EMA 3702 Mechanics and Materials Science (3) – Fall 2004

Professor:

Favel Gov
Office EAS 3234
Office hours: T 1500-1800 2067
Tel: (305) 348-xxxx
e-mail: favelg@hotmail.com (temporary. to be updated)

Textbook:

Beer, Johnston and DeWolf, Mechanics of Materials, 3rd Ed., McGraw Hill
Schaum's Outline - Strength of Materials

References:

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Course Objectives

1. Identify mechanical properties and the characteristics of elastic behavior for material types.
2. Calculate the stress and strain configuration at a point for a specific loading arrangement.
3. Transform plane stress and strain configurations and identify principal stress and Principal Axes.
4. Use the appropriate failure criteria for diverse situation and/or materials (elastic behavior only).
5. Design prismatic beams.

MME Educational Objectives related to this course

1. Broad and in-depth knowledge of engineering science and principles in the major fields of Mechanical Engineering for effective engineering practice, professional growth and as a base for life-long learning.
2. The ability to communicate effectively and to articulate technical matters using verbal, written and graphic techniques.

MME Program Outcomes related to this course

(a) Ability to apply knowledge of mathematics, science, and engineering
(e) Ability to identify, formulate, and solve engineering problems
(f) Understanding of professional and ethical responsibility
(h) Broad education necessary to understand the impact of engineering solutions in a global and societal context
(i) Recognition of the need for, and a ability to engage in life-long learning
(k) Ability to use the techniques, skills and modern engineering tools necessary for engineering practice.
(l) Knowledge of mathematics and of basic engineering science necessary to carry out analysis and
design appropriate to mechanical engineering
(n) Ability to apply advanced mathematics through multivariable calculus and differential equations

Topics of the course

1. Introduction. Statics review. Concept of stress. Normal and Shear stresses. Bearing stresses. Stress on an oblique plane. Stresses under general conditions.
2. Stress and Strain – Axial Loading. Stress vs. strain. Hooke's law. Fatigue and deformations of members under axial load Thermal Stresses, Poisson ratio. Generalized Hooke's law **Quiz 1** → Sept 23
3. Saint Venant's Principle. Stress Concentration. Torsion. Stresses in a shaft Deformations and angle of twist Statically indeterminate shafts. Stress concentration in circular shaft **Quiz 2** → Oct 14
4. Analysis and Design of Beams. Shear and Bending Moment Diagrams. Relation among load, shear and bending moment. Pure Bending. Stresses and deformations in the elastic range Bending on members made of several materials. Stress Concentration. General case of eccentric axial loading **Quiz 3** → Nov 4
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7. Superposition Method. Columns. Euler's formula. Extension Euler's formula. Design of columns: centric and eccentric load. Failure Criteria

Homeworks

Will be assigned and collected. These problems should be worked out at home because you might see them again on your quizzes. **Be neat**. Use the "Given, Required, Solution" format and completely draw appropriate diagrams and coordinate systems. **All numerical answers should have the appropriate units.** You must keep up with the homework in order to do well in class.

HOMEWORK SOLUTION WILL BE FOUND ~~A WEEK~~ (IN A WEBSITE) AFTER THEY ARE COLLECTED)

Grading

Homework	12%
Quizzes (4 * 12% each)	48%
Final Exam	40%
Total	100%

Grades will be assigned based on your performance on the activities above. Final letter grades will be assigned as follows:

100 – 90	A	77-79.99	B-	63-66.99	D+

87-89.99	A-	73-76.99	C+	60-62.99	D
83-86.99	B+	70-72.99	C	Below 60	F
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Student Success in this Course

- Students can achieve success if they do the homeworks assigned in a neat and clean manner, set aside time to study the material learned daily, ask questions, and prepare well in advance of announced quizzes and final exam.
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Class will meet Tuesday 1905-2020 and on Thursday 1905-2145 in Room EAS 1104.

FINAL EXAM (Cumulative): ~~**Tuesday 20 December 2004 at 1230-1515 pm.**~~

Dec 13 - 18

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INTRODUCTION TO THE COURSE

MECHANICS AND MATERIALS SCIENCE (EMA 3702)

INSTRUCTOR: FAVEL GOV

1) ADMINISTRATIVE ISSUES:

1. THE SYLLABUS
2. SEND ME AN EMAIL TO favelg@hotmail.com with the following information:

SUBJECT: EMA 3702

1. first and last name
2. student number
3. Email address you want me to send messages
4. PHONE NUMBER

INTRODUCTION TO MECHANICS AND MATERIALS SCIENCE

SINCE THE 17th CENTURY SCIENTISTS AN ENGINEERS HAVE STUDIED THE PROBLEM OF THE LOAD-CARRYING CAPACITY OF STRUCTURAL MEMBERS AND MACHINE COMPONENTS AND HAVE DEVEPOTED MATHEMATICAL AND EXPERIMENTAL METHODS OF ANALYSIS FOR DETERMINING THE INTERNAL FORCES AND DEFORMATIONS INDUCED BY THE APPLIED LOADS

MECHANICS AND MATERIALS SCIENCE IS A BASIC COURSE FOR MECHANICAL ENGINEERS AS WELL AS FOR CIVIL ENGINEERING, AERONAUTICAL ENGINEERING, AEROSPACE ENGINEERING, THE MECHANICAL BRANCH OF AGRICULTURE ENGINEERING AND ANY FIELD THAT REQUIRES BUILDING STRUCTURES OF ANY KIND.

SIMILAR SUBJETS TO "MECHANICS AND MATERIAL SCIENCE" CAN BE FOUND UNDER TITLES LIKE "SOLID MECHANICS", "MECHANICS OF SOLIDS", "MECHANICS OF MATERIALS" OR "STRENGTH OF MATERIALS"

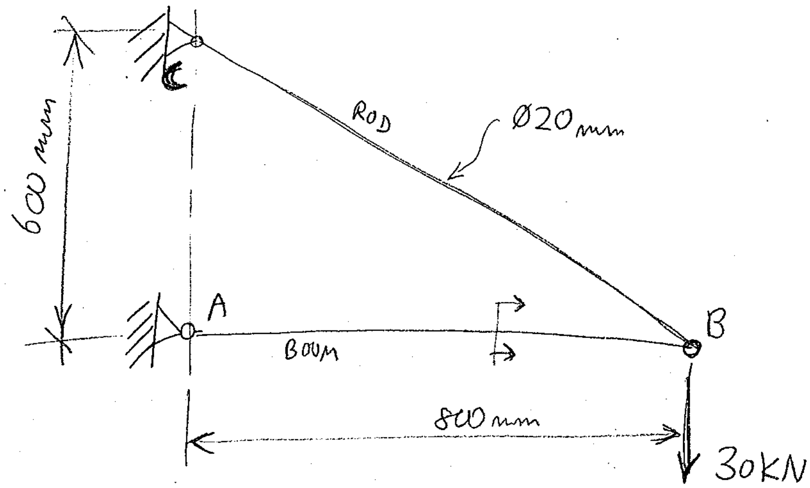
THE COURSE IS BASED ON PRELIMINARY KNOWLEDGE ON <u>STATICS</u>

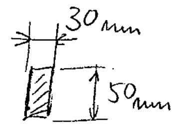
I REMARK - KNOWLEDGE, NOT ONLY CREDIT FOR THE STATICS COURSE.

REVIEW OF THE METHODS OF STATICS

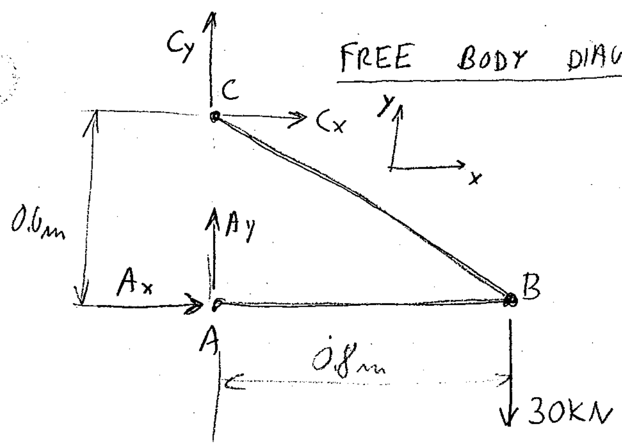
WE ARE GOING TO REVIEW THE BASIC METHODS OF STATICS WHILE DETERMINING THE FORCES IN THE MEMBERS OF A SIMPLE STRUCTURE

CONSIDER THE FOLLOWING STRUCTURE:





FREE BODY DIAGRAM OF THE STRUCTURE



WE KNOW THREE EQUILIBRIUM EQUATIONS:

$+\circlearrowleft \sum M_C = 0 \qquad A_x \cdot 0.6_m - 30kN \cdot 0.8_m = 0$

(TO BE DECIDED) $\qquad A_x = 40\ kN$

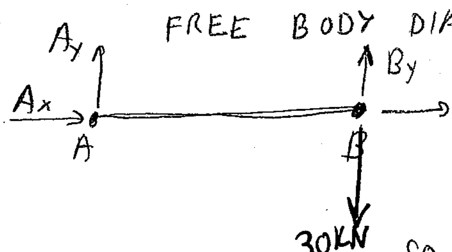
$\xrightarrow{+} \sum F_x = 0 \qquad A_x + C_x = 0 \Leftrightarrow C_x = -40\ kN$

$+\uparrow \sum F_y = 0 \qquad A_y + C_y - 30kN = 0$ (1)

WE CAN'T SOLVE ALL THE REACTIONS BECAUSE WE HAVE FOUR UNKNOWNS AND ONLY THREE EQUATIONS

WE SHOULD DISMEMBER THE STRUCTURE IN ORDER TO SOLVE IT

FREE BODY DIAGRAM OF AB:



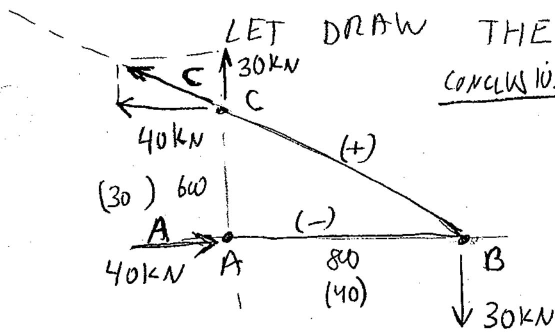
THREE EQUILIBRIUM EQUATIONS:

$B_x \quad \circlearrowleft \sum M_B = 0 \rightarrow A_y = 0$

WE ARE NOT INTERESTED IN B_x, B_y FORCES, SO, WE WILL NOT CONTINUE LOOKING FOR THEM

USING (1) WE CONCLUDE THAT $C_y = +30\ KN$

LET DRAW THE RESULTS:



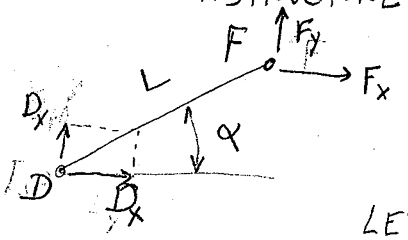
<u>CONCLUSIONS</u>:

THE TOTAL REACTIONS ~~AT~~ A, C ARE IN THE MEMBER DIRECTION (TENSION OR COMPRESSION)

A MEMBER BETWEEN ~~ROLLED SUPPORTS~~ SIMPLE SUPPORT AND IN WHICH FORCES ARE APPLIED ON THE ~~AXLES~~ SUPPORTS, IS CALLED A <u>BAR</u>

<u>LET PROVE IT</u>

CONSIDER A MEMBER AS A PART OF A STRUCTURE WITH A GENERAL FREE BODY DIAGRAM:



1. THE FORCES ARE APPLIED IN THE PINS
2. THERE ARE NOT ADDITIONAL LOADS ON THE MEMBER

LET APPLY THE MOMENT EQUILIBRIUM EQUATION:

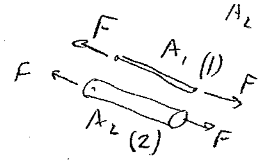
$$\circlearrowleft^{+} \sum M_F = 0 = D_x L \cdot \sin\alpha - D_y L \cdot \cos\alpha$$

$$\frac{D_y}{D_x} = \tan\alpha$$

WE CONCLUDE THAT THE SUM OF D_x AND D_y IS IN THE DIRECTION OF THE MEMBER, SO IT IS A <u>BAR</u>.

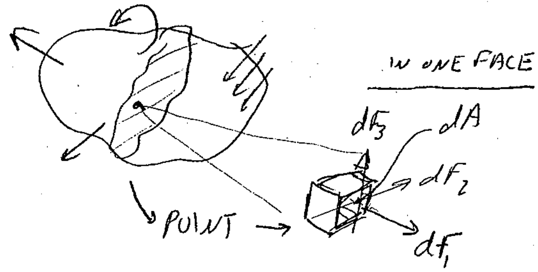
A BAR IS UNDER <u>AXIAL LOADING</u> (TENSION OR COMPRESSION)

$A_2 >> A_1$



STRESSES IN THE MEMBERS OF A STRUCTURE

THE STRESS DESCRIBES HOW A LOAD IS DISTRIBUTED INSIDE THE MATERIAL

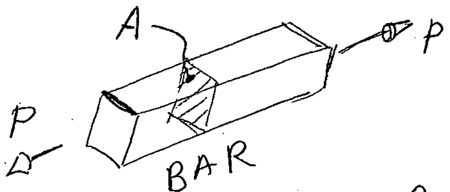


GREEK LETTER SIGMA

$$\sigma_i = \frac{dF_i}{dA} \rightsquigarrow \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

THE STRESS INFORM US HOW SAFE THE STRUCTURE MEMBER IS TAKING THE LOAD.

STRESS IN MEMBERS UNDER AXIAL LOADING

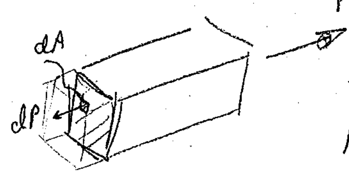


{ AXIAL LOAD P
SECTION AREA A

$$\boxed{\sigma = \frac{P}{A}}$$

ALSO CALLED "NORMAL STRESS"

$$\sigma = \frac{dP}{dA}$$

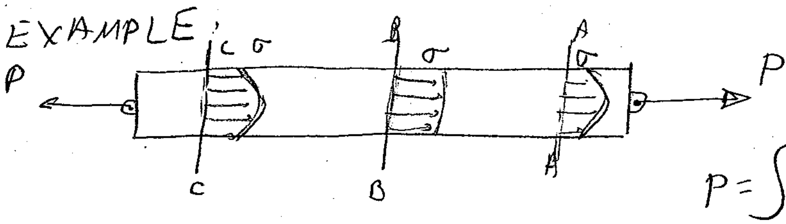


THE ASSUMPTION IS THAT LOAD P IS UNIFORMLY DISTRIBUTED THROW EVERY PERPENDICULAR SECTION

UNITS	P	A	σ
SI	N	m^2	$\frac{N}{m^2} = Pa$
USEFULL	N	mm^2	$\frac{N}{mm^2} = MPa$
US	lbf	in^2	PSI
	kip	in^2	ksi

$10^3 = k$
$10^6 = M$
$10^9 = G$

IN REAL LIFE, A CONCENTRATED LOAD SHOULD CAUSE A NON UNIFORM DISTRIBUTION OF STRESS, AS IN THE FOLLOWING EXAMPLE:



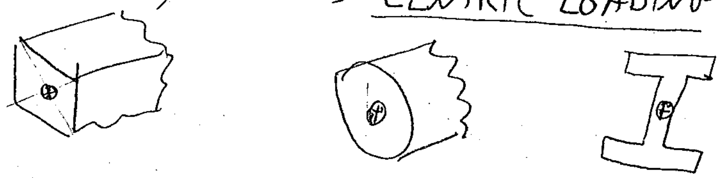
$$P = \int dF = \int \sigma\, dA$$

$$\sigma_{AVE} = \frac{P}{A}$$

IN PRACTICE, IT WILL BE ASSUMED THAT THE DISTRIBUTION OF NORMAL STRESSES IN AN AXIALLY LOADED MEMBER IS UNIFORM

$\sigma = \sigma_{AVE}$

WE ALSO ASSUME THE AXIAL LOAD PASSES TROUGH THE CENTROID OF THE SECTION CONSIDERED, CALLED "CENTRIC LOADING"

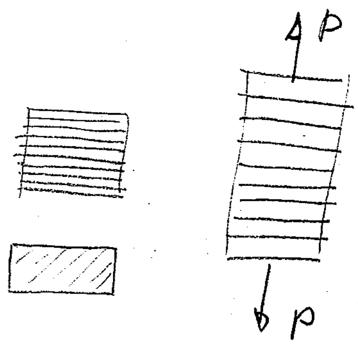


CENTROIDS OF DIFFERENT SECTIONS

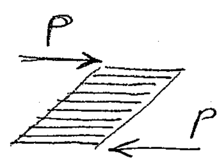
SHEARING STRESS

WHEN WE TALK ABOUT AXIAL STRESS, WE TALK ABOUT OPENING OR CLOSING DISTANCE BETWEEN THE CROSS SECTIONS.

THE SHEARING STRESS IS A DIFFERENT TYPE OF STRESS OBTAINED BY TRYING TO SLIDE BETWEEN THE SECTIONS



AXIAL STRESS
(NORMAL STRESS)



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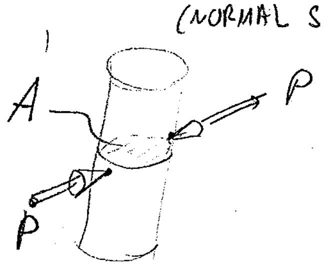
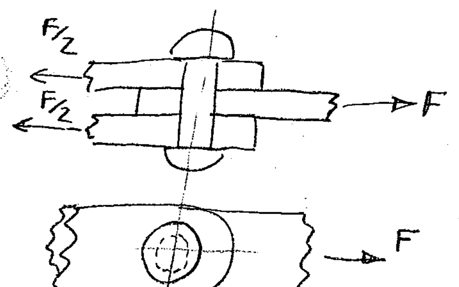
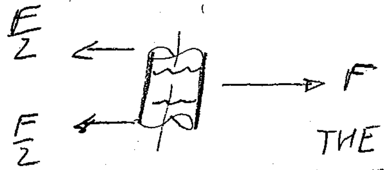


THE AVERAGE SHEARING STRESS IN THE SECTION IS

$$\tau_{AVE} = \frac{P}{A}$$

CONSIDERATE THE FOLLOWING RIVETED JOINT:



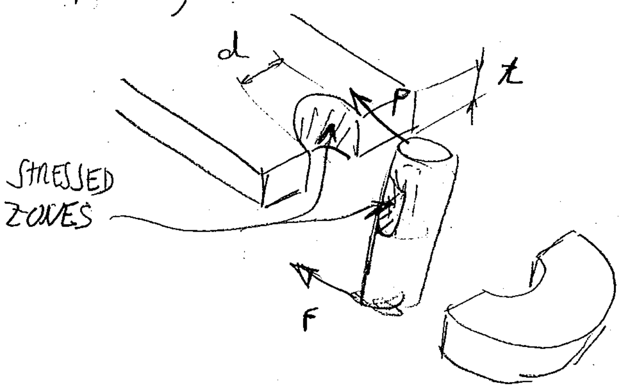


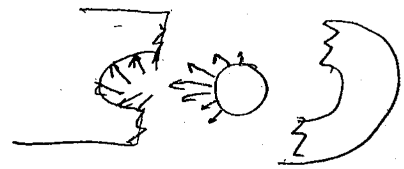
$$\tau = \frac{P}{A} = \frac{F}{2 \cdot A}$$

THE RIVET IS IN DOUBLE SHEAR

BEARING STRESS IN CONNECTIONS

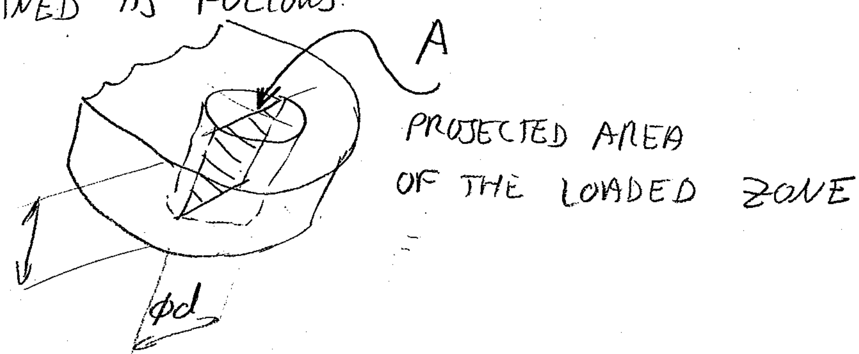
THE CONTACT STRESS BETWEEN A MEMBER AND A BOLT, PIN OR RIVET, AS SHOWN:





THE DISTRIBUTION OF THESE FORCES IS QUITE COMPLICATED, IN PRACTICE AN AVERAGE NOMINAL STRESS σ_b (bearing stress) WILL BE OBTAINED AS FOLLOWS:

$$\sigma_b = \frac{P}{A} = \frac{P}{t\,d}$$

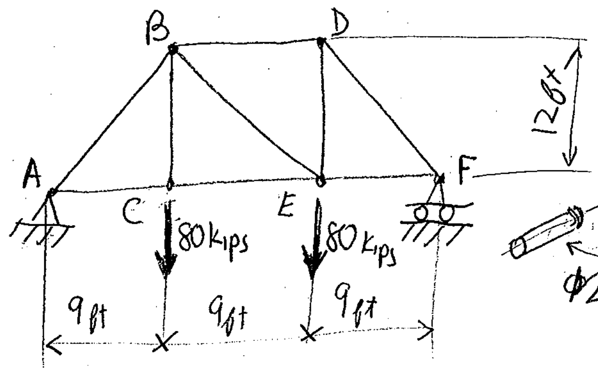


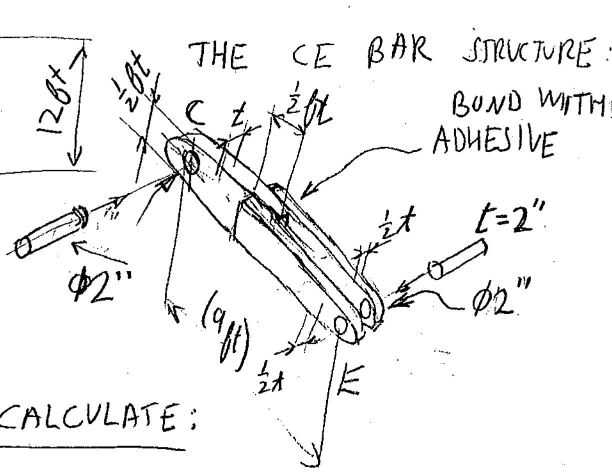
REVIEW THE APPLICATION TO THE ANALYSIS AND DESIGN OF SIMPLE STRUCTURES IN PAGES 12, 13, 14 ON YOUR TEXTBOOK. THE SAMPLE GIVEN IS BASED ON THE STATIC REVIEW EXAMPLE DID IN CLASS. WHILE DOING HOMEWORK

REVIEW ALSO SAMPLE PROBLEM 1.1 (page 16) AND SAMPLE PROBLEM 1.2

SAMPLE PROBLEM

CONSIDER THE FOLLOWING TRUSS: (ON SIMPLE SUPPORTS)





CALCULATE:

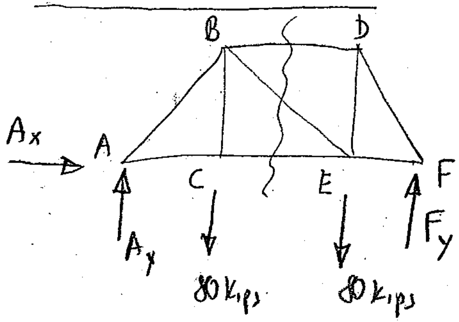
A. .... THE SHEAR STRESS ON ADHESIVE

B. THE LARGEST NORMAL STRESS IN LINK $\overline{CE}$

C. THE BEARING STRESS IN THE LINK AT C

A.

FREE BODY DIAGRAM



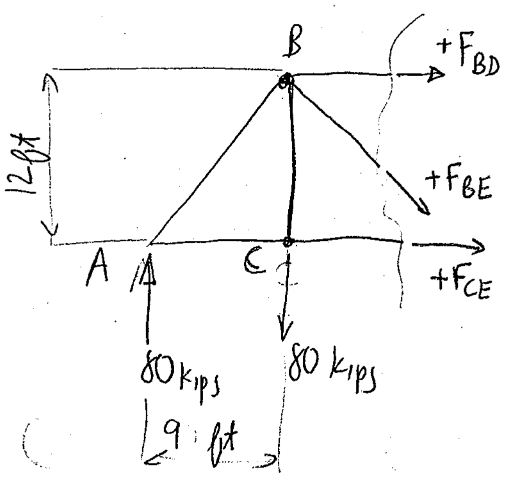
$$\circlearrowleft^{+} \sum M_F = 0 = -A_y \cdot 27\,ft + 80\,kips \cdot 18\,ft + 80\,kips \cdot 9 = 0$$

$$A_y = 80\,kips$$

$$\xrightarrow{+} \sum F_x = A_x = 0$$

WE ARE GOING TO USE THE LEFT SIDE OF THE TRUSS, SO WE DON'T NEET TO CALCULATE F_y, EVEN IT IS NOT DIFFICULT TO GUESS THAT $F_y = 80\,kips$

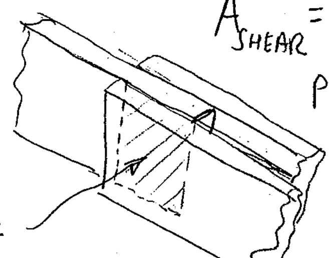
WE WILL BASE OUR MODEL ON THE FACT THAT THE MEMBERS OF THE TRUSS ARE BARS!



$$\circlearrowleft^{+} \sum M_B = 0 = F_{CE} \cdot 12\,ft - 80\,kips \cdot 9\,ft = 0$$

$$F_{CE} = 60\,kips \leftarrow \text{(TENSION)}$$

A. SHEAR STRESS ON ADHESIVE



$\frac{1}{2} \cdot \frac{1}{2}\,ft^2$
TWO AREAS

$$A_{SHEAR} = \underset{SURF.}{2} \cdot \left(\frac{1}{2} \cdot \frac{1}{2}\right) ft^2 = \frac{1}{2}\,ft^2$$

$$P = F_{CE}$$

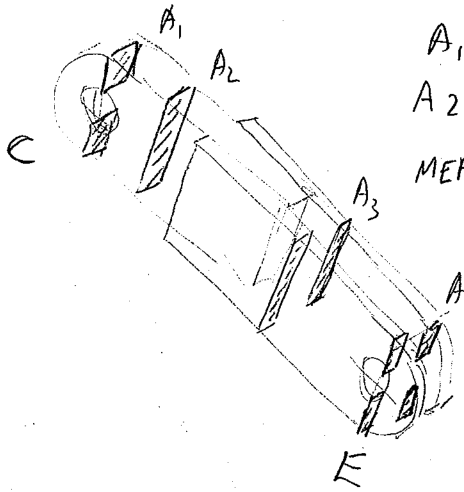
$$\tau = \frac{F_{CE}}{A_{SHEAR}} = 120\,\frac{kips}{ft^2} = \dots$$

$$\dots = 833.3\ PSI$$

B. THE NORMAL STRESS TO BE CALCULATED BY $\sigma = \frac{P}{A}$

$P = F_{CE} = +60$ kips

IN ORDER TO FIND THE MAXIMUM STRESS WE SHOULD LOOK FOR THE MINIMUM SECTION AREA FOR NORMAL STRESSES:



$A_1 = A_4 = t\left(\frac{1}{2}ft - \phi 2''\right) = \cdots = 8\ in^2$

$A_2 = A_3 = t \cdot \frac{1}{2}ft < A_1, A_4$

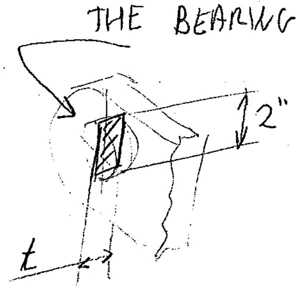
MEANING: IF THE REASON FOR FAILURE IS THE LOW RESISTANCE TO THE AXIAL STRESS, THEN IT SHOULD HAPPEN ON SECTION A_1 OR A_4

$$\sigma = \frac{F_{CE}}{A_{1,4}} = \frac{+60 \cdot 10^3}{8} = +7500\ PSI$$

* IF THE LOAD WAS NOT TENSION BUT COMPRESSION SHOULD WE RECEIVE THE SAME VALUE FOR σ ?

C. THE BEARING STRESS IS GIVEN BY $\sigma_b = \frac{P}{A}$

THE BEARING AREA



$A = t \cdot 2'' = 4\ in^2$, $P = F_{CE}$

$$\sigma_b = \frac{F_{CE}}{A} = \frac{60\ kips}{4\ in^2} = 15\ KSI$$

D. SHEARING STRESS AT PIN C

$$\tau = \frac{P}{A}$$

single shearing $P = F_{FC}$

$A = \frac{\pi d^2}{4} = 3.1415\ in^2$

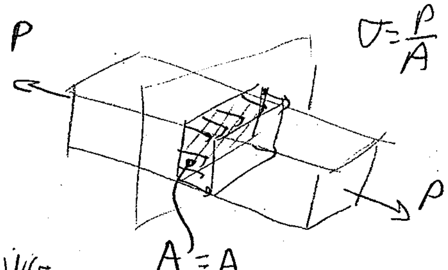
$$\tau = \frac{60 \cdot 10^3}{3.1415} = 19.1\ ksi$$

(For "double shearing", $P = \frac{1}{2} F_{FC} \ldots$)

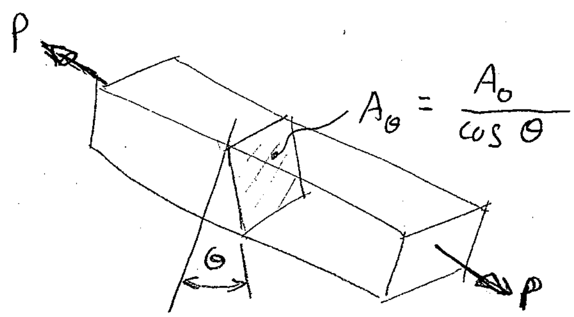
STRESS ON AN OBLIQUE PLANE UNDER AXIAL LOADING

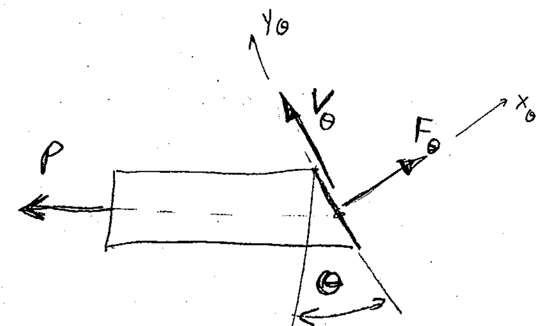
AXIAL FORCES CAUSE BOTH NORMAL AND SHEARING STRESSES ON PLANES WHICH ARE NOT PERPENDICULAR TO THE AXIS OF THE MEMBER.
SIMILARLY, TRANVERSE FORCES APPLIED ON A BOLT OR A PIN CAUSE BOTH NORMAL AND SHEARING STRESSES ON PLANES WHICH ARE NOT PERPENDICULAR TO THE AXIS OF THE BOLT, PIN OR RIVET.

OUR KNOWN AXIAL CASE:



LET ANALYZE A SECTION FORMING AN ANGLE θ WITH A NORMAL SECTION





THE SECTION SHOULD REACT TO THE LOAD WITH AXIAL AND SHEARING REACTIONS

$$\sigma_\theta = \frac{F_\theta}{A_\theta} \qquad \tau = \frac{V_\theta}{A_\theta}$$

> REMARK: THE TEXT BOOK ASSIGNS V IN THE OPPOSITE DIRECTION. WE ARE ALWAYS CONSISTENT WITH THE RIGHT HAND COORDINATE SYSTEM

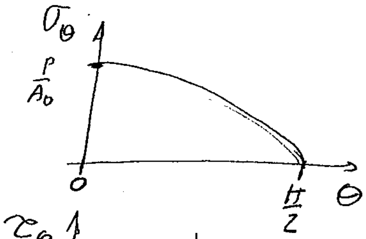
EQUILIBRIUM EQUATIONS:

$$\xrightarrow{+} \sum F_x = 0 = -P - V_\theta \sin\theta + F_\theta \cos\theta$$

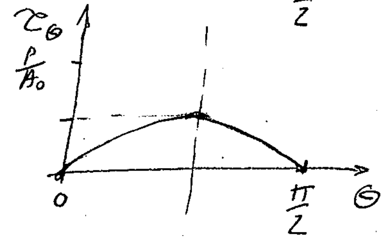
$$+\uparrow \sum F_y = 0 = V_\theta \cos\theta + F_\theta \sin\theta$$

$$V_\theta = -P\sin\theta$$

$$F_\theta = P\cos\theta$$



$$\sigma_\theta = \frac{F_\theta}{A_\theta} = \frac{P\cos\theta}{(A_0/\cos\theta)} = \frac{P}{A_0}\cos^2\theta$$



$$\tau_\theta = \frac{V_\theta}{A_\theta} = \frac{-P\sin\theta}{(A_0/\cos\theta)} = -\frac{P}{A_0}\sin\theta\cos\theta$$

$$= -\frac{P}{2A_0}\sin(2\theta)$$

<u>LET ANALYZE THE RESULTS</u>

FOR $\theta = 0$ $\qquad \sigma_0 = \frac{P}{A_0}$, $\quad \tau_0 = 0$

WE GOT THE ORIGINAL NORMAL STRESS

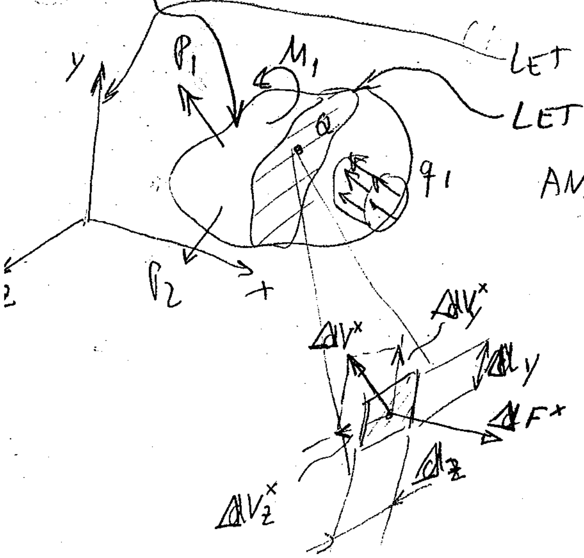
FOR $\theta \to 90^\circ$ $\qquad \sigma_{90} \to 0$, $\quad \tau_{90} \to 0$

APPROACH ZERO.

MAXIMUM SHEARING STRESSES REACHED AT $\theta = \frac{\pi}{4}$ and

STRESS UNDER GENERAL LOADING CONDITIONS; COMPONENTS OF STRESS

CONSIDER A BODY SUBJECTED TO SEVERAL LOADS



- LET ASSUME OUR COORDINATE SYSTEM x, y, z
- LET CUT THE BODY ON A PLANE PARALLEL TO YZ AND ANALYZE A VERY SMALL AREA (SO SMALL, THAT THE CHANGE IN LOAD INSIDE THE AREA IS NEGLEGIBLE)

$\Delta A = \Delta y \cdot \Delta z$

DIVIDING THE MAGNITUDE OF EACH FORCE BY THE AREA;

$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F^x}{\Delta A} \qquad \text{positive for } \Delta F^x \text{ in } x \text{ direction}$$

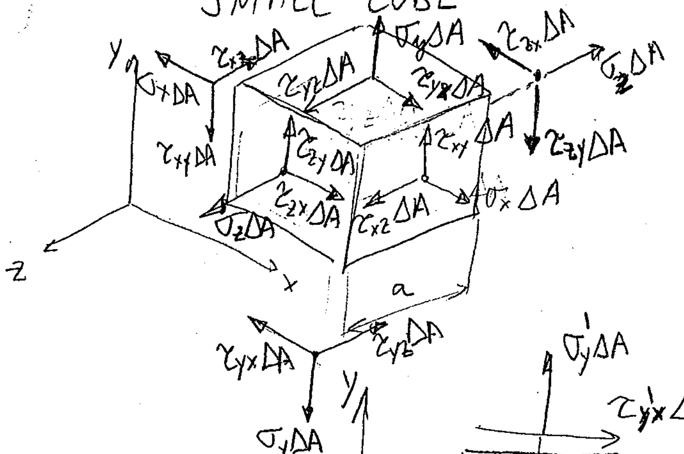
$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta V_y^x}{\Delta A} \qquad \text{positive for } \Delta V_y^x \text{ in } y \text{ direction}$$

$$\tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta V_z^x}{\Delta A} \qquad \text{" } \Delta V_z^x \text{ in } z \text{ "}$$

(1.10)

WE CAN ANALYZE THE DIFFERENT FACES OF THE SAME POINT Q BEING A VERY SMALL CUBE

NO ACCELERATIONS, NO BODY FORCES

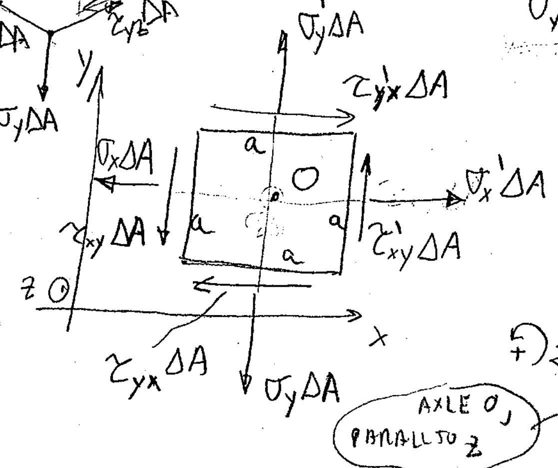


THIS CUBE SHOULD STAY IN STATIC EQULIBRIUM, SO WE CAN FIND SEVERAL RELATIONS BETWEEN THE STRESSES.

LETS LOOK AT PLANE XY

THE CUBE IS VERY LITTLE SO:

$\sigma_y' \approx \sigma_y \; ; \; \tau_{yx}' \approx \tau_{yx} \; ; \; \tau_{xy}' \approx \tau_{xy}$



THE DRAWED LOADS SATISFY

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\circlearrowleft + \; \Sigma M_o = 0 = (\tau_{xy}\,\Delta A)a - (\tau_{yx}\,\Delta A)a = 0$

$\tau_{xy} = \tau_{yx}$

IN A SIMILAR FORM, IT CAN BE PROVED THAT

$\tau_{yz} = \tau_{zy}$

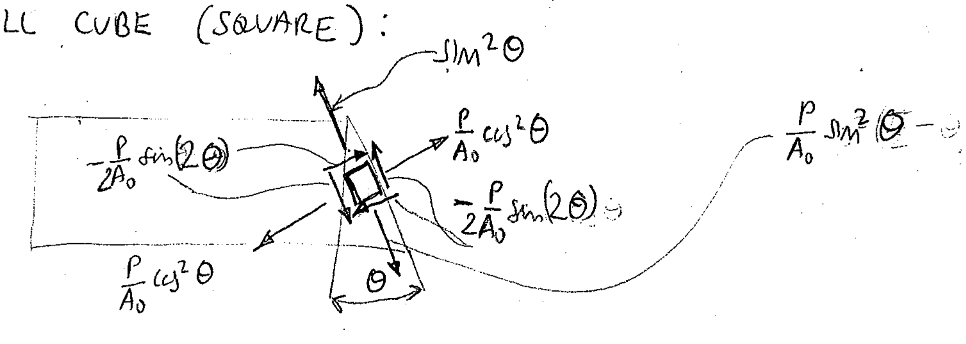
$\tau_{zx} = \tau_{xz}$

CONCLUSIONS:

1. 6 DIFFERENT COMPONENTS OF STRESS ARE ENOUGHT IN ORDER TO DEFINE THE CONDITION OF STRESS AT A GIVEN POINT
$\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}$

2. WHEN WE CALCULATE SHEARING STRESS ON A PIN, BOLT OR RIVET, WE SHOULD UNDERSTAND THAT SIMILAR SHEARING STRESSES ARE CREATED AT PERPENDICULAR FACES.

3. LET CONSIDER THE CASE OF OBLIQUE PLANE UNDER AXIAL LOADING AND GET TWO DIMENSION SOLUTION FOR A SMALL CUBE (SQUARE):



DESIGN CONSIDERATIONS

WE LEARNED SOME SIMPLE WAYS TO CALCULATE DIFFERENT KINDS OF STRESSES. WE WILL LEARN MORE COMPLICATED CASES DURING THIS COURSE.

LET DISCUSS PRELIMINARY DESIGN TOPICS AND TERMS, IN ORDER TO CONNECT OUR ABILITY IN STRUCTURE STATIC ANALYSIS, STRESS CALCULATION WITH THE PROPERTIES OF THE DIFFERNT MATERIALS

ULTIMATE STRENGTH OF A MATERIAL σ_u, τ_u

THE STRESS IN WHICH A SPECIFIC MATERIAL FAILS

THE MOST COMMON AND STANDARD MODE OF MEASSURING THIS STRESS IS BY TENSION OF STANDARD SPECIMEN TO FAILURE

THEN WE GET THE σ_{UTS} (ULTIMATE TENSION STRENGT)

THE ABOVE VALUE CAN BE TRANSLATED TO EQUIVALENT τ_u, WE WILL TALK ABOUT THE NINENTH WEEK OF THE COURSE

THERE ARE ALSO METHODS FOR SHEARING TEST, LEST POPULAR.

ALLOWABLE LOAD AND ALLOWABLE SRESS - FACTOR OF SAFETY

THE MAXIMUM LOAD THAT A COMPONENT IS ALLOWED TO CARRY SAFELY, BEING FAR ENOUGHT FROM FAILURE.

WHAT IS "SAFELY"?

WHAT IS "FAR ENOUGHT"?

THE ABOVE TERMS ARE VALUED BY THE FACTOR OF SAFETY

$$\text{FACTOR OF SAFETY} = F.S. = SF = \frac{\text{ULTIMATE LOAD}}{\text{ALLOWABLE LOAD}} = \frac{\text{ULTIMATE STRESS}}{\text{ALLOWABLE STRESS}} = \frac{\sigma_u}{\sigma_a} \text{ or } \frac{\tau_u}{\tau_a}$$

REMARK: IN THE FOLLOWING CHAPTER WE WILL LEARN ABOUT THE "YIELD STRESS OF THE MATERIAL", AND WILL ADD FOCTOR OF SAFETY TO THE YIELD OF THE MATERIAL

ACTUAL STRESS

SELECTION OF AN APPROPIATE FACTOR OF SAFETY

THE CHOICE OF THE FACTOR OF SAFETY THAT IS APPROPIATE FOR A GIVEN DESIGN APPLICATION REQUIRES ENGINEERING JUDGMENT BASED ON MANY CONSIDERATION:

1. WHAT MAY HAPPEN IF THE STRUCTURE $\overbrace{\text{FAILS}}^{\text{MEMBER}}$?
   - ELEVATOR CABLE ← STANDARD / LAW   8÷10
   - DOOR HANDLE
   - GAS PRESSURISED VESSEL ← STANDARD / LAW
   - AIRCRAFT WING ← STANDARD   1.5÷2.5 (ULTIMATE)
   - DIFFERENT DEVICES   SF ≃ 3 ÷ 5

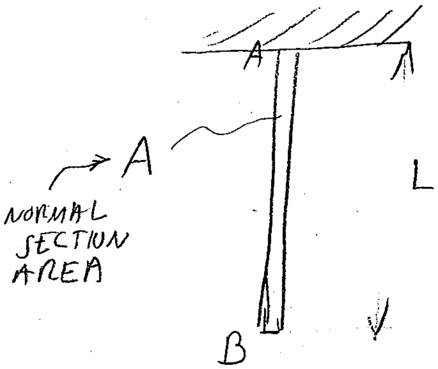
2. WHAT DO YOU KNOW ABOUT THE ENVIRONEMTAL CONDITIONS?
   - HIGH RATE CORROSION?   SHIP'S BODY
   - UNESPECTED DYNAMIC LOADS? $\overbrace{\text{TRASPORTATION}}^{\text{BAD}}$; UNPROPER USE OF THE MACHINE

3. METHODS OF ANALYSIS
   EVERY ANALYSIS CONSIST OF $\overbrace{\text{ASSUMPTIONS}}^{\text{SEVERAL}}$ WE MAKE FOR OUR CAPABILITY TO ANALIZE
   THIS ASSUMTION ARE <u>NEVER</u> THE REAL SITUACION:
   - AXIAL LOAD
   - DIFFERENT DIMENSIONS (GEOMETRY)
   - NOT UNIFORM STRESSES

YOU CAN LOOK AT YOUR TEXTBOOK FOR MORE REASONS FOR SAFETY FACTOR EVALUATION

IN OTHER WORDS, FACTOR OF SAFETY = FACTOR OF IGNORANCE ...

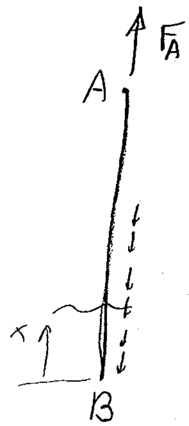
(1) LETS TAKE A MEMBER HOLDED FROM THE TOP



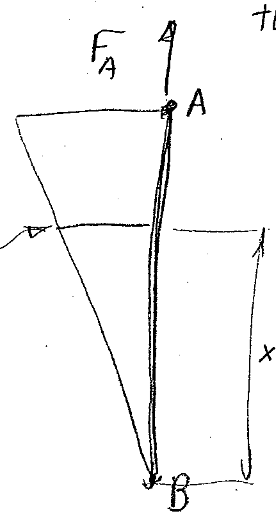
L.C.
Steel → $\sigma_u = \cancel{8}700$ MPa
$\gamma^m = 7850$ kg/m³

WHAT IS THE MAXIMUM LENGTH OF THE MEMBER, BEFORE IT FAILS DUE TO ITS OWN WEIGHT ?

FREE BODY DIAGRAM
(EXTERNAL LOADS)



TENSION FORCE DIAGRAM
AS EXPECTED: MAXIMUM FORCE/STRESS AT $\cancel{x} = L$



$F_x = g \gamma^m \cdot Vol_{(x)}$
$= g \gamma^m \cdot x \cdot A$

$g = 9.807$ m/s²

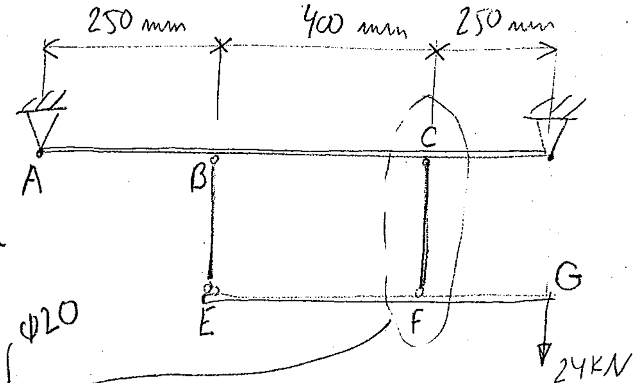
$$F_A = F_x \Big|_{x=L} = \gamma^m \cdot L \cdot A \cdot 9.807$$

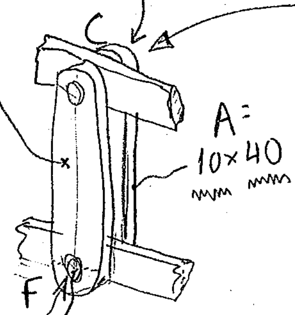
$$\sigma_{MAX} = \frac{F_A}{A} = \underbrace{\frac{\gamma^m \cdot L \cdot \cancel{A}}{\cancel{A}}}_{\cdot 9.807} = \underbrace{\gamma^m \cdot L}_{9.807} = \sigma_u$$

$$L = \frac{\sigma_u}{\gamma^m g} = \frac{700 \cdot 10^6}{7850 \cdot 9.807} = 9092.7 \text{ m}$$
≈ 9.1 Km
≈ 5.6 mi

A MEMBER LONGER THAN 5.6 mi WILL FAILURE, NOT DEPENDING ON THE SECTION AREA VALUE

1.53

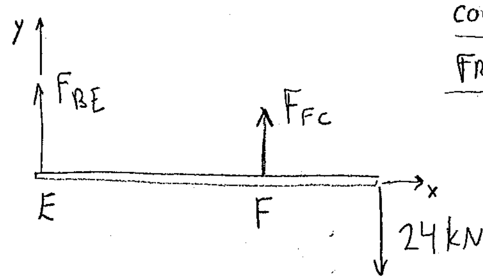




(ACTUAL) OVERALL FACTOR OF SAFETY FOR THE LINKS CF AND THE PINS C, F

COORDINATE SYSTEM

FREE BODY DIAGRAM



$\circlearrowleft + \sum M_E = 0$

$-24\text{ kN} \cdot 650_{mm} + F_{FC} \cdot 400_{mm} = 0$

$F_{FC} = 39\text{ KN} = 39000\text{ N}$ (TENSION)

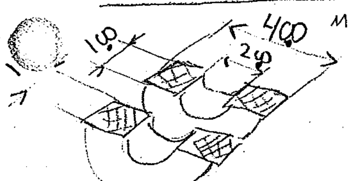
UNITS:

$Pa = \text{Pascal} = N/m^2$

$MPa = 10^6\ N/m^2 = 1\ N/mm^2$

CALCULATING ~~ALLOWABLE~~ ACTUAL ~~(TENSION)~~ NORMAL STRESS ON LINKS CF:

MINIMUM AREA FOR TENSION (MAX. STRESS!)

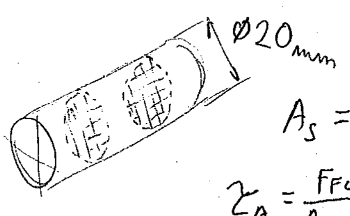


$A_{min} = (10 + 10)(40 - 20) = 400\ mm^2$

$\sigma_A = \dfrac{F_{FC}}{A_{min}} = \dfrac{39000\ N}{400\ mm^2} = 97.5\ MPa$

FACTOR OF SAFETY FOR LINKS FC $\Big\}$ $F.S._{FC} = SF = \dfrac{\text{ULTIMATE STRESS}}{\text{~~ALLOWABLE~~ ACTUAL STRESS}} = \dfrac{400\ MPa}{97.5\ MPa} = \underline{4.1}$

CALCULATION OF (ALLOWABLE) ACTUAL SHEAR STRESS ON PINS



SHEAR AREA:

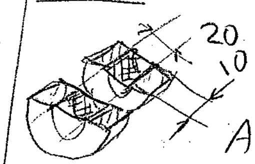
$A_s = 2 \cdot \dfrac{\pi\, 20^2}{4} = 628.3$

$\tau_A = \dfrac{F_{FC}}{A_S} = \dfrac{39000\ N}{628.3\ mm^2} = 62.1\ MPa$

FACTOR OF SAFETY FOR PINS $\Big\}$ $F.S_{PIN} = \dfrac{\tau_v}{\tau_A} = \dfrac{150}{62.1} = \underline{2.42}$

CALCULATION OF ACTUAL BEARING STRESS

$\sigma_{ba} = \dfrac{F_{FC}}{A_b}$



$A_b = 20(10+10) = 400\ mm^2$

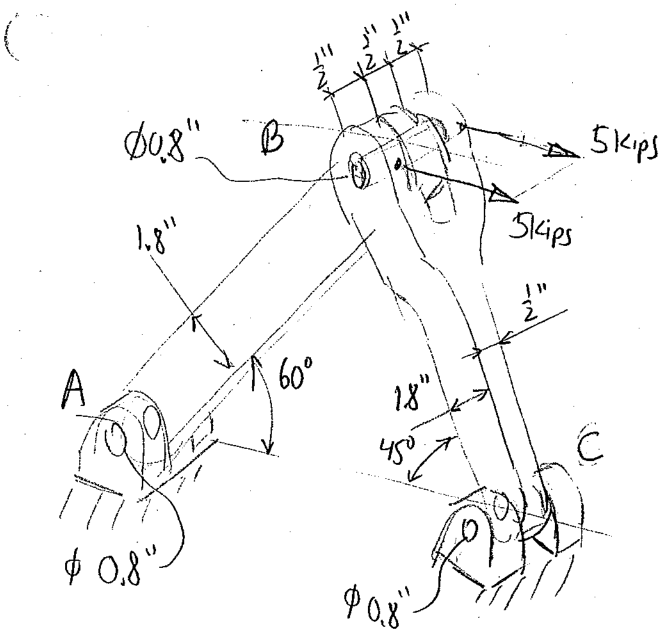
$\sigma_{ba} = \dfrac{39000}{400} = \cdots = 97.5\ MP$

$\underline{F.S_b} = \dfrac{\sigma_{bu}}{\sigma_{ba}} = \dfrac{350}{97.5} = \underline{3.59}$

THE OVERALL FACTOR OF SAFETY FOR THE LINKS CF AND THE PINS IS: $\boxed{2.42}$

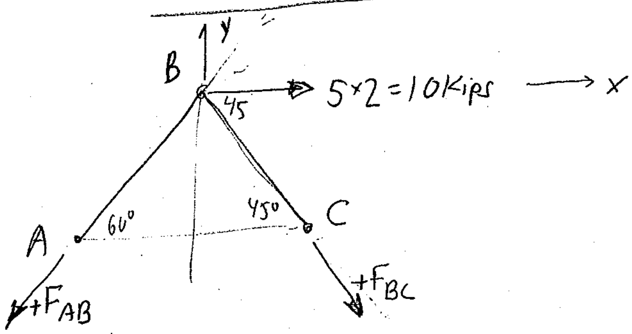
IF A LOAD OF (24kN · 2.42 =) 58.08 kN (ON G) IS APPLIED, THE PINS C, F WILL FAIL DUE TO (HIGH) SHEARING STRESS

PROBEMS 1.9, 1.26



a) NORMAL STRESS IN LINK AB
b) " " " " BC
c) AVERAGE SHEARING STRESS IN THE PIN AT C
d) " BEARING STRESS AT C IN MEMBER BC
e) " " " " B " " BC

FREE BODY DIAGRAM



$\xrightarrow{+} \sum F_x = 0 = 10\text{kips} + F_{BC}\cos 45° - F_{AB}\cos 60°$

$+\uparrow \sum F_y = 0 = -F_{BC}\sin 45° - F_{AB}\sin 60°$

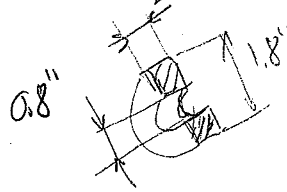
$F_{BC} = -F_{AB}\dfrac{\sin 60}{\sin 45°}$

$10\text{kips} - F_{AB}\left(\dfrac{\sin 60}{\sin 45}\cos 45° + \cos 60°\right) = 0$

$F_{AB} = \dfrac{10 \text{ kips}}{\cos 60° + \sin 60} = 7.3205 \text{ kips}$

$F_{BC} = -7.32\dfrac{\sin 60}{\sin 45} = -8.9658 \text{ kips}$

a) NORMAL STRESS IN LINK AB (+)



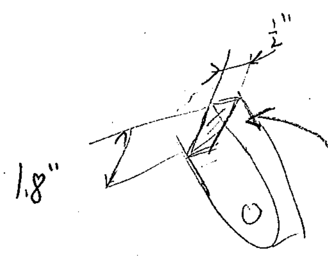
$A = \frac{1}{2}(1.8-0.8) = \frac{1}{2}\text{ in}^2 \qquad \sigma = \dfrac{P}{A}$

$P = F_{AB}$

$\sigma = \dfrac{7.3205}{\frac{1}{2}} = 14.64 \text{ KSI}$

b) NORMAL STRESS IN LINK BC

COMPRESSION



$$\sigma = \frac{P}{A} = \frac{F_{BC}}{\frac{1}{2} \cdot 1.8} = \cdots -9.962 \text{ KSI}$$

SELECTION OF THE CRITICAL AREA IN THE BAR

c) AVERAGE SHEARING STRESS IN THE PIN AT C

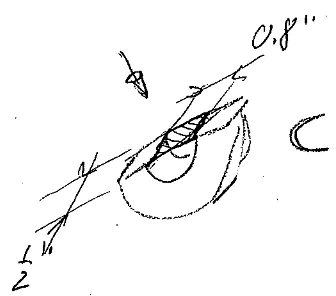
$$\tau = \frac{P}{A} \qquad P = F_{BC} = -8.9658 \text{ kips}$$

$$A = 2 \cdot \frac{\pi 0.8^2}{4} = 1.0053 \text{ in}^2$$

double shearing

$$\tau = \frac{8.9658}{1} = 8.92 \text{ ksi}$$

d) AVERAGE BEARING STRESS AT C IN MEMBER BC

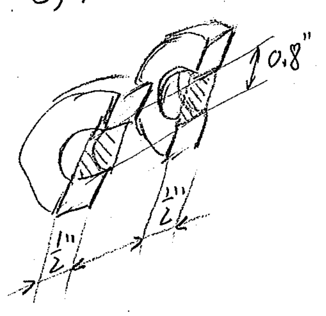


$$\sigma_b = \frac{P}{A} \qquad P = F_{BC}$$

$$A = 0.8 \cdot \tfrac{1}{2} \ (\text{in}^2)$$

$$\sigma_b = \frac{F_{BC}}{0.8 \cdot \frac{1}{2}} = \cdots = 22.41 \text{ KSI}$$

e) AVERAGE BEARING STRESS AT B IN MEMBER BC



$$\sigma_b = \frac{F_{BC}}{2 \cdot \frac{1}{2}'' \cdot 0.8} = \cdots = 11.21 \text{ KSI}$$

FOR PROBLEM 1.53

CALCULATE THE DIAMETER OF THE PIN (d)
IN ORDER TO HAVE THE SAME VALUE OF FACTOR OF SAFETY
IN LINK CF AND IN THE PIN

INCREASING DIAMETER OF THE PIN WILL DECREASE
SHEARING STRESS ON THE PIN BUT WILL INCREASE
NORMAL STRESS ON THE MEMBER (AND DECREASE BEARING
STRESS)

FOR NORMAL STRESS ON CF:

$$A = (10+10)(40-d)$$

$$F.S._{\sigma} = \frac{400}{\left(39000/20(40-d)\right)} = \cdots =$$

FOR SHEARING STRESS ON PIN:

$$A = 2 \cdot \frac{\pi d^2}{4}$$

$$F.S._{\tau} = \cdots = \frac{150}{\left(\frac{39000}{\frac{\pi d^2}{2}}\right)}$$

$$F.S._{\sigma} = F.S._{\tau}$$

$$\vdots$$

$$d^2 + 33.9531d - 1358.124 = 0 \quad \begin{cases} d < 0 \\ d = 23.6 \text{ mm} \end{cases}$$

$$\underline{\underline{F.S._{\sigma}}} = F.S._{\tau} = \cdots = \underline{\underline{3.364}}$$

(1.13)

HOME WORK #1

TEXTBOOK

1.7, 1.14, | 1.32, 1.37, 1.41, 1.59, 1.68

ALSO:

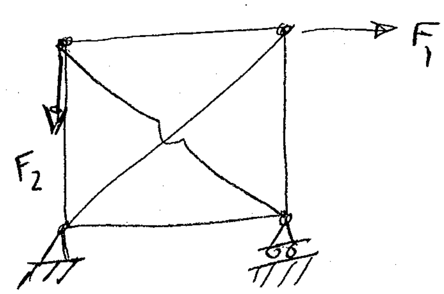
2. CALCULATE SHEARING STRESS AT PIN D ($\theta = 0°, 90°$) (SINGLE SHEAR)
3. CALCULATE BEARING STRESS ON BAR $\overline{BD}$ AT D ($\theta = 0°, 90°$)

CHAPTER 2

STRESS AND STRAIN – AXIAL LOADING

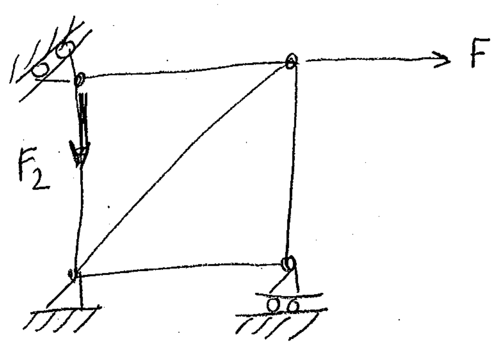
STATICS IS BASED ON THE ASSUMPTION OF UNDEFORMABLE, RIGID STRUCTURES. IT LET US SOLVE CASES THAT ARE NOT OVER-CONSTRAINED

FOR EXAMPLE, A TRUSS WITH TOO MANY BARS



(GRUBER'S FORMULA)
STATIC DETERMINATE

OR AN OVER-SUPPORTED STRUCTURE



IN SUCH CASES, THE LOADS ARE DISTRIBUTED ACCORDING TO THE ELASTICITY PROPERTIES OF THE STRUCTURE.

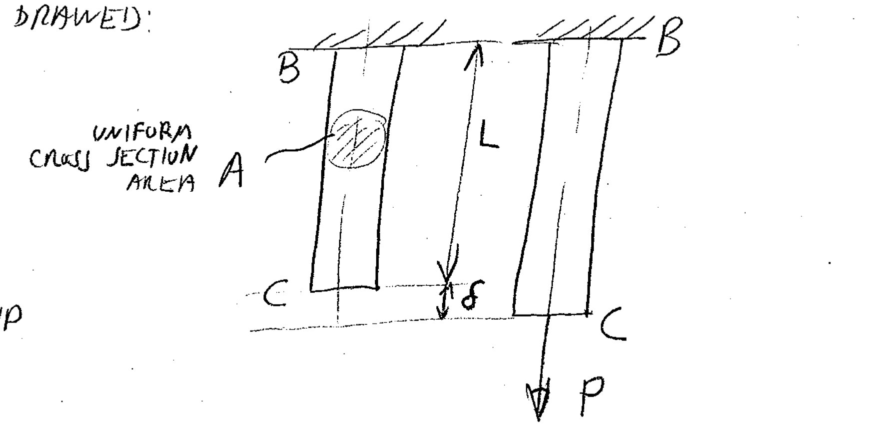
A MORE COMMON EXAMPLE = A DESK WITH 4 LEGS

IT NEEDS ONLY 3 IN ORDER TO BE 3 DIMENSIONALY STATIC DETERMINATE, BUT ~~ALL~~ MOST OF THE DESKS I KNOW HAVE AT LEAST 4 LEGS...

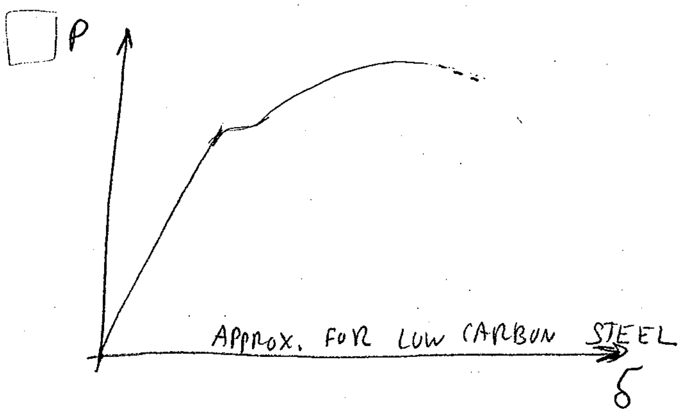
→ IN ORDER TO SOLVE OVER SUPPORTED AND OVER CONSTRAINED CASES, WE NEED TO UNDERSTAND THE ELASTICITY BEAVIOUR OF THE MEMBERS, BEGINING WITH AXIAL LOADING

NORMAL STRAIN UNDER AXIAL LOADING

LET CONSIDER ROD BC AS DRAWED:
AFTER APPLYING AXIAL FORCE P, WE GET AN AXIAL DEFLECTION δ:



IF WE CONTINUE LOADING, WE WILL ~~HAVE~~ FIND THE FOLLOWING RELATIONSHIP BETWEEN P AND δ



IF WE MULTIPLY ~~P~~ AND A BY THE SAME MULTIPLIER, WITHOUT CHANGING L, WE WILL FIND THAT δ IS THE SAME, BUT THE GRAPH P-δ SHOULD BE DIFFERENT (AS P, A CHANGED)

ANY MULTIPLIER

$$\sigma = \frac{P}{A} = \frac{N \cdot P}{N \cdot A} \longrightarrow \text{WE WILL GET } \delta$$

IF WE MUTIPLY L BY ANY MUTIPLYER, ($K \cdot L$) AND STAY WITH P, A
WE WILL GET DEFLECTION MUTIPLYED BY $K \rightarrow K \cdot \delta$
AGAIN, GRAPH P-δ WILL CHANGE

→ IN ORDER TO DISCONNECT BETWEEN EXTERNAL GEOMETRY TO INTERNAL LOADS, LET INTRODUCE THE CONCEPT OF STRAIN:

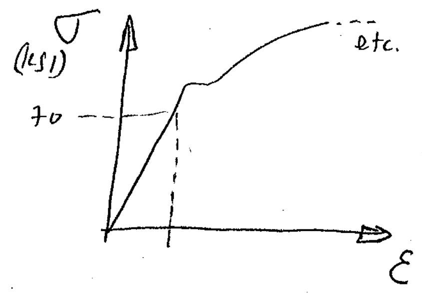
NORMAL STRAIN IS THE NORMAL DEFORMATION PER UNIT LENGHT

$$\boxed{\varepsilon = \frac{\delta}{L}}$$

NOW WE ARE ABLE TO PLOT A CURVE NOT DEPENDING ON THE DIMENSIONS OF THE PARTICULAR SPECIMEN

dimensionless quantity, very little

THIS CURVE IS CALLED STRESS-STRAIN DIAGRAM.



THE CURVE WILL BE SAME FOR DIFFERENT VALUES OF A, L, P

SINCE $\underbrace{\text{THE STRESSES}}_{}$ (IN A GENERAL CASE) ARE NOT CONSTANT INSIDE THE MATERIAL, THERE ARE DIFERENT VALUES OF STRAIN AT THE DIFFERENT POINTS SO THE GENERAL DEFINITION OF THE NORMAL STRAIN IN A POINT IS

$$\boxed{\varepsilon = \lim_{\Delta x \to 0} \frac{\Delta \delta}{\Delta x} = \frac{d\delta}{\delta x}}$$

ε IS DIMENSIONLESS AND VERY LITTLE (THE DEFORMATION IN ENGINEERING MATERIALS ARE MUCH LOWER THAN THE DIMENSION OF THE MEMBER)

WE USE TO MULTIPLY THE STRAIN VALUE BY 10^6 (MILLION) AND CALL "MICROS".

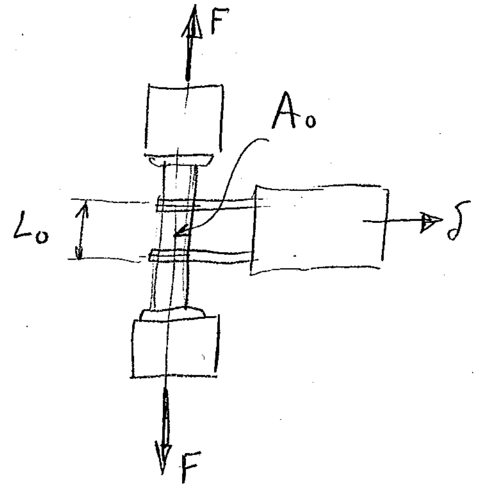
FOR EXAMPLE: IF WE ~~APPLY~~ A STRESS OF 70 KSI IN A STEEL COMPONENT WE SHOULD EXPECT FOR A STRAIN OF $2.5 \cdot 10^{-3}$

THE UNITS USED FOR THE STRAIN:

$$\varepsilon = 2.5 \cdot 10^{-3} = 2500\mu \quad (\text{"micros"}) \quad (= 2500 \cdot 10^{-6})$$

STRESS - STRAIN DIAGRAM

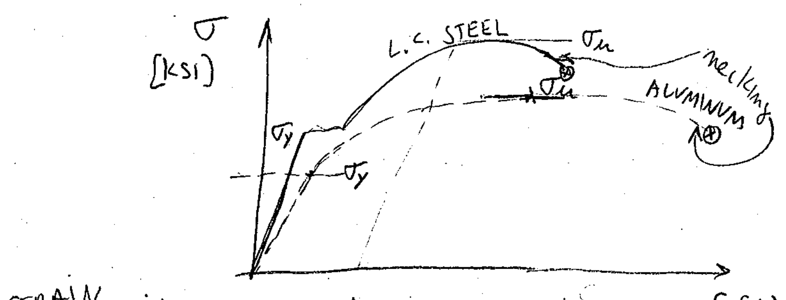
A TENSILE TEST IS CONDUCED IN ORDER TO OBTAIN THE STRESS STRAIN DIAGRAM.



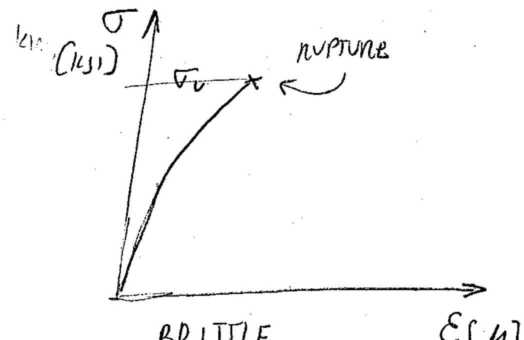
$$\sigma = \frac{F}{A_0} \qquad \varepsilon = \frac{\delta}{L_0}$$

EVERY ALLOY AND EVERY MATERIAL HAS HIS TYPICAL STRESS-STRAIN DIAGRAM.

ENGINEERING MATERIALS CAN BE DEVIDED INTO TWO CATEGORIES ON BASIS OF THESE CHARCTERISTIC, NAMELY THE <u>DUCTILE</u> MATERIALS AND THE <u>BRITTLE</u> MATERIALS



<u>DUCTILE MATERIALS</u>



<u>BRITTLE MATERIAL</u>

1. A NECK IS PRODUCED AT THE HIGHER RATES OF STRESS, THEN, THE STRESS APARENTLY DECREASES. IT IS BECAUSE WE ARE NOT ABLE TO MEASURE CORRECTLY THE SECTION AREA.
2. THE YIELD STRESS IS THE STRESS AT THE END OF THE LINEAR RANGE. EVERY STRESS APLIED, LOWER THAN THE YIELD STRESS, <u>WILL NOT CAUSE PERMANENT</u> DEFORMATION AND THE RELEASE OF THE LOAD RETURNS THE MATERIAL TO ITS PRELIMINAR CONDITION
3. COMPRESSION TEST ARE ALSO PERFORMED. THE MATERIALS BEHAVE THE SAME IN THE LINEAR ZONE, INCLUDING THE MAGNITUDE OF σ_y $\quad \sigma_y|_T = \sigma_y|_C$

MOST OF THE BRITTLE MATERIALS HAVE BETTER PROPERTIES IN COMPRESSION THAN IN TENSION, HAVING σ_u MUCH LARGER.

EXAMPLES = STEEL CASTINGS, CONCRETE.

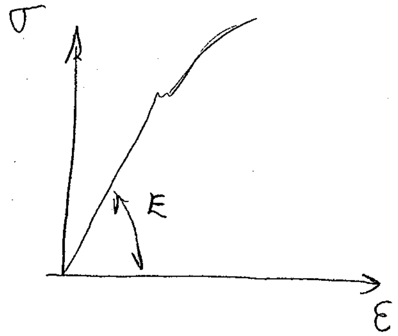
HOOKE'S LAW; MODULUS OF ELASTICITY

THE LINEAR RANGE IN STRESS-STRAIN DIAGRAM HAS A CONSTANT SLOPE, SO THE DIVISION OF STRESS OVER STRAIN IS CONSTANT

$$\boxed{E = \frac{\sigma}{\varepsilon}}$$

THIS RELATION IS KNOWN AS HOOKE'S LAW (ROBERT HOOKE - 17^{th} CENTURY)

E IS CALLED MODULUS OF ELASTICITY OR ALSO MENTIONED AS YOUNG'S MODULUS (THOMAS YOUNG - 18^{th} CENTURY)



ε IS DIMENSIONLESS, THEN E HAS THE SAME UNITS AS STRESS.

THE MODULUS OF ELASTICITY IS IN OTHER WORDS THE "STIFFNESS" OF THE MATERIAL (AS A SPRING CONSTANT OF THE MATERIAL)

THE MODULUS OF ELASTICITY IS DIFFERENT FOR THE DIFFERENT MATERI[AL] BUT VERY SIMILAR FOR THE DIFFERENT ALLOYS OR HEAT TREATMENTS OF THE SAME MATERIAL.

FOR EXAMPLE, THE MODULUS OF ELASTICITY OF ALUMINUM (7075, 6061, 6063, 2024) IS ABOUT 75 GPa

THE MODULUS OF ELASTICITY OF DIFFERENT STEELS (HIGH CARBON ALLOY, TEMPERED, STAINLESS STEEL) IS ABOUT 200 GPa

* WE SHOULD NOT FORGET THAT THE SUBJECT IS AXIAL LOAD.
THE ε AND E ARE TENSORS AS WELL AS σ
THERE ARE DEFORMATIONS IN ALL DIRECTION AS WELL AS DIFFERENT BEHAVIOR OF MATERIALS IN THE DIFFERENT DIRECTION, SUCH AS COMPOSITE MATERIALS.

2.6 ELASTIC VERSUS PLASTIC BEHAVIOR OF A MATERIAL

2.7 REPEATED LOADINGS; FATIGUE

READ THESE SECTIONS

DEFORMATION OF MEMBERS UNDER AXIAL LOADING

THE STRESS AND DEFORMATION OF MEMBER WILL BE UNDER THE PROPORTIONAL LIMIT. IN OTHER WORDS, HOOKE'S LAW WILL BE APPLIED: $\sigma = E \cdot \varepsilon$

CONSIDER THE ROD BC

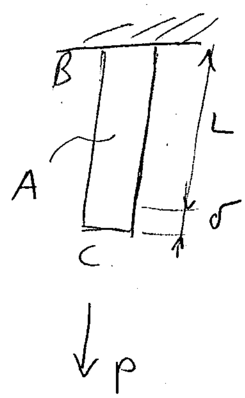
$\sigma = \frac{P}{A}$

$\varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$

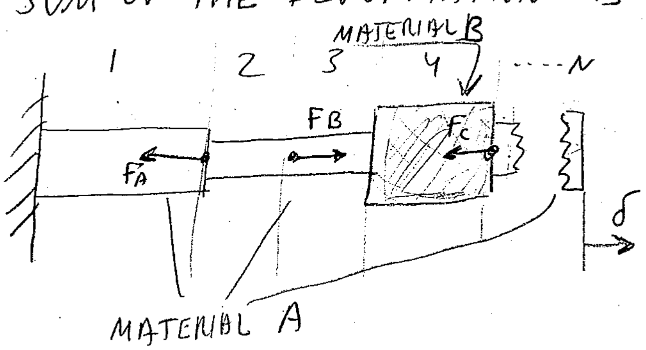
$\varepsilon = \frac{d\delta}{dx} = \frac{\delta}{L}$

$$\left.\begin{array}{l} \varepsilon = \dfrac{\sigma}{E} = \dfrac{P}{AE} \\ \varepsilon = \dfrac{d\delta}{dx} = \dfrac{\delta}{L} \end{array}\right\} \frac{\delta}{L} = \frac{P}{AE}$$

$$\boxed{\delta = \frac{PL}{AE}}$$



IF THERE ARE DIFFERENT INTERNAL FORCES, SECTION AREAS, OR MATERIALS, WE DEVIDE THE PROBLEM INTO HOMOGENOUS COMPONENTS AND USE SUM OF THE DEFORMATION AS FOLLOWS:



AXIAL LOADING

$$\delta = \sum \frac{P_i L_i}{A_i E_i}$$

NOTE: F_i IN THE DRAWING ARE EXTERNAL FORCES

P_i ARE INTERNAL FORCES TO BE FOUND USING AXIAL FORCE DIAGRAM

NON HOMEGENOUS SECTIONS

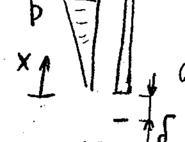
IF THE INTERNAL FORCES AND/OR THE SECTION AREA ARE GIVEN AS A FUNCTION OF x (EXAMPLE: SELF WEIGHT, CENTRIFUGAL ACCELERATION, CONTINOUS CHANGE IN SECTION AREA), $P(x)$, $A(x)$ WE ~~SHOULD~~ WILL INTEGRATE THE GENERAL CASE

$$\varepsilon = \frac{d\delta}{dx} \rightsquigarrow d\delta = \varepsilon\, dx = \frac{P}{AE}\, dx$$

$$\delta = \int_0^L \frac{P\, dx}{AE}$$

SINCE E IS CONSTANT FOR A SPECIFIC MATERIAL,

$$\delta = \frac{1}{E_0} \int_0^L \frac{P}{A}\, dx$$



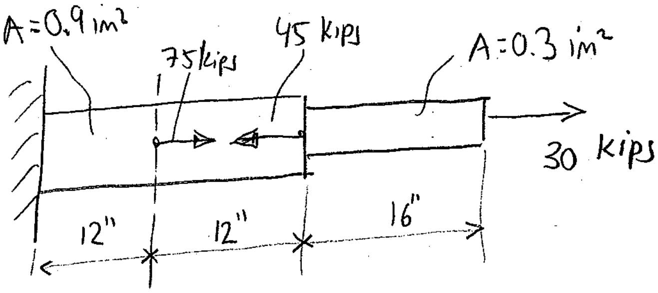
$$\gamma g A \cancel{x} = P$$

$$\delta = \int_0^L \frac{\gamma g A x\, dx}{AE} = \frac{\gamma g}{E} \frac{L^2}{2}$$

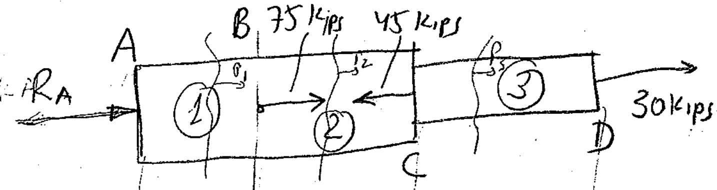
ERROR IN THE BOOK - SIGN

EXAMPLE 2.01, AS WE SHOULD SOLVE

STEEL, $E = 29 \cdot 10^6$ PSI



FREE BODY DIAGRAM

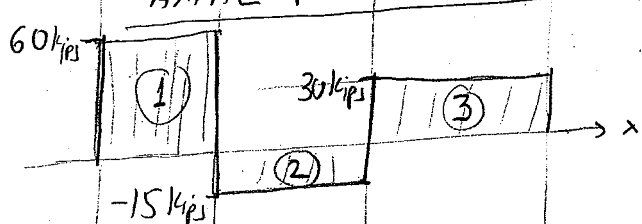


$$\sum F_x = 0 = R_A + 75 - 45 + 30$$

$$R_A = -60 \text{ kips}$$

$$\sum F_x \Big|_{①} = -P_1 + R_A = 0 \rightarrow P_1 = 60 \text{ kips}$$

AXIAL FORCE DIAGRAM



$$\sum F_x \Big|_{②} = R_A + 75\text{ kips} + P_2 = 0 \rightarrow P_2 = -15\text{ k}$$

$$\sum F_x \Big|_{③} = 0 = -P_3 + 30\text{ kips} \rightarrow P_3 = 30 \text{ kips}$$

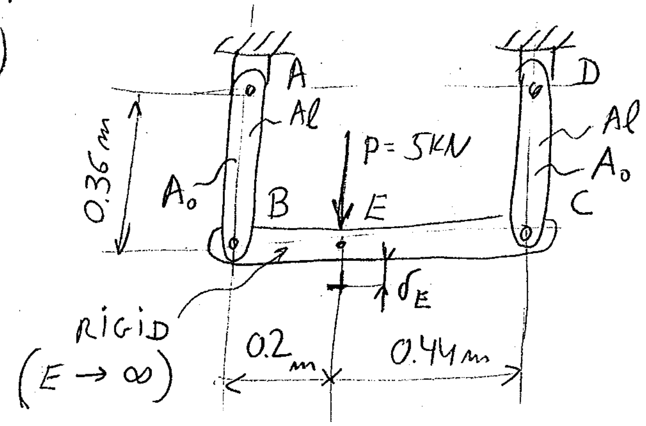
$$\delta = \sum \delta_i = \sum \frac{P_i L_i}{A_i E_i} = \frac{60 \cdot 1 \cdot 10^3 \cdot 12"}{0.9\,\text{in}^2 \cdot 29 \cdot 10^6\,\text{PSI}} + \frac{-15 \cdot 10^3\,\text{lb} \cdot 12"}{0.9\,\text{in}^2 \cdot 29 \cdot 10^6\,\text{PSI}} + \frac{30 \cdot 10^3\,\text{lb} \cdot 16"}{0.3\,\text{in}^2 \cdot 29 \cdot 10^6}$$

$$\delta = 0.07586"$$

(2.8)

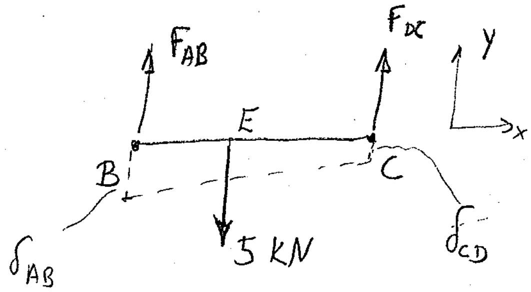
COMBINING DEFORMATIONS + GEOMETRY

p 2.27
(page 69)



$E_{Al} = 75$ GPa (Al)
$A_0 = 125\ mm^2$

δ_E - ?



FREE BODY DIAGRAM

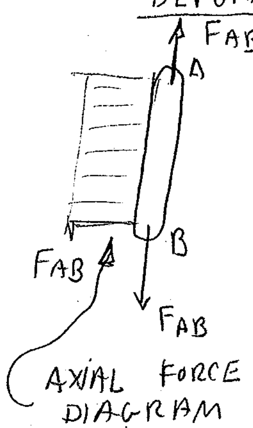
$\circlearrowleft + \sum M_C = 5\ kN \cdot 0.44\ m - F_{AB} \cdot 0.64\ m = 0$

$F_{AB} = 3.4375\ kN$ (TENSION)

$+\uparrow \sum F_y = 0 = F_{AB} + F_{DC} - 5\ kN$

$F_{DC} = \dots = 1.5625\ kN$ (TENSION)

DEFORMATION OF AB:

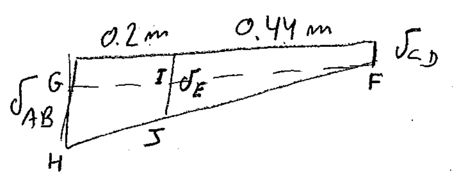


$$\delta_{AB} := \frac{F_{AB}\ L_{AB}}{A_0 \cdot E_{Al}} = \frac{3.4375 \cdot 10^3\ N \cdot 0.36\ m}{125 \cdot 10^{-6}\ m^2 \cdot 75 \cdot 10^9\ Pa} = 1.32 \cdot 10^{-4}\ m = 0.132\ mm$$

DEFORMATION OF CD: SIMILAR ANALYSIS PROCEDURE FOR AB:

$$\delta_{CD} = \frac{F_{CD} \cdot L_{CD}}{A_0 \cdot E_{Al}} = \dots = 0.06\ mm$$

APPLYING GEOMETRY CONSTRAINS



$$\delta_E = \delta_{CD} + \overline{IJ} = 0.06 + 0.0495 = \underline{\underline{0.1095\ mm}}$$

BY SIMILARITY OF TRIANGLES ΔFGH, ΔFIJ

$$\frac{\overline{IJ}}{\overline{IF}} = \frac{\overline{GH}}{\overline{GF}} \rightsquigarrow IJ = \frac{\overline{GH}}{\overline{GF}} \cdot \overline{IF} = \frac{\delta_{AB} - \delta_{CD}}{(0.2 + 0.44)} \cdot 0.44 = \frac{0.132 - 0.06}{(0.2 + 0.44)} \cdot 0.44;\ \underline{IJ = 0.0495\ mm}$$

(2.9)

LOOK AT THE BOOK SAMPLE PROBLEM 2.1 (PAGE 63) – COMBINING TENSION AND COMPRESSION

HOMEWORK TEXTBOOK, PROBLEMS

2.1, 2.8, ~~2.11~~, 2.15, 2.16, 2.25

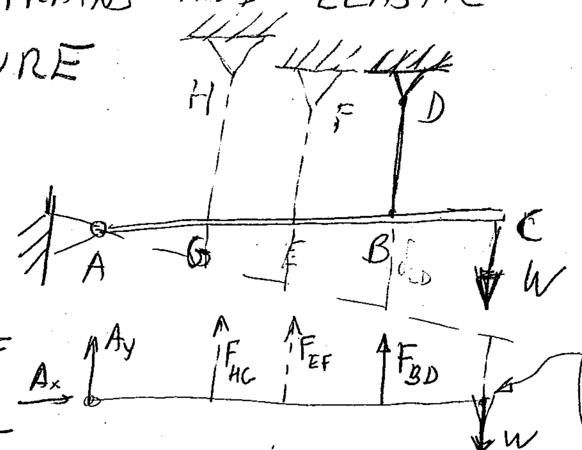
(SEE 2.16)

STATICALLY INDETERMINATE PROBLEMS

- OVER-SUPPORTED OR/AND OVER-MEMBERED STRUCTURE REQUIRES MORE THAN THE EQUILIBRIUM EQUATIONS IN ORDER TO SOLVE ITS REACTIONS OR/AND INTERNAL FORCES
- THE EXTRA EQUATIONS WILL BE FOUND FROM THE GEOMETRICAL CONSTRAINS AND ELASTIC DEFORMATIONS ON THE STRUCTURE

AFTER T

FOR EXAMPLE:



ADDITION OF A GEOMETRICAL CONSTRAIN: LINEAR RELATION BETWEEN THE CABLES DEFORMATIONS

3 EQUATIONS OF EQUILIBRIUM + 3 UNKNOWNS A_x, A_y, F_{BD}

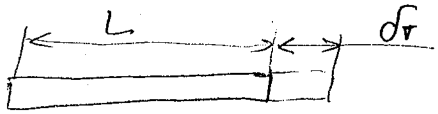
THERMAL EXPANSION AS A CONSTRAIN (IN ADDITION TO EQ. EQ)

INDETERMINATE STRUCTURES CAN BE SENSITIVE TO CHANGES IN TEMPERATURE. DIFFERENT COEFFICIENT OF THERMAL EXPANSION MAY CAUSE INTERNAL FORCES AND STRESSES, WITHOUT APPLYING ANY EXTERNAL LOAD

FOR EXAMPLE: LENS IN ITS CASE, TERMOSTAT

(2.10)

α - COEFFICIENT OF THERMAL EXPANSION $\left[\frac{1}{°C}\right]$ OR $\left[\frac{1}{°F}\right]$



$$\boxed{\delta_T = \alpha(\Delta T)\, L}$$

CHANGE IN TEMPERATURE

$\Delta T = T_A - T_L$ $[°C]$ OR $[°F]$

ACTUAL TEMP.

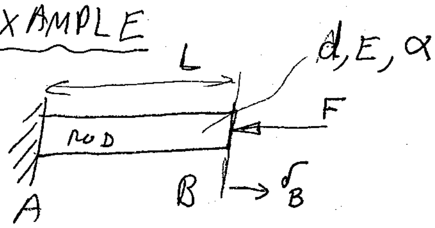
TEMP FOR LENGHT L

$$\boxed{\varepsilon_T = \frac{\delta_T}{L} = \alpha \Delta T}$$

THE THERMAL STRAIN

CAUTION: HOOKE'S LAW IS NOT APPLICABLE FOR ε_T!
THERE IS NO DIRECT RELATIONSHIP BETWEEN THE THERMAL STRAIN TO STRESS
IT IS PREFERABLE NOT TO USE CONCEPT OF THERMAL STRAIN.

EXAMPLE



$L = 0.5\,m$

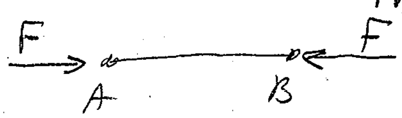
$d = \varnothing 10\,mm$ (~0.4")

$E = 75\,GPa$, $\alpha = 23 \cdot 10^{-6} \frac{1}{°C}$ } ALUMINUM

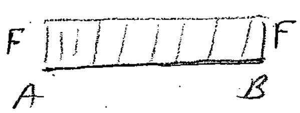
$\Delta T = 80°C$

FIND F IN ORDER TO REMAIN THE ROD WITHOUT DEFORMATION ($\delta_B = 0$)

FREE BODY DIAGRAM



$F_{AB} = F$



AXIAL FORCE DIAGRAM

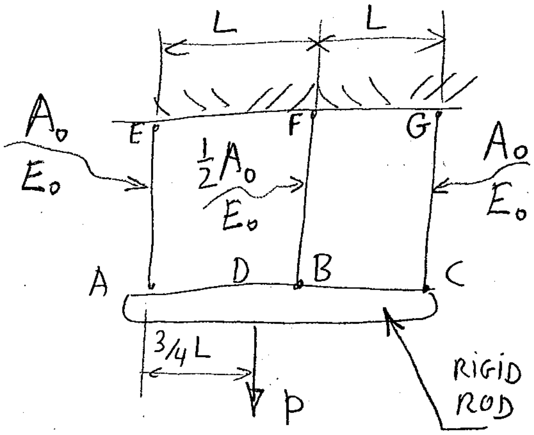
$$\delta_B = \frac{F\cancel{L}}{EA} = \delta_T = \alpha \Delta T \cancel{L}$$

$$F = \alpha \Delta T\, E A = 23 \cdot 10^{-6} \frac{1}{°C} \cdot 80°C \cdot 75 \cdot 10^{9}\, Pa \cdot \frac{(10 \cdot 10^{-3})^2 \cdot \pi}{4}\, m^2$$

$F = 10838\,N$ ($\cong 2440\,lb$)

(2.9) (2.11)

EXERCISE 2.47



SECTION AREA A_0

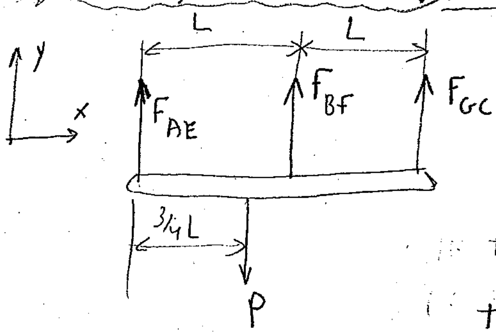
F_{AE} - ?

F_{BF} - ?

F_{GC} - ?

ANSWERS IN THE BOOK ARE WRONG

WIRES, ONLY TENSION! FREE BODY DIAGRAM



$$+\uparrow \sum F_y = F_{AE} + F_{BF} + F_{GC} - P = 0 \quad (1)$$

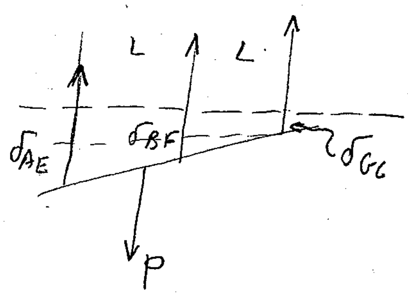
$$\circlearrowleft + \sum M_A = +F_{BF}\cdot L + F_{GC}\cdot 2L - P\cdot \tfrac{3}{4}L = 0 \quad (2)$$

$$\xrightarrow{+} \sum F_x = 0 \longleftarrow \text{TRIVIAL}$$

THREE UNKNOWNS, TWO EQUATIONS

OVER SUPPORTED - INDETERMINATE PROBLEM

THE ADDITIONAL EQUATION IS THE GEOMETRICAL CONSTRAIN: LINEAR RELATION MUST EXIST BETWEEN THE WIRES DEFORMATIONS:



SIMILARITY OF TRIANGLES:

$$\frac{(\delta_{AE} - \delta_{GC})}{2L} = \frac{(\delta_{BF} - \delta_{GC})}{L}$$

$$\delta_{AE} = \frac{F_{AE}\cdot(\text{LENGHT})}{A_0 E_0}$$

$$\delta_{BF} = \frac{F_{BF}(\text{LENGHT})}{\frac{1}{2}A_0\cdot E_0}$$

$$\delta_{GC} = \frac{F_{GC}\ (\text{LENGHT})}{A_0\ E_0}$$

$$\frac{\left(\frac{F_{AE}\ \cancel{(\text{LENGHT})}}{\cancel{A_0 E_0}} - \frac{F_{GC}\ \cancel{(\text{LENGHT})}}{\cancel{A_0 E_0}}\right)}{2\cancel{L}} = \frac{\left(\frac{F_{BF}\ \cancel{(\text{LENGHT})}}{\frac{1}{2}\cancel{A_0 E_0}} - \frac{F_{GC}\ \cancel{(\text{LENGHT})}}{\cancel{A_0 E_0}}\right)}{\cancel{L}}$$

$$\frac{F_{AE} - F_{GC}}{2} = 2F_{BF} - F_{GC} \quad (3)$$

$$\tfrac{1}{2}F_{AE} + \tfrac{1}{2}F_{GC} - 2F_{BF} = 0$$

(2.12)

$$Ax + B = u$$
$$x = A^{-1}[u - B]$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ \frac{1}{2} & -2 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} F_{AE} \\ F_{BF} \\ F_{GC} \end{bmatrix} = \begin{bmatrix} 1 \\ 3/4 \\ 0 \end{bmatrix} P$$

SOLUTION OF A SET OF LINEAR EQUATIONS

$$\begin{bmatrix} F_{AE} \\ F_{BF} \\ F_{GC} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ \frac{1}{2} & -2 & \frac{1}{2} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 3/4 \\ 0 \end{bmatrix} P = \begin{bmatrix} 0.9 & -0.5 & -0.2 \\ 0.2 & 0 & -0.4 \\ -0.1 & 0.5 & -0.2 \end{bmatrix} \begin{bmatrix} 1 \\ 3/4 \\ 0 \end{bmatrix} P = \begin{bmatrix} 0.525 \\ 0.2 \\ 0.275 \end{bmatrix} P$$

IF THE SECTION AREAS WERE EQUAL:

$$-1 \rightsquigarrow \begin{bmatrix} F_{AE} \\ F_{BF} \\ F_{GC} \end{bmatrix} = \begin{bmatrix} 0.4583 \\ 0.3333 \\ 0.2083 \end{bmatrix} P \longrightarrow F_{BF} \text{ IS CARRYING MORE}$$

IF THE SECTION AREA OF BF WAS TWICE THE AREA OF AE, GC

$$-\tfrac{1}{2} \rightsquigarrow \begin{bmatrix} F_{AE} \\ F_{BF} \\ F_{GC} \end{bmatrix} = \begin{bmatrix} 0.375 \\ 0.5 \\ 0.125 \end{bmatrix} P \longrightarrow F_{BF} \text{ IS CARRING MUCH MORE}$$

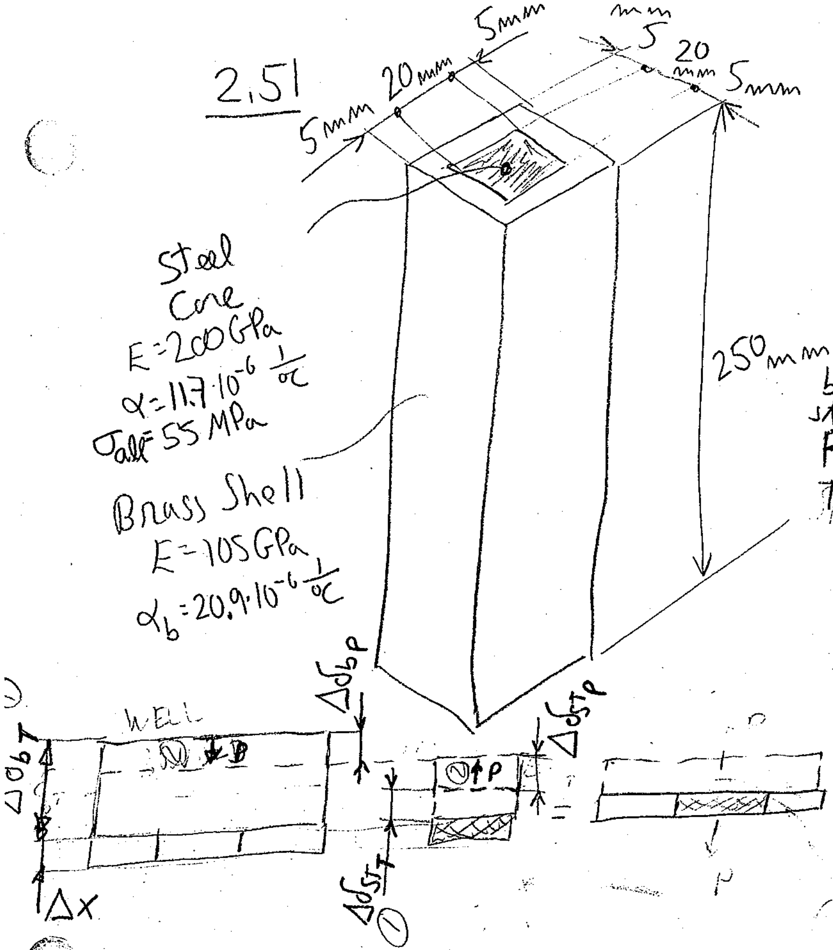
CONCLUSSION

IN INDETERMINATE CASES THE RELATIVE STIFFNESS OF MEMBERS INFLUENCE THE MAGNITUDE OF LOAD TO CARRY

(2.13)

2.51

ΔT - ?
(For σ_{al})



b = brass
st = steel
P = DUE TO INTERNAL FORCE
T = DUE TO TEMP

$$\Delta\delta_{b_T} - \Delta\delta_{b_P} = \Delta\delta_{ST_P} + \Delta\delta_{ST_T}$$

$$\varepsilon_b\Big|_T - \varepsilon_b\Big|_P = \varepsilon_{st}\Big|_T + \varepsilon_{st}\Big|_P$$

$$\varepsilon_{b_T} = \alpha_b \Delta T$$

$$\varepsilon_{bP} = \frac{P}{E_b A_b}$$

$$\varepsilon_{st_T} = \alpha_{st} \Delta T$$

$$\varepsilon_{ST_P} = \frac{P}{E_{ST} A_{ST}}$$

$$\alpha_b \Delta T - \frac{P}{A_b}\cdot\frac{1}{E_b} = \alpha_{st}\Delta T + \frac{P}{A_{st}}\cdot\frac{1}{E_{st}}$$

$$\sigma_{All} = 55\ MPa = \frac{P}{A_{st}} = \frac{P}{20\cdot 20\ mm^2} \rightsquigarrow P = 22000\ N$$

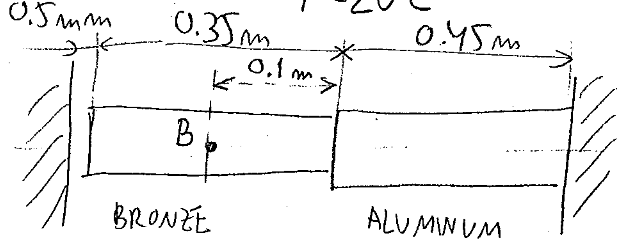
$$\Delta T = \frac{\sigma_{All}\cdot\frac{1}{E_{st}} + \frac{P}{A_b}\cdot\frac{1}{E_b}}{(\alpha_b - \alpha_{st})} = \frac{55_{MPa}\cdot\frac{1}{200\cdot 10^3_{MPa}} + \frac{22000\ N}{(900-400)\ mm^2}\cdot\frac{1}{105\cdot 10^3\ MPa}}{20.9\cdot 10^{-6} - 11.7\cdot 10^{-6}}$$

$$\underline{\Delta T = 75.44\ °C}$$

(2.14)

2.57, 2.58 + (additions)

$T = 20°C$



$A_b = 1500\ mm^2$ $\quad$ $A_{Al} = 1800\ mm^2$

$E_b = 105\ GPa$ $\quad$ $E_{Al} = 73\ GPa$

$\alpha_b = 21.6 \cdot 10^{-6} \frac{1}{°C}$ $\quad$ $\alpha_{Al} = 23.2 \cdot 10^{-6} \frac{1}{°C}$

DETERMINE

(a) THE COMPRESSIVE FORCE IN THE BARS FOR $\Delta T = 96°C$

(b) δ_b - ? (bronze)

(c) $\sigma_{Al}\big|_{ACTUAL} = -90\ MPa \rightsquigarrow T - ?$

(d) L_{Al} - ? AFTER HEATED TO RESULT (c)

(e) FORCE TO BE APPLIED ON B IN ORDER TO KEEP THE GAP CLOSED, BUT WITHOUT LOADING THE LEFT WALL

(a) WE HAVE TWO PROBLEMS IN ONE:

THE GAP OPENED / CLOSED

NO REACTIONS FROM WALLS

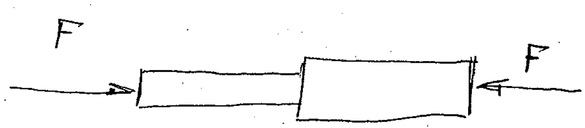
REACTION FROM WALLS

TEMPERATURE RISE NEEDED TO CLOSE THE GAP

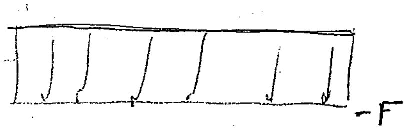
$$\delta = 0.5\ mm = \delta_{bT} + \delta_{Al_T} = \alpha_b \Delta T L_b + \alpha_{Al} \Delta T L_{Al} = (\alpha_b L_b + \alpha_{Al} L_{Al}) \Delta T$$

$$\Delta T = \frac{\delta}{\alpha_b L_b + \alpha_{Al} L_{Al}} = \frac{0.5 \cdot 10^{-3} m}{21.6 \cdot 10^{-6} \frac{1}{°C} \cdot 0.35 m + 23.2 \cdot 10^{-6} \frac{1}{°C} \cdot 0.45 m} = 27.778\ °C$$

THE PROBLEM RISES THE TEMP. BY 96°C, SO FOR $96°C \geq \Delta T > 27.778°C$ THE GAP IS CLOSED AND THE WALLS ARE LOADED



FREE BODY DIAGRAM



AXIAL LOAD DIAGRAM

$$\delta_F = \delta_{b_F} + \delta_{Al_F} = F\left(\frac{L_b}{A_b E_b} + \frac{L_{Al}}{A_{Al} E_{Al}}\right)$$

$$\Delta T\big|_{GAP\ CLOSED} = 96 - 27.778 = 68.222\ °C.$$

GEOMETRICAL CONSTRAIN $\quad \delta_T + \delta_F = 0$

$$\Delta T\big|_{GAP\ CLOSED} \cdot (\alpha_b L_b + \alpha_{Al} L_{Al}) + F\left(\frac{L_b}{A_b E_b} + \frac{L_{Al}}{A_{Al} E_{Al}}\right) = 0 \quad (*)$$

(2.15)

$$F = -\frac{\left(21.6\cdot10^{-6}\frac{1}{°C}\cdot 0.35\,m + 23.2\cdot10^{-6}\frac{1}{°C}\cdot 0.45\,m\right)\cdot 68.222\,°C}{\dfrac{0.35\,m}{1500\cdot10^{-6}\,m^2\cdot 105\cdot10^9\,Pa} + \dfrac{0.45\,m}{1800\cdot10^{-6}\,m^2\cdot 73\cdot10^9\,Pa}}$$

$F = -217465.16\ N = -217.47\ kN$

(b) $(\delta_b - ?)$

$$\delta_b = \alpha_b L_b \Delta T + \frac{F L_b}{A_b E_b} = 21.6\cdot10^{-6}\frac{1}{°C}\cdot \underbrace{0.35\cdot10^3\,mm}_{}\cdot 96\,°C - \underbrace{\frac{217465.16\,N\cdot 0.35\cdot10^3\,mm}{1500\,mm^2\cdot 105\cdot10^3\,MPa}}_{0.48326\ mm}$$

(0.72576) mm

0.5157

$\boxed{\delta_b = 0.2425\,mm}$

NOTE:

$\delta_{b_T}\Big|_{\substack{\text{GAP CLOSED}\\ \Delta T = 68.222°C}} = 0.5157\ mm$

DIFFERENT !!

$\delta_{b_F}\Big|_{\text{GAP CLOSED}} = 0.48326\ mm$

(LIMIT BETWEEN BRONZE AND ALUMINIUM GO TO THE RIGHT)

$\sigma_{Al}|_{ACTUAL} = -90\ MPa \rightarrow T - ?$

(c) $F_{Al} = \sigma_{Al} A_{Al} = -90\,MPa\cdot 1800\,mm^2 = -162000\,N = F_b$

(*) $\rightarrow$ $$\Delta T\Big|_{\text{GAP CLOSED}} = \frac{-F\left(\dfrac{L_b}{A_b E_b} + \dfrac{L_{Al}}{A_{Al} E_{Al}}\right)}{\left(\alpha_b L_b + \alpha_{Al} L_{Al}\right)} = \frac{162000\,(\cdots)}{(\cdots\cdots)} = 50.8219\,°C$$

$$T = T_0 + \Delta T\Big|_{\text{GAP OPENED}} + \Delta T\Big|_{\text{GAP CLOSED}} = 20°C + 27.778°C + 50.822°C$$

ROOM TEMPERATURE

TEMPERATURE RISE IN ORDER TO CLOSE THE GAP

TEMPERATURE RISE DURING STRESS INCREASE

$\underline{\underline{T = 98.6\ °C}}$

(2.16)

(d) $L_{Al_{NEW}}$ - ?

$$\delta_{Al} = \delta_{Al}\Big|_T + \delta_{Al}\Big|_F = \alpha_{Al} \cdot L_{Al} \cdot \Delta T + \frac{F_{Al}\, L_{Al}}{E_{Al}\, A_{Al}}$$

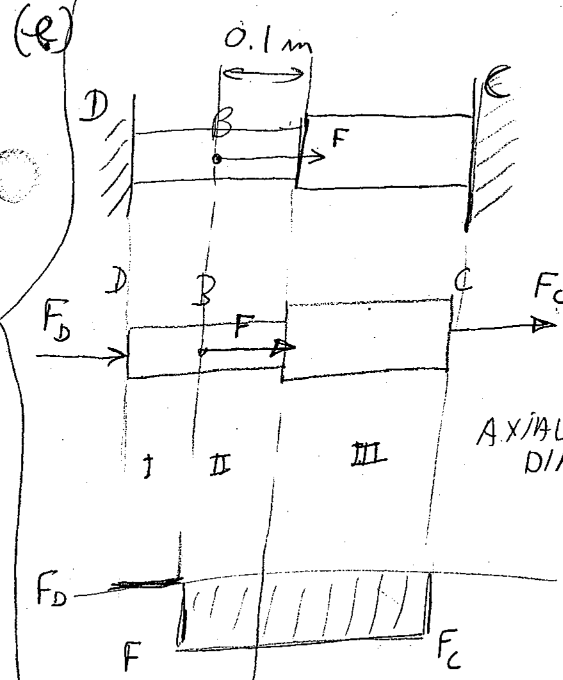
σ_{Al}

$$\delta_{Al} = 23.2 \cdot 10^{-6} \frac{1}{°C} \cdot 450_{mm} \cdot (27.778°C + 50.822°C) + $$

$$+ \frac{-90\,MPa \cdot 450mm}{73 \cdot 10^3\ MPa} = 0.26579\ mm$$

$$\boxed{L_{Al_{NEW}} = L_{Al} + \delta_{Al} = 450.2658\ mm}$$

(e)



WE CAN ASSUME THAT THE LEFT WALL DOESNT EXIST, $F_D = 0$

$$\Delta T = 50.822°C$$

$$\Sigma \delta = 0 = \delta_T + \delta_F$$

$$\delta_T = (\alpha_b L_b + \alpha_{Al} L_{Al})\, \Delta T$$

$$\delta_F = \sum \frac{F_i L_i}{A_i E_i}$$

$$= \underset{\text{ZONE I}}{0} + \underset{\text{ZONE II}}{\frac{F_c \cdot 100mm}{A_b\ E_b}} + \underset{\text{ZONE III}}{\frac{F_c \cdot L_{Al}}{A_{Al}\ E_{Al}}}$$

$$F_c = -\frac{(\alpha_b L_b + \alpha_{Al} L_{Al})\,\Delta T}{\left(\frac{100mm}{A_b\ E_b} + \frac{L_{Al}}{A_{Al}\ E_{Al}}\right)} = \ldots = 225343\ N$$

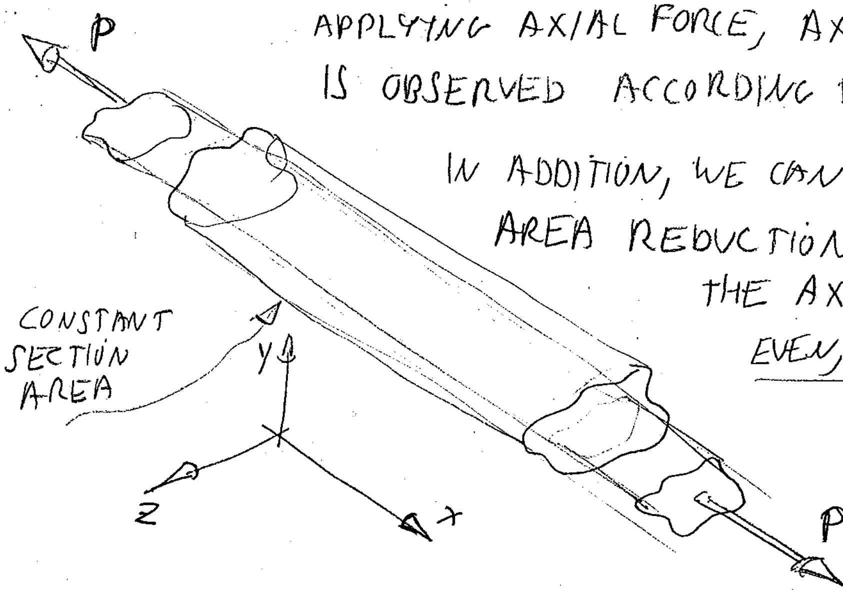
HOMEWORK:

HOOKE'S LAW INDETERMINATE TEMP

2.1, ~~2.11~~, 2.16, 2.25, 2.36, ~~2.45~~, 2.52, 2.41, 2.60,

AXIAL LOAD DIAGRAM (2.45) ? ? ?

POISSON'S RATIO



APPLYING AXIAL FORCE, AXIAL STRAIN IS OBSERVED ACCORDING TO HOOKE'S LAW

IN ADDITION, WE CAN OBSERVE A SECTION AREA REDUCTION BECAUSE OF THE AXIAL LOAD

EVEN, THERE ARE NO LATERAL EXTERNAL LOADINGS

LATERAL STRAIN CAN BE OBSERVED IN AXIAL LOAD. THE RELATION OF LATERAL STRAIN TO AXIAL STRAIN IS CALLED POISSON'S RATIO (IN NAME OF THE FRENCH SCIENTIST):

$$\boxed{\nu = -\frac{\text{LATERAL STRAIN}}{\text{AXIAL STRAIN}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}}$$

NOTES: ① $\varepsilon_y, \varepsilon_z$ ARE NEGATIVE WHEN ε_x POSITIVE. THE MINUS SIGN IN THE DEFINITOON OF POISSON'S RATIO CONVERTS ν TO A POSITIVE NUMBER

② FOR THE MATERIALS WE ARE GOING TO DEAL WITH, $\varepsilon_y = \varepsilon_z$ (homogenous, isotropic)

③ ν IS A PROPIERTY OF THE MATERIAL, AS WELL AS THE MODULUS OF ELASTICITY IS (E)

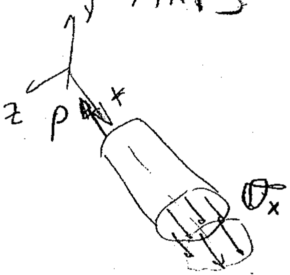
USING HOOKE'S LAW: $\varepsilon_x = \dfrac{\sigma_x}{E}$

$$\varepsilon_y = \varepsilon_z = -\frac{\nu \sigma_x}{E}$$

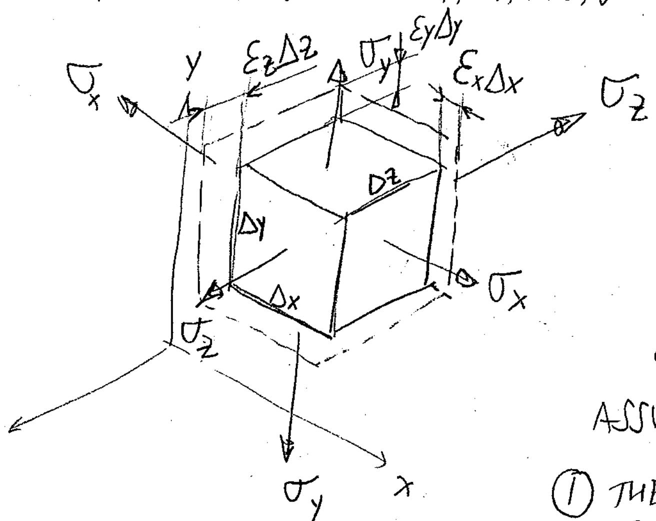
CAN BE PROVED THAT $\Big\}$ $0 < \nu < 0.5$

MULTIAXIAL LOADING; GENERALIZED HOOKE'S LAW

WE REFERED AXIAL LOADING TO A LOAD ACTING IN X AXIS X, CAUSING NORMAL STRESS IN X DIRECTION, σ_x



IN THIS SECTION WE ARE GOING TO APPLY AXIAL STRESSES SUMULTANEOUSLY ON X, Y AND Z AXLE (BUT NOT SHEARING STRESSES) AND RECEIVE THE FOLLOWING SITUATION



THE RECTANGULAR PARALLEPIPOID $\Delta x \cdot \Delta y \cdot \Delta z$

CHANGE DIMENSIONS

$\varepsilon_x \cdot \Delta x$ IN X DIRECTION

$\varepsilon_y \Delta y$ " Y "

$\varepsilon_z \cdot \Delta z$ " Z "

ASSUMPTIONS:

1. THE MATERIAL ISOTROPIC, HOMOGENOUS, CONTINUOUS
2. a) THE EFFECTS OF THE LOADS / STRESSES, ARE LINEARLY RELATED TO THE LOADS THAT PRODUCES IT
   b) THE DEFORMATION RESULTING FROM ANY GIVEN LOAD IS SMALL AND DOES NOT AFFECT THE CONDITIONS OF APPLICATION OF THE OTHER LOADS

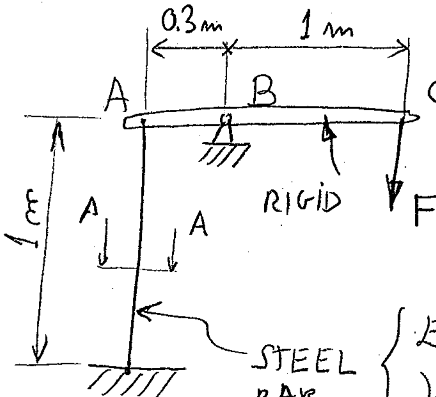
THE PRINCIPLE OF SUPER POSITION CAN BE USED TO ADD THE EFFECTS

3. THE STRESSES DONT EXCEED THE PROPORTIONAL LIMIT (σ_y)

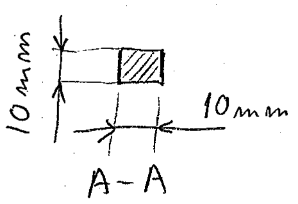
THEN, IN ORDER TO CALCULATE THE STRAINS IN THE DIFFERENT DIRECTIONS:

$$\boxed{\begin{aligned} \varepsilon_x &= +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E} \\ \varepsilon_y &= -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E} \\ \varepsilon_z &= -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E} \end{aligned}}$$

EXERCISE POISSON'S RATIO



STEEL BAR $\begin{cases} E = 200\,GPa \\ \nu = 0.29 \end{cases}$



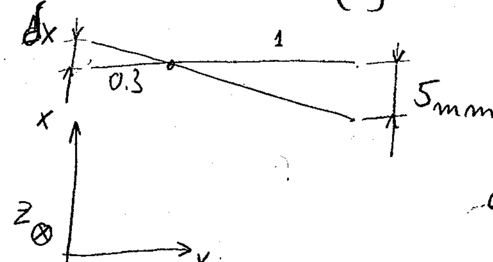
CALCULATE:

(a) THE LOAD F REQUIRED FOR A $\underbrace{\text{DEFLECTION}}_{}$ (5 mm) OF POINT C.

(b) THE SECTION AREA DECREASE RATIO OF THE STEEL BAR

SOLUTION

(a)



SIMILARITY OF TRIANGLES:

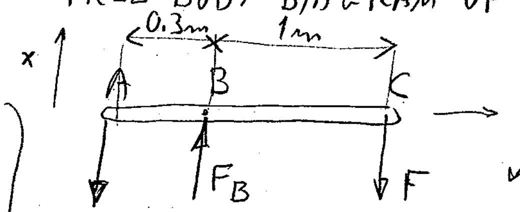
$$\delta_x = 0.3_m \cdot \frac{5\,mm}{1_m} = 1.5\,mm$$

$$\delta_x = \frac{F_A L}{A E} \; ; \; F_A = \frac{1}{L}\delta_x A E$$

$$F_A = \frac{1}{1000_{mm}} \cdot 1.5 \cdot (10 \times 10\,mm^2) \cdot 200 \cdot 10^3_{MPa}$$

$$F_A = 30000\,N \text{ (TENSION)}$$

FREE BODY DIAGRAM OF ABC:



TENSION FORCE APPLIED BY THE STEEL BAR } 30000 N

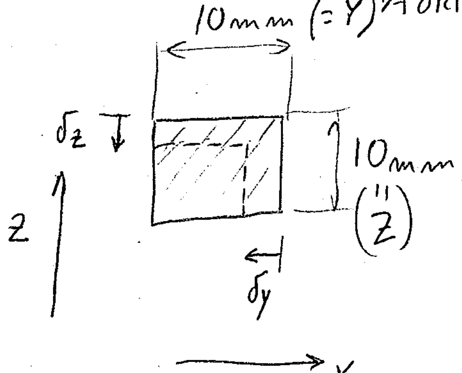
$$\circlearrowleft + \sum M_B = 0 \; \ldots$$

$$F = \frac{30000 \cdot 0.3}{1} = 9000\,N$$

(2.20)

(b) SECTION $\overbrace{\text{DECREASE}}^{\text{AREA}}$ RATIO

$$\text{RATIO} = \frac{A_{NEW}}{A_{ORIG.}} \cdot 100\ \%$$



$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

$$\varepsilon_x = \frac{\delta_x}{L} = \frac{1.5\,mm}{1000\,mm} = 1.5 \cdot 10^{-3} = 1500\,\mu$$

$$\varepsilon_y = \varepsilon_z = -\underset{\text{STEEL}}{\nu}\,\varepsilon_x = -0.29 \cdot 1500 = -435\,\mu \ (\text{OR } -435\,\mu s)$$

$$\delta_y = \varepsilon_y \cdot Y = -435 \cdot 10^{-6} \cdot 10\,mm = -4.35 \cdot 10^{-3}\,mm$$

$$\delta_z = \varepsilon_z \cdot Z = \cdots = -4.35 \cdot 10^{-3}$$

$$Y_{NEW} = Y + \delta_y = 10 - 4.35 \cdot 10^{-3} = 9.99565$$

$$Z_{NEW} = \cdots = 9.99565$$

$$\text{RATIO} = \frac{Y_{NEW} \cdot Z_{NEW}}{10 \times 10\,mm^2} \cdot 100 = \frac{9.99565^2}{100} \cdot 100 = 99.913\ \%$$

BY THE WAY

ACTUAL → $\sigma_x = \varepsilon_x E = 1500 \cdot 10^{-6} \cdot 200\,000\,MPa = 300\,MPa$ ← ACTUAL

NEAR YIELD → $\sigma_y \approx 800\,MPa \rightsquigarrow \varepsilon_x = 4000\,\mu$ ← NEAR YIELD

$\nu = 0.29$

$\varepsilon_y = \varepsilon_z = -1160\,\mu$

↓

RATIO $\approx 99.77\,\%$

CALCULATED $800\,MPa \xrightarrow[\text{STRESS}]{\text{REAL}} 801.86\,MPa$

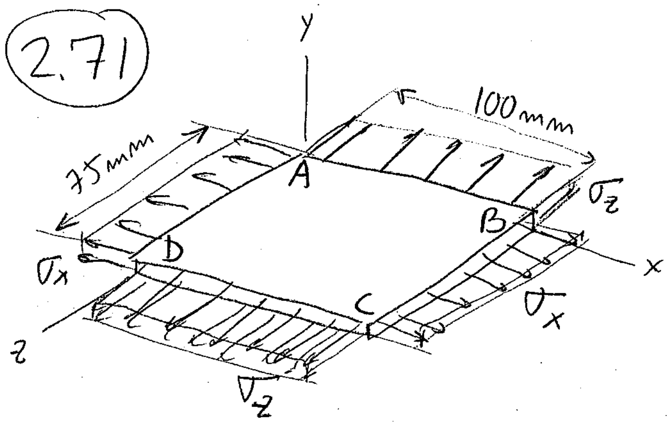
EXAMPLE 2.07, page 85

(2.21)

EXERCISE – MULTIAXIAL LOADING, GENERALIZED HOOKE'S LAW

EXAMPLE 2.08 / PAGE 87

(2.71)



$\sigma_x = 120\ MPa$

$\sigma_z = 160\ MPa$

$E = 87\ GPa$

$\nu = 0.34$

CHANGE IN LENGHT OF

(a) SIDE AB

(b) SIDE BC

(c) DIAGONAL AC

WE IDENTIFY MORE THAN MULTIAXIAL LOADING

$$\varepsilon_x = +\frac{\sigma_x}{E} - \cancel{\frac{\nu\sigma_y}{E}}^{\,0} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \cancel{\frac{\sigma_y}{E}}^{\,0} - \frac{\nu\sigma_z}{E}$$

WE ARE NOT ASKED FOR STRAINS IN Y DIRECTION – – THICKNESS OF THE ELEMENT

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \cancel{\frac{\nu\sigma_y}{E}}^{\,0} + \frac{\sigma_z}{E}$$

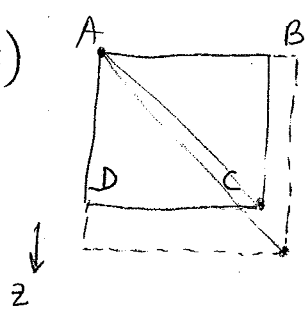
$$\varepsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_z}{E} = \frac{1}{87\cdot 10^3 MPa}\cdot\left(120\ MPa - 0.34\cdot 160\ MPa\right) = 7.54023\cdot 10^{-4} = 754\ \mu$$

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} + \frac{\sigma_z}{E} = \frac{1}{87\cdot 10^3 MPa}\left(-0.34\cdot 120\ MPa + 160\ MPa\right) = 1.37\cdot 10^{-3} = 1370\ \mu$$

(a) $\delta_{AB} = \varepsilon_x \overline{AB} = 754\cdot 10^{-6}\cdot 100\ mm = 0.0754\ mm$

(b) $\delta_{BC} = \varepsilon_z \overline{BC} = 1370\cdot 10^{-6}\cdot 75\ mm = 0.10275\ mm$

(c)



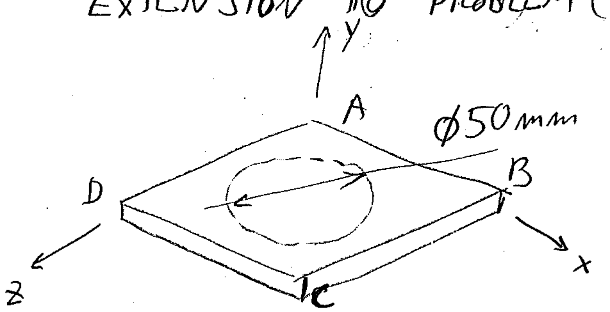
$$\delta_{AC} = \sqrt{(\overline{AB}+\delta_x)^2 + (\overline{BC}+\delta_z)^2} - \sqrt{\overline{AB}^2 + \overline{BC}^2}$$

$$= \sqrt{100.0754^2 + 75.10275^2} - 125$$

$$\boxed{\delta_{AC} = 0.122\ mm}$$

(2.22)

EXTENSION TO PROBLEM (2.71)



BEFORE THE 2.71 ELEMENT WAS LOADED, A CIRCLE OF ϕ50 mm WAS DRAWED.
CALCULATE:
(d) THE DIAMETER OF THE CIRCLE AFTER LOADING
ON PARALLEL TO X AXIS
AN PARALLEL TO Z AXIS
(e) FOR A THICKNESS OF 2 mm
$t = 2$ mm, CALCULATE THE CHANGE IN VOLUME OF THE ELEMENT

(d) $d_x = 50_{mm}(1+\varepsilon_x) = 50_{mm}(1+754\cdot10^{-6}) = 50.0377$ mm
$d_z = 50_{mm}(1+\varepsilon_z) = 50_{mm}(1+1370\cdot10^{-6}) = 50.0685$ mm

(e) $\varepsilon_y = -\frac{\nu\sigma_x}{E} + \underbrace{\frac{\sigma_y}{E}}_{0} - \frac{\nu\sigma_z}{E} = -\frac{0.34\cdot120\,MPa}{87\cdot10^3 MPa} - \frac{0.34\cdot160\,MPa}{87\cdot10^3 MPa}$

$\varepsilon_y = -1094.3\,\mu$

$t_{NEW} = t(1+\varepsilon_y) = 2(1-1094.3\cdot10^{-6}) = 1.9978$ mm

$V_{NEW} = V + \Delta V$

$\Delta V = -100\cdot75\cdot2 + 100.0754\cdot75.10275\cdot1.9978$

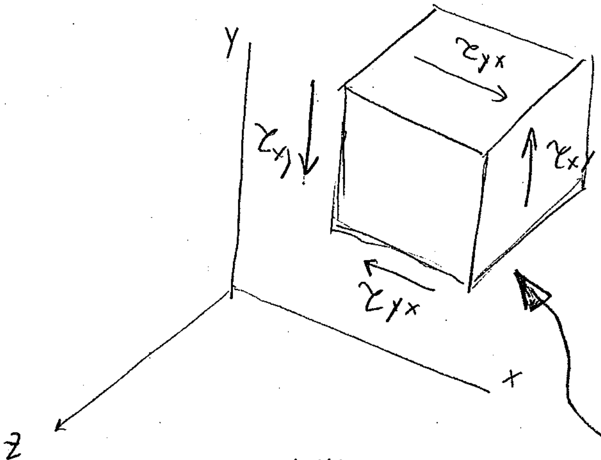
$\Delta V = +15.34\ mm^3$ (INCREASE)

HOMEWORK

2.65, 2.69,

(2.23)

SHEARING STRAIN

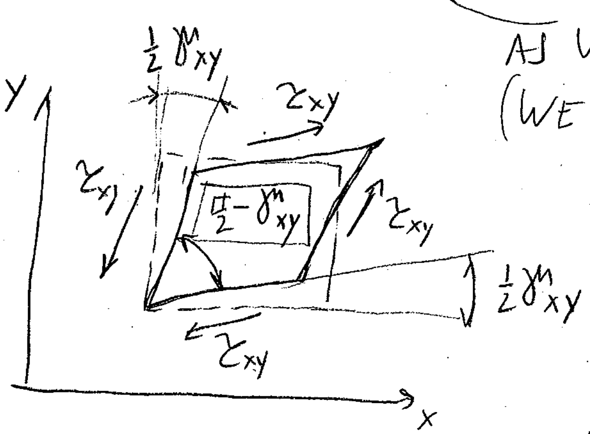


LET APPLY SHEARING STRESSES WITHOUT APPLYING AXIAL STRESS (AS SHEAR OF A PIN OR RIVET)

THE STRESSES APPLIED DO NOT EXCEED THE PROPORTIONAL LIMI

LET DO IT GRADUALY, FIRST

SHEARING ON XY DIRECTIONS



AS WE REMEMBER, $\tau_{xy} = \tau_{yx}$ (WE PROVED IN CLASS)

WE WILL NOTE AN ANGULAR DEFORMATION OF THE ORIGINAL CUBE, DEFINED BY γ = THE CHANGE IN THE ANGLE FORMED BY TO PHASES = = THE SHEARING STRAIN

LET DEFINE THE HOOKE'S LAW FOR SHEARING STRESS AND STRAIN:

MODULUS OF RIGIDITY
SHEAR MODULUS

$$\boxed{G = \frac{\tau_{xy}}{\gamma_{xy}}} \qquad \boxed{\tau_{xy} = G \cdot \gamma_{xy}}$$

γ IS DEFINED AS AN ANGLE IN RADIANS ($\frac{\text{LENGHT}}{\text{LENGHT}}$ = DIMENSIONLESS), THEN G HAS THE SAME UNITS AS τ_{xy}: MPa, Pa, PSI, KSI, etc

AS WELL AS E & ν, G IS A PROPERTY OF THE SPECIFIC MATERIALL, G_{STEEL}, $G_{ALUMINUM}$, etc.

THE SAME ANALYSIS FOR PLANES ZY, ZX:

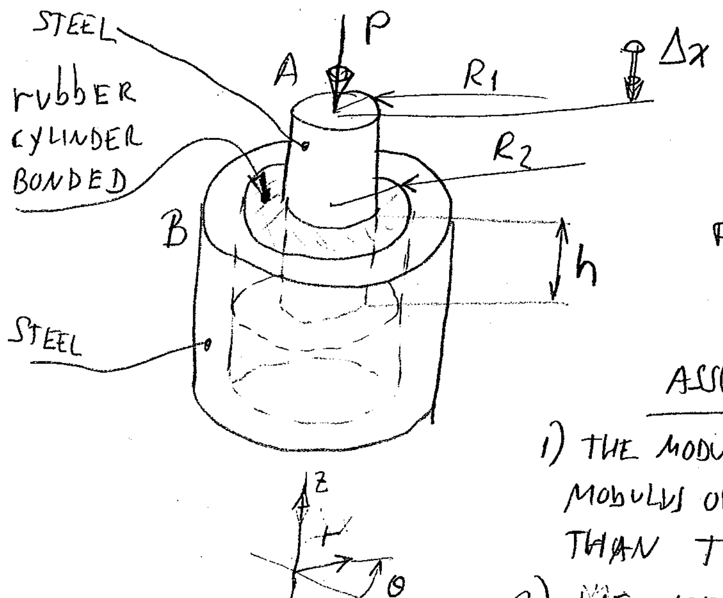
$$\boxed{\tau_{yz} = G\,\gamma_{yz}} \qquad \boxed{\tau_{zx} = G\,\gamma_{zx}}$$

(2.24)

THE SIMPLE

LEARN SOLVED EXAMPLE 2.10 ON THE TEXTBOOK.

EXAMPLE → PROBLEM * 2.87



$R_1 = 3/8"$

$R_2 = 1"$

$G = 1.8\ ksi$

$h = 3"$

FOR $\Delta x_{max} = 0.1"$ OF A, CALCULATE P_{max} ALLOWE.

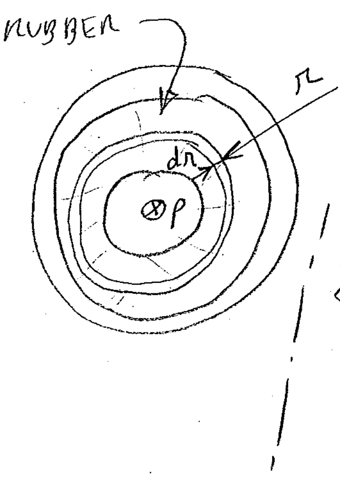
ASSUMPTIONS FOR THIS PROBLEM

1) THE MODULUS OF ELASTICITY AND SHEAR MODULUS OF STEEL ARE MUCH, MUCH BIGGER THAN THOSE FOR RUBBER

2) WE NEGLECT "EDGE PROBLEMS", IN THE SPECIFIC PROBLEM, THE SURFACES OF THE RUBBER IN CONTACT WITH AIR HAVE NO SHEARING REACTION AS WE LEARNED, SO THE WILL NOT REACT TO SHEARING STRESSES

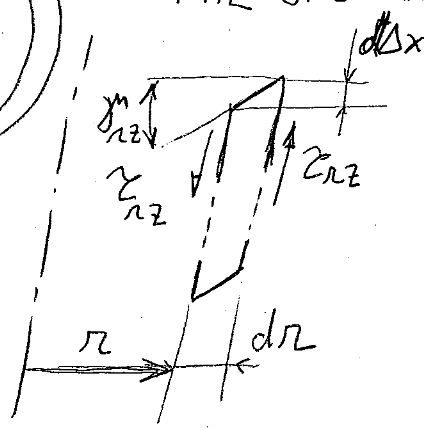
3) γ IS SMALL ENOUGHT, SO $\tan\gamma \approx \sin\gamma \approx \gamma_{rz}$

THE SHEARING STRESS AT ANY RADIUS OF THE RUBBER MAY BY CALCULATED AS FOLLOWS:



$$\tau_{rz} = \frac{P}{A_r} = \frac{P}{2\pi r h} \qquad (1)$$

THE SHEARING STRAIN CAUSED DUE TO τ_{rz}:



$$\boxed{\gamma_{rz} = \frac{\tau_{rz}}{G}} \qquad (2)$$

THE DEFLECTION OF THIS ELEMENT DUE TO γ_{rz}:

$$d\Delta x = \tan(\gamma_{rz}) \cdot dr \approx \gamma_{rz} \cdot dr$$

(2.25)

$$d\Delta x = \gamma^{m}_{rz} \cdot dr = \frac{\tau_{rz}}{G} \cdot dr = \frac{\left(\frac{P}{2\pi r h}\right)}{G} \cdot dr$$

(2) (3)

$$d\Delta x = \frac{P}{2\pi h G} \cdot \frac{1}{r} dr$$

$$\Delta x = \frac{P}{2\pi h G} \int_{R_1}^{R_2} \frac{1}{r} dr = \frac{P}{2\pi h G} \Big[\ln r\Big]_{R_1}^{R_2} = \frac{P}{2\pi h G} \ln \frac{R_2}{R_1}$$

WE GOT THE RELATION BETWEEN THE LOAD AND THE DISPLACEMENT

$$P_{MAX} = \frac{2\pi h G}{\ln\left(\frac{R_2}{R_1}\right)} \cdot \Delta x_{MAX} = \frac{2\pi \cdot 3'' \cdot 1800\,\text{PSI}}{\ln\left(\frac{8}{3}\right)} \cdot 0.1'' = 3459.2\ lb$$

LET SEE HOW ACCURATE IS OUR ASSUMPTION OF γ^m BEING VERY SMALL

$$\gamma^{m}_{rz} = \frac{\tau_{rz}}{G} \rightsquigarrow \gamma^{m}_{rz\ MAX} \quad \text{WHEN } \tau_{rz\ MAX}$$

$$\tau_{rz} = \frac{P}{2\pi r h} \rightsquigarrow \tau_{rz\ MAX} \quad \text{WHEN } r_{min} \text{ AND } P_{MAX}$$

$$r_{min} = R_1 ; \quad P_{MAX} = 3459.2\ lb$$

$$\tau_{rz}\Big|_{R_1, P_{MAX}} = \frac{3459.2\ lb}{2\pi\ 3/8'' \cdot 3''} = 489.38\ PSI$$

$$\gamma^{m}_{rz}\Big|_{R_1, P_{MAX}} = \frac{\tau_{rz}}{G} = \frac{489.38}{1800} = 0.272\ rad$$

ABOUT 2.5% MAX. ERROR

$$\tan(\gamma^{m}_{rz}) = 0.2789$$

THE AVERAGE ERROR IS SMALLER

First and Last name	Panther I. D.		Version
ANTHONY JACKSON-POWNALL	1277169	60	A
Luke Seever	1106526	100	B
HECTOR ROOS	109-63-68	NA	A
Trevor G.A. Harris III	1395585	75	A
NATALIE MARSHALL	1306053	77	A
VERDI M. MAYER	1358420	73	B.
Michael Patterson	213-98-8245	100	B
GRANT MESNER	1366034	72	A
Ruben Galeano	1352635	64	B
KERN WILSON	1589330	76	A
Saleh AlMutawa	1323191	78	B
Shaka Harper	1012782	63	B
Max Bravo	139-68-96	83	A
ERIC INCLAN	1356002	68	B
ANDRES ZAMORA	1397908	94	A
Trevis Moorley	1399021	67	B
CARLOS ARIAS	1371993	73	A
Damian Harper	1399731	56	B
Gustavo Cardona	1398821	77	A
Ernesto Gutierrez	1281667	70	B
YURI LAZZERETTI	1093801	66	A
GEORGETTE MARTINEZ	1339663	59	B
DAMIEN Lloyd	1013644	98	
Robert Jordan	1305753	70	B
Patrick Bellizaire	1629148	66	A
MARIO VALDIVIESO	1369320	68	A
Maribel Garcia	1322452	51	B
Adriana Bonilla	1321865	28	B
Paul Grata	1282007	+8 [illegible]	A
Juan Saluzzio	1372735	88	B
Gustavo Jaramillo	1379120	84	A
MARCELO BEYRA	1370183	63	B
Luis A Sepulveda	1349774	31	A
William Betancourt	1298220	10	B
MICHAEL WOLFF	1103264	90	A
Gonzalo Ocano	1021312	87	B
Alfredo Suarez	1495210	64	A
Felipe Rendon	1334090	50	B
Joseph Nichols	1373806	62	A
Daniel Muguruza	1258280	50	A
Alaa Maatiki	1503723	70	B
AMRIT HARRIPAUL	1644446	73	A.
Francis Fernandez	1026909	76	B
CINDY GOMEZ	1390408	80	A
JORGE TERERO	1014132	77	B
Ottley Brathwaite	1630841	19	A

QUIZ VERSION

AVERAGE - 67

STD. DEV - 21.2

Florida International University
Department of Mechanical and Materials Engineering
Mechanics and Materials Science

EMA 3702 — Quiz 1 - version A — Sept. 23, 2004

Follow the instructions before you begin the quiz:

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2. You should not give nor take any unpermitted aid during the quiz. Violation of this statement will lead to automatic failure of the quiz.
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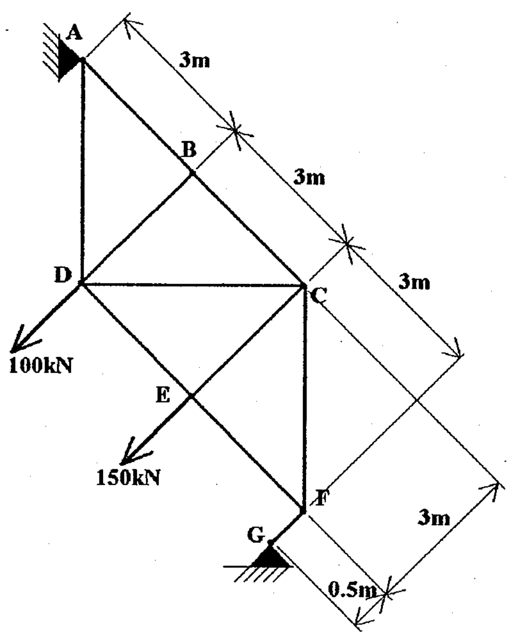
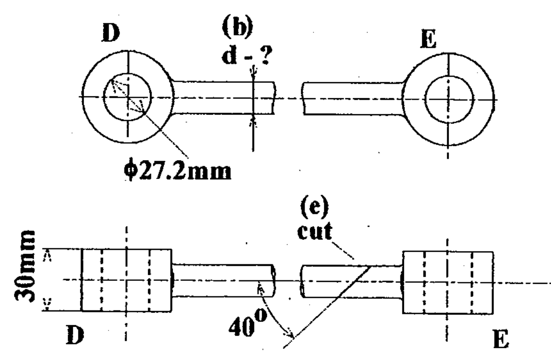
4. Write your first and last name, your Pant. I. D. and the quiz version on the papers you will use for the quiz solution.
5. Explain your steps, use the adequate diagrams.

Good Luck!

A pin-connected truss is loaded and supported as shown in the figure.

a) Determine the load applied on member DE and on member FG (20%)
b) Member DE is a rod (circular section area), made of steel, ultimate normal stress of 800MPa. The required Factor of Safety is 4. Calculate the rod diameter (20%)
c) Member DE is connected at point D with a pin under double shearing. The ultimate shearing stress of the pin is 460MPa. The diameter of the pin is 27.2mm. Calculate the Factor of Safety for the design of the pin at D. (20%)
d) Calculate the Bearing Stress on member DE at joint D. (20%)
e) Member DE is cut and bonded again using a strong loctite adhesive (see figure). Calculate the normal and shearing stress on the adhesive. (20%)





Florida International University
Department of Mechanical and Materials Engineering
Mechanics and Materials Science

EMA 3702 — Quiz 1 - version B — Sept. 23, 2004

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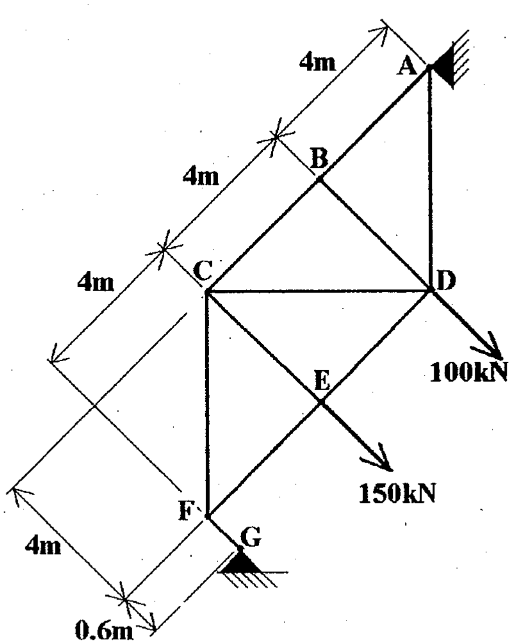

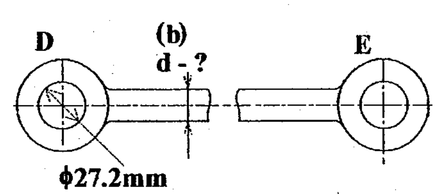

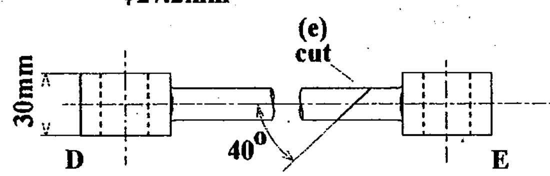

Florida International University
Department of Mechanical and Materials Engineering
Mechanics and Materials Science

EMA 3702 Quiz 1 - version A Sept. 23, 2004

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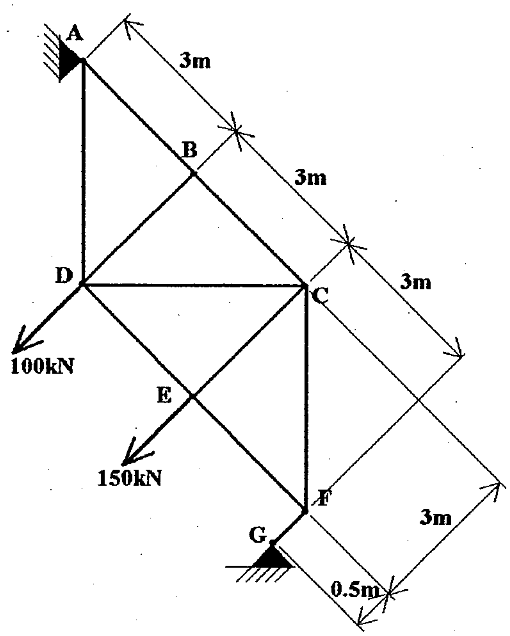

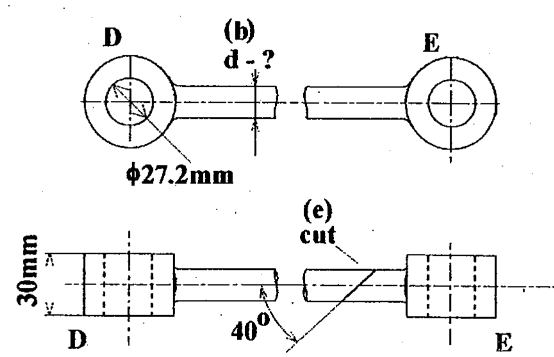

Florida International University
Department of Mechanical and Materials Engineering
Mechanics and Materials Science

EMA 3702 Quiz 1 - version B Sept. 23, 2004

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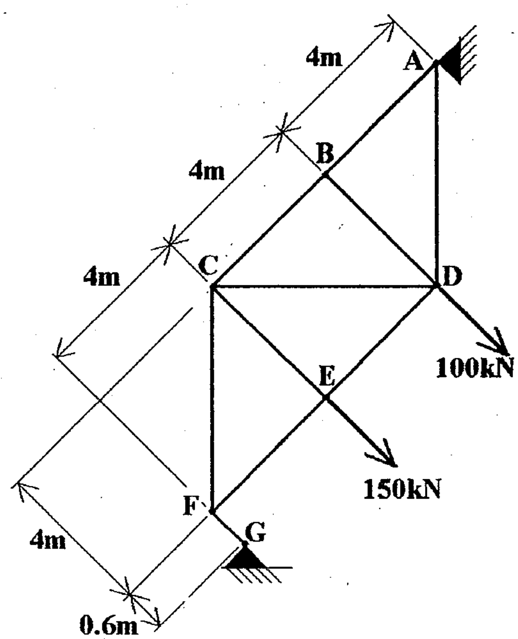
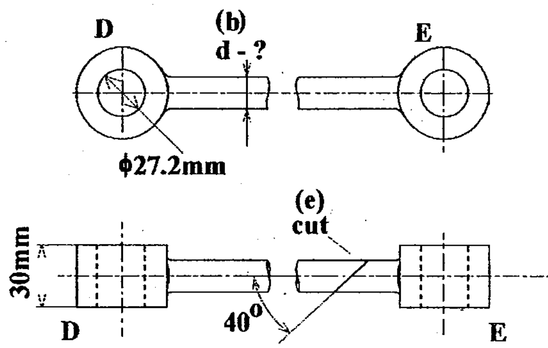




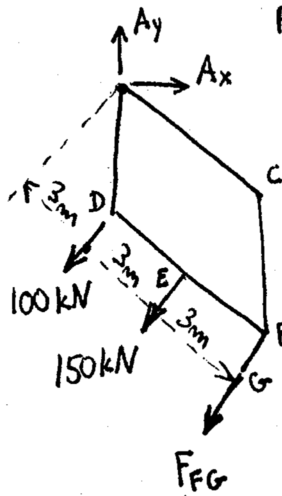
EMA 3702 - MECHANICS AND MATERIAL SCIENCE

<u>QUIZ #1 - SOLUTION</u> SEPT. 23, 2004

(a)

<u>VERSION A</u>

REACTION AT G:

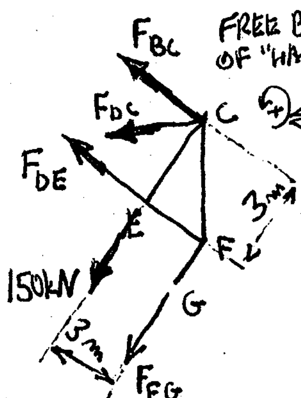


FREE BODY DIAGRAM
FG IS A BAR.
REACTION AT G MUST BE IN FG DIRECTION

$$\circlearrowleft + \sum M_A = -100\,kN \cdot 3 - 150\,kN \cdot 6 - F_{FG} \cdot 9 = 0$$

$$\underline{\underline{F_{FG} = -133.33\ kN}} \text{ (COMPRESSION)}$$

CUT OF THE TRUSS AND F_{DE} CALC.:



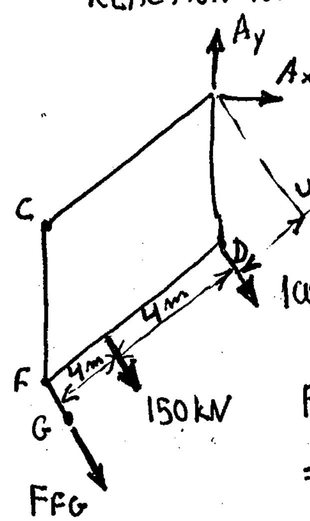
FREE BODY DIAGRAM OF "HALF" A TRUSS

$$\circlearrowleft + \sum M_C = -F_{FG} \cdot 3 - F_{DE} \cdot 3 = 0$$

$$\underline{\underline{F_{DE} = -F_{FG} = +133.33\ kN}} \text{ (TENSION)}$$

<u>VERSION B</u>

REACTION AT G:

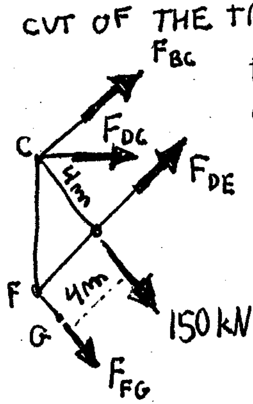


FREE BODY DIAGRAM
FG IS A BAR
REACTION AT G MUST BE IN FG DIRECTION

$$\circlearrowleft + \sum M_A = 100\,kN \cdot 4 + 150\,kN \cdot 8 + F_{FG} \cdot 12 = 0$$

$$\underline{\underline{F_{FG} = -133.33\,kN}} \text{ (COMPRESSION)}$$

CUT OF THE TRUSS AND F_{DE} CALC.:



FREE BODY DIAGRAM OF "HALF" A TRUSS

$$\circlearrowleft + \sum M_C = F_{FG} \cdot 4 + F_{DE} \cdot 4 =$$

$$\underline{\underline{F_{DE} = -F_{FG} = +133.33\,kN}} \text{ (TENSION)}$$

BOTH VERSIONS:

(b) $\sigma_u = 800\,MPa$
$F.S. = 4$

$$\left.\begin{aligned} \sigma_A &= \frac{\sigma_u}{F.S.} = \frac{F_{DE}}{A} \\ A &= \frac{\pi}{4} d_{DE}^2 \end{aligned}\right\} d_{DE} = \sqrt{\frac{4 \cdot F_{DE} \cdot F.S.}{\pi \cdot \sigma_u}} = \sqrt{\frac{4 \cdot 133.33 \cdot 10^3 \cdot 4}{\pi \cdot 800}}$$

$$\underline{\underline{d_{DE} = 29.13\,mm}}$$

(c)
$$\left.\begin{aligned} \tau_a &= \frac{F_{DE}}{2 \cdot A} = \frac{F_{DE}}{2\left(\frac{\pi d^2}{4}\right)} \\ F.S. &= \frac{\tau_u}{\tau_a} \end{aligned}\right\} F.S. = \frac{\tau_u \pi d^2}{2 F_{DE}} = \frac{460\,MPa \cdot \pi \cdot 27.2^2}{2 \cdot 133.33 \cdot 10^3}$$

$$\underline{\underline{F.S. = 4}}$$

(d) $\underline{\underline{\sigma_b}} = \dfrac{F_{DE}}{A_b} = \dfrac{133.33 \cdot 10^3}{27.2 \times 30} = \underline{\underline{163.4\ MPa}}$

(e)
$$\sigma_\theta = \frac{P}{A_0} \cos^2\theta \qquad A_0 = \frac{\pi d_{DE}^2}{4} = \frac{\pi \cdot 29.13^2}{4} = 666.46\ mm^2$$

$$\tau_\theta = -\frac{P}{2 A_0} \sin(2\theta) \qquad \theta = 90^\circ - 40^\circ = 50^\circ$$

$$\underline{\underline{\sigma_\theta}} = \frac{133.33 \cdot 10^3 \cdot \cos^2 50^\circ}{666.46\,mm^2} = \underline{\underline{82.66\ MPa}}$$

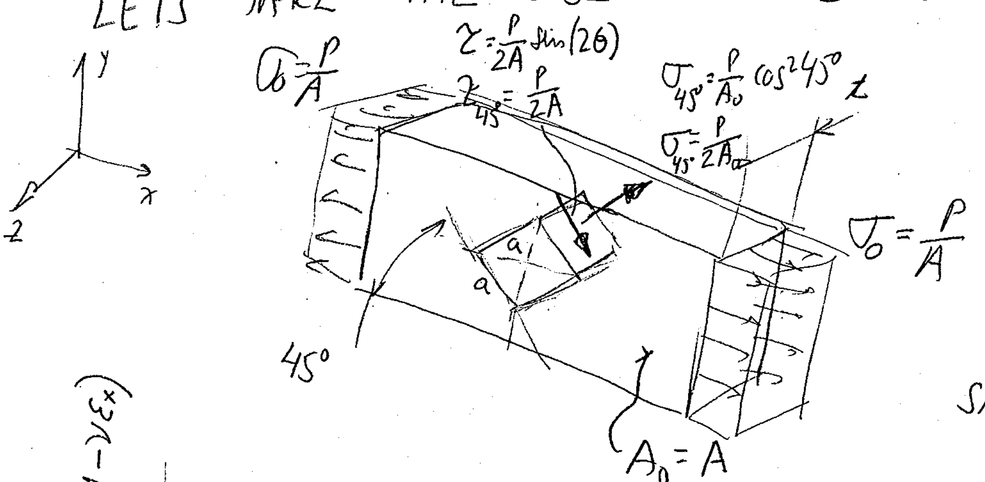
$$\underline{\underline{\tau_\theta}} = -\frac{133.33 \cdot 10^3 \cdot \sin 100^\circ}{2 \cdot 666.46} = \underline{\underline{-98.51\,MPa}}$$

(2.26)

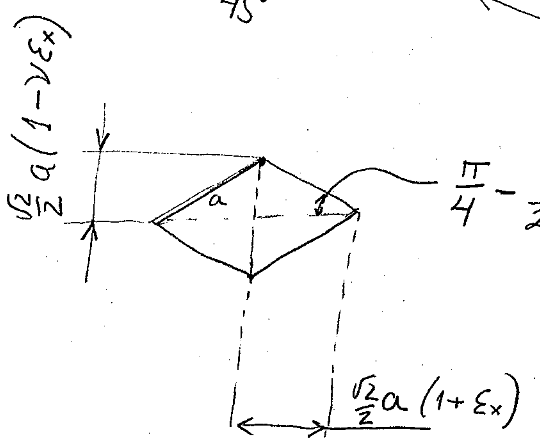
RELATION AMONG E, ν AND G

E, ν, G ARE PROPERTIES OF A SPECIFIC MATERIAL
IT CAN BE FOUND AN ALGEBRIC RELATION BETWEEN THESE CONSTANT BECAUSE ν CONNECTS DEFORMATIONS ON DIFFERENT DIRECTIONS

LETS TAKE THE CASE WE DEVELOPED FOR $\underbrace{\text{}}_{\text{AN}}$ $\overbrace{\text{OBLIQUE PLANE}}^{\text{STRESSES ON,}}$:



SMALL DEFORMATIONS!



$$\tan\left(\frac{\pi}{4} - \frac{\gamma_{45°}}{2}\right) = \frac{\frac{\sqrt{2}}{2}a(1-\nu\varepsilon_x)}{\frac{\sqrt{2}}{2}a(1+\varepsilon_x)}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\frac{\tan\frac{\pi}{4} - \tan\frac{\gamma_{45°}}{2}}{1 + \tan\frac{\pi}{4}\tan\frac{\gamma_{45°}}{2}} = \frac{1-\nu\varepsilon_x}{1+\varepsilon_x}$$

RELATION BETWEEN THE DEFORMATIONS

$$\frac{1 - \frac{\gamma_{45°}}{2}}{1 + \frac{\gamma_{45°}}{2}} = \frac{1-\nu\varepsilon_x}{1+\varepsilon_x}$$

ALGEBRA

ASSUMPTION $1 \gg \frac{1-\nu}{2}\varepsilon_x$

$$\gamma_{45°} = \frac{\varepsilon_x(1+\nu)}{1 + \frac{1-\nu}{2}\varepsilon_x} \approx \varepsilon_x(1+\nu)$$

$$\left.\begin{aligned} \varepsilon_x &= \frac{\sigma_0}{E} = \frac{P}{AE} \\ \gamma^m_{45^\circ} &= \frac{\tau_{45^\circ}}{G} = \frac{P}{2AG} \end{aligned}\right\} \quad \gamma^m_{45^\circ} = \frac{P}{2AG} = \frac{P}{AE}(1+\nu)\ \left(= \varepsilon_x(1+\nu)\right)$$

$$\boxed{\frac{E}{2G} = 1+\nu}$$

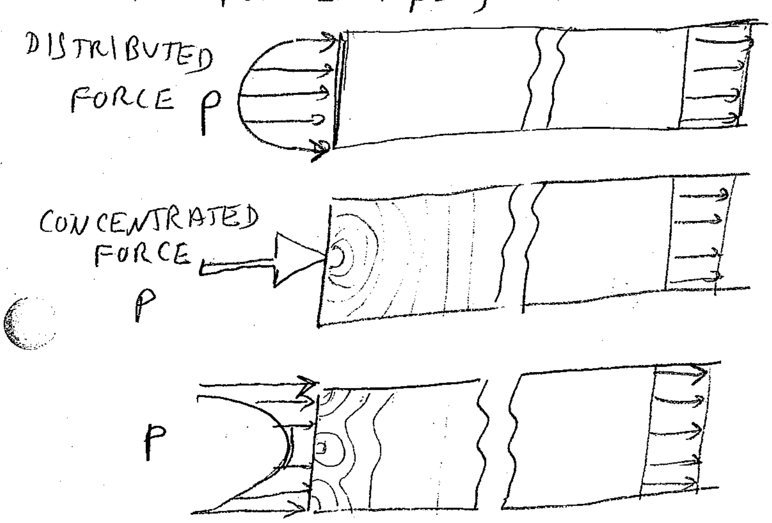
$$\boxed{G = \frac{E}{2(1+\nu)}}$$

STRESS AND STRAIN DISTRIBUTION UNDER AXIAL LOADING, SAINT-VENANT'S PRINCIPLE

BARRE de SAINT-VENANT (1797-1886), A FRENCH MATHEMATICIAN, INVESTIGATED THE DIFFERENCE BETWEEN THE THEORETICAL STRESS DISTRIBUTIONS TO THE ACTUAL (BY OBSERVING LOCALIZED DISTORTIONS) AND STATED:

"AT A POINT FAR ENOUGHT FROM THE APPLIED LOADS ACTION THE THE ACTUAL STRESS FOLLOWS THE THEORETICAL CALCULATED STRESS FOR THE TOTAL STATIC LOADING"

FOR EXAMPLE, FOR UNIAXIAL LOADING



① STATICALLY EQUIVALENT PROBLEMS

\+

② FAR ENOUGHT FROM THE POINTS OF APPLICATION OF THE LOADS

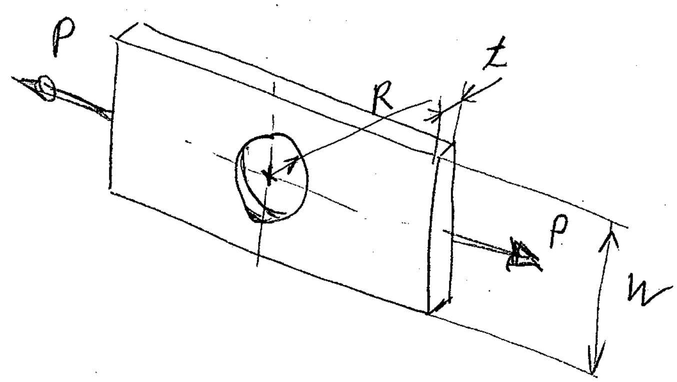
||

THE SAME FIELD OF STRESSES

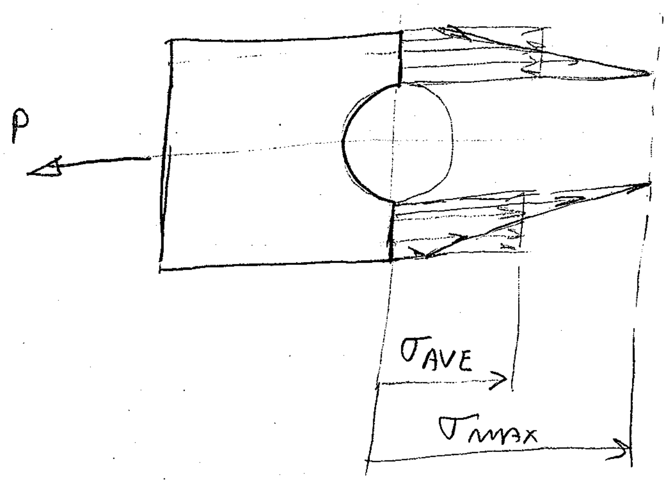
STRESS CONCENTRATION

DISCONTINUITY OF THE SECTION AREA CAUSES AN EFFECT CALLED "STRESS CONCENTRATION"

LETS TAKE A FLAT BAR WITH A CENTRAL HOLE, A UNIAXIAL LOAD APPLIED



WE LEARNED TO CALCULATE THE AVERAGE NORMAL STRESS AS $\sigma_{AVE} = \dfrac{P}{A_{MIN}} = \dfrac{P}{(W-2R)\,t}$



BUT THE REAL SITUATION IS WORST! BECAUSE OF THE DISCONTINUITY OF THE SECTION ARE, WE GET HIGHER STRESS NEAR THE HOLE AND LOWER AT THE EDGES.

THE RATIO $K = \dfrac{\sigma_{MAX}}{\sigma_{AVE}}$ IS DEFINED AS

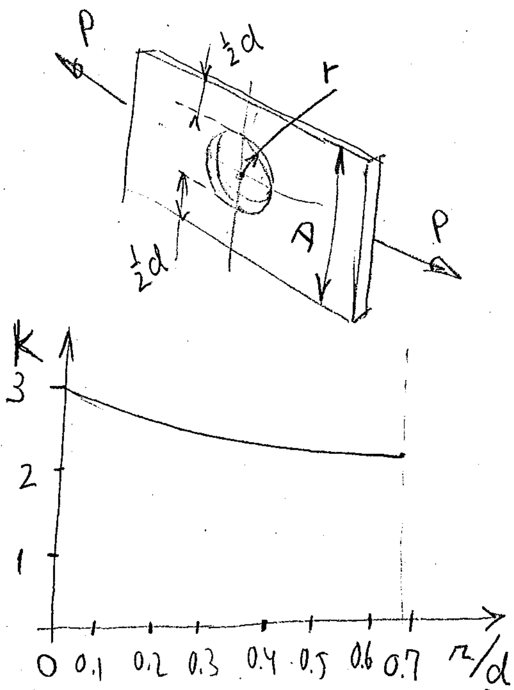
THE STRESS CONCENTRATION RATIO

FOR SPECIFIC CASES, K IS DEVELOPED USING HOOKE'S LAW AND GENERAL EQUATIONS, IN MORE ADVANCED COURSES THAT DEAL WITH "THEORY OF ELASTICITY"

WE LEARN ABOUT STRESS CONCENTRATION IMPLEMENTATION BECAUSE IT IS A SIGNIFICANT EFFECT. K MAY REACH HIGH VALUES OF, 3; 4 OR EVEN 5 IN SPECIFIC CASES

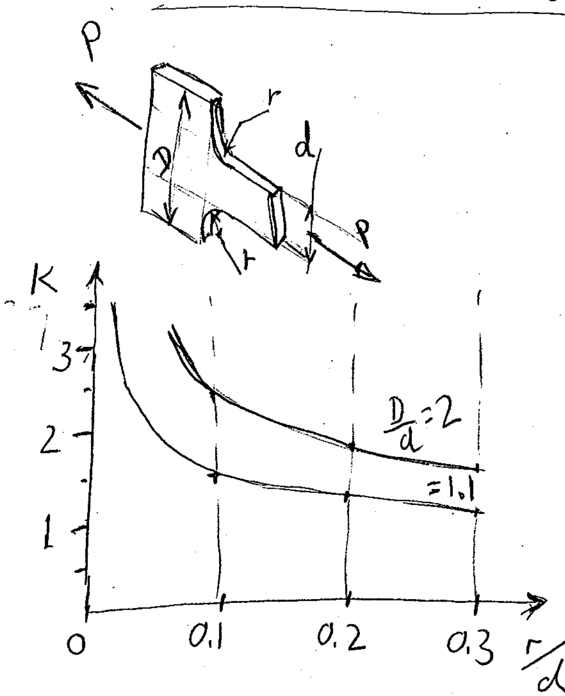
THE TEXT BOOK EXPLAINS ABOUT TWO CASES

FLAT BARS WITH HOLE



(TEXT BOOK, PAGE 108)

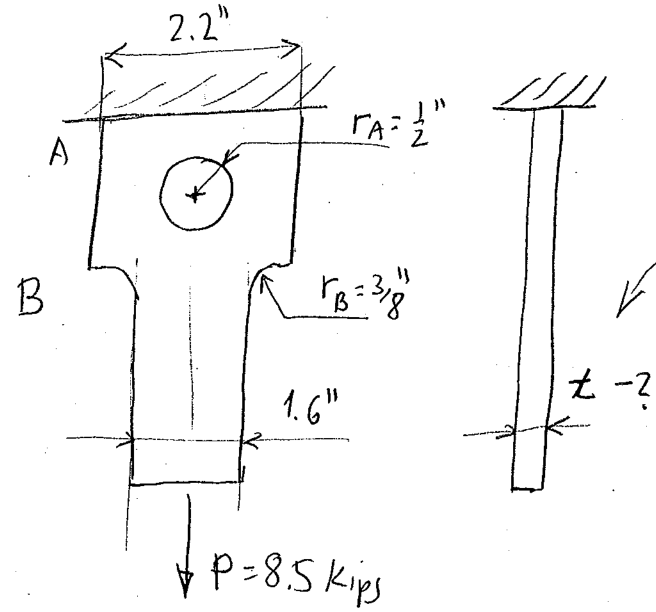
FLAT BAR WITH FILLETS



CONCLUSION: THE STRESS-CONCENTRATION FACTOR IS BIGGER FOR LOWER r'S. A GOOD DESIGN SHOULD AVOID SHARP CORNERS

READ EXAMPLE 2.12, PAGE 108

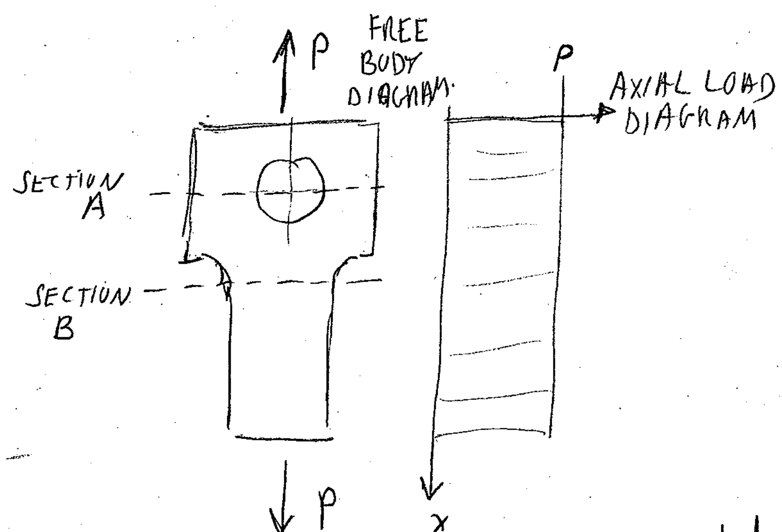
EXAMPLE - PROB 2.98



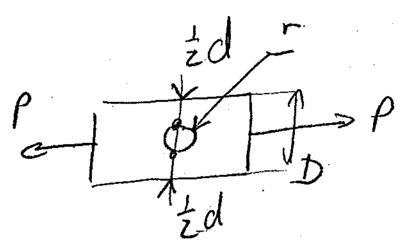
$\sigma_{ALL} = 18\,ksi$

UNIAXIAL LOAD

1. FREEBODY DIAGRAM
2. AXIAL LOAD DIAGRAM
3. IDENTIFY MAXIMUM CONCENTRATION STRESS SECTIONS (A, B)
4. THE PROBLEM ASSUMES THAT THE WALL, SEC A, SEC B AND P ARE "FAR ENOUGHT" SO THERE IS NOT MUTUAL INFLUENCE ON STRESS FIELD BETWEEN THEM (SAINT VENANT'S PRINCIPLE



SECTION A



DEFINITIONS IN ORDER TO USE THE TABLE ON PAGE 108

$d = D - 2r = 2.2" - 1" = 1.2"$

$\frac{r}{d} = \frac{1/2"}{1.2"} = 0.4166$

↓

GRAPH, PAGE 108

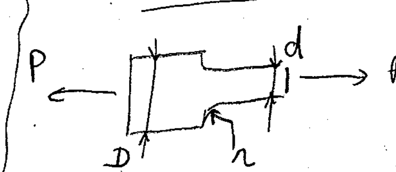
↓

$K \approx 2.25$

$\sigma_{A\,AVE} = \frac{P}{dt} =$

$\sigma_{A\,MAX} = K \cdot \sigma_{A\,AVE} = 2.25 \cdot \frac{8.5}{1.2 \cdot t} = \frac{15.938}{t}$ KSI

SECTION B



$\left.\begin{array}{l} d = 1.6" \\ D = 2.2" \end{array}\right\} \frac{D}{d} = 1.375$

$\frac{r}{d} = \frac{3/8}{1.6} = 0.2344$

GRAPH, PAGE 108

$K \cong 1.75$

$\sigma_{B\,AVE} = \frac{P}{dt}$

$\sigma_{B\,MAX} = K \cdot \sigma_{B\,AVE} = 1.75 \cdot \frac{8.5}{1.6 \cdot t} = \frac{9.3}{t}$ KSI

(2.31)

WE GOT $\sigma_{A\,max} > \sigma_{B\,max}$

THE MAXIMUM STRESS ON SECTION AREA A IS BIGGER THAN IN SECTION AREA B, SO t IS TO BE CALCULATED FOR SECTION A:

$$\sigma_{A\,max} = \sigma_{ALL} = \frac{15.938}{t} = 18 \text{ KSI}$$

$$\underline{t = 0.885 \text{ in}}$$

HOMEWORK #3

$\underbrace{2.45}_{\text{INDETERMINATE}}$, 2.65, 2.69, 2.80, 2.100, 2.127, 2.134

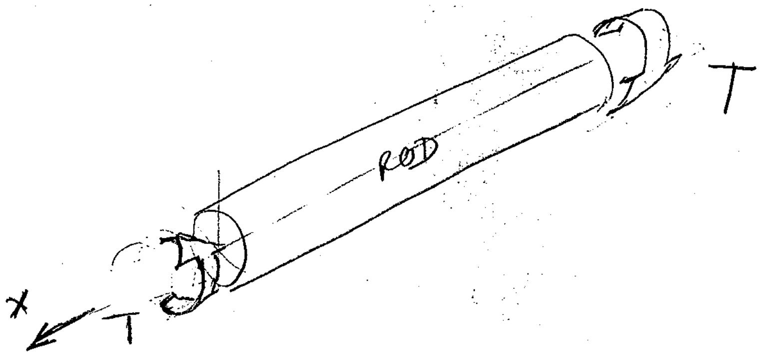
POTN (10 (c))

MULTI AXIAL

G

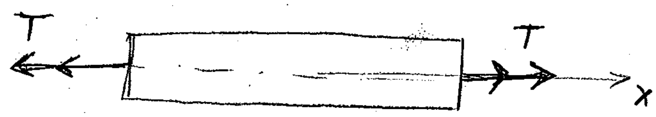
STR. CONC.

TORSION



TORQUE - A MOMENT APPLIED THOUGHT THE MAIN AXIS OF A "LONG" PART - X AXIS

A PART UNDER TORSION. WHEN A TORQUE IS APPLIED

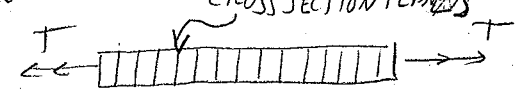


T - N·m, lb·in, etc.

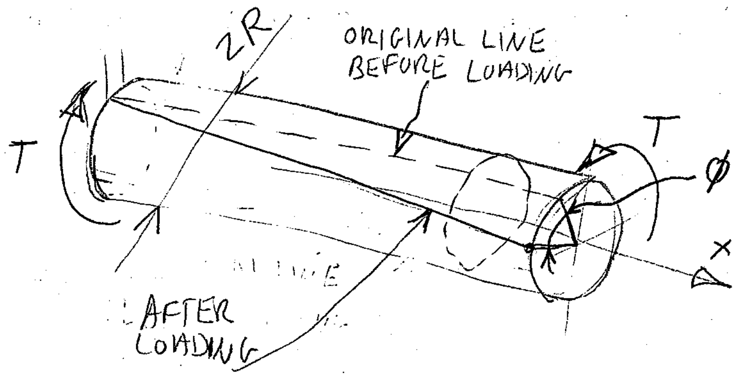
$$\sum M_x = 0 = T - T$$

PROPERTY OF CIRCULAR SHAFTS:
AXI-SYMETRIC SECTION

WHEN A CIRCULAR SHAFT IS SUBJECTED TO TORSION, EVERY CROSS SECTION REMAINS PLANE AND UNDISTORTED, (PURE SHEARING)



THE CROSS SECTION PLANES ROTATE AROUND X AXIS AT DIFFERENT ANGLES.



ANGLE ϕ - ANGULAR DEFORMATION DUE TO TORSION, ANGLE OF TWIST

THE TORQUE IS CONSTANT ALONG X

THE SECTION AREA IS CONSTANT ALONG X

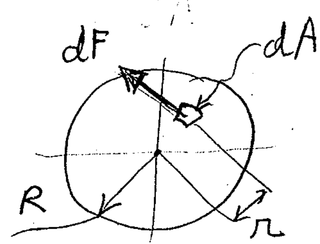
SO $\dfrac{d\phi}{dx}$ = CONSTANT

(THE CHANGE IN ϕ IS CONSTANT ALONG.

LET LOOK AT THE SECTION:

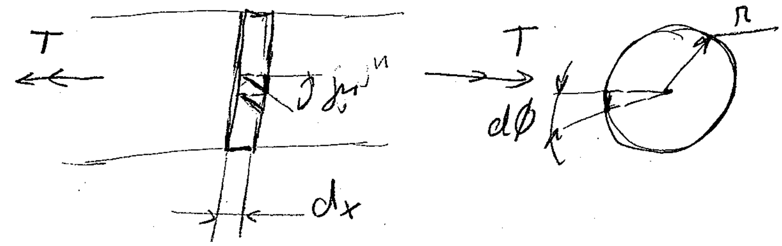
$$dT = dF \cdot r$$

$$T = \int dF \cdot r$$



(3,2)

$$T = \int \tau \, dA \cdot r$$



(1) $\tau = \gamma \cdot G$

ARC COMPARISSON

$dx \tan\gamma = r \cdot d\phi$

FOR SMALL γ:

(2) $\gamma \approx r \dfrac{d\phi}{dx}$

(1)+(2) $\tau = r \dfrac{d\phi}{dx} \cdot G$ (*) { PUT A SIGN BECAUSE WE WILL USE THIS EQUATION LATER

$$T = \int r^2 \frac{d\phi}{dx} G \, dA$$

CONSTANT

$$T = \frac{d\phi}{dx} \cdot G \int r^2 dA$$

$$T = \frac{\tau}{r} \int^R r^2 dA$$

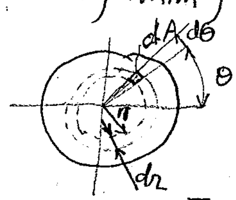
$$\tau = \frac{T \cdot r}{\int r^2 dA}$$

→ POLAR MOMENT OF INERTIA

FEATURE OF THE SECTION AREA

J OR I_0

m^4, mm^4, in^4



FOR A CIRCULAR SECTION AREA:

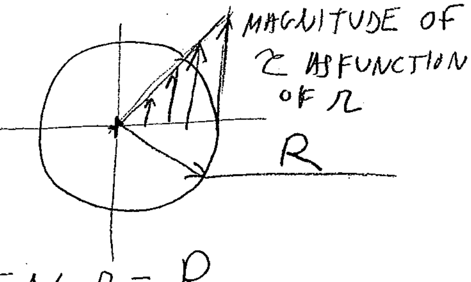
$$dA = dr \cdot r \, d\theta$$

$$J = \int r^2 \, dr \cdot r \cdot d\theta = \int_0^{2\pi} \int_0^R r^3 \, dr \, d\theta$$

$$J = \int_0^{2\pi} d\theta \int_0^R r^3 dr = 2\pi \frac{R^4}{4} = \frac{1}{2} \pi R^4$$

$$\boxed{\text{FOR A CIRCULAR SECTION AREA: } J = \frac{1}{2}\pi R^4 = \frac{1}{32}\pi D^4}$$

$$\tau = \frac{T \cdot r}{J}$$

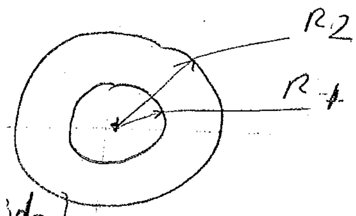


THE MAXIMUM MAGNITUDE OF τ IS ACHIVIED AT THE OUTER SURFACE, WHEN $r = R$

THEN $\tau_{MAX} = \frac{T \cdot R}{J}$; $(\tau_{min} = 0)$

AN ADDITIONAL AXI-SYMETRIC CROSS SECTION IS A RING

THE ANALYSIS IS EXACTLY THE SAME,

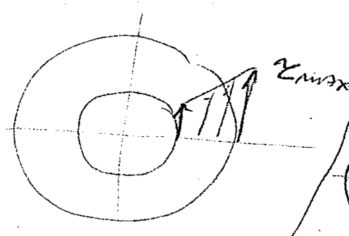


BUT

$$J = \int_0^{2\pi}\int_{R_1}^{R_2} r^3 dr\, d\theta = \int_0^{2\pi} d\theta \left\{ \int_0^{R_2} r^3 dr - \int_0^{R_1} r^3 dr \right\}$$

$$J = J_2 - J_1 = \frac{1}{2}\pi\left(R_2^4 - R_1^4\right)$$

$$\tau_{max} = \frac{T R_2}{J} \qquad \tau_{min} = \frac{T \cdot R_1}{J}$$



FOR A THIN TUBE

$R_2 = R_1 + t$

$J_0 = \frac{1}{2}\pi\left((R_1+t)^4 - R_1^4\right)$

$J_0 = \frac{1}{2}\pi\left(R_1^4 + 4R_1^3 t + 6R_1^2 t^2 + 4R_1 t^3 + t^4 - R_1^4\right)$

$R_1 >> t$

SO t^2 AND UP ARE NEGLECTED

$J_0 \approx \frac{1}{2}\pi\left(4R_1^3 t\right)$

$$\boxed{J_0 \approx 2\pi R^3 t}$$

ERROR $< \frac{3}{2}\frac{t}{R}$

$t = 2\% R$

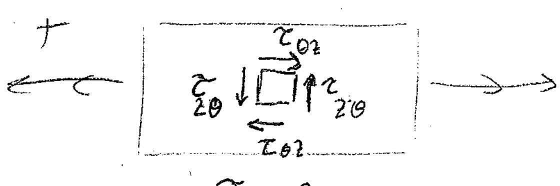
ERROR $< 3\%$

$R_1 \approx R_2 \approx R$

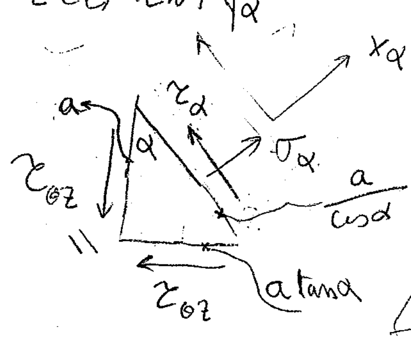
LEARN EXAMPLE 3.01 IN YOUR TEXTBOOK, page 141

3.1 page 143, 3.2 page 144

LETS OBSERVE AN MATERIAL ELEMENT



$\tau_{z\theta} = \tau_{\theta z}$



$$\Sigma F_{x_\alpha} = \sigma_\alpha \cdot \frac{a}{\cos\alpha} - \tau_{\theta z} \cdot a \cdot \sin\alpha - \tau_{\theta z} \cdot a \tan\alpha \cdot \cos\alpha = 0$$

$$\frac{1}{\cos\alpha}\sigma_\alpha = \tau_{\theta z}\left(\sin\alpha + \sin\alpha\right)$$

$$\sigma_\alpha = \tau_{\theta z}\left(2\cos\alpha\sin\alpha\right)$$

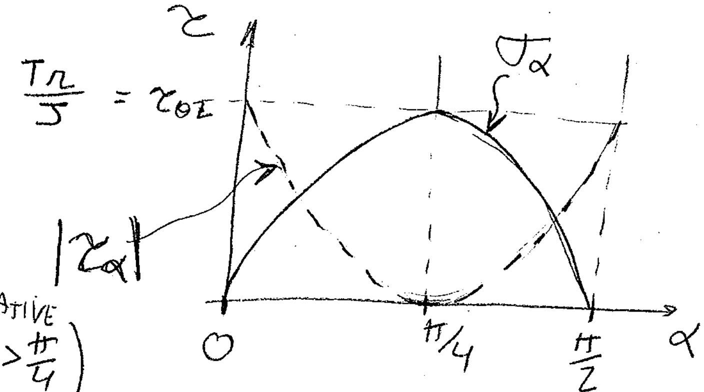
$$\boxed{\sigma_\alpha = \tau_{\theta z}\sin 2\alpha}$$

(3.4)

$$\sum F_{y_\alpha} = \tau_\alpha \cdot \frac{a}{\cos\alpha} - \tau_{\theta z} \cdot a \cdot \cos\alpha + \tau_{\theta z} \cdot a \cdot \tan\alpha \cdot \sin\alpha = 0$$

$$\tau_\alpha = \tau_{\theta z} \cos\alpha \left(\cos\alpha - \frac{\sin^2\alpha}{\cos\alpha}\right) = \tau_{\theta z}\left(\cos^2\alpha - \sin^2\alpha\right)$$

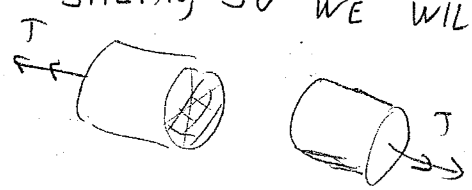
$$\boxed{\tau_\alpha = \tau_{\theta z} \cos(2\alpha)}$$



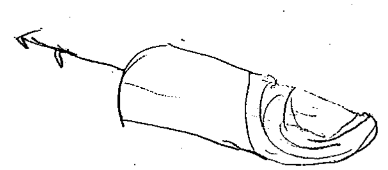
(BECOMES NEGATIVE FOR $\frac{\pi}{2} > \alpha > \frac{\pi}{4}$)

CONCLUSSIONS:

1. $\alpha = 0$, $\sigma_0 = 0$, $\tau_0 = \tau_{rz}$, ORIGINAL SITUATION
2. $\alpha = \frac{\pi}{4}$, $\sigma_{\frac{\pi}{4}} = \tau_{\theta z}$, $\tau_{\frac{\pi}{4}} = 0$, MAXIMUM NORMAL STRESS, NO SHEARING AT $\frac{\pi}{4}$! (PRINCIPAL STRESSES)
3. $\alpha = \frac{\pi}{2}$, $\sigma_{\frac{\pi}{2}} = 0$, $\tau_{\frac{\pi}{2}} = -\tau_{rz}$, ORIGINAL SITUATION
4. DUCTILE MATERIALS GENERALLY FAILS BECAUSE OF SHEAR, SO WE WILL OBSERVE A NORMAL CUT



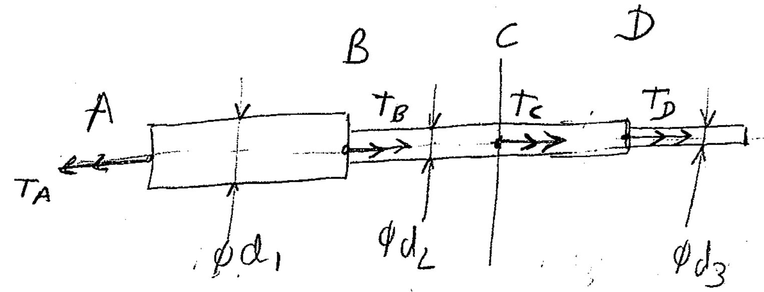
FOR BRITLE MATERIALS, GENERALLY FAILS DUE TO NORMAL STRESSE, WE WILL OBSERVE A 45° CLIMBING (SPIRAL) SURFACE



(3.5)

EXAMPLE 3.13, 3.14, I CHANGED LOADS

FREE BODY DIAGRAM



DATA:

$T_A = 2.4$ kN·m

$T_B = 1.0$ kN·m

$T_C = 0.8$ kN·m

$T_D = 0.6$ kN·m

$d_1 = 54$ mm

$d_2 = 46$ mm

$d_3 = 40$ mm

(a) τ_{AB} - ?

(b) τ_{BC} - ?

(c) τ_{CD} - ?

(d) ϕd_{BC} FOR MAX$(\tau_{AB}, \tau_{BC}, \tau_{CD})$

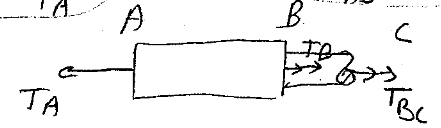
REACTION T_A, GIVEN IN PROBLEM

$$\Sigma M_x = -T_A + T_B + T_C + T_D = 0$$

$$-2.4 + 1.0 + 0.8 + 0.6 = 0 \ \checkmark$$

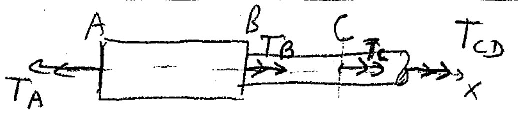
$$\Sigma M_x = T_{AB} - T_A = 0$$

$$\underline{T_{AB} = T_A = 2.4 \text{ kN·m}}$$



$$\Sigma M_x = -T_A + T_B + T_{BC} = 0$$

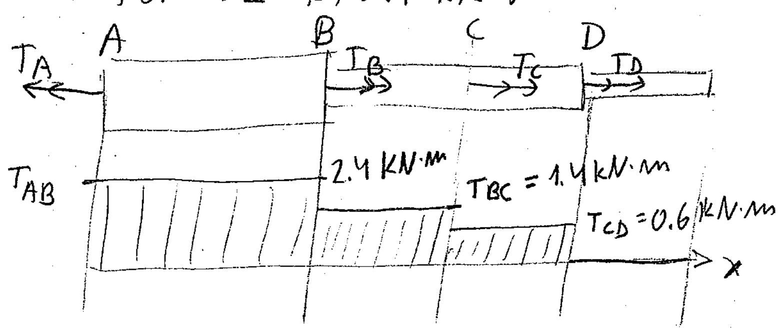
$$\underline{T_{BC}} = T_A - T_B = \underline{1.4 \text{ kNm}}$$



$$\Sigma M_x = -T_A + T_B + T_C + T_{CD} = 0$$

$$T_{CD} = 2.4 - 1.0 - 0.8 = 0.6 \text{ kN·m}$$

TORQUE DIAGRAM:



(a) $$\tau_{AB} = \frac{T_{AB} R_{AB}}{J_{AB}}$$

$$R_{AB} = \frac{d_1}{2} = 27 \text{ mm}$$

$$J_{AB} = \frac{1}{2}\pi R_{AB}^4 = \frac{1}{2}\pi 27^4 = 8.348 \cdot 10^5 \text{ mm}^4$$

$$\tau_{AB} = \frac{2.4 \cdot 10^6 \text{ N·mm} \cdot 27 \text{ mm}}{8.348 \cdot 10^5 \text{ mm}^4} = \underline{\underline{77.62 \text{ MPa}}}$$

(b) $$\tau_{BC} = \frac{T_{BC} R_{BC}}{J_{BC}}$$

$$R_{BC} = \frac{d_2}{2} = 23 \text{ mm}$$

$$J_{BC} = \frac{1}{2}\pi R_{BC}^4 = \frac{1}{2}\pi 23^4 = 4.396 \cdot 10^5 \text{ mm}^4$$

$$\tau_{BC} = \frac{1.4 \cdot 10^6 \text{ N·mm} \cdot 23 \text{ mm}}{4.396 \cdot 10^5 \text{ mm}^4} = \underline{\underline{73.25 \text{ MPa}}}$$

EMA 3702

Oct 4, 2004		HM # 1	Quiz 1	HM # 2	
Adriana Bonilla	1321865	1	28	1	
Alaa Maaliki	150-3723	1	70	1	
Alfredo Suarez	1495210		64		
Amrit Harripaul	1644446	1	73	1	
Andres Zamorra	1397908	1	94	1	
Anthony Jackson-Pownal	1277169	1	60	1	
Arias Carlos	1371993	1	73	1	
Cindy Gomez	1390408	1	80	1	
Damian Harper	1399737	1	56	1	
Damien Hoyd	1013644	1	98	1	
Daniel Mugruza	1258280	1	50	1	
Eric Inclan	1356002	1	68	1	
Ernesto Gutierrez	1281667		70	1	
Felipe Rendon	1334090	1	50	1	
Francis Fernandez	1026909	1	76		
Georgette Martinez	1339663	1	59		
Gonzalo Ocano	1021312	1	87	1	
Grant Mesner	1366034	1	72	1	
Gustavo Barbera	1398821		77	1	
Gustavo Jaramillo	1397120	1	84	1	
Hector Roos	109-63-68	1			
Jorge Tercero	1014132	1	77	1	
Joseph Nichols	1373806	1	62		
Juan Saluzzo	1372735	1	88		
Kern Wilson	1589330	1	76	1	
Luis A Sepulueda	1349774	1	31	1	
Luke Seever	1106526	1	100	1	
Marcelo Beyra	1370183	1	63		
Maribel Garcia	1322452	1	51	1	
Mario Valdivieso	1369320	1	68	1	
Max Brand	139-68-96	1	83	1	
Michael Patterson	1094554	1	100		
Michael Wolff	1103264	1	90		
Natalie Marshal	1306053	1	77	1	
Ottley Brathwaite	1630841	1	19		
Patrick Belizane	1629148	1	66	1	
Paul Girata	1282007	1	8	1	
Robert Jordan	1305753	1	70	1	
Ruben Galeano	1352635	1	64	1	
Saleh Almutawa	1323191		78		
Shaka Harper	1012782	1	63	1	
Trevis Moorley	1399021	1	67	1	
Trevor G. S. Harristo	1395585	1	75	1	
Verdi M. Mayer	1358420	1	73	1	
William Betancourt	1398230	1	10	1	
Yuri Lazzeretti	1093801		66	1	
		41	66.97778	35	

(3.6)

$$\tau_{CD} = \frac{T_{CD} \cdot R_{CD}}{J_{CD}} \quad \left\{ \begin{array}{l} R_{CD} = \frac{d_2}{2} = 23\,mm \ (= R_{BC}) \\ J_{CD} = J_{BC} = 4.396 \cdot 10^5\,mm^4 \end{array} \right.$$

$$\tau_{CD} = \frac{0.6 \cdot 10^6\,N{\cdot}mm \cdot 23\,mm}{4.396 \cdot 10^5\,mm^4} = \underline{\underline{31.39\,MPa}}$$

(d) $\phi\, d_{BC}$ for $\max\{\tau_{AB}, \tau_{BC}, \tau_{CD}\}$

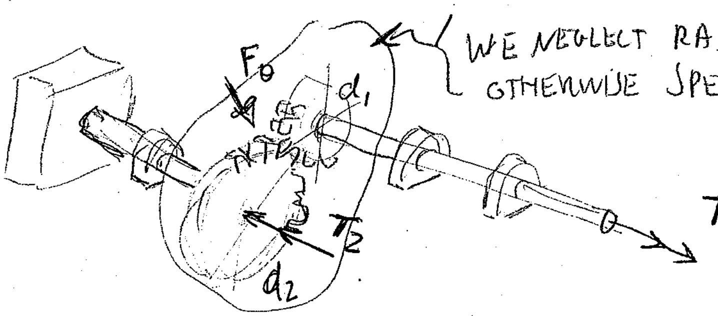
$$\left. \begin{array}{l} \tau_{max} = 77.62\,MPa \\ T_{BC} = 1.4\,kN{\cdot}m \end{array} \right\} \quad \tau_{max} = \frac{T_{BC} \cdot R_{NEW}}{J_{NEW}} = \frac{T_{BC} \cdot \cancel{R_{NEW}}}{\frac{1}{2}\pi R_{NEW}^{4 \to 3}}$$

$$R_{NEW}^3 = \frac{T_{BC}}{\frac{1}{2}\pi\,\tau_{max}} = \left(\frac{1.4 \cdot 10^6}{\frac{1}{2}\pi\ 77.62}\right)^{\frac{1}{3}} = 22.56$$

$$\underline{d_{NEW} = 45.12\,mm}$$

SHOW BEFORE PROB. 3.47

PROBLEMS WITH GEARS:



WE NEGLECT RADIAL LOADS OTHERWISE SPECIFIED

THE RELATIONS BETWEEN T_1, T_2 DUE TO THE GEAR REDUCTION:

$$\frac{T_1}{R_1} = \frac{T_2}{R_2} = F_\theta$$

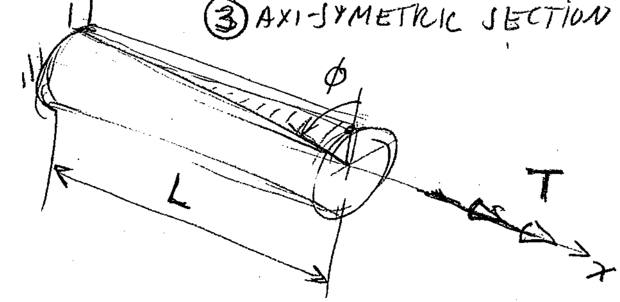
$$\frac{T_1}{d_1} = \frac{T_2}{d_2}$$

(3.7)

ANGLE OF TWIST IN THE ELASTIC RANGE

ASSUMPTIONS:
1. ELASTIC RANGE
2. HOMEGENEOUS, ISOTROPIC CONTINUOUS MATERIAL
3. AXI-SYMETRIC SECTION

LET RECALL (*) $\tau = r \dfrac{d\phi}{dx} \cdot G$



$$d\phi = \frac{\tau}{G r} dx$$

$$\tau = \frac{T r}{J}$$

$$d\phi = \frac{T\,dx}{GJ}$$

ASSUMING T, G, J constant

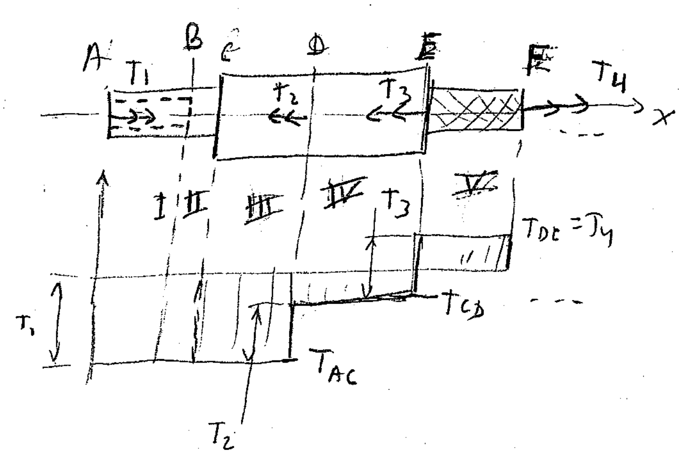
$$\boxed{\phi = \int_0^L \frac{T\,dx}{GJ} = \frac{T \cdot L}{G \cdot J}}$$

APPLICATIONS
- TORSION SPRINGS
-

UNITS

	τ	T	J	G	ϕ
SI	MPa	N·mm	mm^4	MPa	rad
	Pa	N·m	m^4	Pa	rad
US	PSI	lb·in	in^4	PSI	rad
	kSI	kips·in	in^4	kSI	rad

FOR A MULTIPLE $\overbrace{\text{CASE}}^{\text{DISCRETE}}$:



$$\phi = \sum \frac{T_i L_i}{J_i G_i}$$

$$= \frac{T_I L_I}{J_I G_I} + \frac{T_{II} L_{II}}{J_{II} G_{II}} + \ldots\ldots$$

FOR A GENERAL CONTINOUS CHANGING CASE:

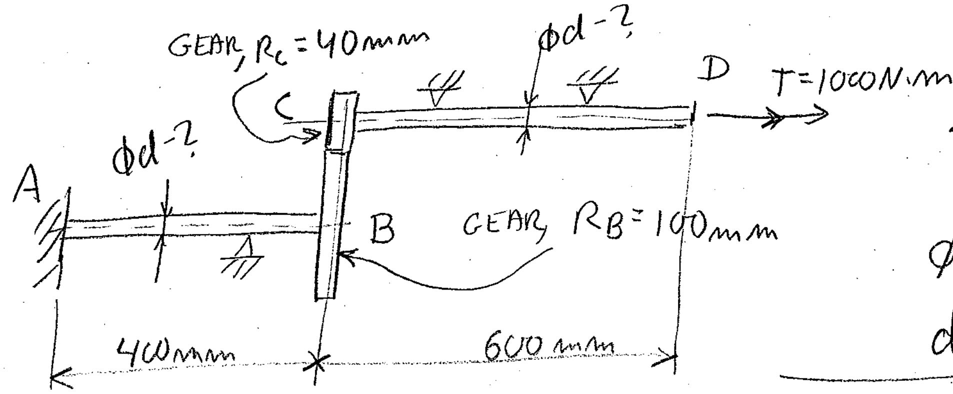
$$\phi = \int_0^L \frac{T\,dx}{JG} \quad \text{FOR } T(x),\ J(x)$$

$$(G(x) - ?)$$

WE WILL SOLVE INDETERMINATE PROBLEMS, USING THE TWIST ANGLE OF THE SHAFT FOR THE CONSTRAIN EQUATION

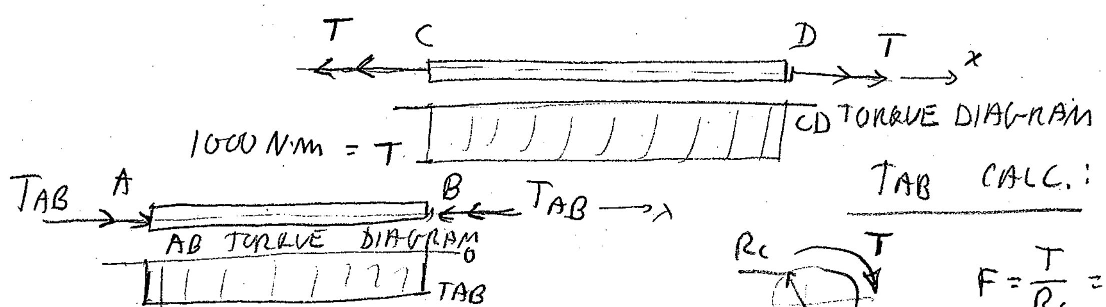
SEE SAMPLE PROBLEM 3.4 IN YOUR TEXTBOOK PAGE 156

PROBLEM 3.47

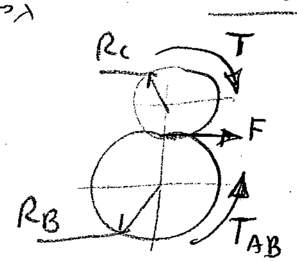


$\tau_{max} = 60$ MPa
$G = 77$ GPa
$\phi_{D\,max} = 1.5°$
d - ?

FREE BODY DIAGRAM



TAB CALC.:



$$F = \frac{T}{R_C} = \frac{T_{AB}}{R_B}$$

$$T_{AB} = \frac{R_B}{R_C} T = \frac{100}{40} \cdot 1000 \text{ N·m}$$

$$T_{AB} = 2500 \text{ N·m}$$

STRESS REQ.:

$$60\text{MPa} \geq \tau_{ACT}\big|_{max} = \frac{T|_{max} \cdot R}{J}$$

$$J = \frac{\pi d^4}{32}$$

$$R = \frac{d}{2}$$

$$T|_{max} = T_{AB} = 2500 \text{N·m}$$

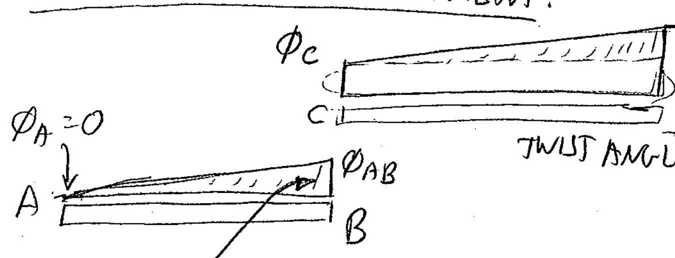
$$60\text{MPa} \geq \frac{2500 \cdot 10^3 \text{N·mm} \cdot 16}{\pi d^3}$$

$$d \geq \sqrt[3]{\frac{2500 \cdot 10^3 \cdot 16}{\pi \cdot 60}}$$

$$\boxed{d \geq 59.65 \text{ mm}}$$

(3.9)

TWIST ANGLE REQUIREMENT:



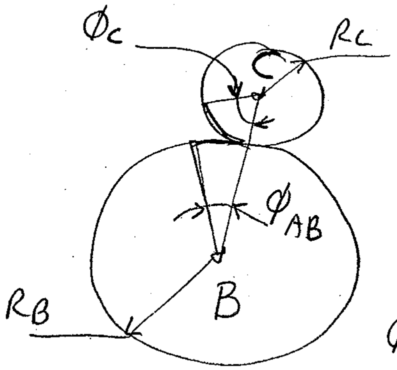
$\phi_D = \phi_C + \phi_{DC}$

TWIST ANGLE DIAGRAM

$$\phi_{AB} = \frac{T_{AB} \cdot L_{AB}}{J G}$$

$$\phi_{DC} = \frac{T \cdot L_{CD}}{J \cdot G}$$

ϕ_C IS TRANSFERED BY THE GEAR, DUE TO ϕ_{AB}. THE GEOMETRIC RELATION AS FOLLOWS:



EQUAL ARCS:

$$R_C \phi_C = R_B \phi_{AB}$$

$$\phi_C = \frac{R_B}{R_C} \cdot \phi_{AB}$$

$$\phi_C = \frac{R_B}{R_C} \cdot \frac{T_{AB} L_{AB}}{J G}$$

$$1{,}5^\circ \geq \phi_D = \phi_C + \phi_{DC} = \frac{R_B}{R_C} \frac{T_{AB} \cdot L_{AB}}{J G} + \frac{T \cdot L_{CD}}{J \cdot G} = \frac{1}{J \cdot G}\left(\frac{R_B}{R_C} T_{AB} L_{AB} + T \cdot L_{CD}\right)$$

$$\rightarrow 0.02618 \text{ rads} \geq \frac{32}{\pi d^4} \cdot \frac{1}{77 \cdot 10^3 \text{ MPa}} \left(\frac{100}{40} 2500 \cdot 10^3 \text{ N} \cdot \text{mm} \cdot 400 \text{ mm} + 1000 \cdot 10^3 \text{ N} \cdot \text{mm} \cdot 600 \text{ mm}\right)$$

$d \geq 62.91 \text{ mm}$

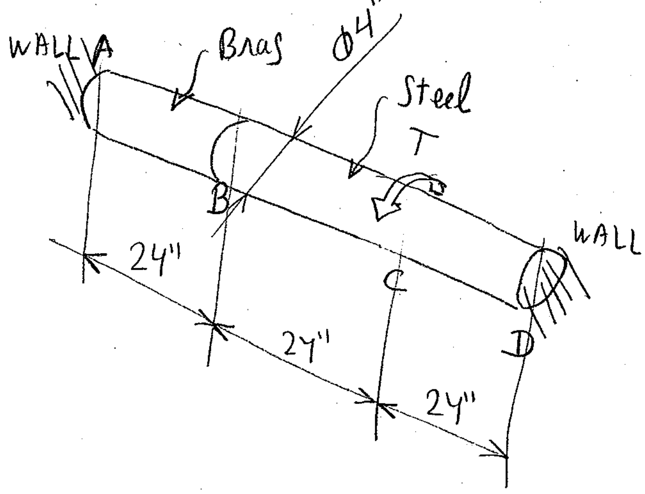
STRESS REQ. – $d \geq 54{,}65$ mm
TWIST ANG. REQ – $d \geq 62.91$ mm

$\left.\right\}$ $d \geq 62.91$ mm

REVIEW EXAMPLE 3.04, 3.05 AND SAMPLES 3.3, 3.4,

EXAMPLE

TORSION - INDETERMINATE PROBLEM



DATA

FOR BRASS:

$G_B = 6000$ ksi

$\tau_{G_{ALL}} = 5$ ksi

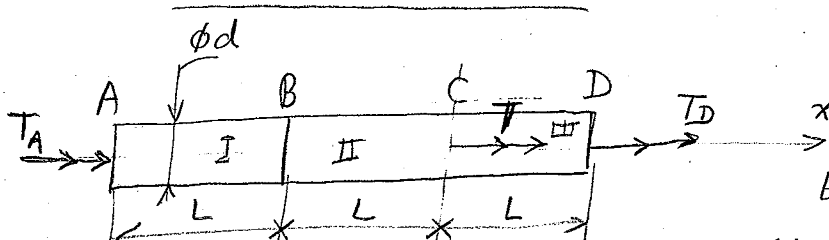
FOR STEEL:

$G_S = 12\,000$ ksi

$\tau_{S_{ALL}} = 12$ ksi

T_{ALL} - ?

FREE BODY DIAGRAM



ASSUME TWO POSITIVE REACTIONS T_D, T_A

EQ. EQ:

$$\sum M_x = 0 = T_A + T + T_D$$

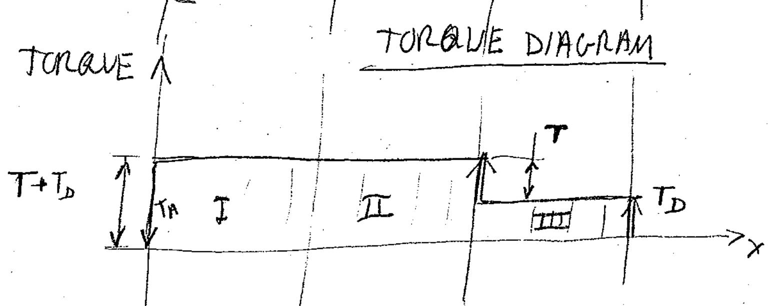
$$T_A = -(T + T_D)$$

THREE UNKNOWNS, ONE EQ. OF EQUILIBRIUM

EQ 2 → COMPARISSON TO ALLOWABLE STRESS

EQ 3 → GEOMETRIC CONSTRAIN (TRY TO RELEASE ONE OF THE SUPPORTS AND SEE THAT THE PROBLEM IS STILL STATICLY STABLE)

TORQUE DIAGRAM



CONSTRAIN EQUATION:

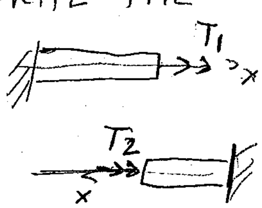
$$\sum \phi_i = \frac{(T+T_D)L}{J \cdot G_B} + \frac{(T+T_D)L}{J \cdot G_S} + \frac{T_D \cdot L}{J \cdot G_S} = 0$$

$$(1) \quad (T+T_D)\left(\frac{1}{G_B} + \frac{1}{G_S}\right) + T_D \frac{1}{G_S} = 0$$

ADDITIONAL OPTION TO WRITE THE CONSTRAIN EQ:

$$\phi_I + \phi_{II} = \phi_{III}$$

$$(T_1 + T_2 = T)$$



(3.11)

THE STRESS REQUIREMENTS:

$$\tau_B = \frac{(T+T_D)(d/2)}{J} \le \tau_{G_{ALL}} \quad (I)$$

$$\tau_S \begin{cases} \dfrac{(T+T_D)(d/2)}{J} \le \tau_{S_{ALL}} & (II) \\ \dfrac{T_D(d/2)}{J} \le \tau_{S_{ALL}} & (III) \end{cases}$$

$\tau_{S_{ALL}} > \tau_{G_{ALL}}$, SO (I) WILL LEAD TO A SMALLER $|T+T_D|$ THAN (II). NOT NEED TO CHECK (II)

FOR SEC. (I):

$$T+T_D = \pm\frac{\tau_{G_{ALL}}\cdot J}{d/2} = \pm\frac{\tau_{G_{ALL}}\cdot\frac{1}{2}\pi\left(\frac{d}{2}\right)^4}{(d/2)} = \pm 5\ \text{ksi}\cdot\frac{1}{2}\cdot\pi\left(\frac{4}{2}\right)^3 \text{in}^3$$

$$T+T_D = \pm 62.832\ \text{kips}\cdot\text{in}$$

RECALL (1) $(T+T_D)\left(\frac{1}{G_B}+\frac{1}{G_S}\right)+T_D\frac{1}{G_S}=0$

$$T_D = -G_S\left(\frac{1}{G_B}+\frac{1}{G_S}\right)(T+T_D) = -12000\left(\frac{1}{6000}+\frac{1}{12000}\right)(\pm 62.832)$$

$$T_D = \mp 188.5\ \text{kips}\cdot\text{in}$$

$$\underline{T} = \pm 62.832\ \text{kips}\cdot\text{in} - (\mp 188.5\ \text{kips}\cdot\text{in}) = \underline{\pm 251.33\ \text{kips}\cdot\text{in}}$$

FOR SEC. (III): $\dfrac{T_D\left(\frac{d}{2}\right)}{J} = \tau_{S_{ALL}}$

$$T_D = \tau_{S_{ALL}}\cdot\frac{\frac{1}{2}\pi\left(\frac{d}{2}\right)^4}{\left(\frac{d}{2}\right)} = 12\ \text{ksi}\cdot\frac{1}{2}\cdot\pi\left(\frac{4''}{2}\right)^3 = \pm 150.8\ \text{kips}\cdot\text{in}$$

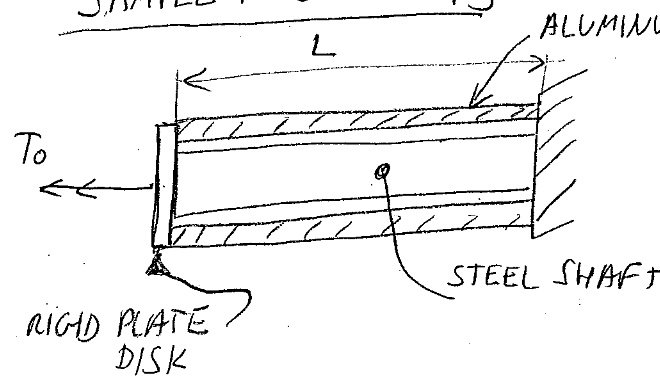
RECALL (1)

$$T = -T_D\left\{\frac{\frac{G_S}{G_B}+2}{\frac{G_S}{G_B}+1}\right\} = \mp 150.8\left\{\frac{\frac{12}{6}+2}{\frac{12}{6}+1}\right\} = \underline{\mp 201.06\ \text{kips}\cdot\text{in}}$$

WE CHOOSE THE MINIMUM $|T|$,

$$\underline{\underline{T_{ALLOWABLE} = 201.06\ \text{kips}\cdot\text{in}}}$$

SAMPLE PROBLEM 3.5



CALCULATE T_0 FOR . . . . .

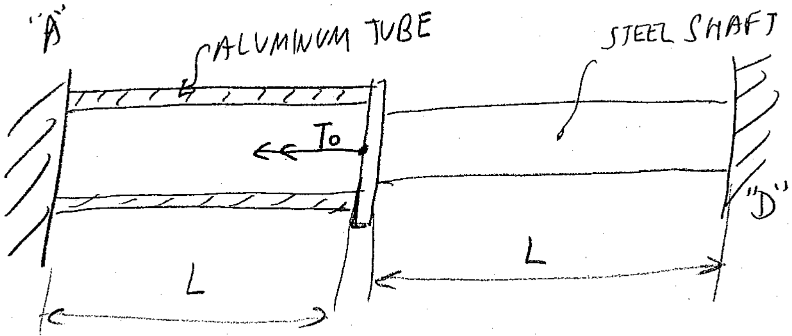
THE CONSTRAIN EQUATION USED IN THE TEXTBOOK:

$$\phi_{Al}\Big|_{x=L} = \phi_{St}\Big|_{x=L}$$

THE EQUIL. EQ. SHOULD GIVE:

$$T_0 = T_{AL} + T_{St}$$

WE CAN CONVERT THIS PROBLEM TO THE EQUIVALET:



AND SOLVE IT AS WE SOLVED THE LAST EXAMPLE

DESIGN OF TRANSMISSION SHAFTS

(FROM DYNAMICS) → FOR A RIGID SHAFT:

POWER TRANSMITED → $\mathbb{P} = T \cdot \omega$

TORQUE (T); ROTATIONAL VELOCITY OR ANGULAR VELOCITY (ω)

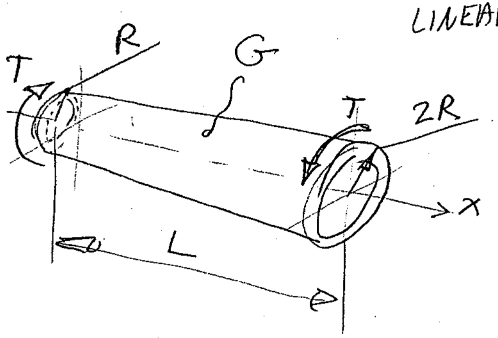
ω IN MEANS OF FREQUENCY → $\omega = 2\pi f$ $\frac{rad}{s}$ (f: $\frac{rev}{sec}$)

	$\mathbb{P}$	T	ω	f
SI	W	N·m	rad/s	1/s = Hz
	KW	kN·m	-"-	-"- = Hz
US	1 hp = 6600 $\frac{in \cdot lb}{s}$	lb·in		1 RPM = $\frac{1}{60}$ Hz

746 W ≃ 1 hp

(3.12_A)

EXERCISE - TORSION ON A BODY WITH ~~CHANGING~~ VARIABLE SECTION AREA



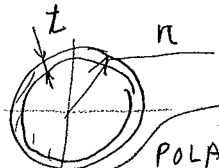
LINEAR CHANGING RADIUS FROM R TO $2R$

THIN WALL, CONSTANT THICKNESS t

$$\phi(T, L, G, t, R)\ -?$$

$$\phi = \int_0^L \frac{\overset{\text{CONSTANT}}{T(x)}\,dx}{J(x)\,G(x)} = \frac{T}{G}\int_0^L \frac{dx}{J(x)}$$

$J(x)$: SECTION AREA CHANGES AS A FUNCTION OF x — $G(x)$: CONSTANT



POLAR MOMENT OF INERTIA FOR THIN WALLED RING

$$J_0 = 2\pi r^3 t$$

$$\left.\begin{aligned} r\big|_{x=0} &= R \\ r\big|_{x=L} &= 2R \end{aligned}\right\}$$

LINEAR CHANGE:

$$r = R\left(1 + \frac{x}{L}\right)$$

$$J_0 = 2\pi R^3\left(1+\frac{x}{L}\right)^3 t$$

$$\phi = \frac{T}{G}\int_0^L \frac{dx}{2\pi R^3 t\left(1+\frac{x}{L}\right)^3} = \frac{T}{2\pi R^3 t\, G}\int_0^L \frac{dx}{\left(1+\frac{x}{L}\right)^3}$$

$$y' = \left(1+\frac{x}{L}\right)^{-3} \rightsquigarrow y = -\frac{1}{2}\cdot L\left(1+\frac{x}{L}\right)^{-2}$$

$$\phi = \frac{T}{2\pi R^3 t\cdot G}\left[-\frac{L}{2}\,\frac{1}{\left(1+\frac{x}{L}\right)^2}\right]_0^L = \frac{TL}{4\pi R^3 t\cdot G}\Bigg[\underbrace{-\frac{1}{\left(1+\frac{L}{L}\right)^2} + \frac{1}{\left(1+\frac{0}{L}\right)^2}}_{-\frac{1}{2^2}+1=\frac{3}{4}}\Bigg]$$

$$\boxed{\phi = \frac{3TL}{16\pi R^3 t\cdot G}}$$

(3,13)

EXAMPLE

A SHAFT CONSISTING OF A STEEL TUBE OF 2" OUTER DIAMETER IS TO TRANSMIT 135 hp OF POWER WHILE ROTATING AT 1200 RPM. DETERMINE THE TUBE THICKNESS WHICH SHOULD BE USED IF THE SHEARING STRESS IS NOT EXCEED 8500 PSI

$\mathbb{P} \rightsquigarrow \frac{in \cdot lb}{s}$ $\quad T \rightsquigarrow lb \cdot in$

$\omega \rightsquigarrow \frac{rad}{s}$

$$T = \frac{\mathbb{P}}{\omega} = \frac{135 \cdot 6600 \ \frac{in \cdot lb}{s}}{2\pi 1200 \cdot \frac{1}{60} \ \frac{1}{s}} = 7090.4 \ in \cdot lb$$

$$\tau = \frac{TR}{J} = \frac{7090.4 \cdot \left(\frac{2''}{2}\right) in}{\frac{1}{2}\pi\left(\left(\frac{2''}{2}\right)^4 - \left(R_{in}\right)^4\right)} \leq 8500 \ PSI$$

$$\frac{7090.4 \cdot (1'')}{\frac{1}{2}\pi \cdot 8500 \ PSI} \leq 1^4 - R_{in}^4$$

$$R_{in}^4 \leq 1 - \frac{7090.4}{\frac{1}{2}\pi \cdot 8500} = 0.469$$

$$R_{in} \leq 0.828''$$

$$t = R_{OUT} - R_{in}$$

$$t \geq 1'' - 0.828'' = 0.172''$$

SEE A SIMILAR PROBLEM IN EXAMPLE 3.07, USING SI UNITS SYSTEM

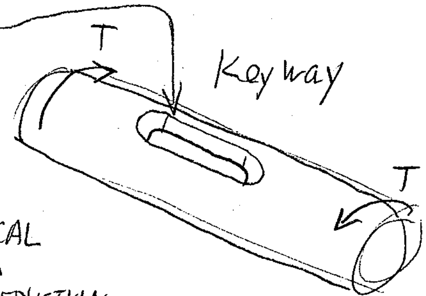
(3.14)

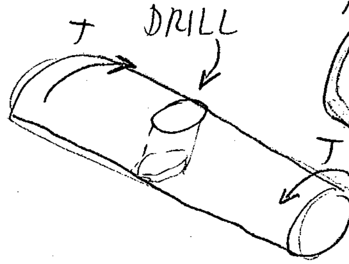
STRESS CONCENTRATIONS IN CIRCULAR SHAFTS

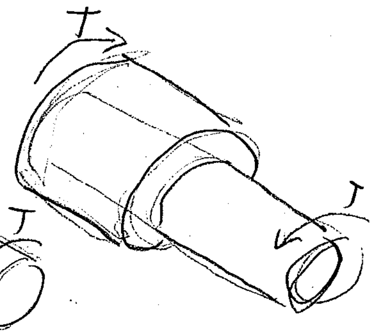
DISCONTINUITY OF THE SECTION AREA CAUSES STRESS CONCENTRATIONS.

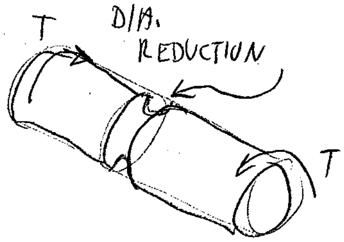
EXAMPLES OF DISCONTINUITIES:

ASME RECOMEND TO DECREASE τ_{ALLOW} BY 25%



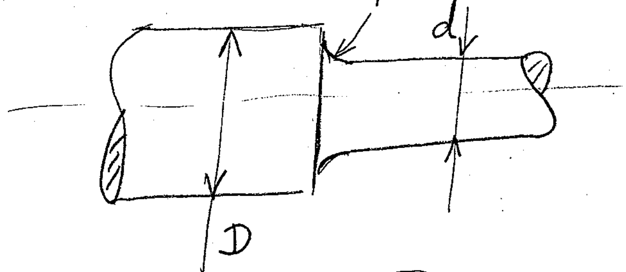


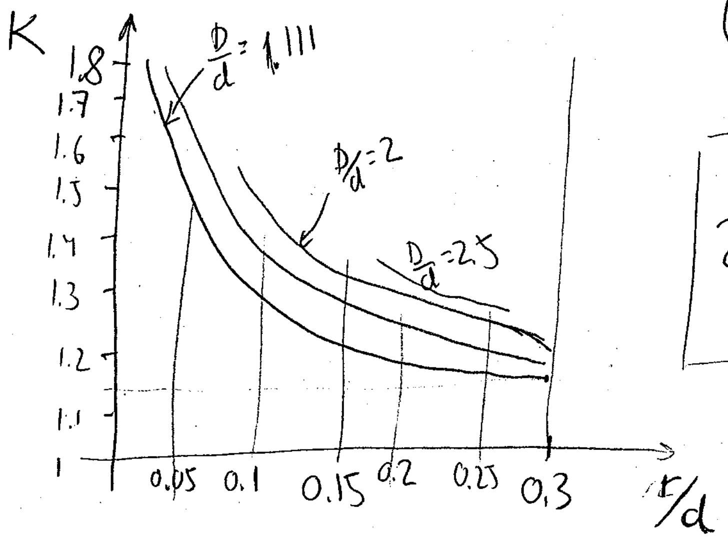




$$\tau_{max} = K \frac{T R^*}{J^*}$$

* OF THE SMALLER SECTION AREA





$$\boxed{\tau_{max} = K \frac{T \cdot \frac{d}{2}}{J_d}}$$

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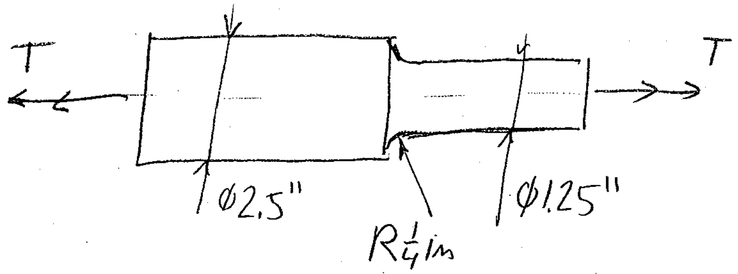
EXAMPLE (PROBLEM 3.90)



$\mathbb{P} = 60\ hp$

$\tau_{ALL} = 6000$ PSI

$f - ?$ MINIMUM

$$\tau_{ALL} \geq K \frac{T \frac{d}{2}}{J_d} = \frac{K \cdot T}{\frac{1}{2}\pi \left(\frac{d}{2}\right)^3}$$

$$T \leq \frac{1}{K} \cdot \frac{1}{2}\pi \left(\frac{d}{2}\right)^3 \tau_{ALL}$$

PAGE 167, $\left.\begin{aligned} \frac{r}{d} &= \frac{1/4}{1.25} = 0.2 \\ \frac{D}{d} &= \frac{2.5}{1.25} = 2 \end{aligned}\right\} \xrightarrow[P.\,167]{\text{GRAPH}} K \approx 1.26$

$$T \leq \frac{1}{1.26} \cdot \frac{1}{2} \cdot \pi \left(\frac{1.25}{2}\right)^3 \cdot 6000 = 1826.17\ lb \cdot in$$

$$\omega = 2\pi f = \frac{\mathbb{P}}{T} \geq \frac{60 \cdot 6600 \ \frac{lb\ in}{s}}{1826.17\ lb \cdot in} = 216.85\ \frac{rad}{s}$$

$$f \geq \frac{1}{2\pi} 216.85 = 34.51\ Hz \Rightarrow f \geq 34.51 \cdot 60\ RPM$$

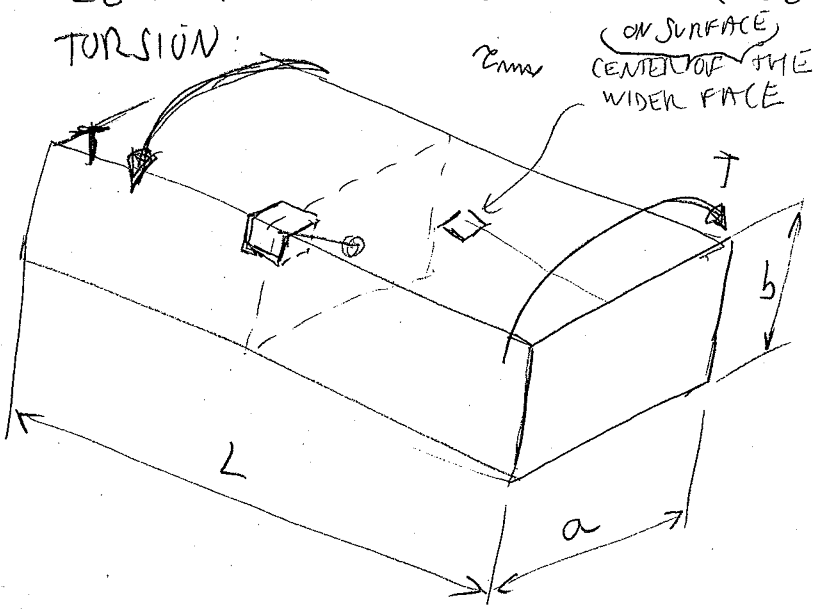
$$\boxed{f \geq 2070.7\ RPM}$$

(3.16)

TORSION OF NON CIRCULAR MEMBERS

WE ASSUMED AXISYMETRIC SECTION AREAS FOR THE DEVELOPMENT OF THE TORSION FORMULAS.

LET'S TAKE A RECTANGULAR SECTION AREA MEMBER UNDER TORSION:



THE MAXIMUM SHEARING STRESS ON THE ELEMENT PLACED AT FAREST DISTAN. FROM THE CENTER OF THE SECTION AREA IS : ZERO

THE CASE OF NON AXI-SYMETRIC SECTION AREA IS DEVELOPED ON THE THEORY OF ELASTICITY

RESULTS FOR UNIFORM RECTANGULAR CROSS SECTION ARE AS FOLLOWS:

(ELASTIC RANGE)

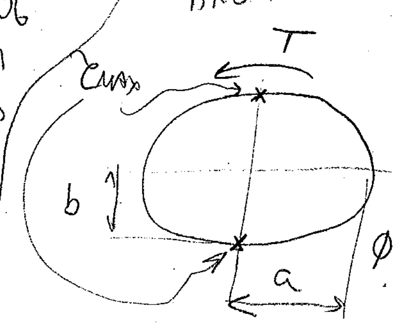
$$\tau_{max} = \frac{T}{c_1 a b^2} \qquad \phi = \frac{T \cdot L}{c_2 a b^3 \cdot G}$$

a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

FOR $a/b \geq 5$:

$$c_1 = c_2 = \frac{1}{3}\left(1 - 0.63\frac{b}{a}\right)$$

FOR ELIPTIC SECTION AREA:



$a > b$

$$\tau_{max} = \frac{2T}{\pi a b^2}$$

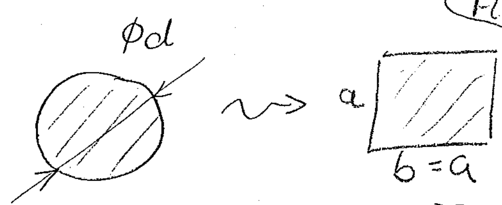
$$\phi = T \cdot L \cdot \frac{a^2 + b^2}{\pi a^3 b^3 \cdot G}$$

(3.17)

EXERCISE: TORSION OF RECTANGULAR SECTION AREA

A CIRCULAR SECTION SHAFT IS GOING TO BE REPLACED BY SQUARE SECTION SHAFT WITH THE SAME LENGTH & SAME MATERIAL. THE TWO SHAFT ARE ABLE TO CARRY THE SAME MAX. TORQUE. HOW HEAVIER IS THE NEW SHAFT IN RELATION TO THE CIRCULAR SECT. SHAFT? WHICH ONE IS MORE FLEXIBLE?

FLEXIBLE = ϕ GREATER FOR THE SAME TORQUE



FOR THE ROD $\rightarrow T_{MAX} = \tau_{ALLOW} \cdot \frac{J}{R} = \tau_{ALLOW} \cdot \frac{\frac{1}{2}\pi\left(\frac{d}{2}\right)^4}{\frac{d}{2}} = \frac{1}{16}\pi\,\tau_{ALLOW}\, d^3$

ON THE SQUARE SHAFT $\rightarrow \tau_{ALLOW} = \frac{T_{MAX}}{c_1\, a\, b^2}$; $a = b$, $a/b = 1 \rightarrow c_1 = 0.208 \rightarrow \tau_{ALLOW} = \frac{T_{MAX}}{0.208 \cdot a^3}$

COMPARING T_{MAX} FOR BOTH SHAFTS:

$$T_{MAX} = \frac{1}{16}\pi\,\tau_{ALLOW}\, d^3 = 0.208\, a^3\, \tau_{ALLOW}$$

$$a = \sqrt[3]{\frac{\pi}{16 \cdot 0.208}}\, d = 0.98097\, d$$

W_O = CIRCULAR SHAFT WEIGHTS $= \rho \cdot A_O \cdot L = \rho L \frac{\pi}{4} d^2$

$W_\square$ = SQUARE SHAFT WEIGHTS $= \rho\, A_\square \cdot L = \rho\, a^2 L = \rho L \cdot 0.98097^2\, d^2 = \rho L \cdot 0.9623\, d^2$

$$\frac{W_\square}{W_O} = \frac{\rho L \cdot 0.9623\, d^2}{\rho L\, \frac{\pi}{4}\, d^2} = 1.2252$$

THE SQUARE SECTION SHAFT WEIGHTS 22.5% MORE THAN THE CIRCULAR SECTION ONE

$$\phi_O = \frac{TL}{JG} = \frac{T \cdot L}{\frac{\pi}{32} d^4\, G} = \frac{T \cdot L}{d^4 \cdot G} \cdot 10.186$$

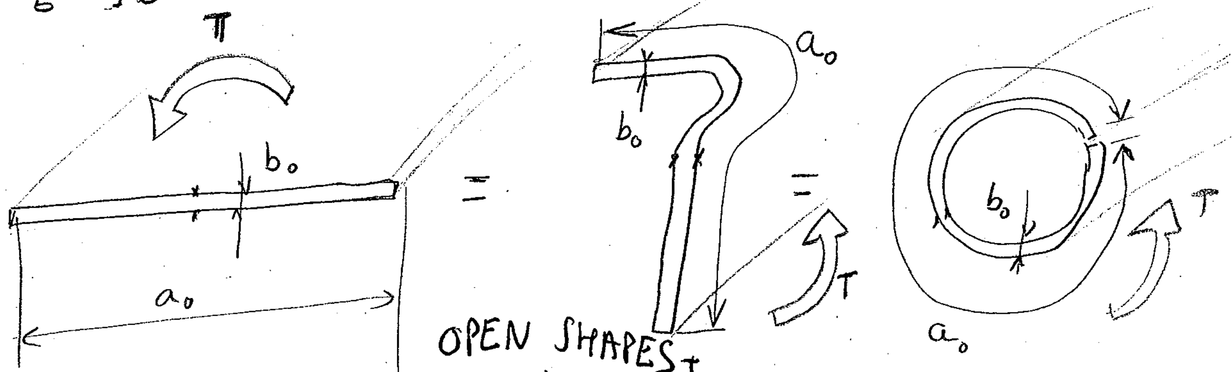
$$\phi_\square = \frac{T \cdot L}{c_2\, a b^3 \cdot G} \quad b = a \rightarrow c_2 = 0.1406 \rightsquigarrow \phi_\square = \frac{T \cdot L}{0.1406 \cdot a^4\, G} = \frac{T \cdot L}{0.1406 \cdot 0.98097^4\, d^4 \cdot G} = \frac{T \cdot L}{d^4 G} \cdot 7.68$$

THE CIRCULAR SEC. SHAFT IS MORE FLEXIBLE

THE SQUARE SEC. SHAFT IS MORE RIGID (HIGHER SPRING CONSTANT)

(3.18)

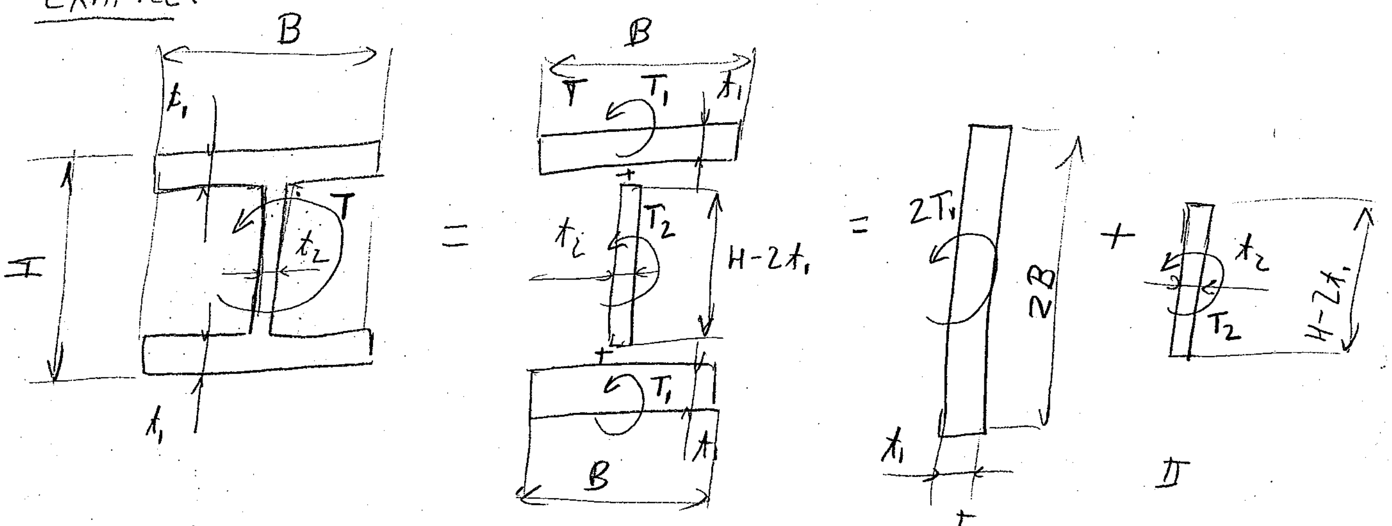
SINCE THE SAME TORQUE IS APPLIED TO EACH MEMBER, FOR A THIN WALLED MEMBER OF UNIFORM THICNESS AND ARBITRARY SHAPE, THE MAXIMUM SHEARING STRESS IS THE SAME AS FOR A RECTANGULAR BAR WITH VERY LARGE VALUE OF a/b AND MAY BE DETERMINATED FOR $\frac{a}{b} > 10$ THE SAME FOR THE ANGLE OF TWIST



OPEN SHAPES

$$\tau_{max} = \frac{T}{c \cdot ab^2} \; ; \; \phi = \frac{T \cdot L}{c \cdot ab^3 \cdot G} \; ; \; \frac{a}{b} > 10 \; ; \; c = \frac{1}{3}\left(1 - 0.63\,\frac{b}{a}\right)$$

EXAMPLE:



$\frac{2B}{t_1} \gg 10$

$\frac{H-2t_1}{t_2} > 10$

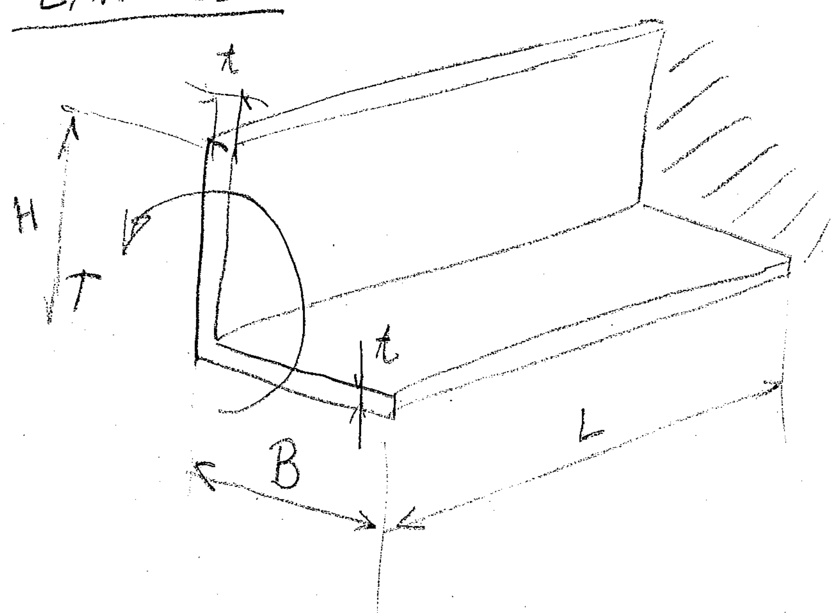
CONSTRAIN $\rightsquigarrow$ $\phi_1 = \phi_2$

$2T_1 + T_2 = T$

AND SOLVE

(3.18 a)

EXAMPLE: TORSION OF THIN RECTANGULAR SECTION



GIVEN

$L = 40''$

$H = 4''$

$B = 3''$

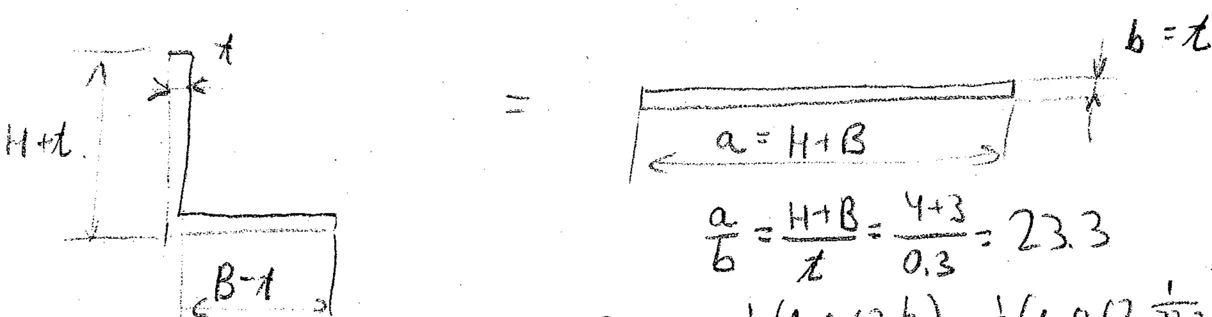
$t = 0.3''$

$T = 2900\ lb \cdot in$

$G = 11 \cdot 10^6\ PSI$

$\tau - ?$

$\phi - ?$



$$\frac{a}{b} = \frac{H+B}{t} = \frac{4+3}{0.3} = 23.3$$

$$c_1 = c_2 = \frac{1}{3}\left(1 - 0.63\frac{b}{a}\right) = \frac{1}{3}\left(1 - 0.63\frac{1}{23.3}\right) = 0.3243$$

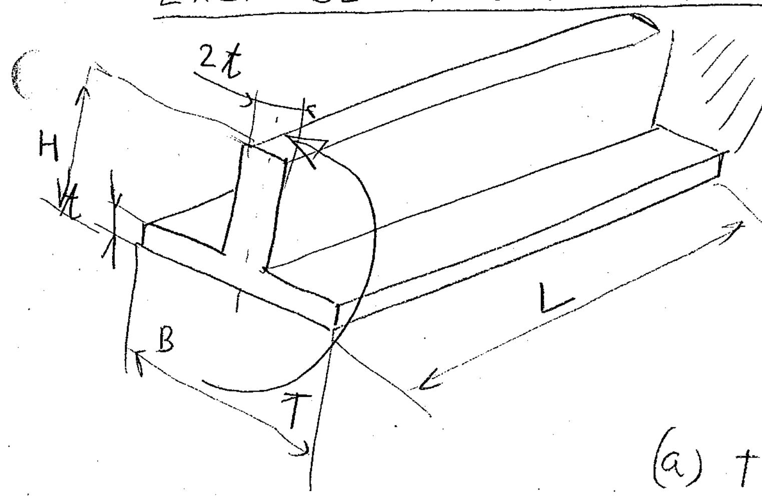
$$\tau_{max} = \frac{T}{c_1\, a\, b^2} = \frac{2900\ lb \cdot in}{0.3243 \cdot (4+3)'' \cdot 0.3^2\ in^2} = 14194\ PSI$$

$$\phi_{max} = \frac{T \cdot L}{c_2\, a b^3 \cdot G} = \frac{2900\ lb \cdot in\ \text{[redacted]}}{0.3243 \cdot (4+3) \cdot 0.3^3 \cdot 11 \cdot 10^6} = \underline{4.3 \cdot 10^{-3}}\ rad$$

$$\phi_{max} = \underline{0.246^\circ}$$

(3.19)

EXERCISE: TORSION OF THIN RECTANGULAR SECTION



$L = 1.5\,m$

$H = 60\,mm$

$B = 80\,mm$

$t = 2\,mm$

$\tau_{ALL} = 50\,MPa$

$G = 77\,GPa$

(a) T_{ALLOW} - ?

(b) $\phi|_{T_{ALLOW}}$ - ?

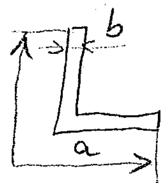
APPENDIX C : STANDARD SHAPES
ASSUMED TO BE THIN WALL STRUCT → I, L

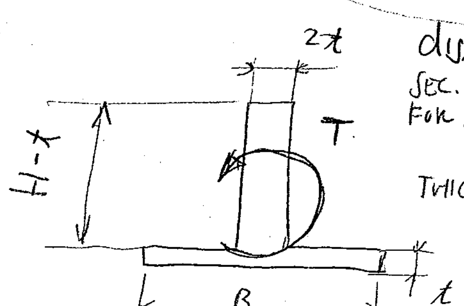
NOT CONSTANT THICKNESS

radius

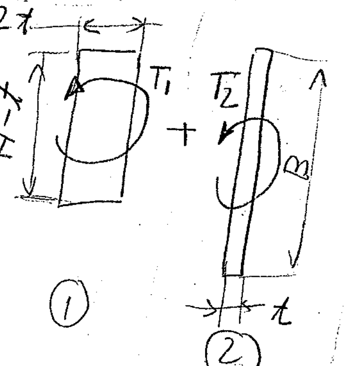
IF THE THICKNESS IS CONSTANT, WE CALCULATE THE OVERALL LENGTH DIVIDING THE SECTION AREA FROM APPENDIX C BY THE THICKNESS OF THE SHAPE

b = THICKNESS
A = STANDARD AREA } $a = \frac{A}{b}$ EQUIVALENT OVERALL LENGHT





dismember 2t
SEC. AREA
FOR DIFFERENT
THICKNESS
b



$; \quad T_1 + T_2 = T$

CONSTRAIN

$\phi|_L = \phi_1 = \phi_2$

$a_1 = H - t = 58\,mm \quad a_2 = B = 80\,mm$

$b_1 = 2t = 4\,mm \quad b_2 = t = 2\,mm$

$\frac{a_1}{b_1} = 14.5 \qquad \frac{a_2}{b_2} = 40$

$c_1 = c_2 = \frac{1}{3}\left(1 - 0.63\frac{b}{a}\right)$

$c_{1_1} = c_{2_1} = 0.31885 \qquad c_{1_2} = c_{2_2} = 0.32808$

(3.20)

$$\phi_1 = \frac{T_1 \cdot L}{c_{21} a_1 b_1^3 \cdot G} = \phi_2 = \frac{T_2 \cdot L}{c_{22} a_2 b_2^3 G}$$

$$\frac{T_1}{0.31885 \cdot 58 \cdot 4^3} = \frac{T_2}{0.32808 \cdot 80 \cdot 2^3}$$

$$T_1 = 5.63683\, T_2$$

$$\tau_1 = \frac{T_1}{c_{11}\, a_1 b_1^2} = \frac{5.63683\, T_2}{0.31885 \cdot 58 \cdot 4^2} = \boxed{\frac{T_2}{52.493\ mm^3} \le \tau_{ALLOW}} \ (*)$$

$$\tau_2 = \frac{T_2}{c_{12}\, a_2 b_2^2} = \frac{T_2}{0.32808 \cdot 80 \cdot 2^2} = \frac{T_2}{105\ mm^3} \le \tau_{ALLOW}$$

$\Downarrow$

(*) $T_2 \le 52.493 \cdot 50\ MPa = 2624.64\ N \cdot mm$

$T_1 = 5.63683\, T_2 \le \dots 14794.64\ N \cdot mm$

$T = T_1 + T_2 \le 14794.64 + 2624.64$

$T \le 17419.28\ N \cdot mm$, $\underline{\underline{T \le 17.42\ N \cdot m}}$

$$\phi|_L = \phi_1 = \frac{T_1 \cdot L}{c_{21} a_1 b_1^3\, G} = \frac{14794.64 \cdot 1500\ mm}{0.31885 \cdot 58 \cdot 4^3 \cdot 77000} = 0.2435\ rad$$

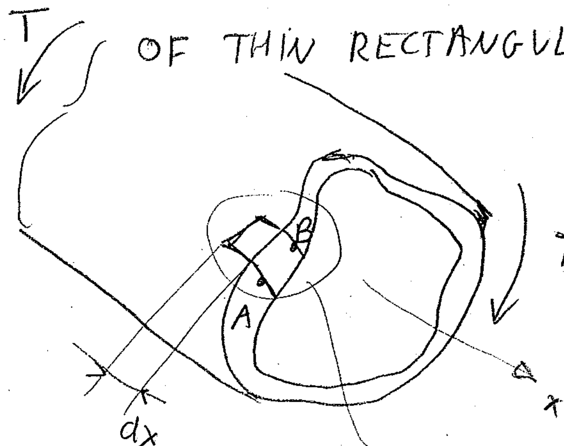
$\underline{\phi|_L = \dots \doteq 14°}$

(3.21)

THIN-WALLED HOLLOW SHAFTS

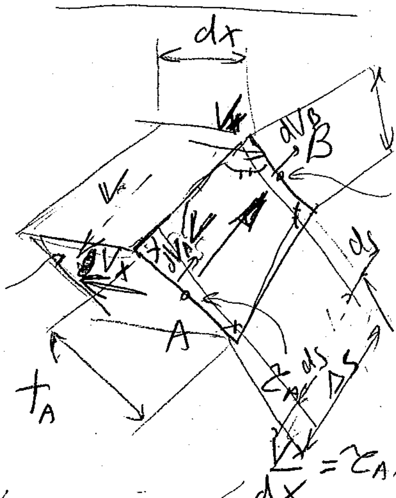
WE ARE GOING TO DISCUSS A THIN WALLED CLOSED SECTION AREA UNDER TORSION

WE TALKED ABOUT AXISYMETRIC SECTIONS (CIRCLES, RINGS), ABOUT RECTANGULAR SECTION AND THE SPECIAL FEATURES OF THIN RECTANGULAR SECTIONS.



THE ACTUAL CASE IS A CLOSED THIN WALLED AND VARIABLE THICKNES SECTION

V MUST BE IN $\overline{AB}$ DIRECTION



τ_B

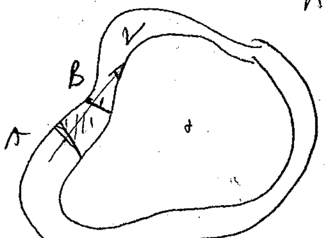
$\tau_A = \frac{V_x}{t_A dx} = \frac{dV_A}{t_A ds}$

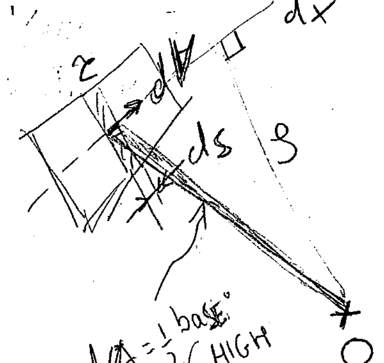
$\tau_B = \frac{V_x}{t_B dx} = \frac{dV_B}{t_B ds}$ $\quad dV_A = dV_B$

$\frac{V_x}{dx} = \tau_A t_A = \tau_B t_B = \tau t = \frac{V_x}{dx} = \frac{dV}{ds} = q$

A ANY WALL THICKNESS PLACE

Shear flow





ASUMES: ① THIN ENOUGHT NO INTERNAL SHEARING STRESSES IN $\overline{AB}$ DIRECTION

② $\overline{AB}$ SHORT ENOUGHT IN ORDER TO DISCUSS A UNI-DIRECTIONAL SHEARING FORCE

③ V_x, AND V FROM BOTH SIDES DUE TO EQUILIBRIUM

$dA = \frac{1}{2}$ base · HIGH

ds $\quad \rho$

$ds\,\rho = 2\,dA$

$dT = dV \cdot \rho$

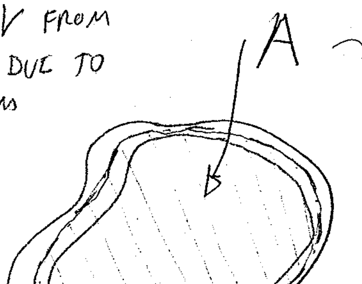
$dT = \tau t\, ds\, \rho$

$dT = \tau t\, 2\, dA$

$T = 2\tau t \int dA$

$T = 2\tau t A$

A



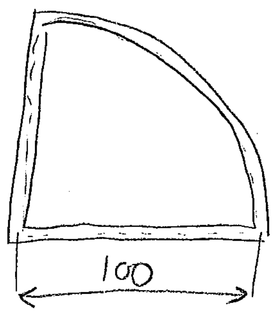
$$\boxed{\tau = \frac{T}{2tA}}$$

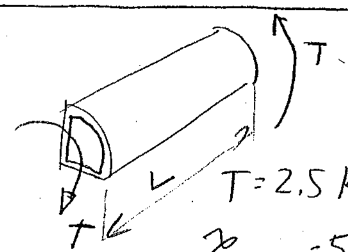
THE ANGLE OF TWIST OF A THIN-WALLED SHAFT MAY BE OBTAINED BY USING THE METHOD OF ENERGY (CHAPT 11) $\phi = \frac{TL}{4A^2G}\oint\frac{ds}{t}$

(3.22)

EXERCISE - THIN WALLED HOLLOW SHAFT

①





$T = 2.5\ kN \cdot m$ $\quad L = 2\ m$

$\tau_{ALLOW} = 50\ MPa$ $\quad G = 77\ GPa$

$t_{min} - ?\ ;\ \phi - ?$

$$\tau_{ALLOW} \geq \frac{T}{2 t A}$$

$$A = \frac{1}{4}(\pi R^2) = \frac{1}{4}(\pi\, 100^2) = 7854\ mm^2$$

$$t \geq \frac{T}{2 \tau A} = \frac{2.5 \cdot 10^6\ N \cdot mm}{2 \cdot 50_{MPa} \cdot 7854\ mm^2} = 3.18\ mm$$

(+)

$$\phi = \frac{TL}{4A^2 G} \oint \frac{ds}{t}$$

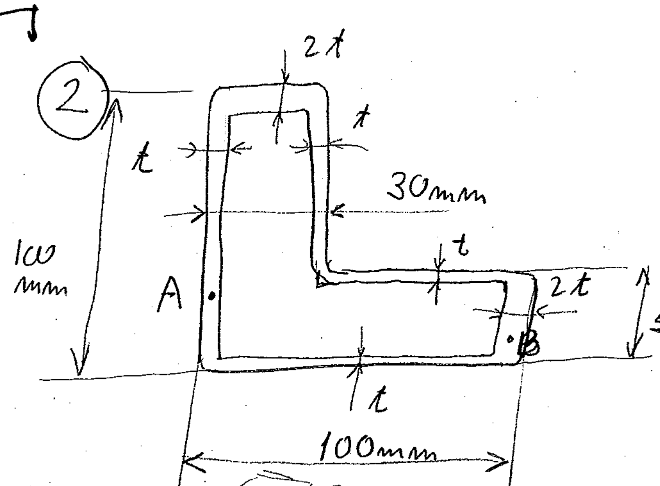
$$= \frac{TL}{4A^2 \cdot G t}\left(2 \cdot 100 + \frac{\pi}{2} \cdot 10\right.$$

$$= \frac{2.5 \cdot 10^6\ N\,mm \cdot 2000\ mm\ (2 \cdot 100 + \frac{\pi}{2}}{4 \cdot 7854^2\ mm^4 \cdot 77000\ MPa \cdot 3.18\ mm}$$

$\phi = 0.0296\ rad$

$\phi = 1.69°$

②



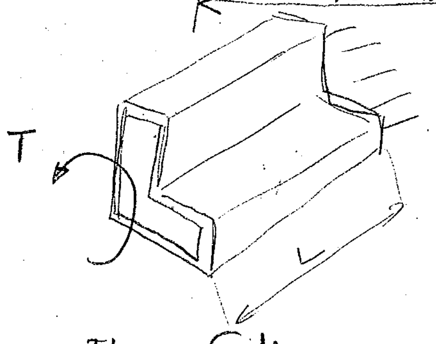
$t = 2\ mm$ $\quad G = 29\ GPa$

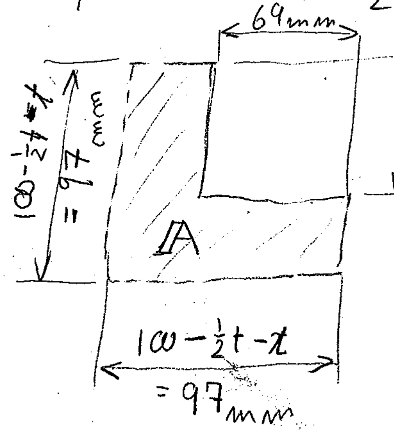
$T = 500\ N \cdot m$ $\quad L = 2\ m$

$\tau_A - ?$

$\tau_B - ?$

$$\tau = \frac{T}{2 t A}$$





$100 - 30 + \frac{1}{2}t - t$

$= 69\ mm$

$$A = 97^2 - 69^2 = 4648\ mm^2$$

$$\phi = \frac{TL}{4A^2 G} \oint \frac{ds}{t}$$

$$= \frac{TL}{4A^2 G}\left\{\int \frac{ds}{t_1} + \int \frac{ds}{2t}\right\}$$

$$= \frac{500 \cdot 10^3\ N \cdot mm \cdot 2000\ mm}{4 \cdot 4648^2\ mm^4 \cdot 24000\ MPa}\left\{\frac{2 \cdot 97 + 2 \cdot 69}{2} + \underbrace{\frac{2 \cdot (97 - 69)}{4}}_{180}\right\}$$

$\phi = 0.0718\ rad$

$\phi = 4.1°$

AT A, $t = 2\ mm$

$$\tau_A = \frac{500 \cdot 10^3\ N \cdot mm}{2 \cdot 2\ mm \cdot 4648\ mm^2} = 26.89\ MPa$$

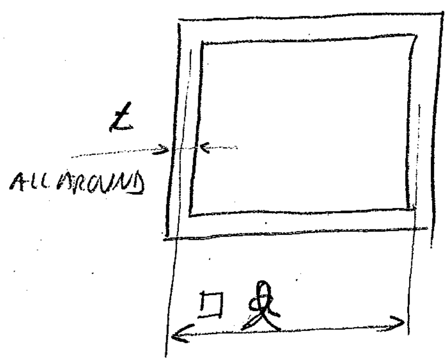
AT B, $2t = 4\ mm$

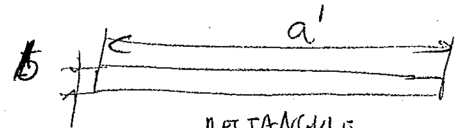
$$\tau_B = \frac{1}{2}\tau_A = 13.45\ MPa$$

SHEARING STRESS (AND TWIST ANGLE) COMPARISSON FOR OPENED AND CLOSED THIN WALL SHAPES (3.23)

SECTION AREAS

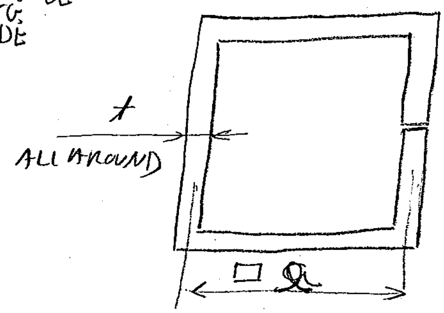
① SQUARE





$\frac{a'}{b} > 10$

REC. WIDTH



$$\tau_{CLOSED} = \frac{T}{2t\,A}$$

$$A = a^2$$

$$\tau_{CLOSED} = \frac{T}{2a^2 t}$$

$$\tau_{OPENED} = \frac{T}{c_1 a' b^2}$$

$$a' = 4a$$

$$b = t$$

$$c_1 < 1/3$$

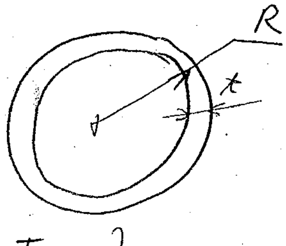
$$\tau_{OPENED}\Big|_{min} = \frac{T}{1/3 \cdot 4a \cdot t^2}$$

$$\frac{\tau_{OPENED}}{\tau_{CLOSED}} \geq \frac{2 \cdot a^2 t}{1/3 \cdot 4 \cdot a t^2} = 1.5\frac{a}{t} = \frac{1.5}{4} \cdot \frac{\overbrace{4a}^{a'}}{t} > \frac{1.5}{4} \cdot 10 = 3.75$$

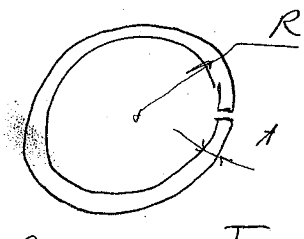
FOR A SQUARE THIN WALLED SHAPE SECTION AREA $\Big\}$ $\frac{\tau_{OPENED}}{\tau_{CLOSED}} \geq 3.75$ (INCREASES AS $\frac{a'}{t}$ INCREASES)

THE OPENED SHAPE IS ALSO MUCH MORE FLEXIBLE !! SEE (3.23a)

② RING



$\frac{a}{b} > 10$



$$\left.\begin{aligned}\tau_{CLOSED} &= \frac{T}{2t\,A}\\ A &= \pi R^2\end{aligned}\right\} \tau_{CLOSED} = \frac{T}{2\pi R^2 t}$$

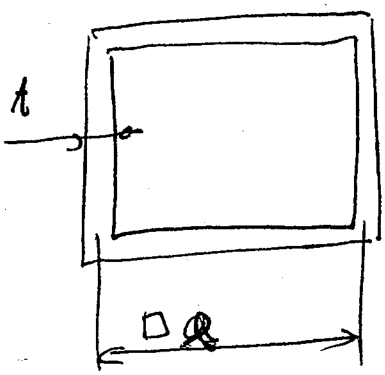
$$\left.\begin{aligned}\tau_{OPENED} &= \frac{T}{c_1 a b^2}\\ a &= 2\pi R\\ b &= t\end{aligned}\right\} \tau_{OPENED}\Big|_{min} = \frac{T}{\frac{1}{3}(2\pi R)\cdot t^2}$$

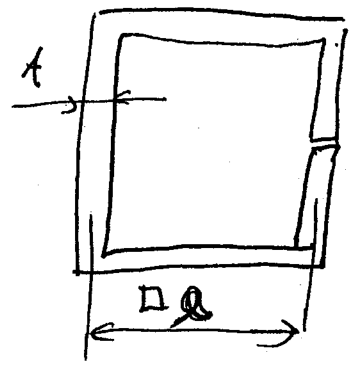
$$\frac{\tau_{OPENED}}{\tau_{CLOSED}} \geq \frac{\overbrace{2\pi R}^{\frac{a^2}{2\pi}} t}{\frac{1}{3}\underbrace{2\pi R}_{a}\, t^2} = \frac{3a^2 t}{2\pi\, a t^2} = \frac{3}{2\pi}\,\frac{a}{t} > \frac{3 \cdot 10}{2\pi} = 4.77$$

$\frac{\tau_{OPENED}}{\tau_{CLOSED}} > 4.77$ (INCREASES AS $\frac{a}{t}$ INCREASES) SEE (3.23a)

(3.23 a)

① SQUARE





$$\phi_{CLOSE} = \frac{TL}{4A^2 \cdot G} \oint \frac{ds}{dt}$$

$$A = a^2$$

$$\oint \frac{ds}{dt} = \frac{4a}{t}$$

$$\phi_{CLOSE} = \frac{TL \cdot 4 \cdot a}{4 \cdot a^4 \cdot G \cdot t} = \frac{TL}{a^3 t G}$$

$$\phi_{OPEN} = \frac{TL}{c_2\, a' b^3 G} \geq \frac{3TL}{a' b^3 G}$$

RECTANGLE LONG SIDE

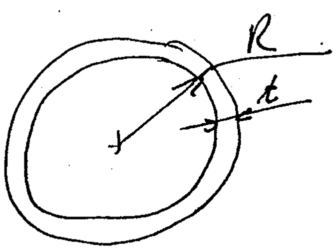
$$a' = 4a, \quad b = t$$

$$\phi_{OPEN} \geq \frac{3TL}{4at^3 \cdot G}$$

THIN RECT REG $\Big\}$ $\frac{a'}{b} > 10, \quad \frac{4a}{t} > 10 \quad \frac{a}{t} > \frac{10}{4}$

$$\frac{\phi_{OPEN}}{\phi_{CLOSE}} \geq \frac{3\,a^2}{4\,t^2} > \frac{3}{4} \cdot \left(\frac{10}{4}\right)^2 = 4.69$$

② RING

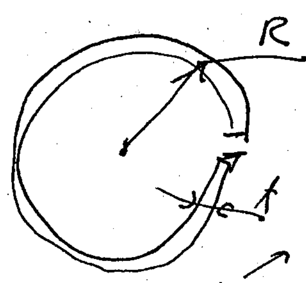


$$\frac{a}{b} > 10$$

$$\downarrow$$

$$\frac{2\pi R}{t} > 10$$

$$\frac{R}{t} > \frac{10}{2\pi}$$



$$c \leq \frac{1}{3}$$

$$\phi_{CLOSE} = \frac{TL}{4A^2 G} \oint \frac{ds}{dt}$$

$$\phi_{CLOSE} = \frac{T \cdot L}{4(\pi R^2)^2 \cdot G} \cdot \frac{2\pi R}{t}$$

$$\phi_{CLOSE} = \frac{TL}{2\pi R^3 t G}$$

$$\phi_{OPEN} = \frac{TL}{c\,a b^3 G} \geq \frac{3TL}{ab^3 G}$$

$$a = 2\pi R \quad b = t$$

$$\phi_{OPEN} \geq \frac{3TL}{2\pi R t^3 G}$$

$$\frac{\phi_{OPEN}}{\phi_{CLOSE}} \geq \frac{3R^2}{t^2} > 3\left(\frac{10}{2\pi}\right)^2 = 7.6$$

Florida International University
Department of Mechanical and Materials Engineering
Mechanics and Materials Science

EMA 3702 Quiz # 2 - **version A** Oct. 19, 2004

Follow the instructions before you begin the quiz:
This test is 40 minutes long. It is not permitted to pass or receive any hardware nor software during the quiz. Use your own papers, pens, pencils, books, notebooks, calculators, etc.
Write your first and last name, your Panther I. D. and the quiz version on the papers you will use for the quiz solution. Explain your steps, use the adequate diagrams.

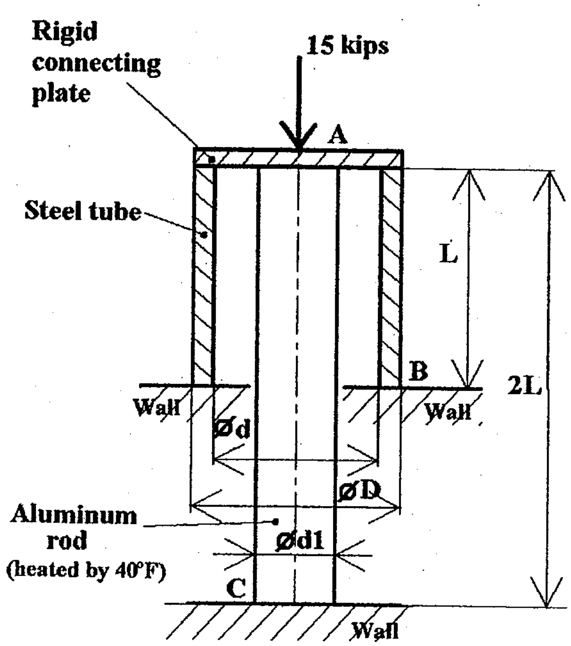
Good Luck!

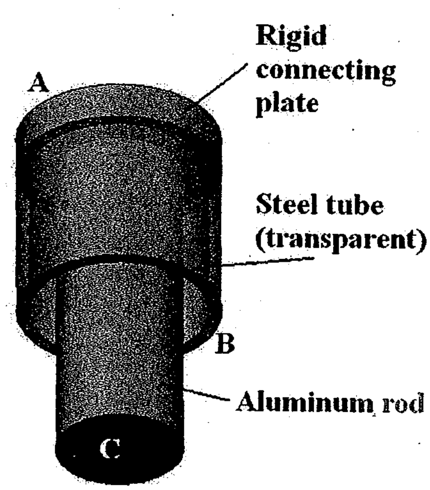
A steel tube and a centered aluminum rod are attached at **A** with a connecting rigid plate. The steel tube is attached to a rigid support at **B** (wall) and the aluminum rod is attached to a rigid support at **C** (wall). The steel tube and the aluminum rod are unstressed until a compression load of 15 kips is applied on the rigid plate and **only** the aluminum rod temperature is raised by 40°F.

Determine:

(a) The load on the aluminum rod and the load on the steel tube **(40%)**
(b) The deflection of the rigid connecting plate (at **A**) **(25%)**
(c) The change in external diameter of the steel tube (magnitude and sign) **(25%)**
(d) Choose the correct answer **(10%)** -> Hooke's law is applicable for:
(I) Strain due to loadings; (II) Strain due to thermal expansion;
(III) The two above; (IV) Not relevant for strains

	Steel tube	Aluminum rod
Young's Modulus	$Es=29\cdot10^6$ PSI	$Ea=10.6\cdot10^6$ PSI
Shear Modulus	$Gs=11.2\cdot10^6$ PSI	$Ga=4\cdot10^6$ PSI
Coefficient of thermal expansion		$\alpha a=13\cdot10^{-6}$ 1/°F
Temperature raise		$\Delta Ta=40$°F
Dimensions: L=10 in.	$\phi D=1.6$ in $\phi d=1.5$ in.	$\phi d1=1.0$ in.





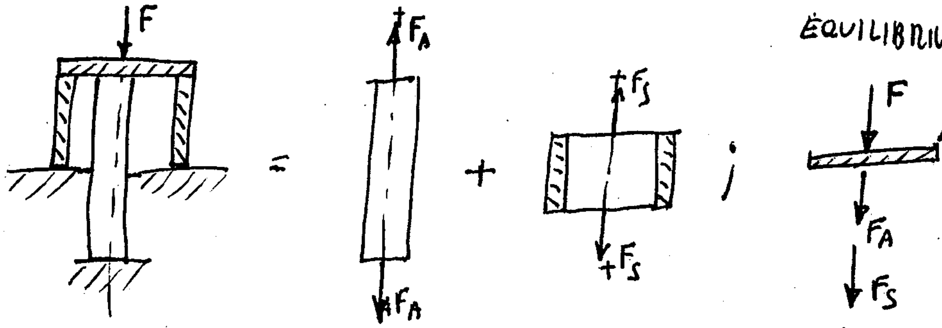
EMA 3702

QUIZ #2 - SOLUTION

Oct. 19, 2004

VERSION A

(a) UNIAXIAL LOAD + 2 REACTIONS ⟶ INDETERMINATE PROBLEM



EQUILIBRIUM: $\sum F = 0 = F_A + F_S + F = 0$

F.B.D OF THE RIGID PLATE

$$F_A + F_S = -F \quad (1)$$

CONSTRAIN:

$$\delta_{\substack{\text{RIGID}\\\text{PLATE}}} = \delta_A = \delta_S$$

$$\delta_A = \delta_{A_{F_A}} + \delta_{A_T}$$

$$\delta_{A_{F_A}} = \frac{F_A \cdot L_A}{A_A E_A}$$

$$\delta_{A_T} = \alpha_A \Delta T L_A$$

$$\delta_S = \delta_{S_{F_S}} = \frac{F_S L_S}{A_S E_S}$$

$$\alpha_A \cdot \Delta T \cdot L_A + \frac{F_A L_A}{A_A E_A} = \frac{F_S L_S}{A_S E_S} \quad (2)$$

$$\alpha_A \cdot \Delta T \cdot L_A = 13 \cdot 10^{-6} \frac{1}{°F} \cdot 40°F \cdot 2 \cdot 10'' = 0.0104''$$

$$A_A = \frac{\pi\, 1''^2}{4} = 0.7854 \text{ in}^2$$

$$A_S = \frac{\pi (1.6^2 - 1.5^2)}{4} = 0.24347 \text{ in}^2$$

$$0.0104'' + \frac{F_A \cdot 2 \cdot 10''}{0.7854 \cdot 10.6 \cdot 10^6} = \frac{F_S \cdot 10''}{0.24347 \cdot 29 \cdot 10^6}$$

$$F_A - 0.5896 F_S = 4329.125$$

INTO EQ. EQ.

$$(-4329.125 + 0.5896 F_S) + F_S = -15000 \text{ lb}$$

$$\underline{F_S = -6713.13 \text{ lb}}$$

$$F_A = -F - F_S = -15000 + 6713.13$$

$$\underline{F_A = -8286.87 \text{ lb}}$$

(b) $$\delta_{\substack{\text{RIGID}\\\text{PLATE}}} = \delta_S = \frac{F_S \cdot L_S}{A_S \cdot E_S} = \frac{-6713.13 \cdot 10''}{0.24347 \cdot 29 \cdot 10^6} = \underline{-9.5078 \cdot 10^{-3} \text{ in}} \text{ (down)}$$

(c) $$\Delta D = -\nu_S \cdot \varepsilon_{x_S} \cdot D$$

$$\nu_S = \frac{E_S}{2 \cdot G_S} - 1 = \frac{29 \cdot 10^6}{2 \cdot 11.2 \cdot 10^6} - 1 = 0.2946$$

$$\varepsilon_{x_S} = \frac{\delta_S}{L_S} = \frac{-9.5078 \cdot 10^{-3}}{10''} = -9.5078 \cdot 10^{-4} \; (= -950\mu)$$

$$\Delta D = -0.2946 \cdot (-9.5078 \cdot 10^{-4}) \cdot 1.6''$$

$$\underline{\underline{\Delta D = +4.4816 \cdot 10^{-4} \text{ in}}} \text{ (EXPANDS)}$$

(d) (I) Hooke's law is applicable for strain due to loadings

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Mechanics and Materials Science

EMA 3702 | Quiz # 2 - <u>**version B**</u> | Oct. 19, 2004

<u>Follow the instructions before you begin the quiz:</u>
This test is 40 minutes long. It is not permitted to pass or receive any hardware nor software during the quiz. Use your own papers, pens, pencils, books, notebooks, calculators, etc.
Write your first and last name, your Panther I. D. and the quiz version on the papers you will use for the quiz solution. Explain your steps, use the adequate diagrams.

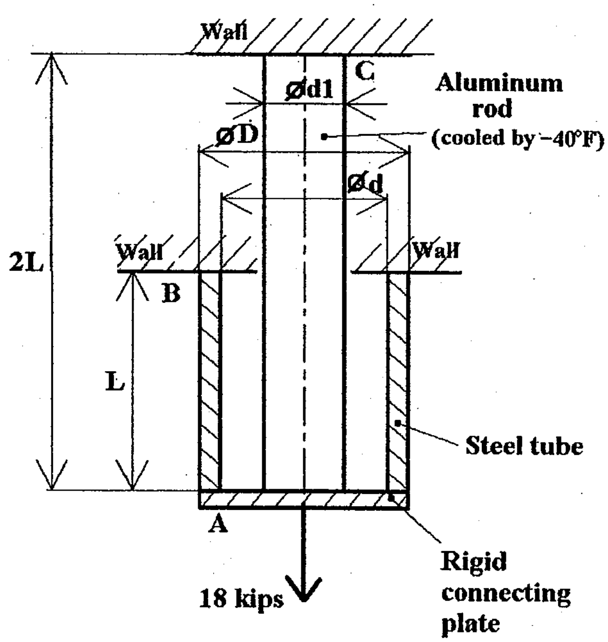
Good Luck!

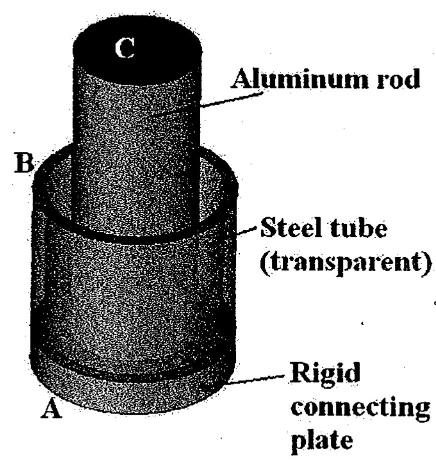
A steel tube and a centered aluminum rod are attached at **A** with a connecting rigid plate. The steel tube is attached to a rigid support at **B** (wall) and the aluminum rod is attached to a rigid support at **C** (wall). The steel tube and the aluminum rod are unstressed until a tension load of 18 kips is applied on the rigid plate and <u>**only**</u> the aluminum rod is cooled by a temperature drop of - 40°F.

<u>Determine</u>:

(a) The load on the aluminum rod and the load on the steel tube **(40%)**
(b) The change in external diameter of the steel tube (magnitude and sign) **(25%)**
(c) The deflection of the rigid connecting plate (at **A**) **(25%)**
(d) Choose the correct answer **(10%)** -> Hooke's law is applicable for:
(I) Strain due to loadings; (II) Strain due to thermal expansion;
(III) The two above; (IV) Not relevant for strains

	Steel tube	Aluminum rod
Young's Modulus	$Es=29\cdot10^6$ PSI	$Ea=10.6\cdot10^6$ PSI
Shear Modulus	$Gs=11.2\cdot10^6$ PSI	$Ga=4\cdot10^6$ PSI
Coefficient of thermal expansion		$\alpha a=13\cdot10^{-6}$ 1/°F
Temperature drop		$\Delta Ta= - 40$°F
Dimensions: L=10 in.	$\phi D=1.6$ in $\phi d=1.5$ in.	$\phi d1=1.0$ in.





B

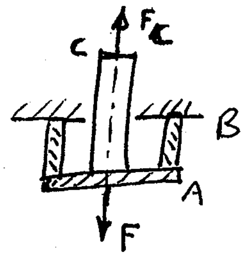
EMA 3702

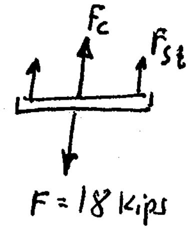
Oct. 19, 2004

QUIZ #2 - SOLUTION

VERSION B

(a) UNIAXIAL LOADING + 2 REACTIONS $\longrightarrow$ INDETERMINATE PROBLEM.
LET RELEASE THE RIGID SUPPORT AT C AND SAY THAT THE GEOMETRICAL CONSTRAIN AT C SHOULD BE $\delta_c = 0$ BY THE REACTING FORCE AT C





F.B.D. OF THE RIGID CONNECTING PLATE

$$+\uparrow \Sigma F = F_c + F_{St} - F = 0$$

$F_c = F_{Al}$ = THE FORCE ACTING ON THE ALUMINUM ROD

$$F_{Al} + F_{St} = F \quad (1)$$

THE CONSTRAIN $\delta_c = 0$

$$\delta_c = \delta_{AB} - \delta_{CA} = \delta_{St} - \delta_{Al} = 0$$

$$0 = F_{St}\frac{L_{St}}{A_{St}E_{St}} - F_{Al}\frac{L_{Al}}{A_{Al}E_{Al}} - \alpha_{Al}\Delta T L_{Al}$$

$$\delta_{St} = \frac{F_{St}\cdot L_{St}}{A_{St}\cdot L_{St}}$$

$$\delta_{Al} = \delta_{Al\,F_{Al}} + \delta_{Al\,T}$$

$$\delta_{Al\,F_{Al}} = \frac{F_{Al}\cdot L_{Al}}{A_{Al}\cdot E_{Al}}$$

$$\delta_{Al\,T} = \alpha_{Al}\cdot\Delta T\cdot L_{Al}$$

$$0 = F_{St}\frac{10''}{\frac{\pi(1.6^2-1.5^2)}{4}\cdot 29\cdot10^6} - F_{Al}\cdot\frac{20''}{\frac{\pi}{4}1^2\cdot 10.6\cdot10^6} - 13\cdot10^{-6}\cdot(-40)\cdot 20''$$

$$F_{Al} = 0.58954 F_{St} + 4329.115 \quad (2)$$

(1)+(2) $\rightsquigarrow$ $0.58954 F_{St} + 4329.115 + F_{St} = 18000$ lb

$$\underline{F_{St} = 8600.53 \text{ lb}}$$

$$\underline{F_{Al}} = F - F_{St} = 18000 - 8600.53 = \underline{9399.47 \text{ lb}}$$

(b) $\Delta D = -\nu_{St}\,\varepsilon_{x_{St}}\cdot D$

$$\nu_{St} = \frac{E_{St}}{2G_{St}} - 1 = \frac{29\cdot10^6}{2\cdot 11.2\cdot10^6} - 1 = 0.2946$$

$$\varepsilon_{x_{St}} = \frac{F_{St}}{A_{St}\cdot E_{St}} = \frac{+8600.53}{\frac{\pi(1.6^2-1.5^2)}{4}\cdot 29\cdot10^6} = 1.21808\cdot10^{-3} \ (=1218.1\mu)$$

$$\Delta D = -0.2946\cdot 1.218\cdot10^{-3}\cdot 1.6''$$

$$\underline{\underline{\Delta D = -5.7415\cdot10^{-4}\,''}} \text{ (SHRINK)}$$

(c) $\underline{\underline{\delta_A}} = \delta_{St} = \varepsilon_{x_{St}}\cdot L_{St} = +1.21808\cdot10^{-3}\cdot 10'' = \underline{\underline{+1.218\cdot10^{-4}\text{ in (down)}}}$

(d) (I) Hooke's law is applicable for strain due to loadings <u>AND NOT</u> for thermal strains

QUIZ # 2	Oct. 19, 2004			
NAME	P. I. D.	Version		
Adriana Bonilla	1321865	B	68	*
Alaa Maaliki	1503723	A	78	
Alfredo Suarez	1495210	B	75	*
Amrit Harripaul	1644446	B.	55	
Andres Zamorra	1397908	B	55	*
Anthony Jackson-Pownal	1277169	B	55	
Arias Carlos	1371993	B	58	*
Cindy Gomez	1390408	A	97	
Damian Harper	1399737	A	36	
Damien Hoyd LLOYD	1013644	A	45	
Daniel Mugruza	1258280	A	51	
Eric Inclan	1356002	A	35	
Ernesto Gutierrez	1281667			
Felipe Rendon	1334090	B	15	
Francis Fernandez	1026909	A	38	
Georgette Martinez	1339663	B	58	
Gonzalo Ocano	1021312	B	70	*
Grant Mesner	1366034	B	+63	*
Gustavo Barbera	1398821	A	+70	
Gustavo Jaramillo	1397120	A	50	
Hector Roos	1096368	A	~~35~~ 43	
Jorge Tercero	1014132	B	83	
Joseph Nichols	1373806	A	85	*
Juan Saluzzo	1372735	B	72	
Kern Wilson	1589330	A	55	
Luis A Sepulueda	1349774	B	+10	
Luke Seever	1106526	A	100	
Marcelo Beyra	1370183	B	66	*
Maribel Garcia	1322452	B	25	
Mario Valdivieso	1369320	A	71	*
Max Brand	1396896	A *	88	
Michael Patterson	1094554	~~A~~ B	95	
Michael Wolff	1103264	A	10	
Natalie Marshal	1306053	B	50	
Ottley Brathwaite	1630841	B	+0	*
Patrick Belizane	1629148	A	+10	
Paul Girata	1282007	A	53	
Robert Jordan	1305753	B	52	
Ruben Galeano	1352635	B	45	
Saleh Almutawa	1323191			
Shaka Harper	1012782	A	30	
Trevis Moorley	1399021	A	15	*
Trevor G. ~~S.~~ A. Harris ~~o~~ II.	1395585	A	73	
Verdi M. Mayer	1358420	A.	20	
William Betancourt	1398230	B	38	*
Yuri Lazzeretti	1093801	B	83	

PLEASE, WRITE THE VERSION OF YOUR QUIZ

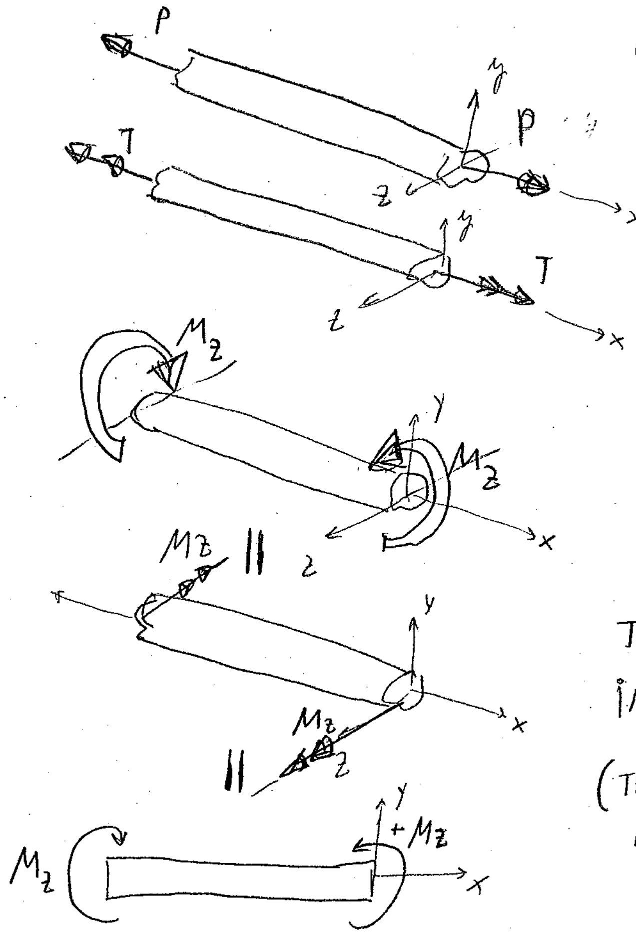
BENDING

INTRODUCTION

"A PRISMATIC MEMBER, IN WHICH IT CROSS SECTION CARRIES A BENDING MOMENT, IS UNDER BENDING"

A PRISMATC MEMBER - IS ALONG CONSTANT SECTION AREA MEMBER

A RELATIVELY LONG MEMBER UNDER BENDING IS CALLED A BEAM.



UNDER UNI-AXIAL LOADING
CROSS SECTION CARRIES AXIAL FORCE

UNDER TORSION
CROSS SECTION CARRIES TORQUE

UNDER BENDING
CROSS SECTION CARRIES BENDING MOMENTS

THE BENDING MOMENT MAY APPEAR IN Z AXIS, Y AXIS OR IN BOTH.

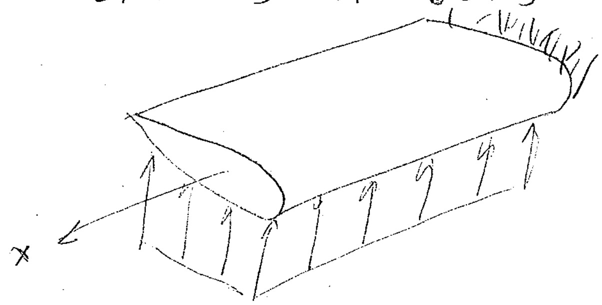
(THE DRAW SHOWS ONLY FOR Z AXIS BENDING MOMENT)

CHAPTERS 4, 5 & 6 DEALS WITH BENDING
WE ARE GOING TO LEARN IT AS FOLLOWS

CHAPTER 5 → REVIEW STATICS (FIRST 3 SECTIONS)
CHAPTER 4 → PURE BENDING
CHAPTER 5 → ANALYSIS AND DESIGN OF BEAMS FOR BENDING
CHAPTER 6 → SHEARING STRESSES OF BEAMS FOR BENDING

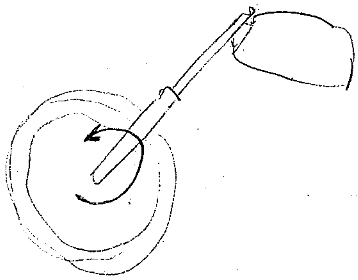
(4-5-2)

EXAMPLES OF $\underbrace{\text{BEAMS}}^{\text{PRISMATIC}}$ UNDER BENDING:

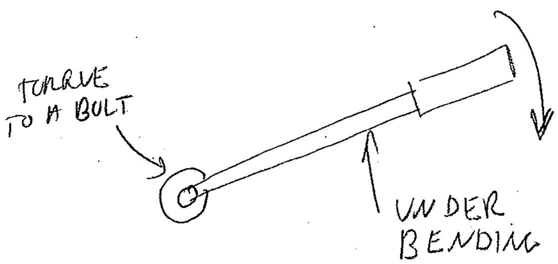


WING, OR A ROTOR BLADE

A TREE UNDER WIND



MOTORBIKE FRONT SUSPENSION SPECIALY WHILE APPLYING BRAKES

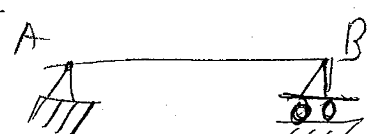


A TOOL HANDLE WHILE APPLYING TORQUE TO A BOLT

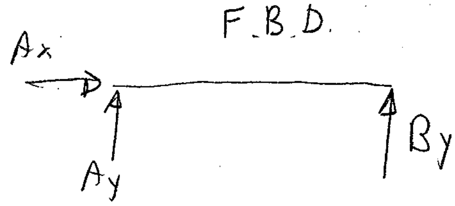
STATICS REVIEW

SUPPORTS

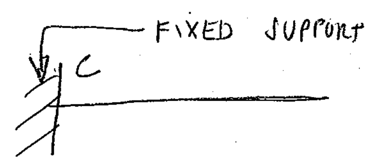
SIMPLY SUPPORTED BEAM



F.B.D.



CANTILEVER BEAM



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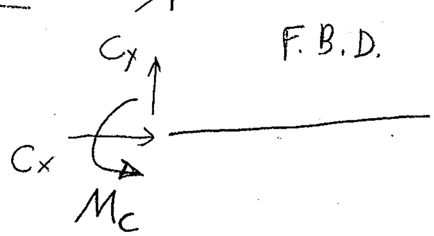
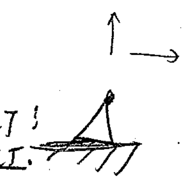
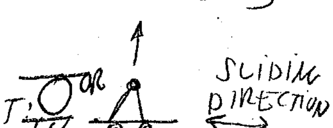


② SIMPLE SUPPORT: PIN CONNECT.



IS ABLE TO REACT WITH FORCE IN ALL DIRECTIONS SO WE DRAW THE COMPONENTS A_x, A_y

① SIMPLE SUPPORT: ROLLER CONNECT



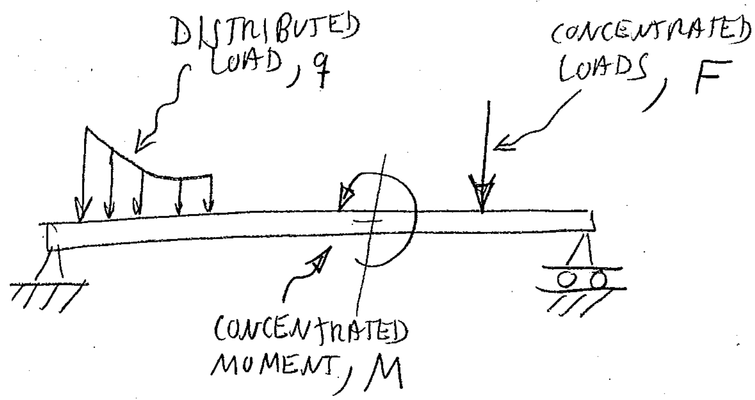
IS ABLE TO REACT NORMAL TO THE SLIDING DIRECTION

③ FIXED SUPPORT IS LIKE A WALL - IS ABLE TO REACT WITH FORCE IN ALL DIRECTIONS AND WITH MOMENTS.

(4-5-3)

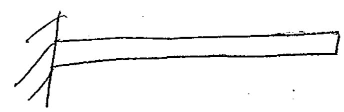
LOADS

EXAMPLES:

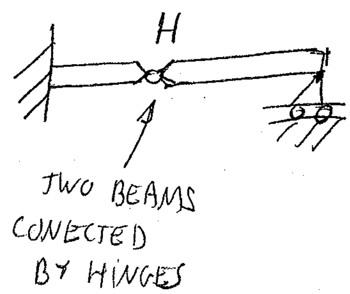


BEAMS

CONTINUOS BEAMS

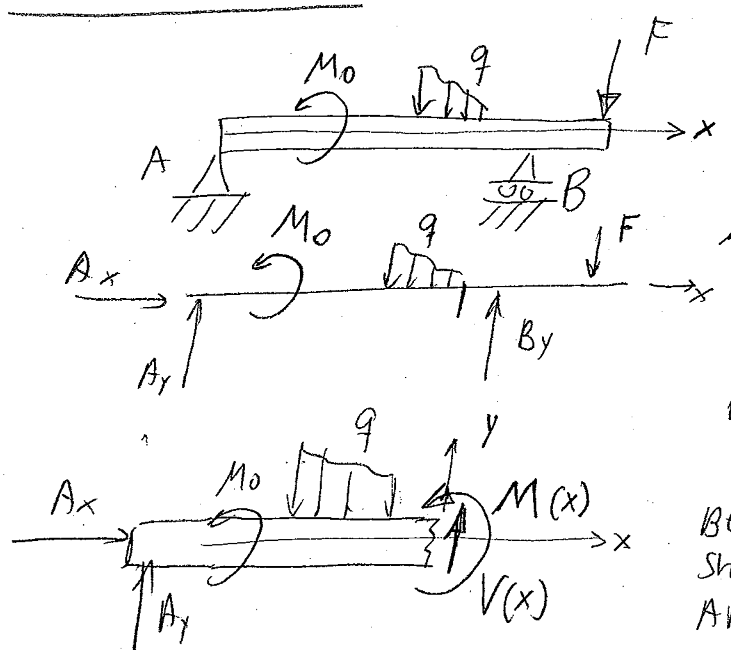


HINGED BEAMS



THE HINGE TRANSFERS FORCES BUT DOESN'T TRANSFER BENDING MOMENTS

INTERNAL LOADS



EXAMPLE

COORDINATE SYSTEM

F.B.D

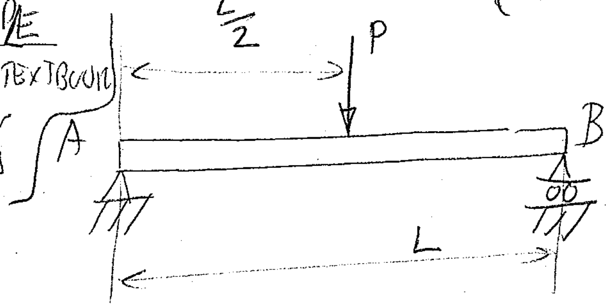
CUT AT ANY x, WE WILL DEFINE THE POSITIVE INTERNAL BENDING MOMENT AND SHEARING FORCES THAT ARE PRODUCED DUE TO THE EXTERNAL LOADS

$M(x)$ & $V(x)$ WILL BE CALCULATED USING THE EQUILIBRIUM EQUATIONS AND BE PLOTED ON SHEAR AND BENDING MOMENT DIAGRAMS

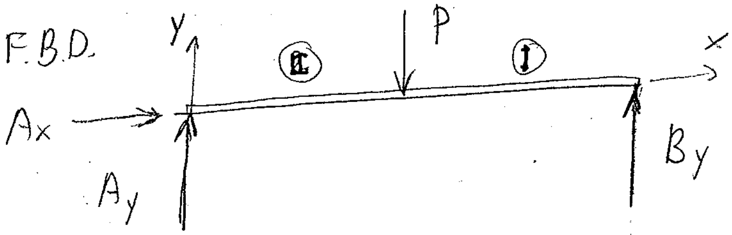
(coming a, b)

EXAMPLE

5.01 IN TEXTBOOK BUT USING OUR PROCESS



F.B.D.



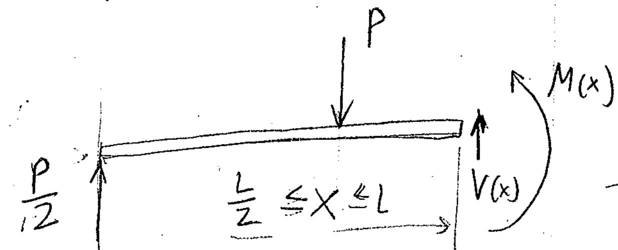
I EQUILIBRIUM EQUATIONS:

$\Sigma F_x = 0,\ A_x = 0$

$\Sigma F_y = A_y + B_y - P = 0$

$\circlearrowleft \Sigma M_A = 0 = B_y L - P\frac{L}{2}$

$B_y = \frac{P}{2}\ ;\ A_y = \frac{P}{2}$



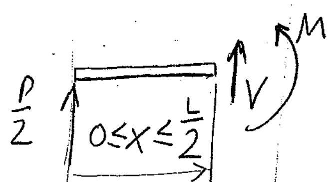
II SECTIONS ON THE BEAM FROM RIGHT TO LEFT

(I) $\Sigma F_y = V_1 + \frac{P}{2} - P = 0$

$V_1 = \frac{P}{2}$

$\circlearrowleft \Sigma M_A = -P\frac{L}{2} + V_1 \cdot x + M_1 = 0$

$M_1 = P\frac{L}{2} - \frac{P}{2}x = \frac{P}{2}(L-x)$



(II) $\Sigma F_y = \frac{P}{2} + V_2 = 0$

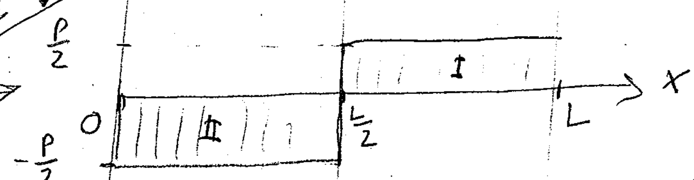
$V_2 = -\frac{P}{2}$

$\circlearrowleft \Sigma M_A = V_2 \cdot x + M_2 = 0$

$M_2 = -V_2 \cdot x = \frac{P}{2}x$

PAY ATTENTION TO OUR DEFINITION OF POSITIVE V

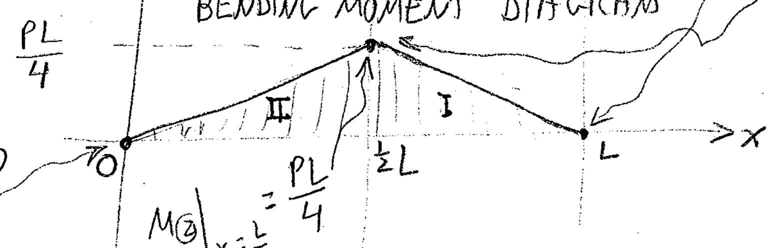
SHEAR FORCE DIAGRAM



$M_1\big|_{x=L} = \frac{P}{2}(L-L) = 0$

$M_1\big|_{x=\frac{L}{2}} = \frac{P}{2}\left(L - \frac{L}{2}\right)$

BENDING MOMENT DIAGRAM



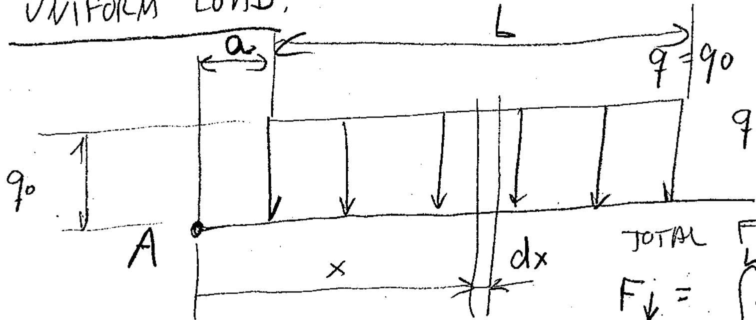
$M_2\big|_{x=0} = \frac{P}{2} \cdot 0$

$M_2\big|_{x=\frac{L}{2}} = \frac{PL}{4}$

(4-5-4 a)

MOMENTS AND SHEAR FORCES PRODUCED BY DISTRIBUTED LOADS:

UNIFORM LOAD:



$q = q_0 = \text{const}$

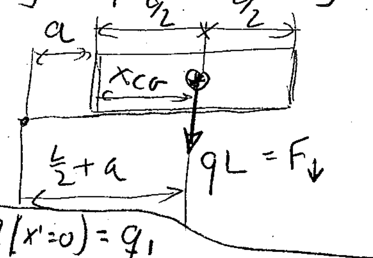
$q = \dfrac{dF}{dx}$

TOTAL FORCE:

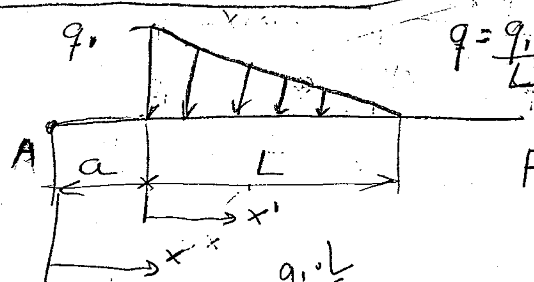
$$F_\downarrow = \int_0^L dF = \int_0^L q\,dx = q_0 \int_0^L dx = q_0 L$$

$$M_A = \int dF \cdot x = \int_a^{a+L} q\,x\,dx = q_0 \left[\frac{x^2}{2}\right]_a^{a+L} = \frac{1}{2} q_0 \left\{(a+L)^2 - a^2\right\} = \frac{1}{2} q_0 \left\{L^2 + 2aL\right\}$$

$$M_A = \underbrace{q_0 L}_{F_\downarrow}\Big(\underbrace{\tfrac{L}{2}}_{x_{CG}} + a\Big) \rightsquigarrow \text{eq. to}$$



LINEAR DISTRIBUTED LOAD:



$$q = \frac{q_1}{L}(L - x') \begin{cases} q(x'=0) = q_1 \\ q(x'=L) = 0 \end{cases}$$

$$F_\downarrow = \int_0^L dF = \int_0^L q\,dx' = \frac{q_1}{L}\int_0^L (L - x')\,dx' = \frac{q_1}{L}\left(Lx' - \frac{x'^2}{2}\right)\Big|_0^L$$

$$A = \frac{q_1 \cdot L}{2} \longleftrightarrow F_\downarrow = \frac{1}{2} q_1 L$$

$x = x' + a$

$dx = dx'$

$$M_A = \int dF \cdot x = \int q\,dx \cdot x$$

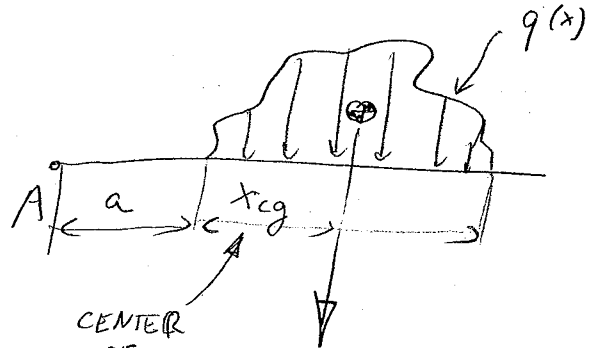
$$M_A = \int dF \cdot x = \int_0^L \frac{q_1}{L}(L - x')(x' + a)\cdot dx' = \frac{q_1}{L}\int_0^L \left(-x'^2 + (L-a)x' + aL\right) dx'$$

$$M_A = \frac{q_1}{L}\left[-\frac{1}{3}x'^3 + (L-a)\frac{x'^2}{2} + aLx'\right]_0^L = \frac{q_1}{L}\left[-\frac{1}{3}L^3 + \frac{1}{2}L^3 - \frac{1}{2}aL^2 + aL^2\right] = \frac{q_1}{L}\left(\frac{1}{6}L^3 + \frac{1}{2}aL^2\right)$$

$$M_A = \underbrace{\tfrac{1}{2}q_1 L}_{F_\downarrow}\Big(\underbrace{\tfrac{1}{3}L}_{x_{CG}\text{ OF A TRIANGLE}} + a\Big) = F_\downarrow (x_{CG} + a)$$

(4-5-4b)

FOR THE GENERAL CASE

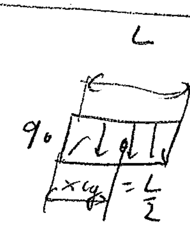
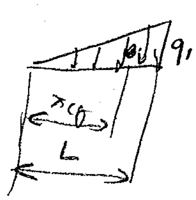


CENTER OF GRAVITY OF THE q(x) AREA

$F = A_q$

AREA $\underbrace{\text{UNDER}}$ q(x)

$$\circlearrowleft M_A = -F(a + x_{cg}) = -A_q(a + x_{cg})$$

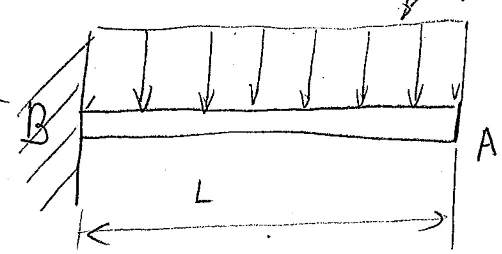
		A_q	x_{cg}
UNIFORM DISTRIBUTED LOAD		$q_0 \cdot L$	$\frac{L}{2}$
LINEAR TRIANGLE		$\frac{q_1 L}{2}$	$\frac{2}{3}L$
⋮	⋮	⋮	⋮

(4-5-5)

EXAMPLE

5.02, BUT USING
OUR METHOD

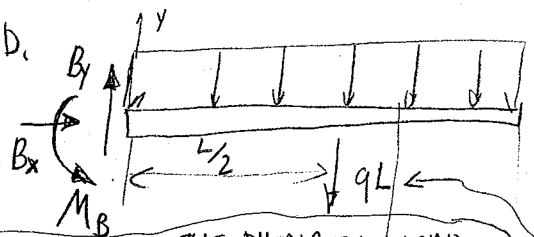
distributed load $q = q_0$ $\quad q = \frac{dF}{dx}$



CHANGE THE FIX SUPPORT

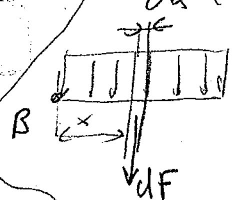
F.B.D.

FIX SUPPORT REACTIONS



((1)) THE DISTRIBUTED LOAD APPLIES A TOTAL FORCE OF $F_q = \int_0^L q \, \underbrace{dx}_{dF} = q_0 \int_0^L dx = q_0 \cdot L$

((3)) AROUND B IT APPLIES A MOMENT OF: $q = \frac{dF}{dx} \rightsquigarrow dF = q\,dx$



$$M_q\Big|_B = \int_0^L (q\,dx)\cdot x = q_0 \left[\frac{x^2}{2}\right]_0^L = \frac{q_0 L^2}{2}$$

OR $M_q\Big|_B = (q_0 L)\cdot \frac{L}{2}$ ← FOR CONSTANT $q = q_0$

TOTAL FORCE — CENTER OF THE DISTRIBUTION

STEP I

REACTIONS CALCULATIONS

EQ. EQ.:

$\Sigma F_x = 0 = B_x$

((2)) $\Sigma F_y = 0 = B_y - q_0 \cdot L$

$B_y = q_0 L$

((4)) $\Sigma M\Big|_B = M_B - (q_0 L)\cdot \frac{L}{2} = 0$

$M_B = q_0 \frac{L^2}{2}$

STEP II

AT x:

$\Sigma F_y = V + B_y - q_0 x = 0$

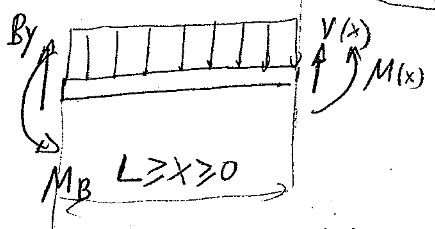
$\underline{\underline{V}} = q_0 x - B_y = \underline{\underline{q_0(x-L)}}$

$\Sigma M\Big|_B = 0 = V\cdot x + M - \frac{q_0 x^2}{2} + M_B$

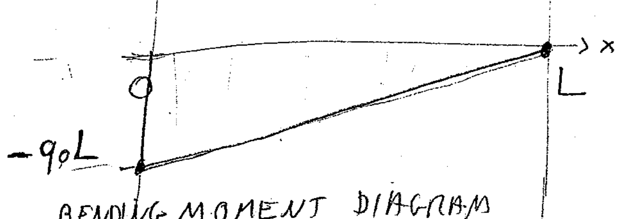
$M = \frac{q_0 x^2}{2} - \underbrace{\frac{q_0 L^2}{2}}_{M_B} - \underbrace{q_0(x-L)}_{V}\cdot x$

$= q_0\left(\frac{1}{2}x^2 - \frac{1}{2}L^2 - x^2 + L\cdot x\right)$

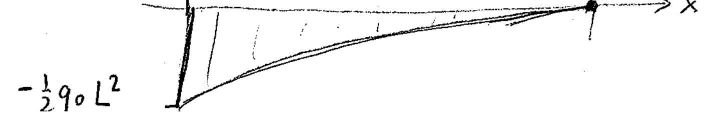
$\underline{\underline{M}} = -\frac{1}{2}q_0\left(x^2 - 2Lx + L^2\right) = \underline{\underline{-\frac{1}{2}q_0(L-x)^2}}$



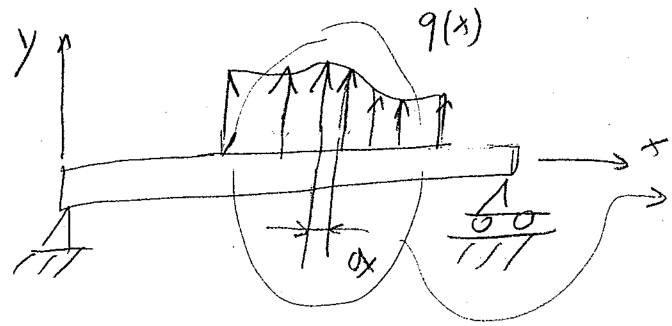
SHEAR FORCE DIAGRAM

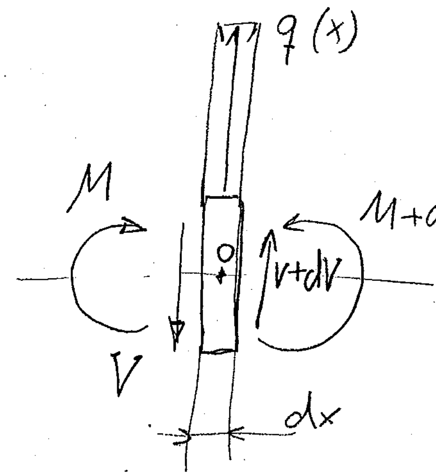


BENDING MOMENT DIAGRAM



RELATIONS AMONG LOAD, SHEAR, AND BENDING MOMENT





dx SMALL ENOUGH IN ORDER TO ASSUME q(x) CONSTANT ALONG dx

EQ. EQ:

$$\sum F_y = q(x)dx - V + V + dV = 0$$

$$dV = -q(x)\,dx$$

$$\boxed{V(x) - V_0 = -\int_{x_0}^{x} q(x)\,dx}$$

$$\circlearrowleft + \quad \sum M_z\Big|_O = -M + V\frac{dx}{2} + (V+dV)\frac{dx}{2} + M + dM = 0$$

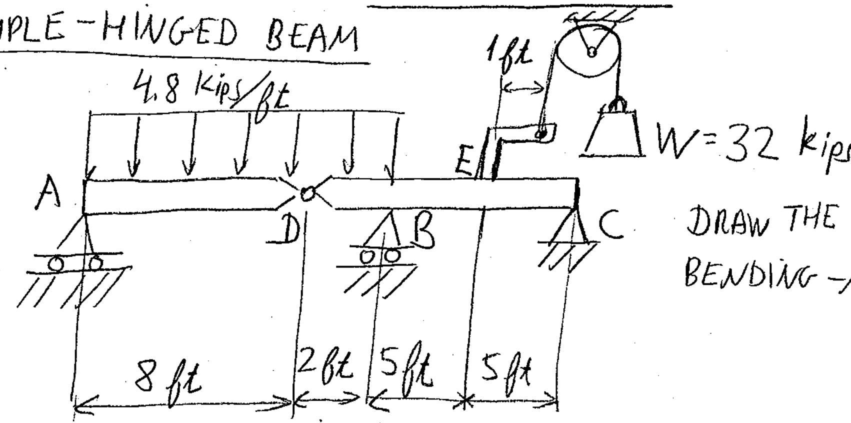
$$V dx + \underbrace{dV\frac{dx}{2}}_{0} + dM = 0$$

$$dM = -V dx$$

$$\boxed{M(x) - M_0 = -\int_{x_0}^{x} V\,dx}$$

USING THE ABOVE RELATIONS, WE DON'T NEED TO USE THE EQUILIBRIUM EQUATIONS EVERY SECTION!

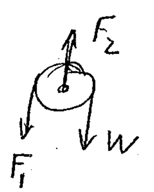
EXAMPLE - HINGED BEAM



DRAW THE SHEAR AND BENDING-MOMENT DIAGRAMS

STEP # 1 - TRANSLATE TO LOADS

SHIFT OF THE ARM AND MECHANISM SHOWN IN E WITH EQUIVALENT LOADS.

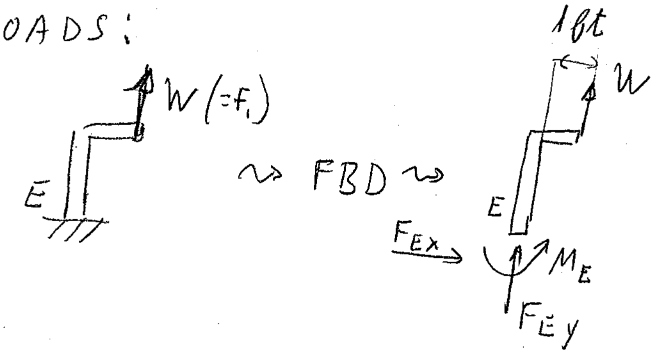


$\Sigma F_y = 0 = F_2 - F_1 - W$

BECAUSE OF THE PULLEY: $\underline{\underline{F_1 = W}}$

$(F_2 = 2W)$

COPYING LOADS:



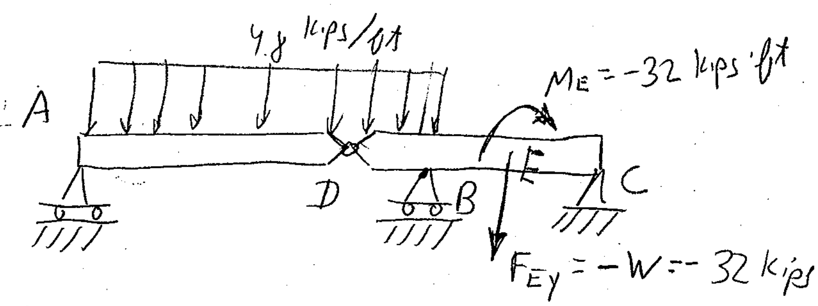
$\Sigma F_x = 0, \ F_{Ex} = 0$

$\Sigma F_y = W + F_{Ey} = 0$

$F_{Ey} = -W$

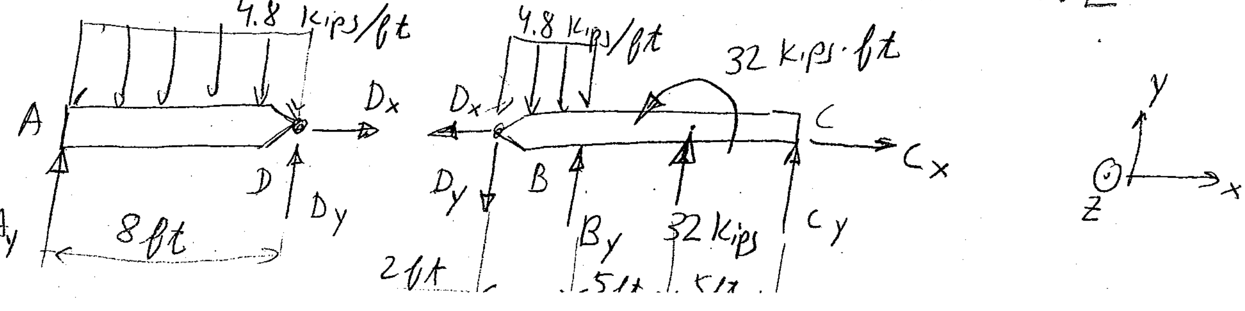
$\Sigma M|_E = M_E + W \cdot 1 ft = 0$

$M_E = -32$ kips·ft



STEP # 2 → F.B.D:

THE HINGED BEAM IS DISMEMBERED INTO TWO BEAMS, AND PIN REACTIONS ARE INTRODUCED AT THE HINGE



STEP # 3 - REACTIONS

EQUILIBRIUM EQUATIONS:

FOR THE TWO BEAMS THERE ARE 6 EQUILIBRIUM EQUATIONS
$2 \times (\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M_z = 0)$ AND 6 UNKNOWNS $(A_y, \underbrace{D_x, D_y}_{\text{IN COMMON}}, B_y, C_x, C_y)$

SO THE PROBLEM IS DETERMINATE AND THE REACTIONS CAN BE CALCULATED BY USING THE EQUILIBRIUM EQUATIONS.

FOR THE LEFT BEAM AD

$$+\Sigma F_x = D_x = 0$$
$$+\Sigma F_y = A_y + D_y - 4.8 \cdot 8' = 0$$
$$\circlearrowleft + \Sigma M_z \Big|_A = D_y \cdot 8 - (4.8 \cdot 8') \cdot \frac{8'}{2} = 0$$

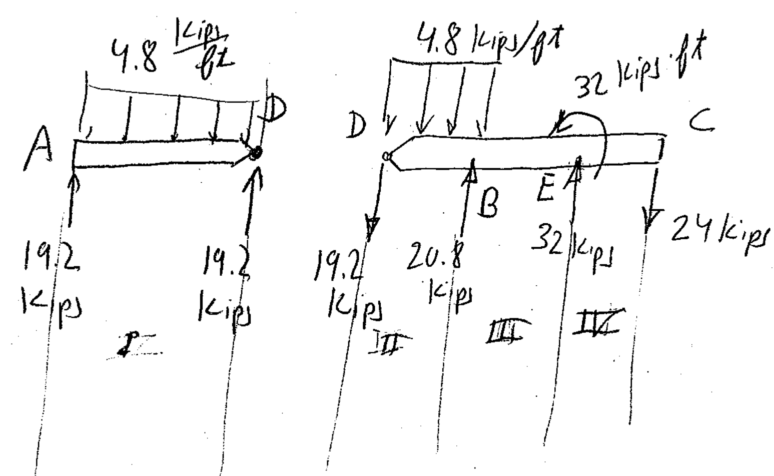
FOR THE RIGHT BEAM DC

$$+\Sigma F_x = -D_x + C_x = 0$$
$$+\Sigma F_y = -D_y - 4.8 \cdot 2' + B_y + 32 \text{ kips} + C_y = 0$$
$$\circlearrowleft + \Sigma M_z \Big|_D = -(4.8 \cdot 2') \cdot \frac{2'}{2} + B_y \cdot 2' + 32 \text{ kips} \cdot 7' + 32 \text{ kips} \cdot \text{ft} + C_y \cdot 12' = 0$$

$D_x = 0$
$D_y = +19.2$ kips
$A_y = +19.2$ kips

$C_x = 0$
$C_y = -24$ kips
$B_y = +20.8$ kips

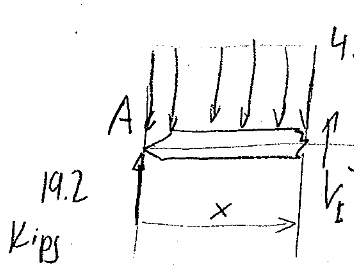
STEP # 4 - DIVIDING INTO SECTIONS



4-5-8

STEP #5 - SHEAR FORCE AND BENDING MOMENT AT THE DIFFERENT SECTIONS

SECTION I $\quad 8' \geq x \geq 0$



USING RELATION

$V_I|_{x=0} = -19.2$ kips

$V - V_0 = -\int q\,dx$

$V + 19.2 = -\int_0^x -4.8\,dx = 4.8x$

$V_I = 4.8x - 19.2$

$M_0 = 0$

$M_I - M_0 = -\int_0^x V\,dx = -\int_0^x (4.8x - 19.2)\,dx$

$M_I = -\frac{4.8}{2}x^2 + 19.2x = -2.4x^2 + 19.2x$

WILL HAVE EXTREMUM WHEN $V_I = 0$, $x = 4$
MAXIMUM BECAUSE q IS NEGATIVE

USING EQ. EQ.

$\Sigma F_y = -4.8 \cdot x + 19.2 + V_I = 0$

$V_I = 4.8x - 19.2$

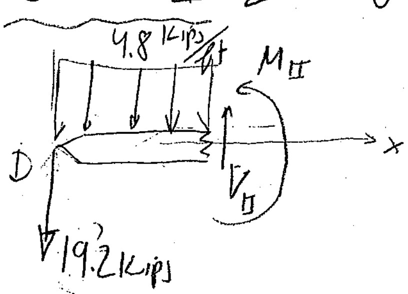
$\Sigma M_z|_A = -4.8x \cdot \frac{x}{2} + V_I + M_I = 0$

$M_I = -2.4x^2 + 19.2x$

CALCULATED VALUES

	A $x=0$	$x=4'$	D $x=8'$
V_I (kips)	-19.2	0	19.2
M_I (kips·ft)	0	38.4	0

SECTION II $\quad 2' \geq x \geq 0$



USING RELATION

$V_{II}|_{x=0} = +19.2$ kips

$V_{II} - 19.2 \text{ kips} = -\int_0^x -4.8\,dx$

$V_{II} = 4.8x + 19.2$ kips

$V_{II} = 0 \Rightarrow x = -4$, THE EXTREMUM FOR M_{II} OUT OF RANGE $2' \geq x \geq 0$

$M_{II}|_{x=0} = 0$

$M_{II} - 0 = -\int_0^x (4.8x + 19.2)\,dx$

$M_{II} = -2.4x^2 - 19.2x$

EQ. EQ.

$+\uparrow \Sigma F_y = -19.2 - 4.8x + V_{II} = 0$

$V_{II} = 19.2 + 4.8x$

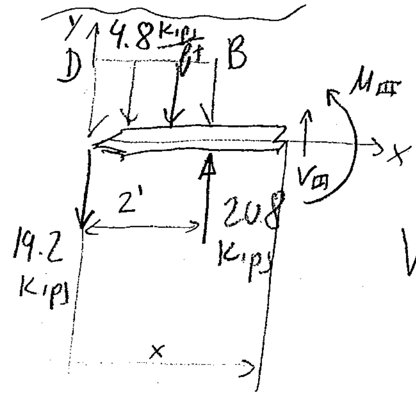
$\circlearrowleft+ \Sigma M_z|_D = -4.8 \cdot x \cdot \frac{x}{2} + V_{II} \cdot x + M_{II} = 0$

$M_{II} = -2.4x^2 - 19.2x$ kips·ft

	D $x=0$	B $x=2'$
V_{II} [kips]	19.2	28.8
M_{II} [kips]	0	-48

4-5-9

SECTION III $7' \geq x \geq 2'$



USING RELATIONS

$$V_{III}(2') = V_{II}(2') - 20.8$$
$$= 28.8 - 20.8 = 8 \text{ kips}$$
$$V_{III} - V_{III}(2') = \int_{2'}^{x} -q \, dx$$
$$q = 0, \int = 0$$
$$\underline{V_{III} = 8 \text{ kips}}$$
$$M_{III}(2') = M_{II}(2') = -48 \text{ kips}\cdot\text{ft}$$
$$M_{III} - M_{III}(2') = \int_{2'}^{x} -V_{III} \, dx = -8x + 8\cdot 2'$$
$$\underline{M_{III}} = -8x - 16 + 48 = \underline{-8x + 32 \text{ kips ft}}$$

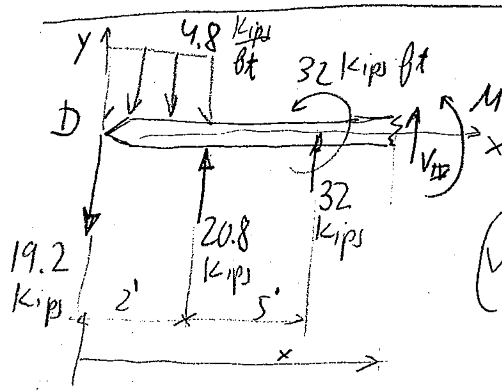
LINEAR LINE WITH NEGATIVE SLOPE BECAUSE V_{III} POSITIVE

EQ. EQ.

$$+\uparrow \sum F_y = -19.2 - 4.8\cdot 2' + 20.8 + V_{III} =$$
$$\underline{V_{III} = 8 \text{ kips}}$$
$$+\circlearrowleft \sum M_z \Big|_D = -4.8\cdot 2'\cdot \frac{2'}{1} + 20.8\cdot 2' +$$
$$+ V_{II}\cdot x + M_{II} = 0$$
$$\underline{M_{II} = -8x - 32 \text{ kips ft}}$$

	B $x = 2'$	E $x = 7'$
V_{III} [kips]	8	8
M_{III} [kips ft]	-48	-88

SECTION IV $7' \geq x \geq 12'$



USING RELATIONS

$$V_{IV}\Big|_{x=7'} = V_{III}\Big|_{x=7'} - 32 \text{ kips}$$
$$V_{IV}\Big|_{x=7'} = 8 - 32 = -24 \text{ kips}$$
$$\left(V_{IV} - V_{IV}\Big|_{x'=7'} = \int_{7'}^{x} -q \, dx = 0\right) \text{ CONSTANT } q = 0$$
$$\underline{V_{IV} = -24 \text{ kips}}$$
$$M_{IV}\Big|_{x=7'} = M_{III}\Big|_{x=7'} - 32 \text{ kips ft}$$
$$= -88 - 32 = -120 \text{ kips ft}$$
$$M_{IV} - (-120) = \int_{7'}^{x} -(-24) \, dx = 24x - 168$$
$$\underline{M_{IV} = 24x - 288}$$

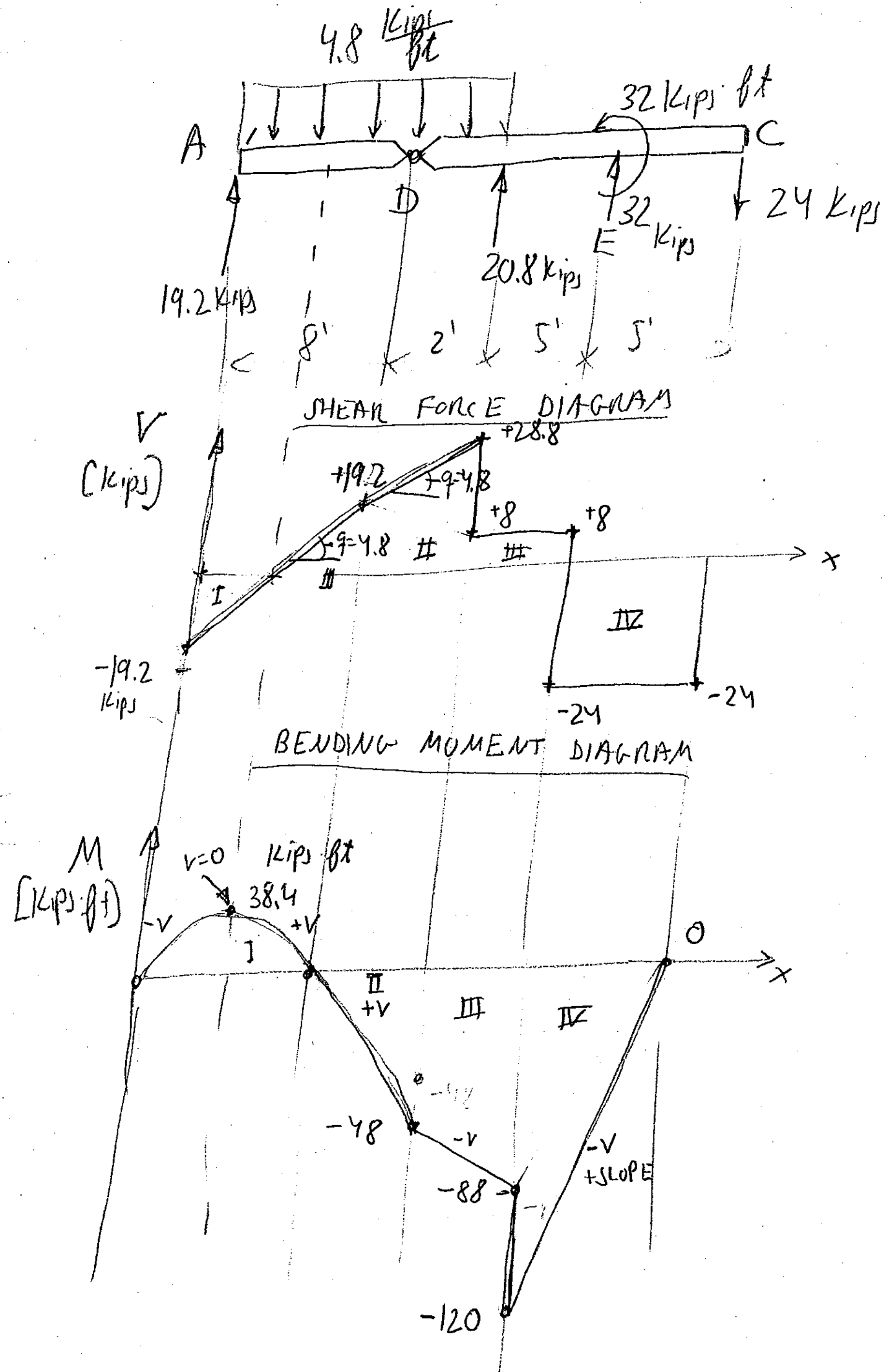
EQ. EQ.

$$+\uparrow \sum F_y = -19.2 - 4.8\cdot 2' + 20.8 + 32 + V_{IV} = 0$$
$$\underline{V_{IV} = -24 \text{ kips}}$$
$$+\circlearrowleft \sum M_z \Big|_D = -4.8\cdot 2'\cdot \frac{2'}{2} + 20.8\cdot 2' + 32\cdot 7' + 32 \text{ kips ft} + V_{IV}\cdot x + M_{IV} = 0$$
$$\underline{M_{IV} = 24x - 288 \text{ kips ft}}$$

	E	C
V_{IV} (kips)	-24	-24
M_{IV} (kips)	-120	0

FORCE 4-5-10

STEP #6 — SHEAR FORCE AND ~~BENDING MOMENT~~ DIAGRAMS



SHEAR FORCE DIAGRAM

1. DRAW THE SHEAR FORCES AT THE LIMITS OF THE DIFFERENT SECTIONS (FROM TABLES WE WROTE IN THIS PROBLEM)

2. COMPLETE CURVES ACCORD. TO q CHARACTERISTICS

$q = 0 \rightsquigarrow V$ CONSTANT

$q = \text{const} \rightsquigarrow V$ LINEAR SLOPE $-q$

$q =$ LINEAR $\rightsquigarrow V$ PARABOLA

MAX V AT $q = 0$

POSITIVE SLOPE FOR NEGATIVE V AND THE OPPOSITE

HOMEWORK

3.139, 3.145, 3.146, 5.6, 5.12, 5.16

FOR Tusday, Nov 2

ADD: CALCULATE THE ANGLE OF TWIST FOR $L = 80\,in$, $G = 11 \cdot 10^6$ PSI

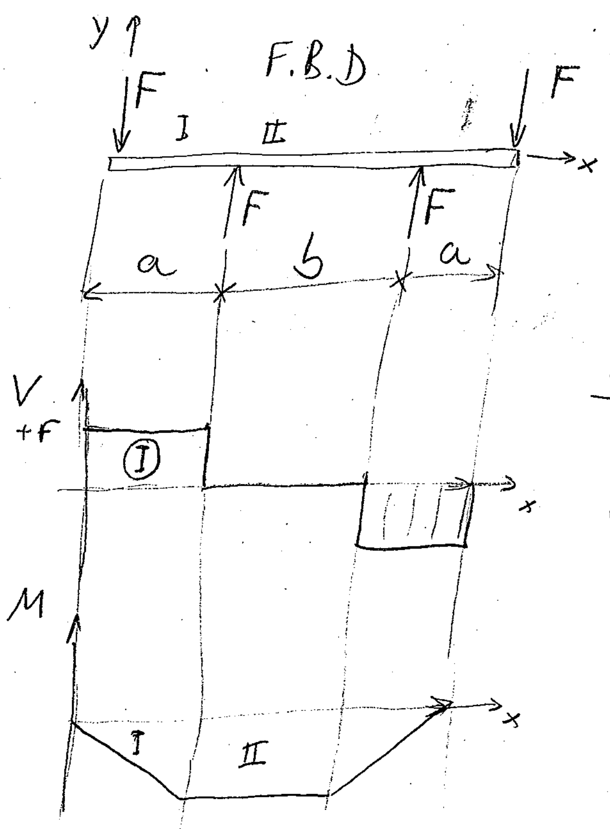
E) CALCULATE THE ANGLE OF TWIST FOR $L = 2\,m$, $G = 77\,GPa$

4. PURE BENDING

PURE BENDING IS A LOADING SITUATION IN WHICH THE ONLY LOAD ON THE CROSS SECTION OF THE BEAM IS A BENDING MOMENT (THERE ARE NO SHEAR FORCES NOR AXIAL FORCES NOR TORQUES)

$F_x, T, V_y, V_z = 0$

THE CLASSIC WAY TO RECEIVE PURE BENDING IS SHOWN BY THE FOLLOWING LOADING:



ALREADY IN EQUILIBRIUM

SYMETRIC

SECTION I:

$V(0) = +F$ $\qquad M(0) = 0$

$V(x) = const$ $\qquad M(x) = LINEAR$

$V(a) = +F$ $\qquad M(a) = -F \cdot a$

SECTION II:

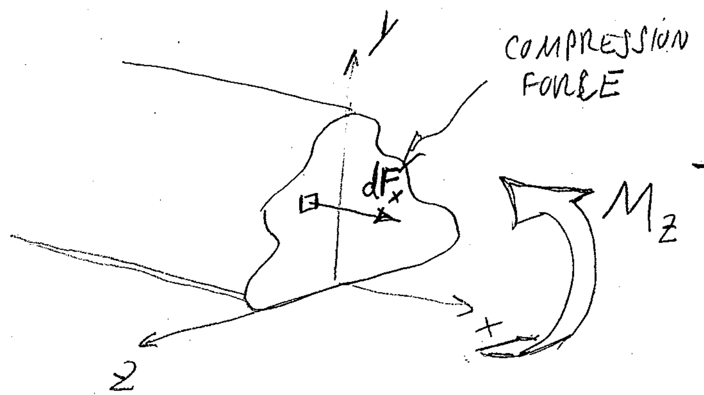
$V(a) = 0$ $\qquad M(a) = -F \cdot a$

$V(x) = const$ $\qquad M(x) = LINEAR$

$V(a+b) = 0$ $\qquad M(a+b) = -F(a+b) + Fb = -F \cdot a$

SECTION II $\rightsquigarrow$ UNDER PURE BENDING

SYMMETRIC MEMBER IN PURE BENDING



NO SHEARING
NO EXTERNAL AXIAL LOADS

EQUILIBRIUM

$$+y \cdot dF_x + dM_z = 0$$

$$dM_z = y\,dF_x$$

$$\sigma_x = -\frac{dF_x}{dA} \quad \text{TENSION}$$

$$dM_z = -y\,\sigma_x\,dA$$

$$M_z = -\int y\,\sigma_x\,dA \qquad (1)$$

NO EXTERNAL AXIAL LOADS $\rightsquigarrow \int dF_x = 0$

$$(-)\int \sigma_x\,dA = 0 \qquad (2)$$

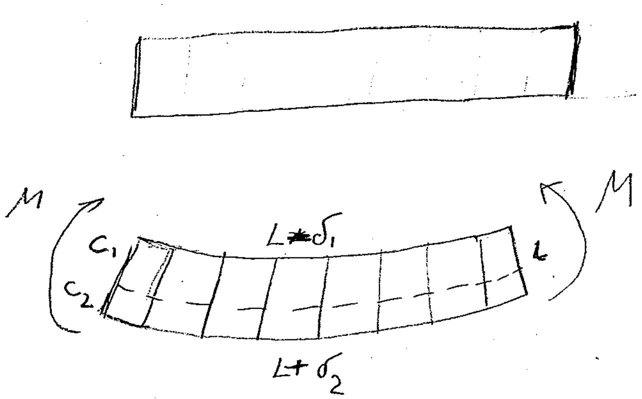
NO MOMENT COMPONENTS AROUND y:

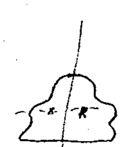
$$dF_x\,z = dM_y$$

FOR SYMETRIC CROSS SECTION, THE PROBLEM SYMMETRIC SO FROM AROUND z

$$(-)\int z\,\sigma_x\,dA = M_y = 0 \qquad (3)$$

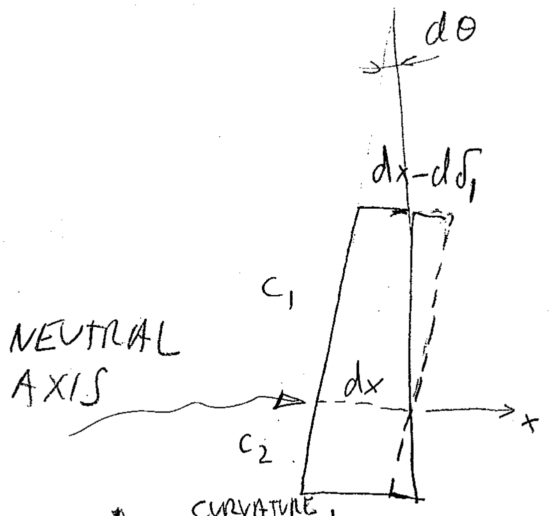
DEFORMATIONS

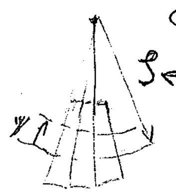




ASSUMPTIONS

1. SYMMETRIC LOADING - SYMMETRY STAYS AFTER DEFORMATION
2. SAME DEFORMATIONS AT ANY x (ONLY BENDING MOMENT APPLIE.
3. NO ANGULAR DISTORSION, THERE IS NO SHEARING RIGHT ANGLES STAYS 90^0





$$\frac{d\delta_1}{c_1} = \frac{d\delta_2}{c_2}$$

$$\frac{d\delta_1}{dx\cdot c_1} = \frac{d\delta_2}{dx\cdot c_2} =$$

$$E\cdot\frac{\varepsilon_{x_1}}{c_1} = \frac{\varepsilon_{x_2}}{c_2}\cdot E = \frac{\varepsilon_{x_y}}{y}\cdot E$$

$$\frac{\sigma_{x1}}{c_2} = \frac{\sigma_{x2}}{c_2} = \frac{\sigma_{x_y}}{\cancel{y}}$$

$$\sigma_{x(y)} = \frac{\sigma_{x_1}}{c_1}\, y$$

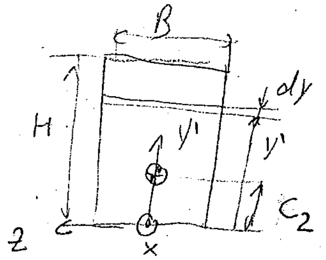
$$\frac{dx+\delta_x}{\rho - y} = \frac{dx}{\rho} = d\theta$$

$$\frac{1+\varepsilon_x}{\rho - y} = \frac{1}{\rho} \rightarrow \boxed{\varepsilon_x = \frac{-y}{\rho}}$$

LET SUBMIT THE ABOVE RELATON ON THE EQUILIBRIUM EQUATIONS (1), (2), (3):

$$(2)\quad \int \sigma_x\, dA = 0 = \frac{\sigma_{x_1}}{c_1}\int y\, dA = 0 \rightsquigarrow \int y\, dA = 0$$

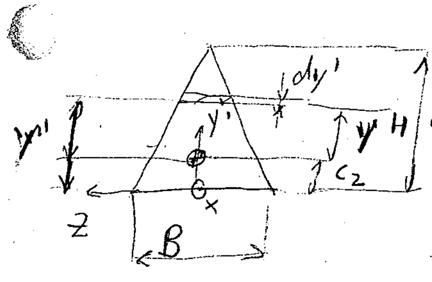
IT IS CALLED THE FIRST MOMENT OF THE CROSS SECTION, IT SAYS US WHAT c_2 SHOULD BE IN ORDER TO CANCEL AXIAL LOADS (CENTROID)



$$\left.\begin{array}{l} dA = dy'\cdot B \\ y = y' - c_2 \end{array}\right\} \int_0^H (y'-c_2)\, dy'\, B;\quad \frac{\cancel{H}'^2}{2} - c_2\cancel{H} = 0$$

$c_2 = \frac{1}{2}H$ THE PLACE OF THE NEUTRAL AXIS ON A RECTANGLE

FOR A TRIANGLE:



$y = y' - c_2$

$dA = dy'\cdot \underbrace{(H-y')\frac{B}{H}}$

$$\int_0^H (y'-c_2)\, dy'\, (H-y')\frac{B}{H} = 0$$

$$\frac{B}{H}\int_0^H \left(-y'^2 - c_2 H + (H+c_2)y'\right) dy' = 0$$

$$\frac{B}{H}\left[-\frac{y'^3}{3} - c_2 H y' + \frac{H+c_2}{2}\, y'^2\right]_0^H = 0$$

4-4

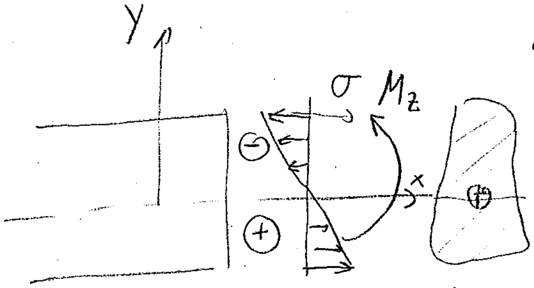
$$\frac{B}{H}\left(-\frac{H'^3}{3}-C_2H^2+\frac{H^3}{2}+\frac{1}{2}C_2H^2\right)=0$$

$$\frac{1}{6}H+\frac{1}{2}C_2=0 \rightsquigarrow \underline{C_2=\frac{1}{3}H}$$

THE NEUTRAL AXIS OF A TRIANGLE CROSS SECTION (BASE DOWN) PASSES AT $\frac{1}{3}H$ FROM IT BASE.

WE PLACE THE COORDINATE SYSTEM AT C_2, TO LET THE CROSS-SECTION TO REACT ~~FOR~~ A PURE BENDING MOMENT, THEN:

$$(1)\quad \left.\begin{aligned} M_z &= -\int y\,\sigma_x\,dA \\ \sigma_x &= \frac{\sigma_{x_1}}{c_1}y \end{aligned}\right\} M_z=-\int\frac{\sigma_{x_1}}{c_1}y^2dA=-\frac{\sigma_{x_1}}{c_1}\int y^2dA$$

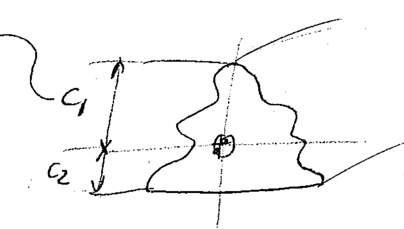


$$\frac{\sigma_{x_1}}{c_1}=\frac{\sigma_x}{y}=-\frac{M_z}{\int y^2dA}$$

$$\sigma_x=-\frac{M_z\,y}{\int y^2dA}=-\frac{M_z\,y}{I_z}$$

MOMENT OF INERTIA OR SECOND MOMENT } OF THE CROSS SECTION AROUN y AXIS, WITH RESPECT TO ITS CENTROID $=I_z$

$$\sigma_{MAX}=|\sigma_x|_{MAX}=\frac{M_z\,C_{MAX}}{I_z}$$



$\boxed{\frac{I_z}{C_{MAX}}}$ IS CALLED THE "ELASTIC SECTION MODULUS" $=S$

$$\sigma_{MAX}=\frac{M_z}{S}$$