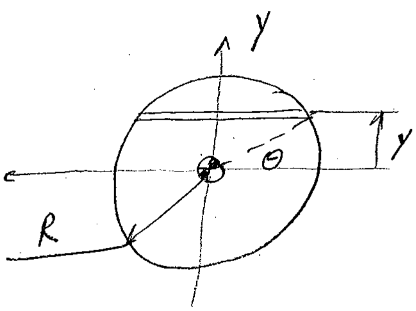
EXAMPLE OF CALCULATION MOMENT OF INERTIA OF A RECTANGLE, SEE TEXTBOOK, APPENDIX A, EXAMPLE A.04

EXAMPLE:

MOMENT OF INERTIA OF A CIRCLE:



$$\left.\begin{aligned} dI_z &= y^2\,dA \\ dA &= 2\sqrt{R^2-y^2}\,dy \\ y &= R\sin\theta \\ dy &= R\cos\theta\,d\theta \end{aligned}\right\} \quad dI_z = R^2\sin^2\theta\cdot 2\sqrt{R^2-R^2\sin^2\theta}\cdot R\cos\theta\,d\theta$$

$$= R^4\cdot 2\cdot\sin^2\theta\cdot\cos^2\theta\cdot d\theta$$

skip

$$I_z = \frac{1}{2}R^4\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^2 2\theta\,d\theta$$

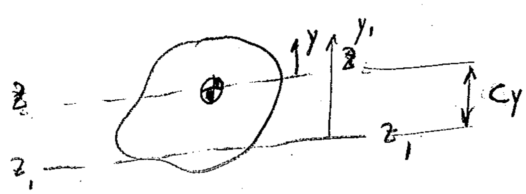
$$= \frac{1}{2}R^4\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1}{2}(1-\cos 4\theta)\,d\theta$$

$$I_z = \frac{1}{4}R^4\left[\theta-\frac{1}{4}\cancelto{0}{\sin 4\theta}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{\pi}{4}R^4$$

$$\boxed{I_z = \frac{\pi}{64}D^4}$$

HALF THE POLAR MOMENT OF INERTIA OF A CIRCLE

PARALLEL-AXIS THEOREM FOR SECOND MOMENT OF AREA



WE KNOW THE MOMENT OF INERTIA I_z (AROUND THE CENTROID OF THE CROSS SECTION)

WE WANT TO CALCULATE THE MOMENT OF INERTIA $I_{z_1 z_1}$

$$I_z = \int y^2 \, dA$$

$$I_{z_{11}} = \int y_1^2 \, dA$$

$$y_1 = y + c_y$$

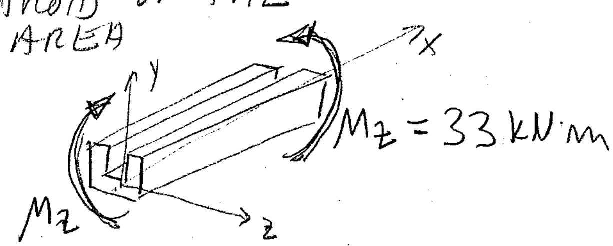
$$I_{z_{11}} = \int (y + c_y)^2 \, dA = \underbrace{\int y^2 dA}_{I_z} + c_y \underbrace{\int dA}_{A} + 2c_y^2 \underbrace{\int y \, dA}_{0} = 0$$

y IS TAKEN FROM THE CENTROID SO $\int y \, dA = 0$

$$\boxed{I_{z_{11}} = I_z + c_y^2 \cdot A}$$

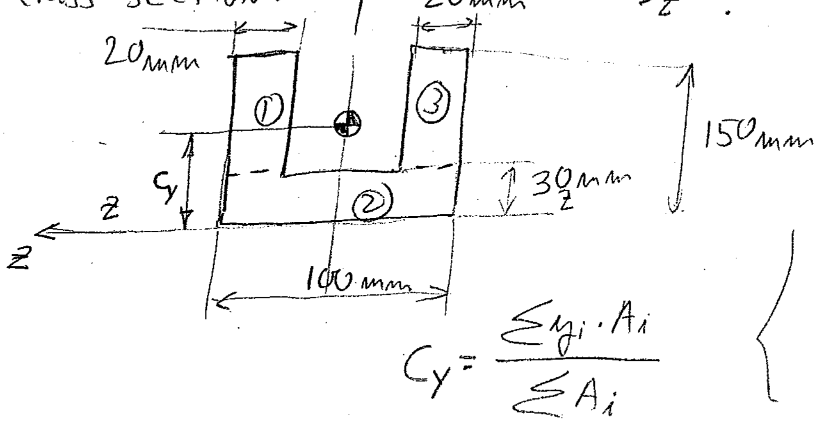
I_z TAKEN FROM THE CENTROID

THE MINIMUM MOMENT OF INERTIA IS REACHED THROUGH THE CENTROID OF THE SECTION AREA



EXAMPLE:

FIND THE MOMENT OF INERTIA BY Z AXIS OF THE FOLLOWING CROSS SECTION: I_z - ?



STEP #1 - CENTROID OF THE SECTION AREA

$$\int y \, dA = 0$$

FOR A MULTIPLE DISCRETE SECTIONS:

$$\sum_{i=1}^{N} (y - c_y) A_i = 0$$

$$\sum_{i=1}^{N} (y_i A_i) - c_y \sum_{i=1}^{N} A_i = 0$$

$$c_y = \frac{\sum y_i \cdot A_i}{\sum A_i}$$

4-4

# AREA	A_i [mm^2]	y_i [mm]
1	$20(150-30) = 2400$	$\frac{1}{2}(150-30)+30=90$
2	$30\cdot 100 = 3000$	$\frac{1}{2}\cdot 30 = 15$
3	$20(150-30) = 2400$	$\frac{1}{2}(150-30)+30=90$

$$C_y = \frac{\sum y_i A_i}{\sum A_i} = \frac{\overbrace{90\cdot 2400}^{①}+\overbrace{15\cdot 3000}^{②}+\overbrace{90\cdot 2400}^{③}}{\underset{①}{2400}+\underset{②}{3000}+\underset{③}{2400}}$$

$$\underline{C_y = 61.154 \text{ mm}}$$

STEP#2 → SECOND MOMENT OF AREA

PARALLEL-AXIS THEOREM

$$I_z = \sum I_{z_i} = \sum I_{z_i'} + C_{y_i}^2 A_i$$

THE MOMENT OF INERTIA OF AREA i (1, 2 or 3 IN OUR CASE) AROUND ITS OWN CENTROID

DISTANCE OF AREA i FROM THE GENERAL CENTROID OF THE CROSS SECTION

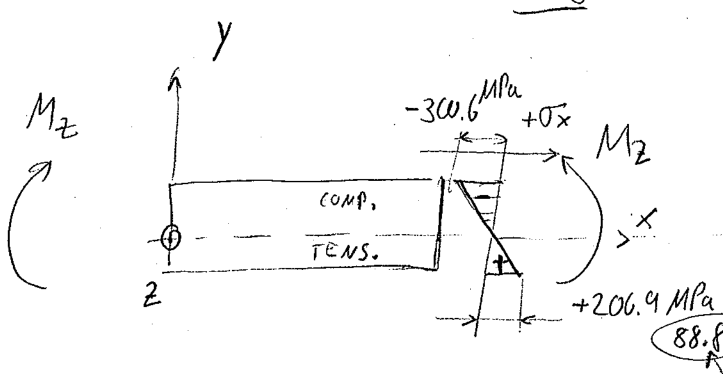
# AREA	$I_{z_i'}$ [mm^4] $\left(=\frac{bh^3}{12}\text{ RECTANGLE}\right)$	C_{y_i} [mm] $(= y_i - C_y)$	$I_{z_i'} + C_{y_i}^2 A_i$ [mm^4]
1	$\frac{20(150-30)^3}{12} = 2.88\cdot 10^6$	$90-61.154 = 28.846$	$2.88\cdot10^6 + 28.846^2\cdot 2400 = 4.877\cdot 10^6$
2	$\frac{100\cdot 30^3}{12} = 0.225\cdot 10^6$	$15-61.154 = -46.154$	$0.225\cdot 10^6 + (-46.154)^2\cdot 3000 = 6.616\cdot 10^6$
3	$\frac{20(150-30)^3}{12} = 2.88\cdot 10^6$	$90-61.154 = 28.846$	$= 4.877\cdot 10^6$

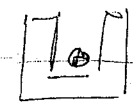
$+ 6.616\cdot 10^6$

$$I_z = \sum I_{z_i} = 9.754\cdot 10^6 \text{ mm}^4$$

$$\sigma_x\Big|_{y=150-C_y} = -\frac{M_z\cdot C_\uparrow}{I_z} = -\frac{33\cdot 10^6 \text{ N}\cdot\text{mm}\cdot \overbrace{(150-61.154)}^{88.846}}{9.754\cdot 10^6 \text{ mm}^4} = -300.6 \text{ MPa}$$

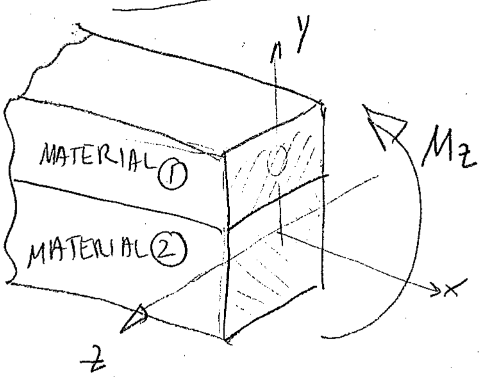
$$\sigma_x\Big|_{y=-C_y} = -\frac{M_z\cdot C_\downarrow}{I_z} = -\frac{33\cdot 10^6 \text{ N}\cdot\text{mm}\cdot(-61.154)}{9.754\cdot 10^6 \text{ mm}^4} = +206.9 \text{ MPa}$$

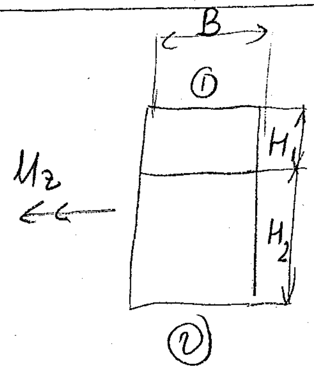




$$\sigma_{max} = \left|\sigma_x\right|_{max} = \frac{M_z\, c_{max}}{I_z} = \dots = 300.6 \text{ MPa}$$

BENDING OF MEMBERS MADE OF SEVERAL MATERIALS





THE SAME DEFORMATION ANALYSIS:

$$\varepsilon_x = -\frac{y}{\rho} \leftarrow \text{CURVATURE RADIUS}$$

$$(2)\ \int \sigma_x \, dA = 0$$

$$= \int_{①} \sigma_{x①}\, dA + \int_{②} \sigma_{x②}\, dA = 0$$

$$\left.\begin{aligned} \sigma_{x_1} &= E_1 \varepsilon_x \\ \sigma_{x_2} &= E_2 \varepsilon_x \end{aligned}\right\} \quad E_{①} \int_{①} \varepsilon_x \, dA + E_{②} \int_{②} \varepsilon_x \, dA = 0$$

$$E_{①} \int_{①} \varepsilon_x \, dA + E_{①} \int_{②} \varepsilon_x \left(\frac{E_{②}}{E_{①}} dA \right) = 0$$

$$\int_{①} \sigma_{x①}\, dA + \int_{②} \sigma_{x①} \left(\frac{E_{②}}{E_{①}} dA \right) = 0$$

IF WE TAKE AN EQUIVALENT SECTION AREA FOR ②, WE CAN TRANSFORM THE CROSS SECTION TO AN SINGLE MATERIAL EQUIVALENT

σ_x IS A FUNCTION OF y, SO IT IS POSSIBLE ONLY IN Z DIRECTION

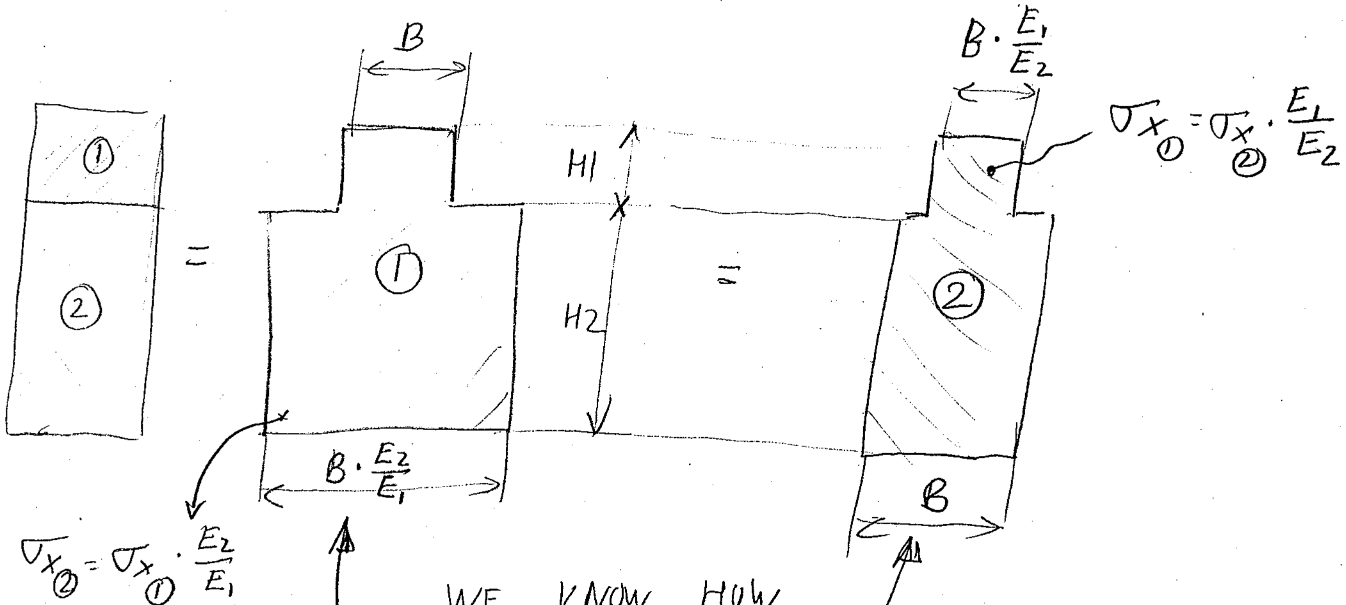
THE SAME FOR THE MOMENT OF INERTIA:

$$(1)\ M_z = -\int y\, \sigma_x \, dA =$$

$$= \dots = -\int_{①} y\, \sigma_{x①}\, dA + \int_{②} y\, \sigma_{x①} \underbrace{\left(\frac{E_{②}}{E_{①}} dA \right)}_{dA'}$$

CHANGED IN Z DIRECTION

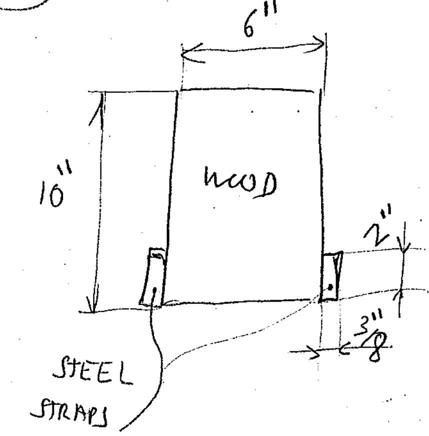
4-9



WE KNOW HOW TO SOLVE

$cg \rightsquigarrow I_z \rightsquigarrow \sigma_{x_1} \rightsquigarrow \sigma_{x_2}$

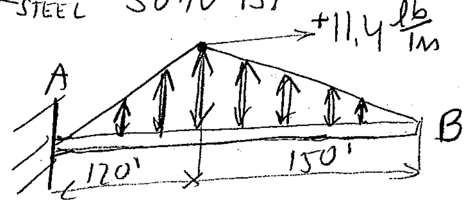
4.48 EXTENDED



$E_{wood} = 1.5 \cdot 10^6$ PSI

$E_{STEEL} = 30 \cdot 10^6$ PSI

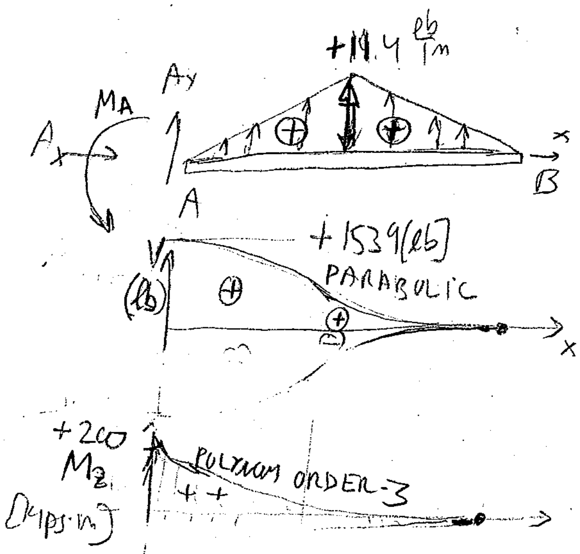
NEGLECT SHEARING EFFECT BECAUSE HIGH BENDING MOMENT



(a) MAXIMUM MOMENT AND WHERE?

(b) $\sigma_{x_{wood}}$ - ? (-1.526 ksi)

(c) $\sigma_{x_{STEEL}}$ - ? (… ksi)



$A_x = 0$

$A_y = \frac{-11.4 \cdot (120' + 150')}{2} = -1539\,(lb) \quad V|_A = 1539\,[lb]$

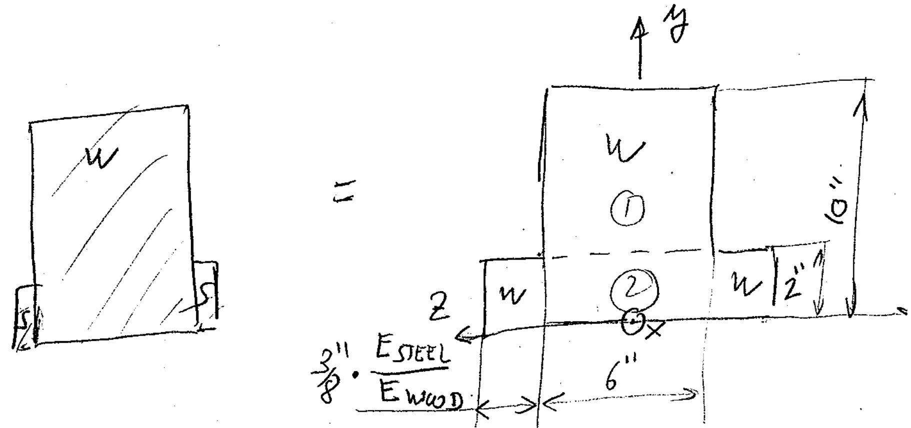
$M_z|_A = \pm \left\{ \frac{11.4 \cdot 120'}{2} \left(\frac{2}{3} 120'\right) + \frac{11.4 \cdot 150'}{2} \cdot \left(120 + \frac{1}{3} 150'\right) \right\} \approx 200070$ lb·in

$M_z|_A \approx 200$ kips·in

$$M_{max} = +200 \text{ kips·in} \quad \text{AT } x = 0$$

4-10

LET TRANSFORM THE CROSS-SECTION TO WOOD



$$\frac{3''}{8} \cdot \frac{E_{STEEL}}{E_{WOOD}} = \frac{3''}{8} \cdot \frac{30 \cdot 10^6}{1.5 \cdot 10^6} = 7.5''$$

AREA #	A_i [in²]	y_i [in]	$I_{z'i}$ [in⁴]	c_{yi} [in]	$I_{z_i} = I_{z'i} + c_{y_i}^2 \cdot A_i$ [in⁴]
1	$(10-2)\cdot 6 = 48$	$\frac{10-2}{2}+2 = 6$	$\frac{bh^3}{12} = \frac{6\cdot 8^3}{12} = 256$	$y_i - c_y$: $6 - 3.667 = 2.333$	$256 + 2.333^2 \cdot 48 = 517.26$
2	$2(7.5+6+7.5) = 42$	1	$\frac{(7.5+6+7.5)2^3}{12} = 14$	$1 - 3.667 = -2.667$	$14 + (-2.667)^2 \cdot 42 = 312.74$

$$c_y = \frac{\sum y_i A_i}{\sum A_i} = \frac{6 \cdot 48 + 1 \cdot 42}{48+42} = 3.667 \text{ in}$$

$$I_z = \sum I_{z_i} = 517.26 + 312.74 = 830 \text{ in}^4$$

$$\sigma_x \Big|_{\substack{WOOD \\ y' = 10 - c_y \\ y' = +6.333}} = -\frac{(+200 \text{ kips}\cdot\text{in}) \cdot 6.333}{830} = -1.526 \text{ KSI}$$

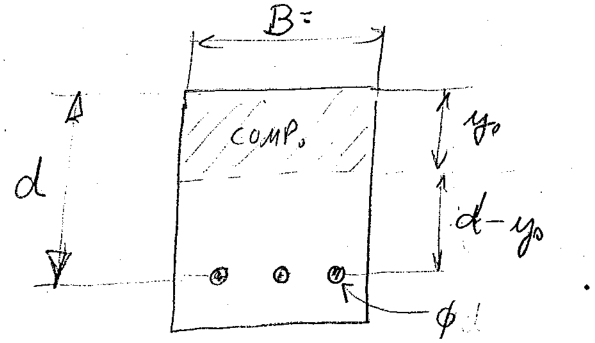
$$\sigma_x \Big|_{\substack{STEEL \\ y' = -c_y \\ y_{CG} = -3.667}} = \underbrace{-\frac{(+200 \text{ kips}\cdot\text{in}) \cdot (-3.667)}{830}}_{\text{TRANS. WOOD}} \cdot \frac{\overset{E_{STEEL}}{30 \cdot 10^6}}{\underset{E_{WOOD}}{1.5 \cdot 10^6}} = +17.67 \text{ KSI}$$

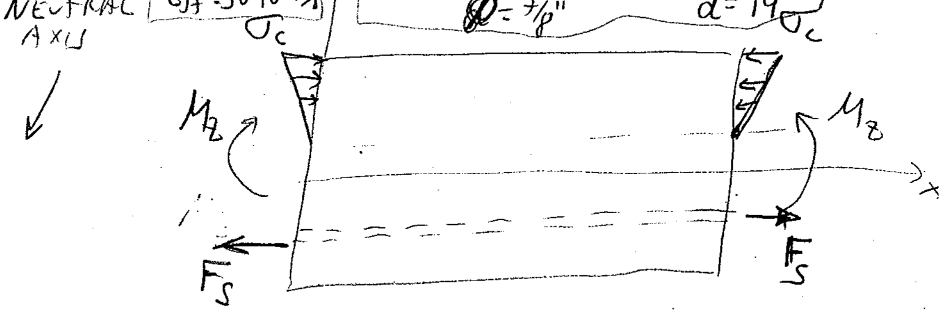
4-11

REINFORCED CONCRETE BEAMS

① WE ASSUME THAT THE CONCRETE IS NOT ABLE TO CARRY ANY TENSION, BUT ITS STEEL REINFORCEMEN DO!

$E_c = 3\cdot10^6$ PSI, $E_{St} = 30\cdot10^6$ PSI, $M_z = 324$ kips·ft, $B = 8''$, $\phi = 7/8''$, $d = 14''$





② WE ASSUME THE DIAMETER OF THE REINFORCED CABLES TO BE SMALL ENOUGH, SO THEIR MOMENT OF INERTIA DON'T COME INTO ACOUNT

CALCULATION OF THE NEUTRAL AXIS PLACE, y_0

$$\underbrace{\int_0^{y_0} \sigma_x \, dA}_{\text{CONCRETE}} + \underbrace{F_s}_{\text{STEEL}} = 0$$ { TOTAL AXIAL FORCES UNDER PURE BENDING = 0

FOR CONCRETE: $\sigma_x = +E_c \varepsilon_x$

$\varepsilon_x = -d\theta y$

FOR STEEL: $F_s = +\varepsilon_x E_{St}$

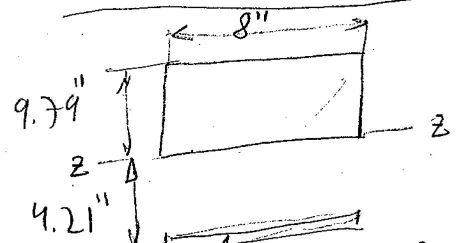
$$\int_0^{y_0} -E_c \cancel{d\theta}\, y \underbrace{dA}_{B\cdot dy} - E_{St} \cancel{d\theta} (y_0 - d) = 0$$

$$\boxed{\frac{1}{2} B y_0^2 + \frac{E_{St}}{E_c} A_{St}\, y_0 - \frac{E_{St}}{E_c} \cdot d = 0} \rightsquigarrow y_0 = \ldots$$

$$\frac{1}{2} 8 y_0^2 + \frac{30\cdot10^6}{3\cdot10^6} \cdot 3 \cdot \frac{\pi \left(\frac{7}{8}\right)^2}{4} \cdot y_0 - \frac{30\cdot10^6}{3\cdot10^6} 14 = 0 \rightsquigarrow y_0 = 9.79 \text{ in}$$

$$y_0^2 + 4.51 y_0 - 140 = 0$$

MOMENT OF INERTIA:



$$I_z = \sum I_{z_i} = \underbrace{\frac{9.79^3 \cdot 8}{12} + \left(\frac{9.79}{2}\right)^2 (9.79 \cdot 8'')}_{I_{z\square}\ 1008.9118} + \underbrace{0 + 4.21''^2 \cdot 18.04}_{I_{z\circ\circ\circ}}$$

$C_\square$; TRANSFORMED AREA

$$\sum A_{St} \cdot \frac{E_{St}}{E_c} = 3 \cdot \frac{\pi \left(\frac{7}{8}\right)^2}{4} \cdot \frac{30\cdot10^6}{3\cdot10^6} = 18.04 \text{ in}^2$$

TRANSFORMED AREA TO CONCRETE

$$I_z = 2821.913 \text{ in}^4$$

4.12

$$\sigma_c = -\frac{M_z\, y_0}{I_z} = -\frac{32.4\cdot 12\,(\text{kips}\cdot\text{in})\cdot \overset{y_0}{9.79''}}{2821.913} = -1.349\ \text{KSI}$$

$$\sigma_{St} = -\frac{M_z\,(y_0-d)}{I_z}\cdot\frac{E_{St}}{E_c} = -\frac{32.4\cdot 12\,(\text{kips}\cdot\text{in})\cdot(9.79-14)}{2821.913}\cdot \underset{\frac{E_{St}}{E_c}}{10} = 5.8\ \text{KSI}$$

REVIEW ALSO SAMPLE PROBLEM 4.4

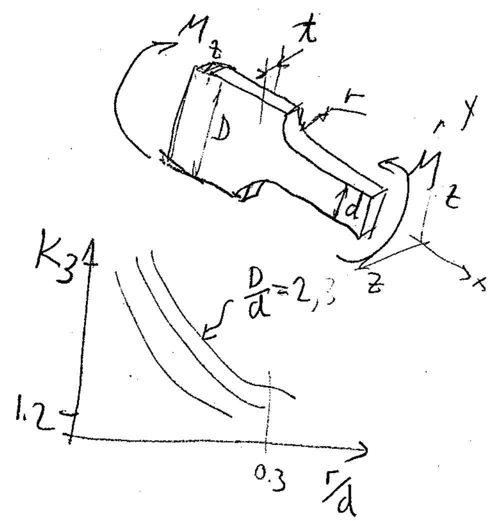
STRESS CONCENTRATION

$$\sigma_m = K\,\frac{Mc}{I}$$

K: STRESS CONCENTRATION FACTOR

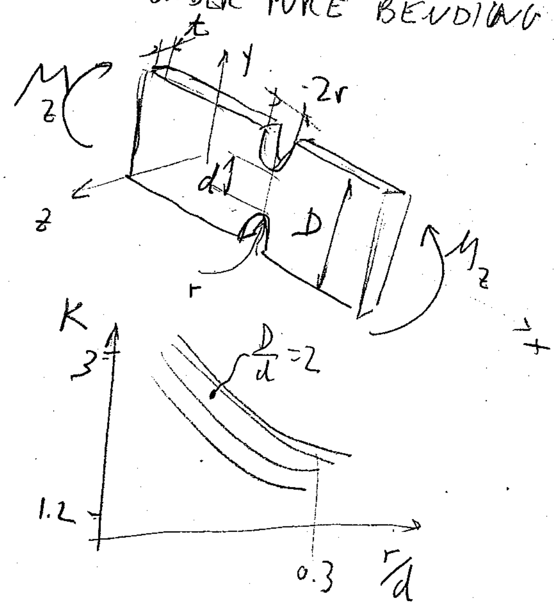
c, I: TAKEN ON THE MINIMUM CROSS SECTION

FLAT BARS WITH FILLETS UNDER PURE BENDING



$$\boxed{I = \frac{d^3 t}{12} \qquad c = \pm\frac{1}{2}d}$$

FLAT BARS WITH GROVES UNDER PURE BENDING



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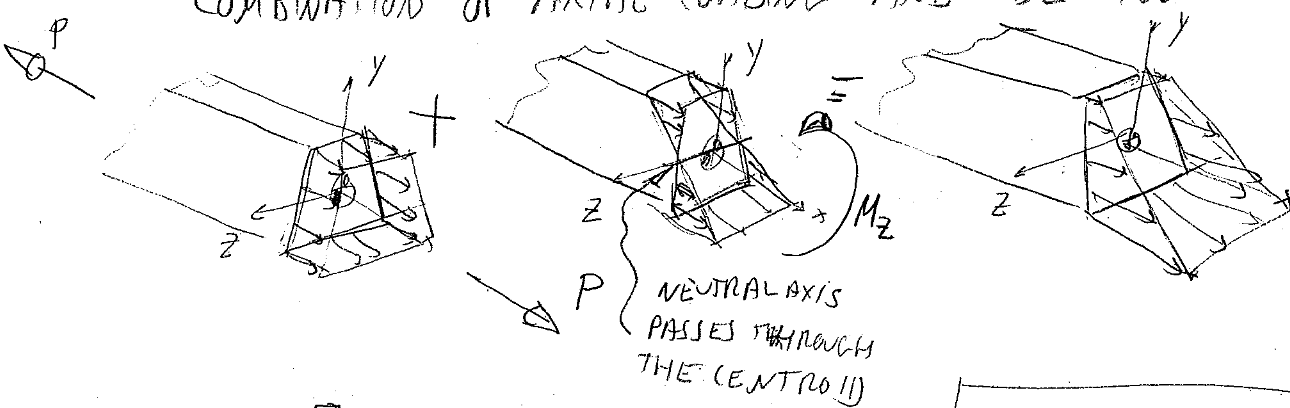
ECCENTRIC AXIAL LOADING IN A PLANE OF SYMMETRY

COMBINATION OF AXIAL LOADING AND BENDING



$\sigma_x = +\dfrac{P}{A}$ AXIAL LOADING

$\sigma_x = -\dfrac{M_z y}{I_z}$ PURE BENDING

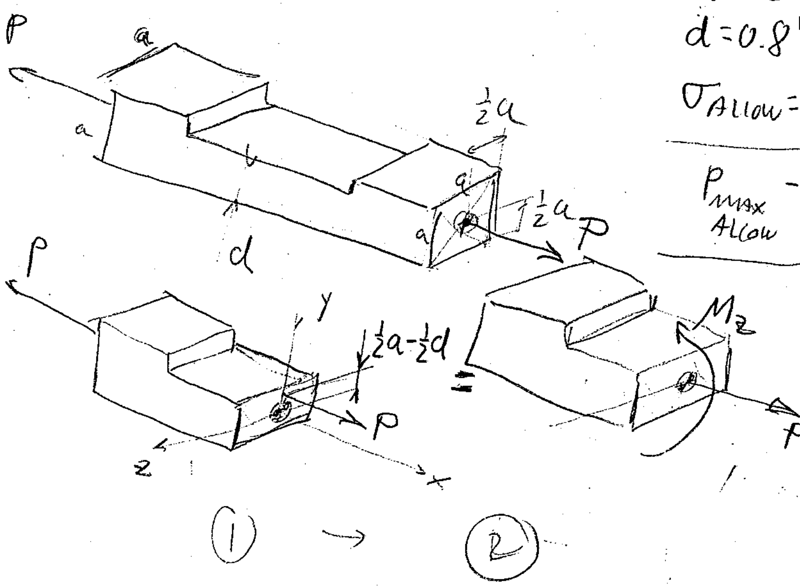
$$\boxed{\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z}}$$

THE NEW NEUTRAL AXIS EXISTS WHERE:

$$\sigma_x = 0 = \frac{P}{A} - \frac{M_z y_N}{I_z} \rightsquigarrow y_N = \dots$$

y_N MAY BE GREATER THAN c!

EXAMPLE 4.127



$a = 1.2''$
$d = 0.8''$
$\sigma_{ALLOW} = 8\,ksi$

$P_{MAX\ ALLOW}$ - ?

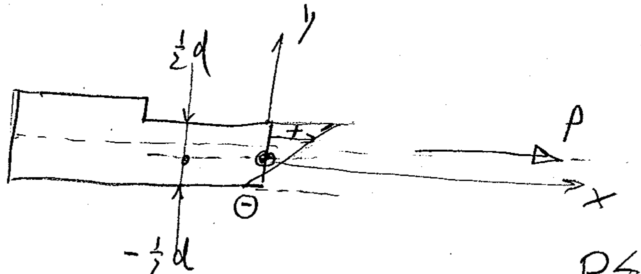
3

$$M_z = -P\left(\tfrac{1}{2}a - \tfrac{1}{2}d\right)$$

$$\sigma_x = \frac{P}{A} - \frac{M_z c}{I_z}$$

$$\sigma_x = +\frac{P}{a \cdot d} - \frac{\left(-\frac{1}{2}P(a-d)\right)\left(+\frac{1}{2}d\right)}{\left(a \cdot d^3/12\right)}$$

LETS DRAW THE STRESSES



$$\sigma_x\Big|_{max\ AT\ +\frac{1}{2}d} = \frac{P}{a\cdot d} + \frac{\left(\frac{1}{2}P(a-d)\right)\cdot\frac{1}{2}d}{\left(\frac{ad^3}{12}\right)} \le \sigma_{ALL}$$

$$P \le \frac{8\,KSI}{\left\{\frac{\frac{1}{2}(1.2-0.8)\cdot\frac{1}{2}\cdot 0.8}{(1.2\cdot 0.8^3/12)} + \frac{1}{1.2\cdot 0.8}\right\}} = 3.072\ Kips$$

UNSYMMETRIC BENDING

WE ASSUMED SYMMETRY AROUND Y AXIS, THEN $\int z\,\sigma_x\,dA = 0$ (3)

AND WE COULD ASSUME THE DEFORMATIONS IN ORDER TO RECEIVE σ_x AS A LINEAR FUNCTION OF y.

LET SUBMIT $\sigma_x = \frac{\sigma_{x_1}}{c_1}\cdot y$ AND FIND THE REQUIREMENTS FOR

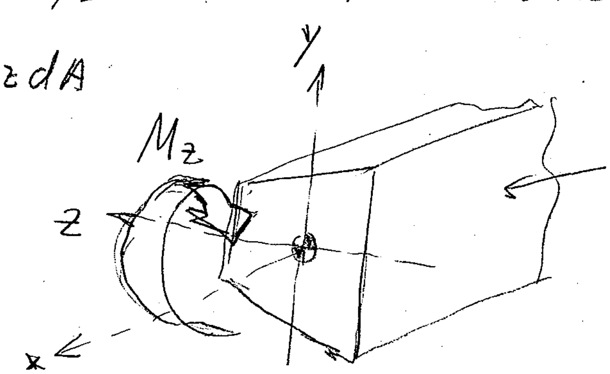
$$\int z\,\sigma_x\,dA = 0 = \frac{\sigma_{x_1}}{c_1}\int z\,y\,dA = 0$$

IT IS CALLED A PRODUCT OF INERTIA I_{yz}. IT IS EQUAL TO ZERO WHEN WE MEASURE IT FROM THE PRINCIPAL CENTROIDAL AXES, OF THE CROSS SECTION. IT CAN BE PROVE THAT WHEN $I_{zy} = I_{yz}$

SO WHEN $I_{yz} = 0$, $I_{zy} = 0$

IF WE CHOOSE yz ON CENTROID + SYMETRY AXES,

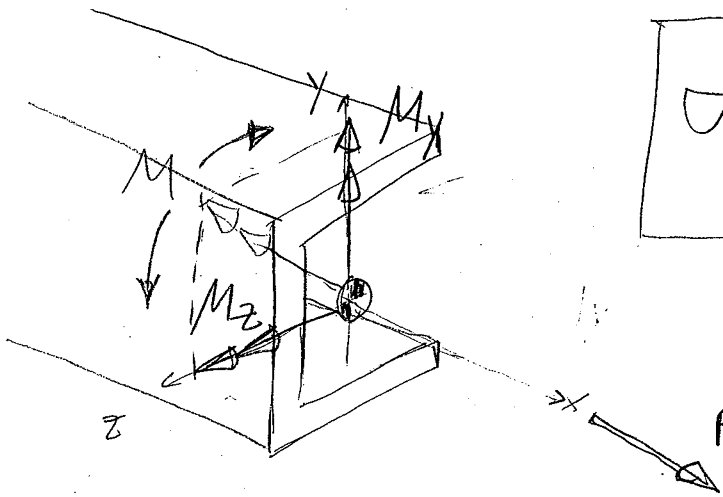
$$\int zy\,dA = 0 = \int yz\,dA$$



THE SYMMETRY ON Z IS ENOUGHT

GENERAL CASE OF ECCENTRIC AXIAL LOADING

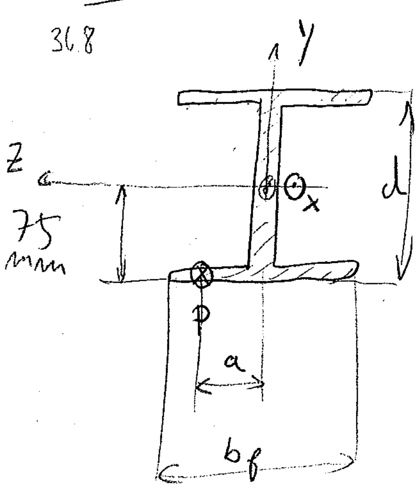
WE CAN COMBINE AXIAL, SYMMETRIC AN UNSYMMETRIC BENDING BY DISCOMPOUNING THE BENDING MOMENT INTO THE PRINCIPAL AXES OF THE CROSS SECTION



$$\boxed{\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y \cdot z}{I_y}}$$

EXAMPLE
4.164 +

36.8



W150×24 → $I_z = 13.4 \cdot 10^6\ mm^4$, $I_y = 1.83 \cdot 10^6\ mm^4$, $A = 3060\ mm^2$, $b_f = 102\ mm$, $d = 160\ mm$

$P = -50\ kN$

$\left.\sigma_{comp}\right|_{MAX} \geq -90\ MPa$

a_{max} - ? NEUTRAL AXIS - ?

$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$M_z = -P \cdot y_p = -(-50 \cdot 10^3) \cdot (-75) = -375 \cdot 10^4\ Nmm$$

$$M_y = +P \cdot z_p = (-50 \cdot 10^3) \cdot a$$

$$\sigma_x = \frac{-50 \cdot 10^3}{3060} + \frac{375 \cdot 10^4 \cdot y}{13.4 \cdot 10^6} - \frac{50 \cdot 10^3 \cdot a \cdot z}{1.83 \cdot 10^6} \geq -90\ MPa$$

4-16

THE HIGHER COMPRESSIVE STRESS IS RECEIVED FOR y NEGATIVE AS POSIBLE AND FOR z POSITIVE HIGHER

$$y = -80\,\text{mm} \left(= -\frac{d}{2}\right) \qquad z = 51 = \frac{b_f}{2}$$

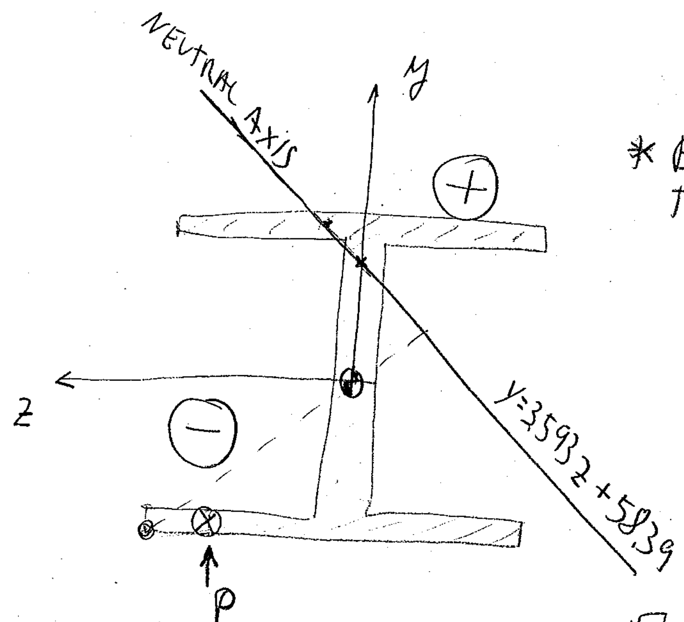
$$-90\,\text{MPa} \leq -\left\{ \frac{50\cdot 10^3}{3060} + \frac{375\cdot 10^4\cdot 80}{13.4\cdot 10^6} + \frac{50\cdot 10^3\cdot 51\cdot}{1.83\cdot 10^6}\cdot a \right\}$$

$\underline{a \leq 36.8\,\text{mm}}$

THE NEUTRAL AXIS :

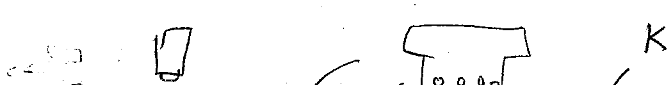
$$\sigma_x = 0 = -\frac{50\cdot 10^3}{3060} + \frac{375\cdot 10^4}{13.4\cdot 10^6}\,y - \frac{50\cdot 10^3\cdot 36.8}{1.83\cdot 10^6}\,z$$

$$y = 3.593\,z + 58.39 \text{ mm}$$



* BECAUSE OF THE AXIAL LOAD, THE NEUTRAL AXIS DOESN'T MATCH WITH THE CENTROID

HOME WORK #7



4.2, 4.7, 4.9, 4.19, 4.26, 4.47, 4.55, 4.57, 4.73

4.17

CHAPTER 5

DESIGN OF PRISMATIC BEAMS FOR BENDING

$$\sigma_m = \frac{|M|_{max}|c|}{I_z} = \frac{|M|_{MAX}}{|S|} \leq \sigma_U$$

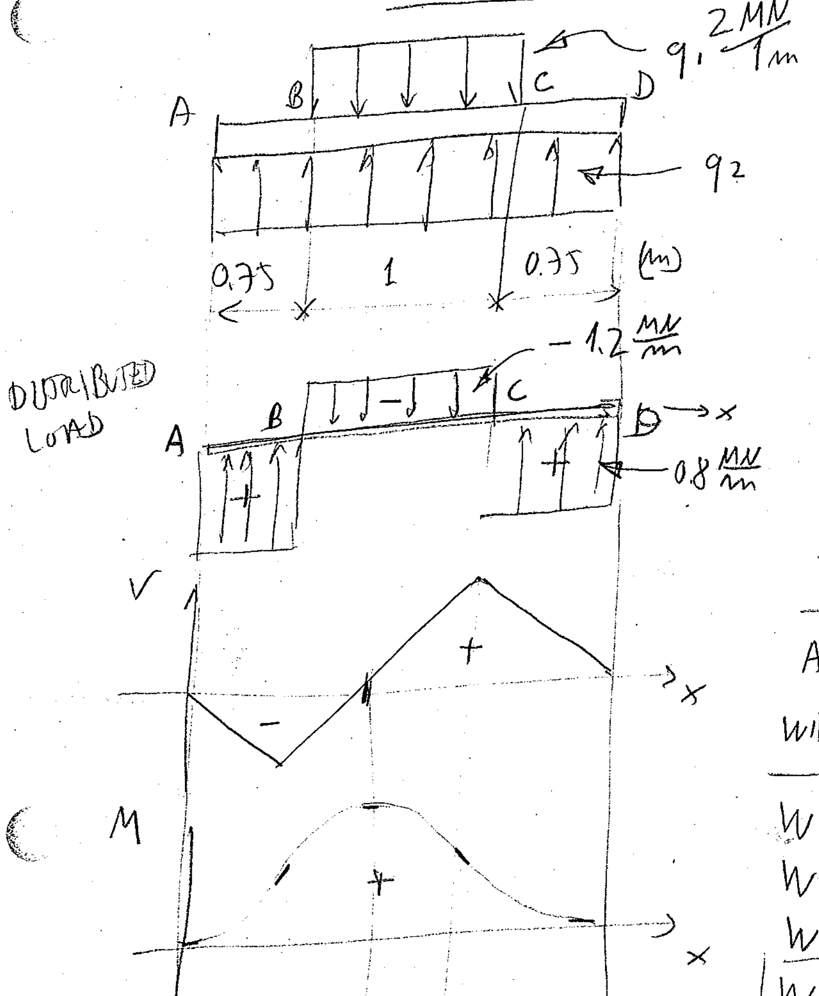
$|S|$ ← SECTION MODULUS

STEPS:

σ_{ALL} ($\sigma_{compression} \overset{?}{=} \sigma_{TENSION}$) → SHEAR FORCE BENDING MOMENT DIAGRAMS → SECTION/S IN BEAM/S TO BE ANALYZED → MINIMUM REQUIRED SECTION MODULUS $S = \frac{I_z}{c} = ?$ → SELECTION / CALCULATION OF CROSS SECTION

WEIGHT ?
COST - ?
REQUIRED GEOMETRY - ?

EXAMPLE PROB. 5.93



$\sigma_{ALL} = 170\,MPa$, THE MOST ECONOMICAL METRIC WIDE-FLANGE I

$$q_1 L_1 = q_2 L_2$$

$$q_2 = \frac{2MN}{1m} \cdot 1m \Big/ 2.5m = 0.8\,\frac{MN}{m}$$

$$M_{MAX}\Big|_{x=1.75m} = 0.8 \cdot 0.75 \cdot \left(\frac{0.75}{2} + 0.5\right) - 1.2 \cdot 0.5 \cdot 0.25$$

$$= 0.375\,MN \cdot m = 375 \cdot 10^6\,N \cdot mm$$

$$S_{min} = \frac{M_{max}}{\sigma_{ALL}} = \frac{375 \cdot 10^6}{170\,MPa} = 2205.9 \cdot 10^3\,mm^3$$

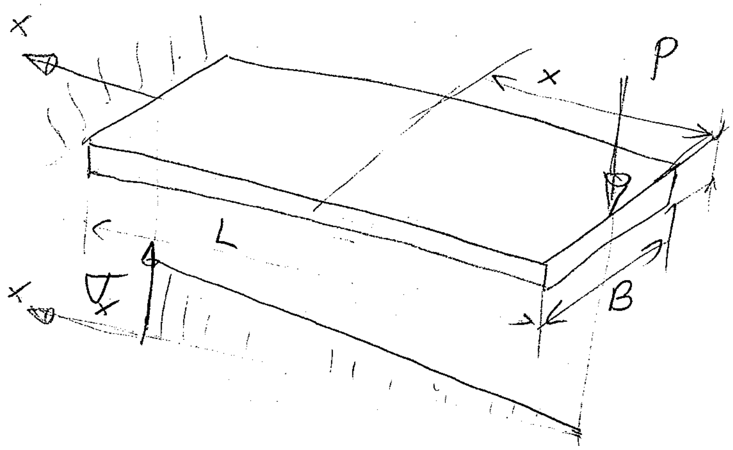
APPENDIX C, PAGE 751:

WIDE FLANGE I	A [mm²]	S_x [$10^3\,mm^3$]
W 360×216	27600	3800
W 410×114	14600	2200
W 460×113	14400	2400
W 610×101	13000	2530 MINIMUM WEIGHT
W 690×125	16000	3510 MINIMUM COST

4.18

DESIGN OF NON PRISMATIC BEAMS

SECTION MODULUS CHANGES AS A FUNCTION OF X.

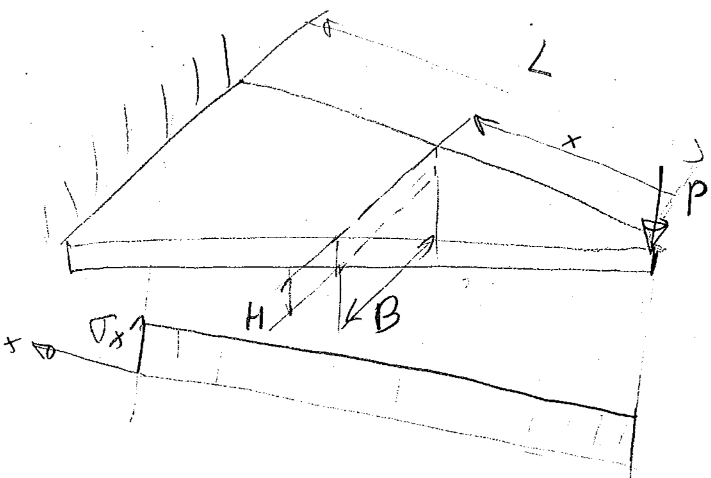


$$\sigma_x = \frac{M}{S} = \frac{Px}{S}$$

$$S = \frac{P\,x}{\sigma_x}$$ LET ASSUME CONSTANT

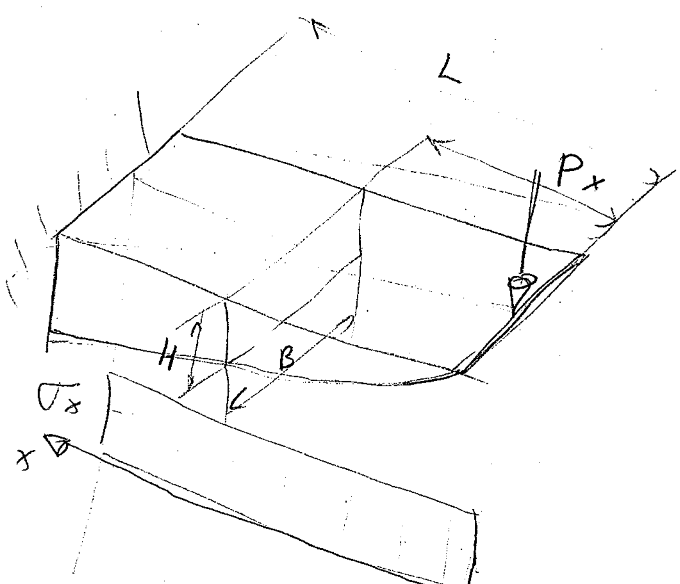
$$S = \frac{I_z}{c} = \frac{\frac{H^3 B}{12}}{\frac{H}{2}} = \frac{H^2 B}{6}$$

$$\frac{H^2 B}{6} = \frac{P \cdot x}{\sigma_x}$$



LET ASSUME CONSTANT H:

$$B = \frac{6\ P}{H^2\ \sigma_x} \cdot x$$



LET ASSUME CONSTANT B:

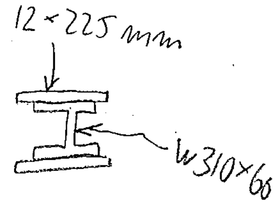
$$H = \sqrt{\frac{6 \cdot P}{B\ \sigma_x}} \cdot \sqrt{x}$$

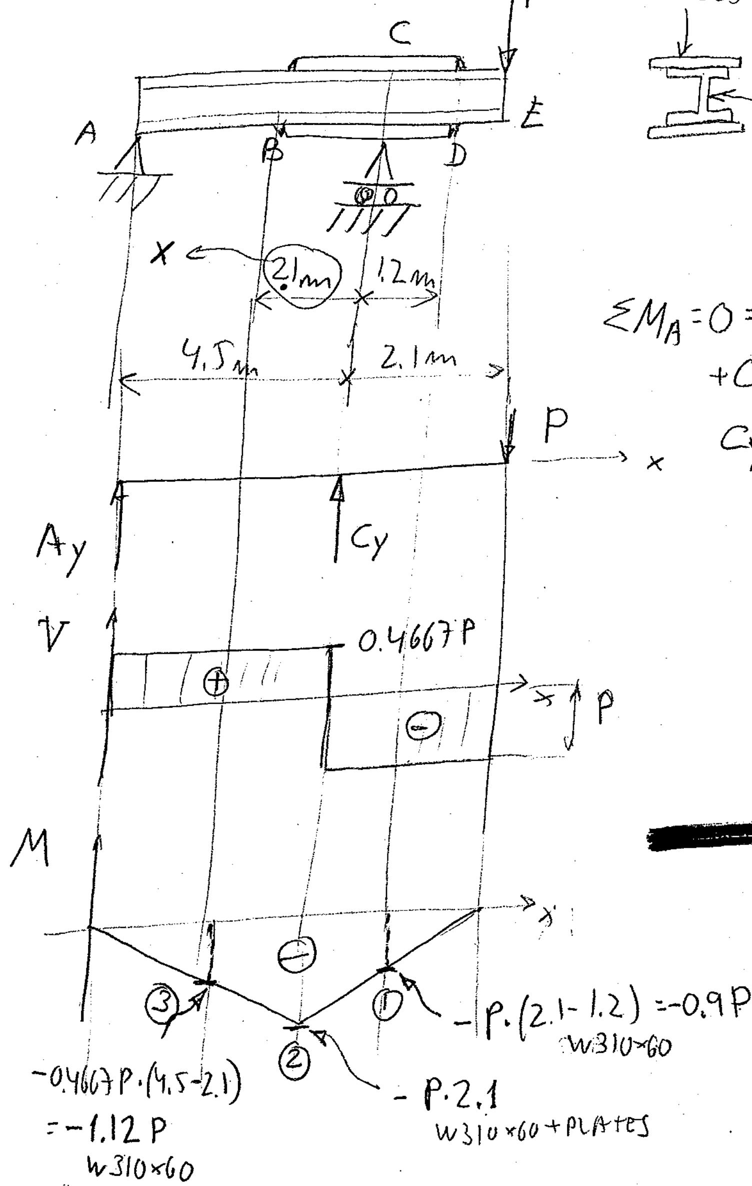
4.19

5.158 (+)

$\sigma_{ALL} = 165\ MPa$

$P - ?$





$\Sigma M_A = 0 = -P\cdot 6.6 + C_y\cdot 4.5$

$C_y = 1.4667\,P$

[redacted]

W310×60

APPENDIX C, PAGE 753

HIGH $d = 303\ mm$

$I_x = 129\cdot 10^6\ mm^4$

$S_x = 851\cdot 10^3\ mm^3$

③ $$|\sigma|_{ALL} = \frac{|M_③|}{S_③} = \frac{1.12\ P\cdot 10^3}{851\cdot 10^3} = 165 \rightsquigarrow P = 125.37\ kN$$

② 

$I_z' = \frac{225\cdot 12^3}{12} = 32400\ mm^4$

$C_y = \frac{303}{2} + \frac{12}{2} = 157.5\ mm$

$I_{z\,PLATE} = 32400 + 157.5^2\cdot(12\cdot 225) = 67009275\ mm^4$

$I_{z②} = 129\cdot 10^6 + 2\times 67009275 = 263.019\cdot 10^6\ mm^4$ "GENERAL"

$$S_② = \frac{I_{z②}}{(303+12)/2} = 1.67\cdot 10^6\ mm^3$$

$$|\sigma|_{ALL} = \frac{|M_②|}{S_②} = \frac{2.1\cdot P\cdot 10^3}{1.67\cdot 10^6} = 165$$

$\to P = 131.2\ kN$

$\to P_{max\ ALLOW} = 125.37\ kN$

4.20

⊕

WHAT SHOULD BE x INSTEAD OF 2.1 m IN ORDER TO ALLOW P = 131.2 kN

$$M_{③} = -0.4667\,P\,(4.5 - x)$$

$P = 131.2 \cdot 10^3$

$$\sigma_{ALL} = \frac{0.4667 \cdot P (4.5 - x) \cdot 10^3}{\underbrace{851 \cdot 10^3}_{S_{③}}} = 165 \rightsquigarrow \boxed{x = 2.2\ m}$$

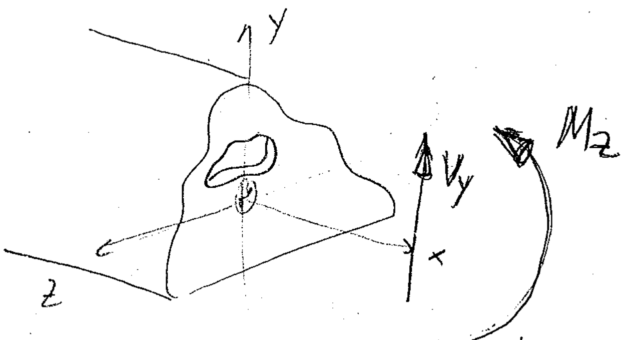
SHEARING STRESSES IN BEAMS AND THIN-WALLED MEMBERS

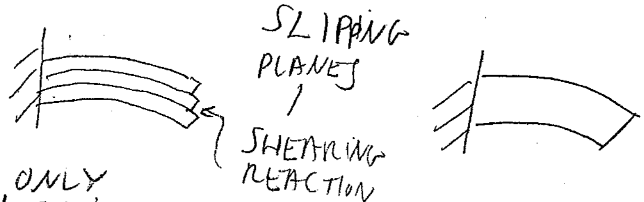
THE CROSS SECTION REACTS TO SHEARING LOADS

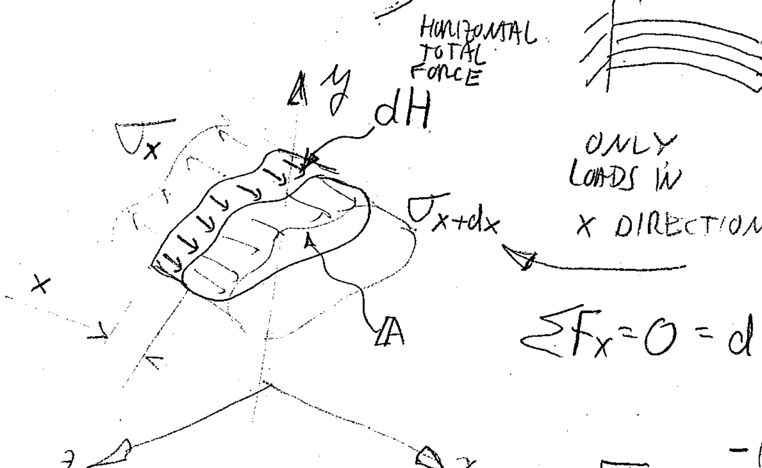
For $M_y = 0$, $V_z = 0$:

$$\int \tau_{xy}\, dA = V_y$$

$$\int \tau_{xz}\, dA = 0$$







$$\sum F_x = 0 = dH + \int_A \sigma_{x+dx}\, dA - \int_A \sigma_x\, dA$$

$$\sigma_{x+dx} = \frac{-(M_z + dM_z)\, y}{I_z}$$

$$\sigma_x = \frac{-M_z\, y}{I_z}$$

SHEARING FORCE ACTING ON THE COMPLETE CROSS-SECTION

$$dH - \left\{ \frac{M_z + dM_z}{I_z} \int_A y\, dA - \frac{M_z}{I_z} \int_A y\, dA \right\} = 0$$

$-V_y$

THE SHEARING FLOW ALONG THE MAIN AXIS OF THE BEAM

$$\frac{dH}{dx} = \frac{dM_z}{dx} \frac{1}{I_z} \int_A y\, dA$$

FIRST MOMENT OF THE CHOSEN AREA

Q [mm³] or [in³] or [m³]

$\left(\frac{N}{m}\right)$ $\left(\frac{N}{mm}\right)$ $\left(\frac{lb}{in}\right)$

q

$$q = +\frac{V_y Q}{I_z}$$

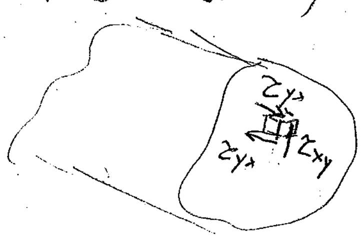
THE AVERAGE SHEARING STRESS:

$$\tau_{AVE} = \frac{dH}{dA} = \frac{\overbrace{dH}^{q}}{dx \cdot \mathbb{P}} = \frac{V_y\, Q}{I_z\, \mathbb{P}}$$

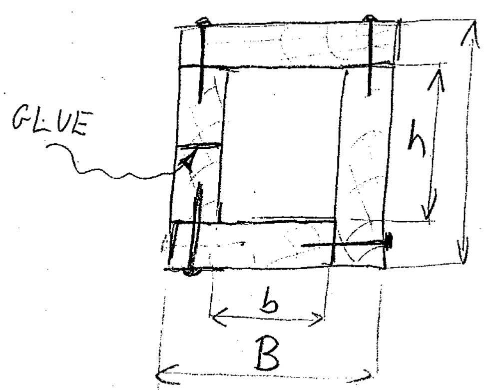
PERIMETER THAT CARRIES LOAD OF THE SELECTED AREA

SELECT AREA IN WHICH H IS CONSTANT, THEN

$$\tau_{AVE} = \tau_{yx} = \tau_{xy}$$



EXAMPLE – NAILS



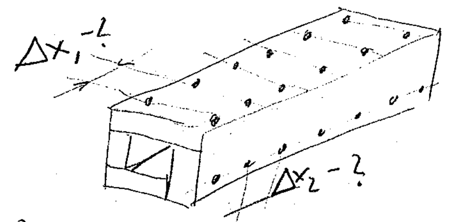
$B = H = 4.5$ in

$h = b = 3$ in

$V = 600$ lb

$F^{SHEAR}_{NAIL\ ALLOW} = 80.8$ lb

(a) SPACING BETWEEN NAILS – ?



(b) SHEARING STRESS ON GLUE ?

STEPS:

1. BEAM ANALYSIS FIND V_y ON CROSS SECTION
2. CENTROID, I_z
3. SELECTION OF AREA FOR H, q CALCULATION
4. Q AND q CALCULATION ← OR τ
5. $q = \dfrac{dH}{dx} = \dfrac{F_{ALLOW} \cdot m}{\Delta x}$ ← NUMBER OF SIMILAR NAILS CARRYING H

CASE I

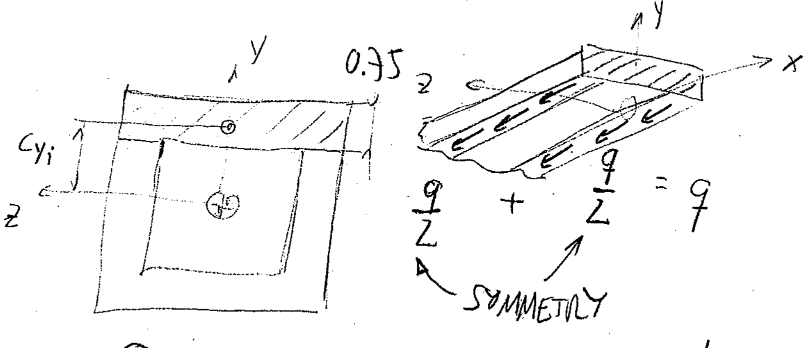
Δx_1 CALCULATION

STEP ① $V_y = 600\ lb$

STEP ② CENTROID AT $y = \frac{H}{2}$ (SQUARE)

$$I_z = \frac{BH^3}{12} - \frac{bh^3}{12} = \frac{4.5^4 - 3^4}{12} = 27.422\ in^4$$

STEP ③ SELECTION OF AREA:



STEP ④

$$Q = \int_A y\,dA = c_{y_i} \cdot A = \left(\underbrace{\frac{3}{2}}_{\frac{h}{2}} + \frac{0.75}{2}\right) \cdot 0.75 \cdot 4.5 = 6.3281\ in^3$$

$$q = \frac{V_y Q}{I_z} = \frac{600\ lb \cdot 6.3281\ in^3}{27.422\ in^4} = 138.458\ \frac{lb}{in}$$

STEP ⑤ $q = \frac{F_{allow}\,(M)}{\Delta x_1}$ → 2 NAILS ARE CARRYING q

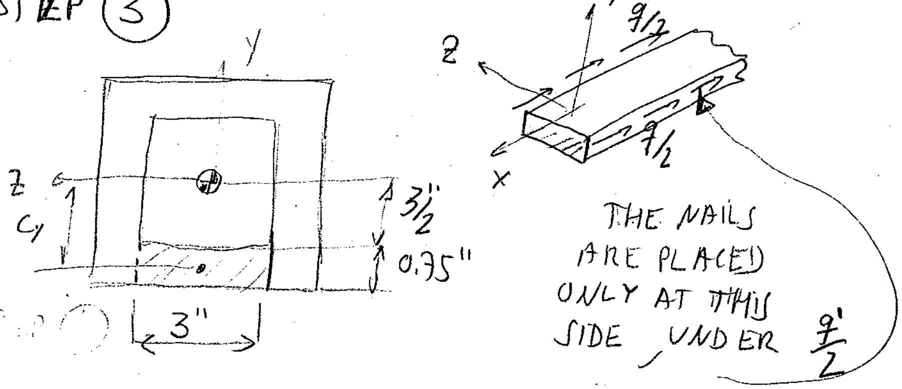
$$\Delta x_1 = \frac{F_{allow} \cdot M}{q} = \frac{80.8\ lb \cdot 2}{138.458\ lb/in} = 1.17\ in$$

CASE II

Δx_2 CALCULATION

STEPS ①, ② AS BEFORE

STEP ③



STEP ④ $Q = c_y \cdot A = \left(\frac{3}{2} + \frac{0.75}{2}\right) \cdot 0.75 \cdot 3 = 4.2188\ in^3$

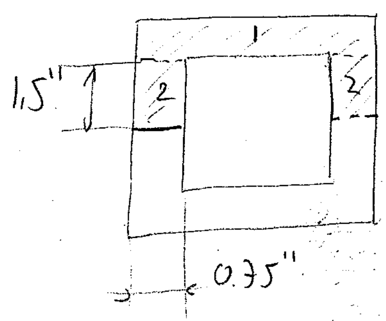
$$|q| = \frac{V_y Q}{I} = \frac{600\ lb \cdot 4.2188\ in^3}{\ldots} = 92.308\ in^3$$

6-4 A

SHEARING STRESS ON GLUE!

STEPS ① AND ② AS BEFORE

STEP ③ - SELECTION



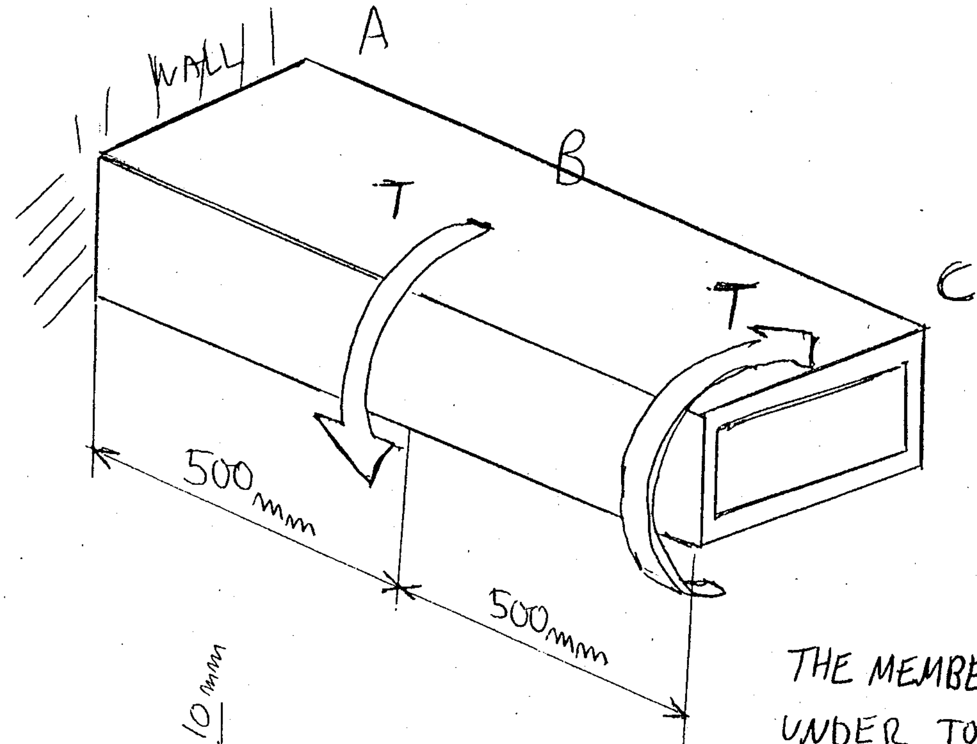
$Q_1 = 6.3281 \, in^3$
FROM THE FIRST PROBLEM

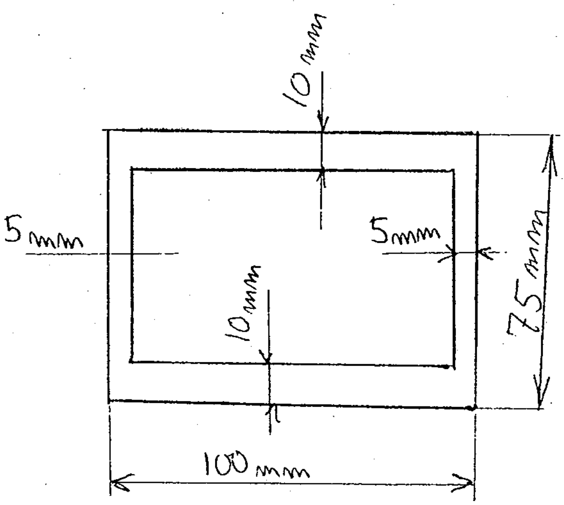
$Q = 6.3281 + 2 \cdot \frac{1.5}{2} \cdot 1.5 \cdot 0.75 = 8.016 \, in^3$

$P = 2 \cdot 0.75'' = 1.5''$

$\tau = \frac{V_y Q}{I_z \cdot P} = \frac{600 \, lb \cdot 8.016 \, in^3}{27.422 \, in^4 \, 1.5 \, in} = 116.92 \text{ PSI}$

QUIZ #3 – SPECIAL VERSION NOV. 22





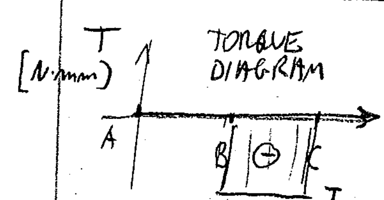
THE MEMBER SHOWN IS UNDER TORSION BY TWO TORQUES PLACED AT B AND AT C.

THE MAXIMUM SHEARING-STRESS IN THE MEMBER IS LIMITED TO 80 MPA

(a) DETERMINE THE MAXIMUM TORQUE T THAT MAY BE APPLIED (75%)

(b) DETERMINE THE TWIST ANGLE AT B (25%)

QUIZ #3 – SPECIAL VERSION – SOLUTION



(a) $\tau = \dfrac{T}{2tA}$ $t = t_{min} = 5\,mm$ $A = (75-10)(100-5) = 6175\,mm^2$

$T = 2 \cdot 5 \cdot 6175 \cdot 80 = \ldots = 4.94\,kN \cdot m$

(b) SINCE $T_{AB} = 0$, $\phi_B = 0$

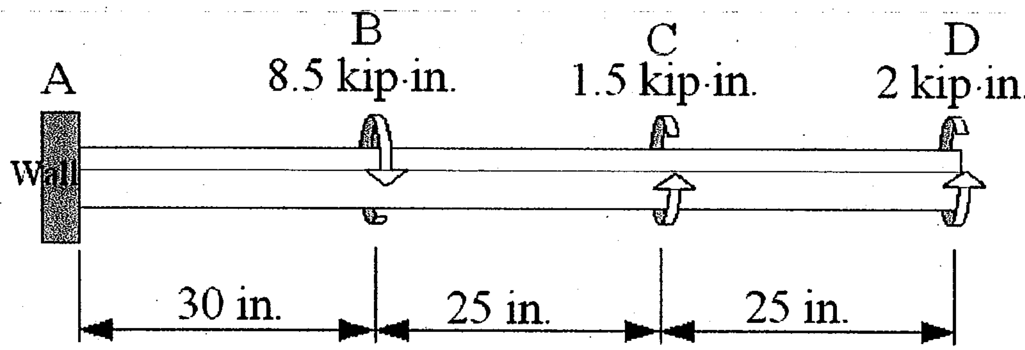
Florida International University
Department of Mechanical and Materials Engineering
Mechanics and Materials Science

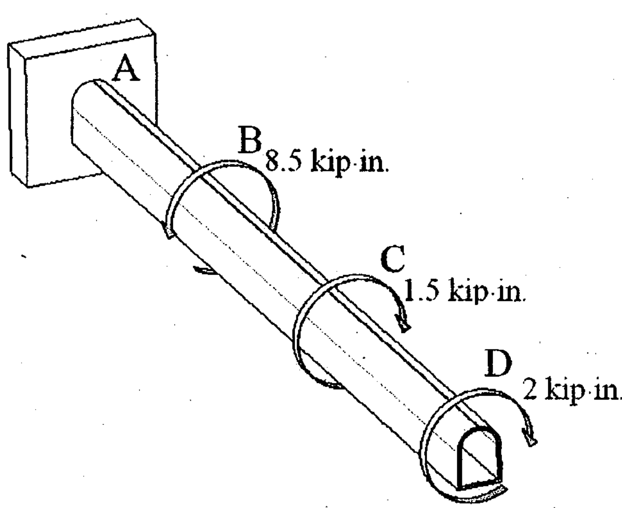
EMA 3702 | Quiz # 3 | Nov. 16, 2004

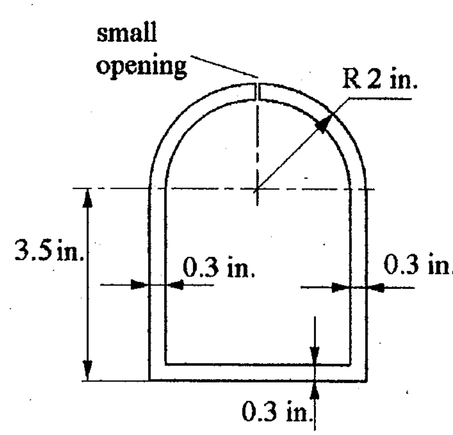
Follow the instructions before you begin the quiz:
This test is 40 minutes long. It is not permitted to pass or receive any hardware nor software during the quiz. Use your own papers, pens, pencils, books, notebooks, calculators, etc.
Write your first and last name and your Panther I. D. Explain your steps, use the adequate diagrams.
Good Luck!

Torques are applied to a non circular member having the cross section shown. Neglect the effects of stress concentration and neglect the dimension of the small opening on the given cross section.
The modulus of rigidity is $11 \cdot 10^6$ PSI

(a) Draw the torque diagram of the given loading (20%)
(b) Determine the maximum shearing stress (40%)
(c) Determine the twist angle at B (40%)

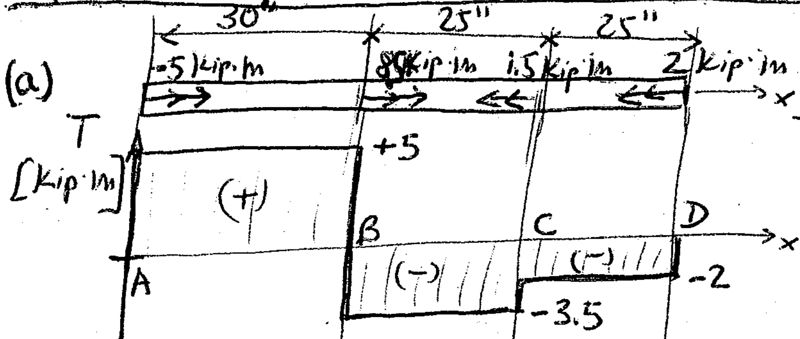






EMA 3702 QUIZ #3 - SOLUTION Nov 16, 2004

(a)



F.B.D

TORQUE DIAGRAM

(b) $\tau_{max} = \dfrac{T_{max}}{c_1 \, a \, b^2}$ $T_{max} = 5 \text{ kip·in}$

$a = 2(3.5'' - 0.15'') + 4'' - 0.3'' + (2'' - 0.15'')\pi = 16.21''$

$b = 0.3'' \rightarrow a/b = 54.04$ ✓

$c_1 = c_2 = \frac{1}{3}\left(1 - 0.63 \frac{b}{a}\right) = 0.32945$

$\tau_{max} = \dfrac{5 \text{ kip·in}}{0.3294 \cdot 16.21'' \cdot 0.3^2 \text{ in}^2} = \underline{10.4 \text{ KSI}}$

(c) $\phi_B = \dfrac{T_{AB} \cdot L_{AB}}{c_2 \, a \, b^3 \, G} = \dfrac{5000 \text{ lb·in} \cdot 30''}{0.3294 \cdot 16.21'' \cdot 0.3^3 \text{ in}^3 \cdot 11 \cdot 10^6 \text{ PSI}} = 0.0946 \text{ rad}$

$\underline{\phi_B = 5.42^\circ}$

	P.I.D.	HM # 1	HM # 2	HM # 3	HM # 4	HM # 5	HM # 6	HM # 7	HM # 8	Quiz 1	Quiz 2	Quiz 3
iana Bonilla	1321865	1	1	1	1	1	1	1	1	28	68	75
Alaa Maaliki	1503723	1	1	1	1	1		1	1	70	78	30
Alfredo Suarez	1495210			1	1	1	1	1	1	64	75	50
Amrit Harripaul	1644446	1	1	1	1	1	1	1		73	55	96
Andres Zamorra	1397908	1	1	1	1	1	1	1	1	94	55	80
Anthony Jackson-Pownal	1277169	1	1	1	1	1	1	1		60	55	55
Arias Carlos	1371993	1	1	1	1		1	1	1	73	58	25
Cindy Gomez	1390408	1	1	1	1	1	1	1		80	97	40
Damian Harper	1399737	1	1	1	1	1	1	1	1	56	30	68
Damien Lloyd	1013644	1	1	1	1	1				98	45	48
Daniel Mugruza	1258280	1	1	1	1	1	1	1	1	50	51	25
Eric Inclan	1356002	1	1	1	1	1	1	1	1	68	35	40
Ernesto Gutierrez	1281667		1	1		1			1	70	85	25
Felipe Rendon	1334090	1	1	1	1	1	1	1	1	50	15	0
Francis Fernandez	1026909	1		1	1	1				76	38	90
Georgette Martinez	1339663	1		1	1		1	1		59	58	20
Gonzalo Ocano	1021312	1	1	1	1	1	1	1	1	87	70	30
Grant Mesner	1366034	1	1	1	1	1	1	1	1	72	63	33
Gustavo Barbera	1398821		1					1	1	77	70	90
Gustavo Jaramillo	1397120	1	1	1	1	1	1	1		84	50	80
Hector Roos	1096368	1			1	1	1	1	1		43	85
Jorge Tercero	1014132	1	1	1	1	1	1	1		77	83	40
Joseph Nichols	1373806	1		1	1	1	1	1		62	85	96
Juan Saluzzio	1372735	1			1					88	72	100
Kern Wilson	1589330	1	1	1	1	1	1	1	1	76	55	80
Luis A Sepulueda	1349774	1	1	1	1	1	1	1	1	31	10	90
e Seever	1106526	1	1	1	1	1		1		100	100	98
Marcelo Beyra	1370183	1		1	1	1	1	1		63	66	96
Maribel Garcia	1322452	1	1	1	1	1	1	1	1	51	25	84
Mario Valdivieso	1369320	1	1	1	1	1	1	1	1	68	71	93
Max Brand	1396896	1	1	1	1	1				83	88	86
Michael Patterson	1094554	1	1	1	1					100	95	100
Michael Wolff	1103264	1								90	10	
Natalie Marshal	1306053	1	1	1	1	1	1	1	1	77	50	75
Ottley Brathwaite	1630841	1		1	1					19	0	
Patrick Belizane	1629148	1	1	1	1	1		1		66	10	15
Paul Girata	1282007	1	1	1	1	1		1	1	8	53	15
Robert Jordan	1305753	1	1	1	1	1	1	1	1	70	52	84
Ruben Galeano	1352635	1	1	1	1	1	1	1	1	64	45	60
Saleh Almutawa	1323191									78		
Shaka Harper	1012782	1	1	1	1	1	1	1	1	63	30	15
Trevis Moorley	1399021	1	1	1	1	1	1	1	1	67	15	63
Trevor G. A. Harris II	1395585	1	1	1	1	1	1	1	1	75	73	86
Verdi M. Mayer	1358420	1	1	1	1	1	1	1	1	73	20	35
William Betancourt	1398230	1	1	1	1	1	1	1	1	10	38	33
Yuri Lazzeretti	1093801	1	1	1	1	1	1	1	1	66	83	35

STEP 5 $\quad \dfrac{q'}{2} \leftarrow q = \dfrac{F_{ALL} \cdot n}{\Delta x}$, $\; n \to 1$

$$q = \frac{q'}{2} = \frac{F_{ALL} \cdot 1}{\Delta x_2} \rightsquigarrow \underline{\underline{\Delta x_2}} = \frac{80.8\ lb \cdot 1}{(92{,}308/2)} = \underline{\underline{1.751\ in}}$$

DISCUSSION

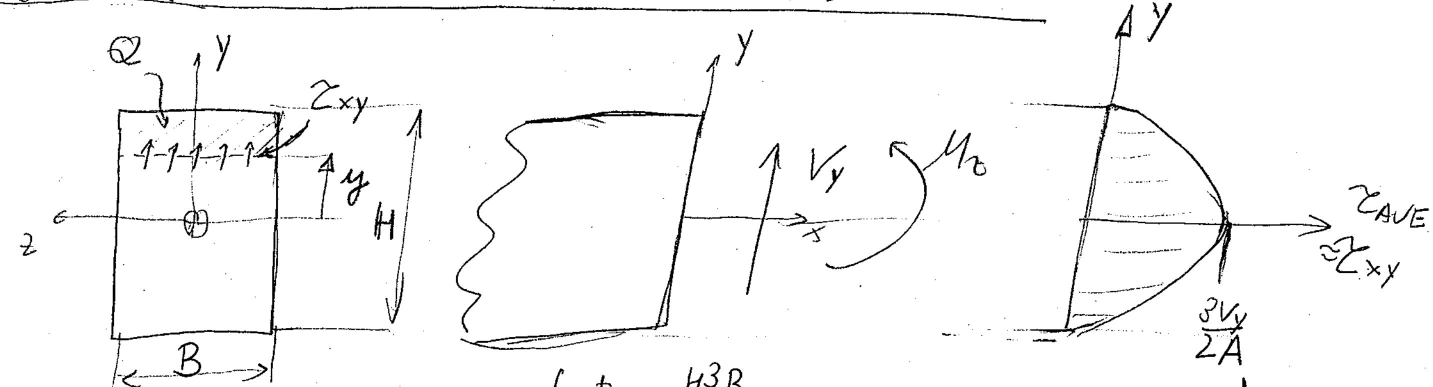
I) $\Delta x_2 > \Delta x$, MORE NAILS PER INCH FOR THE FIRST CASE BECAUSE $Q_I > Q_{II}$

II) Q GROWS AS WE APPROACH THE NEUTRAL AXIS

III) THE MAXIMUM NUMBER OF NAILS PER INCH IS SUPPOSED TO BE AT THE NEUTRAL AXIS

SEE NEXT PAGE → SHEARING STRESS ON GLUE

BENDING - SHEARING STRESS ON RECTANGULAR SECTION



$$\tau_{AVE} = \frac{V_y Q}{I_z\, \mathbb{P}}$$

$$I_z = \frac{H^3 B}{12}$$

$$\mathbb{P} = B$$

$$Q = \underbrace{\left(\frac{\frac{1}{2}H - y}{2} + y\right)}_{c_y}\underbrace{\left(\tfrac{1}{2}H - y\right)(B)}_{A} = \frac{1}{2} B\left(\frac{1}{4}H^2 - y^2\right)$$

$$\tau_{AVE} = \frac{V_y\ \frac{1}{2} B\left(\frac{1}{4}H^2 - y^2\right)}{\frac{H^3 B}{12} \cdot \cancel{B}} = \frac{6}{H^3 B}\left(\frac{1}{4}H^2 - y^2\right) V_y = \frac{3}{2A}\left(1 - \frac{y^2}{c^2}\right) V_y$$

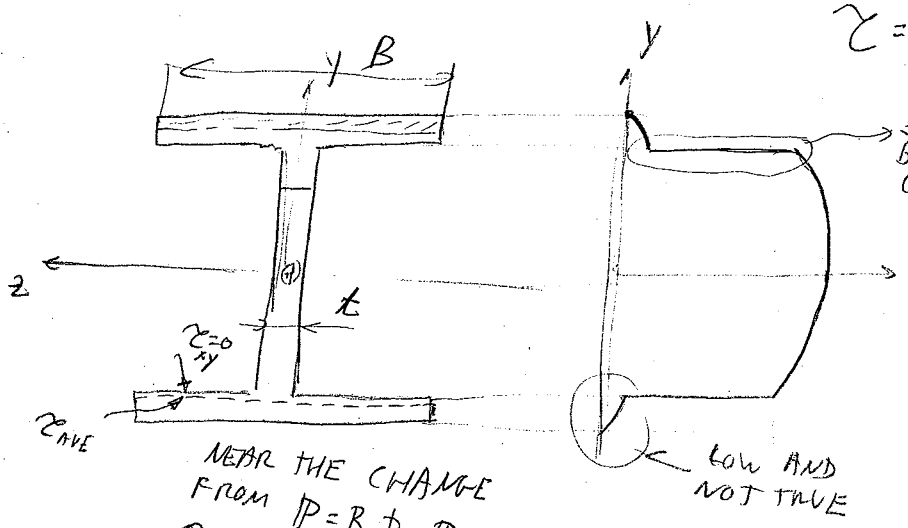
$c = \frac{H}{2}$
$A = H \cdot B$

$$\tau_{AVE}\Big|_{MAX} = \frac{3 V_y}{2A}$$

AT $y = 0$

IN A W-BEAM:

$$\tau = \frac{V_y Q}{I_z P}$$



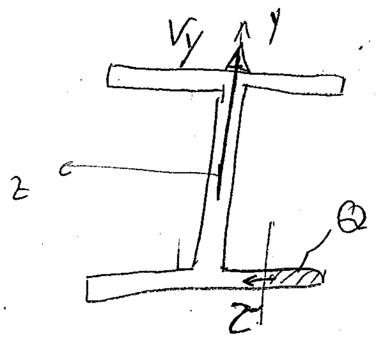
SMALL CHANGE IN Q BUT A SIGNIFICANT DISCRETE CHANGE IN P

τ_{AVE}

LOW AND NOT TRUE

NEAR THE CHANGE FROM $P=B$ to $P=t$ τ_{xy} APPROACHES $0 \neq \tau_{AVE}$

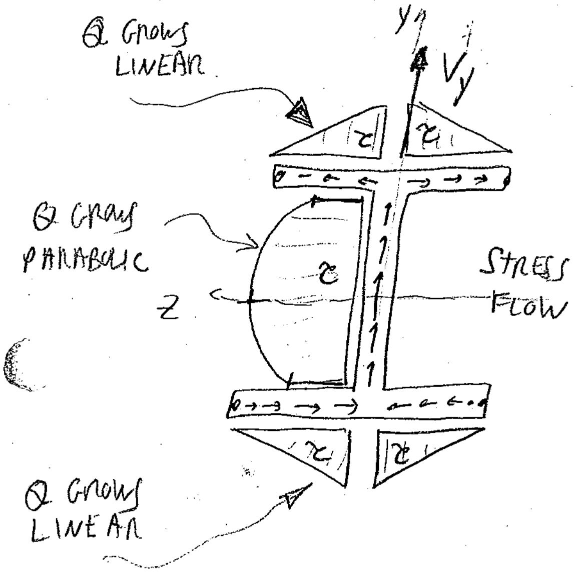
SHEARING STRESSES IN THIN-WALLED MEMBERS

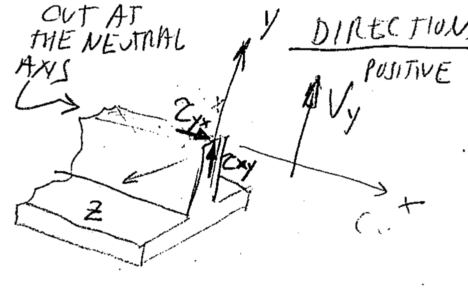


THE SIGNIFICANT SHEAR STRESSES IN THIN-WALLED MEMBERS OCCURS IN PERPENDICULAR TO THE WALL THICKNESS (P MINIMUM)

$$\tau = \frac{V_y Q}{I_z t}$$

t: THE WALL THICKNESS AT THE POINT WHERE τ IS CALCULATED

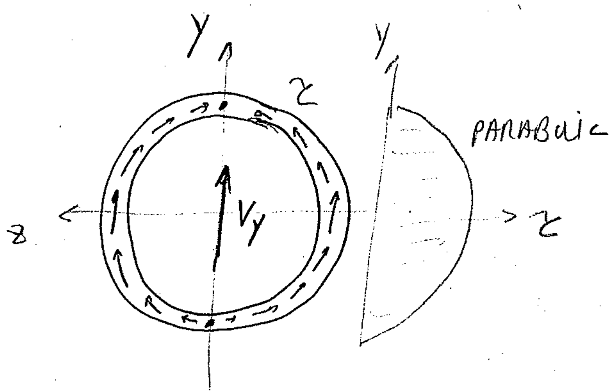




DIRECTIONS POSITIVE

$$\tau_{xy} = +\frac{V_y Q}{I_z t}$$

6-6



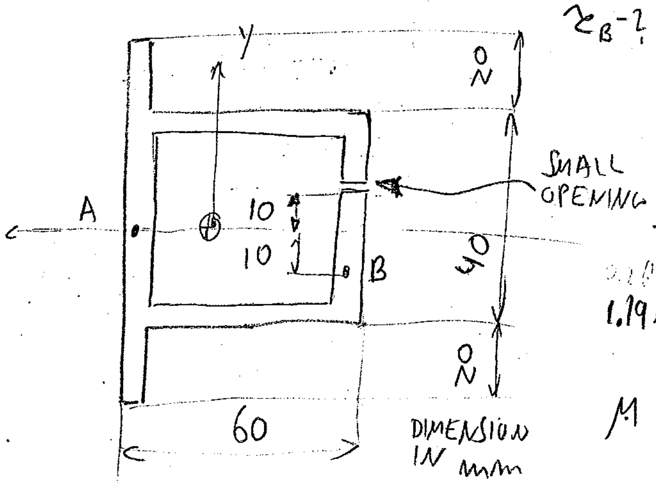
EXAMPLE - SHEARING STRESSES IN THIN-WALLED MEMBERS

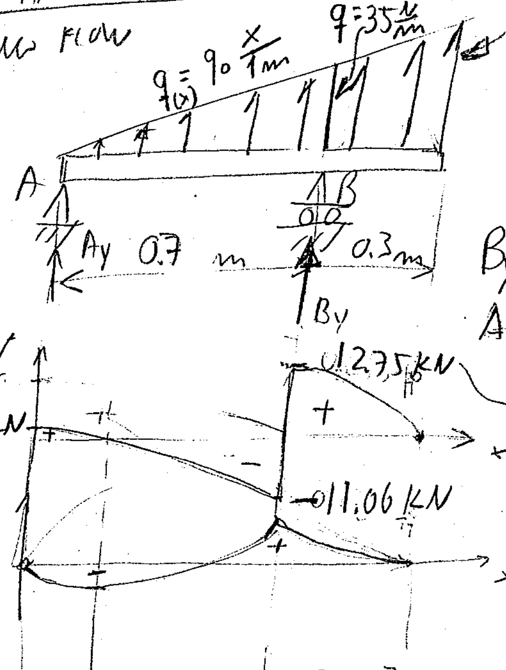
SHEARING FLOW

$t = 3\,mm$

$\tau_A - ?$

$\tau_B - ?$





$B_y = -23.81\,k$

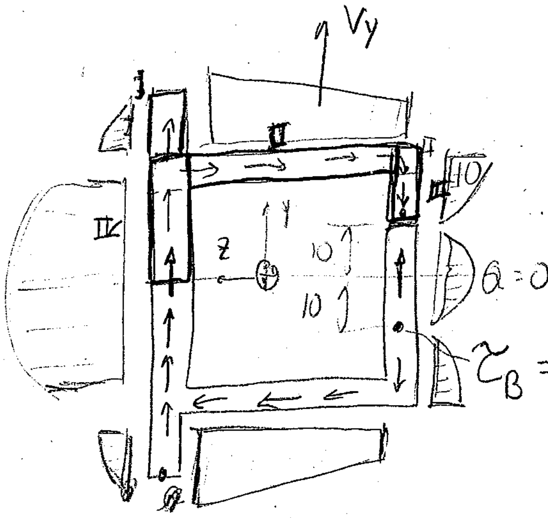
$A_y = -1.19\,k$

$\frac{50.35}{2} \cdot 0.3$

$q_0\left[\frac{N}{m}\right]$

$I_z = 2.55 \cdot 10^5\,mm^4$

$V_y = 12750\,N$



$\tau_B = 0$

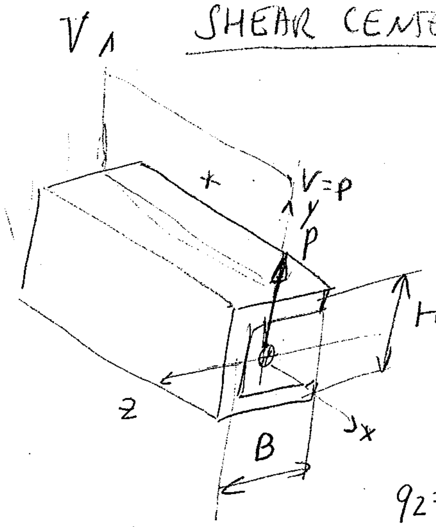
$Q_A = Q_I + Q_{II} + Q_{III} + Q_{IV}$

	A_i mm^2	Cy_i mm	Q_i mm^3
I	3·20	10+20	1800
II	3·54	20-1.5	2997
III	3·10	5+10	450
IV	3·20	10	600

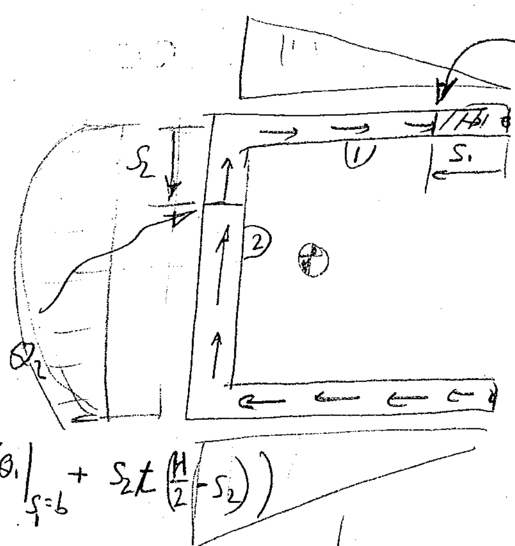
$Q_A = 5847\,mm^3$

$$\tau_A = \frac{V_y Q_A}{I \cdot t} = \frac{12750\,N \cdot 5847\,mm^3}{2.55 \cdot 10^5 \cdot 4 \cdot 3} = 97.5\,MPa$$

UNSYMMETRIC LOADING OF THIN WALLED MEMBERS; SHEAR CENTER



DISTRIBUTION OF $V_y = P$:



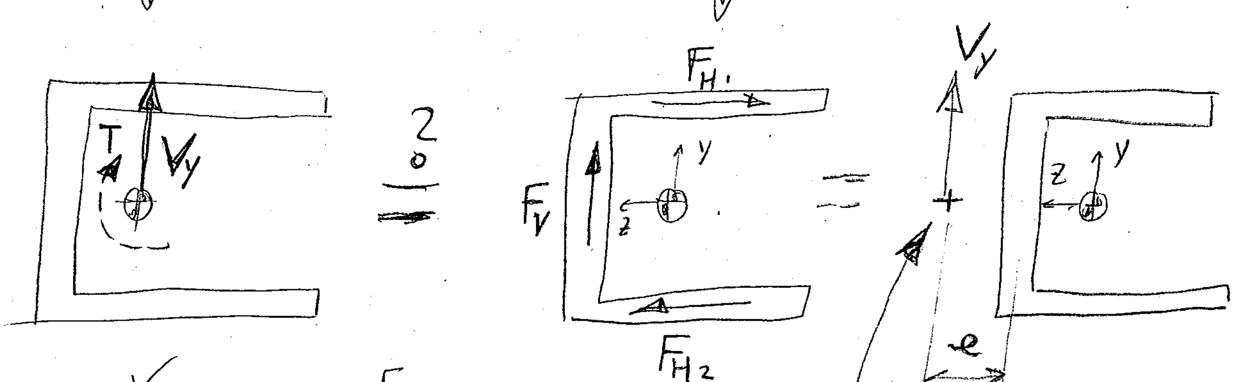
A

$$q_1 = \frac{V_y Q_1}{I_z} = \frac{V_y}{I_z}(s_1 t)\frac{H}{2}$$

LINEAR

$$q_2 = \frac{V_y}{I_z} Q_2 = \frac{V_y}{I_z}\left(Q_1\Big|_{s_1=b} + s_2 t\left(\frac{H}{2} - s_2\right)\right)$$

PARABOLIC



✓	F_x	$F_V = V$
✓	F_y	$F_H = F_H$
?	M_x	$F_H \cdot H$

BUT WE DIDN'T APPLY T,
SO WE ARE TWISTING THE BEAM!!
IN THE OPPOSITE DIRECTION TO T !!

SHEAR CENTER OF THE CROSS SECTION

WE ARE GOING TO FIND e IN ORDER TO EQUILIZE THE APPLIED AND THE DISTRIBUTED LOAD.

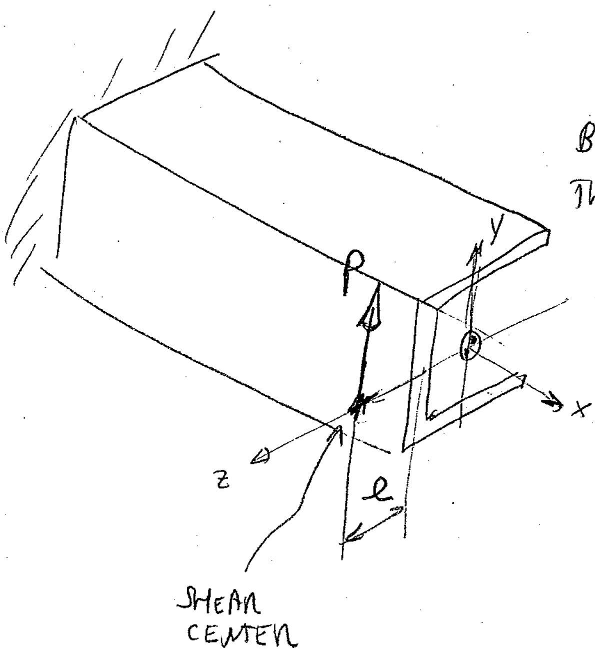
$$F_H \cdot H = V_y \cdot e$$

$$F_H = \int_0^b q_1\,ds_1 = \frac{V_y}{I_z}\frac{tH}{2}\int_0^B s_1\,ds_1 = \frac{V_y}{I_z}\frac{tHB^2}{4}$$

$$\frac{V_y}{I_z}\frac{tH^2B^2}{4} = V_y \cdot e$$

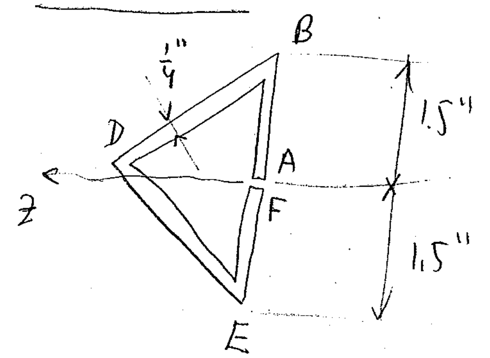
$$\boxed{e = \frac{tH^2B^2}{4 I_z}}$$

SHEAR CENTER:
A FEATURE OF THE CROSS SECTION ONLY
(NO RELATION TO THE MODE OR MAGNITUDE OF LOADING)



BENDING WITHOUT TORSION
THE LOAD APPLIED ON THE SHEAR CENTER OF THE CROSS SECTION

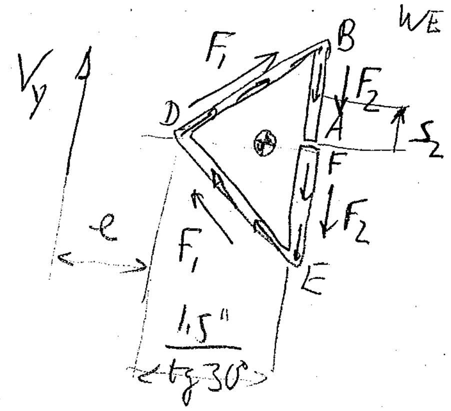
PROB P6.70



DETERMINE THE LOCATION OF THE SHEAR CENTER

$I_z = 1.6875 \text{ in}^4$

MOMENTS AROUND D
WE WILL FIND e



$I_z = 1.6875 \text{ in}^4$

$$q_2 = \frac{V_y}{I_z} Q_2$$

$$Q_2 = S_2 t \cdot \frac{S_2}{2}$$

$$F_2 = \int_A^B q_2 \, dS_2 = \frac{V_y}{I_z} \int_0^{1.5"} S_2 t \frac{S_2}{2} \, dS_2$$

$$F_2 = \frac{V_y}{1.6875} \cdot \underbrace{\frac{1"}{4}}_{t} \cdot \frac{1}{2} \cdot \frac{1.5^3}{3} = \frac{V_y}{12}$$

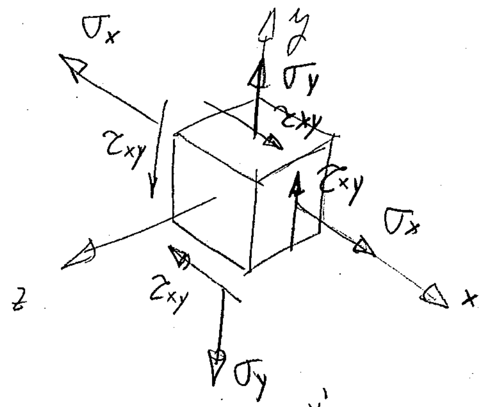
$$\circlearrowleft + \; \Sigma M_D = -2F_2 \cdot \frac{1.5"}{tg 30°} = -V_y e$$

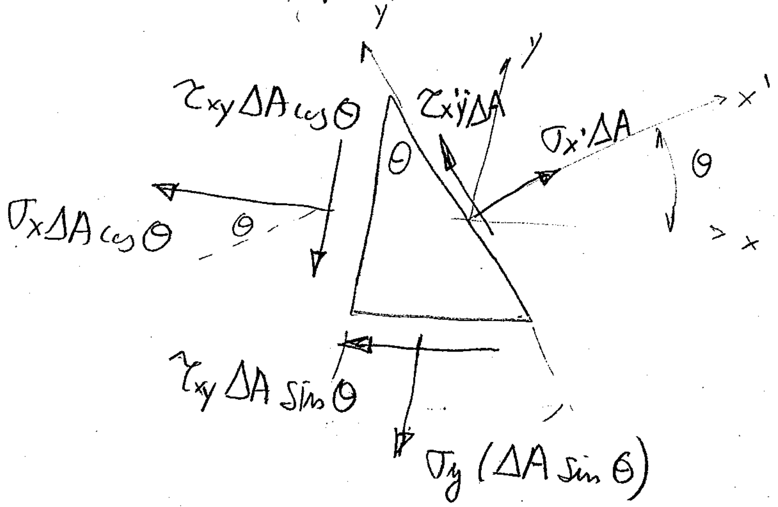
$$2 \frac{V_y}{12} \cdot \frac{1.5"}{tg 30°} = V_y e \rightsquigarrow \underline{\underline{e = 0.433"}}$$

HOME WORK

TRANSFORMATION OF STRESS AND STRAIN

PLANE STRESS TRANSFORMATION





$$\sum F_{x'}=0=\sigma_{x'}\Delta A-\sigma_x(\Delta A\cos\theta)\cos\theta-\tau_{xy}(\Delta A\cos\theta)\sin\theta-$$
$$-\sigma_y(\Delta A\sin\theta)\sin\theta-\tau_{xy}(\Delta A\sin\theta).\cos\theta=0$$

$$\boxed{\sigma_{x'}=\sigma_x\cos^2\theta+\sigma_y\sin^2\theta+2\tau_{xy}\sin\theta\cos\theta}$$

$$\sum F_{y'}=0=\tau_{x'y'}\Delta A+\sigma_x(\Delta A\cos\theta)\sin\theta-\tau_{xy}(\Delta A\cos\theta)\cos\theta-$$
$$-\sigma_y(\Delta A\sin\theta)\cos\theta+\tau_{xy}(\Delta A\sin\theta)\sin\theta$$

$$\boxed{\tau_{x'y'}=-(\sigma_x-\sigma_y)\sin\theta\cos\theta+\tau_{xy}(\cos^2\theta-\sin^2\theta)}$$

7-2

USING TRIGONOMETRIC RELATIONS,

$$\sin 2\theta = 2\sin\theta\cos\theta, \quad \cos 2\theta = \cos^2\theta - \sin^2\theta,$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}, \quad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\sigma_{x'} = \sigma_x \frac{1+\cos 2\theta}{2} + \sigma_y \frac{1-\cos 2\theta}{2} + \tau_{xy}\sin 2\theta$$

OR

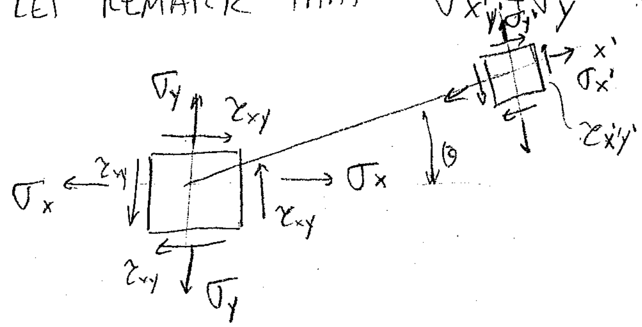
$$\boxed{\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta} \quad (1)$$

$$\boxed{\tau_{x'y'} = -\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta} \quad (2)$$

IN ORDER TO FIND THE RELATION FOR $\sigma_{y'}$, LET REPLACE θ BY $\theta+90$, AN THEN:

$$\boxed{\sigma_{y'} = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta} \quad (3)$$

LET REMARK THAT $\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y$ $\rightsquigarrow$ IN THE PLANE STRESS CASE THE SUM OF σ_x AND σ_y IS CONSTANT AND INDEPENDENT OF IT DIRECTION



LET PERFORM SOME ALGEBRAIC OPERATIONS:

$$(1)\ \left\{\left(\sigma_{x'} - \frac{\sigma_x+\sigma_y}{2}\right)\right\}^2 = \left\{\frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta\right\}^2$$

$$+ \qquad\qquad +$$

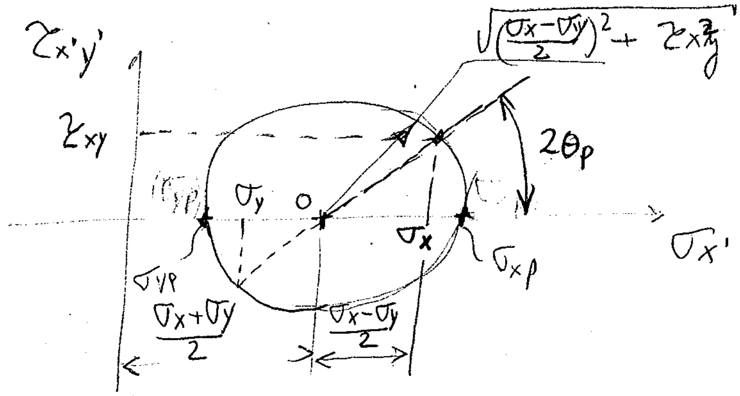
$$(2)\ \left\{\tau_{x'y'}\right\}^2 = \left\{-\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta\right\}^2$$

$$\left(\sigma_{x'} - \frac{\sigma_x+\sigma_y}{2}\right)^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 \overbrace{(\cos^2 2\theta + \sin^2 2\theta)}^{1} + \tau_{xy}^2 \overbrace{(\sin^2 2\theta + \cos^2 2\theta)}^{1} + 0$$

$$\left(\sigma_{x'} - \frac{\sigma_x+\sigma_y}{2}\right)^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$(x - x_0)^2 + y^2 = R^2$$

EQUATION OF A CIRCLE WITH RADIUS R ITS CENTER ON $(x_0, 0)$



MOHR'S CIRCLE FOR PLANE STRESS

σ_x, τ_{xy} SHOULD BE FOUND ON THAT CIRCLE

CONCLUSIONS:

(I) $$\boxed{\text{MAX } \sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}$$

WHEN $\sigma_{x'}$ IS MAX, $\tau_{x'y'}=0$

DEFINITION

THE PLANE IN WHICH $\tau_{x'y'}=0$ IS CALLED THE PRINCIPAL PLANE

IN ORDER TO FIND THE PRINCIPAL PLANE:

(2) $$\tau_{x'y'} = 0 = -\frac{\sigma_x-\sigma_y}{2}\sin 2\theta_p + \tau_{xy}\cos 2\theta_p$$

$$\boxed{\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}}$$

$\theta_p \rightarrow$ ANGLE θ IN WHICH THE PRINCIPAL PLANE IS FOUND

$\sigma_{x'}+\sigma_{y'} = \sigma_x+\sigma_y$ (WE SAW BEFORE)

THEN ON THE PRINCIPAL PLANE

$$\boxed{\sigma_{y'} = (\sigma_x+\sigma_y) - \sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}$$

IT IS THE MINIMAL AVAILABLE VALUE FOR $\sigma_{x'}$

(II) THE MAXIMUM POSSIBLE SHEARING STRESS IS $\tau_{x'y'}\text{ MAX} = \pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$

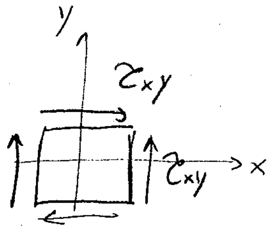
AND IT IS FOUND 45° FROM THE PRINCIPAL PLANE

PRINCIPAL STRESSES

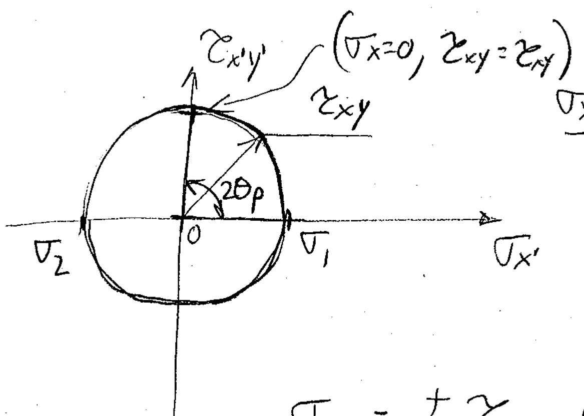
$$\boxed{\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}$$

EXERCISES

① FIND THE PRINCIPAL STRESSES AND PRINCIPAL PLANE FOR THE TORSION CASE (PURE SHEARING)



$\sigma_x = 0$
$\sigma_y = 0$
$\tau_{xy} = \tau_{xy}$

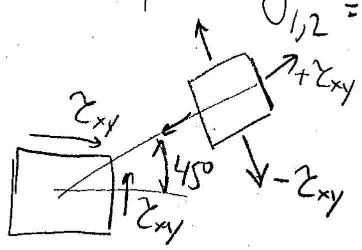


$\frac{\sigma_x+\sigma_y}{2} = 0$, THEN THE CIRCLE ON THE SYSTEM ORIGIN

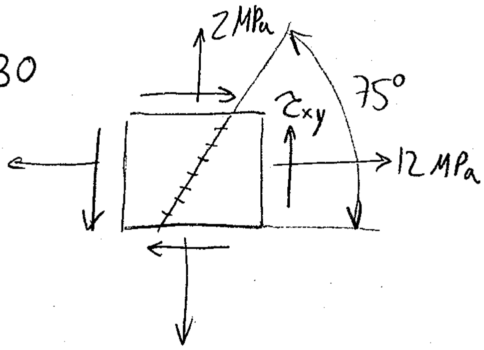
$\tau_{xy'} = 0$ WHEN $2\theta_p = 90°\left(= \operatorname{atan}\left\{\frac{2\cdot\tau_{xy}}{0}\right\}\right)$, $= \infty$

$\theta_p = 45°$

$$\sigma_{1,2} = \pm\tau_{xy}\ \left(= \underbrace{\frac{\sigma_x+\sigma_y}{2}}_{0} \pm \sqrt{\underbrace{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2}_{0} + \tau_{xy}^2}\right)$$

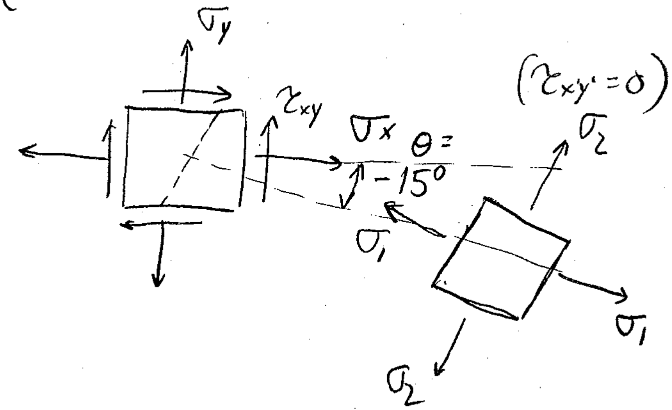


② 7.30



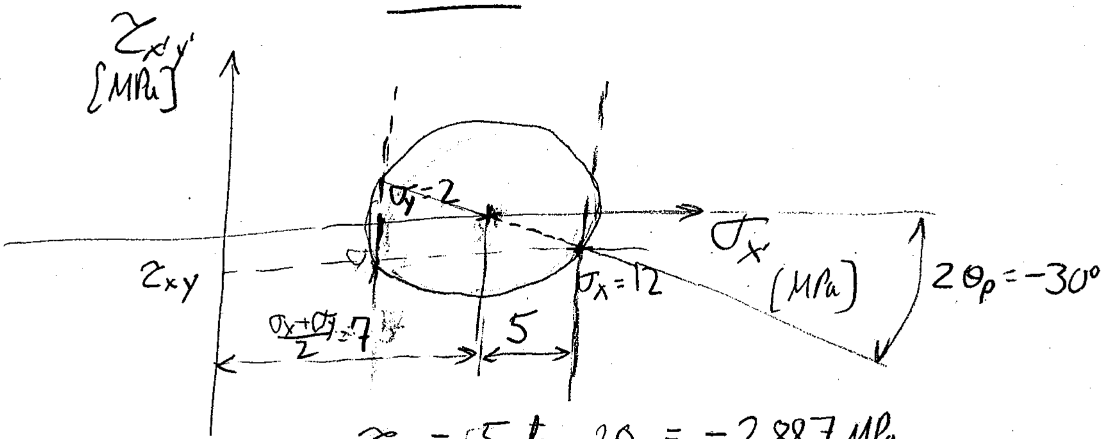
(a) $\tau_{x'y'} = 0$ AT WELDING PLANE; τ_{xy} - ?
(b) σ_1, σ_2 - ? (PRINCIPAL STRESSES - ?)

(a) SHEARING AT THE PLANE OF THE WELDING:



$\sigma_x = 12$ MPa
$\sigma_y = 2$ MPa
τ_{xy} - ?
$\theta_p = -15°$

7-5

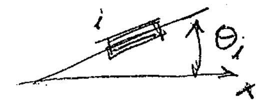


$$\tau_{xy} = 5 \tan \underbrace{2\theta_p}_{-30°} = -2.887 \text{ MPa}$$

(b) $\sigma_{1,2} = \underbrace{\frac{\sigma_x+\sigma_y}{2}}_{7 \text{ MPa}} \pm \underbrace{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}_{\text{RADIUS}}$

$\text{RADIUS} = \sqrt{5^2+2.887^2} = 5.774 \text{ MPa}$

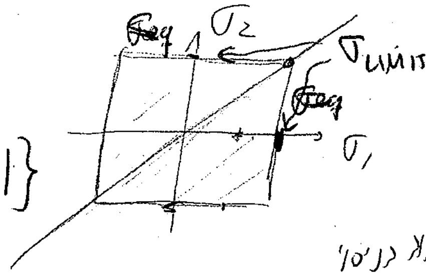
$\varepsilon_i = \varepsilon_x \cos^2\theta_i + \varepsilon_y \sin^2\theta_i + \gamma_{xy} \sin\theta_i \cos\theta_i$



$\sigma_1 = 7 + 5.774 = 12.77 \text{ MPa}$

$\sigma_2 = 7 - 5.774 = 1.226 \text{ MPa}$

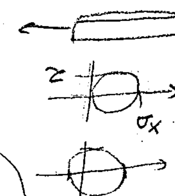
YIELD CRITERION (FAILURE CRITERIA)



FOR BRITLE MATERIALS: $\sigma_{LIMIT} \geq \sigma_{eq} = MAX\{|\sigma_1|, |\sigma_2|\}$

"MAXIMUM-NORMAL STRESS CRITERION"

מתח צירי

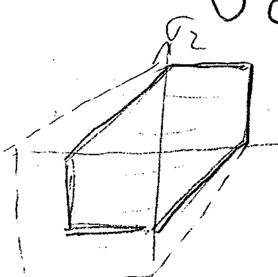


$\tau_{all} = \frac{\sigma_{all}}{2}$

$\tau = \frac{\sigma_1-\sigma_2}{2}$

FOR DUCTILE MATERIALS

"MAXIMUM-SHEARING-STRESS CRITERION"



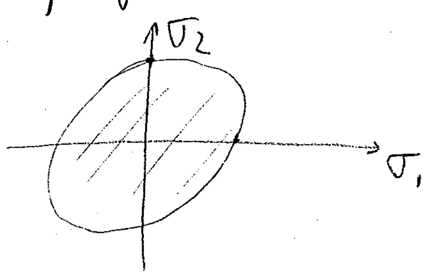
$$\sigma_{LIMIT} \geq \sigma_{eq} = MAX \begin{cases} |\sigma_1 - \sigma_2| \\ |\sigma_1 - \cancel{\sigma_3} \to 0| \\ |\sigma_2 - \cancel{\sigma_3} \to 0| \end{cases} \text{ PLANE STRESS}$$

$\frac{\sigma_{all}}{2} = \frac{\sigma_1-\sigma_2}{2}$

OR

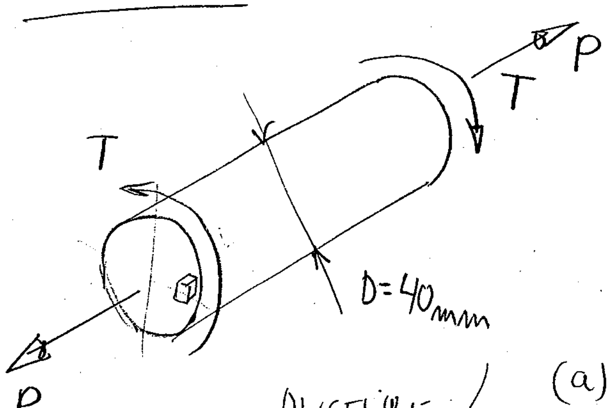
MAXIMUM-DISTORTION-ENERGY CRITERION (VON MISES CRITERION)

$$\sigma_{LIMIT} \geq \sigma_{eq} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$



7-6

EXERCISE – YIELD CRITERION

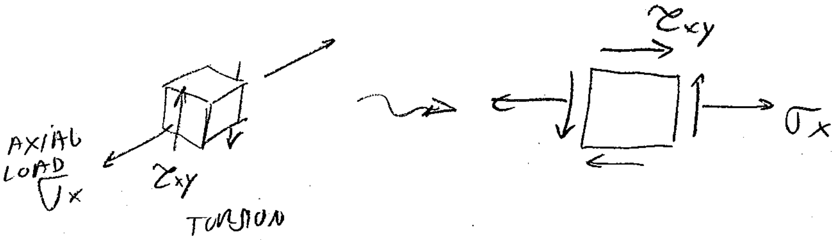


$P = 200\ kN$
$T = 700\ N \cdot m$
$\sigma_{y_{yield}} = 500\ MPa$

CALCULATE THE FACTOR OF SAFETY FROM YIELD

DUCTILE { (a) ACCORDING TO MAXIMUM SHEARING STRESS CRITERI[ON]
(b) " " VON-MISES CRITERION

BRITLE – (c) " " MAXIMUM NORMAL STRESS CRITERI[ON]



$$\sigma_x = \frac{P}{A} = \frac{200 \cdot 1000\,N}{\left\{\frac{\pi \cdot 40^2}{4}\right\} mm^2} = 159.15\ MPa$$

$$\tau_{xy} = \frac{T \cdot r}{J}$$

A LARGER τ_{xy} INCREASES $\sigma_1 \longrightarrow r_{max} = \frac{D}{2}$

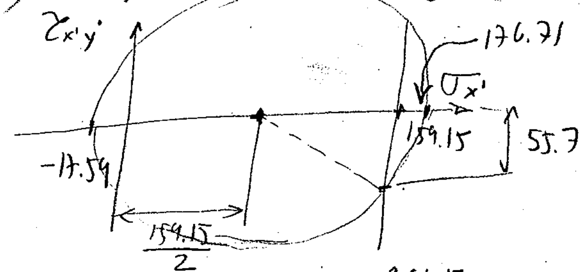
$$\text{SO} \quad \tau_{xy} = \frac{T \cdot \frac{D}{2}}{\left(\frac{\pi D^4}{32}\right)} = \frac{16 \cdot 700 \cdot 1000\,N\,m}{\pi \cdot 40^3\,mm}$$

$$\tau_{xy} = 55.7\ MPa$$

THE PRINCIPAL STRESSES:

$$\sigma_{1,2} = \frac{\sigma_x + \cancel{\sigma_y}^{\,0}}{2} \pm \sqrt{\left(\frac{\sigma_x - \cancel{\sigma_y}^{\,0}}{2}\right)^2 + \tau_{xy}^2} = \frac{159.15}{2} \pm \sqrt{\left(\frac{159.15}{2}\right)^2 + 55.7^2}$$

$\sigma_1 = 176.71\ MPa$
$\sigma_2 = -17.56\ MPa$



MOHR'S CIRCLE

(a) MAXIMUM SHEARING STRESS CRITERION

$$\sigma_{eq_{T(a)}} = MAX \begin{cases} |\sigma_1 - \sigma_2| \\ |\sigma_1| \\ |\sigma_2| \end{cases} = MAX \begin{cases} 176.71 + 17.56 \\ 176.71 \\ 17.56 \end{cases} = 194.27\ MPa$$

7-7

$$SF_{(a)} = \frac{\sigma_y}{\sigma_{eq(a)}} = \frac{500 \text{ MPa}}{194.27 \text{ MPa}} = 2.57$$

(b) VON-MISES CRITERION

$$\sigma_{eq(b)} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} = \sqrt{176.71^2 - 176.71(-17.56) + 17.56^2} = 186.11 \text{ MPa}$$

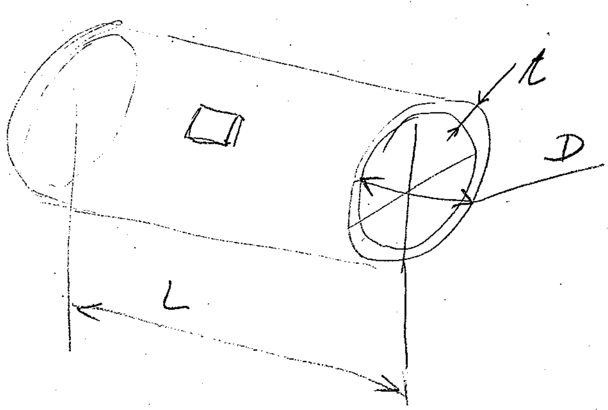
$$SF_{(b)} = \frac{\sigma_y}{\sigma_{eq(b)}} = \frac{500 \text{ MPa}}{186.11 \text{ MPa}} = 2.69$$

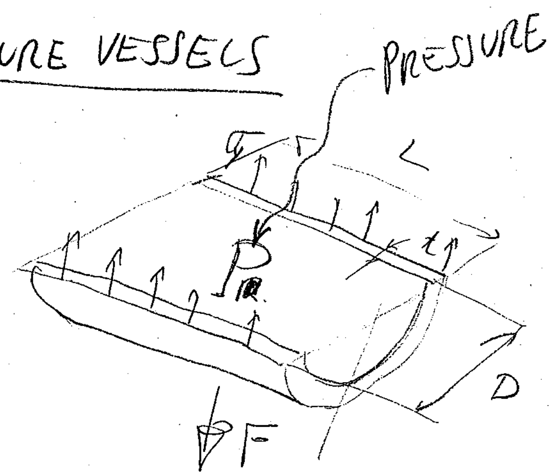
(c) MAXIMUM NORMAL STRESS CRITERION

$$\sigma_{eq(c)} = \text{MAX}\{|\sigma_1|, |\sigma_2|\} = 176.71$$

$$SF_{(c)} = \frac{\sigma_y}{\sigma_{eq(c)}} = \frac{500 \text{ MPa}}{176.71 \text{ MPa}} = 2.84$$

STRESSES IN THIN-WALLED PRESSURE VESSELS

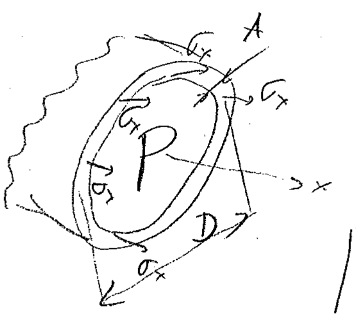




$$\sigma_y \cdot A_t = P_R \cdot A_D$$

$$\sigma_y \cdot 2 \cdot L \cdot t = P_i \cdot D \cdot L$$

$$\boxed{\sigma_y = \frac{PD}{2t} = \frac{PR}{t}}$$



$$\sigma_x \cdot A_t = P \cdot A_D$$

$$\sigma_x \, \pi D t = P \frac{\pi D^2}{4}$$

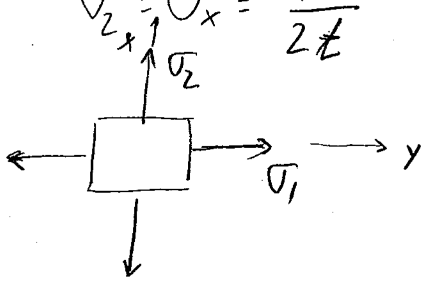
$$\boxed{\sigma_x = \frac{PD}{4t} = \frac{PR}{2t}}$$

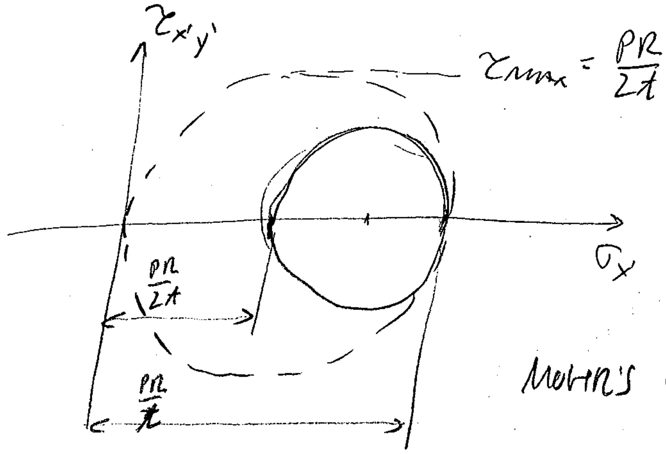
7-8

THERE IS NO LOAD THAT CAUSES τ_{xy}, SO σ_x AND σ_y ARE THE PRINCIPAL STRESSES

$$\sigma_1 = \sigma_y = \frac{PR}{t}$$

$$\sigma_2 = \sigma_x = \frac{PR}{2t}$$



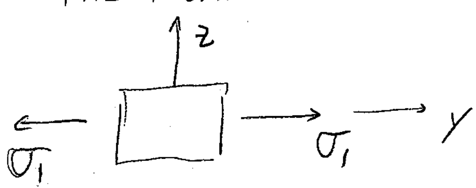


MOHR'S CIRCLE

VESSELS ARE MAKE OF DUCTILE MATERIALS, SO ACCORDING TO MAXIMUM SHEARING STRESS CRITERION

$\sigma_{eq} = \frac{PR}{t}$, TAKING INTO ACCOUNT THAT $\sigma_z = 0$

THE MOHR'S CIRCLE FOR THE ELEMENT IN THAT PROJECTION



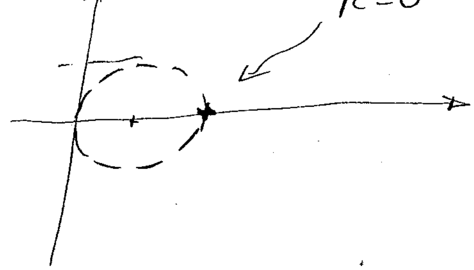
AND IT APPROPIATE MOHR'S CIRCLE IN DASHED LINE.

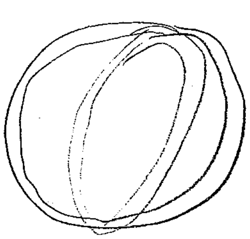
SO $\tau_{max} = \frac{PR}{2t}$

FOR HIGH PRESSURE VESSELS, A SPHERICAL VESSEL IS USED

SINCE $\sigma_1 = \sigma_2 = \frac{PR}{2t}$

MOHR'S CIRCLE
R=0



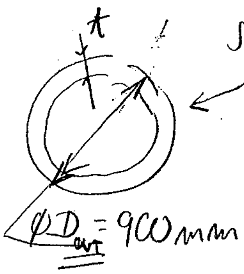


FOR $\sigma_z = 0$:

$$\sigma_{eq} = \frac{PR}{2t}$$

$$\tau_{max} = \frac{PR}{4t}$$

EXAMPLE



SPHERICAL VESSEL

$$\left.\begin{array}{l}\sigma_U = 400\,MPa \\ SF = 4 \\ P_{RES} = 3.5\,MPa\end{array}\right\} t - ?$$

FOR SPHERICAL VESSEL:

$$\sigma_{eq} = \frac{P R_{IN}}{2t}$$

$$R_{IN} = R_{OUT} - t = \frac{D_{OUT}}{2} - t$$

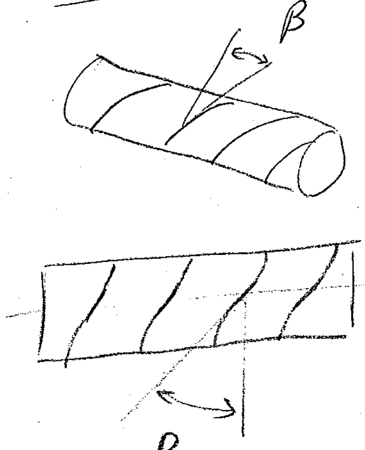
$$\sigma_{eq} = \frac{\sigma_U}{SF} = 100\,MPa$$

$$100_{MPa} = \frac{3.5\,MPa \cdot (450 - t)\,mm}{2 \cdot t\,mm}$$

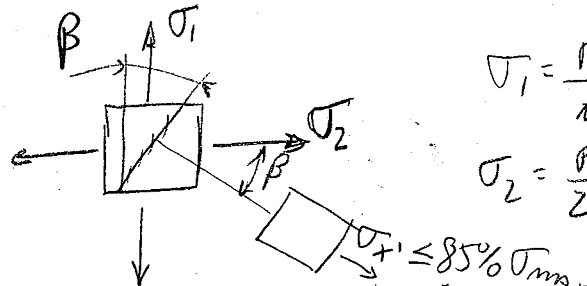
$\vdots$

$$t = 7.74\,mm$$

EXAMPLE 7.115

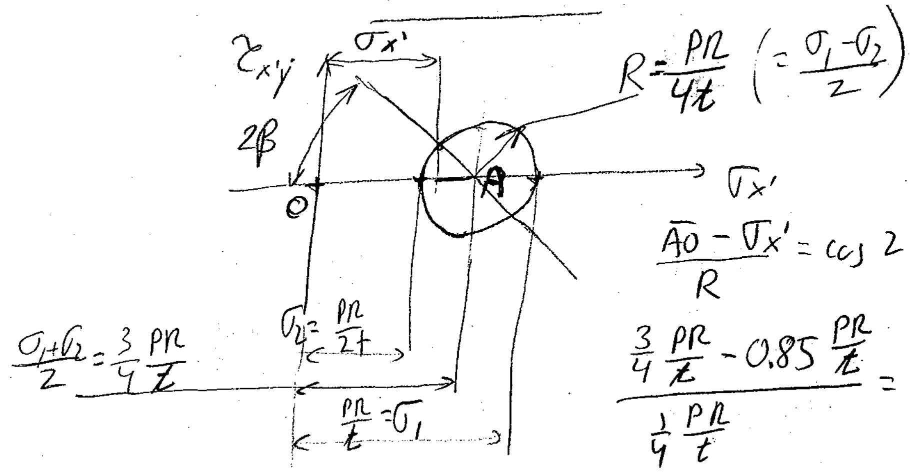


PIPE
FABRICATED BY WELDING STRIPS
HELIX ANGLE β AS SHOWN
DESIGN REQUIREMENT:
- NORMAL STRESS TO THE WELD NOT LARGER THAN 85% OF THE MAXIMUM STRESS IN THE PIPE



$$\left.\begin{array}{l}\sigma_1 = \frac{PR}{t} \\ \sigma_2 = \frac{PR}{2t}\end{array}\right\} \sigma_{max} = \frac{PR}{t}$$

MOHR CIRCLE:



$$\frac{\overline{AO} - \sigma_{x'}}{R} = \cos 2\beta$$

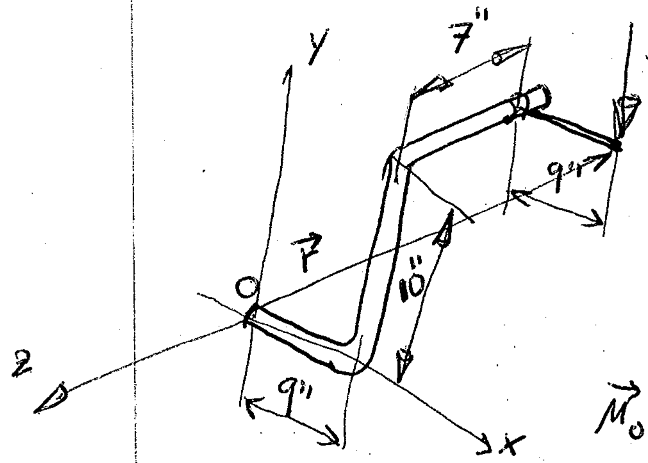
$$\frac{\frac{3}{4}\frac{PR}{t} - 0.85\frac{PR}{t}}{\frac{1}{4}\frac{PR}{t}} = \cos 2\beta$$

7-10

$$\frac{\frac{3}{4}-0.85}{\frac{1}{4}} = -0.4 = \cos 2\beta$$

$$2\beta = 113.578^\circ$$

$$\underline{\underline{\beta = 56.789^\circ}}$$



$$\vec{M}_O = \vec{r}\times\vec{F}$$

$$\vec{r} = (9"+9")\hat{i} + 10"\hat{j} - 7"\hat{k}$$

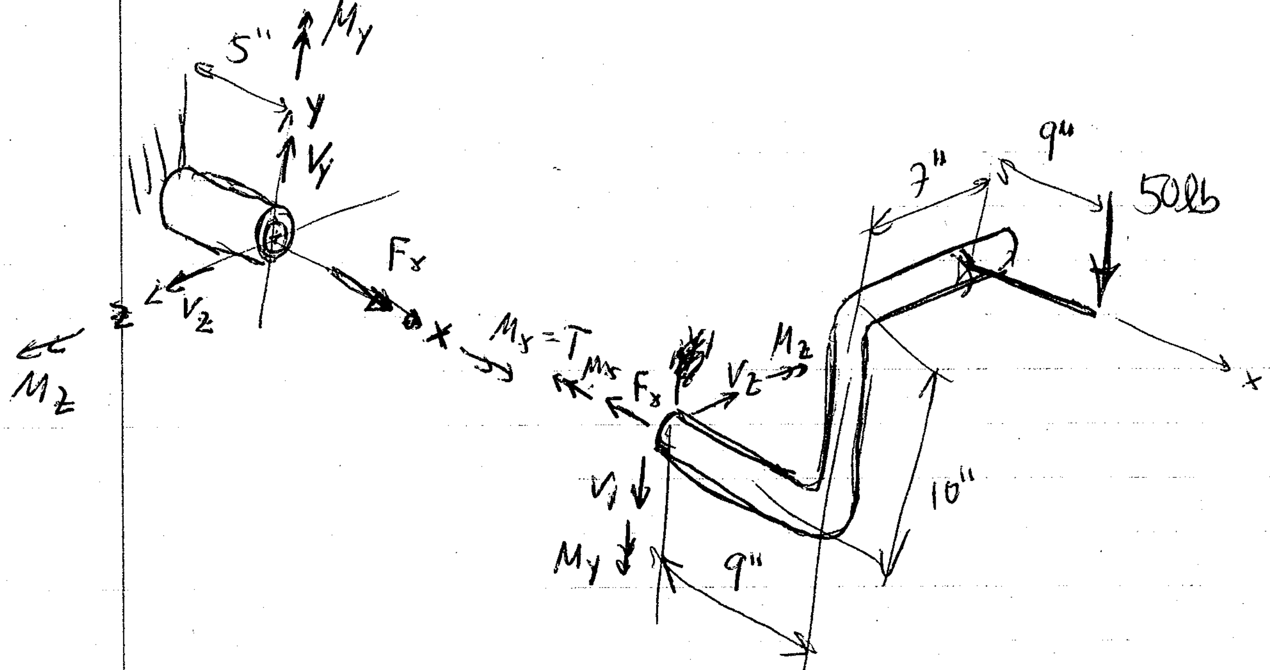
$$\vec{F} = -50\,\hat{j}\ \text{lb}$$

$$\vec{M}_O = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 18" & 10" & -7" \\ 0 & -50\text{lb} & 0 \end{vmatrix} = \overbrace{-50\cdot 7\,\hat{i}}^{M_x = T} - \underbrace{50\cdot 18\,\hat{k}}_{M_z}$$

$$\vec{M}_O = \underbrace{-350\,\hat{i}}_{M_x = T} - \underbrace{900\,\hat{k}}_{M_z}$$

$(M_y = 0)$

FROM THE VECTORIAL MULT. IT CAN BE SEEN THAT THE ARM OF FORCE 50 lb AROUND x AXIS IS 7" (AND NOT $\sqrt{7^2+10^2}$) BECAUSE 7" IS THE DISTANCE FROM THE "LINE OF ACTION" OF 50lb TO THE x AXIS



$$\Sigma F_x = 0 = F_x$$

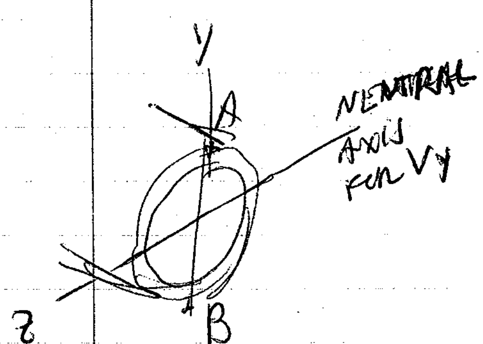
$$\Sigma F_y = -V_y - 50\ lb = 0 \qquad V_y = -50\,lb$$

$$\Sigma F_z = 0 = V_z = 0$$

$$\Sigma M_x = -M_x - 50 \cdot 7'' = 0 \qquad M_x = -350\ in \cdot lb$$

$$\Sigma M_y = 0 = -M_y + 0$$

$$\Sigma M_z = -M_z - 50(9''+9'') = 0 \qquad M_z = -900\ lb\ in$$

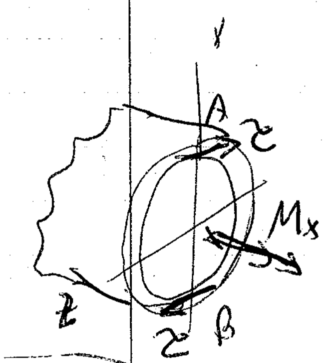


$$V_y = -50\,lb$$

$$\tau = \frac{V_y Q}{I t}$$

$$Q_A = 0, \quad Q_B = 0$$

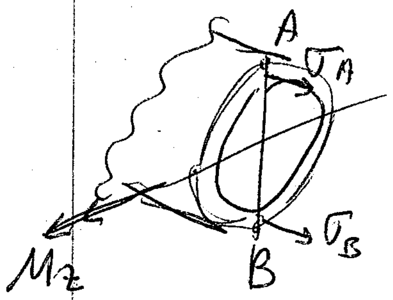
$$\tau = 0$$



$$T = M_x = -350\ lb \cdot in$$

$$\tau = \frac{M_x R}{I_z} = \frac{-350 \cdot 0.5''}{\frac{\pi}{32}\left(1^4 - \left(1 - \frac{1}{4}\right)^4\right)} = 2607.59\ PSI$$

2t



$M_z = -900$ lb in

$$\sigma = \frac{M_z\, c}{I_z}$$

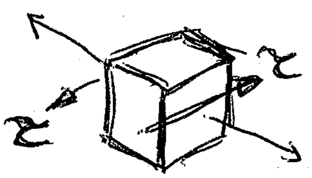
$c_A = +R = +0.5''$

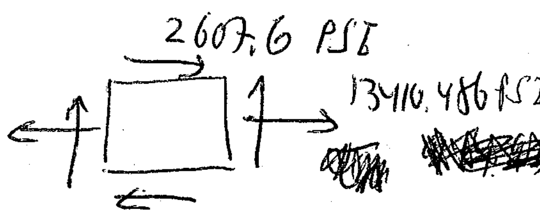
$c_B = -R = -0.5''$

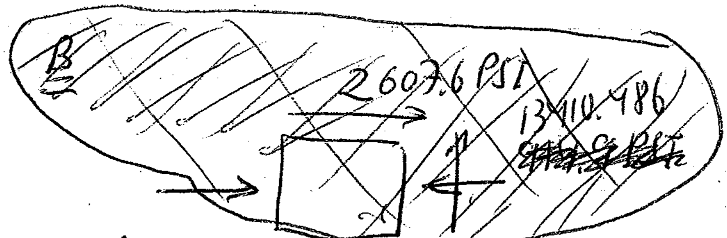
$$\sigma_{A x} = -\frac{(-900)\cdot 0.5}{\frac{\pi}{64}\left(1^4 - 0.75^4\right)} = 13410.486$$

$$\sigma_{B x} = -\sigma_A = -13410.486$$

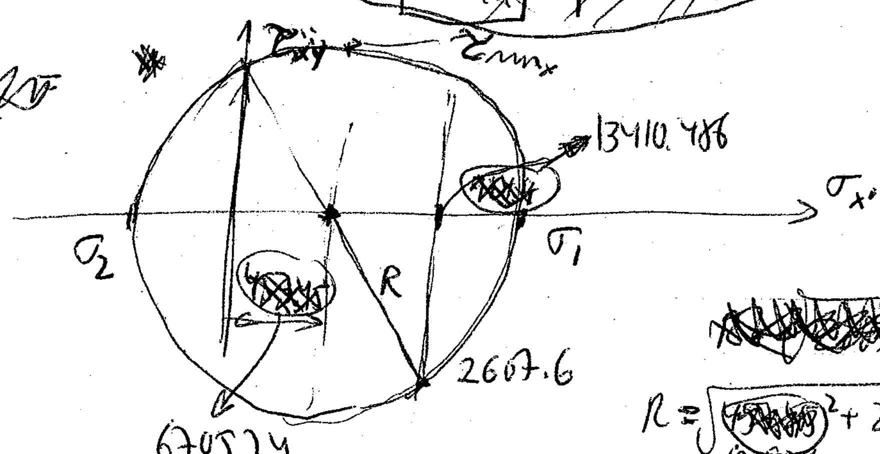
A







POINT A



$R = \sqrt{6705.24^2 + 2607.6^2}$

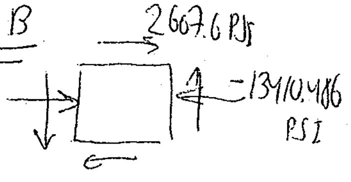
$= 7194.4$ PSI

$13894.7 \text{ PSI} = \sigma_1 = 6705.24 + R$

$-489.2 = \sigma_2 = 6705.24 - R$

$7194.4 = \tau_{max} = R = 7194.4$

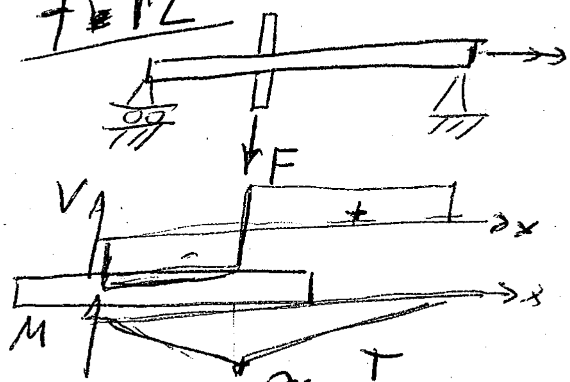
POINT B



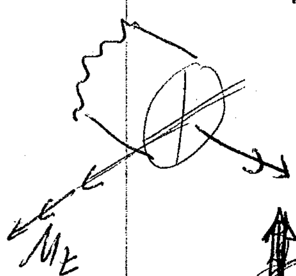
$\sigma_1 = 489.2$ PSI

$\sigma_2 = -13899.7$ PSI

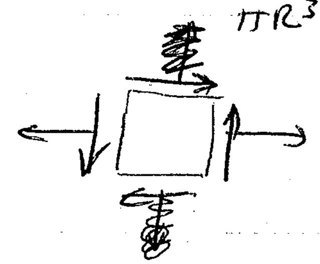
7.12 SHAFTS

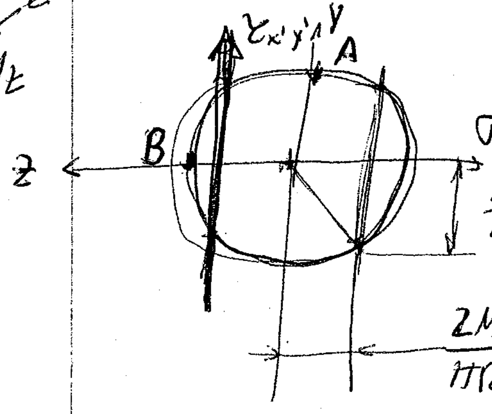


$$\tau = \frac{T}{\frac{1}{2}\pi R^3}$$



$$\sigma_x = \frac{M}{\frac{1}{4}\pi R^3}$$





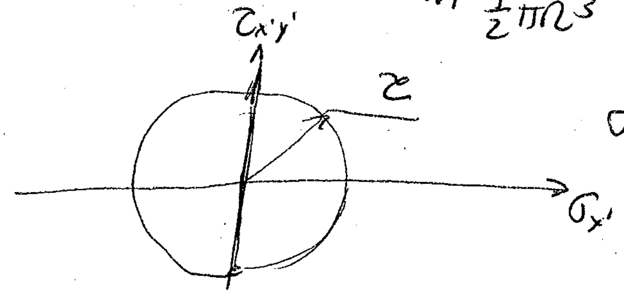
$$\sigma_{1,2} = \frac{2M}{\pi R^3} \pm \frac{2}{\pi R^3}\sqrt{M^2 + T^2}$$

$$\sigma_{1,2} = \frac{2}{\pi R^3}\left(M \pm \sqrt{M^2 + T^2}\right)$$

$$\tau_{max} = \frac{2}{\pi R^3}\sqrt{M^2 + T^2}$$

ON POINT B

$$\tau = \frac{T}{\frac{1}{2}\pi R^3} + \frac{V_y Q}{\frac{1}{4}\pi R^3 (2R)}$$



$$\sigma_{1,2} = \pm \tau$$

NO

~~FOR~~ IN GENERAL:

$$M = \sqrt{M_y^2 + M_z^2}$$

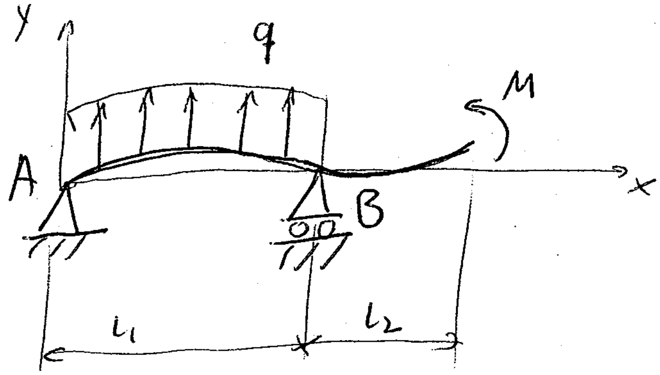
SO:

$$\sigma_{1,2} = \frac{2}{\pi R^3}\left\{\sqrt{M_y^2 + M_z^2} \pm \sqrt{M_y^2 + M_z^2 + T^2}\right\}$$

$$\tau_{max} = \frac{2}{\pi R^3}\sqrt{M_y^2 + M_z^2 + T^2}$$

9-1

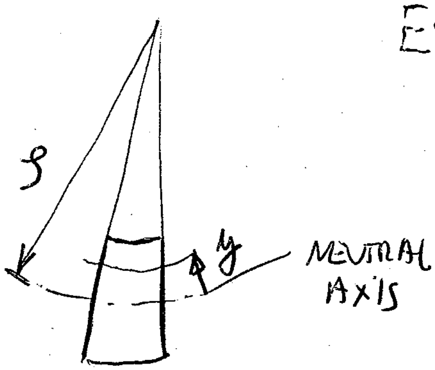
DEFLECTION OF BEAMS



$y(x)$

LET RECALL: (FOR PURE BENDING, NEGLECTING EFFECTS OF SHEARING)

$$E \cdot \varepsilon_x = -\frac{y}{\rho} \cdot E$$



$$\sigma_x = -\frac{y}{\rho} E$$

$$-\frac{M_z y}{I_z} = -\frac{y}{\rho} E$$

$$\frac{1}{\rho} = \frac{M_z}{E I_z}$$

THE CURVATURE OF A PLANE CURVE AT POINT x, y CAN BE EXPRESSED AS

$$\frac{1}{\rho} = \frac{d^2y/dx^2}{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}}$$

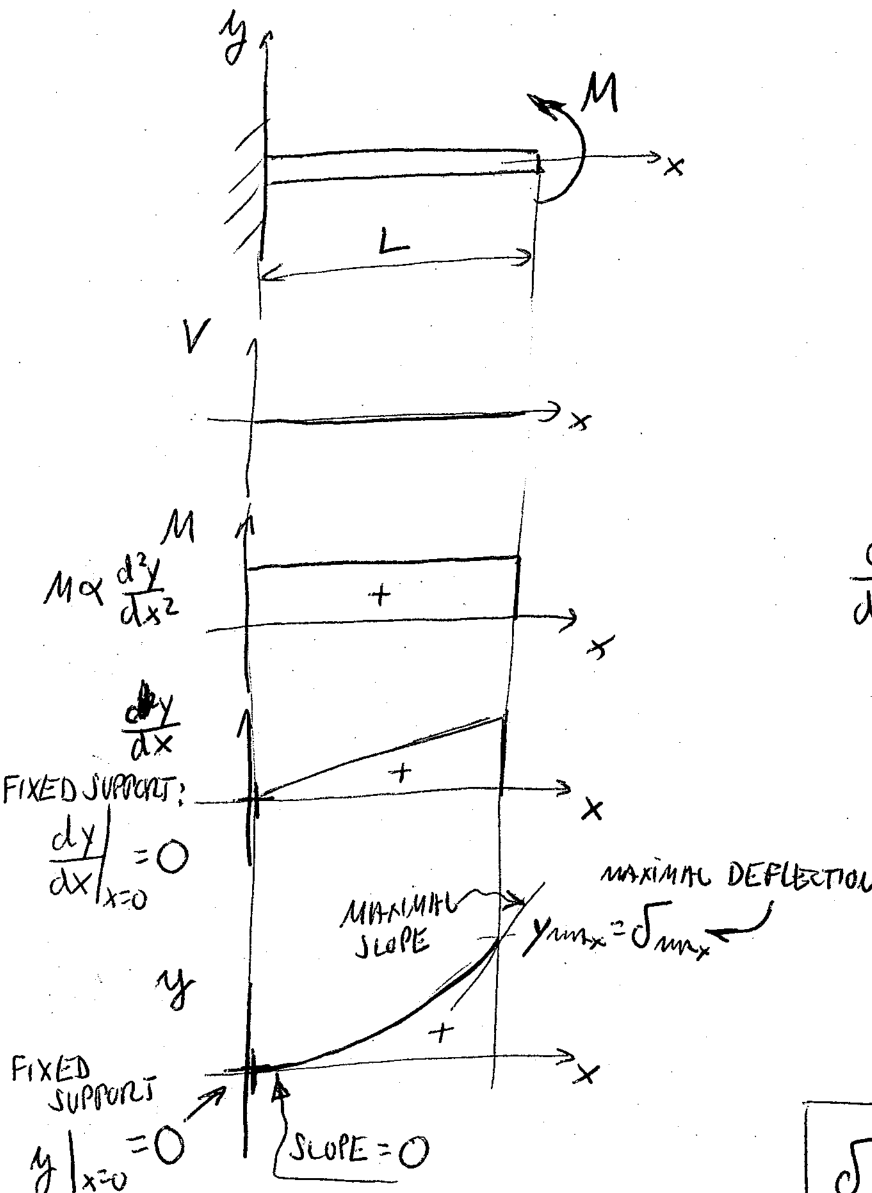
$\frac{dy}{dx}$ IS THE SLOPE OF $y(x)$. WE DISCUSS SMALL DEFORMATIONS, SO THE SLOPE IS VERY SMALL $1 \gg \frac{dy}{dx}$

THEN $$\frac{1}{\rho} \approx \frac{d^2y}{dx^2}$$

So: $$\boxed{\frac{M_z}{E\,I_z} = \frac{d^2y}{dx^2}}$$

$$\frac{dy}{dx} = C_1 + \int \frac{M_z}{EI_z}\,dx$$

$$y = C_2 + \int \left(\frac{dy}{dx}\right) dx$$



$$\boxed{\alpha = \frac{dy}{dx} = \frac{Mx}{EI_z}}$$

$$\alpha_{max} = \frac{ML}{EI_z} = \alpha\Big|_{x=L}$$

CONSTANT IN THIS CASE

$$\frac{dy}{dx} = C_1 + \int_0^x \frac{M}{EI_z}\,dx' = C_1 + \frac{Mx}{EI_z}$$

$$\left.\frac{dy}{dx}\right|_{x=0} = 0 = C_1$$

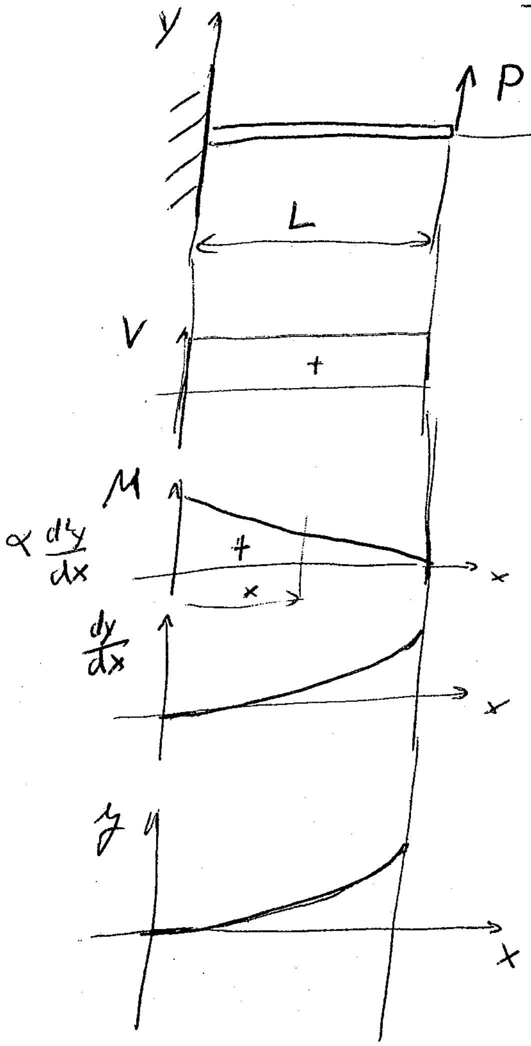
$$y = C_2 + \int_0^x \frac{Mx'}{EI_z}\,dx' = C_2 + \frac{Mx^2}{2EI_z}$$

$$y\Big|_{x=0} = 0 = C_2$$

$$\boxed{\delta = y = \frac{Mx^2}{2EI_z}}$$

$$\delta_{max} = y\Big|_{x=L} = \frac{ML^2}{2EI_z}$$

9-3



$$M_z(x) = P(L-x)$$

$$\frac{dy}{dx} = C_1 + \int_0^x \frac{P(L-x')}{EI_z}\,dx'$$

$$\frac{dy}{dx} = C_1 + \frac{P}{EI_z}\left(Lx - \frac{x^2}{2}\right)$$

$$\left.\frac{dy}{dx}\right|_0 = 0 = C_1$$

$$\frac{dy}{dx} = \alpha = \frac{P}{EI_z}\left(Lx - \frac{x^2}{2}\right)$$

$$\left.\frac{dy}{dx}\right|_L = \frac{PL^2}{2EI_z}$$

$$y = C_2 + \int \left(\frac{dy}{dx}\right) dx = C_2 + \frac{P}{EI_z}\int_0^x \left(Lx' - \frac{x'^2}{2}\right)dx'$$

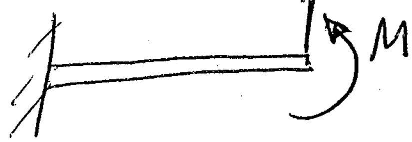
$$= C_2 + \left(L\frac{x^2}{2} - \frac{x^3}{6}\right)\frac{P}{EI_z}$$

$$y|_0 = 0 \rightarrow C_2 = 0$$

$$y = \frac{Px^2}{2EI_z}\left(L - \frac{x}{3}\right)$$

$$y|_L = \frac{PL^3}{3EI_z}$$

SUPERPOSITION

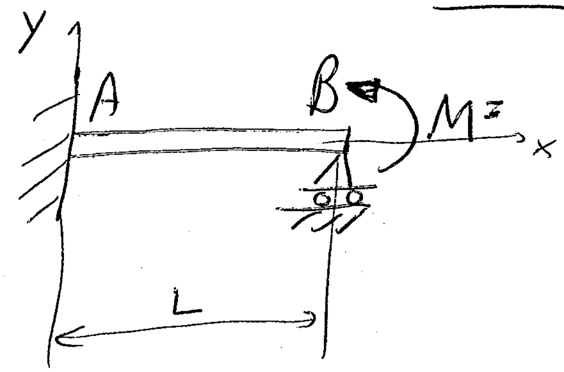


Σ OF LOADS $\longrightarrow$ Σ OF DEFORMATIONS AT ANY POINT:

$$\delta = \delta_P + \delta_M$$

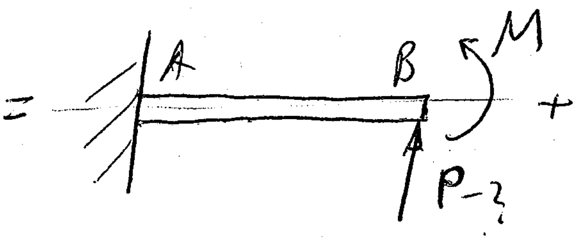
$$\alpha = \alpha_P + \delta_M$$

9-4

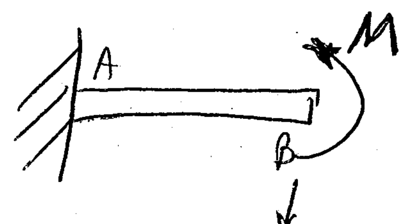


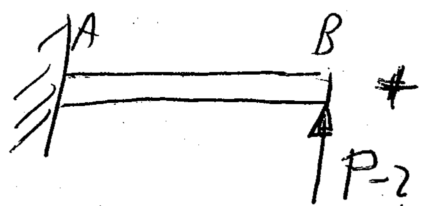
CALCULATE THE REACTIONS AT A AND AT B

M =

= 

+ CONSTRAIN $\delta_{x=L}=0$

= 

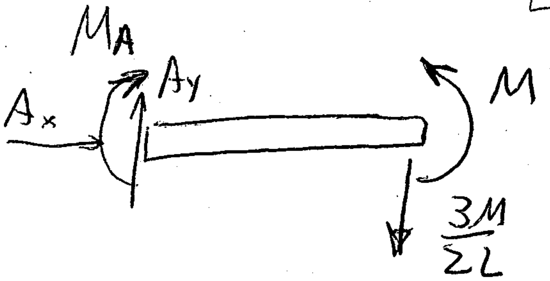
+ 

+ CONSTRAIN $\delta|_{x=L}=0$

$$\delta|_{x=L}=\frac{ML^2}{2EI_z}$$

$$\delta|_{x=L}=\frac{PL^3}{3EI_z} \rightsquigarrow \delta=\frac{ML^2}{2EI_z}+\frac{PL^3}{3EI_z}=0$$

$$\boxed{P=-\frac{M3}{2L}}$$



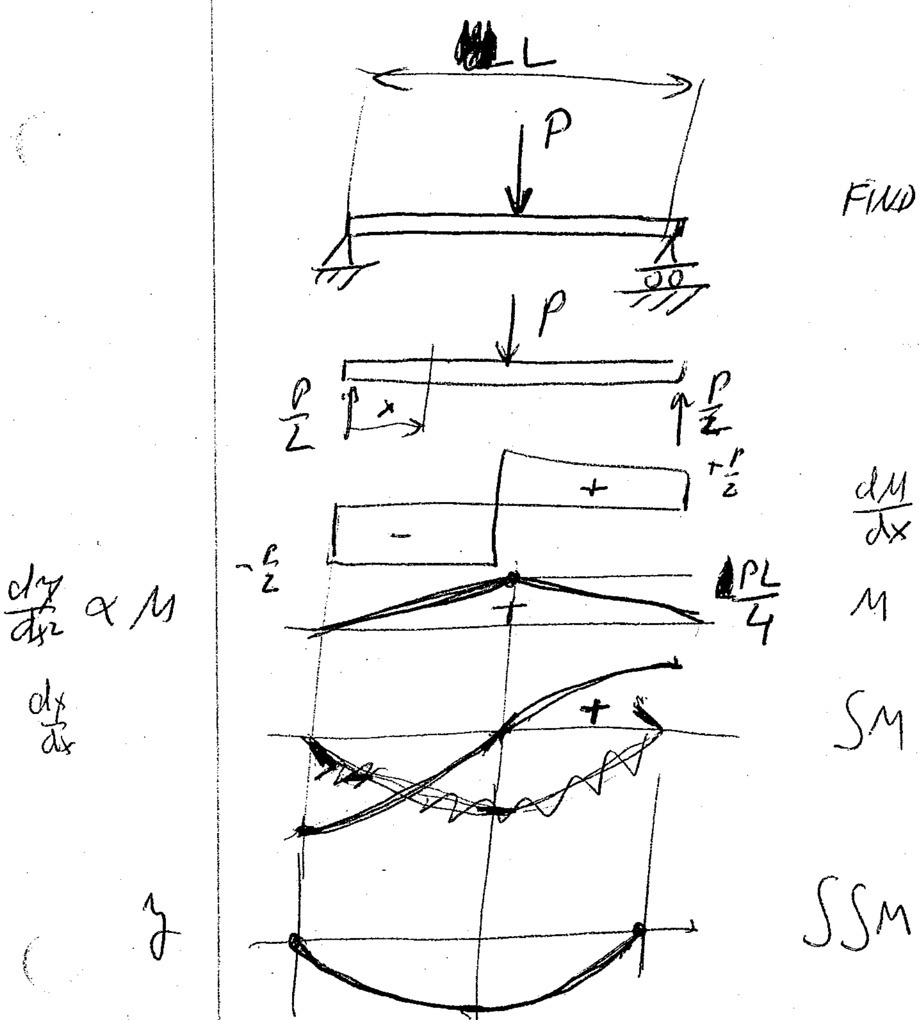
$$\sum F_x=0=A_x$$

$$\sum F_y=A_y-\frac{3M}{2L}=0\ ;\quad A_y=\frac{3M}{2L}$$

$$\sum M_z=0=M-\frac{3M}{2L}\cdot L-M_A=0$$

$$M_A=M\left(1-\frac{3}{2}\right)=-\frac{1}{2}M$$

FIND THE BEAM CURVE DEFORMED CURVE



1 $0 < x < \frac{L}{2}$

$$M_1 = \frac{P}{2}x$$

$$\frac{dy_1}{dx} = C_1 + \frac{P}{2EI}\int_0^x x' \, dx'$$

$$= C_1 + \frac{Px^2}{4EI}$$

$$y_1 = C_2 + C_1 x + \frac{P}{4EI}\int_0^x x'^2 \, dx'$$

$$y_1 = C_2 + C_1 x + \frac{Px^3}{12EI}$$

2 $\frac{L}{2} < x < L$

$$M_2 = \frac{P}{2}(L-x)$$

$$\frac{dy_2}{dx} = C_3 + \frac{P}{2EI}\int_{L/2}^x (L-x')\, dx$$

$$= C_3 + \frac{P}{2EI}\left(Lx - \frac{x^2}{2} \cancel{+ \frac{3L^2}{8}}\right)$$

$$y_2 = C_4 + C_3 x + \frac{P}{2EI}\int_{L/2}^x \left(Lx' - \frac{x'^2}{2}\right) dx$$

$$y_2 = C_4 + C_3 x + \frac{P}{4EI}\left(Lx^2 - \frac{1}{3}x^3 - \frac{3}{8}L^2 x\right)$$

$\frac{20}{344} = \frac{10}{162} = 5$

CONSTRAINS:

$$y_1(0)=0=C_2$$

$$y_2(L)=0=C_4+C_3L+\frac{PL^3}{6EI}$$

$$\frac{dy_1}{dx}\left(\frac{L}{2}\right)=\frac{dy_2}{dx}\left(\frac{L}{2}\right) \rightarrow C_1+\frac{PL^2}{16EI}=C_3+\frac{3PL^2}{16EI}$$

$$y_1\left(\frac{L}{2}\right)=y_2\left(\frac{L}{2}\right) \rightarrow C_1\frac{L}{2}+\frac{PL^3}{8\cdot 12EI}=C_4+C_3\frac{L}{2}+\frac{5PL^3}{96EI}$$

$$\begin{bmatrix} 0 & L & 1 \\ L & -L & 0 \\ \frac{L}{2} & -\frac{L}{2} & 1 \end{bmatrix}\begin{bmatrix} C_1 \\ C_3 \\ C_4 \end{bmatrix}=\begin{bmatrix} \frac{1}{6} \\ \frac{1}{8} \\ \frac{1}{24} \end{bmatrix}\frac{PL^3}{EI}$$

$$y_1=-\frac{1}{16}\frac{PL^2}{EI}x+\frac{Px^3}{12EI}$$

$$y_2=\frac{PL^3}{48EI}-\frac{3}{16}\frac{PL^2}{EI}x+\frac{P}{4EI}\left(Lx^2-\frac{1}{3}x^3\right)$$

$$y\left(\frac{L}{2}\right)=\frac{PL^3}{48EI}$$

$$\begin{bmatrix} \frac{L}{2} & \frac{L}{2} & 2 \\ L & -L & 0 \\ \frac{L}{2} & -\frac{L}{2} & 1 \end{bmatrix}\begin{bmatrix} C_1 \\ C_3 \\ C_4 \end{bmatrix}=\begin{bmatrix} \frac{5}{24} \\ \frac{1}{8} \\ \frac{1}{24} \end{bmatrix}\frac{PL^3}{EI}$$

$$C_4+\cancel{C_3L}+\frac{PL^3}{6EI}=0$$

$$\frac{PL^3}{48EI}+C_3L+\frac{PL^3}{6EI}=0$$

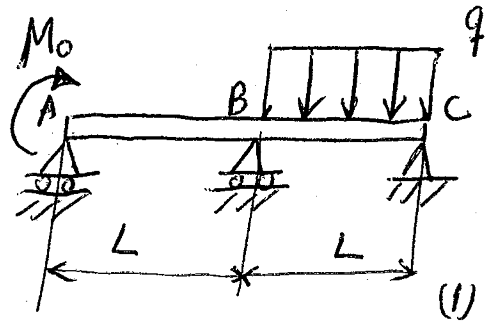
$$\boxed{C_3=-\frac{PL^2 9}{48EI}=-\frac{3}{16}\frac{PL^2}{EI}}$$

$$C_1=C_3+\frac{PL^2}{8EI}$$

$$\cancel{C_3\frac{L}{2}}+\frac{PL^3}{16EI}+\frac{PL^3}{96EI}=C_4+\cancel{C_3\frac{L}{2}}+\frac{5PL^3}{96EI}$$

$$\boxed{C_1=\left(-\frac{3}{16}+\frac{1}{8}\right)\frac{PL^2}{EI}=-\frac{1}{16}\frac{PL^2}{EI}}$$

$$\boxed{\frac{PL^3}{48EI}}=\frac{2PL^3}{96EI}=C_4$$



$L = 3\,m$

$E = 196\,GPa$ (steel)

W150×24 ($I = 13.4 \cdot 10^6\,mm^4$, $S = 168 \cdot 10^3\,mm^3$)

$q = +10$ kN/m ↓

$M_0 = 5$ kN·m

(1) CALCULATE THE REACTIONS AT A, B, C

(2) CALCULATE THE BEAM SLOPE AT B

(3) CALCULATE THE MAXIMUM NORMAL STRESS IN THE BEAM

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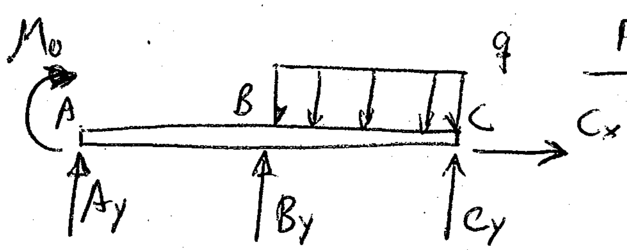
(1)

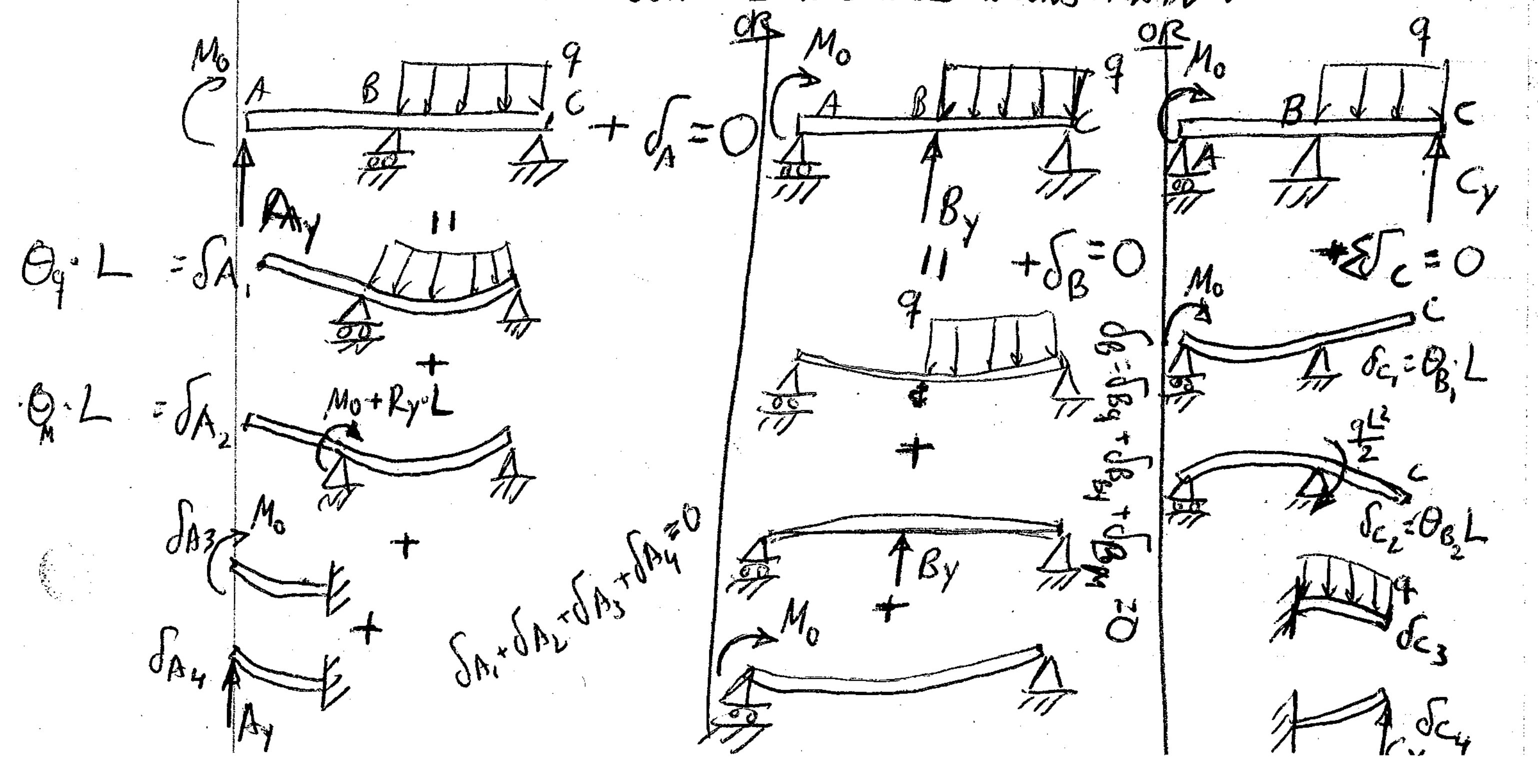


F.B.D

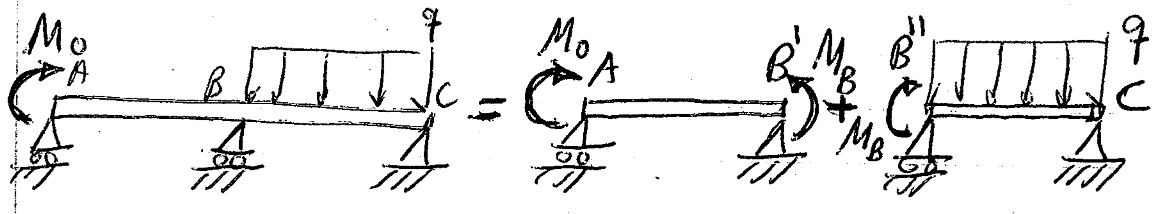
4 UNKNOWNS, 3 EQ. EQ. → INDETERMINATE PROBLEM

THE DEFORMATION OF THE BEAM DISTRIBUTES THE LOADS BETWEEN THE SUPPORTS

RELEASE ONE THE SUPPORTS AND ADD A CONSTRAIN!

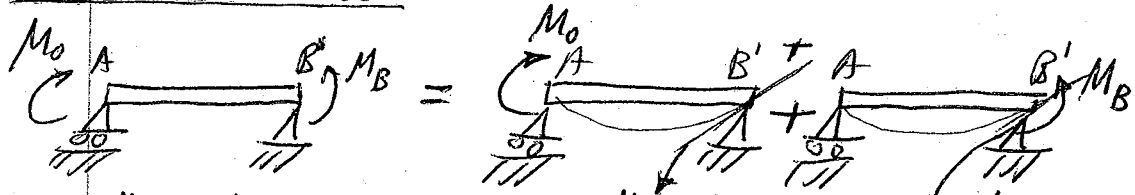


OR DEVIDE THE BEAM INTO TWO BEAMS, WHEN THE INTERNAL BENDING MOMENT IS AN UNKNOWN TRANSFERED BETWEEN THE BEAMS:



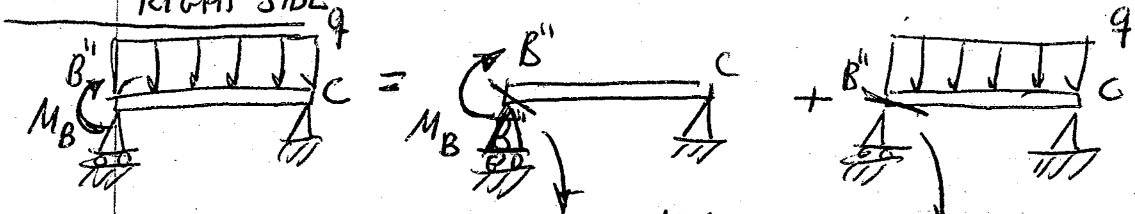
THE CONSTRAIN: $\theta_{B'} = \theta_{B''}$ (CONTINUOUS SLOPE)

LEFT SIDE



$$\theta_{B'} = \frac{M_0 L}{6EI} + \frac{M_B L}{3EI} \longleftarrow \qquad \theta_{B'} = \frac{M_0 L}{6EI} \qquad \theta_{B'} = \frac{M_B L}{3EI}$$

RIGHT SIDE



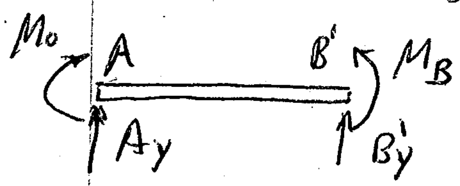
$$\theta_{B''} = -\frac{M_B L}{3EI} - \frac{qL^3}{24EI} \longleftarrow \qquad \theta_{B''} = -\frac{M_B L}{3EI} \qquad \theta_{B''} = -\frac{qL^3}{24EI}$$

$$\theta_{B'} = \theta_{B''}$$

$$\frac{M_0 \cancel{L}}{6\cancel{EI}} + \frac{M_B \cancel{L}}{3\cancel{EI}} = -\frac{M_B \cancel{L}}{3\cancel{EI}} - \frac{qL^{\cancel{3}\,2}}{24\cancel{EI}}$$

$$M_B = -\frac{1}{4}M_0 - \frac{1}{16}qL^2$$

F.B.D. OF LEFT BEAM:



$$\circlearrowleft \sum M_{B'} = M_B - M_0 - A_y \cdot L = 0$$

$$\boxed{A_y = \underbrace{-\frac{1}{4}\frac{M_0}{L} - \frac{1}{16}qL}_{M_B} - \frac{M_0}{L} = -\frac{5}{4}\frac{M_0}{L} - \frac{1}{16}qL}$$

$$\underline{A_y} = -\frac{5}{4}\cdot\frac{5}{3} - \frac{1}{16}\cdot 10\cdot 3 = \underline{-3.9583\ kN}$$

WE CAN CONTINUE WITH THE GENERAL F.B.D AND SOLVE B_y, C_y, $(C_x = 0)$

$$\circlearrowleft^{+} \sum M_B = 0 = C_y\cdot L - \frac{qL^2}{2} - M_o - A_y\cdot L$$

$$\boxed{C_y = -\frac{1}{4}\frac{M_o}{L} + \frac{7}{16}qL} = -\frac{1}{4}\cdot\frac{5}{3} + \frac{7}{16}\cdot 10\cdot 3 = \underline{+12.7083\ kN}$$

$$\sum F_y = 0 = A_y + B_y + C_y - qL = 0$$

$$\boxed{B_y = \frac{3}{2}\frac{M}{L} + \frac{5}{8}qL} = \frac{3}{2}\cdot\frac{5}{3} + \frac{5}{8}\cdot 10\cdot 3 = \underline{+21.25\ kN}$$

(2)

THE BEAM SLOPE AT B

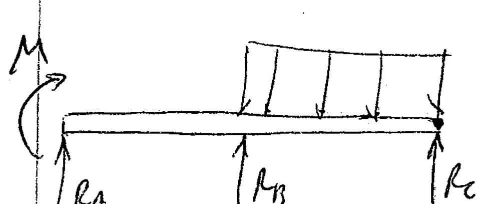
$$\theta_B = \theta_{B'} = \theta_{B''} = \frac{M_o L}{6EI} + \frac{M_B L}{3EI} = \frac{L}{3EI}\left(\frac{M_o}{2} + M_B\right)$$

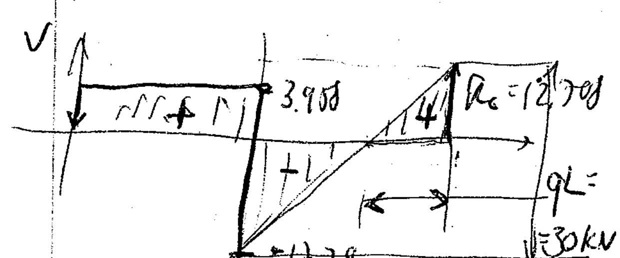
$$M_B = -\frac{1}{4}M_o - \frac{1}{16}qL^2 = -\frac{1}{4}5 - \frac{1}{16}\cdot 10\cdot 3^2 = -6.875\ kN\cdot m$$

$$\underline{\theta_B} = \frac{\overset{(m)}{3}\cdot 1000 (N)(m)}{3\cdot \underset{(Pa)}{196\cdot 10^9}\cdot \underset{(m^4)}{13.4\cdot 10^6\cdot 10^{-12}}}\left(\frac{5}{2} - 6.875\right) = \underline{-1.666\cdot 10^{-3}\ rad}$$

$$= \underline{-0.09544^\circ}$$

(3)



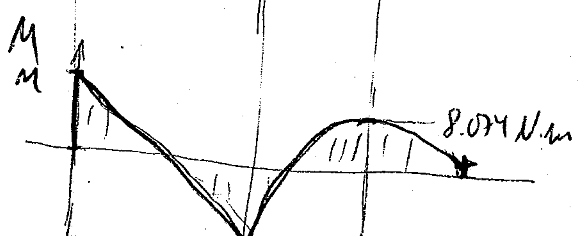


$$\frac{12.708}{x} = \frac{30}{3}$$

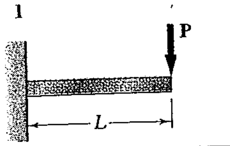
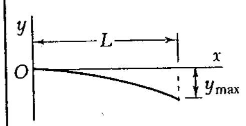
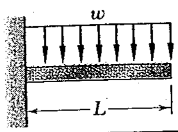
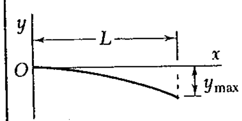
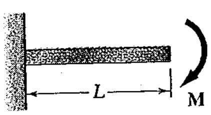
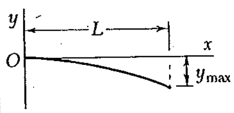
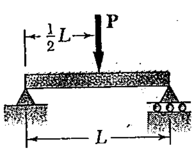
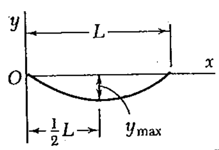
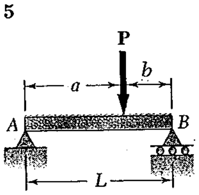
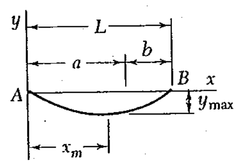
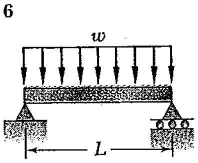
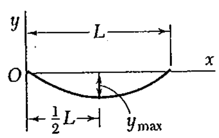
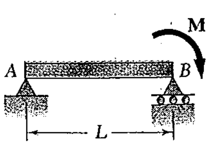
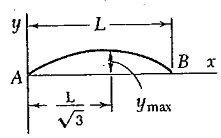
$$x = 1.2708$$

$$\boxed{M_{max} = R_c\cdot 1.2708 - \frac{q\,1.2708^2}{2} = 8.074\ kN\cdot m}$$

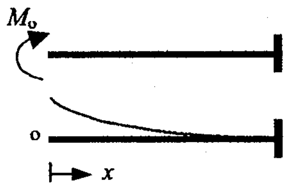
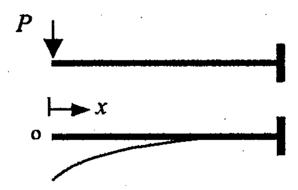
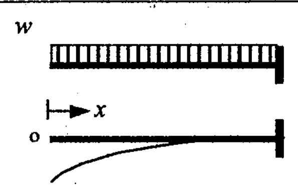
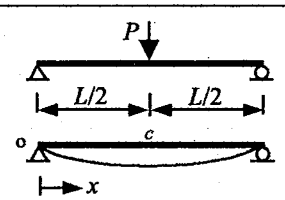
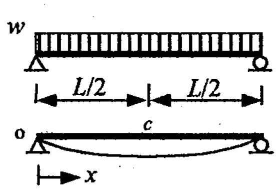
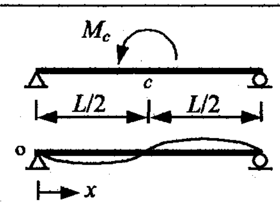
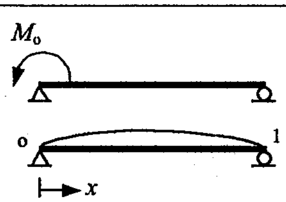
$$R_c\cdot 3 - \frac{q\,3^2}{2} = -6.876\ N\cdot m$$

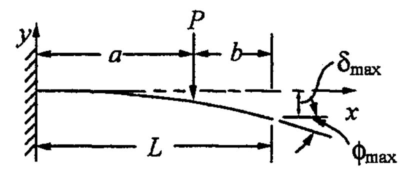
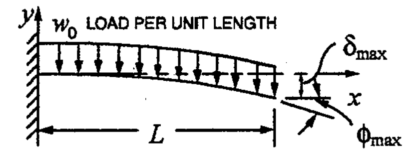
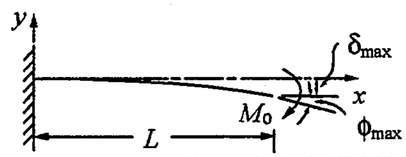
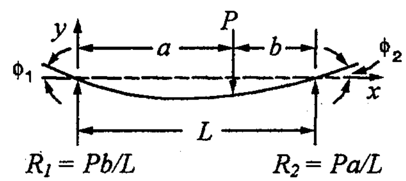
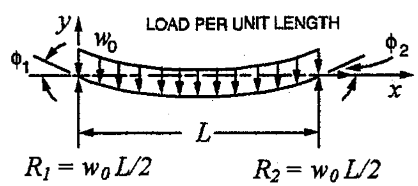
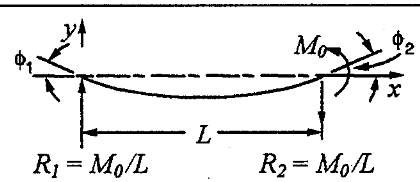


$$\sigma = \frac{M}{\ } = \frac{8.074\ N\cdot m\cdot 10^3}{\ } = 48\ MPa$$

Beam and Loading	Elastic Curve	Maximum Deflection	Slope at End	Equation of Elastic Curve
1 		$-\dfrac{PL^3}{3EI}$	$-\dfrac{PL^2}{2EI}$	$y = \dfrac{P}{6EI}(x^3 - 3Lx^2)$
2 		$-\dfrac{wL^4}{8EI}$	$-\dfrac{wL^3}{6EI}$	$y = -\dfrac{w}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$
3 		$-\dfrac{ML^2}{2EI}$	$-\dfrac{ML}{EI}$	$y = -\dfrac{M}{2EI}x^2$
4 		$-\dfrac{PL^3}{48EI}$	$\pm\dfrac{PL^2}{16EI}$	For $x \le \frac{1}{2}L$: $y = \dfrac{P}{48EI}(4x^3 - 3L^2x)$
5 		For $a > b$: $-\dfrac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}EIL}$ at $x_m = \sqrt{\dfrac{L^2 - b^2}{3}}$	$\theta_A = -\dfrac{Pb(L^2 - b^2)}{6EIL}$ $\theta_B = +\dfrac{Pa(L^2 - a^2)}{6EIL}$	For $x < a$: $y = \dfrac{Pb}{6EIL}[x^3 - (L^2 - b^2)x]$ For $x = a$: $y = -\dfrac{Pa^2b^2}{3EIL}$
6 		$-\dfrac{5wL^4}{384EI}$	$\pm\dfrac{wL^3}{24EI}$	$y = -\dfrac{w}{24EI}(x^4 - 2Lx^3 + L^3x)$
7 		$\dfrac{ML^2}{9\sqrt{3}EI}$	$\theta_A = +\dfrac{ML}{6EI}$ $\theta_B = -\dfrac{ML}{3EI}$	$y = -\dfrac{M}{6EIL}(x^3 - L^2x)$

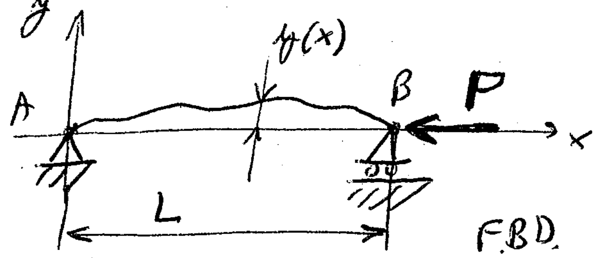
Beam Deflection Formulas

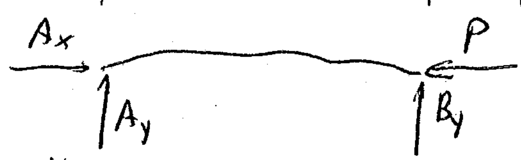
Beam Configuration	Formulas for Any Point	Formulas for Special Points
	$\theta = -\frac{M_o}{EI}(L\text{-}x)$ $v = \frac{M_o}{2EI}(L\text{-}x)^2$	$\theta_o = -\frac{M_oL}{EI}$ $v_o = \frac{M_oL^2}{2EI}$
	$\theta = \frac{P}{2EI}(L^2\text{-}x^2)$ $v = -\frac{PL^3}{3EI} + \frac{Px}{6EI}\ (3L^2\text{-}x^2)$	$\theta_o = \frac{PL^2}{2EI}$ $v_o = -\frac{PL^3}{3EI}$
	$\theta = \frac{w}{6EI}(L^3\text{-}x^3)$ $v = -\frac{w}{6EI}(L^4\text{-}xL^3)$	$\theta_o = \frac{wL^3}{6EI}$ $v_o = -\frac{wL^4}{6EI}$
	$\theta = -\frac{P}{16EI}(L^2\text{-}4x^2),\ x \le \frac{L}{2}$ $v = -\frac{P}{48EI}(3L^2x\ \text{-}4x^3),\ x \le \frac{L}{2}$	$\theta_o = -\frac{PL^2}{16EI}$ $v_c = -\frac{PL^3}{48EI}$
	$\theta = -\frac{w}{24EI}(L^3\text{-}6Lx^2+4x^3)$ $v = -\frac{w}{24EI}\ (L^3x\text{-}2x^3L+x^4)$	$\theta_o = -\frac{wL^3}{24EI}$ $v_c = v_{max} = -\frac{5wL^4}{384EI}$
	$\theta = -\frac{M_c}{24EIL}(L^2\text{-}12x^2),\ x \le \frac{L}{2}$ $v = \frac{M_c}{24EIL}\ (L^2x\text{-}4x^3),\ x \le \frac{L}{2}$	$\theta_o = -\frac{M_cL}{24EI}$ $v_{max} = \frac{M_cL^2}{72\sqrt{3}EI},\ x = \frac{L}{2\sqrt{3}}$
	$\theta = \frac{M_o}{6EIL}(\ 2L^2\text{-}6Lx+3x^2)$ $v = \frac{M_o}{6EIL}\ (2L^2x\text{-}3Lx^2+x^3)$	$\theta_o = \frac{M_oL}{3EI},\ \theta_1 = -\frac{M_oL}{6EI}$ $v_{max} = \frac{M_oL^2}{9\sqrt{3}EI},\ x = \frac{L}{3}(3-\sqrt{3})$
Convention: L=beam length,	positive θ = counter-clockwise ,	positive v= upward

(δ is positive downward)			
	$\delta = \frac{Pa^2}{6EI}(3x-a)$, for $x>a$ $\delta = \frac{Px^2}{6EI}(-x+3a)$, for $x \le a$	$\delta_{max} = \frac{Pa^2}{6EI}(3L-a)$	$\phi_{max} = \frac{Pa^2}{2EI}$
	$\delta = \frac{w_o x^2}{24EI}(x^2+6L^2-4Lx)$	$\delta_{max} = \frac{w_o L^4}{8EI}$	$\phi_{max} = \frac{w_o L^3}{6EI}$
	$\delta = \frac{M_o x^2}{2EI}$	$\delta_{max} = \frac{M_o L^2}{2EI}$	$\phi_{max} = \frac{M_o L}{EI}$
	$\delta = \frac{Pb}{6LEI}\left[\frac{L}{b}(x-a)^3 - x^3 + (L^2-b^2)x\right]$, for $x>a$ $\delta = \frac{Pb}{6LEI}\left[-x^3+(L^2-b^2)x\right]$, for $x \le a$	$\delta_{max} = \frac{Pb(L^2-b^2)^{3/2}}{9\sqrt{3}LEI}$ at $x = \sqrt{\frac{L^2-b^2}{3}}$	$\phi_1 = \frac{Pab(2L-a)}{6LEI}$ $\phi_2 = \frac{Pab(2L-b)}{6LEI}$
	$\delta = \frac{w_o x}{24EI}(L^3-2Lx^2+x^3)$	$\delta_{max} = \frac{5w_o L^4}{384EI}$	$\phi_1 = \phi_2 = \frac{w_o L^3}{24EI}$
	$\delta = \frac{M_o Lx}{6EI}\left(1-\frac{x^2}{L^2}\right)$	$\delta_{max} = \frac{M_o L^2}{9\sqrt{3}EI}$ at $x = \frac{L}{\sqrt{3}}$	$\phi_1 = \frac{M_o L}{6EI}$ $\phi_2 = \frac{M_o L}{3EI}$

BUCKLING (COLUMNS)

LETS ASSUME A SIMPLE SUPPORTED BEAM LOADED BY AN COMPRESSION AXIAL FORCE:





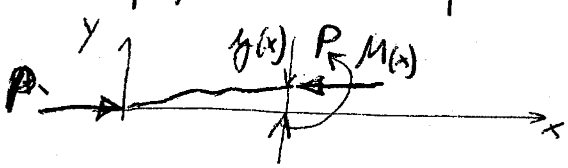
F.B.D.

$\Sigma F_x = A_x - P = 0 \qquad A_x = P$

$\Sigma F_y = 0 = A_y + B_y$

$\circlearrowleft^{+} \Sigma M_z|_A = 0 = B_y \cdot L \qquad B_y = 0$

$A_y = 0$



$\underbrace{\text{BENDING;}}$ THE MOMENT AT x

$\circlearrowleft^{+} \Sigma M_z|_A = 0 = P \cdot y + M(x) = 0$

$M(x) = -P \cdot y$

$$\frac{d^2y}{dx^2} = y'' = \frac{M(x)}{EI} = -\frac{P y}{EI}$$

$y'' + \frac{P}{EI} y = 0$ differential homogeneous equation

$p^2 + \frac{P}{EI} = 0 \rightsquigarrow p = \pm i\sqrt{\frac{P}{EI}}$

$$y = A \sin\sqrt{\frac{P}{EI}}\,x + B\cos\sqrt{\frac{P}{EI}}\,x$$

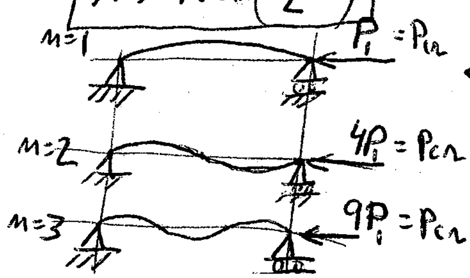
BOUNDARY CONDITIONS:

$y(0) = 0 \rightsquigarrow B = 0$

$y(L) = 0 \rightsquigarrow A\sin\sqrt{\frac{P}{EI}}\,L = 0 \rightarrow A = 0$ * OR $\sqrt{\frac{P}{EI}}\,L = n\pi$

$$\boxed{P = \frac{n^2\pi^2 EI}{L^2}}$$

$$\boxed{y(x) = A\sin\left(\frac{n\pi}{L}x\right)}$$



MINIMUM ENERGY – $\int P_{cr} dx$

CONCLUSSIONS :

1. P_{cr} – DISCRETTE VALUE AT WHICH WE OBSERVE BUCKLING
2. BUCKLING WILL OCURR FOR $n=1$ (IF NO EXTERNAL CONSTRAINS ARE APPLIED)
3. BUCKLING WILL OCURR AROUND THE MINIMUM MOMENT OF INERTIA AXIS ⟶ AXIS OF PRINCIPAL MOMENT OF INERTIA (SYMMETRY IF FOUND)
4. FOR $\sigma_{cr} < \sigma_y$, THE BUCKLING IS ELASTIC ! (THE BEAM WILL RETURN TO $y=0$ AFTER LOAD P RELEASE)

EULER'S ~~EQUATION~~ FORMULA

FOR BUCKLING IN A ~~BEAM~~ SIMPLE SUPPORTED BEAM

$$\boxed{P_{cr} = \frac{\pi^2 EI}{L^2}}$$

THE STRESS DURING BUCKLING $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{L^2}\left(\frac{I}{A}\right)$

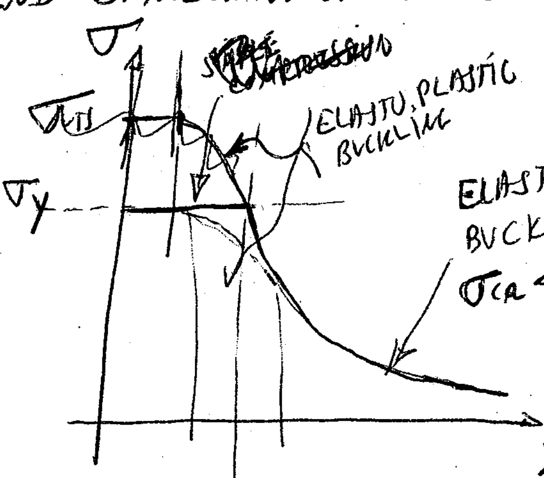
GEOMETRIC DEFINITIONS :

$r^2 = \frac{I}{A}$, $r = \sqrt{\frac{I}{A}}$ radius of gyration about I's axis

"TENDANCY TO BEND BY MEANING OF STRESS"

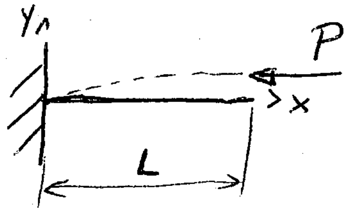
$\lambda = \frac{L}{r}$, slenderness ratio

$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2}$ CRITICAL STRESS A FUNCTION OF MATERIAL + GEOMETRY



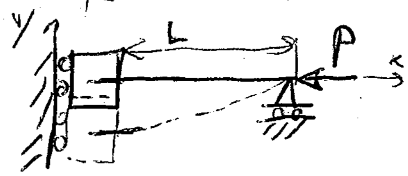
MAXIMUM COMPRESIVE STRESS IN A SIMPLE SUPPORT COLUMN

EXTENSION OF EULER'S FORMULA



FIXED SUPPORT OR AS CALLED FOR COLUMN CASES – FIXED/FREE COLUMN

THE same equation:



$$y'' + \frac{P}{EI} y = 0$$

$$y = A \sin\sqrt{\frac{P}{EI}}\, x + B \cos\sqrt{\frac{P}{EI}}\, x$$

BOUNDARY CONDITIONS } ~~y(0) = 0 = B~~

$$y'(0) = \sqrt{\frac{P}{EI}}\, A \cos\sqrt{\frac{P}{EI}}\cdot 0 - \sqrt{\frac{P}{EI}}\, B \sin\sqrt{\frac{P}{EI}}\cdot 0 = 0$$

$$A = 0$$

$$y(L) = 0 = B \cos\sqrt{\frac{P}{EI}}\, L \;\rightarrow\; B = 0$$

$$\rightarrow \cos\sqrt{\frac{P}{EI}}\, L = 0$$

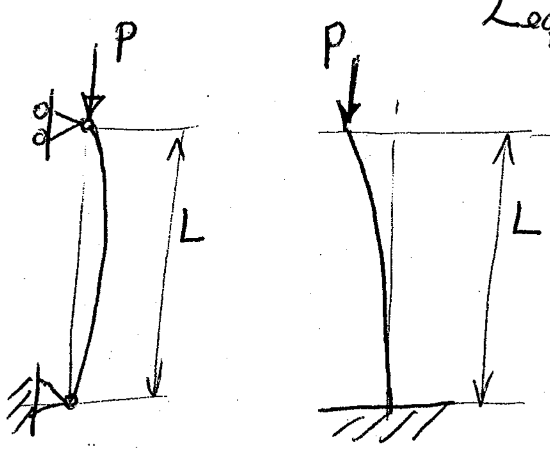
FOR $n=0$ $\quad \sqrt{\frac{P}{EI}}\, L = \frac{\pi}{2} + n\pi$

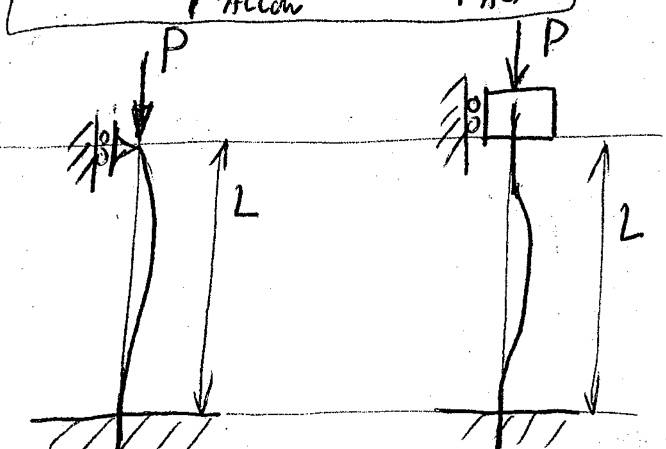
$$P_{cr} = \frac{\pi^2 EI}{2^2 L^2} = \frac{\pi^2 EI}{(2L)^2}$$

EULER CASES

IF WE WRITE $P_{cr} = \dfrac{\pi^2 EI}{L_{eq}^2}$

$$\boxed{F.S. = \frac{P_{cr}}{P_{ALLOW}} \text{ OR } F.S = \frac{P_{cr}}{P_{ACT}}}$$

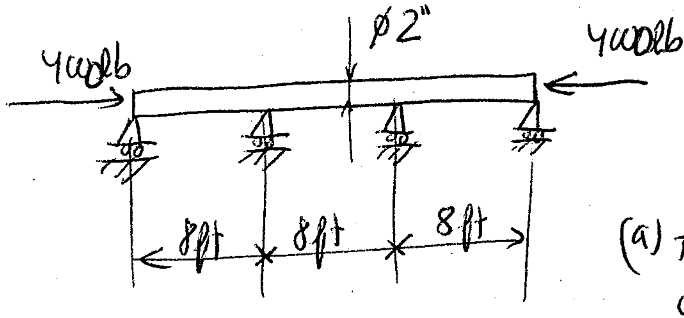




PINNED OR ROUND ENDS	ONE FREE END ONE FIXED END FREE/FIXED ENDS	ONE PINNED END ONE FIXED END PINNED/FIXED ENDS	FIXED ENDS
$L_{eq} = L$	$L_{eq} = 2L$	$L_{eq} \cong 0.7L$	$L_{eq} = \frac{L}{2}$

10-4

EXAMPLE (2.1)



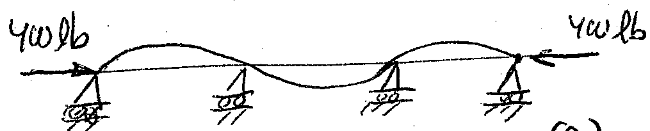
$E = 10000\,KSI$

F.S. - 2 FOR BUCKLING

(a) THREE SIMPLE SUPPORT COLUMNS

OR

(b) ONE COLUMN, WHEN $m=3$



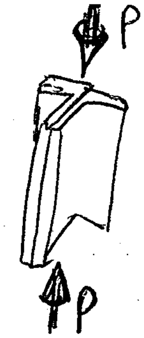
(a) $$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \cdot 10000^{KSI} \cdot \frac{\pi 2^4}{64}}{(8 \cdot 12)^2} =$$

$$P_{cr} = 8411\ lb$$

$$\underline{\underline{F.S}} = \frac{P_{cr}}{P_{ACT}} = \frac{8411}{400} = \underline{\underline{2.103}}$$

(b) $$P_{cr} = \frac{\pi^2 m^2 EI}{L^2} = \frac{\pi^2 \cdot 3^2 \cdot 10000 \cdot 10^3 \cdot \frac{\pi 2^4}{64}}{(24 \cdot 12)^2} = 8411\ lb \ldots$$

10.15

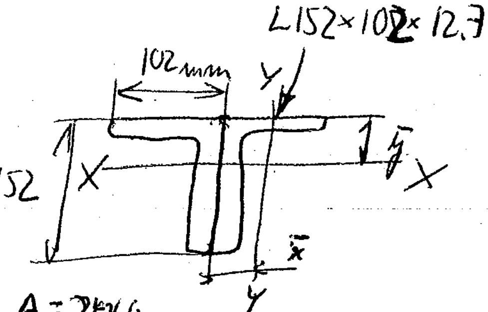


$L_{eq} = 7\,m$

$E = 200\,GPa$

$FS = 2.2$

$P_{all} - ?$



page 761: $L152\times102\times12.7 \begin{cases} A = 3060\,mm^2 \\ I_{xx} = 7.2\cdot10^6\,mm^4 \quad \bar{y} = 50.3\,mm \\ I_{yy} = 2.64\cdot10^6\,mm^4 \quad \bar{x} = 25.3\,mm \end{cases}$

BUCKLING WILL OCURR FOR THE MINIMUM I (FOR PRINCIPAL MOMENTS OF INERTIA)

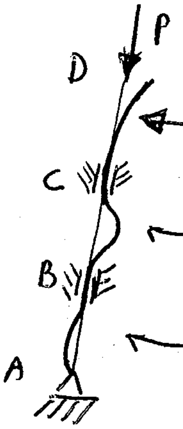
$$I_{xx_{T}} = 2\,I_{xx_{\Gamma}} = 2\cdot7.2\cdot10^6 = 14.4\cdot10^6\,mm^4$$

$$I_{yy_{T}} = 2\left(I_{yy_{\Gamma}} + A\cdot\bar{x}^2\right) = 2\left(2.64\cdot10^6 + 3060\cdot25.3^2\right) = 9.1974\cdot10^6\,mm^4$$

$$I_{min} = I_{yy_{T}} = 9.197\cdot10^6\,mm^4$$

$$P_{all} = \frac{P_{cr}}{FS} = \frac{\pi^2 E\,I_{min}}{FS\;L_{eq}^2} = \frac{\pi^2\cdot200000\,MPa\cdot9.197\cdot10^6\,mm^4}{2.2\cdot(7000)^2} = 168.41\cdot10^3\,N$$

10.23



$I = \frac{25^4}{12} = 32.552\cdot 10^3\ mm^4$ $FS = 2.$

$E = 75\ GPa$

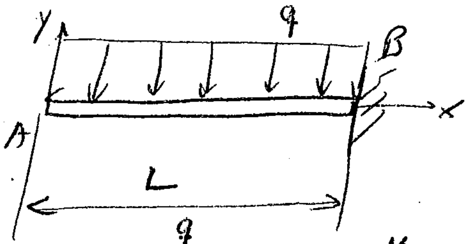
FREE/FIXED
$L_{eq} = 2L_{DC} \rightarrow P = \frac{\pi^2 EI}{SF\, L_{eq}^2} = \frac{\pi^2\cdot 75000\ MPa\cdot 552\cdot 10^3}{2.8\cdot(2.5)^2} = 8605.6N$

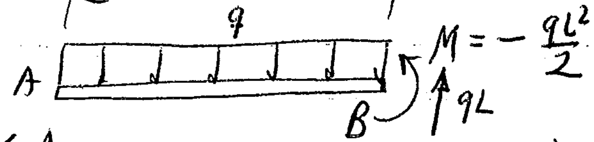
FIXED/FIXED
$L_{eq} = \frac{L_{BC}}{2} \rightarrow P = \frac{\pi^2 EI}{SF\, L_{eq}^2} = \frac{\pi^2\cdot 75000\cdot 32.552\cdot 10}{2.8\,(\frac{1}{2}\cdot 1.250)^2} = 22030.3\,N$

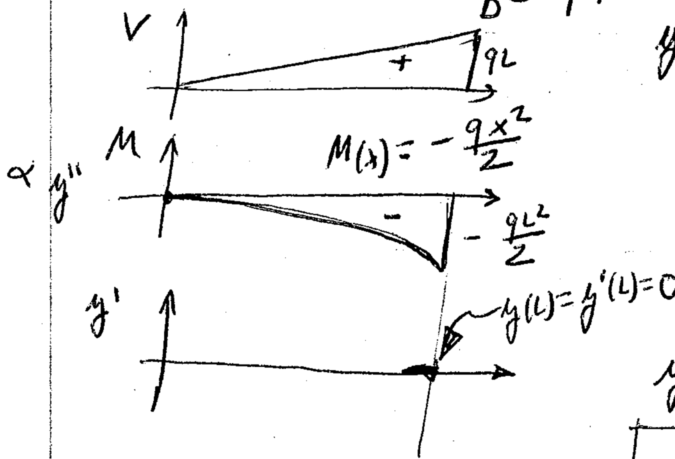
FIXED/ROLLED
$L_{eq} = 0.7 L_{AB} \rightarrow P = \frac{\pi^2 EI}{SF\cdot L_{eq}^2} = \frac{\pi^2\cdot 75000\cdot 32.552\cdot 10^3}{2.8\cdot(0.7\cdot 1)^2} = 17562.4\,N$

$\underline{P = P_{DC} = 8605.6N}$

9.3







$$y' = C_1 + \frac{-q}{2EI}\int x^2\,dx = C_1 - \frac{qx^3}{6EI}$$

$$y'(L) = 0 = C_1 - \frac{qL^3}{6EI},\quad C_1 = \frac{qL^3}{6EI}$$

$$y' = \frac{q}{6EI}(L^3 - x^3)$$

$$y = C_2 + \int y'\,dx = C_2 + \frac{q}{6EI}\left(L^3x - \frac{x^4}{4}\right).$$

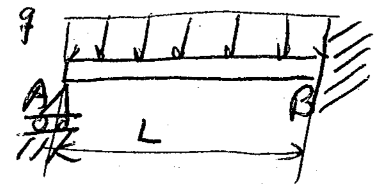
$$y(L) = 0 = C_2 + \frac{qL^4}{8EI} \leadsto C_2 = -\frac{qL^4}{8EI}$$

(a) $\boxed{y = \frac{q}{24EI}\left(4L^3x - x^4 - 3L^4\right)}$

(b) $y(0) = -\frac{qL^4}{8EI}$

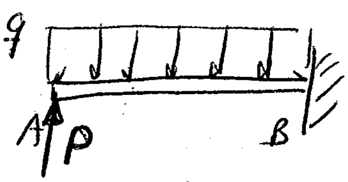
(c) $y'(0) = \frac{q}{6EI}(L^3 - 0^3) = +\frac{qL^3}{6EI}$

9.17



||

$\delta_A = 0 \; +$



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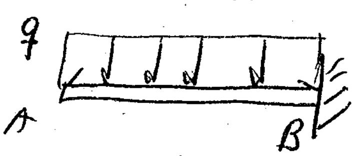
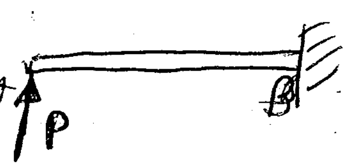
APPENDIX D, CASE #2



$\rightarrow \delta_{A_q} = -\frac{qL^4}{8EI}$

+

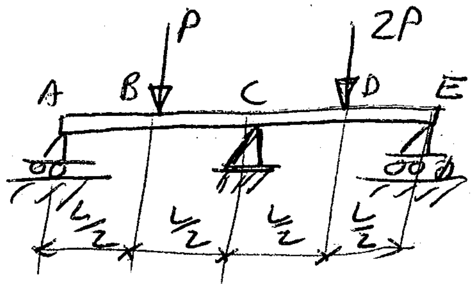


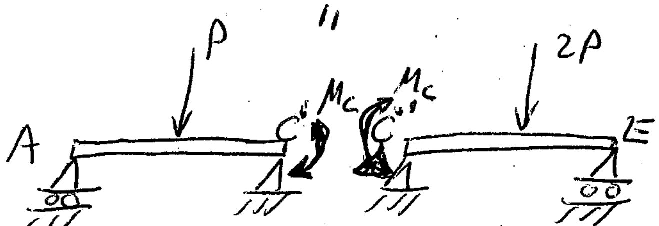
$\rightarrow \delta_{A_P} = +\frac{PL^3}{3EI}$

$$\delta_A = \delta_{A_q} + \delta_{A_P} = -\frac{qL^4}{8EI} + \frac{PL^3}{3EI} = 0$$

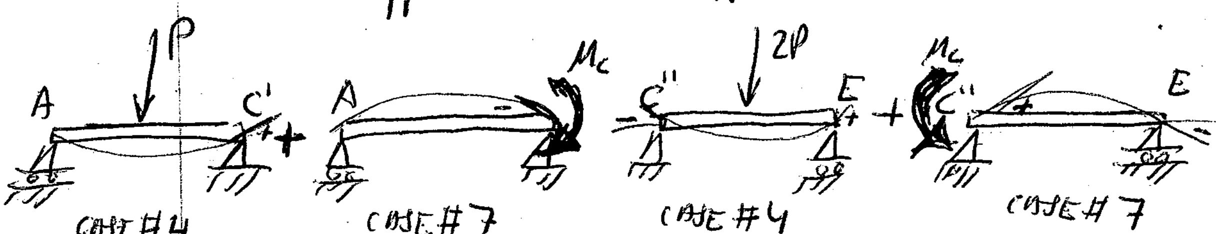
$$\boxed{P = \frac{3}{8}qL}$$

9.83





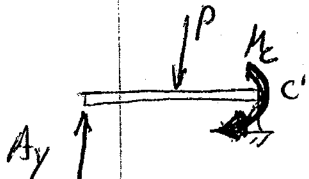
$+\ \phi_{C'} = \phi_{C''}$



$$+\frac{PL^2}{16\cancel{EI}} - \frac{M_c L}{3\cancel{EI}} = -\frac{2PL^2}{16\cancel{EI}} + \frac{M_c L}{3\cancel{EI}}$$

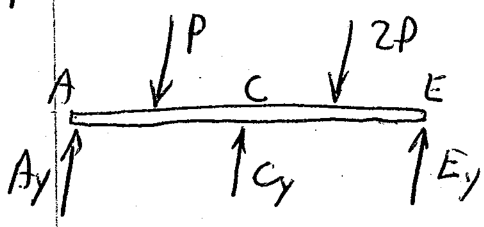
$$\frac{2}{3}M_c = \frac{3}{16}PL$$

$$M_c = \frac{9}{32}PL$$



$\Sigma M_{c'} = -M_c + P\cdot\frac{1}{2}L - A_y\cdot L = 0$ $\quad A_y = \frac{1}{2}P - \frac{M_c}{L}$

$$\boxed{A_y = \frac{7}{32}P}$$



$\Sigma M_c = 0 = -A_y L + P\frac{1}{2}L - 2P\cdot\frac{1}{2}L + E_y\cdot L$

$$\boxed{E_y = A_y + \frac{1}{2}P = \frac{23}{32}P}$$

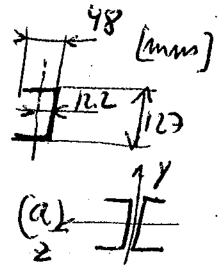
$\Sigma F_y = 0 = A_y + C_y + E_y - P - 2P \rightsquigarrow \boxed{C_y = 3P - \frac{7+23}{32}P = \frac{33}{16}P}$

10.18 $L_{eq} = 3\,m$

$C130 \times 13$

$E = 200\,GPa$

$FS = 2.4$



$A = 1710\,mm^2$

$I_{xx} = 3.7 \cdot 10^6\,mm^4$

$I_{yy} = 0.264\,mm^4 \cdot 10^6$

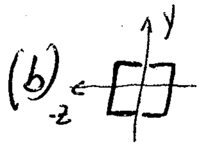
$I_{zz} = 2 \cdot 3.7 \cdot 10^6 = 7.4 \cdot 10^6\,mm^4$

$I_{yy} = 2(0.264 \cdot 10^6 + 1710 \cdot 12.2^2) = 1.037 \cdot 10^6\,mm^4$

CASE (a) WILL BUCKLE AROUND y AXIS BECAUSE $I_{yy} < I_{zz}$

$$\underline{P_{ALL}} = \frac{P_{cr}}{FS} = \frac{\pi^2 E I}{FS\, L_{eq}^2} = \frac{\pi^2 \cdot 200 \cdot 10^3 MPa \cdot 1.037 \cdot 10^6 mm^4}{2.4 \cdot (3000\,mm)^2} = \underline{94766.5\,N}$$

(b)



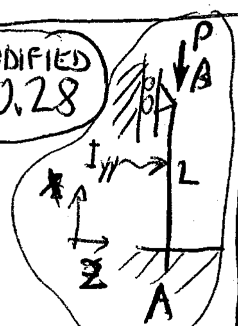
I_{zz} = WITHOUT CHANGE $= 7.4 \cdot 10^6\,mm^4$

$I_{yy} = 2(0.264 \cdot 10^6 + 1710 \cdot (48 - 12.2)^2) = 4.9112 \cdot 10^6\,mm^4$

$I_{yy} < I_{zz} \rightarrow I = I_{yy} = 4.9112 \cdot 10^6\,mm^4$

$$\underline{P_{ALL}} = \frac{P_{cr}}{FS} = \frac{\pi^2 E I}{FS\, L_{eq}^2} = \frac{\pi^2 \cdot 200 \cdot 10^3 MPa \cdot 4.9112 \cdot 10^6}{2.4 (3000)^2} = \underline{448.81 \cdot 10^3 N}$$

MODIFIED 10.28



$P = 72\,kN$

$W250 \times 32.7$

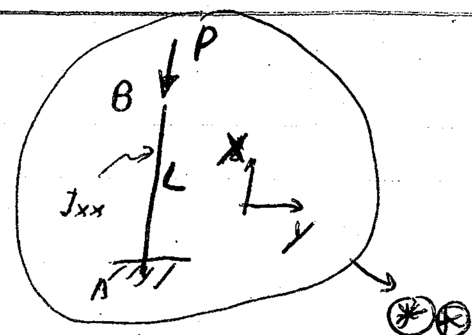
$I_{zz} = 48.9 \cdot 10^6\,mm^4$

$I_{yy} = 4.73 \cdot 10^6\,mm^4$

$FS = 2.3$

$E = 200\,GPa$ *

$\boxed{L - ?}$



XZ BUCKLING

*

PINED / FIX

$$P = \frac{P_{cr}}{FS} = \frac{\pi^2 E I_{yy}}{FS\, L_{eq}^2} \quad ; L_{eq} = 0.7L$$

$$L = \frac{\pi}{0.7}\sqrt{\frac{E I_{yy}}{P\, FS}} = \frac{\pi}{0.7}\sqrt{\frac{200 \cdot 10^3 \cdot 4.73 \cdot 10^6}{72000 \cdot 2.3}} = 10.73\,m$$

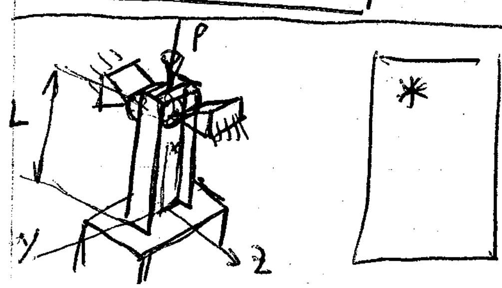
XY BUCKLING

**

$L_{eq} = 2L$

$$L = \frac{\pi}{2}\sqrt{\frac{E I_{zz}}{FS \cdot P}} = \frac{\pi}{2}\sqrt{\frac{200 \cdot 10^3 \cdot 48.9 \cdot 10^6}{2.3 \cdot 72000}} = 12.07\,m$$

$\boxed{L = 10.73\,m}$



① J.124

REVIEW OF STATICS

STRESSES

AXIAL STRESS
SHEARING STRESS
BEARING STRESS
QUIZ #1
STRESS ON AN OBLIQUE PLANE UNDER AXIAL LOADING
DESIGN CONSIDERATIONS → UTS
→ σ_{ALL}
→ F.S.

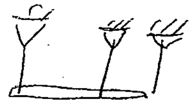
STRESS AND STRAIN – AXIAL LOADING

$\sigma - \varepsilon$ DIAGRAM

$\varepsilon = \frac{\delta}{L}$

→ HOOKE'S LAW, MODULUS OF ELASTICITY $E = \frac{\sigma}{\varepsilon}$

→ AXIAL DEFORMATION $\delta = \frac{PL}{AE}$



~~TEM~~ THERMAL
DETERMINATE
INDETERMINATE
QUIZ #2

→ POISSON'S RATIO

→ MULTIAXIAL LOADING, GENERALIZED HOOKE'S LAW

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\tau_{ij} = G\gamma_{ij}$$

I

→ $G = \frac{E}{2(1+\nu)}$

→ STRESS CONCENTRATION

2

#3 TORSION

II

AXI-SYMMETRIC CROSS SECS:

$\tau = \frac{Tr}{J}$ $\tau_{max} = \frac{TR}{J}$ (DESIGN)

POLAR MOMENT OF INERTIA

DEFORMATIONS:

$\phi = \frac{TL}{GJ}$

- INDETERMINATE PROBLEMS → FIND REACTIONS
- STRESS CONCENTRATION IN CIRCULAR SHAFTS

X

- TORSION OF NON CIRCULAR MEMBERS (STRESS + DEFL) ~~[illegible]~~ QUIZ #3
- " " THIN RECTANGULAR SECTION —"—
- TORSION OF THIN WALLED HOLLOW SHAFTS

BENDING

#5 - STATICS REVIEW

#4 - PURE BENDING:

$\rightarrow \sigma_x = -\frac{M_z y}{I_z}$; $\sigma_{x\,max} = -\frac{M_z y_{max}}{I_z}$

MOMENT OF INERTIA (FOR BENDING)

PARALLEL AXIS THEOREM

X

→ BENDING OF MEMBERS MADE OF SEVERAL MATERIALS
REINFORCED CONCRETE BEAMS

→ STRESS CONCENTRATION ON BENDING

→ UNSYMMETRIC BENDING

→ ECCENTRIC AXIAL LOADING

$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$

#5 DESIGN OF PRISMATIC BEAMS FOR BENDING

" " NON " " " "

#6 SHEARING STRESSES IN BEAMS AND THIN WALLED MEMBERS (NAILS, GLUE, shearing flow)

HIGH

TRANSFORMATION OF STRESS AND STRAIN

#7 PLANE STRESS TRANSFORMATION
MOHR'S CIRCLE
FAILURE ~~CRITERIA~~ CRITERION
MAX. SH. STRESS
VON-MISES } → DUCTILE
MAX. ~~N~~ NORMAL STRESS → BRITTLE
ANALYSIS OF STRESSES IN THIN WALLED PRESSURE VESSELS
} HIGH III

#8 → ~~[illegible]~~

#9 → DEFLECTION OF BEAMS
DEFLECTIONS BY INTEGRATION
DEFLECTIONS BY SUPERPOSITION
DETERMINATE & INDETERMINATE CASES

#10 → BUCKLING ON COLUMNS
EULER'S FORMULA
EXTENSION OF ....
DESIGN OF COLUMS
} BASED HM IV

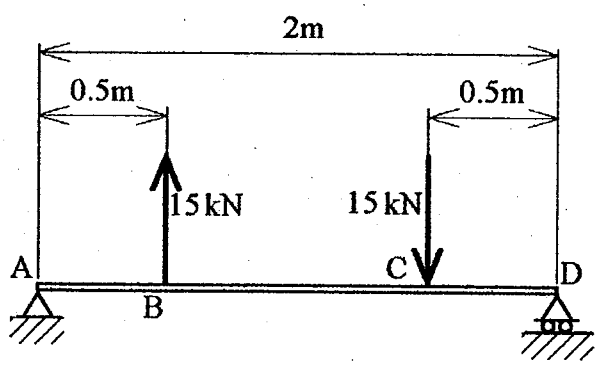
DIDNT LEARN → ELASTO PLASTICITY

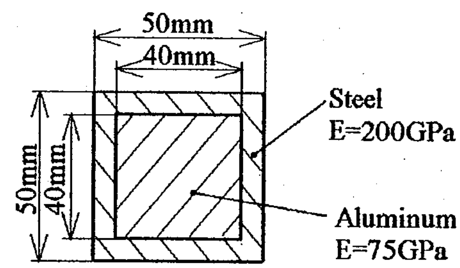
Florida International University
Department of Mechanical and Materials Engineering
Mechanics and Materials Science

EMA 3702 | Quiz # 4 – Version B | Dec. 9, 2004

<u>Follow the instructions before you begin the quiz:</u>
This test is 40 minutes long. It is not permitted to pass or receive any hardware nor software during the quiz. Use your own papers, pens, pencils, books, notebooks, calculators, etc.
Write your first and last name, your Panther I. D. & the quiz version (B)
Explain your steps, use the adequate diagrams.
Good Luck!

a) Given the following beam loaded as shown, draw the shearing force diagram & the bending moment diagram. Find the maximum magnitude of the bending moment and its location on the beam.(40%)
b) Calculate the maximum magnitude of stress σ_x **on Steel** for the given beam's cross section (60%)





Beam's Cross Section

B

Florida International University
Department of Mechanical and Materials Engineering
Mechanics and Materials Science

EMA 3702 | Quiz # 4 – Version A | Dec. 9, 2004

<u>Follow the instructions before you begin the quiz:</u>
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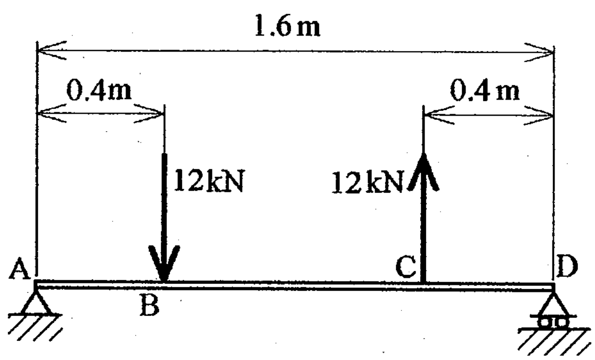
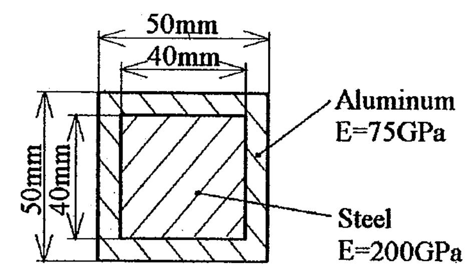
Write your first and last name, your Panther I. D. & the quiz version (A)
Explain your steps, use the adequate diagrams.

Good Luck!

a) Given the following beam loaded as shown, draw the shearing force diagram & the bending moment diagram. Find the maximum magnitude of the bending moment and its location on the beam.(40%)

b) Calculate the maximum magnitude of stress σ_x **on Aluminum** for the given beam's cross section (60%)





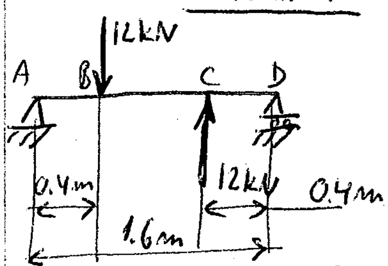
Beam's Cross Section

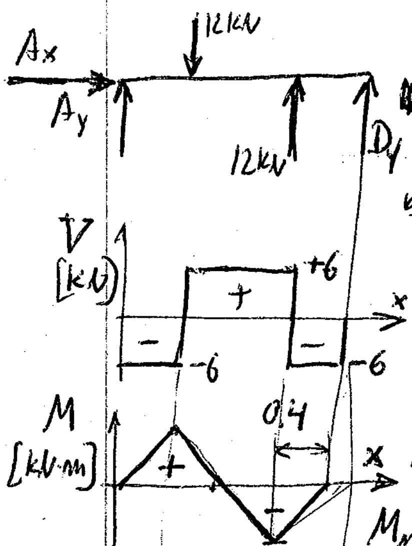
A

QUIZ #4

VERSION A

(a)





F.B.D.

$\Sigma F_x = 0 = A_x$

$\Sigma F_y = A_y - 12 + 12 + D_y = 0$

$A_y + D_y = 0$

$\Sigma M_A = 0 = -12 \cdot 0.4 + 12(1.6 - 0.4) + D_y \cdot 1.6$

$D_y = \frac{-12 \cdot 0.8}{1.6} = -6 kN$

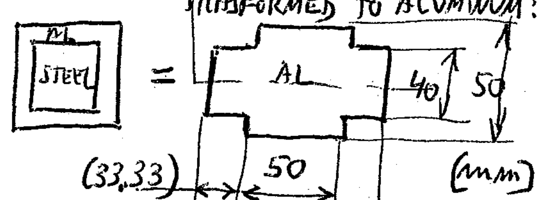
$A_y = +6\ kN$

$M_{max} = |D_y \cdot 0.4| = 2.4 kN{\cdot}m$

LOCATED AT $x = 0.4m$ & AT $x = 1.2m$

TRANSFORMED TO ALUMINUM:

(b)



$10 + \frac{200}{75} \cdot 40 = 116.67$

$I_z = 2 \cdot \frac{40^3 \cdot 33.33}{12} + \frac{50^3 \cdot 50}{12} = 876.39 \cdot 10^3\ mm^4$

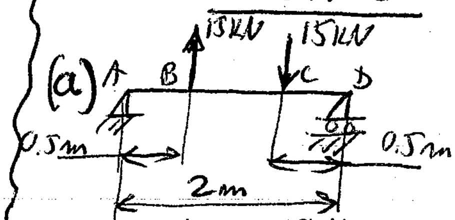
$\sigma_{max} = \left| \frac{M_{max} \cdot y_{max}}{I_z} \right| = \frac{2.4 \cdot 10^6 N{\cdot}mm \cdot 25 mm}{876.39 \cdot 10^3 mm^4}$

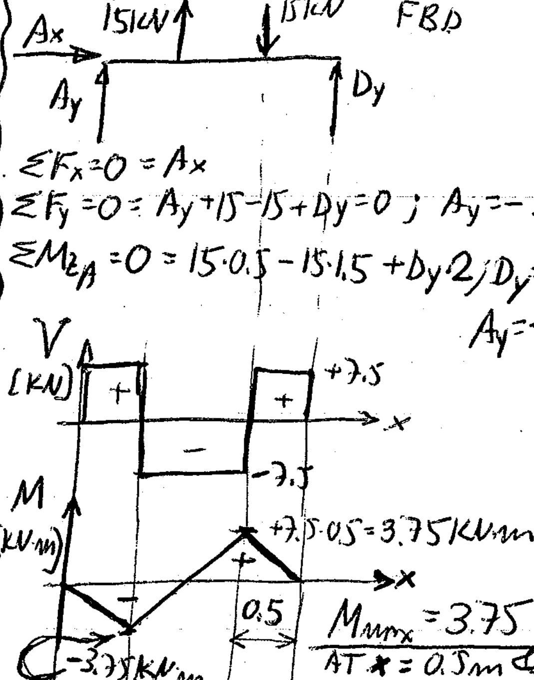
$\sigma_{max} = 68.46\ MPa$

ON ALUMINUM

VERSION B

(a)





$\Sigma F_x = 0 = A_x$

$\Sigma F_y = 0 = A_y + 15 - 15 + D_y = 0$; $A_y = -D_y$

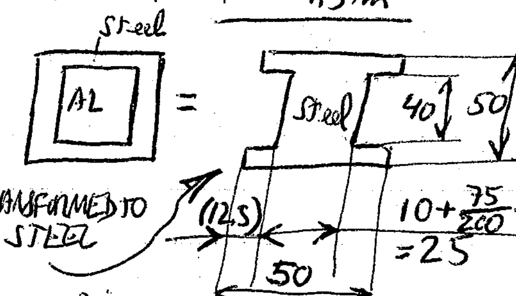
$\Sigma M_A = 0 = 15 \cdot 0.5 - 15 \cdot 1.5 + D_y \cdot 2$; $D_y = +7.5 kN$

$A_y = -7.5 kN$

$M_{max} = 3.75\ kN{\cdot}m$

AT $x = 0.5m$ & AT $x = 1.5m$

(b)



$10 + \frac{75}{200} \cdot 40 = 25$

$I_z = \frac{50^3 \cdot 50}{12} - 2 \cdot \frac{12.5 \cdot 40^3}{12} = 387.5 \cdot 10^3 mm^3$

$\sigma_{max} = \left| \frac{M_{max} \cdot y_{max}}{I_z} \right| = \frac{3.75 \cdot 10^6 N{\cdot}mm \cdot 25 mm}{387.5 \cdot 10^3}$

$\sigma_{max} = 241.94\ MPa$

ON STEEL

Florida International University
Department of Mechanical and Materials Engineering
Mechanics and Materials Science

EMA 3702 FINAL EXAMINATION – Version B Dec. 14, 2004

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Write your first and last name, your Panther I. D. & the exam version (B)
Explain your steps, use the adequate diagrams.

This examination consists of 3 problems with several parts to each of the problems. You are to answer all the problems.

<u>Good Luck!</u>

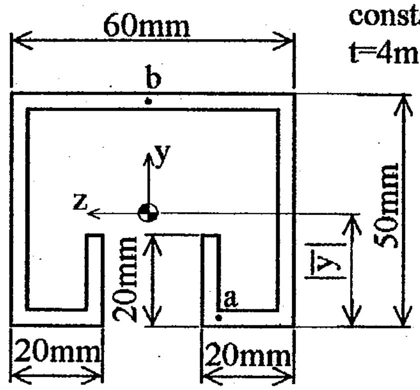
Problem #	Breakdown by Problem	Score
1	35%	
2	40%	
3	25%	

B

PROBLEM 1B

A vertical shear Vy= 6 kN is applied on the beam's cross section shown

a) Determine the shearing stress at **point a** and at **point b** (15%)
b) Calculate the maximum shearing stress (15%)
c) Sketch the shear flow and shearing stress distribution in the cross section (10%)



constant thickness
t=4mm

use :

$I_z = 285852.0\ \text{mm}^4$

$|\bar{y}| = 25.2$ mm

B

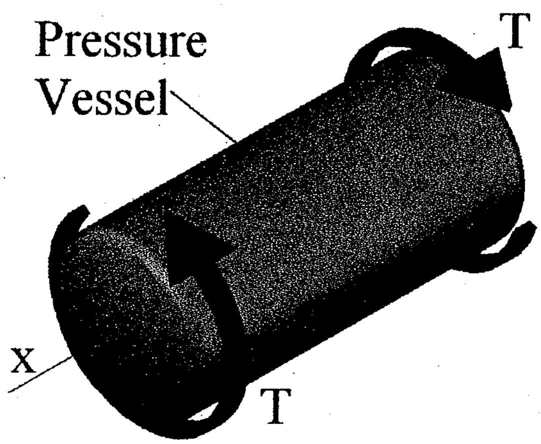
Florida International University
Department of Mechanical and Materials Engineering
Mechanics and Materials Science

EMA 3702 FINAL EXAMINATION – Version B Dec. 14, 2004

PROBLEM 2B

A pressure vessel of 300mm inner diameter and 5mm wall thickness is made of steel (yield stress σy=600MPa). The vessel is loaded by an internal pressure and by an external torque as shown. The pressure is 7 MPa and the magnitude of the torque is T=110kN·m. Analyze the cylindrical section of the vessel as follows:

(a) Draw the state of plane stress and draw its Mohr's circle (including the steps and values used to draw it) (10%)
(b) Determine the principal stresses and maximum shearing stress (10%)
(c) Calculate the angle of the principal plane (the angle between σ_1 and x axis) (10%)
(d) Determine the Factor of Safety according to Maximum-Shearing-Stress criterion (10%)



B

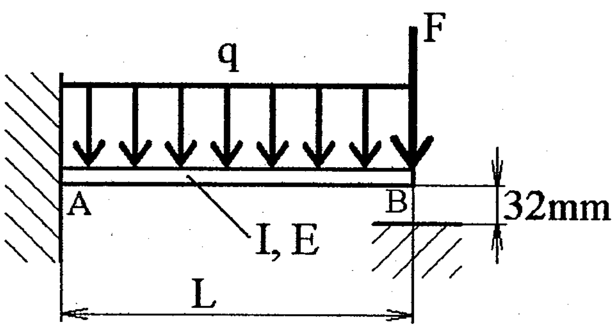
PROBLEM 3B

A fixed beam is loaded by a distributed load q and a force F at point B, as shown. Before loading, the distance of point B from the floor was 32mm. After loading, the total deflection at B **should not be greater than 32mm** (don't touch the floor).

a) Determine the maximum magnitude of force F. (15%)
b) Determine the slope at B for the above loading. (10%)

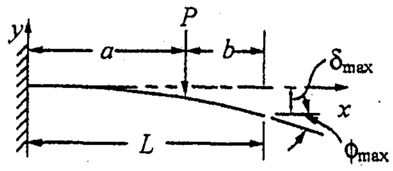
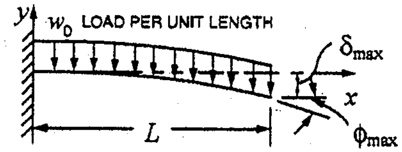
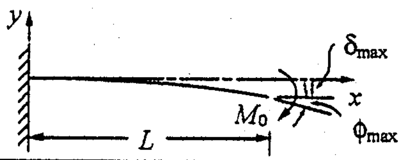
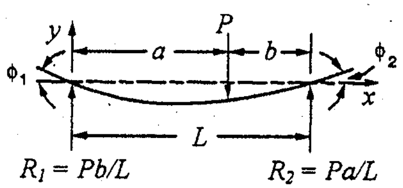
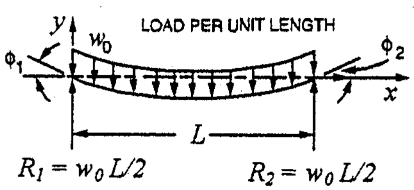
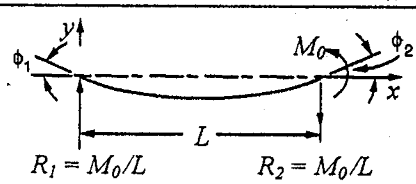
$q = 260 N/m$
$E = 199 GPa$
$I = 1975680 mm^4$
$L = 4 m$

It is recommended to use the super-position method



B

(δ is positive downward)

	$\delta=\frac{Pa^2}{6EI}(3x-a)$, for $x>a$ $\delta=\frac{Px^2}{6EI}(-x+3a)$, for $x\le a$	$\delta_{max}=\frac{Pa^2}{6EI}(3L-a)$	$\phi_{max}=\frac{Pa^2}{2EI}$
	$\delta=\frac{w_o x^2}{24EI}\left(x^2+6L^2-4Lx\right)$	$\delta_{max}=\frac{w_o L^4}{8EI}$	$\phi_{max}=\frac{w_o L^3}{6EI}$
	$\delta=\frac{M_o x^2}{2EI}$	$\delta_{max}=\frac{M_o L^2}{2EI}$	$\phi_{max}=\frac{M_o L}{EI}$
	$\delta=\frac{Pb}{6LEI}\left[\frac{L}{b}(x-a)^3-x^3+\left(L^2-b^2\right)x\right]$, for $x>a$ $\delta=\frac{Pb}{6LEI}\left[-x^3+\left(L^2-b^2\right)x\right]$, for $x\le a$	$\delta_{max}=\frac{Pb\left(L^2-b^2\right)^{3/2}}{9\sqrt{3}LEI}$ at $x=\sqrt{\frac{L^2-b^2}{3}}$	$\phi_1=\frac{Pab(2L-a)}{6LEI}$ $\phi_2=\frac{Pab(2L-b)}{6LEI}$
	$\delta=\frac{w_o x}{24EI}\left(L^3-2Lx^2+x^3\right)$	$\delta_{max}=\frac{5w_o L^4}{384EI}$	$\phi_1=\phi_2=\frac{w_o L^3}{24EI}$
	$\delta=\frac{M_o Lx}{6EI}\left(1-\frac{x^2}{L^2}\right)$	$\delta_{max}=\frac{M_o L^2}{9\sqrt{3}EI}$ at $x=\frac{L}{\sqrt{3}}$	$\phi_1=\frac{M_o L}{6EI}$ $\phi_2=\frac{M_o L}{3EI}$

Florida International University
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EMA 3702 FINAL EXAMINATION – Version A Dec. 14, 2004

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1	35%	
2	40%	
3	25%	

A

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Mechanics and Materials Science

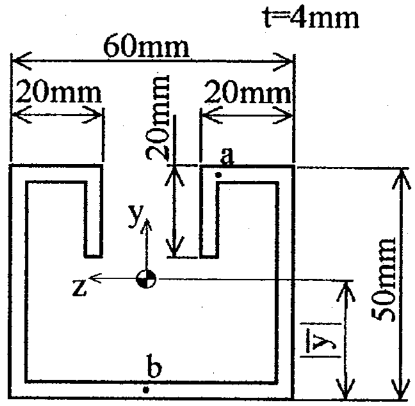
EMA 3702 FINAL EXAMINATION – Version A Dec. 14, 2004

PROBLEM 1A

A vertical shear Vy= 8 kN is applied on the beam's cross section shown

a) Determine the shearing stress at **point a** and at **point b** (15%)
b) Calculate the maximum shearing stress (15%)
c) Sketch the shear flow and shearing stress distribution in the cross section (10%)

constant thickness
t=4mm



use:

$I_z = 285852.0 \text{ mm}^4$

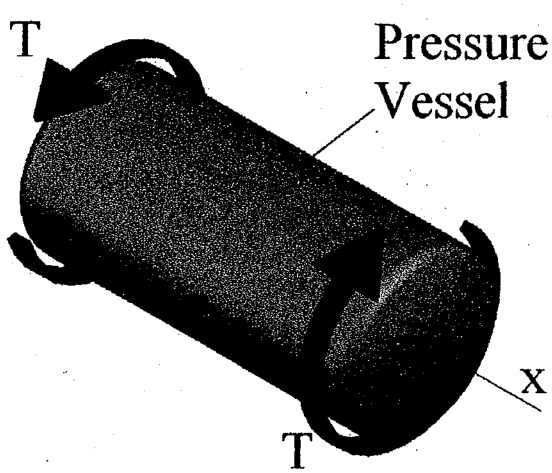
$|\bar{y}| = 24.8 \text{ mm}$

A

PROBLEM 2A

A pressure vessel of 300mm inner diameter and 5mm wall thickness is made of steel (yield stress σy=600MPa). The vessel is loaded by an internal pressure and by an external torque as shown. The pressure is 7 MPa and the magnitude of the torque is T=110kN·m. Analyze the cylindrical section of the vessel as follows:

(a) Draw the state of plane stress and draw its Mohr's circle (including the steps and values used to draw it) (10%)
(b) Determine the principal stresses and maximum shearing stress (10%)
(c) Calculate the angle of the principal plane (the angle between σ_1 and x axis) (10%)
(d) Determine the Factor of Safety according to Maximum-Distortion-Energy criterion (Von Mises criterion) (10%)



A

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EMA 3702 FINAL EXAMINATION – Version A Dec. 14, 2004

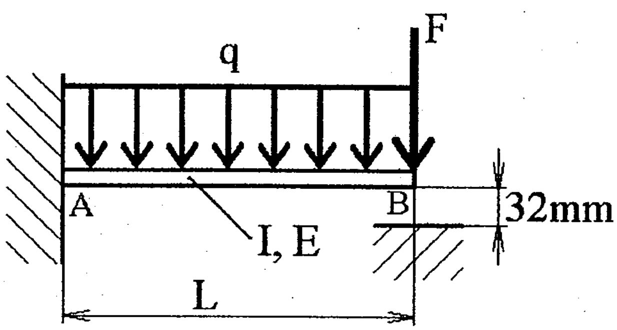
PROBLEM 3A

A fixed beam is loaded by a distributed load q and a force F at point B, as shown. Before loading, the distance of point B from the floor was 32mm. After loading, the total deflection at B **should not be greater than 32mm** (don't touch the floor)

a) Determine the maximum magnitude of the distributed load q. (15%)
b) Determine the slope at B for the above loading. (10%)

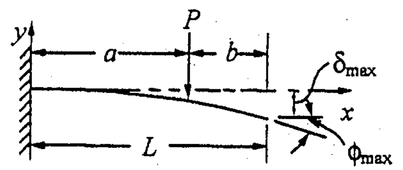
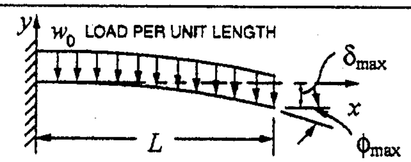
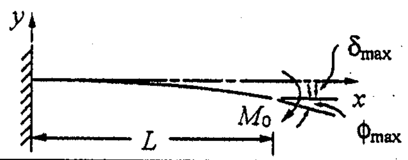
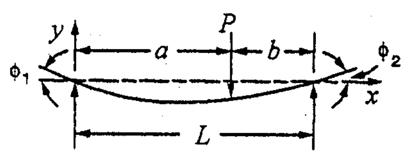
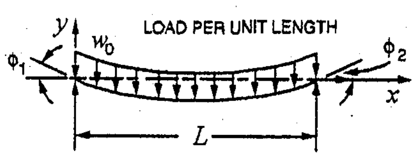
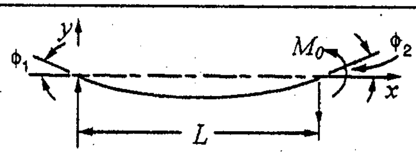
F=200N
E=199GPa
I=1975680mm^4
L=4m

It is recommended to use the super-position method



A

(δ is positive downward)

	$\delta = \frac{Pa^2}{6EI}(3x-a)$, for $x>a$ $\delta = \frac{Px^2}{6EI}(-x+3a)$, for $x \le a$	$\delta_{max} = \frac{Pa^2}{6EI}(3L-a)$	$\phi_{max} = \frac{Pa^2}{2EI}$
	$\delta = \frac{w_o x^2}{24EI}(x^2+6L^2-4Lx)$	$\delta_{max} = \frac{w_o L^4}{8EI}$	$\phi_{max} = \frac{w_o L^3}{6EI}$
	$\delta = \frac{M_o x^2}{2EI}$	$\delta_{max} = \frac{M_o L^2}{2EI}$	$\phi_{max} = \frac{M_o L}{EI}$
 $R_1 = Pb/L$ $R_2 = Pa/L$	$\delta = \frac{Pb}{6LEI}\left[\frac{L}{b}(x-a)^3 - x^3 + (L^2-b^2)x\right]$, for $x>a$ $\delta = \frac{Pb}{6LEI}\left[-x^3 + (L^2-b^2)x\right]$, for $x \le a$	$\delta_{max} = \frac{Pb(L^2-b^2)^{3/2}}{9\sqrt{3}LEI}$ at $x = \sqrt{\frac{L^2-b^2}{3}}$	$\phi_1 = \frac{Pab(2L-a)}{6LEI}$ $\phi_2 = \frac{Pab(2L-b)}{6LEI}$
 $R_1 = w_0 L/2$ $R_2 = w_0 L/2$	$\delta = \frac{w_o x}{24EI}(L^3-2Lx^2+x^3)$	$\delta_{max} = \frac{5w_o L^4}{384EI}$	$\phi_1 = \phi_2 = \frac{w_o L^3}{24EI}$
 $R_1 = M_0/L$ $R_2 = M_0/L$	$\delta = \frac{M_o Lx}{6EI}\left(1-\frac{x^2}{L^2}\right)$	$\delta_{max} = \frac{M_o L^2}{9\sqrt{3}EI}$ at $x = \frac{L}{\sqrt{3}}$	$\phi_1 = \frac{M_o L}{6EI}$ $\phi_2 = \frac{M_o L}{3EI}$

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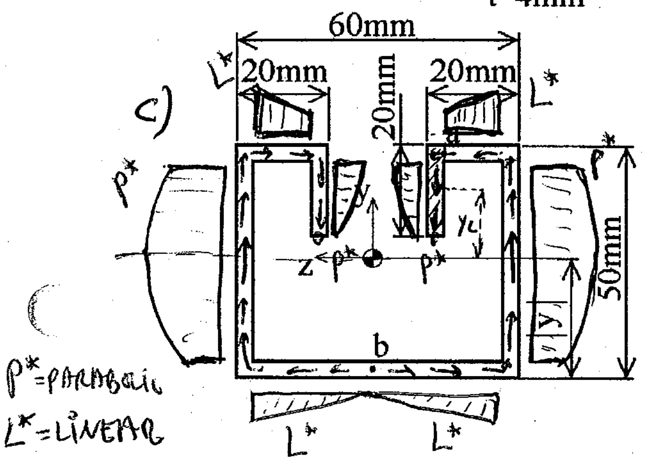
EMA 3702 FINAL EXAMINATION – Version A Dec. 14, 2004

PROBLEM 1A

A vertical shear Vy= 8 kN is applied on the beam's cross section shown

a) Determine the shearing stress at **point a** and at **point b** (15%)
b) Calculate the maximum shearing stress (15%)
c) Sketch the shear flow and shearing stress distribution in the cross section (10%)

constant thickness
t=4mm



a) $\tau_a = \dfrac{V_y Q_a}{I_z \cdot t}$

$Q_a = A_a y_c$

$A_a = 20 \cdot 4 = 80\ mm^2$

$y_c = 50 - |\bar{y}| - 10 = 15.2\ mm$

$Q_a = 80 \cdot 15.2 = 1216\ mm^3$

use :
Iz = 285852.0 mm^4
$|\bar{y}|$ = 24.8 mm

$\tau_a = \dfrac{8000 N \cdot 1216 mm^3}{285852 mm^4 \cdot 4 mm} = 8.51\ MPa$

$\tau_b = 0$

b) AT THE NEUTRAL AXIS:

$\tau_{max} = \dfrac{V_y Q_N}{I_z t}$

$Q_N = Q_a + 16 \cdot 4 \cdot (50 - |\bar{y}| - 2) + \frac{1}{2}(50 - |\bar{y}| - 4)^2 \cdot 4 \cdot \cancel{2} = 3599.68\ mm^3$

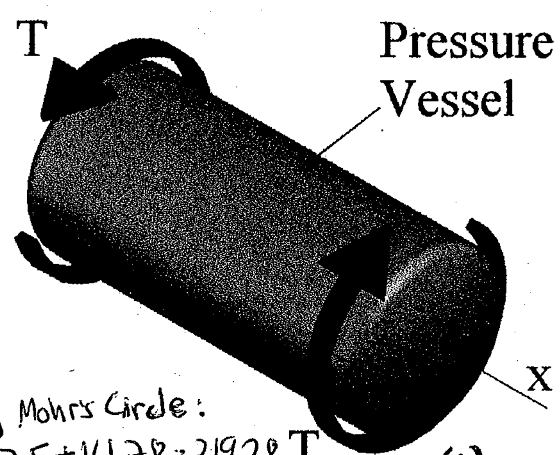
$\tau_{max} = \dfrac{8000 \cdot 3599.68}{285852 \cdot 4} = 25.19\ MPa$

A

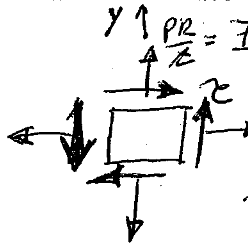
PROBLEM 2A

A pressure vessel of 300mm inner diameter and 5mm wall thickness is made of steel (yield stress σy=600MPa). The vessel is loaded by an internal pressure and by an external torque as shown. The pressure is 7 MPa and the magnitude of the torque is T=110kN·m. Analyze the cylindrical section of the vessel as follows:

(a) Draw the state of plane stress and draw its Mohr's circle (including the steps and values used to draw it) (10%)
(b) Determine the principal stresses and maximum shearing stress (10%)
(c) Calculate the angle of the principal plane (the angle between σ_1 and x axis) (10%)
(d) Determine the Factor of Safety according to Maximum-Distortion-Energy criterion (Von Mises criterion) (10%)



(a)



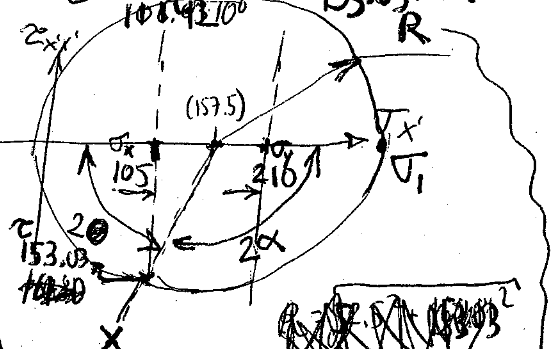
$\frac{PR}{t} = \frac{7 \cdot 150}{5} = 210\ MPa = \sigma_y$

$\frac{PR}{2t} = \frac{7 \cdot 150}{2 \cdot 5} = 105\ MPa = \sigma_x$

$\tau = \frac{T(R+t)}{J}$

$J = 2\pi R^3 t = 2\pi\, 152.5^3\, 5 = 111.42 \cdot 10^6$

$\tau = \frac{110 \cdot 10^6 \cdot (150+5)}{111.42 \cdot 10^6} = 153.03\ MPa$



$R = \sqrt{52.5^2 + 153.03^2}$

$R = 161.78\ MPa$

using Mohr's Circle:

(b) $\sigma_1 = 157.5 + 161.78 = 319.28\ MPa$

$\sigma_2 = 157.5 - 161.78 = -4.28\ MPa$

$\tau_{max} = R = 161.78\ MPa$

(c) $tg\, 2\theta = \frac{161.78}{52.5} = 3.08$

$2\theta = 72°$

$\theta = 36°$

$2\alpha = 180 - 2\theta$

$\alpha = 90 - \theta$

$\alpha = 54°$

(d) $\sigma_{eq} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$

$= (319.28^2 + 319.28 \cdot 4.28 + 4.28^2)^{1/2}$

$\sigma_{eq} = 321.4\ MPa$

$F.S = \frac{600}{321.4} = 1.87$

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EMA 3702 FINAL EXAMINATION – Version A Dec. 14, 2004

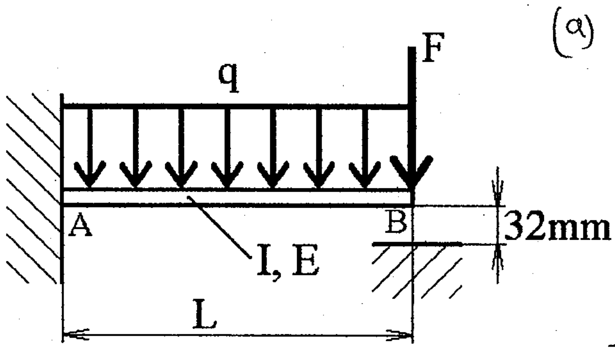
PROBLEM 3A

A fixed beam is loaded by a distributed load q and a force F at point B, as shown. Before loading, the distance of point B from the floor was 32mm. After loading, the total deflection at B **should not be greater than 32mm** (don't touch the floor)

a) Determine the maximum magnitude of the distributed load q. (15%)
b) Determine the slope at B for the above loading. (10%)

F=200N
E=199GPa
I=1975680mm^4
L=4m

It is recommended to use the super-position method



(a) $\delta_B = \delta_{B_F} + \delta_{B_q} = 32\ mm$

$$\delta_{B_F} = \frac{FL^3}{3EI}$$

$$\delta_{B_q} = \frac{qL^4}{8EI}$$

$$q = \frac{8}{L^4}\left\{32EI - \frac{FL^3}{3}\right\} = \frac{8}{(4000)^4}\left\{32\cdot 199000\cdot 1975680 - \frac{200\cdot 4000^3}{3}\right\}$$

$q = 0.26\ \frac{N}{mm} =$

$\underline{q = 260\ \frac{N}{m}}$

b) $\theta_B = \theta_{B_F} + \theta_{B_q}$

$$\left.\begin{aligned}\theta_{B_F} &= -\frac{FL^2}{2EI}\\ \theta_{B_q} &= \frac{-qL^3}{6EI}\end{aligned}\right\}\ \theta_B = \frac{-L^2}{2EI}\left(F + \frac{qL}{3}\right)$$

$$\theta_B = \frac{-4000^2}{2\cdot 199000\cdot 1975680}\left(200 + \frac{260\cdot 4}{3}\right)$$

$\theta_B = -0.0111\ rad$

$\underline{\theta_B = -0.64°}$

A

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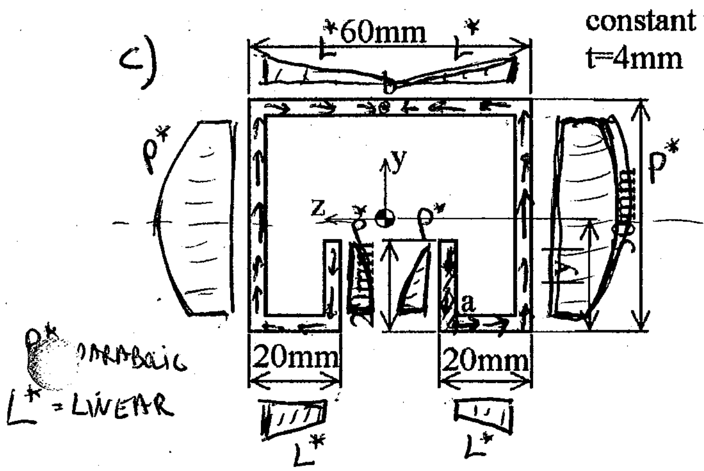
PROBLEM 1B

A vertical shear Vy= 6 kN is applied on the beam's cross section shown

a) Determine the shearing stress at **point a** and at **point b** (15%)
b) Calculate the maximum shearing stress (15%)
c) Sketch the shear flow and shearing stress distribution in the cross section (10%)



constant thickness
t=4mm

a) $\tau_a = \dfrac{V_y Q_a}{I_z \cdot t}$

$Q_a = A_a y_c$

$A_a = 20 \cdot 4 = 80\,mm^2$
$y_c = 25.2 - \frac{20}{2} = 15.2$
$Q_a = 80 \cdot 15.2 = 1216\,mm^3$

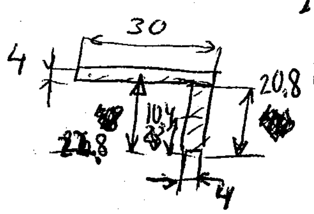
use: $\tau_a = \dfrac{6000 \cdot 1216}{285852 \cdot 4} = 6.38\,MPa$

Iz = 285852.0 mm^4

$|\bar{y}|$ = 25.2 mm

$\tau_b = 0$

b) $\tau_{max} = \dfrac{V_y Q_{max}}{I_z \cdot t}$ AT THE NEUTRAL AXIS



~~$Q = 30 \cdot 4 \cdot 28 + 20.8 \cdot 4 \cdot 23 = 9992\,mm^3$~~

~~$\tau = \frac{6000 \cdot 9992}{4 \cdot 285852} = 52.43\,MPa$~~

$Q_{max} = 30 \cdot 4 \cdot 22.8 + 20.8 \cdot 4 \cdot 10.4 = 3601.3\,mm^3$

$\tau_{max} = \dfrac{6000 \cdot 3601.3}{4 \cdot 285852} = 18.9\,MPa$

B

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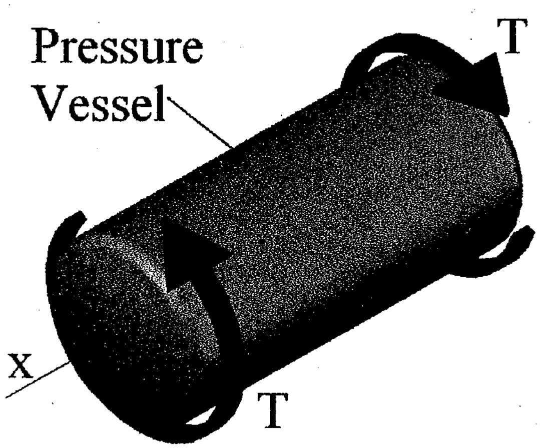
EMA 3702 FINAL EXAMINATION – Version B Dec. 14, 2004

PROBLEM 2B

A pressure vessel of 300mm inner diameter and 5mm wall thickness is made of steel (yield stress σy=600MPa). The vessel is loaded by an internal pressure and by an external torque as shown. The pressure is 7 MPa and the magnitude of the torque is T=110kN·m. Analyze the cylindrical section of the vessel as follows:

(a) Draw the state of plane stress and draw its Mohr's circle (including the steps and values used to draw it) (10%)
(b) Determine the principal stresses and maximum shearing stress (10%)
(c) Calculate the angle of the principal plane (the angle between σ_1 and x axis) (10%)
(d) Determine the Factor of Safety according to Maximum-Shearing-Stress criterion (10%)



(a), (b), (c) – SAME AS VERSION A

(d) $\sigma_{eq} = \max \begin{Bmatrix} |\sigma_1 - \sigma_2| \\ |\sigma_1| \\ |\sigma_2| \end{Bmatrix} = 319.28 + 4.28$

$\sigma_{eq} = 323.56\text{MPa}$

$\underline{F.S.} = \frac{600}{323.56} = \underline{1.85}$

B

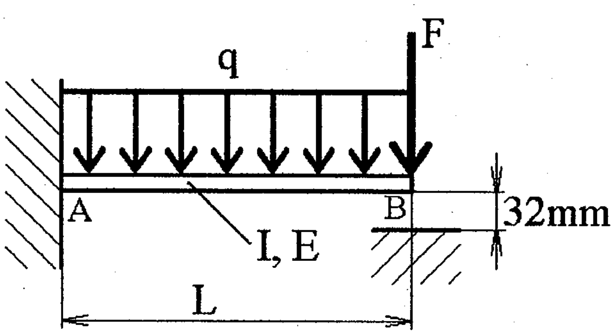
PROBLEM 3B

A fixed beam is loaded by a distributed load q and a force F at point B, as shown. Before loading, the distance of point B from the floor was 32mm. After loading, the total deflection at B **should not be greater than 32mm** (don't touch the floor).

a) Determine the maximum magnitude of force F. (15%)
b) Determine the slope at B for the above loading. (10%)

q=260N/m
E=199GPa
I=1975680mm^4
L=4m

It is recommended to use the super-position method



(a) $\delta_B = \delta_{BF} + \delta_{Bq} = 32mm$

$$\delta_{BF} = \frac{FL^3}{3EI}$$

$$\delta_{Bq} = \frac{qL^4}{8EI}$$

$$\frac{1}{3}F + \frac{1}{8}qL = 32\frac{EI}{L^3}$$

$$F = 3\left\{32\frac{EI}{L^3} - \frac{1}{8}qL\right\}$$

$$= 3\left\{32\frac{199000\cdot 1975680}{(4000)^3} - \frac{1}{8}260\cdot 4\right\} = 200N$$

(b) $\theta_B = \theta_{BF} + \theta_{Bq}$

$$\theta_{BF} = -\frac{FL^2}{2EI}, \quad \theta_{Bq} = -\frac{qL^3}{6EI} \quad \theta_B = -\frac{L^2}{2EI}\left\{F + \frac{qL}{3}\right\}$$

$$\theta_B = \frac{-4000^2}{2\cdot 199000\cdot 1975680}\left(200 + \frac{260\cdot 4}{3}\right)$$

$\theta_B = -0.011$ rad

$\theta_B = -0.637^\circ$

B