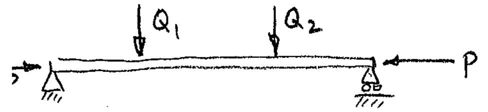
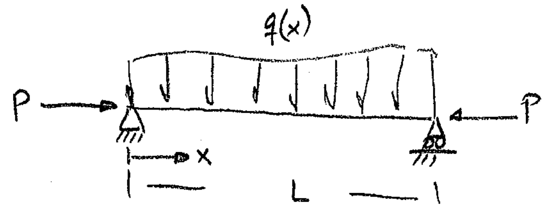
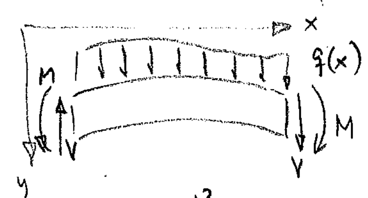


בדרך כלל אנחנו יכולים להשתמש בסופרפוזיציה כדי לקבל את התזוזה של קורה או מוט עם כמה עומסים.
אבל כאשר יש קורה או מוט שבהם יש כוח צירי כמו P שפועל על המוט או קורה עם הכוחות Q_1, Q_2 אי אפשר לקבל את התזוזות בסופרפוזיציה. זה בגלל שהתזוזות מתייחסות בדרך לינארית ל Q_2, Q_1 אבל היחס לא לינארי ל-P.





$$M = EI\frac{d^2v}{dx^2} \quad -V = \frac{dM}{dx} \quad -q = \frac{dV}{dx}$$

אבל הכוח P מכניס לנוסחה מומנט של Pv. המומנט הזה קורה לגזירה של $P\frac{dv}{dx}$ וגם עומס מפורס של $-P\frac{d^2v}{dx^2} = q$. לכן הנוסחה שפועלת במצב הזה היא

$$\frac{d^2M}{dx^2} + \lambda^2 M = q \qquad \lambda^2 = \frac{P}{EI}$$ כפונקציה של מומנט

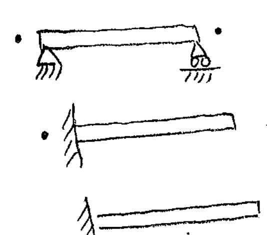
$$\frac{d^4 v}{dx^4} + \lambda^2 \frac{d^2v}{dx^2} = q/EI$$ כפונקציה של התזוזה

אם EI פונקציה של x

$$\frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right) + P\frac{d^2v}{dx^2} = q$$

<u>פיתרון כאשר EI קבוע</u> ו- $q_0 = q(x)$

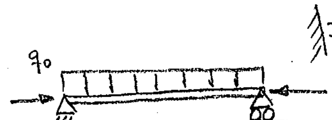
$$v(x) = A\cos\lambda x + B\sin\lambda x + Cx + D + \frac{q_0 x^2}{2\lambda^2 EI}$$



בסמך נייד ונייח: $M = EI\frac{d^2v}{dx^2} = 0, \ v = 0$

בסמך רתום: $\frac{dv}{dx} = 0, \ v = 0$

בצד חופשי: $M = 0, \ V = \frac{d^3v}{dx^3} + \frac{P}{EI}\frac{dv}{dx} = 0$



<u>בעמוד-קורה בקצוות נייח ונייד</u>

$$v = \frac{q_0}{\lambda^4 EI \sin\lambda L}\left\{(1-\cos\lambda L)\sin\lambda x - \left[(1-\cos\lambda x) + \frac{\lambda^2 x}{2}(L-x)\right]\sin\lambda L\right\}$$

<u>כאשר $\lambda \to 0$</u> $v = \frac{q_0}{24EI}(x^4 - 2Lx^3 + L^3x)$ ו- $v(L/2) = \frac{5q_0L^4}{384EI}$

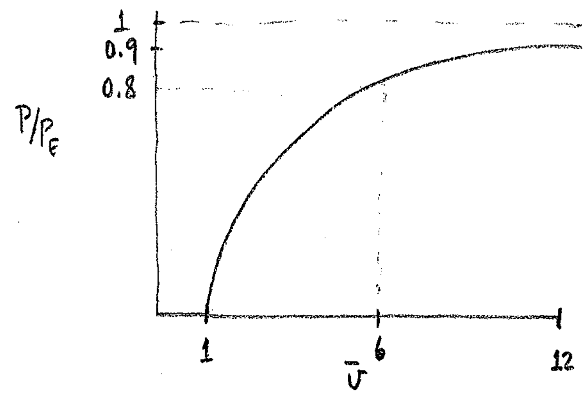
$$384EI \quad L \;\Big/\; \frac{5}{384}(\lambda L)^4 \sin\lambda L$$

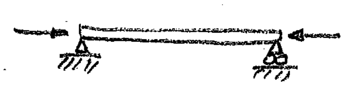
ותזוזה הולכת לאין סוף כאשר $(\lambda L)^4 \sin\lambda L$ הולך לאפס, או כאשר $\sin\lambda L=0$.

$$\sin\lambda L = 0 \Rightarrow \lambda L = n\pi \Rightarrow P = \frac{n^2\pi^2}{L^2}EI$$

לכן התזוזה לא תלויה ב- q_0. ה- P הקטן הוא $P_E = \frac{\pi^2 EI}{L^2}$ וזה העומס הקריטי או <u>עומס הקריסה</u>

אם נסתכל במקדם $P/P_E = \frac{\lambda^2 EI}{\pi^2 EI/L^2} = \left(\frac{\lambda L}{\pi}\right)^2$ כפונקציה של $\bar{v} = \frac{v(L/2,\lambda)}{v(L/2,\lambda=0)}$



אם הסתכלנו רק בקורה  דיינו מקבל $\frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right)+P\frac{d^2v}{dx^2}=0$

והפיתרון למשוואה הזאת הוא ש-

$$v = A\cos\lambda x + B\sin\lambda x + Cx + D$$

אם דיינו משתמשים בתנאי הקצוות, זאת אומרת

$$v(0)=0 \longrightarrow A+D=0$$

$$M(x=0) = EI\frac{d^2v}{dx^2}=0 \longrightarrow -\lambda^2 A = 0$$

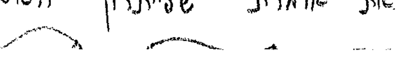
$$v(L)=0 \qquad A\cos\lambda L + B\sin\lambda L + CL + D = 0$$

$$M(x=L) = EI\frac{d^2v}{dx^2}=0 \qquad -\lambda^2 A\cos\lambda L - B\lambda^2\sin\lambda L = 0$$

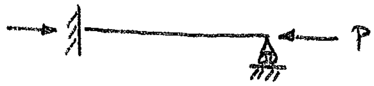
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -\lambda^2 & 0 & 0 & 0 \\ \cos\lambda L & \sin\lambda L & L & 1 \\ -\lambda^2\cos\lambda L & -\lambda^2\sin\lambda L & 0 & 0 \end{bmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

כדי למצוא פיתרון, או $A=B=C=D=0$ (זאת אומרת ש- $v=0$ והוא הפיתרון הפשוט) <u>או</u> ש- ה DETERMINANT שווה לאפס. לכן, לפי העמוד השלישי, ה- DETERMINANT שווה ל-

$$L\begin{vmatrix} 1 & 0 & 1 \\ -\lambda^2 & 0 & 0 \\ -\lambda^2\cos\lambda L & -\lambda^2\sin\lambda L & 0 \end{vmatrix} = L\lambda^4\sin\lambda L = 0$$

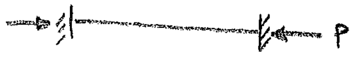
ועוד פעם $\sin\lambda L = 0$. זאת אומרת שפיתרון האפשרי הוא ש- $v = B\sin\lambda x$. לכן אפשרויות לתזוזה יכולה להיות  אבל כולם

התגובה המקסימלית הולכת וגדלה. ואם העומס P הוא P_E, אז העמוד מתחיל לקרוס.



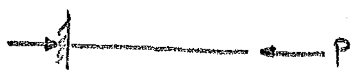
יישום תנאי הקצוות לעמוד-קורה כבציור נותן המשוואה הקריטית $\tan\lambda L = \lambda L$

$$P_{\text{קריסה}} = 2.05\frac{\pi^2 EI}{L^2} = 2.05P_E$$



המשוואה הקריטית $\sin\frac{\lambda L}{2}\left(\sin\frac{\lambda L}{2} - \frac{\lambda L}{2}\cos\frac{\lambda L}{2}\right) = 0$

$$P_{\text{קריסה}} = \frac{4\pi^2 EI}{L^2} = 4P_E$$

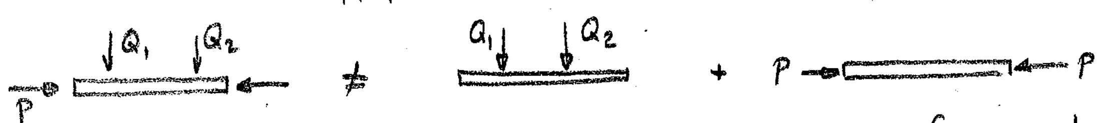


המשוואה הקריטית $\cos kL = 0$

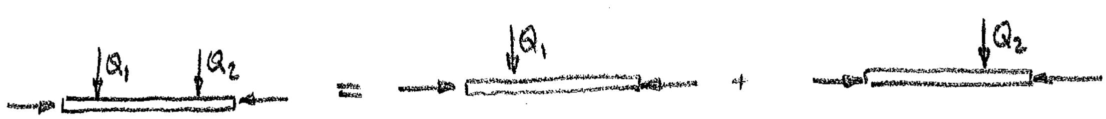
$$P_{\text{קריסה}} = \frac{1}{4}\frac{\pi^2 EI}{L^2} = \frac{1}{4}P_E$$

כאן, לא כללנו את עומס הגזירה. אם הגזירה נכללת העומס הקריטי שגורם לקריסה יהיה יותר נמוך. רואים שתנאי הקצוות משפיעים לעומס הקריטי.

אך על פי ש־אי אפשר להשתמש בסופרפוזיציה כזאת



אפשר לעשות ככה



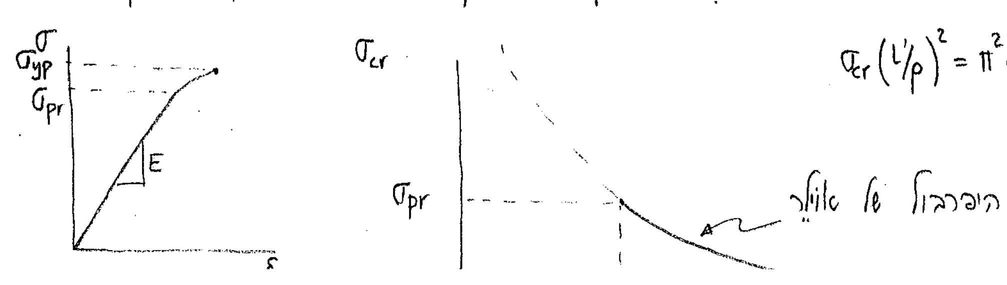
הערות: קבלנו את התוצאות האלה על תנאי שנישאר בחלק האלסטי הלינארי של החומר

בדרך כלל, $P_{cr} = C\frac{\pi^2 EI}{L^2}$ והמספר C תלוי בתנאי הקצוות. אם $C = (L/L')^2$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{AL'^2} = \frac{\pi^2 E(\rho^2 A)}{A(L')^2} = \frac{\pi^2 E}{(L'/\rho)^2}$$

כאן L' = אורך האפקטיבי של הקורה. לדוגמה  $C = 1/4$ לכן $L' = 2L$

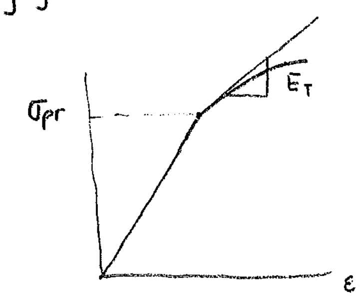
- $I = \rho^2 A$ ו- ρ הוא רדיוס ההסתובבות (רדיוס האינרציה)
- המקדם L/ρ הוא תלוי בגאומטרית הקורה וקוראים לזה מקדם הדקות
- רואים ש- $\sigma_{cr}(L'/\rho)^2 = \pi^2 E$



$$\left(L'/\rho\right)_{cr} = \pi\sqrt{E/\sigma_{pr}}$$

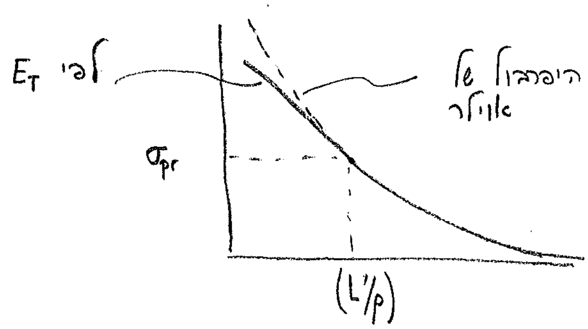
וכולים לקבל את הערך הקריטי. אם $\left(L'/\rho\right)_{cr} > \left(L'/\rho\right)_{גאומ}$ המאמץ הקריטי לא יהיה שווה ל- P_{cr}/A אבל יהיה יותר גדול.

לתקן בזה, כדי לעבוד ב $\left(L'/\rho\right)_{cr} > \left(L'/\rho\right)_{גאומ}$ משתמשים במודול הטנגנטי אחרי נקודת הפרופורציה עד σ_{yp}.

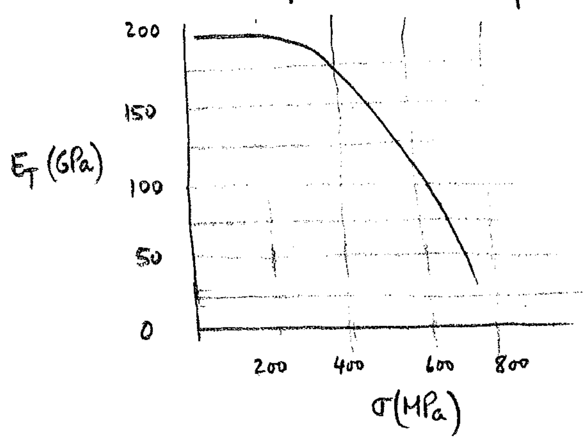


$$\sigma_{t,cr} = \frac{\pi^2 E_T}{\left(L'/\rho\right)^2}$$

ו- בדרך ניסיונות מקבלים את E_T.



לפלדה מסוימת קיבלו את הערך בין E_T למאמץ



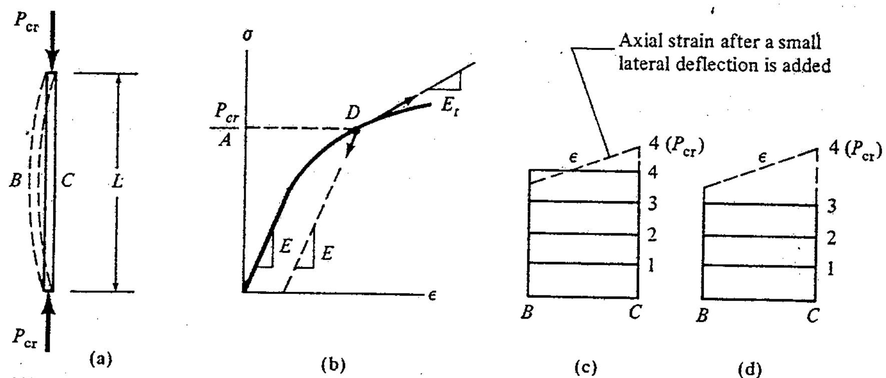


FIGURE 12.4.1. (a) Buckling of a pin-ended column under centroidal axial load. (b) Compressive stress–strain diagram, showing loading and unloading paths from point D, which corresponds to inelastic buckling. (c) Distribution of axial strain across the column at increasing load levels, according to double-modulus theory. (d) Possible distribution of axial strain across the column in tangent modulus theory.

cross section. Therefore, the column must bend before reaching the double-modulus load. But this is in contradiction to a basic assumption in double-modulus theory. The contradiction is resolved by noting that lateral deflection may occur *simultaneously* with application of the last increment of load. There need be no unloading on the convex side, and modulus E_t may prevail all across the section (Fig. 12.4.1d). Under near-perfect test conditions the collapse load slightly exceeds the theoretical tangent-modulus load, but it does not reach the double-modulus load.

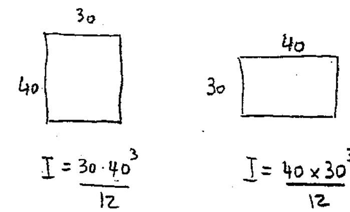
In summary, inelastic buckling of a straight, axially loaded column does not occur at a unique value of axial load P. Instead, buckling begins at the tangent-modulus load and is complete (meaning that collapse takes place) before the theoretical double-modulus load is reached. Tests of real columns, which have larger imperfections than laboratory specimens, are in excellent agreement with tangent modulus theory.

Euler did not realize that bending stiffness EI could be calculated rather than obtained by experiment. However, he anticipated Engesser by remarking in 1757 that EI represents a resistance to bending that need not pertain only to elastic bodies [12.4].

Example 12.4.1. A column has a solid rectangular cross section, 40 mm by 30 mm. It is 200 mm long, free at the top, and fixed at the base. Material properties are shown in Fig. 12.4.2. What centroidal axial compressive load at the top will make the column buckle?

The appropriate equation is $P_{cr} = \pi^2 EI/4L^2$, where

$$I = \frac{bh^3}{12} = \frac{40(30)^3}{12} = 90{,}000 \text{ mm}^4 \qquad (12.4.2)$$



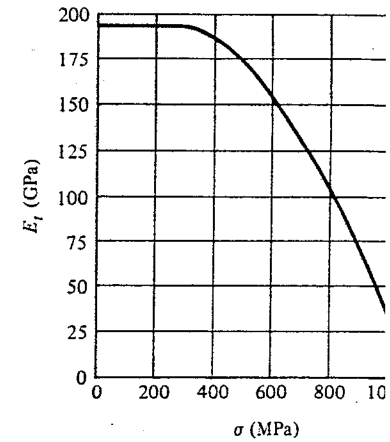


FIGURE 12.4.2. Tangent modulus E_t versus compressive stress σ for a particular steel, obtained from a stress–strain plot.

$P_{cr} = 1077$ kN, or $\sigma_{cr} = P_{cr}/A = 898$ MPa. This stress is consi higher than the proportional limit stress, which appears to be about 32 in Fig. 12.4.2. Therefore, buckling is inelastic, the effective modu pends on load, and an iterative method of calculation is needed to fi as follows.

Assume that σ_{cr} will be, say, 600 MPa. At this stress, Fig. 12.4. $E_t = 160$ GPa. Hence

$$P_{cr} = \frac{\pi^2 E_t I}{4L^2} = 888 \text{ kN} \qquad \frac{P_{cr}}{A} = 740 \text{ MPa} \qquad (1$$

As P_{cr}/A exceeds the assumed σ_{cr} of 600 MPa, another trial is neede

Assume that $\sigma_{cr} = 660$ MPa; then

$$E_t = 142 \text{ GPa} \qquad P_{cr} = \frac{\pi^2 E_t I}{4L^2} = 788 \text{ kN} \qquad \frac{P_{cr}}{A} = 657 \text{ MPa} \qquad (1$$

Now the assumed value of σ_{cr} agrees well enough with the calculated and $P_{cr} = 788$ kN is accepted as the tangent modulus buckling load.

Creep Buckling. As the name implies, creep buckling theory deals material that creeps, that is, a material whose strain changes with time at c stress. A creeping column may display a small but gradually increasing deflection, then fail suddenly by buckling. The phenomenon is explai examination of creep curves (Fig. 12.4.3). One may enter the creep cu a certain time, say t_1, and read the strain for each of several stress Stress–strain data thus obtained is then plotted as a stress–strain curve, s the curve labeled t_1 in Fig. 12.4.3. Repetition of this procedure at severa produces a set of isochronous stress–strain curves (stress versus strain at c time). These curves show that at a given stress level, the tangent m decreases with time. This implies that however light the load, a creeping c

1. Write the second-order differential equation for the bending of the column shown in Fig. P1-2 and use it to determine the critical load of the column. At its lower end the column is completely fixed. At the upper end the column is prevented from rotating, but free to translate laterally. (ans: $P_{cr}=\pi^2 EI/L^2$)

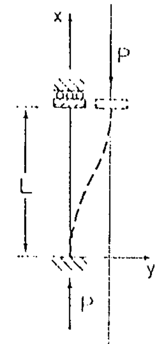
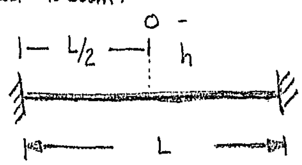


Fig. P1-2

2. Find an expression for the maximum stress when a ball weighing W Newtons is dropped onto a fixed-fixed beam.



3. A linearly elastic beam-column having a flexural rigidity EI, is subjected to a thrust P and a moment M_0 as shown in Fig. A below.

(a) Determine the lateral displacement v(x).

(b) From part (a), write the solution for the system subjected to a force P acting as shown in Fig. B.

(c) Determine Δ_c, the horizontal displacement of point C, assuming small rotations. (Assume also that the horizontal displacement of point B is negligible).

(d) Determine the bending moment M(x).

(g) Explain why, although the beam is made of a linearly elastic material, the results of part (c) are non-linear.

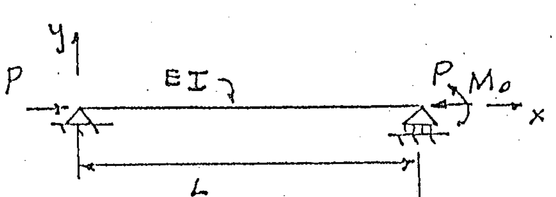


Fig. A

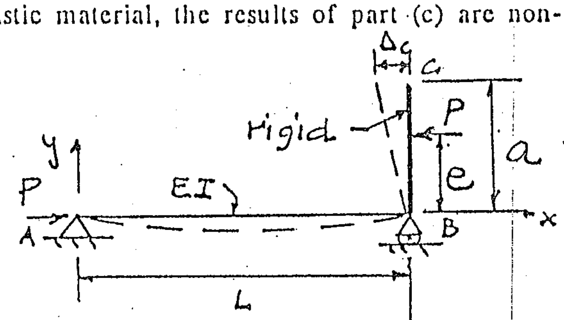


Fig. B

Answers :

(a) $y(x) = -\frac{M_0}{P}\left[\frac{\sin Kx}{\sin KL} - \frac{x}{L}\right]$, $K^2 = \frac{P}{EI}$

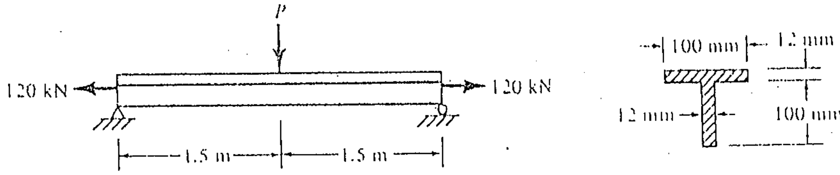
(c) $\Delta_c = \frac{ac}{L}\,[1 - L\sqrt{P/EI}\cot(L\sqrt{P/EI})\,] = \frac{ac}{L}\,(1 - KL\cot KL)$

(d) $M(x)=M_0 \sin kl/\sin kL$

*12.10 A T section carries an axial tensile force of 120 kN, applied through the centroid of the cross section. The allowable stress in tension or compression is 130 MPa. Let $E = 200$ GPa. What transverse force P can be applied at midspan if the beam is

(a) Stem down (as shown)?

(b) Stem up?



PROBLEM 12.10

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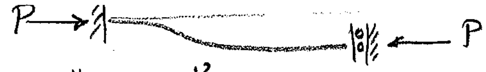


מהנוסחה שמצאנו בשעור שעבר והפיתרון הוא

$$EI\frac{d^4}{dx^4}v+P\frac{d^2v}{dx^2}=0$$

$$v=A\cos\lambda x+B\sin\lambda x+Cx+D$$

$$\lambda^2=P/EI$$

תנאי הקצוות

① תזוזה $v(x=0)=0$

② צווית $\frac{dv}{dx}(x=0)=0$

③ $\frac{dv}{dx}(x=L)=0$ צווית

④ גזירה: $EI\frac{d^3v}{dx^3}(x=L)+P\frac{dv}{dx}(x=L)=0$

מפני ש $\frac{dv}{dx}=0$ ב-$x=L$, לכן ④ נותנת $\underline{\frac{d^3v}{dx^3}=0}$ ב-$x=L$

מקבלים

$$A+D=0 \quad \text{①-מ}$$
$$\lambda B+C=0 \quad ②$$
$$-A(\lambda\sin\lambda L)+B(\lambda\cos\lambda L)+C=0 \quad ③$$
$$A\lambda^3\sin\lambda L-B\lambda^3\cos\lambda L=0 \quad ④$$

לכן

$$\begin{bmatrix}1&0&0&1\\0&\lambda&1&0\\-\lambda\sin\lambda L&\lambda\cos\lambda L&1&0\\\lambda^3\sin\lambda L&-\lambda^3\cos\lambda L&0&0\end{bmatrix}\begin{pmatrix}A\\B\\C\\D\end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\end{pmatrix}$$

כדי לקבל תוצאות לא טריוויאליות הדיטרמיננט צריך להיות אפס. לפי העמוד הרביעי

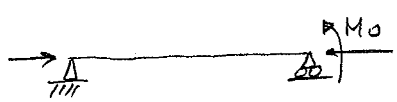
$$-1\begin{pmatrix}0&\lambda&1\\-\lambda\sin\lambda L&\lambda\cos\lambda L&1\\\lambda^3\sin\lambda L&-\lambda^3\cos\lambda L&0\end{pmatrix}=0$$

לכן

$$\lambda^4\sin\lambda L=0$$

$\lambda\neq0$ (זה הפיתרון $v=0$) ; לכן $\sin\lambda L=0 \Leftarrow$ $\lambda L=n\pi$ או $P_E=\left(\frac{\pi}{L}\right)^2EI$

3.



$$v=A\cos\lambda x+B\sin\lambda x+Cx+D \qquad \lambda^2=P/EI$$

① $v(x=0)=0$ ③ $v(x=L)=0$

② $EI\frac{d^2v}{dx^2}(x=0)=M=0$ ④ $EI\frac{d^2v}{dx^2}(x=L)=-M_0$ לפי הסכם הסימנים

$$A+D=0 \quad \text{①-מ}$$
$$-A\lambda^2=0 \quad ②$$
$$A\cos\lambda L+B\sin\lambda L+CL+D=0 \quad ③$$
$$EI(-A\lambda^2\cos\lambda L-B\lambda^2\sin\lambda L)=-M_0 \quad ④$$

אם $\lambda\neq0$ $A\overset{(2)}{=}0 \Rightarrow D\overset{(1)}{=}0 \Rightarrow B\sin\lambda L+CL\overset{(3)}{=}0 \Rightarrow B(EI\lambda^2\sin\lambda L)\overset{(4)}{=}M_0 \Rightarrow B=\frac{M_0}{EI\lambda^2\sin\lambda L}$

$C=-\frac{M_0}{EI\lambda^2L}$

לכן,

$$v=\frac{M_0}{EI\lambda^2}\left(\frac{\sin\lambda x}{\sin\lambda L}-\frac{x}{L}\right)$$

$$\boxed{v=\frac{M_0}{P}\left(\frac{\sin\lambda x}{\sin\lambda L}-\frac{x}{L}\right)}$$

לקבל $M(x)$, גוזר פעמיים לפי x: $EI\frac{d^2v}{dx^2}=M$ $=-EI\frac{M_0\lambda^2}{P}\frac{\sin\lambda x}{\sin\lambda L}$ $\underline{M(x)=-M_0\frac{\sin\lambda x}{\sin\lambda L}}$

הסימן נכון לפי הסכם הסימנים.

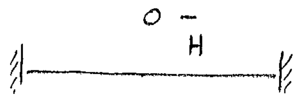
כדי לקבל ... צריך למצוא $\frac{dv}{dx}=\theta$ $=\frac{M_0}{...}\left(\lambda\cos\lambda L-\frac{1}{L}\right)$ ...

לכן $\theta|_{x=L} = \frac{Pe}{P}\left(\lambda\cot\lambda L - \frac{1}{L}\right)$ ו- $a\theta|_{x=L} = \Delta_c$ כי העמוד קשיח וקשור לקורה בזווית אנכית של $90°$. לכן,

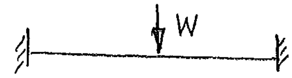
$$\Delta_c = \frac{ae}{L}(\lambda L\cot\lambda L - 1)$$

2.

לפתור את



פותרים את



והתזוזה היא $\Delta_{st} = v = \frac{WL^3}{192EI}$

לפי הנוסחה $P_{dyn} = W\left(1+\sqrt{1+2H/\Delta_{st}}\right)$, בגלל שמצאנו את Δ_{st}, מצאנו את P_{dyn}.

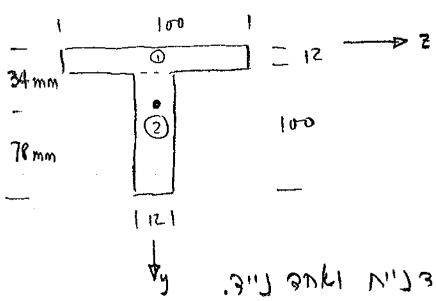
למצוא את המומנט המקס. לפי הנוסחה שנתונה ב- COOK & YOUNG, $v=\frac{P}{48EI}(3Lx^2-4x^3)$

ו- $M = EI\frac{d^2v}{dx^2}$. לכן, $M=\frac{P}{48}(6L-24x)$. ב- $x=0$ $M=\frac{PL}{8}$; ב- $x=L/2$ $M=-\frac{PL}{8}$, וגרף המומנט

לכן יש שלושה מקומות שבהם המומנט הוא מקסימלי והמאמץ המקסימלי הוא $\sigma = \frac{Mc}{I} = \frac{PL}{8}\frac{6}{bh^2}$

$$\frac{3}{4}\frac{LW\left(1+\sqrt{1+\frac{2H}{WL^3/192EI}}\right)}{bh^3} = \frac{3}{4}\frac{PL}{bh^3} = \sigma_{max}$$

12.10



שטח החתך הוא $2(100\times12)\,mm = 2400\,mm^2$

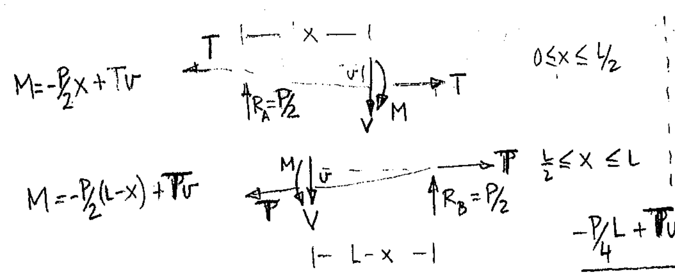
מרכז הכובד הוא

	A	y	Ay
①	100x12	6	7200
②	100x12	62	74400
	2400		81600

$\bar{y} = \frac{\Sigma Ay}{A} = 34\,mm$

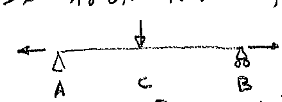
העומס הקריסה הוא $T_E = \frac{\pi^2 EI}{L^2}$ מפני הסמכים, אחד נייח ואחד נייד.

בגלל העומס הצירי T, המומנט בקורה הוא המומנט שנמצא בקורה בלי העומס הצירי נוסף המומנט שבא מהעומס הצירי, Tv; v הוא התזוזה באותו מקום שמחשבים את המומנט



לכן, המומנט המקס. נמצא ב- $x=L/2$ והוא $-\frac{P}{4}L+Tv$

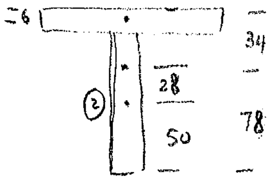
כדי למצוא את $v|_{x=L/2}$, יודעים משיעור בכיתה ש- $v = A\cosh\lambda x + B\sinh\lambda x + Cx + D$ ו- $\lambda^2 = \frac{T}{EI}$ בכל חלק מהקורה.



זאת אומרת שצריך להשתמש ב $v_L = A\cosh\lambda x + B\sinh\lambda x + Cx + D$ בין A-C, ו- $v_R = \bar{A}\cosh\lambda(x-L) + \bar{B}\sinh\lambda(x-L) + \bar{C}(x-L) + \bar{D}$ בין C-B. תנאי השפה: $v_L(x=0)\overset{(1)}{=}0$, $v_L''(x=0)\overset{(2)}{=}0$, $v_R(x=L)\overset{(3)}{=}0$, $v_R''(x=L)\overset{(4)}{=}0$. ב- C: $v_L(x=L/2)\overset{(5)}{=}v_R(x=L/2)$; $v_L'(x=L/2)\overset{(6)}{=}v_R'(x=L/2)$; $v_L''(x=L/2)\overset{(7)}{=}v_R''(x=L/2)$; $EIv_L'''(x=L/2)-Tv_L''(x=L/2)\overset{(8)}{=}EIv_R'''-Tv_R''+P$. אלה תנאי רציפות, אלה שמונה משוואות לקבל $A,B,C,D,\bar{A},\bar{B},\bar{C},\bar{D}$. מ- (1), (2), (3), (4) $\Leftarrow A=D=\bar{A}=\bar{D}=0$. מ- (7) $\Leftarrow B=-\bar{B}$. מ- (7),

$\Leftarrow C=-\bar{C}$; מ- (6) ⟶ [illegible]

$|_{x=L/2}$ [illegible] $2EI\lambda^3\cosh\lambda L/2$ $\overline{2EI\lambda^2}$



I	A	d^2	I_{tt}
$\frac{1}{12}(100)(12)^3$	1200	$(28)^2$	955200
$\frac{1}{12}(12)(100)^3$	1200	$(28)^2$	1940800
			$2.896\times10^6\ mm^4$

$$v|_{x=L/2} = P\left\{\frac{\sinh\lambda L/2}{2T\lambda\cosh\lambda L/2} - \frac{L}{4T}\right\}$$

צריך לדעת λ לחשב את v. גם צריך לדעת I.

$$\lambda = \sqrt{\frac{120{,}000}{(200\times10^9)(2.896\times10^{-6})}} = .4552 \quad , \quad T_E = \frac{\pi^2 EI}{L^2} = 635164N > T.$$

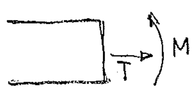
$$v|_{x=L/2} = P\left\{\frac{0.7371}{2(120{,}000)(1.2423)(.4552)} - \frac{3}{4(120000)}\right\}$$

$$= P(8.19\ 10^{-7}m)$$

לכן המומנט יהיה $M = -\frac{PL}{4} + T\cdot P\times 8.19\times10^{-7}$

$$M = P[-0.75 + .09828] = -.65172P$$

התגובות במקום שהמומנט מקס.



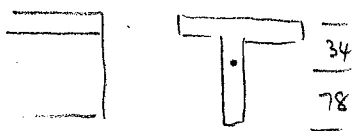
$$\sigma = \frac{T}{A} + \frac{Mc}{I}$$

אם צורת החתך כמו מצויר שמאלה:

$$\sigma_{עליון} = \frac{120{,}000}{2400\times10^{-6}} - \frac{0.65172(.078)P}{2.896\times10^{-6}} = -130\times10^6 \qquad P = 10{,}250\ N$$

$$\sigma_{תחתון} = \frac{120{,}000}{2400\times10^{-6}} + \frac{0.65172(.034)P}{2.896\times10^{-6}} = 130\times10^6 \qquad P = 10{,}451N$$

אם צורת החתך כמו מצויר שמאלה:

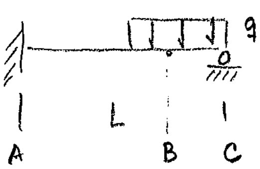


$$\sigma_{עליון} = \frac{120{,}000}{2400\times10^{-6}} - \frac{0.65172(.034)P}{2.896\times10^{-6}} = -130\times10^6 \qquad P = 23{,}515N$$

$$\sigma_{תחתון} = \frac{120{,}000}{2400\times10^{-6}} + \frac{0.65172(.078)P}{2.896\times10^{-6}} = 130\times10^6 \qquad P = 4556\ N$$

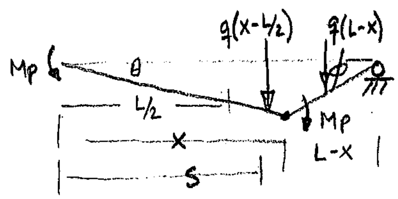
לכן, העומס הגבוה ביותר בא כאשר לשטח החתך יש צורה כמו ⊥

1.26



כאשר שהמומנט מגיע ל-M_p, פרק פלסי קורה בסמך הרתום וגם בנקודה B

צריך למצוא את מקום נקודה B ומהו העומס המפורס שגורם ל-M_p



$$\theta x = \phi(L-x)$$
$$\frac{\theta x}{L-x} = \phi$$

$$s = \tfrac{1}{2}(x + L/2)$$

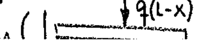
לפי עקרון עבודה וירטואלית

$$M_p\theta + M_p(\theta+\phi) = q(x-L/2)\theta s + q(L-x)\left(\frac{L-x}{2}\right)\phi$$

מחליפים ϕ ו-s ביחסים ופותרים בשביל q

$$q = \frac{8M_p(2L-x)}{(5Lx - 4x^2 - L^2)L}$$

כדי למצוא את ה-q שגורם ל-M_p, גוזרים q לגבי x ומקבלים ש- $0 = 4x^2 - 16Lx + 9L^2$. משם יש תשובה אחת והיא $x = \frac{16-\sqrt{112}}{8}L$. שמים x בנוסחה של q כדי לקבל $q = \frac{19.18M_p}{L^2}$



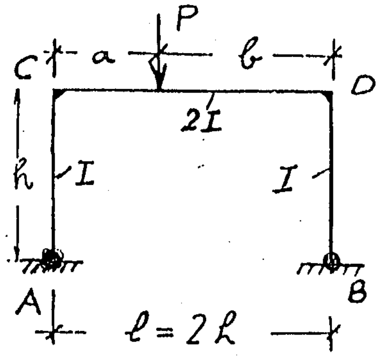
[illegible] BC [illegible] $6.194M_p/L$

המכללה האקדמית יהודה ושומרון

תורת החוזק 1

תרגיל מס' 11

1.

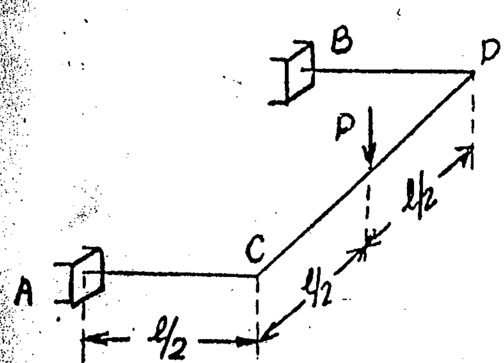


נתון. מסגרת פרטל רתומה עמוסה לפי הציור. $a = \ell/3$.

דרוש. לחשב את הריאקציות ואת מומנטי הריתום ולשרטט את מהלכי N, S ו- M.

תשובה. ~~$H_A = \frac{Pab}{2h\ell}$; $V_A = \frac{64Pb}{63\ell}$; $M_A = \frac{Pab}{7\ell}$~~

2.



נתון. מסגרת סימטרית **אופקית** המתוארת בציור האקסונומטרי רתומה ב-A ו-B ועמוסה עומס אנכי P. המסגרת בעלת קשיחות לכפיפה EI וקשיחות לפיתול GI_T.

דרוש. לחשב את הריאקציות B,A את מומנטי הכפיפה בריתומים M_B, M_A ואת מומנטי הפיתול בריתומים T_B, T_A. ולשרטט את מהלכי M ו- T. (להשרות זווית הכפיפה ϕ_C של EC לזווית הפיתול θ_C של AC).

תשובה. $M_A = M_B = P\ell/4$; $A = B = P/2$

$$T_A = T_B = \frac{P\ell}{8}\,\frac{GI_T}{GI_T + EI}$$

136. For the loading shown, knowing that beams *AB* and *DE* have the same ...al rigidity, determine the reaction (*a*) at *A*, (*b*) at *D*.

ans: (a) 14.07 kN↑ (b) 5.93 kN↑.

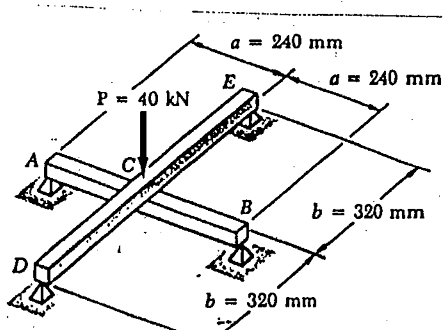


Fig. P8.136

8.138 Before any load is applied, a gap δ_0 = 20 mm exists between the W410 × 60 rolled-steel beam and the support at *C*. Knowing that *E* = 200 GPa, determine the reaction at each support caused by a uniformly distributed load of 24 kN/m.

ans: $R_A = R_B = 76.5$ kN↑
$R_C = 39.0$ kN↓

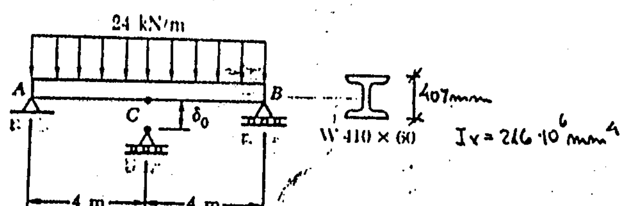
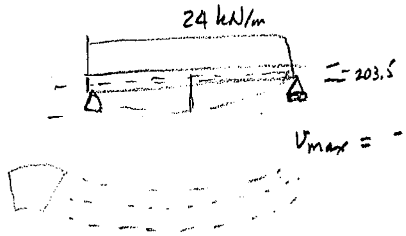


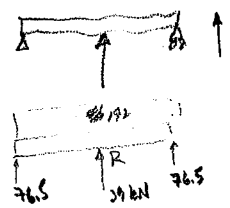
Fig. P8.138 and P8.139

8.138



$$v = \frac{-wx}{24EI}\left[x^3 - 2Lx^2 + L^3\right]$$

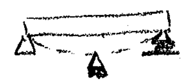
$$v_{max} = -\frac{5wL^4}{384EI} = \frac{-5\,[24000\ N/m]\ 8^4\ m^4}{384\ (200\times10^9 \frac{N}{m^2})[216\times10^{-6}\ m^4]} = .02963\ m = 29.63\ mm$$

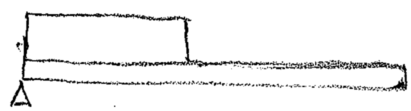


$$v = \frac{PL^3}{48EI} = .00962963\ m$$

$$P = \frac{.00962963\ (48)(200\times10^9 \frac{N}{m^2})(216\times10^{-6}\ m^4)}{8^3} = 39000\ N$$

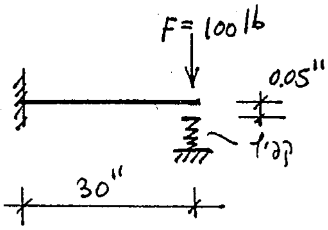
$v_1 + v_2 = \delta$ =





חוזק חומרים - 2 גליון תרגיל כתה מס' 6

12-14

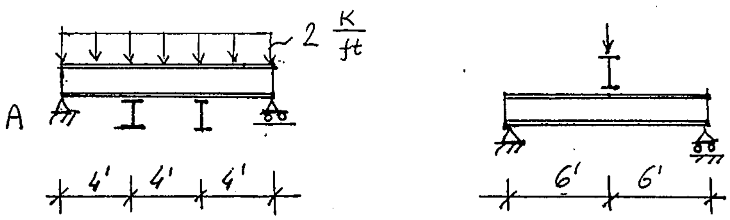


קורה שבתרשים! $EI = 10^7 \ \frac{lb}{in^2}$

קפיץ - קבוע קפיץ $K = 10 \ \frac{kips}{in}$

דרוש: לחשב את הכח בקפיץ בפעולת כח F. (תשובה: 40)

12-15

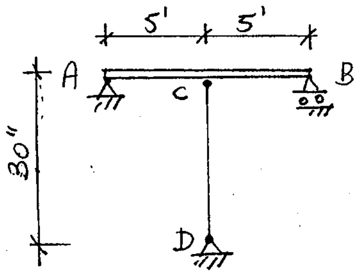


נתונה רשת של קורות

מפרופיל IN°200.

דרוש: לחשב ריאקציה בסמך A. (תשובה 6.74 kips)

12-27

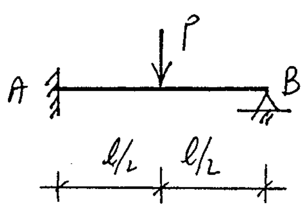


נתון: קורה AB! $EI = 1040 \ lb\text{-}in^2$

מוט CD: $A = 10^{-4} \ in^2, \ E = 30 \cdot 10^6 \ psi, \ \alpha = 6.5 \cdot 10^{-6} \ \frac{1}{°F}$

דרוש: לחשב תוספת הכח במוט CD אם הטמפרטורה יורדת ב- $100°F$.

12-44

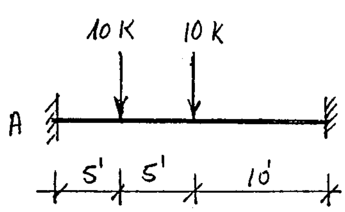


בקורה שבתרשים דרוש: לחשב ריאקציות

לצייר מהלכי כוחות גזירה ומומנט כפיפה

תשובה! $M_A = -\frac{3PL}{16}$

12-65



בקורה המתוארת בתרשים דרוש: לחשב ריאקציות.

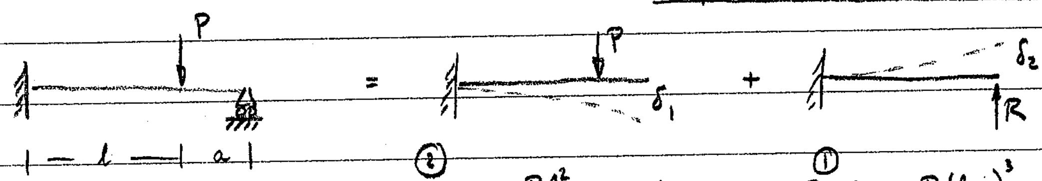
לצייר מהלכי כוחות גזירה ומומנט כפיפה

לחשב שקיעת מרכז הקורה ומאמץ כפיפה מכסימלי.

נתון! $E = 29 \cdot 10^6 \ psi$, $I = 719 \ in^4$, $W = 126 \ in^3$

(תשובה! 13,9 ksi, 0,133 in)

דוגמא שניה . למצוא ראקציות



② $\delta_1 = f_A = \dfrac{Pl^2}{6EI}(2l+3a)$

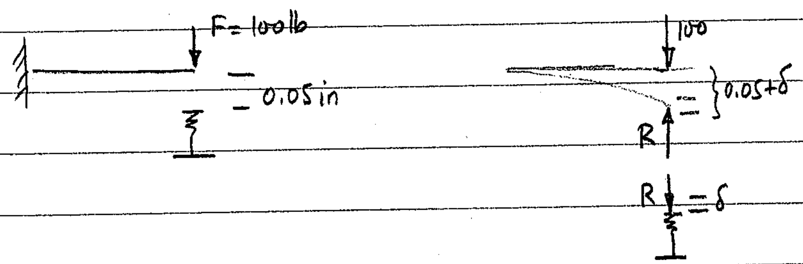
① $\delta_2 = f_A = \dfrac{-R(l+a)^3}{3EI}$

$$\delta_1+\delta_2=0 \qquad \frac{Pl^2(2l+3a)}{6EI} - \frac{R(l+a)^3}{3EI} = 0$$

$$R = P\,\frac{l^2(2l+3a)}{2(l+a)^3}$$

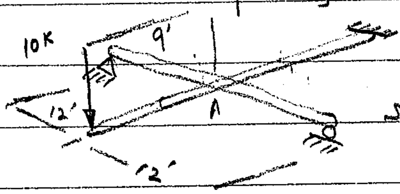
$$R = P\left(\frac{l^2\cdot 5l}{2\cdot 8l^3}\right) = \frac{5}{16}P \qquad \text{אם } a=l$$

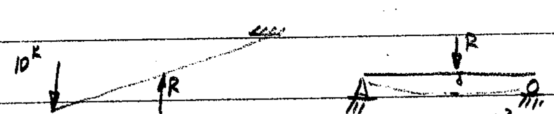
12-14



$$\left.\begin{array}{l} \dfrac{(100-R)L^3}{3EI} = 0.05+\delta \\ R = k\delta \end{array}\right\} \longrightarrow \begin{array}{l} R = 40lb \\ \delta = 0.004in \end{array}$$

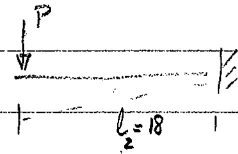
12-15 הנקודה האמצעית של הקורה המקובעת של 18' מונחת בקורה (24' של) הסמוכה בסמכים פשוטים באמצע. מצאו את השקיעה ב A שנגרם מפני יישום כוח 10^k בקצה הקורה המקובעת.





$$\delta = \frac{Rl_1^3}{48EI} \qquad l_1 = 24'$$

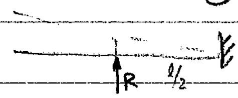
①



$$y = \frac{Pl_2^3}{6EI}\left(2 - \frac{3x}{l_2} + \frac{x^3}{l_2^3}\right)$$

$$y\Big|_{x=l/2} = \frac{5Pl_2^3}{48EI}$$

②



$$y\Big|_{l/2} \quad \frac{Rl^3}{3EI} = \frac{R(l_2^3/8)}{3EI} = \frac{Rl_2^3}{24EI}$$

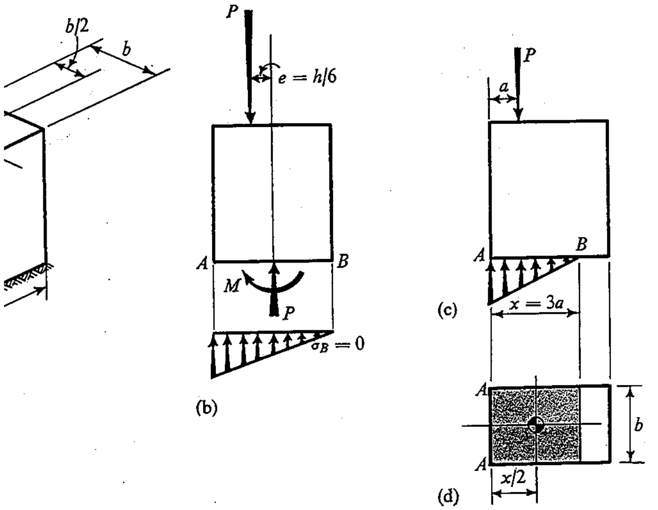
$$\delta = \frac{5Pl_2^3}{48EI} - \frac{Rl_2^3}{24EI} = \frac{Rl_1^3}{48EI}$$

$$\frac{5Pl_2^3}{48EI} = \frac{R}{48EI}\left(2l_2^3 + l_1^3\right)$$

$$\underline{1.1441P} \quad \frac{5P(18)^3}{2\cdot 18^3 + 24^3} = \frac{5Pl_2^3}{2l_2^3 + l_1^3} = \underline{R}$$

$$\delta = \frac{5P(18)^3}{2\cdot 18^3 + 24^3} \cdot \frac{24^3}{48\cdot EI} = \frac{3295}{EI}\ \text{K-ft}^3$$

12-65



trically loaded block; (b) location of P to B; (c) elastic stress distribution between two nable to transmit tensile forces.

the centroidal axis of the shaded contact area, and $bx^2/6$ is its section modulus. Solving for x, one finds that $x = 3a$; the pressure distribution will be "triangular" as in Fig. 8-14(c) (why?). As a decreases, the intensity of pressure on the line A-A increases; when a is zero, the block becomes unstable. Such problems are important in the design of foundations.

8-5. SUPERPOSITION OF SHEARING STRESSES

In the preceding part of the chapter superposition of the normal stresses σ_x was the principal concern. In problems where both the elastic torsional and direct shearing stresses can be determined, the compound shearing stress also may be found by superposition. This corresponds to superposition of the off-diagonal stresses in Eq. 8-1. Here attention will be directed to instances where the shearing stresses being superposed not only act on the same element of area but also have the same line of action.* Only elastic stresses fall within the scope of this treatment.

* Noncolinear shearing stresses acting on the same element of area can be added vectorially.

EXAMPLE 8-7

Find the maximum shearing stress due to the applied forces in the plane A-B of the ½ in.-diameter, high-strength shaft in Fig. 8-15(a).

SOLUTION

The free body of a segment of the shaft is shown in Fig. 8-15(b). The system of forces at the cut necessary to keep this segment in equilibrium consists of a torque $T = 200$ in-lb, a shear $|V| = 60$ lb, and a bending moment $M = 240$ in-lb.

Because of the torque T, the shearing stresses in the cut A-B vary linearly from the axis of the shaft and reach the maximum value given by Eq. 5-4, $\tau_{max} = Tc/J$. These maximum shearing stresses, agreeing in sense with the resisting torque T, are shown at points A, B, D, and E in Fig. 8-15(c).

The "direct" shearing stresses caused by the shearing force V may be obtained by using Eq. 7-6, $\tau = VQ/(It)$. For the elements A and

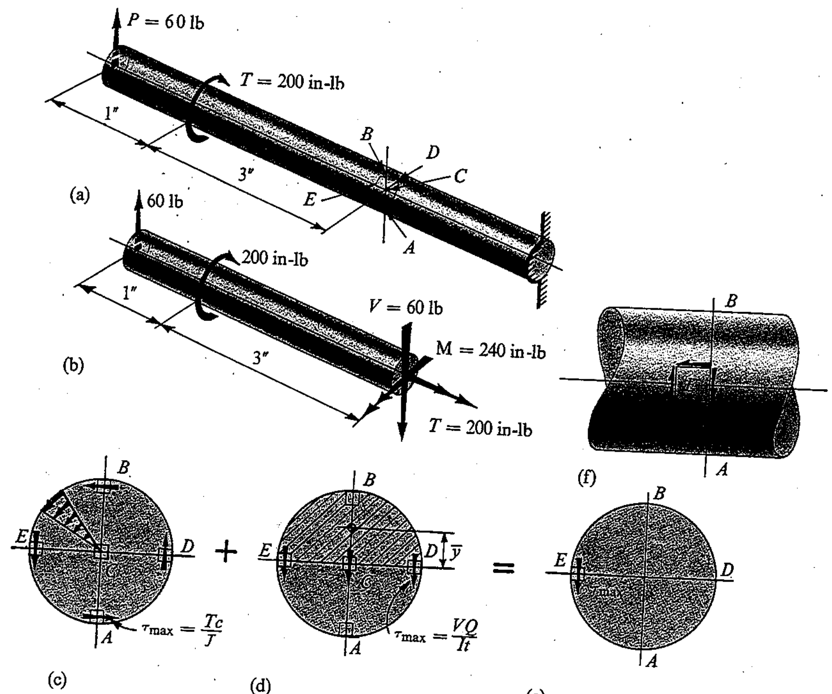


Fig. 8-15

B, Fig. 8-15(d), $Q = 0$, hence $\tau = 0$. The shearing stress reaches its maximum value at the level ED. To determine this, consider Q equal to the shaded area in Fig. 8-15(d) multiplied by the distance from its centroid to the neutral axis. The latter quantity is $\bar{y} = 4c/(3\pi)$, where c is the radius of the cross-sectional area. Hence $Q = (\pi c^2/2)[4c/(3\pi)] = 2c^3/3$. Moreover, since $t = 2c$, and $I = J/2 = \pi c^4/4$, the maximum direct shearing stress is

$$\tau_{max} = \frac{VQ}{It} = \frac{V}{2c}\frac{2c^3}{3}\frac{4}{\pi c^4} = \frac{4V}{3\pi c^2} = \frac{4V}{3A}$$

where A is the entire cross-sectional area of the rod. In Fig. 8-15(d) this shearing stress is shown acting downward on the elementary areas at E, C, and D. This direction agrees with the direction of the shear V.

To find the maximum compound shearing stress in the plane A-B, the stresses shown in Figs. 8-15(c) and (d) are superposed. Inspection shows that the maximum shearing stress is at E since in the two diagrams the shearing stresses at E have the same direction and sense. There are no direct shearing stresses at A and B, and at C there is no torsional shearing stress. The two shearing stresses have an opposite sense at D. The five points A, B, C, D, and E thus considered for the compound shearing stress are all that may be adequately treated by the methods developed in this text. However, this procedure selects the elements where the maximum shearing stresses occur.

$$J = \frac{\pi d^4}{32} = \frac{\pi(0.5)^4}{32} = 0.00614 \text{ in.}^4 \quad \text{and} \quad I = \frac{J}{2} = 0.00307 \text{ in.}^4$$

$$A = \pi d^2/4 = 0.196 \text{ in.}^2$$

$$(\tau_{max})_{torsion} = \frac{Tc}{J} = \frac{200(0.25)}{0.00614} = 8{,}150 \text{ psi}$$

$$(\tau_{max})_{direct} = \frac{VQ}{It} = \frac{4V}{3A} = \frac{4(60)}{3(0.196)} = 408 \text{ psi}$$

$$\tau_E = 8{,}150 + 408 = 8{,}560 \text{ psi}$$

A planar representation of the shearing stress at E with the matching stresses on the longitudinal planes is shown in Fig. 8-15(f). No normal stress acts on this element as it is located on the neutral axis.

8-6. STRESSES IN CLOSELY COILED HELICAL SPRINGS

Helical springs, such as the one shown in Fig. 8-16(a), are often used as elements of machines. With certain limitations, these springs may be analyzed for elastic stresses by a method similar to the one used in the

preceding example. The discussion will be limited* to springs manufactured from rods or wires of circular cross section. Moreover, any one coil of such a spring will be assumed to lie in a plane which is nearly perpendicular to the axis of the spring. This requires that the adjoining coils be close together. With this limitation, a section taken perpendicular to the axis of the spring's rod becomes nearly vertical.† Hence to maintain equilibrium of a segment of the spring, only a shearing force $V = F$ and a torque $T = F\bar{r}$ are required at all sections through the rod, Fig. 8-16(b).‡ Note that $\bar{r}$ is the distance from the axis of the spring to the centroid of the rod's cross-sectional area.

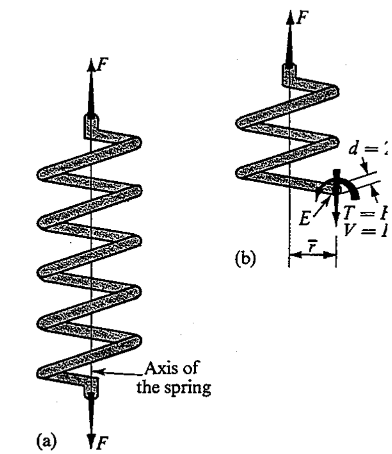


Fig. 8-16. Closely coiled helical spring.

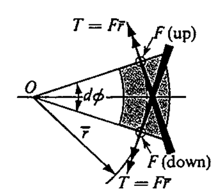
The maximum shearing stress at an arbitrary section through the rod could be obtained as in the preceding example by superposing the torsional and the direct shearing stresses. This maximum shearing stress occurs at the inside of the coil at point E, Fig. 8-16(b). However, in the analysis of springs it has become customary to assume that the shearing stress caused by the direct shearing force is uniformly distributed over the cross-sectional area of the rod. Hence, the nominal direct shearing stress for any point on the cross section is $\tau = F/A$. Superposition of this stress and the torsional shearing stress at E gives the maximum compound shearing stress. Thus since $T = F\bar{r}$, $d = 2c$, and $J = \pi d^4/32$

$$\tau_{max} = \frac{F}{A} + \frac{Tc}{J} = \frac{Tc}{J}\left(\frac{FJ}{ATc} + 1\right) = \frac{16F\bar{r}}{\pi d^3}\left(\frac{d}{4\bar{r}} + 1\right) \qquad \textbf{(8-11)}$$

* For a complete discussion on springs see A. M. Wahl, *Mechanical Springs* (Cleveland, Ohio: Penton Publishing Co., 1944).

† This eliminates the necessity of considering an axial force and a bending moment at the section taken through the spring.

‡ In previous work it has been reiterated that if a shear is present at a section, a change in the bending moment must take place along the member. Here a shear acts at every section of the rod, yet no bending moment nor a change in it occurs. This is so only because the rod is curved. An element of the rod viewed from the top is shown in the figure. At both ends the torques are equal to $F\bar{r}$ and act in the directions shown. The component of these vectors toward the axis of the spring O, resolved at the point of intersection of the vectors, $2F\bar{r}\,d\phi/2 = F\bar{r}\,d\phi$, opposes the couple developed by the vertical shears $V = F$, which are $\bar{r}\,d\phi$ apart.



stituting the values of the sine and cosine functions corresponding to the double angle given by Eq. 9-3 into Eq. 9-1. After this is done and the results are simplified, the expression for the maximum normal stress (denoted by σ_1) and the minimum normal stress (denoted by σ_2) becomes

$$(\sigma_{x'})_{\substack{\max\\ \min}} = \sigma_{1 \text{ or } 2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (9\text{-}4)$$

where the positive sign in front of the radical must be used to obtain σ_1, and the negative sign to obtain σ_2. The planes on which these stresses act can be determined by using Eq. 9-3. A particular root of Eq. 9-3 substituted into Eq. 9-1 will check the result found from Eq. 9-4 and at the same time will locate the plane on which this principal stress acts.

9-5. MAXIMUM SHEARING STRESSES

If σ_x, σ_y, and τ_{xy} are known for an element, the shearing stress on any plane defined by an angle θ is given by Eq. 9-2, and a study similar to the one made above for the normal stresses may be made for the shearing stress. Thus, similarly, to locate the planes on which the maximum or the minimum shearing stresses act, Eq. 9-2 must be differentiated with respect to θ and the derivative set equal to zero. When this is carried out and the results are simplified, the operations yield

$$\tan 2\theta_2 = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} \qquad (9\text{-}5)$$

where the subscript 2 is attached to θ to designate the plane on which the shearing stress is a maximum or a minimum. Like Eq. 9-3, Eq. 9-5 has two roots, which again may be distinguished by attaching to θ_2 a prime or a double prime notation. The two planes defined by this equation are mutually perpendicular. Moreover, the value of $\tan 2\theta_2$ given by Eq. 9-5 is a negative reciprocal of the value of $\tan 2\theta_1$ in Eq. 9-3. Hence the roots for the double angles of Eq. 9-5 are 90° away from the corresponding roots of Eq. 9-3. This means that the angles which locate the planes of maximum or minimum shearing stress form angles of 45° with the planes of the principal stresses. A substitution into Eq. 9-2 of the sine and cosine functions corresponding to the double angle given by Eq. 9-5 and determined in a manner analogous to that in Fig. 9-5 gives the maximum and the minimum values of the shearing stresses. These, after simplifications, are

$$\tau_{\substack{\max\\ \min}} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (9\text{-}6)$$

Thus, the maximum shearing stress differs from the minimum shearing stress only in sign. Moreover, since the two roots given by Eq. 9-5 locate planes 90° apart, this result also means that the numerical values of

the shearing stresses on the mutually perpendicular planes are the same. This concept was repeatedly used after being established in Art. 3-3. In this derivation the difference in sign of the two shearing stresses arises from the convention for locating the planes on which these stresses act. From the physical point of view these signs have no meaning and for this reason the largest shearing stress regardless of sign will be called the *maximum shearing stress*.

The definite sense of the shearing stress may always be determined by direct substitution of the particular root of θ_2 into Eq. 9-2. A positive shearing stress indicates that it acts in the direction assumed in Fig. 9-4(b), and vice versa. The determination of the maximum shearing stress is of utmost importance for materials which are weak in shearing strength. This will be discussed later in the chapter.

Unlike the principal stresses, for which no shearing stresses occur on the principal planes, the maximum shearing stresses act on planes which are usually not free of normal stresses. Substitution of θ_2 from Eq. 9-5 into Eq. 9-1 shows that the normal stresses which act on the planes of the maximum shearing stresses are

$$\sigma' = \frac{\sigma_x + \sigma_y}{2} \qquad (9\text{-}7)$$

Therefore a normal stress acts simultaneously with the maximum shearing stress unless $\sigma_x + \sigma_y$ vanishes.

If σ_x and σ_y in Eq. 9-6 are the principal stresses, τ_{xy} is zero and Eq. 9-6 simplifies to

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} \qquad (9\text{-}8)$$

EXAMPLE 9-2

For the state of stress in Example 9-1, reproduced in Fig. 9-6(a), (*a*) rework the previous problem for $\theta = -22\frac{1}{2}°$, using the general equations for the transformation of stress; (*b*) find the principal stresses and show their sense on a properly oriented element; and (*c*) find the maximum shearing stresses with the associated normal stresses and show the results on a properly oriented element.

SOLUTION

Case (*a*). By directly applying Eqs. 9-1 and 9-2 for $\theta = -22\frac{1}{2}°$, with $\sigma_x = +3$ ksi, $\sigma_y = +1$ ksi, and $\tau_{xy} = +2$ ksi, one has

$$\sigma_{x'} = \frac{3+1}{2} + \frac{3-1}{2}\cos(-45°) + 2\sin(-45°)$$

$$= 2 + 1(0.707) - 2(0.707) = +1.29 \text{ ksi}$$

$$\tau_{x'y'} = -\frac{3-1}{2}\sin(-45°) + 2\cos(-45°)$$

$$= +1(0.707) + 2(0.707) = +2.12 \text{ ksi}$$

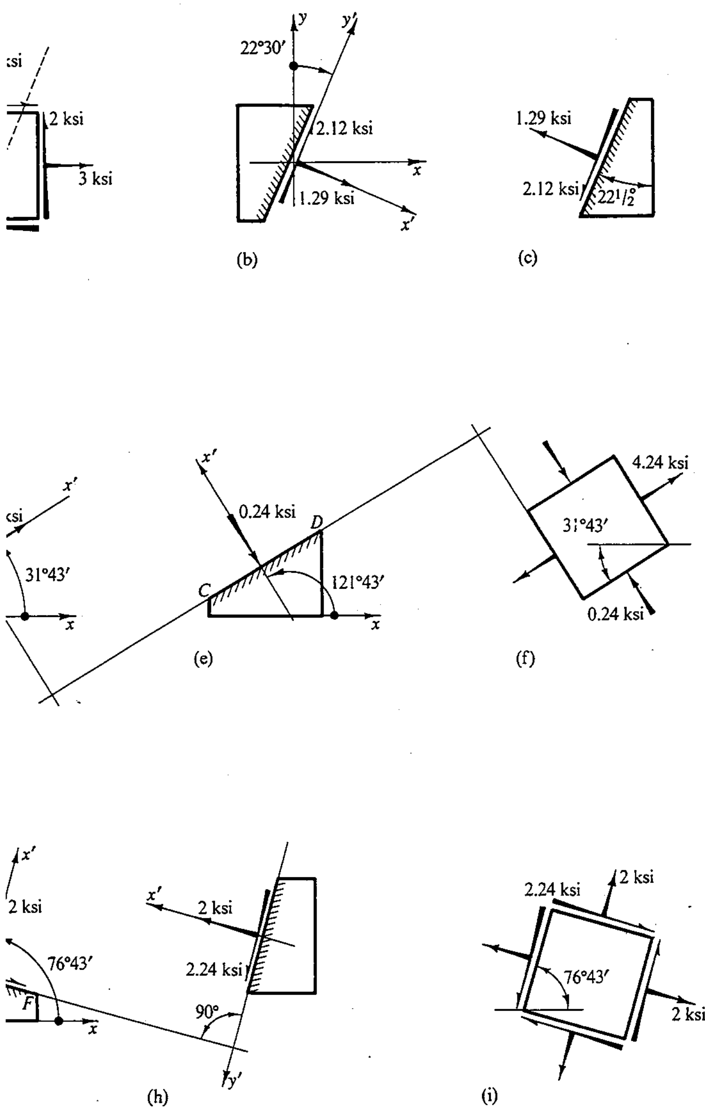


Fig. 9-6

The positive sign of $\sigma_{x'}$ indicates tension; whereas the positive sign of $\tau_{x'y'}$ indicates that the shearing stress acts in the $+y'$ direction, as shown in Fig. 9-4(b). These results are shown in Fig. 9-6(b) as well as in Fig. 9-6(c).

Case (*b*). The principal stresses are obtained by means of Eq. 9-4. The planes on which the principal stresses act are found by using Eq. 9-3.

$$\sigma_{1 \text{ or } 2} = \frac{3+1}{2} \pm \sqrt{\left(\frac{3-1}{2}\right)^2 + 2^2} = 2 \pm 2.24$$

$$\sigma_1 = +4.24 \text{ ksi} \quad \text{(tension)}, \qquad \sigma_2 = -0.24 \text{ ksi} \quad \text{(compression)}$$

$$\tan 2\theta_1 = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{2}{(3-1)/2} = 2$$

$$2\theta_1 = 63°26' \quad \text{or} \quad 63°26' + 180° = 243°26'$$

Hence $\qquad \theta_1' = 31°43' \quad$ and $\quad \theta_1'' = 121°43'$

This locates the two principal planes *AB* and *CD*, Figs. 9-6(d) and (e), on which σ_1 and σ_2 act. On which one of these planes the principal stresses act is unknown. So, Eq. 9-1 is solved by using, for example, $\theta_1' = 31°43'$. The stress found by this calculation is the stress which acts on the plane *AB*. Then, since $2\theta_1' = 63°26'$,

$$\sigma_{x'} = \frac{3+1}{2} + \frac{3-1}{2}\cos 63°26' + 2\sin 63°26' = +4.24 \text{ ksi} = \sigma_1$$

This result, besides giving a check on the previous calculations, shows that the maximum principal stress acts on the plane *AB*. The complete state of stress at the given point in terms of the principal stresses is shown in Fig. 9-6(f).

Case (*c*). The maximum shearing stress is found by using Eq. 9-6. The planes on which these stresses act are defined by Eq. 9-5. The sense of the shearing stresses is determined by substituting one of the roots of Eq. 9-5 into Eq. 9-2. Normal stresses associated with the maximum shearing stress are determined by using Eq. 9-7.

$$\tau_{\max} = \sqrt{[(3-1)/2]^2 + 2^2} = \sqrt{5} = 2.24 \text{ ksi}$$

$$\tan 2\theta_2 = -\frac{(3-1)/2}{2} = -0.500$$

$$2\theta_2 = 153°26' \quad \text{or} \quad 153°26' + 180° = 333°26'$$

Hence $\qquad \theta_2' = 76°43' \quad$ and $\quad \theta_2'' = 166°43'$

These planes are shown in Figs. 9-6(g) and (h). Then, using $2\theta_2' = 153°26'$ in Eq. 9-2,

$$\tau_{x'y'} = -\frac{3-1}{2}\sin 153°26' + 2\cos 153°26' = -2.24 \text{ ksi}$$

which means that the shear along the plane EF has an opposite sense to that in Fig. 9-4(b). From Eq. 9-7

$$\sigma' = \frac{3+1}{2} = 2 \text{ ksi}$$

The complete results are shown in Fig. 9-6(i).

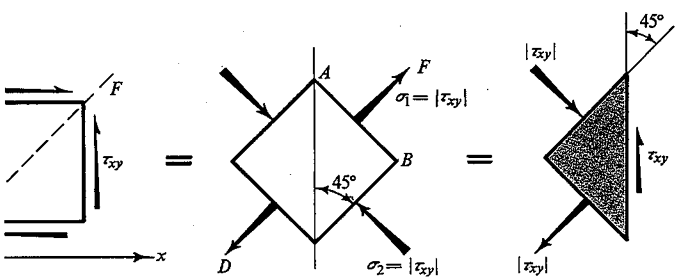
The description of the state of stress now can be exhibited in three alternative forms: as the originally given data, and in terms of the stresses found in parts (*b*) and (*c*) of this problem. In matrix representation of the stress tensors this yields

$$\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix} \text{ or } \begin{pmatrix} 4.24 & 0 \\ 0 & -0.24 \end{pmatrix} \text{ or } \begin{pmatrix} 2 & -2.24 \\ -2.24 & 2 \end{pmatrix} \text{ksi}$$

All these descriptions of the state of stress at the given point are equivalent. Note that in one of the stated forms the matrix is diagonal.

9-6. AN IMPORTANT TRANSFORMATION OF STRESS

A significant transformation of one description of a state of stress at a point to another occurs when pure shearing stress is converted into principal stresses. For this purpose consider an element subjected only to shearing stresses τ_{xy} as in Fig. 9-7(a). Then from Eq. 9-4 the principal stresses $\sigma_{1 \text{ or } 2} = \pm\tau_{xy}$, i.e., numerically σ_1, σ_2, and τ_{xy} are all equal, although σ_1 is a tensile stress and σ_2 is a compressive stress. In this case, from Eq. 9-3 the principal planes are given by $\tan 2\theta_1 = \infty$, i.e., $2\theta_1 = 90°$ or $270°$. Hence $\theta_1' = 45°$ and $\theta_1'' = 135°$; the planes corresponding to these angles are in Fig. 9-7(b). To determine on which plane the tensile stress σ_1 acts, a substitution into Eq. 9-1 is made with $2\theta_1' = 90°$. This computation



(b) (c)

-e shearing stress is equivalent to tension-compression
-; on inclined planes at 45° to the shearing planes.

shows that $\sigma_1 = +\tau_{xy}$, hence the tensile stress acts perpendicular to the plane AB. Both principal stresses which are equivalent to the pure shearing stress are shown in Figs. 9-7(b) and (c). Therefore, whenever pure shearing stress is acting on an element it may be thought of as causing tension along one of the diagonals and compression along the other. The diagonal such as DF in Fig. 9-7(a), along which a tensile stress acts, is referred to as the *positive shear diagonal.*

From the physical point of view, the transformation of stress found completely agrees with intuition. The material "does not know" the manner in which its state of stress is described, and a little imagination should convince one that the tangential shearing stresses combine to cause pull along the positive shear diagonal and compression along the other diagonal.

9-7. MOHR'S CIRCLE OF STRESS

In this article the basic Eqs. 9-1 and 9-2 for the stress transformation at a point will be re-examined in order to interpret them graphically. In doing this, two objectives will be pursued. First, by graphically interpreting these equations a greater insight into the general problem of stress transformation will be achieved. This is the main purpose of this article. Second, with the aid of graphical construction, a quicker solution of stress transformation problems can often be obtained. This will be discussed in the following article.

A careful study of Eqs. 9-1 and 9-2 shows that they represent a circle written in parametric form. That they do represent a circle is made clearer by first rewriting them as

$$\sigma_{x'} - \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \tag{9-9}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \tag{9-10}$$

Then by squaring both these equations, adding, and simplifying

$$\left(\sigma_{x'} - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \tag{9-11}$$

In every given problem σ_x, σ_y, and τ_{xy} are the three known constants, and $\sigma_{x'}$ and $\tau_{x'y'}$ are the variables. Hence Eq. 9-11 may be written in more compact form as

$$(\sigma_{x'} - a)^2 + \tau_{x'y'}^2 = b^2 \tag{9-12}$$

where $a = (\sigma_x + \sigma_y)/2$ and $b^2 = [(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2$ are constants.

This equation is the familiar expression of analytical geometry

1. The stress distribution on the rectangular cross-section shown in Fig. 1 is given by $\sigma_{xx}=1000y-500z+800$ kPa, $\sigma_{xy}=200z$ kPa, $\sigma_{xz}=0$. What is the net internal force system on this cross-section ?

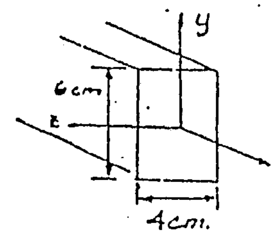


Fig. 1

Answer : $F=1920$ N, $V_y=V_z=0$
$M_y=-1600$ N-cm., $M_z=-7200$ N-cm., $T=-640$ N-cm.

2. Suppose the stress distribution on a cross-section of the circular cylinder of Fig. 2 is given by $\sigma_{xx}=\sigma_{xr}=0$, $\sigma_{x\theta}=k\sqrt{r}$, where k is unknown. What is the value of k in this case?

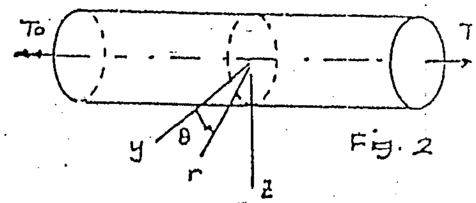


Fig. 2

Answer : $k=7T_0/4\pi R^{7/2}$

6.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium equations of that element, as was done in the derivations presented in Sec. 6.2.

6.6 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

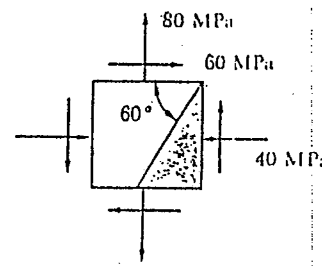


Fig. P6.4

6.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

6.14 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 40° counterclockwise, (*b*) 15° clockwise.

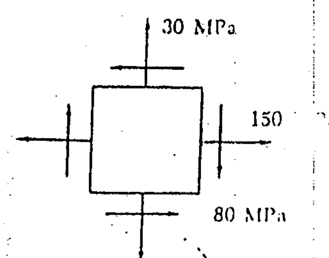
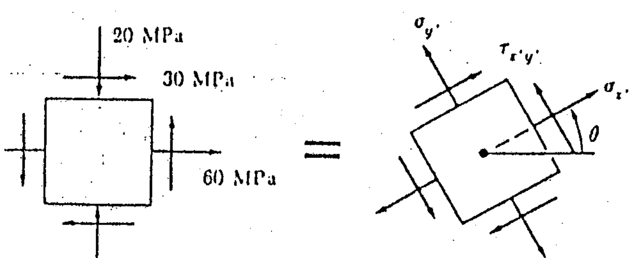


Fig. P6.6 and P6.10

6.56 For the state of plane stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than +65 MPa.



P6.55 and P6.56

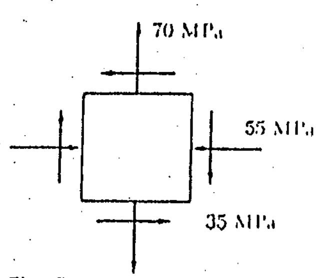


Fig. P6.14

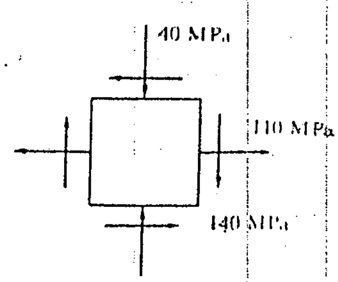


Fig. P6.8 and P6.12

2 5
1/2

1. The stress distribution on the rectangular cross-section shown in Fig. 1 is given by $\sigma_{xx}=1000y-500z+800$ kPa, $\sigma_{xy}=200z$ kPa, $\sigma_{xz}=0$. What is the net internal force system on this cross-section ?

Answer : $F= 1920$ N, $V_y=V_z=0$
$M_y=-1600$ N-cm., $M_z=-7200$ N-cm., $T=-640$ N-cm.

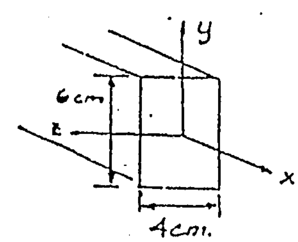


Fig. 1

2. Suppose the stress distribution on a cross-section of the circular cylinder of Fig. 2 is given by $\sigma_{xx}=\sigma_{xr}=0$, $\sigma_{x\theta}=k\sqrt{r}$, where k is unknown. What is the value of k in this case?

Answer : $k = 7T_0/4\pi R^{7/2}$

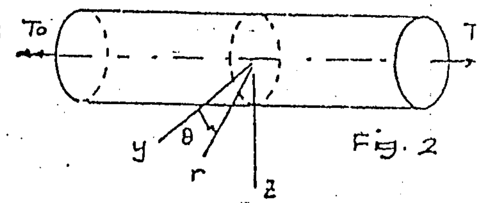


Fig. 2

6.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium equations of that element, as was done in the derivations presented in Sec. 6.2.

6.6 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

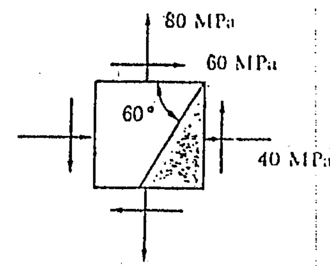


Fig. P6.4

6.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

6.14 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 40° counterclockwise, (*b*) 15° clockwise.

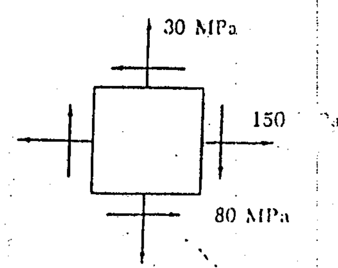
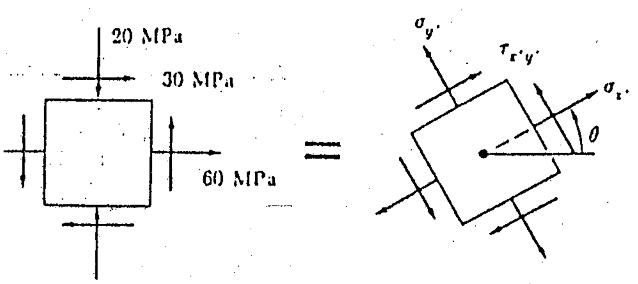


Fig. P6.6 and P6.10

6.56 For the state of plane stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than +65 MPa.



P6.55 and P6.56

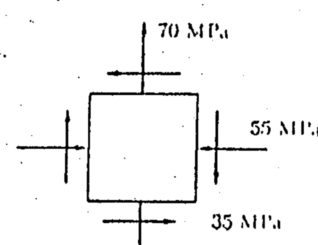


Fig. P6.14

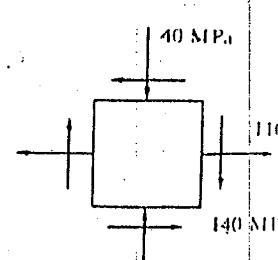
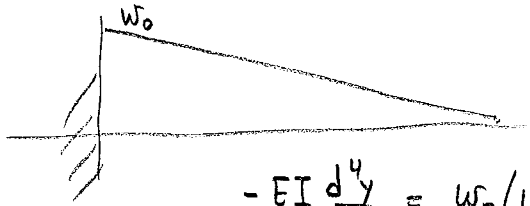


Fig. P6.8 and P6.12

2 5
1/2



$$q = w_0\left(1 - \frac{x}{L}\right)$$

$$-EI\frac{d^4y}{dx^4} = w_0\left(1 - \frac{x}{L}\right)$$

$$V = -EIy''' = -\frac{w_0L}{2}\left(1 - \frac{x}{L}\right)^2 + C_1$$

$$-M = -EIy'' = +\frac{w_0L^2}{6}\left(1 - \frac{x}{L}\right)^3 + C_1x + C_2$$

$$-EIy' = -\frac{w_0L^3}{24}\left(1 - \frac{x}{L}\right)^4 + C_1\frac{x^2}{2} + C_2x + C_3$$

$$-EIy = \frac{w_0L^4}{120}\left(1 - \frac{x}{L}\right)^5 + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3x + C_4$$

$x=0$ $y=0$, $y'=0$ $x=L$ $y=0$, $M=0$

$$\frac{w_0L^4}{120} + C_4 = 0 \qquad C_4 = -\frac{w_0L^4}{120}$$

$$-\frac{w_0L^3}{24} + C_3 = 0 \qquad C_3 = +\frac{w_0L^3}{24}$$

$$\frac{C_1L^3}{6} + \frac{C_2L^2}{2} + \frac{w_0L^4}{24} - \frac{w_0L^4}{120} = 0$$

$$\frac{C_1L^3}{6} + \frac{C_2L^2}{2} + \frac{w_0L^4}{30} = 0$$

$$C_1L + C_2 = 0 \qquad C_2 = -C_1L$$

$$\frac{C_1L^3}{6} - \frac{C_1L^3}{2} + \frac{w_0L^4}{30}$$

$$-\frac{C_1L^3}{3} + \frac{w_0L^4}{30} = 0 \qquad C_1 = \frac{w_0L}{10}$$

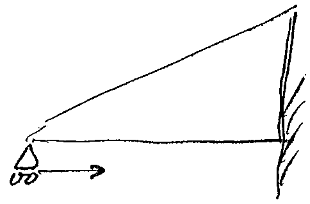
$$C_2 = -\frac{w_0L^2}{10}$$

$$y = -\frac{1}{EI}\left\{\frac{w_0L^4}{120}\left(1 - \frac{x}{L}\right)^5 + \frac{w_0Lx^3}{60} - \frac{w_0L^2x^2}{20} + \frac{w_0L^3x}{24} - \frac{w_0L^4}{120}\right]$$

$$M\Big|_{x=0} = -\frac{w_0L^2}{6}\left(1 - \frac{x}{L}\right)^3 - \frac{w_0L}{10}x + \frac{w_0L^2}{10}\Big|_{x=0} = -\frac{w_0L^2}{6} + \frac{w_0L^2}{10} = -\frac{1}{15}w_0L^2$$

$$y\Big|_{x=L/4} = -\frac{1}{EI}\left\{w_0L^4\frac{1}{120}\left(\frac{3}{4}\right)^5 + w_0L^4\frac{1}{60}\left(\frac{1}{4}\right)^3 - w_0L^4\frac{1}{20}\left(\frac{1}{4}\right)^2 + w_0L^4\frac{1}{24}\cdot\frac{1}{4} - w_0L^4\frac{1}{120}\right]$$

$$V\Big|_{x=L/2} = -\frac{w_0L}{2}\left(1 - \frac{x}{L}\right)^2 + \frac{w_0L}{10}\Big|_{x=L} = -w_0L\frac{1}{2}\left(\frac{1}{4}\right) + w_0L\frac{1}{10} = -\frac{w_0L}{40}$$



$\frac{w_0 x}{L}$

$$-EIy^{IV} = \frac{w_0 x}{L}$$

$$V = -EIy''' = \frac{w_0 x^2}{2L} + C_1$$

$$-EIy'' = -M = \frac{w_0 x^3}{6L} + C_1 x + \cancel{C_2}$$

$$-EIy' = \frac{w_0 x^4}{24L} + C_1 \frac{x^2}{2} + \cancel{C_2 x} + C_3$$

$$-EIy = \frac{w_0 x^5}{120L} + C_1 \frac{x^3}{6} + \cancel{C_2 \frac{x^2}{2}} + C_3 x + \cancel{C_4}$$

$y|_{x=0} = 0$ — $C_4 = 0$

$y|_{x=L} = 0$

$M|_{x=0} = 0$ — $C_2 = 0$

$\frac{dy}{dx}\Big|_{x=L} = 0$

$$0 = \frac{w_0 L^4}{120} + C_1 \frac{L^3}{6} + C_3 L$$

$$0 = \frac{w_0 L^3}{24} + C_1 \frac{L^2}{2} + C_3 \quad \cdot 5L$$

$$\begin{pmatrix} L^3/6 & L \\ L^2/2 & 1 \end{pmatrix} \begin{pmatrix} C_1 \\ C_3 \end{pmatrix} = \begin{pmatrix} -\frac{w_0 L^4}{120} \\ -\frac{w_0 L^3}{24} \end{pmatrix}$$

$$C_1 = \frac{\begin{pmatrix} -\frac{w_0 L^4}{120} & L \\ -\frac{w_0 L^3}{24} & 1 \end{pmatrix}}{-L^3/3} = \frac{\frac{1}{30} w_0 L^4}{-L^3/3} = -\frac{w_0 L}{10}$$

$$C_3 = \frac{\begin{pmatrix} L^3/6 & -w_0 L^4/120 \\ L^2/2 & -w_0 L^3/24 \end{pmatrix}}{-L^3/3} = -w_0 L^6 \left[\frac{10}{144 \cdot 10} - \frac{6}{240 \cdot 6}\right]$$

$6 \cdot 4 \cdot 6 \qquad 6 \cdot 4 \cdot 10 \cdot 6$

$\frac{4}{36 \cdot 40 \cdot 10} = \frac{1}{360}$

$$\frac{-\frac{w_0 L^6}{360}}{-\frac{L^3}{360} \cdot 120} = \frac{w_0 L^3}{120}$$

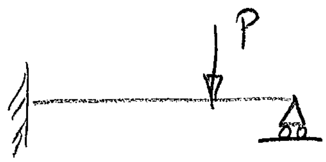
$$y = -\frac{1}{EI}\left[\frac{w_0 x^5}{120L} - \frac{w_0 L x^3}{60} + \frac{w_0 L^3}{120} x\right]$$

$$y\Big|_{x=L/4} = -\frac{w_0 L^4}{EI}\left[\frac{1}{120} \cdot \left(\frac{1}{4}\right)^5 - \left(\frac{1}{4}\right)^3 \frac{1}{60} + \frac{1}{120} \cdot \frac{1}{4}\right]$$

$$M\Big|_{x=L} = -\left[\frac{w_0 L^2}{6} - \frac{w_0 L^2}{10}\right] = -\frac{w_0 L^2}{15} \checkmark$$

$$V\Big|_{x=L/2} = \frac{w_0 L}{8} - \frac{w_0 L}{10} = \frac{w_0 L}{40} \checkmark$$

$= \uparrow \downarrow$



$$EI\frac{d^4y}{dx^4} = -P\langle x-a\rangle^{-1}$$

$$-V = EIy''' = -P\langle x-a\rangle^{0} + C_1$$

$$M = EIy'' = -P\langle x-a\rangle^{1} + C_1x + C_2$$

$$EIy' = -P\frac{\langle x-a\rangle^2}{2} + C_1\frac{x^2}{2} + C_2x + C_3$$

$$EIy = -P\frac{\langle x-a\rangle^3}{6} + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + \cancel{C_3x} + \cancel{C_4}$$

@ $x=0$ $y=0 \Rightarrow C_4=0$

$y'=0 \Rightarrow C_3=0$

@ $x=L$ $y=0 \Rightarrow$ $EIy=0 = -P\frac{(L-a)^3}{6} + C_1\frac{L^3}{6} + C_2\frac{L^2}{2} = 0$

$M=0 \Rightarrow$ $-P(L-a) + C_1L + C_2 = 0$

2 eqs, 2 unknowns
$$\begin{bmatrix} L^3/6 & L^2/2 \\ L & 1 \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} \frac{P(L-a)^3}{6} \\ P(L-a) \end{bmatrix}$$

Cramer's rule
$$C_1 = \frac{\begin{bmatrix} \frac{P(L-a)^3}{6} & L^2/2 \\ P(L-a) & 1 \end{bmatrix}}{\begin{bmatrix} L^3/6 & L^2/2 \\ L & 1 \end{bmatrix}} = \frac{\frac{P(L-a)^3}{6} - P(L-a)L^2/2}{L^3/6 - L^3/2} = \frac{P(L-a)\left[\frac{(L-a)^2}{6} - \frac{L^2}{2}\right]}{-L^3/3}$$

$$C_2 = \frac{\begin{bmatrix} L^3/6 & \frac{P(L-a)^3}{6} \\ L & P(L-a) \end{bmatrix}}{-L^3/3} = \frac{P(L-a)\left[L^3/6 - L\frac{(L-a)^2}{6}\right]}{-L^3/3}$$

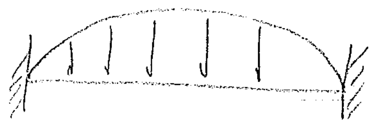
$$y = \frac{1}{EI}\left\{ -P\frac{\langle x-a\rangle^3}{6} + \left\{-\frac{P(L-a)^3}{2L^3} + \frac{3P(L-a)L^2}{2L^3}\right\}\frac{x^3}{6} + \left\{-\frac{P(L-a)L^3}{2L^3} + \frac{P(L-a)^3L}{2L^3}\right\}\frac{x^2}{2}\right\}$$

$y\big|_{x=L} = 0$

$$M\Big|_{x=a} = EIy'' = \cancel{-P\langle x-a\rangle} + \left\{-\frac{P(L-a)^3}{2L^3} + \frac{3P(L-a)L^2}{2L^3}\right\}\frac{a^3}{6} + \left\{-\frac{P(L-a)L^3}{2L^3} + \frac{P(L-a)^3L}{2L^3}\right\}\frac{a^2}{2}$$

$$V = -EIy''' = \cancel{P\langle x-a\rangle^0} + \{+P(L-a)^3 - 3P(L-a)L^2\}$$

$\sin\frac{\pi x}{L}$



$$V = -\frac{dM}{dx} = -EI\frac{d^3y}{dx^3}$$

$$q = \frac{dV}{dx} = -EI\frac{d^4y}{dx^4}$$

$$EI\frac{d^4y}{dx^4} = -\sin\frac{\pi x}{L}$$

$$EI\,y''' = \frac{L}{\pi}\cos\frac{\pi x}{L} + C_1$$

$$EI\,y'' = \frac{L^2}{\pi^2}\sin\frac{\pi x}{L} + C_1 x + C_2$$

$$EI\,y' = -\frac{L^3}{\pi^3}\cos\frac{\pi x}{L} + C_1\frac{x^2}{2} + C_2 x + C_3$$

$$EI\,y = -\frac{L^4}{\pi^4}\sin\frac{\pi x}{L} + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4$$

$y\big|_{x=0} = 0 \implies C_4 = 0$

$y'\big|_{x=0} = 0 \implies -\frac{L^3}{\pi^3} + C_3 = 0 \qquad C_3 = \frac{L^3}{\pi^3}$

$$y = \frac{1}{EI}\left[-\frac{L^4}{\pi^4}\sin\frac{\pi x}{L} + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + \frac{L^3 x}{\pi^3}\right]$$

$y\big|_{x=L} = 0$

$$y = 0 = \frac{1}{EI}\left[0 + C_1\frac{L^3}{6} + C_2\frac{L^2}{2} + \frac{L^4}{\pi^3}\right]$$

$y'\big|_{x=L} = 0 \qquad y' = \frac{1}{EI}\left[\left(-\frac{L^3}{\pi^3}\right)(-1) + C_1\frac{L^2}{2} + C_2 L + \frac{L^3}{\pi^3}\right] = 0$

$$C_1\frac{L^2}{2} + C_2 L = -\frac{2L^3}{\pi^3}$$

$$C_1\frac{L^3}{6} + C_2\frac{L^2}{2} = -\frac{L^4}{\pi^3}$$

Solve for C_1 & C_2

$$\begin{bmatrix} L^2/2 & L \\ L^3/6 & L^2/2 \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} -2L^3/\pi^3 \\ -L^4/\pi^3 \end{bmatrix}$$

Use Cramer's rule

$$C_1 = \frac{\begin{pmatrix} -2L^3/\pi^3 & L \\ -L^4/\pi^3 & L^2/2 \end{pmatrix}}{6\frac{L^4}{24} - 4\frac{L^4}{24}} = \frac{-L^5/\pi^3 + L^5/\pi^3}{L^4/12} \qquad \underline{C_1 = 0}$$

$$C_2 = \frac{\begin{pmatrix} L^2/2 & -2L^3/\pi^3 \\ L^3/6 & -L^4/\pi^3 \end{pmatrix}}{} = \left(\frac{-3L^6/6\pi^3 + 2L^6/6\pi^3}{L^4/12}\right) = -\frac{L^6/6\pi^3}{\frac{L^4}{12}} = \frac{-12L^6}{6\pi^3L^4} = -\frac{2}{\pi^3}L^2$$

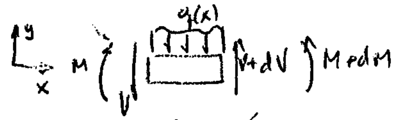
$$y = \frac{1}{EI}\left[-\frac{L^4}{\pi^4}\sin\frac{\pi x}{L} + 0 - \frac{1}{\pi^3}L^2x^2 + \frac{L^3x}{\pi^3}\right]$$

$$y\big|_{x=L/2} = \frac{1}{EI}\left[-\frac{L^4}{\pi^4}\cdot 1 - \frac{L^4}{4\pi^3} + \frac{L^4}{2\pi^3}\right] = \frac{1}{EI}\left[-\frac{L^4}{\pi^4} + \frac{L^4}{4\pi^3}\right] = -\frac{L^4}{EI}(0.0022031)$$

$$M = EI\frac{d^2y}{dx^2} = \frac{L^2}{\pi^2}\sin\frac{\pi x}{L} - \frac{2}{\pi^3}L^2$$

$$M\Big|_{x=L} = \frac{L^2}{\pi^2}\cdot\sin\pi - \frac{2}{\pi^3}L^2 = -\frac{2}{\pi^3}L^2$$

$$V\Big|_{x=L/2} = -EIy''' = -\left[\frac{L}{\pi}\cos\frac{\pi x}{L} + \cancel{C_1}\right] = 0$$

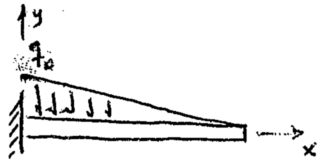


$$V + dV - V + q(x)dx = 0 \qquad \boxed{q(x) = \frac{dV}{dx}}$$

$$M + dM - M + (V + dV)dx - [q(x)dx]\frac{dx}{2} = 0 \qquad \boxed{V = -\frac{dM}{dx}}$$

$$\frac{1}{\rho} = \frac{M}{EI} \approx \frac{d^2v}{dx^2} = \frac{M_z}{EI_{zz}}$$

$-q(x) = EI\frac{d^4v}{dx^4}$, כשנתון כי EI קבוע . $\frac{d^4v}{dx^4} = -\frac{d^2}{dx^2}\left(\frac{1}{EI}M\right)^{-1}$ $\qquad q(x) = -\frac{d^2M}{dx^2}$, כן



$$q(x) = q_0\left(1 - \frac{x}{L}\right)$$

$$-EIv^{IV} = -q_0\left(1 - \frac{x}{L}\right)$$

$$v(x=0) = 0 \qquad -EIv''' = V = 0$$
$$\frac{dv}{dx}(x=0) = 0 \qquad +EIv'' = M = 0 \Big\} x=$$

$$-EIv''' = -q_0\left(x - \frac{x^2}{2L} + C\right)$$

@ $x = L$ $\quad L - \frac{L^2}{2L} + C = 0 \qquad C_1 = -L/2$

$$-EIv''' = -q_0\left(x - \frac{x^2}{2L} - \frac{L}{2}\right) = +V$$

$$EIv'' = -q_0\left(\frac{x^2}{2} - \frac{x^3}{6L} - \frac{Lx}{2} + C_2\right)$$

@ $x = L$ $\quad \frac{L^2}{2} - \frac{L^2}{6} - \frac{L^2}{2} + C_2 = 0 \qquad C_2 = \frac{L^2}{6}$

$$EIv'' = -q_0\left(\frac{x^2}{2} - \frac{x^3}{6L} + \frac{L^2}{6} - \frac{Lx}{2}\right) = M$$

$$v'' = -\frac{q_0}{EI}\left(\frac{x^2}{2} - \frac{x^3}{6L} + \frac{L^2}{6} - \frac{Lx}{2}\right)$$

$$v' = -\frac{q_0}{EI}\left(\frac{x^3}{6} - \frac{x^4}{24L} + \frac{L^2x}{6} - \frac{Lx^2}{4} + C_3\right)$$

@ $x = 0$ $\quad v' = 0 \quad C_3 = 0$

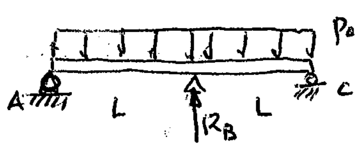
$$v' = -\frac{q_0}{EI}\left(\frac{x^3}{6} - \frac{x^4}{24L} + \frac{L^2x}{6} - \frac{Lx^2}{4}\right)$$

$$v = -\frac{q_0}{EI}\left(\frac{x^4}{24} - \frac{x^5}{120L} + \frac{L^2x^2}{12} - \frac{Lx^3}{12} + C_4\right)$$

@ $x = 0$ $\quad v = 0 \quad C_4 = 0$

$$v = \frac{-q_0}{120EIL}\left(5x^4L - x^5 + 10L^3x^2 - 10L^2x^3\right)$$

$$\langle x - x_0\rangle_*^{+n} = \begin{cases} 0 & x \le \\ (x - x_0)^n & x \end{cases}$$



$$-EIv^{IV} = -p_0 + R_B\langle x - L\rangle_*^{-1}$$

$$EIv''' = -p_0x + R_B\langle x - L\rangle_*^0 + C_1 = -V$$

$$EIv'' = -p_0\frac{x^2}{2} + R_B\langle x - L\rangle_*^1 + C_1x + C_2$$

$$EIv' = -p_0\frac{x^3}{6} + R_B\langle x - L\rangle_*^2 + C_1\frac{x^2}{2} + C_2x + C_3$$

$$EIv = -p_0\frac{x^4}{24} + R_B\langle x - L\rangle_*^3 + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3x + C_4$$

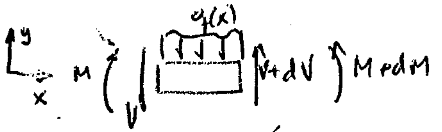
$v(A) = 0 = v(x=0)$ $\qquad M_A = EIv''(x=0) = 0$

$v(B) = 0 = v(x=L)$ $\qquad M_C = EIv''(x=2L) = 0$

$v(C) = 0 = v(x=2L)$

$v(A) = 0 \Rightarrow C_4 = 0 \qquad v''(A) = 0 \Rightarrow C_2 = 0 \qquad R_B = \frac{5}{4}p_0L \quad C_1 = \frac{3}{8}p_0L$

$C_3 = -\frac{1}{48}p_0L^3$

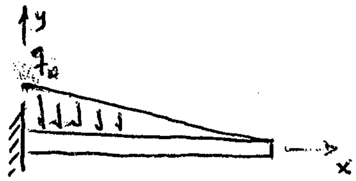


$$V + dV - V + q(x)dx = 0 \qquad \boxed{q(x) = \frac{dV}{dx}}$$

$$M + dM - M + (V + dV)dx - [q(x)dx]\frac{dx}{2} = 0 \qquad \boxed{V = -\frac{dM}{dx}}$$

$$\frac{1}{\rho} = \frac{M}{EI} \approx \underline{\frac{d^2 v}{dx^2} = \frac{M_z}{EI_{zz}}}$$

$$-q(x) = EI\frac{d^4 v}{dx^4}, \text{ הנחנו ש } EI \text{ קבוע} . \quad \frac{d^4 v}{dx^4} = -\frac{d^2}{dx^2}\left(\frac{1}{EI}M\right) \cdot 1 \qquad q(x) = -\frac{d^2 M}{dx^2}$$



$$q(x) = q_0\left(1 - \frac{x}{L}\right)$$

$$EI\,v^{IV} = -q_0\left(1 - \frac{x}{L}\right)$$

$$\left.\begin{array}{ll} v(x=0) = 0 & -EIv''' = V = 0 \\ \frac{dv}{dx}(x=0) = 0 & +EIv'' = M = 0 \end{array}\right\} x = L$$

$$EI\,v''' = -q_0\left(x - \frac{x^2}{2L} + C\right)$$

@ $x = L$ $\quad L - \frac{L^2}{2L} + C = 0 \qquad C_1 = -L/2$

$$EI\,v''' = -q_0\left(x - \frac{x^2}{2L} - \frac{L}{2}\right) = +V$$

$$EI\,v'' = -q_0\left(\frac{x^2}{2} - \frac{x^3}{6L} - \frac{Lx}{2} + C_2\right)$$

@ $x = L$ $\quad \frac{L^2}{2} - \frac{L^2}{6} - \frac{L^2}{2} + C_2 = 0 \qquad C_2 = \frac{L^2}{6}$

$$EI\,v'' = -q_0\left(\frac{x^2}{2} - \frac{x^3}{6L} + \frac{L^2}{6} - \frac{Lx}{2}\right) = M$$

$$v'' = -\frac{q_0}{EI}\left(\frac{x^2}{2} - \frac{x^3}{6L} + \frac{L^2}{6} - \frac{Lx}{2}\right)$$

$$v' = -\frac{q_0}{EI}\left(\frac{x^3}{6} - \frac{x^4}{24L} + \frac{L^2 x}{6} - \frac{Lx^2}{4} + C_3\right)$$

@ $x = 0$ $\quad v' = 0 \quad C_3 = 0$

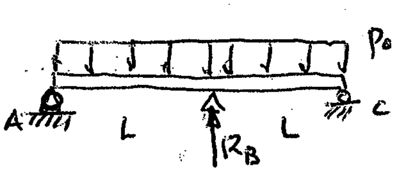
$$v' = -\frac{q_0}{EI}\left(\frac{x^3}{6} - \frac{x^4}{24L} + \frac{L^2 x}{6} - \frac{Lx^2}{4}\right)$$

$$v = -\frac{q_0}{EI}\left(\frac{x^4}{24} - \frac{x^5}{120L} + \frac{L^2 x^2}{12} - \frac{Lx^3}{12} + C_4\right)$$

@ $x = 0$ $\quad v = 0 \quad C_4 = 0$

$$v = \frac{-q_0}{120EIL}\left(5x^4 L - x^5 + 10L^3 x^2 - 10L^2 x^3\right)$$

$$\langle x - x_0\rangle_*^n = \begin{cases} 0 & x \leq \\ (x - x_0)^n & x > \end{cases}$$



$$EI\,v^{IV} = -p_0 + R_B\langle x - L\rangle_*^{-1}$$

$$EI\,v''' = -p_0 x + R_B\langle x - L\rangle_*^0 + C_1 = -V$$

$$EI\,v'' = -p_0\frac{x^2}{2} + R_B\langle x - L\rangle_*^1 + C_1 x + C_2$$

$$EI\,v' = -p_0\frac{x^3}{6} + R_B\langle x - L\rangle_*^2 + C_1\frac{x^2}{2} + C_2 x + C_3$$

Florida International University
Department of Mechanical Engineering

EMA 3702 EXAMINATION NO. 3C April 9, 2002

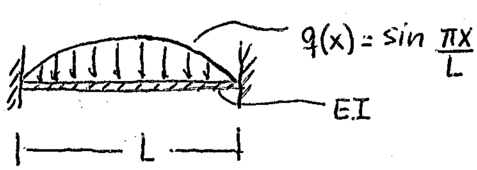
Print your name and sign the following statement:

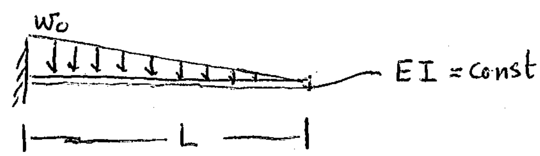
I will not give nor take any unpermitted aid during this examination. I understand that violation of this statement will lead to automatic failure of the examination.

PRINT NAME ______________ SIGN NAME ______________

For the following loading system, which may occur because of build-up of snow on a horizontal flagpole, find:

1) the elastic curve y(x)
2) the displacement at x=L
3) and using the elastic curve equation, find the moment at the location x=0
4) using the elastic curve equation, find the shear at x=L/2





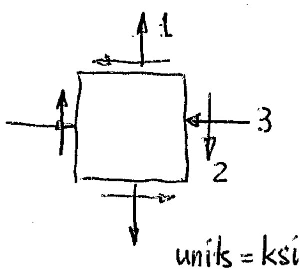
QUIZ 4B   EMA 3702 April 18, 2002

Name:____________________

Student No.________________

Given the following stress state, find

- the principal stresses, σ1 and σ2
- principal stress directions
- maximum shear stress and direct stress, σ, perpendicular to the plane of the shear stress
- maximum shear stress directions



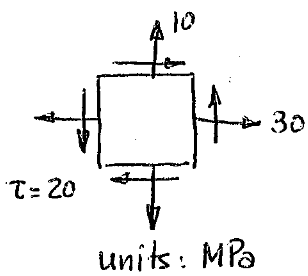
QUIZ 4A   EMA 3702 April 18, 2002

Name:____________________

Student No.________________

Given the following stress state, find

- the principal stresses, σ1 and σ2
- principal stress directions
- maximum shear stress and direct stress, σ, perpendicular to the plane of the shear stress
- maximum shear stress directions



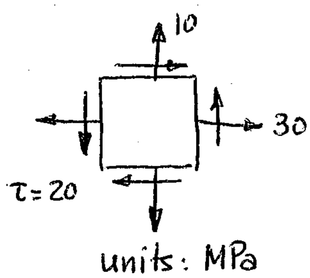
QUIZ 4A EMA 3702 April 18, 2002

Name:____________________

Student No.________________

Given the following stress state, find

- the principal stresses, σ1 and σ2
- principal stress directions
- maximum shear stress and direct stress, σ, perpendicular to the plane of the shear stress
- maximum shear stress directions



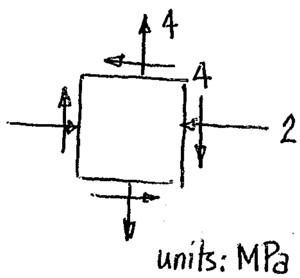
QUIZ 4C EMA 3702 April 18, 2002

Name:______________________

Student No.__________________

Given the following stress state, find

- the principal stresses, σ1 and σ2
- ~~principal stress directions~~
- maximum shear stress and direct stress, σ, perpendicular to the plane of the shear stress
- maximum shear stress directions



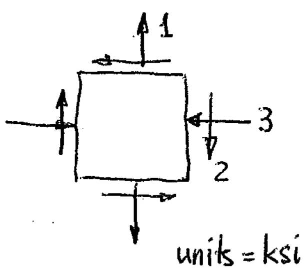
QUIZ 4B EMA 3702 April 18, 2002

Name:____________________

Student No.________________

Given the following stress state, find

- the principal stresses, $\sigma 1$ and $\sigma 2$
- principal stress directions
- maximum shear stress and direct stress, σ, perpendicular to the plane of the shear stress
- maximum shear stress directions



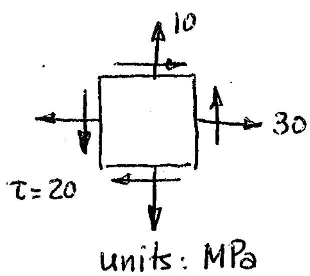
QUIZ 4A EMA 3702 April 18, 2002

Name:_____________________

Student No._________________

Given the following stress state, find

- the principal stresses, σ1 and σ2
- principal stress directions
- maximum shear stress and direct stress, σ, perpendicular to the plane of the shear stress
- maximum shear stress directions

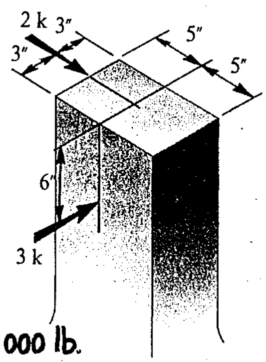


QUIZ 2D EMA 3702 March 14, 2002

Name:____________________

Student No.________________

A cast iron block is loaded as shown in the figure. Neglecting the weight of the block, determine the stresses acting normal to a section taken 18 inches below the top and locate the line of zero stress.



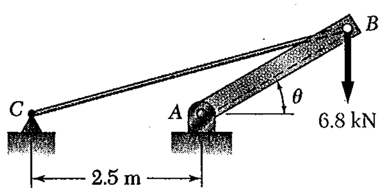
1 k = 1000 lb.

Problem 1.

Member AB consists of a single C130 x 10.4 steel channel of length 2.5 m. Knowing that the pins at A and B pass through the centroid of the cross section of the channel,

a) determine the factor of safety for the load shown with respect to buckling in the plane of the figure when $\theta = 30°$. Let E=200 GPa.
b) Suppose the member CB is a rod also made of the same steel. If we want the structure to buckle and yield simultaneously, what must be the minimum diameter of the rod.

Calculate the safety factor according to the following conditions.
Rod and beam are pin connected at both ends for buckling in their long direction and are considered fixed at both ends for buckling in the direction out of the page.



EMA3702 FALL 2002 DR. C. LEVY

FINAL EXAMINATION-Version B

December 10, 2002

General Instructions -- This examination is 2 hours and 30 minutes long. You are allowed your help aids from previous quizzes and any help aids attached to the examination. SHOW ALL WORK!!!

Please sign the following:

I certify that I will neither receive nor give unpermitted aid on this examination. Violation of this will result in failure of the course and possibly other academic disciplinary actions.

________________________ ________________________

Print your name Sign your name

This examination consists of 3 problems with several parts to each of the problems. You are to answer all the problems!

GOOD LUCK!

Problem #	Breakdown by Problem	Score
1	35%	
2	30%	
3	35%	

TOTAL

Problem 3

Two beams made of steel are joined by means of a rod, also of steel, and the rod's length is 3.75 m. At the beginning no load or moment is acting on the structure. If the rod is now heated by 50 degrees C, find the change in location of point C.

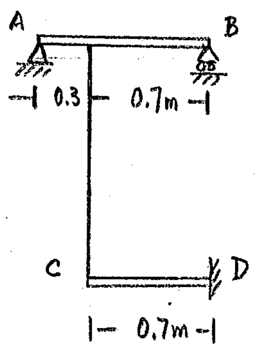
Given: E=206 GPa

coefficient of expansion α= 12 x 10^{-6} mm/mm-degree C.

Both beams have moments of inertia I = 850 cm^4.

The rod has a cross-sectional area of 1.6 cm^2.

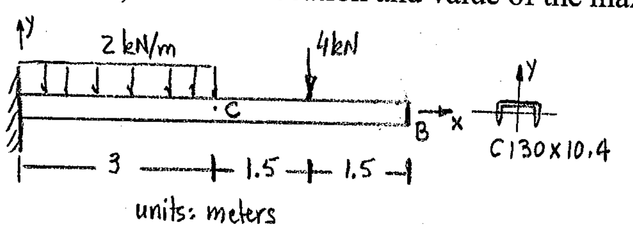
Other helpful information can be found at the back of the examination.



Problem 2.

Given a bar made of wood that is loaded as seen in the figure and the cross-section as given in the figure. The Young's modulus is E=12 GPa

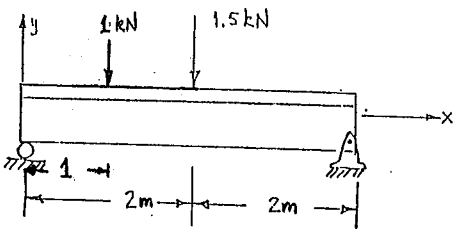
a) Find the equation of the elastic curve and the slope at C and the displacement at B.
b) Draw the moment and shear diagrams for the bar
c) Find the location and value of the maximum stress, τ

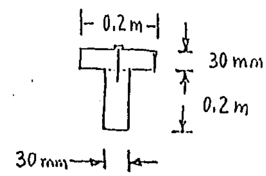


Problem 3.

The T shaped beam is made of 2 wood planks 200 mm x 30 mm which are joined by nails. If the allowable bending stress is 12 MPa, and the allowable shearing stress is 0.8 MPa, find:

a) if the beam is able to support safely the loads shown in the picture
b) the maximum spacing between the nails if each nail is able to support safely 1500 N of shear force.



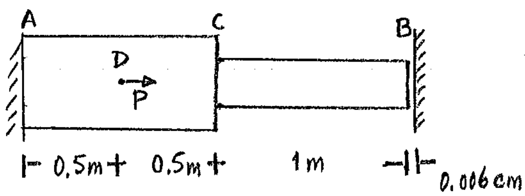


Problem 2.

A composite bar AB is made of steel (section AC) and brass (section BC). The cross-sectional area of AC is 200 cm^2 and that of BC is 100 cm^2. The bar is found between two walls with a space as shown in the diagram. On the bar is placed a load at D and both sections of the bar are heated by 20 degrees C. Young's modulus of the steel is 206×10^9 Pa and that of brass is 103×10^9 Pa. The coefficient of thermal expansion of the steel is 12.5×10^{-6} cm/cm-degree C and that of the brass is 16.5×10^{-6} cm/cm-degree C.

Find the axial stresses in each section AD, DC, CB

Note that P=147 kN

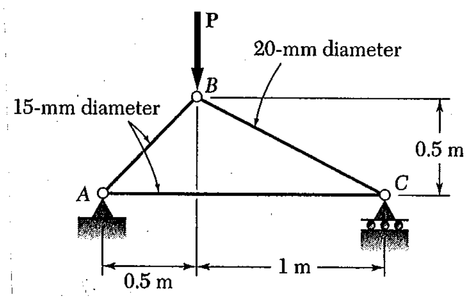


Problem 1.

Knowing that a factor of safety of 2.6 is required,

a) determine the largest load P that can be applied to the structure shown. Use E=200 GPa.
b) If we want the structure to buckle and yield simultaneously, what must be the minimum diameter of the rod that yields first, all else being unchanged. Assume σ_{yp}=360 MPa.

Calculate the safety factor according to the following conditions.
The rods are pin connected at both ends for buckling in their long direction and are considered fixed at both ends for buckling in the direction out of the page.

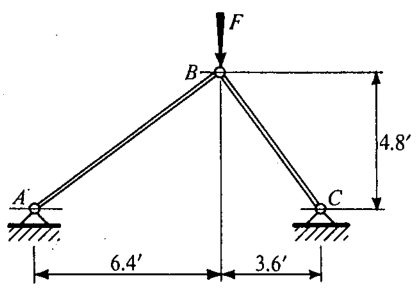


Problem 1.

The pin connected aluminum alloy frame shown carries a concentrated load F.

a) Determine the value of F that will cause buckling. Take $E = 10 \times 10^6$ psi for the alloy. Both members have 2 inch by 2 inch cross-sections.
b) If we want the structure to buckle and yield simultaneously, what must be the minimum dimension of the square member of the rod that yields first, all else being unchanged. Assume σ_{yp}=35 ksi in tension and compression.

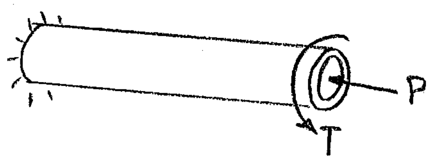
Calculate the load F according to the following conditions.
The rods are pin connected at both ends for buckling in their long direction and are considered fixed at both ends for buckling in the direction out of the page.



Problem 3.

Consider a hollow cylindrical tube of outer radius Ro=140mm and inner radius Ri=125 mm. The tube is fixed at one end and subjected to a torque of 35 kN-m together with an axial compressive force of 68 kN as shown in the diagram.

a) Determine the principal stresses and where they occur
b) Determine the maximum shear stress and the direction in which they occur



EMA3702 Summer 2003 DR. C. LEVY

FINAL EXAMINATION-Version A

June 24, 2003

General Instructions -- This examination is 2 hours long. You are allowed your help aids from previous quizzes and any help aids attached to the examination. SHOW ALL WORK!!!

Please sign the following:

I certify that I will neither receive nor give unpermitted aid on this examination. Violation of this will result in failure of the course and possibly other academic disciplinary actions.

Print your name ____________________ Sign your name ____________________

This examination consists of 3 problems with several parts to each of the problems. You are to answer all the problems!

GOOD LUCK!

Problem #	Breakdown by Problem	Score
1	35%	
2	35%	
3	30%	

TOTAL

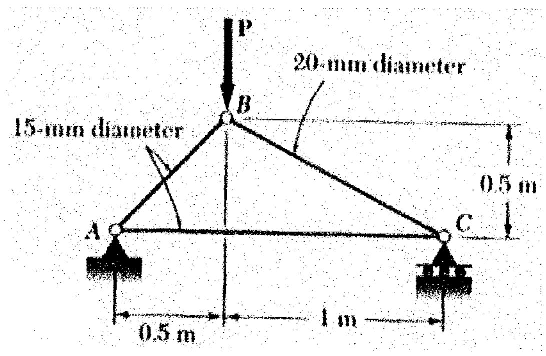
Problem 1a.

Knowing that a factor of safety of 2.6 is required,

a) Determine the largest load P that can be applied to the structure shown.
   Use E=200 GPA.

Calculate the load under the following conditions:
The rods are pin connected at both ends for buckling in their long direction and are considered fixed at both ends for buckling in the out of page direction.

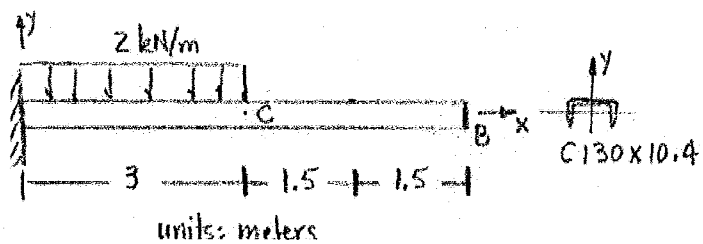
b) For the load found in (a), find the cross-sectional area in the other two members so that their allowable stresses meet the safety factor. Assume that $\sigma_{yp} = 360$ MPa



Problem 2a.

Given a bar made of wood that is loaded as seen in the figure and the cross-section as given in the figure. The Young's modulus is E=12 GPa

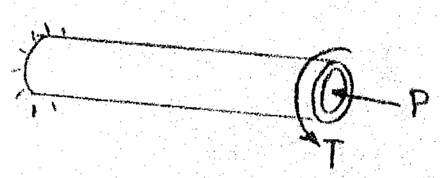
a) Find the equation of the elastic curve
b) Find the location and value of the maximum shear stress, τ



Problem 3c.

Consider a hollow cylindrical tube of outer radius $R_0 = 140$ mm and inner radius $R_i = 125$ mm. The tube is fixed at one end and subjected to a torque of 35 kN-m together with an axial compressive force of 68 kN as shown in the diagram. If the tube is also pressurized to a pressure of 2.1 MPa

Determine the principal stresses and where they occur



EMA 3702 Summer 2003 DR. C. LEVY

FINAL EXAMINATION-Version C

June 24, 2003

General Instructions -- This examination is 2 hours long. You are allowed your help aids from previous quizzes and any help aids attached to the examination. SHOW ALL WORK!!!

Please sign the following:

I certify that I will neither receive nor give unpermitted aid on this examination. Violation of this will result in failure of the course and possibly other academic disciplinary actions.

________________________ ________________________

Print your name Sign your name

This examination consists of 3 problems with several parts to each of the problems. You are to answer all the problems!

GOOD LUCK!

Problem #	Breakdown by Problem	Score
1	35%	
2	35%	
3	30%	

TOTAL

Problem 1c.

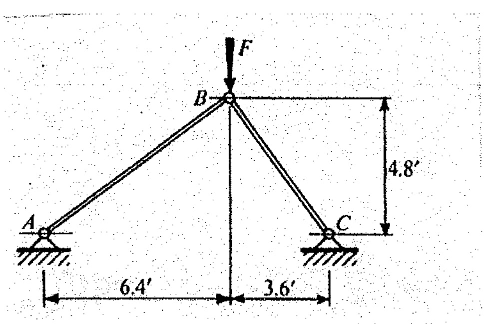
A bar truss, made of steel, is loaded by F as shown in the diagram. If each bar has a rectangular cross section of 2 inches x 2 inches.

a. Find the load F that will cause buckling and in which bar it will happen

Calculate the load F according to the following conditions.
All the bars are pin connected at both ends for buckling in the long direction of the bar and are considered fixed at both ends for buckling in the direction out of the page.

b. For the load, F, found in (a), what is the direct stress in the bar that does not buckle. Has the bar failed in yield.

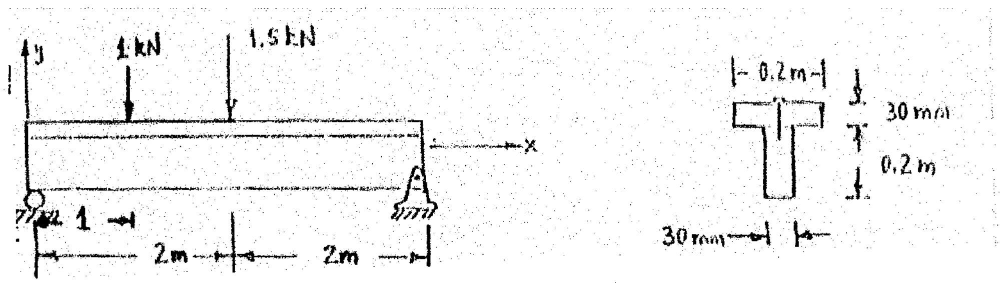
Take $E = 10 \times 10^6$ psi and σ_{yp}=35000 psi in tension and compression. The dimensions of the bars are given in the diagram.



Problem 2c.

The T shaped beam is made of 2 wood planks 200 mm x 30 mm which are joined by nails. If the allowable bending stress is 12 MPa, and the allowable shearing stress is 0.8 MPa, find:

a) if the beam is able to support safely the loads shown in the picture
b) the maximum spacing between the nails if each nail is able to support safely 1500 N of shear force.

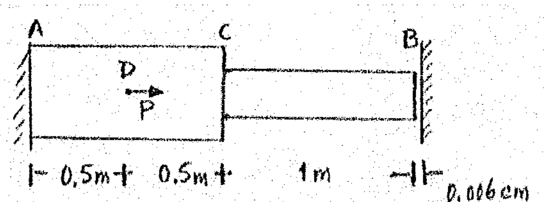


Problem 3c.

A composite bar AB is made of steel (section AC) and brass (section BC). The cross-sectional area of AC is 200 cm^2 and that of CB is 100 cm^2. The bar is found between two walls with a space as shown in the diagram. On the bar is placed a load at D and both sections of the bar are heated by 20 degrees C. Young's modulus of the steel is 206 x 10^9 GPa and that of brass is 103 x 10^9 GPa. The coefficient of thermal expansion of the steel is 12.5 x 10^{-6} cm/cm-deg C and that of the brass is 16.5 x 10^{-6} cm/cm-deg C.

Find the axial stresses in each section AD, DC, CB

Note that P=9800 N



REVIEW PROBLEMS

10.117 A column of 3.5-m effective length is made by welding together two 89 × 64 × 6.4-mm angles as shown. Using $E = 200$ GPa, determine the allowable centric load if a factor of safety of 2.8 is required.

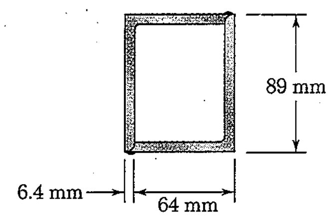


Fig. P10.117

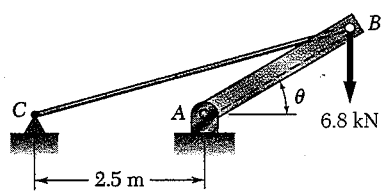


Fig. P10.118

10.118 Member *AB* consists of a single C130 × 10.4 steel channel of length 2.5 m. Knowing that the pins at *A* and *B* pass through the centroid of the cross section of the channel, determine the factor of safety for the load shown with respect to buckling in the plane of the figure when $\theta = 30°$. Use Euler's formula with $E = 200$ GPa.

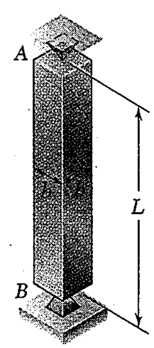
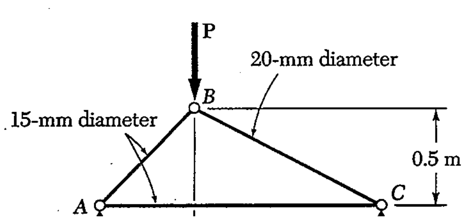


Fig. P10.119

10.119 Supports *A* and *B* of the pin-ended column shown are at a fixed distance *L* from each other. Knowing that at a temperature T_0 the force in the column is zero and that buckling occurs when the temperature is $T_1 = T_0 + \Delta T$, express ΔT in terms of *b*, *L*, and the coefficient of thermal expansion α.



10.121 The steel rod *BC* is attached to the rigid support at *C*. Knowing that $G = 11.2 \times 10^6$ psi, detern *BC* for which the critical load P_{cr} of the system is 80

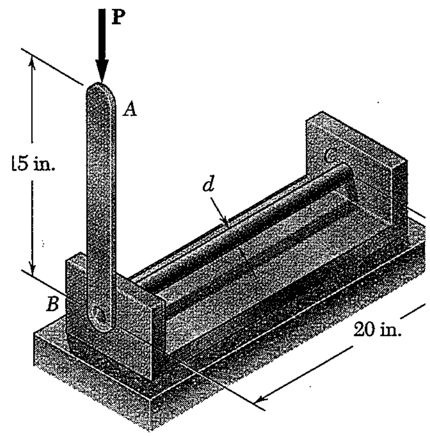


Fig. P10.121

10.122 An axial load **P** of magnitude 560 kN the *x* axis at a distance $e = 8$ mm from the geometric a rolled-steel column *BC*. Using $E = 200$ GPa, determir flection of end *C*, (*b*) the maximum stress in the colur

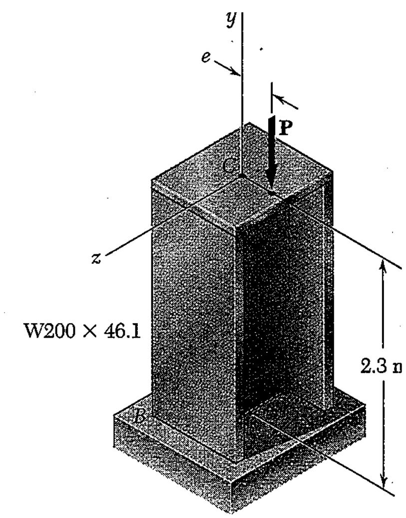


Fig. P10.122

10.123 A column with the cross section showı length. Knowing that $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ able stress design formulas to determine the largest cer plied to the column.

10.124 A column of 4.5-m effective length mu

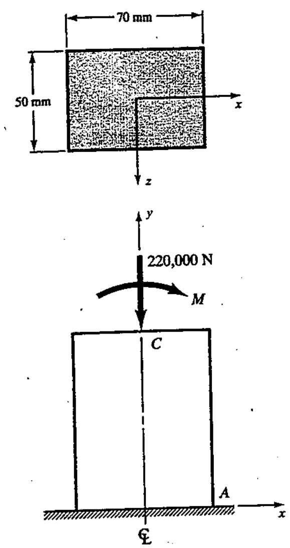


Fig. 17-5

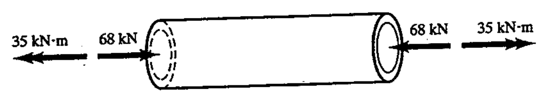


Fig. 17-6

The 68-kN force produces a uniformly distributed compressive stress given by

$$\sigma_1 = \frac{-68{,}000\ \text{N}}{\pi[(0.140\ \text{m})^2 - (0.125\ \text{m})^2]} = -5.44\ \text{MPa}$$

as shown in Fig. 17-7. The torsional shearing stresses due to the 35-kN · m torque were found in Problem 5.2 to be $\tau = T\rho/J$. Here, the polar moment of inertia is

$$J = \frac{\pi}{2}[(0.140\ \text{m})^4 - (0.125)^4] = 0.0002199\ \text{m}^4$$

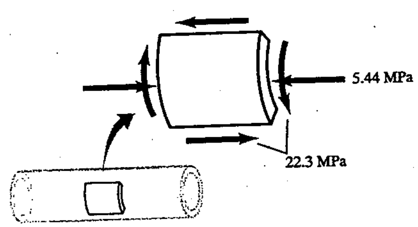


Fig. 17-7

If the approximate expression of Problem 5.6 is used, we find 0.0002191 m^4. Thus, the shearing stresses at the outer fibers of the shell are given by

$$\tau = \frac{T\rho}{J} = \frac{(35{,}000\ \text{N}\cdot\text{m})\,(0.140\ \text{m})}{0.0002199} = 22.3\ \text{MPa}$$

and these are shown in Fig. 17-7.

From Problem 16.13 the principal stresses are found to be

$$\sigma = \frac{-5.44+0}{2} \pm \sqrt{\left(-\frac{5.44-0}{2}\right)^2 + (22.3)^2}$$

$$\sigma_{max} = 19.75\ \text{MPa}$$

$$\sigma_{min} = -25.19\ \text{MPa}$$

and the peak shearing stress is 22.47 MPa.

17.3. Consider a hollow circular shaft whose outside diameter is 3 in and whose inside diameter is equal to one-half the outside diameter. The shaft is subject to a twisting moment of 20,000 lb · in as well as a bending moment of 30,000 lb · in. Determine the principal stresses in the body. Also, determine the maximum shearing stress.

The twisting moment gives rise to shearing stresses that attain their peak values in the outer fibers of the shaft. From Problem 5.2 these shearing stresses are given by $\tau_{xy} = T\rho/J$. From Problem 5.1 it is seen that for the hollow circular area

$$J = \frac{\pi}{32}(D_o^4 - D_i^4) = \frac{\pi}{32}[3^4 - (1.5)^4] = 7.46\ \text{in}^4$$

where D_o denotes the outer diameter of the section and D_i represents the inner diameter. At the outer fibers the torsional shearing stresses are thus

$$\tau_{xy} = \frac{T\rho}{J} = \frac{20{,}000(1.5)}{7.46} = 4000\ \text{lb/in}^2$$

Let the bending moments lie in a vertical plane. Then the upper and lower fibers of the beam are subject to the peak bending stresses. These are found from the expression $\sigma_x = My/I$. The moment of inertia I for the hollow circular cross section may be seen from Problem 7.9 to be

$$I = \frac{\pi}{64}(D_o^4 - D_i^4) = \frac{\pi}{64}[3^4 - (1.5)^4] = 3.73\ \text{in}^4$$

Substituting,

$$\sigma_x = \frac{My}{I} = \frac{30{,}000(1.5)}{3.73} = 12{,}000\ \text{lb/in}^2$$

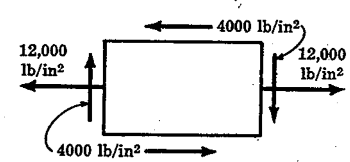


Fig. 17-8

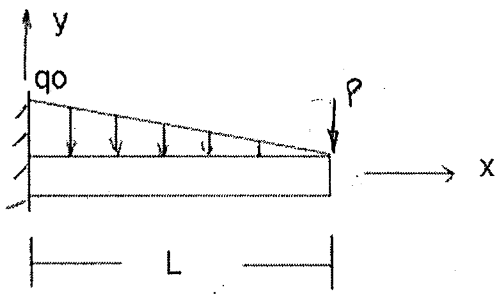
KEY

QUIZ 5A EMA 3702 June 17, 2003

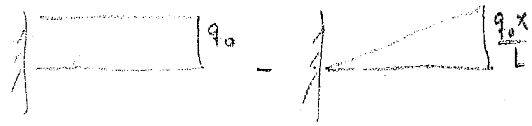
Name:_____________________

Student No.__________________

For the following loading on the beam find the displacement at x=L. Show all work.
Give a mathematical expression for the moment as a function of x.



$q = q_0\left(1-\frac{x}{L}\right)$ =



$\frac{dV}{dx} = -q$

$EIv^{IV} = q = -q_0\left(1-\frac{x}{L}\right)$ 3

$-V = EIv''' = +\frac{q_0 L}{2}\left(1-\frac{x}{L}\right)^2 + C_1$ 3

$EIv'' = -\frac{q_0 L^2}{6}\left(1-\frac{x}{L}\right)^3 + C_1 x + C_2$ 4

$EIv' = +\frac{q_0 L^3}{24}\left(1-\frac{x}{L}\right)^4 + C_1\frac{x^2}{2} + C_2 x + C_3$ 5

$EIv = -\frac{q_0 L^4}{120}\left(1-\frac{x}{L}\right)^5 + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4$ 6

when $v=0$ $x=0$ $\Rightarrow C_4 = \frac{q_0 L^4}{120}$ 2

$v'=0$ $x=0$ $\Rightarrow C_3 = -\frac{q_0 L^3}{24}$ 2

$M=0$ $x=L$ $C_1 L + C_2 = 0$ 2

$V=-P$ $x=L$ $C_1 = P$ $C_2 = -PL$ 2

$\therefore v = \frac{1}{EI}\left\{-\frac{q_0 L^4}{120}\left(1-\frac{x}{L}\right)^5 + \frac{Px^3}{6} - \frac{PLx^2}{2} - \frac{q_0 L^3 x}{24} + \frac{q_0 L^4}{120}\right\} \Rightarrow -\frac{PL^3}{3EI} - \frac{q_0 L^4}{30}$ @ $x=L$ 3

$M = EIv'' = -\frac{q_0 L^2}{6}\left(1-\frac{x}{L}\right)^3 + P(x-L)$ 4

36

$n = 7$

$\bar{x} = 71$

$\sigma = 8.20$

$EIv^{IV} = -q_0\left(1-\frac{x}{L}\right) - P\langle x-L\rangle^{-1}$

$EIv''' = +\frac{q_0 L}{2}\left(1-x/L\right)^2 - P\langle x-L\rangle^0 + C_1$

$EIv'' = -\frac{q_0 L^2}{6}\left(1-\frac{x}{L}\right)^3 - P\langle x-L\rangle^1 + C_1 x + C_2$

$EIv' = +\frac{q_0 L^3}{24}\left(1-x/L\right)^4 - \frac{P}{2}\langle x-L\rangle^2 + C_1\frac{x^2}{2} + C_2 x$

$EIv = +\frac{q_0 L^4}{120}\left(1-x/L\right)^5 - \frac{P}{6}\langle x-L\rangle^3 + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$

@ $x=0$ $v=0$ $+\frac{q_0 L^4}{120} = C_4$

@ $x=0$ $v'=0$ $-\frac{q_0 L^3}{24} = C_3$

@ $x=L$ $-EIv''' = V = 0 = +P - C_1 = 0$

$C_1 = P$

@ $x=L$ $M=0$ $PL + C_2 = 0$

$C_2 = -PL$

$v = \frac{1}{EI}\left\{-\frac{q_0 L^4}{120}\left(1-x/L\right)^5 - P\langle x-L\rangle^3 + \frac{Px^3}{6} - \frac{PLx^2}{2} + \frac{q_0 L^4}{120} - \frac{q_0 L^3 x}{24}\right.$

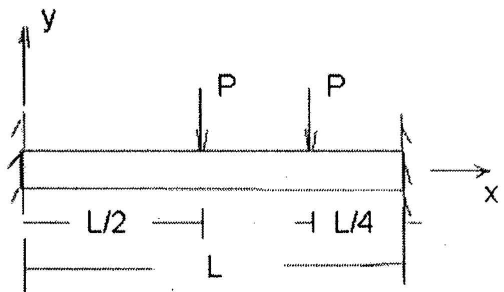
KEY

QUIZ 5B EMA 3702 June 17, 2003

Name:____________________

Student No.__________________

For the following loading on the beam find the displacement at x=L/4. Show all work.
Give a mathematical expression for the shear as a function of x.



$EI v^{IV} = q = -P\langle x - L/2\rangle^{-1} - P\langle x - \frac{3L}{4}\rangle^{-1}$ 3

$-V = EI v''' = -P\langle x - L/2\rangle^{0} - P\langle x - \frac{3L}{4}\rangle^{0} + C_1$ 3

$M = EI v'' = -P\langle x - L/2\rangle^{1} - P\langle x - \frac{3L}{4}\rangle^{1} + C_1 x + C_2$ 4

$EI v' = -\frac{P}{2}\langle x - L/2\rangle^{2} - \frac{P}{2}\langle x - \frac{3L}{4}\rangle^{2} + C_1\frac{x^2}{2} + C_2 x + C_3$ 5

$EI v = -\frac{P}{6}\langle x - L/2\rangle^{3} - \frac{P}{6}\langle x - \frac{3L}{4}\rangle^{3} + C_1 x^3/6 + C_2 x^2/2 + C_3 x + C_4$ 6

$x=0 \quad v=0 \quad \Rightarrow C_4 = 0$ 2

$x=0 \quad v'=0 \quad \Rightarrow C_3 = 0$ 2

$x=L \quad v=0 \quad \Rightarrow \quad -P/6\,(L/2)^3 - P/6\,(L/4)^3 + C_1 L^3/6 + C_2 L^2/2 = 0$ or $-\frac{PL^3}{48} - \frac{PL^3}{384} + C_1 L^3/6 + C_2\frac{L^2}{2} = 0$

$x=L \quad v'=0 \quad \Rightarrow \quad -\frac{P}{2}(L/2)^2 - P/2\,(L/4)^2 + C_1 L^2/2 + C_2 L = 0$ $\quad -\frac{PL^2}{8} - \frac{PL^2}{32} + \frac{C_1 L^2}{2} + C_2 L = 0$

$C_1 L^3/6 + C_2 L^2/2 = \frac{9PL^3}{384}$

$C_1 L^2/2 + C_2 L = \frac{5PL^2}{32} \qquad C_2 = \frac{5PL}{32} - \frac{C_1 L}{2}$

or $C_1\frac{L^3}{6} + \frac{5PL^3}{64} - \frac{C_1 L^3}{4} = \frac{9PL^3}{384}$

$-\frac{C_1 L^3}{12} = \frac{9PL^3}{384} - \frac{30PL^3}{384} = -\frac{21PL^3}{384} = \frac{-7}{128}PL^3$

$C_1 = \frac{84P}{128} = \frac{21P}{32}$ 2

$C_2 = \frac{5PL}{32} - \frac{21PL}{64} = -\frac{11PL}{64}$ 2

$EI v\big|_{L/4} = \frac{21P}{32}\cdot\frac{L^3}{384} + \left(\frac{-11PL}{64}\right)\frac{L^2}{32}$ 4

$v = \frac{-45PL^3}{384\cdot 32}\cdot\frac{1}{EI}$ 2

$V = P\langle x - L/2\rangle^0 + P\langle x - 3L/4\rangle^0 - \frac{21P}{32}$ 3

34

$n=7$

$\bar{X}=$ 60.43

$\sigma=$ 10.11

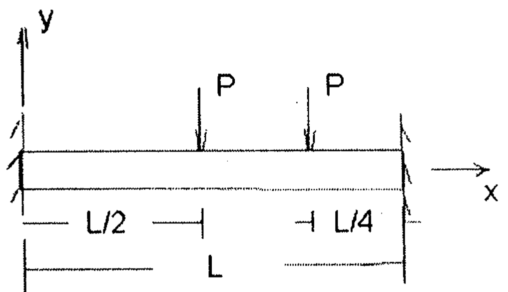
KEY

QUIZ 5B EMA 3702 June 17, 2003

Name:____________________

Student No.________________

For the following loading on the beam find the displacement at x=L/4. Show all work.
Give a mathematical expression for the shear as a function of x.



$$EIv^{IV} = q = -P\langle x-L/2\rangle^{-1} - P\langle x-\tfrac{3L}{4}\rangle^{-1} \quad 3$$

$$-V = EIv''' = -P\langle x-L/2\rangle^{0} - P\langle x-\tfrac{3L}{4}\rangle^{0} + C_1 \quad 3$$

$$M = EIv'' = -P\langle x-L/2\rangle^{1} - P\langle x-\tfrac{3L}{4}\rangle^{1} + C_1x + C_2 \quad 4$$

$$EIv' = -\frac{P}{2}\langle x-L/2\rangle^{2} - \frac{P}{2}\langle x-\tfrac{3L}{4}\rangle^{2} + C_1\frac{x^2}{2} + C_2x + C_3 \quad 5$$

$$EIv = -\frac{P}{6}\langle x-L/2\rangle^{3} - \frac{P}{6}\langle x-\tfrac{3L}{4}\rangle^{3} + C_1 x^3/6 + C_2 x^2/2 + C_3x + C_4 \quad 6$$

$x=0$, $v=0$ $\Rightarrow C_4 = 0$ 2

$x=0$, $v'=0$ $\Rightarrow C_3 = 0$ 2

$x=L$, $v=0$ $\Rightarrow -P/6\,(L/2)^3 - P/6\,(L/4)^3 + C_1L^3/6 + C_2L^2/2 = 0$ or $-\frac{PL^3}{48} - \frac{PL^3}{384} + C_1L^3/6 + C_2\frac{L^2}{2} = 0$

$x=L$, $v'=0$: $-\frac{P}{2}(L/2)^2 - P/2\,(L/4)^2 + C_1L^2/2 + C_2L = 0$ or $-\frac{PL^2}{8} - \frac{PL^2}{32} + \frac{C_1L^2}{2} + C_2L = 0$

$$C_1L^3/6 + C_2L^2/2 = \frac{9PL^3}{384}$$

$$C_1L^2/2 + C_2L = \frac{5PL^2}{32} \qquad C_2 = \frac{5PL}{32} - \frac{C_1L}{2}$$

or $$C_1\frac{L^3}{6} + \frac{5PL^3}{64} - \frac{C_1L^3}{4} = \frac{9PL^3}{384}$$

$$-\frac{C_1L^3}{12} = \frac{9PL^3}{384} - \frac{30PL^3}{384} = -\frac{21PL^3}{384} = \frac{-7}{128}PL^3$$

$$C_1 = \frac{84P}{128} = \frac{21P}{32} \quad 2$$

$$C_2 = \frac{5PL}{32} - \frac{21PL}{64} = -\frac{11PL}{64} \quad 2$$

$$EIv\big|_{L/4} = \frac{21P}{32}\cdot\frac{L^3}{384} + \left(\frac{-11PL}{64}\right)\frac{L^2}{32} \quad 4$$

$$v = \frac{-45PL^3}{384\cdot 32}\cdot\frac{1}{EI} \quad 2$$

$$V = P\langle x-L/2\rangle^0 + P\langle x-3L/4\rangle^0 - \frac{21P}{32} \quad 3$$

34

KEY

QUIZ 5A EMA 3702 June 17, 2003

Name:____________________

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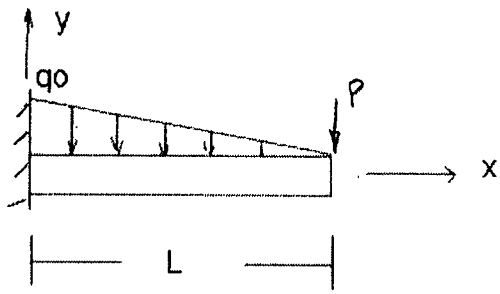
For the following loading on the beam find the displacement at x=L. Show all work.
Give a mathematical expression for the moment as a function of x.



$q = q_0\left(1-\frac{x}{L}\right)$

$EI v^{IV} = q = -q_0\left(1-\frac{x}{L}\right)$ 3

$-V = EI v''' = +\frac{q_0 L}{2}\left(1-\frac{x}{L}\right)^2 + C_1$ 3

$EI v'' = -\frac{q_0 L^2}{6}\left(1-\frac{x}{L}\right)^3 + C_1 x + C_2$ 4

$EI v' = +\frac{q_0 L^3}{24}\left(1-\frac{x}{L}\right)^4 + C_1 \frac{x^2}{2} + C_2 x + C_3$ 5

$EI v = -\frac{q_0 L^4}{120}\left(1-\frac{x}{L}\right)^5 + C_1 \frac{x^3}{6} + C_2 \frac{x^2}{2} + C_3 x + C_4$ 6

when $v=0$ $x=0$ $\Rightarrow C_4 = \frac{q_0 L^4}{120}$ 2

$v'=0$ $x=0$ $\Rightarrow C_3 = -\frac{q_0 L^3}{24}$ 2

$M=0$ $x=L$ $C_1 L + C_2 = 0$ 2

$V=-P$ $x=L$ $C_1 = P$ $C_2 = -PL$ 2

$\therefore v = \frac{1}{EI}\left\{ -\frac{q_0 L^4}{120}\left(1-\frac{x}{L}\right)^5 + \frac{Px^3}{6} - \frac{PLx^2}{2} - \frac{q_0 L^3 x}{24} + \frac{q_0 L^4}{120} \right\}$ 3 $\Rightarrow -\frac{PL^3}{3EI} - \frac{q_0 L^4}{30}$ @ $x=L$

$M = EI v'' = -\frac{q_0 L^2}{6}\left(1-\frac{x}{L}\right)^3 + P(x-L)$ 4

$\overline{36}$

$n = 7$

$\bar{x} = 71$

$\sigma = 8.20$

$\frac{dV}{dx} = -q$

$EI v^{IV} = -q_0\left(1-\frac{x}{L}\right) - P\langle x-L\rangle^{-1}$

$EI v''' = +\frac{q_0 L}{2}\left(1-\frac{x}{L}\right)^2 - P\langle x-L\rangle^0 + C_1$

$EI v'' = -\frac{q_0 L^2}{6}\left(1-\frac{x}{L}\right)^3 - P\langle x-L\rangle^1 + C_1 x + C_2$

$EI v' = +\frac{q_0 L^3}{24}\left(1-\frac{x}{L}\right)^4 - \frac{P}{2}\langle x-L\rangle^2 + C_1\frac{x^2}{2} + C_2 x$

$EI v = +\frac{q_0 L^4}{120}\left(1-\frac{x}{L}\right)^5 - \frac{P}{6}\langle x-L\rangle^3 + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$

@ $x=0$ $v=0$ $+\frac{q_0 L^4}{120} = C_4$

@ $x=0$ $v'=0$ $-\frac{q_0 L^3}{24} = C_3$

@ $x=L$ $EI v''' = V = 0 = +P - C_1 = 0$ $C_1 = P$

@ $x=L$ $M=0$ $PL + C_2 = 0$ $C_2 = -PL$

$v = \frac{1}{EI}\left\{ -\frac{q_0 L^4}{120}\left(1-\frac{x}{L}\right)^5 - P\langle x-L\rangle^3 + \frac{Px^3}{6} - \frac{PLx^2}{2} + \frac{q_0 L^4}{120} - \frac{q_0 L^3 x}{24} \right\}$

KEY

QUIZ 5B EMA 3702 June 17, 2003

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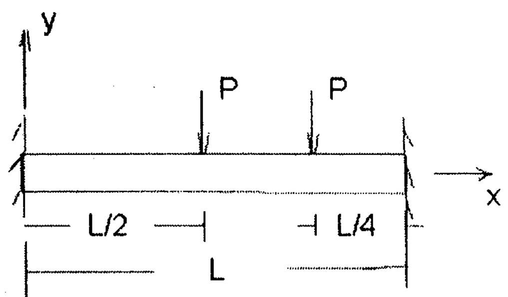
Student No.__________________

For the following loading on the beam find the displacement at x=L/4. Show all work.
Give a mathematical expression for the shear as a function of x.



$EIv^{IV} = q = -P\langle x-L/2\rangle^{-1} - P\langle x-\frac{3L}{4}\rangle^{-1}$ 3

$-V = EIv''' = -P\langle x-L/2\rangle^{0} - P\langle x-\frac{3L}{4}\rangle^{0} + C_1$ 3

$M = EIv'' = -P\langle x-L/2\rangle^{1} - P\langle x-\frac{3L}{4}\rangle^{1} + C_1x + C_2$ 4

$EIv' = -\frac{P}{2}\langle x-L/2\rangle^{2} - \frac{P}{2}\langle x-\frac{3L}{4}\rangle^{2} + C_1\frac{x^2}{2} + C_2x + C_3$ 5

$EIv = -\frac{P}{6}\langle x-L/2\rangle^{3} - \frac{P}{6}\langle x-\frac{3L}{4}\rangle^{3} + C_1 x^3/6 + C_2 x^2/2 + C_3x + C_4$ 6

$x=0 \quad v=0 \Rightarrow C_4 = 0$ 2

$x=0 \quad v'=0 \Rightarrow C_3 = 0$ 2

$x=L \quad v=0 \Rightarrow -P/6\,(L/2)^3 - P/6\,(L/4)^3 + C_1L^3/6 + C_2L^2/2 = 0$ or $-\frac{PL^3}{48} - \frac{PL^3}{384} + C_1\frac{L^3}{6} + C_2\frac{L^2}{2} = 0$

$x=L \quad v'=0$: $-\frac{P}{2}(L/2)^2 - P/2\,(L/4)^2 + C_1L^2/2 + C_2L = 0$ or $-\frac{PL^2}{8} - \frac{PL^2}{32} + C_1\frac{L^2}{2} + C_2L = 0$

$C_1L^3/6 + C_2L^2/2 = \frac{9PL^3}{384}$

$C_1L^2/2 + C_2L = \frac{5PL^2}{32}$ $\qquad C_2 = \frac{5PL}{32} - \frac{C_1L}{2}$

or $C_1\frac{L^3}{6} + \frac{5PL^3}{64} - \frac{C_1L^3}{4} = \frac{9PL^3}{384}$

$-\frac{C_1L^3}{12} = \frac{9PL^3}{384} - \frac{30PL^3}{384} = -\frac{21PL^3}{384} = \frac{-7}{128}PL^3$

$C_1 = \frac{84P}{128} = \frac{21P}{32}$ 2

$C_2 = \frac{5PL}{32} - \frac{21PL}{64} = -\frac{11PL}{64}$ 2

$EIv\Big|_{L/4} = \frac{21P}{32}\cdot\frac{L^3}{384} + \left(\frac{-11PL}{64}\right)\frac{L^2}{32}$ 4

$v = \frac{-45PL^3}{384\cdot 32}\cdot\frac{1}{EI}$ 2

$V = P\langle x-L/2\rangle^{0} + P\langle x-3L/4\rangle^{0} - \frac{21P}{32}$ 3

$\overline{34}$

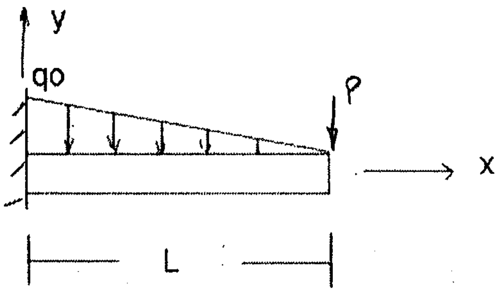
KEY

QUIZ 5A EMA 3702 June 17, 2003

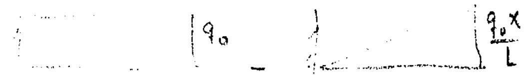
Name:____________________

Student No.__________________

For the following loading on the beam find the displacement at x=L. Show all work.
Give a mathematical expression for the moment as a function of x.



$q = q_0\left(1-\frac{x}{L}\right)$



$EI v^{IV} = q = -q_0\left(1-\frac{x}{L}\right)$ 3

$-V = EI v''' = +\frac{q_0 L}{2}\left(1-\frac{x}{L}\right)^2 + C_1$ 3

$EI v'' = -\frac{q_0 L^2}{6}\left(1-\frac{x}{L}\right)^3 + C_1 x + C_2$ 4

$EI v' = +\frac{q_0 L^3}{24}\left(1-\frac{x}{L}\right)^4 + C_1\frac{x^2}{2} + C_2 x + C_3$ 5

$EI v = -\frac{q_0 L^4}{120}\left(1-\frac{x}{L}\right)^5 + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4$ 6

when $v=0$ $x=0$ $\Rightarrow C_4 = \frac{q_0 L^4}{120}$ 2

$v'=0$ $x=0$ $\Rightarrow C_3 = -\frac{q_0 L^3}{24}$ 2

$M=0$ $x=L$ $C_1 L + C_2 = 0$ 2

$V=-P$ $x=L$ $C_1 = P$ $C_2 = -PL$ 2

$\therefore$ $v = \frac{1}{EI}\left\{-\frac{q_0 L^4}{120}\left(1-\frac{x}{L}\right)^5 + \frac{Px^3}{6} - \frac{PLx^2}{2} - \frac{q_0 L^3 x}{24} + \frac{q_0 L^4}{120}\right\} \Rightarrow -\frac{PL^3}{3EI} - \frac{q_0 L^4}{30}$ @ $x=L$ 3

$M = EI v'' = -\frac{q_0 L^2}{6}\left(1-\frac{x}{L}\right)^3 + P(x-L)$ 4

36

$\frac{dV}{dx} = -q$

$EI v^{IV} = -q_0\left(1-\frac{x}{L}\right) - P\langle x-L\rangle^{-1}$

$EI v''' = +\frac{q_0 L}{2}\left(1-x/L\right)^2 - P\langle x-L\rangle^0 + C_1$

$EI v'' = -\frac{q_0 L^2}{6}\left(1-\frac{x}{L}\right)^3 - P\langle x-L\rangle^1 + C_1 x + C_2$

$EI v' = +\frac{q_0 L^3}{24}\left(1-x/L\right)^4 - \frac{P}{2}\langle x-L\rangle^2 + C_1\frac{x^2}{2} + C_2 x$

$EI v = +\frac{q_0 L^4}{120}\left(1-x/L\right)^5 - \frac{P}{6}\langle x-L\rangle^3 + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$

@ $x=0$ $v=0$ $+\frac{q_0 L^4}{120} = C_4$

@ $x=0$ $v'=0$ $-\frac{q_0 L^3}{24} = C_3$

@ $x=L$ $EI v''' = V = 0$ $= +P - C_1 = 0$ $C_1 = P$

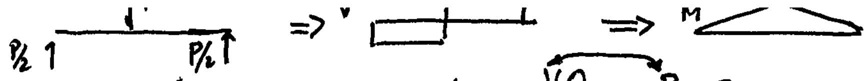
@ $x=L$ $M=0$ $PL + C_2 = 0$ $C_2 = -PL$

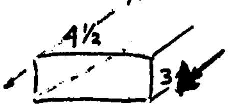
$v = \frac{1}{EI}\left\{-\frac{q_0 L^4}{120}\left(1-x/L\right)^5 - P\langle x-L\rangle^3 + \frac{Px^3}{6} - \frac{PLx^2}{2} + \frac{q_0 L^4}{120} - \frac{q_0 L^3 x}{24}\right.$

$n = 7$

$\bar{x} = 71$

$\sigma = 8.20$





$$\tau_{Glue} = 60 = \frac{VQ}{It} = \frac{P}{2}\cdot\frac{Q}{It}$$

$Q = A\bar{y} = (4.5)(3)(4.5) = 60.75 in^3$

$t = 3 in$

$$I_{zz} = \frac{Bh^3}{12} - \frac{bh^3}{12} = \frac{6\cdot 12^3}{12} - \frac{4.5(6^3)}{12} = 783 in^4$$

since τ acts on 2 surfaces $\tau = \frac{1}{2}\left(\frac{P}{2}\cdot\frac{Q}{It}\right)$

or $P = \frac{4\tau It}{Q} = 9280 lb$

$Q = 6\times3\times4.5 + 2\times 3\times 0.75\times 1.5 = 87.5 in^3$

$t = 0.75 in$

$I = 783 in^4$ $\tau_{shear} = 120$

τ on 2 surfaces

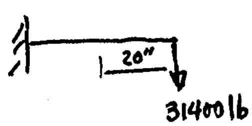
$$\tau = \frac{1}{2}\left(\frac{P}{2}\,\frac{Q}{It}\right)$$

$$P = \frac{4\tau It}{Q} = 3221.49 lb$$

smaller P dictates. so failure at centerline first before glued joints

$$\sigma_{max} = \frac{My}{I} = \frac{(PL/4)\cdot 6in}{783} = 1500\ \frac{lb}{in^2} \quad or \quad 4\,\frac{\sigma_{max}\cdot I}{c\cdot P} = L = 243.06 in$$

#2



$M = 31400 \times 20" = 628000$ lb-in

$\sigma_x = \frac{My}{I}$ @ A $y=0$ $\therefore \sigma_{x\ due\ to\ bending} = 0$

Torque $= W\cdot R = 31400 \times 10" = 314000$ lb-in

$$\tau_{due\ to\ torque} = \frac{T}{2A_{inc}t} = \frac{314000\ lb\text{-}in}{2\cdot\pi(9.875)^2(.25)} = 2049.91\ \frac{lb}{in^2}$$

$\frac{2R}{\pi} = \bar{y}$

$$\tau_{due\ to\ shear} = \frac{VQ}{It} = \frac{1}{2}\,\frac{31400\ lb\ (48.76 in^3)}{756.32\,(.25)} = 4048.20\ \frac{lb}{in^2}$$

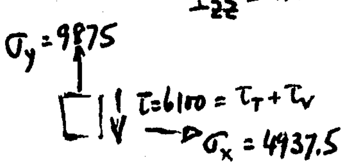
acts on 2 surfaces

$A = \pi R t$

$Q = A\bar{y} = 2R^2 t = 48.76 in^3$

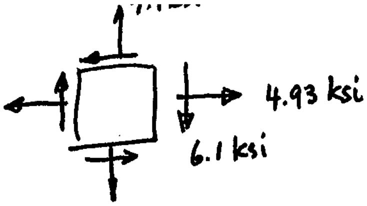
$I_{zz} = \pi R^3 t = 756.32 in^4$

$$\sigma_{y\ pressure} = \frac{pR}{t} = 9875.0\ \frac{lb}{in^2}$$



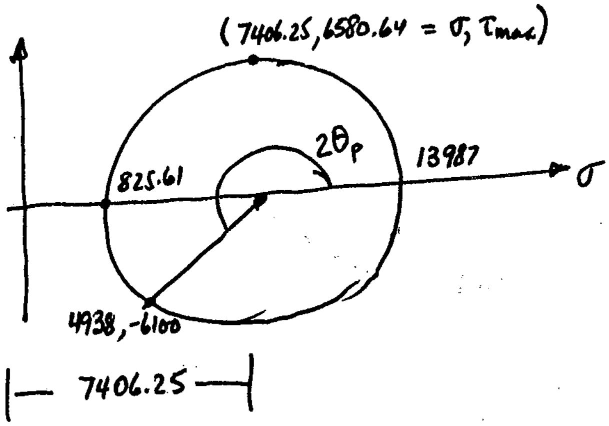
$$\sigma_{x\ pressure} = \frac{pR}{2t} = \frac{250\frac{lb}{in^2}(9.875 in)}{2(0.25)} = 4937.5\ \frac{lb}{in^2}$$

3

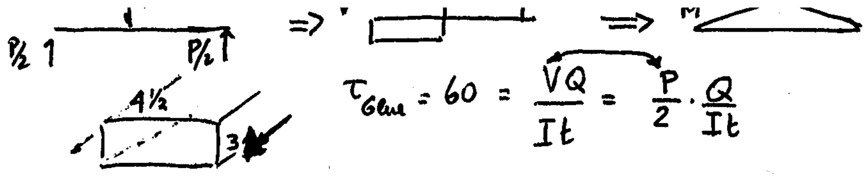


$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(2468.75)^2 + (6100)^2} = 6580.64$$

$$\frac{\sigma_x + \sigma_y}{2} = 7406.25$$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{-6100}{-2468.75} = 248^\circ$$



$$\tau_{Glue} = 60 = \frac{VQ}{It} = \frac{P}{2}\cdot\frac{Q}{It}$$

$$Q = A\bar{y} = (4.5)(3)(4.5) = 60.75\,in^3$$

$$t = 3in$$

$$I_{zz} = \frac{Bh^3}{12} - \frac{bh^3}{12} = \frac{6\cdot 12^3}{12} - \frac{4.5(6^3)}{12} = 783\,in^4$$

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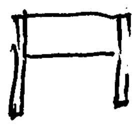
or $P = \frac{4\tau It}{Q} = 9280\,lb$



$$Q = 6\times3\times4.5 + 2\times 3\times0.75\times1.5 = 87.5\,in^3$$

$t = 0.75in$

$I = 783 in^4$ $\tau_{shear} = 120$

τ on 2 surfaces

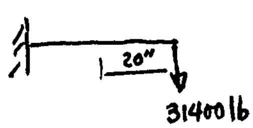
$$\tau = \frac{1}{2}\left(\frac{P}{2}\,\frac{Q}{It}\right)$$

$$P = \frac{4\tau It}{Q} = 3221.49\,lb$$

smaller P dictates. so failure at centerline first before glued joints

$$\sigma_{max} = \frac{My}{I} = \frac{\left(PL/4\right)\cdot 6in}{783} = 1500\,\frac{lb}{in^2} \quad or \quad 4\frac{\sigma_{max}\cdot I}{c\cdot P} = L = 243.06in$$

#2

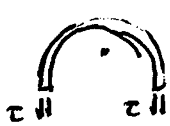


$$M = 31400\times 20'' = 628000\ lb\text{-}in$$

$\sigma_x = \frac{My}{I}$ @ A $y=0$ $\therefore \sigma_x$ due to bending $= 0$

- Torque $= W\cdot R = 31400\times 10'' = 314000\ lb\text{-}in$

$$\tau_{due\ to\ torque} = \frac{T}{2A_{m}t} = \frac{314000\ lb\text{-}in}{2\cdot\pi(9.875)^2(.25)} = 2049.91\,\frac{lb}{in^2}$$



$\frac{2R}{\pi} = \bar{y}$

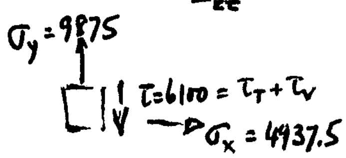
$$\tau_{due\ to\ shear\ acts\ on\ 2\ surfaces} = \frac{VQ}{It} = \frac{1}{2}\,\frac{31400\ lb\ (48.76in^3)}{756.32\,(.25)} = 4048.20\,\frac{lb}{in^2}$$

$A = \pi Rt$

$Q = A\bar{y} = 2R^2t = 48.76in^3$

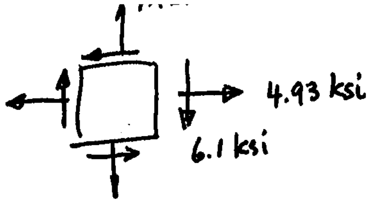
$I_{zz} = \pi R^3 t = 756.32\ in^4$

$$\sigma_{y\ pressure} = \frac{pR}{t} = 9875.0\,\frac{lb}{in^2}$$



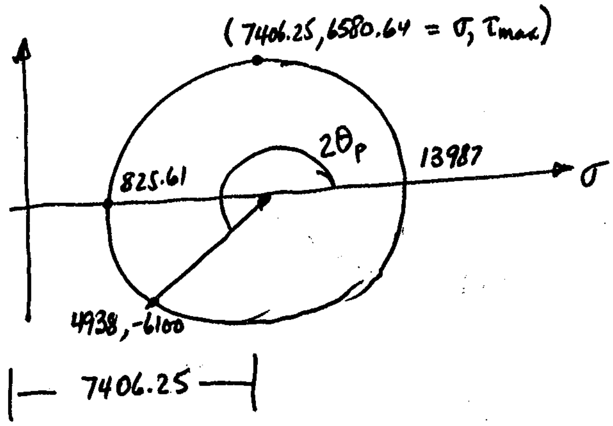
$$\sigma_{x\ pressure} = \frac{pR}{2t} = \frac{250\frac{lb}{in^2}(9.875in)}{2\,(0.25)} = 4937.5\,\frac{lb}{in^2}$$

3

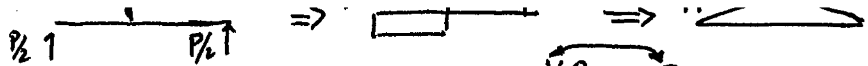


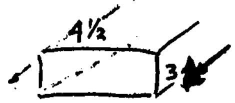
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(2468.75)^2 + (6100)^2} = 6580.64$$

$$\frac{\sigma_x + \sigma_y}{2} = 7406.25$$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{-6100}{-2468.75} = 248^\circ$$





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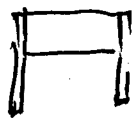
since τ acts on 2 surfaces $\tau = \frac{1}{2}\left(\frac{P}{2}\cdot\frac{Q}{It}\right)$

or $P = \dfrac{4\tau I t}{Q} = 9280\,lb$



$Q = 6\times3\times4.5 + 2\times 3\times 0.75\times 1.5 = 87.5\,in^3$

$t = 0.75\,in$

$I = 783\,in^4$ $\tau_{shear} = 120$

τ on 2 surfaces

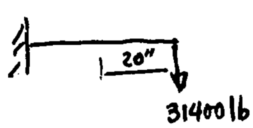
$\tau = \frac{1}{2}\left(\frac{P}{2}\ \frac{Q}{It}\right)$

$P = \dfrac{4\tau I t}{Q} = 3221.49\,lb$

smaller P dictates. so failure at centerline first before glued joints

$\sigma_{max} = \dfrac{My}{I} = \dfrac{(PL/4)\cdot 6in}{783} = 1500\,\dfrac{lb}{in^2}$ or $4\,\dfrac{\sigma_{max}\cdot I}{c\cdot P} = L = 243.06in$

#2



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$\sigma_x = \dfrac{My}{I}$ @ A $y=0$ $\therefore \sigma_x$ due to bending $= 0$

• Torque $= W\cdot R = 31400\times 10'' = 314000$ lb-in

$\tau_{\text{due to torque}} = \dfrac{T}{2A_{me}t} = \dfrac{314000\ lb\text{-}in}{2\cdot\pi(9.875)^2(.25)} = 2049.91\,\dfrac{lb}{in^2}$

$\bar{I}\ \dfrac{2R}{\pi} = \bar{y}$

$A = \pi R t$

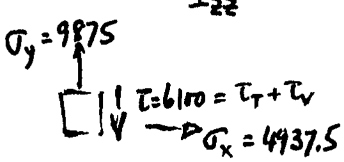
$Q = A\bar{y} = 2R^2t = 48.76\,in^3$

$I_{zz} = \pi R^3 t = 756.32\,in^4$

$\tau_{\text{due to shear}}$ acts on 2 surfaces $= \dfrac{VQ}{It} = \dfrac{1}{2}\ \dfrac{314000\ lb\ (48.76in^3)}{756.32\,(.25)} = 4048.20\,\dfrac{lb}{in^2}$

$\sigma_{y\ pressure} = \dfrac{pR}{t} = 98750\,\dfrac{lb}{in^2}$

$\sigma_{x\ pressure} = \dfrac{pR}{2t} = \dfrac{250\frac{lb}{in^2}(9.875in)}{2(0.25)} = 4937.5\,\dfrac{lb}{in^2}$



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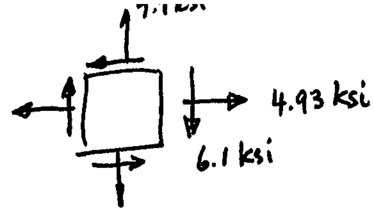

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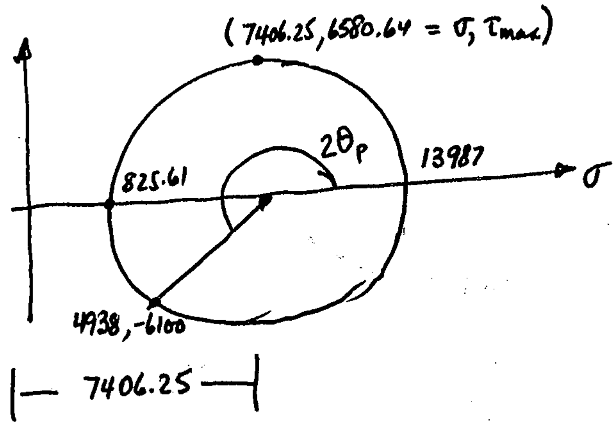


$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{-6100}{-2468.75} = 248^\circ$$

Fig. 9-19, are known as *strain rosettes.* If three strain measurements are taken at a rosette, the information is sufficient to determine the complete state of plane strain at a point.

If the angles θ_1, θ_2, and θ_3, together with the corresponding strains ε_{θ_1}, ε_{θ_2}, and ε_{θ_3} are known from measurements, three simultaneous equations patterned after Eq. 9-13 may be written. In writing these equations it is convenient to employ the following notation: $\varepsilon_{x'} \equiv \varepsilon_{\theta_1}$, $\varepsilon_{x''} \equiv \varepsilon_{\theta_2}$, and $\varepsilon_{x'''} \equiv \varepsilon_{\theta_3}$.

$$\begin{aligned}
\varepsilon_{\theta_1} &= \varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy}\sin\theta_1\cos\theta_1 \\
\varepsilon_{\theta_2} &= \varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy}\sin\theta_2\cos\theta_2 \\
\varepsilon_{\theta_3} &= \varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy}\sin\theta_3\cos\theta_3
\end{aligned} \qquad \textbf{(9-22)}$$

This set of equations may be solved for ε_x, ε_y, and γ_{xy} and the problem reverts back to the cases already considered.

To minimize computational work, the gages in a rosette are usually arranged in an orderly manner. For example, in Fig. 9-19(b), $\theta_1 = 0°$, $\theta_2 = 45°$, and $\theta_3 = 90°$. This arrangement of gage lines is known as the *rectangular* or the 45° *strain rosette.* By direct substitution into Eq. 9-22, it is found that for this rosette

$$\varepsilon_x = \varepsilon_{0°}, \qquad \varepsilon_y = \varepsilon_{90°}, \qquad \varepsilon_{45°} = \frac{\varepsilon_x}{2} + \frac{\varepsilon_y}{2} + \frac{\gamma_{xy}}{2}$$

or

$$\gamma_{xy} = 2\varepsilon_{45°} - (\varepsilon_{0°} + \varepsilon_{90°})$$

Thus ε_x, ε_y, and γ_{xy} become known.

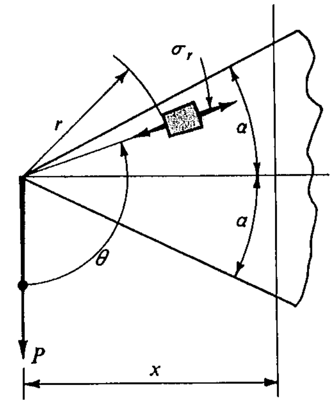
Another arrangement of gage lines is shown in Fig. 9-19(c). This is known as the *equiangular*, or the *delta*, or the 60° *rosette.* Again, by substituting into Eq. 9-22 and simplifying, $\varepsilon_x = \varepsilon_{0°}$, $\varepsilon_y = (2\varepsilon_{60°} + 2\varepsilon_{120°} - \varepsilon_{0°})/3$, and $\gamma_{xy} = (2/\sqrt{3})(\varepsilon_{60°} - \varepsilon_{120°})$.

Other types of rosettes are occasionally used in experiments. The data from all rosettes may be analyzed by applying Eq. 9-22, solving for ε_x, ε_y, and γ_{xy}, and then applying Mohr's circle of strain.*

Sometimes rosettes with more than three lines are used. An additional gage line measurement provides a check on the experimental work. For these rosettes, the invariance of the strains in the mutually perpendicular directions may be used to check the data.

The application of the experimental rosette technique in complicated problems of stress analysis is almost indispensable.

* Convenient graphical solutions for principal strains from measured strains have been developed. See G. Murphy, "A Graphical Method for the Evaluation of Principal Strains from Normal Strains," *Journal of Applied Mechanics*, **12** (1945), A-209.



PROB. 9-26

figure. For such a wedge the elasticity solution shows that only radial stress distribution exists and is given* by

$$\sigma_r = -\frac{P\cos\theta}{r[\alpha - \frac{1}{2}\sin 2\alpha]}$$

Determine the normal and the shearing stresses on a vertical section at distance x from the applied force P and compare with the elementary solutions. If $\alpha = 30°$ find the percentage of discrepancy among the maximum stresses in the alternative solutions.

9-27. Using the stress transformation equations for a three-dimensional state of stress,† one may diagonalize any stress matrix. Suppose this were done and it yields

$$\begin{pmatrix} 12{,}000 & 0 & 0 \\ 0 & -6{,}000 & 0 \\ 0 & 0 & 8{,}000 \end{pmatrix} \text{psi}$$

For this state of stress what is the maximum shearing stress? Illustrate the plane or planes on which it acts in a sketch.

9-28. An investigation of stresses in the plate of a thin-walled pressure vessel indicates that the stress matrix is

$$\begin{pmatrix} 20 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ksi}$$

where it is to be noted that $\sigma_3 \approx 0$. (This state of stress is analogous to that shown in Prob. 4-6.) Are there any shearing stresses in the material? Illustrate with a sketch.

9-29. Let l, m, and n define the direction cosines of a linear element. Using this notation, Eq. 9-18 can be rewritten as

$$\varepsilon_\theta = \varepsilon_x l^2 + \varepsilon_y m^2 + \gamma_{xy} lm$$

Show that for the three-dimensional case

$$\varepsilon_\theta = \varepsilon_x l^2 + \varepsilon_y m^2 + \varepsilon_z n^2 + \gamma_{xy} lm + \gamma_{yz} mn + \gamma_{zx} nl$$

9-30. If the unit strains are $\varepsilon_x = -120 \times 10^{-6}$, $\varepsilon_y = +1{,}120 \times 10^{-6}$, and $\gamma_{xy} = -200 \times 10^{-6}$, what are the principal strains and in which direction do they occur? Use Eqs. 9-20 and 9-21 or Mohr's circle of strain, as directed. *Ans.* $1{,}130 \times 10^{-6}$, -130×10^{-6}.

9-31. If the unit strains are $\varepsilon_x = -800 \times 10^{-6}$, $\varepsilon_y = -200 \times 10^{-6}$, and $\gamma_{xy} = +800 \times 10^{-6}$, what are the principal strains and in which directions do they occur? Use Eqs. 9-20 and 9-21 or Mohr's circle, as directed. *Ans.* 0, $1{,}000 \times 10^{-6}$.

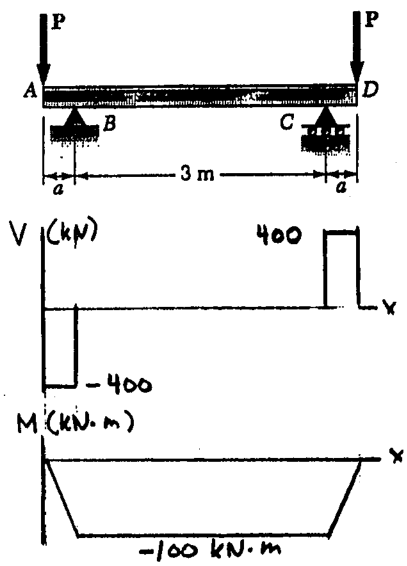
9-32. If the strain measurements given in the above problem were made on a steel member ($E = 29.5 \times 10^6$ psi and $\nu = 0.3$), what are the principal stresses and in which direction do they act?

9-33. The data for a rectangular rosette attached to a stressed steel member are $\varepsilon_{0°} = -220 \times 10^{-6}$, $\varepsilon_{45°} = +120 \times 10^{-6}$, $\varepsilon_{90°} = +220 \times 10^{-6}$. What are the principal stresses and in which directions do they act? $E = 30 \times 10^6$ psi and $\nu = 0.3$. *Ans.* ± 5.76 ksi, 14°18′.

9-34. The data for an equiangular rosette, attached to a stressed, aluminum-alloy member, are $\varepsilon_{0°} = +400 \times 10^{-6}$, $\varepsilon_{60°} = +400 \times 10^{-6}$, and $\varepsilon_{120°} = -600 \times 10^{-6}$. What are the principal stresses and in which directions do they act? $E = 10^7$ psi and $\nu = \frac{1}{4}$. *Ans.* +6.22 ksi, −4.44 ksi, 30°.

* Timoshenko and Goodier, *Theory of Elasticity*, p. 97.
† See any book on elasticity or plasticity. For a brief discussion of this point see Art. 9-9.

PROBLEM 8.1



8.1 An overhanging W250 × 58 rolled-steel beam supports two loads as shown. Knowing that $P = 400$ kN, $a = 0.25$ m, and $\sigma_{all} = 250$ MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of aflange and the web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned..

$|V|_{max} = 400 \text{ kN} = 400 \times 10^3 \text{ N}$

$|M|_{max} = (400 \times 10^3)(0.25) = 100 \times 10^3 \text{ N·m}$

For W 250 × 58 rolled steel section

$d = 252$ mm. $\quad b_f = 203$ mm $\quad t_f = 13.5$ mm

$t_w = 8.0$ mm $\quad I_z = 87.3 \times 10^6 \text{ mm}^4 \quad S_z = 693 \times 10^3 \text{ mm}^3$

$c = \frac{1}{2} d = 126$ mm $\quad y_b = c - t_f = 112.5$ mm.

(a) $\sigma_m = \dfrac{|M|_{max}}{S_z} = \dfrac{100 \times 10^3}{693 \times 10^{-6}} = 144.3 \times 10^6$ Pa $= 144.3$ MPa ◄

$\sigma_b = \dfrac{y_b}{c} \sigma_m = \dfrac{112.5}{126}(144.3) = 128.84$ MPa

$A_f = b_f t_f = (203)(13.5) = 2740.5 \text{ mm}^2$

$\bar{y}_f = \frac{1}{2}(c + y_b) = 119.25 \text{ mm}^3$

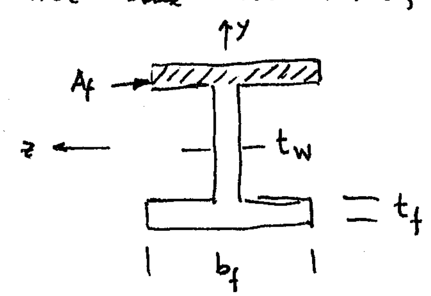
$Q_b = A_f \bar{y}_f = 326.80 \times 10^3 \text{ mm}^3 = 326.80 \times 10^{-6} \text{ m}^3$

$\tau_{xy} = \dfrac{|V|_{max} Q_b}{I_x t_w} = \dfrac{(400 \times 10^3)(326.80 \times 10^{-6})}{(87.3 \times 10^{-6})(8 \times 10^{-3})} = 187.2 \times 10^6 \text{ Pa} = 187.2$ MPa

$R = \sqrt{\left(\dfrac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 197.97$ MPa

(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 262$ MPa ◄

(c) Since $\sigma_{max} > 250$ MPa, W250 × 58 is <u>not acceptable</u>. ◄



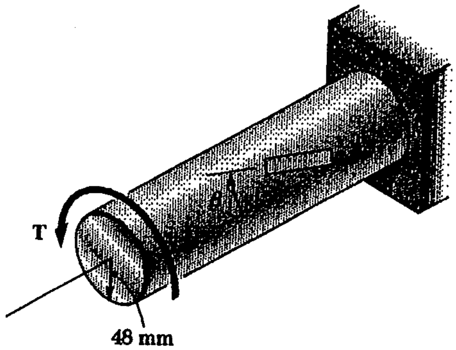
$S_z = \dfrac{I_{zz}}{c} = \dfrac{I_{zz}}{d/2}$

when σ_x is max $\tau = 0$

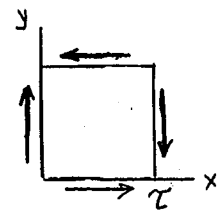
$\therefore \sigma_{max} = \dfrac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$
$= \sigma_x$

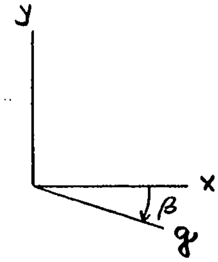
PROBLEM 7.152

7.152 A single strain gage is cemented to a solid 96-mm-diameter aluminum shaft at an angle $\beta = 20°$ with a line parallel to the axis of the shaft. Knowing that $G = 27$ GPa, determine the torque **T** corresponding to a gage reading of 400 μ.



SOLUTION





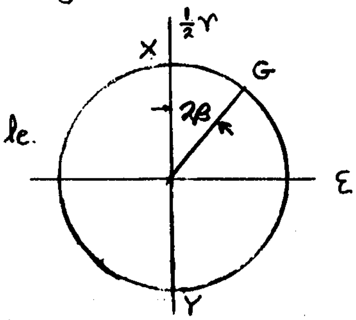
$$\tau = \frac{Tc}{J} \qquad J = \frac{\pi}{2}c^2 \qquad \gamma = \frac{\tau}{G}$$

$$\sigma_x = \sigma_y = 0 \qquad \varepsilon_x = \varepsilon_y = 0$$

Sketch Mohr's circle for strain.

Gage direction is β clockwise from x

Point G is 2β clockwise from X on Mohr's circle.



$$\varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) = 0$$

$$R = \tfrac{1}{2}\gamma_{xy}$$

$$\varepsilon_g = \varepsilon_{ave} + R\sin 2\beta = \tfrac{1}{2}\gamma_{xy}\sin 2\beta = \frac{\tau_{xy}}{2G}\sin 2\beta$$

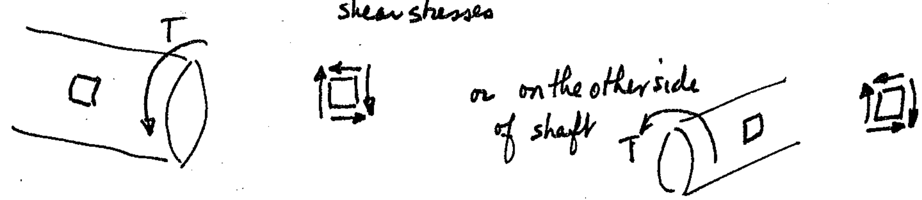
$$= \frac{Tc}{2GJ}\sin 2\beta =$$

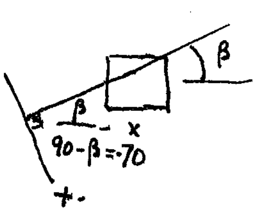
Solving for T $\quad T = \dfrac{2GJ\varepsilon_g}{c\sin 2\beta} = \dfrac{\pi G c^3 \varepsilon_g}{\sin 2\beta}$

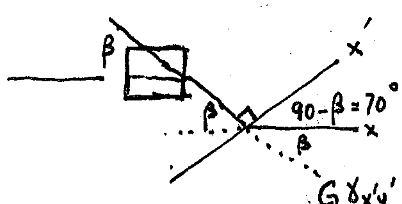
$$T = \frac{\pi(27\times10^9)(48\times10^{-3})^3(400\times10^{-6})}{\sin 40°} = 5.84\times10^3\ \text{N}\cdot\text{m}$$

$$= 5.84\ \text{kN}\cdot\text{m}$$

OR since torsion only produces shear stress then an element on the side only shows shear stresses







$$\tau_{x'y'} = \tau_{xy}\cos 2\theta - \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta$$

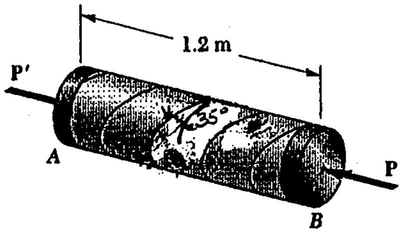
$$G\gamma_{x'y'} = \tau_{x'y'} = \tau_{xy}\cos 140°$$

$$\therefore\ \gamma_{x'y'} = \frac{\tau_{xy}}{G}\cos 140° = \frac{Tc}{JG}\cos 140°$$

$\cos(140) = \cos(180-40) = \cos 180\cos 40 = -\cos 40°$

$$T = \frac{\gamma_{x'y'}}{-\cos 40°}\cdot\frac{JG}{c}$$

PROBLEM 7.120



7.120 A pressure vessel of 250-mm inside diameter and 6-mm wall thickness is fabricated from a 1.2-m section of spirally welded pipe *AB* and is with two rigid end plates. The gage pressure inside the vessel is 2 MPa and 45-kN centric axial forces **P** and **P'** are applied to the end plates.. Determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$r = \frac{1}{2}d = 125 \text{ mm} \qquad t = 6 \text{ mm}$

$\sigma_1 = \frac{pr}{t} = \frac{(2)(125)}{6} = 41.67 \text{ MPa}$ hoop

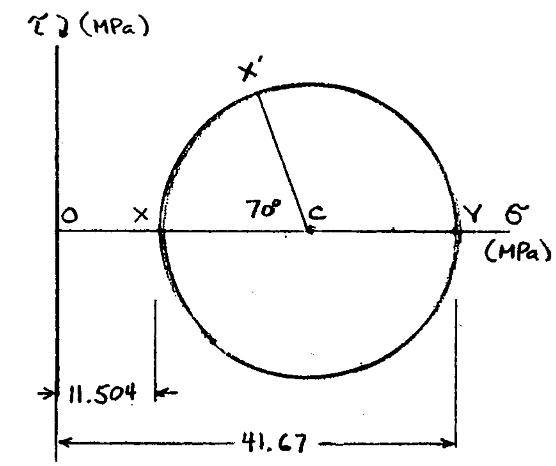
$\sigma_2 = \frac{pr}{2t} = \frac{(2)(125)}{(2)(6)} = 20.83 \text{ MPa}$ axial

$r_o = r + t = 125 + 6 = 131 \text{ mm} \qquad A = \pi(r_o^2 - r^2) = 4.825 \times 10^3 \text{ mm}^2 = 4.825 \times 10^{-3} \text{ m}^2$

$\sigma = -\frac{P}{A} = -\frac{45 \times 10^3}{4.825 \times 10^{-3}} = -9.326 \times 10^6 \text{ Pa} = -9.326 \text{ MPa}$

Total stresses: Longitudinal $\sigma_x = 20.83 - 9.326 = 11.504 \text{ MPa}$

Circumferential $\sigma_y = 41.67 \text{ MPa}$



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 26.585 \text{ MPa}$

$R = \frac{\sigma_x - \sigma_y}{2} = 15.081$

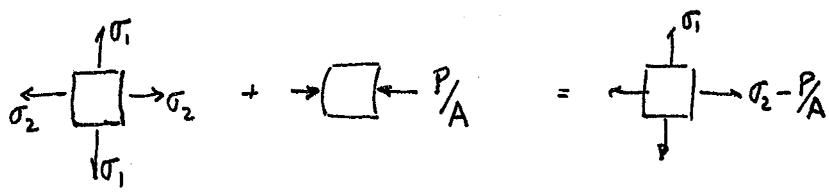
(a) $\sigma_{x'} = \sigma_{ave} + R\cos 70°$

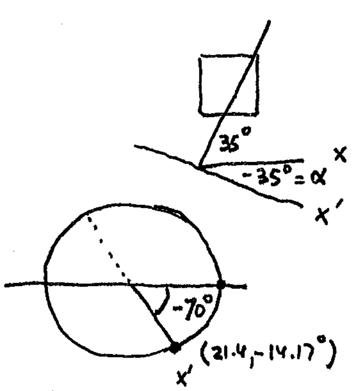
$= 26.585 - 15.081 \cos 70°$

$= 21.4 \text{ MPa}$ ◄

(b) $\tau_{x'y'} = R \sin 70° = 15.081 \sin 70°$

$= 14.17 \text{ MPa}$ ◄





$\sigma_x' = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\alpha + \tau_{xy} \sin 2\alpha$ is stress ⊥ to weld

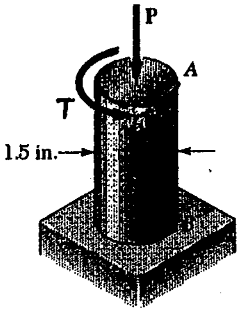
radius of mohr circle $= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_x - \sigma_y}{2}$ when $\tau_{xy} = 0$

so $\sigma_x' = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos(-70°) = 21.4 \text{ MPa}$

$\tau_{x'y'} = \tau_{xy}\cos 2\alpha - \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\alpha = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin(-70°) = 14.17$

PROBLEM 7.87

7.87 The 1.5-in-diameter shaft AB is made of a grade of steel for which the yield strength is $\sigma_Y = 42$ ksi.. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 60$ kips.



SOLUTION

$P = 60$ kips $\qquad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.5)^2 = 1.7671 \text{ in}^2$

$\sigma_x = -\frac{P}{A} = -\frac{60}{1.7671} = -33.953$ ksi

$\sigma_y = 0 \qquad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$

$2\tau_{max} = 2R = \sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_Y$

$4\tau_{xy}^2 = \sigma_Y^2 - \sigma_x^2 \qquad \tau_{xy} = \frac{1}{2}\sqrt{\sigma_Y^2 - \sigma_x^2} = \frac{1}{2}\sqrt{42^2 - 33.953^2}$

$= 12.361$ ksi

From torsion $\qquad \tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c}$

$c = \frac{1}{2}d = 0.75$ in $\qquad J = \frac{\pi}{2}c^4 = 0.49701 \text{ in}^4$

$T = \frac{(0.49701)(12.361)}{0.75} = 8.19$ kip·in ◄

QUIZ 6A EMA 3702 June 16, 2003

Name:______________________

Student No.______________________

This is due on Monday at 540pm in class and no later.

I certify that I will neither receive nor give unpermitted aid on this quiz. Violation of this will result in failure of the quiz.

____

Sign your name

Problem 1. A box beam is fabricated from two pieces of ¾ in. plywood and two 4 ½ in. by 3 in. solid wood pieces as shown in the cross-sectional view. If this beam is to be used to carry a concentrated force in the middle of a simple span,

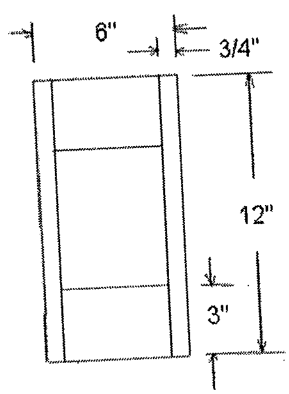
(a) What may the magnitude of the maximum applied load P be?

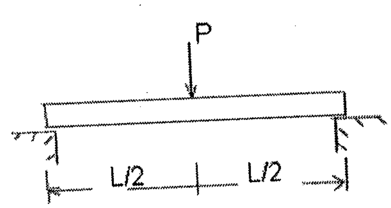
(b) How long may the span be?

Neglect the weight of the beam and assume there is no danger of lateral buckling. The allowable stresses are:

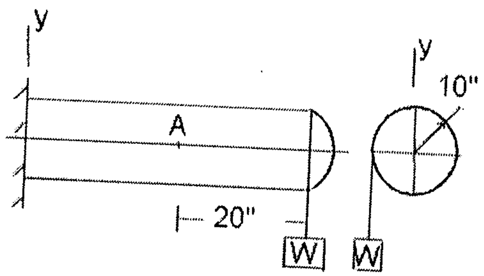
In plywood: 1500 psi in bending, 120 psi in shear

In the glued joint 60 psi in shear





Problem 2. A steel pressure vessel 20 in. in diameter and of 0.25 in. wall thickness acts also as an eccentrically loaded cantilever as in the figure. If the internal pressure is 250 psi and the applied weight W = 31,400 lbs, determine the state of stress at point A. Show the results on an infinitesimal element. Principal stresses are not required. Neglect the weight of the vessel.



Problem 3. Using the information you found in problem 2,

(a) Draw the Mohr's Circle and determine the principal stresses σ_1 and σ_2.
(b) Also determine the maximum shear stress and the accompanying direct stress.
(c) Based on the information you have, find the direction of the maximum direct stress.

QUIZ 6A EMA 3702 June 16, 2003

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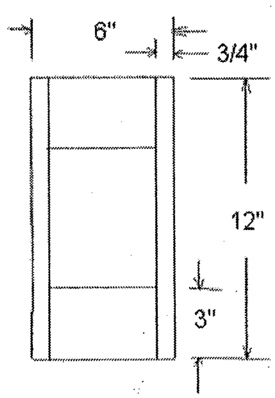

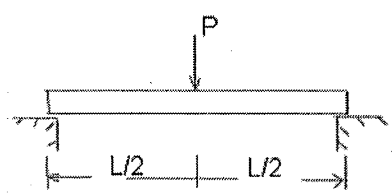

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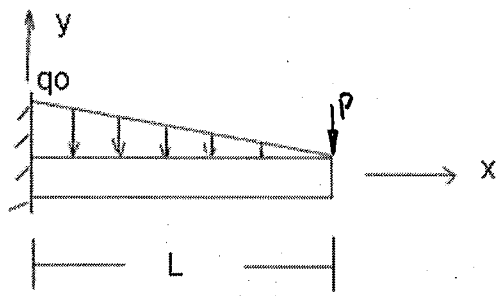
(a) Draw the Mohr's Circle and determine the principal stresses σ_1 and σ_2 .
(b) Also determine the maximum shear stress and the accompanying direct stress.
(c) Based on the information you have, find the direction of the maximum direct stress.

QUIZ 5A EMA 3702 June 17, 2003

Name:____________________

Student No.__________________

For the following loading on the beam find the displacement at x=L. Show all work.
Give a mathematical expression for the moment as a function of x.

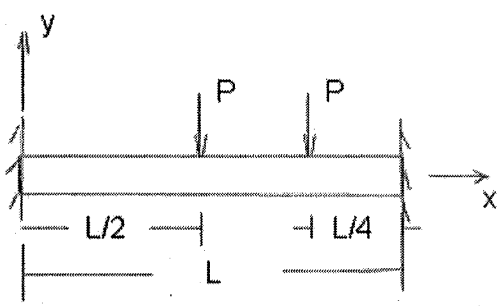


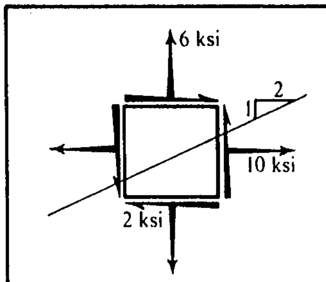
QUIZ 5B EMA 3702 June 17, 2003

Name:______________________

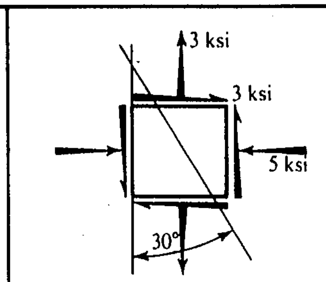
Student No.____________________

For the following loading on the beam find the displacement at x=L/4. Show all work. Give a mathematical expression for the shear as a function of x.

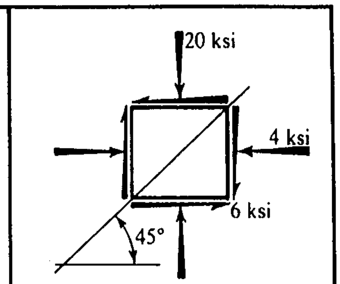




PROB. 9-2, 9-6, 9-7



PROB. 9-3, 9-8, 9-10



PROB. 9-4, 9-9, 9-11

$\sigma_{x'}$ and $\tau_{x'y'}$ as ordinates with θ as abscissa for $0 \le \theta \le 2\pi$. (b) Generalize and discuss the results, especially with regard to the maxima and the minima of the functions.

9-7. Rework Prob. 9-2 using Eqs. 9-1 and 9-2.

9-8. Rework Prob. 9-3 using Eqs. 9-1 and 9-2.

9-9. For the data of Prob. 9-4 find the stresses on $\theta = 45°$ and $\theta = 135°$. Show the complete results on the newly oriented element.

9-10. For the data of Prob. 9-3, (a) find the principal stresses and show their directions and senses on a properly oriented element; (b) determine the maximum shearing stresses and the associated normal stresses. Show the results on a properly oriented element.

9-11. Same as preceding problem for data of Prob. 9-4.

9-12 through 9-15. Draw Mohr's circle of stress for the states of stress given in the figures. (a) Clearly show the planes on which the principal stresses act, and for each stress indicate with arrows its direction and sense. (b) Same as (a) for the maximum shearing stresses and the associated normal stresses. *Ans. Prob.* 9-15. (a) 6 ksi, −4 ksi; (b) 5 ksi, 1 ksi.

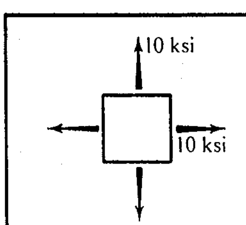
9-16. The state of two-dimensional stress at three different points is given in matrix representation as

(a) $\begin{pmatrix} 12 & 5 \\ 5 & 6 \end{pmatrix}$ ksi (b) $\begin{pmatrix} -6 & 6 \\ 6 & -8 \end{pmatrix}$ ksi

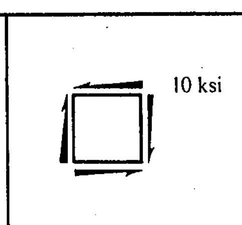
(c) $\begin{pmatrix} 3 & -9 \\ -9 & -12 \end{pmatrix}$ ksi

For each case draw Mohr's circle of stress, and then, using trigonometry, find the principal stresses and show their directions and senses on properly oriented elements. Also find the maximum shearing stresses with the associated normal stresses, and show the results on properly oriented elements. *Ans.* (a) 14.83 ksi, 3.17 ksi; 5.83 ksi, 9 ksi; (b) −0.9 ksi, −13.1 ksi; 6.1 ksi, −7 ksi; (c) 7.2 ksi, −16.2 ksi; 11.7 ksi, −4.5 ksi.

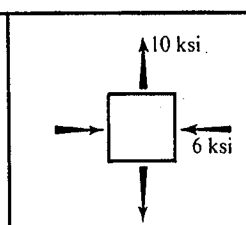
9-17. If $\sigma_x = \sigma_1 = 0$ and $\sigma_y = \sigma_2 = -4{,}000$ psi, using Mohr's circle of stress, find the



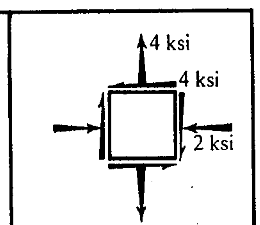
PROB. 9-12



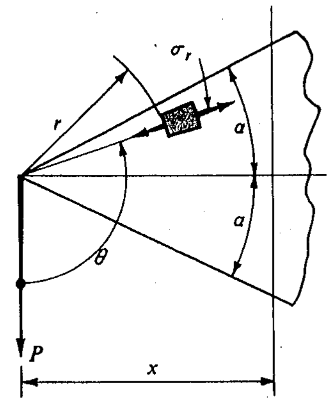
PROB. 9-13



PROB. 9-14



PROB. 9-15



PROB. 9-26

figure. For such a wedge the elasticity solution shows that only radial stress distribution exists and is given* by

$$\sigma_r = -\frac{P\cos\theta}{r[\alpha - \frac{1}{2}\sin 2\alpha]}$$

Determine the normal and the shearing stresses on a vertical section at distance x from the applied force P and compare with the elementary solutions. If $\alpha = 30°$ find the percentage of discrepancy among the maximum stresses in the alternative solutions.

9-27. Using the stress transformation equations for a three-dimensional state of stress,† one may diagonalize any stress matrix. Suppose this were done and it yields

$$\begin{pmatrix} 12{,}000 & 0 & 0 \\ 0 & -6{,}000 & 0 \\ 0 & 0 & 8{,}000 \end{pmatrix} \text{psi}$$

For this state of stress what is the maximum shearing stress? Illustrate the plane or planes on which it acts in a sketch.

9-28. An investigation of stresses in the plate of a thin-walled pressure vessel indicates that the stress matrix is

$$\begin{pmatrix} 20 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ksi}$$

where it is to be noted that $\sigma_3 \approx 0$. (This state of stress is analogous to that shown in Prob. 4-6.) Are there any shearing stresses in the material? Illustrate with a sketch.

9-29. Let l, m, and n define the direction cosines of a linear element. Using this notation, Eq. 9-18 can be rewritten as

$$\varepsilon_\theta = \varepsilon_x l^2 + \varepsilon_y m^2 + \gamma_{xy} lm$$

Show that for the three-dimensional case

$$\varepsilon_\theta = \varepsilon_x l^2 + \varepsilon_y m^2 + \varepsilon_z n^2 + \gamma_{xy} lm + \gamma_{yz} mn + \gamma_{zx} nl$$

9-30. If the unit strains are $\varepsilon_x = -120 \times 10^{-6}$, $\varepsilon_y = +1{,}120 \times 10^{-6}$, and $\gamma_{xy} = -200 \times 10^{-6}$, what are the principal strains and in which direction do they occur? Use Eqs. 9-20 and 9-21 or Mohr's circle of strain, as directed. *Ans.* $1{,}130 \times 10^{-6}$, -130×10^{-6}.

9-31. If the unit strains are $\varepsilon_x = -800 \times 10^{-6}$, $\varepsilon_y = -200 \times 10^{-6}$, and $\gamma_{xy} = +800 \times 10^{-6}$, what are the principal strains and in which directions do they occur? Use Eqs. 9-20 and 9-21 or Mohr's circle, as directed. *Ans.* 0, $1{,}000 \times 10^{-6}$.

9-32. If the strain measurements given in the above problem were made on a steel member ($E = 29.5 \times 10^6$ psi and $\nu = 0.3$), what are the principal stresses and in which direction do they act?

9-33. The data for a rectangular rosette attached to a stressed steel member are $\varepsilon_{0°} = -220 \times 10^{-6}$, $\varepsilon_{45°} = +120 \times 10^{-6}$, $\varepsilon_{90°} = +220 \times 10^{-6}$. What are the principal stresses and in which directions do they act? $E = 30 \times 10^6$ psi and $\nu = 0.3$. *Ans.* ±5.76 ksi, 14°18′.

9-34. The data for an equiangular rosette, attached to a stressed, aluminum-alloy member, are $\varepsilon_{0°} = +400 \times 10^{-6}$, $\varepsilon_{60°} = +400 \times 10^{-6}$, and $\varepsilon_{120°} = -600 \times 10^{-6}$. What are the principal stresses and in which directions do they act? $E = 10^7$ psi and $\nu = \frac{1}{4}$. *Ans.* +6.22 ksi, −4.44 ksi, 30°.

* Timoshenko and Goodier, *Theory of Elasticity*, p. 97.

† See any book on elasticity or plasticity. For a brief discussion of this point see Art. 9-9.