המכללה האקדמית יהודה ושומרון
מחלקה להנדסה אזרחית

תורת החוזק 1 (40126) תשס"א

פרופ' לוי עזרא

פרשיות הלימוד

1. מבוא, מאמצים - מצב מאמצים בנקודה, קשר המאמצים לכוחות פנימיים, טרנצפורמצית המאמצים
2. מאמצים ראשיים (2-D), עיבורים - מצב עיבורים, קשר העיבורים לתזוזה ותוויות
3. טרנצפורמצית העיבורים, עיבורים ראשיים (2-D), התנהגות החומר -
4. קשר בין עיבורים ומאמצים במצב אלסטי לינארי, חוק **HOOKE**, עקרון הסופרפוזיציה
5. עקרון **SAINT VENANT**, עמיסת צירית - בעיות מסוימות ובלתי מסוימות סטטית
6. השפעת הטמפרטורה, פיתול חתכים שונים - בעיות מסוימות ובלתי מסוימות סטטית
7. פיתול חפשי של מוטות בעלי דפנים דקים, פרופילים סגורים ופתוחים
8. כפיפה סימטרית של קורות - כפיפה טהורה, גזירה בשעת הכפיפה בקורות אלסטיות הומוגניות ומורכבות
9. כפיפה סימטרית של קורות-שקיעות בעזרת המישוואה הדיפרנציאלית של הקו האלסטית
10. שקיעות מסוימות ובלתי מסוימות סטטית

ספרות

1. Beer, F.P., and Johnson, E.R., *Mechanics of Materials*, SI Metric Ed., McGraw-Hill
2. Popov, E.P., *Introduction to Mechanics of Solids*, Prentice-Hall, 1968
3. Stippes, M., Wempner, G., Sten, M., and Beckett, R., *Introduction to Mechanics of Deformable Bodies*, Charles Merrill Pub., 1961

תרגילי בית - להשלימם אבל לא לחזירם למרצה. חשוב לעשותם כדי להתכונן לבוחני תרגילי בית
אם לא תוכל להיות נוכח בבוחן תרגילי בית, (כמו מילואים, נסיעות למען עבודתך או מחלות)
עליך להתקשר לפרופ לוי מיד ולהודיע לו. הודעות אחרי בוחן כלשהו לא יתקבלו.

הציונים תלויים בשלוש דברים:

1. 3 בוחנים מתרגילי בית מחושבים כ-**5%** כל אחד
2. בוחן אמצע סמסטר מחושב כ-**35%**
3. מבחן סופי מחושב כ-**50%**

משרדו של פרופ' לוי נמצא בבניין **2** חדר **8.**
שעות קבלה: יום ב בין השעות **12:00-15:00**
מס' הטל: **03-9066211** במשרד, או בזמן חרום **053-771247**
אם יהיו שינויים לסילבוס, קהל הסטודנטים ייוודע בו זמני



מבוא לתורת התוצק

בהנדסת מבנים, שמכירים כמו בטון, קירות, וכד', צריך לדעת להעריך מידות מסוימות כדי לסבול את הכוחות המופעלים והכוחות שיכולים לפעול עליהם.

כשיש כדי שלא להתקלקל הרבה כשהם הרואים פועלים עליו, וכמו כן, המהנדס מעוניין מורכב צריך לעמוד בכוחות, כאלה שאורכו, להיות מספיק

אברים בדק-בוחן יכולים לקרוס כשהם מעמיסים אותם בעומס קטן לפני שיתממשו בדק בפועל.

צריך לקבוע מתמוטטות, ולקורות השלה בלי ביצועים קומר.

- הנושא שלנו מתעסק בחוזק, קשיחות, ויציבות של אברים מהמבנה שאומרים את העומס.

מוטות וקורות פלחיקה ומתיחה, בראשה הדן,
ומתייחס לאופן הפיזיקלי שמאפיינים את התנהגותם של מוטות וקורות בעת שהם נתונים למתיחה או לדחיסה.

מצמן הנושא אך עצמו הוא הרבה חוקרים כמו COULOMB, POISSON, NAVIER, ST. VENANT
ו-CAUCHY

- תורת החוצק עוסקת בכל העומסים, הנדסת אלסטיות, הנדסת אזורים, הנדסת מבנים, הנדסת הכלי, ובכול לחוץ שהם נוגעים מהם

- התנהגות של מוטות וקורות קלות בחוקים הבסיסים של NEWTON שעוסקים בשיווי משקל, שהרי שאפו מכני של החומרים שנמצאים במוטות וקורות בהנחות שלא ניתן לעז לבחור שלך החומר הנבון לקבון המבנים שלנו.
- התנהגות החומרים חשוב מאוד מבחינת ביצועים במערכת. המבנה ניתן

- מתעלים בפיזיקה (review) על מנת שנוכל לגשת ללמוד את הנושא

מכניקה

מכניקה
מכניקת המוצקים
מכניקת הנוזלים
מ. גוף קשיח
מ. גוף נשטף
מ. הגזים
מ. פלזמה
סטטיקה
קינמטיקה
אלסטיות
פלסטיות
זרימה אלסטו-פלסטית
חומרים מורכבים
ת. הכיוון
Vibrations

References

EGOR P. POPOV - Introduction to Mechanics of Solids

E.P. POPOV - Mechanics of Materials

BYARS & SNYDER - Engineering Mechanics of Deformable Bodies

TIMOSHENKO & YOUNG - Elements of Strength of Materials

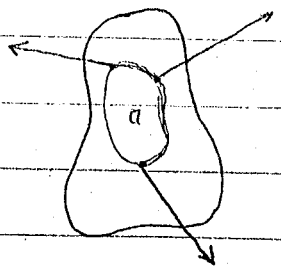
SINGER - Strength of Materials

GRINTER - Engineering Mechanics

BEER & JOHNSTON - AE 100 Book

SEELY & SMITH - Advanced Mechanics of Materials.

H.W. Read Pages 1-53 Prob 21, 2, 3, ~~4~~, 7, 9, 11



BASIC CONCEPTS - STATICS

Full Knowledge (Practical) must be had in:

a) Resolution & Vectoral Manipulation of Forces

b) Equilibrium Problems, equilibrium conditions

c) Free-Body diagram.

Look at structure through strength, stiffness, stability

יכולת לסבול את העומס המיושם מבלי להתמוטט, בקיעה

חוזק Strength - ability to withstand load without breaking

קשיחות stiffness - the ability to withstand change in dimensions (elasticity)

יכולת לסבול שינוים קיימים

יציבות stability - depends on structure & dimensions of body

ability to support a given load w/o experiencing a sudden change in its configuration

We will use Method of Sections - If body in equilibrium, any portion of body in equilibrium.

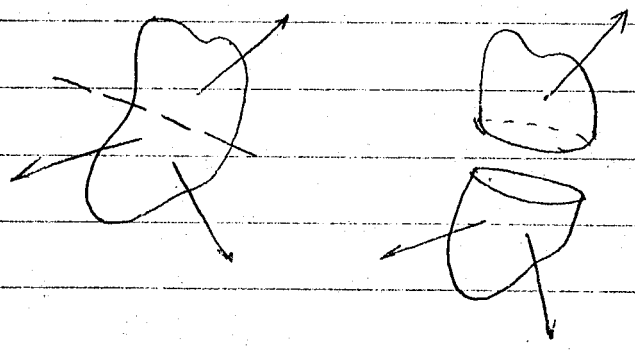
Method of Sections

Approach No.

equilibrium.

2. Pass Arbitrary plane through body separating two parts.

נחקור את ההתנגדות הפנימית והתעוות של הגוף תחת פועלי עומס



MAIN Problem : Approach

שלב?

1) Isolate Member to be analyzed or designed.

מבודדים את האבר

2) Find reactions by

מוצאים את התגובות

a) Statics $\Sigma F_x = \Sigma F_y = \Sigma F_z = \Sigma M_x = \Sigma M_y = \Sigma M_z = 0$

b) Differential Equations & Boundary Conditions.

שלב זה מסוים לפי קינמטיקת של הגוף

c) If Indeterminate, By Kinematic Considerations – displacements are defined, restraints put on member.

צורת אומדת, את התנועות, הזזות וכדה

3) Apply Method of Sections (קרעים)

לישם את שיטת החתכים

4) At Section find internal forces (INTERNAL REACTIONS)

מוצאים את הכוחות הפנימיים

5) Use appropriate formulas to find stresses

השתמש בנוסחאות הנכונות למצוא את הלחצים

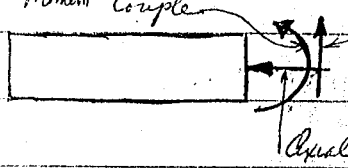
6) In Design, select appropriate material & member.

בוחרים את החומר הנכון עם טיפת החתך הנכון

7) Predict deformations of the body

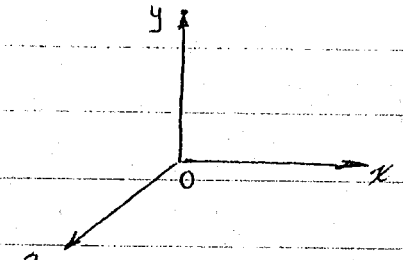
תכה מראש את ההזזות

Internal Forces



Equilibrium

Solid body – $\Sigma F_x = \Sigma F_y = \Sigma F_z = \Sigma M_x = \Sigma M_y = \Sigma M_z = 0$



Plane body – neglect all distances & dimensions in Z direction

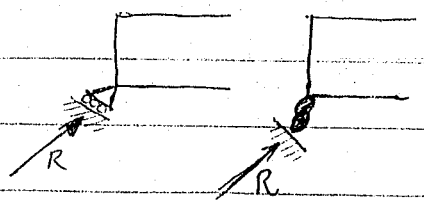
Forces in plane of body.

$\Sigma F_x = \Sigma F_y = \Sigma M_z = 0$

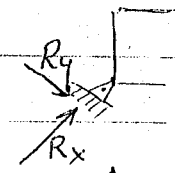
Dimensions will be taken as if before deformation

Supports

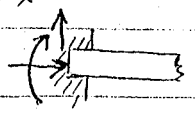
a) Roller or Link

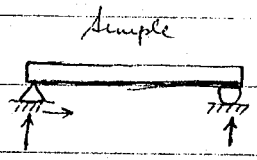


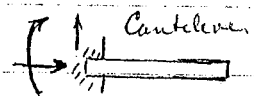
b) Pin (סיכון)

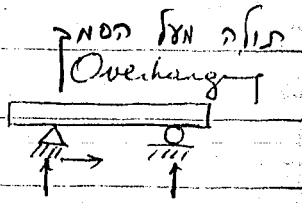


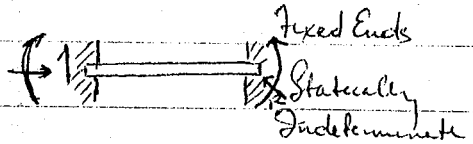
c) Fixed (התנה)

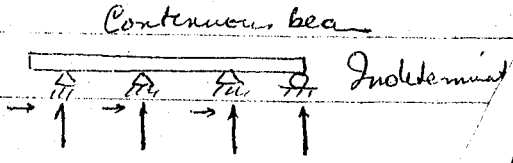


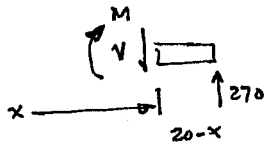








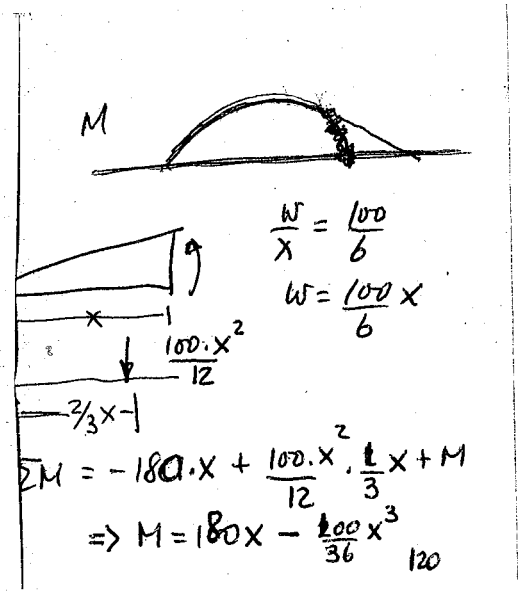




$V = 270$

$270(20-x) - M = 0$

$5400 - 270x = M$



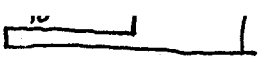
$\frac{w}{x} = \frac{100}{6}$

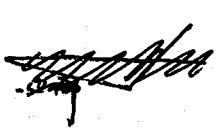
$w = \frac{100}{6}x$

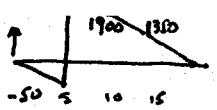
$\Sigma M = -180.x + \frac{100.x^2}{12}.\frac{1}{3}x + M$

$\Rightarrow M = 180x - \frac{100}{36}x^3$

120

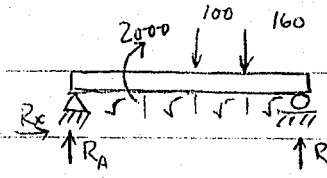






$R_x = 0$

$M = -10x \quad [0,5]$
$M = -10x + 2000 \quad [5,10]$
$M = 5000 - 110x \quad [10,15]$
$M = 5400 - 270x \quad [15,20]$

① 

$\sum F_y = R_A + R_B - 260 = 0$

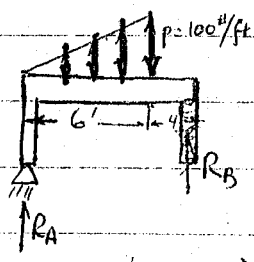
$R_A + R_B = 260$

$\sum M_B = 0 \quad R_A \times 20 + 2000 - 100 \times 10 - 160 \times 5 = 0$

$R_A \times 10 + 100 - 50 - 40 = 0 \quad R_A = -10^{\#} = 10^{\#}\downarrow$

$R_B = 270^{\#} \uparrow$

✓

② 

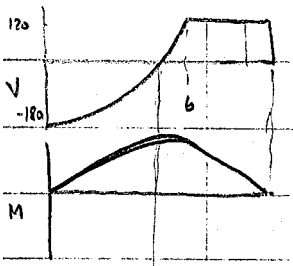
$\sum F_y = 0$

$\sum F_y = R_A + R_B + \frac{1}{2} \times 6 \times 100 = 0$

$R_A + R_B = +300$

$\sum M_B = 0 \quad R_A \times 10 + 300 \times 6 = 0; \quad R_A = +180^{\#} \quad 180^{\#}\uparrow$

$R_B = 120^{\#}\uparrow$

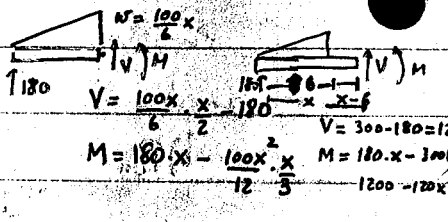


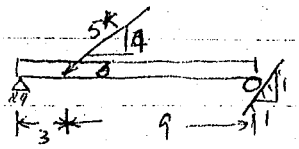
$0 \le x \le 6 \quad V = \frac{100}{12}x^2 - 180$

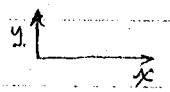
$6 \le x \le 10 \quad V = 120$

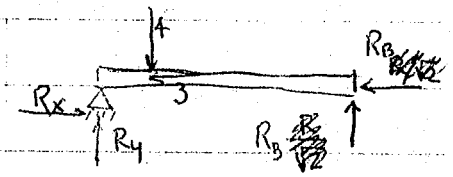
$0 \le x \le 6 \quad M = +R_A x - \frac{50}{18}x^3 \qquad M|_{x=6} = +180 \cdot 6 - \frac{50}{18} \cdot 12 = +480$

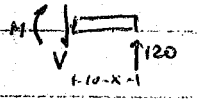
$6 \le x \le 10 \quad M = +480 - 120(x-6)$



③ 







$R_x - 3^K - R_B = 0$

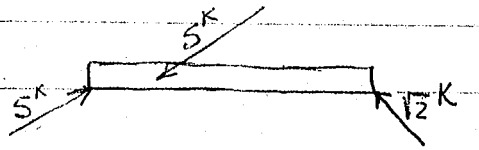
$\boxed{R_x - R_B = 3^K}$

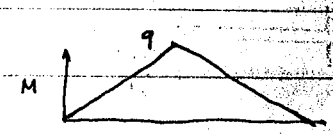
$R_y - 4 + R_B = 0$

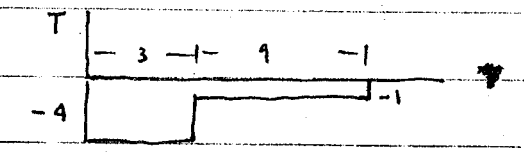
$\boxed{R_y + R_B = 4^K}$

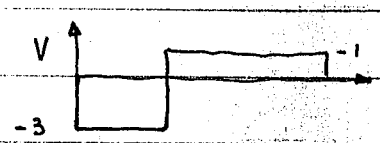
$\sum M_B = 0 \quad R_y(12) - 4(9) = 0 \quad R_y = 3^K \quad R_B = 1^K$

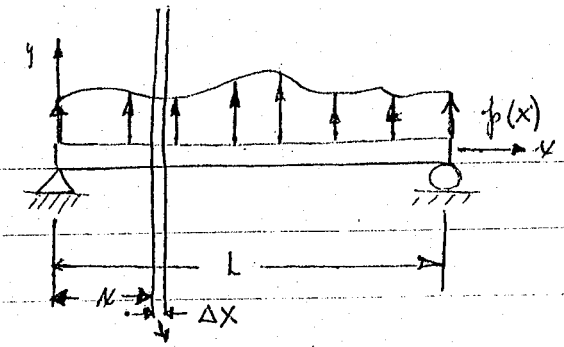
$R_x = 4^K$

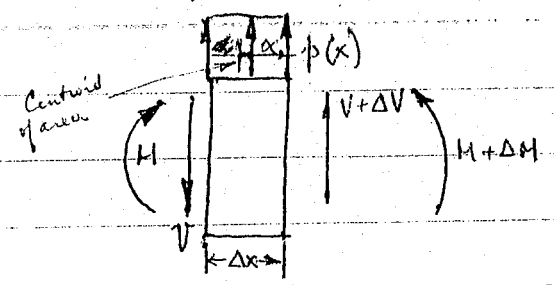












$$\sum F_y = 0$$

$$-V + p(x)\Delta x + V + \Delta V = 0$$

$$\frac{\Delta V}{\Delta x} = -p(x)$$

$$\sum M_R = 0 \qquad M + p(x)\Delta x \cdot \frac{\alpha \Delta x}{2} - V\Delta x - M - \Delta M = 0$$

$$\frac{\Delta M}{\Delta x} = -V + p(x)\alpha\Delta x$$

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = \boxed{\frac{dV}{dx} = -p(x)} \qquad \lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = \boxed{\frac{dM}{dx} = -V}$$

$$\boxed{-\frac{dV}{dx} = p = \frac{d^2M}{dx^2}}$$

$$dV = -p\,dx$$

$$\boxed{V = -\int p(x)\,dx + C_1} \quad (1)$$

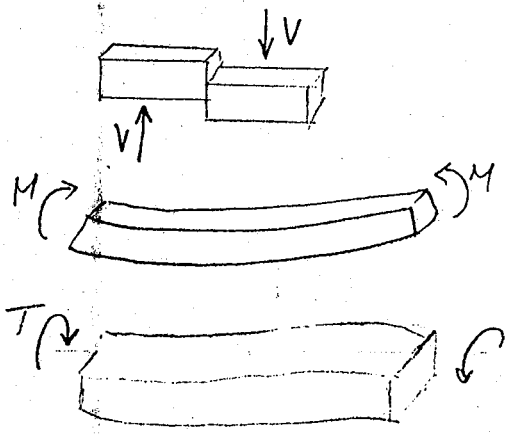
$$\boxed{M = -\int V(x)\,dx + C_2} \quad (2)$$

$$V_b - V_a = -\int_a^b p(x)\,dx$$

$$M_b - M_a = -\int_a^b V(x)\,dx$$

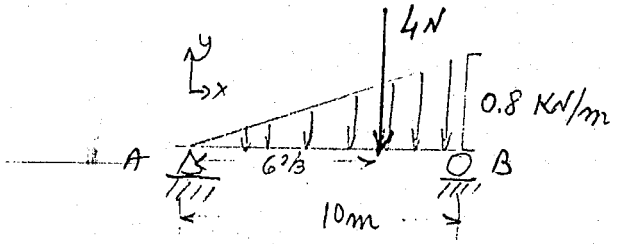
על קורה וכך לפרץ אומס מכוכב, אומס מחולק וו' כל קומבונדה שהוא.

גזירה בפרופה ופתיל



בנשף למחשתה וו' לחיבה ובלה קרה לאומ
מלצרה ברפורה מהפתול כמשורטט.
V נקרא כוח הגלצרה, M מומנט הכפיפה
ו-T מומנט הפתול.

כושי מופרל על קורה אומס מחולק עיכל להשתמש בשקולו כדי
לחשב את שוו המשקל המיוצג של הקורה



פתרון:

חשב את התגובות בסמכים A,B

נבנים מדורים בדמי וכך הכח
המחולק יהיה

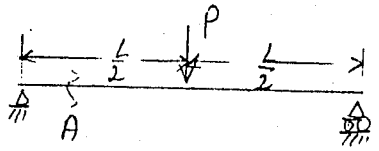
$$w = \frac{0.8}{10} \cdot x \ \text{kN/m} = 0.08x \ [\text{kN/m}]$$

שלמת השקול תהיה $R = \int_0^{10} 0.08x\,dx = 0.04x^2\Big|_0^{10} = 4\ \text{kN}$

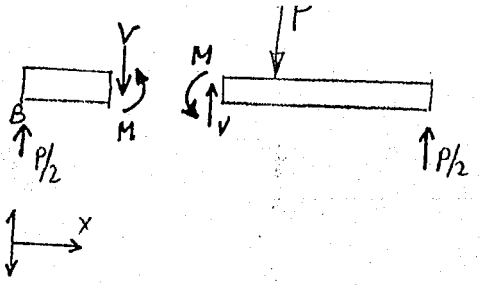
מקום השקול. $R \cdot d = \int_0^{10} w(x) \cdot x\,dx = \int_0^{10} 0.08x^2 = \frac{0.08}{3}x^3\Big|_0^{10} =$

$$4 \cdot d = \frac{80}{3} \qquad d = \frac{20}{3} = 6\tfrac{2}{3}\ \text{m}$$

דוגמאות נוספות לגזירה ומומנטים



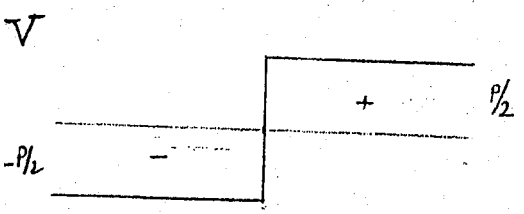
בוא נסתכל בקורה שבשרטוט
ונתחיל בחתך כל שהוא A
כדי לשאול על שוו המשקל יפעל על החתך כוח גזירה V ומומנט
M לשם כך נבצע חתך דמיוני בקורה המגיע

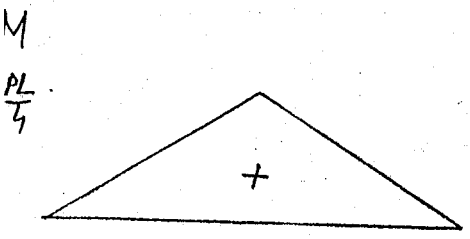


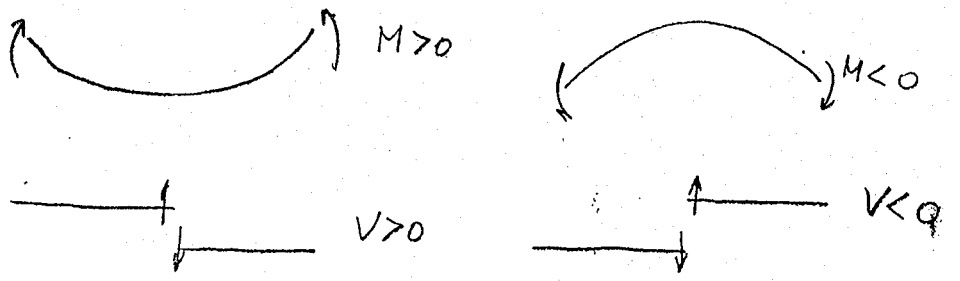
לפי שקילות לחתכים שונים בכל תחב הקורה

$$\Sigma F_y = 0 \quad V = \frac{P}{2}$$

$$\Sigma M_B = 0 \quad M = V \cdot x = \frac{Px}{2}$$

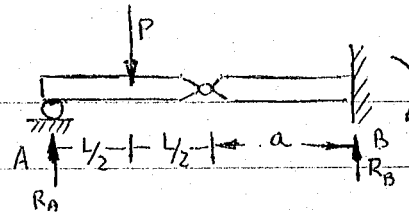




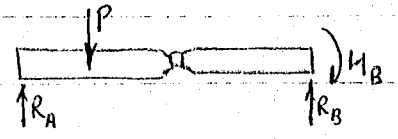


סכם הסימנים

④



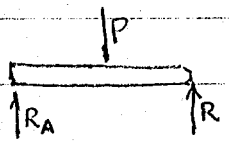
Find R_A, R_B, M_B



$\sum F_y = R_A + R_B - P = 0$

$\boxed{R_A + R_B = P}$

$\sum M_B = R_A(L+a) - P(L/2+a) + M_B = 0$ $\quad \boxed{R_A(L+a) + M_B = P(L/2+a)}$



$R_A - P + R = 0$ $\quad \boxed{R = P/2}$

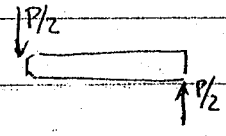
$\sum M_c = 0$ $\quad R_A(L) - PL/2 = 0$ $\quad \boxed{R_A = P/2}$

$\boxed{R_B = P/2}$

$\frac{P}{2}(L+a) + M_B = \frac{P}{2}(L+2a)$

$M_B = P/2\,(L + 2a - L - a) = \frac{Pa}{2}$

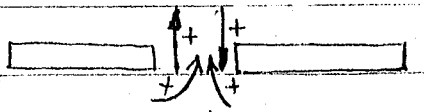
$\boxed{M_B = \frac{Pa}{2}}$

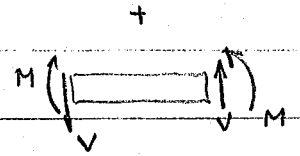


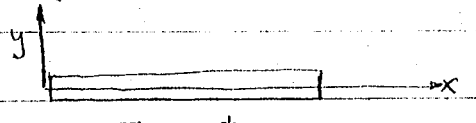
$\sum F_y = 0$ ✓ $\quad \sum M_B = 0$ ✓

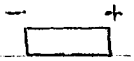
✓

Sign Convention for problems



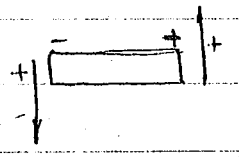






relative to the other end

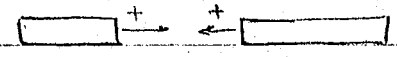
negative face positive face

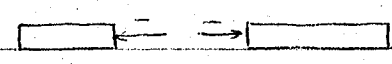


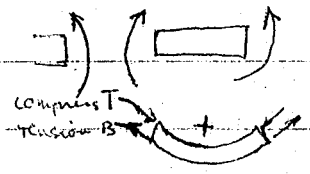
depends on direction & face.

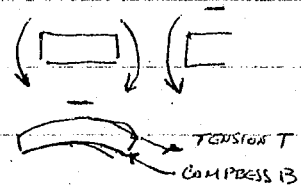
negative y direct.

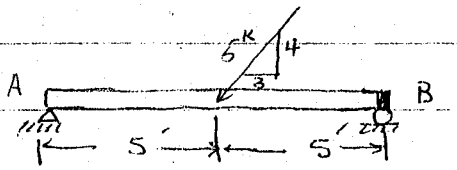
Shear







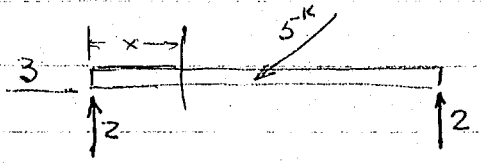


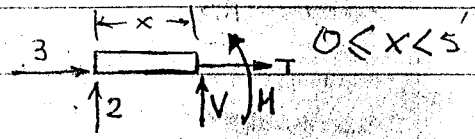
① 

$R_{Ax} = 3^K \rightarrow$

$R_{Ay} - 4 + R_B = 0$

$\circlearrowleft \Sigma M_A = 4(5') - 10' R_B = 0$ $\quad R_B = 2^K \uparrow$ $\quad R_{Ay} = 2^K \uparrow$

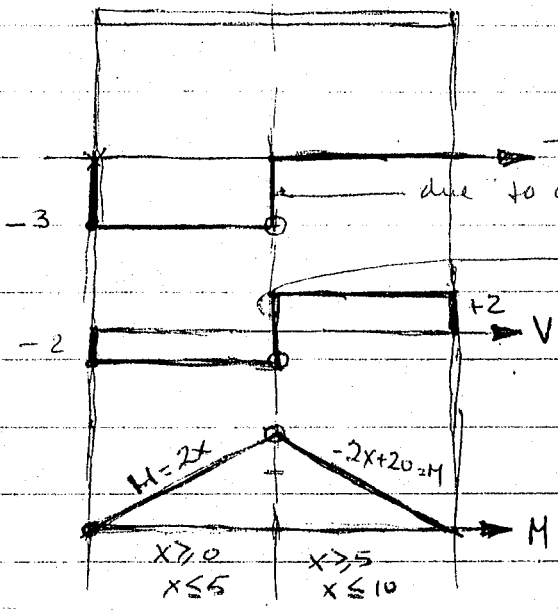


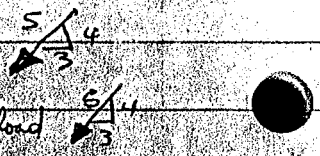


$T = 3^K \leftarrow$ $\quad V = 2^K \downarrow$

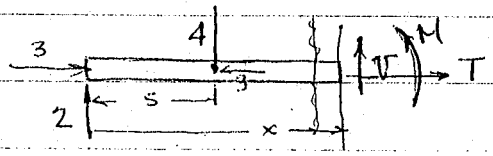
Always take M about cut

$2x - M = 0 \quad M = 2x$





$5 \leq x \leq 10$



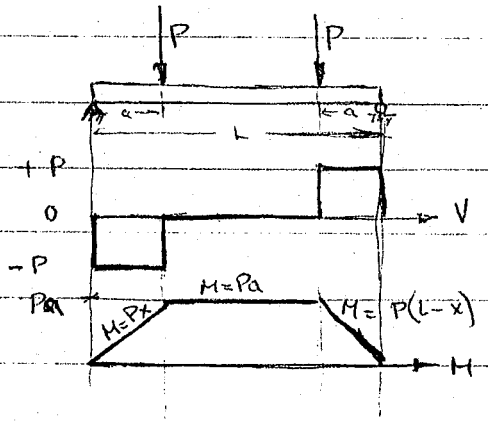
$\sum F_x = 0$ $\quad 3 - 3 + T = 0$ $\quad T = 0$

$\sum F_y = 0$ $\quad 2 - 4 + V = 0$ $\quad V = +2$ kips

$\sum M_C \circlearrowright = 2x - 4(x-5) - M = 0$ $\quad -2x + 20 = M$

Shear curve displaced the curve an equal amount in the opposite direction to the direction of the concentrated load

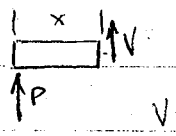
Bend curve is displaced an equal amount in the same direction as the concentrated load.



$R_A + R_B = 2P$; $\sum F_H = 0$

$\sum M_B = 0 \circlearrowright$; $R_A(L) - P(L-a) - P(a) = 0$

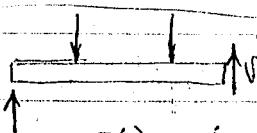
$R_A(L) - PL = 0$ $\quad \boxed{R_A = P}$ $\quad \boxed{R_B = P}$



$0 \leq x \leq a$

$V = -P$

$M = P(x)$ $\quad x \geq 0$ $\quad x \leq a$



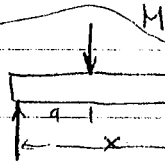
$V = P$

$P(x) - P(x-a) - P(x-(L-a)) - M = 0$

$P(x) - P(x) + P(a) - Px + P(L) + Pa - M = 0$

$\boxed{M = P(L) - P(x)}$ $\quad x \geq L-a$ $\quad x \leq L$

when $x = (L-a)$



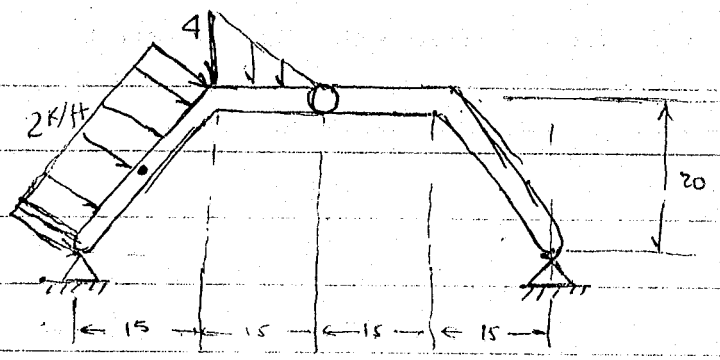
$a \leq x \leq L-a$

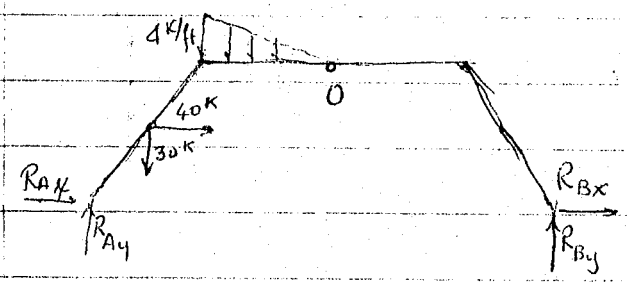
$V = 0$

$P(x) - P(x-a) - M = 0$

$+Pa = M$ $\quad x \geq a$ $\quad x \leq L-a$

#2-11





$\Sigma F_x = 0$

$R_{Ax} + R_{Bx} + 40 = 0$

$R_{Ax} + R_{Bx} = -40$

$\Sigma F_y = 0$

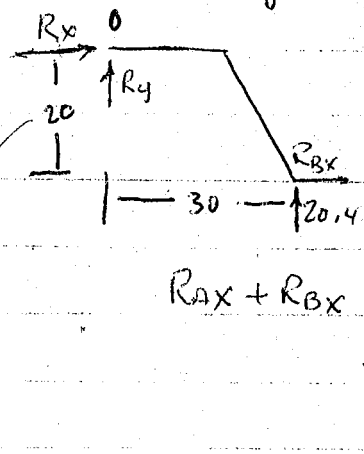
$R_{Ay} - 30 - \frac{1}{2} \times 15 \times 4 + R_{By} = 0$

$R_{Ay} + R_{By} = 60^K$

$\Sigma M_A = 0$ ↻ $\quad 50 \times 12.5 + \frac{1}{2} \times 15 \times 4 \times 20 - R_{By} \times 60 = 0$

$R_{By} = \frac{5}{6} \times 12.5 + 10 = 20.4^K$

$R_{Ay} = 39.6^K$



$\Sigma M_O = 0$ ↻ $\quad -R_{Ax} \times 20 + 39.6 \times 30 - 30 \times 22.5 - 40 \times 10 - \frac{1}{2} \times 4 \times 15 \times 10 + R_{Bx} \times 20 + 20.4 \times 30 = 0$

$R_{Ax} + R_{Bx} = (39.6 - 20.4) \times \frac{30}{20} - \frac{30}{20} \times 22.5 - 35$

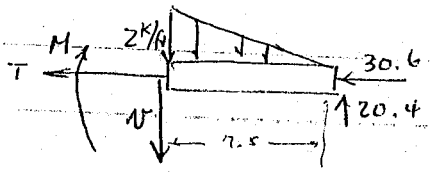
$= (-3.2) \times \frac{30}{20} - 35$

$-35 - 5 = -40^K$

$\Sigma M_O = 0 - 20.4(30) - R_{Bx}(20) = 0 \qquad R_{Bx} = +30.6 \leftarrow$

$R_{Ax} = +9.4 \leftarrow$

✓ Since we know everything about right member we can isolate the smaller member before pin & cut therefore



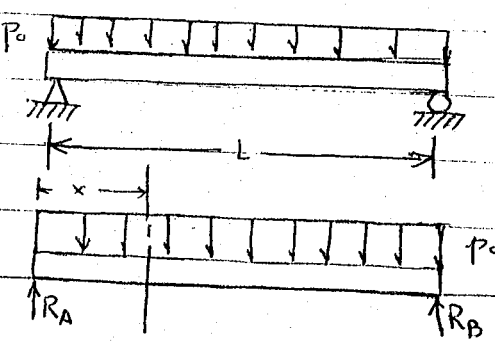
$\sum F_x = 0 \quad -T - 30.6 = 0 \quad T = -30.6\ K$

$\sum F_y = 0 \quad -V + 20.4 - \frac{1}{2}(2)(7.5) = 0 \quad V = 17.9^K$

$\sum M_{\text{at cut}} = 0 \quad M + \frac{1}{2}(2)(7.5)(2.5) - 20.4(7.5) = 0$

$M = (17.9)(7.5) \cong 135\ K\text{-ft.}$

Consider



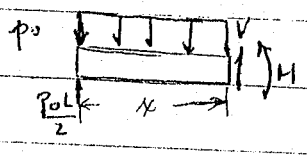
$\sum F_y = R_A - p_o L + R_B = 0$

$\boxed{R_A + R_B = p_o L}$

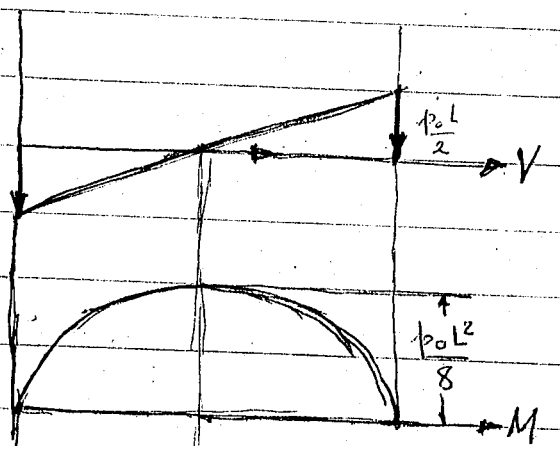
$\sum M_B = R_A L - p_o L \frac{L}{2} = 0$

$\boxed{R_A = \frac{p_o L}{2}}$

$\boxed{R_B = \frac{p_o L}{2}}$



$\sum F_y = 0 \quad \frac{p_o L}{2} - p_o x + V = 0 \quad V = p_o\left(x - L/2\right)$

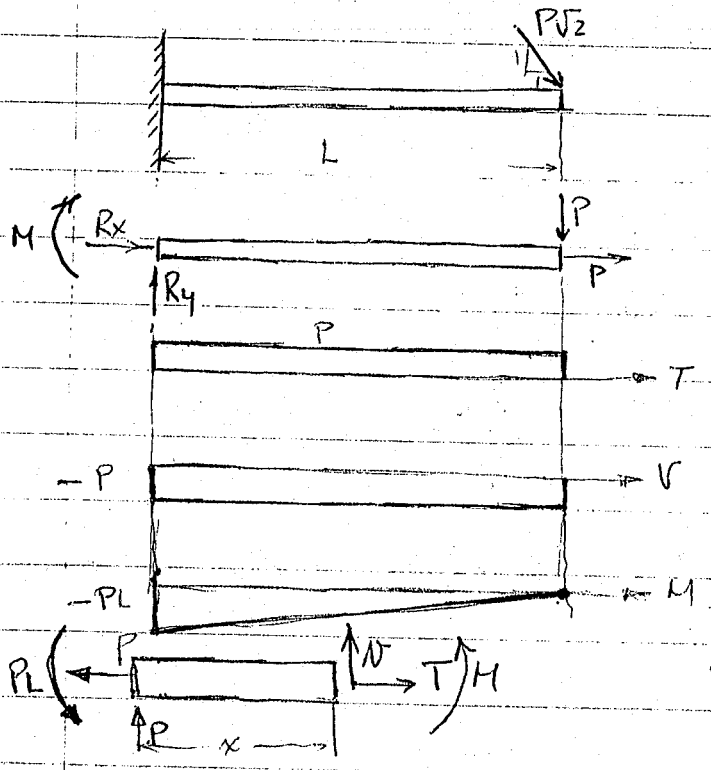


$\sum M_C = 0 \quad \frac{p_o L}{2} x - p_o \frac{x^2}{2} - M = 0$

$M = \frac{p_o x}{2}(L - x)$

$\frac{dM}{dx} = \frac{p_o L}{2} - p_o x = p_o\left(\frac{1}{2} - x\right) = 0$

$x = L/2$



$\Sigma F_x = 0$

$R_x + P = 0$ $\boxed{R_x = -P}$

$\Sigma F_y = 0$

$R_y - P = 0$ $\boxed{R_y = P}$

$\Sigma M_A = 0$ $\quad M + P(L) = 0$

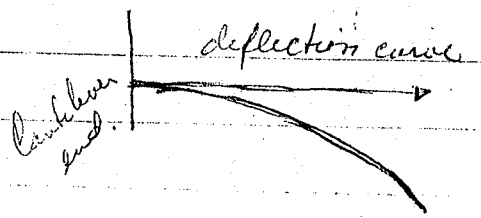
$\boxed{M = -PL}$

$T = P$

$V = -P$

$\Sigma M_C = 0$ $\quad -PL + Px - M = 0$

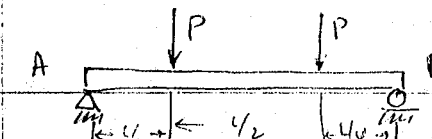
$M = P(x - L)$



✓ Rd Page 30-31 Problem 2-8 $\quad$ 2-15, 17, 19, 21, 25

Steps to solve

1) Draw Sketch of Member - which shows all forces & dimensions
2) Unknown reactions are indicated
3) Resolve inclined forces into components
4) Find reactions
5) Cut arbitrary sections (depending on range that is good for that section)
6) Isolate segment to either side of cut. Show forces acting
7) Indicate internal reactions.
8) Find internal unknown reactions.

① 

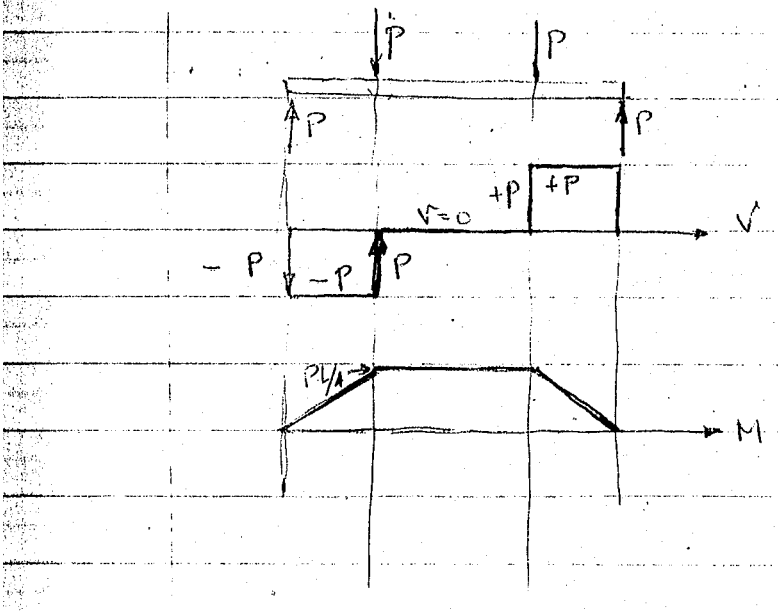
$\Sigma F_y = 0 \uparrow$ Respectively

$R_A + R_B - 2P = 0$

$\boxed{R_A + R_B = 2P}$

$\Sigma M_B = 0 \circlearrowright$

$R_A L - PL = 0 \qquad R_A = P = R_B$



Analytically

$p = 0 \qquad 0 \le x \le L/4 \qquad V_1 = C_{11} \qquad M_1 = -C_{11}x + C_{21}$

$C_{21} = 0$; when $x = 0, M = 0$.

$M_1 = -C_{11}x \qquad$ at $x = L/4 \qquad V_1 = C_{11} \qquad M_1 = -C_{11} L/4$

$L/4 \le x \le 3L/4 \qquad V_2 = C_{12} \qquad M_2 = -C_{12}x + C_{22}$

at $x = L/4 \qquad C_{11} + p = C_{12} \Rightarrow C_{11} = C_{12} - p$

at $x = L/4 \qquad -C_{11} L/4 = -C_{12} L/4 + C_{22} \Rightarrow (-C_{12} + p)L/4 = -C_{12} L/4 + C_{22}$

$\boxed{C_{22} = pL/4}$

at $x = 3L/4 \qquad V_2 = C_{12} \qquad M_2 = -C_{12}\left(\frac{3L}{4}\right) + \frac{pL}{4}$

$3L/4 \le x \le L \qquad V_3 = C_{13} \qquad M_3 = -C_{13}x + C_{23}$

$C_{12} + p = C_{13} \Rightarrow C_{12} = C_{13} - p$

$-C_{12}\frac{3L}{4} + \frac{PL}{4} = -C_{13}\frac{3L}{4} + C_{23}$

$$-C_{13}\frac{3L}{4} + \frac{P_1 3L}{4} + \frac{PL}{4} = -C_{13}\frac{3L}{4} + C_{23}$$

$$\boxed{PL = C_{23}}$$

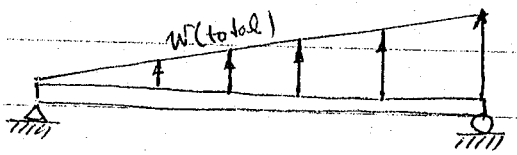
$M_3 = -C_{13}X + PL$ at $X = L$ $M_3 = 0$ $\therefore$ $\boxed{+C_{13} = P}$

$\boxed{C_{12} = 0}$ $\boxed{C_{11} = -P}$

$V_1 = -P$ $V_2 = 0$ $V_3 = P$

$M_1 = +PX$ $M_2 = PL/4$ $M_3 = P(L-X)$

(3)



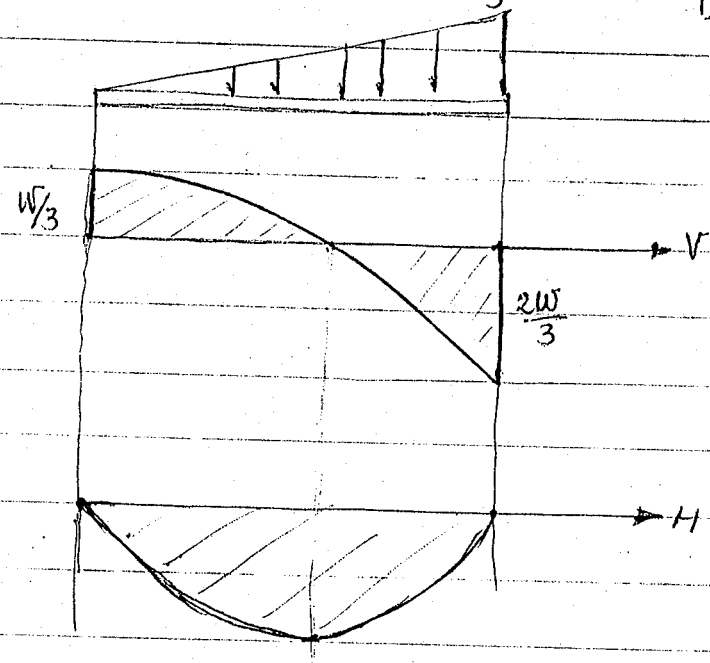
Graphically

$\Sigma F_y = 0 \uparrow$

$R_A + R_B + W = 0$

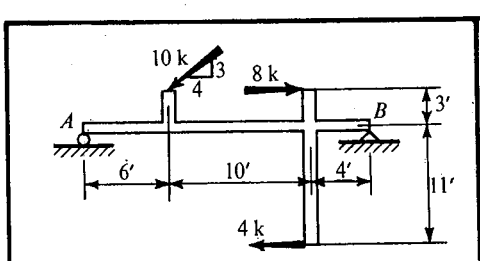
$\boxed{R_A + R_B = -W}$

$\Sigma M_B = 0$ $R_A L + \frac{WL}{3} = 0$ $\boxed{R_A = -\frac{W}{3}}$ $\boxed{R_B = -\frac{2W}{3}}$

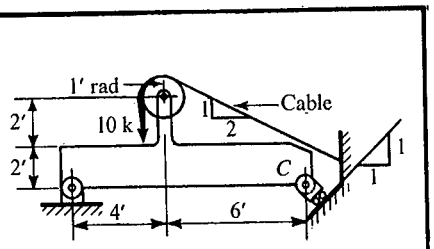


Analytically

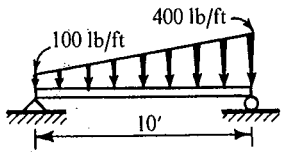
$p = kx$ at L $p = kL$ $W = \frac{1}{2}KL \cdot L$ $k = \frac{2W}{}$



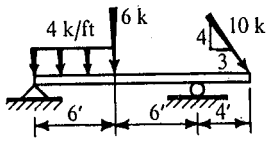
PROB. 2-1



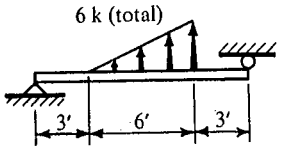
PROB. 2-2



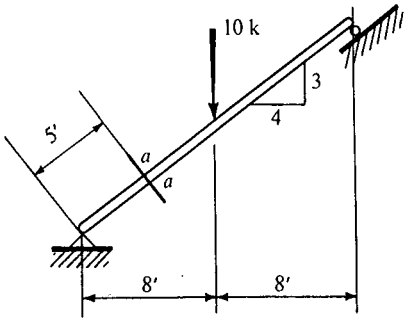
PROB. 2-3



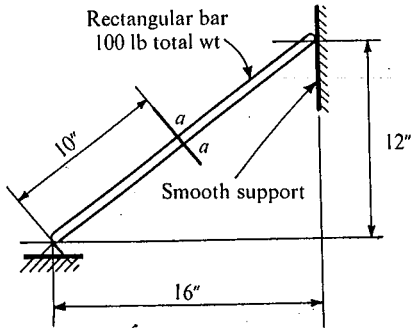
PROB. 2-4



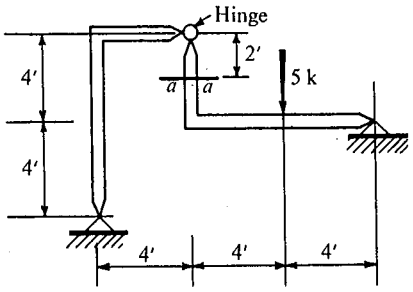
PROB. 2-5



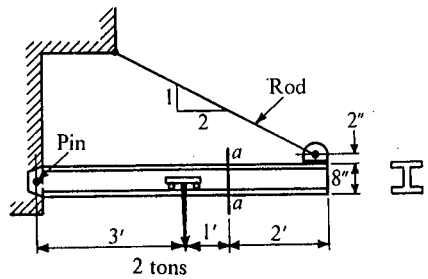
PROB. 2-6



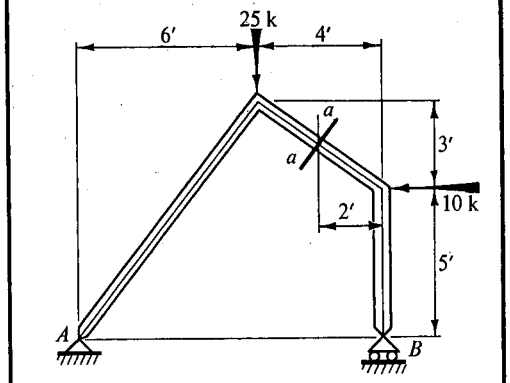
PROB. 2-7



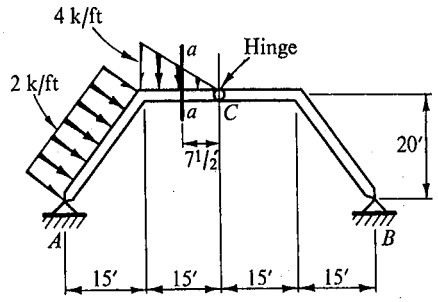
PROB. 2-8



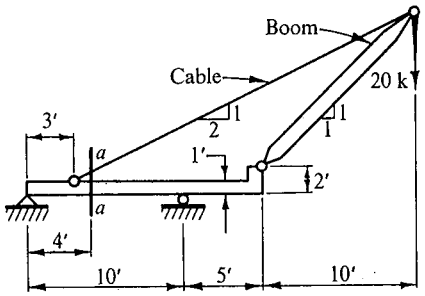
PROB. 2-9



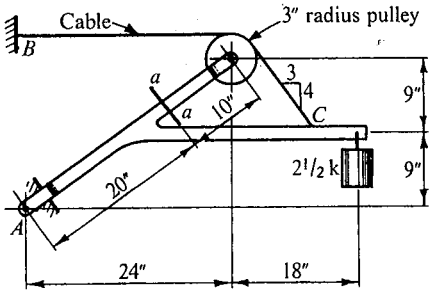
PROB. 2-10



PROB. 2-11



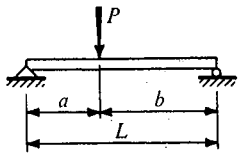
PROB. 2-12



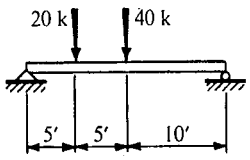
PROB. 2-13

2-14 through 2-19. For the beams loaded as shown in the figures write general expressions for the shear and bending moments for each region over the length of the member. Also plot the corresponding shear and moment diagrams. *Ans.* Maximum moment in parentheses by the figure.

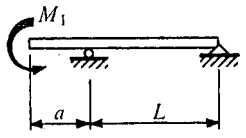
For additional loading conditions see other problems in this chapter.



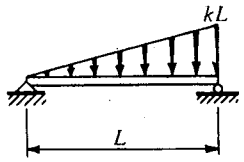
PROB. 2-14 (*Pab*/*L*)



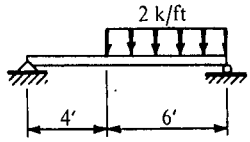
PROB. 2-15 (250)



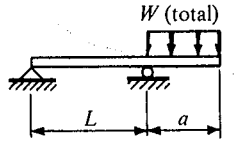
PROB. 2-16 ($-M_1$)



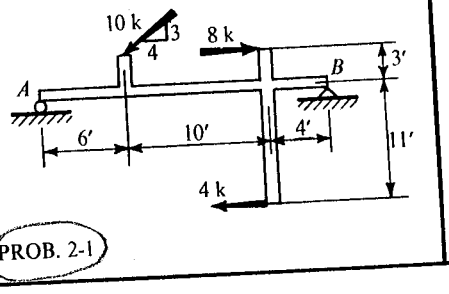
PROB. 2-17 ($kL^3/9\sqrt{3}$)



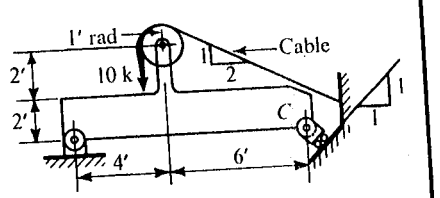
PROB. 2-18 (17.6)



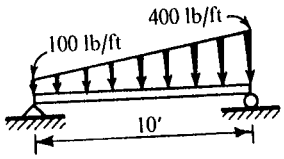
PROB. 2-19 ($-Wa/2$)



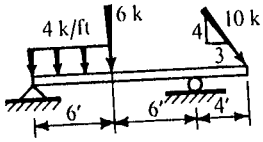
PROB. 2-1



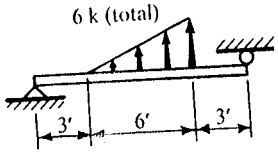
PROB. 2-2



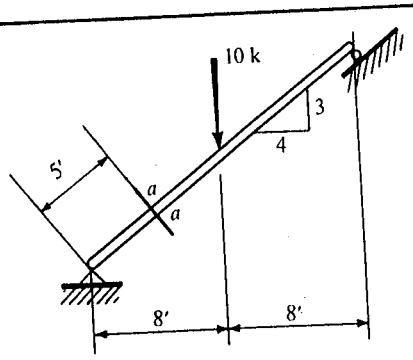
PROB. 2-3



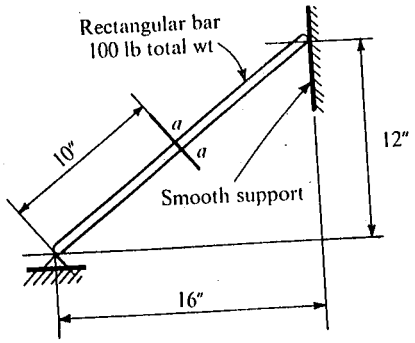
PROB. 2-4



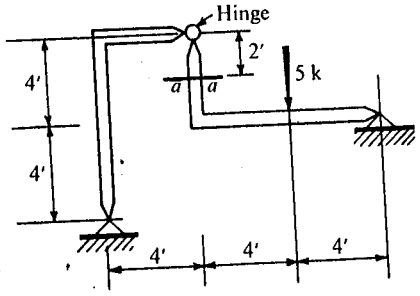
PROB. 2-5



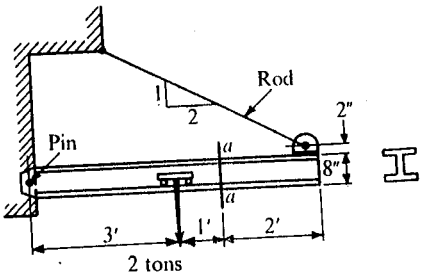
PROB. 2-6



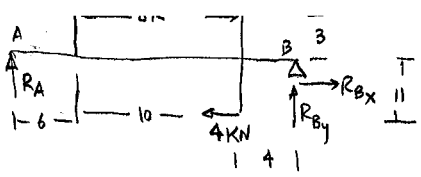
PROB. 2-7



PROB. 2-8



PROB. 2-9



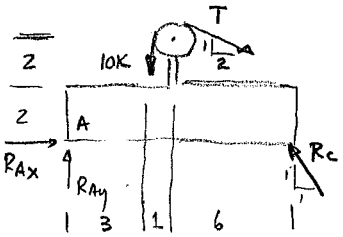
$\Sigma F_y = 0 \Rightarrow R_A - 6 + R_{By} = 0$

$\Sigma M_B \circlearrowleft^+ = 0 \Rightarrow -8\times3 - 4\times11 - R_A\times20 + 6\times14 + 8\times3 = 0$

$R_A = 2\,kN \uparrow$

$R_{By} = 4\,kN \uparrow$

2-2.



$\Sigma F_x = 0 \Rightarrow R_{Ax} - R_{Cx} + T_x = 0$

$\Sigma F_y = 0 \Rightarrow R_{Ay} + R_{By} - 10 - T_y = 0$

$+\circlearrowleft \Sigma M_A = 0 = -10(3) - T\cdot1 - T_x\cdot4 + T_y\cdot4 + \frac{R_C}{\sqrt{2}}(10)$

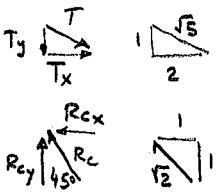
$R_{Cx} = R_C/\sqrt{2}$

$T_x = 2T/\sqrt{5}$

$R_{Cy} = R_C/\sqrt{2}$

$T_y = T/\sqrt{5}$

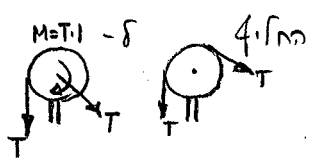
$T = 10\,KN$

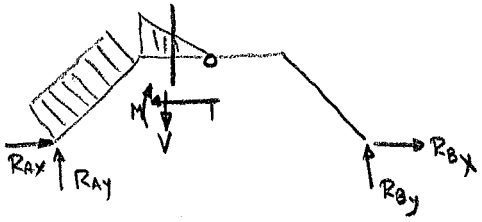


$R_{Ax} = 0.42\,kN \rightarrow$

$R_{Ay} = 5.11\,kN \uparrow$

$R_C = 13.25\,kN$ (1:1)





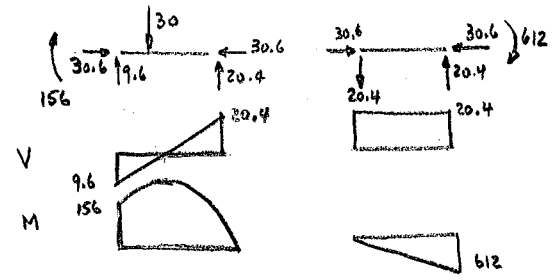
שאלה 11-2 עץ הלוח מישיור על 27 לפרמיכלי

$R_{Bx} = 30.6\,kN \leftarrow$ $R_{Ay} = 39.6\,kN$

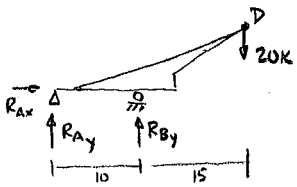
$R_{Ax} = 9.4\,kN \leftarrow$ $R_{By} = 20.4\,kN$

$M = 135\,kN\cdot m$)

$T = 30.6\,kN \rightarrow$ $V = 12.9\,kN \downarrow$



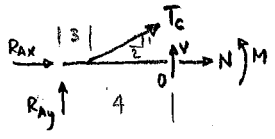
$M = 156$



$\Sigma F_x = R_{Ax} = 0$

$\Sigma F_y = R_{Ay} + R_{By} - 20\,KN = 0$

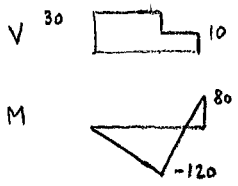
$+\circlearrowleft \Sigma M_A = 0 = -R_{By}\cdot10 + 20\cdot25 = 0$ $R_{By} = 50\,KN$ $R_{Ay} = -30\,KN$

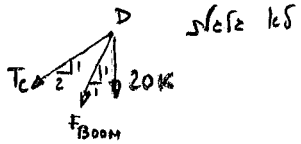


$\Sigma F_x = 0 \Rightarrow N + R_{Ax} + T_c\cdot\frac{2}{\sqrt{5}} = 0$ $N = -40\,KN$

$\Sigma F_y = 0 \Rightarrow V + T_c\cdot\frac{1}{\sqrt{5}} + R_{Ay} = 0$ $V = 10\,KN$

$+\circlearrowleft \Sigma M_0 = 0 \Rightarrow -M + R_{Ay}\cdot4 + T_c\cdot\frac{1}{\sqrt{5}}\cdot1 + T_c\cdot\frac{2}{\sqrt{5}}\cdot0.5 = 0$ $M = -80\,KN\cdot m$ or $80\,KN\cdot m$)





$\Sigma F_x = 0 = -T_c\cdot\frac{2}{\sqrt{5}} - F_{Boom}\cdot\frac{1}{\sqrt{2}} = 0$

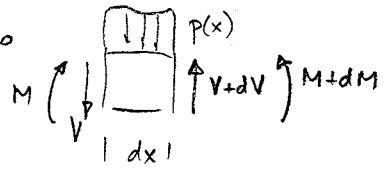
$\Sigma F_y = 0 = -T_c\cdot\frac{1}{\sqrt{5}} - F_{Boom}\frac{1}{\sqrt{2}} - 20\,KN$

$T_c = 20\sqrt{5}\,KN = 44.72\,KN$

$F_{Boom} = -56.57\,KN$

Do 2-11

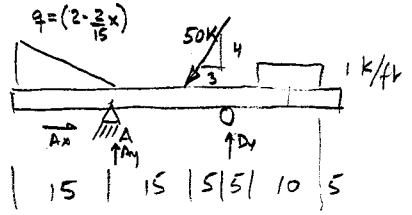
Then do



$\frac{dV}{dx} = P$

$\frac{dM}{dx} = -V$

Do example 2-10



$\rightarrow = \sum F_x \Rightarrow A_x = 30\,k$

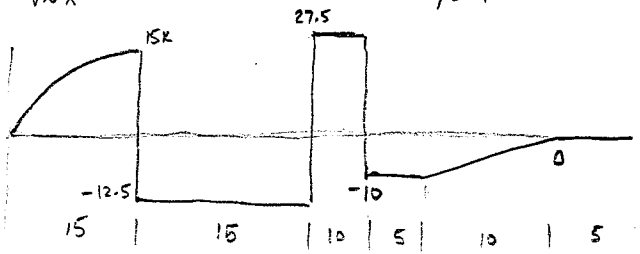
$\uparrow = \sum F_y \Rightarrow -15K + 40K - 10K + A_y + D_y = 0$

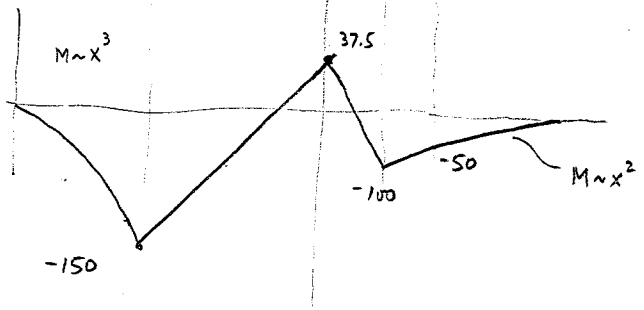
$\sum M_A = 0 \;\rightarrow\; -\frac{2 \cdot 15}{2} \cdot 10 + 40 \cdot 15 - D_y \cdot 20 + 1 \cdot 10 \cdot 30 = 0$

$\frac{-150 + 600 + 300}{20} = D_y = 37.5K$

$A_y = 27.5K$

$V \sim x^2$





$\int q \cdot x\,dx = \left(2 - \frac{2}{15}x\right)x\,dx$

$2\frac{x^2}{2} - \frac{2x^3}{45}\Big|_0^{15}$

$225 - \frac{2}{3} \cdot \frac{75}{225}$

$$V = -\frac{2w}{L^2}x^2 + C_1$$

$$M = +\frac{w}{L^2}\frac{x^3}{3} - C_1 x + C_2 \qquad \text{at } x=0 \; M=0 \; \therefore \boxed{C_2 = 0}$$

$$\text{at } x = L \; M = 0$$

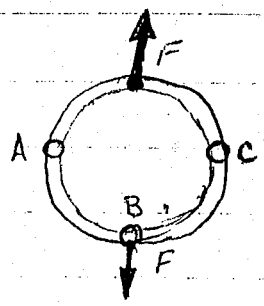
$$M = \frac{wL}{3} - C_1 L = 0 \qquad \boxed{C_1 = \frac{w}{3}}$$

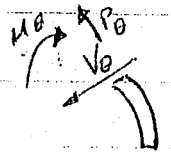
$$\boxed{V = -\frac{w}{L^2}x^2 + \frac{w}{3}} \qquad M = \frac{w}{3}\left[\frac{x^3}{L^2} - x\right]$$

at $V=0$ $\quad x = L/\sqrt{3}$ $\qquad M_{MAX} =$

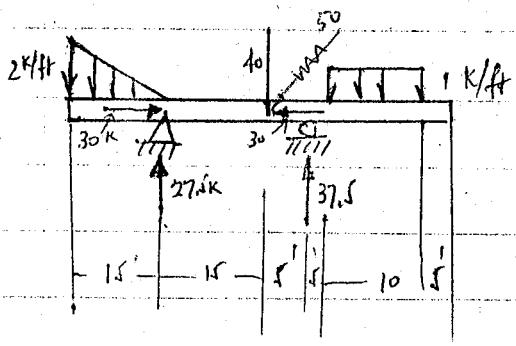
problems: 2-38, 42, 47, 50, 54, 56, 61, 63 Study example 2-10
P. 37-39 Ref. Chapter 3.

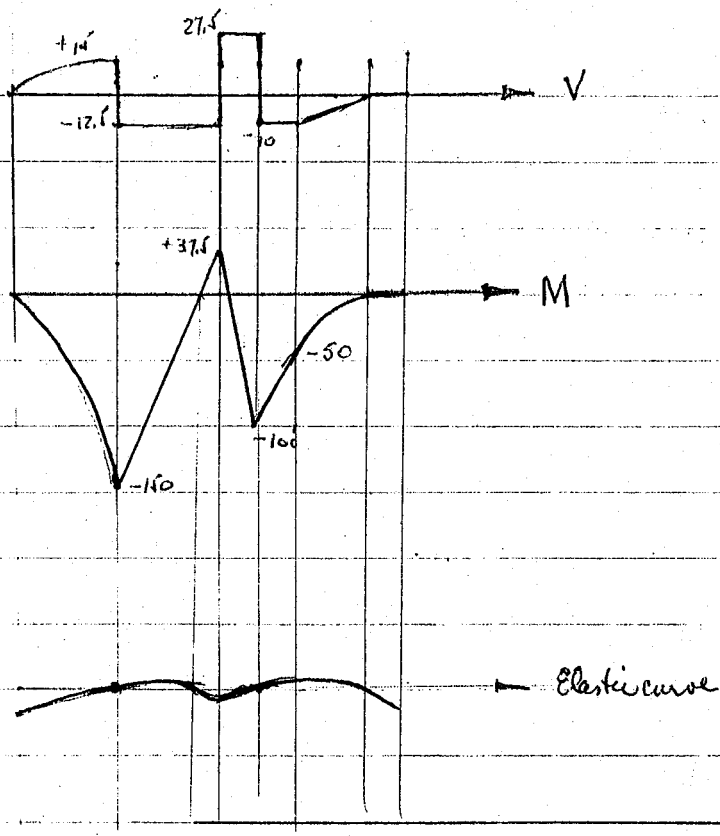
2-29

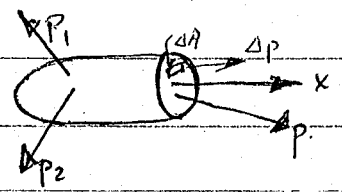


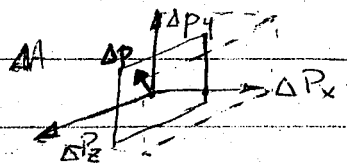


P. 38-9 Example 2-10









ΔP_x - axial (normal)

ΔP_y & ΔP_z - shear.

Stress - force intensity per unit area.

$\#/in^2 = psi$

$K/in^2 = ksi$

$$\tau_{xx} = \lim_{\Delta A \to 0} \frac{\Delta P_x}{\Delta A} \rightarrow \text{compression tension}$$

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta P_y}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta P_z}{\Delta A}$$

$\tau_{a,b}$ a - tells the normal to the plane lies in the a direction

b - direction of force component

when $a = b$ we have an axial type force

$a \neq b$ we have shear-type forces.

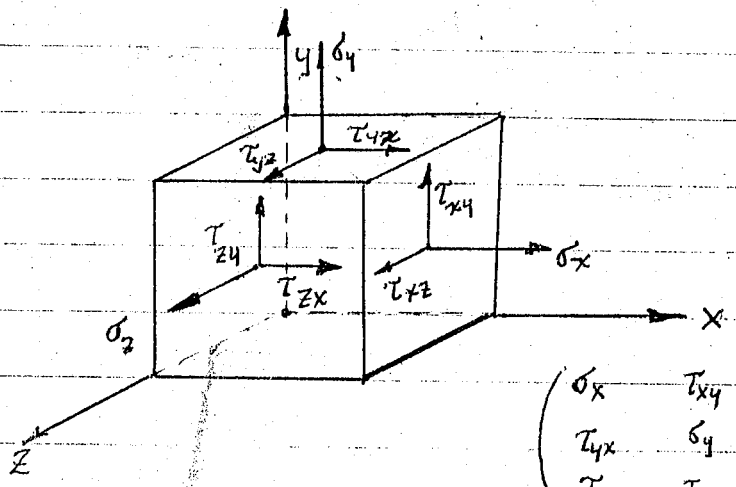
$\tau_{xx} \equiv \sigma_x$, $\tau_{yy} \equiv \sigma_y$, $\tau_{zz} \equiv \sigma_z$

Force = Stress by area



if $\sigma_x A_2 = \sigma_x A_1$ we have equilib.

if $\sigma_x A_2 \neq \sigma_x A_1$ we have motion



$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} \Rightarrow \text{Stress Tensor.}$$

Scalar 0-Rank

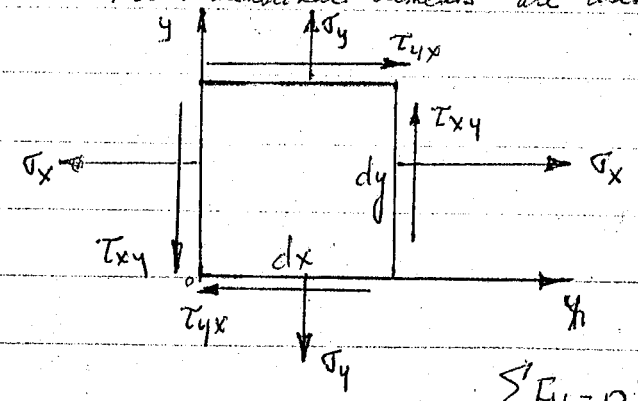
Vector 1st-Rank Stress-Tensor 2nd rank

$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$$

tensorss { Scalar — 0 rank — t — 2^0 — 3^0
vector — 1 rank — V_x, V_y, V_z — 2^1 — 3^1
tensor — 2 rank — 2^2 — 3^2

# of components to describe it in Plane / Free Space

✓ Two dimensional elements are used for the derivation.



Uniform Thickness dz

$\sum F_x = 0$

$\sigma_x\, dy\, dz - \sigma_x\, dy\, dz +$

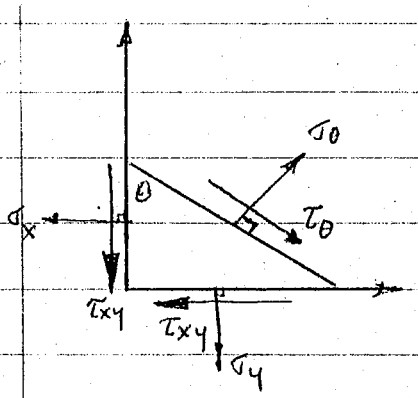
$\tau_{yx}\, dx\, dz - \tau_{yx}\, dy\, dz = 0$

$\sum F_y = 0 \uparrow \quad \sigma_y\, dx\, dz - \sigma_y\, dx\, dz$

$+ \tau_{xy}\, dy\, dz - \tau_{xy}\, dy\, dz = 0$

$\Sigma M_o = 0 \quad \tau_{yx}(dx\,dz)\,dy - \tau_{xy}(dy\,dz)\,dx = 0$

$$\boxed{\tau_{yx} = \tau_{xy}} \iff \tau_{xz} = \tau_{zx} \iff \tau_{yz} = \tau_{zy}$$



$\sigma_\theta = f(\theta)$ since σ_θ depends on $\sigma_x, \sigma_y, \tau_{xy}, \theta$; but σ_x, τ_{xy} have been already calculated & they are constant; thus

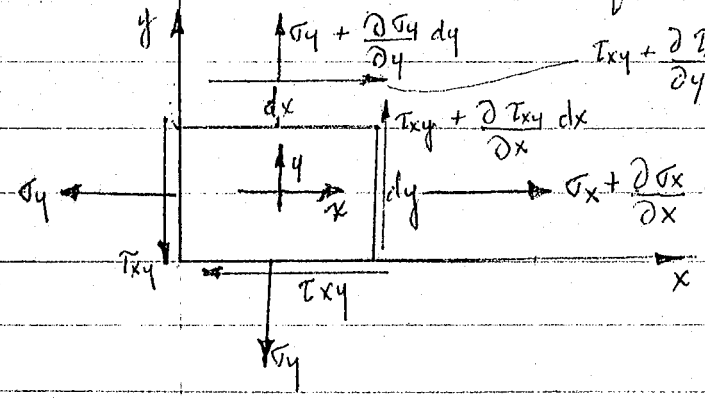
$\tau_\theta = f(\theta)$

σ_θ, and similarly τ_θ, are functions of θ.

we can find some $\theta \Rightarrow \tau_\theta, \sigma_\theta$ are 0 in the xy, yz, xz planes so that we can produce

$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yz} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} \Longrightarrow \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$$

✓ We will derive the equations of unequal stresses



$\Sigma F_x = 0$;

$$\left(\sigma_x + \frac{\partial \sigma_x}{\partial x} dx\right) dy\,dz - \sigma_x\,dy\,dz + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y} dy\right) dx\,dz - \tau_{xy}\,dx\,dz + X\,dy\,dz\,dx = 0$$

$\Sigma F_y = 0$; $\left(\sigma_y + \frac{\partial \sigma_y}{\partial y} dy\right) dx\,dz - \sigma_y\,dx\,dz + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x} dx\right) dy\,dz - \tau_{xy}\,dy\,dz + Y\,dx\,dy\,dz = 0$

$$\boxed{(1) \quad \frac{\partial}{\partial x}(\sigma_x) + \frac{\partial}{\partial y}(\tau_{xy}) + X = 0}$$

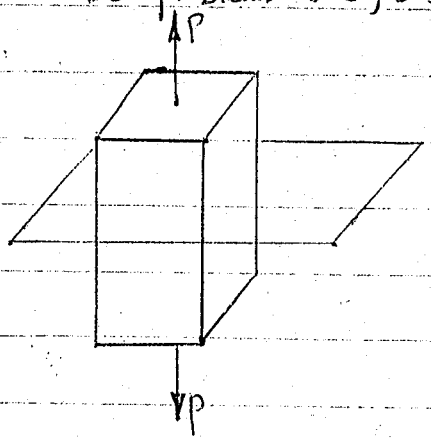
Three dimensionalization

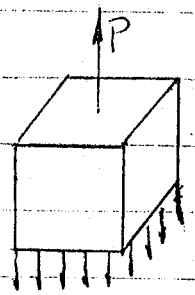
$$\frac{\partial}{\partial x}(\sigma_x) + \frac{\partial}{\partial y}(\tau_{xy}) + \frac{\partial}{\partial z}(\tau_{xz}) + X = 0$$

$$\frac{\partial}{\partial y}(\sigma_y) + \frac{\partial}{\partial x}(\tau_{xy}) + \frac{\partial}{\partial z}(\tau_{yz}) + Y = 0$$

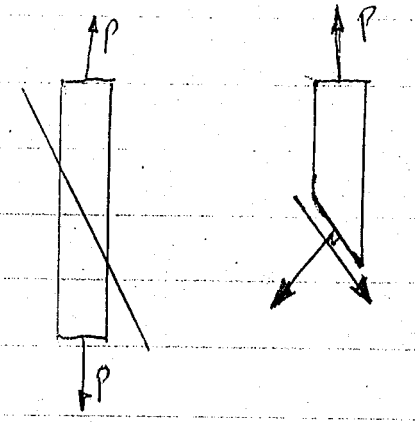
$$\frac{\partial}{\partial z}(\sigma_z) + \frac{\partial}{\partial x}(\tau_{xz}) + \frac{\partial}{\partial y}(\tau_{yz}) + Z = 0$$

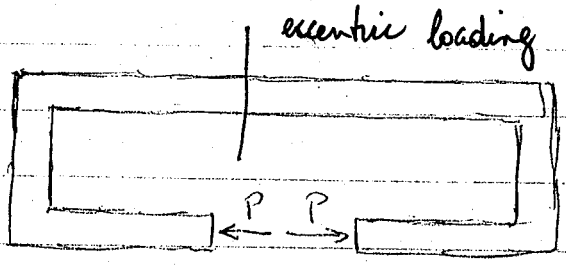
Do problems 3-2, 3-3





$$\sigma = \frac{P}{A}$$





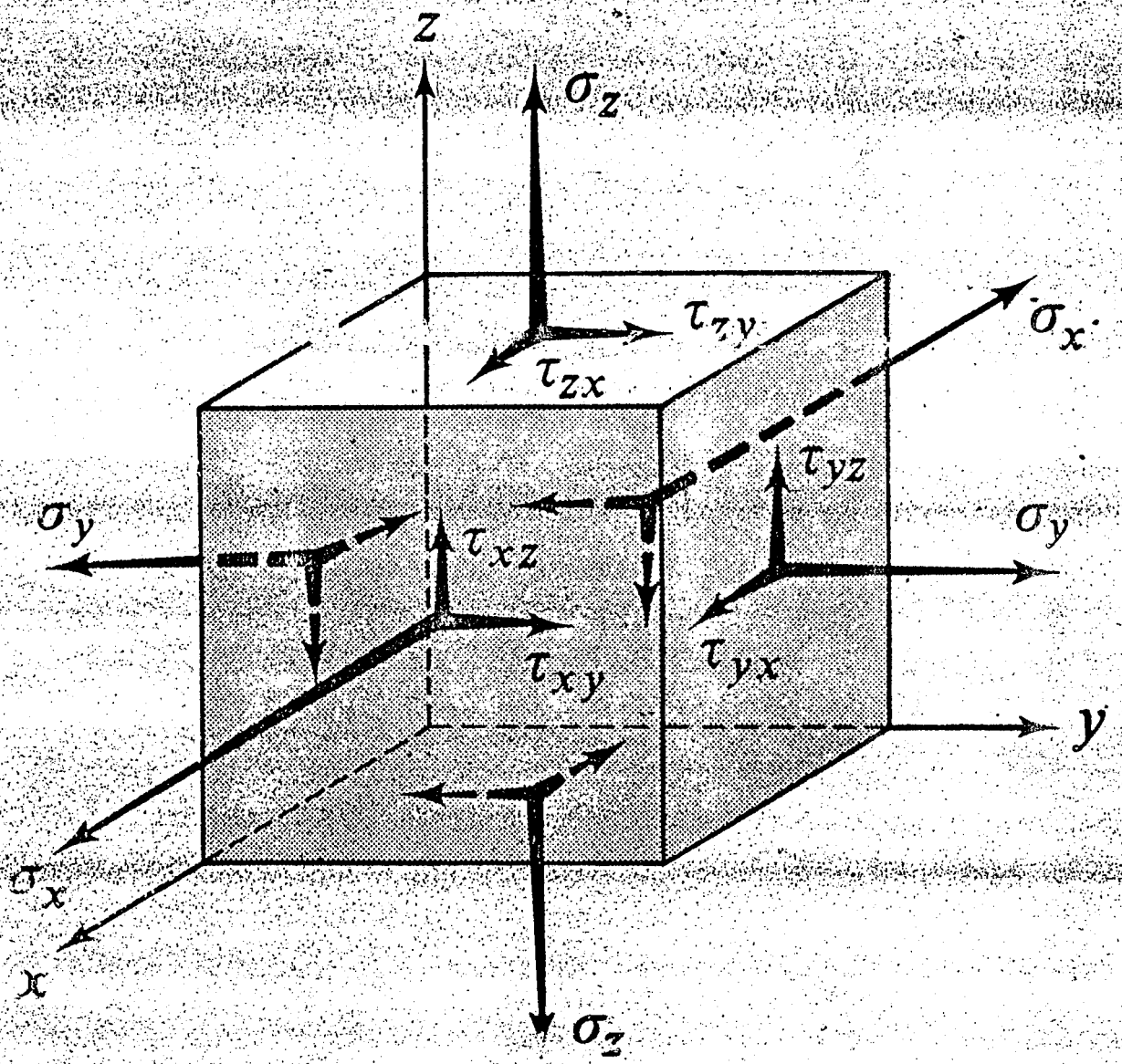


Fig. 1-3. The most general state of stress acting on an element

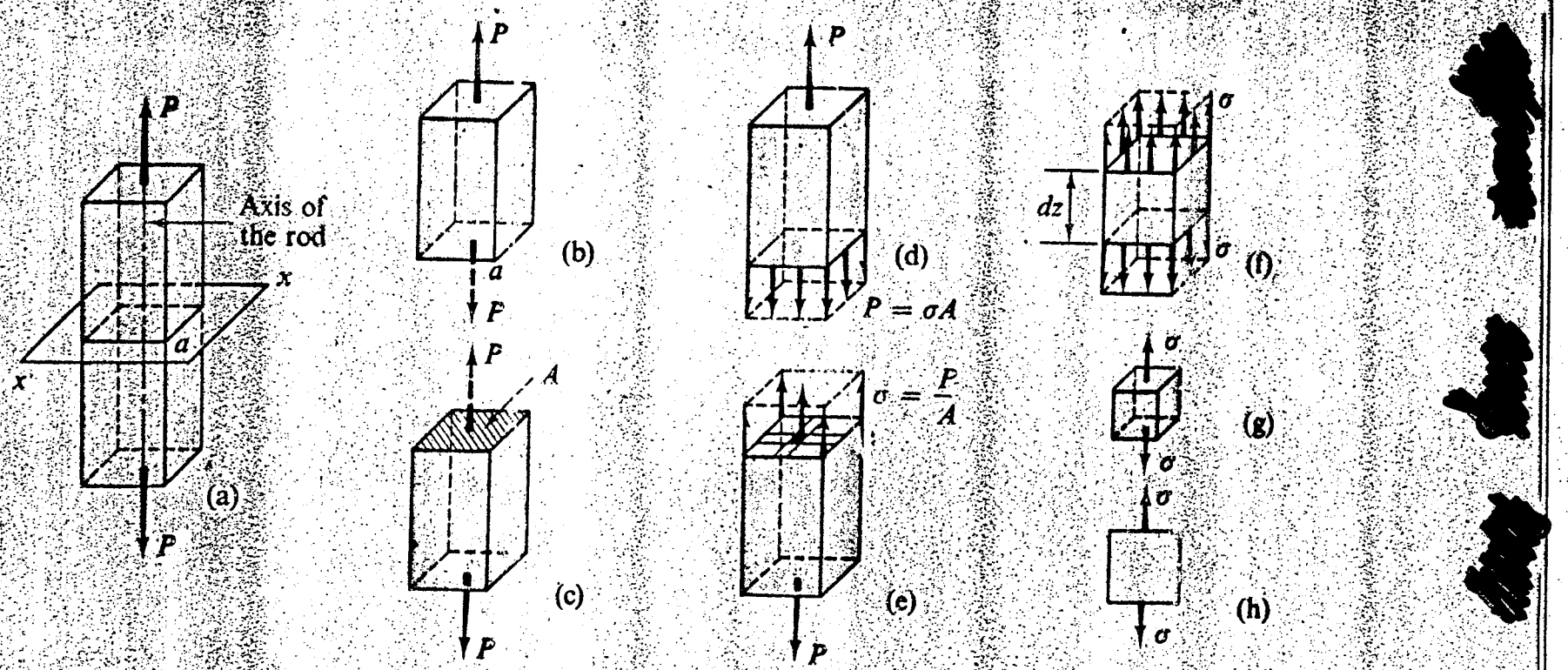
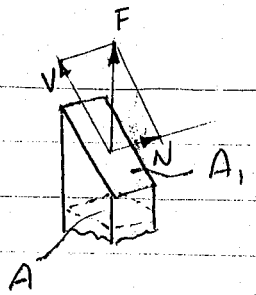


Fig. 4. Successive steps in the analysis of a body for stress





F/A_1 = Intensity of load in axial direction

$F/A_1 < F/A$

$\sigma = \frac{N}{A_1}$ normal stress

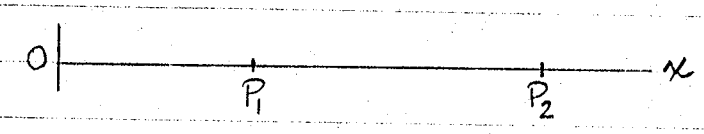
assume that σ & τ are uniformly distributed $\tau = \frac{V}{A_1}$ shear stress

stress at $B = \left(\frac{\Delta F}{\Delta A}\right)_{\Delta A \to 0}$ we define at vector $\vec{n} \perp$ to the plan

↓ need 2 vectors those quantities needing 2 vectors to define it are tensors ∴ stress is a tensor.

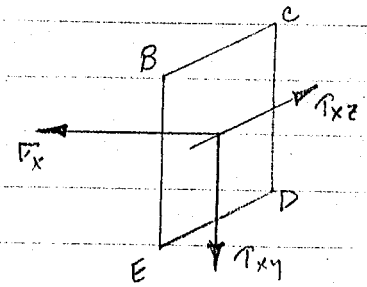
9-29-70

FIG 3 9-29



Positive face of the parallelopiped is defined as the one with the larger coordinate.

FIG. 2a 9-29



σ_x is defined positive

τ_{xy} " " "

Sign convention: σ → tensile stress +

→ compressive " −

τ → pos. direct. for positive

→ neg. " " neg

FIG. 2b 9-29

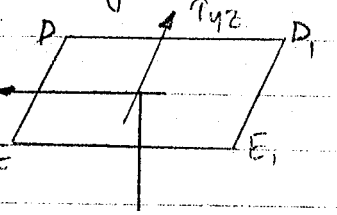
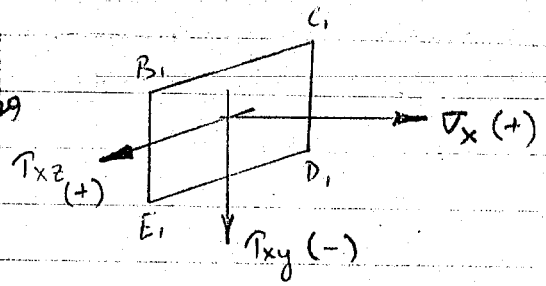


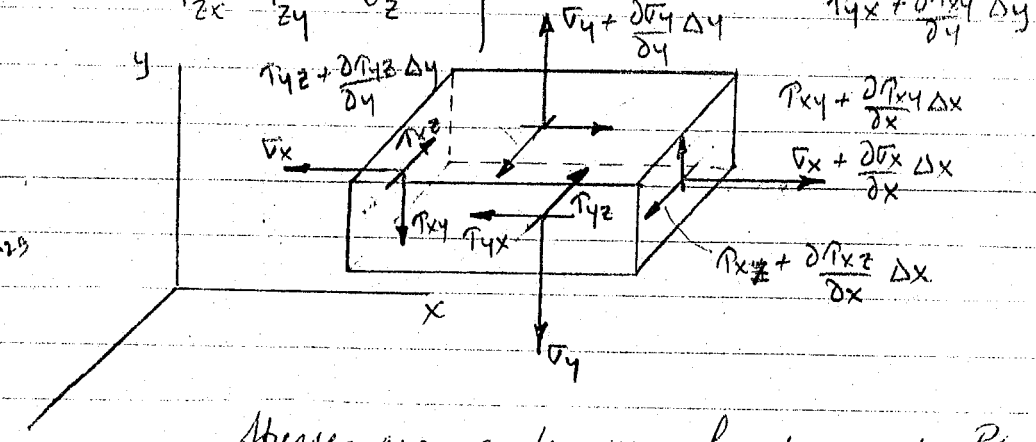
FIG. 2c
9-29



3 planes exist at a pt with 3 stress components each
$\Rightarrow$ 9 stress component all together at a point.

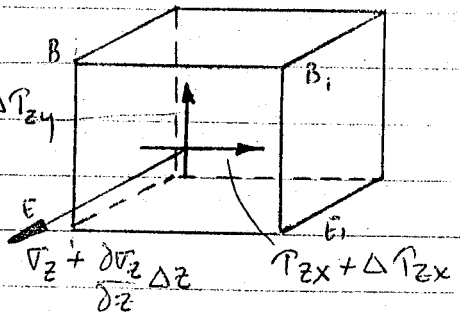
$$\begin{matrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{matrix}$$ Stress tensor

FIG. 1
9-29



Stresses are continuous functions at $P(x,y,z)$
they change from pt to pt.

FIG. 4a
9-29



$$\Delta\tau_{zy} = \frac{\partial \tau_{zy}}{\partial z}\Delta z$$

$$\Delta\tau_{zx} = \frac{\partial \tau_{zx}}{\partial z}\Delta z$$

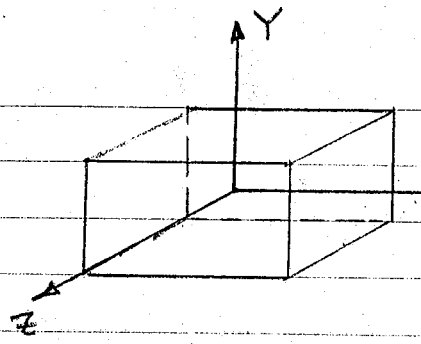


FIG 4b
9-29

Body Forces

$$\sum F_x = \left(\sigma_x + \frac{\partial \sigma_x}{\partial x}\Delta x\right)\Delta y\Delta z - \sigma_x \Delta y\Delta z + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y}\Delta y\right)\Delta x\Delta z$$

$$- \tau_{xy}\Delta x\Delta z + \left(\tau_{xz} + \frac{\partial \tau_{xz}}{\partial z}\Delta z\right)\Delta x\Delta y - \tau_{xz}\Delta x\Delta y$$

$$+ X\Delta x\Delta y\Delta z = 0$$

$$\left[\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + X\right]\Delta x\Delta y\Delta z = 0$$

9-29.1

$$\left.\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + X &= 0 \\ \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + Y &= 0 \\ \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + Z &= 0 \end{aligned}\right\} \text{Equilibrium}$$

look at moment about z-axis

$$\sum M_{cz} = \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x}\Delta x\right)\Delta y\Delta z\,\frac{\Delta x}{2} + \tau_{xy}\Delta y\Delta z\,\frac{\Delta x}{2}$$

$$- \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\Delta y\right)\Delta x\Delta z\,\frac{\Delta y}{2} - \tau_{yx}\Delta x\Delta z\,\frac{\Delta y}{2} = 0$$

$$\cong \left(\tau_{xy} - \tau_{yx}\right)\Delta x\Delta y\Delta z = 0 \longrightarrow \boxed{\tau_{xy} = \tau_{yx}}$$

9-29.2

$$\left.\begin{aligned} \tau_{xy} &= \tau_{yx} \\ \tau_{yz} &= \tau_{zy} \end{aligned}\right\} \text{ from moment eq. Complementary shear theorem}$$

directions assumed for
ree-body diagram.

senses of p, V, and M are
in the figure, derive the
to Eqs. 2-4, 2-5, and 2-6.

For the beams loaded in
in the figures, neglecting
embers, solve as directed:
rmal computations, sketch
diagrams directly below a
n loaded member.
, and, in addition, show
stic curve.
, moment, and, wherever
ce diagrams for the main

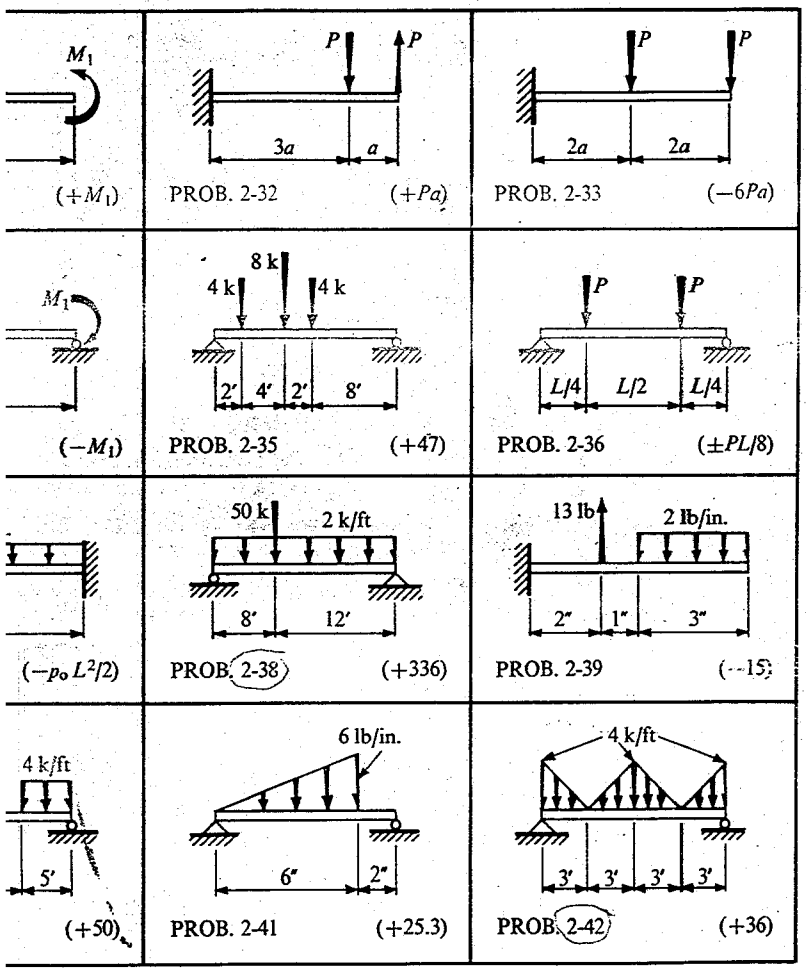
horizontal members. Determine all critical ordinates.

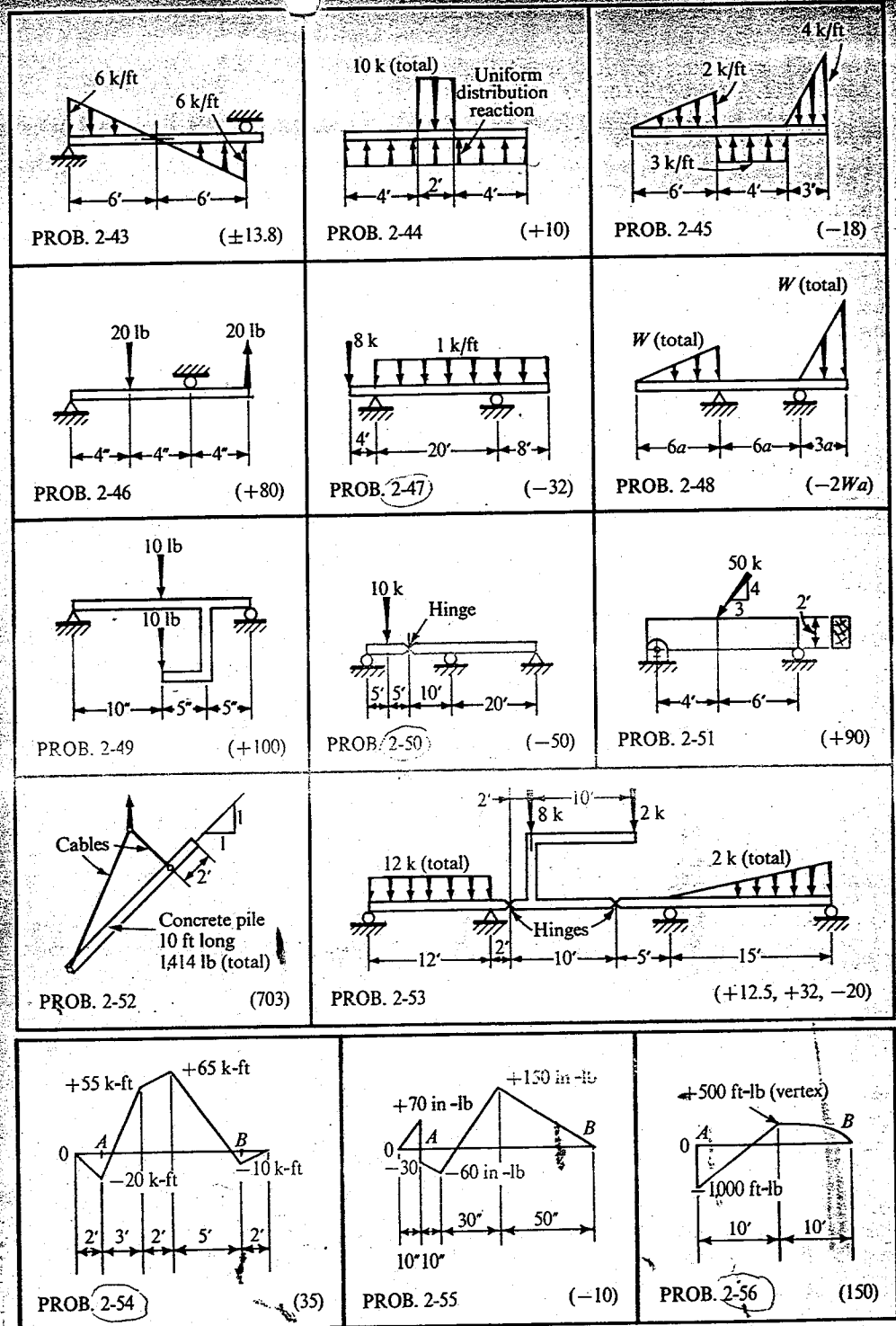
D. Same as **C**, and, in addition, determine the points of inflection and show the shape of the elastic curve.

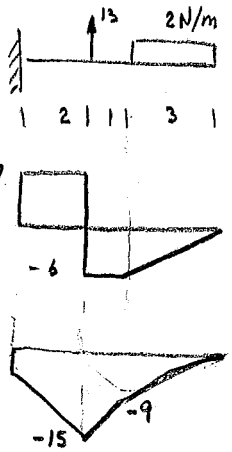
Ans. All shear and moment diagrams must close. The largest moment is given in parentheses by the figures in the units of the problem.

For additional loading conditions see other problems in this chapter.

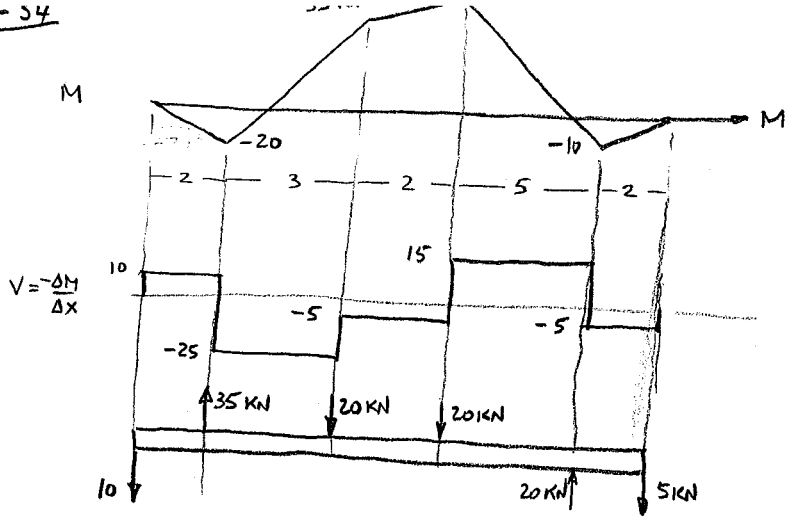
2-54 through 2-56. The moment diagrams for beams supported at A and B are as shown in the figures. How are these beams loaded? All curved lines represent parabolas, i.e., plots of equations of the second degree. (*Hint:* construction of shear diagrams aids the



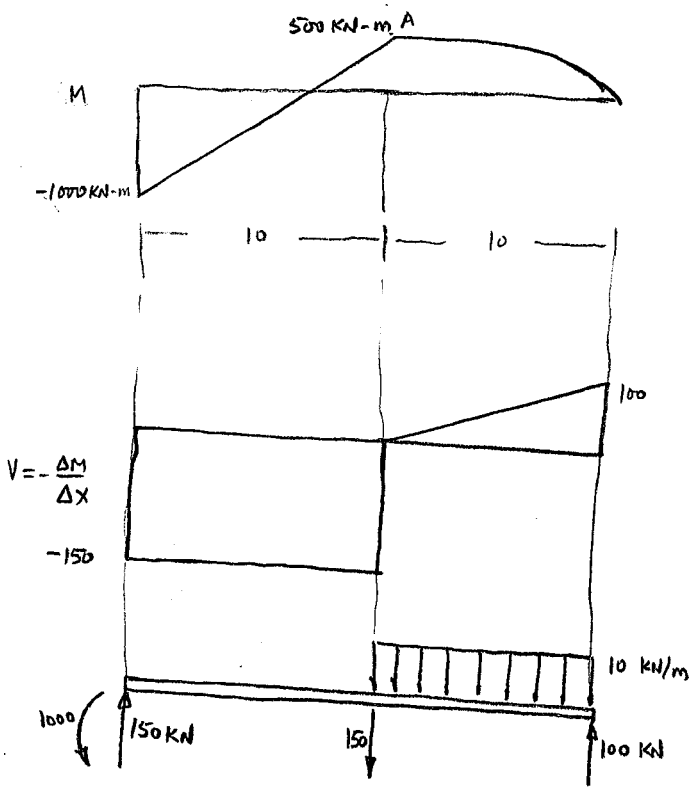




2-54



2-56



מכיוון שהשיפוע של המומנט שווה לאפס ב- A

$\frac{dM}{dx}=-V$ אבל $\frac{dM}{dx}=0$. לכן, $V=0$ ב- A $\Leftarrow$

מכיוון שהשיפוע הוא שלילי, $\frac{dM}{dx}=-V$, לכן

V לינארי בשיפוע חיובי

6 unknowns, 3 equations makes proper problem statically indeterminate.

10-5-70

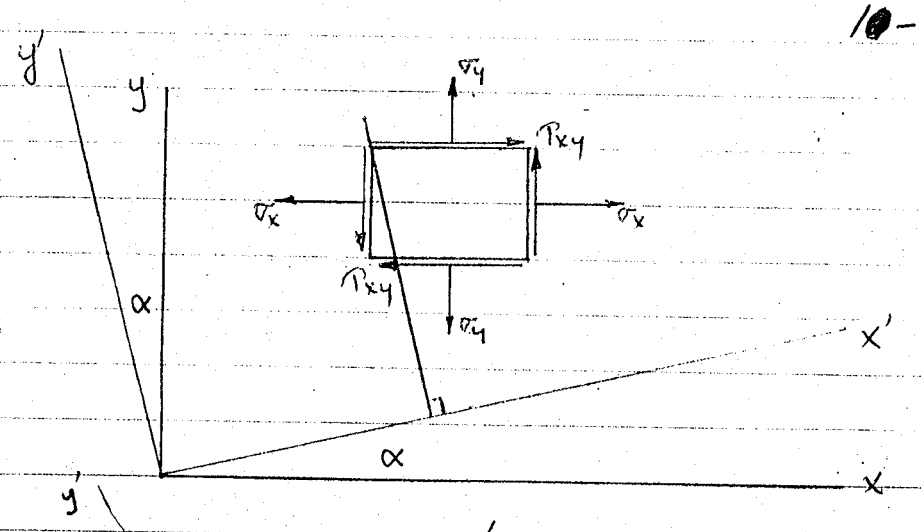


FIG. 1
10-5

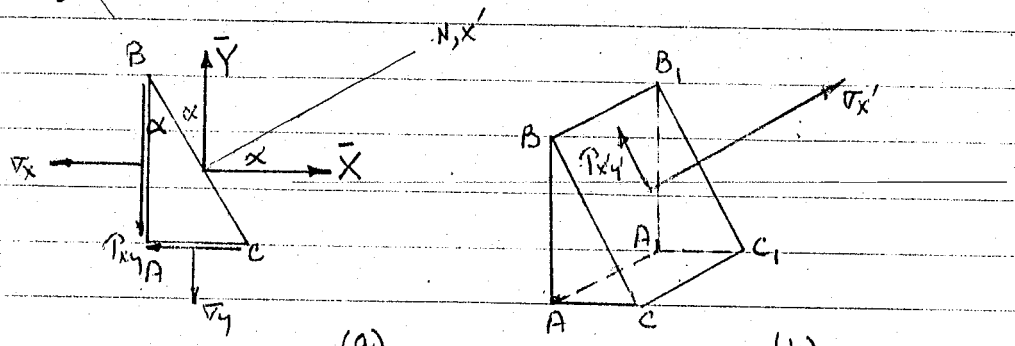


FIG. 2
10-5

$\bar{AB} = \bar{BC}\cos\alpha = A_n \cdot l$

$\bar{AC} = \bar{BC}\sin\alpha = A_m \cdot m$

$ABB_1A_1A = A_x = A_n\cos\alpha$

$ACC_1A_1A = A_y = A_n\sin\alpha$

$BB_1C_1CB = A_n$

$$\bar{X} = \sigma_x A_x + \tau_{xy} A_y = \sigma_x A_n \cos\alpha + \tau_{xy} A_n \sin\alpha$$

✓ $\bar{Y} = \tau_{xy} A_x + \sigma_y A_y = \tau_{xy} A_n \cos\alpha + \sigma_y A_n \sin\alpha$

✓ $X' = A_n \sigma_x' = (\bar{X}\cos\alpha + \bar{Y}\sin\alpha) = (\sigma_x \cos\alpha + \tau_{xy}\sin\alpha) A_n \cos\alpha + (\tau_{xy}\cos\alpha + \sigma_y \sin\alpha) A_n \sin\alpha$

$\sigma_x' = \sigma_x \cos^2\alpha + 2\tau_{xy} \sin\alpha \cos\alpha + \sigma_y \sin^2\alpha$

10-5,1 ✓ $$\boxed{\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\alpha + \tau_{xy} \sin 2\alpha}$$

$Y' = A_n \tau_{x'y'} = \bar{Y}\cos\alpha - \bar{X}\sin\alpha$; (thus $\tau_{x'y'} = \frac{\bar{Y}\cos\alpha - \bar{X}\sin\alpha}{A_n}$)

$A_n \tau_{x'y'} = [\tau_{xy}\cos^2\alpha + (\sigma_y - \sigma_x)\sin\alpha\cos\alpha - \tau_{xy}\sin^2\alpha] A_n$

$= \tau_{xy}(\cos^2\alpha - \sin^2\alpha) - \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\alpha$

10-5,2 ✓ $$\boxed{\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\alpha + \tau_{xy}\cos 2\alpha}$$

✓ for σ_y' use $\alpha = \alpha + 90^\circ$ and get

$$\sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha\right]$$

	X	Y		
x'	l	m	$\cos\alpha = l$	$\bar{X} = (l\sigma_x + m\tau_{xy})A_n$
y'	$l_2 = -m$	$m_2 = l$	$\sin\alpha = m$	$\bar{Y} = (l\tau_{xy} + m\sigma_x)A_n$

$l_2^2 + m^2 = 1$ $\quad \sigma_x' = (l\bar{X} + m\bar{Y})/A_n$

$= l^2\sigma_x + 2lm\tau_{xy} + m^2\sigma_y$

$\frac{\partial \sigma_x'}{\partial l} = (\bar{X} + \bar{Y}\cdot\frac{\partial m}{\partial l})/A_n = [(l\sigma_x + m\tau_{xy}) + (l\tau_{xy} + m\sigma_x)\frac{\partial m}{\partial l}]/A_n$

$\frac{\partial \sigma_x'}{\partial l} = 2\left[l\sigma_x + m\tau_{xy}\right] + 2\left[l\tau_{xy} + m\sigma_y\right]\frac{\partial m}{\partial l} = 0$

if $l^2 + m^2 = 1$ $\quad 2l\,dl + 2m\,dm = 0$ $\quad \frac{dm}{dl} = -l/m$

$$\frac{\overset{(1)}{l\sigma_x + m\tau_{xy}}}{l} = \frac{\overset{(2)}{l\tau_{xy} + m\sigma_y}}{m} = \overset{(3)}{\sigma}$$

$\bar{X}/Al = \bar{Y}/Am = \sigma$

from ① & ③ $\quad (\sigma_x - \sigma)\,l + m\,\tau_{xy} = 0$

from ② & ③ $\quad (\sigma_y - \sigma)\,m + l\,\tau_{xy} = 0$

Therefore, $\begin{vmatrix} \sigma_x - \sigma & \tau_{xy} \\ \tau_{xy} & \sigma_y - \sigma \end{vmatrix} = 0$

or, $\sigma^2 - (\sigma_x + \sigma_y)\sigma + \left[(\sigma_x \sigma_y) - \tau_{xy}^2\right] = 0$

10-5.3 ✓

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

✓ we will say principal stresses σ_1 & σ_2 when $\tau_{x'y'} = 0 \Rightarrow \tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

✓ $\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

$\sin 2\alpha = \frac{\tau_{xy}}{\sqrt{}}$, $\cos 2\alpha = \frac{\sigma_x - \sigma_y}{2\sqrt{}}$

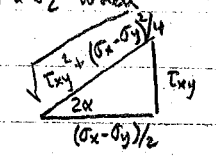
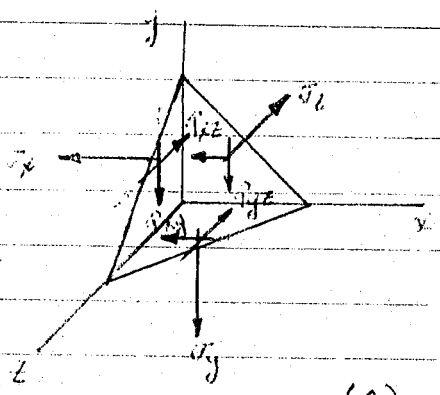
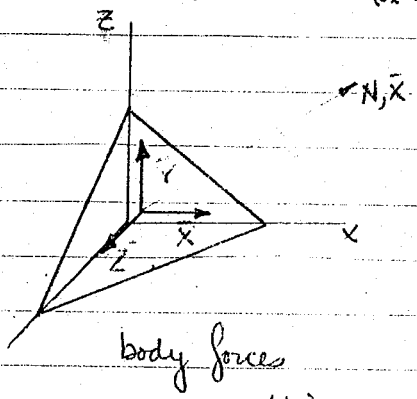


FIG. 3 10-5



(a)



body forces

(b)

$A_x = l_1 A_n$

$A_y = m_1 A_n$

$A_z = n_1 A_n$

$\bar{X} = (l_1 \sigma_x + m_1 \tau_{xy} + n_1 \tau_{zx}) A_n$

$\bar{Y} = (l_1 \tau_{xy} + m_1 \sigma_y + n_1 \tau_{zy}) A_n$

$\bar{Z} = (l_1 \tau_{xz} + m_1 \tau_{yz} + n_1 \sigma_z) A_n$

	x	y	z
$\bar{x}'$	l_1	m_1	n_1
y'			
$\bar{z}'$			

2.1

$\sigma_x = 3x^2y \qquad \sigma_y = 4x^2y^2 \qquad \sigma_z = 7xz^3 \qquad \bar{X}, \bar{Y}, \bar{Z} = 0$

$\tau_{xy} = 8x^2y^2 \qquad \tau_{yz} = 9y^2z^2 \qquad \tau_{zx} = 10x^2y^2$

$6xy + 16x^2y + 0 \neq 0$

$16xy^2 + 8x^2y + 18y^2z \neq 0$

$20xy^2 + 18yz^2 + 21xz^2 \neq 0$

they do not satisfy equil.

2.3. $\sigma_x = 35{,}000$ psi $\quad \sigma_y = -1700$ psi $\quad \tau_{xy} = 13000$ psi

$\theta_c = \arctan 3/4 \qquad \sin\theta_c = 3/5 ; \quad \cos\theta_c = 4/5$

$$\frac{\partial \sigma_x'}{\partial \theta} = -2\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + 2\tau_{xy}\cos 2\theta = 0$$

$$\tau_{xy}\cos 2\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta$$

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

$$\sigma_{1,2} = \left(\frac{33{,}300}{2}\right) \pm \left[\left(18{,}350\right)^2 + \left(13{,}000\right)^2\right]^{1/2}$$

$$\left[3365 \times 10^6 + 169. \times 10^6\right]^{1/2}$$

$$22.5 \times 10^3$$

$$= 16{,}650 \pm 22{,}500$$

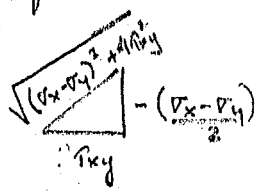
$\sigma_1 = 39{,}150$ psi

$\sigma_2 = -5{,}850$ psi $\qquad \tan 2\theta = \dfrac{2(13000)}{36{,}700} = \dfrac{26.000}{36{,}700} = .707$

$2\theta = 35.3^\circ \qquad \theta_1 = 17.65^\circ \qquad \theta_2 = 107.65^\circ$

b. $\frac{\partial \tau_{x'y'}}{\partial \theta} = -(\sigma_x - \sigma_y)\cos 2\theta + 2\tau_{xy}\sin 2\theta = 0$

$\tan 2\theta = -\frac{(\sigma_x - \sigma_y)}{2\tau_{xy}}$



$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 22500$ psi

$\tan 2\theta = -\frac{(16650)}{13000}$ $\quad 54.7^\circ = 2\theta \quad \theta = 27.35^\circ$

at τ_{max} $\sigma_x' = \left(\frac{\sigma_x + \sigma_y}{2}\right) = 16{,}650$ psi but $\sigma_y' = \left(\frac{\sigma_x + \sigma_y}{2}\right) = 16{,}650$ psi

c. $\theta_c = \arctan 3/1$ $\quad \sin 2\theta_c = 2\sin\theta_c\cos\theta_c = \frac{24}{25}$

$\cos 2\theta_c = \cos^2\theta_c - \sin^2\theta_c = 7/25$

$\sigma_{x'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\left[\cos^2\theta_c - \sin^2\theta_c\right] + 2\tau_{xy}\sin\theta_c\cos\theta_c$

$= 16{,}650 + \left(\frac{36{,}700}{2}\right)\frac{7}{25} + 13{,}000\left(\frac{24}{25}\right)$

$= 16{,}650 + 5140 + 12500 = 34{,}290$ psi

$\sigma_{y'} = 16{,}650 - 17{,}640 = -990$ psi

$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\alpha + \tau_{xy}\cos 2\theta$

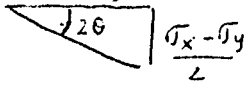
$-17616 + 3640 = -13976$ psi

כדי למצוא את הזוויות (או המישורים) שבהם פועלים המאמצים בציר המקסימלי ומינימלי,

$$\frac{\partial \tau_{xy}}{\partial \theta} = 0 = \frac{\partial}{\partial \theta}\left[\tau_{xy}\cos 2\theta - \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta\right]$$

$$= -2\tau_{xy}\sin 2\theta - 2\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta$$

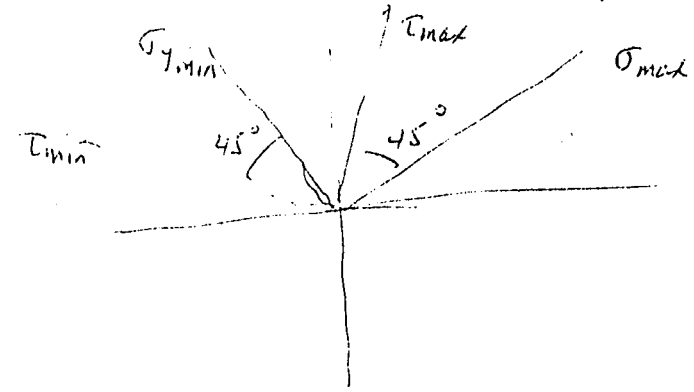
לכן



$$m_2 = \tan 2\theta = -\frac{\sigma_x-\sigma_y}{2\tau_{xy}}$$

$$m_1 m_2 = -1 \quad \Rightarrow \quad 2\theta \perp 2\alpha$$

כלומר זוויות ב- המישורים למאמץ בציר המקסימלי, הוא 45° ל מישורים שם מאמצים מקסימליים ובדיוק באמצע נכון ל מישורים של מאמצים מינימליים



$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \tau_{min} = -\tau_{max}$$

ואם נקח

$$\frac{\sigma_x+\sigma_y}{2} + \sqrt{\quad} = \sigma_1 = \sigma_{max}$$

$$\frac{\sigma_x+\sigma_y}{2} - \sqrt{\quad} = \sigma_2 = \sigma_{min}$$

$$\frac{\sigma_1-\sigma_2}{2} = \tau_{max}$$

אבל, כאן יש מאמצים נורמליים שפועלים מאונך למישורים של מאמצים בציר המקסימליים

$$\sigma' = \frac{\sigma_x+\sigma_y}{2}$$

למצוא את הזוויות שבהם τ_{max}, צריך להוסיף אחד מהזוויות θ_2 ולנסות אותו $\tau_{xy} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$

$$\sigma_x' = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\alpha + \tau_{xy}\sin 2\alpha$$

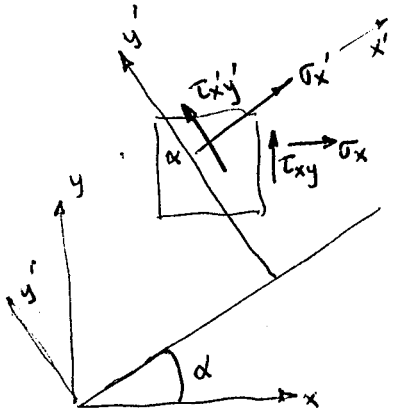
$$\tau_{x'y'} = -\frac{(\sigma_x-\sigma_y)}{2}\sin 2\alpha + \tau_{xy}\cos 2\alpha$$

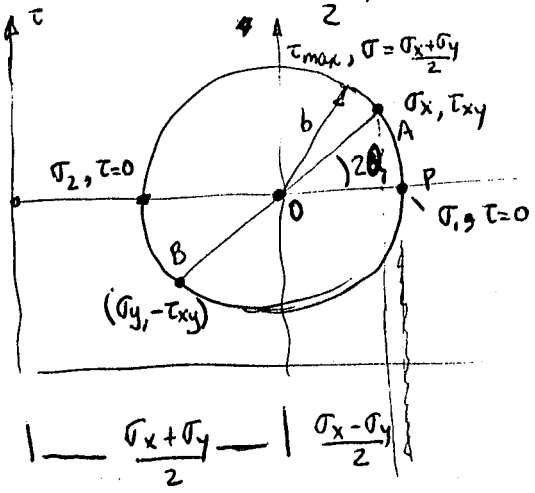
קל להוכיח שגם מתקיים

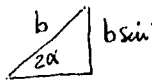
$$\left[\sigma_x' - \left(\frac{\sigma_x+\sigma_y}{2}\right)\right]^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2$$

שהיא משוואה של עיגול $(x-a)^2+y^2=b^2$ שמרכזה של σ_x' ו-$\tau_{x'y'}$

$$a = \frac{\sigma_x+\sigma_y}{2} \qquad b = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$







$b\sin 2\alpha = \tau_{xy}$

$b\cos 2\alpha = \frac{\sigma_x-\sigma_y}{2}$

$A = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right) = \sigma_x$

$B = \frac{\sigma_x+\sigma_y}{2} - \left(\frac{\sigma_x-\sigma_y}{2}\right) = \sigma_y$

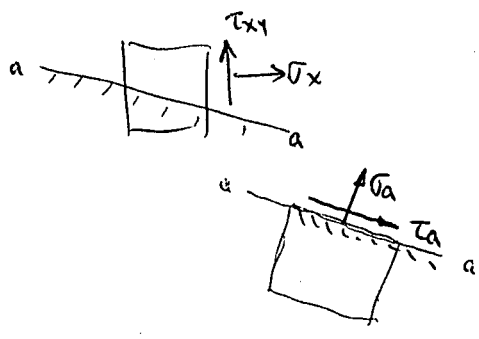
כדי למצוא את המשוואות בכיוון מסוים נניח α

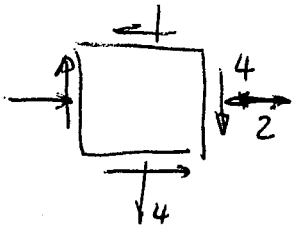
1) מוצאים את a,b לפי הנתונים ושירטטו את העיגול
2) מוצאים 2α ומייחסים את הקוו
3) במקום שהקוו חותך את המעגל אחת הנקודות (σ_x, τ_{xy}), והשניה $(\sigma_y, -\tau_{xy})$

(scale)

1) לפי קנ"מ מושרטטים את המעגל ומציינים את a,b
2) לפי קנ"מ מציינים את $A(\sigma_x, \tau_{xy})$ ← מרחק אופקי בין A למרכז מעגל זה $\frac{\sigma_x-\sigma_y}{2}$
3) לפי קנ"מ מרחק אנכי מ-A למרכז מעגל זה τ_{xy}
4) זווית $2\theta_1 = \angle AOP$ כאשר $\theta = \alpha$
למצוא משוואות לפי משור מסוים, $\sigma_{y'}=\sigma_x$, $\tau_{x'y'}=-\tau_{xy}$

1) מוצאים את a,b לפי הנתונים ושירטטו את העיגול
2) מוצאים את $A(\sigma_x, \tau_{xy})$
3) שרטטו קו ישר שעובר את A ומקביל למישור המסוים עד שקוו חותך את המעגל
4) קרהו את $S(\sigma_a, -\tau_a)$
5) אלה המשוואות שמוצאים על התתך





$\tau_{xy} = -4$

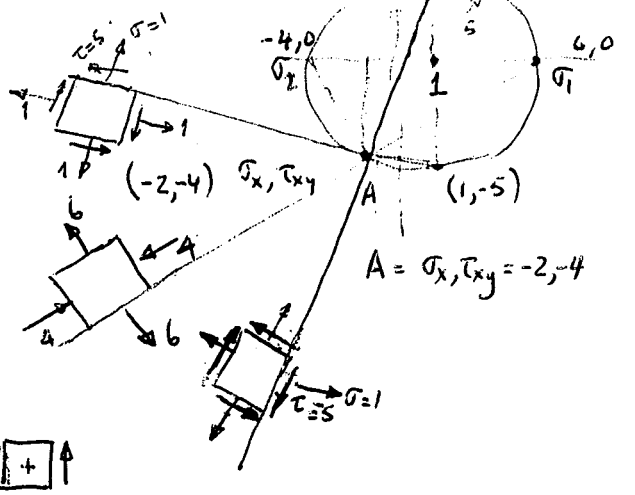
$\sigma_x = -2$

$\sigma_y = 4$

$\frac{\sigma_x + \sigma_y}{2} = \frac{-2+4}{2} = 1$

$\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 5$

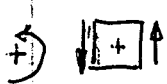
$\sigma_1, \tau_{max} = (1,5)$



$A = \sigma_x, \tau_{xy} = -2, -4$

Draw line from σ_1 through A gives plane for σ_1

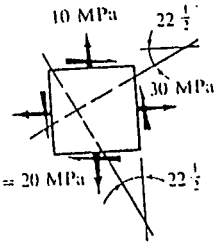
Draw line from σ_2 through A gives plane for σ_2



Direction of shear follows RH rule

- For τ_{max} Find point opposite (ie $\sigma, \tau = 1, -5$)
- Draw line between this pt & A - this gives plane
- choose direction of shear how + shear gives + mom

תרגיל תוספת — שלוש דוגמאות כיצד מצב 3



8-3: לאלמנט המתואר בציור מצד ימין את המאמץ הנורמלי ואת הצפיפה על השטחים הקטו"פ בשרטוט.

Ans: 8-3: $\sigma = -1.2$ MPa, $\tau = 7.1$ MPa; $\sigma = 41.2$ MPa, $\tau = 7.1$ MPa.

8-14. $\sigma_x = -10\ \text{MN/m}^2, \sigma_y = -40\ \text{MN/m}^2, \tau = 120\ \text{MN/m}^2$.

8-14: נתון מצב מאמצים על אלמנט.

דרוש: למצוא באופן אנליטי, למצוא מאמצים ראשיים ואזור שלהם על הקוביה המצורפת.

למצוא מאמצי גזירה מכסימליים ומצב המאמצים על הקוביה המצורפת.

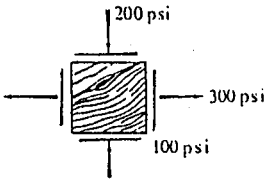
Ans: 0, -50 MN/m²; 25 MN/m².

8-16: נתון שוב כשרטוט מהדר, דרוש לפתור את שאלה 8-14.

8-22: נתן שוב כשרטוט מהדר, למצוא ואזור את הקוביה האלמנטרית, הקוביה עם המאמצים הראשיים והקוביה עם מאמצי גזירה מכסימליים, לציין הכיוון:

8-22. $\sigma_x = +60\,000\ \text{kN/m}^2$, $\sigma_y = +30\,000\ \text{kN/m}^2$, $\tau = +25\,000\ \text{kN/m}^2$. *Ans*: +74.2 MPa, 15.8 MPa, 29½°; 29.2 MPa, 45 MPa.

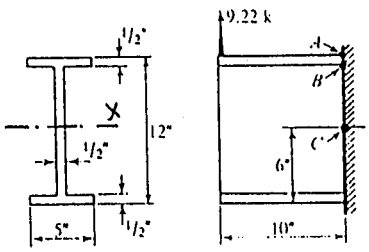
8-33: נתון מצב מאמצים על אלמנט עץ. הסיבים זרמים בזווית 30° עם ציר האנכי. מאמץ בציר המוגבל לסיבים הוא 1500 psi. האם מצב המאמצים המתואר על הקוביה מותר?



PROB. 8 - 33

9-5: בחתך המתואר בשרטוט $I_x = 221\ \text{in}^4$.

דרוש: למצוא את המאמצים הראשיים בנקודות A, B, C.



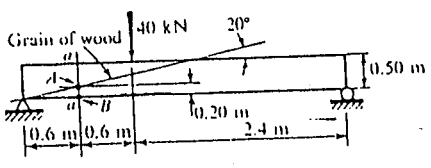
PROB. 9 - 5

Ans: At *A*, 0, −2.50 ksi; at *B*, +0.51 ksi, −2.81 ksi; at *C*, +1.84 ksi.

9-6: קורה עץ בחתך 100/500 מ"מ.

בחתך a-a הסיבים נמצאים ב- 20° לאופק.

דרוש: למצוא מאמצי גזירה בכיוון הסיבים בנקודה A ו- B.

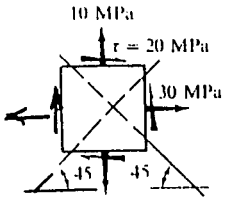


PROB. 9 - 6

Ans: $\tau_A = 835\ \text{kN/m}^2$.

תוכן תומרים - עיון ראשון כתה מס' 3

2-8 : שקול המעגל הבא קריטי, קרוס לחלק מאמץ נורמלי ומאמץ גזירה על המישורים האלכסוניים



Prob. 8-2: $\sigma = +40$ MPa, $\tau = 10$ MPa; $\sigma = 0, \tau = -10$ MPa.

8-10, 8-11 } נתונים המאמצים על אלמנט: $\sigma_x = -3$ ksi, $\sigma_y = +1$ ksi, $\tau = -2$ ksi

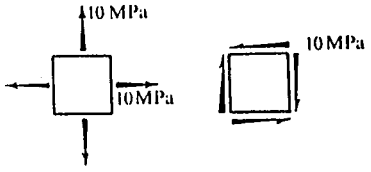
קרוס לחלק מאמצים ראשיים ומצא את כיוונים שעליהם הם פועלים. 1.83 ksi, −3.83 ksi, 22½°.

קרוס בטרנספורמציה מעגל מוהר לחלק מאמץ גזירה מכסימלי ומאמצים נורמליים הנלווים

אותו ולצייר עליהם את הכיוונים שעליהם הם פועלים. *Ans.* 2.83 ksi, −1 ksi, 67½°.

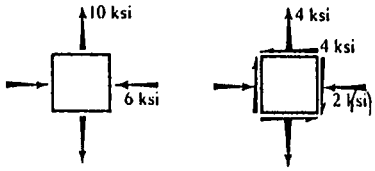
8-18 עד 8-21 } נתון שימוש כמעגל מוהר, לחלק ולצאר מצב מאמצים ראשיים ומצב מאמצי

גזירה מכסימליים באלמנטים הבאים:



PROB. 8 – 18

PROB. 8 – 19



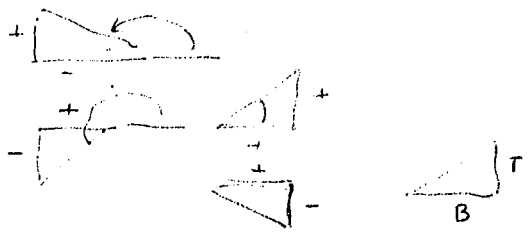
PROB. 8 – 20

PROB. 8 – 21

Ans: Prob. 8-21.
(a) 6 ksi, −4 ksi;
(b) 5 ksi, 1 ksi.

1-9 : מדוע גזרה ריקעי בתאן 50/50 מ"מ נוטן לפקיד כת מהירה.
אם מאמץ הגזירה הכסימלי שנאדד הוא 80 mpa, מה היה
כת המהירה

3-9 בקנה על בתוך מילוי 100/50 מ"מ, הסיבים נטויים ב- 25° לציר
האנך. אם מאמץ הגזירה המותר בחיבור סיבים הוא $600 \frac{kN}{m^2}$,
מה תכת הגזירי המתר (תשובה 83kN)



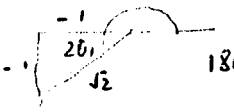
8-10

$$\frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha$$

$$-1 + -2\left(-\frac{1}{\sqrt{2}}\right) + -2\left(-\frac{1}{\sqrt{2}}\right)$$

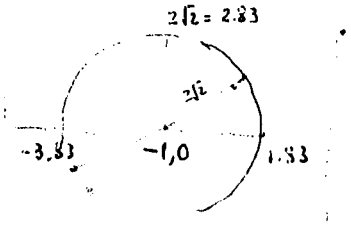
$$-1 + \frac{4}{\sqrt{2}} = -1 + 4(.701) = -1 + 2.83 = 1.83$$

$$\tan 2\theta_1 = \frac{2\tau_{xy}}{\sigma_x-\sigma_y} = \frac{2(-2)}{-3-1} = +1$$

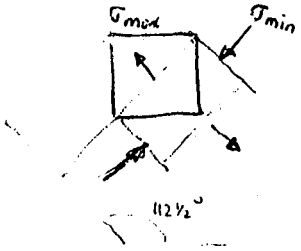


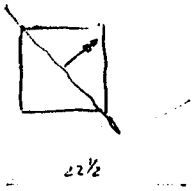
$180+45 = 225^\circ$ $\quad \theta_1 = 112.5^\circ$

8-11



$$\sigma_x' = \frac{\sigma_x+\sigma_y}{2} = \frac{-3+1}{2} = -1 \text{ ksi}$$

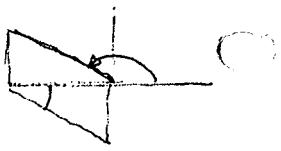




$$\tan 2\theta_1 = \frac{-(\sigma_x-\sigma_y)}{2\tau_{xy}} = \frac{-(-4)}{2(-2)} = -1$$

$$2\theta_1 = 135^\circ$$

$$\theta_1 = 67.5^\circ$$



Ultimate Strength. מאמץ המקסימלי — מאמץ המרבי

strength at which member "failed". failure can be total rupture or excessive deflection.

Factor of Safety – Factor of Ignorance מקדם בטיחות / בלתי מדעי

those factors that can't calculated and those factors which we have doubt.

Allowable Stress – מאמץ המותר

Maximum stress allowed to occur on job and that can't exceeded

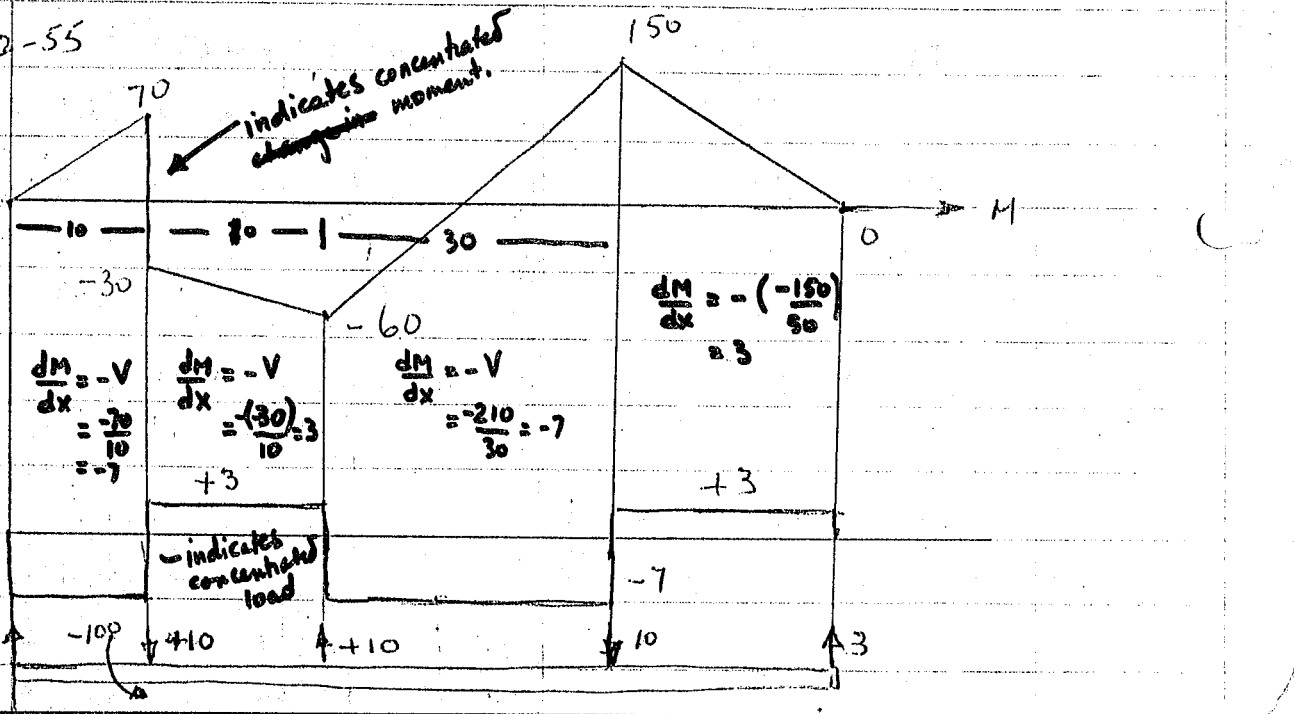
$$\text{Allowable Stress} = \frac{\text{Ult. Strength}}{\text{Factor of Safety}}$$

מאמץ המותר לצורות בלתי מדעי מקדם בטיחות יותר קבוע אחרי בין רב.

זחילה Creep – tendency of material to flow plastically under sustained load (load should produce stress almost = ultimate stress)

יתע"יפות Fatigue – if a material carries a high stress & these stresses are constantly loading & unloading or change of state (T to C. to T to C.)

מאמצים שיריים Residual Stresses –



$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & +6{,}000 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ psi}$$

where it is arbitrarily assumed that the y axis is in the direction of the applied load. In ordinary problems the complete result is implied but is seldom written down in such detail.

EXAMPLE 3-2

The concrete pier shown in Fig. 3-11(a) is loaded at the top with a uniformly distributed load of 600 lb per square foot. Investigate the state of stress at a level of 4 ft above the base. Concrete weighs approximately 150 lb per cubic foot.

SOLUTION

In this problem the weight of the structure itself is appreciable and must be included in the calculations.

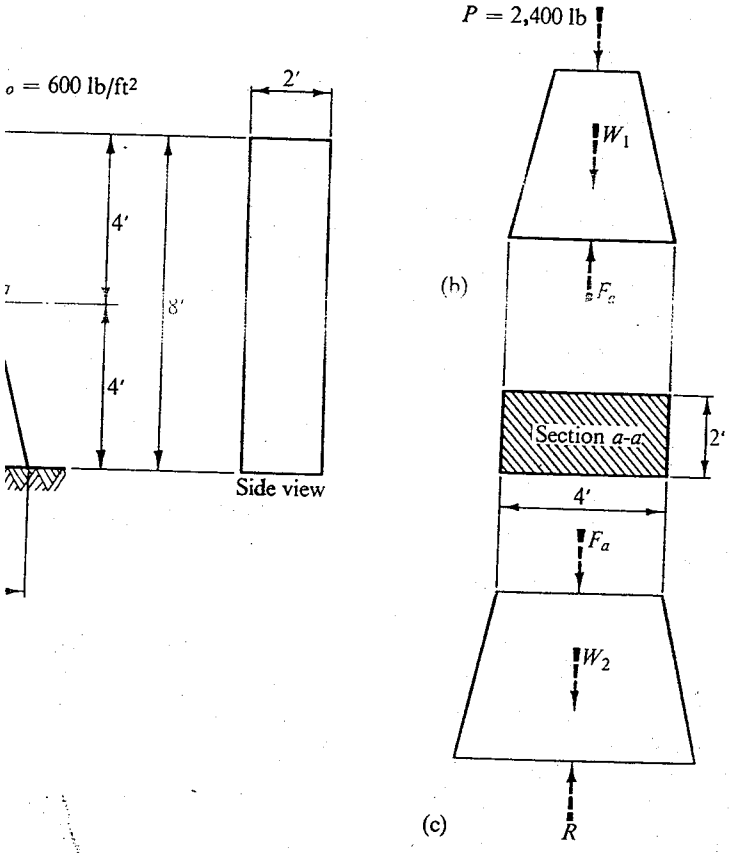


Fig. 3-11

Weight of the whole pier:

$$W = (2 + 6)2(8)150/2 = 9{,}600 \text{ lb}$$

Total applied force:

$$P = 600(2)2 = 2{,}400 \text{ lb}$$

From $\Sigma F_y = 0$, reaction at base:

$$R = W + P = 12{,}000 \text{ lb}$$

These forces are shown in the diagrams schematically as concentrated forces acting through their respective centroids. Then, to determine the stress at the desired level, the body is cut into two separate parts. A free-body diagram for either part is sufficient to solve the problem. For comparison the problem is solved both ways.

Using upper part of the pier as a free body, Fig. 3-11(b), weight of the pier above the cut:

$$W_1 = (2 + 4)2(4)150/2 = 3{,}600 \text{ lb}$$

From $\Sigma F_y = 0$, force at the cut: $F_a = P + W_1 = 6{,}000$ lb. Hence, using Eq. 3-5, the normal stress at the level a-a is

$$\sigma_a = \frac{F_a}{A} = \frac{6{,}000}{2(4)} = 750 \text{ lb per square foot} \quad \text{or} \quad \frac{750}{144} = 5.2 \text{ psi}$$

This stress is compressive as F_a acts on the cut.

Using lower part of the pier as a free body, Fig. 3-11(c), weight of the pier below the cut:

$$W_2 = (4 + 6)2(4)150/2 = 6{,}000 \text{ lb}$$

From $\Sigma F_y = 0$, force at the cut:

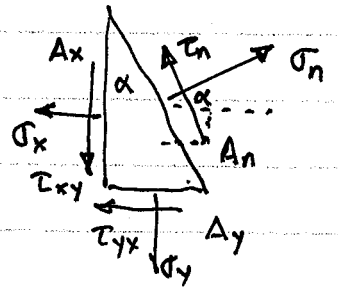
$$F_a = R - W_2 = 6{,}000 \text{ lb}$$

The remainder of the problem is the same as before. The pier considered here has a vertical axis of symmetry, making the application of Eq. 3-5 possible.*

EXAMPLE 3-3

A bracket of negligible weight shown in Fig. 3-12(a) is loaded with a force P of 3 kips. For interconnection purposes the bar ends are clevised (forked). Pertinent dimensions are shown in the figure. Find the normal stresses in the members AB and BC and the bearing and shearing stresses for the pin C. All pins are 0.375 in. in diameter.

* Strictly speaking the solution obtained is not exact as the sides of the pier are sloping. If the included angle between these sides is large, this solution is altogether inadequate. For further details see S. Timoshenko and J. N. Goodier, *Theory of Elasticity* (2nd ed.) (New York: McGraw-Hill Book Company, 1951, p. 96).



$$\frac{A_x}{A_n} = \cos\alpha \qquad \frac{A_y}{A_n} = \sin\alpha \qquad \frac{A_y}{A_x} = \tan\alpha$$

$$\Sigma F_x = 0 \quad -\sigma_x A_x - \tau_{yx} A_y + \sigma_n \cos\alpha A_n + \tau_n A_n \sin\alpha$$

$$0 = -\sigma_x A_n \cos\alpha - \tau_{yx} \sin\alpha A_n + \sigma_n \cos\alpha A_n - \tau_n A_n \sin\alpha$$

$$\Sigma F_y = 0 = -\tau_{xy} A_x + \sigma_y A_y + \sigma_n \sin\alpha A_n + \tau_n \cos\alpha A_n = 0$$

$$-\tau_{xy} \cos\alpha A_n - \sigma_y \cdot \sin\alpha A_n + \sigma_n \sin\alpha A_n + \tau_n \cos\alpha A_n = 0$$

$$-(1)\sin\alpha + (2)\cos\alpha = +\sigma_x A_n \cos\alpha \sin\alpha + \tau_{yx} \sin^2\alpha A_n - \sigma_n \cos\alpha \sin\alpha A_n + \tau_n \cdot A_n \sin^2\alpha$$

$$-\sigma_y A_n \cos\alpha \sin\alpha \quad -\tau_{xy} \cos^2\alpha A_n + \sigma_n \cos\alpha \sin\alpha A_n + \tau_n \cdot A_n \sin^2\alpha = 0$$

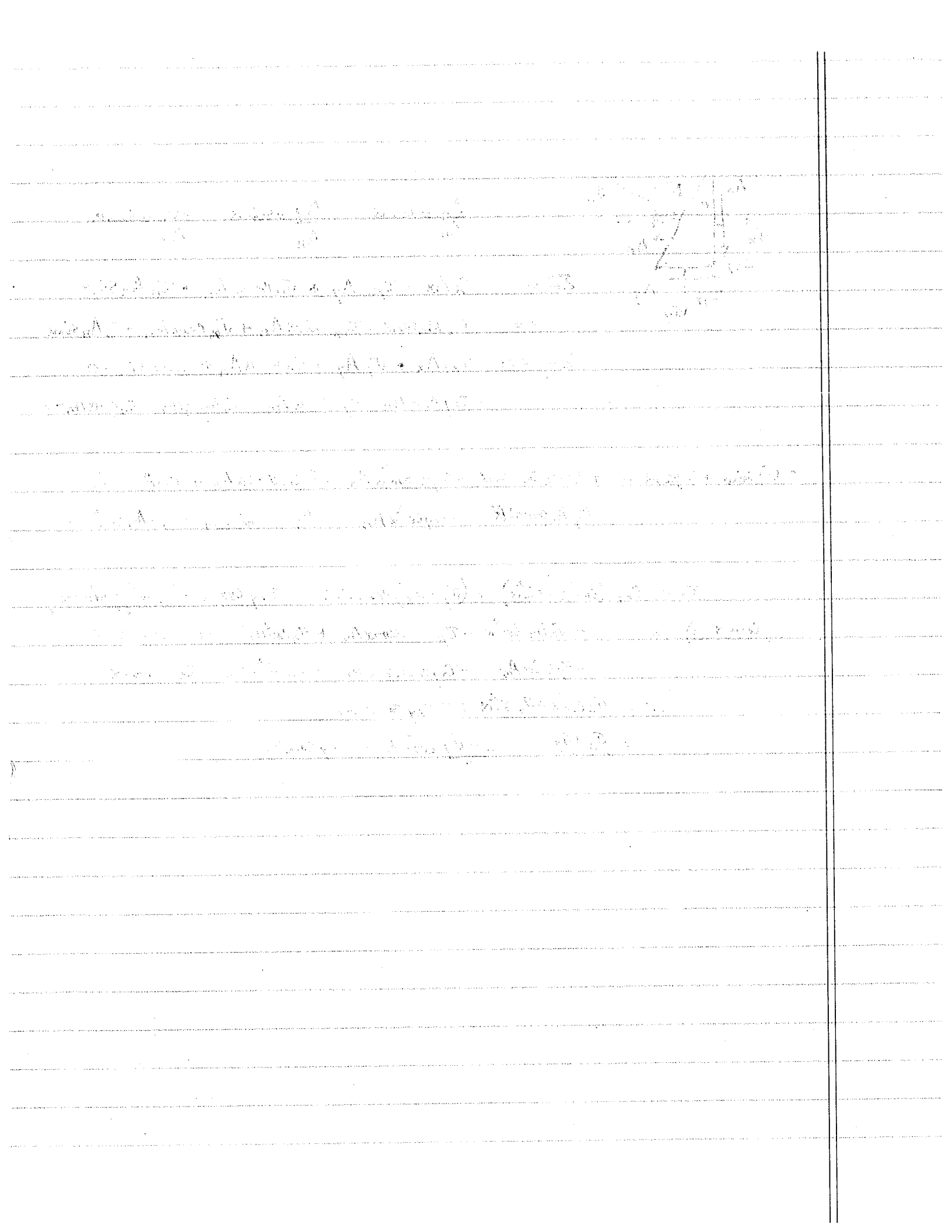
$$\tau_n = \tau_{xy}(\cos^2\alpha - \sin^2\alpha) + (\sigma_y - \sigma_x)\cos\alpha \sin\alpha = \tau_{xy} \cos 2\alpha + \frac{(\sigma_y - \sigma_x)}{2} \sin 2\alpha$$

$$(1)\cos\alpha + (2)\sin\alpha = -\sigma_x A_n \cos^2\alpha - \tau_{yx} \sin\alpha\cos\alpha A_n + \sigma_n \cos^2\alpha A_n - \tau_n A_n \cos\alpha \sin\alpha$$

$$-\sigma_y \sin^2\alpha A_n \quad -\tau_{xy} \cos\alpha \sin\alpha A_n + \sigma_n \sin^2\alpha A_n + \tau_n \cos\alpha \sin\alpha$$

$$\sigma_n = \sigma_x \cos^2\alpha + \sigma_y \sin^2\alpha + 2\tau_{xy} \sin\alpha\cos\alpha$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\alpha + \tau_{xy} \sin 2\alpha$$



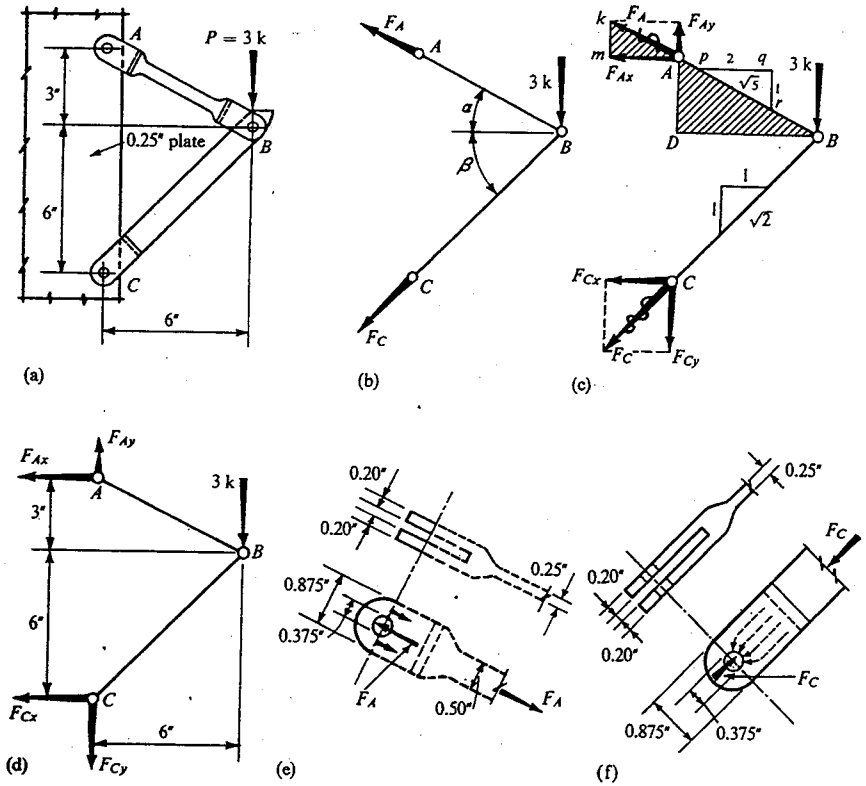


Fig. 3-12

SOLUTION

First an idealized free-body diagram consisting of the two bars pinned at the ends is prepared, Fig. 3-12(b). As there are no intermediate forces acting on the bars and the applied force acts through the joint at B, the forces in the bars are directed along the lines AB and BC, and the bars AB and BC are loaded axially. The magnitudes of the forces are unknown and are labeled F_A and F_C in the diagram.* These forces may be determined graphically by completing a triangle of forces F_A, F_B, and P. These forces may also be found analytically from two simultaneous

* In frameworks it is convenient to assume all unknown forces are tensile. A negative answer in the solution then indicates that the bar is in compression.

equations $\Sigma F_y = 0$ and $\Sigma F_x = 0$, written in terms of the unknowns F_A and F_C, a known force P, and two known angles α and β. Both these procedures are possible. However, in this course usually it will be found advantageous to proceed in a different way. Instead of treating forces F_A and F_C directly, their components are used; and instead of $\Sigma F = 0$, $\Sigma M = 0$ becomes the main tool.

Any force may be resolved into components. For example, F_A may be resolved into F_{Ax} and F_{Ay} as in Fig. 3-12(c). Conversely, if any one of the components of a directed force is known, the force itself may be determined. This follows from similarity of dimension and force triangles. In Fig. 3-12(c) the triangles Akm and BAD are similar triangles (both are shaded in the diagram). Hence, if F_{Ax} is known,

$$F_A = (AB/DB)F_{Ax}$$

Similarly, $F_{Ay} = (AD/DB)F_{Ax}$. However, note further that AB/DB or AD/DB are ratios, hence relative dimensions of members may be used. Such relative dimensions are shown by a little triangle on the member AB and again on BC. In the problem at hand

$$F_A = (\sqrt{5}/2)F_{Ax} \quad \text{and} \quad F_{Ay} = F_{Ax}/2$$

Adopting the above procedure of resolving forces, the revised free-body diagram, Fig. 3-12(d), is prepared. Two components of force are necessary at the pin joints. After the forces are determined by statics, Eq. 3-5 is applied several times, thinking in terms of a free body of an individual member:

$$\sum M_C = 0 \circlearrowleft +, \quad +F_{Ax}(3 + 6) - 3(6) = 0, \quad F_{Ax} = +2 \text{ kips}$$

$$F_{Ay} = F_{Ax}/2 = 2/2 = 1 \text{ kip},$$

$$F_A = 2(\sqrt{5}/2) = +2.23 \text{ kips}$$

$$\sum M_A = 0 \circlearrowleft +, \quad +3(6) + F_{Cx}(9) = 0,$$

$$F_{Cx} = -2 \text{ kips} \quad \text{(compression)}$$

$$F_{Cy} = F_{Cx} = -2 \text{ kips}$$

$$F_C = \sqrt{2}(-2) = -2.83 \text{ kips}$$

Check: $\sum F_x = 0, \quad F_{Ax} + F_{Cx} = 2 - 2 = 0$

$$\sum F_y = 0, \quad F_{Ay} - F_{Cy} - P = 1 - (-2) - 3 = 0$$

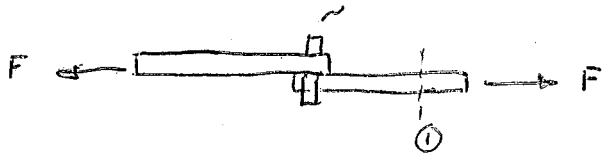
Stress in main bar AB:

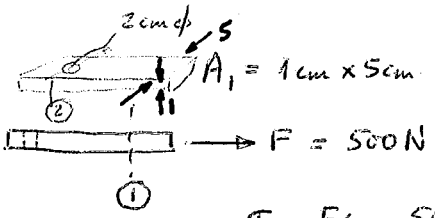
$$\sigma_{AB} = \frac{F_A}{A} = \frac{2.23}{(0.25)(0.50)} = 17.8 \text{ ksi} \quad \text{(tension)}$$

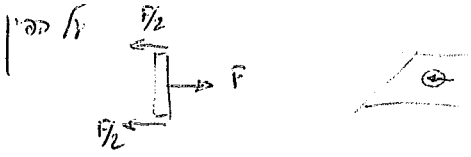
possible.

EXAMPLE 3-3

A bracket of negligible weight shown in Fig. 3-12(a) is loaded with a force P of 3 kips. For interconnection purposes the bar ends are clevised (forked). Pertinent dimensions are shown in the figure. Find the normal stresses in the members AB and BC and the bearing and shearing stresses for the pin C. All pins are 0.375 in. in diameter.







$$\sigma_1 = \frac{F}{A_1} = \frac{500\,N}{5cm^2} = \frac{100N}{cm^2} = \frac{102N}{1\times10^{-4}m^2}$$

$$= 1\,MPa$$

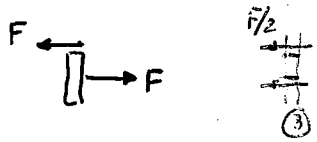
$$\sigma_{2a} = \frac{F}{A} = \frac{500\,N}{2cm \cdot 1cm} = 2.5\,MPa$$

bearing shear on pin

$$\sigma_2 = \frac{F}{A_2} = \frac{500\,N}{(1cm)(5cm-2cm)}$$

average shear at cross section of hole

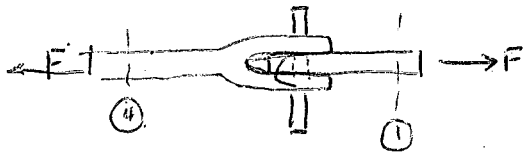
$$= \frac{500}{3cm^2} = 1.67\,MPa$$



average shear on cross-section of pin

$$\tau = \frac{F/2}{A_3} = \frac{F/2}{\pi r^2} = \frac{250 \cdot 2}{3.14(1)^2}$$

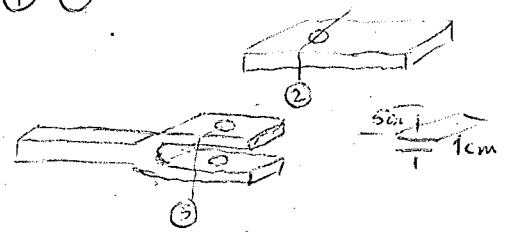
$$\tau = 159.2\,\frac{N}{cm^2} = 1.592\,MPa$$



$$\sigma_1 = F/A_1$$

④ = ①

$$\sigma_2 = F/A_2$$



$$\tau_{//3} = F/2A_3 = .796$$

$$\sigma_4 = \frac{F}{A_4} = \frac{F}{A_1} = 1\,MPa$$

$$\sigma_5 = \frac{F}{A_5} = \frac{F}{2(1cm)(5cm-2cm)} = 0.5\,MPa$$

$$A_5 = 2A_1$$

2-D

$$\sigma_x + \sigma_y = \sigma_x' + \sigma_y' = \sigma_1 + \sigma_2 = I_{1\sigma}$$

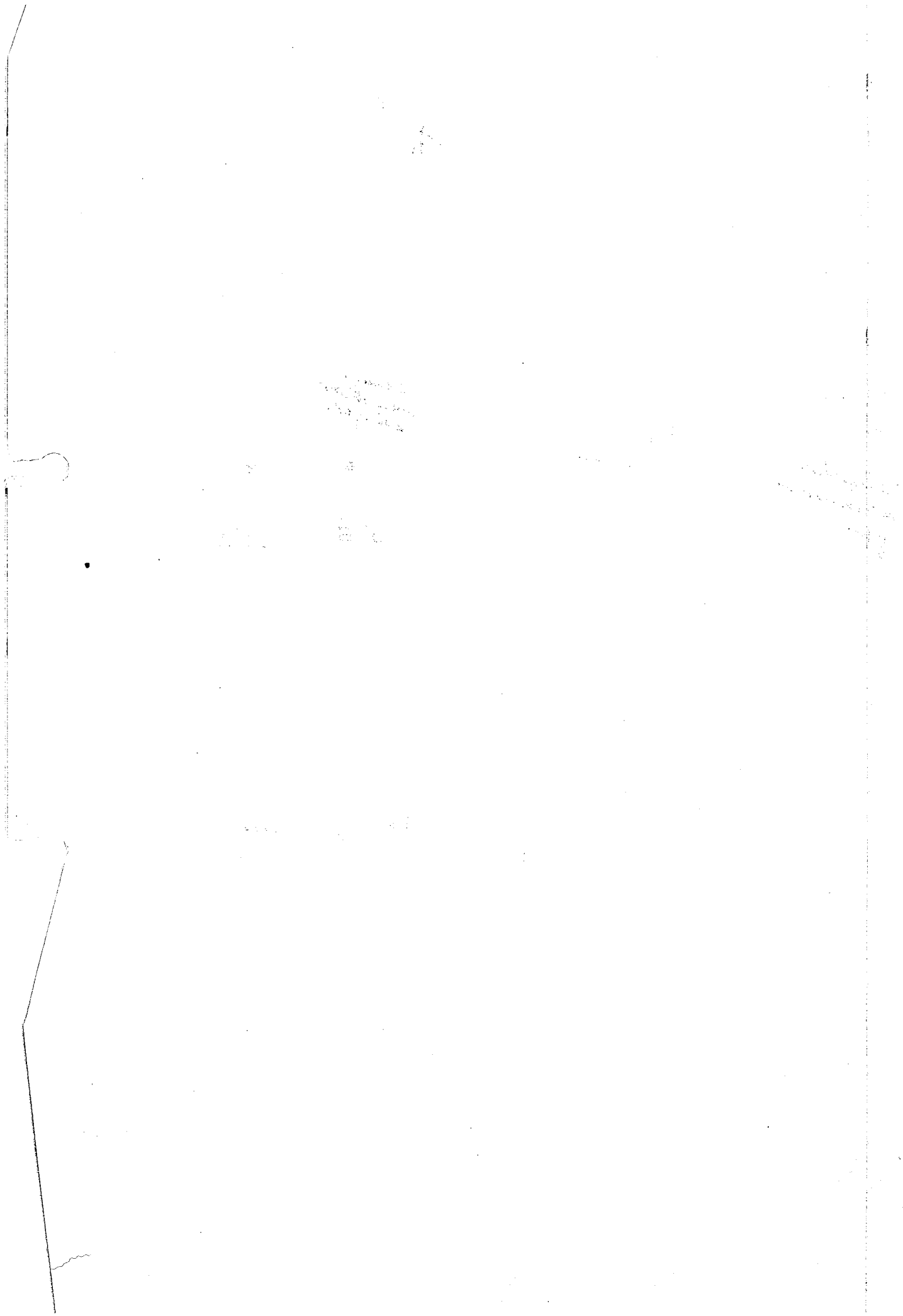
$$\sigma_x\sigma_y - \tau_{xy}^2 = \sigma_1\sigma_2 = \sigma_x'\sigma_y' - \tau_{xy}^2 = I_{2\sigma}$$

3D

$$\sigma_x + \sigma_y + \sigma_z = \sigma_x' + \sigma_y' + \sigma_z' = \sigma_1 + \sigma_2 + \sigma_3 = I_{1\sigma}$$

$$\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) = I_{2\sigma} = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$$

$$\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - (\sigma_x\tau_{yz}^2 + \sigma_y\tau_{zx}^2 + \sigma_z\tau_{xy}^2) = I_{3\sigma} = \sigma_1\sigma_2\sigma_3$$



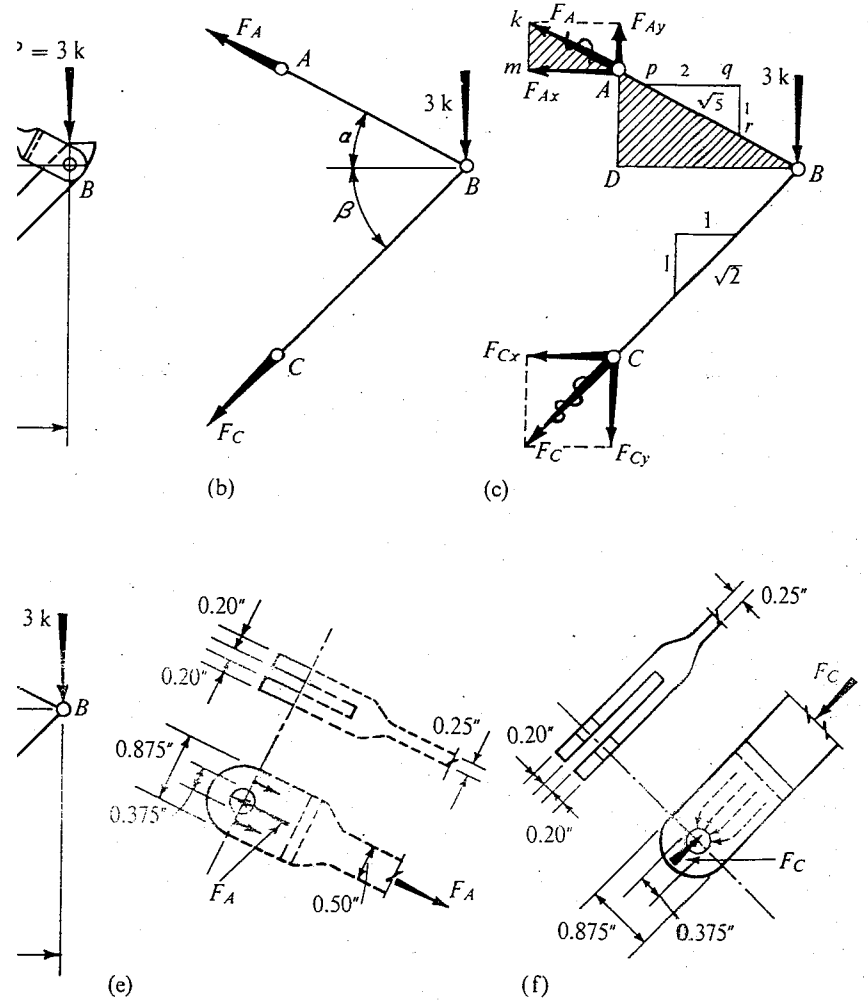


Fig. 3-12

SOLUTION

First an idealized free-body diagram consisting of the two bars pinned at the ends is prepared, Fig. 3-12(b). As there are no intermediate forces acting on the bars and the applied force acts through the joint at B, the forces in the bars are directed along the lines AB and BC, and the bars AB and BC are loaded axially. The magnitudes of the forces are unknown and are labeled F_A and F_C in the diagram.* These forces may be determined graphically by completing a triangle of forces F_A, F_B, and P. These forces may also be found analytically from two simultaneous

* In frameworks it is convenient to assume all unknown forces are tensile. A negative answer in the solution then indicates that the bar is in compression.

equations $\Sigma F_y = 0$ and $\Sigma F_x = 0$, written in terms of the unknowns F_A and F_C, a known force P, and two known angles α and β. Both these procedures are possible. However, in this course usually it will be found advantageous to proceed in a different way. Instead of treating forces F_A and F_C directly, their components are used; and instead of $\Sigma F = 0$, $\Sigma M = 0$ becomes the main tool.

Any force may be resolved into components. For example, F_A may be resolved into F_{Ax} and F_{Ay} as in Fig. 3-12(c). Conversely, if any one of the components of a directed force is known, the force itself may be determined. This follows from similarity of dimension and force triangles. In Fig. 3-12(c) the triangles Akm and BAD are similar triangles (both are shaded in the diagram). Hence, if F_{Ax} is known,

$$F_A = (AB/DB)F_{Ax}$$

Similarly, $F_{Ay} = (AD/DB)F_{Ax}$. However, note further that AB/DB or AD/DB are ratios, hence relative dimensions of members may be used. Such relative dimensions are shown by a little triangle on the member AB and again on BC. In the problem at hand

$$F_A = (\sqrt{5}/2)F_{Ax} \quad \text{and} \quad F_{Ay} = F_{Ax}/2$$

Adopting the above procedure of resolving forces, the revised free-body diagram, Fig. 3-12(d), is prepared. Two components of force are necessary at the pin joints. After the forces are determined by statics, Eq. 3-5 is applied several times, thinking in terms of a free body of an individual member:

$$\sum M_C = 0 \circlearrowleft +, \quad +F_{Ax}(3 + 6) - 3(6) = 0, \quad F_{Ax} = +2 \text{ kips}$$

$$F_{Ay} = F_{Ax}/2 = 2/2 = 1 \text{ kip},$$

$$F_A = 2(\sqrt{5}/2) = +2.23 \text{ kips}$$

$$\sum M_A = 0 \circlearrowleft +, \quad +3(6) + F_{Cx}(9) = 0,$$

$$F_{Cx} = -2 \text{ kips} \quad \text{(compression)}$$

$$F_{Cy} = F_{Cx} = -2 \text{ kips},$$

$$F_C = \sqrt{2}(-2) = -2.83 \text{ kips}$$

Check: $\sum F_x = 0, \quad F_{Ax} + F_{Cx} = 2 - 2 = 0$

$$\sum F_y = 0, \quad F_{Ay} - F_{Cy} - P = 1 - (-2) - 3 = 0$$

Stress in main bar AB:

$$\sigma_{AB} = \frac{F_A}{A} = \frac{2.23}{(0.25)(0.50)} = 17.8 \text{ ksi} \quad \text{(tension)}$$

Stress in clevis of bar AB, Fig. 3-12(e):

$$(\sigma_{AB})_{\text{clevis}} = \frac{F_A}{A_{\text{net}}} = \frac{2.23}{2(0.20)(0.875 - 0.375)} = 11.2 \text{ ksi} \quad \text{(tension)}$$

Stress in main bar BC:

$$\sigma_{BC} = \frac{F_C}{A} = \frac{2.83}{(0.875)(0.25)} = 12.9 \text{ ksi} \quad \text{(compression)}$$

In the compression member the net section at the clevis need not be investigated; see Fig. 3-12(f) for the transfer of forces. The bearing stress at the pin is more critical. Bearing between pin C and clevis:

$$\sigma_b = \frac{F_C}{A_{\text{bearing}}} = \frac{2.83}{(0.375)(0.20)2} = 18.8 \text{ ksi}$$

Bearing between the pin C and the main plate:

$$\sigma_b = \frac{F_C}{A} = \frac{2.83}{(0.375)(0.25)} = 30.1 \text{ ksi}$$

Double shear in the pin C:

$$\tau = \frac{F_C}{A} = \frac{2.83}{2\pi(0.375/2)^2} = 12.9 \text{ ksi}^*$$

For a complete analysis of this bracket, other pins should be investigated. However, it may be seen by inspection that the other pins in this case are stressed the same amount as computed above, or less.

The advantages of the method used in the above example for finding forces in members should now be apparent. It can also be applied with success in a problem such as the one shown in Fig. 3-13. The force F_A transmitted by the curved member AB acts through points A and B since the forces applied at A and B must be collinear. By resolving this force at A', the same procedure may be followed. Wavy lines through F_A and F_C indicate that these forces are replaced by the two components shown. Alternatively, the force F_A may be resolved at A, and since $F_{Ay} = (x/y)F_{Ax}$, the application of $\Sigma M_C = 0$ yields F_{Ax}.

In frames where the applied forces do not act through a joint, proceed as above as far as possible. Then isolate an individual member, and using its free-body diagram, complete the determination of forces.

* Considering the pin in a two-dimensional state of stress $\tau_{xy} = \tau_{yz}$, the tensor representation of the results becomes $\begin{pmatrix} 0 & 12.9 \\ 12.9 & 0 \end{pmatrix}$ ksi

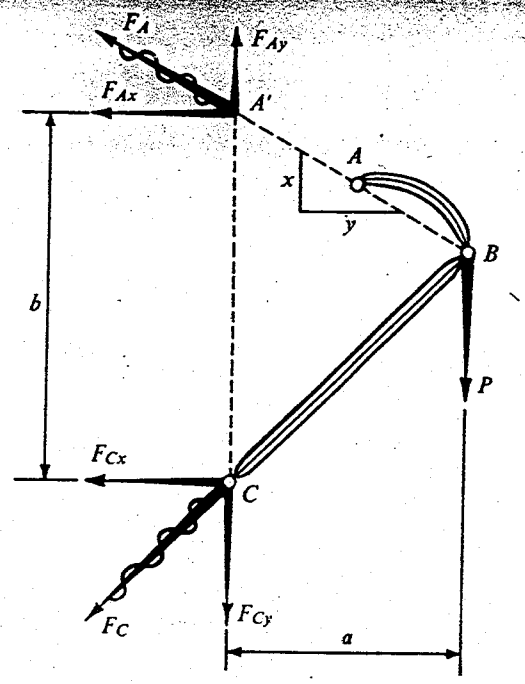
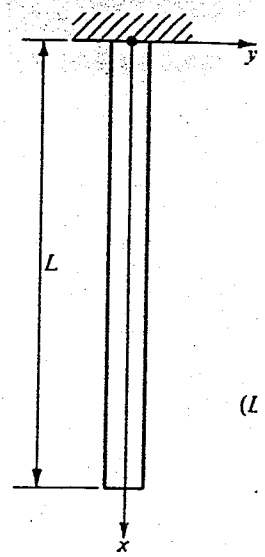


Fig. 3-13



(a)

Fig. 3-14

If inclined forces are acting on the structure, resolve them into convenient components.

EXAMPLE 3-4

A 1-in.^{2} rod L in. long is suspended vertically as shown in Fig. 3-14(a). The unit weight of the material is γ. Determine the normal stress in this rod using differential equations of equilibrium.

SOLUTION

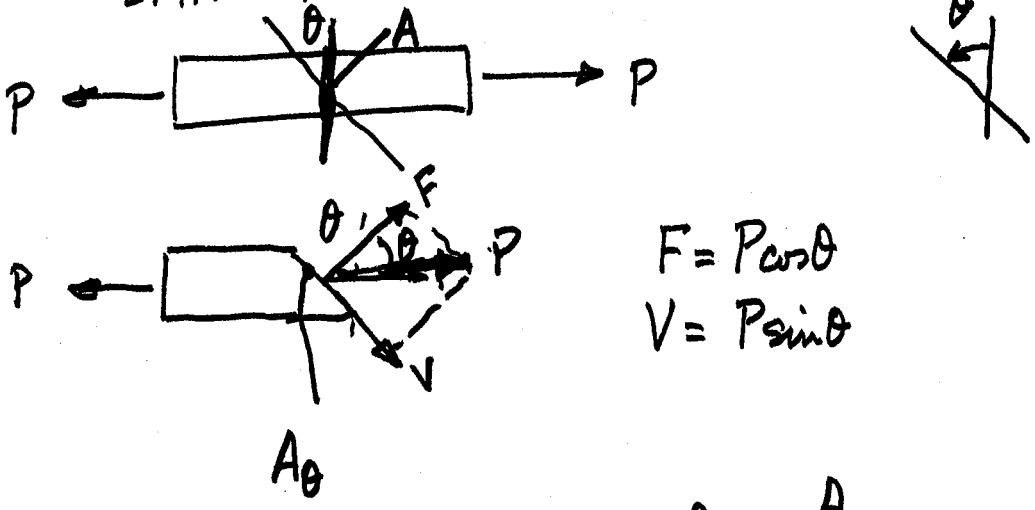
With the axes shown in the figure, $\tau_{xy} = 0$, and only the first part of Eq. 3-3 has relevance. The body force $X = \gamma$. By virtue of the boundary condition at the free end of the rod $\sigma_x(L) = 0$. On this basis, setting up a differential equation, intergating it, and determining the constant of integration from the boundary conditions, one has

$$\frac{d\sigma_x}{dx} + \gamma = 0 \quad \text{and} \quad \sigma_x = -\gamma x + C_1$$

$$\sigma_x(L) = -\gamma L + C_1 = 0 \quad \text{and} \quad \sigma_x = (L - x)\gamma$$

This result can be easily checked by cutting the rod $(L - x)$ above the free end, Fig. 3-14(b), and applying Eq. 3-5. Only very few problems can be analyzed using Eq. 3-3 alone. In more general problems deformations must be considered simultaneously in the analysis.

EMA 3702 (TAPE 2)

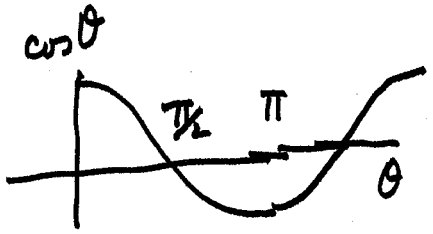


$F = P\cos\theta$

$V = P\sin\theta$

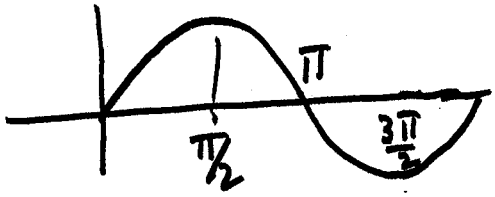
$$A_\theta = \frac{A}{\cos\theta}$$

$$\sigma = \frac{F}{A_\theta} \qquad \tau = \frac{V}{A_\theta}$$

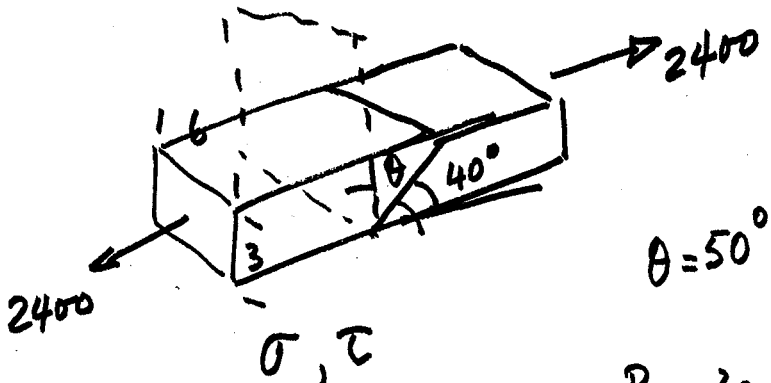


$$\sigma = \frac{P\cos\theta}{A/\cos\theta} = \frac{P}{A}\cos^2\theta = \sigma_x \cos^2\theta$$

$$\tau = \frac{P\sin\theta}{A/\cos\theta} = \frac{P}{A}\sin\theta\cos\theta = \frac{P}{2A}\sin 2\theta = \frac{\sigma_x}{2}\sin 2\theta$$



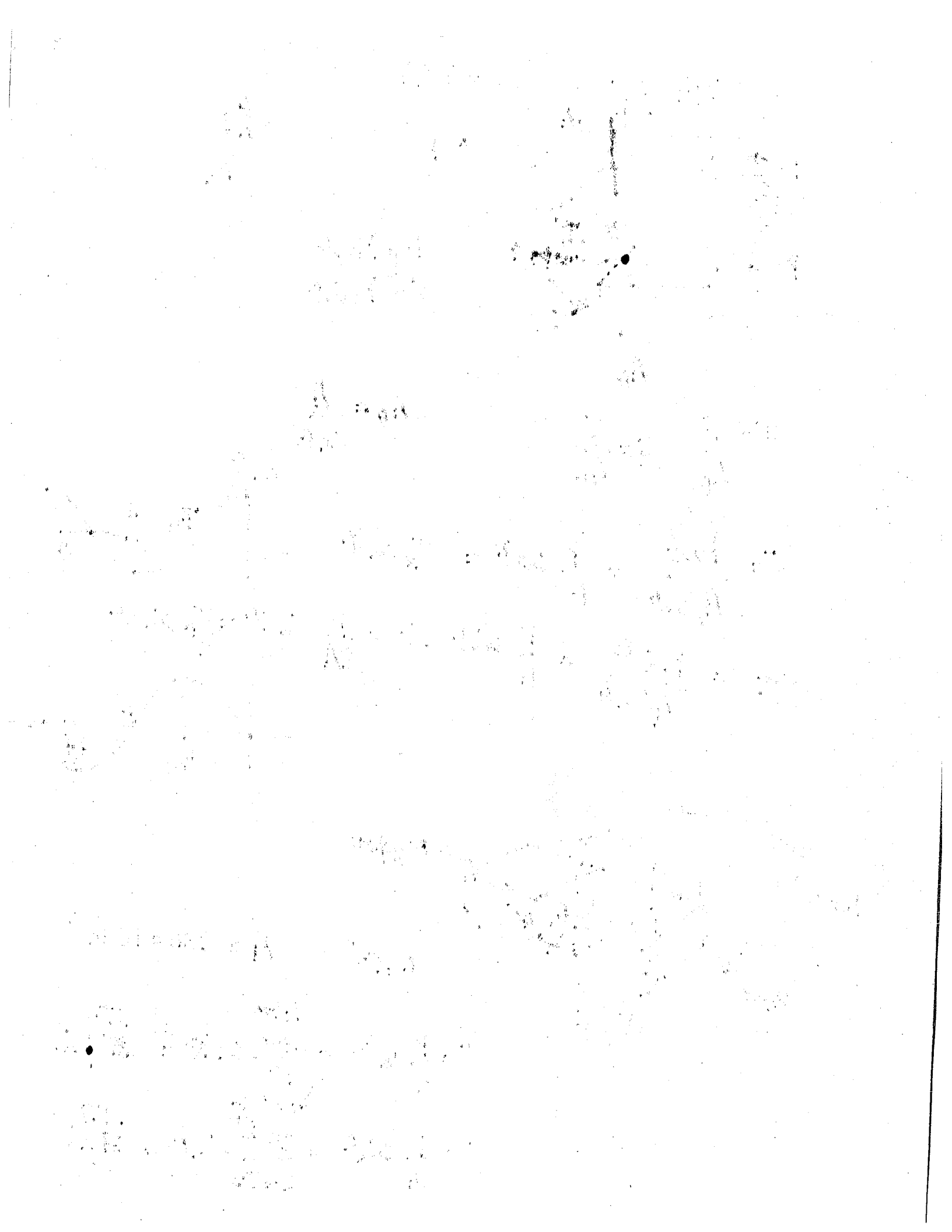
1.32



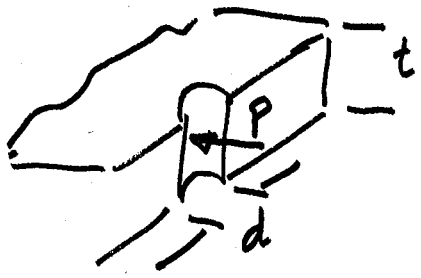
$\theta = 50^\circ$ $\qquad A = 3\times 6 = 18\ in^2$

$$\sigma = \frac{P}{A}\cos^2\theta = \frac{2400}{18}\cos^2 50 \cong 55.1\ psi$$

$$\tau = \frac{P}{2A}\sin 2\theta = \frac{2400}{2\cdot 18\cdot 3}\sin 100 \cong 65.7\ psi$$



BEARING STRESSES (p. 11)



$$\sigma_b = \frac{P}{td} = \frac{P}{A}$$

PROBLEM

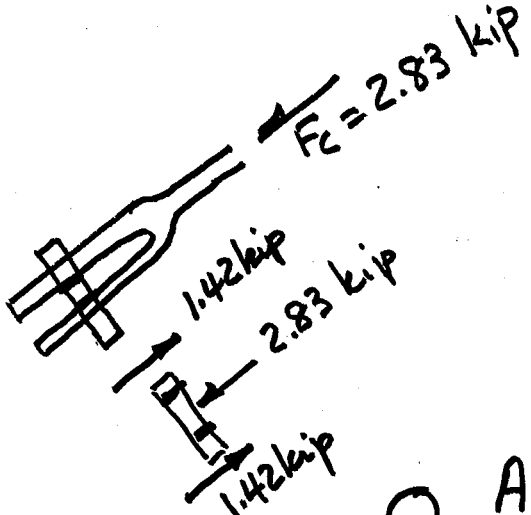
σ_{AB}

σ_{BC}

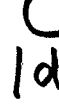
$(\sigma_b)_c$

τ_c

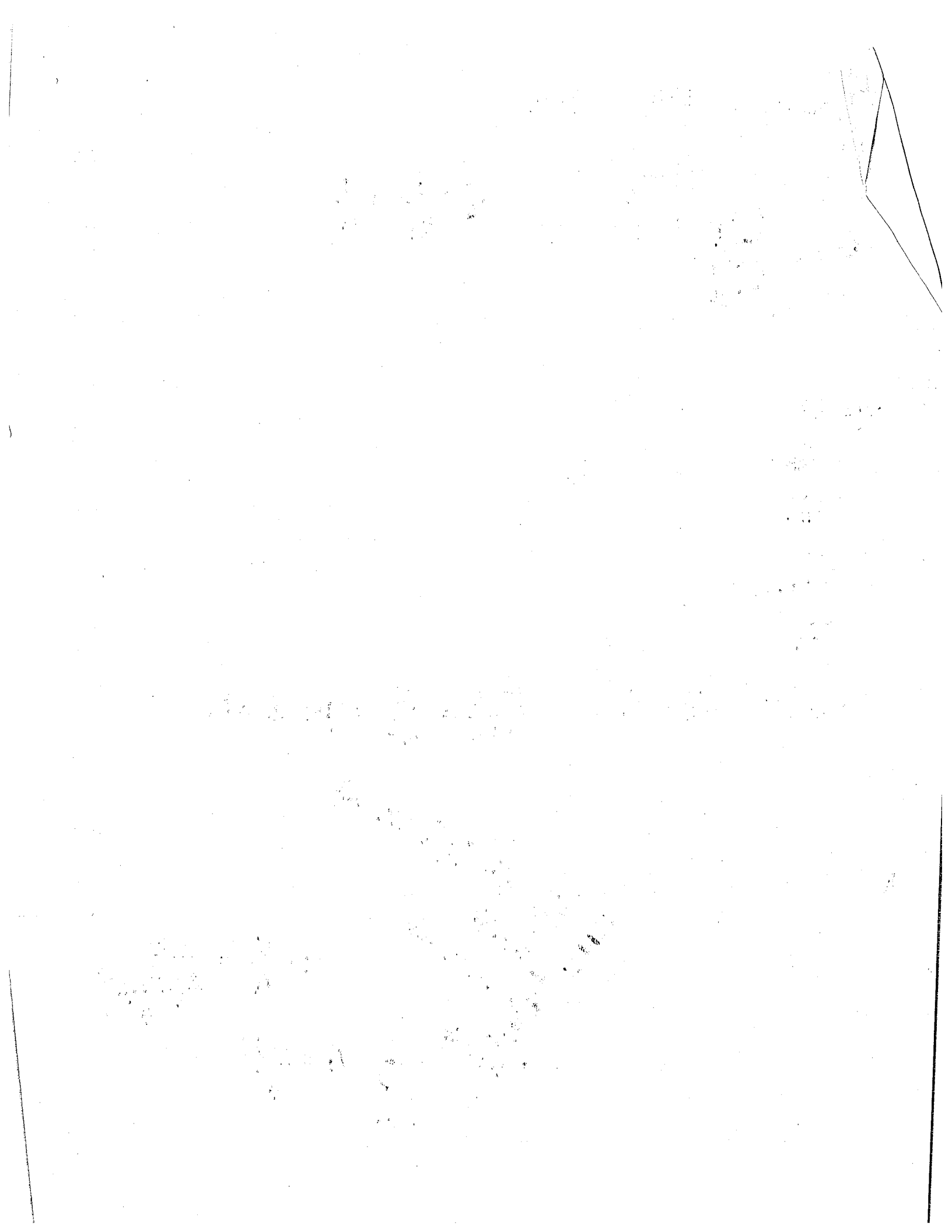
τ or σ units in SI $\frac{\text{force}}{\text{area}} = \frac{N}{m^2} = \text{pascal} = Pa$



$$\tau = \frac{V}{A} = \frac{2.83}{2\left(\pi \frac{.375}{4}\right)^2}$$



$$A = \frac{\pi d^2}{4}$$



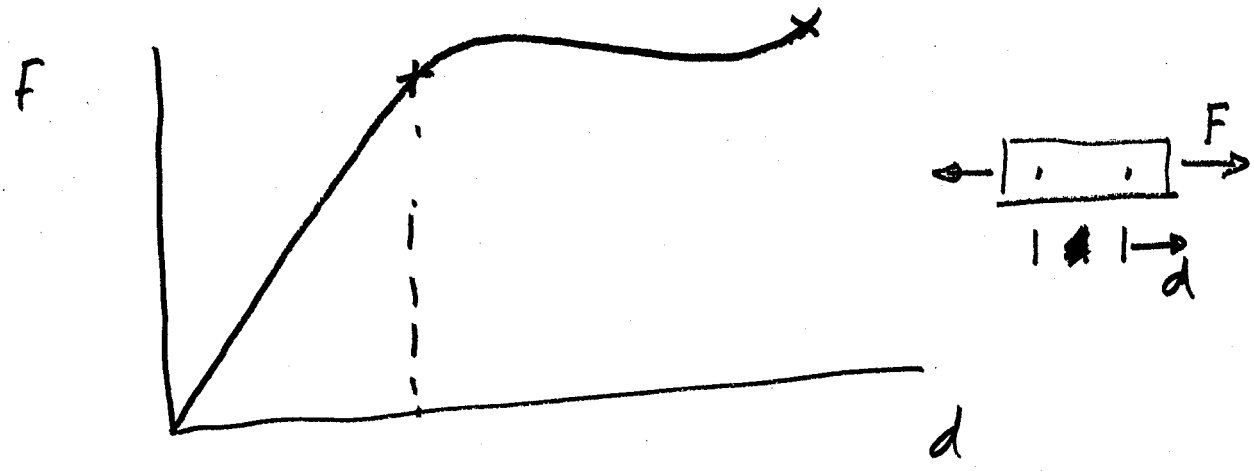
ULTIMATE STRENGTH (STRESS)

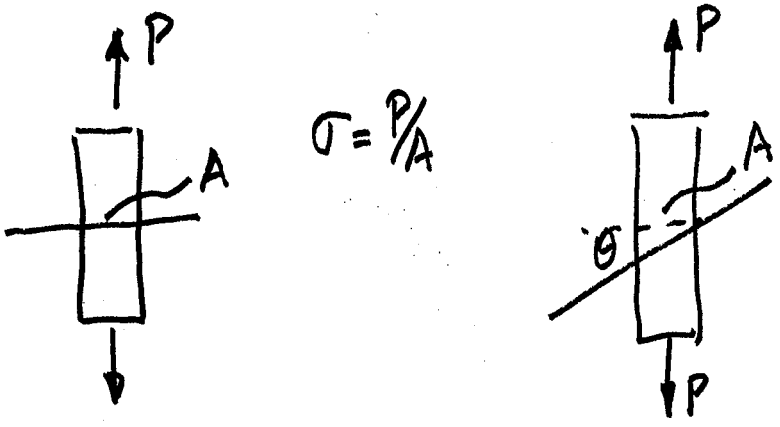
max stress that a material can withstand before failing

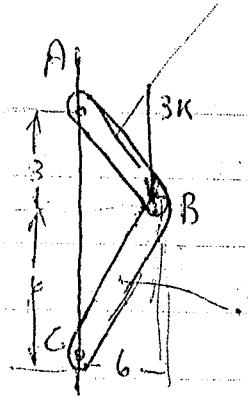
FACTOR OF SAFETY (FACTOR OF IGNORANCE)

ALLOWABLE STRESS - max. stress allowed to occur on the job and that aren't exceeded

$$\text{ALLOWABLE STRESS} = \frac{\text{ULTIMATE STRENGTH}}{\text{F.S.}}$$



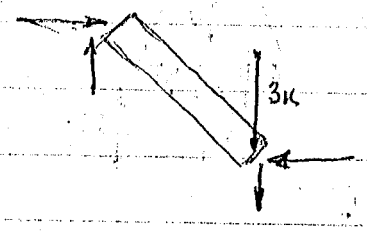


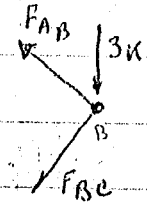


.375" ϕ diameter

Neglect Weight of Members

Consider AB these members must have result in direction of axis of the members.





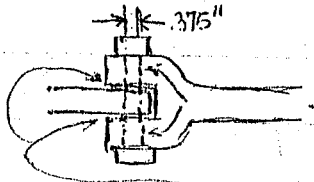
$$\Sigma F_x = 0 \qquad -\frac{2}{\sqrt{5}} F_{AB} + \frac{1}{\sqrt{2}} F_{BC} = 0$$

$$\Sigma F_y = 0 \qquad \frac{1}{\sqrt{5}} F_{AB} - \frac{1}{\sqrt{2}} F_{BC} = 3$$

$$\frac{3}{\sqrt{5}} F_{AB} = 3 \qquad F_{AB} = \sqrt{5}\,K\,T \qquad F_{BC} = +2\sqrt{2}\,K\,C$$

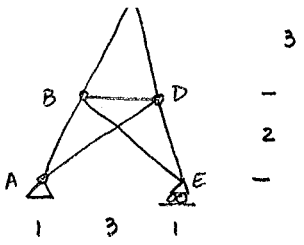
Stresses $\qquad \sigma_{AB} = \dfrac{F_{AB}}{A_{AB}} = \dfrac{\sqrt{5}}{.025 \times .50} \qquad T$

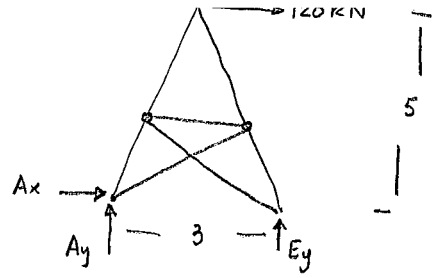
$$\sigma_{BC} = \frac{F_{CB}}{A_{BC}} = \frac{2\sqrt{2}}{.25 \times .875} \qquad C$$



$$\tau_{\text{BOLT A}} = \frac{\sqrt{5}}{\frac{\pi}{4} \times .375^2 \times 2}$$

2 surface shear

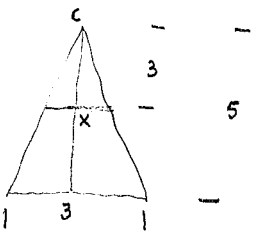




$+\circlearrowleft \sum M_A = -120(5) + E_y \cdot 3 = 0 \quad E_y = 200\ kN$

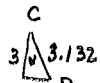
$\pm \rightarrow \sum F_x = 0 \quad 120 + A_x = 0 \Rightarrow A_x = -120 kN$

$+\uparrow \sum F_y = 0 \quad E_y + A_y = 0 \Rightarrow A_y = -200\ kN$

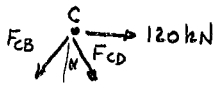


$\frac{5}{3} = \frac{\text{אורך}}{\text{רוחב}} = \frac{3}{x}$

$x = \frac{9}{5} = 1.8m$



צומת C



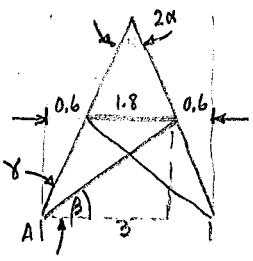
$\pm \rightarrow \sum F_x = 0 \Rightarrow 120 + F_{CD}\sin\alpha - F_{CD}\sin\alpha = 0$

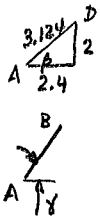
$+\uparrow \sum F_y = 0 \Rightarrow -(F_{CD}\cos\alpha + F_{CB}\cos\alpha) = 0 \Rightarrow F_{CD} = -F_{CB}$

$\Rightarrow 2F_{CD}\sin\alpha = -120$

$2F_{CD}\left(\frac{.9}{3.132}\right) = -120 \quad \underline{F_{CD} = -208.8\ kN}$

$\underline{F_{CB} = 208.8\ kN}$

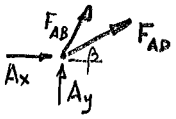




$\gamma = \frac{180 - 2\alpha}{2} = 90 - \alpha \Rightarrow$



צומת A



$\Sigma F_x = 0 \Rightarrow A_x + F_{AD}\cos\beta + F_{AB}\cos\gamma = 0$

$\Sigma F_y = 0 \Rightarrow A_y + F_{AD}\sin\beta + F_{AB}\sin\gamma = 0$

$-120 + F_{AD}\left(\frac{2.4}{3.124}\right) + F_{AB}\left(\frac{.9}{3.132}\right) = 0$

$-200 + F_{AD}\left(\frac{2}{3.124}\right) + F_{AB}\left(\frac{3}{3.132}\right) = 0$

$\underline{F_{AD} = 104.132\ kN}$

$\underline{F_{AB} = 139.201\ kN}$

צומת E



$\pm \rightarrow \sum F_x = -F_{ED}\cos\gamma - F_{EB}\cos\beta = 0$

$\Sigma F_y = F_{ED}\sin\gamma + F_{EB}\sin\beta + E_y = 0$

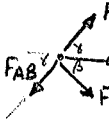
$-F_{ED}\left(\frac{.9}{3.132}\right) - F_{EB}\left(\frac{2.4}{3.124}\right) = 0$

$F_{ED}\left(\frac{3}{3.132}\right) + F_{EB}\left(\frac{2}{3.124}\right) + 200 = 0$

$\underline{F_{ED} = -278.403\ kN}$

$\underline{F_{EB} = 104.135\ kN}$

צומת B



$\Sigma F_x = 0 =$

$F_{CB}\cos\gamma + F_{BD} + F_{BE}\cos\beta - F_{AB}\cos\gamma$

$(F_{CB} - F_{AB})\cos\gamma + F_{BE}\cos\beta + F_{BD} = 0$

$\Sigma F_y = 0 = (F_{CB} - F_{AB})\sin\gamma - F_{BE}\sin\beta$

$69.6\left(\frac{3}{3.132}\right) - 104.135\left(\frac{2}{3.124}\right) = 0$

$69.6\left(\frac{.9}{3.132}\right) + 104.135\left(\frac{2.4}{3.124}\right) = -F_{BD}$

$\underline{F_{BD} = -100\ kN}$

כוח הפנים (kN)	מאמץ המותר (kPa)	שטח הנדרש $= \frac{F}{\sigma_{\text{מותר}}}$ (m^2)	רדיוס
$F_{AB} = 139.201$	15 kPa	9.28	1.719
$F_{AD} = 104.132$	15 kPa	6.945	1.487
$F_{BD} = -100$	-12 kPa	8.33	1.629
$F_{CD} = -208.8$	-12 kPa	17.4	2.354
$F_{CB} = 208.8$	15 kPa	13.92	2.105
$F_{ED} = -278.403$	-12 kPa	23.20	2.718
$F_{EB} = 104.135$	15 kPa	6.95	1.487

- על פי המספרים, שטח המוטות שונים ביותר. אבל התלוי לקבלת עיגון עבור הבנונות. אם המאמץ נתון כ- kN/cm^2, אז המספרים יהיו יותר סבירים. כאשר שוערת, שיחידות השטחים תהיו ב- cm^2 והרדיוסים יהיו ב- cm.

1. The stress distribution on the rectangular cross-section shown in Fig. 1 is given by $\sigma_{xx}=1000y-500z+800$ kPa, $\sigma_{xy}=200z$ kPa, $\sigma_{xz}=0$. What is the net internal force system on this cross-section ?

Answer : $F=1920$ N, $V_y=V_z=0$
$M_y=-1600$ N-cm., $M_z=-7200$ N-cm., $T=-640$ N-cm.

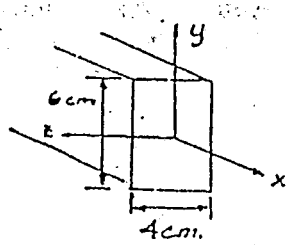


Fig. 1

2. Suppose the stress distribution on a cross-section of the circular cylinder of Fig. 2 is given by $\sigma_{xx}=\sigma_{xr}=0$, $\sigma_{x\theta}=k\sqrt{r}$, where k is unknown. What is the value of k in this case?

Answer : $k=7T_0/4\pi R^{7/2}$

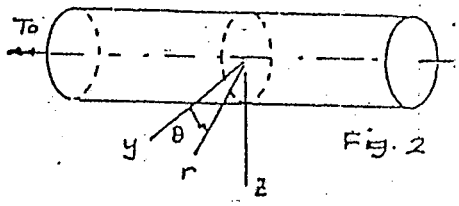


Fig. 2

6.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium equations of that element, as was done in the derivations presented in Sec. 6.2.

6.6 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

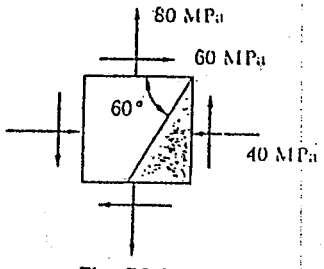


Fig. P6.4

✱6.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

✱6.14 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 40° counterclockwise, (*b*) 15° clockwise.

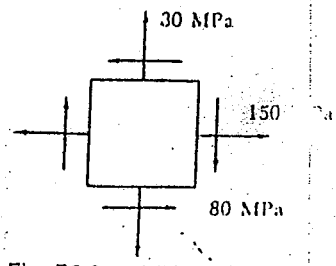
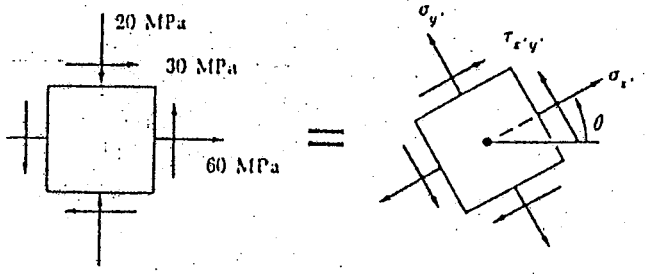


Fig. P6.6 and P6.10

✱6.56 For the state of plane stress shown, determine the range of values of θ for which the normal stress $\sigma_{y'}$ is equal to or less than +65 MPa.



P6.55 and P6.56

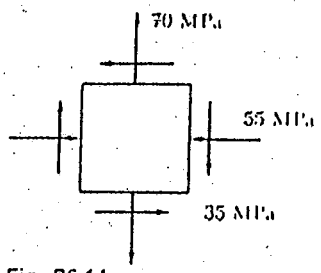


Fig. P6.14

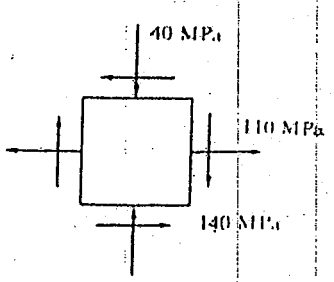


Fig. P6.8 and P6.12

1. The stress distribution on the rectangular cross-section shown in Fig. 1 is given by $\sigma_{xx}=1000y-500z+800$ kPa, $\sigma_{xy}=200z$ kPa, $\sigma_{xz}=0$. What is the net internal force system on this cross-section ?

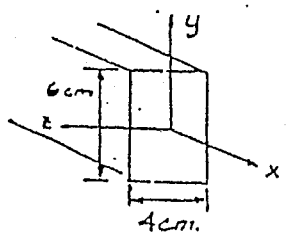


Fig. 1

Answer : $F=1920$ N, $V_y=V_z=0$
$M_y=-1600$ N-cm., $M_z=-7200$ N-cm., $T=-640$ N-cm.

2. Suppose the stress distribution on a cross-section of the circular cylinder of Fig. 2 is given by $\sigma_{xx}=\sigma_{xr}=0$, $\sigma_{x\theta}=k\sqrt{r}$, where k is unknown. What is the value of k in this case?

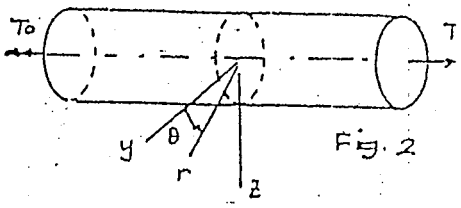


Fig. 2

Answer : $k=7T_0/4\pi R^{7/2}$

6.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium equations of that element, as was done in the derivations presented in Sec. 6.2.

6.6 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

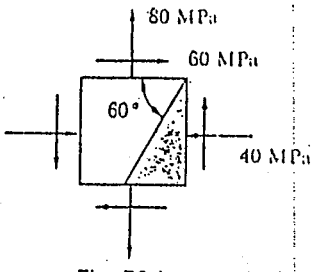


Fig. P6.4

6.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

6.14 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 40° counterclockwise, (*b*) 15° clockwise.

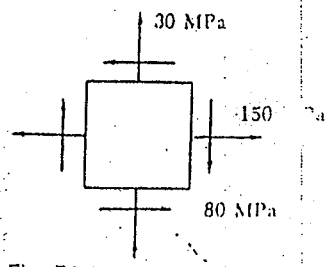
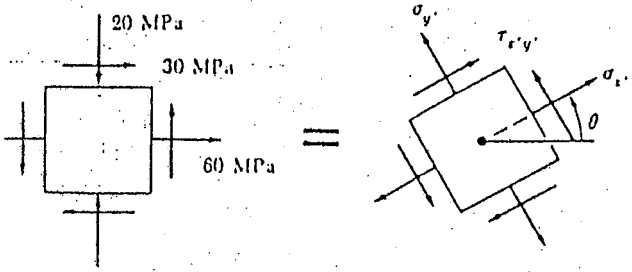


Fig. P6.6 and P6.10

6.56 For the state of plane stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than +65 MPa.



P6.55 and P6.56

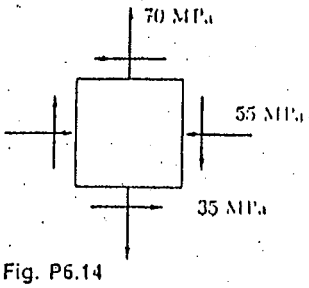


Fig. P6.14

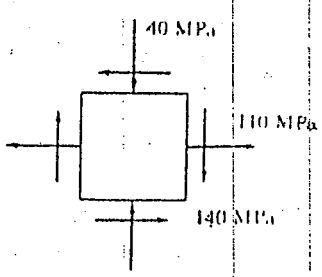


Fig. P6.8 and P6.12

25
12

ב) ✓ $\alpha = -15°$

$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha$$

$$= 7.5 - 62.5\cos(-30) + (-35)\sin(-30) = 7.5 - 62.5(.866) - 35(-\tfrac{1}{2}) = -29.13\ \text{MPa}\ \checkmark$$

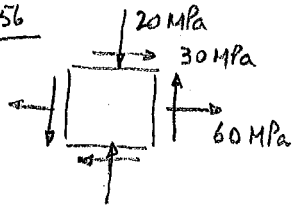
$$\sigma_{y'} = I_{1\sigma} - \sigma_{x'} = (\sigma_x+\sigma_y) - \sigma_{x'} = 15 - (-29.13\ \text{MPa}) = 44.13\ \text{MPa}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\alpha + \tau_{xy}\cos 2\alpha$$

$$= 62.5\sin(-30) + (-35)\cos(-30) = 62.5(-\tfrac{1}{2}) - 35(.866) = -61.56\ \text{MPa}\ \checkmark$$

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6.56



$\sigma_x = 60\ \text{MPa}$
$\sigma_y = -20\ \text{MPa}$
$\tau_{xy} = 30\ \text{MPa}$

$$\sigma_{x'} = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha$$

$$= \frac{60-20}{2} + \frac{60+20}{2}\cos 2\alpha + 30\sin 2\alpha$$

$$\sigma_{x'} = 20 + 40\cos 2\alpha + 30\sin 2\alpha$$

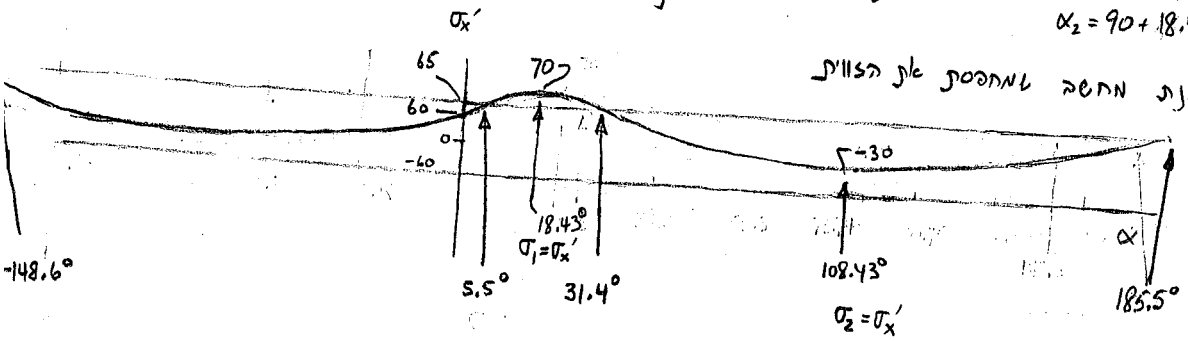
או, אפשר $\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} \pm \left[\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$

$$= \frac{60-20}{2} \pm \left[\left(\frac{60+20}{2}\right)^2 + 30^2\right]^{1/2} = 20 \pm \left[40^2+30^2\right]^{1/2} = 20 \pm 50 = \begin{cases} 70 \\ -30 \end{cases}\ \text{MPa}$$

$$\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x-\sigma_y} = \frac{2(30)}{60+20} = \frac{60}{80} = 0.75$$

$2\alpha = 36.87°$ $\alpha = 18.43°$

$\alpha_2 = 90 + 18.43° = 108.43°$



אלא נשתמש בתוכנת מחשב ונתפנק את הציון

$$\cos 2\alpha = \cos(360+2\alpha) = \cos(-[360-2\alpha])$$

$\alpha = 5.5$ ולכן $2\alpha = 11$ ומצא $185.5°$. רוצים ש- α = $185.5° - 5.5 = 180°$

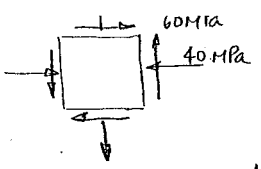
$\alpha = 31.4$ ולכן $-(360-2\alpha) = -297.2$ ומצא $-148.6°$. רוצים ש- α = $31.4 + (-148.6°) = -180°$

הערות: הנקודה (השיפוע של הפונקציה) ב- $31.4°$ ו- $-148.6°$ שוות

הנקודה של $5.5°$ ו- $185.5°$ שוות

לכן, כי $5.5 \le \alpha \le 185.5$ $\sigma_{x'} < 65$

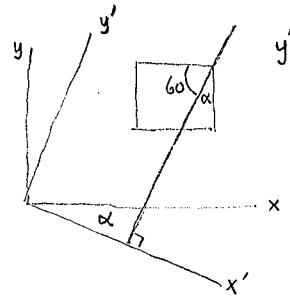
וכי $-148.8° \le \alpha \le 5.5$ $\sigma_{x'} > 65$



$\sigma_x = -40$ MPa
$\tau_{xy} = 60$ MPa
$\sigma_y = 80$ MPa

$$\sigma_x = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\alpha + \tau_{xy}\cos 2\alpha$$

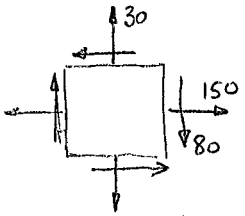


(א) לפי הסימנים האמריקאים $\alpha = -30°$, כי הסיבוב הוא מכיוון y' ל-y

$$\sigma_{x'} = \left(\frac{-40+80}{2}\right) + \left(\frac{-40-80}{2}\right)\cos(-60) + 60\sin(-60)$$
$$= 20 + (-60)\cdot\frac{1}{2} + 60(-.866) = \underline{-61.96\ \text{MPa}}$$

$$\tau_{x'y'} = -\left(\frac{-40-80}{2}\right)\sin(-60) + 60\cos 60$$
$$= +60(-.866) + 60\left(\frac{1}{2}\right) = \underline{-21.96\ \text{MPa}}$$

6.6



$\sigma_x = 150$ MPa ✓
$\sigma_y = 30$ MPa ✓
$\tau_{xy} = -80$ MPa ✓

(ב) למצוא את הכיוונים העיקריים ואת המאמצים העיקריים

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \left[\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

לכן

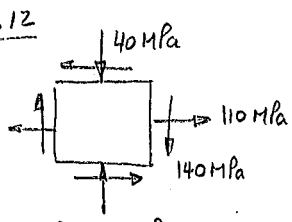
$$\sigma_{1,2} = \left(\frac{150+30}{2}\right) \pm \left[\left(\frac{150-30}{2}\right)^2 + (-80)^2\right]^{1/2}$$
$$= 90 \pm 100 \Rightarrow \quad \sigma_1 = 190.\overline{6}\ \text{MPa} \ ✓$$
$$\sigma_2 = -10.\overline{6}\ \text{MPa} \ ✓$$

$$\tan 2\theta = \frac{2(-80)}{150-30} = -1.333$$

$$2\theta = -53.13 \quad \theta = \underline{-26.57°} \ ✓ \qquad \theta_2 = \theta + 90° = \underline{63.43°} \ ✓$$

7/7

6.12



$\sigma_x = 110$MPa ✓
$\sigma_y = -40$ MPa ✓
$\tau_{xy} = -140$ MPa ✓

(ב) למצוא את הכיוונים של מאמץ הגזירה המקסימלי, והמאמץ הנורמלי

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \left[\left(\frac{110-(-40)}{2}\right)^2 + (-140)^2\right]^{1/2} = \pm 158.82\ \text{MPa} \ ✓$$

$$\tan 2\alpha = -\frac{(\sigma_x-\sigma_y)}{2\tau_{xy}} = -\frac{[110-(-40)]}{2(-140)} = \frac{-150}{-280} \Rightarrow 208.18°$$

מכאן שהזוויות עליהן מתקבל הגזירי:

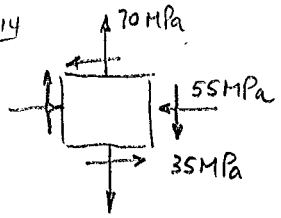
$\alpha = 104.09°$ ✓

$\alpha + 90° = 194.09° = 14.09°$ ✓

$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha$$
$$= \frac{110-40}{2} + \left(\frac{110-(-40)}{2}\right)\cos 208.18° + (-140)\sin 208.18°$$
$$= 35 + 75\cos 208.18 - 140\sin 208.18 = 35\ \text{MPa} = (\sigma_x+\sigma_y)/2 \ ✓$$

7/7

6.14



$\sigma_x = -55$MPa ✓
$\sigma_y = 70$ MPa ✓
$\tau_{xy} = -35$ MPa ✓

(א) למצוא את המאמצים כאשר התלקיק הולך ומסתובב $40°$ נגד כיוון השעון (מאמצים נורמלי וגזירה).

(ב) אותן תגלית כאשר התלקיק הולך ומסובב $15°$ בכיוון השעון.

$\alpha = 40°$ ✓

$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha$$
$$= \frac{-55+70}{2} + \left(\frac{-55-70}{2}\right)\cos 80 - 35\sin 80$$
$$= 7.5 + 62.5\cos 80 - 35\sin 80 = -37.82\ \text{MPa} \ ✓$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\alpha + \tau_{xy}\cos 2\alpha$$

3. The beam shown Fig. 3 has a rectangular cross-section and is subjected to bending couples at its end. The stress distribution on a cross-section is given by :

$$\sigma_{xx} = \begin{cases} -\sigma_0 & , y \geq c \\ -\sigma_0 y/c & , -c \leq y \leq c \\ \sigma_0 & , y \leq -c \end{cases}$$

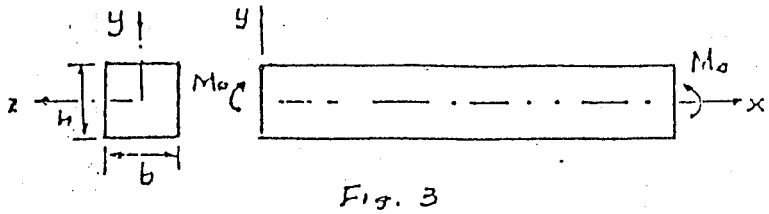


Fig. 3

where c is a constant ($0 \leq c \leq h/2$) and σ_0 is a known number.

(a) Sketch the stress distribution for a given c.
(b) For what value of c will M_0 have a maximum value? Determine $(M_0)_{max}$.
(c) Draw the stress distribution for σ_x when $M=(M_0)_{max}$.

Answer : (b) : $(M_0)_{max} = \sigma_0 bh^2/4$

answers:

6.4 $\sigma = -62.0$ MPa ; $\tau = 22.$ MPa

6.6 (a) $-26.6°$; $+63.4°$

(b) $\sigma_{max} = 190.$ MPa $(\theta = 26.6°)$

$\sigma_{min} = -10.$ MPa

6.10 (a) $18.4°$; $108.4°$

(b) 100. MPa

(c) 90. MPa

6.14 (a) -37.8 MPa ; 52.8 MPa ; 55.5 MPa

(b) -29.1 MPa ; 44.1 MPa ; -61.6 MPa

6.56 $-148.6° \leq \theta \leq 5.52°$

$31.4° \leq \theta \leq 185.5°$

$\sigma_{xx} = 1000y - 500z + 800$

$\sigma_{xy} = 200z$

$\sigma_{xz} = 0$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + X = 0$$

$0 + 0 + 0 + X = 0 \Rightarrow X = 0$

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + Y = 0$$

$0 + ? $

$$\int \sigma_{xx}\, dy\, dz = \left(\frac{1000}{2}y^2 - 500yz + 800y\right)\Big|_{-3}^{3} dz$$

$$= \frac{1000}{2}(9-9) - 500z(3+3) + 800(3+3)$$

$$= (-3000z + 4800)\, dz$$

$$= -\frac{3000}{2}z^2 + 4800z \Big|_{-2}^{2}$$

$$= -\frac{3000}{2}(4-4) + 4800(2+2) = 19200 \text{ kPa-cm}^2$$

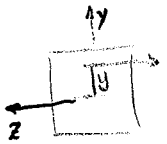
$$= 1.92 \times 10^{7} \text{ Pa} \cdot 1 \times 10^{-4} \text{ m}^2$$

$$= 1920 \text{ N}$$

$$V_y = \int \sigma_{xy}\, dy\, dz = dz(200zy)\Big|_{-3}^{3} = 200z(6) = \int 1200z\, dz$$

$$= \frac{1200}{2}z^2 \Big|_{-2}^{2} = 0$$

$$V_z = \int \sigma_{xz}\, dy\, dz = 0$$



$$M_z = \int \sigma_{xx} \cdot y\, dy\, dz = \left(\frac{1000}{3}y^3 - \frac{500}{2}y^2z + \frac{800}{2}y^2\right)\Big|_{-3}^{3} dz \Big|_{-2}^{2} = \frac{1000}{3}(27+27) - \frac{500}{2}z(9-9) + \frac{800}{2}(9-9) = \frac{1000}{3} \cdot 18z \Big|_{-2}^{2} = 7200 \text{ kPa cm}^3$$

$$= 7.2 \times 10 \times 1 \times 10^{3}$$

$$= 72 \text{ N-m}$$

$$M_y = \int \sigma_{xx} \cdot z\, dy\, dz = \left(\frac{1000}{2}y^2 - 500yz + 800y\right)\Big|_{-3}^{3} \cdot z\, dz = -500 \cdot 6\frac{z^2}{2} + 800(6)\frac{z^2}{2}\Big|_{-2}^{2} = -\frac{3000}{3}(8+8) = -16000 \text{ kPa cm}^3$$

$$= -1.6 \text{ N-m}$$

$$T = M_x = \int \tau_{xy} \cdot z\, dy\, dz = \frac{200z^2}{3}y\Big|_{-3}^{3}\Big|_{-2}^{2} = \frac{200}{3}(8+8)(3+3)$$

$$= 400(16) = 6400 \text{ kPa-cm}^3$$

$$= 6.4 \times 10^{6} \text{ Pa} - 1 \times 10^{-6} \text{ m}^3$$

$$= 6.4 \text{ N-m}$$

2.

1. The stress distribution on the rectangular cross-section shown in Fig. 1 is given by $\sigma_{xx}=1000y-500z+800$ kPa, $\sigma_{xy}=200z$ kPa, $\sigma_{xz}=0$.
What is the net internal force system on this cross-section ?

Answer: $F=1920$ N, $V_y=V_z=0$
$M_y=-1600$ N-cm., $M_z=-7200$ N-cm., $T=-640$ N-cm.

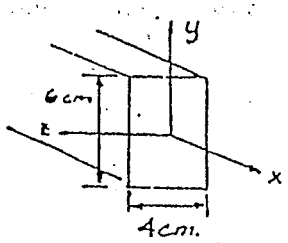


Fig. 1

2. Suppose the stress distribution on a cross-section of the circular cylinder of Fig. 2 is given by $\sigma_{xx}=\sigma_{xr}=0$, $\sigma_{x\theta}=k\sqrt{r}$, where k is unknown. What is the value of k in this case?

Answer: $k=7T_0/4\pi R^{7/2}$

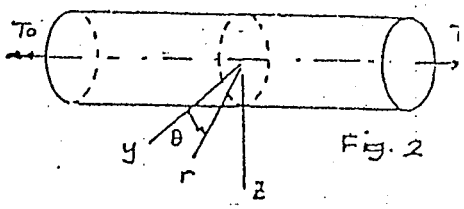


Fig. 2

6.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium equations of that element, as was done in the derivations presented in Sec. 6.2.

6.6 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

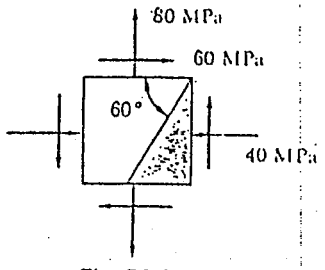


Fig. P6.4

*6.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

*6.14 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 40° counterclockwise, (*b*) 15° clockwise.

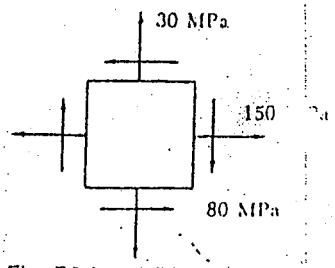
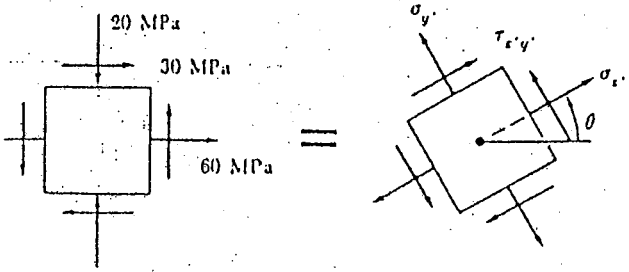


Fig. P6.6 and P6.10

*6.56 For the state of plane stress shown, determine the range of values of θ for which the normal stress σ_x is equal to or less than +65 MPa.



P6.55 and P6.56

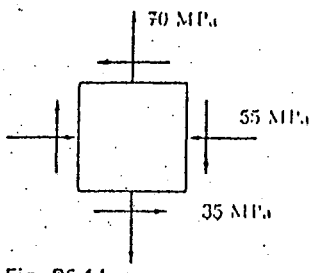


Fig. P6.14

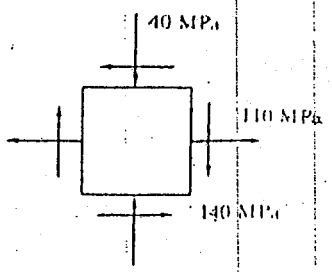


Fig. P6.8 and P6.12

Introduction to Elasticity

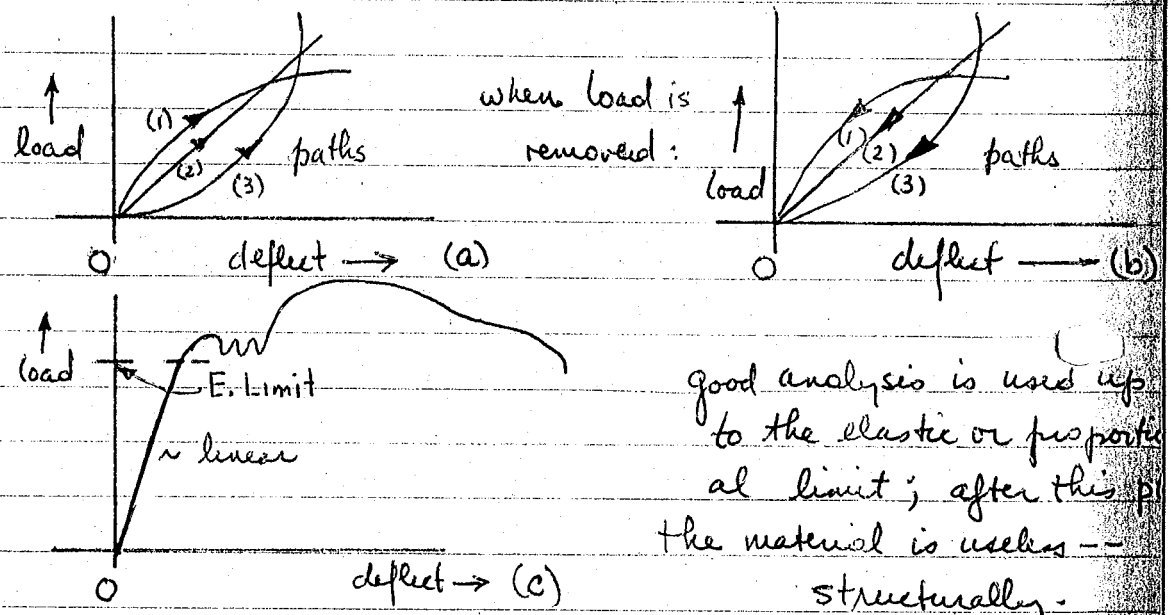
AE 311 Prof. Patel Rm 519

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Elasticity - property of material by which it tends to return to its original state when the applied loads are removed.

Perfectly elastic materials return to original state after load has been removed

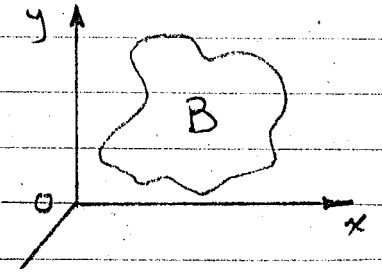
FIG. 1
9-15



good analysis is used up to the elastic or proportional limit; after this pt the material is useless structurally.

Strain - extension of the original element under applied load. It is independent of deflection curve.

FIG. 2
9-15



In studying the body B we make the following assumptions:

B is continuous material

homogeneous

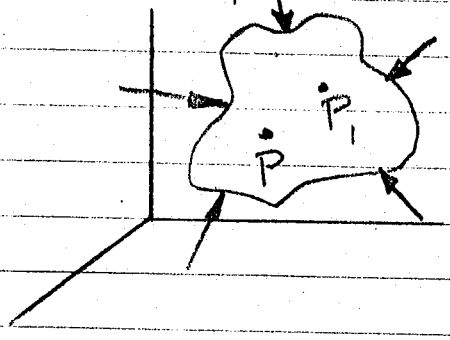
elastic

isotropic - Material properties is independent of direction.

המבנה הגבישי הוא שרירותי ולכן התכונות הממוצעות בכיוון שונות

The material's crystal may not be isotropic but since crystal orientation may be very random in material the average behavior of the material may be isotropic

FIG. 3 9-15



$P(x,y,z)$

$P_1(x+u, y+v, z+w)$

$u = u(x,y,z)$

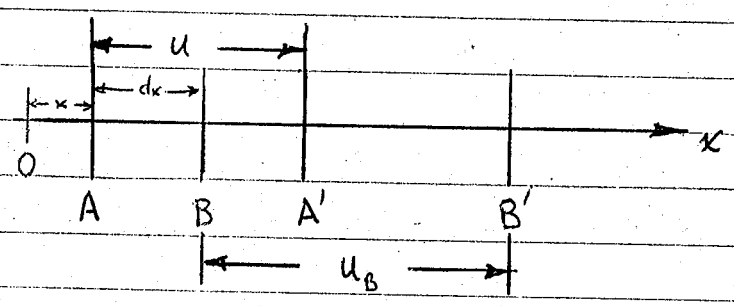
$v = v(x,y,z)$

$w = w(x,y,z)$

u, v, w are continuous functions whose derivatives are also continuous.

1-D Case

FIG. 4 9-15



AB are displaced to A'B' along x

$u_B = u + du = u + \frac{du}{dx} dx$

$A = x$ $\quad A' = A + u = x + u$

$B = x + dx$ $\quad B' = x + dx + u_B$

$AB = dx$ $\quad A'B' = u_B - u \overset{+dx}{=} du/dx\,dx + dx$

$$\epsilon_x = \frac{A'B' - AB}{AB} = \frac{du}{dx} \qquad 9\text{-}15.1$$

If $u(x,y,z)$ we write derivatives $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}$

Stress is defined as the change in force / change in unit area as the change in unit area goes to zero, or.

$$\text{stress} = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

9-17-70

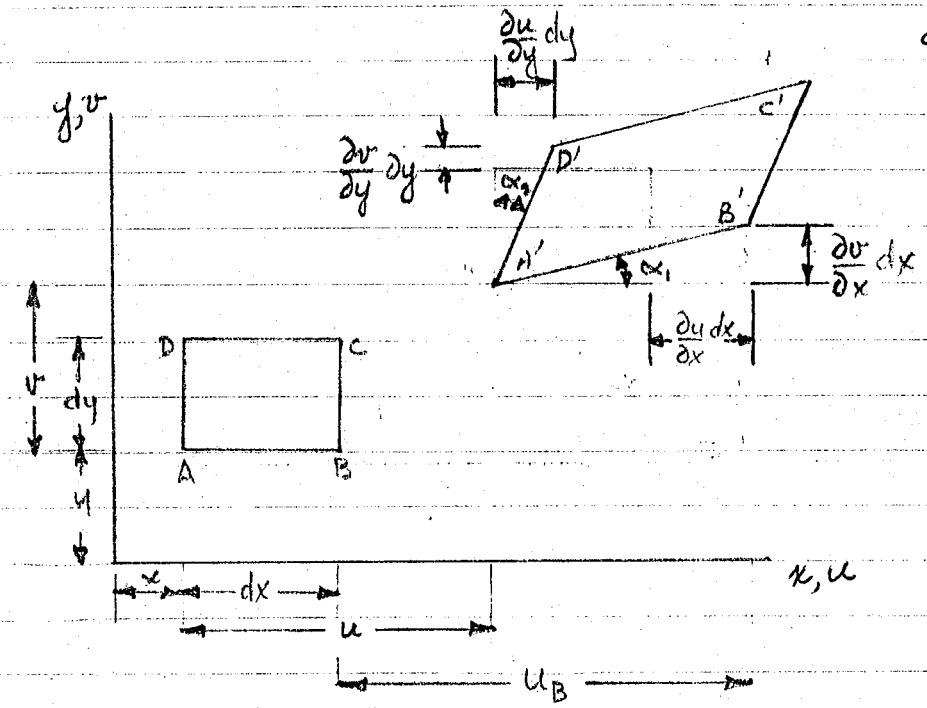


FIG 1
9-17

$$u_B = u + \frac{\partial u}{\partial x} dx \qquad \epsilon = du/dx \text{ in 1-D case.}$$

longitudinal strain

$$\epsilon_x = \frac{A'B' - AB}{AB} \qquad A'B' = (1+\epsilon_x) AB = (1+\epsilon_x) dx$$

$$\overline{A'B'}^2 = \left(dx + \frac{\partial u}{\partial x} dx\right)^2 + \left(\frac{\partial v}{\partial x} dx\right)^2$$

$$\overline{A'B'}^2 = (1 + 2\epsilon_x + \epsilon_x^2) dx^2 = dx^2 + 2\frac{\partial u}{\partial x} dx^2 + \left(\frac{\partial u}{\partial x}\right)^2 dx^2 + \left(\frac{\partial v}{\partial x}\right)^2 dx^2$$

thus $$1 + 2\epsilon_x + \epsilon_x^2 = 1 + 2\frac{\partial u}{\partial x} + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2$$

we can then neglect them. if so we obtain

9-17.1 $\boxed{\epsilon_x = \frac{\partial u}{\partial x}}$ and in the n-direct $\boxed{\epsilon_n = \frac{\partial u_n}{\partial n}}$

$\boxed{\epsilon_y = \frac{\partial v}{\partial y}}$ can be gotten similarly.

Strain, then, along any direction = partial of the displacement along that direction, with respect to that direction (as shown above).

rigid body translation produces no longitudinal strains, $\epsilon_n = 0$.

$$\alpha_1 \approx \tan\alpha_1 = \frac{(\partial v/\partial x)dx}{(1+\frac{\partial u}{\partial x})dx} = \partial v/\partial x$$

because $\left(\frac{\partial u}{\partial x}\right)^2 << \frac{\partial u}{\partial x} << 1$ מכיוון ש $\left(\frac{\partial u}{\partial x}\right)^2$ זניח

$\partial u/\partial x$ can be neglected therefore $\alpha_1 \approx \tan\alpha_1 \approx \frac{\partial v}{\partial x}$

$$\alpha_2 \approx \tan\alpha_2 \approx \frac{(\partial u/\partial y)dy}{(1+\frac{\partial v}{\partial y})dy} \approx \frac{\partial u}{\partial y}$$

$$\alpha_1 + \alpha_2 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad \text{and} \quad \gamma_{xy} = \alpha_1 + \alpha_2$$

עיקר ניתה
עיוור הצירה

9-17.2 $\boxed{\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}}$

2.7. A straight piece of wire PQ lies along the line $y = mx$ as shown in Fig. P2.7. The wire is strained and displaced to the line $y = nx$ in such a way that a point originally at x is displaced to $x^2/2$. Show that the extensional strain at any point along the wire is given by

$$\epsilon = \sqrt{\frac{1 + n^2}{1 + m^2}}\, x - 1$$

(where x is the original coordinate of the point).

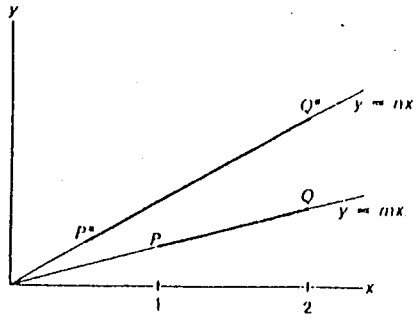


Fig. P2.7

2.16. The point C is displaced to C^* as shown in Fig. P2.16. The horizontal and vertical components of this displacement are u and v. Express the average extensional strains of AC and BC in terms of u, v and L. If u and v are small, what are the approximate expressions for these average extensional strains?

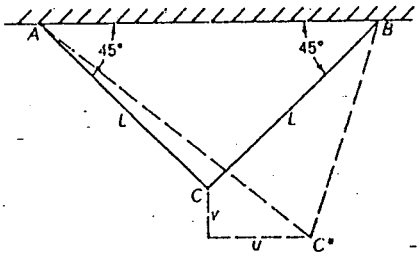


Fig. P2.16

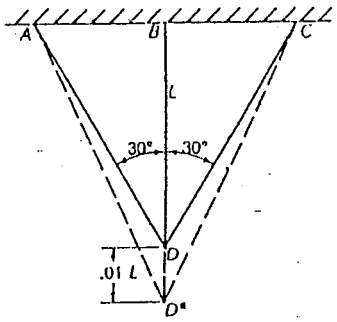


Fig. P2.17

2.17. The point D in Fig. P2.17 is displaced vertically an amount $.01L$. Obtain the approximate (first-order) average extensional strain of AD, BD, and CD.

2.21. The bar AB is rigid and is supported by two wires DB and CB as shown in Fig. P2.21. If the bar rotates through a small angle θ, determine the average extensional strain of DB and CB.

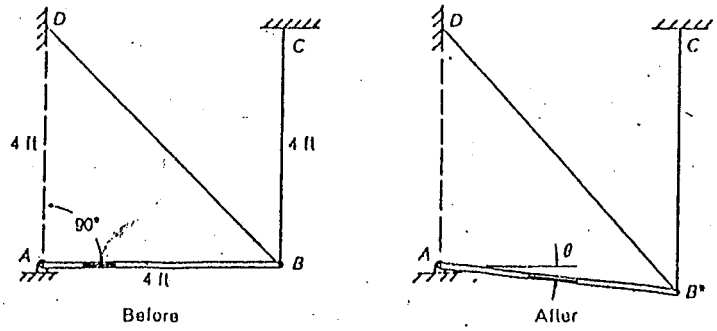


Fig. P2.21

2.32. A thin triangular plate is deformed as indicated in Fig. P2.32. Compute approximately the shear strain $\gamma_{st}(P)$.

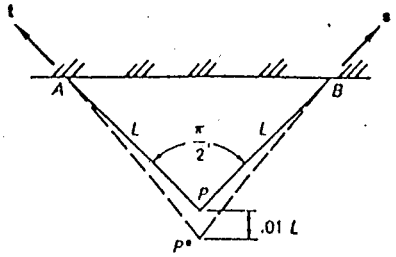


Fig. P2.32

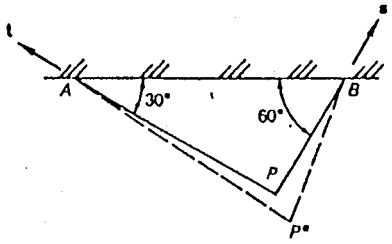


Fig. P2.34

2.34. A thin plate is in the form of a 30°–60°–90° triangle and is supported along its hypotenuse as shown in Fig. P2.34. It is subjected to the uniform strains $\epsilon_s = .004$, $\epsilon_t = .003$. Compute the approximate value of $\gamma_{st}(P)$.

2.37. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.37. It elongates 10 per cent of its original length and contracts 3 per cent of its original width. Compute approximately the shear strain γ_{st}.

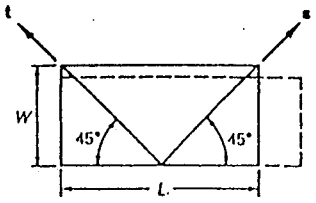


Fig. P2.37

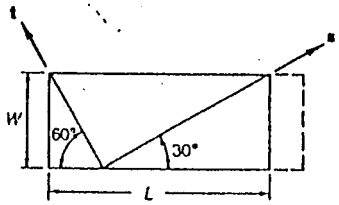


Fig. P2.38

2.38. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.38. The extensional strain along its length is .02 and the width remains unchanged. Compute approximately the shear strain γ_{st}.

Shear strain is by definition an increase in γ or a decrease in the right $\measuredangle$.

Compatibility of Strains. התאמת העיבורים

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = 0$$

Proof: $$\frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial^3 v}{\partial x^2 \partial y} - \left(\frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial^3 v}{\partial x^2 \partial y} \right) \equiv 0$$

9-17.3

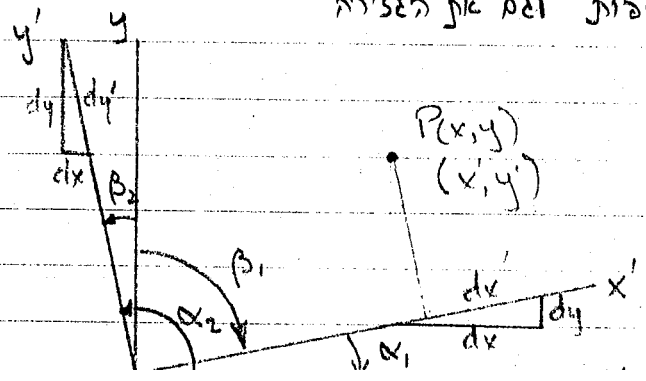
$$\epsilon_x = \frac{\partial u}{\partial x}, \quad \epsilon_y = \frac{\partial v}{\partial y}, \quad \epsilon_z = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \quad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

The assumptions we have made are:

1. Material continuous — תווי רצף
2. homogeneous — הומוגני
3. isotropic — איזוטרופי
4. elastic — אלסטי
5. continuous functions & derivatives. — העיבורים הם פונקציות רציפות וגם את הנגזרת שלהם

FIG. 2
9-17



$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\boxed{\begin{aligned} x' &= x\cos\alpha_1 + y\sin\alpha_1 \\ y' &= y\cos\alpha_1 - x\sin\alpha_1 \end{aligned}} \rightarrow \begin{aligned} x &= x'\cos\alpha - y'\sin\alpha \\ y &= x'\sin\alpha + y'\cos\alpha \end{aligned}$$

if $\cos\alpha_1 = l_1 \qquad \cos\alpha_2 = l_2$

$\cos\beta_1 = m_1 \qquad \cos\beta_2 = m_2$

since $\alpha_1 = -\beta_2 \qquad \beta_2 + \beta_1 + \alpha_1 = -\alpha_2$

$\cos\alpha_1 = \cos(-\beta_2) = \cos\beta_2 \qquad \beta_2 = \alpha_1 \qquad \underline{\underline{l_1 = m_2}}$

since $\beta_2 + \beta_1 + \alpha_1 = 90^\circ + \alpha_1$ then

$\cos(\alpha_2) = \cos(\alpha_2) = \cos(90^\circ + \alpha_1) = -\sin\alpha_1 = -m_1$

$\underline{\underline{l_2 = -m_1}}$

$$\frac{dx}{dx'} = \cos\alpha_1 = l_1 \qquad \frac{dy}{dx'} = \sin\alpha_1 = m_1$$

$$\frac{dx}{dy'} = +\sin\beta_2 = +l_2 = -m_1 \qquad \frac{dy}{dy'} = l_1$$

$$l_1^2 + m_1^2 = 1 \quad \left((\cos\alpha_1)^2 + (\sin\alpha_1)^2 = 1\right)$$

$$l_2^2 + m_2^2 = 1 \quad \left((\cos\alpha_2)^2 + (\sin\alpha_2)^2 = 1\right)$$

$$l_1^2 + l_2^2 = 1$$

$$m_2^2 + m_1^2 = 1$$

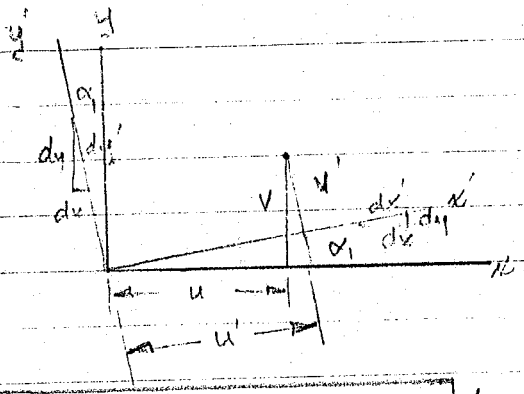
$$l_1 m_1 + l_2 m_2 = 0$$

$$l_1 l_2 + m_1 m_2 = 0$$

9-22-70

	x	y
x'	l_1	m_1
y'	$l_2 = -m_1$	$m_2 = l_1$

FIG 1



9-22

$$u' = l_1 u + m_1 v = \cos\alpha\, u + \sin\alpha\, v$$

$$v' = -m_1 u + l_1 v = -\sin\alpha\, u + \cos\alpha\, v$$

$\cos\alpha_1 = l_1$; $\sin\alpha_1 = m_1$

$$\frac{dx}{dx'} = \frac{dy}{dy'} = l_1 \qquad \frac{dy}{dx'} = -\frac{dx}{dy'} = m_1$$

$$\epsilon_x = \frac{\partial u}{\partial x} \qquad \epsilon_y = \frac{\partial v}{\partial y} \qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\epsilon_{x'} = \frac{\partial u'}{\partial x'} = \frac{\partial}{\partial x'}\left(l_1 u + m_1 v\right)$$

$$\frac{\partial}{\partial x'}(\quad) = \frac{\partial}{\partial x}(\quad)\frac{dx}{dx'} + \frac{\partial}{\partial y}(\quad)\frac{dy}{dx'}$$

$$\epsilon_{x'} = l_1\left(\frac{\partial u}{\partial x}\underbrace{\frac{dx}{dx'}}_{l_1=\cos\alpha} + \frac{\partial u}{\partial y}\underbrace{\frac{dy}{dx'}}_{m_1=\sin\alpha}\right) + m_1\left(\frac{\partial v}{\partial x}\underbrace{\frac{dx}{dx'}}_{l_1=\cos\alpha} + \frac{\partial v}{\partial y}\underbrace{\frac{dy}{dx'}}_{m_1=\sin\alpha}\right)$$

$$= l_1^2\frac{\partial u}{\partial x} + m_1^2\frac{\partial v}{\partial y} + l_1 m_1\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}\right)$$

$$\epsilon_{x'} = l_1^2\epsilon_x + m_1^2\epsilon_y + l_1 m_1 \gamma_{xy}$$

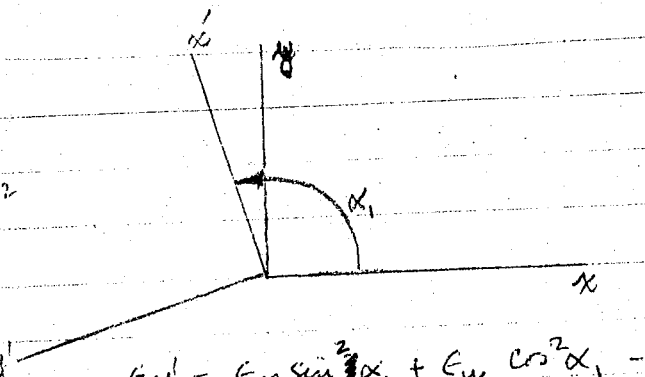
$$\epsilon_{x'} = \epsilon_x \underbrace{\frac{1+\cos 2\alpha}{2}}\cos^2\alpha_1 + \epsilon_y \underbrace{\frac{1-\cos 2\alpha}{2}}\sin^2\alpha_1 + \gamma_{xy}\underbrace{\frac{\sin 2\alpha}{2}}\sin\alpha_1\cos\alpha_1$$

9-22.1

$$\boxed{\epsilon_{x'} = \frac{\epsilon_x+\epsilon_y}{2} + \frac{\epsilon_x-\epsilon_y}{2}\cos 2\alpha_1 + \frac{\gamma_{xy}}{2}\sin 2\alpha_1}$$

ϵ_{xy}

FIG 2



9-22

$$\epsilon_{y'} = \epsilon_x \sin^2\alpha_1 + \epsilon_y \cos^2\alpha_1 - \gamma_{xy} \sin\alpha_1 \cos\alpha_1$$

9-22.2
$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\alpha_1 - \frac{\gamma_{xy}}{2}\sin 2\alpha_1$$

$$\gamma_{x'y'} = \frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'}$$

$$= \frac{\partial u'}{\partial x}\frac{dx}{dy'} + \frac{\partial u'}{\partial y}\frac{dy}{dy'} + \frac{\partial v'}{\partial x}\frac{dx}{dx'} + \frac{\partial v'}{\partial y}\frac{dy}{dx'}$$

$$= -\left(l_1\frac{\partial u}{\partial x} + m_1\frac{\partial v}{\partial x}\right)m_1 + \left(-m_1\frac{\partial u}{\partial x} + l_1\frac{\partial v}{\partial x}\right)l_1$$

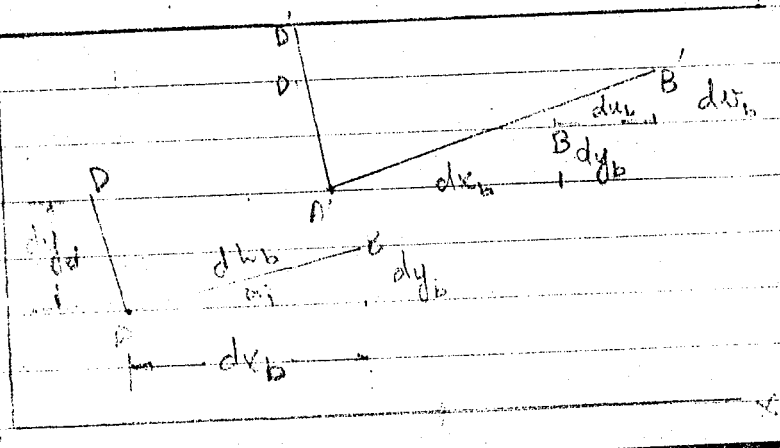
$$+ \left(l_1\frac{\partial u}{\partial y} + m_1\frac{\partial v}{\partial y}\right)l_1 + \left(-m_1\frac{\partial u}{\partial y} + l_1\frac{\partial v}{\partial y}\right)m_1$$

$$= -2l_1m_1\frac{\partial u}{\partial x} + 2l_1m_1\frac{\partial v}{\partial y} + l_1^2\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - m_1^2\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\gamma_{x'y'} = -2(\epsilon_x - \epsilon_y)\sin\alpha_1\cos\alpha_1 + (\cos^2\alpha_1 - \sin^2\alpha_1)\gamma_{xy}$$

9-22.3
$$\gamma_{x'y'} = -(\epsilon_x - \epsilon_y)\sin 2\alpha_1 + \gamma_{xy}\cos 2\alpha_1$$

FIG. 3



9-22

$$\epsilon_x' = \frac{A'B' - AB}{AB}$$

$$(1+\epsilon_x') = A'B'/AB \qquad \overline{A'B'}^2 = (dx_b + du_b)^2 + (dy_b + dv_b)^2$$

$$= dx_b^2 + 2dx_b du_b + du_b^2 + dy_b^2 + 2dy_b dv_b + dv_b^2$$

$$u_B = u + \frac{\partial u}{\partial x} dx_b + \frac{\partial u}{\partial y} dy_b$$

$$(1 + 2\epsilon_x' + \epsilon_x'^2)\, dL_b^2 = dL_b^2 + 2\left(\frac{\partial u}{\partial x} dx_b^2 + \frac{\partial u}{\partial y} dx_b dy_b\right) + 2\left(\frac{\partial v}{\partial x} dx_b dy_b + \frac{\partial v}{\partial y} dy_b^2\right) + \left(\frac{\partial u}{\partial x} dx_b + \frac{\partial u}{\partial y} dy_b\right)^2 + \left(\frac{\partial v}{\partial x} dx_b + \frac{\partial v}{\partial y} dy_b\right)^2$$

neglect these terms since they are small

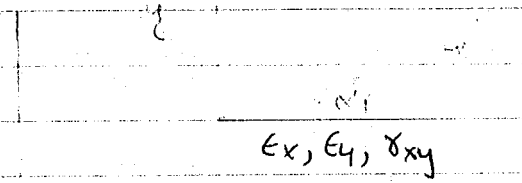
$$\frac{dx_b}{dL_b} = l_1 \qquad \frac{dy_b}{dL_b} = m_1$$

$$\epsilon_x' = l_1^2 \frac{\partial u}{\partial x} + m_1^2 \frac{\partial v}{\partial y} + l_1 m_1 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\epsilon_x' = l_1^2 \epsilon_x + m_1^2 \epsilon_y + \gamma_{xy} (l_1 m_1)$$

FIG. 4

9-22



$$\frac{\partial \epsilon_x'}{\partial(\alpha_1)} = \frac{\epsilon_x - \epsilon_y}{2}(-2\sin 2\alpha_1) + \frac{\gamma_{xy}}{2}(2\cos 2\alpha_1) = 0$$

9-22.4

$$\boxed{\tan 2\alpha_1 = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} \equiv \tan 2\alpha = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}}$$

when no shear stress

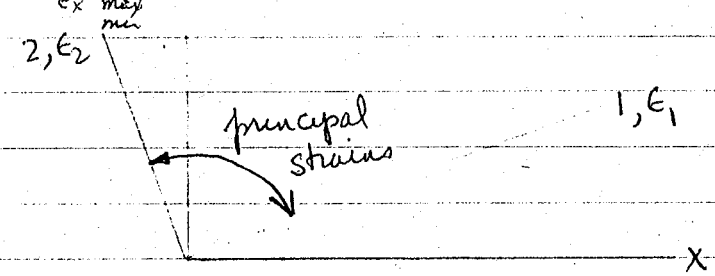
$\tan 2\alpha = \tan(\pi + 2\alpha)$; $\alpha = \frac{\pi}{2} + \alpha$

$$\sin 2\alpha = \pm \gamma_{xy} / \left[(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2 \right]^{1/2}$$

$$(\epsilon_{x'})_{\substack{\max,\min \\ 1,2}} = \frac{\epsilon_x + \epsilon_y}{2} \pm \frac{1}{2}\left[(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2 \right]^{1/2}$$

$$(\gamma_{x'y'})_{\epsilon_{x'}\ \max\ \min} = \cos 2\alpha_1 \left[-(\epsilon_x - \epsilon_y)\tan 2\alpha_1 + \gamma_{xy} \right] = 0$$

FIG. 5
9-22



9-22.5

$$\boxed{\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \frac{1}{2}\left[(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2 \right]^{1/2}} \qquad \tan 2\alpha = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

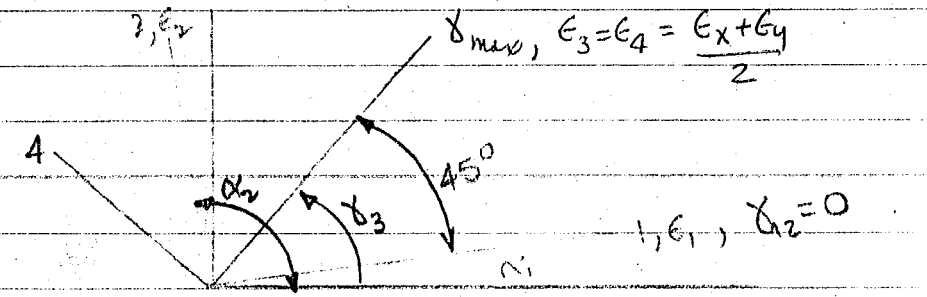
$$\boxed{\gamma_{1,2} = 0}$$

$$\frac{\partial \gamma_{x'y'}}{\partial \alpha_1} = -2(\epsilon_x - \epsilon_y)\cos 2\alpha_1 - 2\gamma_{xy}\sin 2\alpha_1 = 0$$

9-22.6

$$\boxed{\tan 2\alpha_1 = -\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}}}$$

FIG. 6
9-22



$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\cos 2\alpha_1}{2}\left[(\epsilon_x - \epsilon_y) + \gamma_{xy}\tan 2\alpha_1 \right]$$

9-22.7

$$\boxed{\gamma_{max} = \pm \left[(\epsilon_x - \epsilon_y)^2 + (\gamma_{xy})^2 \right]^{1/2}}$$

$$\epsilon_1 = \frac{\epsilon_x + \epsilon_y}{2} + \frac{1}{2}\left[(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2\right]^{1/2}$$

$$\epsilon_2 = \text{"} - \text{"}$$

$$\gamma_{1,2} = 0. \qquad \tan 2\alpha = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

9-22.8

$$\overline{\gamma_{max}} = \overline{\left[(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2\right]^{1/2}}$$

$$\epsilon_x' = \epsilon_y' = \frac{\epsilon_x + \epsilon_y}{2} \qquad \tan 2\alpha = -\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}}$$

$I_{1\epsilon}$ $\boxed{\epsilon_x + \epsilon_y = \epsilon_1 + \epsilon_2}$ unchanged no matter what α_1 is

Invariant at a point.

$I_{2\epsilon}$ $\boxed{\epsilon_x \epsilon_y - \frac{1}{4}\gamma_{xy}^2 = \epsilon_x'\epsilon_y' - \frac{1}{4}\gamma_{x'y'}^2}$ Invariant

9-24-70

2-D $\begin{cases} \epsilon_x, \epsilon_y \\ \gamma_{xy} \end{cases}$ function of u, v

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = 0$$

3 differential eq.
2 unknowns
1 Compatability Eq.

	x	y
x'	l_1	m_1
y'	l_2	m_2

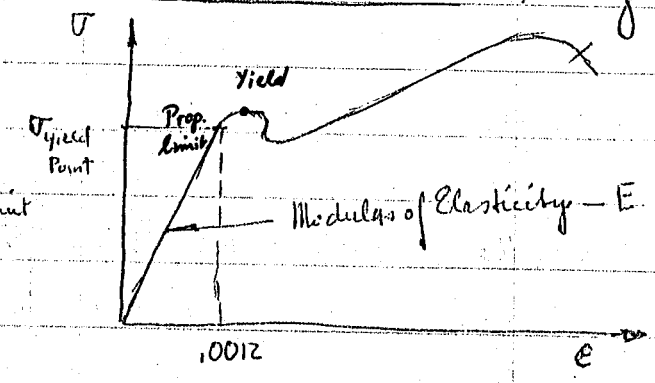
3-D $\begin{cases} \epsilon_x, \epsilon_y, \epsilon_z \\ \gamma_{xy}, \gamma_{yz}, \gamma_{zx} \end{cases}$ function (u, v, w)

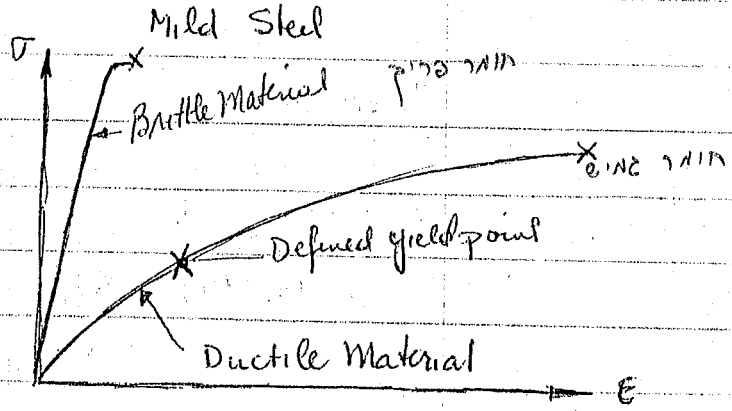
6 differential eq.
3 unknowns
6 Compatability Eq
(1.48)

	x	y	z
x'	l_1	m_1	n_1
y'	l_2	m_2	n_2
z'	l_3	m_3	n_3

$$\begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix} \equiv \begin{pmatrix} \epsilon_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{yx}}{2} & \epsilon_y & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{zx}}{2} & \frac{\gamma_{zy}}{2} & \epsilon_z \end{pmatrix}$$

Constitutive Laws – mostly empirical laws





Hookes Law

$\tau = C\,\epsilon$ $\qquad$ $\epsilon = A\tau$ depends on material

stress.

Principle of Superposition

Can add effects of individual loadings to get effect of resultant loading.

true if 1) each strain must be directly and linearly related to the stress causing the strain (Hooke's Law)

2) Strains (deformation) due to one stress (load) have negligible effect on the conditions of application of the other load. ~~Stress~~ any other stress

$$\epsilon_{x'} = l_1^2 \epsilon_x + m_1^2 \epsilon_y + n_1^2 \epsilon_z + l_1 m_1 \gamma_{xy} + m_1 n_1 \gamma_{yz} + n_1 l_1 \gamma_{zx}$$

$$\epsilon_{y'} = \ldots$$

$$\epsilon_{z'} = \ldots$$

$$\gamma_{x'y'} = -2 l_1 m_1 (\epsilon_x - \epsilon_y) + (l_1 m_2 + m_1 l_2) \gamma_{xy}$$

$$\gamma_{y'z'} = \ldots$$

$$\gamma_{z'x'} = \ldots$$

ϵ_1, ϵ_2 2-D max, min $\quad \gamma_{12} = 0$

$\epsilon_1, \epsilon_2, \epsilon_3$ 3-D max, min, other $\quad \gamma_{12} = \gamma_{23} = \gamma_{31} = 0$

9-24.1

$$\text{2-D} \begin{cases} \epsilon_x + \epsilon_y = I_{1\epsilon} & \text{invariant} \\ \epsilon_x \epsilon_y - \frac{1}{4}\gamma_{xy}^2 = I_{2\epsilon} & \text{invariant} \end{cases}$$

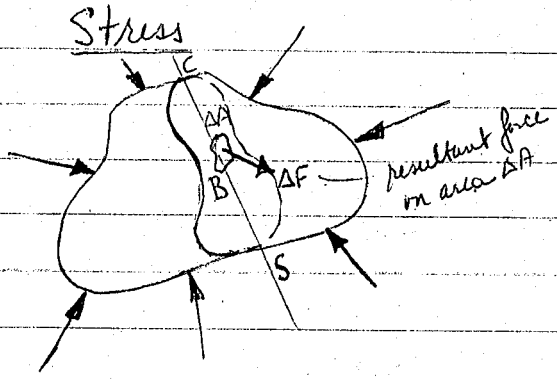
$$\text{3-D} \begin{cases} \epsilon_x + \epsilon_y + \epsilon_z = I_{1\epsilon} & \text{invariant} \\ \epsilon_x \epsilon_y + \epsilon_y \epsilon_z + \epsilon_z \epsilon_x - \frac{1}{4}\left(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2\right) = I_{2\epsilon_2} \\ \epsilon_x \epsilon_y \epsilon_z + \frac{1}{4}\left(\gamma_{xy}\gamma_{yz}\gamma_{zx}\right) - \frac{1}{4}\left(\epsilon_x \gamma_{yz}^2 + \epsilon_y \gamma_{zx}^2 + \epsilon_z \gamma_{xy}^2\right) = I_{3\epsilon} \end{cases}$$

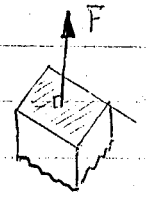
1.139

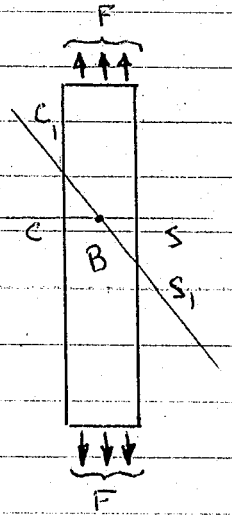
Stress

FIG 1

9-24







$\frac{F}{A}$ = Intensity of internal forces in the axial direction.

10-6-70

from FIG. 1 10-5 we can obtain

$$(\sigma_x')_{\substack{max \\ min}} = \left(\frac{\sigma_x+\sigma_y}{2}\right) \pm \left[\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

$$\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

In 2-D the invariants are

10-6.1

$$I_{1\sigma} = \sigma_x+\sigma_y \quad ; \quad I_{2\sigma} = \sigma_x\sigma_y - \tau_{xy}^2$$

In 3-D the invariants become

$$I_{1\sigma} = \sigma_x+\sigma_y+\sigma_z \quad ; \quad I_{2\sigma} = \sigma_x\sigma_y+\sigma_y\sigma_z+\sigma_z\sigma_x-\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\right)$$

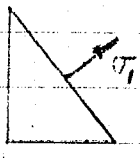
$$I_{3\sigma} = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{zx}\tau_{yz} - \left(\sigma_x\tau_{yz}^2 + \sigma_y\tau_{zx}^2 + \sigma_z\tau_{xy}^2\right)$$

from $\frac{\partial\sigma_x'}{\partial l}$ we had obtained $\frac{l\sigma_x+m\tau_{xy}}{l} = \frac{l\tau_{xy}+m\sigma_y}{m} = \frac{\bar{X}}{l} = \frac{\bar{Y}}{m}$

or $m/l = \bar{Y}/\bar{X} = \tan\alpha$

if ratio holds then we have only σ_1 and no shear

FIG. 1 10-6



From strain displacements we have 6 equations

$$\epsilon_x = \frac{\partial u}{\partial x} \quad ; \quad \epsilon_y = \frac{\partial v}{\partial y} \quad ; \quad \epsilon_z = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} \quad ; \quad \gamma_{yz} = \frac{\partial v}{\partial z}+\frac{\partial w}{\partial y} \quad ; \quad \gamma_{zx} = \frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}$$

From Stress Equilibrium Equations we got

$$\frac{\partial\sigma_x}{} + \frac{\partial\tau_{xy}}{} + \frac{\partial\tau_{xz}}{} + \bar{X} = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \bar{Y} = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + \bar{Z} = 0$$

Total :- 9 equations 15 unknowns (stresses, strains, displ

Stress - Strain Relationships

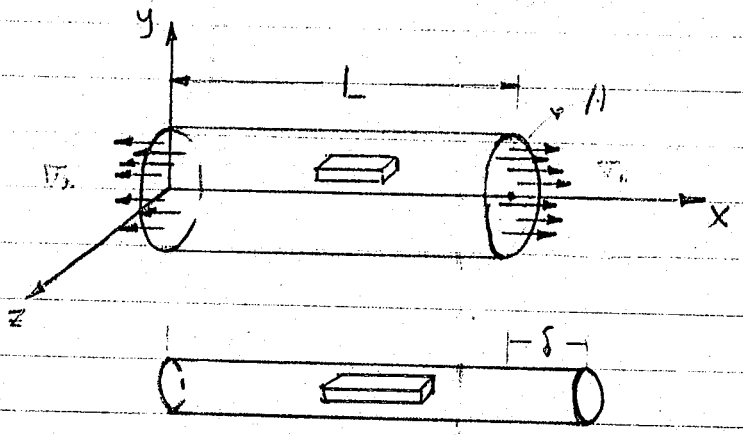


FIG. 2 10-6

cross-section must remain the same - isotropic properties

$\delta = \frac{FL}{AE}$ deformation of bar

$\frac{\delta}{L} = \epsilon_x = \frac{F}{AE} = \frac{\sigma_x}{E}$; E - modulus of elasticity

$\epsilon_y = \epsilon_z = -\nu \epsilon_x$; ν - Poisson's ratio

if $\epsilon_y = \frac{\sigma_y}{E}$ then the effect of this is $\epsilon_x = \epsilon_z = -\nu \epsilon_y$

" $\epsilon_z = \frac{\sigma_z}{E}$ " " " " " " $\epsilon_x = \epsilon_y = -\nu \epsilon_z$

true if we are below Elastic or Proportional limit (FIG. 1(c) 9-15)

Within elastic limit we can use the principle of superposition

$$\therefore \boxed{\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z)$$

10-6.2 $$\epsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y)$$

If load is axial then $\gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 0$ (no change in the angles of the parallelopiped of FIG. 2 10-6)

If the load is at some angle α to the axes then the strain relationships are

$$\epsilon x' = l_1^2 \epsilon_x + m_1^2 \epsilon_y + n_1^2 \epsilon_z + l_1 m_1 \gamma_{xy} + m_1 n_1 \gamma_{yz} + n_1 l_1 \gamma_{zx}$$

$$\gamma_{x'y'} = 2(l_1 l_2 \epsilon_x + m_1 m_2 \epsilon_y + n_1 n_2 \epsilon_z) + (l_1 m_2 + l_2 m_1)\gamma_{xy} + (m_1 n_2 + n_1 m_2)\gamma_{yz} + (n_1 l_2 + l_1 n_2)\gamma_{zx}$$

these 2 equations boil down to

$$\left.\begin{aligned} \epsilon x' &= l_1^2 \epsilon_x + m_1^2 \epsilon_y + n_1^2 \epsilon_z \\ \gamma_{x'y'} &= 2(l_1 l_2 \epsilon_x + m_1 m_2 \epsilon_y + n_1 n_2 \epsilon_z) \end{aligned}\right\} \text{Set 1}$$

iff* $\gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 0$

Similarly the Stress relationship is

$$\left.\begin{aligned} \sigma_{x'} &= l_1^2 \sigma_x + m_1^2 \sigma_y + n_1^2 \sigma_z \\ \tau_{x'y'} &= l_1 l_2 \sigma_x + m_1 m_2 \sigma_y + n_1 n_2 \sigma_z \end{aligned}\right\} \text{Set 2}$$

from 10-6.2 we obtain the following

$$\epsilon_x = \frac{(1+\nu)\sigma_x - \nu(\sigma_x + \sigma_y + \sigma_z)}{E}$$

10-6.3 $$\epsilon_y = \frac{(1+\nu)\sigma_y - \nu(\sigma_x + \sigma_y + \sigma_z)}{E}$$

from Set 1 and 10-6.3 we get

$$\epsilon_{x'} = \frac{(1+\nu)}{E}\left[l_1^2\sigma_x + m_1^2\sigma_y + n_1^2\sigma_z\right] - \frac{\nu}{E}\left(\sigma_x+\sigma_y+\sigma_z\right)\left[l_1^2+m_1^2+n_1^2\right]$$

$$= \frac{(1+\nu)}{E}\sigma_{x'} - \frac{\nu}{E}\left(\sigma_{x'}+\sigma_{y'}+\sigma_{z'}\right) = \frac{\sigma_{x'} - \nu\sigma_{y'} - \nu\sigma_{z'}}{E}$$

$$\gamma_{x'y'} = \frac{2(1+\nu)}{E}\left[l_1 l_2\sigma_x + m_1 m_2\sigma_y + n_1 n_2\sigma_z\right] - \frac{2\nu}{E}\left(\sigma_x+\sigma_y+\sigma_z\right)\left[l_1 l_2 + m_1 m_2 + n_1 n_2\right]$$

$$= \frac{2(1+\nu)}{E}\tau_{x'y'}$$

or

$$\boxed{\gamma_{x'y'} = \frac{2(1+\nu)}{E}\tau_{x'y'} = \frac{\tau_{x'y'}}{G}}$$

10-6.4 $\boxed{G = E/[2(1+\nu)]}$

If the loading is not axial and the angles change they are related in the following manner to the shear stresses

10-6.5 $$\boxed{\gamma_{xy} = \frac{\tau_{xy}}{G}\ ;\ \gamma_{yz} = \frac{\tau_{yz}}{G}\ ;\ \gamma_{xz} = \frac{\tau_{zx}}{G}}$$

we now have 15 unknowns & 15 equations.

10-8-70

$$\epsilon_x = \frac{(1+\nu)\sigma_x - \nu(\sigma_x+\sigma_y+\sigma_z)}{E}$$

$$\epsilon_y = \frac{(1+\nu)\sigma_y - \nu(\sigma_x+\sigma_y+\sigma_z)}{E}$$

$$\epsilon_z = \frac{(1+\nu)\sigma_z - \nu(\sigma_x+\sigma_y+\sigma_z)}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G},\quad \gamma_{yz} = \frac{\tau_{yz}}{G},\quad \gamma_{zx} = \frac{\tau_{zx}}{G}\ ;\ G = \frac{E}{2(1+\nu)}$$

המכללה האקדמית יהודה ושומרון

תורת החוזק 1

תרגיל מס' 3

2.7. A straight piece of wire PQ lies along the line $y = mx$ as shown in Fig. P2.7. The wire is strained and displaced to the line $y = nx$ in such a way that a point originally at x is displaced to $x^2/2$. Show that the extensional strain at any point along the wire is given by

$$\epsilon = \sqrt{\frac{1 + n^2}{1 + m^2}}\, x - 1$$

(where x is the original coordinate of the point).

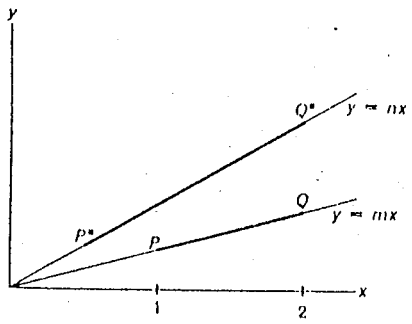


Fig. P2.7

2.16. The point C is displaced to C^* as shown in Fig. P2.16. The horizontal and vertical components of this displacement are u and v. Express the average extensional strains of AC and BC in terms of u, v and L. If u and v are small, what are the approximate expressions for these average extensional strains?

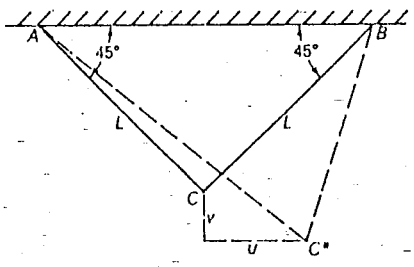


Fig. P2.16

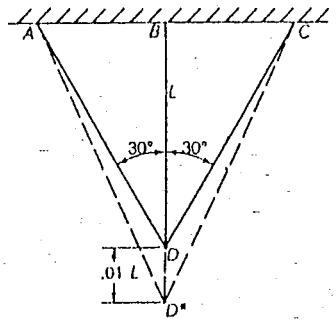
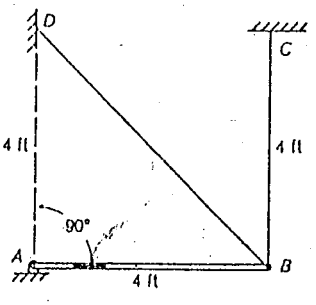
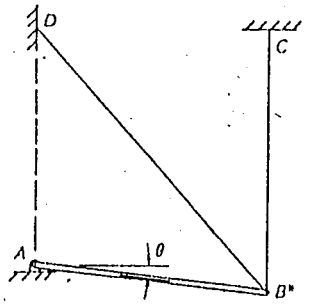


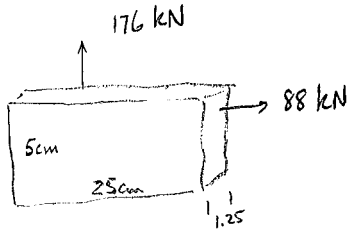
Fig. P2.17

2.17. The point D in Fig. P2.17 is displaced vertically an amount $.01L$. Obtain the approximate (first-order) average extensional strain of AD, BD, and CD.

2.21. The bar AB is rigid and is supported by two wires DB and CB as shown in Fig. P2.21. If the bar rotates through a small angle θ, determine the average extensional strain of DB and CB.







$$\sigma_y = \frac{P_y}{A} = \frac{176000}{25(1.25)} = \frac{176000\ N}{31.25\ cm^2} =$$

2.32. A thin triangular plate is deformed as indicated in Fig. P2.32. Compute approximately the shear strain $\gamma_{st}(P)$.

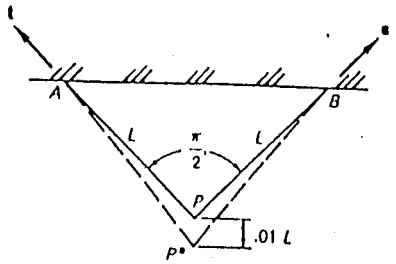


Fig. P2.32

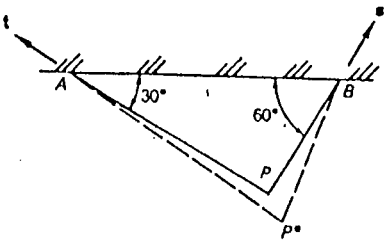


Fig. P2.34

2.34. A thin plate is in the form of a 30°–60°–90° triangle and is supported along its hypotenuse as shown in Fig. P2.34. It is subjected to the uniform strains $\epsilon_s = .004$, $\epsilon_t = .003$. Compute the approximate value of $\gamma_{st}(P)$.

2.37. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.37. It elongates 10 per cent of its original length and contracts 3 per cent of its original width. Compute approximately the shear strain γ_{st}.

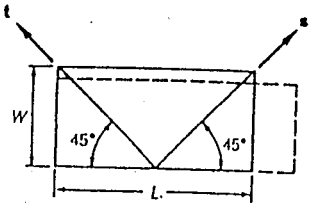


Fig. P2.37

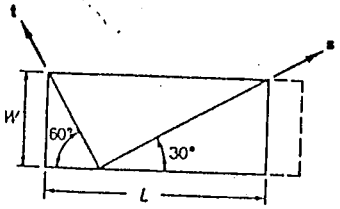


Fig. P2.38

2.38. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.38. The extensional strain along its length is .02 and the width remains unchanged. Compute approximately the shear strain γ_{st}.

Answers:

2.16 $\varepsilon_{AC} = \dfrac{\sqrt{L^2 + \sqrt{2}L(v+u) + u^2 + v^2} - L}{L}$; $\varepsilon_{AC} \approx \dfrac{\sqrt{2}(u+v)}{2L}$ $\varepsilon_{BC} \approx \dfrac{\sqrt{2}(u-v)}{2L}$

2.17 $\varepsilon_{AD} = \varepsilon_{CD} = 0.00754$; $\varepsilon_{BD} = 0.01$

2.21 $\varepsilon_{DB} = \theta/2$; $\varepsilon_{CB} = \theta$

2.32 $\sqrt{2}(0.01)$

2.34 0.0075

2.37 -0.125

2.38 $-0.01\sqrt{3}$

2.32. A thin triangular plate is deformed as indicated in Fig. P2.32. Compute approximately the shear strain $\gamma_{st}(P)$.

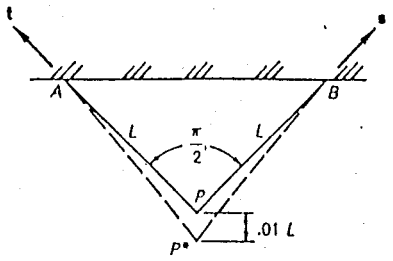


Fig. P2.32

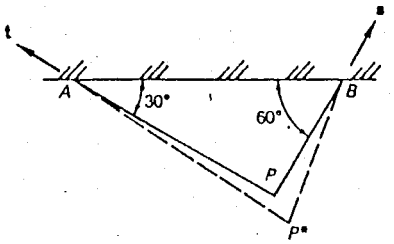


Fig. P2.34

2.34. A thin plate is in the form of a 30°–60°–90° triangle and is supported along its hypotenuse as shown in Fig. P2.34. It is subjected to the uniform strains $\epsilon_s = .004$, $\epsilon_t = .003$. Compute the approximate value of $\gamma_{st}(P)$.

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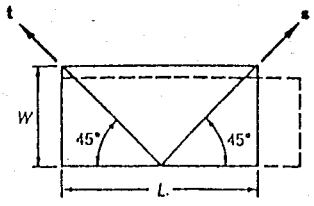


Fig. P2.37

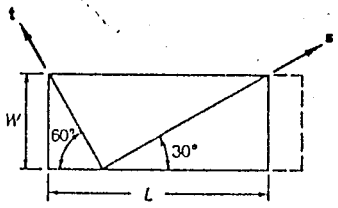


Fig. P2.38

2.38. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.38. The extensional strain along its length is .02 and the width remains unchanged. Compute approximately the shear strain γ_{st}.

Answers:

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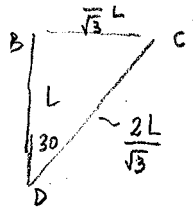
2.21 $\varepsilon_{DB} = \theta/2$; $\varepsilon_{CB} = \theta$

2.32 $\sqrt{2}(0.01)$

2.34 0.0075

2.37 -0.125

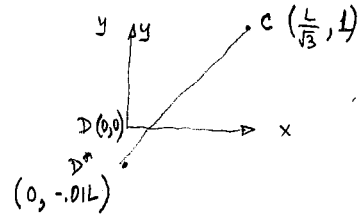
2.38 $-0.01\sqrt{3}$



$\cos 30° = \frac{\overline{BD}}{\overline{CD}} = \frac{L}{\overline{CD}} = .866 = \frac{\sqrt{3}}{2} \Rightarrow \overline{CD} = \frac{2L}{\sqrt{3}}$

$\sin 30° = \frac{\overline{BC}}{\overline{CD}} \Rightarrow \overline{BC} = \overline{CD}\sin 30° = \frac{2L}{\sqrt{3}} \cdot \frac{1}{2} = L/\sqrt{3}$

$\epsilon_{BD} = \frac{\overline{BD}^* - BD}{BD} = \frac{1.01L - L}{L} = .01$



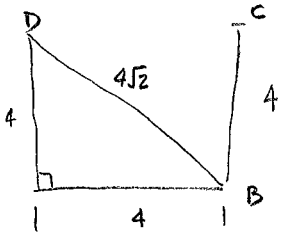
$\epsilon_{CD} = \frac{\overline{CD}^* - CD}{CD} = \frac{1.163L - 1.155L}{1.155L} = .0075$

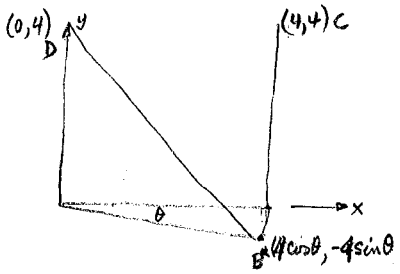
$CD = \frac{L}{\sqrt{3}}\hat{i} + L(1.01)\hat{j} \Rightarrow L\sqrt{\frac{1}{3} + 1.02} = 1.163L$

BY SYMMETRY

$0.0075 = \epsilon_{AD} = \epsilon_{CD}$ מכאן שהמבנה סימטרי

2.21





$\overline{BD} = \left[(0 - 4\cos\theta)^2 + (4 + 4\sin\theta)^2\right]^{1/2}$

$= \left[16\cos^2\theta + 16 + 32\sin\theta + 16\sin^2\theta\right]^{1/2}$

$= \left[32 + 32\sin\theta\right]^{1/2} = 4\sqrt{2}\,(1+\sin\theta)^{1/2}$

$\epsilon_{BD} = \frac{\overline{B^*D} - BD}{BD} = \frac{4\sqrt{2}(1+\sin\theta)^{1/2} - 4\sqrt{2}}{4\sqrt{2}} = (1+\sin\theta)^{1/2} - 1$

$\epsilon_{BD} \cong \left(1 + \frac{\sin\theta}{2} + \ldots\right) - 1 = \frac{\sin\theta}{2} \approx \theta/2$ rad

$\overline{CB}^* = \left[(4\cos\theta - 4)^2 + (-4\sin\theta - 4)^2\right]^{1/2}$

$= \left[16\cos^2\theta - 32\cos\theta + 16 + 16\sin^2\theta + 32\sin\theta + 16\right]^{1/2}$

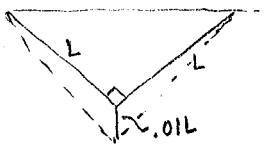
$= \left[48 + 32\sin\theta - 32\cos\theta\right]^{1/2} \cong \left[16 + 32\sin\theta\right]^{1/2}$

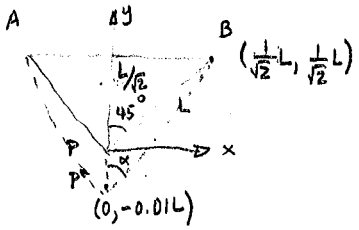
$= 4\left[1 + 2\sin\theta\right]^{1/2}$

$\epsilon_{CB} = \frac{\overline{CB}^* - CB}{CB} = \frac{4(1+2\sin\theta)^{1/2} - 4}{4} = (1+2\sin\theta)^{1/2} - 1$

$= \left(1 + \frac{2\sin\theta}{2} + \ldots\right) - 1 = \sin\theta \cong \theta$

2.32



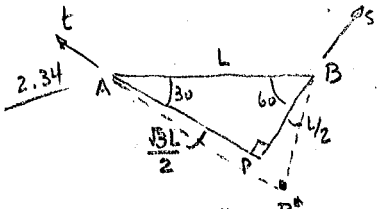


$\tan\alpha = \frac{\frac{1}{\sqrt{2}}L}{\frac{L}{\sqrt{2}} + .01L} = .986 \Rightarrow \alpha = 44.598° = .7781$ rad

$45° = .7854$ rad

$\gamma_{st} = 90° - 2(44.598°) = .8046° = .0140$ rad.

2.34

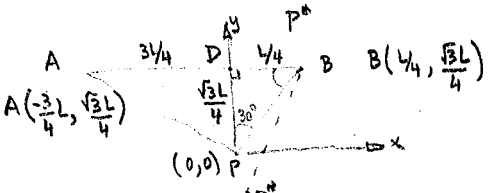


$\epsilon_s = .004$

$\epsilon_t = .003$

ASSUME AB IS OF LENGTH L

אנו מניחים שאורך AB שווה ל-1. זוהי פשוט אחידות מדידה.



$\frac{\sqrt{3}L}{4} = \overline{PD}$, $\frac{\sqrt{3}L}{2} = \overline{AP}$, $1/2 = \overline{BP}$ ⇐ נניח ש- AB שווה ל-1

$\epsilon_s = \frac{\overline{BP}^* - BP}{BP} = \frac{\overline{BP}^* - 1/2}{1/2} \Rightarrow \overline{BP}^* = (\epsilon_s + 1)\frac{1}{2} = (1.004)\frac{1}{2}$

$\epsilon_t = \frac{\overline{AP}^* - AP}{AP} = \frac{\overline{AP}^* - \sqrt{3}L/2}{\sqrt{3}L/2} \Rightarrow \overline{AP}^* = (\epsilon_t + 1)\frac{\sqrt{3}L}{2} = (1.003)\frac{\sqrt{3}L}{2}$



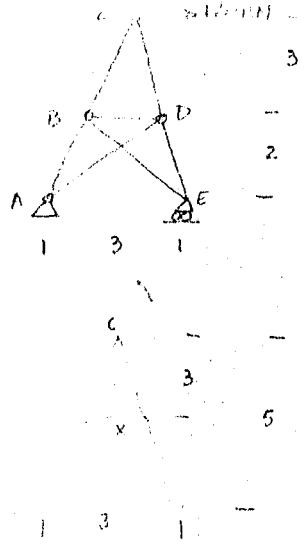
⇐ $\theta = \angle AP^*B$ מזה נמצא

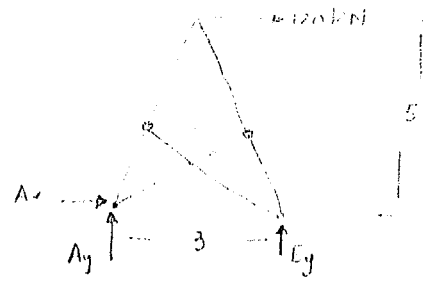
$L^2 = \overline{AP}^{*2} + \overline{BP}^{*2} - 2\overline{AP}^*\overline{BP}^*\cos\theta \Rightarrow \cos\theta = \frac{L^2 - \overline{AP}^{*2} - \overline{BP}^{*2}}{-2\overline{AP}^*\overline{BP}^*}$

$= \frac{1^2 - (1.003)^2\left(\frac{\sqrt{3}}{2}\right)^2 - (1.004)^2\left(\frac{1}{2}\right)^2}{-2(1.003)\frac{\sqrt{3}}{2}(1.004)\frac{1}{2}}$

כאשר θ קרובה ל-$90°$, $90°-\theta$ קרובה ל-$0°$. לכן:

$\cos\theta = \cos(-\theta) = \cos(90° - \theta - 90°) = \cos(90° - \theta)\cos 90° + \sin(90° - \theta)\sin 90°$





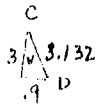
$+\circlearrowleft \sum M_A = -120(5) + E_y \cdot 3 = 0 \quad E_y = 200\,kN$

$\rightarrow \sum F_x = 0 \quad 120 + A_x = 0 \Rightarrow A_x = -120\,kN$

$+\uparrow \sum F_y = 0 \quad E_y + A_y = 0 \Rightarrow A_y = -200\,kN$

$\frac{5}{3} = \frac{\text{אורך}}{\text{רוחב}} = \frac{3}{x}$

$x = \frac{9}{5} = 1.8\,m$



בנקודה C



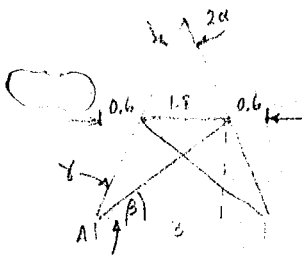
$\rightarrow \sum F_x = 0 \Rightarrow 120 + F_{CD}\sin\alpha - F_{CD}\sin\alpha = 0$

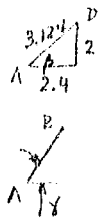
$+\uparrow \sum F_y = 0 \Rightarrow -(F_{CD}\cos\alpha + F_{CB}\cos\alpha) = 0 \Rightarrow F_{CD} = F_{CB}$

$\Rightarrow 2F_{CD}\sin\alpha = -120$

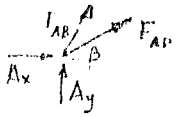
$2F_{CD}\left(\frac{.9}{3.132}\right) = -120 \quad F_{CD} = -208.8\,kN$

$F_{CB} = 208.8\,kN$





בנקודה A



$\gamma = \frac{180 - 2\alpha}{2} = 90 - \alpha \Rightarrow$

$\Sigma F_x = 0 \Rightarrow A_x + F_{AD}\cos\beta + F_{AB}\cos\gamma = 0$

$\Sigma F_y = 0 \Rightarrow A_y + F_{AD}\sin\beta + F_{AB}\sin\gamma = 0$

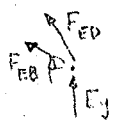
$-120 + F_{AD}\left(\frac{2.4}{3.124}\right) + F_{AB}\left(\frac{.9}{3.132}\right) = 0$

$-200 + F_{AD}\left(\frac{2}{3.124}\right) + F_{AB}\left(\frac{3}{3.132}\right) = 0$

$F_{AD} = 104.132\,kN$

$F_{AB} = 139.201\,kN$

בנקודה E



$\rightarrow \sum F_x = -F_{ED}\cos\gamma - F_{EB}\cos\beta = 0$

$\Sigma F_y = F_{ED}\sin\gamma + F_{EB}\sin\beta + E_y = 0$

$-F_{ED}\left(\frac{.9}{3.132}\right) - F_{EB}\left(\frac{2.4}{3.124}\right) = 0$

$F_{ED}\left(\frac{2}{3.132}\right) + F_{EB}\left(\frac{2}{3.124}\right) + 200 = 0$

$F_{ED} = -278.403\,kN$

$F_{EB} = 104.135\,kN$

בנקודה B



$\Sigma F_x = 0 =$

$F_{CB}\cos\gamma + F_{BD} + F_{BE}\cos\beta - F_{AB}\cos$

$(F_{CB} - F_{AB})\cos\gamma + F_{BE}\cos\beta + F_{BD} = 0$

$\Sigma F_y = 0 = (F_{CB} - F_{AB})\sin\gamma - F_{BE}\sin\beta$

$69.6\left(\frac{3}{3.132}\right) - 104.135\left(\frac{2}{3.124}\right) = 0$

$69.6\left(\frac{.9}{3.132}\right) + 104.135\left(\frac{2.4}{3.124}\right) = -F_{BD}$

$F_{BD} = -100\,kN$

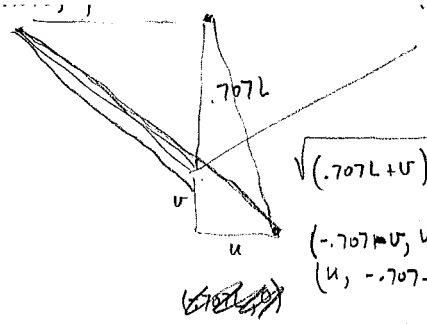
כוח פנימי (kN)	מאמץ מותר (kPa)	שטח חתך $= \frac{F}{\text{מאמץ מותר}}$ (cm^2)	
$F_{AB} = 139.201$	15 kPa	9.28	1.719
$F_{AD} = 104.132$	15 kPa	6.95	1.487
$F_{BD} = -100$	-12 kPa	8.33	1.629
$F_{CD} = -208.8$	-12 kPa	17.4	2.354
$F_{CB} = 208.8$	15 kPa	13.92	2.105
$F_{ED} = -278.403$	-12 kPa	23.20	2.718
$F_{EB} = 104.135$	15 kPa	6.95	1.487

- על פי המספרים, שטח החתך צריך להיות גדול ביותר. אבל התנאים לבעיה עדיין צריכים בקונות. אם המאמץ ניתן כ- kN/cm^2, אז המספרים יהיו יותר סבירים, בשאר שאורכי, שטחי החתך יצאו ב- cm^2 והרדיוסים יהיו ב- cm.

2-16



$\sqrt{u^2+v^2}$ =



$(u+.707L)^2 + (.707L+v)^2$

$u^2 + \frac{2}{\sqrt{2}}uL + \frac{1}{2}L^2 + \frac{1}{2}L^2 + \frac{2}{\sqrt{2}}vL + v^2$

$$\epsilon_{AC} = \frac{\sqrt{u^2+v^2+\frac{2L}{\sqrt{2}}(u+v)+L^2}-L}{L} = \frac{L\left(1+\frac{\sqrt{2}}{L}(u+v)+\frac{u^2+v^2}{L^2}\right)^{\frac{1}{2}}-L}{L} \cong \frac{\sqrt{2}}{2}\left(\frac{u+v}{L}\right)$$

$$\epsilon_{BC} = \frac{\sqrt{(u-.707L)^2+(-.707L-v)^2}}{L} = \frac{\sqrt{u^2+v^2-\frac{2}{\sqrt{2}}L(u-v)+L^2}-L}{L} = \frac{L\left(1-\frac{\sqrt{2}}{L}(u-v)+\frac{u^2+v^2}{L^2}\right)^{\frac{1}{2}}-L}{L} = -\frac{\sqrt{2}}{2}\left(\frac{u-v}{L}\right)$$

$\epsilon_x = 2x^2+5y^2$

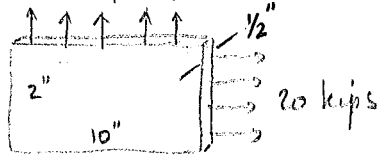
$\epsilon_y = -3x^2+2y^2$

$\gamma_{xy} = 5x^2+9y^2$

$$\frac{\partial^2\epsilon_x}{\partial y^2} + \frac{\partial^2\epsilon_y}{\partial x^2} - \frac{\partial^2\gamma_{xy}}{\partial x\partial y} \neq 0$$

$10 + (-6) - 0 \neq 0$

4-5 Popov



$\sigma_y = \frac{P_y}{A} = \frac{40{,}000}{10\times\frac{1}{2}} = 8{,}000$ psi

$\sigma_x = \frac{P_x}{A} = \frac{20{,}000}{2\times\frac{1}{2}} = 20{,}000$ psi

$\sigma_z \cong 0$ { מפני שמדובר בגוף שמידותיו והרוחק WIDTH HEIGHT THICKNESS ביחס לגודל שטח הפנים קטנות

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_y+\sigma_x) = 0 - \frac{0.25}{30\times10^6\text{psi}}(8{,}000+20{,}000) = -.25\left(\frac{28\times10^3}{30\times10^6}\right) = -.25(.933\times10^{-3}) = -.233\times10^{-3}$$

$$\epsilon_z = \frac{t_{final}-t_{init}}{t_{init}} \Rightarrow \Delta t = t_{final}-t_{init} = \epsilon_z t_{init} = -.233\times10^{-3}\left(\tfrac{1}{2}''\right) = -.1167\times10^{-3}\text{ in}$$

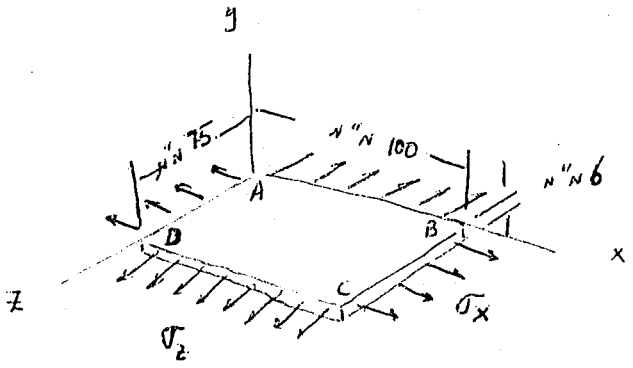
$$\frac{-.1167\times10^{-3}}{\frac{1}{2}} = \frac{\Delta t}{t_{init}} = \epsilon_z = -.233\times10^{-3} = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_y+\sigma_x) = -\frac{\nu}{E}\sigma_x \Rightarrow \sigma_x = -\frac{\epsilon_z E}{\nu} = -\frac{(-.233\times10^{-3})(30\times10^6)}{.25}$$

$$= (.933\times10^{-3})(30\times10^6) = 28000\text{ psi}$$

$P_x = \sigma_x \cdot A = 28000\times(2\times\tfrac{1}{2}) = 28$ kips

A piece of steel 2in x 10in x ½in is loaded by a distributed load along its edges. If P_x = 20 kips P_y = 40 kips what is the change in thickness due to these forces?

... alone to cause the same change in thickness $E = 30\times10^6$ psi $\nu = 0.25$



הלוח המוצג ABCD מצוי במישור
בו-3 כיוונים שונים למאמצים בכיוון
$\sigma_x = 250\,MPa$ ו- $\sigma_z = 200\,MPa$, אם הלוח נעשה
מפלדה הומוגנית בעלת $E = 200\,GPa$ ומקדם
פואסון $\nu = 0.3$, מצאו שינוי אורך של

א) צלע AB

ב) צלע BC

ג) אלכסון AC

$$\sqrt{(l_x+\Delta x)^2+(l_y+\Delta y)^2} = \sqrt{l_x^2+l_y^2+2l_x\Delta x+2l_y\Delta y+\Delta x^2+\Delta y^2}$$

$$\sqrt{l_x^2+l_y^2}\left[1+\frac{2l_x\Delta x}{l_x^2+l_y^2}\right]$$

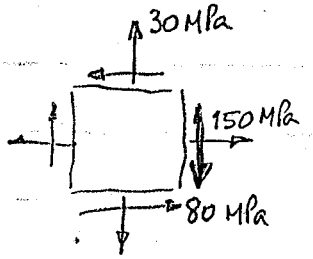
$$\bar{AC}_{new} = \sqrt{(l_x^2+l_y^2)\left[1+\frac{2l_x\Delta x}{l_x^2+l_y^2}+\frac{2l_y\Delta y}{l_x^2+l_y^2}+\frac{\Delta x^2+\Delta y^2}{l_x^2+l_y^2}\right]}$$

$$\sqrt{(l_x^2+l_y^2)}\left[1+\frac{l_x}{l_x^2+l_y^2}\Delta x+\frac{l_y}{l_x^2+l_y^2}\Delta y+\frac{1}{2}\frac{\Delta x^2+\Delta y^2}{l_x^2+l_y^2}\right]$$

$$\sqrt{l_x^2+l_y^2}+\frac{l_x}{\sqrt{}}\Delta x+\frac{l_y}{\sqrt{}}\Delta y+\frac{1}{2}\frac{\Delta x^2+\Delta y^2}{\sqrt{}}$$

$$\bar{AC}_{old} = \sqrt{l_x^2+l_y^2}$$

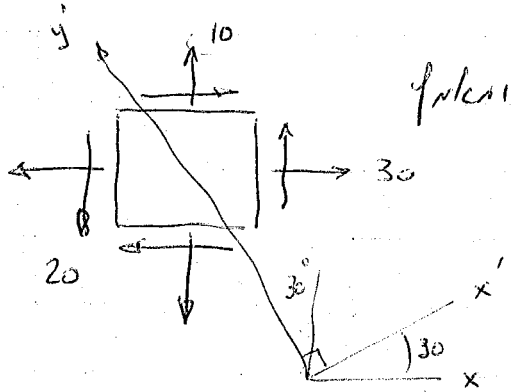
$$\bar{AC}_{new}-\bar{AC}_{old} = \Delta_{AC} = \frac{l_x}{\sqrt{}}\Delta x+\frac{l_y}{\sqrt{}}\Delta y+h.o.t.$$



- לשבב במאמצים הבאים, מצאו את המאמצים העיקריים והמאוזרים שהם פועלים עליו.

$\sigma_1, \sigma_2 = 190, -10$ MPa

$\theta = -26.6°, 63.43°$



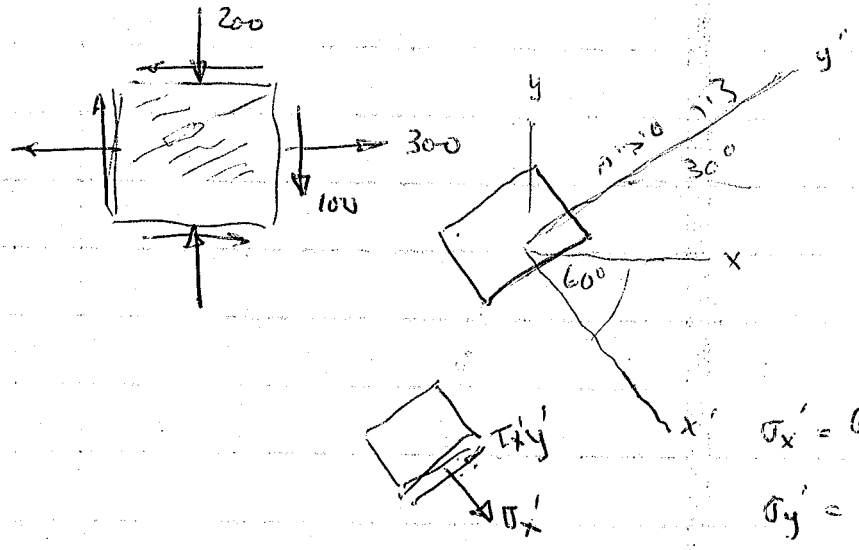
מצאו את מאמצים נורמלי ומשני הפועלים על המישור

$$\sigma_n = \sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\alpha + \tau_{xy}\sin 2\alpha$$

$$= 42.32 \text{ MPa}$$

$$\tau_{x'y'} = \tau_{xy}\cos 2\alpha - \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\alpha$$

$$= 1.34 \text{ MPa}$$



$\alpha = -60°$

$\sigma_x = 300$

$\sigma_y = -200$

$\tau = -100$

$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha$$

$$\sigma_{y'} = \frac{\sigma_x+\sigma_y}{2} - \left[\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha\right]$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\alpha + \tau_{xy}\cos 2\alpha$$

$\sigma_{x'} = 50 + 250\cos(-120) - 100\sin(-120) = 11.6$

$\tau_{x'y'} = -250\sin(-120) - 100\cos(-120) = 266.5$

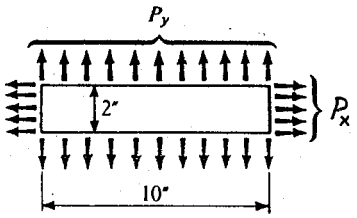
$\sigma_{y'} = 50 - \left[250\cos(-120) - 100\sin(-120)\right] = 88.4$

$\tau_{x'y'} > 150$

$\sigma_n = \sigma_{x'} = 11.6$

עיין אות תרגילים מס. 2.37, 2.38 מדף הקודם

4-5. A piece of 2-in.-by-10-in.-by-½-in. steel plate is subjected to uniformly distributed stresses along its edges (see figure). (a) If $P_x = 20$ kips and $P_y = 40$ kips, what change in thickness occurs due to the application of these forces? (b) To cause the same change in thickness as in (a) by P_x alone, what must be its magnitude? Let $E = 30 \times 10^6$ psi and $\nu = 0.25$.



PROB. 4-5

אם הרכיבים הנמצאים בנקודה שווים ל:

$\varepsilon_x = -800 \times 10^{-6}$

$\varepsilon_y = -200 \times 10^{-6}$

$\gamma_{xy} = 800 \times 10^{-6}$

a) מה הרכיבים המשניים ומה הכיוונים הראשיים?

b) אם המאפיינות האלו מאבר פלדה שמודול יאנג (YOUNG) $E = 29.5 \times 10^6$ psi ויחס פואסון (POISSON) $\nu = 0.3$, מה המשמעות הראשיים והכיוונים הראשיים שלהם

2.32. A thin triangular plate is deformed as indicated in Fig. P2.32. Compute approximately the shear strain $\gamma_{st}(P)$.

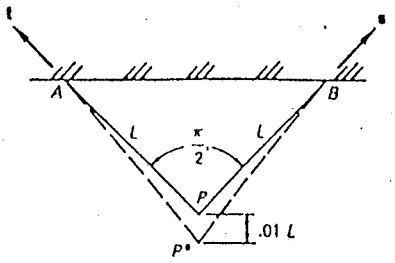


Fig. P2.32

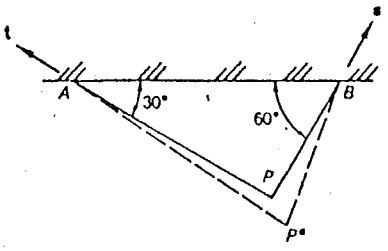


Fig. P2.34

2.34. A thin plate is in the form of a 30°–60°–90° triangle and is supported along its hypotenuse as shown in Fig. P2.34. It is subjected to the uniform strains $\epsilon_s = .004$, $\epsilon_t = .003$. Compute the approximate value of $\gamma_{st}(P)$.

2.37. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.37. It elongates 10 per cent of its original length and contracts 3 per cent of its original width. Compute approximately the shear strain γ_{st}.

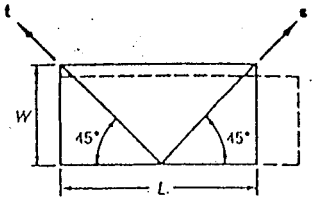


Fig. P2.37

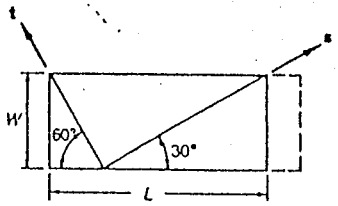


Fig. P2.38

2.38. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.38. The extensional strain along its length is .02 and the width remains unchanged. Compute approximately the shear strain γ_{st}.

2.7. A straight piece of wire PQ lies along the line $y = mx$ as shown in Fig. P2.7. The wire is strained and displaced to the line $y = nx$ in such a way that a point originally at x is displaced to $x^2/2$. Show that the extensional strain at any point along the wire is given by

$$\epsilon = \sqrt{\frac{1 + n^2}{1 + m^2}}\, x - 1$$

(where x is the original coordinate of the point).

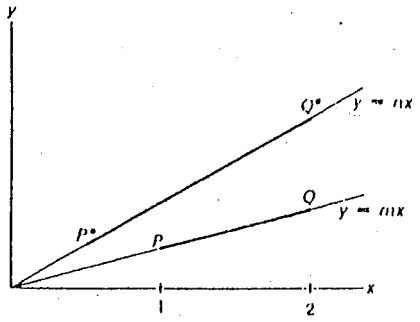


Fig. P2.7

2.16. The point C is displaced to C^* as shown in Fig. P2.16. The horizontal and vertical components of this displacement are u and v. Express the average extensional strains of AC and BC in terms of u, v and L. If u and v are small, what are the approximate expressions for these average extensional strains?

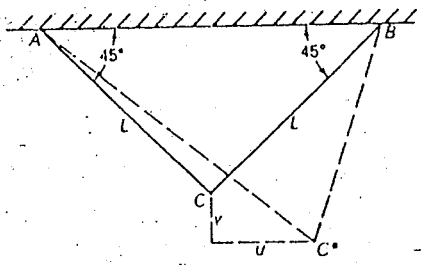


Fig. P2.16

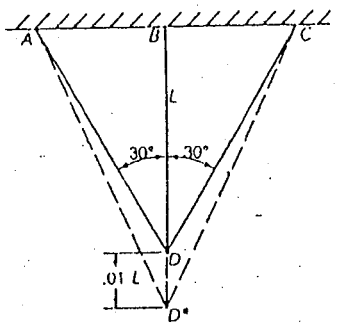


Fig. P2.17

2.17. The point D in Fig. P2.17 is displaced vertically an amount $.01L$. Obtain the approximate (first-order) average extensional strain of AD, BD, and CD.

2.21. The bar AB is rigid and is supported by two wires DB and CB as shown in Fig. P2.21. If the bar rotates through a small angle θ, determine the average extensional strain of DB and CB.

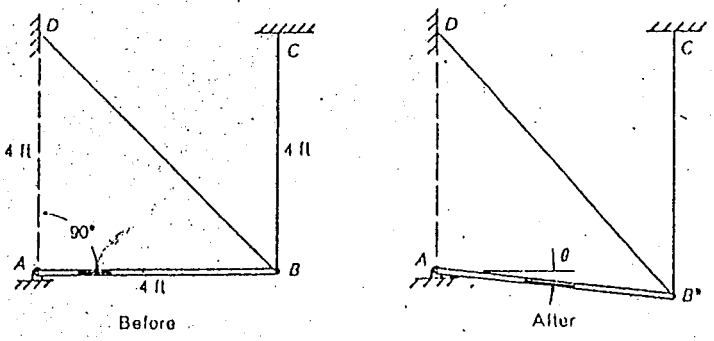


Fig. P2.21

2.56 A homogeneous plate $ABCD$ is subjected to a biaxial loading which results in the normal stresses $\sigma_x = 150$ MPa and $\sigma_z = 100$ MPa. Knowing that the plate is made of steel for which $E = 200$ GPa and $\nu = 0.30$, determine the change in length of (*a*) edge AB, (*b*) edge BC, (*c*) diagonal AC.

$AC = 125$

$d_{AC} = \frac{AD}{AC} d_{AD} + \frac{AB}{AC} d_{AB}$

$= \frac{3}{5} \cdot 20.4 + \frac{4}{5} \cdot 60$

$12.4 + 48 = 60.4$

Answer: $(a)\delta_{AB} = 60\mu m$, $(b)\delta_{BC} = 20.6\mu m$, $(c)\delta_{AC} = 60.4\mu m$

$1\mu m = 10^{-6} m$

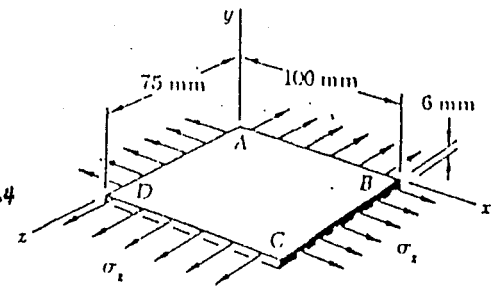


Fig. P2.56 and P2.57

2.57 The homogeneous plate $ABCD$ is subjected to a biaxial loading as shown. It is known that $\sigma_z = \sigma_0$ and that the change in length of the plate in the x direction must be zero, that is, $\epsilon_x = 0$. Denoting by E the modulus of elasticity and by ν Poisson's ration, determine (*a*) the required magnitude of σ_x (*b*) the ratio σ_0/ϵ_z.

$\sigma_y = 0$ $\sigma_z = \sigma_0$ $\epsilon_x = 0 = \frac{\sigma_x}{E} - \frac{\nu}{E}(\cancel{\sigma_y}^{0} + \cancel{\sigma_z}^{\sigma_0})$

$\Rightarrow \sigma_x = \nu\sigma_0$

$\epsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_y + \sigma_x)$

$\epsilon_z = \sigma_0(1-\nu^2)/E$ $\quad \therefore \frac{\sigma_0}{\epsilon_z} = \frac{E}{1-\nu^2}$

Answer: $(a)\sigma_x = \nu\sigma_0$, $(b)\frac{\sigma_0}{\epsilon_z} = \frac{E}{1-\nu^2}$

4.12. A steel member ($E = 30 \times 10^6$ psi, $\nu = .3$) is subjected to the stresses

$$\sigma_x = 15{,}000 \text{ psi}, \quad \sigma_y = -5000 \text{ psi}, \quad \sigma_z = 0,$$

$$\tau_{xy} = -8000 \text{ psi}, \quad \tau_{xz} = 0, \quad \tau_{yz} = 0.$$

Determine the principal strains and the principal directions.

Answer: $\epsilon_1 = 6.66\ 10^{-4}$, $\epsilon_2 = -4.34\ 10^{-4}$, $\theta = 70.7°, 160.7°$

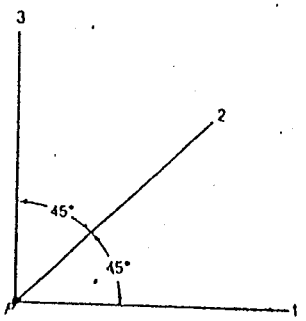
4.13. For a steel, $E = 30 \times 10^6$ psi, $\nu = \frac{1}{3}$. Determine the state of stress which corresponds to the following state of strain if the material obeys Hooke's law:

$$\epsilon_x = .001, \quad \epsilon_y = -.005, \quad \epsilon_z = 0,$$

$$\gamma_{xy} = -.0025, \quad \gamma_{yz} = -.0025, \quad \gamma_{zx} = 0.$$

Answer: $\sigma_{xy} = \sigma_{yz} = -2.81\ 10^4$, $\sigma_{zx} = -6.75\ 10^4$, $\sigma_{yy} = -20.25\ 10^4$, $\sigma_{zz} = -9\ 10^4$ psi

4.17. A state of plane stress (see Prob. 4.7) exists at the free surface of a body. Strain measurements are taken at a point P on such a free surface by the 45° strain rosette shown in Fig. P4.17. If the material obeys Hooke's law, compute the principal stresses in terms of ϵ_1, ϵ_2, ϵ_3, E, and ν.



P4.17

Answer: $\sigma_{1,2} = \frac{E}{2(1-\nu)}(\epsilon_1 + \epsilon_3) \pm \frac{E}{2(1+\nu)}\left[(\epsilon_1 - \epsilon_3)^2 + (2\epsilon_2 - \epsilon_1 - \epsilon_3)^2\right]^{1/2}$

4.18. Repeat Prob. 4.17 for the 60° strain rosette shown in Fig. P4.18.

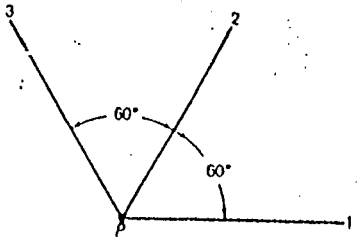


Fig. P4.18

Answer: $\sigma_{1,2} = \frac{E}{3}\left[\frac{\epsilon_1+\epsilon_2+\epsilon_3}{1-\nu} \pm \frac{2}{1+\nu}\sqrt{(\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2) - (\epsilon_1\epsilon_2 + \epsilon_1\epsilon_3 + \epsilon_2\epsilon_3)}\right]$

$\epsilon_y = -200\times10^{-6} = -.0002$

$\gamma_{xy} = 800\times10^{-6} = .0008$

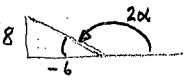
עקרונות ראשיים

$$\epsilon_{1,2} = \frac{\epsilon_x+\epsilon_y}{2} \pm \frac{1}{2}\sqrt{(\epsilon_x-\epsilon_y)^2+\gamma_{xy}^2}$$

$$= \left\{ \frac{-800-200}{2} \pm \frac{1}{2}\sqrt{(-800+200)^2+800^2} \right\}\times10^{-6}$$

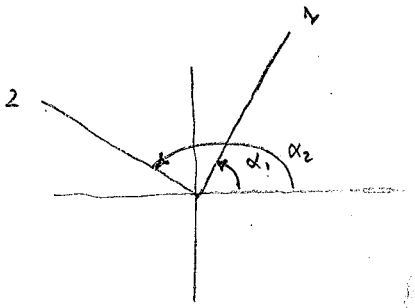
$$= (-500 \pm 500)\times10^{-6} = 0,\ -1000\times10^{-6}$$

$$\tan 2\alpha = \frac{\gamma_{xy}}{\epsilon_x-\epsilon_y} = \frac{800\times10^{-6}}{-800\times10^{-6}+200\times10^{-6}} = \frac{8}{-6} \Rightarrow$$



$2\alpha = 126.86^\circ$

$\alpha_1 = 63.43^\circ$
$\alpha_2 = 153.43^\circ$ } כיוון העקרונות



ב) לחישוב של המאמצים: כדי לחשב $\tau_{xy} = \gamma_{xy}\cdot G$

$$G = \frac{E}{2(1+\nu)} = \frac{29.5\times10^6}{2(1.3)} = \frac{29.5\times10^6}{2.5} = 11.346154\ lb/in^2$$ ברזל

$$\tau_{xy} = (800\times10^{-6})(11.8\times10^6) = 9077\ lb/in^2$$

את מניחים ש $\sigma_z = 0$ כמו בלוח, אז

$$\sigma_1 = \frac{E}{1-\nu^2}(\epsilon_1+\nu\epsilon_2)$$
$$\sigma_2 = \frac{E}{1-\nu^2}(\epsilon_2+\nu\epsilon_1)$$

} אפשר לפתור את אלה מצד שני של

$$\epsilon_1 = \frac{\sigma_1-\nu\sigma_2}{E}$$
$$\epsilon_2 = \frac{\sigma_2-\nu\sigma_1}{E}$$

$\sigma_z = 0 \Rightarrow \sigma_3 = 0$

לכן,

$$\sigma_1 = \frac{29.5\times10^6}{(1-0.3^2)}\left(0+0.3\left(-1000\times10^{-6}\right)\right)$$

$$= -9725\ lb/in^2$$

$$\sigma_2 = \frac{29.5\times10^6}{(1-0.3^2)}\left(-1000\times10^{-6}+0.3(0)\right)$$

$$= -32418\ lb/in^2$$

ג/ה $\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} = 17019231$ $\quad G = \mu = 11346154$, $\sigma_z = 0$ אבל

$$\sigma_z = 0 = \lambda e + 2\mu\epsilon_z = \lambda(\epsilon_x+\epsilon_y)+(\lambda+2\mu)\epsilon_z \quad \text{or} \quad \epsilon_z = -\frac{\lambda}{\lambda+2\mu}(\epsilon_x+\epsilon_y) =$$
$$\epsilon_z = -.42857\left(-1000\times10^{-6}\right) = 428.57\times10^{-6}$$

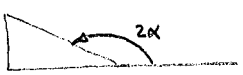
$$\sigma_x = \lambda e + 2\mu\epsilon_x = (\lambda+2\mu)\epsilon_x + \lambda(\epsilon_y+\epsilon_z) = 39711539\left(-800\times10^{-6}\right)+17019231\left(228.57\times10^{-6}\right)$$
$$= -27879\ lb/in^2 \qquad (-33173)$$

$$\sigma_y = \lambda e + 2\mu\epsilon_y = (\lambda+2\mu)\epsilon_y + \lambda(\epsilon_x+\epsilon_z) = 39711539\left(-200\times10^{-6}\right)+17019231\left(-371.43\times10^{-6}\right)$$
$$= -14264\ lb/in^2 \qquad (-21558)$$

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = \frac{-27879-14264}{2} \pm \sqrt{\left(\frac{-27879+14264}{2}\right)^2+(9077)^2}$$

$$= -21071.5 \pm 11346.1 = -9725,\ -32418\ lb/in^2$$
$$(-17019) \quad (-39712)$$

$$\tan 2\alpha = \frac{2\tau_{xy}}{} = -1.333 \Rightarrow$$



$2\alpha = 126.86^\circ$

1. The stress distribution on the rectangular cross-section shown in Fig. 1 is given by $\sigma_{xx}=1000y-500z+800$ kPa, $\sigma_{xy}=200z$ kPa, $\sigma_{xz}=0$. What is the net internal force system on this cross-section ?

Answer : $F= 1920$ N, $V_y=V_z=0$
$M_y=-1600$ N-cm., $M_z=-7200$ N-cm., $T=-640$ N-cm.

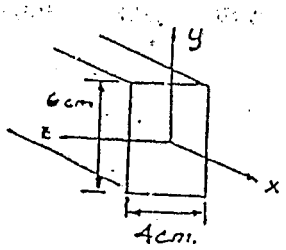


Fig.

2. Suppose the stress distribution on a cross-section of the circular cylinder of Fig. 2 is given by $\sigma_{xx}=\sigma_{xr}=0$, $\sigma_{x\theta}=k\sqrt{r}$, where k is unknown. What is the value of k in this case?

Answer : $k=7T_0/4\pi R^{7/2}$

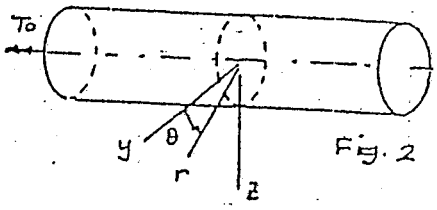


Fig. 2

6.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium equations of that element, as was done in the derivations presented in Sec. 6.2.

6.6 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

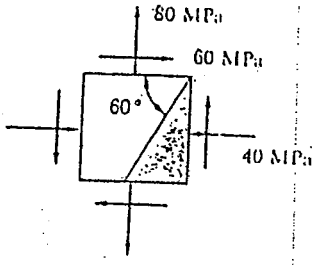


Fig. P6.4

*6.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

*6.14 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 40° counterclockwise, (*b*) 15° clockwise.

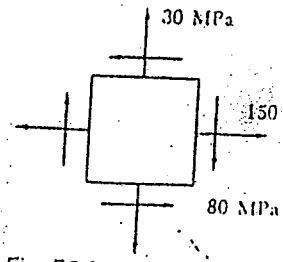
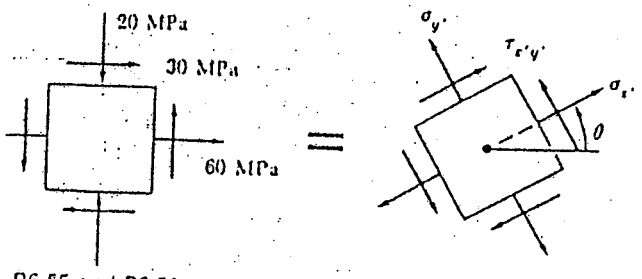


Fig. P6.6 and P6.10

*6.56 For the state of plane stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than +65 MPa.



P6.55 and P6.56

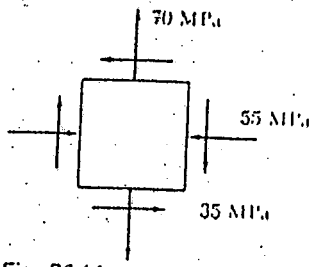


Fig. P6.14

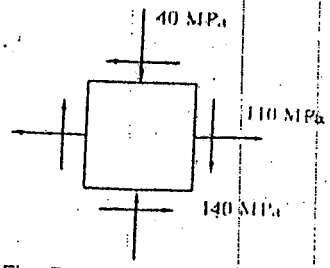


Fig. P6.8 and P6.12

25
/2

2.1. A straight piece of wire PQ lies along the line $y = mx$ as shown in Fig. P2.7. The wire is strained and displaced to the line $y = nx$ in such a way that a point originally at x is displaced to $x^2/2$. Show that the extensional strain at any point along the wire is given by

$$\epsilon = \sqrt{\frac{1 + n^2}{1 + m^2}}\, x - 1$$

(where x is the original coordinate of the point).

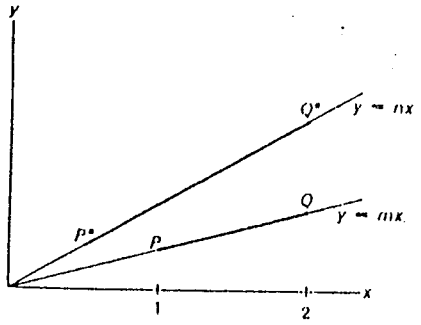


Fig. P2.7

2.16. The point C is displaced to C^* as shown in Fig. P2.16. The horizontal and vertical components of this displacement are u and v. Express the average extensional strains of AC and BC in terms of u, v and L. If u and v are small, what are the approximate expressions for these average extensional strains?

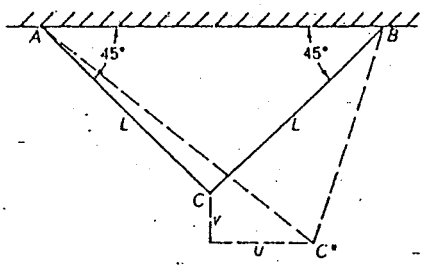


Fig. P2.16

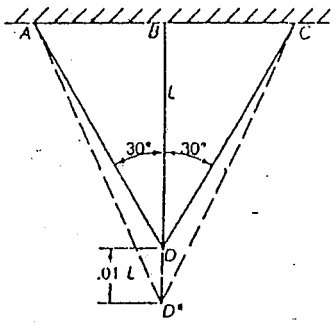


Fig. P2.17

2.17. The point D in Fig. P2.17 is displaced vertically an amount $.01L$. Obtain the approximate (first-order) average extensional strain of AD, BD, and CD.

2.21. The bar AB is rigid and is supported by two wires DB and CB as shown in Fig. P2.21. If the bar rotates through a small angle θ, determine the average extensional strain of DB and CB.

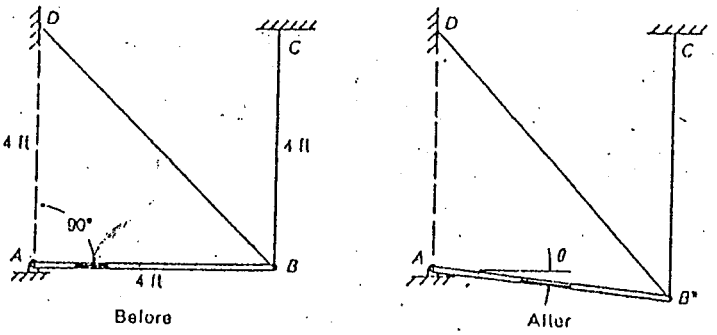


Fig. P2.21

2.32. A thin triangular plate is deformed as indicated in Fig. P2.32. Compute approximately the shear strain $\gamma_{st}(P)$.

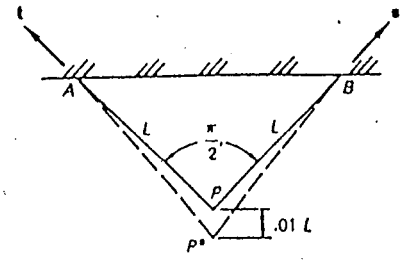


Fig. P2.32

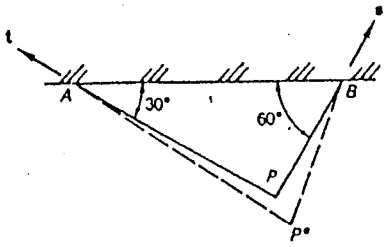


Fig. P2.34

2.34. A thin plate is in the form of a 30°–60°–90° triangle and is supported along its hypotenuse as shown in Fig. P2.34. It is subjected to the uniform strains $\epsilon_s = .004$, $\epsilon_t = .003$. Compute the approximate value of $\gamma_{st}(P)$.

2.37. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.37. It elongates 10 per cent of its original length and contracts 3 per cent of its original width. Compute approximately the shear strain γ_{st}.

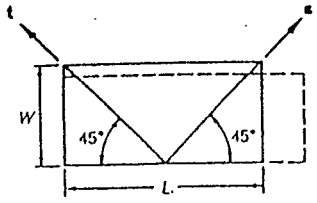


Fig. P2.37

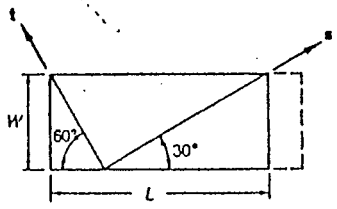
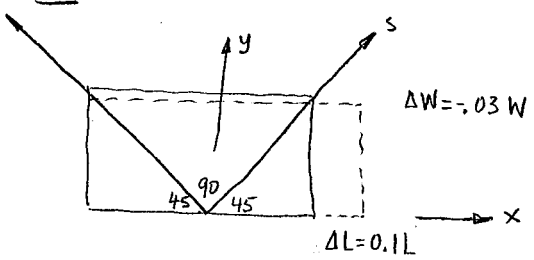


Fig. P2.38

2.38. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.38. The extensional strain along its length is .02 and the width remains unchanged. Compute approximately the shear strain γ_{st}.

2.37



לפי הנתונים $\varepsilon_x = \frac{\Delta L}{L} = 0.1$

$\varepsilon_y = \frac{\Delta W}{W} = -0.03$

כי מתלים במלבן ושואלים מהלבן $\gamma_{xy} = 0$

$\gamma_{st} = -(\varepsilon_x - \varepsilon_y)\sin 2\alpha + \gamma_{xy}\cos 2\alpha$

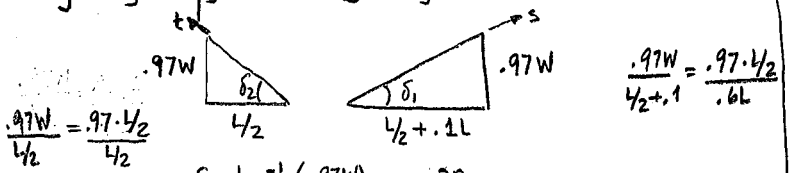
כאן $\alpha = 45°$ זווית s-ציר x

$= -(0.1 - (-0.03))\sin(2\cdot 45) + 2\cdot\cos 0 =$

$= -0.13$

שיטת אחרון: בהתחלה, כאן s ו-t הבוויות $90°$ ו- $W = W/2$

אחרי ההתארכות ב-x והתקצרות ב-y



$\frac{.97W}{L/2} = \frac{.97\cdot L/2}{L/2}$ $\qquad$ $\frac{.97W}{L/2+.1} = \frac{.97\cdot L/2}{.97}$

$\delta_2 = \tan^{-1}\left(\frac{.97W}{L/2}\right) = 44.13°$

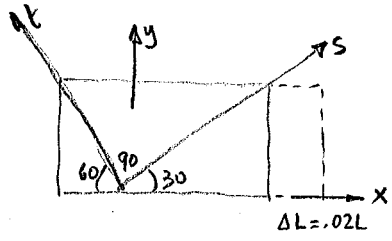
$\delta_1 = \tan^{-1}\left(\frac{.97W}{L/2+.1L}\right) = 38.95°$

$\delta_1 + \delta_2 = 83.08°$

זווית בין ציר s ל-t $180 - 83.08 = 96.92°$

γ_{st} הינו שלילי ו- $-90°$ לכן, $\gamma_{st} = (90° - 96.92°)\frac{\pi}{180} = -.121$

2.38



לפי הנתונים $\varepsilon_x = \frac{\Delta L}{L} = 0.02$

$\varepsilon_y = \frac{\Delta W}{W} = 0$

מתלים ושואלים במלבן $\gamma_{xy} = 0$

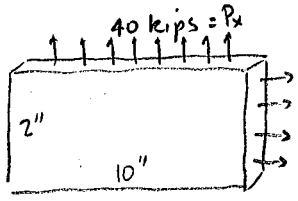
$\gamma_{st} = -(\varepsilon_x - \varepsilon_y)\sin 2\alpha + \gamma_{xy}\cos 2\alpha$

כאן ציר x ל-ציר s $\alpha = 30°$

$= -(0.02 - 0)\sin(2\cdot 30) + 0\cdot\cos 2\cdot 30 =$

$= -0.02\,(.866) = -.01732$

4.5



$E = 30\times10^6$ lb/in²

$\nu = 0.25$

$\sigma_x = \frac{P_x}{A} = \frac{20{,}000}{2\times(0.5)} = 20{,}000$ lb/in²

$\sigma_y = \frac{P_y}{A} = \frac{40{,}000}{10\times(0.5)} = 8{,}000$ lb/in²

מפני שמעוב' קיים מהאומה והרוחב ואין כוח שפועל בכיוון z $\Rightarrow \sigma_z = 0$

$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y) = 0 - \frac{0.25}{30\times10^6}(8000 + 20{,}000) = -0.23\bar{3}\times10^{-3}$

שינוי העובי הוא

$\Delta t = \varepsilon_z \cdot t_{\text{התחלה}} \Leftarrow \varepsilon_z = \Delta t / t_{\text{התחלה}}$ $\quad t_{\text{התחלה}} - t_{\text{סופי}}$

$\Delta t = -0.23\bar{3}\times10^{-3}(0.5) =$

$\Delta t = -0.1\bar{6}\times10^{-3}$ in

לאותו שינוי, מה צריך להיות הכוח P_x אם אין P_y (כלומר P_x פועל לבד)?

Δt הנ"ל נקבע ε_z הנ"ל. אבל מסעיף הקודם $\sigma_z = 0$ כמו קודם וגם $\sigma_y = 0$. לכן,

$\varepsilon_z = -0.23\bar{3}\times10^{-3} = -\frac{\nu}{E}\sigma_x$

$\sigma_x = \frac{-0.23\bar{3}\times10^{-3}(30\times10^6)}{-(0.25)} = 28000$ lb/in²

$= 28$ kips

המכללה האקדמית יהודה ושומרון

תורת החוזק 1

תרגיל מס' 5

2.56 A homogeneous plate $ABCD$ is subjected to a biaxial loading which results in the normal stresses $\sigma_x = 150$ MPa and $\sigma_z = 100$ MPa. Knowing that the plate is made of steel for which $E = 200$ GPa and $\nu = 0.30$, determine the change in length of (*a*) edge AB, (*b*) edge BC, (*c*) diagonal AC.

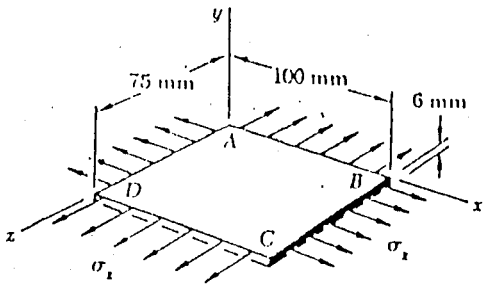


Fig. P2.56 and P2.57

Answer: $(a)\delta_{AB} = 60\mu m, \quad (b)\delta_{BC} = 20.6\mu m, \quad (c)\delta_{AC} = 60.4\mu m$

$1\,\mu m = 10^{-6}\,m$

2.57 The homogeneous plate $ABCD$ is subjected to a biaxial loading as shown. It is known that $\sigma_z = \sigma_0$ and that the change in length of the plate in the x direction must be zero, that is, $\epsilon_x = 0$. Denoting by E the modulus of elasticity and by ν Poisson's ration, determine (*a*) the required magnitude of σ_x (*b*) the ratio σ_0/ϵ_z.

Answer: $(a)\sigma_x = \nu\sigma_0, \quad (b)\frac{\sigma_0}{\epsilon_z} = \frac{E}{1-\nu^2}$

4.12. A steel member ($E = 30 \times 10^6$ psi, $\nu = .3$) is subjected to the stresses

$$\sigma_x = 15{,}000 \text{ psi}, \quad \sigma_y = -5000 \text{ psi}, \quad \sigma_z = 0,$$

$$\tau_{xy} = -8000 \text{ psi}, \quad \tau_{xz} = 0, \quad \tau_{yz} = 0.$$

Determine the principal strains and the principal directions.

Answer: $\epsilon_1 = 6.66\ 10^{-4}, \quad \epsilon_2 = -4.34\ 10^{-4}, \quad \theta = 70.7°,\ 160.7°$

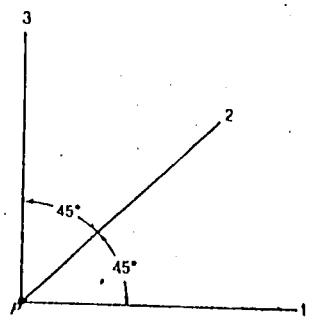
4.13. For a steel, $E = 30 \times 10^6$ psi, $\nu = \frac{1}{3}$. Determine the state of stress which corresponds to the following state of strain if the material obeys Hooke's law:

$$\epsilon_x = .001, \quad \epsilon_y = -.005, \quad \epsilon_z = 0,$$

$$\gamma_{xy} = -.0025, \quad \gamma_{yz} = -.0025, \quad \gamma_{zx} = 0.$$

Answer: $\sigma_{xy} = \sigma_{yz} = -2.81\ 10^4, \quad \sigma_{zx} = -6.75\ 10^4, \quad \sigma_{yy} = -20.25\ 10^4, \quad \sigma_{zz} = -9\ 10^4$ psi

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P 4.17

Answer: $\sigma_{1,2} = \frac{E}{2(1-\nu)}(\epsilon_1 + \epsilon_3) \pm \frac{E}{2(1+\nu)}\left[(\epsilon_1 - \epsilon_3)^2 + (2\epsilon_2 - \epsilon_1 - \epsilon_3)^2\right]^{1/2}$

4.18. Repeat Prob. 4.17 for the 60° strain rosette shown in Fig. P4.18.

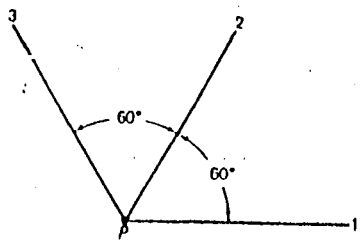


Fig. P4.18

Answer: $\sigma_{1,2} = \frac{E}{3}\left[\frac{\epsilon_1+\epsilon_2+\epsilon_3}{1-\nu} \pm \frac{2}{1+\nu}\sqrt{(\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2) - (\epsilon_1\epsilon_2 + \epsilon_1\epsilon_3 + \epsilon_2\epsilon_3)}\right]$

המכללה האקדמית יהודה ושומרון

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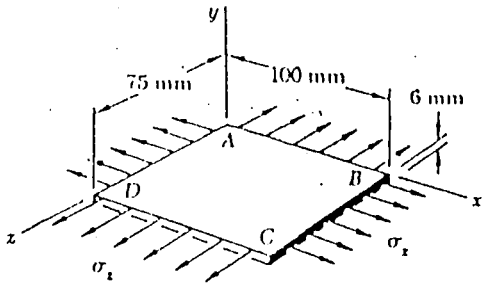


Fig. P2.56 and P2.57

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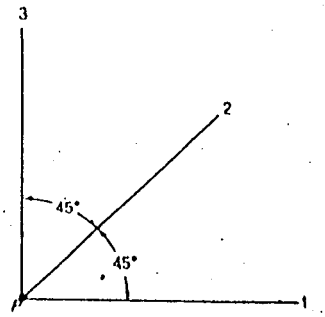
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Answer: $\sigma_{1,2} = \frac{E}{2(1-\nu)}(\epsilon_1 + \epsilon_3) \pm \frac{E}{2(1+\nu)}\left[(\epsilon_1 - \epsilon_3)^2 + (2\epsilon_2 - \epsilon_1 - \epsilon_3)^2\right]^{1/2}$

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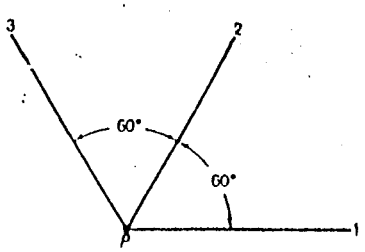


Fig. P4.18

Answer: $\sigma_{1,2} = \frac{E}{3}\left[\frac{\epsilon_1+\epsilon_2+\epsilon_3}{1-\nu} \pm \frac{2}{1+\nu}\sqrt{(\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2) - (\epsilon_1\epsilon_2 + \epsilon_1\epsilon_3 + \epsilon_2\epsilon_3)}\right]$

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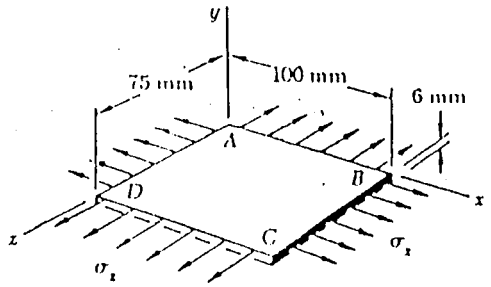


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Answer: $\epsilon_1 = 6.66\ 10^{-4}, \quad \epsilon_2 = -4.34\ 10^{-4}, \quad \theta = 70.7°, 160.7°$

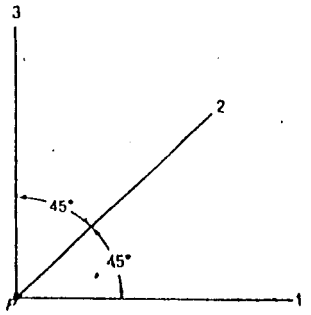
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P4.17

Answer: $\sigma_{1,2} = \frac{E}{2(1-\nu)}(\epsilon_1 + \epsilon_3) \pm \frac{E}{2(1+\nu)}\left[(\epsilon_1 - \epsilon_3)^2 + (2\epsilon_2 - \epsilon_1 - \epsilon_3)^2\right]^{1/2}$

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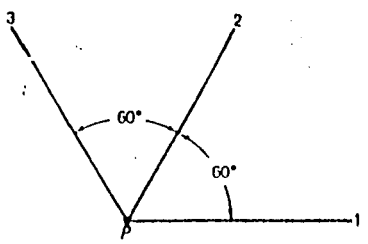


Fig. P4.18

Answer: $\sigma_{1,2} = \frac{E}{3}\left[\frac{\epsilon_1+\epsilon_2+\epsilon_3}{1-\nu} \pm \frac{2}{1+\nu}\sqrt{(\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2) - (\epsilon_1\epsilon_2 + \epsilon_1\epsilon_3 + \epsilon_2\epsilon_3)}\right]$

המכללה האקדמית יהודה ושומרון

תורת החוזק 1

תרגיל מס' 5

2.56 A homogeneous plate $ABCD$ is subjected to a biaxial loading which results in the normal stresses $\sigma_x = 150$ MPa and $\sigma_z = 100$ MPa. Knowing that the plate is made of steel for which $E = 200$ GPa and $\nu = 0.30$, determine the change in length of (*a*) edge AB, (*b*) edge BC, (*c*) diagonal AC.

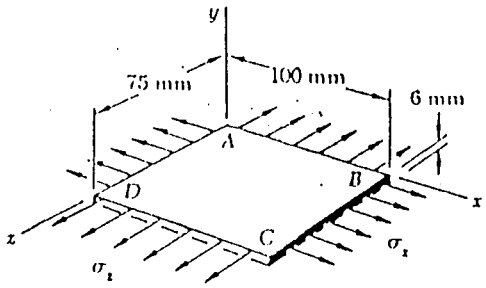


Fig. P2.56 and P2.57

Answer: $(a)\delta_{AB} = 60\mu m$, $(b)\delta_{BC} = 20.6\mu m$, $(c)\delta_{AC} = 60.4\mu m$

$1\mu m = 10^{-6} m$

2.57 The homogeneous plate $ABCD$ is subjected to a biaxial loading as shown. It is known that $\sigma_z = \sigma_0$ and that the change in length of the plate in the x direction must be zero, that is, $\varepsilon_x = 0$. Denoting by E the modulus of elasticity and by ν Poisson's ration, determine (*a*) the required magnitude of σ_x (*b*) the ratio σ_0/ε_z

Answer: $(a)\sigma_x = \nu\sigma_0$, $(b)\frac{\sigma_0}{\varepsilon_z} = \frac{E}{1-\nu^2}$

4.12. A steel member ($E = 30 \times 10^6$ psi, $\nu = .3$) is subjected to the stresses

$$\sigma_x = 15{,}000 \text{ psi}, \quad \sigma_y = -5000 \text{ psi}, \quad \sigma_z = 0,$$

$$\tau_{xy} = -8000 \text{ psi}, \quad \tau_{xz} = 0, \quad \tau_{yz} = 0.$$

Determine the principal strains and the principal directions.

Answer: $\varepsilon_1 = 6.66\ 10^{-4}$, $\varepsilon_2 = -4.34\ 10^{-4}$, $\theta = 70.7°, 160.7°$

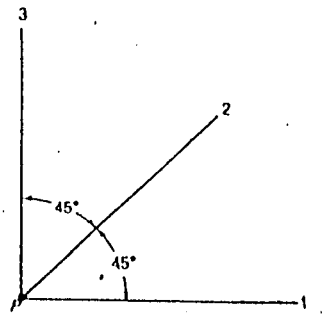
4.13. For a steel, $E = 30 \times 10^6$ psi, $\nu = \frac{1}{3}$. Determine the state of stress which corresponds to the following state of strain if the material obeys Hooke's law:

$$\epsilon_x = .001, \quad \epsilon_y = -.005, \quad \epsilon_z = 0,$$

$$\gamma_{xy} = -.0025, \quad \gamma_{yz} = -.0025, \quad \gamma_{zx} = 0.$$

Answer: $\sigma_{xy} = \sigma_{yz} = -2.81\ 10^4$, $\sigma_{zx} = -6.75\ 10^4$, $\sigma_{yy} = -20.25\ 10^4$, $\sigma_{zz} = -9\ 10^4$ psi

4.17. A state of plane stress (see Prob. 4.7) exists at the free surface of a body. Strain measurements are taken at a point P on such a free surface by the 45° strain rosette shown in Fig. P4.17. If the material obeys Hooke's law, compute the principal stresses in terms of ϵ_1, ϵ_2, ϵ_3, E, and ν.



P 4.17

Answer: $\sigma_{1,2} = \frac{E}{2(1-\nu)}(\varepsilon_1 + \varepsilon_3) \pm \frac{E}{2(1+\nu)}\left[(\varepsilon_1 - \varepsilon_3)^2 + (2\varepsilon_2 - \varepsilon_1 - \varepsilon_3)^2\right]^{1/2}$

4.18. Repeat Prob. 4.17 for the 60° strain rosette shown in Fig. P4.18.

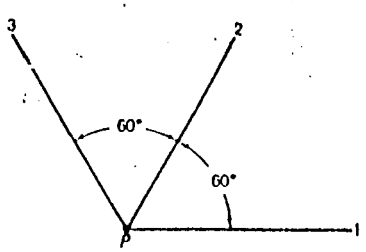


Fig. P4.18

Answer: $\sigma_{1,2} = \frac{E}{3}\left[\frac{\epsilon_1+\epsilon_2+\epsilon_3}{1-\nu} \pm \frac{2}{1+\nu}\sqrt{(\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2) - (\epsilon_1\epsilon_2 + \epsilon_1\epsilon_3 + \epsilon_2\epsilon_3)}\right]$

המכללה האקדמית יהודה ושומרון

תורת החוזק 1

תרגיל מס' 5

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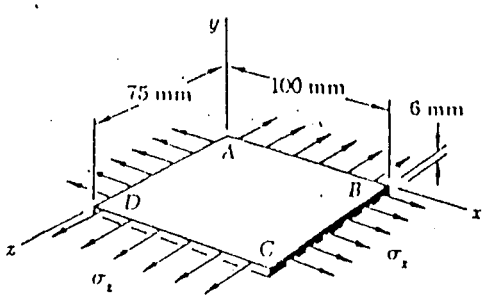


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$$\tau_{xy} = -8000 \text{ psi}, \quad \tau_{xz} = 0, \quad \tau_{yz} = 0.$$

Determine the principal strains and the principal directions.

Answer: $\epsilon_1 = 6.66\ 10^{-4}, \quad \epsilon_2 = -4.34\ 10^{-4}, \quad \theta = 70.7°,\ 160.7°$

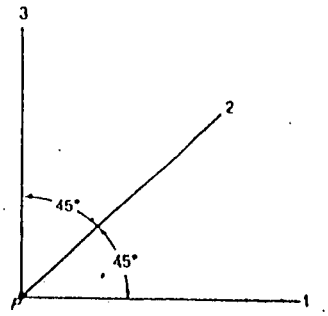
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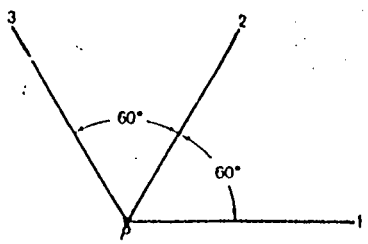
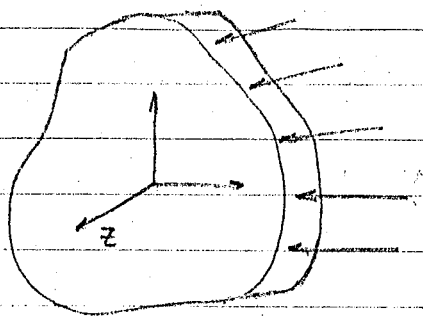


Fig. P4.18

Answer: $\sigma_{1,2} = \frac{E}{3}\left[\frac{\epsilon_1+\epsilon_2+\epsilon_3}{1-\nu} \pm \frac{2}{1+\nu}\sqrt{(\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2) - (\epsilon_1\epsilon_2 + \epsilon_1\epsilon_3 + \epsilon_2\epsilon_3)}\right]$

Plane Stress — כוחות פועלים במישור x-y בלבד
תאוי בכיר z קטן מכל המידות ב-x ו-y



$\sigma_z = 0$

$\tau_{xz} = 0$

$\tau_{yz} = 0$

$$\epsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y)$$

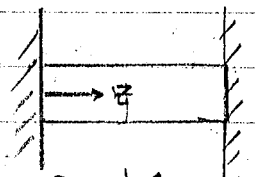
from this we get: $\epsilon_x = \frac{\sigma_x - \nu\sigma_y}{E}$; $\epsilon_y = \frac{\sigma_y - \nu\sigma_x}{E}$; $\gamma_{xy} = \frac{\tau_{xy}}{G}$

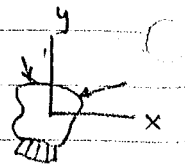
10/20/70

Plane Strain - Assume $\Sigma F = 0$, $\Sigma \bar{M} = 0$

$w = 0$ $\quad \epsilon_z = 0$, $\gamma_{yz} = \gamma_{zx} = 0$

$\tau_{xz} = \tau_{zx} = 0$





$$\sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\epsilon_x = \frac{\partial u}{\partial x}, \quad \epsilon_y = \frac{\partial v}{\partial y}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + Y = 0$$

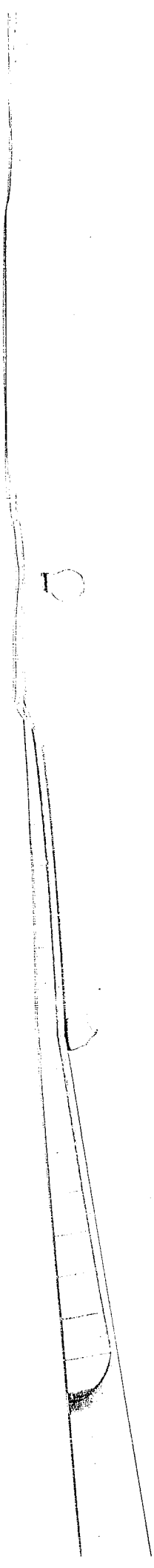
$$\epsilon_x = \frac{1+\nu}{E}\left[(1-\nu)\sigma_x - \nu\sigma_y\right]$$

$$\epsilon_y = \frac{1+\nu}{E}\left[(1-\nu)\sigma_y - \nu\sigma_x\right] \qquad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\epsilon_x + \nu\epsilon_y\right]$$

$$\tau_{xy} = G\,\gamma_{xy}$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\epsilon_y + \nu\epsilon_x\right]$$

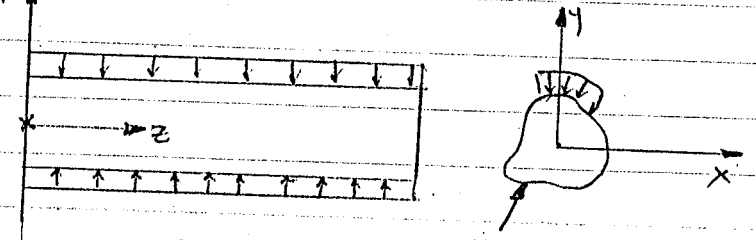


Also

$$\frac{\partial U_{co}}{\partial \sigma_x} = \epsilon_x, \quad \frac{\partial U_{co}}{\partial \sigma_y} = \epsilon_y, \quad \frac{\partial U_{co}}{\partial \sigma_z} = \epsilon_z, \quad \frac{\partial U_{co}}{\partial \tau_{xy}} = \gamma_{xy}, \quad \frac{\partial U_{co}}{\partial \tau_{yz}} = \gamma_{yz}, \quad \frac{\partial U_{co}}{\partial \tau_{zx}} = \gamma_{zx}$$

Plane Strain

עיבור מישורי



Load $\neq f(z)$

Length $\gg$ cross sectional length.

$$w = 0 \quad \epsilon_z = 0, \quad \gamma_{zx} = \gamma_{yz} = 0$$

If $\epsilon_z = 0$ $\quad \sigma_z = \nu(\sigma_x + \sigma_y)$

$$\text{but } \epsilon_x = \frac{(1+\nu)\sigma_x - (\nu+\nu^2)(\sigma_x+\sigma_y)}{E}$$

$$= \frac{(1+\nu)}{E}\left[(1-\nu)\sigma_x - \nu\sigma_y\right]$$

$$\text{similarly } \epsilon_y = \frac{(1+\nu)}{E}\left[(1-\nu)\sigma_y - \nu\sigma_x\right]$$

$$\gamma_{xy} = \tau_{xy}/G$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + Y = 0$$

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\epsilon_x + \nu\epsilon_y\right]$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\epsilon_y + \nu\epsilon_x\right]$$

$$\tau_{xy} = G \cdot \gamma_{xy}$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y - \frac{\nu}{E}\sigma_z$$

$$\varepsilon_y = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_z$$

$$\varepsilon_z = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \frac{\sigma_z}{E}$$

$$\varepsilon_{xy} = \frac{1}{2G}\tau_{xy} \qquad \gamma_{xy} = \frac{1}{G}\tau_{xy} \qquad \gamma_{xz} = \frac{\tau_{xz}}{G}, \quad \gamma_{yz} = \frac{\tau_{yz}}{G}$$

E – modulus of elasticity – Young's modulus.

Mild steel $E \sim 29 \times 10^6$ psi $= 29 \times 10^3$ ksi

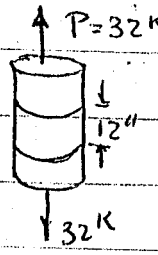
G – shearing modulus of elasticity –

Mild steel $G \sim 10\text{-}12 \times 10^6$ psi $= 10\text{-}12 \times 10^3$ ksi

ν – Poisson's ratio

0.1 – 0.5, 0.25 – 0.33 More practical

Problem:



$2\frac{1}{4}$" ϕ

$\Delta l = .00938$

$\Delta d = -.000585$

Find E & ν

$$E = \frac{\sigma_x}{\varepsilon_x}$$

$$\sigma_x = \frac{32}{\frac{\pi}{4}(2.25)^2}$$

$$\varepsilon_x = \frac{.00938}{12"}$$

$$\nu = -\varepsilon_4/\varepsilon_x \;;\quad \varepsilon_4 = \frac{-.000585}{2.25}$$

$$\nu = \frac{+\frac{.000585}{2.25}}{\frac{.00938}{12}} =$$

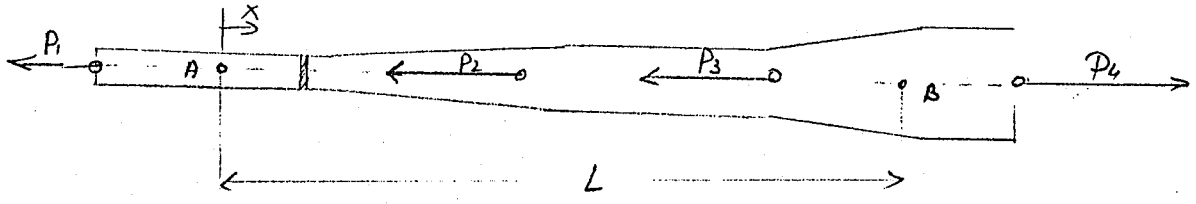
2in cube $\sigma_x = \sigma_y = \sigma_z = 30{,}000$ psi $E = 30 \times 10^6$ psi

$\nu = 1/4$

לעתים מרבות התאורים ההתנהגות בלתי נקוה קרוה לצורת שבמציאות ומוביל ויוצר שגיאה בחישוביהם. ישנם תוארים שכשר לאיזמים שונה מאוד בין לתוכנה למציאות בין במקרה יבוקה.

2.8 התארכות מוטות מהירמים בגורת

נסתכל על שני מוטות 1 ו- 2 בתוספת למוכל מוטות מתוך של התומר מהוטר תשוב ההצבה (התארכות) שמתרי מוט המירמים מוכרות. נסתכל במוט כלשהו כמשוואה

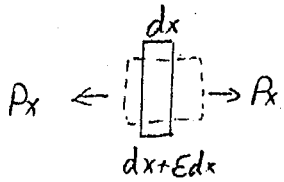


המירמים במצורת P_1, P_2, P_3, P_4. בקצה הירמים היה מוחק $AB=L$ במצב שלתנה המוחק יקבל ערכים לה. ההתארכות לוות הנה סכו ההתארכויות של כל הפולאנטים בין A ל- B.

נקח פולאנט אחד במרחק x מעל פולאנט לה מופעל כוח P_x על שטח חתך A_x לכן

$$\sigma = \frac{P_x}{A_x} \qquad \varepsilon = \frac{\sigma_x}{E}$$

$$\varepsilon = \frac{P_x}{EA_x}$$



כולם ולוקח המוט ההתחלתי הוא dx ולכן אורכו הסופי והוא

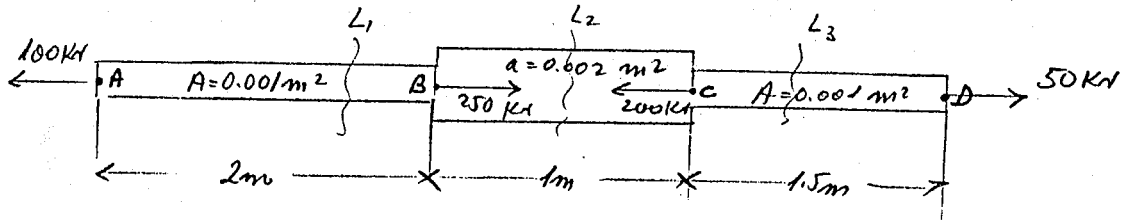
$$dx + \varepsilon dx$$

ושינוי אורכו הוא εdx

$$d\Delta = \varepsilon\, dx = \frac{P_x}{A_x E} dx$$

$$\Delta = \int_A^B \frac{P_x}{A_x E} dx$$ (2.8) ההתארכות

תשם את לבך שהמוט AD עשוי מחומר בעל מודול אלסטיות $E = 200\ GPa$ המתואר בשרטוט



כיון שעומס שונה ושטחי חתך קבוע בכל אזור ואזור.

קטע A-B

חותכים בחתך ב L_1 ומקבלים כי $P_1 = 100\ KN$

$A_1 = 0.001\ m^2$

קטע B-C

חותכים ב L_2 ומקבלים $P_2 = -150\ KN$

$A_2 = 0.002\ m^2$

קטע D-C

חותכים L_3 ומקבלים $P_3 = 50\ KN$

$A_3 = 0.0001\ KN$

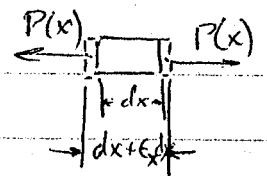
לפי הנוסחה הכללית

$$\Delta = \int_A^D \frac{P_x}{EA_x}\,dx = \frac{1}{E}\left\{ \int_A^B \frac{P_1}{A_1}\,dx + \int_B^C \frac{P_2}{A_2}\,dx + \int_C^D \frac{P_3}{A_3}\,dx \right\} =$$

$$\frac{1}{E}\left\{ \frac{P_1}{A_1}L_1 - \frac{P_2}{A_2}L_2 + \frac{P_3}{A_3}L_3 \right\} = \sum \frac{PL}{EA} = 1\ mm.$$

2.7 מקרים מיוחדים

מכמות ההתארכות בקטע המוצא בכיון אחיד לוח יתן מושגים
ואת מידותיהם בהותו כיון כי אם שני הכוחות הנקבים
לכיון צהה למשל אחד בעל חתך אלכסוני המצוא בשאיות ה
מתוארך ואז צמצמת שטח חתכו קטן. על הצאם בהתאם



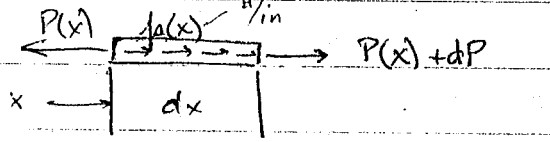
displacement or measure of elongations.

$$du = \epsilon_x dx$$

$$u = \int du = \int \epsilon_x dx = \frac{1}{E}\int \sigma_x dx = \frac{1}{E}\int \frac{P(x)}{A(x)} dx$$

$$\boxed{\frac{du}{dx} = \frac{1}{E}\frac{P(x)}{A(x)}}$$

distributed load



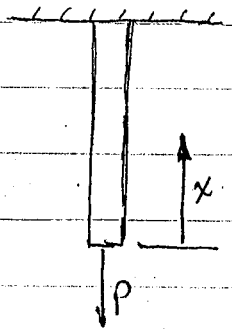
$$\sum F_x = P(x) + dP - P(x) + p_x dx = 0$$

$$\frac{dP}{dx} = -p_x$$

$$\frac{d}{dx}\left(\frac{du}{dx}\right) = \frac{1}{E}\frac{d\left(\frac{P(x)}{A(x)}\right)}{dx}$$ if A is a constant

$$\frac{d^2u}{dx^2} = \frac{1}{EA}\frac{dP}{dx} = -\frac{p_x}{EA}$$

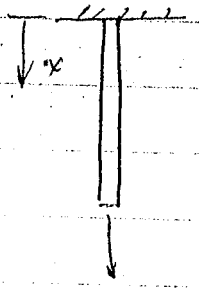
$$\boxed{\frac{d^2u}{dx^2} = -\frac{p_x}{EA}}$$



$$\epsilon_x = \frac{\Delta l}{l} \qquad \sigma_x = P/A \qquad \frac{\sigma_x}{E} = \frac{P}{AE} \qquad \Delta l = \frac{Pl}{AE}$$

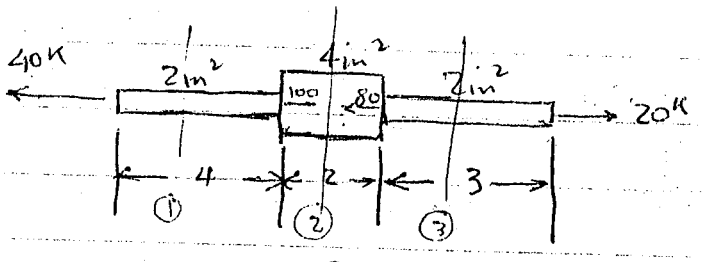
$$u = \frac{1}{E}\int \frac{P}{A} dx = \frac{Px}{AE} + C_1 \qquad u=0 \quad x=0 \Rightarrow C_1 = 0$$

$$u = \frac{Px}{AE} \qquad u_L = \frac{PL}{AE}$$



$$u(L) = \frac{PL}{AE}$$

Problem 2



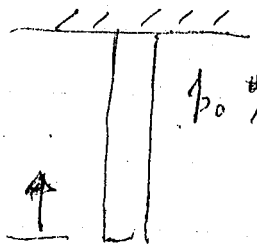
$$u = \sum \frac{PL}{AE}$$

$$u_1 = \frac{40K \times 48in}{2in^2 \times 30\times10^3 K} \qquad u_2 = -\frac{60K \times 24in}{4in^2 \times 30\times10^3}$$

$$u_3 = \frac{20\times36}{2in^2\times30\times10^3 K}$$

$$u_{T.} = 32\times10^{-3} - 12\times10^{-3} + 12\times10^{-3} = 32\times10^{-3}.$$

Problem 3



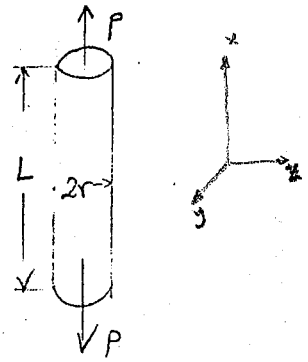
$$p_x = -p_0 \qquad \frac{du}{dx} = -\int \frac{p_x dx}{AE} dx = \frac{P_0 x}{AE} + C_1$$

$$\frac{du}{dx} = \frac{P(x)}{AE} \qquad \frac{du}{dx} = \frac{P_0 x}{AE} \quad \text{since } C_1 = 0$$

והוא מתקבל הוא שטח חתכו קבל. לעומקט לה קלהוא ולעקט פוזון. מסתבר כי עבור חומר איזוטרופי היחס בין הקבוע הרוחבי לקבוע האורכי תלוי במידה רבה במידת הדחיסות של החומר. יחס זה נקרא יחס פואסון ומסומן באות היוונית ν (נו) ונקרא: עבור חומרים בטווח של ν 1- ל $\frac{1}{2}$ שמתקיים:

$$\nu = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}} = -\frac{\varepsilon_{zz}}{\varepsilon_{xx}} \qquad (5.8)$$

ν הוא מקדם פואסון. ערכו האידיאלי 0.5 מספרים ולפונקציה: בהלוים $\nu=0$, פלדה 0.3-0.25, ולאומיניום 0.33, פלסטיקים 0.4 ~ לגומי $\nu=0.5$.



דוגמא

חשב את שינוי הקוטר והאורך של מוט פלדה שהתוכנות:

$E=200GPa$, $\nu=0.25$

$$\varepsilon_{xx} = \frac{P}{EA} = \frac{P}{E\cdot\pi r^2}$$

$$\varepsilon_{yy} = \varepsilon_{zz} = -\nu\varepsilon_{xx} = -\frac{\nu P}{E\pi r^2}$$

$$\varepsilon_{yy} = \frac{\Delta r}{r} \qquad \Delta r = r\cdot\varepsilon_{yy} = -\frac{\nu P}{E\pi r^2}\cdot r = -\frac{\nu P}{E\pi r}$$

$$\varepsilon_{xx} = \frac{\Delta L}{L} \qquad \Delta L = L\frac{P}{E\pi r^2}$$

נפח

$$\frac{\text{נפח סופי}}{\text{נפח התחלתי}} = \frac{\pi(r+\Delta r)^2(L+\Delta L)}{\pi r^2 L} = \frac{\pi r^2 L + \pi(\Delta r)^2(L+\Delta L) + 2\pi r\Delta r(L+\Delta L)}{\pi r^2 L} =$$

$$= \frac{\pi r^2 L + \pi r^2\Delta L + 2\pi r\Delta r L + \pi(\Delta r)^2 L + 2\pi r\Delta r\Delta L + \pi(\Delta r)^2\Delta L}{\pi r^2 L} =$$

$$= \left[2\nu - 1\right]\frac{P}{E\pi r^2} = \varepsilon_{zz} + \varepsilon_{yy} + \varepsilon_{xx} = \frac{\Delta L}{L} + \frac{2\Delta r}{r} =$$

מה קורה כאשר ?

פתרון הוכח הבעלי:

מיוסקט פואסון נאמנו על המקור הרוחבי הנגזר כתוצאה מהעמסה ע"י עבודת הכוח והתוך מעמס בשטוש העמסת תע בזרות כך ותת אחן רוש פוע על העמסה שם בטוחה נוצרה מסה"כ וקבע תוכן הוק מורכל:

reasonably away from load selection is different in the area near the load.

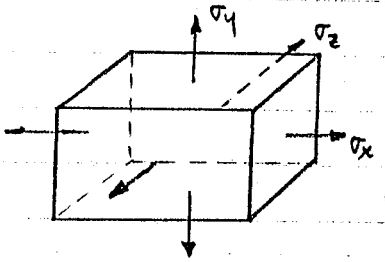
10/13/70

$$\epsilon_x = \frac{\sigma_x - \nu(\sigma_y + \sigma_z)}{E}$$

$$\epsilon_y = \frac{\sigma_y - \nu(\sigma_x + \sigma_z)}{E} \qquad 10-6.2$$

$$\epsilon_z = \frac{\sigma_z - \nu(\sigma_y + \sigma_x)}{E}$$

FIG. 1
10-13



התאוות הקובית

$$\text{cubical dialation} = \frac{\text{new volume} - \text{old vol}}{\text{old vol.}}$$

$$\frac{(1+\epsilon_x)dx(1+\epsilon_y)dy(1+\epsilon_z)dz - dx\,dy\,dz}{dx\,dy\,dz}$$

$$1 + \epsilon_x\epsilon_y + \epsilon_y\epsilon_z + \epsilon_z\epsilon_x + \epsilon_x\epsilon_y\epsilon_z + \epsilon_x + \epsilon_y + \epsilon_z - 1 \;\approx\; \epsilon_x + \epsilon_y + \epsilon_z$$

since $\epsilon_i\epsilon_j$ where $i \neq j \;\ll\; \epsilon_i, \epsilon_j$

$$\therefore \text{ cubic dialation} = \frac{\Delta V}{V} \approx \epsilon_x + \epsilon_y + \epsilon_z = \epsilon_x' + \epsilon_y' + \epsilon_z' = e$$

$$e = \frac{(1-2\nu)(\sigma_x + \sigma_y + \sigma_z)}{E} \qquad 10-13.1$$

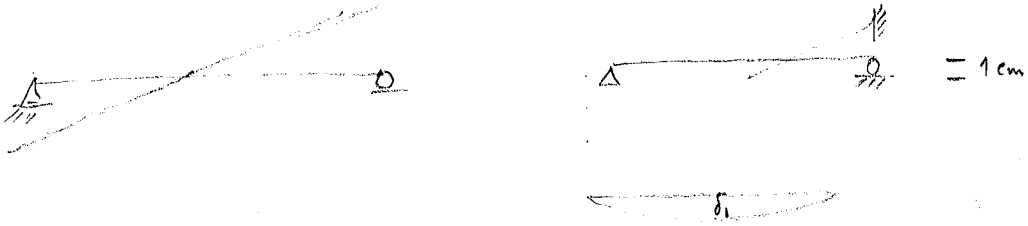
If $\sigma_x = \sigma_y = \sigma_z = -p$ $\qquad e = -3p\frac{(1-2\nu)}{E} = -\frac{p}{E/3(1-2\nu)} = -\frac{p}{K}$

מקדם הנפח
מקדם

K = bulk modulus $= E/3(1-2\nu)$

$$K = E/[3(1-2\nu)] \qquad 10-13.2$$

Strain Energy – consider problems only in statics $KE = 0$



$$\delta_1 = \frac{P l_1^3}{48 E I_1}$$

$$\delta_2 = \frac{P l_2^3}{3 E I_2}$$

$$\delta_1 + \delta_2 = 1\,cm = P\left[\frac{l_1^3}{48 E I_1} + \frac{l_2^3}{3 E I_2}\right]$$

$l_2 = 1m \quad l_1 = 2m$

$E I_1 = 2 E I_2$

A 30" long aluminum rod is enclosed within a steel tube.
The 2 materials are bonded together. If the stress-strain diag. is given as shown, what will be the end deflection for P=80 kip & P=125 kip

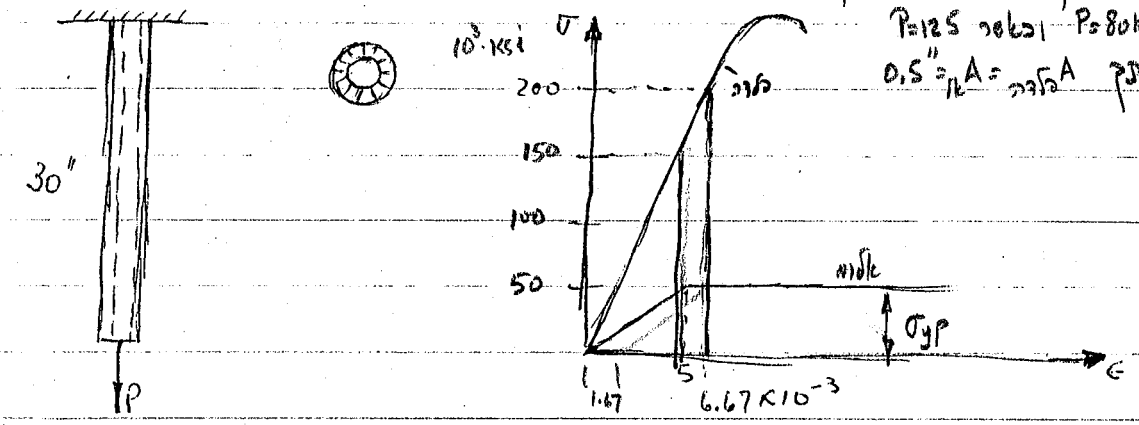
$A_{steel} = A_{al} = 0.5\,in^2$

$$u = \frac{p_0 x^2}{2AE} + C_2 \qquad u=0 \quad x=0 \quad C_2 = 0$$

$$u = \frac{p_0 x^2}{2AE} \qquad u(L) = \frac{p_0 L^2}{2AE}$$

Problem 4-10 Page 128

"סם מוט אלומינום מושפע מישמעת בינוי פלדה שני המוטות קשורים יחד. מצא את הקיטים התוארים כשהעומס $P = 80$ kips וכאשר $P = 125$ שטח החתך $A_{פלדה} = A_{אל} = 0.5$"



$$E_s = \frac{\sigma}{\varepsilon} = \frac{200}{6.67 \times 10^{-3}} = 30 \times 10^3 \text{ ksi}$$ פלדה ואלומיניום

$$E_a = \frac{\sigma}{\varepsilon} = \frac{50}{5 \times 10^{-3}} = 10 \times 10^3 \text{ ksi}$$

① $P_a + P_s = P$ ← equil $\qquad \varepsilon_s = \varepsilon_a$ ← compatability $\qquad$ אחרי $u_s = u_a$ ②

$$\varepsilon_s = \frac{\sigma_s}{E_s} \; ; \; \varepsilon_a = \frac{\sigma_a}{E_a} \leftarrow \text{elastic} \qquad ③$$

$$\varepsilon_s = \frac{\sigma_s}{30 \times 10^3} = \varepsilon_a = \frac{\sigma_a}{10 \times 10^3} \qquad \sigma_s = 3\sigma_a \qquad ④$$

$$P_a = \sigma_a A_a \qquad P_s = \sigma_s A_s \qquad P_a = .5\sigma_a \qquad P_s = .5(3\sigma_a) = .5\sigma_s$$

$$.5\sigma_A + 1.5\sigma_A = P \qquad \sigma_a = P/2.0$$

$$\sigma_s = \frac{3P}{2.0}$$

$$\Delta l = \varepsilon_s l = \varepsilon_a l = \frac{\sigma_a}{10 \times 10^3} \cdot 30 = 3\sigma_a \times 10^{-3} = \sigma_s \times 10^{-3}$$

$P_1 = 80$ k

$\frac{P}{AE_a}$

$\sigma_a = \frac{80}{2.0} = 40$ KSI $\sigma_s = \frac{3.80}{2.0} = 120$ KSI

$\Delta l = 120 \times 10^{-3} = .12$ in

suppose $P_2 = 125$ K $\sigma_a = \frac{125}{2} = 62.5 > 50$

$\sigma_s = \frac{3(125)}{2} = 187.5 < 200$ $\sigma_a \equiv 50$ KSI

$P_a \equiv 75$ K $P_a + P_s = P$

$\therefore \sigma_s = \frac{100}{.5} = 200$ KSI $P_s \equiv 100$ K

$\epsilon_s = \frac{200}{30 \times 10^3} = 6.67 \times 10^{-3}$

$\Delta l = \epsilon_s \times 30 = 200 \times 10^{-3} = .200$ in

4-3

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\epsilon_y = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_z$$

$$\epsilon_z = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \frac{\sigma_z}{E}$$

$\sigma_x = \lambda e + 2\mu\epsilon_x$

$$\sigma_x = \frac{(1-\nu^2)E\,\epsilon_x + \nu E(1+\nu)[\epsilon_y + \epsilon_z]}{1-3\nu^2-2\nu^3}$$

$$= \frac{(1-\nu)E\,\epsilon_x + \nu E[\epsilon_z + \epsilon_y]}{1-\nu-2\nu^2} \qquad (1-2\nu)(1+\nu)$$

$$= \frac{\nu E}{(1-2\nu)(1+\nu)}(\epsilon_x+\epsilon_y+\epsilon_z) + \frac{2E}{2(1+\nu)}\epsilon_x$$

$$= \lambda e + 2\mu\epsilon_x$$

4-8

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} + \alpha\Delta T$$

$$\epsilon_y = \nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} + \alpha\Delta T$$

Can't contract in 3 directions

$\sigma_z = \epsilon_x = \epsilon_y = 0$

They are constrained

$\sigma_x = \sigma_y$

$\therefore 0 = \left(\frac{1-\nu}{E}\right)\sigma + \alpha\Delta T$

$\sigma = \frac{\alpha E \Delta T}{\nu - 1}$

$\epsilon = \frac{\sigma}{E} - \nu\frac{\sigma}{E} - \frac{\nu\sigma}{E}$ $\quad \epsilon = \frac{\sigma}{E}(1-2\nu) = \frac{30000}{30\times106} \therefore \frac{1}{2} = \frac{1}{2}(1000)$

$l = l_0 + \epsilon l_0$ $\quad 2'' + (.0005)2 = 1.999''$

Thermal Strains

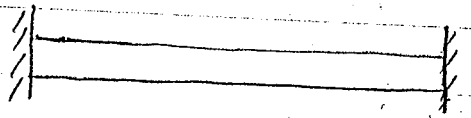
do not affect shear strains in isotropic material

$\epsilon_x = \epsilon_y = \epsilon_z$

α - coefficient of thermal expansion

$\alpha = \frac{\epsilon}{\Delta T}$ $\quad \alpha_{steel} = 6.5\times10^{-6}/°F$

$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} + \alpha\Delta T$



$A = 4in^2$
$L = 20\ ft$

$\Delta T = 100°F$ $\quad \nu =$ $\quad E =$

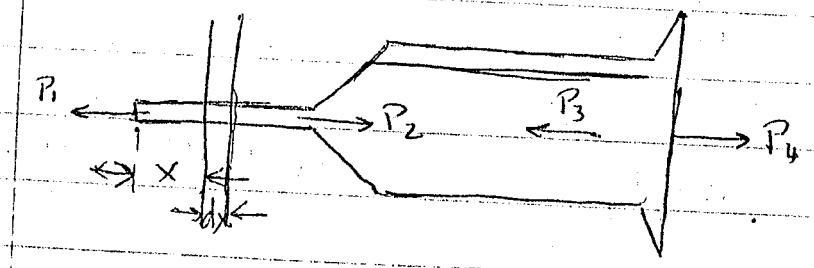
$\epsilon_x = 0 = \frac{\sigma_x}{E} + \alpha\Delta T$ since it is not restrained in y, z

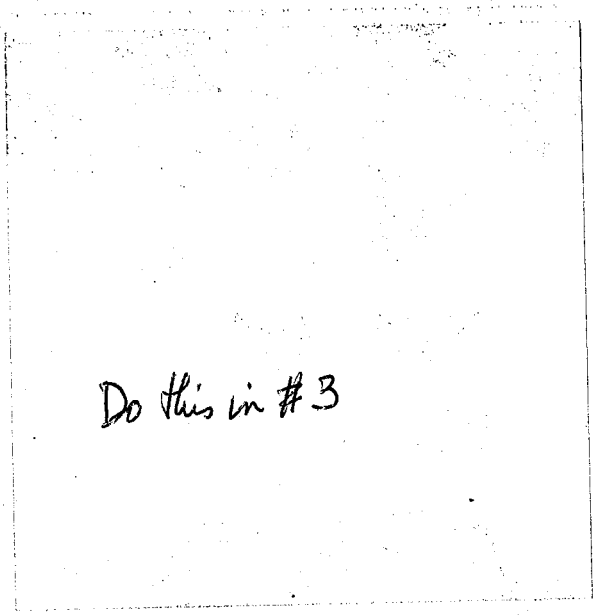
direction $\sigma_y, \sigma_z = 0$

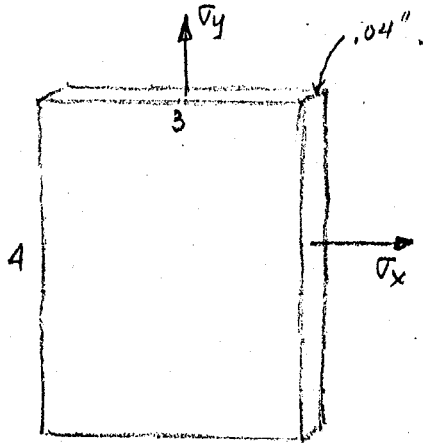
$\sigma_x = -E\alpha\Delta T$ .....

$\epsilon_y = -\frac{\nu\sigma_x}{E} + \alpha\Delta T$

$\epsilon_z = -\frac{\nu\sigma_x}{E} + \alpha\Delta T$




Do this in #3



סריפת 3 × 4 מלבנית בין מסגרת קשיחה
של סגסוגת INVAR, אם יש הקטנות של 200°F, מצאו
המאמצים בסריפת

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y + \alpha\Delta T \qquad 16\times10^6\ psi = E_{brass}$$

$$\epsilon_y = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E} + \alpha\Delta T \qquad 6\times10^6 = G$$

$$11\times10^{-6}\ \frac{in}{in/^\circ F} = \alpha$$

since it is constrained to a rigid invar frame then

$$\epsilon_x = \epsilon_y = 0$$

$$-\alpha\Delta T\, E = \sigma_x - \nu\sigma_y$$

$$-\alpha\Delta T\, E = -\nu\sigma_x + \sigma_y$$

then $-\alpha\Delta T E\nu = \nu\sigma_x - \nu^2\sigma_y$

$-\alpha\Delta T E = -\nu\sigma_x + \sigma_y$

$$-\alpha\Delta T E(1+\nu) = (1-\nu^2)\sigma_y = (1-\nu)(1+\nu)\sigma_y$$

$$\sigma_y = \frac{-\alpha\Delta T E}{1-\nu} = -\left(11\times10^{-6}\ \frac{in}{in/deg\,F}\right)(-200^\circ F)\left(16\times10^6\ psi\right)\Big/\left(\tfrac{2}{3}\right)$$

$$\sigma_y = 52.8\ KSI$$

Substituting back we get $\sigma_x = 26.4$ KSI I made a mistake of computa



$-\alpha\Delta T E = +35.2$ $\qquad 35.2 - 52.8 = -17.6 = -\frac{1}{3}\sigma_x$

$\sigma_x = 52.8$ KSI

A 3×4 in 0.040 in thick rectangular brass diaphragm is stretched between a rigid frame made of invar. Take $E_{br} = 16\times10^6$ psi $G_{br} = 6\times10^6$ psi & $\alpha = 11\times10^{-6}\ \frac{in}{in^\circ F}$ and $\alpha_{invar} = 0$ for range of temp. involved

If a drop in temp of 200°F occurs what are the resulting ~~the~~ normal stresses in the diaphragm

במבנים סטטית ניתן להבחין את ההבדל בין המקרה שבו התארכות המוגבלת שבין נתון לו להתפשט חופשית. במקרים בהם מסוימים לו כך בדיוק. כזכור שהגוף מתחיל מלהתפשט ונקרא גם מאמצים עקבה כך. הערך הנגדי כתלות בהתארכות מהתפשטות חופשית הוא:

$$\Delta = \alpha L (\Delta T) \qquad \text{נוסחה 11.8}$$

כאשר α מקדם ההתפשטות הליניארית ו-ΔT שינוי הטמפ'.

דוגמא 12.5 (114)

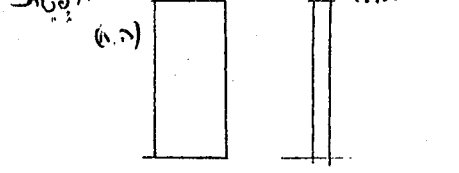
בניה מוטות שאורכו 300mm ובעל חתך של $2000mm^2$ מורכבים בין שני קטעים קשיחות של Invar (מתכת סגסוגת בעלת $\alpha \simeq 0$) המחוברים ביניהם ב-4 ברגים בעלי קוטר של 20mm. המחוברים נמצאים בטמפ' של $70^\circ C$. כמו כן נתון:

$$E_{cu} = 120\ GPa \ ; \ E_{st} = 200\ GPa$$

$$\alpha_{cu} = 16\times10^{-6}/^\circ C \quad \alpha_{st} = 12\times10^{-6}/^\circ C$$

פתרון:

בהתבוננות מהמשוואה ניתן להתפטר ולהתפשט מהפלדה. בגלל הברגים הפלדה מצמצמת אותה.



ומהגדרת שוויון המשקל והתנאי ההתחלתי מתקיים כי הכוחות שווים זה לזה בגודלם ובכיוונם.

$$P_{cu} = P_{st} = P$$

$$\Delta_{cu} = \Delta_s = \Delta$$

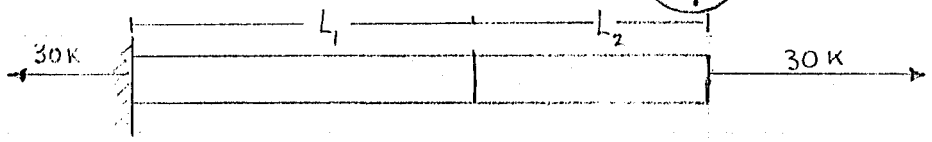
$$\alpha_{cu} L_{cu} \Delta T - \frac{P_{cu} L_{cu}}{E_{cu} A_{cu}} = \alpha_s L_s \Delta_s + \frac{P_s L_s}{A_s E_s}$$

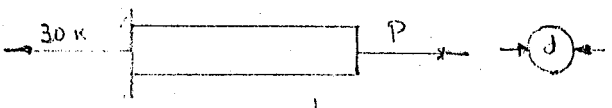
$$\Delta T = 55^\circ$$

$$16\times10^{-6}\cdot 55 \qquad P_{cu} \qquad 12\times10^{-6}\cdot 55 + \frac{P_s}{}$$

(9)

Cesar Leon
CE 201 D
Mr. Blumenthal





$L_1 + L_2 = 36in$

$u_s = u_A$

$$u_s = \frac{1}{E_s}\int_0^L \frac{P_s}{A_s}dx = \frac{1}{E_s}\frac{P}{A}L_1 \qquad u_A = \frac{1}{E_A}\int \frac{P_A}{A_A}dx = \frac{1}{E_A}\frac{P}{A}L_2$$

Since $u_s = u_A$ $\quad \frac{L_1}{L_2} = \frac{E_s}{E_A} = .333$

$L_1 + L_2 = 36in$. $L_2 + 3L_2 = 36in$ $\quad L_2 = 9in$ $\quad L_1 = 27in$

$$u_A = \frac{30{,}000}{10\times10^6(\pi)}(9) = \frac{27}{\pi\times10^{-3}} = .859\times10^{-2} \qquad u_T = 2u_A = 1.718\times10^{-2}\,in$$

23 $$u_T = \frac{1}{E_A}\int\frac{P}{A_A}dx_1 + \frac{1}{E_s}\int\frac{P}{A_s}dx_2$$

$$u_T = \frac{P}{E_A A_A}(15) + \frac{P}{E_s A_s}(12) \qquad E_A = 10\times10^6\,psi \quad E_s = 30\times10^6\,psi \quad A_A = 16\,in^2 \quad A_s = 4\,in^2$$

$$u_T = .101 = \frac{3P + 3.2P}{32\times10^6} \;; \quad P = 51.6\,Kips$$

21. f is given in units of force/length $\therefore$ $F = \int f\,dy = \frac{Ky^3}{3} + C_1$ $\quad$ @ $y=0$ $C_1 = 0$

$$u = \int\frac{F\,dy}{EA} = \frac{1}{EA}\int\frac{Ky^3}{3}dy = \frac{1}{EA}\left[\frac{Ky^4}{12}\right]_0^L = \frac{KL^4}{12EA}$$

$F = \frac{KL^3}{3} \Rightarrow K = \frac{3F}{L^3}$

but $K = \frac{3F}{L^3}$ $\longrightarrow$ $u = \frac{3F}{L^3}\cdot\frac{L^4}{12EA} = \frac{FL}{4EA}$

$F = 96{,}000\,\#$ $\quad L = 40\times12\,in$ $\quad A = 100\,in^2$ $\quad E = 1.5\times10^6\,psi$

$$\frac{(96\times10^3\,\#)(40\times12\,in)}{4\left(1.5\times10^6\frac{\#}{in^2}\right)(100\,in^2)} = \frac{16(40\times12)}{10^5} = 7.682\times10^{-2}\,in$$

2.9 The 36-mm-diameter steel rod ABC and a brass rod CD of the same diameter are joined at point C to form the 7.5-m rod $ABCD$. For the loading shown, and neglecting the weight of the rod, determine the deflection (a) of point C, (b) of point D.

Answer: $\delta_C = 2.95\ mm\downarrow$, $\delta_D = 5.29\ mm\downarrow$.

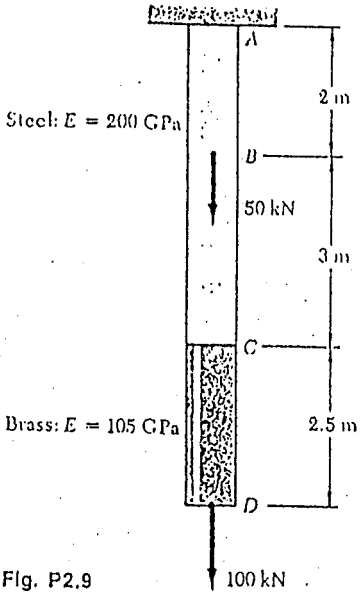


Fig. P2.9

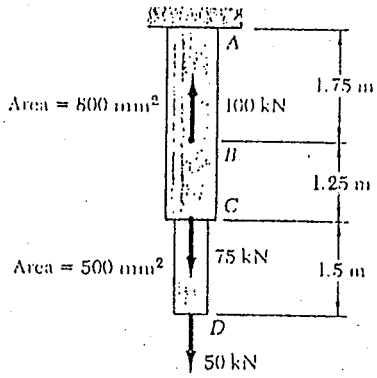


Fig. P2.10

2.10 The rod $ABCD$ is made of an aluminum alloy for which $E = 70$ GPa. For the loading shown, and neglecting the weight of the rod, determine the deflection (a) of point B, (b) of point D.

Answer: $\delta_B = 0.781\ mm\downarrow$, $\delta_D = 5.71\ mm\downarrow$.

2.26 An axial centric force of magnitude $P = 385$ kN is applied to the composite block shown by means of a rigid end plate. Determine the normal stress in (a) the steel core, (b) the aluminum plates.

Answer: $\sigma_S = -175\ MPa$, $\sigma_{Al} = -61.3\ MPa$.

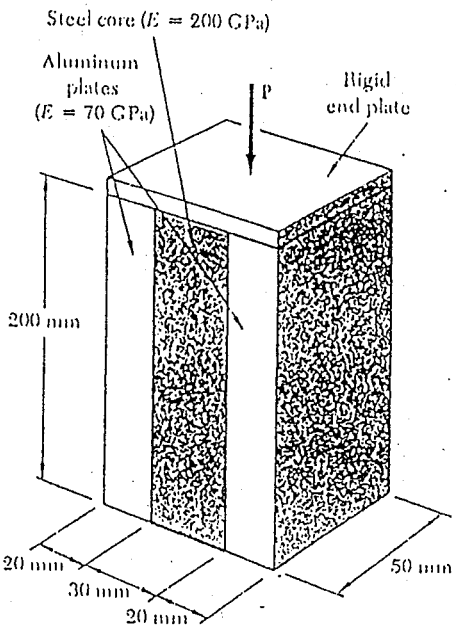


Fig. P2.26

2.37 The rigid rod ABC is suspended by three identical wires. Knowing that $x = \frac{3}{4}L$, determine the tension in each wire due to the force P.

Answer: $T_B = P/3$, $T_C = P/6$, $T_A = P/2$.

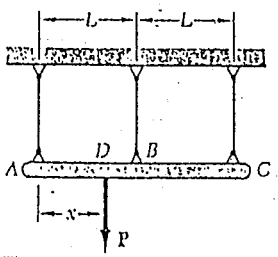


Fig. P2.37 and P2.38

2.39 The rigid bar $ABCD$ is suspended from four identical wires. Determine the tension in each wire caused by the load P shown.

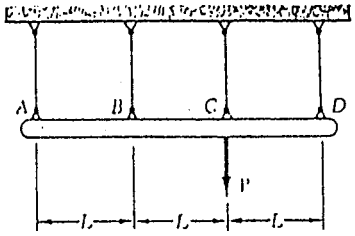
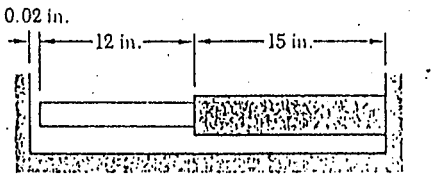


Fig. P2.39

Answer: $T_B = P/5, \quad T_D = 2P/5, \quad T_A = P/10, \quad T_C = 3P/10.$

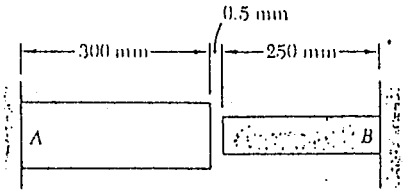
2.45 Determine (*a*) the compressive force in the bars shown after a temperature rise of 200°F, (*b*) the corresponding change in length of the aluminum bar.

Answer: $P = -52\ kips, \quad \delta_{Al} = 0.0124\ in.$



Bronze	Aluminum
$A = 2.5\ in^2$	$A = 3\ in^2$
$E = 15 \times 10^6$ psi	$E = 10 \times 10^6$ psi
$\alpha = 10.1 \times 10^{-6}/°F$	$\alpha = 12.8 \times 10^{-6}/°F$

Fig. P2.45



Aluminum	Stainless steel
$A = 2000\ mm^2$	$A = 800\ mm^2$
$E = 70$ GPa	$E = 190$ GPa
$\alpha = 23 \times 10^{-6}/°C$	$\alpha = 18 \times 10^{-6}/°C$

Fig. P2.46 and P2.47

2.46 At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature reaches 140°C, determine (*a*) the normal stress in the aluminum, (*b*) the exact length of the aluminum rod.

Answer: $\sigma_{Al} = -114.6\ MPa, \quad L = 300.34\ mm.$

2.47 Knowing that a 0.5-mm gap exists between the rods shown when the temperature is 20°C, determine (*a*) the temperature at which the normal stress in the stainless steel rod will be $\sigma = -150$ MPa, (*b*) the corresponding exact length of the stainless steel rod.

Answer: $T = 103.7°C, \quad L = 250.18\ mm.$

המכללה האקדמית יהודה ושומרון

תורת החוזק 1

תרגיל מס' 6

2.9 The 36-mm-diameter steel rod ABC and a brass rod CD of the same diameter are joined at point C to form the 7.5-m rod $ABCD$. For the loading shown, and neglecting the weight of the rod, determine the deflection (a) of point C, (b) of point D.

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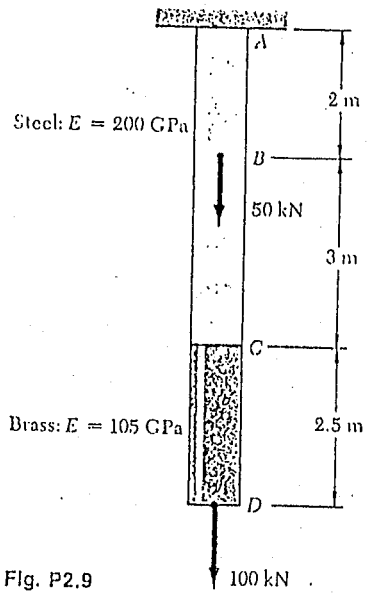


Fig. P2.9

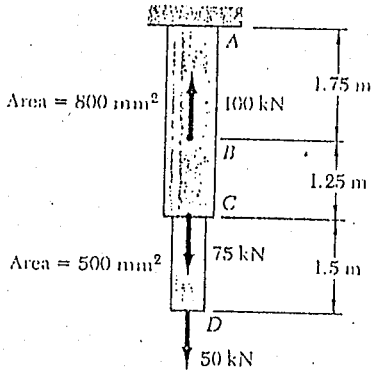


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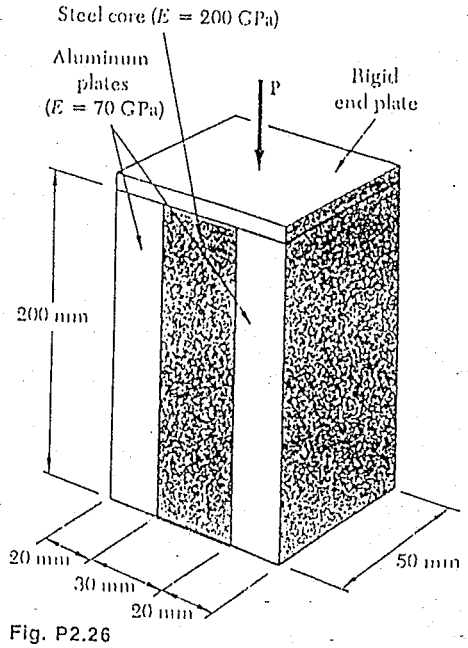


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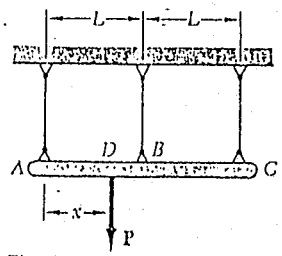


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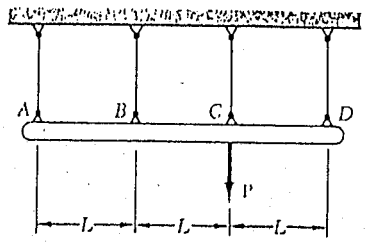
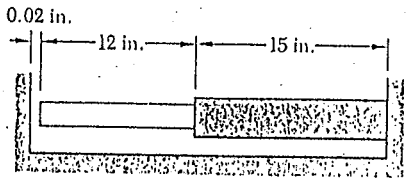


Fig. P2.39

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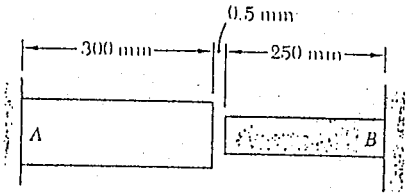
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Aluminum	Stainless steel
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$E = 70$ GPa	$E = 190$ GPa
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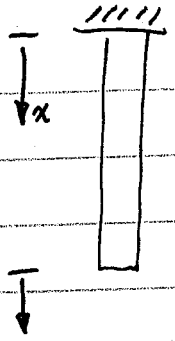
Fig. P2.46 and P2.47

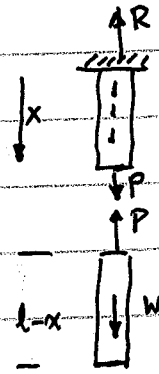
Answer: $\sigma_{Al} = -114.6\ MPa, \quad L = 300.34\ mm.$

2.47 Knowing that a 0.5-mm gap exists between the rods shown when the temperature is 20°C, determine (*a*) the temperature at which the normal stress in the stainless steel rod will be $\sigma = -150$ MPa, (*b*) the corresponding exact length of the stainless steel rod.

Answer: $T = 103.7°C, \quad L = 250.18\ mm.$

a bar A, E, ℓ find u

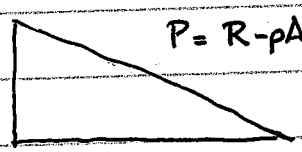




$R = \rho A g \ell$

$P = R - \rho A g x = \rho A g (\ell - x)$

$w' = \rho A g (\ell - x) = P$



$P = R - \rho A g x$

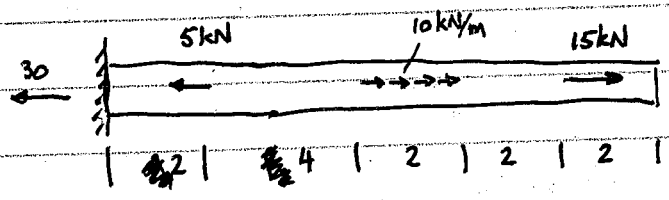
$$u = \int_0^x \frac{P\,dx'}{AE} = \int \frac{\rho A g (\ell - x')\,dx'}{AE}$$

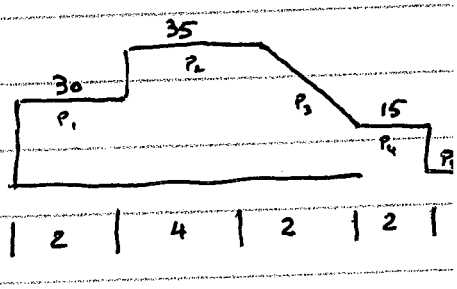
$$= \frac{\rho A g}{AE} \left(\ell - x'\right)^2 \Big|_0^x$$

$$u = -\frac{\rho A g (\ell - x)^2}{2AE} + \frac{\rho A g \ell^2}{AE}\frac{1}{2}$$

when $x = 0$ $\quad u = 0$ ✓

$x = \ell \quad u = \frac{\rho A g}{AE}\frac{\ell^2}{2} = \frac{W\ell}{2AE}$





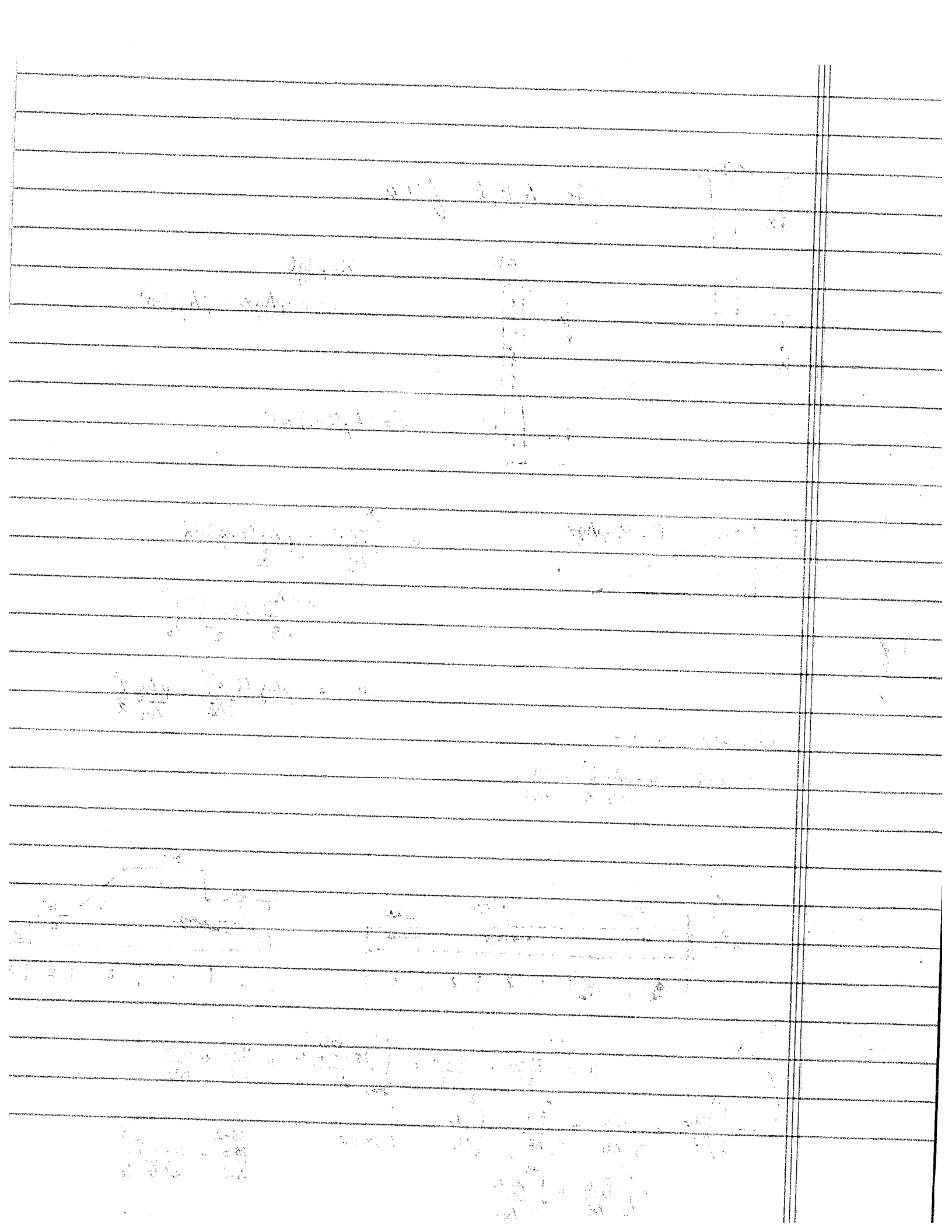
$$u = \frac{30 \cdot 2}{AE} + \frac{35 \cdot 4}{AE} + \int_6^8 \frac{(35 - 10(x-6))}{AE}\,dx + \frac{15 \cdot 2}{AE} + \frac{0 \cdot 2}{AE}$$

$$u = \int_0^{12} \frac{P\,dx}{AE} = \int_0^2 \frac{P_1\,dx}{AE} + \int_2^6 \frac{P_2\,dx}{AE} + \int_6^8 \frac{P_3\,dx}{AE}$$

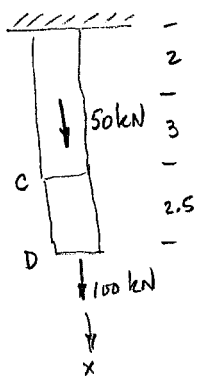
$$+ \int_8^{10} \frac{P_4\,dx}{AE} + \int_{10}^{12} \frac{P_5\,dx}{AE}$$

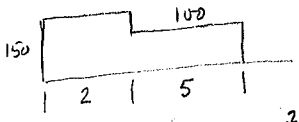
$6 \le x \le 8$

$$\frac{35 \cdot 2}{AE} - \frac{10(x-6)^2}{2AE}\Big|_6^8$$



2.9





$A = \frac{\pi a}{4} = \frac{\pi}{4}(.036)^2$

$E_{\text{פלדה}} = 200\,GPa$ $E_{\text{נחושת}} = 105\,GPa$

$$\downarrow \delta_C = \int_0^5 \frac{P}{AE}dx = \int_0^2 \frac{150000\,dx}{(1.018\times10^{-3})(200\times10^9)} + \int_2^5 \frac{100000dx}{(1.018\times10^{-3})(200\times10^9)}$$

$$= 7.367\times10^{-4}\,x\Big|_0^2 + 4.912\times10^{-4}x\Big|_2^5 =$$

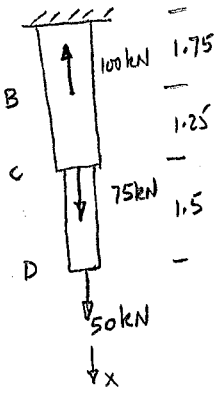
$$= 1.473\times10^{-3} + 1.473\times10^{-3} = \underline{2.947\times10^{-3}\,m}\ \downarrow$$

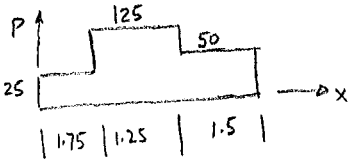
$$\delta_D = \int_0^{7.5} \frac{P}{AE}dx = \int_0^2 \frac{150000\,dx}{(1.018\times10^{-3})(200\times10^9)} + \int_2^5 \frac{100000\,dx}{(1.018\times10^{-3})(200\times10^9)} + \int_5^{7.5} \frac{100000\,dx}{(1.018\times10^{-3})(105\times10^9)}$$

$$= \delta_C + 9.355\times10^{-4}\,x\Big|_5^{7.5} = \delta_C + 2.339\times10^{-3} = \underline{5.286\times10^{-3}\,m}\ \downarrow$$

$A_{AC} = 8\times10^{-4}\,m^2$ $A_{CD} = 5\times10^{-4}\,m^2$ $E = 70\,GPa$

2.10





$$\downarrow \delta_B = \int_0^{1.75} \frac{P}{AE}dx = \int_0^{1.75} \frac{25000\,dx}{(8\times10^{-4})(70\times10^9)} = 4.464\times10^{-4}\,x\Big|_0^{1.75}$$

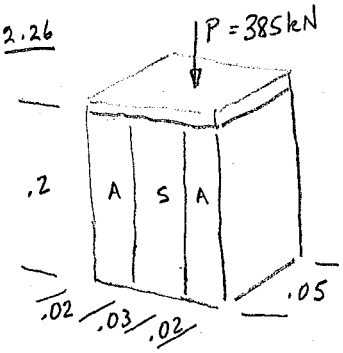
$$= 7.813\times10^{-4}\,m = .781\,mm\ \downarrow$$

$$\downarrow \delta_D = \int_0^{4.5} \frac{P}{AE}dx = \int_0^{1.75} \frac{25000}{(8\times10^{-4})(70\times10^9)}dx + \int_{1.75}^{3} \frac{125000\,dx}{(8\times10^{-4})(70\times10^9)} + \int_3^{4.5} \frac{50,000\,dx}{(5\times10^{-4})(70\times10^9)}$$

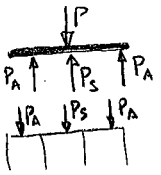
$$= \delta_B + 2.232\times10^{-3}\,x\Big|_{1.75}^{3} + 1.4286\times10^{-3}x\Big|_3^{4.5} = 7.813\times10^{-4} + 2.790\times10^{-3} + 2.1429\times10^{-3}$$

$$= 5.714\times10^{-3}\,m = \underline{5.71\,mm}$$

2.26



$E_A = 70\,GPa$
$E_S = 200\,GPa$



$2P_A + P_S = P = 385\times10^3\,N$ $\Delta u_A = \Delta u_S = \epsilon_A L_A = \epsilon_S L_S$; $L_S = L_A$

$$\epsilon_S = \epsilon_A = \frac{\sigma_A}{E_A} = \frac{\sigma_S}{E_S} \Rightarrow \frac{P_A}{E_A A_A} = \frac{P_S}{E_S A_S}$$

$A_S = .03\times.05 = 1.5\times10^{-3}\,m^2$

$A_A = .02\times.05 = 1.0\times10^{-3}\,m^2$

$$\frac{P_A}{(70\times10^9)(1\times10^{-3})} = \frac{P_S}{(200\times10^9)(1.5\times10^{-3})}$$

$$P_S = \frac{300\times10^6}{70\times10^6}P_A = 4.286\,P_A$$

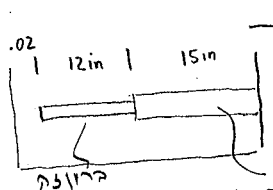
$$P = 2P_A + P_S = 2P_A + 4.286P_A = 6.286\,P_A \Rightarrow P/6.286 = P_A$$

$\sigma_A = P_A/A_A = \frac{6.125\times10^4}{1.0\times10^{-3}} = 61.25\times10^6\,Pa \Leftarrow P_A = 61247.22\,N$ לכן,

$\sigma_S = P_S/A_S = \frac{2.625\times10^5}{1.5\times10^{-3}} = 175\times10^6 \Leftarrow P_S = 262505.66\,N$

הפתרון $\sigma_A = 61.25\,MPa$ ו- $\sigma_S = 175\,MPa$

2.45



$A = 2.5\,in^2$
$E_B = 15\times10^6\,psi$
$\alpha_B = 10.1\times10^{-6}/°F$

$A_A = 3\,in^2$
$E_A = 10\times10^6\,psi$
$\alpha_A = 12.8\times10^{-6}/°F$

התגובות על המוט המורכב הינו $\Delta u = \sum \epsilon_x L$ אבל מכיוון שהמוט מחומם כדי למנוע את התפשטות שהורמת לאחר לקיר. לכן, מכיוון ש $\epsilon_x = \alpha\Delta T$

$$\Delta u = .02\,in = L_B\alpha_B\Delta T + L_A\alpha_A\Delta T = \Delta T\left[(12\times10.1\times10^{-6}) + (15\times12.8\times10^{-6})\right]$$

$$= \Delta T\,(3.132\times10^{-4}) \Rightarrow 63.857\,°F = \Delta T$$

לכן, ע"י עזר $136.14° = \Delta T^*$ שהמוט ניצר לקיר. כדי לשפר פיתרון, לניצר בקיר את התארכות של האלומיניום וההברונזה עד שהמוט מתנתק כאשר המוטות מתחילים להתפשט יותר מאשר המאפשר שם כוח שפועל על המוט

$\Delta u_B = .00774\,in$, $\Delta u_A = L_A\alpha_A(63.86) = .01226\,in$ לכן, אורך האלומיניום בהתחלת הניצרה 15.01226 ושל הברונזה 12.00774

לכן, $\int_{l_A+l_B}^{l_A+l_B} \epsilon_x dx + \int_{l_B}^{l_A+l_B} \epsilon_{x_A} dx$

$$0 = \Delta u = \Delta u_B + \Delta u_A$$

$$\Delta u_A = \int_{l_B}^{l_B+l_A} \epsilon_{x_A}\,dx = \int_{l_B}^{l_B+l_A} \left(\frac{\sigma_{x_A}}{E_A} + \alpha_A \Delta T^*\right) dx = \left(\frac{\sigma_{x_A}}{E_A} + \alpha_A \Delta T^*\right) x \Big|_{l_B}^{l_B+l_A}$$

1- התארכות האלומיניום

כאן, העיבור בכל מהמיומים והלחץ שווה עם הקיר. המאמץ $\sigma_{x_A} = \frac{P_A}{A_A}$, $\sigma_{x_B} = \frac{P_B}{A_B}$. אבל $P_A = P_B$; לכן

$$0 = \Delta u_B + \Delta u_A = \left(\frac{P}{E_B A_B} + \alpha_B \Delta T^*\right) l_B + \left(\frac{P}{E_A A_A} + \alpha_A \Delta T^*\right) l_A$$

או

$$P\left[\frac{l_B}{E_B A_B} + \frac{l_B}{E_A A_A}\right] = -\left(\alpha_B l_B + \alpha_A l_A\right)\Delta T^*$$

$$P\left[\frac{12.00774\ in}{15\times10^6 \frac{lb}{in^2}\cdot 2.5\,in^2} + \frac{15.01226\ in}{10\times10^6 \frac{lb}{in^2}\cdot 3\,in^2}\right] = -\left[10.1\times10^{-6}/{}^\circ F \cdot 12.00774 + 12.8\times10^{-6}/{}^\circ F \cdot 15.01226\ in\right](136.14^\circ F)$$

$$P\left(3.2021\times10^{-7}\frac{in}{lb} + 5.0041\times10^{-7}\frac{in}{lb}\right) = -\left(1.21278\times10^{-4}\frac{in}{{}^\circ F} + 1.92157\times10^{-4}\right)(136.14^\circ F)$$

$$P\left(8.2062\times10^{-7}\frac{in}{lb}\right) = -\left(3.13435\times10^{-4}\frac{in}{{}^\circ F}\right)(136.14^\circ F) = 4.2671\times10^{-2}\,in$$

$$\underline{P = -5.20\times10^4\,lb = -52000\,lb}$$

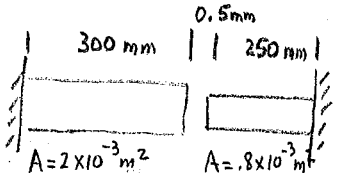
$$\underline{\sigma_{x_A} = \frac{P}{A_A} = -17333\ lb/in^2} \qquad \sigma_{x_B} = \frac{P}{A_B} = -20800\ lb/in^2$$

לחשב את התארכות של האלומיניום

$$\delta_{Al} = \int_{l_A}^{l_B+l_A} \left(\frac{\sigma_{x_A}}{E_A} + \alpha_A\left[\Delta T + \Delta T^*\right]\right) dx$$

$$\underline{\delta_{Al} = \left\{\frac{\sigma_{xA}}{E_A} + \alpha_A\cdot(\Delta T + \Delta T^*)\right\} l_A = \left\{\frac{-17333}{10\times10^6} + 12.8\times10^{-6}(200)\right\}(15) = 1.24\times10^{-2}\,in}$$

2.46



לפי הסיבה הנ"ל

$$\Delta u = 0.5\times10^{-3} = L_A\alpha_A\Delta T + L_S\alpha_S\Delta T$$

$$= (.3\cdot23\times10^{-6} + .25\cdot18\times10^{-6})\Delta T$$

$$0.5\times10^{-3} = 1.14\times10^{-5}\Delta T$$

$$43.86^\circ C = \Delta T$$

$$76.14^\circ = (140-20) - 43.86^\circ = \underline{\Delta T^*}$$

אחרי כת

$$0 = \Delta u_A + \Delta u_S = \left(\frac{P}{E_A A_A} + \alpha_A \Delta T^*\right) l_A + \left(\frac{P}{E_S A_S} + \alpha_S \Delta T^*\right) l_S$$

$$P\left(\frac{l_A}{E_A A_A} + \frac{l_S}{E_S A_S}\right) = -\left(\alpha_A l_A + \alpha_S l_S\right)\Delta T^*$$

$$P\left[\frac{0.3}{(70\times10^9)(2\times10^{-3})} + \frac{0.250}{(190\times10^9)(0.8\times10^{-3})}\right] = -\left((23\times10^{-6})(0.3) + (18\times10^{-6})(0.25)\right)(76.14^\circ)$$

$$P\left(3.7876\times10^{-9}\right) = -\left(1.14\times10^{-5}\right)(76.14^\circ) = -8.6800\times10^{-4}\,m$$

$$\underline{P = -2.29169\times10^5\,N = -229169\,N}$$

$$\sigma_A = \frac{P}{A_A} = \frac{-229169\,N}{2\times10^{-3}\,m^2} = -114.58\,MPa$$

$$\delta_{Al} = \int_0^{l_A}\left(\frac{\sigma_{x_A}}{E_A} + \alpha_A(\Delta T + \Delta T^*)\right)dx = \left\{\frac{\sigma_{x_A}}{E_A} + \alpha_A(\Delta T + \Delta T^*)\right\} l_A = \left\{\frac{-1.1458\times10^8}{70\times10^9} + 23\times10^{-6}(120^\circ C)\right\}0.3$$

$$= 3.37\times10^{-4}\,m$$

$$l_{-1} + \delta = 0.3m + 0.00034m = 0.30034m = \underline{300.34m}$$



לפי המשוואה של תרגיל 2.45 $\sigma_s = 150\times10^6\ N/m^2 = P/A_s = \dfrac{P}{1.8\times10^{-3}\ m^2}$

$$P = -1.2\times10^5\ N$$

$$0 = \Delta u_A + \Delta u_s = \left(\frac{P}{E_A A_A} + \alpha_A \Delta T^*\right) l_A + \left(\frac{P}{E_s A_s} + \alpha_s \Delta T^*\right) l_s$$

$$= P\left[\frac{l_A}{E_A A_A} + \frac{l_s}{E_s A_s}\right] + (\alpha_A l_A + \alpha_s l_s)\Delta T^*$$

$$0 = -1.2\times10^5\left(3.7876\times10^{-9}\right) + \left(1.14\times10^{-5}\right)\Delta T^* = -4.545\times10^{-4} + (1.14\times10^{-5})\Delta T^*$$

המספרים ל $(\alpha_A l_A + \alpha_s l_s)$ ו $\left[\frac{l_A}{E_A A_A} + \frac{l_s}{E_s A_s}\right]$ בשוויון מהתרגיל 2.46

לכן,

$$\Delta T^* = \frac{4.545\times10^{-4}}{1.14\times10^{-5}} = 39.87^\circ C$$

כדי להוסיף את הטמפרטורה לסיבוב של המרחק ש 0.5mm, צריך לחמם ב $\Delta T = 43.86^\circ C$ כמו שמצאנו בתרגיל הקודם. לכן, $\Delta T + \Delta T^* = 43.86 + 39.87 = 83.73^\circ C$. אם נסמן את הטמפרטורה ב $T = 20^\circ C$, T הסופי צריך להיות $20^\circ + 83.73 = 103.73^\circ C$

$$\delta_s = \left[\frac{\sigma_{xs}}{E_s} + \alpha_s (\Delta T + \Delta T^*)\right] l_s = \left[\frac{-150\times10^6}{190\times10^9} + (18\times10^{-6})(43.86 + 39.87)\right](250\ mm)$$

$$= (7.1767\times10^{-4})(250\ mm) = 1.794\times10^{-1}\ mm$$

$$L = L_0 + \delta_s = 250 + 0.1794\ mm = 250.1794\ mm$$

2.39 The rigid bar *ABCD* is suspended from four identical wires. Determine the tension in each wire caused by the load P shown.

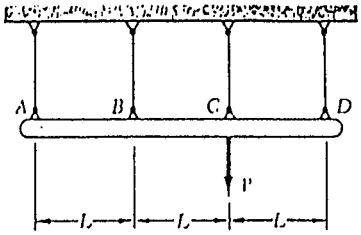
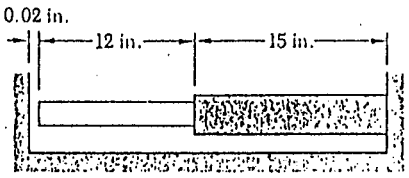


Fig. P2.39

Answer: $T_B = P/5, \quad T_D = 2P/5, \quad T_A = P/10, \quad T_C = 3P/10.$

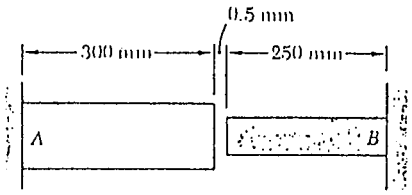
2.45 Determine (*a*) the compressive force in the bars shown after a temperature rise of 200°F, (*b*) the corresponding change in length of the aluminum bar.

Answer: $P = -52\ kips, \quad \delta_{Al} = 0.0124\ in.$



Bronze	Aluminum
$A = 2.5\ in^2$	$A = 3\ in^2$
$E = 15 \times 10^6$ psi	$E = 10 \times 10^6$ psi
$\alpha = 10.1 \times 10^{-6}/°F$	$\alpha = 12.8 \times 10^{-6}/°F$

Fig. P2.45



Aluminum	Stainless steel
$A = 2000\ mm^2$	$A = 800\ mm^2$
$E = 70$ GPa	$E = 190$ GPa
$\alpha = 23 \times 10^{-6}/°C$	$\alpha = 18 \times 10^{-6}/°C$

Fig. P2.46 and P2.47

2.46 At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature reaches 140°C, determine (*a*) the normal stress in the aluminum, (*b*) the exact length of the aluminum rod.

Answer: $\sigma_{Al} = -114.6\ MPa, \quad L = 300.34\ mm.$

2.47 Knowing that a 0.5-mm gap exists between the rods shown when the temperature is 20°C, determine (*a*) the temperature at which the normal stress in the stainless steel rod will be $\sigma = -150$ MPa, (*b*) the corresponding exact length of the stainless steel rod.

Answer: $T = 103.7°C, \quad L = 250.18\ mm.$

2/2

2.9 The 36-mm-diameter steel rod ABC and a brass rod CD of the same diameter are joined at point C to form the 7.5-m rod $ABCD$. For the loading shown, and neglecting the weight of the rod, determine the deflection (a) of point C, (b) of point D.

Answer: $\delta_C = 2.95\ mm\downarrow$, $\delta_D = 5.29\ mm\downarrow$.

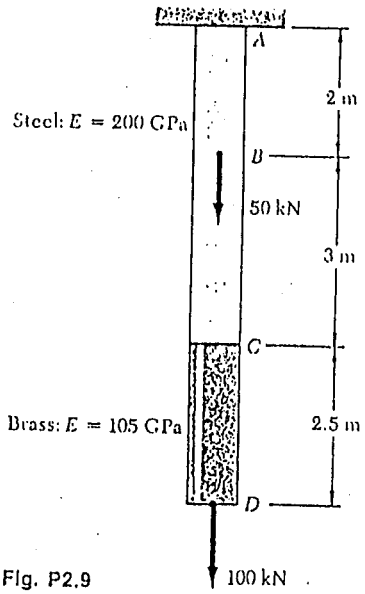


Fig. P2.9

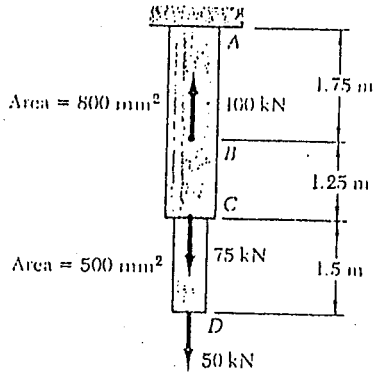


Fig. P2.10

2.10 The rod $ABCD$ is made of an aluminum alloy for which $E = 70$ GPa. For the loading shown, and neglecting the weight of the rod, determine the deflection (a) of point B, (b) of point D.

Answer: $\delta_B = 0.781\ mm\downarrow$, $\delta_D = 5.71\ mm\downarrow$.

2.26 An axial centric force of magnitude $P = 385$ kN is applied to the composite block shown by means of a rigid end plate. Determine the normal stress in (a) the steel core, (b) the aluminum plates.

Answer: $\sigma_S = -175\ MPa$, $\sigma_{Al} = -61.3\ MPa$.

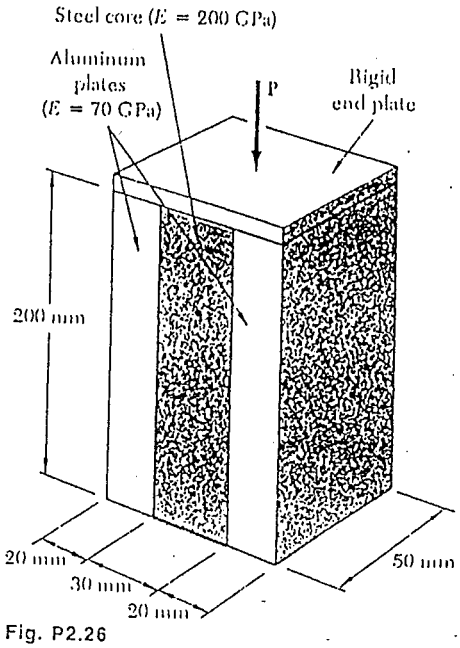


Fig. P2.26

2.37 The rigid rod ABC is suspended by three identical wires. Knowing that $x = \frac{2}{3}L$, determine the tension in each wire due to the force P.

Answer: $T_B = P/3$, $T_C = P/6$, $T_A = P/2$.

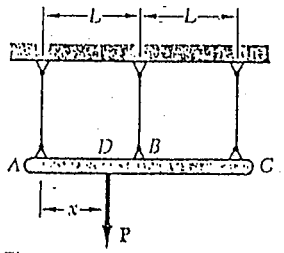
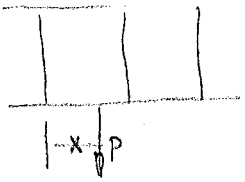
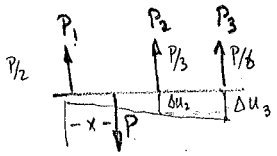


Fig. P2.37 and P2.38



פתרון



$P_1+P_2+P_3=P$ $\quad \Sigma M_{P_1}=P\cdot\frac{2}{3}L-P_2\cdot L-P_3\cdot 2L=0$

$\Delta u_2=\frac{P_2L}{AE}$

$\Delta u_3=\frac{P_3L}{AE}$

$\Delta u_1=\frac{P_1L}{AE}$

$\frac{2Pl}{3}=P_2+2P_3 \qquad P_2=\frac{2P}{3}-2P_3$

$P_1+\left(\frac{2P}{3}-2P_3\right)+P_3=P$

$(P_1-P_3)=P/3$

$P_3-P_2=P_2-P_1$

$\frac{\Delta u_3-\Delta u_2}{L}=\frac{u_3-u_2}{L}=\frac{u_2-u_1}{L}$

$\frac{1}{L}\left(\frac{P_3L}{AE}-\frac{P_2L}{AE}\right)=\frac{1}{L}\left(\frac{P_2}{AE}-\frac{P_1}{AE}\right)$

$P_3-P_2=P_2-P_1$

$P_3-2P_2+P_1=0$ ①

$P_3+P_2+P_1=P$ ②

② - ① $\Rightarrow 3P_2=P \quad P_2=P/3$

$P/3=2P/3-2P_3 \Rightarrow 2P_3=P/3$

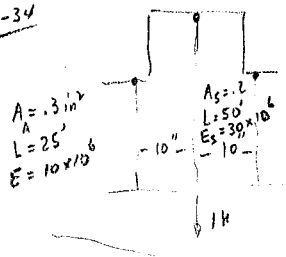
$P_3=P/6$

$P_1+P_2+P_3=P \Longrightarrow P_1=P/2$

$\epsilon_1=$

מה הכוח בכוון תוך.

4-34



$\Sigma M_{P=1k}=0 \Rightarrow P_{A_L}=P_{A_R}$

$2P_A+P_S=P \Leftarrow \Sigma F_y=0$

By symmetry bar moves vertical

$\Delta u_A=\Delta u_S=\epsilon_A L_A=\epsilon_S L_S$

$\frac{\epsilon_A}{\epsilon_S}=\frac{L_S}{L_A}=\frac{2}{1}$

$\epsilon_A=\frac{\sigma_A}{E_A}=\frac{P_A}{E_AA_A}=2$

$\epsilon_S=\frac{\sigma_S}{E_S}=\frac{P_S}{E_SA_S}$

$\frac{P_A}{P_S}\frac{E_SA_S}{E_AA_A}=P_A\frac{30\times10^6}{1\times10^7}\cdot\frac{.2}{.3}=2$

$=P_A\frac{6}{3}\cdot\frac{2}{3}=2$

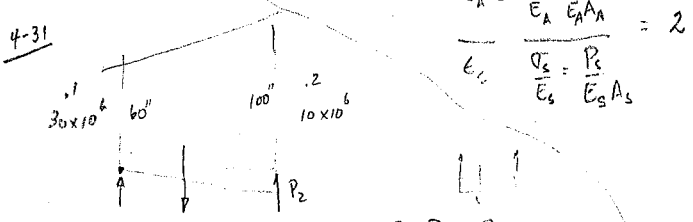
$P_A=\frac{4}{3}P_S$

$P_A=\frac{E_AA_A}{E_SA_S}P_S\cdot 2$

$=\frac{10\times10^6(.3)}{30\times10^6(.2)}\cdot 2P_S$

$P_A=\frac{1}{3}\left(\frac{3}{2}\right)\cdot 2P_S \qquad P_A=P_S \Longrightarrow 3P_A=P \qquad P_A=\frac{1}{3}P=P_S$

4-31



$\Sigma M \quad P\cdot 10-P_2\cdot 20=0 \quad P_2=\frac{1}{2}P \quad P_1=\frac{1}{2}P$

Δu

$\Delta u=\frac{P_1L_1}{AE_1}$

$\frac{P}{2}\frac{60}{.1\times30\times10^6}$

$\frac{1000\ (60)}{30\times10^5}=\frac{6\times10^4}{3\times10^6}$

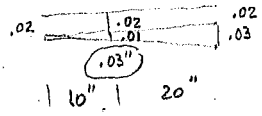
$=2\times10^{-2}$

.02in

$\Delta u_2=\frac{P_2L_2}{AE_{22}}$

$\frac{P}{2}\cdot\frac{10"}{.2\times10\times10^6}$

$\frac{1000\ 00}{2\times10^6}=\frac{1\times10^5}{2\times10^6}\approx 0.5\times10^{-1}=.05$



שני חוטים נקשרים למוט קשיח כמתואר
חוט השמאלי מפלדה בעל שטח חתך של $0.1"^2$ ו $E=30\times10^6$
חוט הימני עשוי מסגסוגת אלומיניום בעל שטח חתך של $0.2"^2$
ו $E=10\times10^6$. שאר מאפיין מוטה במתואר מה יהיה
קביעת ו. מצין התפלגות התזוזות

26. $u_A = u_B$

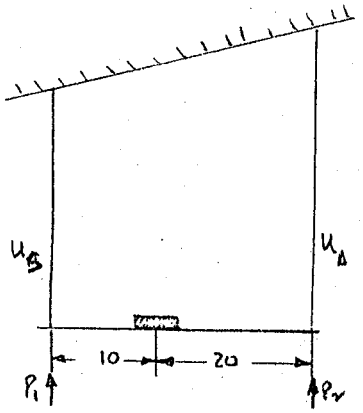
$$u_A = \int_0^{L_A} \frac{1}{E}\frac{P}{A}\,dx = \frac{P}{Ex2}L_A$$

$$u_B = \int_0^{L_B} \frac{1}{E}\frac{P}{A}\,dx = \frac{P}{E}\int \frac{dx}{A_T} = \frac{PL_B}{E}\int_0^{L_B} \frac{dx}{2x + L_B} = \frac{P_{L_B}}{E}\left\{\frac{1}{2}\ln\left(3L_B/L_B\right)\right\} = \frac{P_{L_B}}{E}\left[.55\right]$$

$u_A = u_B \quad \therefore \quad \frac{P}{Ex2}L_A = \frac{P_{L_B}}{E}(.55)$

$L_A/L_B = 1.1$ ✓

31.



$P_1 + P_2 = 2000$

$P_1(10) = P_2(20)$

$3P_2 = 2000$

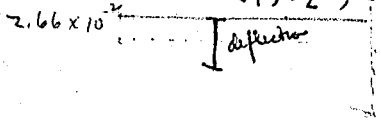
$P_2 = \frac{2000}{3}$ $\quad P_1 = \frac{4000}{3}$

$$u_B = \int_0^{60''} \frac{1}{E_S}\frac{P_1}{A_S}\,dx = \frac{4000 \times 60}{3 \times 30\times10^6 \times .1} = \frac{12(7)}{9\times10^2} = .6667\times10^{-2}\,in \downarrow$$
$$= 2.667$$

$$u_A = \int_0^{100''} \frac{1}{E_A}\frac{P_2}{A_A}\,dx = \frac{2000}{3\times10\times10^6 \times .2} \times 100 = 3.33\times10^{-2}$$

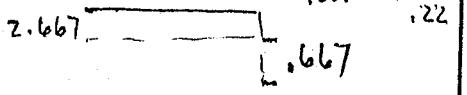
find P_1, P_2

$\sigma_1, \sigma_2, \epsilon_1, \epsilon_2$ $\quad u_1 = \epsilon_1 \Delta L_1$ $\quad u_2 = \epsilon_2 \Delta L_2$



deflection $= .667\times10^{-2}\,in + \frac{1}{3}4(.667\times10^{-2}\,in)$

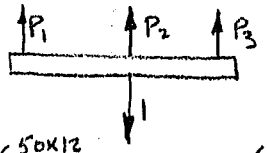
$1.555\times10^{-2}\,in$



34. $u_1 = u_2 = u_3$

$P_1 + P_2 + P_3 = 1\ K$

$P_1 = P_3$



$$u_1 = \int_0^{25\times12} \frac{P_1}{E_1A_1}\,dx_1 = \int_0^{50\times12} \frac{P_2}{E_2A_2}\,dx_2 = \int_0^{25\times12} \frac{P_3}{E_3A_3}\,dx_3$$

$$u_1 = \frac{P_1}{E_1A_1}(25\times12) = \frac{P_2}{E_2A_2}(50\times12)$$

$$\frac{P_1}{E_1A_1} = \frac{2P_2}{E_2A_2}$$

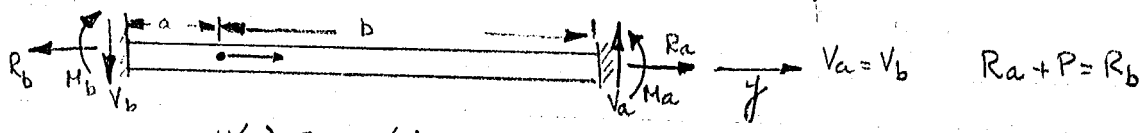
$E_1A_1 = (10\times10^6)(.3) = 3\times10^6$

$E_2A_2 = (30\times10^6)(.2) = 6\times10^6$

$\therefore \quad E_1A_1 = \frac{1}{2}E_2A_2 \quad \therefore 2P_1 = 2P_2$

$\therefore \quad P_1 = P_2 = P_3 \quad \Longrightarrow \quad P_1 = P_2 = P_3 = 1/3\ K$

$$AE\frac{d^2u}{dx^2} = -p_x$$



$u(0)=0 \quad u(L)=0$

$$AE\frac{d^2u}{dx^2} = -p_x = -P(x-a)^{-1}$$

$$AE\frac{du}{dx} = -P(x-a)^0 + C_1$$

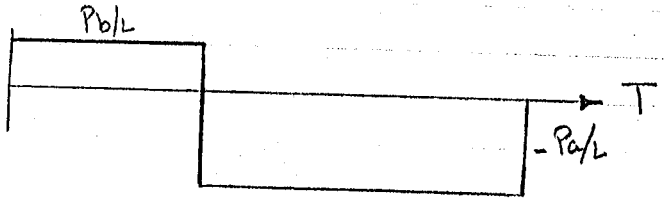
$$AEu = -P(x-a)^1 + C_1x + C_2$$

$$AEu(0) = 0 = C_2$$

$$AEu(L) = 0 = -Pb + C_1L \qquad C_1 = Pb/L$$

$$R_b = P(0) = AE\left(\frac{du}{dx}\right)_{x=0} = Pb/L$$

$$R_a = P(L) = AE\left(du/dx\right)_{x=L} = -P + C_1 = -Pa/L$$



for $x<a$ axial force distrib $= Pb/AL$

for $x=a$ " " " $= P/A$

for $x>a$ axial force dist $= Pa/AL$

מערכות בעלות אפשרויות סטטיות - הרחבה בנושא

8.12 השיטה הכללית

במבנים בעלי אפשרויות סטטיות עלול נתון להשפיע על הכוחות התגובתיים והפנימיים בהתאם למשקלם של חלקים. לצורך הסבר במבנים בעלי אפשרויות נסתייע בחישוב תגובות ובהמחשת התנהגותן כפי שפורט עד כה. מוסיפים שקולים שקולות מתגובות ואת הקינמטיקה של המבנה.

דוגמא 1

נתון המוט שבסרטוט.

1. בהנחה שהמוט חלסטי חשב את התגובות $R_1 - R_2$

סטטיקה ומצא לנו כי

(סטטיקה) $P = R_1 + R_2$

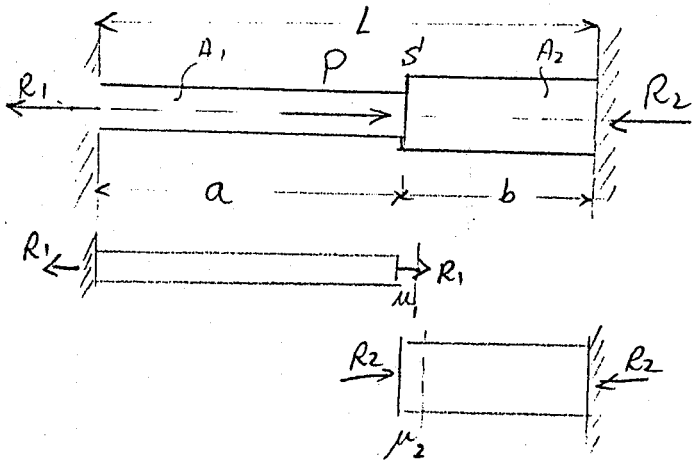
כעת ננתק את המוט ב C ונקרוס שני חלקי המוט ונרשום את התצלה הבאפקית:

(שווי מהות) $u_1 = u_2$

$$\varepsilon = \frac{P}{EA} \qquad \Delta = \frac{PL}{EA}$$

$$\frac{R_1 a}{EA_1} = \frac{R_2 b}{EA_2}$$

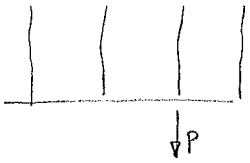
$$R_1 = \frac{P}{1 + \frac{aA_2}{bA_1}} \qquad R_2 = \frac{P}{1 + \frac{bA_1}{aA_2}}$$

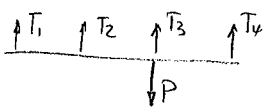


עבור, $E = 200\ GPa$

$a = 0.75\ m$

$R_1 = \frac{P}{4}$; $R_2 = \frac{3P}{4}$

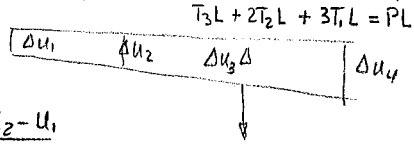




$$T_1+T_2+T_3+T_4 = P$$

$$\Sigma M_{T_4} = 0 = (P-T_3)L - T_2\cdot 2L - T_1\cdot 3L = 0$$

$$T_3L + 2T_2L + 3T_1L = PL$$



$$T_1+T_2+T_3+T_4 = P$$
$$3T_1+2T_2+T_3\ \ 0 = P$$
$$T_1-2T_2+T_3\ 0 = 0$$
$$0\ \ T_2-2T_3+T_4 = 0$$

$$\frac{u_4-u_3}{L} = \frac{u_3-u_2}{L} = \frac{u_2-u_1}{L}$$

$$T_4-T_3 = T_3-T_2 = T_2-T_1$$

$$T_3+2T_2+T_1 = 0$$

$$T_4-2T_3+T_2 = 0$$

$$\begin{matrix} T_3+2T_2+3T_1 = P \\ T_3-2T_2+T_1 = 0 \\ \hline 2T_3+4T_1 = P \end{matrix}$$

$$\begin{matrix} T_4-T_3-T_2+T_1 = 0 \\ T_4+T_3+T_2+T_1 = P \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix} \underline{X} = \begin{bmatrix} P \\ P \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & P \\ 3 & 2 & 1 & 0 & P \\ 4 & 0 & 2 & 0 & P \\ 0 & 1 & -2 & 1 & 0 \end{bmatrix}$$

שורה 3 = שורה 3 + שורה 2

$$\begin{bmatrix} 1 & 1 & 1 & 1 & P \\ 0 & -1 & -2 & -3 & -2P \\ 0 & -4 & -2 & -4 & -3P \\ 0 & 1 & -2 & 1 & 0 \end{bmatrix}$$

שורה 2 = שורה 1 × (-3) + שורה 2
שורה 3 = שורה 3 × (-1) + שורה 3
שורה 4 = שורה 4

$$\begin{bmatrix} 1 & 1 & 1 & 1 & P \\ 0 & -1 & -2 & -3 & -2P \\ 0 & 0 & 6 & 8/6 & 5P/6 \\ 0 & 0 & -2 & -2 & -2P \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & P \\ 0 & -1 & -2 & -3 & -2P \\ 0 & 0 & 1 & 4/3 & 5/6\,P \\ 0 & 0 & 0 & 5/3 & 3/3\,P \end{bmatrix}$$

$$T_4 = 3/5P$$

$$T_3 + 4/3\cdot 3/5P = 5/6\,P$$

$$T_3 + 18/30\,P = 25/30\,P$$

$$T_3 = 9/30P \Rightarrow 3/10P$$

$$3/5P - 3/5P + P_2 = 0 \qquad T_2 = 1/5P$$

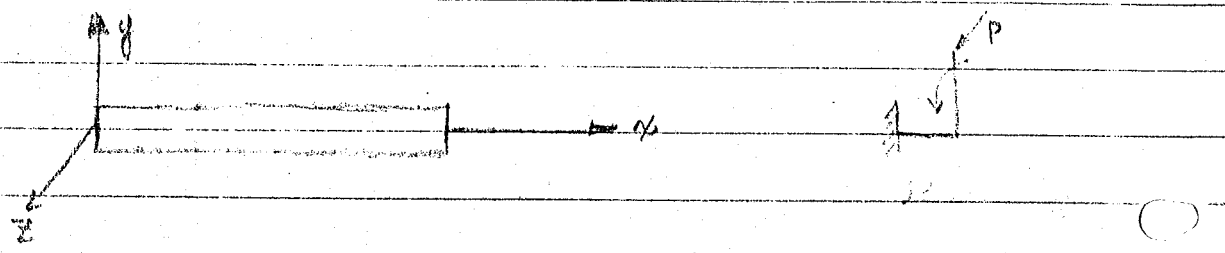
$$3/10P - 4/10P + P_1 = 0$$

$$T_1 = 1/10P$$

$$1/10\,P + 2/10\,P + 3/10\,P + 4/10\,P = P \quad \checkmark\checkmark$$

Chapter 5 and Chapter 6 - Deal with internal reactions

Torsion



1 hp. → 550 ft-lb/sec = 550×12×60 in-lb/min

If crankshaft rotates at N rpm Torque = T

$T \cdot \omega = P$ (power) $= 2\pi N T = 550 \times 12 \times 60 \times$ # hp developed

$\omega = 2\pi N$

$T = \frac{63025\,(hp)}{N}$ lb-in $\qquad T = \frac{5252 \cdot hp}{N}$ lb-ft.

$[N\text{-}m]\ T = \frac{9549 \cdot P}{N}$

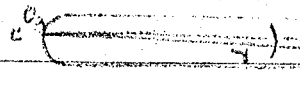
rev/min

$1 \cdot \frac{\text{rev}}{\text{min}} \cdot \frac{1\text{ min}}{60\text{ sec}} = \frac{1}{60}\ \frac{\text{rev}}{\text{sec}}$

Assumptions

1) Plane sections normal to axis remain plane after deformation
No warping or distortion

2) In circular members, shearing strains, vary linearly from central axis (only for solid or hollow circular members



3) Materials remain elastic, Hooke's Law applies, Thus stress proportional to strain.

$\gamma = \frac{\Delta s}{\Delta x} = \frac{R\Delta\theta}{\Delta x} = \frac{\tau_m}{G}$



$\frac{\tau}{\rho} = \frac{\tau_{max}}{c}$

shearing stress are in a plane parallel to the section taken normal to the axis of rotation

$\tau = \frac{\tau_{max}}{c}\rho$

- stresses not constant but max at pt most remote from center

$dT = \tau\, dA\, \rho$ (moment force acts)

$dT = \frac{\tau_{max}}{c}\rho\, dA\, \rho$ (stress, force, moment)

$T = \int \frac{\tau_{max}}{c}\rho^2\, dA = \frac{\tau_{max}}{c}\int \rho^2\, dA$

מומנט אינרציה קוטבי

J (polar mom. of inertia) $= \int_A \rho^2 dA$

$T = \frac{\tau_{max}}{c} J$ $\quad \tau_{max} = Tc/J$ torsion formula for circular shafts.
max shear

$J = \int_A \rho^2 dA$ $\quad dA = 2\pi\rho\, d\rho$

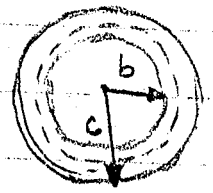


for solid circle $\quad$ ρ3IN 'vic'r 6IN

$$J = \int_0^c \rho^2 \times 2\pi\rho\, d\rho = 2\pi \left.\frac{\rho^4}{4}\right|_0^c = 2\pi \frac{c^4}{4} = \frac{\pi c^4}{2} = \frac{\pi d^4}{32}$$

$\tau = \frac{\tau_{max}\rho}{c}$ $\quad \frac{\tau_{max}}{c} = \frac{\tau c^2}{\rho c} = \frac{Tc}{J}$ $\quad \boxed{\tau = T\rho/J}$
shear stress at a pt

For circular tube :—



$$J = \int_b^c 2\pi\rho^3 d\rho = \frac{2\pi\rho^4}{4} = \frac{\pi c^4}{2} - \frac{\pi b^4}{2}$$

for a thin tube $b \cong c$ $\quad c - b = t$ is assumed

$$J = \frac{\pi}{2}(c^4 - b^4) = \frac{\pi}{2}(c^2 - b^2)(c^2 + b^2) = \frac{\pi}{2}(c^2 + b^2)(c - b)(c + b)$$

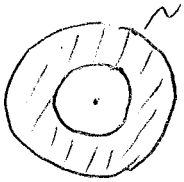
$$\cong \frac{\pi}{2} \cdot 2c^2 t \cdot 2c \cong 2c^3 t\pi$$

$$J \cong 2\pi c^3 t$$

$c = 10''$ $\quad t = 1''$ $\quad J = \frac{\pi}{2}(10^4 - 9^4) = \frac{\pi}{2}(10000 - 6561)$

$J \cong \pi\left(\frac{34.39}{2}\right) = 1719.5\pi$ $\quad J \cong 2000\pi$ $\quad$ if $c = av(c+b) = 9.5$

$J = 1714.75\pi$



$$T = \frac{\tau_{max}}{c}\int_{r_i}^{r_o} \rho^3 d\rho \cdot 2\pi = \frac{2\pi\tau_{max}}{c}\left[\frac{r_o^4}{4} - \frac{r_i^4}{4}\right]$$

$$0.96T = \frac{2\pi\tau_{max}}{c}\left[\frac{r_o^4}{4} - \frac{r_c^4}{4}\right] = \frac{2\pi\tau_{max}}{c}\frac{r_c^2}{4}\left[\frac{r_o^4}{r_c^4} - 1\right]$$

if $r_i = 0$

$$0.04T = \frac{2\pi\tau_{max}}{c}\frac{r_c^4}{4} \qquad .04T\left[\frac{r_o^4}{r_c^4} - 1\right]$$

$$24 = r_o^4/r_c^4 - 1$$

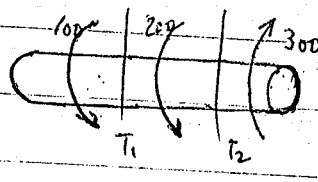
$$.04 = \left(\frac{r_c}{r_o}\right)^4 \qquad .04 = \frac{1}{25} = \left(\frac{r_c}{r_o}\right)^4$$

$$.2 = \left(\frac{r_c}{r_o}\right)^2 \qquad \frac{r_c}{r_o} = \sqrt{.2} \cong .448$$

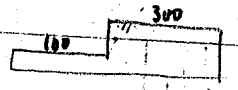
ie that from $r=0$ to $r = .448 r_o$ only 4% of is carried.

3. Apply the material properties + perhaps Hooke's Law.

Prob 1 for shafts find max shear stress



$\phi = 1/2"$



$J = \frac{\pi d^4}{32} = \frac{\pi (1/2)^4}{32} = \frac{\pi}{512}$

$T_1 = 100$

$T_2 = 300$ $\quad \tau_{max} = TJ^{-1}c = Tc/J \quad J = \frac{\pi}{512}$

$\tau_{max} = \frac{512(300)(.25)}{\pi} = 128\frac{(300)}{\pi} \approx 12223$ psi

long tube inner dia = 0.9 in, outer dia = 1 in, twisted about long axis with torque 400 in-lb

Prob 1 find stress inside & outside

1" outside dia $\quad T = 400 \quad$ find $\tau_p \; \tau_c$

.9" outside dia

$J = \frac{\pi}{32}(1^4 - .9^4) = \frac{\pi}{32}(1 - .6561) = \frac{\pi}{32}(.3439) = .0337 in^4$

$J \geq .01\pi$ $\quad$ approx $\rightarrow J_{thin\ tube} = \frac{\pi}{4} 1^3 (.05) = .0125\pi$

$\tau_{max} = \frac{400(0.5)}{.01\pi} \sim 5930$ psi $\quad \tau_p = \frac{400(0.45)}{.01\pi} \sim 5330$ psi

because τ doesn't change much for thin tube, tube has almost uniform τ. For thin tube causes buckling locally

Design

design a steel shaft for 10 hp motor running @ 1800 rpm. Max shear stress is 8000 psi

$\tau_{all} = 8000$ psi steel (annual) $\quad$ p3/N 6/N

$\frac{J}{c} = \frac{T}{\tau_{max}}$ $\quad$ for solid shaft $\frac{\pi c^3}{2} = \frac{J}{c}$

Select a shaft for a ten hp motor at 1800 rpm.
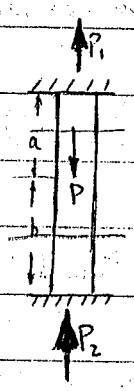
$\tau_{all} = 8000 \quad T = \frac{63000\ Hp}{N} = \frac{630000}{1800} = 350$ in-lb

p3/N 6/N $\quad J/c = \frac{350}{8000} = \frac{T}{\tau_{max}} = \frac{\pi c^3}{2} = \frac{T}{\tau_{max}} = \frac{J}{c} c^3 = \frac{700}{8000\pi} = (.303)^3$

$= .027852$

$c = .3031$ in

4-35



$P = P_1 + P_2$ - equil $\quad \Sigma F_y = 0$

$u = \frac{\sum PL}{AE} = \frac{P_1 a}{AE} - \frac{P_2 b}{AE} = 0$

$P_1 = P - P_2 \quad a = L - b \qquad P_1 = \frac{bP}{L} \quad P_2 = \frac{aP}{L}$

4-31

$\tau_{max} = \frac{Tc}{J}$ solid shaft $J = \frac{\pi d^4}{32}$

hollow shaft $J = \frac{\pi d_o^4}{32} - \frac{\pi d_i^4}{32}$

thin tube $J = 2\pi c^3 t = \frac{\pi d_o^3 t}{4}$

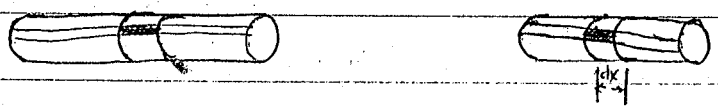
$\frac{J}{\text{Vol. unit length}} = \frac{\pi c^4/2}{\pi c^2} = \frac{c^2}{2}$

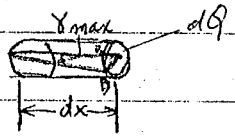
tube: $\frac{J}{V} = \frac{\pi}{2}\left(c_o^4 - c_i^4\right) \Big/ \pi\left(c_o^2 - c_i^2\right) = \frac{c_o^2 + c_i^2}{2}$

thin tube: $\frac{J}{V} = \frac{2\pi c^3 t}{2\pi c t} = c^2$ when $c_o \cong c_i \quad \frac{c_o^2 + c_i^2}{2} = \frac{2c^2}{2} = c^2$

1. Equilibrium - Method of Sections
   Internal Torques

Angle of twist — זווית הפיתול





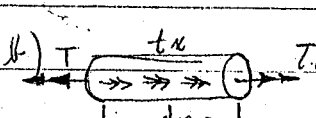
$\overline{BD} = c\,d\varphi$ $\qquad \gamma_{max}\,dx = \overline{BD}$ $\qquad \frac{d\varphi}{dx} = \frac{\gamma_{max}}{c}$

If hookes' law applies $\gamma_{max} = \frac{\tau_{max}}{G}$ $\qquad \tau_{max} = \frac{Tc}{J}$; $\gamma_{max} = \frac{Tc}{JG}$

or $\frac{d\varphi}{dx} = \frac{T}{JG}$ $\qquad \boxed{\varphi = \int_0^L \frac{T}{JG}\,dx}$

1) Suppose $\frac{T}{GJ}$ = constant $\qquad \varphi = \frac{TL}{GJ}$ $\qquad \varphi_T = \sum \frac{TL}{GJ}$

2) Suppose $\frac{T}{JG}$ not constant $\qquad \varphi = \int_0^L \frac{T(x)}{J(x)G}\,dx$



$dT = -t_x\,dx$

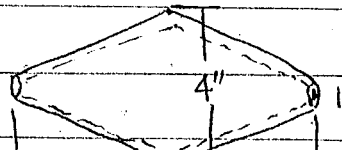
$\frac{dT}{dx} = -t_x$ $\qquad \frac{d\varphi}{dx} = \frac{T}{JG}$

$\frac{d^2\varphi}{dx^2} = \frac{1}{JG}\frac{dT}{dx} + T\frac{d}{dx}\left(\frac{1}{JG}\right)$

if $1/JG$ = const $\quad T\frac{d}{dx}(1/JG) = 0$ $\qquad \boxed{\frac{d^2\varphi}{dx^2} = \frac{-t_x}{JG}}$

$\frac{d\varphi}{dx} = -\int \frac{t_x}{JG}\,dx = -f(x) + C_1 = \frac{T}{JG}$ $\qquad$ JG - torsional stiffness of rod. קשיחות הפיתול של המוט

5-14



$\frac{d\varphi}{dx} = \frac{T}{JG}$ $\qquad t = C_0 - C_i = \frac{1}{2\pi}$ $\qquad C_i = C_0 - t$

$d_o = ax + b$ $\quad 0 \le x \le 12''$ $\quad @\ x=0\ \ d_o = 1$ $\quad @\ x=12\ \ d_o = 4$

$1 = b$

$4 = 12a + 1 \quad a = 1/4$

$$d_o = \frac{x}{4} + 4$$

$x = 12 \quad d_o = 4 \qquad 4 = 12a' + b' \qquad 3 = -12a' \quad a' = -1/4$

$x = 24 \quad d_o = 1 \qquad 1 = 24a' + b' \qquad b' = 28/4$

$$d_o = \frac{-x + 28}{4}$$

$0 \le x \le 12'' \quad d_o = \frac{x}{4} + 4 \qquad c_i = c_o - t$

$12'' + 0 \le x \le 12 + 12'' \quad d_o = -\frac{x}{4} + 7 \qquad J_x = \frac{\pi c_o^4}{2} - \frac{\pi c_i^4}{2}$

$$\frac{\pi c_o^4}{2} - \frac{\pi}{2}(c_o - t)^4 = -\frac{\pi}{2}\left(-4c_o^3 t + 6c_o^2 t^2 - 4c_o t^3 + t^4\right).$$

$$J_x = \frac{1}{4}\left(4c_o^3 + 6c_o^2 t + 4c_o t^2 - t^3\right) = c_o^3 - \frac{3}{2}c_o^2 t + c_o t^2 - \frac{1}{4}t^3$$

What must be the length of a 0.2 in diameter wire so that it could be twisted through one complete revolution without exceeding a shearing stress of 6000 psi

מהו האורך של חוט בקוטר 0.2 אינץ' כך שניתן יהיה לסובב אותו סיבוב שלם אחד מבלי לחרוג מאחץ גזירה של 6000 psi

$\tau_{max} = 6000$

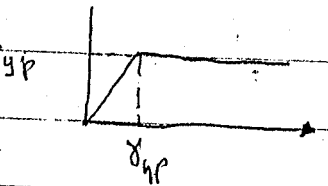
$G = 3.84 \times 10^6\ lb/in^2$

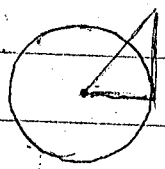
$\phi = \frac{TL}{JG} \qquad \tau_{max} = \frac{Tc}{J} \qquad \frac{\tau_{max} L}{cG} = \phi = 2\pi$

$\tau_{max} = 12\pi \qquad L = \frac{2\pi cG}{\tau_{max}} = \frac{2\pi(.1)(3.84 \times 10^6\ psi)}{6000\ psi} = 402.1\ in$

Relax some of the assumptions

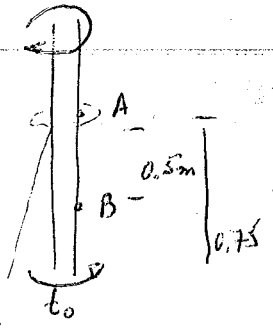
1. relax hookes law





Strain

11.14 Hibbler



A wooden post is half buried in the ground & subjected a torsional moment of 50 N-m that causes the post to rotate at constant angular velocity. This moment is resisted by a linear distribution of torque developed by the soil friction which varies from zero on the ground to to $\frac{N-m}{m}$ at its base. Find the equilib value for to & find τ @ A & B

$d = 100\ mm = .1\ m$

$t = \frac{-t_o}{0.75} y$

$\frac{dT}{dy} = -t = \frac{t_o}{0.75} y$

$T = \int \frac{t_o}{0.75} y\, dy = \frac{t_o}{0.75} \frac{y^2}{2} + C$ @ $y=0$, $T=-50$

$\frac{t_o}{0.75} \frac{(0.75)^2}{2} = \frac{3}{8} t_o = T = 50\ N/m$

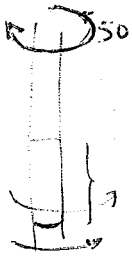
$t_o = 133.3 \frac{N-m}{m}$

$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.05)^4 = 9.817 \times 10^{-6}\ m^4$

$\tau_A = \frac{Tc}{J} = \frac{50\ N-m\ (.05\ m)}{9.817 \times 10^{-6}\ m^4}$

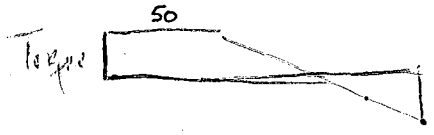
$= 254{,}660 \frac{N}{m^2} = .255\ MPa$

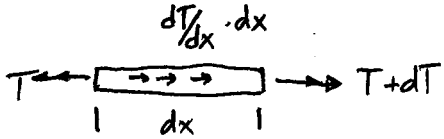
@ B $T = \frac{t_o}{0.75} \frac{(.5)^2}{2} - 50 = \frac{t_o}{6} - 50 = \frac{133.3}{6} \frac{N-m}{m} - 50 = 22.2 \frac{N-m}{m} - 50 = -27.8$



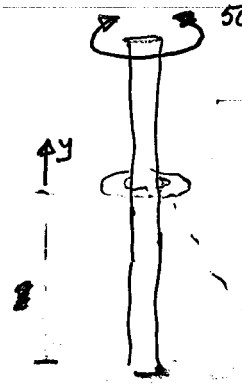
$50 - 22.2 = 27.8$

$\tau_B = \frac{Tc}{J} = \frac{27.8}{50} \cdot (.255\ MPa) = .142\ MPa$





$T + dT + t\,dx - T = 0$ $\quad dT = -\frac{dT}{dx} dx$

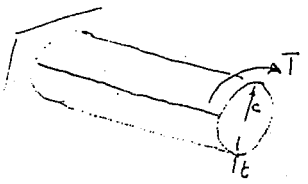


$\frac{dT}{dy} = my$

$50 + \frac{my^2}{2} = 0$

$-178 = -\frac{50.2}{.75^2} = +m$

3.132



closed tube

$$J = \frac{\pi}{2}(a^4 - b^4) = \frac{\pi}{2}(a^2 - b^2)(a^2 + b^2) \; ; \; \tau_1 = \frac{Tc}{J} \; ; \; \phi_1 = \frac{TL}{JG}$$

$$= \frac{\pi}{2}(a-b)(a+b)(a^2+b^2)$$

$$J = \pi/2 \cdot t \cdot 2c \cdot 2c^2 = 2\pi c^3 t \qquad \tau_1 = \frac{T}{2\pi c^2 t} \; ; \; \phi_1 = \frac{TL}{2\pi c^3 t G}$$

Open tube $\quad J_R = \frac{bt^3}{3} = \frac{2\pi c t^3}{3}$

$$; \; \tau_2 = \frac{Tt}{J_R} \; ; \; \beta = \frac{\phi_2}{L} = \frac{T}{GJ_R}$$

$$\tau_2 = \frac{Tt}{2\pi c t^3/3} = \frac{3Tt}{2\pi c t^3} = \frac{3T}{2\pi c t^2} \; ; \; \phi_2 = \frac{TL}{\frac{2\pi c t^3 G}{3}} = \frac{3TL}{2\pi c t^3 G}$$

$$\frac{\tau_1}{\tau_2} = \frac{\frac{T}{2\pi c^2 t} \cdot t}{\frac{3T}{2\pi c t^2} \cdot \frac{c}{c}} = \frac{t}{3c} \; ; \qquad \frac{\phi_1}{\phi_2} = \frac{\frac{TL}{2\pi c^3 t G} \cdot \frac{t^2}{t^2}}{\frac{3TL}{2\pi c t^3 G} \cdot \frac{c^2}{c^2}} = \frac{t^2}{3c^2}$$

$$\left(\frac{\tau_1}{\tau_2}\right)^2 = \frac{t^2}{9c^2} = \frac{1}{3}\left(\frac{\phi_1}{\phi_2}\right)$$

So: $\frac{\tau_2}{\tau_1} = 3\left(\frac{c}{t}\right)$ and $\frac{\phi_2}{\phi_1} = 3\left(\frac{c}{t}\right)^2$

3.132 A thin-walled tube has been fabricated by bending a metal plate of thickness t into a cylinder of radius c and bonding together the edges of the plate. A torque T is then applied to the tube, producing a shearing stress τ_1 and an angle of twist ϕ_1. Denoting by τ_2 and ϕ_2, respectively, the shearing stress and the angle of twist which will develop if the bond suddenly fails, express the ratios τ_2/τ_1 and ϕ_2/ϕ_1 in terms of the ratio c/t.

$\phi_2/\phi_1 = 3\left(\frac{c}{t}\right)^2$

Fig. P3.108 and P3.109

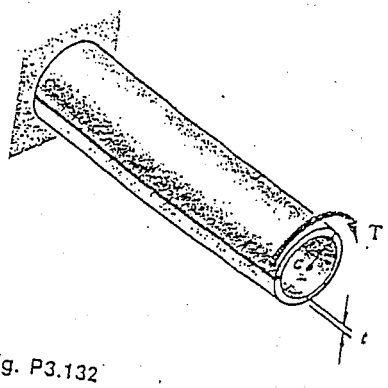


Fig. P3.132

3.122 A 3.5-m-long steel member with a W310 × 143 cross section is subjected to a 4.5 kN · m torque. Knowing that $G = 77$ GPa and referring to Appendix C for the dimensions of the cross section, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the maximum shearing stress along line *b-b*, (*c*) the angle of twist. (*Hint*: Consider the web and the flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

39.7 MPa 24.2 MPa 4.72°

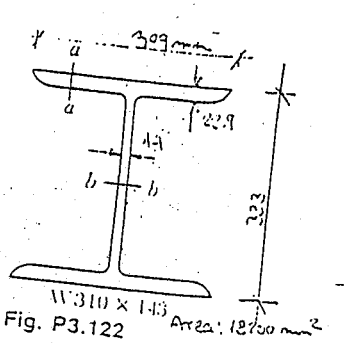
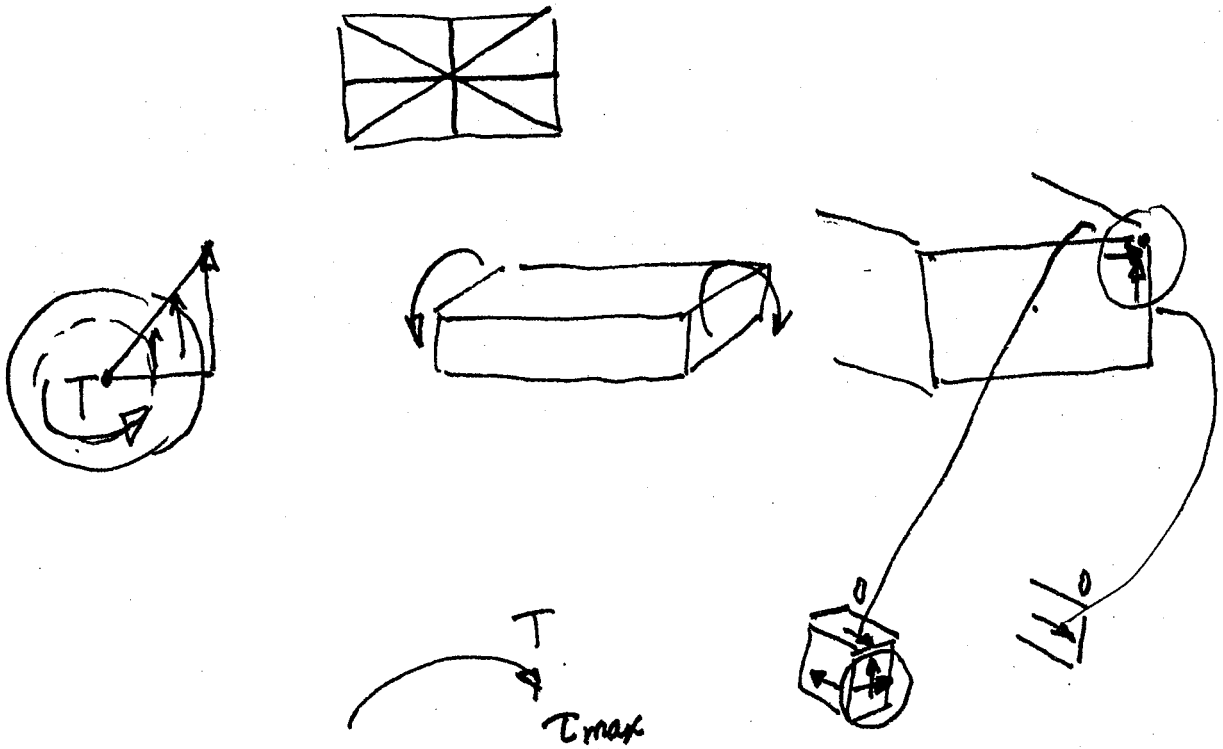
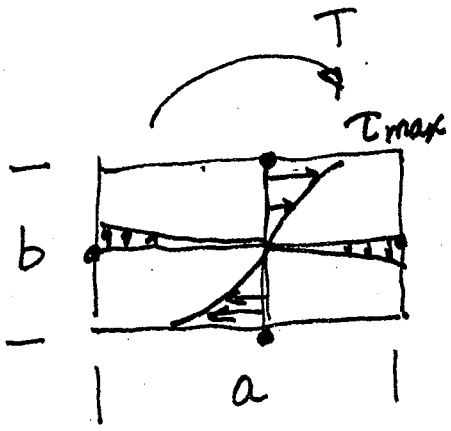


Fig. P3.122

HW: DO 3.21, 3.47 & 3.53

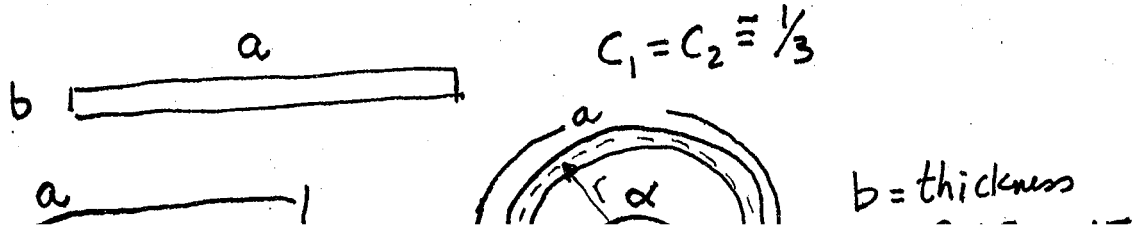




$$\tau_{max} = \frac{T}{c_1 a b^2} \qquad c_1 = f_1(a/b)$$

$$\varphi = \frac{TL}{c_2 a b^3 G} \qquad c_2 = f_2(a/b)$$

Table 2.1 pg 187

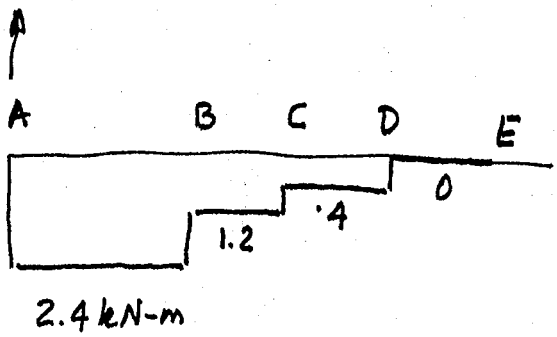


$c_1 = c_2 \equiv 1/3$

b = thickness

EMA 3702

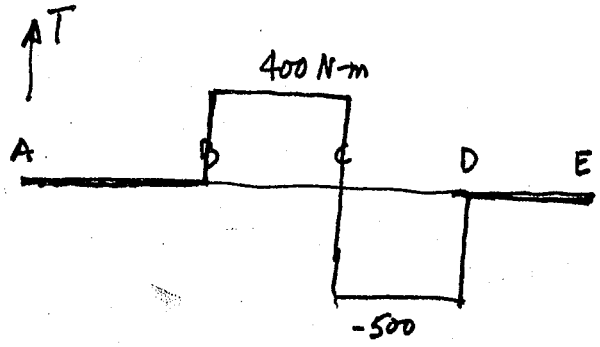
3.13



$$\tau = \frac{Tc}{J} \qquad \frac{2T}{\pi c^3}$$

$$\varphi = \frac{TL}{JG} \qquad \varphi = \int \frac{T\,dx}{JG}$$

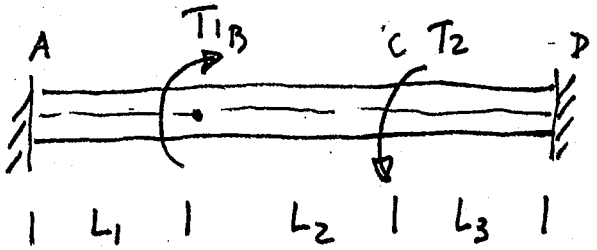
3.36

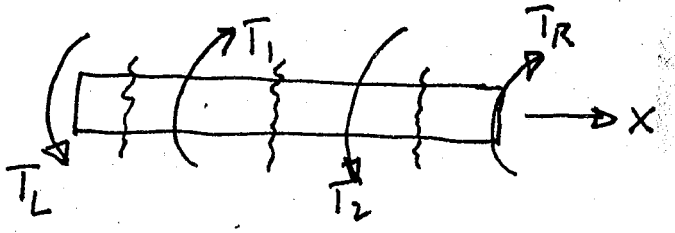


φ_{BC} ?

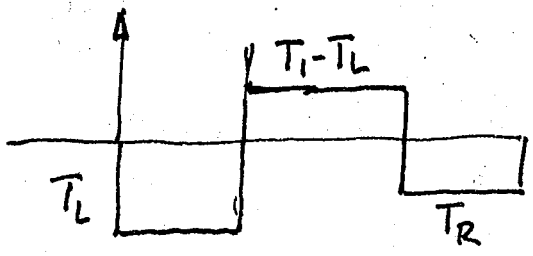
φ_{CD} ?

$\varphi_{AE} = \varphi_{AB} + \varphi_{BC} + \varphi_{CD} + \varphi_{DE}$





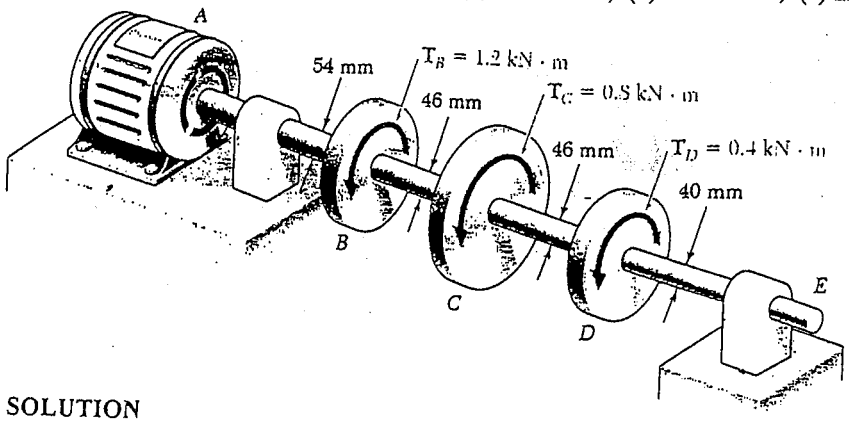
$$\Sigma T = 0 = T_L - T_1 + T_2 - T_R = 0$$



$$\Sigma \varphi = \varphi_{AB} + \varphi_{BC} + \varphi_{CD} = 0$$

PROBLEM 3.13

3.13 Under normal operating conditions, the electric motor exerts a torque of 2.4 kN·m at A. Knowing that each shaft is solid, determine the maximum shearing stress (*a*) in shaft AB, (*b*) in shaft BC, (*c*) in shaft CD.



$$J = \frac{\pi c^4}{2}$$

SOLUTION

Shaft AB: $T_{AB} = 2.4\times10^3$ N·m, $c = \frac{1}{2}d = 0.027$ m

$$\tau_{AB} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2.4\times10^3)}{\pi(0.027)^3} = 77.625\times10^6 \text{ Pa} \qquad 77.6 \text{ MPa}$$

Shaft BC: $T_{BC} = 2.4$ kN·m $- 1.2$ kN·m $= 1.2$ kN·m, $c = \frac{1}{2}d = 0.023$ m

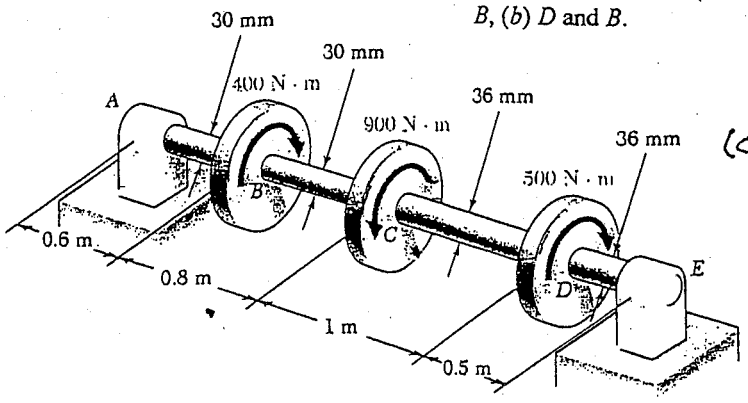
$$\tau_{BC} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1.2\times10^3)}{\pi(0.023)^3} = 62.788\times10^6 \text{ Pa} \qquad 62.8 \text{ MPa}$$

Shaft CD: $T_{CD} = 0.4\times10^3$ N·m $c = \frac{1}{2}d = 0.023$ m

$$\tau_{CD} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(0.4\times10^3)}{\pi(0.023)^3} = 20.929\times10^6 \text{ Pa} \qquad 20.9 \text{ MPa}$$

PROBLEM 3.36

3.36 The torques shown are exerted on pulleys B, C and D. Knowing that the entire shaft is made of steel (G = 27 GPa), determine the angle of twist between (*a*) C and B, (*b*) D and B.



SOLUTION

(a) Shaft BC: $c = \frac{1}{2}d = 0.015$ m

$J_{BC} = \frac{\pi}{2}c^4 = 79.522\times10^{-9}$ m^4

$L_{BC} = 0.8$ m, $G = 27\times10^9$ Pa

$$\varphi_{BC} = \frac{TL}{GJ} = \frac{(400)(0.8)}{(27\times10^9)(79.522\times10^{-9})}$$

$$= 0.149904 \text{ rad} = 8.54°$$

(b) Shaft CD: $c = \frac{1}{2}d = 0.018$ m $J_{CD} = \frac{\pi}{2}c^4 = 164.896\times10^{-9}$ m^4

$L_{CD} = 1.0$ m $T_{CD} = 400 - 900 = -500$ N·m

$$\varphi_{CD} = \frac{TL}{GJ} = \frac{(-500)(1.0)}{(27\times10^9)(164.896\times10^{-9})} = -0.11230 \text{ rad}$$

$$T_L + T_2 - T_1 - T_R = 0$$

$$-T_L \cdot L_1 + (T_1 - T_L) L_2 - T_R \cdot L_3 = 0 \Rightarrow -T_L(L_1+L_2) + T_1 L_2 - T_R L_3 = 0$$

$$T_L - T_R = T_1 - T_2$$

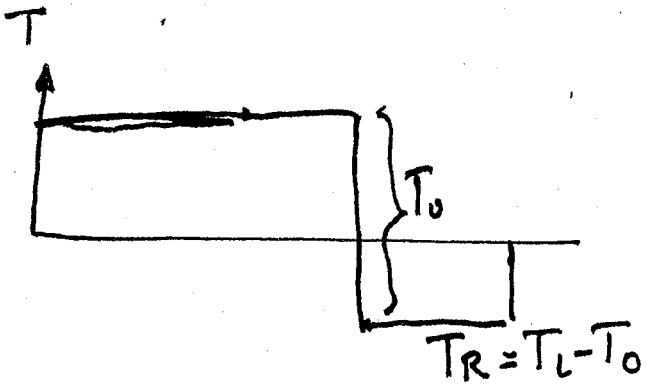
$$-T_L(L_1+L_2) - T_R \cdot L_3 = T_1 L_2$$

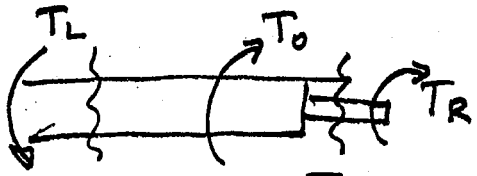
$$T_L = \frac{\begin{vmatrix} T_1 - T_2 & -1 \\ T_1 L_2 & -L_3 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -(L_1+L_2) & -L_3 \end{vmatrix}} = \frac{T_1\left(\frac{L_2+L_3}{L}\right) - T_2\left(\frac{L_3}{L}\right)}{L = L_1 + L_2 + L_3}$$

$$T_R = -T_1\left(\frac{L_1}{L}\right) + T_2\left(\frac{L_1+L_2}{L}\right)$$

$$\varphi_{AB} = \frac{T_{AB} L_1}{J_{AB} G} = \frac{-T_L \cdot L_1}{J_{AB} \cdot G} = [\text{rad.}] \qquad \text{rad} \cdot \frac{180^\circ}{\pi} = (\ )^\circ$$

6.96





$$T_R + T_0 - T_L = 0$$

$$T_0 = T_L - T_R$$

welded together at B as shown in Fig. P6.96. The twisting couples at A and C are equal in magnitude when the couple T_0 is applied at B. Determine the relation between the elastic moduli of the two materials.

Answer: $G_A = 0.592 G_C$

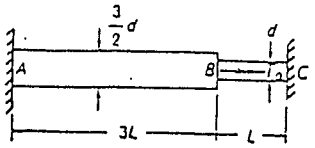


Fig. P6.96

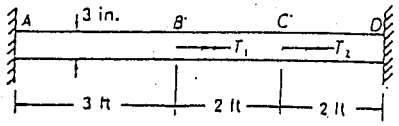


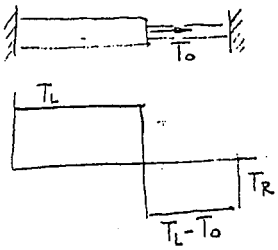
Fig. P6.97

6.97. An aluminum shaft is rigidly restrained at each end and loaded by couples as shown in Fig. P6.97. If $T_1 = 12{,}000$ in.-lb and $T_2 = 10{,}000$ in.-lb; compute the maximum shear stress in each section and the rotations of cross sections at B and C.

Answer: $\tau_{CD} = -2300\ psi, \quad \tau_{BC} = -433\ psi, \quad \tau_{AB} = 1840\ psi,$

A

6.96



$T_L - T_R = T_o$

$T_R = T_L - T_o$

$|T_R| = |T_L|$ ∴

$T_L = T_o/2 \quad T_R = -T_o/2$

$$\varphi_{AB} = \frac{T_L L_{AB}}{J_{AB} G_{AB}} = |\varphi_{BC}| = \left|\frac{T_R L_{BC}}{J_{BC} G_{BC}}\right|$$

$$J_{AB} = \frac{\pi c^4}{2} = \frac{\pi d^4}{32} = \frac{\pi \cdot 81 d^4}{512}; \quad L_{AB} = 3L;$$

$$J_{BC} = \frac{\pi c^4}{2} = \frac{\pi d^4}{32}; \quad L_{BC} = L$$

$$\varphi_{AB} = \frac{T_o/2 \cdot 3L}{\pi 81 d^4/512\, G_{AB}} = \frac{T_o/2 \cdot L}{\frac{\pi d^4}{32} G_{BC}}$$

$$\therefore \frac{G_{AB} \cdot 81}{1536} = \frac{G_{BC}}{32} \quad \text{or} \quad G_{AB} = \frac{1536}{81 \cdot 32} G_{BC} = .5926 G_{BC}$$

Fig. P3.108 and P3.109

3.132 A thin-walled tube has been fabricated by bending a metal plate of thickness t into a cylinder of radius c and bonding together the edges of the plate. A torque T is then applied to the tube, producing a shearing stress τ_1 and an angle of twist ϕ_1. Denoting by τ_2 and ϕ_2, respectively, the shearing stress and the angle of twist which will develop if the bond suddenly fails, express the ratios τ_2/τ_1 and ϕ_2/ϕ_1 in terms of the ratio c/t.

$\phi_2/\phi_1 = 3\left(\frac{c}{t}\right)^2$

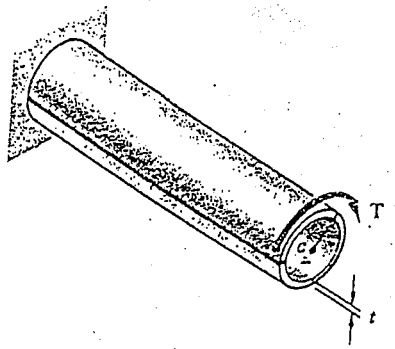
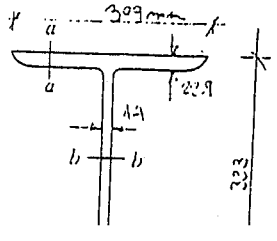
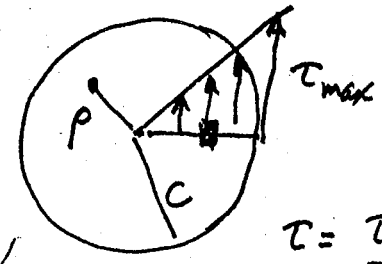
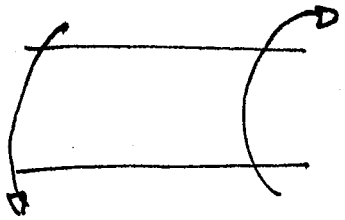


Fig. P3.132

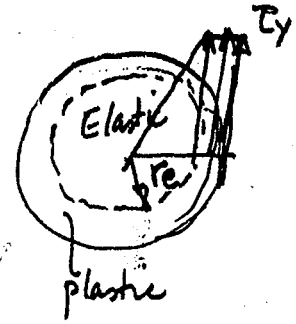
3.122 A 3.5-m-long steel member with a W310 × 143 cross section is subjected to a 4.5 kN · m torque. Knowing that $G = 77$ GPa and referring to Appendix C for the dimensions of the cross section, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the maximum shearing stress along line *b-b*, (*c*) the angle of twist. (*Hint*: Consider the web and the flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting

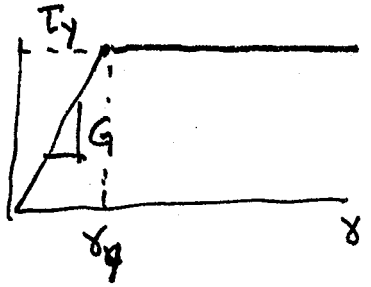






$$\tau = \frac{\tau_{max}}{c} \cdot \rho$$





$\tau = G\gamma$

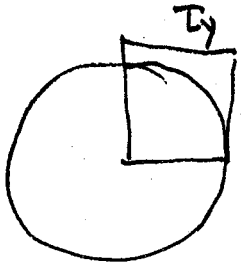
elasto plastic model

$$T = \int \tau \, dA \cdot \rho = \int \frac{\tau_{max}\,\rho}{c} \cdot \rho d\rho d\theta \cdot \rho = \frac{\tau_{max}}{c} \int_0^c \rho^3 d\rho \cdot \int_0^{2\pi} d\theta$$

$$= \frac{\tau_{max}}{c} \cdot \frac{c^4}{4} \cdot 2\pi$$

$$T = \tau_{max} \frac{c^3 \pi}{2}$$

$$T = \int_0^{r_e} \frac{\tau_{max}}{c} \cdot \rho \cdot \rho d\rho d\theta \cdot \rho + \int_{r_e}^{c} \tau_y \cdot \rho d\rho d\theta \cdot \rho$$



$$T_p = \int_0^c \tau_y \cdot \rho d\rho d\theta \cdot \rho$$

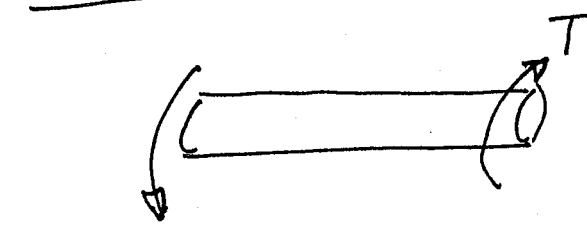
$$= \tau_y \int_0^c \rho^2 d\rho \int_0^{2\pi} d\theta$$

$$= \tau_y \frac{\rho^3}{3} \Big|_0^c \cdot 2\pi = \tau_y \cdot \frac{2\pi c^3}{3}$$

$$T \left(\text{if } \tau = \tau_y \right) = \tau_{max} \cdot c^3 \pi - \tau_y \cdot c^3 \pi$$

EMA 3702 - PT2 (2/14/02)

TORSION



$$P = \underline{F} \cdot \underline{v}$$
$$= \underline{T} \cdot \underline{\omega} = \underline{T} \cdot 2\pi N$$



$$T = 63025 \cdot \frac{P}{N}$$

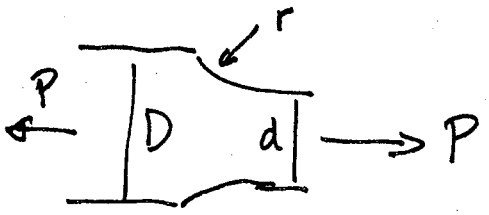
T [lb-in]
P [hp]
N [rpm]

$$T = \frac{9540\ P}{N}$$

T = [N-m]
P [kw]
N [rev per min]

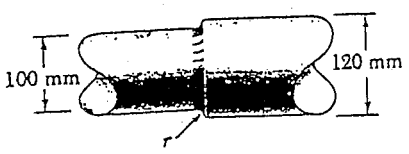
$$Hz \quad [cyc/sec] \times \frac{60 sec}{min} = cyc/min \times \frac{2\pi\ rad}{cyc.}$$

$$\tau = \frac{Tc}{J}$$



$$\sigma_{max} = K \sigma_{ave}$$
$$K = f(D/d, r/d)$$

PROBLEM 3.87



3.87 The stepped shaft shown rotates at 450 rpm. Knowing that $r = 10$ mm, determine the maximum power that can be transmitted without exceeding an allowable shearing stress of 45 MPa.

SOLUTION

$d = 100$ mm, $D = 120$ mm, $r = 10$ mm

$\frac{D}{d} = \frac{120}{100} = 1.2$, $\frac{r}{d} = \frac{10}{100} = 0.10$, From Fig. 3.32 $K = 1.33$.

For smaller shaft $c = \frac{1}{2}d = 0.050$ m $\qquad \tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.050)^3 (45 \times 10^6)}{(2)(1.33)} = 6.643 \times 10^3 \text{ N-m}$$

$f = 450$ rpm $= 7.5$ Hz $= 450/60$

Power $P = 2\pi f T = 2\pi (7.5)(6.643 \times 10^3) = 313 \times 10^3$ W $= 313$ kW

$$T = \frac{9540 P}{N} \Rightarrow P = \frac{T \cdot N}{9540} = 313.35 \text{ kW}$$

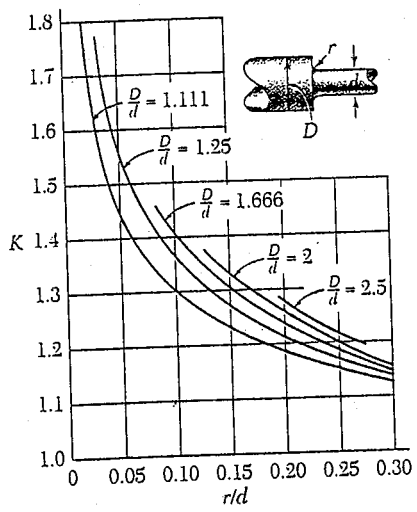


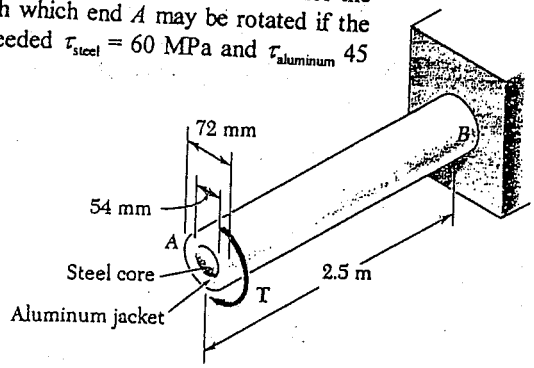
Fig. 3.32 Stress-concentration factors for fillets in circular shafts.†

TABLE 3.1. Coefficients for Rectangular Bars in Torsion

a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

PROBLEM 3.53

3.53 The composite shaft shown is to be twisted by applying a torque T at end *A*. Knowing that the modulus of rigidity is 77 GPa for the steel and 27 GPa for the aluminum, determine the largest angle through which end *A* may be rotated if the following allowable stresses are not to be exceeded $\tau_{steel} = 60$ MPa and $\tau_{aluminum}$ 45 MPa.



SOLUTION

$\tau_{max} = G\gamma_{max} = G c_{max} \frac{\varphi}{L}$

$\frac{\varphi_{all}}{L} = \frac{\tau_{all}}{G c_{max}}$ for each material

Steel core: $\tau_{all} = 60\times10^{6}$ Pa, $c_{max} = \frac{1}{2}d = 0.027$ m, $G = 77\times10^{9}$ Pa

$$\frac{\varphi_{all}}{L} = \frac{60\times10^{6}}{(77\times10^{9})(0.027)} = 28.860\times10^{-3} \text{ rad/m}$$

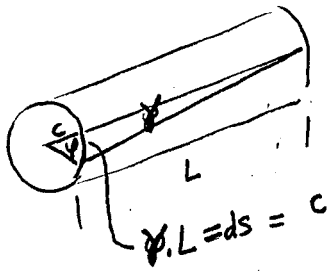
Aluminum jacket: $\tau_{all} = 45\times10^{6}$ Pa, $c_{max} = \frac{1}{2}d = 0.036$ m, $G = 27\times10^{9}$ Pa

$$\frac{\varphi_{all}}{L} = \frac{45\times10^{6}}{(27\times10^{9})(0.036)} = 46.296\times10^{-3} \text{ rad/m}$$

Smaller value governs $\frac{\varphi_{all}}{L} = 28.860\times10^{-3}$ rad/m

Allowable angle of twist $\varphi_{all} = L\frac{\varphi_{all}}{L} = (2.5)(28.860\times10^{-3})$

$= 72.15\times10^{-3}$ rad $= 4.13°$



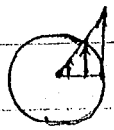
$\gamma \cdot L = ds = c\cdot\varphi$

$\varphi = \frac{\gamma\cdot L}{c} = \frac{\tau_{max}\cdot L}{G\cdot c}$

$\therefore \frac{\varphi}{L}$ must be same for both

Case I $\gamma_{max} \leq \gamma_{yp}$ $\tau_{max} \leq \tau_{yp}$

already discussed.



$\tau_{max} \rightarrow \tau_{yp}$

Telastic = max. stress reaches yield pt $T_{el} = \frac{\tau_{yp} J}{c}$

Case II $\gamma_{max} > \gamma_{yp}$ $\tau_{max} = \tau_{yp}$



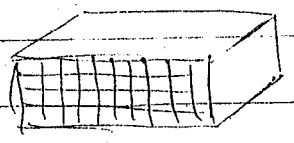
$$T = \int_A \tau dA\rho = \int_r^c \tau_{yp} 2\pi\rho^2 d\rho + \int_0^r \frac{\tau_{yp}}{r} \rho^2 2\pi\rho d\rho$$

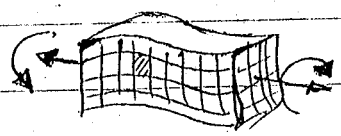
limiting case $r=0$ $T_{plastic} = \int_0^c \tau_{yp} \cdot 2\pi\rho^2 d\rho = \tau_{yp} \frac{2\pi c^3}{3} = \tau_{yp} \frac{4}{3} \frac{\pi c^3}{2}$

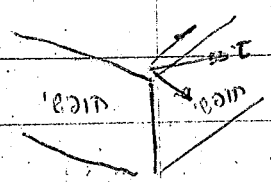
$$= \frac{4}{3} \frac{\tau_{yp}}{c} \cdot \frac{\pi c^4}{2} = \frac{4}{3} \frac{\tau_{yp} J}{c}$$

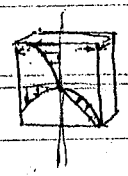
$$T_{plastic} = \tau_{yp} \frac{4}{3} \frac{J}{c}$$

Solid non-circular member

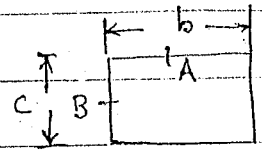








$$\tau_{max} = \frac{T}{\alpha b c^2} \qquad \varphi = \frac{TL}{\beta b c^3 G}$$



$\alpha = f(b/c)$

$$\tau_A = \frac{T}{\alpha b c^2} \qquad \tau_B = \frac{T}{\alpha b^2 c}$$

($\frac{du}{}$ a contains x & constant ter

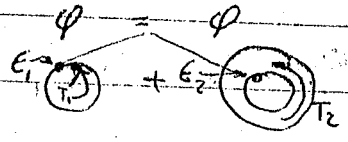
$$\frac{dx}{(ax+b)^4 - c^4} = \frac{1}{a}\int \frac{a\,dx}{(ax+b)^4 - c^4} = \frac{1}{a}\int \frac{du}{u^4 - c^4}$$

let $u = ax - b$

$du = a\,dx$

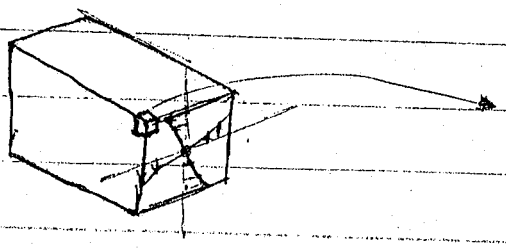
19 

$T_1 + T_2 = T$



$\epsilon_1 = \epsilon_2$

Solid Noncircular Members



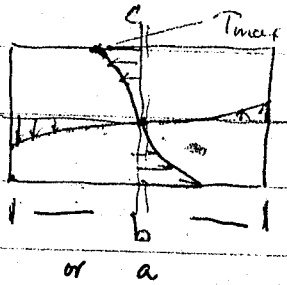
Can resolve shear into components but shears come in pairs. These pairs must be in full symmetry → $\tau = 0$ at corner



Stress at corner = 0

Strain at corner = 0

All assumptions are not valid.



$$\tau_{max} = \frac{T}{\alpha b c^2} \qquad \alpha = f(b/c)$$

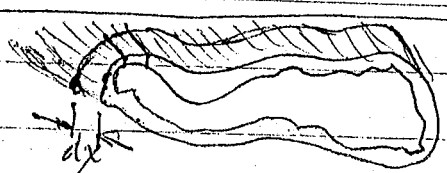
$$c_1 a b^2$$

$$\varphi = \frac{TL}{G(\beta b c^3)} \qquad \beta = f(b/c)$$

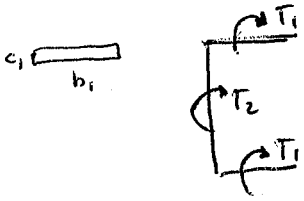
$$c_2 a b^3$$

Table on page 167.

Thin-walled hollow members



Open Sections only — Membrane Analogy — angle of each section the same; each section produces torque



$$2T_1 + T_2 = T$$

$$\phi = \frac{T_1 L}{\beta_1 b_1 c_1^3 G} = \frac{T_2 L}{\beta_2 b_2 c_2^3 G}$$

$$T_1 = \frac{\beta_1}{\beta_2}\left(\frac{b_1}{b_2}\right)\left(\frac{c_1}{c_2}\right)^3 T_2$$

$$\left[2\left\{\frac{\beta_1}{\beta_2}\left(\frac{b_1}{b_2}\right)\left(\frac{c_1}{c_2}\right)^3\right\} + 1\right] T_2 = T$$

$$T_1 = (T - T_2)/2$$

$$\tau = \frac{Tt}{J_R}$$ always irrespective of T_1, T_2

$$\tau_1 = \frac{T_1 t_1}{\beta_1 b_1 c_1^3}$$

$$\tau_F = \frac{T_F}{c_1 a b^2} = \frac{T_F b}{c_1 a b^3} = \frac{T K_F \cdot b}{(2K_F + K_W)\, c_1 a b^3}$$

$$\tau_f = \frac{T \cdot b_f}{J}$$

$$\tau_1 = \frac{T_1 t_1}{J_{R_1}}$$

T_1

$$\phi = \frac{T_1 L}{J_{R_1} G} = \frac{T_2 L}{J_{R_2} G}$$

$$T_2 = \frac{T_1 J_{R_2}}{J_{R_1}} \qquad \frac{J_{R_1} T_2}{J_{R_2}} = T_1$$

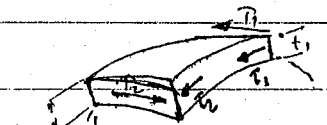
$$\tau = \frac{Tt}{J_R} = \frac{2T_1 + T_2}{J_R} t = \frac{2\left(\frac{J_{R_1}}{J_{R_2}} T_2\right) + T_2}{J_R} t$$

$$= \frac{(2J_{R_1} + J_{R_2})/J_{R_2} \cdot T_2 \cdot t}{J_R}$$

$$\tau = \frac{T_2 t}{J_{R_2}}$$

$$\frac{Tt}{J_R} = \frac{(2T_1 + T_2)t}{J_R} = (2T_1 + T_1 \frac{J_{R_2}}{J_{R_1}})\, t/J_R$$

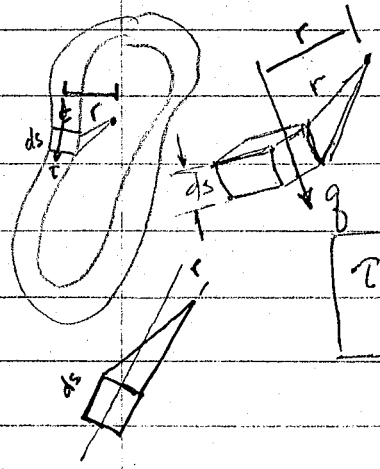
$$= (2J_{R_1} + J_{R_2}) T_1/J_{R_1} \cdot t/J_R$$



since t is small assume that τ is constant across thickness

$$\tau_2(t_2\,dx) - \tau_1(t_1\,dx) = 0$$

$\tau_1 t_1 = \tau_2 t_2 = q$ constant about perimeter. (shear flow) lb/in

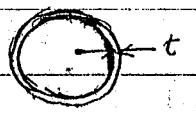


$$\oint q\,ds\,r = T \qquad q\oint r\,ds = T = q\,2A_{enclosed}$$

$$\boxed{\tau = \frac{T}{2A_{enc}t}}$$

$$\boxed{J = 2\pi c^3 t \qquad T = Tc/J \qquad \frac{T}{2\pi c^2 t} = \tau \;\checkmark}$$

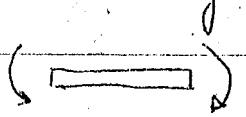
$$\tau = \frac{T}{2A_{enc}t} = \frac{T}{2(\pi c^2)t} \;\checkmark$$

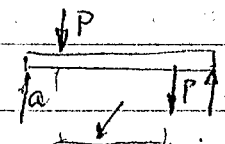


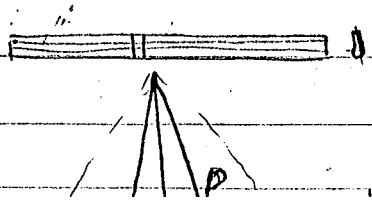
Chapt 6 Flexural (Bending) Stresses in Beams

Straight beams. (homogeneous)

Pure bending = $V = 0 \Rightarrow M$ = constant







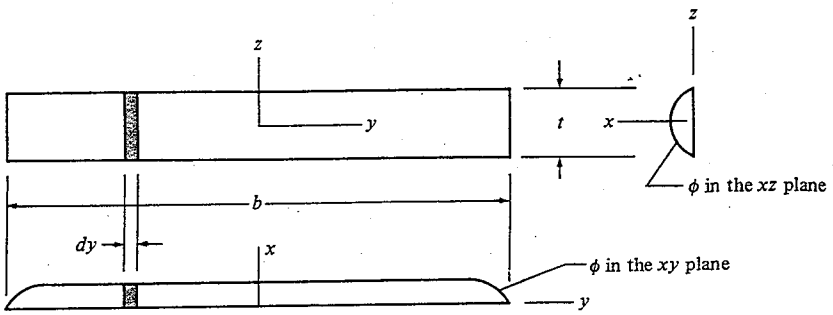


FIGURE 8.4.2. Narrow rectangular cross section ($b >> t$), with views of the stress function (or inflated membrane).

where C is a constant. Substituting into Eq. 8.3.5, we find

$$-2C = -2G\beta \qquad \text{so} \qquad C = G\beta \tag{8.4.4}$$

The torque is

$$T = 2\int_{\text{area}} \phi \, dA = 2\int_{-t/2}^{t/2} \phi b \, dz = G\beta \frac{bt^3}{3} \tag{8.4.5}$$

The maximum shear stress, found along the edges $z = \pm t/2$, is

$$\tau = \left|\frac{\partial \phi}{\partial z}\right|_{z=\pm t/2} = 2C\frac{t}{2} = G\beta t \tag{8.4.6}$$

Substituting for β from Eq. 8.4.5, we find

$$\tau = \frac{Tt}{bt^3/3} = \frac{3T}{bt^2} \qquad \text{at} \quad z = \pm\frac{t}{2} \tag{8.4.7}$$

Equations 8.4.5 and 8.4.7 can be written in the forms

$$\frac{d\theta}{dx} = \beta = \frac{T}{GJ_R} \qquad \tau = \frac{Tt}{J_R} \tag{8.4.8}$$

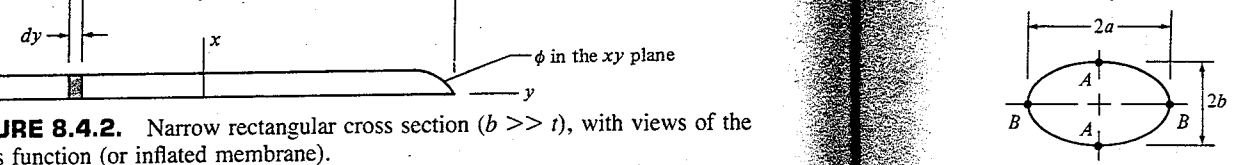
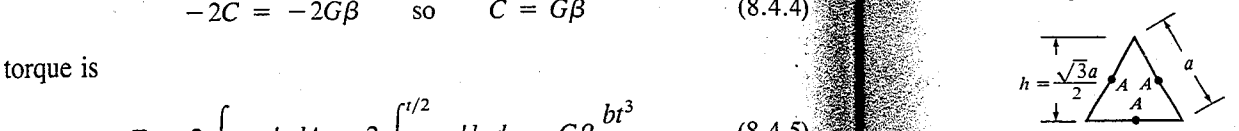
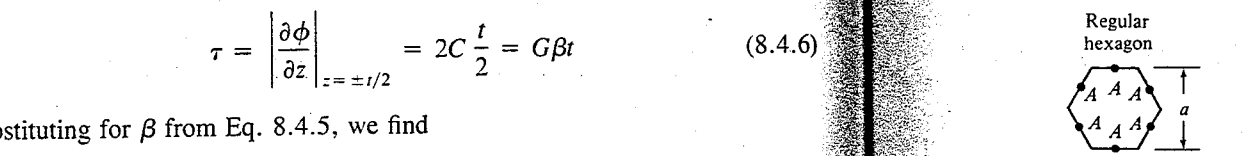
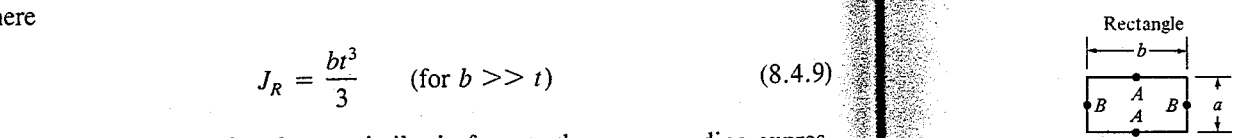
where

$$J_R = \frac{bt^3}{3} \qquad (\text{for } b >> t) \tag{8.4.9}$$

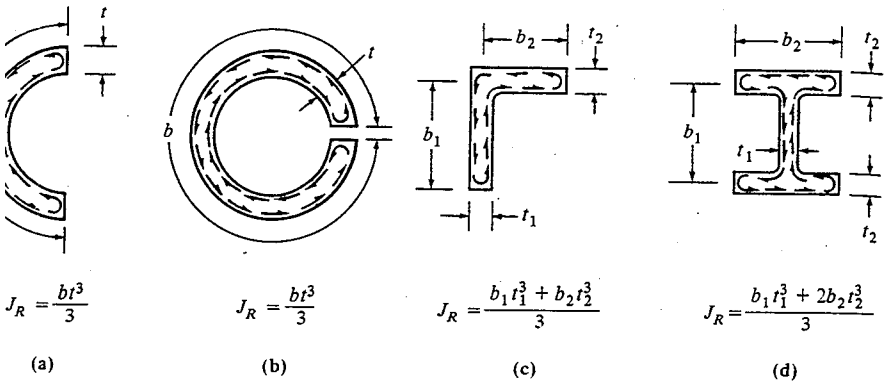
These expressions for β and τ are similar in form to the corresponding expressions for a circular cross section, Eqs. 8.1.2 and 8.1.3. However, J_R is *emphatically NOT* the polar moment of the cross-sectional area about the centroidal x axis.

If (say) $b/t = 10$, Eqs. 8.4.8 give β and τ values that are approximately 6.5 percent low. Accuracy improves as b/t increases. Aspect ratios in the range $1 < b/t < 10$ can be analyzed with tabulated data obtained by other analytical or numerical methods (Table 8.4.1). If the centerline of the bar is curved, as

TABLE 8.4.1 Expressions for Maximum Shear Stress and Rate of Twist in Selected Solid Sections [8.1, 8.4]

Cross Section and Area	Maximum Shear Stress	Rate of Twist
Ellipse  Area = πab	$\tau_A = \frac{2T}{\pi ab^2}$ $(a > b)$ (τ_{max} at B if $b > a$)	$\beta = \frac{a^2 + b^2}{\pi a^3 b^3}\frac{T}{G} = \frac{d\theta}{dx}$
Equilateral triangle  Area = $0.433\,a^2$	$\tau_A = \frac{20T}{a^3}$	$\beta = \frac{46.2}{a^4}\frac{T}{G} = \frac{d\theta}{dx}$
Regular hexagon  Area = $0.866a^2$	$\tau_A = \frac{5.7T}{a^3}$	$\beta = \frac{8.8}{a^4}\frac{T}{G} = \frac{d\theta}{dx}$
Rectangle  Area = ab	$\tau_A = \frac{T}{C_\tau ba^2}$	$\beta = \frac{1}{C_\beta ba^3}\frac{T}{G} = \frac{d\theta}{dx}$

b/a	C_τ	τ_B/τ_A	C_β
1.0	0.208	1.000	0.1406
1.2	0.219	0.935	0.166
1.5	0.231	0.859	0.196
2.0	0.246	0.795	0.229
2.5	0.258	0.766	0.249
3.0	0.267	0.753	0.263
4.0	0.282	0.745	0.281
6.0	0.299	0.743	0.299
10.0	0.312	0.742	0.312
∞	0.333	0.742	0.333



$J_R = \frac{bt^3}{3}$ (a) $J_R = \frac{bt^3}{3}$ (b) $J_R = \frac{b_1 t_1^3 + b_2 t_2^3}{3}$ (c) $J_R = \frac{b_1 t_1^3 + 2b_2 t_2^3}{3}$ (d)

JRE 8.4.3. Thin-walled open sections, showing their shear stresses and torsional con- J_R. In (a) and (b), dimension b is the length of the midline of the cross section.

when a bar is bent to form a helical spring, some corrections to these formulas may be necessary [8.3].

The membrane analogy permits a very useful generalization of the foregoing results to other thin-walled open cross sections. Imagine that the narrow rectangular cross section of Fig. 8.4.2 is distorted into a C or an L shape, or attached to another narrow rectangle to make a T or an I section. By visualizing the inflated membranes for these shapes, we decide that ϕ surfaces of all of them remain parabolic (excepting near ends and near reentrant corners, which we discuss later). The total torque is the sum of torques carried by each part of the cross section. Since $G\beta$ is the same for each part, Eqs. 8.4.5 and 8.4.8 still apply but with the $bt^3/3$ contributions summed to yield

$$J_R = \frac{1}{3}\sum_{i=1}^{n} b_i t_i^3 \tag{8.4.10}$$

where n is the number of parts into which the cross section is divided for purposes of calculation. Examples of this calculation appear in Fig. 8.4.3. In angle and I sections, the part of greater thickness displays greater shear stress. Where there is taper, as in the cross section of a turbine blade or in flanges of a rolled section, we can use

$$J_R = \frac{1}{3}\int t^3\, ds \tag{8.4.11}$$

where ds is an increment of length along the medial line of the cross section. More exact formulas are available [8.3]. These formulas, experimental results [8.5], and coefficient C_β in Table 8.4.1 suggest that, for standard rolled structural shapes, J_R may actually be some 10 percent higher than predicted by Eqs. 8.4.10 and 8.4.11. Omission of this adjustment yields a conservative design under either a stress limit or a deflection limit.

8.5 A HYDRODYNAMIC ANALOGY

There are several hydrodynamic analogies to the torsion problem [8.1]. In this section we outline one, without proof, and use it to draw useful conclusions about shear stresses in twisted bars.

If a fluid without viscosity executes motion in the yz plane with constant vorticity, its motion is described by the equation

$$\frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = -2\omega \tag{8.5.1}$$

where ϕ is the stream function and ω is the (constant) vorticity. Fluid velocities v and w in y and z directions, respectively, are

$$v = \frac{\partial\phi}{\partial z} \qquad w = -\frac{\partial\phi}{\partial y} \tag{8.5.2}$$

Clearly, Eqs. 8.5.1 and 8.5.2 are analogous to Eqs. 8.3.5 and 8.3.1, respectively. Thus fluid velocities are proportional to shear stresses. Lines ϕ = constant are streamlines. The fluid boundary must be a streamline, therefore the boundary condition $d\phi = 0$ of Eq. 8.3.6 is met. Thus the analogy is complete.

Experiments have been done in the following way. Imagine that a square cross section is to be studied. A shallow square tank is painted black and placed on a turntable. A camera, looking down onto the tank, is also attached to the turntable. The tank is filled with water (whose viscosity is low, if not quite zero) and aluminum powder is sprinkled on the water. When all is quiet, the camera shutter is opened while the turntable is rotated about 10°. From the viewpoint of the photograph, the tank is stationary while the fluid rotates with nearly constant vorticity. The aluminum particles show as streaks on the film. Each streak is in the direction of a shear stress. The length of the streak is proportional to the magnitude of the shear stress.

Experiments aside, the hydrodynamic analogy has other uses. One is in aiding visualization of torsion problems. Another is that known solutions for fluid flow can be applied to the torsion problem. For example, consider an elliptical obstacle (Fig. 8.5.1a). Far from the obstacle the flow is uniform and horizontal, so $w = 0$ and $v = v_o$, a constant. Points A are stagnation points, where $v = w = 0$. Fluid theory shows that at points B, where $z = \pm b$, fluid velocities are $v = v_o(1 + b/a)$ and $w = 0$. Applying these results to a twisted bar, Fig. 8.5.1b, we conclude that stress is zero at the sharp external corners C. At the root of the small elliptical notch, shear stress is approximately

$$\tau = \tau_o\left(1 + \frac{b}{a}\right) \tag{8.5.3}$$

where τ_o is the stress that would prevail near the boundary if the notch were not there. The calculation is approximate because the undisturbed stress is not the uniform value τ_o unless the notch is very small. The quantity in parentheses

פרק 9 פתרון

9.1 מבוא

במסגרת הפרקים הקודמים דנו במצבי המאמצים הנוצרים מהשפעה בודדת כגון מצבי הערווי הנדרש. שם התרכזנו בהשפעה אחת ולכן - השפעה המומנט פתרון וגם התרכזנו בקביעת שלמות שלמות מלאות וזו תלולית

9.2 נושא שיטת החתכים

לשם מציאת שקולי המומנט הפועל בחתכ שלם הנתון לפתרון "נחתוך" ונת המגל למשל בחתך A

$\sum M_x = 0$ תלקטמוין $30 - 20 - M_A = 0$

$M = 10\,Nm$ מומנט שיחידות הפעלין (-)

לומר מומנט הפתרון הפועל על פני חתך

והמוט שהוא בתרכז לשקולי המומנטים החיצוניים הפועלים על הגוף להפוקלו במשמעות בצורה פשוטה נתן לאפיין ולת פלוש מומנטים הפתרון לעורך בל הצל ה"ל :

9.3 הגדרת הסימן בפתרון מוט עגול

בנוכי לקבוע ולת התשר בין מומנטים הפתרון בחתך לבין פלוש המומנטים היונדרוש וזהו יש צורך להגדיר מס' הגדרות יסוד בנוסף להגדרת ההומוגניות והאיזוטרופיות של החומר:

1. חתכי רותב נדרוש לצורך המוט ושהו הוא מאורוש לסני הפעלת מומנטים הפתרון שמאורוש ומאורוש ולתרי הפעלת המומנטים.
ויון בל עוד שלל המאורוש הפוקבולוש ה"ל ב ולתך בין מתאבה מותם למשנהו.

ב. ערכו התצורה של משתנה לוקליות המון הנקודה $M=0$ מאיכל הצל
לציר הסיבוב על שפת המוט
כלומר ומומנט שקיבלנו מסתובבות
למשוואות נקודות ישרות.

* הסיבה מצלבת מולות על בניות הסיבוב מ-x שונים

ג. נניח כי התומר לוקלי וולגי מתקיים הקשר

$$\frac{\tau}{G} = \gamma$$

9.4 <u>נוסחת הפתל</u>

מתוך הנחה γ כי נובע כי מומנט
התצורה משתנה לוקליות מהמרכל התוצה

$$\tau = \frac{r}{R}\cdot\tau_{max}$$

נתשה את המומנט שהגרסם פלוס צה ונשלחו למומנט הפתל התוצן הפועל
על המוט T

$$T = \int_A \underbrace{\underbrace{\tau_{max}\frac{r}{R}}_{\text{מומנט}}\cdot\underbrace{dA}_{\text{שטח}}\cdot r}_{\text{כח}} \qquad (9.1)$$

$$T = \frac{\tau_{max}}{R}\int_A r^2 dA$$

לאחר שקיבלנו קבועים מומנט הוולגיה הפולרי של החתך I_p.
ערכו מוט בעל חתך מעגלי יהיה ערכו

$$dA = r\,dr\,d\theta$$

$$\int_0^R\int_0^{2\pi} r^3\,dr\,d\theta = \frac{\pi R^4}{2} = \frac{\pi D^4}{32} \qquad (9.2)$$

ומכאן ש:

$$\tau_{max} = \frac{T \cdot R}{I_p} \qquad (9.3)$$

$$\tau(r) = \frac{\tau_{max} \cdot r}{R} = \frac{T r}{I_p} \qquad (9.3.a)$$

נוסחאות (9.3) ו- (9.3a) תקפות לחתך עגול מלא ולחתך עגול חלול בעל דופן דקה.

לחישוב I_p עבור חתך חלול:

$$I_p = \frac{\pi R_o^4}{2} - \frac{\pi R_i^4}{2} \simeq 2\pi \bar{R}^3 t \qquad (9.4)$$

$$\bar{R} = \frac{R_o + R_i}{2} \qquad t = R_o - R_i$$

הוכיחו שמתקיים כי עקב התנהגות החומר בתחום האלסטי הליניארי החומר החזק מן השניים נתון על ידי $R_o = 1$ ועשוי את קוטר שווה ל- $R_i = 0.9$; $R_o = 1$ עבור החומר הרך יותר.

$$(I_p)_1 = \frac{\pi R^4}{2} = \frac{\pi}{2} \qquad (I_p)_2 = 2\pi R_o^3 \cdot 0.1 = 0.2\pi$$

יחס המשקלים שווה ליחס השטחים

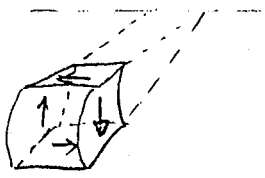
$$\frac{mg_1}{mg_2} = \frac{\pi R_o^2}{2\pi R_o t} = \frac{1}{2 \times 0.1} = 5$$

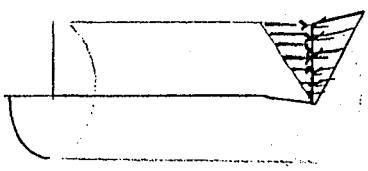
יחס המומנטים ההופכיים

$$\frac{\pi/2}{0.2\pi} = 2.5$$

למרות שיחס המשקלים קבלני מוט כי 5 הוא יותר חזק ועדיף כאמור!

9.5 פתרון מתוחכם לתצורה המוטה





לפעמים תוצאות המחקרים בתחומים שונים לכאורה מתכנסות לכדי הבנה מעמיקה יותר של הדברים. כך למשל הדוגמה הבאה מתחום הסטטיקה חלש ההנדסה מוכיח לנו כיצד שבמחוזק בעזרת תצורה מבורכת עשויים אלף שכשני עמודים בתצורה המוצגת ויצטרפו יחד קושי בקבלת כוח

הוכחת נתחק ומוגש מוט בסיסי

וכי משקלי שווי משקל חובה להפוך לתצורה שם על הסניפים התלויות להתחתנות של המוט

ופלוקה והוא צבוה לצה של הסניפים המקבולים

פתרון 3-3 (המשך)

נתון צינור חלול שמידותיו $d_o = 20mm$, $d_i = 16mm$, התון למומנט פתול $T = 40 Nm$ חסב את τ ב- r_o ו r_i

$$I_p = \frac{\pi r_o^4}{2} - \frac{\pi r_i^4}{2} = 9.27 \times 10^{-9} m^4 \qquad (2\pi r_o^3 \cdot t = 9.16 \times 10^{-9} m^4)$$

$$\tau_{max} = \frac{T R_o}{I_p} = \frac{40 \cdot 0.01}{9.27 \times 10^{-9}} = 43.1 \text{ MPa}$$

$$\tau_{min} = \frac{T R_i}{I_p} = \frac{40 \times 0.008}{9.27 \times 10^{-9}} = 34.5 \text{ MPa}$$

הערה לשאוש במוט חלול הפלוס בי המומנט של המומנט נתון לתכון נבלל התורם בקרבה טובה, במוט מלא אכפל המוט כאיזון ונוונו. נושא עוזם וה תורם בי מבוקבל.

9.6 תכנון מוטות עשויות לפתול

כדי שנוכל לתכנן שלם לפתול עלינו לדעת את כושר העמידות של חומרים שונים מהצרה. ומפורמציה כזות מתקבלת בנסוים על ידי מבחנים קבי דופן המפעילים לכשל. וכלית העמידות של חומרים מהצרה הינו נאוק מצב מאטגה. לפי ASME מחלק כ- Tallow לפקיד מלט מוגבלת והוא הקטן מבין $0.3 S_y$; 0.18 עמ S_u . בכל מקרה שאנחנו בתלקח העמידות לתצופת אז ומומקט יש לאזן הבעיה מוחק את Tallow .

נוסחת התכנון המסוסת תהי:

$$\frac{I}{\tau_{max}} = \frac{I_p}{R}$$

עומס ומומנט אחי — אוומטרי

סוטו לה נתון עמוד מוט עגול בלמך ומחל שאול שלו הוא $\frac{\pi R^3}{2}$. ומתר-

ומלץ ימין מאפשרת קבלה תיד עכבה של R. ערכי מדד חלוץ תיתן נוסחה קומה ל (9.5) ווינסטיר פתחנית ערכה הומחם של שני פרמטרים אידיאליים חומשיים R_o ו- R_i.

בד"כ בתשבץ שלמה לגי נתון האומנט הנדרש כי עם ההספק המועבר קרקן המלך ותדירות סיבוב f הקשר ביניהם לאומנט הינו

$$T = \frac{P}{\omega} \qquad \omega = 2\pi f \qquad (9.6)$$

$$T = \frac{159 \cdot P(kW)}{f} \ [N\times m]$$

$$T = \frac{9540 \cdot P(kW)}{N} \ [N\times m] \qquad (9.7)$$

הוכן ש-N הינו מספר של הסל.

דוגמא: (3.5) (66)

We are to design 2 solid shafts to transmit 200 kw without exceeding a shear stress of 70 MPa one rotates at 20 rpm, the other at 20000 rpm. יש לתכנן שני צירים להעברת 200kW הספק הנמוך שסיבובה 20 rpm ושסיבובה 20,000 rpm

$\tau_{allow} = 70\ MPa$

$$T_1 = \frac{9540 \times 200}{20} = 95{,}400\ N\cdot mm$$

$$T_2 = \frac{9540 \times 200}{20{,}000} = 95.4\ N\cdot mm$$

$$\frac{I_{p_1}}{R_1} = \frac{T_1}{\tau_{max}} = \frac{95{,}400}{70\times10^6} = 1.36\times10^{-3}\ m^3$$

$$\frac{I_{p_2}}{R_2} = \frac{T_2}{\tau_{max}} = \frac{95.4}{70\times10^6} = 1.36\times10^{-6}\ m^3$$

$$\frac{\pi R_1^3}{2} = 1.36\times10^{-3}$$

$$R_1 = 95.3\ mm$$

$$R_2 = 9.53\ mm$$

דוגמא כזו מאחוזה ולא הכנון להעביר הספק קלהרוות סביב שבוש

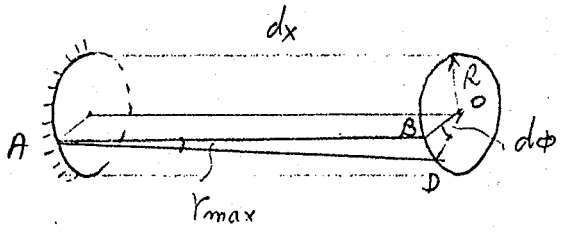
9.7 זווית הסיבוב של חתך עגול

זה כפי שנראה בפרק המתמודד ביותר המאמץ במומנט הפתול. כדי לבין בזווית הסיבוב הנדרשת. הצורך לבדוק גם את זווית הסיבוב נובע מאחד שקולומי:

א. כדי לוודא שהגל יוכל לשמוש את זה ועומד הנדרשת מבלי.

ב. לאזן צה השפעה בזווית התנודות של צלום סובבים

ג. לפעמי אפשר בגל חשובה מבחינת בלתי מסוימת סטטית של צלום



נסתכל על אלמנט אורך dx של המוט הגלגל. בלחץ הפנית מומנט הפתול הזה הקו AB יעבור. עד הסיבוב כל הקו למצבו החדש DA.

כמו ניתן לראות כי $\gamma_{max} = \angle DAB$

$$\widehat{BD} = R\,d\phi = \gamma_{max}\cdot dx$$

$$\gamma_{max}\,dx = R\,d\phi \qquad (9.7)$$

מתוך הזן במצרה נקבל כי $\gamma_{max} = \frac{\tau_{max}}{G}$

ולכן $\tau_{max} = \frac{TR}{I_p}$

הצבה ב (9.7) תתן:

$$d\phi = \frac{T}{I_p G}\,dx$$

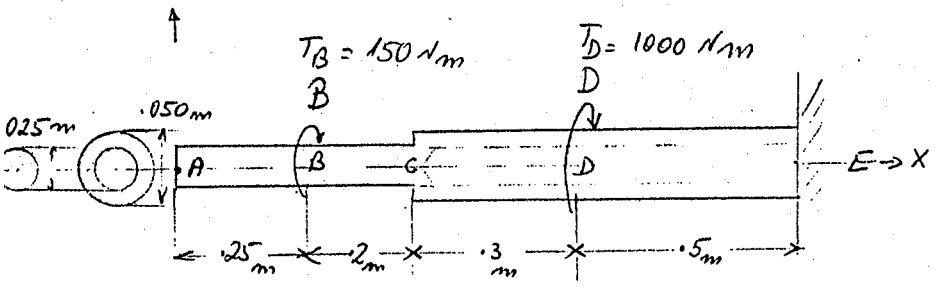
כדי לחשב את הזווית הכוללת בין שני משורות המרוחקים זה מזה dx

$$\phi = \int_A^B \frac{T}{I_p G}\,dx \qquad (9.8)$$

(זווית ϕ קבוע G קבוע) $\phi = \frac{TL}{I_p G}$

ולסיכום לצורך הצבה בנוסחה (9.8) יש לזכור כי

פתרון דוגמא 9-3 (69)



נתון המוט שבסרטוט העשוי מפלדה. חשב את זווית הפתל בקצה A עקב פעולת המומנטים T_B ו- T_D.

Find angle of twist

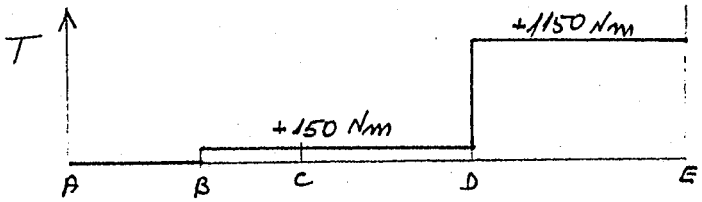
נתון: $G = 80\ GPa$

א. מומנט אינרציה פולרי

$$I_{p1} = I_{pAC} = \frac{\pi d^4}{32} = \frac{\pi (2.5\times10^{-2})^4}{32} = 3.83\times10^{-8}\ m^4$$

$$I_{p2} = I_{pCD} = \frac{\pi}{32}(d_o^4 - d_i^4) = 57.5\times10^{-8}\ m^4$$

ב. פלוט מומנט הפתל לאורך המוט



ג.

$$\phi = \int_E^A \frac{T_x\,dx}{I_{px}G} = \frac{1}{G}\left[\int_E^D \frac{T_{DE}}{I_{p2}}dx + \int_D^C \frac{T_{DC}}{I_{p2}}dx + \int_C^B \frac{T_{BC}}{I_{p1}}dx + \int_B^A \frac{T_{AB}}{I_{p1}}dx\right] =$$

$$\phi = \frac{1}{G}\left[\frac{1}{I_{p2}}(T_{DE}L_{DE} + T_{DC}L_{DC}) + \frac{T_{BC}L_{BC}}{I_{p1}}\right] =$$

$$= +\frac{1}{80\times10^{+9}}\left[\frac{1}{57.5\times10^{-8}}(1150\times0.5 + 150\times0.3) + \frac{150\times0.2}{3.83\times10^{-8}}\right] =$$

$$= +0.0233\ rad \equiv +1.33^\circ$$

9.8 כללי מומנטים מפתלים

הנוסחאות שפיתחנו בכוונת עבור עמוד עגול יציב ברגע מוליך לאורך. המוט ושטח הקטעים חלקות מכל התחתק (מדרגות)

ולכן במקרה הקפדנית ניתן שהמים תמיסה בפועל המומנטים יוכלו לכל
מומנטים גושי שיכמתן תמיד על מקדים לצורת מומנט המחיבה כ:

$$\tau_{max} = K \cdot \frac{Tc}{J_p} \qquad (9.9)$$

c - רדיוס הגל הקטן

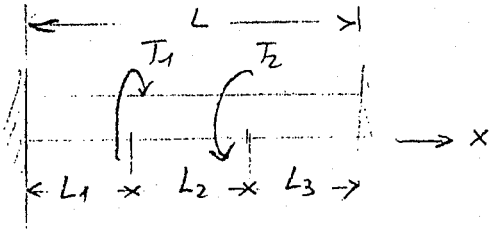
שבגלו של א תלוי בתחס קטרו המינימלי לצורך הרדיוס ההשלמה של
המעבר בהמשך כולה בטבלה 9.1

מקרה שני לכטבי מומנטים לגבי מתרחש לתפיות שאין ומחוץ ל-
שמאפשר במקרה כגון מומלץ ע"י ASME
ל הקטין את Tallow ב 25%.

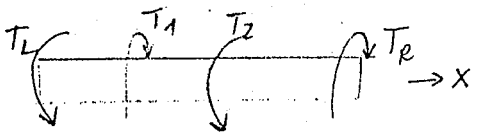
(ולא 2=א לפי תורת הגלסטיות)

9.8 בעיות בלתי מסוימות סטטית - פתרון

תשובה הפתרון של בעיות בלתי מסוימות סטטית בפתרון
דומה לבעיות שבהן מסה צריכה. כפי לפתרון בעיות מקומיות הדומה:



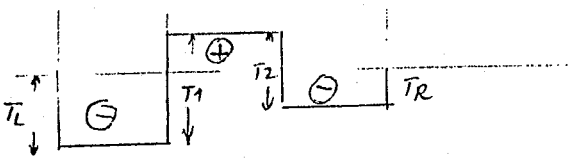
קבע ולא המומנטים את ולקבוע
של חלק העזר הבא:



עשה דוגמה את לפי תפסי:

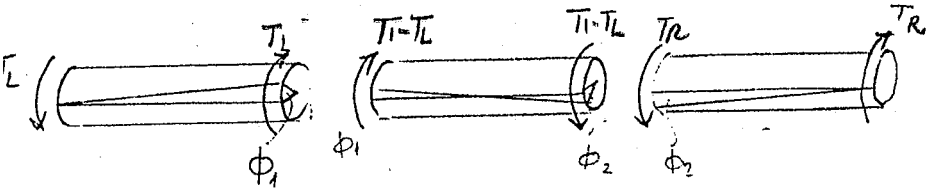
$$\sum M_x = 0 \qquad T_L - T_1 + T_2 - T_R = 0 \qquad (1)$$

ישנו משוו המשקל משוואה אחת בעוד שלושה ולכן לקבוע לקשר נוסף
שיביא להתחשב על ידי ההתעוותות של הגל. לצורך כך נתאר גוף
פרוש המומנט לאורך הגל:



ישנם שלושה וקטורים

$$T_1 = T_L$$
$$T = T_1 - T_L$$
$$T_R = T$$



ולכן ונקבל כי:

$$\phi_1 = \frac{T_L L_1}{GI_p} \qquad (2)$$

$$\phi_2 = \frac{T_R L_3}{GI_p} \qquad (3)$$

$$\phi_1 + \phi_2 = \frac{(T_1 - T_L) L_2}{GI_p} \qquad (4)$$

$$T_L L_1 - T_R L_3 = (T_1 - T_L) L_2 \qquad (5)$$

(1)(2)(3)

פתרון המשוואות (1)-(5) נותן:

$$T_L = T_1\left(\frac{L_2+L_3}{L}\right) - T_2\left(\frac{L_3}{L}\right)$$

$$T_R = -T_1\left(\frac{L_1}{L}\right) + T_2\left(\frac{L_1+L_2}{L}\right)$$

$$(L = L_1+L_2+L_3)$$

ונקבל את תבנית מתח הפעלה.

בדיקת שגיאה לפתרון:

$$T_L - T_1 + T_2 - T_R \equiv 0$$

נסתכל במאזן I ו II

ובמאזן II - III

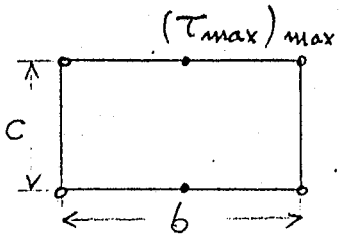
6.9 - פיתול חתכים מלאים ללא יציאות

ניתן אם ניתן ללמוד כיצד מתנהגים חתכים כוללו וגם לחתכים שמכילים יציאות ואפסי יציאות. שתי הנחות היסוד שהנחנו בהתלפה של מוטות יציאות ואת מתקיימות בחתכים ללא יציאות. כלומר קווי שקיעה של המוט $\theta \neq \theta(x)$ כן משורים וקווי עשורים משורים. נקודות כלות בחלות אינן על חתך אחיד.

הפתרון הוגשם ללראשונה ללו אנליזה הוגה ע"י St. Venant ולאחר מכן באמצעות הקורס בולסטרות בשלב זה נבדק באה פתרונות:

חתך מלבני

$b > c$



נקודת חתך זה נהיה המקומית

מאקסימלי

$$\tau_{max} = \frac{T}{\alpha b c^2}$$

וכלות הפתול

$$\phi = \frac{T \cdot L}{\beta b c^3 G}$$

כאן שהקבוע α ו- β תושבו לאורות הנתונים ע"י

b/c	1.00	1.50	2.00	3.00	6.00	10.00	∞
α	0.208	0.231	0.246	0.267	0.299	0.312	1/3
β	0.141	0.196	0.229	0.263	0.299	0.312	1/3

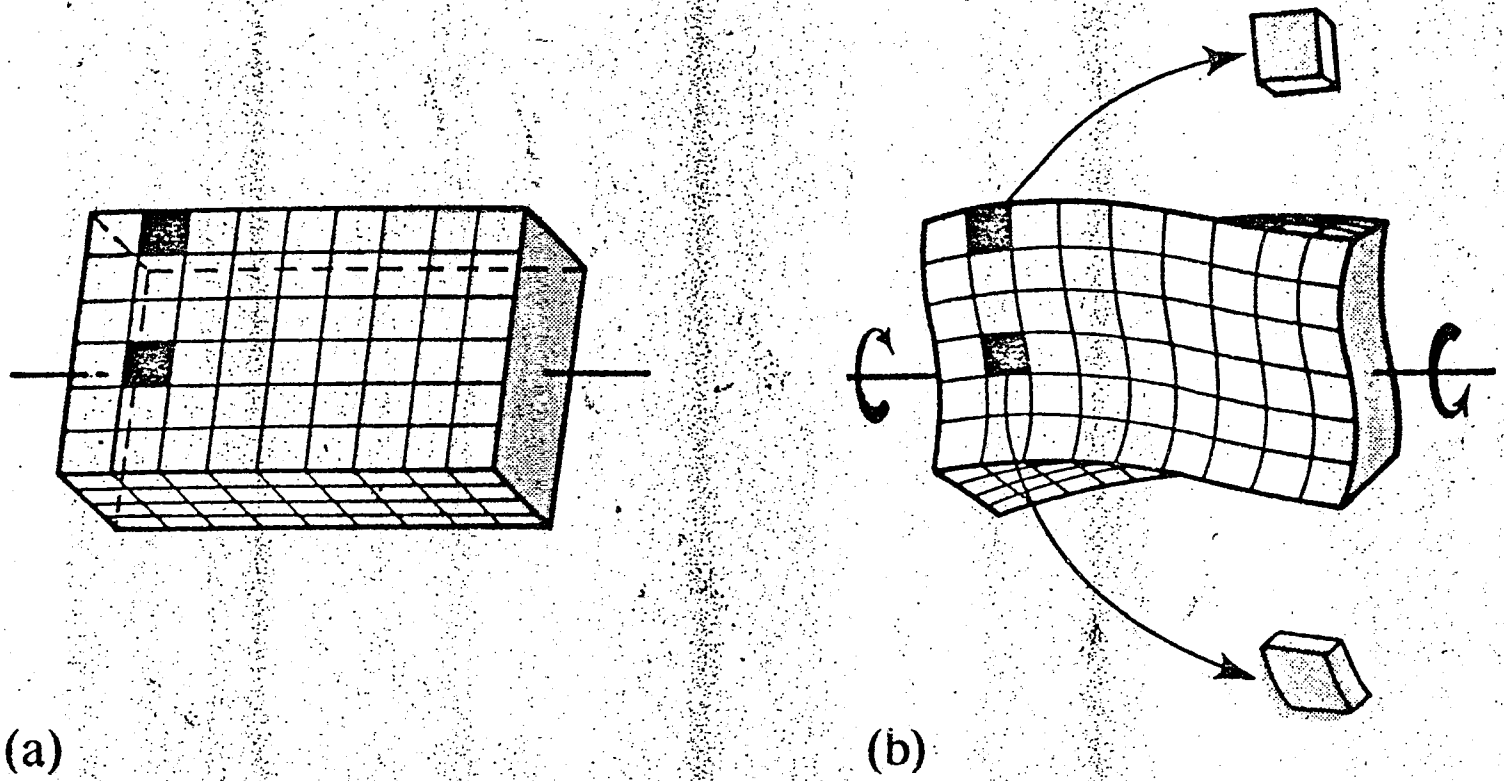
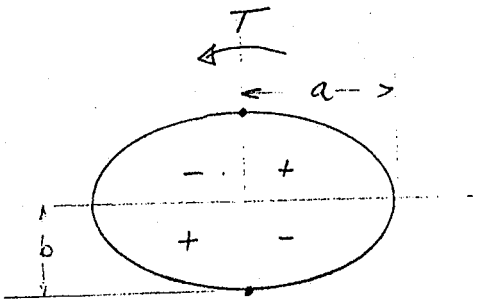


Fig. 3-17. Rectangular shaft (a) before (b) after a torque is applied.

ו. המומנט הפנימי מתקבל מתנאי הקונטור הקשורים במומנט החיצוני ומתנאי הסימטריה.

ב. המומנט הפנימי = 0. כלומר המומנט הפנימי והחיצוני = 0 בהעדר קשרים עודף כלשהו אינו מתאזן.

ג. שקע הקשיש מוט לרוחב אחיד הנתון בקצותיו והנתון למומנט עיתול לקצוה



מתוך אלסטיות:

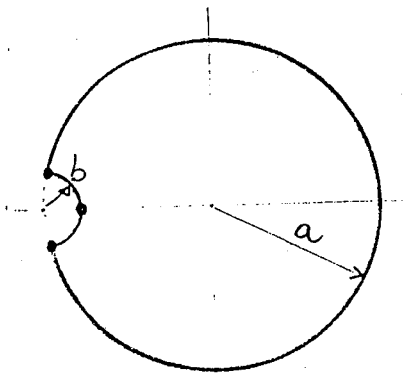
$$\tau_{max} = \frac{2T}{\pi a b^2}$$

$$\left(\text{for } a=b=R \Rightarrow \frac{TR}{I_p}\right)$$

$$\phi = T\cdot L \cdot \frac{a^2+b^2}{\pi a^3 b^3 G} \quad \left(\frac{TL}{GI_p}\right)$$

שימו לב ל (+ −) על ציר המתח.

מתוך אינטגרל עם תנאי שפה



$$\tau_{max} = \frac{T}{I_p}(2a-b)$$

for $b \ll 2a$ $\quad \tau_{max} \simeq 2(\tau_{max})$ ללא שפה.

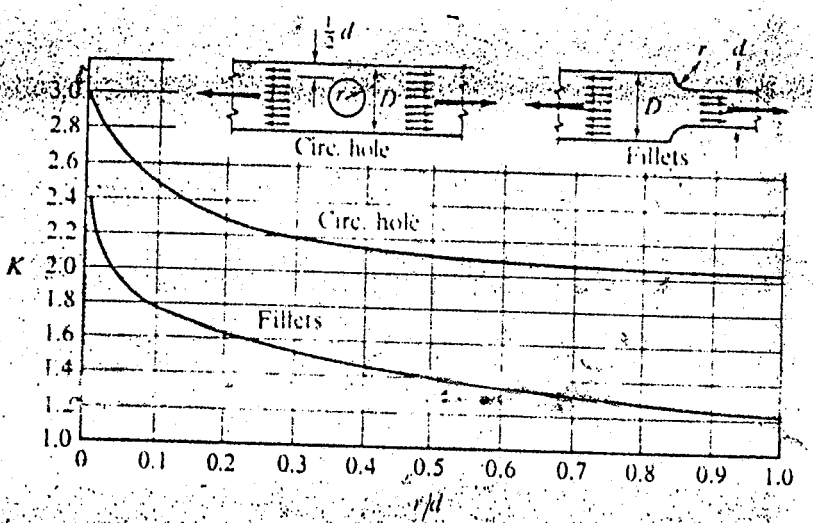


Fig. 2-17. Stress-concentration factors for flat bars in tension

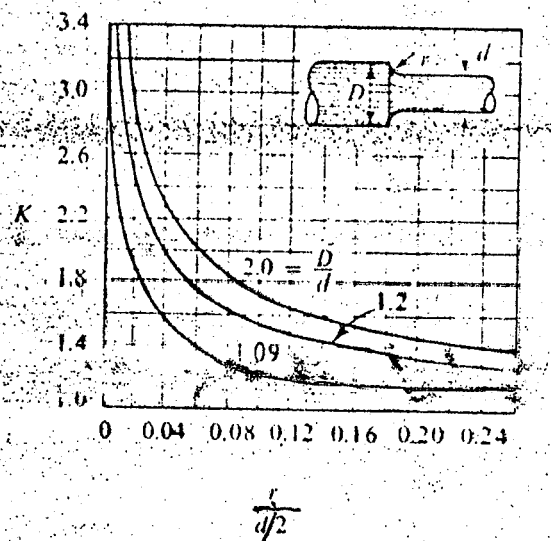


Fig. 3-15. Torsional stress-concentration factors in circular shafts of two diameters.

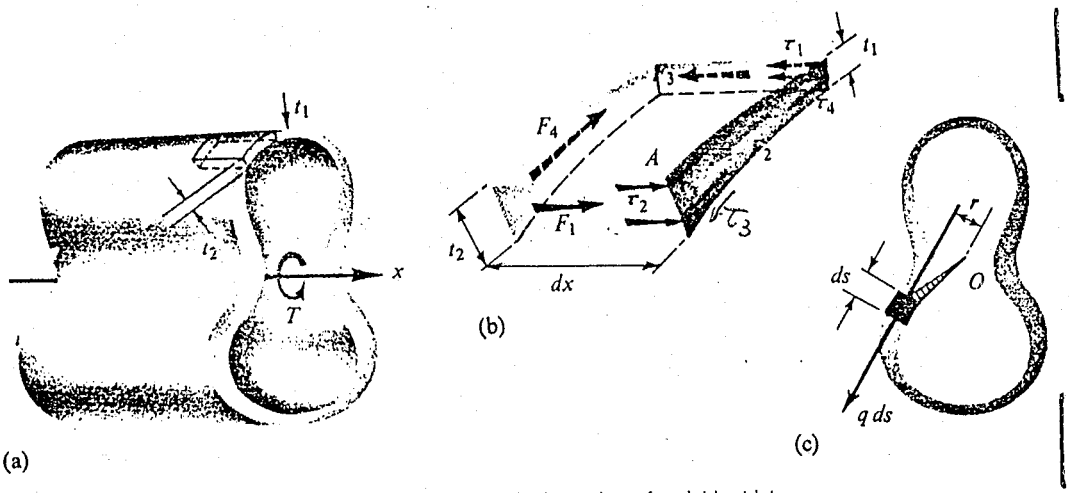


Fig. 3-22. Thin-walled member of variable thickness.

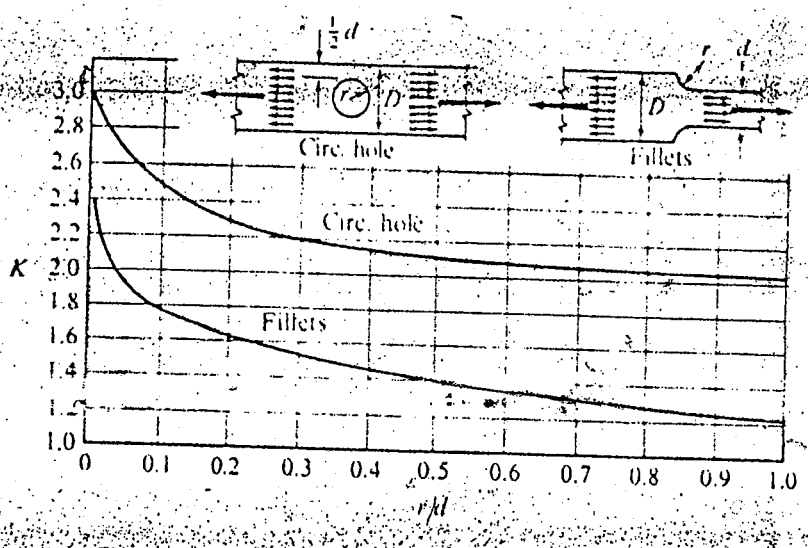


Fig. 2-17. Stress-concentration factors for flat bars in tension

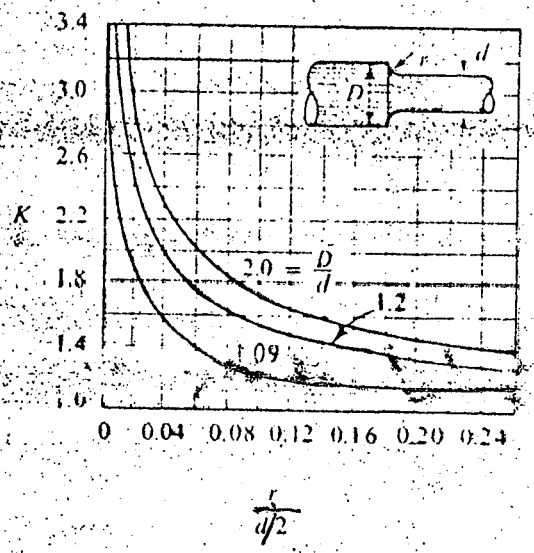


Fig. 3-15. Torsional stress-concentration factors in circular shafts of two diameters.

9.10 פתרון התכסה סלורים קיר קיפן

לצורך הפתרון נניח מתאמי המוביל הדרוש לפתרון בעיית של מוטות לון ערכולים מלוחים מסתתר שקבוע לנוחיות קבל קנון בצל תחתק כל שהו. הפתרון בשלוש נחסות.

נתאר לעני קנור שבצה (שקף 9.2). נתחק מאזן פרוסה קטנה. על פני וריבות בעסנתה פועלים וריבה בוחות: F_1, F_2, F_3, F_4 הנבדלים מאוזני התצורה x שלה מתחק משקלי שוו, משקל עקבו:

(1) $F_1 = F_3$

ולפי

(2) $F_1 = \tau_2 \cdot t_2\, dx$

$F_3 = \tau_1 t_1\, dx$

ומכאן נובע כי

(3) $q = \tau_1 t_1 = \tau_2 t_2$ $\leftarrow 2, 1$

כאשר שניין צה ותקופה עבור כל תחק ורכי שנקבע בקנור.

לפבוע הלה נקבע q.

מלאשר התנחות אומאת התצורה ונני ווקעים כי הלאדוסים הבאים מקיימים את התנע:

(4) $\tau_2 = \tau_3$

$\tau_1 = \tau_4$

ומכאן ינבע (3) ו-(4)

(5) $q = t_1 \tau_4 = t_2 \tau_3$

כלומר q קבוע על פני כל שטח התחק של הקנור.

שימוש בנוסחה זו עלול להוביל לטעויות חמורות. ראשית, הנוסחה תקפה רק עבור חתכים דקי-דופן בעלי עובי קבוע. שנית, הנוסחה מתייחסת רק לזרימת גזירה פנימית (Shear flow) ואינה מתייחסת לגזירה הנוצרת עקב פיתול החתך (Torsion).

נשתמש כעת בשקול שוויון אנרגיה (6) כדי למצוא את המומנט שיוצר זרימת גזירה בחתך חד-תאי סגור עקיבה על שהוא 0 -

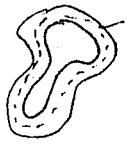
$$dT = \tau t ds \cdot r$$

$$T = \oint \tau t r ds$$

כאשר q קבוע עבור כל החתך ניתן להוציאו אל מחוץ לסימן האינטגרל

$$T = q \oint r ds$$

מסתבר שהאינטגרל שנותר שווה לפי-שטח המשולש הכלוא בתוך קו הממוצע של החתך (עובי דק) כפול $2A$



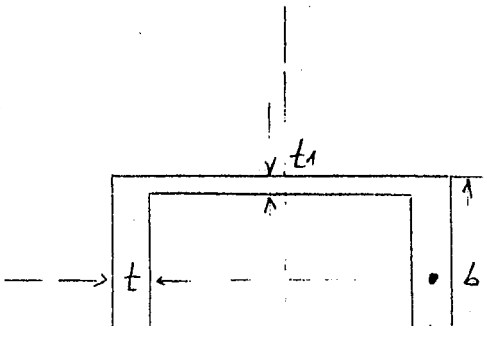
ומכאן:

$$T = 2Aq$$

$$q = \frac{T}{2A}$$

ולכן מאחר והמקסימלי יהיה ה t_{min} שכן $\tau_{max} t_{min} = q$

$$\tau_{max} = \frac{T}{2A t_{min}}$$



דוגמא:

מהו מאמץ הגזירה המקסימלי בחתך הבא:

בגובה t מתקיים $\tau = \frac{T}{2(a-t)(b-t_1)t}$.

בגובה t_1 מתקיים $\tau_1 = \frac{T}{2(a-t)(b-t_1)t_1}$.

שימלה בפועל הפונקציות מלולים להיות כמעט מואחדים לכדופ, ולכן ניתן
כן הערכה הפונקציה בגובה.

קומתיים ניסתה מקומתית לתשובה בעיה הפתיחה ולכן הנחו מערכת
לעבדים בקורס כה.

$$\phi = \frac{TL}{4A_{enc}^2 G} \oint \frac{ds}{t}$$

At this point it is instructive to recall the complete representation of the stress tensor in Eq. 3-1. In this problem the results of Eqs. 5-4 or 5-5 can be stated as*

$$\begin{pmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{5-7}$$

Since only two elements of the stress tensor exist, no ambiguity is caused by having no subscripts on τ in Eq. 5-7. In general elasticity-problem subscripts such as x and θ are attached to τ to designate the axial and the tangential directions. Just as for a Cartesian element, for a cylindrical element $\tau_{x\theta} = \tau_{\theta x}$.

In an isotropic material it makes little difference in which direction the shearing stresses act. However, not all materials used in engineering applications are isotropic. For example, wood exhibits drastically different properties of strength in different directions. The shearing strength of wood on planes parallel to the grain is much less than that on planes perpendicular to the grain. Hence, although equal intensities of shearing stress exist on mutually perpendicular planes, wooden shafts of inadequate size fail longitudinally along axial planes. Wooden shafts are occasionally used in the process industries.

EXAMPLE 5-2

Find the maximum torsional shearing stress in the shaft AC shown in Fig. 5-1(a). Assume the shaft from A to C to be of ½-in. diameter.

SOLUTION

From Example 5-1 the maximum internal torque resisted by this shaft is known to be 300 in-lb. Hence, $T = 300$ in-lb, and $c = d/2 = 0.25$ in.

From Eq. 5-3: $$J = \frac{\pi d^4}{32} = \frac{\pi(0.5)^4}{32} = 0.00614 \text{ in.}^4$$

From Eq. 5-4:

$$\tau_{\max} = \frac{Tc}{J} = \frac{(300)(0.25)}{0.00614} = 12{,}200 \text{ psi}$$

This maximum shearing stress at 0.25 in. from the axis of the rod acts in the plane of a cut perpendicular to the axis of the rod and along the longitudinal planes passing through the axis of the rod (Fig. 5-5(c)).

EXAMPLE 5-3

Consider a long tube of 1-in. outside diameter d_o and of 0.9-in. inside diameter d_i twisted about its longitudinal axis with a torque T of 400 in-lb. Determine the shearing stresses at the outside and the inside of the tube, Fig. 5-6.

* Some readers may find it interesting to look over Example 10-2 at this time.

SOLUTION

From Eq. 5-6: $$J = \frac{\pi(c^4 - b^4)}{2} = \frac{\pi(d_o^4 - d_i^4)}{32}$$

$$= \frac{\pi(1^4 - 0.9^4)}{32} = 0.0337 \text{ in.}^4$$

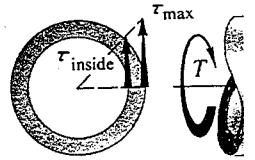


Fig.

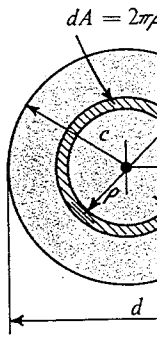
From Eq. 5-4:

$$\tau_{\max} = \frac{Tc}{J} = \frac{(400)(½)}{0.0337} = 5{,}930 \text{ psi}$$

From Eq. 5-5:

$$\tau_{\text{inside}} = \frac{T\rho}{J} = \frac{400(0.9/2)}{0.0337} = 5{,}330 \text{ psi}$$

Since no material works at a low stress, it is important to note that a tube requires less material than a solid shaft to transmit a given torque at the same stress. By making the wall thickness small and the diameter large, nearly uniform shearing stress τ is obtained in the wall. This fact makes thin tubes suitable for experiments where a uniform "field" of pure shearing stress is wanted (Art. 4-15). To avoid local crimping or buckling, the wall thickness, however, cannot be excessively thin.



Fig

EXAMPLE 5-4

Find the energy absorbed by an elastic circular rod subjected to a constant torque in terms of the maximum shearing stress and the volume of the material, Fig. 5-7.

SOLUTION

The shearing stress acting on an element at a distance ρ from the center of the cross section is $\tau_{\max}\rho/c$. Then, using Eq. 4-18 with $\tau_{xy} = \tau$, $\gamma_{xy} = \tau/G$, and integrating over the volume V of the rod L in. long, one obtains

$$U = \int_V \frac{\tau^2}{2G} dV = \int_V \frac{\tau_{\max}^2 \rho^2}{2Gc^2} 2\pi\rho \, d\rho \, L$$

$$= \frac{\tau_{\max}^2}{2G} \frac{2\pi L}{c^2} \int_0^c \rho^3 \, d\rho = \frac{\tau_{\max}^2}{2G} \frac{2\pi L}{c^2} \frac{c^4}{4}$$

$$= \frac{\tau_{\max}^2}{2G} (½ \text{ vol})$$

If instead of the solid shaft a thin-walled tube were used, then

$$U = \frac{\tau_{max}^2}{2G}(\text{vol})$$

For the same level of maximum stress, the uniformly stressed material absorbs energy more efficiently.

5-6. DESIGN OF CIRCULAR MEMBERS IN TORSION

In designing members for strength, allowable shearing stresses must be selected. These depend on the information available from experiments and on the intended application. Accurate information on the capacity of materials to resist shearing stresses comes from tests on thin-walled tubes. Solid shafting is employed in routine tests. Moreover, as torsion members are so often used in power equipment, many fatigue experiments are done. Characteristically, the shearing stress that a material can withstand is lower than the normal stress. The ASME (American Society of Mechanical Engineers) code of recommended practice for transmission shafting gives an allowable value in shearing stress of 8,000 psi for ordinary steel.* In practical designs, suddenly applied and shock loads warrant special considerations.

Once the torque to be transmitted by the shaft is known and the maximum shearing stress is selected, the proportions of the member become fixed. Thus from Eq. 5-4

$$\frac{J}{c} = \frac{T}{\tau_{max}} \tag{5-8}$$

where J/c is the *parameter* on which the elastic strength of a shaft depends. For an axially loaded rod such a parameter is the cross-sectional area of a member. For a solid shaft, $J/c = \pi c^3/2$, where c is the outside radius. By using this expression and Eq. 5-8 the required radius of a shaft may be determined. For a hollow shaft, a number of tubes can provide the same numerical value of J/c, so the problem has an infinite number of possible solutions.

EXAMPLE 5-5

Select a solid shaft for a 10-hp motor operating at 1,800 rpm. The maximum shearing stress is limited to 8,000 psi.

SOLUTION

From Eq. 5-1:

$$T = \frac{63{,}000\ \text{hp}}{N} = \frac{63{,}000(10)}{1{,}800} = 350\ \text{in-lb}$$

From Eq. 5-8:

$$\frac{J}{c} = \frac{T}{\tau_{max}} = \frac{350}{8{,}000} = 0.0438\ \text{in.}^3$$

$$\frac{J}{c} = \frac{\pi c^3}{2} \quad \text{or} \quad c^3 = \frac{2}{\pi}\frac{J}{c} = \frac{2(0.0438)}{\pi} = 0.0279\ \text{in.}^3$$

Hence $c = 0.303$ in. or $d = 2c = 0.606$ in.

For practical purposes a $\frac{5}{8}$-in. shaft would probably be selected.

EXAMPLE 5-6

Select solid shafts to transmit 200 hp each without exceeding a shearing stress of 10,000 psi. One of these shafts operates at 20 rpm and the other at 20,000 rpm.

SOLUTION

Subscript 1 applies to the low-speed shaft; 2 to the high-speed shaft.

From Eq. 5-1:

$$T_1 = \frac{(\text{hp})(63{,}000)}{N_1} = \frac{200(63{,}000)}{20} = 630{,}000\ \text{in-lb}$$

Similarly $T_2 = 630$ in-lb

From Eq. 5-8:

$$\frac{J_1}{c} = \frac{T_1}{\tau_{max}} = \frac{630{,}000}{10{,}000} = 63\ \text{in.}^3$$

$$\frac{J_1}{c} = \frac{\pi d_1^3}{16} \quad \text{or} \quad d_1^3 = \frac{16}{\pi}(63) = 322\ \text{in.}^3$$

and $d_1 = 6.85$ in.

Similarly $d_2 = 0.685$ in.

This example illustrates the reason for the modern tendency to use high-speed machines in mechanical equipment. The difference in size of the two shafts is striking. Further saving in the weight of the material may be effected by making use of hollow tubes.

5-7. ANGLE OF TWIST OF CIRCULAR MEMBERS

So far in this chapter, methods of determining stresses in solid and hollow circular shafts subjected to torque have been discussed. Now attention will be directed to the method of finding the angle of twist for

* Extensive recommendations for other materials may be found in machine-design books. For example, see V. M. Faires, *Design of Machine Elements* (4th ed.) (New York: The Macmillan Company, 1965), p. 580.

The boundary conditions for this equation consist of specifying either φ or T at each boundary. From Eq. 5-10 it should be clear that $T = JG\,d\varphi/dx$. In this case, as in Eq. 4-32, singularity functions can be employed for concentrated moments. Equation 5-12 can be used in the solution of indeterminate problems. A more complete discussion of such problems is given in Chapter 12.

The following two examples illustrate applications of Eq. 5-11.

EXAMPLE 5-7

Find the relative rotation of section B-B with respect to section A-A of the solid shaft shown in Fig. 5-11 when a constant torque T is being transmitted through it. The polar moment of inertia of the cross-sectional area J is constant.

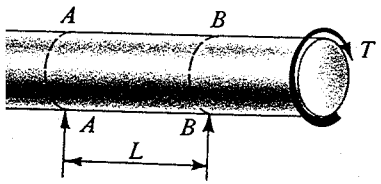


Fig. 5-11

SOLUTION

In this case $T(x) = T$ and $J(x) = J$; hence from Eq. 5-11

$$\Delta\varphi = \int_0^L \frac{T\,dx}{JG} = \frac{T}{JG}\int_0^L dx = \frac{TL}{JG} \qquad (5\text{-}13)$$

Equation 5-13 is an important relation. It can be used in the design of shafts for stiffness, i.e., for limiting the amount of twist that may take place in their length. For such an application T, L, and G are known quantities, and the solution of Eq. 5-13 yields J. This fixes the size of the required shaft (see Eqs. 5-3 and 5-6). Note that for stiffness requirements, J, rather than J/c of the strength requirement, is the significant parameter. This equation is used in torsional vibration analyses. The term JG is referred to as the *torsional stiffness* of the shaft.

Another application of Eq. 5-13 is found in the laboratory. There a shaft may be subjected to a known torque T, J may be computed from the dimensions of the specimen, and the relative angular rotation φ between two planes a distance L apart may be measured. Then, by using Eq. 5-13, the shearing modulus of elasticity in the elastic range can be computed, i.e., $G = TL/J\varphi$.

In using Eq. 5-13 note particularly that the angle φ must be expressed in radians. Also observe the similarity of Eq. 5-13 to Eq. 4-33, $\Delta = PL/AE$, formerly derived for axially loaded rods.

EXAMPLE 5-8

Consider the stepped shaft shown in Fig. 5-12 attached to a wall at E, and determine the rotation of the end A when the two torques at B and at D are applied. Assume the shearing modulus G to be 12×10^6 psi, a typical value for steels.

SOLUTION

From Eq. 5-3:

$$J_{AB} = J_{BC} = \frac{\pi d^4}{32} = \frac{\pi 1^4}{32} = 0.0982 \text{ in.}^4$$

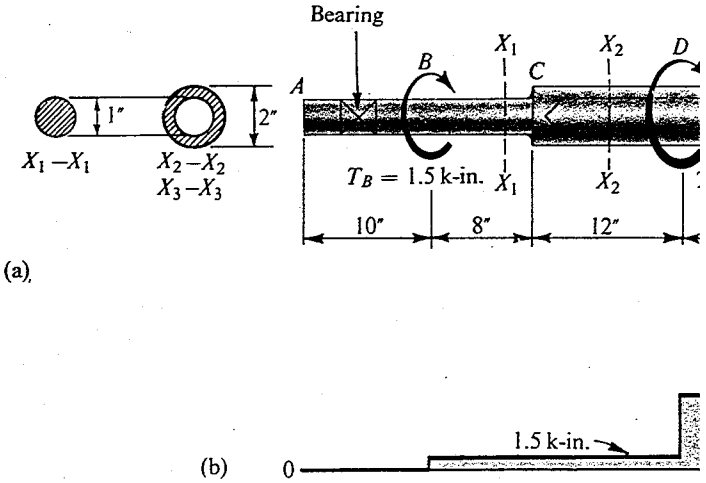


Fig. 5-12

From Eq. 5-6:

$$J_{CD} = J_{DE} = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}(2^4 - 1^4) = 1.47 \text{ in.}^4$$

where subscripts indicate the range of applicability of a given value. Then by passing arbitrary sections X_1-X_1, X_2-X_2, and X_3-X_3, and each time considering a portion of the shaft to the left of such sections, the internal torques for the various intervals are found to be

$$T_{AB} = 0, \quad T_{BD} = T_{BC} = T_{CD} = 1.5 \text{ kip-in.}, \quad T_{DE} = 11.5 \text{ kip-in.}$$

The torque diagram corresponding to these quantities is in Fig. 5-12(b).

To find the rotation of the end A, Eq. 5-11 is applied with the limits of integration broken at points where T or J changes its value abruptly. Integrating from right to left since the right end is built in, one obtains $C_1 = 0$.

$$\varphi = \int_E^A \frac{T(x)\,dx}{J(x)G} = \int_E^D \frac{T_{DE}\,dx}{J_{DE}G} + \int_D^C \frac{T_{CD}\,dx}{J_{CD}G} + \int_C^B \frac{T_{BC}\,dx}{J_{BC}G} + \int_B^A \frac{T_{AB}\,dx}{J_{AB}G}$$

In the last group of integrals T's and J's are constant between the limits considered, so each integral reverts to a known solution, Eq. 5-13. Hence

$$\varphi = \frac{T_{DE}L_{DE}}{J_{DE}G} + \frac{T_{CD}L_{CD}}{J_{CD}G} + \frac{T_{BC}L_{BC}}{J_{BC}G} + \frac{T_{AB}L_{AB}}{J_{AB}G}$$

$$= \frac{11{,}500(20)}{1.47(12)10^6} + \frac{1{,}500(12)}{1.47(12)10^6} + \frac{1{,}500(8)}{0.0982(12)10^6} + 0$$

"imaginary" annulus rebounds elastically. Because of the differences in the strain paths, which cause permanent set in the material, residual stresses develop. This process will be illustrated in one of the examples which follow.

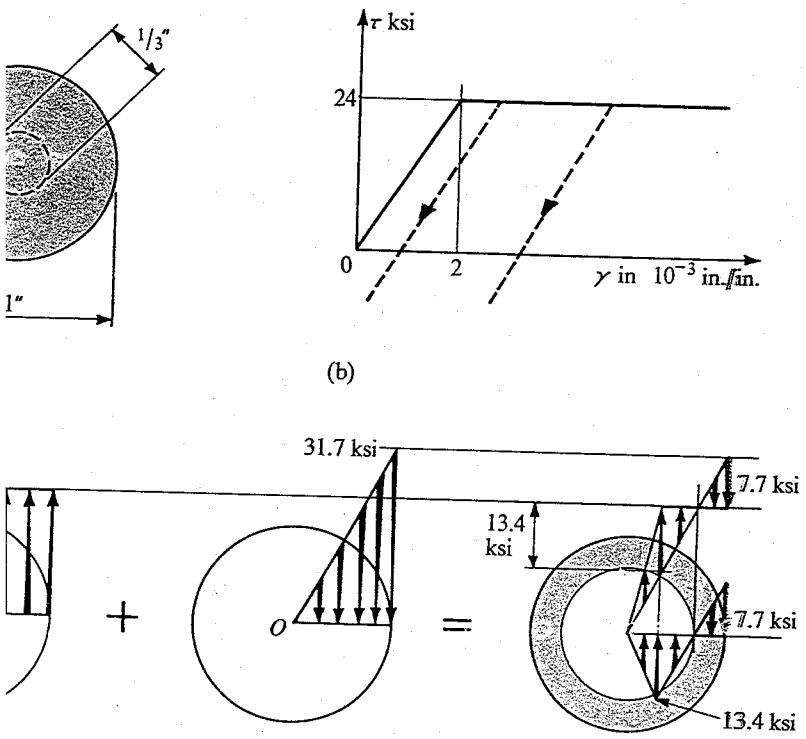
For determining the rate of twist of a circular shaft or tube, Eq. 5-9 can be used in the following form:

$$\frac{d\varphi}{dx} = \frac{\gamma_{max}}{c} = \frac{\gamma_a}{\rho_a} \tag{5-15}$$

Here either the maximum shearing strain at c or the strain at ρ_a determined from the stress-strain diagram must be used.

EXAMPLE 5-9

A solid steel shaft of 1-in. diameter is so severely twisted that only a ⅓-in.-diameter elastic core remains on the inside, Fig. 5-15(a). If the material properties can be idealized as shown in Fig. 5-15(b), what residual stresses and residual rotation will remain upon release of the applied torque?



(b)

c stress on

(d) Elastic rebound stresses

(e) Residual stresses

Fig. 5-15

SOLUTION

To begin, the magnitude of the initially applied torque and the corresponding angle of twist must be determined. The stress distribution corresponding to the given condition is shown in Fig. 5-15(c). The stresses vary linearly from 0 to 24 ksi when $0 \le \rho \le 1/6$ in.; the stress is a constant 24 ksi for $\rho > 1/6$ in. Equation 5-14 can be used to determine the applied torque T. The release of the torque T causes elastic stresses, and Eq. 5-4 applies, Fig. 5-15(d). The difference between the two stress distributions, corresponding to no external torque, gives the residual stresses.

$$T = \int_A \tau\rho\, dA = \int_0^c 2\pi\tau\rho^2\, d\rho = \int_0^{1/6}\left[\frac{\rho}{1/6}24\right]2\pi\rho^2\, d\rho$$

$$+\int_{1/6}^{1/2}(24)\,2\pi\rho^2\, d\rho = 0.17 + 6.05 = 6.22 \text{ kip-in.}$$

(Note the smallness of the contribution of the first integral.)

$$\tau_{max} = \frac{Tc}{J} = \frac{6.22 \times 0.5}{\pi/32} = 31.7 \text{ ksi}$$

At $\rho = 0.5$ in., $\tau_{residual} = 31.7 - 24.0 = 7.7$ ksi. Two diagrams of the residual stresses are shown in Fig. 5-15(e). For clarity the initial results are replotted from the horizontal line. In the entire shaded portion of the diagram, the residual torque is clockwise; an exactly equal residual torque acts in the opposite direction in the inner portion of the shaft.

The initial rotation is best determined by calculating the twist of the elastic core. At $\rho = 1/6$ in., $\gamma = 2 \times 10^{-3}$. The elastic rebound of the shaft is given by Eq. 5-13. The difference between the inelastic and the elastic twists gives the residual rotation per inch of shaft. If the initial torque is re-applied in the same direction, the shaft responds elastically.

Inelastic: $\dfrac{d\varphi}{dx} = \dfrac{\gamma_a}{\rho_a} = \dfrac{2 \times 10^{-3}}{1/6} = 12 \times 10^{-3}$ per inch

Elastic: $\dfrac{d\varphi}{dx} = \dfrac{T}{JG} = \dfrac{6.22}{(\pi/32)12 \times 10^3} = 5.28 \times 10^{-3}$ per inch

Residual: $\dfrac{d\varphi}{dx} = (12 - 5.28)10^{-3} = 6.72 \times 10^{-3}$ radián per inch

EXAMPLE 5-10

Determine the torque carried by a solid circular shaft of mild steel when shearing stresses above the proportional limit are reached essentially everywhere. For mild steel, the shearing stress-strain diagram can be idealized to that shown in Fig. 5-16(a). The shearing yield-point stress τ_{yp} is to be taken as being the same as the proportional limit in shear τ_{pl}.

Cesar Levy CE201A

Mr Blumert.

10

5-1

$\tau_{max} = \frac{Tc}{J}$ $\quad J = \frac{\pi c^4}{2}$ for solid shaft ; $J = \frac{\pi c^4}{2} - \frac{\pi b^4}{2}$ for hollow shaft

$\tau_{max_1} = \tau_{max_2}$; $T_1 = T_2$; $\therefore \quad c_1 J_2 = c_2 J_1$

$c_1 = 1$ in , $c_2 = 1.5$ in , $\quad J_1 = \frac{\pi}{2}$ $\quad J_2 = \frac{\pi}{2}\left(\frac{81}{16} - b^4\right)$

$1.5\left(\frac{\pi}{2}\right) = 1\left(\frac{\pi}{2}\right)\left(\frac{81}{16} - b^4\right)$; $1.5 = \left(\frac{81}{16} - b^4\right)$

$b = \sqrt[4]{5.0625 - 1.5} = \sqrt[4]{3.5625} = 1.375$ in

thickness must be $1.5 - 1.375 = .125$ in.

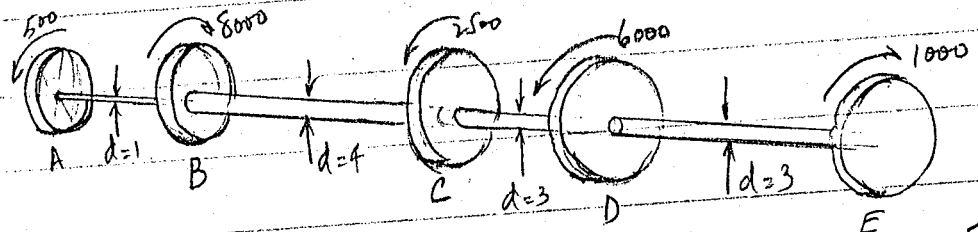
b)

weight of solid / weight of hollow = ρ Volume solid / ρ Volume hollow.

Volume of solid $= \pi r^2 h$ $\quad$ volume of hollow $= \pi R^2 h - \pi r^2 h$

$\frac{W_s}{W_h} = \frac{r^2}{R^2 - r^2}$ $\quad r = 1.375$; $R = 1.5$; $r = 1$; $\quad \frac{W_s}{W_h} = \frac{1}{.365} = 2.74$

5-2



between AB $\quad T = 500 \quad c = 1/2 \quad J = \frac{\pi}{32}$ $\quad \tau_{max} = 2545$ psi

between BC $\quad T = 7500 \quad c = 2 \quad J = 8\pi$ $\quad \tau_{max} = 596$ psi

between CD $\quad T = 5000 \quad c = 1.5 \quad J = \frac{\pi}{2}\left(\frac{81}{16}\right)$ $\quad \tau_{max} = 944$ psi

between DE $\quad T = 1000 \quad c = 1.5 \quad J = \frac{\pi}{2}\left(\frac{81}{16}\right)$ $\quad \tau_{max} = 188.5$ psi

$\tau_{max} = 2545$ psi between A & B

5-3 $T_l = \frac{63000(90)}{63000} = 9000$ in-lb $\quad T_r = \frac{63000}{630,00}(30) = 3000$ in-lb

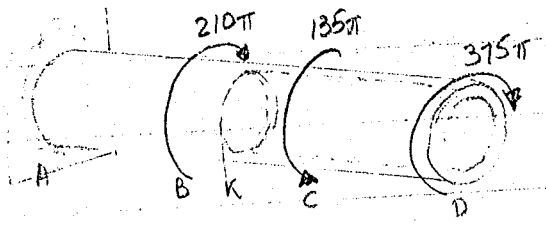
Since $kT = T_{max}$, we must find a rod that will withstand T_{max}. We can therefore say that we must overcome the torque produced by the left machine since it has the greatest torque.

$$\therefore T_{max} = \frac{Tc}{J} \; ; \quad \frac{c}{J} = \frac{c}{\frac{\pi}{2}c^4} = \frac{2}{\pi}\frac{1}{c^3} \; ; \quad \frac{T_{max}}{T} = \frac{2}{\pi}\frac{1}{c^3}$$

$$\frac{\pi}{2}c^3 = \frac{T}{T_{max}} \qquad c = \sqrt[3]{\frac{2T}{\pi T_{max}}} = \sqrt[3]{\frac{18000}{18090}} \approx \sqrt[3]{1.0} = 1.0$$

$\therefore c = 1.0$ $\quad d = 2$ in

5-4



1. below AB $\quad T = 450\pi \quad c = 1'' \quad J = \frac{\pi c^4}{2}$

$$T_{max_1} = \frac{Tc}{J} = \frac{2(450)\pi}{\pi c^3} = \frac{900\pi}{\pi} = 900 \text{ psi}$$

2. between BK $\quad T = 240\pi \quad c = 1 \quad J = \frac{\pi c^4}{2}$

$$T_{max_2} = \frac{Tc}{J} = \frac{2(240)\pi}{\pi c^3} = 480 \text{ psi}$$

3. between KC $\quad T = 240\pi \quad c = 1 \quad b = 1/2 \quad J = \frac{\pi b^4}{2} + \frac{\pi c^4}{2}$

$$J = \frac{\pi}{2}\left(1 - \frac{1}{16}\right) = \frac{\pi}{2}\left(\frac{15}{16}\right) \; ; \quad T_{max_3} = \frac{Tc}{J} = \frac{240\pi(1)2(16)}{\pi(15)} = \frac{480(16)}{15} = 512 \text{ psi}$$

4. between CD $\quad T = 375\pi \quad c = 1 \quad b = 1/2 \quad J = \frac{\pi}{2}\left(\frac{15}{16}\right)$

$$T_{max_4} = \frac{Tc}{J} = \frac{375\pi(1)2(16)}{\pi(15)} = \frac{750(16)}{15} = 800 \text{ psi}$$

$T_{max} = 900$ psi / at AB

$$d\varphi = \frac{T\,dx}{JG} \qquad \varphi = \int_0^x \frac{T(x)}{J(x)G}\,dx + C_1$$

$$J_{AB} = \frac{\pi c^4}{2} = \frac{\pi}{2} = J_{BK} \qquad J_{KC} = J_{CD} = \frac{\pi}{2}\left(c_o^4 - c_i^4\right) = \frac{\pi}{2}\left(\frac{15}{16}\right)$$

$L_{AB} = 10$ in $\quad L_{BK} = 6$ in $\quad L_{KC} = 6$ in $\quad L_{CD} = 10$ in

$T_{AB} = 450\pi$ ↺ $\quad T_{BK} = 240\pi$ ↻ $\quad T_{KC} = 240\pi$ ↺ $\quad T_{CD} = 375\pi$ ↺

$$\varphi = \int_A^D \frac{T(x)}{G(J(x))}\,dx = \int_A^B \frac{T_{AB}}{G\,J_{AB}}\,dx_{AB} + \int_B^K \frac{T_{BK}}{G\,J_{BK}}\,dx_{BK} + \int_K^C \frac{T_{KC}}{G\,J_{KC}}\,dx_{KC} + \int_C^D \frac{T_{CD}}{G\,J_{CD}}\,dx_{CD}$$

$$= \frac{T_{AB}L_{AB}}{G\,J_{AB}} + \frac{T_{BK}L_{BK}}{G\,J_{BK}} + \frac{T_{KC}L_{KC}}{G\,J_{KC}} + \frac{T_{CD}L_{CD}}{G\,J_{CD}}$$

$$= \frac{(450\pi)(10)}{(12\times10^6)(\pi/2)} + \frac{240\pi(6)}{(12\times10^6)(\pi/2)} + \frac{240\pi(6)}{(12\times10^6)\left(\frac{\pi}{32}(15)\right)} + \frac{375\pi(10)}{(12\times10^6)\left(\frac{\pi}{32}(15)\right)}$$

$$= \frac{9000}{12\times10^6} + \frac{480}{2\times10^6} + \frac{240(32)}{(2\times10^6)(15)} + \frac{375(16)}{(6\times10^5)(15)}$$

$$= .75\times10^{-3} + .24\times10^{-3} + .128\times10^{-3} + .667\times10^{-3} = 1.785\times10^{-3}\ \text{rad.}$$

φ in degrees $= .001785\left(\frac{180}{\pi}\right) = .123^\circ$

$\tau_{max} = \frac{Tc}{J}$ $J_1 = \frac{\pi c^4}{2}$ גוש מלא

$J_2 = \frac{\pi b^4}{2} - \frac{\pi a^4}{2}$ גוש נבוב

$\tau_{max_1} = \frac{Tc}{J_1} = \frac{Tb}{J_2}$ שמאפשר התנגנות יחידה שווה לבין המינימום τ_{max} כ. $c=b=3$ יחידה שווה לבין המינימום

$bJ_1 = cJ_2$ ו. נתון $c=1in$ ו $b=1.5in$, $J_1 = \frac{\pi}{2} in^4$ ו- $J_2 = \frac{\pi 81}{32} - \frac{\pi a^4}{2}$ לכן

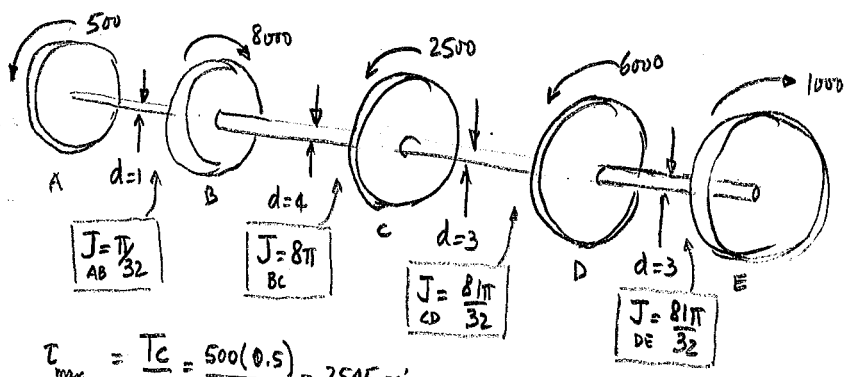
$$bJ_1 = cJ_2 = 1.5\frac{\pi}{2} = 1\left(\frac{\pi \cdot 81}{32} - \frac{\pi}{2}a^4\right) \Rightarrow a = 1.375in$$

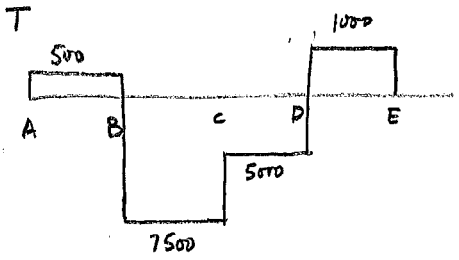
לכן העובי $t = b - a = 0.125$

מחפשים

$$2.74 = \frac{1^2}{1.5^2 - 1.375^2} = \frac{c^2}{b^2 - a^2} = \frac{\pi c^2 L \cdot \rho}{\pi(b^2-a^2)L\rho} = \frac{\text{מסת גוש מלא}}{\text{מסת גוש נבוב}}$$

5-2





$\tau_{max_{AB}} = \frac{Tc}{J} = \frac{500(0.5)}{J_{AB}} = 2545\ psi$

$\tau_{max_{BC}} = \frac{Tc}{J} = \frac{7500(2)}{J_{BC}} = 596\ psi$

$\tau_{max_{CD}} = \frac{Tc}{J} = \frac{5000(1.5)}{J_{CD}} = 944\ psi$

$\tau_{max_{DE}} = \frac{Tc}{J} = \frac{1000(1.5)}{J_{DE}} = 188.6\ psi$

$\tau_{max} = 2545\ psi$

5-3

90hp ← 30hp →

N=630hp

כאן $T = \frac{63000\,hp}{N}$, לכן ימין שמאל

$3000\ in\text{-}lb = T = \frac{63000 \cdot 30}{630}$

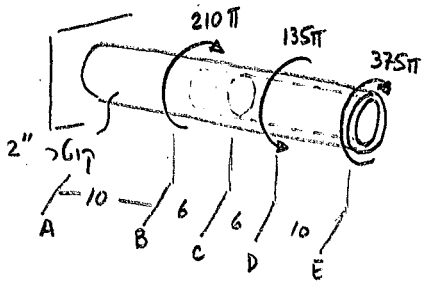
$9000\ in\text{-}lb = T = \frac{63000 \cdot 90}{630}$

כאן למעלה שלח המינימום המקסימלי.

$5750\ psi = \tau_{max} = \frac{T_{max} \cdot c}{J} = \frac{T_{max}\, c}{\pi c^4/2}$

לכן $c = \sqrt{\frac{2}{\pi}\frac{T_{max}}{\tau_{max}}} \cong 1in$

5-4



כאן שיהיה שווי מקסימלי המינימום ניתנת בחירה

לתוך 450π לכל כיוון המעגל.

כיוון AB $T = 450\pi$ $c = 1"$ $J = \pi c^4/2$ $\tau_{max} = \frac{(450\pi)\cdot 1}{\pi/2 \cdot 1^4} = 900psi$

כיוון BC $T = 240\pi$ $c = 1"$ $J = \pi c^4/2$ $\tau_m = \frac{240\pi \cdot 1}{\pi/2 \cdot 1^4} = 480psi$

כיוון C-D $T = 240\pi$ $b=1$ $a=0.5$ $J = \pi/2(b^4 - a^4)$ $\tau_{max} = 512\ psi$

כיוון D-E $T = 375\pi$ $b=1$ $a=0.5$ $J = \pi/2(b^4 - a^4)$ $\tau_{max} = 800\ psi$

לכן, $\tau_{max} = 900\ psi$ כיוון AB

$$\varphi = \int_A^B \frac{T}{GJ}dx + \int_B^C \frac{T}{GJ}dx + \int_C^D \frac{T}{GJ}dx + \int_D^E \frac{T}{GJ}dx$$

$$= \left[\frac{T_{AB}L_{AB}}{G J_{AB}} + \frac{T_{BC}L_{BC}}{G J_{BC}} + \frac{T_{CD}L_{CD}}{G J_{CD}} + \frac{T_{DE}L_{DE}}{G J_{DE}}\right] = \frac{450\pi \cdot 10}{12\times10^6 \cdot \pi/2} + \frac{240\pi \cdot 6}{12\times10^6 \cdot \pi/2} + \frac{240\pi \cdot 6}{12\times10^6 \cdot \frac{15\pi}{32}} + \frac{375\pi \cdot 10}{12\times10^6 \cdot \frac{15\pi}{32}}$$

$$= .75\times10^{-3} + .24\times10^{-3} + .256\times10^{-3} + .667\times10^{-3} = 1.913\times10^{-3} \text{ radians}$$

$$\varphi \text{ במעלות} \quad 0.1096^\circ = 1.913\times10^{-3} \cdot \frac{180}{\pi}$$

5-10 פיתון מקסימלי נמצא ב-A והוא שווה δ 10000π

$$\tau_{max} = \frac{T_{max} c_1}{J} \quad \text{A-B} \quad \text{ין } \tau_{max}$$

$$\tau_{max} = T_{max} \cdot \frac{2}{\pi c_1^3}$$

τ_{max} ין B-C הכי מפיתון של 2500π. לכן,

$$\tau_{max} = \frac{Tc_2}{J_2} = \frac{T\cdot 2}{\pi c_2^3}$$

אם נשמר שהתנאי הכי תמור הוא הכוונת, אז

$$\varphi = \int_A^B \frac{T}{JG}dx = \int_A^B \frac{T}{JG}dx + \int_B^C \frac{T}{JG}dx = \frac{T_{AB}L_{AB}}{J_{AB}G} + \frac{T_{BC}L_{BC}}{J_{BC}G} = \frac{3^\circ \cdot \pi}{180} = \frac{\pi}{60}$$

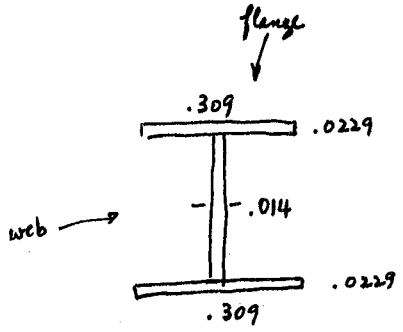
$$\varphi = \frac{10000\pi \cdot 180}{\frac{\pi c^4}{2} \cdot 12\times10^6} + \frac{2500\pi \cdot 60}{\frac{\pi \cdot 2^4}{32} \cdot 12\times10^6} = \pi/60 \qquad c = 1.82 \text{ in}$$

לכן, τ_{max} ין BC $= \frac{2500\pi \cdot 1}{\pi \cdot 1^4/2} = 5000 \text{psi}$

τ_{max} ין AB $= \frac{10000\pi\,(1.82)}{\pi(1.82)^4/2} = 3318 \text{psi}$

אם נאמרנו שהתנאי הכי חמור היה על מנת שקבל ש-C ין AB ויהיה שווה ל 1.4938 ותרומת הקטע AB לכוונת הכללית תיהיה פי 9/4 ממה שהיה וזה שווה לכוונת יותר גדול מ-3°

Pg 753 in 4th ed.



I used $C_1 = C_2 = \frac{1}{3}$

$$J_R = 2\frac{b_1 t_1^3}{3} + \frac{b_2 t_2^3}{3}$$ s/B .3174 ; s/B .3224

$= $ [illegible] $\times 10^{-6}\ m^4$ (s/B 2.601×10^{-6})

T — 277.2 [illegible] to $\mathcal{C}_t$

$2T_1 + T_2 = T$

$$\varphi = \frac{T_1 L}{G_1 [k_1 b_1 t_1^3]} = \frac{T_2 L}{G_1 [k_2 b_2 t_2^3]}$$

$$\frac{T_1}{\text{[illegible]}} = \frac{T_2}{\text{[illegible]} \times 10^{-7}}$$ (s/B 482.45)

s/B 1.178×10^{-6}

$\therefore \left[2\left(\frac{\text{[illegible]}}{4.808}\right) + 1\right] T_2 = T$

[illegible] $T_2 = T$

s/B (1062)

$T_1 = $ [illegible] T_2 (s/B 4.808)

(s/B 423.88) 432.41

$T_2 = $ [illegible] N-m

$T_1 = $ [illegible] N-m

s/B 2038)

$$\varphi = \frac{T_1 L}{G_1 \times 1.237 \times 10^{-6}\ m^4} = .07[\text{illegible}]\ \text{rad} = [\text{illegible}]^\circ\ 4.51$$

$$\tau = \frac{T t_1}{J_R}$$ OK $\frac{T_1 t_1}{J_R} = 3[\text{illegible}]$ MPa 39.62 MPa

$$\tau = \frac{T t_2}{J_R}$$ OK $\frac{T_2 t_2}{J_R} = 23[\text{illegible}]$ MPa 24.22 MPa

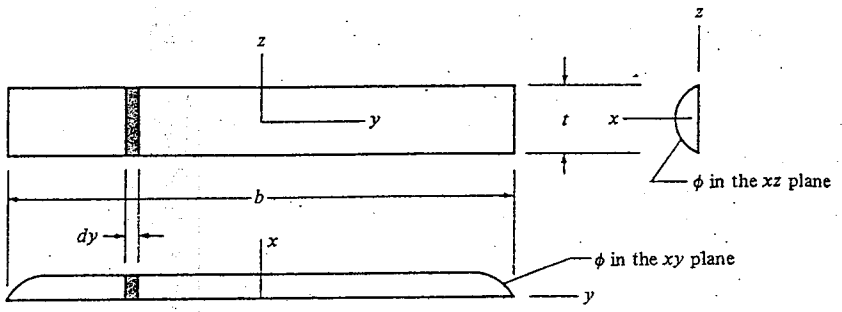


FIGURE 8.4.2. Narrow rectangular cross section ($b >> t$), with views of the stress function (or inflated membrane).

where C is a constant. Substituting into Eq. 8.3.5, we find

$$-2C = -2G\beta \qquad \text{so} \qquad C = G\beta \tag{8.4.4}$$

The torque is

$$T = 2\int_{\text{area}} \phi \, dA = 2\int_{-t/2}^{t/2} \phi b \, dz = G\beta \frac{bt^3}{3} \tag{8.4.5}$$

The maximum shear stress, found along the edges $z = \pm t/2$, is

$$\tau = \left|\frac{\partial \phi}{\partial z}\right|_{z=\pm t/2} = 2C\frac{t}{2} = G\beta t \tag{8.4.6}$$

Substituting for β from Eq. 8.4.5, we find

$$\tau = \frac{Tt}{bt^3/3} = \frac{3T}{bt^2} \qquad \text{at } z = \pm\frac{t}{2} \tag{8.4.7}$$

Equations 8.4.5 and 8.4.7 can be written in the forms

$$\frac{d\theta}{dx} = \beta = \frac{T}{GJ_R} \qquad \tau = \frac{Tt}{J_R} \tag{8.4.8}$$

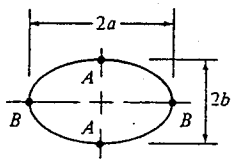
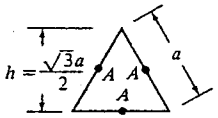
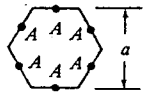
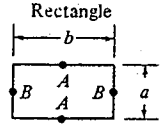
where

$$J_R = \frac{bt^3}{3} \qquad (\text{for } b >> t) \tag{8.4.9}$$

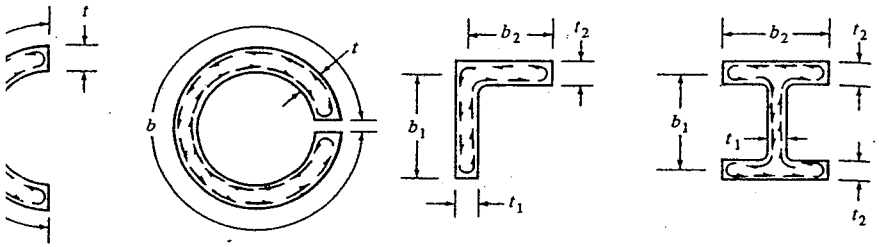
These expressions for β and τ are similar in form to the corresponding expressions for a circular cross section, Eqs. 8.1.2 and 8.1.3. However, J_R is *emphatically NOT* the polar moment of the cross-sectional area about the centroidal x axis.

If (say) $b/t = 10$, Eqs. 8.4.8 give β and τ values that are approximately 6.5 percent low. Accuracy improves as b/t increases. Aspect ratios in the range $1 < b/t < 10$ can be analyzed with tabulated data obtained by other analytical or numerical methods (Table 8.4.1). If the centerline of the bar is curved, as

TABLE 8.4.1 Expressions for Maximum Shear Stress and Rate of Twist in Selected Solid Sections [8.1, 8.4]

Cross Section and Area	Maximum Shear Stress	Rate of Twist
Ellipse  Area = πab	$\tau_A = \dfrac{2T}{\pi ab^2}$ $(a > b)$ (τ_{max} at B if $b > a$)	$\beta = \dfrac{a^2 + b^2}{\pi a^3 b^3}\dfrac{T}{G} = \dfrac{d\theta}{dx}$
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Rectangle  Area = ab	$\tau_A = \dfrac{T}{C_\tau ba^2}$	$\beta = \dfrac{1}{C_\beta ba^3}\dfrac{T}{G} = \dfrac{d\theta}{dx}$

b/a	C_τ	τ_B/τ_A	C_β
1.0	0.208	1.000	0.1406
1.2	0.219	0.935	0.166
1.5	0.231	0.859	0.196
2.0	0.246	0.795	0.229
2.5	0.258	0.766	0.249
3.0	0.267	0.753	0.263
4.0	0.282	0.745	0.281
6.0	0.299	0.743	0.299
10.0	0.312	0.742	0.312
∞	0.333	0.742	0.333



$= \frac{bt^3}{3}$ $\quad J_R = \frac{bt^3}{3}$ $\quad J_R = \frac{b_1 t_1^3 + b_2 t_2^3}{3}$ $\quad J_R = \frac{b_1 t_1^3 + 2b_2 t_2^3}{3}$

(a) (b) (c) (d)

RE 8.4.3. Thin-walled open sections, showing their shear stresses and torsional constant J_R. In (a) and (b), dimension b is the length of the midline of the cross section.

when a bar is bent to form a helical spring, some corrections to these formulas may be necessary [8.3].

The membrane analogy permits a very useful generalization of the foregoing results to other thin-walled open cross sections. Imagine that the narrow rectangular cross section of Fig. 8.4.2 is distorted into a C or an L shape, or attached to another narrow rectangle to make a T or an I section. By visualizing the inflated membranes for these shapes, we decide that ϕ surfaces of all of them remain parabolic (excepting near ends and near reentrant corners, which we discuss later). The total torque is the sum of torques carried by each part of the cross section. Since $G\beta$ is the same for each part, Eqs. 8.4.5 and 8.4.8 still apply but with the $bt^3/3$ contributions summed to yield

$$J_R = \frac{1}{3}\sum_{i=1}^{n} b_i t_i^3 \tag{8.4.10}$$

where n is the number of parts into which the cross section is divided for purposes of calculation. Examples of this calculation appear in Fig. 8.4.3. In angle and I sections, the part of greater thickness displays greater shear stress. Where there is taper, as in the cross section of a turbine blade or in flanges of a rolled section, we can use

$$J_R = \frac{1}{3}\int t^3\, ds \tag{8.4.11}$$

where ds is an increment of length along the medial line of the cross section. More exact formulas are available [8.3]. These formulas, experimental results [8.5], and coefficient C_β in Table 8.4.1 suggest that, for standard rolled structural shapes, J_R may actually be some 10 percent higher than predicted by Eqs. 8.4.10 and 8.4.11. Omission of this adjustment yields a conservative design under either a stress limit or a deflection limit.

8.5 A HYDRODYNAMIC ANALOGY

There are several hydrodynamic analogies to the torsion problem [8.1]. In this section we outline one, without proof, and use it to draw useful conclusions about shear stresses in twisted bars.

If a fluid without viscosity executes motion in the yz plane with constant vorticity, its motion is described by the equation

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = -2\omega \tag{8.5.1}$$

where ϕ is the stream function and ω is the (constant) vorticity. Fluid velocities v and w in y and z directions, respectively, are

$$v = \frac{\partial \phi}{\partial z} \qquad w = -\frac{\partial \phi}{\partial y} \tag{8.5.2}$$

Clearly, Eqs. 8.5.1 and 8.5.2 are analogous to Eqs. 8.3.5 and 8.3.1, respectively. Thus fluid velocities are proportional to shear stresses. Lines ϕ = constant are streamlines. The fluid boundary must be a streamline, therefore the boundary condition $d\phi = 0$ of Eq. 8.3.6 is met. Thus the analogy is complete.

Experiments have been done in the following way. Imagine that a square cross section is to be studied. A shallow square tank is painted black and placed on a turntable. A camera, looking down onto the tank, is also attached to the turntable. The tank is filled with water (whose viscosity is low, if not quite zero) and aluminum powder is sprinkled on the water. When all is quiet, the camera shutter is opened while the turntable is rotated about 10°. From the viewpoint of the photograph, the tank is stationary while the fluid rotates with nearly constant vorticity. The aluminum particles show as streaks on the film. Each streak is in the direction of a shear stress. The length of the streak is proportional to the magnitude of the shear stress.

Experiments aside, the hydrodynamic analogy has other uses. One is in aiding visualization of torsion problems. Another is that known solutions for fluid flow can be applied to the torsion problem. For example, consider an elliptical obstacle (Fig. 8.5.1a). Far from the obstacle the flow is uniform and horizontal, so $w = 0$ and $v = v_o$, a constant. Points A are stagnation points, where $v = w = 0$. Fluid theory shows that at points B, where $z = \pm b$, fluid velocities are $v = v_o(1 + b/a)$ and $w = 0$. Applying these results to a twisted bar, Fig. 8.5.1b, we conclude that stress is zero at the sharp external corners C. At the root of the small elliptical notch, shear stress is approximately

$$\tau = \tau_o\left(1 + \frac{b}{a}\right) \tag{8.5.3}$$

where τ_o is the stress that would prevail near the boundary if the notch were not there. The calculation is approximate because the undisturbed stress is not the uniform value τ_o unless the notch is very small. The quantity in parentheses

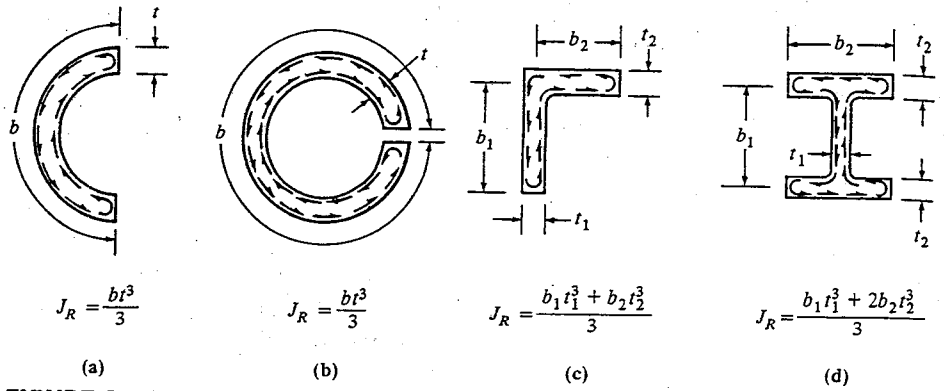


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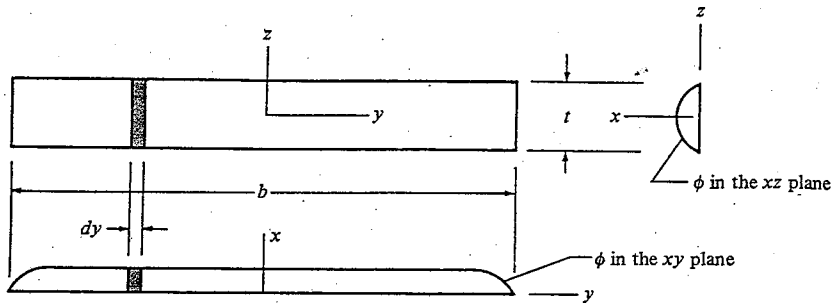


FIGURE 8.4.2. Narrow rectangular cross section ($b >> t$), with views of the stress function (or inflated membrane).

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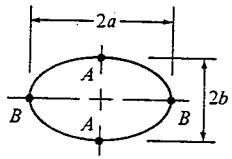
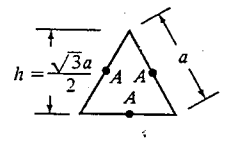
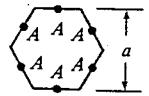
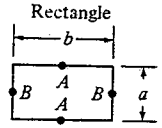
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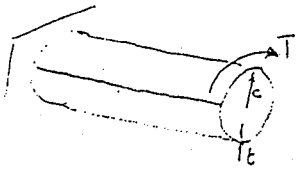
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Equilateral triangle  Area = $0.433\,a^2$	$\tau_A = \dfrac{20T}{a^3}$	$\beta = \dfrac{46.2}{a^4}\dfrac{T}{G}$
Regular hexagon  Area = $0.866a^2$	$\tau_A = \dfrac{5.7T}{a^3}$	$\beta = \dfrac{8.8}{a^4}\dfrac{T}{G}$
Rectangle  Area = ab	$\tau_A = \dfrac{T}{C_\tau ba^2}$	$\beta = \dfrac{1}{C_\beta ba^3}\dfrac{T}{G}$

b/a	C_τ	τ_B/τ_A
1.0	0.208	1.000
1.2	0.219	0.935
1.5	0.231	0.859
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4.0	0.282	0.745
6.0	0.299	0.743
10.0	0.312	0.742
∞	0.333	0.742

3.132



closed tube $J = \frac{\pi}{2}(a^4 - b^4) = \frac{\pi}{2}(a^2 - b^2)(a^2 + b^2)$; $\tau_1 = \frac{Tc}{J}$; $\phi_1 = \frac{TL}{JG}$

$= \frac{\pi}{2}(a-b)(a+b)(a^2+b^2)$

$J = \frac{\pi}{2} \cdot t \cdot 2c \cdot 2c^2 = 2\pi c^3 t$ $\tau_1 = \frac{T}{2\pi c^2 t}$; $\phi_1 = \frac{TL}{2\pi c^3 t G}$

Open tube $J_R = \frac{bt^3}{3} = \frac{2\pi c t^3}{3}$; $\tau_2 = \frac{Tt}{J_R}$; $\beta = \frac{\phi_2}{L} = \frac{T}{GJ_R}$

$\tau_2 = \frac{Tt}{2\pi c t^3/3} = \frac{3Tt}{2\pi c t^3} = \frac{3T}{2\pi c t^2}$; $\phi_2 = \frac{TL}{\frac{2\pi c t^3}{3} G} = \frac{3TL}{2\pi c t^3 G}$

$\frac{\tau_1}{\tau_2} = \frac{\frac{T}{2\pi c^2 t} \cdot \frac{t}{t}}{\frac{3T}{2\pi c t^2} \cdot \frac{c}{c}} = \frac{t}{3c}$; $\frac{\phi_1}{\phi_2} = \frac{\frac{TL}{2\pi c^3 t G} \cdot \frac{t^2}{t^2}}{\frac{3TL}{2\pi c t^3 G} \cdot \frac{c^2}{c^2}} = \frac{t^2}{3c^2}$

$\left(\frac{\tau_1}{\tau_2}\right)^2 = \frac{t^2}{9c^2} = \frac{1}{3}\left(\frac{\phi_1}{\phi_2}\right)$

so: $\frac{\tau_2}{\tau_1} = 3\left(\frac{c}{t}\right)$ and $\frac{\phi_2}{\phi_1} = 3\left(\frac{c}{t}\right)^2$

Fig. P3.108 and P3.109

3.132 A thin-walled tube has been fabricated by bending a metal plate of thickness t into a cylinder of radius c and bonding together the edges of the plate. A torque T is then applied to the tube, producing a shearing stress τ_1 and an angle of twist ϕ_1. Denoting by τ_2 and ϕ_2, respectively, the shearing stress and the angle of twist which will develop if the bond suddenly fails, express the ratios τ_2/τ_1 and ϕ_2/ϕ_1 in terms of the ratio c/t.

$\phi_2/\phi_1 = 3\left(\frac{c}{t}\right)^2$

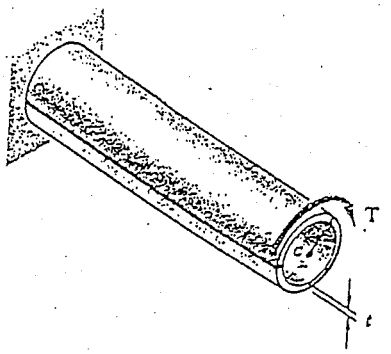


Fig. P3.132

3.122 A 3.5-m-long steel member with a W310 × 143 cross section is subjected to a 4.5 kN · m torque. Knowing that $G = 77$ GPa and referring to Appendix C for the dimensions of the cross section, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the maximum shearing stress along line *b-b*, (*c*) the angle of twist. (*Hint*: Consider the web and the flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

39.7 MPa 24.2 MPa 4.72°

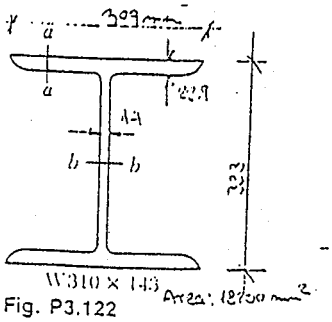
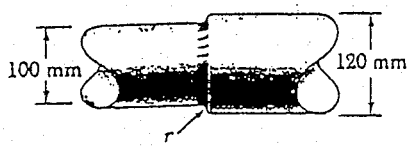


Fig. P3.122

HW: DO 3.21, 3.47 & 3.53

PROBLEM 3.87



3.87 The stepped shaft shown rotates at 450 rpm. Knowing that $r = 10$ mm, determine the maximum power that can be transmitted without exceeding an allowable shearing stress of 45 MPa.

SOLUTION

$d = 100$ mm, $D = 120$ mm, $r = 10$ mm

$\frac{D}{d} = \frac{120}{100} = 1.2$, $\frac{r}{d} = \frac{10}{100} = 0.10$, From Fig. 3.32 $K = 1.33$

For smaller shaft $c = \frac{1}{2}d = 0.050$ m $\qquad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.050)^3 (45 \times 10^6)}{(2)(1.33)} = 6.643 \times 10^3 \text{ N-m}$$

$f = 450$ rpm $= 7.5$ Hz $= 450/60$

Power $P = 2\pi f T = 2\pi (7.5)(6.643 \times 10^3) = 313 \times 10^3$ W $= 313$ kW

$$T = \frac{9540 P}{N} \Rightarrow P = \frac{T \cdot N}{9540} = 313.35 \text{ kW}$$

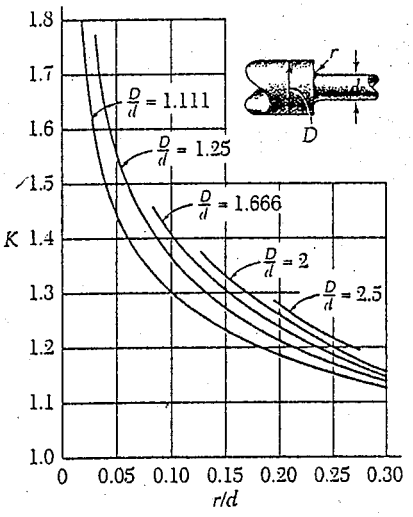


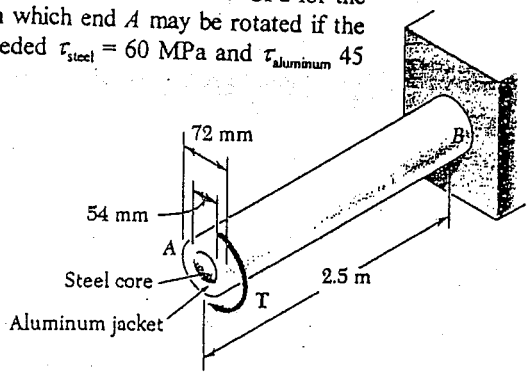
Fig. 3.32 Stress-concentration factors for fillets in circular shafts.†

TABLE 3.1. Coefficients for Rectangular Bars in Torsion

a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

PROBLEM 3.53

3.53 The composite shaft shown is to be twisted by applying a torque T at end *A*. Knowing that the modulus of rigidity is 77 GPa for the steel and 27 GPa for the aluminum, determine the largest angle through which end *A* may be rotated if the following allowable stresses are not to be exceeded τ_{steel} = 60 MPa and $\tau_{aluminum}$ 45 MPa.



SOLUTION

$$\tau_{max} = G\gamma_{max} = -G c_{max}\frac{\varphi}{L}$$

$$\frac{\varphi_{all}}{L} = \frac{\tau_{all}}{G c_{max}} \quad \text{for each material}$$

Steel core: $\tau_{all} = 60\times10^{6}$ Pa, $c_{max} = \frac{1}{2}d = 0.027$ m, $G = 77\times10^{9}$ Pa

$$\frac{\varphi_{all}}{L} = \frac{60\times10^{6}}{(77\times10^{9})(0.027)} = 28.860\times10^{-3} \text{ rad/m}$$

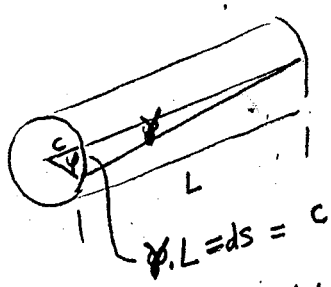
Aluminum jacket: $\tau_{all} = 45\times10^{6}$ Pa, $c_{max} = \frac{1}{2}d = 0.036$ m, $G = 27\times10^{9}$ Pa

$$\frac{\varphi_{all}}{L} = \frac{45\times10^{6}}{(27\times10^{9})(0.036)} = 46.296\times10^{-3} \text{ rad/m}$$

Smaller value governs $\quad \frac{\varphi_{all}}{L} = 28.860\times10^{-3}$ rad/m

Allowable angle of twist $\quad \varphi_{all} = L\frac{\varphi_{all}}{L} = (2.5)(28.860\times10^{-3})$

$$= 72.15\times10^{-3} \text{ rad} = 4.13^{\circ}$$



$\gamma \cdot L = ds = c\cdot\varphi$

$$\varphi = \frac{\gamma\cdot L}{c} = \frac{\tau_{max}\cdot L}{G\cdot c}$$

$\therefore \frac{\varphi}{L}$ must be same for both

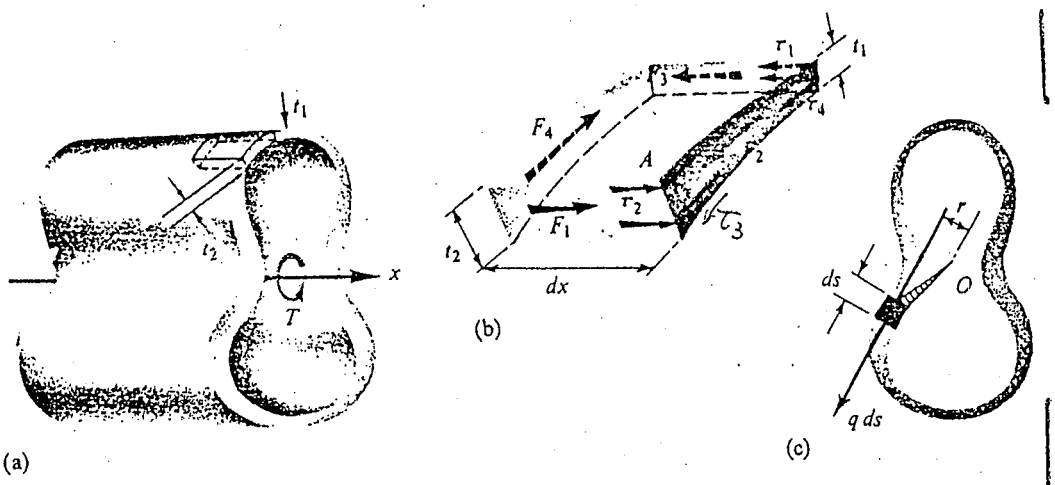


Fig. 3-22. Thin-walled member of variable thickness.

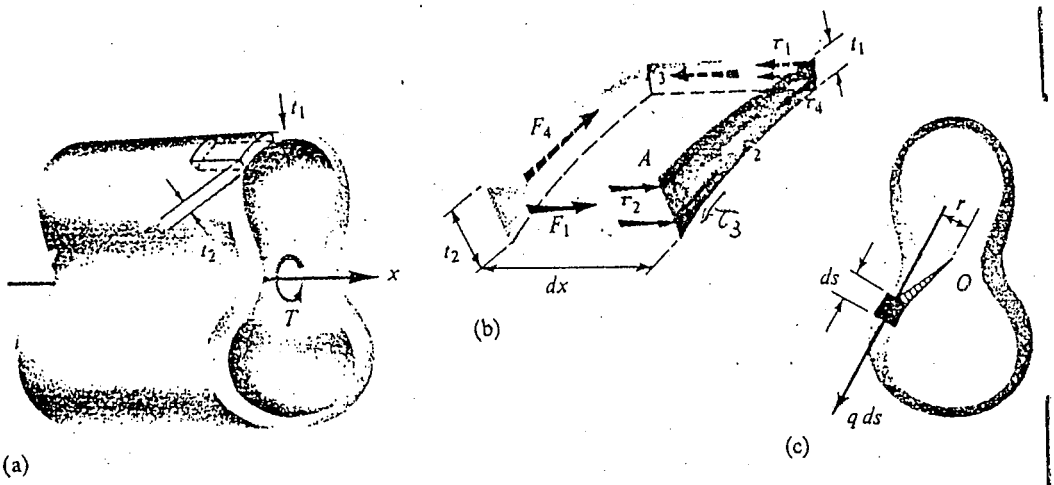


Fig. 3-22. Thin-walled member of variable thickness.

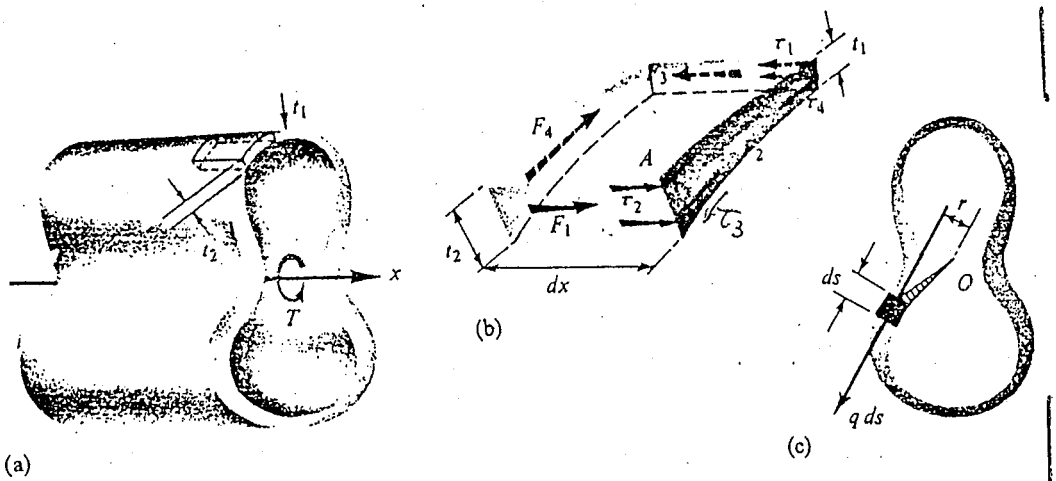


Fig. 3-22. Thin-walled member of variable thickness.

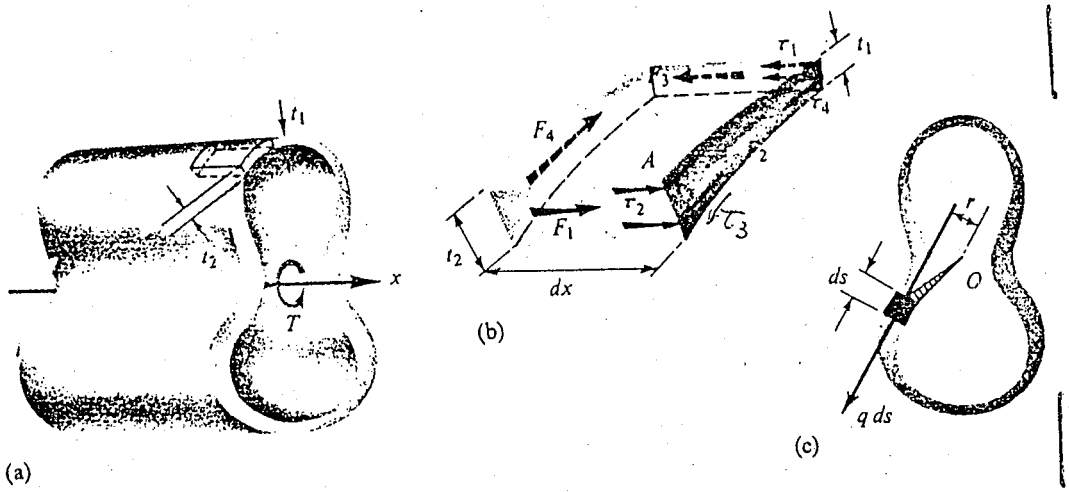


Fig. 3-22. Thin-walled member of variable thickness.

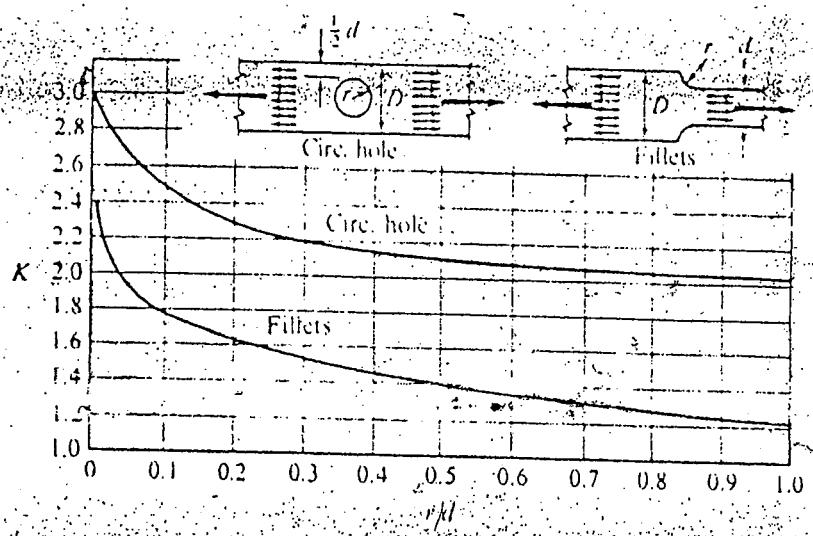


Fig. 2-17. Stress-concentration factors for flat bars in tension

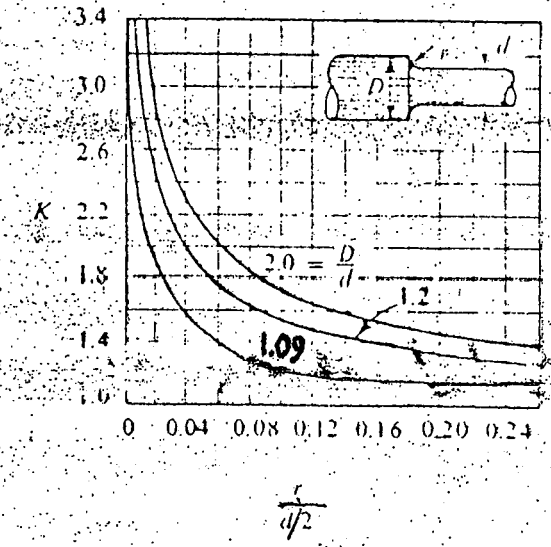


Fig. 3-15. Torsional stress-concentration factors in circular shafts of two diameters.

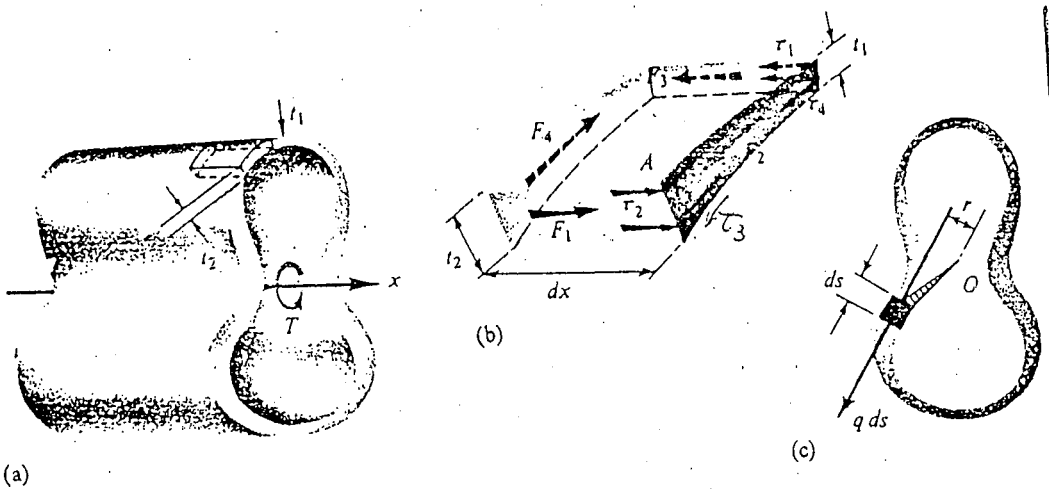
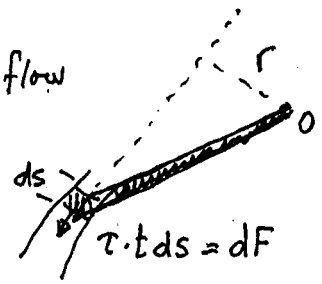


Fig. 3-22. Thin-walled member of variable thickness.

$$F_1 = F_3$$

$$\tau_1 t_1 dx = \tau_2 t_2 dx$$

$$\tau_1 t_1 = \tau_2 t_2 = q = \text{shear flow}$$

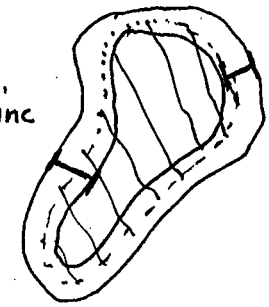


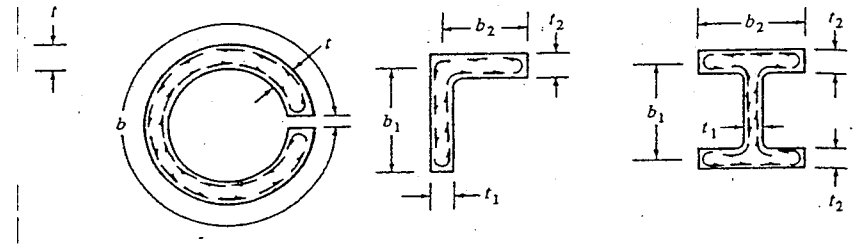
$$T = \oint dT = \oint r \cdot q\, ds = q \oint r\, ds$$

$$T = q \cdot 2A_{inc}$$

$$\tau t = q = \frac{T}{2A_{inc}}$$

$$\tau = \frac{T}{2A_{inc}\, t}$$





$\frac{bt^3}{3}$ (b) $J_R = \frac{bt^3}{3}$ (c) $J_R = \frac{b_1 t_1^3 + b_2 t_2^3}{3}$ (d) $J_R = \frac{b_1 t_1^3 + 2b_2 t_2^3}{3}$

3.4.3. Thin-walled open sections, showing their shear stresses and torsional con- n (a) and (b), dimension b is the length of the midline of the cross section.

/hen a bar is bent to form a helical spring, some corrections to these formulas ıay be necessary [8.3].

The membrane analogy permits a very useful generalization of the foregoing esults to other thin-walled open cross sections. Imagine that the narrow rectan- ular cross section of Fig. 8.4.2 is distorted into a C or an L shape, or attached ɔ another narrow rectangle to make a T or an I section. By visualizing the ıflated membranes for these shapes, we decide that ϕ surfaces of all of them emain parabolic (excepting near ends and near reentrant corners, which we iscuss later). The total torque is the sum of torques carried by each part of the ross section. Since $G\beta$ is the same for each part, Eqs. 8.4.5 and 8.4.8 still pply but with the $bt^3/3$ contributions summed to yield

$$J_R = \frac{1}{3}\sum_{i=1}^{n} b_i t_i^3 \tag{8.4.10}$$

/here n is the number of parts into which the cross section is divided for urposes of calculation. Examples of this calculation appear in Fig. 8.4.3. In ngle and I sections, the part of greater thickness displays greater shear stress. Vhere there is taper, as in the cross section of a turbine blade or in flanges of rolled section, we can use

$$J_R = \frac{1}{3}\int t^3\, ds \tag{8.4.11}$$

/here ds is an increment of length along the medial line of the cross section. Aore exact formulas are available [8.3]. These formulas, experimental results 3.5], and coefficient C_β in Table 8.4.1 suggest that, for standard rolled struc- ıral shapes, J_R may actually be some 10 percent higher than predicted by Eqs. .4.10 and 8.4.11. Omission of this adjustment yields a conservative design nder either a stress limit or a deflection limit.

8.5 A HYDRODYNAMIC ANALOGY

There are several hydrodynamic analogies to the torsion problem [8.1]. In this section we outline one, without proof, and use it to draw useful conclusions about shear stresses in twisted bars.

If a fluid without viscosity executes motion in the yz plane with constant vorticity, its motion is described by the equation

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = -2\omega \tag{8.5.1}$$

where ϕ is the stream function and ω is the (constant) vorticity. Fluid velocities v and w in y and z directions, respectively, are

$$v = \frac{\partial \phi}{\partial z} \qquad w = -\frac{\partial \phi}{\partial y} \tag{8.5.2}$$

Clearly, Eqs. 8.5.1 and 8.5.2 are analogous to Eqs. 8.3.5 and 8.3.1, respectively. Thus fluid velocities are proportional to shear stresses. Lines ϕ = constant are streamlines. The fluid boundary must be a streamline, therefore the boundary condition $d\phi = 0$ of Eq. 8.3.6 is met. Thus the analogy is complete.

Experiments have been done in the following way. Imagine that a square cross section is to be studied. A shallow square tank is painted black and placed on a turntable. A camera, looking down onto the tank, is also attached to the turntable. The tank is filled with water (whose viscosity is low, if not quite zero) and aluminum powder is sprinkled on the water. When all is quiet, the camera shutter is opened while the turntable is rotated about 10°. From the viewpoint of the photograph, the tank is stationary while the fluid rotates with nearly constant vorticity. The aluminum particles show as streaks on the film. Each streak is in the direction of a shear stress. The length of the streak is proportional to the magnitude of the shear stress.

Experiments aside, the hydrodynamic analogy has other uses. One is in aiding visualization of torsion problems. Another is that known solutions for fluid flow can be applied to the torsion problem. For example, consider an elliptical obstacle (Fig. 8.5.1a). Far from the obstacle the flow is uniform and horizontal, so $w = 0$ and $v = v_o$, a constant. Points A are stagnation points, where $v = w = 0$. Fluid theory shows that at points B, where $z = \pm b$, fluid velocities are $v = v_o(1 + b/a)$ and $w = 0$. Applying these results to a twisted bar, Fig. 8.5.1b, we conclude that stress is zero at the sharp external corners C. At the root of the small elliptical notch, shear stress is approximately

$$\tau = \tau_o\left(1 + \frac{b}{a}\right) \tag{8.5.3}$$

where τ_o is the stress that would prevail near the boundary if the notch were not there. The calculation is approximate because the undisturbed stress is not the uniform value τ_o unless the notch is very small. The quantity in parentheses

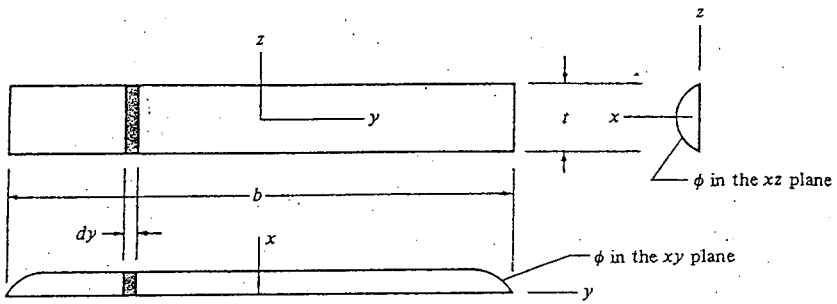


FIGURE 8.4.2. Narrow rectangular cross section ($b >> t$), with views of the stress function (or inflated membrane).

where C is a constant. Substituting into Eq. 8.3.5, we find

$$-2C = -2G\beta \qquad \text{so} \qquad C = G\beta \tag{8.4.4}$$

The torque is

$$T = 2\int_{\text{area}} \phi \, dA = 2\int_{-t/2}^{t/2} \phi b \, dz = G\beta \frac{bt^3}{3} \tag{8.4.5}$$

The maximum shear stress, found along the edges $z = \pm t/2$, is

$$\tau = \left|\frac{\partial \phi}{\partial z}\right|_{z=\pm t/2} = 2C\frac{t}{2} = G\beta t \tag{8.4.6}$$

Substituting for β from Eq. 8.4.5, we find

$$\tau = \frac{Tt}{bt^3/3} = \frac{3T}{bt^2} \qquad \text{at } z = \pm\frac{t}{2} \tag{8.4.7}$$

Equations 8.4.5 and 8.4.7 can be written in the forms

$$\frac{d\theta}{dx} = \beta = \frac{T}{GJ_R} \qquad \tau = \frac{Tt}{J_R} \tag{8.4.8}$$

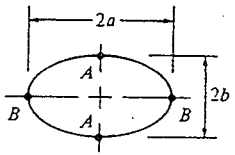
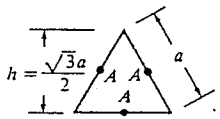
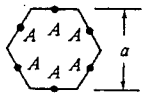
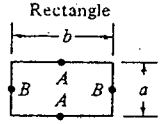
where

$$J_R = \frac{bt^3}{3} \qquad (\text{for } b >> t) \tag{8.4.9}$$

These expressions for β and τ are similar in form to the corresponding expressions for a circular cross section, Eqs. 8.1.2 and 8.1.3. However, J_R is *emphatically NOT* the polar moment of the cross-sectional area about the centroidal x axis.

If (say) $b/t = 10$, Eqs. 8.4.8 give β and τ values that are approximately 6.5 percent low. Accuracy improves as b/t increases. Aspect ratios in the range $1 < b/t < 10$ can be analyzed with tabulated data obtained by other analytical or numerical methods (Table 8.4.1). If the centerline of the bar is curved, as

TABLE 8.4.1 Expressions for Maximum Shear Stress and Rate of Twist in Selected Solid Sections [8.1, 8.4]

Cross Section and Area	Maximum Shear Stress	Rate of Twist
Ellipse  Area = πab	$\tau_A = \frac{2T}{\pi ab^2}$ $(a > b)$ (τ_{max} at B if $b > a$)	$\beta = \frac{a^2 + b^2}{\pi a^3 b^3}\frac{T}{G} = \frac{d\theta}{dx}$
Equilateral triangle  Area = $0.433\,a^2$	$\tau_A = \frac{20T}{a^3}$	$\beta = \frac{46.2}{a^4}\frac{T}{G} = \frac{d\theta}{dx}$
Regular hexagon  Area = $0.866a^2$	$\tau_A = \frac{5.7T}{a^3}$	$\beta = \frac{8.8}{a^4}\frac{T}{G} = \frac{d\theta}{dx}$
Rectangle  Area = ab	$\tau_A = \frac{T}{C_\tau ba^2}$	$\beta = \frac{1}{C_\beta ba^3}\frac{T}{G} = \frac{d\theta}{dx}$

b/a	C_τ	τ_B/τ_A	C_β
1.0	0.208	1.000	0.1406
1.2	0.219	0.935	0.166
1.5	0.231	0.859	0.196
2.0	0.246	0.795	0.229
2.5	0.258	0.766	0.249
3.0	0.267	0.753	0.263
4.0	0.282	0.745	0.281
6.0	0.299	0.743	0.299
10.0	0.312	0.742	0.312
∞	0.333	0.742	0.333

are equal in magnitude when the couple T_0 is applied at B. Determine the relation between the elastic moduli of the two materials.

Answer: $G_A = 0.592G_C$

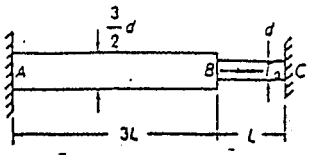


Fig. P6.96

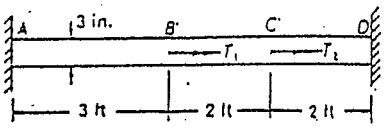


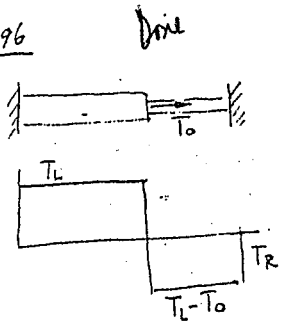
Fig. P6.97

6.97. An aluminum shaft is rigidly restrained at each end and loaded by couples as shown in Fig. P6.97. If $T_1 = 12{,}000$ in.-lb and $T_2 = 10{,}000$ in.-lb; compute the maximum shear stress in each section and the rotations of cross sections at B and C.

Answer: $\tau_{CD} = -2300\ psi, \quad \tau_{BC} = -433\ psi, \quad \tau_{AB} = 1840\ psi,$

A

6.96



$T_L - T_R = T_o$

$T_R = T_L - T_o$

since $|T_R| = |T_L|$

$T_L = T_o/2 \quad T_R = -T_o/2$

$$\varphi_{AB} = \frac{T_L L_{AB}}{J_{AB} G_{AB}} = |\varphi_{BC}| = \left|\frac{T_R L_{BC}}{J_{BC} G_{BC}}\right|$$

$$J_{AB} = \frac{\pi c^4}{2} = \frac{\pi d^4}{32} = \frac{\pi \cdot 81 d^4}{512}; \quad L_{AB} = 3L;$$

$$J_{BC} = \frac{\pi c^4}{2} = \frac{\pi d^4}{32}; \quad L_{BC} = L$$

$$\varphi_{AB} = \frac{T_o/2 \cdot 3L}{\pi 81 d^4/512 \, G_{AB}} = \frac{T_o/2 \cdot L}{\frac{\pi d^4}{32} G_{BC}}$$

$$\therefore \quad \frac{G_{AB} \cdot 81}{1536} = \frac{G_{BC}}{32} \quad \text{or} \quad G_{AB} = \frac{1536}{81 \cdot 32} G_{BC} = .5926 G_{BC}$$

Fig. P3.108 and P3.109

3.132 A thin-walled tube has been fabricated by bending a metal plate of thickness t into a cylinder of radius c and bonding together the edges of the plate. A torque T is then applied to the tube, producing a shearing stress τ_1 and an angle of twist ϕ_1. Denoting by τ_2 and ϕ_2, respectively, the shearing stress and the angle of twist which will develop if the bond suddenly fails, express the ratios τ_2/τ_1 and ϕ_2/ϕ_1 in terms of the ratio c/t.

$\phi_2/\phi_1 = 3\left(\frac{c}{t}\right)^2$

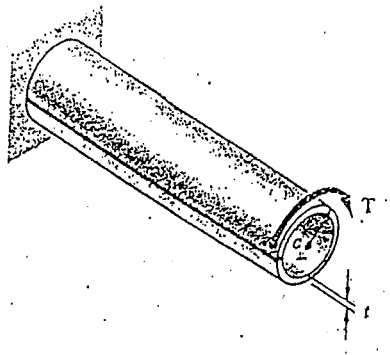


Fig. P3.132

3.122 A 3.5-m-long steel member with a W310 × 143 cross section is subjected to a 4.5 kN · m torque. Knowing that $G = 77$ GPa and referring to Appendix C for the dimensions of the cross section, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the maximum shearing stress along line *b-b*, (*c*) the angle of twist. (*Hint*: Consider the web and the flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

39.7 MPa 24.2 MPa 4.72°

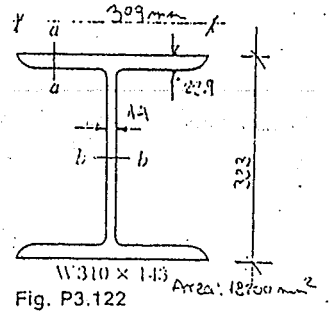


Fig. P3.122

welded together at B as shown in Fig. P6.96. The twisting couples at A and C are equal in magnitude when the couple T_0 is applied at B. Determine the relation between the elastic moduli of the two materials.

Answer: $G_A = 0.592G_C$

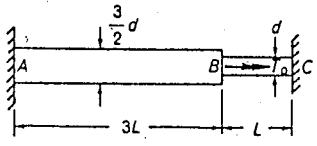


Fig. P6.96

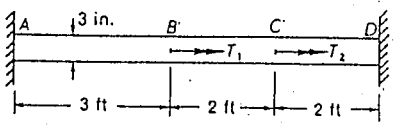


Fig. P6.97

6.97. An aluminum shaft is rigidly restrained at each end and loaded by couples as shown in Fig. P6.97. If $T_1 = 12{,}000$ in.-lb and $T_2 = 10{,}000$ in.-lb, compute the maximum shear stress in each section and the rotations of cross sections at B and C.

Answer: $\tau_{CD} = -2300\ psi, \quad \tau_{BC} = -433\ psi, \quad \tau_{AB} = 1840\ psi,$

3.108 Each of the two aluminum bars shown is subjected to a torque of magnitude $T = 1800$ N · m. Knowing that $G = 26$ GPa, determine for each bar the maximum shearing stress and the angle of twist at B.

Answers: (a) 40.1 MPa 0.653°
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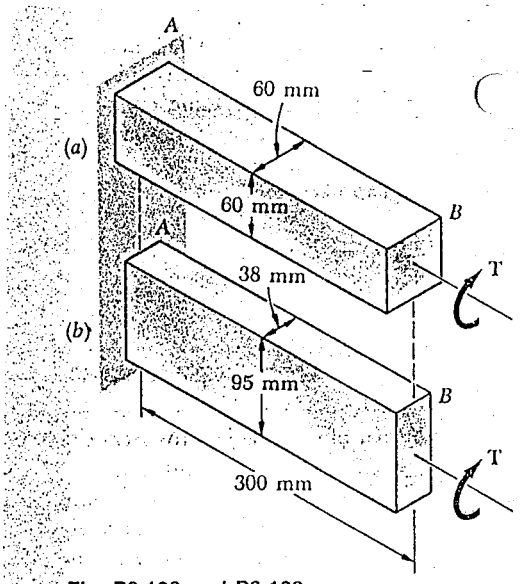


Fig. P3.108 and P3.109

Like 3.144 in 4th ed.

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$\phi_2/\phi_1 = 3\left(\frac{c}{t}\right)^2$

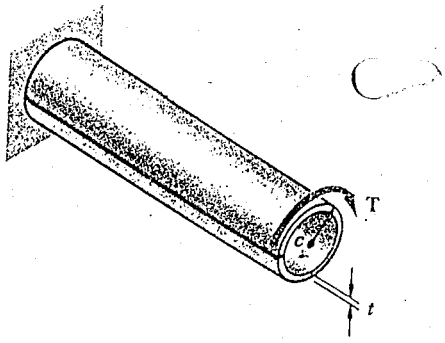


Fig. P3.132

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39.7 MPa 24.2 MPa 4.72°

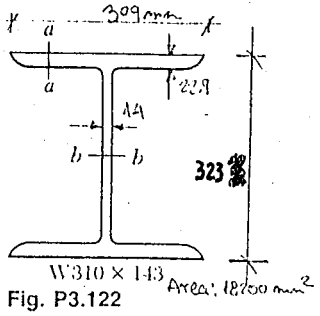


Fig. P3.122

2/2 75

6.96. Shafts AB and BC are made of different materials and unstressed when welded together at B as shown in Fig. P6.96. The twisting couples at A and C are equal in magnitude when the couple T_0 is applied at B. Determine the relation between the elastic moduli of the two materials.

Answer: $G_A = 0.592G_C$

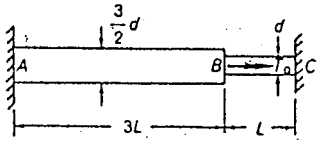


Fig. P6.96

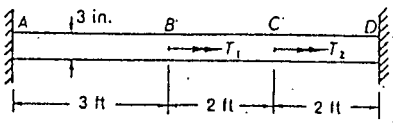


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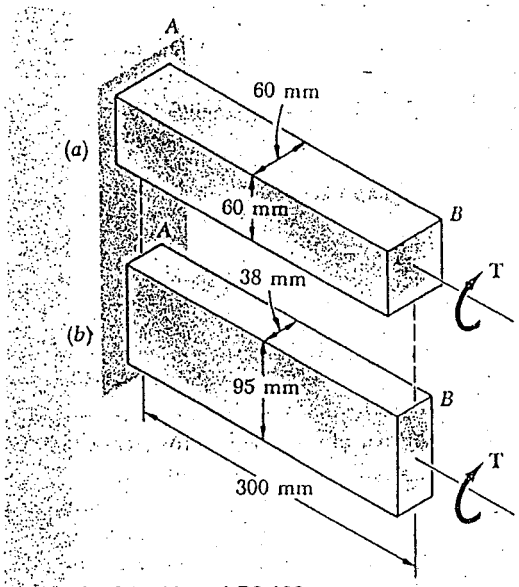


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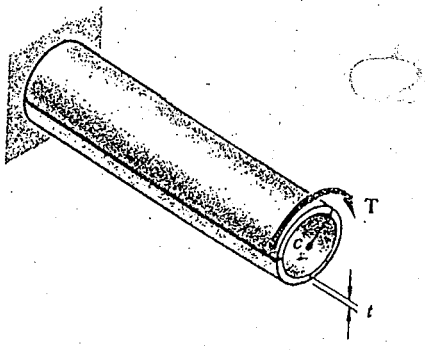
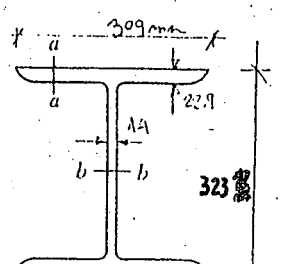


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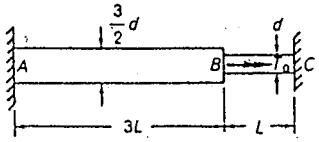


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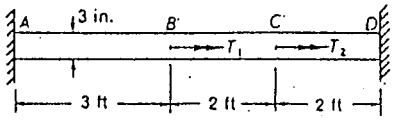


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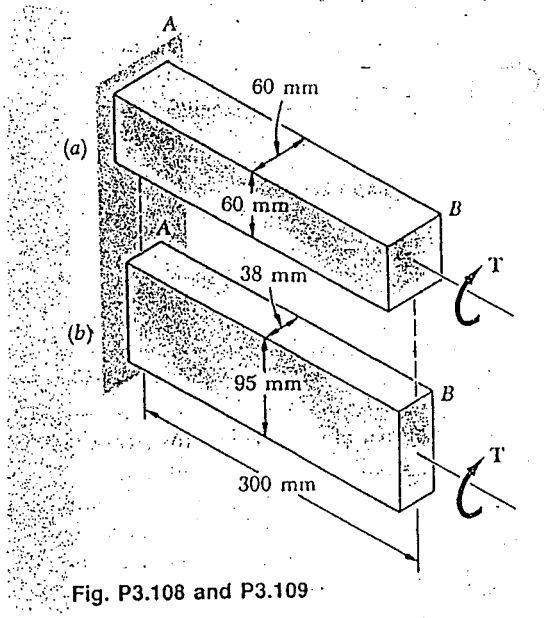


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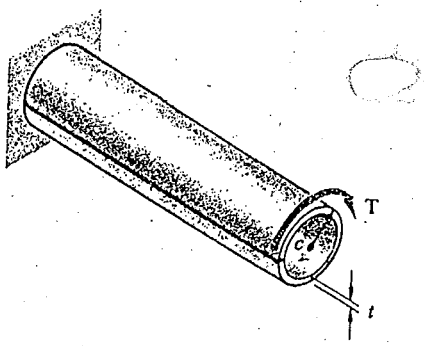
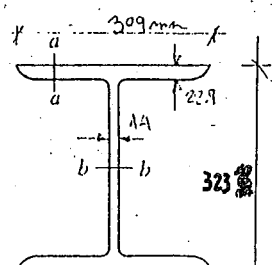


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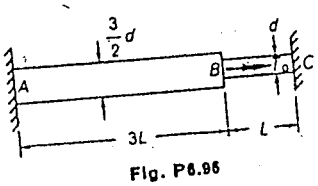


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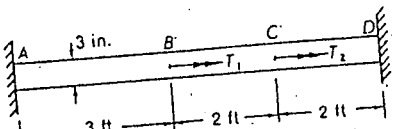


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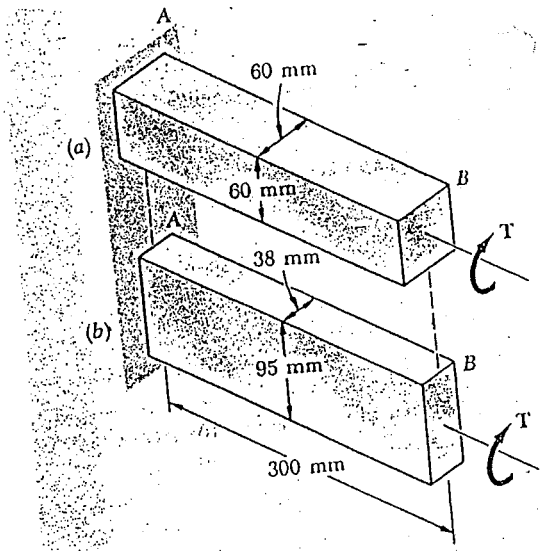
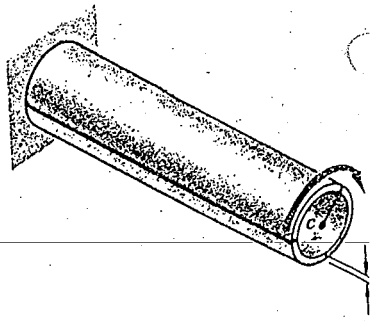


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Like 3.144 in 4th ed.

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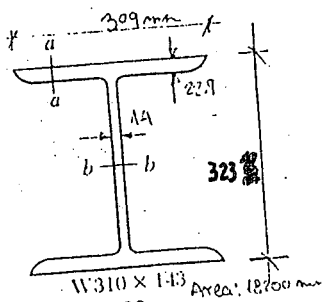


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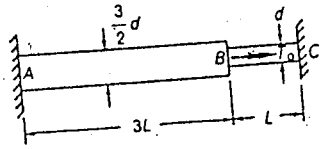


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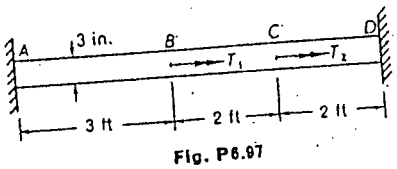


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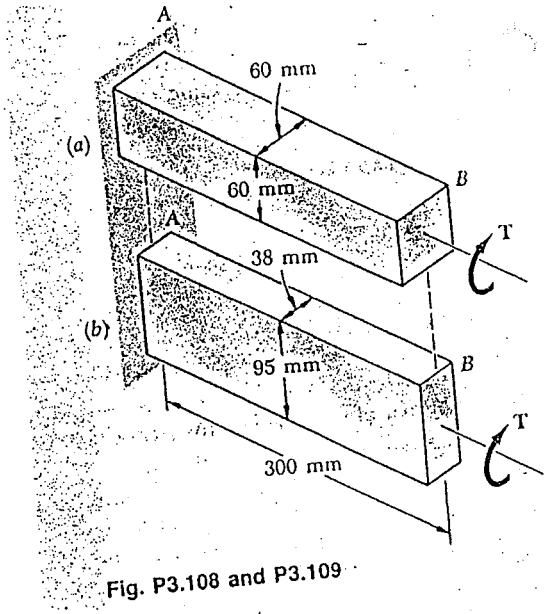
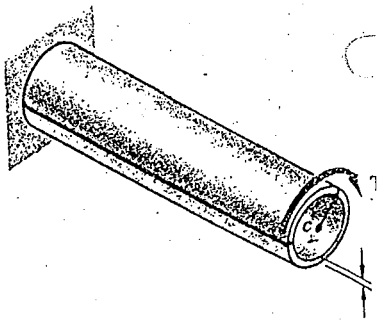


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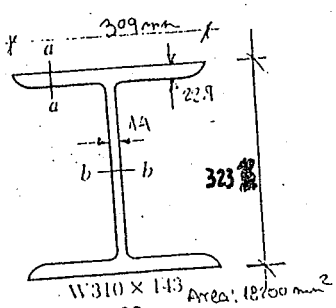


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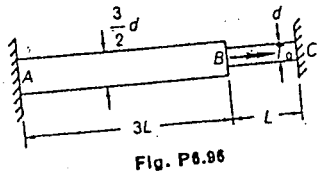


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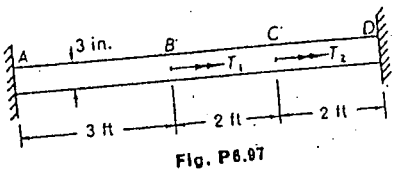


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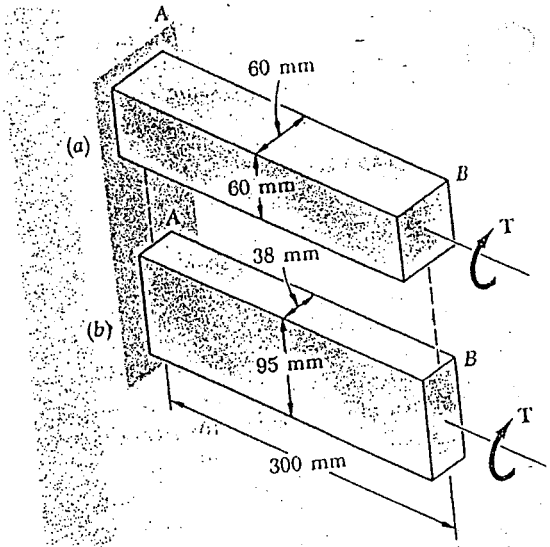
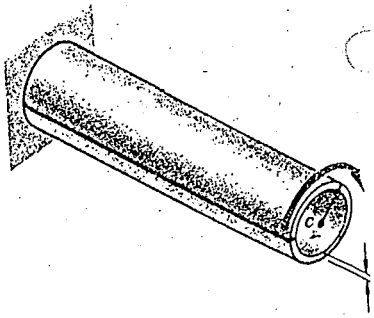


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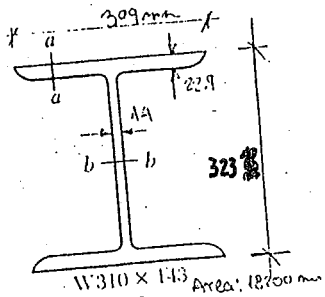


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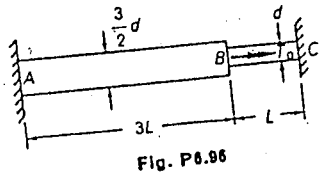


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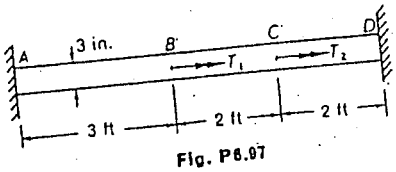


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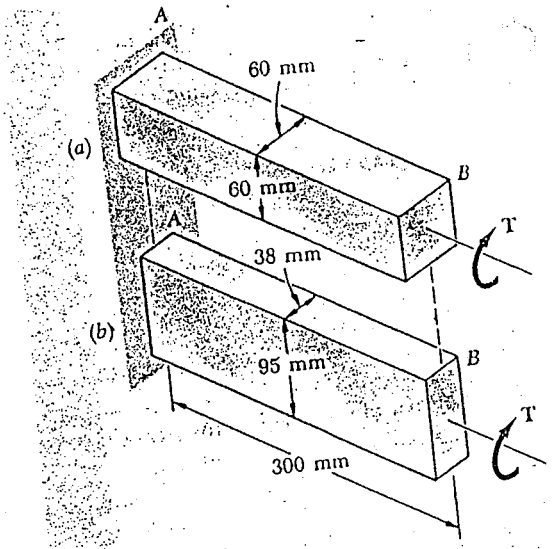
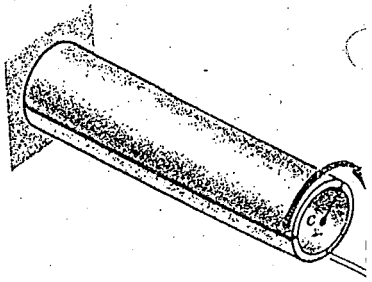


Fig. P3.108 and P3.109

like 3.144 in 4th ed.

3.132 A thin-walled tube has been fabricated by bending a metal plate of thickness t into a cylinder of radius c and bonding together the edges of the plate. A torque $\mathbf{T}$ is then applied to the tube, producing a shearing stress τ_1 and an angle of twist ϕ_1. Denoting by τ_2 and ϕ_2, respectively, the shearing stress and the angle of twist which will develop if the bond suddenly fails, express the ratios τ_2/τ_1 and ϕ_2/ϕ_1 in terms of the ratio c/t.

$\phi_2/\phi_1 = 3\left(\frac{c}{t}\right)^2$


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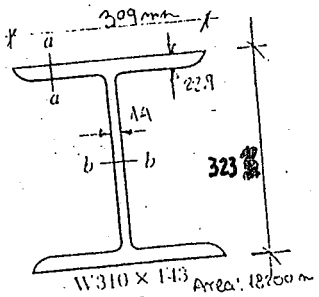


Fig. P3.122

6.96. Shafts AB and [illegible] made of different materials and unstressed when welded together at B as shown in Fig. P6.96. The twisting couples at A and C are equal in magnitude when the couple T_0 is applied at B. Determine the relation between the elastic moduli of the two materials.

Answer: $G_A = 0.592G_C$

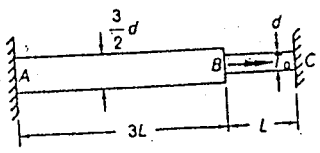


Fig. P6.96

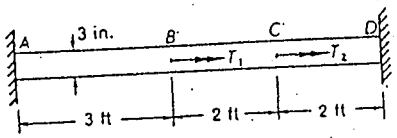


Fig. P6.97

6.97. An aluminum shaft is rigidly restrained at each end and loaded by couples as shown in Fig. P6.97. If $T_1 = 12{,}000$ in.-lb and $T_2 = 10{,}000$ in.-lb, compute the maximum shear stress in each section and the rotations of cross sections at B and C.

Answer: $\tau_{CD} = -2300\ psi, \quad \tau_{BC} = -433\ psi, \quad \tau_{AB} = 1840\ psi,$

3.108 Each of the two aluminum bars shown is subjected to a torque of magnitude $T = 1800\ \text{N} \cdot \text{m}$. Knowing that $G = 26$ GPa, determine for each bar the maximum shearing stress and the angle of twist at B.

Answer: (a) 40.1 MPa 0.653°
(b) 50.9 MPa 0.917°

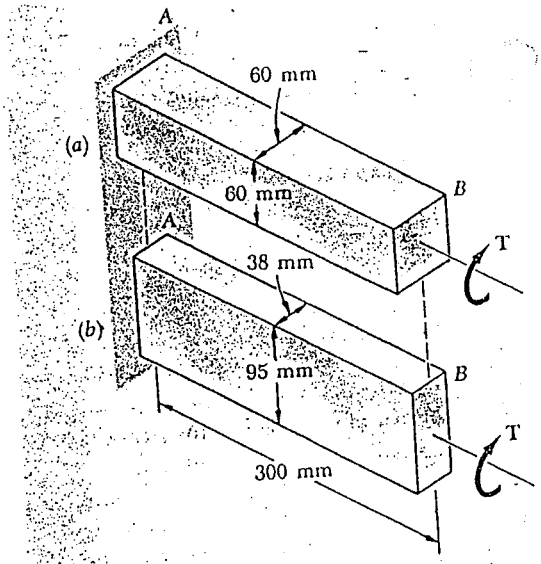


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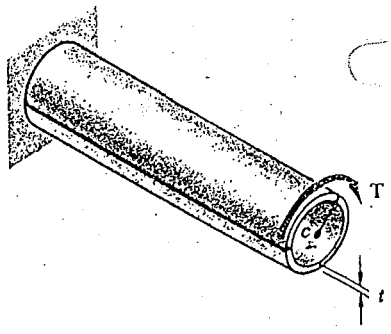


Fig. P3.132

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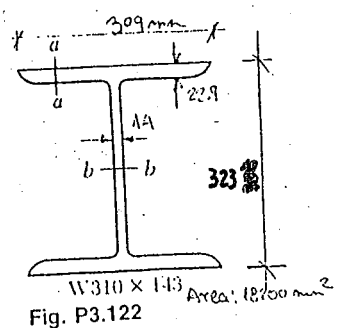


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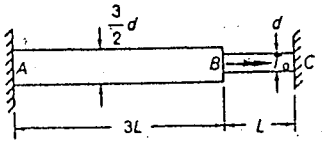


Fig. P6.96

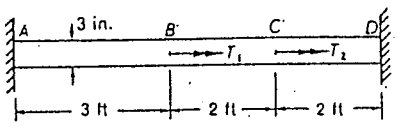


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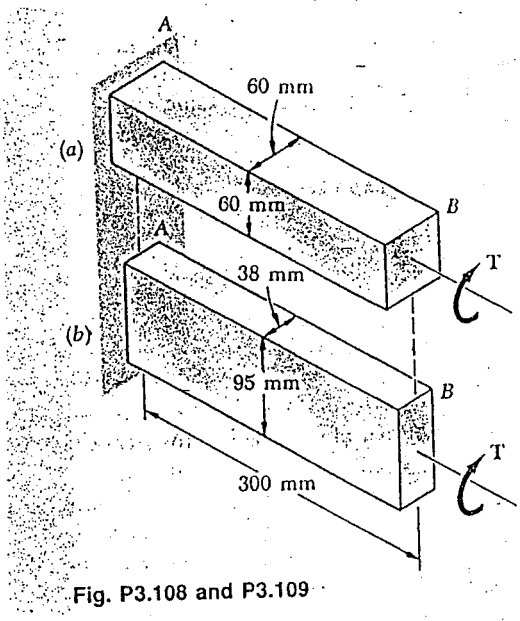


Fig. P3.108 and P3.109

Use 3.144 in 4th ed.

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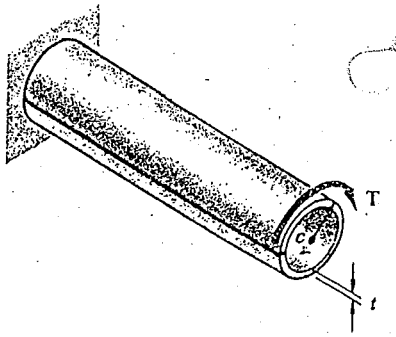
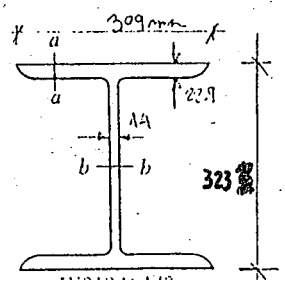


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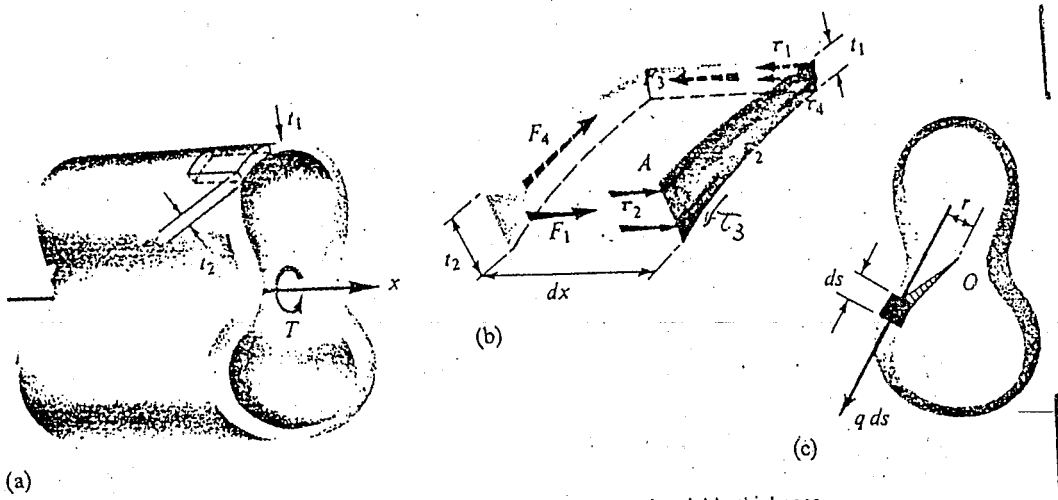


Fig. 3-22. Thin-walled member of variable thickness.

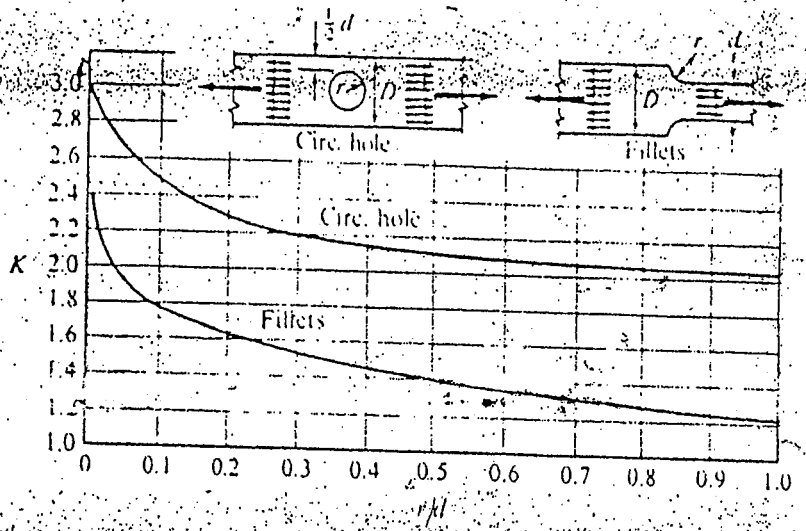


Fig. 2-17. Stress-concentration factors for flat bars in tension

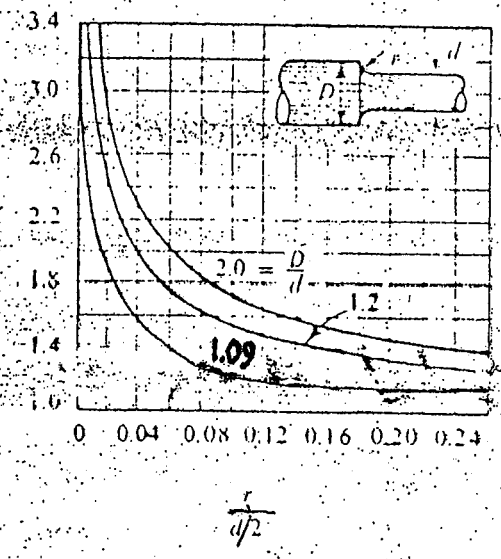
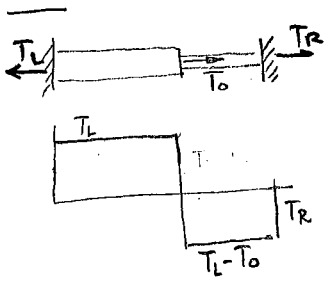


Fig. 3-16. Torsional stress-concentration factors in circular shafts of two diameters.



$T_L - T_R = T_o$

$T_R = T_L - T_o$

$|T_R| = |T_L|$ ולכן

$T_L = T_o/2 \quad T_R = -T_o/2$

$$\varphi_{AB} = \frac{T_L L_{AB}}{J_{AB} G_{AB}} = |\varphi_{BC}| = \left|\frac{T_R L_{BC}}{J_{BC} G_{BC}}\right|$$

$$J_{AB} = \frac{\pi c^4}{2} = \frac{\pi \hat{d}^4}{32} = \frac{\pi \cdot 81 d^4}{512}; \quad L_{AB} = 3L;$$

$$J_{BC} = \frac{\pi c^4}{2} = \frac{\pi d^4}{32}; \quad L_{BC} = L$$

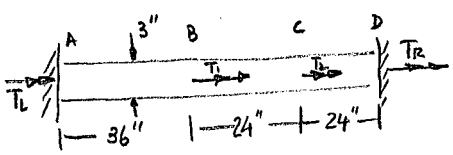
$$\varphi_{AB} = \frac{|T_o/2| \cdot 3L}{\pi 81 d^4/512 \, G_{AB}} = \frac{|T_o/2| \cdot L}{\frac{\pi d^4}{32} \cdot G_{BC}}$$

$$\therefore \frac{G_{AB} \cdot 81}{1536} = \frac{G_{BC}}{32} \quad \text{or} \quad G_{AB} = \frac{1536}{81 \cdot 32} G_{BC} = .5926 G_{BC}$$

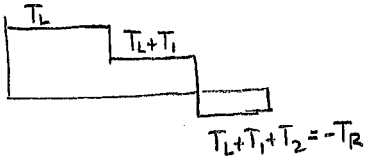
$\Delta\varphi = \varphi_{AB} + \varphi_{BC}$

$$= \frac{T_L L_{AB}}{J_{AB} G_{AB}} + \frac{(T_L - T_o) L_{BC}}{J_{BC} G_{BC}}$$

6.97



$T_L + T_R = -T_1 - T_2$

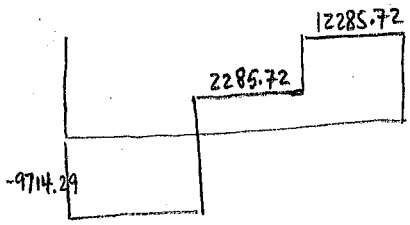


$$\Delta\varphi = \frac{T_L \cdot L_{AB}}{JG} + \frac{(T_L+T_1) L_{BC}}{JG} + \frac{(T_L+T_1+T_2) L_{CD}}{JG} = 0$$

$$T_L \cdot 36 + (T_L + 12000) \cdot 24 + (T_L + 22000) \cdot 24 = 0$$

$$T_L \cdot 84 + 288000 + 528000 = 0$$

$$T_L = -\frac{816000}{84} = -9714.29 \text{ in lb}$$



$J = \frac{\pi c^4}{2}$

$J/c = \frac{\pi c^3}{2}$

$J = 7.952 \text{ in}^4$

$$\tau_{AB} = \frac{T_L \cdot c}{J} = \frac{2T_L}{\pi c^3} = \frac{2(-9714.29)}{\pi (1.5)^3} = -1832.4 \text{ psi}$$

$$\tau_{BC} = \frac{T_{BC} \cdot c}{J} = \frac{2T_{BC}}{\pi c^3} = \frac{2(2285.72)}{\pi (1.5)^3} = +431.15 \text{ psi}$$

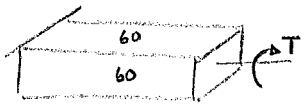
$$\tau_{CD} = \frac{T_{CD} \cdot c}{J} = \frac{2T_{CD}}{\pi c^3} = \frac{2(12285.72)}{\pi (1.5)^3} = +2317.43 \text{ psi}$$

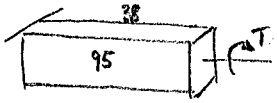
$$\varphi_B = \frac{T_L \cdot L_{AB}}{JG} = -.011 \text{ rad} = -0.63°$$

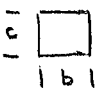
$$\varphi_C = \varphi_B + \frac{T_{BC} \cdot L_{AB}}{JG} = -.009 \text{ rad} = -0.53°$$

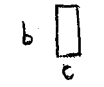
$G = 4 \times 10^6$ psi ולכן הנתון

3.108









	b/c	α	β
	1	0.208	0.141
	2.5	$\frac{(.267+.246)}{2} = .258$	$\frac{(.263+.229)}{2} = .246$

$$\tau_{max} = \frac{T}{\alpha b c^2} \qquad \varphi = \frac{TL}{\beta b c^3 G}$$

$$\frac{1800 \text{ N-m}}{(0.208)(.06)(.06)^2} = 40.064 \text{ MPa}$$

$$\frac{(1800 \text{ N-m})(0.3)}{(0.141)(.06)(.06)^3 26 \times 10^9 \frac{N}{m^2}} = 0.0114 \text{ rad} = .651°$$

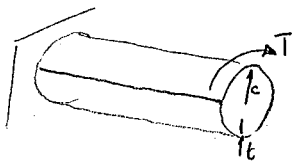
$$\frac{1800 \text{ N-m}}{(.257)(.095)(.038)^2} = 51.156 \text{ MPa}$$

$.249$ $.258$ 50.85

$$\frac{1800 (.03)}{(.246)(.095)(\quad)} \quad .249$$

$= 0.0162 \text{ rad} = .928°$ $.016 \text{ rad}$ $.917°$

3.132



$$J = \frac{\pi}{2}(a^4 - b^4) = \frac{\pi}{2}(a^2 - b^2)(a^2 + b^2); \quad \tau_1 = \frac{Tc}{J}; \quad \phi_1 = \frac{TL}{JG}$$

$$= \frac{\pi}{2}(a-b)(a+b)(a^2+b^2)$$

$$J = \frac{\pi}{2} \cdot t \cdot 2c \cdot 2c^2 = 2\pi c^3 t \qquad \tau_1 = \frac{T}{2\pi c^2 t}; \quad \phi_1 = \frac{TL}{2\pi c^3 t G}$$

$$J_R = \frac{bt^3}{3} = \frac{2\pi c t^3}{3} \qquad ; \tau_2 = \frac{Tt}{J_R}; \quad \beta = \frac{\phi_2}{L} = \frac{T}{G J_R}$$

$$\tau_2 = \frac{Tt}{2\pi c t^3/3} = \frac{3Tt}{2\pi c t^3} = \frac{3T}{2\pi c t^2}; \quad \phi_2 = \frac{TL}{\frac{2\pi c t^3}{3} G} = \frac{3TL}{2\pi c t^3 G}$$

$$\frac{\tau_1}{\tau_2} = \frac{\frac{T}{2\pi c^2 t} \cdot \frac{t}{t}}{\frac{3T}{2\pi c t^2} \cdot \frac{c}{c}} = \frac{t}{3c}; \quad \frac{\phi_1}{\phi_2} = \frac{\frac{TL}{2\pi c^3 t G} \cdot \frac{t^2}{t^2}}{\frac{3TL}{2\pi c t^3 G} \cdot \frac{c^2}{c^2}} = \frac{t^2}{3c^2}$$

$$\left(\frac{\tau_1}{\tau_2}\right)^2 = \frac{t^2}{9c^2} = \frac{1}{3}\left(\frac{\phi_1}{\phi}\right) \quad \text{ולכן}$$

of determining the elastic and the inelastic, or plastic, stress distribution caused by bending moments in straight beams made of homogeneous materials. Discussions of beams made from two or more different materials, curved beams, and stress concentrations are also included.

6-2. SOME IMPORTANT LIMITATIONS OF THE THEORY

Just as in the case of axially loaded rods and in the torsion problem, all forces applied to a beam will be assumed to be delivered without shock or impact. Moreover, all the beams will be assumed to be stable under the applied forces. A similar point was brought out in Chapter 3, where it was indicated that a rod acting in compression cannot be too slender, or its behavior will not be governed by the usual compressive strength criterion. In such cases the stability of the member becomes important. As an example, consider the possibility of using a sheet of paper on edge as a beam. Such a beam has a substantial depth, but even if it is used to carry a force over a small span, it will buckle sidewise and collapse. The same phenomenon may take place in more substantial members which may likewise collapse under an applied force. Such unstable beams do not come within the scope of this chapter. All the beams considered here will be assumed to be sufficiently stable laterally by virtue of their proportions or to be thoroughly braced in the transverse direction. A better understanding of this important phenomenon will result after the study of the chapter on columns. It is indeed fortunate that the majority of beams used in structural framing and as machine elements are such that the flexural theory developed here is applicable; the theory governing the stability of members is more complex.

6-3. THE BASIC KINEMATIC ASSUMPTION

In the technical theory of bending, to establish the relation among the applied bending moment, the cross-sectional properties of a member, and the internal stresses and deformations, the approach applied earlier again employed. This requires first that a plausible deformation assumption reduce the internally statically indeterminate problem to a determinate one; second, that the deformations causing strains be related to stresses through the appropriate stress-strain relations; and, finally, that the equilibrium requirements of external and internal forces be met. The key kinematic assumption for the deformation of a beam as used in the technical theory is discussed in this article. It is important to note that a generalization of this assumption forms the basis for the theories of plates and shells. The apparent mathematical complexity of the latter subjects is due to their two- and three-dimensional aspects, which require the use of the partial differential relations. On the other hand, the static beam problem resolves itself into a dependence on only one independent variable. This will become especially apparent during the study of Chapter 11 on the deflection of beams, where an ordinary differential equation suffices.

Section 6-3
The basic kinematic assumption

For the present consider a horizontal prismatic beam having a cross section with a vertical axis of symmetry, Fig. 6-1(a). Let the line through the centroid of all cross sections be referred to as the axis of the beam. Next, imagine a number of planes passing through the beam perpendicular to its axis, and a number of horizontal planes. In a side view these planes form a rectangular grid, Fig. 6-1(a). When such a beam is subjected to positive bending moments M at its ends as in Fig. 6-1(b), the beam bends, the planes perpendicular to the beam axis tilt slightly, and

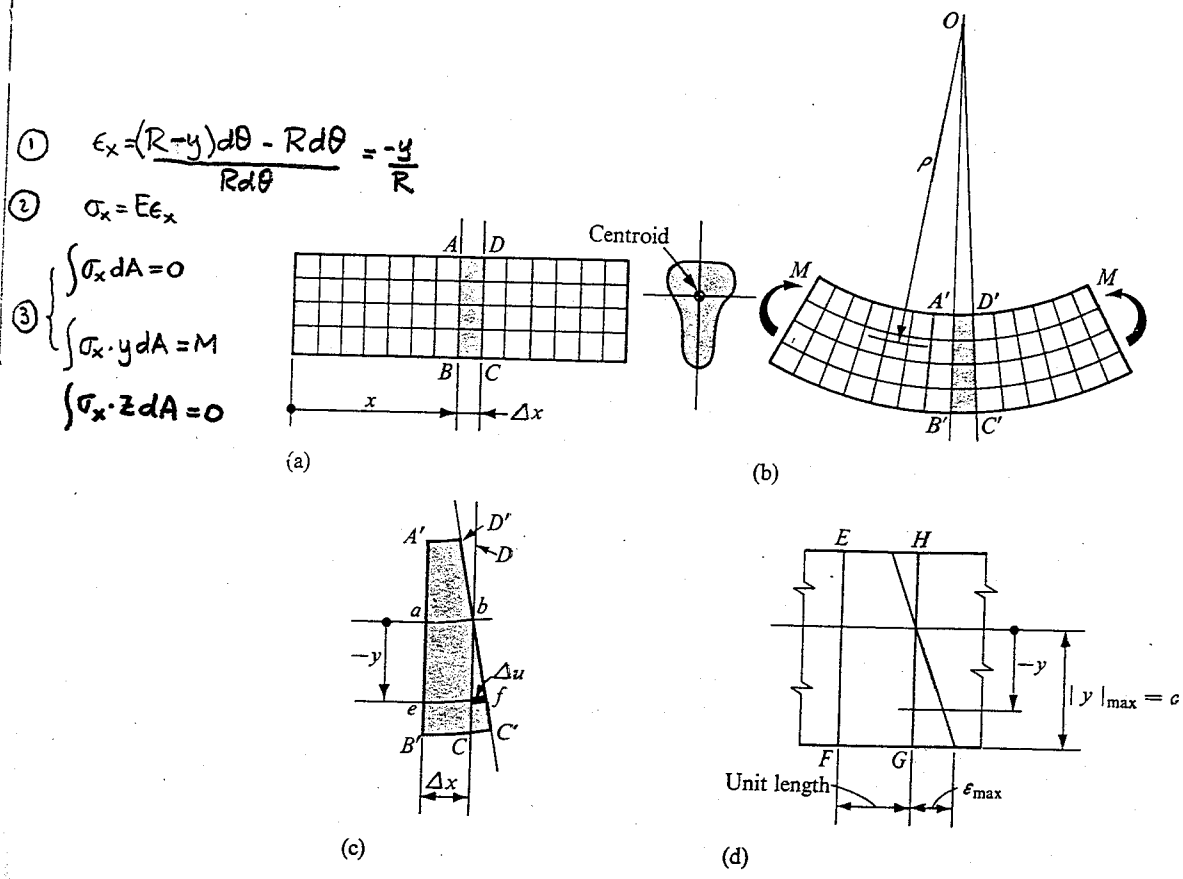
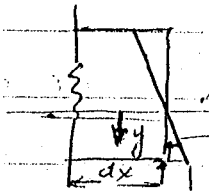


Fig. 6-1. The basic deformation assumption in bending.



neutral axis not planar.

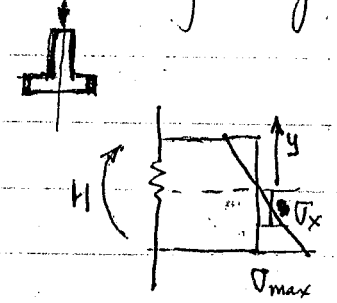
$\epsilon_x = \frac{du}{dx}$ $\quad \epsilon_{max} = \frac{du_{max}}{dx}$

$\epsilon_x = by$ b depends on geometry, moment

Suppose

1) Member has axis of symmetry

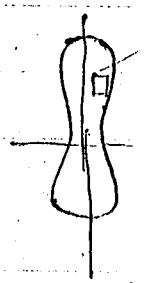
2) loading along axis of symmetry.



$\epsilon_x = \frac{(R-y)d\theta - Rd\theta}{Rd\theta} = \frac{-y}{R}$

$\sigma_x = -Eby$ $\quad \sigma_x = E\epsilon_x = -\frac{Ey}{R}$ (ω)

$\Sigma F_x = 0$



$\Sigma F_x = \int_A \sigma_x dA = 0$ $\quad Eb\int_A y\,dA = 0$

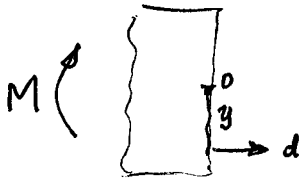
$-\frac{E}{R}\int_A y\,dA = 0 \Rightarrow$ Neutral axis $\equiv$ centroid

$\frac{\int_A y\,dA}{\int_A dA} = \bar{y}$

but since $\int_A y\,dA = 0$ $\quad \bar{y} = 0$

$\therefore$ neutral axis $\equiv$ centroid

σ_x is + y is −
but M_o is +



$dM_o = -\sigma_x \cdot y\,dA$

$M_o = -\int \sigma_x \cdot y\,dA$

$-M + M_o = 0 \qquad M_o = M = \int -\sigma_x y\,dA$

$$M = \int \left(-\frac{E}{R}y\right)(y\,dA) = +\frac{E}{R}\int y^2 dA = +\frac{E}{R}I$$

$$\therefore \frac{M}{I} = +\frac{E}{R} = \frac{-\sigma_x}{y} \qquad \therefore \sigma_x = -\frac{My}{I}$$

$\Sigma M_L = 0$ moment of left in CW because $M>0$ when $y<0$ ✓

$$-M + \int_A \sigma_x \, dA(y) = 0 \qquad M = \int \sigma_x y \, dA$$

$$-M + Eb \int_A y^2 dA = 0 \qquad M = Eb \int_A y^2 dA$$

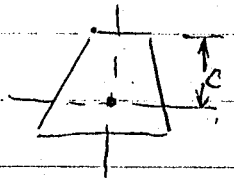
+ stress occurs at $-y$ but produces $+M$

$I_z = \int_A y^2 dA$ moment of inertia $\qquad I_y = \int_A z^2 dA$

$$\boxed{M = EbI} \qquad \text{since } \sigma_x = -Eby \qquad \boxed{\frac{-\sigma_x I}{y} = +M} \qquad \boxed{\sigma_x = \frac{-My}{I}}$$

$$-\frac{\sigma_x}{y} = Eb$$

call c largest distance from one of the free surfaces to NA.

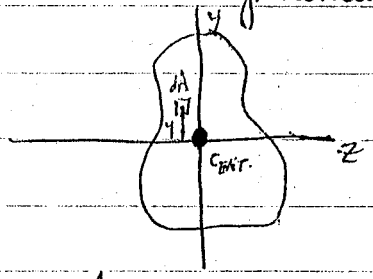


$\sigma_{max} = Mc/I \qquad \boxed{\sigma_{max} = -Mc/I}$

Basic Assumption

1) Deformation based on linear distribution of strain
2) Material properties relate stress – strain. (Hooke's law)
3) Equilibrium $\int \sigma_x dA = 0 \quad \int \sigma_x y dA = M_z \quad \int \sigma_x z dA = 0 = M_y \Rightarrow \int yz dA = I_{yz}$

Unsymmetrical Bending | either member doesn't have axis of symmetry, load is not along axis of symmetry

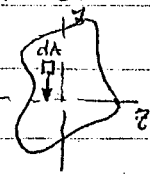


M_z

$$\int dA \, \sigma_x = \int Eby \, dA = F$$

$M_y = 0$

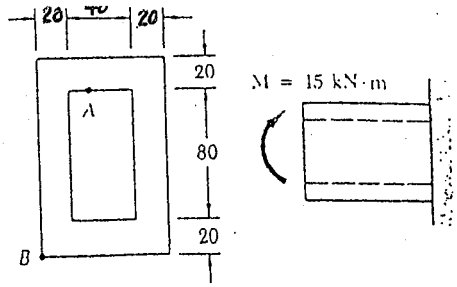
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$M_z \qquad \sigma_x dA$ — force

$$\int_{M_z} dM_z = 0 \Rightarrow M_z = -\int_A \sigma_x dA y$$

4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.



Dimensions in mm

Fig. P4.1

4.9 and 4.10 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in the beam.

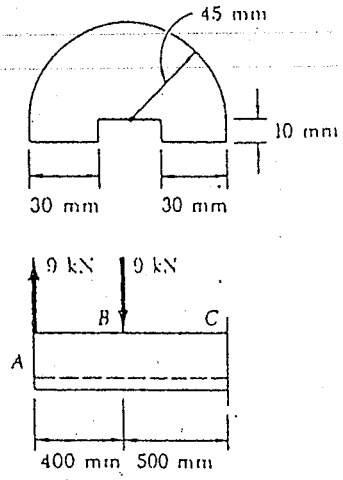


Fig. P4.9

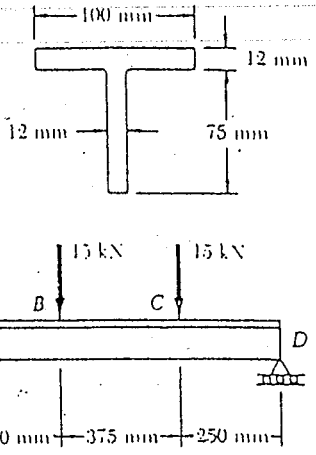


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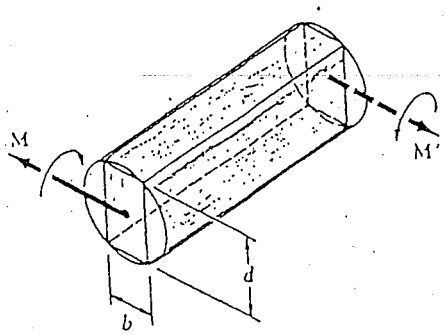
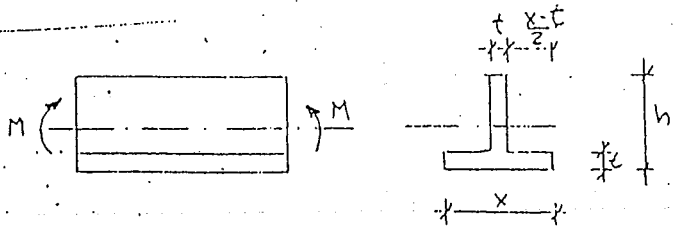


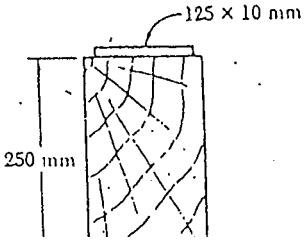
Fig. P4.26

4. What must be the width x of the beam so that the σ_{max} in tension is 3 times the value of σ_{max} in compression?



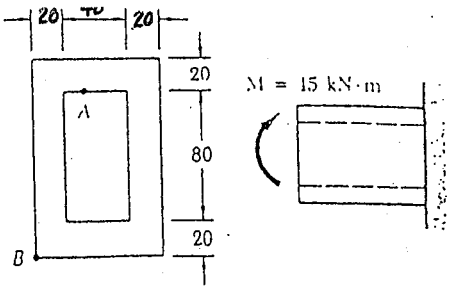
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Wood Steel

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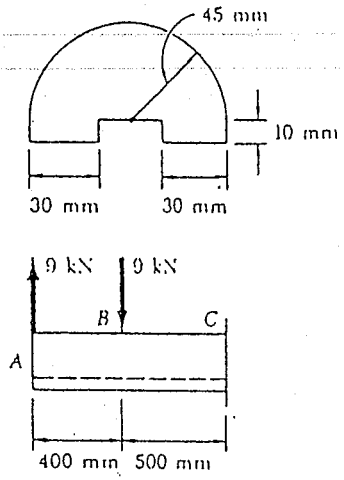


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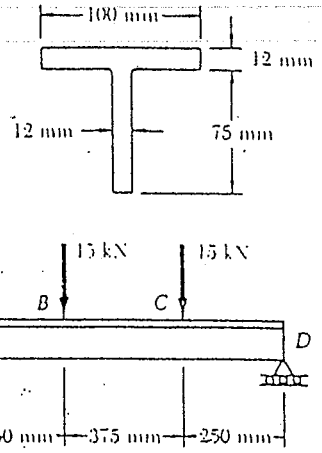


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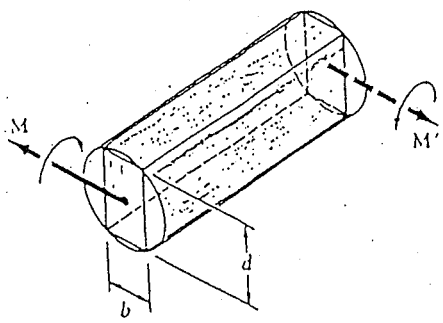
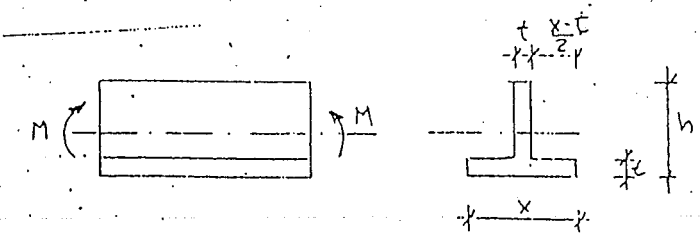


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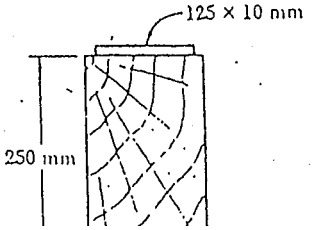
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What must be the width x of the beam so that the σ_{max} in tension is 3 times the value of σ_{max} in compression?



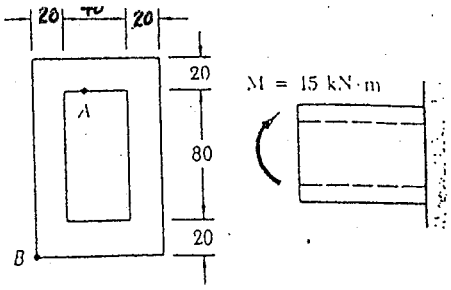
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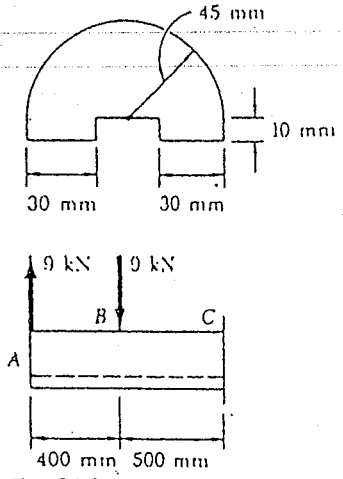


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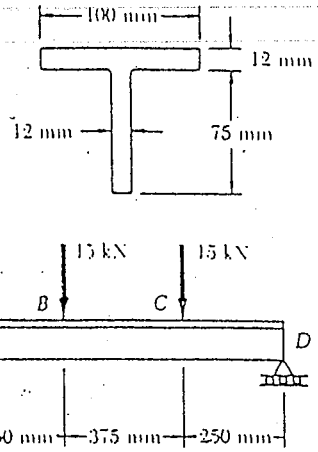


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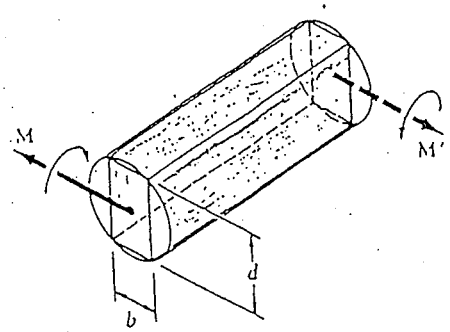
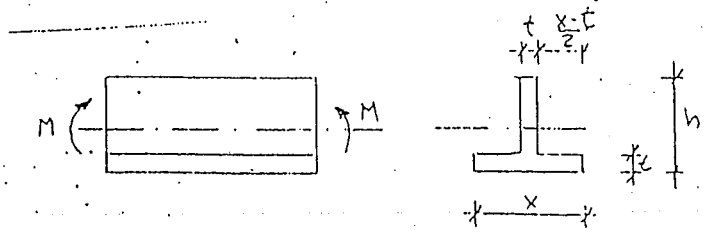


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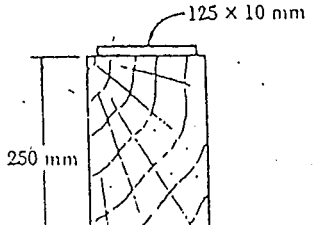
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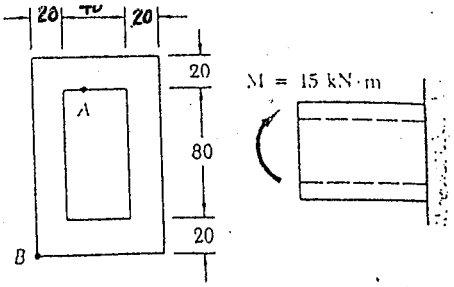
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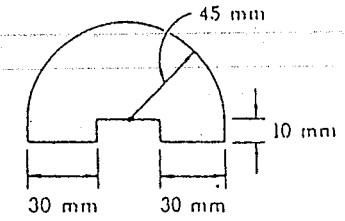
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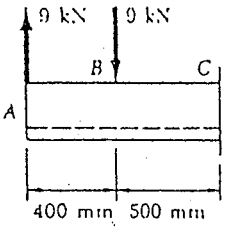
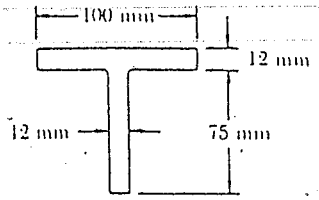


Fig. P4.9



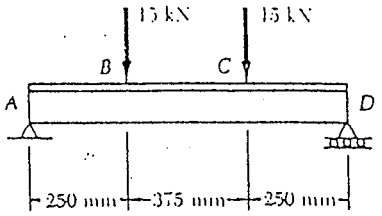


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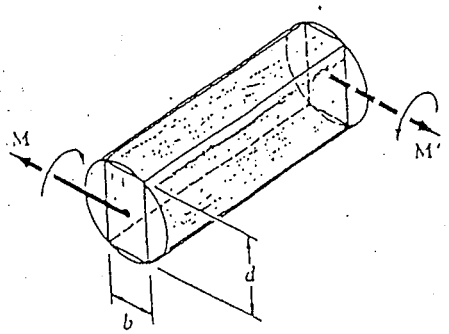
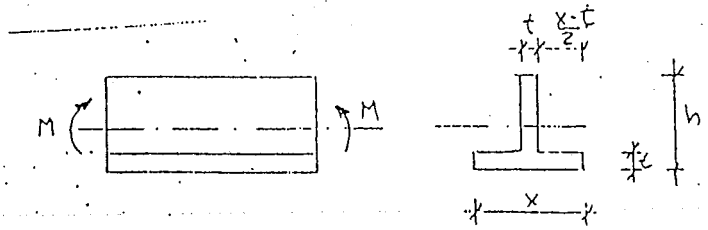


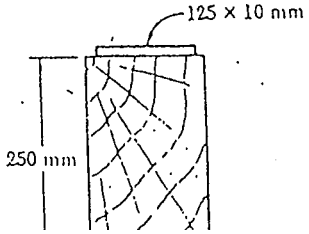
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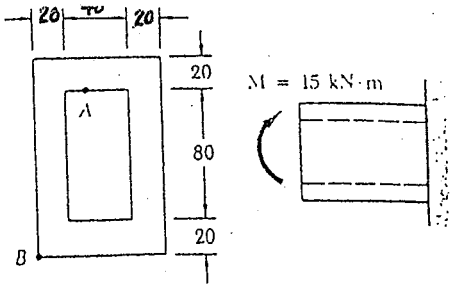


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Wood Steel



Dimensions in mm

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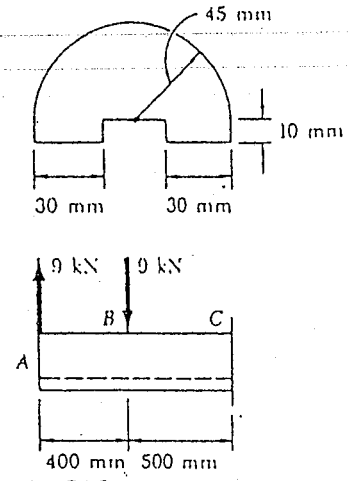


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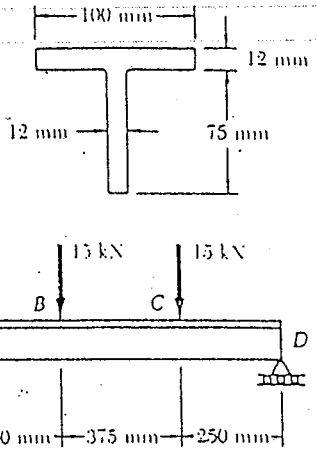


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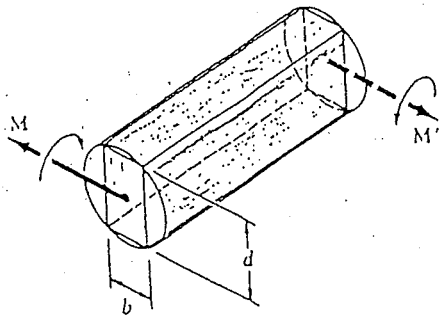
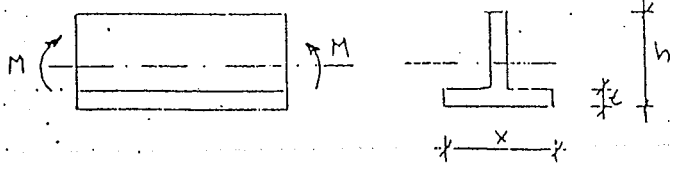


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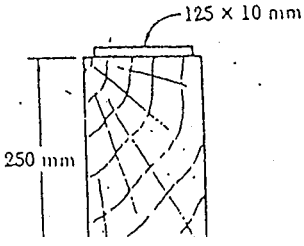
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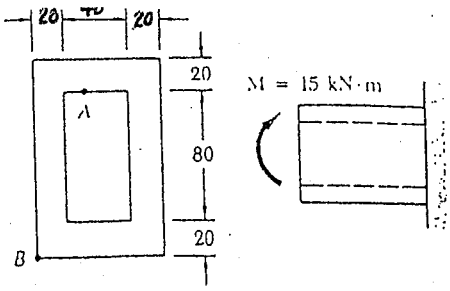
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Wood Steel



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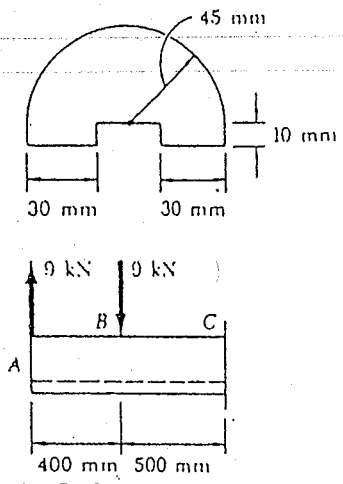


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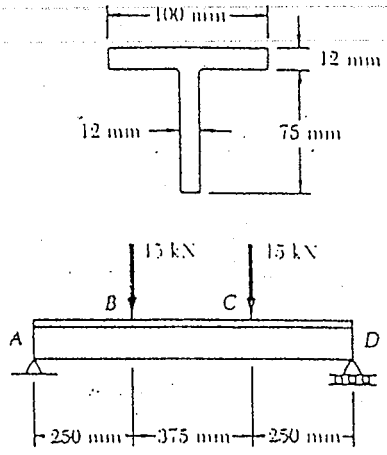


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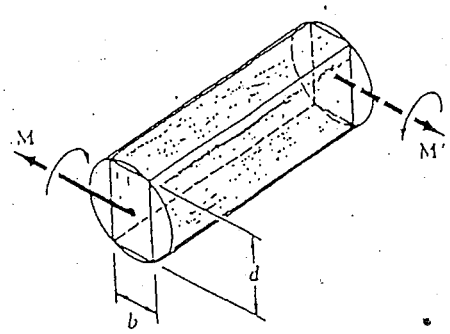
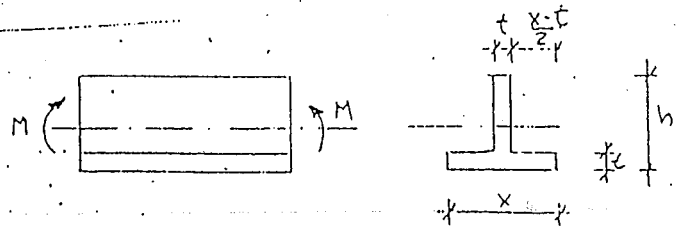


Fig. P4.26

4.

What must be the width x of the beam so that the σ_{max} in tension is 3 times the value of σ_{max} in compression?

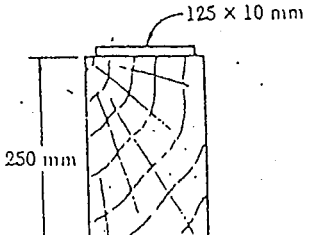


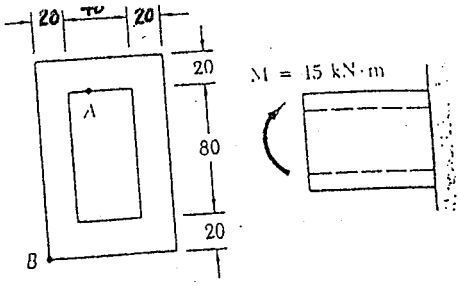
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Wood Steel

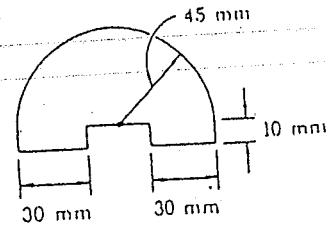




Dimensions in mm
Fig. P4.1

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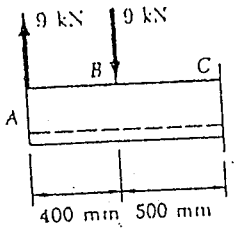
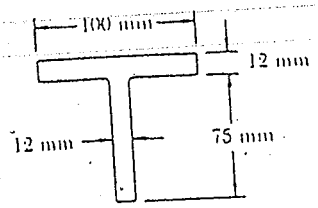


Fig. P4.9



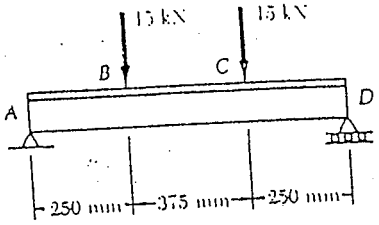


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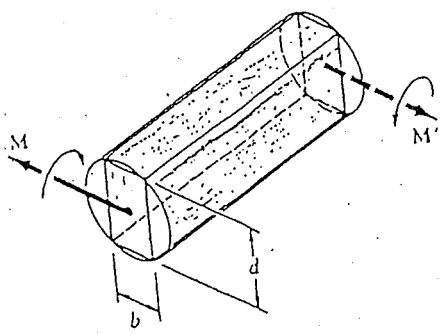
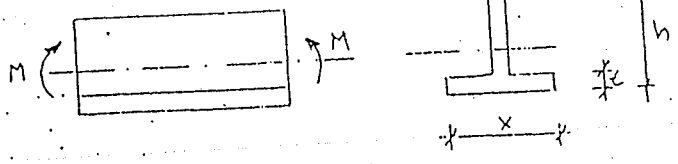


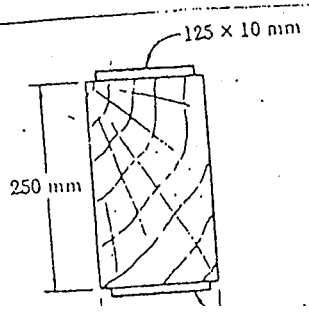
Fig. P4.26

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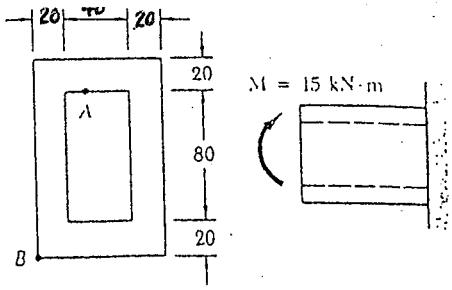
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	Wood	Steel
	13 GPa	200 GPa

4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.



Dimensions in mm

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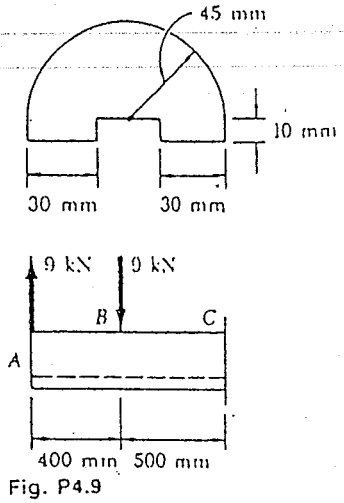


Fig. P4.9

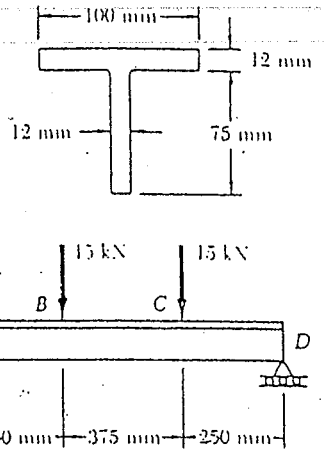


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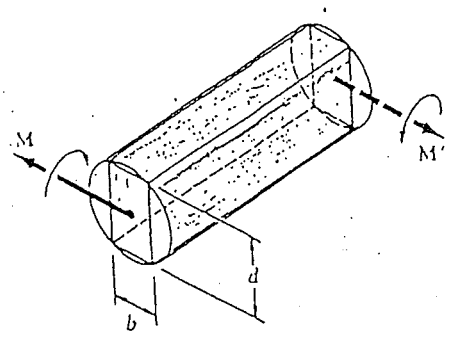
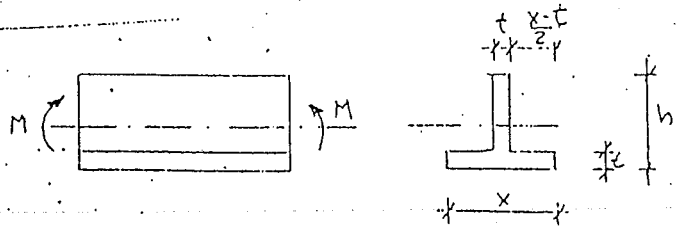


Fig. P4.26

4.

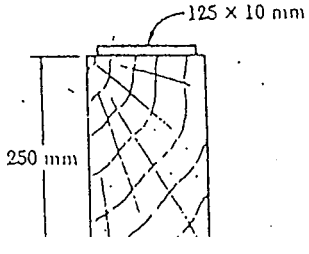
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Wood Steel

$\Sigma M_y = 0 \qquad 0 = \int_A \sigma_x \, dA z$

Suppose $\epsilon_x = by \qquad \sigma_x = Eby$ Define: $\int_A yz\,dA = I_{yz}$ product of inertia

$0 \stackrel{?}{=} Eb \int_A yz\,dA \qquad E \neq b \neq 0$ if $\int_A yz\,dA \neq 0$ the assumption is false

Define If $I_{yz} = 0$ y & z are <u>principal axes</u>.

If $I_{yz} \neq 0$ assumptions are false

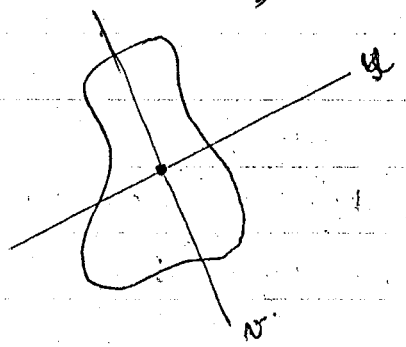
Cor: Axis of symmetry is a principle axis.

Centroidal Axis normal to a principal axis is itself principal

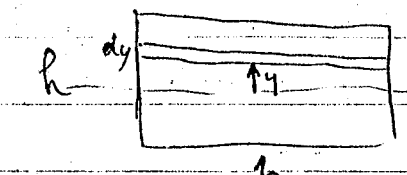
Suppose: $I_{yz} \neq 0 \qquad \sigma_x = E(by + dz)$

$\Sigma M_y = 0 = \int_A \sigma_x \, dA z = E\int (by + dz) z \, dA = E\left[b\int yz\,dA + d\int z^2 \right.$

$E\left[bI_{yz} + dI_{yy}\right] = 0 \qquad E \neq 0$



principle axes for unsymmetric beams

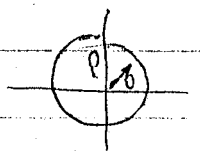


$dA = b\,dy$

$I_{zz} = b\int_A y^2 dy$

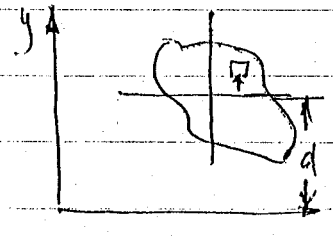
$I_{zz} = b/12 h^3$

$I_{yy} = b^3h/12$



$I_{zz} = \int_A y^2 dA \quad ; \quad I_{yy} = \int_A z^2 dA$

$2I_{zz} = \int_A (y^2+z^2)\,dA = \int_A \rho^2 dA = J \qquad I_{zz} = I_{yy} = J/2$



$$I_{zz} = \int_A (y+d)^2 dA$$

$$\int_A y^2 dA + \int 2yd\,dA + \int d^2 dA$$

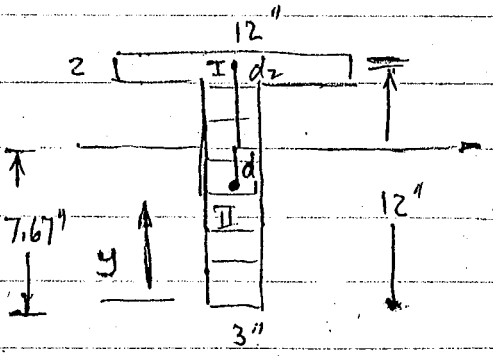
$$\int y^2 dA + 2d\int y\,dA + d^2 \int_A dA$$

if y is measured from centroid $\bar{y} = 0 \therefore \int y\,dA = 0$

$$I_{zz} = I_o + 2d(0) + d^2A$$

The Parallel Axis Theorem:

$I_{zz} = I_o + Ad^2$ ⟶ the take area × d^2 from global axis
⟶ moment about its own centroid



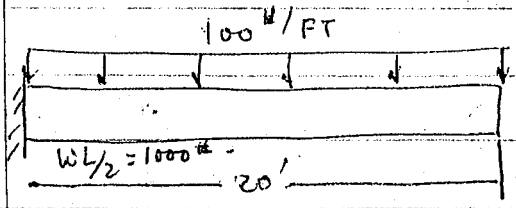
$$I_{zz}_{\text{total}} = \sum I_{o_i} + \sum A_i d_i^2$$

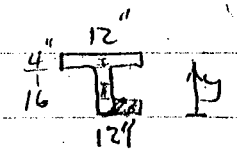
$$\bar{y} = \frac{\sum Ay}{\sum A}$$

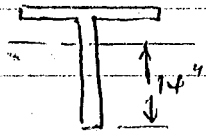
	A	y	Ay	d	Ad^2
I	24	11	264	3.33	
2	30	5	150	2.67	
	54		414		69

$\frac{69}{9} = \frac{414}{54} = \frac{23}{3} = 7.6$

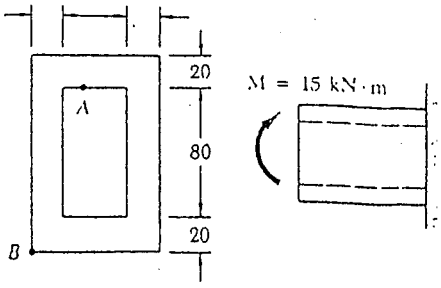
	Ad^2	I_o
I	24×3.3^2	$12 \times 2^3/12$
II	30×2.67^2	$3 \times 10^3/12$







4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.



Dimensions in mm

Fig. P4.1

4.9 and 4.10 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in the beam.

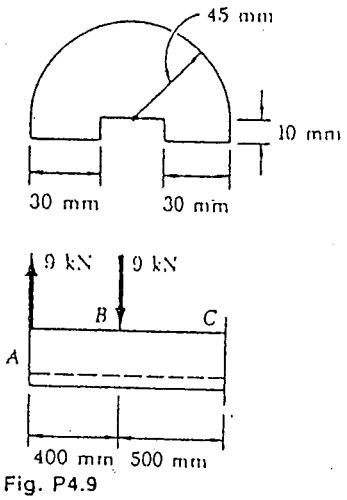
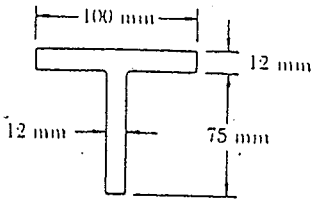


Fig. P4.9



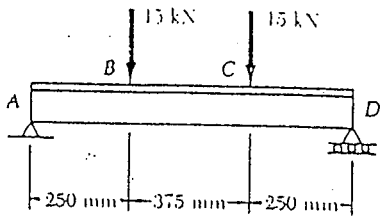


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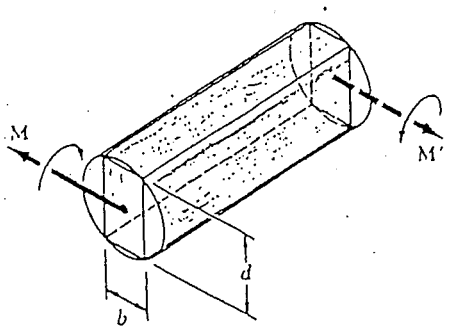
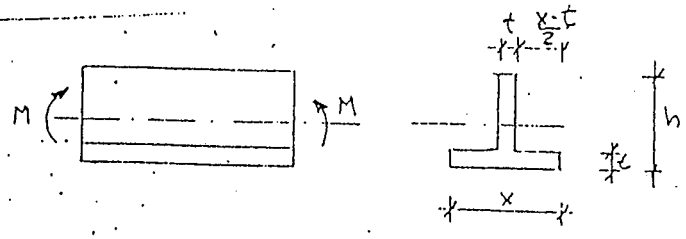


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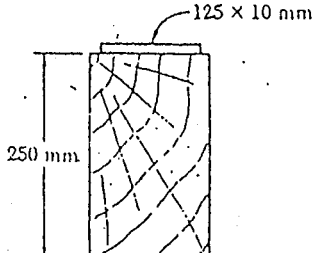
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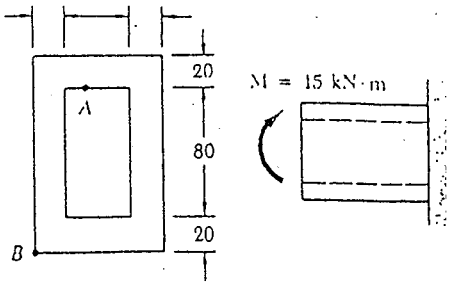
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	Wood	Steel
Modulus of elasticity	13 GPa	200 GPa

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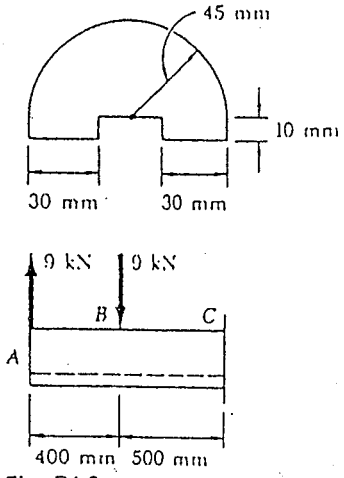
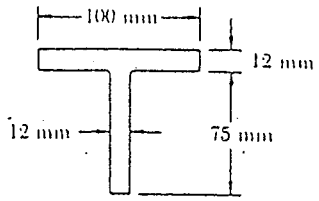


Fig. P4.9



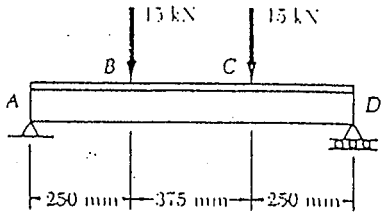


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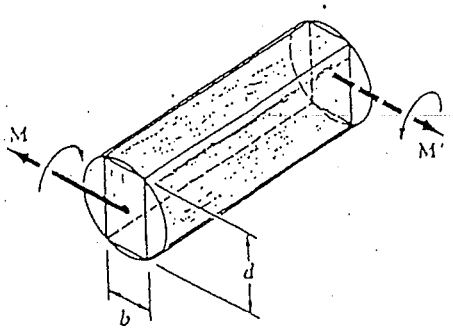
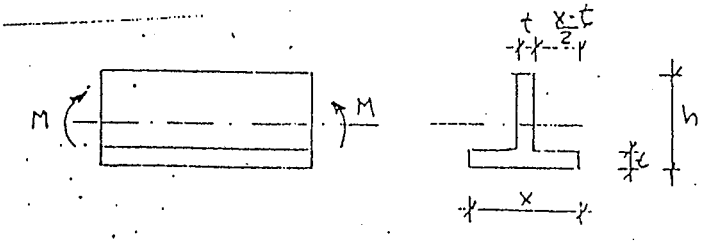


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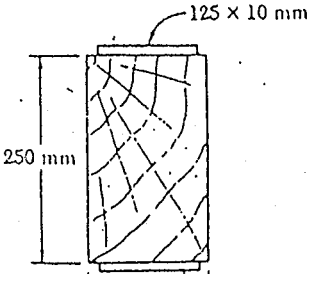
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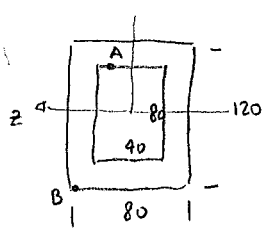
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	Wood	Steel
Modulus of elasticity	13 GPa	200 GPa
Allowable stress	12 MPa	165 MPa

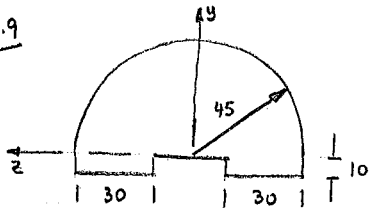


$$I_{zz} = \frac{bh^3}{12} - \frac{bh^3}{12} = \left[\frac{80(120)^3}{12} - \frac{70\cdot80^3}{12}\right] \times 10^{-12} = 9.813\times10^{-6}\ m^4 = 9.813\times10^{6}\ mm^4$$

$$\sigma_{x_A} = -\frac{M_z \cdot y}{I_{zz}} = -\frac{15000\ N\text{-}m \cdot (.040\ m)}{9.813\times10^{-6}\ m^4} = -61.141\ MPa$$

$$\sigma_{x_B} = -\frac{M_z\, y}{I_{zz}} = -\frac{15000\ N\text{-}m\ (-.060m)}{9.813\times10^{-6}\ m^4} = 91.712\ MPa$$

4.9



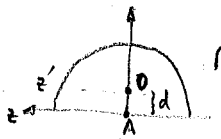
מרכז הכובד של הצורה המורכבת ולאחר מכן למצוא I_{zz} של כל חלק וחלק

		$\bar{y}$	$\bar{z}$	A	$\bar{y}A$	$\bar{z}A$
I	חצי עיגול	$\frac{4R}{3\pi}=\frac{4}{3}\frac{45}{\pi}$	0	$\frac{\pi R^2}{2}=\frac{\pi}{2}(45)^2$	$\frac{2}{3}R^3=\frac{2}{3}(45)^3$	0
II	מלבן	-5	0	10·90	-4500	0
III	מלבן חסר	-5	0	-10·30	1500	0
				3780.863	57750	0

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{57750}{3780.863} = 15.273$$

$$\bar{z} = \frac{\Sigma \bar{z}A}{\Sigma A} = \frac{0}{3780.863} = 0$$

מרכז הכובד של חלק מורכב יהיה אותו דבר אם תבחר ראשית אחרת מאשר מרכז הכובד.



0 - מרכז הכובד של החצי עיגול

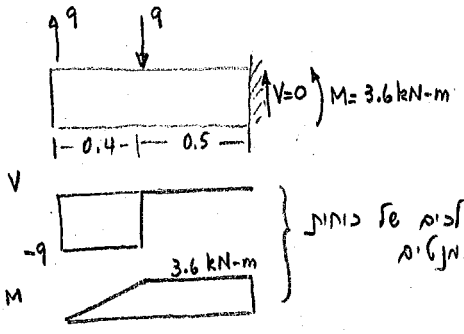
$$I_{zz} = \int y^2 dA = \frac{\pi}{8}R^4$$

$$I_{z'z'} + Ad^2 = I_{zz} \Rightarrow I_{z'z'} + \frac{\pi R^2}{2}\cdot\left(\frac{4}{3}\frac{R}{\pi}\right)^2 = I_{zz}$$

$$I_{z'z'} = I_{zz} - Ad^2 = \left[\frac{\pi}{8} - \frac{\pi}{2}\left(\frac{4}{3\pi}\right)^2\right]R^4 = 0.1098R^4$$

		I_{zz_0}	A	d (mm)	I_{zz}
I		$0.109757R^4$	$\frac{\pi}{2}(45)^2$	19.099-15.273 = 3.826	484418.5 mm^4
II		$\frac{bh^3}{12}=\frac{90(10)^3}{12}$ = 7500	900	-5-15.273 = -20.273	377395.1
III	מלבן חסר	$\frac{bh^3}{12}=\frac{80(10)^3}{12}$ = 1500	300	-5-15.273 = -20.273	-124798.4
					737015.2 mm^4 = $0.737\times10^{-6}\ m^4$

I_{zz_0} הוא מומנט האינרציה מסביב לצירים שעוברים את מרכז הכובד של הצורה I, II או III. I_{zz} הוא מומנט האינרציה מסביב לצירים שעוברים את מרכז הכובד של כל הצורה המורכבת.



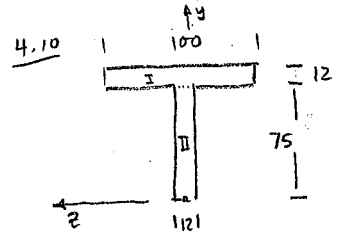
מהלכים של כוחות ומומנטים

המרחק ממרכז הכובד של חלק מרכב לנקודה עליונה הוא 45-15.273 = 29.727 + . לנקודה התחתית מ"ינתך 25.273 -

$$\text{לכן, } \sigma_x = -\frac{M_z\, y}{I_{zz}} = -\frac{3600\ N\text{-}m\ (.029727\ m)}{0.737\times10^{-6}\ m^4}$$

$$= 145.207\ MPa = 145.207\times10^6\ N/m^2$$

4.10

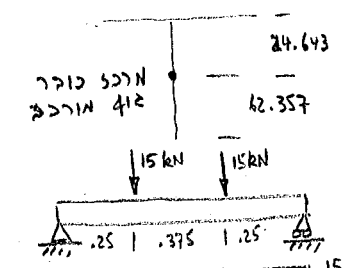


	$\bar{y}$	$\bar{z}$	A	$\bar{y}A$	$\bar{z}A$
I	81	0	1200	97200	0
II	37.5	0	900	33750	0
			2100	130950	0/0

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{130950}{2100} = 62.357\ mm$$

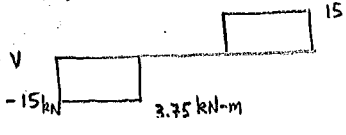
$$\bar{z} = 0$$

	I_{zz_0}	A	d	I_{zz}
I	$\frac{bh^3}{12}=\frac{100(12)^3}{12}$ = 14400	1200	81-62.357 = 18.643	431473.739
II	$\frac{bh^3}{12}=\frac{12(75)^3}{12}$ = 421875	900	37.5-62.357 = -24.857	977958.404
				1409432.143 mm^4 = $1.409\times10^{-6}\ m^4$



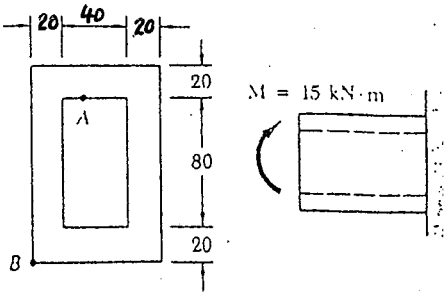
at bottom $\sigma_x = -\frac{M_z\, y}{I_{zz}} = -\frac{3750\ N\text{-}m\ (-.062357\ m)}{1.409\times10^{-6}\ m^4} = 165.961\ MPa = 165961\times10^6\ \frac{N}{m^2}$

at top $\sigma_x = -\frac{M_z\, y}{I_{zz}} = -\frac{3750\ N\text{-}m\ (0.024643)}{1.409\times10^{-6}\ m^4} = -65.57\ MPa$



מהלכים

4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.



Dimensions in mm
Fig. P4.1

4.9 and 4.10 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in the beam.

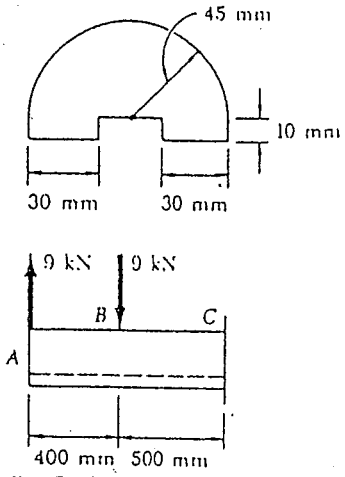


Fig. P4.9

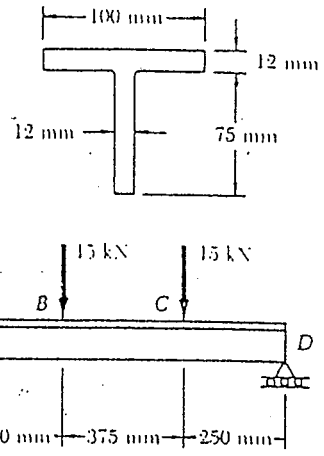


Fig. P4.10

4.26 A couple M will be applied to a beam of rectangular cross section which is to be sawed from a log of circular cross section. Determine the ratio *d*/*b*, for which (*a*) the maximum stress σ_m will be as small as possible, (*b*) the radius of curvature of the beam will be maximum.

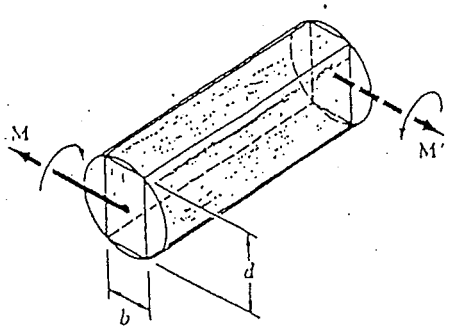
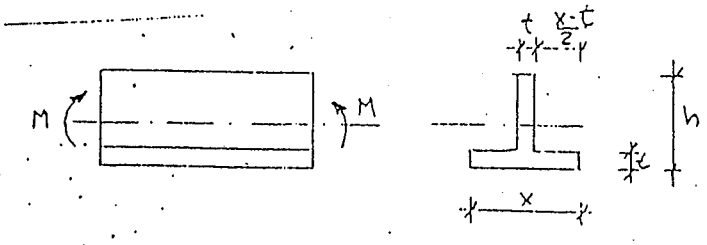


Fig. P4.26

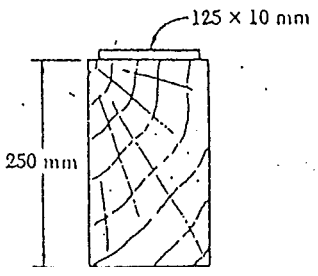
4.

What must be the width x of the beam so that the σ_{max} in tension is 3 times the value of σ_{max} in compression?



h = 10 cm
t = 2,5 cm.

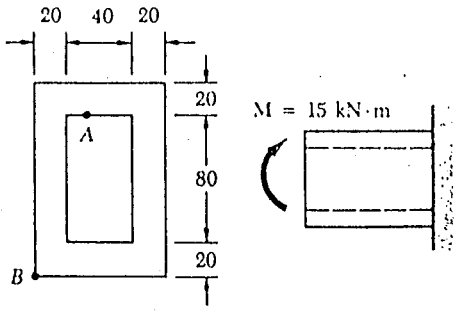
4.38 The 150 × 250 mm timber beam has been strengthened by bolting to it two steel strips to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.



	Wood	Steel
Modulus of elasticity	13 GPa	200 GPa

תורת החוזק 1
תרגיל מס 8

4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.



Dimensions in mm

Fig. P4.1

4.9 and 4.10 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in the beam.

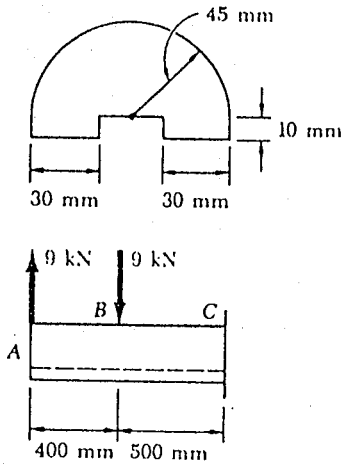


Fig. P4.9

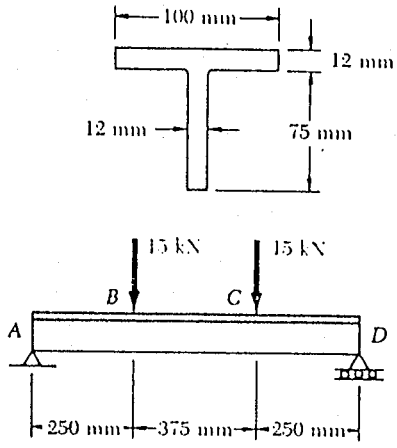


Fig. P4.10

4.26 A couple **M** will be applied to a beam of rectangular cross section which is to be sawed from a log of circular cross section. Determine the ratio *d*/*b*, for which (*a*) the maximum stress σ_m will be as small as possible, (*b*) the radius of curvature of the beam will be maximum.

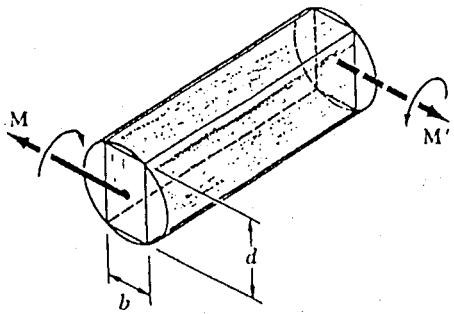
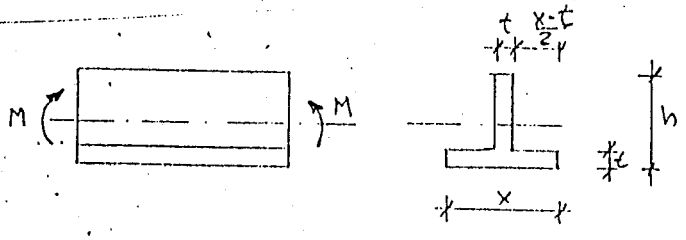


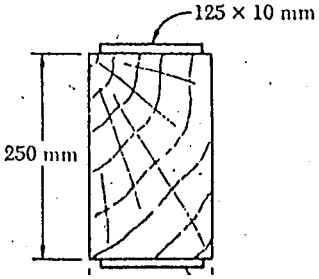
Fig. P4.26

4.

הקורה המתוארת בציור מצויה במצב של כפיפה טהורה. מה צריך להיות עובי האגף x
כך שערכו של מאמץ המתיחה המקסימלי יהיה שליש מערכו של מאמץ הלחיצה המקסימלי?



4.38 The 150 × 250 mm timber beam has been strengthened by bolting to it two steel strips to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.



	Wood	Steel
Modulus of elasticity	13 GPa	200 GPa
Allowable stress	12 MPa	165 MPa

$$\sigma_{max} = \frac{My}{I} = \frac{+M\cdot a/2}{bd^3/12} = \frac{+6M}{bd^2}$$

אבל $b^2+d^2=4R^2$, R הרדיוס של העץ

$$\sigma_{max} = \frac{+6M}{b(R^2-b^2)} = +6M\cdot b^{-1}(R^2-b^2)^{-1}$$

כדי ש σ_{max} יהיה מינימלי ביותר $\frac{d\sigma_{max}}{db} = 0$

$$\frac{d\sigma_{max}}{db} = +6M\left\{-b^{-2}(4R^2-b^2)^{-1} - b^{-1}(4R^2-b^2)^{-2}\cdot(-2b)\right\}$$

$$= +6M\left\{b^{-1}(4R^2-b^2)^{-1}\right\}\left\{-b^{-1}+2b\{4R^2-b^2\}^{-1}\right\}=0$$

$$= \sigma_{max}\left\{-\frac{1}{b}+\frac{2b}{4R^2-b^2}\right\}=0 \Rightarrow \sigma_{max}\left\{\frac{-4R^2+b^2+2b^2}{(4R^2-b^2)b}\right\}=0 \Rightarrow b=\frac{2R}{\sqrt{3}}$$

בדוק לבדוק שזה אכן מינימום ביותר, $\Leftarrow \frac{d^2\sigma_{max}}{db^2} > 0$

$$\frac{d^2\sigma_{max}}{db^2} = +6M\left\{2b^{-3}(4R^2-b^2)^{-1}+2b^{-2}(4R^2-b^2)^{-2}(-2b)+2b^{-1}(4R^2-b^2)^{-3}(4b)+2b^{-1}(4R^2-b^2)^{-2}\right\}$$

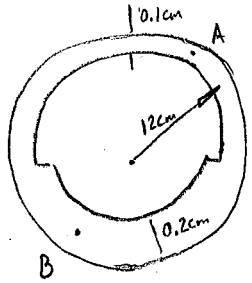
$$= +6M\left\{b^{-1}(4R^2-b^2)^{-1}\right\}\left\{\frac{2}{b^2}-\frac{4b}{b(4R^2-b^2)}+\frac{8b^2}{(4R^2-b^2)^2}+\frac{2}{4R^2-b^2}\right\}$$

$$= \sigma_{max}\left\{\frac{2(4R^2-b^2)^2-2b^2(4R^2-b^2)+8b^4}{b^2(4R^2-b^2)^2}\right\} = \frac{\sigma_{max}}{b^2(4R^2-b^2)^2}\left\{2\cdot\frac{64}{9}R^4-2\cdot\frac{4R^2}{3}\left(\frac{8}{3}R^2\right)+\frac{8\cdot16R^4}{9}\right\}$$

$$= \frac{\sigma_{max}}{4R^2\left(\frac{8}{3}R^2\right)^2}\cdot R^4\left\{\frac{128}{9}-\frac{64}{9}+\frac{128}{9}\right\} = \sigma_{max}\cdot\frac{192/9}{256/27} > 0$$

כלומר, כאשר $b=2R/\sqrt{3}$ הנתון σ_{max} הוא המינימלי ביותר. נמצא את היחס בין d ל b עבור ערך זה.

$$\frac{d}{b} = \frac{\sqrt{4R^2-b^2}}{b} = \frac{2\sqrt{2}R}{\frac{2}{\sqrt{3}}R} = \sqrt{2}$$



$T = 1000\ N\cdot m$

$G = 90\ GPa$

$\tau = \frac{T}{2A_m t}$

$A_m = \pi r^2 = \pi(12)^2 = 452.39\ cm^2 = 452.39\times10^{-4}\ m^2$

$\pi(12)^2/2 + \pi(11.95)^2/2 = 450.51$

או $q = \frac{T}{2A_m}$

$$q = \frac{1000\ N\cdot m}{2(452.39\times10^{-4}\ m^2)} = 11052.43\ \frac{N}{m}$$

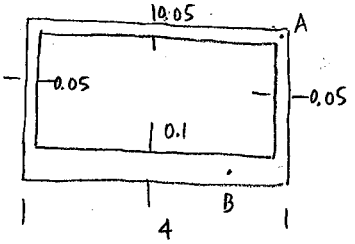
450.51 $\quad$ 11098.53

$\tau_{max} = \tau_A$, לכן $\quad .5526\ MPa = \frac{11052.43}{2\times10^{-3}} = \frac{q}{t_B} = \tau_B$; $11.052\ MPa = \frac{11052.43}{1\times10^{-3}\ m} = \frac{q}{t_A} = \tau_A$

11098 $\quad$ 11.098 $\quad$ $.55493\ MPa$

$$\phi/L = \frac{T}{4A^2G}\oint\frac{ds}{t} = \frac{1000}{4(452.39\times10^{-4})^2\cdot90\times10^9}\left[\frac{\pi(12)}{0.1}+\frac{\pi(11.95)}{0.2}\right] = 7.72\times10^{-4}\ \frac{rad}{m} = .0439^\circ/m$$

7.66×10^{-4} $\quad$ $.0443$



$T = 1000\ N\cdot m$

$G = 90\ GPa$

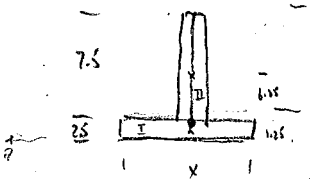
$A_m = (3.95)(1.925) = 7.604\ cm^2 = 7.604\times10^{-4}\ m^2$

$$q = \frac{T}{2A_m} = \frac{1000}{15.21\times10^{-4}} = 657600\ N/m$$

$$1.315\ GPa = 1.315\times10^9\ \frac{N}{m^2} = \frac{657600}{5\times10^{-4}\ m} = \frac{q}{t_A} = \tau_A$$

$$0.658\ GPa = 658\times10^6\ \frac{N}{m^2} = \frac{657600}{1\times10^{-4}\ m} = \frac{q}{t_B} = \tau_B$$

$$\phi/L = \frac{T}{4A^2G}\oint\frac{ds}{t} = \frac{1000}{4(7.604\times10^{-4})^2\cdot90\times10^9}\left[\frac{3.95}{.05}+\frac{1.925}{.05}+\frac{3.95}{.1}+\frac{1.925}{.05}\right] = 0.9392\ \frac{rad}{m} = 53.81^\circ/m$$



$\bar{y} = 2.5$

$\bar{y}$	A	$=$	$\bar{y}A$
$t/2 = 1.25$	$t \cdot x = 2.5x$		$1.25(2.5)x = 3.125x$
$\frac{h+t}{2} = 6.25$	$(h-t)t$		$6.25(18.75) = 117.1875$
	$7.5(2.5) = 18.75$		
	$2.5x + 18.75$		

$$\bar{y}A = \bar{y}_1 A_1 + \bar{y}_2 A_2$$

$$(2.5x + 18.75)(2.5) = 3.125x + 117.1875$$

$$6.25x + 46.875 = 3.125x + 117.1875$$

$$(6.25 - 3.125)x = 117.1875 - 46.875$$

$$3.125x = 70.3125$$

$$x = 22.5 \text{ cm}$$

$$_N\sigma_{max} = +\frac{M_z \cdot y_1}{I_{zz}}$$

$$_T\sigma_{max} = -\frac{M_z \cdot y_2}{I_{zz}}$$

$$\frac{1}{3}{}_T\sigma_{max} = {}_N\sigma_{max}$$

$$\frac{-M_z}{I_{zz}} \cdot \frac{y_2}{3} = \frac{M_z \cdot y_1}{I_{zz}}$$

$$y_1 = -\frac{y_2}{3}$$

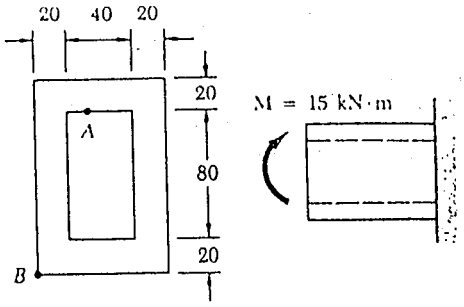
$$-y_1 + y_2 = 10 \Rightarrow \frac{y_2}{3} + y_2 = 10$$

$$\frac{4}{3}y_2 = 10$$

$$y_2 = 7.5$$

$$y_1 = -2.5$$

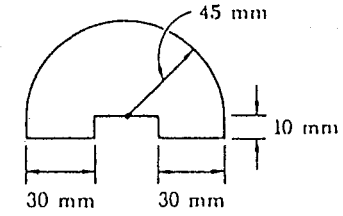
4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.



Dimensions in mm

Fig. P4.1

4.9 and 4.10 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in the beam.



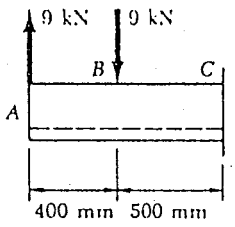
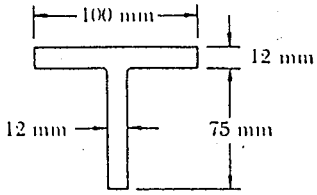


Fig. P4.9



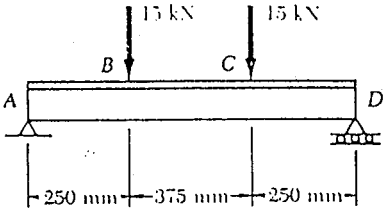


Fig. P4.10

4.26 A couple M will be applied to a beam of rectangular cross section which is to be sawed from a log of circular cross section. Determine the ratio *d*/*b*, for which (*a*) the maximum stress σ_m will be as small as possible, (*b*) the radius of curvature of the beam will be maximum.

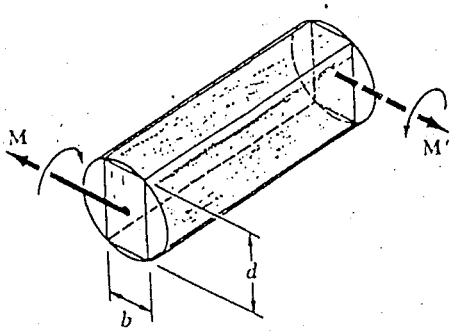
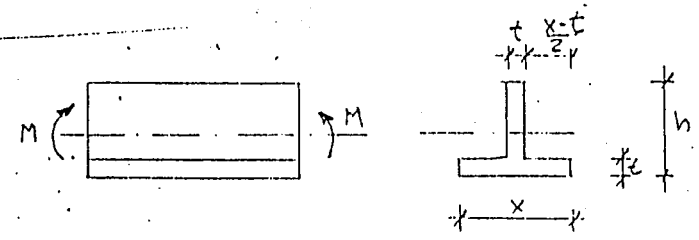


Fig. P4.26

4.

What must be the width x of the beam so that the σ_{max} in tension is 3 times the value of σ_{max} in compression?

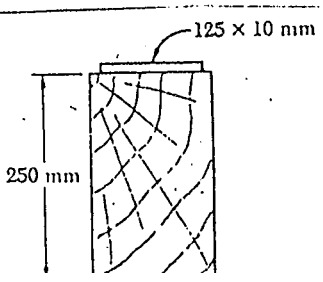


h = 10 cm

t = 2,5 cm.

4.38 The 150 × 250 mm timber beam has been strengthened by bolting to it two steel strips to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

	Wood	Steel
Modulus of elasticity	13 GPa	200 GPa



חתכים בלתי הומוגניים (NON HOMOGENEOUS CROSS-SECTIONS)

בדוגמה מהנדסה אזרחית: קורת בטון – מצויין התמוכה: פלטות בניין

- נניח שיש חתך מורכב משני חומרים

- הידוע משתנה כפי המרחק מהציר הנייטרלי. ממשוך להיות משור אחרי הכפיפה. אזיין פועלת ... שאישור לפני הכפיפה
- על התורה בפועה, הנחה היסודית
- באישור שני החומרים פועלים אחד לשני, הציר הוא אותו
- דבר, אבל המאמצים שונים מפני ש- $E_1 \neq E_2$
- נניח שרוצים לבנות חתך מחומר אחד אבל שינוי למידות חלק ① ובלי שינוי לגובה חלק ②
- נניח ש- $n = E_2/E_1$ ו- n פעמים מאותה.

- מפני שהציר שווה דבר בחלק החדש
- מבחינת לוגיקה הוא הדבר עומס.
- העומס שמופיע ② נוגע הוא $\int_{-y_1}^{y_2} \sigma_x \cdot dy \cdot b_2 = \int_{-y_1}^{y_2} E_2 \varepsilon_2 \cdot b_2 \cdot dy = \int_{-y_1}^{y_2} E_1 \varepsilon \cdot b \, dy$. לכן חומר①

$$E_1 b = E_2 b_2$$
$$b = \frac{E_2}{E_1} b_2 = n b_2$$

- החתך החדש נקרא החתך השקול (Equivalent Section)
- אכשיו יכולים להשתמש ב- $\sigma_{max} = \frac{Mc}{I}$ בחלק הלא נשתנה. בחלק החדש $\sigma_{max} = \frac{Mc}{I} n$
- להתלבט חומר① לחומר②, b_1 ישתנה ל- $a = b_1 \frac{E_1}{E_2} = b_1/n$

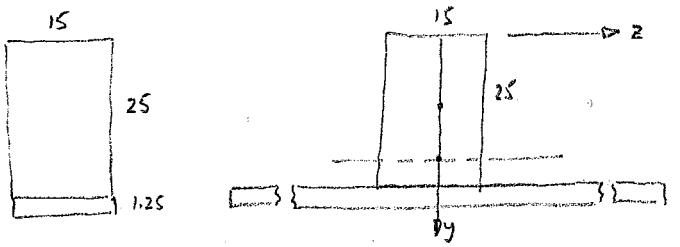
דוגמה composed קורה מורכבת: שני חומרים יש חתך השלם כשיטוק
הקורה תתמוך מומנט כפיפה של 27,115 N-m מסביב הציר האופקי
חלק התחתון הוא מפלדה שהמודול Young $E_s = 2.0686 \times 10^{11}$ Pa
חלק העליון הוא מעץ שהמודול Young $E_w = 1.0343 \times 10^{10}$ Pa
האות מקום תלמוטם כפי הציר האופקי ובקורה המקורית ובקורה השקולה, וחבות השלמנלי ואכן
- מפני שהבות שווה בקורה המקורית ובקורה השקולה אותו דבר.
- להתלבט חומר① לחומר②, b_1 ישתנה ל- $a = b_1 \frac{E_1}{E_2} = b_1/n$
- אכשיו יכולים להשתמש ב- $\sigma_{max} = \frac{Mc}{I}$ בחלק הלא נשתנה. בחלק החדש $\sigma_{max} = \frac{Mc}{I} n$
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- אזיין פועלת ... שאישור לפני הכפיפה
- על התורה בפועה, הנחה היסודית
- נניח שיש חתך מורכב משני חומרים

דוגמה מהנדסה אזרחית: קורת בטון – מצויין התמוכה: פלטות בניין

אזיין פועלת ... שאישור לפני הכפיפה
על התורה בפועה, הנחה היסודית
נניח שיש חתך מורכב משני חומרים

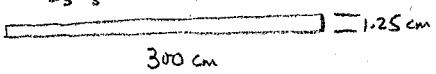


1. לחשב את המרחק של מרכז הכובד מתחתית החתך.
2. לחשב את מומנט התמד הכולל של החתך.
3. לחשב את מומנט ההתנגדות של החתך.
4. לחשב את גודל המאמצים.

1. מתעלמים מעובי הדבק ורואים את החתך כמורכב משני מלבנים.

$E_s/E_w = 20$

$E_s b_s = E_w \cdot b_w$



	$\bar{y}$	A	$\bar{y}A$
①	12.5	375	4687.5
②	25.63	375	9611.25
		750	14298.75

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = 19.065 \text{ cm}$$

$$I_{tt} = I_{zz} + A d^2$$

	I_{zz}	A	d^2	I_{tt}
①	$\frac{1}{12}(15)(25)^3$	375	$(19.065-12.5)^2$	$19531.25 + 16162.21 = 35693.46$
②	$\frac{1}{12}(300)(1.25)^2$	375	$(19.065-25.63)^2$	$48.83 + 16137.6 = 16186.43$
				51879.89 cm^4

- המאמץ המקסימלי בקצה העליון

$$\frac{27115 \cdot 19.065 \cdot 10^{-2}}{51879.89 \times 10^{-8}} = \frac{Mc}{I} = \sigma_{W_{max}}$$

$$9964313 \text{ N/m}^2 = \sigma_{W_{max}}$$

- המאמץ המקסימלי בפלדה הוא מרכך בקצה התיקון וגדל כפול n

$$n \sigma_W = (\sigma_s)_{max}$$

$$\frac{20 \cdot 27115 \cdot 7.185 \times 10^{-2}}{51879.89 \times 10^{-8}} = (\sigma_s)_{max}$$

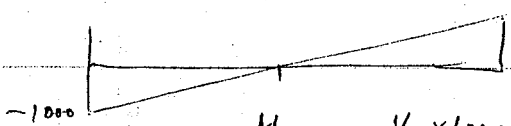
$$75104737 \text{ N/m}^2 = \sigma_{s_{max}}$$

מחשבים המאמץ בגין תואמים (interface)

$$3.102 \times 10^6 \text{ N/m}^2 = \frac{27115 \cdot 5.935 \cdot 10^{-2}}{51879.89 \times 10^{-8}} = \frac{My}{I} = \sigma_W$$

$$62.039 \times 10^6 \text{ N/m}^2 = 20 \cdot \frac{27115 \cdot 5.935 \cdot 10^{-2}}{51879.89 \times 10^{-8}} = \sigma_s$$

$$\sigma_s = 20 \sigma_W$$



$M_{max} = \frac{1}{2} \times 1000 \times 10 = 5000$

$= \frac{1}{2} \times \frac{wL}{2} \times \frac{L}{2}$

$\frac{bh^3}{12}$

	A	y	Ay	d	Ad^2	I_o	$\bar{y} = \frac{\Sigma Ay}{\Sigma A}$
I	48	18	864	4"	768	64	
II	32	8	256	-6"	1152	$682\frac{2}{3}$	$I_{zz} = \Sigma (I_o + Ad^2)$
	80		1120		1920	$746\frac{2}{3}$	

$\bar{y} = \frac{112}{8} = 14$

$I_{zz} = 2666\frac{2}{3}$

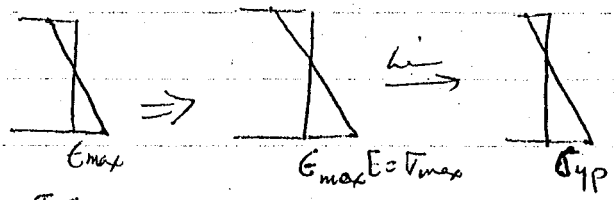
$\sigma_{max} = \frac{5000(12)14}{2666.67}$

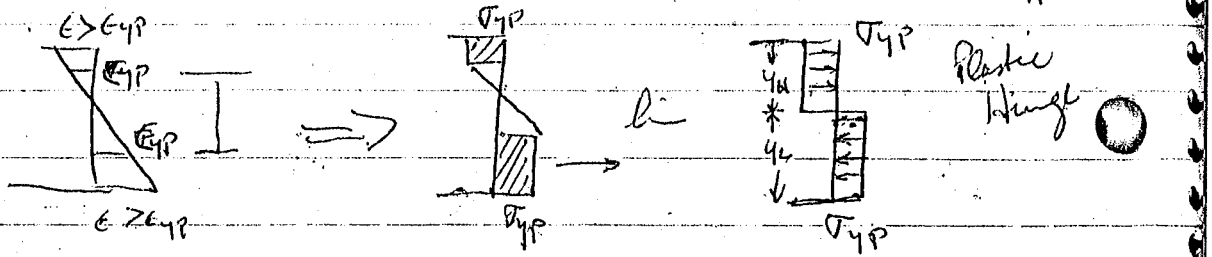
$\sigma_{max} = \frac{5000(12)(6)}{2666.67}$

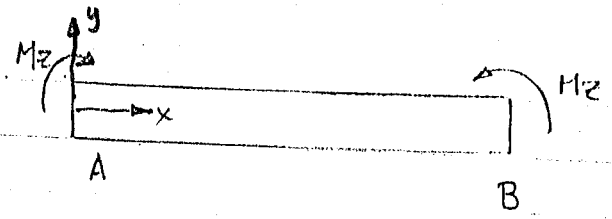
$\sigma_{max} = \frac{Mc}{I}$ let $\frac{I}{c} = S$ - Section modulus

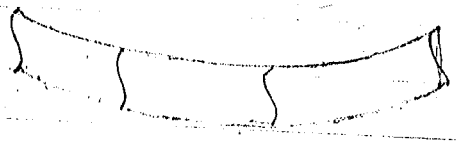
$\sigma_{max} = \frac{M}{S}$

$M_{yp} = \sigma_{yp} S$

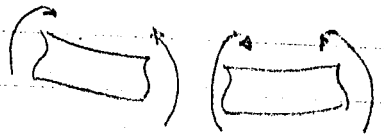








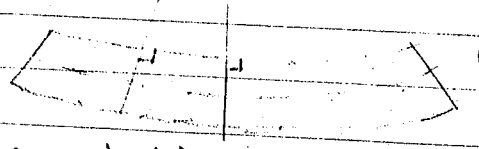
take 2 elements



impossible occurrence

Bernoulli - Navier assumptions

The only possibility is that the cross sections remain plane & normal to the deformed fibers of the beam - for pure bending.

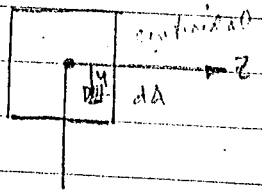


$$\epsilon_x = -k(x)\,y = \frac{\sigma_x}{E} \qquad \therefore \quad \sigma_x = -E\,k(x)\,y$$

$$\int_A \sigma_x\,dA = 0 \qquad -\int_A \sigma_x\,y\,dA = M_z$$

$$\int_A \sigma_x\,dA = -E\,k(x)\int_A y\,dA = 0$$

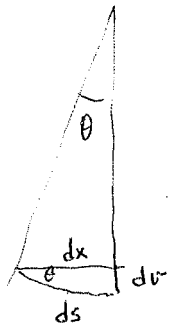
$$-\int_A \sigma_x\,y\,dA = -E\,k(x)\int_A y^2\,dA = M_z = -E\,k(x)\,I_z$$



$$\boxed{k(x) = -\frac{M_z}{EI_z}}$$

$$\epsilon_x = \frac{-y}{R} = \frac{\sigma_x}{E} = \frac{-My}{IE}$$

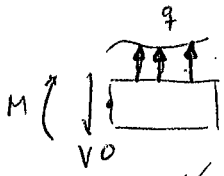
$$\therefore \quad \frac{1}{R} = +\frac{M}{IE} = \frac{\frac{d^2v}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}} \cong \frac{d^2v}{dx^2} \quad \text{if } \left|\frac{dv}{dx}\right| \ll 1$$

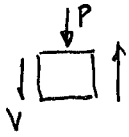


$\tan\theta = \frac{dv}{dx}$

$\sin\theta = \frac{dv}{ds}$

for small θ $\tan\theta \cong \theta$, $\sin\theta \cong \theta$, $ds \cong dx$





$dV + V - P - V$

$dV = P$

$$V + dV + q\,dx - V = 0 \qquad \boxed{-q = \frac{dV}{dx}}$$

$$M + dM + (V+dV)dx - q\,dx\cdot\frac{dx}{2} - M = 0$$

$$\boxed{\frac{dM}{dx} = -V}$$

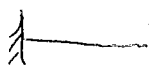
$$\frac{d^2M}{dx^2} = -\frac{dV}{dx} = +q$$

$$M = EIv''$$

$$\frac{d^2M}{dx^2} = (EIv'')'' = +q$$

$$\frac{dM}{dx} = -V = \frac{d}{dx}(EIv'')$$

$$\frac{d^2M}{dx^2} = q = \frac{d^2}{dx^2}(EIv'')$$



Natural $V = V_p$ $M = M_p$

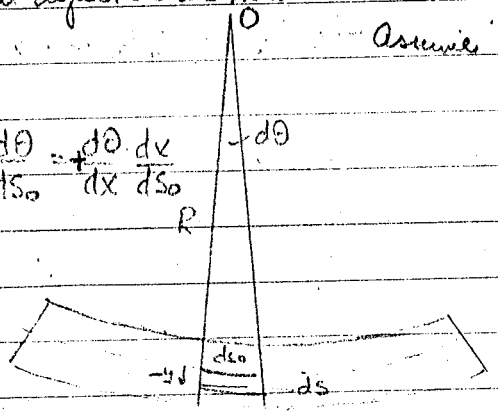
Geometric - $v = 0$, $\frac{dv}{dx} = 0$

$$\sigma_x = -E\,k(x)\,y = -\frac{M_z\,y}{I_z}$$

$$\boxed{\sigma_x = -\frac{M_z\,y}{I_z}}$$

To find deflection we must know how beam is supported.

Assume $v = f(x) \neq f(y,z)$



$$\frac{1}{R} = +\frac{d\theta}{ds_0} = +\frac{d\theta}{dx}\frac{dx}{ds_0}$$

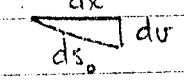
$$ds = (R+y)\,d\theta$$

$$ds_0 = R\,d\theta$$

$$\epsilon_x = \frac{ds - ds_0}{ds_0} = \frac{(R+y)d\theta - (}{R\,d\theta}$$

$$= -y/R = -k(x)\,y$$

$$\therefore \quad \frac{1}{R} = k(x) \quad \text{curvature}$$



$$ds_0^2 = dx^2 + \left(\frac{dv}{dx}\right)^2 dx^2 = \left[1 + \left(\frac{dv}{dx}\right)^2\right] dx$$

$$ds_0 = \sqrt{1 + \left(\frac{dv}{dx}\right)^2}\; dv \qquad \frac{dv}{dx} \ll 1$$

$$\tan\theta = \frac{dv}{dx} \qquad \frac{d^2v}{dx^2} = \sec^2\theta\,\frac{d\theta}{dx} = \left[1 + \left(\frac{dv}{dx}\right)^2\right]\frac{d\theta}{dx}$$

$$\frac{1}{R} = +\frac{\frac{d^2v}{dx^2}}{\left[1 + \left(\frac{dv}{dx}\right)^2\right]^{3/2}}$$

$$\boxed{\frac{1}{R} = k(x) = +\frac{d^2v/dx^2}{\left[1 + \left(\frac{dv}{dx}\right)^2\right]^{3/2}}}$$

Since $\frac{dv}{dx} \ll 1 \qquad \left(\frac{dv}{dx}\right)^2 \ll \frac{dv}{dx} \ll 1 \quad \therefore$

but $K(x) = \frac{M_z}{I_z E} = + \frac{d^2 v}{dx^2}$

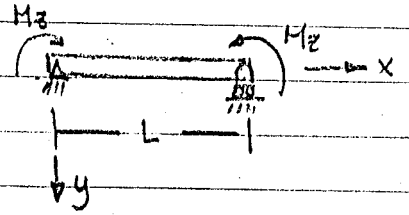
if $M_z = C$

$$\frac{dv}{dx} = + \frac{M_z}{I_z E} x + C_1$$

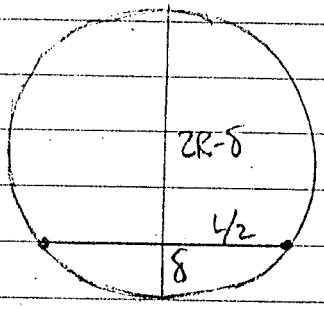
$$v = + \frac{M_z}{2 I_z E} x^2 + C_1 x + C_2$$

if $\begin{cases} v(0) = 0 , & C_2 = 0 \\ v(L) = 0 , & C_1 = \frac{-M_z L}{2 I_z E} \end{cases}$ $\therefore$

$$\boxed{v = \frac{M_z}{2 E I_z} \left(-L x + x^2 \right)}$$



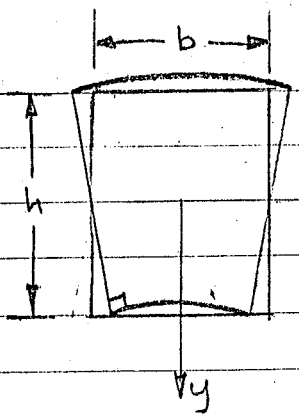
$$v(L/2) = \frac{-M_z L^2}{8 E I_z}$$



$$(2R - \delta)\delta = \frac{L^2}{4}$$

since $\delta << 2R - \delta$ then $2R\delta = \frac{L^2}{4}$ & $\delta = \frac{L^2}{8R} = \frac{M_z L^2}{8 E I_z}$

Cross sections of Beams



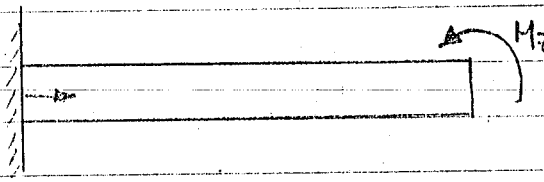
if $\epsilon_x = y/R$ then

$\epsilon_z = -\nu y/R = -y/(R/\nu)$

$\Delta b = \frac{\nu b h}{2R}$ $\quad \epsilon_y = -\frac{\nu y}{R}$

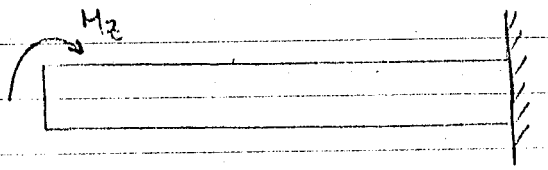
$\Delta h = \int_0^{h/2} \epsilon_y dy = -\frac{\nu y^2}{2R}\Big|_0^{h/2} = -\frac{\nu h^2}{8R}$

Cantilever Beam



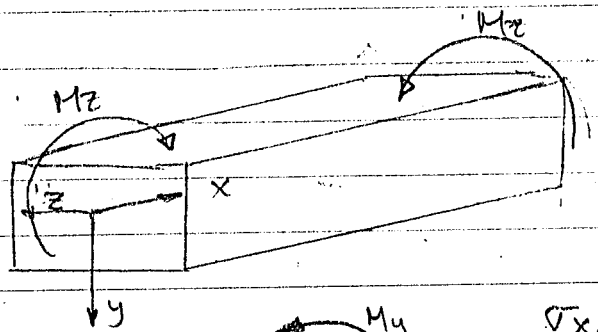
$v(0) = 0$

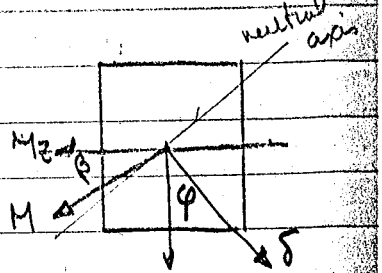
$\frac{dv}{dx}(0) = 0$

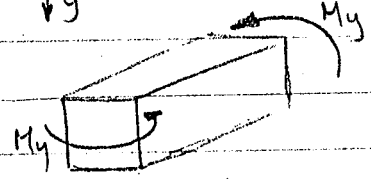


$v(L) = 0$

$\frac{dv}{dx}(L) = 0$



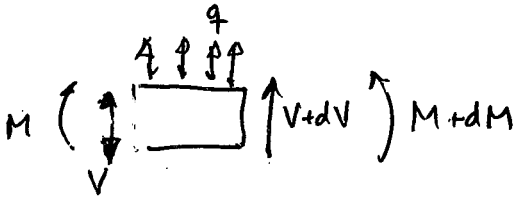




$\sigma_{xx} = \frac{M_y z}{I_y}$

$w = \frac{M_y}{2EI_y}(L-x)x$

$\delta = (v^2+w^2)^{1/2} = \frac{(L-x)x}{2E}\left[\left(\frac{M_z}{I_z}\right)^2 + \left(\frac{M_y}{I_y}\right)^2\right]^{1/2}$



$$+q\,dx + V + dV - V = 0 \qquad q = -\frac{dV}{dx}$$

$$M + dM - M + V\,dx + q\,dx \cdot \frac{dx}{2} = 0 \qquad V = -\frac{dM}{dx} + \cancel{q\frac{dx}{2}}$$

and $\frac{1}{\rho} = \frac{d^2v}{dx^2} = \frac{M}{EI}$

$$\frac{d^3v}{dx^3} = \frac{d}{dx}\left(\frac{M}{EI}\right) = \frac{1}{EI}\frac{dM}{dx} = -\frac{V}{EI}$$

$$\frac{d^4v}{dx^2} = \frac{d^2}{dx^2}\left(\frac{M}{EI}\right) = \frac{-1}{EI}\frac{dV}{dx} = +\frac{q}{EI}$$

$$q = +EI\frac{d^4v}{dx^2}$$

$$\tan\varphi = -I_{yz}/I_y \qquad (12.43 - 12.45)$$

$$K(x) = \frac{M_z\left(I_y^2 + I_{yz}^2\right)^{1/2}}{E\left(I_y I_z - I_{yz}^2\right)} = \frac{M(x)}{EI} \qquad I = \frac{\left(I_y I_z - I_{yz}^2\right)}{\left(I_y^2 + I_{yz}^2\right)^{1/2}}$$

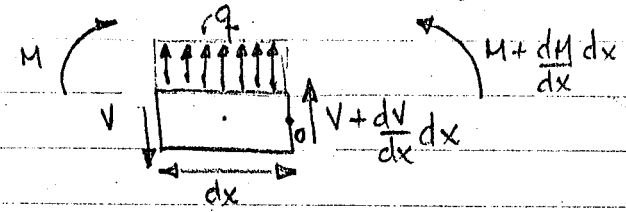
$$\sigma_x = \frac{M\left[I_y y - I_{yz} z\right]}{I_y I_z - I_{yz}^2}$$

Stresses normal xy plane = 0 σ_z small & neglected.; cross section small; cross section retains shape $\rightarrow \gamma_{yz} = 0$ $\sigma_y = -(\ )$ excluded

stresses on boundary act tangentially to boundary.

$\left.\begin{matrix}\tau_{xy}\\ \tau_{xz}\end{matrix}\right\}$ are small $\therefore$ deflect $\neq f(\tau_{xy}, \tau_{xz})$.

deflection will be taken as functions of σ_x only.



$$+\uparrow \sum F_y = 0 \qquad V + \frac{dV}{dx}dx + q\,dx - V = 0$$

$$+\circlearrowleft \sum M_o = M - V\,dx + q\,dx\,\frac{dx}{2} - \left(M + \frac{dM}{dx}dx\right) = 0$$

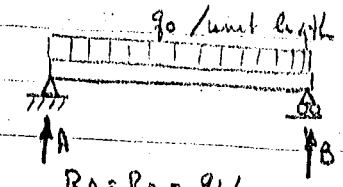
$\frac{dV}{dx} = -q$; now neglect $q\frac{dx^2}{2}$ compared $V\,dx$ & $\frac{dM}{dx}dx$

$$\frac{dM}{dx} = -V \qquad \frac{d^2\delta}{dx^2} = +K(x) = +\frac{M(x)}{EI}$$

$$\frac{d^3\delta}{dx^3} = +\frac{dM}{dx}\Big/(EI) = -\frac{V(x)}{EI} \qquad V(x) = -EI\frac{d^3\delta}{dx^3}$$

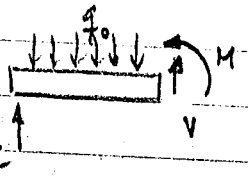
$$\frac{d^4\delta}{dx^4} = -\frac{dV/dx}{EI} = +\frac{q}{EI} \qquad q(x) = +EI\frac{d^4\delta}{dx^4}$$

Conditions

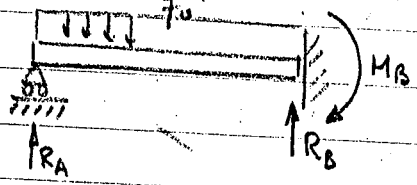


$M_A = 0$ $\quad \delta_A = \delta_B = 0$

$R_A = R_B = qL/2$



$$M(x) = \frac{qL}{2}x - q_0\frac{x^2}{2} \qquad \delta'' = +\frac{M(x)}{EI}$$



$\delta_A = \delta_B = 0 \qquad \frac{d\delta}{dx} = 0$

$$\delta^{IV} = q_0 / EI$$

Simply Supp $\delta = 0 \qquad +EI\frac{d^2\delta}{dx^2} = M_p$

Fixed $\delta = 0 \qquad \frac{d\delta}{dx} = 0$

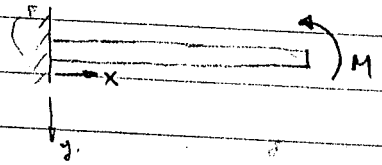
Free $-EI\frac{d^3\delta}{dx^3} = V_p \;;\; +EI\frac{d^2\delta}{dx^2} = M_p$

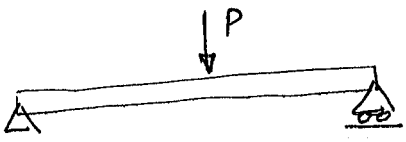


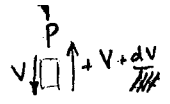
find $M(x)_{max}$, put it into $\sigma_x = \frac{M(I_y y - I_{yz} z)}{I_y I_z - I_{yz}^2}$

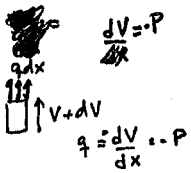
σ_x depends on δ

11/19/70









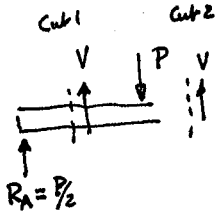
$$q = EI\frac{d^4y}{dx^4} = -P\langle x-L/2\rangle^{-1}$$

$$EIy''' = -P\langle x-L/2\rangle^{0} + C_1$$

$$EIy'' = -P\langle x-L/2\rangle^{1} + C_1x + C_2$$

$$EIy' = -\frac{P}{2}\langle x-L/2\rangle^{2} + C_1\frac{x^2}{2} + C_2x + C_3$$

$$EIy = -\frac{P}{6}\langle x-L/2\rangle^{3} + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3x + C_4$$



$R_A = P/2$

$V + R_A = 0 \quad V = -R_A = -P/2$

$V + R_A - P = 0 \quad V = -R_A + P$

$V = -R_A + P\langle x-L/2\rangle^{0}$

$\therefore EIy''' = -V = +R_A - P\langle x-L/2\rangle^{0}$

$$\left.\begin{matrix} M=0 \\ y=0 \end{matrix}\right|_{x=0} \Rightarrow C_2 = 0 \quad \Rightarrow C_4 = 0$$

$$EIy = -\frac{P}{6}\langle x-L/2\rangle^{3} + C_1\frac{x^3}{6} + C_3x$$

$$\left.\begin{matrix} M=0 \\ y=0 \end{matrix}\right|_{x=L} = -\frac{P}{6}\frac{L^3}{8} + C_1\frac{L^3}{6} + C_3L = 0$$

$$-\frac{P}{1}\frac{L}{2} + C_1L = 0 \quad \therefore C_1 = P/2$$

$$\Rightarrow C_3 = \frac{P}{48}L^2 - \frac{P}{12}L^2 = -\frac{3}{48}PL^2$$

$$EIv = -\frac{P}{6}\langle x-L/2\rangle^{3} + \frac{P}{12}x^3 - \frac{3}{48}PL^2x$$

$$@\ x = \frac{L}{2} \quad EIv = \frac{4}{96}PL^3 - \frac{3PL^3}{96} - \frac{2}{96}PL^3$$

$$= -\frac{PL^3}{48EI}$$

Tuesday?
1230-145pm in EC1105 in mechanics of materials.
Chapter 9, I believe, discusses that.

We are covering the governing equation
d2w/dx2=M/EI with different boundary conditions.

You can solve the equation for several problems that you can find in the book.
Ask Laura to let you into my office and take the book, which is on the middle shelf all the way by the window.

Solve a cantilever beam problem and a simply supported problem. That should be enough.

Just put the book back when you are done and leave me a copy of the materials you covered.

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

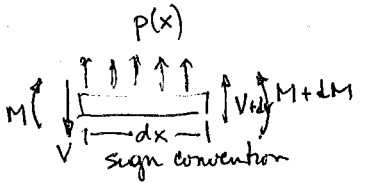
since $\frac{dM}{dx} = -V$

$$\frac{d^3v}{dx^3} = \frac{d}{dx}\left(\frac{M}{EI}\right) = \frac{1}{EI}\frac{dM}{dx} + M\frac{d}{dx}\left(\frac{1}{EI}\right)$$

$$= -\frac{V}{EI} + M\frac{d}{dx}\left(\frac{1}{EI}\right)$$

if EI unchanged $v''' = -\frac{V}{EI}$

since $\frac{dV}{dx} = -p(x)$



$$\frac{d^4v}{dx^4} = \frac{d^2}{dx^2}\left(\frac{M}{EI}\right) = \frac{1}{EI}\frac{d^2M}{dx^2} \quad \text{if } EI = \text{const.}$$

$$= -\frac{1}{EI}\frac{dV}{dx} = \frac{1}{EI}p(x)$$

$$EI\frac{d^4v}{dx^4} = p(x)$$

EMA 3702

3/25/13

EINA 2015

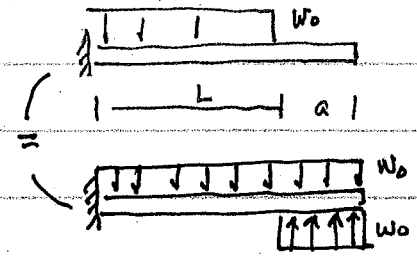
3rd/2nd

Show singularity fns

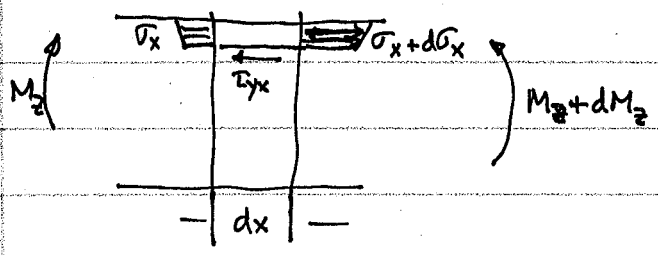
$$\langle x-a\rangle^n = \begin{cases} (x-a)^n & x \geq a \\ 0 & x < a \end{cases}$$

$$\int \langle x-a\rangle^n dx = \begin{cases} \frac{(x-a)^{n+1}}{n+1} & x \geq a \quad n \geq 0 \\ 0 & x < a \\ \langle x-a\rangle^{n+1} & n < 0 \end{cases}$$





$$EIy'''' = -w_0\langle x-a\rangle^0 + w_0\langle x-b\rangle^0$$



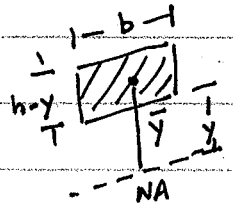
$$\Sigma F_x = \int_y^h (\sigma_x + d\sigma_x) b\, d\bar{y} - \int \sigma_x \cdot b\, d\bar{y} - \tau_{yx} \cdot b\, dx = 0$$

$$= \int d\sigma_x \cdot b\, d\bar{y} - \tau_{yx} b\, dx = 0$$

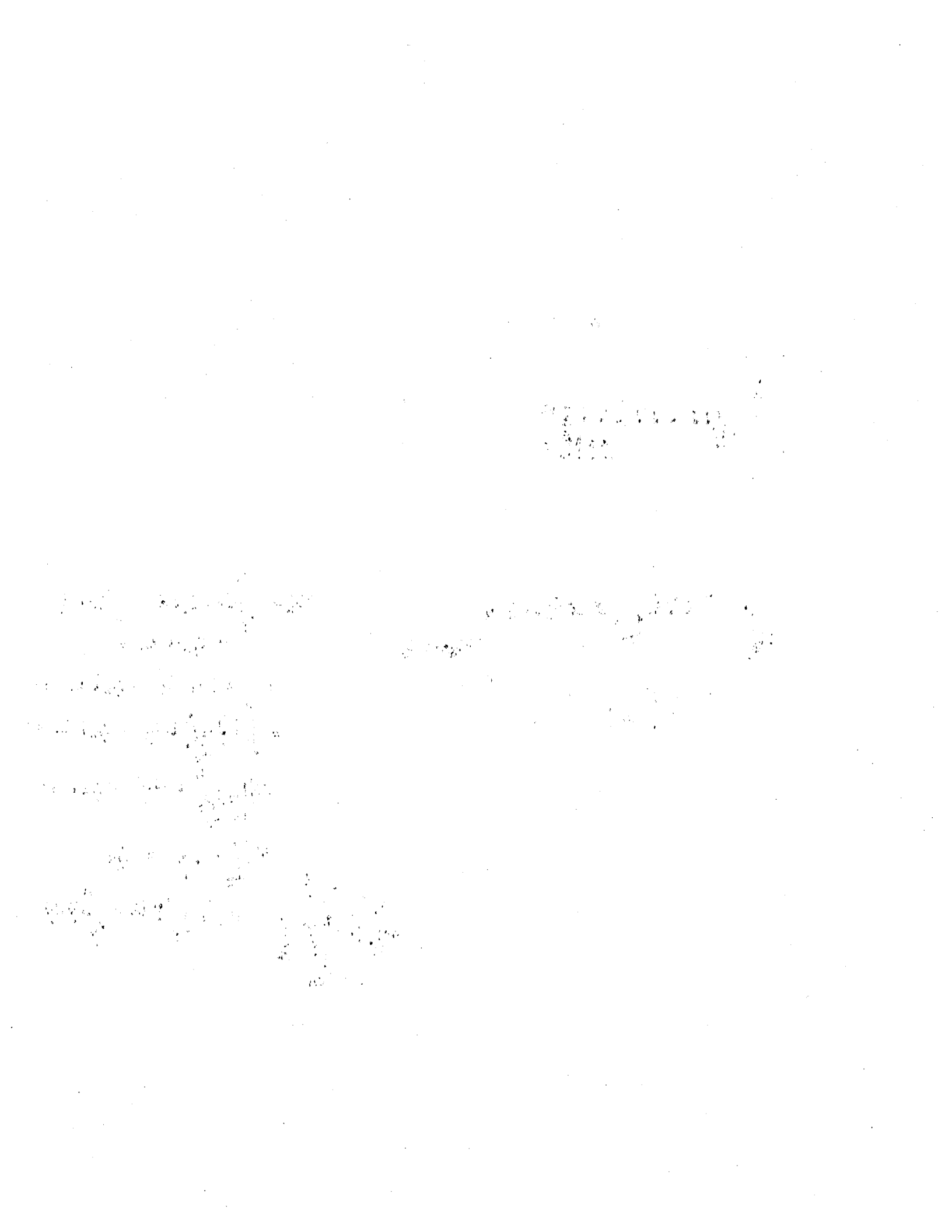
$$= \int d\left(\frac{M_z \bar{y}}{I_z}\right) b\, d\bar{y} - \tau_{yx} b\, dx = 0$$

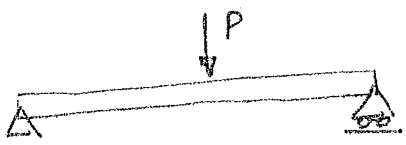
$$= \frac{dM_z}{dx} \frac{1}{I_z} \int_y^h b\, d\bar{y} \cdot \bar{y} - \tau_{yx} b = 0$$

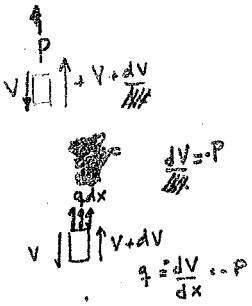
$$\Rightarrow \frac{V}{I_z} \cdot \frac{Q}{b} = \tau_{yx}$$



$$Q = \int_y^h y\, dA = \int_y^h b\bar{y}\, d\bar{y}$$







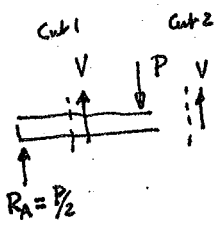
$$q = EI\frac{d^4y}{dx^4} = -P\langle x-\tfrac{1}{2}\rangle^{-1}$$

$$EI\,y''' = -P\langle x-\tfrac{1}{2}\rangle^{0} + C_1$$

$$EI\,y'' = -P\langle x-\tfrac{1}{2}\rangle^{1} + C_1 x + C_2$$

$$EI\,y' = -\frac{P}{2}\langle x-\tfrac{1}{2}\rangle^{2} + C_1\frac{x^2}{2} + C_2 x + C_3$$

$$EI\,y = -\frac{P}{6}\langle x-\tfrac{1}{2}\rangle^{3} + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4$$



$V + R_A = 0 \quad V = -R_A = -P/2$

$V + R_A - P = 0 \quad V = -R_A + P$

$V = -R_A + P\langle x-\tfrac{1}{2}\rangle^{0}$

$\therefore EIy''' = -V = +R_A - P\langle x-\tfrac{1}{2}\rangle^{0}$

$M=0,\ y=0 \Big|_{x=0} \Rightarrow C_2 = 0,\ C_4 = 0$

$$EIy = -\frac{P}{6}\langle x-\tfrac{1}{2}\rangle^{3} + C_1\frac{x^3}{6} + C_3 x$$

$$M=0,\ y=0\Big|_{x=L} = -\frac{P}{6}\frac{L^3}{8} + C_1\frac{L^3}{6} + C_3 L = 0$$

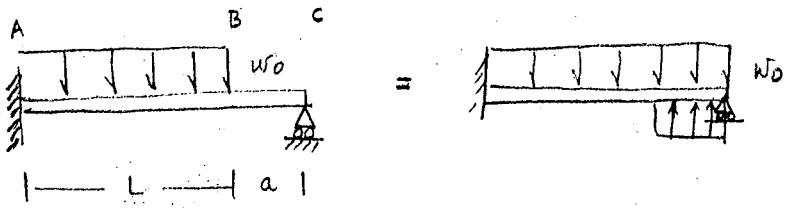
$$-\frac{P}{1}\frac{L}{2} + C_1 L = 0 \quad \therefore C_1 = \frac{P}{2}$$

$$\Rightarrow C_3 = \frac{P}{48}L^2 - \frac{P}{12}L^2 = -\frac{3}{48}PL^2$$

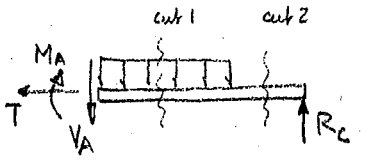
$$EIv = -\frac{P}{6}\langle x-\tfrac{1}{2}\rangle^{3} + \frac{P}{12}x^3 - \frac{3}{48}PL^2 x$$

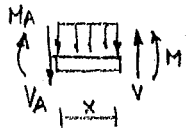
$$@\ x=\frac{L}{2} \quad EIv = \frac{1}{96}PL^3 - \frac{3PL^3}{96} - \frac{2}{96}PL^3$$

$$= -\frac{PL^3}{48EI}$$



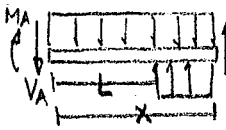
Indeterminate Problem: solve using shear form of equation



at cut 1: 

$V = w_0 x + V_A$

4 unknowns & 3 eqns : 1 extra unknown → choose V_A

at cut 2: 

$V = w_0 x - w_0\langle x-L\rangle + V_A$

$\therefore$ V can be written as $V = w_0 x - w_0\langle x-L\rangle^1 + V_A$ where V_A here is the unknown

Solving from here requires 3 BCs to solve for C_2, C_3, C_4 & 1 more BC for the unknown V_A

Now $EI\,y''' = -V = -w_0 x + w_0\langle x-L\rangle^1 - V_A$.

$$EI\,y'' = -w_0\frac{x^2}{2} + \frac{w_0}{2}\langle x-L\rangle^2 - V_A x + C_2 = M$$

$$EI\,y' = -w_0\frac{x^3}{6} + \frac{w_0}{6}\langle x-L\rangle^3 - V_A\frac{x^2}{2} + C_2 x + C_3$$

$$EI\,y = -w_0\frac{x^4}{24} + \frac{w_0}{24}\langle x-L\rangle^4 - V_A\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4$$

BC's: $y=0$ @ $x=0 \Rightarrow C_4 = 0$

$y'=0$ @ $x=0 \Rightarrow C_3 = 0$

$y=0$ @ $x=L+a \Rightarrow 0 = -w_0\frac{(L+a)^4}{24} + \frac{w_0 a^4}{24} - V_A\frac{(L+a)^3}{6} + C_2\frac{(L+a)^2}{2}$ ①

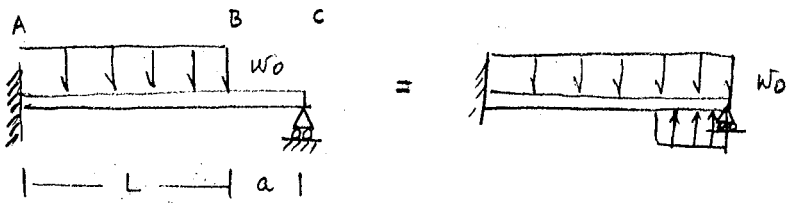
$M=0$ $(EIy''=0)$ @ $x=L+a \Rightarrow 0 = -w_0\frac{(L+a)^2}{2} + \frac{w_0}{2}(a)^2 - V_A(L+a) + C_2$ ②

so
$$\begin{bmatrix} -(L+a)^3/6 & (L+a)^2/2 \\ -(L+a) & 1 \end{bmatrix}\begin{bmatrix} V_A \\ C_2 \end{bmatrix} = \begin{bmatrix} w_0\frac{(L+a)^4}{24} - \frac{w_0 a^4}{24} \\ w_0\frac{(L+a)^2}{2} - \frac{w_0 a^2}{2} \end{bmatrix}$$

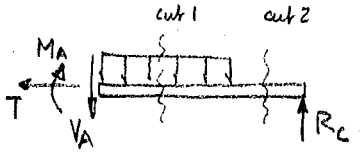
$$V_A = \frac{\begin{bmatrix} \frac{w_0}{24}\left[(L+a)^4 - a^4\right] & (L+a)^2/2 \\ \frac{w_0}{2}\left[(L+a)^2 - a^2\right] & 1 \end{bmatrix}}{\begin{bmatrix} -(L+a)^3/6 & (L+a)^2/2 \\ -(L+a) & 1 \end{bmatrix}} = \frac{\frac{w_0}{24}\left[(L+a)^4 - a^4\right] - \frac{w_0}{4}\left[(L+a)^4 - a^2(L+a)^2\right]}{(L+a)^3/3}$$

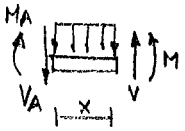
$$C_2 = \frac{\begin{bmatrix} -(L+a)^3/6 & \frac{w_0}{24}\left[(L+a)^4 - a^4\right] \\ -(L+a) & \frac{w_0}{2}\left[(L+a)^2 - a^2\right] \end{bmatrix}}{(L+a)^3/3} = \frac{\frac{-w_0}{12}\left[(L+a)^5 - a^2(L+a)^3\right] + \frac{w_0}{24}\left[(L+a)^5 - a^4(L+a)\right]}{(L+a)^3/3}$$

Note that @ $x=0$ $EIy'' = C_2 = M_A$. Also $R_c = V_A + w_0 L$



Indeterminate Problem: solve using shear form of equation



at cut 1: 

$V = w_0 x + V_A$

4 unknowns & 3 eqns : 1 extra unknown → choose V_A

at cut 2: 

$V = w_0 x - w_0\langle x-L\rangle + V_A$

$\therefore$ V can be written as $V = w_0 x - w_0\langle x-L\rangle^1 + V_A$ where V_A here is the unknown

Now $EI\,y''' = -V = -w_0 x + w_0\langle x-L\rangle^1 - V_A$.

Solving from here requires 3 BCs to solve for C_2, C_3, C_4 & 1 more BC for the unknown V_A.

$$EI\,y'' = -w_0\frac{x^2}{2} + \frac{w_0}{2}\langle x-L\rangle^2 - V_A x + C_2 = M$$

$$EI\,y' = -w_0\frac{x^3}{6} + \frac{w_0}{6}\langle x-L\rangle^3 - V_A\frac{x^2}{2} + C_2 x + C_3$$

$$EI\,y = -w_0\frac{x^4}{24} + \frac{w_0}{24}\langle x-L\rangle^4 - V_A\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4$$

BCs: $y=0$ @ $x=0 \Rightarrow C_4 = 0$

$y'=0$ @ $x=0 \Rightarrow C_3 = 0$

$y=0$ @ $x=L+a \Rightarrow 0 = -w_0\frac{(L+a)^4}{24} + \frac{w_0 a^4}{24} - V_A\frac{(L+a)^3}{6} + C_2\frac{(L+a)^2}{2}$ ①

$M=0$ $(EIy''=0)$ @ $x=L+a \Rightarrow 0 = -w_0\frac{(L+a)^2}{2} + \frac{w_0}{2}(a)^2 - V_A(L+a) + C_2$ ②

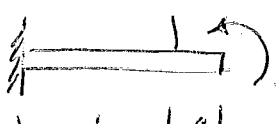
so

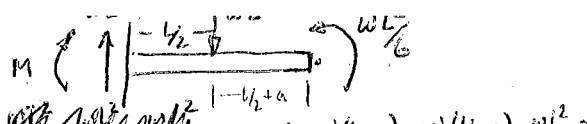
$$\begin{bmatrix} -(L+a)^3/6 & (L+a)^2/2 \\ -(L+a) & 1 \end{bmatrix}\begin{bmatrix} V_A \\ C_2 \end{bmatrix} = \begin{bmatrix} \frac{w_0(L+a)^4}{24} - \frac{w_0 a^4}{24} \\ \frac{w_0(L+a)^2}{2} - \frac{w_0 a^2}{2} \end{bmatrix}$$

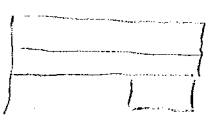
$$V_A = \frac{\begin{bmatrix} \frac{w_0}{24}\left[(L+a)^4 - a^4\right] & (L+a)^2/2 \\ \frac{w_0}{2}\left[(L+a)^2 - a^2\right] & 1 \end{bmatrix}}{\begin{bmatrix} -(L+a)^3/6 & (L+a)^2/2 \\ -(L+a) & 1 \end{bmatrix}} = \frac{\frac{w_0}{24}\left[(L+a)^4 - a^4\right] - \frac{w_0}{4}\left[(L+a)^4 - a^2(L+a)^2\right]}{(L+a)^3/3}$$

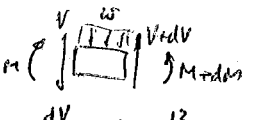
$$C_2 = \frac{\begin{bmatrix} -(L+a)^3/6 & \frac{w_0}{24}\left[(L+a)^4 - a^4\right] \\ -(L+a) & \frac{w_0}{2}\left[(L+a)^2 - a^2\right] \end{bmatrix}}{(L+a)^3/3} = \frac{\frac{-w_0}{12}\left[(L+a)^5 - a^2(L+a)^3\right] + \frac{w_0}{24}\left[(L+a)^5 - a^4(L+a)\right]}{(L+a)^3/3}$$

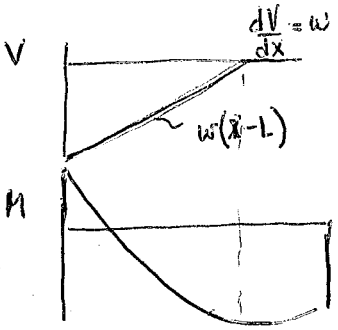
Note that @ $x=0$ $EIy'' = C_2 = M_A$. Also $R_c = V_A + w_0 L$











$$M + \omega L(L+a) - \omega L\left(\tfrac{L}{2}+a\right) - \frac{\omega L^2}{6} = 0$$

$$M = \omega L\tfrac{L}{2} - \omega\tfrac{L^2}{6}$$

$$\frac{dV}{dx} = w = -\frac{d^2M}{dx^2}$$

$$\frac{dM}{dx} = -V$$

$$EI\frac{d^2v}{dx^2} = M \Rightarrow EIv^{IV} = -w$$

$$EIv''' = -V$$

$$EIv^{IV} = -w + w\langle x-L\rangle^0$$

$$EIv''' = -wx + w\langle x-L\rangle^1 + C_1$$

$$EIv'' = -\frac{wx^2}{2} + \frac{w}{2}\langle x-L\rangle^2 + C_1x + C_2$$

$$EIv' = -\frac{wx^3}{6} + \frac{w}{6}\langle x-L\rangle^3 + C_1\frac{x^2}{2} + C_2x + C_3$$

$$EIv = -\frac{wx^4}{24} + \frac{w}{24}\langle x-L\rangle^4 + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3x + C_4$$

① $v(x=0) = 0$ ③ $EIv'''(x=L+a) = 0$

② $v'(x=0) = 0$ $EIv''(x=L) = M_L$

① $\Rightarrow C_4 = 0$

② $\Rightarrow C_3 = 0$

③ $\Rightarrow 0 = -w(L+a) + wa + C_1 = 0 \qquad C_1 = +w(L+a)$

④ $\Rightarrow M_L = -w\frac{(L+a)^2}{2} + \frac{wa^2}{2} + C_1(L+a) + C_2 = -\frac{w(L+a)^2}{2} + \frac{wa^2}{2} + w(L+a)^2 + C_2 = \frac{w}{2}\{-L^2-2aL-a^2+a^2+L^2+2aL\} + C_2$

$$M_L = \frac{w}{2}(L^2+a^2) + C_2$$

$$C_2 = \frac{wL^2}{6} - \frac{w}{2}(L^2+a^2)$$

$$C_2 = M_L + \frac{w(L+a)^2}{2} = \frac{wL^2}{6} + \frac{w(L+a)^2}{2}$$

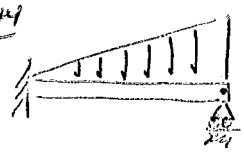
$$v(x) = \frac{1}{EI}\left\{\frac{wx^4}{24} + \frac{w}{24}\langle x-L\rangle^4 + \frac{w}{6}(L+a)(x^3) + \frac{wx^2}{2}\left[\frac{L^2}{6} + \left(\frac{L+a}{2}\right)^2\right]\right\}$$

$$EIv''' = -wx + w\langle x-L\rangle^1 + w(L+a) = -V \quad \checkmark \qquad V = w(x-L)$$

$$EIv'' = M = -\frac{wx^2}{2} + w\langle x-L\rangle^2 + wLx + \frac{wL^2}{6} - \frac{w}{2}(L^2+a^2)$$

$$-\frac{w}{2}(x^2 - 2xL + L^2) + wx^2 + w(xL - L^2) + \frac{wL^2}{6} - \frac{wL^2}{2} - \frac{wa^2}{2}$$

(9.19 in Bean, Johnston & deWolf 3rd ed 8-44



$$EIv^{IV} = -q_0\left(\frac{x}{L}\right)$$

$$EIv''' = -\frac{q_0}{L}\frac{x^2}{2} + C_1$$

$$EIv'' = -\frac{q_0}{L}\frac{x^3}{6} + C_1x + C_2$$

$$EIv' = -\frac{q_0}{L}\frac{x^4}{24} + C_1\frac{x^2}{2} + C_2x + C_3$$

$$EIv = -\frac{q_0}{L}\frac{x^5}{120} + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3x + C_4$$

$v(x=0) = 0 \Rightarrow C_4 = 0$

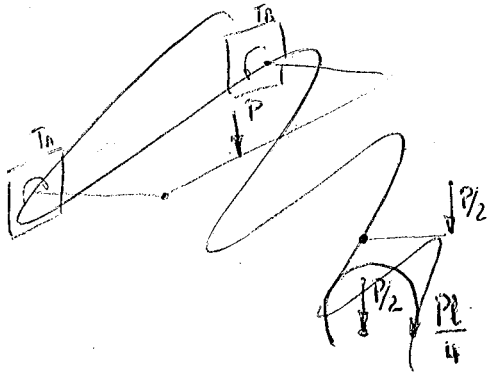
$v'(x=0) = 0 \Rightarrow C_3 = 0$

$v(x=L) = 0 \qquad -\frac{q_0L^4}{120} + \frac{C_1L^3}{6} + \frac{C_2L^2}{2} = 0$

$EIv''(x=L) = 0 \qquad -\frac{q_0L^2}{6} + C_1L + C_2 = 0$

$$C_2 = -C_1L + \frac{q_0L^2}{180} = -\frac{7q_0L^2}{120}$$

$$-q_0L^4 + C_1L^3 - \frac{7}{2}C_1L^3 + \frac{7}{12}q_0L^4 = 0$$



$$v = \frac{1}{EI}\left\{-\frac{q_0x^5}{120L} + \frac{27q_0Lx^3}{720} - \frac{7q_0L^2x^2}{240}\right\}$$

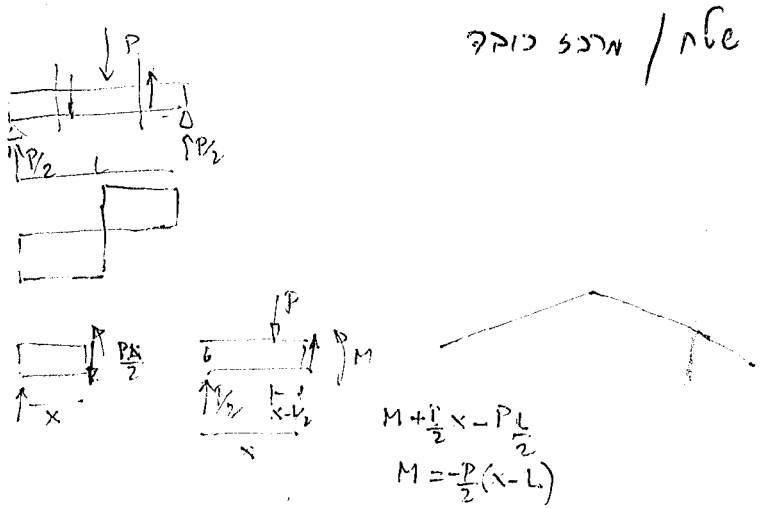
$$EIv'''(x=L) = -V = -\frac{q_0}{L}\frac{L^2}{2} + \frac{27q_0L}{120} = -\frac{33}{120}q_0L \qquad V = \frac{11}{40}q_0L$$

שיטת המאמצים, שיטת התזוזות

חיכון

מהלכים של מומנטים, כוחות שניה, כוח ציר.

מומנט אינרציה ומרכז שלט / מרכז כובד



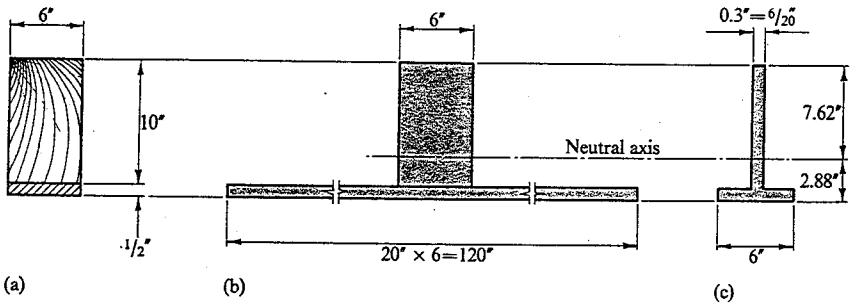


Fig. 6-22

EXAMPLE 6-9

Consider a composite beam of the cross-sectional dimensions shown in Fig. 6-22(a). The upper 6-in.-by-10-in. (full-sized) part is wood, $E_w = 1.5 \times 10^6$ psi; the bottom ½-in.-by-6-in. strap is steel, $E_s = 30 \times 10^6$ psi. If this beam is subjected to a bending moment of 20,000 ft-lb around a horizontal axis, what are the maximum stresses in the steel and wood?

SOLUTION

The ratio of the elastic moduli $E_s/E_w = 20$. Hence, using a transformed section of wood, the width of the bottom strip is $6(20) = 120$ in. The transformed area is shown in Fig. 6-22(b). Its centroid and moment of inertia around the centroidal axis are

$$y = \frac{6(10)5 + (0.5)120(10.25)}{6(10) + (0.5)120} = 7.62 \text{ in.} \qquad \text{from the top}$$

$$I_{zz} = \frac{6(10)^3}{12} + (6)10(2.62)^2 + \frac{120(0.5)^3}{12} + (0.5)120(2.63)^2 = 1{,}328 \text{ in.}^4$$

The maximum stress in the wood is

$$(\sigma_w)_{max} = \frac{Mc}{I} = \frac{(20{,}000)12(7.62)}{1{,}328} = 1{,}380 \text{ psi}$$

The maximum stress in the steel is

$$(\sigma_s)_{max} = n\sigma_w = 20\,\frac{(20{,}000)12(2.88)}{1{,}328} = 10{,}400 \text{ psi}$$

ALTERNATE SOLUTION

A transformed area in terms of steel may be used instead. Then the equivalent width of wood is $b/n = 6/20$, or 0.3 in. This transformed area is shown in Fig. 6-22(c).

$$y = \frac{(0.3)10(5.5) + 6(0.5)(0.25)}{(0.3)10 + 6(0.5)} = 2.88 \text{ in.} \qquad \text{from the bottom}$$

$$I_{zz} = \frac{(0.3)10^3}{12} + (0.3)10(2.62)^2 + \frac{6(0.5)^3}{12} + (0.5)6(2.63)^2 = 66.5 \text{ in.}^4$$

$$(\sigma_s)_{max} = \frac{(20{,}000)12(2.88)}{66.5} = 10{,}400 \text{ psi}$$

$$(\sigma_w)_{max} = \frac{\sigma_s}{n} = \left(\frac{1}{20}\right)\frac{(20{,}000)12(7.62)}{66.5} = 1{,}380 \text{ psi}$$

Note that if the transformed section is an equivalent wooden section, the stresses in the actual wooden piece are obtained directly. Conversely, if the equivalent section is steel, stresses in steel are obtained directly. The stress in a material stiffer than the material of the transformed section is increased since to cause the same unit strain a higher stress is required.

EXAMPLE 6-10

Determine the maximum stress in the concrete and the steel for a reinforced-concrete beam with the section shown in Fig. 6-23(a) if it is subjected to a positive bending moment of 50,000 ft-lb. The reinforcement consists of two #9 steel bars. (These bars are 1⅛ in. in diameter and have a cross-sectional area of 1 in.²) Assume the ratio of E for steel to that of concrete to be 15, i.e., $n = 15$.

SOLUTION

Plane sections are assumed to remain plane in a reinforced-concrete beam. Strains vary linearly from the neutral axis as shown in Fig. 6-23(b) by the line *ab*. A transformed section in terms of concrete is used to solve this problem. However, concrete is so weak in tension that there is no assurance that minute cracks will not occur in the tension zone of the beam. For this reason no credit is given to concrete for resisting

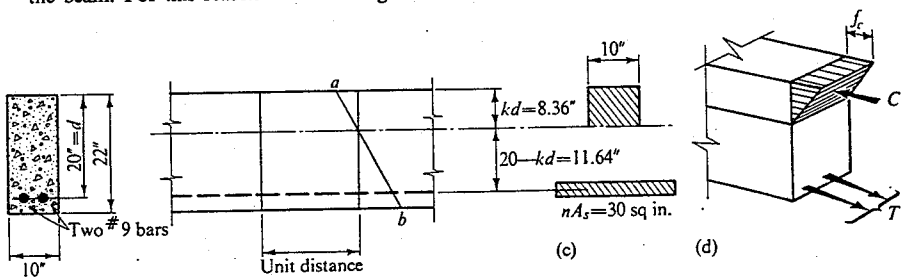


Fig. 6-23

occur in the plastic zones, no unbalance in longitudinal forces occurs and no shearing stresses are developed.

This elementary solution has been refined by using a more carefully formulated criterion of yielding caused by the simultaneous action of normal and shearing stresses.* Some fundamental aspects of the interaction of such stresses will be considered in Chapter 9.

EXAMPLE 7-6

An I beam is loaded as in Fig. 7-14(a). If it has the cross section shown in Fig. 7-14(c), determine the shearing stresses at the levels indicated. Neglect the weight of the beam.

SOLUTION

A free-body diagram of a segment of the beam is in Fig. 7-14(b). It is seen from this diagram that the vertical shear at every section is 50 kips. Bending moments do not enter directly into the present problem. The shear flow at the various levels of the beam is computed in the table below using Eq. 7-5. Since $\tau = q/t$ (Eq. 7-6), the shearing stresses are obtained by dividing the shear flows by the respective widths of the beam.

$$I = \frac{6(12)^3}{12} - \frac{(5.5)(11)^3}{12} = 254 \text{ in.}^4$$

For use in Eq. 7-5 the ratio $V/I = -50{,}000/254 = -197$ lb/in.^{4}

Level	A_{fghj}*	$\bar{y}$**	$Q = A_{fghj}\bar{y}$	$q = VQ/I$	t	τ, psi
1-1	0	6	0	0	6.0	0
2-2	(0.5)6 = 3.00	5.75	17.25	−3,400	6.0 0.5	−570 −6,800
3-3	(0.5)6 = 3.00 (0.5)(0.5) = 0.25	5.75 5.25	17.25 1.31 } 18.56	−3,650	0.5	−7,300
4-4	(0.5)6 = 3.00 (0.5)(5.5) = 2.75	5.75 2.75	17.25 7.56 } 24.81	−4,890	0.5	−9,780

* A_{fghj} is the partial area of the cross section above a given level in square inches.
** $\bar{y}$ is the distance from the neutral axis to the centroid of the partial area in inches.

The negative signs of τ show that, for the section considered, the stresses act downward on the right face of the elements. The sense of the shearing stresses acting on the section coincides with the sense of the shearing force V. For this reason a strict adherence to the sign convention is often unnecessary. It is always true that $\int_A \tau\, dA$ is equal to V and has the same sense.

* D. C. Drucker, "The Effect of Shear on the Plastic Bending of Beams," *Journal of Applied Mechanics*, 23 (1956), pp. 509–14.

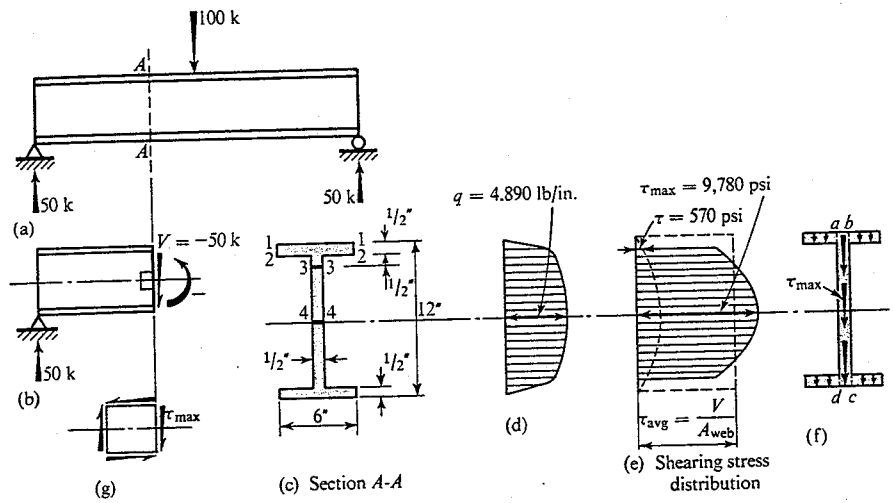


Fig. 7-14

Note that at the level 2-2 two widths are used to determine the shearing stress—one just above the line 2-2, and one just below. A width of 6 in. corresponds to the first case, and 0.5 in. to the second. This transition point will be discussed in the next article. The results obtained, which by virtue of symmetry are also applicable to the lower half of the section, are plotted in Fig. 7-14(d) and (e). By a method similar to the one used in the preceding example, it may be shown that the curves in Fig. 7-14(e) are parts of a second-degree parabola.

The variation of the shearing stress indicated by Fig. 7-14(e) may be interpreted as is shown in Fig. 7-14(f). The maximum shearing stress occurs at the neutral axis; the vertical shearing stresses throughout the web of the beam are nearly of the same magnitude. The shearing stresses occurring in the flanges are very small. For this reason the maximum shearing stress in an I beam is often approximated by dividing the total shear V by the cross-sectional area of the web (area *abcd* in Fig. 7-11(f)). Hence

$$(\tau_{max})_{approx} = V/A_{web} \tag{7-9}$$

In the example considered this gives

$$(\tau_{max})_{approx} = \frac{50{,}000}{(0.5)12} = 8{,}330 \text{ psi}$$

This stress differs by about 15 per cent from the one found by the accurate formula. For most cross sections a much closer approximation

best approach. For example, if for the beam segment in Fig. 7-17(b) positive bending moments increase toward the reader, larger normal forces act on the nearer cross section. For the elements shown, $\tau t\,dx$ or $q\,dx$ must aid the smaller force acting on the partial area of the cross section. This determines the sense of the shearing stresses in the longitudinal cuts. Numerically equal shearing stresses act on the mutually perpendicular planes of an infinitesimal element, and the shearing stresses on such planes either meet or part with their directional arrowheads at a corner. In this manner the sense of the shearing stresses in the plane of the cross section becomes known.

The magnitude of the shearing stresses varies for the different vertical cuts. For example, if the cut *c-c* in Fig. 7-17(a) is at the edge of the beam, the shaded area of the beam's cross section is zero. However, if the thickness of the flange is constant, and the cut *c-c* is made progressively closer to the web, the shaded area increases from zero at a linear rate.

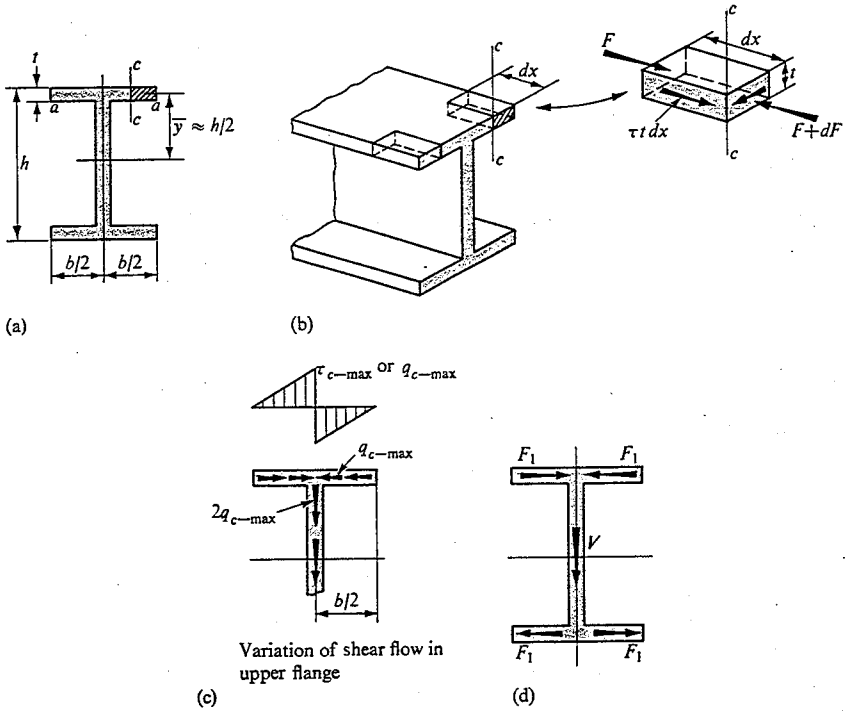


Fig. 7-17. Existence of shearing forces in the flange of an *I* beam which act perpendicularly to the axis of symmetry.

Moreover, as $\bar{y}$ remains constant for any such area, Q also increases linearly from zero toward the web. Therefore, since V and I are constant at any section through the beam, the shear flow $q_c = VQ/I$ follows the same variation. If the thickness of the flange remains the same, the shearing stress $\tau_c = VQ/It$ varies similarly. The same variation of q_c and τ_c exists on either side of the axis of symmetry of the cross section. However, these quantities in the plane of the cross section act in opposite directions on the two sides as may be determined by isolating another flange element to the left side of the web in Fig. 7-17(b). The variation of these shearing stresses or shear flows is represented in Fig. 7-17(c), where it is assumed that the web is very thin.

In common with all stresses, the shearing stresses in Fig. 7-17(c), when integrated over the area on which they act, are equivalent to a force. The magnitude of the horizontal force F_1 for one-half the flange, Fig. 7-17(d), is equal to the average shearing stress multiplied by one-half the flange area, i.e.,

$$F_1 = (\tau_{c\text{-max}}/2)(bt/2) \qquad \text{or} \qquad F_1 = (q_{c\text{-max}}/2)(b/2)$$

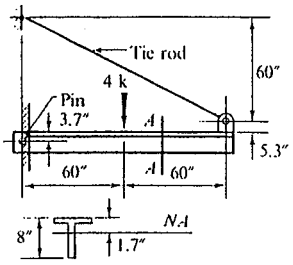
These horizontal forces act in the upper and lower flanges. Because of the symmetry of the cross section, these equal forces occur in pairs and oppose each other; thus they cause no apparent external effect.

To determine the shear flow at the juncture of the flange and the web (cut *a-a* in Fig. 7-17(a)), the whole area of the flange times $\bar{y}$ must be used in computing the value of Q. However, since in finding $q_{c\text{-max}}$ one-half the flange area times the same $\bar{y}$ has already been used, the sum of the two horizontal shear flows coming in from opposite sides gives the vertical shear flow at the cut *a-a*. Hence, figuratively speaking, the horizontal shear flows turn through 90° and merge to become the vertical shear flow. Then the shear flows at the various horizontal cuts through the web may be determined in the manner explained in the preceding articles. Moreover, as the resistance to the vertical shear V in thin-walled I beams is developed mainly in the web, it is so shown in Fig. 7-17(d). The sense of the shearing stresses and shear flows in the web coincides with the direction of the shear V. Note that the vertical shear flow "splits" upon reaching the lower flange. This is represented in Fig. 7-17(d) by the two forces F_1, which are the result of the horizontal shear flows in the flanges.

The shearing forces which act at a section of an I beam are shown in Fig. 7-17(d), and, for equilibrium, the applied vertical forces must act through the centroid of the cross-sectional area to be coincident with V. If the forces are so applied, no torsion of the member will occur. This is true for all sections having cross-sectional areas with an axis of symmetry. Thus, to avoid torsion of such members, the applied forces must act in the plane of symmetry of the cross section and the axis of the beam. A beam with an unsymmetrical section will be discussed next.

תוכן מומחים 2 - שלון עיצוב כיתה מס 2

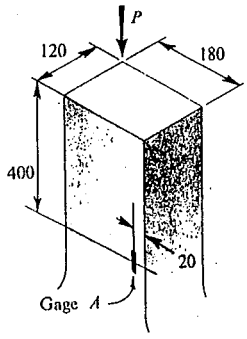
10-50. Plot shear, moment, and axial force diagrams for the jib-crane loaded as shown in the figure. Neglect the weight of the beam. *Ans:* 128.6 k-in.



PROB. 10 – 50

8-29

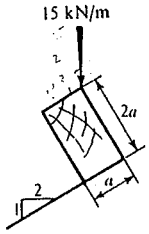
7-35. An aluminum-alloy block is loaded as shown in the figure. The application of this load produces a tensile strain of 500×10^{-6} mm. per mm at A as measured by means of an electrical strain gage. Compute the magnitude of the applied force P. Let $E = 10 \times 10^6$ kN/m². All dimensions given in the figure are in mm.



PROB. 7 – 35

8-19 (N>

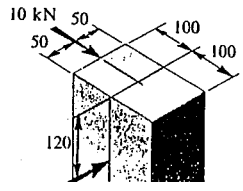
7-44. A tilted, simply supported beam with a depth to width ratio of 2 to 1 is to span 4 m and is to carry a uniformly distributed load of 15 kN per linear meter, including its own weight, applied as shown in the figure. (a) Determine the required dimensions of the beam so that the maximum stress due to bending does not exceed 10 MN/m². (b) Locate the neutral axis of the beam and show its position on the sketch.



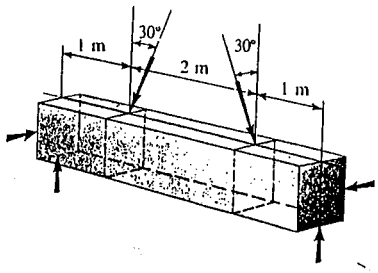
PROB. 7 – 44

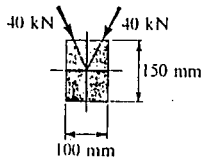
8-33

7-34. A cast iron block is loaded as shown in the figure. Neglecting the weight of the block, determine the stresses acting normal to a section taken 0.5 m below the top and locate the line of zero stress. All dimensions given in the figure are in mm. *Ans:* ±24.6 MPa, ±9.6 MPa.



10-91. A 4 m long beam is loaded as shown in the figure. The two applied forces act perpendicularly to the long axis of the beam and are inclined 30° with the vertical. If these forces act through the centroid of the cross-sectional area find the location and magnitude of the maximum bending stress. Neglect the weight of the beam. *Ans:* ±132 MN/m².

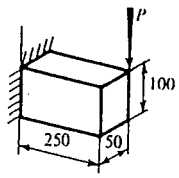




PROB. 10 – 91

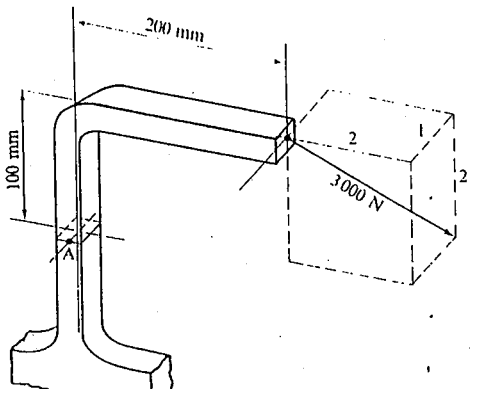
8-28

7-45. A rectangular cantilever 250 mm long is loaded with $P = 50$ kN at the free end as shown in the figure. Determine the maximum shearing stress at the built-in end due to the direct shear and the torque. Show the result on a sketch analogous to Fig. 7-17(e). All dimensions shown in the figure are in mm. *Ans:* 35.3 MPa.



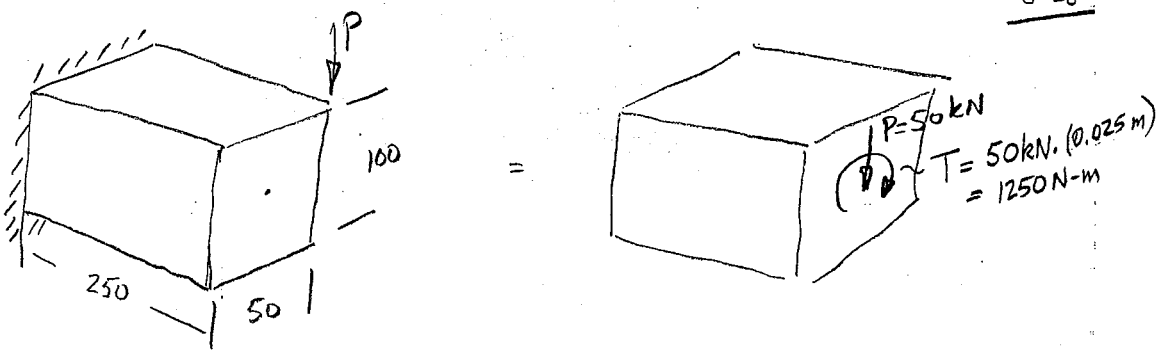
PROB. 7 – 45

9-26. A bent rectangular bar is subjected to an inclined force of 3 000 N as shown in the figure. The cross-section of the bar is 12 mm by 12 mm. (a) Determine the state of stress at point A caused by the applied force and show the results on an element. (b) Find the maximum principal stress. *Ans:* (a) +333 MPa, 578 MPa

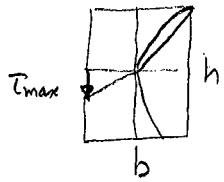


קורה רחבה מצד אחד באורך 250mm עומסה בכוח P=50kN בקצה חופשי כמתואר בציור. מצאו את המאמץ הגזירה המקסימלי בקצה המקובע מבין בזה הגזירה הישר והפיתול. הרשמו את התוצאה בציור. כל המידות ב מ"מ.

8-28

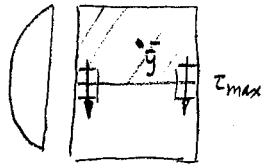


h/b	1.00	1.50	2.00	3.00	∞
α	0.208	0.231	0.246	0.267	0.333



$\frac{h}{b} = \frac{100}{50} = 2 \qquad \alpha = 0.246$

$$\tau_{max}\text{ פיתול} = \frac{T}{\alpha h b^2} = \frac{1250\ N\text{-}m}{0.246(0.10)(0.05)^2} = 20.3252\ MPa$$



$$\tau_{max} = \frac{VQ}{It} = \frac{V}{\frac{bh^3}{12}\cdot b}\cdot\frac{bh^2}{8}$$

$$Q = \int y\,dA = \bar{y}\cdot A = \frac{h}{4}\cdot\frac{hb}{2} = \frac{h^2 b}{8}$$

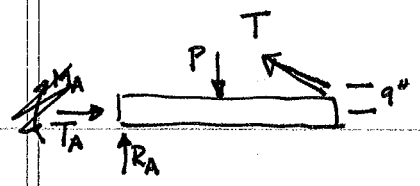
$$\tau_{max}\text{ גזירה} = \frac{3V}{2A} = \frac{3\cdot 50000N}{2(0.10)(0.05)} = 15\ MPa$$

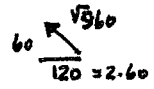
$$\tau_{max} = 20.33\ MPa + 15\ MPa = 35.33\ MPa$$

$$\sigma_{max} = +\frac{Pl\cdot y}{I_{zz}} = \frac{50000\cdot(0.25)(0.050)}{4.167\times10^{-6}} = 150\ MPa \qquad I_{zz} = \frac{bh^3}{12} = \frac{(0.05)(0.1)^3}{12} = 4.166\times10^{-6}\ m^4$$

$\sigma_{min} = 0 \qquad 0 = y$ ←

10.50



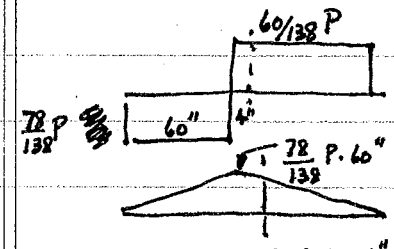


$\Sigma M_A = P \cdot 60 + T \cdot \frac{1}{\sqrt{5}} \cdot 120 + T \cdot \frac{2}{\sqrt{5}} \cdot 9 = \frac{138}{\sqrt{5}} T - P \cdot 60 = 0 \quad T = P \cdot \frac{60}{138} \sqrt{5}$

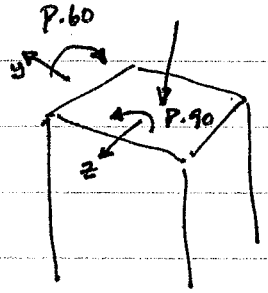
$T = P \cdot 60 \cdot \frac{\sqrt{5}}{120} = P \cdot \frac{\sqrt{5}}{2}$

$\Sigma F_x = T_A - T \cdot \frac{2}{\sqrt{5}} = 0 \quad T_A = T \cdot \frac{2}{\sqrt{5}} = P \cdot \frac{60}{138}\sqrt{5} \cdot \frac{2}{\sqrt{5}} = P \cdot \frac{120}{138}$

$\Sigma F_y = R_A - P + T \cdot \frac{1}{\sqrt{5}} = 0 \quad R_A = P - T \cdot \frac{1}{\sqrt{5}} = P - P \cdot \frac{60}{138} \frac{\sqrt{5}}{\sqrt{5}} = \frac{78}{138} P$



@ A-A = 4" for center 128.6 k-in



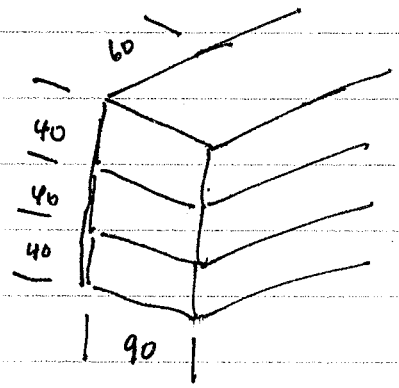
$\sigma_x = -P/A + \frac{P \cdot 90\, y}{I_z} - \frac{P \cdot 60\, z}{I_y}$

$I_z = \frac{bh^3}{12} = \frac{120 \cdot 180^3}{12}$

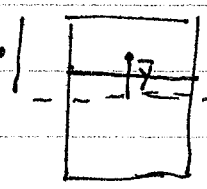
$I_y = \frac{hb^3}{12} = \frac{180 \cdot 120^3}{12}$

@ A $\quad \sigma_x = -\frac{P}{A} + \frac{P \cdot 90}{I_z}(-70) - \frac{P \cdot 60\,(60)}{I_z}$

$\frac{\sigma_x}{E} = \epsilon_x = 300 \times 10^{-6}$ mm

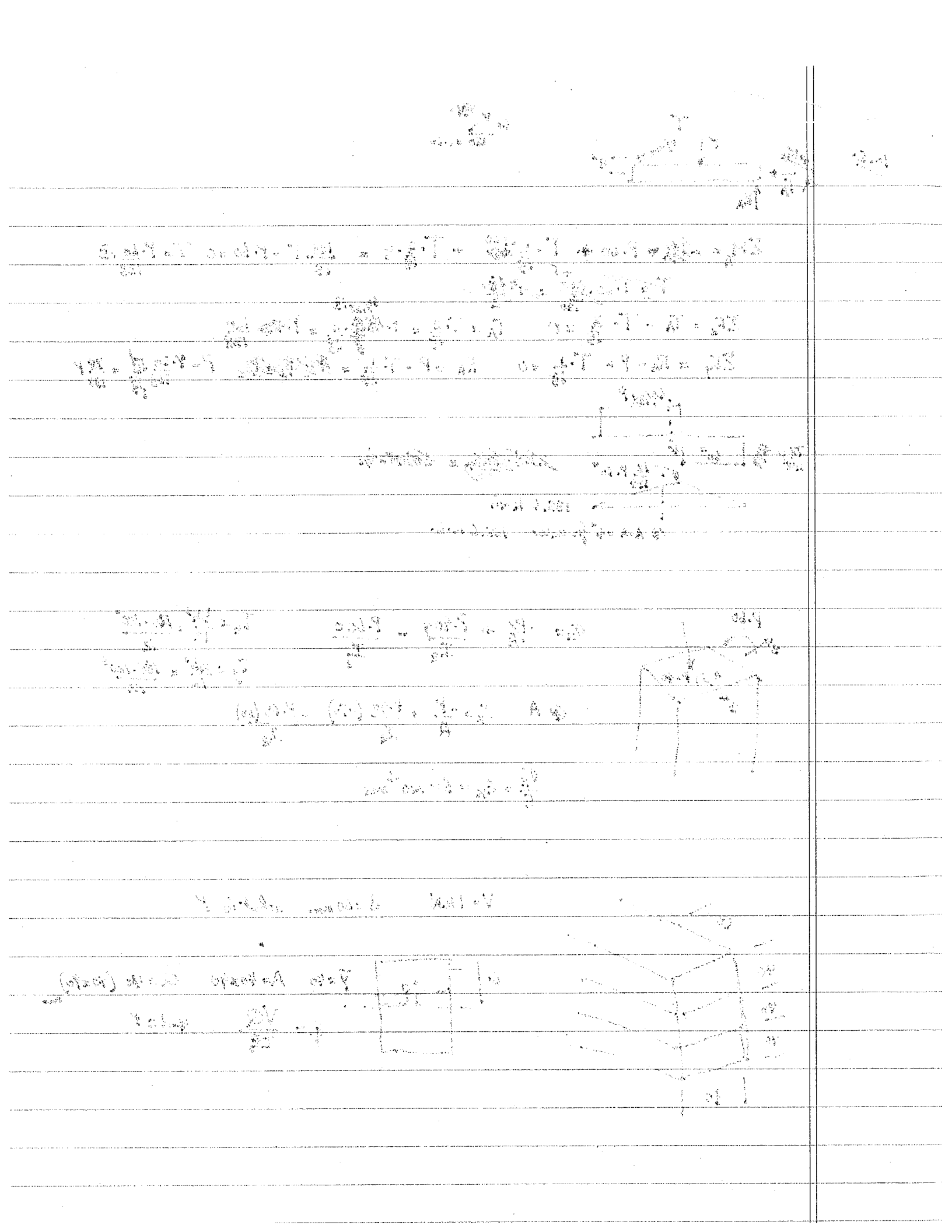


V = 1 kN $\quad$ d = 60 mm $\quad$ what is P



$\bar{y} = 40 \quad A = 40 \times 90 \quad Q = 40\,(40 \times 90)$ mm

$q = \frac{VQ}{I}$ $\quad q \cdot d = P$



חתכים בלתי הומוגניים (NON HOMOGENEOUS CROSS-SECTIONS)

עד עתה דיברנו על חומר שהוא הומוגני, מה קורה כאשר גוף מורכב מכמה חומרים שונים זה מזה?

דוגמה מהנדסה אזרחית: קורת בטון מזוין המחוזק במוטות ברזל

- נניח שיש בחתך מוביל מעל חומרים שונים זה מזה.
- שלם הקורה בפועל, הנחנה היסודית הבסיסית ... שאיזור נשאר אחרי הכפיפה.
- העיבור משתנה כפי המרחק מהציר הנייטרלי.
- מאחר להיות משור אחרי הכפיפה.
- במישור שני החומרים פוגש אחד לשני, העיבור הוא אותו דבר, אבל המאמצים שונים מפני ש- $E_1 \neq E_2$
- נניח שרוצים לבנות חתך מחומר אחד בלי שינוי למידות חלק ① ובלי שינוי לצורה לחלק ②.
- נניח ש- $n = E_2/E_1$ ו- n פעמים המאמץ.
- מכיוון שחומר ② נוכל הוא $\int_{-y_1}^{y_2} \sigma_x \cdot dy \cdot b_2 = \int_{-y_1}^{y_2} E_2 \varepsilon_2 \cdot b_2 \cdot dy = \int_{-y_1}^{y_2} E_1 \varepsilon \cdot b \, dy$. לכן חומר ① צריך לעטוף אותו עומס.
- מפני שהעיבור אותו דבר בחלק החדש

$$E_1 b = E_2 b_2$$
$$b = \frac{E_2}{E_1} b_2 = n b_2$$

- החתך החדש נקרא החתך האקוויוולנטי (Equivalent Section)
- עכשיו יכולים להשתמש $\sigma = \frac{Mc}{I}$ בחלק הלא נשתנה. בחלק החדש $\sigma = \frac{Mc}{I} = \frac{nMc}{I}$
- להתלהלך חומר ① לחומר ②, b ישתנה ל- $b = b_1 \frac{E_1}{E_2} = b_1 \frac{n}{1}$

באותו מקום תקבלו כפי הנמוך אותו דבר.

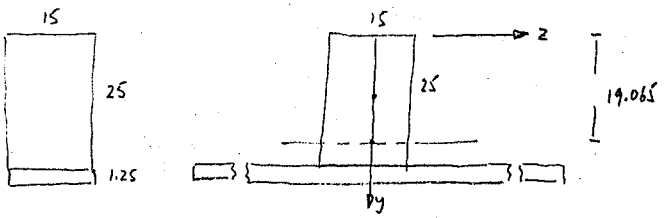
- מפני שהכוח שווה בקורה המקורית והקורה האקוויוולנטי, והכוח השלמנלי נשאר

דוגמה

קורה תיבה מונחת כפיפה עם $M = 27{,}115\ \text{N-m}$ מסביב הציר האופקי החלק התחתון הוא מפלדה שהמודול Young $E_s = 2.0686 \times 10^{11}\ \text{Pa}$
החלק העליון הוא מעץ שהמודול Young $E_w = 1.0343 \times 10^{10}\ \text{Pa}$
קורה מורכבת. מפני חומרים יש חתך השלם כשיטרוק

פתרון

1. לחשב את המיקום של מרכז הכובד
2. לחשב את מומנט האינרציה
3. לחשב את המאמצים



1. מתלים את הפלדה לתוך

$E_s/E_w = 20$

$E_s b_s = E_w \cdot b_w$

לכן מידת החלק המבוקש $= 1.25 \text{ cm}$

300 cm

	$\bar{y}$	A	$\bar{y}A$
①	12.5	375	4687.5
②	25.63	375	9611.25
		750	14298.75

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = 19.065 \text{ cm}$$

$$I_{tt} = I_{zz} + A d^2$$

	I_{zz}	A	d^2	I_{tt}
①	$\frac{1}{12}(15)(25)^3$	375	$(19.065-12.5)^2$	$19531.25 + 16162.21 = 35693.46$
②	$\frac{1}{12}(300)(1.25)^2$	375	$(19.065-25.63)^2$	$48.83 + 16137.6 = 16186.43$

$$51879.89 \text{ cm}^4$$

• המאמץ המקסימלי בקרן הוא

$$\frac{27115 \cdot 19.065 \cdot 100}{51879.89 \times 10^{-8}} = \frac{Mc}{I} = \sigma_{W_{max}}$$

$19.065 \times 10^{-2} \text{ m}$

$$996.431 \text{ N/cm}^2 =$$

$$9964313 \text{ N/m}^2 = \sigma_{W_{max}}$$

• המאמץ המקסימלי בפלדה הוא
מתרכז בקרן הקלה ונקרן כפול n

$$n \sigma_W = (\sigma_s)_{max}$$

$$\frac{20 \cdot 27115 \cdot 100 \cdot 7.185}{51879.89} = (\sigma_s)_{max}$$

$$7510.47 \text{ N/cm}^2 =$$

$$75104737 \text{ N/m}^2 = \sigma_{s_{max}}$$

במישור המגע בין חומרים (interface)

$$310.19 \frac{N}{cm^2} = \frac{27115 \cdot 5.935 \cdot 100}{51879.89} = \frac{My}{I} = \sigma_W$$

$$6203.85 \frac{N}{cm^2} = 20 \cdot \frac{27115 \cdot 5.935 \cdot 100}{51879.89} = \sigma_s$$

$$\sigma_s = 20 \sigma_W$$

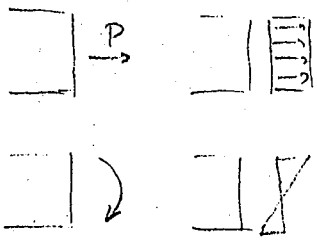
$$3.x10^{-4} = \frac{3101900}{1.0343 \times 10^{10}} = \sigma_W / E_W = \epsilon_W \qquad \sigma_s / E_s = \epsilon_s = \frac{62038500}{2.0686 \times 10^{11}} = 3.x10^{-4}$$

$$\epsilon_W = \epsilon_s$$

תסבולת אלסטית של חתכים (ELASTIC LOAD CARRYING CAPACITY OF SECTIONS)

כדי למצוא את התסבולת האלסטית של חתכים צריך לדעת את כל הגורמים הפועלים על שטח-החתך. (TO KNOW LOAD CARRYING CAPACITY, NEED TO KNOW LOADS ACTING ON CROSS-SECTION)

אבל, יכולים למצוא את המאמצים דרך המשוואות הרגילות. כאשר אומרת ש-לגורם סיבי המאמץ הוא



$$\sigma_x = P/A$$

לגורם שגורם כפיפה

$$\sigma_x = -\frac{My}{I}$$

ביחד שניהם נותנים הגורמים שפועלים על שטח-החתק. אנחנו יכולים לתכנן (DESIGN) את שטח-החתק לפי המאמץ המקסימלי.

The two give the load working on the cross-section

what if axial load only $\sigma_x = P/A$? מה המאמץ המקסימלי? נניח שרק הגורם הסיבי פועל על הקורה.

where will it start? ואיפה הוא פועל? יכול מקום בשטח-החתק. לכן עד כמה המאמץ מגיע ל-מאמץ

lower down define for ductile בהנחה, הקורה פועלת כ-קורה אלסטית ו- $\sigma_{x_{max}} = \sigma_{yp} = P/A$

- לכן, אם יש שטח-החתק מסוים שאנחנו צריכים להשתמש בו, יכולים למצוא את הגורם המקסימלי: $P = \sigma_{yp} A$

To find max load for given A

- אם יש גורם מסוים שאנחנו צריכים לתמוך, יכולים לחשב את שטח-החתק: $A = P/\sigma_{yp}$

To find cross-section to given load.

בדרך כלל, למי רוצים שהמאמץ מגיע ל-σ_{yp}. אנחנו מגדירים (DEFINE) מאמץ מסוים

Defining safety factor

$$\sigma_{allow} = f\,\sigma_{yp}$$

σ_{allow} - מאמץ המותר

f - מספר הגדרת בחות מאחד שמגדיר את המאמץ המקסימלי להשגה בתחום האלסטי - המקדם בטיחות (SAFETY FACTOR)

לכן: צריכים להחליף σ_{yp} בכול המשוואות ב-σ_{allow}

replace all σ_{yp} by σ_{allow}

$$P_{max} = f\sigma_{yp} A = \sigma_{allow} \cdot A$$

$$A_{\text{תכנון}} = P/f\sigma_{yp} = P/\sigma_{allow}$$

for bending

עם קורות כפיפות צריך לעשות את החישוב והתכנון הזה

נניח ויש רק גורם שקורת כפיפה. מהו המאמץ המקסימלי ואיפה הוא פועל

c - כאן הוא מרחק הסיבי המקסימלי לציר הנטרלי - התיק

- כאשר המאמץ המקסימלי מגיע למאמץ הכניעה, σ_{yp}, הקורה עדיין בתחום אלסטי ומתחילה להתעוות (BECOMES DUCTILE)

$$\sigma_{max} = \sigma_x = \frac{Mc}{I} = \frac{M}{S} = \sigma_{yp}$$

SECTION MODULUS

$S = I/c$ - מודול החתך

- לכן אם יש שטח-חתך מסוים שאנחנו צריכים להשתמש בו, S יפה לנו ויכולים למצוא את מומנט הכפיפה המקסימלי

$$M_{max} = S\sigma_{yp}$$

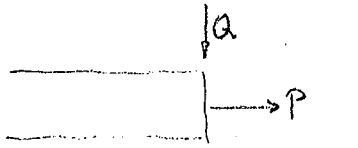
והעומס שגורם ל- M_{max} הוא $Q = M_{max}/l$

- אם יש מומנט מסוים, אפשר למצוא את ה- מודול החתך $S = \frac{M}{\sigma_{yp}}$

- כמו שציינו, לא משתמשים בדרך כלל, במאמץ הכניעה אלא במאמץ המותר σ_{allow}

לכן $M_{max} = S\cdot\sigma_{yp} = S\cdot\sigma_{allow}$ או $Q_{כפיפה\ מקס} = \frac{S\cdot\sigma_{allow}}{l}$

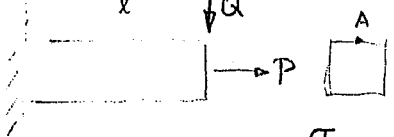
$S_{נדרש} = \sigma_{yp} f/M = \sigma_{allow}/M$ או $S_{נדרש} = \frac{l\cdot Q_{כפיפה}}{\sigma_{allow}}$



נניח ש- $Q + P$ פועלים יחד

לכן $\sigma_{x\ max} = P/A \pm \frac{Mc}{I} = P/A \pm \frac{Qlc}{I}$

כאשר $\sigma_{x\ max} = \sigma_{yp}$ הגענו לקריטריון הראשוני.



דוגמא: קורה בשטח חתך מלבני

$$\sigma_{yp} = \frac{6Ql}{h\cdot A} + \frac{P}{A} = \frac{Ql\cdot h/2}{A\cdot h^2/12} + \frac{P}{A} = \frac{Mc}{I} + \frac{P}{A} = \sigma_{x\ max}$$

ר.ב. - יש חומרים שבירים (BRITTLE) שלא סובלים מאיבוד זרוע בכלל שינוי קטן במאמץ. לכן התעוותם או נכשלם כאשר המאמץ מגיע למאמץ המתמתי (ULTIMATE STRESS) המתמתי, משתמשים בחומרים השבירים

לכן -

$\sigma_x = \sigma_{UTS} = P/A$ מקס'

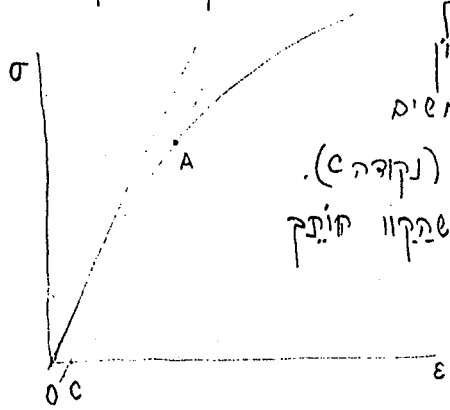
σ_{UTS}

או

$\sigma_x = \sigma_{UTS} = \frac{Mc}{I}$ מקס'

σ_{UTS}

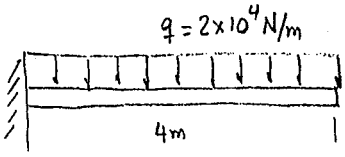
צריך להבהיר שמומר שלפנינו וכול להיות לפי ניסיוני וכאן צריך למצוא את הפיזור שנמצא ב-נקודה 0. חומר פרוץ ומומרים אחרים הולכים אחרי הדוגמא הנשאת, כמו בטון כדי למצוא את המאמץ הכניעה לחומרים הללה משתמשים בשיטת היסוט (OFFSET METHOD) שהוא 0.2% לימין הנקודה 0 (נקודה c). משרטטין קו לינארי מקביל לפיזור שנמצא מנקודה 0. הנקודה שהקו חותך את הברך (y) קורטום לנו את נקודת הכניעה.



אלף על פי שהכנתי את הקורה לפי התאוגן בהצלחה ובמתח ובלחץ, לפי הבחנתי את הקורה לפי התאוגן הפך הצורה, על פי נוסן בעיה.

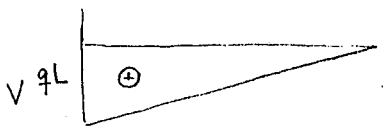
נדבר על שני מדדים שאנו צריכים לפתור בעיית כפיפה קורה.

① **שלילה**: למצוא הערום המקסימלי שאוכל לפגוע על הקורה עבור קורה נתונה ומומר נתון.

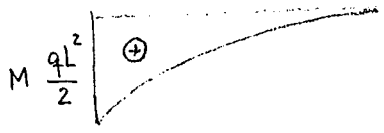




$\sigma_{allow} = \pm 1.2 \times 10^8 Pa$



$$M_{max} = \frac{qL^2}{2} = \frac{2\times10^4(4)^2}{2} = 16\times10^4\ N\text{-}m$$



$$I = \frac{bh^3}{12} \quad c = \frac{h}{2} \quad S = I/c = \frac{bh^3/12}{h/2} = \frac{bh^2}{6} = \frac{10\times(20)^2}{6} = 6.67\times10^2\ cm^3 = 6.67\times10^{-4}\ m^3$$

$$\sigma_{max} = \frac{M}{S} = \frac{16\times10^4\ N\text{-}m}{6.67\times10^{-4}\ m^3} = 2.4\times10^8\ N/m^2 > \sigma_{allow}$$

לכן הקורה תתמוגן !

$S = \frac{M}{\sigma} = \frac{16\times10^4}{12\times10^7} = 1.33\times10^{-3}\ m^3 = 1.33\times10^6\ mm^3$

$S \geq 1333\times10^3\ mm^3$

תרגיל: לאיזו עומס מקסימלי, שאותו ספרוץ על קורה שאורך שלה נתונים.
והשלו מטרייה שלה נתונים.

$$M_{max} = \frac{qL^2}{2} = \sigma_{allow} \cdot S$$

$$q_{allow} = \frac{2\sigma_{allow} \cdot S}{L^2} = \frac{2(1.2\times10^8)(6.67\times10^{-4})}{4^2} = 1.0\times10^4 N/m$$

(העומס המפורס distributed load)

$$q_{allow} = q/2$$

שהקורה לפי תכנון, העומס המפורס בריק לפיות ½ מ-מה שנמצא.
לכן רואים שהתכנון כזה יותר מאה שאותה והקורה תתמוגג. כלי

תיכון: לקדוח הטכנולוגיה על הקורה לפי הנתונים - העומס וכך המותר.

מהו q על הקורה שאפשר שיופיע העומס הנתון

$$S_{allow} = \frac{M_{max}}{\sigma_{allow}} = \frac{b(2b)^2}{6} = \frac{4}{6}b^3$$

לכן

$$b = \sqrt[3]{\frac{6}{4}\frac{M_{max}}{\sigma_{allow}}}$$

$$\sqrt[3]{\frac{3}{2}\cdot\frac{qL^2}{2\sigma_{allow}}} = \sqrt[3]{\frac{0.75\times2\times10^4\cdot4^2}{1.2\times10^8}} = 12.6\ cm$$

פתרון ב'

$c_1 = 8.72cm$
$c_2 = 3.28\ cm$
$I_{zz} = 366\ cm^4$

נתונים המאמצים המותרים
אין הפרופיל זו ⊥
נתון: הקורה הרשו"ה

σ_{allow} מתח $= 10^8 N/m^2 = 100\ MPa$

σ_{allow} לחץ $= 1.8\times10^8 N/m^2 = 180\ MPa$

$$\text{לחץ}\left(\sigma_{max}\right) = \frac{Mc_1}{I_{zz}} = \frac{5600\times8.72\times10^{-2}\ m}{366\times10^{-8}\ m^4} =$$

$$= 1.334\times10^{+8} Pa$$

$$\text{מתח}\left(\sigma_{max}\right) = \frac{Mc_2}{I_{zz}} = \frac{5600\times3.28\times10^{-2}\ m}{366\times10^{-8}\ m^4} =$$

תשב התקבולת הפרופיל ⊥ עם הנתונים

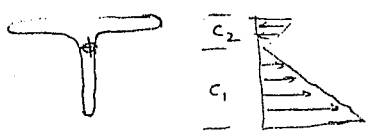
$$(M_{allow})_{\text{לחץ}} = \frac{(\sigma_{allow})_{\text{לחץ}} \cdot I_{zz}}{c_1} = \frac{(1.8\times10^8)(366\times10^{-8})}{8.72\times10^{-2}} = 7555\ Nm$$

$$(M_{allow})_{\text{מתח}} = \frac{(\sigma_{allow})_{\text{מתח}} I_{zz}}{c_2} = \frac{10^8\times(366\times10^{-8})}{3.28\times10^{-2}} = 11,159\ Nm$$

ומה יהיה אם הכיוון הפרופיל?

$$(M_{allow}) = \frac{(\sigma_{allow})_{\text{לחץ}} \cdot I_{zz}}{c_2} = \frac{(1.8\times10^8)(366\times10^{-8})}{3.28\times10^{-2}} = 20,085\ Nm$$

$$(M_{allow}) = \frac{(\sigma_{allow})_{\text{מתח}} I_{zz}}{c_1} = \frac{(10^8)\times(366\times10^{-8})}{8.72\times10^{-2}} = 4,197\ Nm$$



בדוגמא הראשונה: $\frac{P_{allow}L}{4} = M_{allow} = 7555\ N\text{-}m$

והכוח שנוכל למרוח הכי הרבה $P_{allow} = \frac{4M}{L} = 15,110\ N$

בדוגמא השניה: $\frac{P_{allow}L}{4} = M_{allow} = 4,197\ Nm$

והכוח הזה

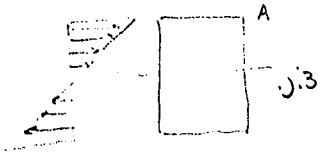
$$P_{allow} = 4M/L = 8,394\ N$$

לכן, הפרופיל ⊥ יכול לסבול כוח יותר גדול מהפרופיל על ⊥

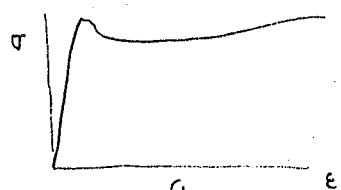
התנהגות פלסטית של תקבים.

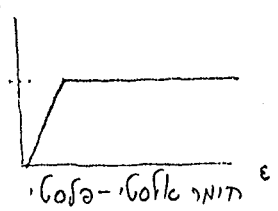
נאמרנו ש-שלת התקף מתרחש לראשונה כאשר המאמץ המקסימלי, מגיע למאמץ הכניעה: $\sigma_{max} = \frac{Mc}{I} = \sigma_{yp}$

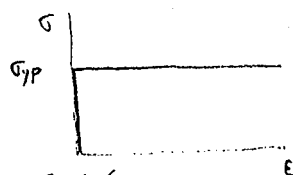
בדוגמן ההוא, המאמצים בשלת-התקף עדיין לינאריים



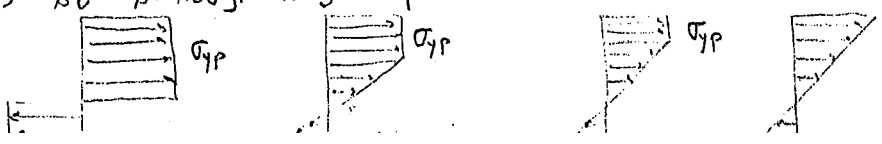
תלוי בהגדרה תיאורטית:







בנית שמשתמשים בתיאוריה אלסטי-פלסטי. לכן, כאשר המאמץ מגיע למאמץ הכניעה המאמץ נשאר בשלב הזה. וכאשר העומס ממשיך לגדול, המאמצים מתחת המיתר A ממשיכים לגדול עד שמגיעים למאמץ הכניעה ונשארים שם כאשר מגדילים את העומס.



תיאוריות הנזק

יש כמה אפשרויות שנכון להתמודד עם בעיה זו. נתאר כמה תיאוריות שנכון להתמודד:

1. התמוטטות קשה בלי שינוי צורה משמעותי או אפילו שאנחנו מתייחסים לחומר את צורתו
   - שבר פריך (BRITTLE) שאין תגובה פלסטית (BRITTLE FRACTURE – NO PLASTICITY)
   - פריכות בחלק השטח-החתך של השבר הוא מישור עמוד. σ ⟵ ▭
   - לפריכות בחתך השטח-החתך של השבר הוא אלכסוני לעומת. σ ⟵ ▱
   - וכולי להתגבר מתחת לעומס מתמיד ומצטבר.

2. התמוטטות יכולה לקרות בגלל בעיית היציבות כאשר שטח החלק הולך ומצטמצם את התנגדות שלו יותר ממה שמתחזק. למשל קטן במאמץ יש שינוי גדול לעומר YIELD

3. התמוטטות יכולה לקרות בגלל קריסה. המוט נכון להתכופף בצורה גדולה בכיוון הניצב לעומס הצירי. σ ⟵ ▭ σ ⟵ BUCKLING

4. בגלל טמפרטורות גבוהות יכול להיות פגיעה. (CREEP) התנגדות יכולות להשתנות אך שחלק קרוב לו כמו במנוע (בוכנה, טבעות בוכנה) PISTON

נראה בדיוק בדקת ומה לקחת מתי חלק נכשל

נדון בשיטות שונות על מנת TRESCA, VON MISES, קריטריון המאמץ השני מקסימלי, קריטריון MOHR, קריטריון מקסימלי, ועוד. אבל יש הרבה תיאוריות בשוק. NORRIS

1. קריטריון TRESCA: נניח יהיה כי - התיאוריה על המאמץ הצירה המקסימלי, ותיאוריית COULOMB
   התנגדות מתחילה כאשר שהמאמץ הצירה המקסימלי מגיע לערך מסוים
   התיאוריה הטובה ביותר מפני שיש תוצרים מתמתים (מחומים) SOFT MAT ← $\sigma_1 = \sigma_y$
   שבחלקם הולך דרך מישור שהוא 45 מעלות לציר של עומס מתיח. מישורים בשבירה (FAILURE ON 45° PLANES TO TENSILE LOAD)
   מקסימלית הם 45 מעלות למישור העומס שנסיון תחילי ביריד $\sigma_1 = \sigma_y$ ההתנגדות המתוכננת $\tau_{max} = \frac{\sigma_y}{2} = \frac{\sigma_1 - \sigma_2}{2}$

2. קריטריון VON MISES: נניח יהיה כי - התיאוריה על MAXWELL. מיכוסס בירעיון אנרגיה.
   התנגדות מתחילה כאשר המאמץ האפקטיבי (EFFECTIVE) מגיע לערך מסוים σ_e
   הציר המרוחק על המאמצים.
   הציר הנמצאת על מישור שהוא זוויות של 45° לכול σ_e - (OCTOHEDRAL SHEAR)
   שבו יש ואם לשבירה אוקטהדרלי
   $\sigma_e = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$ τ_{oct} σ_3 σ_2

הבנייה מתקיימת.
אז"ה $\tau_{max} < \tau_{oct}$, ויהיו מתאימים את המבחן הכללי על המאמצים כאשר

$$\sigma_{eq} = \sqrt{\frac{1}{2}\left((\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)\right)} = \sigma_e \qquad \text{שגיאת התאוריה} \quad \frac{1}{12G}\sigma_1^2 = \frac{2\sigma_y^2}{12G}$$

הציר המשווים על המאמצים.
הציר הנמצאת על מישור שזוית זוויות של 45° לכל
שבו יש ואם לציר אוקטהדרלי (OCTOHEDRAL SHEAR) ל- σ_e
תתאוששת מתחילה כאשר המאמץ האפקטיבי (EFFECTIVE) מגיע לערך מסוים, σ_e

2. קריטריון VON MISES: בא ידוע כ- התיאוריה של MAXWELL. מיכאוס בריטון אנרגיה.
ניסיון הפ-ברי $\sigma_1 = \sigma_y$ בהתאלת התאוששת מישר הזווית 45 מעלות $\tau_{max} = \frac{\sigma_1-\sigma_3}{2} = \frac{\sigma_y}{2}$
מקסימלית היא
שבשלונע תוכל דרך מטור שהוא 45 מעלות לציר של עוות מתיח. מיוחים לציר:
(FAILURE ON 45° PLANES TO TENSILE LOAD)

התיאוריה הטובה ביותר מפני שיש חומרים מתכתיים (מיוחים) SOFT MAT
תתאוששת מתחילה כאשר שהמאמץ הציר המקסימלי מגיע לערך מסוים
1. קריטריון TRESCA: בא ידוע כ- התיאוריה על המאמץ הציר המקסימלי, ותיאוריית COULOMB

עיבור השני מקסימלי, קריטריון MOHR, קריטריון NORRIS. אבל יש הרבה תיאוריות בשימוש.
נדון בשיטות שונות אך שם TRESCA, VON MISES, קריטריון המאמץ השני מקסימלי, קריטריון
לבן בריק לבדק ומם לקבור אתי חוק נכשל.

להשתנות אך שהחוק מגיע לחוק קרוב לו כמו במנוע (בוכנה, טבעות בוכנה) PISTON
4. בשלל מטכ"טורות בעבודות בכוך להיות בעיה. (CREEP) המידות יכולות

BUCKLING
בכיוון המלבי לעומס הציר. P ← [ ] → P
3. התתמוטטות יכולה לקרות בשלל קריסה. המוט וכול להציג בתצוגה בעבודה

YIELD
ומאריך את המידות שלו יותר ממה שמקבל. לשינוי קטן במאמץ יש שינוי גדול לעומר
2. התתמוטטות יכולה לקרות בשלל בניגוד הגיבוי פעמי שלומרת שהחלק הולך

לפריכות בלתי השטח-חתך של השבר הוא משפך לעומס. P ← ⊏
לפריכות בשתי השטח-חתך של השבר הוא אנכי לעומס. P ← ⊏

- לשבר פריך (BRITTLE) אין תגובה פלסטית (BRITTLE FRACTURE – NO PLASTICITY)
- וכולים להשתבר מתחת לעומס מתמיד ומצטברות.)

1. התתמוטטות קורה בשל שמופיע סדקים או שסדקים שנמצאים מתחילים לפתח את גודלם
גודל שמביא שמי מתכנן כהלכה מתמוטט. יש כמה אפשרויות שגורם להתמוטטות:

3. קריטריון של המאמץ הראשי המקסימלי: כשטר המאמץ הראשי σ_1 מגיע לערך מסוים, מסמנים מישור שנוצר σ_1

$\longrightarrow$ $\{$ $\longrightarrow$ במקומות. רשלים בה מתוארים שְׁבִירָה (פריכים) BRITTLE שהמישור הפריק הוא

4. קריטריון מור: התמוטטות מתחילה כשאר עיגל מור הגדול ביותר $(\sigma_3-\sigma_1)$ סמוך לעקומת

התמוטטות. מור עובד טוב מאוד פְּרִיכִים. (LARGEST MOHR'S CIRCLE $(\sigma_1-\sigma_3)$ TOUCHES A FAILURE ENVELOPE)

לשרטט את העקומה התמוטטות - לעשות ניסויות-שלב בעומס גדול מתות והשני מעומס

גדול לתוך

קרוב לדרך הגבירה במישור מטופח מצוות על $45^\circ+\frac{\phi}{2}$ לכיוון של σ_3.

התמוטטות מתחילה <- $1 \leq \frac{\sigma_3}{|\sigma_c|} - \frac{\sigma_1}{\sigma_t}$. כש- הלחץ הגדל דומיננטי (DOMINANT), ההתמוטטות

5. במאמר אורתוטרופי (ORTHOTROPIC MATERIAL), בריכות או התנהגות ירושה קרוב כשאר

שהם מהמשוואות הללה מוסקת. כה הקריטריון של NORRIS

$$\frac{\sigma_x^2}{S_x^2} - \frac{\sigma_x\sigma_y}{S_x S_y} + \frac{\sigma_y^2}{S_y^2} + \frac{\tau_{xy}^2}{S_{xy}^2} \leq 1$$

$$\frac{\sigma_y^2}{S_y^2} - \frac{\sigma_y\sigma_z}{S_y S_z} + \frac{\sigma_z^2}{S_z^2} + \frac{\tau_{yz}^2}{S_{yz}^2} \leq 1$$

$$\frac{\sigma_z^2}{S_z^2} - \frac{\sigma_z\sigma_x}{S_z S_x} + \frac{\sigma_x^2}{S_x^2} + \frac{\tau_{zx}^2}{S_{zx}^2} \leq 1$$

כאן x,y,z הם הכיוונים הראשיים של התומר.

כאן $S_x, S_y, S_{xy}, S_{yz}, S_{zx}$ הם הערכים שמקבלים מניסויות מיוחדים.

S_x is DESIGNED FROM TENSILE TEST IF σ_x IS TENSILE

S_y " " " COMPRESSION TEST IF σ_x IS COMPRESSIVE

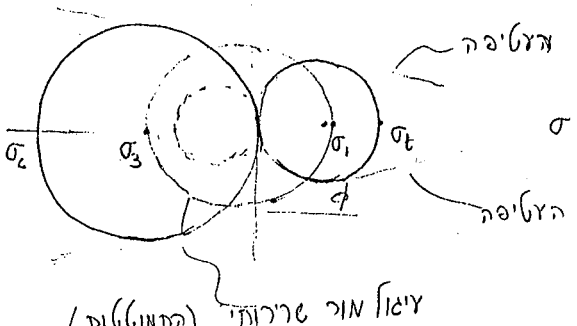
6. תיאוריה שמניחים לתומרים אִיזוֹטְרוֹפִיים (apply) בדרך כלל (SOFT) בדרך כלל לא עובדת בתומרים ברצים

7. היתוך מניחים לתשוריות הללו כאשר יש מצב מאמץ דו-ביריי, כי התשוריות

הללו עובדים, בדרך כלל, במצב מאמץ תלת-ביריי. LEAST MAJOR

מיתוחות. רשום כמה בחומרים שְׁבִירִים (פריכים) BRITTLE שהמישור הקריטי הוא
מישור שנוצר σ_1

4. קריטריון מור: התמוטטות מתחילה כאשר עיגול מור הגדול ביותר $(\sigma_1-\sigma_3)$ נוגע במעטפת
התמוטטות. מור עובד טוב מאוד בחומר שְׁבִירִי. (LARGEST MOHR'S CIRCLE $(\sigma_1-\sigma_3)$ TOUCHES A FAILURE ENVELOPE).
לשרטט את המעטפה מהתמוטטות - לעשות ניסויים - אחד ביאקס בירו מתות והשני מיאקס
בירו למחץ



התמוטטות מתחילה $\leftarrow$ $1 \leq \frac{\sigma_3}{|\sigma_c|} - \frac{\sigma_1}{\sigma_t}$. כש- הלחץ הוא דומיננטי (DOMINANT), ההתמוטטות
קורית דרך הצטרות במישור מאונך בערות על $45^\circ+\frac{\phi}{2}$ לכיוון של σ_3.

5. בחומר אורתוטרופי (ORTHOTROPIC MATERIAL), בריכות או התלת ניצה קורת כאשר
אחת מהאשוואות הבאה מספקת. כך הקריטריון של NORRIS

$$1 \leq \frac{\sigma_x^2}{S_x^2} - \frac{\sigma_x\sigma_y}{S_xS_y} + \frac{\sigma_y^2}{S_y^2} + \frac{\tau_{xy}^2}{S_{xy}^2}$$

$$1 \leq \frac{\sigma_y^2}{S_y^2} - \frac{\sigma_y\sigma_z}{S_yS_z} + \frac{\sigma_z^2}{S_z^2} + \frac{\tau_{yz}^2}{S_{yz}^2}$$

$$1 \leq \frac{\sigma_z^2}{S_z^2} - \frac{\sigma_z\sigma_x}{S_zS_x} + \frac{\sigma_x^2}{S_x^2} + \frac{\tau_{xz}^2}{S_{xz}^2}$$

כאן x,y,z הם הכיוונים הראשיים של החומר.
כאן $S_x, S_y, S_{xy}, S_{yz}, S_z$ הם הערכים שמקבלים מניסויות מיוחדים.
S_x is DESIGNED FROM TENSILE TEST IF σ_x IS TENSILE
S_y " " " COMPRESSION TEST IF σ_x IS COMPRESSIVE

6. תיאוריה שמיישמים (apply) לחומרים אִיזוֹטְרוֹפִיים (SOFT) בדרך כלל לא עובדת בחומרים פריכים

7. הקשר מיישמים לתיאוריות הללו כאשר יש מצב מאומץ דו-צירי, כי התיאוריות
הללו עובדות, בדרך כלל, במצב מאומץ חד-צירי. HOW TO USE FOR BIAXIAL WHEN MOST THEORIES ONLY WORK FOR UNIAXIAL STATES

8. צריך לבדוק את הערכים של σ_1 או τ_{max} או σ_3 לכול המצבים ובפרוק
מהובו המצב ההכי גרוע, אלא לכל היו תיאוריות התמוטטות IF NO FAILURE THEORY, NEED TO CHECK ALL STRESS STATES

מכאן רואים שפיזיקאי מהתאוריות נותנות לנו שיש התאוטות. לכן צריכים לתקן את הצורה ולתכנן (DESIGN) את התכונות בהצרה.

לפי TRESCA

לפי VON MISES

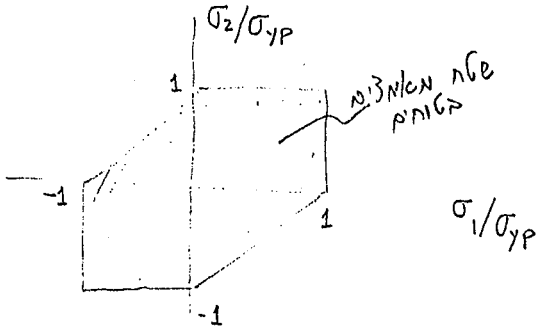
ר.ב.: מאותו הכיוון בלחץ ובמאותו אותו ערך

לפי MOHR

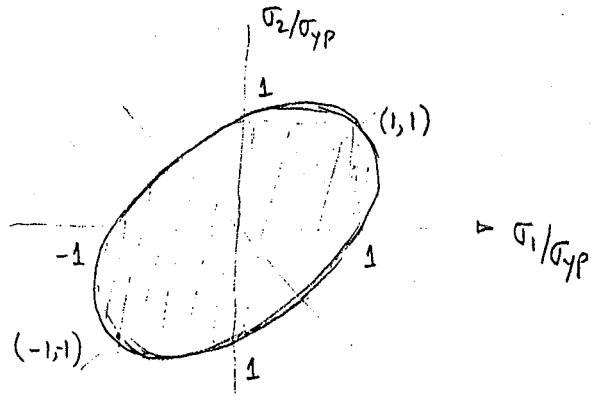
המאומץ המתאי- σ_{ult}

לפי המאומץ הראשי המקסימלי.

בתיאוריות פלסטיות מתמשים עם שני התיאוריות האלו - VON MISES או TRESCA קריטריון TRESCA כלול בשל VON MISES. תכונות מניסויים מסכימות יותר עם VON MISES.

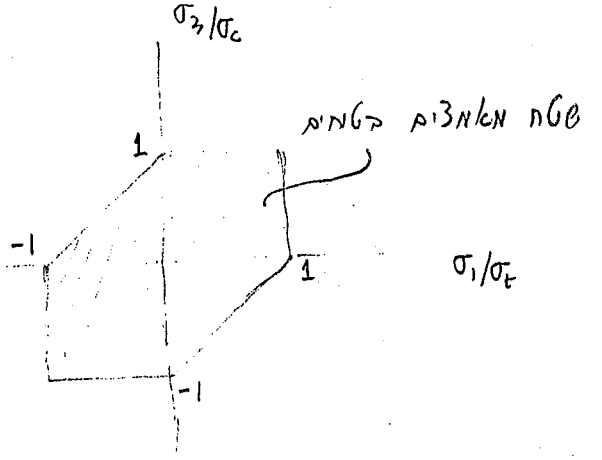


לפי TRESCA

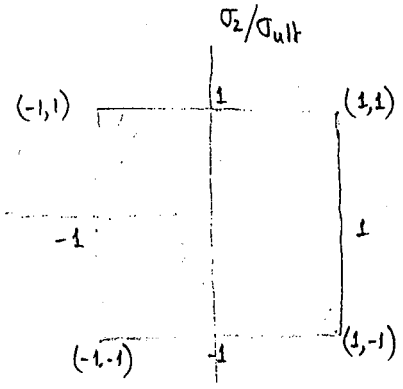


לפי VON MISES

ר.ק.: המאמץ הכניעה בלחץ ובמאמץ אותו ערך



לפי MOHR



המאמץ המתאי- σ_{ult}

לפי המאמץ הראשי המקסימלי.

קריטריון TRESCA כלול בשל VON MISES. תוצאות ניסויים מסכימות יותר עם VON MISES.
בתיאוריות פלסטיות נתמקד עם שני התיאוריות הללו - TRESCA או VON MISES.

Experimental data follow VON MISES. IN THEORY OF PLASTICITY, WE WILL CONCENTRATE ON THESE 2.

נתרכז בשניהם

- כי צה יקח הרבה כסף, והרבה זמן, אז אשאר למבוק את כל המבחים המאושרים

CANT CHECK ALL STRESS STATE
USE THEORY W/ UNIAXIAL TENSION TEST. ASSUME THAT FAILURE IN TENSION INDICATES FAILURE IN ALL OTHERS

לכן משתמשים בתיאוריות ואשים ניסויים במתח חד-צירי ומקבלים על יציאני שהתוצאות בכל שיטה אחרת מובא מהתוצאות במבחה מתוח חד-צירי

קריטריון - Mohr & Young

- TENSION & COMPRESSION TESTS OF A BRITTLE MATERIAL GIVE $\sigma_t = 14$ MPa & $\sigma_c = 120$ MPa
- UNDER LOADING CONDITION, A CERTAIN STATE OF STRESS EXISTS $\sigma_x = 0$, $\sigma_y = -18$ MPa, $\tau_{xy} = 20$ MPa

IS THIS STATE OF STRESS SAFE? CONSIDER PLANE STRESS

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2}$$

$$= \frac{0 - 18}{2} + \sqrt{\frac{(18)^2}{4} + (20)^2} = -9.0 + 21.9 = 12.9 \text{ MPa}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2} = -9.0 - 21.9 = -30.9 \text{ MPa}$$

$$\sigma_e = \frac{1}{\sqrt{2}}\left[(0+18)^2 + (-18-0)^2 + (0-0)^2 + 6\left(20^2 + 0^2 + 0^2\right)\right]^{1/2} = 39.04 \text{ MPa}$$

קריטריון המאמץ העיקרי המקסימלי

מכיוון ש- $(\sigma_1 < \sigma_t)$ המבנה במבחה טוב בקריטריון הזה.

MAX STRESS SHOWS SAFE

קריטריון TRESCA

$$12.9 > 0 > -30.9 \quad \Leftarrow \quad \sigma_1 > \sigma_2 > \sigma_3$$

TRESCA & VON MISES SHOW UNSAFE, BUT NORMALLY DON'T WORK FOR BRITTLE

$$\tau_{max} = \frac{12.9 + 30.9}{2} = 21.9 \text{ MPa} = \frac{\sigma_1 - \sigma_3}{2}$$

מכיוון ש- בניסוי המאמץ מתוח חד-צירי הוא $\sigma_t = 14$MPa

ו- τ_{max} שווה ל- $\sigma_{yp}/2 = \sigma_t/2 = 7$MPa

לפי TRESCA $\tau_{max} > \sigma_{yp}/2$ לכן המבנה במבחה לא טוב והתיל להתמוטט.

קריטריון VON MISES

מכיוון ש- $\sigma_e = 39.04$ ובניסוי המאמץ מתוח חד-צירי הוא $\sigma_t = \sigma_{yp}$

ו $(\sigma_{yp} < \sigma_e)$. לכן לפי VON MISES המבנה מתחיל להתמוטט.

קריטריון MOHR

USE THEORY W/ UNIAXIAL TENSION TEST. ASSUME THAT FAILURE IN TENSION INDICATES FAILURE IN ALL OTHERS

COOK & YOUNG - דוגמה

- TENSION & COMPRESSION TESTS OF A BRITTLE MATERIAL GIVE $\sigma_t = 14$ MPa & $\sigma_c = 120$ MPa
- UNDER LOADING CONDITION, A CERTAIN STATE OF STRESS EXISTS $\sigma_x = 0$, $\sigma_y = -18$ MPa, $\tau_{xy} = 20$ MPa

IS THIS STATE OF STRESS SAFE? CONSIDER PLANE STRESS

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2}$$

$$= \frac{0 - 18}{2} + \sqrt{\frac{(18)^2}{4} + (20)^2} = -9.0 + 21.9 = 12.9 \text{ MPa}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2} = -9.0 - 21.9 = -30.9 \text{ MPa}$$

$$\sigma_e = \frac{1}{\sqrt{2}}\left[(0+18)^2 + (-18-0)^2 + (0-0)^2 + 6\left(20^2 + 0^2 + 0^2\right)\right]^{1/2} = 39.04 \text{ MPa}$$

קריטריון המאמץ הראשי המקסימלי

MAX STRESS SHOWS SAFE

מכיוון ש- $(\sigma_1 < \sigma_t)$ התואר במצב טוב בקריטריון הזה.

קריטריון TRESCA

$$12.9 > 0 > -30.9 \quad \Leftarrow \quad \sigma_1 > \sigma_2 > \sigma_3$$

TRESCA & VON MISES SHOW UNSAFE, BUT NORMALLY DON'T WORK FOR BRITTLE

$$\tau_{max} = \frac{12.9 + 30.9}{2} = 21.9 \text{ MPa} = \frac{\sigma_1 - \sigma_3}{2}$$

מכיוון ש- $\sigma_t = 14$MPa המאמץ המותר בחומר הוא

τ_{max} - שווה ל- $\sigma_{t/2} = \sigma_{yp/2} = 7$MPa

לפי TRESCA $\tau_{max} > \sigma_{yp/2}$ לכן התואר במצב על טוב והתרסק להתמוטט.

קריטריון VON MISES

מכיוון ש- $\sigma_e = 39.04$ והקריטריון המאמץ המותר בחומר הוא $\sigma_t = \sigma_{yp}$

$(\sigma_{yp} < \sigma_e)$ - לכן לפי VON MISES התואר מתחיל להתמוטט.

קריטריון MOHR

במצב כזה התמוטטות $\frac{\sigma_1}{\sigma_t} - \frac{\sigma_3}{|\sigma_c|} \geq 1 \Leftarrow \frac{12.9}{14} - \frac{(-30.9)}{120} = 1.18$

לכן יש התמוטטות

תורת החוזק 1
תרגיל מס 9

5.16 and 5.17 For the wide-flange beam and loading shown, determine in a section located halfway between points D and E, (a) the largest normal stress, (b) the largest shearing stress.

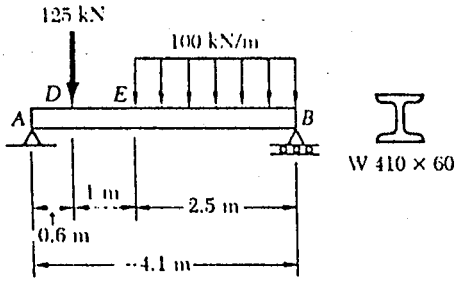


Fig. P5.16

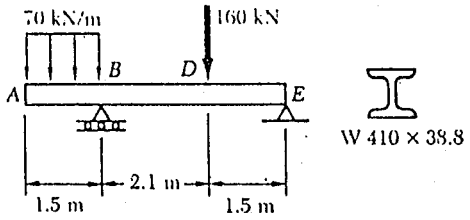


Fig. P5.17

5.18 Two rectangular plates are welded to the 310-mm-wide-flange beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 90 MPa.

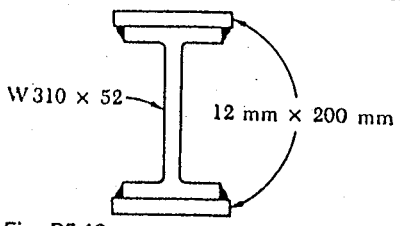
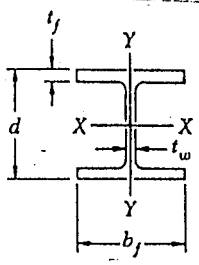


Fig. P5.18

Designation†	Area A, mm²	Depth d, mm	Flange Width b_f, mm	Flange Thickness t_f, mm	Web Thickness t_w, mm	Axis X-X I_x 10⁶ mm⁴	Axis X-X S_x 10³ mm³	Axis X-X r_x mm
W310 × 52	6650	317	167	13.2	7.6	118.6		
W410 × 114	14600	420	261	19.3	11.6	462	2200	177.8
85	10800	417	181	18.2	10.9	316	1516	170.7
60	7610	407	178	12.8	7.7	216	1061	168.4
46.1	5880	403	140	11.2	7.0	156.1	775	162.8
38.8	4950	399	140	8.8	6.4	125.3	628	159.0



3. For a timber beam having the cross section shown, determine the dimension w if the maximum allowable vertical shear force is 7.67kN, and the shearing stress is not to exceed 1.MPa.

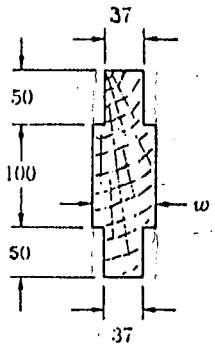
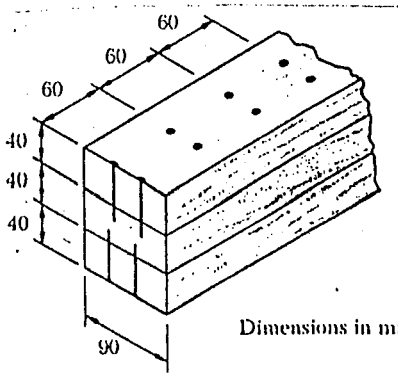


Fig. P5.20 Dimensions in mm

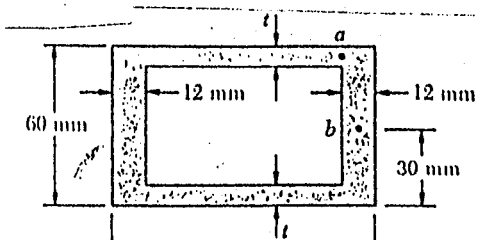
5.1 Three boards, each of 40 mm × 90 mm rectangular cross section, are nailed together to form a beam which is subjected to a vertical shear of 1 kN. Knowing that the spacing between each pair of nails is 60 mm, determine the shearing force in each nail.



Dimensions in mm

Fig. P5.1

5.32 An extruded beam has the cross section shown and is subjected to a vertical shear of 50 kN. For $t = 6$ mm, determine the shearing stress at (a) point a, (b) point b.



לשיטת הנוחות: $t_1 = t_2 = 2\,cm$

$b = 10\,cm$

$h = 10\,cm$

כדי למצוא את מרכז הכובד ביחס לתחתית הקורה נחלק את החתך לשני מלבנים.

$$\bar{y}\,(10\times2 + 8\times2) = 10\times2\times9 + 2\times8\times4 =$$

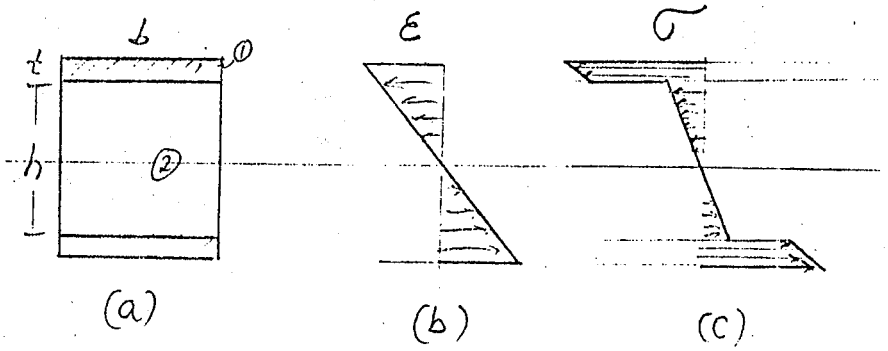
$$\bar{y} = 6.777\,cm$$

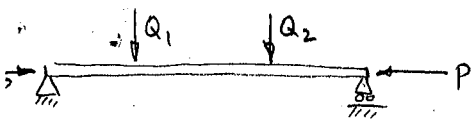
כאשר פלסטי מלא, מתקיים כי η הוא המרחק שבין ציר הניטרלי ומרכז הכובד של החלק העליון.

$$8\times2 + (\eta-8)\times10 = (10-\eta)10$$
$$16 + 10\eta - 80 = 100 - 10\eta$$
$$20\eta = 164$$
$$\eta = 8.2\,cm$$

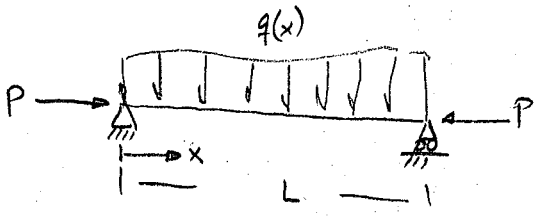
11.11 בסעיף הקודם מצאנו את חתך הטרנזיסטור

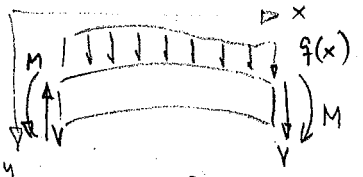
הקורה מורכבת מהחומרים הבאים: הקורה בנויה ממתכת חומרים לא מוליכים על המוליכים המחוברים אל פני הקורה, קורת בטון המחוברת במוליכים בהדק רפי. במשורת מעגל בה עוברת קורה הטרנזיסטור שבכאשר נסתכל בקורת הטרנזיסטור הרשתה שני חומרים ① ו ②. שלב E_2 ו E_1 נניח כי $E_1 > E_2$





דרך כלל אנחנו יכולים להשתמש בסופרפוזיציה כדי לקבל את התגובה על קורה זו מול עם כמה עומסים.
אבל כאשר יש קורה זו מול שהיא יש כוח צירי כמו P שפועל על המוט זו קורה עם הכוחות Q_1, Q_2 זו אפשר לקבל את התזוזות בסופרפוזיציה. מה בזו בכלל שהתגובות מיומסים בדרך לניטרית ו Q_1, Q_2 אבל היחס שלו לניטרי ל- P.





$$q = -\frac{dV_p}{dx} \qquad V = -\frac{dM_p}{dx} \qquad M = EI\frac{d^2v_p}{dx^2}$$

אבל הכוח P מכניס לנוסחה מומנט של Pv. המומנט הזה קורה לשינוי של $\frac{dv_p}{dx}$ ולכן יומס מפורש של $q = -P\frac{d^2v_p}{dx^2}$. לכן הנוסחה שפורטת במצב הזה היא

$$\lambda^2 = \frac{P}{EI} \qquad q = M\lambda^2 + \frac{d^2M_p}{dx^2}$$ כפונקציה של מומנט

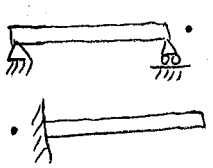
$$q/EI = \lambda^2\frac{d^2v_p}{dx^2} + \frac{d^4v_p}{dx^4}$$ כפונקציה של התצוגה

אם EI פונקציה של x

$$q = P\frac{d^2v_p}{dx^2} + \frac{d^2}{dx^2}\left(EI\frac{d^2v_p}{dx^2}\right)$$

ביטרון כאשר EI קבוע $q = q_0(1-x)$

$$v(x) = A\cos\lambda x + B\sin\lambda x + Cx + D + \frac{q_0x^2}{EI\lambda^2 2}$$

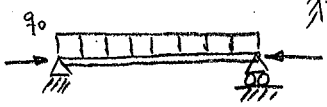


כמוך נייד וניית: $v=0$, $M=EI\frac{d^2v_p}{dx^2}=0$

כמוך כתוב: $v=0$, $\frac{dv_p}{dx}=0$

בקצה חופשי: $M=0$, $V=\frac{EI}{P}\frac{d^3v_p}{dx^3}+\frac{d^2v_p}{dx^2}=0$

בצמוד - קורה בקצוות נייח וניידי



$$v = \frac{q_0}{\lambda^4 EI\sin\lambda L}\left\{\sin\lambda L\left[\frac{2}{\lambda^2}(x-1) + (x\lambda\cos\lambda x - 1)\right] - x\lambda\sin\lambda(\lambda\cos\lambda - 1)\right\}$$

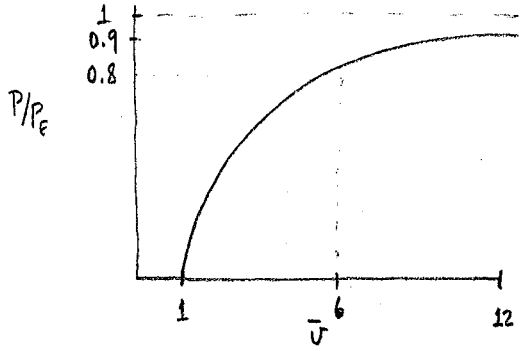
$$\sqrt{\frac{1}{\frac{5}{384}(\lambda L)^4 \sin\lambda L}} \cdot \sqrt{\frac{EI}{384}}$$

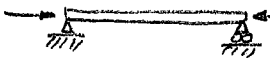
ותצורה הולכת לשלון 0 לפ כשהי $(\lambda L)^4 \sin\lambda L$ הולך לאפס, או כשהי $\sin\lambda L = 0$.

$$\sin\lambda L = 0 \Rightarrow \lambda L = n\pi \Rightarrow P = \frac{n^2\pi^2}{L^2} EI$$

לכן התצורה של תלויה ב- q_0. ה- P הקטן הוא $P_E = \frac{\pi^2 EI}{L^2}$ וזה הנקרא הקריטי או עומס הקריסה

ולא נסתכל במקרים $P/P_E = \frac{P}{\pi^2 EI/L^2} = \left(\frac{\lambda L}{\pi}\right)^2$ כפונקציה של $\bar{v} = \frac{v(\frac{1}{2},\lambda)}{v(\frac{1}{2},\lambda=0)}$



אם הסתכלנו כך בקורה  היינו מקבלים $\frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right) + P\frac{d^2v}{dx^2} = 0$

והפיתרון למשוואה הכללית היה מ-

$$v = A\cos\lambda x + B\sin\lambda x + Cx + D$$

את היינו מציבים בתנאי השפה, כלומר

$$v(0) = 0 \quad \rightarrow \quad A + D = 0$$
$$M(x=0) = EI\frac{d^2v}{dx^2} = 0 \quad \rightarrow \quad -\lambda^2 A = 0$$
$$v(L) = 0 \quad \rightarrow \quad A\cos\lambda L + B\sin\lambda L + CL + D = 0$$
$$M(x=L) = EI\frac{d^2v}{dx^2} = 0 \quad \rightarrow \quad -\lambda^2 A\cos\lambda L - \lambda^2 B\sin\lambda L = 0$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -\lambda^2 & 0 & 0 & 0 \\ \cos\lambda L & \sin\lambda L & L & 1 \\ -\lambda^2\cos\lambda L & -\lambda^2\sin\lambda L & 0 & 0 \end{bmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

כדי למצוא פיתרון, אז $A=B=C=D=0$ (צלצלת אומרת ש- $v=0$ והוא הפיתרון הפשוט) או שזה ה- DETERMINANT שווה לאפס. לכן, לפי הנאמר הפשוט, ה- DETERMINANT שווה ל-

$$L\begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ -\lambda^2 & 0 & 0 \\ 0 & -\lambda^2\sin\lambda L & -\lambda^2\cos\lambda L \end{vmatrix} = -\lambda^4 L\sin\lambda L = 0$$

זאת אומרת שפיתרון השפה הוא ש- $v = B\sin\lambda x$. לכן אפשרויות לתצורה ובולות להיות ~~~~~ אבל כולם

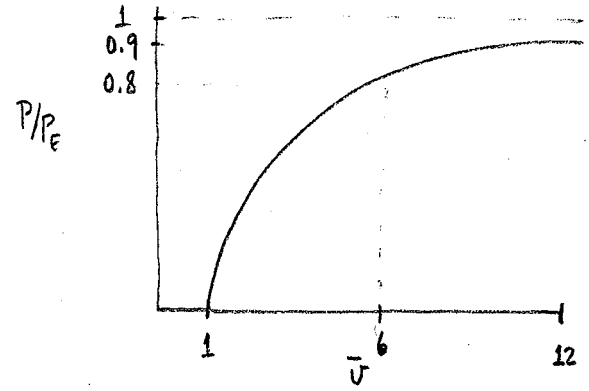
$$\left[\frac{\frac{5}{384}(\lambda L)^4}{\lambda L \sin \lambda L}\right]\frac{L}{EI}$$

ותצורה הולכת לאיזון סוף כאשר $(\lambda L)^4 \sin \lambda L$ הולך לאפס, או כאשר $\sin \lambda L = 0$.

$$\sin \lambda L = 0 \Rightarrow \lambda L = n\pi \Rightarrow P = \frac{n^2\pi^2}{L^2}EI$$

לכן התצורה של תלויה ב- q_0. ה- P הקטן הוא $P_E = \frac{\pi^2 EI}{L^2}$ וזה העומס הקריטי או עומס הקריסה

על ניתן מאקדם $P/P_E = \frac{P}{\pi^2 EI/L^2} = \left(\frac{\lambda L}{\pi}\right)^2$ כפונקציה של $\bar{v} = \frac{v(\lambda, y_2)}{v(\lambda=0, y_2)}$



אם הסתכלנו רק בקורה ← היינו מקבלים $\frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right)+P\frac{d^2v}{dx^2}=0$

והפיתרון למשוואה הכללית הוא ש-

$$v = A\cos\lambda x + B\sin\lambda x + Cx + D$$

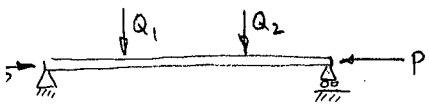
אם היינו מציבים את התנאי הקצה, כלומר את

$$v(0)=0 \rightarrow A+D=0$$
$$M(x=0) = EI\frac{d^2v}{dx^2} = 0 \rightarrow -\lambda^2 A = 0$$
$$v(L)=0 \qquad A\cos\lambda L + B\sin\lambda L + CL + D = 0$$
$$M(x=L) = EI\frac{d^2v}{dx^2} = 0 \qquad -\lambda^2 A\cos\lambda L - \lambda^2 B\sin\lambda L = 0$$

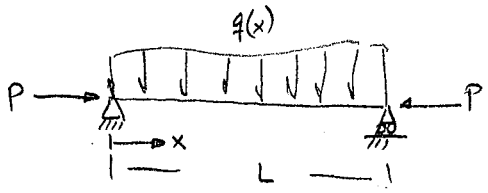
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -\lambda^2 & 0 & 0 & 0 \\ \cos\lambda L & \sin\lambda L & L & 1 \\ -\lambda^2\cos\lambda L & -\lambda^2\sin\lambda L & 0 & 0 \end{bmatrix}\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

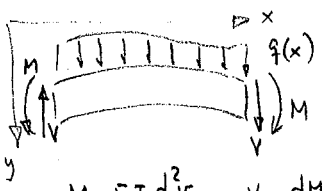
כדי למצוא פיתרון, אז $A=B=C=D=0$ (כלומר אומרת ש- $v=0$ והוא הפיתרון הפשוט) אז ה- DETERMINANT שווה לאפס. לכן, לפי הראוי הנ"ל, ה- DETERMINANT שווה ל-

$$L\begin{vmatrix} 1 & 0 & 1 \\ -\lambda^2 & 0 & 0 \\ 0 & \lambda\sin\lambda L & -\lambda^2\cos\lambda L \end{vmatrix} = \lambda L\sin\lambda L = 0$$



בדרך כלל אנחנו יכולים להשתמש בסופרפוזיציה כדי לקבל את התגובה על קורה אז מול עם כמה עומסים.
אבל במשפר יש קורה אז מול שהתא יש כות בירי כמו P
שפוגע על המוט אז קורה עם הכוחות Q_1, Q_2 אז אפשר לקבל
את התוכונות בסופרפוזיציה. מה בטו בכלל שהתגובות מיומסים בדרך לינטרית
ו Q_2, Q_1 אבל היחס לכל לינטרי ל- P.





$$\frac{dV_p}{dx} = q \qquad -\frac{dM_p}{dx} = V \qquad M = EI\frac{d^2v_p}{dx^2}$$

אבל הכוח P מכניס לנוסחה מומנט על עד. המומנט הזה קורה לשניה על $\frac{dv_p}{dx}$
ושם עומס מפורס על $q = -P\frac{d^2v_p}{dx^2}$. לכן הנוסחה שפורלת מאבד הזה היא

$$\frac{P}{EI} = \lambda^2 \qquad q = M\lambda^2 + \frac{d^2M_p}{dx^2} \qquad \text{כפונקציה על מומנט}$$

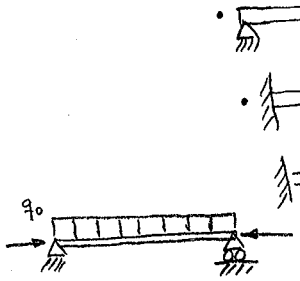
$$q/EI = \frac{d^2v_p}{dx^2}\lambda^2 + \frac{d^4v_p}{dx^4} \qquad \text{כפונקציה על התגובה}$$

אם EI פונקציה על x

$$q = \frac{d^2v_p}{dx^2}P + \frac{d^2}{dx^2}\left(EI\frac{d^2v_p}{dx^2}\right)$$

ביטרון כאשר EI קבוע $q = q_0 = q(x)$ - 1

$$v(x) = A\cos\lambda x + B\sin\lambda x + Cx + D + \frac{q_0x^2}{2\lambda^2EI}$$



בקצה חופשי: $M = 0$, $\frac{d^2v_p}{dx^2}EI = M$, $0 = v$:קצה נייח וניית
קצה כלוא: $\frac{dv_p}{dx} = 0$, $v = 0$
$\frac{d^3v_p}{dx^3}EI + P\frac{dv_p}{dx} = V$, $0 = M$ בקצה חופשי:

בעמוד- קורה בקצוות נייח וניית

$$v = \frac{q_0}{\lambda^4 EI\sin\lambda L}\left\{\sin\lambda L\left[\frac{\lambda^2}{2}x(L-x) + (\lambda x\cos\lambda x - 1)\right] - \lambda x\sin\lambda x(\lambda L\cos\lambda L - 1)\right\}$$

כאשר $q = q_0$ $v = \frac{q_0}{24EI}(x^4 - 2Lx^3 + L^3x)$ -1

$v(L/2) = \frac{5q_0L^4}{384EI}$:נקודת האמצע

התצורה המקסימלית הולכת וגדלת. ואם העומס P הוא P_E, אז העומד מתחיל לקרוס.

משוע תנאי הקצוות ל"עמוד-קורת כבורה

$$P_{\text{קריסה}} = 2.05\frac{\pi^2 EI}{L^2} = 2.05P_E$$

נותן המשוואה הקריטית $\lambda L = \lambda L \tan\lambda L$

$$P_{\text{קריסה}} = 4\frac{\pi^2 EI}{L^2} = 4P_E$$

המשוואה הקריטית $\left(\frac{\lambda L}{2}\cos\frac{\lambda L}{2} - \frac{\lambda L}{2}\sin\frac{\lambda L}{2}\right)\frac{\lambda}{2}\sin\frac{\lambda L}{2} = 0$

$$P_{\text{קריסה}} = \frac{1}{4}\frac{\pi^2 EI}{L^2} = \frac{1}{4}P_E$$

המשוואה הקריטית $\cos kL = 0$

בשלב זה, כדי שנוכל את עומס הסגירה. שלב הסגירה נקרא העומס הקריטי, שגורם לקריסה יהיה יותר נמוך. רוטית שתנאי הקצוות משפיעים על העומס הקריטי.

על פי זה אפשר להשתמש בסופרפוזיציה כמו את

$$P + \quad \neq \quad Q_1 \quad Q_2 \qquad \text{אפשר לעשות כך}$$

$$Q_1 \quad Q_2 \;=\; Q_1 \;+\; Q_2$$

הערות קבלנו את התוצאות הללו על תנאי שניסיון בחלק האלמנטרי הניחו על העומד כדרך כלל, $P_{cr} = C\frac{\pi^2 EI}{L^2}$ והמספר C נקרא בתנאי הקצוות. שלב $C = (L/L')^2$

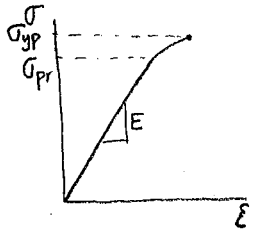
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{AL^2} = \frac{\pi^2 E(\rho^2 A)}{A(L')^2} = \frac{\pi^2 E}{(L'/\rho)^2}$$

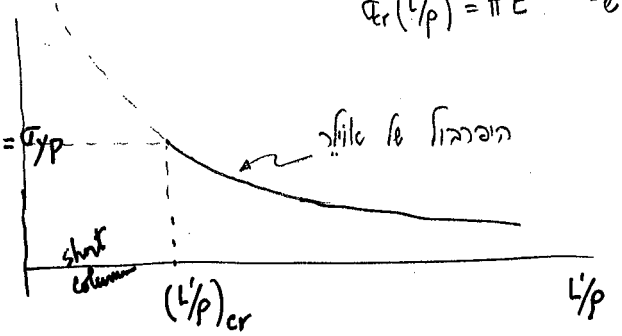
כאן L' = אורך האפקטיבי של הקורה. לדוגמה $C = 1/4$ לכן $L' = 2L$

ρ - רדיוס הג'ירציה (radius of gyration) הוא $I = A\rho^2$.

L'/ρ נקרא יחס הדקות (slenderness ratio) של הקורה וקובע את סוג הקריסה.

רואים ש- $\sigma_{cr} = \frac{\pi^2 E}{(L'/\rho)^2}$





$$\left(L/\rho\right)_{cr} = \pi\sqrt{E/\sigma_{pr}}$$

שווה ל- P_{cr}/A אבל יהיה יותר גדול. $\left(L/\rho\right)_{actual} < \left(L/\rho\right)_{cr}$ המאומץ הקריטי לפני יהיה

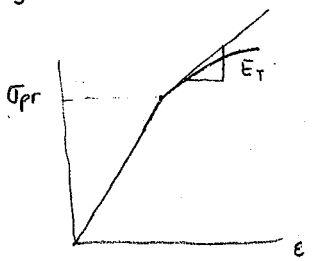
יכולים לקבל את הערך הקריטי. אם $\left(L/\rho\right)_{actual} > \left(L/\rho\right)_{cr}$ המאומץ הקריטי

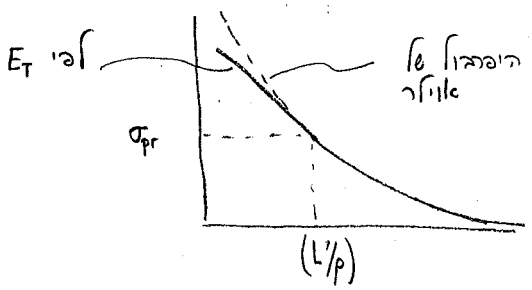
לתקן בצורה כזו, לעבוד ב $\left(L/\rho\right)_{actual} < \left(L/\rho\right)_{cr}$ משתמשים במודול הטנגנטי שהרי נקודת

הפרופורציה זה σ_{pr}.

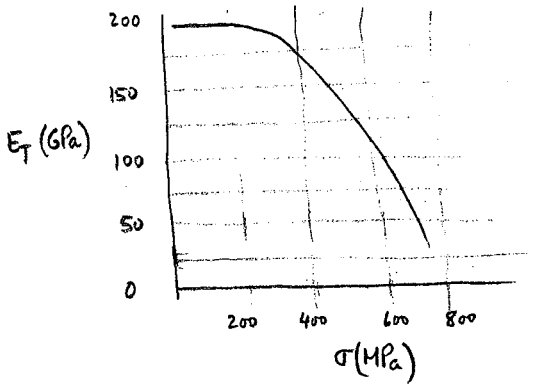
$$\sigma_{t,cr} = \frac{\pi^2 E_T}{\left(L/\rho\right)^2}$$

1- בדרך ניסיונית מקבלים את E_T





לפעמים מעוניינים לקבוע את היחס בין E_T למאומץ



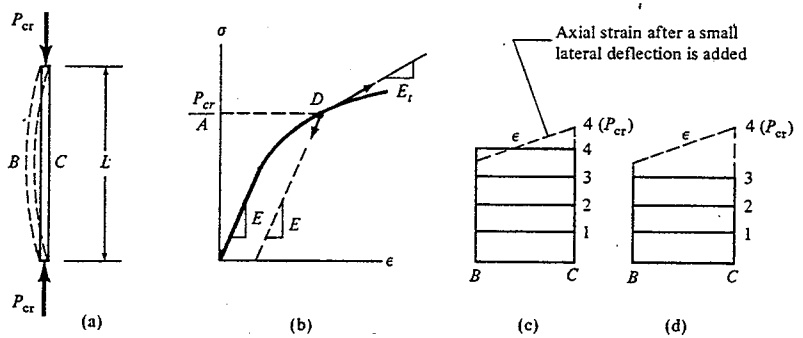


FIGURE 12.4.1. (a) Buckling of a pin-ended column under centroidal axial load. (b) Compressive stress–strain diagram, showing loading and unloading paths from point D, which corresponds to inelastic buckling. (c) Distribution of axial strain across the column at increasing load levels, according to double-modulus theory. (d) Possible distribution of axial strain across the column in tangent modulus theory.

cross section. Therefore, the column must bend before reaching the double-modulus load. But this is in contradiction to a basic assumption in double-modulus theory. The contradiction is resolved by noting that lateral deflection may occur *simultaneously* with application of the last increment of load. There need be no unloading on the convex side, and modulus E_t may prevail all across the section (Fig. 12.4.1d). Under near-perfect test conditions the collapse load slightly exceeds the theoretical tangent-modulus load, but it does not reach the double-modulus load.

In summary, inelastic buckling of a straight, axially loaded column does not occur at a unique value of axial load P. Instead, buckling begins at the tangent-modulus load and is complete (meaning that collapse takes place) before the theoretical double-modulus load is reached. Tests of real columns, which have larger imperfections than laboratory specimens, are in excellent agreement with tangent modulus theory.

Euler did not realize that bending stiffness EI could be calculated rather than obtained by experiment. However, he anticipated Engesser by remarking in 1757 that EI represents a resistance to bending that need not pertain only to elastic bodies [12.4].

Example 12.4.1. A column has a solid rectangular cross section, 40 mm by 30 mm. It is 200 mm long, free at the top, and fixed at the base. Material properties are shown in Fig. 12.4.2. What centroidal axial compressive load at the top will make the column buckle?

The appropriate equation is $P_{cr} = \pi^2 EI/4L^2$, where

$$I = \frac{bh^3}{12} = \frac{40(30)^3}{12} = 90{,}000 \text{ mm}^4 \qquad (12.4.2)$$

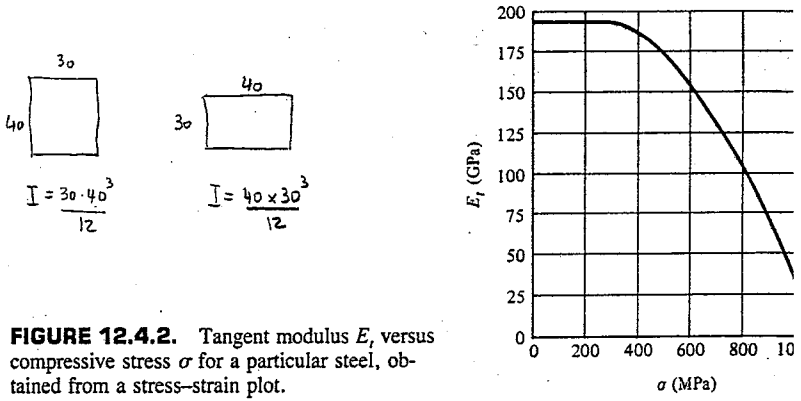


FIGURE 12.4.2. Tangent modulus E_t versus compressive stress σ for a particular steel, obtained from a stress–strain plot.

P_{cr} = 1077 kN, or $\sigma_{cr} = P_{cr}/A$ = 898 MPa. This stress is consi higher than the proportional limit stress, which appears to be about 32 in Fig. 12.4.2. Therefore, buckling is inelastic, the effective modu pends on load, and an iterative method of calculation is needed to fi as follows.

Assume that σ_{cr} will be, say, 600 MPa. At this stress, Fig. 12.4.: E_t = 160 GPa. Hence

$$P_{cr} = \frac{\pi^2 E_t I}{4L^2} = 888 \text{ kN} \qquad \frac{P_{cr}}{A} = 740 \text{ MPa} \qquad (:$$

As P_{cr}/A exceeds the assumed σ_{cr} of 600 MPa, another trial is neede Assume that σ_{cr} = 660 MPa; then

$$E_t = 142 \text{ GPa} \qquad P_{cr} = \frac{\pi^2 E_t I}{4L^2} = 788 \text{ kN} \qquad \frac{P_{cr}}{A} = 657 \text{ MPa} \qquad (1$$

Now the assumed value of σ_{cr} agrees well enough with the calculated and P_{cr} = 788 kN is accepted as the tangent modulus buckling load.

Creep Buckling. As the name implies, creep buckling theory deals material that creeps, that is, a material whose strain changes with time at c stress. A creeping column may display a small but gradually increasing deflection, then fail suddenly by buckling. The phenomenon is explai examination of creep curves (Fig. 12.4.3). One may enter the creep cu a certain time, say t_1, and read the strain for each of several stress Stress–strain data thus obtained is then plotted as a stress–strain curve, the curve labeled t_1 in Fig. 12.4.3. Repetition of this procedure at severa produces a set of isochronous stress–strain curves (stress versus strain at c time). These curves show that at a given stress level, the tangent m decreases with time. This implies that however light the load, a creeping

משקל W שנופל על קורה שורם למאומצים דינמיים והמאומצים הנלווים יותר גדולים ממצב שמשקל W נותן על הקורה בלבד

העומס הדינמי נתון ע"י

$$P_{dyn} = W\left(1 + \sqrt{1 + 2H/\Delta_{st}}\right)$$

Δ_{st} - זה התכונה הסטטית של הקורה שהיא מישום המשקל על הקורה.

h - זה מרחק הנפילה בין המשקל W לקורה.

דוגמא

$$\Delta_{st} = \frac{WL^3}{48EI}$$

המאמץ הדינמי כאן -

① למצוא העומס הדינמי
② למצוא הרגע המקס.
③ למצוא המאמץ

$$\sigma_{מקס} = \frac{My}{I} = \frac{P_{dyn} L \cdot (h/2)}{4 \cdot b h^3/12} = \frac{3 P_{dyn} \cdot L}{2bh^2}$$

הערות לישום הנוסחה הנ"ל.

- אין שום הנחה בנקודת הנקיפה או בסמיכים
- אינרציית המערכת נשארת כשהיה (משקל המערכת לא התחשב)
- יש לנו עומס סטטי או דינמי
- התכונה על המערכת יכולה להיות מיושם ואין אפשרות שום

1. Write the second-order differential equation for the bending of the column shown in Fig. P1-2 and use it to determine the critical load of the column. At its lower end the column is completely fixed. At the upper end the column is prevented from rotating, but free to translate laterally. (ans: $P_{cr}=\pi^2EI/L^2$)

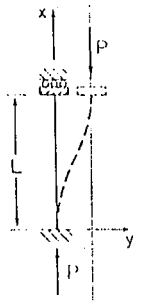
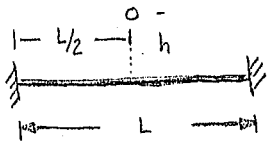


Fig. P1-2

2. Find an expression for the maximum stress when a ball weighing W Newtons is dropped onto a fixed-fixed beam.



3. . A linearly elastic beam-column having a flexural rigidity EI, is subjected to a thrust P and a moment M_0 as shown in Fig. A below.

(a) Determine the lateral displacement v(x).

(b) From part (a), write the solution for the system subjected to a force P acting as shown in Fig. B.

(c) Determine Δ_c, the horizontal displacement of point C, assuming small rotations. (Assume also that the horizontal displacement of point B is negligible).

(d) Determine the bending moment M(x).

(g) Explain why, although the beam is made of a linearly elastic material, the results of part (c) are non-linear.

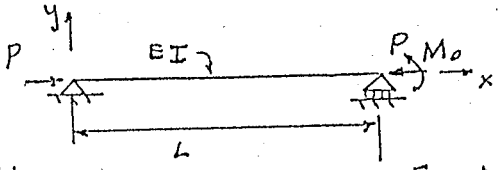


Fig. A

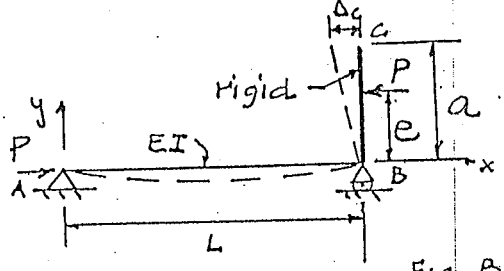


Fig. B

Answers :

(a) $y(x)=-\frac{M_0}{P}\left[\frac{\sin Kx}{\sin KL}-\frac{x}{L}\right]$, $K^2=\frac{P}{EI}$

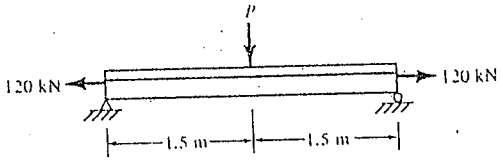
(c) $\Delta_c=\frac{ac}{L}\left[1-L\sqrt{P/EI}\cot\left(L\sqrt{P/EI}\right)\right]=\frac{ac}{L}(1-KL\cot KL)$

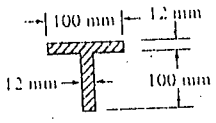
(d) $M(x)=M_0 \sin kl/\sin kL$

*12.10 A T section carries an axial tensile force of 120 kN, applied through the centroid of the cross section. The allowable stress in tension or compression is 130 MPa. Let $E = 200$ GPa. What transverse force P can be applied at midspan if the beam is

(a) Stem down (as shown)?

(b) Stem up?





PROBLEM 12.10



מהנוסחה שמצאנו בשיעור שעבר

$$EI\frac{d^4v}{dx^4}+P\frac{d^2v}{dx^2}=0$$

והפיתרון הוא

$$v=A\cos\lambda x+B\sin\lambda x+Cx+D$$

$$\lambda^2=P/EI$$

תנאי השפה:

① $v(x=0)=0$ תצוגה

② $\frac{dv}{dx}(0)=0$ צוויות

③ $\frac{dv}{dx}(x=L)=0$ צוויות

④ $EI\frac{d^3v}{dx^3}(x=L)+P\frac{dv}{dx}(x=L)=0$: צגירה

מתוך ③ $\frac{dv}{dx}=0$ כאשר $x=L$, לכן ④ נותנת $\frac{d^3v}{dx^3}=0$ כאשר $x=L$

מקבלים

$$\begin{aligned}
&A+D=0 && ①\\
&\lambda B+C=0 && ②\\
&-A(\lambda\sin\lambda L)+B(\lambda\cos\lambda L)+C=0 && ③\\
&A\lambda^3\sin\lambda L-B\lambda^3\cos\lambda L=0 && ④
\end{aligned}$$

לכן

$$\begin{bmatrix}1&0&0&1\\0&\lambda&1&0\\-\lambda\sin\lambda L&\lambda\cos\lambda L&1&0\\\lambda^3\sin\lambda L&-\lambda^3\cos\lambda L&0&0\end{bmatrix}\begin{pmatrix}A\\B\\C\\D\end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\end{pmatrix}$$

כדי לקבל תובאות לא טריוויאליות הדיטרמיננט צריך להיות אפס. לפי הצמוד הראשי:

$$1\cdot\begin{pmatrix}0&\lambda&1\\-\lambda\sin\lambda L&\lambda\cos\lambda L&1\\\lambda^3\sin\lambda L&-\lambda^3\cos\lambda L&0\end{pmatrix}=0$$

לכן

$$\lambda^4\sin\lambda L=0$$

לכן $\sin\lambda L=0$; $v=0$ (פיתרון טריוויאלי) $\Leftarrow$ $\lambda=n\pi$ או $P_E=\left(\frac{n\pi}{L}\right)^2EI$

3.



$$v=A\cos\lambda x+B\sin\lambda x+Cx+D \qquad \lambda^2=P/EI$$

① $v(x=0)=0$ ③ $v(x=L)=0$

② $EI\frac{d^2v}{dx^2}(x=0)=M=0$ ④ $EI\frac{d^2v}{dx^2}(x=L)=-M_0$ לפי הסכם הסימנים

$$\begin{aligned}
&A+D=0 && ①\\
&-A\lambda^2=0 && ②\\
&A\cos\lambda L+B\sin\lambda L+CL+D=0 && ③\\
&EI(-A\lambda^2\cos\lambda L-B\lambda^2\sin\lambda L)=-M_0 && ④
\end{aligned}$$

$A\overset{(2)}{=}0 \Leftarrow D\overset{(1)}{=}0 \Leftarrow C\overset{(3)}{=}0$ שלא $\lambda\neq0$

$$B(EI\lambda^2\sin\lambda L)\overset{(4)}{=}M_0 \Leftarrow B=\frac{M_0}{EI\lambda^2\sin\lambda L}$$

לכן,

$$v=\frac{M_0}{EI\lambda^2}\left(\frac{\sin\lambda x}{\sin\lambda L}-\frac{x}{L}\right)$$

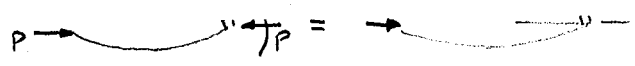
$$\boxed{v=\frac{M_0}{P}\left(\frac{\sin\lambda x}{\sin\lambda L}-\frac{x}{L}\right)}$$

לקבל $M(x)$, צוגר בראיית לפי x: $M=EI\frac{d^2v}{dx^2}=-EI\frac{M_0\lambda^2}{P}\frac{\sin\lambda x}{\sin\lambda L}=-M_0\frac{\sin\lambda x}{\sin\lambda L}$

$M(x)=-M_0\frac{\sin\lambda x}{\sin\lambda L}$

נכון לפי הסכם הסימנים,

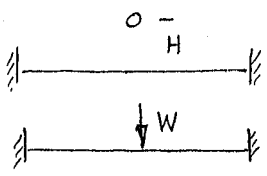
כדי לקבל θ. צריך לחשב $\theta=\frac{dv}{dx}\Big|_{x=L}=\frac{M_0}{P}\left(\frac{\lambda\cos\lambda L}{\sin\lambda L}-\frac{1}{L}\right)=\frac{M_0}{P}\left(\frac{1}{L}-\lambda\cot\lambda L\right)$



לכן $\theta\big|_{x=L} = \frac{Pe}{D}\left(\frac{1}{L} - \lambda\cot\lambda L\right)$ ו- $\Delta_c = a\theta\big|_{x=L}$ כי הזווית קטנה וקטור לקורה בצורת אנכית על 90°. לכן,

$$\Delta_c = \frac{ae}{L}\left(1 - \lambda L\cot\lambda L\right)$$

2.

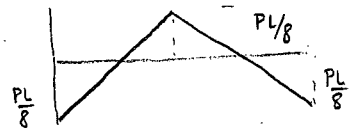


מפתור את מיתרים את והתגובה היא $\Delta_{st} = v = \frac{WL^3}{192EI}$

לפי הנוסחה $P_{dyn} = W\left(1 + \sqrt{1 + 2H/\Delta_{st}}\right)$, כאשר מציבים את Δ_{st}, מוצאים את P_{dyn}.

למצוא את המומנט המקסי. לפי הנוסחה שנתונה ב- Cook & Young $v = \frac{P}{48EI}(3Lx^2 - 4x^3)$

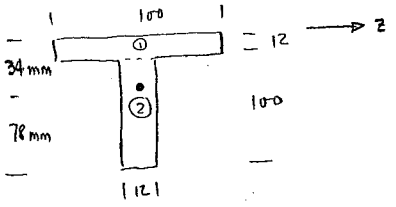
ו- $M = EI\frac{d^2v}{dx^2}$. לכן, $M = \frac{P}{48}(24x - 9L)$. ב- $x = 0$ $M = -\frac{PL}{8}$; ב- $x = L/2$ $M = \frac{PL}{8}$, וגרף המומנט



לכן יש שלושה מקומות שבהם המומנט הוא מקסימלי, והמאמץ המקסימלי, הוא $v = \frac{Mc}{I} = \frac{PL}{8}\frac{b}{bh^2}$

$$\sigma_{max} = \frac{3}{4}\frac{PL}{bh^3} = \frac{\frac{3}{4}\frac{LW(1+\sqrt{1+2H/WL^3/192EI})}{bh^3}}{}$$

12.10

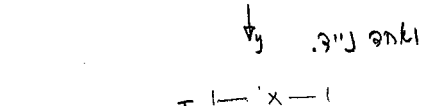


שטח החתך הוא $2400\ mm^2 = 2(100\times12)\ mm$

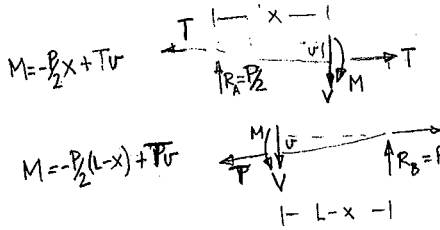
מרכז הכובד הוא

	A	y	Ay
①	100×12	6	7200
②	100×12	62	74400
	2400		81600

$\bar{y} = \frac{\Sigma Ay}{A} = 34\ mm$



העומס הקריטי פרש $T_E = \frac{\pi^2 EI}{L^2}$ אפני הסאכים, אחד נייח ואחד נייד.



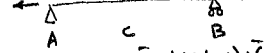
$0 \le x \le L/2$ $M = -\frac{P}{2}x + Tv$

$L \ge x \ge L/2$ $M = -\frac{P}{2}(L-x) + Tv$

כאשר העומס הקריטי, T, גורם לעמוד קריסה הוא המומנט שגורם לעמוד קריסה בלי העומס הצירי נוסף המומנט שבגלל מהעומס הצירי, Tv; ו השני התגובה הנשלחת מקום שמשפיעים את המומנט

לכן, המומנט המקסי נמצא ב- $x = L/2$ והוא $-\frac{P}{4}L + Tv$

כדי למצוא את $v\big|_{x=L/2}$, ופרים משוואה דיפרנציאלית ו- $v = A\cosh\lambda x + B\sinh\lambda x + Cx + D$ הכל תלוי ב- $\lambda^2 = \frac{T}{EI}$



מהקורה.

לכן AC, $v_L = A\cosh\lambda x + B\sinh\lambda x + Cx + D$ ו- CB. הקצוות השפה הוא $v_R = \bar{A}\cosh\lambda(x-L) + \bar{B}\sinh\lambda(x-L) + \bar{C}(x-L) + \bar{D}$

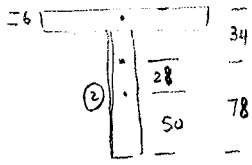
$v_L(x=0) = 0$ (1), $v_L''(x=0) = 0$ (2), $v_R(x=L) = 0$ (3), $v_R''(x=L) = 0$ (4). ב- $x = L/2$ $v_L(x=L/2) = v_R(x=L/2)$ (5); $v_L'(x=L/2) = v_R'(x=L/2)$ (6); $v_L''(x=L/2) = v_R''(x=L/2)$ (7); $EIv_L'''(x=L/2) - Tv_L''(x=L/2) = EIv_R'''(x=L/2) - Tv_R'' + P$ (8). שלוש הקצוות

רבועות, שאנו מחוות לקבל $A, B, C, D, \bar{A}, \bar{B}, \bar{C}, \bar{D}$. N- (1), (2), (3), (4) $\Rightarrow A = D = \bar{A} = \bar{D} = 0$. N- (7) $\Rightarrow B = -\bar{B}$. N- (6) $\Rightarrow C = -\bar{C}$: N- (6) ←

$x=1/2$ עבור

$$\Delta\Big|_{x=1/2} = P\left\{\frac{\sinh\lambda L/2}{2T\lambda\cosh\lambda L/2} - \frac{L}{4T}\right\} \qquad \frac{2EI\lambda^3\sinh\lambda L/2}{2EI\lambda^2}$$

כדי לדעת λ נחשב את T. ראשית כל נדרש לדעת I.



	I	A	d^2	I_{tt}
	$\frac{1}{12}(100)(12)^3$	1200	$(28)^2$	952800
	$\frac{1}{12}(12)(100)^3$	1200	$(28)^2$	1940800
				$2,896\times10^{-6}\ m^4$

$$\lambda = \sqrt{\frac{120,000}{(200\times10^9)(2.896\times10^{-6})}} = .4552\ , \qquad T_\varepsilon = \frac{\pi^2 EI}{L^2} = 635164N > T .$$

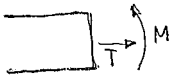
$$\Delta\Big|_{x=1/2} = P\left\{\frac{0.7371}{2(120,000)(1.2423)(.4552)} - \frac{3}{4(120000)}\right\}$$

$$= P(8.19\ 10^{-7}\ m)$$

לכן המומנט יהיה $M = -\frac{PL}{4} + T\cdot P\times 8.19\times10^{-7}$

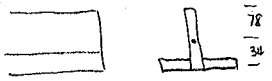
$$M = P\left[-0.75 + .09828\right] = -.65172P$$

התפלגות הלחצים שהמומנט מקים.



$$\sigma = \frac{T}{A} + \frac{Mc}{I}$$

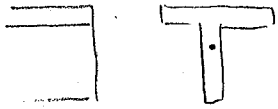
שלב בדיקת התכן כאן מושלם שאלה:



$$\sigma_{\text{עליון}} = \frac{120,000}{2400\times10^{-6}} - \frac{0.65172(.078)P}{2.896\times10^{-6}} = -130\times10^6 \qquad P = 10,250\ N$$

$$\sigma_{\text{תחתון}} = \frac{120,000}{2400\times10^{-6}} + \frac{0.65172(.034)P}{2.896\times10^{-6}} = 130\times10^6 \qquad P = 10,451N$$

שלב בדיקת התכן כאן מושלם שאלה:



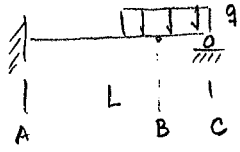
$$\sigma_{\text{עליון}} = \frac{120,000}{2400\times10^{-6}} - \frac{0.65172(.034)P}{2.896\times10^{-6}} = -130\times10^6 \qquad P = 23,515N$$

$$\sigma_{\text{תחתון}} = \frac{120,000}{2400\times10^{-6}} + \frac{0.65172(.078)P}{2.896\times10^{-6}} = 130\times10^6 \qquad P = 4556\ N$$

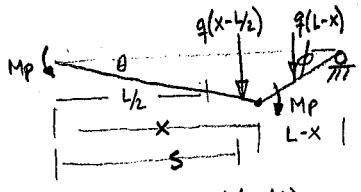
לכן, הרווח הגבוה ביותר בו כאשר לשלב התכן יש צורת בצד ⊥

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92. כאשר שהמומנט מגיע M_p, מרק פרסי קורה תמיד התנהג ובמ בנקודה B



מרק למצוא את מקום נקודת B ומהו הרווח המפורט שנוצר ל- M_p



$$\theta x = \phi(x-1)$$

$$\frac{\theta x}{x-1} = \phi$$

$$s = \tfrac{1}{2}(x+\tfrac{1}{2})$$

מתוך עקרון העבודה הוירטואלית

$$\phi\tfrac{1}{2}(x-1)q + s\theta\,(\tfrac{1}{2}-x)q = (\phi+\theta)M_p + \theta M_p$$

מציבים ϕ ו-s ביחסים ופותרים בשביל q

$$q = \frac{8M_p(x-12)}{7(2^1 - 4x^2 - x15)}$$

כדי למצוא את ה-q שבורת M_p, אוברית q בשביל x.

ואז קבלים $\frac{dq}{dx}=0$. מצה יש פתרות שלת $2^1+x19-4x^2=0$

והינו $x=\frac{19-\sqrt{2111}}{8}$. שמים x בנוסחה של q כדי לקבל $q=19.18\frac{M_p}{1^2}$

מוות המומנט בתוך BC $\frac{(x-1)q}{2}$ ומהו ה- R הנתון $6.194\frac{M_p}{1}$

$$v = A\cos\lambda x + B\sin\lambda x + Cx + D$$

לפי תנאי השפה על הקורה.

כפי לקבל את הדרך לפתור את הבעיה נציב z-

$$EI\frac{d^2v}{dx^2} + Pv = M_t$$

כאשר $P=0$

$$EI\frac{d^2v_t}{dx^2} = M_t$$

פה M_t הוא המומנט שבא מעומסים אנכיים בלבד, ו- v_t הוא התגובה הבזון y שנובה מהעומס האנכיים. מניחים שבגרת עיקום התגובה יהיה אותו סדר עם או בלי הכוח האופקי P.

לכן, לפי תנאי השפה, כאן קורה על סמך נייד וסמך נייד, מניחים ש- $v = \bar{v}\sin\frac{\pi x}{L}$

ו- $v_t = \bar{v}_t\sin\frac{\pi x}{L}$. $\bar{v}$, ו- $\bar{v}_t$ הם ה- AMPLITUDE של v ו- v_t. לפי הנוסחאות למעלה

$$EI\frac{\pi^2}{L^2}\bar{v}_t = -EI\frac{\pi^2}{L^2}\bar{v} + P\bar{v}$$

מפני ש- $P_E = \frac{\pi^2 EI}{L^2}$, או $\bar{v}(P-P_E) = -P_E\bar{v}_t$

$$\bar{v} = \frac{\bar{v}_t}{1 - P/P_E}$$

המומנט גם כן עיקב אחרי הנוסחא דומה לזה

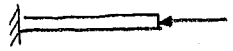
$$M = \frac{M_t}{1 - P/P_E}$$

אם P חיובי, $\bar{v} > \bar{v}_t$ ו- $M > M_t$. אם P שלילי, $\bar{v} < \bar{v}_t$ ו- $M < M_t$

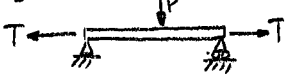
TACHLIS



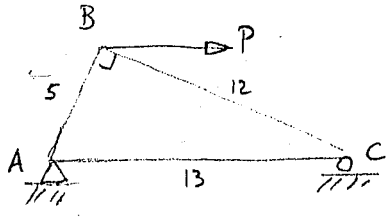
פיתר  וקבל את v מתיק העומס או בכול מקום אחר. קורא לזה $\bar{v}_t$.



v יהיה $v = \frac{\bar{v}_t}{1 - P/P_E}$ קבל P_E לקורה עמוסה כמו בדוגמה

בתרגיל בית 12.10  $T_E = \frac{\pi^2 EI}{L^2}$, $\bar{v}_t\Big|_{x=L/2} = \frac{PL^3}{48EI}$, ו- $v = \frac{\bar{v}_t}{1 + T/T_E}$

כי P היה שלילי.

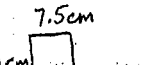


לפי שיטת הצמתים (רשמו את הפירוק של הכוחות)

$F_{AB} = \frac{5}{13}P$ דחיסות

$F_{AC} = \frac{144}{169}P$ מתיחות

$F_{BC} = -\frac{12}{13}P$ בלחיצה



אם שטח החתך ..., מהו העומס שיגרום לכיתמוטות ? קחו $\sigma_y = 300\ MPa$, $E = 200GPa$

מבין ש- F_{AB} ו- F_{AC} דחיסות צריך לבדוק מהו העומס שגורם לכניעה. מבין ש- F_{BC} בלחיצה צריך לבדוק מהו העומס שגורם לכניעה בלחיצה וגם בקריסה.

$A = .1(.075) = 7.5\times10^{-3}\ m^2$

$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{5}{13}\frac{P}{(7.5\times10^{-3})} = 300\times10^6\ \frac{N}{m^2}$ $\quad P = (300\times10^6)(7.5\times10^{-3})\frac{13}{5}$

$= 5.4\times10^6 N$

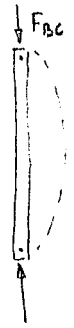
$\sigma_{AC} = \frac{F_{AC}}{A} = \frac{144}{169}\frac{P}{7.5\times10^{-3}m^2} = 300\times10^6\ \frac{N}{m^2}$ $\quad P = (300\times10^6)(7.5\times10^{-3})\frac{169}{144}$

$= 2.64\times10^6 N$

$\sigma_{BC} = \frac{F_{AC}}{A} = -\frac{12}{13}\frac{P}{7.5\times10^{-3}m^2} = -300\times10^6\ \frac{N}{m^2}$ $\quad P = (300\times10^6)(7.5\times10^{-3})\cdot\frac{13}{12}$

$= 2.44\times10^6 N$

כדי לבדוק קריסה יש שתי אפשרויות — או — לכן, צריך למצוא את מומנט האינרציה הגדל ביותר



$I = \frac{(7.5)10^3}{12}\times10^{-8} = 6.25\times10^{-6}\ m^4$

$F_{BC} = \frac{\pi^2(200\times10^9)(6.25\times10^{-6})}{(12)^2} = 85674 N$

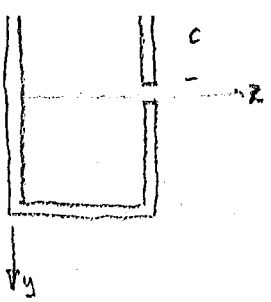
$F_{BC} = \frac{12}{13}P \rightarrow \underline{P = 92813.\ N}$

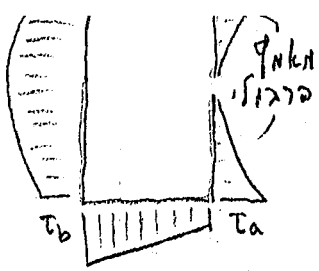
$I = \frac{10(7.5)^3}{12}\times10^{-8} = 3.515\times10^{-6}\ m^4$

$F_{BC} = \frac{4\pi^2 EI}{L^2} = \frac{4(\pi^2)(200\times10^9)(3.515\times10^{-6})}{(12)^2} = 192766 N$

$F_{BC} = \frac{12}{13}P \rightarrow P = 208830 N$

לכן BC תקרוס בהשוואה כאשר $P = 92813 N$





גורם מומנט הפיזרה שפועל על החתך

$$\frac{}{It}$$

$$\tau_b = (\tau_b - \tau_a) + \tau_a = \frac{VQ_b}{It} + \frac{VQ_a}{It}$$



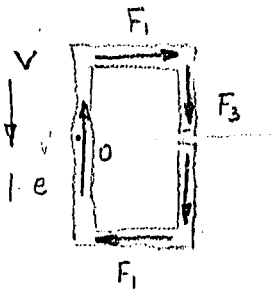
$$I = \frac{t(2c)^3}{12} + 2(bt)c^2 + 2\left[\frac{tc^3}{12} + tc\left(\frac{c}{2}\right)^2\right]$$

$$Q_a = \int y\,dA = \int_{b-t/2}^{b+t/2}\int_0^c y\,(dy\,dz) = \frac{y^2}{2}t\Big|_0^c = \frac{c^2}{2}t$$

$$Q_b = \int y\,dA = \bar{y}A = c\cdot bt$$

$$F_3 = \int \tau_a\,dA = \frac{V}{It}\int_0^c \frac{y^2 t}{2}\cdot t\,dy = \frac{V}{I}\frac{y^3}{6}\Big|_0^c = \frac{Vtc^3}{6I}$$

$$F_1 = \frac{\tau_a+\tau_b}{2}(b\cdot t) = \frac{Vtbc}{2I}(b+c)$$



$$\Sigma M_o = 2F_3\cdot b + F_1\cdot 2c - Ve = 0$$

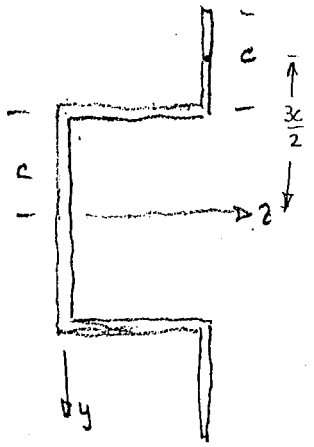
$$= 2\tau_a\frac{ct}{3}\cdot b + \left(\frac{\tau_a+\tau_b}{2}\right)bt\cdot 2c - Ve = 0$$

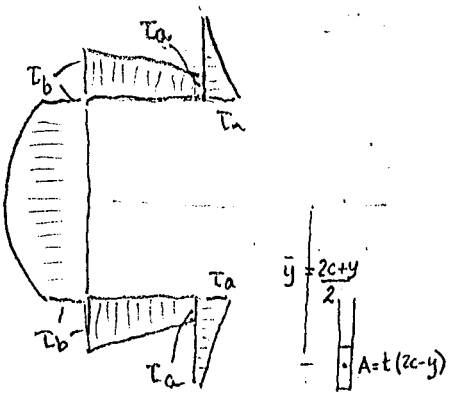
$$= \frac{2VQ_a}{It}\cdot\frac{ct}{3}\cdot b + \left(\frac{VQ_b}{2It} + \frac{2VQ_a}{2It}\right)bt\cdot 2c - Ve = 0$$

$$= \frac{V}{It}\left\{2\frac{c^2t}{2}\cdot\frac{ct}{3}\cdot b + \left(cbt + \frac{c^2t}{2}\right)bt\cdot 2c\right\} - Ve = 0$$

$$= \frac{V}{It}\left\{\frac{c^3t^2b}{3} + c^2b^2t^2 + c^3bt^2\right\} - Ve = 0$$

$$e = \frac{1}{It}\left\{\frac{4}{3}c^3t^2b + c^2b^2t^2\right\} = \frac{c^2bt}{I}\left(b+\frac{4}{3}c\right) = \frac{c^2bt\left(b+\frac{4}{3}c\right)}{c^2t\left(\frac{4}{3}c+2b\right)} = \frac{b\left(b+\frac{4}{3}c\right)}{\left(\frac{4}{3}c+2b\right)}$$





$$\tau_a = \frac{VQ_a}{It} \qquad \tau_b = (\tau_b-\tau_a)+\tau_a = \frac{VQ_b}{It} + \frac{VQ_a}{It}$$

$$I = \frac{t(2c)^3}{12} + 2bt\,c^2 + 2\left[\frac{tc^3}{12} + tc\left(\frac{3c}{2}\right)^2\right]$$

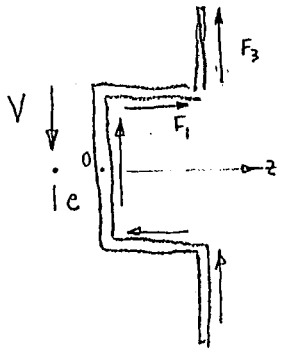
$$Q_a = \int y\,dA = t\frac{(2c-y)(2c+y)}{2} = \frac{t}{2}(4c^2-y^2)\Big|_{y=c} = \frac{3c^2t}{2}$$

$$Q_b = cbt$$

כאן נתרכז הקודמת

$$F_3 = \int \tau_a\,dA = \frac{V}{It}\int_c^{2c}\frac{t}{2}(4c^2-y^2)\cdot t\,dy = \frac{Vt}{2I}\left(4c^2y - \frac{y^3}{3}\right)_c^{2c} = \frac{5}{6I}Vtc^3$$

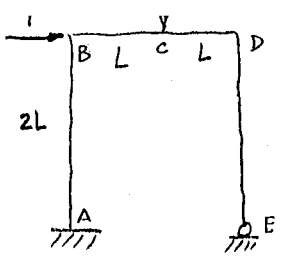
$$F_1 = \frac{\tau_b+\tau_a}{2}bt = \frac{1}{2}\left(\frac{VQ_b}{It} + \frac{2VQ_a}{It}\right)bt = \frac{Vtbc}{2I}(b+3c)$$

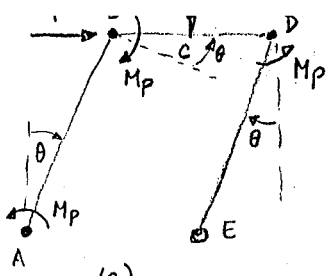


$$\Sigma M_o = -Ve + F_1\cdot 2c - 2F_3\cdot b = 0$$

$$= -Ve + \frac{Vtbc}{2I}(b+3c)\cdot 2c - 2\cdot\frac{5}{6I}Vtc^3\cdot b$$

$$e = \frac{tbc^2(b+3c) - \frac{5}{3}tc^3b}{\frac{64}{12}tc^3 + 2btc^2} = \frac{b}{2}\frac{\left(b+\frac{4}{3}c\right)}{\left(b+\frac{8}{3}c\right)}$$





(a)

$$P\cdot 2\theta L = M_p\theta + M_p\theta + M_p\theta$$

$$P = \frac{3M_p}{2L}$$

M_C לגדרר את בדיוק כאשר

$$M_A = M_D = M_p$$

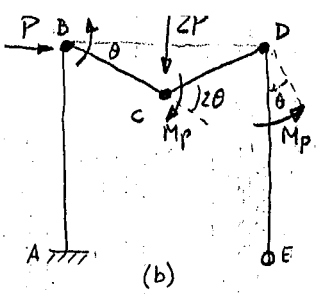
$$M_B = -M_p$$

$$M_A - 2M_C + 2M_D = 4PL$$

$$M_p - 2M_C + 2M_p = 4\left(\frac{3M_p}{2L}\right)L$$

$$M_C = -\frac{3}{2}M_p$$

המינימלי הוא $|M_C| > M_p$



(b)

$$2P\cdot L\theta = M_p\theta + M_p\cdot 2\theta + M_p\theta$$

$$P = 2M_p/L$$

M_A לגדרון את בדיוק כאשר

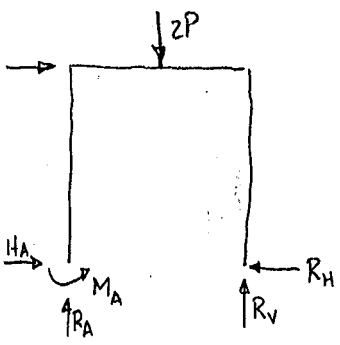
$$M_B = M_D = M_p \;,\; M_C = -M_p$$

$$M_A - 2M_C + 2M_D = 4PL$$

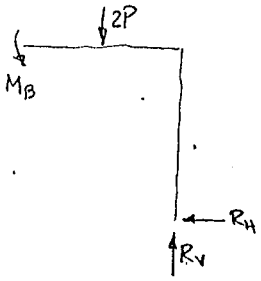
$$M_A + 2M_p + 2M_p = 4\left(\frac{2M_p}{L}\right)L$$

$$M_A = 3M_p$$

המינימלי הוא כאשר $M_A > M_p$



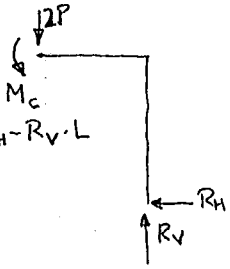
$$M_A = -R_V\cdot 2L + P(2L) + 2P\cdot L$$



$$M_B = 2P\cdot L + R_H\cdot 2L - R_V\cdot 2L$$



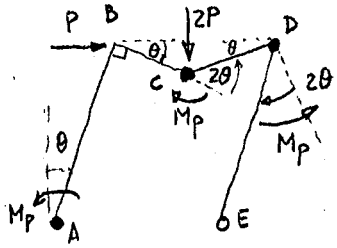
$$M_D = +2L\cdot R_H$$



$$M_C = 2L\cdot R_H - R_V\cdot L$$

$$\Rightarrow M_A - 2M_C + 2M_D = 4PL$$

$$\Rightarrow M_B - 2M_C + M_D = 2PL$$



$$P\cdot 2L\theta + 2P\cdot L\theta = M_p\theta + M_p\cdot 2\theta + M_p\cdot 2\theta$$

$$P = \frac{5M_p}{4L}$$

M_B לגדרון את בדיוק כאשר

$$M_C = -M_p \;,\; M_A = M_D = M_p$$

$$M_B - 2M_C + M_D = 2PL$$

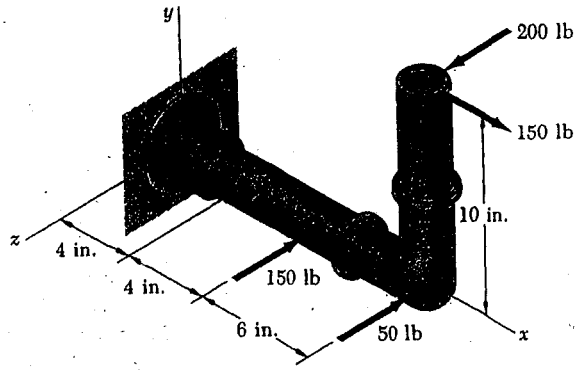
$$M_B + 2M_p + M_p = 2\left(\frac{5M_p}{4L}\right)L$$

$$M_B = -\frac{M_p}{2}$$

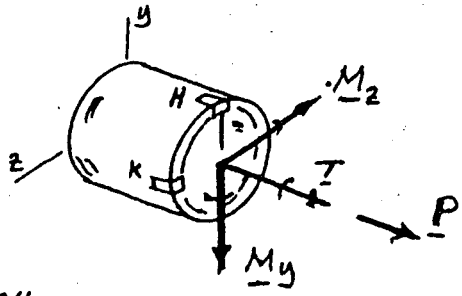
מבין ה- M_B בחות ה- M_p ו- P הוגו התחות ביותר תאורוץ הפה יכול לקרות

Problem 8.39

8.39 Several forces are applied to the pipe assembly shown. Knowing that the pipe has inner and outer diameters equal to 1.61 and 1.90 in., respectively, determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.



=



$P = 150\text{ lb}$

$T = (200\text{ lb})(10\text{ in.}) = 2000\text{ lb·in.}$

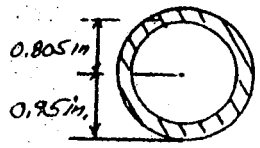
$M_z = (150\text{ lb})(10\text{ in.}) = 1500\text{ lb·in.}$

$M_y = (200\text{ lb} - 50\text{ lb})(10\text{ in.}) - (150\text{ lb})(4\text{ in.}) = 900\text{ lb·in.}$

$V_z = 200 - 150 - 50 = 0$

$V_y = 0$

Section properties



$A = \pi(0.95^2 - 0.805^2) = 0.79946\text{ in}^2$

$I = \frac{\pi}{4}(0.95^4 - 0.805^4) = 0.30989\text{ in}^4$ $\quad J = 2I = 0.61979\text{ in}^4$

(a) POINT H: $\sigma_H = \frac{P}{A} + \frac{M_z c}{I} = \frac{150\text{ lb}}{0.79946\text{ in}^2} + \frac{(1500\text{ lb·in.})(0.95\text{ in.})}{0.30989\text{ in}^4}$

$= 187.6\text{ psi} + 4593\text{ psi}$

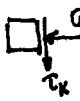
$\sigma_H = 4.79\text{ ksi}$



$\tau_H = \frac{Tc}{J} = \frac{(2000\text{ lb·in.})(0.95\text{ in.})}{0.61979\text{ in}^4} = 3065.6\text{ psi}$

$\tau_H = 3.07\text{ ksi}$

(b) POINT K: $\sigma_K = \frac{P}{A} + \frac{M_y c}{I} = \frac{150\text{ lb}}{0.79946\text{ in}^2} - \frac{(900\text{ lb·in.})(0.95\text{ in.})}{0.30989\text{ in}^4}$



$= 187.6\text{ psi} - 2759\text{ psi}$

$\sigma_K = -2.57\text{ ksi}$

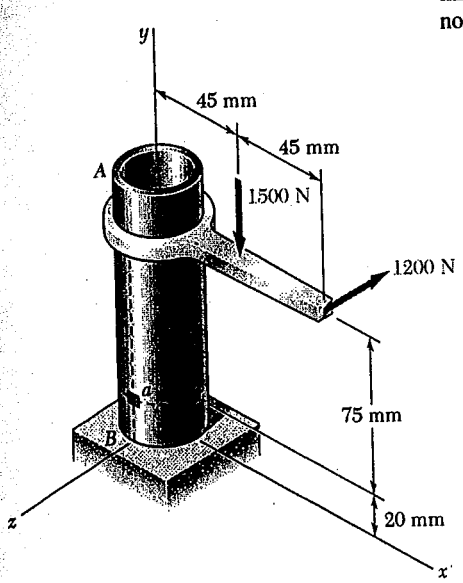
$\tau_K = \frac{Tc}{J}$ = SAME AS FOR τ_H

$\tau_K = 3.07\text{ ksi}$

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Problem 8.40

8.40 Two forces are applied to the pipe AB as shown. Knowing that the pipe has inner and outer diameters equal to 35 and 42 mm, respectively, determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*.

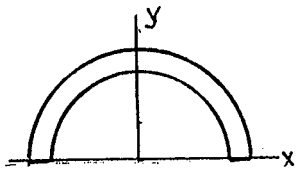


$c_o = \frac{d_o}{2} = 21\text{ mm}$, $c_i = \frac{d_i}{2} = 17.5\text{ mm}$

$A = \pi(c_o^2 - c_i^2) = 423.33\text{ mm}^2$

$J = \frac{\pi}{2}(c_o^4 - c_i^4) = 158.166 \times 10^3\text{ mm}^4$

$I = \frac{1}{2}J = 79.083 \times 10^3\text{ mm}^4$



For semicircle with semicircular cutout

$Q = \frac{2}{3}(c_o^3 - c_i^3)$

$Q = 2.6011 \times 10^3\text{ mm}^3 = \frac{4r}{3\pi} \cdot \frac{\pi r^2}{2} = \frac{2}{3}r^3$ (1)

At the section containing points a and b

$P = -1500\text{ N}$ $\quad V_z = -1200\text{ N}$ $\quad V_x = 0$

$M_z = -(45\times10^{-3})(1500) = -67.5\text{ N}\cdot\text{m}$

$M_x = -(75\times10^{-3})(1200) = -90\text{ N}\cdot\text{m}$

$T = (90\times10^{-3})(1200) = 108\text{ N}\cdot\text{m}$

(a) $\sigma = \frac{P}{A} - \frac{M_x c}{I} = \frac{-1500}{423.33\times10^{-6}} - \frac{(-90)(21\times10^{-3})}{79.083\times10^{-9}}$ $\quad \sigma = 20.4\text{ MPa}$

$\tau = \frac{Tc}{J} + \frac{V_x Q}{It} = \frac{(108)(21\times10^{-3})}{158.166\times10^{-9}} + 0$ $\quad \tau = 14.34\text{ MPa}$

(b) $\sigma = \frac{P}{A} + \frac{M_z c}{I} = \frac{-1500}{423.33\times10^{-6}} + \frac{(-67.5)(21\times10^{-3})}{79.083\times10^{-9}}$ $\quad \sigma = -21.5\text{ MPa}$

$\tau = \frac{Tc}{J} + \frac{|V_z|Q}{It} = \frac{(108)(21\times10^{-3})}{158.166\times10^{-9}} + \frac{(1200)(2.6011\times10^{-6})}{(79.083\times10^{-9})(7\times10^{-3})}$

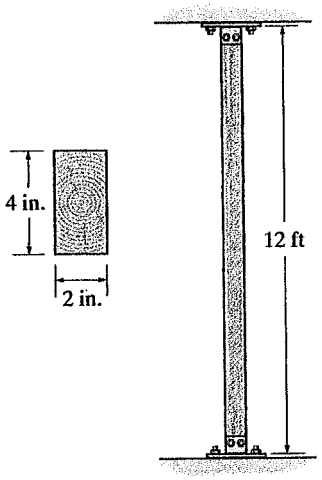
$\tau = 19.98\text{ MPa}$

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13

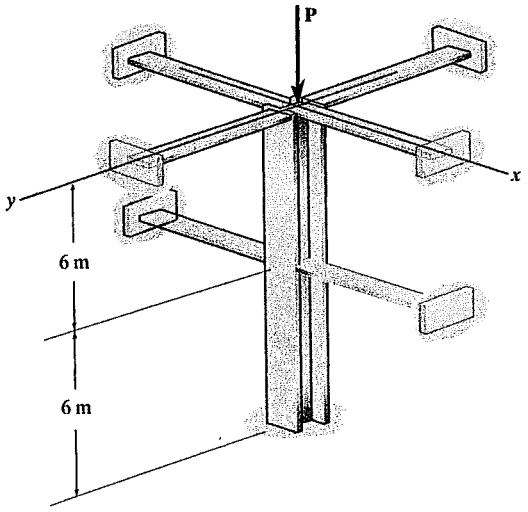
F13–1. A 50-in.-long rod is made from a 1-in.-diameter steel rod. Determine the critical buckling load if the ends are fixed supported. $E = 29(10^3)$ ksi, $\sigma_Y = 36$ ksi.

F13–2. A 12-ft wooden rectangular column has the dimensions shown. Determine the critical load if the ends are assumed to be pin-connected. $E = 1.6(10^3)$ ksi. Yielding does not occur.



F13–2

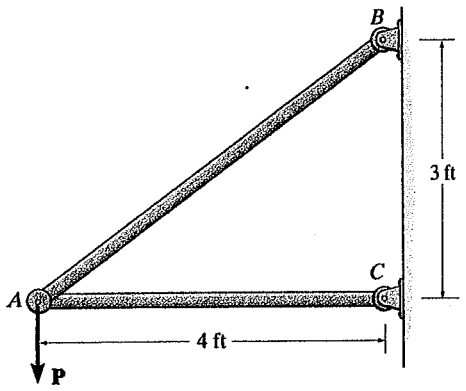
F13–3. The A-36 steel column can be considered pinned at its top and bottom and braced against its weak axis at the mid-height. Determine the maximum allowable force **P** that the column can support without buckling. Apply a F.S. = 2 against buckling. Take $A = 7.4(10^{-3})$ m^2, $I_x = 87.3(10^{-6})$ m^4, and $I_y = 18.8(10^{-6})$ m^4.



F13–3

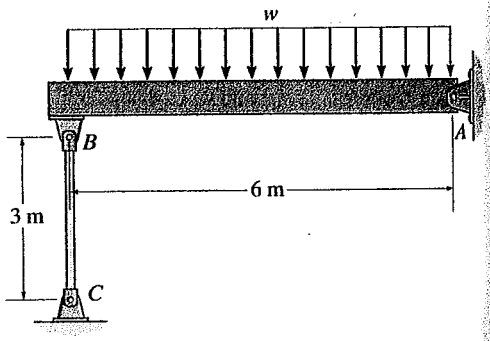
F13–4. A steel pipe is fixed supported at its ends. If it is 5 m long and has an outer diameter of 50 mm and a thickness of 10 mm, determine the maximum axial load P that it can carry without buckling. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.

F13–5. Determine the maximum force **P** that can be supported by the assembly without causing member AC to buckle. The member is made of A-36 steel and has a diameter of 2 in. Take F.S. = 2 against buckling.



F13–5

F13–6. The A-36 steel rod BC has a diameter of 50 mm and is used as a strut to support the beam. Determine the maximum intensity w of the uniform distributed load that can be applied to the beam without causing the strut to buckle. Take F.S. = 2 against buckling.



F13–6

תרגיל: למצוא את העומס המקסימלי, שאותו לפרוץ על קורה שחומר שלה נתונים והמשוואה שלה נתונים.

$$M_{max} = \frac{qL^2}{2} = \sigma_{allow} \cdot S$$

$$q_{allow} = \frac{2\sigma_{allow} \cdot S}{L^2} = \frac{2(1.2\times10^8)(6.67\times10^{-4})}{4^2} = 1.0\times10^4 \text{ N/m}$$

(העומס המפוזע distributed load)

$$q_{allow} = q/2$$

לכן רואים שהתכולת כאן יותר מאה שאותר והקורה תתמוגג, כלי שהקורה על תכולה, העומס המפורס בריק להיות ½ ממה שנמצא.

תיכון: לקבוע המשוואה על הקורה לפי הנתונים - העומס וגם המומר.

מהו q על הקורה שאפשר ~~שאפשר~~ העומס הנתון
גם לשמוך

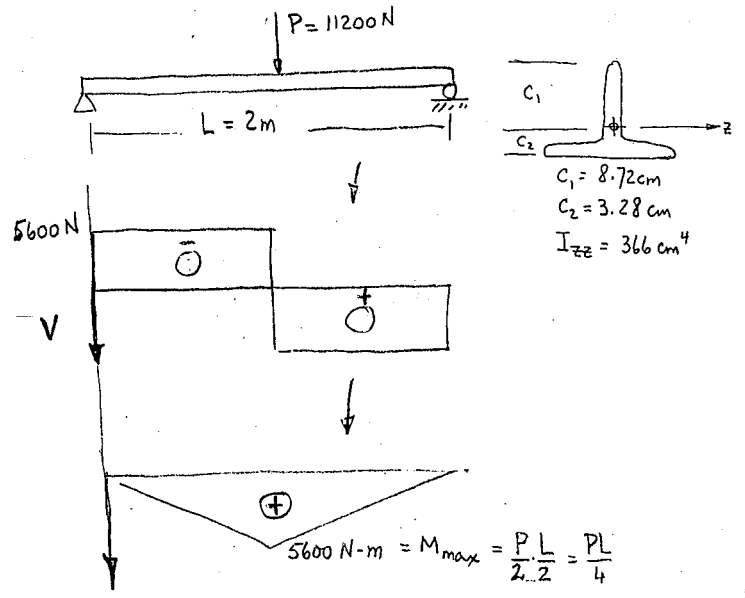
$$S_{allow} = \frac{M_{max}}{\sigma_{allow}} = \frac{b(2b)^2}{6} = \frac{4}{6}b^3$$

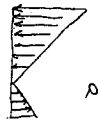
לכן

$$b = \sqrt[3]{\frac{6}{4}\frac{M_{max}}{\sigma_{allow}}}$$

$$\sqrt[3]{\frac{3}{2}\cdot\frac{qL^2}{2\sigma_{allow}}} = \sqrt[3]{\frac{0.75\times2\times10^4\cdot4^2}{1.2\times10^8}} = 12.6 \text{ cm}$$

פתרון כי





נתון: הקורה הנשוייה
מן הפרופיל 12 ⊥
נתונים המאמצים המותרים
מתח $\sigma_{allow} = 10^8 \text{ N/m}^2 = 100 \text{ MPa}$
לחץ $\sigma_{allow} = 1.8\times10^8 \text{ N/m}^2 = 180 \text{ MPa}$

$$\text{לחץ } (\sigma_{max}) = \frac{Mc_1}{I_{zz}} = \frac{5600\times8.72\times10^{-2}\text{ m}}{366\times10^{-8}\text{ m}^4} =$$

$$= 1.334\times10^{+8}\text{ Pa}$$

$$\text{מתח } (\sigma_{max}) = \frac{Mc_2}{I_{zz}} = \frac{5600\times3.28\times10^{-2}\text{ m}}{366\times10^{-8}\text{ m}^4}$$

$$= 0.502\times10^8 \text{ Pa}$$

שניהם בסדר!

לכן -

$\sigma_x = \sigma_{UTS} = P/A$ מקס'

או

$\sigma_x = \sigma_{UTS} = \frac{Mc}{I}$ מקס'

כדרוש להדגיש שחומר אלסטי וכול להיות לעל לינארי וכאן כדרוש למצוא את השיפוע שנמצא ב-נקודה 0. חומר פרוק ומוריש אחרים הולכים אחרי הדוגמא הצאת, כמו בטון.

כדי למצוא את המאמץ הכניעה לחומרים הללה משתמשים בשיטת היסוט (OFFSET METHOD) שהוא 0.2% לימין הנקודה 0 (נקודה c).

את הערך (A) קורסטם לצה את נקודת הכניעה.

משרטטין קו לינארי מקביל לשיפוע שנמצא מנקודה 0. הנקודה שבה קו זה חותך את העקומה, לא תהיה נקודת הכניעה.

את הקורה לא להתמוטט תחת העומס, אלא צה נדון בהמשך.

נדגיש על שני מדדים שאנו מריכים לפקור בעיית כפיפה קורה.

① **אלסטיות**: למצוא העומס המקסימלי שאותו לפעול על הקורה לפני קורה נתונה ומומר נתון.

$\sigma_{allow} = \pm 1.2 \times 10^8 Pa$

$M_{max} = \frac{qL^2}{2} = \frac{2\times10^4(4)^2}{2} = 16\times10^4\ N\text{-}m$

$I = \frac{bh^3}{12} \quad c = \frac{h}{2} \quad S = I/c = \frac{bh^3/12}{h/2} = \frac{bh^2}{6} = \frac{10\times(20)^2}{6} = 6.67\times10^{-4} m^3$

$\sigma_{max} = \frac{M}{S} = \frac{16\times10^4\ N\text{-}m}{6.67\times10^{-4} m^3} = 2.4\times10^8 N/m^2 > \sigma_{allow}$

לכן הקורה תתמוטט!

$M = -\frac{q(L-x)^2}{2}$

$V = -q(L-x)$

at $x=0$ $M = max = \frac{qL^2}{2}$

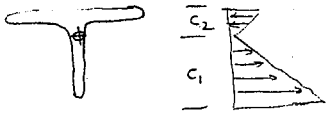
$\sigma_{max} = -\frac{My}{I} = -\frac{M\cdot \pm h/2}{bh^3/12} = -\left(-\frac{qL^2}{2}\right)\left(\pm\frac{h}{2}\right) =$

תשב התשובות המרומזים ⊥ 21 עם הנתונים

$$(M_{allow})_{\text{מתח}} = \frac{(\sigma_{allow})_{\text{מתח}} \cdot I_{zz}}{c_1} = \frac{(1.8\times10^8)(366\times10^{-6})}{8.72\times10^{-2}} = 7555\ Nm$$

$$(M_{allow})_{\text{מתח}} = \frac{(\sigma_{allow})_{\text{מתח}} I_{zz}}{c_2} = \frac{10^8\times(366\times10^{-6})}{3.28\times10^{-2}} = 11,159\ Nm$$

ומה יהיה אם מחברים המרומזים?



$$(M_{allow}) = \frac{(\sigma_{allow})_{\text{מתח}} \cdot I_{zz}}{c_2} = \frac{(1.8\times10^8)(366\times10^{-6})}{3.28\times10^{-2}} = 20,085\ Nm$$

$$(M_{allow}) = \frac{(\sigma_{allow})_{\text{מתח}} I_{zz}}{c_1} = \frac{(10^8)\times(366\times10^{-6})}{8.72\times10^{-2}} = 4,197\ Nm$$

בדוגמא הראשונה: $M_{allow} = 7555\ N\text{-}m = \frac{P_{allow}\,L}{4}$

והכוח שניתן לתמוך הוא $P_{allow} = \frac{4M}{L} = 15,110\ N$

בדוגמא השניה: $M_{allow} = 4,197\ Nm = \frac{P_{allow}\,L}{4}$

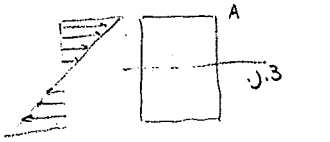
והכוח הוא $P_{allow} = 4M/L = 8,394\ N$

לכן, המרומז ⊥ יכול לסבול כוח יותר גדול מהמרומז על ⊥

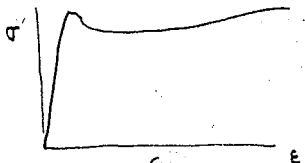
התנהגות פלסטית של חתכים.

נזכיר ש-שלת החתך מתאר לחיבור הנדסי כאשר המאמץ המקסימלי מגיע למאמץ הכניעה: $\sigma_{max} = \frac{Mc}{I} = \sigma_{yp}$

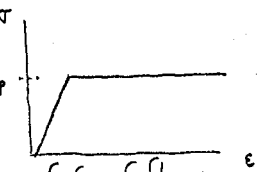
כלומן ההומג, המאמצים בשלת-החתך עדיין נמצאים



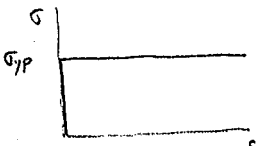
תלוי בהגדרה תיאורטית:



תאור רגיל

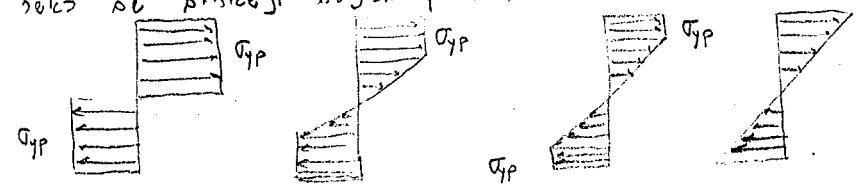


תאור אלסטי-פלסטי



תאור קשית-פלסטי טהור

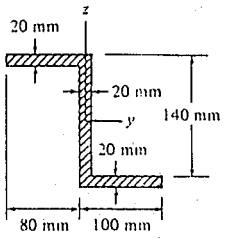
בגלל שמאמצים בתיאוריה אלסטי-פלסטי. לכן, כאשר המאמץ מגיע למאמץ הכניעה המאמץ נישאר בשלב הזה. וכאשר העומס ממשיך לגדול, המאמצים מתפתחים האיזור A ממשיכים לגדל עד שמגיעים למאמץ הכניעה וגישארים שם כאשר מגדילים את



תורת החוזק 2

תרגיל מס' 2

*9.3 (a) For the Z section shown, calculate I_y, I_z, and I_{yz}. Also locate the principal centroidal axes and calculate the principal moments of inertia.



(a)

PROBLEM 9.3

9.9 What centroidal axial force must be applied to the beam of Fig. 9.2.3 to make $|\sigma_{xA}| = |\sigma_{xB}|$ after bending and axial stresses are superposed?

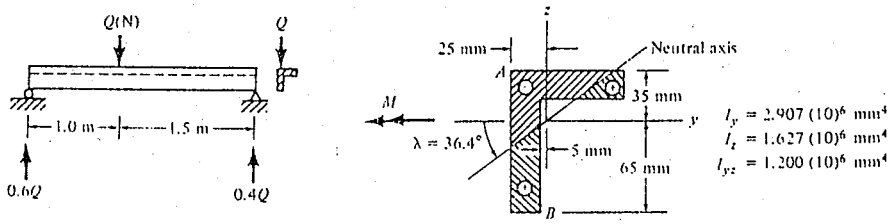
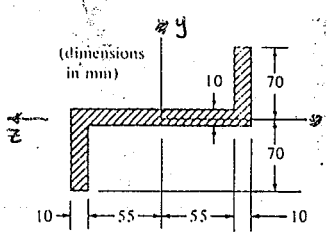


FIGURE 9.2.3.

*9.13 (a) A cantilever beam 1.6 m long has the Z section shown. It carries a load P = 2500 N in the $-y$ direction at the free end. Find the maximum magnitude of flexural stress (either tensile or compressive). Where does this stress appear?

(b) Repeat part (a) with load P redirected so that it acts in the z direction.

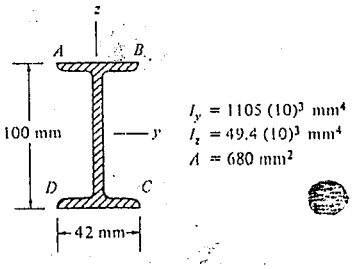
Answer: (a) 184 MPa y = 55mm z = 70mm



PROBLEM 9.13

*9.15 A column has the I section shown. A compressive force P parallel to the axis of the column is applied at corner A. Find axial stress σ_x in terms of P at the remaining three corners B, C, and D.

$\sigma_x = 0.00972P$ (MPa if P in N).



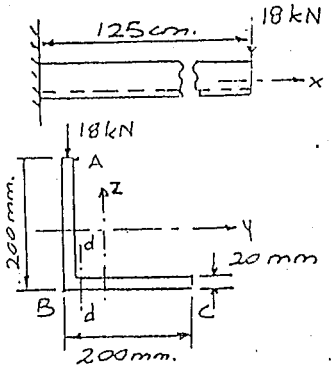
PROBLEM 9.15

5. A structural angle (L200 x 200 x 20) is used as a horizontal cantilever beam 125 cm long, and is subjected to a vertical concentrated load of 18,000 N acting through the shear center of the cross section as shown below.

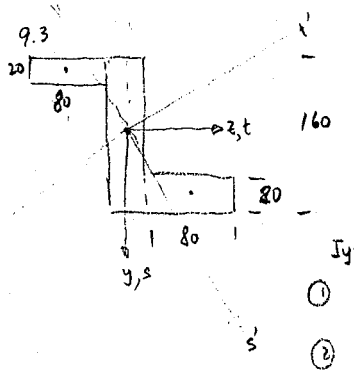
(a) Calculate the flexural stress σ_x at the points A, B and C at the fixed end of the beam, as well as the average shear stress τ acting along the line d-d of this cross section (see figure).

(b) Find the position of the Neutral Axis and the direction of the deflection. Show on a sketch of the cross-section, indicating the orientation with respect to the y-axis.

Given : $I_{zz} = I_{yy} = 2875$ cm⁴.



Answer: σ_{xA} = 145.6 MPa, σ_{xB} = −109.3MPa, $|\tau|$ = 2.9MPa



$I_{zz} = 22.613\times10^6\ mm^4$

$I_{yy} = 9.813\times10^6\ mm^4$

$I_{yz} = +11.2\times10^6\ mm^4$

	I_{zz}	A	d^2
①	$\frac{1}{12}80\cdot20^3$	1600	70^2
②	$\frac{1}{12}20\cdot(160)^3$	3200	0
③	$\frac{1}{12}\cdot80\,(20)^3$	1600	70^2

	I_{yy}	A	e^2
	$\frac{1}{12}20\cdot80^3$	1600	50^2
	$\frac{1}{12}160\cdot20^3$	3200	0
	$\frac{1}{12}\cdot20\cdot80^3$	1600	50^2

	I_{yz}	A	e d	
①	0	20×80	50·70	$= 5.6\times10^6$
②	0	160×20	0	
③	0	20×80	−50·70	$= 5.6\times10^6$
				$11.2\times10^6\ mm^4$

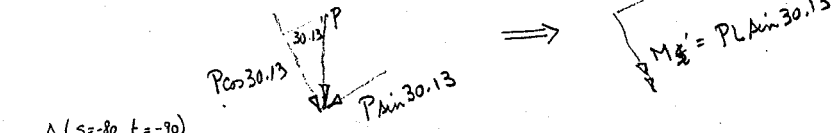
$$\tan 2\theta = \frac{2\cdot11.2\times10^6}{(22.613-9.813)\times10^6} = \frac{2I_{st}}{I_{tt}-I_{ss}} = 1.75$$

$2\theta = 60.255$

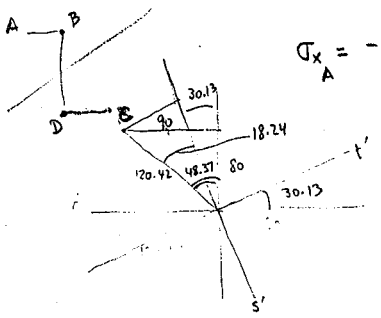
$\theta = 30.13^\circ$

$$I_{\substack{max\\min}} = \frac{I_{tt}+I_{ss}}{2} \pm \sqrt{\left(\frac{I_{tt}-I_{ss}}{2}\right)^2 + I_{yz}^2} = (16.21 \pm 12.90)\times10^6 = \begin{matrix}29.11\times10^6\\3.31\times10^6\end{matrix}$$

אזור המתיחות המקסימלי:



A (s=-80 t=-90)



$$\sigma_{x_A} = -\frac{M_t s'}{I_{tt}} + \frac{M_s' t'}{I_{ss}'} = -\frac{PL\cos30.13(-114.37)}{29.11\times10^6} + \frac{PL\sin30.13(-37.69)}{3.31\times10^6}$$

$$= PL\,(3.398 - 5.716)\times10^{-6} = -2.318\,PL\times10^{-6}$$

$\sigma_{x_C}(s'=114.37, t=37.69) = -\sigma_{x_A}$

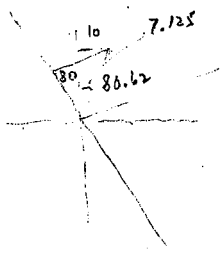
$= 2.318\,PL\times10^{-6}$

$\beta = 18.24^\circ$

$t' = -120.42\sin 18.24^\circ = -37.69$

$s' = -120.42\cos 18.24^\circ = -114.37$

B(s=-80, t=10) σ_{x_B} לחשב



$\beta = 30.13 + 7.13 = 37.26^\circ$

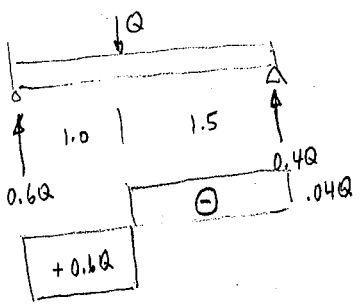
$t' = 80.62\sin 37.26 = 48.81$

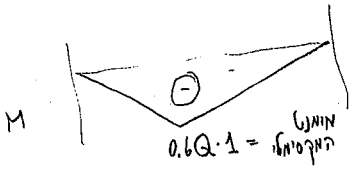
$s' = \ "\ \cos 37.26 = -64.17$

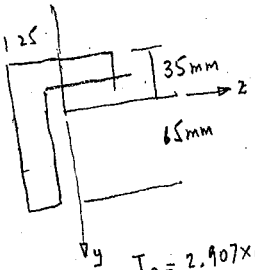
$$\sigma_{x_B} = -\frac{PL\cos30.13(-64.17)}{29.11\times10^6} + \frac{PL\sin30.13(48.81)}{3.31\times10^6}$$

$\sigma_{x_B} = PL\,(1.907+7.402)\times10^{-6} = 9.309PL\times10^{-6}$

$\sigma_{x_D} = -\sigma_{x_B} = -9.309PL\times10^{-6}$







$I_z = 2.907\times10^6$

$I_y = 1.627\times10^6$

$I_{yz} = -1.2\times10^6$

$I_{t't'} = 3.627\times10^6\ mm^4$

$I_{s's'} = 0.907\times10^6\ mm^4$

$$\tan 2\theta = \frac{2I_{yz}}{I_z - I_y} = \frac{2(-1.2)}{2.907-1.627} = -1.93$$

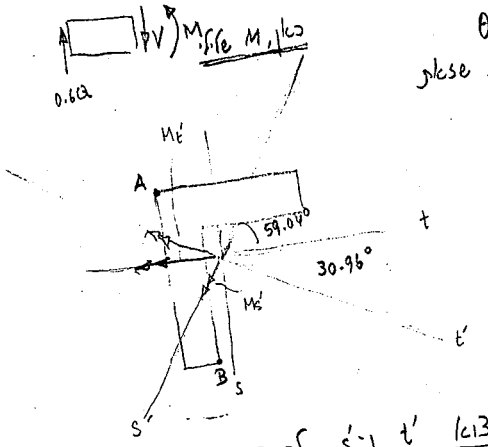
$\theta = -30.96^\circ$

כדי לדעת את הזוויות המדויקות של $I_{t't'}$ נשאל את השאלה: באיזו זווית t' יהיה t?

$M_t = -0.6Q$ N-m
$= -600Q$ N-mm

$M_{t'} = M_t \cos -30.96 = -514.3Q$

$M_{s'} = +M_t \sin -30.96 = +309.0Q$



$A(s=-35,\ t=-25)$

$t' = t\cos(-30.96^\circ) - s\sin(-30.96^\circ) = -39.46$ mm

$s' = t\sin(-30.96^\circ) + s\cos(-30.96^\circ) = -17.12$ mm

$$\sigma_{x_A} = -\frac{M_{t'}s'}{I_{t't'}} + \frac{M_{s'}t'}{I_{s's'}} = -\frac{(-514.3Q)(-17.12)}{3.627\times10^6} + \frac{(+309.0Q)(-39.46)}{0.907\times10^6} = -0.0159$$

$B(s=65,\ t=-5)$

$t' = t\cos\theta - s\sin\theta = 29.15$

$s' = t\sin\theta + s\cos\theta = 58.31$

$$\sigma_{x_B} = -\frac{M_{t'}s'}{I_{t't'}} + \frac{M_{s'}t'}{I_{s's'}} = -\frac{(-514.3Q)(58.31)}{3.627\times10^6} + \frac{(309.0Q)(29.15)}{.907\times10^6} = 0.0182Q$$

$$\sigma_x = -\frac{(M_t I_{ss}\cdot s) + M_t I_{st}\cdot t}{I_{ss}I_{tt} - I_{st}^2} = \frac{M_t(-I_{ss}s + I_{st}\cdot t)}{I_{ss}I_{tt} - I_{st}^2}$$

$$= \frac{(-600Q)\left[-1.627\times10^6\cdot(-35) + (-1.2\times10^6)(-25)\right]}{(1.627\times10^6)(2.907\times10^6) - (-1.2\times10^6)^2} = -0.0159Q \text{ MPa}$$

$s=65, t=-5$ עם σ_{x_B} נמצא

אותנטיות בשוויון שיכת

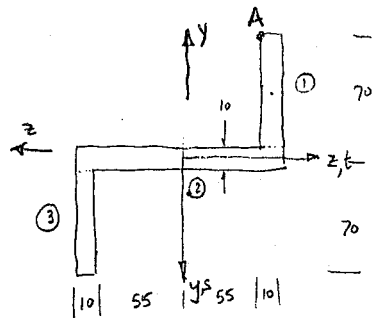
אם יש שווי 3 יחי N, המשוואה היא $N/3200$, שהם הכיסוי הנוסף $3200\ mm^2$. לכן

$$-\left(\sigma_{x_A} + N/3200\right) = \left(\sigma_{x_B} + N/3200\right)$$

פותרים את המשוואה הנתונה עם הערך של N

$N = -8.68Q$

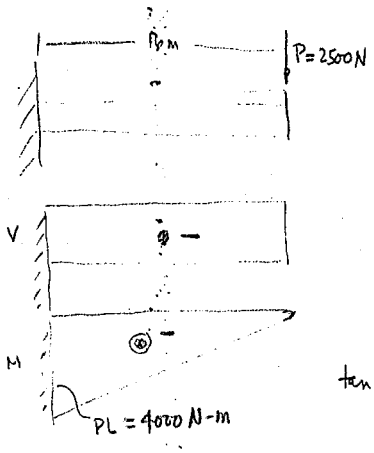
כ.ב. הערוך N נותן על הכיסוי, אפשר של Q הנוסף תיובי.



	I_{zz_o}	A	d^2	I_{tt}
①	$\frac{1}{12}(10)\cdot 65^3$	10×65	$(37.5)^2$	11.43×10^5
②	$\frac{1}{12}(130)\cdot 10^3$	10×130	0	$.1083\times10^5$
③	$\frac{1}{12}(10)\times 65^3$	10×65	$(-37.5)^2$	11.43×10^5
				2.297×10^6

	I_{yy_o}	A	e^2	I_{ss}
①	$\frac{1}{12}\cdot 65\cdot 10^3$	10×65	$(-60)^2$	23.45×10^5
②	$\frac{1}{12}\cdot 10\cdot 130^3$	10×130	0	18.31×10^5
③	$\frac{1}{12}\cdot 65\cdot 10^3$	10×65	$(60)^2$	23.45×10^5
				6.522×10^6

	I_{xy}	A	$e\cdot d$	I_{st}
①	0	10×65	$37.5\times(-60)$	-14.63×10^5
②	0	10×130	0.0	0
③	0	10×65	$(-37.5)(60)$	-14.63×10^5
				-2.925×10^6



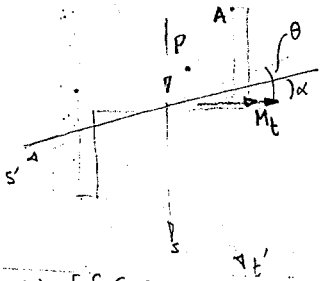
לחישוב הכיוונים הראשיים

$$\tan 2\theta = \frac{2I_{st}}{I_{tt}-I_{ss}} = \frac{2(-2.925\times10^6)}{(2.297-6.522)\times10^6} = 1.3846$$

$$2\theta = 54.16^\circ$$
$$\theta = 27.08^\circ$$

$$I_{\substack{t't'\\s's'}} = \left(\frac{I_{ss}+I_{tt}}{2}\right) \pm \sqrt{\left(\frac{I_{tt}-I_{ss}}{2}\right)^2 + I_{st}^2} = 4.41 \pm 3.608 = \begin{cases} 8.018\times10^6 \\ .8015\times10^6 \end{cases}$$

אם נרצה לבדוק את הערכים של $I_{t't'} = I_{tt}\sin^2\theta + I_{ss}\cos^2\theta - I_{st}\sin 2\theta$, נחשב את הכיוונים הנ"ל ונציב בביטוי: $\left(\frac{I_{tt}+I_{ss}}{2}\right) + \left(\frac{I_{tt}-I_{ss}}{2}\right)\cos 2\theta$



P גורם למומנט מסביב ל-t, וערכו הוא $M_t = PL = 4000\ N\text{-}m$

אם נשתמש במשוואה הבדולה, $M_s = 0$ ו-

$$\sigma_x = -\frac{M_t I_{ss} \cdot s + M_t I_{st} \cdot t}{I_{tt}I_{ss} - I_{st}^2}$$

רואים שנקודה A היא ביתר רחוקה מהציר הנייטרלי, לכן, לפי הכיוונים s ו-t $\sin\alpha = -1$ ו- $\cos\alpha = 0$

$$\sigma_{x_A} = (4000\times1000)\left[\frac{-I_{ss}\cdot s + I_{st}\cdot t}{I_{ss}I_{tt} - I_{st}^2}\right] = 4\times10^6\left[\frac{-6.522\times10^6(-2.925)(55)}{2.297\times6.522 - (-2.925)^2}\right]$$

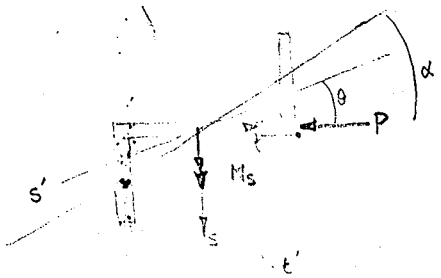
$$= 184.06\ N\text{-}mm \times mm^4/mm^8$$
$$= 184.06\ N/mm^2 = 184.06\ MPa$$

מנוסחת הכיוון הנייטרלי, לחלק (a):

$$\tan\alpha = \frac{I_{tt}\tan\theta + I_{st}}{I_{ss} + I_{st}\tan\theta}$$

$$\tan\theta = M_s/M_t = 0$$

$$-24.16^\circ = \alpha \leftarrow -.448 = \frac{I_{st}}{I_{ss}} = \tan\alpha$$



(b) P גורם למומנט מסביב ל-s, וערכו $M_s = 4000\ N\text{-}m$

$$\sigma_x = -\frac{M_s I_{st}\cdot s + M_s I_{tt}\cdot t}{I_{ss}I_{tt} - I_{st}^2}$$
$$= (4000\times1000)\left[\frac{-I_{st}\cdot s + I_{tt}\cdot t}{(I_{ss}I_{tt} - I_{st}^2)}\right]$$

יש בקורה המורכבת מאלומת הנתונה בתרשים וועלי פלדה ב 3 (ב) הראו
עיוות האומאנים תלוי ם ב- ε ($\sigma = E\varepsilon$) ולכן פלדה נקבע אחיז
פונקציה לינארית ב (c). שטח כלשהו נתון כאיזון מהתנגדות תורמות בסה"כ
את יותר לעבוד עם שטח שווה מאיזה היחידה בשטח התחתון
הטרנספורמציה. על פי שטח ניתן מאזן את החוזרים לחומר השני ולכן
יחידות אחיזת מתקבל כך שהרכיב y התחתון הטרנספורמציה A_{eq} בדומה
לגבה של התחתון המקורי A השטח.

צורת התחתון של החוזר המאזן תלויה וגם תבוטל על פי הרחב E_1/E_2
רק במכון מתקבל לשבר הנוסחה $-x$ לפחות ניתן מקור הקיצוני

$E_{פלדה} = 210$ GPa

$E_{אלומ} = 70$ GPa

פתרון

נתונה הקורה ההטרוגנית שבשרטוט

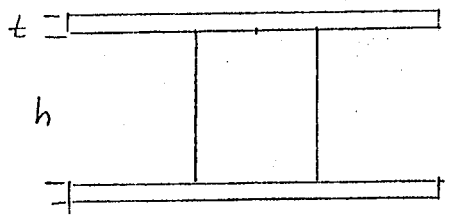
נדרש: לחשב ולשרטט פלדה σ

ולהבין עיוות כשהיון

$$\sigma_{a.Al} = \pm 9\times10^7 \text{ N/m}^2$$

$$\sigma_{a.Fe} = \pm 18\times10^7 \text{ N/m}^2$$

$b_1 = b\frac{E_2}{E_1} = 15cm$



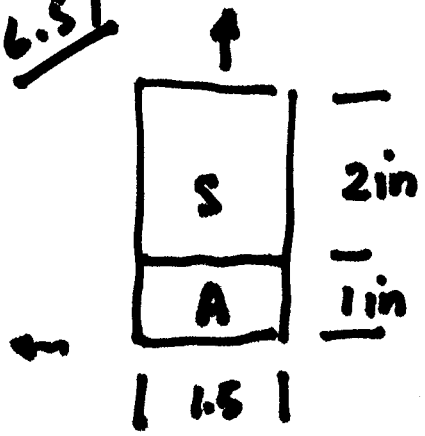
$$I_{2eq} = \frac{1}{12}\left[b_1 h^3 - 2b(h-2t)^3\right] = 823\,cm^4$$

פלדה $\sigma_{max} = \frac{M_{max}\cdot 4}{I_{eq}} = \frac{10^4 \cdot 4}{823\times10^{-8}} = 4.86\times10^7\ N/m^2 < \sigma_{all}$

אלומיניום $\sigma_{max} = \frac{E_2}{E_{AL}} \cdot \frac{Mc}{I} = 3\cdot\frac{10^4\cdot 5}{823\times10^{-8}} = 1.823\times10^8\ N/m^2 >$

$\sigma_{all\,e}$

6.57



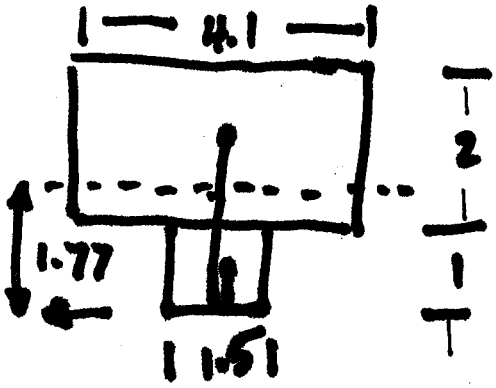
	$\bar{y}$	A	$\bar{y}A$
S	2	8.21	16.42
A	0.5	1.5	0.75
		9.71	17.17

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = 1.77 \text{ in}$$

$$\frac{E_S}{E_A} = \frac{29 \times 10^6}{10.6 \times 10^6} = 2.736$$

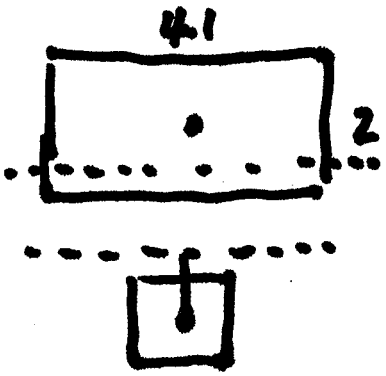
convert steel to aluminum

8.21 × 2.736
1.5 × 2.736
9.0



$$\tau = \frac{VQ}{It}$$

	$I_{z'}$	A	d	$I_z = I_{z'} + Ad^2$
S	$\frac{1}{12}(4.1)(2)^3$	8.21	-.23	.441 + 2.736 = 3.177
A	$\frac{1}{12}(1.5)(1)^3$	1.5	1.27	2.413 + .125 = 2.538
				5.715 in^4



At the interface

$Q = \bar{y}A = (.23)(4.1)(2) = 1.902 \text{ in}^3$ from upper section

$Q = \bar{y}A = (1.27)(1.5)(1) = 1.902 \text{ in}^3$ from lower section

$$Q = \bar{v}A = (1.53)(2.5)(1) = 10.95\ m^3/s$$

$$Q = \bar{v}A = (2.5)(4)(1)(5) = 10.0\ m^3/s$$

	I_x	A	d	$I_x = I_x' + Ad^2$
2	$\frac{1}{12}(4)(1)^3(5)$	5.0	2.5	$0.42 + 31.25 = 31.67$
4	$\frac{1}{12}(2)(1)^3(2)$	2.0	0.5	$0.17 + 0.50 = 0.67$
				$32.34\ m^4$

$$\tau = \frac{QV}{gI}$$

$$\frac{E^2}{E^1} = \frac{10.1 \times 10^6}{2 \times 10^6} = 5.05$$

$$\frac{E^A}{E^1} = 0.76$$

$$\bar{v} = \frac{\Sigma Av}{\Sigma A} = 1.77\ m/s$$

$\bar{v}$	A	$\bar{v}A$	$\bar{v}^2A$
2	5	8.75	14.44
0.5	2	1.0	0.25
			14.69

$$\tau = \frac{VQ}{It} = \frac{(4000\,\text{lb})(1.902\,\text{in}^3)}{(5.715\,\text{in}^4)(1.5)} = 888\,\frac{\text{lb}}{\text{sq in}}$$

$$\tau = \frac{VQ}{It'} = \frac{(4000\,\text{lb})(1.902\,\text{in}^3)}{(5.715\,\text{in}^4)(4.1\,\text{in})}$$ is not way to get τ for steel

$$\tau_{al} = G_{al}\gamma = G_{al}\,\gamma$$

$$\tau_s = G_{st}\,\gamma$$

$$\tau_s = \tau_{al}\cdot\frac{G_s}{G_{al}} = \tau_{al}\cdot\frac{\left[\frac{E_s}{2(1+\nu)}\right]}{\left[\frac{E_{al}}{2(1+\nu)}\right]}$$

$$\tau_s = \tau_{al}\cdot\frac{E_s}{E_{al}}$$

$$\frac{\tau_{al}}{G_{al}} = \gamma = \frac{\tau_s}{G_s}$$

$$\tau_s = \tau_{al}\cdot\frac{G_s}{G_{al}} \cong \tau_{al}\cdot\frac{E_s}{E_{al}}$$

$$\tau_s = \frac{VQ}{It'}\cdot\frac{E_s}{E_{al}} = \frac{(4000\,\text{lb})(1.902\,\text{in}^3)}{(5.715\,\text{in}^4)(4.1)}\cdot\frac{29\times10^6}{10.6\times10^6}$$

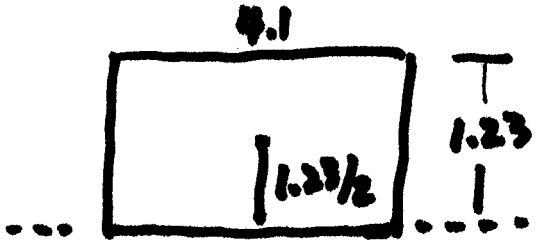
2.736

$$\frac{1}{t'}\cdot\frac{E_s}{E_{al}} = \frac{1}{t}$$ means that $t' = 2.736\times1.5$

$$\tau_s = \frac{VQ}{It_{steel}}$$

$$\sigma_b = \frac{Mc}{I} = \frac{(4000\ \text{lb})(1.625\ \text{in})}{(5.375\ \text{in}^4)(1.0)} = 1208\ \frac{\text{lb}}{\text{in}^2}$$

τ at neutral axis (ie τ_{max})



$$Q = \bar{y}A = \frac{1.23}{2}(4.1)(1.23) = 3.113\ in^3$$

$$\tau = \frac{VQ}{It'} \cdot \frac{E_s}{E_a} = \frac{VQ}{It} = \frac{(4000\ lb)(3.113\ in^3)}{(5.715\ in^4)(1.5\ in)}$$

$$= 1453\ lb/in^2$$

to as the *shear flow*. Since force is usually measured in pounds, shear flow q has units of pounds per inch. Then, recalling that $dM/dx = -V$, one obtains the following expression for the shear flow in beams:

$$q = \frac{dF}{dx} = -\frac{dM}{dx}\frac{1}{I}\int_{\substack{\text{area}\\fghj}} y\,dA = \frac{V\,A_{fghj}\bar{y}}{I} = \frac{V\,Q}{I} \qquad (7\text{-}5)$$

In this equation I stands for the moment of inertia of the entire cross-sectional area around the neutral axis, just as it does in the flexure formula from which it came. The total shearing force at the section investigated is represented by V, and the integral of $y\,dA$ for determining Q extends only over the cross-sectional area of the beam to one side of this area at which q is investigated.

In retrospect, note carefully that Eq. 7-5 was derived on the basis of the elastic flexure formula, but no term for a bending moment appears in the final expressions. This resulted from the fact that only the change in the bending moments at the adjoining sections had to be considered, and the latter quantity is linked with the shear V. The shear V was substituted for $-dM/dx$, and this masks the origin of the established relations. Equation 7-5 is very useful in determining the necessary interconnection between the elements making up a beam. This will be illustrated by examples.

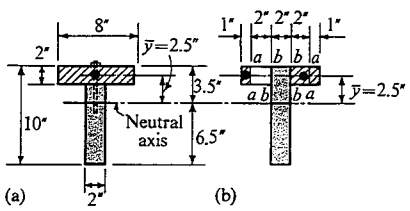


Fig. 7-6

EXAMPLE 7-1

Two long wooden planks form a T section of a beam as shown in Fig. 7-6(a). If this beam transmits a constant vertical shear of 690 lb, find the necessary spacing of the nails between the two planks to make the beam act as a unit. Assume that the allowable shearing force per nail is 150 lb.

SOLUTION

In attacking such problems the analyst must ask: What part of a beam has a tendency to slide longitudinally from the remainder? Here it is the plane of contact of the two planks; Eq. 7-5 must be applied to determine the shear flow in this plane. To do this the neutral axis of the whole section and its moment of inertia around the neutral axis must be found. Then as V is known and Q is defined as the statical moment of the area of the upper plank around the neutral axis, q may be determined. The distance y_c from the top to the neutral axis is

$$y_c = \frac{2(8)1 + 2(8)6}{2(8) + 2(8)} = 3.5 \text{ in.}$$

$$I = \frac{8(2)^3}{12} + (2)8(2.5)^2 + \frac{2(8)^3}{12} + (2)8(2.5)^2 = 291 \text{ in.}^4$$

$$Q = A_{fghj}\bar{y} = (2)8(3.5 - 1) = 40 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{690(40)}{291} = 95 \text{ lb per in.}$$

Thus, a force of 95 lb must be transferred from one plank to the other in every linear inch along the length of the beam. However, from the data given, each nail is capable of resisting a force of 150 lb, hence one nail can take care of 150/95 = 1.59 linear inches along the length of the beam. As shear remains constant at the consecutive sections of the beam, the nails should be spaced throughout at 1.59-in. intervals. In a practical problem a 1.5-in. spacing would probably be used.

SOLUTION FOR AN ALTERNATE ARRANGEMENT OF PLANKS

If, instead of using the two planks as above, a beam of the same cross section were made from five pieces, Fig. 7-6(b), a different nailing schedule would be required.

To begin, the shear flow between one of the 1-in.-by-2-in. pieces and the remainder of the beam is found, and although the contact surface a-a is vertical, the procedure is the same as before. The push or pull on an element is built up in the same manner as formerly:

$$Q = A_{fghj}\bar{y} = (1)2(2.5) = 5 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{690(5)}{291} = 11.8 \text{ lb per in.}$$

If the same nails as before are used to join the 1-in.-by-2-in. piece to the 2-in.-by-2-in. piece, they may be 150/11.8 = 12.7 in. apart. This nailing applies to both sections a-a.

To determine the shear flow between the 2-in.-by-10-in. vertical piece and either one of the 2-in.-by-2-in. pieces, the whole 3-in.-by-2-in. area must be used to determine Q. It is the difference of pushes (or pulls) on this whole area that causes the unbalanced force which must be transferred at the surface b-b:

$$Q = A_{fghj}\bar{y} = (3)2(2.5) = 15 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{690(15)}{291} = 35.6 \text{ lb per in.}$$

Space nails at 150/35.6 = 4.2 in. or, in practice, 4-in. intervals along the length of the beam in both sections b-b. These nails could be driven in first and then the 1-in.-by-2-in. pieces put on.

EXAMPLE 7-2

A simple beam on a 20-ft span carries a load of 200 lb per foot including its own weight. The beam cross section is to be made from several full-sized wooden pieces as in Fig. 7-7(a). Specify the spacing of the ½-in.

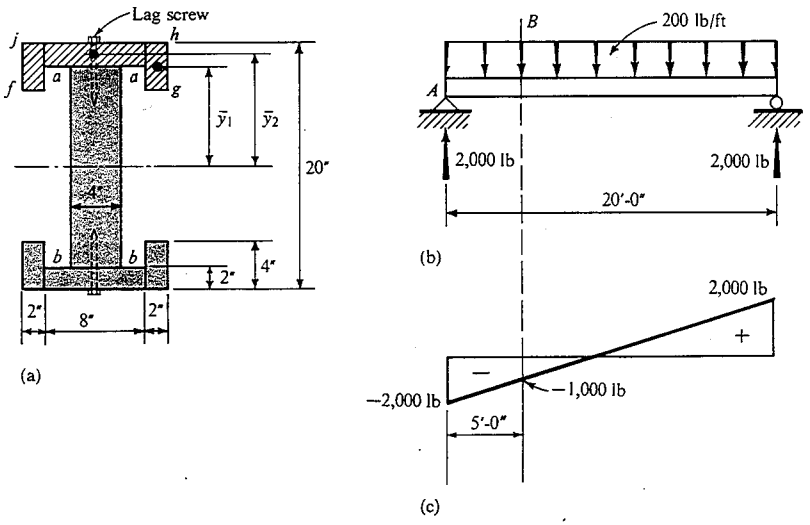


Fig. 7-7

lag screws shown which is necessary to fasten this beam together. Assume that one ½-in. lag screw, as determined by laboratory tests, is good for 500 lb when transmitting a lateral load parallel to the grain of the wood. For the entire section I is equal to 6,060 in.[4]

SOLUTION

To find the spacing of the lag screws, the shear flow at section *a-a* must be determined. The loading on the given beam is shown in Fig. 7-7(b); to show the variation of the shear along the beam, the shear diagram is constructed in Fig. 7-7(c). Then, to apply the shear flow formula, Q must be determined. This is done by considering the shaded area to one side of the cut *a-a* in Fig. 7-7(a). The statical moment of this area is most conveniently computed by multiplying the area of the two 2-in.-by-4-in. pieces by the distance from their centroid to the neutral axis of the beam and adding to this product a similar quantity for the 2-in.-by-8-in. piece. The largest shear flow occurs at the supports, as the largest vertical shears V of 2,000 lb act there:

$$Q = A_{fghj}\bar{y} = \sum A_i\bar{y}_i = 2A_1\bar{y}_1 + A_2\bar{y}_2$$

$$= 2(2)4(8) + 2(8)9 = 272 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{2{,}000(272)}{6{,}060} = 90 \text{ lb per in.}$$

At the supports the spacing of the lag screws must be 500/90 = 5.56 in. apart. This spacing of the lag screws applies only at a section where the shear V is equal to 2,000 lb. Similar calculations for a section where V = 1,000 lb gives q = 45 lb per inch, and the spacing of the lag screws becomes 500/45 = 11.12 in. Thus it is proper to specify the use of ½-in. lag screws at 5½ in. on centers for a distance of 5 ft nearest both the supports and at 11 in. for the middle half of the beam. A greater refinement in making the transition from one spacing of fastenings to another may be desirable in some problems. The same spacing of lag screws should be used at the section *b-b* as at the section *a-a*.



(a)

Fig. 7-8. Typical beam sec of several components: (a) I beam reinforced with plat

In a manner analogous to the above, the spacing of rivets or bolts in fabricated beams made from continuous angles and plates, Fig. 7-8, may be determined. Welding requirements are established similarly. The nominal shearing stress in a rivet is determined by dividing the total shearing force transmitted by the rivet (shear flow times spacing of the rivets) by the cross-sectional area of the rivet.

7-4. THE SHEARING STRESS FORMULA FOR BEAMS

The shearing stress formula for beams may be obtained from the shear flow formula. Analogously to the earlier procedure, an element of a beam may be isolated between two adjoining sections taken perpendicular to the axis of the beam. Then by passing another imaginary section through this element parallel to the axis of the beam, a new element is obtained, which corresponds to the element of one "plank" used in the earlier derivations. A side view of such an element is shown in Fig. 7-9(a), where the imaginary longitudinal cut is made at a distance y_1 from the neutral axis.* The cross-sectional area of the beam is shown in Fig. 7-9(c).

If shearing forces exist at the sections through the beam, a different bending moment acts at section A than at B. Hence more push or pull is developed on one side of the area *fghj* than on the other, and, as before, the difference in the longitudinal forces in a distance dx is

$$dF = -\frac{dM}{I}\int_{\substack{\text{area}\\fghj}} y\,dA = -\frac{dM}{I}A_{fghi}\bar{y} = -\frac{dM}{I}Q$$

The force equilibrating dF is developed in the plane of the longi-

* Since $dM/dx = -V$, for a positive V the change in moment $dM = -V\,dx$. For this reason $M_A > M_B$ and the magnitudes of the normal stresses in Fig. 7-9(a) are shown accordingly.

occur in the plastic zones, no unbalance in longitudinal forces occurs and no shearing stresses are developed.

This elementary solution has been refined by using a more carefully formulated criterion of yielding caused by the simultaneous action of normal and shearing stresses.* Some fundamental aspects of the interaction of such stresses will be considered in Chapter 9.

EXAMPLE 7-6

An I beam is loaded as in Fig. 7-14(a). If it has the cross section shown in Fig. 7-14(c), determine the shearing stresses at the levels indicated. Neglect the weight of the beam.

SOLUTION

A free-body diagram of a segment of the beam is in Fig. 7-14(b). It is seen from this diagram that the vertical shear at every section is 50 kips. Bending moments do not enter directly into the present problem. The shear flow at the various levels of the beam is computed in the table below using Eq. 7-5. Since $\tau = q/t$ (Eq. 7-6), the shearing stresses are obtained by dividing the shear flows by the respective widths of the beam.

$$I = \frac{6(12)^3}{12} - \frac{(5.5)(11)^3}{12} = 254 \text{ in.}^4$$

For use in Eq. 7-5 the ratio $V/I = -50{,}000/254 = -197$ lb/in.^{4}

Level	A_{rsh_j}*	$\bar{y}$**	$Q = A_{rsh_j}\bar{y}$	$q = VQ/I$	t	τ, psi
1-1	0	6	0	0	6.0	0
2-2	(0.5)6 = 3.00	5.75	17.25	−3,400	6.0 0.5	−570 −6,800
3-3	(0.5)6 = 3.00 (0.5)(0.5) = 0.25	5.75 5.25	17.25 1.31 | 18.56	−3,650	0.5	−7,300
4-4	(0.5)6 = 3.00 (0.5)(5.5) = 2.75	5.75 2.75	17.25 7.56 | 24.81	−4,890	0.5	−9,780

* A_{rsh_j} is the partial area of the cross section above a given level in square inches.
** $\bar{y}$ is the distance from the neutral axis to the centroid of the partial area in inches.

The negative signs of τ show that, for the section considered, the stresses act downward on the right face of the elements. The sense of the shearing stresses acting on the section coincides with the sense of the shearing force V. For this reason a strict adherence to the sign convention is often unnecessary. It is always true that $\int_A \tau\, dA$ is equal to V and has the same sense.

* D. C. Drucker, "The Effect of Shear on the Plastic Bending of Beams," *Journal of Applied Mechanics*, **23** (1956), pp. 509–14.

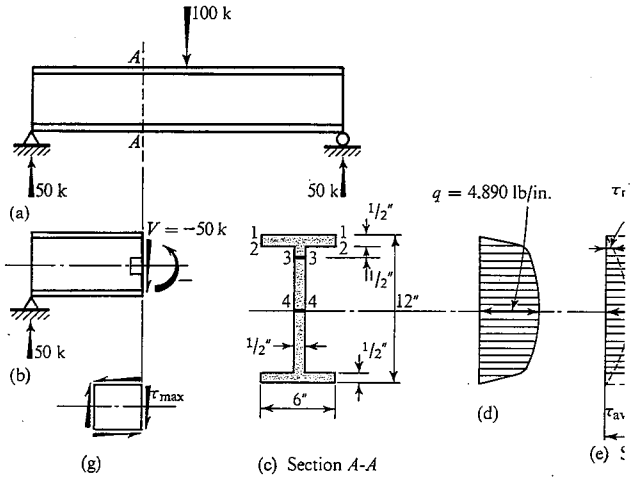


Fig. 7-14

Note that at the level 2-2 two widths are used to determine the shearing stress—one just above the line 2-2, and one just below. A width of 6 in. corresponds to the first case, and 0.5 in. to the second. This transition point will be discussed in the next article. The results obtained, which by virtue of symmetry are also applicable to the lower half of the section, are plotted in Fig. 7-14(d) and (e). By a method similar to the one used in the preceding example, it may be shown that the curves in Fig. 7-14(e) are parts of a second-degree parabola.

The variation of the shearing stress indicated by Fig. 7-14(e) may be interpreted as is shown in Fig. 7-14(f). The maximum shearing stress occurs at the neutral axis; the vertical shearing stresses throughout the web of the beam are nearly of the same magnitude. The shearing stresses occurring in the flanges are very small. For this reason the maximum shearing stress in an I beam is often approximated by dividing the total shear V by the cross-sectional area of the web (area *abcd* in Fig. 7-14(f)). Hence

$$(\tau_{\max})_{\text{approx}} = V/A_{\text{web}} \qquad \textbf{(7-9)}$$

In the example considered this gives

$$(\tau_{\max})_{\text{approx}} = \frac{50{,}000}{(0.5)12} = 8{,}330 \text{ psi}$$

This stress differs by about 15 per cent from the one found by the accurate formula. For most cross sections a much closer approximation

best approach. For example, if for the beam segment in Fig. 7-17(b) positive bending moments increase toward the reader, larger normal forces act on the nearer cross section. For the elements shown, $\tau t\,dx$ or $q\,dx$ must aid the smaller force acting on the partial area of the cross section. This determines the sense of the shearing stresses in the longitudinal cuts. Numerically equal shearing stresses act on the mutually perpendicular planes of an infinitesimal element, and the shearing stresses on such planes either meet or part with their directional arrowheads at a corner. In this manner the sense of the shearing stresses in the plane of the cross section becomes known.

The magnitude of the shearing stresses varies for the different vertical cuts. For example, if the cut *c-c* in Fig. 7-17(a) is at the edge of the beam, the shaded area of the beam's cross section is zero. However, if the thickness of the flange is constant, and the cut *c-c* is made progressively closer to the web, the shaded area increases from zero at a linear rate.

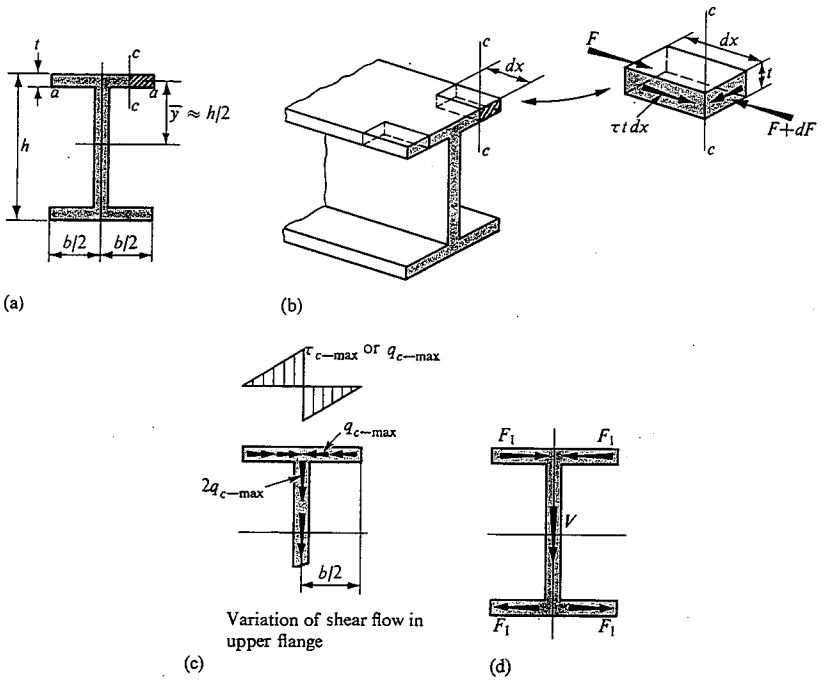


Fig. 7-17. Existence of shearing forces in the flange of an *I* beam which act perpendicularly to the axis of symmetry.

Moreover, as $\bar{y}$ remains constant for any such area, Q also increases linearly from zero toward the web. Therefore, since V and I are constant at any section through the beam, the shear flow $q_c = VQ/I$ follows the same variation. If the thickness of the flange remains the same, the shearing stress $\tau_c = VQ/It$ varies similarly. The same variation of q_c and τ_c exists on either side of the axis of symmetry of the cross section. However, these quantities in the plane of the cross section act in opposite directions on the two sides as may be determined by isolating another flange element to the left side of the web in Fig. 7-17(b). The variation of these shearing stresses or shear flows is represented in Fig. 7-17(c), where it is assumed that the web is very thin.

In common with all stresses, the shearing stresses in Fig. 7-17(c), when integrated over the area on which they act, are equivalent to a force. The magnitude of the horizontal force F_1 for one-half the flange, Fig. 7-17(d), is equal to the average shearing stress multiplied by one-half the flange area, i.e.,

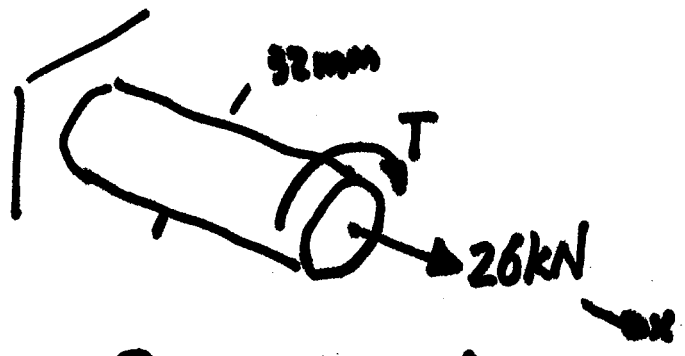
$$F_1 = (\tau_{c\text{-max}}/2)(bt/2) \qquad \text{or} \qquad F_1 = (q_{c\text{-max}}/2)(b/2)$$

These horizontal forces act in the upper and lower flanges. Because of the symmetry of the cross section, these equal forces occur in pairs and oppose each other; thus they cause no apparent external effect.

To determine the shear flow at the juncture of the flange and the web (cut *a-a* in Fig. 7-17(a)), the whole area of the flange times $\bar{y}$ must be used in computing the value of Q. However, since in finding $q_{c\text{-max}}$ one-half the flange area times the same $\bar{y}$ has already been used, the sum of the two horizontal shear flows coming in from opposite sides gives the vertical shear flow at the cut *a-a*. Hence, figuratively speaking, the horizontal shear flows turn through 90° and merge to become the vertical shear flow. Then the shear flows at the various horizontal cuts through the web may be determined in the manner explained in the preceding articles. Moreover, as the resistance to the vertical shear V in thin-walled I beams is developed mainly in the web, it is so shown in Fig. 7-17(d). The sense of the shearing stresses and shear flows in the web coincides with the direction of the shear V. Note that the vertical shear flow "splits" upon reaching the lower flange. This is represented in Fig. 7-17(d) by the two forces F_1, which are the result of the horizontal shear flows in the flanges.

The shearing forces which act at a section of an I beam are shown in Fig. 7-17(d), and, for equilibrium, the applied vertical forces must act through the centroid of the cross-sectional area to be coincident with V. If the forces are so applied, no torsion of the member will occur. This is true for all sections having cross-sectional areas with an axis of symmetry. Thus, to avoid torsion of such members, the applied forces must act in the plane of symmetry of the cross section and the axis of the beam. A beam with an unsymmetrical section will be discussed next.

7.96



$$\frac{\sigma_1}{\sigma_t} - \frac{\sigma_3}{|\sigma_c|} = 1$$

$$\sigma_x = P/A = \frac{26000\,N}{\pi (32\times10^{-3})^2/4} = 32.38\ MPa = 32.38\times10^6\ Pa$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 16.16\ MPa$$

$$\sigma_1 = \sigma_{max} = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{ave} + \tau_{max}$$

$$\sigma_3 = \sigma_{min} = \text{"} - \text{"} = \sigma_{ave} - \tau_{max}$$

$$\frac{\sigma_{ave}+\tau_{max}}{\sigma_t} - \frac{(\sigma_{ave}-\tau_{max})}{|\sigma_c|} = 1$$

$$\tau_{max}\cdot\left[\frac{1}{\sigma_t} + \frac{1}{|\sigma_c|}\right] = 1 - \frac{\sigma_{ave}}{\sigma_t} + \frac{\sigma_{ave}}{|\sigma_c|}$$

$$\tau_{max} = \frac{1 - \sigma_{ave}\left[\frac{1}{\sigma_t} - \frac{1}{|\sigma_c|}\right]}{\left[\frac{1}{\sigma_t} + \frac{1}{|\sigma_c|}\right]}$$

$$= \frac{1 - 16.16\left[\frac{1}{75} - \frac{1}{120}\right]}{\left[\frac{1}{75} + \frac{1}{120}\right]} = 34.61\ MPa$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 34.61 \qquad \frac{\sigma_x - \sigma_y}{2} = 16.16 \text{ MPa}$$

$$\tau_{xy} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{max}^2} = 30.61 \text{ MPa}$$

$$\tau = \frac{Tc}{J} \qquad J = \frac{\pi c^4}{2}$$

$$= \frac{2T}{\pi c^3} \qquad T = \frac{\pi c^3 \tau}{2}$$

$$= \frac{\pi (16 \times 10^{-3})^3 \cdot 30.61 \times 10^6}{2}$$

$$= 196.9 \text{ N-m}$$

$$\sigma_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 34.81 \text{ MPa} \qquad \frac{\sigma_x + \sigma_y}{2} = 15.19 \text{ MPa}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.61 \text{ MPa}$$

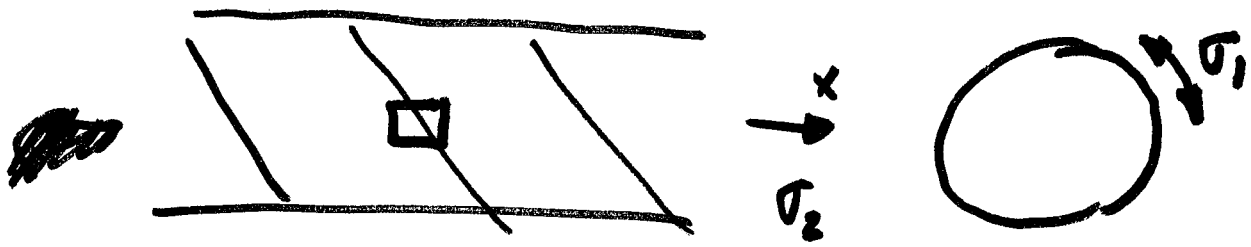
$$f = \frac{1}{T} \qquad T = \frac{2\pi}{\omega}$$

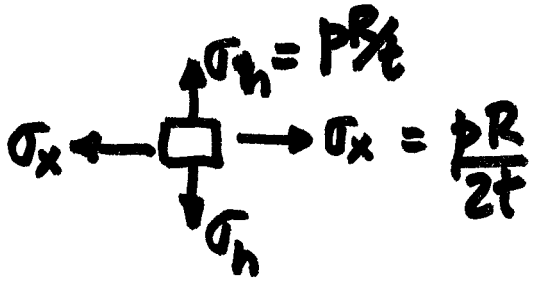
$$\omega = \frac{2\pi}{T_c} \qquad T = \frac{2\pi r_c^2}{v}$$

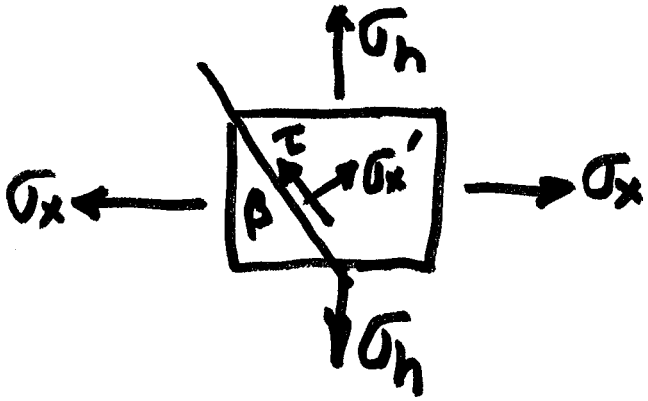
$$T = \frac{2\pi (15 \times 10^{-3})^2 \cdot 30.61 \times 10^6}{2}$$

$$= 1.44 \text{ N-m}$$

7.117



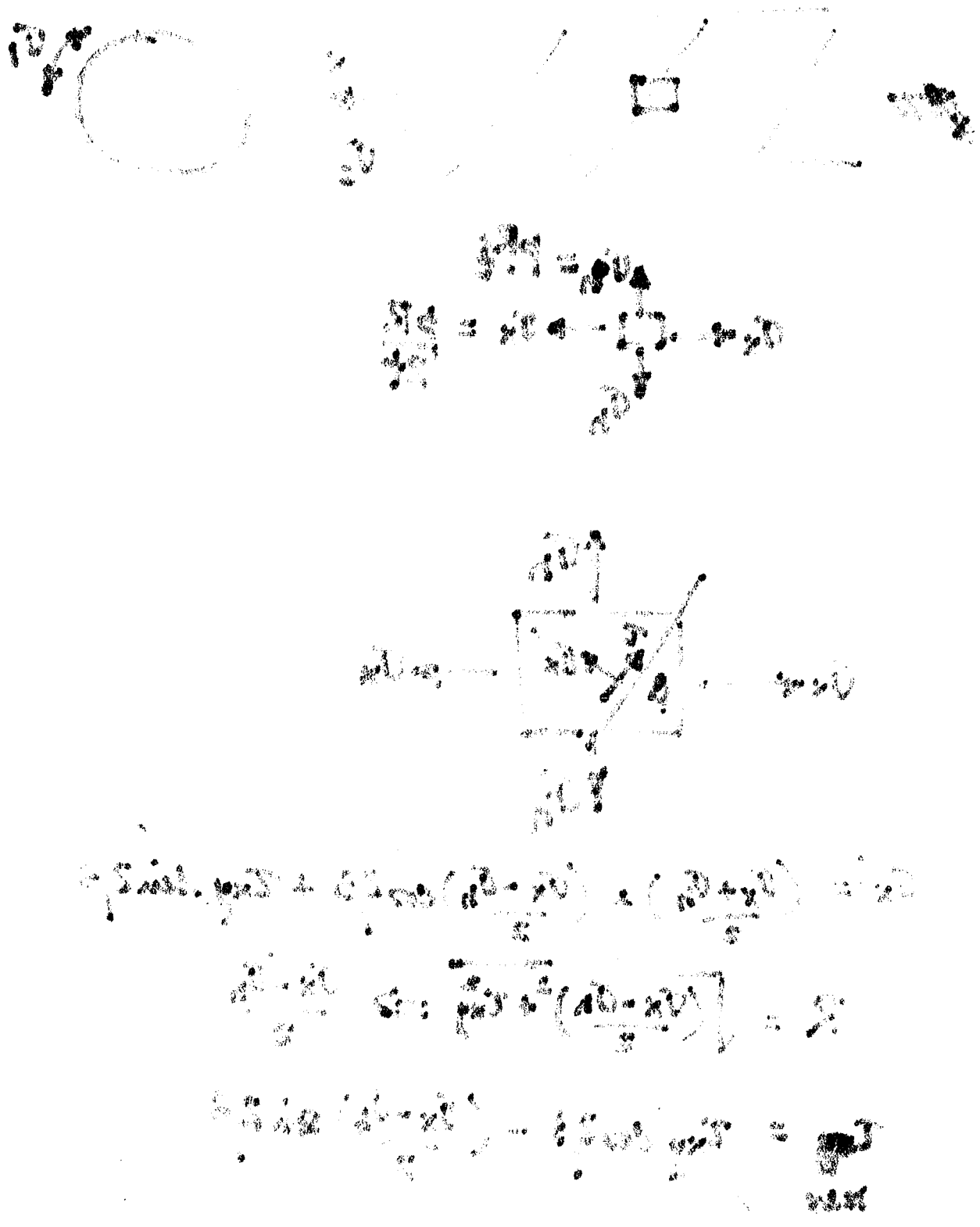




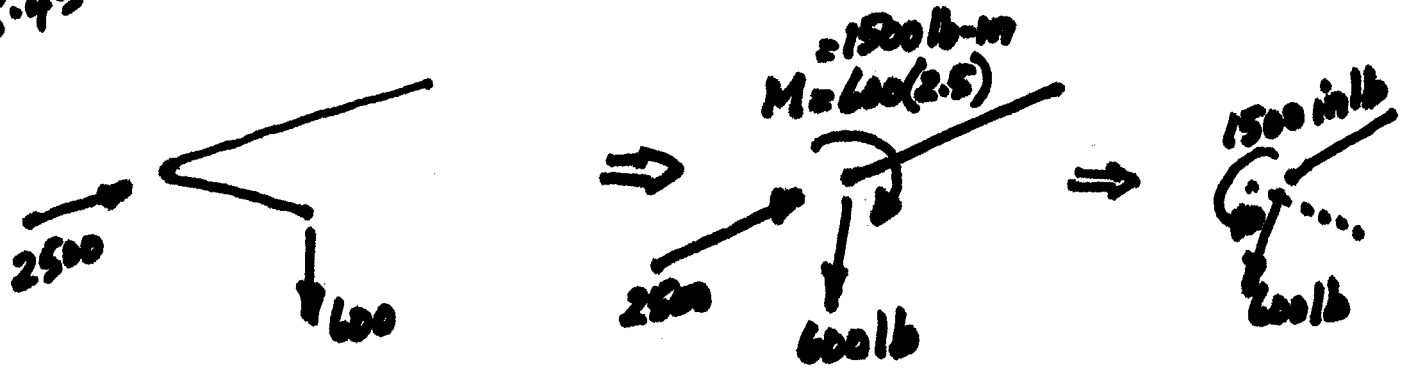
$$\sigma_x' = \left(\frac{\sigma_x+\sigma_h}{2}\right) + \left(\frac{\sigma_x-\sigma_h}{2}\right)\cos 2\beta + \tau_{xy}\sin 2\beta$$

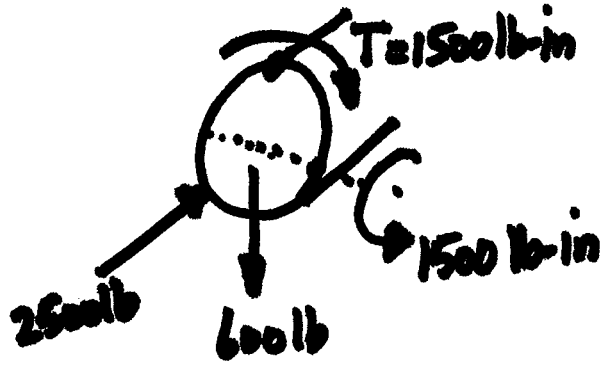
$$R = \sqrt{\left(\frac{\sigma_x-\sigma_h}{2}\right)^2 + \tau_{xy}^2} \Rightarrow \frac{\sigma_x-\sigma_h}{2}$$

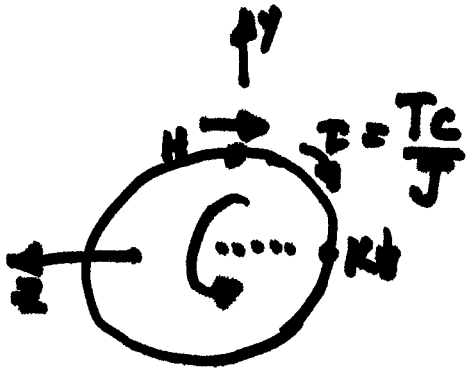
$$\tau_{max} = \tau_{xy}\cos 2\beta - \left(\frac{\sigma_x-\sigma_h}{2}\right)\sin 2\beta$$



8.43







$$\sigma_x = \frac{P}{A} = \frac{-2500\text{lb}}{\pi d^2/4} \quad @ H \& K$$

@ H τ due to 600lb = 0

K " " " " $= \frac{VQ}{It}$

$$\sigma_x = \frac{-My}{I} = \frac{-(-1500)(d/2)}{\frac{1}{4}\pi(d/2)^4} \quad @ H$$

$= 0$ @ K

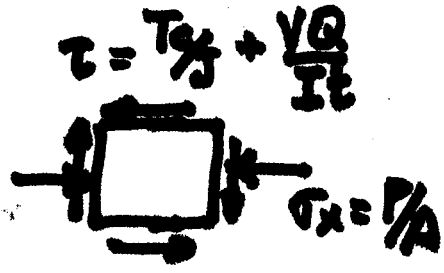
at H



$$\sigma_x = +P/A - \frac{My}{I}$$

at K





$Q = \bar{y}A$

$\bar{y} = \frac{4r}{3\pi}$ $A = \pi r^2/2$ $t = d$

$$\sigma_{\substack{max \\ min}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

@K $\sigma_x = P/A$

$\sigma_y = 0$

$\tau_{xy} = \frac{Tc}{J} + \frac{VQ}{It}$

@H $\sigma_x = P/A - \frac{My}{I}$

$\sigma_y = 0$

$\tau_{xy} = \frac{Tc}{J}$

$$\int_{-\infty}^{\infty} \left[\left(\frac{x}{2} - \frac{x^2}{2} \right) + \left(\frac{x}{2} - \frac{x^2}{2} \right) \right] dx = \text{area}$$

$$\int_{-\infty}^{\infty} \left[\left(\frac{x}{2} - \frac{x^2}{2} \right) + \left(\frac{x}{2} - \frac{x^2}{2} \right) \right] = \text{area}$$

(G) $x = 0$

$$\frac{\pi}{2} + \frac{\pi}{2} = \text{area}$$

$$\frac{\pi}{2} = \text{area}$$

(H) $x = 0$

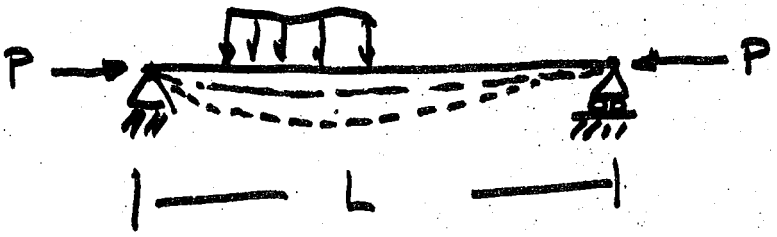
MOHR CRITERIA

$$\frac{\sigma_1}{\sigma_t} - \frac{\sigma_3}{|\sigma_c|} \geq 1 \Rightarrow \text{FAIL}$$

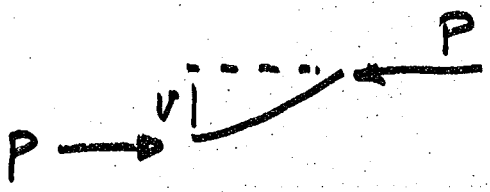
$$\frac{12.9}{14} - \frac{(-30.9)}{120} = 1.18 \overset{?}{\geq} 1 \quad \text{YES} \qquad \times$$

FAILURE – DUE TO EXCESSIVE DISPLACEMENT

BUCKLING



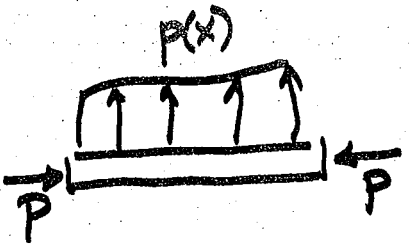
$$\frac{v}{L} \ll 1$$



Normally we have $EI\,v'' = M$

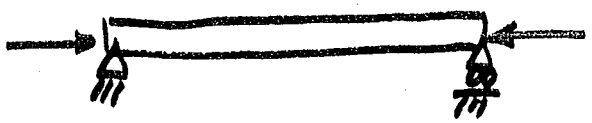
$$EI\,v'' + Pv = M$$

$$(EI\,v'')'' + (Pv)'' = +p(x)$$



IF $EI = \text{const.}$ $\quad v'' + \frac{P}{EI} v'' = \frac{p(x)}{EI}$

$$\frac{P}{EI} = \lambda^2$$



$$v^{IV} + \lambda^2 v'' = 0$$

$$v''' + \lambda^2 v' = C_1$$

$$v'' + \lambda^2 v = C_1 x + C_2$$

homogeneous $v'' + \lambda^2 v = 0 \Rightarrow A\cos\lambda x + B\sin\lambda x = v_h$

particular $v'' + \lambda^2 v = C_1 x + C_2$

$\rightarrow v_p = D$

$\rightarrow v_p = Cx$

$v_p = D : v_p'' + \lambda^2 v_p = 0 + \lambda^2 D = C_2 \qquad D = C_2/\lambda^2$

$v_T = v_h + v_p = A\cos\lambda x + B\sin\lambda x + Cx + D$

$v_T'' = -\lambda^2 A\cos\lambda x - \lambda^2 B\sin\lambda x$

$v(x=0) = 0 \qquad v(x=L) = 0$

$v''(x=0) = 0 \qquad v''(x=L) = 0$

$$\begin{aligned} A + 0B + 0C + D &= 0 \\ -\lambda^2 A + 0\cdot B + 0C + 0\cdot D &= 0 \\ A\cos\lambda L + B\sin\lambda L + CL + D &= 0 \\ -\lambda^2 A\cos\lambda L - \lambda^2 B\sin\lambda L + 0\cdot C + 0\cdot D &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -\lambda^2 & 0 & 0 & 0 \\ \cos\lambda L & \sin\lambda L & L & 1 \\ -\lambda^2\cos\lambda L & -\lambda^2\sin\lambda L & 0 & 0 \end{bmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

det matrix $= L\lambda^4 \sin\lambda L = 0 \Rightarrow \sin\lambda L = 0$

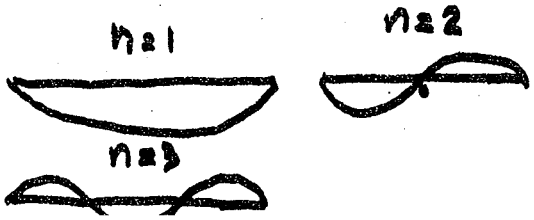
$\Rightarrow \lambda L = n\pi$

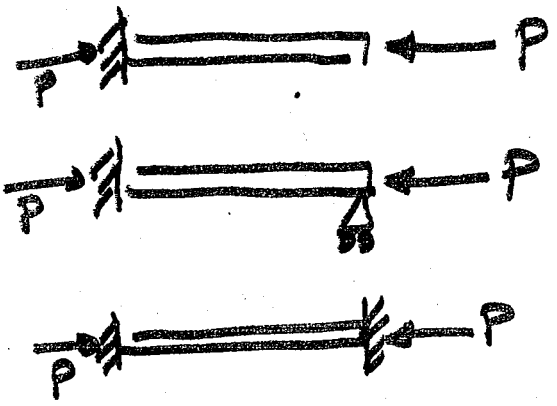
$$\sqrt{\frac{P}{EI}} = \lambda = \frac{n\pi}{L}$$

$$P = \frac{n^2\pi^2}{L^2} \cdot EI \qquad n=1 \quad P = \frac{\pi^2 EI}{L^2}$$ Euler Buckling Load.



$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

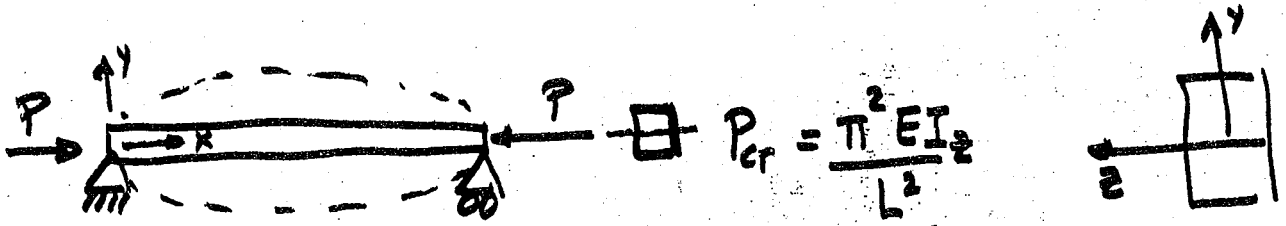




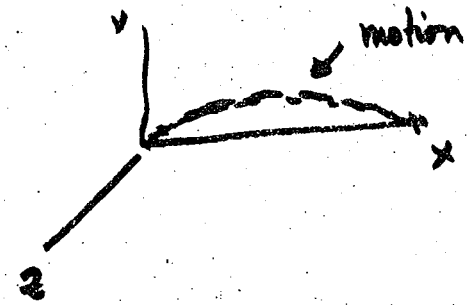
$$P = \frac{1}{4} P_{cr} = \frac{1}{4} \frac{\pi^2 E I}{L^2}$$

$$P_{cr} = 2.05\, P_{cr} = 2.05\, \frac{\pi^2 E I}{L^2}$$

$$P = 4 P_{cr} = \frac{4 \pi^2 E I}{L^2}$$

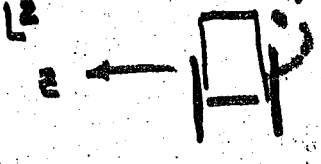


$$P_{cr} = \frac{\pi^2 E I_z}{L^2}$$



motion in x-z plane

$$P_{cr} = \frac{4 \pi^2 E I_y}{L^2} \quad >> \text{ if } I_z = I_y$$



if $4 I_y < I_z$ then buckling is out of the plane of the paper

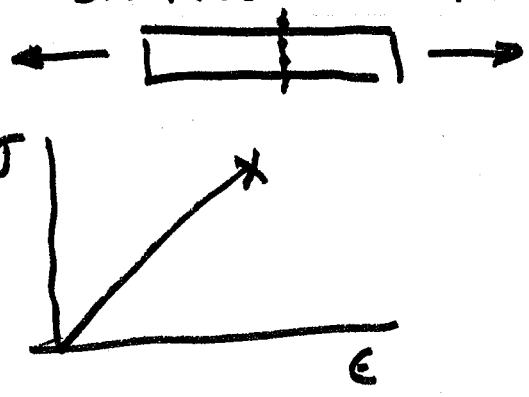


EMA 3702

EXTRA SESSIONS

1. FAILURE CRITERION - Chpt 7
2. BUCKLING - Chpt 10

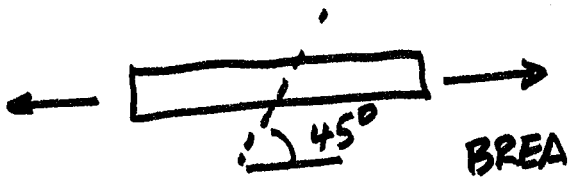
BRITTLE MATERIAL



BREAK ⊥ TO AXIS OF LOAD $\sigma_1 > \sigma_{max}$

MATERIALS W/ PLASTICITY





BREAK SURFACE IS 45° TO APPLIED LOAD.

TRESCA - MATERIALS W/ PLASTICITY

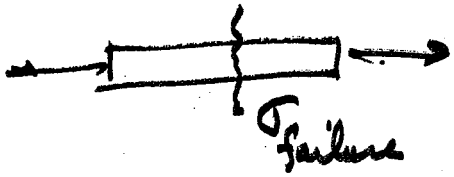
$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\sigma_{1\,max} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{2\,min} = (\ \ ) - \sqrt{\ \ \ \

PRINCIPAL STRESS - BRITTLE

$\sigma_1 > \sigma_{failure} \Rightarrow$ FAILED



IN 2-D

$$\sigma_{max} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} > \sigma_{fail}$$

FOR EXAMPLE $\sigma_1 \geq \sigma_{yp}$ is failure

$\sigma_{max} = \geq \sigma_{yp}$

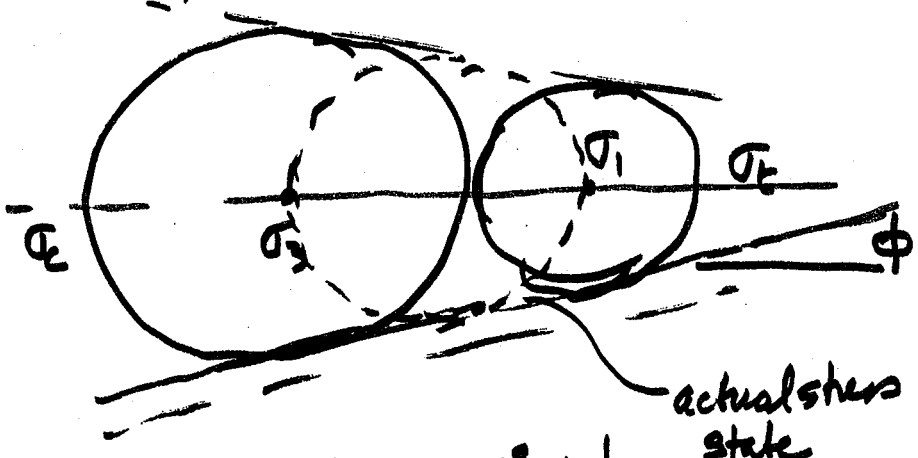
PRINC. STRAIN



$$\epsilon_1 > \epsilon_{failure} \Rightarrow \frac{1}{2}\left[(\epsilon_x + \epsilon_y) + \sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}\right] > \epsilon_{fail}$$

FOR EXAMPE $\epsilon_{fail} = \frac{\sigma_{yp}}{E}$

MOHR CRITERION - BRITTLE MATERIAL



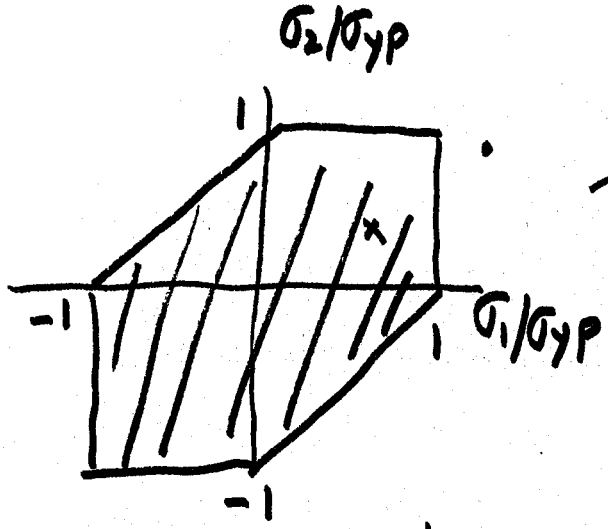
plane of failure $45° + \frac{1}{2}\phi$

FAILURE IF $\frac{\sigma_1}{\sigma_t} - \frac{\sigma_3}{|\sigma_c|} \geq 1$

σ_t - max failure stress in tension

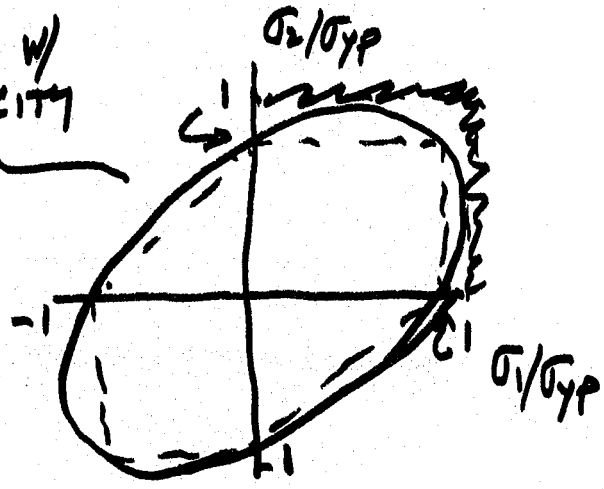
σ_c - max failure stress in compression

MATERIALS W/ PLASTICITY



inside you are safe

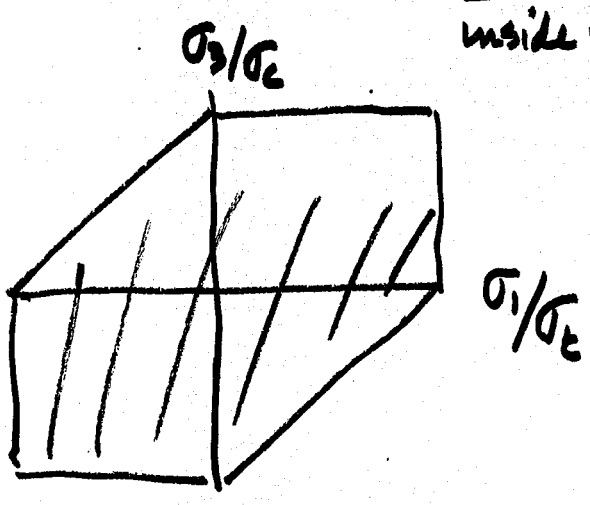
TRESCA



VON MISES

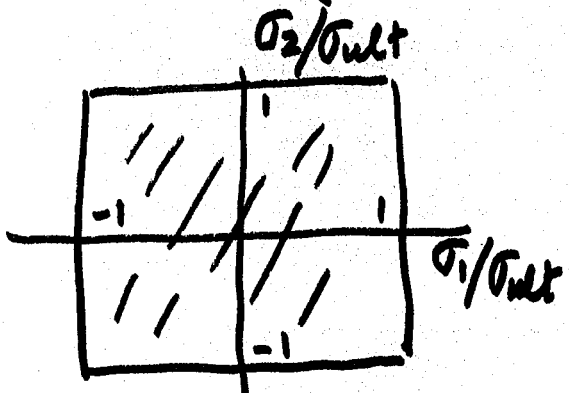
inside you are safe

$$\begin{pmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_y & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



MOHR

inside you are safe



PRINCIPAL STRESS

inside safe

BRITTLE

TENSION & COMP. TESTS FOR BRITTLE MATERIAL

$\sigma_t = 14$ MPa $\qquad \sigma_c = 120$ MPa

UNDER A CERTAIN LOADING : $\sigma_x = 0$, $\sigma_y = -18$ MPa, $\tau_{xy} = 20$ MPa

ARE WE SAFE ? CONSIDER PLANE STRESS ($\sigma_z, \tau_{xz}, \tau_{yz} = 0$)

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0-18}{2} + \sqrt{\left(\frac{0-(-18)}{2}\right)^2 + (20)^2} = -9 + 21.9 = 12.9 \text{ MPa}$$

$$\sigma_2 = \left(\frac{\sigma_x - \sigma_y}{2}\right) - \sqrt{\qquad} = -9 - 21.9 = -30.9 \text{ MPa}$$

$$\sigma_1 > 0 > \sigma_2 \implies 12.9 > 0 > -30.9$$

$$\sigma_e = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right]^{1/2}$$

$$= \frac{1}{\sqrt{2}}\left[(0+18)^2 + (-18-0)^2 + (0-0)^2 + 6(20^2 + 0^2 + 0^2)\right]^{1/2}$$

$$= 39.04 \text{ MPa}$$

PRINC. STRESS CRIT: $\sigma_1 > \sigma_t \Rightarrow$ FAIL $\qquad 12.9 \overset{?}{>} 14$ NO ✓

TRESCA : $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{12.9 - (-30.9)}{2} = 21.9$ MPa

$\Rightarrow$ 1-D TEST $\quad \sigma_{yp} = \sigma_t = 14$ MPa , $\tau_{max} = \dfrac{\sigma_{yp}}{2} = 7$ MPa

$\tau_{max} \overset{?}{>} \dfrac{\sigma_{yp}}{2} \qquad 21.9 > 7$ MPa YES ×

MISES : $\sigma_e = 39.04$ MPa $\qquad \sigma_e \geq \sigma_{yp} \Rightarrow$ FAIL $\qquad \sigma_{yp} = \sigma_t = 14$ MPa

$39.04 \overset{?}{>} 14$ YES ×

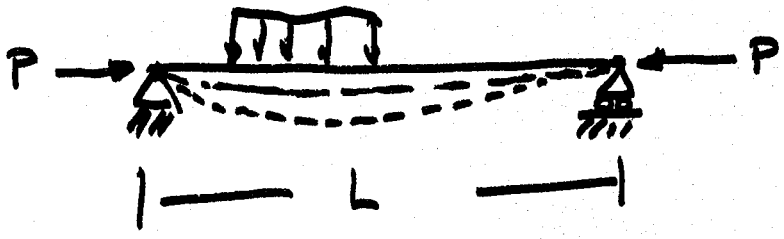
MOHR CRITERIA

$$\frac{\sigma_1}{\sigma_t} - \frac{\sigma_3}{|\sigma_c|} \geq 1 \Rightarrow \text{FAIL}$$

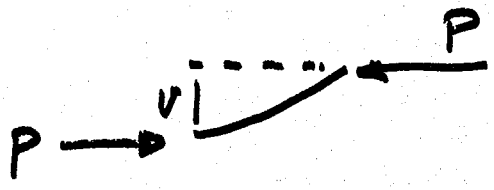
$$\frac{12.9}{14} - \frac{(-30.9)}{120} = 1.18 \overset{?}{\geq} 1 \quad \text{YES} \qquad \text{X}$$

FAILURE – DUE TO EXCESSIVE DISPLACEMENT

BUCKLING



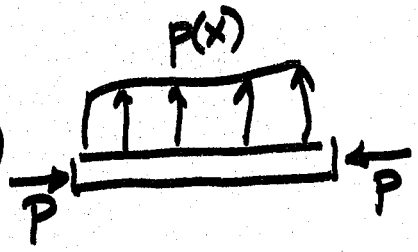
$$\frac{v}{L} \ll 1$$



Normally we have $EI\,v'' = M$

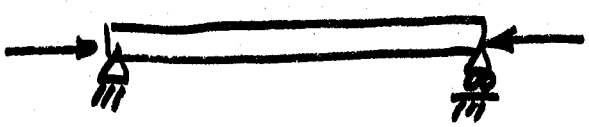
$$EI\,v'' + Pv = M$$

$$(EI\,v'')'' + (Pv)'' = +p(x)$$



IF $EI = \text{const.}$ $\quad v^{IV} + \frac{P}{EI} v'' = \frac{p(x)}{EI}$

$$\frac{P}{EI} = \lambda^2$$



$$v^{IV} + \lambda^2 v'' = 0$$

$$v''' + \lambda^2 v' = C_1$$

$$v'' + \lambda^2 v = C_1 x + C_2$$

homog soln $v'' + \lambda^2 v = 0 \Rightarrow A\cos\lambda x + B\sin\lambda x = v_h$

Particular $v'' + \lambda^2 v = C_1 x + C_2$

$\rightarrow v_p = D$

$v_p = Cx$

$v_p = D$: $v_p'' + \lambda^2 v_p = 0 + \lambda^2 D = C_2 \quad D = C_2/\lambda^2$

$v_T = v_h + v_p = A\cos\lambda x + B\sin\lambda x + Cx + D$

$v_T'' = -\lambda^2 A\cos\lambda x - \lambda^2 B\sin\lambda x$

$v(x=0) = 0 \qquad v(x=L) = 0$

$v''(x=0) = 0 \qquad v''(x=L) = 0$

$$A + 0B + 0C + D = 0$$
$$-\lambda^2 A + 0\cdot B + 0C + 0\cdot D = 0$$
$$A\cos\lambda L + B\sin\lambda L + CL + D = 0$$
$$-\lambda^2 A\cos\lambda L - \lambda^2 B\sin\lambda L + 0\cdot C + 0\cdot D = 0$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -\lambda^2 & 0 & 0 & 0 \\ \cos\lambda L & \sin\lambda L & L & 1 \\ -\lambda^2\cos\lambda L & -\lambda^2\sin\lambda L & 0 & 0 \end{bmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

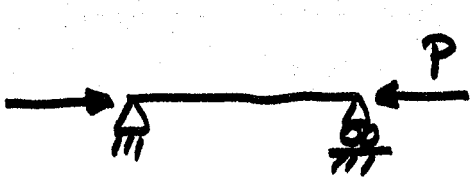
$\det$ matrix $= L\lambda^4 \sin\lambda L = 0 \Rightarrow \sin\lambda L = 0$

$\Rightarrow \lambda L = n\pi$

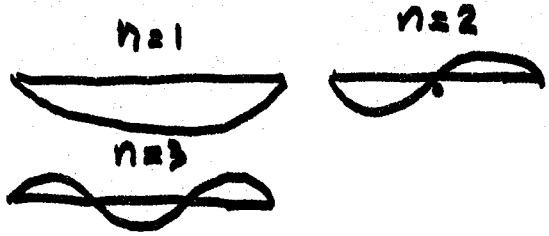
$$\sqrt{\frac{P}{EI}} = \lambda = \frac{n\pi}{L}$$

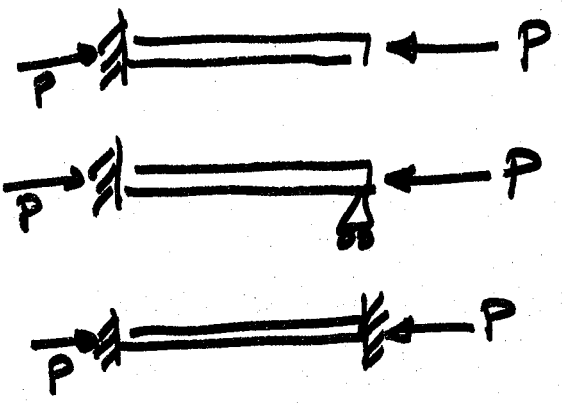
$$P = \frac{n^2\pi^2}{L^2} \cdot EI \qquad n=1 \quad P = \frac{\pi^2 EI}{L^2}$$

Euler Buckling Load.



$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

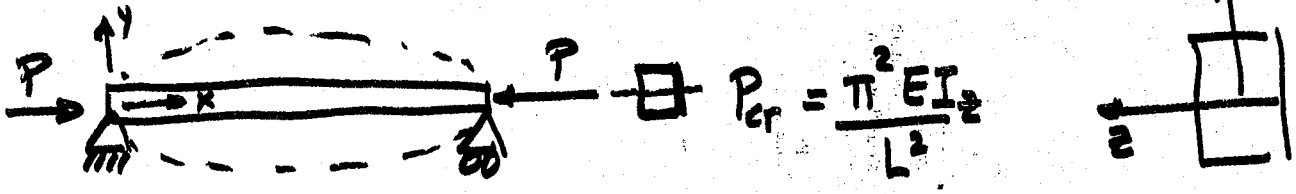




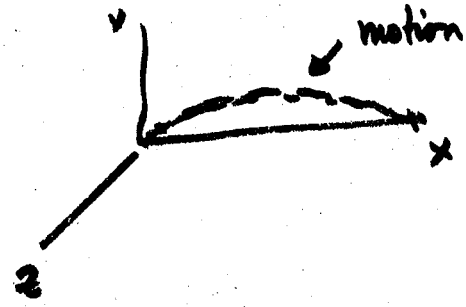
$$P = \frac{1}{4} P_{cr} = \frac{1}{4} \frac{\pi^2 EI}{L^2}$$

$$P = 2.05\, P_{cr} = 2.05\, \frac{\pi^2 EI}{L^2}$$

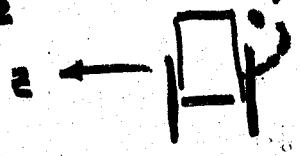
$$P = 4 P_{cr} = \frac{4\pi^2 EI}{L^2}$$



$$P_{cr} = \frac{\pi^2 E I_z}{L^2}$$



$$P_{cr} = \frac{4\pi^2 E I_y}{L^2} \quad >> \text{ if } I_z = I_y$$

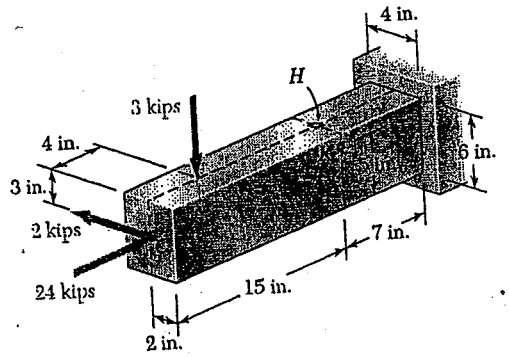


if $4I_y < I_z$ then buckling is out of the plane of the paper



Problem 8.51

8.51 Three forces are applied to the cantilever beam shown. Determine the principal stresses and the maximum shearing stress at point *H*.



At the section containing points H and K, the axial and shearing forces are:

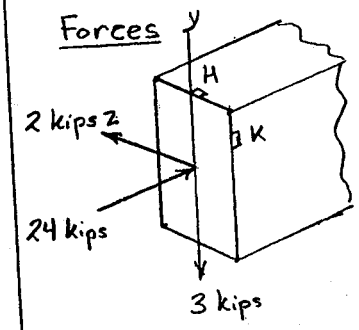
$P = 24$ kips, $V =$ 3 kips vertical, 2 kips horizontal

The bending moment components are:

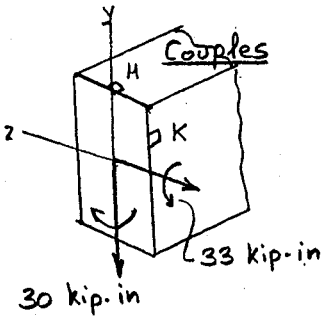
about horizontal axis: $M = (15-4)(3) = 33$ kip·in

about vertical axis $M = (15)(2) = 30$ kip·in

Forces



Couples



Section properties

$A = (4)(6) = 24$ in^2

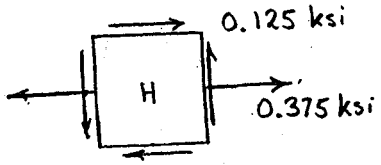
$I_z = \frac{1}{12}(4)(6)^3 = 72$ in^4

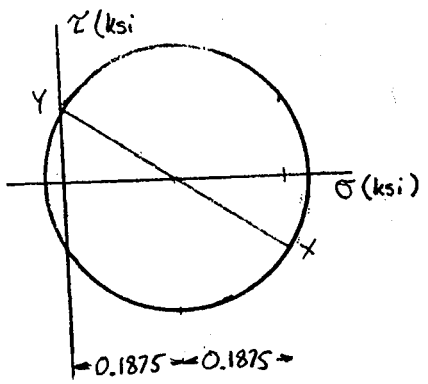
$I_y = \frac{1}{12}(6)(4)^3 = 32$ in^4

At point H:

$$\sigma = -\frac{P}{A} + \frac{Mc}{I} = -\frac{24}{24} + \frac{(33)(3)}{72} \qquad \sigma = 0.375 \text{ ksi}$$

$$\tau = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2}{24} \qquad \tau = 0.125 \text{ ksi}$$





$$\sigma_c = \frac{0.375}{2} = 0.1875 \text{ ksi}$$

$$R = \sqrt{\left(\frac{0.375}{2}\right)^2 + (0.125)^2} = 0.2253 \text{ ksi}$$

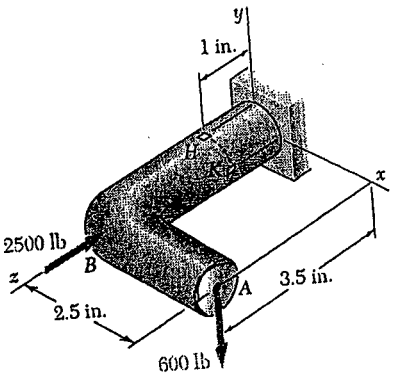
$\sigma_a = \sigma_c + R$ $\qquad \sigma_a = 0.413$ ksi

$\sigma_b = \sigma_c - R$ $\qquad \sigma_b = -0.0378$ ksi

$\tau_{max} = R$ $\qquad \tau_{max} = 0.225$ ksi

Problem 8.43

8.43 Forces are applied at points *A* and *B* of the solid cast-iron bracket shown. Knowing that the bracket has a diameter of 0.8 in., determine the principal stresses and the maximum shearing stress (*a*) point *H*, (*b*) point *K*.



At the section containing points H and K

$P = 2500$ lb (compression).

$V_y = -600$ lb. $\quad V_x = 0$.

$M_x = (3.5-1)(600) = 1500$ lb·in.

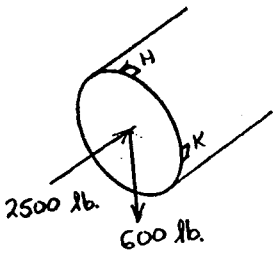
$M_y = 0 \quad M_z = -(2.5)(600) = -1500$ lb·in.

$c = \frac{1}{2}d = 0.4$ in

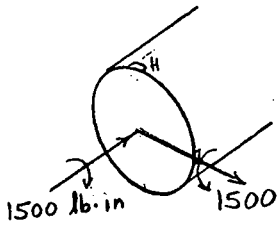
$A = \pi c^2 = 0.50265$ in^2.

$I = \frac{\pi}{4}c^4 = 20.106\times10^{-3}$ in^4.

$J = 2I = 40.212\times10^{-3}$ in^4.



Forces

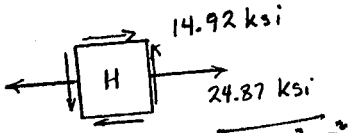


Couples

For semi circle $Q = \frac{2}{3}c^3 = 42.667\times10^{-6}$ in^3.

(a) At point H: $\sigma_H = \frac{P}{A} + \frac{Mc}{I} = -\frac{2500}{0.50265} + \frac{(1500)(0.4)}{20.106\times10^{-3}} = 24.87\times10^3$ psi.

$\tau_H = \frac{Tc}{J} = \frac{(1500)(0.4)}{40.212\times10^{-3}} = 14.92\times10^3$ psi.



$\sigma_{max} = \frac{(\sigma_1+\sigma_2)}{2} \pm \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2+\tau_0^2}$

$\sigma_{ave} = \frac{24.87}{2} = 12.435$ ksi

$R = \sqrt{\left(\frac{24.87}{2}\right)^2 + (14.92)^2} = 19.423$ ksi

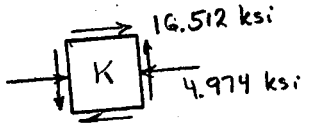
$\sigma_{max} = \sigma_{ave} + R$ $\quad \sigma_{max} = 31.9$ ksi

$\sigma_{min} = \sigma_{ave} - R$ $\quad \sigma_{min} = -6.99$ ksi

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$ $\quad \tau_{max} = 19.42$ ksi

(b) At point K: $\sigma_K = \frac{P}{A} = -\frac{2500}{0.50265} = -4.974\times10^3$ psi.

$\tau_K = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(1500)(0.4)}{40.212\times10^{-3}} + \frac{(600)(42.667\times10^{-6})}{(20.106\times10^{-3})(0.8)} = 16.512\times10^3$ psi.



$\sigma_{ave} = -\frac{4.974}{2} = -2.487$ ksi

$R = \sqrt{\left(-\frac{4.974}{2}\right)^2 + (16.512)^2} = 16.698$ ksi

$\sigma_{max} = \sigma_{ave} + R$ $\quad \sigma_{max} = 14.21$ ksi

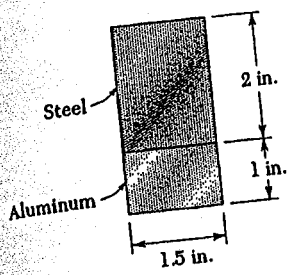
$\sigma_{min} = \sigma_{ave} - R$ $\quad \sigma_{min} = -19.18$ ksi

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$ $\quad \tau_{max} = 16.70$ ksi

$\frac{1}{2}pt + 20 = 10$

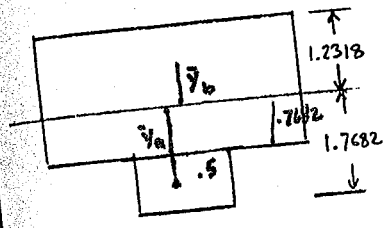
Problem 6.57

6.57 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum shearing stress in the beam. (*Hint*: Use the method indicated in Prob. 6.55.)



$n = 1$ in aluminum $\qquad n = \frac{29\times10^6 \text{ psi}}{10.6\times10^6 \text{ psi}} = 2.7358$ in steel

Part	nA (in^2)	$\bar{y}$ (in)	$nA\bar{y}$ (in^3)	d (in)	nAd^2 (in^4)	$n\bar{I}$ (in^4)
Steel	8.2074	2.0	16.4148	0.2318	0.4410	2.7358
Alum	1.5	0.5	0.75	1.2682	2.4125	0.1250
Σ	9.7074		17.1648		2.8535	2.8608



$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma A} = \frac{17.1648}{9.7074} = 1.7682$ in.

$I = \Sigma nAd^2 + \Sigma n\bar{I} = 5.7143$ in^4.

$Q = (1)(1.5)(1.2682)\left(\frac{h}{1}\right)\left(\frac{w}{1}\right)\left(\frac{\bar{y}}{1}\right) = 1.9023$ in^3.

(a) At the bonded surface:

$$\tau = \frac{VQ}{It} = \frac{(4)(1.9023)}{(5.7143)(1.5)} = 0.888 \text{ ksi}$$

$Q = (2.7358)(1.5)(1.2318)\left(\frac{1.2318}{2}\right) = 3.1133$ in^3.

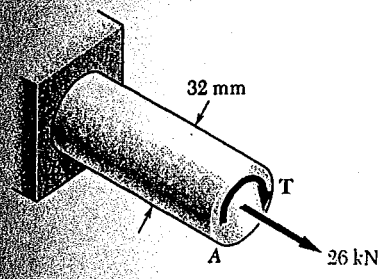
(b) At the neutral axis:

$$\tau_{max} = \frac{VQ}{It} = \frac{(4)(3.1133)}{(5.7143)(1.5)} = 1.453 \text{ ksi}$$

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Problem 7.96

7.96 The cast-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 60$ MPa and $\sigma_{UC} = 120$ MPa. Using Mohr's criterion, determine the magnitude of the torque **T** for which failure should be expected.



$P = 26 \times 10^3$ N

$A = \frac{\pi}{4}(32)^2 = 804.25\ mm^2 = 804.25 \times 10^{-6}\ m^2$

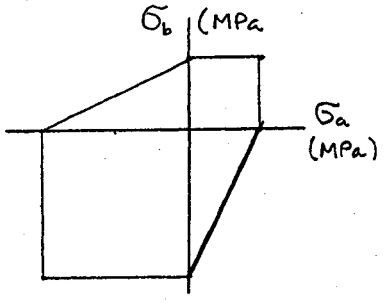
$\sigma_x = \frac{P}{A} = \frac{26 \times 10^3}{804.25 \times 10^{-6}} = 32.328 \times 10^6$ Pa $= 32.328$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(32.328 + 0) = 16.164$ MPa

$\frac{\sigma_x - \sigma_y}{2} = \frac{1}{2}(32.328 - 0) = 16.164$ MPa

$\sigma_a = \sigma_{ave} + R = 16.164 + R$ MPa, $\sigma_b = \sigma_{ave} - R = 16.164 - R$ MPa

Since $|\sigma_{ave}| < R$, stress point lies in the 4th quadrant. Equation of the 4th quadrant is



$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{16.164 + R}{60} - \frac{16.164 - R}{120} = 1$$

$$\left(\frac{1}{60} + \frac{1}{120}\right)R = 1 - \frac{16.164}{60} + \frac{16.164}{120}$$

$R = 34.612$ MPa

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right) + \tau_{xy}^2}$$

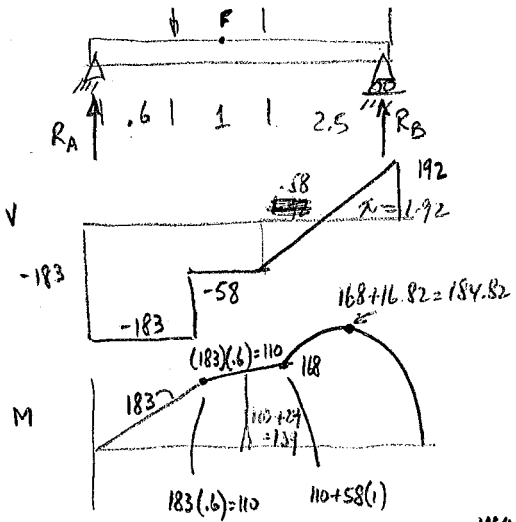
$$\tau_{xy} = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{34.612^2 - 16.164^2} = 30.606\ \text{MPa}$$

$= 30.606 \times 10^6$ Pa

For torsion $\tau_{xy} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$ where $c = \frac{1}{2}d = 16\ mm = 16 \times 10^{-3}$ m

$T = \frac{\pi}{2} c^3 \tau_{xy} = \frac{\pi}{2}(16 \times 10^{-3})^3 (30.606 \times 10^6)$ $\quad T = 196.9$ N·m

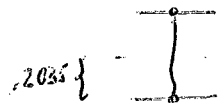
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$\Sigma M_A \circlearrowleft = 0 \quad 125(.6) + 250(2.85) - R_B(4.1) = 0$

$R_B = 192.1$ kN $\quad R_A = 183$ kN

$\frac{192}{x} = \frac{58}{2.5-x} \quad x = 1.92$



$\frac{My}{I} = \sigma$

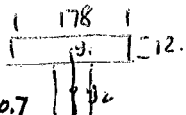
$I_{zz} = 2.16 \times 10^{-4}$ m^4

W410x60
pg 751 4th ed.

$\frac{\text{kN-m}\ (184.82)(.2035)}{2.16 \times 10^{-4}} = 174.1$ MPa max

@ F 130.96 MPa

$\tau = \frac{VQ}{Ib}$

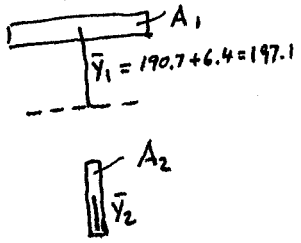


$Q = \int y\,dA = \bar{y}_1 A_1 + \bar{y}_2 A_2$

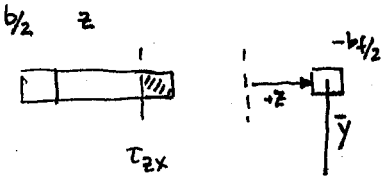
$= [178(12.8)][203.5 - 6.4] + [7.1][203.5 - 12.8][203.5 - 12.8]/2 = 5898.4$ mm^3

$= 5.87 \times 10^{-4}$ m^3

$\tau = \frac{(58\text{ kN})(5.89 \times 10^{-4})}{2.16 \times 10^{-4}(.0077)} = 20.543$ MPa



$Q = \int y\,dA = \bar{y}_1 A_1 + \bar{y}_2 A_2$



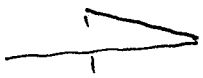
$dA = t\,dz$

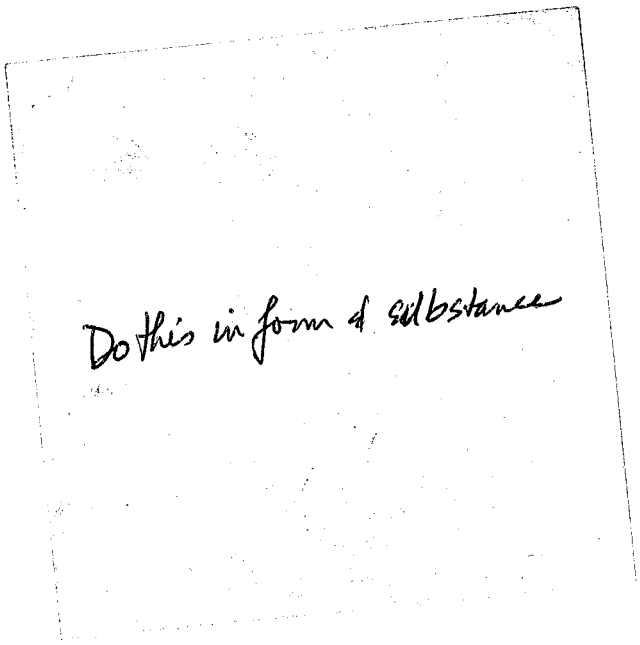
~~$\int y\,dA = \bar{y} \cdot t \cdot z + b_t/2$~~

$\int y\,dA = \int_{-b_t/2}^{z} \bar{y}\,t\,d\bar{z} = \bar{y}\,t\,\bar{z}\Big|_{-b_t/2}^{z}$

$= \bar{y}t \cdot (z + b_t/2)$

$\int y\,dA = \bar{y}t \cdot b_t/2$




Do this in form of subbstance

המכללה האקדמית יהודה ושומרון

תורת החוזק 1

תרגיל מס' 10

8.17 and 8.18 For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.

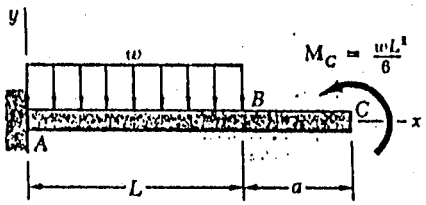


Fig. P8.18

8.25 and 8.26 For the beam and loading shown, determine (*a*) the equation of the elastic curve for portion *BD*, (*b*) the slope at *B*, (*c*) the deflection at the center *C* of the beam.

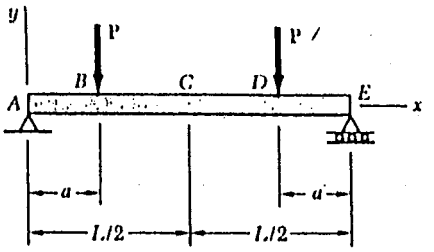


Fig. P8.26

8.41 through 8.44 For the beam and loading shown, determine the reaction at the supports, and the equation of the elastic curve.

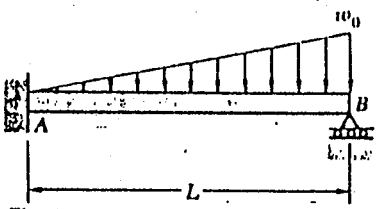


Fig. P8.44

8.45 through 8.48 Determine the reaction at the supports and draw the bending-moment diagram for the beam and loading shown.

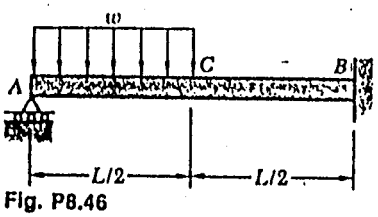


Fig. P8.46

8.55 and 8.56 Determine the reaction at *A* and draw the bending-moment diagram for the beam and loading shown. Determine the equation of the elastic curve.

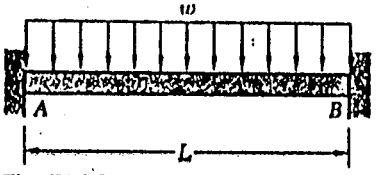
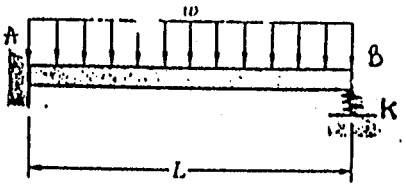


Fig. P8.55

8.42. Determine the equation of the elastic curve and draw it.



We must now find a new neutral axis since Hooke's law does not hold any more

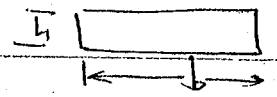
$$\Sigma F_x = 0 \quad \int_A \sigma_x \, dA = 0$$

$$\Sigma M_{\text{Neutral axis}} = 0 \quad M_{\text{plast.}} + \int_A \sigma_x \, dA \, y = 0$$

$$\Sigma F_x = 0 = \int_A \sigma_x \, dA = \sigma_{yp} \int_{\text{lower area}} dA - \sigma_{yp} \int_{\text{upper area}} dA = \sigma_{yp} \left(\int_{y_L}^{0} dA - \int_{0}^{y_u} dA \right)$$

$$\int_{y_L}^{0} dA = \int_{0}^{y_u} dA$$ areas above & below the neutral axis = equal.

For rectangle

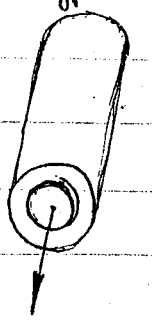


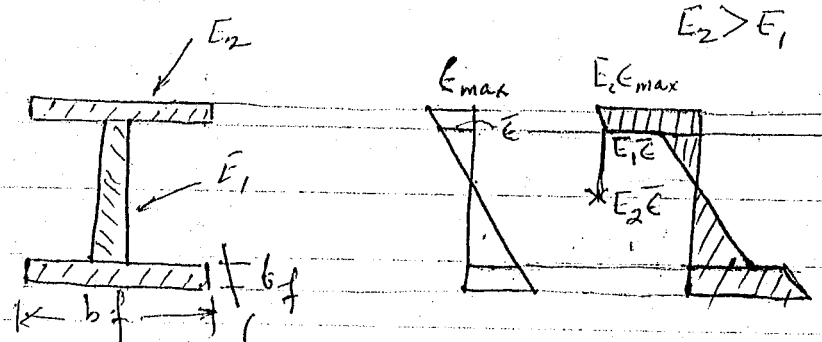
$$M_{\text{plastic}} = -\sigma_{yp} b \int_{-h/2}^{0} y \, dy + \sigma_{yp} b \int_{0}^{h/2} y \, dy$$

$$M_{\text{plastic}} = \frac{b h^2}{4} (\sigma_{yp})$$

$M_{\text{plastic}} = \sigma_{yp} Z$ Z is the plastic modulus.

$$\frac{M_{\text{plastic}}}{M_{yp}} = \frac{Z}{S} = \text{Shape Factor.}$$





$E_2 > E_1$

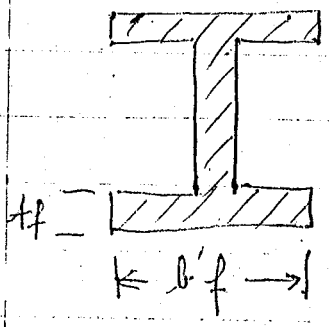
$\eta = \frac{E_2}{E_1}$

$$M = -\int_A \sigma_x \, dA(y) \qquad \sigma = \frac{My}{I}$$

Equivalent beam

web same

$$F_{in\ flange} = \int_{A_{flange}} \sigma_x \, dA = \frac{E_2(\epsilon_{max} + \bar{\epsilon})}{2} t_f \, b_f$$



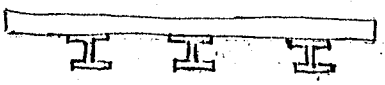
$$F'_{flange} = t_f \, b'_f \, \frac{E_1(\epsilon_{max} + \bar{\epsilon})}{2}$$

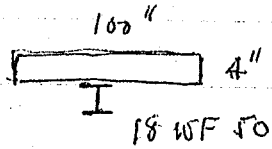
$$b'_f E_1 = b_f E_2$$

$$\boxed{b'_f = \eta \, b_f}$$

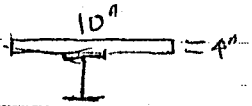
σ_{max} in original flange = $\eta \, \sigma_{max}$ of equiv. material

1) 





$$\frac{E_s}{E_c} = 10 = \eta$$



	A	y	Ay	d	Ad^2	I_o
PL	40	20	800	3	360	53.33
WF	14.71	9	132.39	−8	942	800.6
	54.71		932.39		1302	853.93

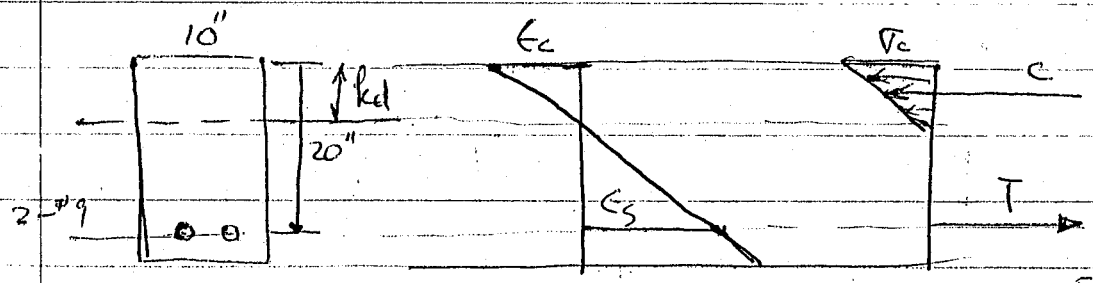
$$\bar{y} = \frac{932.39}{54.71} = 17$$

1) $\sigma^T_{max} = \frac{M}{I}\,17''$

2) $\sigma^c_{max} = \frac{M}{I}\,1''$

} WF

σ^c_{max} conc $= \frac{M}{I}\frac{5''}{n} = \frac{M}{2I}$ Concrete



A = 1" sq. $\quad n = \frac{E_s}{E_c} = 15$

$T = C \qquad \epsilon_s = \epsilon_c \frac{(20 - kd)}{kd}$

$\epsilon_s = \frac{\sigma_s}{E_s} = \frac{T}{A_s E_s}$; $\quad \epsilon_c = \frac{\sigma_c}{E_c}$; $\quad \frac{T}{A_s E_s} = \frac{\sigma_c}{E_c}\frac{(20 - kd)}{kd}$

$T = \sigma_c n A_s \frac{(20 - kd)}{kd} \qquad T = \frac{1}{2}\sigma_c kd \cdot 10$

$T = 5\sigma_c kd$

$nA_s = 30$

$I = \sum (Ad^2 + I_o) \qquad \frac{bh^3}{12} = \frac{Ah^2}{12}$

$= \frac{10 \cdot kd^3}{12} + \frac{10 kd^3}{4} + 30 \cdot \frac{(1\frac{1}{8})^2}{12}$ ← not needed.

$+ 30(20 - kd)^2 \qquad \sigma_c = \frac{M}{I} kd$

מכח 2: $\frac{dM}{dx} = -V$

$dF = \tau dx \cdot b$

$\frac{dF}{dx} = \tau b \Rightarrow \tau b = -\frac{dM}{dx} \cdot \frac{Q}{I} = \frac{VQ}{I}$

$\tau = \frac{VQ}{Ib} = \tau_{xy}$

τ_{yx} τ_{xy}

$\tau = \frac{VQ}{I} = \frac{V}{Ib}\int y\,dA$

$= \frac{V}{Ib}\int_{y}^{h/2} y \cdot b\,dy = \frac{V}{I}\frac{1}{2}y^2\Big|_{y}^{h/2} = \frac{V}{2I}\left(\frac{h^2}{4} - y^2\right)$



$\tau_{max}\Big|_{y=0} = \frac{Vh^2}{8I} = \frac{Vh^2}{8\frac{bh^3}{12}} = \frac{3}{2}\frac{V}{bh}$

$$5kd = \frac{30(20-kd)}{kd}$$

$$\sigma_s = \frac{M}{I}(20-kd)\eta = \frac{15M}{I}(20-kd)$$

$$T = A_s \sigma_s$$

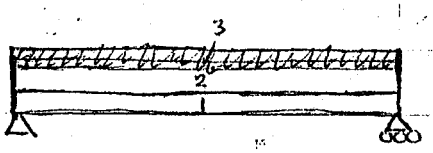
18.28
-2.54
-11.5
18.28
14.44
3.84

Shearing Stresses in Beams.

Stress tensor $\tau = \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & 0 \end{pmatrix}$

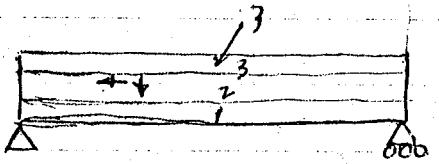
$$-\frac{dM}{dx} = V$$

$$dM = -V dx$$





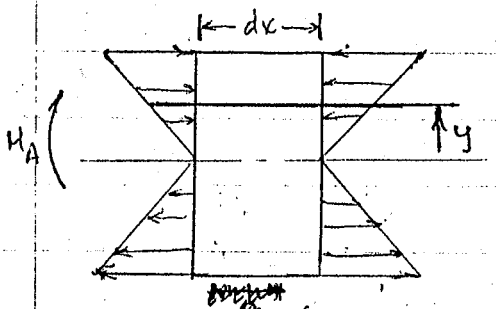
if bonded adequately
3 same laminar layer

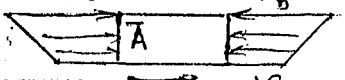


if not bonded









$= dF = \tau \cdot b \cdot dx$

$$F_a = \int_A \sigma_A dA \qquad \sigma_a = -\frac{M_A y}{I}$$

$$F_a = \int_A -\frac{M_A}{I}(y)dA = -\frac{M_A}{I}\int_A y dA = -\frac{M_A}{I} Q_y$$

$$F_B = \int_A -\frac{M_B}{I}(y)dA = -\frac{M_B Q}{I} = -(M_A + dM)Q/I$$

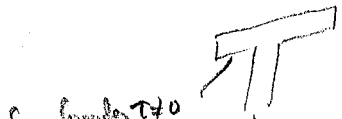
$$dF = F_a - F_B = -\frac{dM Q}{I} \qquad \frac{dF}{dx} = -\frac{dM}{dx}$$

Limitations

material Elastic

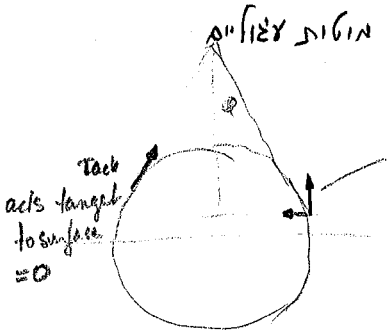
E same in tension & compression

applies to straight beams



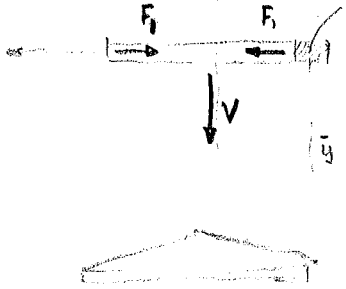
from formula $\tau \neq 0$

but $= 0$ since free surface



but this comes from $\frac{VQ}{It}$

$$\tau_{act} = \frac{\tau}{\cos\varphi} \neq 0$$



$$\tau = \frac{V\int \bar{y}\,dA}{Ib} = \frac{V\bar{y}^2\int dz}{Ib} = \frac{V\bar{y}^2}{Ib}\left(\frac{b}{2}-z\right)$$

(Q)

$\tau \sim$ linear fn. $Q \sim$ linear fn.

parabola (in flange)

$F_1 = \int \tau\,dA$

$$5kd = \frac{30(20-kd)}{kd}$$

$$\nabla_s = \frac{M}{I}(20-kd)\eta = \frac{15M}{I}(20-kd)$$

$$T = A_s \nabla_s$$

18.78
2.94
11.5

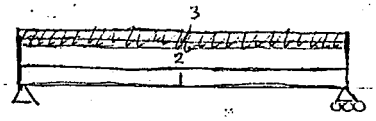
18.78
14.44
3.8

Shearing Stresses in Beams.

Stress tensor $\tau = \begin{pmatrix} \nabla_x & \tau_{xy} \\ \tau_{yx} & 0 \end{pmatrix}$

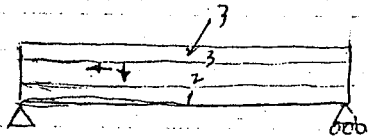
$$-\frac{dM}{dx} = V$$

$$dM = -V dx$$





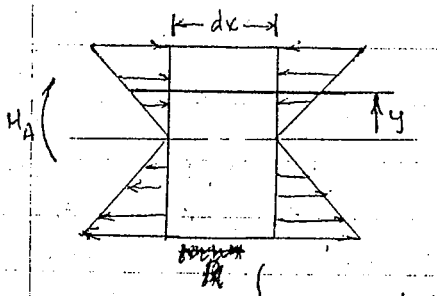
if bonded adequately
3 some laminar layer

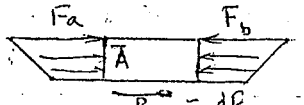


if not bonded









$$F_a = \int_A \nabla_a dA \qquad \nabla_a = -\frac{M_A y}{I}$$

$$F_a = \int_A -\frac{M_A}{I}(y)dA = -\frac{M_A}{I}\int_A y dA = -\frac{M_A Q_y}{I}$$

$$F_B = \int_A \frac{M_B}{I}(y)dA = -\frac{M_B Q}{I} = -(M_A+dM)Q/I$$

$$dF = F_a - F_B = -\frac{dM Q}{I} \qquad \frac{dF}{dx} = -\frac{dM}{dx}\cdot\frac{Q}{I}$$

$$F_A + R - F_b = 0$$

$$R = F_b - F_a$$

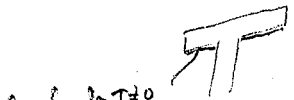
$$\frac{dF}{dx} = \tau b = \frac{VQ}{I} \quad \text{since } \frac{dM}{dx} = -V$$

Limitations

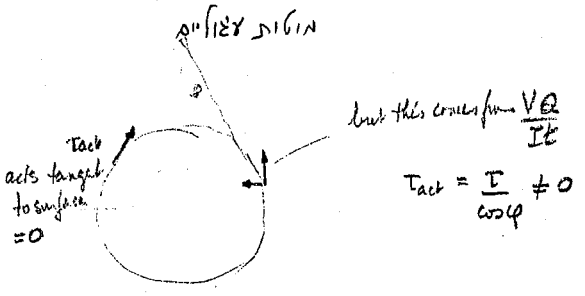
material Elastic

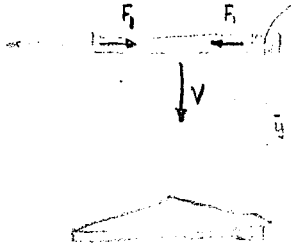
E same in tension & compression

applies to straight beams



from formula $\tau \neq 0$
but $=0$ since free surface





$$\tau = \frac{V\int \bar{y}\,dA}{Ib} = \frac{V\bar{y}^2\int dz}{Ib} = \frac{V\overbrace{\bar{y}^2\left(\frac{b}{2}-z\right)}^{Q}}{Ib}$$

$\tau \sim$ linear fn. $Q \sim$ linear fn. (in flange)

$F_1 = \int \tau\,dA$

$$\tau = \frac{V\int_{-h/2}^{h/2} y\,dA}{Ib} = \frac{V\cdot b}{Ib\cdot 2}\left(\left(\frac{h}{2}\right)^2 - y^2\right)$$

$$\tau = \frac{V}{2I}\left[\frac{h^2}{4} - y^2\right]$$



max @ $y=0$

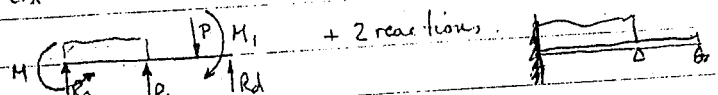
min @ $y=h/2$

$$\tau = \frac{V}{2\frac{bh^3}{12}}\cdot\frac{h^2}{4} = \frac{3}{2}\frac{V}{\underbrace{(bh)}_{A}}$$

$$\therefore \quad \tau_{max} = \frac{3}{2}\frac{V}{A}$$



$\frac{d^2\delta}{dx^2} = -\frac{M(x)}{EI}$ → 2 const of int / each span n total of 6 const



+ 2 reactions

A: $\delta(0)=0$, $\delta'(0)=0$

B: $\delta(L_b^-) = \delta(L_b^+) = 0$

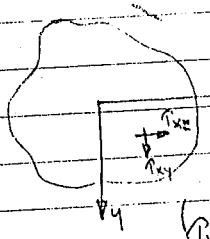
$\delta'(L_b^-) = \delta'(L_b^+)$

C: $\delta(L_c^-) = \delta(L_c^+)$

$\delta'(L_c^-) = \delta'(L_c^+)$

D: $\delta(L_D) = 0$

11/24/70

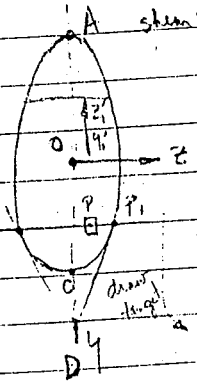


$$\sigma_x = \frac{M(I_y y - I_{yz} z)}{I_y I_z - I_{yz}^2}$$

Since q acts along x-y plane V acts along y axis

$\int \tau_{xy}\, dA = V_y$

$\int \tau_{xz}\, dA = V_z = 0$

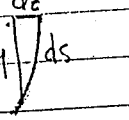


Shear stress = 0

$I_{yz} = 0$ since it is symmetric about y axis.

$\sigma_x = \frac{My}{I_z}$

let boundary $z = z_1(y)$



$\frac{\tau_{zx}}{\tau_{xy}} = \frac{dz_1}{dy}$ at boundary

if at p resume resultant will pass through D.

we are saying $\tau_{xy} \neq f(z)$ at any y.

$\frac{\tau_{zx}}{\tau_{z_1 x}} = \frac{z}{z_1} \Rightarrow \tau_{zx} = \tau_{z_1 x}\frac{z}{z_1}$; $\tau_{zx} = \tau_{xy}\frac{dz_1}{dy}\frac{z}{z_1}$

$\tau_{zx} = 0$ at y axis components of τ_{zx} will cancel each other out $\therefore$

$\int \tau_{zx}\, dA = V_z = 0$

to find τ_{zx} any where

$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0$ $\qquad \frac{d\sigma_x}{dx} = \frac{Vy}{I}$ since $\sigma_x = \frac{My}{I}$

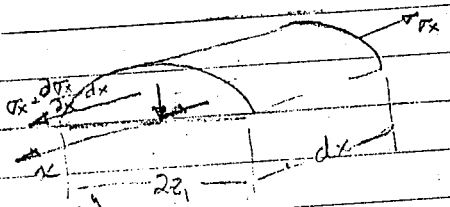
$\frac{Vy}{I} + \frac{d\tau_{yx}}{dy} + \frac{1}{z_1}\frac{dz_1}{dy}\tau_{xy} = 0$

$\frac{Vy}{I} + \frac{1}{z_1}\frac{d}{dy}(z_1 \tau_{xy}) = 0$

$z_1 \tau_{xy} = -\int_{y_a}^{y} \frac{V y' z_1'}{I}\, dy' = -\frac{V}{I}\int_{y_a}^{y} y' z_1' dy'$

$\tau_{xy} = \int_{y_a}^{y} -\frac{V}{2Iz_1} \cdot 2z_1' y' dy' = \frac{-V}{2Iz_1} Q$ $\qquad Q$ is the static moment

$\sigma_x = \frac{My}{I}$



$2z_1\, dx\, (\tau_{xy}) + \int_{y_a}^{y} \left(\sigma_x + \frac{d\sigma_x}{dx} dx\right) 2z_1' dy' - \int_{y_a}^{y} \sigma_x\, 2z_1' dy' = 0$

$\tau_{xy} = -\frac{V}{2z_1 I} Q$

occur in the plastic zones, no unbalance in longitudinal forces occurs and no shearing stresses are developed.

This elementary solution has been refined by using a more carefully formulated criterion of yielding caused by the simultaneous action of normal and shearing stresses.* Some fundamental aspects of the interaction of such stresses will be considered in Chapter 9.

EXAMPLE 7-6

An I beam is loaded as in Fig. 7-14(a). If it has the cross section shown in Fig. 7-14(c), determine the shearing stresses at the levels indicated. Neglect the weight of the beam.

SOLUTION

A free-body diagram of a segment of the beam is in Fig. 7-14(b). It is seen from this diagram that the vertical shear at every section is 50 kips. Bending moments do not enter directly into the present problem. The shear flow at the various levels of the beam is computed in the table below using Eq. 7-5. Since $\tau = q/t$ (Eq. 7-6), the shearing stresses are obtained by dividing the shear flows by the respective widths of the beam.

$$I = \frac{6(12)^3}{12} - \frac{(5.5)(11)^3}{12} = 254 \text{ in.}^4$$

For use in Eq. 7-5 the ratio $V/I = -50{,}000/254 = -197$ lb/in.^{4}

Level	A_{part}*	$\bar{y}$**	$Q = A_{\text{part}}\bar{y}$	$q = VQ/I$	t	τ, psi
1-1	0	6	0	0	6.0	0
2-2	(0.5)6 = 3.00	5.75	17.25	−3,400	6.0 0.5	−570 −6,800
3-3	(0.5)6 = 3.00 (0.5)(0.5) = 0.25	5.75 5.25	17.25 1.31 | 18.56	−3,650	0.5	−7,300
4-4	(0.5)6 = 3.00 (0.5)(5.5) = 2.75	5.75 2.75	17.25 7.56 | 24.81	−4,890	0.5	−9,780

* A_{part} is the partial area of the cross section above a given level in square inches.
** $\bar{y}$ is the distance from the neutral axis to the centroid of the partial area in inches.

The negative signs of τ show that, for the section considered, the stresses act downward on the right face of the elements. The sense of the shearing stresses acting on the section coincides with the sense of the shearing force V. For this reason a strict adherence to the sign convention is often unnecessary. It is always true that $\int_A \tau\, dA$ is equal to V and has the same sense.

* D. C. Drucker, "The Effect of Shear on the Plastic Bending of Beams," *Journal of Applied Mechanics*, 23 (1956), pp. 509–14.

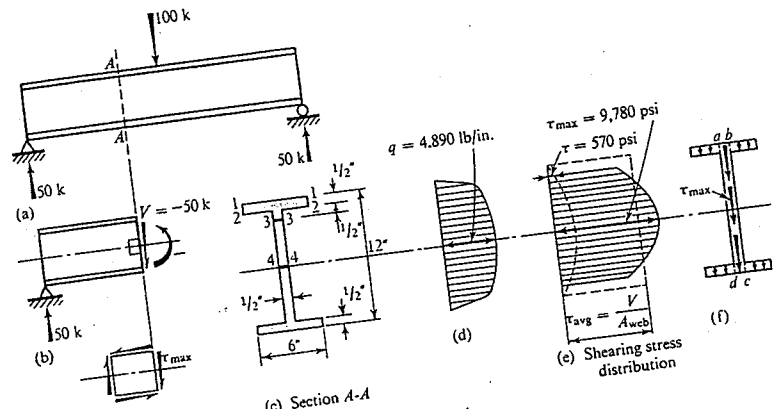


Fig. 7-14

Note that at the level 2-2 two widths are used to determine the shearing stress—one just above the line 2-2, and one just below. A width of 6 in. corresponds to the first case, and 0.5 in. to the second. This transition point will be discussed in the next article. The results obtained, which by virtue of symmetry are also applicable to the lower half of the section, are plotted in Fig. 7-14(d) and (e). By a method similar to the one used in the preceding example, it may be shown that the curves in Fig. 7-14(e) are parts of a second-degree parabola.

The variation of the shearing stress indicated by Fig. 7-14(e) may be interpreted as is shown in Fig. 7-14(f). The maximum shearing stress occurs at the neutral axis; the vertical shearing stresses throughout the web of the beam are nearly of the same magnitude. The shearing stresses occurring in the flanges are very small. For this reason the maximum shearing stress in an I beam is often approximated by dividing the total shear V by the cross-sectional area of the web (area *abcd* in Fig. 7-11(f)). Hence

$$(\tau_{\max})_{\text{approx}} = V/A_{\text{web}} \tag{7-9}$$

In the example considered this gives

$$(\tau_{\max})_{\text{approx}} = \frac{50{,}000}{(0.5)12} = 8{,}330 \text{ psi}$$

This stress differs by about 15 per cent from the one found by the accurate formula. For most cross sections a much closer approximation

best approach. For example, if for the beam segment in Fig. 7-17(b) positive bending moments increase toward the reader, larger normal forces act on the nearer cross section. For the elements shown, $\tau t\,dx$ or $q\,dx$ must aid the smaller force acting on the partial area of the cross section. This determines the sense of the shearing stresses in the longitudinal cuts. Numerically equal shearing stresses act on the mutually perpendicular planes of an infinitesimal element, and the shearing stresses on such planes either meet or part with their directional arrowheads at a corner. In this manner the sense of the shearing stresses in the plane of the cross section becomes known.

The magnitude of the shearing stresses varies for the different vertical cuts. For example, if the cut c-c in Fig. 7-17(a) is at the edge of the beam, the shaded area of the beam's cross section is zero. However, if the thickness of the flange is constant, and the cut c-c is made progressively closer to the web, the shaded area increases from zero at a linear rate.

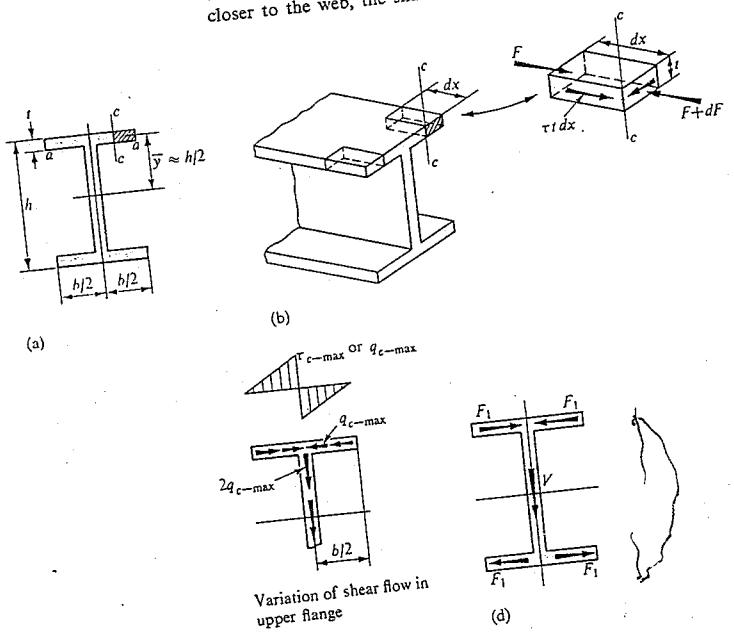


Fig. 7-17. Existence of shearing forces in the flange of an I beam which act perpendicularly to the axis of symmetry.

Moreover, as $\bar{y}$ remains constant for any such area, Q also increases linearly from zero toward the web. Therefore, since V and I are constant at any section through the beam, the shear flow $q_c = VQ/I$ follows the same variation. If the thickness of the flange remains the same, the shearing stress $\tau_c = VQ/It$ varies similarly. The same variation of q_c and τ_c exists on either side of the axis of symmetry of the cross section. However, these quantities in the plane of the cross section act in opposite directions on the two sides as may be determined by isolating another flange element to the left side of the web in Fig. 7-17(b). The variation of these shearing stresses or shear flows is represented in Fig. 7-17(c), where it is assumed that the web is very thin.

In common with all stresses, the shearing stresses in Fig. 7-17(c), when integrated over the area on which they act, are equivalent to a force. The magnitude of the horizontal force F_1 for one-half the flange, Fig. 7-17(d), is equal to the average shearing stress multiplied by one-half the flange area, i.e.,

$$F_1 = (\tau_{c\text{-max}}/2)(bt/2) \qquad \text{or} \qquad F_1 = (q_{c\text{-max}}/2)(b/2)$$

These horizontal forces act in the upper and lower flanges. Because of the symmetry of the cross section, these equal forces occur in pairs and oppose each other; thus they cause no apparent external effect.

To determine the shear flow at the juncture of the flange and the web (cut a-a in Fig. 7-17(a)), the whole area of the flange times $\bar{y}$ must be used in computing the value of Q. However, since in finding $q_{c\text{-max}}$ one-half the flange area times the same $\bar{y}$ has already been used, the sum of the two horizontal shear flows coming in from opposite sides gives the vertical shear flow at the cut a-a. Hence, figuratively speaking, the horizontal shear flows turn through 90° and merge to become the vertical shear flow. Then the shear flows at the various horizontal cuts through the web may be determined in the manner explained in the preceding articles. Moreover, as the resistance to the vertical shear V in thin-walled I beams is developed mainly in the web, it is so shown in Fig. 7-17(d). The sense of the shearing stresses and shear flows in the web coincides with the direction of the shear V. Note that the vertical shear flow "splits" upon reaching the lower flange. This is represented in Fig. 7-17(d) by the two forces F_1, which are the result of the horizontal shear flows in the flanges.

The shearing forces which act at a section of an I beam are shown in Fig. 7-17(d), and, for equilibrium, the applied vertical forces must act through the centroid of the cross-sectional area to be coincident with V. If the forces are so applied, no torsion of the member will occur. This is true for all sections having cross-sectional areas with an axis of symmetry. Thus, to avoid torsion of such members, the applied forces must act in the plane of symmetry of the cross section and the axis of the beam. A beam with an unsymmetrical section will be discussed next.

REVIEW AND SUMMARY FOR CHAPTER 6

This chapter was devoted to the analysis of beams and thin-walled members under transverse loadings.

Stresses on a beam element

In Sec. 6.1 we considered a small element located in the vertical plane of symmetry of a beam under a transverse loading (Fig. 6.2) and found that normal stresses σ_x and shearing stresses τ_{xy} were exerted on the transverse faces of that element, while shearing stresses τ_{yx}, equal in magnitude to τ_{xy}, were exerted on its horizontal faces.

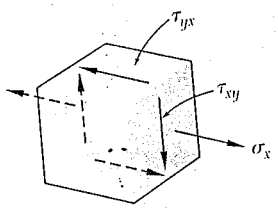


Fig. 6.2

In Sec. 6.2 we considered a prismatic beam AB with a vertical plane of symmetry supporting various concentrated and distributed loads (Fig. 6.5). At a distance x from end A we detached from the beam an element $CDD'C'$ of length Δx extending across the width of the beam from the upper surface of the beam to a horizontal plane located at a distance y_1 from the neutral axis (Fig. 6.6). We found

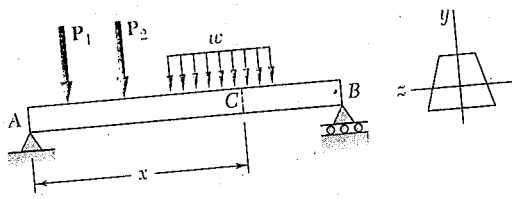


Fig. 6.5

Horizontal shear in a beam

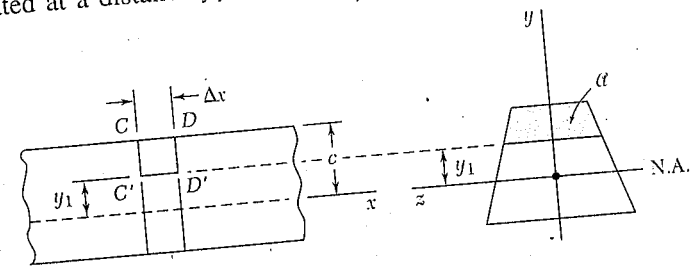


Fig. 6.6

that the magnitude of the shearing force $\Delta\mathbf{H}$ exerted on the lower face of the beam element was

$$\Delta H = \frac{VQ}{I}\Delta x \tag{6.4}$$

where V = vertical shear in the given transverse section
Q = first moment with respect to the neutral axis of the shaded portion $\mathcal{A}$ of the section
I = centroidal moment of inertia of the entire cross-sectional area

The *horizontal shear per unit length*, or *shear flow*, which was denoted by the letter q, was obtained by dividing both members of Eq. (6.4) by Δx:

Shear flow

$$q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I} \tag{6.5}$$

Dividing both members of Eq. (6.4) by the area ΔA of the horizontal face of the element and observing that $\Delta A = t\,\Delta x$, where t is the width of the element at the cut, we obtained in Sec. 6.3 the following expression for the *average shearing stress* on the horizontal face of the element

Shearing stresses in a beam

$$\tau_{ave} = \frac{VQ}{It} \tag{6.6}$$

We further noted that, since the shearing stresses τ_{xy} and τ_{yx} exerted respectively on a transverse and a horizontal plane through D' are equal, the expression in (6.6) also represents the average value of τ_{xy} along the line $D'_1 D'_2$ (Fig. 6.12).

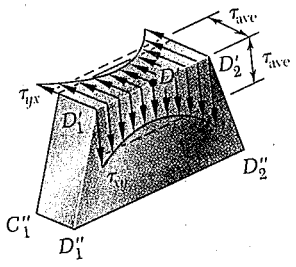


Fig. 6.12

In Secs. 6.4 and 6.5 we analyzed the shearing stresses in a beam of rectangular cross section. We found that the distribution of stresses is parabolic and that the maximum stress, which occurs at the center of the section, is

Shearing stresses in a beam of rectangular cross section

$$\tau_{max} = \frac{3}{2}\frac{V}{A} \tag{6.10}$$

where A is the area of the rectangular section. For wide-flange beams, we found that a good approximation of the maximum shearing stress can be obtained by dividing the shear V by the cross-sectional area of the web.

In Sec. 6.6 we showed that Eqs. (6.4) and (6.5) could still be used to determine, respectively, the longitudinal shearing force ΔH and the shear flow q exerted on a beam element if the element was bounded by an arbitrary curved surface instead of a horizontal plane (Fig. 6.24). This made it possible for us in Sec. 6.7 to extend the use

Longitudinal shear on curved surface

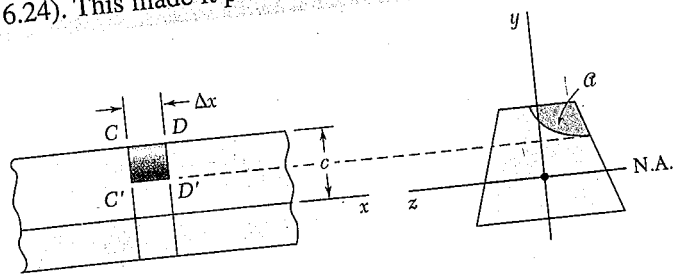
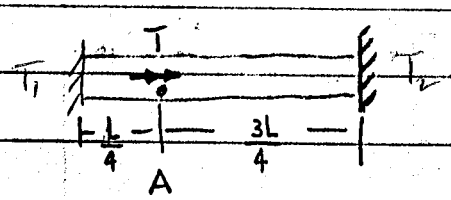
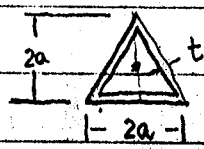


Fig. 6.24

ת.ז. 46922870

קורה עם שטח חתך בצורת משולש שווה צלעות נתונה בתרשים.
הקורה חתומה בשני קצותיה ומפעילים מומנט פיתול T בנקודה A. מודול גזירה שווה
ל- G וקטן ז. $\frac{a}{t} \ll 10$





• מצאו את הפיתולים בסמכים התחומים
• מצאו ביטויים למאומץ גזירה שגרא מהפיתולים T_1 ו- T_2 לצורך למצוא
• מצאו את זווית הפיתול ב- A.

$a = b \Rightarrow b/a = 1.0$

מהטבלה: $c_\beta = 0.1406$, $c_T = 0.208$

$\frac{a}{t} \gg 10$,

$A = \frac{4a^2}{2} = 2a^2$ (m

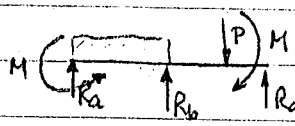
$\tau_{max} = \frac{T}{c_T \cdot b a^2} = \frac{T}{0.208 \cdot 1}$

$c_\beta = 0.1406$

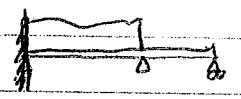
$\theta = \frac{T \cdot L}{c_\beta \cdot b \cdot a^3 \cdot G}$ ← $\frac{1/4 L \cdot$

$\theta = \frac{T_1 L}{G \cdot 4A^2}$

$\frac{d^2\delta}{dx^2} = -\frac{M(x)}{EI}$ → 2 const of int / each span → total of 6 const



+ 2 reactions.



A: $\delta(0) = 0$, $\delta'(0) = 0$

B: $\delta(L_b^-) = \delta(L_b^+) = 0$

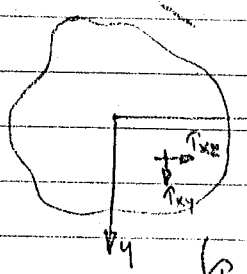
$\delta'(L_b^-) = \delta'(L_b^+)$

C: $\delta(L_c^-) = \delta(L_c^+)$

$\delta'(L_c^-) = \delta'(L_c^+)$

D: $\delta(L_D) = 0$

11/24/70

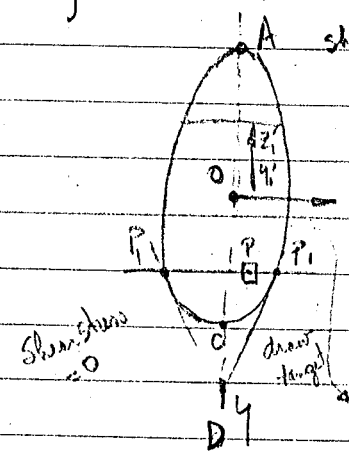


$$\sigma_x = \frac{M(I_y y - I_{yz} z)}{I_y I_z - I_{yz}^2}$$

Since q acts along x-y plane V acts along y axis

$\int \tau_{xy}\, dA = V_y$

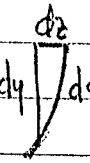
$\int \tau_{xz}\, dA = V_z = 0$



$I_{yz} = 0$ since it is symmetric about y axis.

$\sigma_x = \frac{My}{I_z}$

let boundary $z = z_1(y)$



$$\frac{\tau_{zx}}{\tau_{xy}} = \frac{dz_1}{dy}$$ at boundary

if at p assume resultant will pass through D.

we are saying $\tau_{xy} \neq f(z)$ at any y.

$$\frac{\tau_{zx}}{\tau_{z_1 x}} = \frac{z}{z_1} \Rightarrow \tau_{zx} = \tau_{z_1 x}\frac{z}{z_1} \; ; \; \tau_{zx} = \tau_{xy}\frac{dz_1}{dy}\frac{z}{z_1}$$

$\tau_{zx} = 0$ at y axis components of τ_{zx} will cancel each other out $\therefore$

$\int \tau_{zx}\, dA = V_z = 0$

to find τ_{zx} anywhere

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0 \qquad \frac{d\sigma_x}{dx} = \frac{Vy}{I} \text{ since } \sigma_x = \frac{My}{I}$$

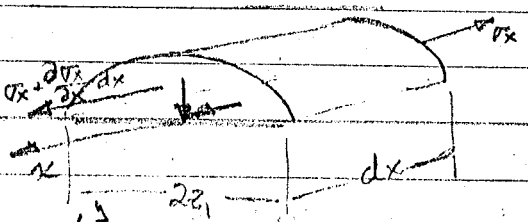
$$\frac{Vy}{I} + \frac{d\tau_{yx}}{dy} + \frac{1}{z_1}\frac{dz_1}{dy}\tau_{xy} = 0$$

$$\frac{Vy}{I} + \frac{1}{z_1}\frac{d}{dy}(z_1 \tau_{xy}) = 0$$

$$z_1 \tau_{xy} = -\int_{y_a}^{y} \frac{V y' z_1'}{I}\, dy' = -\frac{V}{I}\int_{y_a}^{y} y' z_1'\, dy'$$

$$\tau_{xy} = \int_{y_a}^{y} -\frac{V}{2Iz_1} \cdot 2z_1' y'\, dy' = -\frac{V}{2Iz_1} Q$$ Q is the static moment

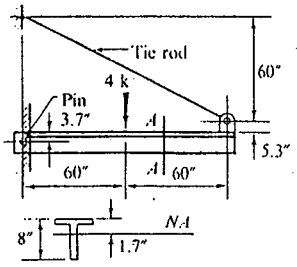
$\sigma_x = \frac{My}{I}$



$$2z_1\, dx\,(\tau_{xy}) + \int_{y_a}^{y}\left(\sigma_x + \frac{d\sigma_x}{dx}dx\right) 2z_1' dy' - \int_{y_a}^{y} \sigma_x\, 2z_1' dy' = 0$$

$$\tau_{xy} = -\frac{V}{2z_1 I} Q$$

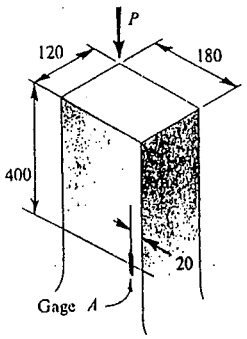
10-50. Plot shear, moment, and axial force diagrams for the jib-crane loaded as shown in the figure. Neglect the weight of the beam. *Ans:* 128.6 k-in.



PROB. 10 – 50

8-29

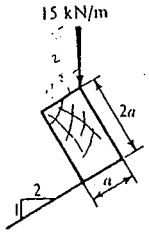
7-35. An aluminum-alloy block is loaded as shown in the figure. The application of this load produces a tensile strain of 500×10^{-6} mm. per mm at A as measured by means of an electrical strain gage. Compute the magnitude of the applied force P. Let $E = 10 \times 10^6$ kN/m². All dimensions given in the figure are in mm.



PROB. 7 – 35

8-19 (a)

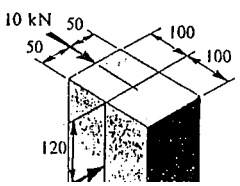
7-44. A tilted, simply supported beam with a depth to width ratio of 2 to 1 is to span 4 m and is to carry a uniformly distributed load of 15 kN per linear meter, including its own weight, applied as shown in the figure. (a) Determine the required dimensions of the beam so that the maximum stress due to bending does not exceed 10 MN/m². (b) Locate the neutral axis of the beam and show its position on the sketch.



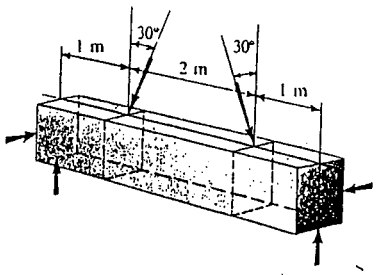
PROB. 7 – 44

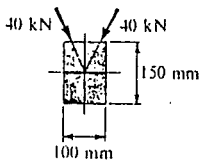
8-23

7-34. A cast iron block is loaded as shown in the figure. Neglecting the weight of the block, determine the stresses acting normal to a section taken 0.5 m below the top and locate the line of zero stress. All dimensions given in the figure are in mm. *Ans:* ±24.6 MPa, ±9.6 MPa.



10-91. A 4 m long beam is loaded as shown in the figure. The two applied forces act perpendicularly to the long axis of the beam and are inclined 30° with the vertical. If these forces act through the centroid of the cross-sectional area find the location and magnitude of the maximum bending stress. Neglect the weight of the beam. *Ans:* ±132 MN/m².

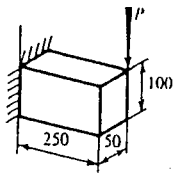




PROB. 10 – 91

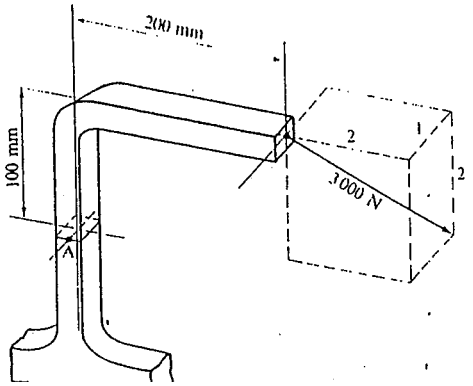
8-28

7-45. A rectangular cantilever 250 mm long is loaded with $P = 50$ kN at the free end as shown in the figure. Determine the maximum shearing stress at the built-in end due to the direct shear and the torque. Show the result on a sketch analogous to Fig. 7-17(e). All dimensions shown in the figure are in mm. *Ans:* 35.3 MPa.



PROB. 7 – 45

9-26. A bent rectangular bar is subjected to an inclined force of 3 000 N as shown in the figure. The cross-section of the bar is 12 mm by 12 mm. (a) Determine the state of stress at point A caused by the applied force and show the results on an element. (b) Find the maximum principal stress. *Ans:* (a) +333 MPa, 578 MPa





9.163 The rigid bar *BDE* is welded at point *B* to the rolled-steel beam *AC*. For the loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *B*. Use $E = 200$ GPa.

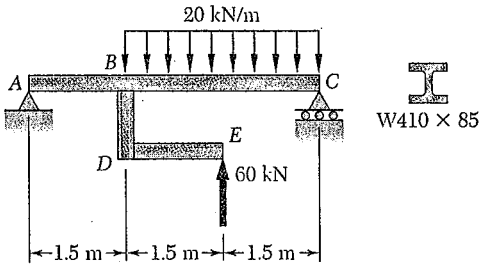


Fig. P9.163

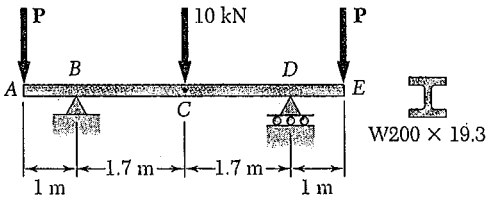


Fig. P9.164

9.164 Knowing that $P = 8$ kN, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 200$ GPa.

9.165 The two beams shown have the same cross section and are joined by a hinge at *C*. For the loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *B*. Use $E = 29 \times 10^6$ psi.

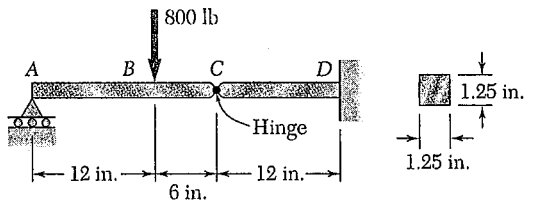


Fig. P9.165

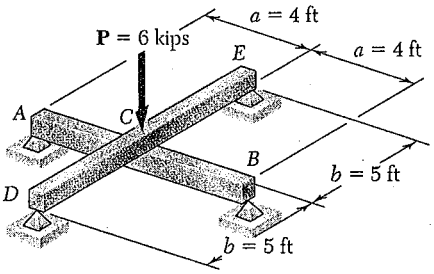


Fig. *P9.166*

9.166 For the loading shown, and knowing that beams *AB* and *DE* have the same flexural rigidity, determine the reaction (*a*) at *B*, (*b*) at *E*.

9.167 A hydraulic jack can be used to raise point *B* of the cantilever beam *ABC*. The beam was originally straight, horizontal, and unloaded. A 5-kip load was then applied at point *C*, causing this point to move down. Determine (*a*) how much point *B* should be raised to return point *C* to its original position, (*b*) the final value of the reaction at *B*. Use $E = E = 29 \times 10^6$ psi.

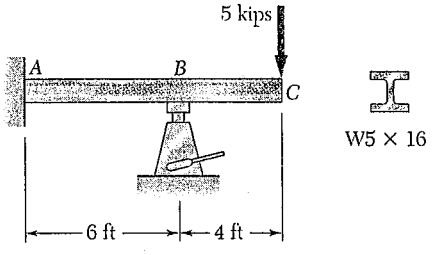


Fig. P9.167

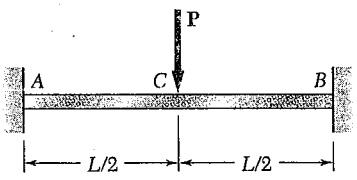
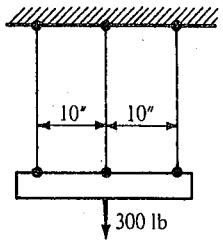


Fig. *P9.168*

9.168 Determine the reaction at *A* and draw the bending moment diagram for the beam and loading shown.

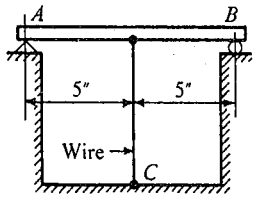
and $\alpha_a = 12.0 \times 10^{-6}$ per °F; $E_s = 30 \times 10^6$ psi and $\alpha_s = 6.5 \times 10^{-6}$. *Ans.* 1,650 lb.

12-24. Three steel wires attached to a rigid bar support a load of 300 lb. Initially this load is equally distributed between the three wires. What will each wire carry if the temperature of the right wire raises 84°F? Assume for all wires $E = 30 \times 10^6$ psi, $A = 0.011$ in.^{2}, and $\alpha = 6.5 \times 10^{-6}$ per °F. *Ans.* 70 lb, 160 lb, 70 lb.



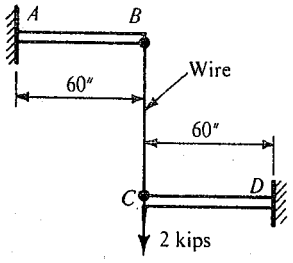
PROB. 12-24

12-25. A steel piano wire 30 in. long is stretched from the middle of an aluminum beam AB to a rigid support at C as shown in the figure. What is the increase in unit stress in the wire if the temperature drops 100°F? The cross-sectional area of the wire is 0.0001 in.^{2}; $E = 30 \times 10^6$ psi. For the aluminum beam $EI = 1{,}040$ lb-in.^{2} Let $\alpha_s = 6.5 \times 10^{-6}$ per °F, $\alpha_a = 12.9 \times 10^{-6}$ per °F. *Ans.* 6,500 psi.



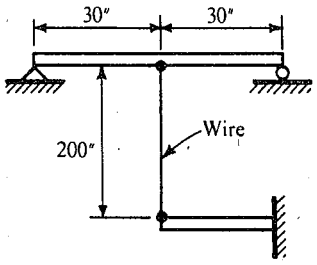
PROB. 12-25

12-26. Two steel cantilever beams AB and CD are connected by a taut steel wire BC having a length equal to 150 in. under initial no-load conditions, see figure. Determine the stress in the wire produced by a 2-kip load applied at C and a temperature drop, in the wire only, of 100°F. For beams AB and CD: $E = 30 \times 10^6$ psi, and $I = 24$ in.^{4} For wire BC: $E = 30 \times 10^6$ psi, $A = 0.1$ in.^{2}, and $\alpha = 6.0 \times 10^{-6}$ per °F. *Ans.* 1.16 k.

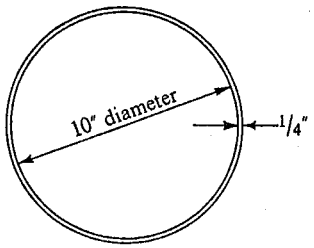


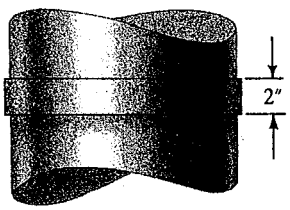
PROB. 12-26

12-27. A steel wire 200 in. in length with cross-sectional area equal to 0.25 in.^{2} is stretched tightly between the midpoint of the simple beam and the free end of the cantilever as shown in the figure. Determine the deflection of the end of the cantilever as a result of a temperature drop of 50°F. For steel wire: $E = 30 \times 10^6$ psi, $\alpha = 6.5 \times 10^{-6}$ per °F. For both beams: $I = 21.3$ in.^{4} and $E = 1.5 \times 10^6$ psi. *Ans.* 4.08×10^{-2} in.



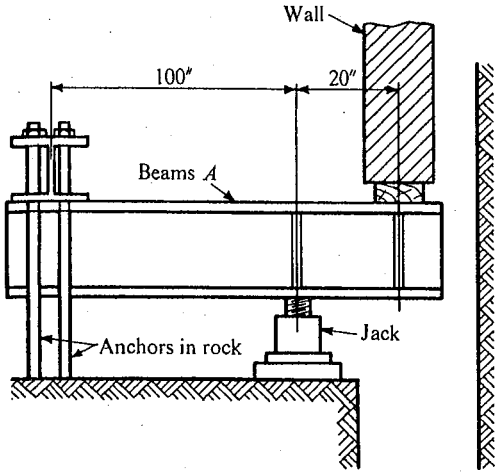
PROB. 12-27





PROB. 12-28

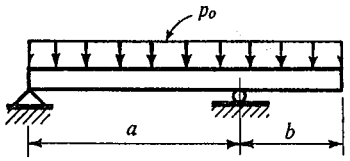
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PROB. 10-36

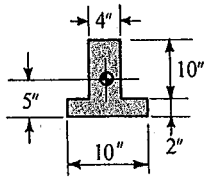
size be? Use steel *WF* or *I* beams, whichever are lighter. The allowable bending stress is 20 ksi, and the allowable shearing stress is 10 ksi.

10-37. Select the required cross section for a rectangular wooden beam that is to carry a load of $p_o = 1.0$ k per foot, including its own weight, for the span in the figure. The allowable bending stress is 1,300 psi and the allowable shearing stress is 150 psi. The beam is to be twice as high as it is wide. Let $a = 10$ ft and $b = 4$ ft.



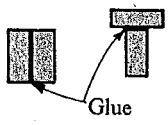
PROB. 10-37

10-38. A *T* beam is supported in the same manner as the beam in the preceding problem; however, $a = 16$ ft and $b = 8$ ft. The cross-sectional dimensions of the *T* are in the figure; the moment of inertia around the centroidal axis *I* is 820 in.^{4} If the allowable stresses are 1,200 psi in bending and 100 psi in shear, what is the largest load p_o in lb per foot that this beam can carry?



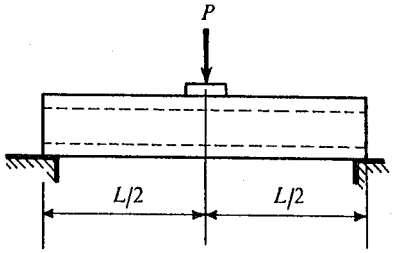
PROB. 10-38

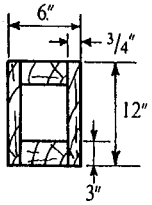
10-39. A plastic beam is to be made from two 1-in.-by-3-in. pieces to span 24 in. and to carry an intermittently applied, uniformly distributed load *p*. The pieces can be arranged in two alternative ways as in the figure. The allowable stresses are 500 psi in flexure, 100 psi shear in plastic, and 75 psi shear in glue. Which arrangement of pieces should be used, and what load *p* may be applied?



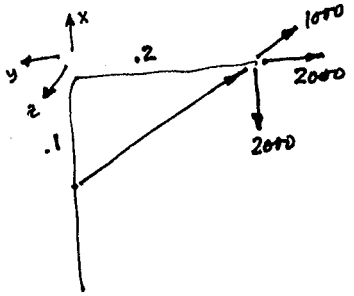
PROB. 10-39

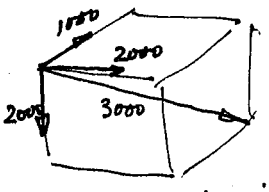
10-40. A box beam is fabricated from two pieces of ¾-in. plywood and two 4½-in.-by-3-in. (actual size) solid wood pieces as shown





PROB. 10-40





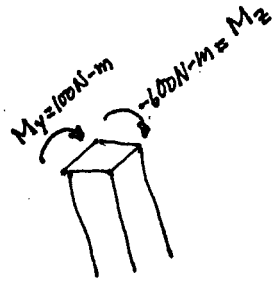
$d = \sqrt{2^2+1^2+2^2} = 3$

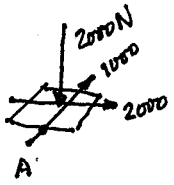
$\underline{M} = \underline{r} \times F$

$$\begin{vmatrix} i & j & k \\ .1 & -.2 & 0 \\ -2 & -2 & -1 \end{vmatrix} = i\begin{vmatrix} -.2 & 0 \\ -2 & -1 \end{vmatrix} - j\begin{vmatrix} .1 & 0 \\ -2 & -1 \end{vmatrix} + k\begin{vmatrix} .1 & -.2 \\ -2 & -2 \end{vmatrix}$$

$\underline{M} = +.2\ \text{kN-m}\,\underline{i} + .1\ \text{kN-m}\,\underline{j} + .6\ \text{kN-m}\,\underline{k}$

$\underline{M} = T\underline{i}$





$$\sigma_x = \pm\frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$P = -2000\ N$

$M_z = -600\ N\text{-}m$

$M_y = 100\ N\text{-}m$

$V_y = -2000\ N$

$V_z = -2000\ N$

$T = 200\ N\text{-}m$

$$I_z = \frac{1}{12}(.012)(.012)^3 = I_y = 1.728 \times 10^{-9}$$

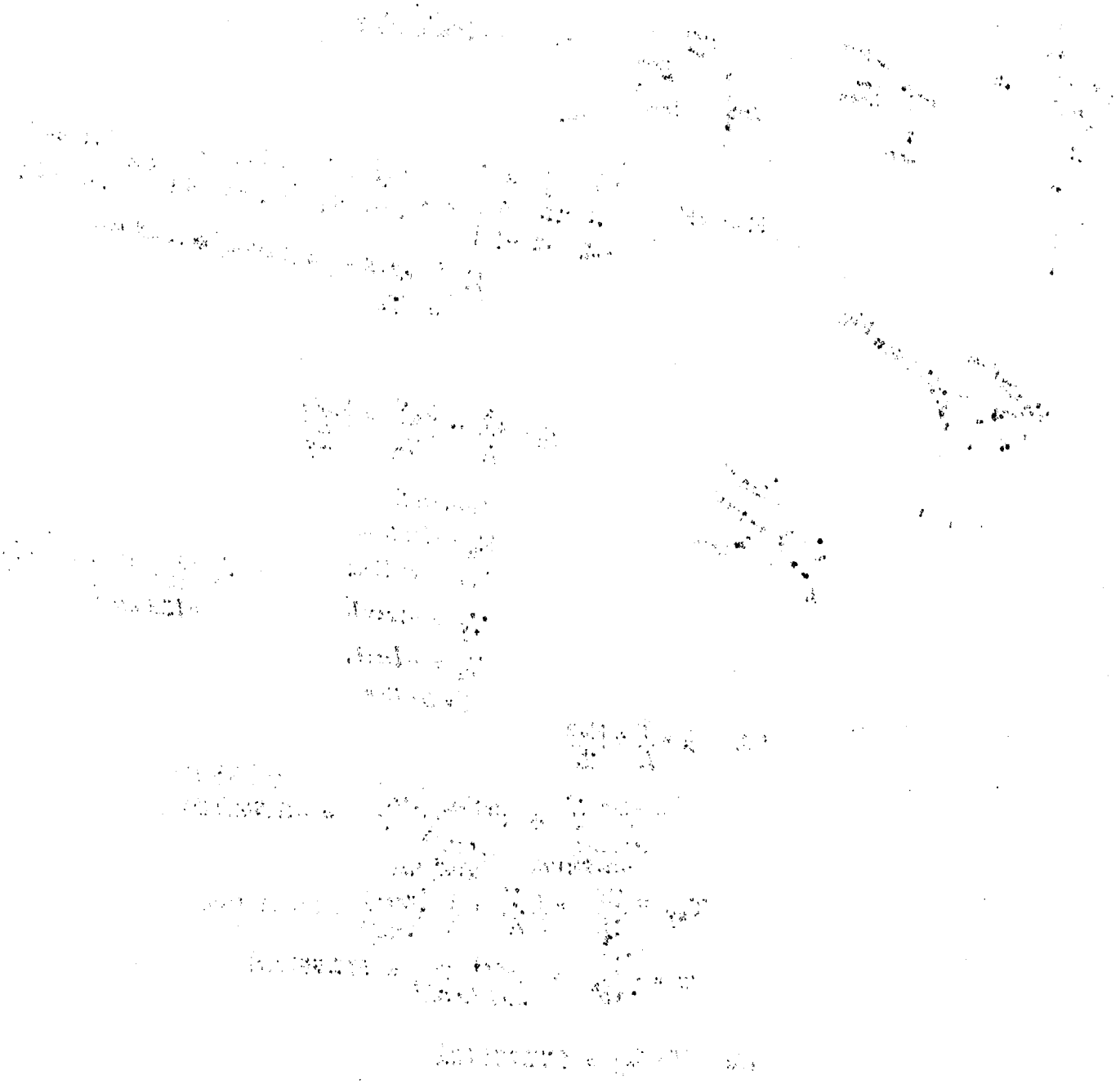
@A $\sigma_x = \frac{P}{A} + \frac{M_y z}{I_y}$

$$= \underbrace{\frac{-2000\ N}{.012 \times .012}}_{-13.889\ MPa} + \underbrace{\frac{(100\ N\text{-}m)(.006)}{(.012)^3}}_{+347.\ MPa} = +333\ MPa$$

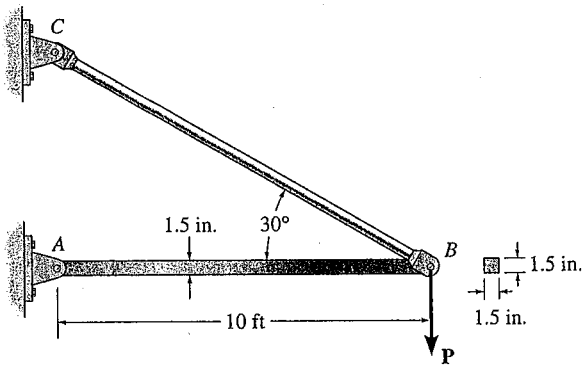
$$\tau_{xy} = \frac{VQ}{It} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{(2000)}{(.012)^2} = 20.833\ MPa$$

$$\tau = \frac{T}{cab^2} = \frac{200 N\text{-}m}{.208\,(.012)^3} = 556.446\ MPa$$

@A: $\tau + \tau_{xy} = 577.279\ MPa$

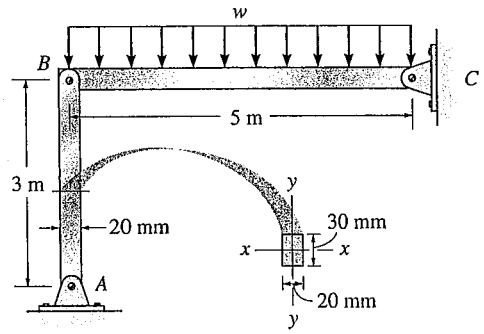


13-35. The A-36 steel bar AB has a square cross section. If it is pin-connected at its ends, determine the maximum allowable load P that can be applied to the frame. Use a factor of safety with respect to buckling of 2.



Prob. 13–35

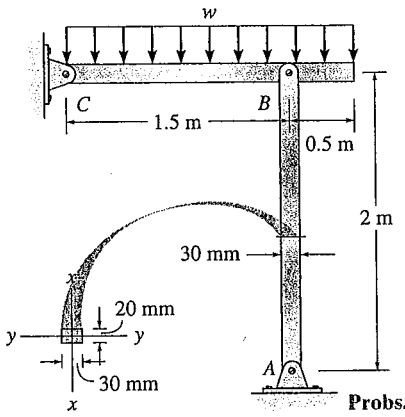
***13-36.** The steel bar AB has a rectangular cross section. If it is pin connected at its ends, determine the maximum allowable intensity w of the distributed load that can be applied to BC without causing bar AB to buckle. Use a factor of safety with respect to buckling of 1.5. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



Prob. 13–36

13-37. Determine the maximum allowable intensity w of the distributed load that can be applied to member BC without causing member AB to buckle. Assume that AB is made of steel and is pinned at its ends for x–x axis buckling and fixed at its ends for y–y axis buckling. Use a factor of safety with respect to buckling of 3. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.

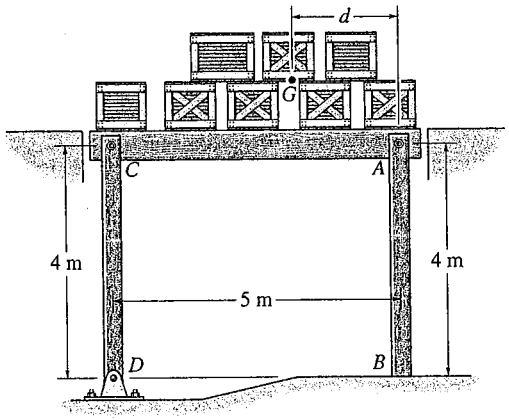
13-38. Determine if the frame can support a load of $w = 6$ kN/m if the factor of safety with respect to buckling of member AB is 3. Assume that AB is made of steel and is pinned at its ends for x–x axis buckling and fixed at its ends for y–y axis buckling. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



Probs. 13–37/38

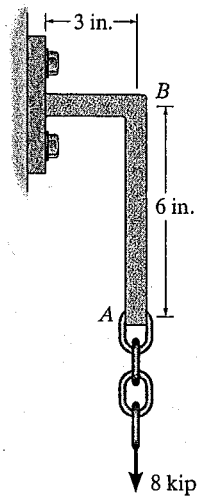
13-39. The deck is supported by the two 40-mm-square columns. Column AB is pinned at A and fixed at B, whereas CD is pinned at C and D. If the deck is prevented from sidesway, determine the greatest weight of the load that can be applied without causing the deck to collapse. The center of gravity of the load is located at $d = 2$ m. Both columns are made from Douglas Fir.

***13-40.** The deck is supported by the two 40-mm-square columns. Column AB is pinned at A and fixed at B, whereas CD is pinned at C and D. If the deck is prevented from sidesway, determine the position d of the center of gravity of the load and the load's greatest magnitude without causing the deck to collapse. Both columns are made from Douglas Fir.



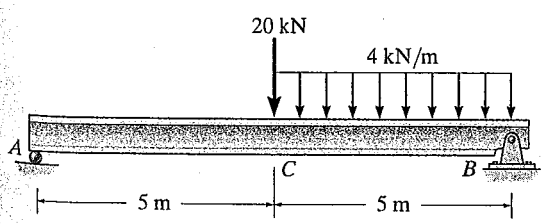
Probs. 13–39/40

12–94. Determine the vertical deflection and slope at the end A of the bracket. Assume that the bracket is fixed supported at its base, and neglect the axial deformation of segment AB. EI is constant.



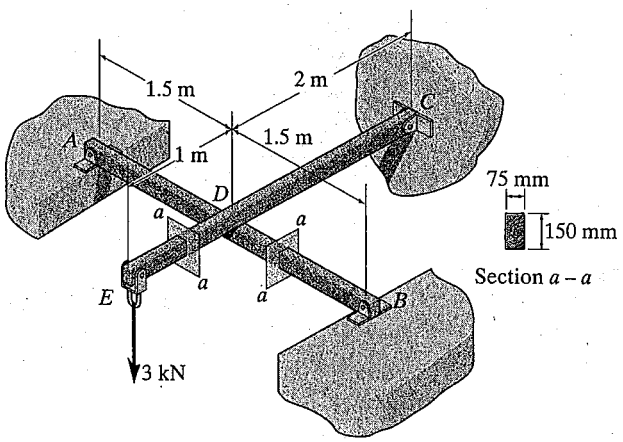
Prob. 12–94

12–95. The simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its center C. $I = 0.1457(10^{-3})\ \text{m}^4$.



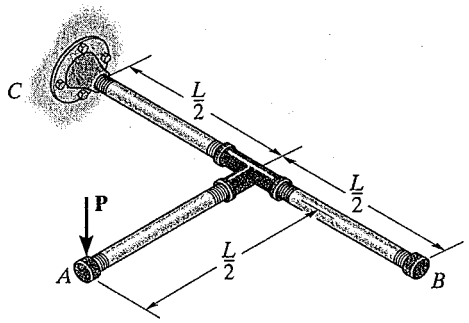
Prob. 12–95

***12–96.** Determine the deflection at end E of beam CDE. The beams are made of wood having a modulus of elasticity of $E = 10$ GPa.

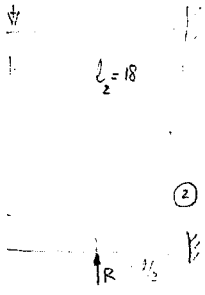


Prob. 12–96

•12–97. The pipe assembly consists of three equal-sized pipes with flexibility stiffness EI and torsional stiffness GJ. Determine the vertical deflection at point A.



Prob. 12–97



$$y = \frac{P\ell_2^3}{6EI}\left(2 - 3\frac{x}{\ell_2} + \frac{x^3}{\ell_2^3}\right)$$

$$y\Big|_{x=1/2} = \frac{5P\ell_2^3}{48EI}$$

②

$$y\Big|_{1/2} \frac{R\ell^3}{3EI} = \frac{R\left(\ell^3/8\right)}{3EI} \therefore \frac{R\ell_2^3}{24EI}$$

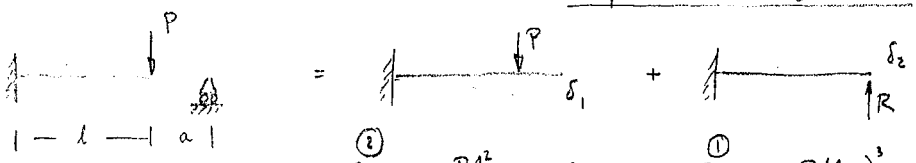
$$\delta = \frac{5P\ell_2^3}{48EI} - \frac{R\ell_2^3}{24EI} = \frac{R\ell_1^3}{48EI}$$

$$\frac{5P\ell_2^3}{48EI} = \frac{R}{48EI}\left(2\ell_2^3 + \ell_1^3\right)$$

$$1.1441P \quad \frac{5P(18)^3}{2\cdot 18^3 + 24^3} = \frac{5P\ell_2^3}{2\ell_2^3 + \ell_1^3} = R$$

$$\delta = \frac{5P(18)^3}{2\cdot 18^3 + 24^3} \cdot \frac{24^3}{48\cdot EI} = \frac{3295}{EI} \text{ K-ft}^3$$

דוגמא שניה. לאמוץ השלכויות



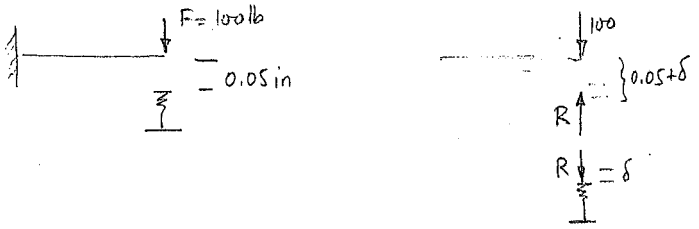
$$\delta_1 = f_A = \frac{Pl^2}{6EI}(2l+3a) \qquad \delta_2 = f_A = \frac{-R(l+a)^3}{3EI}$$

$$\delta_1+\delta_2=0 \qquad \frac{Pl^2(2l+3a)}{6EI} - \frac{R(l+a)^3}{3EI} = 0$$

$$R = P\,\frac{l^2(2l+3a)}{2(l+a)^3}$$

$$R = P\left(\frac{l^2\cdot 5l}{2\cdot 8l^3}\right) = \frac{5}{16}P \qquad a=l \text{ אם}$$

12-14



$$\frac{(100-R)L^3}{3EI} = 0.05+\delta$$

$$R = k\delta$$

$$\Rightarrow \quad R = 40\text{lb}, \quad \delta = 0.004\text{in}$$

15-12 התקדה השאלה של הקורה המקורית על 18' שנמת בקורה המסומנת המסומנת על 24'

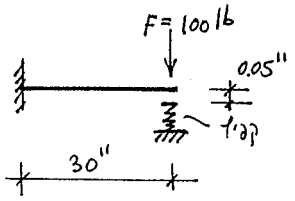


משוים השאלה, מוחלפ את הקורה ב A

שרשא מפני שזום כוח 10K בקצה הקורה המקורית



12-14

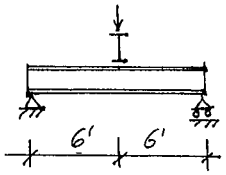


נתונה קורה חסרת משקל: $EI = 10^7 \frac{lb}{in^2}$

נתון: קפיץ קשיחות $K = 10 \frac{kips}{in}$

דרוש: למצוא את הכח בקפיץ כאשר כח F. (תשובה: 9k)

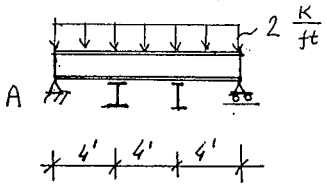
12-15



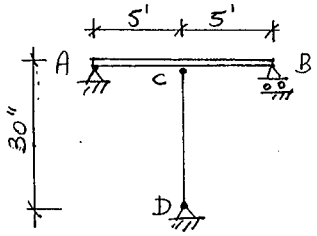
נתונה קורה חסרת משקל

מחוברת ב- INP200.

דרוש: למצוא ריאקציה בסמך A. (תשובה 6.74 kips)



12-27

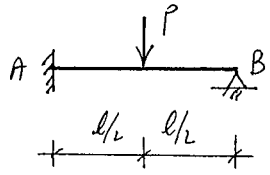


נתון: קורה AB: $EI = 1040 \text{ lb-in}^2$

מוט CD: $\alpha = 5.9 \cdot 10^{-6} \frac{1}{°F}$, $E = 30 \cdot 10^6 \text{ psi}$, $A = 10^{-4} \text{ in}^2$

דרוש: למצוא באמה הכח בשוט CD אם הטמפרטורה עולה ב- $100°F$.

12-44



בקורה שבתרשים דרוש: למצוא ריאקציה.

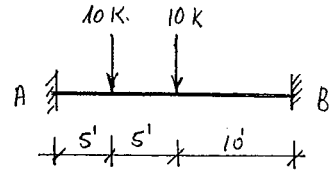
לצורך חישוב כוחות ציר ואורכן כפיפה

ראשית חישוב מרכז הקורה ואחרי כפיפה מפעילים.

נתון: $E = 29 \cdot 10^6 \text{ psi}$, $I = 716 \text{ in}^4$, $W = 126 \text{ in}^3$

(תשובה: 13.9 ksi, 0.133 in)

12-65



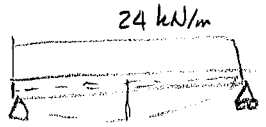
בקורה בתרשים דרוש: למצוא ריאקציה.

לצורך חישוב כוחות ציר ואורכן כפיפה

ראשית חישוב מרכז הקורה ואחרי כפיפה מפעילים.

נתון: $E = 29 \cdot 10^6 \text{ psi}$, $I = 716 \text{ in}^4$, $W = 126 \text{ in}^3$

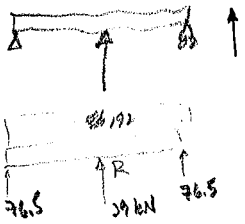
(תשובה: 13.9 ksi, 0.133 in)



$$v = \frac{-Wx}{24EI}\left[x^3 - 2Lx^2 + L^3\right]$$

$$v_{max} = -\frac{5wL^4}{384EI} = \frac{-5\,[24000\ N/m]\,8^4\ m^4}{384\,(200\times10^9\ \frac{N}{m^2})\,[216\times10^{-6}\ m^4]} = -.02963\ m = -29.63\ mm$$





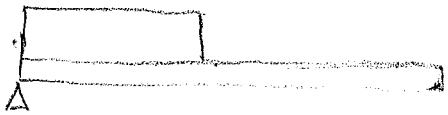
$$v_w + v_p = \delta_o$$

$$v = \frac{PL^3}{48EI} = .00962963\ m$$

$$P = \frac{.00962963\,(48)\left(200\times10^9\ \frac{N}{m^2}\right)\left(216\times10^{-6}\ m^4\right)}{8^3} = 39000\ N$$



$$v_1 + v_2 = \delta$$

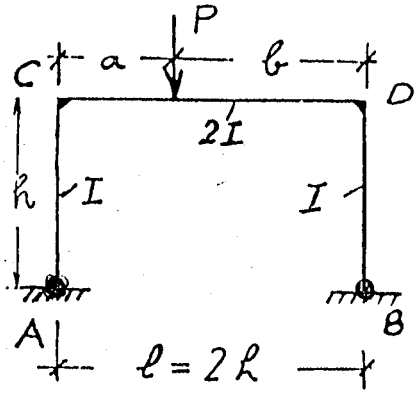


המכללה האקדמית יהודה ושומרון

תורת החוזק 1

תרגיל מס' 11

1.



נתון. מסגרת פורסל רתומה עמוסה לפי

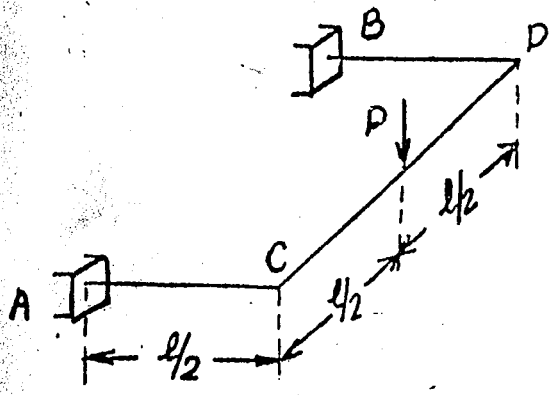
הציור. $a = \ell/3$.

דרוש. לחשב את הריאקציות ואת מומנטי

הריתום ולשרטט את מהלכי N, S ו- M.

תשובה. ~~$H_A = \frac{Pab}{2h\ell}$~~ ; ~~$V_A = \frac{64Pb}{63\ell}$~~ ; ~~$M_A = \frac{Pab}{7\ell}$~~

2.



נתון. מסגרת סימטרית **ארפקית** המיוארת בציור

האקסונומטרי רתומה ב-A ו-B ועמוסה עומס

אנכי P. המסגרת בעלת קשיחות לכפיפה EI

וקשיחות לפיתול GI_T.

דרוש. לחשב את הריאקציות A, B את מומנטי

הכפיפה בריתומים M_A, M_B ואת מומנטי הפיתול

בריתומים T_A, T_B. ולשרטט את מהלכי M ו- T.

(להשוות

זווית הכפיפה ϕ_C של DC לזווית הפיתול θ_C

של AC).

תשובה.

$$M_A = M_B = P\ell/4 \; ; \; A = B = P/2$$

$$T_A = T_B = \frac{P\ell}{8}\frac{GI_T}{GI_T+EI}$$

8.136 For the loading shown, knowing that beams *AB* and *DE* have the same flexural rigidity, determine the reaction (*a*) at *A*, (*b*) at *D*.

ans: (a) 4.07 kN↑ (b) 5.93 kN↑.

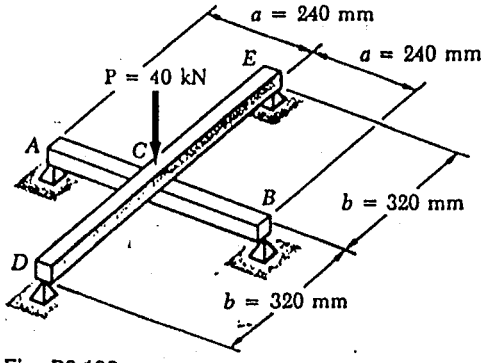


Fig. P8.136

8.138 Before any load is applied, a gap $\delta_0 = 20$ mm exists between the W410 × 60 rolled-steel beam and the support at *C*. Knowing that $E = 200$ GPa, determine the reaction at each support caused by a uniformly distributed load of 24 kN/m.

ans: $R_A = R_B = 76.5$ kN↑

$R_C = 39.0$ kN↓

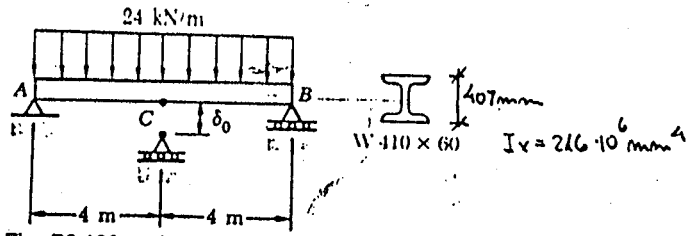


Fig. P8.138 and P8.139

נתון: מבנה מרחבי המתואר בציור עשוי מחומר אחיד ואלסטי בעל התכונות הבאות.
$E = 200\ GPa$, $G = 80\ GPa$. חלקים FK ו-CD קשיחים אינסופית.

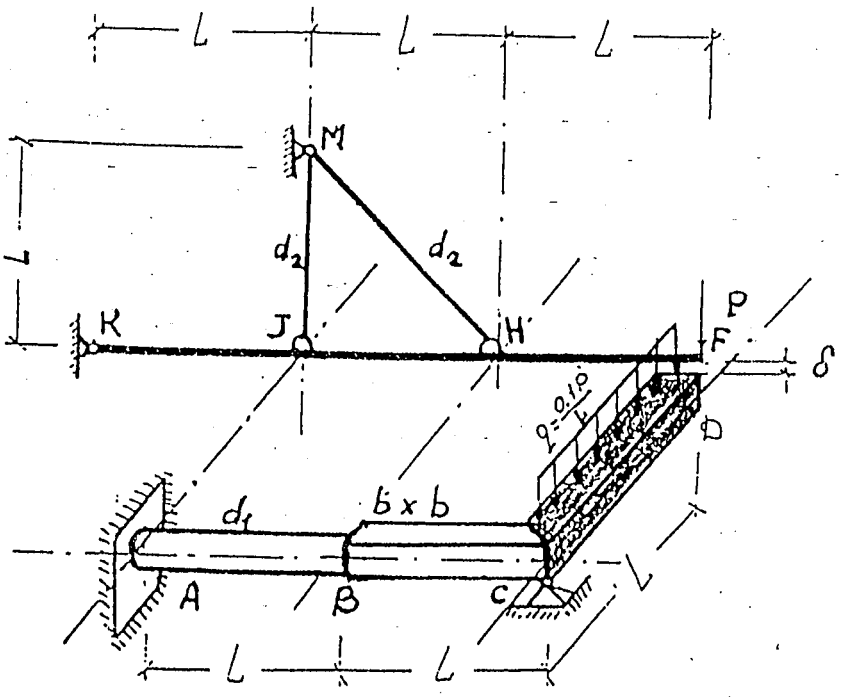
חתכי המוטות MH ו-MJ עגולים בעלי קוטר $d_2 = 0.2cm$; חתך המוט AB עגול בעל קוטר $d_1 = 4cm$; חתך המוט BC הוא מרובע b x b , $b = 4cm$, $L = 1m$.

הרווח בין נקודות F ו-D 0.3cm= δ.

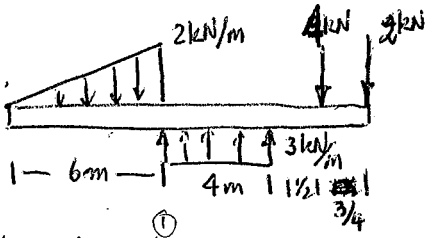
על המבנה פועלים עומס מרוכז P ועומס מפורס שווה $q = 0.1P/L$

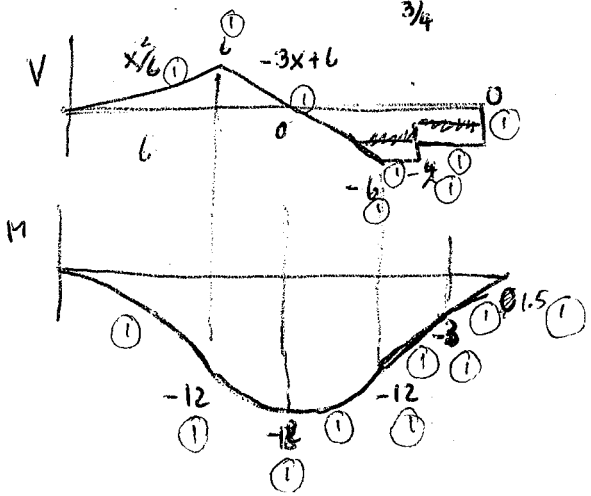
דרוש:

1. לחשב את השקיעות בנקודות D ו- F כאשר $P = 1.\ kN$.
2. מה הוא ה-P_δ הדרוש לסגירת הרווח δ? מה היא השקיעה בנקודה F כאשר $P = P_\delta$
3. לחשב את השקיעה בנקודה F כאשר $P = 5.\ kN$.
4. לבנות מהלכי מומנטי כפיפה ופיתול בחלקים DC, KF ו- AC כאשר $P = 5.\ kN$



הרשו את הדיאגרמות המקסימליות והמינימליות.
ציינו את הנקודות החיוניות של M ו-V





$\frac{dM}{dx} = -V$

$\Delta M = -\int V\,dx$

שלח 7:

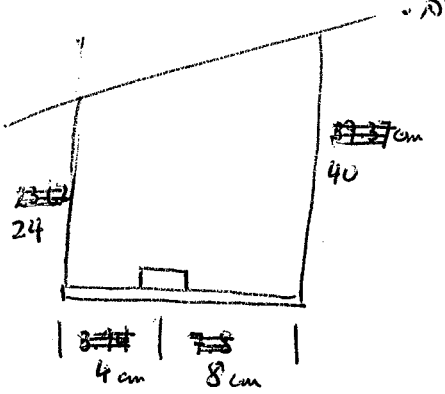
$\frac{2}{6}x = q$

$\int q\,dx = \frac{x^2}{6}\Big|_0^6 = 6$

$M \int \frac{2}{6}x^2\,dx = \frac{2}{6}\frac{x^3}{3} = \frac{1}{9}x^3\Big|_0^6 = 24$

(17)

שני חוטים דקורים למוט קשיח כמתואר בציור. לחוט האמצעי יש שטח חתך של $0.65\,cm^2$, ומודול יאנג שווה ל-210GPa. לחוט הימני, שנעשה מסגסוגת אלומיניום יש שטח חתך של $1.29\,cm^2$ ומודול יאנג שווה ל-70GPa. אם משקל שלמטקל W=8kN מיושם בנקודה שמאל המשקל, מה תהיה התארכות כל החוטים.



$A_1 = .65$ $E_1 = 210GPa$

$A_2 = 1.29$ $E_2 = 70GPa$

בציור



$P = P_1 + P_2$ (3)

(2) $u_1 = \frac{P_1 L_1}{A_1 E_1} = \frac{\frac{2}{3}8920 \cdot 24}{.65 \cdot 210} = 2.6$

(4) $\frac{u_w - u_1}{\Delta L_1} = \frac{u_2 - u_w}{\Delta L_2}$

(2) $u_2 = \frac{P_2 L_2}{A_2 E_2} = \frac{\frac{1}{3}8920 \cdot 40}{1.29 \cdot 70}$

$\Sigma M_1 = P \cdot 4 - P_2 \cdot 12 = 0$ (4)

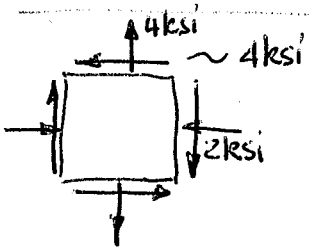
$P_2 = \frac{1}{3}P \quad P_1 = \frac{2}{3}P$ (2)

$u_w(\Delta L_2 + \Delta L_1) = (u_2 \Delta L_1 + u_1 \Delta L_2)/(\Delta L_2 + \Delta L_1)$

$u_w = \frac{3.76\times10^{-4}(.04) + 2.63\times10^{-4}(.08)}{.12} = 2.83\times10^{-4}\,m = .283\,mm$

(4)

18



מצאו את מצבות רשויות וציירו רשויות למלקין המשורטט שמולה.
שם המלקין קטן מלות בין מיבאו העבורים בשאלה.

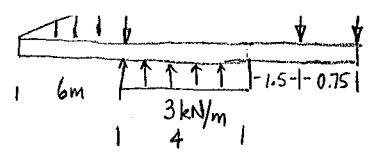
(4) $\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

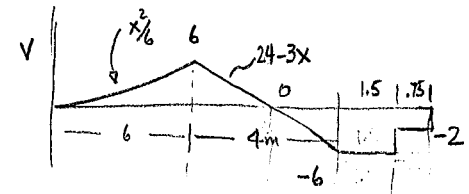
(4) $\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

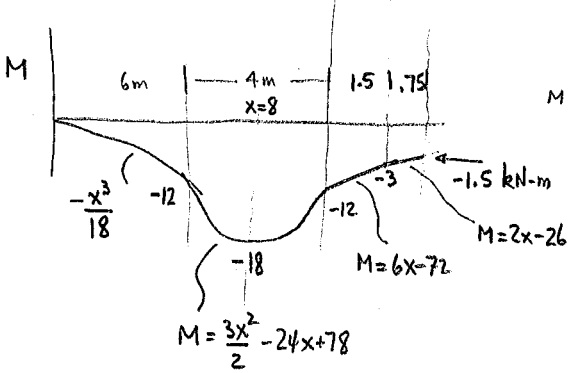
(1) $\sigma_z = 0$ $\quad \epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y)$ (2)

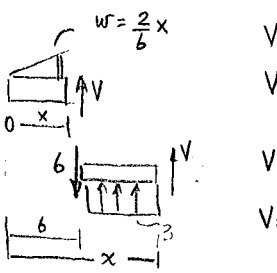
$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y)$ (2)

18









$V=\int w\,dx = 0$

$V=\int \frac{2}{6}x\,dx = \frac{2}{6}\frac{x^2}{2} = \frac{x^2}{6}$ $\quad 0\le x\le 6$

$V-6+\int_6^{10} w\,dx$

$V = 6-3(x-6) = 24-3x$ $\quad 6\le x\le 10$

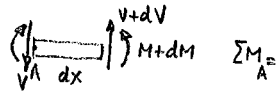
$6-4+V=0$

$V=4-6=-2$

$V=-6 \quad 10\le x\le 11.5$

$V=-2 \quad 11.5\le x\le 12.25$

$V=0 \quad x=12.25$



$\Sigma M_A = M+dM-M+(V+dV)dx = 0 \Rightarrow dM+Vdx=0$

$dM=-Vdx \qquad \frac{dM}{dx}=-V$

$M=-\int V dx$

$0\le x\le 6 \quad M=-\int \frac{x^2}{6}dx = -\frac{x^3}{18} \qquad M|_{x=6}=-12$

$6\le x\le 10 \quad M=-\int_6^x (24-3\bar{x})d\bar{x} = -\left(24x-\frac{3x^2}{2}+C\right) \qquad C=78$

שיטת הקו הישר

$M=\frac{3x^2}{2}-24x+78 \qquad M|_{x=8}=-18 \qquad M|_{x=10}=-12$

$10\le x\le 11.5 \quad M=-\int_{10}^x -6\,d\bar{x} = 6x+C \qquad C=-72$

שיטת הקו הישר

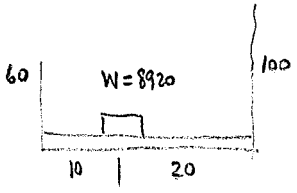
$M=6x-72 \qquad M|_{x=10}=-12 \qquad M|_{x=11.5}=-3$

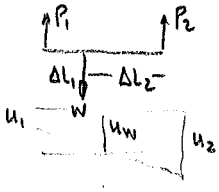
$11.5\le x\le 12.25$

$M=-\int_{11.5}^x -2\,d\bar{x} = 2x+C \qquad C=-26$

שיטת הקו הישר

$M=2x-26 \qquad M|_{x=11.5}=-3 \qquad M|_{12.25}=-1.5$





$A=.65\,cm^2$ $\quad E=210\,GPa$

$A=1.29\,cm^2$ $\quad E=70\,GPa$

$P_1+P_2=W \qquad \Sigma M_1=0=W\cdot 10-P_2\cdot 30=0 \qquad P_2=W/3 \qquad P_1=\frac{2}{3}W$

$u_1=\frac{P_1L_1}{A_1E_1} \qquad u_2=\frac{P_2L_2}{A_2E_2}$ קורסן ל קבוצה שתקבלת u_W

$\frac{u_W-u_1}{\Delta L_1}=\frac{u_2-u_W}{\Delta L_2} \Rightarrow u_W=(u_2\Delta L_1+u_1\Delta L_2)/(\Delta L_1+\Delta L_2)$

$u_1=\frac{2(8920/3)\cdot .60}{0.65\times10^{-4}\times 210\times10^9}=2.614\times10^{-4}\,m=.261\,mm$

$u_2=\frac{1/3\,(8920)(1.0)}{1.29\times10^{-4}\times 70\times10^9}=3.293\times10^{-4}\,m=.329\,mm$

$u_W=\left(3.293\times10^{-4}(.1)+2.614\times10^{-4}(.2)\right)(.3)=2.84\times10^{-4}\,m=.284\,mm$

$\sigma_x=-2\,ksi$

$\sigma_y=4\,ksi$

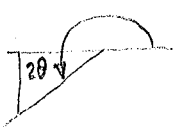
$\tau_{xy}=-4\,ksi$

$\sigma_{1,2}=\frac{\sigma_x+\sigma_y}{2}\pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$

$=1\pm\sqrt{(-3)^2+(-4)^2}=1\pm5$

$\sigma_1=6\,ksi \qquad \sigma_2=-4\,ksi$

$\tan 2\theta=\frac{2(-4)}{-6}=\frac{2\tau_{xy}}{\sigma_x-\sigma_y}=\frac{8}{6}$



$2\theta=233.13 \qquad \theta_1=116.57°$

$\theta_2=26.57°$ או $206.57°$

$\gamma_{xy}=\tau_{xy}\frac{2(1+\nu)}{E}\tau_{xy}$ $\quad \varepsilon_x=\frac{1}{E}(\sigma_x-\nu(\sigma_y+\sigma_z))=(-2-0.3(4))\times1000/_{30\times10^6}=-1.07\times10^{-4}$

$\sigma_z=0$ $\Leftarrow$ נתון

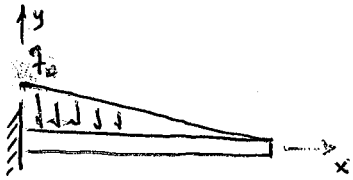


$V+dV - V + q(x)dx = 0 \quad \boxed{q(x) = \frac{dV}{dx}}$

$M+dM - M + (V+dV)dx - [q(x)dx]\frac{dx}{2} = 0 \quad \boxed{V = -\frac{dM}{dx}}$

$\frac{1}{\rho} = \frac{M}{EI} \approx \frac{d^2v}{dx^2} = \frac{M_z}{EI_{zz}}$

$-q(x) = EI\frac{d^4v}{dx^4}$, נגזרת של EI אפס. $\frac{d^4v}{dx^4} = -\frac{d^2}{dx^2}\left(\frac{1}{EI}M\right)$ $\quad q(x) = -\frac{d^2M}{dx^2}$



$q(x) = q_0\left(1 - \frac{x}{L}\right)$

$EIv^{IV} = -q_0\left(1 - \frac{x}{L}\right)$

$v(x=0) = 0 \quad -EIv''' = V \neq 0$

$\frac{dv}{dx}(x=0) = 0 \quad +EIv'' = M = 0$ $\Big\}\ x=L$

$EIv''' = -q_0\left(x - \frac{x^2}{2L} + C\right)$

@ $x=L \quad L - \frac{L^2}{2L} + C = 0 \quad C_1 = -\frac{1}{2}$

$EIv''' = -q_0\left(x - \frac{x^2}{2L} - \frac{L}{2}\right) = +V$

$EIv'' = -q_0\left(\frac{x^2}{2} - \frac{x^3}{6L} - \frac{Lx}{2} + C_2\right)$

@ $x=L \quad \frac{L^2}{2} - \frac{L^2}{6} - \frac{L^2}{2} + C_2 = 0 \quad C_2 = \frac{L^2}{6}$

$EIv'' = -q_0\left(\frac{x^2}{2} - \frac{x^3}{6L} + \frac{L^2}{6} - \frac{Lx}{2}\right) = M$

$v'' = -\frac{q_0}{EI}\left(\frac{x^2}{2} - \frac{x^3}{6L} + \frac{L^2}{6} - \frac{Lx}{2}\right)$

$v' = -\frac{q_0}{EI}\left(\frac{x^3}{6} - \frac{x^4}{24L} + \frac{L^2x}{6} - \frac{Lx^2}{4} + C_3\right)$

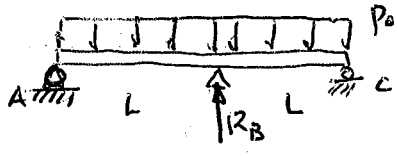
@ $x=0 \quad v'=0 \quad C_3 = 0$

$v' = -\frac{q_0}{EI}\left(\frac{x^3}{6} - \frac{x^4}{24L} + \frac{L^2x}{6} - \frac{Lx^2}{4}\right)$

$v = -\frac{q_0}{EI}\left(\frac{x^4}{24} - \frac{x^5}{120L} + \frac{L^2x^2}{12} - \frac{Lx^3}{12} + C_4\right)$

@ $x=0 \quad v=0 \quad C_4 = 0$

$v = -\frac{q_0}{120EIL}\left(5x^4L - x^5 + 10L^3x^2 - 10L^2x^3\right)$



$EIv^{IV} = -p_0 + R_B\langle x-L\rangle_*^{-1}$

$EIv''' = -p_0x + R_B\langle x-L\rangle_*^{0} + C_1 = -V$

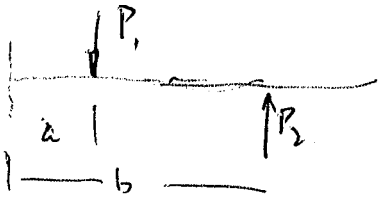
$EIv'' = -p_0\frac{x^2}{2} + R_B\langle x-L\rangle_*^{1} + C_1x + C_2$

$EIv' = -p_0\frac{x^3}{6} + R_B\langle x-L\rangle_*^{2} + C_1\frac{x^2}{2} + C_2x + C_3$

$EIv = -p_0\frac{x^4}{24} + R_B\langle x-L\rangle_*^{3} + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3x + C_4$

$\langle x-x_0\rangle_*^n = \begin{cases} 0 & x \leq x_0 \\ (x-x_0)^n & x > x_0 \end{cases}$

$v(A) = 0 = v(x=0) \quad M_A = EIv''(x=0) = 0$


P1
a
P2
b

תורת החוזק 1 | בוחן מס 1 | מאי 2000 14

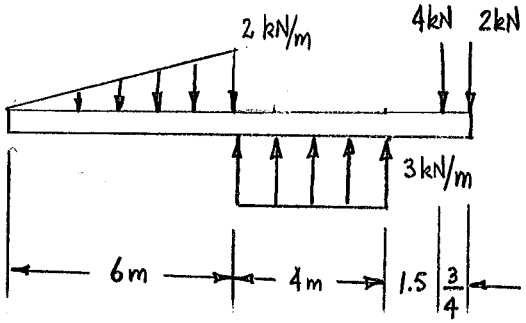
פרופ עזרא לוי

אורך הבוחן הזה יהיה 90 דקות. לבוחן יש שלושה שאלות. עליכם לענות בכל השאלות.
רק להשתמש בתפסים הרשאים, ומחשבון, ואם יש, דברים הנתונים לכם עם הבוחן.

לחתום על סעיף הבא בבקשה:
לאורך הבוחן אני לא רשאי(ת) לקבל עזרה מאף אחד או לתת עזרה לאף אחד. אם אני אעבור על אלה, אני אכשל את הבוחן.

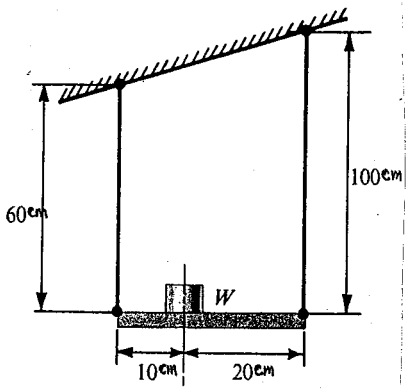
חתימת הסטודנט_____ מס' הסטודנט_____

תרגיל מס. 1
נתון לכם מוט עם עומסים מפורסים ושני עומסים יחידים כמשורטט בציור בצד השמאל. שירטטו את מהלך מומנט ומהלך כוח הגזירה לפי שיטת כלשהו.



תרגיל מס. 2

נתון שני חוטים קשורים למוט קשיח כמשורטט בציור. לחוט השמאלי שנעשה מפלדה, יש שטח חתך של $0.65\ cm^2$, ו-$E=210\ GPa$. לחוט הימני שנעשה מסגסגת אלומיניום, יש שטח חתך של $1.29\ cm^2$, ו-$E=70\ GPa$. אם משקל של $W=8920\ N$ מיושם למוט כהוראה, מה תהיה תזוזתו המשקל מפני התארכותם של החוטים. לא להכניס משקל המוט החשבון.

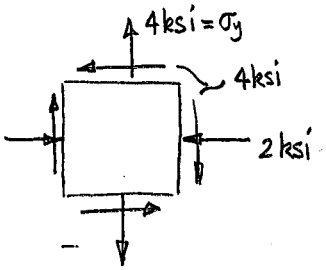


תרגיל מס. 3

מיצאו את המאמצים הראשיים והצירים הראשיים לחלקיק המשורטט שמואלה.

אם החלקיק הזה בא מלוח דק בכיוון מאונך לדף, מיצאו את העיבורים.

הלוח נעשה מפלדה עם E=210 GPa ויחס פואסון (ν) שווה ל-0.3



7 AM

תורת החוזק 1
פרופ עזרא לוי

מבחן סופי

21 ביוני 2000

אורך המבחן הזה יהיה 180 דקות. למבחן יש ארבע שאלות. עליכם לענות בכל השאלות.
רק להשתמש בתפסים הרשאים, ומחשבון. ואם יש, דברים הנתונים לכם עם המבחן.

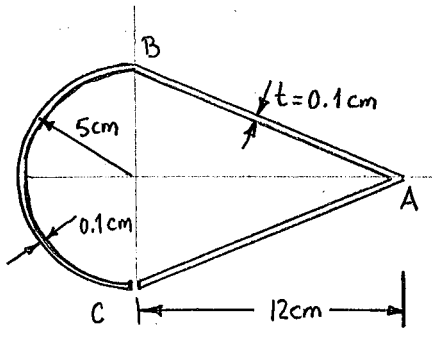
לחתום על סעיף הבא בבקשה:
לאורך הבוחן אני לא רשאי(ת) לקבל עזרה מאף אחד או לתת עזרה לאף אחד. אם אני אעבור על אלה, אני אכשל את הבוחן.

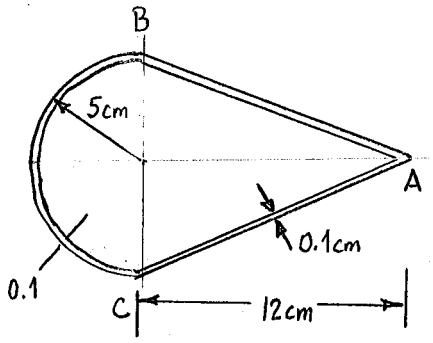
חתימת הסטודנט_____ מס' הסטודנט_____

תרגיל מס. 1
נתון לכם מוט בעל דק־דופן שטח חתך סגור כמשורטט בציור בצד השמאל.

א) השווה את זווית הגזירה וגם המאמץ הגזירה המקסימלי של המוט הזה עם אלה של מוט באותה צורה אבל עם פתיחה קטנה לאורך המוט בכיוון ציר x בנקודה C.

שני המוטות עמוסים בפיתול T. עשוים מאותו חומר. ואורכם המוטות שווים ל-L.

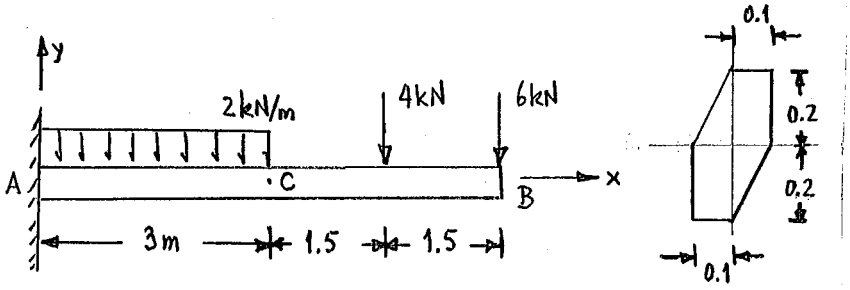




תרגיל מס. 2

נתון מוט מסוג עץ המעומס כמשורטט בציור ושטח החתך כמשורטט עם E=12 GPa.

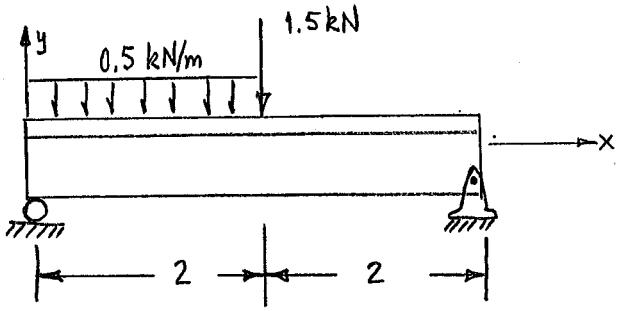
א) מיצאו את המישוואה של הקו האלסטי וגם השיפוע בנקודה C והשקיעה בנקודה B .
ב) שירטטו את מהלך מומנט ומהלך כוח הגזירה של המוט.
ג) מיצאו את מיקום וערך מאמץ המקסימלי.

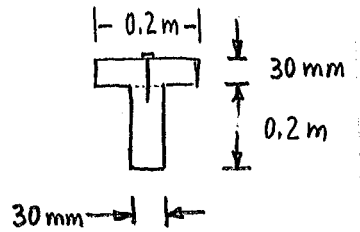


תרגיל מס. 3

הקורה בפרופיל T הנעשתה מעץ מחוברת דרך מסמרים משני לוחות של 200 mm x 300 mm .
אם מאמץ הכפיפה המותר שווה ל - 12 MPa ומאמץ הגזירה המותר שווה ל - 0.8MPa.

א) מיצאו אם הקורה יכולה לתמוך בבטיחות את העומסים כמשורטט בציור.
ב) מיצאו הרווח המקסימלי בין המסמרים אם כל מסמר ומסמר יכול לתמך בבטיחות 1500 N בגזירה.

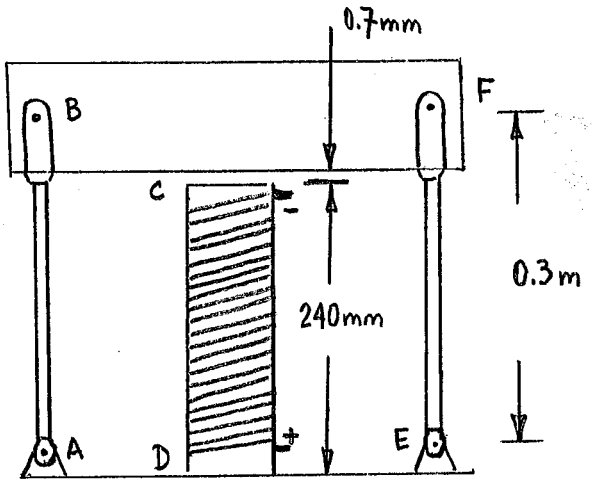




תרגיל מס. 4

המוט המרכזי CD של ההרכבה מחומם מ-T_1=30 C ל-T_2=180 C דרך חמום התנגדות חשמלי.
כאשר הטמפרטורה שווה ל- T_1 הרווח בין C והמוט הקשיח הוא 0.7 mm

א) מיצאו את הכוח במוטות AB ו- EF שבא משינוי הטמפרטורה של מוט CD.
מוטות AB ו- EF עשוים מפלדה, ולכל אחד יש שטח חתך של $125\ mm^2$. המוט גב נעשה מאלומיניום ולו יש שטח חתך של $375\ mm^2$.
ב) מה תהיה ההיתארכות/ההיתכווצות הסופית של המוט CD



to as the *shear flow*. Since force is usually measured in pounds, shear flow q has units of pounds per inch. Then, recalling that $dM/dx = -V$, one obtains the following expression for the shear flow in beams:

$$q = \frac{dF}{dx} = -\frac{dM}{dx}\frac{1}{I}\int_{\substack{\text{area}\\ fghj}} y\,dA = \frac{V\,A_{fghj}\bar{y}}{I} = \frac{V\,Q}{I} \qquad (7\text{-}5)$$

In this equation I stands for the moment of inertia of the entire cross-sectional area around the neutral axis, just as it does in the flexure formula from which it came. The total shearing force at the section investigated is represented by V, and the integral of $y\,dA$ for determining Q extends only over the cross-sectional area of the beam to one side of this area at which q is investigated.

In retrospect, note carefully that Eq. 7-5 was derived on the basis of the elastic flexure formula, but no term for a bending moment appears in the final expressions. This resulted from the fact that only the change in the bending moments at the adjoining sections had to be considered, and the latter quantity is linked with the shear V. The shear V was substituted for $-dM/dx$, and this masks the origin of the established relations. Equation 7-5 is very useful in determining the necessary interconnection between the elements making up a beam. This will be illustrated by examples.

EXAMPLE 7-1

Two long wooden planks form a T section of a beam as shown in Fig. 7-6(a). If this beam transmits a constant vertical shear of 690 lb, find the necessary spacing of the nails between the two planks to make the beam act as a unit. Assume that the allowable shearing force per nail is 150 lb.

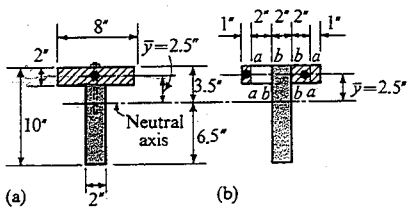


Fig. 7-6

SOLUTION

In attacking such problems the analyst must ask: What part of a beam has a tendency to slide longitudinally from the remainder? Here it is the plane of contact of the two planks; Eq. 7-5 must be applied to determine the shear flow in this plane. To do this the neutral axis of the whole section and its moment of inertia around the neutral axis must be found. Then as V is known and Q is defined as the statical moment of the area of the upper plank around the neutral axis, q may be determined. The distance y_c from the top to the neutral axis is

$$y_c = \frac{2(8)1 + 2(8)6}{2(8) + 2(8)} = 3.5 \text{ in.}$$

$$I = \frac{8(2)^3}{12} + (2)8(2.5)^2 + \frac{2(8)^3}{12} + (2)8(2.5)^2 = 291 \text{ in.}^4$$

$$Q = A_{fghj}\bar{y} = (2)8(3.5 - 1) = 40 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{690(40)}{291} = 95 \text{ lb per in.}$$

Thus, a force of 95 lb must be transferred from one plank to the other in every linear inch along the length of the beam. However, from the data given, each nail is capable of resisting a force of 150 lb, hence one nail can take care of 150/95 = 1.59 linear inches along the length of the beam. As shear remains constant at the consecutive sections of the beam, the nails should be spaced throughout at 1.59-in. intervals. In a practical problem a 1.5-in. spacing would probably be used.

SOLUTION FOR AN ALTERNATE ARRANGEMENT OF PLANKS

If, instead of using the two planks as above, a beam of the same cross section were made from five pieces, Fig. 7-6(b), a different nailing schedule would be required.

To begin, the shear flow between one of the 1-in.-by-2-in. pieces and the remainder of the beam is found, and although the contact surface a-a is vertical, the procedure is the same as before. The push or pull on an element is built up in the same manner as formerly:

$$Q = A_{fghj}\bar{y} = (1)2(2.5) = 5 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{690(5)}{291} = 11.8 \text{ lb per in.}$$

If the same nails as before are used to join the 1-in.-by-2-in. piece to the 2-in.-by-2-in. piece, they may be 150/11.8 = 12.7 in. apart. This nailing applies to both sections a-a.

To determine the shear flow between the 2-in.-by-10-in. vertical piece and either one of the 2-in.-by-2-in. pieces, the whole 3-in.-by-2-in. area must be used to determine Q. It is the difference of pushes (or pulls) on this whole area that causes the unbalanced force which must be transferred at the surface b-b:

$$Q = A_{fghj}\bar{y} = (3)2(2.5) = 15 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{690(15)}{291} = 35.6 \text{ lb per in.}$$

Space nails at 150/35.6 = 4.2 in. or, in practice, 4-in. intervals along the length of the beam in both sections b-b. These nails could be driven in first and then the 1-in.-by-2-in. pieces put on.

EXAMPLE 7-2

A simple beam on a 20-ft span carries a load of 200 lb per foot including its own weight. The beam cross section is to be made from several full-sized wooden pieces as in Fig. 7-7(a). Specify the spacing of the ½-in.

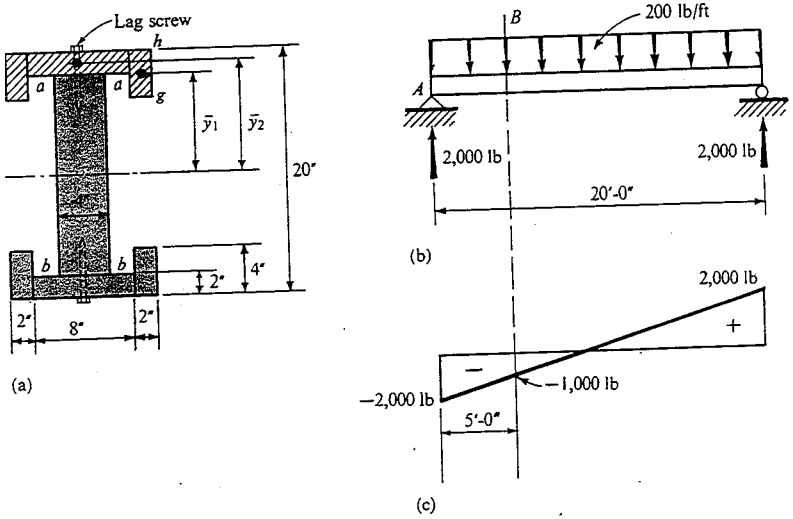


Fig. 7-7

lag screws shown which is necessary to fasten this beam together. Assume that one ½-in. lag screw, as determined by laboratory tests, is good for 500 lb when transmitting a lateral load parallel to the grain of the wood. For the entire section I is equal to 6,060 in.^{4}

SOLUTION

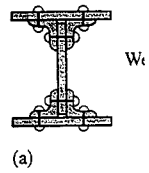
To find the spacing of the lag screws, the shear flow at section *a-a* must be determined. The loading on the given beam is shown in Fig. 7-7(b); to show the variation of the shear along the beam, the shear diagram is constructed in Fig. 7-7(c). Then, to apply the shear flow formula, Q must be determined. This is done by considering the shaded area to one side of the cut *a-a* in Fig. 7-7(a). The statical moment of this area is most conveniently computed by multiplying the area of the two 2-in.-by-4-in. pieces by the distance from their centroid to the neutral axis of the beam and adding to this product a similar quantity for the 2-in.-by-8-in. piece. The largest shear flow occurs at the supports, as the largest vertical shears V of 2,000 lb act there:

$$Q = A_{fghj}\bar{y} = \sum A_i\bar{y}_i = 2A_1\bar{y}_1 + A_2\bar{y}_2$$

$$= 2(2)4(8) + 2(8)9 = 272 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{2{,}000(272)}{6{,}060} = 90 \text{ lb per in.}$$

At the supports the spacing of the lag screws must be 500/90 = 5.56 in. apart. This spacing of the lag screws applies only at a section where the shear V is equal to 2,000 lb. Similar calculations for a section where V = 1,000 lb gives q = 45 lb per inch, and the spacing of the lag screws becomes 500/45 = 11.12 in. Thus it is proper to specify the use of ½-in. lag screws at 5½ in. on centers for a distance of 5 ft nearest both the supports and at 11 in. for the middle half of the beam. A greater refinement in making the transition from one spacing of fastenings to another may be desirable in some problems. The same spacing of lag screws should be used at the section *b-b* as at the section *a-a*.



(a)

Fig. 7-8. Typical beam sec of several components: (a) *I* beam reinforced with plat

In a manner analogous to the above, the spacing of rivets or bolts in fabricated beams made from continuous angles and plates, Fig. 7-8, may be determined. Welding requirements are established similarly. The nominal shearing stress in a rivet is determined by dividing the total shearing force transmitted by the rivet (shear flow times spacing of the rivets) by the cross-sectional area of the rivet.

7-4. THE SHEARING STRESS FORMULA FOR BEAMS

The shearing stress formula for beams may be obtained from the shear flow formula. Analogously to the earlier procedure, an element of a beam may be isolated between two adjoining sections taken perpendicular to the axis of the beam. Then by passing another imaginary section through this element parallel to the axis of the beam, a new element is obtained, which corresponds to the element of one "plank" used in the earlier derivations. A side view of such an element is shown in Fig. 7-9(a), where the imaginary longitudinal cut is made at a distance y_1 from the neutral axis.* The cross-sectional area of the beam is shown in Fig. 7-9(c).

If shearing forces exist at the sections through the beam, a different bending moment acts at section A than at B. Hence more push or pull is developed on one side of the area *fghj* than on the other, and, as before, the difference in the longitudinal forces in a distance dx is

$$dF = -\frac{dM}{I}\int_{\substack{\text{area}\\fghi}} y\,dA = -\frac{dM}{I}A_{fghi}\bar{y} = -\frac{dM}{I}Q$$

The force equilibrating dF is developed in the plane of the longi-

* Since $dM/dx = -V$, for a positive V the change in moment $dM = -V\,dx$. For this reason $M_A > M_B$ and the magnitudes of the normal stresses in Fig. 7-9(a) are shown accordingly.

occur in the plastic zones, no unbalance in longitudinal forces occurs and no shearing stresses are developed.

This elementary solution has been refined by using a more carefully formulated criterion of yielding caused by the simultaneous action of normal and shearing stresses.* Some fundamental aspects of the interaction of such stresses will be considered in Chapter 9.

EXAMPLE 7-6

An I beam is loaded as in Fig. 7-14(a). If it has the cross section shown in Fig. 7-14(c), determine the shearing stresses at the levels indicated. Neglect the weight of the beam.

SOLUTION

A free-body diagram of a segment of the beam is in Fig. 7-14(b). It is seen from this diagram that the vertical shear at every section is 50 kips. Bending moments do not enter directly into the present problem. The shear flow at the various levels of the beam is computed in the table below using Eq. 7-5. Since $\tau = q/t$ (Eq. 7-6), the shearing stresses are obtained by dividing the shear flows by the respective widths of the beam.

$$I = \frac{6(12)^3}{12} - \frac{(5.5)(11)^3}{12} = 254 \text{ in.}^4$$

For use in Eq. 7-5 the ratio $V/I = -50{,}000/254 = -197$ lb/in.^{4}

Level	A_{fohj}*	$\bar{y}$**	$Q = A_{fohj}\bar{y}$	$q = VQ/I$	t	τ, psi
1-1	0	6	0	0	6.0	0
2-2	(0.5)6 = 3.00	5.75	17.25	−3,400	6.0 0.5	−570 −6,800
3-3	(0.5)6 = 3.00 (0.5)(0.5) = 0.25	5.75 5.25	17.25 1.31 | 18.56	−3,650	0.5	−7,300
4-4	(0.5)6 = 3.00 (0.5)(5.5) = 2.75	5.75 2.75	17.25 7.56 | 24.81	−4,890	0.5	−9,780

* A_{fohj} is the partial area of the cross section above a given level in square inches.
** $\bar{y}$ is the distance from the neutral axis to the centroid of the partial area in inches.

The negative signs of τ show that, for the section considered, the stresses act downward on the right face of the elements. The sense of the shearing stresses acting on the section coincides with the sense of the shearing force V. For this reason a strict adherence to the sign convention is often unnecessary. It is always true that $\int_A \tau\, dA$ is equal to V and has the same sense.

* D. C. Drucker, "The Effect of Shear on the Plastic Bending of Beams," *Journal of Applied Mechanics*, 23 (1956), pp. 509–14.

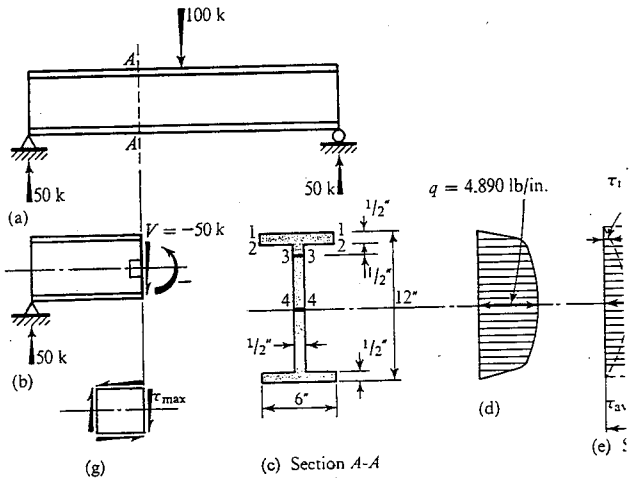


Fig. 7-14

Note that at the level 2-2 two widths are used to determine the shearing stress—one just above the line 2-2, and one just below. A width of 6 in. corresponds to the first case, and 0.5 in. to the second. This transition point will be discussed in the next article. The results obtained, which by virtue of symmetry are also applicable to the lower half of the section, are plotted in Fig. 7-14(d) and (e). By a method similar to the one used in the preceding example, it may be shown that the curves in Fig. 7-14(e) are parts of a second-degree parabola.

The variation of the shearing stress indicated by Fig. 7-14(e) may be interpreted as is shown in Fig. 7-14(f). The maximum shearing stress occurs at the neutral axis; the vertical shearing stresses throughout the web of the beam are nearly of the same magnitude. The shearing stresses occurring in the flanges are very small. For this reason the maximum shearing stress in an I beam is often approximated by dividing the total shear V by the cross-sectional area of the web (area *abcd* in Fig. 7-14(f)). Hence

$$(\tau_{max})_{approx} = V/A_{web} \tag{7-9}$$

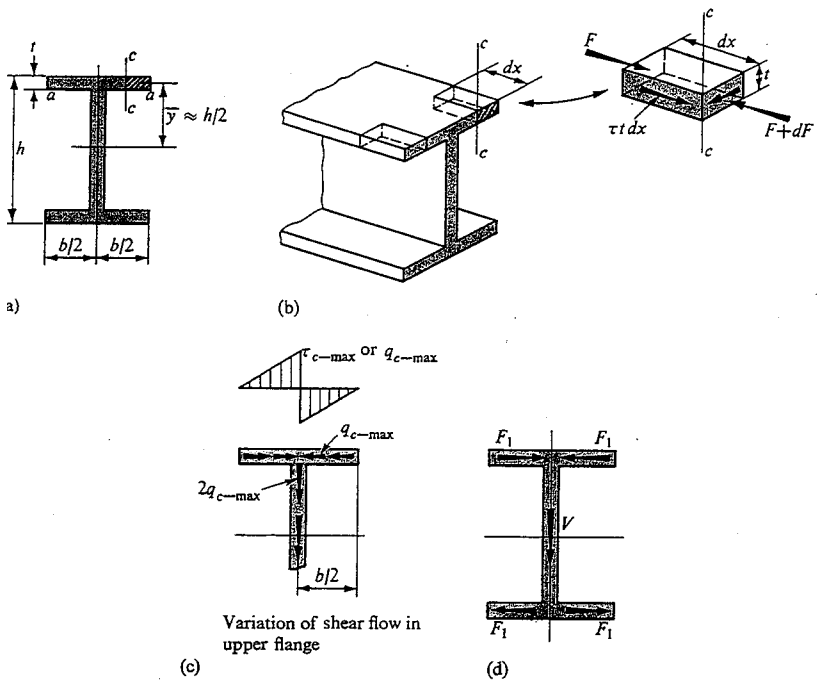
In the example considered this gives

$$(\tau_{max})_{approx} = \frac{50{,}000}{(0.5)12} = 8{,}330 \text{ psi}$$

This stress differs by about 15 per cent from the one found by the accurate formula. For most cross sections a much closer approximation

best approach. For example, if for the beam segment in Fig. 7-17(b) positive bending moments increase toward the reader, larger normal forces act on the nearer cross section. For the elements shown, $\tau t\,dx$ or $q\,dx$ must aid the smaller force acting on the partial area of the cross section. This determines the sense of the shearing stresses in the longitudinal cuts. Numerically equal shearing stresses act on the mutually perpendicular planes of an infinitesimal element, and the shearing stresses on such planes either meet or part with their directional arrowheads at a corner. In this manner the sense of the shearing stresses in the plane of the cross section becomes known.

The magnitude of the shearing stresses varies for the different vertical cuts. For example, if the cut *c-c* in Fig. 7-17(a) is at the edge of the beam, the shaded area of the beam's cross section is zero. However, if the thickness of the flange is constant, and the cut *c-c* is made progressively closer to the web, the shaded area increases from zero at a linear rate.



ig. 7-17. Existence of shearing forces in the flange of an *I* beam hich act perpendicularly to the axis of symmetry.

Moreover, as $\bar{y}$ remains constant for any such area, Q also increases linearly from zero toward the web. Therefore, since V and I are constant at any section through the beam, the shear flow $q_c = VQ/I$ follows the same variation. If the thickness of the flange remains the same, the shearing stress $\tau_c = VQ/It$ varies similarly. The same variation of q_c and τ_c exists on either side of the axis of symmetry of the cross section. However, these quantities in the plane of the cross section act in opposite directions on the two sides as may be determined by isolating another flange element to the left side of the web in Fig. 7-17(b). The variation of these shearing stresses or shear flows is represented in Fig. 7-17(c), where it is assumed that the web is very thin.

In common with all stresses, the shearing stresses in Fig. 7-17(c), when integrated over the area on which they act, are equivalent to a force. The magnitude of the horizontal force F_1 for one-half the flange, Fig. 7-17(d), is equal to the average shearing stress multiplied by one-half the flange area, i.e.,

$$F_1 = (\tau_{c\text{-max}}/2)(bt/2) \qquad \text{or} \qquad F_1 = (q_{c\text{-max}}/2)(b/2)$$

These horizontal forces act in the upper and lower flanges. Because of the symmetry of the cross section, these equal forces occur in pairs and oppose each other; thus they cause no apparent external effect.

To determine the shear flow at the juncture of the flange and the web (cut *a-a* in Fig. 7-17(a)), the whole area of the flange times $\bar{y}$ must be used in computing the value of Q. However, since in finding $q_{c\text{-max}}$ one-half the flange area times the same $\bar{y}$ has already been used, the sum of the two horizontal shear flows coming in from opposite sides gives the vertical shear flow at the cut *a-a*. Hence, figuratively speaking, the horizontal shear flows turn through 90° and merge to become the vertical shear flow. Then the shear flows at the various horizontal cuts through the web may be determined in the manner explained in the preceding articles. Moreover, as the resistance to the vertical shear V in thin-walled I beams is developed mainly in the web, it is so shown in Fig. 7-17(d). The sense of the shearing stresses and shear flows in the web coincides with the direction of the shear V. Note that the vertical shear flow "splits" upon reaching the lower flange. This is represented in Fig. 7-17(d) by the two forces F_1, which are the result of the horizontal shear flows in the flanges.

The shearing forces which act at a section of an I beam are shown in Fig. 7-17(d), and, for equilibrium, the applied vertical forces must act through the centroid of the cross-sectional area to be coincident with V. If the forces are so applied, no torsion of the member will occur. This is true for all sections having cross-sectional areas with an axis of symmetry. Thus, to avoid torsion of such members, the applied forces must act in the plane of symmetry of the cross section and the axis of the beam. A beam with an unsymmetrical section will be discussed next.

תרגיל מס. 3

הקורה בפרופיל T הנעשתה מעץ מחוברת דרך מסמרים משני לוחות של 200 mm x 300 mm.
אם מאמץ הכפיפה המותר שווה ל - 12 MPa ומאמץ הגזירה המותר שווה ל - 0.8MPa.

א) מיצאו אם הקורה יכולה לתמוך בבטיחות את העומסים כמשורטט בציור.
ב) מיצאו הרווח המקסימלי בין המסמרים אם כל מסמר ומסמר יכול לתמך בבטיחות 1500 N בגזירה.

תרגיל מס. 4

המוט הנרכזי CD של ההרכבה מחומם מ-$T_1=30\ C$ ל-$T_2=180\ C$ דרך חמום התנגדות חשמלי.
כאשר הטמפרטורה שווה ל-T_1 הרווח בין C והמוט הקשיח הוא 0.7 mm

א) מיצאו את הכוח במוטות AB ו- EF שבא משינוי הטמפרטורה של מוט CD.
מוטות AB ו- EF עשוים מפלדה, ולכל אחד יש שטח חתך של $125\ mm^2$. המוט **CD** נעשה מאלומיניום ולו יש שטח חתך של $375\ mm^2$.
ב) מה תהיה ההיתארכות/ההיתכווצות הסופית של המוט CD

