מפתח מונחים (מילון עברי-אנגלי) *

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מפתח מונחים (מלון עברי – אנגלי)

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05A-844305

מכללת יהודה ושומרון
חטיבה עצמאית-אקדמית

מכלול 4.0
ראשים - תכנון וביצוע מערכות מידע בע"מ

תאריך עברי: כ"ג חשון תשס"א
ת.לועזי...: 18:3321/11/2000
דף........: 58

שעור...: 4010005-01 תורת החוזק 2 תשסא הרצאה
נ.זיכוי:4.00
ש"ש....:2.00 כיתה:

תא	מרצה	חדר	עד שעה	משעה	יום	סמסטר
	ד"ר לוי עזרא	ראאב-2.12	13:00	9:00	ד	א

רשימת התלמידים בשעור למילוי נוכחות

	שם	מס.זיהוי	שלי'	22/11 כ"ד חשון	29/11 ב' כסלו	6/12 ט' כסלו	13/12 ט"ז כסלו	20/12 כ"ג כסלו	27/12 א' טבת	3/01 ח' טבת	10/01 ט"ו טבת	17/01 כ"ב טבת	24/01 כ"ט טבת				
1	אופק שגיא	2574879-9	2														
2	איידלמן איתן	31162609-7	2														
3	בן-חי אמיר	2512047-8	2														
4	זק"ש קרן	3752443-6	2														
5	מור רויטל	3768424-8	2														
6	סלומה אסנת	3898553-7	2														
7	עמרוסי אייל	3209262-9	2														
8	עסיס גיל	2881934-0	2														

המכללה האקדמית יהודה ושומרון
הפקולטה להנדסה אזרחית

תורת החוזק 2 (40411)

פרופ' לוי עזרא

פרשיות הלמוד

0. מושוואות דיפרנציאליות למצוא תזוזה אנכית לקורה

1. טרנספורמציה מומנטי אינרציה

2. כפיפה משופעת. פעולה משולבת של כוח צירי ומומנט כפיפה, גרעין החתך

3. התנהגות פלסטית של חתכים, תסבולת פלסטית

4. אנרגיה במבנים, עבודה של עומסים חיצוניים, אנרגיה אלסטית, משפט קלפירון, משפט מקצוול-בטי,
עקרון מינימום האנרגיה

5. עקרון קסטגליאנו (מינימום האנרגיה האלסטית), משפט קסטגליאנו ושימושו לחישובים שקיעות,
אינטגרל מור

6. עקרון שימור האנרגיה ושימושו לחישוב כוחות נגיפה

7. קריסה צירת, שיטת אוילר לקביעת עומס הקריטי (משוואת הקריסה) לעמוד ולמסגרת, השפעת
תפיסה על משוואות הקריסה וכוח קריטי

8. עומס הקריטי של מערכת עמודים וקורות, עמוד-קורה, שיטת האנרגיה למציאת עומס הקריטי.

9. מושגים ראשונים של קריסה צירית, קריסה מקומית, סירוס דופן

10. חתכים בלתי הומוגניים

11. תיאוריות חוזק, תסבולת אלסטית של חתכים

ספרות

1. Timoshenko, S., *Strength of Materials I and II*, van Nostrand Publishers, 1955
2. Timoshenko, S., and Gere, J., *Theory of Elastic Stability*, McGraw-Hill, 1961
3. Shames, I., H., *Mechanics of Deformable Bodies*, Prentice-Hall 1964
4. Oden, J.T., *Mechanics of Elastic Structures*, McGraw-Hill, 1967
5. Chajes, A., *Principles of Structural Stability Theory*, Waneland Press, Inc., 1993
6. Beer, F.P., and Johnston, E.R., *Mechanics of Materials* (SI Ed) McGraw-Hill, 1994
7. Cook, R.D., and Young, W.C., *Advanced Mechanics of Materials*, Macmillan, 1985
8. Popov, E.P., *Introduction to Mechanics of Solids*, Prentice-Hall, 1968

הסטודנט חייב להיות בכל השיעורים, לפי תקנון המכללה. אם לא תוכל/י להיות בשיעור, חיובך לקבל את החומר וללמוד אותו. תרגילי בית - חובה להשלימם ולחזירם למרצה **7** ימים אחרי שהתרגילים נתונים לכם. אם יש משהו שמשפיע על הגשת תרגילי בית שלך (כמו מילואים, נסיעות למען עבודתך או מחלות), יש להתקשר לפרופ לוי מיד ולהודיע לו.

הציונים תלויים בשלוש דברים:
1. תרגילי בית מחושב כ-**10%**
2. בוחן אמצע הסמסטר מחושב כ-**35%**
3. מבחן סופי מחושב כ-**55%**

משרדו של פרופ׳ לוי נמצא בבניין **2** חדר **8**.

שעות קבלה: יום ב בין השעות **12:00-13:00**

יום ד בין השעות **14:00-16:00**

מס׳ הטל: **03-9066211** במשרד, או בזמן חרום **053-771247**

אם יהיו שינויים לסילבוס, קהל הסטודנטים ייודע בו זמני

המכללה האקדמית יהודה ושומרון

הנדסה אזרחית

תורת החוזק 2 (40411)

ד"ר רבקה גילת

פרשיות הלמוד:

אנרגיה - עקרונות, ישומים בניתוח מבנים.

energy - principles, applic in analysis of structures

התנהגות אלסטו-פלסטית - עמיסה צירית, פיתול וכפיפה.

bending torque axis loading

elastic-plastic behavior, loading axis

E.P. Behavior

torque פיתול

bending כפיפה

buckling קריסה

תאוריות חוזק.

Theory of Strength

כפיפה משופעת, מרכז הגזירה.

shear center ~~influenced~~ bending

קריסה - עמודים בעלי תנאי שפה שונים ומסגרות.

column buckling

of different edge conditions

קורות-עמוד.

beam columns,

& limiting conditions

תנאי מסגרות -

limiting conditions

ספרות:

1. Timoshenko, S., *Strength of materials*, I and II, Van Nostrand, 1955.
2. Timoshenko, S. and Gere, J., *Theory of elatic stability*, McGraw-Hill, 1961.
3. Shames, I. H., *Mechanics of deformable solids*, prentice-Hall, 1964.
4. Oden, J. T., *Mechanics of elastic strucrures*, McGraw-Hill, 1967.
5. Chajes, A., *Principles of structural stability theory*, Waneland Press, Inc, 1993.
6. Beer, F. P. and Johnston, E. R., *Mechanics of Materials.* (SI Ed.)McGraw-Hill, 1994.
7. Cook, R. D., Young, W. C., *Advanced mechanics of materials*, Macmillan, 1985

שעות קבלה: יום ג 13:30 - 12:30.

עבודות בית - חובה להגיש

ref:aria6sil

פרופ' בלוך

עבודה 04-8326685

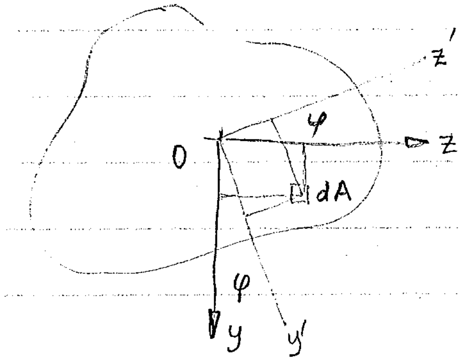
בבית 04-8231177

טרנספורמציה מומנט אינרציה

SOMETIMES THE MOMENTS OF INERTIA ARE GIVEN ABOUT A SET OF AXES, BUT WE WANT TO KNOW THE MOMENTS OF INERTIA ABOUT A DIFFERENT SET OF AXES

O IS CENTROID

הגופים הומוגניים מרכז השטח=מרכז הכובד
Homogeneous bodies CG = CM



DEFINE מגדירים

$$I_{zz} = \int y^2 dA$$

$$y' = y\cos\varphi + z\sin\varphi$$

$$z' = -y\sin\varphi + z\cos\varphi$$

$$I_{z'z'} = \int y'^2 dA = \int (y\cos\varphi + z\sin\varphi)^2 dA$$
$$= \int y^2\cos^2\varphi\, dA + 2\int yz\cos\varphi\sin\varphi\, dA$$
$$+ \int z^2\sin^2\varphi\, dA$$
$$= I_{zz}\cos^2\varphi + I_{yz}\sin 2\varphi + I_{yy}\sin^2\varphi$$
$$= (I_{zz} + I_{yy})/2 + (I_{zz} - I_{yy})\frac{\cos 2\varphi}{2} + I_{yz}\sin 2\varphi$$

HW#1
Prove these identities

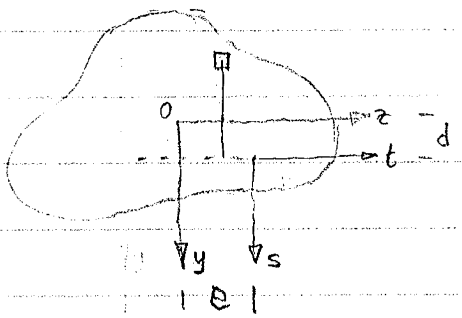
$$I_{y'y'} = \int z'^2 dA = \int (-y\sin\varphi + z\cos\varphi)^2 dA = \int y^2\sin^2\varphi\, dA - 2\int yz\cos\varphi\sin\varphi\, dA + \int z^2\cos^2\varphi\, dA$$
$$= I_{zz}\sin^2\varphi - I_{yz}\sin 2\varphi + I_{yy}\cos^2\varphi$$
$$= (I_{zz} + I_{YY})/2 - (I_{zz} - I_{YY})\frac{\cos 2\varphi}{2} - I_{yz}\sin 2\varphi$$

IDENTITIES (זהויות) $\cos^2\varphi = \frac{1+\cos 2\varphi}{2}$

$$I_{y'z'} = \int z'y' dA = \int (-y\sin\varphi + z\cos\varphi)(y\cos\varphi + z\sin\varphi)\, dA = \int -y^2\sin\varphi\cos\varphi\, dA + \int yz(\cos^2\varphi - \sin^2\varphi)\, dA$$
$$+ \int z^2\sin\varphi\cos\varphi\, dA$$
$$= (-I_{zz} + I_{yy})\frac{\sin 2\varphi}{2} + I_{yz}\cos 2\varphi$$

PARALLEL AXIS THEOREM

תורת הציר המקביל
(משפט שטיינר)



$$I_{tt} = \int (y+d)^2 dA = \int (y^2 + 2yd + d^2)\, dA$$
$$= \int y^2 dA + 2d\int y\, dA + d^2\int dA$$
$$= I_{zz} + 2d\bar{y}A + d^2A \qquad \bar{y} = \frac{\int z\, dA}{\int dA}$$

$$I_{ss} = I_{yy} + 2e\bar{z}A + e^2A$$

משתמשים בחתכים מורכבים

IF O CENTER OF MASS $\bar{z} = 0$, $\bar{y} = 0$

$$I_{tt} = I_{zz_0} + d^2A$$

$$I_{ss} = I_{yy_0} + e^2A$$

(USE FOR COMPOSITE BODIES)

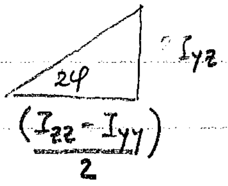
$$I_{st} = \int (z+e)(y+d)\, dA = \int (zy + zd + ye + de)\, dA = \int zy\, dA + d\int z\, dA + e\int y\, dA + de\int dA$$
$$= I_{yz} + (d\bar{z} + e\bar{y} + ed)A$$

FOR PRINCIPAL AXES (צירים ראשיים) לחשב את הצירים הראשיים

TAKE $dI_{y'y'}/d\varphi$ or $\dfrac{dI_{z'z'}}{d\varphi}=0 \Rightarrow$ זה נותן את הקיצוניים

$$\frac{dI_{z'z'}}{d\varphi} = 0 = (I_{zz}-I_{yy})(-\sin 2\varphi) + I_{yz}2\cos 2\varphi$$

or $\tan 2\varphi = 2I_{yz}/(I_{zz}-I_{yy})$



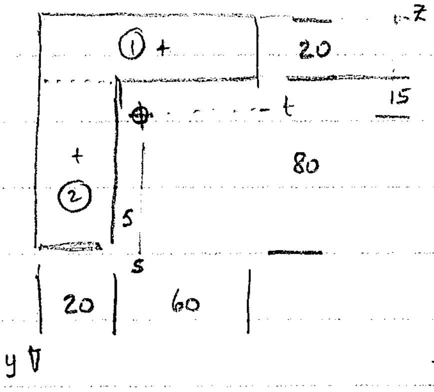
(יֶתֶר) HYPOTENUSE IS $\sqrt{\left(\frac{I_{zz}-I_{yy}}{2}\right)^2 + I_{yz}^2}$

AND $I_{z'z'} = (I_{zz}+I_{yy})/2 + \sqrt{\left(\frac{I_{zz}-I_{yy}}{2}\right)^2 + I_{yz}^2} = I_{max}$

$I_{y'y'} = (I_{zz}+I_{yy})/2 - \sqrt{\left(\frac{I_{zz}-I_{yy}}{2}\right)^2 + I_{yz}^2} = I_{min}$

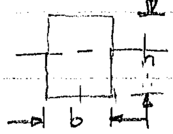
$I_{y'z'} = 0$

לשטח-חתך עגולי (CIRCULAR CROSS-SEC.), אם $I_{max} = I_{min}$ הצירים
הם צירים ראשיים



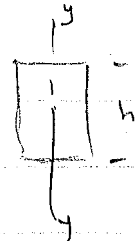
	A	$\bar{y}$ יחיד	$A\bar{y}$	$\bar{z}$ יחיד	$A\bar{z}$
①	1600	10	16000	40	64000
②	1600	60	96000	10	16000
Σ	3200		112,000		80,000

$\bar{y}_{מרוכב} = \frac{112,000}{3,200} = 35mm$ $\bar{z}_{מרוכב} = \frac{80000}{3200} = 25mm$



$I_{zz_0} = \frac{1}{12}bh^3$ $I_{yy_0} = \frac{1}{12}hb^3$ $I_{yz} = 0$

	I_{zz_0}	A	d^2	I_{tt}
①	$\frac{1}{12}(80)(20)^3$	1600	$(35-10)^2$	1053333.33
			$(35-60)^2$	1853333.33



$$I_{yy_0} = \frac{1}{12} h b^3$$

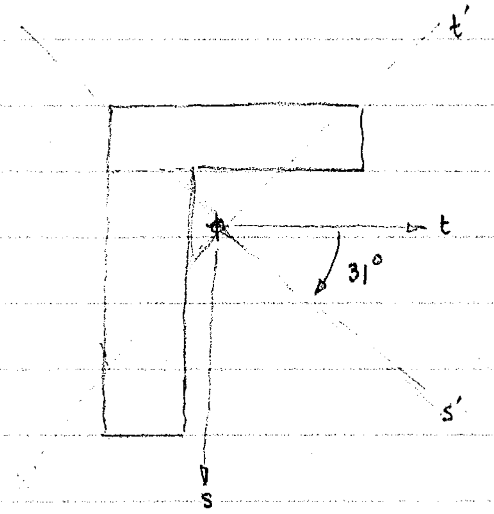
	I_{yy_0}	A	e^2	I_{ss}
①	$\frac{1}{12}(20)(80)^3$	1600	$(25-40)^2$	1213333.33
②	$\frac{1}{12}(80)(20)^3$	1600	$(25-10)^2$	413333.33
				1626666.67 mm^4

	I_{yz_0}	A	de	I_{st}
①	0	1600	$(35-10)(25-40)$	$-600,000.$
②	0	1600	$(35-60)(25-10)$	$-600,000$
				$-1200000.$ mm^4

$$\tan 2\varphi = \frac{2I_{st}}{I_{tt} - I_{ss}} = \frac{2(-1.2\times10^6)}{2.907\times10^6 - 1.627\times10^6} = -1.875$$

$$2\varphi = -61.928^\circ \qquad \varphi = -30.96^\circ$$

זאת הזווית לציר הראשי המינימלי



$$I_{t't'} = \frac{(I_{tt} + I_{ss})}{2} + \sqrt{\left(\frac{I_{tt} - I_{ss}}{2}\right)^2 + I_{st}^2}$$

$$= 3.627\times10^6 \text{ mm}^4$$

$$I_{s's'} = \frac{(I_{tt} + I_{ss})}{2} - \sqrt{\left(\frac{I_{tt} - I_{ss}}{2}\right)^2 + I_{st}^2}$$

$$= 0.907\times10^6 \text{ mm}^4$$

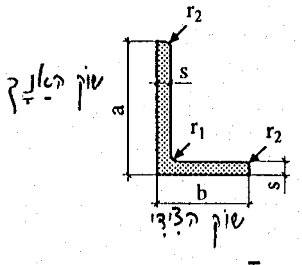
WATCHOUT FOR SIGN of e & d in I_{yz}

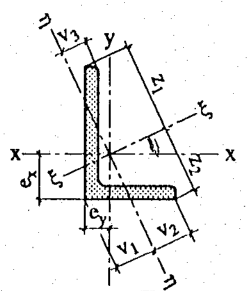
נ.ב. : צריך להיזהר בסימן + או - בחישוב ב- d וגם ב-ש שפועל בחשיבות בחישוב של I_{yz} וגם הזווית φ

יש טבלאות שכוללות כל החישובים שעשינו ומקום אחד והנה אחת מהן:

פרופילי פלדה LPN – זוויתנים שוני שוקיים

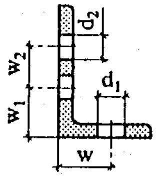
תקן: DIN 1028

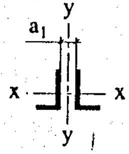




A - שטח החתך
G - משקל של 1 מטר אורך
U - שטח המעטפת
I - מומנט ההתמד (אינרציה)
W - מודול החתך (מומנט התנגדות) *
i - רדיוס ההתמד (אינרציה) = $\sqrt{I/A}$
a1 - המרחק בין זוויתנים הנותן $I_x = I_y$

סימון	(17) I_x cm^4	$\frac{I_x}{a-e_x}$ (18) W_x cm^3	$\sqrt{I_x/A}$ (19) i_x cm	(20) I_y cm^4	$\frac{I_y}{b-e_y}$ (21) W_y cm^3	$\sqrt{I_y/A}$ (22) i_y cm	(23) I_ζ cm^4	$\sqrt{I_\zeta/A}$ (24) i_ζ cm	(25) I_η cm^4	$\sqrt{I_\eta/A}$ (26) i_η cm	(27) I_{xy} cm^4	(28) a_1 mm	קוטר החורים (29) d_1 mm	(30) d_2 mm	(31) w mm	(32) w_2 mm	(33) w_1 mm
* 30X20X3	1.25	0.62	0.94	0.44	0.29	0.56	1.43	1.00	0.25	0.42	0.43	5.2	6.4	8.4	12	17	---
* 30X20X4	1.59	0.81	0.93	0.55	0.38	0.55	1.81	0.99	0.33	0.42	0.53	4.2	6.4	8.4	12	17	---
* 40X20X3	2.79	1.08	1.27	0.47	0.30	0.52	2.96	1.31	0.30	0.42	0.64	14.6	6.4	11.0	12	22	---
* 40X20X4	3.59	1.42	1.26	0.60	0.39	0.52	3.79	1.30	0.39	0.42	0.81	13.8	6.4	11.0	12	22	---
(40X25X4)	3.89	1.47	1.26	1.16	0.62	0.69	4.35	1.33	0.70	0.53	1.22	10.4	6.4	11.0	12	22	---
(45X30X3)	4.47	1.46	1.43	1.60	0.70	0.86	5.15	1.53	0.93	0.65	1.55	9.0	8.4	11.0	17	25	---
* 45X30X4	5.78	1.91	1.42	2.05	0.91	0.85	6.65	1.52	1.18	0.64	2.01	8.0	8.4	11.0	17	25	---
* 45X30X5	6.99	2.35	1.41	2.47	1.11	0.84	8.02	1.51	1.44	0.64	2.39	7.2	8.4	11.0	17	25	---
* 50X30X4	7.71	2.33	1.59	2.09	0.91	0.82	8.53	1.67	1.27	0.64	2.29	13.1	8.4	13.0	17	30	---
* 50X30X5	9.41	2.88	1.58	2.54	1.12	0.82	10.40	1.66	1.56	0.64	2.77	12.2	8.4	13.0	17	30	---
(50X40X4)	8.54	2.47	1.57	4.86	1.64	1.19	10.90	1.78	2.46	0.84	3.82	---	11.0	13.0	22	30	---
* 50X40X5	10.40	3.02	1.56	5.89	2.01	1.18	13.30	1.76	3.02	0.84	3.60	---	11.0	13.0	22	30	---
* 60X30X5	15.60	4.04	1.90	2.60	1.12	0.78	16.50	1.96	1.69	0.63	3.55	21.4	8.4	17.0	17	35	---
* 60X40X5	17.20	4.25	1.89	6.11	2.02	1.13	19.80	2.03	3.50	0.86	5.98	11.2	11.0	17.0	22	35	---
* 60X40X6	20.10	5.03	1.88	7.12	2.38	1.12	23.10	2.02	4.12	0.85	6.94	10.2	11.0	17.0	22	35	---
(60X40X7)	22.00	5.79	1.87	8.07	2.74	1.11	26.30	2.00	4.73	0.85	7.81	9.2	11.0	17.0	22	35	---
* 65X50X5	23.10	5.11	2.04	11.90	3.18	1.47	28.80	2.28	6.21	1.06	9.810	3.6	13.0	21.0	30	35	---
(65X50X7)	31.00	6.99	2.02	15.80	4.31	1.44	38.40	2.25	8.37	1.06	13.00	1.8	13.0	21.0	30	35	---
(65X50X9)	38.20	8.77	2.00	19.40	5.39	1.42	47.00	2.22	10.50	1.05	15.70	---	13.0	21.0	30	35	---
* 70X50X6	33.50	7.04	2.21	14.30	3.81	1.44	39.90	2.41	7.94	1.07	12.67	---	13.0	21.0	30	40	---
* 75X50X7	46.40	9.24	2.36	16.50	4.39	1.41	53.30	2.53	9.56	1.07	15.90	13.0	13.0	23.0	30	40	---
(75X50X9)	57.40	11.60	2.34	20.20	5.49	1.39	65.70	2.50	11.90	1.07	19.40	11.0	13.0	23.0	30	40	---
* 75X55X5	35.50	6.84	2.37	16.20	3.89	1.60	43.10	2.61	8.68	1.17	14.20	8.4	17.0	23.0	30	40	---
* 75X55X7	47.90	9.39	2.35	21.80	5.32	1.59	57.90	2.59	11.80	1.17	18.90	6.6	17.0	23.0	30	40	---
(75X55X9)	59.40	11.80	2.33	26.80	6.66	1.57	71.30	2.55	14.80	0.86	23.10	5.0	17.0	23.0	30	40	---
* 80X40X6	44.90	8.73	2.55	7.59	2.44	1.05	47.60	2.63	4.90	0.84	10.40	29.0	11.0	23.0	22	45	---
* 80X40X8	57.60	11.40	2.53	9.68	3.18	1.04	60.90	2.60	6.41	0.84	13.00	27.2	11.0	23.0	22	45	---
* 80X60X7	59.00	10.70	2.51	28.40	6.34	1.74	72.00	2.77	15.40	1.28	23.81	5.7	11.0	23.0	35	45	---
* 80X65X8	68.10	12.30	2.49	40.10	8.41	1.91	88.00	2.82	20.30	1.36	30.80	---	21.0	23.0	35	45	---
(80X65X10)	82.20	15.10	2.46	48.30	10.30	1.89	106.00	2.79	24.80	1.35	36.80	---	21.0	23.0	35	45	---
* 90X60X6	71.70	11.70	2.87	25.80	5.61	1.72	82.80	3.09	14.60	1.30	25.20	17.8	17.0	25.0	35	50	---
* 90X60X8	92.50	15.40	2.85	33.00	7.31	1.70	107.00	3.06	19.00	1.29	32.20	16.0	17.0	25.0	35	50	---





density דְּחִיסוּת $\rho=\frac{G}{A}$

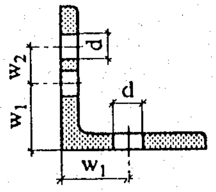
סימון	(1) a mm	(2) b mm	(3) s mm	(4) r_1 mm	(5) r_2 mm	(6) A cm^2	(7) G kg/m	(8) U m^2/m	(9) c_x cm	(10) c_y cm	(11) z_1 cm	(12) z_2 cm	(13) v_1 cm	(14) v_2 cm	(15) v_3 cm	(16) tg α
* 30X20X3	30	20	3	3.5	2.0	1.42	1.11	0.097	0.99	0.50	2.04	1.51	0.86	1.04	0.56	0.431
* 30X20X4	30	20	4	3.5	2.0	1.85	1.45	0.097	1.03	0.54	2.02	1.52	0.91	1.03	0.58	0.423
* 40X20X3	40	20	3	3.5	2.0	1.72	1.35	0.117	1.43	0.44	2.61	1.77	0.79	1.19	0.46	0.259
* 40X20X4	40	20	4	3.5	2.0	2.25	1.77	0.117	1.47	0.48	2.57	1.80	0.83	1.18	0.50	0.252
(40X25X4)	40	25	4	4.0	2.0	2.46	1.93	0.127	1.36	0.62	2.69	1.90	1.10	1.35	0.68	0.381
(45X30X3)	45	30	3	4.5	2.0	2.19	1.72	0.146	1.43	0.70	3.09	2.23	1.21	1.59	0.80	0.436
* 45X30X4	45	30	4	4.5	2.0	2.87	2.25	0.146	1.48	0.74	3.07	2.26	1.27	1.58	0.83	0.436
* 45X30X5	45	30	5	4.5	2.0	3.53	2.77	0.146	1.52	0.78	3.05	2.27	1.32	1.58	0.85	0.430
* 50X30X4	50	30	4	4.5	2.0	3.07	2.41	0.156	1.68	0.70	3.36	2.35	1.24	1.67	0.78	0.356
* 50X30X5	50	30	5	4.5	2.0	3.78	2.96	0.156	1.73	0.74	3.33	2.38	1.28	1.66	0.80	0.353
(50X40X4)	50	40	4	4.0	2.0	3.46	2.71	0.177	1.52	1.03	3.50	2.85	1.67	1.84	1.26	0.629
* 50X40X5	50	40	5	4.0	2.0	4.27	3.35	0.177	1.56	1.07	3.49	2.88	1.73	1.84	1.27	0.625
* 60X30X5	60	30	5	6.0	3.0	4.29	3.37	0.175	2.15	0.68	3.90	2.67	1.20	1.77	0.72	0.256
* 60X40X5	60	40	5	6.0	3.0	4.79	3.76	0.195	1.96	0.97	4.08	3.01	1.68	2.09	1.10	0.437
* 60X40X6	60	40	6	6.0	3.0	5.68	4.46	0.195	2.00	1.01	4.06	3.02	1.72	2.08	1.12	0.433
(60X40X7)	60	40	7	6.0	3.0	6.55	5.14	0.195	2.04	1.05	4.04	3.03	1.77	2.07	1.14	0.429
* 65X50X5	65	50	5	6.0	3.0	5.54	4.35	0.224	1.99	1.25	4.52	3.61	2.08	2.38	1.50	0.583
(65X50X7)	65	50	7	6.0	3.0	7.60	5.97	0.224	2.07	1.33	4.50	3.62	2.19	2.37	1.52	0.574
(65X50X9)	65	50	9	6.0	3.0	9.58	7.52	0.224	2.15	1.41	4.48	3.63	2.28	2.36	1.57	0.567
* 70X50X6	70	50	6	6.0	3.0	6.88	5.40	0.235	2.24	1.25	4.82	3.68	2.20	2.52	1.42	0.497
* 75X50X7	75	50	7	6.5	3.5	8.30	6.51	0.244	2.48	1.25	5.10	3.77	2.13	2.63	1.38	0.433
(75X50X9)	75	50	9	6.5	3.5	10.50	8.23	0.244	2.56	1.32	5.06	3.80	2.22	2.62	1.44	0.427
* 75X55X5	75	55	5	7.0	3.5	6.30	4.95	0.254	2.31	1.33	5.19	4.00	2.27	2.71	1.58	0.530
* 75X55X7	75	55	7	7.0	3.5	8.66	6.80	0.254	2.40	1.41	5.16	4.02	2.37	2.70	1.62	0.525
(75X55X9)	75	55	9	7.0	3.5	10.90	8.59	0.254	2.47	1.48	5.14	4.04	2.46	2.70	1.66	0.518
* 80X40X6	80	40	6	7.0	3.5	6.89	5.41	0.234	2.85	0.88	5.21	3.53	1.55	2.42	0.89	0.259
* 80X40X8	80	80	40	8.0	3.5	9.01	7.07	0.234	2.94	0.95	5.15	3.57	1.65	2.38	1.04	0.253
* 80X60X7	80	60	7	8.0	4.0	9.38	7.36	0.274	2.51	1.52	5.55	4.42	2.70	2.92	1.68	0.546
* 80X65X8	80	65	8	8.0	4.0	11.00	8.66	0.283	2.47	1.73	5.59	4.65	2.79	2.94	2.05	0.645
(80X65X10)	80	65	10	8.0	4.0	13.60	10.70	0.283	2.55	1.81	5.56	4.68	2.90	2.95	2.11	0.640
* 90X60X6	90	60	6	7.0	3.5	86.90	6.82	0.294	2.89	1.41	6.14	4.50	2.46	3.16	1.60	0.442
* 90X60X8	90	60	8	7.0	3.5	11.40	8.96	0.294	2.97	1.49	6.11	4.54	2.56	3.15	1.69	0.437

* זוויתנים מועדפים בהזמנה

פרופילי פלדה LPN – זוויתנים שווי שוקיים

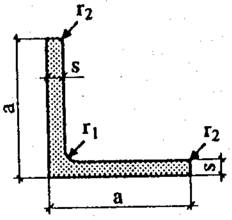
תקן: DIN 1028

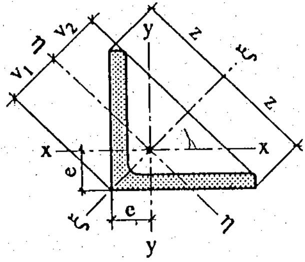
A - שטח החתך
G - משקל של מטר אורך אחד
U - שטח המעטפת
I - מומנט ההתמד (אינרציה)
W - מודול החתך (מומנט התנגדות)
i - רדיוס ההתמד (אינרציה)
a_1 - המרחק בין זוויתנים הנותן $I_x = I_y$



	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
סימון	$I_x=I_y$ cm^4	$W_x=W_y$ cm^3	$i_x=i_y$ cm	I_ζ cm^4	i_ζ cm	I_η cm^4	i_η cm	I_{xy} cm^4	d mm	w_1 mm	w_2 mm
* 20x3	0.39	0.28	0.59	0.62	0.74	0.15	0.37	0.24	6.4	12	---
* 25x3	0.79	0.45	0.75	1.27	0.95	0.31	0.47	0.48	8.4	15	---
25x4	1.01	0.58	0.74	1.61	0.93	0.40	0.47	0.61	8.4	15	---
* 30x3	1.41	0.65	0.90	2.24	1.14	0.57	0.57	0.84	8.4	17	---
30x4	1.81	0.86	0.89	2.85	1.12	0.76	0.58	1.05	8.4	17	---
(30x5)	2.16	1.04	0.88	3.41	1.11	0.91	0.57	1.25	8.4	17	---
* 35x4	2.96	1.18	1.05	4.68	1.33	1.24	0.68	1.72	11.0	20	---
35x5	3.56	1.45	1.04	5.63	1.31	1.49	0.67	2.07	11.0	20	---
* 40x4	4.48	1.55	1.21	7.09	1.52	1.86	0.78	2.62	11.0	22	---
40x5	5.43	1.91	1.20	8.64	1.51	2.22	0.77	3.20	11.0	22	---
45x4	6.43	1.97	1.36	10.20	1.71	2.68	0.88	3.85	11.0	25	---
* 45x5	7.83	2.43	1.35	12.40	1.70	3.25	0.87	4.58	11.0	25	---
* 50x5	11.00	3.05	1.51	17.40	1.90	4.59	0.98	6.41	13.0	30	---
50x6	12.80	3.61	1.50	20.40	1.89	5.24	0.96	7.56	13.0	30	---
50x7	14.60	4.15	1.49	23.10	1.88	6.02	0.96	8.58	13.0	30	---
(55x6)	17.30	4.40	1.66	27.40	2.08	7.24	1.07	10.10	17.0	30	---
60x5	19.40	4.45	1.82	30.70	2.30	8.03	1.17	11.30	17.0	35	---
* 60x6	22.80	5.29	1.82	36.10	2.29	9.43	1.17	13.40	17.0	35	---
60x8	29.10	6.88	1.80	46.10	2.26	12.10	1.16	17.00	17.0	35	---
(65x7)	33.40	7.18	1.96	53.00	2.47	13.80	1.26	19.60	21.0	35	---
(70x6)	36.90	7.27	2.13	58.50	2.68	15.30	1.37	21.60	21.0	40	---
* 70x7	42.40	8.43	2.12	67.10	2.67	17.60	1.37	24.80	21.0	40	---
70x9	52.60	10.60	2.10	83.10	2.64	22.00	1.36	30.60	21.0	40	---
75x7	52.40	9.67	2.28	83.60	2.88	21.10	1.45	31.30	23.0	40	---
75x8	58.90	11.00	2.26	93.30	2.85	24.40	1.46	34.50	23.0	40	---
80x6	55.80	9.57	2.44	88.50	3.08	23.10	1.57	32.70	23.0	45	---
* 80x8	72.30	12.60	2.42	115.00	3.06	29.60	1.55	42.70	23.0	45	---
80x10	87.50	15.50	2.41	139.00	3.03	35.90	1.54	51.60	23.0	45	---
90x7	92.60	14.10	2.75	147.00	3.46	38.30	1.77	54.40	25.0	50	---
* 90x9	116.00	18.00	2.74	184.00	3.45	47.80	1.76	68.20	25.0	50	---

* זוויתנים מועדפים בהזמנה



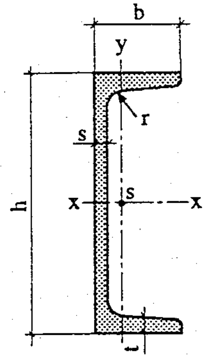


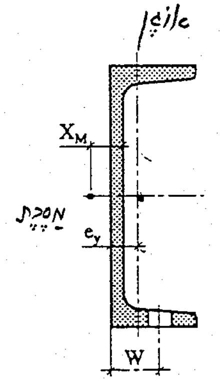
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
סימן	a	s	r_1	r_2	A	G	U	e	z	v_1	v_2
	mm	mm	mm	mm	cm^2	kg/m	m^2/m	cm	cm	cm	cm
* 20X3	20	3	3.5	2.0	1.12	0.88	0.077	0.60	1.41	0.85	0.70
* 25X3	25	3	3.5	2.0	1.42	1.12	0.097	0.73	1.77	1.03	0.87
25X4	25	4	3.5	2.0	1.85	1.45	0.097	0.76	1.77	1.08	0.89
* 30X3	30	3	5.0	2.5	1.74	1.36	0.116	0.84	2.12	1.18	1.04
30X4	30	4	5.0	2.5	2.27	1.78	0.116	0.89	2.12	1.24	1.05
(30X5)	30	5	5.0	2.5	2.78	2.18	0.116	0.92	2.12	1.30	1.07
* 35X4	35	4	5.0	2.5	2.67	2.10	0.136	1.00	2.47	1.41	1.24
35X5	35	5	5.0	2.5	3.28	2.57	0.136	1.04	2.47	1.47	1.25
* 40X4	40	4	6.0	3.0	3.08	2.42	0.155	1.12	2.83	1.58	1.40
40X5	40	5	6.0	3.0	3.79	2.97	0.155	1.16	2.83	1.64	1.42
45X4	45	4	7.0	3.5	3.49	2.74	0.174	1.23	3.18	1.75	1.57
* 45X5	45	5	7.0	3.5	4.30	3.38	0.174	1.28	3.18	1.81	1.58
* 50X5	50	5	7.0	3.5	4.80	3.77	0.194	1.40	3.54	1.98	1.76
50X6	50	6	7.0	3.5	5.69	4.47	0.194	1.45	3.54	2.04	1.77
50X7	50	7	7.0	3.5	6.56	5.15	0.194	1.49	3.54	2.11	1.78
(55X6)	55	6	8.0	4.0	6.31	4.95	0.213	1.56	3.89	2.21	1.94
60X5	60	5	8.0	4.0	5.82	4.57	0.233	1.64	4.24	2.32	2.11
* 60X6	60	6	8.0	4.0	6.91	5.42	0.233	1.69	4.24	2.39	2.11
60X8	60	8	8.0	4.0	9.03	7.09	0.233	1.77	4.24	2.50	2.14
(65X7)	65	7	9.0	4.5	8.70	6.83	0.252	1.85	4.60	2.62	2.29
(70X6)	70	6	9.0	4.5	8.13	6.38	0.272	1.93	4.95	2.73	2.46
* 70X7	70	7	9.0	4.5	9.40	7.38	0.272	1.97	4.95	2.79	2.47
70X9	70	9	9.0	4.5	11.90	9.34	0.272	2.05	4.95	2.90	2.50
75X7	75	7	10.0	5.0	10.10	7.94	0.291	2.09	5.30	2.95	2.63
75X8	75	8	10.0	5.0	11.50	9.03	0.291	2.13	5.30	3.01	2.65
80X6	80	6	10.0	5.0	9.35	7.34	0.311	2.17	5.66	3.07	2.80
* 80X8	80	8	10.0	5.0	12.30	9.66	0.311	2.26	5.66	3.20	2.82
80X10	80	10	10.0	5.0	15.10	11.90	0.311	2.34	5.66	3.31	2.85
90X7	90	7	11.0	5.5	12.20	9.61	0.351	2.45	6.36	3.47	3.16
* 90X9	90	9	11.0	5.5	15.50	12.20	0.351	2.54	6.36	3.59	3.18

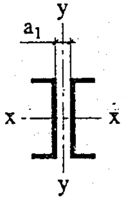
* זוויתנים מועדפים בהזמנה

פרופיל-פלדה UPB U

תקן: DIN 1026







A - שטח החתך
G - משקל של מטר אורך אחד
I - מומנט ההתמד (אינרציה)
W - מודול החתך (מומנט התנגדות)
i - רדיוס ההתמד (אינרציה)
a_1 - המרחק בין צירי הפרופילים הנותן $I_x=I_y$

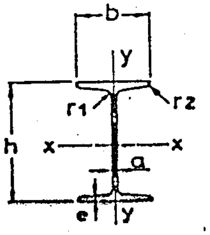
סימון	h mm	b mm	s mm	$t=r_1$ mm	A cm^2	G kg/m	I_x cm^4	W_x cm^3	i_x cm	I_y cm^4	W_y cm^3	i_y cm	e_y cm	x_M cm	Z_x cm^3	Z_y cm^3	a_1 mm	d_1 mm	w mm	סימון
50	50	38	5.0	7.0	7.12	5.59	26.4	10.6	1.92	9.12	3.75	1.13	1.37	2.47	—	---	4	11	20	50
60	60	30	6.0	6.0	6.46	5.07	31.6	10.5	2.21	4.51	2.16	0.84	0.91	1.50	—	---	---	—	—	60
65	65	42	5.5	7.5	9.03	7.09	57.5	17.7	2.52	14.10	5.07	1.25	1.42	2.60	23.4	5.1	16	11	25	65
80	80	45	6.0	8.0	11.00	8.64	106.0	26.5	3.10	19.40	6.36	1.33	1.45	2.67	31.8	16.6	28	13	25	80
100	100	50	6.0	8.5	13.50	10.60	206.0	41.2	3.91	29.30	8.49	1.47	1.55	2.93	49.0	22.3	42	13	30	100
120	120	55	7.0	9.0	17.00	13.40	364.0	60.7	4.62	43.20	11.10	1.59	1.60	3.03	72.6	30.2	56	17	30	120
140	140	60	7.5	10.5	20.40	16.00	605.0	86.4	5.45	62.70	14.80	1.75	1.75	3.37	102.8	40.0	70	17	35	140
160	160	65	7.5	10.0	24.00	18.80	925.0	116.0	6.21	85.30	18.30	1.89	1.84	3.56	137.6	50.2	82	21	35	160
180	180	70	8.0	11.0	28.00	22.00	1350.0	150.0	6.95	114.00	22.40	2.02	1.92	3.75	179.2	62.4	96	21	40	180
200	200	75	8.5	11.5	32.20	25.30	1910.0	191.0	7.70	148.00	27.00	2.14	2.01	3.94	228.0	72.6	108	23	40	200
220	220	80	9.0	12.5	37.40	29.40	2690.0	245.0	8.48	197.00	33.60	2.30	2.14	4.20	292.0	94.2	122	23	45	220
240	240	85	9.5	13.0	42.30	33.20	3600.0	300.0	9.22	248.00	39.60	2.42	2.23	4.39	358.0	112.0	134	25	45	240
260	260	90	10.0	14.0	48.30	37.90	4820.0	371.0	9.99	317.00	47.70	2.56	2.36	4.66	442.0	136.0	146	25	50	260
280	280	95	10.0	15.0	53.30	41.80	6280.0	448.0	10.90	399.00	57.20	2.74	2.53	5.02	532.0	160.0	160	25	50	280
300	300	100	10.0	16.0	58.80	46.20	8030.0	535.0	11.70	495.00	67.80	2.90	2.70	5.41	632.0	188.0	174	25	55	300
320	320	100	14.0	17.5	75.80	59.50	10870.0	679.0	12.10	597.00	80.60	2.81	2.60	4.82	826.0	215.0	182	25	55	320
350	350	100	14.0	16.0	77.30	60.60	12840.0	734.0	12.90	570.00	75.00	2.72	2.40	4.45	918.0	205.0	204	25	55	250
380	380	102	13.5	16.0	80.40	63.10	15760.0	829.0	14.00	615.00	78.70	2.77	2.38	4.58	1014.0	214.0	230	25	55	380
400	400	110	14.0	18.0	91.50	71.80	20350.0	1020.0	14.90	846.00	102.00	3.04	2.65	5.11	1240.0	279.0	240	25	60	400

טבלאות פרופילים: פ-1 IPN

?

IPN	Weight	Depth	Width	Thickness		Toe Rad	Area	Moment of Inertia		Rad of Gyration		Elastic Modulus		Plastic Modulus						
	G	D	W	Web	Flange	R2	F	Ixx	Iyy	rxx	ryy	Zxx	Zyy	Sxx	Syy	iw	J	Ct	Cw,e	Cw,c
	kg/m	h (mm)	b (mm)	t w(mm)	t f(mm)	mm	cm^2	cm^4	cm^4	cm	cm	cm^3	cm^3	cm^3	cm^3	cm	cm^4	kN*m^2	kN*m^3	kN*m^3
80	5.95	80	42	3.9	5.9	2.3	7.58	77.8	6.29	3.2	0.91	19.5	3	22.8	5	1.04	0.869	9.34	4.72	4.25
100	8.34	100	50	4.5	6.8	2.7	10.6	171	12.2	4.01	1.07	34.2	4.88	39.8	8.1	1.23	1.6	17.65	11.50	10.43
120	11.1	120	58	5.1	7.7	3.1	14.2	328	21.5	4.81	1.23	54.7	7.41	10.5	12.4	1.42	2.71	30.49	24.43	22.22
140	14.3	140	66	5.7	8.6	3.4	18.2	573	35.2	5.61	1.4	81.9	10.7	95.4	17.9	1.60	4.32	49.25	46.79	42.62
160	17.9	160	74	6.3	9.5	3.8	22.8	935	54.7	6.4	1.55	117	14.8	136	24.8	1.79	6.57	75.72	83.28	76.07
180	21.9	180	82	6.9	10.4	4.1	27.9	1450	81.3	7.2	1.71	161	19.8	187	33.3	1.98	9.58	111.47	139.49	127.83
200	26.2	200	90	7.5	11.3	4.5	33.4	2140	117	8	1.87	214	26	250	43.6	2.17	13.5	158.74	223.35	204.47
220	31.1	220	98	8.1	12.2	4.9	39.5	3060	162	8.8	2.02	278	33.1	324	55.7	2.36	18.6	219.25	340.55	312.13
240	36.2	240	106	8.7	13.1	5.2	46.1	4250	221	9.59	2.2	354	41.7	412	70	2.55	25	296.88	507.28	465.94
260	41.9	260	113	9.4	14.1	5.6	53.3	5740	288	10.4	2.32	442	51	514	85.9	2.71	33.5	392.32	716.43	658.22
280	47.9	280	119	10.1	15.2	6.1	61	7590	364	11.1	2.45	542	61.2	632	103	2.86	44.2	506.62	975.09	895.41
300	54.2	300	125	10.8	16.2	6.5	69	9800	451	11.9	2.56	653	72.2	762	122	3.00	56.8	639.26	1294.83	1188.54
320	61	320	131	11.5	17.3	6.9	77.7	12510	555	12.7	2.67	782	84.7	914	143	3.14	72.5	801.18	1699.53	1560.65
340	68	340	137	12.2	18.3	7.3	86.7	15700	674	13.5	2.8	923	98.4	1080	166	3.28	90.4	985.90	2193.48	2014.52
360	76.1	360	143	13	19.5	7.8	97	19610	818	14.2	2.9	1090	114	1280	194	3.43	115	1225.02	2817.70	2588.96
380	84	380	149	13.7	20.5	8.2	107	24010	975	15	3.02	1260	131	1480	222	3.57	141	1480.91	3545.91	3246.12
400	92.4	400	155	14.4	21.6	8.6	118	29210	1160	15.7	3.13	1460	149	1710	254	3.71	170	1773.66	4440.51	4075.10
425	104.	425	163	15.3	23	9.2	132	36970	1440	16.7	3.3	1740	176			3.91	216	2227.54	5856.15	5375.42
450	115	450	170	16.2	24.3	9.7	147	45850	1730	17.7	3.43	2040	203	2400	345	4.07	267	2714.54	7450.29	6846.43
475	128	475	178	17.1	25.6	10.3	163	56480	2090	18.6	3.6	2380	235			4.26	329	3311.98	9501.74	8729.22
500	141	500	185	18	27	10.8	179	68740	2480	19.6	3.72	2750	268	3240	456	4.42	402	3988.01	11866.88	10894.28
550	166	550	200	19	30	11.9	212	99180	3490	21.6	4.02	3610	349			4.81	544	5503.38	18359.14	16877.71
600	199	600	215	21.6	32.4	13	254	139000	4670	23.4	4.3	4630	434			5.12	813	7782.53	26815.31	24597.27

IPN



$$M_{cr,t} = \frac{C_t}{l_e} \qquad C_t = \frac{\pi E \sqrt{I_y J}}{\sqrt{2.6}}$$

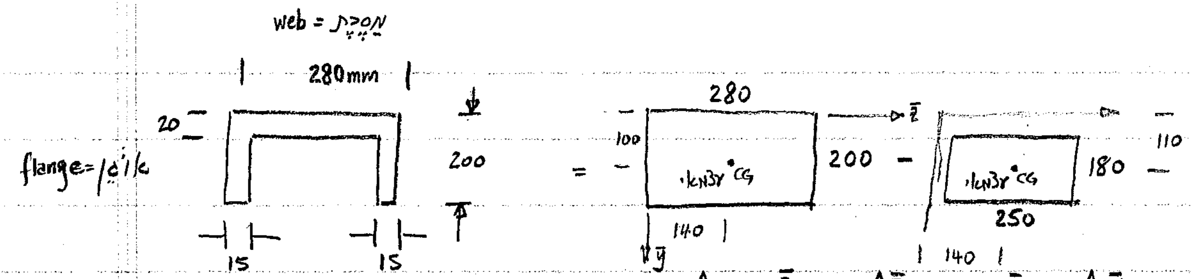
$$M_{cr,wc} = \frac{C_{w,c}}{l_e^2} \qquad C_{w,c} = \pi^2 E i_w^2 W_{el,x}$$

$$M_{cr,we} = \frac{C_{w,e}}{l_e^2} \qquad C_{w,e} = \frac{\pi^2 E I_y (D - t_f)}{2}$$

$$i_w = \sqrt{\frac{I_y}{2(A_f + \frac{1}{6} A_w)}}$$

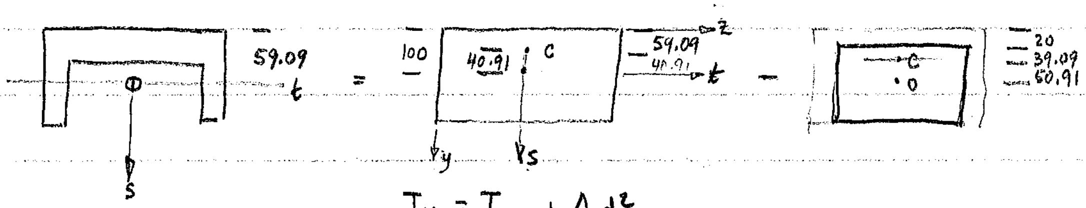
דוגמא - ב

web = מסכת



	A	$\bar{z}$	$A\bar{z}$	$\bar{y}$	$A\bar{y}$
①	56000	140	7.84×10^6	100	56×10^5
− ②	45000	140	6.3×10^6	110	49.5×10^5
Σ	11000		1.54×10^6		6.5×10^5

$$\bar{z}=\frac{\sum A\bar{z}}{\sum A}=140 \qquad \bar{y}=\frac{\sum A\bar{y}}{\sum A}=59.09\text{mm}$$



$$I_{tt}=I_{zz_0}+Ad^2$$

$$I_{zz_0}=\frac{1}{12}bh^3$$

	I_{zz_0}	A	d^2	I_{tt}
①	$\frac{1}{12}(280)(200)^3$	56000	$(-40.91)^2$	$2.804\times10^8\ mm^4$
− ②	$\frac{1}{12}(250)(180)^3$	45000	$(-50.91)^2$	$-2.381\times10^8\ mm^4$
				$42.258\times10^6\ mm^4$

SINCE y AXIS IS AXIS OF SYMMETRY

$I_{st}=0$

$$I_{yy_0}=\frac{1}{2}hb^3$$

$$I_{ss}=I_{yy_0}+Ae^2\ ;\ =I_{yy_0}$$

מכיוון ש-e שווה לאפס

①	$\frac{1}{12}(200)(280)^3$	56000	0	3.659×10^8
− ②	$\frac{1}{12}(180)(250)^3$	45000	0	-2.344×10^8
				$1.315\times10^8\ mm^4$

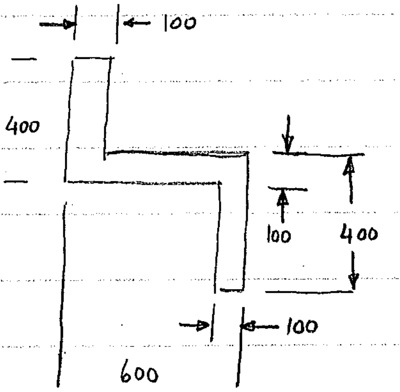
THUS PRINC. AXES ARE SAME AS s,t AND $\varphi=0$

מפני שהציר y או s ציר סימטרי ⇐ $I_{st}=0$. לכן הצירים הראשיים $(t'-s')$ הם אותם צירים כמו s ו-t וזווית φ שווה לאפס

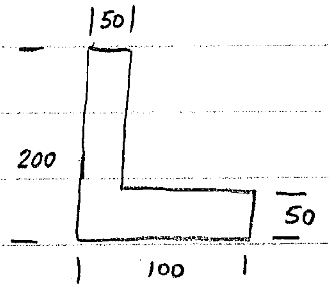
24.10.99

תרגילי בית #2

כל מדידות ב- mm

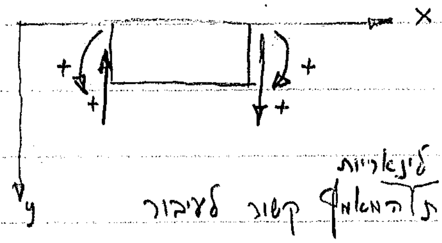


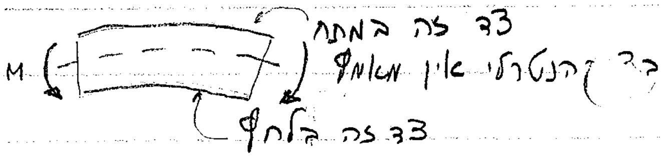
למצוא: מרכז הכובד, מומנט אינרציה מסביב מרכז הכובד, וצירים הראשיים



להגיש ב- 2.11.99

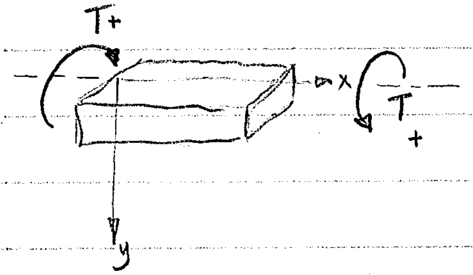
הסכם הסימנים למומנט הכפיפה ולכוח הגזירה





בציר הנטרלי אין עיבור ובבעיות אלסטיות ~~המאמץ~~ ליניאריות המאמץ קשור לעיבור בחוק הוק ($\sigma=E\epsilon$)

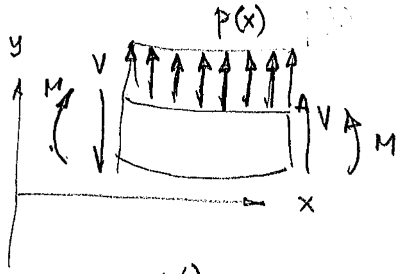
הסכם הסימנים למומנט הפיתול



הנחות

- מישורים לפני כפיפה נשארים מישורים אחרי הכפיפה PLANE SECTIONS REMAIN PLANE
- ציר הנטרלי עובר דרך מרכז הכובד NEUTRAL AXIS PASSES THROUGH CG.
- הגוף הוא הומוגני הצפיפות לא משתנה HOMOG. DENSITY

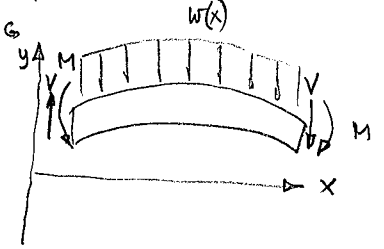
Popov



$$\sigma_x = \frac{-My}{I}$$

$$\frac{dV}{dx} = -p(x) \qquad \frac{dM}{dx} = -V$$

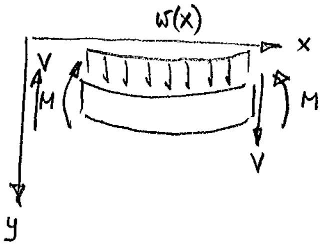
Cook & Young



$$\sigma_x = \frac{My}{I}$$

$$\frac{dV}{dx} = -w(x) \qquad \frac{dM}{dx} = \div V$$

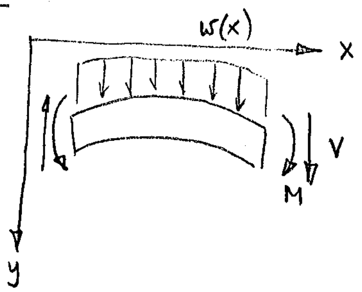
Patel & Venkatraman



$$\sigma_x = \frac{My}{I}$$

$$\frac{dV}{dx} = -w(x) \qquad \frac{dM}{dx} = V$$

שלנו

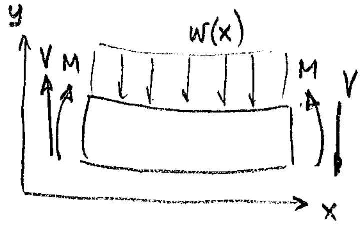


$$\sigma_x = -\frac{My}{I}$$

אותו דבר כמו **אבל**
הציר- y, +V, +M, w מסומנים
הפוך לכיוונים ב- POPOV

$$\frac{dV}{dx} = -w(x) \qquad \frac{dM}{dx} = -V$$

Beer & Johnston
Hibbeler

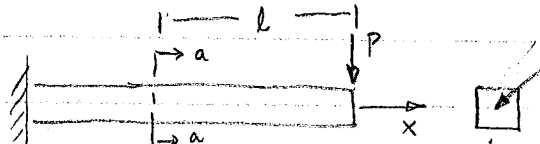


$$\sigma_x = -\frac{My}{I}$$

$$\frac{dV}{dx} = -w(x) \qquad \frac{dM}{dx} = V$$

כפיפה משופעת Skewed or Angled Bending

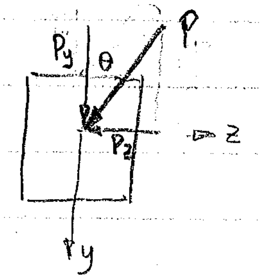
IF SUCH A FORCE ACTS - RESOLVE FORCES



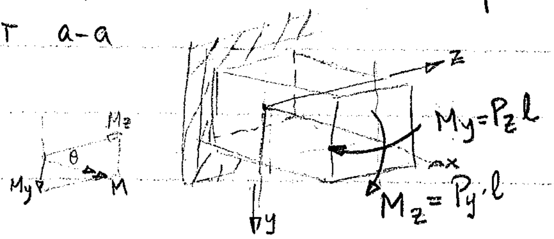
נניח שעומס כזה פועל על המוט

הקטע של a-a רוצים את המאמצים

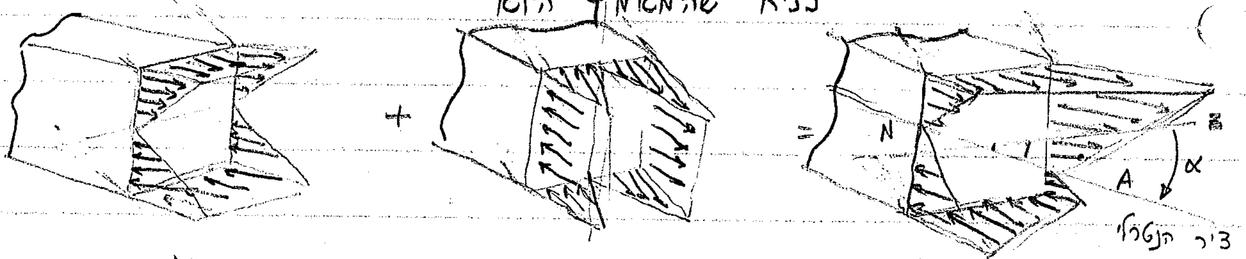
צריך לפרק את העומסים דרך הצירים הראשיים



AT a-a



נניח שהמאמץ הוא



$\sigma_{x_{①}} = \frac{-M_z y}{I_{zz}}$ $\qquad$ $\sigma_{x_{②}} = \frac{M_y \cdot z}{I_{yy}}$ $\qquad$ $\sigma_x = \frac{-M_z y}{I_{zz}} + \frac{M_y z}{I_{yy}}$

SET $\sigma_x = 0$ TO GET NEUTRAL AXIS. AXIS LIES IN y-z plane

בציר הניטרלי $\sigma_x = 0$. לכן, יכולים לקבל את ההשוואה של הקו. הציר שוכב במישור ה y-z.

לכן,
$$y = \frac{M_y z I_{zz}}{M_z I_{yy}}$$

נניח שהציר הניטרלי עושה זווית של α עם הציר - z הפוזיטיבי. מכיון שלמרכז הכובד אין מאמץ, הציר צריך לעבור דרך הנקודה הזאת. לכן מפני שהשיפוע (slope) של הקו הוא $m = \frac{Y - Y_0}{z - z_0}$, ומרכז הכובד הוא נקודת הקו ומרכז הכובד הוא מקור הצירים

לכן,
$$m = \tan\alpha = y/z$$

ומכוון ש
$$M_y = P_z l = P \sin\theta \cdot l$$
$$M_z = P_y l = P \cos\theta \cdot l$$

לכן

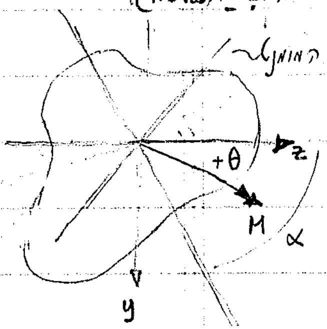
$$\frac{M_y}{M_z} = \tan\theta$$

$$\tan\alpha = \frac{I_{zz}}{I_{yy}} \tan\theta \qquad -1$$

או שהמומנט עושה עם הציר-z. רק אם $I_{yy}=I_{zz}$ שהציר הנטרלי עושה זווית שווה לזווית של העומס. במצב הזה, מישור הכפיפה הוא מאונך למישור הנטרלי.

THUS $\theta \neq \alpha$. ONLY... WHEN $I_z=I_y \Rightarrow \theta=\alpha$

- כמה דברים נוספים:
- בשטח-החתך עיגולי, בגלל ש $I_{yy}=I_{zz}$, $\alpha=\theta$.
- אם צורת שטח-החתך שרירותית (ARBITRARY), פירי בספר שלו Aircraft Struct. הוכיח (PROVED) שהמאמץ שווה ל-



$$\sigma_x = -\left(\frac{M_z I_{yy} + M_y I_{yz}}{I_{yy} I_{zz} - I_{yz}^2}\right) y + \left(\frac{M_y I_{zz} + M_z I_{yz}}{I_{yy} I_{zz} - I_{yz}^2}\right) z$$

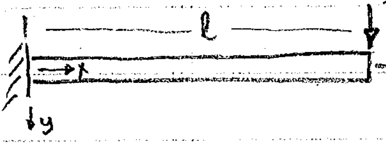
והזווית α מיוחסת ל-θ והיחס הוא

$$\frac{y}{z} = \tan\alpha = \frac{I_{zz}\tan\theta + I_{yz}}{I_{yy} + I_{yz}\tan\theta}$$

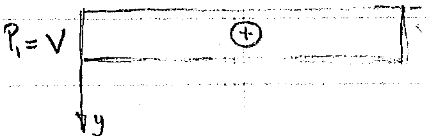
- הצירים z ו-y חייבים לעבור דרך מרכז הכובד אבל אינם מחויבים להיות ראשיים
- $\theta \neq \alpha$ והמישור כפיפה אינו מאונך למישור הנטרלי
- כדי שלא יהיו מאמצים פיתוליים, צריך לשים את הכוחות במרכז הגזירה (נדון על זה בעתיד). כאשר יש שטח-חתך דו-סימטרי, מרכז הגזירה הוא מרכז הכובד.

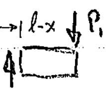
— פעולה משולבת של כוח צירי ומומנט כפיפה (Combined action of axial force + bending moments)

כאשר יש כפיפה טהורה, ציר הנטרלי הולך דרך מרכז הכובד.

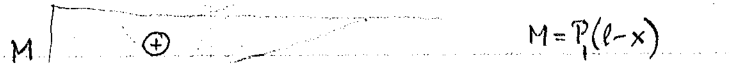


נסתכל במוט שעליו יש שני עומסים - אחד שגורם לכפיפה והעומס השני שגורם למתח בכיוון ציר ה-x החיובי. מה קורה לציר הנטרלי ואיך לחשב את השוואת הקוו? מהו המאמץ המקסימלי?

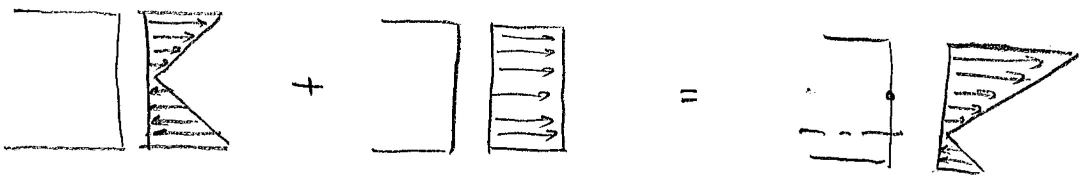




+M +V כוח הגזירה הוא מתמיד



המומנט של העומס P_1 הוא חיובי $M=P_1(\ell-x)$



$\sigma_{x_①} = \frac{-My}{I_{zz}}$ + $\sigma_{x_②} = \frac{P_2}{A}$ סך-הכל

הציר הנטרלי זז ממקומו וזה תלוי בגודל σ_{x_2}. פה הציר לא עובר דרך מרכז הכובד

- יכולים לעשות זה אך ורק כאשר התזוזות כלפי, בכיוון -y הן קטנות

displ small

- יכולים לעשות זה כאשר העיבורים הם עדיין אלסטיים ויש קשר לינארי למאמצים.

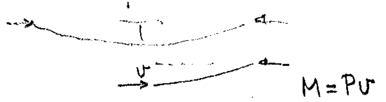
ε still elastic

- אם התזוזות לא קטנות, הן מגדילות את המומנטים לפעמים (אם העומס P_2 הוא עומס לחיצה).

COMPRESSIVE LOAD INCREASE DISPLACEMENT

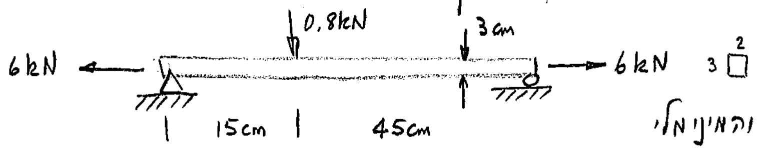
- השרטוטים האלה לא מציגים את המאמצים במקום קרוב למקום שבו פועלים את העומסים.

stresses represent region away from load



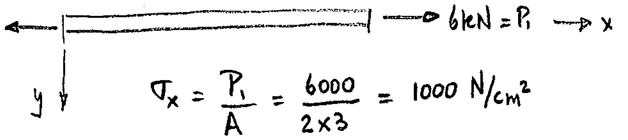
דוגמא

מוט בן שטח-חתך 2cm x 3cm, באורך 60cm, ללא משקל, עמוס ומורכב בשירטוט



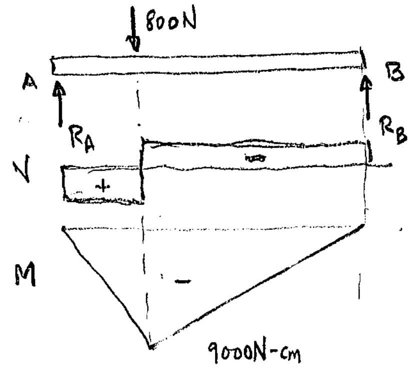
למצוא: את המאמץ המקסימלי והמינימלי

הנחות: המוט פועל כמוט אלסטי, המוט הומוגני



מכוח הצירי:

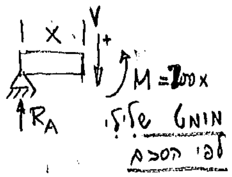
$$\sigma_x = \frac{P_1}{A} = \frac{6000}{2\times3} = 1000\ N/cm^2$$



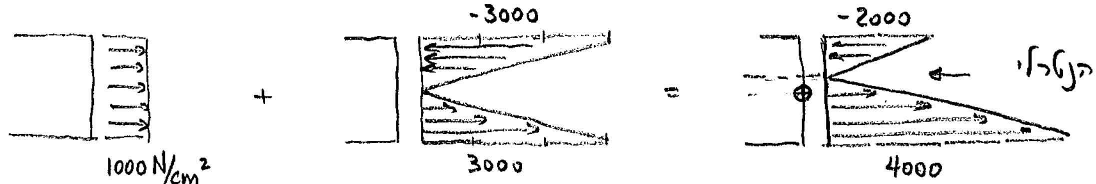
$$\Sigma M_A = 800\times15 - R_B\cdot60 = 0 \quad R_B = 200N$$

$$\Sigma F = -(R_A+R_B)+800=0 \quad R_A = 600N$$

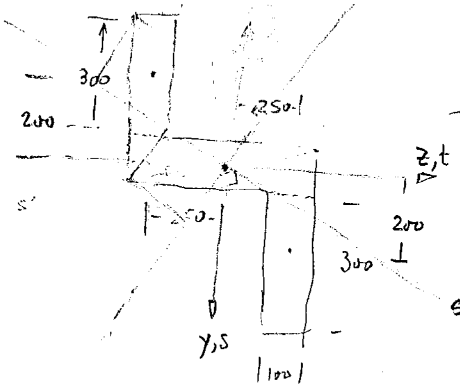
לחשב את המאמץ שבא מהכפיפה



$$\sigma_x = \frac{Mc}{I} = \frac{(9000\ N\text{-}cm)\cdot1.5\ cm}{2\cdot3^3/12} = \pm 3000\ N/cm^2$$



- מאמץ הכפיפה גדול משל מאמץ הכוח הצירי.



	A	y	$A\bar{y}$	$\bar{z}$	$A\bar{z}$
①	300×100	-200	-105×10^5	-250	-75×10^5
②	600×100	0	0	0	0
③	300×100	200	105×10^5	250	75×10^5
ΣA	12×10^4		0		0

$$\bar{y}=\frac{\Sigma A\bar{y}}{\Sigma A}=0 \qquad \bar{z}=0$$

	I_{zz_0}	A	d^2	I_{tt}
①	$\frac{1}{12}(100)(300)^3$	3×10^4	$(+200)^2$	$2.25\times10^8+12\times10^8=14.25\times10^8$
②	$\frac{1}{12}(600)(100)^3$	6×10^4	0	0.5×10^8
③	$\frac{1}{12}(100)(300)^2$	3×10^4	$(-200)^2$	14.25×10^8
				29×10^8

	I_{yy_0}	A	e^2	I_{ss}
①	$\frac{1}{12}(300)(100)^3$	3×10^4	$(250)^2$	$25\times10^6+1875\times10^6=19\times10^8$
②	$\frac{1}{12}(100)(600)^3$	6×10^4	$(0)^2$	18×10^8
③	$\frac{1}{12}(300)(100)^3$	3×10^4	$(-250)^2$	19×10^8
				56×10^8

I_{yz_0}	A	ed	I_{st}
0	3×10^4	$(250)(200)$	15×10^8
0	6×10^4	$0\cdot0$	0
0	3×10^4	$(-250)(-200)$	15×10^8
			30×10^8

$$I_{t't'}=\frac{I_{ss}+I_{tt}}{2}+\sqrt{\frac{(I_{tt}-I_{ss})^2}{4}+I_{st}^2}$$

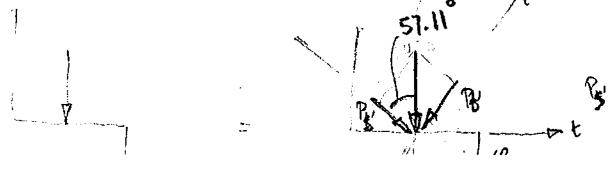
$$=42.5\times10^8+\sqrt{\frac{(27\times10^8)^2}{4}+(30\times10^8)^2}=42.5\times10^8+32.898\times10^8=75.398\times10^8$$

$$I_{s's'}=\frac{I_{ss}+I_{tt}}{2}-\sqrt{\left(\frac{I_{tt}-I_{ss}}{2}\right)^2+I_{st}}=42.5\times10^8-32.898\times10^8=9.602\times10^8$$

$$\tan 2\varphi=\frac{2I_{st}}{I_{tt}-I_{ss}}=\frac{2\cdot30\times10^8}{29\times10^8-56\times10^8}=\frac{60}{-27}=-2.222$$

$2\varphi=-65.77^\circ \qquad \varphi=-32.89^\circ$

$I_{s't'}=0$



$P_{s'}=P\cos 57.11^\circ$

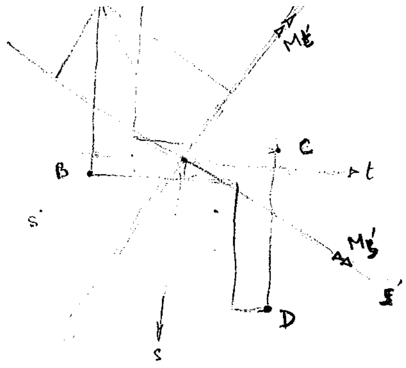
$P_{t'}=P\sin 57.11^\circ$

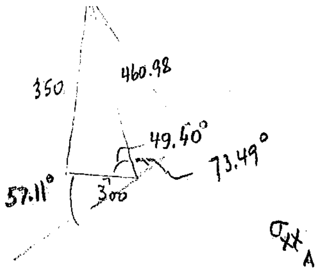
$M_{t'}=PL\cos 57.11^\circ=P_{t'}l$

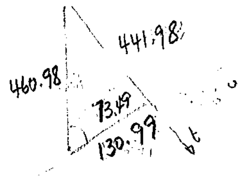
$M_{s'}=PL\sin 57.11^\circ$

$t' = -s\sin\varphi + t\cos\varphi$

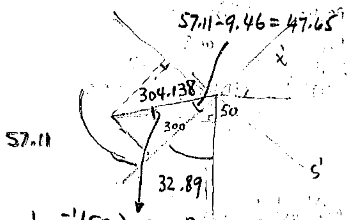
$= -52.89$
$s = -350$
$t = -300$







$$\sigma_{xx_A} = \frac{+PL\sin 57.11^\circ\cdot(+130.99)}{9.602\times10^8} - \frac{PL\cos 57.11^\circ(441.98)}{75.398\times10^8} = PL\cdot 14.64\times10^{-8}$$



$s' = 304.138\sin 47.65 = 224.77$
$t' = 304.138\cos 47.65 = 204.88$

$$\sigma_x = -\frac{M_t s'}{I_{t't'}} + \frac{M_s t'}{I_{s's'}}$$

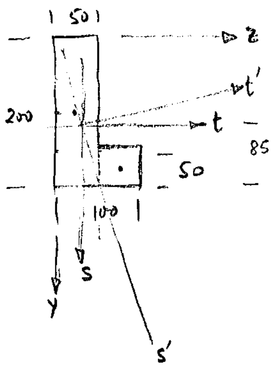
$\tan^{-1}\left(\frac{50}{300}\right) = 9.46^\circ$

$$\sigma_{xx_B} = \frac{+PL\sin 57.11^\circ}{9.602\times10^8}(+204.88) - \frac{PL\cos 57.11^\circ(-224.77)}{75.398\cdot10^8}$$

$$\sigma_{xx_C} = -\sigma_{xx_B}$$

$$\sigma_{xx_A} = -\sigma_{xx_D}$$

#2 פתרון



	A	$\bar{y}$	$A\bar{y}$	$\bar{z}$	$A\bar{z}$
①	50x200	100	1×10^6	25	2.5×10^5
②	50x50	175	4.375×10^5	75	1.875×10^5
	12500		14.375×10^5		4.375×10^5

$$\bar{y} = 115 = \frac{\Sigma A\bar{y}}{\Sigma A} \qquad \bar{z} = 35 = \frac{\Sigma A\bar{z}}{\Sigma A}$$

	I_{zz_0}	A	d^2	I_{tt}
①	$\frac{1}{12}(50)(200)^3$	1×10^4	$(15)^2$	$33.33\times10^6 + 2.25\times10^6 = 35.583\times10^6$
②	$\frac{1}{12}(50)(50)^3$	$.25\times10^4$	$(-60)^2$	$.52083\times10^6 + 9\times10^6 = 9.521\times10^6$
				45.104×10^6

	I_{yy_0}	A	e^2	I_{ss}
①	$\frac{1}{12}(200)(50)^3$	1×10^4	10^2	$2.0833\times10^6 + 1\times10^6 = 3.0833\times10^6$
②	$\frac{1}{12}(50)(50)^3$	$.25\times10^4$	$(-40)^2$	$.52083\times10^6 + 4\times10^6 = 4.5208\times10^6$
				7.6041×10^6

	I_{yz_0}	A	e d	I_{st}
①	0	1×10^4	10(15)	1.5×10^6
②	0	$.25\times10^4$	(-60)(-40)	6×10^6
				7.5×10^6

$$\tan 2\varphi = \frac{2I_{st}}{I_{tt} - I_{ss}} = \frac{2\times7.5\times10^6}{(45.104 - 7.604)\times10^6} = 0.4$$

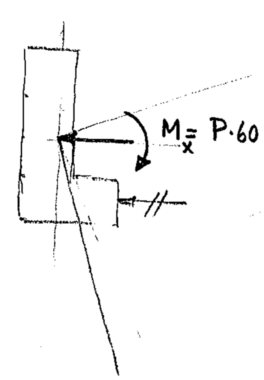
$2\varphi = 21.80^\circ$

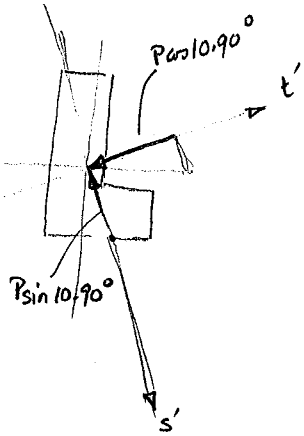
$$I_{t't'} = \frac{I_{ss}+I_{tt}}{2} + \sqrt{\left(\frac{I_{tt}-I_{ss}}{2}\right)^2 + I_{st}^2}$$

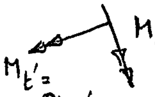
$$= \left(\frac{17.604+45.104}{2} + \sqrt{\left(\frac{37.5}{2}\right)^2+(7.5)^2}\right) \times 10^6 = 26.354 + 20.194 = 46.548 \times 10^6$$

$$I_{s's'} = \frac{I_{ss}+I_{tt}}{2} - \sqrt{\quad} = 26.354 - 20.194 = 6.16 \times 10^6$$

$$I_{s't'} = 0$$



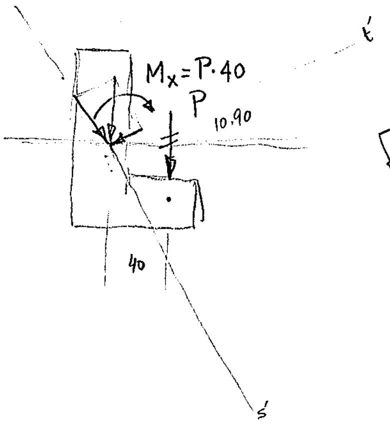




$M_{s'} = PL\cos 10.90°$

$M_{t'} = PL\sin 10.90°$

$$\sigma_{xx'} = \frac{PL\sin 10.90°}{I_{t't'}} s' + \frac{PL\cos 10.90°}{I_{s's'}} t'$$



$P_{s'} = +P\cos 10.90$

$P_{t'} = -P\sin 10.90$

$M_{t'} = PL\cos 10.90$

$M_{s'} = +PL\sin 10.90$

$$\sigma_{xx} = \frac{-M_{t'}}{I_{t't'}} s' + \frac{M_s}{I_{s's'}} t'$$

$$= -\frac{PL\cos 10.90 \cdot s'}{46.548 \times 10^6} + \frac{PL\sin 10.90 \cdot t'}{6.16 \times 10^6}$$

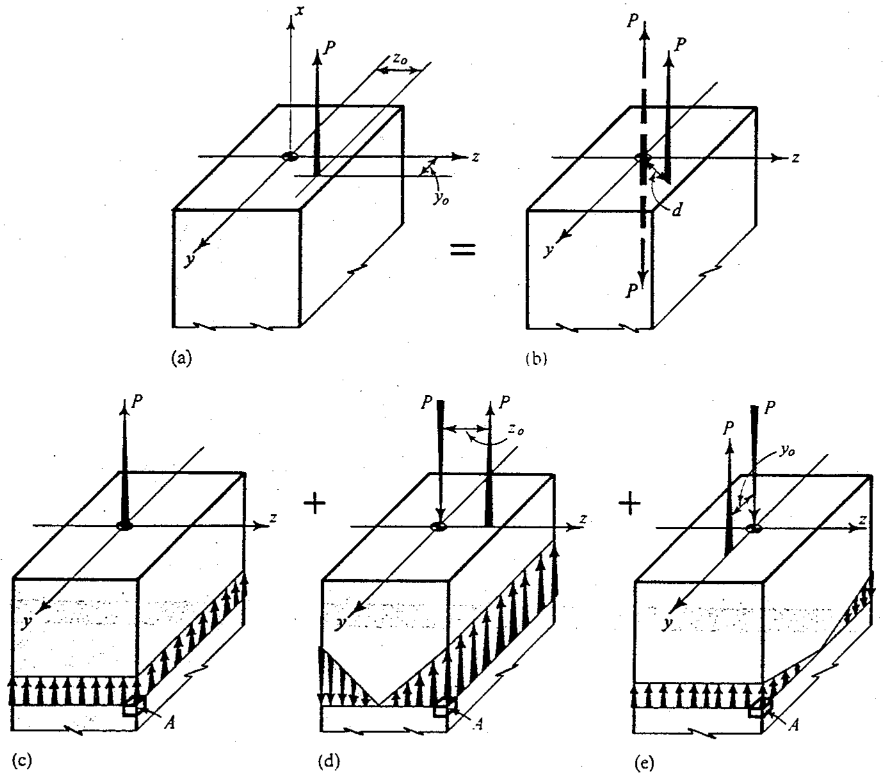


Fig. 8-11. Resolution of a problem into three problems, each one of which may be solved by the methods previously discussed.

are the principal axes, Eq. 8-8 is applicable to prismatic members of any cross-sectional shape.

For a given loading condition Eq. 8-8 can be rewritten as

$$\sigma_x = A + By + Cz \tag{8-9}$$

where A, B, and C are constants. This is seen to be an equation of a plane; it clearly shows the nature of stress distribution. For the linearly elastic case under discussion, dividing through Eq. 8-9 by the elastic modulus E recovers the basic kinematic assumption of the technical theory, i.e.,

$$\varepsilon_x = a + by + cz \tag{8-10}$$

where a, b, and c are constants.

KERN - גרעין

Section 8-4
Eccentrically loaded members

In some eccentrically loaded members it is possible to locate the line of zero stress within the cross-sectional area of a member by determining a line where $\sigma_x = 0$. This line is analogous to the neutral axis occurring in pure bending. Unlike the former case, however, with $P \neq 0$ this line does not pass through the centroid of a section. For large axial loads and small moments, it lies outside the cross section. Its significance lies in the fact that the normal stresses vary linearly from it.

This method is applicable for compression members providing their length is small in relation to their transverse dimensions. Slender bars in compression require special treatment (Chapter 14). Also, near the point of application of the force, the analysis developed here is incorrect. There the stress distribution is greatly disturbed and is similar to a local stress concentration (see Art. 4-18 and especially Fig. 4-30).

EXAMPLE 8-5

Find the stress distribution at the section $ABCD$ for the block shown in Fig. 8-12(a) if $P = 14.4$ kips. At the same section, locate the line of zero stress. Neglect the weight of the block.

SOLUTION

The forces acting on the section $ABCD$, Fig. 8-5(c), are $P = -14.4$ kips, $M_{yy} = -14.4(6) = -86.4$ kip-in., and $M_{zz} = -14.4(3 + 3) = -86.4$

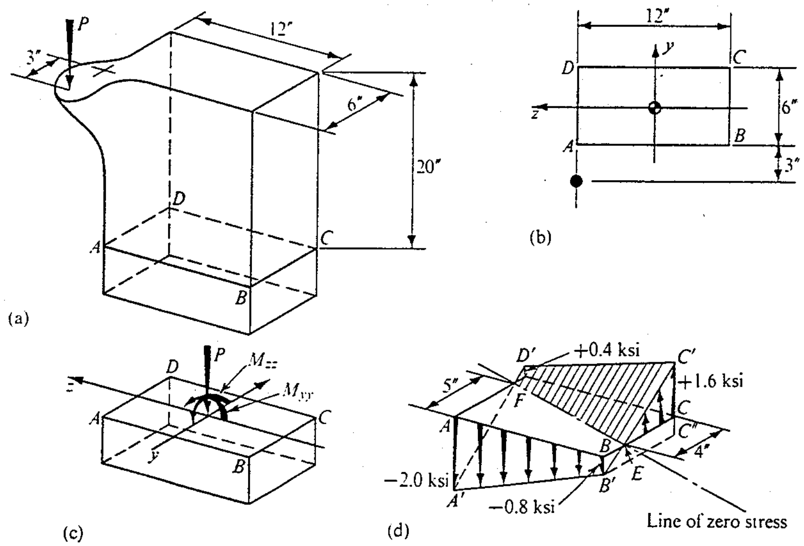


Fig. 8-12

kip-in. The cross section of the block $A = 6(12) = 72$ in.^{2}, and the respective section moduli are $S_{zz} = 12(6)^2/6 = 72$ in.^{3} and $S_{yy} = 6(12)^2/6 = 144$ in.^{3} Hence, using a relation equivalent to Eq. 8-8 gives the compound normal stresses for the corner elements:

$$\sigma = \frac{P}{A} \mp \frac{M_{zz}}{S_{zz}} \pm \frac{M_{yy}}{S_{yy}} = -\frac{14.4}{72} \pm \frac{86.4}{72} \mp \frac{86.4}{144} = -0.2 \pm 1.2 \mp 0.6$$

Here the units of stress are kips per square inch. The sense of the forces shown in Fig. 8-12(c) determines the signs of stresses. Therefore, if the subscript of the stress signifies its location, the corner normal stresses are:

$$\sigma_A = -0.2 - 1.2 - 0.6 = -2.0 \text{ ksi}$$

$$\sigma_B = -0.2 - 1.2 + 0.6 = -0.8 \text{ ksi}$$

$$\sigma_C = -0.2 + 1.2 + 0.6 = +1.6 \text{ ksi}$$

$$\sigma_D = -0.2 + 1.2 - 0.6 = +0.4 \text{ ksi}$$

These stresses are shown in Fig. 8-12(d). The ends of these four stress vectors at *A′*, *B′*, *C′*, and *D′* lie in the plane *A′B′C′D′*. The vertical distance between the planes *ABCD* and *A′B′C′D′* defines the compound stress at any point on the cross section. The intersection of the plane *A′B′C′D′* with the plane *ABCD* locates the line of zero stress *FE*.

By drawing a line *B′C″* parallel to *BC*, similar triangles *C′B′C″* and *C′EC* are obtained: thus the distance $CE = [1.6/(1.6 + 0.8)]6 = 4$ in. Similarly, the distance *AF* is found to be 5 in. Points *E* and *F* locate the line of zero stress.

EXAMPLE 8-6

Find the zone over which the vertical downward force P_o may be applied to the rectangular weightless block shown in Fig. 8-13(a) without causing any tensile stresses at the section *A-B*.

SOLUTION

The force $P = -P_o$ is placed at an arbitrary point in the first quadrant of the *y-z* coordinate system shown. Then the same reasoning used in the preceding example shows that with this position of the force the greatest tendency for a tensile stress exists at *A*. With $P = -P_o$, $M_{zz} = +P_oy$ and $M_{yy} = -P_oz$, setting the stress at *A* equal to zero fulfills the limiting condition of the problem. Using Eq. 8-8 allows the stress at *A* to be expressed as:

$$\sigma_A = 0 = \frac{(-P_o)}{A} - \overset{M_{zz} \cdot y/I_{zz}}{\frac{(P_oy)(-b/2)}{I_{zz}}} + \overset{M_{yy} \cdot z/I_{yy}}{\frac{(-P_oz)(-h/2)}{I_{yy}}}$$

or

$$-\frac{P_o}{A} + \frac{P_oy}{b^2h/6} + \frac{P_oz}{bh^2/6} = 0$$

M_z-I M_y-N $\sigma_x>0$

At A $(y=-b/2, z=-h/2)$

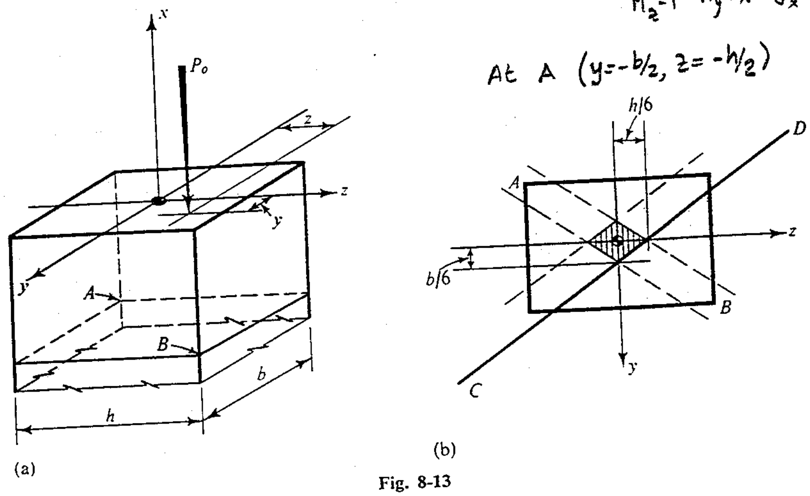


Fig. 8-13

Simplifying $[z/(h/6)] + [y/(b/6)] = 1$, which is an equation of a straight line. It shows that when $z = 0$, $y = b/6$; and when $y = 0$, $z = h/6$. Hence this line may be represented by the line *CD* in Fig. 8-13(b). A vertical force may be applied to the block anywhere on this line and the stress at *A* will be zero. Similar lines may be established for the other three corners of the section; these are shown in Fig. 8-13(b). If the force *P* is applied on any one of these lines or on any line parallel to such a line toward the centroid of the section, there will be no tensile stress at the corresponding corner. Hence the force *P* may be applied anywhere within the shaded area in Fig. 8-13(b) without causing tensile stress at any of the four corners or anywhere else. This zone of the cross-sectional area is called the *kern* of a section.

If for a rectangular block the location of the force *P* is limited to one of the lines of symmetry, the maximum eccentricity $e = h/6$ to give zero stress along one of the edges, Figs. 8-14(a) and (b). This leads to a practical rule, much used in the past by designers of masonry structures: If the resultant of vertical forces acts within the middle third of a rectangular section, there is no tension in the material at that section. If, further, the applied load *P* acts outside the middle third and the contact surfaces cannot transmit tensile forces, one has the case shown in Figs. 8-14(c) and (d). Here, assuming elastic action, the normal stress at *B* may be expressed as

$$\sigma_B = -\frac{P}{xb} + P\left(\frac{x}{2} - a\right)\frac{6}{bx^2} = 0$$

where $(x/2) - a$ is the eccentricity of the applied force with respect to

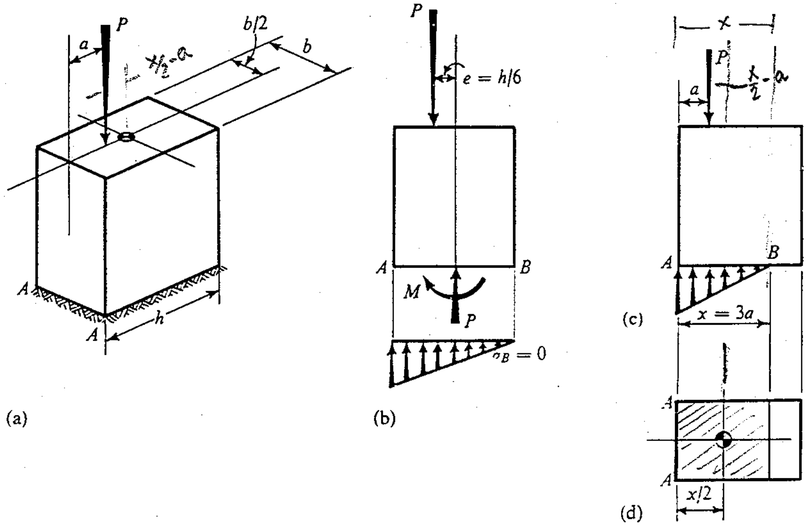


Fig. 8-14. (a) Eccentrically loaded block; (b) location of P to cause zero stress at B; (c) elastic stress distribution between two surfaces which are unable to transmit tensile forces.

$I_{eff} = \frac{bx^3}{12}$

$c = x/2$

$\frac{c}{I_{eff}} = \frac{bx^2}{6}$

the centroidal axis of the shaded contact area, and $bx^2/6$ is its section modulus. Solving for x, one finds that $x = 3a$; the pressure distribution will be "triangular" as in Fig. 8-14(c) (why?). As a decreases, the intensity of pressure on the line A-A increases; when a is zero, the block becomes unstable. Such problems are important in the design of foundations.

8-5. SUPERPOSITION OF SHEARING STRESSES

In the preceding part of the chapter superposition of the normal stresses σ_x was the principal concern. In problems where both the elastic torsional and direct shearing stresses can be determined, the compound shearing stress also may be found by superposition. This corresponds to superposition of the off-diagonal stresses in Eq. 8-1. Here attention will be directed to instances where the shearing stresses being superposed not only act on the same element of area but also have the same line of action.*

Only elastic stresses fall within the scope of this treatment.

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* Noncolinear shearing stresses acting on the same element of area can be added vectorially.

EXAMPLE 8-7

Find the maximum shearing stress due to the applied forces in the plane A-B of the ½ in.-diameter, high-strength shaft in Fig. 8-15(a).

SOLUTION

The free body of a segment of the shaft is shown in Fig. 8-15(b). The system of forces at the cut necessary to keep this segment in equilibrium consists of a torque $T = 200$ in-lb, a shear $|V| = 60$ lb, and a bending moment $M = 240$ in-lb.

Because of the torque T, the shearing stresses in the cut A-B vary linearly from the axis of the shaft and reach the maximum value given by Eq. 5-4, $\tau_{\max} = Tc/J$. These maximum shearing stresses, agreeing in sense with the resisting torque T, are shown at points A, B, D, and E in Fig. 8-15(c).

The "direct" shearing stresses caused by the shearing force V may be obtained by using Eq. 7-6, $\tau = VQ/(It)$. For the elements A and

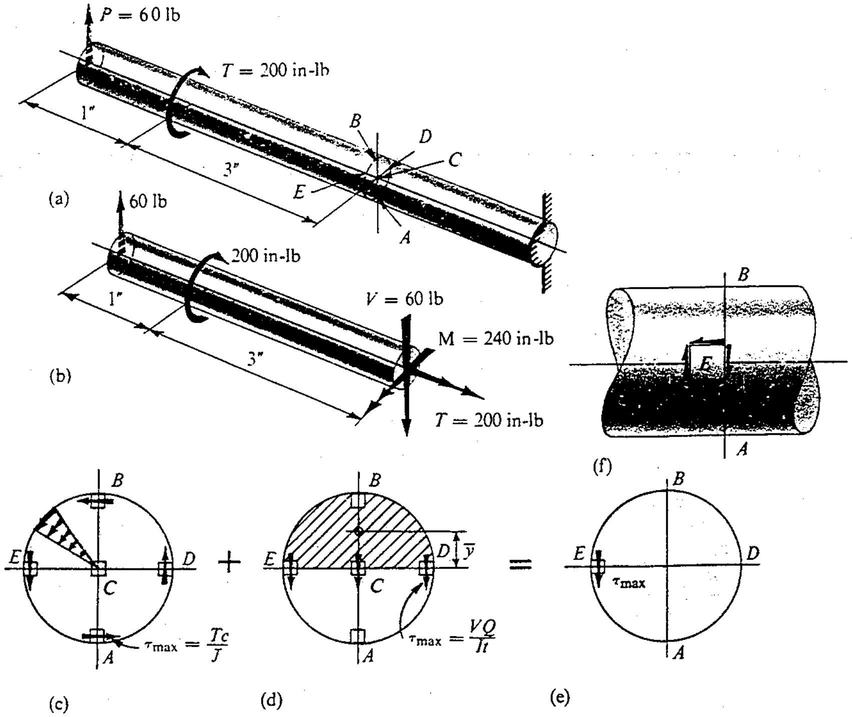

Fig. 8-15

B, Fig. 8-15(d), $Q = 0$, hence $\tau = 0$. The shearing stress reaches its maximum value at the level ED. To determine this, consider Q equal to the shaded area in Fig. 8-15(d) multiplied by the distance from its centroid to the neutral axis. The latter quantity is $\bar{y} = 4c/(3\pi)$, where c is the radius of the cross-sectional area. Hence $Q = (\pi c^2/2)[4c/(3\pi)] = 2c^3/3$. Moreover, since $t = 2c$, and $I = J/2 = \pi c^4/4$, the maximum direct shearing stress is

$$\tau_{\max} = \frac{VQ}{It} = \frac{V}{2c}\frac{2c^3}{3}\frac{4}{\pi c^4} = \frac{4V}{3\pi c^2} = \frac{4V}{3A}$$

where A is the entire cross-sectional area of the rod. In Fig. 8-15(d) this shearing stress is shown acting downward on the elementary areas at E, C, and D. This direction agrees with the direction of the shear V.

To find the maximum compound shearing stress in the plane A-B, the stresses shown in Figs. 8-15(c) and (d) are superposed. Inspection shows that the maximum shearing stress is at E since in the two diagrams the shearing stresses at E have the same direction and sense. There are no direct shearing stresses at A and B, and at C there is no torsional shearing stress. The two shearing stresses have an opposite sense at D. The five points A, B, C, D, and E thus considered for the compound shearing stress are all that may be adequately treated by the methods developed in this text. However, this procedure selects the elements where the maximum shearing stresses occur.

$$J = \frac{\pi d^4}{32} = \frac{\pi(0.5)^4}{32} = 0.00614 \text{ in.}^4 \qquad \text{and} \qquad I = \frac{J}{2} = 0.00307 \text{ in.}^4$$

$$A = \pi d^2/4 = 0.196 \text{ in.}^2$$

$$(\tau_{\max})_{\text{torsion}} = \frac{Tc}{J} = \frac{200(0.25)}{0.00614} = 8{,}150 \text{ psi}$$

$$(\tau_{\max})_{\text{direct}} = \frac{VQ}{It} = \frac{4V}{3A} = \frac{4(60)}{3(0.196)} = 408 \text{ psi}$$

$$\tau_E = 8{,}150 + 408 = 8{,}560 \text{ psi}$$

A planar representation of the shearing stress at E with the matching stresses on the longitudinal planes is shown in Fig. 8-15(f). No normal stress acts on this element as it is located on the neutral axis.

8-6. STRESSES IN CLOSELY COILED HELICAL SPRINGS

Helical springs, such as the one shown in Fig. 8-16(a), are often used as elements of machines. With certain limitations, these springs may be analyzed for elastic stresses by a method similar to the one used in the

preceding example. The discussion will be limited* to springs manufactured from rods or wires of circular cross section. Moreover, any one coil of such a spring will be assumed to lie in a plane which is nearly perpendicular to the axis of the spring. This requires that the adjoining coils be close together. With this limitation, a section taken perpendicular to the axis of the spring's rod becomes nearly vertical.† Hence to maintain equilibrium of a segment of the spring, only a shearing force $V = F$ and a torque $T = F\bar{r}$ are required at all sections through the rod, Fig. 8-16(b).‡ Note that $\bar{r}$ is the distance from the axis of the spring to the centroid of the rod's cross-sectional area.

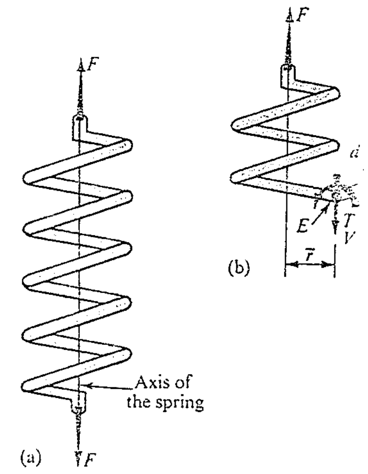

Fig. 8-16. Closely coiled helical spring.

The maximum shearing stress at an arbitrary section through the rod could be obtained as in the preceding example by superposing the torsional and the direct shearing stresses. This maximum shearing stress occurs at the inside of the coil at point E, Fig. 8-16(b). However, in the analysis of springs it has become customary to assume that the shearing stress caused by the direct shearing force is uniformly distributed over the cross-sectional area of the rod. Hence, the nominal direct shearing stress for any point on the cross section is $\tau = F/A$. Superposition of this stress and the torsional shearing stress at E gives the maximum compound shearing stress. Thus since $T = F\bar{r}$, $d = 2c$, and $J = \pi d^4/32$

$$\tau_{\max} = \frac{F}{A} + \frac{Tc}{J} = \frac{Tc}{J}\left(\frac{FJ}{ATc} + 1\right) = \frac{16F\bar{r}}{\pi d^3}\left(\frac{d}{4\bar{r}} + 1\right) \tag{8-11}$$

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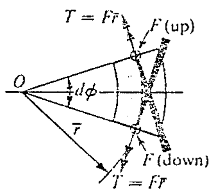
* For a complete discussion on springs see A. M. Wahl, *Mechanical Springs* (Cleveland, Ohio: Penton Publishing Co., 1944).

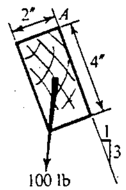
† This eliminates the necessity of considering an axial force and a bending moment at the section taken through the spring.

‡ In previous work it has been reiterated that if a shear is present at a section, a change in the bending moment must take place along the member. Here a shear acts at every section of the rod, yet no bending moment nor a change in it occurs. This is so only because the rod is curved. An element of the rod viewed from the top is shown in the figure. At both ends the torques are equal to $F\bar{r}$ and act in the directions shown. The component of these vectors toward the axis of the spring O, resolved at the point of intersection of the vectors, $2F\bar{r}\,d\phi/2 = F\bar{r}\,d\phi$, opposes the couple developed by the vertical shears $V = F$, which are $\bar{r}\,d\phi$ apart.



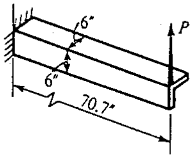
Determine the required dimensions of the beam so that the maximum stress due to bending does not exceed 1,500 psi. (b) Locate the neutral axis of the beam and show its position on the sketch.

8-20. A full-sized, 2-in.-by-4-in. cantilever projects 4 ft from a wall in a tilted position as shown in the figure. At the free end a vertical force of 100 lb is applied which acts through the centroid of the section. Determine the maximum flexural stress, caused by the applied force, in the beam at the built-in end and locate the neutral axis. Neglect the weight of the beam. *Ans.* ±1,423 psi, 3.34 in.



PROB. 8-20

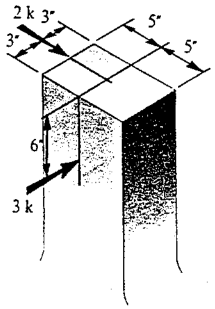
8-21. A 6-in.-by-6-in.-by-½-in. steel angle with one of its legs placed in a horizontal position and its other leg directed downward is used as a cantilever 70.7 in. long, see figure. If an upward force of 1,000 lb is applied at the end of this cantilever through the shear center, what are the maximum tensile and compressive stresses at the built-in end? Neglect the weight of the angle. (See hint for Prob. 6-22.) *Ans.* 14.7 ksi, −18.4 ksi.



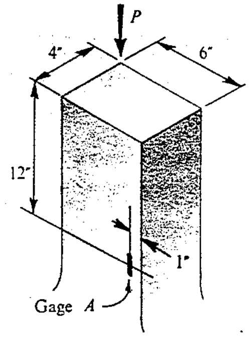
PROB. 8-21

8-22. If the block shown in Fig. 8-12 is made from steel weighing 0.283 lb per cubic inch, find the magnitude of the force P necessary to cause zero stress at D. Neglect the weight of the small bracket supporting the load. For the same condition, locate the line of zero stress at the section $ABCD$. *Ans.* 203 lb.

8-23. A cast iron block is loaded as shown in the figure. Neglecting the weight of the block, determine the stresses acting normal to a section taken 18 in. below the top and locate the line of zero stress.



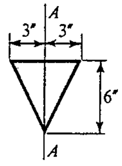
PROB. 8-23



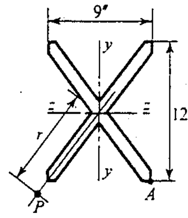
PROB. 8-24

8-24. An aluminum-alloy block is loaded as shown in the figure. The application of this load produces a tensile strain of 500×10^{-6} in. per inch at A as measured by means of an electrical strain gage. Compute the magnitude of the applied force P. Let $E = 10 \times 10^6$ psi. *Ans.* 30 k.

8-25. A short block has cross-sectional dimensions in plan view as shown in the figure. Determine the range along the line A–A over which a downward vertical force could be applied to the top of the block without causing any tension at the base. Neglect the weight of the block. *Ans.* Between 3 in. and 4½ in. from the apex.

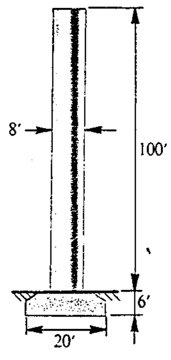


PROB. 8-25



PROB. 8-26

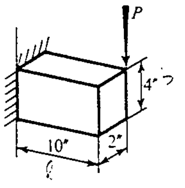
8-26. A short compression member has the proportions shown in the figure; its $A = 72.9$ in.², $I_{zz} = 1{,}199$ in.⁴, and $I_{yy} = 633$ in.⁴ Determine the distance r along the diagonal where a longitudinal force P should be applied so that point A lies on the line of zero stress. Neglect the weight of the member. *Ans.* 53 in.



PROB. 8-28

8-27. Determine the kern for a member having a solid, circular cross section. *Ans.* $c/4$.

8-28. An 8-ft-diameter steel stack, partially lined with brick on the inside, together with a 20-ft-by-20-ft concrete foundation pad weighs 160,000 lb. This stack projects 100 ft above the ground level, as shown in the figure, and is anchored to the foundation. If the horizontal wind pressure is assumed to be 20 lb per square foot of the projected area of the stack and the wind blows in the direction parallel to one of the sides of the square foundation, what is the maximum foundation pressure? *Ans.* 1.21 k per square foot.



PROB. 8-29

8-29. A rectangular cantilever 10 in. long is loaded with $P = 10$ kips at the free end as shown in the figure. Determine the maximum shearing stress at the built-in end due to the direct shear and the torque. Show the result on a sketch analogous to Fig. 8-15(e).

8-30. A helical compression spring is made from ⅛-in.-diameter phosphor-bronze wire and has an outside diameter of 1¼ in. If the allowable shearing stress is 30,000 psi, what force may be applied to this spring? Correct the answer for stress concentrations.

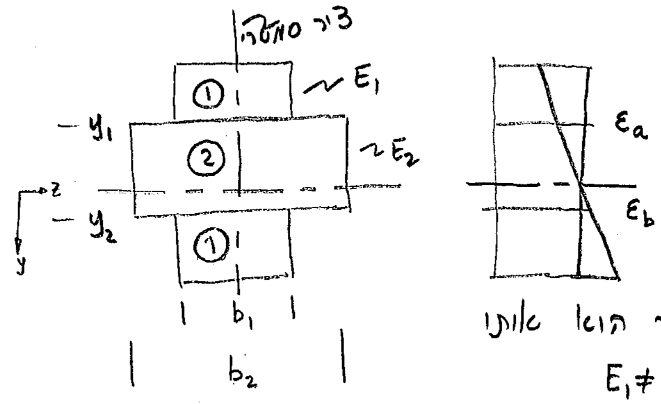
8-31. A helical spring is made of ½-in.-diameter steel wire by winding it on a 5-in.-diameter mandrel. If there are 10 active coils, what is the spring constant? $G = 12 \times 10^6$ psi. What force must be applied to the spring to elongate it 1¾ in?

8-32. A helical valve spring is made of ¼-in.-diameter steel wire and has an outside diameter of 2 in. In operation the compressive force applied to this spring varies from 20 lb minimum to 70 lb maximum. If there are eight active coils, what is the valve lift (or travel) and what is the maximum shearing

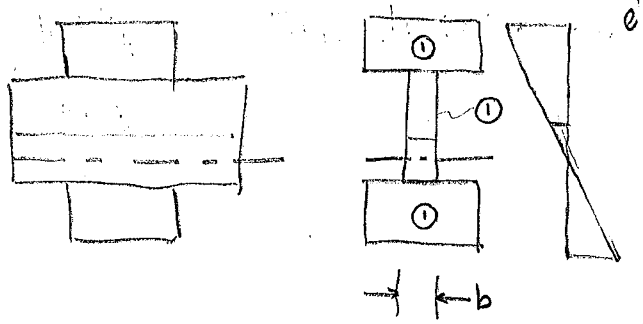
חתכים בלתי הומוגניים (NON HOMOGENEOUS CROSS-SECTIONS)

עד עתה דברנו על חומר שהוא הומוגני. מה קורה כאשר גוף מורכב מכמה חומרים לדוגמא משני חומרים.

דוגמה מהנדסה אזרחית: קורה מבטון מזוין - בטון המחוזק במוטות ברזל

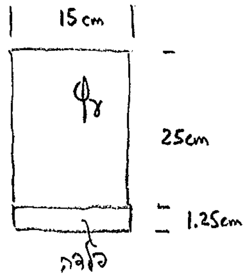


- נניח שיש חתך מורכב משני חומרים
- אם הקורה כפופה, ההנחה היסודית עדיין פועלת... שמישור לפני הכפיפה ממשיך להיות מישור אחרי הכפיפה.
- העיבור משתנה כפי המרחק מהציר הניטרלי
- במישור ששני החומרים סמוכים אחד לשני, העיבור הוא אותו דבר, אבל המאמצים שונים מפני ש- $E_1 \neq E_2$
- נניח שרוצים לבנות חתך מחומר מספר אחד בלי שינוי למידות חלק ① ובלי שינוי לגובה לחלק ②
- נניח ש- $n = E_2/E_1$ ו- n פחות מאחד.
- העומס שחומר ② נושא הוא $\int_{-y_1}^{y_2}\sigma_x\cdot dy\cdot b_2 = \int_{-y_1}^{y_2}E_2\epsilon_2\cdot b_2\cdot dy = \int_{-y_1}^{y_2}E_1\epsilon\cdot b\,dy$. לכן חומר ① צריך לשאת אותו עומס.
- מפני שהעיבור אותו דבר בחלק החדש



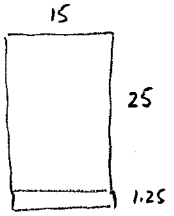
$$E_1 b = E_2 b_2$$
$$b = \frac{E_2}{E_1} b_2 = n b_2$$

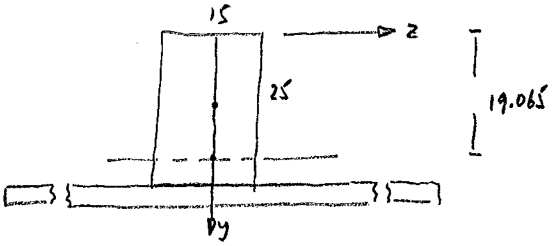
- החתך החדש נקרא החתך האקוויולנטי (Equivalent Section)
- עכשיו יכולים להשתמש ב- $\sigma_{max} = \frac{Mc}{I}$ בחלק הלא נשתנה. בחלק החדש $\sigma_{max} = n\frac{Mc}{I}$
- להחליף חומר ① לחומר ②, b_1 ישתנה ל- $b_1/n = \frac{E_1}{E_2} b_1 = b$
- מפני שהכוח שווה בקורה המקורית ובקורה האקוויולנטי, והכוח האלמנטרי נמצא באותו מקום תוצאותם כלפי המומנט אותו דבר.



דוגמא

לקורה מורכבת משני חומרים יש חתך השטח כבשרטוט
החלק העליון הוא מעץ שהמודול YOUNG $1.0343\times10^{10}\,Pa = E_W$
החלק התחתון הוא מפלדה שהמודול YOUNG $2.0686\times10^{11}\,Pa = E_s$
הקורה תרשה מומנט כפיפה של 27,115 N-m מסביב הציר האופקי HORIZONTAL
למצוא: המאמצים המקסימלי בעץ ובפלדה





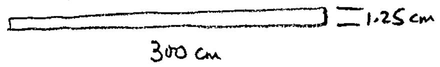
1. למצוא המידות של החלק האקוויולנטי
2. למצוא את הציר הנטרלי
3. למצוא מומנט האינרציה
4. למצוא את המאמצים

$E_s/E_w = 20$

1, מחליפים את הפלדה לעץ

$E_s b_s = E_w \cdot b_w$

לכן מידות החלק החדש



	$\bar{y}$	A	$\bar{y}A$
①	12.5	375	4687.5
②	25.63	375	9611.25
		750	14298.75

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = 19.065\,cm$$

$$I_{tt} = I_{zz} + A d^2$$

	I_{zz}	A	d^2	I_{tt}
①	$\frac{1}{12}(15)(25)^3$	375	$(19.065-12.5)^2$	$19531.25 + 16162.21 = 35693.46$
②	$\frac{1}{12}(300)(1.25)^2$	375	$(19.065-25.63)^2$	$48.83 + 16137.6 = 16186.43$
				$51879.89\ cm^4$

- המאמץ המקסימלי בעץ הוא

$$\frac{27115 \cdot 19.065 \cdot 100}{51879.89} = \frac{Mc}{I} = \sigma_{w_{max}}$$

$$996.431\ N/cm^2 =$$
$$9964313\ N/m^2 = \sigma_{w\ max}$$

- המאמץ המקסימלי בפלדה הוא
הערך בעץ האקוויולנט כפול n

$$n\,\sigma_w = (\sigma_s)_{max}$$

$$\frac{20 \cdot 27115 \cdot 100 \cdot 7.185}{51879.89} = (\sigma_s)_{max}$$

$$7510.47\ N/cm^2 =$$

$$75104737\ N/m^2 = \sigma_{s_{max}}$$

במישור הסמוך לשני חומרים (interface)

$$310.19\ \frac{N}{cm^2} = \frac{27115 \cdot 5.935 \cdot 100}{51879.89} = \frac{My}{I} = \sigma_w$$

$$6203.85\ \frac{N}{cm^2} = 20 \cdot \frac{27115 \cdot 5.935 \cdot 100}{51879.89} = \sigma_s$$

$$\underline{\underline{\sigma_s = 20\,\sigma_w}}$$

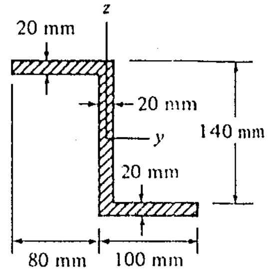
$3.x10^{-4} = \frac{3101900}{} = \sigma_w / = \epsilon_w \qquad \sigma_s / - \epsilon - \frac{62038500}{} = 3.x10^{-4}$

מיכאל סלבוטין

תורת החוזק 2

תרגיל מס 2

*9.3 (a) For the Z section shown, calculate I_y, I_z, and I_{yz}. Also locate the principal centroidal axes and calculate the principal moments of inertia.



(a)

PROBLEM 9.3

9.9 What centroidal axial force must be applied to the beam of Fig. 9.2.3 to make $|\sigma_{xA}| = |\sigma_{xB}|$ after bending and axial stresses are superposed?

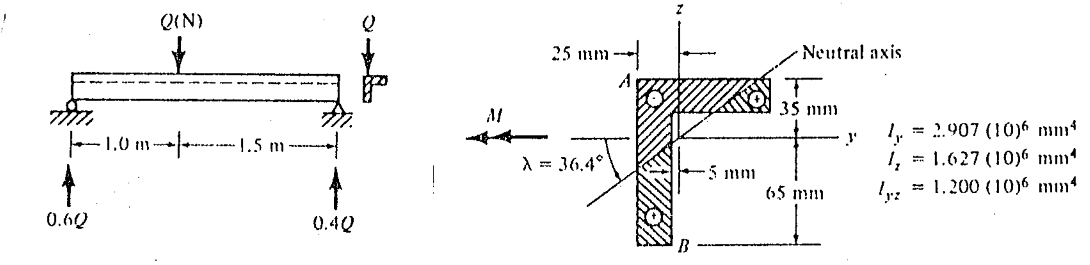
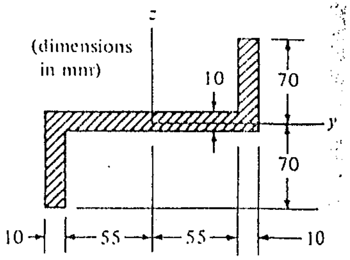


FIGURE 9.2.3.

*9.13 (a) A cantilever beam 1.6 m long has the Z section shown. It carries a load P = 2500 N in the $-z$ direction at the free end. Find the maximum magnitude of flexural stress (either tensile or compressive). Where does this stress appear?

(b) Repeat part (a) with load P redirected so that it acts in the $-y$ direction.

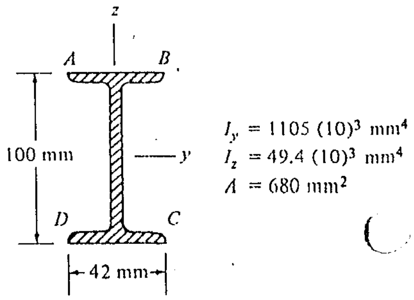
Answer: (a) 184 MPa $y = 55$mm $z = 70$mm



PROBLEM 9.13

*9.15 A column has the I section shown. A compressive force P parallel to the axis of the column is applied at corner A. Find axial stress σ_x in terms of P at the remaining three corners B, C, and D.

$\sigma_c = 0.00972P$ (MPa if P in N).

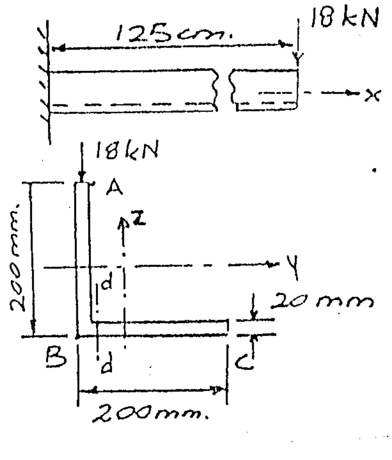


PROBLEM 9.15

5. A structural angle (L200 x 200 x 20) is used as a horizontal cantilever beam 125 cm long, and is subjected to a vertical concentrated load of 18,000 N acting through the shear center of the cross section as shown below.

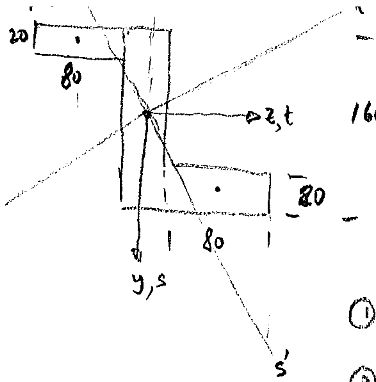
(a) Calculate the flexural stress σ_x at the points A, B and C at the fixed end of the beam, as well as the average shear stress τ acting along the line d-d of this cross section (see figure).

(b) Find the position of the Neutral Axis and the direction of the deflection. Show on a sketch of the cross-section, indicating the orientation with respect to the y-axis.



Given : $I_{zz} = I_{yy} = 2875$ cm^4,

Answer: $\sigma_{xA} = 145.6$ MPa, $\sigma_{xB} = -109.3$MPa, $|\tau| = 2.9$MPa



$I_{zz} = 22.613 \times 10^6 mm^4$

$I_{yy} = 9.813 \times 10^6 mm^4$

$I_{yz} = +11.2 \times 10^6 mm^4$

	I_{zz}	A	d^2
①	$\frac{1}{12} 80 \cdot 20^3$	1600	70^2
②	$\frac{1}{12} 20 \cdot (160)^3$	3200	0
③	$\frac{1}{12} \cdot 80 (20)^3$	1600	70^2

I_{yy}	A	e^2
$\frac{1}{12} 20 \cdot 80^3$	1600	50^2
$\frac{1}{12} 160 \cdot 20^3$	3200	0
$\frac{1}{12} \cdot 20 \cdot 80^3$	1600	50^2

	I_{yz}	A	e d	
①	0	20x80	50.70	$= 5.6 \times 10^6$
②	0	160x20	0	
③	0	20 x 80	-50.70	$= 5.6 \times 10^6$
				$11.2 \times 10^6 mm^4$

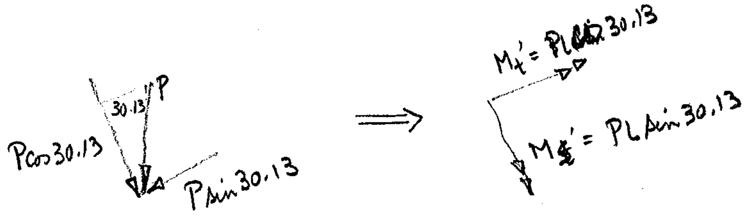
$$\tan 2\theta = \frac{2 \cdot 11.2 \times 10^6}{(22.613 - 9.813) \times 10^6} = \frac{2 I_{st}}{I_{tt} - I_{ss}} = 1.75$$

$2\theta = 60.255$

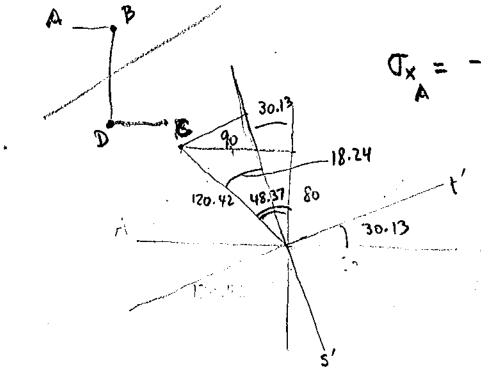
$\theta = 30.13°$

$$I_{\substack{max \\ min}} = \frac{I_{tt} + I_{ss}}{2} \pm \sqrt{\left(\frac{I_{tt} - I_{ss}}{2}\right)^2 + I_{yz}^2} = (16.21 \pm 12.90) \times 10^6 = \begin{matrix} 29.11 \times 10^6 \\ 3.31 \times 10^6 \end{matrix}$$

איפה המאמץ המקסימלי :



A (s=-80 t=-90)



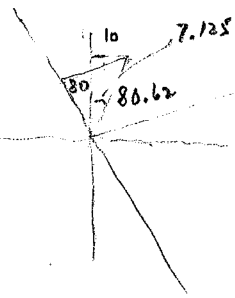
$$\sigma_{x_A} = -\frac{M_t s'}{I_{tt}'} + \frac{M_s' t'}{I_{ss}'} = -\frac{PL\cos 30.13(-114.37)}{29.11 \times 10^6} + \frac{PL\sin 30.13(-37.69)}{3.31 \times 10^6}$$

$= PL\,(3.398 - 5.716) \times 10^{-6} = -2.318\,PL \times 10^{-6}$

$\sigma_{x_C}\,(s' = 114.37, t = 37.69) = -\sigma_{x_A}$
$= 2.318\,PL \times 10^{-6}$

$\beta = 18.24°$
$t' = -120.42 \sin 18.24° = -37.69$
$s' = -120.42 \cos 18.24° = -114.37$

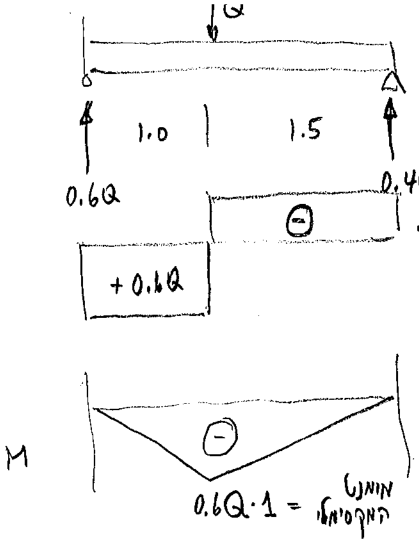
B(s=-80, t=10) σ_{x_B} |c13Nf

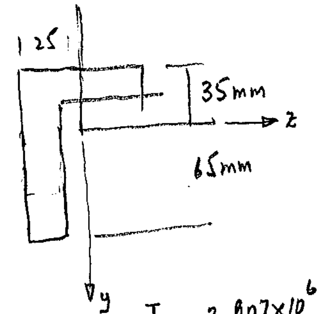


$\beta = 30.13 + 7.13 = 37.26°$
$t' = 80.62 \sin 37.26 = 48.81$
$s' =$ " $\cos 37.26 = -64.17$

$$\sigma_{x_B} = \frac{-PL\cos 30.13(-64.17)}{29.11 \times 10^6} + \frac{PL\sin 30.13(48.81)}{3.31 \times 10^6}$$

$\sigma_{x_B} = PL\,(1.907 + 7.402) \times 10^{-6} = 9.309\,PL \times 10^{-6}$





$I_z = 2.907\times10^6$
$I_y = 1.627\times10^6$
$I_{yz} = -1.2\times10^6$

$I_{t't'} = 3.627\times10^6\ mm^4$
$I_{s's'} = 0.907\times10^6\ mm^4$

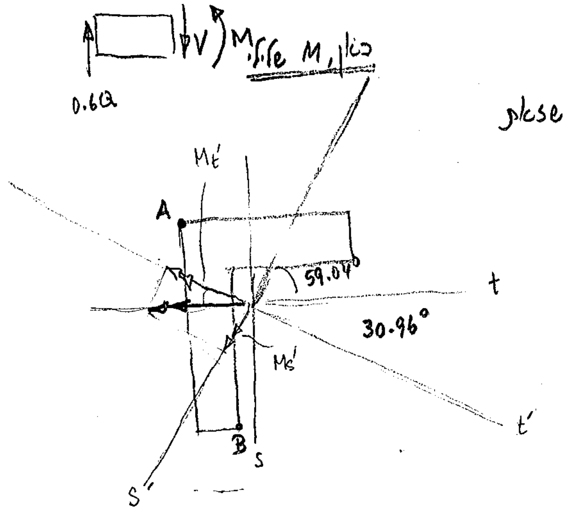
$$\tan 2\theta = \frac{2 I_{yz}}{I_z - I_y} = \frac{2(-1.2)}{2.907-1.627} = -11.93$$

$\theta = -30.96°$

צריך לשים את הזווית הזאת במשוואות של $I_{t't'}$ כדי לדעת שאלת
הזווית בין t ו-t'. לכן:

$M_t = -0.6Q$ N-m
$= -600Q$ N-mm

$M_{t'} = -M_t \cos -30.96 = -514.3Q$
$M_{s'} = +M_t \sin -30.96 = +309.0Q$



$A(s=-35,\ t=-25)$

$t' = t\cos(-30.96°) - s\sin(-30.96°) = -39.46$ mm

$s' = t\sin(-30.96°) + s\cos(-30.96°) = -17.12$ mm

$$\boxed{\sigma_{x_A} = -\frac{M_{t'} s'}{I_{t't'}} + \frac{M_{s'} t'}{I_{s's'}} = -\frac{(514.3Q)(-17.12)}{3.627\times10^6} + \frac{(+309.0Q)(+39.46)}{0.907\times10^6} = -0.0159Q\ MPa}$$

למצוא t' ו-s' ב-B:

$B(s=65,\ t=-5)$
$t' = t\cos\theta - s\sin\theta = 29.15$
$s' = t\sin\theta + s\cos\theta = 58.31$

$$\sigma_{x_B} = \frac{-M_{t'} s'}{I_{t't'}} + \frac{M_{s'} t'}{I_{s's'}} = -\frac{(-514.3Q)(58.31)}{3.627\times10^6} + \frac{(309.0Q)(29.15)}{.907\times10^6} = 0.0182Q$$

או יכולים להשתמש ב-

$$\sigma_x = \frac{-(M_t I_{ss}\cdot s) + M_t I_{st}\cdot t}{I_{ss} I_{tt} - I_{st}^2} = \frac{M_t(-I_{ss}s + I_{st}\cdot t)}{I_{ss} I_{tt} - I_{st}^2}$$

$$= \frac{(-600Q)\left[-1.627\times10^6\cdot(-35) + (-1.2\times10^6)(-25)\right]}{(1.627\times10^6)(2.907\times10^6) - (-1.2\times10^6)^2} = -0.0159Q\ MPa$$

משתמשים באותה שיטה למצוא σ_{x_B} עם $s=65, t=-5$

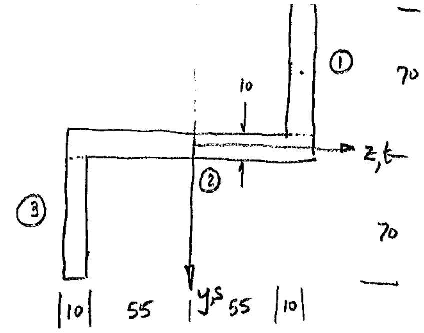
אם יש עומס צירי N, המאמץ שלו $= N/3200$, ואם השטח הוא $3200\ mm^2$. לכן

$$-\left(\sigma_{x_A} + N/3200\right) = \left(\sigma_{x_B} + N/3200\right)$$

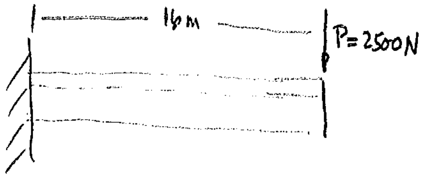
פותרים את המשוואה הזאת נותן את הערך ל-N

$$\underline{\underline{N = -3.68Q}}$$

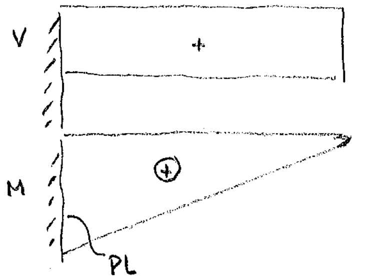
נ.ב. העומס N לוחץ על השטח, מפני ש-Q הוא חיובי.



				I_{tt}
①	$\frac{1}{12}(10)\cdot 65^3$	10x65	$(37.5)^2$	11.43×10^5
②	$\frac{1}{12}(130)\cdot 10^3$	10x130	0	$.1083\times10^5$
③	$\frac{1}{12}(10)\times 65^3$	10x65	$(-37.5)^2$	11.43×10^5
				2.297×10^6



	I_{yy}	A	e^2	I_{ss}
①	$\frac{1}{12}\cdot 65\cdot 10^3$	10x65	$(-60)^2$	23.45×10^5
②	$\frac{1}{12}\cdot 10\cdot 130^3$	10x130	0	18.31×10^5
③	$\frac{1}{12}\cdot 65\cdot 10^3$	10x65	$(60)^2$	23.45×10^5
				6.522×10^6



	I_{yz}	A	e·d	I_{st}
①	0	10x65	37.5x(-60)	-14.63×10^5
②	0	10x130	0.0	0
③	0	10x65	(-37.5)(60)	-14.63×10^5
				-2.925×10^6

למצוא הצירים הראשיים

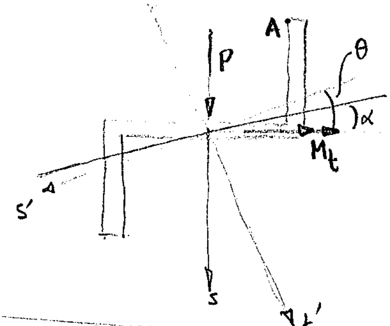
$$\tan 2\theta = \frac{2I_{st}}{I_{tt}-I_{ss}} = \frac{2(-2.925\times10^6)}{(2.297-6.522)\times10^6} = 1.3846$$

$$2\theta = 54.16^\circ$$

$$\theta = 27.08^\circ$$

$$\begin{matrix} I_{t't'} \\ I_{s's'} \end{matrix} = \left(\frac{I_{ss}+I_{tt}}{2}\right) \pm \sqrt{\left(\frac{I_{tt}-I_{ss}}{2}\right)^2 + I_{st}^2} = 4.41 \pm 3.608 = \begin{cases} 8.018\times10^6 \\ .8015\times10^6 \end{cases}$$

אם בודקים את הערך של $\left(\frac{I_{tt}+I_{ss}}{2}\right) + \left(\frac{I_{tt}-I_{ss}}{2}\right)\cos 2\theta + I_{ts}\sin 2\theta = I_{t't'}$, נמצא שהצירים הולכים ככה:



P גורם למומנט מסביב ל-t, וערכו הוא $4000 N\text{-}m = PL$ ▹t

אם נשתמש במשוואה הגדולה, $0 = M_s$ ו-

$$\sigma_x = \frac{-M_t I_{ss}\cdot s + M_t I_{st}\cdot t}{I_{ss}I_{tt} - I_{st}^2}$$

רואים שנקודה A היא יותר רחוקה מהציר הנייטרלי

לכן, לפי הצירים s ו-t $t=55$ ו- $s=-70$

$$\sigma_{x_A} = (4000\times1000)\left[\frac{-I_{ss}\cdot s + I_{st}\cdot t}{I_{ss}I_{tt} - I_{st}^2}\right] = 4\times10^6\left[\frac{-6.522\times(-70)+(-2.925)(55)}{2.297\times6.522-(-2.925)^2}\right]\times10^{-6}$$

$$= 184.06\ N\text{-}mm\times mm^4\times m/mm^8$$

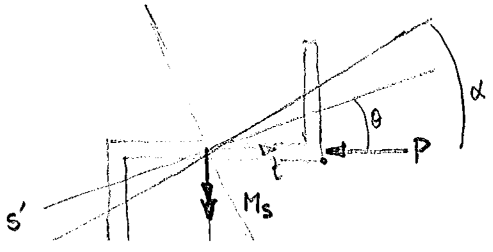
$$= 184.06\ N/mm^2 = 184.06\ MPa$$

למצוא ציר הנייטרלי לחלק (a):

$$s/t = \tan\alpha = \frac{I_{tt}\tan\theta + I_{st}}{I_{ss} + I_{st}\tan\theta}$$

$$\tan\theta = M_s/M_t = 0$$

$$-24.16^\circ = \alpha \Leftarrow -.448 = \frac{I_{st}}{I_{ss}} = \tan\alpha$$



(b) P גורם למומנט מסביב ל-s, וערכו 4000-Nm

$$\sigma_x = -M_s I_{st}\cdot s + M_s I_{tt}\cdot t$$

M_t $\qquad$ $I_{ss} + I_{st} \tan\theta$

ו- $\theta = 90^\circ$

לכן, $\frac{s}{t} = \tan\alpha = \frac{I_{tt}}{I_{st}}$ ו- $\tan\alpha = -.785 = \frac{I_{tt}}{I_{st}} \Leftarrow \alpha = 38.14^\circ$

נמצא שנקודה B הכי יותר רחוקה מהציר הניטרלי, ולכן $t = 65$ ו- $s = 5$.

$$\sigma_x = 4\times10^6 \left[-\frac{(-2.925)(5) + (2.297)\times(65)}{2.297\times 6.522 - (11.2)^2} \right] \times 10^{-6} = 4\times10^6 (25.51\times10^{-6}) = 102.06\,MPa$$

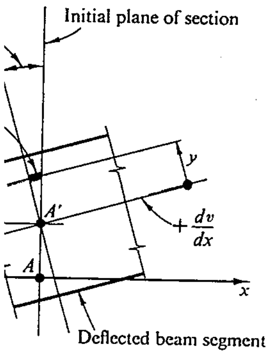
It is important and interesting to note that for the elastic curve, at the level of accuracy of Eq. 11-10, one has $ds = dx$. This follows from the fact that, as before, the square of the slope dv/dx is negligibly small compared with unity, and

$$ds = \sqrt{dx^2 + dv^2} = \sqrt{1 + (v')^2}\, dx \approx dx \qquad \textbf{(11-12)}$$

Therefore in the small deflection theory no difference in length is said to exist between the initial length of the beam axis and the arc of the elastic curve. Stated alternatively, there is no horizontal displacement u of the points lying on the neutral surface, i.e., at $y = 0$. This approximation can be made the basis of an alternative derivation of Eq. 11-10, which follows.

11-4. AN ALTERNATIVE DERIVATION OF EQUATION 11-10

In the classical theories of plates and shells which deal with small deflections, equations analogous to Eq. 11-10 must be established. The characteristic approach can be illustrated on the beam problem.



3. Longitudinal displacements in due to rotation of a plane section.

In a deformed condition a point A on the axis of an unloaded beam, Fig. 11-3, according to Eq. 11-12 is directly above its initial position. The tangent to the elastic curve at the same point rotates through an angle dv/dx. A plane section with the centroid at A' also rotates through the same angle dv/dx as during bending deformation sections remain normal to the bent axis of a beam. Therefore, the displacement u of a material point at a distance* y from the elastic curve is

$$u = -y\frac{dv}{dx} \qquad \textbf{(11-13)}$$

where the negative sign shows that for positive y and v' the displacement u is toward the origin. For $y = 0$, there is no displacement u, as required by Eq. 11-12.

Next, recall Eq. 4-3, which states that $\varepsilon_x = \partial u/\partial x$. Therefore, from Eq. 11-13, $\varepsilon_x = -y\, d^2v/dx^2$ since v is only a function of x.

The same linear strain also can be found from Eqs. 4-11 and 6-3 yielding $\varepsilon_x = -My/EI$. On equating the two alternative expressions for ε_x and eliminating y from both sides of the equation, one has

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

which is the previously derived Eq. 11-10.

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* As the angle dv/dx is small its cosine can be taken as unity.

11-5. ALTERNATIVE DIFFERENTIAL EQUATIONS OF ELASTIC BEAMS

In Chapter 2 a number of differential relations were shown among shear, moment, and the applied load (Eqs. 2-4, 2-5, and 2-6). These can be combined with Eq. 11-10 to yield the following useful sequence of equations:

$$v = \text{deflection of the elastic curve}$$

$$0 = \frac{dv}{dx} = v' = \text{slope of the elastic curve}$$

$$M = EI\frac{d^2v}{dx^2} = EIv''$$

$$\textbf{(11-14)}$$

$$V = -\frac{dM}{dx} = -\frac{d}{dx}\left(EI\frac{d^2v}{dx^2}\right) = -(EIv'')'$$

$$p = -\frac{dV}{dx} = \frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right) = (EIv'')''$$

For beams with constant flexural rigidity EI, Eqs. 11-14 simplifies into three alternative equations for determining the deflection of a loaded beam:

$$EI\frac{d^2v}{dx^2} = M(x) \qquad \textbf{(11-15)}$$

$$EI\frac{d^3v}{dx^3} = -V(x) \qquad \textbf{(11-16)}$$

$$EI\frac{d^4v}{dx^4} = p(x) \qquad \textbf{(11-17)*}$$

The choice of equation for a given case depends on the case with which an expression for load, shear, or moment can be formulated. Fewer constants of integration are needed in the lower-order equations.

11-6. BOUNDARY CONDITIONS

For the solution of beam deflection problems, in addition to the differential equations, boundary conditions must be prescribed. Several types of homogeneous boundary conditions are as follows:

(A) CLAMPED OR FIXED SUPPORT: In this case the displacement v

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* If in Eq. 11-17 in accordance with the d'Alembert principle one sets $p = -m\ddot{v}$, where m is the mass of the beam per unit length and $\ddot{v} = \partial^2v/\partial t^2$, the basic equation for the free lateral vibration of a beam is obtained. This equation is $EI\, \partial^4v/\partial x^4 + m\, \partial^2v/\partial t^2 = 0$.

L and of constant flexural rigidity EI, Fig. 11-6(a). Find the equation of the elastic curve.

SOLUTION

The boundary conditions are recorded near the figure from inspection of the conditions at the ends. At $x = L$, $M(L) = +M_1$, a nonhomogeneous condition.

From a free-body diagram of Fig. 11-6(b) it can be observed that throughout the beam the bending moment is $+M_1$. By applying Eq. 11-15, integrating successively, and making use of the boundary conditions, one obtains the solution for v:

$$EI\frac{d^2v}{dx^2} = M = M_1$$

$$EI\frac{dv}{dx} = M_1x + C_3$$

But $\theta(0) = 0$; hence at $x = 0$ one has $EIv'(0) = C_3 = 0$ and

$$EI\frac{dv}{dx} = M_1x$$

$$EIv = \tfrac{1}{2}M_1x^2 + C_4$$

But $v(0) = 0$; hence $EIv(0) = C_4 = 0$ and

$$v = M_1x^2/(2EI) \qquad \textbf{(11-24)}$$

The positive sign of the result indicates that the deflection due to M_1 is upward. The largest value of v occurs at $x = L$. The slope of the elastic curve at the free end is $+M_1L/(EI)$ radians.

Equation 11-24 shows that the elastic curve is a parabola. However, every element of the beam experiences equal moments and deforms alike. Therefore the elastic curve should be a part of a circle. The inconsistency results from the use of an approximate relation for the curvature $1/\rho$. It can be shown that the error committed is in the ratio of $(\rho - v)^3$ to ρ^3. As the deflection v is much smaller than ρ, the error is not serious.

It is important to associate the above successive integration procedure with a graphical solution or interpretation. This is shown in the sequence of Figs. 11-6(c) through (f). First the conventional moment diagram is shown. Then from Eqs. 11-9 and 11-10, $1/\rho \approx d^2v/dx^2 = M/(EI)$, the curvature diagram is plotted in Fig. 11-6(d). For the elastic case this is simply a plot of $M/(EI)$. By integrating the curvature diagram one obtains the θ diagram. In the next integration the elastic curve is obtained. In this problem since the beam is fixed at the origin, the conditions $\theta(0) = 0$, and $v(0) = 0$ are used in constructing the diagrams. This graphical approach or its numerical equivalents are very useful in the solution of problems with variable EI.

EXAMPLE 11-3

A simple beam supports a uniformly distributed downward load p_o. The flexural rigidity EI is constant. Find the elastic curve by the following three methods: (*a*) Use the second-order differential equation to obtain the deflection of the beam. (*b*) Use the fourth-order equation instead of the one in (*a*). (*c*) Illustrate a graphical solution of the problem.

SOLUTION

Case (*a*). A diagram of the beam together with the implied boundary conditions is in Fig. 11-7(a). The expression for M for use in the second-order differential equation has been found in Example 2-6. From Fig. 2-22

$$M = \frac{p_oLx}{2} - \frac{p_ox^2}{2}$$

Substituting this relation into Eq. 11-15, integrating it twice in succession, and making use of the boundary conditions, one finds the equation of the elastic curve:

$$EI\frac{d^2v}{dx^2} = M = \frac{p_oLx}{2} - \frac{p_ox^2}{2}$$

$$EI\frac{dv}{dx} = \frac{p_oLx^2}{4} - \frac{p_ox^3}{6} + C_3$$

$$EIv = \frac{p_oLx^3}{12} - \frac{p_ox^4}{24} + C_3x + C_4$$

But $v(0) = 0$; hence $EIv(0) = 0 = C_4$; and, since also $v(L) = 0$,

$$EIv(L) = 0 = \frac{p_oL^4}{24} + C_3L \quad \text{and} \quad C_3 = -\frac{p_oL^3}{24}$$

$$v = -\frac{p_o}{24EI}(L^3x - 2Lx^3 + x^4) \qquad \textbf{(11-25)}$$

By virtue of symmetry, the largest deflection occurs at $x = L/2$. On substituting this value of x into Eq. 11-25 one obtains

$$|v|_{\max} = 5p_oL^4/(384EI) \qquad \textbf{(11-26)}$$

The condition of symmetry could also have been used to determine the constant C_3. As it is known that $v'(L/2) = 0$, one has

$$EIv'(L/2) = \frac{p_oL(L/2)^2}{4} - \frac{p_o(L/2)^3}{6} + C_3 = 0$$

and, as before, $C_3 = -(1/24)p_oL^3$.

Case (*b*). Application of Eq. 11-17 to the solution of this problem is direct. The constants are found from the boundary conditions.

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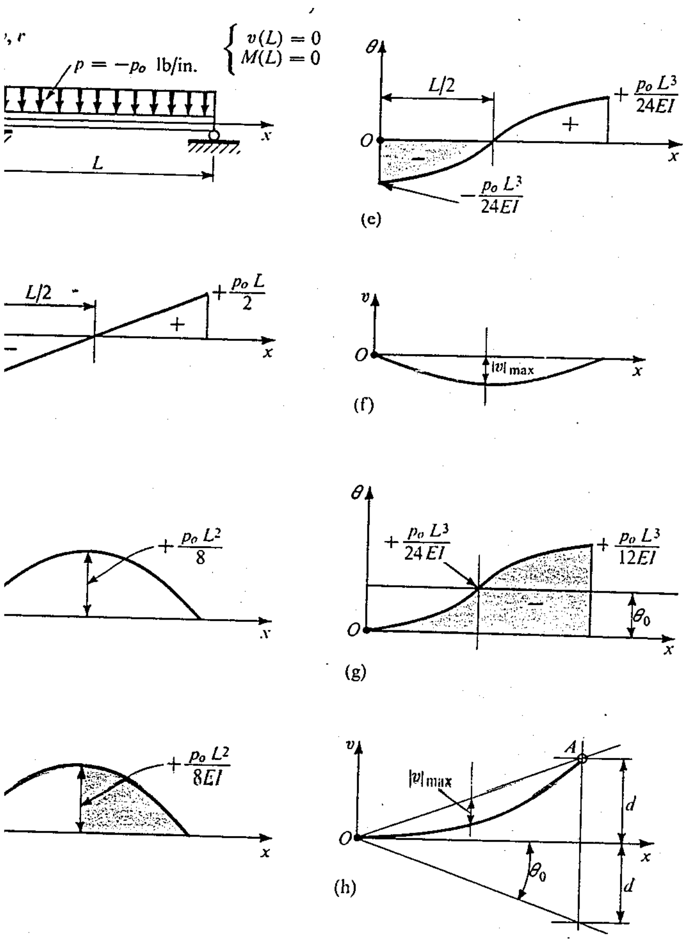

Fig. 11-7

$$EI\frac{d^4v}{dx^4} = p = -p_o$$

$$EI\frac{d^3v}{dx^3} = -p_o x + C_1$$

$$EI\frac{d^2v}{dx^2} = -\frac{p_o x^2}{2} + C_1 x + C_2$$

But $M(0) = 0$; hence $EIv''(0) = 0 = C_2$; and, since also $M(L) = 0$,

$$EIv''(L) = 0 = -\frac{p_o L^2}{2} + C_1 L \quad \text{or} \quad C_1 = \frac{p_o L}{2}$$

hence
$$EI\frac{d^2v}{dx^2} = \frac{p_o Lx}{2} - \frac{p_o x^2}{2}$$

The remainder of the problem is the same as in Case (*a*). In this approach no preliminary calculation of reactions is required. As will be shown later, this is advantageous in some statically indeterminate problems.

Case (*c*). The steps needed for a graphical solution of the complete problem are in Figs. 11-7(b) through (f). In Figs. 11-7(b) and (c) the conventional shear and moment diagrams are shown. The curvature diagram is obtained by plotting $M/(EI)$, as in Fig. 11-7(d).

Since by virtue of symmetry the slope to the elastic curve at $x = L/2$ is horizontal, $\theta(L/2) = 0$. Therefore, the construction of the θ diagram can be begun from the center. In this procedure, the right ordinate in Fig. 11-7(e) must equal the shaded area of Fig. 11-7(d), and vice versa. By summing the θ diagram, one finds the elastic deflection v. The shaded area of Fig. 11-7(e) is equal numerically to the maximum deflection. In the above, the condition of symmetry was employed. A generally applicable procedure follows.

After the curvature diagram is established as in Fig. 11-7(d), the θ diagram can be constructed with an assumed initial value of θ at the origin. For example, let $\theta(0) = 0$ and sum the curvature diagram to obtain the θ diagram, Fig. 11-7(g). Note that the shape of the curve so found is identical to that of Fig. 11-7(e). Summing the area of the θ diagram gives the elastic curve. In Fig. 11-7(h) this curve extends from O to A. This violates the boundary condition at A, where the deflection must be zero. Correct deflections are given, however, by measuring them vertically from a straight line passing through O and A. This inclined line corrects the deflection ordinates caused by the incorrectly assumed $\theta(0)$. In fact, after constructing Fig. 11-7(h), one knows that $\theta(0) = -d/L = -p_o L^3/(24EI)$. When this value of $\theta(0)$ is used, the problem reverts to the preceding solution (Figs. 11-7(e) and (f)). In Fig. 11-7(h) inclined measurements have no meaning. The procedure described is applicable for beams with overhangs. In such cases the base line for measuring deflections must pass through the support points.

EXAMPLE 11-4

A simple beam supports a concentrated downward force P at a distance a from the left support, Fig. 11-8(a). The flexural rigidity EI is constant. (*a*) Find the equation of the elastic curve without the use of operational notation. (*b*) Rework the problem using singularity functions.

SOLUTION

Case (*a*). The solution will be made by using the second-order differential equation. The reactions and boundary conditions are noted in

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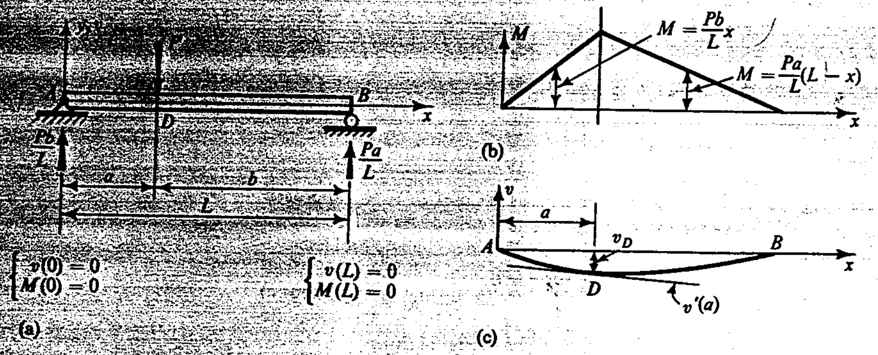

Fig. 11-8

Fig. 11-8(a). The moment diagram plotted in Fig. 11-8(b) clearly shows that a discontinuity at $x = a$ exists in $M(x)$, requiring two different functions for it. At first the solution proceeds independently for each segment of the beam.

For segment AD:

$$\frac{d^2v}{dx^2} = \frac{M}{EI} = \frac{Pb}{EIL}x$$

$$\frac{dv}{dx} = \frac{Pb}{EIL}\frac{x^2}{2} + A_1$$

$$v = \frac{Pb}{EIL}\frac{x^3}{6} + A_1x + A_2$$

For segment DB:

$$\frac{d^2v}{dx^2} = \frac{M}{EI} = \frac{Pa}{EI} - \frac{Pa}{EIL}x$$

$$\frac{dv}{dx} = \frac{Pa}{EI}x - \frac{Pa}{EIL}\frac{x^2}{2} + B_1$$

$$v = \frac{Pa}{EI}\frac{x^2}{2} - \frac{Pa}{EIL}\frac{x^3}{6} + B_1x + B_2$$

To determine the four constants A_1, A_2, B_1, and B_2, two boundary and two continuity conditions must be used.

For segment AD: $\quad v(0) = 0 = A_2$

For segment DB: $\quad v(L) = 0 = \frac{PaL^2}{3EI} + B_1L + B_2$

Equating deflections for both segments at $x = a$:

$$v_D = v(a) = \frac{Pa^3b}{6EIL} + A_1a = \frac{Pa^3}{2EI} - \frac{Pa^4}{6EIL} + B_1a + B_2$$

Equating slopes for both segments at $x = a$:

$$\theta_D = v'(a) = \frac{Pa^2b}{2EIL} + A_1 = \frac{Pa^2}{EI} - \frac{Pa^3}{2EIL} + B_1$$

Upon solving the above four equations simultaneously, one finds

$$A_1 = -\frac{Pb}{6EIL}(L^2 - b^2) \qquad A_2 = 0$$

$$B_1 = -\frac{Pb}{6EIL}(2L^2 + a^2) \qquad B_2 = \frac{Pa^3}{6EI}$$

With these constants, for example, the elastic curve for the left segment AD of the beam becomes

$$v = [(Pb/(6EIL)][x^3 - (L^2 - b^2)x] \tag{11-27}$$

The largest deflection occurs in the longer segment of the beam. If $a > b$, the point of maximum deflection is at $x = \sqrt{a(a + 2b)/3}$, which follows from setting the expression for the slope equal to zero. The deflection at this point is

$$|v|_{\max} = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}EIL} \tag{11-28}$$

Usually the deflection at the center of the span is very nearly equal to the numerically largest deflection. Such a deflection is much simpler to determine, recommending its use in practice. If the force P is applied at the middle of the span, i.e., $a = b = L/2$, it can be shown by direct substitution into Eq. 11-27 or 11-28 that at $x = L/2$

$$|v|_{\max} = PL^3/(48EI) \tag{11-29}$$

Case (*b*). The solution of the same problem using singularity functions is very direct and follows the procedure used previously:

$$EI\frac{d^4v}{dx^4} = p = -P\langle x - a\rangle_*^{-1}$$

$$EI\frac{d^3v}{dx^3} = -P\langle x - a\rangle^0 + C_1$$

$$EI\frac{d^2v}{dx^2} = -P\langle x - a\rangle^1 + C_1x + C_2$$

But $M(0) = 0$; hence $EIv''(0) = 0 = C_2$; and, since also $M(L) = 0$,

$$EIv''(L) = -Pb + C_1L = 0 \qquad \text{or} \qquad C_1 = Pb/L$$

$$EI\frac{dv}{dx} = -\frac{P}{2}\langle x - a\rangle^2 + \frac{Pb}{2L}x^2 + C_3$$

$$EIv = -\frac{P}{6}\langle x - a\rangle^3 + \frac{Pb}{6L}x^3 + C_3x + C_4$$

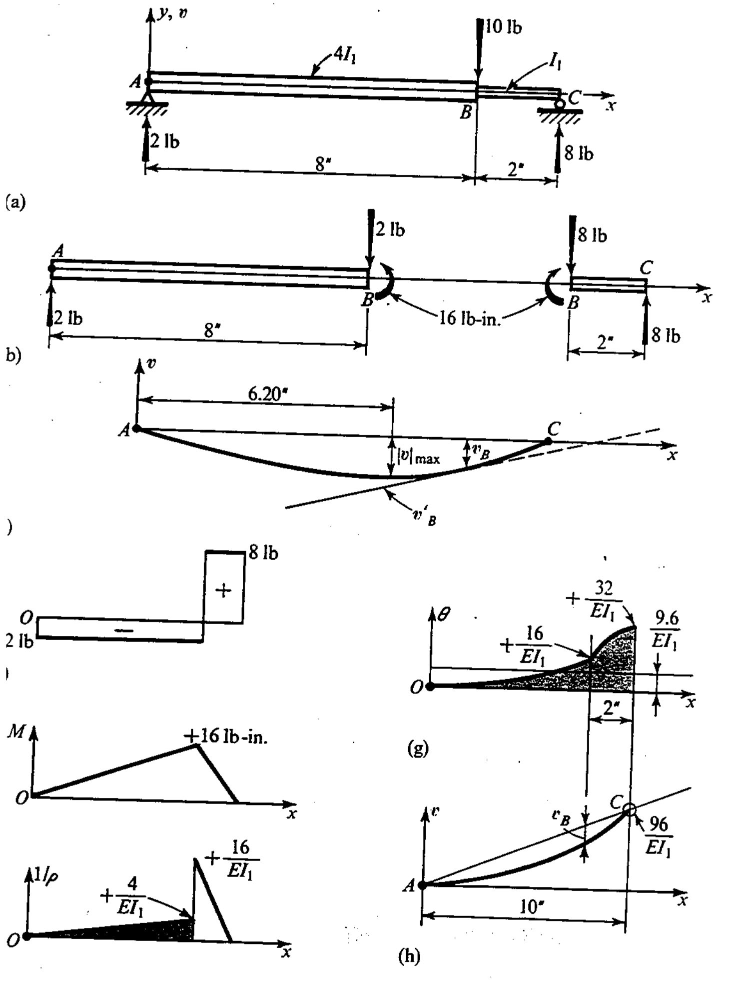

Fig. 11-9

But $v(0) = 0$; hence $EIv(0) = 0 = C_4$. Similarly, from $v(L) = 0$,

$$EIv(L) = 0 = -\frac{Pb^3}{6} + \frac{PbL^2}{6} + C_3L \quad \text{or} \quad C_3 = -\frac{Pb}{6L}(L^2 - b^2)$$

$$v = \frac{Pb}{6EIL}\left[x^3 - (L^2 - b^2)x - \frac{L}{b}\langle x - a\rangle^3\right] \tag{11-30}$$

For segment AD this general equation is the same as Eq. 11-27.

EXAMPLE 11-5

A simply supported beam 10 in. long is loaded with a 10-lb downward force 8 in. from the left support, Fig. 11-9(a). The cross section of the beam is such that in the segment AB the moment of inertia is $4I_1$; in the remainder of the beam it is I_1. Determine the elastic curve.

SOLUTION

An analytical solution of this problem can be achieved by either one of the two methods used in the previous example. Since, however, a special procedure must be used with singularity functions, this approach will be illustrated here.*

The beam is separated at the point of discontinuity in I and the forces necessary for the equilibrium of segments are computed, Fig. 11-9(b). Then the solution is commenced independently for each segment of the beam with the same location of the origin at A. The successive integrations are carried out until the moment-curvature equations are found. No constants of integration appear in the first two integrations as the reactive forces are computed beforehand. For segment AB:

$$\frac{d^4v}{dx^4} = \frac{p}{EI} = \frac{1}{4EI_1}[+2\langle x\rangle_*^{-1} - 2\langle x - 8\rangle_*^{-1} - 16\langle x - 8\rangle_*^{-2}]$$

$$\frac{d^3v}{dx^3} = -\frac{V}{EI} = \frac{1}{2EI_1}\langle x\rangle^0 - \frac{1}{2EI_1}\langle x - 8\rangle^0 - \frac{4}{EI_1}\langle x - 8\rangle_*^{-1}$$

$$\frac{d^2v}{dx^2} = \frac{M}{EI} = \frac{1}{2EI_1}\langle x\rangle^1 - \frac{1}{2EI_1}\langle x - 8\rangle^1 - \frac{4}{EI_1}\langle x - 8\rangle^0$$

$$\frac{dv}{dx} = \theta = \frac{1}{4EI_1}\langle x\rangle^2 - \frac{1}{4EI_1}\langle x - 8\rangle^2 - \frac{4}{EI_1}\langle x - 8\rangle^1 + \theta_o$$

where θ_o is an unknown constant of integration. For segment BC:

$$\frac{d^4v}{dx^4} = \frac{p}{EI} = \frac{1}{EI_1}[+16\langle x - 8\rangle_*^{-2} - 8\langle x - 8\rangle_*^{-1} + 8\langle x - 10\rangle_*^{-1}]$$

$$\frac{d^3v}{dx^3} = -\frac{V}{EI} = \frac{16}{EI_1}\langle x - 8\rangle_*^{-1} - \frac{8}{EI_1}\langle x - 8\rangle^0 + \frac{8}{EI_1}\langle x - 10\rangle^0$$

$$\frac{d^2v}{dx^2} = \frac{M}{EI} = \frac{16}{EI_1}\langle x - 8\rangle^0 - \frac{8}{EI_1}\langle x - 8\rangle^1 + \frac{8}{EI}\langle x - 10\rangle^1$$

At this stage of integration it must be recognized that by virtue of the continuity requirements, the terminal value of θ for the segment AB is the initial one for the segment BC. Moreover, this expression of θ for

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* Several schemes for using singularity functions for problems with variable I can be devised. For the procedure used here the author is indebted to Professor E. L. Wilson.

the segment AB for $x \geq 8$ remains constant. Therefore it is possible to integrate the last expression above for the segment BC and add to it the θ for segment AB. This yields a complete continuous function of θ for the whole beam AC. On subsequent integrations the boundary conditions can be used for determining the constants. For segment BC:

$$\frac{dv}{dx} = \theta = \frac{16}{EI_1}\langle x - 8\rangle^1 - \frac{4}{EI_1}\langle x - 8\rangle^2$$

Here the last term of the earlier expression, having no relevance, has been dropped. Adding this expression to the one found earlier for the segment AB, one has for the entire beam AC:

$$\frac{dv}{dx} = \theta = \frac{1}{4EI_1}\langle x\rangle^2 - \frac{4.25}{EI_1}\langle x - 8\rangle^2 + \frac{12}{EI_1}\langle x - 8\rangle^1 + \theta_o$$

and $$v = \frac{1}{12EI_1}\langle x\rangle^3 - \frac{4.25}{3EI_1}\langle x - 8\rangle^3 + \frac{6}{EI_1}\langle x - 8\rangle^2 + \theta_o x + v_o$$

But since $v(0) = 0$, one has $v_o = 0$. The condition that $v(10) = 0$ yields $\theta_o = -9.6/(EI_1)$. This completes the solution of the problem.

The equation for the slope in segment AB is $\theta = x^2/(4EI_1) - 48/(5EI_1)$. Upon setting this quantity equal to zero, x is found to be 6.20 in. The largest deflection occurs at this value of x, and $|v|_{\max} = 39.7/(EI_1)$. Characteristically, the deflection at the center of the span—i.e., at $x = 5$ in.—is nearly the same, being $37.6/(EI_1)$.

A self-explanatory graphical procedure is in Figs. 11-9(d) through (h). Variation in I causes virtually no complications in the graphical solution, a great advantage in complex problems.

11-9. STATICALLY INDETERMINATE ELASTIC BEAM PROBLEMS

In a large and important class of beam problems reactions cannot be determined using the conventional procedures of statics. For example, for the beam shown in Fig. 11-10(a), four reaction components are unknown. The three vertical components cannot be found from equations of static equilibrium. Further examination of Fig. 11-10(a) shows that any one of the vertical reactions can be removed and the beam would remain in equilibrium. Therefore any one of these reactions may be said to be *superflous*, or *redundant*, for maintaining equilibrium. Problems with extra or redundant reactive forces and/or moments are called (*externally*) *statically indeterminate*.

When the number of unknown reactions exceeds by one that which may be determined by statics, the member is said to be indeterminate to the *first degree*. As the number of unknowns increases, the degree of indeterminacy also increases. For example, by providing one more support than shown for the beam in Fig. 11-10(a), the beam would become indeterminate to the second degree. The beam of Fig. 11-10(b) is indeterminate to the first degree since either M_A or R_C can be considered redundant.

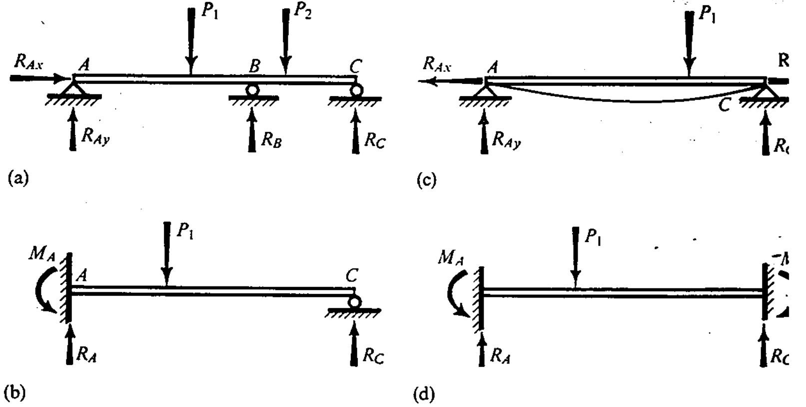

Fig. 11-10. Illustrations of statical indeterminacy of beams. and (b) the beams are indeterminate to the first degree. assumed that the horizontal components of the reactio negligible, the beam in (c) is determinate, and in (d) indeter to the second degree.

As according to Eq. 11-12 for small deflections $ds \approx dx$, no significant axial strain can develop in a transversely loaded beam.* Therefore, the horizontal components of the reactions in situations with immovable supports, such as shown in Figs. 11-10(c) and (d), are negligible. On this basis, the beam shown in Fig. 11-10(c), with pins at both ends, is a determinate beam. The beam of Fig. 11-10(d) is indeterminate to the second degree.

To determine the elastic deflection of statically indeterminate beams, the procedure of solving the differential equations is practically the same as that discussed above for determinate beams. The only difference is that kinematic boundary conditions replace some of the static ones. As the degree of indeterminacy increases, as in continuous members, the number of simultaneous equations for determining the constants increases. In such problems the number of constants to be found is no longer limited to a maximum of four.

A simple example of a statically indeterminate beam follows. After solving the problem using the fourth-order differential equation, an approach for applying the second-order equation is given.

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* The horizontal force becomes important in thin plates. See S. Timoshenko and S. Woinowsky-Krieger, *Theory of Plates and Shells* (2nd. ed.) (New York: McGraw-Hill Book Company, 1959), p. 6.

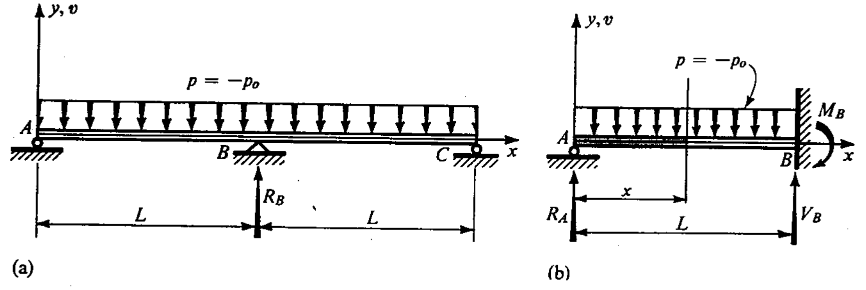


Fig. 11-11

EXAMPLE 11-6

Find the equation of the elastic curve for the uniformly loaded, two-span continuous beam shown in Fig. 11-11(a). The EI is constant.

SOLUTION

Equation 11-17, $EIv^{\text{iv}} = p$, can be used here. In addition to the boundary conditions of zero moment and zero deflection at the ends A and C, the deflection at B is also zero. The reaction R_B at B must be treated as an unknown force. On this basis

$$EI\frac{d^4v}{dx^4} = p = -p_o + R_B\langle x - L\rangle_*^{-1}$$

$$EI\frac{d^3v}{dx^3} = -p_o x + R_B\langle x - L\rangle^0 + C_1$$

$$EI\frac{d^2v}{dx^2} = -\frac{p_o x^2}{2} + R_B\langle x - L\rangle^1 + C_1 x + C_2$$

$$EI\frac{dv}{dx} = -\frac{p_o x^3}{6} + \frac{R_B}{2}\langle x - L\rangle^2 + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$$EIv = -\frac{p_o x^4}{24} + \frac{R_B}{6}\langle x - L\rangle^3 + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$

The five constants R_B, C_1, C_2, C_3, and C_4 are found from three deflection and two moment conditions:

$$v_A = v(0) = 0, \qquad v_B = v(L) = 0, \qquad v_C = v(2L) = 0$$

$$M_A = EIv''(0) = 0 \qquad \text{and} \qquad M_C = EIv''(2L) = 0$$

The boundary conditions at $x = 0$ yield directly $C_4 = 0$ and $C_2 = 0$. The remaining three conditions $v_B = v_C = M_C = 0$ give the following three simultaneous equations:

$$\left.\begin{aligned} +\tfrac{1}{6}L^3C_1 + LC_3 &= \tfrac{1}{24}\,p_oL^4 \\ +\tfrac{1}{6}L^3R_B + \tfrac{4}{3}L^3C_1 + 2LC_3 &= \tfrac{2}{3}\,p_oL^4 \\ +LR_B + 2LC_1 &= 2p_oL^2 \end{aligned}\right\}$$

from which $R_B = \tfrac{5}{4}p_oL$, $C_1 = \tfrac{3}{8}p_oL$, and $C_3 = -\tfrac{1}{48}p_oL^3$. Therefore

$$v = -[p_o/(48EI)][2x^4 - 3Lx^3 + L^3x - 10L\langle x - L\rangle^3] \qquad \textbf{(11-31)}$$

ALTERNATE SOLUTIONS

The stated problem is symmetrical around the support at B. Therefore the tangent to the elastic curve at B is horizontal, and an equivalent problem involving one-half of the original beam shown in Fig. 11-11(b) can be analyzed instead. This new problem can be solved using the fourth-order differential equation with the following four boundary conditions:

$$v_A = 0, \qquad M_A = EIv''(0) = 0, \qquad v_B = 0, \qquad \text{and } v_B' = 0$$

Alternatively, on designating the unknown reaction at A as R_A, one may state the bending moment within the span as

$$M = R_A x - p_o x^2/2$$

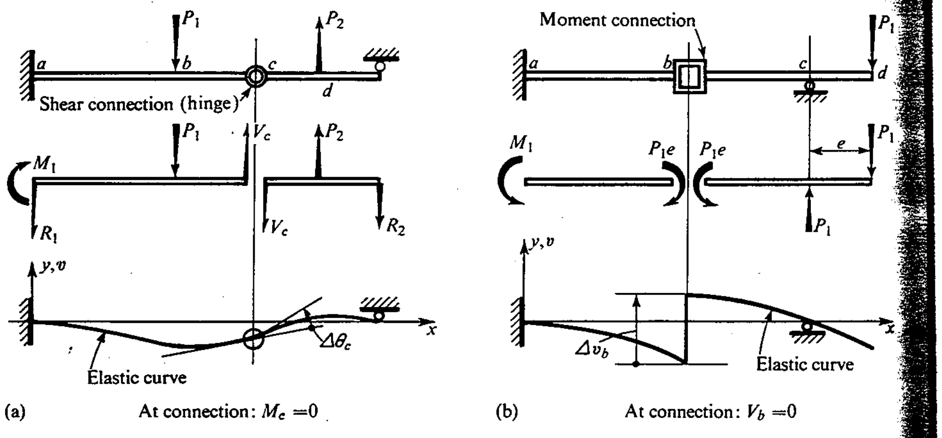
By substituting this relation into Eq. 11-15, integrating it twice, and making use of three of the kinematic boundary conditions stated above, one finds the unknown constants R_A, C_3, and C_4.

11-10. TWO ADDITIONAL SINGULARITY FUNCTIONS

In some assemblies of beams it is necessary to introduce connections which permit movement. One commonly used type, a hinge, cannot resist bending moment, but can transmit shear. Thus it is a shear connection. Another, basically different connection, a moment connection, can transmit moment but not shear. The elastic curve at such connections is not continuous. At a hinge a local concentrated angle change between the tangents to the elastic curve takes place. In a moment connection the adjoining ends of the elastic curve undergo relative translations. The characteristic discontinuities in the elastic curves at such connections are shown in Fig. 11-12. An abrupt change of slope at a hinge is shown as $\Delta\theta_c$; an abrupt change in deflection at a moment connection as Δv_b.

For the analysis of beams having the above connections it is convenient to introduce two new singularity functions expressing a fictitious load acting on a beam as

$$p = \Delta\theta_a EI\langle x - a\rangle_*^{-3} \qquad \text{[lb/in.]} \qquad \textbf{(11-32)}$$



(a) At connection: $M_c = 0$ (b) At connection: $V_b = 0$

Fig. 11-12. Discontinuities in the elastic curves at connections.

for a concentrated change of slope $\Delta\theta_a$ at $x = a$; and as

$$p = \Delta v_a EI\langle x - a\rangle_*^{-4} \quad \text{[lb/in.]} \tag{11-33}$$

for a concentrated or sudden change in deflection Δv_a at $x = a$.

These functions together with the previously defined ones, $P\langle x - a\rangle_*^{-1}$ for a concentrated force and $M_a\langle x - a\rangle_*^{-2}$ for a concentrated moment, integrate according to the following rule:

$$\int_0^x \langle x - a\rangle_*^n \, dx = \langle x - a\rangle_*^{n+1} \quad \text{for} \quad n < 0 \tag{11-34}$$

As before, Eq. 2-16 applies for $n \geq 0$.

The negative exponents in Eqs. 11-32 and 11-33 are so taken that on successive integration of $EIv^{\text{iv}} = p$, one obtains the correct quantities for slope and deflections.

Having available the set of four singularity functions permits remarkable versatility in the analysis of beams. The beams can be either determinate or indeterminate. In either case the boundary conditions at both ends must be used in the solution of problems. If connections exist, then an additional condition of $M = 0$ holds true at each hinge and of $V = 0$ at every moment connection. In indeterminate beams, for every constraint caused by a redundant reaction, a kinematic condition on the deflection becomes available. The solution of beams with the aid of

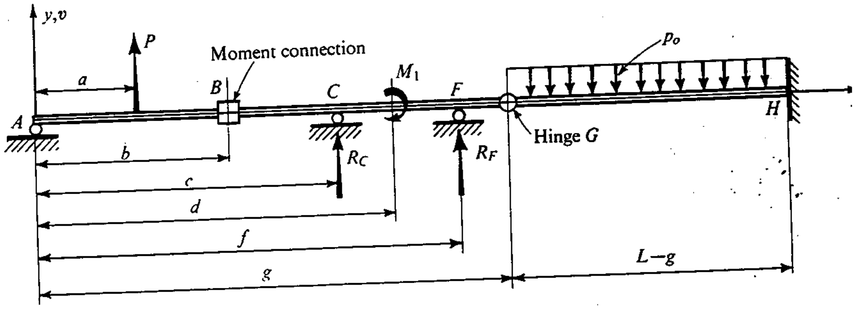


Fig. 11-13

singularity functions is completely general.* The method becomes, however, very cumbersome for stepped beams.

As an example of general approach, consider the beam shown in Fig. 11-13. The pertinent equation here is

$$EI\frac{d^4v}{dx^4} = p = P\langle x - a\rangle_*^{-1} + \Delta v_B EI\langle x - b\rangle_*^{-4} + R_C\langle x - c\rangle_*^{-1}$$
$$+ M_1\langle x - d\rangle_*^{-2} + R_F\langle x - f\rangle_*^{-1} + \Delta\theta_G EI\langle x - g\rangle_*^{-3} - p_o\langle x - g\rangle^0$$

where the loads P, M_1, and p_o are given. The boundary conditions are

$$v_A = v(0) = 0, \qquad M(0) = EIv''(0) = 0,$$
$$v_H = v(L) = 0, \qquad \text{and} \qquad v'(L) = 0$$

The conditions at the connections are

$$V_B = V(b) = 0 \qquad \text{and} \qquad M_G = M(g) = 0$$

And the two restraining conditions are

$$v_C = v(c) = 0 \qquad \text{and} \qquad v_F = v(f) = 0$$

The above information is sufficient for solving this complex problem, which, however, is indeterminate only to the first degree.

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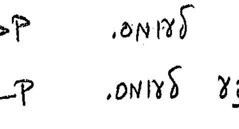
* Some of the presentation in this article follows R. J. Brungraber, "Singularity Functions in the Solution of Beam-Deflection Problems," *Journal of Engineering Education*, **55**, no. 9 (May 1965), 278–80. Although no cases can be found in literature, these functions can be used very effectively in constructing the influence lines for beams. This topic, however, is beyond the scope of this book.

תיאורית החוזק

ידוע שמבנה שלא מתוכנן כהוגן מתמוטט. יש כמה אפשרויות שגורם להתמוטטות:

1. ההתמוטטות קורה בגלל שמופיע בגוף סדקים או שסדקים שנמצאים מתחילים להתארך בגודלם
- יכולים להשבר תחת עומס קבוע ומעייפות.
- לשבר פריך (BRITTLE) אין תזוזה פלסטית (BRITTLE FRACTURE – NO PLASTICITY)
- לפריכות במתח השטח-החתך של השבר הוא מאונך לעומס.
- לפריכות בלחץ השטח-החתך של השבר הוא משופע לעומס.

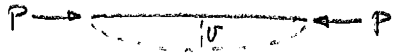


YIELD

2. ההתמוטטות יכולה לקרות בגלל כניעת העיבור זאת אומרת שהגוף הולך ומאריך את המידות שלו יותר ממה שמקובל. לשינוי קטן במאמץ יש שינוי גדול לעיבור

3. ההתמוטטות יכולה לקרות בגלל קריסה. המוט יכול להזיז בתזוזה גדולה בכיוון מאונך לעומס הצירי.

BUCKLING

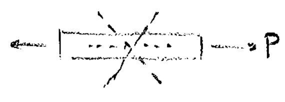


4. בגלל טמפרטורות גבוהות יכול להיות זחילה. (CREEP) המידות יכולות להשתנות עד שהגוף מפריע לגוף קרוב לו כמו במנוע (בוכנה, טבעות בוכנה)
PISTON

• לכן צריך לדעת וגם לתאר מתי גוף נכשל.

נדון בשיטות שונות על שם TRESCA, VON MISES, קריטריון מאמץ ראשי מקסימלי, קריטריון עיבור ראשי מקסימלי, קריטריון MOHR, קריטריון NORRIS. אבל יש הרבה תיאוריות בשוק.

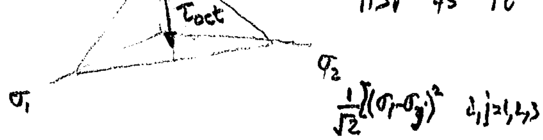
1. קריטריון TRESCA: נקרא גם כ- התיאוריה של המאמץ הגזירה המקסימלי, ותיאורית OULOMB
התמוטטות מתחילה כאשר שהמאמץ הגזירה המקסימלי מגיע לערך מסויים
התיאוריה הזאת באה מפני שיש חומרים מתמתחים (גמישים) SOFT MAT. שכשלונם הוא דרך מישור שהוא 45 מעלות לציר של עומס המתיחה. מישורים לגזירה מקסימלית הם 45 מעלות למישור העומס



(FAILURE ON 45° PLANES TO TENSILE LOAD)

$$\tau_{max} = \frac{\sigma_y}{2} = \frac{\sigma_1 - \sigma_3}{2}$$

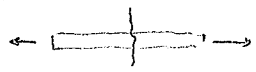
2. קריטריון VON MISES: נקרא גם כ- התיאוריה של MAXWELL.
התמוטטות מתחילה כאשר המאמץ האפקטיבי (EFFECTIVE מגיע לערך מסויים, σ_e
אבל יש יחס בין גזירה אוקטהידרלי σ_e-ל (OCTOHEDRAL SHEAR)
הגזירה הנמצאת על מישור שווה זוויות של 45° לכל הצירים הראשיים של המאמצים.



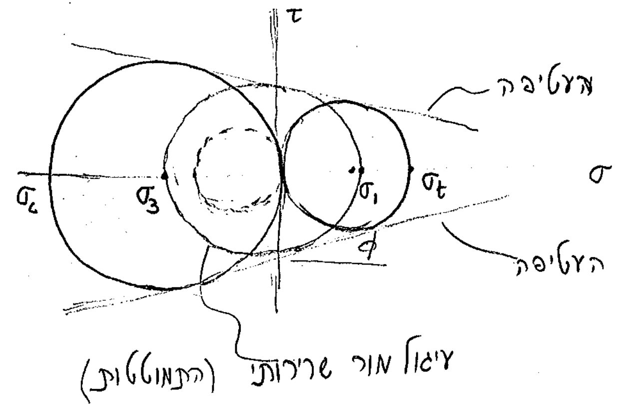
$$\frac{1}{\sqrt{2}}\sum(\sigma_i-\sigma_j)^2 \quad i,j=1,2,3 = \sigma_e = \frac{3}{\sqrt{2}}\tau_{oct} = \frac{1}{\sqrt{2}}\left((\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)\right)^{1/2} = \sigma_{yp}$$

אך על פי ש- $\tau_{max} \geq \tau_{oct}$, הוא מתארת את המצב הכללי של המאמצים כאשר

3. קריטריון של המאמץ הראשי המקסימלי: כאשר המאמץ הראשי σ_1 מגיע לערך מסויים, מסגרתי במתיחות. רואים זה בחומרים שבירים (פריכים) BRITTLE שהמישור הפריק הוא מישור שנוטל σ_1



4. קריטריון מור: התמוטטות מתחילה כאשר עיגל מור הגדול ביותר $(\sigma_1-\sigma_3)$ סמוך לעטיפת התמוטטות. מור עובד טוב בחומר פריך (LARGEST MOHR'S CIRCLE $(\sigma_1-\sigma_3)$ TOUCHES A FAILURE ENVELOPE).
לשרטט את העטיפה ההתמוטטות - לעשות ניסיונות - אחד בעומס צירי מתוח והשני בעומס צירי לחוץ



התמוטטות מתחילה כאשר $\frac{\sigma_1}{\sigma_t}-\frac{\sigma_3}{|\sigma_c|}\geqslant 1$. כש- הלחץ הוא דומיננטי (DOMINANT), ההתמוטטות קורת דרך הגזירה במישור משופע בזווית של $45°+\frac{1}{2}\phi$ לכיוון של σ_3.

5. בחומר אורטוטרופי (ORTHOTROPIC MATERIAL), פריכות או התחלת כניעה קורת כאשר אחת מהמשוואות הללה מסופקת. זה הקריטריון של NORRIS

$$\frac{\sigma_x^2}{S_x}-\frac{\sigma_x\sigma_y}{S_xS_y}+\frac{\sigma_y^2}{S_y^2}+\frac{\tau_{xy}^2}{S_{xy}^2}\geqslant 1$$

$$\frac{\sigma_y^2}{S_y^2}-\frac{\sigma_y\sigma_z}{S_yS_z}+\frac{\sigma_z^2}{S_z^2}+\frac{\tau_{yz}^2}{S_{yz}^2}\geqslant 1$$

$$\frac{\sigma_z^2}{S_z^2}-\frac{\sigma_z\sigma_x}{S_zS_x}+\frac{\sigma_x^2}{S_x^2}+\frac{\tau_{zx}^2}{S_{zx}^2}\geqslant 1$$

כאן x,y,z הם הצירים הראשיים של החומר.
כאן $S_x, S_y, S_{xy}, S_{yz}, S_{zx}, S_z$ הם הערכים שמקבלים מניסיונות מיוחדים.

S_x is DESIGNED FROM TENSILE TEST IF σ_x IS TENSILE
S_y " " " COMPRESSION TEST IF σ_x IS COMPRESSIVE

6. תיאוריה שמיישמים (apply) לחומרים מתמתחים (SOFT) בדרך כלל לא עובדת בחומרים פריכים

7. איך מיישמים לתיאוריות האלו כאשר יש מצב מאמץ דו-צירי, כי התיאוריות האלו עובדים, בדרך כלל, במצב מאמץ חד-צירי.
HOW TO USE FOR BIAXIAL WHEN MOST THEORIES ONLY WORK FOR UNIAXIAL STATES

8. צריך לקבל את הערכים של σ_1, או τ_{max} או σ_e לכול המצבים ולבדוק מהוא המצב ההכי גרוע, אם לא היו תיאוריות

-- כי זה יקח הרבה כסף, והרבה זמן, אי אפשר לבדוק את כל המצבים המאמציים

CANT CHECK ALL STRESS STATE
USE THEORY W/ UNIAXIAL TENSION TEST. ASSUME THAT FAILURE IN TENSION INDICATES FAILURE IN ALL OTHERS

לכן משתמשים בתיאוריות ועושים ניסיונות במתח חד-צירי
ומקבלים על עצמינו שהתמוטטות בכל שיטה אחרת מוצג
בהתמוטטות במצב מתוח חד-צירי

דוגמא - COOK & YOUNG

- TENSION & COMPRESSION TESTS OF A BRITTLE MATERIAL GIVE $\sigma_t = 14$ MPa & $\sigma_c = 120$ MPa
- UNDER LOADING CONDITION, A CERTAIN STATE OF STRESS EXISTS $\sigma_x = 0$, $\sigma_y = -18$ MPa, $\tau_{xy} = 20$ MPa

IS THIS STATE OF STRESS SAFE? CONSIDER PLANE STRESS

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2}$$

$$= \frac{0-18}{2} + \sqrt{\frac{(18)^2}{4} + (20)^2} = -9.0 + 21.9 = 12.9 \text{ MPa}$$

$$\sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2} = -9.0 - 21.9 = -30.9 \text{ MPa}$$

$$\sigma_e = \frac{1}{\sqrt{2}}\left[(0+18)^2 + (-18-0)^2 + (0-0)^2 + 6\left(20^2 + 0^2 + 0^2\right)\right]^{1/2} = 39.04 \text{ MPa}$$

קריטריון המאמץ הראשי המקסימלי

MAX STRESS SHOWS SAFE

מפני ש- $(\sigma_1 < \sigma_t)$ החומר במצב טוב בקריטריון הזה.

קריטריון TRESCA

$$12.9 > 0 > -30.9 \Leftarrow \sigma_1 > \sigma_2 > \sigma_3$$

TRESCA & VON MISES SHOW UNSAFE, BUT NORMALLY DON'T WORK FOR BRITTLE

$$\tau_{max} = \frac{12.9 + 30.9}{2} = 21.9 \text{ MPa} = \frac{\sigma_1 - \sigma_3}{2}$$

מפני ש- בניסיון מאמץ מתוח חד-צירי הוא $14MPa = \sigma_t$

ו- τ_{max} שווה ל- $7MPa = \sigma_{t/2} = \sigma_{yp/2}$

לפי TRESCA $\tau_{max} > \sigma_{yp/2}$ לכן החומר במצב לא טוב
והתחיל להתמוטט.

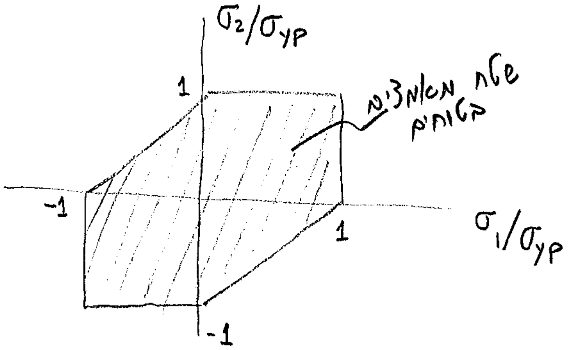
קריטריון VON MISES

מפני ש- $\sigma_e = 39.04$ ובניסיון מאמץ מתוח חד-צירי הוא $\sigma_{yp} = \sigma_t$
ו- $(\sigma_e > \sigma_{yp})$. לכן לפי VON MISES החומר מתחיל להתמוטט.

קריטריון MOHR

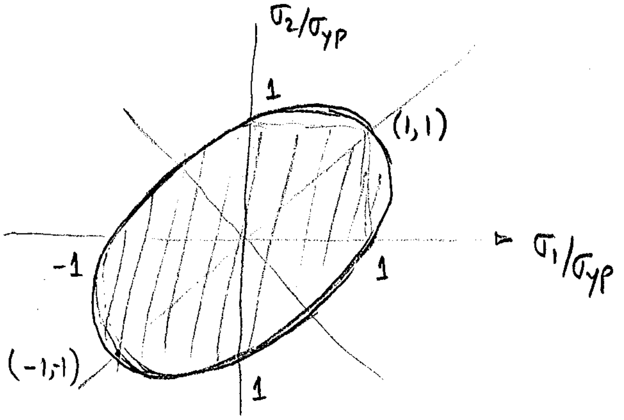
במצב ללא התמוטטות $1.18 = \frac{12.9}{14} - \frac{(-30.9)}{120} \Leftarrow \frac{\sigma_1}{\sigma_t} - \frac{\sigma_3}{|\sigma_c|} \le 1$

מכאן רואים ששימוש מהתיאוריות נותנות לנו שש המשמעויות. וכן [illegible] ניתן את הבדיקה
ולתכנן (DESIGN) את התיכנון בחברה.

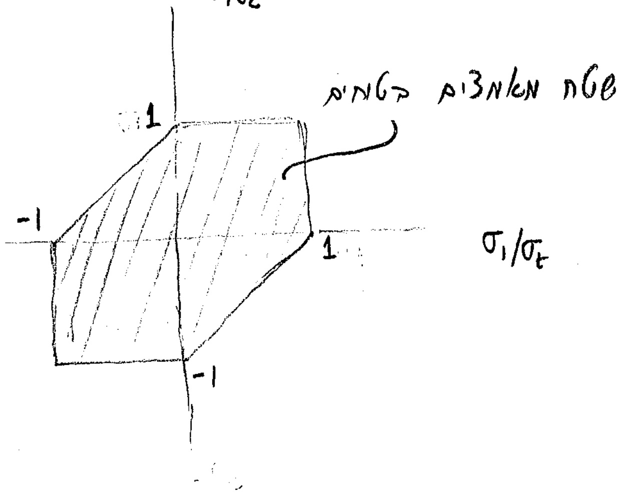


לפי TRESCA

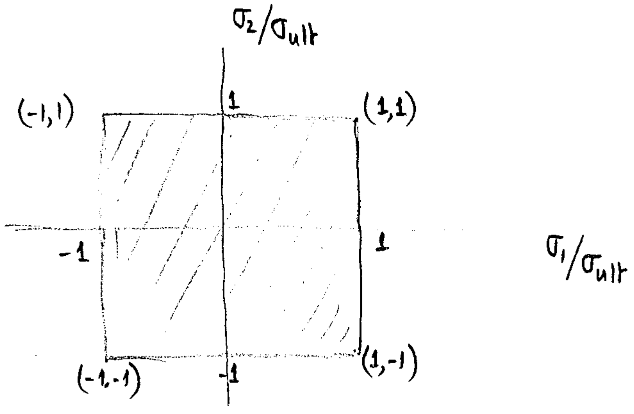
נ.ב.: מאמץ הכניעה בלחץ ובמאמץ אותו ערך



לפי VON MISES



לפי MOHR



המאמץ המרבי - σ_{ult}

לפי המאמץ הראשי המקסימלי

קריטריון TRESCA כלול בשל VON MISES. תוצאות ניסיוניות מסכימות יותר עם VON MISES. בתיאוריות פלסטיות משתמשים עם שני התיאוריות האלו - VON MISES או TRESCA

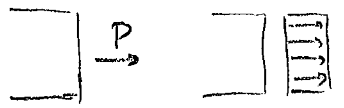
Experimental data follow VON MISES. IN THEORY OF PLASTICITY, WE WILL CONCENTRATE ON THESG 2.

נתרכז בשניהם

תסבולת אלסטית של חתכים

(ELASTIC LOAD CARRYING CAPACITY OF SECTIONS)

כדי למצוא את התסבולת האלסטית של חתכים צריך לדעת את כל העומסים הפועלים על שטח-החתך.
(TO KNOW LOAD CARRYING CAPACITY, NEED TO KNOW LOADS ACTING ON CROSS-SECTION)
מכאן, יכולים למצוא את המאמצים דרך המשוואות הרגילות, זאת אומרת ש-לעומס צירי המאמץ הוא



$\sigma_x = P/A$

לעומס שעושה כפיפה

$\sigma_x = -\frac{My}{I}$

ביחד שניהם נותנים העומסים שפועלים על שטח-החתך. אנחנו יכולים לתכנן (DESIGN) את שטח-החתך לפי המאמץ המקסימלי
The two give the load working on the cross-section

what if axial load only
נניח שרק העומס הצירי פועל על הקורה. $\sigma_x = P/A$ מה המאמץ המקסימלי? ואיפה הוא פועל? בכל מקום בשטח-החתך. לכן עד ש-המאמץ מגיע ל-מאמץ הכניעה, הקורה פועלת כ-קורה אלסטית. ו- $\sigma_{x_{\text{מקס}}} = \sigma_{yp} = P/A$
where is it
how do we define for ductile

- לכן, אם יש שטח-החתך מסוים שאנחנו צריכים להשתמש בו, יכולים למצוא את העומס המקסימלי: $P = \sigma_{yp}A$
To find max load for given A
- אם יש עומס מסוים שאנחנו צריכים לתמוך, יכולים לחשב את שטח-החתך: $A = \sigma_{yp}/P$
To find cross-section to given load.

בדרך כלל, לא רוצים שהמאמץ מגיע ל-σ_{yp}. אנחנו מגדירים (DEFINE) מאמץ מסויים

Defining safety factor

<u>מאמץ המותר</u> $\sigma_{allow} = f\,\sigma_{yp}$

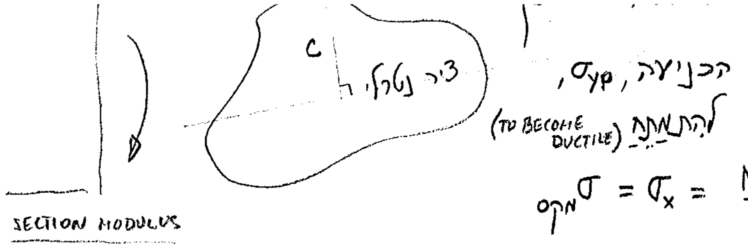
σ_{allow} - מאמץ המותר
f - מספר הגדרה פחות מאחד שמגדיר את המאמץ המקסימלי להישאר בתחום האלסטי - המקדם בטיחות (SAFETY FACTOR)

לכן: צריכים להחליף σ_{yp} בכול המשוואות ב-σ_{allow}
replace all σ_{yp} by σ_{allow}

$$\sigma_{allow}\cdot A = f\sigma_{yp}A = P_{\text{מקס}}$$

$$P/\sigma_{allow} = P/f\sigma_{yp} = A_{\text{תיכנון}}$$

for bending
עם קורות בכפיפה גם עושים את החישוב והתיכנון הזה
נניח שיש רק עומס שקורה כפיפה. מהו המאמץ המקסימלי ואיפה הוא פועל



SECTION MODULUS

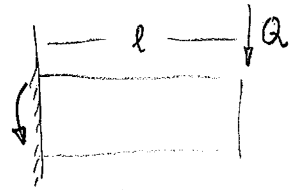
- כאשר המאמץ המקסימלי מגיע ל-מאמץ הכניעה, σ_{yp}, הקורה עדיין בתחום אלסטי ומתחילה להתמתח (TO BECOME DUCTILE)

$$\sigma_{\text{מקס}} = \sigma_x = \frac{Mc}{I} = \frac{M}{S} = \sigma_{yp}$$

S - מודול החתך $S = I/c$

- לכן אם יש שטח-החתך מסויים שאנחנו צריכים להשתמש בו, S ידוע לנו ויכולים למצוא את מומנט הכפיפה המקסימלי

$$M_{\text{מקס}} = \sigma_{yp} S$$



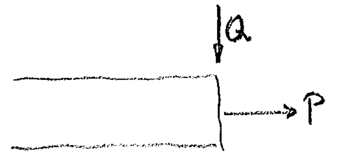
והעומס שגורם ל- $M_{\text{מקס}}$ הוא $\cancel{Q} = M_{\text{מקס}}/l$

- אם יש מומנט מסויים, אפשר למצוא את ה- מודול החתך $S = \dfrac{M}{\sigma_{yp}}$

- כמו שביארנו, לא משתמשים, בדרך כלל, במאמץ הכניעה אבל במאמץ המותר, σ_{allow}

לכן $M_{\text{מקס}} = f\sigma_{yp} \cdot S = \sigma_{allow} \cdot S$ או $Q_{\text{כפיפה מקס}} = \dfrac{\sigma_{allow} \cdot S}{l}$

$S_{\text{תיכנון}} = M/f\sigma_{yp} = M/\sigma_{allow}$ או $S_{\text{תיכנון}} = \dfrac{Q_{\text{כפיפה}} \cdot l}{\sigma_{allow}}$

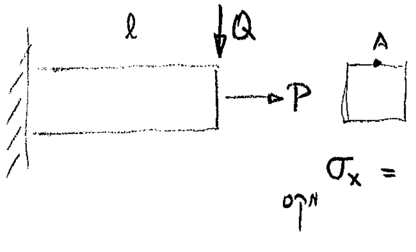


נניח ש- Q + P פועלים יחד

לכן $\sigma_{x\,\text{מקס}} = P/A \pm \dfrac{Mc}{I} = P/A \pm \dfrac{Qlc}{I}$

כאשר $\sigma_{x\,\text{מקס}} = \sigma_{yp}$ הגענו לקיבולת האלסטית.

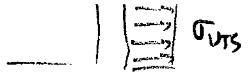
דוגמא: קורה בשטח צורת מלבן



$$\sigma_{x\,\text{מקס}} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Ql \cdot h/2}{A \cdot h^2/12} = \frac{P}{A} + \frac{6Ql}{A \cdot h} = \sigma_{yp}$$

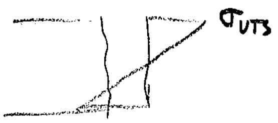
נ.ב. - יש חומרים שבירים (BRITTLE) שלא סובלים מעיבור גדול בגלל שינוי קטן במאמץ. לכן התמוטטותם או ~~כשלם~~ כשלונם באו כאשר המאמץ מגיע למאמץ המרבי (ULTIMATE STRESS) ואז כמו אנחנו משתמשים ב- מאמץ הכניעה בחומרים המתמתחים (גמישים), משתמשים במאמץ המרבי בחומרים השבירים.

לכן -



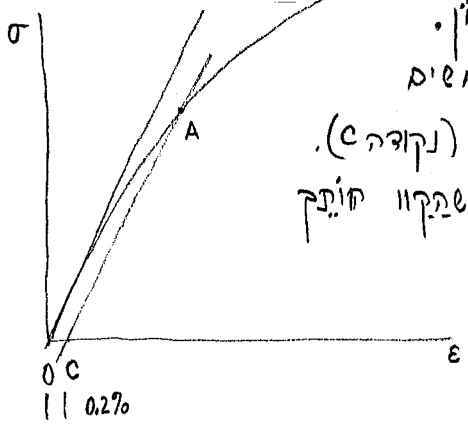
$$\sigma_{x\,מקס} = \sigma_{UTS} = P/A$$

או



$$\sigma_{x\,מקס} = \sigma_{UTS} = \frac{Mc}{I}$$

צריך להדגיש שחומר אלסטי יכול להיות לא ליניארי וכאן צריך למצוא את השיפוע שנמצא ב-נקודה 0. חומר פרוק וחומרים אחרים הולכים אחרי הדוגמא הזאת, כמו בטון.

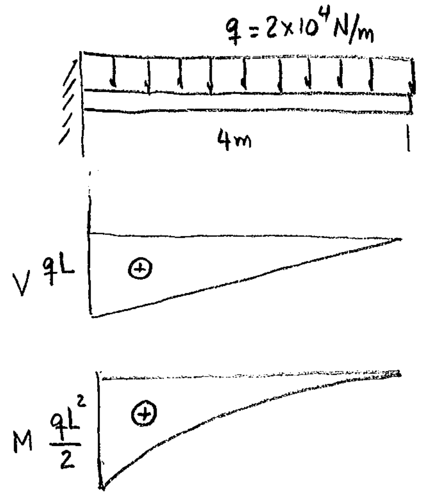


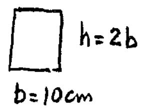
כדי למצוא את המאמץ הכניעה לחומרים האלה משתמשים בשיטת הביזון (OFFSET METHOD) שהוא 0.2% לימין הנקודה 0 (נקודה C). משרטטין קו ליניארי מקביל לשיפוע שנמצא מנקודה 0. בנקודה שהקו חותך את הגרף (A) קוראים לזה את <u>נקודת הכניעה</u>.

אף על פי שתכננו את הקורה לא להתמוטט בכפיפה ובמתח ובלחץ, לא תכננו את הקורה לא להתמוטט נגד הגזירה, על זה נדון בעתיד.

נדבר על שני מבנים שאנו צריכים לפתור בעיית כפיפה קורה.

① <u>אנליזה</u>: למצוא העומס המקסימלי שמותר לפעול על הקורה עבור קורה נתונה וחומר נתון.





$\sigma_{allow} = \pm 1.2\times10^8 Pa$

$$M_{max} = \frac{qL^2}{2} = \frac{2\times10^4(4)^2}{2} = 16\times10^4\ N\text{-}m$$

$$I = \frac{bh^3}{12} \quad c = \frac{h}{2} \quad S = I/c = \frac{bh^3/12}{h/2} = \frac{bh^2}{6} = \frac{10\times(20)^2}{6} = 6.67\times10^2\ cm^3 = 6.67\times10^{-4}\ m^3$$

$$\sigma_{max} = \frac{M}{S} = \frac{16\times10^4\ N\text{-}m}{6.67\times10^{-4}\ m^3} = 2.4\times10^8\ N/m^2 > \sigma_{allow}$$

לכן הקורה תתמוטט!

__תסבולת:__ למצוא את העומס המקסימלי שמותר
והגיאומטריה שלה נתונים.

$$M_{max} = q\frac{L^2}{2} = \sigma_{allow} \cdot S$$

(העומס המפורס distributed load)

$$q_{allow} = \frac{2\sigma_{allow} \cdot S}{L^2} = \frac{2(1.2\times10^8)(6.67\times10^{-4})}{4^2} = 1.0\times10^4 N/m$$

$$q_{allow} = q/2$$

לכן רואים שהתסבולת כאן יותר ממה שמותר והקורה תתמוטט. כדי שהקורה לא תכשל, העומס המפורס צריך להיות ½ מ-מה שנמצא.

__תיכון:__ לקבוע הגיאומטריה של הקורה לפי הנתונים - העומס וגם החומר.

מהו b של הקורה שמאפשר ~~עמוד~~ לה לעמוד בעומס הנתון

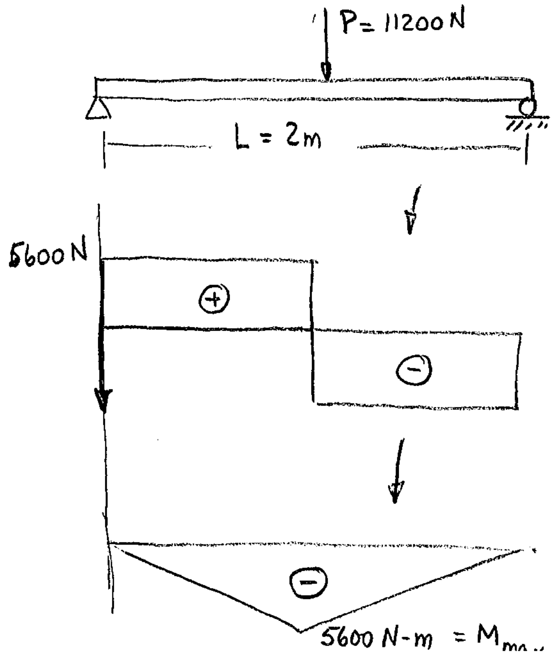
$$S_{allow} = \frac{M_{max}}{\sigma_{allow}} = \frac{b(2b)^2}{6} = \frac{4}{6}b^3$$

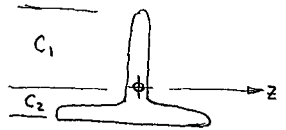
לכן

$$\sqrt[3]{\frac{6}{4}\frac{M_{max}}{\sigma_{allow}}} = b$$

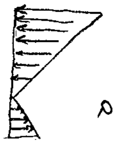
$$\sqrt[3]{\frac{3}{2}\cdot\frac{qL^2}{2\sigma_{allow}}} = \sqrt[3]{\frac{0.75\times2\times10^4\cdot4^2}{1.2\times10^8}} = 12.6 cm$$

__דוגמא ב'__





$c_1 = 8.72cm$
$c_2 = 3.28 cm$
$I_{zz} = 366 cm^4$



נתון: הקורה העשויה
מן הפרופיל ⊥ 12
נתונים המאמצים המותרים

$$_{מתח}\sigma_{allow} = 10^8 N/m^2 = 100 MPa$$

$$_{לחץ}\sigma_{allow} = 1.8\times10^8 N/m^2 = 180 MPa$$

$$_{לחץ}(\sigma_{max}) = \frac{Mc_1}{I_{zz}} = \frac{5600\times8.72\times10^{-2}m}{366\times10^{-8}m^4} = $$

$$= 1.334\times10^{+8}Pa$$

$$_{מתח}(\sigma_{max}) = \frac{Mc_2}{I_{zz}} = \frac{5600\times3.28\times10^{-2}m}{366\times10^{-8}m^4} = $$

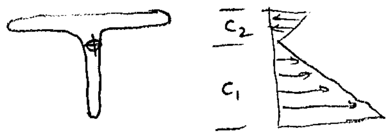
$$_{\text{לחץ}}(M_{allow}) = \frac{(\sigma_{allow})_{\text{לחץ}} \cdot I_{zz}}{c_1} = \frac{(1.8\times10^8)(366\times10^{-8})}{8.72\times10^{-2}} = 7555\ Nm$$

$$_{\text{מתח}}(M_{allow}) = \frac{(\sigma_{allow})_{\text{מתח}}\ I_{zz}}{c_2} = \frac{10^8\times(366\times10^{8})}{3.28\times10^{-2}} = 11,159\ Nm$$

ומה יהיה אם מהפכים הפרופיל?

$$(M_{allow}) = \frac{(\sigma_{allow})_{\text{לחץ}} \cdot I_{zz}}{c_2} = \frac{(1.8\times10^8)(366\times10^{-8})}{3.28\times10^{-2}} = 20,085\ Nm$$

$$(M_{allow}) = \frac{(\sigma_{allow})_{\text{מתח}}\ I_{zz}}{c_1} = \frac{(10^8)\times(366\times10^{-8})}{8.72\times10^{-2}} = 4,197\ Nm$$



בדוגמא הראשונה:

$$\frac{P_{allow}L}{4} = M_{allow} = 7555 N\text{-}m$$

והכוח שגורם למומנט הזה הוא

$$15,110\ N = P_{allow} = \frac{4M}{L}$$

בדוגמא השניה:

$$\frac{P_{allow}L}{4} = M_{allow} = 4,197\ Nm$$

והכוח הוא

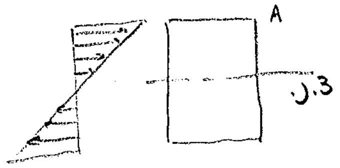
$$8,394\ N = P_{allow} = 4M/L$$

לכן, הפרופיל ⊥ יכול לסבול כוח יותר גדול מהפרופיל של T

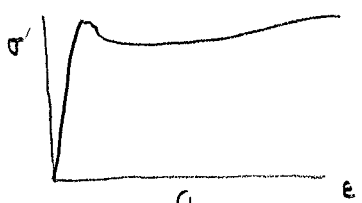
התנהגות פלסטית של חתכים.

אמרנו ש-שטח החתך מתחיל להיכנע לגבי הכפיפה כאשר המאמץ המקסימלי מגיע למאמץ הכניעה: $\sigma_{yp} = \sigma_{max} = \frac{Mc}{I}$

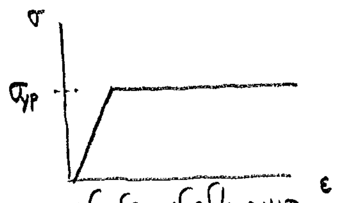
בזמן ההוא, המאמצים בשטח-החתך עדיין ליניאריים



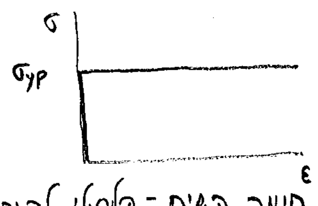
תלוי בצורה תיאורית:



חומר ריאלי

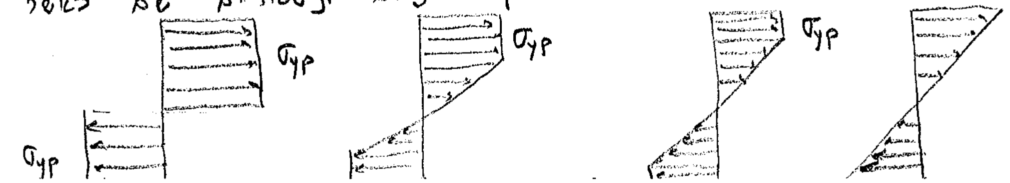


חומר אלסטי-פלסטי

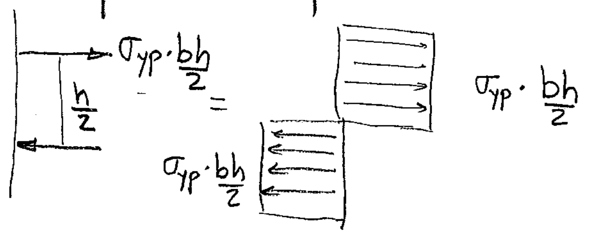


חומר קשיח-פלסטי טהור

נניח שמשתמשים בתיאוריה אלסטי-פלסטי. לכן, כאשר המאמץ מגיע למאמץ הכניעה המאמץ נשאר בשלב הזה. וכאשר המומנט ממשיך לגדל, המאמצים מתחת המישור A ממשיכים לגדל עד שמגיעים למאמץ הכניעה ונשארים שם כאשר מגדילים את המומנט.



כדי לחשב את התפסולת הפלסטית על שטח חתך בצורת מלבן



$$M_p = \sigma_{yp} \cdot \frac{bh}{2} \cdot \frac{h}{2} = \sigma_{yp} \cdot \frac{bh^2}{4}$$

אבל המומנט שנמצא כאשר המאמץ המקסימלי בכל השטח הוא σ_{yp}

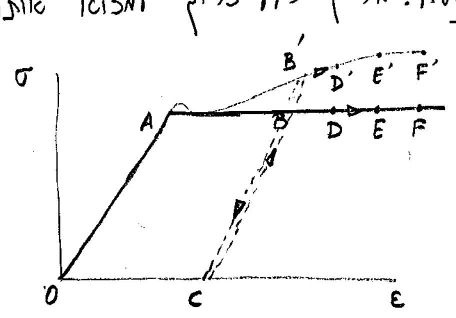
$$M_{yp} = \frac{\sigma_{yp} \cdot I}{c} = \frac{\sigma_{yp} \cdot bh^3/12}{h/2} = \sigma_{yp} \cdot \frac{bh^2}{6}$$

לכן

$$\underline{\underline{M_p = \frac{3}{2} M_{yp}}}$$

והעומס המקסימלי כאשר כל השטח סובל את מאמץ הכניעה הוא 50% יותר ממה שנמצא בהתחלת הכניעה.

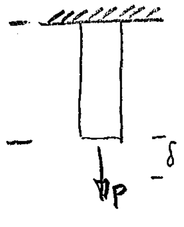
נ.ב.: כאן הציר הנטרלי לא זז ממקומו בגלל סימטריותו של השטח. בדרך כלל צריך למצוא אותו



מה קורה כאשר מסירים (remove) את העומס?

1. יש מדד קבועה שנשארת ⇐ $\overline{OC}$
2. המסלול בין המאמץ והעיבור הוא מקביל למסלול הליניארי ⇐ OA//BC
3. כאשר מעמיסים את הגוף בחזרה, המסלול הוא של BC בחזרה עד B ואחר-כך הוא ממשיך כרגיל.
4. איך מחשבים את מדד הבליזוני (offset distance)

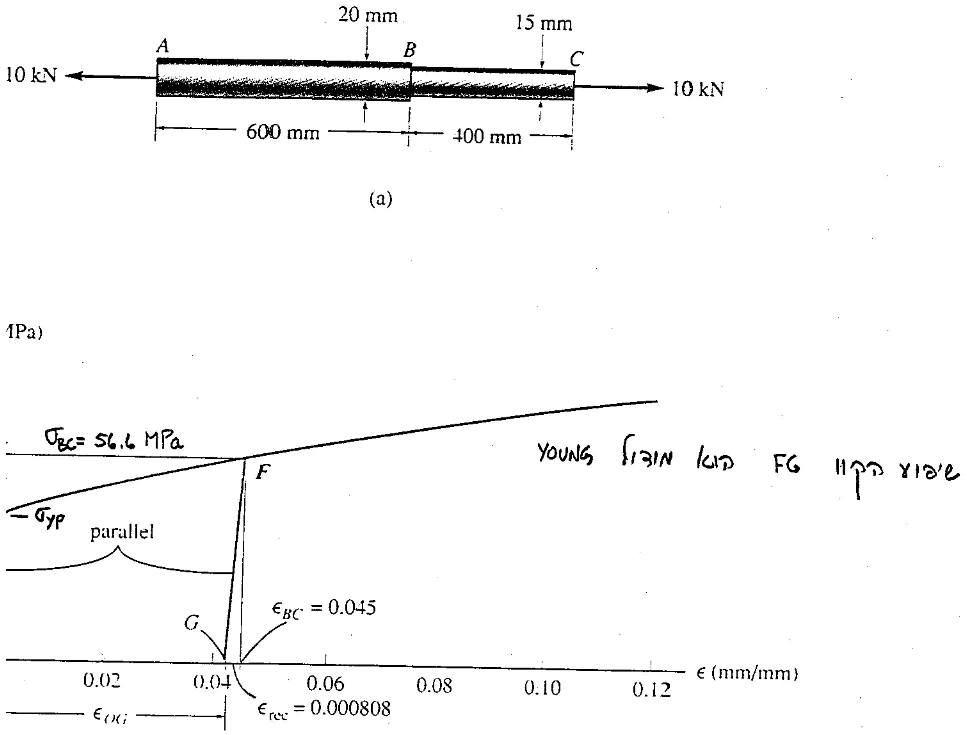
$$\delta = \frac{PL}{AE} = \frac{\sigma}{E} \cdot L$$



- במצב שמגיעים לכניעה $\delta_{yp} = \frac{\sigma_{yp} \cdot L}{E}$ ⇐
- אם מניחים שהחומר הוא אלסטי - פלסטי אידיאלי אז"פ שהעומס הולך וגודל במצב רגיל, המאמץ נשאר כ- σ_{yp}

והמדד הבליזוני הוא ההבדל בין δ ו- δ_{yp} ; δ מקבלים מניסיונות

An aluminum rod shown in Fig. 9–19*a* has a circular cross section and is subjected to an axial load of 10 kN. If a portion of the stress–strain diagram for the material is shown in Fig. 9–19*b*, determine the approximate elongation of the rod when the load is applied. If the load is removed, does the rod return to its original length? Take $E_{al} = 70$ GPa.



(a)

(b)

Fig. 9–19

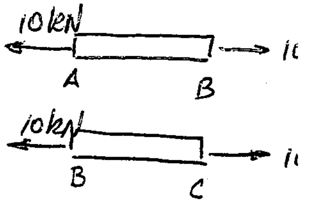
SOLUTION

For the analysis we will neglect the *localized deformations* at the point of load application and where the rod's cross-sectional area suddenly changes. (These effects will be discussed in Sec. 10.1.) Throughout the midsection of each segment the normal stress and deformation are uniform.

In order to study the deformation of the rod, we must obtain the strain. This is done by first calculating the stress, then using the stress–strain diagram to obtain the strain. The normal stress within each segment is

$$\sigma_{AB} = \frac{P}{A} = \frac{10(10^3)\text{ N}}{\pi\,(0.01\text{ m})^2} = 31.8\text{ MPa}$$

$$\sigma_{BC} = \frac{P}{A} = \frac{10(10^3)\text{ N}}{\pi\,(0.0075\text{ m})^2} = 56.6\text{ MPa}$$



From the stress–strain diagram, the material in region *AB* is strained *elastically* since σ_{YP}= 40 MPa > 31.8 MPa. Using Hooke's law,

$$\epsilon_{AB} = \frac{\sigma_{AB}}{E_{al}} = \frac{31.8(10^6)\text{ Pa}}{70(10^9)\text{ Pa}} = 0.0004543\text{ mm/mm}$$

The material within region *BC* is strained plastically, since σ_{YP}= 40 MPa < 56.6 MPa. From the graph, for $\sigma_{BC} = 56.6$ MPa,

$$\epsilon_{BC} \approx 0.0450\text{ mm/mm}$$

The approximate elongation of the rod is therefore

$$\Delta = \Sigma\epsilon L = 0.0004543(600\text{ mm}) + 0.045(400\text{ mm})$$
$$= 18.3\text{ mm}$$

Ans.

כאשר מסירים את הכוח

When the 10-kN load is removed, segment *AB* of the rod will be restored to its original length. Why? On the other hand, the material in segment *BC* will recover elastically along line *FG*, Fig. 9–19*b*. Since the slope of *FG* is E_{al}, the elastic strain recovery is

$$\epsilon_{rec} = \frac{\sigma_{BC}}{E_{al}} = \frac{56.6(10^6)\text{ Pa}}{70(10^9)\text{ Pa}} = 0.000808\text{ mm/mm}$$

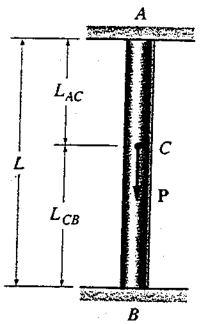
The remaining plastic strain in segment *BC* is then

$$\epsilon_{OG} = 0.0450 - 0.000808 = 0.0442\text{ mm/mm}$$

Therefore, when the load is removed the rod remains elongated by an amount

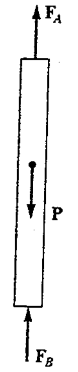
מוט אמוס צירי, יתר סטטי.

4 Statically Indeterminate Axially Loaded Member



(a)

יש רק משוואה אחת לקבל את
שתי התגובות. צריך להשק
בתנאי התאמה שאומר שתנו
היחסית בין A ל-B צריכה



(b)

Fig. 10–11

When a bar is fixed-supported at one end and is subjected to an axial force, the force equilibrium equation applied along the axis of the bar is *sufficient* to find the reaction at the fixed support. A problem such as this, where the reactions can be determined strictly from the equations of equilibrium, is called *statically determinate*. If the bar is fixed at *both ends*, however, as in Fig. 10–11*a*, then two unknown reactions occur, Fig. 10–11*b*, and the force equilibrium equation becomes

$$+\uparrow \Sigma F = 0; \qquad F_B + F_A - P = 0$$

In this case the bar in Fig. 10–11*a* is called *statically indeterminate*, since the equilibrium equation(s) are not sufficient to determine the reactions.

In order to establish an additional equation needed for solution, it is necessary to consider the geometry of the deformation. Specifically, an equation that specifies the conditions for displacement is referred to as a *compatibility* or *kinematic condition*. For the bar in Fig. 10–11*a*, a suitable compatibility condition would require the relative displacement of one end of the bar with respect to the other end to be equal to zero, since the end supports are fixed. Hence, we can write

$$\Delta_{A/B} = 0$$

This equation can be expressed in terms of the applied loads by using a *load-displacement relationship*, which depends on the material behavior. For example, if linear-elastic behavior occurs, Eq. 10–2 can be used. Realizing that the internal force in segment AC is $+F_A$, and in segment CB the internal force is $-F_B$, the compatibility equation can be written as

$$\frac{F_A L_{AC}}{AE} - \frac{F_B L_{CB}}{AE} = 0$$

Assuming that AE is constant, we can solve the above two equations for the reactions, which gives

$$F_A = P\left(\frac{L_{CB}}{L}\right) \quad \text{and} \quad F_B = P\left(\frac{L_{AC}}{L}\right)$$

To summarize the above procedure, the unknown forces in statically indeterminate problems must be determined by satisfying both equilibrium and compatibility requirements for the bar. To do so it is necessary to transform the compatibility conditions, which involve displacement, into equations that involve force. This is done by using load-displacement relations that describe the material behavior.

Superposition of Forces. For some types of problems it may be easier to write the compatibility equation using the superposition of the forces acting on the free-body diagram. This method of solution is often referred to as the *flexibility* or *force method of analysis*. To show how it is applied, consider again the bar in Fig. 10–11*a*. In order to write the necessary equation of compatibility, we will first choose any one of the two supports as "redundant" and temporarily remove its effect on the bar. The word *redundant*, as used here, indicates that the support is not needed to hold the bar in stable equilibrium, so that when it is removed, the bar becomes statically determinate. Here we will choose the support at B as redundant. By using the principle of superposition, the bar having its original loading on it, Fig. 10–11*c*, is then equivalent to the bar subjected only to the external load **P**, Fig. 10–11*d*, plus the bar subjected only to the redundant load $\mathbf{F}_B$, Fig. 10–11*e*. Although not shown, notice that the reaction at the support A satisfies force equilibrium, $F_A = P - F_B$, as determined either from Fig. 10–11*b* or from the *sum* of the reactions at A in Fig. 10–11*d* and 10–11*e*.

No displacement at B

(c)

If the load **P** causes B to be displaced *downward* by an amount Δ_B, Fig. 10–11*d*, the reaction $\mathbf{F}_B$ must be capable of displacing the end B of the bar *upward* by an amount δ_B, Fig. 10–11*e*, such that no displacement occurs at B, Fig. 10–11*c*, when the two loadings are superimposed.

$$(+\downarrow) \qquad 0 = \Delta_B - \delta_B$$

This equation represents the compatibility equation for displacements at point B, for which we have assumed that displacements are positive downward.

Displacement at B when redundant force at B is removed

(d)

Applying the load–displacement relationship, we have $\Delta_B = PL_{AC}/AE$ and $\delta_B = F_B L/AE$. Consequently,

$$0 = \frac{PL_{AC}}{AE} - \frac{F_B L}{AE}$$

$$F_B = P\left(\frac{L_{AC}}{L}\right)$$

From the free-body diagram of the bar, Fig. 10–11*b*, the reaction at A can now be determined from the equation of equilibrium,

$$+\uparrow \Sigma F_y = 0; \qquad P\left(\frac{L_{AC}}{L}\right) + F_A - P = 0$$

Since $L_{CB} = L - L_{AC}$, then

$$F_A = P\left(\frac{L_{CB}}{L}\right)$$

Displacement at B when only the redundant force at B is applied

(e)

These results are the same as those obtained previously, except that here we have applied the condition of compatibility and then the equilibrium condition to obtain the solution. Also note that the principle of superposition can be used here since the displacement and the load are linearly related ($\Delta = PL/AE$), which assumes, of course, that the material behaves in a linear-elastic manner.

תורת החוזק 2

תרגיל מס 3

2.100 The cylindrical rod AB has a length $L = 2$ m and a 32 mm diameter; it is made of a mild steel which is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod until its end A has moved down by an amount δ_m. Determine the maximum value of the force **P** and the permanent set of the rod after the force has been removed, knowing that (*a*) $\delta_m = 3$ mm, (*b*) $\delta_m = 6$ mm.

(a) 201 kN; 0.5 mm
(b) 201 kN; 3.5 mm

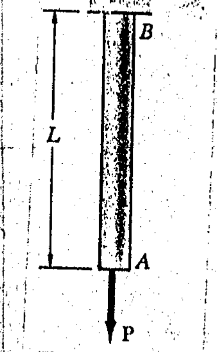


Fig. P2.100 and P2.101

2.102 Rod ABC consists of two cylindrical portions AB and BC; it is made of a mild steel which is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod and then removed to give it a permanent set $\delta_p = 2$ mm. Determine the maximum value of the force **P** and the maximum amount δ_m by which the rod should be stretched to give it the desired permanent set.

176.7 KN 3.84 mm.

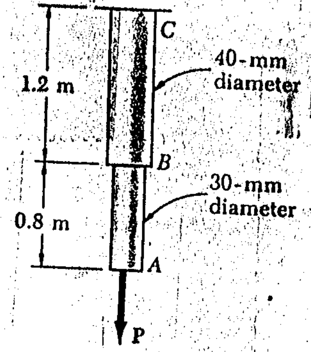


Fig. P2.102 and P2.103

2.104 Rod AB consists of two portions, AC and CB, each 200 mm long and of 1.9×10^{-3} m^2 cross-sectional area. Portion AC is made of a mild steel with $E = 200$ GPa and $\sigma_Y = 250$ MPa, and portion CB of a high-strength steel with $E = 200$ GPa and $\sigma_Y = 350$ MPa. A load **P** is applied at C as shown. (*a*) Assuming both steels to be elastoplastic, draw the load-deflection diagram for point C. (*b*) If P is gradually increased from zero to 1.080 MN and then reduced back to zero, determine the maximum deflection of C, the maximum stress in each portion of rod, and the permanent deflection of C.

$\delta_m = 0.318$ mm & $\sigma_{AC} = 250$ MPa $\sigma_{CB} = -318$ MPa
$\delta_p = 0.0342$ mm

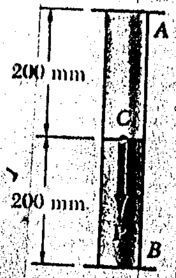


Fig. P2.104

3.76 The solid shaft shown is made of a mild steel which is assumed to be elastoplastic with $G = 77$ GPa and $\tau_Y = 145$ MPa. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (*a*) $T = 10$ kN · m, (*b*) $T = 15$ kN · m.

(a) 120.7 MPa, 37.5 mm
(b) 145 MPa, 23.7 mm

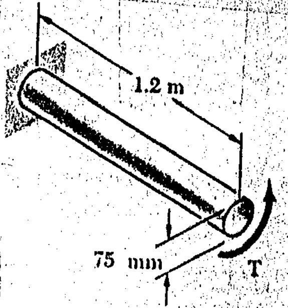


Fig. P3.76

3.84 The hollow shaft shown is made of a steel which is assumed to be elastoplastic with $G = 77$ MPa and $\tau_Y = 145$ MPa. Determine the magnitude T of the torque and the corresponding angle of twist (*a*) at the onset of yield, (*b*) when the plastic zone is 10 mm deep.

(a) 5.96 KN m, 17.98°
(b) 7.31 KN·m, 27.°

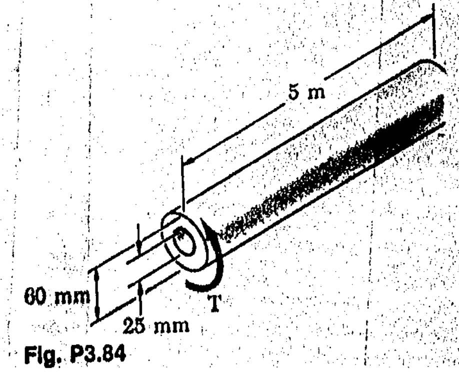


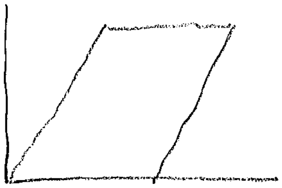
Fig. P3.84

$\sigma_y = P/A$

$P = \sigma_y \cdot A = \sigma_y \cdot \frac{\pi d^2}{4} = 250\times10^6 Pa \left(\frac{3.14159 \times (.032)^2}{4}\right) = 201,062 N$

$A = \frac{\pi d^2}{4} = 8.042 \times 10^{-4} m^2$

לפי התאוריה של אלסטו-פלסטי אידאלי $P = 201$ kN



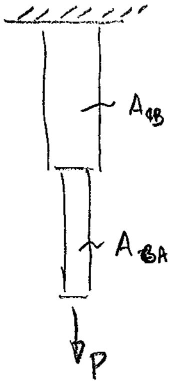
העיבור האלסטי הוא $\epsilon = \frac{\sigma_y}{E} = \frac{250 MPa}{200 GPa} = 1.25\times10^{-3}$

ההתארכות האלסטית $= \epsilon \cdot L = 1.25\times10^{-3} \cdot 2000 mm = 2.5mm = \delta_{yp}$

אם ההתארכות היתה 3mm, אז $\delta_{פלסטי}$ הנשאר הוא $3 - 2.5 = 0.5$ mm

אם ההתארכות היתה 6mm, אז האיזון הוא $6mm - \delta_{ym} = 3.5$ mm $= \delta_{פלסטי}$

2.102



A_{BA} $\frac{\pi d_{BA}^2}{4} = 7.069\times10^{-4} m^2 = \frac{\pi(.03)^2}{4}$

$A_{CB} = \frac{\pi d_{CB}^2}{4} = 1.257 \times 10^{-3} m^2 = \frac{\pi(.04)^2}{4}$

מפני ש- $A_{CB} > A_{BA}$ $\frac{P}{A_{BA}}$ מגיע ל- σ_{yp} קודם

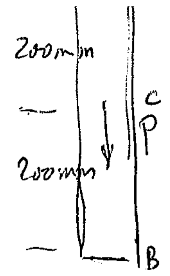
לכן $P = A_{BA} \cdot \sigma_{yp} = 7.069\times10^{-4} \times 250\times10^6 = \underline{176.725 kN}$

$$\delta = \frac{PL_{BA}}{A_{BA}E} + \frac{PL_{CB}}{A_{CB} \cdot E}$$

$$= \sigma_{yp} \cdot \frac{L_{BA}}{E} + \frac{P}{A_{CB}} \cdot \frac{L_{BC}}{E}$$

$$\delta = 250\times10^6 \left(\frac{0.8}{200\times10^9} + \frac{A_{BA}}{A_{CB}} \cdot \frac{1.2}{200\times10^9}\right) = 1.844\times10^{-3} m$$

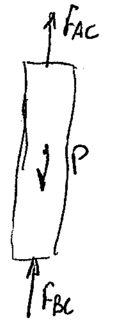
שיהיה $\delta_{פלסטי} = 2mm$, $\delta_{סה"כ} = \delta + 2_{mm} = 3.84mm$



$A_{AC} = A_{BC} = 1.9\times10$ mm

$\sigma_{yp_{AC}} = 250$ MPa $E_{AC} = E_{BC} = 200$ GPa

$\sigma_{yp_{BC}} = 350$ MPa



$F_{AC} + F_{BC} = P$

$$\frac{F_{AC}L_{AC}}{E_{AC}A_{AC}} - \frac{F_{BC}L_{BC}}{E_{BC}A_{BC}} = 0$$

$F_{AC} = \frac{PL_{BC}}{L}$ $F_{BC} = \frac{PL_{AC}}{L}$

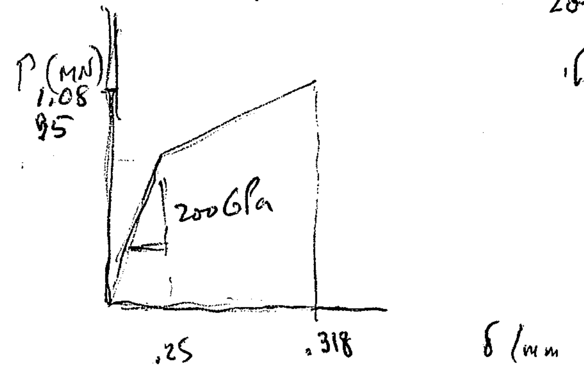
$F_{AC} = P/2$ $F_{BC} = P/2$

$\sigma_{AC} = \frac{F_{AC}}{A_{AC}}$ $\sigma_{BC} = \frac{F_{BC}}{A_{BC}}$

$\epsilon_{AC} = \frac{\sigma_{AC}}{E_{AC}}$ $\epsilon_{BC} = \frac{\sigma_{BC}}{E_{BC}}$

כאשר שניהם באזור אלסטי, $(\sigma_{AC} < \sigma_{yp_{AC}} = 250MPa)$, לכן $F_{AC} = \sigma_{yp_{AC}} \cdot A_{AC} = (250MPa)(1.9\times10^{-3})$

- $F_{AC} = .475$ MN ו- $\delta_{AC} = \frac{\sigma_{yp} L_{AC}}{E_{AC}} = \frac{250MPa \cdot .200}{200\ GPa} = 2.5\times10^{-4}m = .25mm$
- $P = 2F_{AC} = .95MN$
- כשמגיעים מעל 0.9MN חלק AC מתחיל להיות פלסטי
- והמאמץ הוא $250MPa = \sigma_{yp}$ ו- $\epsilon_{AC} = 1.25\times10^{-4} = \frac{\sigma_{AC}}{E_{AC}}$
- $F_{BC} = P/2 = .475MN$ ו- $\sigma_{BC} = \sigma_{AC} = -250$ MPa
- $\epsilon_{BC} = \frac{\sigma_{BC}}{E_{BC}} = -1.25\times10^{-4}$



אצל $P = .95MN$ לחלק AC, המאמץ $= \sigma_{yp} = 250$ MPa ו- $F_{AC} = .475MN$ אבל חלק BC עדיין באזור אלסטי. לכן $F_{BC} = P - F_{AC}$. כאשר $P = 1.08$ MN, $\sigma_{BC} < \sigma_{yp_{BC}}$, ו- $F_{BC} = 1.08 - .475MN = .605MN \Rightarrow \sigma_{BC} = \frac{F_{BC}}{A_{BC}} = -318.42 MPa$, $\sigma_{AB} = \sigma_{yp_{AB}} = 250MPa$

ו- העיבור של BC הוא $\frac{\sigma_{BC}}{E_{BC}} = 1.592\times10^{-4} \frac{mm}{mm}$ והתכווצות של BC הוא $\epsilon_{BC} \cdot L_{BC} = 0.318\times10^{-3} m$ וזה צריך להיות גם תזוזת נקודה C. מפני רצופו של הגוף, חלק AC גם כן 0.318mm. כאשר מורידים את המאמץ, חלק AC מאבד את העיבור $1.25\times10^{-3} \frac{\sigma_{yp}}{}$ וההתכווצות היא $.25mm = \epsilon_{AC} L_{AC}$ לכן פלסטי $= -.318mm + .25$

<u>תורת החוזק 2</u>

<u>תרגיל מס 4</u>

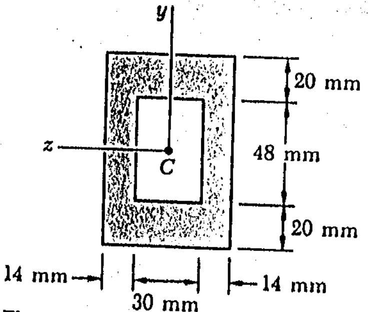


Fig. P4.78

4.78 and 4.79 A beam of the cross section shown is made of a steel which is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 300$ MPa. For bending about the z axis, determine the bending moment and the radius of curvature at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the beam are 20 mm thick.

(a) 20.6 kNm ; 29.3m (b) 26.9 kN·m ; 16. m.

שאלה: רדיוס העקמומיות ועומק הכפיפה

רדיוס העקמומיות $\frac{EI}{M}$ לחלק האלסטי

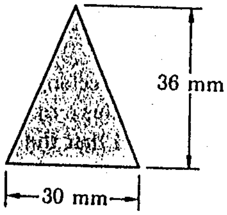
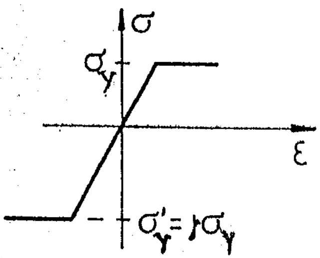


Fig. P4.88

4.88 and 4.89 Determine the plastic moment M_p of a steel beam of the cross section shown when the beam is bent about a horizontal axis. Assume the steel to be elastoplastic with a yield strength of 200 MPa. 759 N·m.



2.6 <u>נתון</u>: חתך מלבני (bxh) נתון לכפיפה.
החומר – אלסטי פלסטי – אידיאלי, אבל
עם גבול בזילה σ'_y בלחיצה גדול פי γ
מגבול הבזילה σ_y במשיכה.

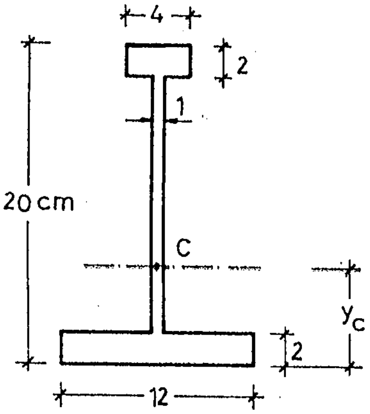
<u>דרוש</u>:

א) לחשב את מקומו של קו האפס ואת M_{pl}

ב) האם גודלו של M_{pl} שוה במקרה של מומנט חיובי ושלילי?

<u>תשובה</u>:

א) $x = h/(\gamma + 1)$; $M_{pl} = \sigma_y \frac{bh^2}{2} \frac{\gamma}{\gamma+1}$



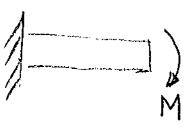
2.16 <u>נתון</u>: קורת פלדה על 2 סמכים
במיפתח ℓ=4.0m עמוסה עומס
מחולק שוה q. חתך הקורה לפי
הציור. $\sigma_y = \pm 240$ MPa ;

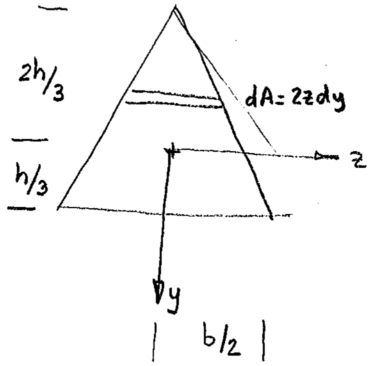
<u>דרוש</u>:

א) לחשב את q המותר לפי השיטה
האלסטית,

ב) כנ"ל, לפי השיטה הפלסטית.

[illegible]





$y=-\frac{2h}{3}$ $z=0$

$y=h/3$ $z=b/2$

$\frac{2h}{b}=\frac{h/3-(-\frac{2h}{3})}{\frac{b}{2}-0}=$ (שיפוע) m

$$\boxed{y=\frac{2h}{b}z-\frac{2h}{3}}$$

<u>הכוח בחלק מעל הציר z</u>

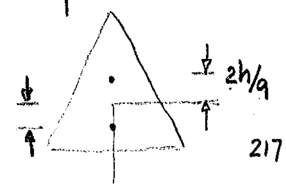
$\iint\sigma dA=\iint\frac{\sigma_{max}y}{(-2h/3)}2z\,dy$ מפני שהמאמץ לינארי ביחס ל-y. לכן $\sigma=\frac{\sigma_{max}\cdot y}{-(2h/3)}$

$$=\int_{-2h/3}^{0}\frac{2\sigma_{max}}{(-2h/3)}y\left[\frac{y+\frac{2h}{3}}{2h/b}\right]dy=\frac{2\sigma_{max}}{-4h^2/3b}\left(\frac{y^3}{3}+\frac{2h}{3}\frac{y^2}{2}\right)\Big|_{-\frac{2h}{3}}^{0}=\frac{2\sigma_{max}}{-4h^2/3b}\left[-\left(\frac{8h^3}{81}\right)-\frac{4h^3}{27}\right]=\frac{2\sigma_{max}3bh}{81}=\frac{2\sigma_{max}bh}{27}$$

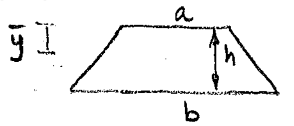
הכוח בחלק מתחת הציר z

$$\iint\sigma dA=\iint\frac{\sigma_{max}\cdot y\cdot 2z\,dy}{(-2h/3)}=\int_0^{h/3}\frac{2\sigma_{max}y}{(-2h/3)}\frac{(y+\frac{2h}{3})dy}{(2h/b)}=\frac{2\sigma_{max}}{-4h^2/3b}\left(y^3/3+\frac{h}{3}y^2\right)_0^{h/3}=\frac{2\sigma_{max}}{-4h^2/3b}\left[\frac{h^3}{81}+\frac{h^3}{27}\right]=-\frac{2\sigma_{max}bh}{27}$$

לכן סכ"ה של הכוחות הוא אפס, ומרכז הכובד גם הציר הניטרלי. הכוח בכול חלק הוא $\frac{2\cdot\sigma_y\cdot bh}{27}=16\text{ kN}$ והמומנט בהתחלת הכניעה הוא הכוח כפול הזרוע $\left(\frac{8}{5}\frac{h}{9}+\frac{2h}{9}=\right)\frac{3.4h}{9}$, לכן המומנט הוא $217.6\text{ N·m}=\frac{6.8}{243}\sigma_y\cdot bh^2$



לטרפז



מרכז הכובד $\bar y$ נמצא $\frac{h}{3}\frac{2b+a}{b+a}$ מהבסיס העליון.

כאשר המאמץ בכל נקודה מגיע ל-σ_{yp}, כדי שאין כוח על שטח-החתך, הציר הניטרלי צריך לחתוך השטח לשניים, זאת אומרת שהשטח המתוח הוא $\frac{bh}{4}$.

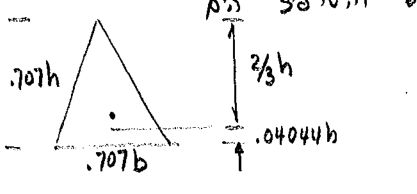
$$\frac{b}{h}\left(\frac{y^2}{2}+\frac{2h}{3}y+\frac{4h^2}{18}\right)=\frac{b}{h}\left(\frac{y^2}{2}+\frac{2h}{3}y-\frac{4h^2}{18}+\frac{4h^2}{9}\right)=\frac{b}{h}\left(\frac{\bar y^2}{2}+\frac{2h}{3}\bar y\right)_{-2h/3}^{y}=\int_{-2h/3}^{y}\frac{2(\bar y+\frac{2h}{3})d\bar y}{2h/b}=\int 2z\,dy=\iint dA=\frac{bh}{4}$$

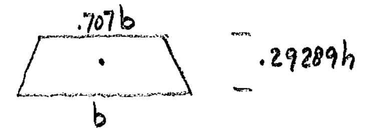
לכן $\frac{b}{h}\left(\frac{y^2}{2}+\frac{2hy}{3}+\frac{4h^2}{18}\right)=\frac{bh}{4}$ ו- $\frac{y^2}{2}+\frac{2h}{3}y-\frac{2h^2}{72}=0$ ו- $y=-\frac{2h}{3}\pm\sqrt{\frac{4h^2}{9}+\frac{4}{2}\left(\frac{2h^2}{72}\right)}$

$=-\frac{2h}{3}\pm\sqrt{h^2/2}\Rightarrow .04044h$

$\Rightarrow -1.37377h$ התשובה השנייה <u>בלתי אפשרית</u>

מפני ש $z=\frac{b}{2h}(y+2h/3)=\frac{b}{2h}\sqrt{\frac{h^2}{2}}=\frac{b}{2\sqrt2}$, כאשר $y=0.04044h$ לכן המשולש והטרפז הם

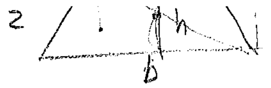




- מרכז הכובד של המשולש הוא 0.2357h מהבסיס. מרכז הכובד של הטרפז הוא 0.1548 h מהבסיס העליון והמרחק בין שני המרכזים הוא .3905h.
- לכן הכוח של כל חלק הוא $\frac{bh}{4}\cdot\sigma_y=54\text{ kN}$ והמומנט הוא

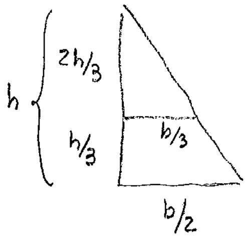
$M_{pl}=759.18\text{ Nm}=\frac{\sigma_y\cdot bh}{4}(.3905h)$

מקדם הצורה $3.5\approx\frac{759.18}{217.6}=\frac{M_{pl}}{M_y}$ Shape factor



$$\frac{1}{2}\left(\frac{2a+b}{a+b}\right)h$$

A	d	=
$\frac{1}{2}a\cdot h$	$h/3$	$\frac{1}{6}ah^2$
$\frac{1}{2}b\cdot h$	$2h/3$	$\frac{2bh^2}{6}$
$\frac{1}{2}(a+b)h$		$\frac{1}{6}(2b+a)h^2$



$h : b/2 :: 2h/3 : b/3$

To find for trapezoid

$$\frac{3\cancel{\frac{1}{6}}(2b+a)h^{\cancel{2}}}{\cancel{\frac{1}{2}}(a+b)\cancel{h}}$$

$$\boxed{\frac{1}{3}\,\frac{2b+a}{a+b}\cdot h} = \frac{1}{3}\left[\frac{2\cdot b/2 + b/3}{b/3+b/2}\right]\frac{h}{3}$$

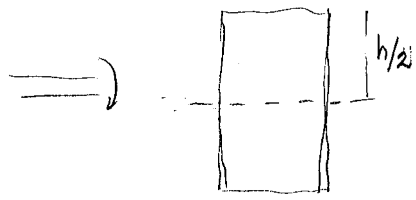
$$\frac{1}{3}\quad\frac{60+20}{50}\cdot 12 = \frac{32}{5} = 6.4 = \frac{1}{3}\left[\frac{4b/3}{5b/6}\right]h/3 = \frac{8}{5}\,\frac{h}{9}$$

$$\frac{2\frac{1}{\sqrt{2}}\cancel{b}+\cancel{b}1}{\frac{1}{\sqrt{2}}\cancel{b}+1} = \frac{\sqrt{2}+1}{1+\sqrt{2}}\cdot\sqrt{2}$$

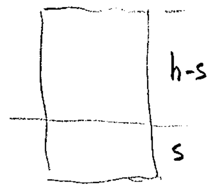
$$\frac{.29289h}{3}\left(\frac{2\left(\frac{1}{\sqrt{2}}b\right)+b}{\frac{1}{\sqrt{2}}b+b}\right) = \frac{\sqrt{2}}{3}(.29289h)$$

$$\frac{2b+\frac{1}{\sqrt{2}}b}{\frac{1}{\sqrt{2}}+b} = \frac{2+\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}+1} = \frac{2\sqrt{2}+1}{\sqrt{2}+1}$$

2.6



כאשר המאמץ מגיע ל-σ_y למעלה בהתחלה המלבן מתחיל להיכנע. אבל למטה מקוו הנייטרלי הוא עדיין אלסטי.
הכוח למעלה הוא = $\int \sigma b\,dy$ והציר הנייטרלי
יהיה כמו מלבן שאין הבדל מצד המתיחה לצד הלחוץ
כאשר כל השטח הוא פלסטי, לחלק מעל הציר הכוח יהיה $\sigma_y\cdot b\cdot(h-s)$
והכוח לחלק התחתון יהיה $\sigma_y'\cdot b\cdot s$ ושניהם צריכים להיות שווים
אבל בסימן הפוך. לכן



$$+\gamma\cancel{\sigma_y}\cancel{b}\cdot s = -\left(-\sigma_y' b\cdot s\right) = \cancel{\sigma_y}\cancel{b}(h-s)$$

$$+\gamma s = h-s$$

$$s = h/_{\gamma+1}$$

בין מרכזי הכובד של שני החלקים המרחק הוא $\frac{h}{2} = \frac{h\gamma}{2(\gamma+1)} + \frac{h}{2(\gamma+1)}$

לכן $M_{pl} = \frac{h}{2}\cdot\frac{\gamma\sigma_y\cdot b\cdot h}{\gamma+1} = \frac{\gamma}{2(\gamma+1)}\sigma_y bh^2$. בהתחלת הכניעה $M_{yp} = \sigma_y\cdot\frac{bh^2}{6}$

3γ M_{pl}

CONCEPT OF ELASTIC STRAIN ENERGY - מושג אנרגיה של עיבור האלסטי

- DEFORMATION OF BODY UNDER EXTERNAL LOADS - LOADS DO WORK
- ASSUME - NO KINETIC OR HEAT EXCHANGE
  - GRADUAL INCREASE IN LOAD FROM INITIAL TO FINAL STATE
  - CONSERVATION OF ENERGY ⇒ STRAIN ENERGY IS POTENTIAL ENERGY
- FROM STATICS REMEMBER

  מכפלה סקלרית

  WORK DONE ON UNIT VOLUME IS $\vec{F}\cdot d\vec{x} = \bar{\sigma}A \cdot d\bar{\epsilon}\, l = \bar{\sigma}\cdot d\bar{\epsilon}\, Al$
- TOTAL WORK DONE BY FORCE IS STORED AS STRAIN ENERGY GIVEN BY

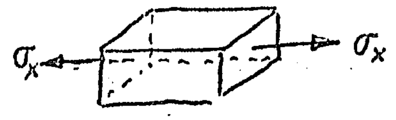
$$\int_V \left[\int \bar{\sigma}\cdot d\bar{\epsilon}\right] dV = \int_V \left[\frac{\sigma^2}{2E}\right] dV = \int_V \left[\frac{E}{2}\epsilon^2\right] dV = \int \frac{\bar{\sigma}\cdot\bar{\epsilon}}{2}\, dV$$

יכולים להוסיף עבודה על עבודה

- REMEMBER WORK IS ADDITIVE AND DEPENDS ON FINAL & INITIAL STATES AND NOT ON PATH BETWEEN STATES

  מציית
- IF WE HAVE A BODY THAT OBEYS HOOKE'S LAW AND WE APPLY A... FORCE IN THE X - DIRECTION ONLY

  $\sigma_x = E\epsilon_{x_1}$, $\epsilon_{y_1} = -\nu\epsilon_{x_1}$; $\epsilon_{z_1} = -\nu\epsilon_{x_1}$ ν - POISSON RATIO
- SINCE σ_x , ϵ_{x_1} ARE PARALLEL TO EACH OTHER , WORK IS DONE
- SINCE σ_x , ϵ_{y_1} OR ϵ_{z_1} ARE ⊥ TO EACH OTHER , NO WORK DONE
- WORK DUE TO σ_x IS $\int \frac{\sigma_x \epsilon_{x_1}}{2}\, dV$



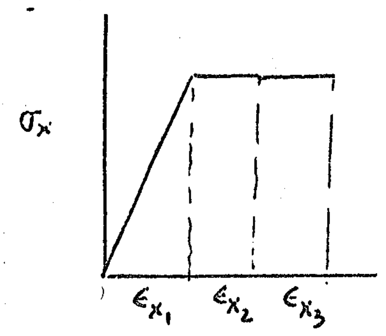
- IF WE NOW APPLY TO THIS STATE AN ADDITIONAL FORCE IN THE Y-DIRECTION KEEPING σ_x FIXED

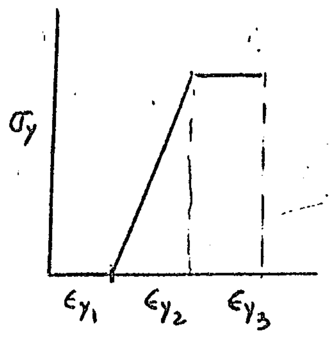
  $\sigma_y = E\epsilon_{y_2}$ $\epsilon_{x_2} = -\nu\epsilon_{y_2}$ $\epsilon_{z_2} = -\nu\epsilon_{y_2}$
- SINCE σ_y , ϵ_{y_2} ARE PARALLEL , WORK IS DONE
- SINCE σ_y , ϵ_{x_2} OR ϵ_{y_2} ARE ⊥ TO EACH OTHER , NO WORK DONE
- BUT

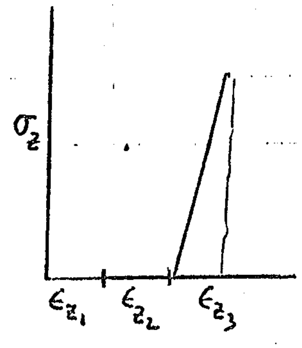
  σ_x FROM BEFORE DOES DO WORK DUE TO ϵ_{x_2}

- THUS ADDITIVE WORK DUE TO σ_y IS -

$$\int_V \left[\frac{\sigma_y \epsilon_{y_2}}{2} + \sigma_x \epsilon_{x_2}\right] dV$$







- SIMILARLY IF WE APPLY A FORCE IN THE Z DIRECTION ONLY KEEPING σ_x, σ_y FIXED : $\sigma_z = \bar{E}\epsilon_{z_3}$, $\epsilon_{x_3} = -\nu\epsilon_{z_3}$, $\epsilon_{y_3} = -\nu\epsilon_{z_3}$
- A SIMILAR ARGUMENT YIELDS THE ADDITIVE WORK DONE

נימוק דומה

$$\int_V \left[\frac{\sigma_z \epsilon_{z_3}}{2} + \sigma_x \epsilon_{x_3} + \sigma_y \epsilon_{y_3}\right] dV$$

- BY ADDING THE THREE TERMS WE GET THE TOTAL WORK DONE

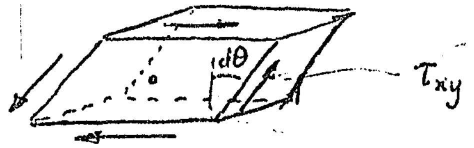
$$\int\left[\frac{\sigma_x \epsilon_{x_1}}{2} + \frac{\sigma_y \epsilon_{y_2}}{2} + \sigma_x \epsilon_{x_2} + \frac{\sigma_z \epsilon_{z_3}}{2} + \sigma_x \epsilon_{x_3} + \sigma_y \epsilon_{y_3}\right] dV$$

- NOTE THAT $\frac{1}{2}\sigma_x \epsilon_{x_2} = \frac{1}{2}E\epsilon_{x_1}\cdot(-\nu\epsilon_{y_2}) = \frac{1}{2}E\epsilon_{y_2}\cdot(-\nu\epsilon_{x_1}) = \sigma_y \epsilon_{y_1}/2$
- SIMILARLY $\frac{1}{2}\sigma_x \epsilon_{x_3} = \frac{1}{2}\sigma_z \epsilon_{z_1}$ & $\frac{1}{2}\sigma_y \epsilon_{y_3} = \frac{1}{2}\sigma_z \epsilon_{z_2}$

- THUS TOTAL WORK DONE IS $\frac{1}{2}\int\left[\sigma_x \epsilon_x + \sigma_y \epsilon_y + \sigma_z \epsilon_z\right] dV$

  WHERE $\epsilon_x = \epsilon_{x_1} + \epsilon_{x_2} + \epsilon_{x_3}$, $\epsilon_y = \epsilon_{y_1} + \epsilon_{y_2} + \epsilon_{y_3}$ ...

- LOOK AT SHEAR FORCES THEY DO WORK THRU SHEAR STRAINS $d\gamma$



זוג כוח

- CAN DO WORK BY FORCE COUPLE CAUSING BODY TO ROTATE ABOUT AN AXIS THROUGH 0
- FROM STATICS WORK DONE BY MOMENT IS $\vec{M}\cdot d\bar{\theta} = \bar{T}\ell\cdot d\bar{\theta} = \bar{\tau}A\ell\cdot d\bar{\theta}$

$$= \bar{\tau}\cdot d\bar{\gamma}\ A\ell$$

- WORK DONE $= \int_V \left[\int \bar{\tau}\cdot d\bar{\gamma}\right] dV$
- FOR A BODY OBEYING HOOKE'S LAW $\int \bar{\tau}\cdot d\bar{\gamma} = \frac{\tau^2}{2G} = \frac{G\gamma^2}{2} = \frac{\tau\cdot\gamma}{2}$

- BY SIMILAR MANNER WORK DONE BY SHEAR FORCES ARE

$$\int_V \left[\frac{\tau_{xy}\gamma_{xy}}{2} + \frac{\tau_{xz}\gamma_{xz}}{2} + \frac{\tau_{yz}\gamma_{yz}}{2}\right] dV$$

- TOTAL WORK DONE DUE TO ALL STRESSES IS SUM OF THE TWO

$$\int_V \frac{1}{2}\left[\sigma_x\epsilon_x + \sigma_y\epsilon_y + \sigma_z\epsilon_z + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{xz}\gamma_{xz}\right] dV = \int_V U_{s_0}\, dV = U_s$$

אנרגית העיבור הסגולית

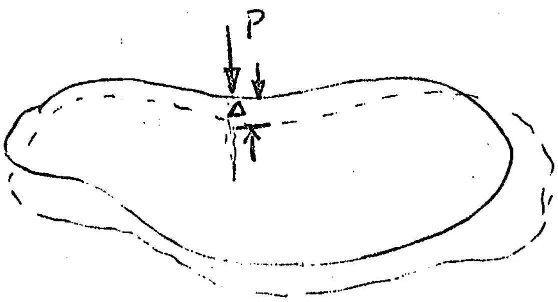
- U_{s_0} IS THE STRAIN ENERGY DENSITY ; ALWAYS ≥ 0
- U_{s_0} IS ZERO ONLY WHEN ALL σ's, ϵ's, τ's AND γ's $= 0$

- NOTE THAT SINCE $dU_{s_0} = \sigma_x d\epsilon_x + \sigma_y d\epsilon_y + \sigma_z d\epsilon_z + \tau_{xy} d\gamma_{xy} + \tau_{xz} d\gamma_{xz} + \tau_{yz} d\gamma_{yz}$ AND dU_{s_0} is perfect DIFFERENTIAL, MEANING THAT $U_{s_0} = f(\epsilon_x, \epsilon_y, \epsilon_z, \gamma_{xy}, \ldots)$

$$dU_{s_0} = \frac{\partial U_{s_0}}{\partial \epsilon_x} d\epsilon_x + \frac{\partial U_{s_0}}{\partial \epsilon_y} d\epsilon_y + \frac{\partial U_{s_0}}{\partial \epsilon_z} d\epsilon_z + \frac{\partial U_{s_0}}{\partial \gamma_{xy}} d\gamma_{xy} + \ldots$$

$\Rightarrow \frac{\partial U_{s_0}}{\partial \epsilon_x} = \sigma_x$ AND $\frac{\partial U_{s_0}}{\partial \epsilon_y} = \sigma_y$ ETC.

- SINCE σ IS RELATED TO A LOAD AND ϵ IS RELATED TO A DISPLACEMENT IN THE DIRECTION OF THAT LOAD $\Rightarrow$ WE CAN DETERMINE THE LOAD IF WE KNOW HOW THE STRAIN ENERGY VARIES WITH THE DISPLACEMENT IN THE DIRECTION OF THAT LOAD



$$U_s = U_s(\Delta) \Rightarrow \frac{\partial U_s}{\partial \Delta} = P$$

THIS FACT WILL BE USED LATER

- WE HAVE CONSIDERED WORK DONE = $\vec{F} \cdot d\vec{x}$

  IF $\vec{F}$ is CONSTANT $\vec{F} \cdot d\vec{x} = d(\vec{F} \cdot \vec{x})$ — MOST GENERAL EXPRESSION FOR WORK

- WHAT IF $\underset{\sim}{X}$ IS NOW CONSTANT AND $\underset{\sim}{F}$ CHANGES ie $d(\vec{F} \cdot \vec{x}) = \vec{x} \cdot d\vec{F}$
  - THIS ALSO CAUSE WORK TO BE DONE

$$d\vec{F} = d\vec{\sigma} \cdot A \qquad \vec{X} = \ell \vec{\epsilon} \qquad \Rightarrow \quad \vec{x} \cdot d\vec{F} = \vec{\epsilon} \cdot d\vec{\sigma} \quad (\ell A)$$

- JUST AS BEFORE WE CAN GO THROUGH THE PROCESS AND SHOW THAT

$$\text{TOTAL WORK DONE} = \int_V \frac{1}{2} \int \left(\bar{\epsilon}_x \cdot d\bar{\sigma}_x + \bar{\epsilon}_y \cdot d\bar{\sigma}_y + \bar{\epsilon}_z \cdot d\bar{\sigma}_z + \bar{\gamma}_{xy} \cdot d\bar{\tau}_{xy} + \bar{\gamma}_{xz} \cdot d\bar{\tau}_{xz} + \bar{\gamma}_{yz} \cdot d\bar{\tau}_{yz} \right) dV$$

- THE INNER INTEGRAL IS THE COMPLEMENTARY ENERGY OF THE BODY : U_{co} (האנרגיה המשלמתית)
  - $dU_{co} = \bar{\epsilon}_x \cdot d\bar{\sigma}_x + \bar{\epsilon}_y \cdot d\bar{\sigma}_y + \bar{\epsilon}_z \cdot d\bar{\sigma}_z + \ldots$
  - IT IS ALSO A PERFECT DIFFERENTIAL SO THAT $U_{co} = U_{co}(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \ldots)$

$$dU_{co} = \frac{\partial U_{co}}{\partial \sigma_x} d\sigma_x + \frac{\partial U_{co}}{\partial \sigma_y} d\sigma_y + \frac{\partial U_{co}}{\partial \sigma_z} d\sigma_z + \frac{\partial U_{co}}{\partial \tau_{xy}} d\tau_{xy} + \cdots$$

$$\text{AND} \quad \frac{\partial U_{co}}{\partial \sigma_x} = \epsilon_x \quad \frac{\partial U_{co}}{\partial \sigma_y} = \epsilon_y \quad \text{etc.} \quad \frac{\partial U_{co}}{\partial \tau_{xy}} = \gamma_{xy}$$

- HERE IF WE KNOW HOW THE COMPLEMENTARY ENERGY CHANGES AS LOAD CHANGES THE WE CAN FIND THE DISPLACEMENT, DUE TO THAT LOAD, IN DIRECTION OF LOAD

- REMEMBER σ_x CAN BE RELATED TO LOAD IN X-DIRECTION

  ϵ_x CAN BE RELATED TO DISPLACEMENT IN DIRECTION OF LOAD

- WHAT WE SAID ABOUT U_{s_0} & U_{c_0} IS TRUE EVEN IF BODY DOESN'T OBEY HOOKE'S LAW.
- IF BODY IS LINEALY ELASTIC : $U_{c_0} = U_{s_0}$

- NOTE : ALWAYS WRITE U_s IN TERMS OF STRAINS / DISPLACEMENTS
  U_c IN TERMS OF STRESSES / LOADS

- CLARIFY SOME PTS
  - FROM STATICS BODY WAS RIGID - NO WORK DONE DUE TO INTERNAL LOADS
  - FOR A BODY THAT DEFORMS WORK IS DONE BY INTERNAL FORCES (U_c , U_s)
  - WHAT WE'VE JUST DISCUSSED IS WORK DONE BY INTERNAL FORCES
  - BOTH FOR NON DEFORMABLE & DEFORMABLE BODIES, THE EXTERNAL FORCES ALSO DO WORK
  - THE EXPRESSIONS FOR U_c, U_{c_c} , U_s, U_{s_0} HOLD FOR NON-LINEARLY ELASTIC BODIES AS WELL
  - TOTAL WORK DONE BY BODY THAT HAS EXTERNAL LOADS APPLIED AND UNDERGOES DEFORMATION IS $W_i + W_e = \Pi$

- $$W_e = \int_S (X u + Y v + Z w) dS$$

  S - surface of body
  u, v, w - displacements undergone by forces X, Y, Z applied to surface

- ASSUMPTION - BODY FORCES (LIKE WEIGHT) CAN BE ACCOUNTED FOR THROUGH W_e TERM.
- WHEN AN ELASTIC BODY IS AT REST, THE EXTERNAL FORCES + BODY FORCES + INTERNAL FORCES ARE IN A STATE OF EQUILIBRIUM

שלושתן קבוצות הכוחות

- WORK DONE BY THESE THREE SET OF FORCES IS AT A MINIMUM
- THUS $\delta \pi = \delta W_i + \delta W_e = 0$
- ALSO $\delta W_i = - \delta U_s$ REMEMBER FROM STATICS POTENTIAL ENERGY IS NEGATIVE OF WORK DONE
- THUS FOR ANY CHANGE IN DISPLACEMENTS OF THE BODY THAT KEEPS IT IN EQUILIBRIUM

שינוי עבודת הגוף
ביחס לתזוזות הגוף

$$\delta \pi / \delta \text{ displacements} = 0 = - \frac{\delta U_s}{\delta \text{displ.}} + \frac{\delta W_e}{\delta \text{displ.}}$$

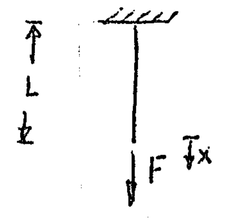
- BUT $\frac{\delta U_s}{\delta \text{displ}} = \frac{\delta W_e}{\delta \text{displ}}$ = load, due to that displ, IN DIRECTION OF DISPL.
- THIS IS CASTIGLIANO'S THEOREM (FIRST)
- SIMILARLY SINCE $\delta W_i = - \delta U_s = - \delta U_c$
- FOR ANY CHANGE IN LOADS OF THE BODY THAT KEEPS IT IN EQUILIBRIUM

$$\delta \pi / \delta_{\text{LOADS}} = 0 = - \frac{\delta U_c}{\delta \text{ LOAD}} + \frac{\delta W_e}{\delta \text{ LOAD}}$$

- BUT $\frac{\delta U_c}{\delta \text{ LOAD}} = \frac{\delta W_e}{\delta \text{ LOAD}}$ = DISPL, DUE TO THAT LOAD, IN DIR. OF LOAD
- THIS IS CASTIGLIANO'S THEOREM (SECOND)

→ USING U_c TO DETERMINE DISPLACEMENTS ←

EXAMPLE #1 EXTENSIBLE ROD



$\sigma = F/A$ $\epsilon = \frac{\sigma}{E} = \frac{F}{AE} = \frac{x}{L}$

$$\overset{U_{c_0} \cdot V}{\downarrow} \qquad \overset{U_{s_0} \cdot V}{\downarrow}$$

$$U_c = \frac{\sigma^2}{2E} \cdot AL \qquad U_s = \frac{E\epsilon^2}{2} \cdot AL$$

$$= \frac{F^2 L}{2EA} \qquad = \frac{E x^2}{2L} A$$

THUS $U_c = \frac{F^2 L}{2EA}$ $U_s = \frac{E x^2}{2L} A$

TO FIND F, ASSUMING x IS KNOWN, DEFINE π IN TERMS OF U_s & W_e

$$\pi = -\frac{Ex^2}{2L}A + Fx$$

$$\frac{\partial \pi}{\partial x} = -\frac{2ExA}{2L} + F = 0 \qquad F = x\frac{EA}{L} \qquad \left(\frac{\partial U_s}{\partial x} = F\right)$$

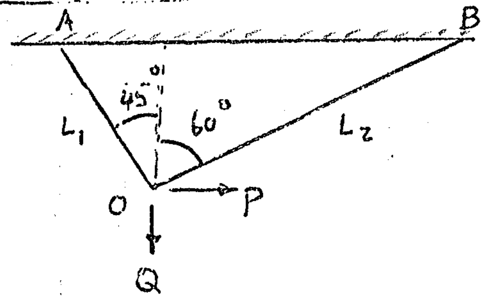
TO FIND x, ASSUMING F IS KNOWN, DEFINE π IN TERMS OF U_c & W_e

$$\pi = -\frac{F^2L}{2EA} + Fx$$

$$\frac{\partial \pi}{\partial F} = -\frac{2FL}{2EA} + x = 0 \qquad x = \frac{FL}{AE} \qquad \left(\frac{\partial U_c}{\partial F} = x\right)$$

- WE WILL LOOK AT TRUSSES WHERE EXTENSION/COMPRESSION IS PRIMARY LOADING

EXAMPLE #2 : TWO-ROD TRUSS

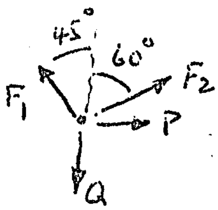


STATICALLY DETERMINATE SYSTEM

LOOK AT TWO BARS CONNECTED AT O, BARS ARE EXTENSIBLE — HAVE THE SAME CROSS SECTION, A, AND YOUNG'S MODULUS, E.

→ WANT TO FIND DISPLACEMENTS u, v GIVEN P AND Q

1) USE STATICS TO FIND FORCES IN OB & OA
2) DETERMINE U_c
3) $\frac{\partial U_c}{\partial P} = u \qquad \frac{\partial U_c}{\partial Q} = v$



$$\left.\begin{array}{l} Q = F_2\cos 60° + F_1 \cos 45° \\ P = F_1 \sin 45° - F_2 \sin 60° \end{array}\right\} \quad \begin{array}{l} F_1 = \frac{\sqrt{3}-1}{\sqrt{2}}(P+\sqrt{3}Q) \\ F_2 = (\sqrt{3}-1)(Q-P) \end{array}$$

NOTE F_1 & F_2 ARE FNS OF P & Q

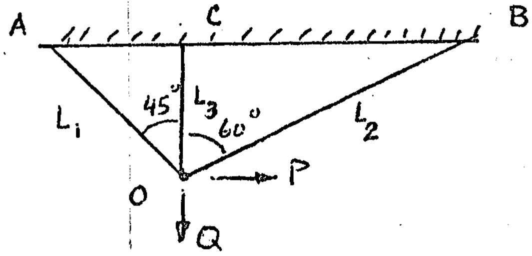
$$\text{Now} \quad U_c = \sum\frac{F^2L}{2AE} = \frac{F_1^2L_1}{2AE} + \frac{F_2^2L_2}{2AE} = \frac{1}{2AE}\left\{\left[\frac{\sqrt{3}-1}{\sqrt{2}}(P+\sqrt{3}Q)\right]^2 L_1 + \left[(\sqrt{3}-1)(Q-P)\right]^2 L_2\right.$$

$$u = \frac{\partial U_c}{\partial P} = \frac{2F_1L_1}{2AE}\frac{\partial F_1}{\partial P} + \frac{2F_2L_2}{2AE}\frac{\partial F_2}{\partial P} = \frac{\sqrt{3}-1}{\sqrt{2}}(P+\sqrt{3}Q)\frac{L_1}{AE}\cdot\frac{\sqrt{3}-1}{\sqrt{2}} + (\sqrt{3}-1)(Q-P)\frac{L_2}{AE}\left\{-(\sqrt{3}-1)\right\}$$

$$v = \frac{\partial U_c}{\partial Q} = \frac{2F_1L_1}{2AE}\frac{\partial F_1}{\partial Q} + \frac{2F_2L_2}{2AE}\frac{\partial F_2}{\partial Q} = \frac{L_1}{AE}\left\{\frac{\sqrt{3}-1}{\sqrt{2}}(P+\sqrt{3}Q)\cdot\frac{\sqrt{3}}{\sqrt{2}}(\sqrt{3}-1)\right\} + \frac{L_2}{AE}\left\{(\sqrt{3}-1)^2(Q-P)\right\}$$

מסבך סטטי בלתי מסויים

EXAMPLE #3 - INDETERMINATE (STATICALLY) TRUSS



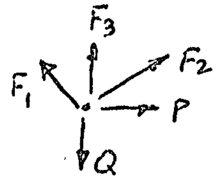
$L_3 = L$ $\quad L_1 = \sqrt{2}\,L$ $\quad L_2 = 2L$

GIVEN: A, E SAME FOR ALL THREE

WANT TO FIND DISPLACEMENTS u, v

GIVEN P & Q

STATICALLY INDETERMINATE : 3 FORCES, 2 EQS.



$$Q = F_3 + F_1\cos 45° + F_2\cos 60°$$
$$P = F_1\sin 45° - F_2\sin 60°$$

} NOTE P & Q ARE FNS OF F_1, F_2, F_3

- NOTE THIS WILL GIVE SAME SOLUTION FOR F_1 & F_2 IF $Q - F_3$ REPLACES Q

TO FIND F_3 : 1) FIND U_c FIRST $\quad U_c = \sum \frac{F^2L}{2AE} = \frac{F_1^2L_1}{2AE} + \frac{F_2^2L_2}{2AE} + \frac{F_3^2L_3}{2AE}$

SO THAT U_c is minimum when system is in equilibrium

2) TAKE $\frac{\partial U_c}{\partial F_3} = 0$ THIS GIVES 3^{rd} EQ. NEEDED

$$U_c = \frac{1}{2AE}\left\{\left[\frac{\sqrt{3}-1}{\sqrt{2}}\left(P+\sqrt{3}(Q-F_3)\right)\right]^2 L_1^2 + \left[(\sqrt{3}-1)(Q-F_3-P)\right]^2 L_2^2 + F_3^2 L_3^2\right\}$$

$$\frac{\partial U_c}{\partial F_3} = \frac{F_1L_1}{AE}\frac{\partial F_1}{\partial F_3} + \frac{F_2L_2}{AE}\frac{\partial F_2}{\partial F_3} + \frac{F_3L_3}{AE} = \frac{\sqrt{3}-1}{\sqrt{2}}\left[P+\sqrt{3}(Q-F_3)\right]\frac{L_1}{AE}\left[\frac{-\sqrt{3}(\sqrt{3}-1)}{\sqrt{2}}\right] -$$

$$+ (\sqrt{3}-1)(Q-F_3-P)\frac{L_2}{AE}\left[-(\sqrt{3}-1)\right] + \frac{F_3L_3}{AE} = 0$$

3) SOLVE FOR F_3 IN TERMS OF KNOWN FORCES P & Q

$$F_3 = -0.01295\,P + 0.6883\,Q$$

4) PUT THIS INTO $F_1 = \frac{\sqrt{3}-1}{\sqrt{2}}\left(P+\sqrt{3}(Q-F_3)\right)$; $F_2 = (\sqrt{3}-1)(Q-F_3-P)$

TO FIND F_1 & F_2 IN TERMS OF P & Q $\rightarrow F_1 = .6337P + .2794Q$

$\rightarrow F_2 = -.6373P + .2282Q$

5) TAKE $\frac{\partial U_c}{\partial P}$ TO GET u & $\frac{\partial U_c}{\partial Q}$ TO GET v $\rightarrow u = \frac{L}{AE}(1.3969P - .1295Q)$

ASIDE

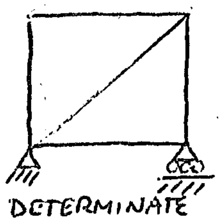
- WHEN IS TRUSS (מסבך) STATICALLY DETERMINATE
  - MUST CHECK BOTH EXTERNAL & INTERNAL CONDITIONS
  - FOR A TRUSS HAVING j JOINTS (מפרקים) & n BARS (מוטות)

EQNS OF EQUILIB.

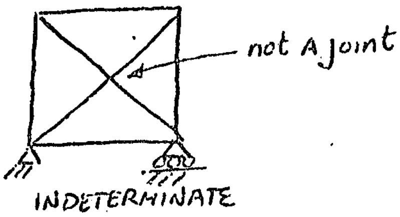
IF $2j-3=n$ STATICALLY DETERMINATE INTERNALLY (הוא מסוים סטטית)

IF $<n$ STATICALLY INDETERMINATE INTERNALLY (פנימי המסבך סטטי בלתי מסויים)

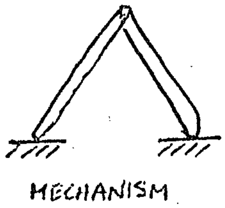
IF $>n$ IT IS A MECHANISM AND IT IS STATICALLY DETERMINATE (המסבך בלתי קשיח ודאי סטטי מסויים)



DETERMINATE



INDETERMINATE



MECHANISM

- FOR A SPACE TRUSS $3j-6=n$ (3-D TRUSS)

TO SATISFY - לספק

- EXTERNAL EQNS OF EQUILIB. $\Sigma F_x=0 \quad \Sigma F_y=0 \quad \Sigma M=0$

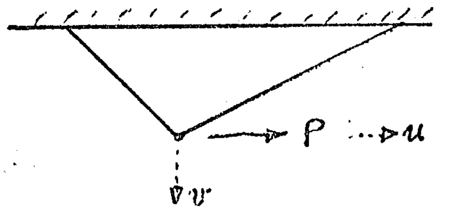
IF NO. OF UNKNOWNS > NO. OF EQNS STATICALLY INDETERMINATE

IF " " " $\leq$ " " " DETERMINATE

- FOR A SPACE TRUSS EQS OF EQUILIB. $\Sigma F_x, \Sigma F_y, \Sigma F_z, \Sigma M_x, \Sigma M_y, \Sigma M_z = 0$

- WE SEE WE CAN FIND DISPLACEMENT IN DIRECTION OF FORCE USING CASTIGLIANO'S THEOREM $\left(\frac{\partial U_c}{\partial P} = u\right)$

- "WHAT IF WE WANT DISPLACEMENT OF A POINT WHERE THERE IS NO FORCE APPLIED?



$$\frac{\partial U_c}{\partial P} = u$$

HOW DO WE GET v?

" " ∂U_c

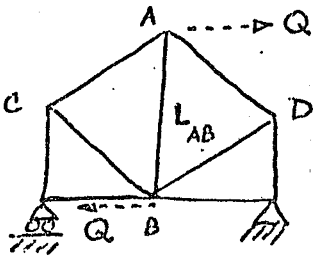
- CAN WE USE CASTIGLIANO'S THEOREM TO DETERMINE ROTATIONS OF A BAR IN A TRUSS ? YES

- HOW ? 1) APPLY A <u>COUPLE</u> WHOSE FORCES ARE ⊥ TO BAR

  2) FIND U_c DUE TO THAT COUPLE, AFTER FINDING FORCES IN BARS OF TRUSS USING EQUILIB.

  3) TAKE $\frac{\partial U_c}{\partial (\text{FORCE OF COUPLE})}$; THEN TAKE limit AS THAT FORCE GOES TO ZERO

  4) TAKE THE RESULT AND DIVIDE BY LENGTH OF BAR. THIS GIVES ROTATION OF BAR <u>IN RADIANS</u>



FIND $U_c = \sum \frac{F_i^2 L_i}{2A_i E_i}$

- HERE THE F_i's WOULD BE FUNCTIONS OF Q

- NOW TAKE $\frac{\partial U_c}{\partial Q}$; TAKE $\lim\limits_{Q \to 0} \frac{\partial U_c}{\partial Q} = L_{AB}\,\theta_{AB}$

- NOW TAKE $\frac{1}{L_{AB}} \cdot (L_{AB}\,\theta_{AB}) = \theta_{AB}$ IN RADIANS

- ALSO NOTE THAT $L_{AB}\,Q$ REPRESENTS THE MOMENT OF THE COUPLE
- WHAT IF <u>AT A</u> A FORCE P EXISTED ALREADY AND YOU WANTED THE ROTATION OF BAR AB ? <u>MUST ADD THE COUPLE IN ADDITION TO THE EXISTING FORCE SYSTEM.</u>

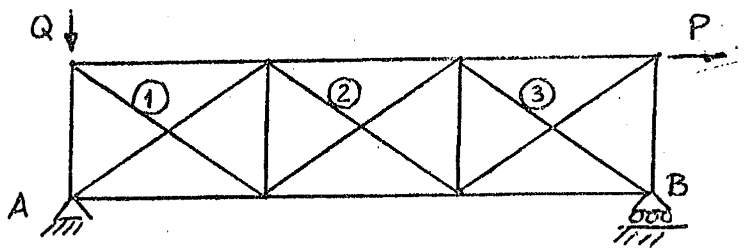
- THE ABOVE SYSTEM IS DETERMINATE. WHAT IF A BAR WERE PLACED ACROSS CD MAKING THE SYSTEM INDETERMINATE. HOW WOULD YOU PROCEED ? 1) WRITE EQUILIBRIUM EQS AND FIND FORCES IN TERMS OF THE INDETERMINATE FORCE F_{CD} (AS WE DID BEFORE)

  2) FIND U_c ; TAKE $\frac{\partial U_c}{\partial F_{CD}} = 0$ TO GET F_{CD}

  3) SUBSTITUTE THE RESULT INTO THE FORCES FOUND FROM EQUILIBRIUM. FIND U_c ; TAKE $\partial U_c \big| = L\,\theta$ ETC

- THE DEGREE OF INDETERMINACY IS THE DIFFERENCE BETWEEN n AND $2j-3$ FOR A PLANE TRUSS OR n AND $3j-6$ FOR A SPACE TRUSS THIS IS THE NUMBER OF EQUATIONS NEEDED VIA CASTIGLIANO'S THEOREM

EXAMPLE



8 JOINTS

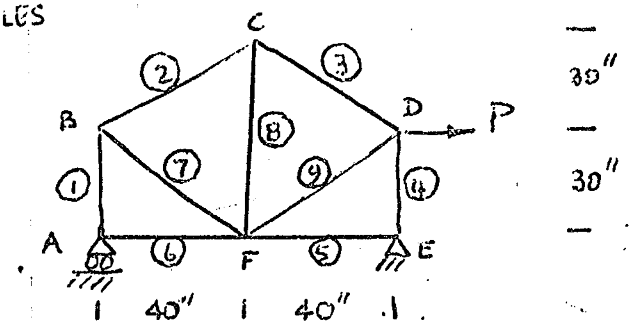
$2j-3=13$

$n=16$

$n-(2j-3)=3$

- THIS IS DEGREE OF INDETERMINACY 3
- THUS AFTER YOU FIND U_c THEN $\frac{\partial U_c}{\partial F_1}=0$; $\frac{\partial U_c}{\partial F_2}=0$; $\frac{\partial U_c}{\partial F_3}=0$
- THESE GIVE THE REQUIRED EQ'S TO SOLVE FOR F_1, F_2, F_3
- TO FIND THE REACTION FORCES AT A & B: USE TRUSS METHODS LEARNED IN STATICS OR USE CASTIGLIANO'S THEOREM WITH $u_A = v_A = 0$ AND $v_B = 0$

EXAMPLES



$P = 4000$ lb

$A = .1$ in^2

$E = 30 \times 10^6$ psi

CF = 60″ BC, BF, FD, CD = 50″

BA = DE = 30″ = L

THIS IS STATICALLY DETERMINATE: EXTERNAL/INTERNAL

USE EXTERNAL EQUILIBRIUM TO FIND AT A $V_A = \frac{3P}{8}\downarrow$

E $H_E = P \leftarrow$ $V_E = \frac{3P}{8}\uparrow$

USE JOINT METHOD OF STATICS TO FIND

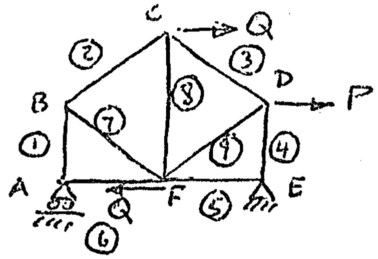
$F_1 = -F_4 = \frac{3P}{8} = -F_8$; $F_2 = F_3 = -F_7 = \frac{5P}{16}$; $F_6 = 0$; $F_5 = -P$; $F_9 = \frac{15P}{16}$

$$U_c = \sum \frac{F_i^2 L_i}{2A_iE_i} = \frac{L}{2AE}\left\{\left(\frac{3P}{8}\right)^2\cdot 1 + \left(\frac{5P}{16}\right)^2\cdot \frac{5}{3} + \left(\frac{5P}{16}\right)^2 \frac{5}{3} + \left(-\frac{3P}{8}\right)^2\cdot 1 + (-P)^2\cdot\frac{4}{3}\right.$$

$$\left. + (0)^2\cdot \frac{4}{3} + \left(-\frac{5P}{16}\right)^2\cdot\frac{5}{3} + \left(-\frac{3P}{8}\right)^2\cdot 2 + \left(\frac{15P}{16}\right)^2\cdot\frac{5}{3}\right\} = \frac{739}{384}\frac{P^2L}{AE}$$

$$u_d = \frac{\partial U_c}{\partial P} = \frac{739}{192}\frac{PL}{AE} = \frac{733\,(4000)(30'')}{192 \quad .1 \quad (30\times 10^6)} = .154\text{ in}$$ DISPL OF D DUE TO P

TO FIND ROTATION OF CF, ASSUME LOADS Q AT C & F



USE EQUILIB. EQNS TO FIND

$V_E = \frac{3P}{8} + \frac{3Q}{4}$ ↑ $\quad V_A = \frac{3P}{8} + \frac{3Q}{4}$ ↓

$H_E = P$ ←

USE JOINT EQUIL. TO FIND : $F_1 = -F_4 = \frac{3P}{8} + \frac{3Q}{4}$; $F_2 = -F_7 = \frac{5P}{16} + \frac{5Q}{8}$; $F_3 = \frac{5P}{16} - \frac{5Q}{8}$

$F_5 = -P + 0$; $F_6 = 0 + 0$; $F_8 = -\frac{3P}{8} + 0$; $F_9 = \frac{15P}{16} + \frac{5Q}{8}$

$$U_c = \sum \frac{F_i^2 L_i}{2A_iE_i} = \frac{L}{2AE}\left\{\left(\frac{3P}{8}+\frac{3Q}{4}\right)^2\cdot 1 + \left(\frac{5P}{16}+\frac{5Q}{8}\right)^2\cdot\frac{5}{3} + \left(\frac{5P}{16}-\frac{5Q}{8}\right)^2\cdot\frac{5}{3} + \left(-\frac{3P}{8}-\frac{3Q}{4}\right)^2\cdot 1\right.$$

$$+ (-P+0)^2\cdot\frac{4}{3} + (0+0)^2\cdot\frac{4}{3} + \left(-\frac{5P}{16}-\frac{5Q}{8}\right)^2\cdot\frac{5}{3} + \left(-\frac{3P}{8}+0\right)^2\cdot 2$$

$$\left. + \left(\frac{15P}{16}+\frac{5Q}{8}\right)^2\cdot\frac{5}{3}\right\}$$

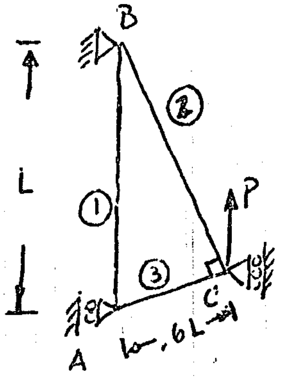
$$\left.\frac{\partial U_c}{\partial Q}\right|_{Q=0} = \frac{L}{AE}\left\{\left(\frac{3P}{8}\right)\cdot 1\cdot\frac{3}{4} + \left(\frac{5P}{16}\right)\cdot\frac{5}{3}\cdot\frac{5}{8} + \left(\frac{5P}{16}\right)\cdot\frac{5}{3}\cdot -\frac{5}{8} + \left(-\frac{3P}{8}\right)\cdot 1\cdot -\frac{3}{4} + 0 + 0 +\right.$$

$$\left.\left(-\frac{5P}{16}\right)\cdot\frac{5}{3}\cdot -\frac{5}{8} + 0 + \left(\frac{15P}{16}\right)\cdot\frac{5}{3}\cdot\frac{5}{8}\right\} = \frac{179}{96}\frac{PL}{AE}$$

Now

$$\theta_{CF} = \frac{1}{L_{CF}}\left.\frac{\partial U_c}{\partial Q}\right|_{Q=0} = .00124\text{ radians} \quad \text{or } .0712^\circ$$

- REMEMBER: TRUSSES ASSUME LOAD AT JOINTS, WEIGHTLESS AND ONLY EXTEND BUT DO <u>NOT</u> BEND
- WHAT ABOUT IF WE WANT TO FIND LOADS GIVEN THE DISPLACEMENTS



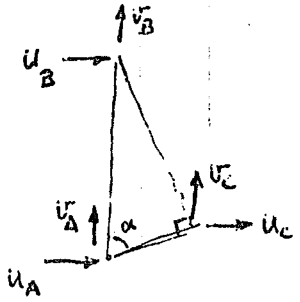
$P = 2500$ LB $A = .1\,in^2$ $E = 30 \times 10^6 psi$

$L_1 = L$, $L_2 = .8L$, $L_3 = .6L$

DFN: ELONGATION - CHANGE IN LENGTH OF BAR ALONG ITS LINE OF ACTION DUE TO LOAD IN BAR

EXAMPLE : $e_1 = \epsilon_1 L_1 = \frac{\sigma_1}{E_1} L_1 = \frac{F_1 L_1}{A_1 E_1}$

- CONSIDER ELONGATION POSITIVE IN DIRECTION OF POSITIVE (TENSILE FORCE)



$u_B, v_B, u_C, u_A = 0$ BY BOUNDARY CONDITIONS

$e_1 = -v_A$

$e_2 = -v_C \sin\alpha = -.8v_C$

$e_3 = v_C \cos\alpha - v_A \cos\alpha = (v_C - v_A).6$

MUST WRITE ELONGATIONS INTERMS OF DISPLACEMENTS

- HERE WE ASSUME POSITIVE DISPLACEMENT IN POSITIVE X & Y DIRECTION

$$U_s = \sum \frac{A_i E_i}{2} \frac{e_i^2}{L_i} = \sum \frac{AE}{2} \left\{ \frac{(-v_A)^2}{L} + \frac{(-.8v_C)^2}{.8L} + \frac{(.6[v_C - v_A])^2}{.6L} \right\}$$

- BUT AT A THERE IS NO VERTICAL LOAD ∴ $\frac{\partial U_s}{\partial v_A} = 0 = \frac{AE}{L}\left[(-v_A)(-1) + .6[v_C - v_A](-1)\right]$

$$\boxed{0 = \frac{AE}{L}(1.6v_A - .6v_C)}$$

BUT AT C $\frac{\partial U_s}{\partial v_C} = P \longrightarrow P = AE\left[\frac{(-.8v_C)(-.8)}{.8L} + \frac{.6(v_C - v_A)(.6)}{.6L}\right]$

$$\boxed{P = \frac{AE}{L}[1.4v_C - .6v_A]}$$

SOLUTION OF THESE GIVE

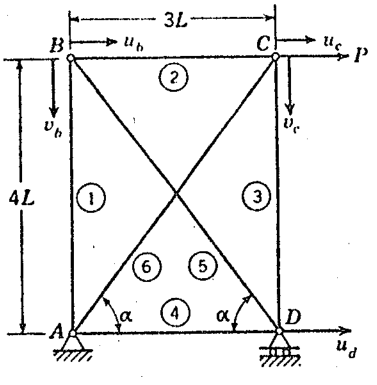
$v_A = \frac{15PL}{47AE}$ $v_C = \frac{40}{47}\frac{PL}{AE}$

NOW: $e_1 = -v_A = -\frac{15PL}{47AE}$ $e_2 = -.8v_C = \frac{-32PL}{47AE}$ $e_3 = .6(v_C - v_A) = \frac{15PL}{47AE}$

THUS $F_1 = \frac{AEe_1}{L_1} = \frac{-15P}{47}$; $F_2 = \frac{AEe_2}{L_2} = \frac{-32PL}{47(.8L)} = -\frac{40P}{47}$; $F_3 = \frac{AEe_3}{L_3} = \frac{15PL}{47(.6L)} = \frac{25P}{47}$

Example 15.4 For the six-bar truss supported and loaded in its own plane as shown in Fig. 15.6a, determine the forces in the bars and the displacement components of the joints. All the bars of the truss have the same cross-sectional area A and the same elastic modulus E.

STATICALLY
INDETERMINATE
TRUSS $2j-3<n$



(a)

$$\frac{L_iF_i}{A_iE_i}=e_i$$

$$\frac{4LF_1}{AE}=e_1=-v_b \qquad \frac{3LF_4}{AE}=e_4=u_d$$

$$\frac{3LF_2}{AE}=e_2=u_c-u_b \qquad \frac{5LF_5}{AE}=e_5=(u_d-u_b)\cos\alpha-v_b\sin\alpha$$

$$\frac{4LF_3}{AE}=e_3=-v_c \qquad \frac{5LF_6}{AE}=e_6=u_c\cos\alpha-v_c\sin\alpha$$

NOTE: $u_A, v_A, v_d = 0$ BOUNDARY CONDITIONS

$$U_s=\sum\frac{A_iE_ie_i^2}{2L_i} \; ; \quad \frac{\partial U_s}{\partial u_c}=P \; ; \quad \frac{\partial U_s}{\partial u_b}=\frac{\partial U_s}{\partial v_b}=\frac{\partial U_s}{\partial u_d}=0=\frac{\partial U_s}{\partial v_c}$$

$$U_s=\frac{AE}{2}\left\{\frac{v_b^2}{4L}+\frac{(u_c-u_b)^2}{3L}+\frac{v_c^2}{4L}+\frac{(u_d)^2}{3L}+\frac{[(u_d-u_b)\cos\alpha-v_b\sin\alpha]^2}{5L}\right.$$
$$\left.+\frac{[u_c\cos\alpha-v_c\sin\alpha]^2}{5L}\right\}$$

$$\frac{\partial U_s}{\partial u_c}=P=\frac{AE}{L}\left\{\frac{(u_c-u_b)}{3}\cdot 1+\frac{(u_c\cos\alpha-v_c\sin\alpha)}{5}\cos\alpha\right\}=\frac{-AE}{375L}(125u_b-152u_c+v_c\cdot 36)$$

$$\frac{\partial U_s}{\partial u_d}=0=\frac{AE}{L}\left\{\frac{u_d}{3}\cdot 1+\frac{[(u_d-u_b)\cos\alpha-v_b\sin\alpha]}{5}\cos\alpha\right\}=\frac{-AE}{375L}(27u_b+36v_b-152u_d)$$

$$\frac{\partial U_s}{\partial u_b}=0=\frac{AE}{L}\left\{\frac{(u_c-u_b)}{3}(-1)+\frac{[(u_d-u_b)\cos\alpha-v_b\sin\alpha]}{5}(-\cos\alpha)\right\}=\frac{-AE}{375L}(27u_d+125u_c-36v_b-152u)$$

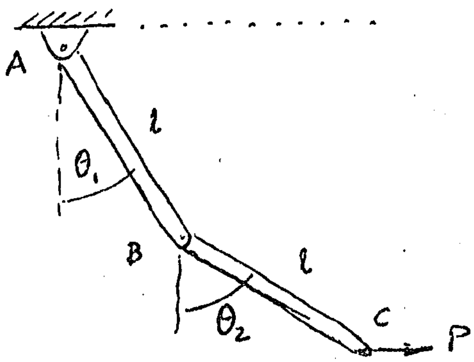
$$\frac{\partial U_s}{\partial v_b}=0=\frac{AE}{L}\left\{\frac{v_b}{4}\cdot 1+\frac{[(u_d-u_b)\cos\alpha-v_b\sin\alpha]}{5}(-\sin\alpha)\right\}=\frac{-AE}{375L}\left(-36u_b+\frac{567}{4}v_b+36u_d\right)$$

$$\frac{\partial U_s}{\partial v_c}=0=\frac{AE}{L}\left\{\frac{v_c}{4}\cdot 1+\frac{[u_c\cos\alpha-v_c\sin\alpha]}{5}(-\sin\alpha)\right\}=\frac{-AE}{375L}\left(\frac{567}{4}v_c-36u_c\right)$$

FROM THESE WE GET $u_b=\frac{21PL}{2AE}$ $u_c=\frac{189}{16}\frac{PL}{AE}$ $u_d=\frac{21PL}{16AE}$ $v_b=-\frac{7PL}{3AE}$

$$v_c=+\frac{3PL}{AE} \; ;$$

AND $F_1=\frac{7P}{12}$, $F_2=\frac{7P}{16}$, $F_3=-\frac{3P}{4}$, $F_4=\frac{7P}{16}$, $F_5=\frac{15P}{16}$; $F_6=-\frac{35P}{48}$



- THIS IS A 2 DEGREE OF FREEDOM SYSTEM SINCE EACH BAR CAN MOVE INDEPENDENTLY OF THE OTHER
- DEGREES OF FREEDOM

BAR #1 $= \cdot x_A, y_A, x_B, y_B$ $\qquad x_A=0, y_A=0$

$l = \sqrt{(x_B-x_A)^2+(y_B-y_A)^2}$

- #UNKNOWNS − # EQNS ⟶ 1 DEGREE OF FREEDOM (1 DOF.)

BAR #2 $= x_B, y_B, x_c, y_c$ $\qquad x_B, y_B$ KNOWN

$l = \sqrt{(x_c-x_B)^2+(y_c-y_B)^2}$

- # UNKNOWNS − # EQNS ⟶ 1 DEGREE OF FREEDOM (1D.O.F.)
- FOR SYSTEM 2 D.O.F. CHOOSE θ_1, θ_2
- TO FIND W_e : WORK DUE TO WEIGHTS & LOAD P

W_i : ZERO SINCE RIGID BARS

AT A : REACTIONS FORCES DO <u>NO</u> WORK, WHY? AT B : <u>NO</u> WORK, W.

- CONSIDER WORK DONE WITH RESPECT TO POSITION $\theta_1=\theta_2=0$

$$\Pi = W_e = -W\left[\tfrac{l}{2}(1-\cos\theta_1)\right] - W\left[l(1-\cos\theta_1)+\tfrac{l}{2}(1-\cos\theta_2)\right] + P\left[l\sin\theta_1 + l\sin\theta_2\right]$$

$$\delta\Pi = \delta W_e = 0 = -\frac{Wl}{2}\sin\theta_1\,\delta\theta_1 - Wl\sin\theta_1\delta\theta_1 - \frac{Wl}{2}\sin\theta_2\,\delta\theta_2 + Pl\left[\cos\theta_1\,\delta\theta_1 + \cos\theta_2\,\delta\theta_2\right]$$

$$0 = \left[-\frac{3Wl}{2}\sin\theta_1 + Pl\cos\theta_1\right]\delta\theta_1 + \left[-\frac{Wl}{2}\sin\theta_2 + Pl\cos\theta_2\right]\delta\theta_2$$

$$\Rightarrow \quad \tan\theta_1 = \frac{2P}{3W} \qquad \tan\theta_2 = \frac{2P}{W}$$

ALSO $\quad P[x_1+(x_2-x_1)] - Wy_1/_2 - W(y_1+[y_2-y_1]/2) = \Pi$

$$P[\delta x_1 + \delta(x_2-x_1)] - \frac{W}{2}\delta y_1 + W\left(\delta y_1 + \frac{1}{2}\delta(y_2-y_1)\right) = \delta\Pi = 0$$

with $x_1^2+y_1^2=l^2$ and $(x_2-x_1)^2+(y_2-y_1)^2=l^2 \Rightarrow x_1\delta x_1 + y_1\delta y_1 = 0$

AND $(x_2-x_1)\delta(x_2-x_1)+(y_2-y_1)\delta(y_2-y_1)$

now $\delta x_1 = -\frac{y_1}{x_1}\delta y_1 = \cot\theta_1\delta y_1 \qquad \delta(x_2-x_1) = -\frac{(y_2-y_1)}{x_2-x_1}\delta(y_2-y_1) = \cot\theta_2\,\delta(y_2-y_1)$

THUS

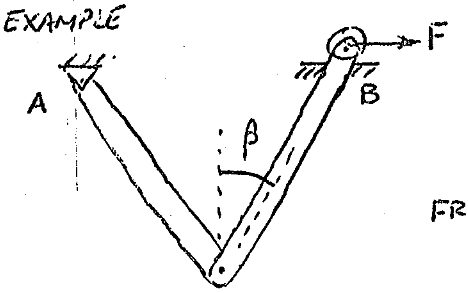
$$P(\cot\theta_1\delta y_1 + \cot\theta_2\,\delta(y_2-y_1)) - \frac{3W}{2}\delta y_1 - \frac{W}{2}\delta(y_2-y_1) = 0 \Rightarrow \tan\theta_1 = \frac{2P}{3W},\ \tan\theta_2 = \frac{2P}{W}$$

- WHAT IF A STRUCTURE IS <u>RIGID</u>? — <u>STRAIN ENERGY</u> & $\underline{W_i = 0}$

$\Rightarrow \quad \Pi = W_e$ (work DONE BY BODY FORCES, EXTERNAL FORCES ONLY

$\Rightarrow \quad \delta\Pi = \delta W_e = 0$ PRINCIPLE OF VIRTUAL WORK

EXAMPLE



TWO-RIGID BARS OF WEIGHT W AND LENGTH ℓ

WHAT IS β FOR EQUILIB. IF F IS APPLIED

FROM EQUILIB - REACTIONS AT A & B DO NO WORK

AT A : FORCES AT FIXED PT

AT B : NORMAL FORCE IS ⊥ TO DIRECTION OF MOTION

WORK DONE: IF $\beta = 0$ AND WE CONSIDER WORK DONE WITH RESPECT TO THIS POSITION

$$F \cdot 2\ell \sin\beta - 2W\left(\frac{\ell}{2}\left[1-\cos\beta\right]\right) = \Pi = W_e$$

$$2\ell F \cos\beta\, \delta\beta - W\ell \sin\beta\, \delta\beta = \delta W_e = \delta\Pi = 0$$

$$\Rightarrow \tan\beta = \frac{2F}{W}$$

ANOTHER WAY $\quad F(2x) - 2W(y/2) = \Pi \quad$ WITH $\quad x^2 + y^2 = L^2$

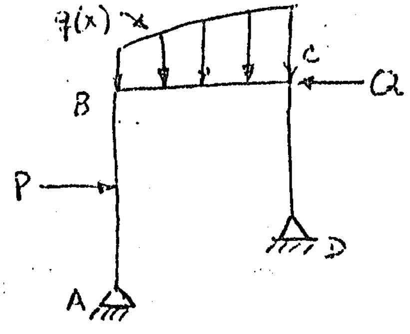
$$F(2\delta x) - W\delta y = \delta\Pi = 0 \quad \text{AND} \quad x\delta x + y\delta y = 0 \Rightarrow \delta y = \frac{-x}{y}\delta x$$

now $\frac{x}{-y} = \tan\beta \quad \Rightarrow \quad 2F/W = \frac{\delta y}{\delta x} = -\frac{x}{y} = \tan\beta$

<u>GO TO NEXT PAGE</u>

- WHAT IF STRUCTURE BENDS? BEAM UNDER END LOADING
  FRAME UNDER TRANSVERSE LOADING

∴ HERE <u>BENDING IS PRIMARY MODE</u> OF LOADING.



ACTUALLY <u>TOTAL</u> SOLUTION WILL INVOLVE EXTENSIONAL, BENDING AND SHEARING EFFECTS

REMEMBER $\quad U_{s_e} = \int_V \frac{E\epsilon^2}{2} dV \qquad U_{c_e} = \int \frac{\sigma^2}{2E} dV$

DUE TO TENSION

- SIMILARLY $U_{s_S} = \int_V \frac{G\gamma^2}{2} dV$ and - $U_{c_s} = \int_V \frac{\tau^2}{2G} dV$ DUE TO SHEAR

- FOR BENDING ENERGY $\sigma = \frac{My}{I}$ AND $\epsilon = Ky = -\frac{d^2v}{dx^2} y$ K is CURVATURE

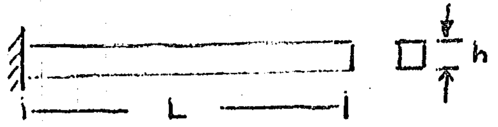
FOR A BEAM

$$U_{s_b} = \int_V \frac{E}{2} \epsilon^2 dV = \int_V \frac{E}{2} \left(-\frac{d^2v}{dx^2} y\right)^2 dV = \int_0^L \frac{E}{2} (v'')^2 \left(\int_A y^2 dA\right) dx$$

$$= \int_0^L \frac{EI}{2} (v'')^2 dx$$

$$U_{c_b} = \int_V \frac{\sigma^2}{2E} dV = \int_V \frac{M^2 y^2}{2EI^2} dV = \int_0^L \frac{M^2}{2EI^2} \left(\int_A y^2 dA\right) dx = \int_0^L \frac{M^2}{2EI} dx$$

- LOOK AT SHEAR EFFECTS - IF BEAM IS <u>NOT LONG</u> (SHORT BEAM), MUST ACCOUNT FOR THEM



IF $h/L \sim 1$ MUST ACCOUNT FOR SHEAR

OTHER WISE - NO.

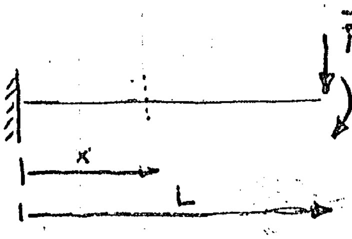
- IF NECESSARY $\int_V \frac{\tau^2}{2G} dV = \int_0^L \int_A \frac{\tau^2 dA}{2G} dx = \int_0^L k \frac{V^2}{2GA} dx$

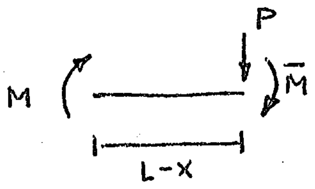
k IS REPORTED IN TABLE 7.2.1 PG 231; V IS SHEAR FORCE FOUND FROM

$$\frac{dM}{dx} = V$$

A IS CROSS-SECTIONAL AREA

- LOOK AT SEVERAL CASES





$$M + \bar{P}(L-X) + \bar{M} = 0$$

$$M = -\left[\bar{M} + \bar{P}(L-X)\right]$$

AT POSITION X

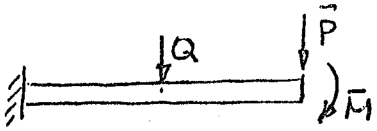
$$\text{BY} \quad U_{c_b} = \int_0^L \frac{M^2}{2EI}\,dx = \int_0^L \frac{(-[\bar{M} + \bar{P}(l-x)])^2}{2EI}\,dx$$

$$\frac{\partial U_{c_b}}{\partial \bar{P}} = \text{DISPL (VERTICAL) WHERE } \bar{P} \text{ IS APPLIED} = \int_0^L \frac{1}{EI}\Big(-[\bar{M}+\bar{P}(L-x)]\Big)\underset{\uparrow \frac{\partial M}{\partial \bar{P}}}{(-(l-x))}\,dx$$

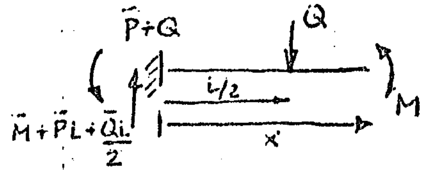
$$\frac{\partial U_{c_b}}{\partial \bar{M}} = \text{ROTATION WHERE } \bar{M} \text{ IS APPLIED} = \int_0^L \frac{1}{EI}\Big(-[\bar{M}+\bar{P}(L-x)]\Big)\underset{\uparrow \frac{\partial M}{\partial \bar{M}}}{(-1)}\,dx$$

- THIS IS A STATICALLY DETERMINATE PROBLEM. NOTE THAT LOAD & MOMENT GIVE THE TWO EQUATIONS OF EQUILIBRIUM

- WHAT IF WE WANTED TO FIND DISPLACEMENT AT $x = L/2$?
- PUT LOAD Q THERE, FIND U_{c_b} ; THEN TAKE $\left.\frac{\partial U_{c_b}}{\partial Q}\right|_{Q=0}$

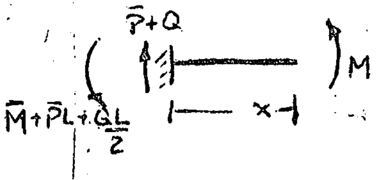


DEFINE M $\quad 0 \le x \le L/2 \quad$ & $\quad L/2 \le x \le L$



$$+M + (Q)(x - L/2) - (\bar{P}+Q)x + (\bar{M} + \bar{P}L + \frac{QL}{2}) = M - \bar{P}(x-L) + \bar{M} = 0$$

$$M = +\bar{P}(x-L) - \bar{M} \qquad \text{FOR} \quad L/2 \le x \le L$$



$$+M - (\bar{P}+Q)x + (\bar{M} + \bar{P}L + \frac{QL}{2}) = M + \bar{P}(L-x) + Q(L/2 - x) + \bar{M} = 0$$

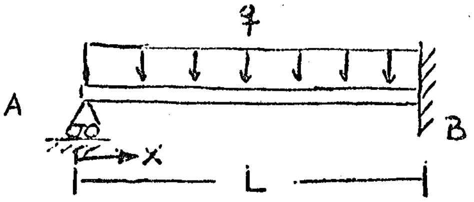
$$M = -[\bar{M} + \bar{P}(L-x) + Q(L/2 - x)] \qquad \text{FOR} \quad 0 \le x \le L$$

$$U_{c_b} = \int_0^L \frac{M^2}{2EI}\,dx = \int_0^{L/2} \frac{(-[\bar{M} + \bar{P}(L-x) + Q(L/2 - x)])^2}{2EI}\,dx + \int_{L/2}^{L} \frac{(-[\bar{M}+\bar{P}(L-x)])^2}{2EI}\,dx$$

$$\frac{\partial U_{c_b}}{\partial Q} = \int_0^{L/2} \Big(\frac{-[\bar{M} + \bar{P}(L-x) + Q(L/2 - x)]}{EI}\Big)\underset{\nwarrow \partial M/\partial Q}{(-[L/2 - x])}\,dx$$

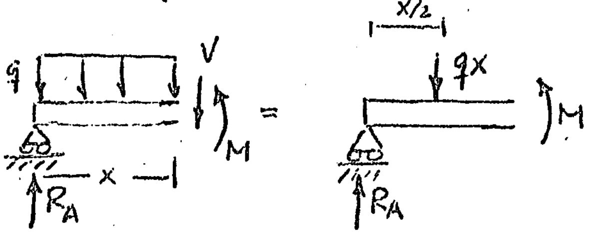
$$v\Big|_{x=L/2} = \left.\frac{\partial U_{c_b}}{\partial Q}\right|_{Q=0} = \int_0^{L/2} \Big(\frac{-[\bar{M} + \bar{P}(L-x)]}{EI}\Big)(-[L/2 - x])\,dx$$

STATICALLY INDETERMINATE BEAMS



AT A REACTION UPWARD
B MOMENT + REACTION } 3 UNKNOWNS
2 EQNS

LOOK AT BEAM AT DISTANCE x FROM LEFT END



$$M + q\frac{x^2}{2} - R_A x = 0$$

$$M = R_A x - q\frac{x^2}{2} \qquad 0 \le x \le L$$

$$U_{c_b} = \int_0^L \frac{M^2}{2EI}\,dx = \int_0^L \frac{\left(R_A x - q\frac{x^2}{2}\right)^2}{2EI}\,dx$$

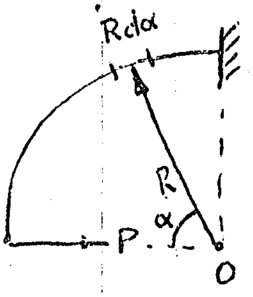
- WHAT IS $\frac{\partial U_{c_b}}{\partial R_A} = U_A^* = 0 = \int_0^L \frac{\left(R_A x - q\frac{x^2}{2}\right)}{EI}\, \underbrace{x}_{\frac{\partial M}{\partial R_A}}\, dx \Longrightarrow R_A = \frac{3qL}{8}$

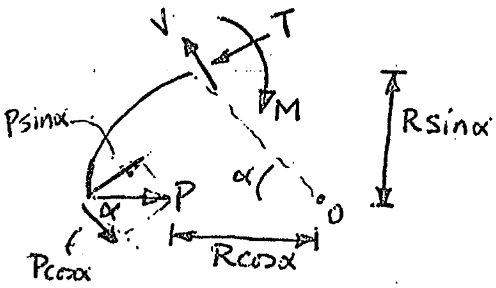
- WHAT IF WE WANTED TO FIND THE ROTATION OF THE BEAM AT $x = L/4$?

  SUPERPOSE A MOMENT M_1 AT $x = L/4$, FIND U_{c_b}, TAKE $\left.\frac{\partial U_{c_b}}{\partial M_1}\right|_{M_1=0} = \theta\big|_{L/4}$

- WHAT IF THERE ARE TWO LOADS CALLED P ? WHAT DOES $\frac{\partial U_{c_b}}{\partial P}$ MEAN ?
- HOW WOULD I FIND THE DISPLACEMENT UNDER ONE OF THESE LOADS ?

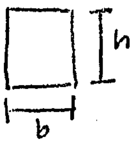
LET'S LOOK AT A CIRCULAR CANTILEVERED BEAM UNDER HORIZONTAL LOAD





T = Psinα
V = Pcosα
M = PRsinα

FROM EQUILIBRIUM -

IF THE CROSS-SECTION IS RECTANGULAR 

$A = bh \quad I = \frac{bh^3}{12}$

$$\sigma = T/A = \frac{P\sin\alpha}{A} \qquad \tau = \frac{3V(h^2-4y^2)}{2bh^3} = \frac{3P\cos\alpha\,(h^2-4y^2)}{2bh^3}$$

$$U_{c_t} = U_{c_e} + U_{c_s} + U_{c_b} \qquad U_{c_e} = \int_V \frac{\sigma^2}{2E}\,dV = \int_0^{\pi/2} \frac{P^2\sin^2\alpha}{2A^2E}\,A\cdot R\,d\alpha$$

$$= \int_0^{\pi/2} \frac{P^2R}{2AE}\sin^2\alpha\,d\alpha = \frac{\pi P^2R}{8AE}$$

$$U_{c_s} = \int_V \frac{\tau^2}{2G}\,dV = \int_0^{\pi/2} \frac{R\,d\alpha}{2G}\int_{-h/2}^{h/2} \frac{9V^2}{(2bh^3)^2}\cdot(h^2-4y^2)^2dy$$

$$= \frac{3\pi P^2R}{20GA}$$

$$U_{c_b} = \int_V \frac{\sigma^2}{2E}\,dV = \int_0^{\pi/2} \frac{M^2}{2EI}\,R\,d\alpha = \int_0^{\pi/2} \frac{P^2R^3}{2EI}\sin^2\alpha\,d\alpha = \frac{\pi P^2R^3}{8EI}$$

$$U_{c_t} = \frac{\pi P^2R^3}{8EI}\left[1 + \frac{1}{12}\left(h/R\right)^2 + \frac{E}{10G}\left(\frac{h}{R}\right)^2\right]$$

BENDING EXTENSION SHEAR

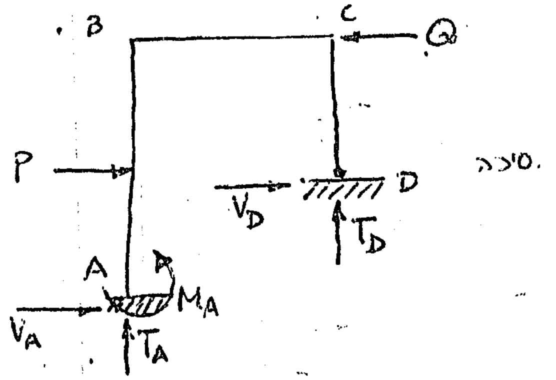
AND $\frac{\partial U_{c_t}}{\partial P}$ = DISPL IN DIRECTION OF $P = \frac{\pi PR^3}{4EI}\left[1 + \frac{1}{12}\left(\frac{h}{R}\right)^2 + \frac{E}{10G}\left(\frac{h}{R}\right)^2\right]$

IF $\frac{E}{G} = 2(1+\nu)$ AND $\nu = .3$ $\quad \frac{E}{10G} = .26$

IF $h/R = \frac{1}{10}$ $\quad \frac{1}{12}\left(\frac{h}{R}\right)^2 = .00083$ $\quad \frac{E}{10G} = .0026$

NOTE EXTENSION & SHEAR CONTRIB. MUCH SMALLER

WHAT ABOUT A FRAME?



1) CONSIDER FRAME AS FREE BODY
2) THE REACTION COMPONENTS ARE V_A, T_A, M_A AT A AND V_D, T_D (PINNED END) AT D
3) SINCE 3 EQNS OF EQUILIB ONLY
$\Rightarrow$ 2 OF THESE UNKNOWNS CAN BE WRITTEN IN TERMS OF OTHER 3
e.g. $V_A = V_A(V_D, T_A, T_D)$ $\quad M_A = M_A(V_D, T_A, T_D)$

4) NOW FIND M IN TERMS OF THESE QUANTITIES OVER EACH RANGE

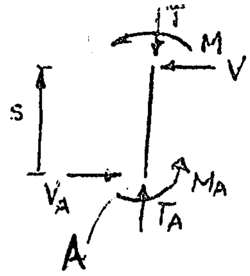
FROM A TO P   P TO B   B TO C   C TO D

5) USE $$U_{c_b} = \Sigma \int \frac{M^2}{2EI}ds = \int_A^P \frac{M^2}{2EI}ds + \int_P^B \frac{M^2 ds}{2EI} + \int_B^C \frac{M^2 ds}{2EI} + \int_C^D \frac{M^2 ds}{2EI}$$

6) THEN TAKE $\frac{\partial U_{c_b}}{\partial T_D} = 0$ $\frac{\partial U_{c_b}}{\partial T_A} = 0$ SINCE THESE ARE THE INDETERMINATE QUANTITIES

7) THESE TWO EQUATIONS AND THE EQUILIBRIUM EQUATIONS GIVE 5 EQNS & 5 UNKN

EXAMPLE : BETWEEN A - P



$\Sigma F_H = 0 \Rightarrow V = V_A$

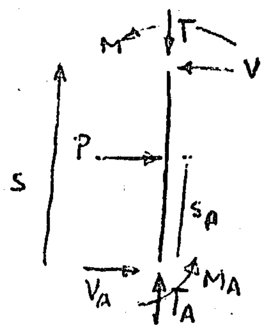
$\Sigma F_V = 0 \Rightarrow T = T_A$

$\Sigma M_a = 0 = M + Vs + M_A = 0$

$M = -(V_A s + M_A)$

$$\therefore \int_A^P \frac{M^2 ds}{2EI} = \int_0^{S_P} \frac{[-(V_A s + M_A)]^2 ds}{2EI}$$

BETWEEN P - B



$V = P + V_A$

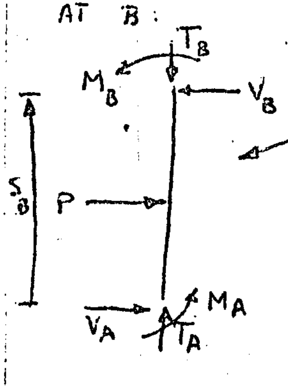
$T = T_A$

$M = -P(s - s_p) - V_A s - M_A$

$$\int_P^B \frac{M^2 ds}{2EI} = \int_{S_p}^{S} \frac{[-(P\{s - s_p\} + V_A s + M_A)]^2 ds}{2EI}$$

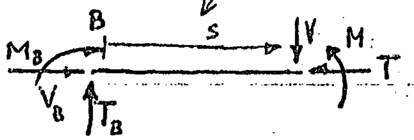
BETWEEN B - C

AT B :



$$\begin{cases} V_B = P + V_A \\ T_B = T_A \\ M_B = -P(s_B - s_p) - V_A s_B - M_A \end{cases}$$

SECTION B - C



$T = V_B = P + V_A$

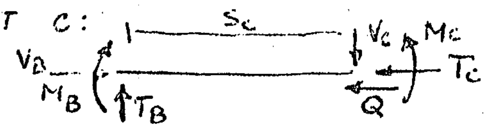
$V = T_B = T_A$

$M = T_B s + M_B = T_A s - [P(s_B - s_P) + V_A s_B + M_A]$

$$\int_B^C \frac{M^2 ds}{2EI} = \int_0^S \frac{(T_A s + M_B)^2}{2EI} ds$$

BETWEEN C – D

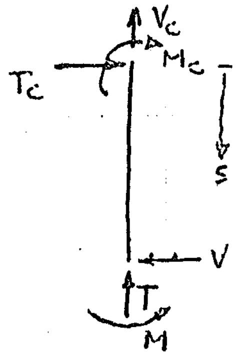
AT C:



$T_c = V_B - Q = P + V_A - Q$

$V_c = T_B = T_A$

$$\underbrace{M_c = T_B s_c + M_B = T_A s_c - \left[P(s_B - s_P) + V_A s_B + M_A\right]}$$



$T = -V_c = -T_A$

$V = T_c = P + V_A - Q$

$M = M_c + T_c s = M_c - T_A s$

$$\int_c^D \frac{M^2 ds}{2EI} = \int_0^{S_D} \frac{(M_c - T_A s)^2 ds}{2EI}$$

NOW $U_{c_b} = \sum \int \frac{M^2}{2EI} ds$ AND TAKE $\underbrace{\frac{\partial U_{c_b}}{\partial M_A} = 0 \quad \& \quad \frac{\partial U_{c_b}}{\partial V_A} = 0}$

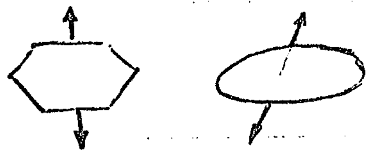
FROM EQUILIB.

$$\left.\begin{aligned} &V_A + P + Q + V_D = 0 \\ &T_A + T_D = 0 \\ &M_A - P s_P + Q s_B - V_D (s_B - s_D) + T_D S_c = 0 \end{aligned}\right\}$$

GIVE 5 EQNS

5 UNKNOWNS V_A, V_D, T_A, T_D
M_A

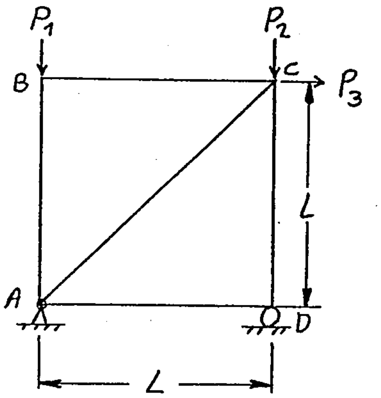
- WHAT ABOUT CLOSED PLANE FRAMES & RINGS SUBJECTED TO LOADS THAT ARE SELF EQUILIBRATING ?



- YOU DO SAME THING AS WITH A FRAME. THERE WILL BE 3 STATICALLY INDETERMINATE REACTIONS – THEY ARE INTERNAL REACTIONS M, T, V. THESE REACTIONS MUST MINIMIZE $\Pi = W_i = -U_{c_b} = -U_{c_s}$
- FORM $U_{c_b} = \sum \int \frac{M^2}{2EI} ds$ AROUND THE CLOSED PERIMETER
- THEN TAKE $\frac{\partial U_{c_b}}{\partial M} = 0 \quad \frac{\partial U_{c_b}}{\partial V} = 0 \quad \frac{\partial U_{c_b}}{\partial T} = 0$

תורת החוזק 2
תרגיל מס' 5

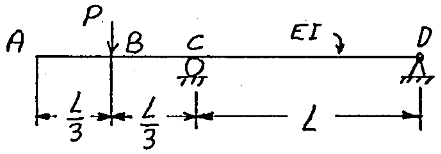
1. For the pin-connected truss loaded as shown (where AE is the same for all members) determine, by means of Castigliano's second theorem :



(a) the vertical displacement of point B,
(b) the horizontal displacement of point C,
(c) the vertical displacment of point C.

Answers : (a) $\frac{P_1L}{AE}$, (b) $[P_2 + (1+2\sqrt{2})P_3]\ \frac{L}{AE}$, (c) $(P_2+P_3)\ \frac{L}{AE}$

2. By means of Castigliano's Second Theorem, determine (in terms of E, I, L and P), for the linear elastic beam shown :

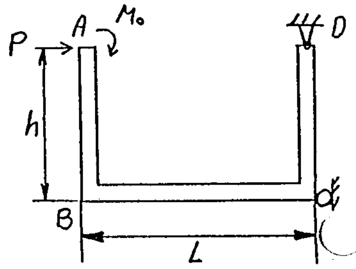


(a) the vertical displacement of point B.
(b) the vertical displacement of point A.
(c) the rotation of point A.

Answers : (a) $\Delta_B = \frac{4PL^3}{81EI} \downarrow$, (b) $\Delta_A = \frac{17PL^3}{162EI} \downarrow$, (c) $\theta_A = \frac{PL^2}{6EI}$ $\curvearrowright$

3. An elastic structural member ABCD, pinned at C and D, is subjected to an applied moment M_0 at A and a horizontal force P.

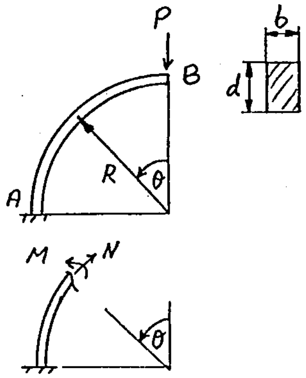
(a) Using Castigliano's theorem, determine the horizontal component of displacement of point A in terms of M_0, P, E, I, L and h. Consider only flexural effects.



(b) From the answer to part (a) above, determine the angle which member AB makes with the vertical at point A due to a horizontal force P = 1 applied at A.

Answer : (a) $\Delta_A = \frac{1}{EI}\left[\frac{M_0h}{6}(5h+6L) + \frac{Ph^2}{3}(3L+2h)\right]$

4. For a curved beam whose lateral dimensions are small with respect to the radius of curvature, the flexural and normal strain energy may be expressed as



$$U = \frac{1}{2}\int \frac{M^2}{EI}\,ds + \frac{1}{2}\int \frac{N^2}{AE}\,ds$$

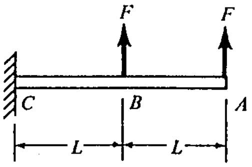
where A and I are the area and moment of inertia of the cross section and where $M = M(\theta)$, $N = N(\theta)$ are the moments and axial forces at a cross-section.

Note : The above integrals represent the elastic strain energy due to flexural and axial deformation respectively.

(a) Using Castigliano's theorem, determine the horizontal component of deflection of point B due to a vertical load.

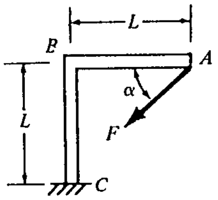
Answer : $\Delta_B = \frac{PR^3}{2EI} - \frac{PR}{2AE}$

7.27 For the uniform beam shown, a student expresses the bending moment in terms of F and writes $\Delta_A = \partial U^*/\partial F$ to find the deflection of point A. Why is this incorrect? What is the Δ_A thus computed, and what is its physical meaning? Also compute Δ_A by correct application of Castigliano's second theorem.



PROBLEM 7.27

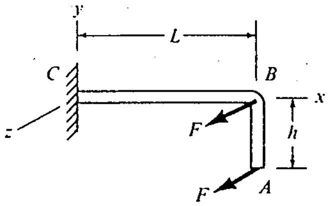
*7.39 The angled beam shown has the same bending stiffness EI throughout. What should be angle α if the displacement of point A is to be in the direction of force F? Force F lies in the plane of ABC.



PROBLEM 7.39

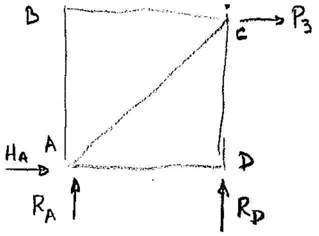
7.28 Forces F act normal to the plane of the uniform bent bar shown.

(a) Compute translational deflection components Δ_x, Δ_y, and Δ_z at point A by use of Castigliano's second theorem (Eq. 7.9.2).



PROBLEM 7.28

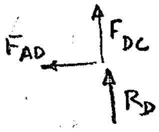
$m = 2j-3$ מסבך מיוחד



משוואות
שיווי משקל
ושיווי מומנטים

$$\left.\begin{aligned}\Sigma F_x=0 &\Rightarrow H_A = -P_3\\ \Sigma M_A=0 &\Rightarrow R_D = P_2+P_3\\ \Sigma F_y=0 &\Rightarrow R_A = P_1-P_3\end{aligned}\right\}$$

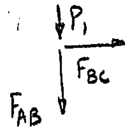
בצומת D



$\Sigma F_x=0 \Rightarrow \underline{0 = F_{AD}}$ לכן

$\Sigma F_y=0 \Rightarrow \underline{(-P_2-P_3) = -R_D = F_{DC}}$

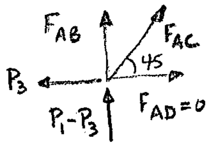
בצומת B



$\Sigma F_x=0 \Rightarrow \underline{0 = F_{BC}}$ לכן

$\Sigma F_y=0 \Rightarrow \underline{-P_1 = F_{AB}}$

בצומת A



$\underline{F_{AC} = \sqrt{2}P_3} \Leftarrow P_3 = F_{AC}\cos 45$ לכן

$\Downarrow \quad 0 = +(P_1-P_3) + F_{AB} + F_{AC}\sin 45$

$$U_c = \frac{\Sigma F_i^2 l_i}{2A_iE_i} = \frac{1}{2AE}\left\{P_1^2L + 0^2L + (P_2+P_3)^2L + 0^2\cdot L + (\sqrt{2}P_3)^2\sqrt{2}L\right\}$$

גזירת U_c לגבי P_1 נותנת את השקיעה בנקודה B בכיוון העומס P_1

$$\frac{\partial U_c}{\partial P_1} = U_B = \frac{1}{AE}(P_1L)\downarrow$$

גזירת U_c לגבי P_2 נותנת את השקיעה בנקודה C בכיוון העומס P_2

$$\frac{\partial U_c}{\partial P_2} = U_c = \frac{1}{AE}(P_2+P_3)L\downarrow$$

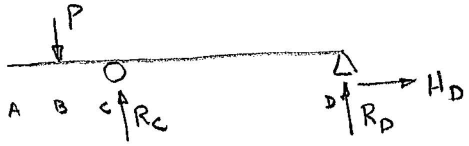
$$\frac{\partial U_c}{\partial P_3} = u_3 = \frac{1}{2AE}\left(2(P_2+P_3)L + 2(\sqrt{2}P_3\cdot\sqrt{2}L)\cdot\sqrt{2}\right) = \frac{1}{AE}\left[P_2L + P_3L(1+\overset{2}{\cancel{\ }}\sqrt{2})\right]\rightarrow$$

2.

$\Sigma F_x=0 \Rightarrow 0 = H_D$ לכן

$\Sigma M_c=0 \Rightarrow R_D = -\frac{P}{3}$

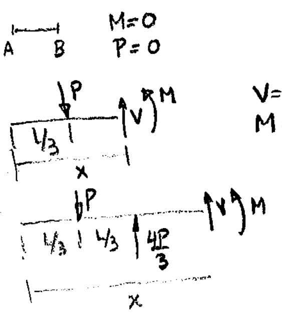
$\Sigma F_y=0 \Rightarrow R_c = 4P/3$



$$U_c = \int_0^{\frac{5L}{3}} \frac{M^2}{2EI}dx$$

$$= \int_0^{L/3}\frac{M^2}{2EI}dx + \int_{L/3}^{2L/3}\frac{M^2}{2EI}dx + \int_{2L/3}^{\frac{5L}{3}}\frac{M^2}{2EI}dx$$

$$U_c = 0 + \int_{L/3}^{2L/3}\frac{P^2}{2EI}(x-L/3)^2dx + \int_{2L/3}^{5L/3}\frac{\left[P\left(\frac{x}{3}-\frac{5L}{9}\right)\right]^2dx}{2EI}$$



$M=0$
$P=0$ $\quad 0 \le x \le L/3$

$V=P$
$M=-P(x-L/2)$ $\quad L/3 \le x \le 2L/3$

$V=-P/3$
$M=-P(x-L/3)+\frac{4P}{3}(x-2L/3)$
$=\frac{P}{3}x - \frac{5PL}{9}$ $\quad 2L/3 \le x \le 5L/3$

$$\frac{\partial U_c}{\partial P} = U_B = \frac{P}{EI}\int_{L/3}^{2L/3}(x-L/3)^2dx + \frac{P}{EI}\int_{2L/3}^{5L/3}\left(\frac{x}{3}-\frac{5L}{9}\right)^2dx$$

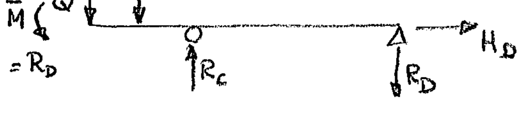
$$= \frac{P}{EI}\left\{\frac{(x-L/3)^3}{3}\Bigg|_{L/3}^{2L/3} + \left(\frac{x}{3}-\frac{5L}{9}\right)^3\Bigg|_{\frac{2L}{3}}^{5L/3}\right\} = \frac{P}{EI}\left\{\frac{L^3}{81} - \left(-\frac{L^3}{27}\right)\right\} = \frac{4PL^3}{81}$$

השקיעה בכיוון P ולכן ↓

כדי למצוא את השקיעה והזווית בנקודה A, צריך לשים כוח ומומנט וירטואליים שם

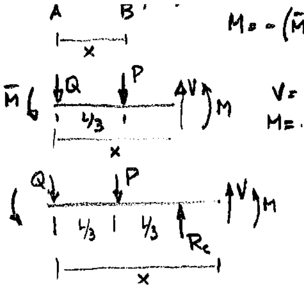
$\Sigma F_x=0 \Rightarrow H_D=0$

$\Sigma M_c=0 \Rightarrow \frac{\bar{M}}{L}+\frac{2Q}{3}+P/3 = R_D$



$- \quad \int_0 \; 2EI \qquad \int_0 \; 2EI \qquad \int_{L/3} 2EI \qquad \int_{2L/3} 2EI$

$$= \int_0^{L/3}\frac{(\bar{M}+Qx)^2dx}{2EI} + \int_{L/3}^{2L/3}\frac{[\bar{M}+Qx+P(x-L/3)]^2}{2EI}dx + \int_{2L/3}^{\frac{5L}{3}}\frac{[\bar{M}+Qx+P(x-L/3) - R_c(x-2L/3)]^2}{2EI}$$



$M = -(\bar{M}+Qx)$

$V = P+Q$

$M = -[\bar{M}+Qx+P(x-L/3)]$

$V = P+Q-R_c = -R_D$

$M = -[\bar{M}+Qx+P(x-L/3) - R_c(x-2L/3)]$

כדי לקבל את התזוזה בנקודה A, קח $\bar{M} = 0$ וחשב U_c

$$U_c = \int_0^{L/3}\frac{Q^2x^2dx}{2EI} + \int_{L/3}^{2L/3}\frac{[Qx+P(x-L/3)]^2}{2EI}dx + \int_{2L/3}^{5L/3}\frac{\left[-\frac{2Q}{3}x+\frac{10QL}{9}-\frac{Px}{3}+\frac{5PL}{9}\right]^2dx}{2EI}$$

$$v_A = \lim_{Q\to 0}\frac{\partial U_c}{\partial Q} = \int_0^{L/3}\frac{Qx^{\cancel{2}}dx}{\cancel{2}EI} + \int_{L/3}^{2L/3}\frac{P(x-L/3)x\,dx}{EI} + \int_{2L/3}^{5L/3}\frac{\left[\frac{-Px}{3}+\frac{5PL}{9}\right]}{EI}\left(-\frac{2x}{3}+\frac{10L}{9}\right)dx$$

$$= \frac{P}{EI}\left(\frac{x^3}{3}-\frac{Lx^2}{6}\right)_{L/3}^{2L/3} + \frac{P}{EI}\left[+\frac{2x^3}{27}-\frac{10x^2L}{27}+\frac{50L^2x}{81}\right]_{2L/3}^{5L/3} = \frac{P}{EI}\left(\frac{8L^3}{81}-\frac{4}{54}L^3-\frac{L^3}{81}+\frac{L^3}{54}\right)+\frac{P}{EI}\left(\frac{250L^3}{729}-\frac{250L^3}{27\cdot9}+\frac{250L^3}{243}-\frac{16L^3}{729}+\frac{40L^3}{243}-\frac{100L^3}{243}\right)$$

$$= \frac{P}{EI}\left(\frac{7L^3}{81}-\frac{1}{18}L^3\right)+\frac{P}{EI}\left(\frac{234}{729}L^3-\frac{60}{243}L^3\right) = \frac{PL^3}{EI}\left(\frac{7}{81}+\frac{6}{81}-\frac{1}{18}\right)$$

$$= \frac{PL^3}{EI}\left(\frac{13}{9\cdot9}-\frac{1}{9\cdot2}\right) = \underline{\underline{\frac{PL^3}{EI}\cdot\frac{17}{162}}}\downarrow$$

כדי לקבל את הזווית בנקודה A, קח $Q = 0$ וחשב את U_c

$$U_c = \int_0^{L/3}\frac{\bar{M}^2dx}{2EI} + \int_{L/3}^{2L/3}\frac{[\bar{M}+P(x-L/3)]^2}{2EI}dx + \int_{2L/3}^{5L/3}\frac{\left[\bar{M}\left(\frac{5}{3}-\frac{x}{L}\right)-\frac{Px}{3}+\frac{5PL}{9}\right]^2}{EI}dx$$

$$\theta_A = \lim_{\bar{M}\to 0}\frac{\partial U_c}{\partial \bar{M}} = \int_0^{L/3}\frac{\bar{M}d\cancel{x}}{\cancel{2}EI} + \int_{L/3}^{2L/3}\frac{P(x-L/3)\,dx}{EI} + \int_{2L/3}^{5L/3}\frac{\left(-\frac{Px}{3}+\frac{5PL}{9}\right)\left(\frac{5}{3}-\frac{x}{L}\right)}{EI}dx$$

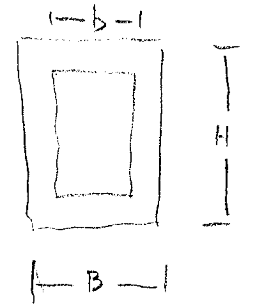
$$= \frac{P}{EI}\left.\frac{(x-L/3)^2}{2}\right|_{L/3}^{2L/3} + \frac{P}{EI}\left[\frac{-5x^2}{18}+\frac{x^3}{9L}+\frac{25}{27}Lx-\frac{5x^2}{18}\right]_{2L/3}^{5L/3}$$

$$= \frac{P}{EI}\left(\frac{L^2}{18}\right)+\frac{P}{EI}\left(-\frac{5}{9}x^2+\frac{x^3}{9L}+\frac{25Lx}{27}\right)_{2L/3}^{5L/3} = \frac{PL^2}{18EI}+\frac{PL^2}{EI}\left(-\frac{5}{9}\cdot\frac{25}{9}+\frac{125}{9\cdot27}+\frac{125}{27\cdot3}+\frac{5}{9}\cdot\frac{4}{9}-\frac{8}{9\cdot27}-\frac{50}{3\cdot27}\right)$$

$$\left(-\frac{105}{9\cdot9}+\frac{117}{9\cdot27}+\frac{25}{27}\right)$$

$$= \frac{PL^2}{18EI}+\frac{PL^2}{EI}\left(\frac{-315+117+225}{27\cdot9}\right) = \frac{PL^2}{18EI}+\frac{PL^2}{EI}\left(\frac{27}{9\cdot27}\right) = \frac{PL^2}{6EI}$$

4.78



$$\frac{58\cdot88^3}{12}-\frac{30.48^3}{12} = \frac{BH^3}{12}-\frac{bh^3}{12}$$

מומנט האינרציה של החתך הוא

$$3.017\times10^6\,mm^4 = I_{zz}$$

$$\sigma = \frac{\sigma_y}{c}\cdot y$$

$$\int_{-H/2}^{H/2}\frac{\sigma_y}{c}\cdot y^2\cdot B\cdot dy = \frac{\sigma_y}{c}\cdot\frac{BH^3}{12} = \frac{\sigma_y}{c}\frac{BH^3}{12}$$

$$-\int_{-h/2}^{h/2}\frac{\sigma_y}{c}\cdot y^2b\,dy = \frac{\sigma_y}{c}\cdot\frac{bh^3}{12} = \frac{\sigma_y}{c}\frac{bh^3}{12}$$

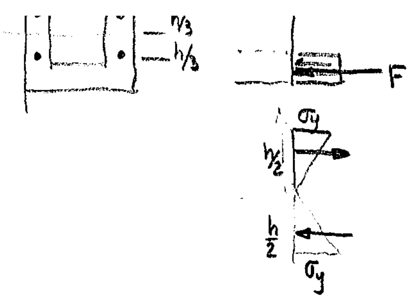
בהתחלת הכניעה $M = \frac{\sigma_y}{c}\cdot I_{zz}$

בהתחלת הכניעה

$$M = \frac{300\times10^6\,N/m^2\cdot3.017\times10^{-6}\,m^4}{.044\,m}$$

$$\boxed{M = 20.573\,kN\text{-}m}$$

$\boxed{\text{רדיוס}}$, הוא שווה $\rho = \frac{EI}{M} = \frac{200\times10^9\,N/m^2\cdot3.017\times10^{-6}\,m^4}{20572.5\,N\text{-}m} = \boxed{29.33\,m}$



הכוח בחלק העליון והתחתון הוא $\pm\sigma_y(\frac{H-h}{2})B$ ומרוכז ב $\frac{H-h}{4}$

בצדדים הכוחות נמצאים בארבע הנקודות $h/3$ מציר הניטרלי מלמעלה ולמטה. כל כוח הוא $\sigma_y(\frac{B-b}{2})\cdot\frac{h}{4}$

לכן המומנט הוא $2\left\{\frac{2h}{3}\cdot\sigma_y(\frac{B-b}{2})\frac{h}{4}\right\}+\left\{H-(\frac{H-h}{2})\right\}\cdot\sigma_y(\frac{H-h}{2})B$

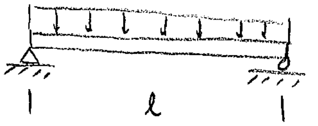
$$\sigma_y\left\{\frac{4h^2(B-b)}{24}+\frac{H+h}{2}\left(\frac{H-h}{2}\right)B\right\}=\sigma_y\cdot 89632\ mm^3=300\times10^6\frac{N}{m^2}\ (89632\times10^{-9}m^3$$

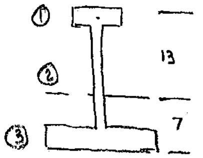
$$\boxed{M=26889.6\ N\text{-}m}$$

הרדיוס הוא $\frac{EI}{M}$, <u>אבל</u> <u>רק</u> <u>לחלק</u> <u>האלסטי</u>. זאת אומרת ש-I_{zz} הוא של החלק האלסטי, $\frac{Bh^3}{12}-\frac{bh^3}{12}=258048\ mm^4$, והמומנט הוא רק של החלק האלסטי $\frac{\sigma_y\cdot4h^2(B-b)}{24}=3225.6\ N\text{-}m$

לכן, הרדיוס הוא $\frac{EI}{M}=\frac{200\times10^9\frac{N}{m^2}\times.25805\times10^{-6}m^4}{3225.6\ N\text{-}m}=\boxed{16\ m}$

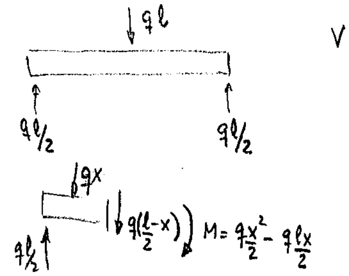
2.16



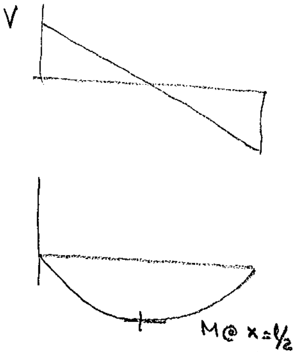


	A	$\bar{y}$	$A\bar{y}$
①	8	1	8
②	16	10	160
③	24	19	456
	48		624

$\bar{y}=\frac{\Sigma A\bar{y}}{\Sigma A}=13$



$M=q\frac{x^2}{2}-q\frac{lx}{2}$



$-\frac{ql^2}{8}$ לכן המומנט המקסימלי הוא $\frac{ql^2}{8}$

חישוב I_{zz}

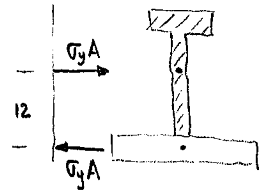
	I_{zz_0}	A	d^2	I_{tt}
①	$4\cdot2^3/12$	8	$(12)^2$	1154.67
	$1\cdot16^3/12$	16	$(3)^2$	485.33
③	$12\cdot2^3/12$	24	$(-6)^2$	872
				2512 cm^4

לפי שיטת אלסטית $\sigma_y=\frac{M\cdot c}{I_{tt}}=\frac{\frac{ql^2}{8}\cdot.13m}{2512\times10^{-8}m^4}$

לכן $q=\frac{\sigma_y I_{tt}\cdot8}{c\ \ l^2}=\frac{(240\times10^6)(2512\times10^{-8})8}{(0.13)(4)^2}$ $\boxed{q=23187.69\ N/m}$

והמומנט $\frac{ql^2}{8}=46375.4\ N\text{-}m$

לפי שיטת פלסטית כל השטח בכניעה. לכן השטח נחלק לשניים והציר הניטרלי בצומת חלק ② וחלק ③. מרכז הכובד של חלק ①+② הוא 7cm ו-12cm מרכז הכובד של חלק ③. המומנט הוא $12cm\cdot\sigma_y\cdot24cm^2=q\frac{l^2}{8}$

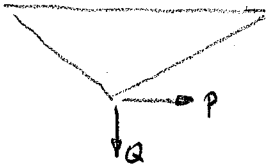


לכן $q=\frac{288\times10^{-6}m^3\cdot\sigma_y\cdot8}{l^2}=\frac{288\times10^{-6}(240\times10^6)(8)}{4^2}=\boxed{q=34560\ N/m}$

והמומנט שווה ל $\frac{ql^2}{8}=69120\ N\text{-}m$

לכן $\frac{q_{\text{שיטת פלסטית}}}{q_{\text{שיטת אלסטית}}}=1.49$

IF WE LOOK AT THE CASE



WE FOUND THAT

$$u = \frac{L}{AE}(1.3969P - .1295Q)$$

$$v = \frac{L}{AE}(-.1295P + .6883Q)$$

נוכל לכתוב את זה כ-

$u = \delta_{uu}P + \delta_{uv}Q$

$v = \delta_{vu}P + \delta_{vv}Q$

קוראים ל- $\delta_{uu}, \delta_{uv}=\delta_{vu}, \delta_{vv}$

מקדמים גמישות (flexibility coeffs)

נותנים את הקשר בין התזוזה לעומס

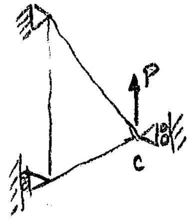
ס . מקדם גמישות

δ

כוח P_j . ורואים ש- $\delta_{ji}=\delta_{ij}$ -

לניארי.

IN THE FOLLOWIN



קוראים ל- $S_{uu}, S_{uv}, S_{vu}=S_{uv}$

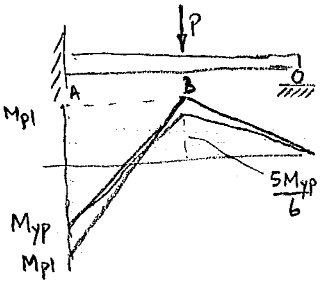
ם את הקשר בין העומס לבין התזוזה

קשיחות

u . רואים ש- $S_{ji} = S_{ij}$

TO CALCULATE COLLAPSE LOAD

ות



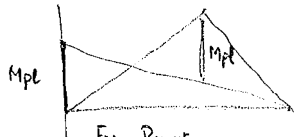
HIGHEST MO

MO

- WHEN P
- P CONTINUE
  & M = M
- WHEN THIS
- TRYING TO INCREASE
- NO ADDITIONAL HINGES AS LONG AS NO STRAIN HARDENING
- IT IS A MECHANISM W/ 2 HINGES המבנה הוא מנגנון עם שני פרקים

S PLATICITY

A & B



From Popov

המכללה האקדמית יהודה ושומרון

החטיבה העצמאית

תש"ס

סמסטר: א' מועד: ______

המחלקה: עצמאי

שם המרצה: פרופ' לוי עזרא

מס' הקורס: ב'ח

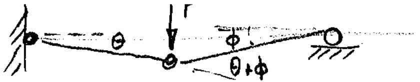
נושא הקורס: תורת החוזק 2

תאריך: 4.1.00 שעה: 18⁰⁰

מס' החדר: 2.3 דף: 1 מתוך: 1

	מס' מחברת	ציון בחינה	ציון תרגיל	ציון סופי
.1	3526			
.2	3415			
.3	3512			
.4	07066			
.5	0695			
.6	3511			
.7	3411			
.8	3617			
.9	3510			
.10	3517			
.11	01082			
.12	0346			
.13	05888			
.14	3514			
.15				
.16				
.17				
.18				
.19				
.20				

חתימה: ______



מעקרון עבודת ווירטואלית - כאשר העומס עב המרחק
$\Delta = \theta L/2$, האנרגיה נספג בפרקים כאשר הם מסובבים בזוויות
θ ו- ϕ דרך המומנט M_{pl}

$$M_{pl}\theta + M_{pl}(\theta+\phi) = P\frac{L}{2}\theta$$

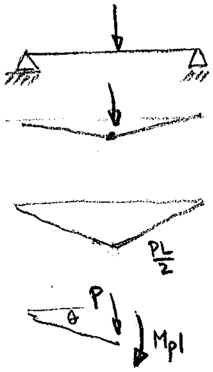
כאן $\theta=\phi$ כי P באמצע אורך הקורה

$$3M_{pl}\theta = \frac{PL}{2}\theta$$

$$P_{התמוטטות} = \frac{6M_{pl}}{L}$$

HERE θ IS A VIRTUAL DISPL. THAT TAKES PLACE AFTER COLLAPSE MECHANISM OCCURS

IF

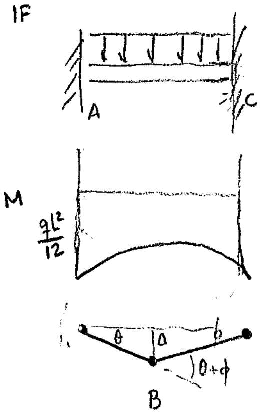


HIGHEST MOMENT IS $\frac{PL}{2}$ AT B FOR SIMPLY SUPPORTED

$$M_{pl}\cdot 2\theta = \frac{PL\theta}{2}$$

$$P = \frac{M_{pl}\cdot 4}{L}$$

IF



$\Delta = X_B\theta$
$\Delta = \phi(L-X_B)$

- HIGHEST MOMENT IS $\frac{qL^2}{12}$ @ A & C , 3 HINGES FORM
- ASSUME YOU DON'T KNOW WHERE

$$qX_B\cdot\theta\frac{X_B}{2} + q(L-X_B)\phi\frac{(L-X_B)}{2} = M_{pl}\theta + M_{pl}(\theta+\phi) + M_{pl}\phi$$

- LET LOAD IN EACH SECTION BE CONCENTRATED AT CENTROID
- DUE TO SYMMETRY $\phi = \theta$ & $X_B = L/2$

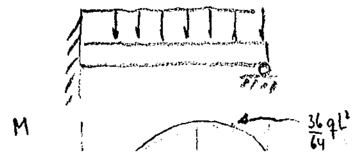
$$q\frac{L^2}{8} + q\frac{L^2}{8} = 4M_{pl}$$

$$q_{pl} = \frac{16M_{pl}}{L^2}$$

$$q_{yp}\frac{L^2}{12} = M_{yp} \qquad q_{yp} = \frac{12M_{yp}}{L^2}$$

גבול עליון לעומס התמוטטות - עומס מחושב דרך מנגנון התמוטטות בלתי ראלי, והוא יותר גדול או לפחות שווה לעומס התמוטטות שפועל.

גבול תחתון לעומס התמוטטות - עומס מחושב דרך מומנט, שבא מגרף המומנט שגודלו פחות מ M_{pl}, הוא פחות מעומס התמוטטות או לפחות שווה לו.



IF $X_B = L/2$ $\qquad q_{PL} = \frac{12M_{pl}}{L^2}$

$X_B = 5/8L$ WHERE MAX DEFLECTION $\qquad q_{PL} = \frac{11.733M_{pl}}{L^2}$

$X_B = .586L$ WHERE $\frac{dq}{\ldots} = 0$ $\qquad q_{PL} = 11.657M_{pl}$

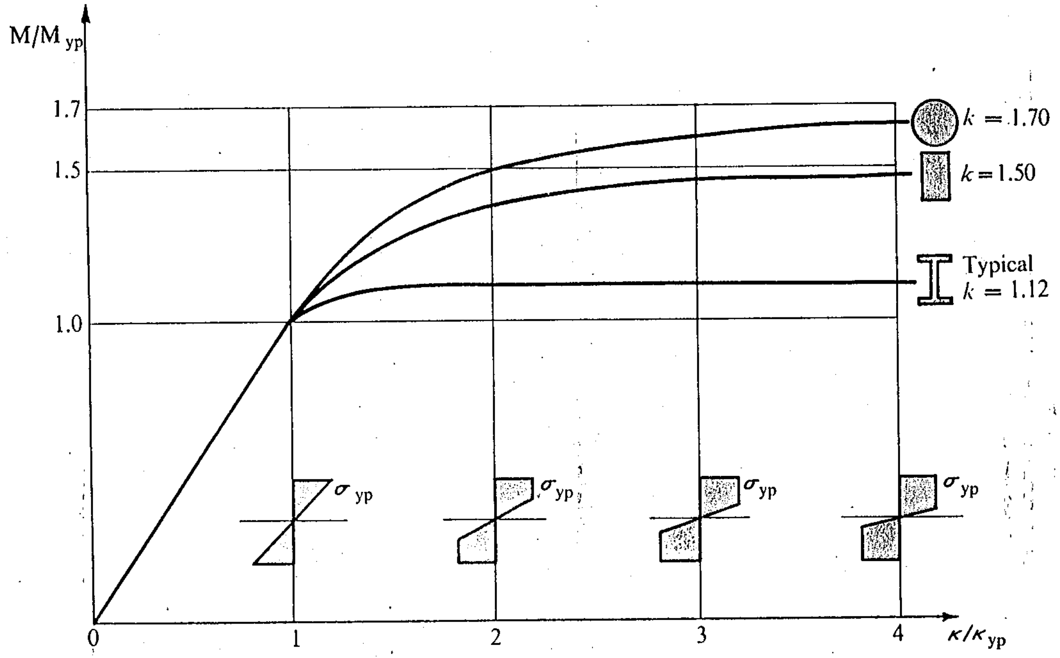


Fig. 12-23. Moment-curvature relations for circular, rectangular, and I cross sections. $M_p/M_{yp} = k$, the shape factor.

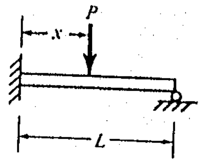
Fig. 11-18.) Note especially the rapid ascent of the curves toward their respective asymptotes as the cross sections plastify. This means that very soon after exhausting the elastic capacity of a beam, a rather constant moment is both achieved and maintained. This condition is likened to a plastic hinge. In contrast to a frictionless hinge capable of permitting large rotations at no moment, the plastic hinge allows large rotations to occur at a constant moment. This constant moment is approximately M_p, the ultimate or plastic moment for a cross section.

הסבר על פרק פלסטי
וההבדל בינו ובין
פרק בלתי חיכוכי

Using plastic hinges, a sufficient number may be inserted into a structure at the points of maximum moments to create a kinematically admissible collapse mechanism. Such a mechanism, permitting unbounded movement of a system, enables one to determine the ultimate or limit carrying capacity of a beam or of a frame. This approach will now be illustrated by several examples, confining the discussion to beams.

When the limit analysis approach is used for the selection of members, the working loads are multiplied by a load factor larger than unity to obtain the limit loads for which the calculations are performed. This is analogous to the use of the factor of safety in elastic analyses. In structural steel work the term *plastic design* is commonly applied to this approach.

***11.18–** For each beam shown, find an expression for collapse load P_p versus its location
11.20 x. For what value of x is P_p a minimum? X = 0.586L



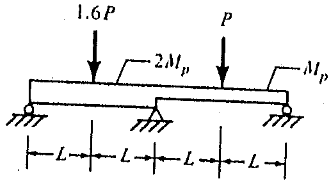
PROBLEM 11.18

11.25, *11.26, 11.27, *11.28, 11.29, 11.30, *11.31–11.33, *11.34, 11.35, *11.36–11.38, *11.39–11.41, *11.42, *11.43, *11.44

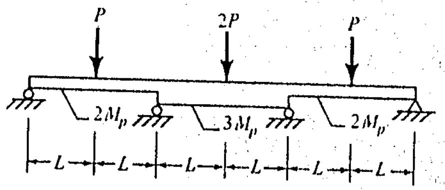
In the structures sketched, the fully plastic bending moment capacity is M_p throughout the structure unless otherwise noted.

(a) Find the collapse load by examining various assumed mechanisms. That is, find the least upper bound.

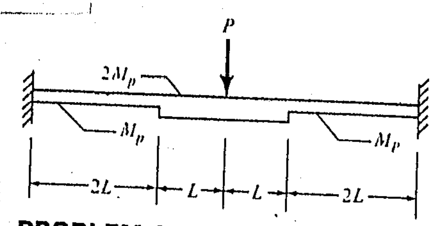
(b) Check your answer by seeing if all bending moments associated with your proposed collapse mechanism are at or below the fully plastic bending moment.



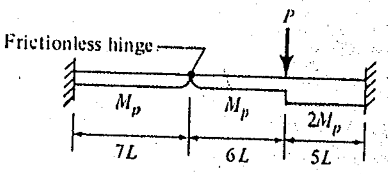
PROBLEM 11.27



PROBLEM 11.30



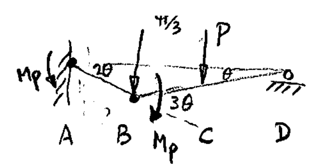
PROBLEM 11.29



× **PROBLEM 11.34**

$P_p = 0.714 M_p/L$

25
2/

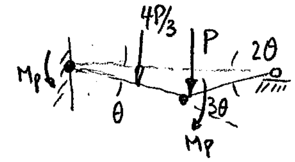


אם הפרק הפלסטי ב- A ו- B

$$\frac{4P}{3}\cdot\left(2\theta\cdot\frac{L}{3}\right) + P\cdot\frac{L}{3}\cdot\theta = M_p\cdot 2\theta + M_p\cdot 3\theta$$

$$P = \frac{45}{11}\,\frac{M_p}{L} = 4.09\,M_p/L$$

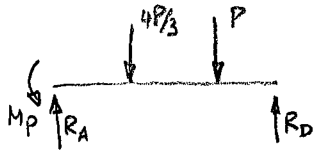
אם הפרק הפלסטי ב- A ו- C



$$\frac{4P}{3}\cdot\left(\theta\cdot\frac{L}{3}\right) + P\cdot 2\theta\cdot\frac{L}{3} = M_p\cdot\theta + M_p\cdot 3\theta$$

$$\underline{\underline{P = 3.60\,\frac{M_p}{L}}}$$

לכן $P_{\text{מסגרתי}} = 3.60\frac{M_p}{L}$



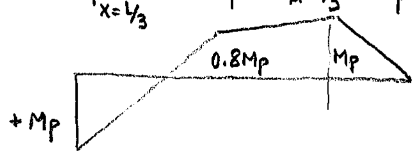
לבדוק המומנט ב $x=L/3$

$$+\circlearrowleft \Sigma M_D = 0 \quad -M_p - \frac{4P}{3}\cdot\frac{2L}{3} - P\cdot\frac{L}{3} + R_A\cdot L = 0$$

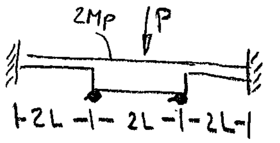
$$R_A = \frac{M_p}{L} + \frac{11P}{9} = \frac{M_p}{L} + \frac{11}{9}\left(3.60\frac{M_p}{L}\right) = 5.4\frac{M_p}{L}$$

$$M\Big|_{x=L/3} = M_p - R_A\cdot\frac{L}{3} = M_p - \frac{5.4}{3}M_p = -0.8\,M_p \quad < M_p$$

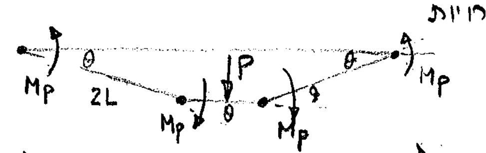
מפני של גודל ~~ה~~ מוחלט של המומנט ב $x=L/3$ פחות מ-M_p,
לכן זה המנגנון שנותן את הכוח המסגרתי.



11.29

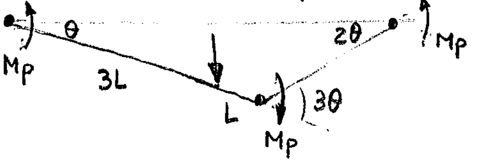


אפשרויות



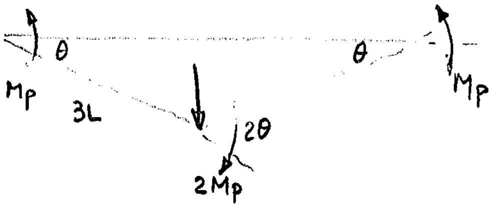
$$P\cdot 2L\cdot\theta = M_p\cdot\theta + M_p\theta + M_p\theta + M_p\theta$$

$$\underline{P = 2M_p/L}$$



$$P\cdot 3L\theta = M_p\cdot\theta + M_p\cdot 3\theta + M_p\cdot 2\theta$$

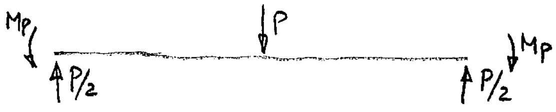
$$\underline{P = 2M_p/L}$$



$$P\cdot 3L\theta = M_p\theta + 2M_p\cdot 2\theta + M_p\cdot\theta$$

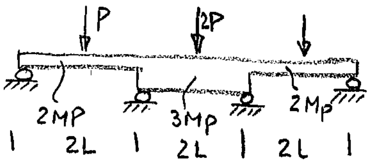
$$\underline{P = 2M_p/L}$$

מפני הסימטריות

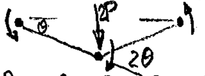


ויכולים להראות ש-|M| בחלקים
הדקים פחות מ-M_p ובחלק העבה
|M| פחות מ- $2M_p$

11.30



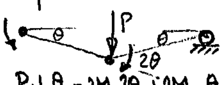
אם הפרק מופיע במרכז



$$2P\cdot L\theta = 2M_p\theta + 3M_p\cdot 2\theta + 2M_p\theta$$

$$P = \frac{5M_p}{L}$$

אם הפרקים מופיעים בחלק הימני



$$P\cdot L\theta = 2M_p 2\theta + 2M_p\theta$$

$$P = 6M_p/L$$

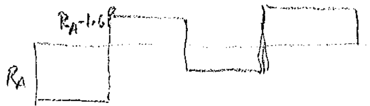
העומס המסגרתי

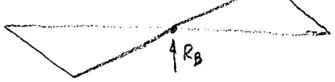
$2.6P = R_A + R_B + R_c$

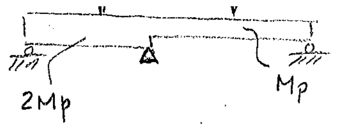
$\Sigma M_B = 0 \quad R_A \cdot 2L - P(1.6)L + PL - R_c \cdot 2L$

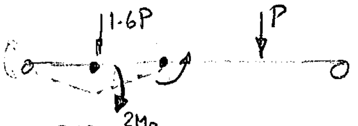
$\Rightarrow \quad (R_A - R_c)2L = .6PL$

$R_A - R_c = .3P$



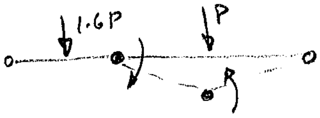






$1.6Pl\theta = \overset{2M_p}{2M_p} \cdot 2\theta + M_p \cdot \theta$

$P = 3.125 M_p/l$



$P \cdot l\theta = M_p\theta + M_p 2\theta$

$Pl\theta = 3M_p\theta$

$P = 3M_p/l$

$$U = \frac{1}{2EI}\left\{\int_0^L \frac{R_A^2 x^2}{2} dx + \int_0^L \frac{[R_A L + (R_A - 1.6P)x]^2}{2} dx + \int_0^L \{(2R_A L + 1.6PL) + (R_A - 1.6P + R_B)x\}^2 dx + \int_0^L \{3R_A L - 3.2PL + R_B L + (R_A - 2.6P + R_B)x\}^2 dx\right\}$$

$$\frac{\partial U_c}{\partial R_A} = 0 = \frac{1}{EI}\int_0^L [(2R_A L - 1.6PL) + (R_A - 1.6P + R_B)x]\,x\,dx + \int_0^L \{3R_A L - 3.2PL + R_B L + (R_A - 2.6P + R_B)x\}(L+x)\,dx$$

$$\frac{1}{EI}\left[(2R_A L - 1.6PL)\frac{L^2}{2} + (R_A - 1.6P + R_B)\frac{L^3}{3} + (3R_A L + 3.2PL + R_B L)\frac{(L+x)^2}{2}\Bigg|_0^L + (R_A - 2.6P + R_B)\left(\frac{Lx^2}{2} + \frac{x^3}{3}\right)\Bigg|_0^L\right] = 0$$

$\frac{3L^2}{2}$ $\quad \left(\frac{5}{6}L^3\right)$

$$\frac{L^3}{EI}\left\{R_A - 0.8P + \frac{R_A}{3} - \frac{1.6P}{3} + \frac{R_B}{3} + \frac{9R_A}{2} + 4.8P + 1.5R_B + \frac{5}{6}R_A - \frac{13}{6}P + \frac{5}{6}R_B\right\} = 0$$

$R_A(1 + 1/3 + 4.5 + 5/6) - P(4/5 + \frac{1.6}{3} + 4.8 + 13/6) + R_B(1/3 + 1.5 + 5/6) = 0$

$\frac{20}{3}R_A - 8.3P + R_B\, 8/3 = 0$

$20R_A + 8R_B = 24.9P$

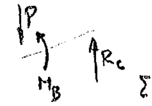
$20R_A + 10R_B = 38P$

$2R_B = 13.1P$

$R_B = 6.55P$

$R_A =$

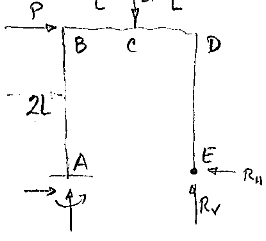
$2R_A + R_B = 3.8P$



ΣM ~~$M_B = P_p L - R_c \cdot 2L = \frac{3M_p L}{L} - R_c \cdot 2L = 2M_p$~~ ~~$R_c = \frac{2M_p}{L}$~~

$\Sigma M_B = 0 \qquad P_p \cdot L - M_B - R_c \cdot 2L = 0$

$\frac{3M_p - M_B}{2} = R_c \qquad \frac{3M_p - M_p}{2} = R_c = M_p$ ✓



$M_A = -R_V \cdot 2L + 4PL$

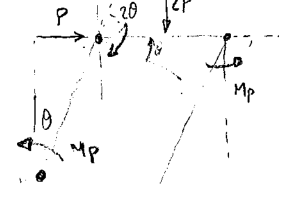
$M_B = -R_V \cdot 2L + R_H \cdot 2L + 2PL$

$M_D = 2R_H L$

$M_c = 2R_H L - R_V L$

$\Rightarrow \quad M_A - 2M_c + 2M_D = 4PL$

$M_B - 2M_c + M_D = 2PL$



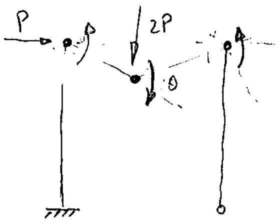
$P \cdot 2l\theta = M_p\theta + M_p \cdot \theta + M_p\theta$

$P = \frac{3M_p}{2L}$

צריך לבדוק את המומנטים
והתגובות ב ABCDE

$M_A = M_p = M_D \quad M_c = ? \quad P = \frac{3M_p}{2L} \Rightarrow M_c = -\frac{3}{2}M_p$

מנגנון לא יתקיים



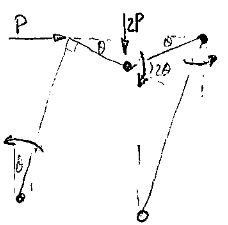
$2P \cdot \theta l = M_p\theta + M_p \cdot 2\theta + M_p\theta$

$P = \frac{2M_p}{L}$

$M_B = M_D = M_p$

$M_c = -M_p, \quad P = \frac{2M_p}{L}$

$M_A = ? \Rightarrow M_A = 4M_p$



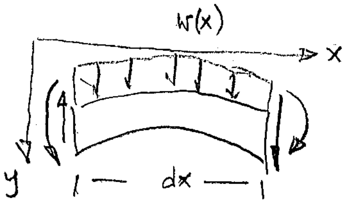
$P \cdot 2\theta l + 2P \cdot \theta l = M_p\theta + M_p \cdot 2\theta + M_p 2\theta$

$P = \frac{5M}{4L}$

$M_A = M_D = M_p \quad M_c = -M_p$

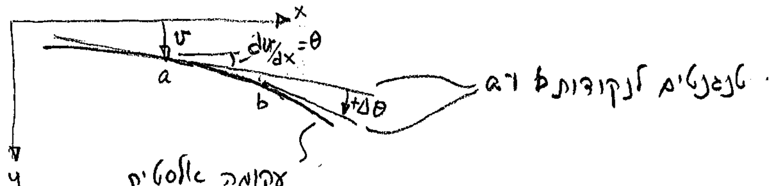
$M_B = ?, \quad P = \frac{5M_p}{4L}$

$M_B = -\frac{M_p}{2}$



$\sigma_x = -\frac{M_z y}{I_{zz}}$ $\frac{dV}{dx} = -w(x)$ $\frac{dM}{dx} = -V$

$\frac{d^2v}{dx^2} = \frac{M_z}{EI_{zz}}$ $K = \frac{1}{\rho} \approx \frac{d^2v}{dx^2}$ (העיקומיות)



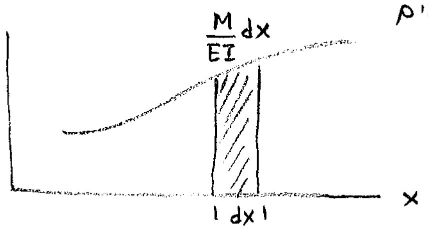
עקומה אלסטית

עם $0<\frac{1}{\rho}$ (העיקומיות)

כאשר התזוזה בקורה במקרים ספציפיים, יש עוד שיטה שנוכל להשתמש בה. השיטה עובדת כאשר יש שינויים בשטח-החתך, או העומס מסובך, קורות מתחדדות (TAPERED) מוגדרות כדוגמא במבנה גשרים. נשתמש בגרף של העיקומיות (CURVATURE) של הקורה לקבל את התזוזה.

השיטה עובדת גם כאשר יש אי-רציפות העומסים או אי-רציפות של שטחי-החתך.

$K = \frac{M}{EI}$

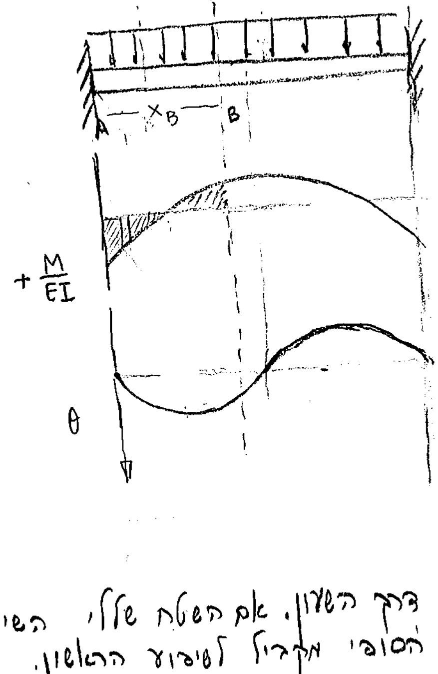


מפני ש- $\frac{d^2v}{dx^2} = \frac{M}{EI}$ ו- $\frac{d\theta}{dx} = \frac{d}{dx}\left(\frac{dv}{dx}\right)$, לכן נוכל לכתוב ש-

$$d\theta = \frac{M}{EI}dx$$

לכן, שינוי זווית של העקומה האלסטית שווה לשטח תחת הגרף של העיקומיות של הקורה.

כאשר זווית קטנה מוכפלת במרחק x מנקודת התחלת מערכת הקואורדינטות, $dt=xd\theta$, או $dt=\frac{M}{EI}x\,dx$. אם מתחילים ממקום שזווית ידוע לנו, ומחשבים את השטח תחת גרף העיקומיות, נקבל את הזווית בנקודה אחרת על העקומה האלסטית



__דוגמה__ - קורה בעומס מפורס של $q\ N/m$ קבועה בסמכים מקובעים.

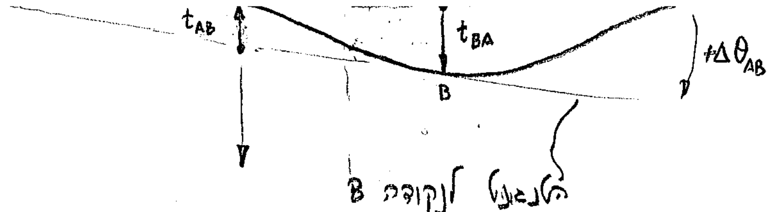
מצאנו ש- M בסמך הוא $qL^2/12$. לכן $\frac{M}{EI} = \frac{qL^2}{12EI}$. בגלל הסמך, בנקודה A, $\theta_A = \underline{0}$ ו- בנקודה B שבמרחק של x_B

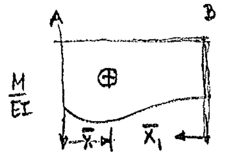
$$\theta_B - \theta_A = \int_A^B d\theta = \int_{x=0}^{x=x_B} \frac{M}{EI}dx$$

$$\theta_B = \int_0^{x_B} \frac{M}{EI}dx + \theta_A$$

אם השטח חיובי, השיפוע הולך וגודל וסובב דרך השעון. אם השטח שלילי, השיפוע הולך וקטן סובב נגד דרך השעון. אם השטח שווה לאפס השיפוע הסופי מקביל לשיפוע הראשון.

הכמות dt בה מהעיקומיות של האלמנט dx. אם נקח את כל ההשפעות מכל האלמנטים מ-A ל-B נקבל את המרחק האנכי t_{AB}. זה הסטייה הטנגנטית של נקודה A מטנגנט בנקודה B





אם נשתמש בהגדרת מרכז הכובד של שטח

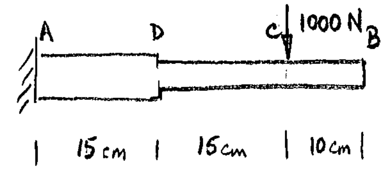
$$t_{AB} = \int_A^B \frac{M}{EI}\,x\,dx = \left(\int \frac{M}{EI}\,dx\right)\bar{x}$$

$\bar{x}$ המרחק האופקי מ-A למרכז הכובד של $\int_A^B \frac{M}{EI}dx$

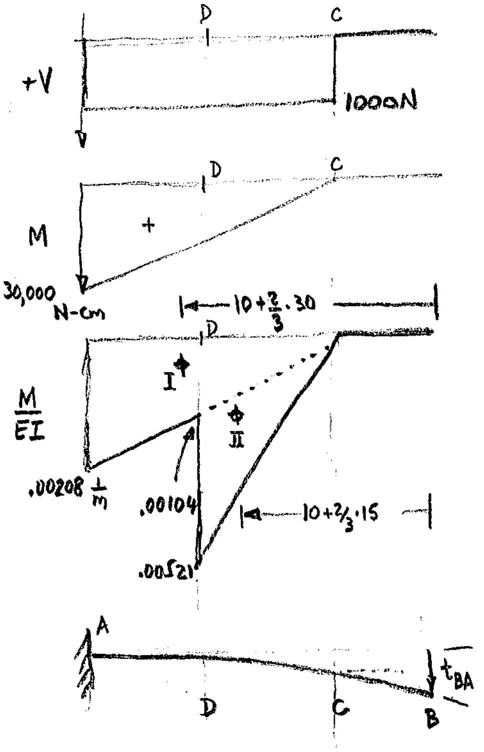
t_{BA} זה ההטייה הטנגנטי של נקודה B מהטנגנט בנקודה A והוא שווה ל $\left(\int \frac{M}{EI}dx\right)\bar{x}_1$

ו-$\bar{x}_1$ הוא המרחק מ-קו האנכי דרך נקודה B למרכז הכובד של $\int_A^B \frac{M}{EI}dx$

בדרך כלל, ההטיה הטנגנטית איננה התזוזה הסופית.



__דוגמה__ קורה של אלומיניום E=72 GPa בצורה כנראה מעומסת מכוח של 1000N, 10cm מהקצה הימני.
אם I_{zz} של AD = 200 cm^4, ו-I_{zz} של DB הוא 40 cm^4, מצוא את התזוזה של קצה B וגם הסיבוב של נקודה B. לא לכלול משקל הקורה.

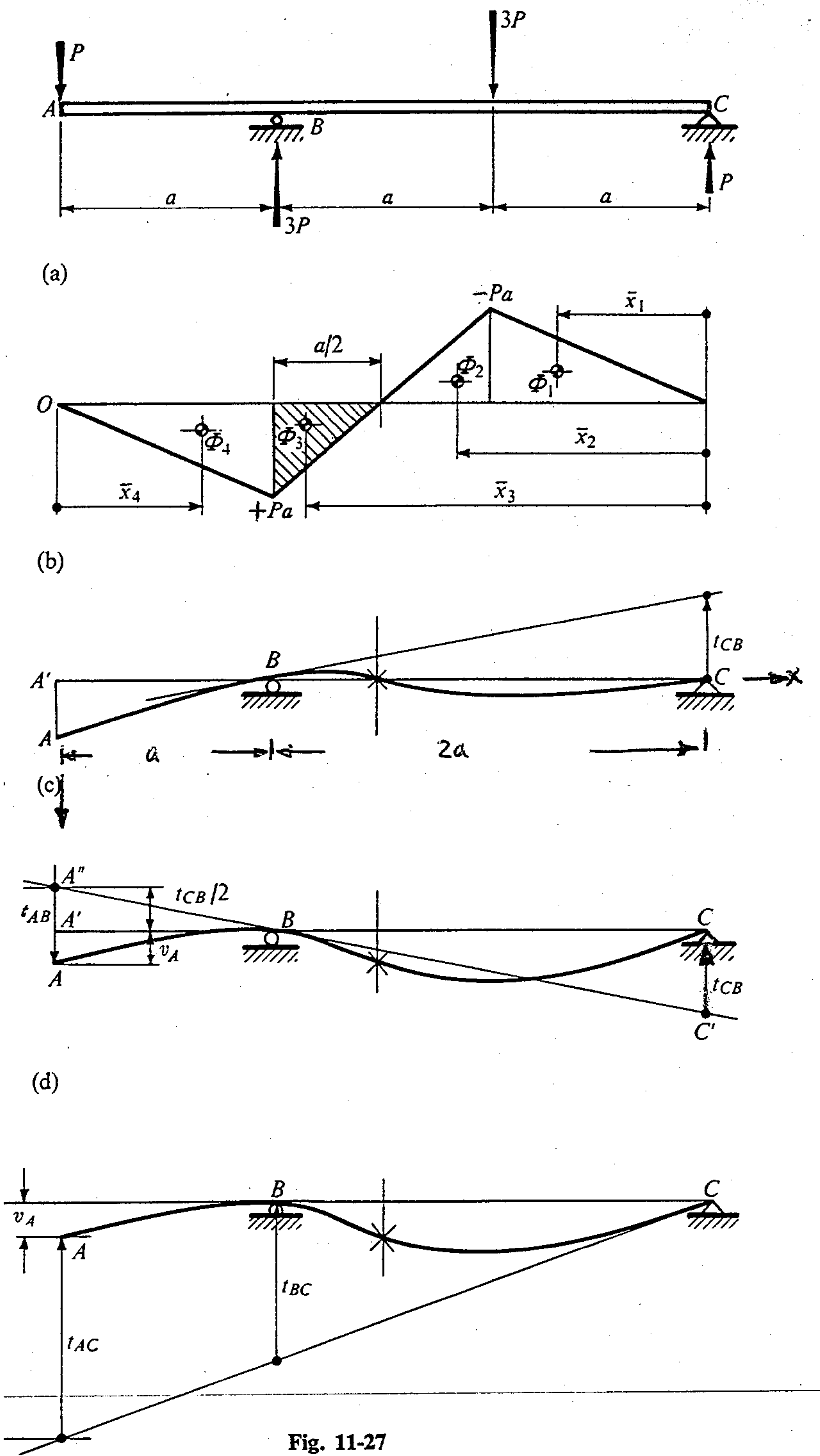


מפני שקצה A הוא סמך מקובע, $0 = \theta_A$

$$\theta_B = \Delta\theta_{AB} = \int_0^{40} \frac{M}{EI}dx = 0 + II + I \text{ שטחי משולשים}$$
$$= (.00208)\left(\frac{.30}{2}\right) + (.00521-.00104)\left(\frac{.15}{2}\right)$$
$$= .000625 \text{ radians}$$

$$t_{BA} = 2\cdot0 + \bar{x}_2 \times II \text{ שטח משולש} + \bar{x}_1 \times I \text{ שטח משולש}$$
$$= 2\cdot0 + (.20)(.000313) + (.30)(.000312)$$
$$= .000156 \text{ m}$$

t_{BA} הוא ההטיה הטנגנטית מנקודה B לטנגנט ב-A
וזה התזוזה $v_B = 0.156$ מ"מ.



למצוא את v_A

$$\frac{\overline{A'A}}{\overline{A'B}} = \frac{t_{CB}}{\overline{BC}}$$

$$A'A = t_{CB}\frac{\overline{A'B}}{\overline{BC}} = \frac{t_{CB}}{2}$$

הציור הנכון

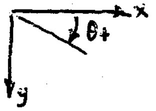


Fig. 11-27

SOLUTION

The bending-moment diagram for the applied forces is in Fig. 11-27(b). The bending moment changes sign at $a/2$ from the left support. At this point an inflection in the elastic curve takes place. Corresponding to the positive moment, the curve is concave up, and vice versa. The elastic curve is so drawn and passes over the supports at B and C, Fig. 11-27(c). To begin, the inclination of the tangent to the elastic curve at the support B is determined by finding t_{CB} as the statical moment of the areas with the proper signs of the $M/(EI)$ diagram between the verticals through C and B about C.

$$t_{CB} = \Phi_1\bar{x}_1 + \Phi_2\bar{x}_2 + \Phi_3\bar{x}_3$$

$$= \frac{1}{EI}\left[\frac{a}{2}(-Pa)\frac{2a}{3} + \frac{1}{2}\frac{a}{2}(-Pa)\left(a + \frac{1}{3}\frac{a}{2}\right) + \frac{1}{2}\frac{a}{2}(+Pa)\left(\frac{3a}{2} + \frac{2}{3}\frac{a}{2}\right)\right]$$

$$= -\frac{Pa^3}{6EI}$$

The ~~positive~~ negative sign of t_{CB} indicates that the point C is ~~above~~ below the tangent through B. Hence a corrected sketch of the elastic curve is made, Fig. 11-27(d), where it is seen that the deflection sought is given by the distance AA' and is equal to $AA'' - A'A''$. Further, since the triangles $A'A''B$ and $CC'B$ are similar, the distance $A'A'' = t_{CB}/2$. On the other hand, the distance AA'' is the deviation of the point A from the tangent to the elastic curve at the support B. Hence

$$v_A = AA' = \overrightarrow{AA''} + \overleftarrow{A'A''} = t_{AB} + (t_{CB}/2)$$

$$t_{AB} = \frac{1}{EI}(\Phi_4\bar{x}_4) = \frac{1}{EI}\left[\frac{a}{2}(+Pa)\frac{2a}{3}\right] = +\frac{Pa^3}{3EI}$$

where the ~~negative~~ positive sign means that point A is ~~below~~ above the tangent through B. This sign is not used henceforth as the geometry of the elastic curve indicates the direction of the actual displacements. Thus the deflection of point A below the line passing through the supports is

$$v_A = \frac{Pa^3}{3EI} - \frac{1}{2}\frac{Pa^3}{6EI} = \frac{Pa^3}{4EI}$$

This example illustrates the necessity of watching the signs of the quantities computed in the applications of the moment-area method, although usually less difficulty is encountered than in the above example. For instance, if the deflection of the end A is established by first finding the rotation of the elastic curve at C, no ambiguity in the direction of tangents occurs. This scheme of analysis is shown in Fig. 11-27(e), where $v_A = \frac{3}{2} t_{BC} - t_{AC}$.

The foregoing examples illustrate the manner in which the moment-area method may be used to obtain the deflection of any statically deter-

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$$\frac{1}{EI}\left(\frac{1{,}000 + \frac{1}{3}(100)M_B}{10}\right) = -\frac{1}{EI}\left(\frac{2{,}880 + 108M_B + 54M_C}{18}\right)$$

or $$(\tfrac{28}{3})M_B + 3M_C = -260$$

Using condition (b) for the span BC provides another equation, $t_{BC} = 0$, or

$$\frac{1}{EI}\left[\frac{(18)}{2}(+40)\frac{(18+12)}{3} + \frac{(18)(+M_B)}{2}\frac{(18)}{3} + \frac{(18)(+M_C)}{2}\frac{2(18)}{3}\right] = 0$$

or $$3M_B + 6M_C = -200$$

Solving the two reduced equations simultaneously,

$$M_B = -20.4 \text{ ft-lb} \qquad \text{and} \qquad M_C = -23.3 \text{ ft-lb}$$

where the signs agree with the convention of signs used for beams. These moments with their proper sense are shown in Fig. 12-17(b).

After the redundant moments M_A and M_C are found, no new techniques are necessary to construct the moment and shear diagrams. However, particular care must be exercised to include the moments at the supports while computing shears and reactions. Usually, isolated beams as shown in Fig. 12-17(b) are the most convenient free bodies for determining shears. Reactions follow by adding the shears on the adjoining beams. In units of kips and feet, for free body AB:

$$\sum M_B = 0 \circlearrowleft +, \quad 2.4(10)5 - 20.4 - 10R_A = 0, \quad R_A = \ \ 9.96 \text{ kips} \uparrow$$

$$\sum M_A = 0 \circlearrowright +, \quad 2.4(10)5 + 20.4 - 10V_B' = 0, \quad V_B' = 14.04 \text{ kips} \uparrow$$

For free body BC:

$$\sum M_C = 0 \circlearrowleft +, \qquad 10(6) + 20.4 - 23.3 - 18V_B'' = 0,$$

$$V_B'' = 3.17 \text{ kips} \uparrow$$

$$\sum M_B = 0 \circlearrowright +, \qquad 10(12) - 20.4 + 23.3 - 18V_C = 0,$$

$$V_C = R_C = 6.83 \text{ kips} \uparrow$$

Check:

$$R_A + V_B' = 24 \text{ kips} \uparrow \qquad \text{and} \qquad V_B'' + R_C = 10 \text{ kips} \uparrow$$

From above, $R_B = V_B' + V_B'' = 17.21$ kips ↑.

The complete shear and moment diagrams are in Figs. 12-17(e) and (f), respectively.

12-6. THE THREE-MOMENT EQUATION

Generalizing the procedure used in the preceding example, a recurrence formula, i.e., an equation which may be repeatedly applied for every two adjoining spans, may be derived for continuous beams. For any n number of spans, $n - 1$ such equations may be written. This gives enough simultaneous equations for the solution of redundant moments over the supports. This recurrence formula is called the *three-moment equation* because three unknown moments appear in it.

משפט קלפירון

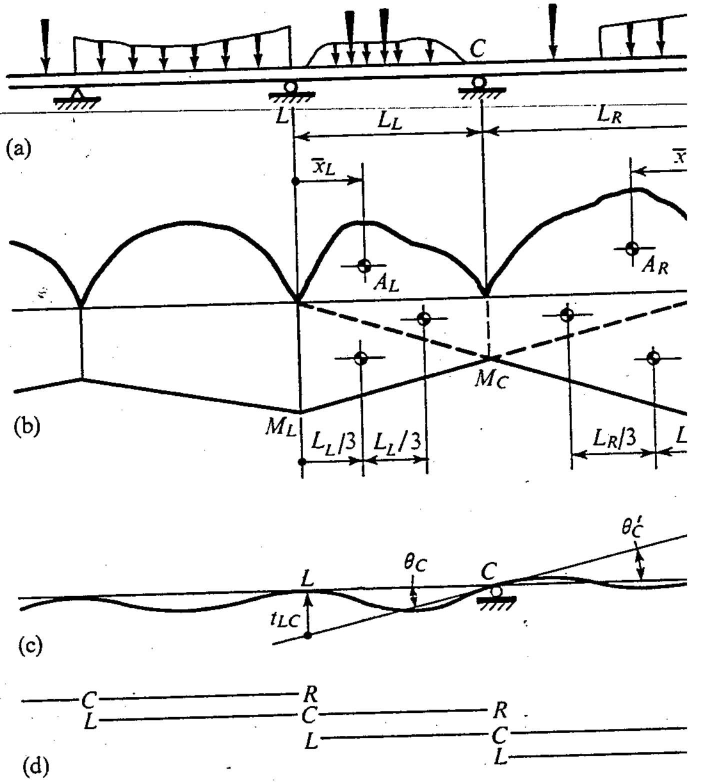

Fig. 12-18. Derivation of the three-mom

Consider a continuous beam, such as shown in Fig. 12-18(a), subjected to any transverse loading. For any two adjoining spans, as LC and CR, the bending-moment diagram is considered to consist of two parts. The areas A_L and A_R to the left and to the right of the center support C, Fig. 12-18(b), correspond to the bending-moment diagrams in the respective spans if these spans are treated as being simply supported. These moment diagrams depend entirely upon the nature of the known forces applied within each span. The other part of the moment diagram of known shape is due to the unknown moments M_L at the left support, M_C at the center support, and M_R at the right support.

Next, the elastic curve in Fig. 12-18(c) must be considered. This curve is continuous for any continuous beam. Hence the angles θ_C and θ_C', which define, from the respective sides, the inclination of the same tangent to the elastic curve at C, are equal. By using the second moment-area theorem to obtain t_{LC} and t_{RC}, these angles are defined as $\theta_C = t_{LC}/L_L$ and $\theta_C' = -t_{RC}/L_R$, where L_L and L_R are span lengths on the left and on

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מניחים שהאבנה
כים כאו ב־ C,R,L
ל המומנט ב־ A_L,A_R
ורתי ידוע ורה
M_R, M_C, M_L ם
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the right of C, respectively. The negative sign for the second angle is necessary since the tangent from point C is above the support R and a positive deviation of t_{RC} locates a tangent below the same support. Hence, following the steps outlined,

$$\theta_C = \theta'_C \qquad \text{or} \qquad t_{LC}/L_L = -t_{RC}/L_R$$

and

$$\frac{1}{L_L}\frac{1}{EI_L}\left(A_L\bar{x}_L + \frac{L_LM_L}{2}\frac{L_L}{3} + \frac{L_LM_C}{2}\frac{2L_L}{3}\right)$$
$$= -\frac{1}{L_R}\frac{1}{EI_R}\left(A_R\bar{x}_R + \frac{L_RM_R}{2}\frac{L_R}{3} + \frac{L_RM_C}{2}\frac{2L_R}{3}\right)$$

מומנט שלקט

where I_L and I_R are the respective moments of inertia of the cross-sectional area of the beam in the left and the right spans. Throughout each span, I_L and I_R are assumed constant. The term $\bar{x}_L$ is the distance from the left support L to the centroid of the area A_L, and $\bar{x}_R$ is a similar distance for A_R measured from the right support R. The terms M_L, M_C, and M_R denote the unknown moments at the supports.

Simplifying the above expression, the three-moment equation* is

$$L_LM_L + 2\left(L_L + \frac{I_L}{I_R}L_R\right)M_C + \frac{I_L}{I_R}L_RM_R$$
$$= -\frac{6A_L\bar{x}_L}{L_L} - \frac{6A_R\bar{x}_R}{L_R}\frac{I_L}{I_R} \qquad \textbf{(12-12)}$$

קורות רצופות

This equation 12-12 applies to continuous beams on unyielding supports, with the beam in each span of constant I. In a particular problem, all terms, with the exception of the redundant moments at the supports, are constant. A sufficient number of simultaneous equations for the unknown moments is obtained by successively imagining the supports of the adjoining spans as L, C, and R as shown in Fig. 12-18(d). However, in these equations the subscripts of the M's must correspond to the actual designation of the supports, such as A, B, C, etc. Also note that at pinned ends of beams the moments are known to be zero. Likewise, if a continuous beam has an overhang, the moment at the first support is known from statics. Fixed supports will be discussed in Example 12-16. For symmetrical beams symmetrically loaded, work may be minimized by noting that moments at symmetrically placed supports are equal.

נקבל משוו: למצ. נעלו.

In deriving the three-moment equation, the moments at the supports were assumed positive. Hence an algebraic solution of simultaneous equations automatically gives the correct sign of moments according to the convention for beams.

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* The three-moment equation was originally derived by E. Clapeyron, a French engineer, in 1857, and sometimes is referred to as Clapeyron's equation.

12-7. SPECIAL CASES (אפשרויות מיוחדות)

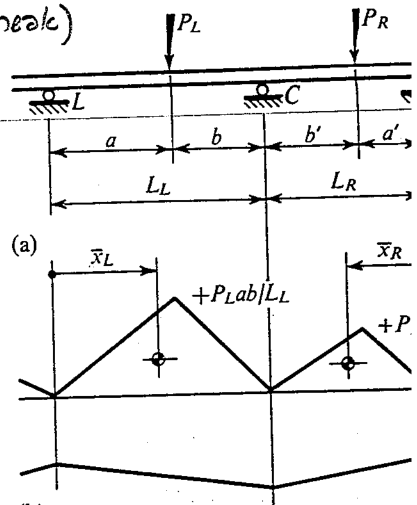


Fig. 12-19. Establishing the constants on the right hand of the three-moment equation for concentrated loads.

As a specific example of the evaluation of the constant terms on the right side of the three-moment equation, consider two adjoining spans loaded with the concentrated forces P_L and P_R, Fig. 12-19. Considering these spans simply supported, since the maximum moment in the left span is $+P_Lab/L_L$, and $\bar{x}_L = (L_L + a)/3$ one writes

כוחות מרוכזים

$$-6A_L\frac{\bar{x}_L}{L_L} = -6\overbrace{\left(\frac{L_L}{2}\right)\frac{P_Lab}{L_L}}^{A_L}\frac{(L_L + a)}{3L_L}$$
$$= -P_Lab\left(1 + \frac{a}{L_L}\right) \qquad \textbf{(12-13)}$$

Similarly, by interchanging the role of the dimensions a and b in the right span, i.e., by always measuring a's from the outside support toward the force,

$$-6A_R\frac{I_L}{I_R}\frac{\bar{x}_R}{L_R} = -P_Ra'b'\left(1 + \frac{a'}{L_R}\right)\frac{I_L}{I_R} \qquad \textbf{(12-14)}$$

If a number of concentrated forces occurs within a span, the contribution of each one of them to the above constant may be treated separately. Hence a constant term for the right side of the three-moment equation applicable for any number of concentrated forces applied within the spans is

כוחות מרוכזים

קיאני של (12-14) (12-15)

$$-\sum_i P_La_ib_i\left(1 + \frac{a_i}{L_L}\right) - \sum_i P_Ra'_ib'_i\left(1 + \frac{a'_i}{L_R}\right)\frac{I_L}{I_R} \qquad \textbf{(12-15)}$$

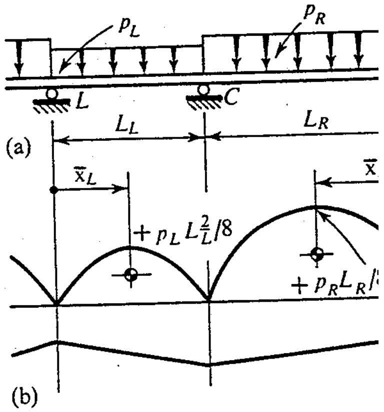


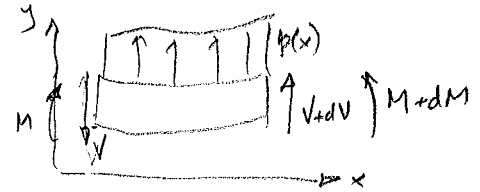
Fig. 12-20. Establishing the constants on the right hand of the three-moment equation for uniformly distributed loads.

where the summation sign designates the fact that a separate term appears for every concentrated force P_L in the left span, and similarly, for every force P_R in the right span. In both cases, a or a' is the distance from the outside support to the particular concentrated force, and b or b' is the distance to the force from the center support. If any one of these forces acts upward, the term contributed to the constant by such a force is of opposite sign.

The constant for the right side of the three-moment equation, when uniformly distributed loads are applied to a beam, is determined similarly. Thus, using the diagram in Fig. 12-20,

$$-6A_L\frac{\bar{x}_L}{L_L} = -6\left(\frac{2L_L}{3}\right)\left(\frac{p_LL_L^2}{8}\frac{L_L}{2L_L}\right) = -\frac{p_LL_L^3}{4} \qquad \textbf{(12-16)}$$

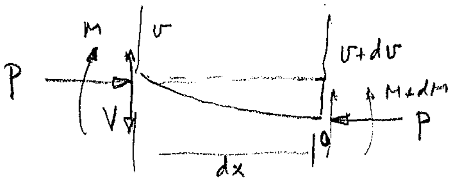
$$-6A_R\frac{I_L}{I_R}\frac{\bar{x}_R}{L_R} = -\frac{p_RL_R^3}{4}\frac{I_L}{I_R}$$

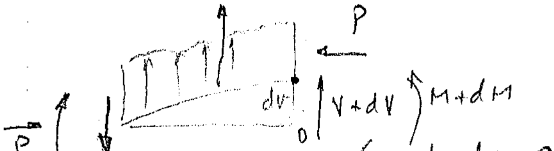


$$\cancel{V}+dV+pdx-\cancel{V}=0$$

$$\frac{dV}{dx}=-p$$

$$M=EI\,\frac{d^2v}{dx^2} \qquad -V=\frac{dM}{dx} \qquad -p=\frac{dV}{dx}$$





$$+Vdx+\cancel{M}+dM+Pdv-\cancel{M}-pdx\cdot\frac{dx}{2}=0$$

$$-\frac{dM}{dx}+\frac{Pdv}{dx}+p\cancel{\frac{dx^2}{2}}=+V$$

$$\overset{+}{\ominus}\frac{d^2M}{dx^2}\overset{+}{\ominus}\frac{Pd^2v}{dx^2}=\frac{dV}{dx}=\ominus p$$

$$EI\frac{d^4v}{dx^4}+\frac{Pd^2v}{dx^2}=p \qquad p\uparrow+$$

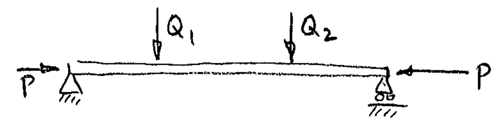
$$\Sigma M_0= \quad M+dM+Vdx-M-(-pdv)=0$$

$$\boxed{-p\frac{dv}{dx}}-\frac{dM}{dx}=V$$

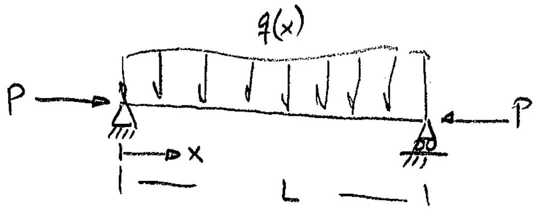
$$\frac{d}{dx}\left(-p\frac{dv}{dx}\right)-\frac{d^2M}{dx^2}=\frac{dV}{dx}=-q \quad\rightarrow\quad \frac{d^2M}{dx^2}+\frac{P}{EI}\left(EI\frac{d^2v}{dx^2}\right)=q \rightarrow \frac{d^2M}{dx^2}+\lambda^2M=q$$

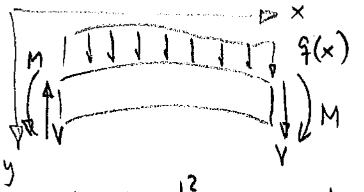
$$\frac{d}{dx}\left(-p\frac{dv}{dx}\right)-EI\,\frac{d^4v}{dx^4}=-q$$

$$EI\,v^{IV}+pv''=q \quad\rightarrow\quad v^{IV}+\lambda^2v''=q$$



בדרך כלל אנחנו וכולים להשתמש בסופרפוזיציה כדי לקבל את התגובה של קורה או מוט עם כמה עומסים. אבל כאשר יש קורה או מוט שבהם יש כוח צירי כמו P שפועל על המוט או קורה עם הכוחות Q_1, Q_2 אי אפשר לקבל את התגובות בסופרפוזיציה. זה בגלל שהתגובות מיוחסים בדרך לינארית ל Q_2, Q_1 אבל היחס לא לינארי ל-P.





$$M = EI\frac{d^2v}{dx^2} \quad -V = \frac{dM}{dx} \quad -q = \frac{dV}{dx}$$

אבל הכוח P מכניס לנוסחה מומנט של Pv. המומנט הזה קורה לגזירה של $P\frac{dv}{dx}$ וגם עומס מפורס של $-P\frac{d^2v}{dx^2} = q$. לכן הנוסחה שפועלת במצב הזה היא

כפונקציה של מומנט $\quad \frac{d^2M}{dx^2} + \lambda^2 M = q \qquad \lambda^2 = \frac{P}{EI}$

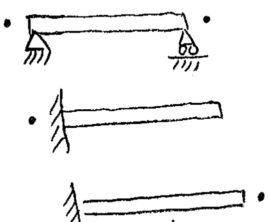
כפונקציה של התגובה $\quad \frac{d^4v}{dx^4} + \lambda^2\frac{d^2v}{dx^2} = q/EI$

אם EI פונקציה של x

$$\frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right) + P\frac{d^2v}{dx^2} = q$$

<u>פיתרון כאשר EI קבוע</u> ו- $q_0 = q(x)$

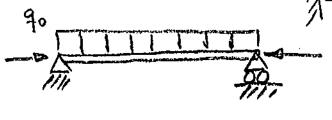
$$v(x) = A\cos\lambda x + B\sin\lambda x + Cx + D + \frac{q_0x^2}{2\lambda^2 EI}$$



בסמך נייד ונייח: $M = EI\frac{d^2v}{dx^2} = 0, \ v = 0$

בסמך רתום: $\frac{dv}{dx} = 0, \ v = 0$

בקצה חופשי: $M = 0$, $V = \frac{d^3v}{dx^3} + \frac{P}{EI}\frac{dv}{dx} = 0$



<u>בעמוד-קורה בקצוות נייח ונייד</u>

$$v = \frac{q_0}{\lambda^4 EI \sin\lambda L}\left\{(1-\cos\lambda L)\sin\lambda x - \left[(1-\cos\lambda x) + \frac{\lambda^2 x}{2}(L-x)\right]\sin\lambda L\right\}$$

<u>כאשר $\lambda \to 0$</u> $\quad v = \frac{q_0}{24EI}(x^4 - 2Lx^3 + L^3x) \quad$ ו- $\quad v(L/2) = \frac{5q_0L^4}{384EI}$

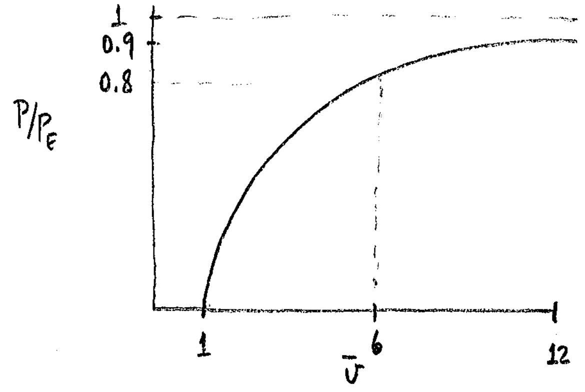
$$\frac{\quad}{384EI}\left\{\frac{\quad}{\frac{5}{384}(\lambda L)^4\sin\lambda L}\right\}$$

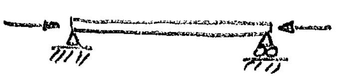
ותזוזה הולכת לאין סוף כאשר $(\lambda L)^4\sin\lambda L$ הולך לאפס, או כאשר $\sin\lambda L=0$.

$$\sin\lambda L=0 \Rightarrow \lambda L=n\pi \Rightarrow P=\frac{n^2\pi^2}{L^2}EI$$

לכן התזוזה לא תלויה ב- q_0. ה- P הקטן הוא $P_E=\frac{\pi^2EI}{L^2}$ וזה העומס הקריטי או עומס הקריסה

אם נסתכל במקדם $P/P_E=\frac{\lambda^2EI}{\pi^2EI/L^2}=\left(\frac{\lambda L}{\pi}\right)^2$ כפונקציה של $\bar{v}=\frac{v(L/2,\lambda)}{v(L/2,\lambda=0)}$



אם הסתכלנו רק בקורה  היינו מקבל $\frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right)+P\frac{d^2v}{dx^2}=0$

והפיתרון למשוואה הזאת הוא ש-

$$v=A\cos\lambda x+B\sin\lambda x+Cx+D$$

אם היינו משתמשים בתנאי הקצוות, זאת אומרת

$$v(0)=0 \longrightarrow A+D=0$$

$$M(x=0)=EI\frac{d^2v}{dx^2}=0 \longrightarrow -\lambda^2A=0$$

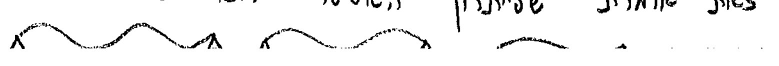
$$v(L)=0 \qquad A\cos\lambda L+B\sin\lambda L+CL+D=0$$

$$M(x=L)=EI\frac{d^2v}{dx^2}=0 \qquad -\lambda^2A\cos\lambda L-B\lambda^2\sin\lambda L=0$$

$$\begin{bmatrix}1 & 0 & 0 & 1\\ -\lambda^2 & 0 & 0 & 0\\ \cos\lambda L & \sin\lambda L & L & 1\\ -\lambda^2\cos\lambda L & -\lambda^2\sin\lambda L & 0 & 0\end{bmatrix}\begin{pmatrix}A\\B\\C\\D\end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\end{pmatrix}$$

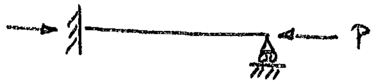
כדי למצוא פיתרון, או $A=B=C=D=0$ (זאת אומרת ש- $v\equiv0$ והוא הפיתרון הפשוט), או ש- ה DETERMINANT שווה לאפס. לכן, לפי העמוד השלישי, ה- DETERMINANT שווה ל-

$$L\begin{vmatrix}1 & 0 & 1\\ -\lambda^2 & 0 & 0\\ -\lambda^2\cos\lambda L & -\lambda^2\sin\lambda L & 0\end{vmatrix}=L\lambda^4\sin\lambda L=0$$

ועוד פעם $\sin\lambda L=0$. זאת אומרת שפיתרון האפשרי הוא ש- $v=B\sin\lambda x$. לכן אפשרויות לתזוזה הולות להיות  אבל כולם

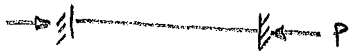
$I = \rho^2 A$

התזוזה המקסימלית הולכת וגודלת. ואם העומס P הוא P_E, אז העומד מתחיל לקרוס.



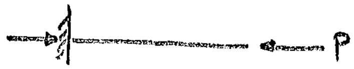
ישום תנאי הקצוות לעמוד-קורה כבצורה
נותן המשוואה הקריטית $\tan\lambda L = \lambda L$

$$2.05P_E = 2.05\frac{\pi^2EI}{L^2} = P_{\text{קריסה}}$$



המשוואה הקריטית $0 = \sin\frac{\lambda L}{2}\left(\sin\frac{\lambda L}{2} - \frac{\lambda L}{2}\cos\frac{\lambda L}{2}\right)$

$$4P_E = \frac{4\pi^2EI}{L^2} = P_{\text{קריסה}}$$

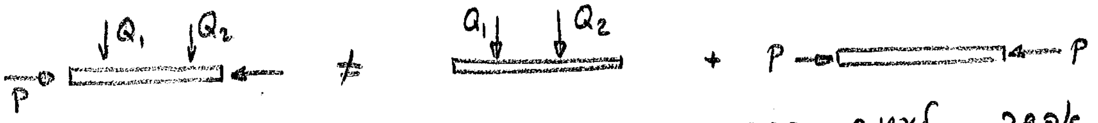


המשוואה הקריטית $\cos kL = 0$

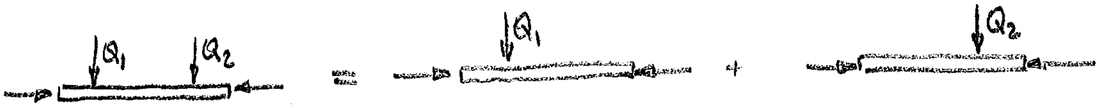
$$\frac{1}{4}P_E = \frac{1}{4}\frac{\pi^2EI}{L^2} = P_{\text{קריסה}}$$

<u>כאן, לא כללנו את עומס הגזירה</u>. אם הגזירה נכללת העומס הקריטי שגורם לקריסה יהיה יותר נמוך. רואים שתנאי הקצוות משפיעים לעומס הקריטי.

אך על פי ש-טו אפשר להשתמש בסופרפוזיציה כזאת



אפשר לעשות ככה

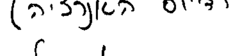


הערות קבלנו את התוצאות האלה על תנאי שנישאר בחלק האלסטי הלינארי של החומר

בדרך כלל, $P_{cr} = C\frac{\pi^2EI}{L^2}$ והמספר C תלוי בתנאי הקצוות. אם $C=(L/L')^2$

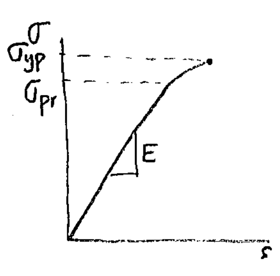
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2EI}{AL'^2} = \frac{\pi^2E(\rho^2A)}{A(L')^2} = \frac{\pi^2E}{(L'/\rho)^2}$$

כאן L' = אורך האפקטיבי של הקורה. לדוגמה



$C=1/4$ לכן $L'=2L$

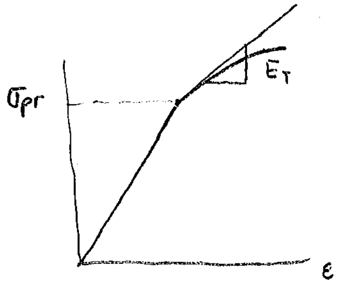
- $I = \rho^2A$ ו-ρ הוא רדיוס ההסתובבות (רדיוס האנרציה)
- המקדם L/ρ הוא תלוי בגאומטרית הקורה וקוראים לזה מקדם הדקות
- רואים ש- $\sigma_{cr}(L'/\rho)^2 = \pi^2E$



$$\left(L'/\rho\right)_{cr} = \pi\sqrt{E/\sigma_{pr}}$$

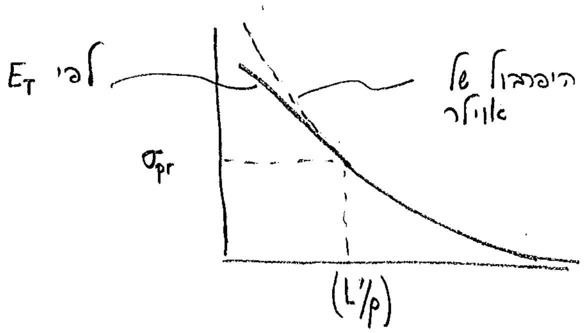
וכולים לקבל את הערך הקריטי. אם $\left(L'/\rho\right)_{cr} > \left(L'/\rho\right)_{\text{גאומ}}$ המאמץ הקריטי לפי: יהיה שווה ל- P_{cr}/A אבל יהיה יותר גדול.

לתקן בזה, כדי לעבוד ב $\left(L'/\rho\right)_{cr} > \left(L'/\rho\right)_{\text{גאומ}}$ משתמשים במודול הטנגנטי אחרי נקודת הפרופורציה עד σ_{yp}.

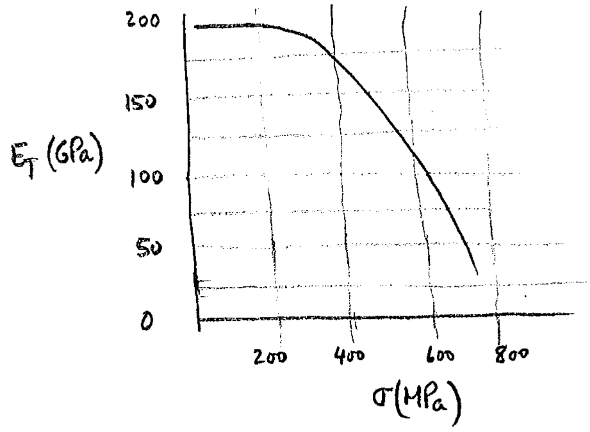


$$\sigma_{t,cr} = \frac{\pi^2 E_T}{\left(L'/\rho\right)^2}$$

1- בדרך ניסיונות מקבלים את E_T.



לפלדה מסוימת קיבלו את הגרף בין E_T למאמץ



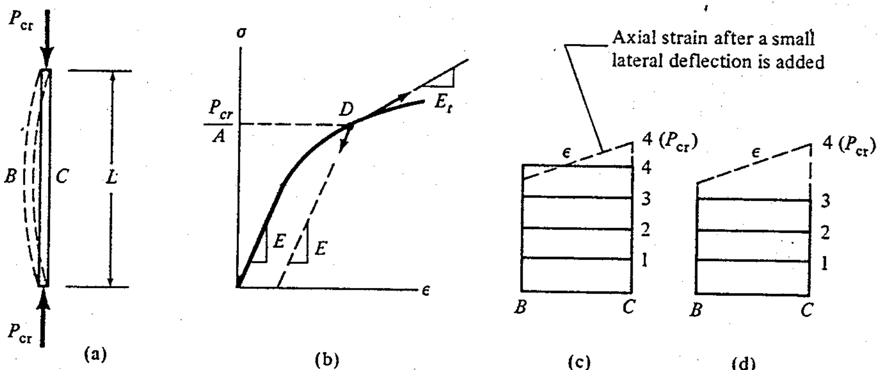


FIGURE 12.4.1. (a) Buckling of a pin-ended column under centroidal axial load. (b) Compressive stress–strain diagram, showing loading and unloading paths from point D, which corresponds to inelastic buckling. (c) Distribution of axial strain across the column at increasing load levels, according to double-modulus theory. (d) Possible distribution of axial strain across the column in tangent modulus theory.

cross section. Therefore, the column must bend before reaching the double-modulus load. But this is in contradiction to a basic assumption in double-modulus theory. The contradiction is resolved by noting that lateral deflection may occur *simultaneously* with application of the last increment of load. There need be no unloading on the convex side, and modulus E_t may prevail all across the section (Fig. 12.4.1d). Under near-perfect test conditions the collapse load slightly exceeds the theoretical tangent-modulus load, but it does not reach the double-modulus load.

In summary, inelastic buckling of a straight, axially loaded column does not occur at a unique value of axial load P. Instead, buckling begins at the tangent-modulus load and is complete (meaning that collapse takes place) before the theoretical double-modulus load is reached. Tests of real columns, which have larger imperfections than laboratory specimens, are in excellent agreement with tangent modulus theory.

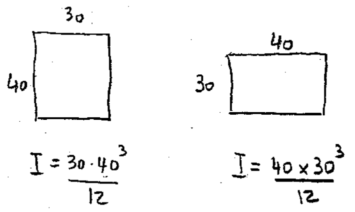
Euler did not realize that bending stiffness EI could be calculated rather than obtained by experiment. However, he anticipated Engesser by remarking in 1757 that EI represents a resistance to bending that need not pertain only to elastic bodies [12.4].

Example 12.4.1. A column has a solid rectangular cross section, 40 mm by 30 mm. It is 200 mm long, free at the top, and fixed at the base. Material properties are shown in Fig. 12.4.2. What centroidal axial compressive load at the top will make the column buckle?

The appropriate equation is $P_{cr} = \pi^2 EI/4L^2$, where

$$I = \frac{bh^3}{12} = \frac{40(30)^3}{12} = 90{,}000 \text{ mm}^4 \qquad (12.4.2)$$

because buckling will take place about the weaker axis of the cross section



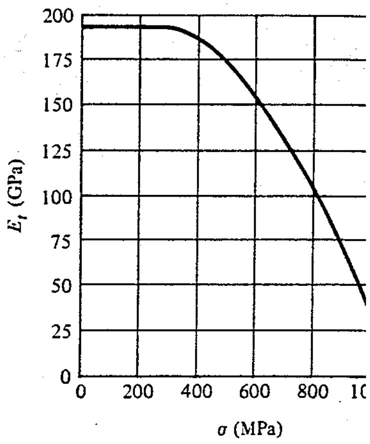


FIGURE 12.4.2. Tangent modulus E_t versus compressive stress σ for a particular steel, obtained from a stress–strain plot.

P_{cr} = 1077 kN, or $\sigma_{cr} = P_{cr}/A$ = 898 MPa. This stress is consi higher than the proportional limit stress, which appears to be about 3 in Fig. 12.4.2. Therefore, buckling is inelastic, the effective modu pends on load, and an iterative method of calculation is needed to fi as follows.

Assume that σ_{cr} will be, say, 600 MPa. At this stress, Fig. 12.4. E_t = 160 GPa. Hence

$$P_{cr} = \frac{\pi^2 E_t I}{4L^2} = 888 \text{ kN} \qquad \frac{P_{cr}}{A} = 740 \text{ MPa} \qquad ($$

As P_{cr}/A exceeds the assumed σ_{cr} of 600 MPa, another trial is neede

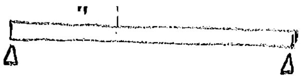
Assume that σ_{cr} = 660 MPa; then

$$E_t = 142 \text{ GPa} \qquad P_{cr} = \frac{\pi^2 E_t I}{4L^2} = 788 \text{ kN} \qquad \frac{P_{cr}}{A} = 657 \text{ MPa} \qquad ($$

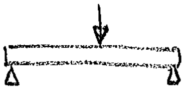
Now the assumed value of σ_{cr} agrees well enough with the calculated and P_{cr} = 788 kN is accepted as the tangent modulus buckling load.

Creep Buckling. As the name implies, creep buckling theory deals material that creeps, that is, a material whose strain changes with time at c stress. A creeping column may display a small but gradually increasing deflection, then fail suddenly by buckling. The phenomenon is explai examination of creep curves (Fig. 12.4.3). One may enter the creep cu a certain time, say t_1, and read the strain for each of several stress Stress–strain data thus obtained is then plotted as a stress–strain curve, s the curve labeled t_1 in Fig. 12.4.3. Repetition of this procedure at severa produces a set of isochronous stress–strain curves (stress versus strain at c time). These curves show that at a given stress level, the tangent m decreases with time. This implies that however light the load, a creeping c

מאמץ הדינמי [illegible]



משקל W שנופל על קורה גורם למאמצים דינמיים והמאמצים האלה יותר גבוהים ממצב שמשקל W לוחץ על הקורה בלבד



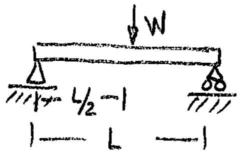
העומס הדינמי נחשב N

$$P_{dyn} = W\left(1 + \sqrt{1 + 2H/\Delta_{st}}\right)$$

h - זה מרחק האנכי בין המשקל W לקורה.

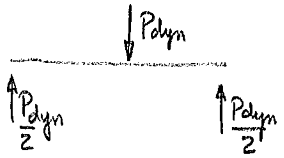
Δ_{st} - זה התזוזה הסטטית של הקורה שבאה מיישום המשקל על הקורה.

דוגמא ...

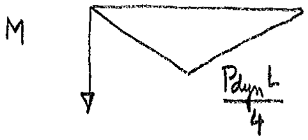


$$\Delta_{st} = \frac{WL^3}{48EI}$$

המאמץ הדינמי כאן -



① למצוא העומס הדינמי
② למצוא המומנט המקס.
③ למצוא המאמץ



$$\sigma_{\text{מקס}} = \frac{My}{I} = \frac{P_{dyn}L\cdot(h/2)}{4\cdot bh^3/12} = \frac{3P_{dyn}\cdot L}{2bh^2}$$

<u>הערות</u> ליישום הנוסחה הזאת.

- אין איבוד אנרגיה בנקודת הנגיפה או בסמכים
- אנרגית המערכת נחשבת כאפס (משקל המערכת לא התחשב)
- לתזוזה של המערכת יש יחס לינארי לגובה העומס המיושם ואין אכפת אם זה עומס סטטי או דינמי

1. Write the second-order differential equation for the bending of the column shown in Fig. P1-2 and use it to determine the critical load of the column. At its lower end the column is completely fixed. At the upper end the column is prevented from rotating, but free to translate laterally. (ans: $P_{cr}=\pi^2EI/L^2$)

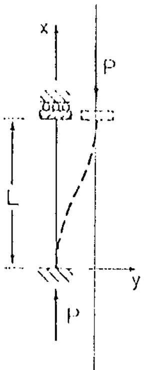
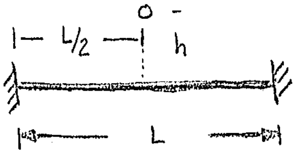


Fig. P1-2

2. Find an expression for the maximum stress when a ball weighing W Newtons is dropped onto a fixed-fixed beam.



3. . A linearly elastic beam-column having a flexural rigidity EI, is subjected to a thrust P and a moment M_0 as shown in Fig. A below.

(a) Determine the lateral displacement v(x).

(b) From part (a), write the solution for the system subjected to a force P acting as shown in Fig. B.

(c) Determine Δ_c, the horizontal displacement of point C, assuming small rotations. (Assume also that the horizontal displacement of point B is negligible).

(d) Determine the bending moment M(x).

(g) Explain why, although the beam is made of a linearly elastic material, the results of part (c) are non-linear.

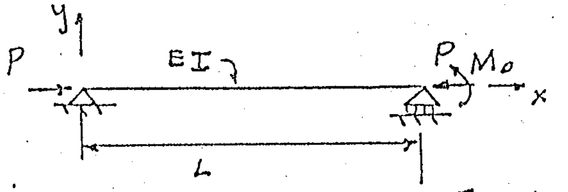


Fig. A

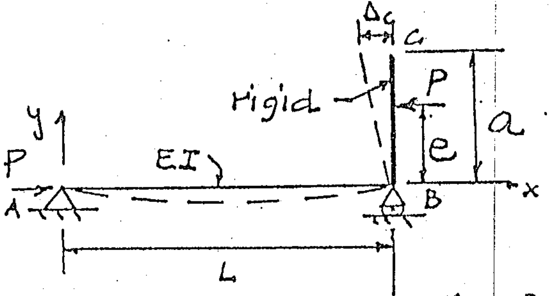


Fig. B

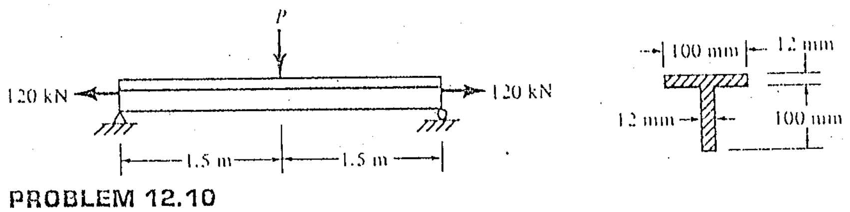
Answers :

$$(a)\ y(x) = -\frac{M_0}{P}\left[\frac{\sin Kx}{\sin KL} - \frac{x}{L}\right],\ K^2 = \frac{P}{EI}$$

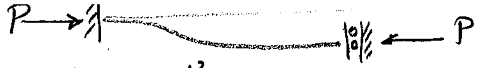
$$(c)\ \Delta_c = \frac{ac}{L}\ [1 - L\sqrt{P/EI}\ \cot(L\sqrt{P/EI})\ ] = \frac{ac}{L}\ (1 - KL\ \cot KL\ )$$

(d) $M(x)=M_0 \sin kl/\sin kL$

*12.10 A T section carries an axial tensile force of 120 kN, applied through the centroid of the cross section. The allowable stress in tension or compression is 130 MPa. Let $E = 200$ GPa. What transverse force P can be applied at midspan if the beam is
(a) Stem down (as shown)?
(b) Stem up?



PROBLEM 12.10



מהנוסחה שמצאנו בשעור שעבר והפיתרון הוא

$$EI\frac{d^4}{dx^4}v + P\frac{d^2v}{dx^2} = 0$$

$$v = A\cos\lambda x + B\sin\lambda x + Cx + D$$

$$\lambda^2 = P/EI$$

תנאי הקצוות

① תזוזה $v(x=0)=0$

② זווית $\frac{dv}{dx}(x=0)=0$

③ זווית $\frac{dv}{dx}(x=L)=0$

④ גזירה : $EI\frac{d^3v}{dx^3}(x=L) + P\frac{dv}{dx}(x=L)=0$

מפני ש $\frac{dv}{dx}=0$ ב-$x=L$, לכן ④ נותנת $\frac{d^3v}{dx^3}=0$ ב-$x=L$

מקבלים

$A+D=0$ מ-①

$\lambda B + C = 0$ ②

$-A(\lambda\sin\lambda L) + B(\lambda\cos\lambda L) + C = 0$ ③

$A\lambda^3\sin\lambda L - B\lambda^3\cos\lambda L = 0$ ④

לכן

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \lambda & 1 & 0 \\ -\lambda\sin\lambda L & \lambda\cos\lambda L & 1 & 0 \\ \lambda^3\sin\lambda L & -\lambda^3\cos\lambda L & 0 & 0 \end{bmatrix}\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

כדי לקבל תוצאות לא טריוויאליות הדיטרמיננט צריך להיות אפס. לפי העמוד הרביעי

$$-1\begin{pmatrix} 0 & \lambda & 1 \\ -\lambda\sin\lambda L & \lambda\cos\lambda L & 1 \\ \lambda^3\sin\lambda L & -\lambda^3\cos\lambda L & 0 \end{pmatrix} = 0$$

לכן

$$\lambda^4\sin\lambda L = 0$$

$\lambda\neq 0$ (זה הפיתרון $v=0$) ; לכן $\sin\lambda L = 0 \Rightarrow \lambda L = n\pi$ או $P_E = \left(\frac{\pi}{L}\right)^2 EI$

3.

$v = A\cos\lambda x + B\sin\lambda x + Cx + D$ $\quad\lambda^2 = P/EI$

לפי הסכם הסימנים

① $v(x=0)=0$ ③ $v(x=L)=0$

② $EI\frac{d^2v}{dx^2}(x=0) = M = 0$ ④ $EI\frac{d^2v}{dx^2}(x=L) = -M_0$

$A+D=0$ מ-①

$-A\lambda^2 = 0$ ②

$A\cos\lambda L + B\sin\lambda L + CL + D = 0$ ③

$EI(-A\lambda^2\cos\lambda L - B\lambda^2\sin\lambda L) = -M_0$ ④

אם $\lambda\neq 0$ $\overset{(2)}{A=0} \Rightarrow \overset{(1)}{D=0} \Rightarrow B\sin\lambda L + CL \overset{(3)}{=} 0 \Rightarrow B(EI\lambda^2\sin\lambda L) \overset{(4)}{=} M_0 \Rightarrow B = \frac{M_0}{EI\lambda^2\sin\lambda L}$

ו $C = -\frac{M_0}{EI\lambda^2 L}$

לכן,

$$v = \frac{M_0}{EI\lambda^2}\left(\frac{\sin\lambda x}{\sin\lambda L} - \frac{x}{L}\right)$$

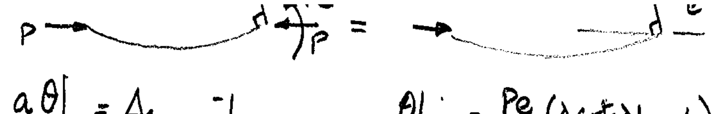
$$\boxed{v = \frac{M_0}{P}\left(\frac{\sin\lambda x}{\sin\lambda L} - \frac{x}{L}\right)}$$

לקבל $M(x)$, גוזר פעמיים לפי x: $EI\frac{d^2v}{dx^2} = M = -EI\frac{M_0\lambda^2}{P}\frac{\sin\lambda x}{\sin\lambda L} = M(x) = -M_0\frac{\sin\lambda x}{\sin\lambda L}$

הסימן נכון לפי הסכם הסימנים.

כדי לקבל Δ_c, צריך למצוא $\theta|$. $\theta| = \frac{dv}{dx} = M_0\left(\frac{\lambda\cos\lambda L}{} - \frac{1}{}\right) = M_0(\lambda\cot\lambda L - 1)$

לפי הקורה

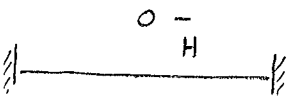


לכן $\theta\big|_{x=L} = \frac{Pe}{P}\left(\lambda\cot\lambda L - \frac{1}{L}\right)$ ו- $a\theta\big|_{x=L} = \Delta_c$ כי העמוד קשיח וקשור לקורה בזווית אנכית של 90°. לכן,

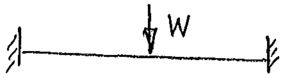
$$\Delta_c = \frac{ae}{L}\left(\lambda L\cot\lambda L - 1\right)$$

2.

לפתור את



פותרים את



והתזוזה היא $\Delta_{st} = v = \frac{WL^3}{192EI}$

לפי הנוסחה $P_{dyn} = W\left(1+\sqrt{1+2H/\Delta_{st}}\right)$, בגלל שמצאנו את Δ_{st}, מצאנו את P_{dyn}.

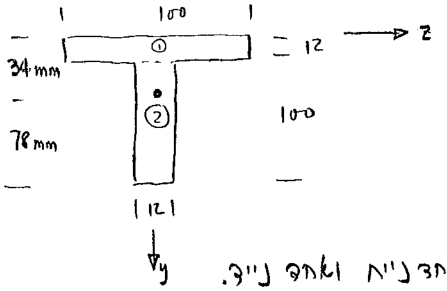
למצוא את המומנט המקס. לפי הנוסחה שנתונה ב- Cook & Young, $v=\frac{P}{48EI}(3Lx^2-4x^3)$

ו- $M = EI\frac{d^2v}{dx^2}$. לכן, $M = \frac{P}{48}(6L-24x)$. ב- $x=0$ $M=\frac{PL}{8}$; ב- $x=L/2$ $M=-\frac{PL}{8}$, וגרף המומנט

לכן יש שלושה מקומות שבהם המומנט הוא מקסימלי והמאמץ המקסימלי הוא $\sigma = \frac{Mc}{I} = \frac{PL}{8}\frac{6}{bh^2}$

$$\frac{3LW\left(1+\sqrt{1+\frac{2H}{WL^3/192EI}}\right)}{4bh^3} = \frac{3}{4}\frac{PL}{bh^3} = \sigma_{max}$$

12.10



שטח החתך הוא $2(100\times12)\,mm = 2400\,mm^2$

מרכז הכובד הוא

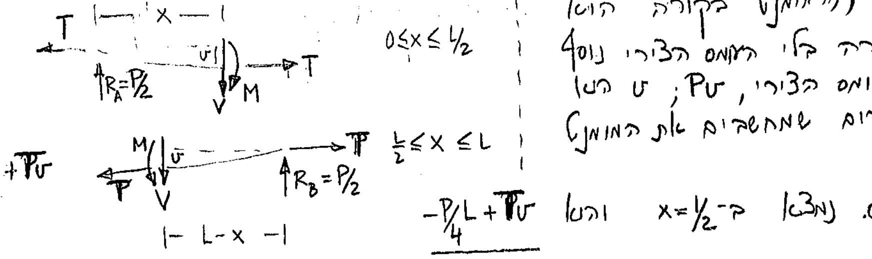
	A	y	Ay
①	100x12	6	7200
②	100x12	62	74400
	2400		81600

$\bar{y} = \frac{\Sigma Ay}{A} = 34\,mm$

העומס הקריסה הוא $T_E = \pi^2\frac{EI}{L^2}$ מפני הסמכים, אחד נייח ואחד נייד.

בגלל העומס הצירי, T, המומנט בקורה הוא המומנט שנמצא בקורה בלי העומס הצירי נוסף המומנט שבא מהעומס הצירי, Pv; v הוא התזוזה באותו מקום שמחשבים את המומנט

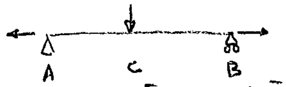
$M = -\frac{P}{2}x + Tv$



$M = -\frac{P}{2}(L-x) + Pv$

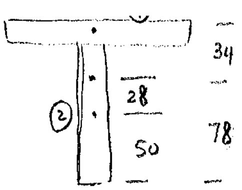
לכן, המומנט המקס. נמצא ב- $x = L/2$ והוא $-\frac{P}{4}L + Pv$

כדי למצוא את $v\big|_{x=L/2}$, יודעים משיעור בכיתה ש- $v = A\cosh\lambda x + B\sinh\lambda x + Cx + D$ ו- $\lambda^2 = \frac{T}{EI}$ בכל חלק מהקורה.



זאת אומרת שצריך להשתמש ב $v_L = A\cosh\lambda x + B\sinh\lambda x + Cx + D$ בין A-C, ו- $v_R = \bar{A}\cosh\lambda(x-L) + \bar{B}\sinh\lambda(x-L) + \bar{C}(x-L) + \bar{D}$ בין C-B. תנאי השפה $v_L(x=0)\overset{(1)}{=}0$, $v_L''(x=0)\overset{(2)}{=}0$, $v_R(x=L)\overset{(3)}{=}0$, $v_R''(x=L)\overset{(4)}{=}0$. ב- C $v_L(x=L/2)\overset{(5)}{=}v_R(x=L/2)$; $v_L'(x=L/2)\overset{(6)}{=}v_R'(x=L/2)$; $v_L''(x=L/2)\overset{(7)}{=}v_R''(x=L/2)$; $EIv_L'''(x=L/2) - Tv_L''(x=L/2)\overset{(8)}{=}EIv_R''' - Tv_R'' + P$. אלה תנאי רציפות, אלה שמונה משוואות לקבל $A,B,C,D,\bar{A},\bar{B},\bar{C},\bar{D}$. מ- (1), (2), (3), (4) $\Leftarrow A=D=\bar{A}=\bar{D}=0$. מ- (7) $\Leftarrow B=-\bar{B}$. מ- (7),

$|_{x=L/2}$ … $/EI$ … $2EI\,\lambda^3\cosh\lambda L/2$ … $2EI\lambda^2$



I	A	d^2	I_{tt}
$\frac{1}{12}(100)(12)^3$	1200	$(28)^2$	955200
$\frac{1}{12}(12)(100)^3$	1200	$(28)^2$	1940800

$2{,}896\times10^6\,mm^4$

$$v\Big|_{x=L/2} = P\left\{\frac{\sinh\lambda L/2}{2T\lambda\cosh\lambda L/2} - \frac{L}{4T}\right\}$$

צריך לדעת λ לחשב את v. גם צריך לדעת I.

$$\lambda = \sqrt{\frac{120{,}000}{(200\times10^9)(2.896\times10^{-6})}} = .4552 \quad , \quad T_E = \pi^2\frac{EI}{L^2} = 635164N > T.$$

$$v\Big|_{x=L/2} = P\left\{\frac{0.7371}{2(120{,}000)(1.2423)(.4552)} - \frac{3}{4(120000)}\right\}$$

$$= P(8.19\ 10^{-7}m)$$

לכן המומנט יהיה $M = -\frac{PL}{4} + T\cdot P\times8.19\times10^{-7}$

$M = P\left[-0.75 + .09828\right] = -.65172P$

התגובות במקום שהמומנט מקס.

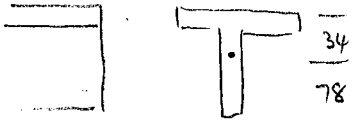
$$\sigma = T/A + \frac{Mc}{I}$$

אם צורת החתך כמו משורטט משמאלה:

$\sigma_{עליון} = \frac{120{,}000}{2400\times10^{-6}} - \frac{0.65172(.078)P}{2.896\times10^{-6}} = -130\times10^6$ $\underline{\underline{P = 10{,}250\ N}}$

$\sigma_{תחתון} = \frac{120{,}000}{2400\times10^{-6}} + \frac{0.65172(.034)P}{2.896\times10^{-6}} = 130\times10^6$ $P = 10{,}451N$

אם צורת החתך כמו משורטט משמאלה:

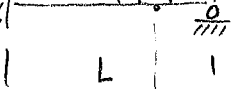


$\sigma_{עליון} = \frac{120{,}000}{2400\times10^{-6}} - \frac{0.65172(.034)P}{2.896\times10^{-6}} = -130\times10^6$ $P = 23{,}515N$

$\sigma_{תחתון} = \frac{120{,}000}{2400\times10^{-6}} + \frac{0.65172(.078)P}{2.896\times10^{-6}} = 130\times10^6$ $\underline{\underline{P = 4556\ N}}$

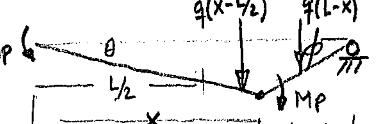
לכן, העומס הגבוה ביותר בא כאשר לשטח החתך יש צורה כזה ⊥

11.26



כאשר שהמומנט מגיע ל-M_p, פרק פלסי קורה בסמך הרתום וגם בנקודה B

צריך למצוא את מקום נקודה B ומהוא העומס המפורס שגורם ל-M_p



$\theta x = \phi(L-x)$

$\frac{\theta x}{L-x} = \phi$

$s = \frac{1}{2}(x+L/2)$

לפי עקרון עבודת ווירטואלית

$$M_p\theta + M_p(\theta+\phi) = q(x-L/2)\,\theta s + q(L-x)\left(\frac{L-x}{2}\right)\phi$$

מחליפים φ ו-s ביחסים ופותרים בשביל q

$$q = \frac{8M_p(2L-x)}{(5Lx-4x^2-L^2)L}$$

כדי למצוא את ה-q שגורם ל-M_p, גוזרים q לגבי x

ומקבלים ש- $0 = 4x^2-16Lx+9L^2$. מזה יש תשובה אחת

והיא $x = \frac{16-\sqrt{112}}{8}L$. שמים x בנוסחה של q כדי לקבל $q = \underline{\frac{19.18M_p}{L^2}}$

בחלק BC $M\big($ $\downarrow q(L-x)$ ונתן ש-R שווה $\underline{6.194M_p}/L$

$$v = A\cos\lambda x + B\sin\lambda x + Cx + D$$

לפי תנאי השפות של הקורה.

כדי לקצר את הדרך לפתור את הבעיה נזכיר ש-

$$EI\frac{d^2v}{dx^2} + Pv = M_t$$

כאשר $P=0$

$$EI\frac{d^2v_t}{dx^2} = M_t$$

פה M_t הוא המומנט שבא מעומסים אנכיים בלבד, ו- v_t היא התזוזה בכוון y שבאה מהעומסים האנכיים. מניחים שצורת עיקום התזוזה יהיה אותו דבר עם או בלי הכוח האופקי P.

לכן, לפי תנאי השפה, כמו קורה על סמך נייד וסמך נייח, מניחים ש- $v = \bar{v}\sin\frac{\pi x}{L}$ ו- $v_t = \bar{v}_t\sin\frac{\pi x}{L}$. $\bar{v}$, ו- $\bar{v}_t$ הם ה- AMPLITUDE של v ו- v_t. לפי הנוסחאות למעלה

$$-EI\frac{\pi^2}{L^2}\bar{v} + P\bar{v} = -EI\frac{\pi^2}{l^2}\bar{v}_t$$

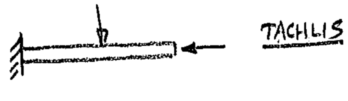
מפני ש- $P_E = \frac{\pi^2 EI}{L^2}$, $\bar{v}(P-P_E) = -P_E\bar{v}_t$ או

$$\bar{v} = \frac{\bar{v}_t}{1 - P/P_E}$$

המומנט גם כן עוקב אחרי נוסחא דומה לזה

$$M = \frac{M_t}{1 - P/P_E}$$

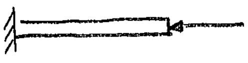
אם P חיובי, $\bar{v}_t < \bar{v}$ ו- $M_t < M$. אם P שלילי, $\bar{v}_t > \bar{v}$ ו- $M_t > M$



פותר

וקבל את v מתחת העומס או בכול מקום אחר. קורא לזה $\bar{v}_t$.

קבל P_E

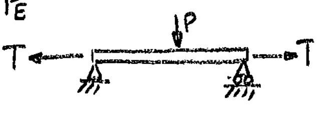


לקורה עמוסה כמו בצורה

v יהיה

$$v = \frac{\bar{v}_t}{1 - P/P_E}$$

בתרגיל בית 12.10



כי P היא שלילי.

$$v = \frac{\bar{v}_t}{1 + T/T_E} \quad \text{ו-} \quad , \bar{v}_t\Big|_{x=L/2} = \frac{PL^3}{48EI} \quad , T_E = \frac{\pi^2 EI}{L^2}$$

כמו שמצאנו בעבר שסך כל האנרגיה הוא האנרגיה שבאה מהמאמצים הפנימיים והאנרגיה שבאה מהכוחות החיצוניים. הפוטנציאל הטוטלי

$$\Pi = U - W_e = U + \Omega$$

U - האנרגיה העיבורית ו- W_e העבודה מהכוחות החיצוניים. לשיווי משקל

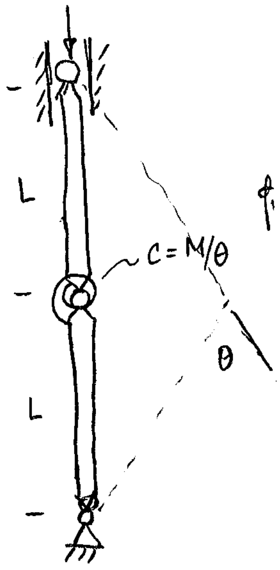
$$\delta\Pi = \delta U - \delta W_e = 0$$

למצב יציב (STABILITY), צריך לבדוק $\delta^2\Pi$, כי לפי TAYLOR SERIES

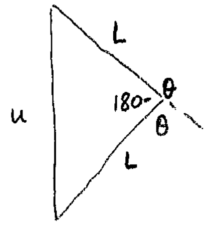
$$\Delta\Pi = \delta\Pi + \frac{1}{2!}\delta^2\Pi + \frac{1}{3!}\delta^3\Pi + \dots$$

שיווי משקל דורש $\delta\Pi = 0$ ומצב יציב נטרלי דורש ש- $\delta^2\Pi = 0$ וזה נותן את העומס הקריטי

דוגמא: למצוא העומס הקריטי לדוגמא דרגה חופשי אחת.
שתי" המוטות קשיחים עם תנאי השפה נ"ד.
קפיץ ליניארי סיבובי בקשיחות $C=M/\theta$ קושר
את שתי המוטות, θ זווית בין המוטות, ו-M המומנט הקפיץ



אם הכוח P זז למטה



$$u = \sqrt{2L^2 - 2L^2\cos(180-\theta)}$$
$$= 2L\cos\theta/2$$

מפני שהמוטות קשיחים $\Leftarrow \delta U = 0$

עבודת הכוח P והמומנט M $\Leftarrow P(2L-2L\cos\theta/2) - \frac{1}{2}M\theta$; עבודת הקפיץ הוא - הפוטנציאל

ו- $\cos\theta/2 \sim 1 - (\theta/2)^2 1/2! + \dots$

לכן $2LP(1-\cos\theta/2) - \frac{1}{2}C\theta^2 = 2LP(\theta^2/8) - \frac{1}{2}C\theta^2 = \Pi$

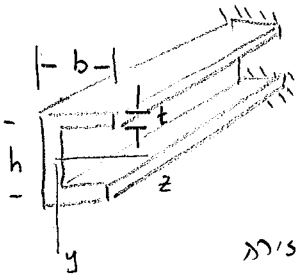
$$0 = (\frac{LP\theta}{2} - C\theta)\delta\theta = \delta\Pi$$

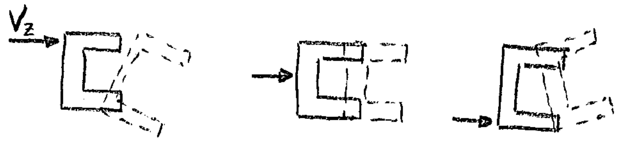
לכן, או $\theta=0$ או $P=2C/L$. צריך לבדוק $\delta^2\Pi$

$$0 = (\frac{LP}{2} - C)\delta\theta^2 + \theta(\frac{LP}{2}-C)\delta^2\theta = \delta^2\Pi$$

מפני ש- $\delta^2\theta \neq 0$ כאשר $\theta=0$ $\Leftarrow$ $0 = \frac{LP}{2} - C$ וזה $P_{\text{קריטי}} = \frac{2C}{L}$

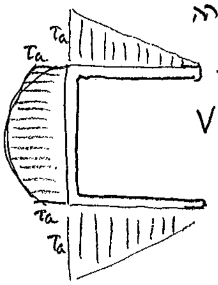
מרכז גזירה - מקום שבו הגזירה צריך לעבור כדי שיקויים [illegible] 1
פיתול



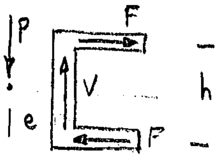


אם לפלטה יש שני צירים סימטריים, צומת הצירים הוא המרכז הגזירה

אם יש רק ציר אחד כמו בפרופיל כאן מפני שמאמץ הגזירה הוא כמו בצורה



- מניחים שהקירות דקים, שהמאמץ הגזירה ברגלי האופקיות הוא ליניארי מצד החופשי ובמסכת המאמץ פרבולי. הכוח בכול רגל הוא $\frac{\tau_a}{2}bt$ והגזירה האנכית $V=\int_{-h/2}^{h/2}\tau t\,dy$
- מניחים שיכולים למדוד הרגליים והמסכת בקו האמצעי; ומאמץ הגזירה טנגנט לקיר
- מאמץ הכפיפה מתמיד בכיוון העובי;



לכן, הפיתול שבא מהכוחות F הוא $F\cdot h$. כדי למנוע את הפיתול, צריך לשים כוח P, במרחק e מהמסכת. לפי שווי משקל $P=V$ ו- $Pe=Fh$

$$e=\frac{Fh}{V}=\frac{\tau_a bt/_2\cdot h}{V}$$

$\frac{\tau_a}{2}$ - מאמץ הגזירה הממוצע של הרגל וכמו שלמדנו בתורת החוזק 1

$$\tau_a=\frac{VQ}{It}=\frac{q}{t}$$

$Q = \int ydA$ של הרגל $= \int yt\,ds = \bar{y}A = \frac{h}{2}\cdot bt$. קוראים לזה המומנט הסטטי של חלק השטח המסוים

- $\bar{y}$ - המרחק ממרכז הכובד של הרגל לציר הניטרלי של הגוף
- q - זרם הגזירה

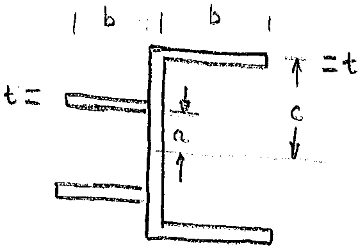
$$e=\frac{V(h/_2\cdot bt)}{It}\cdot\frac{bt/_2\cdot h}{V}$$

אם הרגל דקה מאוד $2bt\cdot\left(\frac{h}{2}\right)^2+\frac{th^3}{12}\cong I$ אז. $e=\frac{b^2h^2t}{4I}$

- המרחק e תלוי בשטח ולא בעומס ולא במקום שהעומס מיושם.

דוגמה

$$F_1\cdot 2c-F_3\cdot 2a=Pe$$
$$P=V$$



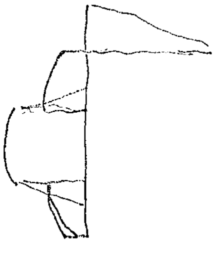
$$\frac{\tau_a}{2}\cdot bt\cdot 2c-\frac{\tau_b}{2}\cdot bt\cdot 2a=Ve$$

$$\frac{1}{2}\frac{VQ_a}{It}\cdot bt\cdot 2c-\frac{1}{2}\frac{VQ_b}{It}\cdot bt\cdot 2a=Ve$$

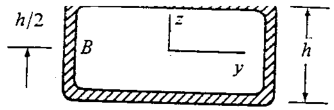
$$\frac{1}{2}V\frac{(bt\cdot c)}{It}\cdot bt\cdot 2c-\frac{1}{2}V\frac{(bt\cdot a)}{It}\cdot bt\cdot 2a=Ve$$

$$e=\frac{1}{2}\left\{\frac{2t^2(b^2c^2-b^2a^2)}{It}\right\}$$

$$I=\frac{t(2c)^3}{12}+2bt\cdot c^2+2bt\cdot a^2$$



$$e = 3b^2(c^2-a^2)$$

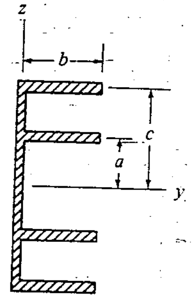


PROBLEM 9.31

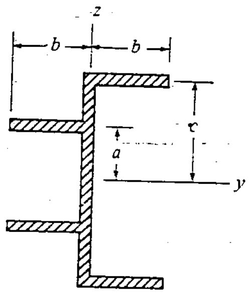
n 9.7

***9.33, 9.34, 9.35, *9.36, 9.37–9.40, *9.41, *9.42, 9.43**

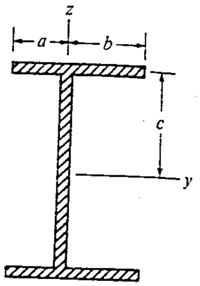
The open cross sections shown are all thin-walled, of constant thickness, and have one axis of symmetry. Assume a frictionless contact where overlaps appear. In each case find an expression for the location of the shear center. Check your answer by examining limiting cases where possible (as by letting dimension b approach zero, for example).



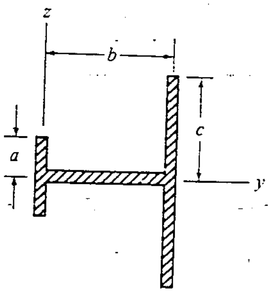
PROBLEM 9.32



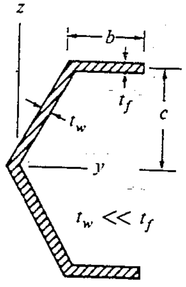
PROBLEM 9.33



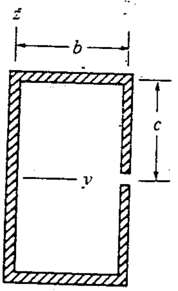
PROBLEM 9.34



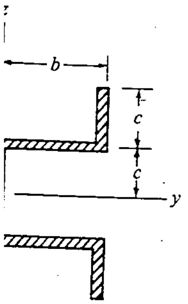
PROBLEM 9.35



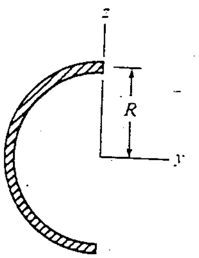
PROBLEM 9.36



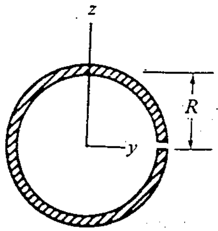
PROBLEM 9.37



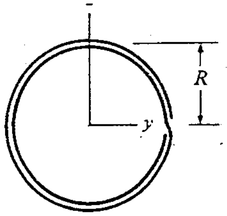
ROBLEM 9.38



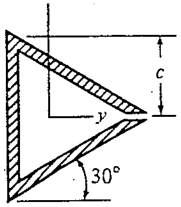
PROBLEM 9.39



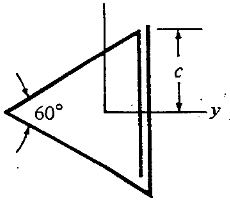
PROBLEM 9.40



PROBLEM 9.41



PROBLEM 9.42

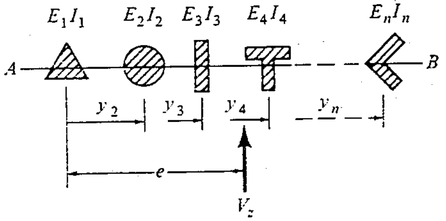


PROBLEM 9.43

9.44 (a) Several beams of arbitrary cross section have their centroids in a common plane AB. If somehow coupled together so that they share the same neutral axis, the composite beam will bend without twisting if e is as stated in the sketch. Derive this expression for e, in which y_i is the distance to the ith shear center.

(b) Use this formula to solve Problem 9.35.

(c) Use this formula to solve Problem 9.43.



$$e = \frac{E_2I_2y_2 + E_3I_3y_3 + \cdots + E_nI_ny_n}{E_1I_1 + E_2I_2 + E_3I_3 + \cdots + E_nI_n}$$

PROBLEM 9.44

Section 9.8

9.45 For an arbitrary open cross section, prove that $S_{\omega y} = S_{\omega z} = 0$ if axes yz are centroidal and pole P coincides with shear center S, as claimed above Eq. 8.10.5.

Suggestion: Set $e_y = e_z = 0$ in Eqs. 9.8.5.

9.46 Use Eqs. 9.8.5 to show that point P is the shear center in Figs. 9.7.2 and 9.7.3.

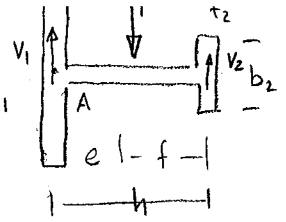
9.47 Use the method of Section 9.8 to locate the shear centers of the following cross sections.

(a) Problem 9.35.
(b) Problem 9.36.
(c) Problem 9.40
(d) Problem 9.42.

9.48 Imagine that the box section of Problem 9.31 is cut open at the lower left corner. Locate the shear center by the method of Section 9.8.

9.49 Locate the y and z coordinates of the shear center of the cross section shown. Dimensions are in millimeters. Thickness t is constant. Axes yz centroidal, and $I_y = 10.5(10)^6$ mm^4, $I_z = 20.8(10)^6$ mm^4, $I_{yz} = 6.00(10)^6$ mm^4.

***9.50** Imagine that the channel section of Fig. 9.7.1 is thin-walled, of constant thickness, with $h = b$, but that the upper flange has a modulus three times as great as that of the rest of the section. Locate the shear center.



יודעים ש- $V_1+V_2=P$

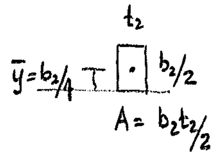
כדי שאין פיתול, אם ניקח $\Sigma M_A=0$ אזי $V_2 \cdot h = Pe$

מהו V_2. מפני שהמאמץ הגזירה הוא פרבולי, $\frac{2}{3} b_2 q_{t_{2max}} = V_2$

$$\frac{PQ}{I} = \frac{VQ}{I} = q_{t_{2max}}$$

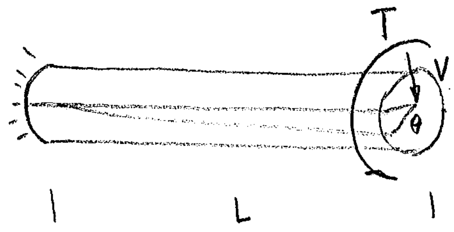
לכן, $V_2 h = \frac{2}{3} b_2 q_{t_{2max}} h = \frac{2}{3} b_2 \frac{PQ}{I} h = Pe$

$$e = \frac{2}{3} b_2 \frac{Qh}{I} = \frac{2}{3} \frac{b_2 h}{I} \left(\frac{b_2 t_2}{2} \cdot \frac{b_2}{4} \right) = \frac{b_2^3 t_2 h}{12 I}$$



נזכיר ש- Q המומנט הסטטי של חלק השטח המסוים עד הציר הנייטרלי,

אם הגזירה לא נמצאת במרכז הגזירה, פיתול קורה. אם רוצים זווית הפיתול:



בפיתול

$T = Ve$

זווית הפיתול היא

$$\theta = \frac{TL}{JG}$$

$$J = I_{zz} + I_{yy}$$

G - מודול הגזירה $\frac{E}{2(1+\nu)}$ בדרך כלל

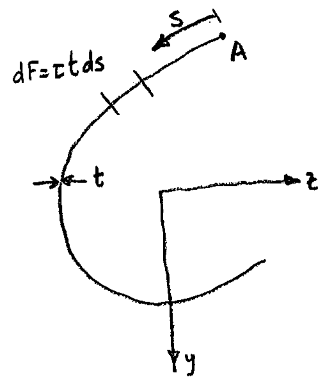
בדרך כלל זרם הגזירה q לקטע פתוח

$$q_{פתוח} = - \frac{(I_{yy} Q_z - I_{yz} Q_y) V_y + (I_{zz} Q_y - I_{yz} Q_z) V_z}{I_y I_z - I_{yz}^2}$$

$$\int_0^s t y \, dS = Q_z$$

$$\int_0^s t z \, dS = Q_y$$

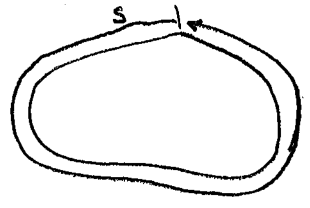
אם t משתנה ממקום למקום כפונקציה של s

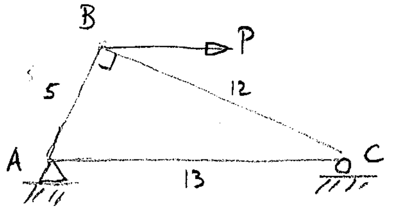


לקטע סגור $q = q_0 + q_{פתוח}$

ו- q_0

$$q_0 = - \frac{\oint (q_{פתוח}/t) \, dS}{\oint 1/t \, dS}$$





לפי שיטת הצמתים (ראו את הפתרון של הבוחן)

$F_{AB} = \frac{5}{13}P$ במתיחות

$F_{AC} = \frac{144}{169}P$ במתיחות

$F_{BC} = -\frac{12}{13}P$ בלחיצה

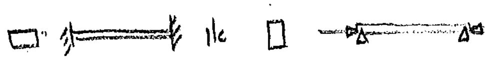
אם שטח החתך מהו העומס שיגרום להתמוטטות? קחו $E=200GPa, \pm 300 MPa = \sigma_y$
מפני ש- F_{AB} ו- F_{AC} במתיחות צריך לבדוק מהו העומס שגורם לכניעה. מפני ש- F_{BC} בלחיצה
צריך לבדוק מהו העומס שגורם לכניעה בלחיצה וגם בקריסה.

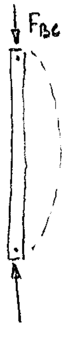
$$A = .1(.075) = 7.5\times10^{-3} m^2$$

$$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{5}{13}\frac{P}{(7.5\times10^{-3})} = 300\times10^6 \frac{N}{m^2} \qquad P = (300\times10^6)(7.5\times10^{-3})\cdot\frac{13}{5} = 5.4\times10^6 N$$

$$\sigma_{AC} = \frac{F_{AC}}{A} = \frac{144}{169}\frac{P}{7.5\times10^{-3}m^2} = 300\times10^6\frac{N}{m^2} \qquad P = (300\times10^6)(7.5\times10^{-3})\frac{169}{144} = 2.64\times10^6 N$$

$$\sigma_{BC} = \frac{F_{BC}}{A} = -\frac{12}{13}\frac{P}{7.5\times10^{-3}m^2} = -300\times10^6\frac{N}{m^2} \qquad P = (300\times10^6)(7.5\times10^{-3})\cdot\frac{13}{12} = 2.44\times10^6 N$$

כדי לבדוק בקריסה יש שתי אפשרויות או

לכן, צריך למצוא את מומנט האינרציה בשני כיוונים



$$I = \frac{(7.5)10^3}{12}\times10^{-8} = 6.25\times10^{-6} m^4$$

$$F_{BC} = \frac{\pi^2(200\times10^9)(6.25\times10^{-6})}{(12)^2} = 85674N$$

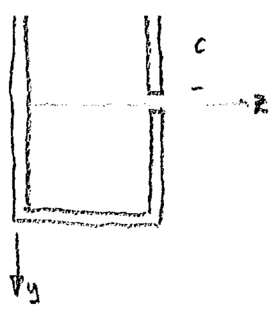
$$F_{BC} = \frac{12}{13}P \rightarrow \underline{P = 92813.\ N}$$

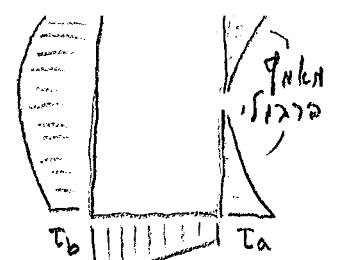
$$I = \frac{10(7.5)^3}{12}\times10^{-8} = 3.515\times10^{-6} m^4$$

$$F_{BC} = \frac{4\pi^2 EI}{L^2} = \frac{4(\pi^2)(200\times10^9)(3.515\times10^{-6})}{(12)^2} = 192766N$$

$$F_{BC} = \frac{12}{13}P \rightarrow P = 208830 N$$

לכן BC תקרוס בראשונה כאשר $P=92813N$

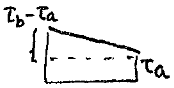




צורת מאמץ הגזירה שפועל על החתך

$$\tau_a = \frac{VQ_a}{It}$$

$$\tau_b = (\tau_b - \tau_a) + \tau_a = \frac{VQ_b}{It} + \frac{VQ_a}{It}$$



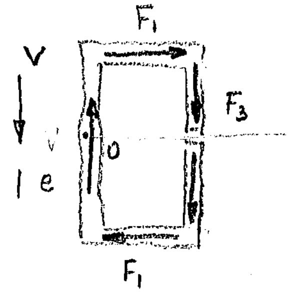
$$I = \frac{t(2c)^3}{12} + 2(bt)c^2 + 2\left[\frac{tc^3}{12} + tc\left(\frac{c}{2}\right)^2\right]$$

$$Q_a = \int y\,dA = \int_{b-t/2}^{b+t/2}\int_0^c y\,(dy\,dz) = \frac{y^2}{2}t\Big|_0^c = \frac{c^2t}{2}$$

$$Q_b = \iint y\,dA = \bar{y}A = c\cdot bt$$

$$F_3 = \int \tau_a\,dA = \frac{V}{It}\int_0^c \frac{y^2t}{2}\cdot t\,dy = \frac{V}{I}\frac{y^3t}{6}\Big|_0^c = \frac{Vtc^3}{6I}$$

$$F_1 = \frac{\tau_a + \tau_b}{2}(b\cdot t) = \frac{Vtbc}{2I}(b+c)$$



$$\Sigma M_o = 2F_3\cdot b + F_1\cdot 2c - Ve = 0$$

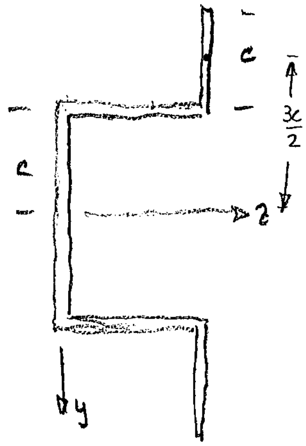
$$= 2\tau_a\frac{ct}{3}\cdot b + \left(\frac{\tau_a+\tau_b}{2}\right)bt\cdot 2c - Ve = 0$$

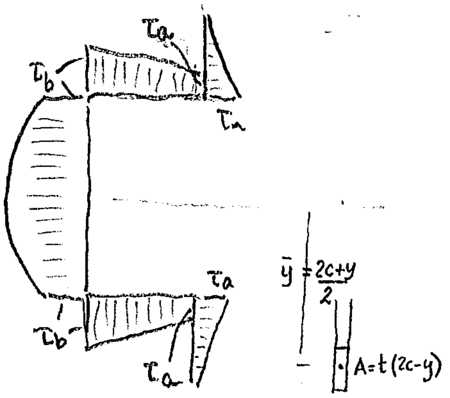
$$= \frac{2VQ_a}{It}\cdot\frac{ct}{3}\cdot b + \left(\frac{VQ_b}{2It} + \frac{2VQ_a}{2It}\right)bt\cdot 2c - Ve = 0$$

$$= \frac{V}{It}\left\{\frac{2c^2t}{2}\cdot\frac{ct}{3}\cdot b + \left(\frac{cbt}{2} + \frac{c^2t}{2}\right)bt\cdot 2c\right\} - Ve = 0$$

$$= \frac{V}{It}\left\{\frac{c^3t^2b}{3} + c^2b^2t^2 + c^3bt^2\right\} - Ve = 0$$

$$e = \frac{1}{It}\left\{\frac{4}{3}c^3t^2b + c^2b^2t^2\right\} = \frac{c^2bt}{I}\left(b + \frac{4}{3}c\right) = \frac{c^2bt\left(b+\frac{4}{3}c\right)}{c^2t\,(4/3c + 2b)} = \frac{b\,(b + 4/3c)}{(4/3c + 2b)}$$





$$\tau_a = \frac{VQ_a}{It} \qquad \tau_b = (\tau_b - \tau_a) + \tau_a = \frac{VQ_b}{It} + \frac{VQ_a}{It}$$

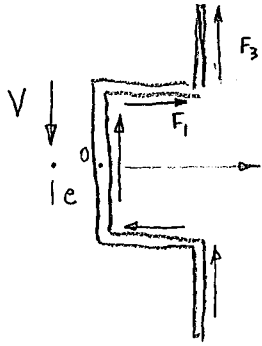
$$I = \frac{t(2c)^3}{12} + 2btc^2 + 2\left[\frac{tc^3}{12} + tc\left(\frac{3c}{2}\right)^2\right]$$

$$Q_a = \int y\,dA = t(2c-y)\left(\frac{2c+y}{2}\right) = \frac{t}{2}(4c^2 - y^2)\Big|_{y=c} = \frac{3c^2t}{2}$$

$Q_b = cbt$ כמו בתרגיל הקודם

$$F_3 = \int \tau_a\,dA = \frac{V}{It}\int_c^{2c}\frac{t}{2}(4c^2 - y^2)\cdot t\,dy = \frac{Vt}{2I}\left(4c^2y - \frac{y^3}{3}\right)_c^{2c} = \frac{5}{6I}Vtc^3$$

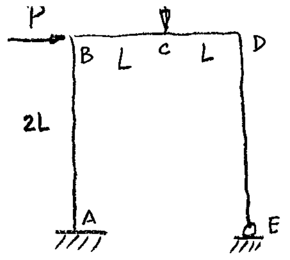
$$F_1 = \frac{\tau_b + \tau_a}{2}bt = \frac{1}{2}\left(\frac{VQ_b}{It} + \frac{2VQ_a}{It}\right)bt = \frac{Vtbc}{2I}(b+3c)$$

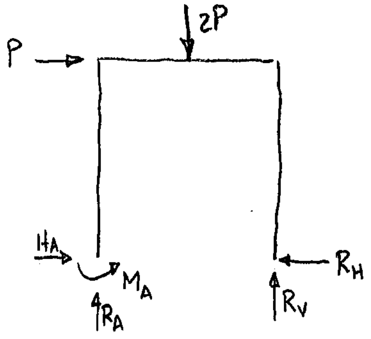


$$\Sigma M_o = -Ve + F_1\cdot 2c - 2F_3\cdot b = 0$$

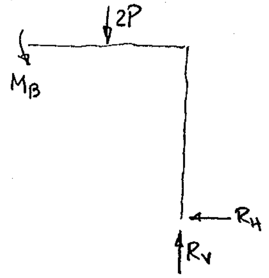
$$= -Ve + \frac{Vtbc}{2I}(b+3c)\cdot 2c - 2\cdot\frac{5}{6I}Vtc^3\cdot b$$

$$e = \frac{tbc^2(b+3c) - 5/3\,tc^3b}{\frac{64}{12}tc^3 + 2btc^2} = \frac{b}{2}\frac{(b + 4/3c)}{(b + 8/3c)}$$



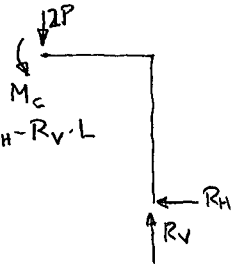


$M_A = -R_V \cdot 2L + P(2L) + 2P \cdot L$



$M_B = 2P \cdot L + R_H \cdot 2L - R_V \cdot 2L$

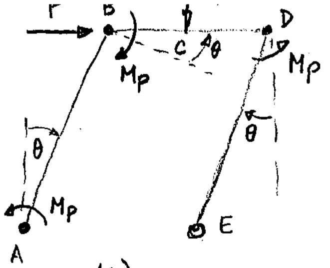
$M_D = +2L \cdot R_H$



$M_c = 2L \cdot R_H - R_V \cdot L$

$\Longrightarrow M_A - 2M_c + 2M_D = 4PL$

$\Longrightarrow M_B - 2M_c + M_D = 2PL$



(a)

$P \cdot 2\theta L = M_p\theta + M_p\theta + M_p\theta$

$P = \frac{3M_p}{2L}$

צריך לברר את M_c כאשר

$M_A = M_D = M_p$

$M_B = -M_p$

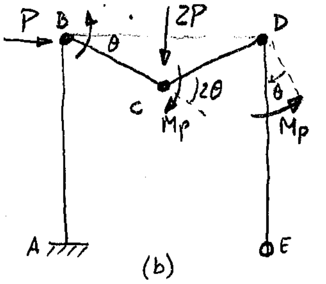
$M_A - 2M_c + 2M_D = 4PL$

$M_p - 2M_c + 2M_p = 4\left(\frac{3M_p}{2L}\right)L$

$M_c = -\frac{3}{2}M_p$

המנגנון בלתי אפשרי

$|M_c| > M_p$



(b)

$2P \cdot L\theta = M_p\theta + M_p \cdot 2\theta + M_p\theta$

$P = 2M_p/L$

צריך לבדוק את M_A כאשר

$M_B = M_D = M_p \ , \ M_c = -M_p$

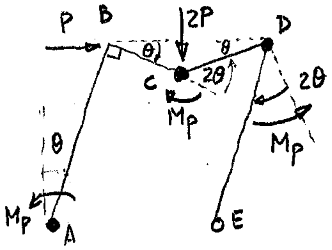
$M_A - 2M_c + 2M_D = 4PL$

$M_A + 2M_p + 2M_p = 4\left(\frac{2M_p}{L}\right)L$

$M_A = 3M_p$

המנגנון בלתי אפשרי

$M_A > M_p$



$P \cdot 2L\theta + 2P \cdot L\theta = M_p\theta + M_p \cdot 2\theta + M_p \cdot 2\theta$

$P = \frac{5M_p}{4L}$

צריך לבדוק את M_B כאשר

$M_c = -M_p \ , \ M_A = M_D = M_p$

$M_B - 2M_c + M_D = 2PL$

$M_B + 2M_p + M_p = 2\left(\frac{5M_p}{4L}\right) \cdot L$

$M_B = -\frac{M_p}{2}$

מפני ש- M_B פחות מ- M_p ו- P הוא הפחות ביותר
המאורע הזה יכול לקרות

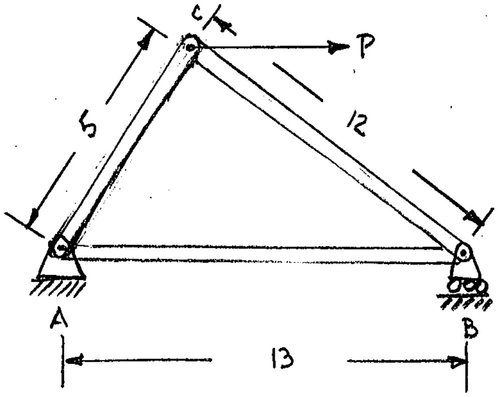
תורת החוזק 2 בוחן מס 1 ינואר 2 2000
פרופ עזרא לוי

אורך הבוחן הזה יהיה 90 דקות. בבוחן יש ~~שלושה~~ שתי שאלות. עליכם לענות בכל השאלות. רק להשתמש בתפסים הרשאים (כשלוש-עשרי דפים), ומחשבון. ואם יש. דברים הנתונים לכם עם הבוחן.

לחתום על סעיף הבא בבקשה:
לאורך הבוחן אני לא רשאי(ת) לקבל עזרה מאף אחד או לתת עזרה לאף אחד. אם אני אעבור על אלה, אני אכשל את הבוחן.

חתימת הסטודנט________________________ ________________________
שם הסטודנט מס. זהות

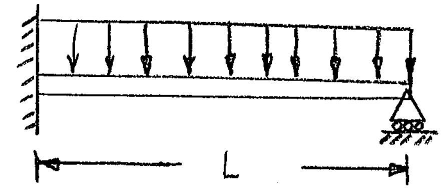
תרגיל מס. 1
נתון לכם מסבך המשורטט בציור בצד השמאל. שטח החתך הוא A, ומודול יאנג הוא E. הסמכים כנמצאים על הציור. בנקודה C, המסבך עמוס בעומס האופקי P. לא לכלול את משקלם של המוטות.



נא למצוא:
התזוזה האופקית, u_C
שינוי אורך המוט AC

תרגיל מס. 2

נתון לכם קורה עמוסה המשורטטת בציור בצד השמאל. העומס הוא עומס מפורס בגודל של q N/m.
בצד שמאל סמך רתום, ובצד ימין יש סמך נייד.



נא למצוא:

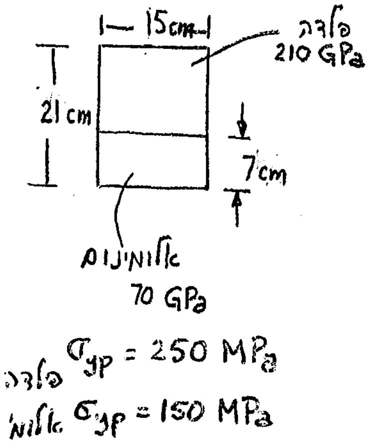
(a) התגובות בסמכים.

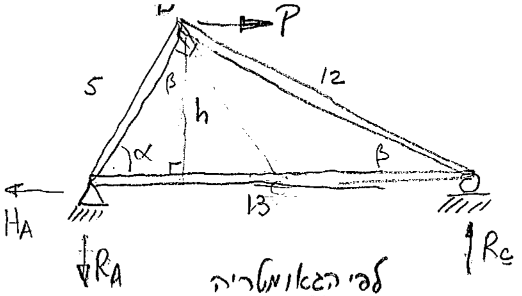
(b) המומנט המקסימלי ומקומו.

(c) נתון לכם ששטח החתך כבציור (B). למצוא את המאמץ המקסימלי.

(d) כאשר המאמץ המקסימלי מגיע ל-σ_{yp}, מהוא העומס המפורס שגורם למאמץ הזה.

(e) לפי שיטת הפלסטית, מהוא המומנט, M_{pl} והעומס המפורס שהקורה יכולה לסבול.





לפי הגאומטריה

$\sin\beta = \cos\alpha = 5/13$

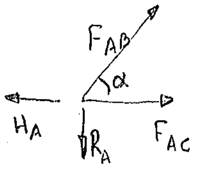
$\sin\alpha = 12/13 = \frac{h}{5}$ $\qquad h = \frac{60}{13} = 4\,8/13$

$\Sigma F_x = 0 \Rightarrow \underline{H_A = P}$

$\Sigma M_A = 0 \Rightarrow P.h = R_C \cdot 13$

$P.\frac{60}{13} = R_C \cdot 13$ $\qquad \underline{R_C = P\,\frac{60}{169}}$ $\qquad \underline{R_A = +P.\frac{60}{169} = .3550P} \Leftarrow \Sigma F_y = 0$

בשיטת הצמתים



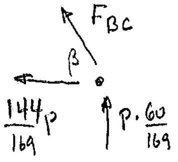
$\Sigma F_y = 0 \Rightarrow F_{AB}\sin\alpha + R_A = 0$ $\qquad 12/13\,F_{AB} + P\cdot\frac{60}{169} = 0$

$\Sigma F_x = 0 \Rightarrow F_{AB}\cos\alpha + F_{AC} - H_A = 0$ $\qquad \underline{F_{AB} = +\frac{5}{13}P}$

$\longrightarrow +\frac{5}{13}P\cdot 5/13 + F_{AC} - P = 0$

$\underline{F_{AC} = \frac{144}{169}P} = 0.85207P$

בדיקת השיטה



$\Sigma F_y = 0 \qquad F_{BC}\sin\beta + \frac{60P}{169} = 0$ $\qquad \Sigma F_x = 0 \Rightarrow F_{BC}\cos\beta + \frac{144P}{169} = 0$

$F_{BC}\,\frac{h}{12} + \frac{60P}{169} = 0$ $\qquad -\frac{12}{13}P\cdot\frac{12}{13} + \frac{144P}{169} = 0$ ✓

$F_{BC}\,\frac{\overset{5}{\cancel{60}}}{13\cdot\cancel{12}} + \frac{60P}{169} = 0$ $\qquad \underline{F_{BC} = -\frac{12}{13}P}$

מפני ש- E,A שווים בכל מוט:

$$U_c = \frac{1}{2}\sum\frac{F_i^2 L_i}{E_i A_i} = \frac{1}{2EA}\left[F_{AC}^2 L_{AC} + F_{BC}^2 L_{BC} + F_{AB}^2 L_{AB}\right]$$

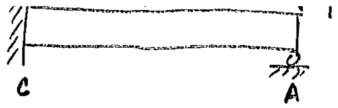
$$= \frac{1}{2EA}\left[\left(\frac{144}{169}P\right)^2 13 + \frac{144}{169}P^2\cdot 12 + \frac{25}{169}P^2\cdot 5\right]$$

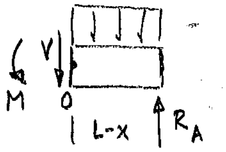
$$u_B = \frac{\partial U_c}{\partial P} = \frac{1}{EA}\left[\left(\frac{144}{169}\right)^2 P\cdot 13 + \frac{144P.12}{169} + \frac{125}{169}P\right]$$

$$= \frac{P}{EA}\left\{20.4028\right\}$$

$$\Delta AC = \frac{F_{AC}}{AE}L_{AC} = \frac{0.85207P\cdot 13}{AE} = 11.07691\,\frac{P}{AE}$$

לִמצוא את התגובות
למצוא את המומנט המקסימלי
למצוא את המאמץ המקסימלי
מהוא q שגורם ש $\sigma_{\text{מקס}} = \sigma_{yp}$
מהוא M_{pl} ו- q שגורם ל- M_{pl}





$$\Sigma M_o = M + R_A(L-x) - q\frac{(L-x)^2}{2}$$

$$M = -R_A(L-x) + q\frac{(L-x)^2}{2} \qquad 0 \le x \le L$$

$$U_c = \frac{1}{2EI}\int_0^L M^2 dx = \frac{1}{2EI}\int_0^L \left\{-R_A(L-x) + q\frac{(L-x)^2}{2}\right\}^2 dx$$

$$\frac{\partial U_c}{\partial R_A} = 0 = \frac{1}{EI}\int_0^L \left\{-R_A(L-x) + q\frac{(L-x)^2}{2}\right\}\left(-(L-x)\right)dx$$

$$= \frac{1}{EI}\left\{\frac{R_A(L-x)^3}{3} + q\frac{(L-x)^4}{8}\right\}\Bigg|_0^L$$

$$= \frac{1}{EI}\left\{\frac{R_A L^3}{3} - q\left(\frac{L^4}{8}\right)\right\} = 0 \qquad R_A = \frac{3qL}{8}$$

לכן, $R_c = qL - R_A = \frac{5qL}{8}$

כאשר $x=0$, $M = -R_A L + \frac{qL^2}{2}$

$$M_c = \frac{qL^2}{8}$$

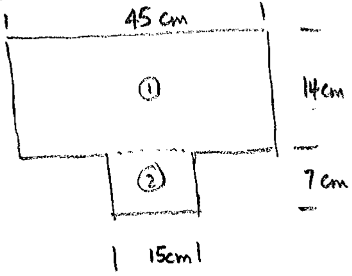
המומנט המקס. יכול לקרות כש- $\frac{dM}{dx} = 0$

$\frac{dM}{dx} = R_A - \frac{2q}{2}(L-x)$, $0 =$ ו $x = L - \frac{R_A}{q} = \frac{5L}{8}$

לכן, $M\Big|_{x=\frac{5L}{8}} = -\frac{3qL}{8}\left(\frac{3L}{8}\right) + \frac{9qL^2}{128} = \frac{-9qL^2}{128} = -.0703qL^2$

מפני זה המומנט המקס. קורה ב- $x=0$ והוא $\frac{qL^2}{8}$

קורה מחומר יחיד ... צריך להחליף את שטח החתך המורכב לשטח חומר יחידי. מחליפים את השטח לאלומיניום



לכן $E_{\text{פל}} b_{\text{פל}} = E_{\text{אל}} b_{\text{אל}}$

$b_{\text{אל}} = \frac{E_{\text{פל}} b_{\text{פל}}}{E_{\text{אל}}}$

$b_{\text{אל}} = 45_{cm}$

עכשיו למצוא מומנט האינרציה ומרכז הכובד

	A	$\bar{y}$	$A\bar{y}$
①	(45)14	7	4410
②	7·15	17.5	1837.5
	735		6247.5

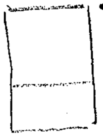
$$\bar{y} = \frac{6247.5}{735} = 8.5\ cm$$

I_{zz}	A	d^2	I_{tt}
$45 \cdot 14^3/12$	630	$(1.5)^2$	11707.5
$15 \cdot 7^3/12$	105	$(-9)^2$	8933.75
			20641.25 cm^4

בפלדה $\sigma_{max} = \frac{Mc}{I} = \frac{E_{\text{פלדה}}}{E_{\text{אלומ}}} \cdot \left(\frac{qL^2}{8}\right) \cdot \frac{(0.085)m}{20641.25 \times 10^{-8} m^4}$

באלומיניום $\sigma_{max} = \frac{Mc}{I} = \frac{-\frac{qL^2}{8}(0.125m)}{20641.25 \times 10^{-8} m^4}$

לכן המקסימום נמצא בפלדה כאן



אם σ_{yp} מגיע בפלדה
העומס המפורס שגורם ל σ_{yp} $\Leftarrow$ $q = \frac{\sigma_{yp}^{\text{פ}} \cdot I}{\frac{L^2}{8} \cdot 3(0.085)} = \frac{1618921.6}{L^2}$

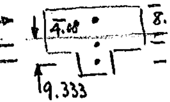
ב- q הזה, המאמץ באלומיניום הוא

$$\frac{-\frac{qL^2}{8}(0.125)}{I} = -\left(\frac{\sigma_{yp}^{\text{פ}} \cdot I}{\frac{L^2}{8} \cdot 3(0.085)}\right) \cdot \frac{L^2}{8}\frac{(0.125m)}{I} = -\sigma_{yp}^{\text{פ}}(.4902)$$

$$= -122.55\ MPa$$

אם σ_{yp} הגיע באלומיניום קודם, המאמץ המקס. בפלדה והיה יותר גבוה מ $\sigma_{yp}^{\text{פ}}$. אז האפשרות הזאת לא קורת!

לפי שיטת הפלסטית, ציר הניטרלי מחלק השטח לשניים. לכן, אם השטח הוא מיוצר מאלומיניום,
הציר הוא $8\frac{1}{16}$ ס"מ תחת המישור העליון



$$(9.333 \times 15 \times 7 + 2.917 \times 45 \times 5.833 + 4.083 \times 45 \times 8.17)\,\sigma_{yp}^{\text{אל}} = M_{pl}$$

$M_{pl} = 486937.5\ N\text{-}m$. פה המומנט הוא מסביב הציר הניטרלי.

תורת החוזק 2
פרופ עזרא לוי

מבחן סופי

3 פברואר 2000

אורך המבחן הזה יהיה 180 דקות. במבחן יש ארבעה שאלות. עליכם לענות לכל השאלות. רק להשתמש ברשמות הכיתה, תרגילי בית שלכם, ומחשבון. ואם יש דברים הנתונים לכם עם המבחן בלבד.

לחתום על סעיף הבא בבקשה:
לאורך המבחן אני לא רשאי(ת) לקבל עזרה מאף אחד או לתת עזרה לאף אחד. אם אני אעבור על אלה, אני אכשול את המבחן.

חתימת הסטודנט________________

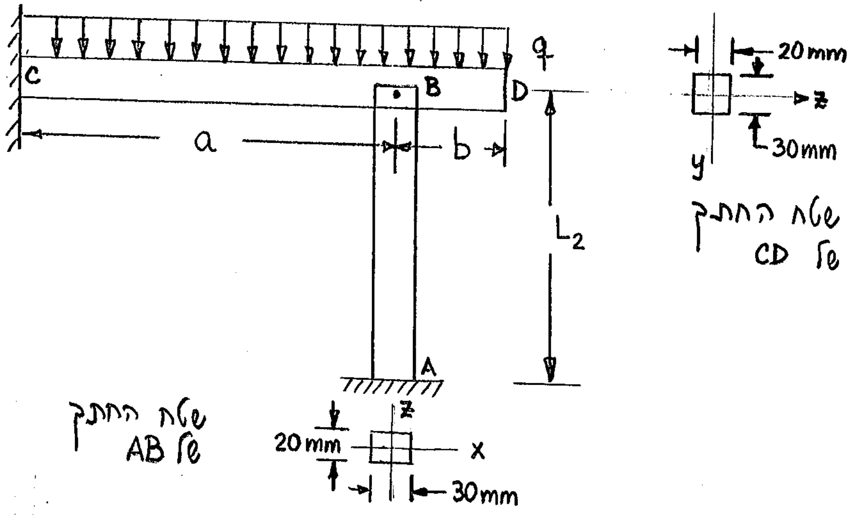
תרגיל מס. 1 (40 נקודות)

שתי קורות מחוברות ועמוסות כמשורטט בציור. שתי הקורות עשויות מפלדה. הניחו ש- $E = 200$ GPa ו- $\sigma_{yp}=360$ MPa. $L_2=2$ m, $a=1.5$ m, $b=0.5$ m.

א. מיצאו את העומס המפורס המקסימלי, q, שיכול לפעול על הקורה CD לפני שקורה AB תקרוס.
ב. מהוא q כאשר קורה CD מתחילה להיכנע.
ג. לפי מה שמצאתם ב- (א.) וב- (ב.), איזה קורה תתמוטט בראשונה ומה תהיה הסיבה.

חשבו את q לפי התנאים הבאים: על הקורה AB שפה A רתומה, ומחוברת בסיכה ב-B נגד קריסה בכוון x, ורתומה ב-A וב-B נגד קריסה בכוון z. המדדים של הקורות נתונים בצורה.

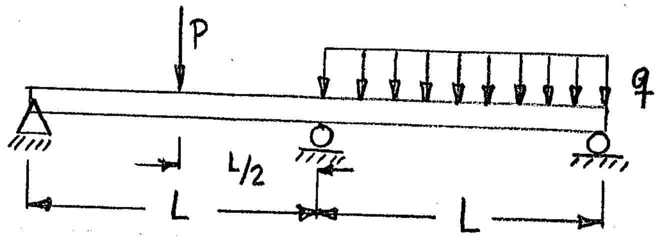
הערה אם זאת בעיה בלתי מסוימת, תשתמשו במשפט קסטגליאנו למצוא כמה מהנעלמים.



תרגיל מס. 2 (30 נקודות)

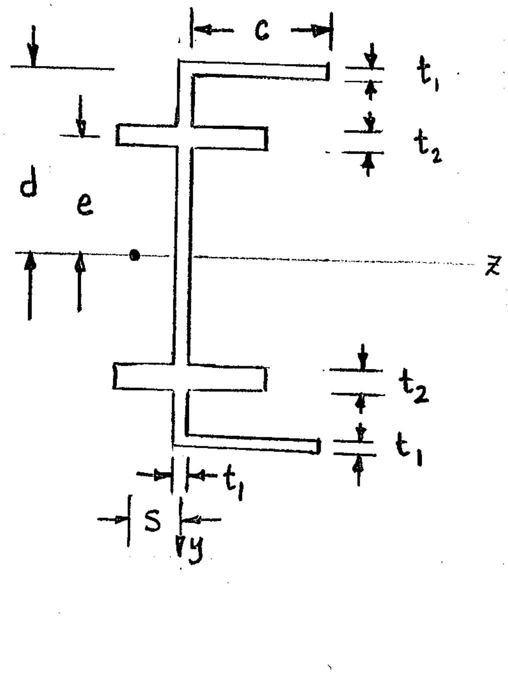
נתון גשר רצוף מורכב משני חלקים AB ו- BC ונושא עומס P ועומס מפורס, q, כמשורטט בצורה. לפי שיטת עומס הגבולי (זאת אומרת, מומנט הפלסטי). מיצאו את היחס בין qL ל- P כדי ששני חלקי הגשר יתמוטטו בו-זמני.

רמז - מיצאו את מומנט הפלסטי לכל האפשרויות ויחסו את התוצאות.



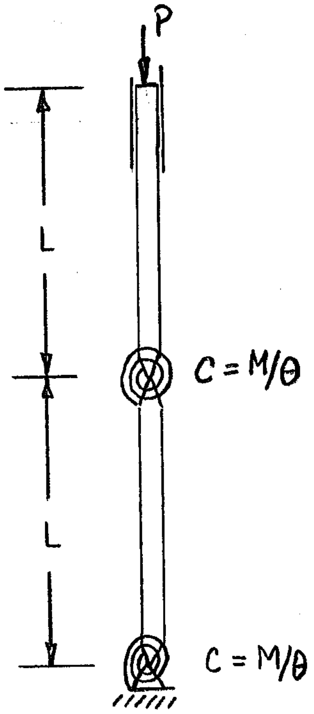
תרגיל מס. 3 (16 נקודות)

נתון חתך בדק-דופן כמשורטט בצורה הבאה. מיצאו את מרכז הגזירה לחתך, s.



תרגיל מס. 4 (14 נקודות)

למערכת המשורטטת, עליכם למצוא את הכוח הקריטי, לפי שיטת אנרגייה, שגורם להתמוטטות המערכת. הניחו בחישוב הכוח הקריטי שהמערכת הזאת היא מערכת של דרגה חופשית יחידית, כמו שעשינו בשיעור האחרון.



תורת החוזק 2 מועד ב | מבחן סופי | אפריל 3 2000
פרופ עזרא לוי

אורך המבחן הזה יהיה 180 דקות. במבחן יש ארבעה שאלות. עליכם לענות לכל השאלות. רק להשתמש ברשמות הכיתה, תרגילי בית שלכם, ומחשבון, ואם יש דברים הנתונים לכם עם המבחן בלבד.

לחתום על סעיף הבא בבקשה:

לאורך המבחן אני לא רשאי(ת) לקבל עזרה מאף אחד או לתת עזרה לאף אחד. אם אני אעבור על אלה, אני אכשול את המבחן.

חתימת הסטודנט____________________________

תרגיל מס. 1 (40)

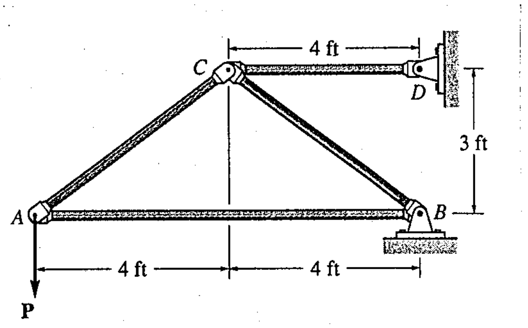
מסבך מוטות, עשוים מפלדה, מעומס בכוח P כמשורטט בציור. אם לכל מוט יש שטח החתך עגול בקוטר של 3 in.

א. מיצאו את הכוח שיגרום לקריסה ובאיזה מוט זה יקרה.

ב. מה צריך להיות הקוטר כדי שיהיה קריסה וכניעה בו-זמנית במסבך.

חשבו את P לפי התנאים הבאים: כל המוטות מחוברות בסיכה בשני הצדדים נגד קריסה בכוון x, ורתומים בשני הצדדים נגד קריסה בכוון z.

הניחו ש- $E = 29 \times 10^6$ lb/in^2 ו- σ_{yp}=36000 lb/in^2. המדדים של הקורות נתונים בצורה, ויש לזכור ש- 12 in.= 1 ft.



תרגיל מס. 2

נתון גשר רצוף מורכב משני חלקים AB ו- BC ונושא עומס P ועומס מפורס, q, כמשורטט בצורה. לפי שיטת עומס הגבולי (זאת אומרת, מומנט הפלסטי), מיצאו את היחס בין qL ל- P כדי ששני חלקי הגשר יתמוטטו בו-זמני.

רמז - מיצאו את מומנט הפלסטי לכל האפשרויות ויחסו את התוצאות.

תרגיל מס. 3

נתון חתך בדק-דופן כמשורטט בצורה הבאה. מיצאו את מרכז הגזירה לחתך.

תרגיל מס. 4 (14 נקודות)

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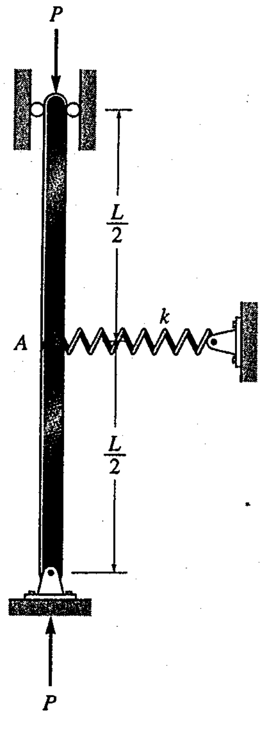

(a)

אפריל 3 2000

מבחן סופי

תורת החוזק 2 מועד ב
פרופ עזרא לוי

אורך המבחן הזה יהיה 180 דקות. במבחן יש ארבעה שאלות. עליכם לענות לכל השאלות. רק להשתמש ברשמות הכיתה, תרגילי בית שלכם, ומחשבון. ואם יש דברים הנתונים לכם עם המבחן בלבד.

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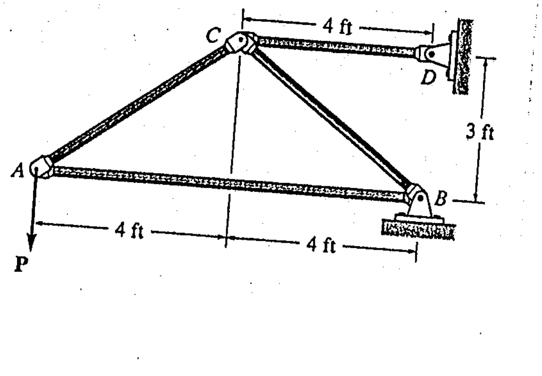

תרגיל מס. 2 (30 נקודות)

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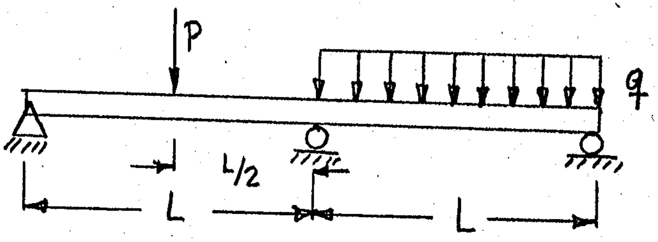

תרגיל מס. 3 (16 נקודות)

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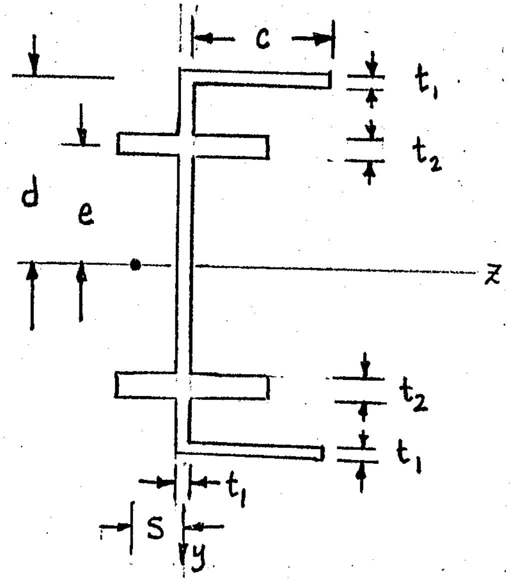

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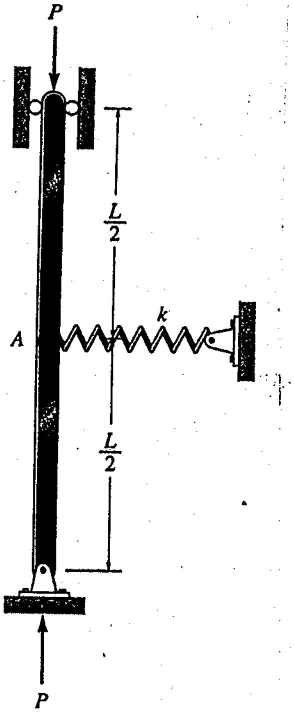
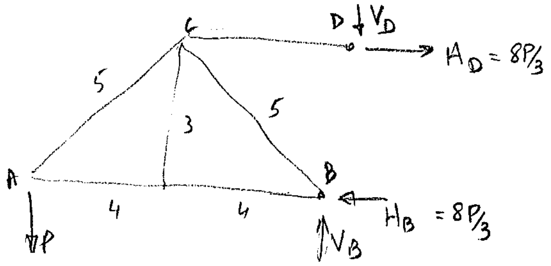


(a)

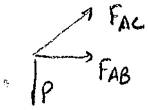


$\sum F_x = 0 \qquad H_D - H_B = 0 \qquad H_D = 8P/3$ ②

$\sum F_y = 0 \qquad V_B - V_D - P = 0$

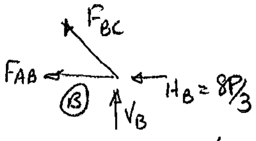
$+\circlearrowleft \sum M_D = 0 \qquad H_B \cdot 3 - P \cdot 8 = 0 \qquad H_B = \frac{8P}{3}$ ②

Ⓐ



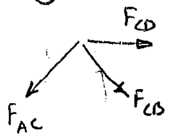
$F_{AC} \cdot 3/5 - P = 0 \qquad \frac{5}{3}P = F_{AC}$ ②

$F_{AC} \cdot \frac{4}{5} + F_{AB} = 0 \qquad \underline{F_{AB}} = -\frac{4}{5}F_{AC} = -\frac{4}{5} \cdot \frac{5}{3}P = \underline{\frac{-4}{3}P}$ ②



$\underline{V_B = P}$ ②

Ⓒ

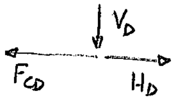


$F_{CD} - F_{AC} \cdot 4/5 + F_{CB}\,4/5 = 0$

$F_{AC} \cdot 3/5 + F_{CB} \cdot 3/5 = 0 \qquad F_{AC} = -F_{CB} \qquad \underline{F_{CB} = -5P/3}$ ②

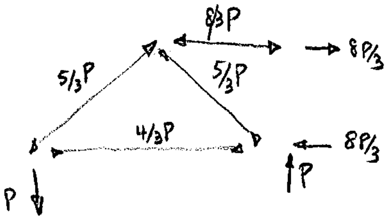
$F_{CD} - \frac{8}{5}F_{AC} = 0 \qquad \underline{F_{AC}} = \frac{5}{8}F_{CD} = \frac{5}{8} \cdot \frac{8}{3}P = \underline{\frac{5P}{3}}$ ②

Ⓓ



$-F_{CD} + H_D = 0 \qquad \underline{F_{CD} = 8P/3 = H_D}$ ②

$\underline{V_D = 0}$ ②



$F_{crit} = \frac{4}{3}P = \frac{\pi^2 EI}{l^2}$ ② in AB ②

$$I = \int y^2 dA = \int r^2 \sin^2\theta \cdot r\,dr\,d\theta$$
$$= \int r^3 dr \int \sin^2\theta\, d\theta$$
$$= r^4/4 \Big|_0^R \; \overset{\frac{1-\cos 2\theta}{2}}{\theta/2 - \frac{\sin 2\theta}{4}} \Big|_0^{2\pi} = \frac{R^4}{4}\pi$$ ②

$$P = \frac{3}{4}\pi^2 \frac{EI}{l^2} = \frac{3}{4}\pi^2 \frac{(29\times10^6 \frac{lb}{in^2})(\frac{\pi}{4}(1.5)^4)}{(8\cdot12)^2}$$
$$= \underline{92613 \text{ lb}}$$ ②

$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{4/3P}{\pi r^2} = \frac{4/3(92613)}{\pi(1.5)^2} = 17470 \text{ psi} < \sigma_y$ ②

since $|F_{BC}| > |F_{AB}|$ F_{AB} buckles first

$\sigma_{CD} = \frac{F_{CD}}{A} = \frac{2F_{AB}}{A} = 34939 \text{ psi} < \sigma_y$ ②

out of plane buckling $F_{crit} = 4/3P = 4\pi^2 EI/L^2$ or $P_{FF} = 3\pi^2 EI/L^2 = 370452 \text{ lb} > P_{S.S.}$ ②

For simultaneous buckle & yield
yield at CD first

$$\sigma_y = \frac{F_{CD}}{A} = \frac{8/3\,P_{crit}}{\pi r^2} = \frac{\cancel{8/3}\,^{2}\left[\cancel{3/4}\,\pi^2 E \frac{\cancel{\pi r^4}^{\,2}}{L^2_{AB}\,\cancel{4}\,2}\right]}{\cancel{\pi r^2}} = \frac{\pi^2 E r^2}{2L^2_{AB}}$$ ②

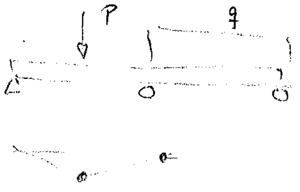
$$36000 = \frac{\pi^2 E r^2}{2L^2_{CD}} \qquad r = \sqrt{\frac{36000 \cdot 2\,(8\cdot12)^2}{29\times10^6 \cdot \pi^2}} = 1.523 \text{ in}$$

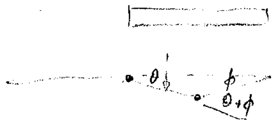
$$r = \sqrt{\frac{\sigma_y \cdot 2L^2_{AB}}{\pi^2 E}} \qquad d = 3.045 \text{ in}$$ ②

and $$P_{crit} = \frac{3}{4}\frac{\pi^2 EI}{L^2} = \frac{3}{4}\pi^2 \frac{(29\times10^6 \frac{lb}{in^2})(\frac{\pi}{4}(1.523)^4)}{(8\cdot12)^2} = 98325 \text{ lb}$$ ②

שיכול לפעול על קורה BC לפני שקורה AB תקרוס. שתי הקורות עשויות מפלדה. תחשבו את q,
רתום בנקודה A, ומחובר בסיכה בנקודה B נגד קריסה בכיוון x, ורתום ב-A וב-B
נגד קריסה בכיוון z. $E_{פלדה} = 200\,GPa$, $\sigma_{YP} = 360\,MPa$

לפי התנאי
הבאים
על קורה AB





$M_p 2\theta + M_p\cdot\Theta = P\frac{L}{2}\theta$

$6\frac{M_p}{L} = P$ ⑤

$qx_B\cdot\frac{x_B}{2}\theta + q(L-x_B)\left(\frac{L-x_B}{2}\right)\phi$

$(L-x_B)\phi = x_B\theta$

$\phi = x_B\frac{(\theta)}{L-x_B}$ ②

$2+2+3=⑦$

$\frac{qx_B^2\theta}{2}$ $\quad q(L-x_B)\frac{x_B}{2}\theta = M\theta + M(\theta+\phi)$

$2M\theta + \frac{Mx_B}{L-x_B}\Theta$

$q\frac{Lx_B}{2} = 2M + \frac{Mx_B}{L-x_B}$

$q\frac{Lx_B}{2}(L-x_B) = 2ML - Mx_B$

$q = \frac{4ML-2Mx_B}{L^2x_B - Lx_B^2} = 2M\frac{(2L-x_B)}{L^2x_B - Lx_B^2}$ ②

$\frac{dq}{dx} = 2M\left\{\frac{-1}{L^2x_B - Lx_B^2} - \frac{(2L-x_B)}{(L^2x_B-Lx_B^2)}\left(L^2-2Lx_B\right)\right\} = 0$ ④

$\left\{-(L^2x_B - Lx_B^2) - (2L^3 - 4L^2x_B - L^2x_B + 2Lx_B^2)\right.$

$-\cancel{L^2x_B} + \cancel{Lx_B^2} - 2L^3 + \cancel{5L^2}x_B - 2L\cancel{x_B^2} = 0$

$+4L^2x_B + Lx_B^2 - 2L^3 = 0$ ③

$-L(x_B^2 - 4Lx_B + 2L^2) = 0$

$x_B = \frac{4L \pm\sqrt{16L^2 - 8\cdot 1L^2}}{2} = \frac{4L \pm L2\sqrt{2}}{2} = L(2\pm\sqrt{2})$ ③

$= (2-1.414)L = .586L$

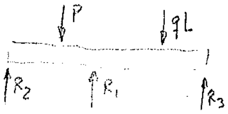
$(2+1.414)L = 3.414L$ X

$q = \frac{2M(2L-(2-\sqrt{2})L)}{L^3((2-\sqrt{2}) - L^3(2-\sqrt{2})^2} = \frac{2M(\sqrt{2}L)}{L^3(2-\sqrt{2} - (4-4\sqrt{2}+2))}$

$(-4+3\sqrt{2})$

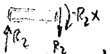
$q = \frac{2M\sqrt{2}L}{L^2(3\sqrt{2}-4)} = \frac{2.82843M}{.24264L^2} = .11.657\frac{M}{L^2}$

2



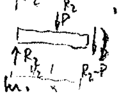
$R_1L + R_3 2L = P\frac{L}{2} + qL\frac{3}{2}$

$R_2 + R_1 + R_3 = P + qL$ $\quad R_2L+$



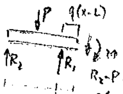
$M = -R_2x$

פרק פלסטי.



$P\frac{L}{2} + (R_2-P)x + M = 0$

$P(x-\frac{L}{2}) - R_2x = M$



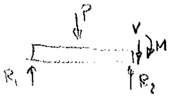
$R_2 - P + R_1 - q(x-L) = V$

$q(x-L)\frac{(x+L)}{2} - R_1(x-L) + P(x-\frac{L}{2}) - R_2(x) = M$

$U = \int_0^{L/2}\frac{(R_2x)^2}{2EI} + \int_{L/2}^{L}\frac{\left[P(x-\frac{L}{2}) - R_2x\right]^2}{2EI} + \int_L^{2L}\frac{\left[q\frac{(x^2-L^2)}{2} - R_1(x-L) + P(x-\frac{L}{2}) - R_2(x)\right]^2}{2EI}$

$\frac{\partial U}{\partial R_2} = \int_0^{L/2}\frac{R_2(x^2}{EI} + \int_0^{L}\frac{\left[P(x-\frac{L}{2}) - R_2x\right](-x)}{EI} + \int_L^{2L}\frac{\left[q\frac{(x^2-L^2)}{2} - R_1(x-L) + P(x-\frac{L}{2}) - R_2x\right](-x)}{EI} = 0$

$\frac{\partial U}{\partial R_1} = \int_L^{2L}\left\{q\frac{(x^2-L^2)}{2} - R_1(x-L) + P(x-\frac{L}{2}) - R_2(x)\right\}(L-x) = 0$

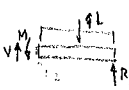


$P\frac{L}{2} + V\cdot L + M - R_2\cdot L = 0$

$R_2 = \frac{P}{2} + V + \frac{M}{L}$

$R_1 + R_2 = P + V$

$R_1 = \frac{P}{2} - \frac{M}{L}$



$V + R_3 = qL$ $\quad R_3 = qL - V$

$M = q\frac{L^2}{2} - R_3L$

$= VL - q\frac{L^2}{2}$

$M = \frac{.08579\,qL^2}{}$

$M = \frac{PL}{6}$

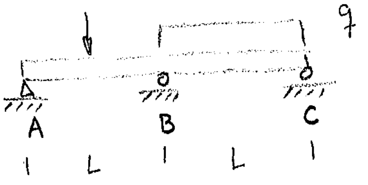
$.5147\frac{qL}{P}$

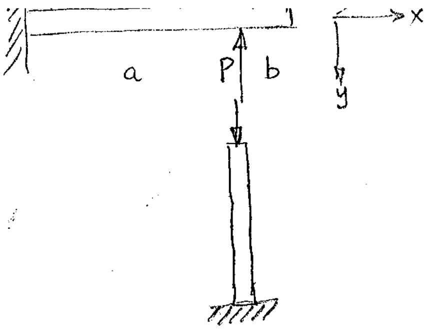
$P = .5147\,qL$

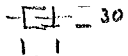
②

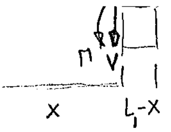
30

גשר רצוף מורכב משני חלקים AB ו-BC ונושא עומס P ועומס מפוזר q, כמשורטט בצורה
לפי שיטת הפלסטית תמצאו את היחס בין qL ל-P כדי ששני החלקים, AB ו-BC, יתמוטטו
באותו זמן.

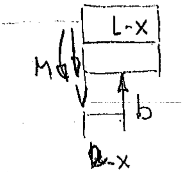




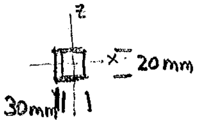




$M = q\left(\frac{L_1-x}{2}\right)$ ②



$M = q\left(\frac{L-x}{2}\right)^2 - P(a-x)$ ④



$I_{zz} = \frac{20(30)^3}{12} = 45000\ mm^4$
$= 4.5\times10^{-8} m^4$ ②

$J_{xx} = \frac{30(20)^3}{\cancel{12}12} = 20000\ mm^4$
$= 2.0\times10^{-8} m^4$ ②

$$U = \frac{1}{2EI}\left\{\int_0^a \left[q\left(\frac{L-x}{2}\right)^2 - P(a-x)\right]^2 dx + \int_a^L q\left(\frac{L-x}{2}\right)^2 dx\right\}$$

$$\frac{\partial U}{\partial P} = \frac{1}{EI}\int_0^a \left[q\left(\frac{L-x}{2}\right)^2 - P(a-x)\right](a-x)dx$$

$$= \frac{1}{EI}\int_0^a \left\{\frac{q}{2}(L^2-2Lx+x^2)(x-a) + P(a-x)^2\right\}dx$$

$$= \frac{q}{2EI}\int_0^a (L^2x-2Lx^2+x^3-aL^2+2aLx-ax^2) + \frac{P}{EI}\left(\frac{a-x}{3}\right)^3\Big|_0^a$$

$$\frac{L^2x^2}{2}-\frac{2Lx^3}{3}+\frac{x^4}{4}-aL^2x+aLx^2-\frac{ax^3}{3}\Big|_0^b \quad -\frac{P}{3EI}(a-x)^3\Big|_0^a$$

$$\frac{L^2a^2}{2}-\frac{2La^3}{3}+\frac{a^4}{4}-a^2L^2+a^3L-a^4/3 + \frac{P}{3EI}a^3$$

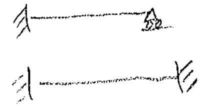
Castigliano
Buckling

1 hr.

$$0 = \frac{q}{2\cancel{EI}}\left(-\frac{a^2L^2}{2}+\frac{La^3}{3}-a^4/12\right) + \frac{Pa^3}{3\cancel{EI}}$$ ④

$$P = \frac{3}{2}q\left(\frac{L_1^2}{2a}+\frac{L_1}{3}+\frac{a}{12}\right) = 1.1875q = 0.59375qL_1$$ ②

$V_c = qL - P = .40625qL_1$



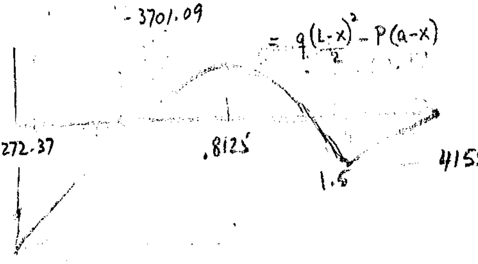
$P_E = 2.05\frac{\pi^2 EI_{zz}}{L_2^2} = P = 45,523.55\ N$ for z-z buckling ②

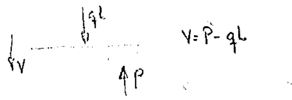
$P_E = 4\pi^2E_2I_2/L_2^2 = P : 39478.42N$ for xx buckling lower ②

$$q = \frac{\pi^2E_2I_{2zz\,xx}}{L_2^2}\left(\frac{2}{3}\frac{1}{\frac{L_1^2}{2a}-\frac{L_1}{3}+a/12}\right)$$

$45523.55N\ (.8421) = 38335.38\ N/m$ buckling in zz ②

$q = 39478.42N(.8421) = 33245\ N/m$ ← buckling in xx ②





$M = q\,L_1^2/2 - P(a)$ ②

@ buckling: $(33245)\frac{4}{2} - 39478.42(1.5) = 7272.37$

② 7272.37

$\frac{dM}{dx} = -q(L-x)+P = 0$ ②

$-qL+qx+P=0 \qquad x = L_1 - \frac{P}{q} = 0.8125$ ② $\qquad M|_{x=0.8125} = -3701.09$ ②

$\sigma = \frac{Mc}{I} = \frac{(7272.37)(.015)}{4.5\times10^{-8}} = 2424.12\ MPa > \sigma_y = 360\ MPa$ ②

$M = 1080\ N/m$ when $\sigma = \sigma_y$ ②

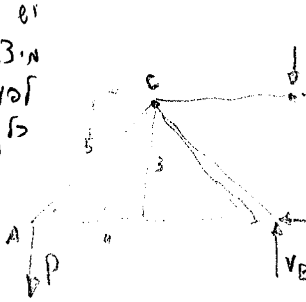
$= q\frac{L_1^2}{2} - \frac{3}{2}q\left(\frac{L_1^2}{2a} - L_1/3 + a/12\right) = q\left\{L_1^2/2 - \frac{3L_1^2}{2}/2a + L_1a/2 - 3a^2/24\right\}$

⇒ q = 4927.[illegible] N/m   P = [illegible] N ①

40

יש שלא חתך עגול ברדיוס של 1.90cm
מיצאו את הכוח המקסימלי P שיכול
לפעול כדי שאף אחד מהאיברים יקרוס.
כל איבר ואיבר נתמך בסיכה בקצותיו. ⑤

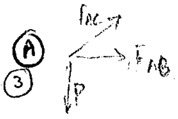
$E = 200\ GPa \qquad \sigma_{yp} = 250\ MPa$



$8P/3$ $(+\Sigma M_D = -H_B \cdot 3 + P \cdot 8 = 0 \quad H_B = \frac{8P}{3}$ ③

$H_B = \frac{8P}{3} \quad \Sigma F_y \cdot 4 = V_D \cdot 4 - V_B \cdot 4 + P \cdot 4 = 0$

$P + V_D - V_B = 0$ ③

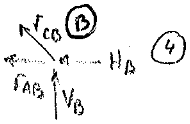


$F_{AC} \cdot \frac{3}{5} = P$ ②

$F_{AC} \cdot \frac{4}{5} + F_{AB} = 0$

$F_{AB} = -\frac{4}{5} F_{AC} = -\frac{4}{5} \cdot \frac{5}{3} P$

$\underline{F_{AB} = -\frac{4P}{3}}$ ②



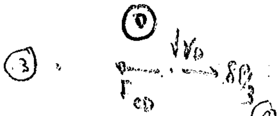
$F_{CB} \cdot \frac{4}{5} + F_{AB} + H_B = 0$

$F_{CB} \cdot \frac{3}{5} + V_B = 0$

$H_B = -F_{CB} \cdot \frac{4}{5} - F_{AB} = +\frac{5}{3} \cdot \frac{4}{5} P + \frac{4P}{3} = \frac{8P}{3}$ ②

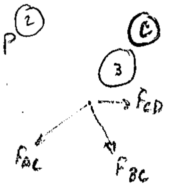
$\underline{F_{CB} = -\frac{5}{3}P}$ ②

FBD	5
Ⓐ	4
Ⓑ	4
Ⓒ	4
Ⓓ	4



$V_D = 0$ ② $\Rightarrow V_B = P$ ②

$\underline{F_{CD} = 8P/3}$ ②



$F_{AC} = F_{BC}$ ②

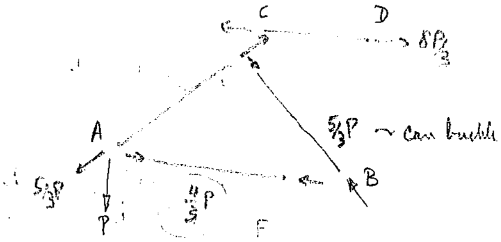
$F_{CD} - F_{AC} \cdot \frac{4}{5} + F_{BC} \cdot \frac{4}{5} = 0$

$\underline{F_{CD} = 2F_{AC} \cdot \frac{4}{5}} = 2 \cdot \frac{5}{3} P \cdot \frac{4}{5} = \frac{8P}{3}$ ②

longest $F = \frac{4}{3}P = \frac{\pi^2 EI}{L^2}$ ②

$P = 5789\ N$ ②

$\sigma = 3272$ psi ②



$F_{fail} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 EI}{25} = \frac{5}{3}P$

$P = \frac{3\pi^2 EI}{125}$

$F_{fail} = \frac{\pi^2 EI}{L^2}$ ①

①

$\frac{4}{3}P$

$P = \frac{3}{4}\frac{\pi^2 EI}{L^2}$

ⓐ

רתום - רתום $F = \frac{4\pi^2 EI}{L^2}$

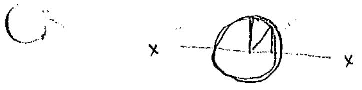
$I = \frac{1}{4}\pi r^4$ ②

$P = \frac{3\pi^2 (29 \times 10^3 ksi)\left(\frac{1}{4}\pi (1.5)^4\right) in^4}{4 \cdot (8 \cdot 12)^2 in^2}$

$\sigma = \frac{8}{3}\frac{P}{A} = \frac{\frac{3}{4}\frac{\pi^2 E}{L^2}\frac{\pi r^4}{4} \cdot \frac{8}{3}}{\pi r^2}$

$= \frac{\cancel{8}}{\cancel{4}}\frac{\pi^2 E}{2L^2}\frac{r^2}{\cancel{4}} \cdot \frac{\cancel{8}}{\cancel{3}}$

100836



$r\sin\theta = y$

$\int y^2 dA = \int_0^{2\pi}\int_0^R r^3 \sin^2\theta \, dr\, d\theta = \int_0^R r^3 dr \int_0^{2\pi} \frac{1-\cos 2\theta}{2} d\theta$

$= \quad \frac{r^4}{4} \quad \frac{\theta}{2} - \frac{\sin 2\theta}{4}\Big|_0^{2\pi}$

π

$I_{xx} = \int y^2 dA$

$\sqrt{R^2 - x^2}\, dxdy$

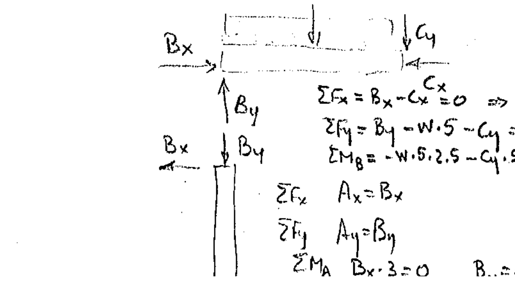
$I_{xx} = \frac{\pi R^4}{4}$

$P = 92.6$ kip. ②

$\sigma = P/A = \frac{P}{\pi r^2} \cdot \frac{5.789 \times 10^3\ lb}{\pi (1.5)^2 in^2} = \frac{3272}{~~[illegible]~~}\frac{lb}{in^2}$ ②

$\sigma_{CD} = \frac{8736}{~~[illegible]~~}\frac{lb}{in^2} < \sigma_y$ ②

$\sigma_{AB} = \frac{4368}{~~[illegible]~~}\frac{lb}{in^2} < \sigma_y$ ②



$\Sigma F_x = B_x - C_x = 0 \Rightarrow C_x = 0$

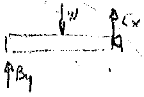
$\Sigma F_y = B_y - W \cdot 5 - C_y = 0$

$\Sigma M_B = -W \cdot 5 \cdot 2.5 - C_y \cdot 5 = 0$

$\Sigma F_x \quad A_x = B_x$

$\Sigma F_y \quad A_y = B_y$

$\Sigma M_A \quad B_x \cdot 3 = 0 \quad B_x = 0 \quad A_x = 0$



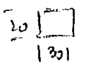
$C_y = -W(2.5)$

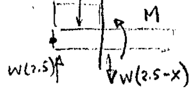
$B_y = W(2.5)$

$W(2.5) = \left(\frac{\pi^2 EI}{L^2}\right)/1.5$

$5W = \pi^2 EI$

$FS = \frac{P_{fail}}{P_{allow}}$



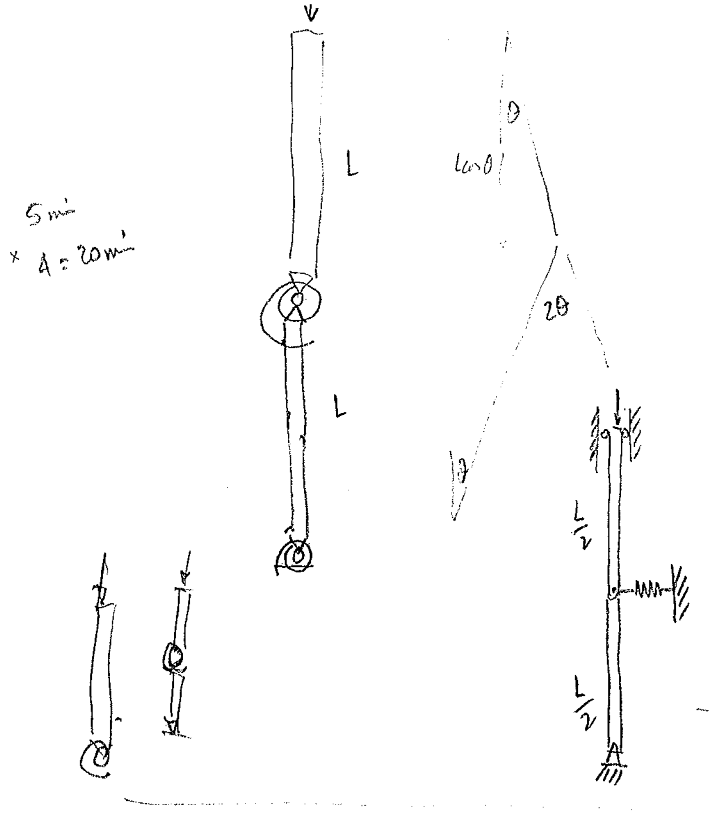


$w(x) \cdot \frac{x}{2} + w(2.5-x) \cdot x - M = 0$

$M = W(2.5)x - W\frac{x^2}{2}$

$\sigma = \frac{cM}{I} = \frac{(.01) W (2.5)^2}{2 \cdot 2 \times 10^{-8}}$

$= .01(1170(6.25)) \quad 10^8$



5 min
× 4 = 20 min

$$2L\left(1-\left(1-\theta^2/2\right)\right)$$

$$P(2L-2L\cos\theta)-\frac{1}{2}M\cdot 2\theta-\frac{1}{2}M\theta$$ 4

$$2LP\left(\theta^2/2\right) - \frac{M\cdot 3\theta}{2} = \frac{3C}{2}\theta^2$$

$$U = PL\theta^2 - \frac{3}{2}C\theta^2 =$$

4

$$\frac{\partial U}{\partial\theta} = 2PL\theta - \frac{3}{\cancel{2}}C\theta\,\cancel{\delta\theta}\ 0 \qquad \underline{\theta=0} \text{ or } P=\frac{3C}{2L} \text{ if } \delta\theta\neq 0$$

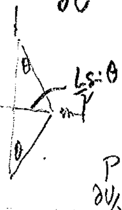
$$\frac{\partial^2 U}{\partial\theta^2} = (2PL-3C)\delta\theta^2 + (2PL-3C)\theta\delta^2\theta = 0$$

$\frac{4}{\cancel{12}}$

$$2PL-3C=0$$

$$\boxed{P=\frac{3C}{2L}}$$

$\frac{2}{1.5}$

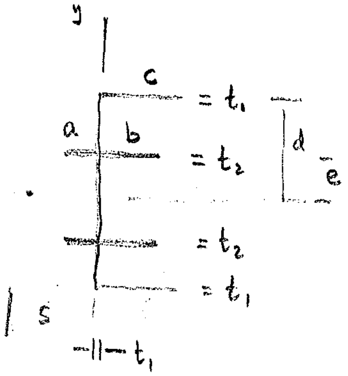


$$\frac{1}{2}k\left(\frac{L}{2}\sin\theta\right)^2$$

$$P\left(2\frac{L}{2}-2\frac{L}{2}\cos\theta\right)-\frac{1}{2}k\frac{L^2}{4}\sin^2\theta = U$$ ④

$$\partial U/\partial\theta = \frac{1}{2}\left(2PL\sin\theta - \frac{kL^2}{4}\sin\theta\cos\theta\right)\delta\theta = 0 \sim \left(2PL\theta - \frac{kL^2}{4}\theta\right)\delta\theta = 0 \qquad \theta=0 \text{ or } \underline{P=kL/4}$$ ④

$$\partial^2 U/\partial\theta^2 = \frac{1}{2}\left(2PL\cos\theta - \frac{kL^2}{4}\cdot 2\cos 2\theta\right)\delta\theta^2 + (\quad)\delta^2\theta = 0$$ ④



$$I_z = \frac{t_1(2d)^3}{12} + 2t_1c_1\cdot d^2 + 2t_2(b+a)e^2$$ 4

$$2d\tau_c\cdot\frac{t_1c}{2} + 2e\tau_b\cdot\frac{t_2b}{2} - \tau_a\cdot\frac{t_2a}{2}\cdot 2e = Vs$$ 4

10 min
×3 30 min

$$\tau_c = \frac{VQ}{I_zt_1} = \frac{Vct_1\cdot d}{I_zt_1} \qquad F_c = \frac{Vc^2dt_1}{2I_z}$$ 2

$$\tau_b = V\cdot\frac{bt_2\cdot e}{I_zt_2} = \frac{Vb^2et_2}{2I_z}$$ 2

$$\tau_a = V\cdot\frac{at_2\cdot e}{I_zt_2} = \frac{Va^2t_2e}{2I_z}$$ 2

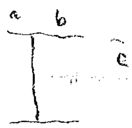
$$2d\cdot\frac{Vct_1\cdot d}{I_zt_1}\cdot\frac{t_1c}{2} + 2e\cdot\frac{Vbt_2e}{I_zt_2}\cdot\frac{t_2b}{2} - 2e\cdot\frac{Vat_2e}{I_zt_2}\cdot\frac{t_2a}{2}Vs$$

$$\frac{d^2c^2t_1^2}{I_zt_1} + \frac{e^2t_2^2b^2}{I_zt_2} - \frac{e^2a^2t_2^2}{I_zt_2} = s$$

$$\frac{d^2c^2t_1 + e^2t_2(b^2-a^2)}{\frac{2}{3}t_1d^3 + 2t_1cd^2 + 2t_2(a+b)e^2} = s$$ 2

$\frac{}{16}$

if e + d disappear



$$\frac{e^2(b^2-a^2)}{2e^2(a+b)} = \frac{b-a}{2}$$

if $e = d/2$ & $t_1 = t_2$

$$\frac{td^2\left(4c^2+(b^2-a^2)\right)}{td^2\left(\frac{8}{3}d+8c+2(a+b)\right)} = \frac{4c^2+(b^2-a^2)}{8/3d+8c+2(a+b)}$$

20	10	13	7	50	62
15	15	11	9	50	62
12	10	14	7	43	55
15	15	11	9	50	62
15	21	14	9	59	71
30	30	15	9	64	76
10	10	14	6	40	52
26	10	14	5	55	67
13	13	13	7	46	58
20	10	14	9	53	65
18	20	14	7	59	71
10	10	12	8	40	52
8	5	6	2	50.75	21
			7.67	±7.36	
17	12.83		±1.3		
+5.91	3.91				
+7	+5				

3	10	7	0	20	32
10	15	15	7	47	59
26	10	16	9	61	73
14	10	16	7	47	59

תורת החוזק 2 מועד מיוחד
פרופ עזרא לוי

מבחן סופי

אורך המבחן הזה יהיה 300 דקות. במבחן יש <u>חמישה</u> שאלות. עליכם לענות <u>לכל</u> השאלות.
רק להשתמש ברשמות הכיתה, תרגילי בית שלכם, ומחשבון. ואם יש דברים הנתונים לכם עם המבחן בלבד.

לחתום על סעיף הבא בבקשה:

לאורך המבחן אני לא רשאי(ת) לקבל עזרה מאף אחד או לתת עזרה לאף אחד. אם אני אעבור על אלה, אני אכשול את המבחן.

חתימת הסטודנט_עדי בן סיניור____________________

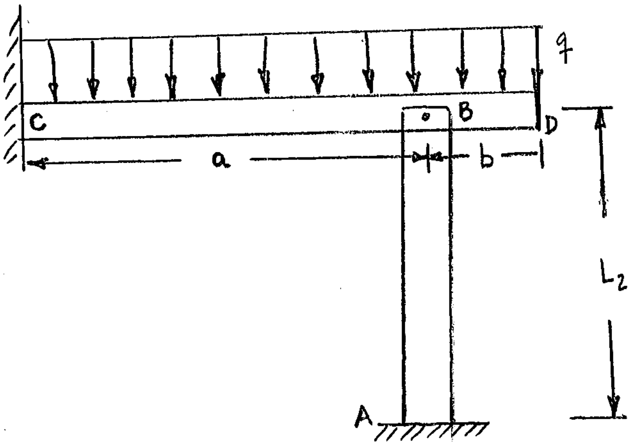
תרגיל מס. 1

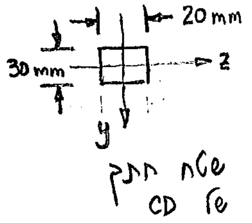
שתי קורות מחוברות ועמוסות כמשורטט בציור. שתי הקורות עשויות מפלדה. הניחו ש- $E = 200$ GPa ו- $\sigma_{yp}=360$ MPa, $L_2=2$ m, $a=1.5$ m, $b=0.5$ m.

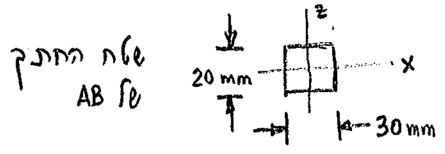
א. מיצאו את העומס המפורס המקסימלי, q, שיכול לפעול על הקורה CD לפני שקורה AB תקרוס.
ב. מהוא q כאשר קורה CD מתחילה להיכנע.
ג. לפי מה שמצאתם ב- (א.) וב- (ב.), איזה קורה תתמוטט בראשונה ומה תהיה הסיבה.

חשבו את q לפי התנאים הבאים: על הקורה AB שפה A רתומה, ומחוברת בסיכה ב-B נגד קריסה בכוון x, ורתומה ב-A וב-B נגד קריסה בכוון z. המדדים של הקורות נתונים בצורה.

הערה: אם זאת בעיה בלתי מסוימת, תשתמשו במשפט קסטגליאנו למצוא כמה מהנעלמים.







תרגיל מס. 2

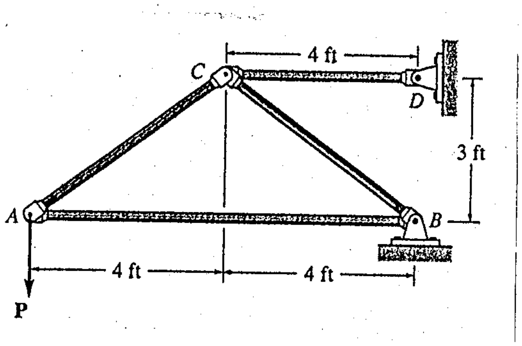
מסבך מוטות, עשוים מפלדה, מעומס בכוח P כמשורטט בציור. אם לכל מוט יש שטח החתך עגול בקוטר של 3 in.

א. מיצאו את הכוח שיגרום לקריסה ובאיזה מוט זה יקרה.

ב. מה צריך להיות הקוטר כדי שיהיה קריסה וכניעה בו-זמנית במסבך.

חשבו את P לפי התנאים הבאים: כל המוטות מחוברות בסיכה בשני הצדדים נגד קריסה בכוון x, ורתומים בשני הצדדים נגד קריסה בכוון z.

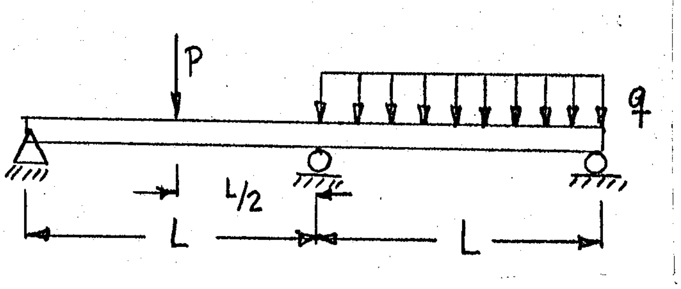
הניחו ש- $E = 29 \times 10^6$ lb/in^2 ו- σ_{yp}=36000 lb/in^2. המדדים של הקורות נתונים בצורה, ויש לזכור ש- 12 in.= 1 ft.



תרגיל מס. 3

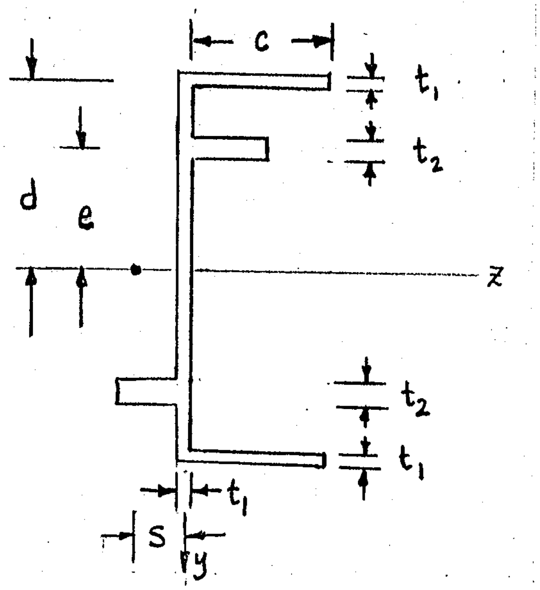
נתון גשר רצוף מורכב משני חלקים AB ו- BC ונושא עומס P ועומס מפורס, q, כמשורטט בצורה. לפי שיטת עומס הגבולי (זאת אומרת, מומנט הפלסטי). מיצאו את היחס בין qL ל- P כדי ששני חלקי הגשר יתמוטטו בו-זמני.

רמז - מיצאו את מומנט הפלסטי לכל האפשרויות ויחסו את התוצאות.



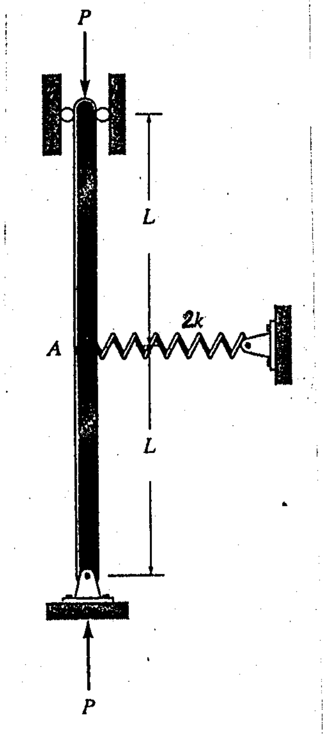
תרגיל מס. 4

נתון חתך בדק-דופן כמשורטט בצורה הבאה. מיצאו את מרכז הגזירה לחתך, s.



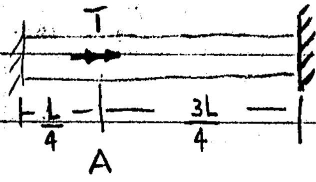
תרגיל מס. 5

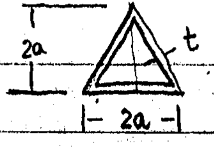
למערכת המשורטטת, עליכם למצוא את הכוח הקריטי, לפי שיטת אנרגייה, שגורם להתמוטטות המערכת.



(a)

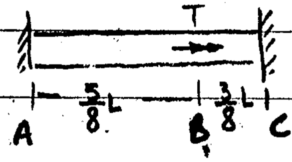
קורה עם שטח חתך בצורת משולש דק דופן ניתנת בתרשים. הקורה רתומה בשני צדדיה ומפעילים מומנט פיתול T בנקודה A. מודול גזירה שווה ל-G וקחו ש- $\frac{a}{t} >> 10$

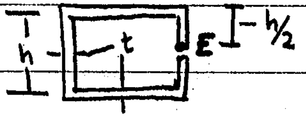




- מצאו את הפיתולים בסמכים הרתומים
- מצאו ביטוים למאמץ גזירה שבא מהפיתולים
- מצאו את זווית הפיתול ב-A.

קורה עם שטח חתך בצורת מלבן דק דופן ניתנת בתרשים. בשטח החתך של המלבן יש רווח קטן בנקודה E לאורך כל הקורה. הקורה רתומה משני הצדדים ומומנט פיתול T פועל בנקודה B. מודול גזירה של הקורה שווה ל-G וקחו ש- $\frac{h}{t} >> 10$ ו- $\frac{b}{t} >> 10$





- מצאו את הפיתולים בסמכים הרתומים
- מצאו ביטוים למאמץ גזירה שבא מהפיתולים
- מצאו את זווית הפיתול ב-B.

$$\varphi = \frac{TL}{c_2 ab^3 G}$$

$a_{new} = 2h+2b$
$b = t$
$c_2 = .333$

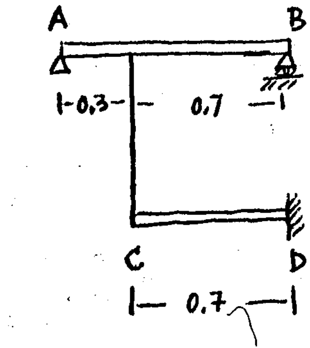
$$\varphi_{AB} = \frac{T_1 L_1}{c_2 (ab^3 G)} \qquad \varphi_{BC} = \frac{T_2 L_2}{c_2 (ab^3 G)} \qquad \varphi_{AB} + \varphi_{BC} = 0$$

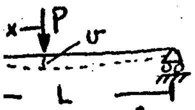
$T_A + T + T_C = 0$
$T_1 = -T_A$

ג.ג.

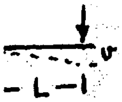
שתי קורות פלדה מחוברות בחבל מתוח, גם מפלדה, ואורכו 3.75m. בהתחלה,
אין כוח ומומנטים שפועלים על המערכת. אם מקררים את
החבל ב- 50°C, מצאו את שינוי מיקום של C.

נתונים: $E= 206GPa$, $\alpha= 12\times10^{-6}$ 1/°C, לשתי הקורות $I= 850 cm^4$
שטח החתך לחבל = $22\ cm^2$. גם נתון הקשר בין כוח P
והשקיעה (התזוזה) לכול קורה בצד שמאל





$$= \frac{P(x-L)}{6EIL}\left[x^2(2L-x)-x^3\right]$$



$$v = -\frac{PL^3}{3EI}$$

$$u = \alpha\Delta T\cdot l + \frac{Pl}{AE} = -(|v_2|+|v_1|)$$

$$-2.25\times10^{-3} + 8.27\times10^{-9}P = -(8.4\times10^{-9}P + 6.5\times10^{-8}P)$$

$$-2.25\times10^{-3} = -8.167\times10^{-8}P$$

$$27550\ N = P$$

$$u = -2.02\times10^{-3}$$

$v_2 = -2.3142\times10^{-4}$ T ✓

C עולה ב- $1.791\times10^{-3}m = .0018mm$

$v_1 = 1.79075\times10^{-3}$ ⊥ ✓

2

$G = \frac{206\,GPa}{2.6} = 79.23$ $\quad \nu = 0.3 \quad E = 206\,GPa \qquad \gamma_{x'y'} = \frac{\tau_{x'y'}}{G} = \frac{\tau_{x'y'}}{E/2(1+\nu)}$ הוא עיבור הגזירה

$$\underline{\gamma_{x'y'}} = \frac{-16.16\,MPa}{206\,GPa/2.6} = \underline{-2.04\times10^{-4}} \quad 2$$

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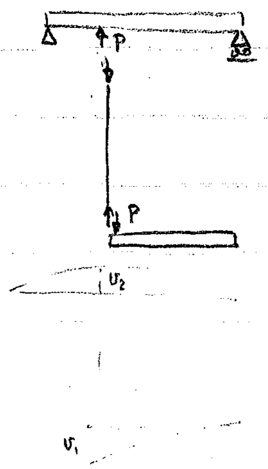

$x=.3 \quad 1=L$

$$v_2 = \frac{-P(.7)\,m}{6[206\times10^9][850\times10^{-8}][1]}\left[.3^2(1.7) - .3^3\right] = +8.4\times10^{-9}P \quad 2 \quad (6$$

$\frac{N}{m^2}, m^4$

$$v_1 = \frac{-PL^3}{3EI} = \frac{-P(.7)^3}{3[206\times10^9][850\times10^{-8}]} = -6.5\times10^{-8}P \quad 2$$

$$v_2 - v_1 = \alpha\Delta T l - \frac{Pl}{AE} \qquad [N] \quad 4$$

$$73.69\times10^{-9}P = 12\times10^{-6}(50)(3.75) - \frac{P\,(3.75)}{(1.6\times10^{-4})(206\times10^9)}$$

$$= 2.25\times10^{-3} - 11.4\times10^{-8}P$$

$$1.875\times10^{-7}P = 2.25\times10^{-3} \qquad \underline{P = 12002\,N} \quad 2$$

2 נקודת C זז ב- $\underline{-0.8\,mm = -.0008\,m = -6.5\times10^{-8}P}$

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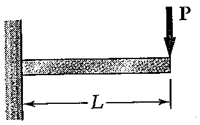
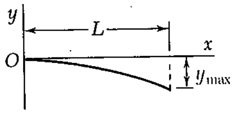
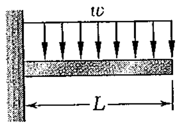
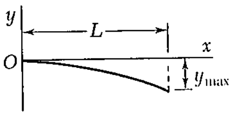
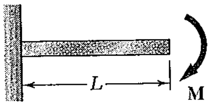
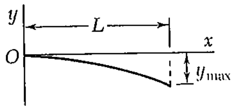
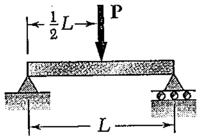
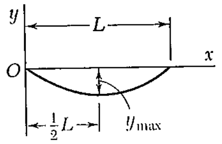
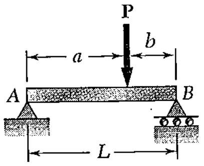
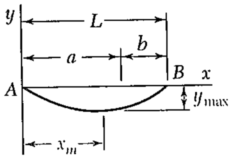
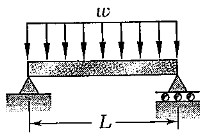
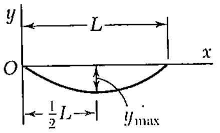
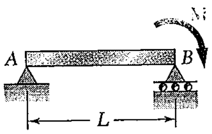
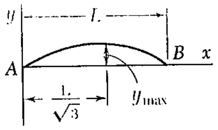
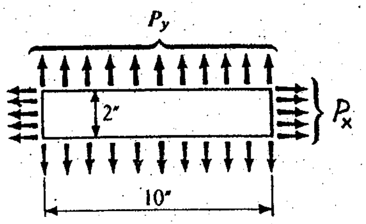
Beam and Loading	Elastic Curve	Maximum Deflection	Slope at End	Equation of Elastic Curve
1 		$-\frac{PL^3}{3EI}$	$-\frac{PL^2}{2EI}$	$y = \frac{P}{6EI}(x^3 - 3Lx^2)$
2 		$-\frac{wL^4}{8EI}$	$-\frac{wL^3}{6EI}$	$y = -\frac{w}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$
3 		$-\frac{ML^2}{2EI}$	$-\frac{ML}{EI}$	$y = -\frac{M}{2EI}x^2$
4 		$-\frac{PL^3}{48EI}$	$\pm\frac{PL^2}{16EI}$	For $x \leq \frac{1}{2}L$: $y = \frac{P}{48EI}(4x^3 - 3L^2x)$
5 		For $a > b$: $-\frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}EIL}$ at $x_m = \sqrt{\frac{L^2 - b^2}{3}}$	$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL}$ $\theta_B = +\frac{Pa(L^2 - a^2)}{6EIL}$	For $x < a$: $y = \frac{Pb}{6EIL}[x^3 - (L^2 - b^2)x]$ For $x = a$: $y = -\frac{Pa^2b^2}{3EIL}$
6 		$-\frac{5wL^4}{384EI}$	$\pm\frac{wL^3}{24EI}$	$y = -\frac{w}{24EI}(x^4 - 2Lx^3 + L^3x)$
7 		$\frac{ML^2}{9\sqrt{3}EI}$	$\theta_A = +\frac{ML}{6EI}$ $\theta_B = -\frac{ML}{3EI}$	$y = -\frac{M}{6EIL}(x^3 - L^2x)$

עשו את תרגילים מס. 2.37, 2.38 מדף הקודם

4-5. A piece of 2-in.-by-10-in.-by-½-in. steel plate is subjected to uniformly disributed stresses along its edges (see figure). (a) If $P_x = 20$ kips and $P_y = 40$ kips, what change in thickness occurs due to the application of these forces? (b) To cause the same change in thickness as in (a) by P_x alone, what must be its magnitude? Let $E = 30 \times 10^6$ psi and $\nu = 0.25$.



PROB. 4-5

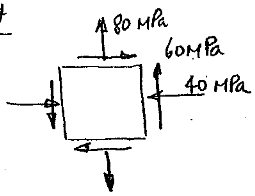
אם העיבורים הנמצאים בנקודה שווים ל:

$\varepsilon_x = -800 \times 10^{-6}$
$\varepsilon_y = -200 \times 10^{-6}$
$\gamma_{xy} = 800 \times 10^{-6}$

a) מה העיבורים הראשיים ומה הכיוונים הראשיים?

b) אם המדידות האלו באות מאבר פלדה שמודול יאנג (YOUNG) = $E = 29.5 \times 10^6$ psi
ו יחס פואסון (POISSON) $\nu = 0.3$, מה המאמצים הראשיים והכיוונים הראשיים שלהם

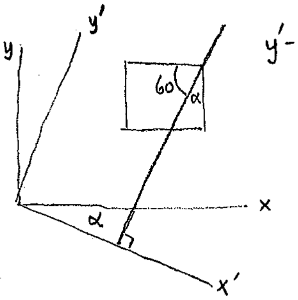
6.4



$\sigma_x = -40$ MPa
$\tau_{xy} = 60$ MPa
$\sigma_y = 80$ MPa

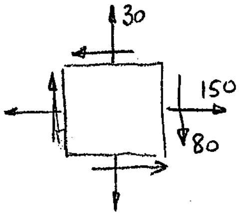
$$\sigma_x' = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha$$
$$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\alpha + \tau_{xy}\cos 2\alpha$$

(א) לפי הדיאגרמה $\alpha = -30°$, כי האלכסון הוא מקביל ל-y'



$$\sigma_x' = \left(-\frac{40+80}{2}\right) + \left(\frac{-40-80}{2}\right)\cos(-60) + 60\sin(-60)$$
$$= 20 + (-60)\cdot\tfrac{1}{2} + 60(-.866) = \underline{-61.96 \text{ MPa}}$$
$$\tau_{x'y'} = -\left(\frac{-40-80}{2}\right)\sin(-60) + 60\cos(-60)$$
$$= +60(-.866) + 60\left(\tfrac{1}{2}\right) = \underline{-21.96 \text{ MPa}}$$

6.6



$\sigma_x = 150$ MPa
$\sigma_y = 30$ MPa
$\tau_{xy} = -80$ MPa

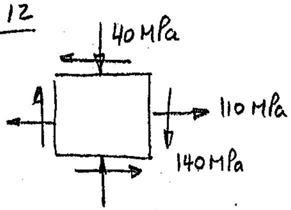
(ב) למצוא את הצירים הראשיים וגם המאמצים הראשיים

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \left[\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$
$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

לכן

$$\sigma_{1,2} = \left(\frac{150+30}{2}\right) \pm \left[\left(\frac{150-30}{2}\right)^2 + (-80)^2\right]^{1/2}$$
$$= 90 \pm 100 \Rightarrow \quad \sigma_1 = 190.\text{[illegible]} \text{ MPa} \quad \sigma_2 = -10\text{[illegible]}5 \text{ MPa}$$
$$\tan 2\theta = \frac{2(-80)}{150-30} = -1.333$$
$$2\theta = -53.13 \quad \theta = \underline{-26.57°} \qquad \theta_2 = \theta + 90° = \underline{63.43°}$$

6.12



$\sigma_x = 110$ MPa
$\sigma_y = -40$ MPa
$\tau_{xy} = -140$ MPa

למצוא את הצירים של מאמץ הגזירה המקסימלי והמאמץ המקס.

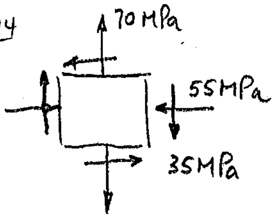
$$\tau_{\text{מקס}} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \left[\left(\frac{110-(-40)}{2}\right)^2 + (-140)^2\right]^{1/2} = \pm 158.82 \text{ MPa}$$
$$\tan 2\alpha = \frac{-(\sigma_x-\sigma_y)}{2\tau_{xy}} = \frac{-[110-(-40)]}{2(-140)} = \frac{-150}{-280} = 208.18°$$

מפני שהיחס
הוא שלילי מחלקי שלילי

$\alpha = 104.09°$
$\alpha + 90° = 194.09° = 14.09°$

$$\sigma_x' = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha$$
$$= \frac{110-40}{2} + \left(\frac{110-(-40)}{2}\right)\cos 208.18 + (-140)\sin 208.18°$$
$$= 35 + 75\cos 208.18 - 140\sin 208.18 = 35 \text{ MPa} = (\sigma_x+\sigma_y)/2$$

6.14



$\sigma_x = -55$ MPa

בסעיף א) למצוא את המאמצים כאשר החלקיק הולך ומסתובב 40° נגד השעון (מאמצים נורמלי וגזירה).
בסעיף ב) אותו תהליך כאשר החלקיק הולך ומסובב 15° בכיוון השעון.

$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha \qquad \alpha = 40°$$
$$= \frac{-55+70}{2} + \left(\frac{-55-70}{2}\right)\cos 80 + (-35)\sin 80$$

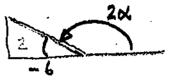
עבורים ראשיים

$$\epsilon_1,\epsilon_2 = \frac{\epsilon_x+\epsilon_y}{2} \pm \frac{1}{2}\sqrt{(\epsilon_x-\epsilon_y)^2+\gamma_{xy}^2}$$

$$= \left\{-\frac{800-200}{2} \pm \frac{1}{2}\sqrt{(-800+200)^2+800^2}\right\}\times 10^{-6}$$

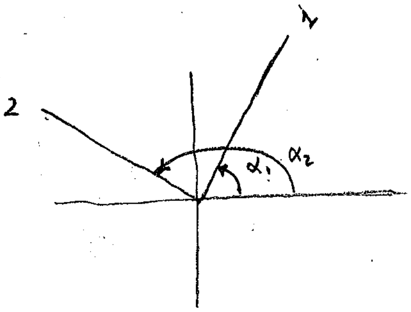
$$= (-500 \pm 500)\times10^{-6} = 0 \, , -1000\times10^{-6}$$

$$\tan 2\alpha = \frac{\gamma_{xy}}{\epsilon_x-\epsilon_y} = \frac{800\times10^{-6}}{-800\times10^{-6}+200\times10^{-6}} = \frac{8}{-6} \Rightarrow$$



$2\alpha = 126.86^\circ$

$$\left.\begin{array}{l}\alpha_1 = 63.43^\circ \\ \alpha_2 = 153.43^\circ\end{array}\right\}$$ זווית הראשיות



ב) למצוא את המאמצים: כדי למצוא $\tau_{xy} = \gamma_{xy}\cdot G$

$$G = \frac{E}{2(1+\nu)} = \frac{29.5\times10^6}{2(1.3)} = \frac{29.5\times10^6}{2.5} = 11346154 \ \text{lb/in}^2$$ בדיוק

$$\tau_{xy} = (800\times10^{-6})(11.8\times10^6) = 9077 \ \text{lb/in}^2$$

אם מניחים ש $\sigma_z=0$ כמו בלוח, אז

$$\left.\begin{array}{l}\sigma_1 = \frac{E}{1-\nu^2}(\epsilon_1+\nu\epsilon_2) \\ \sigma_2 = \frac{E}{1-\nu^2}(\epsilon_2+\nu\epsilon_1)\end{array}\right\}$$ אפשר לפתור את אלה הפוך $$\left\{\begin{array}{l}\epsilon_1 = \frac{\sigma_1-\nu\sigma_2}{E} \\ \epsilon_2 = \frac{\sigma_2-\nu\sigma_1}{E}\end{array}\right.$$

$$\sigma_z=0 \Longrightarrow \sigma_3=0$$

לכן,

$$\sigma_1 = \frac{29.5\times10^6}{(1-0.3^2)}\left(0+0.3\left(-1000\times10^{-6}\right)\right)$$

$$= -9725 \ \text{lb/in}^2$$

$$\sigma_2 = \frac{29.5\times10^6}{(1-0.3^2)}\left(-1000\times10^6 + 0.3(0)\right)$$

$$= -32418 \ \text{lb/in}^2$$

או אם $\sigma_z=0$, $G=\mu=11346154$ ו- $\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} = 17019231$ אז

$$\sigma_z = 0 = \lambda e + 2\mu\epsilon_z = \lambda(\epsilon_x+\epsilon_y)+(\lambda+2\mu)\epsilon_z \quad \text{or} \quad \epsilon_z = -\frac{\lambda}{\lambda+2\mu}(\epsilon_x+\epsilon_y) =$$

$$\epsilon_z = -.42857\left(-1000\times10^{-6}\right) = 428.57\times10^{-6}$$

$$\sigma_x = \lambda e + 2\mu\epsilon_x = (\lambda+2\mu)\epsilon_x + \lambda(\epsilon_y+\epsilon_z) = 39711539\left(-800\times10^{-6}\right)+17019231\left(228.57\times10^{-6}\right)$$

$$= -27879 \ \text{lb/in}^2 \qquad (-35173)$$

$$\sigma_y = \lambda e + 2\mu\epsilon_y = (\lambda+2\mu)\epsilon_y + \lambda(\epsilon_x+\epsilon_z) = 39711539\left(-200\times10^{-6}\right)+17019231\left(-371.43\times10^{-6}\right)$$

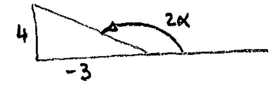
$$= -14264 \ \text{lb/in}^2 \qquad (-21558)$$

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = \frac{-27879-14264}{2} \pm \sqrt{\left(\frac{-27879+14264}{2}\right)^2+(9077)^2}$$

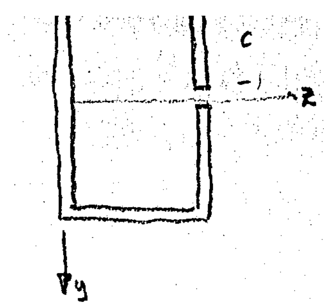
$$= -21071.5 \pm 11346.1 = -9725 \, , -32418 \ \text{lb/in}^2$$

$$(-17019) \qquad (-39712)$$

$$\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x-\sigma_y} = -1.333 \Rightarrow$$



$2\alpha = 126.86^\circ$



צורת מאמץ הגזירה שפועל על החתך

$$It$$

$$\tau_b = (\tau_b - \tau_a) + \tau_a = \frac{VQ_b}{It} + \frac{VQ_a}{It}$$

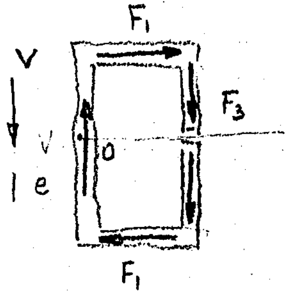
$$I = \frac{t(2c)^3}{12} + 2(bt)c^2 + 2\left[\frac{tc^3}{12} + tc\left(\frac{c}{2}\right)^2\right]$$

$$Q_a = \int y\,dA = \int_{b-t/2}^{b+t/2}\int_0^{0} y\,(dy\,dz) = \frac{y^2}{2}t\Big|_0^c = \frac{c^2t}{2}$$

$$Q_b = \iint y\,dA = \bar{y}A = c \cdot bt$$

$$F_3 = \int \tau_a\,dA = \frac{V}{It}\int_0^c \frac{y^2t}{2} \cdot t\,dy = \frac{V}{I}\frac{y^3t}{6}\Big|_0^c = \frac{Vtc^3}{6I}$$

$$F_1 = \frac{\tau_a + \tau_b}{2}(b \cdot t) = \frac{Vtbc}{2I}(b+c)$$



$$\Sigma M_o = 2F_3 \cdot b + F_1 \cdot 2c - Ve = 0$$

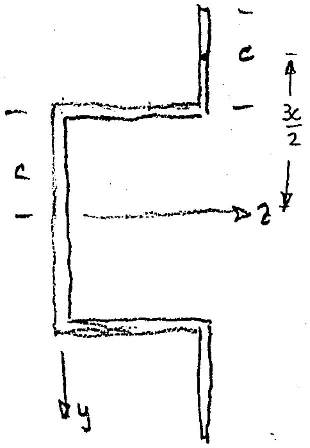
$$= 2\tau_a \frac{ct}{3} \cdot b + \left(\frac{\tau_a + \tau_b}{2}\right) bt \cdot 2c - Ve = 0$$

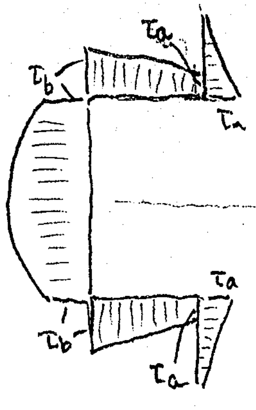
$$= \frac{2VQ_a}{It} \cdot \frac{ct}{3} \cdot b + \left(\frac{VQ_b}{2It} + \frac{2VQ_a}{2It}\right) bt \cdot 2c - Ve = 0$$

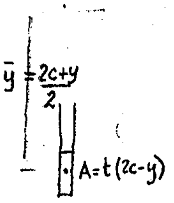
$$= \frac{V}{It}\left\{\frac{2c^2t}{2} \cdot \frac{ct}{3} \cdot b + \left(\frac{cbt}{2} + \frac{c^2t}{2}\right) bt \cdot 2c\right\} - Ve = 0$$

$$= \frac{V}{It}\left\{\frac{c^3t^2b}{3} + cb^2t^2 + c^3bt^2\right\} - Ve = 0$$

$$e = \frac{1}{It}\left\{\frac{4}{3}c^3t^2b + c^2b^2t^2\right\} = \frac{c^2bt}{I}\left(b + \frac{4}{3}c\right) = \frac{c^2bt\left(b + \frac{4}{3}c\right)}{c^2t\left(\frac{4}{3}c + 2b\right)} = \frac{b\left(b + \frac{4}{3}c\right)}{\left(\frac{4}{3}c + 2b\right)}$$







$$\tau_a = \frac{VQ_a}{It} \qquad \tau_b = (\tau_b - \tau_a) + \tau_a = \frac{VQ_b}{It} + \frac{VQ_a}{It}$$

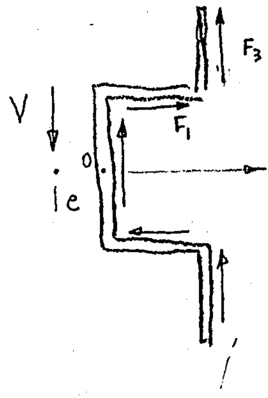
$$I = \frac{t(2c)^3}{12} + 2btc^2 + 2\left[\frac{tc^3}{12} + tc\left(\frac{3c}{2}\right)^2\right]$$

$$Q_a = \int y\,dA = t(2c-y)\left(\frac{2c+y}{2}\right) = \frac{t}{2}(4c^2 - y^2)\Big|_{y=c} = \frac{3c^2t}{2}$$

$Q_b = cbt$ כמו בתרגיל הקודם

$$F_3 = \int \tau_a\,dA = \frac{V}{It}\int_c^{2c} \frac{t}{2}(4c^2 - y^2) \cdot t\,dy = \frac{Vt}{2I}\left(4c^2y - \frac{y^3}{3}\right)_c^{2c} = \frac{5}{6I}$$

$$F_1 = \frac{\tau_b + \tau_a}{2} bt = \frac{1}{2}\left(\frac{VQ_b}{It} + \frac{2VQ_a}{It}\right) bt = \frac{Vtbc}{2I}(b + 3c)$$

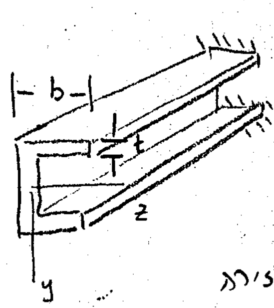


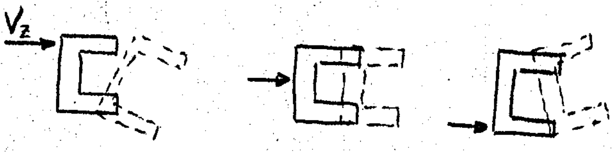
$$\Sigma M_o = -Ve + F_1 \cdot 2c - 2F_3 \cdot b = 0$$

$$= -Ve + \frac{Vtbc}{2I}(b+3c) \cdot 2c - 2 \cdot \frac{5}{6I}Vtc^3 \cdot b$$

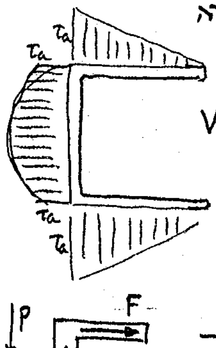
$$e = \frac{tbc^2(b+3c) - \frac{5}{3}tc^3b}{\frac{64}{12}tc^3 + 2btc^2} = \frac{b}{2}\frac{\left(b + \frac{4}{3}c\right)}{\left(b + \frac{8}{3}c\right)}$$

מרכז גזירה - מקום שבו הגזירה כרוך [illegible] פיתול



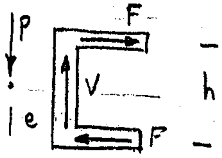


אם לשטח יש שני צירים סימטריים, צומת הצירים הוא המרכז הגזירה



אם יש רק ציר אחד כמו בפרופיל כאן מפני שמאמץ הגזירה הוא כמו בצורה

- מניחים שהקירות דקים, שהמאמץ הגזירה ברגלי האופקיות הוא ליניארי מצד החופשי והמסכת המאמץ פרבולי: הכוח בכול רגל הוא $\frac{\tau_a}{2}bt$ והגזירה האנכית $V=\int_{-h/2}^{h/2}\tau t\,dy$
- מניחים סיבולים למרוד הרגליים והמסכת בקו האמצעי; ומאמץ הגזירה טנגנט לקיר
- מאמץ הכפיפה מתמיד בכיוון העובי;

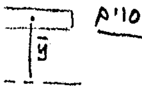


לכן, הפיתול שבא מהכוחות F הוא F·h. כדי למנוע את הפיתול, צריך לשים כוח P, במרחק e מהמסכת. לפי שווי משקל $P=V$ ו- $Pe=Fh$

$$e=\frac{Fh}{V}=\frac{\tau_a bt/_2\cdot h}{V}$$

$\frac{\tau_a}{2}$ - מאמץ הגזירה הממוצע של הרגל וכמו שלמדנו בתורת החוזק 1

$$\tau_a=\frac{VQ}{It}=\frac{q}{t}$$



$Q = \int y\,dA$ של הרגל $=\int yt\,ds = \bar{y}A = \frac{h}{2}.bt$. קוראים לזה המומנט הסטטי של חלק השטח המסוים

- $\bar{y}$ - המרחק ממרכז הכובד של הרגל לציר הנטרלי של הגוף
- q - זרם הגזירה

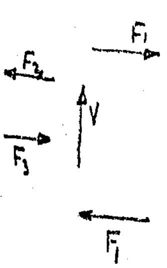
$$e=\frac{V(h/_2\cdot bt)}{It}\cdot\frac{bt/_2\cdot h}{V}$$

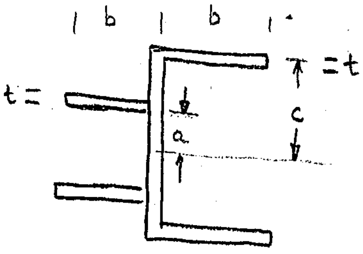
אם הרגל דקה מאוד $2bt\cdot\left(\frac{h}{2}\right)^2+\frac{th^3}{12}\cong I$ פה. $e=\frac{b^2h^2t}{4I}$

- המרחק e תלוי בשטח ולא בעומס ולא במקום שהעומס מיושם.

דוגמה

$$F_1\cdot 2c-F_3\cdot 2a=Pe$$
$$P=V$$





$$\frac{\tau_a}{2}.bt.2c-\frac{\tau_b}{2}.bt.2a=Ve$$

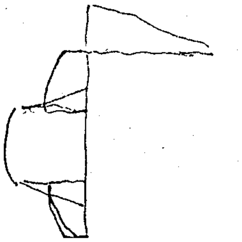
$$\frac{1}{2}\frac{VQ_a}{It}.bt\cdot 2c-\frac{1}{2}\frac{VQ_b}{It}.bt\cdot 2a=Ve$$

$$\frac{1}{2}\frac{V(bt\cdot c)}{It}.bt\cdot 2c-\frac{1}{2}V\frac{(bt\cdot a)}{It}.bt\cdot 2a=Ve$$

$$e=\frac{1}{2}\left\{\frac{2t^2(b^2c^2-b^2a^2)}{It}\right\}$$

$$I\cong\frac{t(2c)^3}{12}+2bt\cdot c^2+2bt.a^2$$

$$e=\quad 3b^2(c^2-a^2)$$



המכללה האקדמית יהודה ושומרון

בוחן ל חוזק חומרים 2

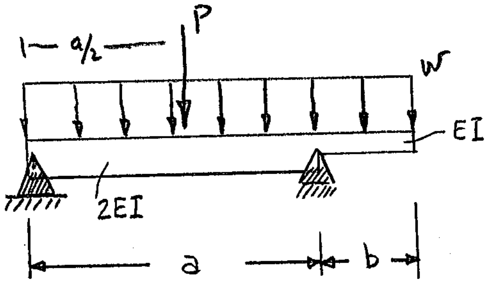
סמסטר א' מועד א' התשס"א

פרופ' עזרא לוי

<u>הערות: חומר פתוח. 3 שעות בלבד</u>.

1) לקורה שבולטת מעל סמך B , מצאו את קוו אלסטיות ,$v(x)$, והשקיעה בקצה החופשי. מקדם קשיחות קבוע בכל חלק וחלק.

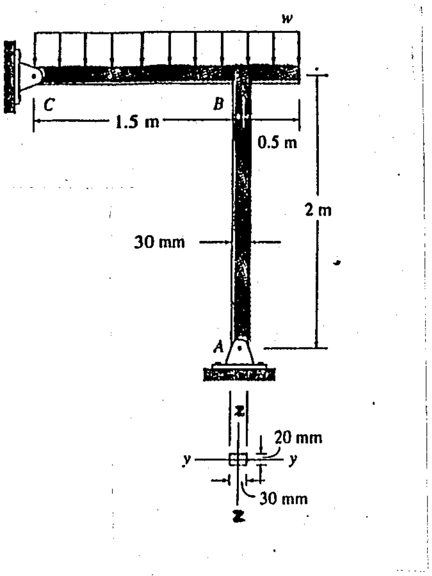
- מה יהיה המומנט המקסימלי בקורה AB, ומהו המיקום שלו.



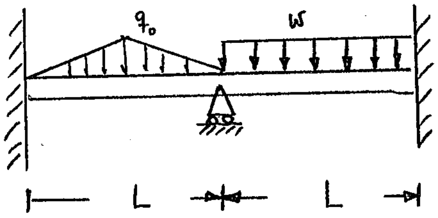
2) מצאו את העוצמה המקסימלית המותרת, w, של הכוח המפורס שנוכל ליישם על הקורה BC ללא גרימת קריסה לקורה AB.

הניחו שהקורה AB נעשתה מפלדה והיא באופן פרקי-פרקי בקצותיה לקריסה מסביב ציר z-z ובאופן רתום-רתום בקצותיה לגבי קריסה מסביב ציר y-y.

השתמשו במקדם בטיחות לגבי קריסה שווה ל-3.0. קחו $E_{\text{פלדה}}=200GPa$, $\sigma_{yp}=360$ MPa

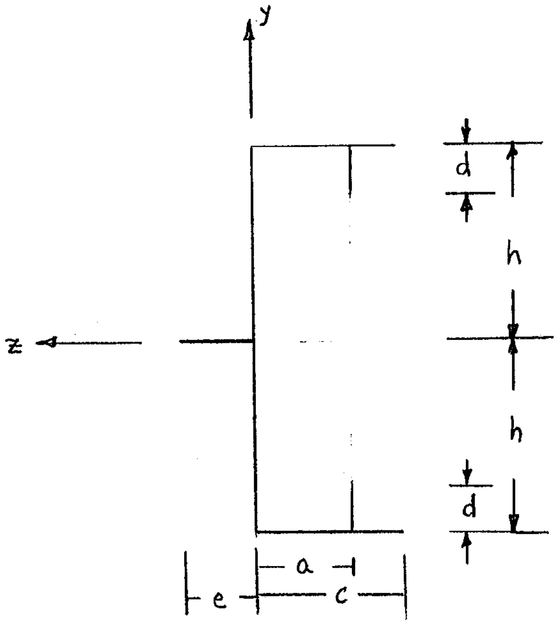


3). לקורה שנמצאת בתרשים, מצאו את הכוחות המפורסים הגבוליים, w ו-q_0, שיגרמו להתמוטטות בו-זמנית.



4) לשטח חתך בעל דק-דופן, מצאו את מרכז הגזירה שלו.

עובי הדפנים = t



תורת החוזק 2 בוחן מס 1 ינואר 2 2000
פרופ עזרא לוי

אורך הבוחן הזה יהיה 90 דקות. בבוחן יש ~~שלושה~~ שתי שאלות. עליכם לענות בכל השאלות.
רק להשתמש בתפסים הראשים (כשלוש-עשרי דפים), ומחשבון, ואם יש, דברים הנתונים לכם עם הבוחן.

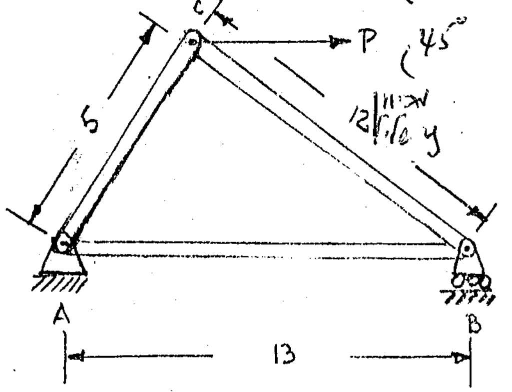
לחתום על סעיף הבא בבקשה:
לאורך הבוחן אני לא רשאי(ת) לקבל עזרה מאף אחד או לתת עזרה לאף אחד. אם אני אעבור על אלה,
אני אכשל את הבוחן.

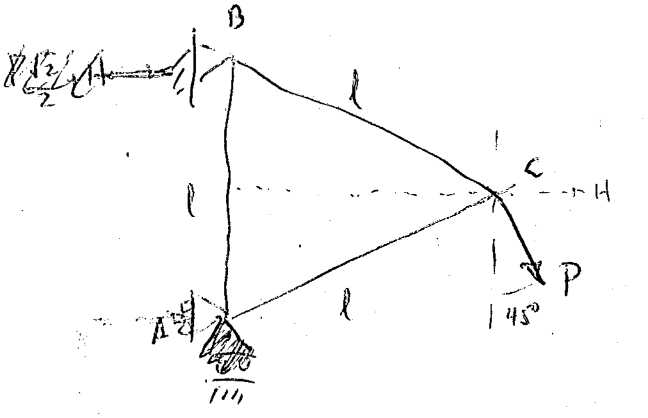
חתימת הסטודנט____________________ ____________________
שם הסטודנט מס. זהות

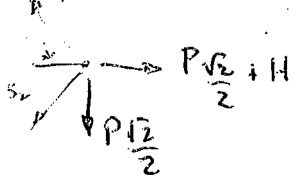
תרגיל מס. 1 לכל צלע

נתון ~~לכם~~ מסבך ~~המשורטט בציור~~ [illegible] בצד השמאל. שטח החתך הוא A, ומודול יאנג הוא E. הסמכים כנמצאים על הציור. בנקודה C, המסבך עמוס בעומס ~~האופקי~~ P. לא לכלול את משקל של המוטות.

נא למצוא:
התזוזה האופקית, u_C.
שינוי אורך המוט AC







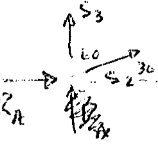
$R_{B_y}+R_A=\frac{P\sqrt{2}}{2}$

$S_1\cos30^\circ+S_2\cos30^\circ=\frac{P\sqrt{2}}{2}+H$

$S_1\sin30-S_2\sin30=\frac{P\sqrt{2}}{H}$

$S_1=\frac{\left(\frac{P\sqrt{2}}{2}+H\right)\sin30^\circ+\frac{P\sqrt{2}}{2}\cos30}{\sin60^\circ}$

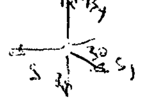
$S_2=\frac{\left(\frac{P\sqrt{2}}{2}+H\right)\sin30-\frac{P\sqrt{2}}{2}\cos30}{\sin60^\circ}$



$0=S_2\cos30^\circ+S_3$ $S_3=S_2\sin30^\circ$

$S_2\cos30^\circ+R_A=0$

$R_A=-S_2\cos30^\circ$



$S_1\cos30=\frac{P\sqrt{2}}{2}+H$

$S_1\sin30+S_3=R_{B_y}$

$\Sigma M_B=R_A\cdot l\cdot+\left(\frac{P\sqrt{2}}{2}+H\right)\frac{l}{2}-\frac{P\sqrt{2}}{2}\frac{l\sqrt{3}}{2}=0$

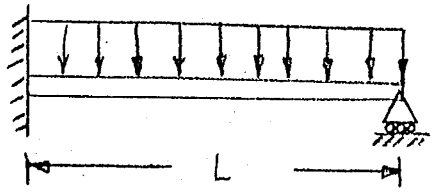
$R_A=P\left(\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}\right)-\frac{H}{2}$

$R_A+R_{B_x}+\frac{P\sqrt{2}}{2}+H=0$ $R_{B_x}=-\left(\frac{P\sqrt{2}}{2}+H\right)+P\left(\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}\right)+\frac{H}{2}$

$=-P\sqrt{2}-P\sqrt{2}-H$

תרגיל מס. 2

נתון לכם קורה עמוסה המשורטטת בציור בצד השמאל. העומס הוא עומס מפורס בגודל של q N/m. בצד שמאל סמך רתום, ובצד ימין יש סמך נייד.



נא למצוא:

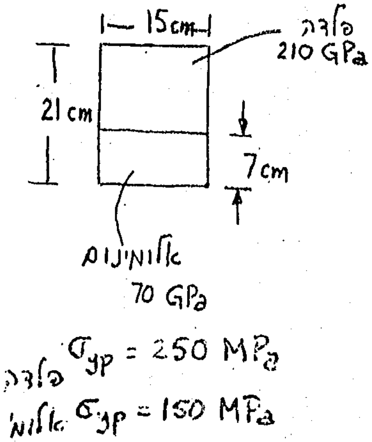
(a) התגובות בסמכים.

(b) המומנט המקסימלי ומקומו.

(c) נתון לכם ששטח החתך כבציור (B). למצוא את המאמץ המקסימלי.

(d) כאשר המאמץ המקסימלי מגיע ל-σ_{yp}, מהוא העומס המפורס שגורם למאמץ הזה.

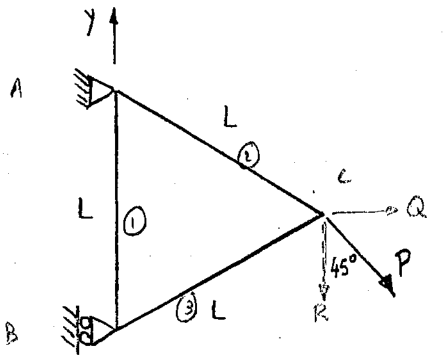
~~(e) לפי שיטת הפלסטית, מהוא המומנט, M_{pl}, והעומס המפורס שחקורה יכולה לסבול.~~

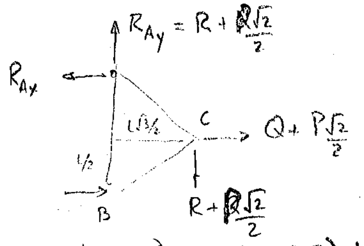


המכללה האקדמית יהודה ושומרון
בוחן ל סטטיקת מבנים
פרופ' עזרא לוי

<u>הערות: חומר פתוח. 2 שעות בלבד.</u>

1) עבור EA=const <u>דרוש לחשב את תזוזה אופקית ואנכית</u> הבאות מכוח P, בשיטת אנרגיה. הכוח פועל כבתרשים ואורכו כל צלע וצלע הוא L.



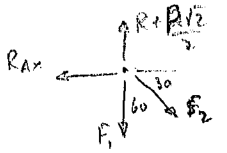


$$R_B \cdot L + \left(Q + \frac{P\sqrt{2}}{2}\right) \cdot \frac{L}{2} \mp \left(R + \frac{P\sqrt{2}}{2}\right) \frac{L\sqrt{3}}{2} = 0$$

$$R_B = \left(R + \frac{P\sqrt{2}}{2}\right)\frac{\sqrt{3}}{2} - \left(Q + \frac{P\sqrt{2}}{2}\right) \cdot \frac{1}{2} \quad ; \quad R_{Ax} = R_B + Q + \frac{P\sqrt{2}}{2}$$

$$R_{A_X} = \left(R + \frac{P\sqrt{2}}{2}\right)\frac{\sqrt{3}}{2} + \left(Q + \frac{P\sqrt{2}}{2}\right)\frac{1}{2}$$

$\cos 30 = \frac{\sqrt{3}}{2}$

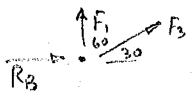


$F_2 \cos 30° = R_{Ax}$

$$\boxed{F_2 = R_{Ax}/\cos 30° = \left(R + \frac{P\sqrt{2}}{2}\right) + \left(Q + \frac{P\sqrt{2}}{2}\right)\frac{1}{\sqrt{3}}}$$

$F_1 + F_2 \sin 30° = R + \frac{P\sqrt{2}}{2}$

$$F_1 = -\frac{R_{Ax}}{\cos 30°} \sin 30° + R + \frac{P\sqrt{2}}{2} = -R_{Ax} \tan 30° + \left(R + \frac{P\sqrt{2}}{2}\right)$$
$$= -\left(R + \frac{P\sqrt{2}}{2}\right)\frac{1}{2} - \left(Q + \frac{P\sqrt{2}}{2}\right)\frac{1}{2\sqrt{3}} + R + \frac{P\sqrt{2}}{2}$$



$F_3 \cos 30 + R_{B} = 0$

$$\boxed{F_3 = -R_B/\cos 30° = -\left(R + \frac{P\sqrt{2}}{2}\right) + \left(Q + \frac{P\sqrt{2}}{2}\right)\frac{1}{\sqrt{3}}} \quad \boxed{F_1 = \left(R + \frac{P\sqrt{2}}{2}\right)\frac{1}{2} - \left(Q + \frac{P\sqrt{2}}{2}\right)\frac{1}{2\sqrt{3}}}$$

$$\sum \frac{F_i^2 L_i}{2 A_i E_i} = \frac{F_1^2 L}{2AE} + \frac{F_2^2 L}{2AE} + \frac{F_3^2 L}{2AE} \qquad u_c = \left.\frac{\partial U_c}{\partial Q}\right|_{\substack{Q=0\\R=0}} = \frac{1}{AE}\left[F_1 \frac{\partial F_1}{\partial Q} + F_2 \frac{\partial F_2}{\partial Q} + F_3 \frac{\partial F_3}{\partial Q}\right]$$

$$\frac{1}{AE}\left\{\left[\left(\cancel{R} + \frac{P\sqrt{2}}{2}\right)\frac{1}{2} - \left(\cancel{Q} + \frac{P\sqrt{2}}{2}\right)\frac{1}{2\sqrt{3}}\right]\left(-\frac{1}{2\sqrt{3}}\right) + \left[\left(\cancel{R} + \frac{P\sqrt{2}}{2}\right) + \left(\cancel{Q} + \frac{P\sqrt{2}}{2}\right)\frac{1}{\sqrt{3}}\right]\left(\frac{1}{\sqrt{3}}\right) + \left[-\left(\cancel{R} + \frac{P\sqrt{2}}{2}\right) + \left(\cancel{Q} + \frac{P\sqrt{2}}{2}\right)\frac{1}{\sqrt{3}}\right]\left(+\frac{1}{\sqrt{3}}\right)\right]$$

$$u_c = \frac{P}{AE}\left[\left(\frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{4\sqrt{3}}\right)\left(-\frac{1}{2\sqrt{3}}\right) + \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right) + \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)\right] = \frac{P}{AE} 0.428$$

$$v_c = \left.\frac{\partial U_c}{\partial R}\right|_{\substack{Q=0\\R=0}} = \frac{1}{AE}\left[F_1 \frac{\partial F_1}{\partial R} + F_2 \frac{\partial F_2}{\partial R} + F_3 \frac{\partial F_3}{\partial R}\right] = \frac{1}{AE}\left\{\left[\left(\cancel{R} + \frac{P\sqrt{2}}{2}\right)\frac{1}{2} - \left(\cancel{Q} + \frac{P\sqrt{2}}{2}\right)\frac{1}{2\sqrt{3}}\right]\left(\frac{1}{2}\right) + \left[\left(\cancel{R} + \frac{P\sqrt{2}}{2}\right) + \left(\cancel{Q} + \frac{P\sqrt{2}}{2}\right)\frac{1}{\sqrt{3}}\right] 1 +\right.$$
$$\left.\left[-\left(\cancel{R} + \frac{P\sqrt{2}}{2}\right) + \left(\cancel{Q} + \frac{P\sqrt{2}}{2}\right)\frac{1}{\sqrt{3}}\right](-1)\right\}$$

$$= \frac{P}{AE}\left[\left(\frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{4\sqrt{3}}\right)\frac{1}{2} + \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2\sqrt{3}}\right) + \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2\sqrt{3}}\right)\right] \approx \frac{P}{AE} 1.489$$

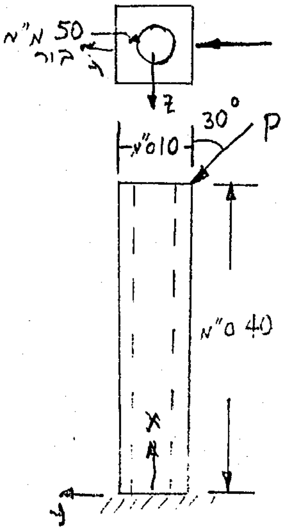
רואים שבכל לא אותן תשובות אם נקח $\frac{\partial U_c}{\partial T}$ כאשר $T = \frac{P\sqrt{2}}{2}$

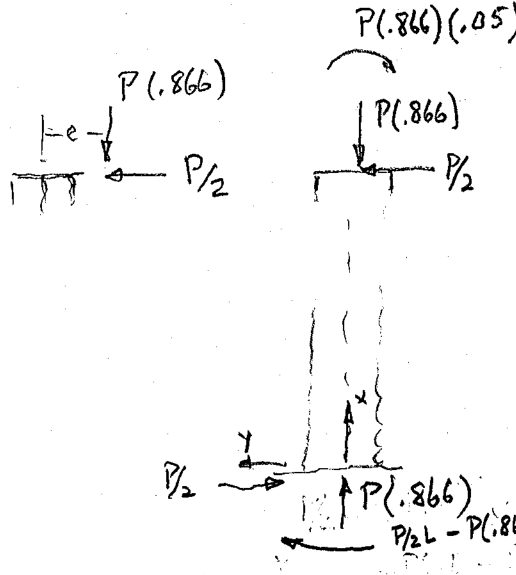
$$v_c = u_c = \frac{\partial}{\partial T} \sum \frac{F_i^2 L_i}{2 A_i E_i} = \frac{1}{AE}\left[F_1 \frac{\partial F_1}{\partial T} + F_2 \frac{\partial F_2}{\partial T} + F_3 \frac{\partial F_3}{\partial T}\right]$$

אבל זה כן, נכון אם מבדילים בין $T = \frac{P_1\sqrt{2}}{2}$ אופקי ו- $T = \frac{P_2\sqrt{2}}{2}$ אנכי

2) לעמוד המתואר בתרשים, בעל שטח חתך רבועי 10 ס"מ x 10 ס"מ ובו קדח בור בקוטר 50 מ"מ. הכוח P פועל כבתרשים והבסיס של העמוד הוא סמך רתום.

דרוש : לחשב את גודל הכוח P שבגללו המאמץ הנורמלי המקסימלי לא יעלה מעל 140 MPa. לא להתייחס למשקלו של העמוד.





$$P/2L - P(.866)(0.05) = M_{max}$$

$$I_{zz} = \frac{bh^3}{12} - \frac{\pi}{8} r^4$$

$$= \frac{(0.1)(0.1)^3}{12} - \frac{\pi}{8}(0.025)^4$$

$$= 0.0833 \times 10^{-4} - 1.53 \times 10^{-7}$$

$$\boxed{I = 0.0818 \times 10^{-4} m^4}$$

$$A = bh - \frac{\pi d^2}{4} = (0.1)(0.1) - \frac{\pi (0.05)^2}{4}$$

$$= 0.01 - .002$$

$$A = .8037 \times 10^{-2} m^2$$

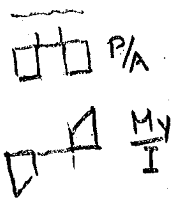
$$\sigma = P/A - \frac{My}{I} = -\frac{P(.866)}{0.8037 \times 10^{-2} m^2} - \frac{[P/2 L - P(.866)(.05)]\,[\pm .05 m]}{.0588 \times 10^{-4} m^4}$$

$$\sigma = -107.76P - P\frac{[0.2 - .866(.05)][\pm .05]}{.0818 \times 10^{-4}} = P\left[-107.76 - \frac{0.1567(\pm 0.05)}{.0818 \times 10^{-4}}\right] = P[-107.76 \mp 958.22]$$

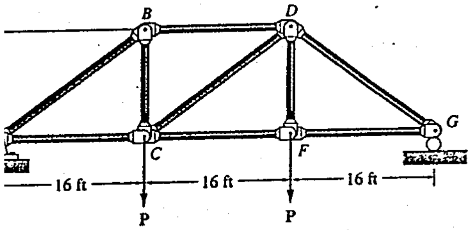
בלחיצה $\sigma = -140 MPa = P[-107.76 - 958.22] \Rightarrow$ $P = 131334 N$

במתיחה $\sigma = 140 MPa = P[-107.76 + 958.22] \Rightarrow$ $P = 164617 N$

לכן, לוקחים את ה-P הקטן ביותר ⇐ $\underline{P = 131334 N}$

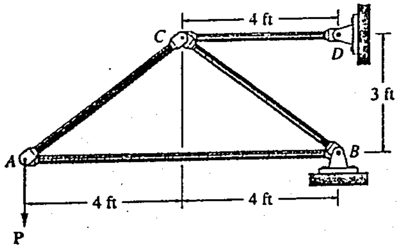


). The members of the truss are assumed to be pin-
cted. If member BD is a steel rod of radius $r = 2$ in., deter-
he maximum load P that can be supported by the truss with-
using the member to buckle. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 36$ ksi.



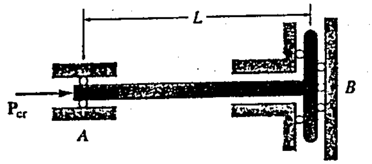
Prob. 17–10

. The truss is made from steel bars, each of which has a
ır cross section with a diameter of 1.5 in. Determine the
ıum force P that can be applied without causing any of the
ers to buckle. The members are pin-supported at their ends.
$29(10^3)$ ksi, $\sigma_Y = 36$ ksi.



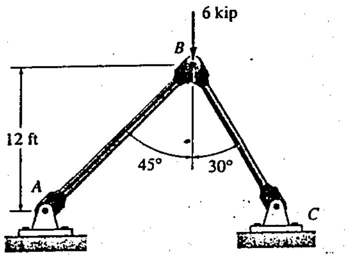
Prob. 17–11

2. The column is supported at B by a support that does not
t rotation but allows vertical deflection. Determine the criti-
ad P_{cr}. Assume elastic buckling.



Prob. 17–12

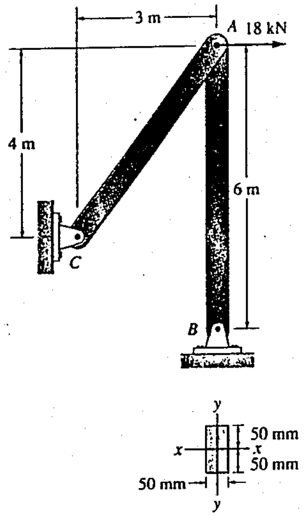
17–13. The linkage is made using two steel rods, each having a circular cross section. Determine the diameter of each rod to the nearest $\frac{1}{8}$ in. that will support the 6-kip load. Assume that the rods are pin-connected at their ends. Use a factor of safety with respect to buckling of F.S. = 1.8. $E_{st} = 29(10^3)$, $\sigma_Y = 36$ ksi.



Prob. 17–13

17–14. The steel bar AB of the frame is pin-connected at its ends. Determine the factor of safety with respect to buckling about the y–y axis due to the applied loading. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.

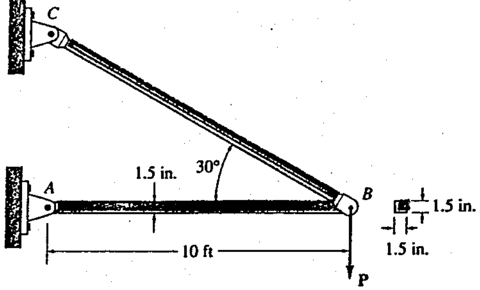
FS 2.35 ✓



Prob. 17–14

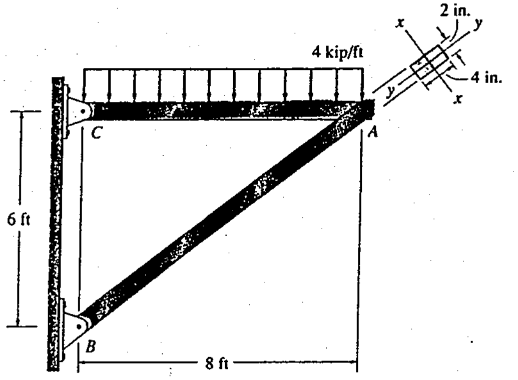
17–15. The steel bar AB has a square cross section. If it is pin-connected at its ends, determine the maximum allowable load P that can be applied to the frame. Use a factor of safety with respect to buckling of F.S. = 2. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 36$ ksi.

P=2.42 Kf



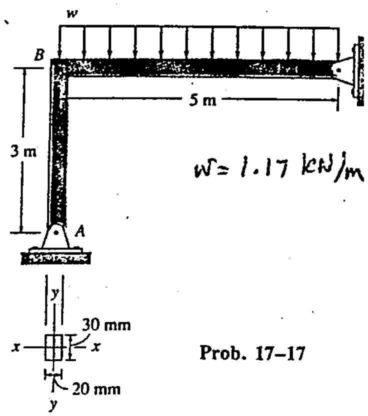
Prob. 17–15

***17–16.** The steel bar AB of the frame is pin-connected at its ends. Determine the factor of safety with respect to buckling about the y–y axis due to the applied loading. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 36$ ksi.



Prob. 17–16

17–17. The steel bar AB has a rectangular cross se
pin-connected at its ends, determine the maximum
tensity w of the distributed load that can be applied t
causing bar AB to buckle. Use a factor of safety wi
buckling of F.S. = 1.5. $E_{st} = 200$ GPa, $\sigma_Y = 360$ M



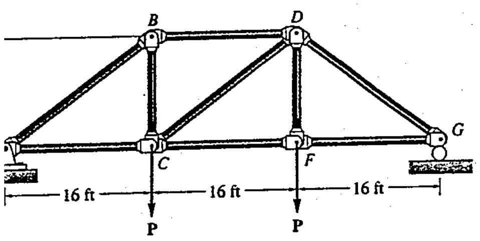
w= 1.17 kN/m

Prob. 17–17

17–18. Determine the maximum allowable intensit
distributed load that can be applied to member BC with
member AB to buckle. Assume that AB is made of
pinned at its ends for x–x axis buckling and fixed at
y–y axis buckling. Use a factor of safety with respect
of F.S. = 3. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.

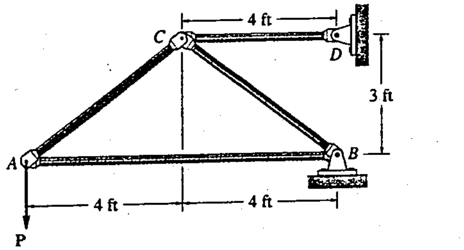
Prob. 17–18

0. The members of the truss are assumed to be pin-
cted. If member BD is a steel rod of radius $r = 2$ in., deter-
the maximum load **P** that can be supported by the truss with-
ausing the member to buckle. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 36$ ksi.



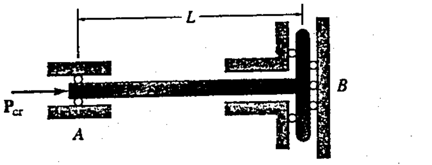
Prob. 17–10

1. The truss is made from steel bars, each of which has a
lar cross section with a diameter of 1.5 in. Determine the
mum force **P** that can be applied without causing any of the
bers to buckle. The members are pin-supported at their ends.
: $29(10^3)$ ksi, $\sigma_Y = 36$ ksi.



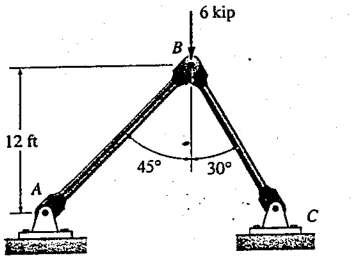
Prob. 17–11

12. The column is supported at B by a support that does not
nit rotation but allows vertical deflection. Determine the criti-
oad P_{cr}. Assume elastic buckling.



Prob. 17–12

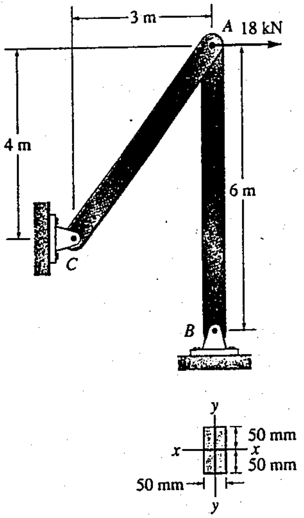
17–13. The linkage is made using two steel rods, each having a circular cross section. Determine the diameter of each rod to the nearest $\frac{1}{8}$ in. that will support the 6-kip load. Assume that the rods are pin-connected at their ends. Use a factor of safety with respect to buckling of F.S. = 1.8. $E_{st} = 29(10^3)$, $\sigma_Y = 36$ ksi.



Prob. 17–13

17–14. The steel bar AB of the frame is pin-connected at its ends. Determine the factor of safety with respect to buckling about the y–y axis due to the applied loading. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.

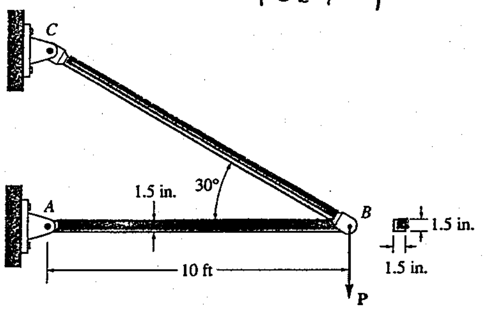
FS 2.38 ✓



Prob. 17–14

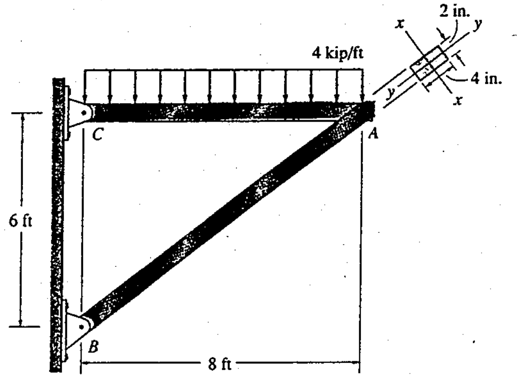
17–15. The steel bar AB has a square cross section. If it is pin-connected at its ends, determine the maximum allowable load **P** that can be applied to the frame. Use a factor of safety with respect to buckling of F.S. = 2. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 36$ ksi.

P=2.42 Kf



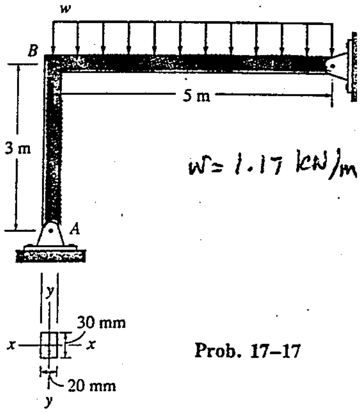
Prob. 17–15

***17–16.** The steel bar AB of the frame is pin-connected at its ends. Determine the factor of safety with respect to buckling about the y–y axis due to the applied loading. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 36$ ksi.



Prob. 17–16

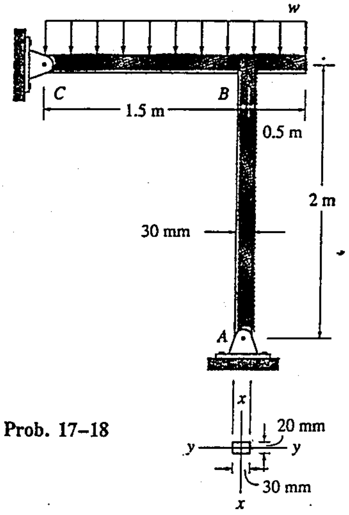
17–17. The steel bar AB has a rectangular cross se
pin-connected at its ends, determine the maximum
tensity w of the distributed load that can be applied t
causing bar AB to buckle. Use a factor of safety wi
buckling of F.S. = 1.5. $E_{st} = 200$ GPa, $\sigma_Y = 360$ M



w= 1.17 kN/m

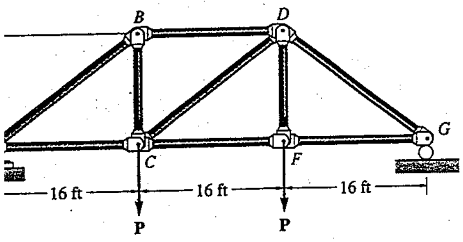
Prob. 17–17

17–18. Determine the maximum allowable intensi
distributed load that can be applied to member BC wit
member AB to buckle. Assume that AB is made of
pinned at its ends for x–x axis buckling and fixed at
y–y axis buckling. Use a factor of safety with respect
of F.S. = 3. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



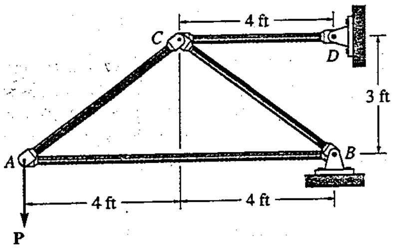
Prob. 17–18

The members of the truss are assumed to be pin-ted. If member BD is a steel rod of radius $r = 2$ in., deter-e maximum load P that can be supported by the truss with-sing the member to buckle. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 36$ ksi.



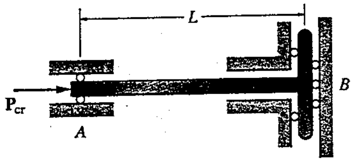
Prob. 17–10

. The truss is made from steel bars, each of which has a r cross section with a diameter of 1.5 in. Determine the um force P that can be applied without causing any of the ers to buckle. The members are pin-supported at their ends. 29(10³) ksi, $\sigma_Y = 36$ ksi.



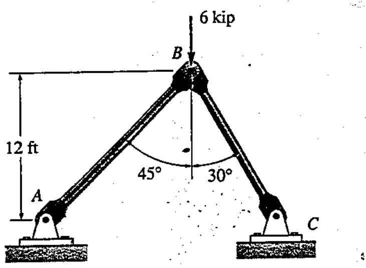
Prob. 17–11

2. The column is supported at B by a support that does not t rotation but allows vertical deflection. Determine the criti-ad P_{cr}. Assume elastic buckling.



Prob. 17–12

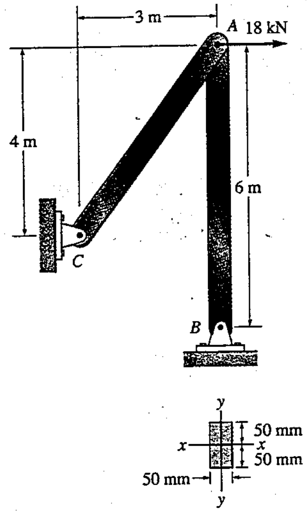
17–13. The linkage is made using two steel rods, each having a circular cross section. Determine the diameter of each rod to the nearest $\frac{1}{8}$ in. that will support the 6-kip load. Assume that the rods are pin-connected at their ends. Use a factor of safety with respect to buckling of F.S. = 1.8. $E_{st} = 29(10^3)$, $\sigma_Y = 36$ ksi.



Prob. 17–13

17–14. The steel bar AB of the frame is pin-connected at its ends. Determine the factor of safety with respect to buckling about the y–y axis due to the applied loading. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.

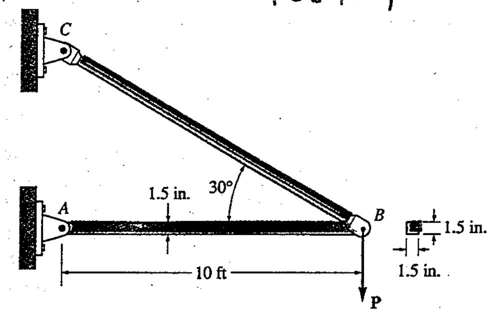
FS 2.38 ✓



Prob. 17–14

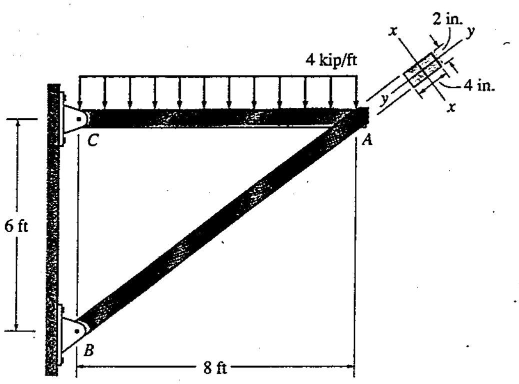
17–15. The steel bar AB has a square cross section. If it is pin-connected at its ends, determine the maximum allowable load P that can be applied to the frame. Use a factor of safety with respect to buckling of F.S. = 2. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 36$ ksi.

P=2.42 Kip



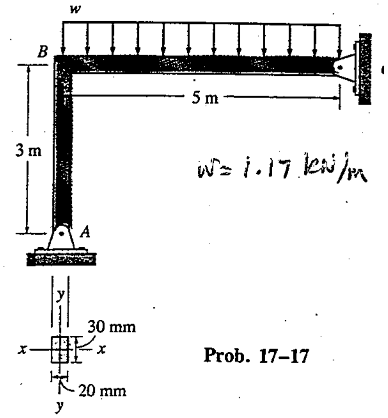
Prob. 17–15

***17–16.** The steel bar AB of the frame is pin-connected at its ends. Determine the factor of safety with respect to buckling about the y–y axis due to the applied loading. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 36$ ksi.



Prob. 17–16

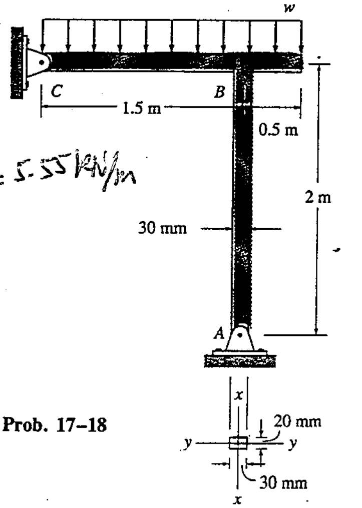
17–17. The steel bar AB has a rectangular cross secti pin-connected at its ends, determine the maximum all tensity w of the distributed load that can be applied to B causing bar AB to buckle. Use a factor of safety with buckling of F.S. = 1.5. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MP



w= 1.17 kN/m

Prob. 17–17

17–18. Determine the maximum allowable intensity distributed load that can be applied to member BC withou member AB to buckle. Assume that AB is made of ste pinned at its ends for x–x axis buckling and fixed at its y–y axis buckling. Use a factor of safety with respect to of F.S. = 3. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



w= 5.55 kN/m

Prob. 17–18

ינואר 2 2000 -

בוחן מס 1

תורת החוזק 2
פרופ עזרא לוי

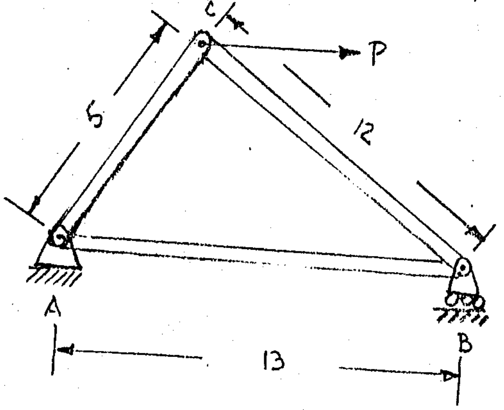
אורך הבוחן הזה יהיה 90 דקות. בבוחן יש ~~שלושה~~ שתי שאלות. עליכם לענות בכל השאלות. רק להשתמש בתפסים הרשאים (כשלוש-עשרי דפים), ומחשבון, ואם יש, דברים הנתונים לכם עם הבוחן.

לחתום על סעיף הבא בבקשה:
לאורך הבוחן אני לא רשאי(ת) לקבל עזרה מאף אחד או לתת עזרה לאף אחד. אם אני אעבור על אלה, אני אכשל את הבוחן.

חתימת הסטודנט.................................. שם הסטודנט מס. זהות

תרגיל מס. 1

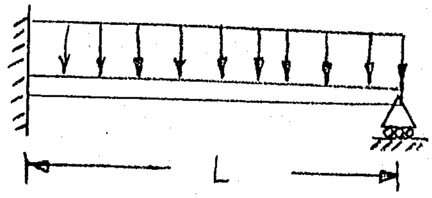
נתון לכם מסבך המשורטט בציור בצד השמאל. שטח החתך הוא A, ומודול יאנג הוא E. לא לכלול את משקלם של המוטות. המסבך עמוס בעומס האופקי P. בנקודה C. הסמכים כנמצאים על הציור.



נא למצוא:
התזוזה האופקית, u_C.
שינוי אורך המוט AC

תרגיל מס. 2

נתון לכם קורה עמוסה המשורטטת בציור בצד השמאל. העומס הוא עומס מפורס בגודל של q N/m. בצד שמאל סמך רתום, ובצד ימין יש סמך נייד.



נא למצוא:

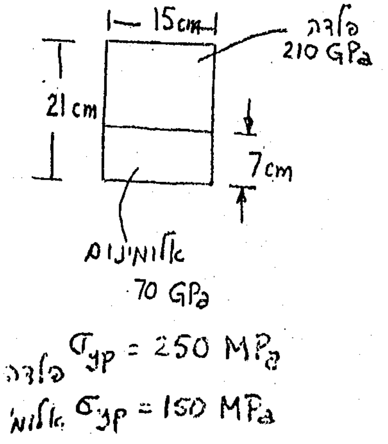
(a) התגובות בסמכים.

(b) המומנט המקסימלי ומקומו.

(c) נתון לכם ששטח החתך כבציור (B). למצוא את המאמץ המקסימלי.

(d) כאשר המאמץ המקסימלי מגיע ל-σ_{yp}, מהוא העומס המפורס שגורם למאמץ הזה.

(e) לפי שיטת הפלסטית, מהוא המומנט. M_{pl} והעומס המפורס שהקורה יכולה לסבול.



תורת החוזק 2
פרופ עזרא לוי

בוחן מס 1

ינואר 2 2000

אורך הבוחן הזה יהיה 90 דקות. בבוחן יש ~~שלושה~~ שתי שאלות. עליכם לענות בכל השאלות. רק להשתמש בתפסים הרשאים (כשלוש-עשרי דפים), ומחשבון. ואם יש. דברים הנתונים לכם עם הבוחן.

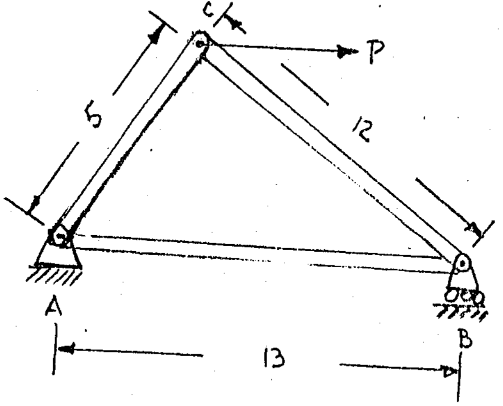
לחתום על סעיף הבא בבקשה:

לאורך הבוחן אני לא רשאי(ת) לקבל עזרה מאף אחד או לתת עזרה לאף אחד. אם אני אעבור על אלה. אני אכשל את הבוחן.

חתימת הסטודנט________________ ________________
שם הסטודנט מס. זהות

תרגיל מס. 1

נתון לכם מסבך המשורטט בציור בצד השמאל. שטח החתך הוא A, ומודול יאנג הוא E. לא לכלול את משקלם של הסמכים כנמצאים על הציור. בנקודה C, המסבך עמוס בעומס האופקי P. לא לכלול את משקלם של המוטות.



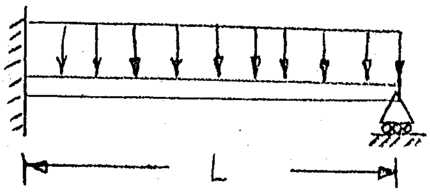
נא למצוא:

התזוזה האופקית, u_C.

שינוי אורך המוט AC

תרגיל מס. 2

נתון לכם קורה עמוסה המשורטטת בציור בצד השמאל. העומס הוא עומס מפורס בגודל של q N/m.
בצד שמאל סמך רתום, ובצד ימין יש סמך נייד.



נא למצוא:

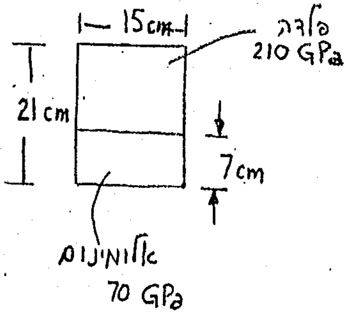
(a) התגובות בסמכים.

(b) המומנט המקסימלי ומקומו.

(c) נתון לכם ששטח החתך כבציור (B). למצוא את המאמץ המקסימלי.

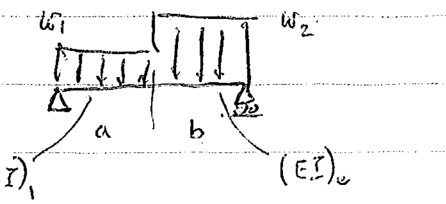
(d) כאשר המאמץ המקסימלי מגיע ל-σ_{yp}, מהוא העומס המפורס שגורם למאמץ הזה.

(e) לפי שיטת הפלסטית, מהוא המומנט, M_{pl} והעומס המפורס שהקורה יכולה לסבול.



פלדה $\sigma_{yp} = 250\ MPa$

אלומ' $\sigma_{yp} = 150\ MPa$



$$\frac{dM}{dx} = -V \quad \frac{dV}{dx} = -p \Bigg\} \quad \frac{d^2M}{dx^2} = p$$

תנאים

$$(EI)_* \frac{d^4v}{dx^4} = -w_1 - (w_2-w_1)\langle x-a\rangle^0 \qquad (EI)_* = (EI)_1 + \left[(EI)_2-(EI)_1\right]\langle x-a\rangle^0$$

$v=0$ @ $x=0$ (1

$v=0$ @ $x=L$ (2

$v''=0$ @ $x=0$ (3

$v''=0$ @ $x=L$ (4

$$(EI)_* v''' = -w_1x - (w_2-w_1)\langle x-a\rangle^1 + C_1$$

$$(EI)_* v'' = -w_1 x^2/2 - \frac{(w_2-w_1)}{2}\langle x-a\rangle^2 + C_1x + C_2$$

$C_2 = 0 \Leftarrow$ (3) תנאי

$$(EI)_* v''(x=L) = 0 = -w_1 L^2/2 - \frac{(w_2-w_1)}{2} b^2 + C_1 \cdot L = 0 \Leftarrow \text{(4) תנאי}$$

$$\underline{C_1 = w_1 L/2 + \frac{w_2-w_1}{2L} b^2}$$

$$(EI)_* v' = -w_1x^3/6 - \frac{(w_2-w_1)}{6}\langle x-a\rangle^3 + C_1 x^2/2 + C_3$$

$$(EI)_* v = -w_1x^4/24 - \frac{(w_2-w_1)}{24}\langle x-a\rangle^4 + C_1x^3/6 + C_3x + C_4$$

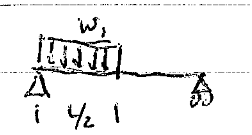
$C_4 = 0 \Leftarrow$ (1) תנאי

$$(EI)_* v(x=L) = 0 = -w_1 L^4/24 - \frac{(w_2-w_1)}{24} b^4 + C_1 L^3/6 + C_3 L \Leftarrow \text{(2) תנאי}$$

$$C_3 = w_1\frac{L^3}{24} + \frac{(w_2-w_1)}{24}\frac{b^4}{L} - C_1\frac{L^2}{6}$$

$0<x<\infty$

$$v(x) = \frac{1}{EI_1}\left[-w_1\frac{x^4}{24} + w_1\frac{x^3L}{12} + \frac{w_2-w_1}{12L}b^2x^3 + \frac{w_1L^3x}{24} + \frac{(w_2-w_1)}{24L}b^4x - \frac{w_1L^3x}{12} - \frac{(w_2-w_1)}{12}b^2Lx\right]$$



$-w_1 = w_2 - w_1$ ו- , $a=b=L/2$ אם

$$v(x) = \frac{1}{EI_1}\left[-w_1\frac{x^4}{24} + w_1\frac{x^3L}{12} - \frac{w_1x^3L}{48} + w_1\frac{L^3x}{24} - \frac{w_1L^3x}{24\cdot16} - w_1\frac{L^3x}{12} + \frac{w_1L^3x}{48}\right]$$

$$v(x) = \frac{1}{EI_1}\left[-w_1\frac{x^4}{24} + \frac{w_1x^3L}{16} - \frac{3}{128}w_1L^3x\right]; \quad \underline{C_1 = \frac{3w_1L}{8}}, \quad C_3 = \frac{w_1L^3}{24} - \frac{w_1L^3}{24\cdot16} - \frac{3w_1L^3}{48} = \underline{-\frac{9}{384}w_1L^3}$$

$$v(x=\tfrac{L}{2}) = \frac{1}{EI_1}\left[-\frac{w_1L^4}{24\cdot16} + \frac{w_1L^4}{8\cdot16} - \frac{3w_1L^4}{16\cdot16}\right] = -\frac{5}{768}\frac{w_1L^4}{EI}$$

1. Write the second-order differential equation for the bending of the column shown in Fig. P1-2 and use it to determine the critical load of the column. At its lower end the column is completely fixed. At the upper end the column is prevented from rotating, but free to translate laterally. (ans: $P_{cr}=\pi^2 EI/L^2$)

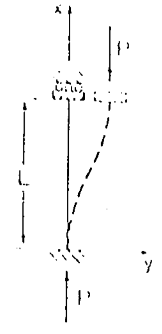
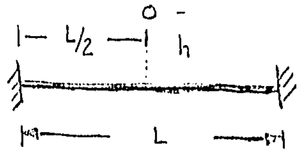


Fig. P1-2

2. Find an expression for the maximum stress when a ball weighing W Newtons is dropped onto a fixed-fixed beam.



3. . A linearly elastic beam-column having a flexural rigidity EI, is subjected to a thrust P and a moment M_0 as shown in Fig. A below.

(a) Determine the lateral displacement v(x).

(b) From part (a), write the solution for the system subjected to a force P acting as shown in Fig. B.

(c) Determine Δ_c, the horizontal displacement of point C, assuming small rotations. (Assume also that the horizontal displacement of point B is negligible).

(d) Determine the bending moment M(x).

(g) Explain why, although the beam is made of a linearly elastic material, the results of part (c) are non-linear.

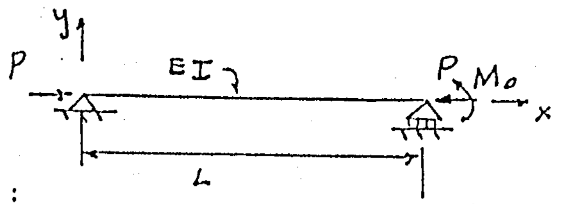


Fig. A

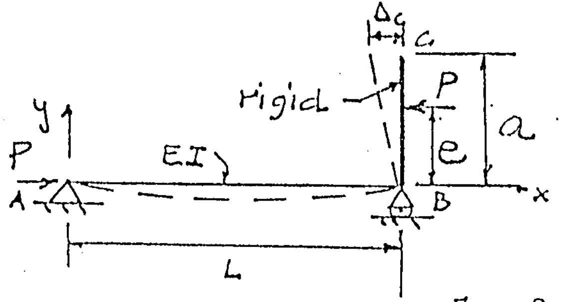


Fig. B

Answers :

(a) $y(x)=-\frac{M_0}{P}\left[\frac{\sin Kx}{\sin KL}-\frac{x}{L}\right], K^2=\frac{P}{EI}$

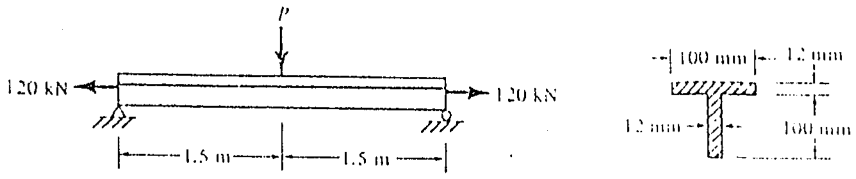
(c) $\Delta_c=\frac{ac}{L}\,[1-L\sqrt{P/EI}\cot(L\sqrt{P/EI})\,]=\frac{ac}{L}\,(1-KL\cot KL\,)$

(d) $M(x)=M_0 \sin kl/\sin kL$

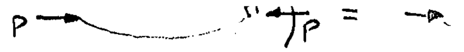
*12.10 A T section carries an axial tensile force of 120 kN, applied through the centroid of the cross section. The allowable stress in tension or compression is 130 MPa. Let $E = 200$ GPa. What transverse force P can be applied at midspan if the beam is

(a) Stem down (as shown)?

(b) Stem up?



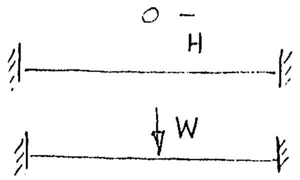
PROBLEM 12.10



לכן $\theta|_{x=L} = \frac{Pe}{P}\left(\lambda\cot\lambda L - \frac{1}{L}\right)$ ו- $a\theta|_{x=L} = \Delta_c$ כי העמוד קשיח וקשור לקורה בזווית אנכית של 90°. לכן,

$$\Delta_c = \frac{ae}{L}\left(\lambda L\cot\lambda L - 1\right)$$

2.



לפתור את

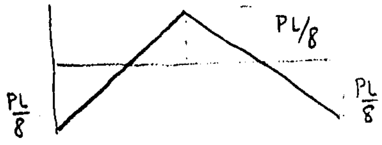
פותרים את

$\Delta_{st} = v = \frac{WL^3}{192EI}$ והתזוזה היא

לפי הנוסחה $P_{dyn} = W\left(1+\sqrt{1+2H/\Delta_{st}}\right)$, בגלל שמצאנו את Δ_{st}, מצאנו את P_{dyn}.

למצוא את המומנט המקס. לפי הנוסחה שנתונה ב- YOUNG & COOK, $v=\frac{P}{48EI}(3Lx^2-4x^3)$

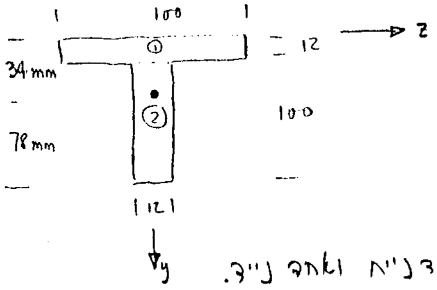
ו- $M=EI\frac{d^2v}{dx^2}$. לכן, $M=\frac{P}{48}(6L-24x)$. ב- $x=0$ $M=\frac{PL}{8}$; ב- $x=L/2$ $M=-\frac{PL}{8}$, וגרף המומנט



לכן יש שלושה מקומות שבהם המומנט הוא מקסימלי והמאמץ המקסימלי הוא $\sigma = \frac{Mc}{I} = \frac{PL}{8}\frac{6}{bh^2}$

$$\frac{3LW\left(1+\sqrt{1+\frac{2H}{WL^3/192EI}}\right)}{4bh^3} = \frac{3}{4}\frac{PL}{bh^3} = \sigma_{max}$$

12.10



שטח החתך הוא $2(100\times12)$ mm $= 2400$ mm^2

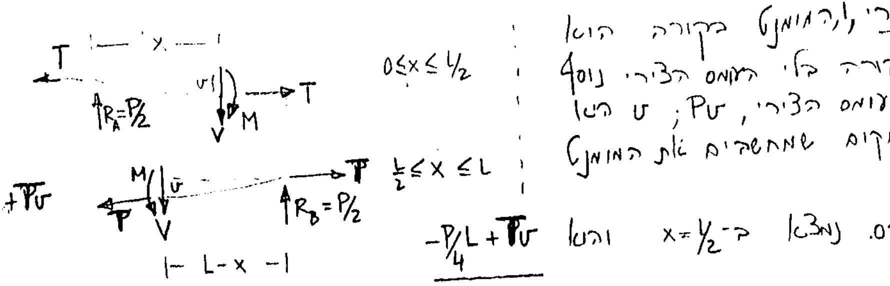
מרכז הכובד הוא

	A	y	Ay
①	100x12	6	7200
②	100x12	62	74400
	2400		81600

$\bar{y} = \frac{\Sigma Ay}{A} = 34$ mm

העומס הקריסה הוא $T_E = \pi^2\frac{EI}{L^2}$ מפני הסמכים, אחד נייח ואחד נייד.

בגלל העומס הצירי, T, המומנט בקורה הוא המומנט שנמצא בקורה בלי העומס הצירי נוסף המומנט שבא מהעומס הצירי, Pv; v הוא התזוזה באותו מקום שמחשבים את המומנט



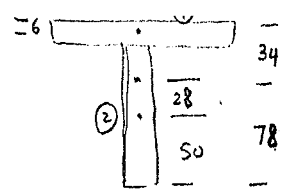
$M=-\frac{P}{2}x+Tv$ $0\le x\le L/2$

$M=-\frac{P}{2}(L-x)+Pv$ $\frac{L}{2}\le x\le L$

לכן, המומנט המקס. נמצא ב- $x=L/2$ והוא $-\frac{P}{4}L+Pv$

כדי למצוא את $v|_{x=L/2}$, יודעים משיעור בכיתה ש- $v=A\cosh\lambda x+B\sinh\lambda x+Cx+D$ ו- $\lambda^2=\frac{T}{EI}$ בכל חלק מהקורה.

זאת אומרת שצריך להשתמש ב $v_L=A\cosh\lambda x+B\sinh\lambda x+Cx+D$ בין A-C, ו- $v_R=\bar{A}\cosh\lambda(x-L)+\bar{B}\sinh\lambda(x-L)+\bar{C}(x-L)+\bar{D}$ בין C-B. תנאי השפה $v_L(x=0)\overset{(1)}{=}0$, $v_L''(x=0)\overset{(2)}{=}0$, $v_R(x=L)\overset{(3)}{=}0$, $v_R''(x=L)\overset{(4)}{=}0$. ב- C $v_L(x=L/2)\overset{(5)}{=}v_R(x=L/2)$; $v_L'(x=L/2)\overset{(6)}{=}v_R'(x=L/2)$; $v_L''(x=L/2)\overset{(7)}{=}v_R''(x=L/2)$; $EIv_L'''(x=L/2)-Tv_L''(x=L/2)\overset{(8)}{=}EIv_R'''-Tv_R''+P$. אלה תנאי רציפות. אלה שמונה משוואות לקבל $A,B,C,D,\bar{A},\bar{B},\bar{C},\bar{D}$. מ- (1),(2),(3),(4) $\Leftarrow A=D=\bar{A}=\bar{D}=0$. מ- (7) $\Leftarrow B=-\bar{B}$. מ- (5),(7) $\Leftarrow C=-\bar{C}$; מ- (6) $\Leftarrow C=-B\lambda\cosh\lambda L$.



I	A	d^2	I_{tt}
$\frac{1}{12}(100)(12)^3$	1200	$(28)^2$	955200
$\frac{1}{12}(12)(100)^3$	1200	$(28)^2$	1940800
			$2,896\times10^6 mm^4$

$v|_{x=L/2}$ … λ … EI … $2EI\;\lambda^3\cosh\lambda L/2\;\;2EI\lambda^2$

$$v\Big|_{x=L/2} = P\left\{\frac{\sinh\lambda L/2}{2T\lambda\cosh\lambda L/2} - \frac{L}{4T}\right\}$$

צריך לדעת λ לחשב את v. גם צריך לדעת I.

$$\lambda = \sqrt{\frac{120{,}000}{(200\times10^9)(2.896\times10^{-6})}} = .4552 \quad , \quad T_E = \frac{\pi^2 EI}{L^2} = 635164N > T.$$

$$v\Big|_{x=L/2} = P\left\{\frac{0.7371}{2(120{,}000)(1.2423)(.4552)} - \frac{3}{4(120000)}\right\}$$

$$= P(8.19\;\;10^{-7}m)$$

לכן המומנט יהיה $M = -\frac{PL}{4} + T\cdot P\times8.19\times10^{-7}$

$M = P[-0.75 + .09828] = -.65172P$

התגובות במקום שהמומנט מקס.

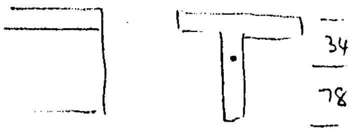
$$\sigma = T/A + \frac{Mc}{I}$$

אם צורת החתך כמו משורטט משמאלה:

$\sigma_{\text{עליון}} = \frac{120{,}000}{2400\times10^{-6}} - \frac{0.65172(.078)P}{2.896\times10^{-6}} = -130\times10^6$ $\quad$ $\underline{\underline{P = 10{,}250\ N}}$

$\sigma_{\text{תחתון}} = \frac{120{,}000}{2400\times10^{-6}} + \frac{0.65172(.034)P}{2.896\times10^{-6}} = 130\times10^6$ $\quad$ $P = 10{,}451N$

אם צורת החתך כמו משורטט שמאלה:

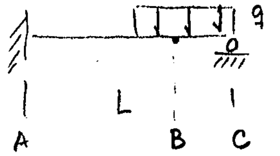


$\sigma_{\text{עליון}} = \frac{120{,}000}{2400\times10^{-6}} - \frac{0.65172(.034)P}{2.896\times10^{-6}} = -130\times10^6$ $\quad$ $P = 23{,}515N$

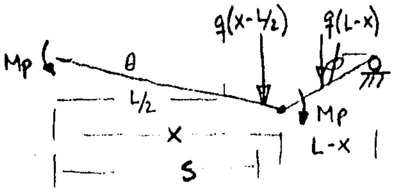
$\sigma_{\text{תחתון}} = \frac{120{,}000}{2400\times10^{-6}} + \frac{0.65172(.078)P}{2.896\times10^{-6}} = 130\times10^6$ $\quad$ $\underline{\underline{P = 4556N}}$

לכן, העומס הגבוה ביותר בא כאשר לשטח החתך יש צורה ככה ⊥

11.26



כאשר שהמומנט מגיע ל-M_p, פרק פלסי קורה בסמך הרתום וגם בנקודה B
צריך למצוא את מקום נקודה B ומהוא העומס המפורס שגורם ל-M_p



$\theta x = \phi(L-x)$

$\frac{\theta x}{L-x} = \phi$

$s = \frac{1}{2}(x+L/2)$

לפי עקרון עבודה ווירטואלית

$$M_p\theta + M_p(\theta+\phi) = q(x-L/2)\,\theta s + q(L-x)\left(\frac{L-x}{2}\right)\phi$$

מחליפים ϕ ו-s ביחסים ופותרים בשביל q

$$q = \frac{8M_p(2L-x)}{(5Lx-4x^2-L^2)L}$$

כדי למצוא את ה-q שגורם ל-M_p, גוזרים q לגבי x
ומקבלים ש- $0 = 4x^2-16Lx+9L^2$. מזה יש תשובה אחת
והיא $x = \frac{16-\sqrt{112}}{8}L$. שמים x בנוסחה של q כדי לקבל $q = \frac{19.18M_p}{L^2}$

… BC … $\downarrow q(L-x)$ … ומה ש-R שווה ל $\frac{6.194M_p}{L}$

$$v = A\cos\lambda x + B\sin\lambda x + Cx + D$$

לפי תנאי השפות של הקורה.

כדי לקצר את הדרך לפתור את הבעיה נזכיר ש-

$$EI\frac{d^2v}{dx^2} + Pv = M_t$$

כאשר P=0

$$EI\frac{d^2v_t}{dx^2} = M_t$$

פה M_t הוא המומנט שבא מעומסים אנכיים בלבד, ו- v_t היא התזוזה בכוון y שבאה מהעומסים האנכיים. מניחים שצורת עיקום התזוזה יהיה אותו דבר <u>עם או בלי</u> הכוח האופקי P.

לכן, לפי תנאי השפה, כמו קורה על סמך נייד וסמך נייד, מניחים ש- $v = \bar{v}\sin\frac{\pi x}{L}$

ו- $v_t = \bar{v}_t\sin\frac{\pi x}{L}$. $\bar{v}$, ו- $\bar{v}_t$ הם ה- AMPLITUDE של v ו- v_t. לפי הנוסחאות למעלה

$$-EI\frac{\pi^2}{L^2}\bar{v} + P\bar{v} = -EI\frac{\pi^2}{l^2}\bar{v}_t$$

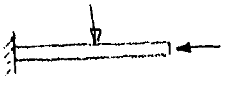
מפני ש- $P_E = \frac{\pi^2 EI}{L^2}$, $\bar{v}(P-P_E) = -P_E\bar{v}_t$ או

$$\underline{\underline{\bar{v} = \frac{\bar{v}_t}{1 - P/P_E}}}$$

המומנט גם כן עוקב אחרי נוסחא דומה לזה

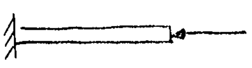
$$\underline{\underline{M = \frac{M_t}{1 - P/P_E}}}$$

אם P חיובי, $\bar{v}_t < \bar{v}$ ו- $M_t < M$. אם P שלילי. $\bar{v}_t > \bar{v}$ ו- $M_t > M$



פותר

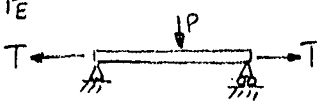
וקבל את v מתחת העומס או בכול מקום אחר קורא לזה $\bar{v}_t$.



קבל P_E לקורה עמוסה כמו בצורה

v יהיה $$v = \frac{\bar{v}_t}{1 - P/P_E}$$

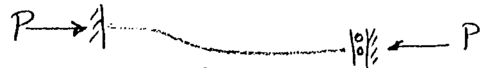
בתרגיל בית 12.10



$T_E = \frac{\pi^2 EI}{L^2}$, $\bar{v}_t\Big|_{x=L/2} = \frac{PL^3}{48EI}$, ו- $v = \frac{\bar{v}_t}{1 + T/T_E}$

כי P היה שלילי.

1.



מהנוסחה שמצאנו בשעור שעבר והפיתרון הוא

$$EI\frac{d^4}{dx^4}v+P\frac{d^2v}{dx^2}=0$$

$$v=A\cos\lambda x+B\sin\lambda x+Cx+D$$

$$\lambda^2=P/EI$$

תנאי הקצוות

① תזוזה $v(x=0)=0$

② זווית $\frac{d v}{dx}(x=0)=0$

③ זווית $\frac{dv}{dx}(x=L)=0$

④ גזירה: $EI\frac{d^3v}{dx^3}(x=L)+P\frac{dv}{dx}(x=L)=0$

מפני ש $\frac{dv}{dx}=0$ ב-$x=L$, לכן ④ נותנת $\underline{\frac{d^3v}{dx^3}=0}$ ב-$x=L$

מקבלים

$A+D=0$ ①-מ

$\lambda B+C=0$ ②

$-A(\lambda\sin\lambda L)+B(\lambda\cos\lambda L)+C=0$ ③

$A\lambda^3\sin\lambda L-B\lambda^3\cos\lambda L=0$ ④

לכן

$$\begin{bmatrix}1&0&0&1\\0&\lambda&1&0\\-\lambda\sin\lambda L&\lambda\cos\lambda L&1&0\\\lambda^3\sin\lambda L&-\lambda^3\cos\lambda L&0&0\end{bmatrix}\begin{pmatrix}A\\B\\C\\D\end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\end{pmatrix}.$$

כדי לקבל תוצאות לא טריוויאליות הדיטרמיננט צריך להיות אפס. לפי העמוד הרביעי

$$-1\begin{pmatrix}0&\lambda&1\\-\lambda\sin\lambda L&\lambda\cos\lambda L&1\\\lambda^3\sin\lambda L&-\lambda^3\cos\lambda L&0\end{pmatrix}=0$$

לכן

$$\lambda^4\sin\lambda L=0$$

$\lambda\neq0$ (זה הפיתרון $v=0$) ; לכן $\sin\lambda L=0$ $\Rightarrow$ $\lambda L=n\pi$ או $P_E=\left(\frac{\pi}{L}\right)^2EI$

3.

$$v=A\cos\lambda x+B\sin\lambda x+Cx+D\qquad\lambda^2=P/EI$$

לפי הסכם הסימנים

① $v(x=0)=0$

② $EI\frac{d^2v}{dx^2}(x=0)=M=0$

③ $v(x=L)=0$

④ $EI\frac{d^2v}{dx^2}(x=L)=-M_0$

$A+D=0$ ①-מ

$-A\lambda^2=0$ ②

$A\cos\lambda L+B\sin\lambda L+CL+D=0$ ③

$EI(-A\lambda^2\cos\lambda L-B\lambda^2\sin\lambda L)=-M_0$ ④

אם $\lambda\neq0$ $\overset{(2)}{A=0}$ $\Rightarrow$ $\overset{(1)}{D=0}$ $\Rightarrow$ $B\sin\lambda L+CL\overset{(3)}{=}0$ $\Rightarrow$ $B(EI\lambda^2\sin\lambda L)\overset{(4)}{=}M_0$ $\Rightarrow$ $B=\frac{M_0}{EI\lambda^2\sin\lambda L}$

$C=-\frac{M_0}{EI\lambda^2L}$

לכן,

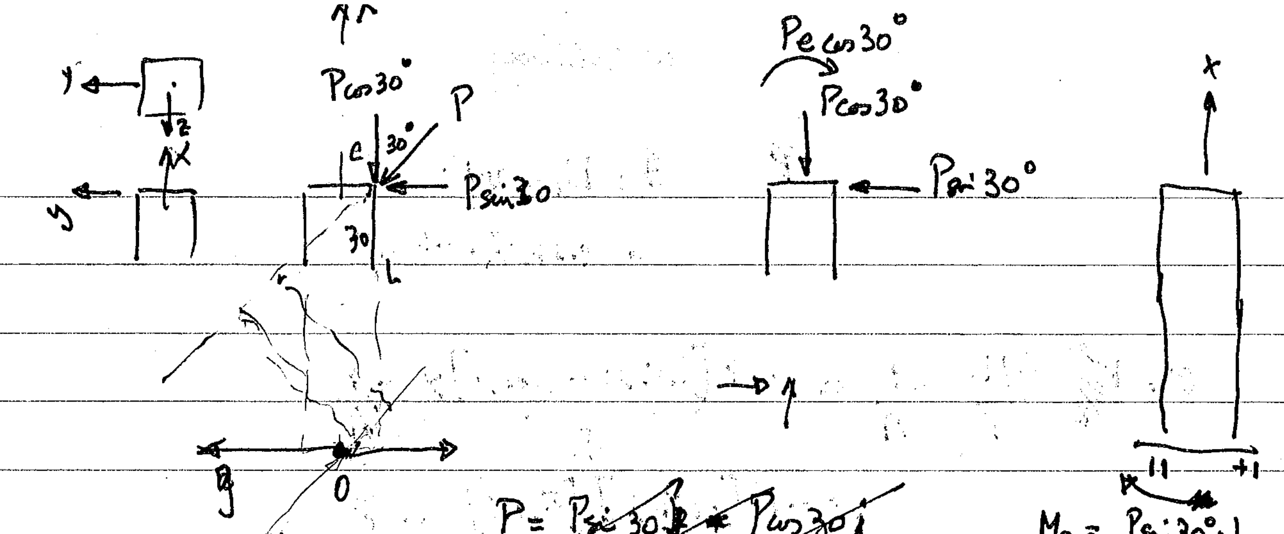
$$v=\frac{M_0}{EI\lambda^2}\left(\frac{\sin\lambda x}{\sin\lambda L}-\frac{x}{L}\right)$$

$$\boxed{v=\frac{M_0}{P}\left(\frac{\sin\lambda x}{\sin\lambda L}-\frac{x}{L}\right)}$$

לקבל $M(x)$, גוזר פעמיים לפי x: $EI\frac{d^2v}{dx^2}=M$ $=-EI\frac{M_0\lambda^2}{P}\frac{\sin\lambda x}{\sin\lambda L}$ $=$ $\underline{M(x)=-M_0\frac{\sin\lambda x}{\sin\lambda L}}$

הסימן נכון לפי הסכם הסימנים.

כדי לקבל Δ_c, צריך למצוא $\theta|=\frac{dv}{}=M_0(\lambda\cos\lambda L-\frac{1}{}$



~~$P = P\sin 30 \, i + P\cos 30 \, j$~~

~~$r = -e k + L j$~~

$M_z = P\sin 30° \cdot L$

$Pe\cos 30$

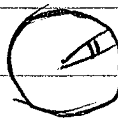
$\underline{P} = P\sin 30 j - P\cos 30 i$

$\underline{r} = -e\underline{j} + L\underline{i}$

~~$\underline{r} \times \underline{P} = \begin{bmatrix} i & j & k \\ 0 & L & -e \\ 0 & -P\cos & P\sin 30 \end{bmatrix} = i\left[PL\sin 30 - Pe\cos 30\right]$~~

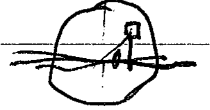
$$\begin{bmatrix} i & j & k \\ L & -e & 0 \\ -P\cos & P\sin & 0 \end{bmatrix} = \underline{k}\left[LP\sin - Pe\cos\right] = \Big[$$

$$I_{zz} = \frac{bh^3}{12} - \frac{1}{8}\pi d^4$$



$$\int_0^{2\pi}\int_0^{r} \rho^2 \cdot \rho \, d\rho \, d\theta = \frac{\rho^4}{4}\theta \quad \frac{\pi}{2}\rho^4 = \frac{\pi}{2} \frac{d^4}{16} = \frac{\pi}{32} d^4 = J = I_{xx} + I_{yy}$$

$$= 2I_{zz}$$



$$y^2 dA = \rho^2 \sin^2\theta \cdot \rho \, d\rho \, d\theta$$

$$\frac{\rho^4}{4}\int \sin^2\theta \, d\theta = \int \frac{1-\cos 2\theta}{2} d\theta$$

$$\frac{\pi d^4}{64} + \frac{\pi d^2}{4} \cdot \frac{d^2}{4}$$

$$\frac{\theta}{2} - \frac{\sin 2\theta}{4}\Big|_0^{2\pi}$$

$$I = \frac{5\pi d^4}{64}$$

$$\frac{\pi \rho^4}{4} = \frac{\pi d^4}{64}$$

$$\Longrightarrow \frac{.1(.1)^3}{12} - \frac{\pi(.050)^4}{8} = I$$

$$.0833 \times 10^{-4} - .0245 \times 10^{-4} = .0588 \times 10^{-4} \checkmark$$

$$3(1+.0472)$$

$$\frac{\pi}{128} \times 10^{-4} = \frac{3.14159}{120(1+\frac{2}{30})} \times 10^{-4}$$

$$\sigma = -\frac{My}{I} = \frac{(PL\sin 30° - Pe\cos 30°)_{M_z} y}{I} \quad P\left[0.4\left(\frac{1}{2}\right) - 0.05(.866)\right]$$

$$.2 + (.0433) = .1567$$

$$\frac{1}{40}(1+.0472-.0667) \times 10^{-4}$$

$$(1-.0195)$$

$$10^{-4} \times .025(1-.02) = \frac{.025-.0005}{10000} = .0245 \times 10^{-4} \checkmark$$

$$\sigma = 140 \times 10^6 \frac{N}{m} = \frac{-P\left[.1566\right](.05)}{I}$$

$$P = \frac{140 \times 10^6 (.0588 \times 10^{-4})}{.1566(.05)} \doteq$$

105.067 kN

M.. P

$$A = bh - \frac{\pi d^2}{4}$$

$$= 9.375 \times 10^{-3}\ m^2$$

$$\sigma = -\frac{My}{I} - \frac{P}{A} = -\frac{P[L\sin 30 - e\cos 30]y}{I} - \frac{P}{A}$$

$$= -\frac{P[.1567][\pm .05]}{.0588 \times 10^{-4}} - \frac{P}{9.375 \times 10^{-3}}$$

$$\pm 1332.48P - 106.67P = \begin{matrix} 1225.82P \\ -1439.07P \end{matrix}$$

9ns $P = 97.285$ kN (N) 161kN 869.9P

^^sn $P = 114.209$ kN (S) 129.28kN 1083.24P

minate beam. No matter how complex the $M/(EI)$ diagrams may become, the above procedures are applicable. In practice, any $M/(EI)$ diagram whatsoever may be approximated by a number of rectangles and triangles. It is also possible to introduce concentrated angle changes at hinges to account for discontinuities in the directions of the tangents at such points. The magnitudes of the concentrations can be found from kinematic requirements.*

For complicated loading conditions, deflections of elastic beams determined by the moment-area method are often best found by superposition. In this manner the areas of the separate $M/(EI)$ diagrams may become simple geometrical shapes. In the next chapter superposition will be used in solving statically indeterminate problems.

The method described here can be used very effectively in determining the inelastic deflection of beams, providing the $M/(EI)$ diagrams are replaced by the appropriate curvature diagrams.

PROBLEMS FOR SOLUTION

11-1. A long, flat, rectangular bar of aluminum alloy is ½ in. thick by 3 in. wide. (a) Determine the smallest diameter of the cylinder around which this bar could be wrapped so that the elastic limit of the material would not be exceeded. Let $\sigma_{yp} = 24{,}000$ psi, and $E = 10 \times 10^6$ psi. (b) What bending moment would develop in the bar for the above condition?

11-2. Assume that a straight, rectangular bar after severe cold working has a residual stress distribution such as was found in Example 6-7, see Fig. 6-16. (a) If one-sixth of the thickness of this bar is machined off on the top and on the bottom, reducing the bar to two-thirds of its original thickness, what will the curvature ρ of the machined bar be? Assign the necessary parameters to solve this problem in general terms. (b) For the above conditions, if the bar is 1 in.² and 40 in. long, what will the deflection of the bar at the center from the chord through the end be? Let $\sigma_{yp} = 54$ ksi, and $E = 27 \times 10^6$ psi. Note that for small deflections the maximum deflection from a chord L long of a curve bent into a circle of radius R is approximately† $L^2/(8R)$. (*Hint:* The machining operation removes the internal microresidual stresses.)

11-3. Consider a pipe of a linearly viscoelastic material which spans horizontally across a distance L. This pipe is empty for 16 hr a day, and is filled 8 hr. The ratio of the weight of the filled pipe to its empty weight is 5. (a) Assuming that the material is Maxwellian, sketch a diagram showing the center deflection as a function of time for two typical days. (b) For the same conditions of input, sketch another diagram for a material having the properties of a Standard Solid (see Prob. 4-17).

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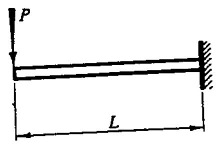
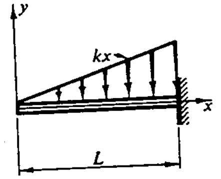


PROB. 11-6



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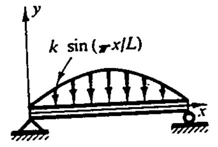

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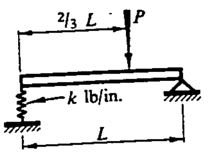

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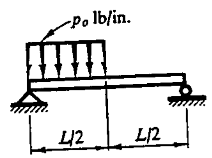

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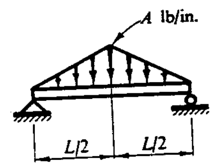
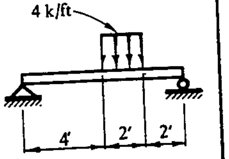
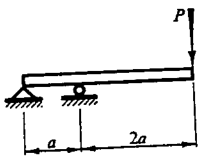


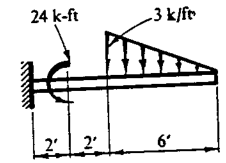
PROB. 11-11



PROB. 11-12



PROB. 11-13



PROB. 11-14

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[*] For a systematic treatment of more complex problems see for example A. C. Scordelis and C. M. Smith, "An Analytical Procedure for Calculating Truss Displacements," *Proceedings of the American Society of Civil Engineers*, paper no. 732, **81** (July 1955).

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minate beam. No matter how complex the $M/(EI)$ diagrams may become, the above procedures are applicable. In practice, any $M/(EI)$ diagram whatsoever may be approximated by a number of rectangles and triangles. It is also possible to introduce concentrated angle changes at hinges to account for discontinuities in the directions of the tangents at such points. The magnitudes of the concentrations can be found from kinematic requirements.*

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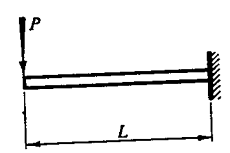
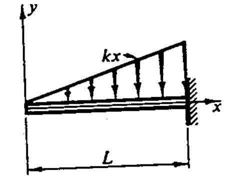


PROB. 11-6



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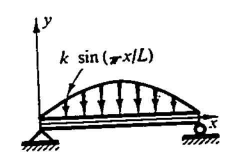

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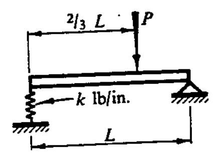

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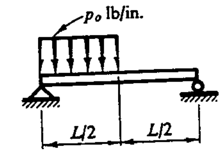

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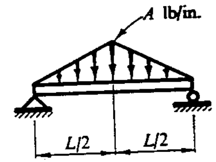
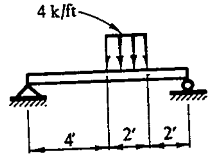
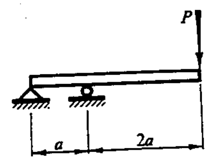


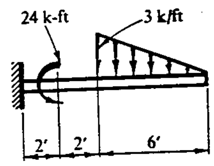
PROB. 11-11



PROB. 11-12



PROB. 11-13



PROB. 11-14

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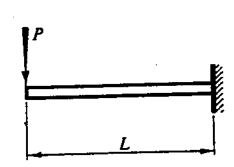
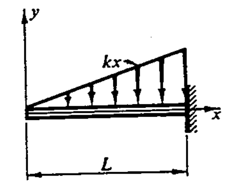


PROB. 11-6



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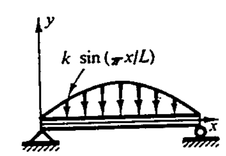
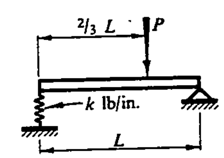


PROB. 11-8



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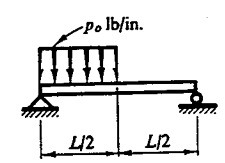

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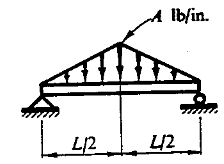

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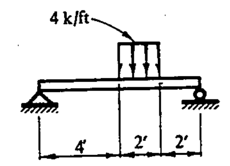
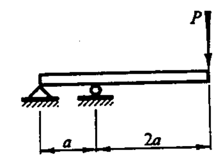
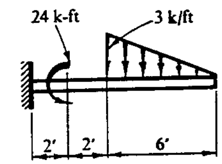


PROB. 11-12



PROB. 11-13



PROB. 11-14

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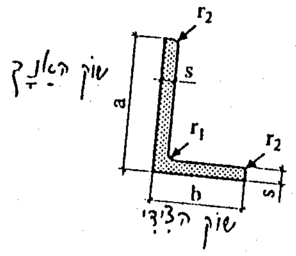
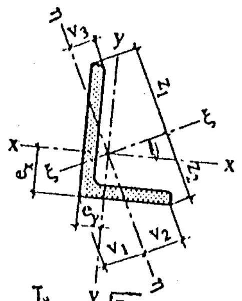
פרופילי פלדה LPN – זוויתנים שוני שוקיים

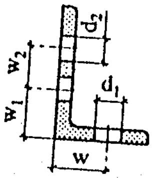
תקן: DIN 1028

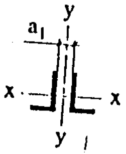




A - שטח החתך
G - משקל של 1 מטר אורך
U - שטח המעטפת
I - מומנט ההתמד (אינרציה)
W - מודול החתך (מומנט התנגדות)
i - רדיוס ההתמד (אינרציה) = $\sqrt{I/A}$
a1 - המרחק בין זוויתנים הנותן $I_x = I_y$

סימון	(17) I_x cm⁴	$\frac{I_x}{a-e_x}$ (18) W_x cm³	$\sqrt{I_x/A}$ (19) i_x cm	(20) I_y cm⁴	$\frac{I_y}{b-e_y}$ (21) W_y cm³	$\sqrt{I_y/A}$ (22) i_y cm	(23) I_ζ cm⁴	$\sqrt{I_\zeta/A}$ (24) i_ζ cm	(25) I_η cm⁴	$\sqrt{I_\eta/A}$ (26) i_η cm	(27) I_{xy} cm⁴	(28) a_1 mm	קוטר החורים (29) d_1 mm	(30) d_2 mm	(31) w mm	(32) w_2 mm	(33) w_1 mm
* 30X20X3	1.25	0.62	0.94	0.44	0.29	0.56	1.43	1.00	0.25	0.42	0.43	5.2	6.4	8.4	12	17	--
* 30X20X4	1.59	0.81	0.93	0.55	0.38	0.55	1.81	0.99	0.33	0.42	0.53	4.2	6.4	8.4	12	17	--
* 40X20X3	2.79	1.08	1.27	0.47	0.30	0.52	2.96	1.31	0.30	0.42	0.64	14.6	6.4	11.0	12	22	--
* 40X20X4	3.59	1.42	1.26	0.60	0.39	0.52	3.79	1.30	0.39	0.42	0.81	13.8	6.4	11.0	12	22	--
(40X25X4)	3.89	1.47	1.26	1.16	0.62	0.69	4.35	1.33	0.70	0.53	1.22	10.4	6.4	11.0	12	22	--
(45X30X3)	4.47	1.46	1.43	1.60	0.70	0.86	5.15	1.53	0.93	0.65	1.55	9.0	8.4	11.0	17	25	--
* 45X30X4	5.78	1.91	1.42	2.05	0.91	0.85	6.65	1.52	1.18	0.64	2.01	8.0	8.4	11.0	17	25	--
* 45X30X5	6.99	2.35	1.41	2.47	1.11	0.84	8.02	1.51	1.44	0.64	2.39	7.2	8.4	11.0	17	25	--
* 50X30X4	7.71	2.33	1.59	2.09	0.91	0.82	8.53	1.67	1.27	0.64	2.29	13.1	8.4	13.0	17	30	--
* 50X30X5	9.41	2.88	1.58	2.54	1.12	0.82	10.40	1.66	1.56	0.64	2.77	12.2	8.4	13.0	17	30	--
(50X40X4)	8.54	2.47	1.57	4.86	1.64	1.19	10.90	1.78	2.46	0.84	3.82	---	11.0	13.0	22	30	--
* 50X40X5	10.40	3.02	1.56	5.89	2.01	1.18	13.30	1.76	3.02	0.84	3.60	---	11.0	13.0	22	30	--
* 60X30X5	15.60	4.04	1.90	2.60	1.12	0.78	16.50	1.96	1.69	0.63	3.55	21.4	8.4	17.0	17	35	--
* 60X40X5	17.20	4.25	1.89	6.11	2.02	1.13	19.80	2.03	3.50	0.86	5.98	11.2	11.0	17.0	22	35	--
* 60X40X6	20.10	5.03	1.88	7.12	2.38	1.12	23.10	2.02	4.12	0.85	6.94	10.2	11.0	17.0	22	35	--
(60X40X7)	22.00	5.79	1.87	8.07	2.74	1.11	26.30	2.00	4.73	0.85	7.81	9.2	11.0	17.0	22	35	--
* 65X50X5	23.10	5.11	2.04	11.90	3.18	1.47	28.80	2.28	6.21	1.06	9.810	3.6	13.0	21.0	30	35	--
(65X50X7)	31.00	6.99	2.02	15.80	4.31	1.44	38.40	2.25	8.37	1.06	13.00	1.8	13.0	21.0	30	35	--
(65X50X9)	38.20	8.77	2.00	19.40	5.39	1.42	47.00	2.22	10.50	1.05	15.70	---	13.0	21.0	30	35	--
* 70X50X6	33.50	7.04	2.21	14.30	3.81	1.44	39.90	2.41	7.94	1.07	12.67	---	13.0	21.0	30	40	--
* 75X50X7	46.40	9.24	2.36	16.50	4.39	1.41	53.30	2.53	9.56	1.07	15.90	13.0	13.0	23.0	30	40	--
(75X50X9)	57.40	11.60	2.34	20.20	5.49	1.39	65.70	2.50	11.90	1.07	19.40	11.0	13.0	23.0	30	40	--
* 75X55X5	35.50	6.84	2.37	16.20	3.89	1.60	43.10	2.61	8.68	1.17	14.20	8.4	17.0	23.0	30	40	--
* 75X55X7	47.90	9.39	2.35	21.80	5.32	1.59	57.90	2.59	11.80	1.17	18.90	6.6	17.0	23.0	30	40	--
(75X55X9)	59.40	11.80	2.33	26.80	6.66	1.57	71.30	2.55	14.80	0.86	23.10	5.0	17.0	23.0	30	40	--
* 80X40X6	44.90	8.73	2.55	7.59	2.44	1.05	47.60	2.63	4.90	0.84	10.40	29.0	11.0	23.0	22	45	--
* 80X40X8	57.60	11.40	2.53	9.68	3.18	1.04	60.90	2.60	6.41	0.84	13.00	27.2	11.0	23.0	22	45	--
* 80X60X7	59.00	10.70	2.51	28.40	6.34	1.74	72.00	2.77	15.40	1.28	23.81	5.7	11.0	23.0	35	45	--
* 80X65X8	68.10	12.30	2.49	40.10	8.41	1.91	88.00	2.82	20.30	1.36	30.80	---	21.0	23.0	35	45	--
(80X65X10)	82.20	15.10	2.46	48.30	10.30	1.89	106.00	2.79	24.80	1.35	36.80	---	21.0	23.0	35	45	--
* 90X60X6	71.70	11.70	2.87	25.80	5.61	1.72	82.80	3.09	14.60	1.30	25.20	17.8	17.0	25.0	35	50	
* 90X60X8	92.50	15.40	2.85	33.00	7.31	1.70	107.00	3.06	19.00	1.29							





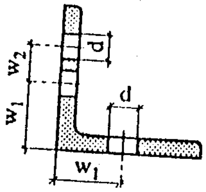
density צפיפות $\rho=\frac{G}{A}$

סימון	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
	a mm	b mm	s mm	r_1 mm	r_2 mm	A cm^2	G kg/m	U m^2/m	c_x cm	c_y cm	z_1 cm	z_2 cm	v_1 cm	v_2 cm	v_3 cm	tg α
* 30X20X3	30	20	3	3.5	2.0	1.42	1.11	0.097	0.99	0.50	2.04	1.51	0.86	1.04	0.56	0.431
* 30X20X4	30	20	4	3.5	2.0	1.85	1.45	0.097	1.03	0.54	2.02	1.52	0.91	1.03	0.58	0.423
* 40X20X3	40	20	3	3.5	2.0	1.72	1.35	0.117	1.43	0.44	2.61	1.77	0.79	1.19	0.46	0.259
* 40X20X4	40	20	4	3.5	2.0	2.25	1.77	0.117	1.47	0.48	2.57	1.80	0.83	1.18	0.50	0.252
(40X25X4)	40	25	4	4.0	2.0	2.46	1.93	0.127	1.36	0.62	2.69	1.90	1.10	1.35	0.68	0.381
(45X30X3)	45	30	3	4.5	2.0	2.19	1.72	0.146	1.43	0.70	3.09	2.23	1.21	1.59	0.80	0.436
* 45X30X4	45	30	4	4.5	2.0	2.87	2.25	0.146	1.48	0.74	3.07	2.26	1.27	1.58	0.83	0.436
* 45X30X5	45	30	5	4.5	2.0	3.53	2.77	0.146	1.52	0.78	3.05	2.27	1.32	1.58	0.85	0.430
* 50X30X4	50	30	4	4.5	2.0	3.07	2.41	0.156	1.68	0.70	3.36	2.35	1.24	1.67	0.78	0.356
* 50X30X5	50	30	5	4.5	2.0	3.78	2.96	0.156	1.73	0.74	3.33	2.38	1.28	1.66	0.80	0.353
(50X40X4)	50	40	4	4.0	2.0	3.46	2.71	0.177	1.52	1.03	3.50	2.85	1.67	1.84	1.26	0.629
* 50X40X5	50	40	5	4.0	2.0	4.27	3.35	0.177	1:56	1.07	3.49	2.88	1.73	1.84	1.27	0.625
* 60X30X5	60	30	5	6.0	3.0	4.29	3.37	0.175	2.15	0.68	3.90	2.67	1.20	1.77	0.72	0.256
* 60X40X5	60	40	5	6.0	3.0	4.79	3.76	0.195	1.96	0.97	4.08	3.01	1.68	2.09	1.10	0.437
* 60X40X6	60	40	6	6.0	3.0	5.68	4.46	0.195	2.00	1.01	4.06	3.02	1.72	2.08	1.12	0.433
(60X40X7)	60	40	7	6.0	3.0	6.55	5.14	0.195	2.04	1.05	4.04	3.03	1.77	2.07	1.14	0.429
* 65X50X5	65	50	5	6.0	3.0	5.54	4.35	0.224	1.99	1.25	4.52	3.61	2.08	2.38	1.50	0.583
(65X50X7)	65	50	7	6.0	3.0	7.60	5.97	0.224	2.07	1.33	4.50	3.62	2.19	2.37	1.52	0.574
(65X50X9)	65	50	9	6.0	3.0	9.58	7.52	0.224	2.15	1.41	4.48	3.63	2.28	2.36	1.57	0.567
* 70X50X6	70	50	6	6.0	3.0	6.88	5.40	0.235	2.24	1.25	4.82	3.68	2.20	2.52	1.42	0.497
* 75X50X7	75	50	7	6.5	3.5	8.30	6.51	0.244	2.48	1.25	5.10	3.77	2.13	2.63	1.38	0.433
(75X50X9)	75	50	9	6.5	3.5	10.50	8.23	0.244	2.56	1.32	5.06	3.80	2.22	2.62	1.44	0.427
* 75X55X5	75	55	5	7.0	3.5	6.30	4.95	0.254	2.31	1.33	5.19	4.00	2.27	2.71	1.58	0.530
* 75X55X7	75	55	7	7.0	3.5	8.66	6.80	0.254	2.40	1.41	5.16	4.02	2.37	2.70	1.62	0.525
(75X55X9)	75	55	9	7.0	3.5	10.90	8.59	0.254	2.47	1.48	5.14	4.04	2.46	2.70	1.66	0.518
* 80X40X6	80	40	6	7.0	3.5	6.89	5.41	0.234	2.85	0.88	5.21	3.53	1.55	2.42	0.89	0.259
* 80X40X8	80	80	40	8.0	3.5	9.01	7.07	0.234	2.94	0.95	5.15	3.57	1.65	2.38	1.04	0.253
* 80X60X7	80	60	7	8.0	4.0	9.38	7.36	0.274	2.51	1.52	5.55	4.42	2.70	2.92	1.68	0.546
* 80X65X8	80	65	8	8.0	4.0	11.00	8.66	0.283	2.47	1.73	5.59	4.65	2.79	2.94	2.05	0.645
(80X65X10)	80	65	10	8.0	4.0	13.60	10.70	0.283	2.55	1.81	5.56	4.68	2.90	2.95	2.11	0.640
* 90X60X6	90	60	6	7.0	3.5	86.90	6.82	0.294	2.89	1.41	6.14	4.50	2.46	3.16	1.60	0.442
* 90X60X8	90	60	8	7.0	3.5	11.40	8.96	0.294	2.97	1.49	6.11	4.54	2.56	3.15	1.69	0.437

* זוויתנים מועדפים בהזמנה

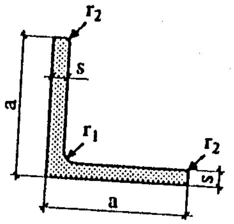
פרופילי פלדה LPN – זוויתנים שווי שוקיים

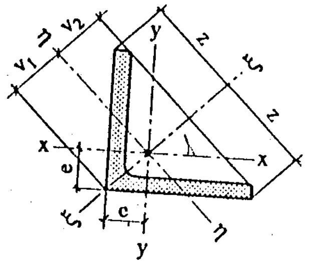
תקן: DIN 1028



A - שטח החתך
G - משקל של מטר אורך אחד
U - שטח המעטפת
I - מומנט ההתמד (אינרציה)
W - מודול החתך (מומנט התנגדות)
i - רדיוס ההתמד (אינרציה)
a_1 - המרחק בין זוויתנים הנותן $I_x = I_y$

סימון	(12) $I_x=I_y$ cm^4	(13) $W_x=W_y$ cm^3	(14) $i_x=i_y$ cm	(15) I_ζ cm^4	(16) i_ζ cm	(17) I_η cm^4	(18) i_η cm	(19) I_{xy} cm^4	(20) d mm	(21) w_1 mm	(22) w_2 mm
* 20x3	0.39	0.28	0.59	0.62	0.74	0.15	0.37	0.24	6.4	12	—
* 25x3	0.79	0.45	0.75	1.27	0.95	0.31	0.47	0.48	8.4	15	—
25x4	1.01	0.58	0.74	1.61	0.93	0.40	0.47	0.61	8.4	15	—
* 30x3	1.41	0.65	0.90	2.24	1.14	0.57	0.57	0.84	8.4	17	—
30x4	1.81	0.86	0.89	2.85	1.12	0.76	0.58	1.05	8.4	17	—
(30x5)	2.16	1.04	0.88	3.41	1.11	0.91	0.57	1.25	8.4	17	—
* 35x4	2.96	1.18	1.05	4.68	1.33	1.24	0.68	1.72	11.0	20	—
35x5	3.56	1.45	1.04	5.63	1.31	1.49	0.67	2.07	11.0	20	—
* 40x4	4.48	1.55	1.21	7.09	1.52	1.86	0.78	2.62	11.0	22	—
40x5	5.43	1.91	1.20	8.64	1.51	2.22	0.77	3.20	11.0	22	—
45x4	6.43	1.97	1.36	10.20	1.71	2.68	0.88	3.85	11.0	25	—
* 45x5	7.83	2.43	1.35	12.40	1.70	3.25	0.87	4.58	11.0	25	—
* 50x5	11.00	3.05	1.51	17.40	1.90	4.59	0.98	6.41	13.0	30	—
50x6	12.80	3.61	1.50	20.40	1.89	5.24	0.96	7.56	13.0	30	—
50x7	14.60	4.15	1.49	23.10	1.88	6.02	0.96	8.58	13.0	30	—
(55x6)	17.30	4.40	1.66	27.40	2.08	7.24	1.07	10.10	17.0	30	—
60x5	19.40	4.45	1.82	30.70	2.30	8.03	1.17	11.30	17.0	35	—
* 60x6	22.80	5.29	1.82	36.10	2.29	9.43	1.17	13.40	17.0	35	—
60x8	29.10	6.88	1.80	46.10	2.26	12.10	1.16	17.00	17.0	35	—
(65x7)	33.40	7.18	1.96	53.00	2.47	13.80	1.26	19.60	21.0	35	—
(70x6)	36.90	7.27	2.13	58.50	2.68	15.30	1.37	21.60	21.0	40	—
* 70x7	42.40	8.43	2.12	67.10	2.67	17.60	1.37	24.80	21.0	40	—
70x9	52.60	10.60	2.10	83.10	2.64	22.00	1.36	30.60	21.0	40	—
75x7	52.40	9.67	2.28	83.60	2.88	21.10	1.45	31.30	23.0	40	—
75x8	58.90	11.00	2.26	93.30	2.85	24.40	1.46	34.50	23.0	40	—
80x6	55.80	9.57	2.44	88.50	3.08	23.10	1.57	32.70	23.0	45	—
* 80x8	72.30	12.60	2.42	115.00	3.06	29.60	1.55	42.70	23.0	45	—
80x10	87.50	15.50	2.41	139.00	3.03	35.90	1.54	51.60	23.0	45	—
90x7	92.60	14.10	2.75	147.00	3.46	38.30	1.77	54.40	25.0	50	—
* 90x9	116.00	18.00	2.74	184.00	3.45	47.80	1.76	68.20	25.0	50	—



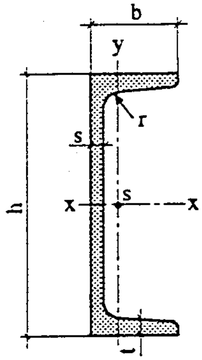


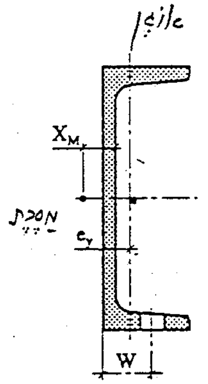
סימן	(1) a mm	(2) s mm	(3) r_1 mm	(4) r_2 mm	(5) A cm^2	(6) G kg/m	(7) U m^2/m	(8) c cm	(9) z cm	(10) v_1 cm	(11) v_2 cm
* 20X3	20	3	3.5	2.0	1.12	0.88	0.077	0.60	1.41	0.85	0.70
* 25X3	25	3	3.5	2.0	1.42	1.12	0.097	0.73	1.77	1.03	0.87
25X4	25	4	3.5	2.0	1.85	1.45	0.097	0.76	1.77	1.08	0.89
* 30X3	30	3	5.0	2.5	1.74	1.36	0.116	0.84	2.12	1.18	1.04
30X4	30	4	5.0	2.5	2.27	1.78	0.116	0.89	2.12	1.24	1.05
(30X5)	30	5	5.0	2.5	2.78	2.18	0.116	0.92	2.12	1.30	1.07
* 35X4	35	4	5.0	2.5	2.67	2.10	0.136	1.00	2.47	1.41	1.24
35X5	35	5	5.0	2.5	3.28	2.57	0.136	1.04	2.47	1.47	1.25
* 40X4	40	4	6.0	3.0	3.08	2.42	0.155	1.12	2.83	1.58	1.40
40X5	40	5	6.0	3.0	3.79	2.97	0.155	1.16	2.83	1.64	1.42
45X4	45	4	7.0	3.5	3.49	2.74	0.174	1.23	3.18	1.75	1.57
* 45X5	45	5	7.0	3.5	4.30	3.38	0.174	1.28	3.18	1.81	1.58
* 50X5	50	5	7.0	3.5	4.80	3.77	0.194	1.40	3.54	1.98	1.76
50X6	50	6	7.0	3.5	5.69	4.47	0.194	1.45	3.54	2.04	1.77
50X7	50	7	7.0	3.5	6.56	5.15	0.194	1.49	3.54	2.11	1.78
(55X6)	55	6	8.0	4.0	6.31	4.95	0.213	1.56	3.89	2.21	1.94
60X5	60	5	8.0	4.0	5.82	4.57	0.233	1.64	4.24	2.32	2.11
* 60X6	60	6	8.0	4.0	6.91	5.42	0.233	1.69	4.24	2.39	2.11
60X8	60	8	8.0	4.0	9.03	7.09	0.233	1.77	4.24	2.50	2.14
(65X7)	65	7	9.0	4.5	8.70	6.83	0.252	1.85	4.60	2.62	2.29
(70X6)	70	6	9.0	4.5	8.13	6.38	0.272	1.93	4.95	2.73	2.46
* 70X7	70	7	9.0	4.5	9.40	7.38	0.272	1.97	4.95	2.79	2.47
70X9	70	9	9.0	4.5	11.90	9.34	0.272	2.05	4.95	2.90	2.50
75X7	75	7	10.0	5.0	10.10	7.94	0.291	2.09	5.30	2.95	2.63
75X8	75	8	10.0	5.0	11.50	9.03	0.291	2.13	5.30	3.01	2.65
80X6	80	6	10.0	5.0	9.35	7.34	0.311	2.17	5.66	3.07	2.80
* 80X8	80	8	10.0	5.0	12.30	9.66	0.311	2.26	5.66	3.20	2.82
80X10	80	10	10.0	5.0	15.10	11.90	0.311	2.34	5.66	3.31	2.85
90X7	90	7	11.0	5.5	12.20	9.61	0.351	2.45	6.36	3.47	3.16
* 90X9	90	9	11.0	5.5	15.50	12.20	0.351	2.54	6.36	3.59	3.18

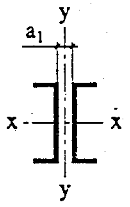
* זוויתנים מועדפים בהזמנה

פרופיל-פלדה UPB U

תקן: DIN 1026







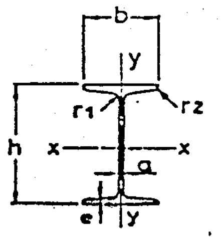
A. - שטח החתך
G - משקל של מטר אורך אחד
I - מומנט ההתמד (אינרציה)
W - מודול החתך (מומנט התנגדות)
i - רדיוס ההתמד (אינרציה)
a_1 - המרחק בין צירי הפרופילים הנותן $I_x=I_y$

סימון	h mm	b mm	s mm	$t=r_1$ mm	A cm^2	G kg/m	I_x cm^4	W_x cm^3	i_x cm	I_y cm^4	W_y cm^3	i_y cm	e_y cm	x_M cm	Z_x cm^3	Z_y cm^3	a_1 mm	d_1 mm	w mm	סימון
50	50	38	5.0	7.0	7.12	5.59	26.4	10.6	1.92	9.12	3.75	1.13	1.37	2.47	—	—	4	11	20	50
60	60	30	6.0	6.0	6.46	5.07	31.6	10.5	2.21	4.51	2.16	0.84	0.91	1.50	—	—	—	—	—	60
65	65	42	5.5	7.5	9.03	7.09	57.5	17.7	2.52	14.10	5.07	1.25	1.42	2.60	23.4	5.1	16	11	25	65
80	80	45	6.0	8.0	11.00	8.64	106.0	26.5	3.10	19.40	6.36	1.33	1.45	2.67	31.8	16.6	28	13	25	80
100	100	50	6.0	8.5	13.50	10.60	206.0	41.2	3.91	29.30	8.49	1.47	1.55	2.93	49.0	22.3	42	13	30	100
120	120	55	7.0	9.0	17.00	13.40	364.0	60.7	4.62	43.20	11.10	1.59	1.60	3.03	72.6	30.2	56	17	30	120
140	140	60	7.5	10.5	20.40	16.00	605.0	86.4	5.45	62.70	14.80	1.75	1.75	3.37	102.8	40.0	70	17	35	140
160	160	65	7.5	10.0	24.00	18.80	925.0	116.0	6.21	85.30	18.30	1.89	1.84	3.56	137.6	50.2	82	21	35	160
180	180	70	8.0	11.0	28.00	22.00	1350.0	150.0	6.95	114.00	22.40	2.02	1.92	3.75	179.2	62.4	96	21	40	180
200	200	75	8.5	11.5	32.20	25.30	1910.0	191.0	7.70	148.00	27.00	2.14	2.01	3.94	228.0	72.6	108	23	40	200
220	220	80	9.0	12.5	37.40	29.40	2690.0	245.0	8.48	197.00	33.60	2.30	2.14	4.20	292.0	94.2	122	23	45	220
240	240	85	9.5	13.0	42.30	33.20	3600.0	300.0	9.22	248.00	39.60	2.42	2.23	4.39	358.0	112.0	134	25	45	240
260	260	90	10.0	14.0	48.30	37.90	4820.0	371.0	9.99	317.00	47.70	2.56	2.36	4.66	442.0	136.0	146	25	50	260
280	280	95	10.0	15.0	53.30	41.80	6280.0	448.0	10.90	399.00	57.20	2.74	2.53	5.02	532.0	160.0	160	25	50	280
300	300	100	10.0	16.0	58.80	46.20	8030.0	535.0	11.70	495.00	67.80	2.90	2.70	5.41	632.0	188.0	174	25	55	300
320	320	100	14.0	17.5	75.80	59.50	10870.0	679.0	12.10	597.00	80.60	2.81	2.60	4.82	826.0	215.0	182	25	55	320
350	350	100	14.0	16.0	77.30	60.60	12840.0	734.0	12.90	570.00	75.00	2.72	2.40	4.45	918.0	205.0	204	25	55	250
380	380	102	13.5	16.0	80.40	63.10	15760.0	829.0	14.00	615.00	78.70	2.77	2.38	4.58	1014.0	214.0	230	25	55	380
400	400	110	14.0	18.0	91.50	71.80	20350.0	1020.0	14.90	846.00	102.00	3.04	2.65	5.11	1240.0	279.0	240	25	60	400

טבלאות פרופילים: פ-1 IPN

IPN	Weight	Depth	Width	Thickness		Toe Rad	Area	Moment of Inertia		Rad of Gyration		Elastic Modulus		Plastic Modulus		iw	J	Ct	Cw,e	Cw,c
	G	D	W	Web	Flange	R2	F	Ixx	Iyy	rxx	ryy	Zxx	Zyy	Sxx	Syy					
	kg/m	h (mm)	b (mm)	t w(mm)	t f(mm)	mm	cm ^ 2	cm ^ 4	cm ^ 4	cm	cm	cm ^ 3	cm ^ 3	cm ^ 3	cm ^ 3	cm	cm ^ 4	kN*m ^ 2	kN*m ^ 3	kN*m ^ 3
80	5.95	80	42	3.9	5.9	2.3	7.58	77.8	6.29	3.2	0.91	19.5	3	22.8	5	1.04	0.869	9.34	4.72	4.25
100	8.34	100	50	4.5	6.8	2.7	10.6	171	12.2	4.01	1.07	34.2	4.88	39.8	8.1	1.23	1.6	17.65	11.50	10.43
120	11.1	120	58	5.1	7.7	3.1	14.2	328	21.5	4.81	1.23	54.7	7.41	10.5	12.4	1.42	2.71	30.49	24.43	22.22
140	14.3	140	66	5.7	8.6	3.4	18.2	573	35.2	5.61	1.4	81.9	10.7	95.4	17.9	1.60	4.32	49.25	46.79	42.62
160	17.9	160	74	6.3	9.5	3.8	22.8	935	54.7	6.4	1.55	117	14.8	136	24.8	1.79	6.57	75.72	83.28	76.07
180	21.9	180	82	6.9	10.4	4.1	27.9	1450	81.3	7.2	1.71	161	19.8	187	33.3	1.98	9.58	111.47	139.49	127.83
200	26.2	200	90	7.5	11.3	4.5	33.4	2140	117	8	1.87	214	26	250	43.6	2.17	13.5	158.74	223.35	204.47
220	31.1	220	98	8.1	12.2	4.9	39.5	3060	162	8.8	2.02	278	33.1	324	55.7	2.36	18.6	219.25	340.55	312.13
240	36.2	240	106	8.7	13.1	5.2	46.1	4250	221	9.59	2.2	354	41.7	412	70	2.55	25	296.88	507.28	465.94
260	41.9	260	113	9.4	14.1	5.6	53.3	5740	288	10.4	2.32	442	51	514	85.9	2.71	33.5	392.32	716.43	658.22
280	47.9	280	119	10.1	15.2	6.1	61	7590	364	11.1	2.45	542	61.2	632	103	2.86	44.2	506.62	975.09	895.41
300	54.2	300	125	10.8	16.2	6.5	69	9800	451	11.9	2.56	653	72.2	762	122	3.00	56.8	639.26	1294.83	1188.54
320	61	320	131	11.5	17.3	6.9	77.7	12510	555	12.7	2.67	782	84.7	914	143	3.14	72.5	801.18	1699.53	1560.65
340	68	340	137	12.2	18.3	7.3	86.7	15700	674	13.5	2.8	923	98.4	1080	166	3.28	90.4	985.90	2193.48	2014.52
360	76.1	360	143	13	19.5	7.8	97	19610	818	14.2	2.9	1090	114	1280	194	3.43	115	1225.02	2817.70	2588.96
380	84	380	149	13.7	20.5	8.2	107	24010	975	15	3.02	1260	131	1480	222	3.57	141	1480.91	3545.91	3246.12
400	92.4	400	155	14.4	21.6	8.6	118	29210	1160	15.7	3.13	1460	149	1710	254	3.71	170	1773.66	4440.51	4075.10
425	104.	425	163	15.3	23	9.2	132	36970	1440	16.7	3.3	1740	176			3.91	216	2227.54	5856.15	5375.42
450	115	450	170	16.2	24.3	9.7	147	45850	1730	17.7	3.43	2040	203	2400	345	4.07	267	2714.54	7450.29	6846.43
475	128	475	178	17.1	25.6	10.3	163	56480	2090	18.6	3.6	2380	235			4.26	329	3311.98	9501.74	8729.22
500	141	500	185	18	27	10.8	179	68740	2480	19.6	3.72	2750	268	3240	456	4.42	402	3988.01	11866.88	10894.28
550	166	550	200	19	30	11.9	212	99180	3490	21.6	4.02	3610	349			4.81	544	5503.38	18359.14	16877.71
600	199	600	215	21.6	32.4	13	254	139000	4670	23.4	4.3	4630	434			5.12	813	7782.53	26815.31	24597.27

IPN



$$M_{cr,t} = \frac{C_t}{l_e} \qquad C_t = \frac{\pi E \sqrt{I_y J}}{\sqrt{2.6}}$$

$$M_{cr,wc} = \frac{C_{w,c}}{l_e^2} \qquad C_{w,c} = \pi^2 E\, i_\omega^2 W_{el,x}$$

$$M_{cr,we} = \frac{C_{w,e}}{l_e^2} \qquad C_{w,e} = \frac{\pi^2 E I_y (D - t_f)}{2}$$

$$i_\omega = \sqrt{\frac{I_y}{2\,(A_f + \frac{1}{6} A_w)}}$$

תורת החוזק 2 מועד ב		מבחן סופי		אפריל 3 2000

פרופ עזרא לוי

אורך המבחן הזה יהיה 180 דקות. במבחן יש ארבעה שאלות. עליכם לענות לכל השאלות.
רק להשתמש ברשמות הכיתה, תרגילי בית שלכם, ומחשבון. ואם יש דברים הנתונים לכם עם המבחן בלבד.

לחתום על סעיף הבא בבקשה:
לאורך המבחן אני לא רשאי(ת) לקבל עזרה מאף אחד או לתת עזרה לאף אחד. אם אני אעבור על אלה, אני אכשול את המבחן.

חתימת הסטודנט______________________

תרגיל מס. 1 (40)

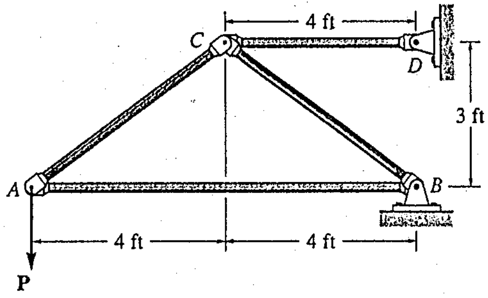
מסבך מוטות, עשוים מפלדה, מעומס בכוח P כמשורטט בציור. אם לכל מוט יש שטח החתך עגול בקוטר של 3 in.

א. מיצאו את הכוח שיגרום לקריסה ובאיזה מוט זה יקרה.

ב. מה צריך להיות הקוטר כדי שיהיה קריסה וכניעה בו-זמנית במסבך.

חשבו את P לפי התנאים הבאים: כל המוטות מחוברות בסיכה בשני הצדדים נגד קריסה בכוון x, ורתומים בשני הצדדים נגד קריסה בכוון z.

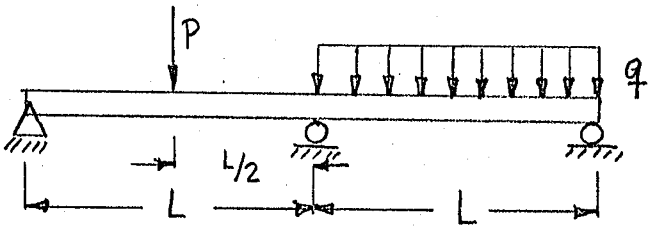
הניחו ש- $E = 29 \times 10^6$ lb/in^2 ו- σ_{yp}=36000 lb/in^2. המדדים של הקורות נתונים בצורה, ויש לזכור ש- 12 in.= 1 ft.



תרגיל מס. 2 (30 נקודות)

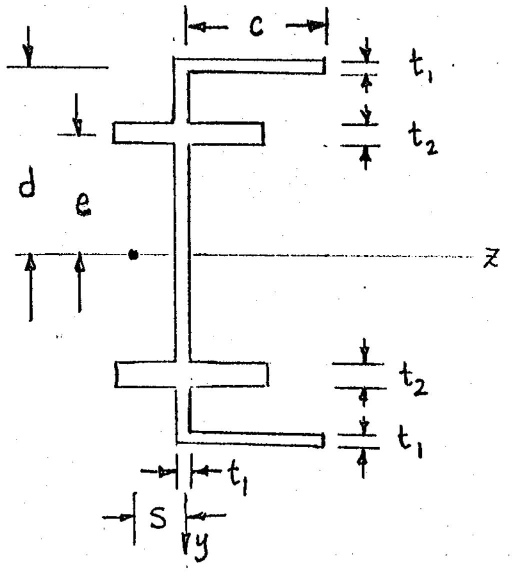
נתון גשר רצוף מורכב משני חלקים AB ו- BC ונושא עומס P ועומס מפורס, q, כמשורטט בצורה. לפי שיטת עומס הגבולי (זאת אומרת, מומנט הפלסטי). מיצאו את היחס בין qL ל- P כדי ששני חלקי הגשר יתמוטטו בו-זמני.

רמז - מיצאו את מומנט הפלסטי לכל האפשרויות ויחסו את התוצאות.



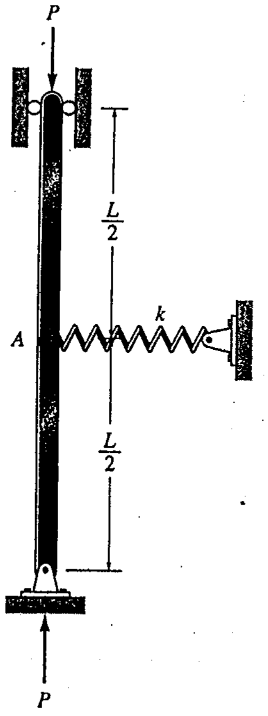
תרגיל מס. 3 (16 נקודות)

נתון חתך בדק-דופן כמשורטט בצורה הבאה. מיצאו את מרכז הגזירה לחתך, s.



תרגיל מס. 4 (14 נקודות)

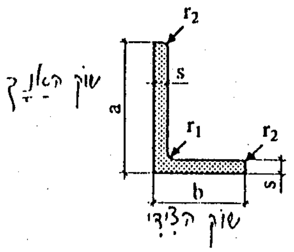
למערכת המשורטטת, עליכם למצוא את הכוח הקריטי, לפי שיטת אנרגייה, שגורם להתמוטטות המערכת.

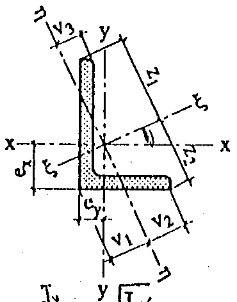


(a)

פרופילי פלדה LPN – זוויתנים שוני שוקיים

תקן: DIN 1028

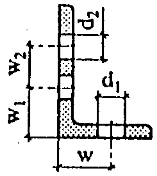


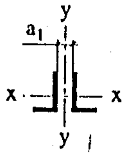


A - שטח החתך
G - משקל של 1 מטר אורך
U - שטח המעטפת
I - מומנט ההתמד (אינרציה)
W - מודול החתך (מומנט התנגדות) *
i - רדיוס ההתמד (אינרציה) = $\sqrt{I/A}$
a_1 - המרחק בין זוויתנים הנותן $I_x = I_y$

	$\frac{I_x}{a-e_x}$	$\sqrt{I_x/A}$		$\frac{I_y}{b-e_y}$	$\sqrt{I_y/A}$		$\sqrt{I_\zeta/A}$		$\sqrt{I_\eta/A}$			קוטר החורים				
	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)
סימון	I_x cm^4	W_x cm^3	i_x cm	I_y cm^4	W_y cm^3	i_y cm	I_ζ cm^4	i_ζ cm	I_η cm^4	i_η cm	I_{xy} cm^4	a_1 mm	d_1 mm	d_2 mm	w mm	w_2 mm	w_1 mm
* 30X20X3	1.25	0.62	0.94	0.44	0.29	0.56	1.43	1.00	0.25	0.42	0.43	5.2	6.4	8.4	12	17	---
* 30X20X4	1.59	0.81	0.93	0.55	0.38	0.55	1.81	0.99	0.33	0.42	0.53	4.2	6.4	8.4	12	17	---
* 40X20X3	2.79	1.08	1.27	0.47	0.30	0.52	2.96	1.31	0.30	0.42	0.64	14.6	6.4	11.0	12	22	---
* 40X20X4	3.59	1.42	1.26	0.60	0.39	0.52	3.79	1.30	0.39	0.42	0.81	13.8	6.4	11.0	12	22	---
(40X25X4)	3.89	1.47	1.26	1.16	0.62	0.69	4.35	1.33	0.70	0.53	1.22	10.4	6.4	11.0	12	22	---
(45X30X3)	4.47	1.46	1.43	1.60	0.70	0.86	5.15	1.53	0.93	0.65	1.55	9.0	8.4	11.0	17	25	---
* 45X30X4	5.78	1.91	1.42	2.05	0.91	0.85	6.65	1.52	1.18	0.64	2.01	8.0	8.4	11.0	17	25	---
* 45X30X5	6.99	2.35	1.41	2.47	1.11	0.84	8.02	1.51	1.44	0.64	2.39	7.2	8.4	11.0	17	25	---
* 50X30X4	7.71	2.33	1.59	2.09	0.91	0.82	8.53	1.67	1.27	0.64	2.29	13.1	8.4	13.0	17	30	---
* 50X30X5	9.41	2.88	1.58	2.54	1.12	0.82	10.40	1.66	1.56	0.64	2.77	12.2	8.4	13.0	17	30	---
(50X40X4)	8.54	2.47	1.57	4.86	1.64	1.19	10.90	1.78	2.46	0.84	3.82	---	11.0	13.0	22	30	---
* 50X40X5	10.40	3.02	1.56	5.89	2.01	1.18	13.30	1.76	3.02	0.84	3.60	---	11.0	13.0	22	30	---
* 60X30X5	15.60	4.04	1.90	2.60	1.12	0.78	16.50	1.96	1.69	0.63	3.55	21.4	8.4	17.0	17	35	---
* 60X40X5	17.20	4.25	1.89	6.11	2.02	1.13	19.80	2.03	3.50	0.86	5.98	11.2	11.0	17.0	22	35	---
* 60X40X6	20.10	5.03	1.88	7.12	2.38	1.12	23.10	2.02	4.12	0.85	6.94	10.2	11.0	17.0	22	35	---
(60X40X7)	22.00	5.79	1.87	8.07	2.74	1.11	26.30	2.00	4.73	0.85	7.81	9.2	11.0	17.0	22	35	---
* 65X50X5	23.10	5.11	2.04	11.90	3.18	1.47	28.80	2.28	6.21	1.06	9.810	3.6	13.0	21.0	30	35	---
(65X50X7)	31.00	6.99	2.02	15.80	4.31	1.44	38.40	2.25	8.37	1.06	13.00	1.8	13.0	21.0	30	35	---
(65X50X9)	38.20	8.77	2.00	19.40	5.39	1.42	47.00	2.22	10.50	1.05	15.70	---	13.0	21.0	30	35	---
* 70X50X6	33.50	7.04	2.21	14.30	3.81	1.44	39.90	2.41	7.94	1.07	12.67	---	13.0	21.0	30	40	---
* 75X50X7	46.40	9.24	2.36	16.50	4.39	1.41	53.30	2.53	9.56	1.07	15.90	13.0	13.0	23.0	30	40	---
(75X50X9)	57.40	11.60	2.34	20.20	5.49	1.39	65.70	2.50	11.90	1.07	19.40	11.0	13.0	23.0	30	40	---
* 75X55X5	35.50	6.84	2.37	16.20	3.89	1.60	43.10	2.61	8.68	1.17	14.20	8.4	17.0	23.0	30	40	---
* 75X55X7	47.90	9.39	2.35	21.80	5.32	1.59	57.90	2.59	11.80	1.17	18.90	6.6	17.0	23.0	30	40	---
(75X55X9)	59.40	11.80	2.33	26.80	6.66	1.57	71.30	2.55	14.80	0.86	23.10	5.0	17.0	23.0	30	40	---
* 80X40X6	44.90	8.73	2.55	7.59	2.44	1.05	47.60	2.63	4.90	0.84	10.40	29.0	11.0	23.0	22	45	---
* 80X40X8	57.60	11.40	2.53	9.68	3.18	1.04	60.90	2.60	6.41	0.84	13.00	27.2	11.0	23.0	22	45	---
* 80X60X7	59.00	10.70	2.51	28.40	6.34	1.74	72.00	2.77	15.40	1.28	23.81	5.7	11.0	23.0	35	45	---
* 80X65X8	68.10	12.30	2.49	40.10	8.41	1.91	88.00	2.82	20.30	1.36	30.80	---	21.0	23.0	35	45	---
(80X65X10)	82.20	15.10	2.46	48.30	10.30	1.89	106.00	2.79	24.80	1.35	36.80	---	21.0	23.0	35	45	---
* 90X60X6	71.70	11.70	2.87	25.80	5.61	1.72	82.80	3.09	14.60	1.30	25.20	17.8	17.0	25.0	35	50	---
* 90X60X8	92.50	15.40	2.85	33.00	7.31	1.70	107.00	3.06	19.00	1.29	32.20	16.0	17.0	25.0	35	50	---

* זוויתנים מועדפים בהזמנה





density דחיסות $\rho=\frac{G}{A}$

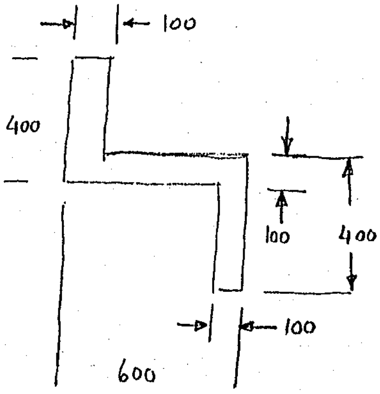
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
סימון	a mm	b mm	s mm	r_1 mm	r_2 mm	A cm^2	G kg/m	U m^2/m	e_x cm	e_y cm	z_1 cm	z_2 cm	v_1 cm	v_2 cm	v_3 cm	tg α
* 30X20X3	30	20	3	3.5	2.0	1.42	1.11	0.097	0.99	0.50	2.04	1.51	0.86	1.04	0.56	0.431
* 30X20x4	30	20	4	3.5	2.0	1.85	1.45	0.097	1.03	0.54	2.02	1.52	0.91	1.03	0.58	0.423
* 40X20X3	40	20	3	3.5	2.0	1.72	1.35	0.117	1.43	0.44	2.61	1.77	0.79	1.19	0.46	0.259
* 40X20X4	40	20	4	3.5	2.0	2.25	1.77	0.117	1.47	0.48	2.57	1.80	0.83	1.18	0.50	0.252
(40X25X4)	40	25	4	4.0	2.0	2.46	1.93	0.127	1.36	0.62	2.69	1.90	1.10	1.35	0.68	0.381
(45X30X3)	45	30	3	4.5	2.0	2.19	1.72	0.146	1.43	0.70	3.09	2.23	1.21	1.59	0.80	0.436
* 45X30X4	45	30	4	4.5	2.0	2.87	2.25	0.146	1.48	0.74	3.07	2.26	1.27	1.58	0.83	0.436
* 45X30X5	45	30	5	4.5	2.0	3.53	2.77	0.146	1.52	0.78	3.05	2.27	1.32	1.58	0.85	0.430
* 50X30X4	50	30	4	4.5	2.0	3.07	2.41	0.156	1.68	0.70	3.36	2.35	1.24	1.67	0.78	0.356
* 50X30X5	50	30	5	4.5	2.0	3.78	2.96	0.156	1.73	0.74	3.33	2.38	1.28	1.66	0.80	0.353
(50X40X4)	50	40	4	4.0	2.0	3.46	2.71	0.177	1.52	1.03	3.50	2.85	1.67	1.84	1.26	0.629
* 50X40X5	50	40	5	4.0	2.0	4.27	3.35	0.177	1:56	1.07	3.49	2.88	1.73	1.84	1.27	0.625
* 60X30X5	60	30	5	6.0	3.0	4.29	3.37	0.175	2.15	0.68	3.90	2.67	1.20	1.77	0.72	0.256
* 60X40X5	60	40	5	6.0	3.0	4.79	3.76	0.195	1.96	0.97	4.08	3.01	1.68	2.09	1.10	0.437
* 60X40X6	60	40	6	6.0	3.0	5.68	4.46	0.195	2.00	1.01	4.06	3.02	1.72	2.08	1.12	0.433
(60X40X7)	60	40	7	6.0	3.0	6.55	5.14	0.195	2.04	1.05	4.04	3.03	1.77	2.07	1.14	0.429
* 65X50X5	65	50	5	6.0	3.0	5.54	4.35	0.224	1.99	1.25	4.52	3.61	2.08	2.38	1.50	0.583
(65X50X7)	65	50	7	6.0	3.0	7.60	5.97	0.224	2.07	1.33	4.50	3.62	2.19	2.37	1.52	0.574
(65X50X9)	65	50	9	6.0	3.0	9.58	7.52	0.224	2.15	1.41	4.48	3.63	2.28	2.36	1.57	0.567
* 70X50X6	70	50	6	6.0	3.0	6.88	5.40	0.235	2.24	1.25	4.82	3.68	2.20	2.52	1.42	0.497
* 75X50X7	75	50	7	6.5	3.5	8.30	6.51	0.244	2.48	1.25	5.10	3.77	2.13	2.63	1.38	0.433
(75X50X9)	75	50	9	6.5	3.5	10.50	8.23	0.244	2.56	1.32	5.06	3.80	2.22	2.62	1.44	0.427
* 75X55X5	75	55	5	7.0	3.5	6.30	4.95	0.254	2.31	1.33	5.19	4.00	2.27	2.71	1.58	0.530
* 75X55X7	75	55	7	7.0	3.5	8.66	6.80	0.254	2.40	1.41	5.16	4.02	2.37	2.70	1.62	0.525
(75X55X9)	75	55	9	7.0	3.5	10.90	8.59	0.254	2.47	1.48	5.14	4.04	2.46	2.70	1.66	0.518
* 80X40X6	80	40	6	7.0	3.5	6.89	5.41	0.234	2.85	0.88	5.21	3.53	1.55	2.42	0.89	0.259
* 80X40X8	80	80	40	8.0	3.5	9.01	7.07	0.234	2.94	0.95	5.15	3.57	1.65	2.38	1.04	0.253
* 80X60X7	80	60	7	8.0	4.0	9.38	7.36	0.274	2.51	1.52	5.55	4.42	2.70	2.92	1.68	0.546
* 80X65X8	80	65	8	8.0	4.0	11.00	8.66	0.283	2.47	1.73	5.59	4.65	2.79	2.94	2.05	0.645
(80X65X10)	80	65	10	8.0	4.0	13.60	10.70	0.283	2.55	1.81	5.56	4.68	2.90	2.95	2.11	0.640
* 90X60X6	90	60	6	7.0	3.5	86.90	6.82	0.294	2.89	1.41	6.14	4.50	2.46	3.16	1.60	0.442
* 90X60X8	90	60	8	7.0	3.5	11.40	8.96	0.294	2.97	1.49	6.11	4.54	2.56	3.15	1.69	0.437

* זוויתנים מועדפים בהזמנה

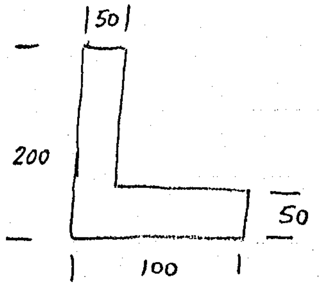
24.10.99

כל מדידות ב- mm

תרגילי בית #2

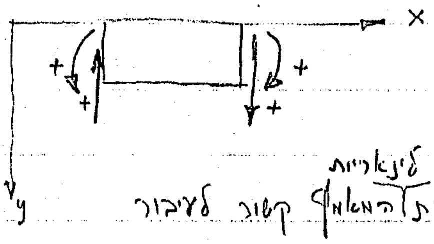


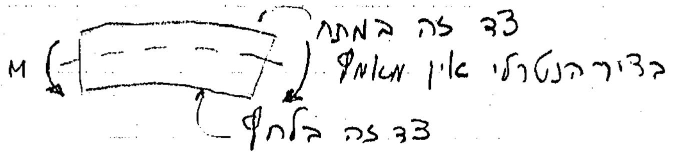
למצוא: מרכז הכובד, מומנט אינרציה
מסביב מרכז הכובד, וצירים
הראשיים



להגיש ב- 2.11.99

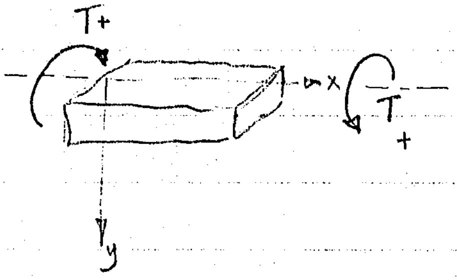
הסכם הסימנים למומנט הכפיפה ולכוח הגזירה





בציר הנטרלי אין עיבור ובבעיות אלסטיות ליניאריות המאמץ קשור לעיבור
בחוק הוק ($\sigma=E\epsilon$)

הסכם הסימנים למומנט הפיתול



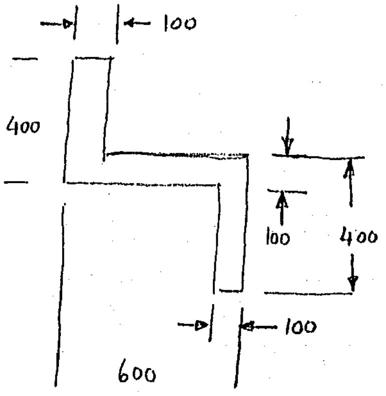
הנחות

- מישורים לפני כפיפה נשארים מישורים אחרי הכפיפה — PLANE SECTIONS REMAIN PLANE
- ציר הנטרלי עובר דרך מרכז הכובד — NEUTRAL AXIS PASSES THROUGH CG.
- הגוף הוא הומוגני הצפיפות לא משתנה — HOMOG. DENSITY

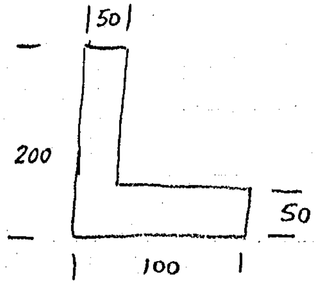
24.10.99

תרגילי בית #2

כל מדידות ב- mm

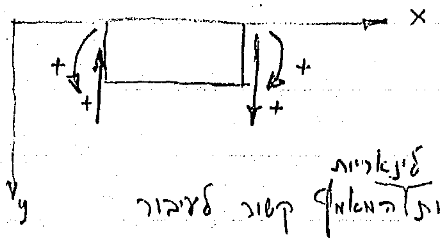


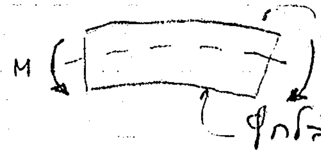
למצוא: מרכז הכובד, מומנט אינרציה
מסביב מרכז הכובד, וצירים
הראשיים



להגיש ב- 2.11.99

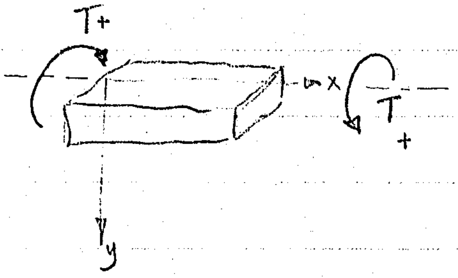
הסכם הסימנים למומנט הכפיפה ולכוח הגזירה





בציר הנטרלי אין עיבור ובבעיות אלסטיות לינאריות המאמץ קשור לעיבור
בחוק הוק ($\sigma = E\epsilon$)

הסכם הסימנים למומנט הפיתול



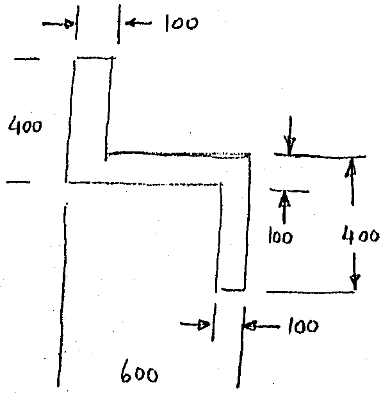
הנחות

- מישורים לפני כפיפה נשארים מישורים אחרי הכפיפה — PLANE SECTIONS REMAIN PLANE
- ציר הנטרלי עובר דרך מרכז הכובד — NEUTRAL AXIS PASSES THROUGH CG.
- הגוף הוא הומוגני הצפיפות (DENSITY) לא משתנה — HOMOG.

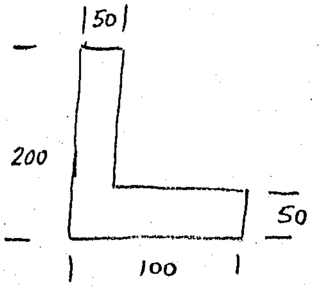
24.10.99

תרגילי בית #2

כל מדידות ב- mm

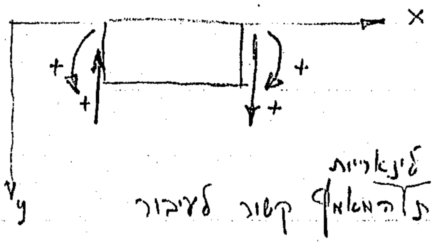


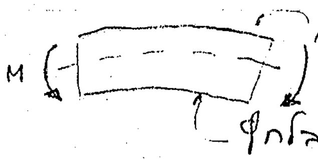
למצוא: מרכז הכובד, מומנט אינרציה מסביב מרכז הכובד, וצירים הראשיים



להגיש ב- 2.11.99

הסכם הסימנים למומנט הכפיפה ולכוח הגזירה





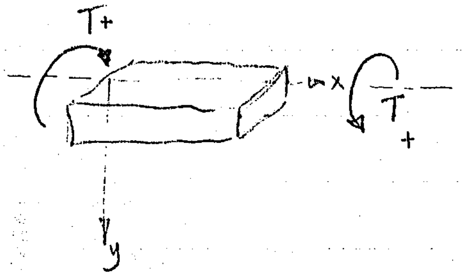
צד זה במתח

בציר הנטרלי אין מאמץ

צד זה בלחץ

בציר הנטרלי אין עיבור ובבעיות אלסטיות לינאריות המאמץ קשור לעיבור בחוק הוק ($\sigma = E\epsilon$)

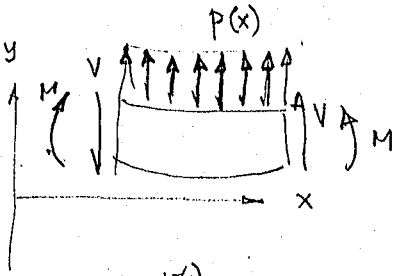
הסכם הסימנים למומנט הפיתול



הנחות

- מישורים לפני כפיפה נשארים כמישורים אחרי הכפיפה PLANE SECTIONS REMAIN PLANE
- ציר הנטרלי עובר דרך מרכז הכובד NEUTRAL AXIS PASSES THROUGH C.G.
- הגוף הוא הומוגני הדחיסות לא משתנה DENSITY HOMOG.

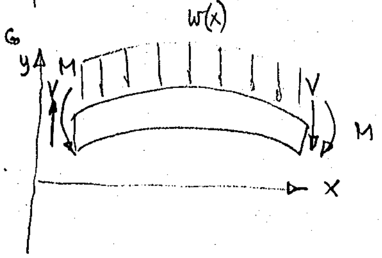
Popov



$$\sigma_x = \frac{-My}{I}$$

$$\frac{dV}{dx} = -p(x) \qquad \frac{dM}{dx} = -V$$

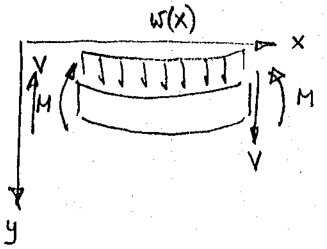
Cook & YOUNG



$$\sigma_x = \frac{My}{I}$$

$$\frac{dV}{dx} = -w(x) \qquad \frac{dM}{dx} = -V$$

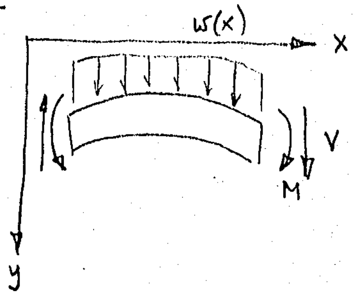
Patel & Venkatraman



$$\sigma_x = \frac{My}{I}$$

$$\frac{dV}{dx} = -w(x) \qquad \frac{dM}{dx} = V$$

שלנו

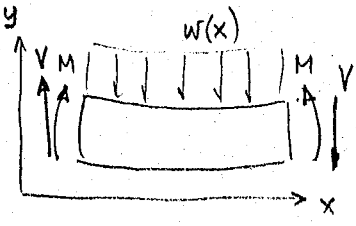


$$\sigma_x = -\frac{My}{I}$$

אותו דבר כמו POPOV <u>אבל</u>
הציר- y, +V, +M, w הפוך
בכיוונם בי POPOV

$$\frac{dV}{dx} = -w(x) \qquad \frac{dM}{dx} = -V$$

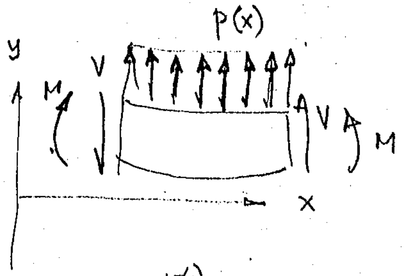
Beer & Johnston
Hibbeler



$$\sigma_x = -\frac{My}{I}$$

$$\frac{dV}{dx} = -w(x) \qquad \frac{dM}{dx} = V$$

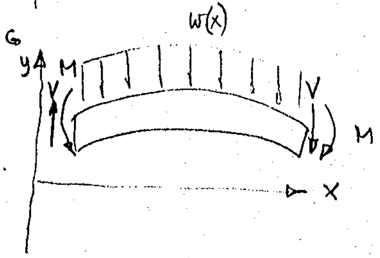
Popov



$$\sigma_x = \frac{-My}{I}$$

$$\frac{dV}{dx} = -p(x) \qquad \frac{dM}{dx} = -V$$

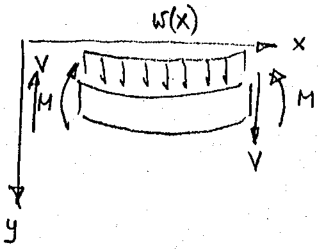
Cook & Young



$$\sigma_x = \frac{My}{I}$$

$$\frac{dV}{dx} = -w(x) \qquad \frac{dM}{dx} = -V$$

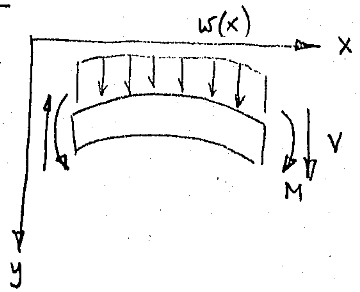
Patel & Venkatraman



$$\sigma_x = \frac{My}{I}$$

$$\frac{dV}{dx} = -w(x) \qquad \frac{dM}{dx} = V$$

שלנו

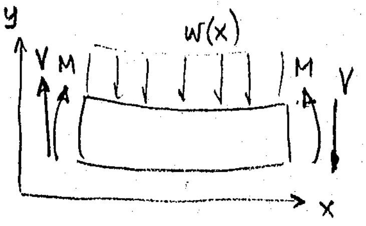


$$\sigma_x = -\frac{My}{I}$$

אותו דבר כמו **אבל**
הציר- y, +V, +M, w הפוך
בכיוונים ב- POPOV

$$\frac{dV}{dx} = -w(x) \qquad \frac{dM}{dx} = -V$$

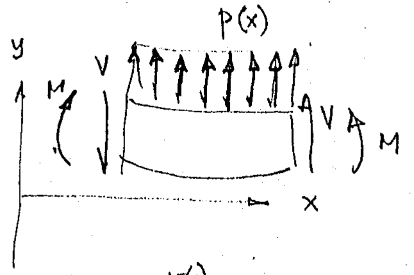
Beer & Johnston
Hibbeler



$$\sigma_x = -\frac{My}{I}$$

$$\frac{dV}{dx} = -w(x). \qquad \frac{dM}{dx} = V$$

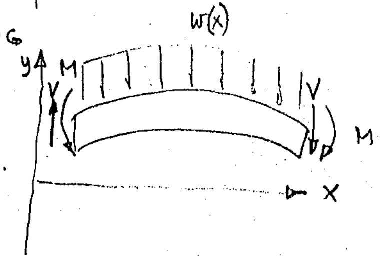
Popov



$$\sigma_x = \frac{-My}{I}$$

$$\frac{dV}{dx} = -p(x) \qquad \frac{dM}{dx} = -V$$

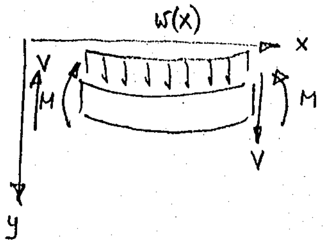
Cook & Young



$$\sigma_x = \frac{My}{I}$$

$$\frac{dV}{dx} = -w(x) \qquad \frac{dM}{dx} = -V$$

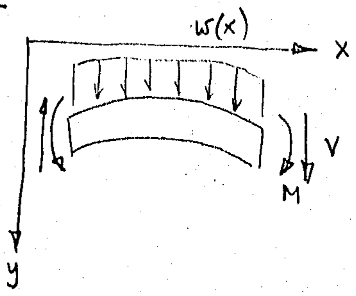
Patel & Venkatraman



$$\sigma_x = \frac{My}{I}$$

$$\frac{dV}{dx} = -w(x) \qquad \frac{dM}{dx} = V$$

שלנו

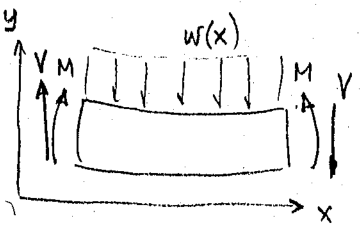


$$\sigma_x = -\frac{My}{I}$$

$$\frac{dV}{dx} = -w(x) \qquad \frac{dM}{dx} = -V$$

אותו דבר כמו **אבל**
הציר- y, +V, +M, w הפוך
בכיוונים ב- POPOV

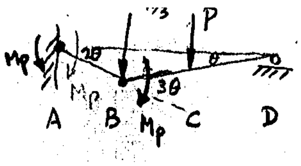
Beer & Johnston
Hibbeler



$$\sigma_x = -\frac{My}{I}$$

$$\frac{dV}{dx} = -w(x) \qquad \frac{dM}{dx} = V$$

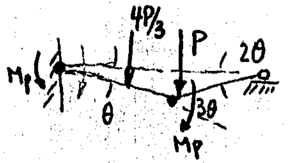
אם הפרק הפלסטי ב ו



$$\frac{4P}{3}\cdot\left(2\theta\cdot\frac{L}{3}\right)+P\cdot\frac{L}{3}\cdot\theta=M_p\cdot 2\theta+M_p\cdot 3\theta$$

$$P=\frac{45}{11}\frac{M_p}{L}=4.09\,M_p/L$$

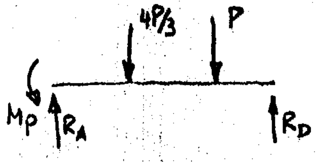
אם הפרק הפלסטי ב- A ו- C



$$\frac{4P}{3}\cdot\left(\theta\cdot\frac{L}{3}\right)+P\cdot 2\theta\cdot\frac{L}{3}=M_p\cdot\theta+M_p\cdot 3\theta$$

$$\underline{\underline{P=3.60\,\frac{M_p}{L}}}$$

לכן $P_{\text{מסגרתי}}=3.60\frac{M_p}{L}$

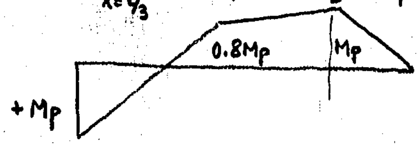


לבדוק המומנט ב $x=L/3$

$$\circlearrowleft\!+\ \Sigma M_D=0\qquad -M_p-\frac{4P}{3}\cdot\frac{2L}{3}-P\cdot\frac{L}{3}+R_A\cdot L=0$$

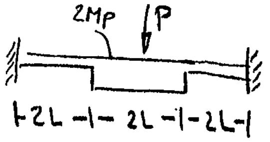
$$R_A=\frac{M_p}{L}+\frac{11P}{9}=\frac{M_p}{L}+\frac{11}{9}\left(3.60\frac{M_p}{L}\right)=5.4\frac{M_p}{L}$$

$$M\Big|_{x=L/3}=M_p-R_A\cdot\frac{L}{3}=M_p-\frac{5.4}{3}M_p=-0.8\,M_p\quad<M_p$$

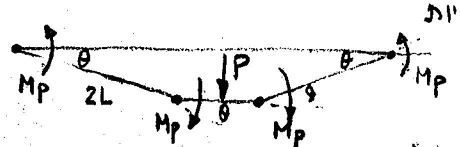


מפני של גודל רב של המומנט ב $x=L/3$ פחות מ-M_p,
לכן זה המנגנון שניתן את הכוח במסגרתי.

11.29

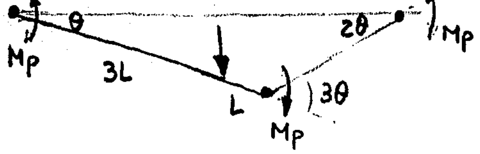


אפשרויות



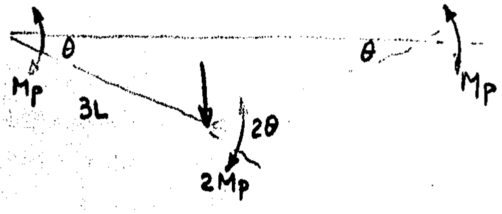
$$P\cdot 2L\cdot\theta=M_p\cdot\theta+M_p\theta+M_p\theta+M_p\theta$$

$$\underline{P=2M_p/L}$$



$$P\cdot 3L\theta=M_p\cdot\theta+M_p\cdot 3\theta+M_p\cdot 2\theta$$

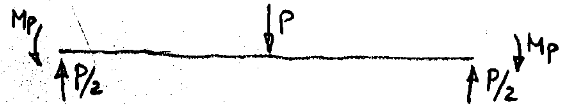
$$\underline{P=2M_p/L}$$



$$P\cdot 3L\theta=M_p\theta+2M_p\cdot 2\theta+M_p\cdot\theta$$

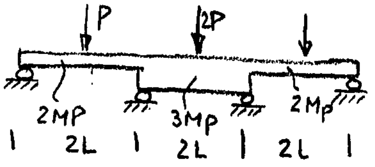
$$\underline{P=2M_p/L}$$

ויכולים להראות ש-$|M|$ בחלקים הדקים פחות מ-M_p ובחלק העבה $|M|$ פחות מ-$2M_p$

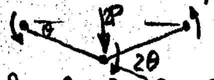


מפני הסימטריות

11.30



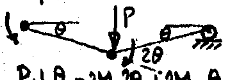
אם הפרק מופיע במרכז



$$2P\cdot L\theta=2M_p\theta+3M_p\cdot 2\theta+2M_p\theta$$

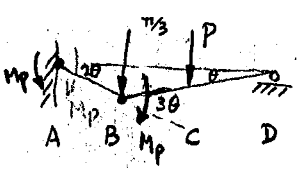
$$P=\frac{5M_p}{L}$$

אם הפרקים מופיעים בחלק הימני



$$P\cdot L\theta=2M_p 2\theta+2M_p\theta$$

$$P=6M_p/L$$

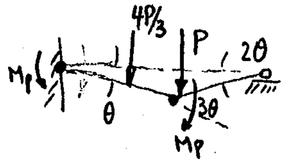


L/3 L/3 L/3 אם הפרק הפלסטי ב- B ו C

$$\frac{4P}{3}\cdot\left(2\theta\cdot\frac{L}{3}\right)+P\cdot\frac{L}{3}\cdot\theta=M_p\cdot2\theta+M_p\cdot3\theta$$

$$P=\frac{45}{11}\frac{M_p}{L}=4.09\,M_p/L$$

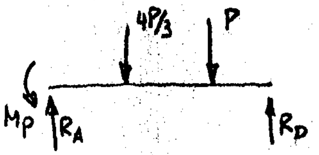
אם הפרק הפלסטי ב- A ו-C



$$\frac{4P}{3}\cdot\left(\theta\cdot\frac{L}{3}\right)+P\cdot2\theta\cdot\frac{L}{3}=M_p\cdot\theta+M_p\cdot3\theta$$

$$\underline{\underline{P=3.60\,\frac{M_p}{L}}}$$

לכן $P_{\text{מסגרתי}}=3.60\frac{M_p}{L}$

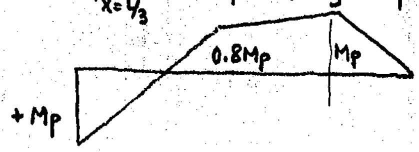


לבדוק המומנט ב $x=L/3$

$$+\circlearrowleft\ \Sigma M_D=0\quad -M_p-\frac{4P}{3}\cdot\frac{2L}{3}-P\cdot\frac{L}{3}+R_A\cdot L=0$$

$$R_A=\frac{M_p}{L}+\frac{11P}{9}=\frac{M_p}{L}+\frac{11}{9}\left(3.60\frac{M_p}{L}\right)=5.4\frac{M_p}{L}$$

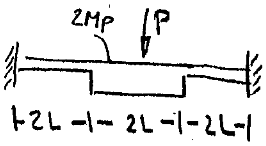
$$M\Big|_{x=L/3}=M_p-R_A\cdot\frac{L}{3}=M_p-\frac{5.4}{3}M_p=-0.8\,M_p\quad<M_p$$

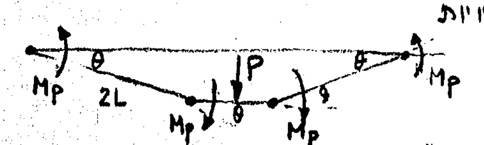


מפני של גודל רב של המומנט ב $x=L/3$ פחות מ-M_p,
לכן זה המנגנון שנותן את הכוח המסגרתי.

11.29

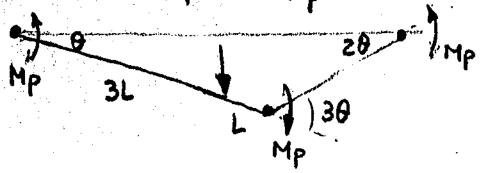
אפשרויות





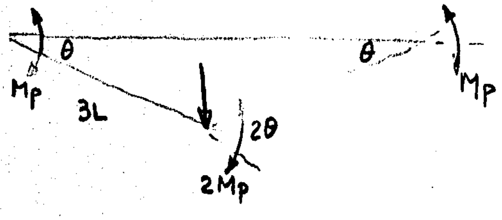
$$P\cdot2L\cdot\theta=M_p\cdot\theta+M_p\theta+M_p\theta+M_p\theta$$

$$\underline{P=2M_p/L}$$



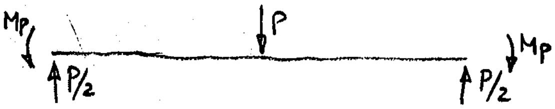
$$P\cdot3L\theta=M_p\cdot\theta+M_p\cdot3\theta+M_p\cdot2\theta$$

$$\underline{P=2M_p/L}$$



$$P\cdot3L\theta=M_p\theta+2M_p\cdot2\theta+M_p\cdot\theta$$

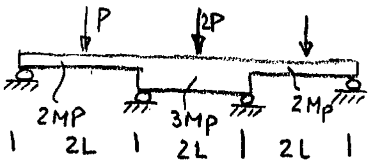
$$\underline{P=2M_p/L}$$



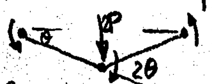
מפני הסימטריות

ויכולים להראות ש-|M| בחלקים הדקים פחות מ-M_p ובחלק העבה |M| פחות מ-$2M_p$

11.30



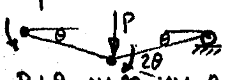
אם הפרק מופיע במרכז



$$2P\cdot L\theta=2M_p\theta+3M_p\cdot2\theta+2M_p\theta$$

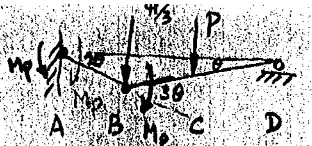
$$P=\frac{\underline{5M_p}}{L}$$

אם הפרקים מופיעים בחלק הימני



$$P\cdot L\theta=2M_p2\theta+2M_p\theta$$

$$P=6M_p/L$$



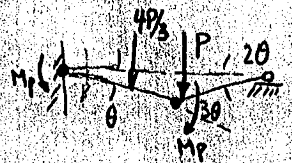
L/3 L/3 L/3

אם הפרק הפלסטי ב- B ו C

$$\frac{4P}{3}\cdot\left(2\theta\cdot\frac{L}{3}\right)+P\cdot\frac{L}{3}\cdot\theta=M_p\cdot2\theta+M_p\cdot3\theta$$

$$P=\frac{45}{11}\frac{M_p}{L}=4.09\,M_p/L$$

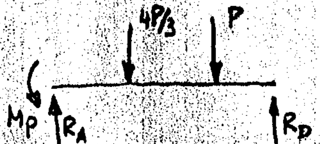
אם הפרק הפלסטי ב- A ו- C



$$\frac{4P}{3}\cdot\left(\theta\cdot\frac{L}{3}\right)+P\cdot2\theta\cdot\frac{L}{3}=M_p\cdot\theta+M_p\cdot3\theta$$

$$\underline{\underline{P=3.60\,\frac{M_p}{L}}}$$

לכן $P_{\text{מסגרתי}}=3.60\frac{M_p}{L}$

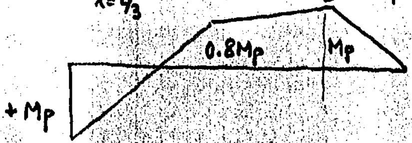


לבדוק המומנט ב $x=L/3$

$$\overset{+}{\curvearrowright}\ \Sigma M_D=0\qquad -M_p-\frac{4P}{3}\cdot\frac{2L}{3}-P\cdot\frac{L}{3}+R_A\cdot L=0$$

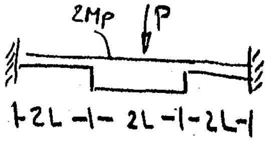
$$R_A=\frac{M_p}{L}+\frac{11P}{9}=\frac{M_p}{L}+\frac{11}{9}\left(3.60\frac{M_p}{L}\right)=5.4\frac{M_p}{L}$$

$$M\Big|_{x=L/3}=M_p-R_A\cdot\frac{L}{3}=M_p-\frac{5.4}{3}M_p=-0.8\,M_p\quad<M_p$$

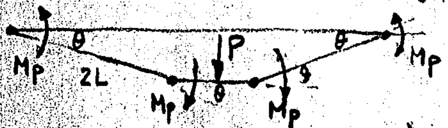


מפני של גודל רב של המומנט ב $x=L/3$ פחות מ- M_p,
לכן זה המנגנון שנותן את הכוח המסגרתי.

11.29

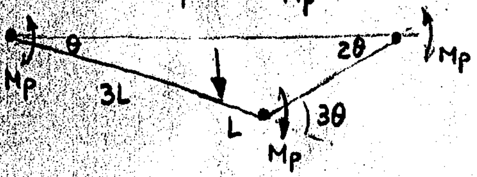


אפשרויות



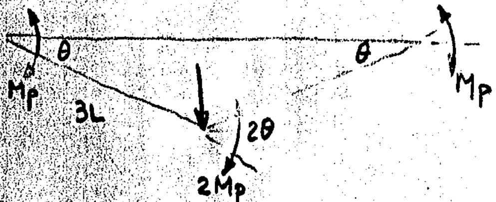
$$P\cdot2L\theta=M_p\cdot\theta+M_p\theta+M_p\theta+M_p\theta$$

$$\underline{P=2M_p/L}$$



$$P\cdot3L\theta=M_p\cdot\theta+M_p\cdot3\theta+M_p\cdot2\theta$$

$$\underline{P=2M_p/L}$$



$$P\cdot3L\theta=M_p\theta+2M_p\cdot2\theta+M_p\cdot\theta$$

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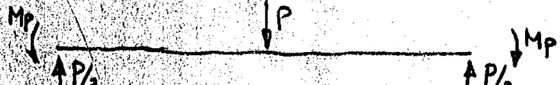
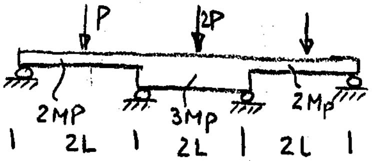


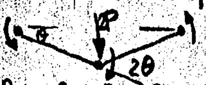
מפני הסימטריות

ויכולים להראות ש|M| בחלקים הדקים פחות מ- M_p ובחלק העבה |M| פחות מ- $2M_p$

11.30



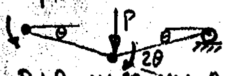
אם הפרק מופיע במרכז



$$2P\cdot L\theta=2M_p\theta+3M_p\cdot2\theta+2M_p\theta$$

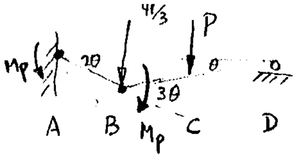
$$P=\frac{5M_p}{L}$$

אם הפרקים מופעים בחלק הימני



$$P\cdot L\theta=2M_p2\theta+2M_p\theta$$

$$P=6M_p/L$$



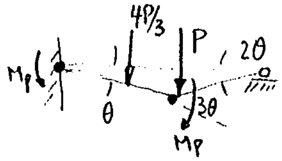
$L/3 \quad L/3 \quad L/3$

אם הפרק הפלסטי ב- A ו D

$$4P/3 \cdot (2\theta \cdot L/3) + P \cdot L/3 \cdot \theta = M_p \cdot 2\theta + M_p \cdot 3\theta$$

$$P = \frac{45}{11}\frac{M_p}{L} = 4.09\, M_p/L$$

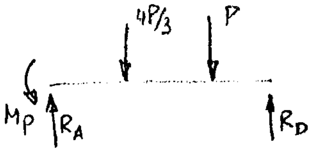
אם הפרק הפלסטי ב- A ו- C



$$\frac{4P}{3}\cdot\left(\theta \cdot L/3\right) + P \cdot 2\theta \cdot L/3 = M_p \cdot \theta + M_p \cdot 3\theta$$

$$P = 3.60\,\frac{M_p}{L}$$

לכן $P_{\text{מסגרתי}} = 3.60\frac{M_p}{L}$

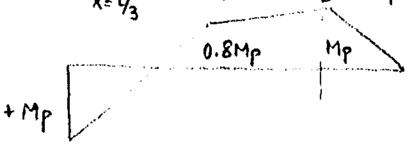


לבדוק המומנט ב $x=L/3$

$$+\circlearrowleft \Sigma M_D = 0 \qquad -M_p - \frac{4P}{3}\cdot\frac{2L}{3} - P\cdot\frac{L}{3} + R_A \cdot L = 0$$

$$R_A = \frac{M_p}{L} + \frac{11P}{9} = \frac{M_p}{L} + \frac{11}{9}\left(3.60\frac{M_p}{L}\right) = \frac{5.4M_p}{L}$$

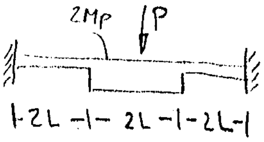
$$M\Big|_{x=L/3} = M_p - R_A \cdot L/3 = M_p - \frac{5.4}{3}M_p = -0.8\,M_p \quad < M_p$$



מפני של גודל [illegible] של המומנט ה $x=L/3$ פחות מ- M_p,
לכן זה המנגנון שניתן את הכוח המסגרתי.

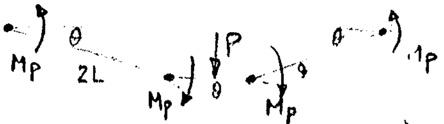
11.29

אפשרויות



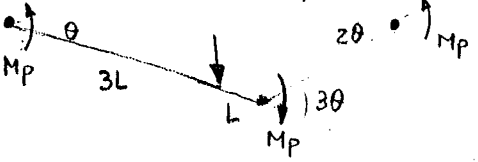
$$P \cdot 2L \cdot \theta = M_p\cdot\theta + M_p\theta + M_p\theta + M_p\theta$$

$$P = 2M_p/L$$



$$P \cdot 3L\theta = M_p\cdot\theta + M_p\cdot 3\theta + M_p \cdot 2\theta$$

$$P = 2M_p/L$$



$$P \cdot 3L\theta = M_p\theta + 2M_p \cdot 2\theta + M_p \cdot \theta$$

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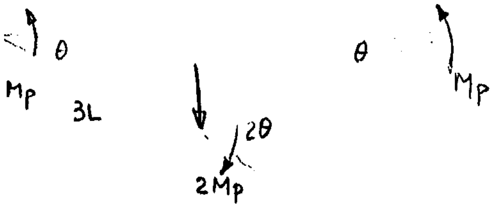
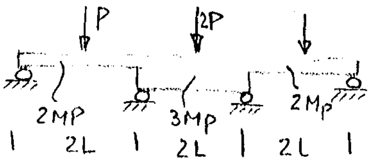


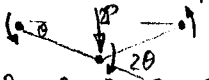
ויכולים להראות ש- $|M|$ בחלקים הדקים פחות מ- M_p ובחלק העבה $|M|$ פחות מ- $2M_p$

מפני הסימטריות

11.30



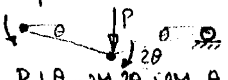
אם הפרק מופיע במרכז



$$2P \cdot L\theta = 2M_p\theta + 3M_p \cdot 2\theta + 2M_p\theta$$

$$P = \frac{5M_p}{L}$$

אם הפרקים מופעים בחלק הימני



$$P \cdot L\theta = 2M_p 2\theta + 2M_p\theta$$

$$P = 6M_p/L$$

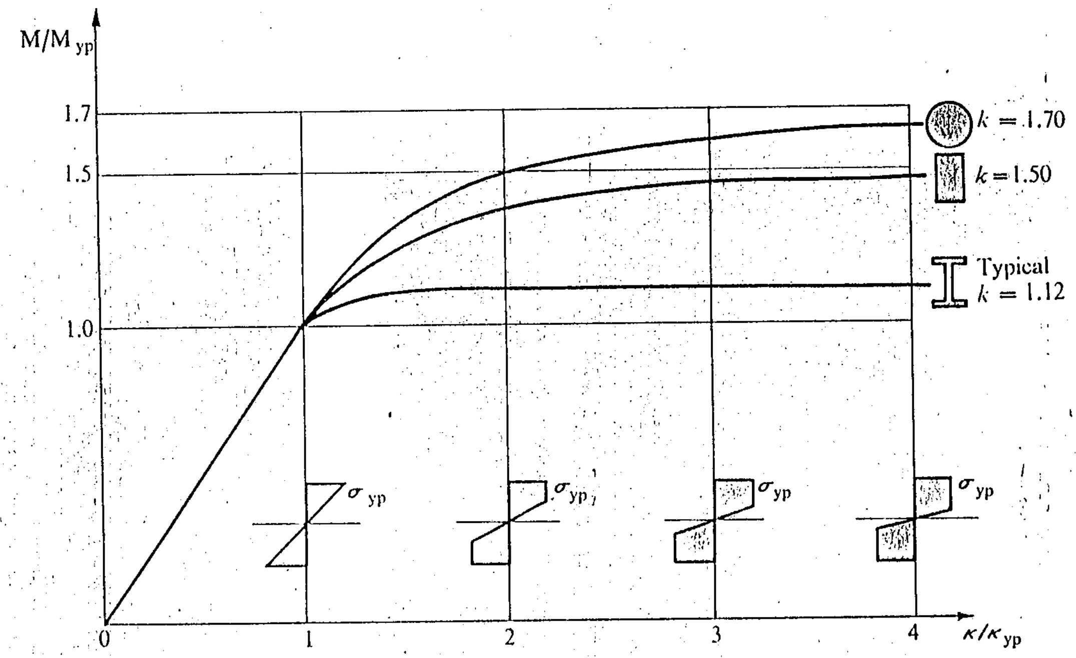


Fig. 12-23. Moment-curvature relations for circular, rectangular, and I cross sections. $M_p/M_{yp} = k$, the shape factor.

Fig. 11-18.) Note especially the rapid ascent of the curves toward their respective asymptotes as the cross sections plastify. This means that very soon after exhausting the elastic capacity of a beam, a rather constant moment is both achieved and maintained. This condition is likened to a plastic hinge. In contrast to a frictionless hinge capable of permitting large rotations at no moment, the plastic hinge allows large rotations to occur at a constant moment. This constant moment is approximately M_p, the ultimate or plastic moment for a cross section.

הסבר על פרק פלסט
וההבדל בינו ובין
פרק בלתי חיכוכ.

Using plastic hinges, a sufficient number may be inserted into a structure at the points of maximum moments to create a kinematically admissible collapse mechanism. Such a mechanism, permitting unbounded movement of a system, enables one to determine the ultimate or limit carrying capacity of a beam or of a frame. This approach will now be illustrated by several examples, confining the discussion to beams.

When the limit analysis approach is used for the selection of members, the working loads are multiplied by a load factor larger than unity to obtain the limit loads for which the calculations are performed. This is analogous to the use of the factor of safety in elastic analyses. In structural steel work the term *plastic design* is commonly applied to this approach.

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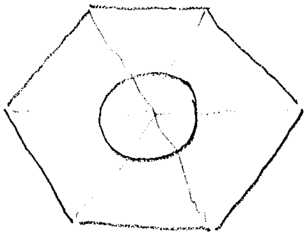
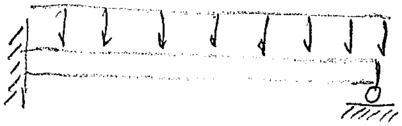


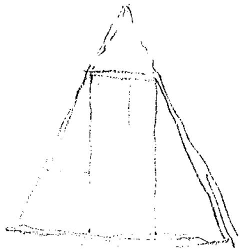


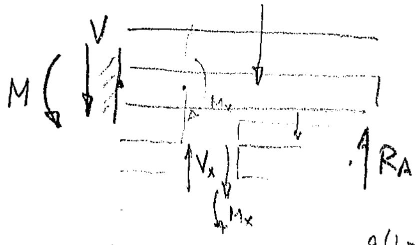
בלת מסויים

למצוא את התגובות
בסמכים.

למצוא את המומנט
המקסימלי

למצוא את המאמץ
המקסימלי





$q(L-x)^2/2$

$$V + qx = V_A$$

~~$M + qx\cdot\frac{x}{2} + M_x$~~

$$M = q\frac{x^2}{2} + M_A$$

~~$U_c = \int \frac{\left(q\frac{x^2}{2} - R_A\cdot L\right)^2}{2EI} dx$~~

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$$\frac{\partial U_c}{\partial R_A} = 0 = \frac{1}{EI}\int_0^L \left(q\frac{x^2}{2} - R_A\cdot L\right)(-L)\,dx$$

$$= -\frac{L}{EI}\left(q\frac{x^3}{6} - R_A L x\right)\Big|_0^L$$

$$0 = -\frac{L}{EI}\left(q\frac{L^3}{6} - R_A L^2\right)$$

$$R_A = \frac{qL}{6}$$

$$M_x \curvearrowleft \cdot \curvearrowright q\frac{(L-x)^2}{2} \curvearrowright R_A(L-x)$$

$$M_x = -q\frac{(L-x)^2}{2} + R_A(L-x)$$

$$U_c = \frac{1}{2EI}\int\left[-q\frac{(L-x)^2}{2} + R_A(L-x)\right]^2 dx$$

$$\frac{\partial U_c}{\partial R_A} = \frac{1}{EI}\int\left[-q\frac{(L-x)^2}{2} + R_A(L-x)\right](L-x)$$

$$= \frac{1}{EI}\quad q\frac{(L-x)^4}{8} + R_A\frac{(L-x)^3}{3}\Big|_0^L$$

$$= \frac{1}{EI}\left(\frac{qL^4}{8} - \frac{R_A L^3}{3}\right)$$

$$\underline{R_A = \frac{3qL}{8}}$$

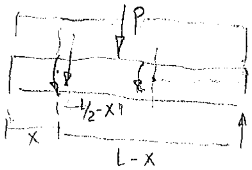
$$M_B = -qL^2/2 + R_A L = -\frac{qL^2}{2} + \frac{3qL^2}{8}$$

$$= -\frac{qL^2}{8}$$

$$\frac{\partial M}{\partial x} = 0 = +\frac{\cancel{2}q}{\cancel{2}}(L-x) + R_A = 0$$

$$x = -\frac{R_A}{q} + L$$

$$= -\frac{3}{8}L + L = \frac{5}{8}L$$



$$+R_A(L-x) + P(L/2 - x) + q\frac{(L-x)^2}{2}$$

$$\int_0^{L/2}\left[+R_A(L-x) + P(L/2-x) + q\frac{(L-x)^2}{2}\right]^2 + \int_{L/2}^{L}\left[+R_A(L-x) + q\frac{(L-x)^2}{2}\right]^2$$

U_c

$$\frac{\partial U_c}{\partial R_A} = \frac{1}{EI}\int_0^{L/2}\left\{+R_A(L-x) + P(L/2-x) + q\frac{(L-x)^2}{2}\right\}(L-x) + \frac{1}{EI}\int_{L/2}^{L}\left\{+R_A(L-x) + q\frac{(L-x)^2}{2}\right\}(+(L-x))$$

$$M\Big|_{x=\frac{5L}{8}} = -\frac{q}{2}\left(\frac{9}{64}L^2\right) + \frac{9}{64}qL$$

$$= \frac{9}{128}qL^2$$

~~$\int_0^L R_A(L-x) + q\left(\frac{L-x}{2}\right)^2$~~

$$\int_0^L\left\{+R_A(L-x) + q\frac{(L-x)^2}{2}\right\}(L-x) + \int_0^{L/2} +P(L/2-x)(L-x)\,dx$$

$$P\left(\frac{L^2}{2}x - \frac{3}{4}Lx^2 + \frac{x^3}{3}\right)\Big|_0^{L/2}$$

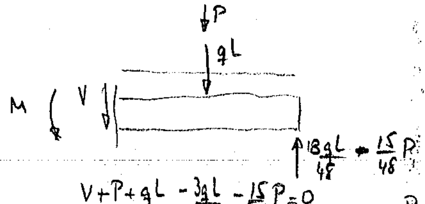
$$-R_A\frac{(L-x)^3}{3} + q\frac{(L-x)^4}{8}\Big|_0^L$$

$$+R_A L^3/3 + qL^4/8 + P\left(12L^3/48 - 9L^3/48 + 2L^3/48\right) = 0$$

$$\frac{3}{8\cdot 2}\qquad \frac{5}{48}PL^3$$

$$R_A = 3qL^4/8 - 15/48\,P$$

$$\underline{= 3qL/8 - 15/48\,P}$$



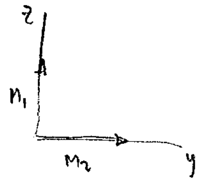
$$V + P + qL - \frac{3qL}{8} - \frac{15}{48}P = 0$$

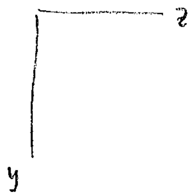
$$P = \frac{6}{5}qL$$

$$\underline{V = -\frac{5qL}{8} - \frac{33}{48}P}$$

$$M = 24PL/ + 4qL^2/ - (3qL - 15P)L$$

$$\frac{-M_z I_y y - M_z I_{yz} z}{I_y I_z - I_{yz}^2} = \frac{-PL\,(9.813 \cdot -80) - PL\,(11.2 \times -90)}{22.613 \cdot 9.813 - 11.2^2}$$

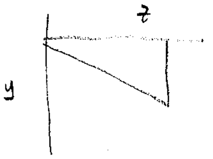




C Y to PAV

$$\sigma_x = \frac{(-M_z I_{yz} - M_1 I_z)\, z + (M_z I_y + M_1 I_{yz})(-y)}{}$$

$$= \frac{-(M_z I_{yz} - M_y I_z)\, z + (M_z I_y + M_y I_{zy})(-y)}{}$$



$$(-M_z I_{yz} + M_y I_z) z + (-M_y I_{yz} + M_z I_y) y$$

$$\sigma_x = \frac{-(M_z I_y + M_y I_{yz})\, y + (M_z I_{yz} + M_y I_z)\, z}{I_y I_z - I_{yz}^2}$$

$$\frac{y}{z} = \frac{M_z I_{yz} + M_y I_z}{M_z I_y + M_y I_{yz}}$$

$$\tan\alpha = \frac{y}{z} = \frac{I_z \tan\theta + I_{yz}}{I_y + \tan\theta\, I_{yz}}$$

$$\frac{-M_z I_y \cdot y - M_z I_{yz} \cdot z}{I_y I_z - I_{yz}^2}$$

$$-M_z\,(I_y y + I_{yz} \cdot z)$$

$$+PL\,\frac{(56\,(350) + 30\,(300))}{}$$

39.50

15 9000

$$\frac{-M_t I_{ss} \cdot s + M_t I_{st} \cdot t}{I_{ss} I_{tt} - I_{st}}$$

$$\frac{+PL\left(9.813 \cdot (-80) + -11.2 \times (-90)\right) \times 10^6}{9.813 \times 22.613 - (11.2)^2 \times 10^{12}} =$$

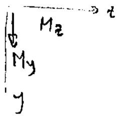
$$I'_{zz} = \frac{I_{zz} + I_{yy}}{2} + \frac{I_{zz} - I_{yy}}{2} \cos 2\varphi + I_{yz} \sin 2\varphi$$

$$\frac{29+56}{2} + \frac{29-56}{2} \cos 65.77 + 30 \sin(-65.77)$$

$$42.5 + 13.5\,(.4104) + 30\,(.9119) = 9.603$$

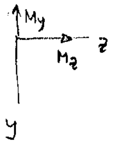
$$\frac{45.104 + 7.604}{2} + \frac{(7.604 + 45.104)}{2} \cos 21.8 + 7.5 \times \sin 21.8$$

$$26.354 + (+18.75)(.9285) + 7.5\,(.3714)$$



$$\sigma_x = \frac{\left(-M_z(-I_{yz}) + M_y I_z\right) z + \left(M_z I_y + M_y I_{yz}\right)(-y)}{}$$

$$\sigma_x = \frac{\left(M_z I_{yz} + M_y I_z\right) z - y\left(M_z I_y + M_y I_{yz}\right)}{I_y I_z - I_{yz}^2} \checkmark$$



$$\sigma = \frac{-\left(M_z I_y + M_y I_{yz}\right) y + \left(M_z I_{yz} + M_y I_z\right) z}{I_y I_z - I_{yz}^2}$$

$$= \frac{-M_z I_y y + M_z I_{yz} z}{I_y I_z - I_{yz}^2}$$

$$= \frac{PL\left[-I_y(-350) + I_{yz}(-300)\right]}{} = PL\left[\frac{-56(-350) + 30(-300)}{56 \cdot 29 - 30^2}\right.$$

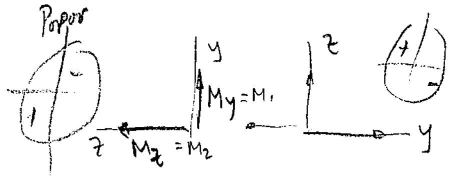
14.64

$$= \cancel{34302} \times 10^{-8} PL$$

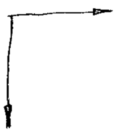
$$\frac{-M_y I_{yz} y + M_y I_z \cdot z}{I_y I_z - I_{yz}^2}$$

$$\frac{-PL(-30) \cdot (-300) + PL\, 56(350)}{56 \times 29 - 30^2}$$

14.64



$$I_{yz}\cos\varphi + I_y \sin\varphi = M_y$$
$$I_z \cos\varphi + I_{yz}\sin\varphi = -M_z$$



covert popov to CAY

$$\sigma_x = \underset{-}{} \frac{\mp M_y I_z \pm M_z I_{yz}}{I_y I_z - I_{yz}^2} z + \frac{M_z I_y \pm M_y I_{yz}}{I_y I_z - I_{yz}^2}(-y)$$

$$\tan\beta = \frac{M_y}{-M_z}$$

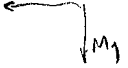
$$\sigma_x = \underset{-}{} \frac{M_y I_z + M_z I_{yz}}{(\quad)} z - \frac{M_z I_y - M_y I_{yz}}{I_y I_z - I_{yz}^2} y$$

$$-\left[\qquad \bigcirc \underline{\qquad} \right]$$

?

Conv. popov to PAV.

$$\sigma_x = \pm \frac{M_z I_y \mp M_y I_{yz}}{I_y I_z - I_{yz}}(-y) \pm \frac{-M_y I_z - M_z I_{yz}}{(\quad)}(-z)$$



$$\sigma_x = + \frac{M_z I_y + M_y I_{yz}}{(\quad)} y + \frac{+M_y I_z + M_z I_{yz}}{(\quad)} z$$

1. Write the second-order differential equation for the bending of the column shown in Fig. P1-2 and use it to determine the critical load of the column. At its lower end the column is completely fixed. At the upper end the column is prevented from rotating, but free to translate laterally. (ans: $P_{cr}=\pi^2 EI/L^2$)

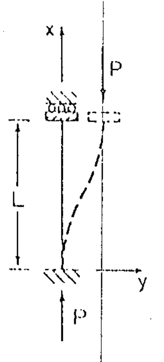
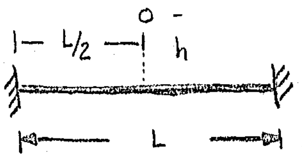


Fig. P1-2

2. Find an expression for the maximum stress when a ball weighing W Newtons is dropped onto a fixed-fixed beam.



3. A linearly elastic beam-column having a flexural rigidity EI, is subjected to a thrust P and a moment M_0 as shown in Fig. A below.

(a) Determine the lateral displacement v(x).

(b) From part (a), write the solution for the system subjected to a force P acting as shown in Fig. B.

(c) Determine Δ_c, the horizontal displacement of point C, assuming small rotations. (Assume also that the horizontal displacement of point B is negligible).

(d) Determine the bending moment M(x).

(g) <u>Explain</u> why, although the beam is made of a linearly elastic material, the results of part (c) are non-linear.

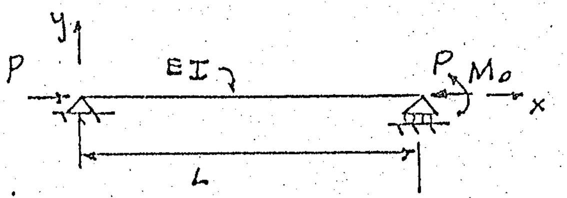


Fig. A

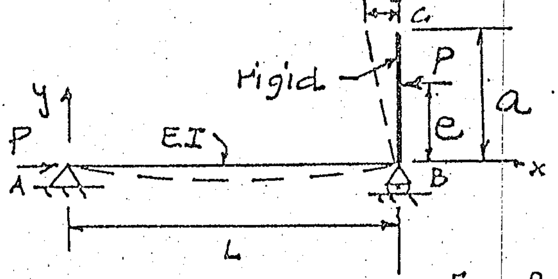


Fig. B

Answers :

(a) $y(x) = -\dfrac{M_0}{P}\left[\dfrac{\sin Kx}{\sin KL} - \dfrac{x}{L}\right],\ K^2 = \dfrac{P}{EI}$

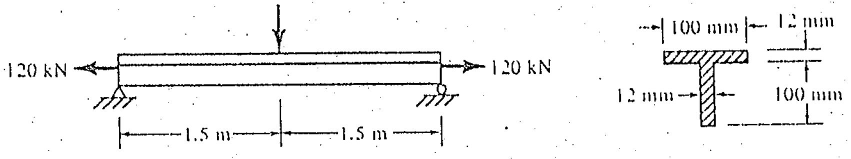
(c) $\Delta_c = \dfrac{ac}{L}\left[1 - L\sqrt{P/EI}\cot\left(L\sqrt{P/EI}\right)\right] = \dfrac{ac}{L}\left(1 - KL\cot KL\right)$

(d) $M(x)=M_0 \sin kl/\sin kL$

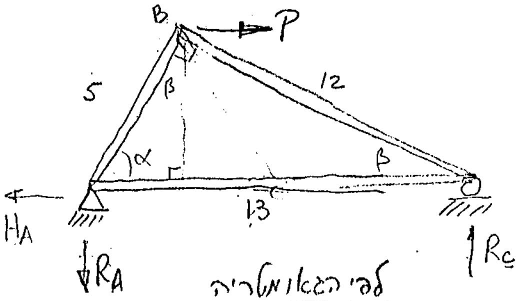
*12.10 A T section carries an axial tensile force of 120 kN, applied through the centroid of the cross section. The allowable stress in tension or compression is 130 MPa. Let E = 200 GPa. What transverse force P can be applied at midspan if the beam is

(a) Stem down (as shown)?

(b) Stem up?



PROBLEM 12.10



לפי הגאומטריה

$\cos\alpha = 5/13$

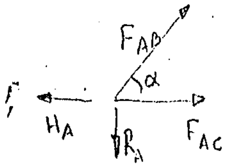
$\sin\alpha = 12/13 = \frac{h}{5}$ $\qquad h = \frac{60}{13} = 4\,8/13$

$\Sigma F_x = 0 \Rightarrow \underline{H_A = P}$

$\Sigma M_A = 0 \Rightarrow P \cdot h = R_C \cdot 13$

$P \cdot \frac{60}{13} = R_C \cdot 13 \qquad \underline{R_C = P\,\frac{60}{169}} \qquad \underline{R_A = +P \cdot \frac{60}{169} = .3550P} \Longleftarrow \Sigma F_y = 0$

שיטת הצמתים

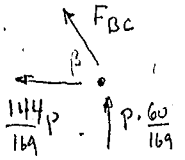


$\Sigma F_y = 0 \Rightarrow F_{AB} \sin\alpha + R_A = 0 \qquad 12/13\, F_{AB} + P \cdot \frac{60}{169} = 0$

$\Sigma F_x = 0 \Rightarrow F_{AB} \cos\alpha + F_{AC} - H_A = 0 \qquad \underline{F_{AB} = +\frac{5}{13} P}$

$+\frac{5}{13} P \cdot 5/13 + F_{AC} - P = 0$

$\underline{F_{AC} = \frac{144}{169} P} = 0.85207\,P$



$\Sigma F_y = 0 \qquad F_{BC} \sin\beta + \frac{60P}{169} = 0$

$F_{BC} \frac{h}{12} + \frac{60P}{169} = 0$

$F_{BC} \frac{\overset{5}{\cancel{60}}}{13 \cdot \cancel{12}} + \frac{60P}{169} = 0 \qquad \underline{F_{BC} = -\frac{12}{13} P}$

בדיקת השיטה

$\Sigma F_x = 0 \Rightarrow F_{BC} \cos\beta + \frac{144P}{169} = 0$

$-\frac{12}{13} P \cdot \frac{12}{13} + \frac{144P}{169} = 0$ ✓

מפני ש- E,A שווים בכל מוט:

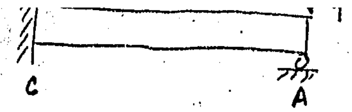
$$U_c = \frac{1}{2} \sum \frac{F_i^2 L_i}{E_i A_i} = \frac{1}{2EA}\left[F_{AC}^2 L_{AC} + F_{BC}^2 L_{BC} + F_{AB}^2 L_{AB} \right]$$

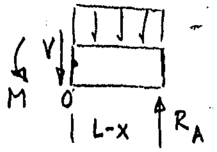
$$= \frac{1}{2EA}\left[\left(\frac{144}{169}P\right)^2 13 + \frac{144}{169}P^2 \cdot 12 + \frac{25}{169}P^2 \cdot 5 \right]$$

$$u_B = \frac{\partial U_c}{\partial P} = \frac{1}{EA}\left[\left(\frac{144}{169}\right)^2 P \cdot 13 + \frac{144P}{169} \cdot 12 + \frac{125}{169} P \right]$$

$$= \frac{P}{EA}\left\{ 20.4028 \right\}$$

למצוא את התגובות
למצוא את המומנט המקסימלי
למצוא את המאמץ המקסימלי
מהוא q שגורם ש- $\sigma_{מקס} = \sigma_{yp}$
מהוא M_{pl} ו- q שגורם ל- M_{pl}





$$\Sigma M_0 = M + R_A(L-x) - q\frac{(L-x)^2}{2}$$

$$M = -R_A(L-x) + q\frac{(L-x)^2}{2} \qquad 0 \le x \le L$$

$$U_c = \frac{1}{2EI}\int_0^L M^2 dx = \frac{1}{2EI}\int_0^L \left\{-R_A(L-x) + q\frac{(L-x)^2}{2}\right\}^2 dx$$

$$\frac{\partial U_c}{\partial R_A} = 0 = \frac{1}{EI}\int_0^L \left\{-R_A(L-x) + q\frac{(L-x)^2}{2}\right\}\left(-(L-x)\right)dx$$

$$= \frac{1}{EI}\left\{R_A\frac{(L-x)^3}{3} + q\frac{(L-x)^4}{8}\right\}\Bigg|_0^L$$

$$= \frac{1}{EI}\left\{\frac{R_A L^3}{3} - q\left(\frac{L^4}{8}\right)\right\} = 0 \qquad \underline{\underline{R_A = \frac{3qL}{8}}}$$

לכן, $R_c = qL - R_A = \underline{\underline{\frac{5qL}{8}}}$

כאשר x=0, $M = -R_A L + \frac{qL^2}{2}$

$$\underline{\underline{M_c = \frac{qL^2}{8}}}$$

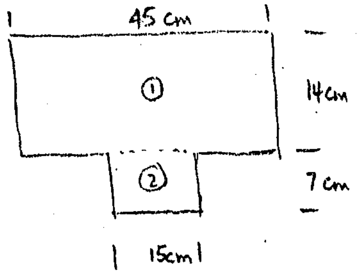
המומנט המקס. יכול לקרות כש- $\frac{dM}{dx} = 0$

$\frac{dM}{dx} = R_A - \frac{2q}{2}(L-x)$, $0 =$ ו-ה $\underline{x = L - \frac{R_A}{q} = \frac{5}{8}L}$

לכן, $M\Big|_{x=\frac{5L}{8}} = -\frac{3qL}{8}\left(\frac{3L}{8}\right) + \frac{9qL^2}{128} = \frac{-9qL^2}{128} = -.0703qL^2$

מפני זה המומנט המקס. קורה ב- x=0 והוא $\underline{\underline{qL^2/8}}$

קורה מחומר יחיד: צריך להחליף את שטח החתך המורכב לשטח חומר יחידי. מחליפים את השטח לאלומיניום



לכן $E_{פלדה} b_{פלדה} = E_{אל} b_{אל}$

$b_{אל} = \frac{E_{פלדה} b_{פלדה}}{E_{אל}}$

$b_{אל} = 45_{cm}$

עכשיו למצוא מומנט האינרציה ומרכז הכובד

	A	$\bar{y}$	$A\bar{y}$
①	(45)14	7	4410
②	7·15	17.5	1837.5
	735		6247.5

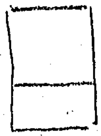
$$\bar{y} = \frac{6247.5}{735} = 8.5\text{ cm}$$

I_{zz}	A	d^2	I_{tt}
$45\cdot14^3/12$	630	$(1.5)^2$	11707.5
$15\cdot7^3/12$	105	$(-9)^2$	8933.75
			20641.25 cm^4

בפלדה $\sigma_{max} = \frac{Mc}{I} = \frac{E_{פלדה}}{E_{אל}}\cdot\left(qL^2/8\right)\cdot\frac{(0.085)m}{20641.25\times10^{-8}m^4}$

באלומיניום $\sigma_{max} = \frac{Mc}{I} = -qL^2/8\,\frac{(0.125m)}{20641.25\times10^{-8}m^4}$

לכן המקסימום נמצא בפלדה כאן



אם σ_{yp} מגיע בפלדה ← העומס המפורס שגורם ל- σ_{yp}

$$q = \frac{{}_{פ}\sigma_{yp}\cdot I}{L^2/8\cdot3(0{,}085)} = \frac{1618921.6}{L^2}$$

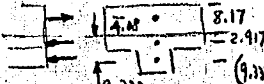
ב-q הזה, המאמץ באלומיניום הוא

$$\frac{-qL^2/8(0.125)}{I} = -\left(\frac{{}_{פ}\sigma_{yp}\cdot I}{L^2/8\cdot3(0.085)}\right)\cdot L^2/8\,\frac{(0.125m)}{I} = -{}_{פ}\sigma_{yp}(.4902)$$

$$= -122.55\text{ MPa}$$

אם σ_{yp} הגיע באלומיניום קודם, המאמץ המקס. בפלדה יהיה יותר גבוה מ ${}_{פ}\sigma_{yp}$. אז האפשרות הזאת לא קורית!

לפי שיטת הפלסטית, ציר הניטרלי מחלק השטח לשניים. לכן, אם השטח הוא מיוצר מאלומיניום,
הציר הוא 8/16 ס"מ תחת המישור העליון



$$M_{pl} = \sigma_{yp\,אל}\,(9.333\times15\times7 + 2.917\times45\times5.833 + 4.083\times45\times8.17)$$

$M_{pl} = 486937.5$ N-m . פה המומנט הוא מסביב הציר הניטרלי.

$$v = A\cos\lambda x + B\sin\lambda x + Cx + D$$

לפי תנאי השפות של הקורה.

כדי לקצר את הדרך לפתור את הבעיה נזכיר ש-

$$EI\frac{d^2v}{dx^2} + Pv = M_t$$

כאשר P=0

$$EI\frac{d^2v_t}{dx^2} = M_t$$

פה M_t הוא המומנט שבא מעומסים אנכיים בלבד, ו- v_t היא התזוזה בכיוון y שבאה מהעומסים האנכיים. מניחים שצורת עיקום התזוזה יהיה אותו דבר עם או בלי הכוח האופקי P.

לכן, לפי תנאי השפה, כמו קורה על סמך נייד וסמך נייד, מניחים ש- $v = \bar{v}\sin\frac{\pi x}{L}$ ו- $v_t = \bar{v}_t\sin\frac{\pi x}{L}$. $\bar{v}$, ו- $\bar{v}_t$ הם ה- AMPLITUDE של v ו- v_t. לפי הנוסחאות למעלה

$$-EI\frac{\pi^2}{l^2}\bar{v} + P\bar{v} = -EI\frac{\pi^2}{l^2}\bar{v}_t$$

מפני ש- $P_E = \frac{\pi^2 EI}{l^2}$, $\bar{v}(P-P_E) = -P_E\bar{v}_t$ או

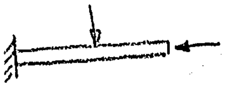
$$\bar{v} = \frac{\bar{v}_t}{1 - P/P_E}$$

המומנט גם כן עוקב אחרי נוסחא דומה לזה

$$M = \frac{M_t}{1 - P/P_E}$$

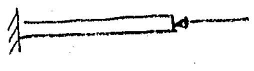
אם P חיובי, $\bar{v}_t < \bar{v}$ ו- $M_t < M$. אם P שלילי, $\bar{v}_t > \bar{v}$ ו- $M_t > M$

TACHLIS



פותר

וקבל את v מתחת העומס או בכל מקום אחר. קורא לזה $\bar{v}_t$.

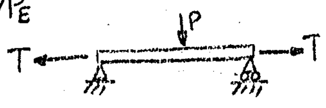


קבל P_E לקורה עמוסה כמו בצורה

v יהיה

$$v = \frac{\bar{v}_t}{1 - P/P_E}$$

בתרגיל בית 12.10



כי P היה שלילי.

$$v = \frac{\bar{v}_t}{1 + T/T_E} \quad \text{ו-} \quad , \bar{v}_t\Big|_{x=l/2} = \frac{PL^3}{48EI} \quad , T_E = \frac{\pi^2 EI}{L^2}$$

כמו שמצאנו בעבר שסך כל האנרגיה הוא האנרגיה שבאה מהמאמצים הפנימיים והאנרגיה שבאה מהכוחות החיצוניים. הפוטנציאל הטוטלי,

$$\Pi = U - W_e = U + \Omega$$

U - האנרגיה העיבורית ו- W_e העבודה מהכוחות החיצוניים. לשיווי משקל

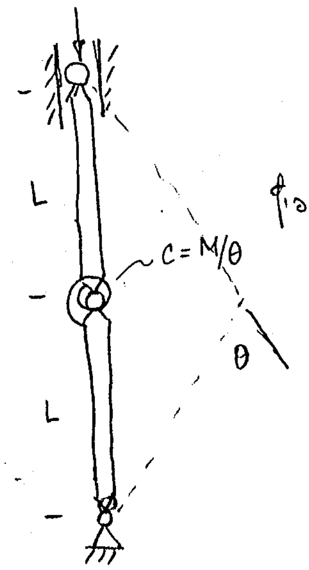
$$\delta\Pi = \delta U - \delta W_e = 0$$

למצב יציב (STABILITY), צריך לבדוק $\delta^2\Pi$, כי לפי TAYLOR SERIES

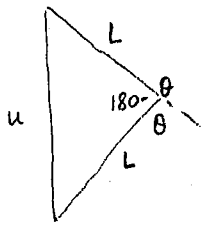
$$\Delta\Pi = \delta\Pi + \frac{1}{2!}\delta^2\Pi + \frac{1}{3!}\delta^3\Pi + \dots$$

שיווי משקל דורש $\delta\Pi = 0$ ומצב יציב נטרלי דורש ש- $\delta^2\Pi = 0$ וזה נותן את העומס הקריטי,

דוגמא: <u>למצוא העומס הקריטי</u> לדוגמא דרגת חופשי אחת.
שתי המוטות קשיחים עם תפסי השפה נ"ד.
קפיץ ליניארי סיבובי בקשיחות $C=M/\theta$ קושר
את שתי המוטות, θ זווית בין המוטות, ו-M מומנט הקפיץ



אם הכוח P זז למטה



$$u = \sqrt{2L^2 - 2L^2\cos(180-\theta)}$$
$$= 2L\cos\theta/2$$

מפני שהמוטות קשיחים $\Rightarrow \delta U = 0$

עבודת הכוח P והמומנט M $\Leftarrow P(2L - 2L\cos\theta/2) - \frac{1}{2}M\theta$; עבודת הקפיץ הוא - הפוטנציאל

ו- $\cos\theta/2 \sim 1 - (\theta/2)^2 1/2! + \dots$

לכן $2LP(1-\cos\theta/2) - \frac{1}{2}C\theta^2 = 2LP(\theta^2/8) - \frac{1}{2}C\theta^2 = \Pi$

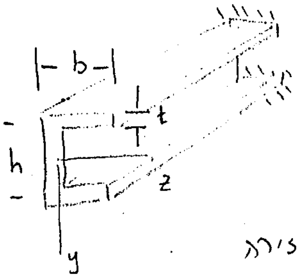
$$0 = \left(\frac{LP\theta}{2} - C\theta\right)\delta\theta = \delta\Pi$$

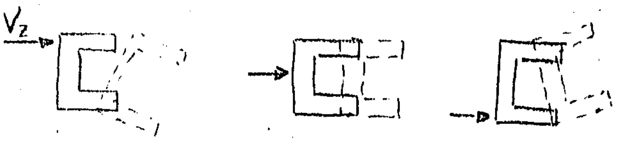
לכן, או $\theta = 0$ או $P = 2C/L$. צריך לבדוק $\delta^2\Pi$

$$0 = \left(\frac{LP}{2} - C\right)\delta\theta^2 + \theta\left(\frac{LP}{2} - C\right)\delta^2\theta = \delta^2\Pi$$

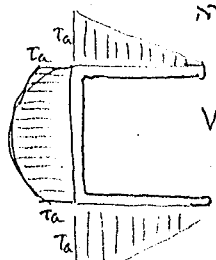
מפני ש- $\delta^2\theta \neq 0$ כאשר $\theta = 0$ $\Leftarrow$ $0 = \frac{LP}{2} - C$ וזה P קריטי $= \frac{2C}{L}$

מרכז גזירה - מקום שבו הגזירה צריך לעבור כדי שקורה ישרה יכווץ וכפוף בלי פיתול



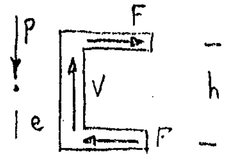


אם לשטח יש שני צירים סימטריים, צומת הצירים הוא המרכז הגזירה



אם יש רק ציר אחד כמו בפרופיל כאן מפני שמאמץ הגזירה הוא כמו בצורה

- מניחים שהקירות דקים, שהמאמץ הגזירה ברגלי האופקיות הוא לינארי מצב החופשי ובמסכת המאמץ פרבולי. הכוח בכול רגל הוא $\frac{\tau_a}{2}bt$ והגזירה האנכית $V=\int_{-h/2}^{h/2}\tau t\,dy$
- מניחים שיכולים למדוד הרגליים והמסכת בקו האמצעי; ומאמץ הגזירה טנגנט לקיר
- מאמץ הפסיפה מתמיד בכיוון העובי;

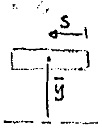


לכן, הפיתול שבא מהכוחות F הוא $F\cdot h$. כדי למנוע את הפיתול, צריך לשים כוח P, במרחק e מהמסכת. לפי שווי משקל $P=V$ ו- $Pe=Fh$

$$e=\frac{Fh}{V}=\frac{\tau_a bt/_2\cdot h}{V}$$

τ_a - מאמץ הגזירה האמצעי של הרגל וכפי שלמדנו בתורת החוזק 1

$$\tau_a=\frac{VQ}{It}=\frac{q}{t}$$



$Q=\int ydA$ של הרגל $=\int yt\,ds=\bar{y}A=\frac{h}{2}.bt$. קוראים לזה המומנט הסטטי של חלק השטח המסוים

- $\bar{y}$ - המרחק ממרכז הכובד של הרגל לציר הנטרלי של הגוף
- q - זרם הגזירה

$$e=\frac{V(h/_2\cdot bt)}{It}\cdot\frac{bt/_2\cdot h}{V}$$

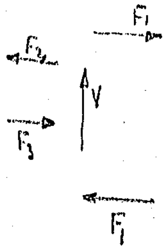
אם הרגל דקה מאוד $I\cong 2bt\cdot\left(\frac{h}{2}\right)^2+\frac{th^3}{12}$ סה. $e=\frac{b^2h^2t}{4I}$

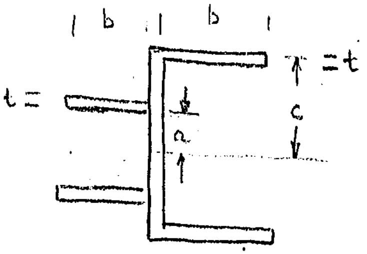
- המרחק e תלוי בשטח ולא בעומס ולא במקום שהעומס מיושם.

דוגמה

$$F_1\cdot 2c-F_3\cdot 2a=Pe$$

$$P=V$$





$$\frac{\tau_a}{2}.bt.2c-\frac{\tau_b}{2}.bt.2a=Ve$$

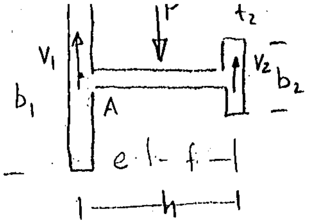
$$\frac{1}{2}\frac{VQ_a}{It}.bt\cdot 2c-\frac{1}{2}\frac{VQ_b}{It}.bt\cdot 2a=Ve$$

$$\frac{1}{2}V\frac{(bt\cdot c)}{It}.bt\cdot 2c-\frac{1}{2}V\frac{(bt\cdot a)}{It}.bt\cdot 2a=Ve$$

$$e=\frac{1}{2}\left\{\frac{2t^2(b^2c^2-b^2a^2)}{It}\right\}$$

$$I=\frac{t(2c)^3}{12}+2bt\cdot c^2+2bt.a^2$$

$2b^2(c^2-a^2)$



יודעים ש- $V_1+V_2=P$

כדי שאין פיתול, אם ניקח $\Sigma M_A=0$ אז $V_2 \cdot h = Pe$

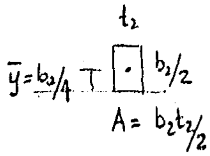
מהו V_2. מפני שהמאמץ הגזירה הוא פרבולי, $\frac{2}{3} b_2 q_{2_{max}} = V_2$

$$\frac{PQ}{I} = \frac{VQ}{I} = q_{2_{max}}$$

לכן,

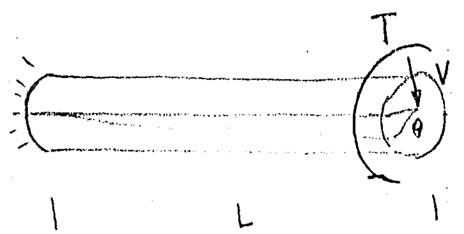
$$V_2 h = \frac{2}{3} b_2 q_{2_{max}} h = \frac{2}{3} b_2 \frac{PQ}{I} h = Pe$$

$$e = \frac{2}{3} b_2 \frac{Qh}{I} = \frac{2}{3} \frac{b_2 h}{I} \left(\frac{b_2 t_2}{2} \cdot \frac{b_2}{4} \right) = \frac{b_2^3 t_2 h}{12 I}$$



נזכיר ש- Q המומנט הסטטי של חלק השטח המסוים עד הציר הניטרלי,

אם הגזירה לא נמצאת במרכז הגזירה, פיתול קורה. אם רוצים זווית הפיתול:



בפיתול

$T = Ve$

זווית הפיתול היא

$$\theta = \frac{TL}{JG}$$

$$J = I_{zz} + I_{yy}$$

G - מודול הגזירה $\frac{E}{2(1+\nu)}$ בדרך כלל

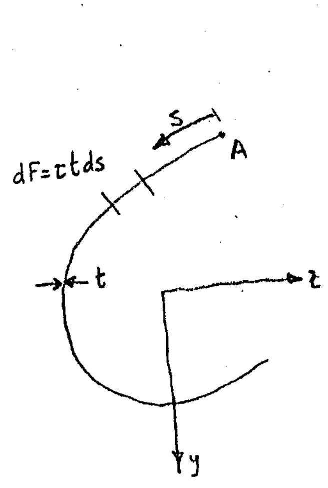
בדרך כלל זרם הגזירה q לקטע פתוח

$$q_{\text{פתוח}} = -\frac{(I_{yy} Q_z - I_{yz} Q_y) V_y + (I_{zz} Q_y - I_{yz} Q_z) V_z}{I_y I_z - I_{yz}^2}$$

$\int_0^s t y \, dS = Q_z$

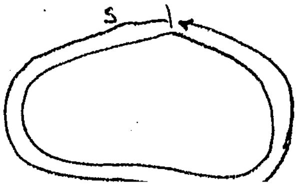
$\int_0^s t z \, ds = Q_y$

אם t משתנה ממקום למקום כפונקציה של s

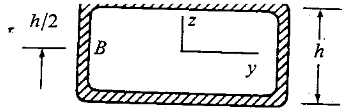


לקטע סגור $q = q_0 + q_{\text{פתוח}}$

ו- q_0



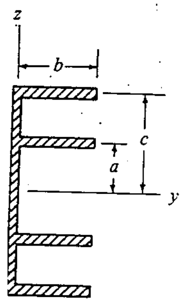
$$q_0 = -\frac{\oint (q_{\text{פתוח}}/t) \, dS}{\oint 1/t \, dS}$$



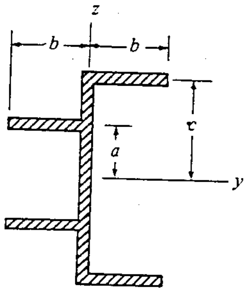
PROBLEM 9.31

9.7

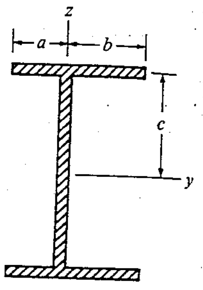
9.33, 9.34, 9.35, *9.36, 9.37–9.40, *9.41, *9.42, 9.43 The open cross sections shown are all thin-walled, of constant thickness, and have one axis of symmetry. Assume a frictionless contact where overlaps appear. In each case find an expression for the location of the shear center. Check your answer by examining limiting cases where possible (as by letting dimension b approach zero, for example).



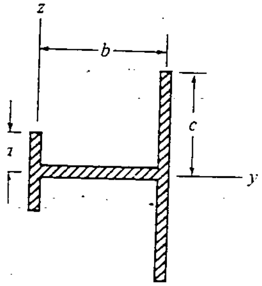
PROBLEM 9.32



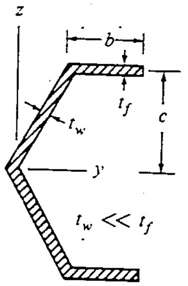
PROBLEM 9.33



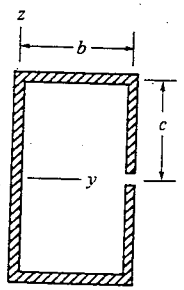
PROBLEM 9.34



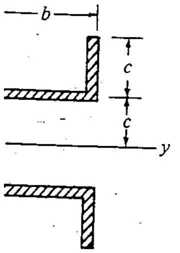
ROBLEM 9.35



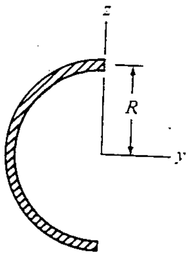
PROBLEM 9.36



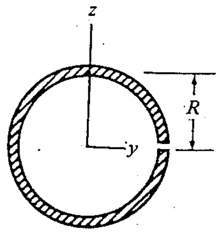
PROBLEM 9.37



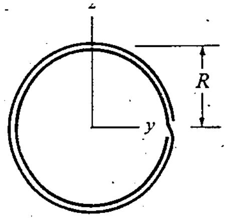
ROBLEM 9.38



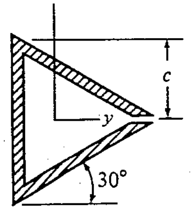
PROBLEM 9.39



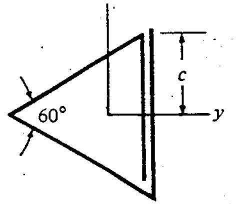
PROBLEM 9.40



PROBLEM 9.41



PROBLEM 9.42

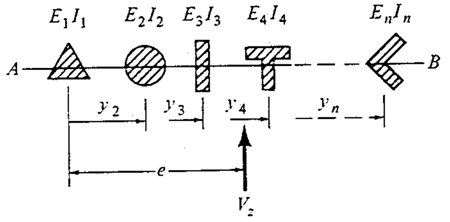


PROBLEM 9.43

9.44 (a) Several beams of arbitrary cross section have their centroids in a common plane AB. If somehow coupled together so that they share the same neutral axis, the composite beam will bend without twisting if e is as stated in the sketch. Derive this expression for e, in which y_i is the distance to the ith shear center.

(b) Use this formula to solve Problem 9.35.

(c) Use this formula to solve Problem 9.43.



$$e = \frac{E_2I_2y_2 + E_3I_3y_3 + \cdots + E_nI_ny_n}{E_1I_1 + E_2I_2 + E_3I_3 + \cdots + E_nI_n}$$

PROBLEM 9.44

Section 9.8

9.45 For an arbitrary open cross section, prove that $S_{\omega y} = S_{\omega z} = 0$ if axes yz are centroidal and pole P coincides with shear center S, as claimed above Eq. 8.10.5.
Suggestion: Set $e_y = e_z = 0$ in Eqs. 9.8.5.

9.46 Use Eqs. 9.8.5 to show that point P is the shear center in Figs. 9.7.2 and 9.7.3.

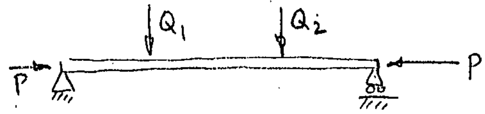
9.47 Use the method of Section 9.8 to locate the shear centers of the following cross sections.

(a) Problem 9.35.
(b) Problem 9.36.
(c) Problem 9.40
(d) Problem 9.42.

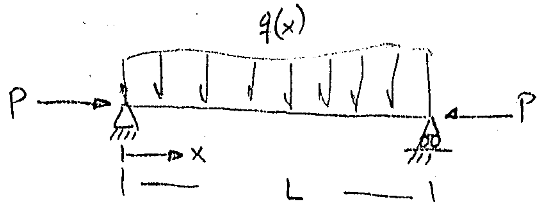
9.48 Imagine that the box section of Problem 9.31 is cut open at the lower left corner. Locate the shear center by the method of Section 9.8.

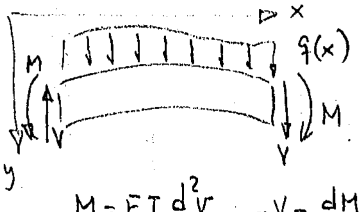
9.49 Locate the y and z coordinates of the shear center of the cross section shown. Dimensions are in millimeters. Thickness t is constant. Axes yz centroidal, and $I_y = 10.5(10)^6\ \text{mm}^4$, $I_z = 20.8(10)^6\ \text{mm}^4$, $I_{yz} = 6.00(10)^6\ \text{mm}^4$.

***9.50** Imagine that the channel section of Fig. 9.7.1 is thin-walled, of constant thickness, with $h = b$, but that the upper flange has a modulus three times as great as that of the rest of the section. Locate the shear center.



בדרך כלל אנחנו יכולים להשתמש בסופרפוזיציה כדי לקבל את התגובה של קורה או מוט עם כמה עומסים. אבל כאשר יש קורה או מוט שבהם יש כוח צירי כמו P שפועל על המוט או קורה עם הכוחות Q_1,Q_2 אי אפשר לקבל את התגובות בסופרפוזיציה. זה בא בגלל שהתגובות מיוחסים בדרך לינארית ל Q_2,Q_1 אבל היחס לא לינארי ל-P.





$$M = EI\frac{d^2v}{dx^2} \quad -V = \frac{dM}{dx} \quad -q = \frac{dV}{dx}$$

אבל הכוח P מכניס לנוסחה מומנט של Pv. המומנט הזה קורה לגזירה של $P\frac{dv}{dx}$ וגם עומס מפורס של $q = -P\frac{d^2v}{dx^2}$. לכן הנוסחה שפועלת במצב הזה היא

$$\frac{d^2M}{dx^2} + \lambda^2 M = q \qquad \lambda^2 = \frac{P}{EI}$$ כפונקציה של מומנט

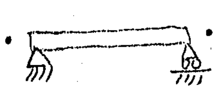
$$\frac{d^4v}{dx^4} + \lambda^2\frac{d^2v}{dx^2} = q/EI$$ כפונקציה של התגובה

אם EI פונקציה של x

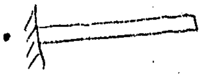
$$\frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right) + P\frac{d^2v}{dx^2} = q$$

<u>פיתרון כאשר EI קבוע</u> ו- $q_0 = q(x)$

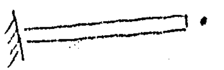
$$v(x) = A\cos\lambda x + B\sin\lambda x + Cx + D + \frac{q_0x^2}{2\lambda^2 EI}$$



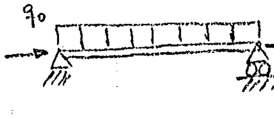
בסמך נייד ונייח: $M = EI\frac{d^2v}{dx^2} = 0,\ v = 0$



בסמך רתום: $\frac{dv}{dx} = 0,\ v = 0$



בצד חופשי: $M = 0$, $V = \frac{d^3v}{dx^3} + \frac{P}{EI}\frac{dv}{dx} = 0$



<u>בעמוד-קורה בקצוות נייח ונייד</u>

$$v = \frac{q_0}{\lambda^4 EI\sin\lambda L}\left\{(1-\cos\lambda L)\sin\lambda x - \left[(1-\cos\lambda x) + \frac{\lambda^2 x}{2}(L-x)\right]\sin\lambda L\right\}$$

<u>כאשר $\lambda \to 0$</u> $v = \frac{q_0}{24EI}(x^4 - 2Lx^3 + L^3x)$ ו- $v(1/2) = \frac{5q_0L^4}{\;}$

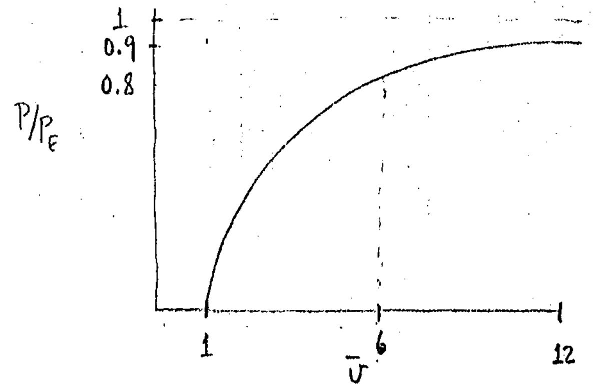
$v(L/2) = \frac{-q_0}{384EI}\left\{\frac{\quad 2 \quad}{\frac{5}{384}(\lambda L)^4\sin\lambda L}\right\}$ כאשר $\lambda\neq 0$

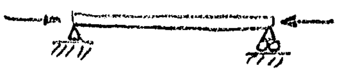
ותזוזה הולכת לאין סוף כאשר $(\lambda L)^4\sin\lambda L$ הולך לאפס, או כאשר $\sin\lambda L=0$.

$$\sin\lambda L = 0 \Rightarrow \lambda L = n\pi \Rightarrow P = \frac{n^2\pi^2}{L^2}EI$$

לכן התזוזה לא תלויה ב- q_0. ה- P הקטן הוא $P_E = \frac{\pi^2 EI}{L^2}$ וזה העומס הקריטי או עומס הקריסה

אם נסתכל במקדם $\left(\frac{\lambda L}{\pi}\right)^2 = \frac{\lambda^2 EI}{\pi^2 EI/L^2} = P/P_E$ כפונקציה של $\bar{v} = \frac{v(L/2,\lambda)}{v(L/2,\lambda=0)}$



אם הסתכלנו רק בקורה  היינו מקבל $\frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right)+P\frac{d^2v}{dx^2}=0$

והפיתרון למשוואה הזאת הוא ש-

$$v = A\cos\lambda x + B\sin\lambda x + Cx + D$$

אם היינו משתמשים בתנאי הקצוות, זאת אומרת

$$v(0)=0 \longrightarrow A+D=0$$

$$M(x=0) = EI\frac{d^2v}{dx^2}=0 \longrightarrow -\lambda^2 A = 0$$

$$v(L)=0 \qquad A\cos\lambda L + B\sin\lambda L + CL + D = 0$$

$$M(x=L) = EI\frac{d^2v}{dx^2}=0 \qquad -\lambda^2 A\cos\lambda L - B\lambda^2\sin\lambda L = 0$$

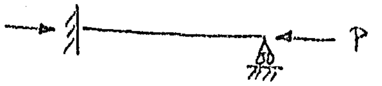
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -\lambda^2 & 0 & 0 & 0 \\ \cos\lambda L & \sin\lambda L & L & 1 \\ -\lambda^2\cos\lambda L & -\lambda^2\sin\lambda L & 0 & 0 \end{bmatrix}\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

כדי למצוא פיתרון או $A=B=C=D=0$ (זאת אומרת ש- $v=0$ הפיתרון הפשוט), או ש- ה DETERMINANT שווה לאפס. לכן, לפי העמוד השלישי, ה- DETERMINANT שווה ל-

$$L\begin{vmatrix} 1 & 0 & 1 \\ -\lambda^2 & 0 & 0 \\ -\lambda^2\cos\lambda L & -\lambda^2\sin\lambda L & 0 \end{vmatrix} = L\lambda^4\sin\lambda L = 0$$

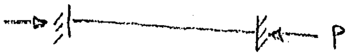
ועוד פעם $\sin\lambda L=0$. זאת אומרת שפיתרון האפשרי הוא ש- $v=B\sin\lambda x$. לכן אפשרויות

התזוזה המקסימלית הולכת וגודלת. ואם העומס P הוא P_E, אז העומד מתחיל לקרוס.



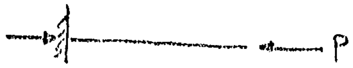
יישום תנאי הקצוות לעמוד-קורה כבצורה
נותן המשוואה הקריטית $\tan\lambda L = \lambda L$

$$2.05P_E = 2.05\frac{\pi^2EI}{L^2} = P_{\text{קריסה}}$$



המשוואה הקריטית $0 = \sin\frac{\lambda L}{2}\left(\sin\frac{\lambda L}{2} - \frac{\lambda L}{2}\cos\frac{\lambda L}{2}\right)$

$$4P_E = \frac{4\pi^2EI}{L^2} = P_{\text{קריסה}}$$

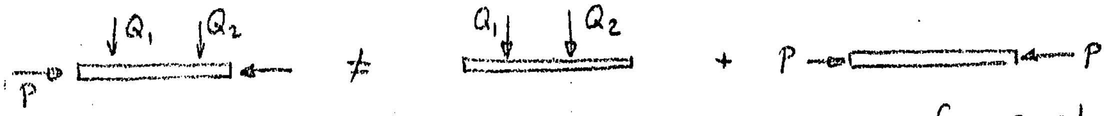


המשואה הקריטית $\cos kL = 0$

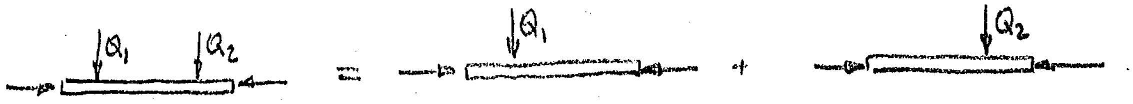
$$\frac{1}{4}P_E = \frac{1}{4}\frac{\pi^2EI}{L^2} = P_{\text{קריסה}}$$

כאן, לא כללנו את עומס הגזירה. אם הגזירה נכללת העומס הקריטי שגורם לקריסה יהיה יותר נמוך. רואים שתנאי הקצוות משפיעים לעומס הקריטי.

אף על פי ש־לא אפשר להשתמש הסופרפוזיציה כזאת



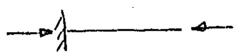
אפשר לעשות ככה



הערות קבלנו את התוצאות האלה על תנאי שנישאר בחלק האלסטי הליניארי של החומר בדרך כלל, $P_{cr} = C\frac{\pi^2EI}{L^2}$ והמספר C תלוי בתנאי הקצוות אם $C = (L/L')^2$

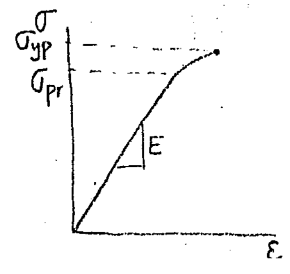
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2EI}{AL'^2} = \frac{\pi^2E(\rho^2A)}{A(L')^2} = \frac{\pi^2E}{(L'/\rho)^2}$$

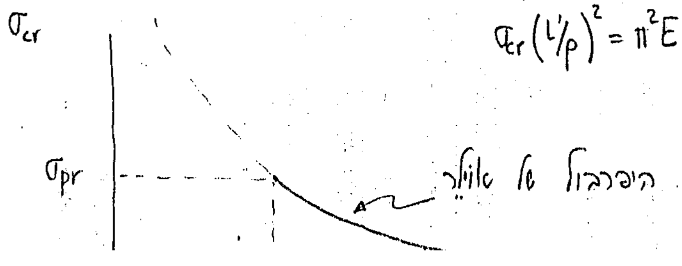
כאן L' = אורך האפקטיבי של הקורה. לדוגמא



$C = 1/4$ לכן $L' = 2L$

- $\rho^2A = I$ ו- ρ הוא רדיוס ההסתובבות
- המקדם L'/ρ הוא תלוי בגאומטרית הקורה וקוראים לזה מקדם הדקות
- רואים ש- $\sigma_{cr}(L'/\rho)^2 = \pi^2E$



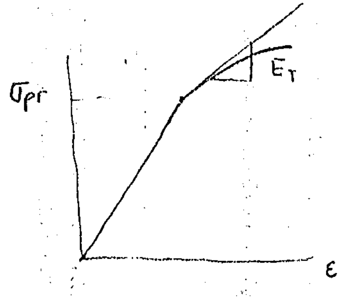


גאומ

$$\left(L'/\rho\right)_{cr} = \pi\sqrt{E/\sigma_{pr}}$$

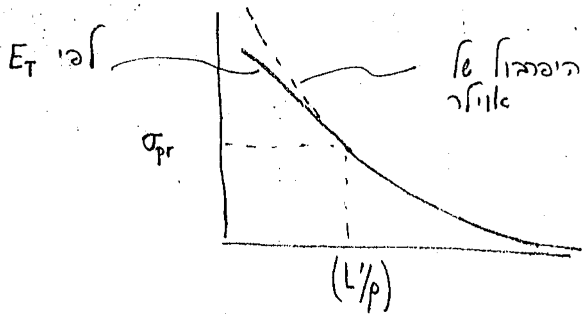
וכולים לקבל את הערך הקריטי. אם $\left(L'/\rho\right)_{cr} > \left(L'/\rho\right)_{\text{גאומ}}$ המאמץ הקריטי לא יהיה שווה ל- P_{cr}/A אבל יהיה יותר גדול.

לתקן בזה, כדי לעבוד ב $\left(L'/\rho\right)_{cr} > \left(L'/\rho\right)_{\text{גאומ}}$ משתמשים במודול הטנגנטי אחרי נקודת הפרופורציה עד σ_{yp}.

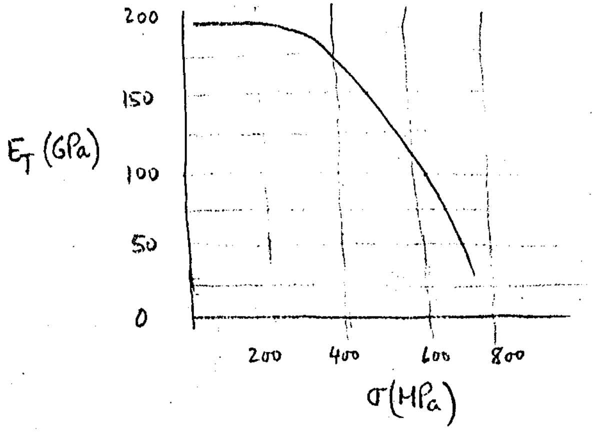


$$\sigma_{t,cr} = \frac{\pi^2 E_T}{\left(L'/\rho\right)^2}$$

ו- בדרך ניסיונות מקבלים את E_T.



לפלדה מסוימת קיבלו את הקשר בין E_T למאמץ



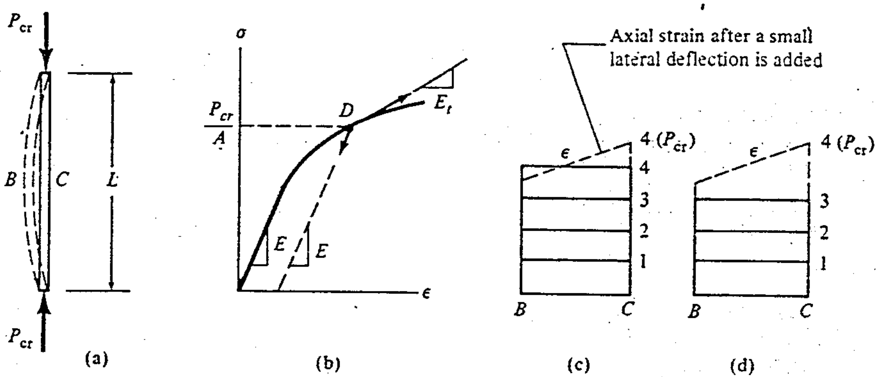


FIGURE 12.4.1. (a) Buckling of a pin-ended column under centroidal axial load. (b) Compressive stress–strain diagram, showing loading and unloading paths from point D, which corresponds to inelastic buckling. (c) Distribution of axial strain across the column at increasing load levels, according to double-modulus theory. (d) Possible distribution of axial strain across the column in tangent modulus theory.

cross section. Therefore, the column must bend before reaching the double-modulus load. But this is in contradiction to a basic assumption in double-modulus theory. The contradiction is resolved by noting that lateral deflection may occur *simultaneously* with application of the last increment of load. There need be no unloading on the convex side, and modulus E_t may prevail all across the section (Fig. 12.4.1d). Under near-perfect test conditions the collapse load slightly exceeds the theoretical tangent-modulus load, but it does not reach the double-modulus load.

In summary, inelastic buckling of a straight, axially loaded column does not occur at a unique value of axial load P. Instead, buckling begins at the tangent-modulus load and is complete (meaning that collapse takes place) before the theoretical double-modulus load is reached. Tests of real columns, which have larger imperfections than laboratory specimens, are in excellent agreement with tangent modulus theory.

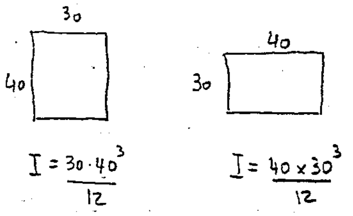
Euler did not realize that bending stiffness EI could be calculated rather than obtained by experiment. However, he anticipated Engesser by remarking in 1757 that EI represents a resistance to bending that need not pertain only to elastic bodies [12.4].

Example 12.4.1. A column has a solid rectangular cross section, 40 mm by 30 mm. It is 200 mm long, free at the top, and fixed at the base. Material properties are shown in Fig. 12.4.2. What centroidal axial compressive load at the top will make the column buckle?

The appropriate equation is $P_{cr} = \pi^2 EI/4L^2$, where

$$I = \frac{bh^3}{12} = \frac{40(30)^3}{12} = 90{,}000 \text{ mm}^4 \qquad (12.4.2)$$

because buckling will take place about the weaker axis of the cross section.



$I = \frac{30 \cdot 40^3}{12}$ $I = \frac{40 \times 30^3}{12}$

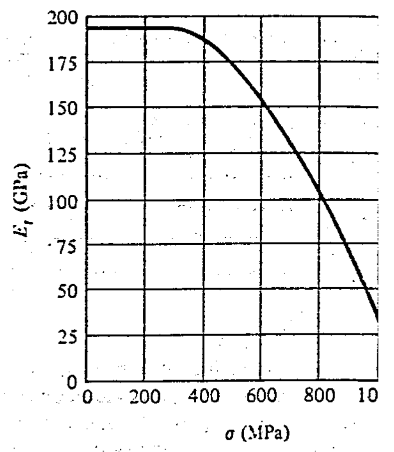


FIGURE 12.4.2. Tangent modulus E_t versus compressive stress σ for a particular steel, obtained from a stress–strain plot.

P_{cr} = 1077 kN, or $\sigma_{cr} = P_{cr}/A$ = 898 MPa. This stress is consic
higher than the proportional limit stress, which appears to be about 32
in Fig. 12.4.2. Therefore, buckling is inelastic, the effective modu
pends on load, and an iterative method of calculation is needed to fi
as follows.

Assume that σ_{cr} will be, say, 600 MPa. At this stress, Fig. 12.4.
E_t = 160 GPa. Hence

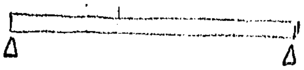
$$P_{cr} = \frac{\pi^2 E_t I}{4L^2} = 888 \text{ kN} \qquad \frac{P_{cr}}{A} = 740 \text{ MPa} \qquad (1$$

As P_{cr}/A exceeds the assumed σ_{cr} of 600 MPa, another trial is neede
Assume that σ_{cr} = 660 MPa; then

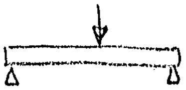
$$E_t = 142 \text{ GPa} \qquad P_{cr} = \frac{\pi^2 E_t I}{4L^2} = 788 \text{ kN} \qquad \frac{P_{cr}}{A} = 657 \text{ MPa} \qquad (1$$

Now the assumed value of σ_{cr} agrees well enough with the calculated
and P_{cr} = 788 kN is accepted as the tangent modulus buckling load.

Creep Buckling. As the name implies, creep buckling theory deals
material that creeps, that is, a material whose strain changes with time at c
stress. A creeping column may display a small but gradually increasing
deflection, then fail suddenly by buckling. The phenomenon is explai
examination of creep curves (Fig. 12.4.3). One may enter the creep cu
a certain time, say t_1, and read the strain for each of several stress
Stress–strain data thus obtained is then plotted as a stress–strain curve,
the curve labeled t_1 in Fig. 12.4.3. Repetition of this procedure at severa
produces a set of isochronous stress–strain curves (stress versus strain at c
time). These curves show that at a given stress level, the tangent m
decreases with time. This implies that however light the load, a creeping
will eventually buckle if we wait long enough for its tangent modulus to d



משקל W שנופל על קורה גורם למאמצים דינמיים והמאמצים האלה
יותר גבוהים ממצב שמשקל W לוחץ על הקורה בלבד



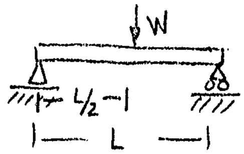
העומס הדינמי נחשב W

$$P_{dyn} = W\left(1+\sqrt{1+2H/\Delta_{st}}\right)$$

h - זה מרחק האנכי בין המשקל W לקורה.

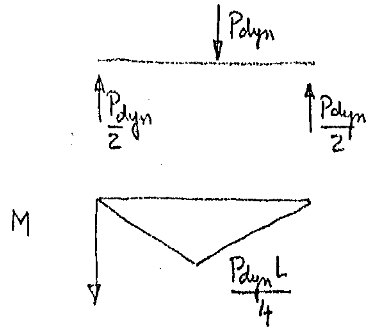
Δ_{st} - זה התזוזה הסטטית של הקורה שבאה מיישום המשקל על הקורה.

<u>דוגמא</u> ..



$$\Delta_{st} = \frac{WL^3}{48EI}$$

המאמץ הדינמי כאן -



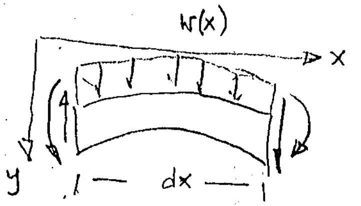
① למצוא העומס הדינמי
② למצוא המומנט המקס.
③ למצוא המאמץ

$$\sigma_{מקס} = \frac{My}{I} = \frac{P_{dyn}L\cdot(h/2)}{4\cdot bh^3/12}$$
$$=\frac{3P_{dyn}\cdot L}{2bh^2}$$

<u>הערות</u> ליישום הנוסחה הבאות.

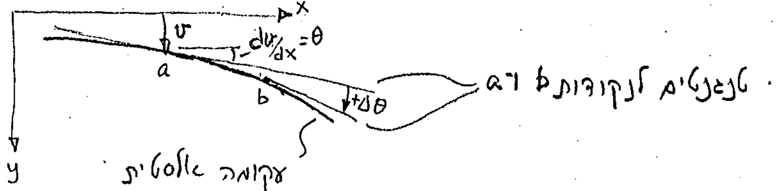
- אין איבוד אנרגיה בנקודת הנגיפה או בסמכים
- אינרצית המערכת נחשבת כאפס (משקל המערכת לא התחשב
- לתזוזה של המערכת יש יחס לינארי לגובה העומס המיושם ואין אכפת אם זה עומס סטטי או דינמי

אינטגרל מור - השיטה [illegible]



$\sigma_x = -\frac{M_z}{I_{zz}} y \qquad \frac{dV}{dx} = -w(x) \qquad \frac{dM}{dx} = -V$

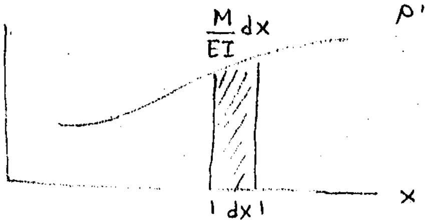
$\frac{d^2v}{dx^2} = \frac{M_z}{EI_{zz}} \qquad K = \frac{1}{\rho} \approx \frac{d^2v}{dx^2}$ (העקומיות)



עם $0 < \frac{1}{\rho}$ (העקומיות)

כאשר התצורה דקוקה במקרים ספציפיים, יש עוד <u>שיטה</u> שתכלו להשתמש בה. השיטה עובדת כאשר יש שינויים בשטח-החתך, או העומס מסובך, קורות (TAPERED) התחדדותיות מוגדרות. כדוגמא במבנה גשרים. נשתמש בגרף של העקומיות (CURVATURE) של הקורה לקבל את התצורה.

$K = \frac{M}{EI}$



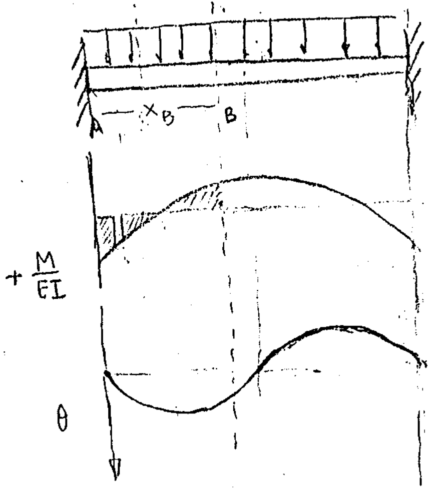
השיטה עובדת גם כאשר יש אי-צירוף העומסים או אי-צירוף של שטחי-החתך.

מפני ש- $\frac{d^2v}{dx^2} = \frac{M}{EI}$ ו- $\frac{d\theta}{dx} = \frac{d}{dx}\left(\frac{dv}{dx}\right)$, לכן נוכל לכתוב ש-

$$d\theta = \frac{M}{EI}dx$$

לכן, שינוי זווית של העקומה האלסטית שווה לשטח תחת הגרף של העקומיות של הקורה.

כאשר זווית קטנה מכופלת במרחק x מנקודת התחלת מערכת הקואורדינטות, $dt = x\,d\theta$, או $\underline{dt = \frac{M}{EI} x\,dx}$. אם מתחילים ממקום שזווית ידוע לנו, ומחשבים את השטח תחת גרף העקומיות, נקבל את הזווית בנקודה אחרת על העקומה האלסטית



<u>דוגמה</u> - קורה בעומס מפורס של $q\ N/m$ קבועה בסמכים מקובעים.

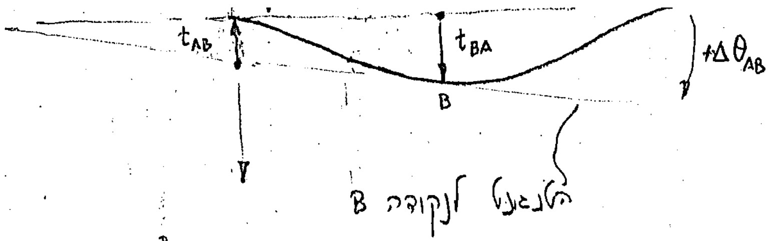
מצאנו ש- M בסמך הוא $ql^2/12$. לכן $\frac{ql^2}{12EI} = \frac{M}{EI}$. בגלל הסמך, בנקודה A, $\theta_A = \underline{0}$ ו- בנקודה B שבמרחק של x_B

$$\theta_B - \theta_A = \int_A^B d\theta = \int_{x=0}^{x=x_B} \frac{M}{EI}dx$$

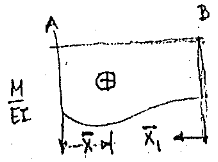
$$\theta_B = \int_0^{x_B} \frac{M}{EI}dx + \theta_A$$

אם השטח חיובי, השיפוע הולך וגודל וסובב דרך השעון. אם השטח שלילי השיפוע הולך ומקטין סובב נגד דרך השעון. אם השטח שווה לאפס השיפוע הסופי מקביל לשיפוע הראשון.

הכמות dt היא מהעקומיות של האלמנט dx. אם נקח את כל ההשפעות מכל האלמנטים מ-A ל-B נקבל את המרחק האנכי t_{AB}. זה <u>הסטייה הטנגנטית</u> של נקודה A מטנגנט בנקודה B



אם נשתמש בהגדרת מרכז הכובד של שטח



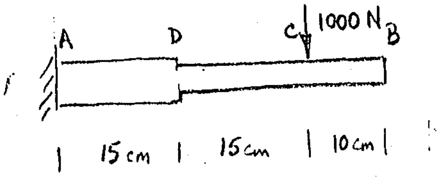
$$t_{AB} = \int_A^B \frac{M}{EI} x\,dx = \left(\int \frac{M}{EI}\,dx\right)\bar{x}$$

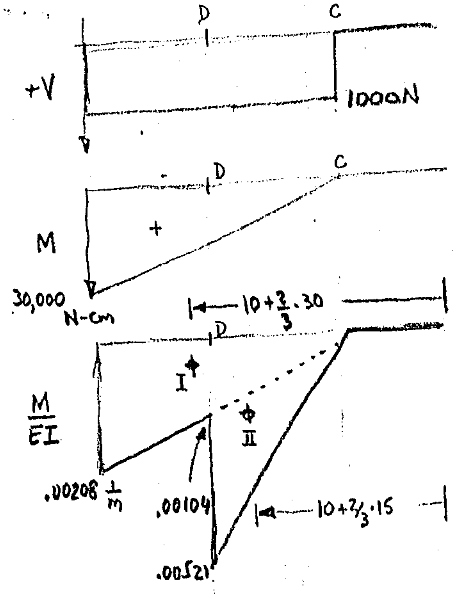
$\bar{x}$ המרחק האופקי מ-A למרכז הכובד של $\int_A^B \frac{M}{EI}dx$

t_{BA} זה הסטייה הטנגנטי של נקודה B מהטנגנט בנקודה A והוא שווה ל $\left(\int \frac{M}{EI}dx\right)\bar{x}_1$
ו-$\bar{x}_1$ הוא המרחק מ-קוו האנכי דרך נקודה B למרכז הכובד של $\int_A^B \frac{M}{EI}dx$

בדרך כלל, הסטיה הטנגנטית איננה התזוזה הסופית.

דוגמה קורה של אלומיניום $E = 72$ GPa בצורה כזאת מעומסת מכוח של 1000N, 10cm מהקצה הימני. אם I_{zz} של AD = 200 cm^4, ו-I_{zz} של DB הוא 40 cm^4, מצוא את התזוזה של קצה B וגם הסיבוב של נקודה B. לא לכלול משקל הקורה.





מפני שקצה A הוא סמך מקובע, $\theta_A = 0$

$\theta_B = \Delta\theta_{AB} = \int_0^{40} \frac{M}{EI}dx =$ שטחי משולשים $\mathrm{I} + \mathrm{II} + 0$

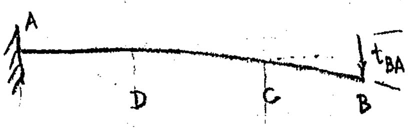
$$= (.00208)\left(\frac{.30}{2}\right) + (.00521 - .00104)\left(\frac{.15}{2}\right).$$

$= .000625$ radians

$t_{BA} = 2\cdot 0 + \bar{x}_2 \times$ שטח משולש II $+\ \bar{x}_1 \times$ שטח משולש I

$$= 2\cdot 0 + (.20)(.000313) + (.30)(.000312)$$

$$= .000156\ m$$



t_{BA} הוא הסטיה הטנגנטית מנקודה B לטנגנט ב-A
וזה התזוזה $v_B = 0.156$ מ"מ.

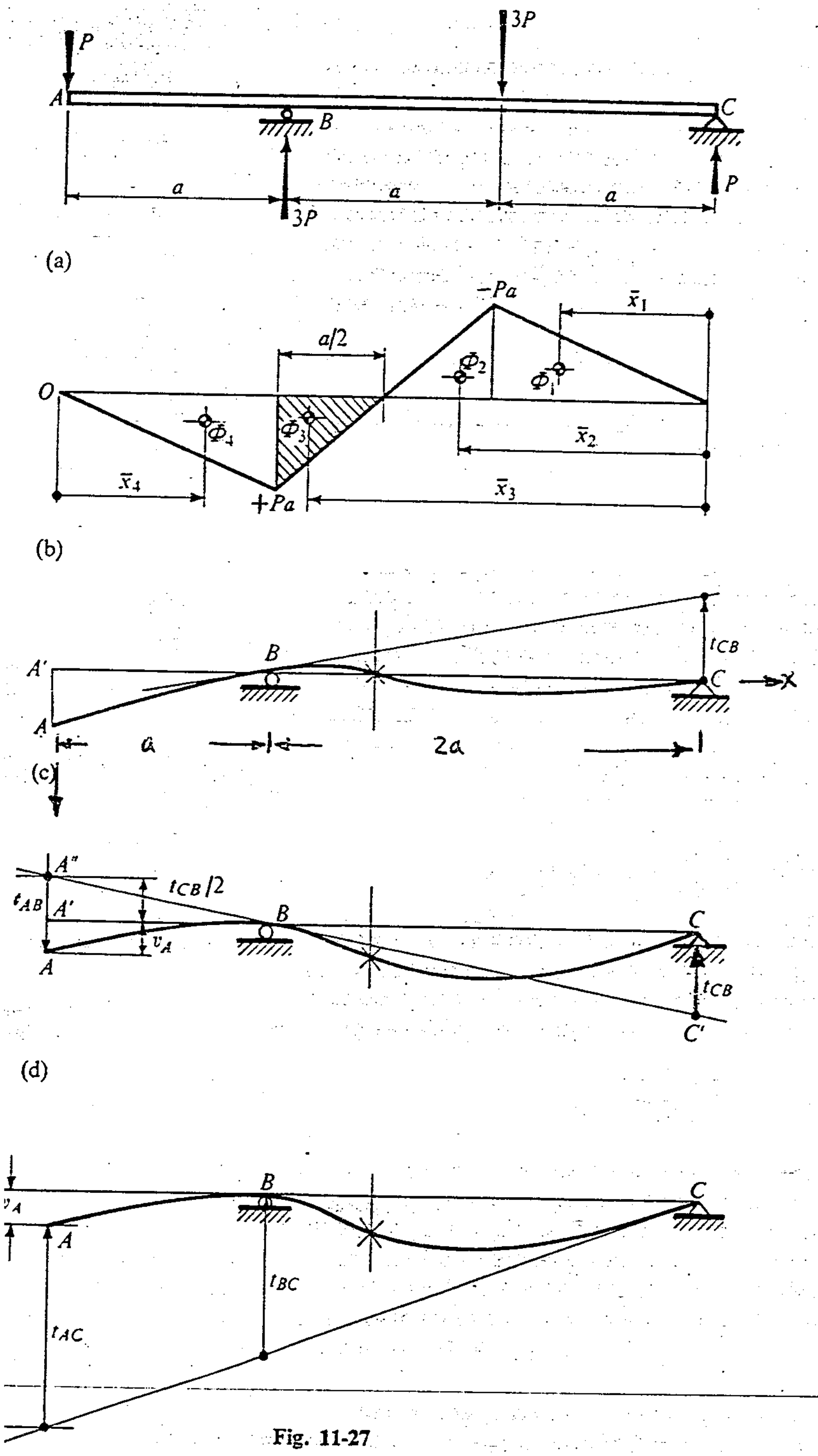


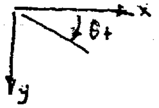
Fig. 11-27

להציב את v_A

$$\frac{\overline{A'A}}{\overline{A'B}} = \frac{t_{cB}}{\overline{BC}}$$

$$A'A = t_{cB}\frac{\overline{A'B}}{\overline{BC}} = \frac{t_{cB}}{2}$$

הציר הנכון



SOLUTION

The bending-moment diagram for the applied forces is in Fig. 11-27(b). The bending moment changes sign at $a/2$ from the left support. At this point an inflection in the elastic curve takes place. Corresponding to the positive moment, the curve is concave up, and vice versa. The elastic curve is so drawn and passes over the supports at B and C, Fig. 11-27(c). To begin, the inclination of the tangent to the elastic curve at the support B is determined by finding t_{CB} as the statical moment of the areas with the proper signs of the $M/(EI)$ diagram between the verticals through C and B about C.

$$t_{CB} = \Phi_1\bar{x}_1 + \Phi_2\bar{x}_2 + \Phi_3\bar{x}_3$$

$$= \frac{1}{EI}\left[\frac{a}{2}(-Pa)\frac{2a}{3} + \frac{1}{2}\frac{a}{2}(-Pa)\left(a + \frac{1}{3}\frac{a}{2}\right) + \frac{1}{2}\frac{a}{2}(+Pa)\left(\frac{3a}{2} + \frac{2}{3}\frac{a}{2}\right)\right]$$

$$= -\frac{Pa^3}{6EI}$$

The ~~positive~~ negative sign of t_{CB} indicates that the point C is ~~above~~ below the tangent through B. Hence a corrected sketch of the elastic curve is made, Fig. 11-27(d), where it is seen that the deflection sought is given by the distance AA' and is equal to $AA'' - A'A''$. Further, since the triangles $A'A''B$ and $CC'B$ are similar, the distance $A'A'' = t_{CB}/2$. On the other hand, the distance AA'' is the deviation of the point A from the tangent to the elastic curve at the support B. Hence

$$v_A = AA' = \overrightarrow{AA''} \dotplus \overleftarrow{A'A''} = t_{AB} + (t_{CB}/2)$$

$$t_{AB} = \frac{1}{EI}(\Phi_4\bar{x}_4) = \frac{1}{EI}\left[\frac{a}{2}(+Pa)\frac{2a}{3}\right] = +\frac{Pa^3}{3EI}$$

where the ~~negative~~ positive sign means that point A is ~~below~~ above the tangent through B. This sign is not used henceforth as the geometry of the elastic curve indicates the direction of the actual displacements. Thus the deflection of point A below the line passing through the supports is

$$v_A = \frac{Pa^3}{3EI} - \frac{1}{2}\frac{Pa^3}{6EI} = \frac{Pa^3}{4EI}$$

This example illustrates the necessity of watching the signs of the quantities computed in the applications of the moment-area method, although usually less difficulty is encountered than in the above example. For instance, if the deflection of the end A is established by first finding the rotation of the elastic curve at C, no ambiguity in the direction of tangents occurs. This scheme of analysis is shown in Fig. 11-27(e), where $v_A = \frac{3}{2}t_{BC} - t_{AC}$.

The foregoing examples illustrate the manner in which the moment-area method may be used to obtain the deflection of any statically deter-

ally
nate
ems

$$\frac{1}{EI}\left(\frac{1,000 + \frac{1}{3}(100)M_B}{10}\right) = -\frac{1}{EI}\left(\frac{2,880 + 108M_B \quad 54M_C}{18}\right)$$

or $$(\tfrac{28}{3})M_B + 3M_C = -260$$

Using condition (b) for the span BC provides another equation, $t_{BC} = 0$, or

$$\frac{1}{EI}\left[\frac{(18)}{2}(+40)\frac{(18+12)}{3} + \frac{(18)(+M_B)}{2}\frac{(18)}{3} + \frac{(18)(+M_C)}{2}\frac{2(18)}{3}\right] = 0$$

or $$3M_B + 6M_C = -200$$

Solving the two reduced equations simultaneously,

$$M_B = -20.4 \text{ ft-lb} \quad \text{and} \quad M_C = -23.3 \text{ ft-lb}$$

where the signs agree with the convention of signs used for beams. These moments with their proper sense are shown in Fig. 12-17(b).

After the redundant moments M_A and M_C are found, no new techniques are necessary to construct the moment and shear diagrams. However, particular care must be exercised to include the moments at the supports while computing shears and reactions. Usually, isolated beams as shown in Fig. 12-17(b) are the most convenient free bodies for determining shears. Reactions follow by adding the shears on the adjoining beams. In units of kips and feet, for free body AB:

$$\sum M_B = 0 \circlearrowright +, \quad 2.4(10)5 - 20.4 - 10R_A = 0, \quad R_A = 9.96 \text{ kips} \uparrow$$

$$\sum M_A = 0 \circlearrowleft +, \quad 2.4(10)5 + 20.4 - 10V_B' = 0, \quad V_B' = 14.04 \text{ kips} \uparrow$$

For free body BC:

$$\sum M_C = 0 \circlearrowright +, \quad 10(6) + 20.4 - 23.3 - 18V_B'' = 0,$$

$$V_B'' = 3.17 \text{ kips} \uparrow$$

$$\sum M_B = 0 \circlearrowleft +, \quad 10(12) - 20.4 + 23.3 - 18V_C = 0,$$

$$V_C = R_C = 6.83 \text{ kips} \uparrow$$

Check:

$$R_A + V_B' = 24 \text{ kips} \uparrow \quad \text{and} \quad V_B'' + R_C = 10 \text{ kips} \uparrow$$

From above, $R_B = V_B' + V_B'' = 17.21$ kips ↑.

The complete shear and moment diagrams are in Figs. 12-17(e) and (f), respectively.

12-6. THE THREE-MOMENT EQUATION

Generalizing the procedure used in the preceding example, a recurrence formula, i.e., an equation which may be repeatedly applied for every two adjoining spans, may be derived for continuous beams. For any

יוסט קלפירון

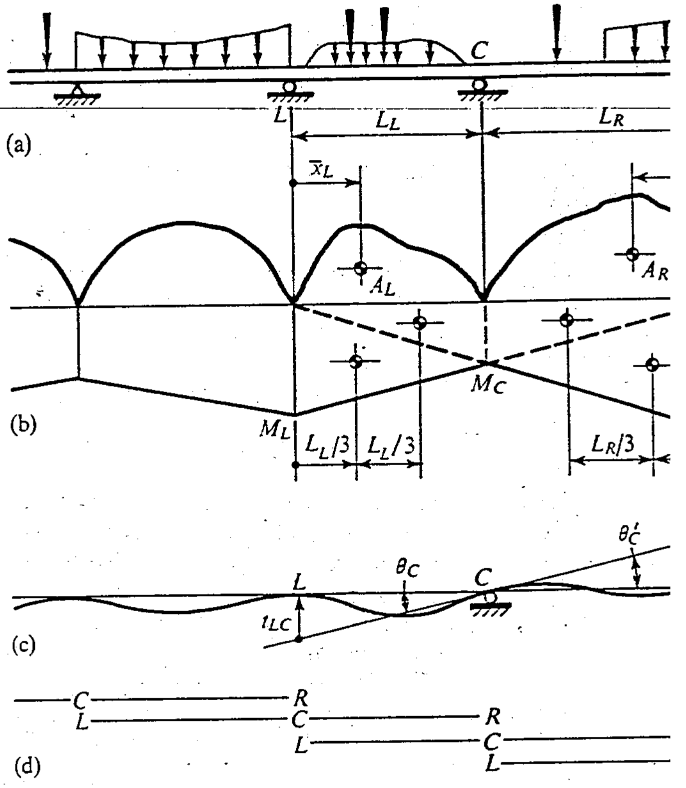


Fig. 12-18. Derivation of the three-mor

n number of spans, $n - 1$ such equations may be written. This gives enough simultaneous equations for the solution of redundant moments over the supports. This recurrence formula is called the *three-moment equation* because three unknown moments appear in it.

Consider a continuous beam, such as shown in Fig. 12-18(a), subjected to any transverse loading. For any two adjoining spans, as LC and CR, the bending-moment diagram is considered to consist of two parts. The areas A_L and A_R to the left and to the right of the center support C, Fig. 12-18(b), correspond to the bending-moment diagrams in the respective spans if these spans are treated as being simply supported. These moment diagrams depend entirely upon the nature of the known forces applied within each span. The other part of the moment diagram of known shape is due to the unknown moments M_L at the left support, M_C at the center support, and M_R at the right support.

Next, the elastic curve in Fig. 12-18(c) must be considered. This curve is continuous for any continuous beam. Hence the angles θ_C and θ_C', which define, from the respective sides, the inclination of the same tangent to the elastic curve at C, are equal. By using the second moment-area theorem to obtain t_{LC} and t_{RC}, these angles are defined as $\theta_C = t_{LC}/L_L$ and $\theta_C' = -t_{RC}/L_R$, where L_L and L_R are span lengths on the left and on

ות המומנט -
ה
מניחים שהקורה
כ״ח כמו ב L,R,C
ה המומנט ב A_L, A_R
רתי ידוע ובה
M_R, M_C, M_L ה
העקומה הכללית

the right of C, respectively. The negative sign for the second angle is necessary since the tangent from point C is above the support R and a positive deviation of t_{RC} locates a tangent below the same support. Hence, following the steps outlined,

$$\theta_C = \theta'_C \qquad \text{or} \qquad t_{LC}/L_L = -t_{RC}/L_R$$

and

$$\frac{1}{L_L}\frac{1}{EI_L}\left(A_L\bar{x}_L + \frac{L_LM_L}{2}\frac{L_L}{3} + \frac{L_LM_C}{2}\frac{2L_L}{3}\right)$$

$$= -\frac{1}{L_R}\frac{1}{EI_R}\left(A_R\bar{x}_R + \frac{L_RM_R}{2}\frac{L_R}{3} + \frac{L_RM_C}{2}\frac{2L_R}{3}\right)$$

מומנט [illegible]

where I_L and I_R are the respective moments of inertia of the cross-sectional area of the beam in the left and the right spans. Throughout each span, I_L and I_R are assumed constant. The term $\bar{x}_L$ is the distance from the left support L to the centroid of the area A_L, and $\bar{x}_R$ is a similar distance for A_R measured from the right support R. The terms M_L, M_C, and M_R denote the unknown moments at the supports.

Simplifying the above expression, the three-moment equation* is

$$L_LM_L + 2\left(L_L + \frac{I_L}{I_R}L_R\right)M_C + \frac{I_L}{I_R}L_RM_R$$

$$= -\frac{6A_L\bar{x}_L}{L_L} - \frac{6A_R\bar{x}_R}{L_R}\frac{I_L}{I_R} \qquad \textbf{(12-12)}$$

קורה רציפה

This equation 12-12 applies to continuous beams on unyielding supports, with the beam in each span of constant I. In a particular problem, all terms, with the exception of the redundant moments at the supports, are constant. A sufficient number of simultaneous equations for the unknown moments is obtained by successively imagining the supports of the adjoining spans as L, C, and R as shown in Fig. 12-18(d). However, in these equations the subscripts of the M's must correspond to the actual designation of the supports, such as A, B, C, etc. Also note that at pinned ends of beams the moments are known to be zero. Likewise, if a continuous beam has an overhang, the moment at the first support is known from statics. Fixed supports will be discussed in Example 12-16. For symmetrical beams symmetrically loaded, work may be minimized by noting that moments at symmetrically placed supports are equal.

[illegible]

In deriving the three-moment equation, the moments at the supports were assumed positive. Hence an algebraic solution of simultaneous equations automatically gives the correct sign of moments according to the convention for beams.

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[*] The three-moment equation was originally derived by E. Clapeyron, a French engineer, in 1857, and sometimes is referred to as Clapeyron's equation.

12-7. SPECIAL CASES (אפשרויות מיוחדות)

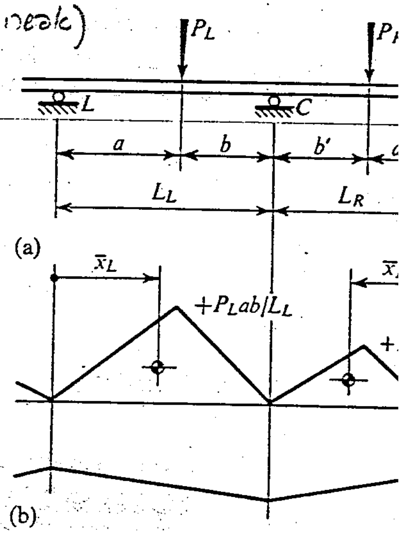


Fig. 12-19. Establishing the constants on the right hand of the three-moment equation for concentrated loads.

כוחות מרוכזים

As a specific example of the evaluation of the constant terms on the right side of the three-moment equation, consider two adjoining spans loaded with the concentrated forces P_L and P_R, Fig. 12-19. Considering these spans simply supported, since the maximum moment in the left span is $+P_Lab/L_L$, and $\bar{x}_L = (L_L + a)/3$ one writes

$$-6A_L\frac{\bar{x}_L}{L_L} = -6\left(\frac{L_L}{2}\right)\frac{P_Lab}{L_L}\frac{(L_L+a)}{3L_L}$$

$$= -P_Lab\left(1 + \frac{a}{L_L}\right) \qquad \textbf{(12-13)}$$

Similarly, by interchanging the role of the dimensions a and b in the right span, i.e., by always measuring a's from the outside support toward the force,

$$-6A_R\frac{I_L}{I_R}\frac{\bar{x}_R}{L_R} = -P_Ra'b'\left(1 + \frac{a'}{L_R}\right)\frac{I_L}{I_R} \qquad \textbf{(12-14)}$$

If a number of concentrated forces occurs within a span, the contribution of each one of them to the above constant may be treated separately. Hence a constant term for the right side of the three-moment equation applicable for any number of concentrated forces applied within the spans is

כוחות מרוכזים

$$-\sum_i P_{L_i}a_ib_i\left(1 + \frac{a_i}{L_L}\right) - \sum_i P_{R_i}a'_ib'_i\left(1 + \frac{a'_i}{L_R}\right)\frac{I_L}{I_R} \qquad \textbf{(12-15)}$$

הימני של (12-14)
(12-15)

where the summation sign designates the fact that a separate term appears for every concentrated force P_L in the left span, and similarly, for every force P_R in the right span. In both cases, a or a' is the distance from the outside support to the particular concentrated force, and b or b' is the distance to the force from the center support. If any one of these forces acts upward, the term contributed to the constant by such a force is of opposite sign.

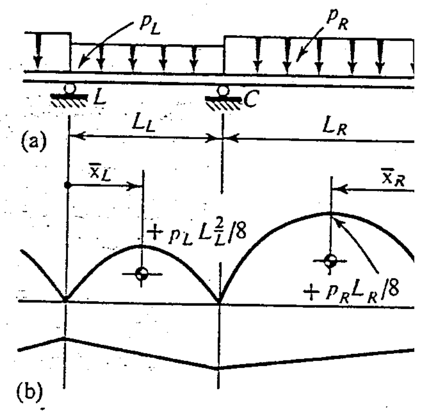
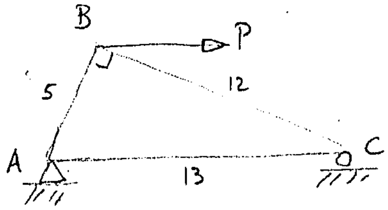


Fig. 12-20. Establishing the constants on the right hand of the three-moment equation for uniformly distributed loads.

The constant for the right side of the three-moment equation, when uniformly distributed loads are applied to a beam, is determined similarly. Thus, using the diagram in Fig. 12-20,

$$-6A_L\frac{\bar{x}_L}{L_L} = -6\left(\frac{2L_L}{3}\right)\left(\frac{p_LL_L^2}{8}\frac{L_L}{2L_L}\right) = -\frac{p_LL_L^3}{4} \qquad \textbf{(12-16)}$$

$$-6A_R\frac{I_L}{I_R}\frac{\bar{x}_R}{L_R} = -p_R\frac{L_R^3}{4}\frac{I_L}{I_R}$$

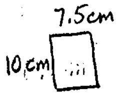


לפי שיטת הצמתים (ראו את הפתרון של הבוחן)

$F_{AB} = \frac{5}{13}P$ במתיחות

$F_{AC} = \frac{144}{169}P$ במתיחות

$F_{BC} = -\frac{12}{13}P$ בלחיצה



אם שטח החתך ...... מהו העומס שיגרום להתמוטטות? קחו $E=200GPa, \pm 300 MPa = \sigma_y$

מפני ש- F_{AB} ו- F_{AC} במתיחות צריך לבדוק מהו העומס שגורם לכניעה. מפני ש- F_{BC} בלחיצה צריך לבדוק מהו העומס שגורם לכניעה בלחיצה וגם בקריסה.

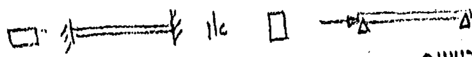
$$A = .1(.075) = 7.5 \times 10^{-3} m^2$$

$$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{5}{13}\frac{P}{(7.5 \times 10^{-3})} = 300 \times 10^6 \frac{N}{m^2} \qquad P = (300 \times 10^6)(7.5 \times 10^{-3}) \cdot \frac{13}{5} = 5.4 \times 10^6 N$$

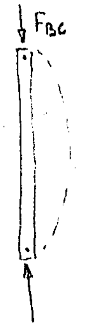
$$\sigma_{AC} = \frac{F_{AC}}{A} = \frac{144}{169}\frac{P}{7.5 \times 10^{-3} m^2} = 300 \times 10^6 \frac{N}{m^2} \qquad P = (300 \times 10^6)(7.5 \times 10^{-3})\frac{169}{144} = 2.64 \times 10^6 N$$

$$\sigma_{BC} = \frac{F_{AC}}{A} = -\frac{12}{13}\frac{P}{7.5 \times 10^{-3} m^2} = -300 \times 10^6 \frac{N}{m^2} \qquad P = (300 \times 10^6)(7.5 \times 10^{-3}) \cdot \frac{13}{12} = 2.44 \times 10^6 N$$

כדי לבדוק בקריסה יש שתי אפשרויות



לכן, צריך למצוא את מומנט האינרציה בשני כיוונים



$$I = \frac{(7.5)10^3}{12} \times 10^{-8} = 6.25 \times 10^{-6} m^4$$

$$F_{BC} = \frac{\pi^2 (200 \times 10^9)(6.25 \times 10^{-6})}{(12)^2} = 85674 N$$

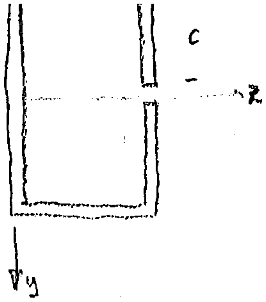
$$F_{BC} = \frac{12}{13}P \rightarrow \underline{P = 92813.\ N}$$

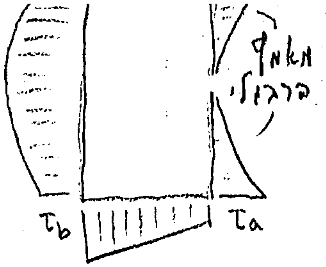
$$I = \frac{10(7.5)^3}{12} \times 10^{-8} = 3.515 \times 10^{-6} m^4$$

$$F_{BC} = \frac{4\pi^2 EI}{L^2} = \frac{4(\pi^2)(200 \times 10^9)(3.515 \times 10^{-6})}{(12)^2} = 192766 N$$

$$F_{BC} = \frac{12}{13}P \rightarrow P = 208830 N$$

לכן BC תקרוס בראשונה כאשר P=92813N

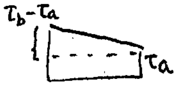




צורת מאמץ הגזירה
שפועל על החתך

$$\overline{It}$$

$$\tau_b = (\tau_b - \tau_a) + \tau_a = \frac{VQ_b}{It} + \frac{VQ_a}{It}$$



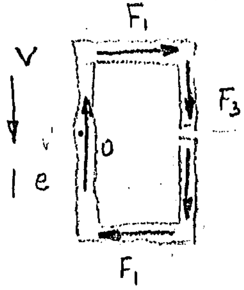
$$I = \frac{t(2c)^3}{12} + 2(bt)c^2 + 2\left[\frac{tc^3}{12} + tc\left(\frac{c}{2}\right)^2\right]$$

$$Q_a = \int y\,dA = \int_{b-t/2}^{b+t/2}\int_0^c y\,(dy\,dz) = \frac{y^2}{2}t\Big|_0^c = \frac{c^2t}{2}$$

$$Q_b = \iint y\,dA = \bar{y}A = c\cdot bt$$

$$F_3 = \int \tau_a\,dA = \frac{V}{It}\int_0^c \frac{y^2t}{2}\cdot t\,dy = \frac{V}{I}\frac{y^3t}{6}\Big|_0^c = \frac{Vtc^3}{6I}$$

$$F_1 = \frac{\tau_a + \tau_b}{2}(b\cdot t) = \frac{Vtbc}{2I}(b+c)$$



$$\Sigma M_0 = 2F_3\cdot b + F_1\cdot 2c - Ve = 0$$

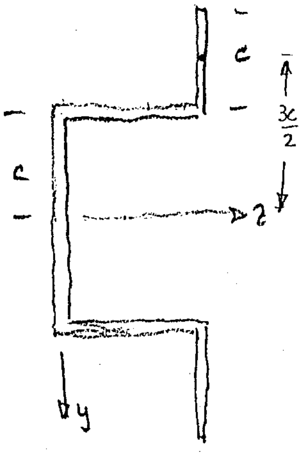
$$= 2\tau_a\frac{ct}{3}\cdot b + \left(\frac{\tau_a+\tau_b}{2}\right)bt\cdot 2c - Ve = 0$$

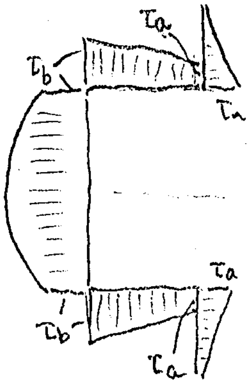
$$= \frac{2VQ_a}{It}\cdot\frac{ct}{3}\cdot b + \left(\frac{VQ_b}{2It} + \frac{2VQ_a}{2It}\right)bt\cdot 2c - Ve = 0$$

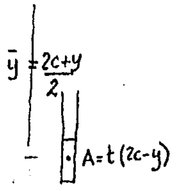
$$= \frac{V}{It}\left\{2\frac{c^2t}{2}\cdot\frac{ct}{3}\cdot b + \left(\frac{cbt}{2} + \frac{c^2t}{2}\right)bt\cdot 2c\right\} - Ve = 0$$

$$= \frac{V}{It}\left\{\frac{c^3t^2b}{3} + c^2b^2t^2 + c^3bt^2\right\} - Ve = 0$$

$$e = \frac{1}{It}\left\{\frac{4}{3}c^3t^2b + c^2b^2t^2\right\} = \frac{c^2bt}{I}\left(b + \frac{4}{3}c\right) = \frac{c^2bt\left(b+\frac{4}{3}c\right)}{c^2t\left(4/3c + 2b\right)} = \frac{b\left(b+4/3c\right)}{\left(4/3c+2b\right)}$$







$$\tau_a = \frac{VQ_a}{It} \qquad \tau_b = (\tau_b - \tau_a) + \tau_a = \frac{VQ_b}{It} + \frac{VQ_a}{It}$$

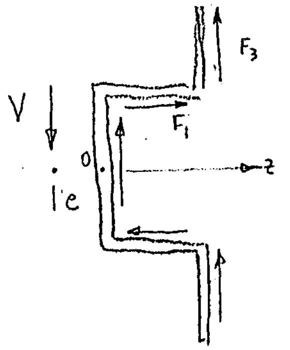
$$I = \frac{t(2c)^3}{12} + 2btc^2 + 2\left[\frac{tc^3}{12} + tc\left(\frac{3c}{2}\right)^2\right]$$

$$Q_a = \int y\,dA = t(2c-y)\left(\frac{2c+y}{2}\right) = \frac{t}{2}(4c^2-y^2)\Big|_{y=c} = \frac{3c^2t}{2}$$

$Q_b = cbt$ כמו בתרגיל הקודם

$$F_3 = \int \tau_a\,dA = \frac{V}{It}\int_c^{2c}\frac{t}{2}(4c^2-y^2)\cdot t\,dy = \frac{Vt}{2I}\left(4c^2y - \frac{y^3}{3}\right)_c^{2c} = \frac{5}{6I}Vt$$

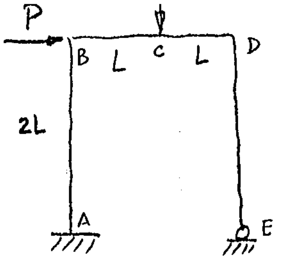
$$F_1 = \frac{\tau_b+\tau_a}{2}bt = \frac{1}{2}\left(\frac{VQ_b}{It} + \frac{2VQ_a}{It}\right)bt = \frac{Vtbc}{2I}(b+3c)$$

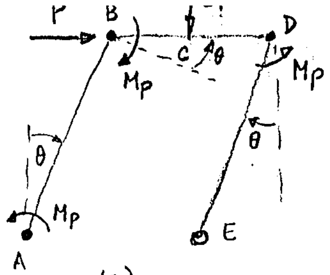


$$\Sigma M_0 = -Ve + F_1\cdot 2c - 2F_3\cdot b = 0$$

$$= -Ve + \frac{Vtbc}{2I}(b+3c)\cdot 2c - 2\cdot\frac{5}{6I}Vtc^3\cdot b$$

$$e = \frac{tbc^2(b+3c) - 5/3\,tc^3b}{\frac{64}{12}tc^3 + 2btc^2} = \frac{b}{2}\frac{\left(b+4/3c\right)}{\left(b+8/3c\right)}$$





(a)

$$P\cdot 2\theta L = M_p\theta + M_p\theta + M_p\theta$$

$$P = \frac{3M_p}{2L}$$

צריך לברר את M_c כאשר

$M_A = M_D = M_p$

$M_B = -M_p$

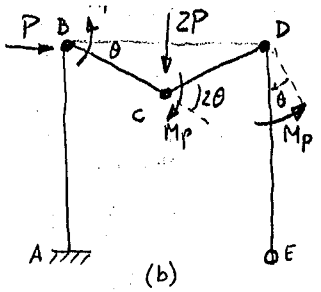
$$M_A - 2M_c + 2M_D = 4PL$$

$$M_p - 2M_c + 2M_p = 4\left(\frac{3M_p}{2L}\right)L$$

$$M_c = -\frac{3}{2}M_p$$

המנגנון בלתי אפשרי

$$|M_c| > M_p$$



(b)

$$2P\cdot L\theta = M_p\theta + M_p\cdot 2\theta + M_p\theta$$

$$P = 2M_p/L$$

צריך לבדוק את M_A כאשר

$M_B = M_D = M_p$, $M_c = -M_p$

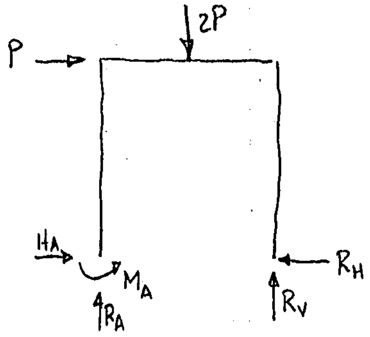
$$M_A - 2M_c + 2M_D = 4PL$$

$$M_A + 2M_p + 2M_p = 4\left(\frac{2M_p}{L}\right)L$$

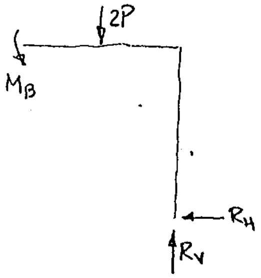
$$M_A = 3M_p$$

המנגנון בלתי אפשרי

$$M_A > M_p$$

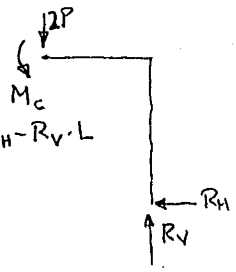


$$M_A = -R_V\cdot 2L + P(2L) + 2P\cdot L$$



$$M_B = 2P\cdot L + R_H\cdot 2L - R_V\cdot 2L$$

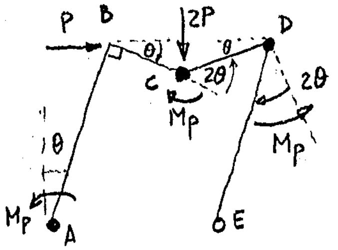
$$M_D = +2L\cdot R_H$$



$$M_c = 2L\cdot R_H - R_V\cdot L$$

$$\Rightarrow M_A - 2M_c + 2M_D = 4PL$$

$$\Rightarrow M_B - 2M_c + M_D = 2PL$$



$$P\cdot 2L\theta + 2P\cdot L\theta = M_p\theta + M_p\cdot 2\theta + M_p\cdot 2\theta$$

$$P = \frac{5M_p}{4L}$$

צריך לבדוק את M_B כאשר

$M_c = -M_p$, $M_A = M_D = M_p$

$$M_B - 2M_c + M_D = 2PL$$

$$M_B + 2M_p + M_p = 2\left(\frac{5M_p}{4L}\right)\cdot L$$

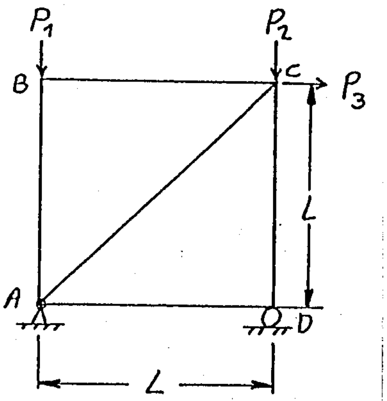
$$M_B = -\frac{M_p}{2}$$

מפני ש- M_B פחות מ- M_p ו- P הוא הפחות ביותר
האירוע הזה יכול לקרות

תורת החוזק 2

תרגיל מס 1

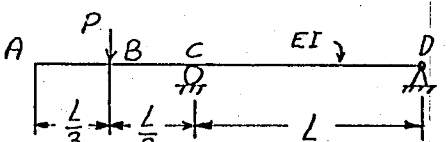
1. For the pin-connected truss loaded as shown (where AE is the same for all members) determine, by means of Castigliano's second theorem :



(a) the vertical displacement of point B,
(b) the horizontal displacement of point C,
(c) the vertical displacment of point C.

Answers : (a) $\frac{P_1L}{AE}$, (b) $[P_2+(1+2\sqrt{2})P_3]\ \frac{L}{AE}$, (c) $(P_2+P_3)\ \frac{L}{AE}$

2. By means of Castigliano's Second Theorem, determine (in terms of E, I, L and P), for the linear elastic beam shown :

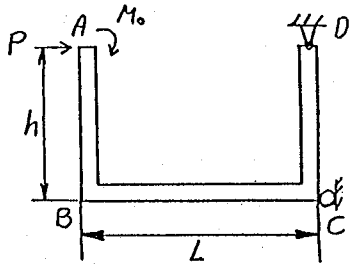


(a) the vertical displacement of point B.
(b) the vertical displacement of point A.
(c) the rotation of point A.

Answers : (a) $\Delta_B = \frac{4PL^3}{81EI}\downarrow$, (b) $\Delta_A = \frac{17PL^3}{162EI}\downarrow$, (c) $\theta_A = \frac{PL^2}{6EI}$ ⤸

3. An elastic structural member ABCD, pinned at C and D, is subjected to an applied moment M_0 at A and a horizontal force P.

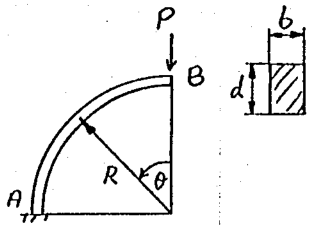
(a) Using Castigliano's theorem, determine the horizontal component of displacement of point A in terms of M_0, P, E, I, L and h. Consider only flexural effects.



(b) From the answer to part (a) above, determine the angle which member AB makes with the vertical at point A due to a horizontal force P = 1 applied at A.

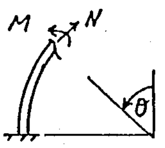
Answer : (a) $\Delta_A = \frac{1}{EI}\left[\frac{M_0h}{6}(5h+6L) + \frac{Ph^2}{3}(3L+2h)\right]$

4. For a curved beam whose lateral dimensions are small with respect to the radius of curvature, the flexural and normal strain energy may be expressed as



$$U = \frac{1}{2}\int\frac{M^2}{EI}ds + \frac{1}{2}\int\frac{N^2}{AE}ds$$

where A and I are the area and moment of inertia of the cross section and where M = M(θ), N = N(θ) are the moments and axial forces at a cross-section.



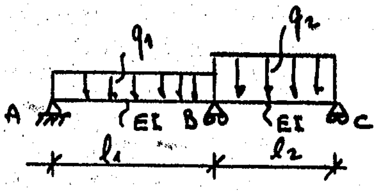
Note : The above integrals represent the elastic strain energy due to flexural and axial deformation respectively.

(a) Using Castigliano's theorem, determine the horizontal component of deflection of point B due to a vertical load.

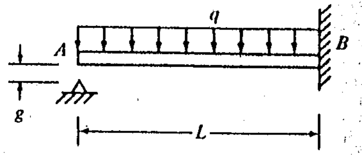
Answer : $\Delta_B = \frac{PR^3}{2EI} - \frac{PR}{2AE}$

תורת החוזק 2
תרגיל מס 2

1. קורה על שלושה סמכים עמוסה לפי הציור.
חשב\י את הריאקציות ו M_B בשני אופנים : א. הנעלם הוא R_A
ב. הנעלם הוא M_B

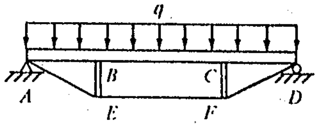


7.45 The uniform cantilever beam shown makes contact with the support at *A* only after gap *g* is closed by application of load. What is the reaction at *A* in terms of *q*, *L*, *g*, and *EI*? What is the largest *g* for which your answer is valid?

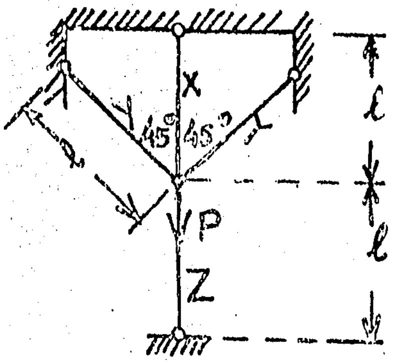


PROBLEM 7.45

7.67 The queen-post truss shown consists of beam *ABCD*, compression members *BE* and *CF*, and a cable *AEFD* that is attached to ends *A* and *D* of the beam. A stress analysis under uniform load *q* is required. Outline an analysis procedure in detail. State what effects you will ignore and why.



PROBLEM 7.67



4. נתון. המסבך הסימטרי המתואר בציור.
חתכי כל המוטות (והחומר) שווים.
דרוש. א) לחשב את הכוחות במוטות
ב) כנ"ל כאשר אורך המוט Z הוא רק $\ell/2$.

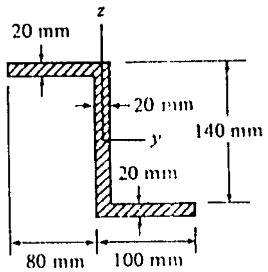
תשובה. א) $Y=P/(3\sqrt{2});\ X = -Z = P/3$

ב) $X = P/4;\ Z = -2X;\ Y = P\sqrt{2}/8$

תורת החוזק 2

תרגיל מס 2

*9.3 (a) For the Z section shown, calculate I_y, I_z, and I_{yz}. Also locate the principal centroidal axes and calculate the principal moments of inertia.



(a)

PROBLEM 9.3

9.9 What centroidal axial force must be applied to the beam of Fig. 9.2.3 to make $|\sigma_{xA}| = |\sigma_{xB}|$ after bending and axial stresses are superposed?

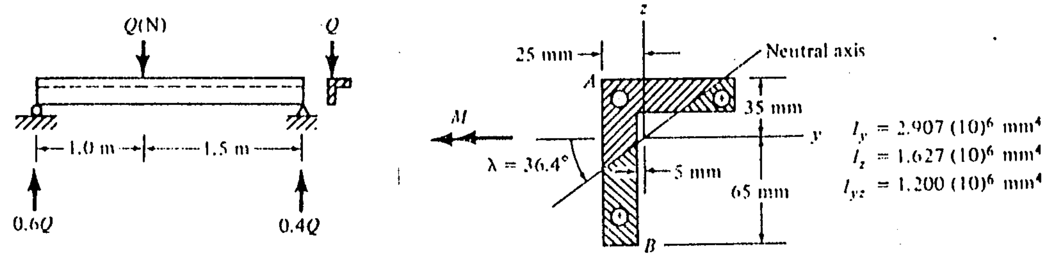
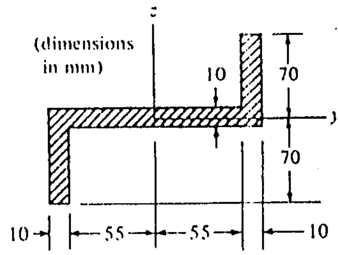


FIGURE 9.2.3.

*9.13 (a) A cantilever beam 1.6 m long has the Z section shown. It carries a load P = 2500 N in the $-z$ direction at the free end. Find the maximum magnitude of flexural stress (either tensile or compressive). Where does this stress appear?

(b) Repeat part (a) with load P redirected so that it acts in the $-y$ direction.

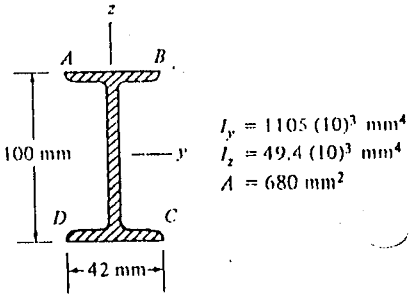
Answer: (a) 184 MPa y = 55mm z = 70mm



PROBLEM 9.13

*9.15 A column has the I section shown. A compressive force P parallel to the axis of the column is applied at corner A. Find axial stress σ_x in terms of P at the remaining three corners B, C, and D.

σ_c = 0.00972P (MPa if P in N).

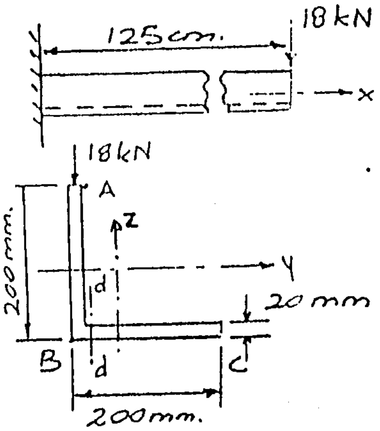


PROBLEM 9.15

5. A structural angle (L200 x 200 x 20) is used as a horizontal cantilever beam 125 cm long, and is subjected to a vertical concentrated load of 18,000 N acting through the shear center of the cross section as shown below.

(a) Calculate the flexural stress σ_x at the points A, B and C at the fixed end of the beam, as well as the average shear stress τ acting along the line d-d of this cross section (see figure).

(b) Find the position of the Neutral Axis and the direction of the deflection. Show on a sketch of the cross-section, indicating the orientation with respect to the y-axis.



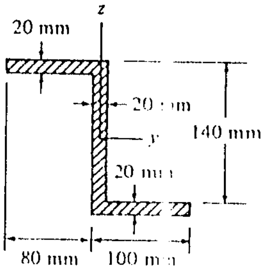
Given : $I_{zz} = I_{yy}$ = 2875 cm^4,

Answer: σ_{xA} = 145.6 MPa, σ_{xB} = - 109.3MPa, |τ| = 2.9MPa

תורת החוזק 2

תרגיל מס 2

*9.3 (a) For the Z section shown, calculate I_y, I_z, and I_{yz}. Also locate the principal centroidal axes and calculate the principal moments of inertia.



(a)

PROBLEM 9.3

9.9 What centroidal axial force must be applied to the beam of Fig. 9.2.3 to make $|\sigma_{xA}| = |\sigma_{xB}|$ after bending and axial stresses are superposed?

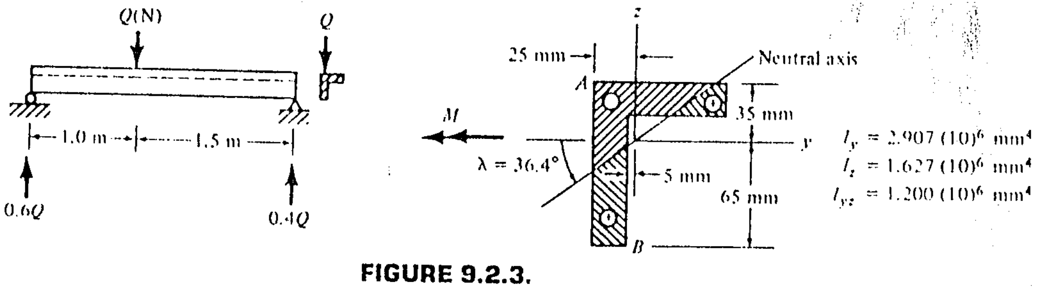
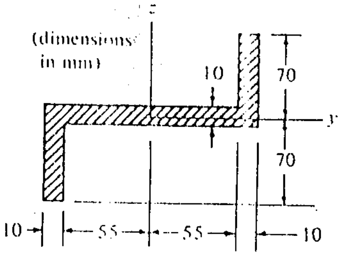


FIGURE 9.2.3.

*9.13 (a) A cantilever beam 1.6 m long has the Z section shown. It carries a load $P = 2500$ N in the $-z$ direction at the free end. Find the maximum magnitude of flexural stress (either tensile or compressive). Where does this stress appear?

(b) Repeat part (a) with load P redirected so that it acts in the $-y$ direction.

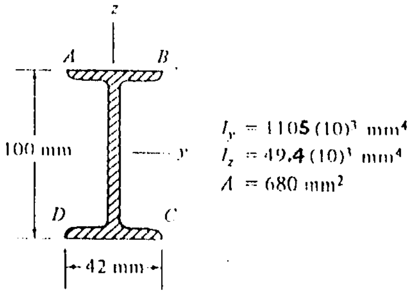
Answer: (a) 184 MPa $y = 55$mm $z = 70$mm



PROBLEM 9.13

*9.15 A column has the I section shown. A compressive force P parallel to the axis of the column is applied at corner A. Find axial stress σ_x in terms of P at the remaining three corners B, C, and D.

$\sigma_c = 0.00972P$ (MPa if P in N).

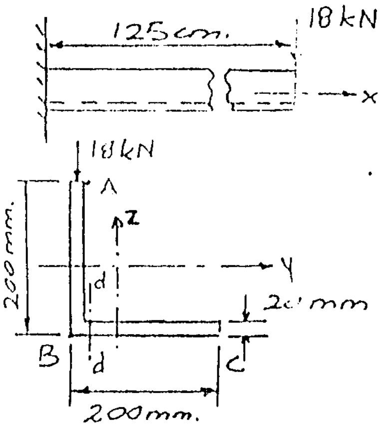


PROBLEM 9.15

5. A structural angle (L200 x 200 x 20) is used as a horizontal cantilever beam 125 cm long, and is subjected to a vertical concentrated load of 18,000 N acting through the shear center of the cross section as shown below.

(a) Calculate the flexural stress σ_x at the points A, B and C at the fixed end of the beam, as well as the average shear stress τ acting along the line d-d of this cross section (see figure).

(b) Find the position of the Neutral Axis and the direction of the deflection. Show on a sketch of the cross-section, indicating the orientation with respect to the y-axis.



Given : $I_{zz} = I_{yy} = 2875$ cm^4,

Answer: $\sigma_{xA} = 145.6$ MPa, $\sigma_{xB} = -109.3$MPa, $|\tau| = 2.9$MPa

1. Write the second-order differential equation for the bending of the column shown in Fig. P1-2 and use it to determine the critical load of the column. At its lower end the column is completely fixed. At the upper end the column is prevented from rotating, but free to translate laterally... (ans: $P_{cr}=\pi^2EI/L^2$)

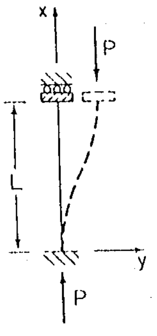
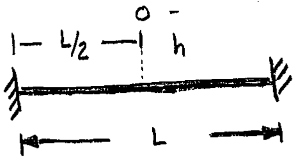


Fig. P1-2

2. Find an expression for the maximum stress when a ball weighing W Newtons is dropped onto a fixed-fixed beam.



3. A linearly elastic beam-column having a flexural rigidity EI, is subjected to a thrust P and a moment M_0 as shown in Fig. A below.

(a) Determine the lateral displacement v(x).

(b) From part (a), write the solution for the system subjected to a force P acting as shown in Fig. B.

(c) Determine Δ_c, the horizontal displacement of point C, assuming small rotations. (Assume also that the horizontal displacement of point B is negligible).

(d) Determine the bending moment M(x).

(g) Explain why, although the beam is made of a linearly elastic material, the results of part (c) are non-linear.

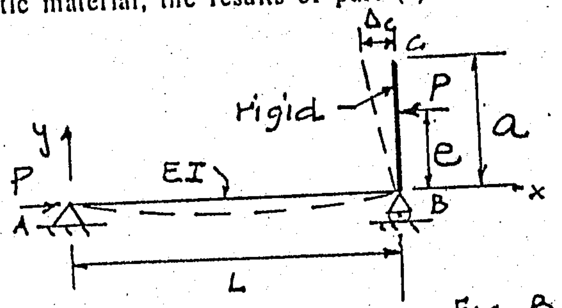


Fig. B

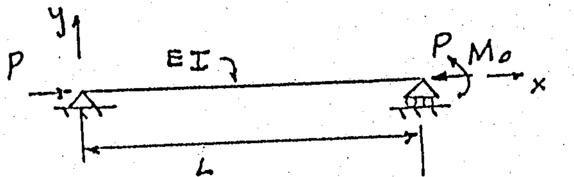


Fig. A

Answers :

$$(a)\ y(x)=-\frac{M_0}{P}\left[\frac{\sin Kx}{\sin KL}-\frac{x}{L}\right],\ K^2=\frac{P}{EI}$$

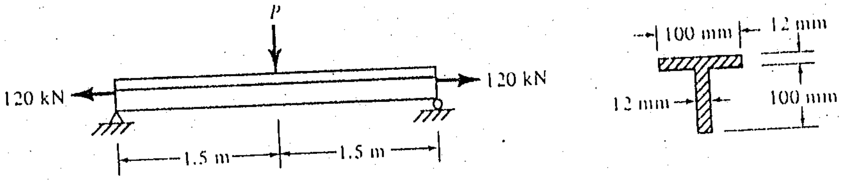
$$(c)\ \Delta_c=\frac{ac}{L}\left[1-L\sqrt{P/EI}\cot\left(L\sqrt{P/EI}\right)\right]=\frac{ac}{L}(1-KL\cot KL)$$

(d) $M(x)=M_0 \sin kl/\sin kL$

*12.10 A T section carries an axial tensile force of 120 kN, applied through the centroid of the cross section. The allowable stress in tension or compression is 130 MPa. Let E = 200 GPa. What transverse force P can be applied at midspan if the beam is

(a) Stem down (as shown)?

(b) Stem up?



PROBLEM 12.10

תורת החוזק 2
תרגיל מס 9

3.1 Obtain expressions for the maximum deflection and maximum moment of a beam column whose ends are built in and that is loaded with a concentrated load at midspan as shown in Fig. P3-1.

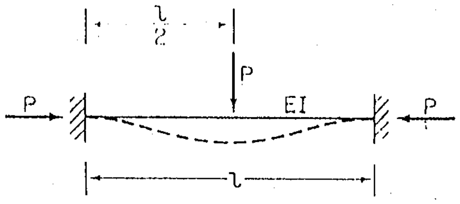


Fig. P3-1

2. A linearly elastic beam-column having a flexural rigidity EI, is subjected to a thrust P and a moment M_0 as shown in Fig. A below.

(a) Determine the lateral displacement v(x).

(b) From part (a), write the solution for the system subjected to a force P acting as shown in Fig. B.

(c) Determine Δ_c, the horizontal displacement of point C, assuming small rotations. (Assume also that the horizontal displacement of point B is negligible).

(d) Determine the bending moment M(x).

(g) Explain why, although the beam is made of a linearly elastic material, the results of part (c) are non-linear.

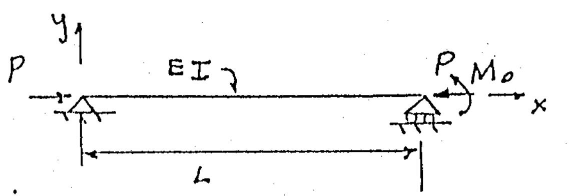


Fig. A

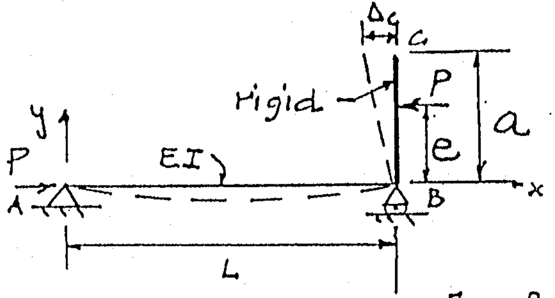


Fig. B

Answers :

(a) $y(x) = -\frac{M_0}{P}\left[\frac{\sin Kx}{\sin KL} - \frac{x}{L}\right], K^2 = \frac{P}{EI}$

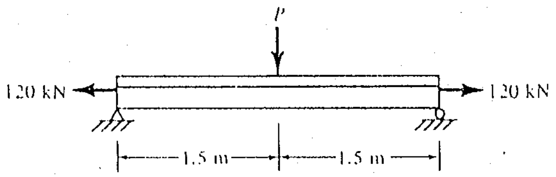
(c) $\Delta_c = \frac{ac}{L}\,[1 - L\sqrt{P/EI}\,\cot(L\sqrt{P/EI})\,] = \frac{ac}{L}\,(1 - KL\cot KL\,)$

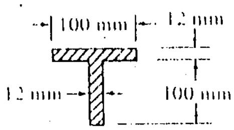
(d) $M(x)=M_0 \sin kl/\sin kL$

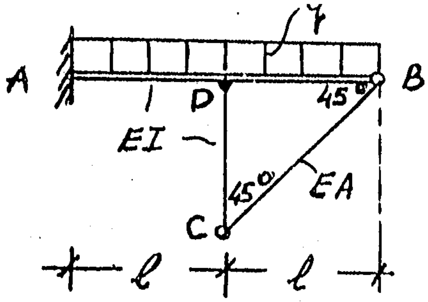
*12.10 A T section carries an axial tensile force of 120 kN, applied through the centroid of the cross section. The allowable stress in tension or compression is 130 MPa. Let E = 200 GPa. What transverse force P can be applied at midspan if the beam is

(a) Stem down (as shown)?

(b) Stem up?







נתון. המבנה המתואר בציור מורכב מקונסול AB אליו מחובר ב-D באופן קשיח העמוד DC והמוט הפרקי BC. הקונסול עמוס עומס q .

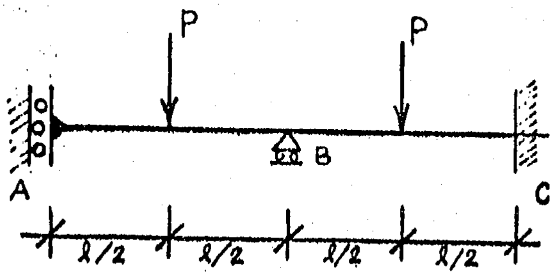
דרוש. א) לחשב את הכוח X הפועל במוט BC בהנחת $\frac{I}{A\ell^2} = 0.04$

ב) לחשב ולשרטט מהלך M , S ו- N עבור המבנה; 'לבדוק שווי משקל של הצומת D .

ג) מה גודלו של X כאשר העומס היחיד הוא עומס אנכי P הפועל על הקונסול ב- D . הסבר .

תשובה. א) $X = -\frac{q\ell\sqrt{2}}{16(\frac{1}{3}+\frac{I\sqrt{2}}{\ell^2 A})} = -0.23\,q\ell$

ב) $M_{DC} = 0.163\,q\ell^2; \quad -M_{DB} = 0.337\,q\ell^2; \quad -M_{DA} = \frac{q\ell^2}{2}$ ג) $X = 0$.



תון. הקורה הנמשכת לפי הציור.
- A סמך הנע בכיוון אנכי ויחד
ם זה שומר על משיק אופקי של הקו
האלסטי של הקורה (ז"א קיים שם כעין
ריתום נייד). ב- C ריתום רגיל.

רוש: א) לחשב ולשרטט מהלכי S ו- M
ולשרטט את צורת הקורה לאחר הדיפורמציה.
) לחשב הזזת A .

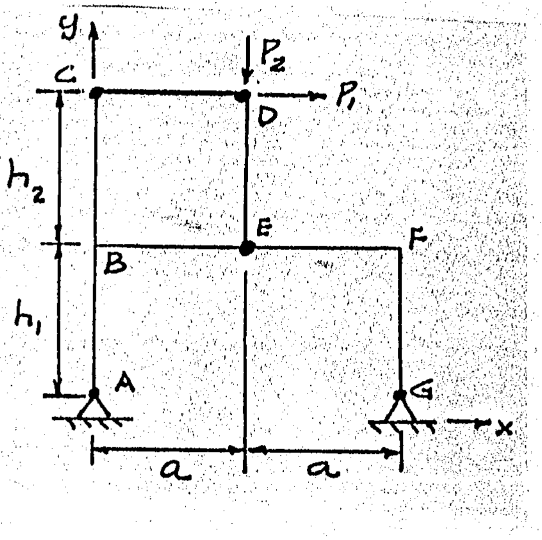
שובה. א) $M_A = +7P\ell/40\ ; \ M_C = -P\ell/40 \quad (H_A = 0)$

) $\downarrow\delta_A = 1\,P\ell^3/(45\,EI)$

The statically determinate structure shown consists of elements rigidly connected at B and F and connected by hinges at A, C, D, E and G. Loads P_1 and P_2 are applied at point D.

a) Using the Principle of Virtual Work, determine
  (i) the reactions R_{xA} and R_{yA},
  (ii) the moment M_B in member AB at point B,
  (iii) the shear force V_F in member FG at point F.

Note: For each case show, by means of a sketch, the virtually displaced structure which is used (with respect to the original structure) such that only the desired unknown appears in the virtual work expression.



תורת החוזק 2

תרגיל מס 8

4.2 Using reasoning similar to that employed in Article 4.2*, determine the limiting values for the symmetric and sidesway buckling loads of the frame in Fig. P4-2.

4.3 Letting $l_c = l_b = l$ and $I_c = I_b = I$, determine the critical load of the frame in Fig. P4-2 for
(a) the sidesway mode,
(b) the symmetric mode.
Find the critical load by setting up and solving the governing differential equations.

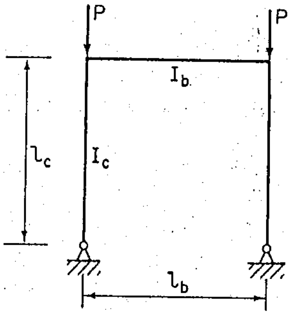
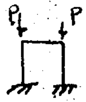


Fig. P4-2

* presented in class for



Moment of Inertia transformation

1. טרנספורמציה מומנטי אינרציה

bending moment & axial force combined action

2. כפיפה משולבת, פעולה משולבת של כוח צירי ומומנט כפיפה, גרעין החתך

infinitesimal section

non homogeneous sections

3. חתכים בלתי הומוגניים

פלסטית

elastic load carrying strength, theories

4. תורות חוזק, נשיאות ~~אלסטית~~ של חתכים

capacity of sections

5. התנהגות פלסטית של חתכים, נשיאות פלסטית

plastic load carrying behavior plastic behavior of sections

6. אנרגיה במבנים, עבודה של עומסים חיצוניים, אנרגיה אלסטית,

elastic energy work of outer loads energy in structures

משפט קלפירון, משפט מקסוול-בטי, עקרון מינימום האנרגיה

minimum Energy principle Maxwell Betti Clapeyron's law (3moment law in Popov)

7. עקרון קסטיליאנו (מינימום האנרגיה האלסטית), משפט קסטיליאנו

Castigliano's law min. elastic energy castigliano principle

אינטגרל מוהר ושימושו לחישוב העתקות

Mohr Integral and its use to calculate displacements

8. עקרון שימור האנרגיה ושימושו לחישוב כוחות נגיפה

and its use to calculate ~~[illegible]~~ impact force energy plane principle

9. קריסה צירית, שיטת אוילר לקביעת העומס הקריטי (משוואת הקריסה)

Buckling Equation Euler Method to fix critical load Axial Buckling

עמודים ומסגרות, השפעת תנאים על משוואות הקריסה וכוח קריטי,

to effect of fixity conditions on the buckling equation & crit. load For column & frame

~~משוואות קריסה ללא קורה, קריסה ללא משוואת קריסה~~

10 ? עומס קריטי של מערכות ~~עמודים וקורות~~, עמוד-קורה, ~~שיטת האנרגיה~~

energy method to find critical load beam column for column & beam arrangements crit. load

~~למציאת העומס הקריטי~~

~~11.~~ שיטות חישוב לקריסה צירית עמודים ועמודים-קורות לפי מפרטי (ת"י 1225

according to tech. spec. & beam col. columns for axial buckling Calculation method.

(AISC , DIN

12. מושגים ראשוניים של קריסה צידית, קריסה מקומית, סיכום קורס

local buckling sideways buckling first ~~[illegible]~~ of Concepts

22.03.98

2. בלוק איטונג - 2
3. בלוק בטון מלא - 2
4. לבני חול - 10
5. בלוק עפ פומיס - 2 קלון עם מילוי

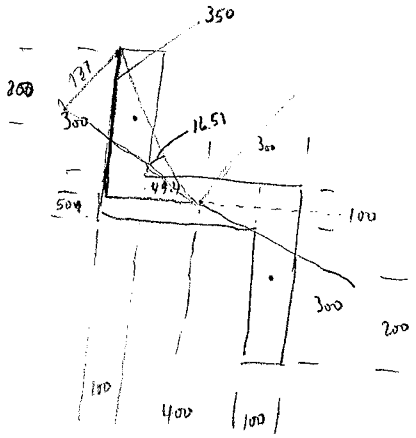
II כלי עבודה

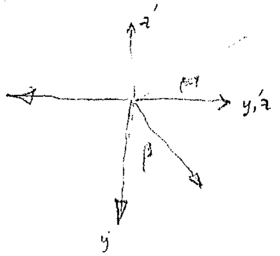
1. סרגלי פלדה 300mm - 5
2. מסור אבניט
3. מברשת ברונזה
4. מברשת ברונזה למקדחה Ø50-60 mm
5. שמן הידראולי — 1 ליטר

$\int d$

$$= -\int y \frac{My}{I} dA = -\frac{M}{I}\int y^2 dA$$

$\frac{My}{I} = \sigma$





$$\frac{(M_z I_z + M_y I_{yz})y + (M_y I_y + M_z I_{yz})z}{I_y I_z - I_{yz}^2}$$

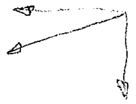
PdV	C+Y
y	-z
z	y

$I_y \to I_z$

$I_z \to I_y$

$I_{yz} \to -I_{yz}$

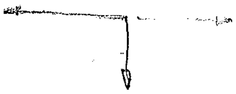
$$\frac{PL(\overset{9.83}{\cancel{\;\;}}\times\cancel{10^{6}})\,80 - PL\,\cancel{\;\;}3\times\cancel{10^{6}}\,70}{[22.613\cdot 9.813 - (11.2)^2]} \times 10^{-6}$$



$$+\frac{(+M_y I_z + M_{+z} I_{yz})(z) + (-M_{+z} I_y + M_y I_{yz})(y)}{I_z I_y - I_{yz}^2}$$

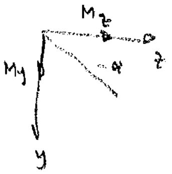
~~$-(M_z I_y + M_y I_{yz})y + (M_y I_z$~~

$$\frac{PL\; 9.813\cdot 80 - PL\; 11.2\cdot 90}{22.613\times 9.813 - 11.2^2}$$



$$-\frac{(M_z I_y + M_y I_{yz})y + (M_y I_z + M_z I_{yz})z}{I_y I_z - I_{yz}^2}$$

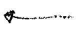
when we replace $\frac{M_z y}{I}$ by $-\frac{M_z y}{I}$



$-\left(\underline{M_{zz} I_{yy} - M_{yy} I_{yz}}\right) y + \left(\underline{M_{yy} I_{zz} - M_{zz} I_{yz}}\right) z$

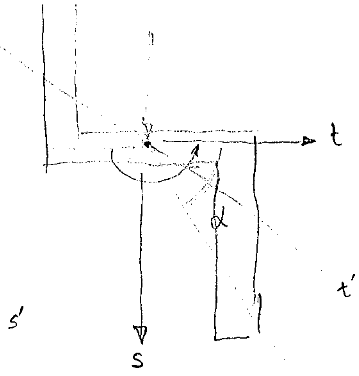
$$\frac{+(M_{zz} I_{yy} - M_{yy} I_{zz})\, y + (M_{yy} \qquad)\, z}{I_y I_z - I_{yz}}$$

=



$$-\frac{\sin 57.11}{I_{t't'}} s' + \frac{\cos 57.11}{I_{s's'}} t' = 0$$

$$t' = -\tan 57.11 \frac{I_{s's'}}{I_{t't'}} s$$



$\longrightarrow M_t = PL \qquad \theta = \cancel{180}°\ 0°$

$$\sigma_x = -\left(\frac{M_t I_{ss} + M_s I_{st}}{I_{ss} I_{tt} - I_{st}^2}\right)s + \left(\frac{M_s I_{tt} + M_t I_{st}}{I_{ss} I_{tt} - I_{st}^2}\right)t$$

$M_s = 0$

$$\sigma_x = -\frac{\left(M_t I_{ss}\cdot s + M_t I_{st}\cdot t\right)}{I_{ss} I_{tt} - I_{st}^2}$$

$-M_t I_{ss} s + M_t I_{st}\cdot t$

$M_t\left(-I_{ss}\cdot s + I_{st}\cdot t\right)$

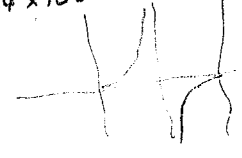
-56·

$$= -PL\,\frac{\left(56\times10^8 s + 30\times10^8 t\right)}{\left(56\times29 - 30^2\right)10^{16}}$$

$$= -PL\,\frac{\left(56s + 30t\right)}{\left(56\times29 - 30^2\right)10^8} = -PL\,\frac{\left(56s - 30t\right)}{724\times108} = -PL\left(.07735s - .04144t\right)\times10^{-8}$$

$(-350) - .04144(300)$

$+14.64\,PL$



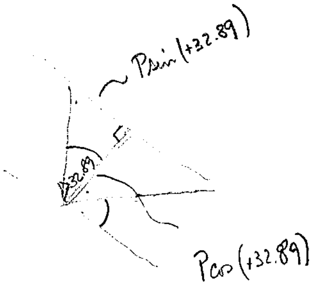
$$\tan\alpha = \frac{I_{tt}\tan\theta + I_{st}}{I_{ss} + I_{st}\tan\theta} = \frac{+I_{st}}{I_{ss}} = \frac{+30\times10^8}{56\times10^8}$$

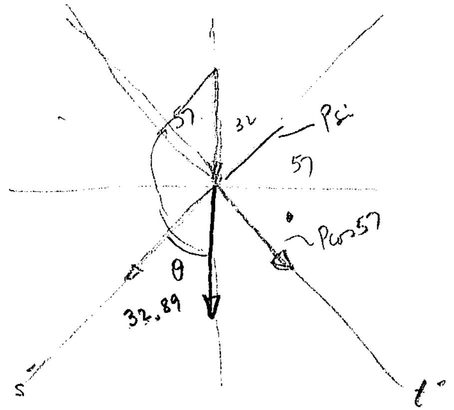
$$\alpha = \tan^{-1}\left(\frac{+30}{56}\right) = +28.11°$$

$-PL\left(.07735(400)\ \overset{-}{\oplus}\ .04144(-300)\right)$

$-PL\begin{pmatrix} -18.508 \\ \cancel{18.508} \\ -\cancel{43.372}\end{pmatrix}\times10^{-8}$

~~-42.427~~





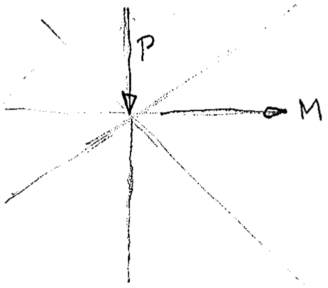
$122.89 = \theta$

$\therefore\ M_t = PL\cos \cancel{1}22.89 = -.543\,PL$

$M_s = PL\sin 122.89 = .223\,PL$

$$\sigma_x = -\frac{\left(M_t I_{ss}\right)s + M_s I_{tt}\cdot t}{I_{ss} I_{tt}}$$

$$= \frac{-M_t\ s}{I_{tt}} + \frac{M_s\cdot t}{I_{ss}}$$



$PL\sin 122.89 = PL\sin 57.11$

$PL\cos 122.89 = -PL\cos 57.11$

$\sin(90+\alpha) = \cos\alpha = \cos 32.$

$\cos(90-\beta) = \sin\beta = \sin 57.11$

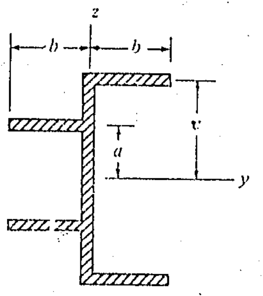
$\cos(90+\alpha) = -\sin 32.89$

$-\sin(90-\beta) = -\cos 57.11$

תורת החוזק 2
תרגיל מס 6.

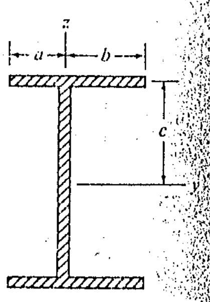
9.32, *9.33, 9.34, 9.35, *9.36, 9.37–9.40, *9.41, *9.42, 9.43

The open cross sections shown are all thin-walled, of constant thickness, and have one axis of symmetry. Assume a frictionless contact where overlaps appear. In each case find an expression for the location of the shear center. Check your answer by examining limiting cases where possible (as by letting dimension b approach zero, for example).

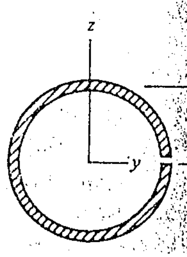


PROBLEM 9.33

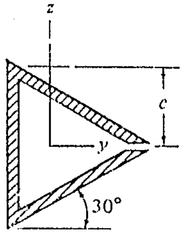
ans. $y = -\dfrac{3b^2(c^2-a^2)}{2(c^3+3b(a^2+c^2))}$



PROBLEM 9.34



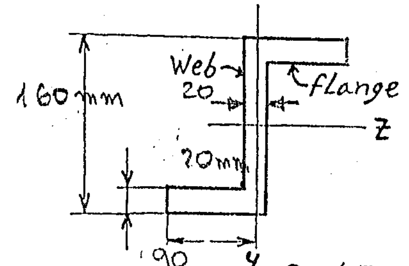
PROBLEM 9.40



PROBLEM 9.42

at 0.577c left to the vertical web

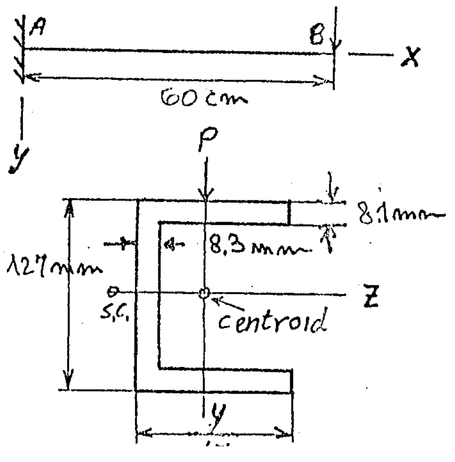
2. (a) Calculate the shear stress distribution acting on a Z-section as shown when subjected to a vertical shear force V_y which passes through the shear center.
(b) Plot the variation of the shear stress τ/V_y along the flanges and the web. Indicate the shear flow by means of a figure.
(c) What is the location of the shear center of this section ? Explain the reasoning which justifies your answer.



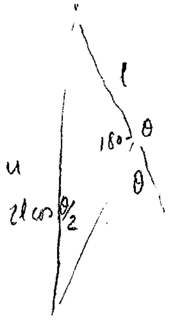
Given : $I_{zz} = 20.25 \cdot 10^6 \, mm^4$, $I_{yy} = 9.71 \cdot 10^6 \, mm^4$, $I_{yz} = -8.9 \cdot 10^6 \, mm^4$

3. An aluminium channel is used as a cantilever 60cm long and has an end load of 4.5 kN passing through the centroid as shown.

(a) Locate the shear center
(b) Determine the angle of twist at the end B of the beam.
(c) Find the maximum shear stress in the section x=0 including the effects of both torsion and flexure.



Given : G = 26 GPa, $I_{zz} = 3.7 \cdot 10^6 \, mm^4$ $\bar{z} = 12.14$ mm



$\frac{1}{2}M\theta$

$$u = \sqrt{2l^2 - 2l^2\cos(180-\theta)}$$
$$2l^2 + 2l^2\cos\theta$$
$$\sqrt{4l^2\left(\frac{1+\cos\frac{2\theta}{2}}{2}\right)} = 2l\cos\theta/2$$
$$\sqrt{\frac{1+\cos\theta}{2}} = \cos\theta/2$$

$$P(2l^{\cancel{2}} - 2l\cos\theta/2) - \frac{1}{2}M\theta$$

עבודת העומס ועבודת הקפיץ

$$2lP\left(1 - \left[1 - \theta^2/8\right]\right) - \frac{1}{2}C\theta^2$$

$\cos\theta/2 \sim 1 - (\theta/2)^2/2! + \cdots$

$$2lP\left(\theta^2/8\right) - \frac{1}{2}C\theta^2$$
$$lP\theta^2/4 - \frac{1}{2}C\theta^2$$
$$\left[\frac{lP}{4} - \frac{C}{2}\right]\theta^2 = \Pi$$
$$\frac{\partial\Pi}{\partial\theta} = \left(\frac{lP}{4} - \frac{C}{2}\right)2\theta = 0 \qquad \theta = 0 \qquad P = \frac{2C}{l}$$
$$\frac{\partial^2\Pi}{\partial\theta^2} = 2\left(\frac{lP}{4} - \frac{C}{2}\right) > 0 \qquad P > \frac{2C}{l} \quad \text{stable}$$

תורת החוזק 2

תרגיל מס 7

1.3 Find the critical load of the one-degree-of-freedom model of a column shown in Fig. P1-3. The model consists of two rigid bars pin connected to each other and to the supports. A linear rotational spring of stiffness $C = M/\theta$, where M is the moment at the spring and θ is the angle between the two bars, also connects the two bars to each other. ($P_{cr} = 2C/L$)

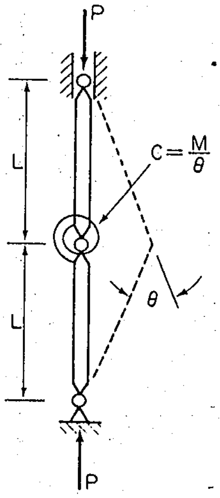


Fig. P1-3

2. Write the second-order differential equation for the bending of the column shown in Fig. P1-2 and use it to determine the critical load of the column. At its lower end the column is completely fixed. At its upper end the column is prevented from rotating, but free to translate laterally. (ans: $P_{cr}=\pi^2EI/L^2$)

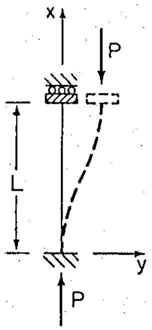
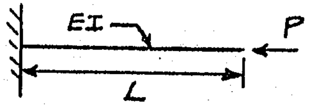
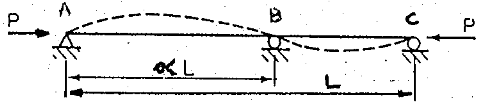


Fig. P1-2

3. Determine the critical load of instability for the elastic column shown.
(ans: $P_{cr}=n^2\pi^2EI/4L^2$)



4. An elastic road ABC having flexural rigidity EI is simply supported as shown and
is subjected to an axial load P.



a) Obtain the following characteristic equation for the eigenvalue k corresponding to the critical load P

$$\alpha kL(1-\alpha)\sin(kl)-\sin(\alpha kL)\sin[(1-\alpha)kL]=0, \quad k^2=P/EI$$

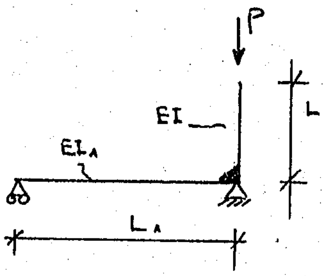
(Hint: Write separate equations for each span and make use of the conditions of continuity at the center support).

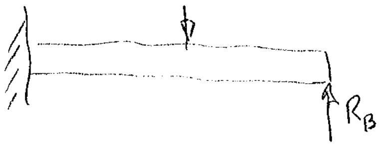
b) From the above characteristic equation, show that $P_{cr}=4\pi^2EI/L^2$ when $\alpha=0.5$, that is when the support B is at the midspan.

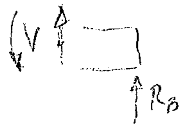
c) show that as $\alpha\rightarrow0$, the characteristic equation reduces to $\tan(kL)=kL$. Verify that the first root is $kL\approx4.49$ and show that the corresponding critical load is given by $P_{cr}=\pi^2EI/(0.7L)^2$.

5.

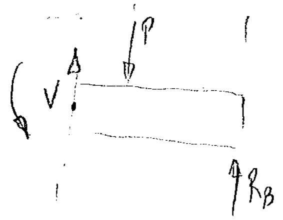
עמוד AB עם קצה חופשי A רתום באופן קשיח בקורה BC ועמוס בכוח צירי P כמתואר בציור.
מצא\י את עומס הקריסה ואת אורך הקריסה (האורך האפקטיבי Le) של המוט.
(ת. כאשר $L_1=L$, $Le=2.65L$)







$M = -R_A(L-x)$ $\quad \frac{L}{2} \le x \le L$



$M = -R_B(L-x) + P(L/2 - x)$

$0 < x \le L/2$

$$\frac{1}{2EI}\int_{L/2}^{L}\left[-R_A(L-x)\right]^2 + \int_0^{L/2}\left[-R_B(L-x)+P(L/2-x)\right]^2 dx$$

$$\int_{L/2}^{L}\left[R_B(L-x)\right](L-x)\,dx + \int_0^{L/2}\left[\qquad\right](-(L-x)\,dx$$

$$-R_B\frac{(L-x)^3}{3}\Bigg|_{L/2}^{L} + \int_0^{L/2}\left[R_B(L-x)^2 - P(L/2-x)(L-x)\right]dx$$

$$L^2/2 - \tfrac{3}{2}Lx + x^2$$

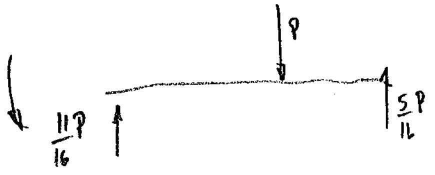
$$+\frac{R_B}{3}\left(\frac{L^3}{8}\right) + -R_B\frac{(L-x)^3}{3}\Bigg|_0^{L/2} - P\left(\frac{L^2}{2}x - \frac{3}{4}Lx^2 + \frac{x^3}{3}\right)\Bigg|_0^{L/2}$$

$$-R_B\left(\cancel{\frac{L^3}{24}} - \frac{L^3}{3}\right) - P\left(\frac{L^3}{4} - \frac{3L^3}{16} + \frac{L^3}{24}\right)$$

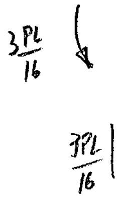
$$R_B\frac{L^3}{3} = \frac{L^3}{24}P\left(6 - \frac{9}{2} + 1\right)$$

$$R_B\frac{16L^3}{48} = \frac{5}{48}PL^3 \quad \frac{5}{[illegible]}$$

$$R_B = \frac{5}{16}L^3$$



$$M = 8PL/16 - \frac{5}{16}PL = \frac{3PL}{16}$$



$$-R_A(L) + \frac{qL^2}{2}$$

$$-\frac{3qL^2}{8} + \frac{qL^2}{2} = \frac{qL^2}{8} = \frac{16qL^2}{128}$$

$$\frac{1}{EI}\left(\frac{1{,}000 + \frac{1}{3}(100)M_B}{10}\right) = -\frac{1}{EI}\left(\frac{2{,}880 + 108M_B + 54M_C}{18}\right)$$

or

$$(\tfrac{28}{3})M_B + 3M_C = -260$$

Using condition (b) for the span BC provides another equation, $t_{BC} = 0$, or

$$\frac{1}{EI}\left[\frac{(18)}{2}(+40)\frac{(18+12)}{3} + \frac{(18)(+M_B)}{2}\frac{(18)}{3} + \frac{(18)(+M_C)}{2}\frac{2(18)}{3}\right] = 0$$

or

$$3M_B + 6M_C = -200$$

Solving the two reduced equations simultaneously,

$$M_B = -20.4 \text{ ft-lb} \qquad \text{and} \qquad M_C = -23.3 \text{ ft-lb}$$

where the signs agree with the convention of signs used for beams. These moments with their proper sense are shown in Fig. 12-17(b).

After the redundant moments M_A and M_C are found, no new techniques are necessary to construct the moment and shear diagrams. However, particular care must be exercised to include the moments at the supports while computing shears and reactions. Usually, isolated beams as shown in Fig. 12-17(b) are the most convenient free bodies for determining shears. Reactions follow by adding the shears on the adjoining beams. In units of kips and feet, for free body AB:

$$\sum M_B = 0 \circlearrowleft +, \quad 2.4(10)5 - 20.4 - 10R_A = 0, \quad R_A = 9.96 \text{ kips} \uparrow$$

$$\sum M_A = 0 \circlearrowright +, \quad 2.4(10)5 + 20.4 - 10V_B' = 0, \quad V_B' = 14.04 \text{ kips} \uparrow$$

For free body BC:

$$\sum M_C = 0 \circlearrowleft +, \qquad 10(6) + 20.4 - 23.3 - 18V_B'' = 0,$$

$$V_B'' = 3.17 \text{ kips} \uparrow$$

$$\sum M_B = 0 \circlearrowright +, \qquad 10(12) - 20.4 + 23.3 - 18V_C = 0,$$

$$V_C = R_C = 6.83 \text{ kips} \uparrow$$

Check:

$$R_A + V_B' = 24 \text{ kips} \uparrow \qquad \text{and} \qquad V_B'' + R_C = 10 \text{ kips} \uparrow$$

From above, $R_B = V_B' + V_B'' = 17.21$ kips ↑.

The complete shear and moment diagrams are in Figs. 12-17(e) and (f), respectively.

12-6. THE THREE-MOMENT EQUATION

Generalizing the procedure used in the preceding example, a recurrence formula, i.e., an equation which may be repeatedly applied for every two adjoining spans, may be derived for continuous beams. For any

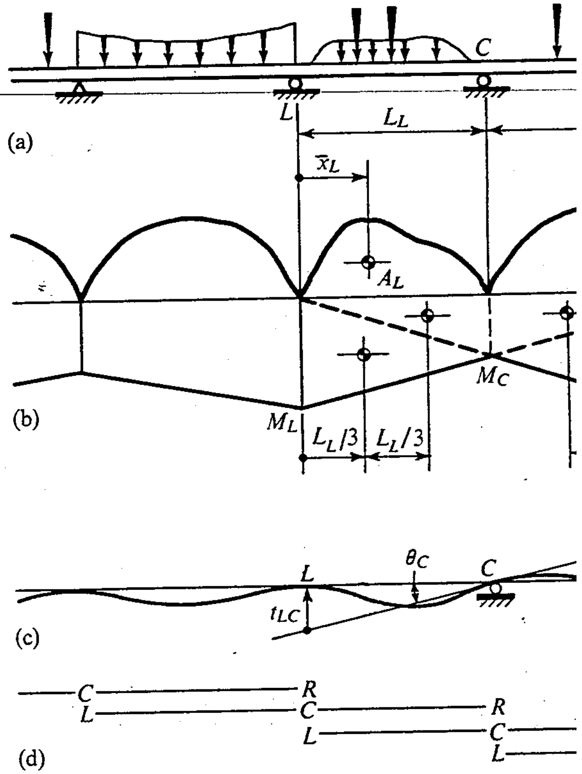


Fig. 12-18. Derivation of the

n number of spans, $n - 1$ such equations may be written. This gives enough simultaneous equations for the solution of redundant moments over the supports. This recurrence formula is called the *three-moment equation* because three unknown moments appear in it.

Consider a continuous beam, such as shown in Fig. 12-18(a), subjected to any transverse loading. For any two adjoining spans, as LC and CR, the bending-moment diagram is considered to consist of two parts. The areas A_L and A_R to the left and to the right of the center support C, Fig. 12-18(b), correspond to the bending-moment diagrams in the respective spans if these spans are treated as being simply supported. These moment diagrams depend entirely upon the nature of the known forces applied within each span. The other part of the moment diagram of known shape is due to the unknown moments M_L at the left support, M_C at the center support, and M_R at the right support.

Next, the elastic curve in Fig. 12-18(c) must be considered. This curve is continuous for any continuous beam. Hence the angles θ_C and θ_C', which define, from the respective sides, the inclination of the same tangent to the elastic curve at C, are equal. By using the second moment-area theorem to obtain t_{LC} and t_{RC}, these angles are defined as $\theta_C = t_{LC}/L_L$ and $\theta_C' = -t_{RC}/L_R$, where L_L and L_R are span lengths on the left and on

$$\frac{1}{EI}\left(\frac{1{,}000 + \frac{1}{3}(100)M_B}{10}\right) = -\frac{1}{EI}\left(\frac{2{,}880 + 108M_B + 54M_C}{18}\right)$$

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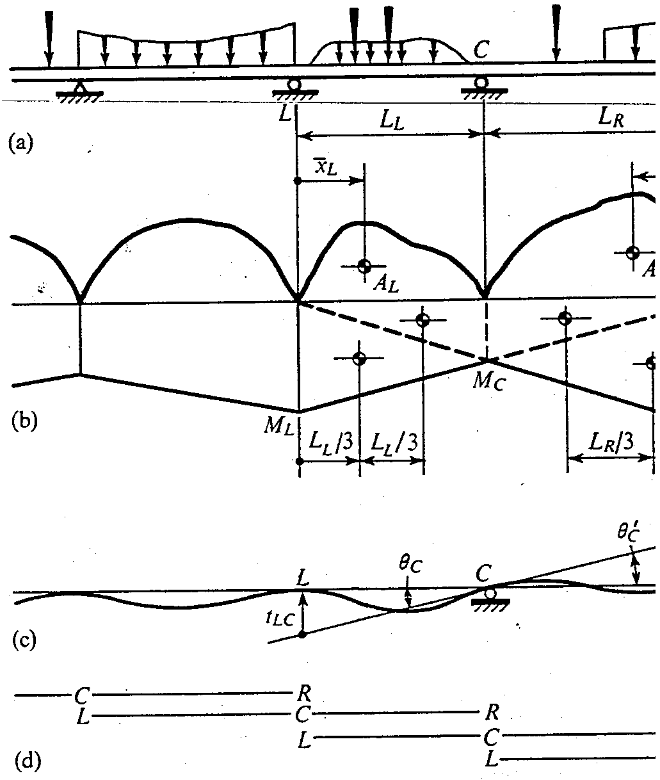

Fig. 12-18. Derivation of the three-m

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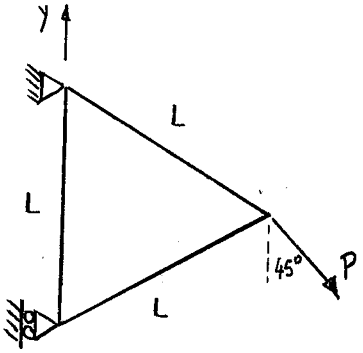
המכללה האקדמית יהודה ושומרון
בוחן ל סטטיקת מבנים
פרופ' עזרא לוי

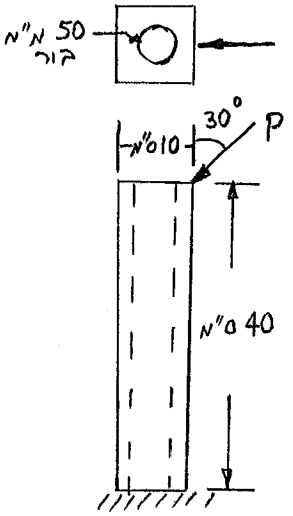
הערות: חומר פתוח. 2 שעות בלבד.

1) עבור EA=const דרוש לחשב את תזוזה אופקית ואנכית הבאות מכוח P, בשיטת אנרגיה. הכוח פועל כבתרשים ואורכו כל צלע וצלע הוא L.



2) לעמוד המתואר בתרשים, בעל שטח חתך רבועי 10 ס"מ x 10 ס"מ ובו קדח בור בקוטר 50 מ"מ. הכוח P פועל כבתרשים והבסיס של העמוד הוא סמך רתום.

דרוש : לחשב את גודל הכוח P שבגללו המאמץ הנורמלי המקסימלי לא יעלה מעל 140 MPa. לא להתייחס למשקלו של העמוד.



Picture

R_{Ax}, R_{Ay}, R_B

F_1, F_2, F_3, U_c

$u_c = \frac{\partial U}{\partial Q}$ $u_c = \frac{\partial U}{\partial R}$

Moment = $(P\cos 30^\circ)e$

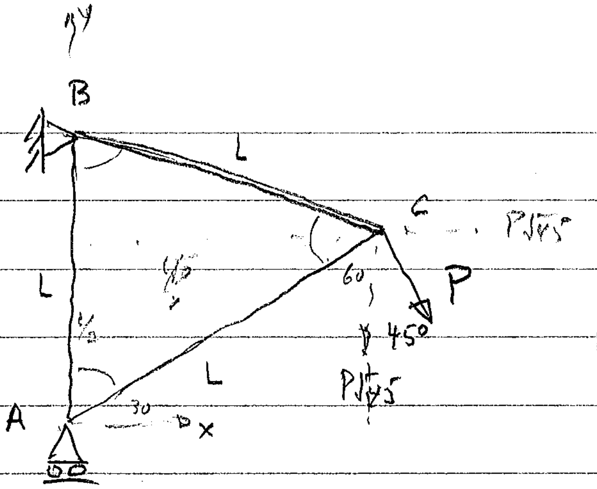
Picture: M, P, T @ fixed end.

$I_{zz} = I_{zz}\,\square\,,\ I_{zz}\,\bigcirc\,,\ I_{zz_t}$

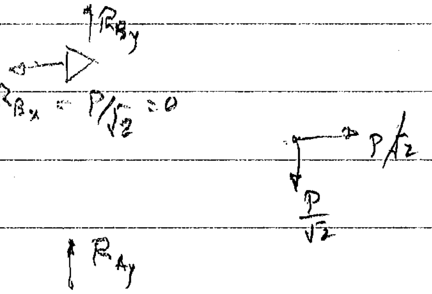
$A = A\,\square \quad A\,\bigcirc \quad A_t$

$\sigma = P/A - \frac{My}{I}$

P



$\Sigma M_B = \frac{P}{\sqrt{2}}\cdot\frac{L}{2} - \frac{P}{\sqrt{2}}\cdot\frac{L\sqrt{3}}{2}$



$R_{Bx} - P/\sqrt{2} = 0$

$(R_{By} + R_{Ay})\cdot\frac{L\sqrt{3}}{2} - R_{Bx}\frac{L}{2}$

$(R_{By} + R_{Ay})\cdot\sqrt{3} = R_{Bx} = P/\sqrt{2}$

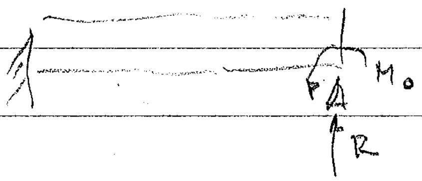
$(R_{By} + R_{Ay}) = P/\sqrt{2}$

$\frac{dV}{dx} = -w$

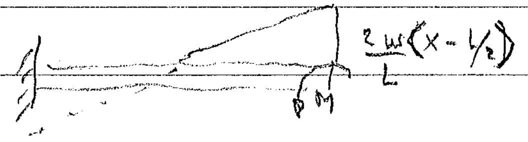
$\frac{dM}{dx} = -V$

$-V = EIv'''$

$w = EIv^{IV}$



$EI\,v^{IV} = -w + M_0\langle x-L\rangle^{-2}$



$EI\,v^{IV} = -\frac{2w}{L}\langle x-L/2\rangle^{1} + M_0\langle x-L\rangle^{-2}$

$EI\,v''' = -\frac{2w}{2L}\langle x-L/2\rangle^{2} + M_0\langle x-L\rangle^{-1} + C_1$

$M = M_0$

$EI\,v'' = -\frac{w}{3L}\langle x-L/2\rangle^{3} + M_0\langle x-L\rangle^{0} + C_1x + C_2$

$EI\,v' = -\frac{w}{12L}\langle x-L/3\rangle^{4} + M_0\langle x-L\rangle^{1} + C_1\frac{x^2}{2} + C_2x + \cancel{C_3}$

@ x=0

$EI\,v = -\frac{w}{60L}\langle \quad \rangle^{5} + \frac{M_0}{2}\langle x-L\rangle^{2} + C_1\frac{x^3}{6} + C_3\frac{x^2}{2} + \cancel{C_4}$

@ x=0

@ x=L $= -\frac{w}{60L}\frac{L^5}{32} + C_1\frac{L^3}{6} + C_3\frac{L^2}{2} = 0$