5)} = 1.453 \text{ ksi}$$

50

Florida International University
Department of Mechanical and Materials Engineering

EMA 3702 QUIZ 3A April 10, 2008

You are allowed five sheet of 8 ½ x 11 inch paper with whatever you wish on the sheet

Print your name and sign the following statement:

I will not give nor take any unpermitted aid during this quiz. I understand that violation of this statement will lead to automatic failure of the quiz.

BERNARD SUPERSTEIN
PRINT NAME

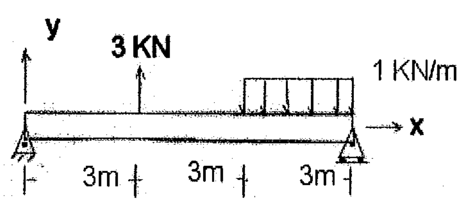
[signature]
SIGN NAME

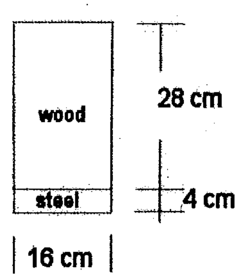
Problem 1.

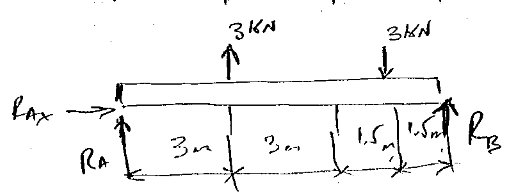
a) Given the following beam loaded as shown, find the elastic curve of the beam in terms of E and I_z. DO NOT DETERMINE I_z for this problem.

b) Determine the slope of the elastic curve at x=6m, just at the start of the distributed load.

STATICALLY DETERMINATE







$\Sigma F_x = 0 \Rightarrow R_{Ay} = 0$

$\Sigma M_A = 0$ $3KN \cdot 3 = 9 \; KN \cdot m$

$3KN \cdot 7.5 = 22.5 \; KN \cdot m$

$R_B \cdot 9 = 9 R_B$

$9_m R_B + 9 \; KN \cdot m = 22.5 \; KN \cdot m$

$9 R_{B\,m} = 13.5 \; KN \cdot m$

$R_B = \frac{13.5}{9} = 1.5 \; KN$

$\boxed{R_B = 1.5 \; KN}$

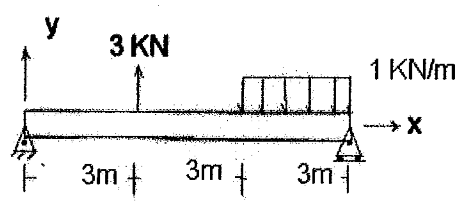
$\Sigma F_y = 0$ $R_A + 3KN - 3KN + 1.5KN = 0$

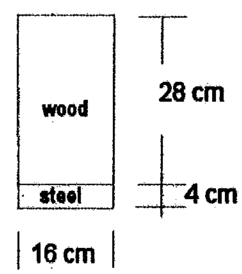
$\boxed{R_A = -1.5 \; KN}$

Problem 2

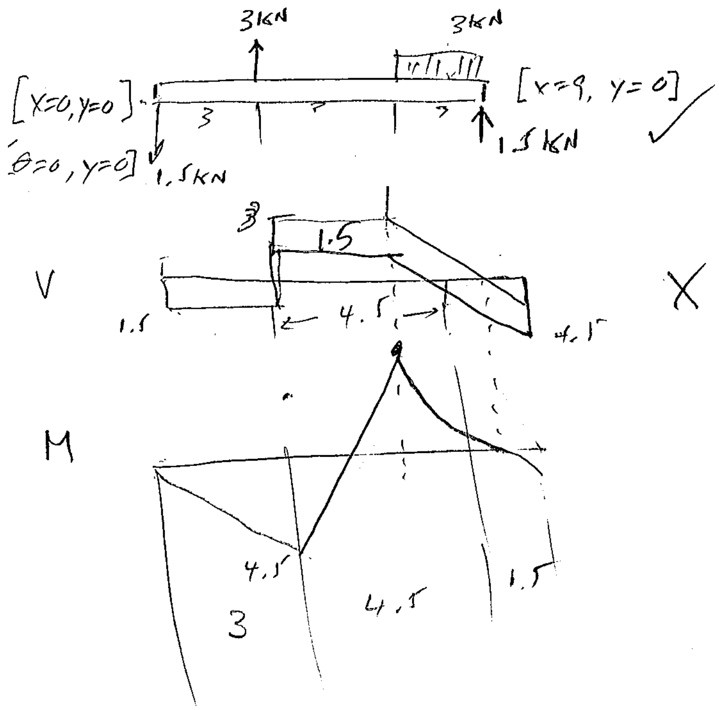
a) For the beam given below, what is the maximum shear stress τ_{xy} and where can it be found, given the following information:
$E_{steel} = 206$ GPa, $E_{wood} = 10.3$ GPa, for the cross-section below.

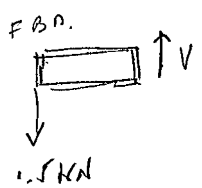
b) Find the shear stress τ_{xy} at the interface between the wood and steel





① cont'd





$V = 1.5\,KN \uparrow$

$M = 1.5x$

$$\frac{M(x)}{EI} = \frac{d^2y}{dx^2}$$

$$EI\frac{d^2y}{dx^2} = M(x) = 1.5x$$

$$EI\frac{dy}{dx} = \frac{1.5x^2}{2} + C_1$$

$$EI\,y = \frac{1.5x^3}{6} + C_1x + C_2 \quad \text{in first section}$$

B.C. $[x=0,\ y=0]$ $\quad 0 = C_2$ $\quad \boxed{C_2 = 0}$

$[x=0,\ \theta=0]$ $\quad 0 = 0 + C_1$ *this is not fixed into wall* $\quad \boxed{C_1 = 0}$

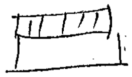
ELASTIC CURVE $\quad \boxed{EI\,y = \frac{1.5x^3}{6}} \rightarrow \boxed{y = \frac{1.x^3}{4EI}}\ ;\ x < 3$

X

what about other sections

(b) @ x=6 m

$w(x) = -(1 kN)\,x$ $\qquad w(x) = -x$



$V = \int w(x)\,dx = \frac{-x^2}{2} + C_1$

$M = \int \frac{-x^2}{2}\,dx = \frac{-x^3}{6} + C_1 x + C_2$

$\frac{dy}{dx} = \theta = \int \frac{-x^3}{6}\,dx = -\frac{x^4}{24} + \frac{C_1 x^2}{2} + C_2 x + C_3$

$\frac{dy}{dx} @ x=6 = \frac{-6^4}{24} = \frac{-1296}{24} = -54$

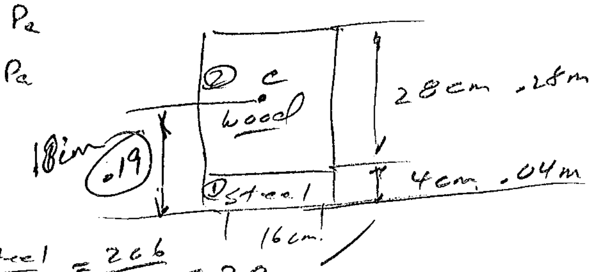
$\frac{dy}{dx} @ x=6 = -54$ X

PROBLEM 2

(a) $\tau_{xy\,MAX}$

$E_{steel} = 206$ GPa

$E_{wood} = 10.3$ GPa



$n = \frac{E_{steel}}{E_{wood}} = \frac{206}{10.3} = 20$

	A	$\bar{y}$	$A\bar{y}$	I ($\frac{1}{12}bh^3$)
①(20)	.0064 m²	.02 m	.00256	$\frac{1}{12}(.16\times20)(.04)^3 = 1.71\times10^{-5}$
②	.0448 m²	.16 m	.00717 m³	$\frac{1}{12}(.16)(.28)^3 = 2.93\times10^{-4}$

Must convert steel to wood. You didn't do it correctly.

see me after class.

$\Sigma A = .0512\ m^3$ $\Sigma A\bar{y} = .00973$

$\Sigma I = 3.098 \times 10^{-4}$ X

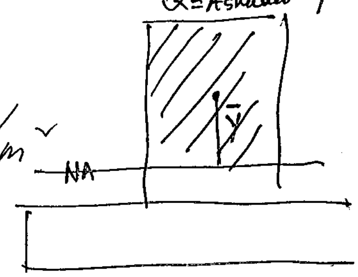
N.A. $\bar{y} = \frac{.00973}{.0512} = .190$ m X

τ_{MAX} @ C furthest from centroid (LOWER EDGE OF BEAM)

$\tau_{MAX} = \frac{VQ}{It}$

↳ when V is max (see problem 1). between 3 and 6 m.

Q of shaded
$Q = A_{shaded} \cdot \bar{y}$



where

$\tau_{MAX} = \frac{(3kN)((.19)(.16)(.095))}{(3.098\times10^{-4})(.16)} = 174.79\ kN/m^2$ X X X

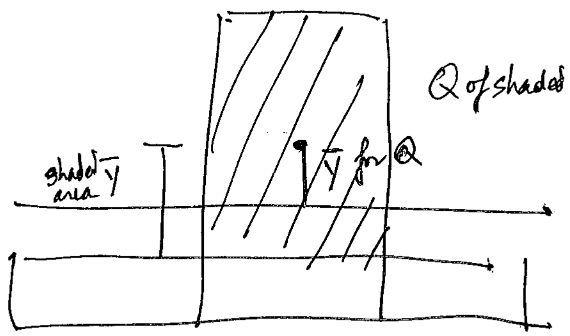
$\tau_{MAX} = 174.79\ kN/m^2$ X

30

(b) τ_{xy} @ INTERFACE

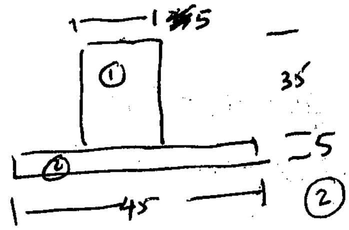
change Q by changing $\bar{y}$ from .16 to .12

$$\tau_{interface} = \frac{(3kN)[(.28)(.16)]}{} \; X$$



$\frac{E_{al}}{E_W} = \frac{92.7}{10.3} = 9$ (1)

Support forces (25)

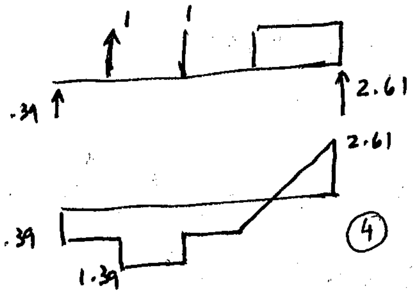


(2)

	$\bar{y}$	A	$\bar{y}A$
①	22.5	175	3937.5
②	2.5	225	562.5
		400	4500

(9)

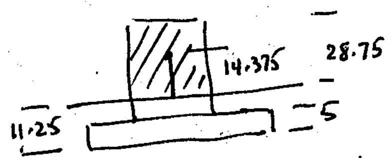
$\bar{y} = \frac{4500}{400} = 11.25$



(4)

V_{max} is at 9 m.
= 2.61 kN

max shear stress is at 11.25 (2)



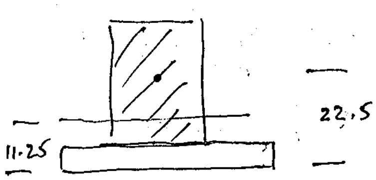
$Q = A\bar{y}$ $A = (40 - 11.25) \cdot 5$ $\bar{y} = 14.375$
$= 28.75 \cdot 5$
$= 143.75\,cm$

$Q = 143.75 \cdot 14.375$
$= 2066.41\,cm^3$ (4)

$I =$		A	d	
①	$\frac{1}{12}(5)(35)^3$	175	22.5 − 11.25	40013.02
②	$\frac{1}{12}(45)(5)^3$	225	2.5 − 11.25	17695.31
				57708.33

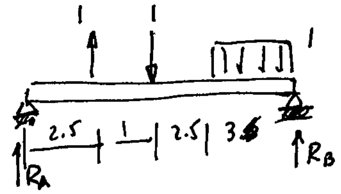
(9)

$$\tau_{max} = \frac{(2610)\ 2066.41 \times 10^{-6}}{57708.33 \times 10^{-8} \cdot 5 \times 10^{-2}}$$ (5)
$= 186916.87\ Pa = .187\ MPa$



$A = (5 \times 35)$ $\bar{y} = 22.5 - 11.25 = 11.25$
$Q = 175(11.25) = 1968.75$ (4)

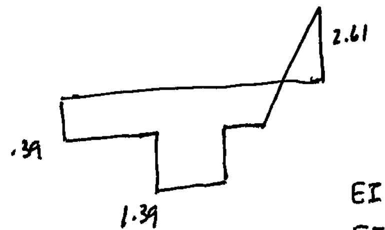
$$\tau_{int} = \frac{(2610)(1968.75)}{(57708.33 \times 10^{-8})(5 \times 10^{-2})} = .178\ MPa$$ (5)



$1\cdot 2.5 - 1\cdot 3.5 - 1\cdot 3\cdot 7.5 + R_B\cdot 9$

$-1.0 - 22.5 + R_B\cdot 9 = 0 \quad R_B = \frac{23.5}{9} = 2.61\ kN$

$R_A = .39\ kN$



$$q(x) = -w_0 \langle x - \frac{2L}{3}\rangle^0 + P_1 \langle x - \frac{2.5}{9}L\rangle^{-1} - P_2 \langle x - \frac{3.5}{9}L\rangle^{-1}$$

$$EI v^{IV} = q(x) = -w_0 \langle x - \frac{2L}{3}\rangle^0 + P_1 \langle x - \frac{2.5}{9}L\rangle^{-1} + P_2 \langle x - \frac{3.5}{9}L\rangle^{-1}$$

$$EI v''' = -V = -w_0 \langle x - \frac{2L}{3}\rangle^1 + P_1 \langle x - \frac{2.5}{9}L\rangle^0 - P_2 \langle x - \frac{3.5}{9}L\rangle^0 + C_1 = R_A \qquad C_1 = R_A = .39\ kN$$

$$EI v'' = M = -\frac{w_0}{2}\langle x - \frac{2L}{3}\rangle^2 + P_1 \langle x - \frac{2.5}{9}L\rangle^1 - P_2 \langle x - \frac{3.5}{9}L\rangle^1 + C_1 x + C_2$$

$@\ x=0 \quad M=0 \quad \Rightarrow \quad 0 + 0 + 0 + 0 + C_2 = 0 \Rightarrow C_2 = 0$

$x = L \quad M = 0 \quad \Rightarrow \quad -\frac{w_0}{2}\frac{L^2}{9} + P_1 \frac{6.5}{9}L - P_2 \frac{5.5}{9}L + C_1 L = 0 \qquad C_1 = P_2\cdot\frac{5.5}{9} - P_1\cdot\frac{6.5}{9} + \frac{w_0 L}{18} = 388.89\ N$

$$EI v'' = M = -\frac{w_0}{2}\langle x - \frac{2L}{3}\rangle^2 + P_1 \langle x - \frac{2.5}{9}L\rangle^1 - P_2 \langle x - \frac{3.5}{9}L\rangle^1 + \frac{5.5}{9}P_2 x - \frac{6.5}{9}P_1 x + \frac{w_0 L x}{18}$$

$$EI v' = -\frac{w_0}{6}\langle x - \frac{2L}{3}\rangle^3 + \frac{P_1}{2}\langle x - \frac{2.5}{9}L\rangle^2 - \frac{P_2}{2}\langle x - \frac{3.5}{9}L\rangle^2 + \frac{5.5}{18}P_2 x^2 - \frac{6.5}{18}P_1 x^2 + \frac{w_0 L x^2}{36} + C_3$$

$$EI v = -\frac{w_0}{24}\langle x - \frac{2L}{3}\rangle^4 + \frac{P_1}{6}\langle x - \frac{2.5}{9}L\rangle^3 - \frac{P_2}{6}\langle x - \frac{3.5}{9}L\rangle^3 + \frac{5.5}{54}P_2 x^3 - \frac{6.5}{54}P_1 x^3 + \frac{w_0 L}{108}x^3 + C_3 x + C_4$$

$@\ x=0 \quad v=0 \quad = \quad 0 + 0 - 0 + 0 - 0 + 0 + 0 + C_4 \quad \Rightarrow C_4 = 0$

$$@\ x=L \quad v=0 \quad = -\frac{w_0}{24}\frac{L^4}{81} + \frac{P_1}{6}\left(\frac{6.5}{9}\right)^3 L^3 - \frac{P_2}{6}\left(\frac{5.5}{9}\right)^3 L^3 + \frac{5.5}{54}P_2 L^3 - \frac{6.5}{54}P_1 L^3 + \frac{w_0 L^4}{108} + C_3 L = 0$$

$$= \frac{17 w_0 L^4}{24\cdot 81} + P_1 L^3 \left\{\left(\frac{6.5}{9}\right)^3\cdot\frac{1}{6} - \frac{6.5}{54}\right\} - P_2 L^3 \left\{\left(\frac{5.5}{9}\right)^3\frac{1}{6} - \frac{5.5}{54}\right\} + C_3 L = 0$$

$$C_3 = P_2 L^2 \left\{\left(\frac{5.5}{9}\right)^3\cdot\frac{1}{6} - \frac{5.5}{54}\right\} - P_1 L^2 \left\{\left(\frac{6.5}{9}\right)^3\cdot\frac{1}{6} - \frac{6.5}{54}\right\} - \frac{17 w_0 L^3}{24\cdot 81}$$

$-.0638 \qquad -.0576$

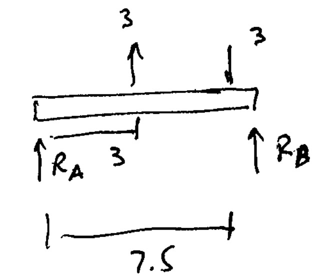
$-5.169 P_2 \qquad +4.6644 P_1$

$$EI v' = -\frac{w_0}{6}\langle x - \frac{2L}{3}\rangle^3 + \frac{P_1}{2}\langle x - \frac{2.5}{9}L\rangle^2 - \frac{P_2}{2}\langle x - \frac{3.5}{9}L\rangle^2 + \frac{5.5}{18}P_2 x^2 - \frac{6.5}{18}P_1 x^2 + w_0\frac{L x^2}{36} + C_3 \qquad -.0638 P_2 L^2 + .0576 P_1 L^2 - \frac{17}{1944}w_0 L^3$$

$$@\ x=3.5 \quad v' = \frac{1}{EI}\left\{ 0 + \frac{P_1}{2}1^2 - \frac{P_2}{2}\cdot 0^2 + \frac{5.5}{18}P_2\cdot 3.5^2 - \frac{6.5}{18}P_1\cdot 3.5^2 + w_0\cdot 9\cdot\frac{3.5^2}{36} - 5.169 P_2 + 4.6644 P_1 - \frac{17}{24}w_0\cdot 9\right.$$

$$= \frac{1}{EI}\left\{ 500 + 3743.056 - 4423.611 + 3062.5 - 5169 + 4664.4 - 6375 \right\} = -\frac{3997.655}{EI}$$

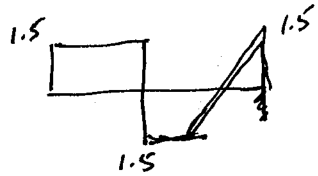
$$EI v = -\frac{1000}{24}\langle x-6\rangle^4 + \frac{1000}{6}\langle x-2.5\rangle^3 - \frac{1000}{6}\langle x-3.5\rangle^3 + \underbrace{101.85x^3 - 120.37x^3 + 83.3x^3 - 53387.4x}_{64.78x^3 - 53387.4x}$$



$$\not{3}\cdot 3 - \not{3}\cdot 7.5 + R_B \cdot \not{9}^{\,3} = 0$$

$$R_B = \frac{4.5}{3} = 1.5$$

$$R_A = -1.5$$



$$q(x) = -1000\langle x-6\rangle^0 + 3000\langle x-3\rangle^{-1}$$

$$EIv^{IV} = -1000\langle x-6\rangle^0 + 3000\langle x-3\rangle^{-1}$$

$$EIv''' = -V = -1000\langle x-6\rangle^1 + 3000\langle x-3\rangle^0 + C_1$$

$V = -R_A$

$V = -R_A - 3000\langle x-3\rangle^0$

$\frac{dV}{dx} = -q = -3000\langle x-3\rangle^{-1}$

$$EIv'' = M = -500\langle x-6\rangle^2 + 3000\langle x-3\rangle^1 + C_1x + C_2$$

$@\ x=0\quad M=0 \Rightarrow C_2 = 0 \qquad x=9 \quad M=0$

$$-500\langle 9\rangle + 3000\langle 6\rangle + C_1 \cdot 9 = 0$$

$$-\frac{13500}{9} = C_1 = -1500$$

$$EIv'' = M = -500\langle x-6\rangle^2 + 3000\langle x-3\rangle^1 + 1500x$$

$$EIv' = -\frac{500}{3}\langle x-6\rangle^3 + 1500\langle x-3\rangle^2 - 750x^2 + C_3$$

$$EIv = -\frac{500}{12}\langle x-6\rangle^4 + 500\langle x-3\rangle^3 - 250x^3 + C_3x + C_4$$

$x=0 \quad v=0 \Rightarrow C_4 = 0$

$x=9 \quad v=0 \qquad -\frac{500}{12}\cdot 81 + 500\cdot 6^3 - 250\cdot 9^3 + C_3\cdot 9 = 0$

$$C_3 = 500\cdot\frac{9}{12} - 500\cdot 24 + 250\cdot 81$$

$@\ x=6 \qquad v' = \frac{1}{EI}\left\{0 + 1500\cdot 9 - 750\cdot 36 + 500\cdot\frac{9}{12} - 500\cdot 24 + 250\cdot 81\right\}$

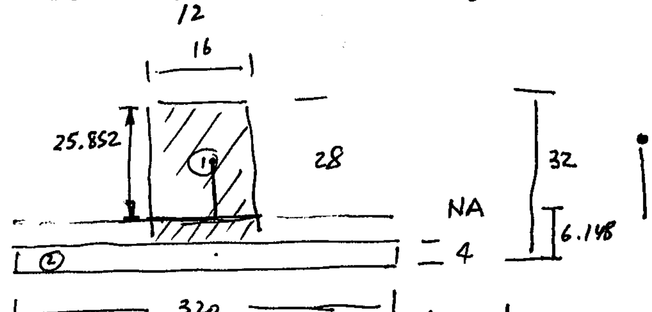
3A Prob 1

$n = 20$ steel wood to ~~steel~~

	$\bar{y}$	A	$\bar{y}A$
1	18	16·28 = 448	8064
2	2	320·4 = 1280	2560
		1728	10624

8960 − 896 = 8064

$$\bar{y} = \frac{10624}{1728} = 6.148$$



I	A	d	
$\frac{1}{12}16\cdot 28^3$	16·28	18−6.148	92199.85 cm^4
$\frac{1}{12}\cdot 320\cdot 4^3$	320·4	6.148−2	23730.22
			115930.07 cm^4 or .00116 m^4

max shear stress is where max shear is. Max shear is anywhere betwe $x=0$ & $x=6$ & at $x=9$

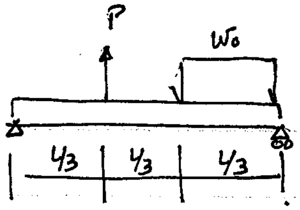
$$\tau_{max} = \frac{VQ}{I_z t} = \frac{1500\,Q}{I_z t} = \frac{1500\left[16\left(32-6.148\right)\left(25.852/2\right)\right]}{I_z\cdot 16\,cm} = 4.324\ \frac{N}{cm^2} = 43.24\ kPa$$

18−6.148

$A = 413.632\ cm^2$

$\bar{y} = 12.926$

$Q = 5346.607\ cm$



$$\Sigma F_y = 0 = R_A + P - w_0 \cdot L/3 + R_B = 0$$

$$\Sigma M_A = 0 = P \cdot L/3 - \frac{w_0 \cdot L}{3} \cdot \frac{5L}{6} + R_B \cdot L = 0 \qquad R_B = -P/3 + \frac{w_0 \cdot 5L}{18}$$

$$R_A = -\frac{2P}{3} + \frac{w_0 L}{18}$$

$$q(x) = -w_0 \langle x - \tfrac{2L}{3} \rangle^0 + P \langle x - L/3 \rangle^{-1}$$

$$EIv^{IV} = q(x) = -w_0 \langle x - \tfrac{2L}{3} \rangle^0 + P \langle x - L/3 \rangle^{-1}$$

$$EIv''' = -V = -w_0 \langle x - 2L/3 \rangle^1 + P \langle x - L/3 \rangle^0 + C_1$$

$$EIv'' = M = -\frac{w_0}{2} \langle x - 2L/3 \rangle^2 + P \langle x - L/3 \rangle^1 + C_1 x + C_2$$

$$@\ x=0 \quad M=0 \quad = \quad 0 \quad + \quad 0 \quad + \quad 0 + C_2 \quad \Rightarrow \quad C_2 = 0$$

$$@\ x=L \quad M=0 \quad = -\frac{w_0}{2} (L/3)^2 + P \langle 2/3 L \rangle + C_1 \cdot L = 0 \qquad C_1 = \frac{w_0}{18} L - P \cdot 2/3$$

$$EIv'' = M = -\frac{w_0}{2} \langle x - 2L/3 \rangle^2 + P \langle x - L/3 \rangle^1 + \frac{w_0 L x}{18} - \frac{2Px}{3}$$

$$EIv' = -\frac{w_0}{6} \langle x - 2L/3 \rangle^3 + \frac{P}{2} \langle x - L/3 \rangle^2 + \frac{w_0 L x^2}{36} - \frac{Px^2}{3} + C_3$$

$$EIv = -\frac{w_0}{24} \langle x - 2L/3 \rangle^4 + \frac{P}{6} \langle x - L/3 \rangle^3 + \frac{w_0 L x^3}{108} - \frac{Px^3}{9} + C_3 x + C_4$$

$$@\ x=0 \quad v=0 \quad = \quad 0 \quad + \quad 0 \quad + 0 - 0 + 0 + C_4 \Rightarrow C_4 = 0$$

$$@\ x=L \quad v=0 \quad = \frac{-w_0}{24} \frac{L^4}{81} + \frac{P}{6} \cdot \frac{8}{27} L^3 + \frac{w_0 L^4}{108} - \frac{PL^3}{9} + C_3 L$$

$$= \frac{17 w_0 L^4}{24 \cdot 81} \quad \frac{-10 PL^3}{162} + C_3 L \qquad \text{or} \quad C_3 = \frac{10}{162} PL^2 - \frac{17 w_0 L^3}{24 \cdot 81}$$

$$\therefore \quad v = \frac{1}{EI} \left\{ \frac{-w_0}{24} \langle x - \tfrac{2L}{3} \rangle^4 + \frac{P}{6} \langle x - L/3 \rangle^3 + \frac{w_0 L x^3}{108} - \frac{Px^3}{9} + \frac{10}{162} PL^2 x - \frac{17}{1944} w_0 L^3 x \right\}$$

$$R_A = -2000 + 1000 \cdot 9/18 = -1500$$
$$R_B = -1000 + \frac{1000 \cdot 5 \cdot 9}{18} = 1500$$

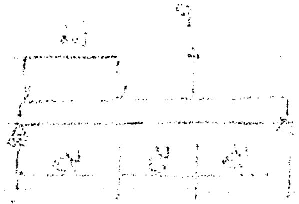
In the case $w_0 = 1000 \frac{N}{m}$ $\quad P = 3000\ N \quad L = 9$

$$\frac{1}{EI} \left\{ -41.67 \langle x-6 \rangle^4 + 500 \langle x-3 \rangle^3 + 83.33 x^3 - 333.33 x^3 + 15000x - 6375x \right\}$$

$$v = \frac{1}{EI} \left\{ -41.67 \langle x-6 \rangle^4 + 500 \langle x-3 \rangle^3 - 250 x^3 + 8625x \right\}$$

$$v' = \frac{1}{EI} \left\{ -4 \cdot 41.67 \langle x-6 \rangle^3 + 1500 \langle x-3 \rangle^2 - 750 x^2 + 8625 \right\}$$

$$@\ x=6 \qquad v' = \frac{1}{EI} \left\{ \quad 0 \quad + 13500 - 27000 + 8625 \right\} = \frac{-4875}{EI}$$



$$\Sigma F_y = -R_A + P - w_0 \cdot L/3 + R_B = 0$$

$$\Sigma M_A = 0 = -P \cdot \frac{L}{3} - \frac{w_0 L}{3} \cdot \frac{5L}{6} + R_B \cdot L = 0 \quad R_B = \frac{P}{3} + \frac{5 w_0 L}{18}$$

$$R_A = \frac{2P}{3} + \frac{w_0 L}{18}$$

$$q(x) = -w_0 \langle x - 2L/3 \rangle^0 + P \langle x - L/3 \rangle^{-1}$$

$$EIv'''' = q(x) = -w_0 \langle x - 2L/3 \rangle^0 + P \langle x - L/3 \rangle^{-1}$$

$$EIv''' = -V = -w_0 \langle x - 2L/3 \rangle^1 + P \langle x - L/3 \rangle^0 + C_1$$

$$EIv'' = M = -\frac{w_0}{2} \langle x - 2L/3 \rangle^2 + P \langle x - L/3 \rangle^1 + C_1 x + C_2$$

@ $x = 0$ $M = 0 = 0 + 0 + 0 + C_2 \Rightarrow C_2 = 0$

@ $x = L$ $M = 0 = -\frac{w_0}{2}(L/3)^2 + P(2L/3) + C_1 L = 0 \quad C_1 = \frac{w_0 L}{18} - \frac{2P}{3}$

$$EIv'' = M = -\frac{w_0}{2} \langle x - 2L/3 \rangle^2 + P \langle x - L/3 \rangle^1 + \frac{w_0 L x}{18} - \frac{2Px}{3}$$

$$EIv' = -\frac{w_0}{6} \langle x - 2L/3 \rangle^3 + \frac{P}{2} \langle x - L/3 \rangle^2 + \frac{w_0 L x^2}{36} - \frac{Px^2}{3} + C_3$$

$$EIv = -\frac{w_0}{24} \langle x - 2L/3 \rangle^4 + \frac{P}{6} \langle x - L/3 \rangle^3 + \frac{w_0 L x^3}{108} - \frac{Px^3}{9} + C_3 x + C_4$$

@ $x = 0$ $v = 0 = 0 + 0 + 0 - 0 + 0 + C_4 \Rightarrow C_4 = 0$

@ $x = L$ $v = 0 = -\frac{w_0}{24}\frac{L^4}{81} + \frac{P}{6}\frac{8L^3}{27} + \frac{w_0 L^4}{108} - \frac{PL^3}{9} + C_3 L$

[illegible]

$$v = \frac{1}{EI}\left\{ -\frac{w_0}{24} \langle x - 2L/3 \rangle^4 + \frac{P}{6} \langle x - L/3 \rangle^3 + \frac{w_0 L x^3}{108} - \frac{Px^3}{9} + [illegible] \right\}$$

[illegible]

$$\frac{1}{EI}\left\{ [illegible] \right\}$$

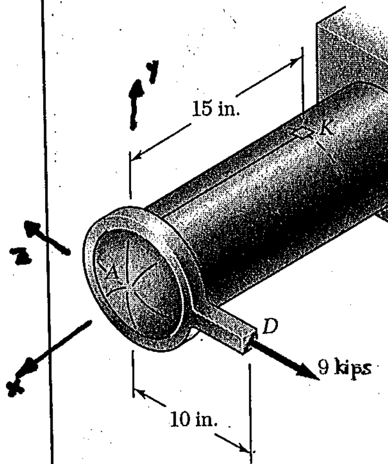
$$v' = \frac{1}{EI}\left\{ [illegible] \right\}$$

$$v'' = \frac{1}{EI}\left\{ [illegible] \right\}$$

[illegible]

Problem 7.162

7.162 The cylindrical tank *AB* has an 8-in. inner diameter and a 0.32-in. wall thickness. Knowing that the pressure inside the tank is 600 psi, determine the maximum normal stress and the maximum shearing stress at point *K* located on the top of the tank.



$r_i = \frac{d_i}{2} = 4 \text{ in} \qquad r_o = r_i + t = 4.32 \text{ in.}$

$\sigma_1 = \frac{p r_i}{t} = \frac{(600)(4)}{0.32} = 7500 \text{ psi} = 7.50 \text{ ksi}$

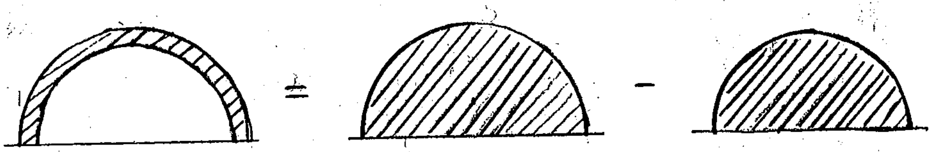
$\sigma_2 = \frac{1}{2}\sigma_1 = 3.75 \text{ ksi}$

Torsion: No applied torque

Bending: Point K lies on neutral axis.

Transverse shear: $V = 9 \text{ kips}$

For semicircle: $A = \frac{\pi}{2}r^2$, $\bar{y} = \frac{4r}{3\pi}$, $Q = \frac{2}{3}r^3$



$Q = Q_o - Q_i = \frac{2}{3}r_o^3 - \frac{2}{3}r_i^3 = \frac{2}{3}(4.32^3 - 4^3) = 11.081 \text{ in}^3$

$t = (2)(0.32) = 0.64 \text{ in}$

$I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}(4.32^4 - 4^4) = 72.481 \text{ in}^4$

$\tau = \frac{VQ}{It} = \frac{(9)(11.081)}{(72.481)(0.64)} = 2.15 \text{ ksi}$

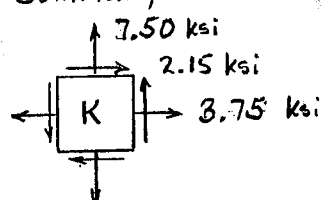
Summary of stresses: L = Longitudinal, C = Circumferential, Shear

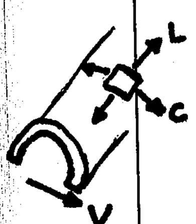
$\sigma_x = \sigma_1 = 3.75 \text{ ksi}$

$\sigma_y = \sigma_2 = 7.50 \text{ ksi}$

$\tau_{xy} = 2.15 \text{ ksi}$

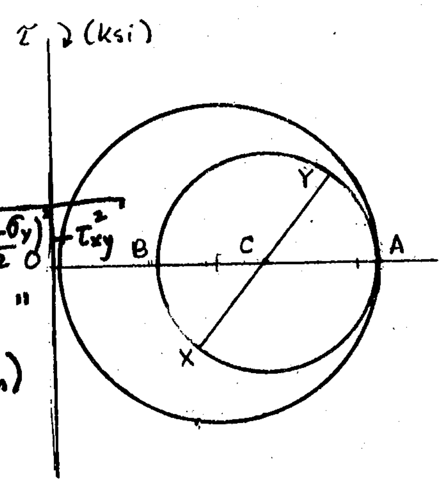
Draw stress state





$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 5.625 \text{ ksi}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 2.853 \text{ ksi}$



$\sigma_{max} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\sigma_{min} = \text{''} - \text{''}$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$

$\sigma_a = \sigma_{ave} + R = 8.48 \text{ ksi}$

$\sigma_b = \sigma_{ave} - R = 2.77 \text{ ksi}$

$\sigma_z = 0$

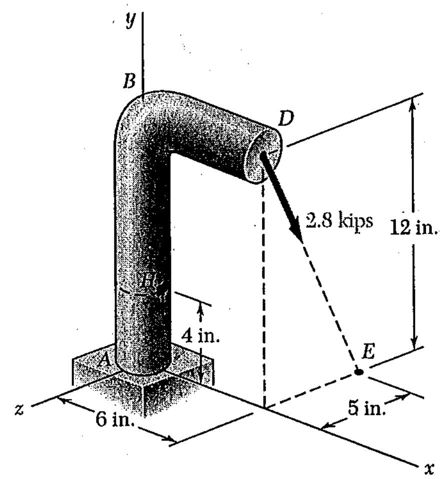
$\sigma_{max} = 8.48 \text{ ksi}$

$\sigma_{min} = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 4.24 \text{ ksi}$

Problem 8.44

8.44 A 2.8-kip force is applied as shown to the 2.4-in.-diameter cast-iron post *ABD*. At point *H*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.



$DE = \sqrt{15^2 + 12^2} = 13$ in.

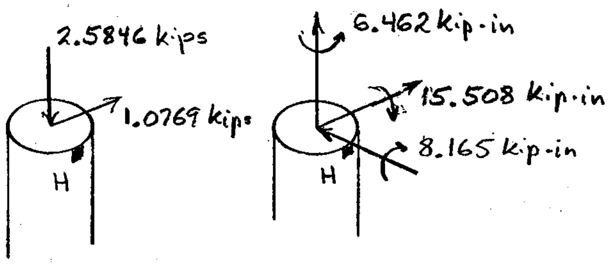
At point D $\quad F_x = 0$

$F_y = -\left(\frac{12}{13}\right)(2.8) = -2.5846$ kips

$F_z = -\left(\frac{5}{13}\right)(2.8) = -1.0769$ kips

Moment of equivalent force-couple system at C, the centroid of the section containing point H

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 8 & 0 \\ 0 & -2.5846 & -1.0769 \end{vmatrix} = 8.165\,\vec{i} + 6.462\,\vec{j} - 15.508\,\vec{k} \text{ kip}\cdot\text{in}$$



Section properties

$d = 2.4$ in. $\quad c = \frac{1}{2}d = 1.2$ in.

$A = \pi c^2 = 4.5239\ \text{in}^2$

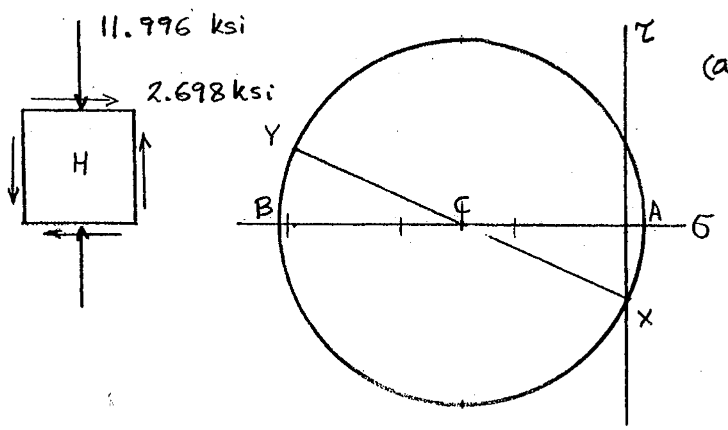
$I = \frac{\pi}{4}c^4 = 1.6286\ \text{in}^4$

$J = 2I = 3.2572\ \text{in}^4$

For a semicircle $\quad Q = \frac{2}{3}c^3 = 1.152\ \text{in}^3$

At point H $\quad \sigma_H = -\frac{P}{A} - \frac{Mc}{I} = -\frac{2.5846}{4.5239} - \frac{(15.508)(1.2)}{1.6286} = -11.996$ ksi

$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(6.462)(1.2)}{3.2572} + \frac{(1.0769)(1.152)}{(1.6286)(2.4)} = 2.698$ ksi



(a) $\sigma_c = \frac{\sigma_H}{2} = -5.998$ ksi

$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_H^2} = 6.677$ ksi

$\sigma_a = \sigma_c + R = 0.579$ ksi ◄ =3.93 MPa

$\sigma_b = \sigma_c - R = -12.58$ ksi ◄ −86.73

$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = 0.4497$

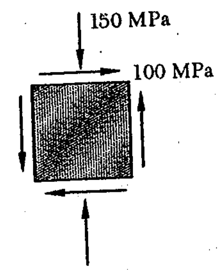
$\theta_a = 12.1°, \quad \theta_b = 102.1°$ ◄

(b) $\tau_{max} = R = 6.58$ ksi ◄

46.7 MPA

Problem 7.161

7.161 The state of plane stress shown is expected to occur in a cast-iron machine base. Knowing that for the grade of cast iron used $\sigma_{UT} = 160$ MPa and $\sigma_{UC} = 320$ MPa and using Mohr's criterion, determine whether rupture of the component will occur.

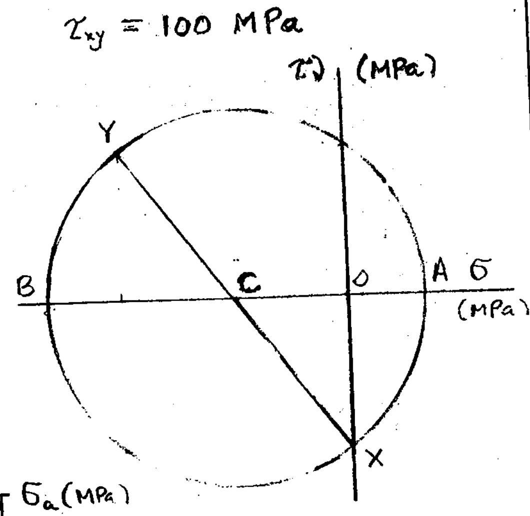


$\sigma_x = 0 \qquad \sigma_y = -150 \text{ MPa} \qquad \tau_{xy} = 100 \text{ MPa}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -75 \text{ MPa}$

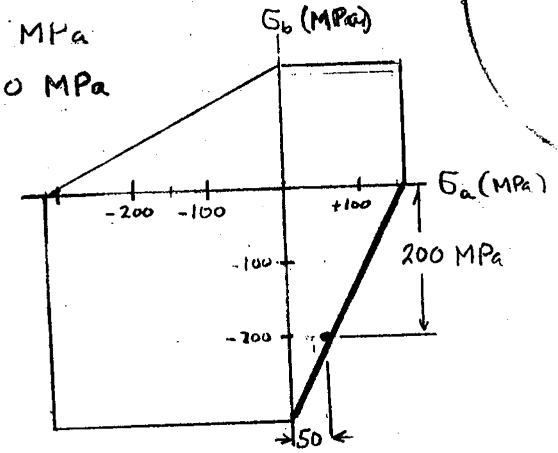
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{75^2 + 100^2} = 125 \text{ MPa}$



$\sigma_a = \sigma_{ave} + R = 50 \text{ MPa}$

$\sigma_b = \sigma_{ave} - R = -200 \text{ MPa}$

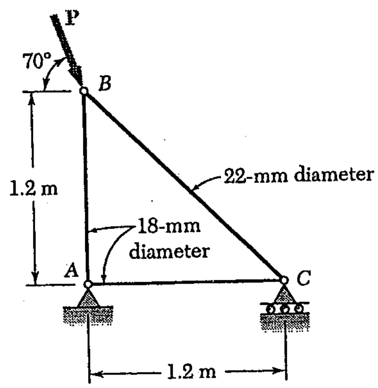


Equation of the 4th quadrant boundary is $\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$

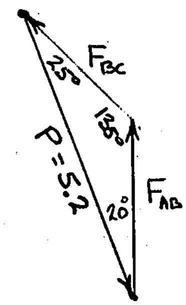
$\frac{50}{160} - \frac{(-200)}{320} = 0.9375 < 1$, No rupture.

Problem 10.20

10.20 Knowing that $P = 5.2$ kN, determine the factor of safety for the structure shown. Use $E = 200$ GPa and consider only buckling in the plane of the structure.



Joint B: From force triangle,



$$\frac{F_{AB}}{\sin 25^\circ} = \frac{F_{BC}}{\sin 20^\circ} = \frac{5.2}{\sin 135^\circ}$$

$F_{AB} = 3.1079$ kN (comp)

$F_{BC} = 2.5152$ kN (comp)

Member AB: $I_{AB} = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{18}{2}\right)^4 = 5.153\times10^3 \text{ mm}^4 = 5.153\times10^{-9} \text{ m}^4$

$$F_{AB,cr} = \frac{\pi^2 E I_{AB}}{L_{AB}^2} = \frac{\pi^2(200\times10^9)(5.153\times10^{-9})}{(1.2)^2} = 7.0636\times10^3 \text{ N} = 7.0636 \text{ kN}$$

$$F.S. = \frac{F_{AB,cr}}{F_{AB}} = \frac{7.0636}{3.1079} = 2.27$$

Member BC: $I_{BC} = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{22}{2}\right)^4 = 11.499\times10^3 \text{ mm}^4 = 11.499\times10^{-9} \text{ m}^4$

$L_{BC}^2 = 1.2^2 + 1.2^2 = 2.88 \text{ m}^2$

$$F_{BC,cr} = \frac{\pi^2 E I_{BC}}{L_{BC}^2} = \frac{\pi^2(200\times10^9)(11.499\times10^{-9})}{2.88} = 7.8813\times10^3 \text{ N} = 7.8813 \text{ kN}$$

$$F.S. = \frac{F_{BC,cr}}{F_{BC}} = \frac{7.8813}{2.5152} = 3.13$$

Smallest F.S. governs. F.S. = 2.27 ◄

Florida International University
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EMA 3702 QUIZ 4C April 8, 2004

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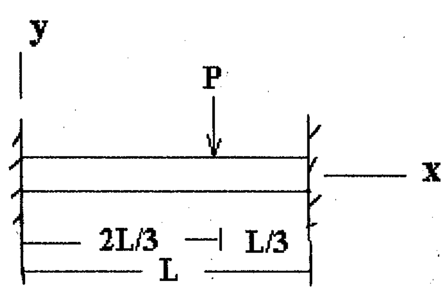
____________________ ____________________
PRINT NAME SIGN NAME

Problem 1.

a) Given the following beam loaded as shown, find the deflection v as a function of x.

b) What is the moment at x=L ?



$q(x) = +P\langle x - \frac{2L}{3}\rangle^{-1}$

$EIv'''' = -q = -P\langle x - \frac{2L}{3}\rangle^{-1}$ (1)

$EIv''' = -P\langle x - \frac{2L}{3}\rangle^{0} + C_1$ (2)

$EIv'' = -P\langle x - \frac{2L}{3}\rangle^{1} + C_1 x + C_2$ (3)

$EIv' = -\frac{P}{2}\langle x - \frac{2L}{3}\rangle^{2} + C_1 \frac{x^2}{2} + C_2 x + C_3$ (4)

$EIv = -\frac{P}{6}\langle x - \frac{2L}{3}\rangle^{3} + C_1 \frac{x^3}{6} + C_2 \frac{x^2}{2} + C_3 x + C_4$ (5)

@ $x=0$ $v=0 \Rightarrow C_4 = 0$ (1)

$v'=0 \Rightarrow C_3 = 0$ (1)

@ $x=L$ $v=0 \Rightarrow -\frac{P}{6}(L/3)^3 + C_1 L^3/6 + C_2 L^2/2 = 0$ (1)

$v'=0 \Rightarrow -\frac{P}{2}(L/3)^2 + C_1 \cdot \frac{L^2}{2} + C_2 L = 0$ (1)

27 grades x2

$n=10$
$\bar{x} = 52$
$\sigma = 22.32$

$-\frac{PL^2}{18} + C_1\frac{L^2}{2} + C_2 L = 0$ $C_2 L = -C_1 L^2/2 + \frac{PL^2}{18}$

$\therefore -\frac{2 \cdot PL^3}{2 \cdot 162} + \frac{2C_1 L^3}{2 \cdot 6} + \frac{L}{2}\left[-C_1 L^2/2 + \frac{PL^2}{18}\right] = 0$

$+\frac{9PL^3}{9 \cdot 36}$ $-\frac{3C_1 L^3}{3 \cdot 4}$

$+\frac{7PL^3}{324} - \frac{C_1 L^3}{12} = 0$ (2)

$C_1 = +\frac{84}{324}P = \frac{+28}{108}P = \frac{+7}{27}P$

$v = \frac{1}{EI}\left\{\frac{-P}{6}\langle x - \frac{2L}{3}\rangle^3 + \frac{7}{27}\frac{Px^3}{6} - \frac{PL^2}{27}x^2\right\}$ (3)

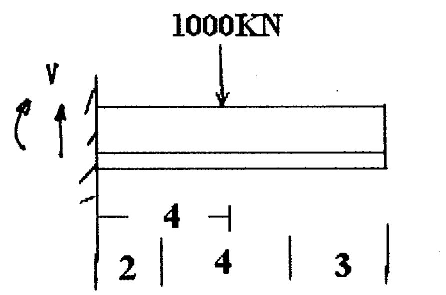
$M = EIv'' = -P\langle x - \frac{2L}{3}\rangle^1 + (+\frac{7}{27}P)x - \frac{2PL}{27} = -\frac{9 PL}{...} + \frac{7}{27}PL - \frac{2PL}{27} = -\frac{4PL}{...}$

$C_2 = -\frac{7}{...}PL + \frac{3PL}{...} = -\frac{4PL}{...} = -\frac{2PL}{...}$

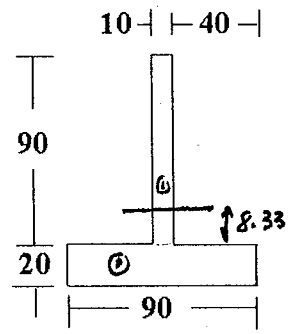
Problem 2c.

a) The figure gives the beam with the loads applied to it, as well as its cross-section. Find the maximum shear stress on the beam and where the maximum shear stress is located on the cross section. **Units of length for the cross section are in cm.**

b) What is the shear stress one meter from the left end of the beam and at a distance of 45 cm from the top of the cross section ?

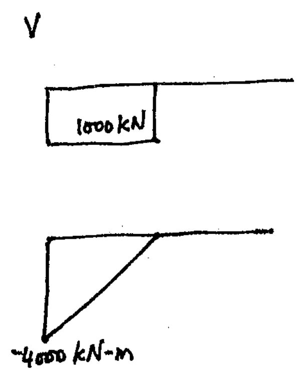


units of length in meters



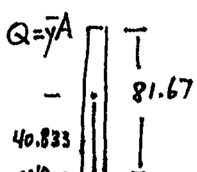
	A	$\bar{y}$	$A\bar{y}$
①	90·10	65	58500
②	20·90	10	18000
	2700 cm²		76500 cm³

$\bar{y} = 28.33$ cm ⑩



τ_{max} @ support $\tau = \frac{VQ}{It}$

	I_{zz}	A	d	I_{zz}'
①	$\frac{1}{12}\cdot 10\cdot 90^3$	900	65-28.33	1817500.22 cm⁴ .018175 m⁴
②	$\frac{1}{12}\cdot 90\cdot 20^3$	1800	10-28.33	664997.8 cm⁴ .00665 m⁴
				.024825 m⁴ ⑩



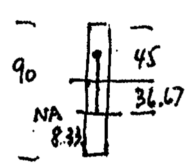
$Q = (81.67\cdot 10)\cdot 40.833 = 33350\ cm^3$

$= .03335\ m^3$ ③

$\tau_{max} = \frac{1000\times 10^3 N\cdot (.03335\ m^3)}{.024825\ m^4 \cdot 0.1\ m}$

$= 13.434$ MPa ⑤

ⓑ



$Q = \bar{y}A$ $A = 45\times 10 = 450 cm^2$

$\bar{y} = 36.67 + 22.5 = 59.17$ cm

$Q = 26626.5\ cm^3$

$= .026627\ m^4$ ③

$\tau = \frac{1\times 10^6 N\,(.026627\ m^4)}{.024825\ m^4\,(0.1\ m)} = 10.726$ MPa

⑤

36 grad

× $\frac{4}{3}$

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EMA 3702 QUIZ 4A April 8, 2004

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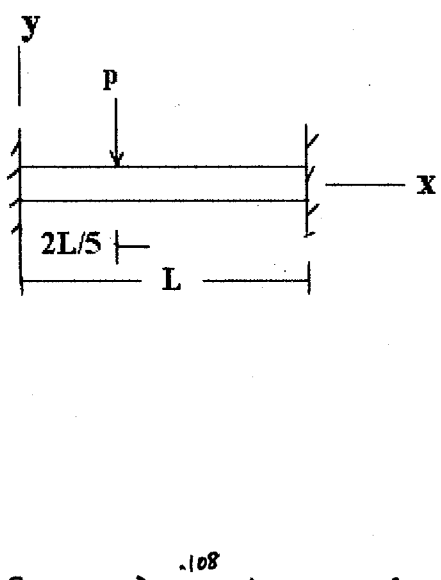
PRINT NAME SIGN NAME

Problem 1.

a) Given the following beam loaded as shown, find the deflection v as a function of x.

b) What is the moment at x=0 ?



$q(x) = P\langle x - 2L/5\rangle^{-1}$

$EIv'''' = -q = -P\langle x-\frac{2L}{5}\rangle^{-1}$ (1)

$EIv''' = -V = -P\langle x-\frac{2L}{5}\rangle^{0} + C_1$ (2)

$EIv'' = M = -P\langle x-2L/5\rangle^{1} + C_1x + C_2$ (3)

$EIv' = -\frac{P}{2}\langle x-2L/5\rangle^{2} + C_1 x^2/2 + C_2 x + C_3$ (4)

$EIv = -P/6\langle x-\frac{2L}{5}\rangle^{3} + C_1 x^3/6 + C_2 x^2/2 + C_3 x + C_4$ (5)

@ $x=0$ $v=0 \Rightarrow C_4 = 0$ (1)

$x=0$ $v'=0 \Rightarrow C_3 = 0$ (1)

@ $x=L$ $v=0 \Rightarrow -\frac{P}{6}\left(\frac{3L}{5}\right)^3 + C_1 L^3/6 + C_2 L^2/2 = 0$ (1)

$v'=0 \Rightarrow -P/2\left(\frac{3L}{5}\right)^2 + C_1 L^2/2 + C_2 L = 0$ (1)

$$C_1 = \frac{\begin{pmatrix} +P/6 \cdot \frac{27L^3}{125} & L^2/2 \\ P/2 \cdot 9L^2/25 & L \end{pmatrix}}{\begin{pmatrix} L^3/6 & L^2/2 \\ L^2/2 & L \end{pmatrix}} = \frac{\frac{2\cdot 9PL^4}{2\cdot 250} - \frac{5\cdot 9PL^4}{5\cdot 100}}{\frac{2L^4}{12} - \frac{3L^4}{12}} = \frac{\frac{-27}{500}PL^4}{\frac{-L^4}{12}} = \frac{324}{500}P \quad (2)$$

$$C_2 = \frac{\begin{pmatrix} L^3/6 & P/6\left(\frac{27L^3}{125}\right) \\ L^2/2 & P/2\left(9L^2/25\right) \end{pmatrix}}{-L^4/12} = \frac{\frac{5\cdot 9PL^5}{5\cdot 12\cdot 25} - \frac{27PL^5}{60\cdot 25}}{-L^4/12} = \frac{\frac{18PL^5}{60\cdot 25}}{-L^4/12} \quad (2)$$

$v = -\frac{P}{6}\langle x-\frac{2L}{5}\rangle^3 + \frac{.648Px^3}{6} - .072PLx^2$ (3) [.108]

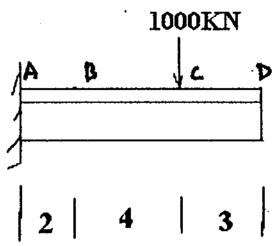
@ $x=0$ $M = EIv'' = C_2 = -.144PL$ (1)

$n = 11$
$\bar{x} = 66.18$
$\sigma = 18.86$

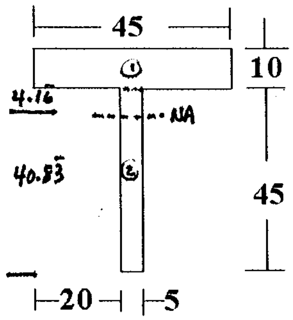
Problem 2a.

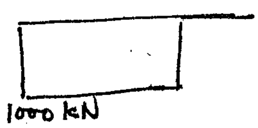
a) The figure gives the beam with the loads applied to it, as well as its cross-section. Find the largest shear stress 2 meters from the left end and its location on the cross section. **Units of length for the cross section are in cm.**

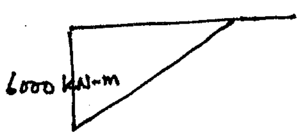
b) What is the shear stress at a distance of 5 cm from the top of the cross section ?



units of length in meters







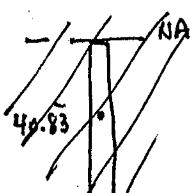
	A	$\bar{y}$	$A\bar{y}$
①	45·10	50	22500
②	45×5	22.5	5062.5
	675		27562.5

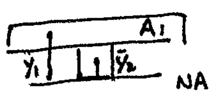
$\bar{y} = 40.8\bar{3}$ (10)

	I_{zz}	A	d	I_{zz}'
①	$\frac{1}{12}\cdot 45\cdot 10^3$	450	$50 - 40.8\bar{3}$	41562.78
②	$\frac{1}{12}\cdot 5\cdot 45^3$	225	$22.5 - 40.8\bar{3}$	113593.48
				155156.25 cm⁴
				.00155156 m⁴

(10)

@ 2 meters V = 1000 kN & largest shear stress is at N.A.





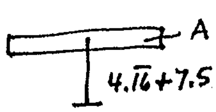
$A_1 = 45\cdot 10$ $\bar{y}_1 = 4.1\bar{6} + 5 = 9.1\bar{6}$

$A_2 = 4.1\bar{6}\times 10$ $\bar{y}_2 = 4.1\bar{6}/2$

$Q = A_1\bar{y}_1 + A_2\bar{y}_2 =$ 4[illegible]1 cm³ $= .004$[illegible] m³ (3)

$\tau = \frac{VQ}{It} = \frac{1000\times 10^3 N\ (.004\text{[illegible]} m^3)}{(.00155156\ m^4)(.05 m)} = 53.739$ MPa (5)

@ 5cm from top



$4.1\bar{6} + 7.5 = 11.6\bar{6}$ cm $= \bar{y}$ A = 45×5 = 225 cm² (3)

$Q = 2625$ cm³ $= .002625$ m³

between A & B $\tau = \frac{VQ}{It} = \frac{1000\times 10^3 N\ (.002625 m^3)}{.00155156 m^4\ (.05m)} = 3.756$ MPa (5)

between B & D $\tau = 0$

36 gradables ×4/3

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Department of Mechanical and Materials Engineering

EMA 3702 QUIZ 4B April 8, 2004

You are allowed three sheets of 8 ½ x 11 inch paper, two new sheets and one with whatever you wish except solutions

Print your name and sign the following statement:

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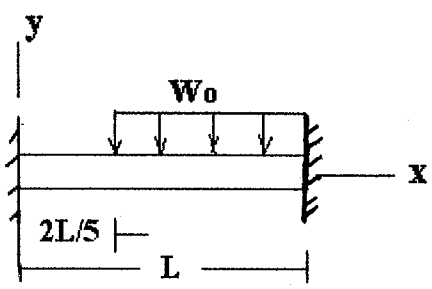
PRINT NAME SIGN NAME

Problem 1.

a) Given the following beam loaded as shown, find the deflection v as a function of x.

b) What is the shear at x=0 ?



$$EI v^{IV} = -q = -w_0 \langle x - \tfrac{2L}{5} \rangle^0 \quad (1)$$

$$EI v''' = -V = -w_0 \langle x - 2L/5 \rangle^1 + C_1 \quad (2)$$

$$EI v'' = M = -\frac{w_0}{2} \langle x - 2L/5 \rangle^2 + C_1 x + C_2 \quad (3)$$

$$EI v' = -\frac{w_0}{6} \langle x - 2L/5 \rangle^3 + C_1 x^2/2 + C_2 x + C_3 \quad (4)$$

$$EI v = -\frac{w_0}{24} \langle x - 2L/5 \rangle^4 + C_1 x^3/6 + C_2 x^2/2 + C_3 x + C_4 \quad (5)$$

@ $x=0$ $v=0 \Rightarrow C_4 = 0$ (1)
$v'=0 \Rightarrow C_3 = 0$ (1)

@ $x=L$ $v=0 \Rightarrow -\frac{w_0}{24}\left(\frac{3L}{5}\right)^4 + C_1 L^3/6 + C_2 L^2/2 = 0$ (1)
$v'=0 \Rightarrow -\frac{w_0}{6}(3L/5)^3 + C_1 L^2/2 + C_2 L = 0$ (1)

$$C_1 = \frac{\begin{pmatrix} \frac{w_0}{24}\cdot\frac{81}{625}L^4 & L^2/2 \\ \frac{w_0}{6}\cdot\frac{27}{125}L^3 & L \end{pmatrix}}{\begin{pmatrix} L^3/6 & L^2/2 \\ L^2/2 & L \end{pmatrix}} = \frac{\frac{w_0}{24}\frac{81}{625}L^5 - \frac{w_0}{12}\cdot\frac{27}{125}L^5}{-\frac{L^4}{12}} = \frac{-.0126 w_0 L}{-.0833 L^4} = -.1512 w_0 L \quad (2)$$

$$C_2 = \frac{\begin{matrix} \frac{L^3}{6} & \frac{w_0}{24}\cdot\frac{81}{625}L^4 \\ L^2/2 & \frac{w_0}{6}\cdot 27/125 L^3 \end{matrix}}{-\frac{L^4}{12}} = \frac{\frac{w_0 L^6 \cdot 27}{36\cdot 125} - \frac{w_0 L^6 \cdot 81}{48\cdot 625}}{-\frac{L^4}{12}} = \frac{.0033 w_0 L^6}{-.0833 L^4}$$

$$v = \frac{1}{EI}\left\{ \frac{-w_0}{24}\langle x - \tfrac{2L}{5}\rangle^4 - \frac{.1512 w_0 L x^3}{6} + \frac{.0396 w_0 L^2 x^2}{2} \right\} \quad (3)$$

$$EI v''' = -V = -w_0 \langle x - \tfrac{2L}{5} \rangle^1 + C_1 = -.1512 w_0 L \text{ @ } x=0$$

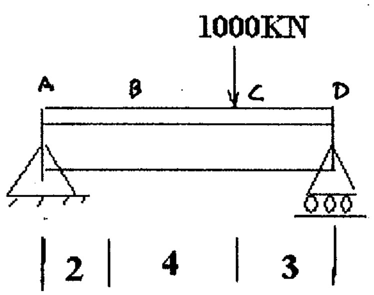
$V = .1512 w_0 L$ (1)

$n = 11$
$\bar{x} = 63.73$
$\sigma = 14.88$

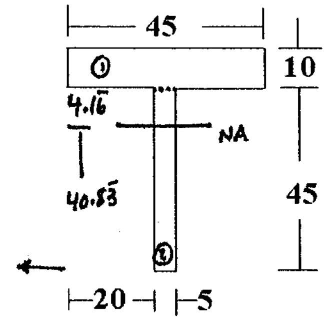
Problem 2b.

a) The figure gives the beam with the loads applied to it, as well as its cross-section. Find the maximum shear stress on the beam and where it is located on the cross section. **Units of length for the cross section are in cm.**

b) What is the largest shear stress two meters from the left end of the beam and where is it located?



units of length in meters



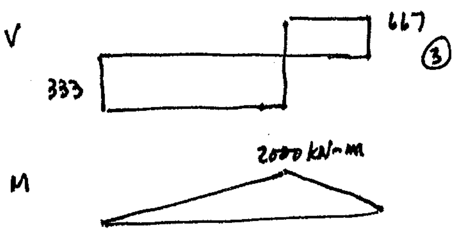
$\Sigma M_A = -1000 \cdot 6 + R_D \cdot 9 = 0$

$R_D = 667$ kN

$R_A = 333$ kN

	A	$\bar{y}$	$A\bar{y}$
①	450	50	22500
②	225	22.5	5062.5
	675		27562.5

$\bar{y} = 40.8\bar{3}$ (10)

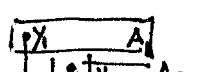


(3)

	I_{zz}	A	d	$I_{z'z'}$
①	$\frac{1}{12} 45 \cdot 10^3$	450	$50 - 40.8\bar{3}$	41562.78
	$\frac{1}{12} \cdot 5 \cdot 45^3$	225	$22.5 - 40.8\bar{3}$	113593.48
				$.00155156\ m^4$

(10)

τ_{max} is at N.A. between CD   V there is 667 kN



$A_1 = 45 \times 10$ $\bar{y}_1 = 4.1\bar{6} + 5 = 9.1\bar{6}$

$A_2 = 4.1\bar{6} \times 5$ $\bar{y}_2 = \frac{4.1\bar{6}}{2}$

$Q = A_1\bar{y}_1 + A_2\bar{y}_2 = 4169.81\ cm^3$
$= .004169\ m^3$ (5)

$\tau = \frac{VQ}{It} = \frac{667 \times 10^3 N \cdot (.004169 m^3)}{.00155156\ m^4\ (.05 m)} = 35.826$ MPa (5)

2 meters from the left end $V = 333.33$ kN & τ_{max} is at N.A.   Q is the same, I same, t same

$\therefore$ since $V = \frac{1}{2}$ of V_{CD} then $\tau = \frac{1}{2} \tau_{CD} = 17.913$ MPa (5)

36 grad

$\times 4/3$

SOLUTIONS

Florida International University
Department of Mechanical and Materials Engineering

EMA 3702 QUIZ 4D April 8, 2004

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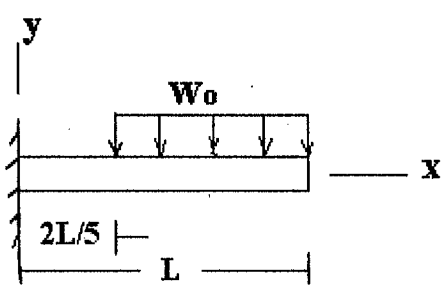
PRINT NAME SIGN NAME

Problem 1.

a) Given the following beam loaded as shown, find the deflection v as a function of x.

c) What is the shear at x=L ?



$$EI\,v^{IV} = -q = -w_0\langle x-\tfrac{2L}{5}\rangle^0 \quad (1)$$

$$-V = EI\,v''' = -w_0\langle x-\tfrac{2L}{5}\rangle^1 + C_1 \quad (2)$$

$$M = EI\,v'' = -\frac{w_0}{2}\langle x-\tfrac{2L}{5}\rangle^2 + C_1 x + C_2 \quad (3)$$

$$EI\,v' = -\frac{w_0}{6}\langle x-\tfrac{2L}{5}\rangle^3 + C_1\frac{x^2}{2} + C_2 x + C_3 \quad (4)$$

$$EI\,v = -\frac{w_0}{24}\langle x-\tfrac{2L}{5}\rangle^4 + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4 \quad (5)$$

@ $x=0$ $v=0 \Rightarrow C_4 = 0$ (1)

$v'=0 \Rightarrow C_3 = 0$ (1)

@ $x=L$ $M=0 \Rightarrow -\frac{w_0}{2}(3L/5)^2 + w_0\cdot\frac{3L}{5}\cdot L + C_2 = 0$ (1) $C_2 = -\frac{21}{50}w_0L^2$ (2)

$V=0 \Rightarrow -w_0\cdot\frac{3L}{5} + C_1 = 0$ (1) $C_1 = w_0\cdot\frac{3L}{5}$ (2)

$$v = \frac{1}{EI}\left\{-\frac{w_0}{24}\langle x-\tfrac{2L}{5}\rangle^4 + \frac{3w_0}{30}Lx^3 - \frac{21}{100}w_0L^2x^2\right\} \quad (3)$$

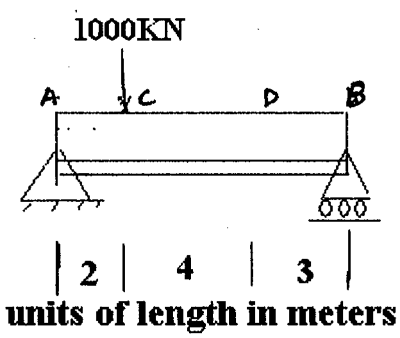
b) shear = 0 @ $x=L$ (1)

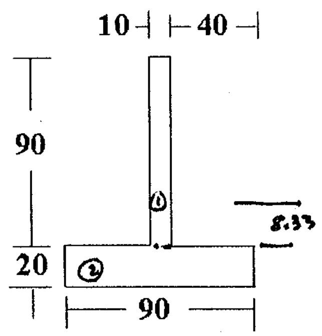
$n=10$
$\bar{x} = 60.3$
$\sigma = 13.89$

Problem 2d.

a) The figure gives the beam with the loads applied to it, as well as its cross-section. Find the maximum shear stress and its location on the cross section. **Units of length for the cross section are in cm.**

b) What is the largest shear stress six meters from the left end and where is it located ?

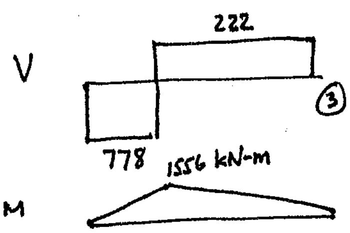




$\Sigma M_A = -1000 \times 2 + R_B \cdot 9$ $R_B = 2000/9$ $R_A = 7000/9$

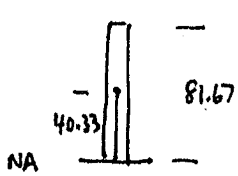
	A	$\bar{y}$	$A\bar{y}$
①	900	65	58500
②	1800	10	18000
	2700 cm²		76500 cm³

$\bar{y} = 28.33$ cm (10)



	I_{zz}	A	d	$I_{z'z'}$
①	$\frac{1}{12}\cdot 10 \cdot 90^3$	900	65−28.33	.018175 m⁴
②	$\frac{1}{12}\, 90 \cdot 20^3$	1800	10−28.33	.00665 m⁴
				.024825 m⁴ (10)

τ max occurs in section AC



$Q = \bar{y}A$
$= 40.33 \cdot (81.67 \cdot 10)\ cm^3$
$= .03335\ m^3$ (3)

$\tau_{max} = \frac{VQ}{It} = \frac{778 \times 10^3 N \cdot .03335\ m^3}{.024825 m^4\ (0.1 m)}$
$= 10.452$ MPa (5)

(b) at 6 meters V = 222 kN Q is the same as in (a) since τ_{max} is at neutral axis

$\tau = \frac{222 \times 10^3 N\ (.03335\ m^3)}{.024825\ m^4\ (0.1 m)} = 2.982$ MPa

(5)

36 grad
×4/3

Florida International University
Department of Mechanical and Materials Engineering

EMA 3702 QUIZ 2B February 25, 2004

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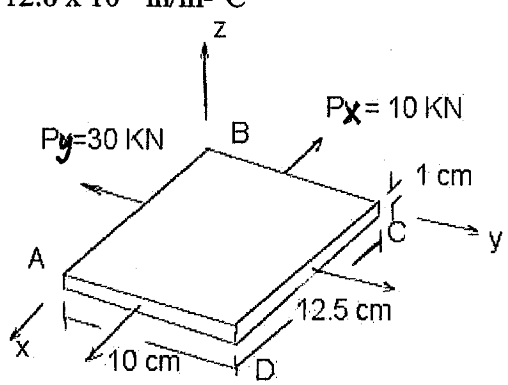
a) For the plate shown, find the maximum elongation that the side AB will see if the plate is acted upon by the loads shown and has the dimensions shown. The plate has a Young's Modulus, E = 100 GPa.

b) Find the strain in the Z direction when, on top of any loads, the plate sees a temperature **decrease** ΔT of 20 °C.

You are given that the shear modulus, G = 38 GPa and the coefficient of thermal expansion, α, is 12.8×10^{-6} in/in-°C



$$\sigma_x = \frac{P_x}{A} = 10\ MPa$$

$$\sigma_y = \frac{P_y}{A} = 24\ MPa$$

$$\sigma_z = 0$$

$$\nu = \frac{E}{2G} - 1 = .316$$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z) = \frac{10 \times 10^6}{100 \times 10^9} - \frac{.316}{100 \times 10^9}(24 \times 10^6) = 2.416 \times 10$$

$$u = \epsilon_x \cdot 12.5cm = 3.02 \times 10^{-6} m = 3.02 \times 10^{-3} mm$$

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y) + \alpha\Delta T = \frac{-.316}{100 \times 10^9}(34 \times 10^6) + 12.8 \times 10^{-6}(-$$

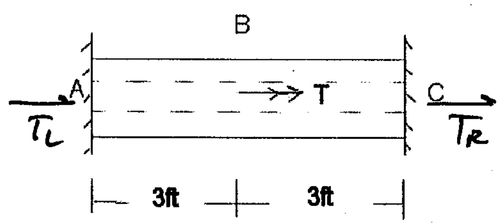
$$= -3.64 \times 10^{-4} m = -3.64 \times 10^{-1}$$

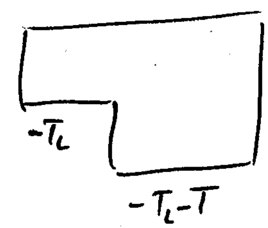
n=10
$\bar{x}$ = 62.6
σ = 21.03

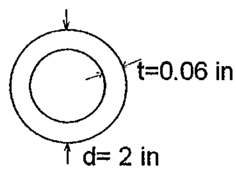
Problem.

The member, having the **cylindrical** cross-section shown, is to be formed from the sheet metal of 0.06 in. thickness. If the member is fixed between two walls and has an applied torque, T, of 2500 lb-in., determine the maximum angle of twist and its location. The Young's Modulus, E = 30 x 10^6 psi and the Poisson ratio, ν =0.3.

$$\Sigma T = T_L + T + T_R = 0$$







$$\Delta\varphi = \frac{-T_L \cdot 36}{JG} + \frac{(-T_L - T)\cdot 36}{JG} = 0$$

$$-T_L \cdot 72 - 2500 \cdot 36 = 0 \qquad T_L = -1250$$
$$T_R = -1250$$

$$\Delta\varphi_{max} = \frac{1250 \cdot 36}{J \cdot G}$$

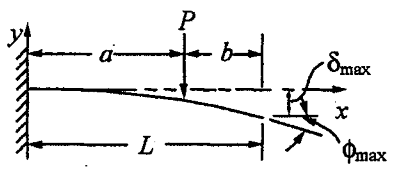
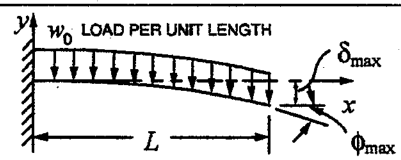
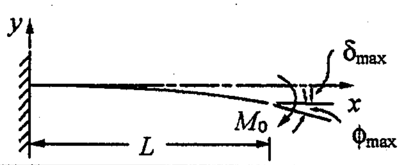
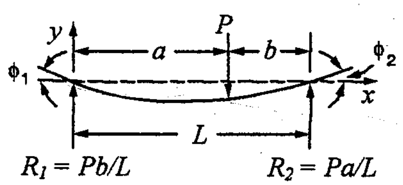
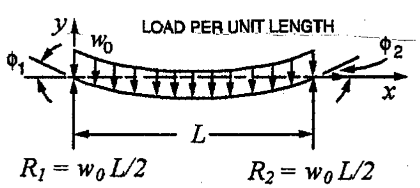
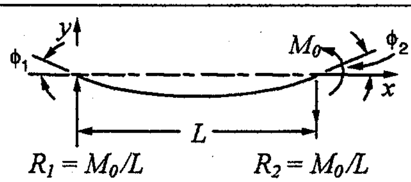
$$G = \frac{E}{2(1+\nu)} = \frac{30}{2.6} = 11.54 \times 10^6$$

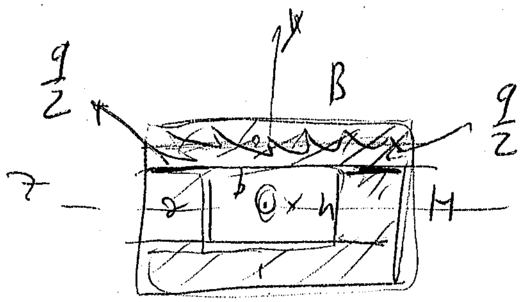
$$\frac{\pi}{2}(c_o^4 - c_i^4) = \frac{\pi}{2}(c_o^2 + c_i^2)(c_o^2 - c_i^2)$$
$$2c_o^2 \cdot 2c_o t$$
$$= 2\pi c_o^3 t$$

$$J = 2\pi c^3 t = 2\pi(1)(.06)$$
$$= .377 in^4$$

$$\Delta\varphi_{max} = \frac{1250(36)}{.38 \times 11.7 \times 10^6} = .0103 rad \approx .593°$$

(δ is positive downward)

	$\delta=\frac{Pa^2}{6EI}(3x-a)$, for $x>a$ $\delta=\frac{Px^2}{6EI}(-x+3a)$, for $x\le a$	$\delta_{max}=\frac{Pa^2}{6EI}(3L-a)$	$\phi_{max}=\frac{Pa^2}{2EI}$
	$\delta=\frac{w_o x^2}{24EI}\left(x^2+6L^2-4Lx\right)$	$\delta_{max}=\frac{w_o L^4}{8EI}$	$\phi_{max}=\frac{w_o L^3}{6EI}$
	$\delta=\frac{M_o x^2}{2EI}$	$\delta_{max}=\frac{M_o L^2}{2EI}$	$\phi_{max}=\frac{M_o L}{EI}$
	$\delta=\frac{Pb}{6LEI}\left[\frac{L}{b}(x-a)^3-x^3+\left(L^2-b^2\right)x\right]$, for $x>a$ $\delta=\frac{Pb}{6LEI}\left[-x^3+\left(L^2-b^2\right)x\right]$, for $x\le a$	$\delta_{max}=\frac{Pb\left(L^2-b^2\right)^{3/2}}{9\sqrt{3}LEI}$ at $x=\sqrt{\frac{L^2-b^2}{3}}$	$\phi_1=\frac{Pab(2L-a)}{6LEI}$ $\phi_2=\frac{Pab(2L-b)}{6LEI}$
	$\delta=\frac{w_o x}{24EI}\left(L^3-2Lx^2+x^3\right)$	$\delta_{max}=\frac{5w_o L^4}{384EI}$	$\phi_1=\phi_2=\frac{w_o L^3}{24EI}$
	$\delta=\frac{M_o Lx}{6EI}\left(1-\frac{x^2}{L^2}\right)$	$\delta_{max}=\frac{M_o L^2}{9\sqrt{3}EI}$ at $x=\frac{L}{\sqrt{3}}$	$\phi_1=\frac{M_o L}{6EI}$ $\phi_2=\frac{M_o L}{3EI}$



$$\frac{BH^3}{12} - \frac{bh^3}{12}$$

$$q = \frac{VQ}{I_t}$$

1. Write the second-order differential equation for the bending of the column shown in Fig. P1-2 and use it to determine the critical load of the column. At its lower end the column is completely fixed. At the upper end the column is prevented from rotating, but free to translate laterally. (ans: $P_{cr}=\pi^2 EI/L^2$)

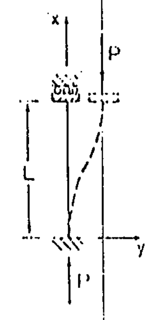
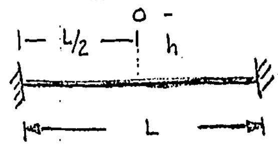


Fig. P1-2

2. Find an expression for the maximum stress when a ball weighing W Newtons is dropped onto a fixed-fixed beam.



3. A linearly elastic beam-column having a flexural rigidity EI, is subjected to a thrust P and a moment M_0 as shown in Fig. A below.

(a) Determine the lateral displacement v(x).

(b) From part (a), write the solution for the system subjected to a force P acting as shown in Fig. B.

(c) Determine Δ_c, the horizontal displacement of point C, assuming small rotations. (Assume also that the horizontal displacement of point B is negligible).

(d) Determine the bending moment M(x).

(g) Explain why, although the beam is made of a linearly elastic material, the results of part (c) are non-linear.

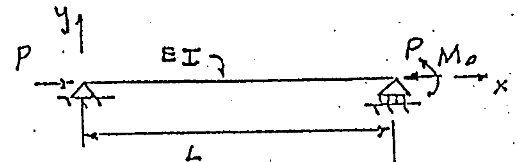


Fig. A

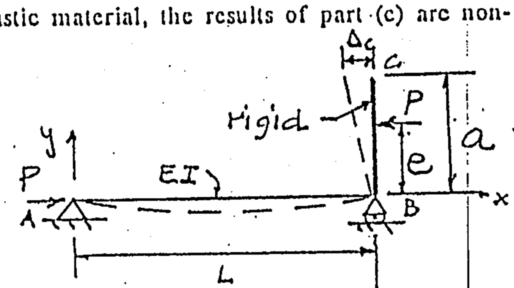


Fig. B

Answers :

(a) $y(x) = -\frac{M_0}{P}\left[\frac{\sin Kx}{\sin KL} - \frac{x}{L}\right], K^2 = \frac{P}{EI}$

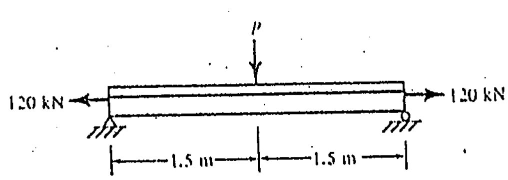
(c) $\Delta_c = \frac{ac}{L}\left[1 - L\sqrt{P/EI}\cot\left(L\sqrt{P/EI}\right)\right] = \frac{ac}{L}(1 - KL\cot KL)$

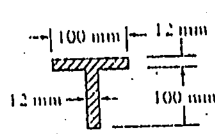
(d) $M(x)=M_0 \sin kl/\sin kL$

*12.10 A T section carries an axial tensile force of 120 kN, applied through the centroid of the cross section. The allowable stress in tension or compression is 130 MPa. Let $E = 200$ GPa. What transverse force P can be applied at midspan if the beam is

(a) Stem down (as shown)?

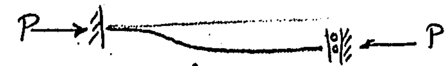
(b) Stem up?





PROBLEM 12.10

1.



מהנוסחה שמצאנו בשעור שעבר והפיתרון הוא

$$EI\frac{d^4}{dx^4}v + P\frac{d^2v}{dx^2} = 0$$

$$v = A\cos\lambda x + B\sin\lambda x + Cx + D$$

$$\lambda^2 = P/EI$$

תנאי הקצוות

① תמוכה $v(x=0)=0$

② זווית $\frac{d^2v}{dx^2}(x=0)=0$

③ זווית $\frac{dv}{dx}(x=L)=0$

④ גזירה: $EI\frac{d^3v}{dx^3}(x=L) + P\frac{dv}{dx}(x=L) = 0$

מפני ש $\frac{dv}{dx}=0$ ב-$x=L$, לכן ④ נותנת $\underline{\frac{d^3v}{dx^3}=0}$ ב-$x=L$

מקבלים

$A+D=0$ ①-מ

$\lambda B + C = 0$ ②

$-A(\lambda\sin\lambda L) + B(\lambda\cos\lambda L) + C = 0$ ③

$A\lambda^3\sin\lambda L - B\lambda^3\cos\lambda L = 0$ ④

לכן

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \lambda & 1 & 0 \\ -\lambda\sin\lambda L & \lambda\cos\lambda L & 1 & 0 \\ \lambda^3\sin\lambda L & -\lambda^3\cos\lambda L & 0 & 0 \end{bmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

כדי לקבל תוצאות לא טריוויאליות הדיטרמיננט צריך להיות אפס. לפי העמוד הרביעי

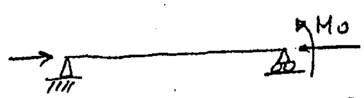
$$-1\begin{pmatrix} 0 & \lambda & 1 \\ -\lambda\sin\lambda L & \lambda\cos\lambda L & 1 \\ \lambda^3\sin\lambda L & -\lambda^3\cos\lambda L & 0 \end{pmatrix} = 0$$

לכן

$$\lambda^4\sin\lambda L = 0$$

$\lambda\neq 0$ (זה הפיתרון $v=0$) ; לכן $\sin\lambda L = 0 \Leftarrow \lambda L = n\pi$ או $P_E = \left(\frac{\pi}{L}\right)^2 EI$

3.



$$v = A\cos\lambda x + B\sin\lambda x + Cx + D \qquad \lambda^2 = P/EI$$

לפי הסכם הסימנים

① $v(x=0)=0$ ③ $v(x=L)=0$

② $EI\frac{d^2v}{dx^2}(x=0) = M = 0$ ④ $EI\frac{d^2v}{dx^2}(x=L) = -M_0$

$A+D=0$ ①-מ

$-A\lambda^2 = 0$ ②

$A\cos\lambda L + B\sin\lambda L + CL + D = 0$ ③

$EI(-A\lambda^2\cos\lambda L - B\lambda^2\sin\lambda L) = -M_0$ ④

אם $\lambda\neq 0 \Rightarrow A \overset{(2)}{=} 0 \Rightarrow D \overset{(1)}{=} 0 \Rightarrow B\sin\lambda L + CL \overset{(3)}{=} 0 \Rightarrow B(EI\lambda^2\sin\lambda L) \overset{(4)}{=} M_0 \Rightarrow B = \frac{M_0}{EI\lambda^2\sin\lambda L}$

ו- $C = -\frac{M_0}{EI\lambda^2 L}$

לכן,

$$v = \frac{M_0}{EI\lambda^2}\left(\frac{\sin\lambda x}{\sin\lambda L} - \frac{x}{L}\right)$$

$$\boxed{v = \frac{M_0}{P}\left(\frac{\sin\lambda x}{\sin\lambda L} - \frac{x}{L}\right)}$$

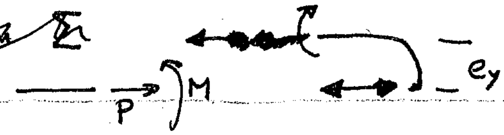
לקבל $M(x)$, גוזר פעמיים לפי x: $EI\frac{d^2v}{dx^2} = M = -EI\frac{M_0\lambda^2}{\ldots}\frac{\sin\lambda x}{\ldots}$ $M(x) = -M_0\sin\lambda x \ldots$

הסימן נכון לפי הסכם הסימנים.

[illegible]

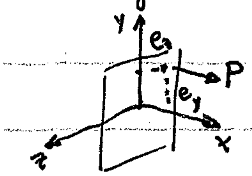
~~First define I_{yz} coming from Σ~~

Taping - Eccentric loading causing



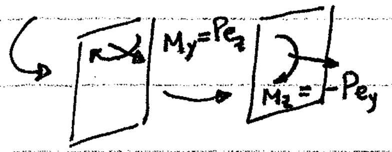
$$\sigma_x = P/A - \frac{M_z y}{I} \qquad M_z = P e_y$$

N.A. No longer where centroid might be



move to center

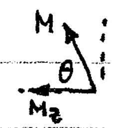
$$\sigma_x = \frac{P}{A} + \frac{M_z y}{I_{zz}} + \frac{M_y z}{I_{yy}} \qquad M_y = +Pe_z \quad M_z = +Pe_y$$



I_{yy} & I_{zz} about centroidal axis of body

what about if x & y are not centroidal

$$\sigma_x = -\frac{y I_y - z I_{yz}}{I_y I_z - I_{yz}^2} M_z + \frac{z I_z - y I_{yz}}{I_y I_z - I_{yz}^2} M_y$$

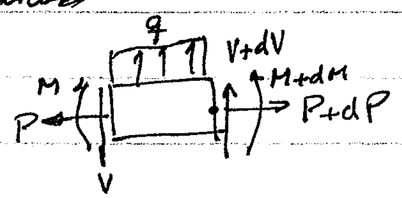


$M_z = M\cos\theta \quad M_y = M\sin\theta$

also stress concentration

~~Class stress concentrations~~

Class ~~day~~ derive



$$\epsilon_x = \frac{-y}{R} \qquad \sigma_x = E\epsilon_x = \ldots$$

$$\sigma_x = \frac{-M_z y}{I_{zz}}$$

$$\therefore \frac{M_z}{I_{zz}} = \frac{E}{R} \qquad \frac{M_z}{EI_{zz}} = \frac{1}{\rho} = \frac{d^2v/dx^2}{\sqrt{\left(1 + \left(\frac{dv}{dx}\right)^2\right)^3}}$$

since $\left|\frac{dv}{dx}\right| << 1$ $\quad \frac{d^2v}{dx^2} \cong \frac{M_z}{EI_{zz}} \qquad \frac{dv}{dx} = \theta = \frac{M_z}{EI_{zz}} x + C_1$

$$v = \frac{M_z}{EI_{zz}} \frac{x^2}{2} + C_1 x + C_2$$

$\Sigma F_y = V + dV - V + q\,dx = 0 \qquad \underline{\frac{dV}{dx} = -q}$

also ΣM_0

$= M + dM - M + V dx - q\,dx \cdot \frac{dx}{2} = 0 \qquad \underline{\frac{dM}{dx} = -V} \qquad \underline{\frac{d^2M}{dx^2} = -\frac{dV}{dx} = q}$

now $\frac{d^2v}{dx^2} = \frac{M_z}{EI_{zz}}$

IF $EI_{zz} = \text{const} \neq \text{fn of } x$ $\qquad \frac{d^4v}{dx^4} = \frac{1}{EI_{zz}} \frac{d^2M_z}{dx^2} = q/EI_{zz}$ or $\underline{EIv^{IV} = q}$ $\quad$ q+↑ q-↓

$$\frac{d^3v}{dx^3} = \frac{1}{EI_{zz}} \frac{dM_z}{dx} = -\frac{V}{EI_{zz}} \quad \text{or} \quad \underline{-EIv''' = V}$$

$$EI\frac{d^2v}{dx^2} = M_z$$