SELF-PACED COURSES

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SCOPE

The scope of this report is to present a comprehensive approach to the preparation of self-paced courses in basic engineering science subjects for college students as well as to overview the work already done and the available building blocks of this program.

SELF-PACED COURSES

Aim

The self-paced courses are aimed at all levels of the student population, responding to the particular needs of each group. The top students will be able to study independently at an accelerated pace, while the weaker students will either use the self-paced courses as their primary way to study, enabling them to devote longer periods of time to problematic topics; or in the case of failure in a regular classroom course, the self-paced course will be providing them with a second chance to fulfill the requirements. To most students, the self-paced courses can serve as a review or an enhancing tool, or under certain circumstances, as a primary means of learning in a particularly difficult subject or a missed class.

Course Framework

All self-paced courses are based on written and visual materials. These two kinds of information sources complement each other, thus improving the efficiency of the learning process. The major building blocks of such a course include:

a) *Study Guide*

The study guide should be a fully written structured course. In its introduction, it should contain the rationale behind the need to learn a particular course and the course's relation with previous and future courses. Furthermore, the introduction should contain a short historical overview and present clearly the various sections of the study guide. Last, but not least, the introduction should provide the student with accurate and detailed instructions on how to use the self-paced course to enjoy its maximum benefits.

Each of the following sections of the study guide should cover one particular topic in the form of lecture notes supplemented by worked examples and a set of problems for the student to work on.

b) *Electronic Notes*

The electronic notes are to complement the written notes of the study guide presenting selected topics in a visually more animated form, including multi-colored graphs and animation. In the future, the electronic notes might be enhanced by incorporating short video clips presenting experiments or any other real life situations.

c) *Video Tapes*

Video tapes, containing problem solving sessions given in a class type atmosphere by a professor, would serve to present the worked examples appearing in the study guide in a visual form. The ability of creating a class type situation using these videos undoubtedly will improve the student's ability to follow the solution process of the worked examples.

d) *Review Materials*

Towards the mid-term and/or the final of the self-paced course, additional written and visual material will be supplied to help the students prepare themselves for the particular exam. The review material should also provide the student with a more general perception of the course subject.

e) *Testing Material*

A computerized testing capability should accompany each self-paced course. It should include mock tests as well as real tests.

The Learning Process

Only a comprehensive integration of all of the above means will provide the student using the self-paced course with a most effective learning tool. The learning instructions, as well as

detailed referencing between all the above means, will provide the real integration of all these venues of learning.

Present Available Building Blocks

At present, only part of the building blocks are available:

a) *Written notes*

Preliminary notes were written for the following subjects:

1. Statics
2. Fluid Mechanics

b) *Written examples*

Solved examples were prepared for:

1. Mathematics
2. Statics
3. Mechanics of Materials
4. Thermodynamics
5. Material Science
6. Electrical Engineering

c) *Electronic notes*

Electronic notes were prepared in the following area:

1. Introduction to Engineering

2. Statics
3. Dynamics
4. Fluid Mechanics

d) *Video tapes*

Video tapes containing sessions of problem solving were taped in the following areas:

1. Mathematics
2. Chemistry
3. Statics
4. Dynamics
5. Mechanics of Materials
6. Material Science
7. Thermodynamics
8. Fluid Mechanics
9. Electrical Engineering
10. Magnetic Fields
11. Engineering Economics

SELF-PACED MATERIAL AT TECHNION

Presently, videotapes of all basic science and engineering courses taken in the classroom are the only available self-paced material at the Technion-Israel Institute of Technology. These tapes are mainly used for our students who miss classes for reason of military service during the

semester. Currently at the Technion, there is an increasing demand for more diverse self-paced material of the sort described above. Thus, we will be looking forward to cooperate with Florida International University in developing such self-paced material for the benefit of the students at both institutions.

August 25th 1986

Statics - Introductory Remarks.

1. Instructor - Dr. Mordechai Perl
   Visiting Research Professor
   Technion - Israel Institute of Technology
   Research Fields: Fracture Mechanics, Bio-Mechanics

2. Classes: Two periods a week: alltogether 28 meetings:
   No Meetings on: M Sept 1st - Labor Day
   M Oct 13th - Yom Kippur

3 Sillabus - Preliminary
   Book - Meriam

4. Homework - will be assigned each period (4 ex)
   will not be collected. Student is advised to solve MOR
   Any particular problematic problem - handout
   with the detailed solution.

5. Grading Policy: 7 quizzes × 10% + 30% Final = 100
   Absence from quiz - to be discussed as
   a personal problem with the instructor.

6. Quizzes: simillar but not identical with homework.

7. Office Hours: to be announced.

Chapter 1 Introduction To Statics

Mechanics is the oldest and probably the most important branch of physics. Mechanics deals with the state of rest or motion of bodies under the action of forces. Mechanics can be divided into two main fields: <u>Statics</u> and <u>Dynamics</u>. Since mechanics is an essential part of any modern R&D field such as strength and stability of structures and machines, vibrations, robotics, aircraft rocket and spacecraft design etc, its understanding is the basic prerequisite to work in these and many other fields.

Some of the historical milestones in Mechanics are

212 B.C.E Archimedes - principles of lever and buoyancy

1548-1620 Stevinus - Vector combination - parallelogram law
Basic Principles of Statics

1564-1642 Galileo - Dynamics

1642-1727 Newton - Law of Gravitation

Others : Da Vinci, Varignon, Euler, D'Alembert, Lagrange, Laplace and others.

Mechanics has relatively, a few basic principles but they have exceedingly wide application

1.2 Basic Concepts

* Space - the geometric region occupied by bodies
* Time - the measure of succession of events
* Mass - the measure of the resistance of a body to a change in its velocity.
* Force - the action of one body on another. It tends to move the body, upon which it is acting in the direction of its action.
  characterized by: a) magnitude
  b) direction
  c) point of application
* particle - a body of negligible dimensions
* Rigid body - a body whose deformation under the action of force is negligible as compared to its dimension

1.3 Scalar & Vectors

Mechanics deals with various kinds of quantities two of which are scalars & vectors.

To a scalar only its magnitude is associated e.g: time volume, density, speed, mass, energy etc.

Vectors posses a direction as well as a magnitude e.g: force, velocity, acceleration moment & momentum.

Vectors fall into one of the following categories:

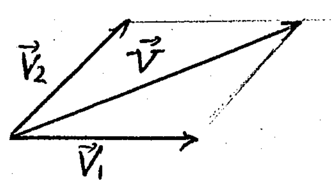
* Free Vector - its action is not confined to a unique line in space e.g.: displacement vector

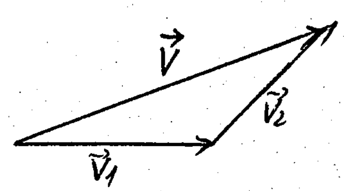
* Sliding Vector - is confined to a unique line in space but might be slided along it - principle of transmissibility - e.g. External force acting on a rigid body

* Fixed Vector - is confined to a unique point of application e.g. a force acting on a deformable body

A vector is designated by $\vec{F}$ or $\underline{F}$, its magnitude ("length") is $|F|$ or F.

The parallelogram law



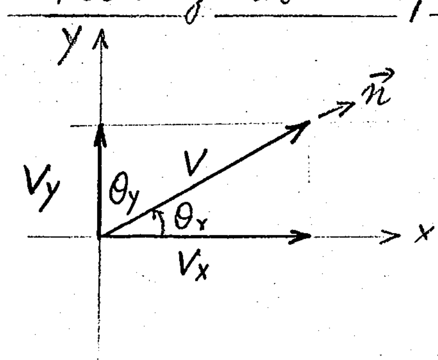


Triangle Law
head-to-tail addition

$\vec{V} = \vec{V_1} + \vec{V_2}$

$\vec{V_1}$ and $\vec{V_2}$ are called the components of $\vec{V}$

Rectangular Components



$\vec{V} = \vec{V_x} + \vec{V_y}$ $\quad |V| = \sqrt{V_x^2 + V_y^2}$ $\quad V_x = V\cos\theta_x$

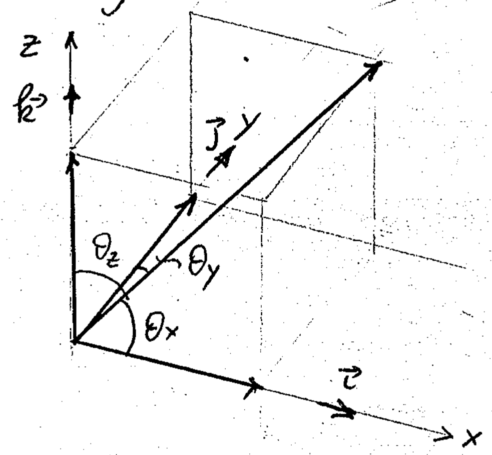
$\theta = \tan^{-1}\frac{V_y}{V_x}$ $\quad V_y = V\cos\theta_y$

$\vec{n}$ - unit vector

$$\vec{V} = V\,\vec{n}$$

In many cases, particularly in 3-D ones, it is convinient to express a vector in terms of its rectangular components and a set of unit vectors i, j, k along the three axes

$\vec{V} = V_x \vec{i} + V_y \vec{j} + V_z \vec{k}$



we can define the direction cosines

$l = \cos\theta_x = \frac{V_x}{V}$; $m = \cos\theta_y = \frac{V_y}{V}$; $n = \cos\theta_z = \frac{V_z}{V}$

$V^2 = V_x^2 + V_y^2 + V_z^2 \qquad l^2 + m^2 + n^2 = 1$

1.4 Newton's Laws

Newton, in his Principia (1687), stated the three basic Laws governing the motion of particles:

Law I : A particle remains at rest or continues to move in a straight line with a uniform velocity if the resultant force acting on it is nill

Law II : The acceleration of a particle is directly propotional to the resultant force acting on it and is in the direction of that force $F = ma$

Law III The forces of action and reaction between interacting bodies are: equal in magnitude, opposite in direction and collinear.

It is important to be careful in interpreting the third law.

1.5. Units

The basic four quantities in Mechanics are: length, mass force and time. In order for Newton's 2nd law to read

$$F = ma$$

and not $F = kma$ the units for these four quantities are to be chosen consistently. There are various units systems that comply with this demand will discuss two of them the SI (Système International d'Unités) and the British system FPS

Quantity	Dimensional symbol	SI Unit	SI Symbol	FPS Unit	FPS Symbol
Mass	M	Kilogram	Kg	Slug	—
Length	L	meter	m	foot	ft
Time	T	second	s	second	sec
Force	F	Newton	N	pound	lb

remember

$$N = Kg \cdot \frac{m}{sec^2} \qquad [F] = [M]\frac{[L]}{[T^2]}$$

$$lb = slug \frac{ft}{sec^2}$$

Standards

Mass - platinum iridium cylinder at Sèvre (near Paris)

Length - 1,650,763.73 λ (wavelength) Kr-86

Time - 9,192,631,770 τ (period) Cs-133

1.6 Law of Gravitation

Newton's law of gravitation states that two particles of mass m_1 and m_2 are mutually attracted with equal and opposite forces of magnitude F

$$F = G \cdot \frac{m_1 m_2}{r^2}$$

where: r - the distance between the particles
G - the universal constant of Gravitation.

for a partical on the earth (M) surface:

$$W = \frac{GM}{R^2} \cdot m \quad \Rightarrow \quad g = \frac{GM}{R^2}$$

1.7 Accuracy and Approximations

The number of SIGNIFICANT figures in an answer depends upon two items:

a) the accuracy of the given data
b) the accuracy of the computation performed

the solution can not be more accurate then the less accurate of these two items.

example: 3 cars are weighed together and their weight is found to be 100,000 N ± 1% = 100,000 ± 1000

How much weighs one car?

$$\frac{100{,}000}{3} = 33{,}333\,[N] \text{ but } 1\% \text{ of error} \sim 333$$

thus 1car = 33,333 ± 333 [N] = 33,300 ± 333

error ≠ mistake

Engineers use 3-4 digits of accuracy.

<u>approximations</u>

In many cases, in order to simplify the analysis, we approximate various physical quantities e.g. when small angles are involved we tend to use the following approximation

$$\sin\theta \simeq tg\,\theta \simeq \theta \;\&\; \cos\theta \simeq 1$$

this approximation retains only the first term in a series expansion for these functions:

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$$

$$tg\,\theta = \theta + \frac{\theta^3}{3!} + \dots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!}$$

nevertheless one should be carefull e.g

$$1-\cos\theta \simeq 1-\left(1-\frac{\theta^2}{2!}\right) = \frac{\theta^2}{2} \text{ and not } 0$$

Chapter 2 Force Systems

2.1 Introduction

In this chapter we'll discuss the topic of Force Systems, in its wide context, in details.

2.2 Force

Two definitions for a force:

a. The action of one body on another

* b. An action that tends to maintain or to change the state of a body and/or to deform it.

* Force can be applied directly by mechanical contact or remotely by e.g a gravitational or an electric field

* There are concentrated or distributed forces.

* Surface Tractions - Body Forces.

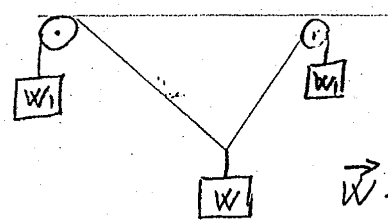
The action of a force on a body results in two effects: external and internal. The external effect involves reactions while the internal creates stresses and strains. Statics is concerned only with the external effect

The Basic Principles of Statics

statics has three basic principles or axiomes:

* The Parallelogram law

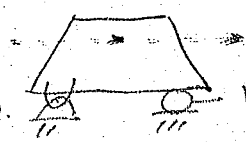
Experimental proof:



$\vec{W} = \vec{W_1} + \vec{W_2}$

* The Action & Reaction law (Newton III)

* The principle of Transmissibility



2.3 Rectangul components

$F_x = F\cos\theta_x$ $\quad F_x = F\cos\theta_x$ $\quad F_x = \vec{F}\cdot\vec{i}$

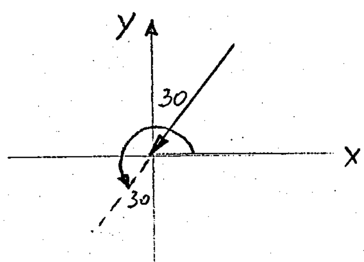
$F_y = F\sin\theta$ $\quad F_y = F\cos\theta_y$ $\quad F_y = \vec{F}\cdot\vec{j}$

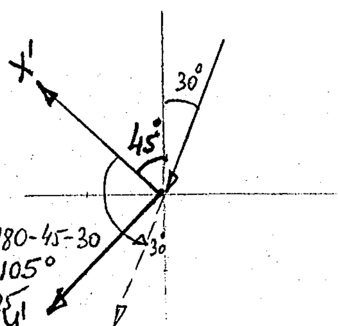
$F_y = F\cos\theta_y$

$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k} = F(l\vec{i} + m\vec{j} + n\vec{k})$

$\vec{F}\cdot\vec{i} = F_x \cdot i \cdot i = F_x$

example A





$\theta = 180 - 45 - 30$

$\theta_x = 105°$

$\theta_y = 25$

$\theta_x = 240°$ $\quad \theta_y = 150$

$F_x = 100\cos 240 = -50\,N$

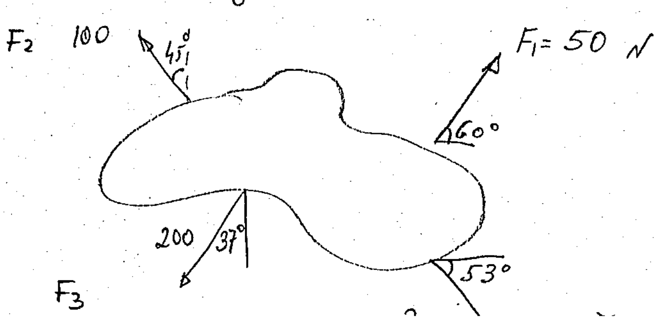
$F_y = 100\sin 150 = -86.6\,N$

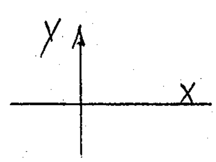
$F_x = 100\cos 105 = -25.9\,N$

$F_y = 100\cos 25 = 96.5\,N$

example C

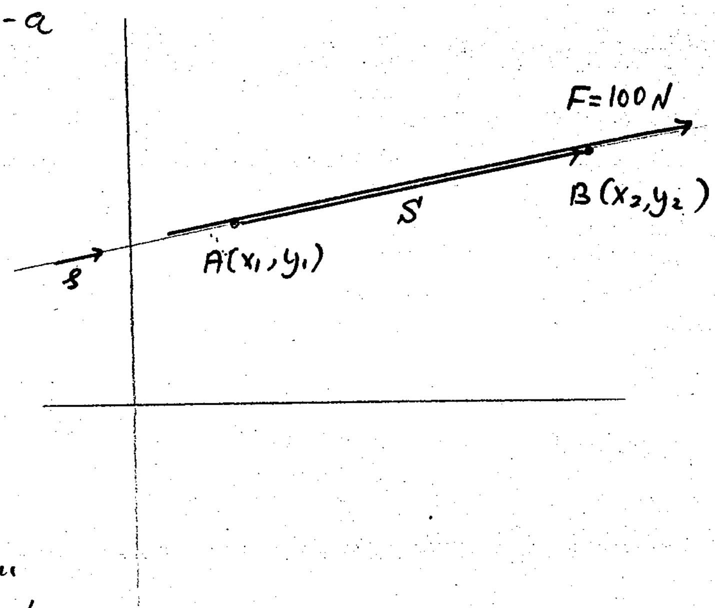
Find the resultant force and its orientation





example D (for article 2.3)

The force F= 100 N acts along line t, which goes through point $A(x_1, y_1)$ and $B(x_2, y_2)$ express the vector F.



Step 1

determine a vector $\vec{S}$ which is a position vector of point B with respect to point A

$$\vec{S} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j}$$

Step 2

determine a unit vector $\vec{s}$ in the same direction

$$\vec{s} = \frac{\vec{S}}{|S|} = \frac{1}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}\left[(x_2-x_1)\vec{i} + (y_2-y_1)^2\right]$$

step 3

$$\vec{F} = F \cdot \vec{s} = \frac{100}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}\left[(x_2-x_1)\vec{i} + (y_2-y_1)^2\right]$$

example: A (5, 9) ; B (8, 12)

$$\vec{S} = (8-5)\vec{i} + (12-9)\vec{j} = 3\vec{i} + 3\vec{j}$$

$$\vec{s} = \frac{3\vec{i} + 3\vec{j}}{\sqrt{3^2+3^2}} = \frac{3i + 3j}{3\sqrt{2}} = \frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j} = \frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j}$$

$$\vec{F} = 100\frac{\sqrt{2}}{2}(\vec{i} + \vec{j})$$

The force	Magnitude	θ_x	θ_y	$F_x = F\cos\theta_x$	$F_y = F\cos\theta_y$
F_1	50 N	60°	30°	+25.0 N	+43.3 N
F_2	100 N	135°	45°	−70.7 N	+70.7 N
F_3	200 N	233°	143°	−120 N	−160. N
F_4	300 N	307°	217°	+180 N	−240. N
				$R_x =$ 14.3 N	−286 N

$$\theta = tg^{-1}\ \frac{-286}{14.3} = tg^{-1}\ \frac{-20}{+1} = -87.1^\circ$$

$$\theta = 273^\circ$$

2.4 Moment

In addition to the tendency to move a body in the direction of its application a force may also tend to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the force's line of action. This rotational tendency is known as the MOMENT M of the force

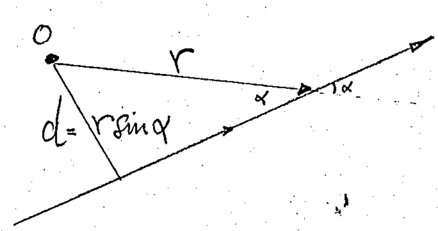
* Magnitude $M = Fd$ $\quad d =$ moment arm
* The moment is a vector RIGHT-HAND-RULE (fingers curled in the direction of rotation, Thumb is $\vec{M}$)
* M is a sliding vector

In the plane we speak about the moment about a point! Actually we mean an axis normal to the plane!

CCW (+) CW (−)

* $\vec{M} = \vec{r} \times \vec{F}$ (cross-product)

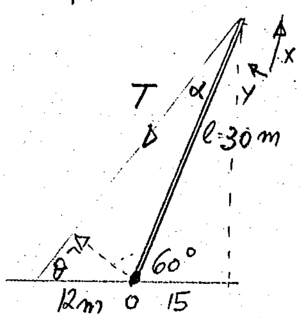
sequence $r \times F$!



Varignon's Theorem

The moment of a force about any point (2-D) is equal to the sum of the moments of the components of the force about the same point. $r \times \vec{R} = r \times (\vec{P} + \vec{Q}) = r \times \vec{P} + r \times \vec{Q}$ (distributive law)

example b. (p 2-36)



The tension T in the cable MUST supply a moment of 72 KN.m about O to raise The flagpole. Determine T

(a) $\theta = tg^{-1} \dfrac{30 \times 0.866}{12+15} = 43.9°$

$\alpha = 90° - (30 + \theta) = 16.1°$ $\alpha + \theta = 60°$ $\alpha = 60 - 43.9 = 16.1$

$M = 72\ KN \cdot m = T \sin 16°.1 \times 30 = 8.32\,T$

$T = \dfrac{72}{8.32} = 8.65\ KN$

(b) $r = 30\vec{i}$

$T = -T\cos\alpha\,\vec{i} + T\sin\alpha\,\vec{j} + 0$

$$M = \begin{vmatrix} i & j & k \\ 30i & 0 & 0 \\ -T\cos\alpha & T\sin\alpha & 0 \end{vmatrix} = +30\,T\sin\alpha = 72 \rightarrow T = 8.65\ KN$$

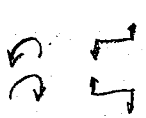
2.5 Couples

The moment produced by two equal and opposite and noncollinear forces is known as a couple.

Its resultant is null. Its moment is invariably equal about any axis and it depends solely on the Forces magnitude F and the distance between them d. Thus M is a free vector.

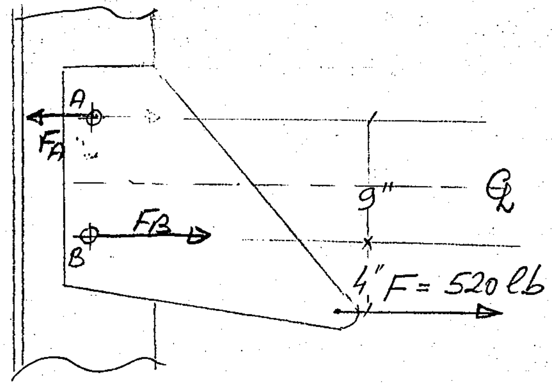
The dual effect of force to pull (or push) and to rotate a body can be represented by replacing any force by an equal and parallel force and a couple ≡ force-couple system

A couple can be represented by one of the following conventions:



<u>example b</u> (prob 2/60 p. 45)

A bracket is fastened to a girder by two rivets A and B and supports a 520 lb force.



(a) Replace F by a force acting along the centerline ℄ and a couple

(b) Replace (a) by two forces acting at A & B

Sol 1: (a) $d = 4 + \frac{9}{2} = 8.5''$

$M_0 = 520 \times 8.5''$ CCW $= 4420$ lb·in ccw

(b) F_2 due to couple $= \frac{4420}{9} = 491.1$ lb.

$F_A = \frac{F}{2} - F_2 = \frac{520}{2} - 491.1 = -231$ lb

$F_B = \frac{F}{2} + F_2 = 751$ lb

Sol 2: $\underset{\text{due to } F}{\Sigma M_A} \equiv \underset{\text{due to } F_A \,\&\, F_B}{\Sigma M_A}$ thus $520 \times 13 = F_B \cdot 9 \Rightarrow F_B = 751$ lb.

Vaugnon's theorem.

$F_A = F - F_B = -231$ lb.

2.6 <u>Resultants</u>

In real problems we deal with bodies subjected to systems of forces and moments. In order to reduce these systems to their simplest form without alterating their external effect we replace them by their resultants. When these resultants are null the body is in equilibrium.

Algorithem for determining $\vec{R}$ (2-D)

(a) Magnitude : $R_x = \Sigma F_{ix}$ $R_y = \Sigma (F_i)_y$ $R = \sqrt{R_x^2 + R_y^2}$

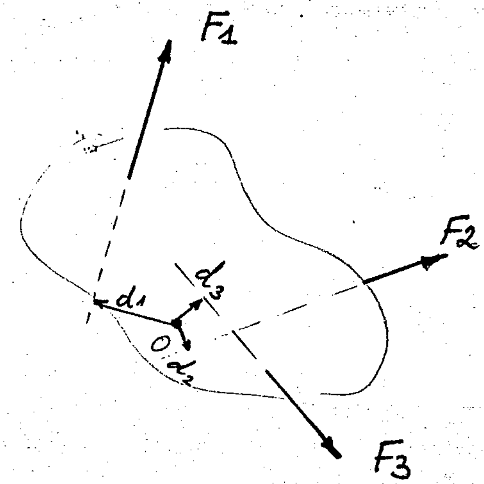
(b) Location $R d_0 = \Sigma (M_i)_0$

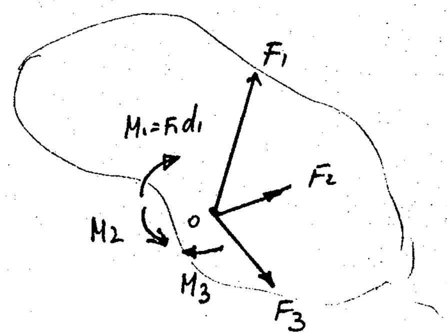
(c) Orientation $\theta_x = \cos^{-1} \frac{R_x}{R}$ $\theta_y = \cos^{-1} \frac{R_y}{R}$

for 2-D $\theta = tg^{-1} \frac{R_y}{R_x}$

statement (b) is an extension of Varignon's principles for NONCONCURRENT force systems and is known as PRINCIPLE of MOMENTS

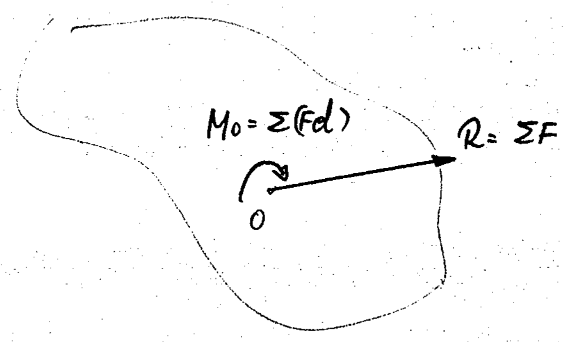
Resultants :



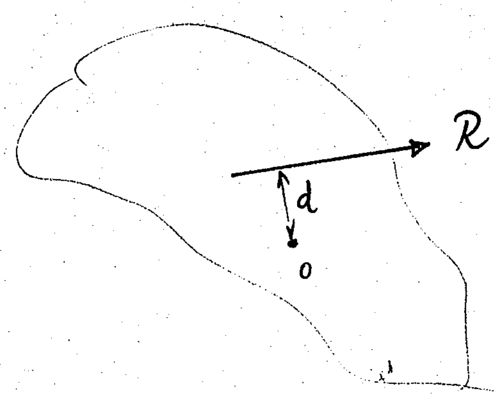


$R = \Sigma F$

$M_0 = \Sigma M = \Sigma (Fd)$



$R d = M_0$

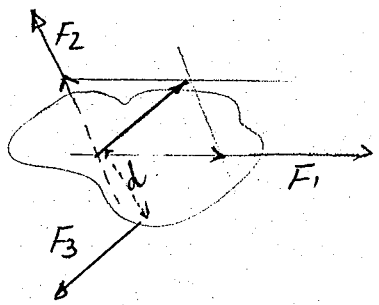


$d = \frac{M_0}{R}$

* For concurrent systems eq. (b) is identical zero since the Resultant passes through the common point "O"

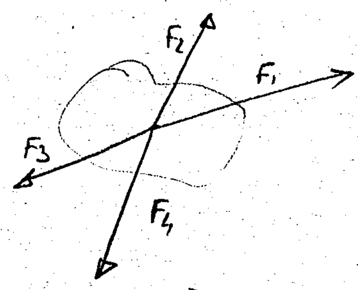
* For a parallel force system eq. (c) is "obvious".

** R and M_o are independent:



$\Sigma F = 0$

$\Sigma M = F_3 d$



$\Sigma F_i = \vec{F}$

$\Sigma M_o \equiv 0$

SECTION B - 3D FORCE SYSTEMS

2.7 Rectangular Components

$F_x = F\cos\theta_x = Fl$

$F_y = F\cos\theta_y = Fm$

$F_z = F\cos\theta_z = Fn$

$$\vec{F} = F(l\vec{i} + m\vec{j} + n\vec{k})$$

$F_n = \vec{F}\cdot\vec{n}$ if $\vec{n} = \alpha\vec{i} + \beta\vec{j} + \gamma\vec{k}$

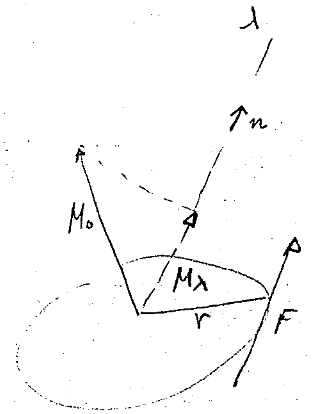
$$F_n = F(l\vec{i} + m\vec{j} + n\vec{k})\cdot(\alpha\vec{i} + \beta\vec{j} + \gamma\vec{k}) =$$

$$= F(l\alpha + m\beta + n\gamma)$$

2.8 3-D Moments & Couples

$$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = \vec{i}\,(r_y F_z - r_z F_y) + \vec{j}\,(r_z F_x - r_x F_z) + \vec{k}\,(r_x F_y - r_y F_x)$$

Show the Figure.



$|M_\lambda| = \vec{M_0} \cdot \vec{n} = (\vec{r} \times \vec{F}) \cdot \vec{n}$ Triple Scalar Product!

$$\vec{M}_\lambda = \left[(\vec{r} \times \vec{F}) \cdot \vec{n}\right] \vec{n}$$

$$M_\lambda = \begin{vmatrix} r_x & r_y & r_z \\ F_x & F_y & F_z \\ n_x & n_y & n_z \end{vmatrix}$$

$\alpha \quad \beta \quad \gamma$

Varignon's Theorem.

$$\vec{M}_0 = \Sigma \vec{r} \times \vec{F} = \vec{r} \times \vec{R} \quad \text{where } \vec{R} = \Sigma \vec{F}$$

* Replacing a force by a force-couple system

2.9 Resultants

Magnitude: $R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R_z = \Sigma F_z$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

orientation $\theta_x = \cos^{-1} \frac{R_x}{R} \qquad \theta_y = \cos^{-1} \frac{R_y}{R} \qquad \theta_z = \cos^{-1} \frac{R_z}{R}$

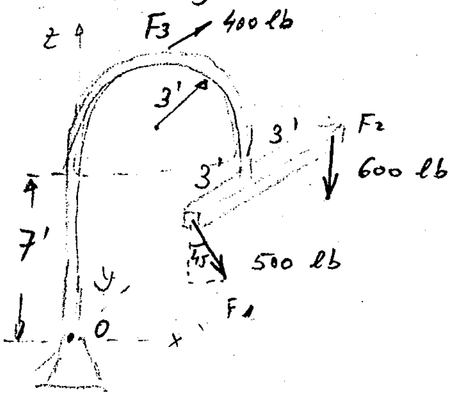
location $M_x = \Sigma (r \times F)_x \qquad M_y = \Sigma (r \times F)_y \qquad M_z = \Sigma (r \times F)_z$

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

equilibrium $\vec{R} = 0$

$\vec{M} = 0$

concluding example 2/122 p. 80



Determine $|R|$ & $|M|$ at "O".

$$F_1 = 500\left(\frac{\sqrt{2}}{2}\vec{i} - \frac{\sqrt{2}}{2}\vec{k}\right) = 353.6\,(\vec{i} - \vec{k})$$

$$F_2 = -600\,\vec{k}$$

$$F_3 = +400\,\vec{j}$$

$$\vec{R} = 353.6\,\vec{i} + 400\,\vec{j} - 953.6\,\vec{k}$$

$$|R| = \sqrt{353.6^2 + 400^2 + 953.6^2} = 1092.87 \sim 1093\ N$$

$$\vec{r}_1 = 6\vec{i} - 3\vec{j} + 7\vec{k}$$

$$\vec{r}_2 = 6\vec{i} + 3j + 7\vec{k}$$

$$r_3 = 3i + 10\,\vec{k}$$

$$M = \vec{r}_i \times \vec{F}_i = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3 =$$

$$= \begin{vmatrix} i & j & k \\ 6 & -3 & 7 \\ 353.6 & 0 & -353.6 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 6 & 3 & 7 \\ 0 & 0 & -600 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 3 & 0 & +10 \\ 0 & 400 & 0 \end{vmatrix} =$$

$$\vec{i}\,(1060.8) - \vec{j}\,(-4596.8) + \vec{k}\,(+1060.8)$$

$$+\ \vec{i}\,(-1800) - \vec{j}\,(-3600)$$

$$+\ \vec{i}\,(-4000) + \qquad \vec{k}\,(1200)$$

$$\vec{M} = -4739.2\,\vec{i} + 8196.8\,\vec{j} + 2260.8\,\vec{k}$$

$$|M| = \sqrt{4739.2^2 + 8196.8^2 + 2260.8^2} = 9734.4\ lb \cdot ft$$

Chapter 3 Equilibrium

3.1 Introduction

Statics deals primarily with the NECESSARY and SUFFICIENT conditions to maintain the state of EQUILIBRIUM of engineering structures:

$$\vec{R} = 0 \qquad \vec{M} = 0 \tag{3.1}$$

Equilibrium in 2-D

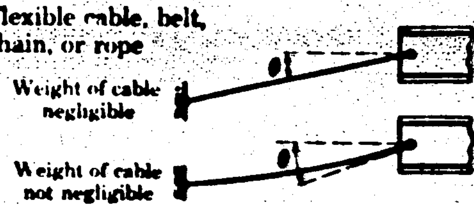
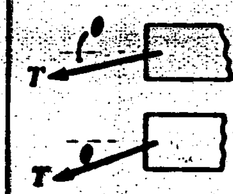
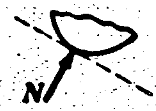
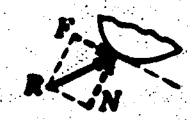
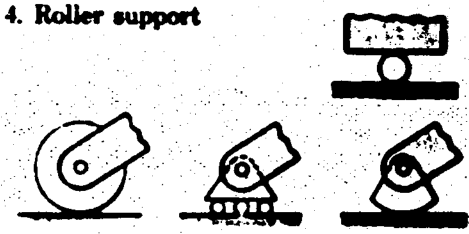
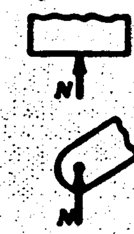
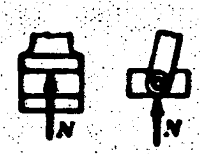
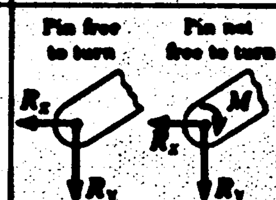
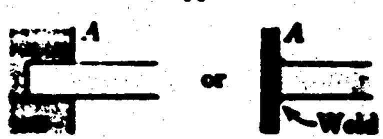
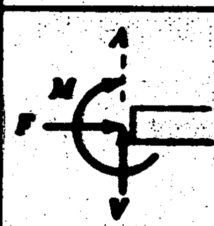
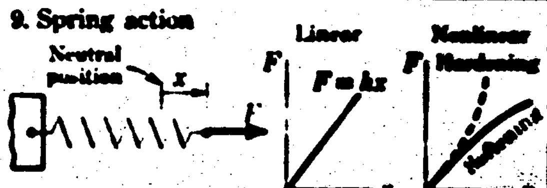
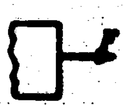
3.2 Mechanical System Isolation

Before we can apply the above eqs. we have to DEFINE unambigously the particular body (or mechanical system) to be analyzed. Once this body has been defined it has to be ISOLATED from all surrounding bodies. This isolation is accomplished by means of a FREE-BODY-DIAGRAM (FBD). The FBD represents the isolated body + all forces originally applied to it + all reaction forces applied to it by all surrounding bodies that are imagined to be removed.

The Free body diagram is the most important single step in the solution of problems in mechanics.

There are various ways by which force is transmitted from one body to another. Trans. 3.1 represents some basic forms.

MECHANICAL ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS

Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.
4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.
6. Pin connection 	 A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the axis; usually shown as two components R_x and R_y. A pin not free to turn may also support a couple M.
7. Built-in or fixed support 	 A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 	 The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center of mass G.
9. Spring action 	 Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

Figur

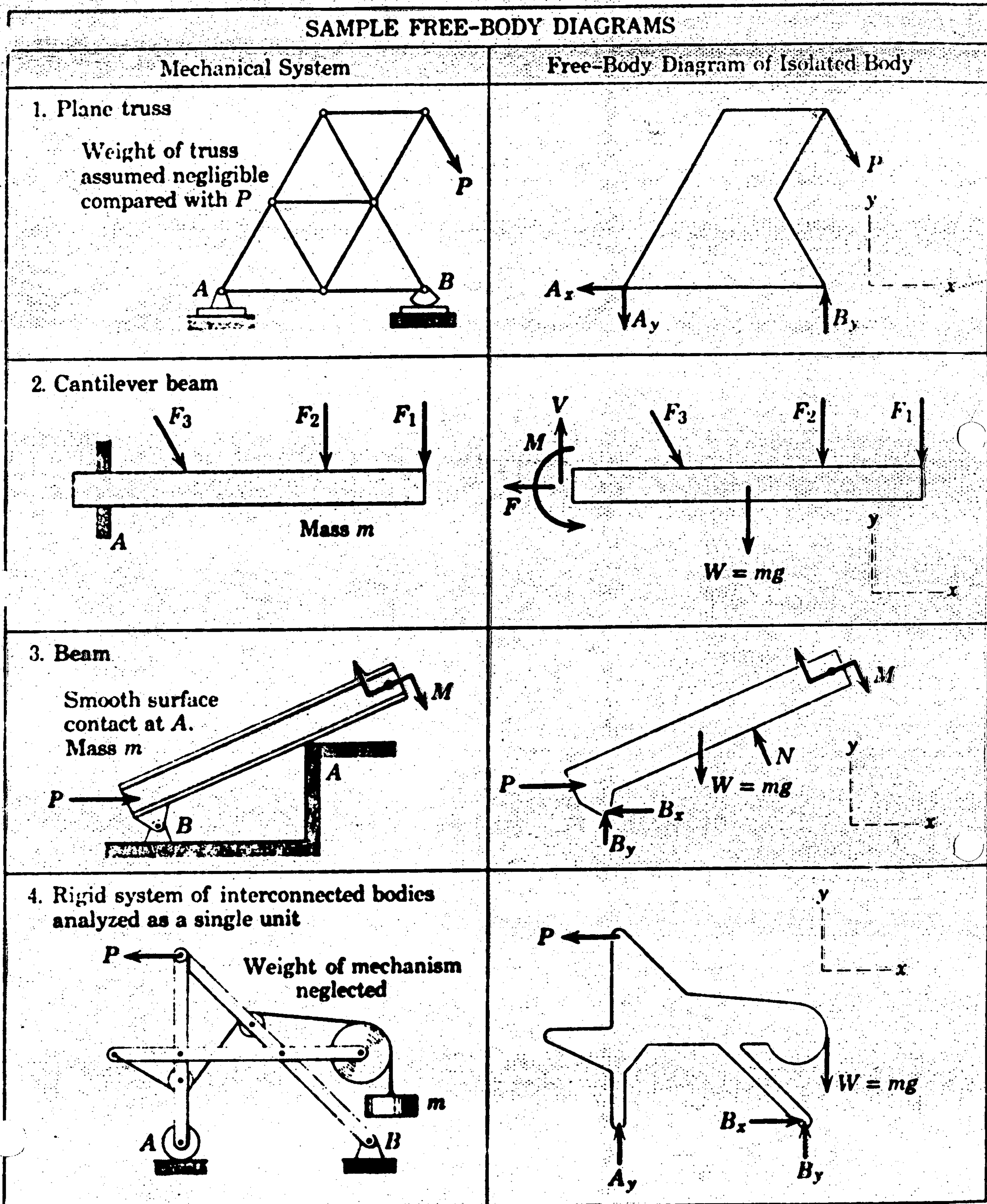

SAMPLE FREE-BODY DIAGRAMS
Mechanical System
Free-Body Diagram of Isolated Body
1. Plane truss
Weight of truss assumed negligible compared with P
P
A
B
P
y
x
A_x
A_y
B_y
2. Cantilever beam
F_3
F_2
F_1
A
Mass m
V
M
F
F_3
F_2
F_1
W = mg
y
x
3. Beam
Smooth surface contact at A.
Mass m
M
A
P
B
M
N
y
P
W = mg
B_x
B_y
x
4. Rigid system of interconnected bodies analyzed as a single unit
P
Weight of mechanism neglected
m
A
B
y
P
x
W = mg
B_x
A_y
B_y

Figure 3/2

Construction of Free body diagrams

Step 1. Chose the body (ies) to be isolated. It will involve one or more of the desired unknown quantities

Step 2. Draw a sketch of the CONTOUR-LINE of the isolated body

Step 3. Represent all forces acting on the isolated body:
(a) Original forces.
(b) Reaction forces applied by the REMOVED contacting and attracting bodies.

* Known forces represent by Blue Vector Arrows
* Unknown forces should be represented with TENTATIVE sense by RED Vector Arrows. The calculation will AUTOMATICALLY reveal their correct sense.

Step 4. Indicate on the diagram a coordinate system as well as all pertinent dimensions.

Examples Trans 3, 2.

3.3 Equilibrium Conditions

For 2-D problems the general eqs. take the form

$$\sum F_x = 0$$
$$\sum F_y = 0 \qquad (3.2)$$
$$\sum M_o = 0$$

CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS

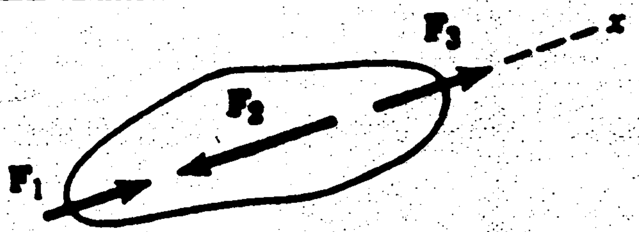
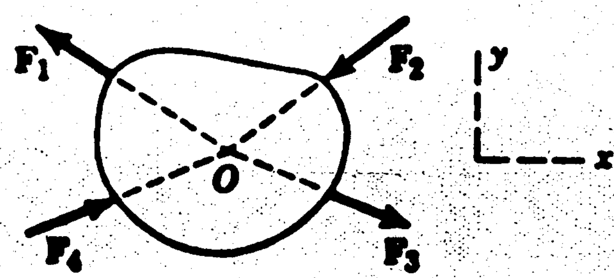
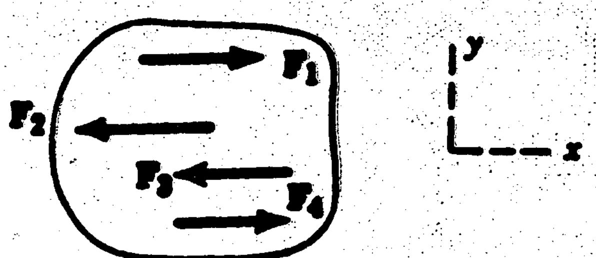
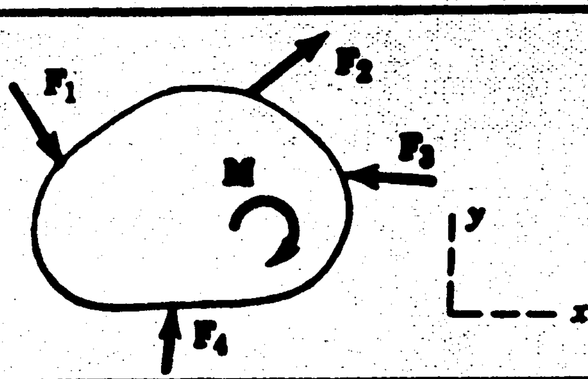
Force System	Free-Body Diagram	Independent Equations
Collinear		$\Sigma F_x = 0$
Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
Parallel		$\Sigma F_x = 0$ $\quad$ $\Sigma M_z = 0$
General		$\Sigma F_x = 0$ $\quad$ $\Sigma M_z = 0$ $\Sigma F_y = 0$

Figure 3/3

(a) categories of 2-D equilibrium: Trans 3.3

There are two other situations that occur frequently to which the student should alerted:

two face members.

three force members.

(b) Alternative equilibrium eqs.

$\sum F_x = 0$	$\sum F_x = 0$	$\sum M_A = 0$
$\sum F_y = 0$	$\sum M_A = 0$	$\sum M_B = 0$
$\sum M_0 = 0$	$\sum M_B = 0$	$\sum M_C = 0$
	$\overline{AB}$ should not be perpendicular to x	A, B, C not on one straight line

(c) Constraints and Statical determinacy:

The eqs. of equilibrium are the necessary and sufficient conditions to maintain equilibrium BUT do not necessarily provide all the information required to calculate ALL the unknown forces that may act on a body in equilibrium. The question of ADEQUACY lies in the characteristics of the constraints to movement provided by the supports. Example: The truss in Fig 3.2

A body that possesses more Constraints than necessary to maintain equilibrium is called STATICALLY INDETERMINATE. The supports that can be removed without altering equilibrium are called REDUNDENT. The DEGREE of STATICAL INDETERMINACY ≡ № of Redundent unknowns ≡ Total unknowns − № of available Eqs of Eq.

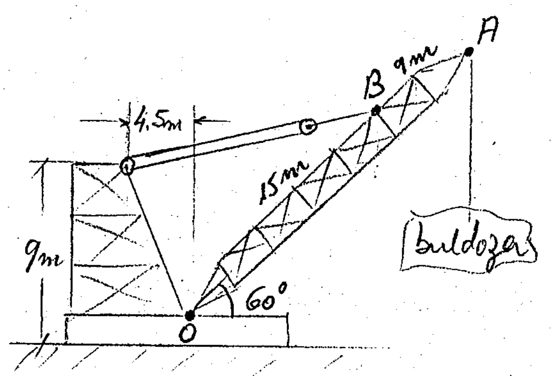
<u>Example (a)</u> (3/4) To Determine the Tension in the cable T and the force on the pin A for a JIB CRANE

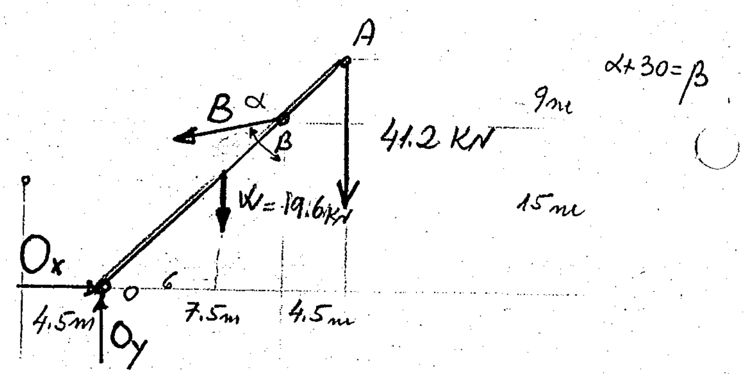
<u>Example (b)</u> (pr. 3/58 p 119)

A crane is hoisting a 4.2 Mg buldozer. The mass center of the 2 Mg boom is at its mid-span. Calculate:

a) the Tension T in the cable attached at B

b) the magnitude of the force supported by the boom at hinge O





evaluation of the angle β:

$$\beta = tg^{-1} \frac{12}{15 \sin 30 - 9m} = 71.6^\circ$$

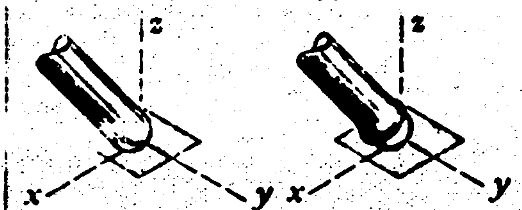
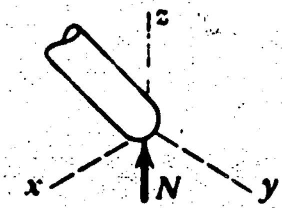
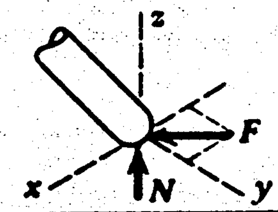
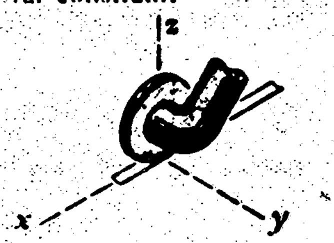
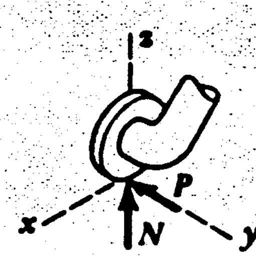
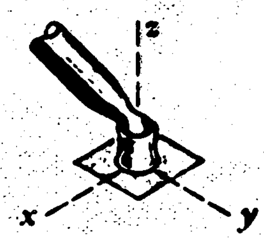
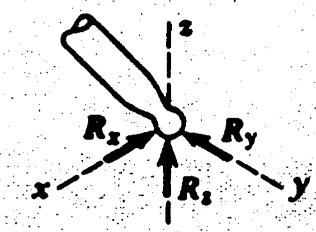
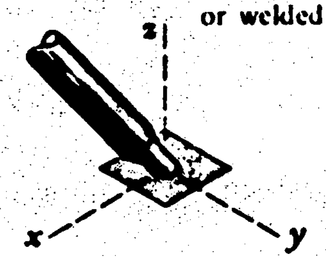
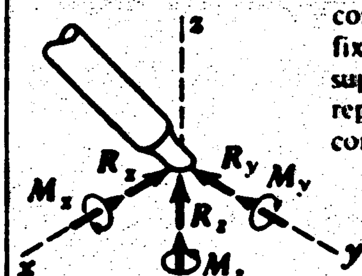
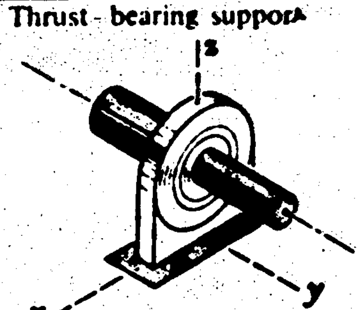
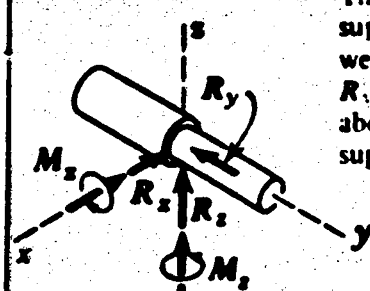
$\Sigma M_o = 0 \qquad -41.2 \times 12 - 19.6 \times 6 - B \cos 71.6 \cdot 7.5 + B \sin 71.6 \cdot (15 \sin 60) = 0$

$$B = \frac{494.4 + 117.6}{15 \sin 71.6 \cdot \sin 60 - 7.5 \cos 71.6} = 61.45 \cong 61.5 \text{ KN}$$

$\Sigma F_x = 0 \qquad -B \sin 71.6 + O_x = 0 \quad \Rightarrow O_x = 58.3 \text{ KN}$

$\Sigma F_y = 0 \qquad -41.2 - 19.6 - B \cos 71.6 + O_y = 0 \qquad O_y = 80.2 \text{ KN}$

MECHANICAL ACTION OF FORCES IN THREE-DIMENSIONAL ANALYSIS

Type of Contact and Force Origin	Action on Body to be Isolated
1. Member in contact with smooth surface, or ball-supported member 	 Force must be normal to the surface and directed toward the member.
2. Member in contact with rough surface 	 The possibility exists for a force F tangent to the surface (friction force) to act on the member, as well as a normal force N.
3. Roller or wheel support with lateral constraint 	 A lateral force P exerted by the guide on the wheel can exist, in addition to the normal force N.
4. Ball-and-socket joint 	 A ball-and-socket joint free to pivot about the center of the ball can support a force **R** with all three components.
5. Fixed connection (embedded or welded) 	 In addition to three components of force, a fixed connection can support a couple **M** represented by its three components.
6. Thrust-bearing support 	 Thrust bearing is capable of supporting axial force R_y as well as radial forces R_x and R_z. Unless bearing is pivoted about x- and z-axes, it can support couples M_x and M_z.

CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS

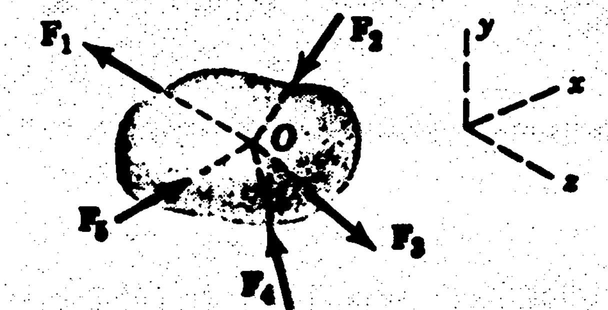
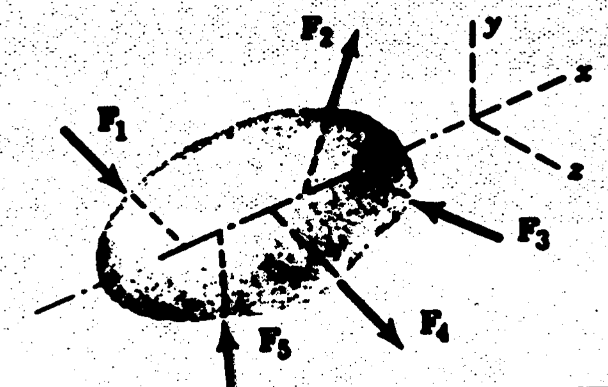
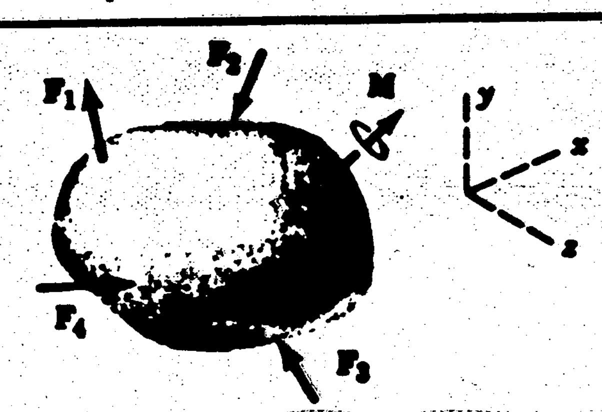
Force System	Free-Body Diagram	Independent Equations
1. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
2. Concurrent with a line		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma F_y = 0$ $\Sigma M_z = 0$ $\Sigma F_z = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_x = 0$ $\Sigma F_y = 0$ $\Sigma M_y = 0$ $\Sigma F_z = 0$ $\Sigma M_z = 0$

Figure 3/9

$O = \sqrt{O_x^2 + O_y^2} = 99.15$ KN

Part B- Equilibrium in 3-D.

3.4 Equilibrium Conditions

The necessary and sufficient conditions to maintain equilibrium in 3-D are

$$\sum \vec{F} = 0 \text{ or } \begin{array}{l} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{array} \quad \& \quad \sum \vec{M}_o = 0 \text{ or } \begin{array}{l} \sum M_x = 0 \\ \sum M_y = 0 \\ \sum M_z = 0 \end{array}$$

all these six conditions are independent.

(a) Free Body Diagram

Force action in 3-D - Trans 3.4

FBD as in the 2-D case.

(b) Categories of Equilibrium

Trans 3.5

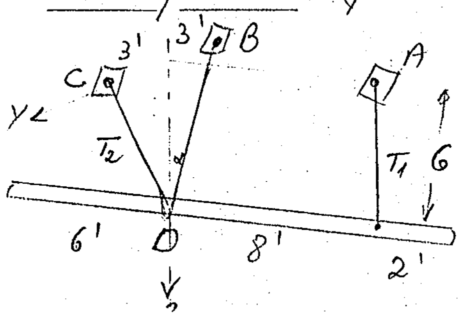
(c) Constraints and statical Determinacy

As for 2-D.

example a

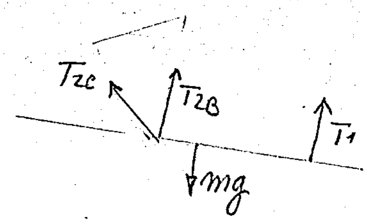
A uniform 7m steel shaft has a mass of 200 kg The shaft is suppoted by a ball & socket joint at A and rests against two smooth walls at B

example b (pr 3.65 pg. 128)



The horizontal steel shaft weights 620 lb is suspended by a vertical cable from A and by two cables in a VERTICAL TRAVERSE plane from B and C. Calculate the tensions T_1 & T_2

(a) FBD



$\vec{T_1} = -T_1 \vec{k}$

$\vec{T}_{2B} = -T_2 \sin\alpha \vec{j} - T_2 \cos\alpha \vec{k}$ $\qquad$ $\vec{T}_{2C} = T_2 \sin\alpha \vec{j} - T_2 \cos\alpha \vec{k}$

Conclusions: (1) no force in $\vec{i}$ thus $\Sigma F_x = 0$ is trivial

(2) T_1, T_2 are parallel to z axis $\Sigma M_z = 0$ is trivial.

(3) T_1 produces no M_x moment.

Solution

$\overset{\curvearrowleft}{\Sigma} M_D = 0$ $\qquad$ $T_1 \cdot 8 - 620 \times 2 = 0$ $\qquad$ $T_1 = 155$ lb

since $|T_{2B}| \equiv |T_{2C}|$ and $\sin\alpha_1 = \sin\alpha_2 \Rightarrow (T_{2B})_z = (T_{2C})_z$

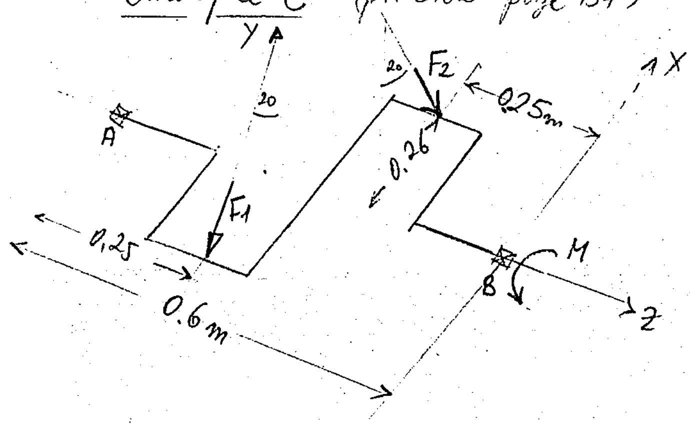
$\Sigma F_z = 0$ $\qquad$ $620 - 155 - 2(T_{2B})_z = 0$

$(T_{2B})_z = 232.5$

$(T_{2B})_z = T_{2B} \cos\alpha$ $\qquad$ $T_{2B} = \dfrac{(T_{2B})_z}{\cos\alpha}$ 260 lb.

$\alpha = \tan^{-1} \frac{1}{2}$ $\qquad$ $\alpha = 26.6$

example C (pr. 3.82 page 134)

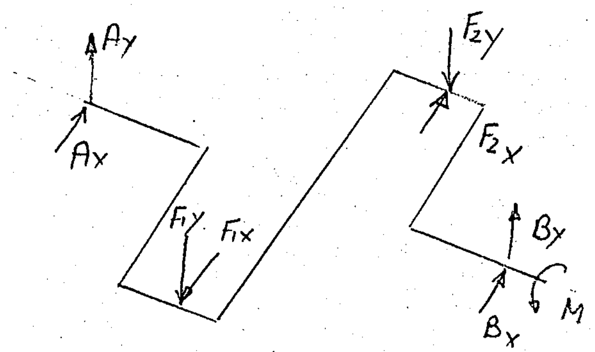


The crankshaft for a two cylinder compressor is subjected at this position to $F_1 = 16$ KN and $F_2 = 8$ KN for an equilib. condition determine

(a) the total suported force by bearings A and B

(b) the Torque M applied to the shaft

(a) FBD



$\overset{\curvearrowleft}{\sum} M_z = 0 \qquad M + F_{1y} \times 0.26 - F_{2y}\, 0.26 = 0$

$M = 0.26\,(F_{2y} - F_{1y}) = 0.26\,(F_2 - F_1)\cdot \cos 20° =$

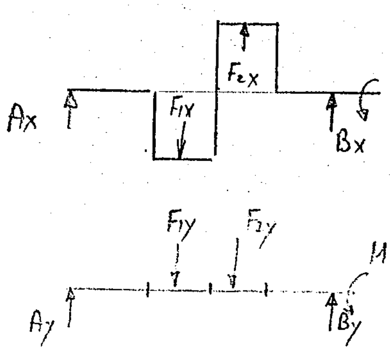
$M = 0.26 \cdot (-16 + 8)\cdot \cos 20° = -1.955 \text{ KNm}$

$\sum F_y = 0 \qquad -F_{1y} - F_{2y} + A_y + B_y = 0$

$A_y + B_y = (F_1 + F_2)\cos 20° = 22.55 \text{ KN}$

$\sum F_x = 0 \qquad F_{2x} - F_{1x} + A_x + B_x = 0$

$A_x + B_x = (F_1 - F_2)\sin 20° = 2.736 \text{ KN}$



$(x\text{-}z) \quad \sum M_A \quad -F_{1x}\cdot 0.25 + F_{2x}\, 0.35 + B_x 0.6 = 0$

$$B_x = \frac{16 \sin 20 \cdot 0.25 - 0.35\, 8 \cdot \sin 20}{0.6} = 0.684 \text{ KN}$$

$A_x = 2.736 - 0.684 = 2.05 \text{ KN}$

$(y\text{-}z) \quad \sum M_A \quad -F_{1y}\cdot 0.25 - 0.35 F_{2y} + B_y \cdot 0.6$

$$B_y = \frac{(16 \times 0.25 + 8 \times 0.35)\cos 20°}{0.6} \quad 10.65 \text{ KN}$$

$A_y = 22.55 - B_y = 11.90 \text{ KN}$

$A = \sqrt{A_x^2 + A_y^2} = 12.08 \text{ KN} \qquad B = \sqrt{B_x^2 + B_y^2} = 10.67 \text{ KN}$

Chapter 4 Structures

4.1 Introduction

In ch. 3 we were mainly concerned with the external equilibrium of a single rigid body. In Ch. 4 our attention is directed toward the determination of the internal forces in the various members of a given structure. In order to do so we need to "DISMEMBER" the structure and to separately analyze individual or groups of members. While doing so we shall frequently use two principles:

(a) Newton's third Law.

(b) The fact that if a body is in equilibrium then each part of it is in equil. as well.

We shall consider two basic groups of structures

(a) Trusses (2-D & 3-D)

(b) $\left\{ \begin{array}{l} \text{Frames} \\ \quad + \\ \text{Machines} \end{array} \right.$ F/M

4.2 Plane Trusses

A framework composed of members joined at their ends to form a RIGID structure is known as a TRUSS: e.g Bridges, Roof supports, Cranes etc.

* The members are typically made of I, T, U, L etc Beams
* When the members lie essentially in one plane — Plane Truss.

* The basic element of a plane truss is a TRIANGLE
* Higher polygon constitute a non-rigid frame

NON RIGID = COLLAPSIBLE

RIGID = Noncollapsible + Small Deformations.

* When more members are added to a truss than are needed to prevent it from collapse - The truss is Internally - Statically - Indeterminate
* All members are Two - Force members C or T
* All forces are applied at joints. If member's weight is to be considered split it $W/2$ at each end
* Welded or riveted joints may be assumed pin-joints as long as all the members centerlines are concurrent.
* There are two methods of analysis:

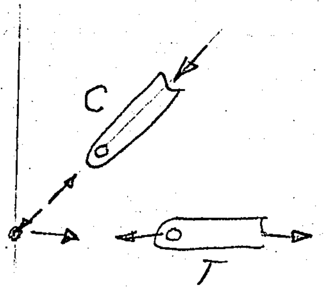
(a) Method of Joints

(b) Method of Sections

4.3 Method of Joints

This method is based on satisfying equi. cond. at each joint where we have 2 independent eqs:

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$



example a.

Indeterminacy: External Indeterminate: more supports than are necessary to ensure equilibrium

Internal Indeterminate: more internal members than are necessary to prevent callapse.

For a truss externally determined there exist a necessary though not sufficient condition for internal determinacy:

$m + 3 = 2j$ — m - № of members, j - № of joints.

$m+3 > 2j$ statically indeterminate - internally

$m+3 < 2j$ truss unstable will callapse

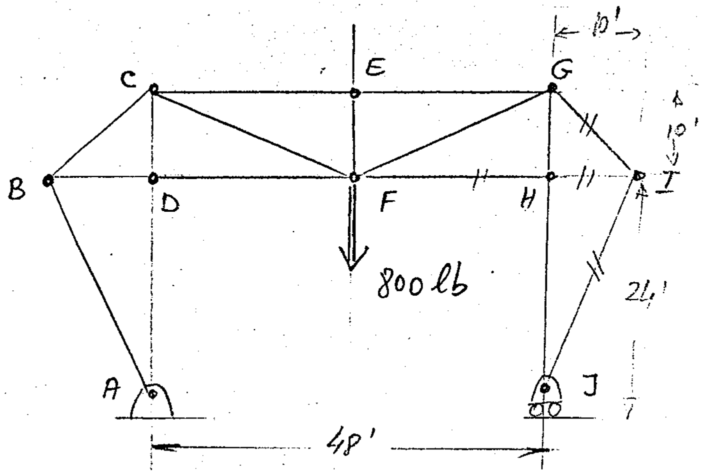
Special Cases

* Three members joined, out of which two are callinear at an unloaded joint
* Two noncollinear members joined at an unloaded joint.
* Two pairs of callinear members joined at an unloaded joint.

example 4.1

example b (Hibbeler 6/7)

solve the following truss:



$m = 17$
$j = 10$
$s = 3$

Since the truss is symmetric only half should be analyzed.

$(F_{IJ})_x = 0 \Rightarrow F_{Ij} = 0$

$F_{JH} = 400\ lb$ (C)

$(F_{GI})_y = 0 \Rightarrow F_{GI} = 0$

JOINT (I) $F_{HI} = 0$

JOINT (H) $F_{FH} = 0$

$\Sigma F_y = 0$ $F_{HG} = 400\ lb$

JOINT (G):

$\Sigma F_y = 0 \Rightarrow (F_{FG})_y = 400$

$F_{GF} = 1040\ lb$ (T)

$\Sigma F_x = 0$ $F_{EG} = 960\ lb$ (C)

JOINT (E) $F_{EF} = 0$

4.4 Method of Sections

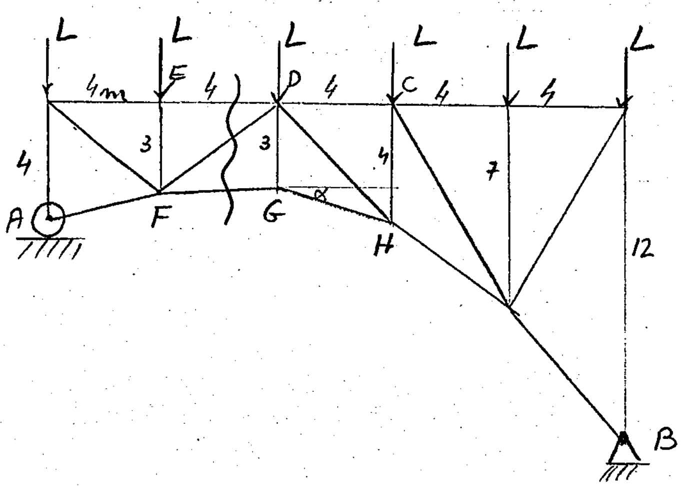
While with the method of joints we can employ only 2 eqs of equi., the method of sections, that considers the equilibrium of an entire portion of a truss, makes use of all three eqs. The main advantage of this method is that the force in almost any desired member can be found directly by analysing a section which has "cut" that member.

* when "cutting" a section we have to make sure that there are no-more than 3 unknowns.

example a

example b.

example c (p. 4-32 pp. 167)



* Determine the force in member DG

phase 1

* external determinacy ✓
* internal determinacy : $m = 21$ $j = 12$

$21 + 3 = 2 \times 12$ ✓

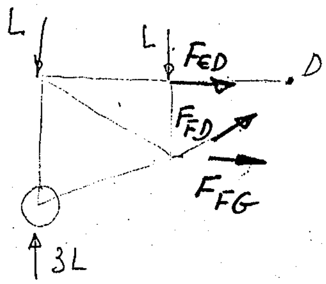
phase 2

$B_x = 0$

$A_x = B_x = 3L$ ✓

phase 3

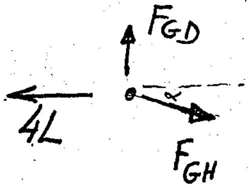
we "cut" along blue line and perform moments about D



$$-3L \times 4 + F_{FG} \cdot 3 = 0$$

$$F_{FG} = 4L$$

joint G



$$F_{GH} \cdot \cos\alpha = 4L$$

$$F_{GH} = \frac{4L}{\cos\alpha}$$

$$F_{GD} = F_{GH} \sin\alpha = 4L \tan\alpha$$

$$\tan\alpha = \frac{1}{4}$$

$$F_{GD} = L \quad T$$

4.5 Space Trusses

A space truss is the 3-D counterpart of the plane-truss. The basic unit is a tetrahedron made of 4 rigid links conected at their end by ball-and-socket joints.

* External Determinacy – no more than 6 unknowns at the supports.
* A sufficent condition for an externally determinate 3-D truss to be internally determinate is:

$$m + 6 = 3j$$

$$m + 6 > 3j \quad \text{indeterminate}$$

$$m + 6 < 3j \quad \text{callapsible.}$$

* Methods of solution: a) Joints
  b) Sections.

example a

4.6 Frames and Machines

A structure is considered a FRAME or a MACHINE if at least one of its individual members bears <u>more</u> than 2 forces (a multiforce member)

* Frames are structures which are designed to support applied forces and are fixed in position.
* Machines are structures that contain moving parts and are designed to transmit forces & couples.

Since frames and machines contain multiforce members the force in these members are no longer colliniear with the members thus the methods developed for solving trusses are not applicable.

The method of solution involves the isolation of all the members of a given F/M and by considering the equilibrium of each member separatly.

If the F/M is a non collapsible structure, once the supports are removed than the external equil can be considered first. Otherwise the external and internal equil are to be considered simulaneously.

<u>algorithm</u>

(a) static determinacy

if F/M non collapsible

(b) external equil.

(c) internal equil.

if collapssible

(b) external + internal equil.

<u>example a</u>

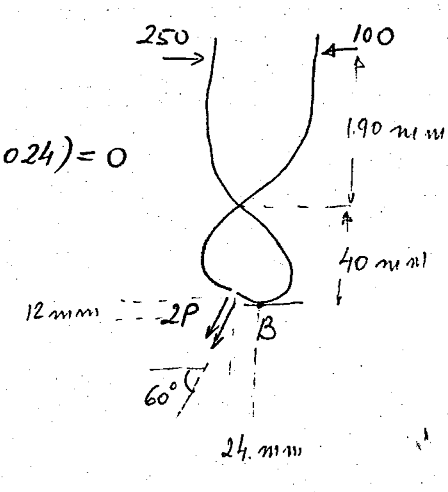
<u>example b.</u>

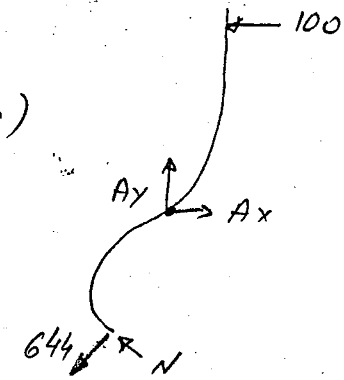
<u>example c</u> pr. 4/99 pp. 195

$\Sigma M_B = 0$: $100(0.23) - 250(0.23)$

$+ 2P\cos 60\,(0.012) + 2P\sin 60\,(0.024) = 0$

$P = 644\,N$.





$$\Sigma M_A = 0: \quad 100(0.19) + 644\cos 60 \cdot (0.028)$$
$$- 644\sin 60\,(0.024) - N\cos 60 \cdot (0.024)$$
$$- N\sin 60\ (0.028) = 0$$

$$N = 645\ N$$

$$\Sigma F_x = 0 \qquad -100 + A_x + 644\cos 60 - 645\sin 60 = 0$$
$$A_x = 980\ N$$

$$\Sigma F_y = 0 \qquad A_y - 644\cos 30 + 645\cos 60 = 0$$
$$A_y = 235\ N. \qquad \therefore A = \sqrt{A_x^2 + A_y^2} = 1008\ N.$$

The two sharp jaws of the nail puller exert equal pulling forces parallel to the nail and two equal normal forces N to the nail. At B there is enough friction! Determine: the pulling force P, the normal force N, and the force on the pin A

Chapter 5 Distributed Forces

5.1 Introduction

In our previous presentation all forces were assumed to be concentrated. Actually concentrated forces do not exist since every external force applied to a body is distributed over a finite area - the contact area. The contact area can be large - a book on the desk, medium - between a tire and the pavement and very small as between the ball and the race in a ball bearing. When the contact area is small (negligible compared to other pertinent dimensions one can approximate the force by a concentrated one.

* Line distribution N/m
* Area distribution N/m^2 Pa
* Volume distribution N/m^3

Centers of Mass & Centroids

5.2 Center of Mass

To determine the resultant of the gravitational force we suspend the body and the resultant is collinear with the chord. Repeating the process enables the determination of the Center of Gravity CG. CG is of real meaning only in a PARALLEL gravitational field.

To evaluate the location of the resultant of the gravitational force we apply Varignon's Principle:

$$\bar{X} = \frac{\int x_c \, dmg}{\int dmg} \; ; \quad \bar{Y} = \frac{\int y_c \, dmg}{\int dmg} \; ; \quad \bar{z} = \frac{\int z_c \, dmg}{\int dmg}$$

$(\bar{X}, \bar{Y}, \bar{z})$ is the C.G. location.

If g is assumed constant we can define the center of mass C.M.

$$\bar{X} = \frac{\int x_c \rho \, dv}{\int \rho \, dv} \qquad \bar{Y} = \frac{\int y_c \rho \, dv}{\int \rho \, dv} \qquad \bar{z} = \frac{\int z_c \rho \, dv}{\int \rho \, dv}$$

If the material is homogeneous we can define the geometrical center - the CENTROID as:

$$\bar{x} = \frac{\int x_c \, dv}{v} \qquad \bar{y} = \frac{\int y_c \, dv}{v} \qquad \bar{z} = \frac{\int z_c \, dv}{v}$$

the centroid always lies on lines and planes of symmetry.

5.3 Centroids of Lines, Planes and Volumes

The centroid is a pure geometrical property of an homogeneous body. The calculations of centroids fall within three distinct categories: line, area & volume

(a) line - slender rods or wires of constant cross section A

(b) area - thin sheet of constant thickness t

(c) volume - all the rest.

<u>lines</u>: $\bar{x} = \dfrac{\int x\,dL}{L}$ $\bar{y} = \dfrac{\int y\,dL}{L}$ $\bar{z} = \dfrac{\int z\,dL}{L}$

* in General C will not lie on the line itself

<u>Areas</u>: $\bar{x} = \dfrac{\int x\,dA}{A}$; $\bar{y} = \dfrac{\int y\,dA}{A}$; $\bar{z} = \dfrac{\int z\,dA}{A}$

* C in general will not be on the surface

<u>Volume</u>: $\bar{x} = \dfrac{\int x\,dv}{v}$ $\bar{y} = \dfrac{\int y\,dv}{v}$ $\bar{z} = \dfrac{\int z\,dv}{v}$

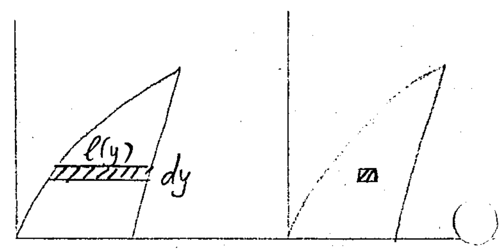
<u>Choice of Integration Element</u>

The main difficulties encountered with centroids evaluation lies in the procedure rather than in the concept there are five guidelines to be observed:

(a) <u>Order of element</u>

a lower order differential element is preferable over a higher order.

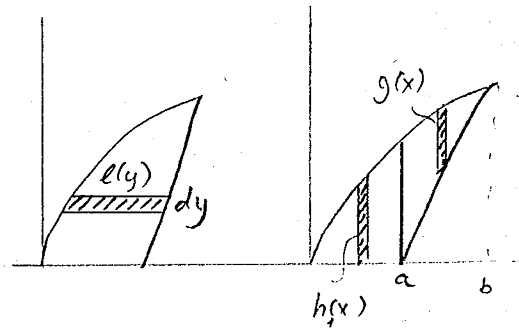
$\int l(y)\,dy \sim \iint dx\,dy$



$l(y)\,dx$ $dx\,dy$

(b) <u>Continuity</u>

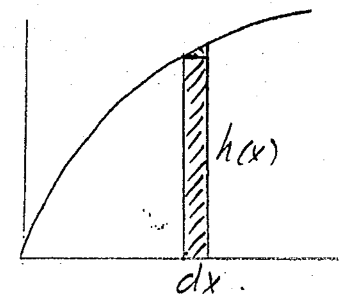
Whenever possible choose an element which can be integrated in one continuous integration.



$\int l(y)\,dy \sim \int_0^a h_1(x)\,dx + \int_a^b g(x)\,dx$

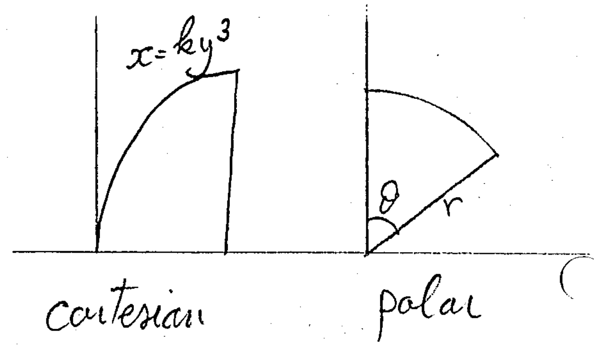
(c) Discard High order Terms

$\frac{1}{2} dx\, dy \ll h(x)\, dx$



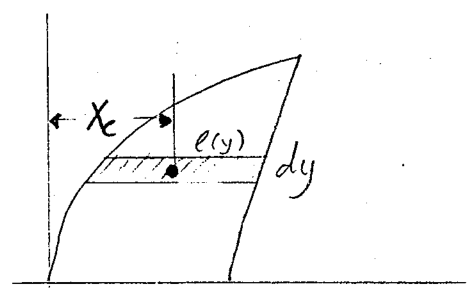
(d) Choice of Coordinates

Choose a coordinate system which best matches the boundaries of the figure:



(e) Centroid of the Differential Element

For first or second order elements be careful to choose the centroid coordinates for the moment arm

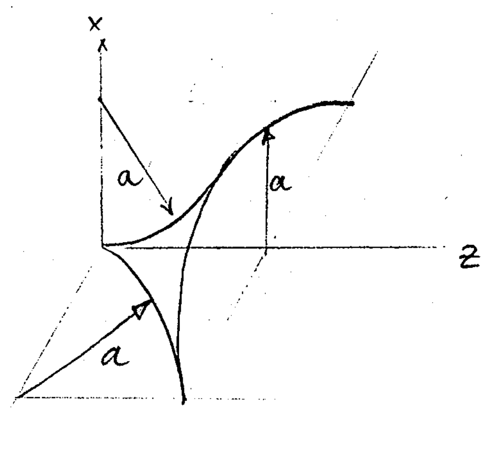


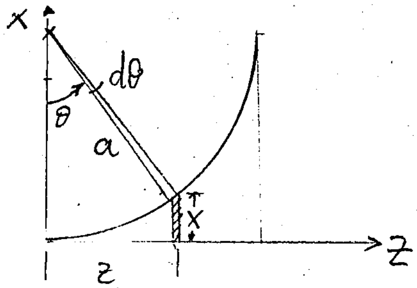
$$\bar{x} = \frac{\int \ell(y)\, dy \cdot x_c}{\int \ell(y)\, dy}$$

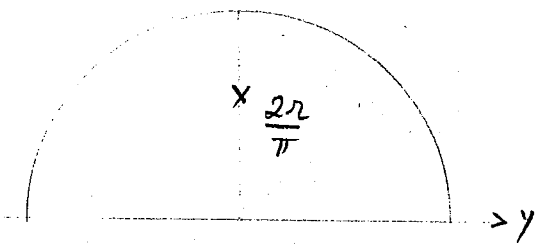
example a circular arc
example b circular sector
example c (pr 5.35 p. 226)

Determine the x coordinate of the centroid of the top half of a bell-shaped shell

lets project the body onto the xz plane:







$$dA = (\pi x)\, a\, d\theta$$

$$x = a - a\cos\theta$$

$$x_c = \frac{2x}{\pi}$$

$$\bar{X} = \frac{\int x_c\, dA}{\int dA}$$

$$\int dA = \int_0^{\pi/2} \pi a^2 (1-\cos\theta)\, d\theta = \pi a^2 \int_0^{\pi/2} (1-\cos\theta)\, d\theta = \pi a^2 \Big[\theta - \sin\theta\Big]_0^{\pi/2}$$

$$= \pi a^2 \left[\frac{\pi}{2} - 1\right] =$$

$$\int_0^{\pi/2} x_c\, dA = \int_0^{\pi/2} \pi x\, a\, d\theta \cdot \frac{2x}{\pi} = \int_0^{\pi/2} 2x^2 a\, d\theta = 2a^3 \int_0^{\pi/2} (1-\cos\theta)^2\, d\theta$$

$$= 2a^3 \int_0^{\pi/2} (1 - 2\cos\theta + \cos^2\theta)\, d\theta = 2a^3 \left[\theta - 2\sin\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]\Big|_0^{\pi/2}$$

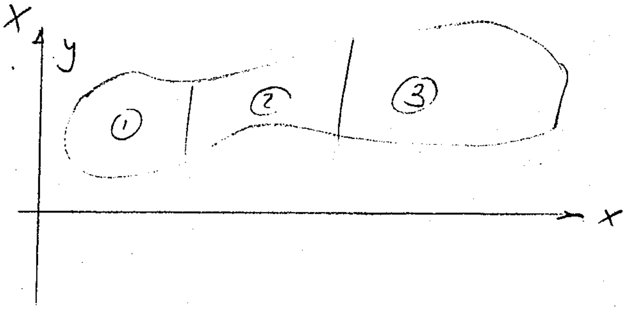
$$2a^3 \left\{\left(\frac{\pi}{2} - 2 + \frac{\pi}{4} + 0\right) - (0 - 0 + 0 + 0)\right\} = 2a^3\left(\frac{3}{4}\pi - 2\right)$$

$$\bar{X} = \frac{2a^3\left(\frac{3}{4}\pi - 2\right)}{\pi a^2\left(\pi/2 - 1\right)} = \frac{a}{\pi}\,\frac{(3\pi - 8)}{(\pi - 2)}$$

5.4 Composite Bodies and Figures - Approximations

when a given 2-D or 3-D body can be conviniently divided into several parts of simple shape we can use the principle of moments to find the centhoid or center of mass of the composite body:

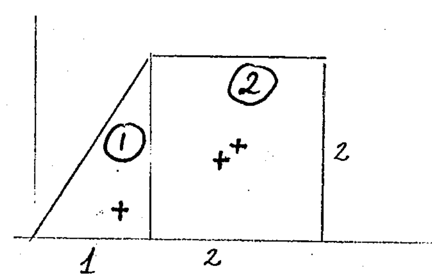
$$m_1\bar{x}_1 + m_2\bar{x}_2 + m_3\bar{x}_3 = (m_1+m_2+m_3)\bar{x}$$



and in general

$$\bar{X} = \frac{\Sigma m_i\bar{x}_i}{\Sigma m_i} \qquad \bar{Y} = \frac{\Sigma m_i\bar{y}_i}{\Sigma m_i} \qquad \bar{Z} = \frac{\Sigma m_i\bar{z}_i}{\Sigma m_i}$$

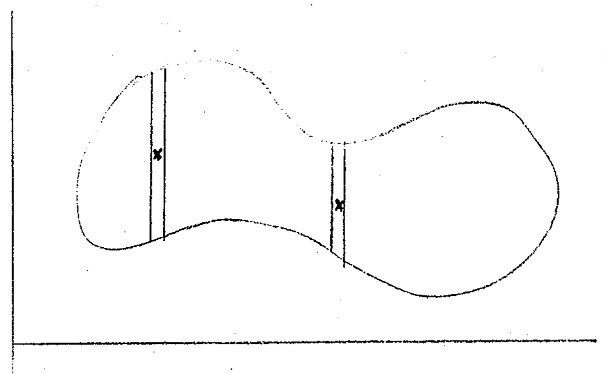
example



body	$\bar{X}$	$\bar{Y}$	A	$\bar{x}A$	$\bar{y}A$
1	$\frac{2}{3}$	$\frac{1}{3}$	1	$\frac{2}{3}$	$\frac{1}{3}$
2	2	1	4	8	4
		Σ	5	$8\frac{2}{3}$	$4\frac{1}{3}$

$$\bar{X} = \frac{8\frac{2}{3}}{5} = 1.73 \qquad \bar{y} = \frac{4\frac{1}{3}}{5} = 0.866$$

As for irregular shapes and bodies we can use the same approximation as for composite bodies. This is acheived by deviding the body into "differential bodies" whose C.Ms or centroids are known

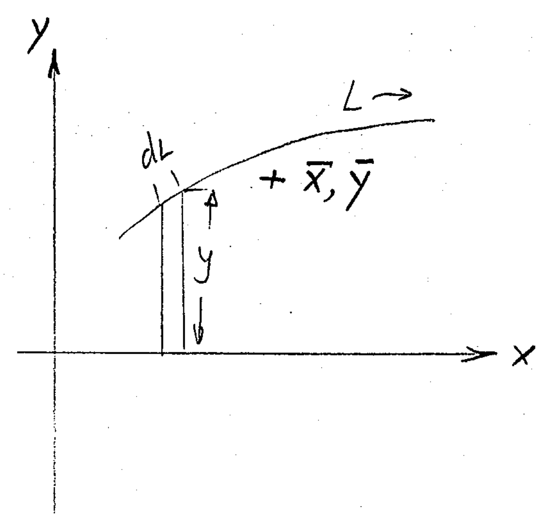


5.5 Pappus Theorems

Pappus was a Greek geometer who lived in the third century A.D.

These theorems enable the evaluation of surface areas and volumes of bodies of revolution;

Surfaces



$$dA = 2\pi y \, dL$$

$$A = 2\pi \int y \, dL$$

$$\bar{y} = \frac{\int y \, dL}{L}$$

$$A = 2\pi \bar{y} L$$

you can revolve it about the y axis

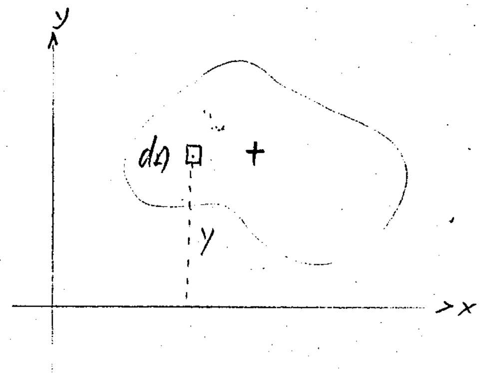
$$A_y = 2\pi \bar{x} L.$$

Volumes

$$dV = 2\pi y \, dA$$

$$V = 2\pi \int y \, dA$$

$$V = 2\pi \bar{y} A$$



if this configuation is revolved about the y axis

$$V_x = 2\pi \bar{x} A$$

Chapter 6 Friction

6.1 Introduction

In the preceding chapters the forces of action and reaction between contacting surfaces have been assumed to be normal to the surfaces. This assumption characterize smooth surfaces. A tangential force ≡ friction force developes whenever there is a tendency of one contacting surface to slide along another surface; we always find the friction forces that develope to be in a direction to oppose the sliding tendency.

In certain type of machines we try to minimize the effect of friction: bearings, gears, power screws etc. In other situations we wish to maximize the friction effect: car brakes, clutches, the friction between the tires and the pavement, the friction between our shoes and the ground etc. Whenever friction has to be taken into account in the analysis we consider that machine to be REAL. The development of these friction forces result in energy loss which is dissipated in the form of heat.

6.2 Types of Friction

Dry friction: developes between unlubricated surfaces. this is the most common type.

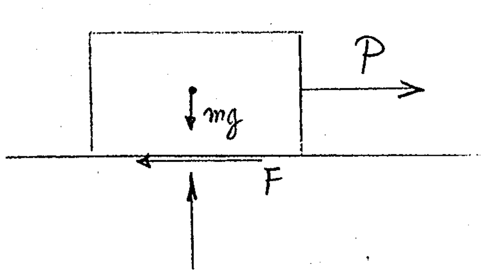
Fluid friction: developes between adjacent layers of fluids traveling with different velocities. Fluid friction is propotional to grad $\vec{v}$ and to the viscosity of the fluid

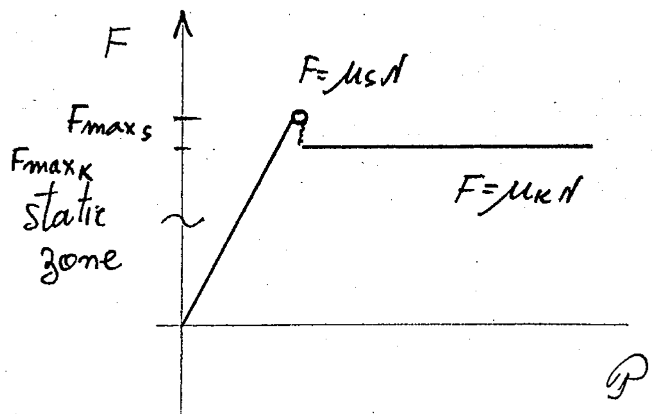
Internal friction: developes within solids undergoing cyclic loading = plastic defformation.

* We shall consider in the following presentation Dry Friction only.

6.3 Dry Friction

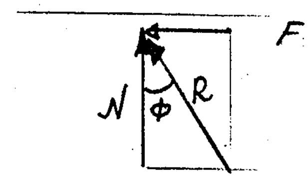
the experiment





$\Phi_s = \tan^{-1} \mu_s$

$\Phi_k = \tan^{-1} \mu_k$



$\tan \phi = \frac{F}{N}$

$\tan \phi_s = \frac{F_{max}}{N} = \frac{N\mu_s}{N} = \mu_s$

Types of Friction Problems

Type I : $F_{s(max)} = N\mu_s$

The body in equilibrium is on the VERGE of SLIPPING

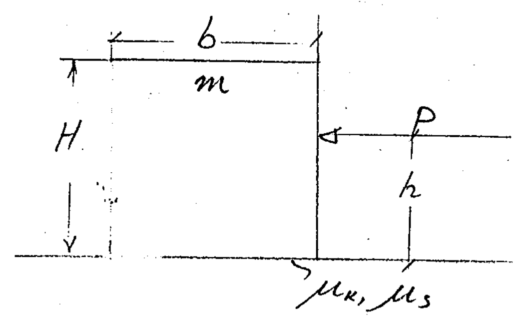
Type II : F = determined by the equilibrium conditions

F develops to maintain equilibrium.
We might find out that the F necessary to maintain equilibrium is larger than F_{max}
Thus motion occurs as well as acceleration

Type III : $F_{(max)} = N\mu_k$

relative motion is known to exist between the contacting surfaces.

problem 1

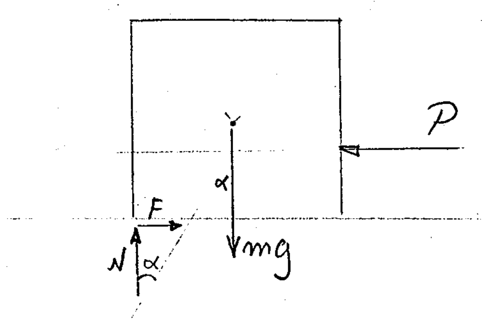


Given the block in the fig:

(a) For a given h determine the friction force and R location as a function of P

(I)

$$F = P \qquad \tan\alpha = \frac{F}{N} = \frac{P}{mg} \quad \Bigg\} \text{ for } F \leq N\mu_s \leq mg\mu_s$$



(II) $F_{max} = \mu_s N = \mu_s mg$

$\alpha = \phi_s =$

as long as: $\frac{b/2}{h} > \tan\phi_s = \mu_s$ the block will not tip over

if $\frac{b/2}{H} \geq \mu_s$ it will never tip over

problem 2

Three flat blocks are positioned on a 30° incline as shown. a force P parallel to the incline is applied to the middle block. The upper block is prevented from moving by a wire attached to a fixed support. Determine the maximum value of P before any slipping occurs.

6.4 Wedges

The wedge is one of the oldest simplest and most important machines:

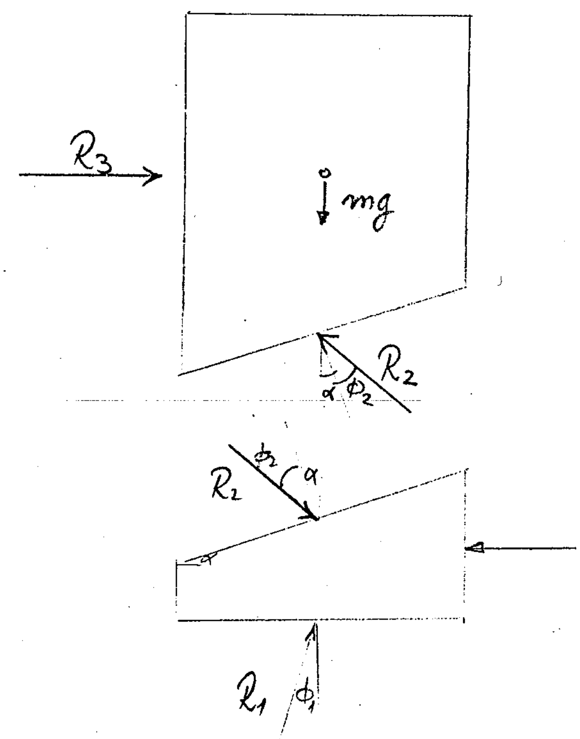
On the verge of movement the resultant force on each of the wedges surfaces makes an angle ϕ_s with respect to the normal to the surface:

$$R_2 \cos(\phi_2 + \alpha) = mg$$

$$R_1 \cos\phi_1 = R_2 \cos(\phi_2 + \alpha) = mg$$

$$R_1 = \frac{mg}{\cos\phi_1}$$

$$R_1 \sin\phi_1 + R_2 \sin(\phi_2 + \alpha) \equiv P$$



$$P = mg\tan\phi_1 \quad mg\tan(\phi_2 + \alpha)$$

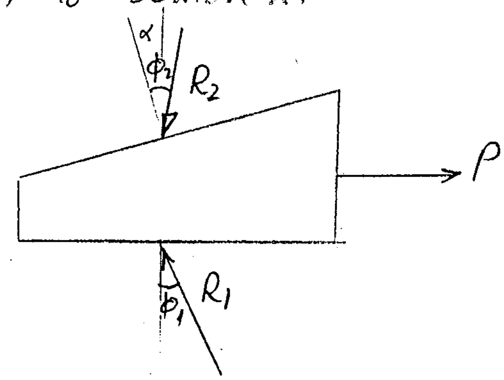
$$= mg\left[\mu_i + \tan(\phi_2 + \alpha)\right]$$

Prob. 1

when P is removed does the wedge remain in its place? Lets assume we need to apply a force P to remove it.

$$R_2 \cos(\phi_2 - \alpha) = mg.$$

$$R_1 \cos\phi_1 = R_2 \cos(\phi_2 - \alpha) = mg.$$



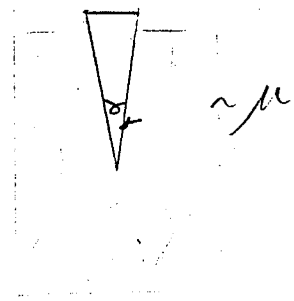
$$P = mg\left[\tan\phi_1 + \tan(\phi_2 - \alpha)\right]$$

for P to be zero $\quad \alpha = \phi_1 + \phi_2$

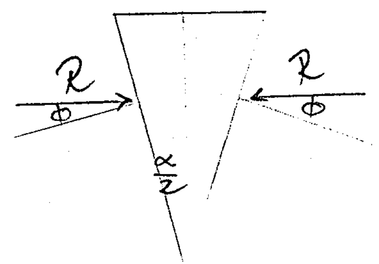
$\alpha < \phi_1 + \phi_2$ self locking

<u>example</u> p. 6.45 pp. 322

The coefficent of friction between the steel wedge and the moist fibers of the newly cut stump is $\mu = 0.2$ determine the max. angle α which the wedge may have so as not to pop out of the wood



* we demand that R will have no y component! thus



$$\frac{\alpha}{2} = \phi = tg^{-1}\mu$$

$$\alpha_{max} = 2tg^{-1}\mu$$

Chapter 7 Virtual Work

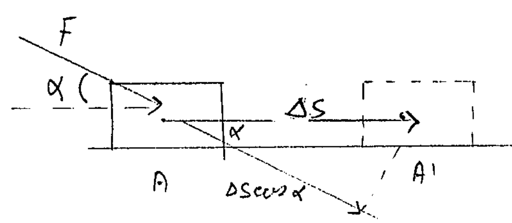
7.1 Introduction

In the previous chapters we analyzed the state of equilibrium of a body by using a FBD and by solving the six equil. eqs.

There exists a separate class of prob-lems in which bodies are composed of interconnected member that allow relative motion between the parts, thus permitting various possible equil. configurations to be considered. For this class of problems there exists a direct approach of solution known as the method of virtual work.

7.2 Work

(a) Work of a Force



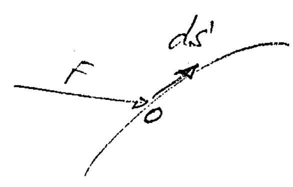
$$U = (F\cos\alpha)\,\Delta S$$

$$= F \cdot (\Delta S \cos\alpha) \quad \text{(or)}$$

* Possitive & Negative Work

To generalize the definition

$$dU = F \cdot ds$$



$$dU = (\vec{i}F_x + \vec{j}F_y + \vec{k}F_z)\cdot(\vec{i}dx + \vec{j}dy + \vec{k}dz)$$

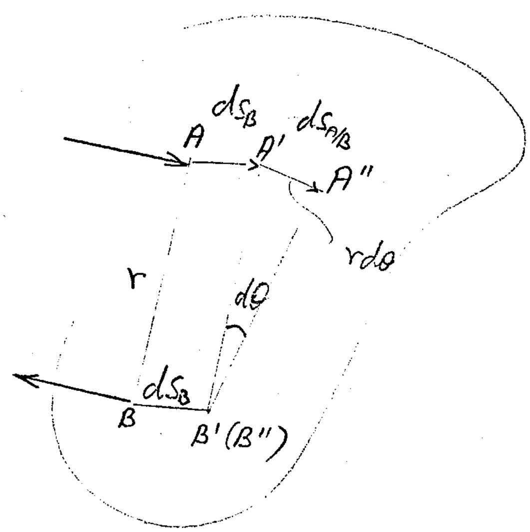
$$= F_x dx + F_y dy + F_z dz$$

If the process continues along a given path than we can integrate :

$$U = \int_a^b \vec{F}\cdot d\vec{s}$$

(b) <u>Work of a Couple</u>

In order to determine the work done by a couple we represent the couple M by two arbitrary forces F (where $F = \frac{M}{r}$) at a distance r. During the infinitismal movement line A'B' moves to A"B" we shall consider this movement into steps:



<u>AB→A'B'</u> - translation

$$dU = F ds_B - F ds_B = 0$$

a couple performs <u>no</u> work during pure translation

<u>A'B'→A"B"</u> – rotation

$$dU = F\cdot ds_{A/B} = F\cdot(r d\theta) = (Fr)\, d\theta = M d\theta$$

$$U = \int M d\theta$$

* If M and $d\theta$ are in the same sense the work is possitive else its negative.
* Work is a scalar! bearing the same units as a moment (which is a vector!)

SI :	Joule	N·m
B :	ft·lb	lb·ft

(c) Virtual Work

Given a particle in equilibrium under the action of a given set of forces at a given Position. Any arbitrary small permissible displacement δs away from this position is called a Virtual Displacement.

The term VIRTUAL indicate that this is an IMAGINARY displacement ment to enable the analysis of various equilibrium positions. The associated work with this displacement is known as the virtual work:

$$\delta U = \vec{F}\cdot\delta\vec{s} \quad \text{or} \quad \delta U = F\,\delta s\cos\alpha$$

or for a moment:

$$\delta U = \vec{M}\cdot\delta\vec{\theta}$$

during the virtual process $\vec{F}$ and/or $\vec{M}$ are assumed to remain constant.

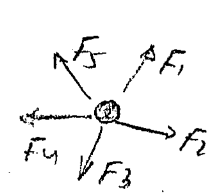
* Show the difference between δs and ds.

7.3 Equilibrium

We shall express now the equilibrium conditions for a Particle, a Rigid Body and a system of connected rigid bodies in terms of Virtual Work.

(a) Particle

Consider a particle in equilibrium.
If we apply a virtual displacement δs:



$$\delta U = F_1 \cdot \delta s + F_2 \cdot \delta s + F_3 \cdot \delta s + \dots F_n \delta s =$$

$$= (\Sigma F_i)\, \delta s \equiv 0 \qquad \text{as } \Sigma F = 0.$$

$$\text{or} \quad = (\vec{i}\, \Sigma F_x + \vec{j}\, \Sigma F_y + \vec{k}\, \Sigma F_z)(\vec{i}\, \delta x + \vec{j}\, \delta y + \vec{k}\, \delta z)$$

$$= R_x\, \delta x + R_y\, \delta y + R_z\, \delta z = 0$$

$$\therefore \quad R_x = 0 \qquad R_y = 0 \qquad R_z = 0$$

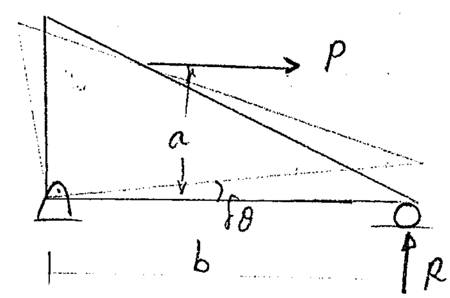
The application of the principle of virtual work to a particle results in the 3 eqs of equil. — NO ADVANTAGE

(b) Rigid Body

A rigid body can be considered as a system of Particles. The inter particle forces will contribute no virtual work as they appear in pairs of equal opposite and collinear forces. Thus only external forces

contribute to the virtual work.

example:



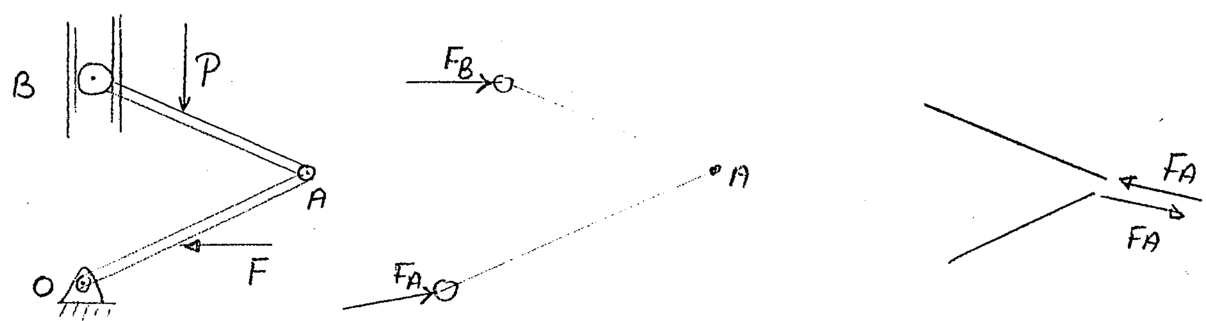
$$-Pa\,\delta\theta + Rb\,\delta\theta = 0$$

$$R = \frac{a}{b}P$$

this again the eq of equilibrium — NO-ADVANTAGE

(c) Ideal Systems of Rigid Bodies

We consider the equil. of an interconnected system of rigid bodies. We shall consider only IDEAL SYSTEMS composed of two or more bodies incapable of absorbing energy due to elongation or compression and in which friction is negligible:



Active Forces Reactive Forces Internal Forces.

* Internal forces do no work because they appear in pairs of equal opposite & collinear forces.
* Reactive forces do no work because they are in the direction of a constraint that can not be virtually displaced.

thus only the active external forces contribute to the virtual work. Now this principle can be stated:

"The Virtual work done by external active forces on an ideal mechanical system in equilibrium is zero for any and all virtual displacements consistent with the constraints"

$$\delta U = 0$$

the advantage of using this approach are:

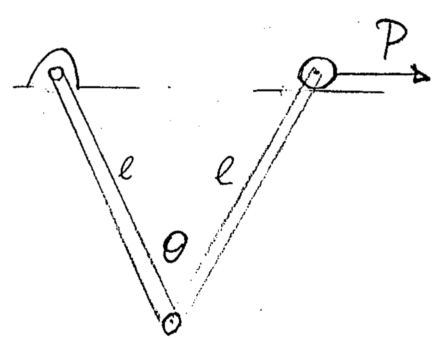
(a) We don't have to dismember the system

(b) We don't have to evaluate the reactive forces.

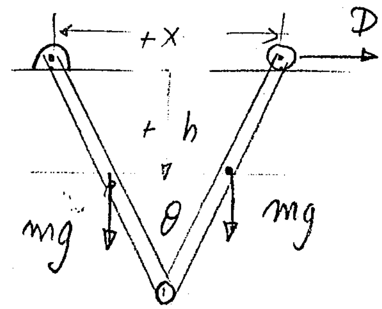
* In solving a problem we need to draw an active-force diagram

example 7.1

Each of the two uniform hinged bars has a mass m and a length l. For a given force P determine θ



First we draw an active force diagram



(1) $\delta U = 0 \Rightarrow \quad P \cdot \delta x + 2mg\,\delta h = 0$

now we express δx and δh in terms of θ:

(2)
$$\frac{x}{2} = l \sin\frac{\theta}{2} \qquad \delta x = l \cos\frac{\theta}{2}\,\delta\theta$$
$$h = \frac{l}{2}\cos\frac{\theta}{2} \qquad \delta h = -\frac{l}{4}\sin\frac{\theta}{2}\cdot\delta\theta$$

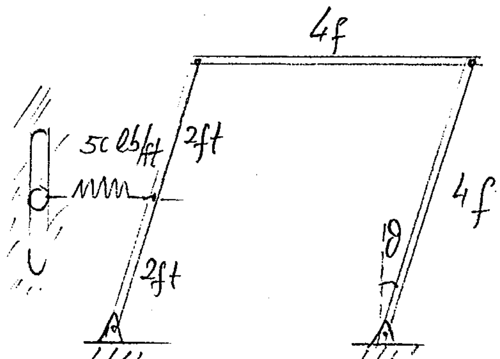
substituting in (1)

$$P \cdot l \cos\frac{\theta}{2}\,\delta\theta - 2mg\,\frac{l}{4}\sin\frac{\theta}{2}\,\delta\theta = 0$$

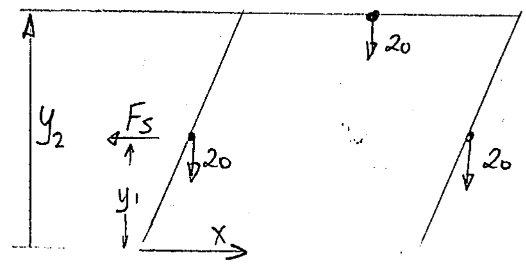
$$tg\,\frac{\theta}{2} = \frac{2P}{mg} \qquad \theta = 2\tan^{-1}\frac{2P}{mg}.$$

<u>example 6 Hibbeler 11/5</u> p(4..)

If each of the three links of the mechanism weights 20lb determine the angle θ for equilibrium. The Spring remains horizontal and is unstretched for $\theta = 0°$.



We draw first an active force diagram:



$x = 2\sin\theta \qquad \delta x = 2\cos\theta\,\delta\theta$

$y_1 = 2\cos\theta \qquad \delta y_1 = -2\sin\theta\,\delta\theta$

$y_2 = 4\cos\theta \qquad \delta y_2 = -4\sin\theta\,\delta\theta$

$\Delta x = 2\sin\theta$

the force in the spring is:

$$F_s = K\Delta x = 50\times 2\sin\theta = 100\sin\theta$$

$\delta U = 0$

$-20\,\delta y_2 - 2\cdot(20)\,\delta y_1 - F_s\,\delta x = 0$

$80\sin\theta\,\delta\theta + 80\sin\theta\,\delta\theta - F_s\cdot 2\cos\theta\,\delta\theta = 0$

$[160\sin\theta - 200\sin\theta\cos\theta]\,\delta\theta = 0$

either $\quad \sin\theta = 0 \qquad \theta = 0°$

or $\quad \cos\theta = \frac{160}{200} \qquad \theta = 36.9°$

there are <u>two</u> equilibrium positions.

* stability $\delta^2 U$.

Assigned Problems.

Ch. 1

1/1

1/3 (with $-6j$)

1/8 99.72 lb

1/10 (sin & tangent) for sin 2.06% for tan 4.09

Ch. 2

2/2

2/7

2/14 forces

2/19 $R = 50.2$ lb ; $\theta = 84.3°$

2/22

2/29 $M_o = 0.268$ lb-in ccw

2/30

2/32 moments

2/38 $P = 63.7$ lb

2/44

2/52 $R = 50\,N$ $\theta = 110°$ $M_o = 17.3\,N \cdot m$ ccw

2/56 couples.

2/61 $\vec{R} = 7247\,N$ $\theta = 105°$ $M_G = -7938\,N \cdot m$ cw

---

2/66

2/75 $M_B = 1220$ lb-in $d = 9.59$ in (משמאל לחור) Resultants

2/77

2/83 $\theta_x = 66.1°$; $\theta_y = 139.5°$; $\theta_z = 59.5°$

2/92 3-D Components

2/99 $\vec{M} = 0.457\,(-3\vec{i} + 4.8\vec{j} + 2.4\vec{k})$ KN·m

2/112 Moments & Couples 3-D

2/114

2/119 $\vec{M} = -780\,i + 290\vec{j} + 200\,\vec{k}$ N·m

2/120 3-D Resultants

2/121 $\vec{M}_A = -44.7\,\vec{i} + 30\vec{j} + 20\,\vec{k}$ KN·m

Assigned Problems (con't)

Ch. 3

3/6

3/9 $T = 134.2$ lb

3/12 $T_1 = 69.8$ Kips $T_2 = 56.6$ Kips

3/16 $T = \frac{2}{7} mg$

2-D Concurrent

3/34

3/40 $T = 141.6$ KN

3/43 $F_A = 929$ N ($A_x = 500$ N ; $A_y = 783$ N)

3/60

2-D Nonconcurrent

3/66

3/71

3/77

Con

3/80

3/88

3/92

Noncon.

3/96

3/99 $T_A = 147.2$ N ; $T_B = 245.2$ N ; $T_C = 196.2$ N

3/110

3/112

Ch. 4

4/3

4/5

4/14

4/16 $BE = 559$ lb T $BC = 300$ lb T

4/25

Method of Joints

4/28 $CG = 14.14$ Kips T

4/33 $FG = 150$ lb C $BC = 150$ lb T Method of Sections

4/39

4/44

4/56 $F_{AB} = \frac{4}{27} mg$; $F_{AC} = -\frac{5}{54} mg$ 3-D trusses

---

4/61

4/69

4/71 Frames & Machines

4/89 $A_x = 6.57$ KN $A_y = 44.2$ KN $A = 44.7$ KN

4/109

4/112

4/124

Ch 5

5.2

5.6 analytical

5.17

5.21

5.42

5.44 Composites

5.63

5.77 Pappus

5.84

Ch 6

6.3

6.5 $F_A = F_B = 126.6$ N ; $x = 86.2$ mm

6.15

6.24

6.27 $\mu_s = 0.268$

6.48

. 6.51 $P = 4.53$ KN Wedges.

. 6.59 $M = 7.3$ N·m

Ch. 7

7.12

7.16

7.36

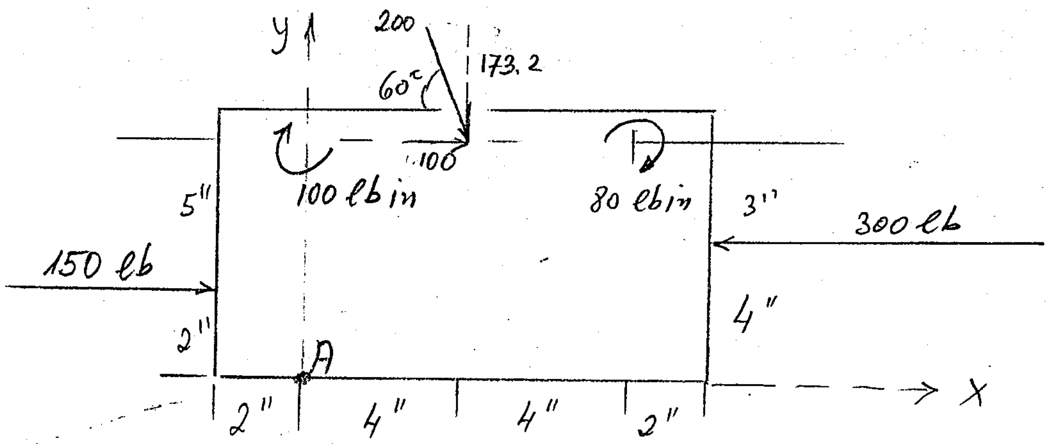
. 7.38

Problem # 1 Equivalent Systems and Resultants

Three forces and two couples act on the plate as shown. Determine :

(a) The equivalent force-couple system at A

(b) The resultant of the system and its point of intersection with the X-axis.



Solution

(a) An equivalent system consists of a single force + a couple-moment which have the same effect on the body as the system of forces and moments that they replace.

Step 1 specify the point at which the equivalent system is to be located :

in our case point A

step 2 decompose all forces into their cartesian components

200 lb force : in the x $200 \cdot \cos 60^\circ = 100$ lb

y $-200 \sin 60^\circ = -173.2$ lb

step 3 Calculate the equivalent force by summing all force components respectively

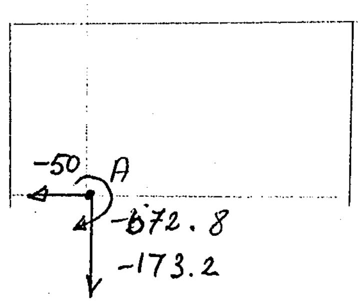
$R_x = \Sigma F_x = 100 + 150 - 300 = \underline{-50 \text{ lb}}$

$R_y = \Sigma F_y = \underline{-173.2 \text{ lb}}$

step 4 Calcutate the equivalent moment by suming moments at the specified point

$M_{RA} = \Sigma M_A = -80 - 100 + 300 \times 4 - 173.2 \times 4 - 100 \times 3 - 150 \times 2$

$= -80 - 100 + 1200 - 692.8 - 700 - 300 = \underline{-672.8}$

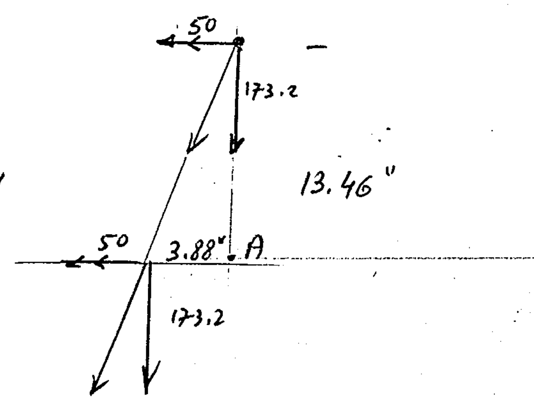


(b) A resultant consists of a single equivalent force UNIQUELY located on the body so that the equivalent moment is zero

<u>step 1</u> Locate an equivalent system at any specified point (<u>convinient</u>)

point A

<u>step 2</u> Relocat either R_x or R_y to a parallel location (usually along one of the axes) such that the introduced couple-moment cancels out the equivalent moment



<u>case I</u> moving R_x upwards:

$$d = \frac{672.8}{50} \cong 13.46''$$

<u>case II</u> moving R_y to the left:

$$d = \frac{672.8}{173.2} \cong 3.88''$$

<u>step 3</u> Relocate the other component using the principle of transmissibility

Problem #2

Equivalent moment calculation – using vectors

$\underline{F_1} = 60\,j \qquad \underline{r}_1 = 8\underline{i}$

$\underline{F_2} = -60\,j \qquad \underline{r_2} = 8\underline{i} - 8\underline{k}$

$F_3 = -50\,i \qquad r_3 = 8\underline{j} - 8\underline{k}$

$F_4 = 70\,i \qquad r_4 = 0$

$$\underline{M}_A = \Sigma \underline{r}_i \times F_i = (8\underline{i})\times(60\underline{j}) + (8\underline{i}-8\underline{k})\times(-60\underline{j}) + (8\underline{j}-8\underline{k})\times(-50\underline{i})$$

$$= \cancel{480\,\underbrace{\underline{i}\times\underline{j}}_{\underline{k}}} - \cancel{480\,\underbrace{\underline{i}\times\underline{j}}_{\underline{k}}} + 480\,\underbrace{\underline{k}\times\underline{j}}_{-\underline{i}} - 400\,\underbrace{\underline{j}\times\underline{i}}_{-\underline{k}} + 400\,\underbrace{\underline{k}\times\underline{i}}_{\underline{j}}$$

$$= (-480\,\underline{i} + 400\,\underline{j} + 400\,\underline{k})$$

Recall the cross product calculation!

$$\Sigma F_y = 0 \qquad A_y - 900 - 4000 - T\cos\theta = 0$$

$$A_y = 4900 + 630 \cdot \cos\theta = 5472.8 \text{ N}$$

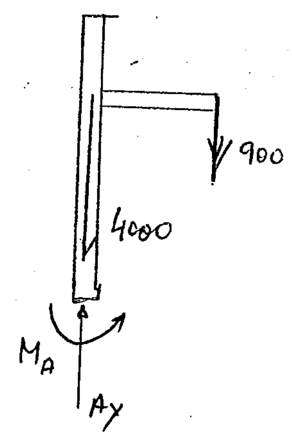
בדיקה עבור מומנטים סביב כח 900 (נקודה D)

$$\Sigma M = \qquad A_x \cdot 7.2 - A_y \cdot 2.1 + (T\cos\theta + 4000) \cdot 2.1 \overset{?}{=} 0$$

$$262.5 \times 7.2 - 5472.8 \cdot 2.1 + (572.8 + 4000) \cdot 2.1 \overset{?}{=} 0$$

$$1890 \quad - \quad 11{,}492.9 \qquad 9602.9 = 0 \ \checkmark\checkmark$$

<u>Case II</u>



$$\Sigma F_x = 0$$

$$\Sigma F_y = 0 \qquad A_y - 4000 - 900 = 0$$

$$A_y = 4900 \text{ N}$$

$$\Sigma M_A = 0 \qquad M_A - 900 \times 2.1 = 0$$

$$M_A = 1890 \text{ Nm}$$

בדיקה: מומנטים סביב D

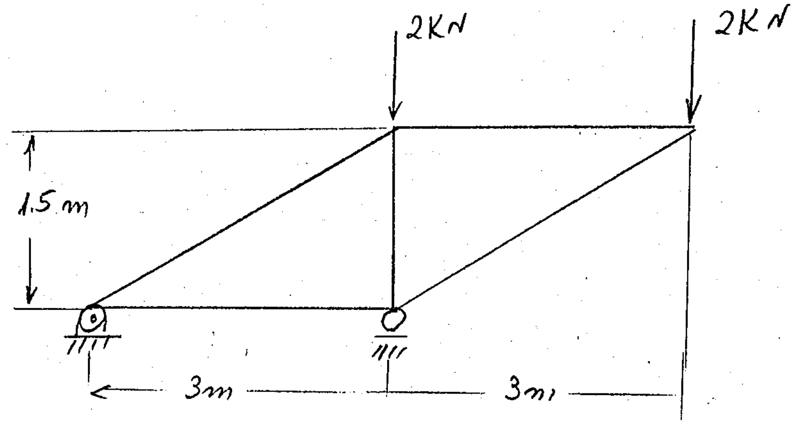
$$M_D = 0 \qquad 1890 + 4000 \times 2.1 - A_y \times 2.1 = 0$$

$$1890 + 8400 - 10{,}290 = \checkmark\checkmark\ 0$$

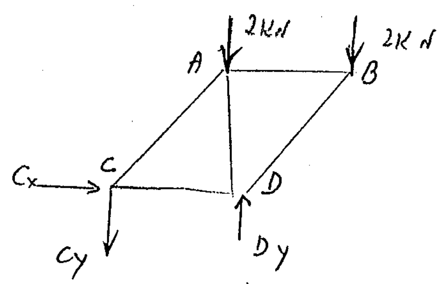
Case 3

Problem # 5

Determine the force in each member of the truss shown. State wheather each member is in tension or compression



Step 1 FBD of the entire interconected structure



Step 2 solve external equilibrium

$\Sigma F_x = 0 \qquad C_x = 0$

$\Sigma M_c = 0 \qquad D_y \cdot 3 - 2 \cdot 3 - 2 \times 6 = 0$

$D = 6 \text{ KN}$

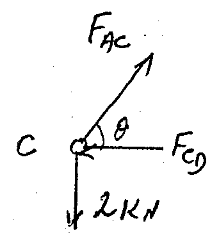
$\Sigma F_y = 0 \qquad D_y - C_y - 2 - 2 = 0$

$C_y = 6 - 4 = 2 \text{KN} \downarrow$

Step 3 Identify a joint which has no more than two unknowns acting on it. (The force in each member is an un-known)

Step 4 FBD of the Joint (Point). Since each member of a truss is a TWO FORCE MEMBER the unknown member force must act along the direction of the member

$\theta = tg^{-1} \frac{1.5}{3.0} = 26.6°$



$\Sigma F_y = 0$ $F_{AC} \sin\theta - 2 = 0$

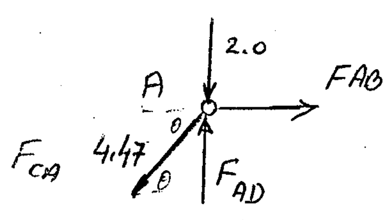
$F_{AC} = 4.47$ KN (T) FORCE POINTS AWAY FROM JOINT = TENSION

$\Sigma F_x = 0$ $F_{AC} \cos\theta - F_{CD} = 0$

$F_{CD} = 4.0$ KN (C) FORCE POINTS TOWARDS THE JOINT = COMPRESION

Step 5 Repeat step 3-4 for all the joints

JOINT A



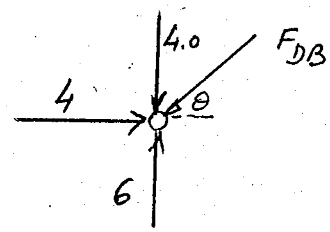
$\Sigma F_x = 0$ $- F_{CA} \cos\theta + F_{AB} = 0$

$F_{AB} = 4.0$ KN (T)

$\Sigma F_y = 0 \qquad F_{AD} - 2 - 4.47 \sin\theta = 0$

$F_{AD} = 4.0 \text{ KN}$ (C)

Joint D



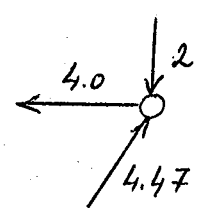
$\Sigma F_x = 0 \qquad 4 - F_{DB} \cos\theta = 0$

$F_{DB} = 4.47 \text{ KN}$ (C)

$\Sigma F_y \stackrel{?}{=} 0 \qquad 6 - 4 - 4.47 \sin\theta = 0$ ✓✓

Step 6 Verify analysis on the last joint

Joint B

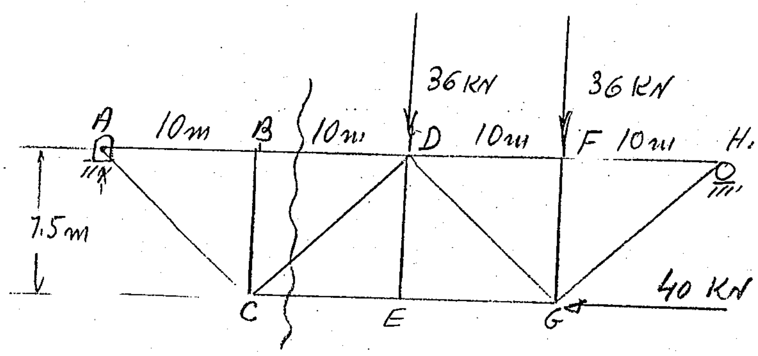


$\Sigma F_x = 0 \qquad 4.47 \cos\theta - 4 = 0$ ✓

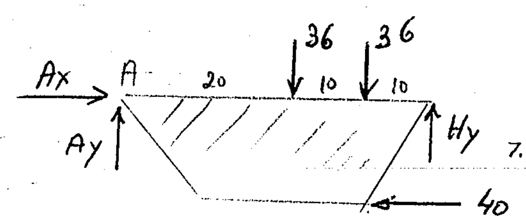
$4.47 \sin\theta - 2 = 0$ ✓

Problem 6 TRUSS ANALYSIS BY THE METHOD OF SECTION

Determine the force acting in members BD, CD & CE of the truss shown. Specify wheather they are in tension or compression



Step 1: Construct a FBD of entire structure



Step 2 Perform a coplanar equilibrium analysis

$\Sigma F_x = 0$ $\qquad A_x - 40 = 0$

$$A_x = 40 \text{ KN}$$

$\Sigma M_A = 0$ $\qquad -36 \times 20 - 36 \times 30 - 40 \times 7.5 + H_y \times 40 = 0$

$$H_y = 52.5 \text{ KN}$$

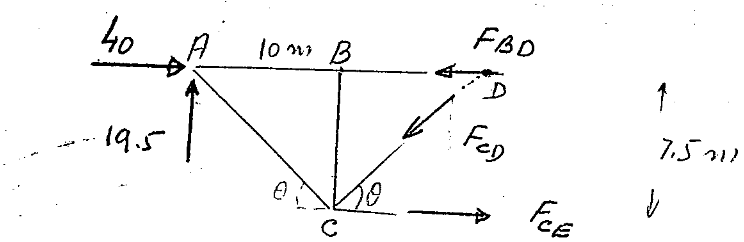
$\Sigma F_y = 0$ $\qquad 52.5 + A_y - 36 - 36 = 0$

$$A_y = 19.5 \text{ KN}$$

Step 3 Draw a section line through the truss intersecting at least one of the members of interest; However NO MORE than three unknowns may be cut by the section line!

See Fig

Step 4 Draw a FBD of one section of the truss. The forces in the cut members become unknowns. The forces are directed along the members and their direction may be ASSUMED



Step 5 Coplanar equilibrium

$\Sigma M_C = 0 \qquad F_{BD} \cdot 7.5 - 40 \times 7.5 - 19.5 \cdot 10 = 0$

$F_{BD} = 66 \text{ KN}$ (C)

$\Sigma M_D = 0 \qquad -19.5 \times 20 + F_{CE} \times 7.5 = 0$

$F_{CE} = 52 \text{ KN}$ (T)

$\Sigma F_y = 0 \qquad 19.5 - F_{CD} \sin\theta = 0 \qquad \theta = tg^{-1}\frac{7.5}{10} = 36.87°$

$F_{CD} = \frac{19.5}{\sin\theta} = 32.5 \text{ KN}$ (C)

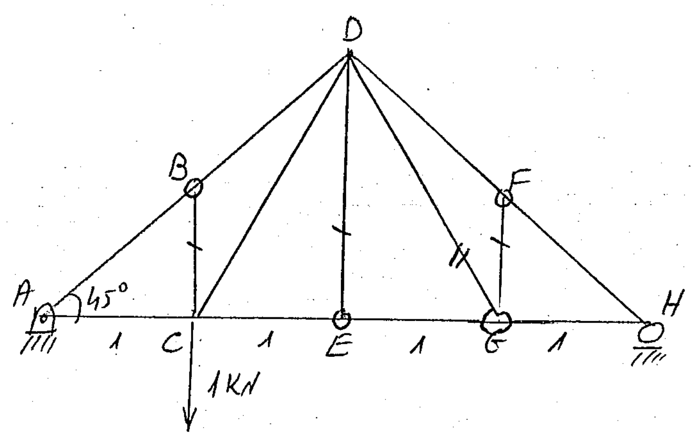
* This process can be repeated to determine the desired number of forces.

You can use now the method of sections to determine the force in members DF, DG, EG and FG of this problem.

Problem # 7

Inspect the truss below and identify zero-force members.

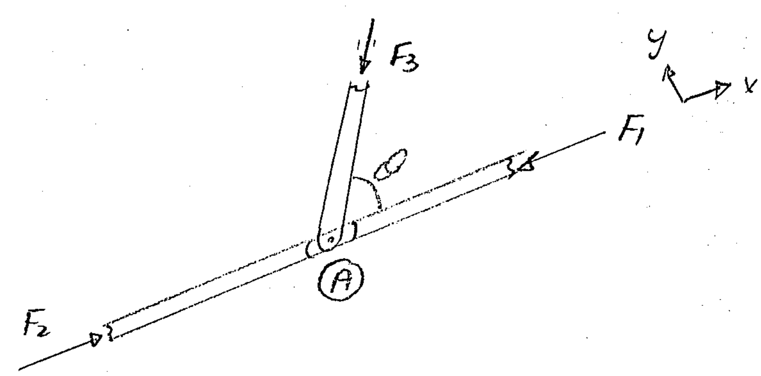
Do NOT actually analyze the truss.



In order to identify zero-force members we need to be reminded of a special condition that frequently occurs in the analysis of trusses. When two collinear members are in compression it is necessary to add a third member to maintain allignment of the two members and to prevent buckling

THE JOINT IS UNLOADED

Three member joint



By summing forces in the y direction at joint A we can readily see that

$$F_3 = 0$$

$$F_1 = F_2$$

This is true for any θ and in tension as well

Thus by inspecting all the unloaded joints of the above truss we readily conclude that the force in members:

$$F_{FG} = 0 \qquad F_{FH} = F_{FD}$$

$$F_{DE} = 0 \quad \text{and} \quad F_{CE} = F_{EG}$$

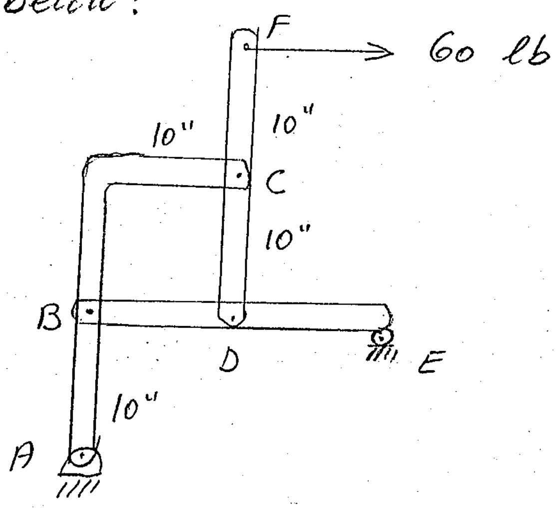
$$F_{BC} = 0 \qquad F_{AB} = F_{BD}$$

As a result of this first conclusion re-examining joint G yield

$$F_{GD} = 0 \quad \& \quad F_{EG} = F_{GH}$$

Problem # 8 FRAME AND MACHINE ANALYSIS

Determine the force acting on all pins of the loaded frame shown below:



A structure is considered a frame or a machine if at least one of its members is a multiforce member e.i. bears more than 2 forces.

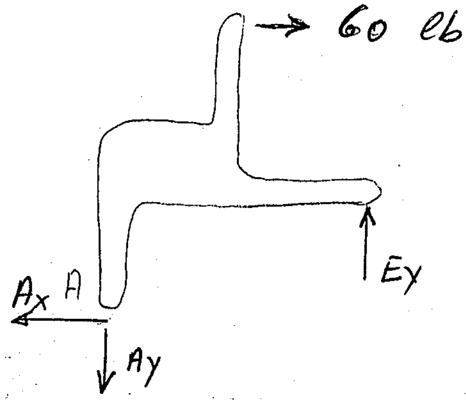
step 1 check if structure is externally determinate.

2 unknowns at A
1 " at E
= 3 unknowns ≡ 3 eq. of EQ.

step 2 solve external equilibrium

(a) draw FBD

(b) solve



$\Sigma F_x = 0 \qquad 60 - A_x = 0$

$A_x = 60 \text{ lb}$

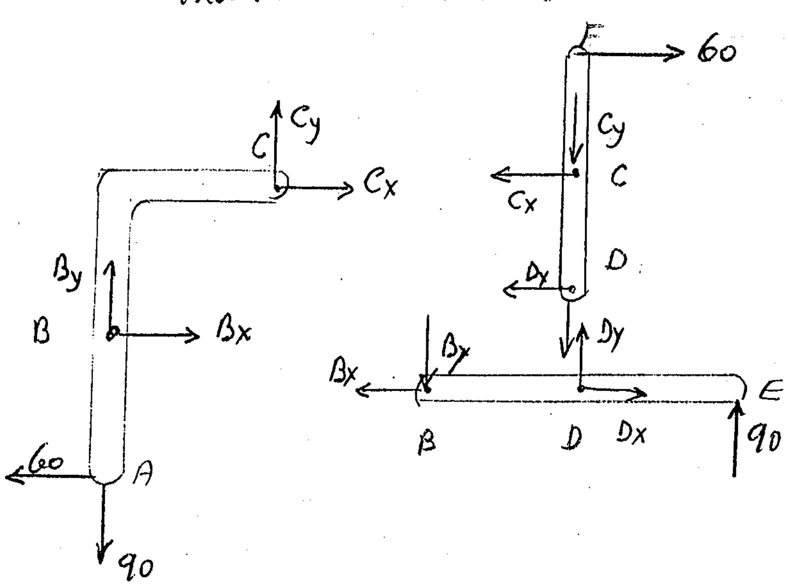
$\Sigma M_A = 0 \qquad -60 \times 30 + E_y \times 20 = 0$

$E_y = 90 \text{ lb}$

$\Sigma F_y = 0 \qquad E_y - A_y = 0$

$A_y = 90 \text{ lb}$

<u>Step 3</u> Dismember the frame and draw FBD of each individual member



Step 4 Solve for all unknowns by performing coplanar eq. analysis on all members.

member BE

1) $\Sigma M_B = 0$ $\quad D_y \cdot 10 + 90 \times 20 = 0$

$$\underline{D_y = -180 \text{ lb}} \quad \downarrow$$

2) $\Sigma F_y = 0$ $\quad D_y - B_y + 90 = 0$

$$\underline{B_y = -90 \text{ lb}} \quad \uparrow$$

3) $\Sigma F_x = 0$ $\quad D_x - B_x = 0$

$$\underline{D_x = B_x}$$

member DF

4) $\Sigma M_D = 0$ $\quad -60 \times 20 + C_x \cdot 10 = 0$

$$\underline{C_x = 120 \text{ lb}} \quad \leftarrow$$

5) $\Sigma F_x = 0$ $\quad -C_x - D_x + 60 = 0$

$$\underline{D_x = -C_x + 60 = -60} \text{ lb} \rightarrow$$

6) from 3, 5 $\quad \underline{B_x = -60} \quad \rightarrow$

7) $\Sigma F_y = 0$ $\quad -C_y - D_y = 0$

$$\underline{C_y = -D_y = 180 \text{ lb}} \quad \downarrow$$

Step 5 We solved for all unknowns without refering to the last member AC. This member can serve to check the analysis

member AC

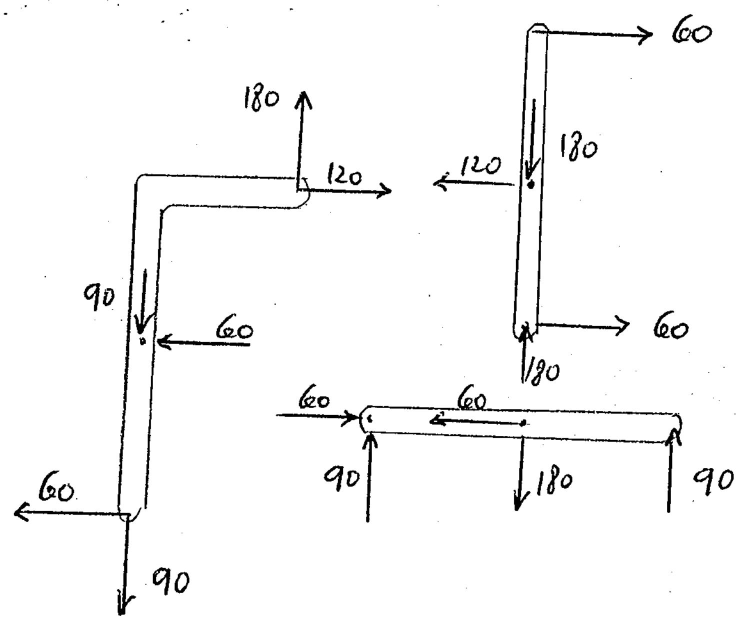
$\Sigma F_x = 0 \qquad C_x + B_x - 60 = 120 - 60 - 60 = 0$ ✓✓

$\Sigma F_y = 0 \qquad C_y + B_y - 90 = 180 - 90 - 90 = 0$ ✓✓

$\Sigma M_A = 0 \qquad -C_x \cdot 20 + C_y \cdot 10 - B_x \cdot 10 =$

$\qquad -60 \times 20 + 180 \cdot 10 + 60 \cdot 10 = 0$ ✓✓

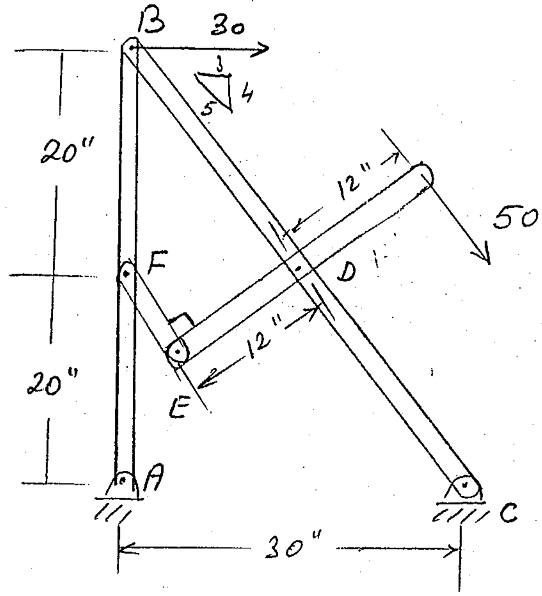
Step 5 Draw a correct FBD



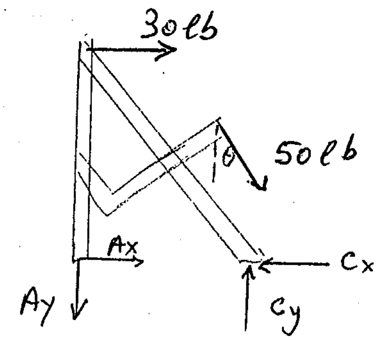
✓✓

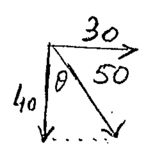
Problem #8A

Determine the forces acting on all the pins of the loaded structure:



Step 1 We note by inspection that if the supports at A & C are removed the frame is not rigid - or callapsible. Thus the external reactions can not be completely determined until individual members are analyzed. However by drawing a FBD of the entire structure we can still evaluate the vertical reactions at A & C ;



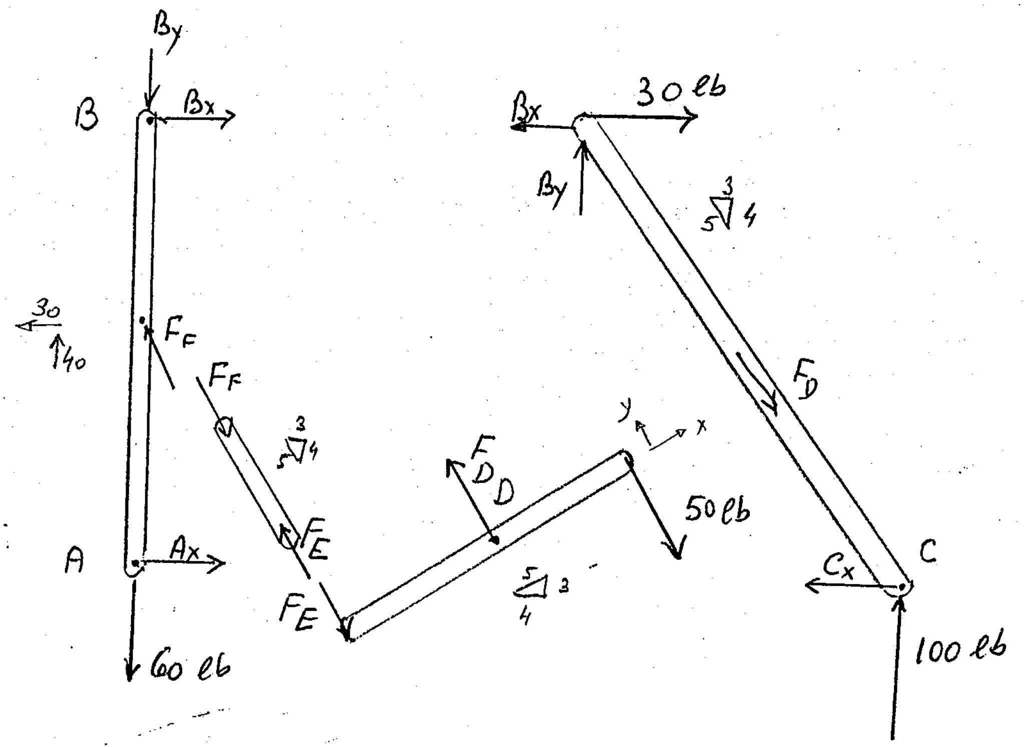


$\Sigma M_C = 0 \qquad -50 \cdot 12 - 30 \cdot 40 + A_y \cdot 30 = 0 \qquad \underline{A_y = 60 \text{ lb}}$

$\Sigma F_y = 0 \qquad C_y - A_y - 50 \times \frac{4}{5} = 0 \qquad \underline{C_y = 100 \text{ lb}}$

$50 \cos\theta \qquad \cos\theta = \frac{4}{5}$

Step 2 Dismember the frame and draw FBDs for all members



1. Note as FE is a two force member F_E & F_F have to be collinear and

$$F_F = F_E$$

2. As F_E and the 50 lb are perpendicular to member ED so has to be F_D

Analysis

member ED

$\Sigma M_D = 0$ $\quad F_E \cdot 12 - 50 \cdot 12 = 0$ $\quad \underline{F_E = 50 \text{ lb}}$

$\Sigma F_N = 0$ $\quad F_D - 50 - 50 = 0$ $\quad \underline{F_D = 100 \text{ lb}}$

member EF

$\Sigma F_N = 0 \qquad F_F - F_E = 0 \qquad \underline{F_F = 50\ lb}$

member AB

$\Sigma M_A = 0 \qquad -B_x \cdot 40 + (50 \cdot \frac{3}{5})\, 20 = 0 \qquad \underline{B_x = 15\ lb}$

$\Sigma F_x = 0 \qquad A_x + 15 - 50 \times \frac{3}{5} = 0 \qquad \underline{A_x = 15 lb}$

$\Sigma F_y = 0 \qquad 50 \times \frac{4}{5} - 60 - B_y = 0 \qquad \underline{B_y = -20 lb} \uparrow\ !$

member BC

$\Sigma F_x = 0 \qquad 30 + 100\,\frac{3}{5} - 15 - C_x = 0 \qquad \underline{C_x = 75\ lb}$

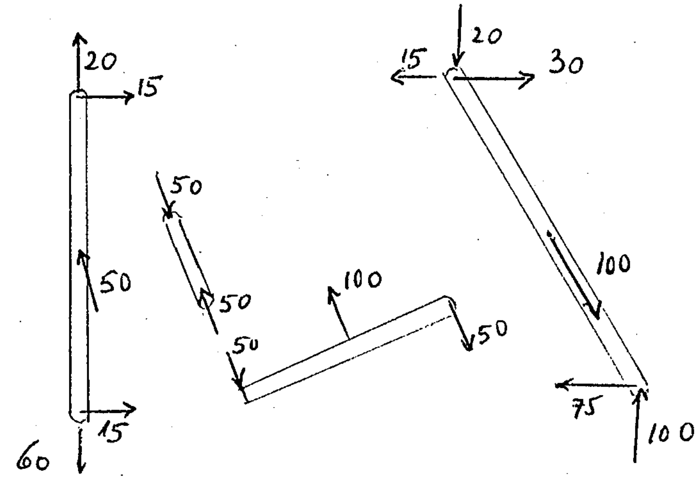
Step 3

we may apply the two remaining eq. of EQ on member BC to check the solution

$\Sigma F_y = 0 \qquad 100 + (-20) - 100 \times \frac{4}{5} = 0$ ✓✓

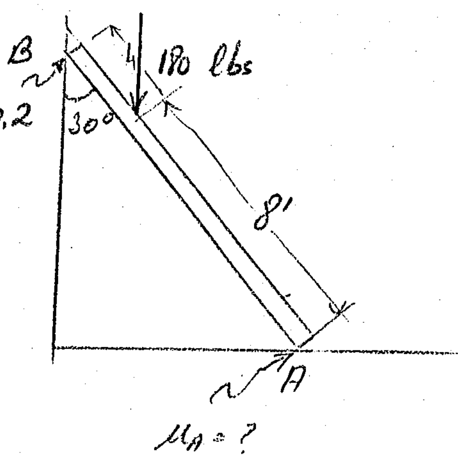
$\Sigma M_C = 0 \quad -(30 - 15) \cdot (40) + 20 \times 30 = 0$ ✓✓

Step 4 Redraw a FBD



Problem # 9 FRICTION - IMPEDING MOTION

A 12 ft ladder weighing 40 lbs holds a 180 lbs man who has just stepped to the shown position. At this point the ladder is on the verge of slipping. If the coeficient of friction at B is $\mu_B = 0.2$ what is the coefficient of friction at A μ_A ?

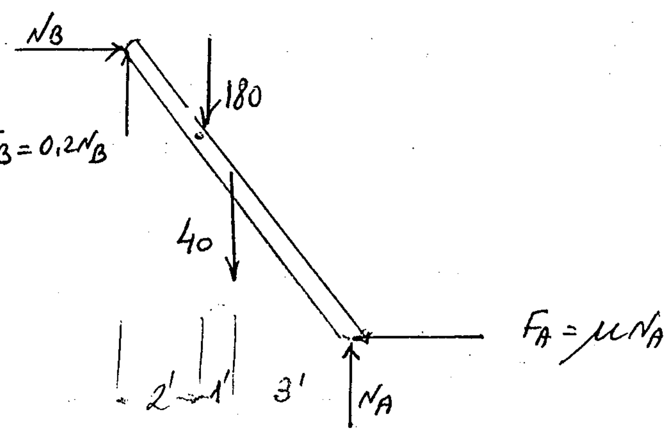


Step 1 Construct a FBD including friction forces and normal forces at each rough surface of contact.

Remember: at the point of impeding motion:

1. The magnitude of the friction force is $F_f = \mu \cdot N$ coefficient of friction × normal force to the surface
2. The direction of the friction force is OPPOSITE to the direction of slippage

$12\cos 30° = 10.39$



Step 2 Analysis

$\Sigma M_A = 0$

$$40 \times \underbrace{6\cos 60°}_{3} + 180 \times \underbrace{8\cos 60}_{4} - N_B \times \underbrace{12\cos 30}_{10.39} - 0.2\, N_B \times \underbrace{12 \sin 30°}_{6} = 0$$

$$120 + 720 - 10.39 N_B - 1.2 N_B = 0$$

$$11.59\, N_B = 840$$

$$N_B = 72.48$$

$\Sigma F_y = 0$

$$\underbrace{0.2 \times 72.48}_{\sim 14.5} + N_A - 40 - 180 = 0$$

$$N_A = 205.5$$

$\Sigma F_x = 0$

$$N_B - \mu N_A = 0$$

$$\mu = \frac{N_B}{N_A} = \frac{72.48}{205.5} = 0.35 \quad \text{vv}$$

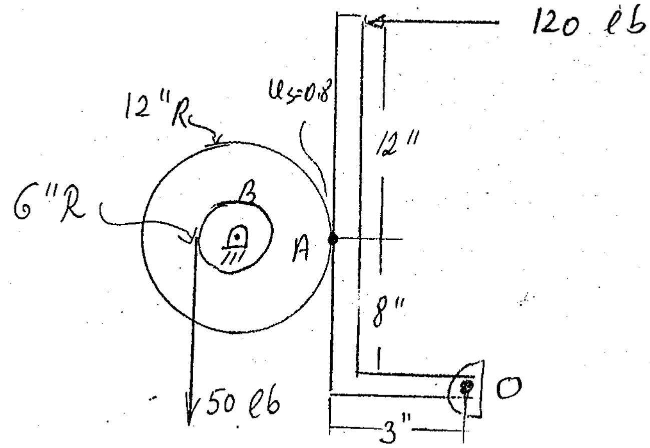
בדיקה מומנטים סביב B

$\Sigma M_B = 0$ $\quad -180 \times 2 - 40 \times 3 - 0.35 \times 205.5 \times 10.39 + 205.5 \times 6 = 0$ vv

Problem # 10 FRICTION – IMPEDING MOTION – UKNOWN

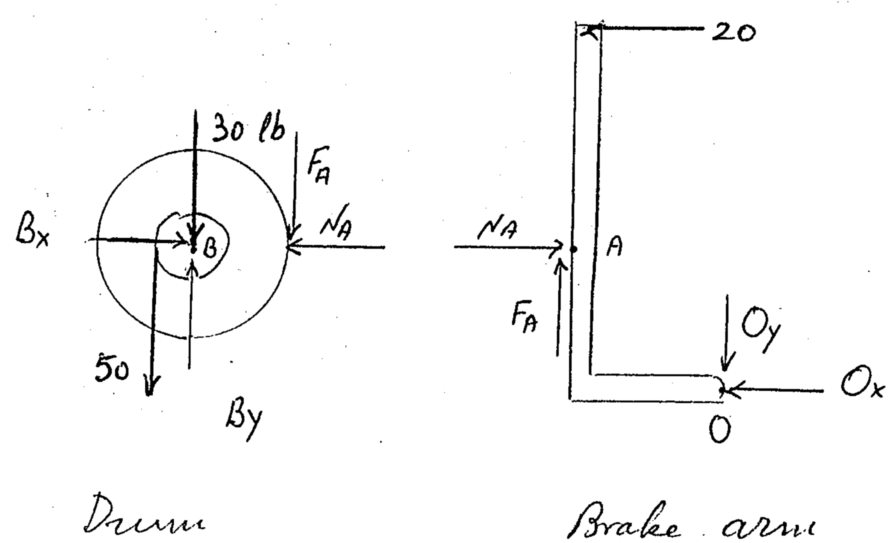
The brake shown bears against the outer surface of a 30 lb drum for which $\mu_s = 0.80$

Determine wheather the 20 lb force prevents the drum from rotating:



If impeding motion is unknown than:

Step 1 Construct FBD assume friction face at rough surface: the direction must be assumed and the magnitude not related to the normal force!



Step 2 Coplanar EQ analysis

On Drum

$\Sigma M_B = 0$ $\quad\quad 50 \times 6 - F_A \times 12 = 0$

$$F_A = 25 \text{ lb}$$

On Brake Arm

$\Sigma M_o = 0$ $\quad\quad 20 \times 20 - F_A \times 3 - N_A \times 8 = 0$

$$N_A = \frac{400 - 75}{8} = 40.6 \text{ lb}$$

Check condition of friction

$$F_A \leq \mu N_A$$

$$25 \leq 0.8 \times 40.6 = 32.5 \text{ lb}$$

Thus drum is in rest.

CASE B

If the pulling force on the drum is increased to 100 lb will the drum be still at rest?

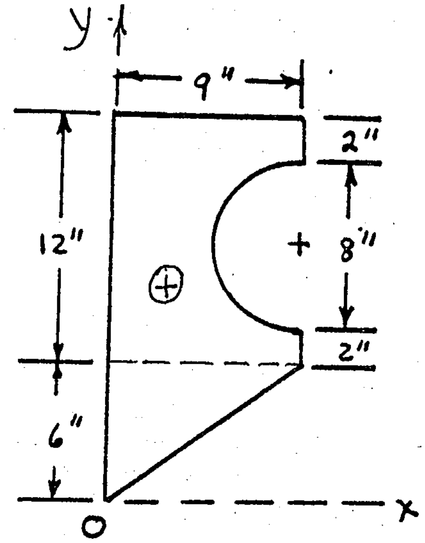
$$F_A = 50 \text{ lb}$$

$$N_A = \frac{400 - 150}{8} = 31.3 \text{ lb}$$

$$50 \overset{?}{\leq} 0.8 \cdot 31.3 = 25 \quad \underline{NO}$$

drum rotates

PROBLEM 11
CENTROID



$\bar{X} = 3.68"$

$\bar{y} = 10.0"$

Locate the centroid of the area shown.

Solution: The process for locating the centroid or center of gravity of a composite body is:

Step 1 - Divide the composite body into elements of standard geometric shapes.

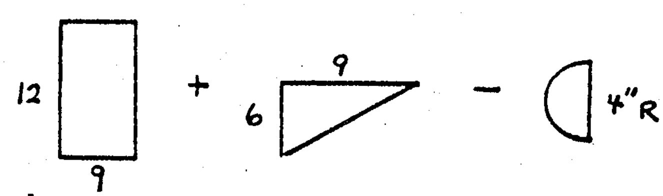
Step 2 - Construct a table to tabulate the required information for each element.

Step 3 - Calculate and tabulate either the weight, volume, area or length of each element.

Step 4 - Locate an origin and tabulate the coordinates to the centroid or center of gravity of each element with respect to this origin.

Step 5 - Form the necessary products and summations to calculate the centroid or center of gravity.

Calculations



Complete the table below

Element	a_i	x_i	$a_i x_i$	y_i	$a_i y_i$
Rectangle +Triangle -1/2 Circle	108 27 25.13				
$\sum$					

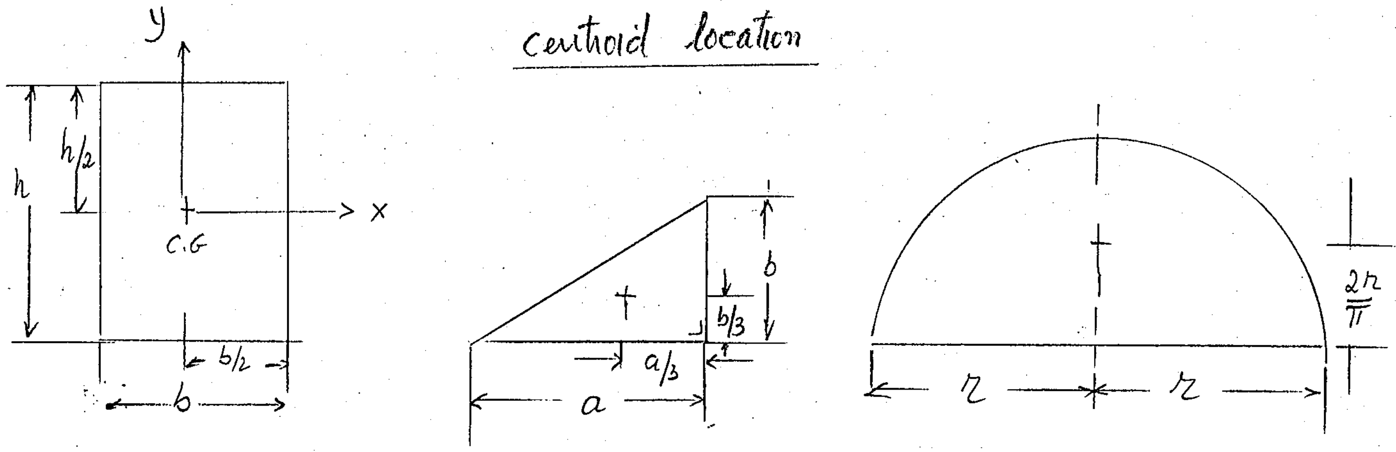
$\sum a\ x$ $\qquad$ $\sum a\ y$

Problem # 11

Step 1

Divide the composite body into elements of standard geometries and use tables to find their centroids

centroid location



Area of "Holes" is taken as "negative"

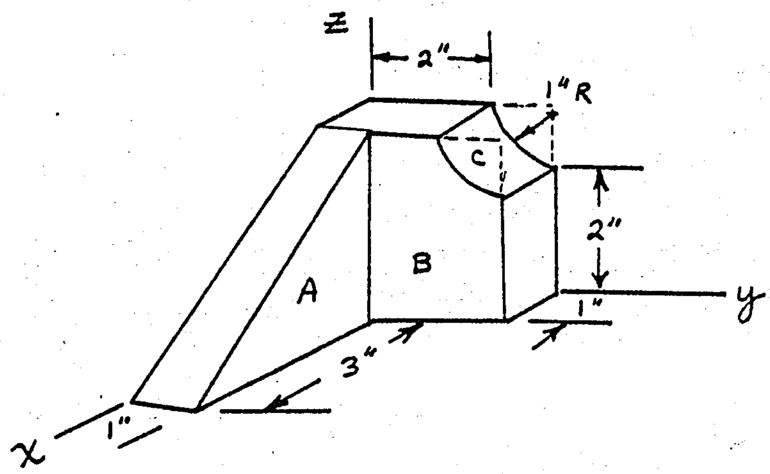
Element	A_i	$\bar{X}_i$	$A_i \bar{X}_i$	$\bar{y}_i$	$A_i \bar{y}_i$
ctangle	108	4.5	486	12"	1296
iangle	27	3"	81	4"	108
mi circle	−25.13	6.45	−162.2	12"	301.6
Σ	109.9		404.8		1102.4

$$\bar{X} = \frac{\Sigma A_i \bar{X}_i}{\Sigma A_i} = \frac{404.8}{109.9} = 3.68''$$

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{1102.4}{109.9} = 10.0''$$

PROBLEM 12
CENTER OF GRAVITY

Determine the center of gravity of the composite part shown below. The density of materials A and B are .1 lb/in^3 and .2 lb/in^3 respectively.



Solution

Complete the table:

Element	lb $w_i = v_i p_i$	x_i	$w_i x_i$	y_i	$w_i y_i$	z_i	$w_i z_i$
Wedge A	.45	2	0.9	.5	0.225	1	0.45
Block B	1.8	.5	0.9	1.5	2.7	1.5	2.70
1/4 Cylinder C	− .16	.5	−0.08	2.576	−0.412	2.576	−0.412
$\sum$	2.09		1.72		2.52		2.7~~5~~4

$$x = \frac{\sum w_i x_i}{\sum w_i} = \frac{1.72}{2.09} = 0.82''$$

$$y = \frac{\sum w_i y_i}{\sum w_i} = \frac{2.52}{2.09} = 1.21$$

$$z = \frac{\sum w_i z_i}{\sum w_i} = \frac{2.75}{2.09} = 1.32$$

Problem # 12

In order to determine the center of gravity of this composed body we need first to divide it into standard geometrical shapes and to determine their weights:

Element	Volume	density ρ	Weight
Wedge A	$3\times3\times\frac{1}{2}\times1 = 4.5\ in^3$	$0.1\ lb/in^3$	0.45 lb
Block B	$3\times3\times1 = 9.0\ in^3$	0.2 "	1.80 lb
$-\frac{1}{4}$ cylinder C	$-\frac{\pi\times1^2\times1}{4} = -0.785\ in^3$	0.2	−0.16 lb

Now we can proceed in the same manner as in the previous problem though we have a 3-D calculation vs. a 2-D one

element	W_i	$\bar{X}_i$	$W_i\bar{X}_i$	$\bar{y}_i$	$W_i\bar{y}_i$	$\bar{z}_i$	$W_i\bar{z}_i$
Wedge A	0.45	2.0"	0.9	0.5"	0.23	1"	0.45
Block B	1.80	0.5"	0.9	1.5"	2.70	1.5"	2.70
$-\frac{1}{4}$ cylinder C	−0.16	0.5"	−0.08	2.576"	−0.41	2.576"	−0.41
Σ	2.09		1.72		2.52		2.74

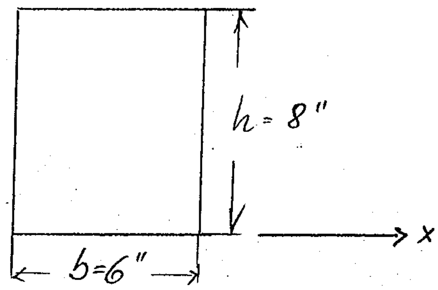
$$\bar{X} = \frac{\Sigma W_i \bar{X}_i}{\Sigma W_i} = \frac{1.72}{2.09} = 0.82''$$

$$\bar{y} = \frac{\Sigma W_i \bar{y}_i}{\Sigma W_i} = \frac{2.52}{2.09} = 1.21''$$

$$\bar{z} = \frac{\Sigma W_i \bar{z}_i}{\Sigma W_i} = \frac{2.74}{2.09} = 1.31''$$

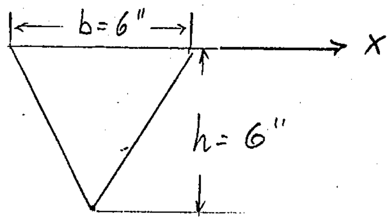
roblem # 13

Rectangle



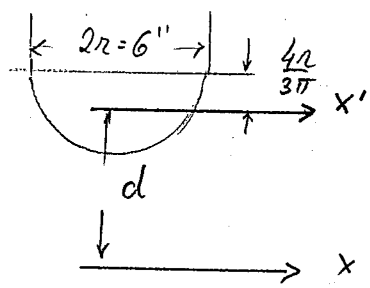
$$I_{xx} = \frac{bh^3}{3} = \frac{6\times 8^3}{3} = 1024 \text{ in}^4$$

Triangle



$$I_{xx} = \frac{bh^3}{12} = \frac{6\cdot 6^3}{12} = 108 \text{ in}^4$$

Semi Circle (missing)



$$I_{x'x'} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4 = 0.1098 r^4$$

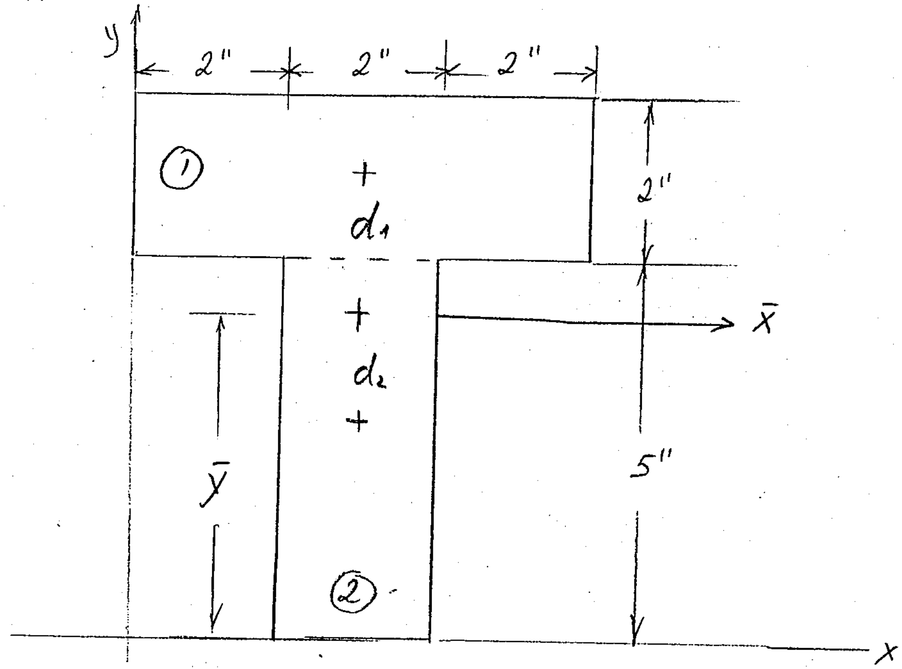
$$I_{xx} = I_{x'x'} + Ad^2 = 0.1098 r^4 + \frac{\pi r^2}{2} \cdot d^2$$ Parallel transfer of axes theorem

$$I_{xx} = 0.1098\times 3^4 + \frac{\pi}{2}\cdot 3^2 \cdot \left(8 - \frac{4\cdot 3}{3\pi}\right)^2 = 8.894 + 639.7 = 648.6 \text{ in}^4$$

$$(I_x)_{composite} = I_{x_\square} + I_{x_\triangledown} - I_{x_\circ} = 1024 + 108 - 648.6 = \underline{483.4} \text{ in}^4$$

Problem # 14

Determine the moment of inertia of the T cross section with respect to the x-centroidal axis



Step 1 Locate $\bar{y}$ with respect to x

element	A_i	$\bar{y}_i$	$A_i y_i$
rectangle 1	12 in²	6"	72
rectangle 2	10 in²	2.5"	25
Σ	22		97

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{97}{22} = 4.41''$$

$$\bar{I}_{xx} = (\bar{I}_{xx})_1 + (\bar{I}_{xx})_2$$

$$(\bar{I}_{xx})_1 = \frac{b_1 h_1^3}{12} + A_1 d_1^2 = \frac{6\times2^3}{12} + (6\cdot2)(6-4.41)^2 = 4 + 30.3 = 34.3 \text{ in}^4$$

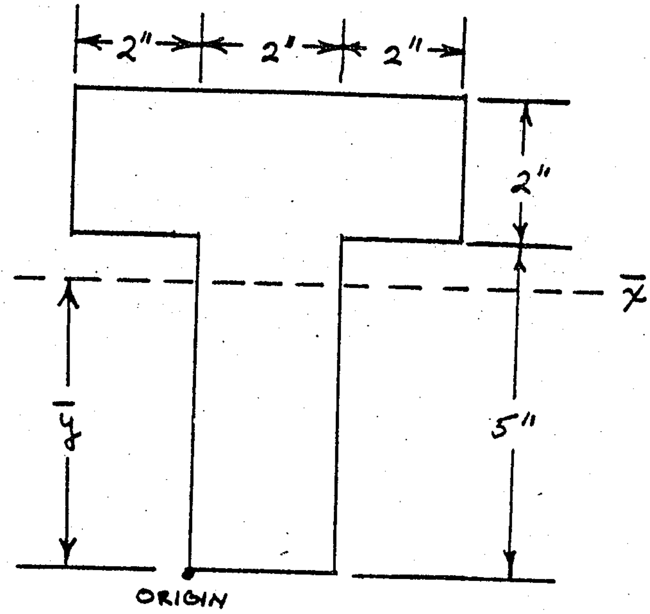
$$(\bar{I}_{xx})_2 = \frac{b_2 h_2^3}{12} + A_2 d_2^2 = \frac{2\times5^3}{12} + (5\cdot2)(4.41-2.5)^2 = 20.8 + 36.48 = 57.3 \text{ in}^4$$

$$(\bar{I}_{xx})_{composite} = 34.3 + 57.3 = \underline{\underline{91.6 \text{ in}^4}}$$

PROBLEM 14

CENTROIDAL MOMENT OF INERTIA

Determine the moment of inertia of the composite area with respect to the x-centroidal axis.



Solution:

Step 1 - Locate the centroid of the composite area.

Step 2 - Repeat the process of locating the moment of inertia of the composite body.

Calculations

Find $\bar{y}$

	area	y	ay
Rectangle 1	12	6	72
Rectangle 2	10	2.5	25
$\sum$	22		97

$$\bar{y} = \frac{\sum ay}{\sum a} = \frac{97}{22} = 4.41\text{" above the origin}$$

$$\bar{I}_{x\,composite} = I_{1x} + I_{2x}$$

$$\bar{I}_{1x} = \frac{bh^3}{12} + A_1 d_1^2$$

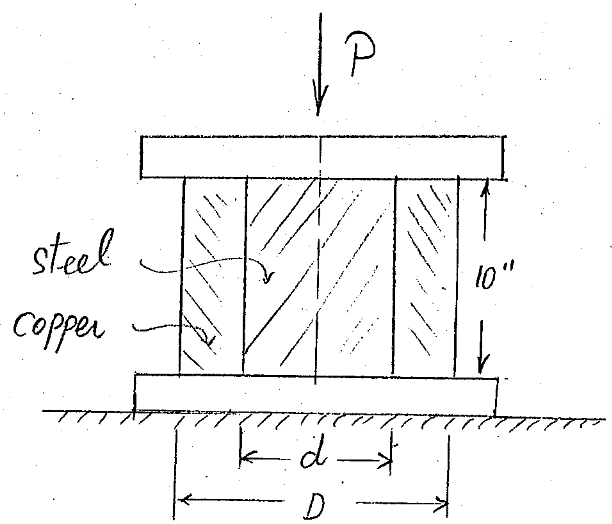
$$= \frac{(6)(2)^3}{12} + (2)(6)(6-4.41)^2$$

$$\bar{I}_{2x} = \frac{bh^3}{12} + A_2 d_2^2$$

$$= \frac{(2)(5)^3}{12} + (2)(5)(4.41 - 2.5)^2$$

Problem # 1 (13)

A steel cylinder surrounded by a copper tube is subjected to compression between two parallel plates as shown: Determine the stress in the copper and the steel



$d = 4"$

$D = 8"$

$P = 100{,}000$ lb

$E_s = 30 \times 10^6$ psi

$E_c = 16 \times 10^6$ psi

Cross Sectional Areas:

Steel: $A_s = \frac{\pi}{4} d^2 = \frac{\pi}{4} \cdot 4^2 = 4\pi \ in^2$

Copper: $A_c = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(64 - 16) = 12\pi \ in^2$

As the deformation of the two materials is equal, thus from Hooke's law we have

$$\Delta = \frac{PL}{EA}$$

and thus: $$\frac{P_s L}{E_s A_s} = \frac{P_c L}{E_c A_c}$$

we can express the force P_c in terms of the force P_s

$$P_c = P_s\left(\frac{A_c E_c}{A_s E_s}\right) = P_s\left(\frac{12\pi \cdot 16\times10^6}{4\pi \cdot 30\times10^6}\right) = 1.6\,P_s$$

3. From statics we know that:

$$P_s + P_c = 100{,}000$$

$$P_s + 1.6\,P_s = 100{,}000$$

$$P_s = 38{,}500 \text{ lb}$$

$$P_c = 61{,}500 \text{ lb}$$

As we know the force and the cross-sectional area of each material we can evaluate the stress

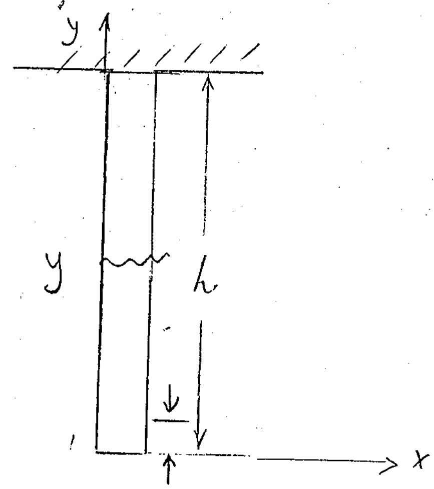
$$\sigma = \frac{P}{A}$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{38{,}500}{4\pi} = 3061 \text{ psi}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{61{,}500}{12\pi} = 1630 \text{ psi}$$

Problem # 2 (25)

A homogenous round bar of diameter D, length L and total weight W is hung vertically from one end. If E is the modulus of elasticity, what is the total elongation of the bar?



1. The force acting on each cross section of the bar equals the weight of the portion of the bar beneath it
Thus:

$$P(y) = W \cdot \frac{y}{L}$$

2. We have learned that when a varying longitudinal force acts on a bar the total elongation is given by

$$\Delta = \int_0^L \frac{P\,dy}{AE}$$

3. Substituting and integrating:

$$\Delta = \int_0^L \left(y\frac{W}{L}\right)\cdot\frac{dy}{EA} = \frac{W}{LEA}\int_0^L y\,dy = \frac{W}{LEA}\cdot\left(\frac{y^2}{2}\right)\Big|_0^L =$$

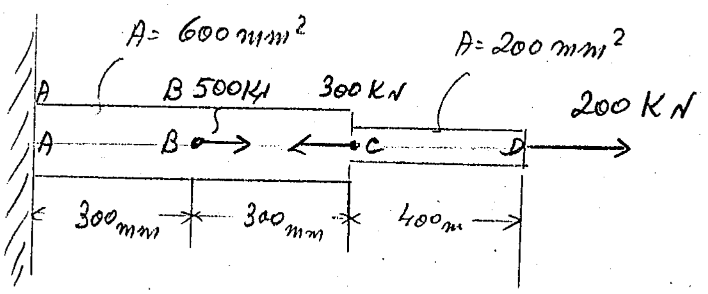
$$\Delta = \frac{W}{LEA}\cdot\frac{L^2}{2} = \frac{WL}{2EA}$$

but $\quad A = \dfrac{\pi D^2}{4}$

thus $\quad \Delta = \dfrac{2WL}{E\pi D^2}$

Problem # 3 . (B&J E 2.01)

Determine the total deformation and the stress level in the steel rod shown in the figure $(E = 200\ GPa)$



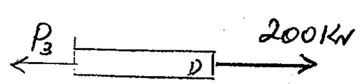
1. We devide the rod into three parts

Part AB $\quad L_1 = 0.3\ m \quad A_1 = 6\times10^{-4}\ m^2 \quad (600\times10^{-6})$

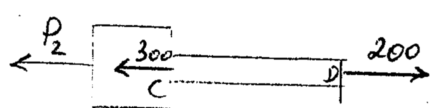
Part BC $\quad L_2 = 0.3\ m \quad A_2 = 6\times10^{-4}\ m^2$

Part CD $\quad L_3 = 0.4\ m \quad A_3 = 2\times10^{-4}\ m^2$

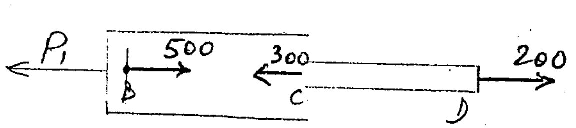
2. Using the method of sections we determine the internal force in each portion of the rod : Let us construct FBD.



$P_3 = 200\ KN$ (T) $= 2\times10^5\ N$



$P_2 = -100\ KN$ (C) $= -1\times10^5\ N$



$P_1 = 400\ KN$ (T) $= +4\times10^5\ N$

3. Carrying the values obtained we use eq.

$$\delta = \sum_{i=1}^{3} \frac{P_i L_i}{A_i E_i} = \frac{1}{E}\left[\frac{P_1 L_1}{A_1} + \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3}\right] =$$

$$= \frac{1}{2\times10^{11}}\left[\frac{400\times10^{3}\cdot 0.3}{6\times10^{-4}} - \frac{100\times10^{3}\cdot 0.3}{6\times10^{-4}} + \frac{200\times10^{3}\, 0.4}{2\times10^{-4}}\right]$$

$$= \frac{1}{2\times10^{11}}\left[2\cdot10^{8} - 0.5\times10^{8} + 4\times10^{8}\right] = \frac{5.5\times10^{8}}{2\times10^{11}} =$$

$$\delta = 2.75\times10^{-3}\,m = 2.75\ mm$$

4. The stress level in each portion is given by

$$\sigma = \frac{P}{A}$$

Portion AB $\quad \sigma_1 = \dfrac{400\times10^{3}}{6\times10^{-4}} = 6.67\times10^{8} = 666.7\ MPa$ (T)

Portion BC $\quad \sigma_2 = \dfrac{-100\times10^{3}}{6\times10^{-4}} = \ominus 166.7\ MPa$ (C)

Portion CD $\quad \sigma_3 = \dfrac{200\times10^{3}}{2\times10^{-4}} = 1000\ MPa$

Problem # 4 (B&J 6.107)

Determine the largest internal pressure which may be applied to a cylindrical tank of 1.8 m outer diameter and 14-mm wall thickness if the ultimate stress of the steel used is 400 MPa and a factor of safety of 6.0 is desired.

The largest stress in a cylindrical pressure vessel is the hoop stress which for a thin-walled case is given by

$$\sigma_1 = \frac{pr}{t}$$

thus

$$\sigma_1 = \sigma_{max} = \frac{\sigma_U}{F.S} = \frac{400}{6}\ MPa = 66.67\ MPa$$

$$t = 14\,mm = 0.014\ m$$

$$r = \frac{1.8}{2} - 0.014 = 0.886\ m$$

$$\sigma_1 = \frac{pr}{t} = \sigma_{max}$$

$$P_{max} = \sigma_{max}\,\frac{t}{r} = \frac{66.67 \cdot 0.014}{0.886} = 1.053\ MPa$$

Problem #5 (B&J 6.101)

A spherical gas container made of steel is 20 ft in external diameter and has a uniform wall thickness of $\frac{7}{16}$". Knowing that the internal pressure is 75 psi determine the maximum normal stress and maximum shearing stress in the container.

The maximum normal stress for a thin spherical pressure vessel is given by

$$\sigma_1 = \frac{pr}{2t}$$

Thus:

$$t = \frac{7}{16}"$$

$$r = \frac{20 \times 12}{2} - \frac{7}{16} = 119.56"$$

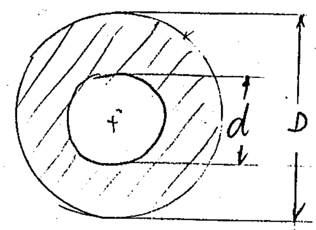
$$\sigma_1 = \frac{75 \times 119.56}{2 \times (7/16)} = \underline{10.25 \text{ ksi}}$$

The maximum shearing stress which is out of the plane of the surface of the vessel

$$\tau_{max} = \frac{\sigma_1}{2} = \underline{\underline{5.12 \text{ ksi}}}$$

Problem # 6

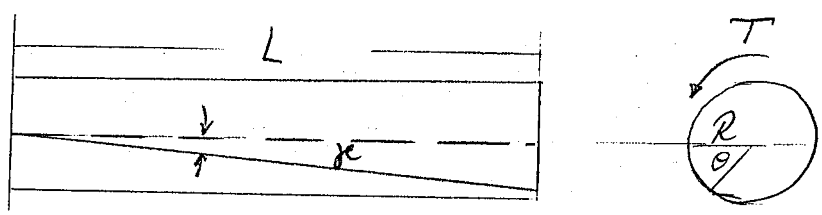
A hollow circular steel shaft is 100" long and has to transmit a torque of $T = 300{,}000$ in-lb. The total angle of twist must not exceed 3 degrees. The maximum shearing stress must not exceed 16,000 psi. Find the inner diameter (d) and the outer diameter (D) of the shaft that meets these conditions. The shear modulus for steel is $G = 12 \times 10^6$ psi



Solution

To solve this problem we need three formulas:

(a)



(1) $$\gamma = \frac{R\theta}{L}$$

where: γ - the shearing strain [radians]

θ - twist angle [radians]

L - shaft's length [in]

R - shaft's outer radius [in]

(2) $$\tau = G\gamma$$

(b)

$$(3) \qquad \tau_{max} = \frac{TR}{J} = \frac{TD}{2J} \quad \text{(elastic torsion formula)}$$

where: τ_{max} - masimum shearing stress [psi]

T - applied torque [lb-in]

J - polar moment of inertia in^4

for a hollow circular shaft

$$J = \frac{1}{2}\pi (R^2 - r^2)$$

r - inner radius

Step 2

Applying first eq. 1:

$$\theta_{max} = 3° = \frac{3}{57.3} = 0.0524 \text{ rad.}$$

$$\gamma = \frac{R\theta}{L} = \frac{D}{2} \frac{0.0524}{100} = 2.62 \times 10^{-4} D$$

substituting this value into eq. (2) yields

$$\tau_{max} = G\gamma^e$$

thus: $16{,}000 \text{ psi} = 12 \times 10^6 \cdot 2.62 \times 10^{-4} D$

$$D = 5.1''$$

This outer diameter that meets the allowable twist angle and shear stress may not necessarily carry the required torque.

Step 3 if this were a solid shaft it could transmit the following torque

$$T = \frac{\tau_{max} \cdot J}{R} =$$

for solid shaft $J = \frac{\pi R^4}{2}$

$$T_{max} = \tau_{max} \cdot \frac{\pi R^3}{2} = \frac{16000 \cdot \pi}{2} \left(\frac{5.1}{2}\right)^3$$

$$T_{max} = 416{,}735 \text{ lb-in}$$

As this is more than required the shaft may be hollowed out for weight reduction

Step 4

substituting in eq. (3) for a hollow shaft:

$$T = \frac{2\tau_{max}}{D} \cdot \pi\left(\frac{D^4 - d^4}{32}\right)$$

$$300{,}000 = \frac{2 \times 16{,}000\pi}{5.1 \times 32}\left(5.1^4 - d^4\right)$$

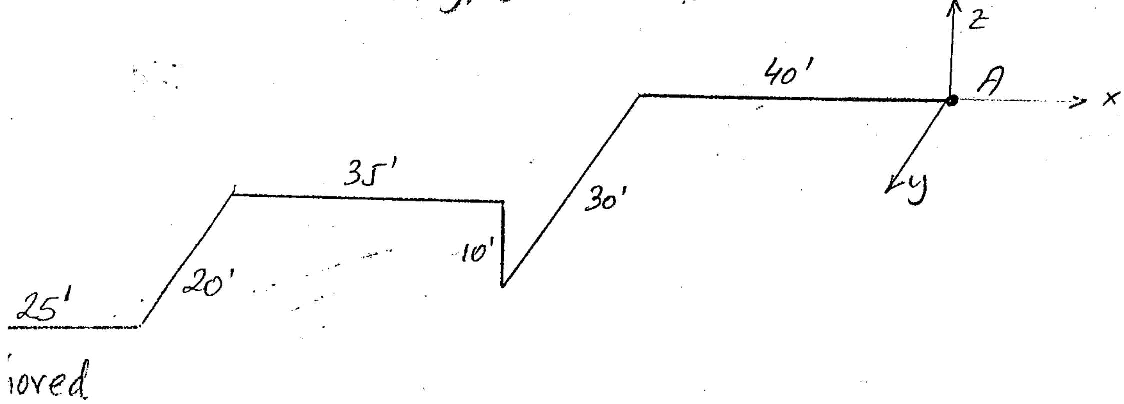
$$\underline{\underline{d = 3.70''}}$$

Problem # 7

A steam line has the dimensions as shown below at $70°F$. What is the total distance the end A will move when steam at $450°F$ flows in the line?

the coef. of thermal expansion is $\alpha = 6.5\times10^{-6}\ \frac{ft}{ft\,°F}$

Assume all the distances are measured along the coordinate (x, y, z) axes.



step 1 The elongation due to thermal expansion is given by

$$\Delta = L\alpha \cdot \Delta T$$

Thus the three movements are:

$$\Delta_x = L_x \alpha\, \Delta T = (25' + 35' + 40')(450° - 70°)\times 6.5\times10^{-6}$$

$$= 100 \times 380 \times 6.5\times10^{-6} = 0.247'$$

$$\Delta_y = L_y \alpha\, \Delta T = (20' + 30')\cdot 380\times 6.5\times10^{-6} = 0.124'$$

$$\Delta_z = L_z\, \alpha\, \Delta T = 10\times 380\times 6.5\times10^{-6} = 0.025'$$

The resulting movement of end A

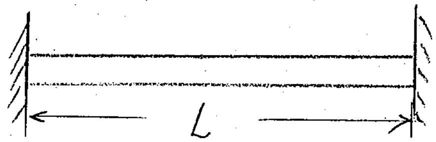
$$\Delta_A = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} = \sqrt{0.247^2 + 0.124^2 + 0.025^2}$$

$$\underline{\Delta_A = 0.278'}$$

assuming perfect conditions frictionless supports

Problem # 8 (40)

A steel bar is mounted between unyielding supports. If the temperature of the bar increases by ΔT, find the stress that develops in the bar in terms of the coefficient of thermal expansion α and the modulus of elasticity E



1. If the bar was free to expand, due to the temperature raise it would have expanded

$$(1) \quad \Delta L = L \alpha \Delta T$$

2. As the unyielding surface prevented it from expanding, a force was created in the support such that the negative deformation due to this force is equal the one due to the thermal expansion thus yielding a nil change in the length of the bar:

$$(2) \quad |\Delta| = \frac{P}{A}\frac{L}{E} = \sigma \cdot \frac{L}{E}$$

Substituting in (1)

$$L \alpha \Delta T = \sigma \frac{L}{E}$$

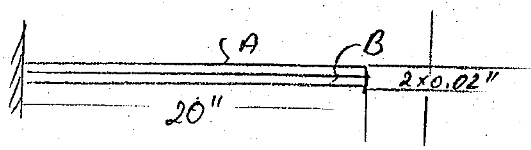
$$\underline{\sigma = E \alpha \Delta T}$$

note: 1. The stress caused by thermal expansion is independent of the length L of the body

2. For positive ΔT σ = Compression for negative ΔT σ = Tension

Problem # 9

A 20" long bimetallic strip is made of metals A & B each 0.02" thick. The strip is straight at 68°F, fixed at one end and free to move at the other. Compute the deflection of the free end at a temperature of 86°F if the curvature of the strip is in the form of a circle.



$$\alpha_A = 24\times10^{-6} \text{ in/in/°C}$$

$$\alpha_B = 14\times10^{-6} \text{ in/in/°C}$$

Step 1: Converting temperatures to Centigrades

$$°C = \frac{5}{9}(°F - 32)$$

$$68°F \Rightarrow \quad \frac{5}{9}(68-32) = 20°C$$

$$86°F \Rightarrow \quad \frac{5}{9}(86-32) = 30°C$$

Step 2 the length of the strips once temp is raised

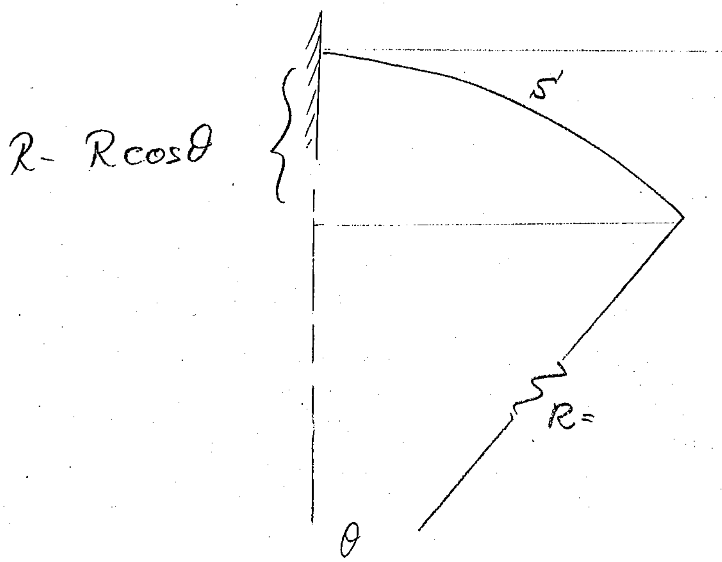
$$L_A = L(1+\alpha\Delta T) = 20''(1+24\times10^{-6}\times10) = 20''(1+0.24\times10^{-3})$$

$$L_B = L(1+\alpha\Delta T) = 20''(1+14\times10^{-6}\times10) = 20''(1+0.14\times10^{-3})$$

the difference in length is

$$\Delta L = L_A - L_B = 20''(0.24-0.4)\times10^{-3} = 0.002''$$

Step 3



the two strips are located 0.01" away on both sides of a circle of radius R. For a circular segment

$$S' = R\theta \qquad \theta \text{ in rad.}$$

thus:

$$L_A = (R + 0.01)\,\theta$$

$$L_B = (R - 0.01)\,\theta$$

subtracting the two eqs:

$$\Delta L = L_A - L_B = 0.02\,\theta$$

$$\theta = \frac{0.002}{0.02} = 0.1 \text{ rad} = 5.73^\circ$$

$$R \cong \frac{20''}{0.1} = 200''$$

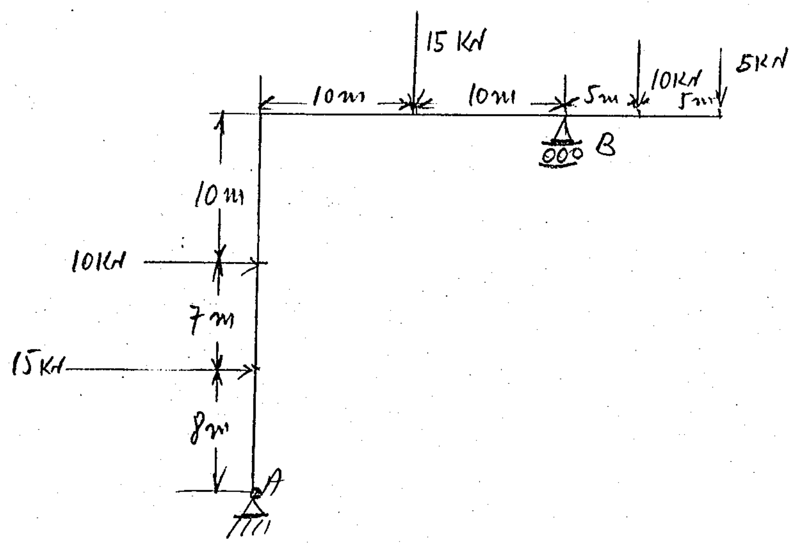
Therefore the deflection is

$$\delta = R - R\cos\theta = 200\,(1 - 0.995) = 1''$$

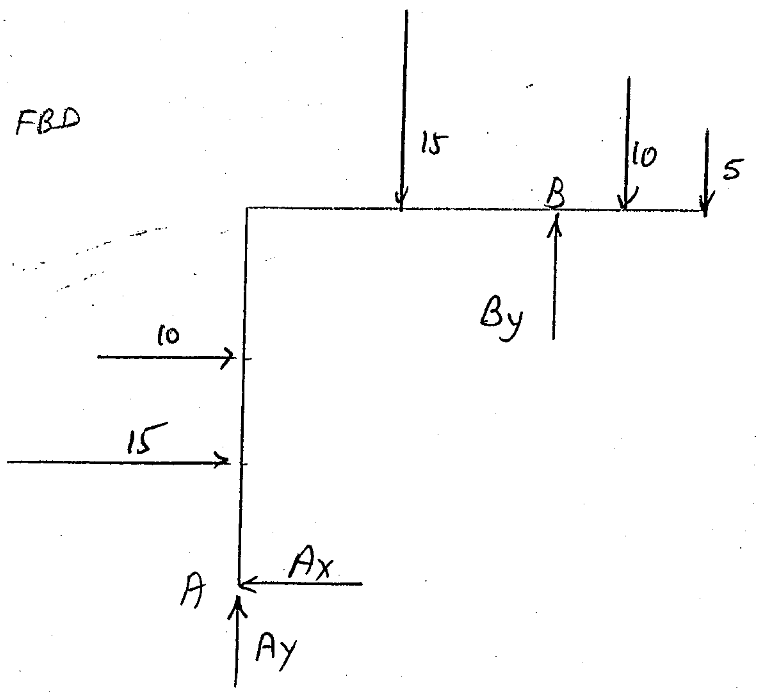
$$\underline{\underline{\delta = 1''}}$$

Problem # 10

Determine the reactions at the supports for the given frame



Step 1 FBD



Step 2

Coplanar EQ analysis

$\Sigma M_A = 0 \qquad -15\times10 - 10\times25 - 5\times30 + B_y\times20 - 10\times15 - 15\times8$

$$B_y = \frac{820}{20} = 41.0\ KN$$

$\Sigma F_y = 0 \qquad A_y + 41 - 15 - 10 - 5 = 0$

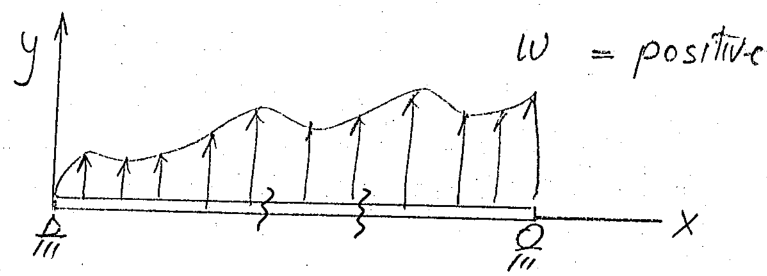
$$\underline{\underline{A_y = -11\ KN}}$$

$\Sigma F_x = 0 \qquad 10 + 15 - A_x = 0$

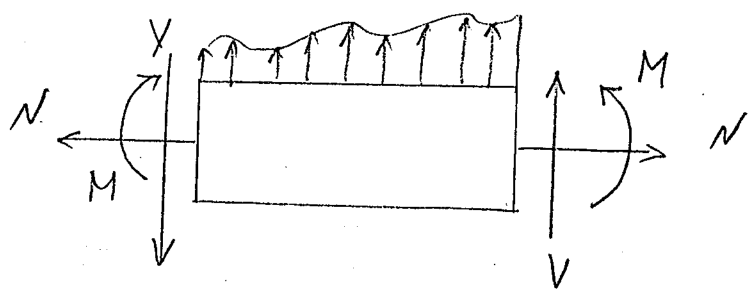
$$\underline{\underline{A_x = 25\ KN}}$$

SIGN CONVENTION FOR BENDING OF BEAMS

COORDINATE SYSTEM



NVM



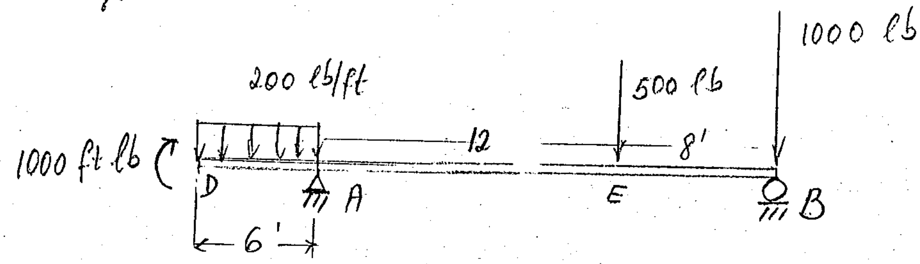
deflection

↓ negative

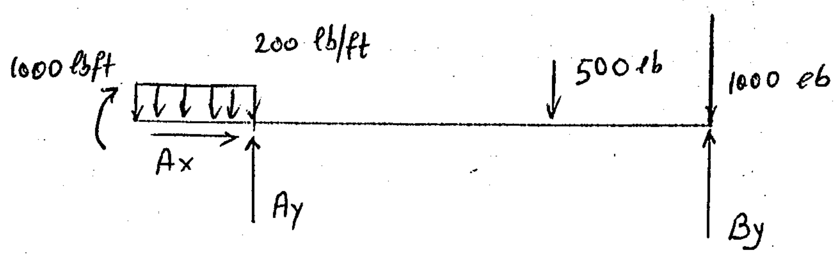
↑ positive

Prob-lem # 11 (49)

Calculate the reactions A and B and draw the shear and moment diagrams



step Calculate the reactions : FBD + Coplanar EQ analysis



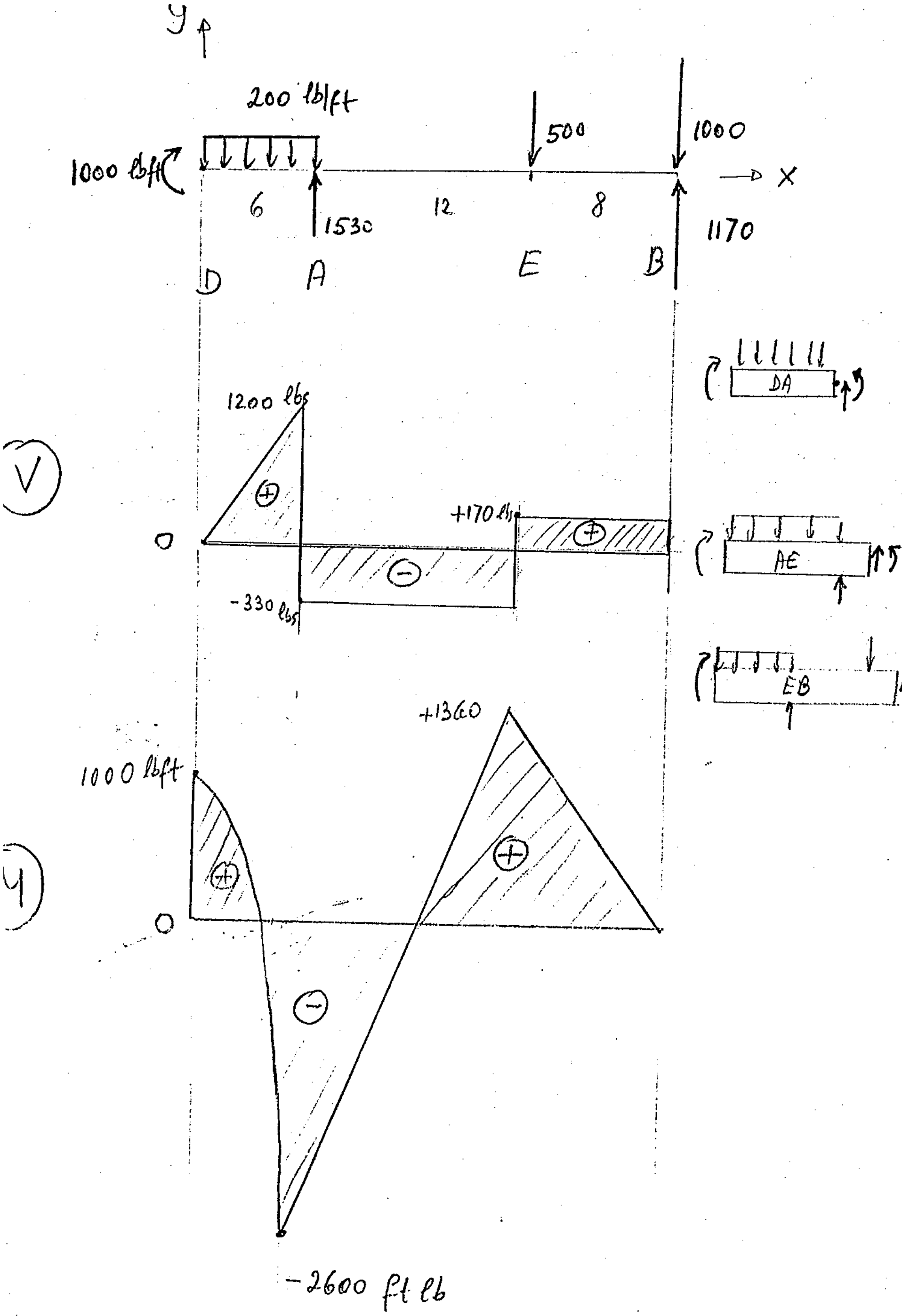
$\Sigma F_x = 0$ $\quad$ $\underline{\underline{A_x = 0}}$

$\Sigma M_B = 0$ $\quad$ $+500 \times 8 - A_y \times 20 + (200 \times 6) \times 23 - 1000 = 0$

$$\underline{\underline{A_y = 1530 \text{ lbs}}}$$

$\Sigma F_y = 0$ $\quad$ $B_y + 1530 - 200 \times 6 - 500 - 1000 = 0$

$$\underline{\underline{B_y = 1170 \text{ lbs}}}$$



$$\underline{V_{DA}} = 200x$$

$x = 0 \qquad V = 0$

$x = 6' \qquad V = +1200$

$$V_{AE} = 1200 - 1530 = -330$$

$$\underline{V_{EB}} = 1200 - 1530 + 500 = +170$$

$$V_B = 0$$

$$M_{DA} = 1000 - (200 \cdot x)\frac{x}{2}$$

$$\underline{M_{DA}} = 1000 - 100x^2$$

$$M_{AE} = 1000 - 1200(x-3) + 1530(x-6)$$

$$= 1000 - 1200x + 3600 + 1530x - 9180$$

$$\underline{M_{AE}} = 330x - 4580$$

$x = 6' \qquad M_{AE} = -2600$

$x = 18' \qquad M_{AE} = +1360$

$$\underline{M_{EB}} = M_{AE} - 500(x - 18)$$

$$= -170x + 4420$$

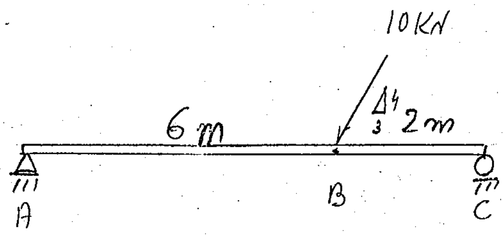
$x = 18'' \qquad M_{EB} = 1360$

$x = 26'' \qquad M_{EB} = 0$ ✓✓

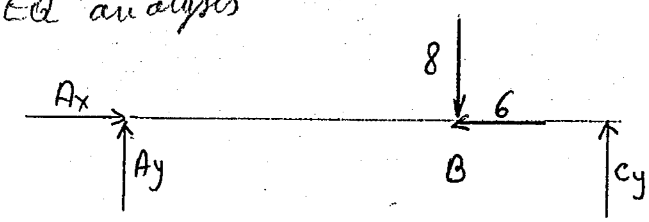
CROSS - HATCHING

Problem #12 (d)

Calculate the reactions A and B and draw diagrams for the axial load N, the shear force V and the bending moment M



1. FBD + Coplanar EQ analysis



$\Sigma M_A = 0$ $\qquad - 6 \times 8 + C_y \times 8 = 0$

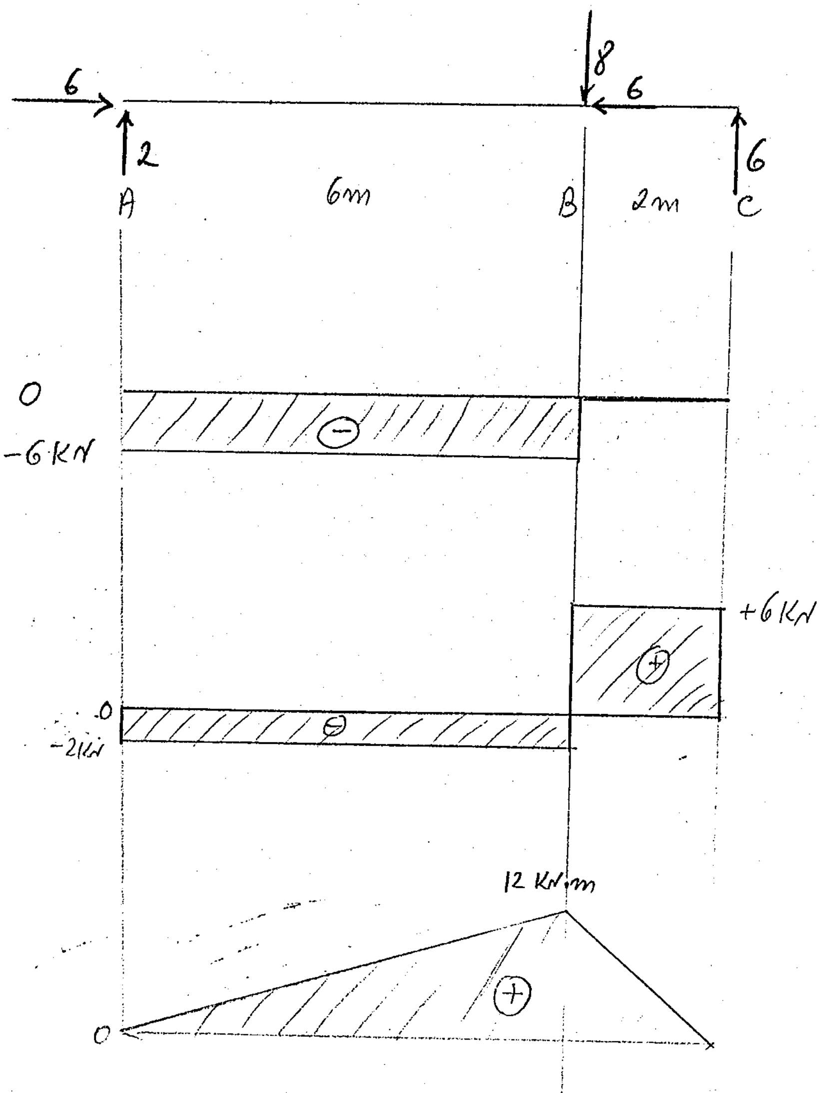
$C_y = 6\ KN$

$\Sigma F_x = 0$ $\qquad A_x - 6 = 0$

$A_x = 6\ KN$

$\Sigma F_y = 0$ $\qquad A_y + 6 - 8 = 0$

$A_y = 2$



$N_{AB} = -6\ KN$

$N_{BC} = 0$

$V_{AB} = -2\ KN$

$V_{BC} = +6\ KN$

$\underline{M_{AB}} = 2 \cdot x$

$M_{BC} = 2x - 8(x-6)$

$= 2x - 8x + 48$

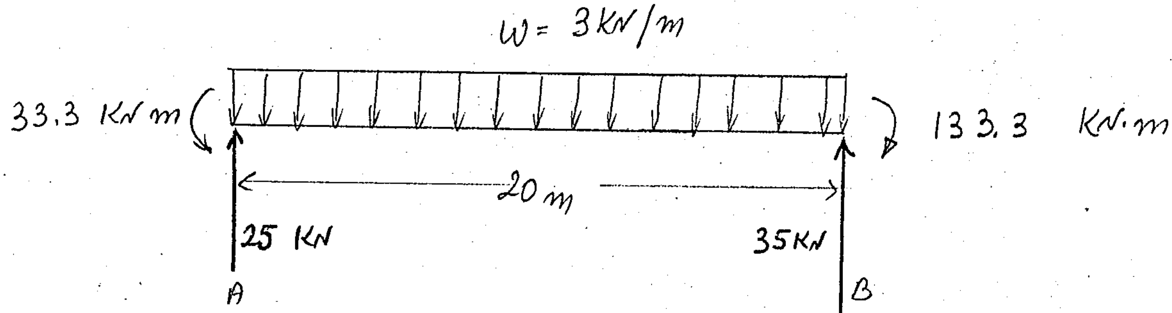
$\underline{M_{BC}} = 48 - 6x$

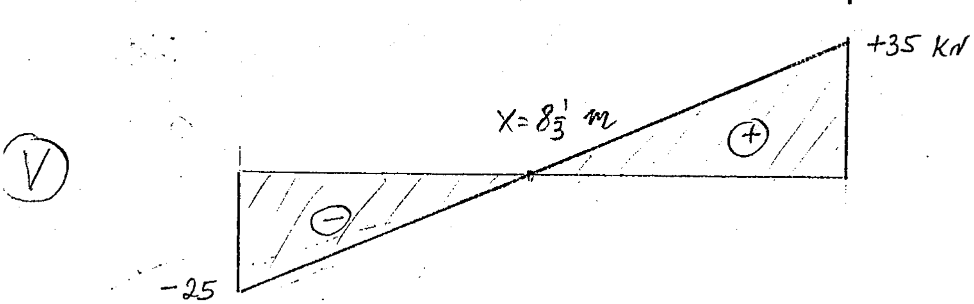
$x = 6 \qquad M_{BC} = 12$

$x = 8 \qquad M_{BC} = 0$

Problem #13 (18)

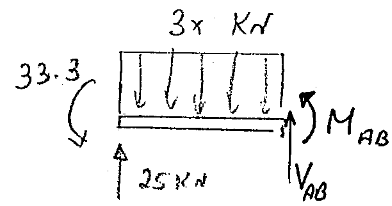
The free body diagram of a certain beam is shown. Draw the shear and moment diagrams and locate the point of maximum moment.

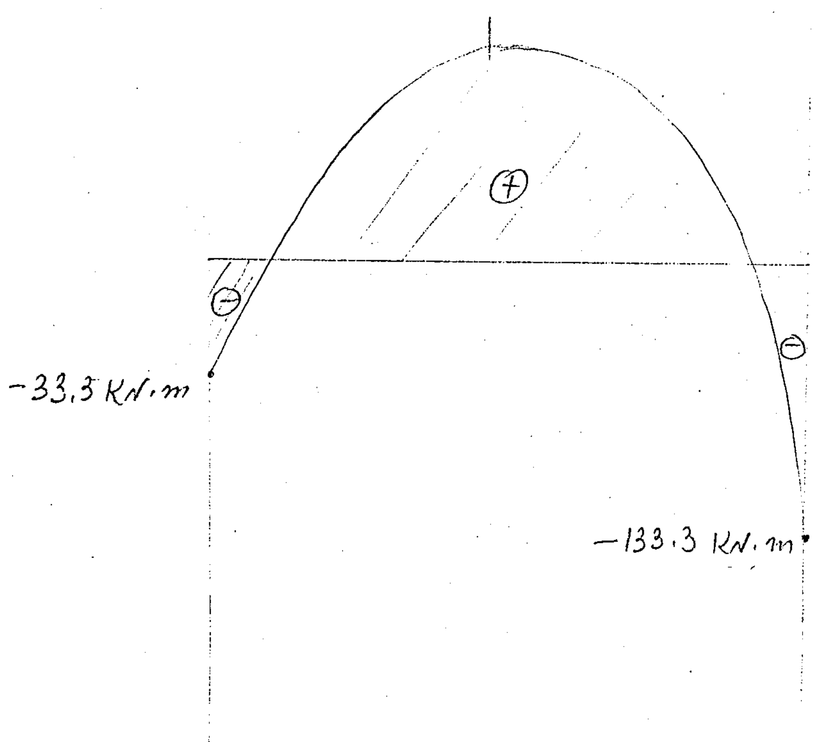




$$\underline{V_{AB}} = -25 + \frac{60}{20}x = -25 + 3x$$

$$V = 0 \qquad x = \frac{25}{3} = 8\tfrac{1}{3} \text{ m}$$





$$M_{AB} = -33.3 - 3x \cdot \frac{x}{2} + 25x$$

$$\underline{M_{AB}} = -\frac{3}{2}x^2 + 25x - 33.3$$

$$x = 0 \qquad M_{AB} = -33.3 \text{ KN.m}$$

$$x = 20 \qquad M_{AB} = -600 + 500 - 33.3 = -133.3 \text{ K}$$

$$\frac{dM_{AB}}{dx} = 0 \qquad -3x + 25 = 0$$

$$x = \frac{25}{3} = 8\tfrac{1}{3} \text{ m}$$

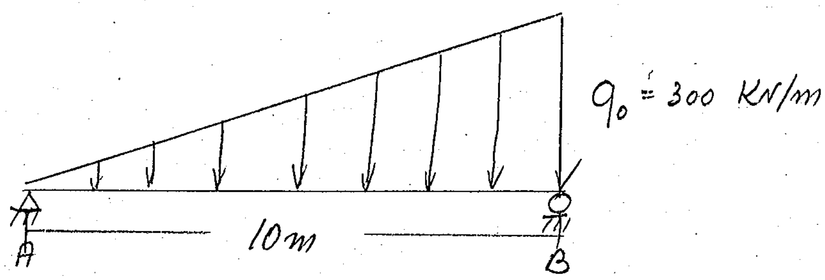
$$M_{max} = -\frac{3}{2}\left(\frac{25}{3}\right)^2 + \frac{25 \times 25}{3} - 33.3 =$$

$$= -\frac{1}{6}25^2 + \frac{1}{3}25^2 - 33.3 = \frac{625}{6} - 33.3 =$$

$$M_{max} = 70.9 \text{ KN.m}$$

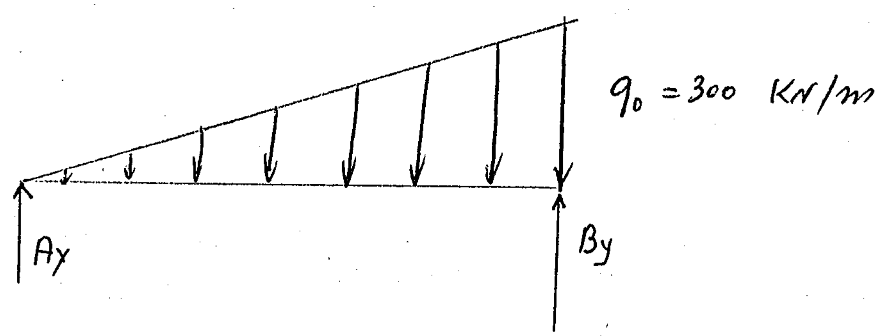
Problem # 14 (Sol)

Determine the reactions at A and B, draw shear and moment diagrams, and locate and evaluate the maximum bending moment



Step 1

FBD + Coplanar EQ analysis



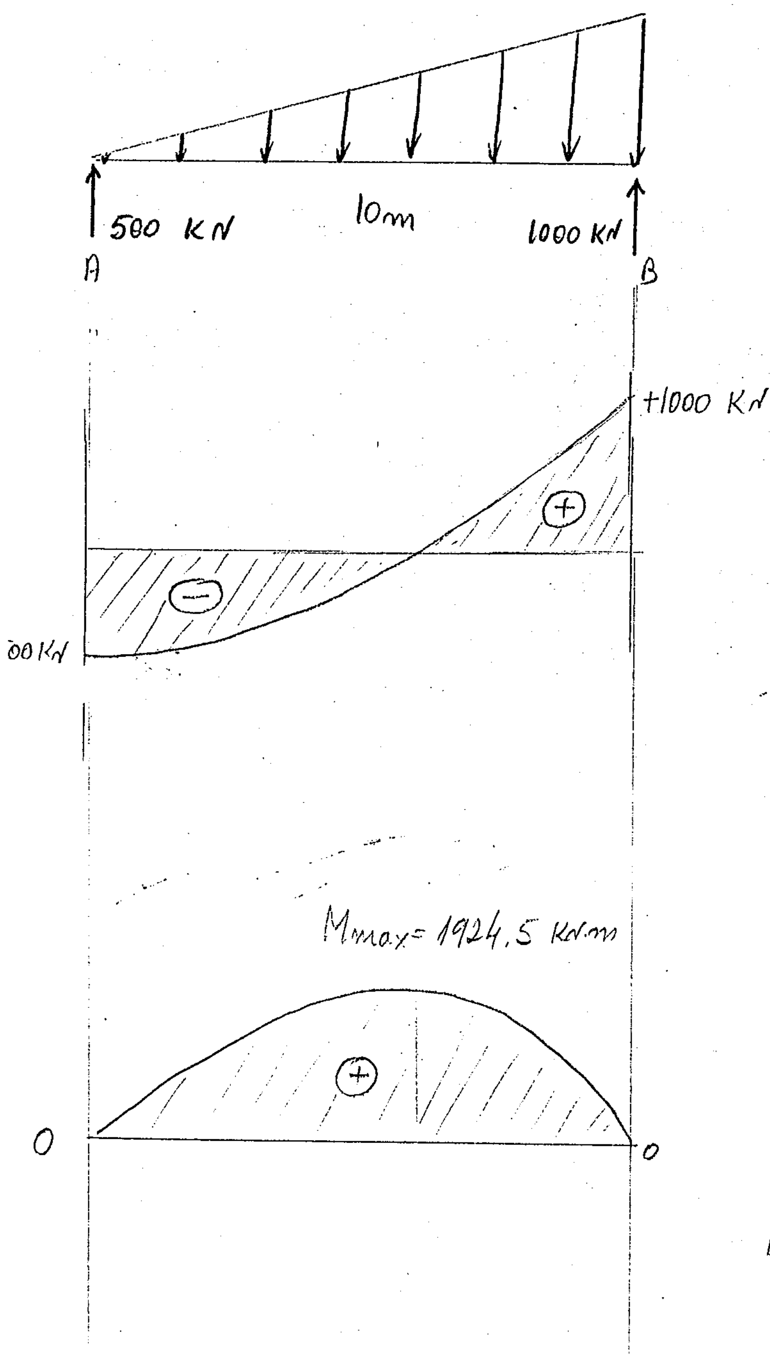
$\Sigma M_B = 0$ $\qquad -Ay \cdot 10 + \left(\frac{300 \times 10}{2}\right) \times \frac{10}{3}$

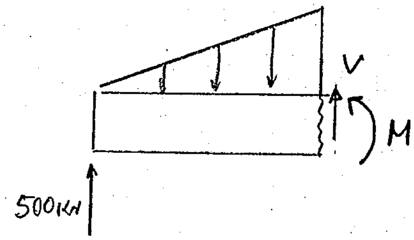
$\underline{Ay = 500 \text{ KN}}$

$\Sigma F_y = 0$ $\qquad Ay + By - \frac{300 \times 10}{2} = 0$

$\underline{By = 1000 \text{ KN}}$

step 2





$$q(x) = q_0 \cdot \frac{x}{L}$$

$$V_{AB} = -500 + \frac{q(x) \cdot x}{2} = -500 + q_0 \frac{x^2}{2L} = -500 + 15x^2$$

$$V = 0 \qquad x^2 = \frac{500}{15} \qquad x = 5.77\,m$$

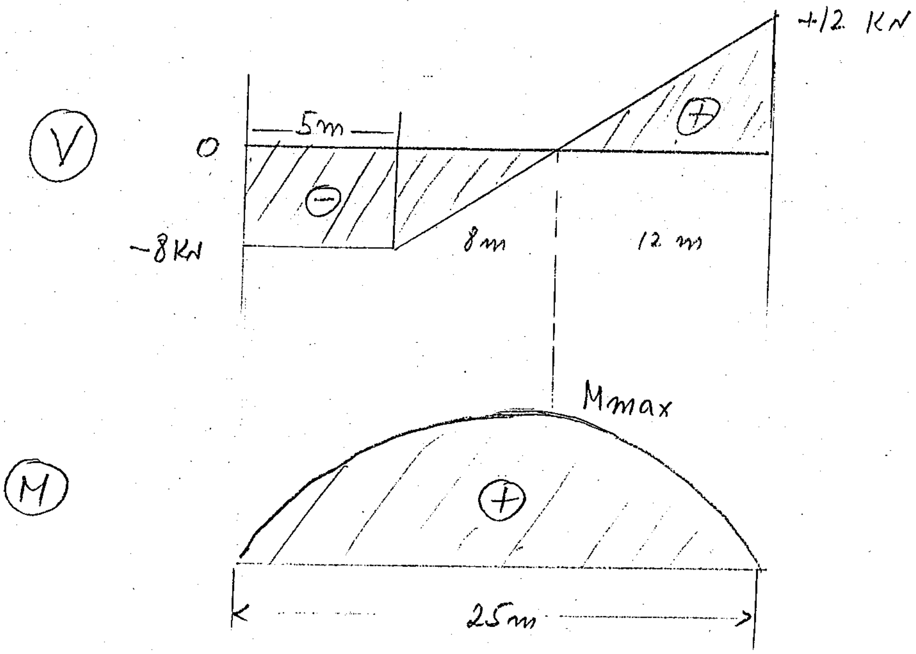
$$M_{AB} = 500 \cdot x - \frac{q(x) \cdot x}{2} \cdot \frac{x}{3} = 500x - q_0 \frac{x^3}{6L}$$

$$M_{AB} = 500x - 5x^3$$

$$M(x = 5.77) = 1924.5 \quad KN\,m$$

Problem #15 (39)

What is the maximum moment of a loaded beam given ONLY the following diagrams:



The solution is based on two facts:

a. The maximum bending moment occurs where the shear passes through zero. M_{max} occurs when $V=0$

b. The change in moment between two sections of a beam equals the negative value of the area of the shear diagram between those sections, taking the shear sign into account

$$M_B = M_A - \int_A^B V(x)\,dx$$

Solution

1. We first determine the location of zero shear from the right end of the beam:

$$\frac{12}{20}(25m - 5m) = 12\ m$$

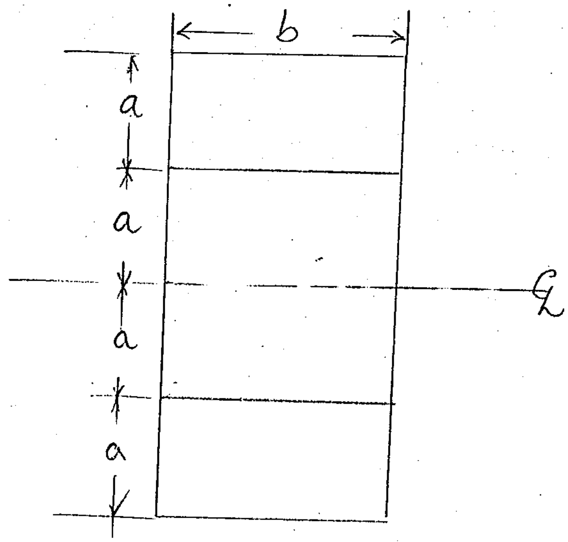
the moment at $x = 13m$

$$M_{x=13} = M_{x=0} - (\text{Area of V diagram } x=0-13)$$

$$= 0 - \left(-8 \times 5 - \frac{8 \times 8}{2}\right) = 72 \text{ KN}\cdot\text{m}$$

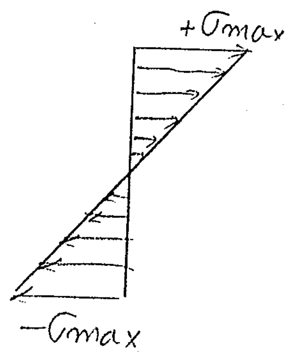
Problem # 16 (41)

A beam of rectangular cross section is subjected to pure bending as shown. How many times the resisting moment of area A (the middle two quarters of the beam) would area B (the outer two quarters) have.



Solution

The stresses in the beam vary linearly from zero at the center of the cross section to a maximum at the extreme fibers



according to the flexural formula

$$\sigma(y) = \frac{My}{I} =$$

where: I is the moment of inertia of the cross-section

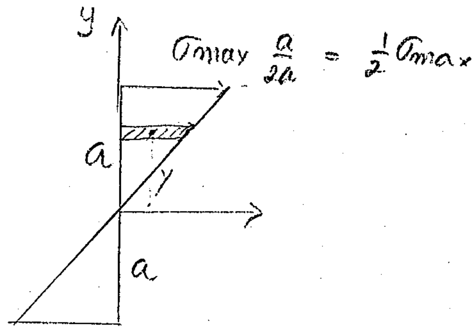
and:

$$\sigma_{max} = \frac{Mc}{I}$$

where: c is the distance of the outer fiber from the neutral axis. in our case $c = 2a$

step 2

We can evaluate the resisting moment of the central portion



$$\sigma(y) = \sigma_{max}\,\frac{y}{2a}$$

$$M_A = \int_{-a}^{a} b\,\sigma(y)\cdot y\,dy = \int_{-a}^{a}\left(\sigma_{max}\cdot\frac{y}{2a}\right) b\,y\,dy = \frac{\sigma_{max} \times b}{2a}\int_{-a}^{a} y^2\,dy = \frac{\sigma_{max}}{2a}\cdot\frac{2a^3}{3} = \frac{\sigma_{max}\,a^2 b}{3}$$

For part B

$$M_B = 2\int_{a}^{2a}\left(\sigma_{max}\,\frac{y}{2a}\right) b\,y\,dy = \frac{\sigma_{max} \times b}{a}\int_{a}^{2a} y^2\,dy = \frac{\sigma_{max}}{a}\cdot\left(\frac{y^3}{3}\right)_{a}^{2a} = \frac{7}{3} b\,\sigma_{max}\,a^2$$

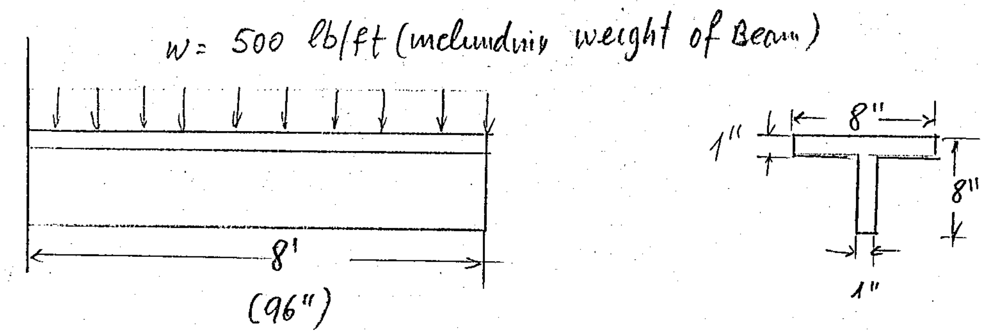
thus the ratio of the resisting moments is

$$\frac{M_B}{M_A} = \frac{7}{1} = 7.$$

Thus we see the the parts of the beam that are farther from the (center)- neutral axis carry more load. I-beams and wide flange beams are two types of beams taking advantage of this fact and save weight.

Problem # 17 (51)

Determine the maximum tensile and compressive bending stresses in the cantilever T-beam shown below



Step 1

The maximum stress is given by

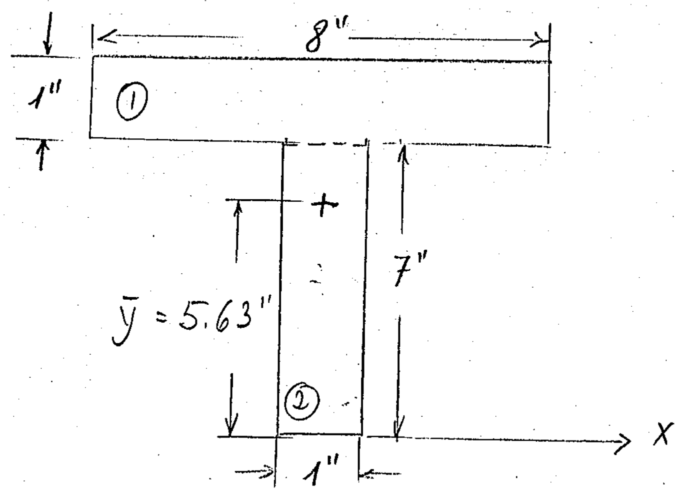
$$\sigma_{max} = \frac{Mc}{I}$$

The maximum stress will occur at the cross-section where the moment M is maximum. For a cantilever beam it is at the face of the wall

$$M_{max} = wL \cdot \frac{L}{2} = \frac{500 \cdot 8^2}{2} = 16{,}000 \text{ lb-ft}$$

Step 2

As the T section is not symmetrical with respect to the X axis we need to locate the neutral axis that passes through the centroid.



Composite Area

to locate the centroid with respect to x we devide the T into two rectangles:

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{(8)(1)(7.5) + (7)(1)(3.5)}{(8)(1) + (7)(1)} = 5.63"$$

Step 3

To use the flexural equation we need to evaluate the moment of inertia of the T cross section using the parallel shift of coordinates:

$$I_{x'} = I_{\bar{x}} + Ad^2$$

$$I_{\bar{x}} = \frac{bh^3}{12}$$

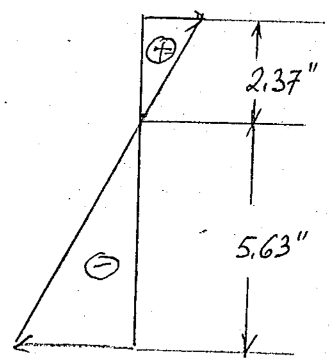
thus:

$$I_x = \frac{8\times1^3}{12} + 8(7.5-5.63)^2 + \frac{1\times7^3}{12} + 7(5.63-3.5)^2 =$$

$$= 0.67 + 28.0 + 28.58 + 31.75 = 89.0 \text{ in}^4$$

Step 3

The stress distribution along the cross section of the beam is:



Maximum tensile stress

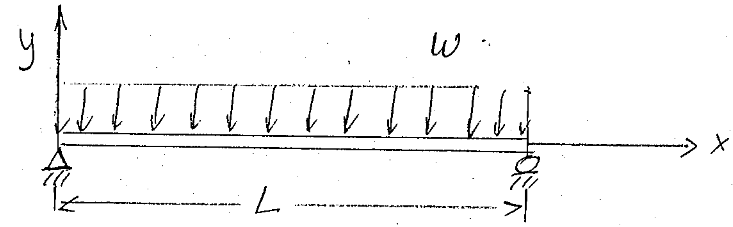
$$\sigma_T = \frac{Mc_1}{I} = \frac{(16{,}000\times12)\,2.37''}{89} = +5112 \text{ psi} = \underline{\underline{5.1}} \text{ ksi}$$

Maximum Compressive stress

$$\sigma_c = -\frac{Mc_2}{I} = -\frac{(16{,}000\times12)\,5.63''}{89} = -12{,}146 \text{ psi} = \underline{\underline{-12.1}} \text{ ksi}$$

Problem # 18 (12)

Given the beam below find the equation the describes the deflection of the beam



Solution

The differential equation for the deflection y is:

$$EI \frac{d^2y}{dx^2} = M \qquad (1)$$

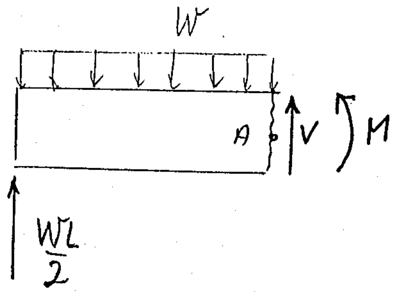
where : E - Young's modulus
I - moment of inertia
M = the bending moment

step 1 solve for the reactions

$$R_A = R_B = \frac{WL}{2} \qquad (2)$$

step 2 determine the bending moment in the beam as a function of (x)

draw a FBD of a section of the beam:



$$M + wx \cdot \frac{x}{2} - \frac{wL}{2} \cdot x = 0$$

$$M(x) = \frac{wL}{2} x - \frac{wx^2}{2} \qquad (3)$$

Step 3

substituting the moment into eq. of deflection:

$$EI \frac{d^2y}{dx^2} = \frac{wL}{2} x - \frac{wx^2}{2} \qquad (4)$$

integrating once

$$EI \frac{dy}{dx} = \frac{wLx^2}{4} - \frac{wx^3}{6} + C_1 \qquad (5)$$

$\frac{dy}{dx}$ is the slope of the deflection curve. Since our beam is symmetric y_{max} will occur at mid-span thus

$$\left(\frac{dy}{dx}\right)_{x=\frac{L}{2}} = y'(x=\tfrac{L}{2}) = 0 \qquad (6)$$

substituting this value into eq. (5) yields

$$\frac{wL}{4}\left(\frac{L}{2}\right)^2 - \frac{w}{6}\left(\frac{L}{2}\right)^3 + C_1 = 0$$

$$C_1 = -\frac{wL^3}{24} \qquad (7)$$

thus

$$EI\frac{dy}{dx} = \frac{wLx^2}{4} - \frac{wx^3}{6} - \frac{wL^3}{24} \qquad (8)$$

integrating exp. (8)

$$EIy = \frac{wLx^3}{12} - \frac{wx^4}{24} - \frac{wL^3x}{24} + C_2 \qquad (9)$$

as at $x = 0 \Rightarrow y = 0$ $\quad y_{(x=0)} = 0$

$\therefore \quad C_2 = 0$

Rearanging eq. (9) yields the desired expression

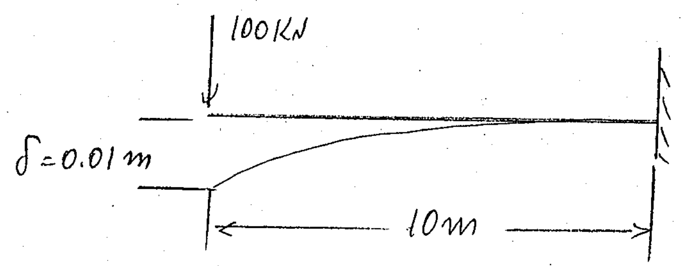
$$y = \frac{w}{24EI}\left[2Lx^3 - x^4 - L^3x\right]$$

$$= -\frac{w}{24EI}\left[L^3x - 2Lx^3 + x^4\right]$$

$$y_{max} = y_{(x=\frac{L}{2})} = -\frac{5}{384}\frac{wL^4}{EI}$$

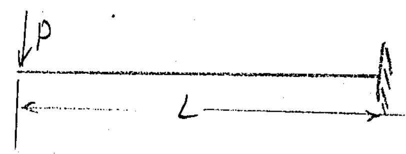
Problem # 19 (417)

A simple cantilever beam is 10 m long and has a concentrated load of 100 KN at its free end. The deflection at the free end is 1 cm = 0.01 m. If the same beam is extended to 20 m and the load is reduced to 50 KN what will be the deflection at the free end (neglect the weight of the beam)



Step 1

Let's solve the deflection for the general problem and an expression for the end deflection



$M(x) = -Px$ P, M $M + Px = 0$ .1

$EI\,y'' = -Px$ .2

$EI\,y' = -\frac{Px^2}{2} + C_1$ .3

at $y = L$ $y' = 0$ thus:

$$-\frac{PL^2}{2} + C_1 = 0$$

$$C_1 = +\frac{PL^2}{2}$$ .4

subs. (4) into (3)

$$EIy' = -\frac{Px^2}{2} + \frac{PL^2}{2} \qquad .5$$

integrating

$$EIy = -\frac{Px^3}{6} + \frac{PL^2x}{2} + C_2 \qquad .6$$

at $x = L$ $y' = 0$ built-in end

$$-\frac{PL^3}{6} + \frac{PL^3}{2} + C_2 = 0$$

$$C_2 = -\frac{PL^3}{3} \qquad .7$$

Thus:

$$y = -\frac{P}{6EI}\left[2L^3 - 3L^3x + x^3\right] \qquad .8$$

Hence $y(x=0) = \delta = -\frac{PL^3}{3EI}$

$$\frac{\delta_1}{\delta_2} = \frac{P_1 L_1^3}{P_2 L_2^3} = \frac{100 \times 10^3}{50 \times 20^3} = \frac{10^5}{4 \times 10^5} = \frac{1}{4}$$

$$\delta_2 = 0.4 \text{ m}.$$