**EML 3702 Mechanics of Materials (3)**

**Professor:**

Cesar Levy
Office EAS 3462
Office hours: to be announced
Tel: (305) 348-3643 (voice mail)
e-mail: levy@eng.fiu.edu

**Textbook:**

Egor P. Popov, Introduction to Mechanics of Solids. 1968 Prentice Hall

**References:**

There are plenty of books at the FIU Library on the subject. Here is a list of a few books on the subject available at the library.

Engineesing mechanics of solids. Egor P. Popov. Englewood Cliffs, N.J. : Prentice Hall, c1990.

Mechanics of materials. James M. Gere, Stephen P. Timoshenko. 3rd ed. Boston : PWS-KENT Pub. Co., c1990.

Problem solver in strength of materials and mechanics of solids staff of Research and Education Association ; M. Fogiel, director. Rev. print. Piscataway, N.J. : The Association, 1988, c1980.

Engineering considerations of stress, strain, and strength, Robert C. Juvinall, New York, McGraw-Hill [1967].

Elements of mechanics of materials Gerner A. Olsen. Englewood Cliffs, N.J. Prentice-Hall, c1982.

**Course Objectives**

1. Identify mechanical properties and the characteristics of elastic behavior for material types.
2. Calculate the stress and strain configuration at a point for a specific loading arrangement.
3. Transform plane stress and strain configurations and identify principal stress and Principal Axes.
4. Use the appropriate failure criteria for diverse situation and/or materials (elastic behavior only).
5. Design prismatic beams.

Topics By Week

7-11 Jan 1. Introduction. Shear and Bending Moment Diagrams. Relation among load, shear and bending moment

14-18 Jan 2. Normal and Shear stresses. Bearing stresses. Stress on an oblique plane. Stresses under general conditions

21-25 Jan 3. Stress and Strain – Axial Loading. Stress vs. strain. Hooke's law. Fatigue and deformations of members under axial load

28 J-1 Feb 4. Thermal Stresses, Poisson ratio. Generalized Hooke's law

4-8 Feb 5. Saint Venant's Principle. Stress Concentration. Torsion. Stresses in a shaft Deformations and angle of twist

11-15 Feb 6. Statically indeterminate shafts. Stress concentration in circular shaft

18-22 Feb 7. **EXAM 1.** Analysis and Design of Beams. Pure Bending. Stresses and deformations in the elastic range

25Feb-1M 8. Bending on members made of several materials. Stress Concentration. General case of eccentric axial loading

4-8 Ma 9. Prismatic beams for bending. **EXAM 2.**

11-15 10. Transformation of Stresses and Strains. Principal Stresses. Mohr's Circle: Plane Stress. General State of Stress

25M-29 11. Stresses in Thin-Walled pressure vessels. Mohr's Circle: Plane Strain. Strain Rosette

1-5 A 12. Deflection of Beams. Beam deflection by integration. Statically indeterminate beam.

8-12 A 13. Superposition Method. **EXAM 3.** Columns. Euler's formula.

15-19 A 14. Extension Euler's formula. Design of columns: centric and eccentric load. Failure Criteria

Homeworks are due a week after being assigned. These problems should be neatly worked out, preferably on engineering paper. Use the "Given, Required, Solution" format and completely draw appropriate diagrams and coordinate systems. All numerical answers should have the appropriate units. Note in each exam there will be one problem quite similar to the assigned problems. Problems submitted after the class hour the due day or the next day will be penalized with 15% of the total grade for that assignment. No homework will be accepted after two days without a medical or any other documented excuse.You must keep up with the homework in order to do well in class

**Grading**

| | |
|---|---|
| Homework | 15% |
| Exams (3 15% each) | 45% |
| Final Exam | 40% |
| Total | 100% |

Grades will be assigned based on your performance on the activities above. Final letter grades will be assigned as follows:

| | | | | | |
|---|---|---|---|---|---|
| 100 – 90 | A | 77-79.99 | B- | 60-64.99 | D |
| 87-89.99 | A- | 73-76.99 | C+ | Below 60 | F |
| 84-86.99 | B+ | 70-72.99 | C | | |
| 80-83.99 | B | 65-69.99 | C- | | |

**FINAL EXAM (Cumulative): Time and Place to be announced.**

**NOTE:** ***This is a preliminary syllabus and it might be changed during the semester. Any change will be announced in class.***

| | Aug | S | | | |
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| 6 | Oct 3 | 8 | | | |
| 7 | 10 | 15 | | | |
| 8 | 17 | 22 | | | |
| 9 | 24 | 29 | | | |
| 10 | 31 | N 5 | | | |
| 11 | N 7 | 12 | | | |

**EMA 3702 Mechanics of Materials (3)**
**Spring 2012**

A mid-level course addressing the selection of engineering materials based on static and dynamic loadings, environmental analysis and the experimental analysis of mechanical systems. Emphasis on metals and composite materials.

**Professor:** Cesar Levy
Office EC3474
Class T 1230-315pm and R 1230-145pm in EC1107
Office hours: TBD. Use my office hours as a resource as well.
Tel: (305) 348-3643 (voice mail)
e-mail: levyez@fiu.edu

**Textbook:** Beer, Johnston, DeWolf and Mazurek, Mechanics of Materials, 5th Ed., McGraw Hill

**References:** There are plenty of books at the FIU Library on the subject. Here is a list of a few books on the subject available at the library.

Engineering mechanics of solids. Egor P. Popov. Englewood Cliffs, N.J. : Prentice Hall, c1990.

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Problem solver in strength of materials and mechanics of solids staff of Research and Education Association ; M. Fogiel, director. Rev. print. Piscataway, N.J. : The Association, 1988, c1980.

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**Course Objectives**

1. Identify mechanical properties and the characteristics of elastic behavior for material types.
2. Calculate the stress and strain configuration at a point for a specific loading arrangement.
3. Transform plane stress and strain configurations and identify principal stress and Principal Axes.
4. Use the appropriate failure criteria for diverse situation and/or materials (elastic behavior only).
5. Design prismatic beams.
6. Determine the deflection at any point of a beam, given the loading and end conditions
7. Determine the load, shear and moment diagrams for a beam given the loading and end conditions.

Topics By Week

1. Introduction. Shear and Bending Moment Diagrams. Relation among load, shear and bending moment
2. Normal and Shear stresses. Bearing stresses. Stress on an oblique plane. Stresses under general conditions
3. Transformation of Stresses. Principal Stresses.
4. Stress and Strain – Axial Loading. Stress vs. strain. Hooke's law. Fatigue and deformations of members under axial load. Transformation of Strains. **EXAM 1**
5. Thermal Stresses, Poisson ratio. Generalized Hooke's law
6. Saint Venant's Principle. Stress Concentration. Torsion. Stresses in a shaft Deformations and angle of twist
7. Statically indeterminate shafts. Stress concentration in circular shaft
8. Analysis and Design of Beams. Pure Bending. Stresses and deformations in the elastic range
9. Bending on members made of several materials. Stress Concentration. General case of eccentric axial loading
10. Prismatic beams for bending. **EXAM 2.**

11. Quick Review of Transformation of Stresses and Strains. Principal Stresses. Mohr's Circle: Plane Stress. General State of Stress
12. Stresses in Thin-Walled pressure vessels. Mohr's Circle: Plane Strain. Strain Rosette
13. Deflection of Beams. Beam deflection by integration. Statically indeterminate beam.
14. Superposition Method. **EXAM 3.** Columns. Euler's formula.
15. Extension Euler's formula. Design of columns: centric and eccentric load. Failure Criteria

Homeworks are going to be assigned and some problems will be collected, unless specified in class. These problems should be neatly worked out, on engineering paper/graph paper, one problem to a page. One or more of the assigned problems will be chosen randomly for grading. In preparing the homework, use the "Given, Required, Solution" format and completely draw appropriate diagrams and coordinate systems. All numerical answers should have the appropriate units. The collected HW grade may be used to replace your Exam 1 quiz grade and it would behoove you to do all problems as the exam questions will be similar in nature. You must keep up with the homework in order to do well in class.

Exams: as you see above we have 3. No make-up exams will be given except for medical reasons. If you know you will be out, for example for work reasons, you need to tell me BEFORE the exam, not after. You can inform me (or have a parent or wife or best friend inform me) by email (levyez@fiu.edu) or by phone (305-348-3643).

**FINAL EXAM (Cumulative): Will be announced as soon as the new exam schedule is released.**

**CHEATING: Cheating will not be tolerated; it is an ethics violation. Also, you don't learn anything by cheating. All instances of cheating will automatically result in a zero grade, especially exams, and will result in disciplinary action against ALL cheaters.** Please read the header on http://web.eng.fiu.edu/levy.

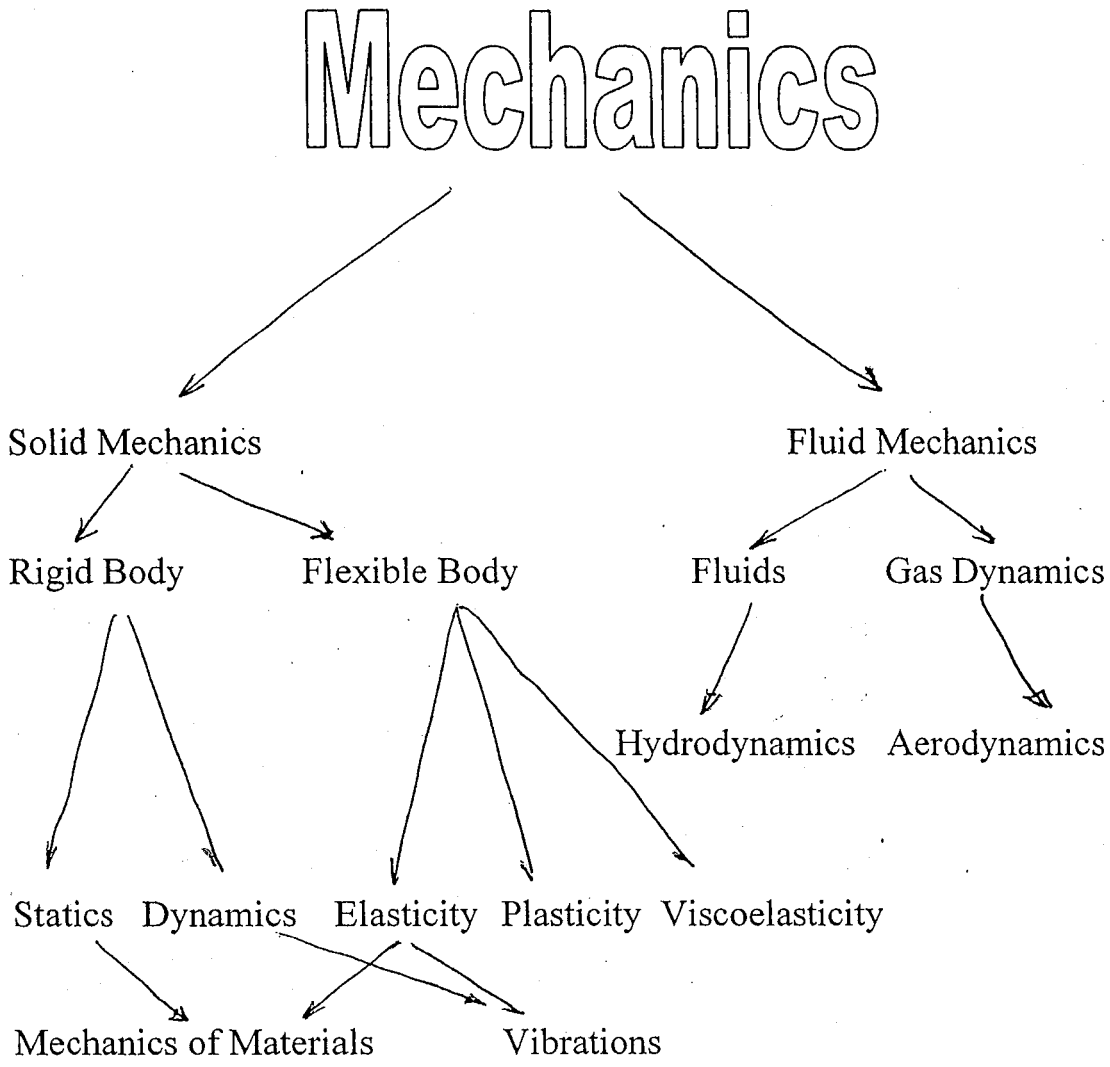
**Grading**

| | |
|---|---|
| Exams (1-3: 20%, 23%, 23% each) | 66% |
| Final Exam | 34% |
| Total | 100% |

Grades will be assigned based on your performance on the activities above. Final letter grades will be assigned as follows:

| | | | | | |
|---|---|---|---|---|---|
| 100 – 93 | A | 77-79.99 | B- | 63-66.99 | D+ |
| 87-92.99 | A- | 73-76.99 | C+ | 60-62.99 | D |
| 84-86.99 | B+ | 70-72.99 | C | Below 60 | F |
| 80-83.99 | B | 67-69.99 | C- | | |

**NOTE:** ***This is a preliminary syllabus and it might be changed during the semester. Any change will be announced in class.***


Mechanics
Solid Mechanics
Fluid Mechanics
Rigid Body
Flexible Body
Fluids
Gas Dynamics
Hydrodynamics
Aerodynamics
Statics
Dynamics
Elasticity
Plasticity
Viscoelasticity
Mechanics of Materials
Vibrations

Introduction to Mechanics and Materials Science

In structural engineering, one has to give dimensions to the particular members, such as beams, floors, walls, shafts and the like, so that they can support the forces that act upon them, both the known ones as well as those that could act upon them.

Therefore, the parts of a composite structure, must be able to withstand the forces, i.e., be strong enough in order not to deflect much when the forces act upon them.

Thin wall members can buckle when they are loaded by small loads before they fail in bending.

One must also design bars and beams without waste of materials.

Our subject deals with strength, stiffness and stability of members of our structure that carry the loads.

Galileo, in the $17^{th}$ Century, first investigated the behavior of materials that were loaded in a rational manner, bars, beams in both tension and compression.

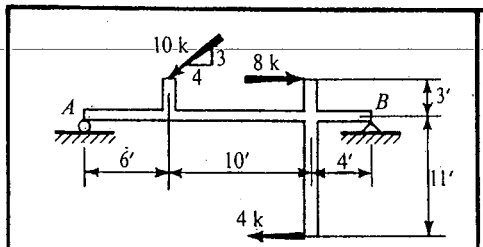
From that time until now, there were many investigators such as Coulomb, Poisson, Navier, St. Venant and Cauchy.

Mechanics and Materials Science crosses all realms-- from Civil Engineering, to Aeronautical Engineering, mechanical Engineering, Aerospace Engineering and any field that requires building structures of any kind.

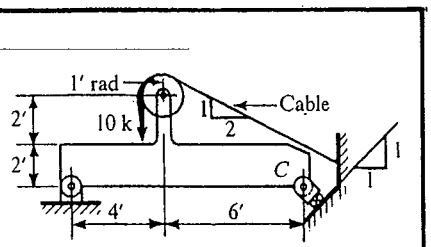
The behavior of bars and beams depend not only on the basic laws of Newton that require equilibrium, but also upon the mechanical characteristics of materials that make up the bars and beams.

Materials behavior are obtained from many rational experiments done in laboratories. The knowledge has given us the information to choose the material that is proper to design our structures.

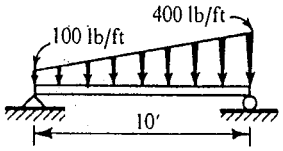
We will begin with a review of statics and after that we will begin to learn our subject matter.



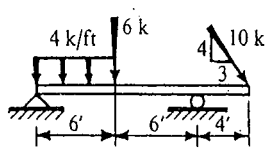

PROB. 2-1



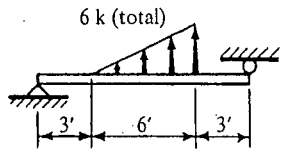

PROB. 2-2



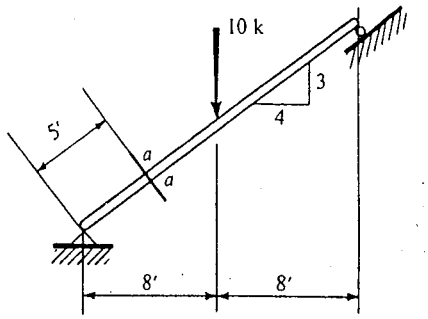

PROB. 2-3



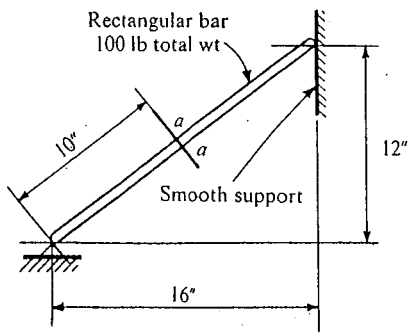

PROB. 2-4



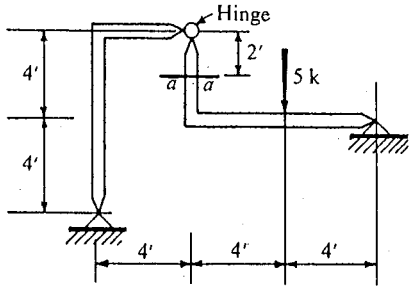

PROB. 2-5



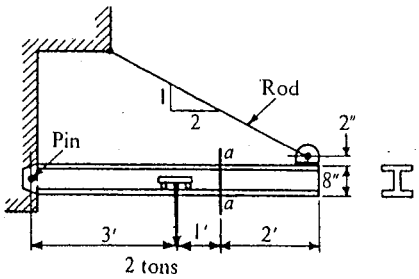

PROB. 2-6



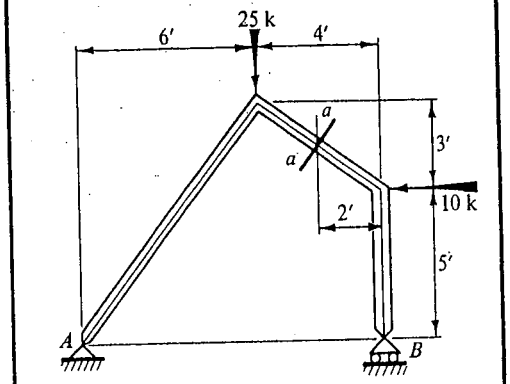

PROB. 2-7



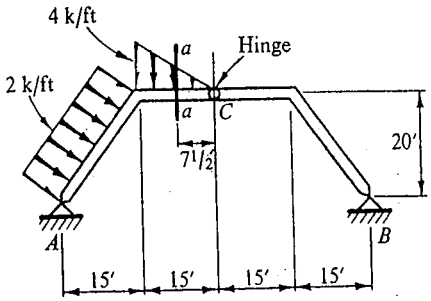

PROB. 2-8



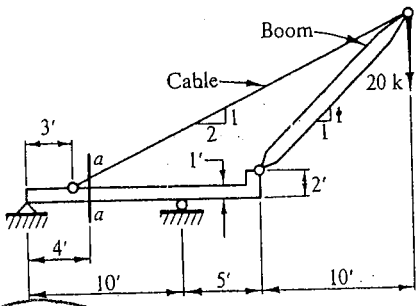

PROB. 2-9



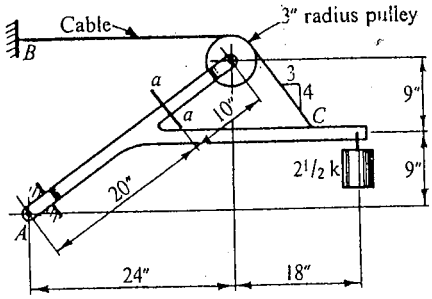

PROB. 2-10




PROB. 2-11



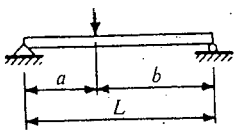

PROB. 2-12



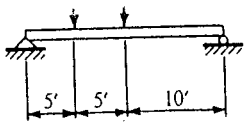

PROB. 2-13

**2-14 through 2-19.** For the beams loaded as shown in the figures write general expressions for the shear and bending moments for each region over the length of the member. Also plot the corresponding shear and moment diagrams. *Ans.* Maximum moment in parentheses by the figure.

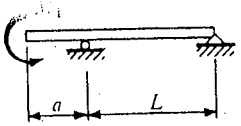
For additional loading conditions see other problems in this chapter.



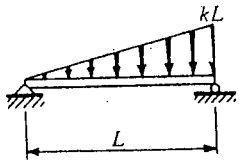

PROB. 2-14 ($Pab/L$)



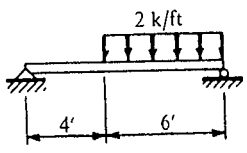

PROB. 2-15 (250)



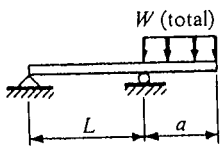

PROB. 2-16 ($-M_1$)




PROB. 2-17 ($kL^3/9\sqrt{3}$)




PROB. 2-18 (17.6)




PROB. 2-19 ($-Wa/2$)

ow positive directions assumed for
id $M$ on a free-body diagram.

the positive senses of $p$, $V$, and $M$ are
as shown in the figure, derive the
s equivalent to Eqs. 2-4, 2-5, and 2-6.
. 11-2(b).)

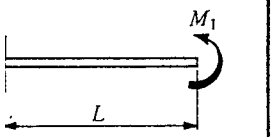
**ough 2-53.** For the beams loaded in
ie as shown in the figures, neglecting
ht of the members, solve as directed:
Without formal computations, sketch
id moment diagrams directly below a
of the given loaded member.
Same as **A**, and, in addition, show
e of the elastic curve.
Plot shear, moment, and, wherever
nt, axial force diagrams for the main
horizontal members. Determine all critical ordinates.

**D.** Same as **C**, and, in addition, determine the points of inflection and show the shape of the elastic curve.

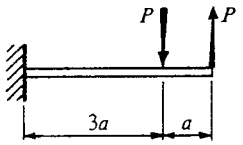
*Ans.* All shear and moment diagrams must close. The largest moment is given in parentheses by the figures in the units of the problem.

For additional loading conditions see other problems in this chapter.

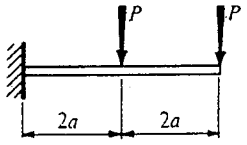
**2-54 through 2-56.** The moment diagrams for beams supported at $A$ and $B$ are as shown in the figures. How are these beams loaded? All curved lines represent parabolas, i.e., plots of equations of the second degree. (*Hint:* construction of shear diagrams aids the



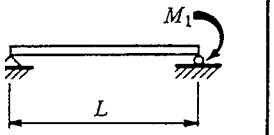
B. 2-31 $(+M_1)$



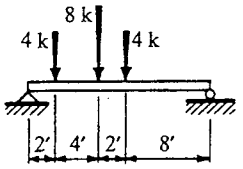
PROB. 2-32 $(+Pa)$



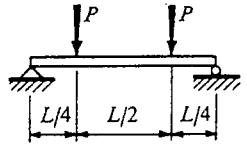
PROB. 2-33 $(-6Pa)$



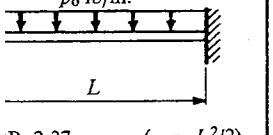
B. 2-34 $(-M_1)$



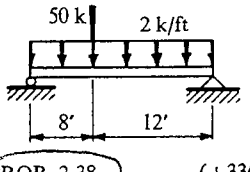
PROB. 2-35 (+47)



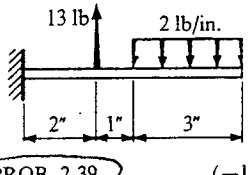
PROB. 2-36 $(\pm PL/8)$



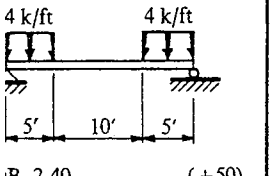
B. 2-37 $(-p_o L^2/2)$



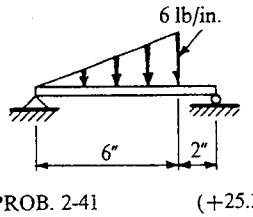
PROB. 2-38 (+336)



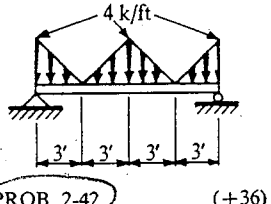
PROB. 2-39 (−15)



B. 2-40 (+50)

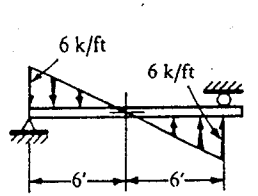


PROB. 2-41 (+25.3)

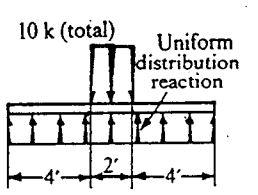


PROB. 2-42 (+36)

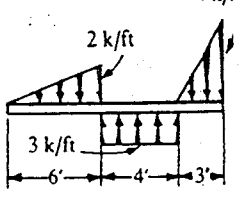




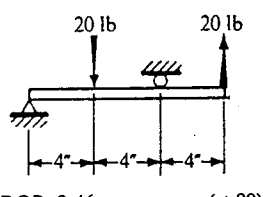
PROB. 2-43 $(\pm 13.8)$



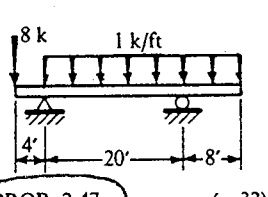
PROB. 2-44 (+10)



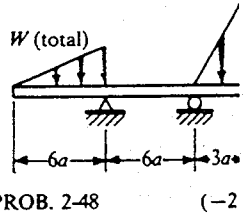
PROB. 2-45 (−



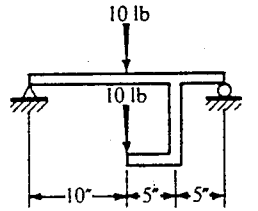
PROB. 2-46 (+80)



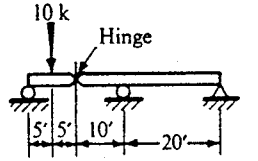
PROB. 2-47 (−32)



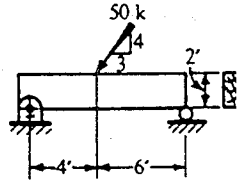
PROB. 2-48 (−2



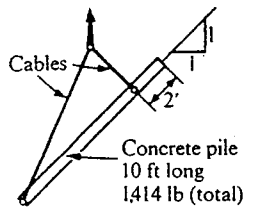
PROB. 2-49 (+100)



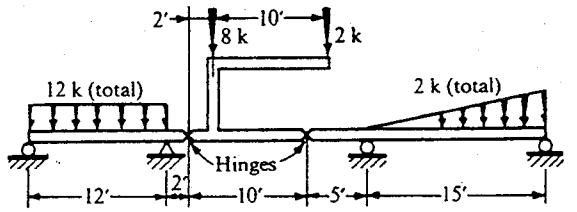
PROB. 2-50 (−50)



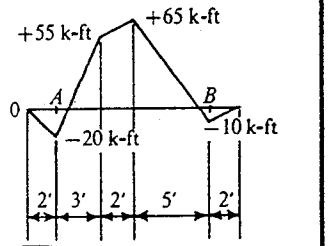
PROB. 2-51 (+



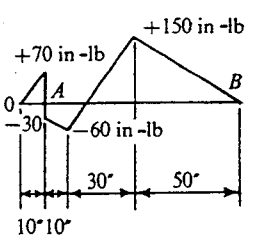
PROB. 2-52 (703)



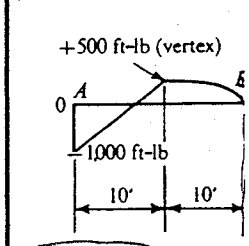
PROB. 2-53 (+12.5, +32, −



PROB. 2-54 (35)



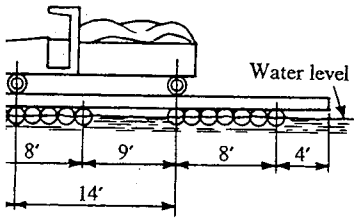
PROB. 2-55 (−10)



PROB. 2-56 (

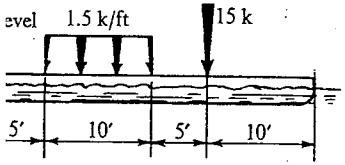
*Ans.* Reaction at $A$ in parentheses
ure.

ruck is standing on a raft; it weighs
loaded. Assume that 0.1 of the total
ried by each of the front wheels, and
ch of the rear wheels. Assume that
ain longitudinal beams of the raft
art, i.e., each beam carries one-half
ck. Also assume that each of the
pontoons provide reactions which
ated as being uniformly distributed.
r and moment diagrams for each
the truck in the position shown.
he critical values using foot-pound
. +18,000 ft-lb (maximum).




PROB. 2-57

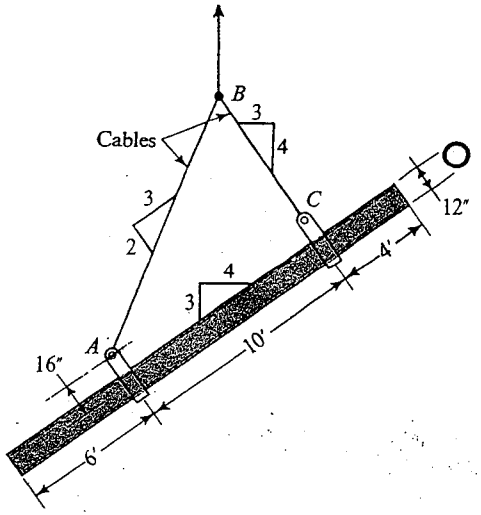
mall narrow barge is loaded as
the figure. Plot shear and moment
for the applied loading. *Ans.*
imum), +50 k-ft (maximum).



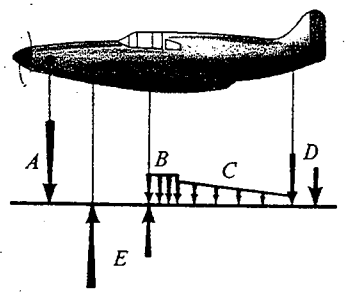

PROB. 2-58

2-in. O.D. steel pipe weighing
foot is held by means of rigidly
okes in an inclined position as
he figure. Plot shear and moment
or this pipe giving the values of all
linates. Joints $A$, $B$, and $C$ are

load distribution for a small,
ne airplane in flight may be
is shown in the figure. In this
e vector $A$ represents the weight of




PROB. 2-59

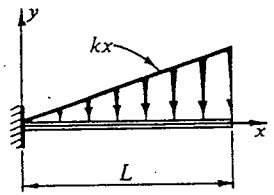



PROB. 2-60

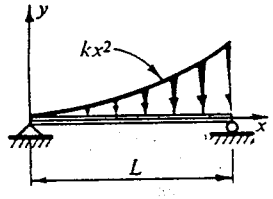
the engine, $B$ the uniformly distributed cabin weight, $C$ the weight of the aft fuselage, and $D$ the forces from the tail control surfaces. The upward forces $E$ are developed by the two longerons from the wings. Using this data construct plausible, qualitative shear and moment diagrams for the fuselage.

**2-61 through 2-63.** For the beams loaded as shown in the figures, using Eq. 2-6, (a) find $V(x)$ and $M(x)$. Check reactions by conventional statics. (b) Plot the shear and moment diagrams.

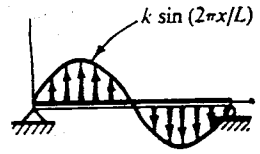
**2-64 through 2-72.** For the beams loaded as shown in the figures, using singularity functions and Eq. 2-6, (a) find $V(x)$ and $M(x)$. Check reactions by conventional statics. (b) Plot the shear and moment diagrams.



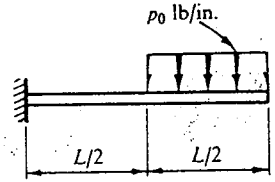

PROB. 2-61



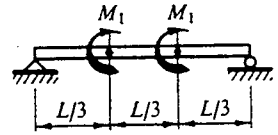

PROB. 2-62



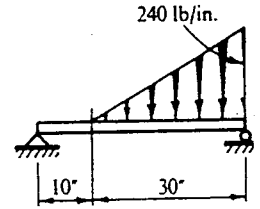

PROB. 2-63



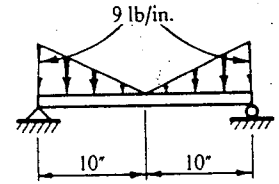

PROB. 2-64



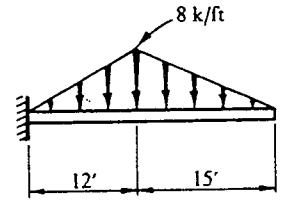

PROB. 2-65



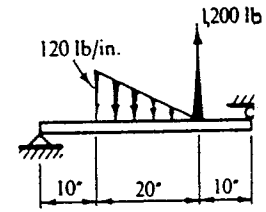

PROB. 2-66



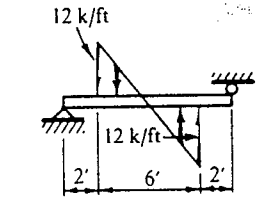

PROB. 2-67



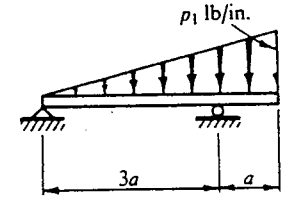

PROB. 2-68



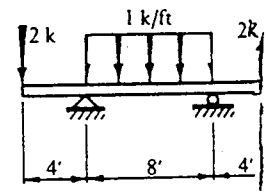

PROB. 2-69



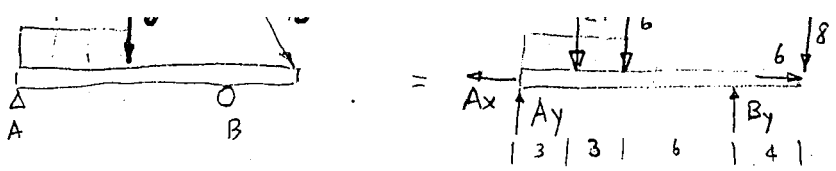

PROB. 2-70




PROB. 2-71




PROB. 2-72




Only for reactions

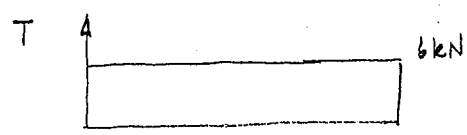
$A_x = 6\ kN \Leftarrow \Sigma F_x = 0$

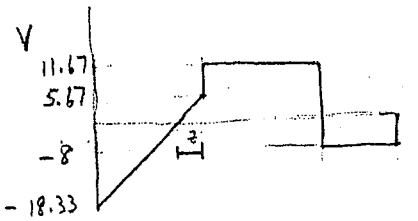
$A_y + B_y = 38\ kN \Leftarrow \Sigma F_y = 0$

$-24.3 - 6\cdot 6 + B_y \cdot 12 - 8\cdot 16 = \Sigma M_A = 0$

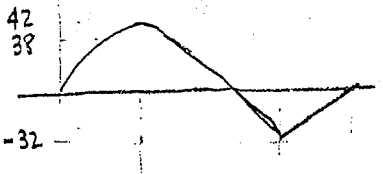
$B_y = 19.7\ kN$

$A_y = 18.3\ kN$

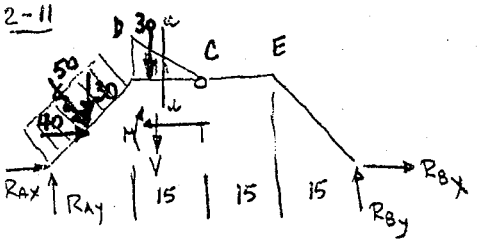


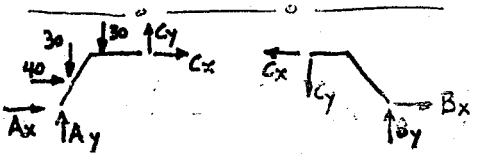




$\frac{5.67}{18.33} = \frac{z}{6-z}$ $\quad z = 1.4167$ $\quad 6 - z = 4.5833$




2-11



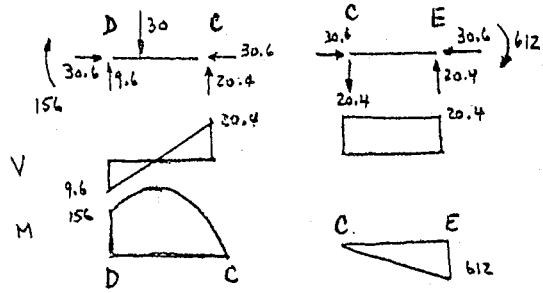




$R_{Bx} = 30.6\ kN \leftarrow$ $\qquad R_{Ay} = 39.6\ kN$

$R_{Ax} = 9.4\ kN \leftarrow$ $\qquad R_{By} = 20.4\ kN$

at a-a $M = 135\ kN\text{-}m$

$T = 30.6\ kN \rightarrow$ $\quad V = 12.9\ kN \downarrow$



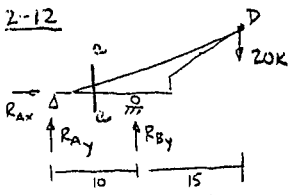




$M = 156$

whole system

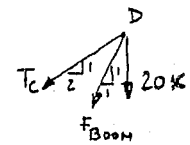
2-12




$\Sigma F_x = R_{Ax} = 0$

$\Sigma F_y = R_{Ay} + R_{By} - 20\ kN = 0$

$+\circlearrowleft \Sigma M_A = 0 = -R_{By}\cdot 10 + 20\cdot 25 = 0 \quad R_{By} = 50\ kN \quad R_{Ay} = -30\ kN$



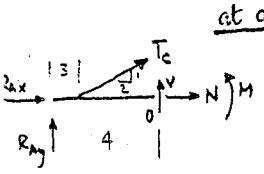

$\Sigma F_x = 0 = -T_c \cdot \frac{2}{\sqrt{5}} - F_{Boom}\cdot \frac{1}{\sqrt{2}} = 0$

$\Sigma F_y = 0 = -T_c \cdot \frac{1}{\sqrt{5}} - F_{Boom}\frac{1}{\sqrt{2}} - 20\ kN$

$T_c = 20\sqrt{5}\ kN = 44.72\ kN$

$F_{BOOM} = -56.57\ kN$

at a-a




$\Sigma F_x = 0 \Rightarrow N + R_{Ax} + T_c \cdot \frac{2}{\sqrt{5}} = 0 \qquad N = -40\ kN$

$\Sigma F_y = 0 \Rightarrow V + T_c \cdot \frac{1}{\sqrt{5}} + R_{Ay} = 0 \qquad V = 10\ kN$

$+\circlearrowleft \Sigma M_0 = 0 \Rightarrow -M + R_{Ay}\cdot 4 + T_c \cdot \frac{1}{\sqrt{5}}\cdot 1 + T_c \cdot \frac{2}{\sqrt{5}}\cdot 0.5 = 0 \qquad M = -80\ kN\text{-}m \text{ or } 80\ kN\text{-}m \circlearrowright$

**2.9** The 36-mm-diameter steel rod $ABC$ and a brass rod $CD$ of the same diameter are joined at point $C$ to form the 7.5-m rod $ABCD$. For the loading shown, and neglecting the weight of the rod, determine the deflection ($a$) of point $C$, ($b$) of point $D$.

Answer: $\delta_C = 2.95\ mm\downarrow$, $\delta_D = 5.29\ mm\downarrow$.

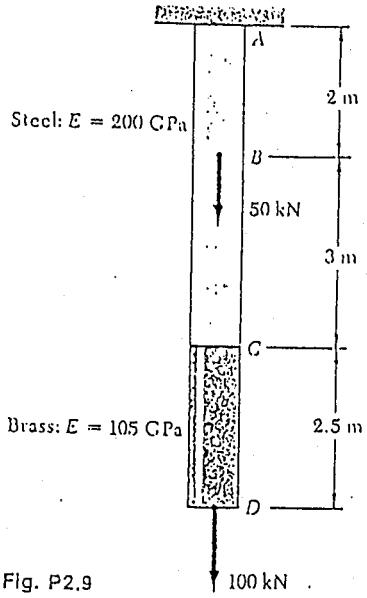


Fig. P2.9

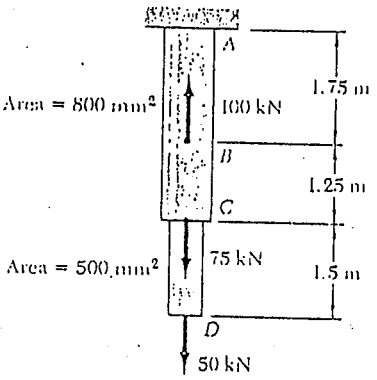


Fig. P2.10

**2.10** The rod $ABCD$ is made of an aluminum alloy for which $E = 70$ GPa. For the loading shown, and neglecting the weight of the rod, determine the deflection ($a$) of point $B$, ($b$) of point $D$.

Answer: $\delta_B = 0.781\ mm\downarrow$, $\delta_D = 5.71\ mm\downarrow$.

**2.26** An axial centric force of magnitude $P = 385$ kN is applied to the composite block shown by means of a rigid end plate. Determine the normal stress in ($a$) the steel core, ($b$) the aluminum plates.

Answer: $\sigma_S = -175\ MPa$, $\sigma_{Al} = -61.3\ MPa$.

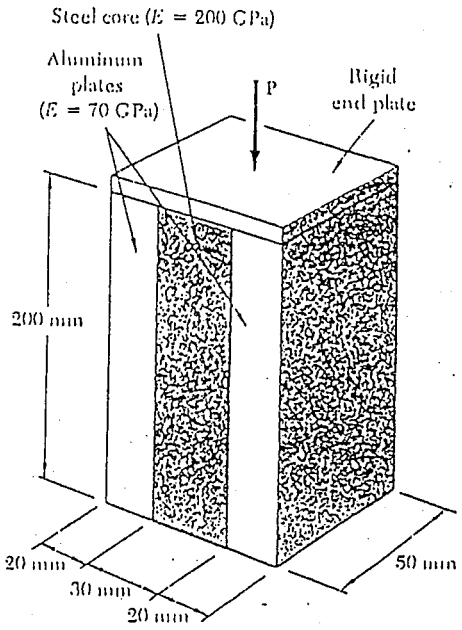


Fig. P2.26

**2.37** The rigid rod $ABC$ is suspended by three identical wires. Knowing that $x = \frac{2}{3}L$, determine the tension in each wire due to the force P.

Answer: $T_B = P/3$, $T_C = P/6$, $T_A = P/2$.

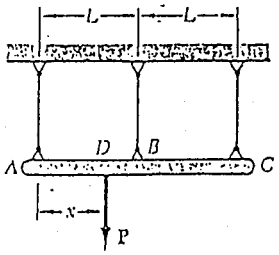
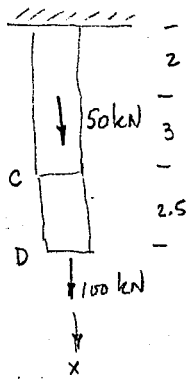
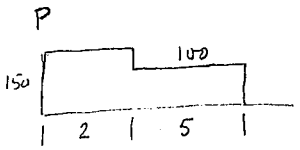


Fig. P2.37 and P2.38

2.9




$$A = \frac{\pi d^2}{4} = \frac{\pi}{4}(.036)^2 = 1.018\times10^{-3}\,m^2$$

פלדה $E = 200\,GPa$ פלסטיק $E = 10\,GPa$

$$\downarrow \delta_C = \int_0^5 \frac{P}{AE}dx = \int_0^2 \frac{150000\,dx}{(1.018\times10^{-3})(200\times10^9)} + \int_2^5 \frac{100000\,dx}{(1.018\times10^{-3})(200\times10^9)}$$

$$= 7.367\times10^{-4}\,x\Big|_0^2 + 4.912\times10^{-4}\,x\Big|_2^5 =$$

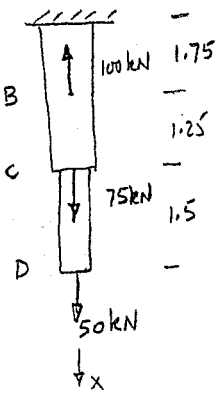
$$= 1.473\times10^{-3} + 1.473\times10^{-3} = \underline{2.947\times10^{-3}\,m}\ \downarrow$$

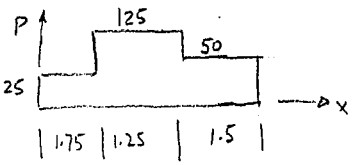
$$\delta_D = \int_0^{7.5} \frac{P}{AE}dx = \int_0^2 \frac{150000\,dx}{(1.018\times10^{-3})(200\times10^9)} + \int_2^5 \frac{100000\,dx}{(1.018\times10^{-3})(200\times10^9)} + \int_5^{7.5} \frac{100000\,dx}{(1.018\times10^{-3})(10\times10^9)}$$

$$= \delta_C + 9.355\times10^{-4}\,x\Big|_5^{7.5} = \delta_C + 2.339\times10^{-3} = \underline{5.286\times10^{-3}\,m}\ \downarrow$$

---

2.10




$A_{AC} = 8\times10^{-4}\,m^2$ $A_{CD} = 5\times10^{-4}\,m^2$ $E = 70\,GPa$

$$\downarrow \delta_B = \int_0^{1.75} \frac{P}{AE}dx = \int_0^{1.75} \frac{25000\,dx}{(8\times10^{-4})(70\times10^9)} = 4.464\times10^{-4}\,x\Big|_0^{1.75}$$

$$= 7.813\times10^{-4}\,m = .781\,mm\ \downarrow$$

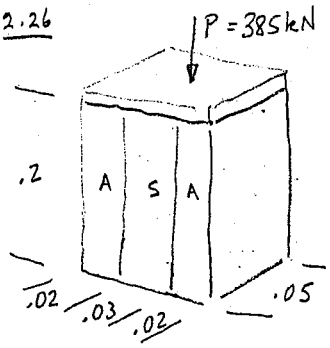
$$\downarrow \delta_D = \int_0^{4.5} \frac{P}{AE}dx = \int_0^{1.75} \frac{25000}{(8\times10^{-4})(70\times10^9)}dx + \int_{1.75}^{3} \frac{125000\,dx}{(8\times10^{-4})(70\times10^9)} + \int_3^{4.5} \frac{50,000\,dx}{(5\times10^{-4})(70\times10^9)}$$

$$= \delta_B + 2.232\times10^{-3}\,x\Big|_{1.75}^{3} + 1.4286\times10^{-3}\,x\Big|_3^{4.5} = 7.813\times10^{-4} + 2.790\times10^{-3} + 2.1429\times10^{-3}$$

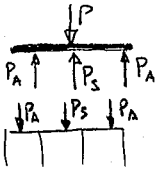
$$= 5.714\times10^{-3}\,m = \underline{5.71\,mm}$$

---

2.26




$E_A = 70\,GPa$
$E_S = 200\,GPa$




$2P_A + P_S = P = 385\times10^3\,N$ $\Delta u_A = \Delta u_S = \epsilon_A L_A = \epsilon_S L_S$; $L_S = L_A$

$$\epsilon_S = \epsilon_A = \frac{\sigma_A}{E_A} = \frac{\sigma_S}{E_S} \Rightarrow \frac{P_A}{E_A A_A} = \frac{P_S}{E_S A_S}$$

$A_S = .03\times.05 = 1.5\times10^{-3}\,m^2$

$A_A = .02\times.05 = 1.0\times10^{-3}\,m^2$

$$\frac{P_A}{(70\times10^9)(1\times10^{-3})} = \frac{P_S}{(200\times10^9)(1.5\times10^{-3})}$$

$$P_S = \frac{300\times10^6}{70\times10^6}P_A = 4.286\,P_A$$

$$P = 2P_A + P_S = 2P_A + 4.286P_A = 6.286\,P_A \Rightarrow P/6.286 = P_A$$

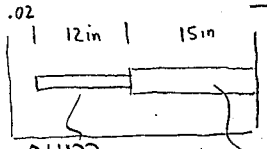
$\sigma_A = P_A/A_A = \frac{6.125\times10^4}{1.0\times10^{-3}} = 61.25\times10^6\,Pa \Leftarrow P_A = 61247.22\,N$ ,לכן

$\sigma_S = P_S/A_S = \frac{2.625\times10^5}{1.5\times10^{-3}} = 175\times10^6 \Leftarrow P_S = 262505.66\,N$

כמו כן $\sigma_A = 61.25\,MPa$ בלחץ ו- $\sigma_S = 175\,MPa$ בלחץ

---

2.45




$A_B = 2.5\,in^2$
$E_B = 15\times10^6\,psi$
$\alpha_B = 10.1\times10^{-6}/°F$

$A_A = 3\,in^2$
$E_A = 10\times10^6\,psi$
$\alפ_A = 12.8\times10^{-6}/°F$

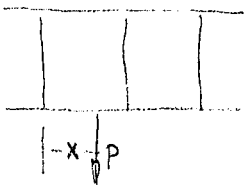
התארכות על המוט המורכב היא $\Delta u = L_x\epsilon_x$ אבל מפני שהמוט מחומם בתוך למבנה אלת הלא מרוטורה שגורמת למתח קיים. לכן, מפני ש $\epsilon_x = \alpha\Delta T$

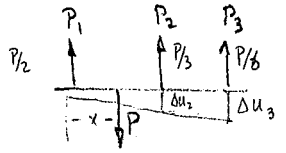
$$\Delta u = .02\,in = L_B\alpha_B\Delta T + L_A\alpha_A\Delta T = \Delta T\left[(12\times10.1\times10^{-6}) + (15\times12.8\times10^{-6})\right]$$

$$= \Delta T\,(3.132\times10^{-4}) \Rightarrow 63.857\,°F = \Delta T$$

לכן, יש עוד $\Delta T^* = 136.14°$ שהמוט ניצב בקיר. כדי לאפשר פינוי, צריך למצוא את התארכות על האלומיניום והברונזה עד שהמוט מתנתק לגמרי בקיר $\Delta u_A = L_A\alpha_A(63.86) = .01226\,in$, $\Delta u_B = .00774\,in$. לכן, אורך האלומיניום בהתחלת הגזירה 15.01226 ושל הברונזה 12.00774

כאשר המוט מתנתק




$$P_1+P_2+P_3=P \qquad \Sigma M_{P_1}=P\cdot\frac{2}{3}L-P_2\cdot L-P_3\cdot 2L=0$$

$$\Delta u_2=\frac{P_2L}{AE}$$
$$\Delta u_3=\frac{P_3L}{AE}$$
$$\Delta u_1=\frac{P_1L}{AE}$$

$$\frac{2P}{3}=P_2+2P_3 \qquad P_2=\frac{2P}{3}-2P_3$$

$$P_1+\left(\frac{2P}{3}-2P_3\right)+P_3=P$$
$$\left(P_1-P_3\right)=P/3$$

$$P_3-P_2=P_2-P_1$$
$$\frac{\Delta u_3-\Delta u_2}{L}=\frac{u_2-u_2}{L}=\frac{u_2-u_1}{L}$$
$$\frac{1}{L}\left(\frac{P_3L}{AE}-\frac{P_2L}{AE}\right)=\frac{1}{L}\left(\frac{P_2}{AE}-\frac{P_1}{AE}\right)$$
$$P_3-P_2=P_2-P_1$$
$$P_3-2P_2+P_1=0 \quad (1)$$
$$P_3+P_2+P_1=P \quad (2)$$

(2)-(1) $\Rightarrow 3P_2=P \quad P_2=P/3$

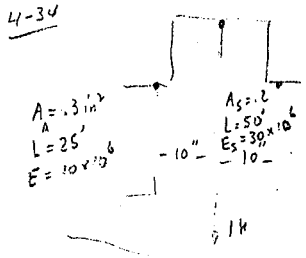
$$P/3=2P/3-2P_3 \Rightarrow 2P_3=P/3$$
$$P_3=P/6$$

$$P_1+P_2+P_3=P \Longrightarrow P_1=P/2$$

מה הכוח בכול חוט.

$\epsilon_1=$

4-34

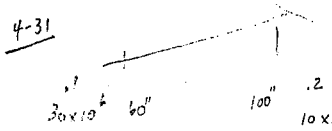



$$\Sigma M_{P=14'}=0 \Rightarrow P_{AL}=P_{AR}$$
$$2P_A+P_S=P \Leftarrow \Sigma F_y=0$$

By symmetry bar moves vertical

$$\Delta u_A=\Delta u_S=\epsilon_A L_A=\epsilon_S L_S$$
$$\frac{\epsilon_A}{\epsilon_S}=\frac{L_S}{L_A}=\frac{2}{1}$$

4-31




$$\frac{\epsilon_A}{\epsilon_S}=\frac{\sigma_A/E_A}{\sigma_S/E_S}=\frac{P_A/E_AA_A}{P_S/E_SA_S}=2$$

$$\frac{P_A}{P_S}\frac{E_SA_S}{E_AA_A}=\frac{P_A}{P_S}\frac{30\times10^6}{1\times10^7}\cdot\frac{.2}{.3}=2$$
$$=\frac{P_A}{P_S}\cdot\frac{6}{3}=2$$




$$\Sigma M \quad P\cdot 10-P_2\cdot 20=0 \quad P_2=\frac{1}{2}P \quad P_1=\frac{1}{2}P$$

$$P_A=\frac{E_AA_A}{E_SA_S}P_S\cdot 2 \qquad P_A=\frac{4}{3}P_S$$
$$=\frac{10\times10^6(.3)}{30\times10^6(.2)}\cdot 2\,P_S$$

$$P_A=\frac{1}{3}\left(\frac{3}{2}\right)\cdot 2P_S \qquad P_A=P_S \Longrightarrow 3P_A=P \qquad P_A=\frac{1}{3}P=P_S$$

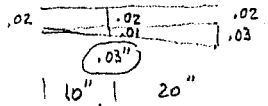
$\Delta u$

$$\Delta u=\frac{P_1L_1}{A_1E_1} \qquad \Delta u_2=\frac{P_2L_2}{A_2E_2}$$

$$\frac{P}{2}\frac{60}{.1\times30\times10^6} \qquad \frac{P}{2}\cdot\frac{10''}{.2\times10\times10^6}$$

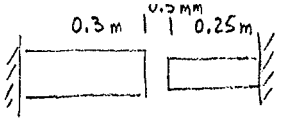
$$\frac{1000\,(60)}{30\times10^5}=\frac{6\times10^4}{3\times10^6}=2\times10^{-2}=.02'$$

$$\frac{100000}{2\times10^6}=\frac{1\times10^5}{2\times10^6}=0.5\times10^{-1}=.05'$$




שני חוטים נקשרים לגוף קשיח כמתואר. חוט השמאלי מפלדה בעל שטח חתך $0.2''^2$ ו $E=30\times10^6$. חוט הימני נעשה מאלומיניום בעל שטח חתך $0.1''^2$ ו $E=10\times10^6$. שאר מאפיין מיושם כמתואר. מה יהיה זווית הגיבוי, אבני הקשיחות הזוויתי?

2.47




$$\sigma_s = -150\times10^6\ N/m^2 = P/A_s = \frac{P}{.8\times10^{-3}\ m^2}$$

$$P = -1.2\times10^5\ N$$

$$0 = \Delta u_A + \Delta u_s = \left(\frac{P}{E_A A_A} + \alpha_A \Delta T^*\right) l_A + \left(\frac{P}{E_s A_s} + \alpha_s \Delta T^*\right) l_s$$

$$= P\left[\frac{l_A}{E_A A_A} + \frac{l_s}{E_s A_s}\right] + (\alpha_A l_A + \alpha_s l_s)\Delta T^*$$

$$0 = -1.2\times10^5\left(3.7876\times10^{-9}\right) + \left(1.14\times10^{-5}\right)\Delta T^* = -4.545\times10^{-4} + (1.14\times10^{-5})\Delta T^*$$

המספרים ל $(\alpha_A l_A + \alpha_s l_s)$ ו $\left[\frac{l_A}{E_A A_A} + \frac{l_s}{E_s A_s}\right]$ בשאלה 2.46 שקיבלנו מהבוחנים

$$\Delta T^* = \frac{4.545\times10^{-4}}{1.14\times10^{-5}} = 39.87^\circ C$$

לכן,

כדי לדרוש להוסיף את הטמפרטורה כדי לסגור את המרחק של 0.5mm, כלומר $\Delta T = 43.86^\circ C$ כמו שמצאנו בדיוק התקדמה. לכן, $\Delta T + \Delta T^* = 43.86 + 39.87 = 83.73^\circ C$. אפשר שאומרים את שינוי הטמפרטורה מ $T = 20^\circ C$, $T$ הסופי דרוש להיות $20^\circ + 83.73 = 103.73^\circ C$

$$\delta_s = \left[\frac{\sigma_{xs}}{E_s} + \alpha_s(\Delta T + \Delta T^*)\right] l_s = \left[\frac{-150\times10^6}{190\times10^9} + (18\times10^{-6})(43.86 + 39.87)\right](250\ mm)$$

$$= (7.1767\times10^{-4})(250\ mm) = 1.794\times10^{-1}\ mm$$

$$L = L_0 + \delta_s = 250 + 0.1794\ mm = 250.1794\ mm$$

$$0 = \Delta u = \Delta u_B + \Delta u_A$$

$$\Delta u_B = \int_0^{0} \epsilon_{x_B} dx = \int_0 \left(\frac{\sigma_{x_B}}{E_B} + \alpha_B \Delta T\right) dx = \left(\frac{\sigma_{x_B}}{E_B} + \alpha_B \Delta T\right) x \Big|_0$$

1- תגובת הקרונות

$$\Delta u_A = \int_{l_B}^{l_B+l_A} \epsilon_{x_A} dx = \int_{l_B}^{l_B+l_A} \left(\frac{\sigma_{x_A}}{E_A} + \alpha_A \Delta T^*\right) dx = \left(\frac{\sigma_{x_A}}{E_A} + \alpha_A \Delta T^*\right) x \Big|_{l_B}^{l_B+l_A}$$

1- תגובת האלומינום

כאן, הציור בא מהחומר והלחץ מאוזן עם הקיר. המאמץ $\sigma_{x_A} = \frac{P_A}{A_A}$, $\sigma_{x_B} = \frac{P_B}{A_B}$. אבל $P_A = P_B$ ; לכן

$$0 = \Delta u_B + \Delta u_A = \left(\frac{P}{E_B A_B} + \alpha_B \Delta T^*\right) l_B + \left(\frac{P}{E_A A_A} + \alpha_A \Delta T^*\right) l_A$$

או

$$P\left[\frac{l_B}{E_B A_B} + \frac{l_B}{E_A A_A}\right] = -\left(\alpha_B l_B + \alpha_A l_A\right) \Delta T^*$$

$$P\left[\frac{12.00774\ in}{15\times10^6 \frac{lb}{in^2} \cdot 2.5 in^2} + \frac{15.01226\ in}{10\times10^6 \frac{lb}{in^2} \cdot 3 in^2}\right] = -\left[10.1\times10^{-6}/^\circ F \cdot 12.00774 + 12.8\times10^{-6}/^\circ F \cdot 15.01226\ in\right](136.14^\circ F)$$

$$P\left(3.2021\times10^{-7} \frac{in}{lb} + 5.0041\times10^{-7} \frac{in}{lb}\right) = -\left(1.21278\times10^{-4} \frac{in}{^\circ F} + 1.92157\times10^{-4}\right)(136.14^\circ F)$$

$$P\left(8.2062\times10^{-7} \frac{in}{lb}\right) = -\left(3.13435\times10^{-4} \frac{in}{^\circ F}\right)(136.14^\circ F) = 4.2671\times10^{-2} in$$

$$P = -5.20\times10^4\ lb = -52000\ lb$$

$$\sigma_{x_A} = \frac{P}{A_A} = -17333\ lb/in^2 \qquad \sigma_{x_B} = \frac{P}{A_B} = -20800\ lb/in^2$$

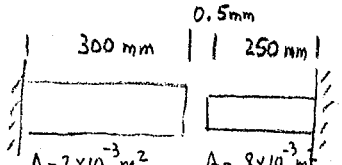
למצוא את התארכות של האלומינום

$$\delta_{Al} = \int_{l_A}^{l_B+l_A} \left(\frac{\sigma_{x_A}}{E_A} + \alpha_A\left[\Delta T + \Delta T^*\right]\right) dx$$

$$\delta_{Al} = \left\{\frac{\sigma_{x_A}}{E_A} + \alpha_A \cdot (\Delta T + \Delta T^*)\right\} l_A = \left\{\frac{-17333}{10\times10^6} + 12.8\times10^{-6}(200)\right\}(15) = 1.24\times10^{-2} in$$

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2.46




לפי הסימה הנ"ל

$$\Delta u = 0.5\times10^{-3} = L_A \alpha_A \Delta T + L_S \alpha_S \Delta T$$

$$= (.3 \cdot 23\times10^{-6} + .25 \cdot 18\times10^{-6}) \Delta T$$

$$0.5\times10^{-3} = 11.4\times10^{-5} \Delta T$$

$$43.86^\circ C = \Delta T$$

אתרי כת $\Delta T^* = (140-20) - 43.86^\circ = 76.14^\circ$

לכן,

$$0 = \Delta u_A + \Delta u_S = \left(\frac{P}{E_A A_A} + \alpha_A \Delta T^*\right) l_A + \left(\frac{P}{E_S A_S} + \alpha_S \Delta T^*\right) l_S$$

$$P\left(\frac{l_A}{E_A A_A} + \frac{l_S}{E_S A_S}\right) = -\left(\alpha_A l_A + \alpha_S l_S\right) \Delta T^*$$

$$P\left[\frac{0.3}{(70\times10^9)(2\times10^{-3})} + \frac{0.250}{(190\times10^9)(0.8\times10^{-3})}\right] = -\left((23\times10^{-6})(0.3) + (18\times10^{-6})(0.25)\right)(76.14^\circ)$$

$$P\left(3.7876\times10^{-9}\right) = -\left(1.14\times10^{-5}\right)(76.14^\circ) = -8.6800\times10^{-4}\ m$$

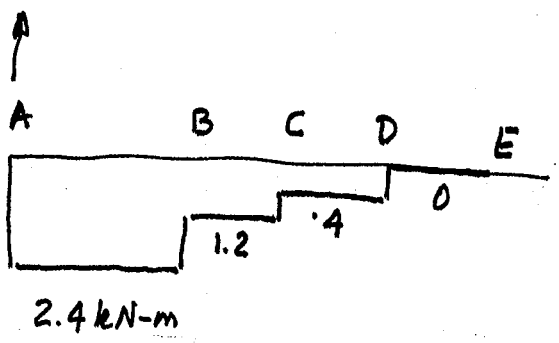
$$P = -2.29169\times10^5\ N = -229169\ N$$

$$\sigma_A = \frac{P}{A_A} = \frac{-229169\ N}{2\times10^{-3} m^2} = -114.58\ MPa$$

$$\delta_{Al} = \int_0^{l_A} \left(\frac{\sigma_{x_A}}{E_A} + \alpha_A (\Delta T + \Delta T^*)\right) dx = \left\{\frac{\sigma_{x_A}}{E_A} + \alpha_A (\Delta T + \Delta T^*)\right\} l_A = \left\{\frac{-1.1458\times10^8}{70\times10^9} + 23\times10^{-6}(120^\circ C)\right.$$

# EMA 3702

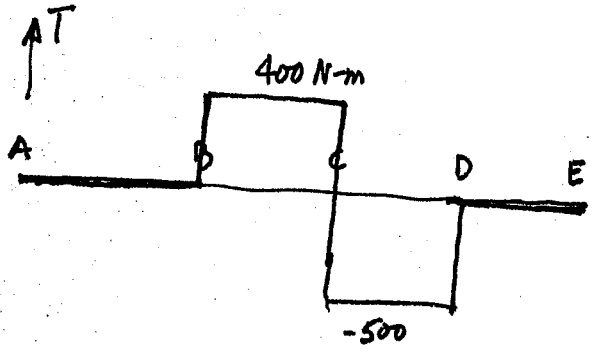
3.13




$$\tau = \frac{Tc}{J} \qquad \frac{2T}{\pi c^3}$$

$$\varphi = \frac{TL}{JG} \qquad \varphi = \int \frac{T\,dx}{JG}$$

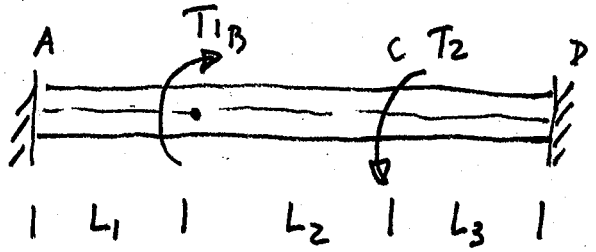
3.36

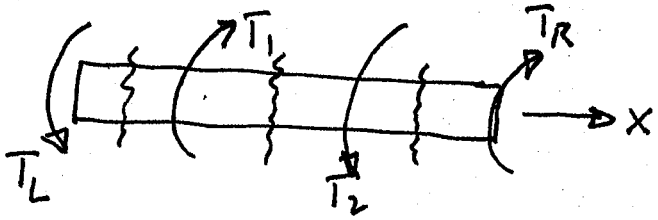



$\varphi_{BC}$?

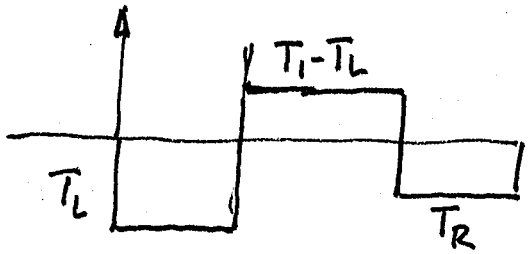
$\varphi_{CD}$?

$\varphi_{AE} = \varphi_{AB} + \varphi_{BC} + \varphi_{CD} + \varphi_{DE}$



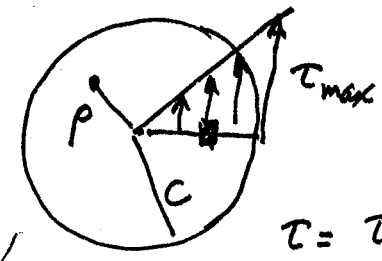

$$\Sigma T = 0 = T_L - T_1 + T_2 - T_R = 0$$

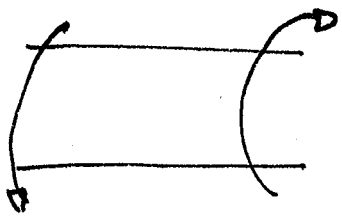



$$\Sigma\varphi = \varphi_{AB} + \varphi_{BC} + \varphi_{CD} = 0$$

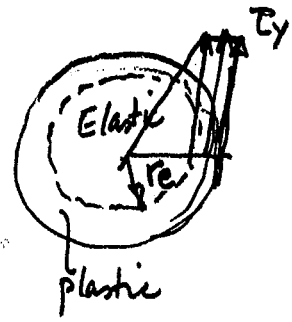
$$= \frac{T_{AB} L_1}{J_{AB} \cdot G} + \frac{T_{BC} L_2}{J_{BC} \cdot G} + \frac{T_{CD} L_3}{J_{CD} \cdot G} = 0 \Rightarrow \quad T_{AB} \cdot L_1 + T_{BC} \cdot L_2 + T_{CD} \cdot L_3 = 0$$

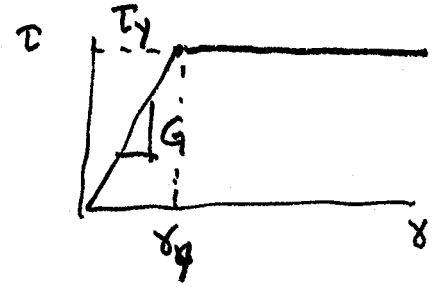
$$-T_L \cdot L_1 + (T_1 - T_L) L_2 - T_R \cdot L_3 = 0$$



$$\tau = \frac{\tau_{max}}{c} \cdot \rho$$




$\tau = G\gamma$

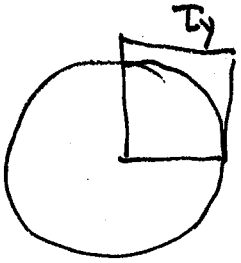
elasto plastic model

$$T = \int \tau \, dA \cdot \rho = \int \frac{\tau_{max}\rho}{c} \cdot \rho d\rho d\theta \cdot \rho = \frac{\tau_{max}}{c} \int_0^c \rho^3 d\rho \cdot \int_0^{2\pi} d\theta$$

$$= \frac{\tau_{max}}{c} \frac{c^4}{4} \cdot 2\pi$$

$$T = \tau_{max} \frac{c^3 \pi}{2}$$

$$T = \int_0^{r_e} \frac{\tau_{max}}{c} \cdot \rho \cdot \rho d\rho d\theta + \int_{r_e}^{c} \tau_y \cdot \rho d\rho d\theta \cdot \rho$$



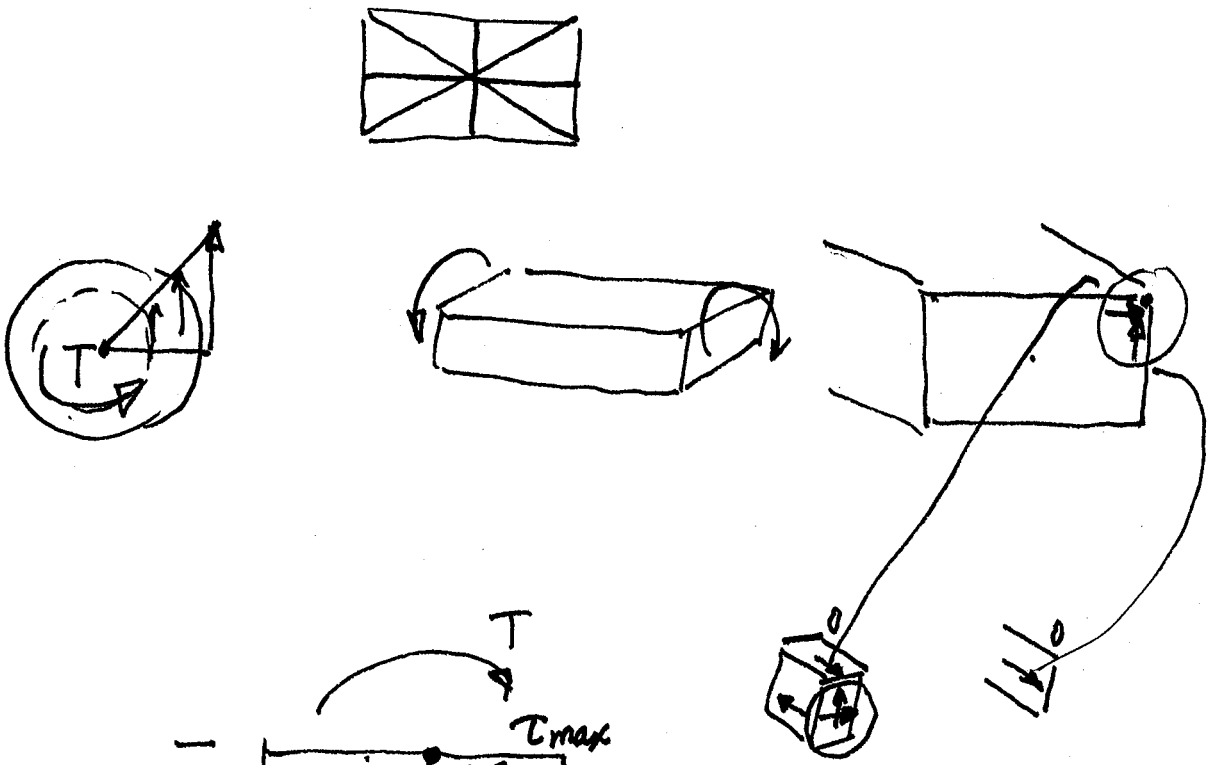

$$T_p = \int_0^c \tau_y \cdot \rho d\rho d\theta \cdot \rho$$

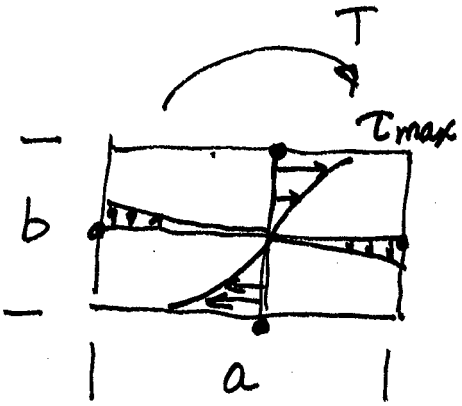
$$= \tau_y \int_0^c \rho^2 d\rho \int_0^{2\pi} d\theta$$

$$= \tau_y \frac{\rho^3}{3} \Big|_0^c \cdot 2\pi = \tau_y \cdot \frac{2\pi c^3}{3}$$

$$T_y \left(\text{when } \tau_{max} = \tau_y\right) = \tau_{max} \frac{c^3 \pi}{2} = \tau_y \cdot \frac{c^3 \pi}{2}$$

$$T_p = \frac{4}{3} T_y$$

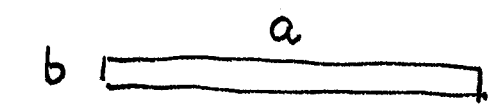


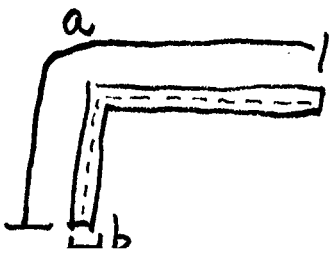

$$\tau_{max} = \frac{T}{c_1 a b^2} \qquad c_1 = f_1(a/b)$$

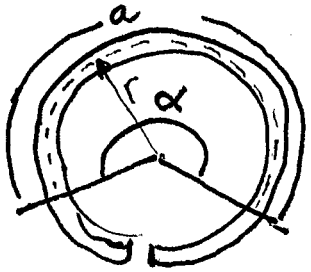
$$\varphi = \frac{TL}{c_2 a b^3 G} \qquad c_2 = f_2(a/b)$$

Table 2.1 pg 187




$c_1 = c_2 \cong 1/3$



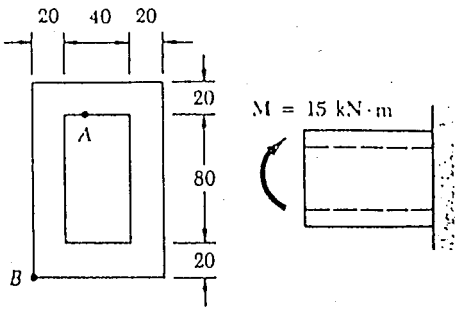




b = thickness

$a = 2\pi r = \alpha r$

## תורת החוזק 1
## תרגיל מס 8

**4.1 and 4.2** Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.




Dimensions in mm
Fig. P4.1

**4.9 and 4.10** Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in the beam.

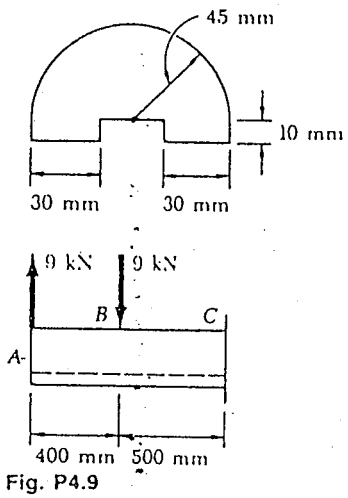
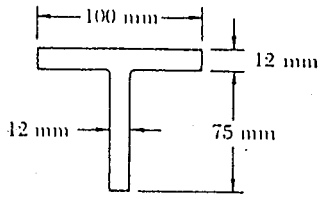



Fig. P4.9



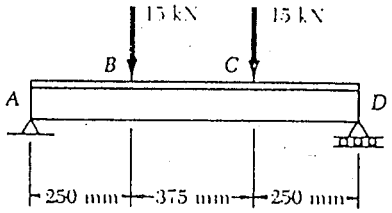




Fig. P4.10

**4.26** A couple **M** will be applied to a beam of rectangular cross section which is to be sawed from a log of circular cross section. Determine the ratio *d*/*b*, for which (*a*) the maximum stress $\sigma_m$ will be as small as possible, (*b*) the radius of curvature of the beam will be maximum.

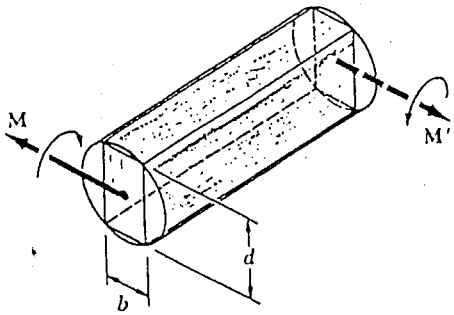
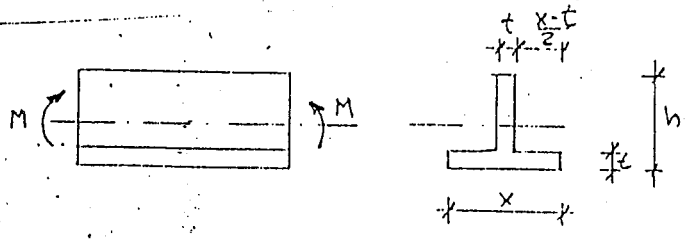



Fig. P4.26

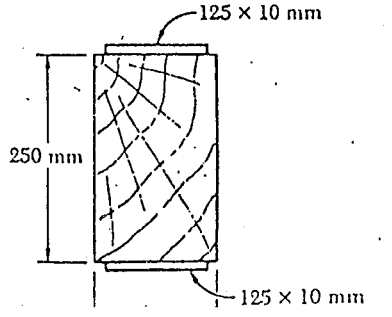
4.

הקורה המתוארת בציור מצויה במצב של כפיפה טהורה. מה צריך להיות עובי האגף x
כך שערכו של מאמץ המתיחה המקסימלי יהיה שליש מערכו של מאמץ הלחיצה המקסימלי?




**4.38** The 150 × 250 mm timber beam has been strengthened by bolting to it two steel strips to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

| | Wood | Steel |
|---|---|---|
| Modulus of elasticity | 13 GPa | 200 GPa |
| Allowable stress | 12 MPa | 165 MPa |



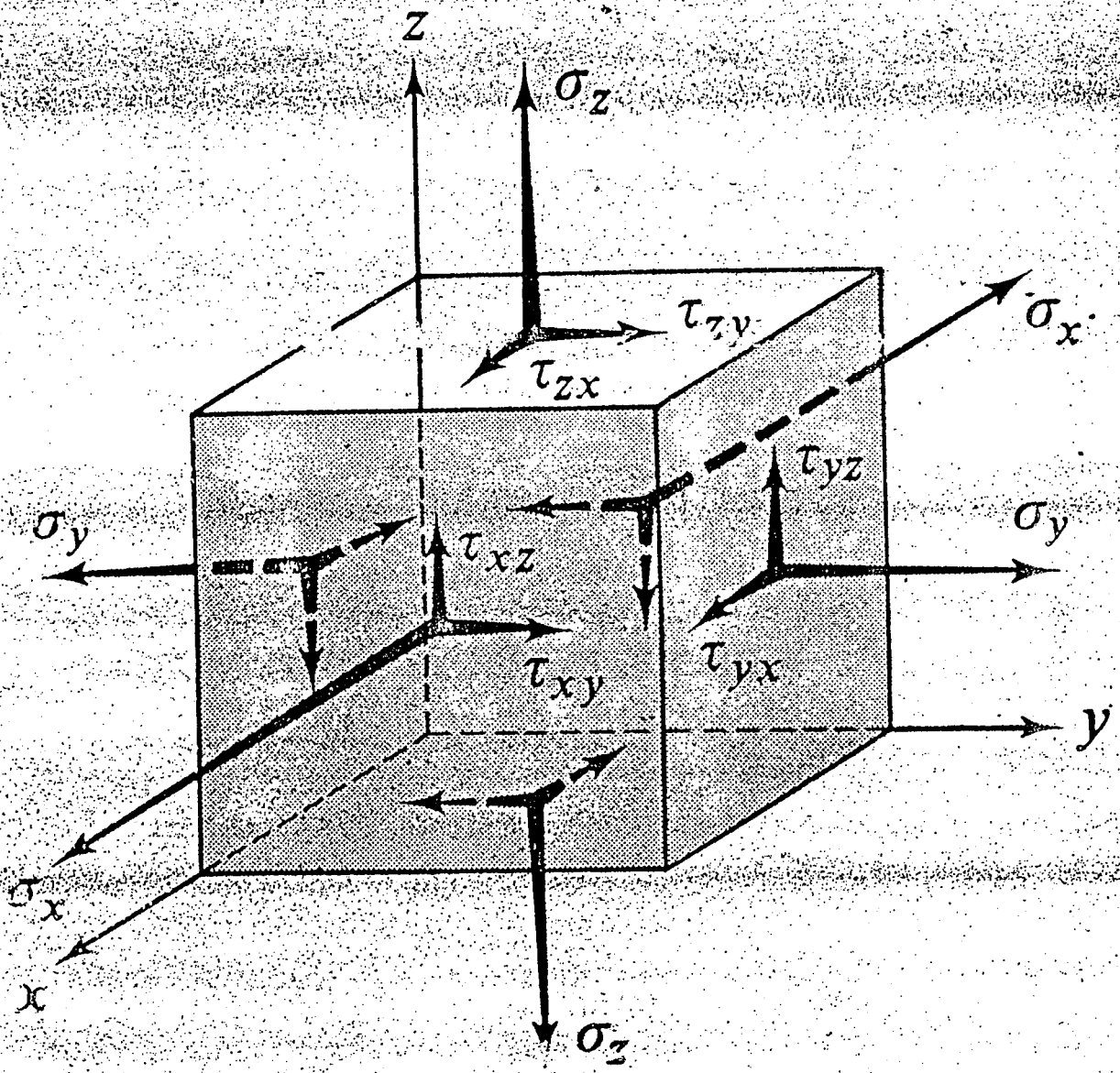



**Fig. 1-3.** The most general state of stress acting on an element

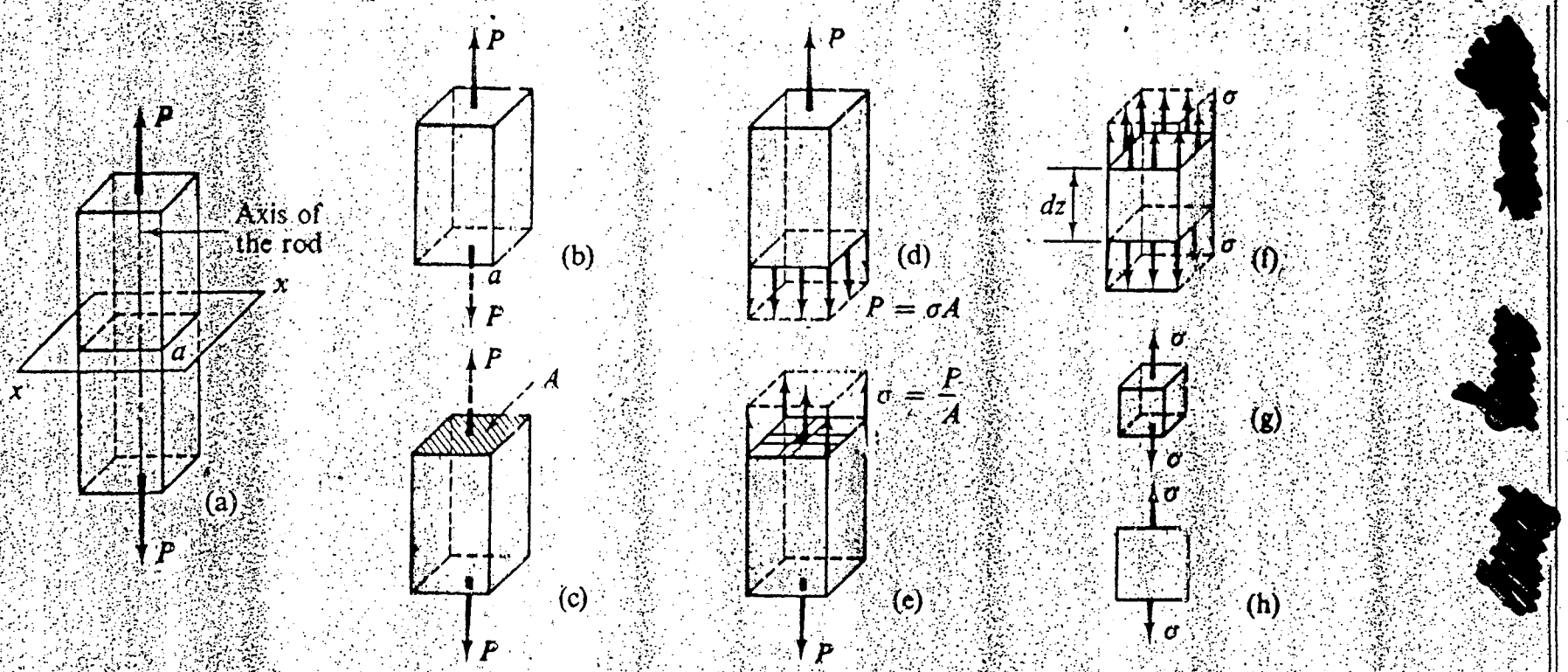



Fig. 4. Successive steps in the analysis of a body for stress

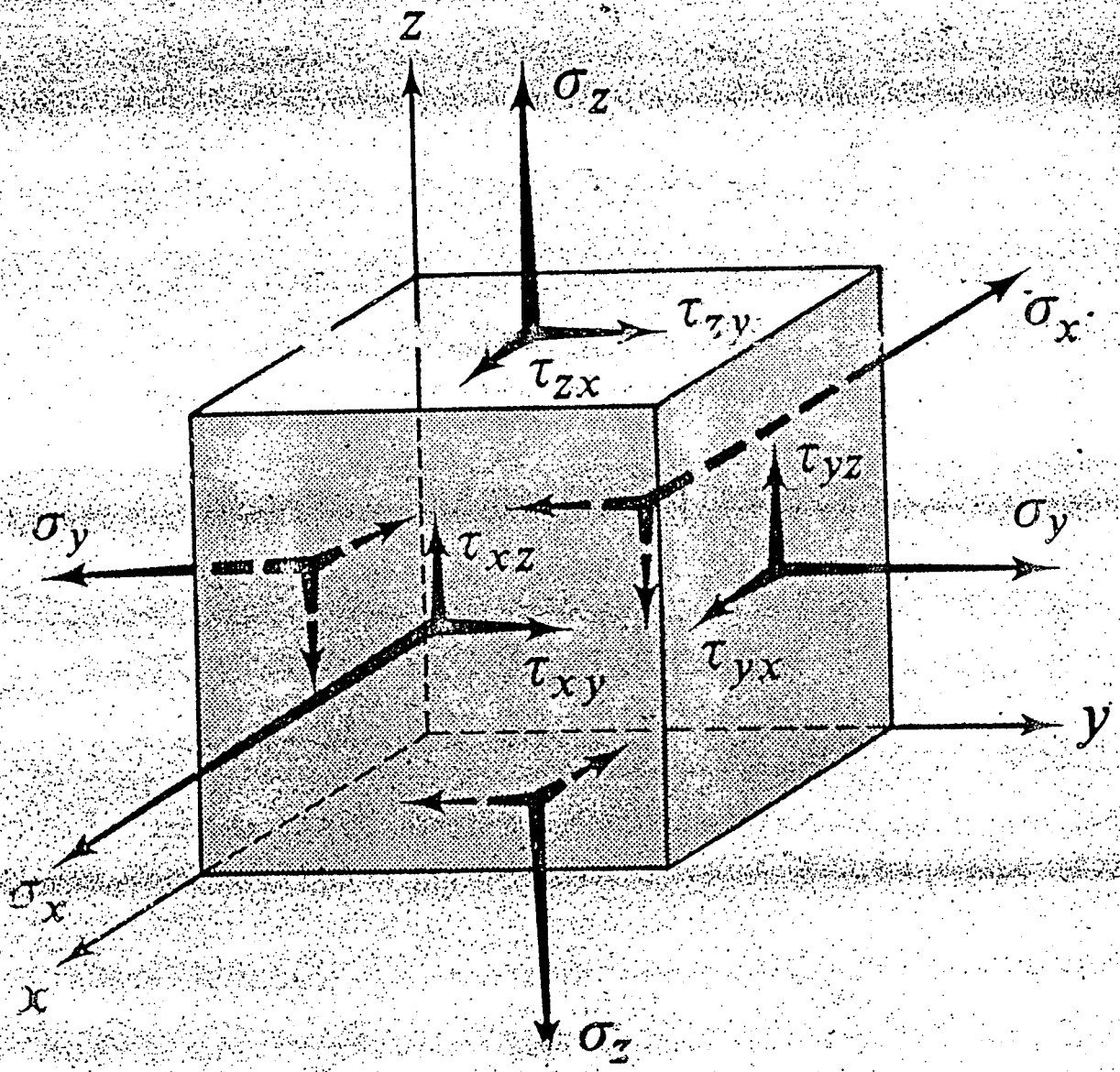



**Fig. 1-3.** The most general state of stress acting on an element

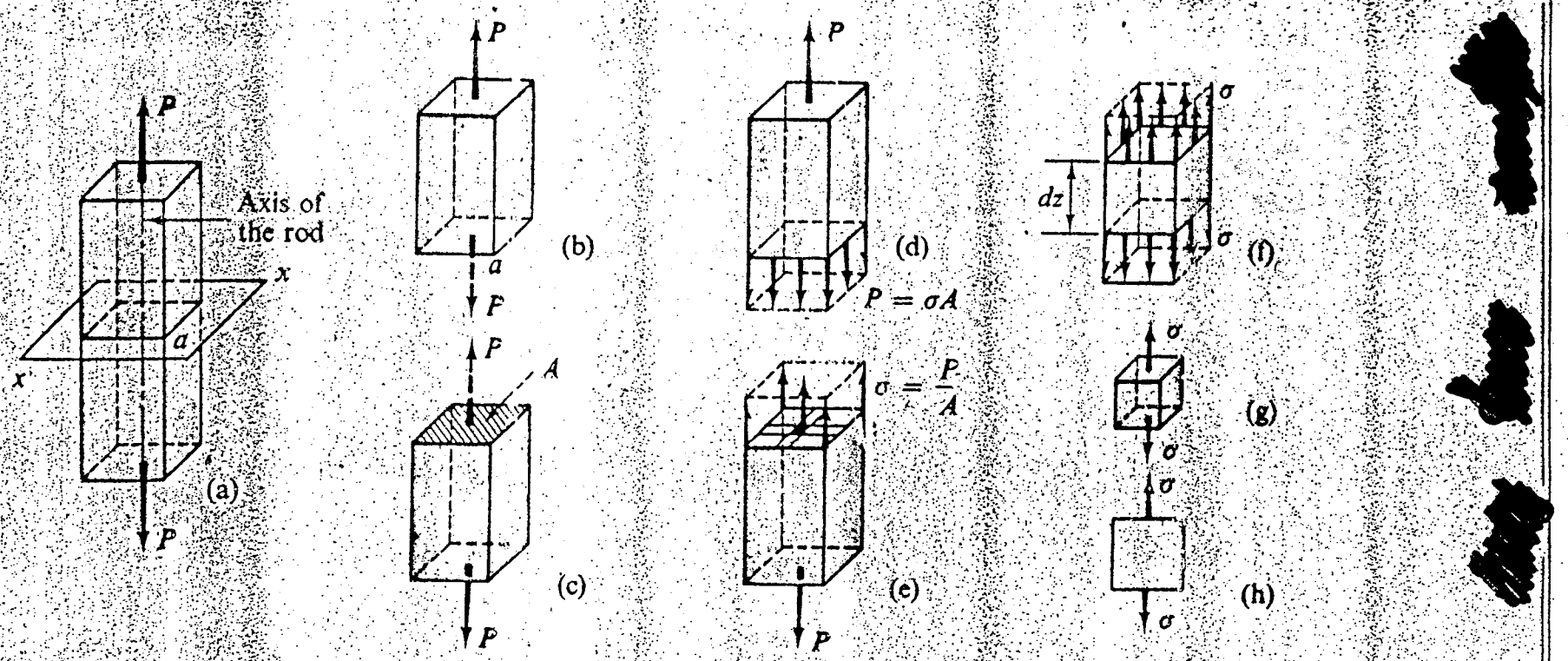
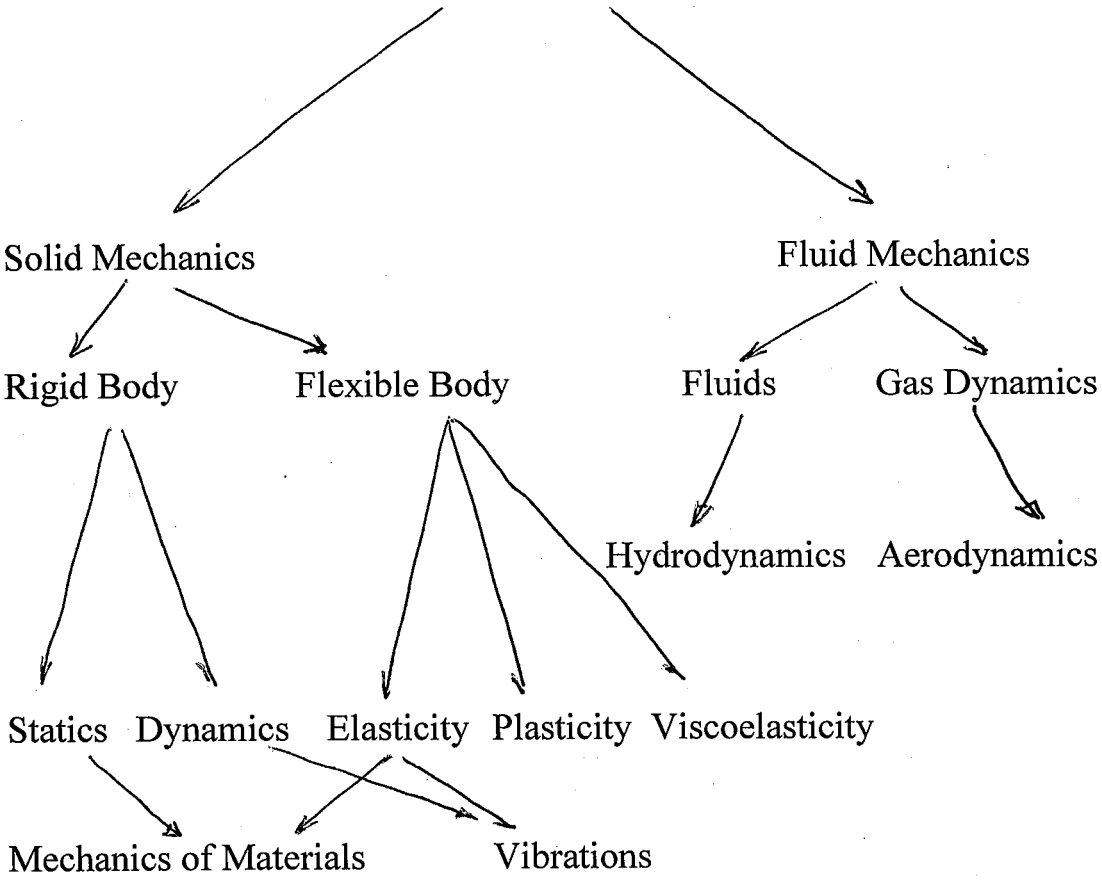



Fig. 1. Successive steps in the analysis of a body for stress

# Mechanics


Solid Mechanics
Fluid Mechanics
Rigid Body
Flexible Body
Fluids
Gas Dynamics
Hydrodynamics
Aerodynamics
Statics
Dynamics
Elasticity
Plasticity
Viscoelasticity
Mechanics of Materials
Vibrations

Introduction to Mechanics and Materials Science

In structural engineering, one has to give dimensions to the particular members, such as beams, floors, walls, shafts and the like, so that they can support the forces that act upon them, both the known ones as well as those that could act upon them.

Therefore, the parts of a composite structure, must be able to withstand the forces, i.e., be strong enough in order not to deflect much when the forces act upon them.

Thin wall members can buckle when they are loaded by small loads before they fail in bending.

One must also design bars and beams without waste of materials.

Our subject deals with strength, stiffness and stability of members of our structure that carry the loads.

Galileo, in the $17^{th}$ Century, first investigated the behavior of materials that were loaded in a rational manner, bars, beams in both tension and compression.

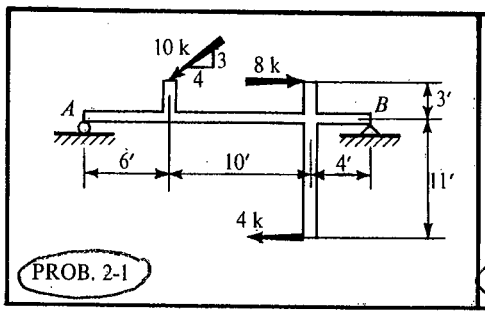
From that time until now, there were many investigators such as Coulomb, Poisson, Navier, St. Venant and Cauchy.

Mechanics and Materials Science crosses all realms-- from Civil Engineering, to Aeronautical Engineering, mechanical Engineering, Aerospace Engineering and any field that requires building structures of any kind.

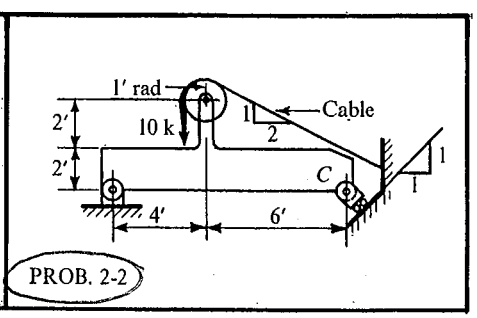
The behavior of bars and beams depend not only on the basic laws of Newton that require equilibrium, but also upon the mechanical characteristics of materials that make up the bars and beams.

Materials behavior are obtained from many rational experiments done in laboratories. The knowledge has given us the information to choose the material that is proper to design our structures.

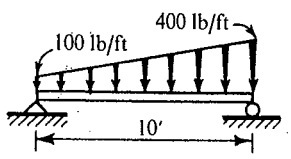
We will begin with a review of statics and after that we will begin to learn our subject matter.



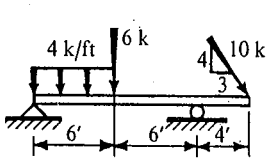

PROB. 2-1



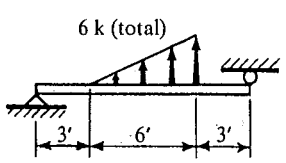

PROB. 2-2



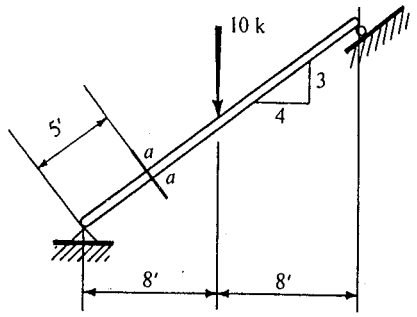

PROB. 2-3



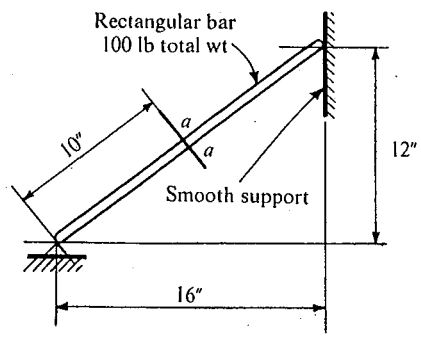

PROB. 2-4



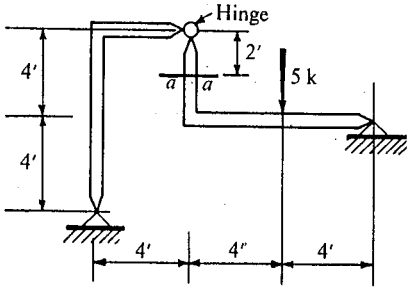

PROB. 2-5



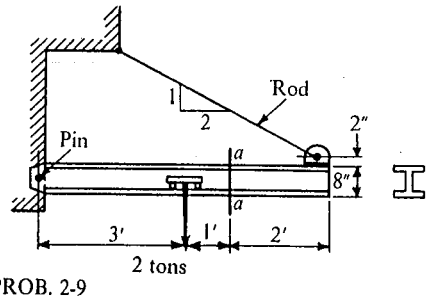

PROB. 2-6



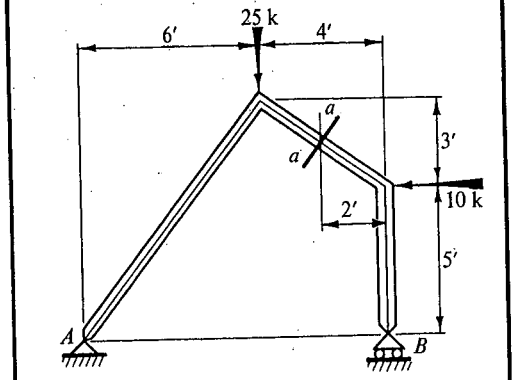

PROB. 2-7



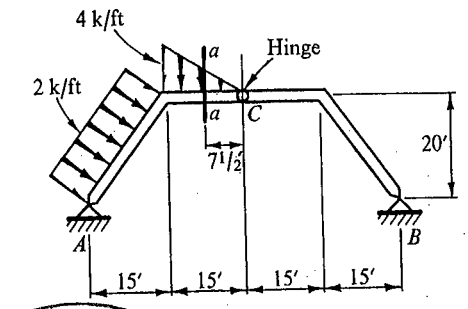

PROB. 2-8



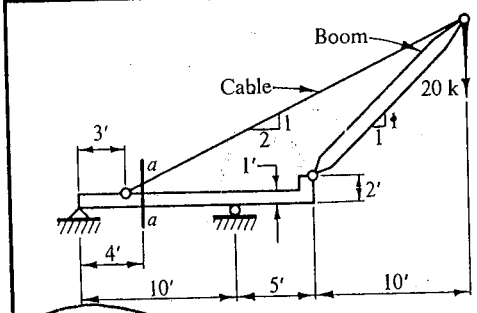

PROB. 2-9



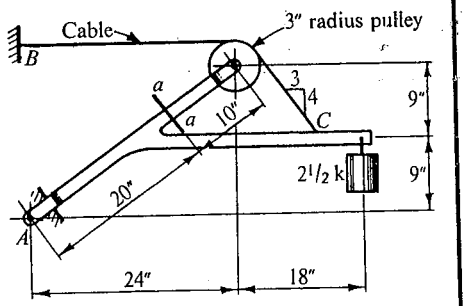

PROB. 2-10




PROB. 2-11



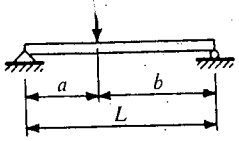

PROB. 2-12



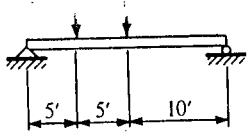

PROB. 2-13

**2-14 through 2-19.** For the beams loaded as shown in the figures write general expressions for the shear and bending moments for each region over the length of the member. Also plot the corresponding shear and moment diagrams. *Ans.* Maximum moment in parentheses by the figure.

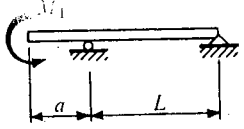
For additional loading conditions see other problems in this chapter.



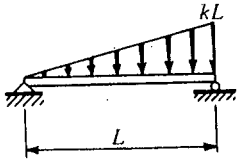

PROB. 2-14 ($Pab/L$)



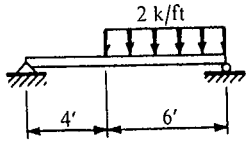

PROB. 2-15 (250)



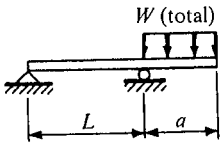

PROB. 2-16 ($-M_1$)




PROB. 2-17 ($kL^3/9\sqrt{3}$)




PROB. 2-18 (17.6)




PROB. 2-19 ($-Wa/2$)

Show positive directions assumed for and $M$ on a free-body diagram.

If the positive senses of $p$, $V$, and $M$ are ed as shown in the figure, derive the ons equivalent to Eqs. 2-4, 2-5, and 2-6. Fig. 11-2(b).)

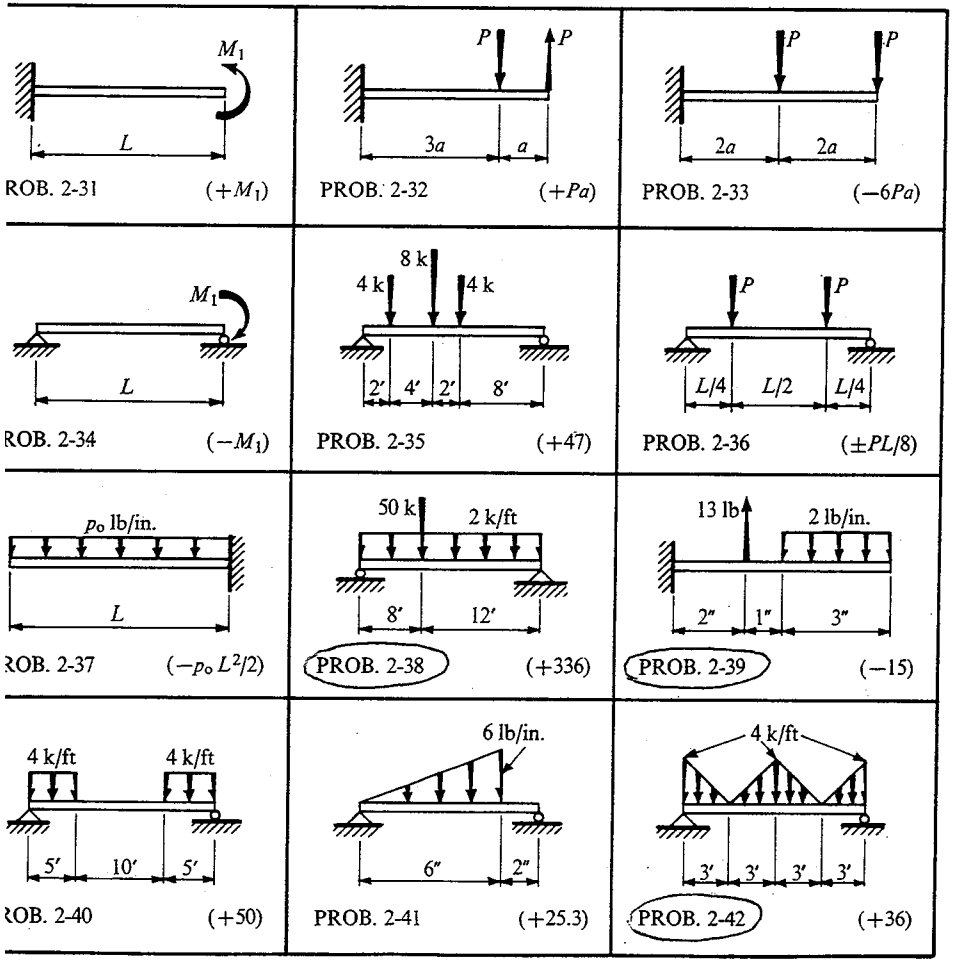
**through 2-53.** For the beams loaded in lane as shown in the figures, neglecting eight of the members, solve as directed:
**A.** Without formal computations, sketch and moment diagrams directly below a am of the given loaded member.
**B.** Same as **A**, and, in addition, show iape of the elastic curve.
**C.** Plot shear, moment, and, wherever cant, axial force diagrams for the main horizontal members. Determine all critical ordinates.

**D.** Same as **C**, and, in addition, determine the points of inflection and show the shape of the elastic curve.

*Ans.* All shear and moment diagrams must close. The largest moment is given in parentheses by the figures in the units of the problem.

For additional loading conditions see other problems in this chapter.

**2-54 through 2-56.** The moment diagrams for beams supported at $A$ and $B$ are as shown in the figures. How are these beams loaded? All curved lines represent parabolas, i.e., plots of equations of the second degree. (*Hint:* construction of shear diagrams aids the



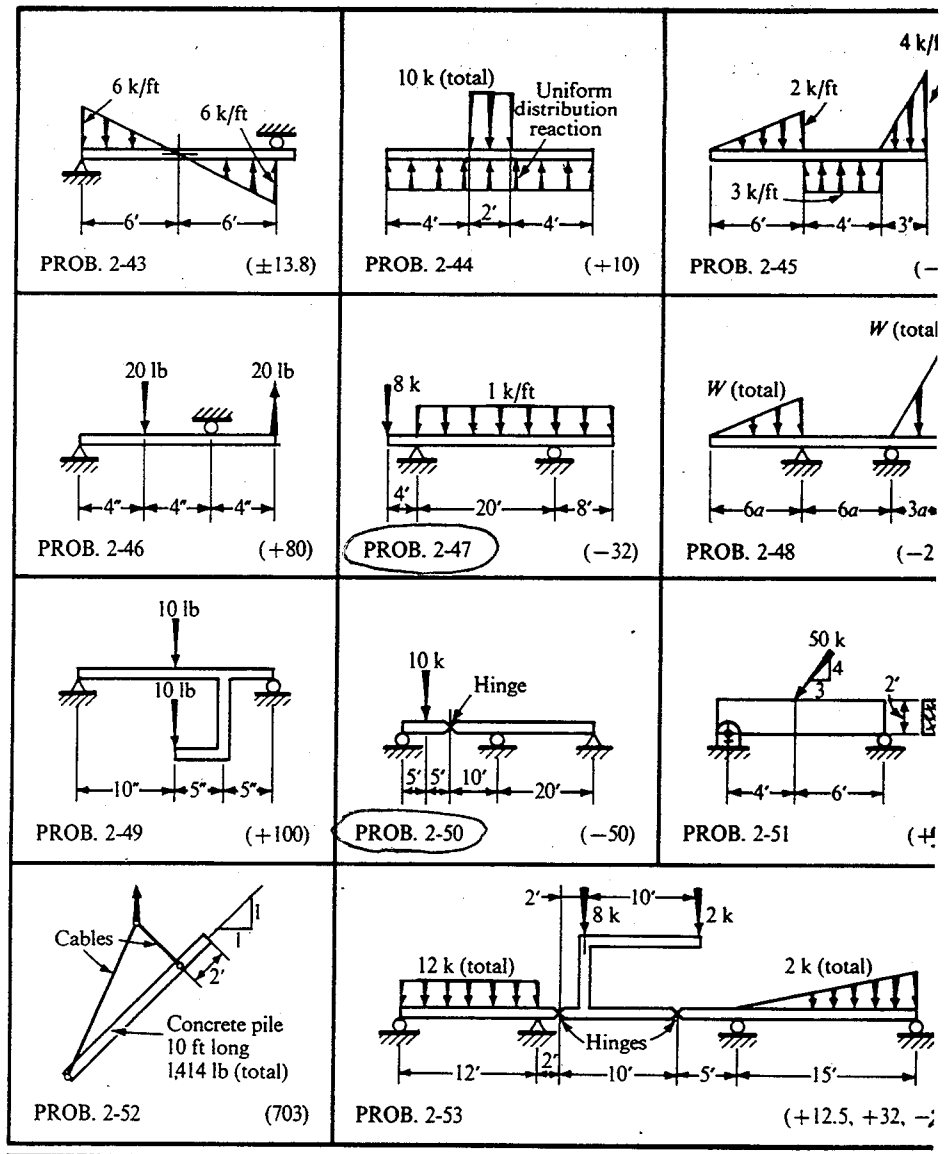

ROB. 2-31 (+$M_1$) PROB. 2-32 (+$Pa$) PROB. 2-33 (−$6Pa$)

ROB. 2-34 (−$M_1$) PROB. 2-35 (+47) PROB. 2-36 (±$PL/8$)

ROB. 2-37 (−$p_o L^2/2$) PROB. 2-38 (+336) PROB. 2-39 (−15)

ROB. 2-40 (+50) PROB. 2-41 (+25.3) PROB. 2-42 (+36)



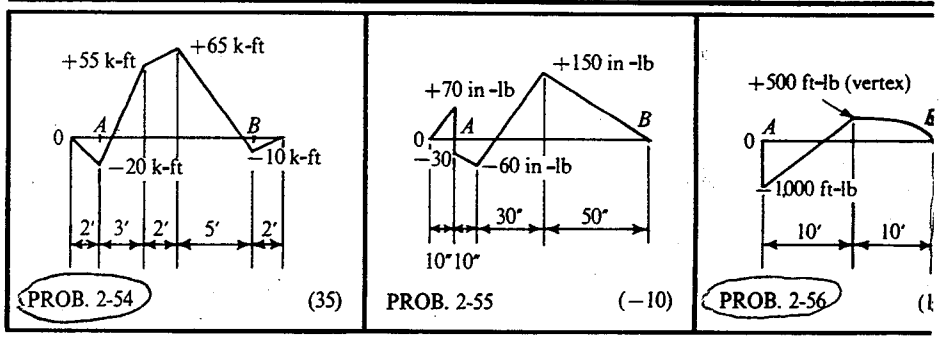



PROB. 2-43 (±13.8) PROB. 2-44 (+10) PROB. 2-45 (−

PROB. 2-46 (+80) PROB. 2-47 (−32) PROB. 2-48 (−2

PROB. 2-49 (+100) PROB. 2-50 (−50) PROB. 2-51 (+

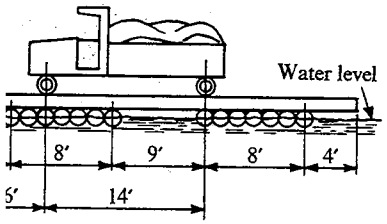
PROB. 2-52 (703) PROB. 2-53 (+12.5, +32, −




PROB. 2-54 (35) PROB. 2-55 (−10) PROB. 2-56 (

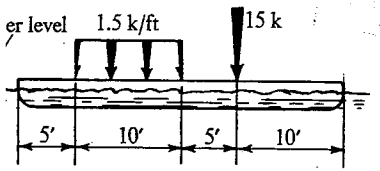
n.) *Ans.* Reaction at $A$ in parentheses
figure.

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or the truck in the position shown.
e the critical values using foot-pound
*Ans.* +18,000 ft-lb (maximum).




PROB. 2-57

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in the figure. Plot shear and moment
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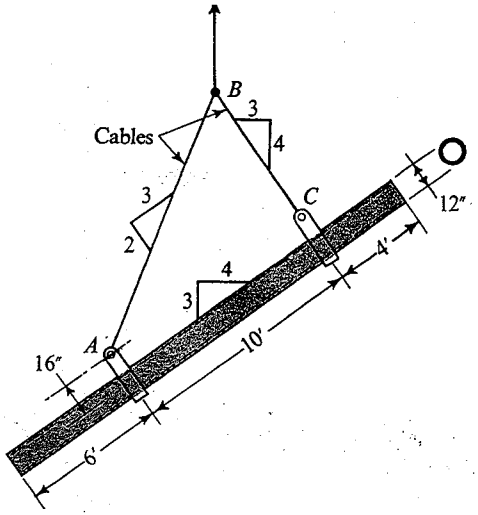



PROB. 2-58

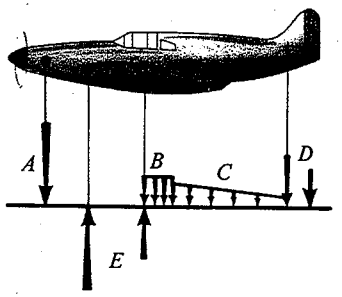
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PROB. 2-59

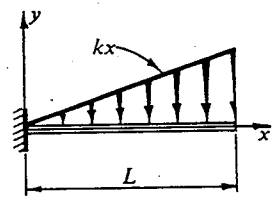



PROB. 2-60

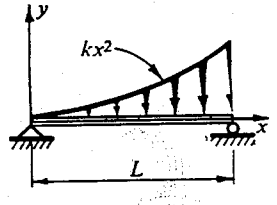
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**2-61 through 2-63.** For the beams loaded as shown in the figures, using Eq. 2-6, (a) find $V(x)$ and $M(x)$. Check reactions by conventional statics. (b) Plot the shear and moment diagrams.

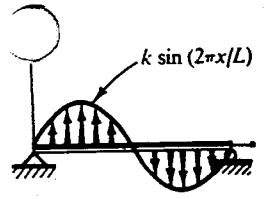
**2-64 through 2-72.** For the beams loaded as shown in the figures, using singularity functions and Eq. 2-6, (a) find $V(x)$ and $M(x)$. Check reactions by conventional statics. (b) Plot the shear and moment diagrams.



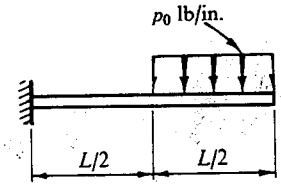

PROB. 2-61



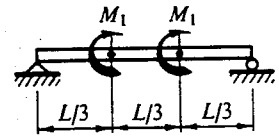

PROB. 2-62



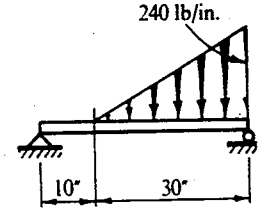

PROB. 2-63



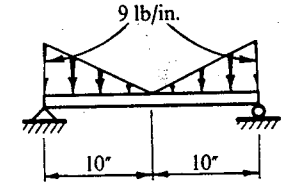

PROB. 2-64



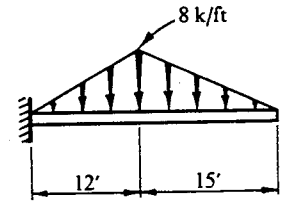

PROB. 2-65



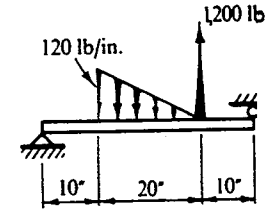

PROB. 2-66



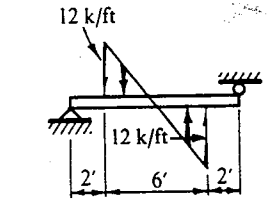

PROB. 2-67



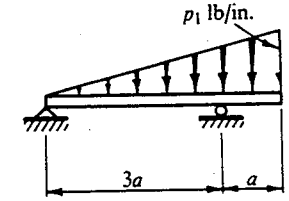

PROB. 2-68



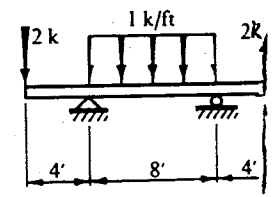

PROB. 2-69




PROB. 2-70



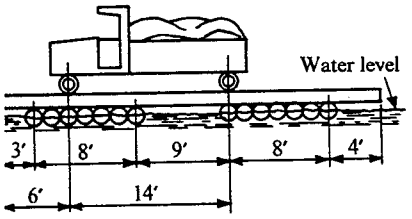

PROB. 2-71




PROB. 2-72

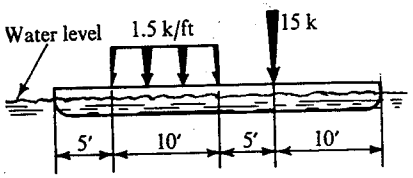
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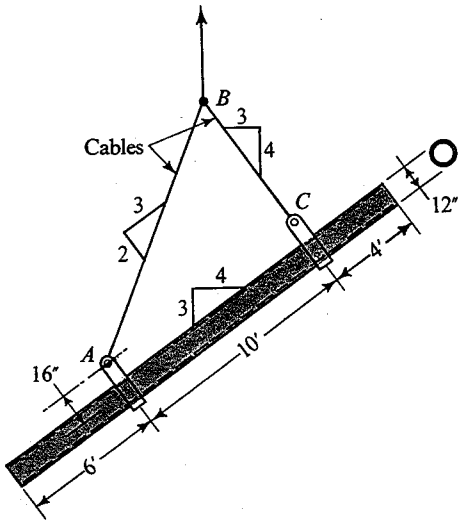
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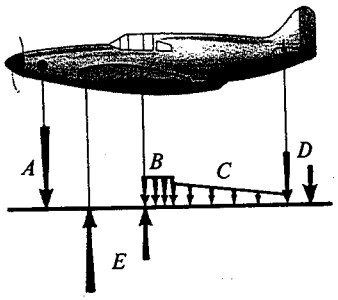

PROB. 2-58

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PROB. 2-59

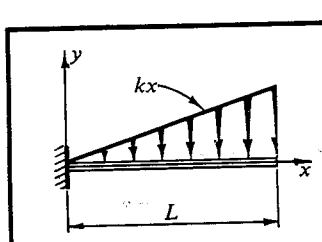



PROB. 2-60

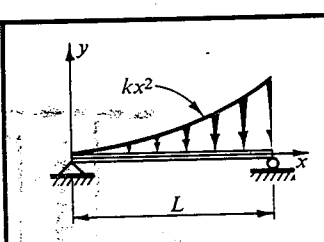
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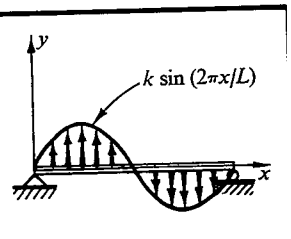
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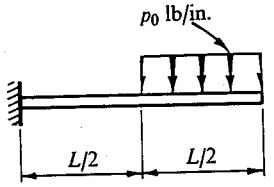

PROB. 2-61



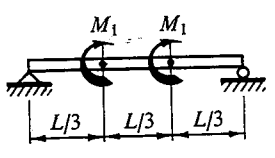

PROB. 2-62



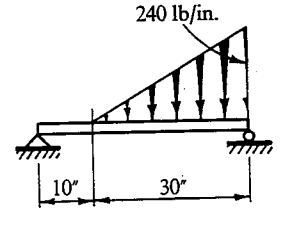

PROB. 2-63



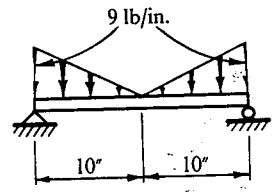

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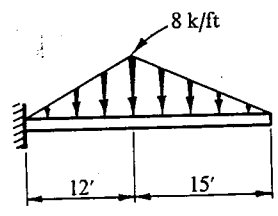

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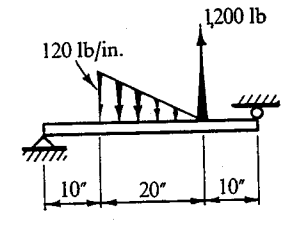

PROB. 2-66



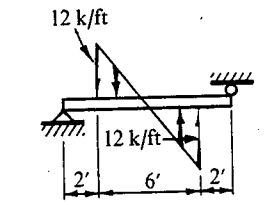

PROB. 2-67



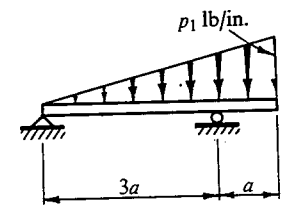

PROB. 2-68



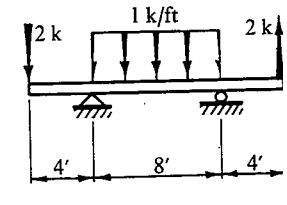

PROB. 2-69




PROB. 2-70




PROB. 2-71




PROB. 2-72

References

EGOR P. POPOV - Introduction to Mechanics of Solids

E.P. POPOV - Mechanics of Materials

BYARS & SNYDER - Engineering Mechanics of Deformable Bodies

TIMOSHENKO & YOUNG - Elements of Strength of Materials

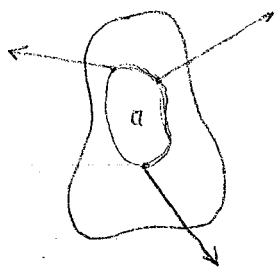
SINGER - Strength of Materials

GINTER - Engineering Mechanics

BEER & JOHNSTON - AE 100 Book

SEELY & SMITH - Advanced Mechanics of Materials.

H.W. Read Pages 1-53 Prob 21, 2, 3, 5, 7, 9, 11




BASIC CONCEPTS - STATICS

Full Knowledge (Practical) must be had in:

a) Resolution & Vectorial Manipulation of Forces

b) Equilibrium Problems, equilibrium conditions

c) Free-Body diagram.

Look at structure through strength, stiffness, stability

Strength - ability to withstand load without breaking

stiffness - the ability to withstand change in dimensions (elasticity)

stability - depends on structure & dimensions of body

We will use Method of Sections - If body in equilibrium, any portion of body in equilibrium.

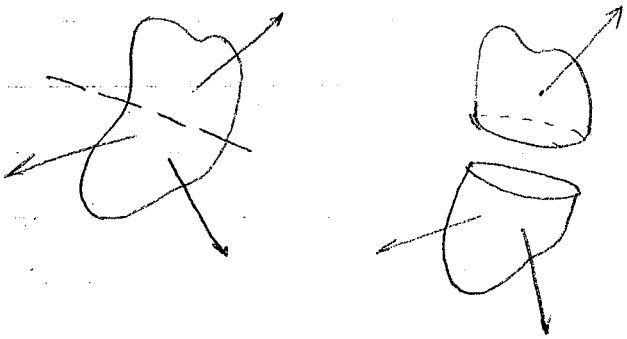
## Method of Sections

Approach No.

1. Free-Body diagram showing all forces - body must be in

equilibrium.

2. Pass Arbitrary plane through body separating two pieces.



MAIN Problem : Approach

1) Isolate Member to be analyzed or designed.

2) Find reactions by

a) Statics $\Sigma F_x = \Sigma F_y = \Sigma F_z = \Sigma M_x = \Sigma M_y = \Sigma M_z = 0$

b) Differential Equations & Boundary Conditions.

c) If Indeterminate, By Kinematic Considerations – displacements are defined, restraints put on member.

3) Apply Method of Sections

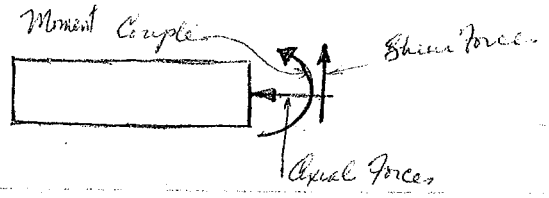
4) At Section find internal forces (INTERNAL REACTIONS)

5) Use appropriate formulas to find stresses

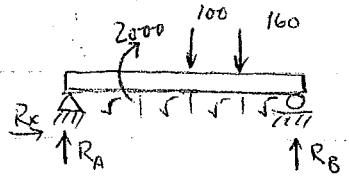
6) In Design, select appropriate material & member.

7) Predict deformations of the body

Internal Forces



(1)




$R_x = 0$

$\sum F_y = R_A + R_B - 260 = 0$

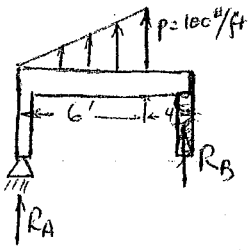
$R_A + R_B = 260$

$\sum M_B \circlearrowleft = 0$ $\quad R_A \times 20 + 2000 - 100 \times 10 - 160 \times 5 = 0$

$R_A \times 10 + 100 - 50 - 40 = 0$ $\quad R_A = -10^{\#} = 10^{\#} \downarrow$

$R_B = 270^{\#} \uparrow$

(2)




$\sum F_y = 0$

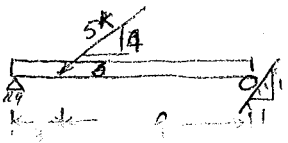
$\sum F_y = R_A + R_B + \frac{1}{2} \times 6 \times 100 = 0$

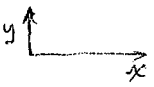
$R_A + R_B = -300$

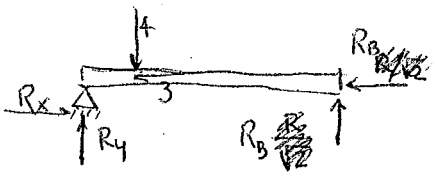
$\sum M_B \circlearrowleft = 0$ $\quad R_A \times 10 + 300 \times 6 = 0$; $R_A = -180^{\#}$ $\quad 180^{\#} \downarrow$

$R_B = 120^{\#} \downarrow$

(3)




$R_x - 3^K - R_B = 0$

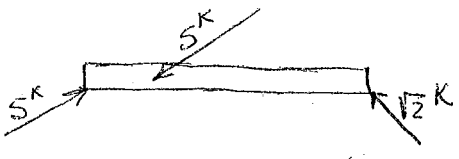
$\boxed{R_x - R_B = 3^K}$

$R_y - 4 + R_B = 0$

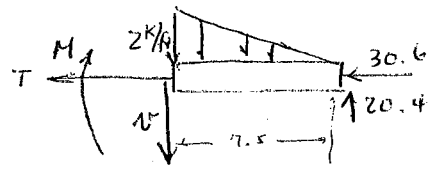
$\boxed{R_y + R_B = 4^K}$

$\sum M_B \circlearrowleft = 0$ $\quad R_y(12) - 4(9) = 0$ $\quad R_y = 3^K$ $\quad R_B = 1^K$

$R_x = 4^K$



Since we know everything about right member we can isolate the smaller member between pin & cut therefore



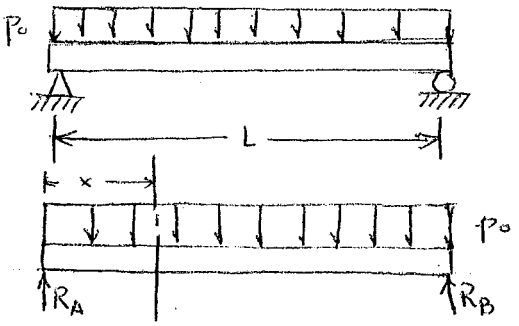

$\Sigma F_x = 0 \quad -T - 30.6 = 0 \quad T = -30.6\ K$

$\Sigma F_y = 0 \quad -V + 20.4 - \frac{1}{2}(2)(7.5) = 0 \quad V = 12.9$

$\Sigma M_{at\,cut} = 0 \quad M + \frac{1}{2}(2)(7.5)(2.5) - 20.4(7.5) = 0$

$M = (17.9)(7.5) \cong 135\ K\text{-}ft.$

Consider

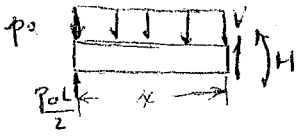



$\Sigma F_y = R_A - p_o L + R_B = 0$

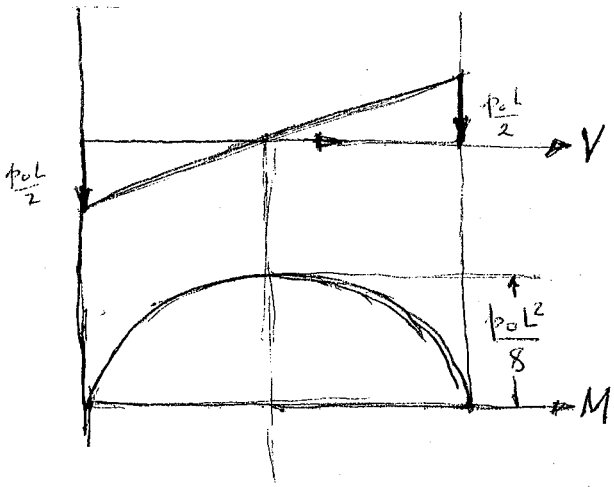
$\boxed{R_A + R_B = p_o L}$

$\Sigma M_B = R_A L - \frac{p_o L \cdot L}{2} = 0$

$\boxed{R_B = \frac{p_o L}{2}}$




$\Sigma F_y = 0 \quad \frac{p_o L}{2} - p_o x + V = 0 \quad V = p_o\left(x - L/2\right)$

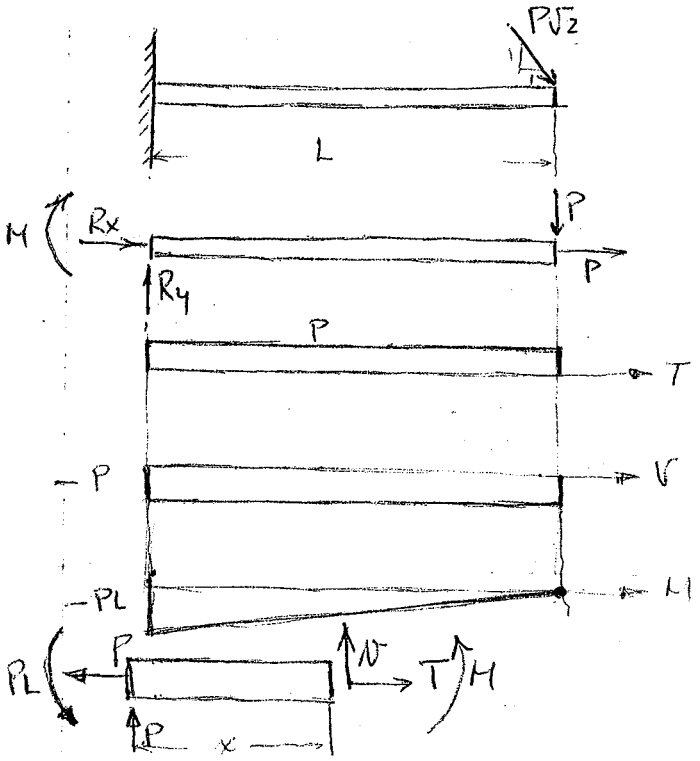



$\Sigma M_c = 0 \quad \frac{p_o L}{2} x - p_o \frac{x^2}{2} - M = 0$

$M = \frac{p_o x}{2}(L - x)$

$\frac{dM}{dx} = \frac{p_o L}{2} - p_o x = p_o\left(\frac{1}{2} - x\right) =$

$x = L/2$




$\Sigma F_x = 0$

$R_x + P = 0$ $\boxed{R_x = -P}$

$\Sigma F_y = 0$

$R_y - P = 0$ $\boxed{R_y = P}$

$\Sigma M_A = 0$ $\quad M + P(L) = 0$

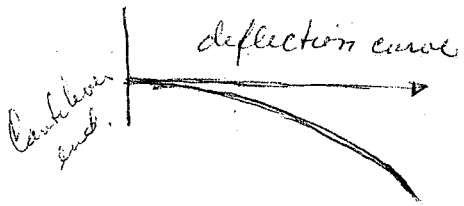
$\boxed{M = -PL}$

$T = P$

$V = -P$

$\Sigma M_c = 0$ $\quad -PL + Px - M = 0$

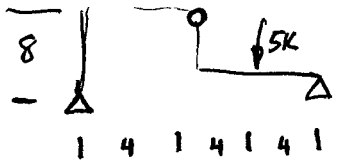
$M = P(x - L)$

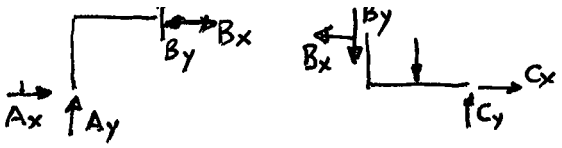



Rd Page 30-31 Problem 2-8 2-15, 17, 19, 21, 25

Steps to solve

1) Draw Sketch of Member - which shows all forces & dimensions
2) Unknown reactions are indicated
3) Resolve inclined forces into components
4) Find reactions
5) Cut arbitrary sections (depending on range that is good for that section)
6) Isolate segment to either side of cut. Show forces acting
7) Indicate internal reactions.
8) Find internal unknown reactions




$\Sigma F_y = A_y + B_y = 0$

$\Sigma F_x = A_x + B_x = 0$

$+\circlearrowleft \Sigma M_A = -B_x \cdot 8 + B_y \cdot 4 = 0$

$B_x = +\frac{1}{2} B_y$ (*)

$\Sigma F_x = 0 = B_x + C_x = 0$

$\Sigma F_y = 0 = C_y + B_y - 5000 = 0$

$+\circlearrowleft \Sigma M_C = 0 = +B_x \cdot 4 + B_y 8 + 5000(4) = 0$

$+B_x + 2B_y + 5000 = 0$ (**)

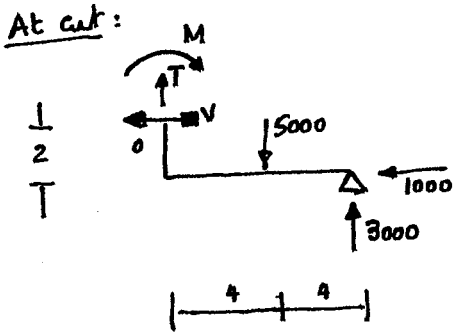
Put (*) into (**)

$+B_y/2 + 2B_y + 5000 = 0$ $\quad B_y = -2000$ lb

$B_x = -1000 = +C_x = -A_x$ $\quad A_y = -B_y$

$C_y = 3000$

At cut:




$\Sigma F_x = 0 = -V - 1000$

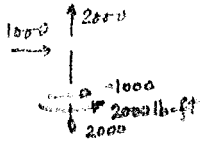
$V = -1000$ lb

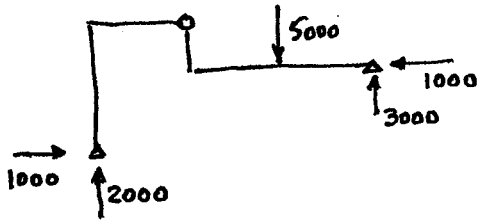
$\Sigma F_y = 0 = T - 5000 + 3000$

$T = 2000$ lb

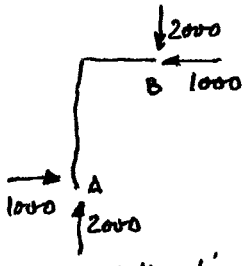
$+\circlearrowleft \Sigma M_o = -M - 5000(4) + 1000(2) + 3000(8) = 0$

$M = +12000$ lb-ft

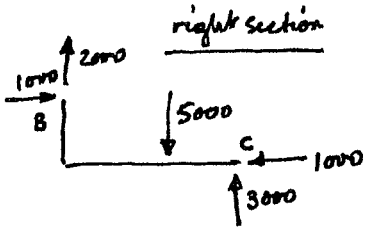


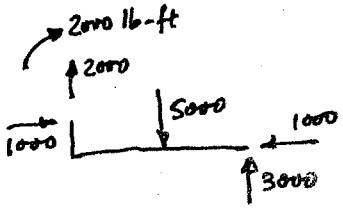




Loading on the system

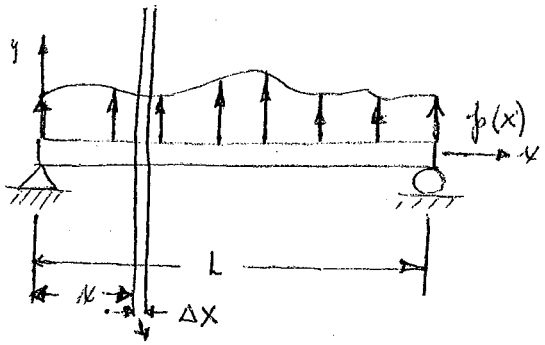


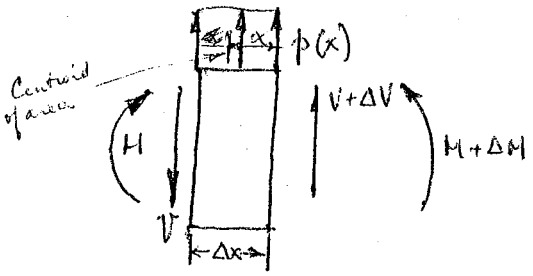

left section




at cut




$$\Sigma F_y = 0$$

$$-V + p(x)\Delta x + V + \Delta V = 0$$

$$\frac{\Delta V}{\Delta x} = -p(x)$$

$$\Sigma M_R = 0 \qquad H + p(x)\Delta x \cdot \frac{\Delta x}{2} - V\Delta x - H - \Delta M = 0$$

$$\frac{\Delta M}{\Delta x} = -V + p(x)\frac{\Delta x}{2}\Delta x$$

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = \boxed{\frac{dV}{dx} = -p(x)} \qquad \lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = \boxed{\frac{dM}{dx} = -V}$$

$$\boxed{-\frac{dV}{dx} = p = \frac{d^2M}{dx^2}}$$

$$dV = -p\,dx$$

$$\boxed{V = -\int p(x)\,dx + C_1} \quad (1)$$

$$\boxed{M = -\int V(x)\,dx + C_2} \quad (2)$$

$$V_b - V_a = -\int_a^b p(x)\,dx$$

$$M_b - M_a = -\int_a^b V(x)\,dx$$

$$V = -\frac{2w}{L^2}x^2 + C_1$$

$$M = +\frac{w}{L^2}\frac{x^3}{3} - C_1 x + C_2$$

at $x=0$ $M=0$ $\therefore \boxed{C_2 = 0}$

at $x=L$ $M=0$

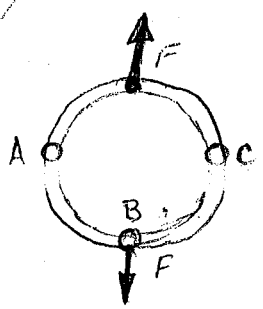
$$M = \frac{wL}{3} - C_1 L = 0 \qquad \boxed{C_1 = \frac{w}{3}}$$

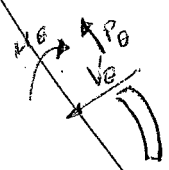
$$\boxed{V = -\frac{w}{L^2}x^2 + \frac{w}{3}} \qquad M = \frac{w}{3}\left[\frac{x^3}{L^2} - x\right]$$

at $V=0$ $x = L/\sqrt{3}$ $M_{max} =$

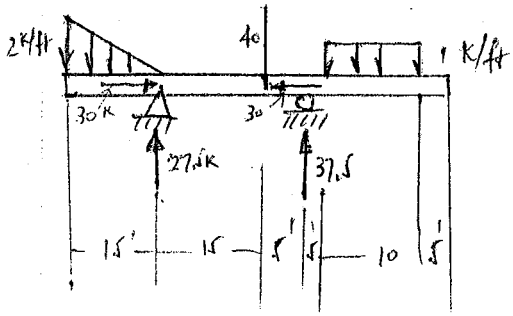
problem: 2-38, 42, 47, 50, 54, 56, 61, 63 Study example 2-10
P. 37-39 Rd. Chapter 3.

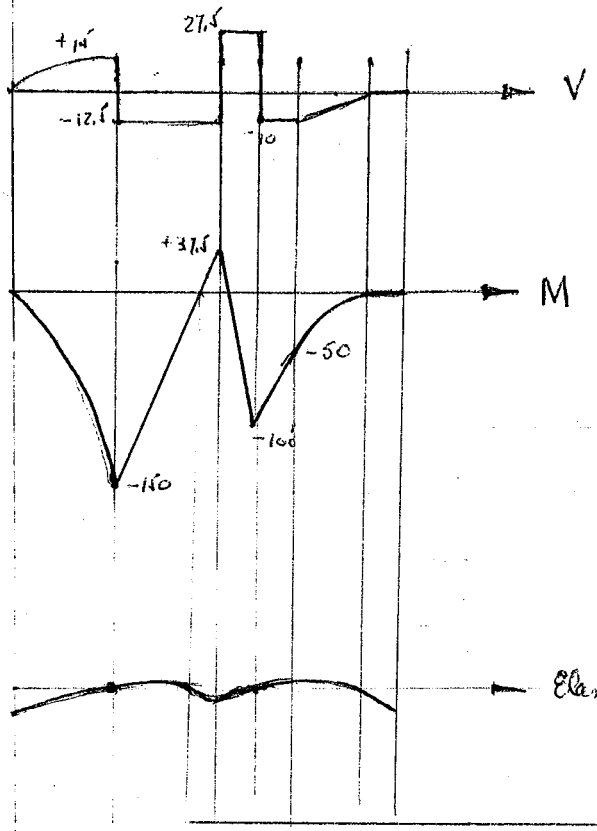
2-29

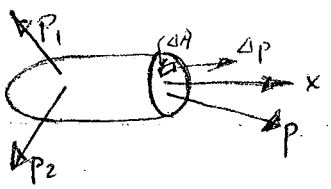


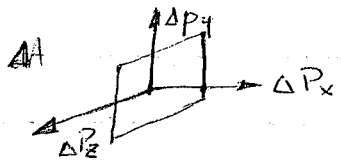




P. 38-9 Example 2-10






$\Delta P_x$ - axial (norm
$\Delta P_y$ & $\Delta P_z$ - shear

Stress - [illegible]

$K/in^2 =$ Ksi

$$\tau_{xx} = \lim_{\Delta A \to 0} \frac{\Delta P_x}{\Delta A} \rightarrow \text{compression tension}$$

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta P_y}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta P_y}{\Delta A}$$

$\tau_{a,b}$ a - tells the normal to the plane lies in the a direct
b - direction of force

when $a = b$ we have an axial type force
$a \neq b$ we have shear-type forces

$\tau_{xx} \equiv \sigma_x$ , $\tau_{yy} \equiv \sigma_y$ , $\tau_{zz} \equiv \sigma_z$

Force = Stress by area

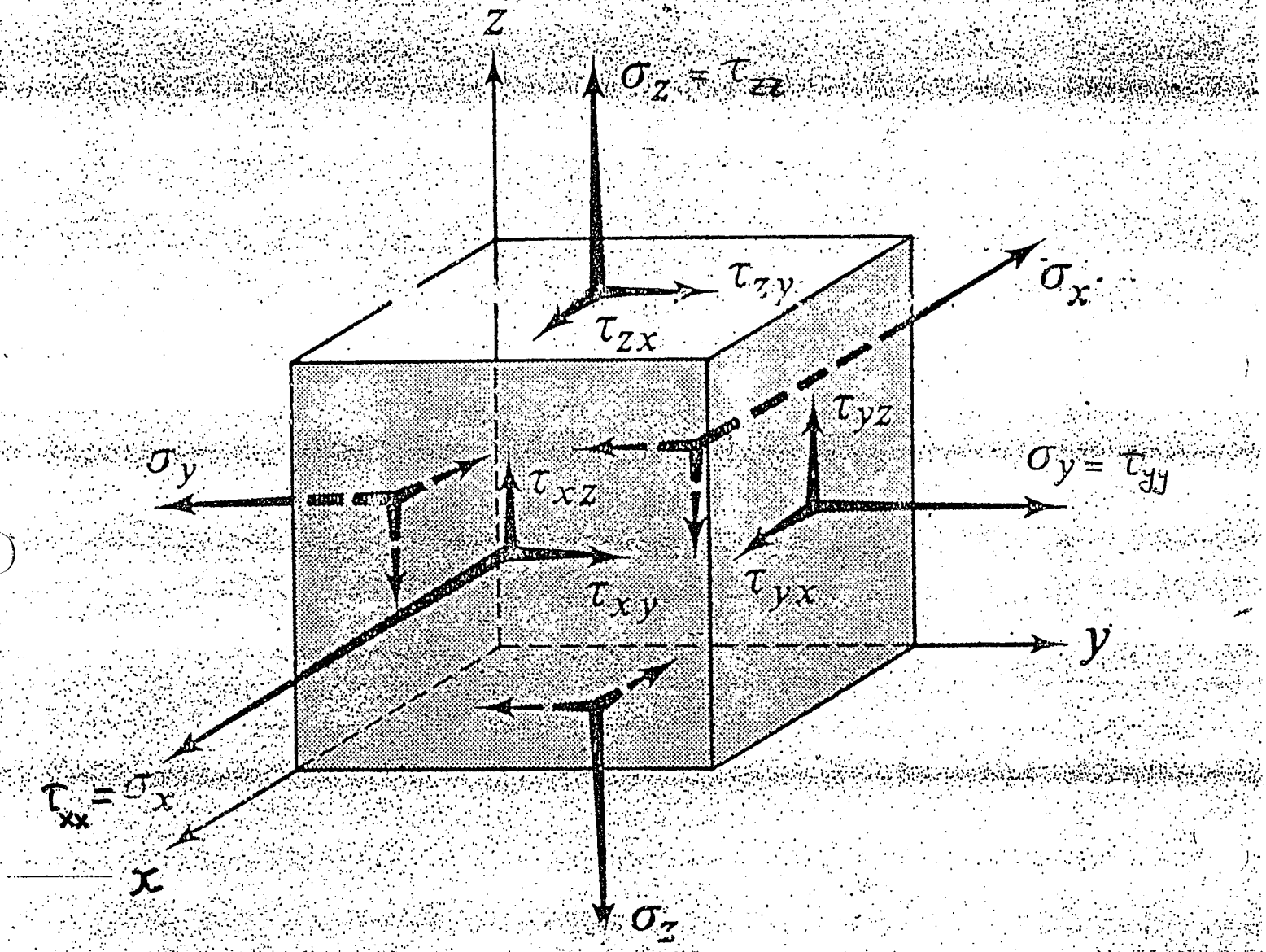



Fig. 1-3. The most general state of stress acting on an element

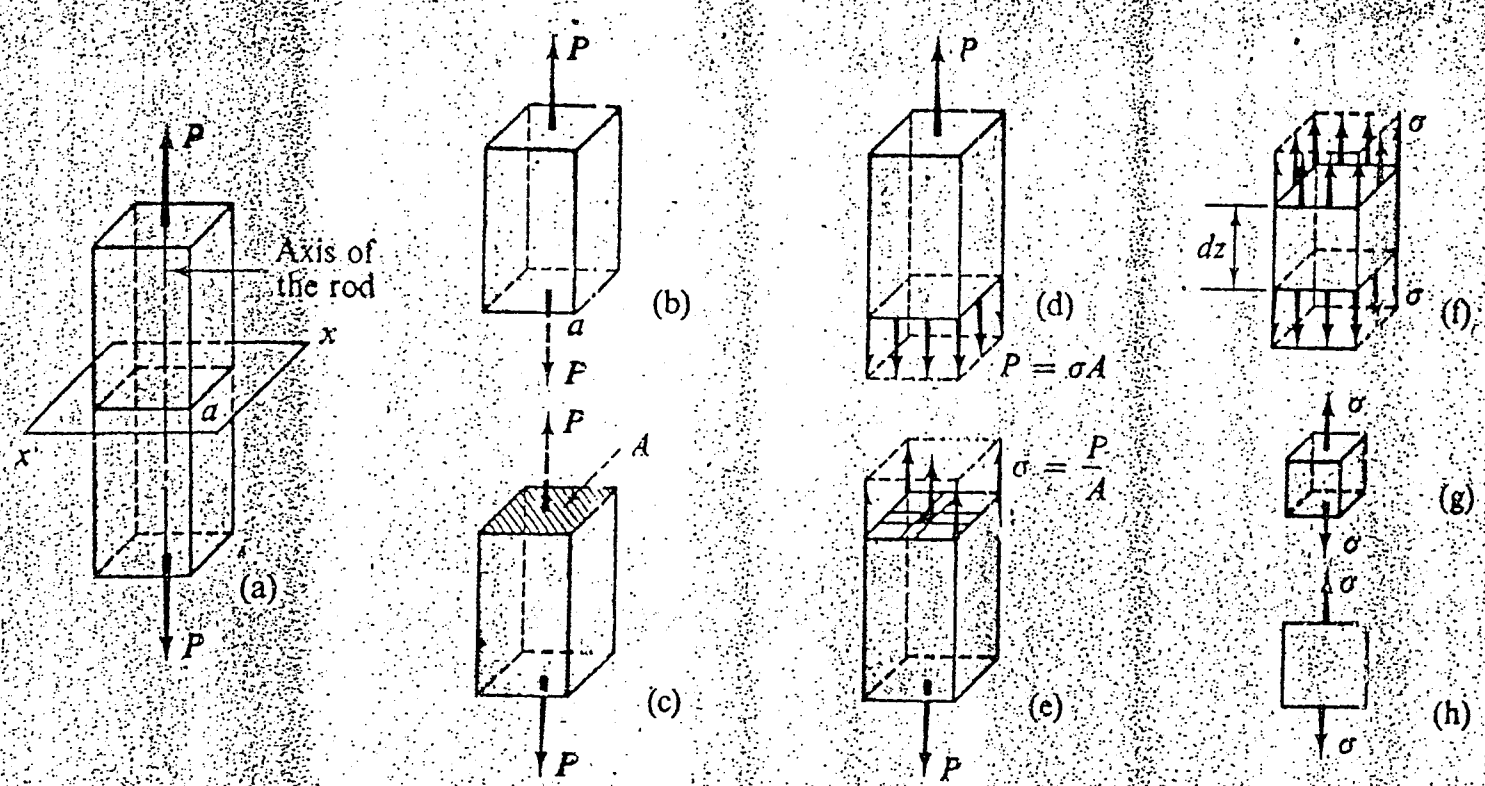



Fig. 1. Successive steps in the analysis of a body for stress

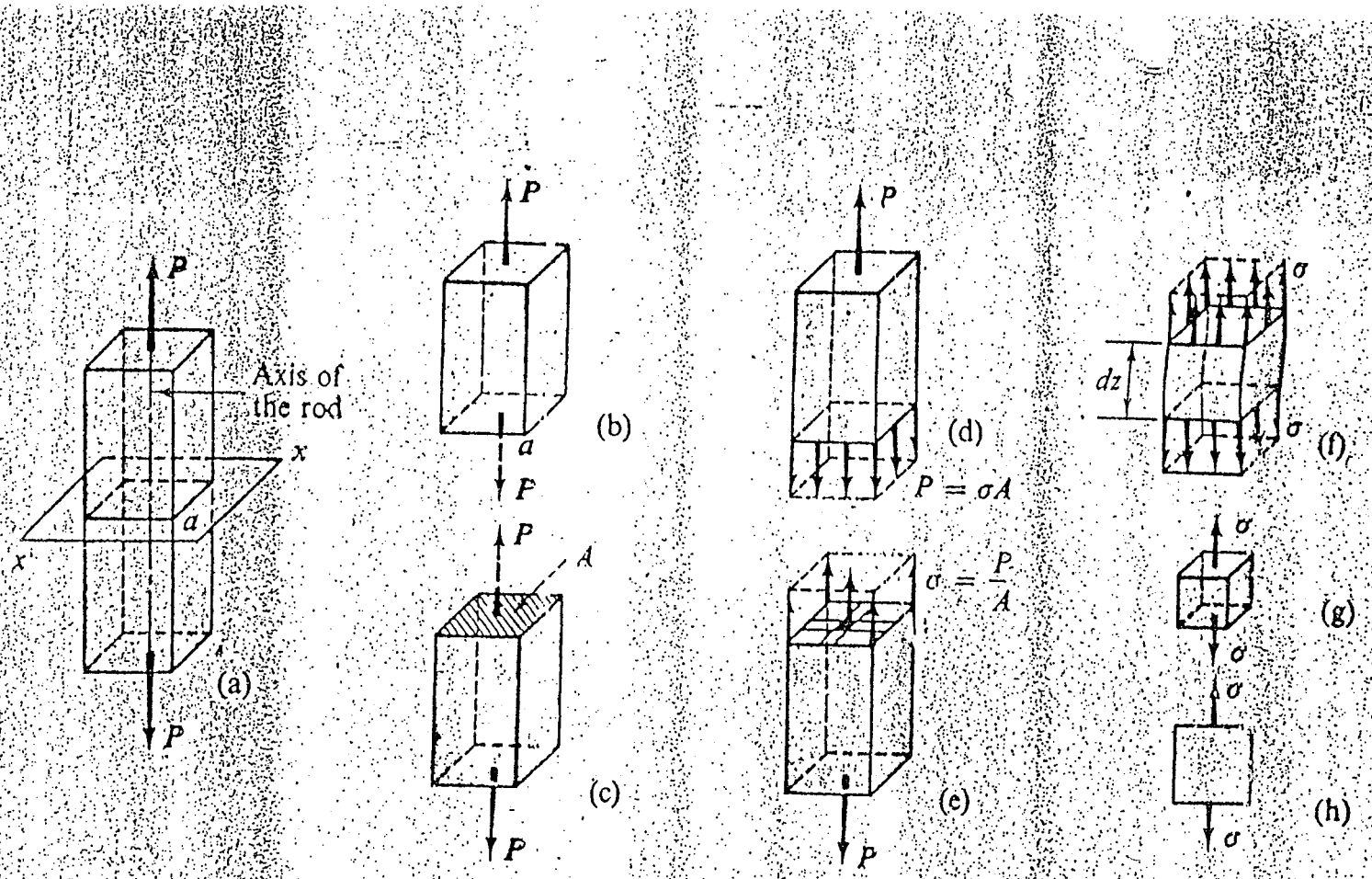



Fig. 1. Successive steps in the analysis of a body for stress

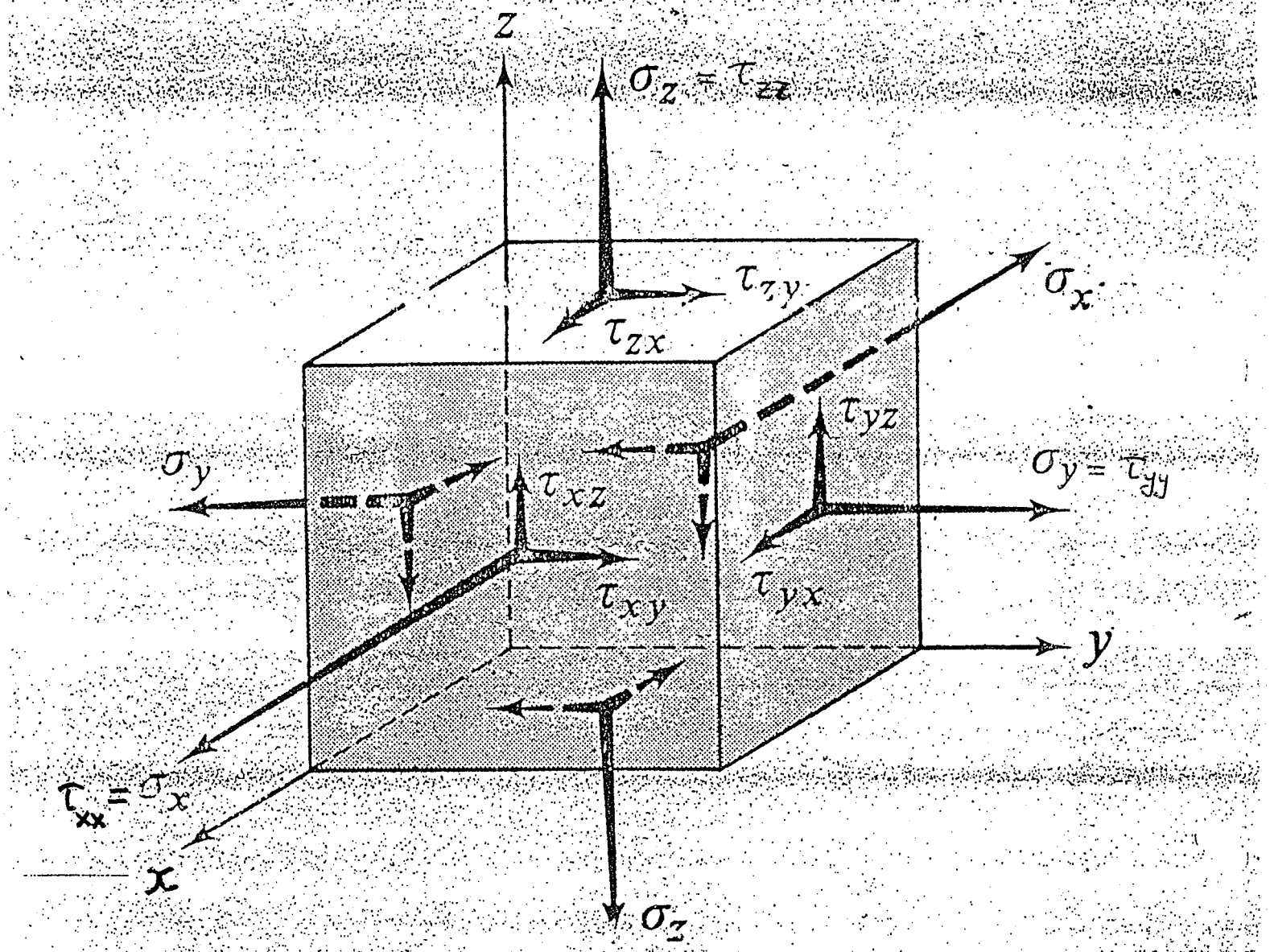



Fig. 1-3. The most general state of stress acting on an element

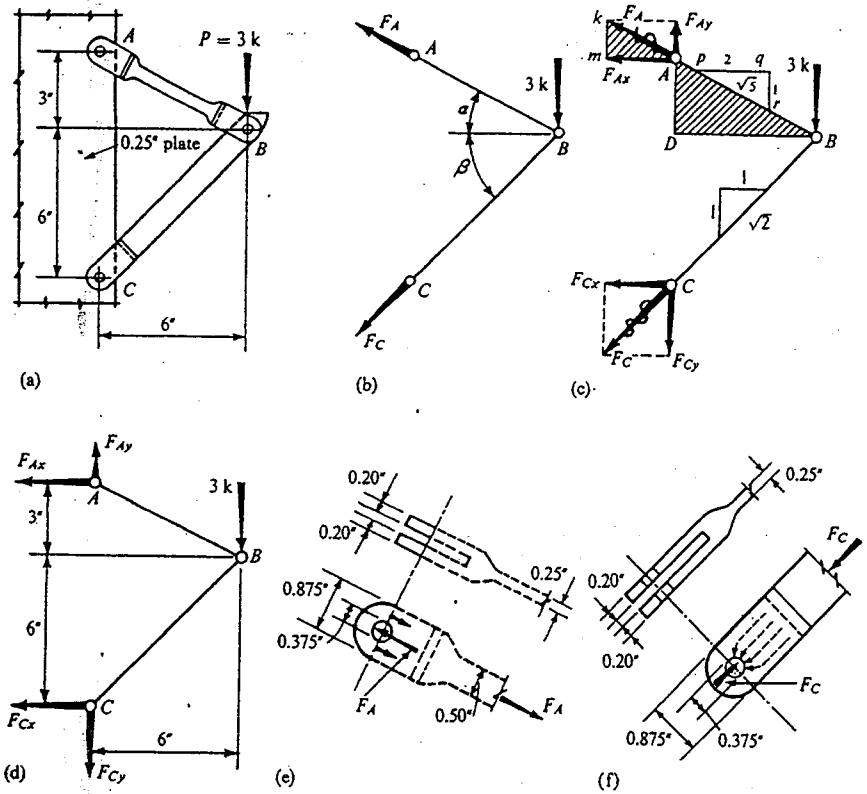



Fig. 3-12

SOLUTION

First an idealized free-body diagram consisting of the two bars pinned at the ends is prepared, Fig. 3-12(b). As there are no intermediate forces acting on the bars and the applied force acts through the joint at $B$, the forces in the bars are directed along the lines $AB$ and $BC$, and the bars $AB$ and $BC$ are loaded axially. The magnitudes of the forces are unknown and are labeled $F_A$ and $F_C$ in the diagram.* These forces may be determined graphically by completing a triangle of forces $F_A$, $F_B$, and $P$. These forces may also be found analytically from two simultaneous

* In frameworks it is convenient to assume all unknown forces are tensile. A negative answer in the solution then indicates that the bar is in compression.





equations $\Sigma F_y = 0$ and $\Sigma F_x = 0$, written in terms of the unknowns $F_A$ and $F_C$, a known force $P$, and two known angles $\alpha$ and $\beta$. Both these procedures are possible. However, in this course usually it will be found advantageous to proceed in a different way. Instead of treating forces $F_A$ and $F_C$ directly, their components are used; and instead of $\Sigma F = 0$, $\Sigma M = 0$ becomes the main tool.

Any force may be resolved into components. For example, $F_A$ may be resolved into $F_{Ax}$ and $F_{Ay}$ as in Fig. 3-12(c). Conversely, if any one of the components of a directed force is known, the force itself may be determined. This follows from similarity of dimension and force triangles. In Fig. 3-12(c) the triangles $Akm$ and $BAD$ are similar triangles (both are shaded in the diagram). Hence, if $F_{Ax}$ is known,

$$F_A = (AB/DB)F_{Ax}$$

Similarly, $F_{Ay} = (AD/DB)F_{Ax}$. However, note further that $AB/DB$ or $AD/DB$ are ratios, hence relative dimensions of members may be used. Such relative dimensions are shown by a little triangle on the member $AB$ and again on $BC$. In the problem at hand

$$F_A = (\sqrt{5}/2)F_{Ax} \quad \text{and} \quad F_{Ay} = F_{Ax}/2$$

Adopting the above procedure of resolving forces, the revised free-body diagram, Fig. 3-12(d), is prepared. Two components of force are necessary at the pin joints. After the forces are determined by statics, Eq. 3-5 is applied several times, thinking in terms of a free body of an individual member:

$$\sum M_C = 0 \circlearrowleft +, \quad +F_{Ax}(3 + 6) - 3(6) = 0, \qquad F_{Ax} = +2 \text{ kips}$$

$$F_{Ay} = F_{Ax}/2 = 2/2 = 1 \text{ kip},$$

$$F_A = 2(\sqrt{5}/2) = +2.23 \text{ kips}$$

$$\sum M_A = 0 \circlearrowleft +, \quad +3(6) + F_{Cx}(9) = 0,$$

$$F_{Cx} = -2 \text{ kips} \quad \text{(compression)}$$

$$F_{Cy} = F_{Cx} = -2 \text{ kips}$$

$$F_C = \sqrt{2}(-2) = -2.83 \text{ kips}$$

*Check:* $\sum F_x = 0, \quad F_{Ax} + F_{Cx} = 2 - 2 = 0$

$$\sum F_y = 0, \quad F_{Ay} - F_{Cy} - P = 1 - (-2) - 3 = 0$$

Stress in main bar $AB$:

$$\sigma_{AB} = \frac{F_A}{A} = \frac{2.23}{(0.25)(0.50)} = 17.8 \text{ ksi} \quad \text{(tension)}$$



possible.

EXAMPLE 3-3

A bracket of negligible weight shown in Fig. 3-12(a) is loaded with a force $P$ of 3 kips. For interconnection purposes the bar ends are clevised (forked). Pertinent dimensions are shown in the figure. Find the normal stresses in the members $AB$ and $BC$ and the bearing and shearing stresses for the pin $C$. All pins are 0.375 in. in diameter.

Stress in clevis of bar $AB$, Fig. 3-12(e):

$$(\sigma_{AB})_{\text{clevis}} = \frac{F_A}{A_{\text{net}}} = \frac{2.23}{2(0.20)(0.875 - 0.375)} = 11.2 \text{ ksi} \quad \text{(tension)}$$

Stress in main bar $BC$:

$$\sigma_{BC} = \frac{F_C}{A} = \frac{2.83}{(0.875)(0.25)} = 12.9 \text{ ksi} \quad \text{(compression)}$$

In the compression member the net section at the clevis need not be investigated; see Fig. 3-12(f) for the transfer of forces. The bearing stress at the pin is more critical. Bearing between pin $C$ and clevis:

$$\sigma_b = \frac{F_C}{A_{\text{bearing}}} = \frac{2.83}{(0.375)(0.20)2} = 18.8 \text{ ksi}$$

Bearing between the pin $C$ and the main plate:

$$\sigma_b = \frac{F_C}{A} = \frac{2.83}{(0.375)(0.25)} = 30.1 \text{ ksi}$$

Double shear in the pin $C$:

$$\tau = \frac{F_C}{A} = \frac{2.83}{2\pi(0.375/2)^2} = 12.9 \text{ ksi}^*$$

For a complete analysis of this bracket, other pins should be investigated. However, it may be seen by inspection that the other pins in this case are stressed the same amount as computed above, or less.

The advantages of the method used in the above example for finding forces in members should now be apparent. It can also be applied with success in a problem such as the one shown in Fig. 3-13. The force $F_A$ transmitted by the curved member $AB$ acts through points $A$ and $B$ since the forces applied at $A$ and $B$ must be collinear. By resolving this force at $A'$, the same procedure may be followed. Wavy lines through $F_A$ and $F_C$ indicate that these forces are replaced by the two components shown. Alternatively, the force $F_A$ may be resolved at $A$, and since $F_{Ay} = (x/y)F_{Ax}$, the application of $\Sigma M_C = 0$ yields $F_{Ax}$.

In frames where the applied forces do not act through a joint, proceed as above as far as possible. Then isolate an individual member, and using its free-body diagram, complete the determination of forces.

* Considering the pin in a two-dimensional state of stress $\tau_{xy} = \tau_{yz}$, the tensor representation of the results becomes $\begin{pmatrix} 0 & 12.9 \\ 12.9 & 0 \end{pmatrix}$ ksi



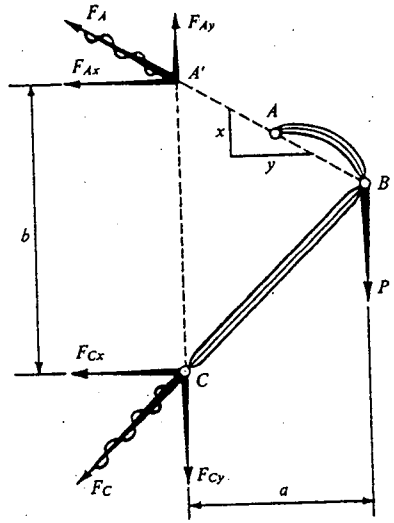



Fig. 3-13

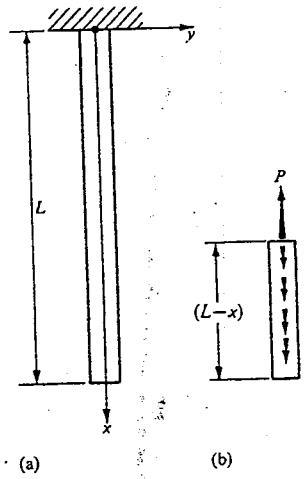



Fig. 3-14

If inclined forces are acting on the structure, resolve them into convenient components.

EXAMPLE 3-4

A 1-in.$^2$ rod $L$ in. long is suspended vertically as shown in Fig. 3-14(a). The unit weight of the material is $\gamma$. Determine the normal stress in this rod using differential equations of equilibrium.

SOLUTION

With the axes shown in the figure, $\tau_{xy} = 0$, and only the first part of Eq. 3-3 has relevance. The body force $X = \gamma$. By virtue of the boundary condition at the free end of the rod $\sigma_x(L) = 0$. On this basis, setting up a differential equation, intergating it, and determining the constant of integration from the boundary conditions, one has

$$\frac{d\sigma_x}{dx} + \gamma = 0 \quad \text{and} \quad \sigma_x = -\gamma x + C_1$$

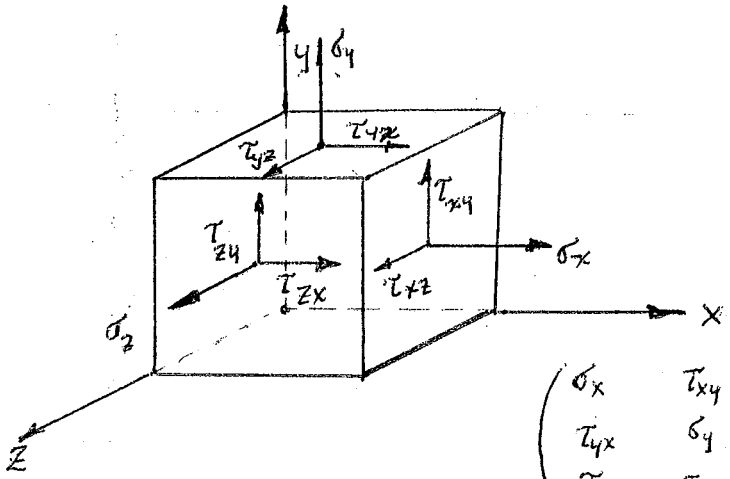
$$\sigma_x(L) = -\gamma L + C_1 = 0 \quad \text{and} \quad \sigma_x = (L - x)\gamma$$

This result can be easily checked by cutting the rod $(L - x)$ above the free end, Fig. 3-14(b), and applying Eq. 3-5. Only very few problems can be analyzed using Eq. 3-3 alone. In more general problems deformations must be considered simultaneously in the analysis.




if $\sigma_x A_2 = \sigma_x A_1$ we have equilib.

if $\sigma_x A_2 \neq \sigma_x A_1$ we have motion




$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} \Rightarrow \text{Stress Tensor.}$$

Scalar 0-Rank

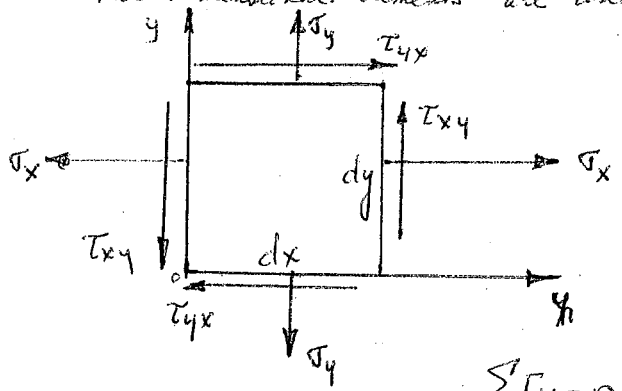
Vector 1st-Rank  Stress-Tensor 2nd rank

$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$$

tensors { Scalar — 0 rank — $t$ — $2^0$ — $3^0$
vector — 1 rank — $V_x, V_y, V_z$ — $2^1$ — $3^1$
[illegible] — $2^2$ — $3^2$

# of components to describe it in Plane / Free Space

Two dimensional elements are used for the derivation.




Uniform Thickness $dz$

$\sum F_x = 0$

$\sigma_x\, dy\, dz - \sigma_x\, dy\, dz +$

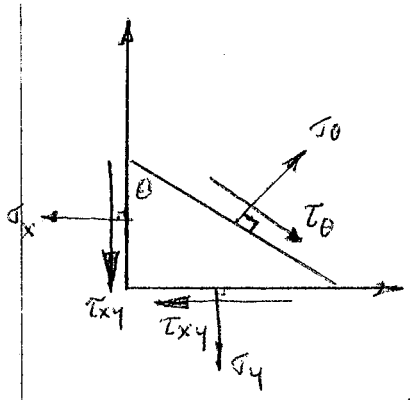
$\tau_{yx}\, dx\, dz - \tau_{yx}\, dy\, dz = 0$

$\sum F_y = 0 \uparrow$  $\sigma_y\, dx\, dz - \sigma_y\, dx\, dz$

$+ \tau_{xy}\, dy\, dz - \tau_{xy}\, dy\, dz = 0$

$\Sigma M_o \circlearrowright = 0 \qquad \tau_{yx}(dx\,dz)\,dy - \tau_{xy}(dy\,dz)\,dx = 0$

$$\boxed{\tau_{yx} = \tau_{xy}} \iff \tau_{xz} = \tau_{zx} \iff \tau_{yz} = \tau_{zy}$$




$\sigma_\theta = f(\theta)$ since $\sigma_\theta$ depends on $\sigma_x, \sigma_y, \tau_{xy}, \theta$; but $\sigma_x, \sigma_y, \tau_{xy}$ have been already calculated & they are constant; thus

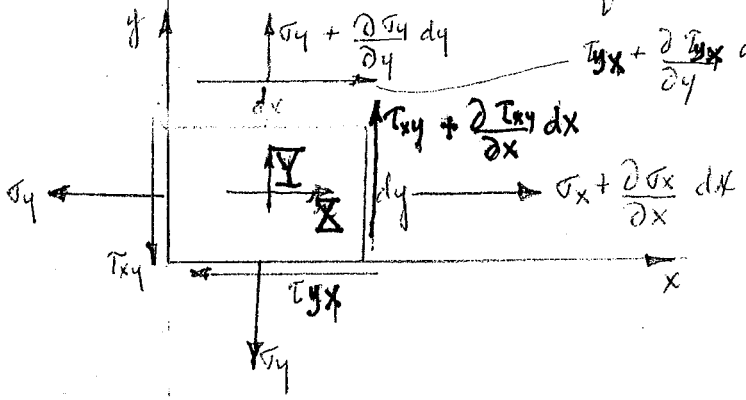
$\tau_\theta = f(\theta)$

$\sigma_\theta$, and, similarly $\tau_\theta$, are functions of $\theta$.

we can find some $\theta \Rightarrow \tau_\theta, \sigma_\theta$ are 0 in the xy, yz, xz planes so that we can produce

$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} \Longrightarrow \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$$

We will derive the equations of unequal stress




$\Sigma F_x = 0$;

$$\left(\sigma_x + \frac{\partial \sigma_x}{\partial x} dx\right) dy\,dz - \sigma_x\,dy\,dz + \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} dy\right) dx\,dz - \tau_{yx}\,dx\,dz + X\,dy\,dz\,dx = 0$$

$\Sigma F_y = 0$;

$$\left(\sigma_y + \frac{\partial \sigma_y}{\partial y} dy\right) dx\,dz - \sigma_y\,dx\,dz + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x} dx\right) dy\,dz - \tau_{xy}\,dy\,dz + Y\,dx\,dy\,dz = 0$$

$$\boxed{\begin{aligned} &(1) \quad \frac{\partial}{\partial x}(\sigma_x) + \frac{\partial}{\partial y}(\tau_{yx}) + X = 0 \\ &(2) \quad \frac{\partial}{\partial y}(\sigma_y) + \frac{\partial}{\partial x}(\tau_{xy}) + Y = 0 \end{aligned}}$$

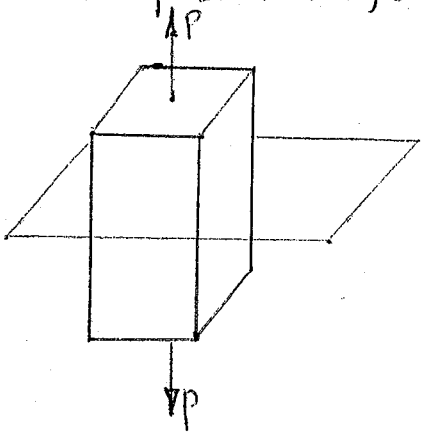
Three dimensionalization

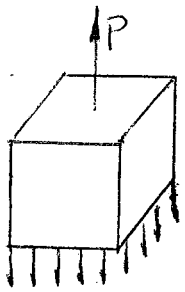
$$\frac{\partial}{\partial x}(\sigma_x) + \frac{\partial}{\partial y}(\tau_{yx}) + \frac{\partial}{\partial z}(\tau_{zx}) + X = 0$$

$$\frac{\partial}{\partial y}(\sigma_y) + \frac{\partial}{\partial x}(\tau_{xy}) + \frac{\partial}{\partial z}(\tau_{zy}) + Y = 0$$

$$\frac{\partial}{\partial z}(\sigma_z) + \frac{\partial}{\partial x}(\tau_{xz}) + \frac{\partial}{\partial y}(\tau_{yz}) + Z = 0$$

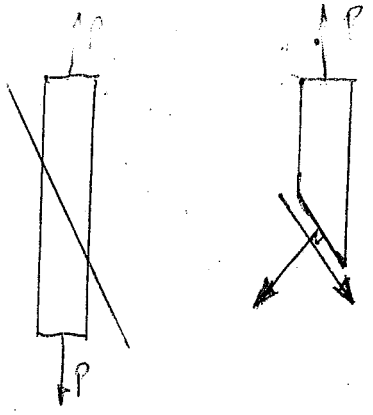
Do problems 3-2, 3-3

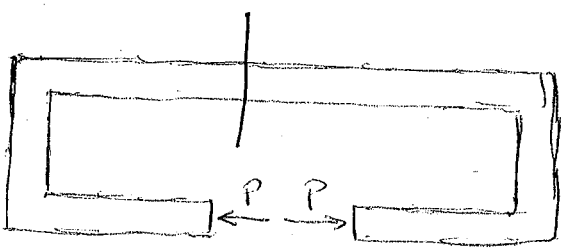



$\sigma = \frac{P}{A}$

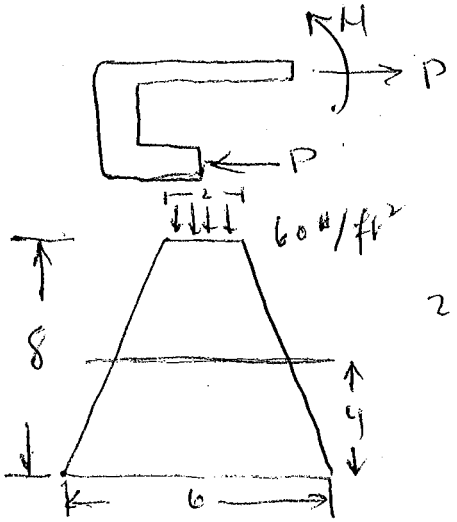
centric load




eccentric load

eccentric load




distribution is not uniform

wt of concrete = 120 #/ft³

2 ft wide




Since we have resultant passing thru centroid we can say $\sigma_y$ must be uniform.

$b_y = 6 - y/8\,(6-2) = (12-y)/2$

$W_y = \left(2 + \frac{12-y}{2}\right)(8-y)\,2 \cdot .150 = 75\,(8-y)(16-y)$

$P_y = W_y + 600\,(4) = 2400 + 75(8-y)(16-y)$

$\sigma_y = \dfrac{2400 + 75\,(8-y)\,(16-y)}{2(12-y)}$ if $P_y = \sigma_y \cdot A$

2nd way

$\dfrac{\partial \sigma_y}{\partial y} + \dfrac{\partial \tau_{xy}}{\partial x} + Y = 0$ since $F$'s are 0 we can

there is a mistake

then say $\dfrac{\partial \sigma_y}{\partial y} + Y = 0$ $Y$ #/ft³

$\dfrac{\partial \sigma_y}{\partial y} = -150$

$\int d\sigma_y = \int -150\,dy$

$\boxed{\sigma_y = -150y + f(x)}$

at $x = 8$ $\sigma = 600$ #/ft²

$600 = -1200 + f(x)$ $f(x) = 1800$ $(\sigma_y = -150y + 1800)$

6 unknowns, 3 equations makes proper problem statically indeterminate.

10-5-70

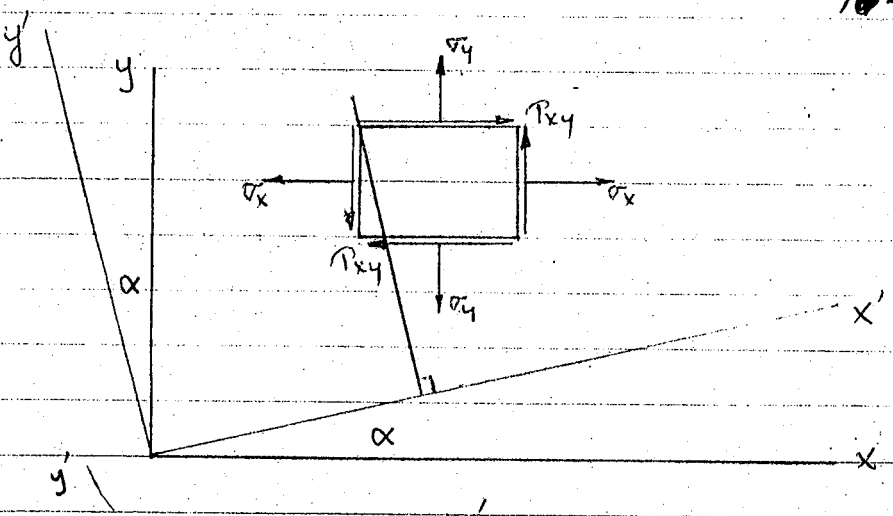



FIG. 1
10-5

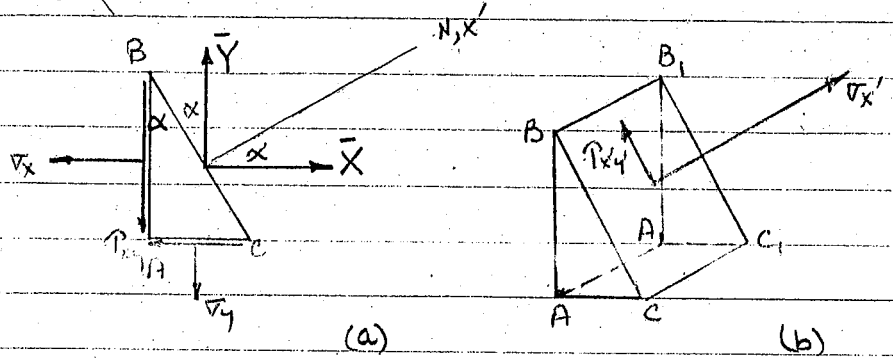



FIG. 2
10-5

$\bar{AB} = \bar{BC}\cos\alpha = A_n \cdot l$

$\bar{AC} = \bar{BC}\sin\alpha = A_m \cdot m$

$ABB_1A_1A = A_x = A_n\cos\alpha$

$ACC_1A_1A = A_y = A_n\sin\alpha$

$BB_1C_1CB = A_n$

$$\bar{X} = \sigma_x A_x + \tau_{xy} A_y = \sigma_x A_n \cos\alpha + \tau_{xy} A_n \sin\alpha$$

$$\bar{Y} = \tau_{xy} A_x + \sigma_y A_y = \tau_{xy} A_n \cos\alpha + \sigma_y A_n \sin\alpha$$

$$X' = A_n \sigma_x' = \left(\bar{X}\cos\alpha + \bar{Y}\sin\alpha\right) = (\sigma_x\cos\alpha + \tau_{xy}\sin\alpha) A_n \cos\alpha + (\tau_{xy}\cos\alpha + \sigma_y \sin\alpha) A_n \sin\alpha$$

$$\sigma_x' = \sigma_x \cos^2\alpha + 2\tau_{xy}\sin\alpha\cos\alpha + \sigma_y \sin^2\alpha$$

10-5,1 $$\boxed{\sigma_x' = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha}$$

$$Y' = A_n \tau_{x'y'} = \bar{Y}\cos\alpha - \bar{X}\sin\alpha \;;\; (\text{thus } \tau_{x'y'} = \bar{Y}\cos\alpha - \bar{X}\sin\alpha)$$

$$A_n \tau_{x'y'} = \left[\tau_{xy}\cos^2\alpha + (\sigma_y - \sigma_x)\sin\alpha\cos\alpha - \tau_{xy}\sin^2\alpha\right] A_n$$

$$= \tau_{xy}\left(\cos^2\alpha - \sin^2\alpha\right) - \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\alpha$$

10-5,2 $$\boxed{\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\alpha + \tau_{xy}\cos 2\alpha}$$

for $\sigma_y'$ use $\alpha = \alpha + 90^\circ$ and get

$$\sigma_y' = \frac{\sigma_x+\sigma_y}{2} - \left[\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha\right]$$

| | $\bar{X}$ | $\bar{Y}$ | | |
|---|---|---|---|---|
| $x'$ | $l$ | $m$ | $\cos\alpha = l$ | $\bar{X} = (l\sigma_x + m\tau_{xy})A_n$ |
| $y'$ | $l_2 = -m$ | $m_2 = l$ | $\sin\alpha = m$ | $\bar{Y} = (l\tau_{xy} + m\sigma_x)A_n$ |

$$l^2 + m^2 = 1 \qquad \sigma_x' = (l\bar{X} + m\bar{Y})/A_n$$

$$= l^2\sigma_x + 2lm\tau_{xy} + m^2\sigma_y$$

$$\frac{\partial \sigma_x'}{\partial l} = (\bar{X} + \bar{Y}\cdot\frac{\partial m}{\partial l})/A_n = \left[(l\sigma_x + m\tau_{xy}) + (l\tau_{xy} + m\sigma_x)\frac{\partial m}{\partial l}\right]/A_n$$

$$\frac{\partial \sigma_x'}{\partial l} = 2\left[l\sigma_x + m\tau_{xy}\right] + 2\left[l\tau_{xy} + m\sigma_y\right]\frac{\partial m}{\partial l} = 0$$

if $l^2 + m^2 = 1$ $\quad 2l\,dl + 2m\,dm = 0$ $\quad \frac{dm}{dl} = -l/m$

$$\frac{\overset{(1)}{l\sigma_x + m\tau_{xy}}}{l} = \frac{\overset{(2)}{l\tau_{xy} + m\sigma_y}}{m} = \overset{(3)}{\sigma}$$

$$\bar{X}/A_n l = \bar{Y}/A_n m = \sigma$$

from ① & ③ $\quad (\sigma_x - \sigma)\, l + m\, \tau_{xy} = 0$

from ② & ③ $\quad (\sigma_y - \sigma)\, m + l\, \tau_{xy} = 0$

therefore, $\begin{vmatrix} \sigma_x - \sigma & \tau_{xy} \\ \tau_{xy} & \sigma_y - \sigma \end{vmatrix} = 0$

or, $\sigma^2 - (\sigma_x + \sigma_y)\sigma + \left[(\sigma_x \sigma_y) - \tau_{xy}^2\right] = 0$

10-5.3 ✓

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

✓ we will say principal stresses $\sigma_1$ & $\sigma_2$ when

$\tau_{x'y'} = 0 \Rightarrow \tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

✓ $\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

$\sin 2\alpha = \frac{\tau_{xy}}{\sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2}}$, $\cos 2\alpha = \frac{\frac{\sigma_x - \sigma_y}{2}}{\sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2}}$

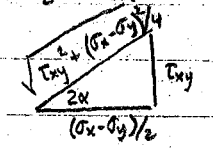
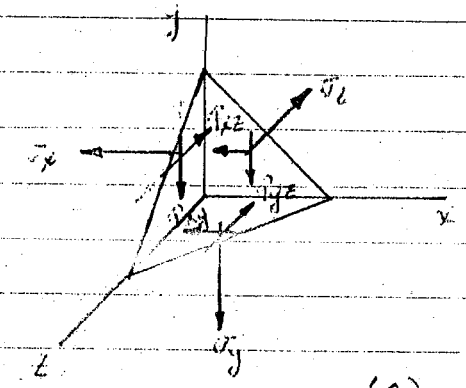
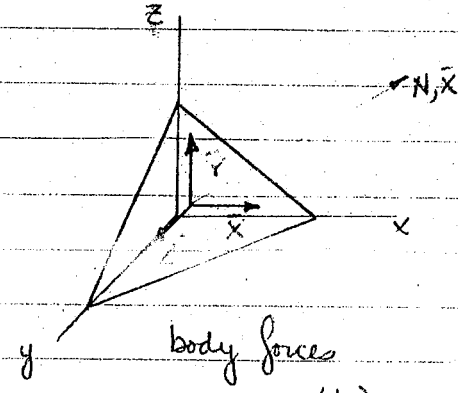



FIG. 3
10-5




(a)




(b)

$A_x = l_1 A_n$

$A_y = m_1 A_n$

$A_z = n_1 A_n$

$\bar{X} = (l_1 \sigma_x + m_1 \tau_{xy} + n_1 \tau_{zx}) A_n$

$\bar{Y} = (l_1 \tau_{xy} + m_1 \sigma_y + n_1 \tau_{zy}) A_n$

$\bar{Z} = (l_1 \tau_{xz} + m_1 \tau_{yz} + n_1 \sigma_z) A_n$

| | x | y | z |
|---|---|---|---|
| $\bar{x}'$ | $l_1$ | $m_1$ | $n_1$ |
| $y'$ | | | |
| $z'$ | | | |

César Levy
AE311

✓

2.1

$\sigma_x = 3x^2y \qquad \sigma_y = 4x^2y^2 \qquad \sigma_z = 7xz^3 \qquad \bar{X}, \bar{Y}, \bar{Z} = 0$

$\tau_{xy} = 8x^2y^2 \qquad \tau_{yz} = 9y^2z^2 \qquad \tau_{zx} = 10x^2y^2$

$6xy + 16x^2y + 0 \neq 0$

$16xy^2 + 8x^2y + 18y^2z \neq 0$

$20xy^2 + 18yz^2 + 21xz^2 \neq 0$

they do not satisfy equil.

2.3. $\sigma_x = 35{,}000$ psi $\quad \sigma_y = -1700$ psi $\quad \tau_{xy} = 13000$ psi

$\theta_2 = \arctan 3/4 \qquad \sin\theta_2 = 3/5 \; ; \; \cos\theta_2 = 1/5$

$$\frac{\partial \sigma_{x'}}{\partial \theta} = -2\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + 2\tau_{xy}\cos 2\theta = 0$$

$$\tau_{xy}\cos 2\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta$$

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

$$\sigma_{1,2} = \left(\frac{33{,}300}{2}\right) \pm \left[\left(18{,}350\right)^2 + \left(13{,}000\right)^2\right]^{1/2}$$

$$\left[3365 \times 10^6 + 169. \times 10^6\right]^{1/2}$$

$$22.5 \qquad \times 10^3$$

$$= 16{,}650 \pm 22{,}500$$

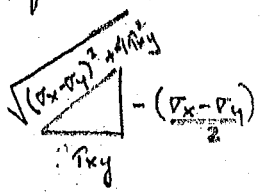
$\sigma_1 = 39{,}150$ psi

$\sigma_2 = -5{,}850$ psi $\qquad \tan 2\theta = \dfrac{2(13000)}{36{,}700} = \dfrac{26.000}{36{,}700} = .707$

$2\theta = 35.3^\circ \qquad \theta_1 = 17.65^\circ \qquad \theta_2 = 107.65^\circ$

b. $\frac{\partial \tau_{x'y'}}{\partial \theta} = -(\sigma_x - \sigma_y)\cos 2\theta + 2\tau_{xy}\sin 2\theta = 0$

$\tan 2\theta = -\frac{(\sigma_x - \sigma_y)}{2\tau_{xy}}$




$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 22500$ psi

$\tan 2\theta = -\frac{(16650)}{13000}$ $\quad 54.7^\circ = 2\theta \quad \theta = 27.35^\circ$

at $\tau_{max}$ $\sigma_x' = \left(\frac{\sigma_x + \sigma_y}{2}\right) = 16{,}650$ psi but $\sigma_y' = \left(\frac{\sigma_x + \sigma_y}{2}\right) = 16{,}650$ psi

c. $\theta_c = \arctan 3/1$ $\quad \sin 2\theta_c = 2\sin\theta_c\cos\theta_c = \frac{24}{25}$

$\cos 2\theta_c = \cos^2\theta_c - \sin^2\theta_c = 7/25$

$\sigma_{x'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\left[\cos^2\theta_c - \sin^2\theta_c\right] + 2\tau_{xy}\sin\theta_c\cos\theta_c$

$= 16{,}650 + \left(\frac{36{,}700}{2}\right)\frac{7}{25} + 13{,}000\left(\frac{24}{25}\right)$

$= 16{,}650 + 5140 + 12{,}500 = 34{,}290$ psi

$\sigma_{y'} = 16{,}650 - 17{,}640 = -990$ psi

$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\alpha + \tau_{xy}\cos 2\theta$

$-17616 + 3640 = -13976$ psi

Ultimate Strength.

strength at which member "failed". failure can be total rupture or excessive deflection.

Factor of Safety - Factor of Ignorance

those factors that can't calculated and those factors which we have doubt.

Allowable Stress -

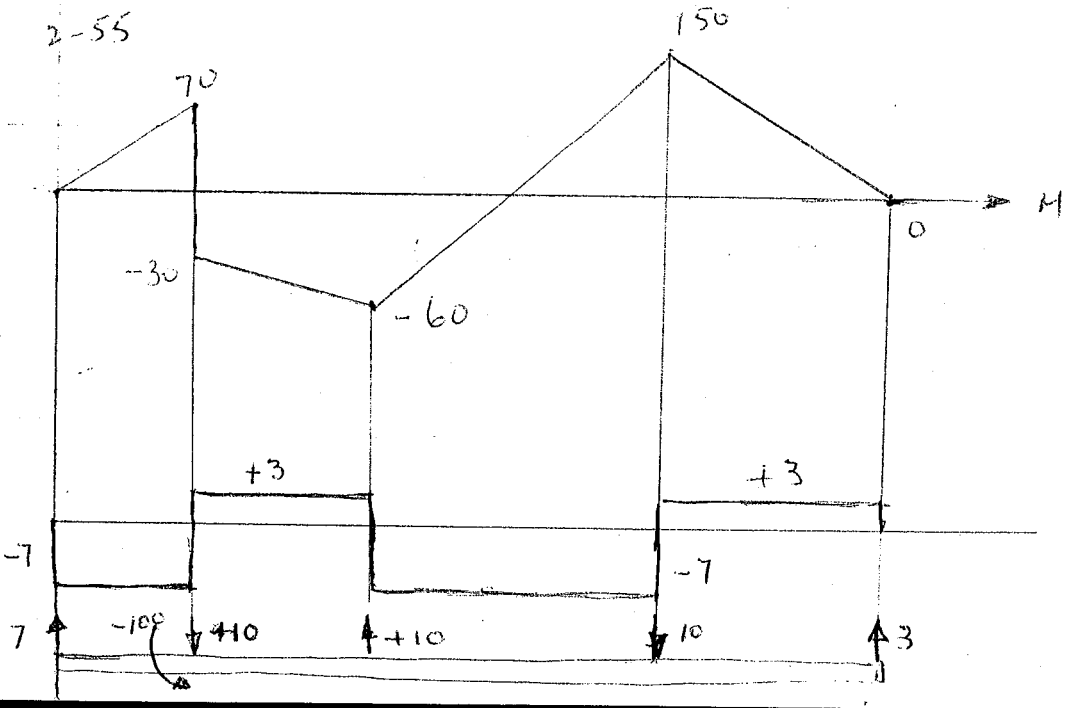
Maximum stress allowed to occur on job and that can't exceeded

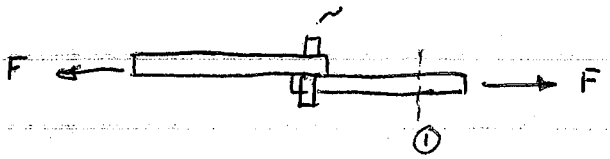
$$\text{Allowable Stress} = \frac{\text{Ult. Strength}}{\text{Factor of Safety}}$$

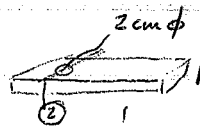
Creep - tendency of material to flow plastically under sustained load (load should produce stress almost = ultimate stress)

Fatigue - if a material carries a high stress & these stresses are constantly loading & unloading or change of state (T to C to T to C)

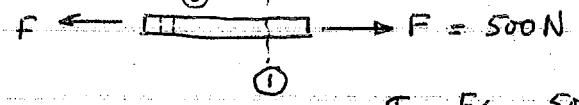
Residual stresses -

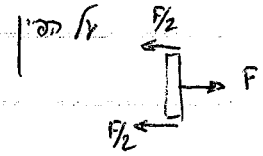





$A_1 = 1\,cm \times 5\,cm$






$$\sigma_1 = \frac{F}{A_1} = \frac{500\,N}{5cm^2} = \frac{100N}{cm^2} = \frac{100N}{1\times10^{-4}m^2}$$

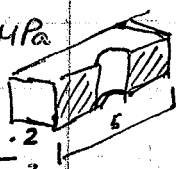
$$= 1\,MPa$$

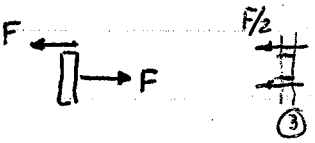
$$\sigma_{2a} = \frac{F}{A} = \frac{500\,N}{2cm \cdot 1cm} = 2.5\,MPa$$

bearing stress on pin

$$\sigma_2 = \frac{F}{A_2} = \frac{500\,N}{(1cm)(5cm-2cm)}$$

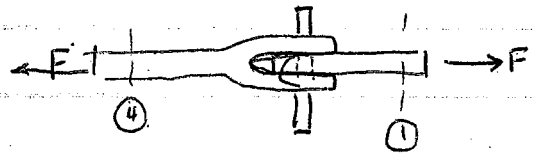
$$= \frac{500}{3cm^2} = 1.67\,MPa$$






$$\tau = \frac{F/2}{A_3} = \frac{F/2}{\pi r^2} = \frac{250 \cdot 2}{3.14(1)^2}$$

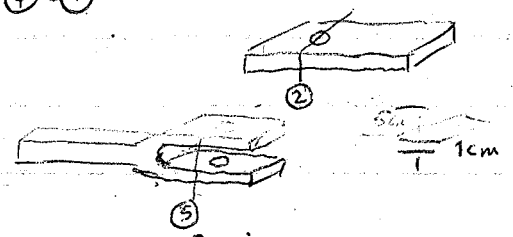
$$\tau = 159.2\,\frac{N}{cm^2} = 1.592\,MPa$$




④ = ①

$$\sigma_1 = F/A_1$$

$$\sigma_2 = F/A_2$$




$$\tau_{\phi 3} = F/2A_3$$

$$\sigma_4 = \frac{F}{A_4} = \frac{F}{A_1} = 1\,MPa$$

$$\sigma_5 = \frac{F}{A_5} = \frac{F}{2(1cm)(5cm-2cm)} = 0.5\,MPa$$

$$A_5 = 2A_1$$

2-D

$$\sigma_x + \sigma_y = \sigma_x' + \sigma_y' = \sigma_1 + \sigma_2 = I_{1\sigma}$$

$$\sigma_x\sigma_y - \tau_{xy}^2 = \sigma_1\sigma_2 = \sigma_x'\sigma_y' - \tau_{xy}'^2 = I_{2\sigma}$$

3D

$$\sigma_x + \sigma_y + \sigma_z = \sigma_x' + \sigma_y' + \sigma_z' = \sigma_1 + \sigma_2 + \sigma_3 = I_{1\sigma}$$

$$\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) = I_{2\sigma} = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$$

$$\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - (\sigma_x\tau_{yz}^2 + \sigma_y\tau_{zx}^2 + \sigma_z\tau_{xy}^2) = I_{3\sigma} = \sigma_1\sigma_2\sigma_3$$

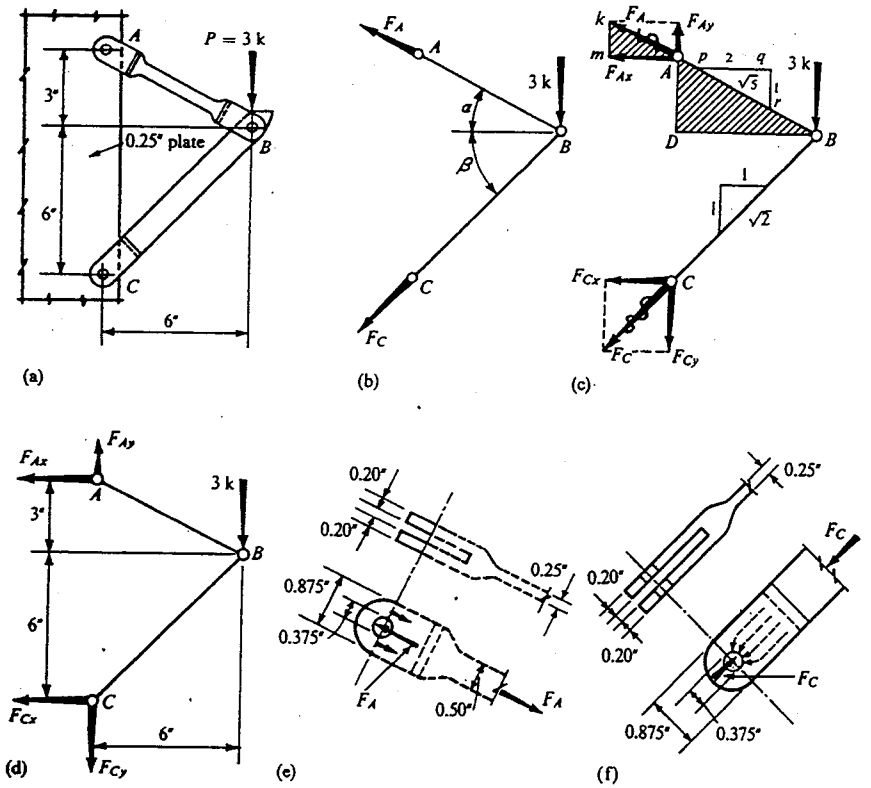



**Fig. 3-12**

SOLUTION

First an idealized free-body diagram consisting of the two bars pinned at the ends is prepared, Fig. 3-12(b). As there are no intermediate forces acting on the bars and the applied force acts through the joint at $B$, the forces in the bars are directed along the lines $AB$ and $BC$, and the bars $AB$ and $BC$ are loaded axially. The magnitudes of the forces are unknown and are labeled $F_A$ and $F_C$ in the diagram.* These forces may be determined graphically by completing a triangle of forces $F_A$, $F_B$, and $P$. These forces may also be found analytically from two simultaneous

* In frameworks it is convenient to assume all unknown forces are tensile. A negative answer in the solution then indicates that the bar is in compression.





equations $\Sigma F_y = 0$ and $\Sigma F_x = 0$, written in terms of the unknowns $F_A$ and $F_C$, a known force $P$, and two known angles $\alpha$ and $\beta$. Both these procedures are possible. However, in this course usually it will be found advantageous to proceed in a different way. Instead of treating forces $F_A$ and $F_C$ directly, their components are used; and instead of $\Sigma F = 0$, $\Sigma M = 0$ becomes the main tool.

Any force may be resolved into components. For example, $F_A$ may be resolved into $F_{Ax}$ and $F_{Ay}$ as in Fig. 3-12(c). Conversely, if any one of the components of a directed force is known, the force itself may be determined. This follows from similarity of dimension and force triangles. In Fig. 3-12(c) the triangles $Akm$ and $BAD$ are similar triangles (both are shaded in the diagram). Hence, if $F_{Ax}$ is known,

$$F_A = (AB/DB)F_{Ax}$$

Similarly, $F_{Ay} = (AD/DB)F_{Ax}$. However, note further that $AB/DB$ or $AD/DB$ are ratios, hence relative dimensions of members may be used. Such relative dimensions are shown by a little triangle on the member $AB$ and again on $BC$. In the problem at hand

$$F_A = (\sqrt{5}/2)F_{Ax} \quad \text{and} \quad F_{Ay} = F_{Ax}/2$$

Adopting the above procedure of resolving forces, the revised free-body diagram, Fig. 3-12(d), is prepared. Two components of force are necessary at the pin joints. After the forces are determined by statics, Eq. 3-5 is applied several times, thinking in terms of a free body of an individual member:

$$\sum M_C = 0 \text{ Q} +, \quad +F_{Ax}(3 + 6) - 3(6) = 0, \quad F_{Ax} = +2 \text{ kips}$$

$$F_{Ay} = F_{Ax}/2 = 2/2 = 1 \text{ kip},$$

$$F_A = 2(\sqrt{5}/2) = +2.23 \text{ kips}$$

$$\sum M_A = 0 \text{ Q} +, \quad +3(6) + F_{Cx}(9) = 0,$$

$$F_{Cx} = -2 \text{ kips} \quad \text{(compression)}$$

$$F_{Cy} = F_{Cx} = -2 \text{ kips}$$

$$F_C = \sqrt{2}(-2) = -2.83 \text{ kips}$$

*Check:* $\sum F_x = 0, \quad F_{Ax} + F_{Cx} = 2 - 2 = 0$

$\sum F_y = 0, \quad F_{Ay} - F_{Cy} - P = 1 - (-2) - 3 = 0$

Stress in main bar $AB$:

$$\sigma_{AB} = \frac{F_A}{A} = \frac{2.23}{(0.25)(0.50)} = 17.8 \text{ ksi} \quad \text{(tension)}$$



possible.

EXAMPLE 3-3

A bracket of negligible weight shown in Fig. 3-12(a) is loaded with a force $P$ of 3 kips. For interconnection purposes the bar ends are clevised (forked). Pertinent dimensions are shown in the figure. Find the normal stresses in the members $AB$ and $BC$ and the bearing and shearing stresses for the pin $C$. All pins are 0.375 in. in diameter.

Stress in clevis of bar $AB$, Fig. 3-12(e):

$$(\sigma_{AB})_{\text{clevis}} = \frac{F_A}{A_{\text{net}}} = \frac{2.23}{2(0.20)(0.875 - 0.375)} = 11.2 \text{ ksi} \qquad \text{(tension)}$$

Stress in main bar $BC$:

$$\sigma_{BC} = \frac{F_C}{A} = \frac{2.83}{(0.875)(0.25)} = 12.9 \text{ ksi} \qquad \text{(compression)}$$

In the compression member the net section at the clevis need not be investigated; see Fig. 3-12(f) for the transfer of forces. The bearing stress at the pin is more critical. Bearing between pin $C$ and clevis:

$$\sigma_b = \frac{F_C}{A_{\text{bearing}}} = \frac{2.83}{(0.375)(0.20)2} = 18.8 \text{ ksi}$$

Bearing between the pin $C$ and the main plate:

$$\sigma_b = \frac{F_C}{A} = \frac{2.83}{(0.375)(0.25)} = 30.1 \text{ ksi}$$

Double shear in the pin $C$:

$$\tau = \frac{F_C}{A} = \frac{2.83}{2\pi(0.375/2)^2} = 12.9 \text{ ksi}^*$$

For a complete analysis of this bracket, other pins should be investigated. However, it may be seen by inspection that the other pins in this case are stressed the same amount as computed above, or less.

The advantages of the method used in the above example for finding forces in members should now be apparent. It can also be applied with success in a problem such as the one shown in Fig. 3-13. The force $F_A$ transmitted by the curved member $AB$ acts through points $A$ and $B$ since the forces applied at $A$ and $B$ must be collinear. By resolving this force at $A'$, the same procedure may be followed. Wavy lines through $F_A$ and $F_C$ indicate that these forces are replaced by the two components shown. Alternatively, the force $F_A$ may be resolved at $A$, and since $F_{Ay} = (x/y)F_{Ax}$, the application of $\Sigma M_C = 0$ yields $F_{Ax}$.

In frames where the applied forces do not act through a joint, proceed as above as far as possible. Then isolate an individual member, and using its free-body diagram, complete the determination of forces.

* Considering the pin in a two-dimensional state of stress $\tau_{xy} = \tau_{yx}$, the tensor representation of the results becomes $\begin{pmatrix} 0 & 12.9 \\ 12.9 & 0 \end{pmatrix}$ ksi



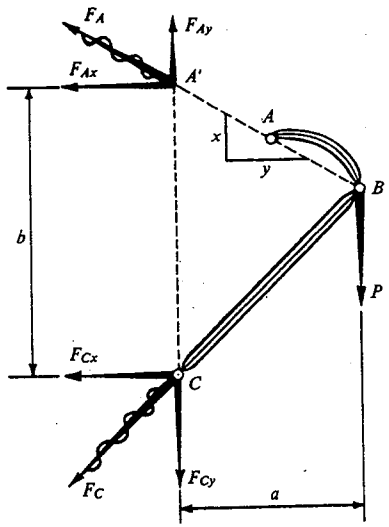



Fig. 3-13

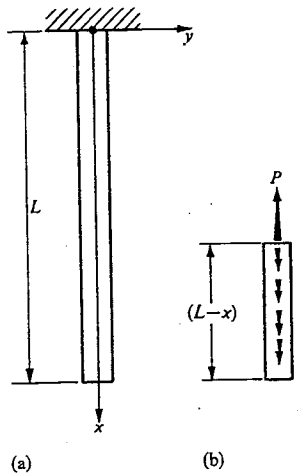



Fig. 3-14

If inclined forces are acting on the structure, resolve them into convenient components.

EXAMPLE 3-4

A 1-in.$^2$ rod $L$ in. long is suspended vertically as shown in Fig. 3-14(a). The unit weight of the material is $\gamma$. Determine the normal stress in this rod using differential equations of equilibrium.

SOLUTION

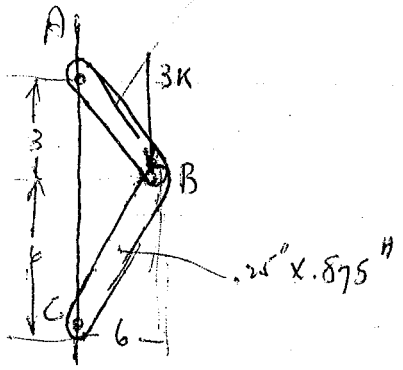
With the axes shown in the figure, $\tau_{xy} = 0$, and only the first part of Eq. 3-3 has relevance. The body force $X = \gamma$. By virtue of the boundary condition at the free end of the rod $\sigma_x(L) = 0$. On this basis, setting up a differential equation, intergating it, and determining the constant of integration from the boundary conditions, one has

$$\frac{d\sigma_x}{dx} + \gamma = 0 \qquad \text{and} \qquad \sigma_x = -\gamma x + C_1$$

$$\sigma_x(L) = -\gamma L + C_1 = 0 \qquad \text{and} \qquad \sigma_x = (L - x)\gamma$$

This result can be easily checked by cutting the rod $(L - x)$ above the free end, Fig. 3-14(b), and applying Eq. 3-5. Only very few problems can be analyzed using Eq. 3-3 alone. In more general problems deformations must be considered simultaneously in the analysis.

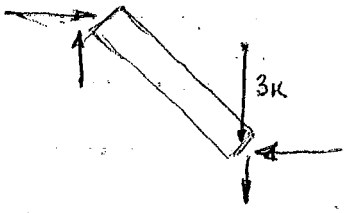
.25"X.50" - cross sectional area.

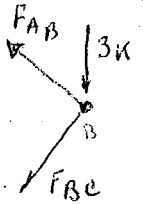



.375" $\phi$ diameter

Neglect Weight of Members

Consider AB these members must have results in direction of axis of the members.




$\Sigma F_x = 0 \quad -\frac{2}{\sqrt{5}} F_{AB} - \frac{1}{\sqrt{2}} F_{BC} = 0$

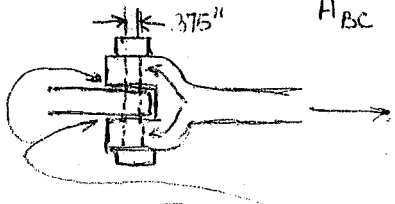
$\Sigma F_y = 0 \quad \frac{1}{\sqrt{5}} F_{AB} - \frac{1}{\sqrt{2}} F_{BC} = 3$

$\frac{3}{\sqrt{5}} F_{AB} = 3 \qquad F_{AB} = \sqrt{5}\,KT \qquad F_{BC} = +2\sqrt{2}\,KC$

2.828 KN

Stresses $\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{\sqrt{5}}{.025 \times .50}$ T

$\sigma_{BC} = \frac{F_{CB}}{A_{BC}} = \frac{2\sqrt{2}}{.25 \times .875}$ C




$\tau_{\text{Bolt A}} = \frac{\sqrt{5}}{\frac{\pi}{4} \times .375^2 \times 2}$ 2 surface shear

EMA 3702 - 1/16/02

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

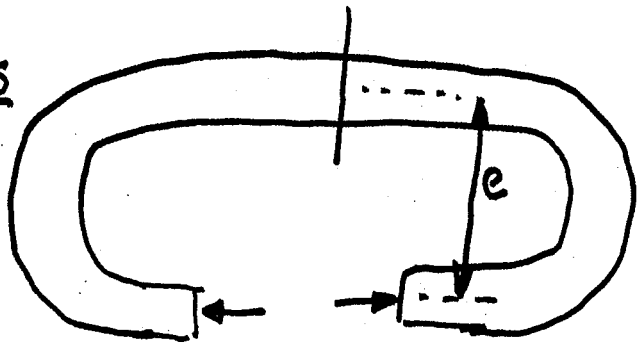
STRESS TENSOR



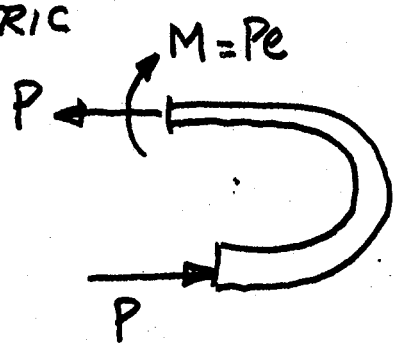

DIRECT STRESS

$\sigma_{ave} = P/A$

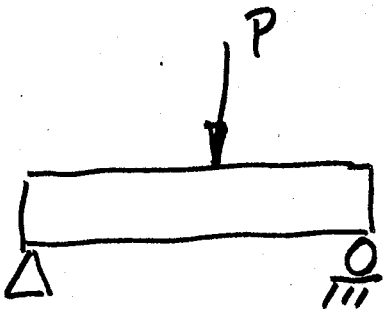
CENTRIC STRESS

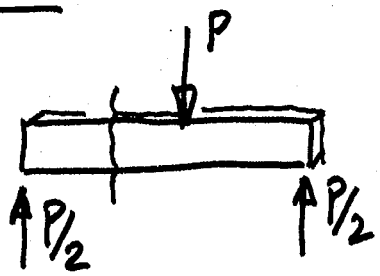


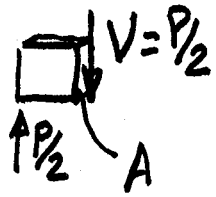

ECCENTRIC



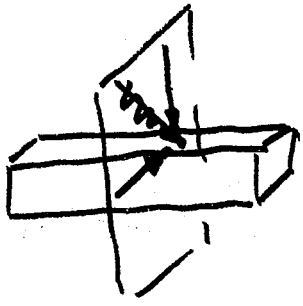

SHEAR STRESSES

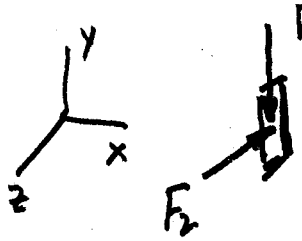



$\tau_{ave} = \frac{V}{A}$






$\tau_{xy} = F_1/A$

$\tau_{xz} = F_2/A$

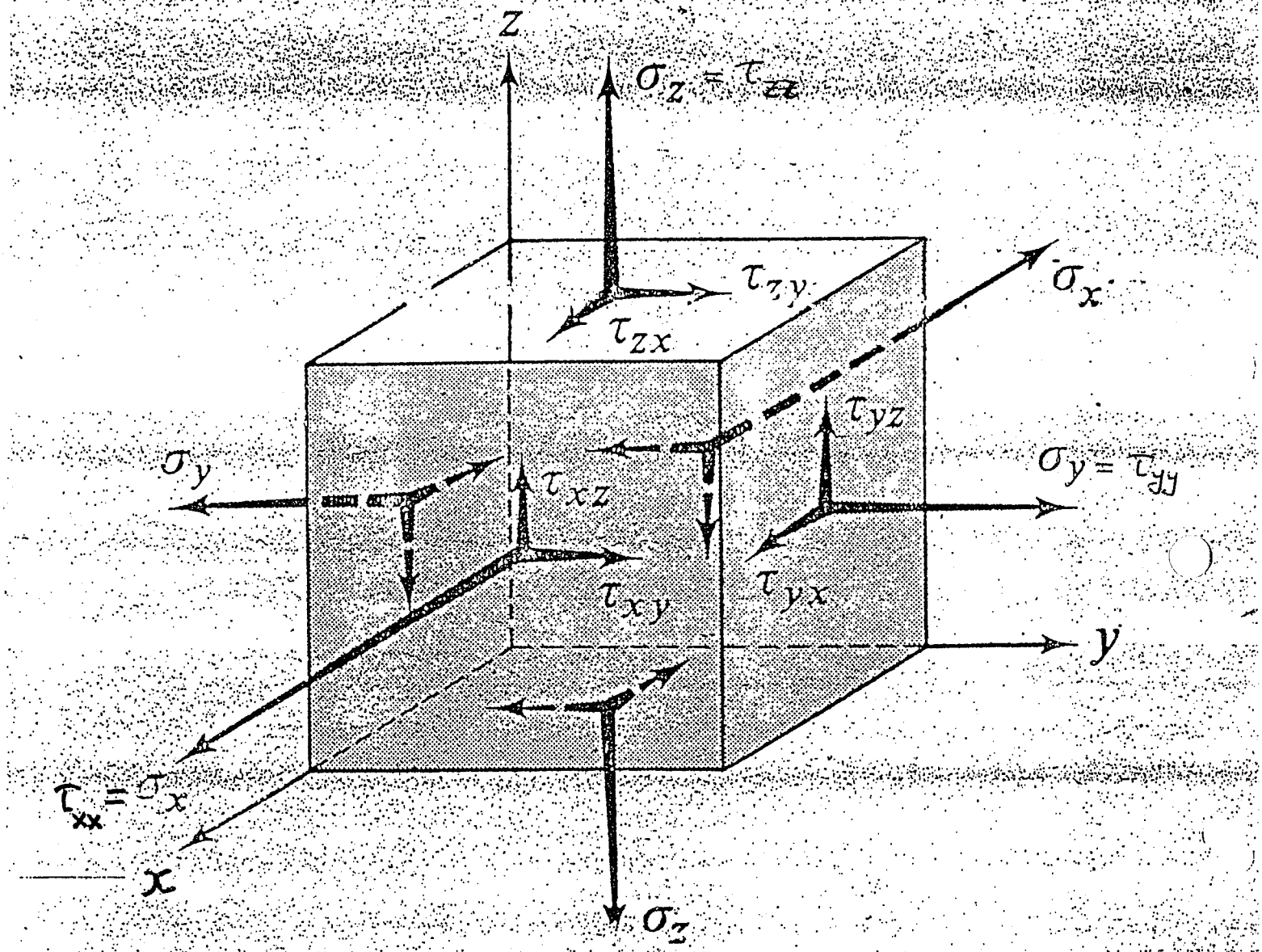



Fig. 1-3. The most general state of stress acting on an element

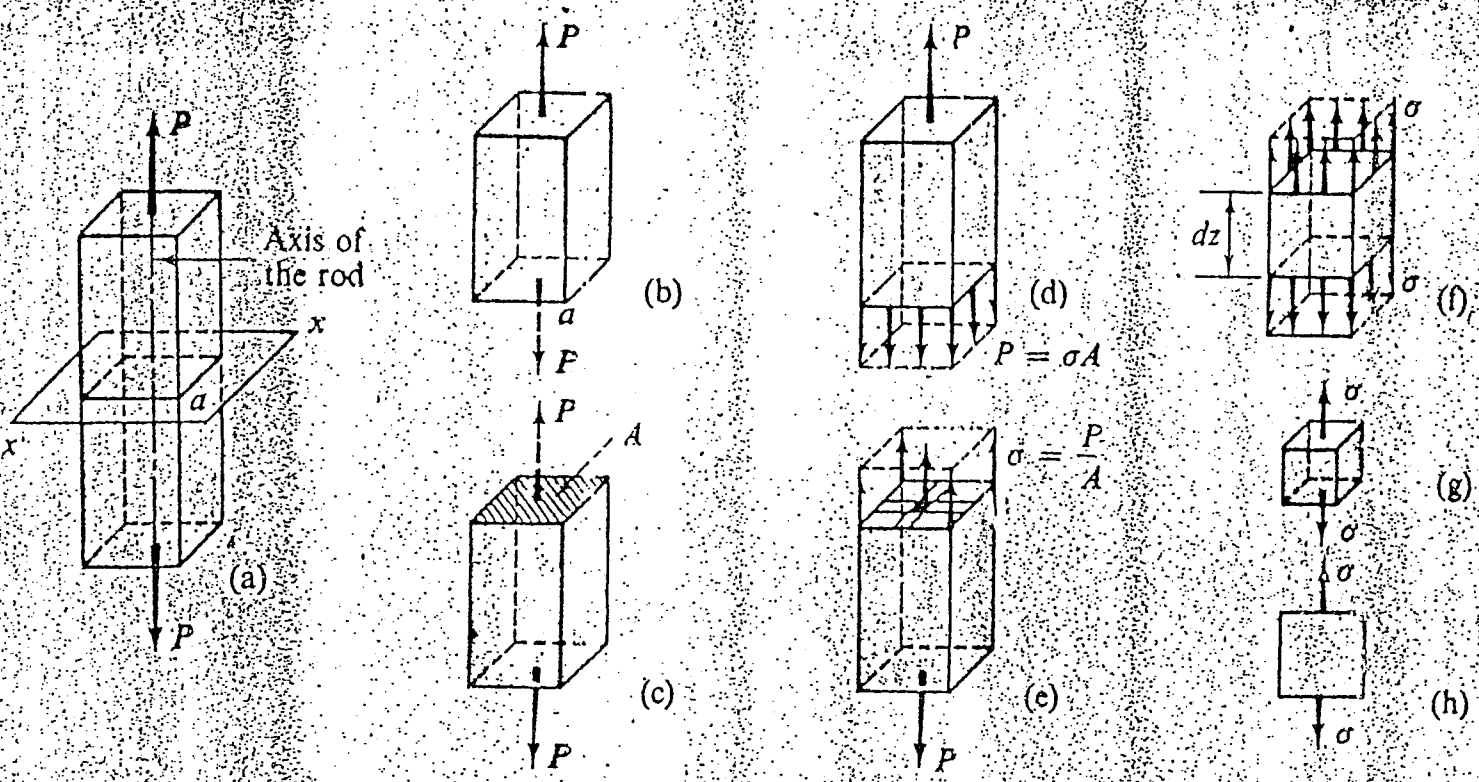
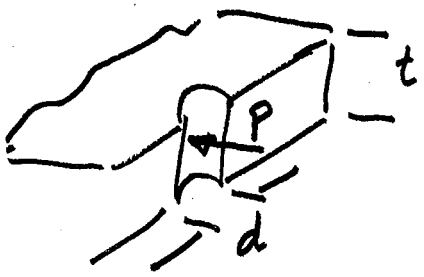



Fig. 1. Successive steps in the analysis of a body for stress

# BEARING STRESSES (p. 11)




$$\sigma_b = \frac{P}{td} = \frac{P}{A}$$

PROBLEM

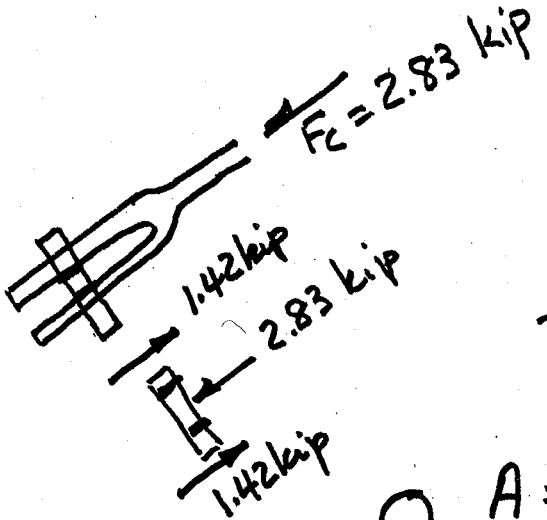
$\sigma_{AB}$

$\sigma_{BC}$

$(\sigma_b)_c$

$\tau_c$

$\tau$ or $\sigma$ units in SI $\frac{\text{force}}{\text{area}} = \frac{N}{m^2} = \text{pascal} = Pa$

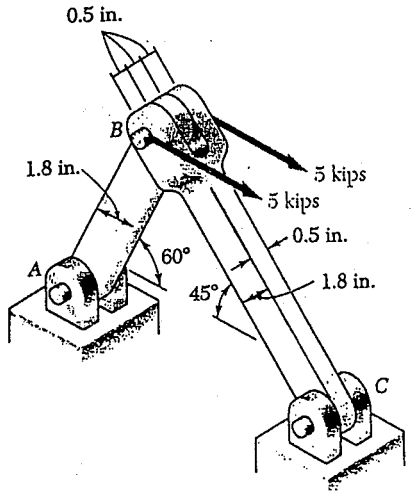



$$\tau = \frac{V}{A} = \frac{2.83}{2\left(\pi \frac{.375^2}{4}\right)}$$

$$A = \frac{\pi d^2}{4}$$

DO THIS OR POPOV PROB.

**PROBLEM 1.26**

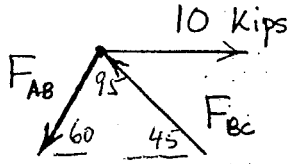


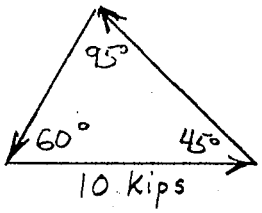

**1.9** Two horizontal 5-kip forces are applied to pin $B$ of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress ($a$) in link $AB$, ($b$) in link $BC$.

**1.26** For the assembly and loading of Prob. 1.9, determine ($a$) the average shearing stress in the pin at $C$, ($b$) the average bearing stress at $C$ in member $BC$, ($c$) the average bearing stress at $B$ in member $BC$.

**SOLUTION**

Use joint B as free body




Force triangle

Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{10}{\sin 95^\circ} \qquad F_{BC} = 8.9658 \text{ kips}$$

(a) Shearing stress in pin at C $\qquad \tau = \frac{F_{BC}}{2A_p}$

$$A_p = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.8)^2 = 0.5026 \text{ in}^2$$

$$\tau = \frac{8.9658}{(2)(0.5026)} = 8.92 \qquad 8.92 \text{ ksi}$$

(b) Bearing stress at C in member BC $\qquad \sigma_b = \frac{F_{BC}}{A}$



$$A = t\,d = (0.5)(0.8) = 0.4 \text{ in}^2$$

$$\sigma_b = \frac{8.9658}{0.4} = 22.4 \qquad 22.4 \text{ ksi}$$

(c) Bearing stress at B in member BC $\qquad \sigma_b = \frac{F_{BC}}{A}$

$$A = 2t\,d = 2(0.5)(0.8) = 0.8 \text{ in}^2$$

$$\sigma_b = \frac{8.9658}{0.8} = 11.21 \qquad 11.21 \text{ ksi}$$

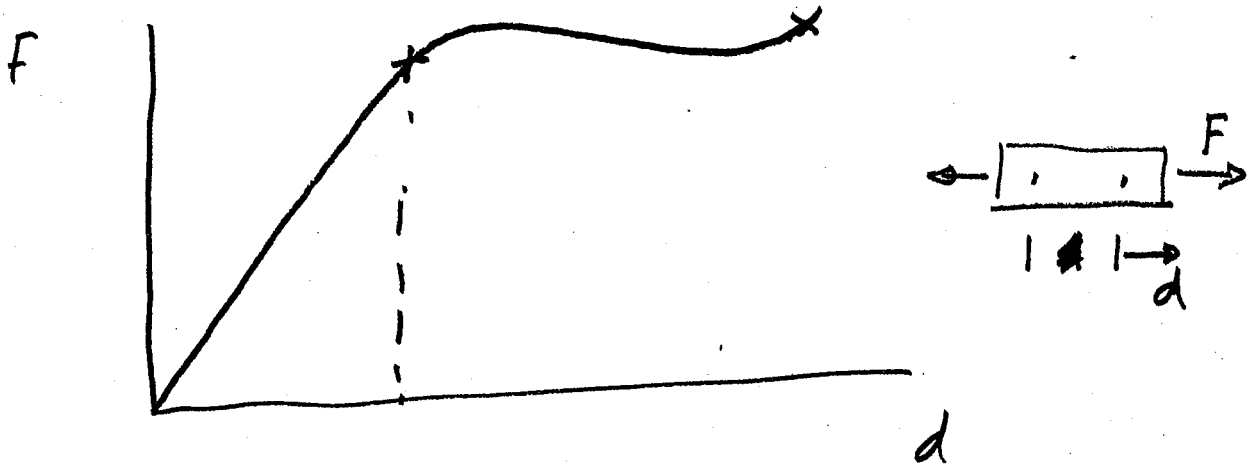
ULTIMATE STRENGTH (STRESS)

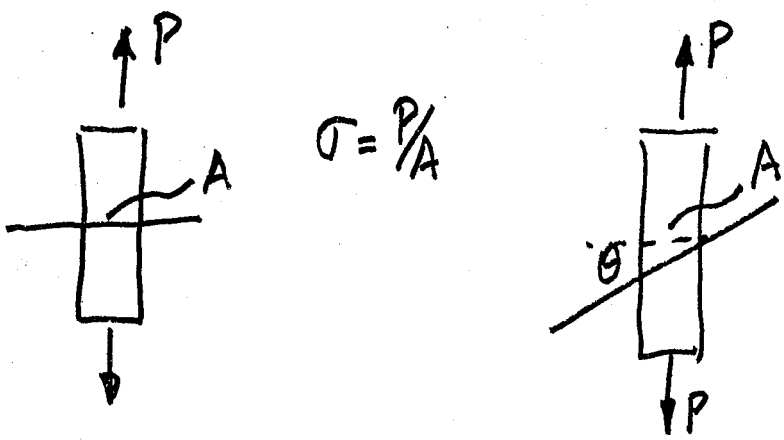
max stress that a material can withstand before failing

FACTOR OF SAFETY (FACTOR OF IGNORANCE)

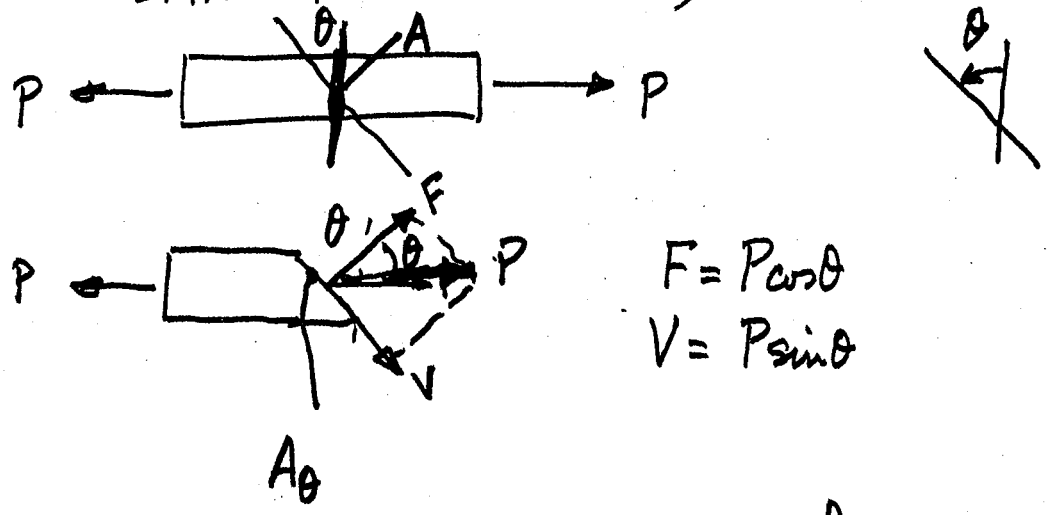
ALLOWABLE STRESS - max. stress allowed to occur on the job and that aren't exceeded

$$\text{ALLOWABLE STRESS} = \frac{\text{ULTIMATE STRENGTH}}{\text{F.S.}}$$



# EMA 3702 (TAPE 2)

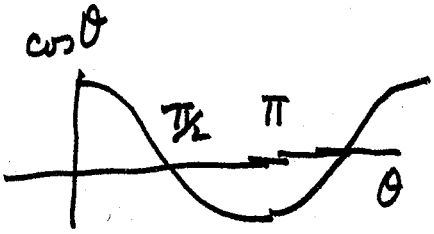



$F = P\cos\theta$

$V = P\sin\theta$

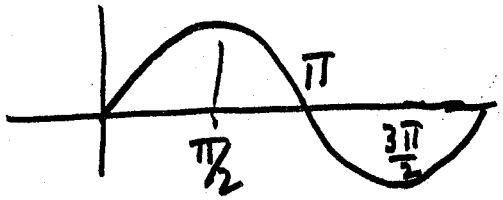
$\sigma = \frac{F}{A_\theta} \quad \tau = \frac{V}{A_\theta}$

$A_\theta = \frac{A}{\cos\theta}$

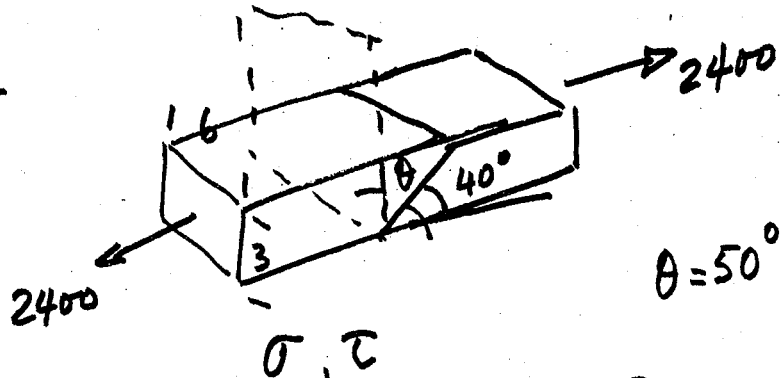



$$\sigma = \frac{P\cos\theta}{A/\cos\theta} = \frac{P}{A}\cos^2\theta = \sigma_x \cos^2\theta$$

$$\tau = \frac{P\sin\theta}{A/\cos\theta} = \frac{P}{A}\sin\theta\cos\theta = \frac{P}{2A}\sin 2\theta = \frac{\sigma_x}{2}\sin 2\theta$$




1.32



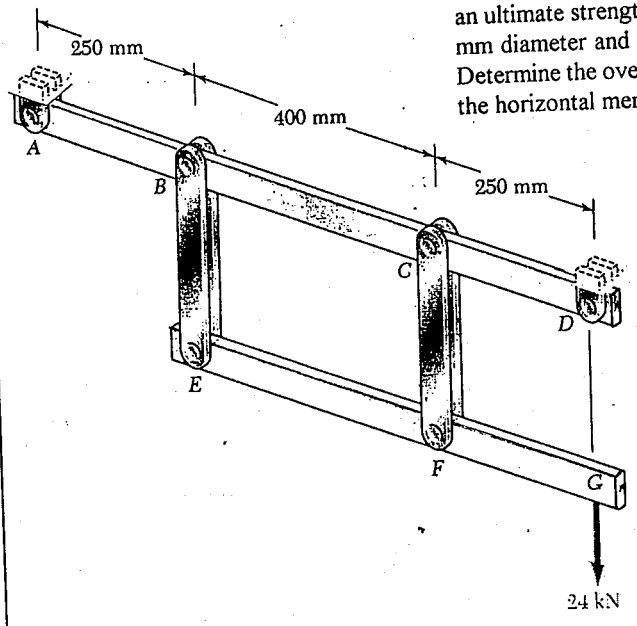

$\theta = 50^\circ \qquad A = 3\times 6 = 18\ in^2$

$$\sigma = \frac{P}{A}\cos^2\theta = \frac{2400}{18}\cos^2 50 \cong 55.1\ psi$$

$$\tau = \frac{P}{2A}\sin 2\theta = \frac{2400}{2\cdot 18\cdot 3}\sin 100 \cong 65.7\ psi$$

Do on 10/17 Problem 1.67 in 4th ed.

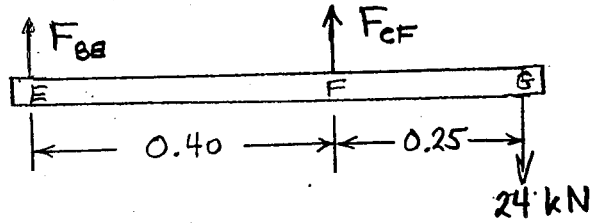
**PROBLEM 1.53**




**1.53** Each of the two vertical links $CF$ connecting the two horizontal members $AD$ and $EG$ has a 10 × 40- mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at $C$ and $F$ has a 20-mm diameter and is made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links $CF$ and the pins connecting them to the horizontal members.

**SOLUTION**

Use member EFG as free body.




$+\circlearrowleft \Sigma M_E = 0$

$0.40\, F_{CF} - (0.65)(24 \times 10^3) = 0$

$F_{CF} = 39 \times 10^3$ N, $F_{BE} = -15$ kN

Based on tension in links CF

$A = (b - d)t = (0.040 - 0.02)(0.010) = 200 \times 10^{-6}$ m² (one link)

$F_U = 2\sigma_U A = (2)(400 \times 10^6)(200 \times 10^{-6}) = 160.0 \times 10^3$ N

Based on double shear in pins

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}$ m²

$F_U = 2\tau_U A = (2)(150 \times 10^6)(314.16 \times 10^{-6}) = 94.248 \times 10^3$ N

Actual $F_U$ is smaller value, i.e. $F_U = 94.248 \times 10^3$ N

Factor of safety $F.S. = \frac{F_U}{F_{CF}} = \frac{94.248 \times 10^3}{39 \times 10^3} = 2.42$

$\tilde{E}_1 \quad \frac{.3}{4} \qquad \tilde{E}_2 \quad \frac{.5}{10}$

$$\frac{1}{E} = \frac{1}{E_1} + \frac{1}{E_2}$$

$$\frac{.5}{4} \qquad \frac{4}{.3} + \frac{10}{.5} = \frac{5}{.15} = \frac{1}{.03}$$

$$\frac{E_2 + E_1}{E_1 E_2} \qquad E = \frac{E_1 E_2}{E_1 + E_2} = \frac{\frac{.15}{40}}{\frac{5}{40}} = \frac{.15}{5} = \frac{.03}{1}$$

# Introduction to Elasticity

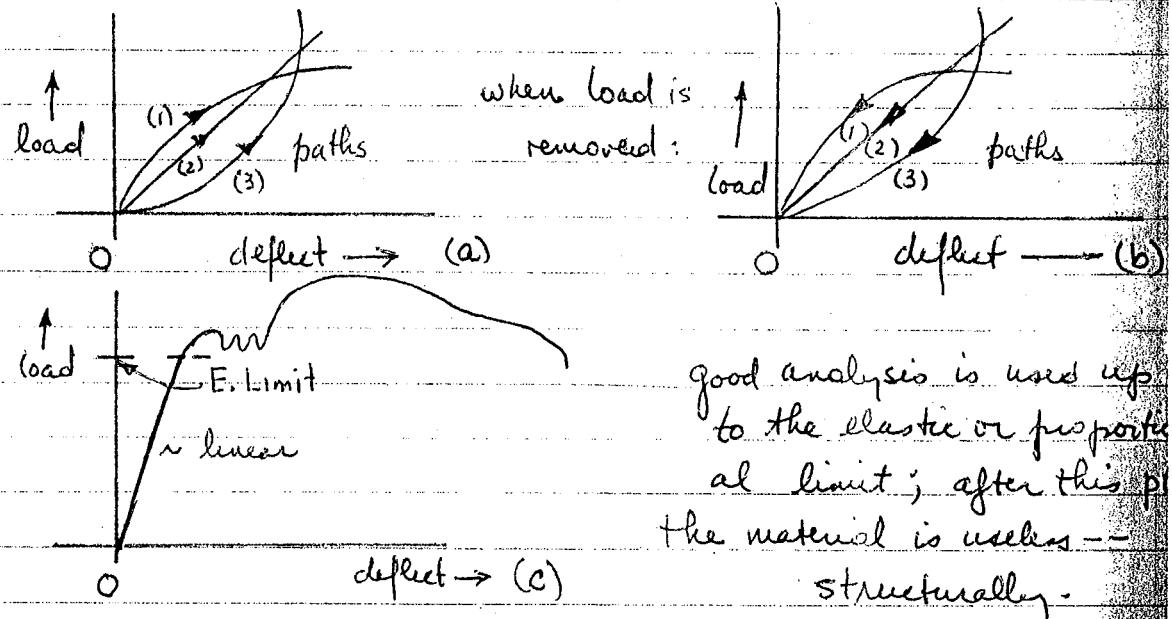
AE 311 Prof. Patel Rm 519

9/15/70

Elasticity - property of material by which it tends to return to its original state when the applied loads are removed.

Perfectly elastic materials return to original state after load has been removed

FIG. 1
9-15



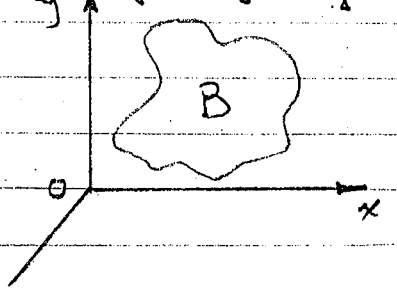

good analysis is used up to the elastic or proportional limit; after this p the material is useless -- structurally.

deformation

Strain - extension of the original element under applied load. It is independent of deflection curve.

strain = change in length to original length

FIG. 2
9-15




In studying the body B we make the following assumptions:

B is continuous material

homogeneous

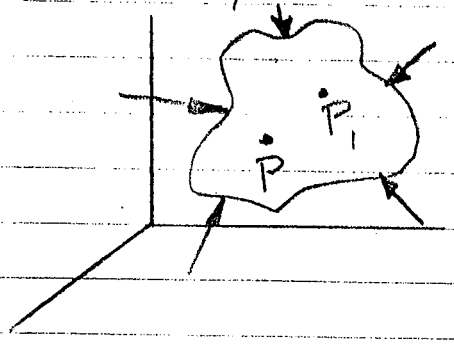
elastic

isotropic - Material properties is independent of direction.

מתכת הבנוי מגוש שריר ולכן התכונות האיזוטרופיות הינו אופייני

The material's crystal may not be isotropic but since crystal orientation may be very random in material the average behavior of the material may be isotropic

FIG. 3 9-15




$P(x,y,z)$

$P_1(x+u, y+v, z+w)$

$u = u(x,y,z)$

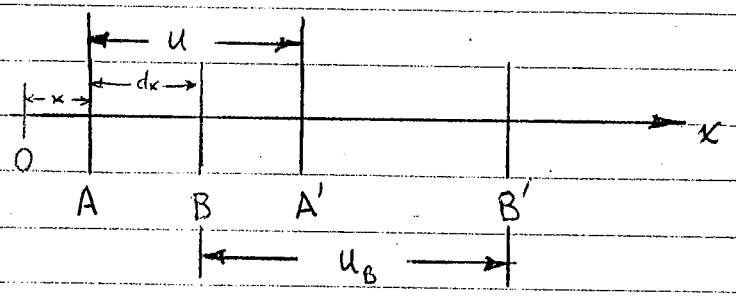
$v = v(x,y,z)$

$w = w(x,y,z)$

$u, v, w$ are continuous functions whose derivatives are also continuous.

## 1-D Case

FIG. 4 9-15




AB are displaced to A'B' along x

$u_B = u + du = u + \frac{du}{dx}dx$

$A = x$ $\quad A' = A + u = x + u$

$B = x + dx$ $\quad B' = x + dx + u_B$

$AB = dx$ $\quad A'B' = u_B - u = du = \frac{du}{dx}dx$

$u_B + dx - u = dx = u_B - u$

$$\epsilon_x = \frac{A'B' - AB}{AB} = \frac{du}{dx} \qquad 9\text{-}15.1$$

If $u(x,y,z)$ we write derivatives $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}$

Stress is defined as the change in force / change in unit area as the change in unit area goes to zero, or.

$$\text{stress} = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

9-17-70

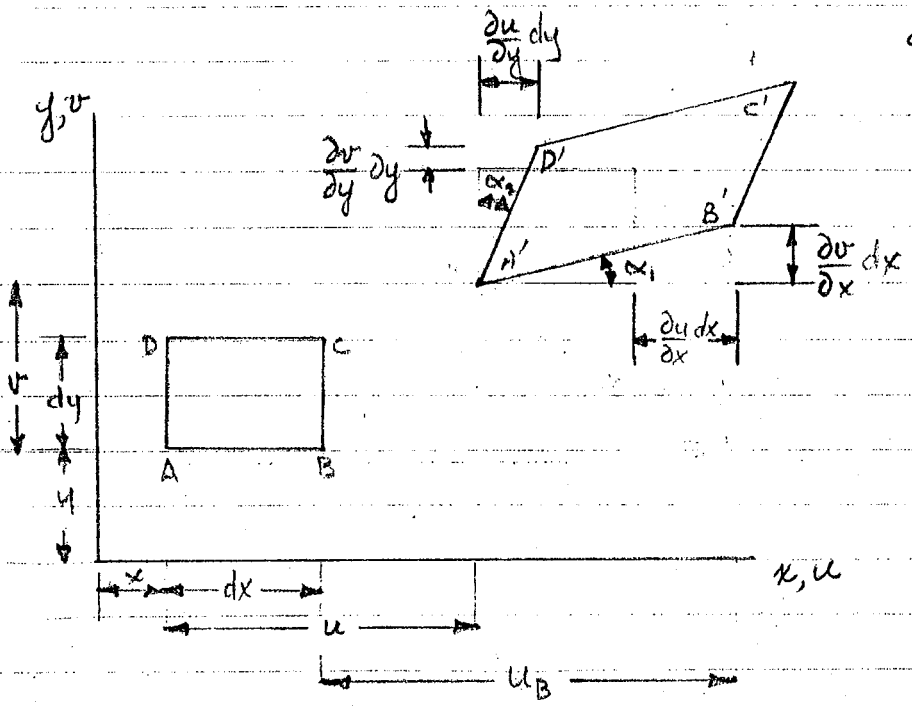



FIG 1
9-17

$$u_B = u + \frac{\partial u}{\partial x} dx \qquad \epsilon = du/dx \text{ in 1-D case.}$$

longitudinal strain

$$\epsilon_x = \frac{A'B' - AB}{AB} \qquad A'B' = (1+\epsilon_x) AB = (1+\epsilon_x) dx$$

$$\overline{A'B'}^2 = \left(dx + \frac{\partial u}{\partial x} dx\right)^2 + \left(\frac{\partial v}{\partial x} dx\right)^2$$

$$\overline{A'B'}^2 = (1 + 2\epsilon_x + \epsilon_x^2) dx^2 = dx^2 + 2\frac{\partial u}{\partial x} dx^2 + \left(\frac{\partial u}{\partial x}\right)^2 dx^2 + \left(\frac{\partial v}{\partial x}\right)^2 dx^2$$

thus $1 + 2\epsilon_x + \epsilon_x^2 = 1 + 2\frac{\partial u}{\partial x} + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2$ and $\overline{AB} = \sqrt{AB}$

$= \sqrt{1 + \frac{1}{2}(2$

assuming $\epsilon_x^2 \ll \epsilon_x$ and $\left|\frac{\partial u}{\partial x}\right|^2, \left|\frac{\partial v}{\partial x}\right|^2 \ll \left|\frac{\partial u}{\partial x}\right| \left|\frac{\partial v}{\partial x}\right|$ respectively

we can then neglect them. if so we obtain

9-17.1

$$\epsilon_x = \frac{\partial u}{\partial x} \qquad \epsilon_y = \frac{\partial v}{\partial y}$$

and in the n-direct $\epsilon_n = \frac{\partial u_n}{\partial n}$ can be gotten similarly.

Strain, then, along any direction = partial of the displacement along that direction, with respect to that direction (as shown above).

rigid body translation produces no longitudinal strains, $\epsilon_n = 0$.

$$\alpha_1 \approx \tan\alpha_1 = \frac{(\partial v/\partial x)\,dx}{\left(1+\frac{\partial u}{\partial x}\right)dx} = \partial v/\partial x$$

because $\left(\frac{\partial u}{\partial x}\right)^2 << \frac{\partial u}{\partial x} << 1$ מכיוון ש $\left(\frac{\partial u}{\partial x}\right)^2$ זניח ביחס ל $\left(\frac{\partial u}{\partial x}\right)$

$\partial u/\partial x$ can be neglected therefore $\alpha_1 \approx \tan\alpha_1 \approx \frac{\partial v}{\partial x}$

$$\alpha_2 \approx \tan\alpha_2 \approx \frac{(\partial u/\partial y)\,dy}{\left(1+\frac{\partial v}{\partial y}\right)dy} \approx \frac{\partial u}{\partial y}$$

$$\alpha_1 + \alpha_2 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad \text{and} \quad \gamma_{xy} = \alpha_1 + \alpha_2$$

עיקר ניתה
עיבור הצירים

9-17.2

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

Shear strain is by definition an increase in $\gamma$ or a decrease in the right $\measuredangle$.

## Compatebility of Strains. תנאים התאמת

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = 0$$

Proof: $$\frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial^3 v}{\partial x^2 \partial y} - \left( \frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial^3 v}{\partial x^2 \partial y} \right) \equiv 0$$

9-17.3

$$\epsilon_x = \frac{\partial u}{\partial x}, \quad \epsilon_y = \frac{\partial v}{\partial y}, \quad \epsilon_z = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \quad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

The assumptions we have made are:

1. Material continuous — תווי רצף
2. homogeneous — הומוגני
3. Isotropic — איזוטרופי
4. Elastic — אלסטי
5. continuous functions & derivatives. הנגזרות הם פונקציות רציפות וגם אין הגבירה שלהם

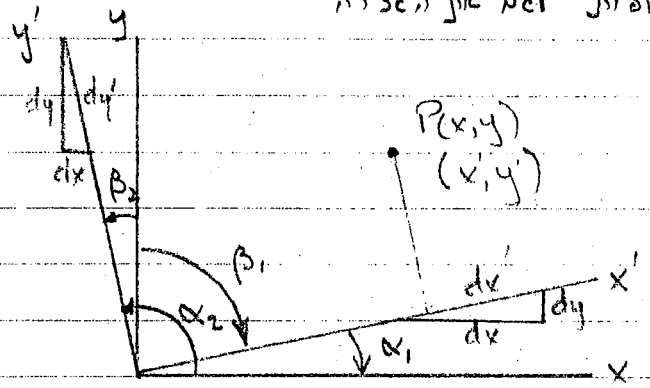



FIG. 2
9-17

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\boxed{\begin{aligned} x' &= x\cos\alpha_1 + y\sin\alpha_1 \\ y' &= y\cos\alpha_1 - x\sin\alpha_1 \end{aligned}} \rightarrow \begin{aligned} x &= x'\cos\alpha - y'\sin\alpha \\ y &= x'\sin\alpha + y'\cos\alpha \end{aligned}$$

if $\cos\alpha_1 = l_1 \qquad \cos\alpha_2 = l_2$

$\cos\beta_1 = m_1 \qquad \cos\beta_2 = m_2$

since $\alpha_1 = -\beta_2 \qquad \beta_2 + \beta_1 + \alpha_1 = -\alpha_2$

$\cos\alpha_1 = \cos(-\beta_2) = \cos\beta_2 \qquad \beta_2 = \alpha_1 \qquad \underline{l_1 = m_2}$

since $\beta_2 + \beta_1 + \alpha_1 = 90^\circ + \alpha_1$ then

$\cos(\alpha_2) = \cos(\alpha_2) = \cos(90^\circ + \alpha_1) = -\sin\alpha_1 = -m_1$

$\underline{l_2 = -m_1}$

$$\frac{dx}{dx'} = \cos\alpha_1 = l_1 \qquad \frac{dy}{dx'} = \sin\alpha_1 = m_1$$

$$\frac{dx}{dy'} = +\sin\beta_2 = +l_2 = -m_1 \qquad \frac{dy}{dy'} = l_1$$

$l_1^2 + m_1^2 = 1 \quad ((\cos\alpha_1)^2 + (\sin\alpha_1)^2 = 1)$

$l_2^2 + m_2^2 = 1 \quad ((\cos\alpha_2)^2 + (\sin\alpha_2)^2 = 1)$

$l_1^2 + l_2^2 = 1$

$m_2^2 + m_1^2 = 1$

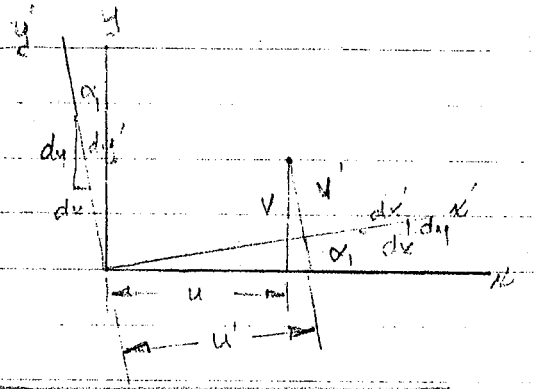
$l_1 m_1 + l_2 m_2 = 0$

$l_1 l_2 + m_1 m_2 = 0$

9-22-70

| | x | y |
|---|---|---|
| x' | $l_1$ | $m_1$ |
| y' | $l_2 = -m_1$ | $m_2 = l_1$ |

FIG 1 9-22




$$u' = l_1 u + m_1 v = \cos\alpha\, u + \sin\alpha\, v$$
$$v' = -m_1 u + l_1 v = -\sin\alpha\, u + \cos\alpha\, v$$

$\cos\alpha_1 = l_1$ ; $\sin\alpha_1 = m_1$

$$\frac{dx}{dx'} = \frac{dy}{dy'} = l_1 \qquad \frac{dy}{dx'} = -\frac{dx}{dy'} = m_1$$

$$\epsilon_x = \frac{\partial u}{\partial x} \qquad \epsilon_y = \frac{\partial v}{\partial y} \qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\epsilon_{x'} = \frac{\partial u'}{\partial x'} = \frac{\partial}{\partial x'}(l_1 u + m_1 v)$$

$$\frac{\partial}{\partial x'}(\quad) = \frac{\partial}{\partial x}(\quad)\frac{dx}{dx'} + \frac{\partial}{\partial y}(\quad)\frac{dy}{dx'}$$

$$\epsilon_{x'} = \underbrace{l_1}_{\cos\alpha}\left(\frac{\partial u}{\partial x}\underbrace{\frac{dx}{dx'}}_{l=\cos\alpha} + \frac{\partial u}{\partial y}\underbrace{\frac{dy}{dx'}}_{m=\sin\alpha}\right) + \underbrace{m_1}_{\sin\alpha}\left(\frac{\partial v}{\partial x}\underbrace{\frac{dx}{dx'}}_{l=\cos\alpha} + \frac{\partial v}{\partial y}\underbrace{\frac{dy}{dx'}}_{m=\sin\alpha}\right)$$

$$= l_1^2\frac{\partial u}{\partial x} + m_1^2\frac{\partial v}{\partial y} + l_1 m_1\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}\right)$$

$$\epsilon_{x'} = l_1^2\epsilon_x + m_1^2\epsilon_y + l_1 m_1 \gamma_{xy}$$

$$\epsilon_{x'} = \epsilon_x \underbrace{\cos^2\alpha_1}_{\frac{1+\cos 2\alpha}{2}} + \epsilon_y \underbrace{\sin^2\alpha_1}_{\frac{1-\cos 2\alpha}{2}} + \gamma_{xy}\underbrace{\sin\alpha_1\cos\alpha_1}_{\frac{\sin 2\alpha}{2}}$$

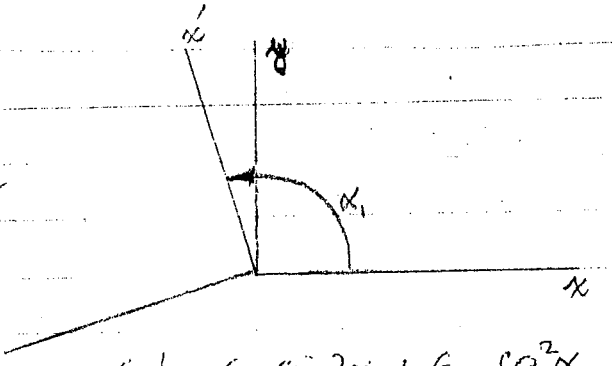
9-22.1

$$\epsilon_{x'} = \frac{\epsilon_x+\epsilon_y}{2} + \frac{\epsilon_x-\epsilon_y}{2}\cos 2\alpha_1 + \frac{\gamma_{xy}}{2}\sin 2\alpha_1$$

$\epsilon_{xy}$

FIG. 2

9-22




$$\epsilon_{y'} = \epsilon_x \sin 2\alpha_1 + \epsilon_y \cos^2\alpha_1 - \gamma_{xy} \sin\alpha_1 \cos\alpha_1$$

9-22.2

$$\boxed{\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\alpha_1 - \frac{\gamma_{xy}}{2}\sin 2\alpha_1}$$

$$\gamma_{x'y'} = \frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'}$$

$$= \frac{\partial u'}{\partial x}\frac{dx}{dy'} + \frac{\partial u'}{\partial y}\frac{dy}{dy'} + \frac{\partial v'}{\partial x}\frac{dx}{dx'} + \frac{\partial v'}{\partial y}\frac{dy}{dx'}$$

$$= -\left(l_1\frac{\partial u}{\partial x} + m_1\frac{\partial v}{\partial x}\right)m_1 + \left(-m_1\frac{\partial u}{\partial x} + l_1\frac{\partial v}{\partial x}\right)l_1$$

$$+ \left(l_1\frac{\partial u}{\partial y} + m_1\frac{\partial v}{\partial y}\right)l_1 + \left(-m_1\frac{\partial u}{\partial y} + l_1\frac{\partial v}{\partial y}\right)m_1$$

$$= -2l_1m_1\frac{\partial u}{\partial x} + 2l_1m_1\frac{\partial v}{\partial y} + l_1^2\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - m_1^2\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

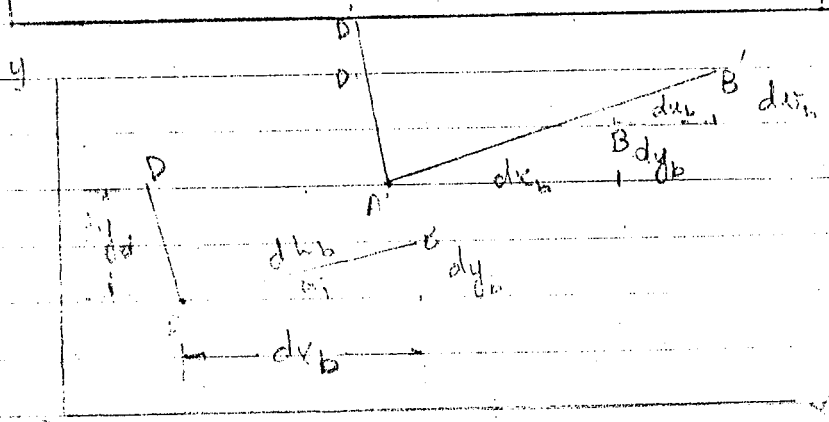
$$\gamma_{x'y'} = -2(\epsilon_x - \epsilon_y)\sin\alpha_1\cos\alpha_1 + (\cos^2\alpha_1 - \sin^2\alpha_1)\gamma_{xy}$$

9-22.3

$$\boxed{\gamma_{x'y'} = -(\epsilon_x - \epsilon_y)\sin 2\alpha_1 + \gamma_{xy}\cos 2\alpha_1}$$

FIG. 3

9-22



$$\epsilon_{x'} = \frac{A'B' - AB}{AB}$$

$$(1 + \epsilon_{x'}) = A'B'/AB \qquad \overline{A'B'}^2 = (dx_b + du_b)^2 + (dy_b + dv_b)^2$$

$$= dx_b^2 + 2dx_b dy_b + du_b^2 + dy_b^2 + 2dy_b dv_b + dv_b^2$$

$$u_B = u + \frac{\partial u}{\partial x} dx_b + \frac{\partial u}{\partial y} dy_b$$

$$(1 + 2\epsilon_{x'} + \epsilon_{x'}^2) dL_b^2 = dL_b^2 + 2\left(\frac{\partial u}{\partial x} dx_b^2 + \frac{\partial u}{\partial y} dx_b dy_b\right) + 2\left(\frac{\partial v}{\partial x} dx_b dy_b + \frac{\partial v}{\partial y} dy_b^2\right) + \left(\frac{\partial u}{\partial x} dx_b + \frac{\partial u}{\partial y} dy_b\right)^2 + \left(\frac{\partial v}{\partial x} dx_b + \frac{\partial v}{\partial y} dy_b\right)^2$$

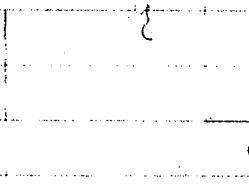
neglect these terms since they are small

$$\frac{dx_b}{dL_b} = l_1 \qquad \frac{dy_b}{dL_b} = m_1$$

$$\epsilon_{x'} = l_1^2 \frac{\partial u}{\partial x} + m_1^2 \frac{\partial v}{\partial y} + l_1 m_1 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\epsilon_{x'} = l_1^2 \epsilon_x + m_1^2 \epsilon_y + \gamma_{xy} (l_1 m_1)$$

FIG. 4




9-22

$$\frac{\partial \epsilon_{x'}}{\partial(\alpha_1)} = \frac{\epsilon_x - \epsilon_y}{2} (-2\sin 2\alpha_1) + \frac{\gamma_{xy}}{2} (2\cos 2\alpha_1) = 0$$

9-22.4

$$\boxed{\tan 2\alpha_1 = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} \equiv \tan 2\alpha = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}}$$

when no shear strain

$$\tan 2\alpha = \tan(\pi + 2\alpha); \quad \alpha = \frac{\pi}{2} + \alpha$$

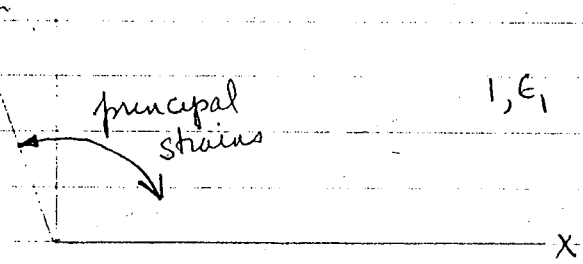
$$\sin 2\alpha = \pm (\epsilon_x - \epsilon_y) / \left[(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2\right]^{1/2}$$

$$\sin 2\alpha = \pm \gamma_{xy} / \left[ (\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2 \right]^{1/2}$$

$$(\epsilon_{x'})_{\substack{\text{max, min} \\ 1,2}} = \frac{\epsilon_x + \epsilon_y}{2} \pm \frac{1}{2} \left[ (\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2 \right]^{1/2}$$

$$(\gamma_{x'y'})_{\epsilon_{x'}\ \text{max, min}} = \cos 2\alpha_1 \left[ -(\epsilon_x - \epsilon_y) \tan 2\alpha_1 + \gamma_{xy} \right] = 0$$

FIG. 5
9-22




9-22.5

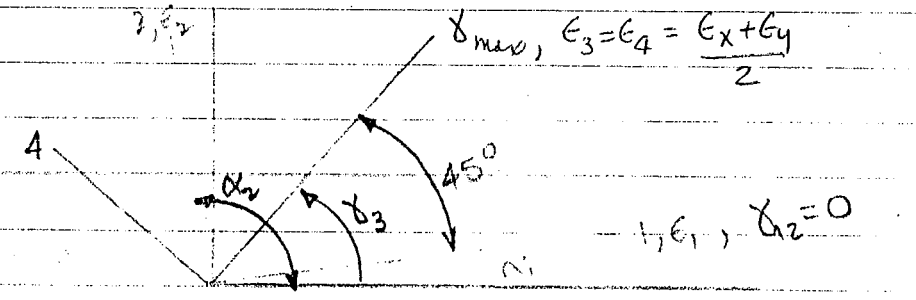
$$\boxed{\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \frac{1}{2} \left[ (\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2 \right]^{1/2} \qquad \gamma_{1,2} = 0} \qquad \tan 2\alpha = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

$$\frac{\partial \gamma_{x'y'}}{\partial \alpha_1} = -2(\epsilon_x - \epsilon_y)\cos 2\alpha_1 - 2\gamma_{xy} \sin 2\alpha_1 = 0$$

9-22.6

$$\boxed{\tan 2\alpha_1 = -\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}}}$$

FIG. 6
9-22




$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\cos 2\alpha_1}{2} \left[ (\epsilon_x - \epsilon_y) + \gamma_{xy} \tan 2\alpha_1 \right]$$

9-22.7

$$\boxed{\gamma_{max} = \pm \left[ (\epsilon_x - \epsilon_y)^2 + (\gamma_{xy})^2 \right]^{1/2}}$$

$$\epsilon_1 = \frac{\epsilon_x + \epsilon_y}{2} + \frac{1}{2}\left[(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2\right]^{1/2}$$

$\epsilon_2 =$ " $-$ "

$\gamma_{1,2} = 0.$ $\qquad \tan 2\alpha = \dfrac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$

9-22.8

$$\overline{\gamma_{max}} = \left[(\epsilon_x - \epsilon_y)^2 + \overline{\gamma_{xy}^2}\right]^{1/2}$$

$$\epsilon_x' = \epsilon_y' = \frac{\epsilon_x + \epsilon_y}{2} \qquad \tan 2\alpha = -\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}}$$

$I_{1\epsilon}$ $\boxed{\epsilon_x + \epsilon_y = \epsilon_1 + \epsilon_2}$ unchanged no matter what $\alpha_1$ is

Invariant at a point.

$I_{2\epsilon}$ $\boxed{\epsilon_x \epsilon_y - \frac{1}{4}\gamma_{xy}^2 = \epsilon_x'\epsilon_y' - \frac{1}{4}\gamma_{x'y'}^2}$ Invariant

9-24-70

2-D $\begin{cases} \epsilon_x, \epsilon_y \\ \gamma_{xy} \end{cases}$ function of $u, v$

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = 0$$

3 differential eq.
2 unknowns
1 Compatability Eq.

| | x | y |
|---|---|---|
| x' | $l_1$ | $m_1$ |
| y' | $l_2$ | $m_2$ |

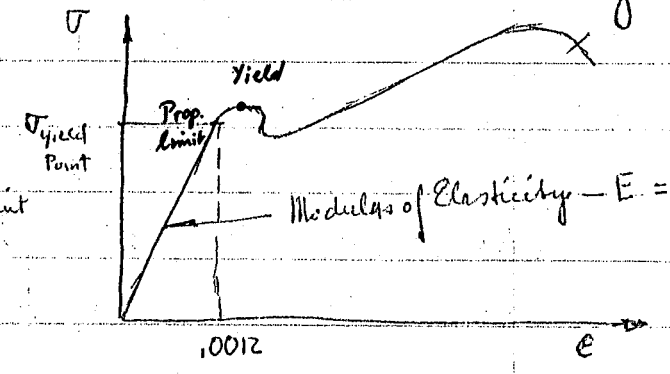
3-D $\begin{cases} \epsilon_x, \epsilon_y, \epsilon_z \\ \gamma_{xy}, \gamma_{yz}, \gamma_{zx} \end{cases}$ function $(u, v, w)$

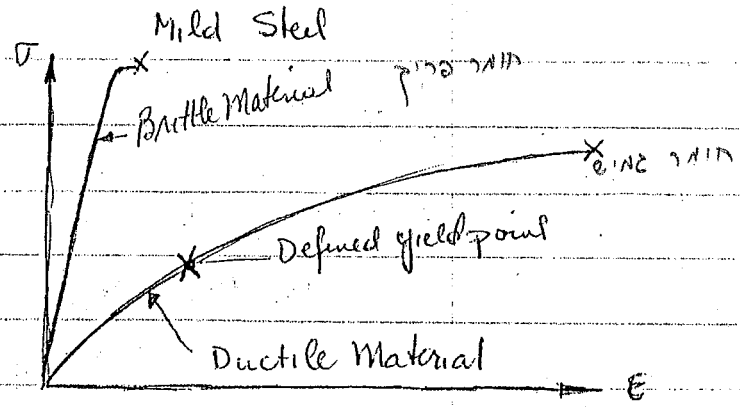
6 differential eq.
3 unknowns
6 Compatability Eq.
(1.48)

| | x | y | z |
|---|---|---|---|
| x' | $l_1$ | $m_1$ | $n_1$ |
| y' | $l_2$ | $m_2$ | $n_2$ |

$$\begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix} \equiv \begin{pmatrix} \varepsilon_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{yx}}{2} & \varepsilon_y & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{zx}}{2} & \frac{\gamma_{zy}}{2} & \varepsilon_z \end{pmatrix}$$

Constitutive Laws – mostly empirical laws




## Hooke's Law

$\tau = C\varepsilon$ (stress.) $\qquad \varepsilon = A\tau$ depends on material

## Principle of Superposition

Can add effects of individual loadings to get effect of resultant loading.

true if 1) each strain must be directly and linearly related to the stress causing the strain (Hooke's law)

2) Strain (deformation) due to one load has negligible effect on the conditions of application of the other load. ~~Stress~~ any other stress

**2.7.** A straight piece of wire $PQ$ lies along the line $y = mx$ as shown in Fig. P2.7. The wire is strained and displaced to the line $y = nx$ in such a way that a point originally at $x$ is displaced to $x^2/2$. Show that the extensional strain at any point along the wire is given by

$$\epsilon = \sqrt{\frac{1 + n^2}{1 + m^2}}\, x - 1$$

(where $x$ is the original coordinate of the point).

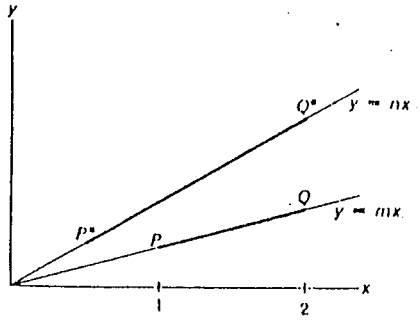



Fig. P2.7

2.16. The point $C$ is displaced to $C^*$ as shown in Fig. P2.16. The horizontal and vertical components of this displacement are $u$ and $v$. Express the average extensional strains of $AC$ and $BC$ in terms of $u$, $v$ and $L$. If $u$ and $v$ are small, what are the approximate expressions for these average extensional strains?

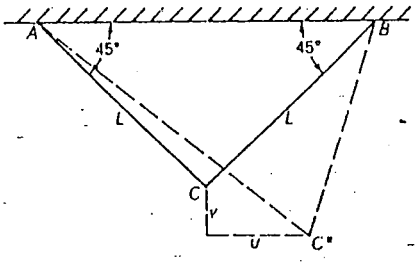



Fig. P2.16

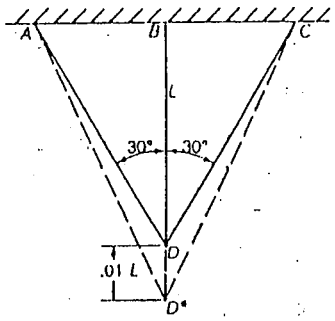



Fig. P2.17

2.17. The point $D$ in Fig. P2.17 is displaced vertically an amount $.01L$. Obtain the approximate (first-order) average extensional strain of $AD$, $BD$, and $CD$.

2.21. The bar $AB$ is rigid and is supported by two wires $DB$ and $CB$ as shown in Fig. P2.21. If the bar rotates through a small angle $\theta$, determine the average extensional strain of $DB$ and $CB$.

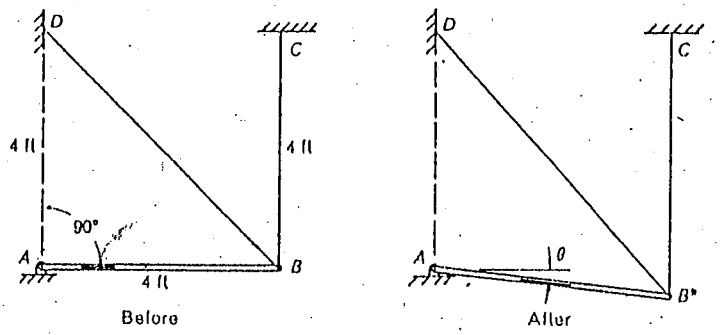



Fig. P2.21

2.32. A thin triangular plate is deformed as indicated in Fig. P2.32. Compute approximately the shear strain $\gamma_{st}(P)$.

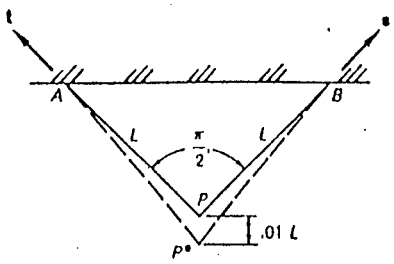



Fig. P2.32

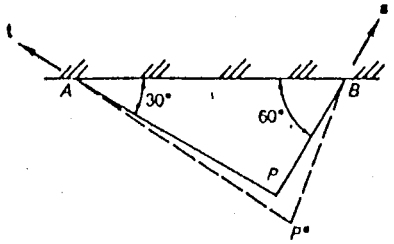



Fig. P2.34

2.34. A thin plate is in the form of a 30°–60°–90° triangle and is supported along its hypotenuse as shown in Fig. P2.34. It is subjected to the uniform strains $\epsilon_s = .004$, $\epsilon_t = .003$. Compute the approximate value of $\gamma_{st}(P)$.

2.37. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.37. It elongates 10 per cent of its original length and contracts 3 per cent of its original width. Compute approximately the shear strain $\gamma_{st}$.

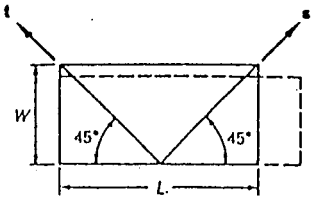



Fig. P2.37

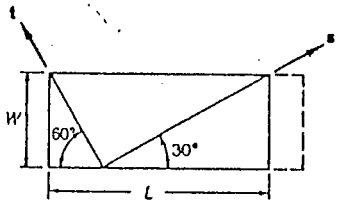



Fig. P2.38

2.38. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.38. The extensional strain along its length is .02 and the width remains unchanged. Compute approximately the shear strain $\gamma_{st}$.

Answers:

2.16 $\epsilon_{AC} = \dfrac{\sqrt{L^2+\sqrt{2}L(v+u)+u^2+v^2}-L}{L}$; $\epsilon_{AC} \approx \dfrac{\sqrt{2}(u+v)}{2L}$, $\epsilon_{BC} \approx \dfrac{\sqrt{2}(u-v)}{2L}$

2.17 $\epsilon_{AD} = \epsilon_{CD} = 0.00754$; $\epsilon_{BD} = 0.01$

2.21 $\epsilon_{DB} = \theta/2$; $\epsilon_{CB} = \theta$

2.32 $\sqrt{2}(0.01)$

2.34 0.0075

2.37 $-0.125$ $\epsilon_x = .1$ $\epsilon_y = -.03$ $\gamma_{xy} = 0$, $\alpha = \frac{\pi}{4}$ $\gamma_{st} = -\gamma_{xy}\cos 2\theta - (\epsilon_x - \epsilon_y)\sin 2\theta = -.125$

2.38 $-0.01\sqrt{3}$

1. The stress distribution on the rectangular cross-section shown in Fig. 1 is given by $\sigma_{xx}=1000y-500z+800$ kPa, $\sigma_{xy}=200z$ kPa, $\sigma_{xz}=0$.
What is the net internal force system on this cross-section ?

Answer : $F=1920$ N, $V_y=V_z=0$
$M_y=-1600$ N-cm., $M_z=-7200$ N-cm., $T=-640$ N-cm.

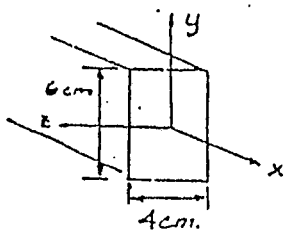


Fig. 1

2. Suppose the stress distribution on a cross-section of the circular cylinder of Fig. 2 is given by $\sigma_{xx}=\sigma_{xr}=0$, $\sigma_{x\theta}=k\sqrt{r}$, where k is unknown. What is the value of k in this case?

Answer : $k=7T_0/4\pi R^{7/2}$

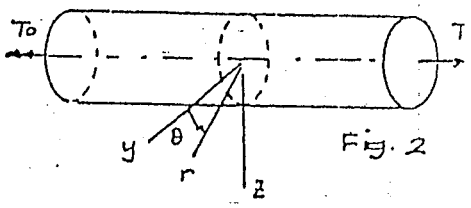


Fig. 2

6.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium equations of that element, as was done in the derivations presented in Sec. 6.2.

6.6 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

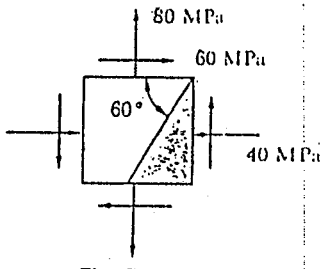


Fig. P6.4

6.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

6.14 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 40° counterclockwise, (*b*) 15° clockwise.

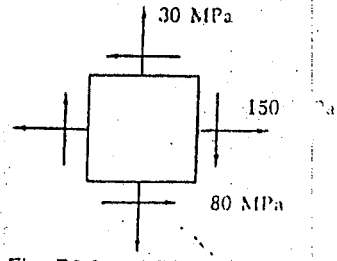
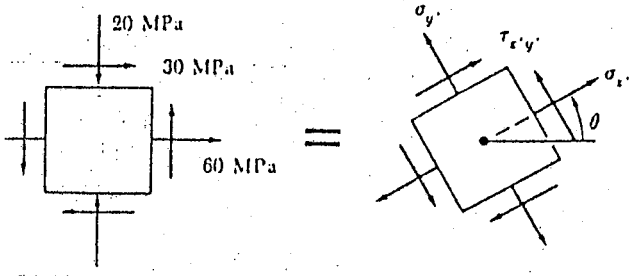


Fig. P6.6 and P6.10

6.56 For the state of plane stress shown, determine the range of values of $\theta$ for which the normal stress $\sigma_x$ is equal to or less than +65 MPa.



P6.55 and P6.56

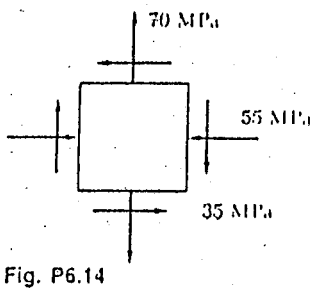


Fig. P6.14

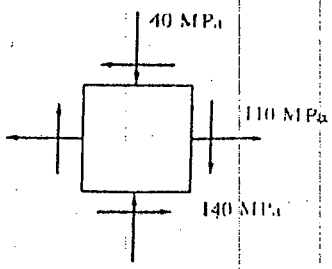


Fig. P6.8 and P6.12

25
1/2

**2.7.** A straight piece of wire $PQ$ lies along the line $y = mx$ as shown in Fig. P2.7. The wire is strained and displaced to the line $y = nx$ in such a way that a point originally at $x$ is displaced to $x^2/2$. Show that the extensional strain at any point along the wire is given by

$$\epsilon = \sqrt{\frac{1 + n^2}{1 + m^2}}\, x - 1$$

(where $x$ is the original coordinate of the point).

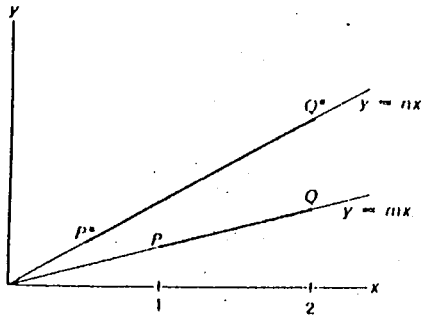



Fig. P2.7

**2.16.** The point $C$ is displaced to $C^*$ as shown in Fig. P2.16. The horizontal and vertical components of this displacement are $u$ and $v$. Express the average extensional strains of $AC$ and $BC$ in terms of $u$, $v$ and $L$. If $u$ and $v$ are small, what are the approximate expressions for these average extensional strains?

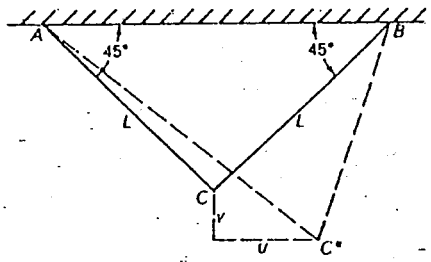



Fig. P2.16

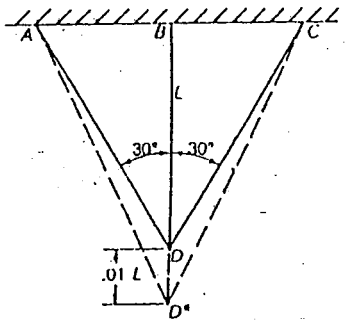
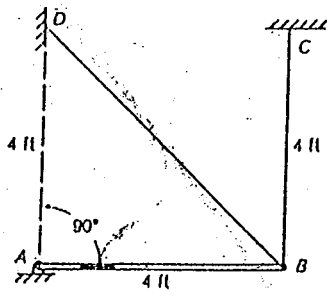


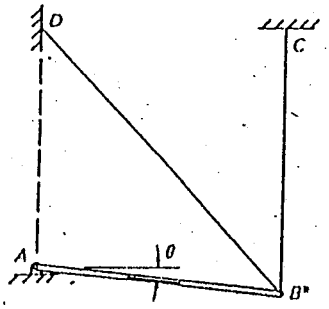

Fig. P2.17

**2.17.** The point $D$ in Fig. P2.17 is displaced vertically an amount $.01L$. Obtain the approximate (first-order) average extensional strain of $AD$, $BD$, and $CD$.

**2.21.** The bar $AB$ is rigid and is supported by two wires $DB$ and $CB$ as shown in Fig. P2.21. If the bar rotates through a small angle $\theta$, determine the average extensional strain of $DB$ and $CB$.




Before




After

Fig. P2.21

2.32. A thin triangular plate is deformed as indicated in Fig. P2.32. Compute approximately the shear strain $\gamma_{st}(P)$.

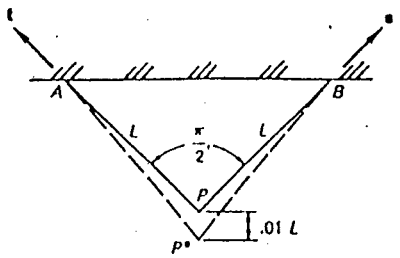



Fig. P2.32

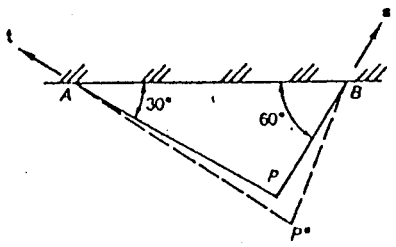



Fig. P2.34

2.34. A thin plate is in the form of a 30°–60°–90° triangle and is supported along its hypotenuse as shown in Fig. P2.34. It is subjected to the uniform strains $\epsilon_s = .004$, $\epsilon_t = .003$. Compute the approximate value of $\gamma_{st}(P)$.

2.37. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.37. It elongates 10 per cent of its original length and contracts 3 per cent of its original width. Compute approximately the shear strain $\gamma_{st}$.

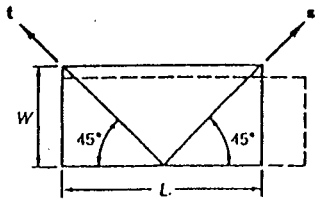



Fig. P2.37

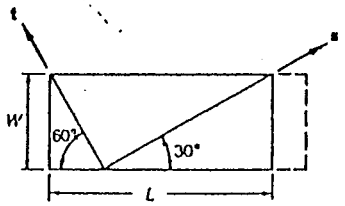



Fig. P2.38

2.38. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.38. The extensional strain along its length is .02 and the width remains unchanged. Compute approximately the shear strain $\gamma_{st}$.

2.32. A thin triangular plate is deformed as indicated in Fig. P2.32. Compute approximately the shear strain $\gamma_{st}(P)$.

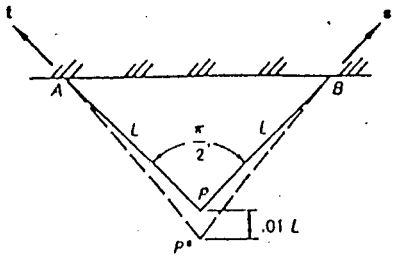


Fig. P2.32

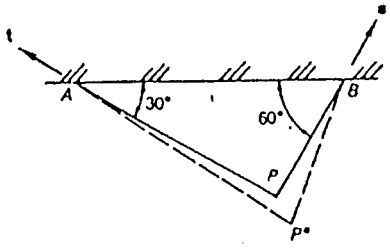


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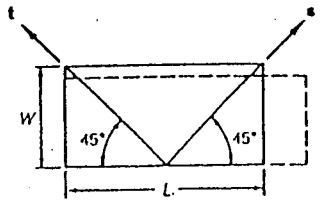


Fig. P2.37

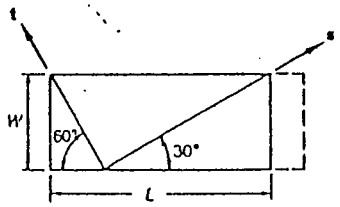


Fig. P2.38

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(where $x$ is the original coordinate of the point).

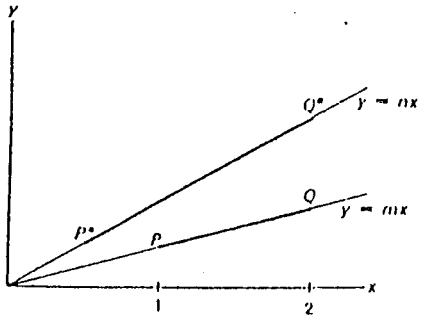



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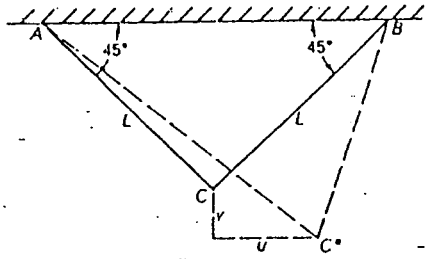



Fig. P2.16

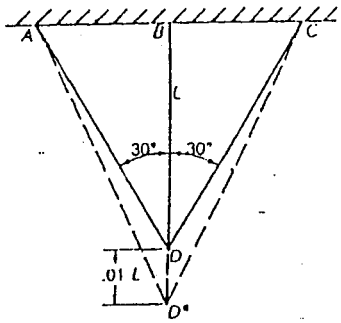
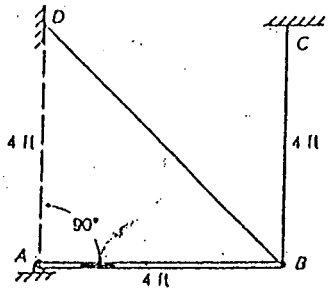


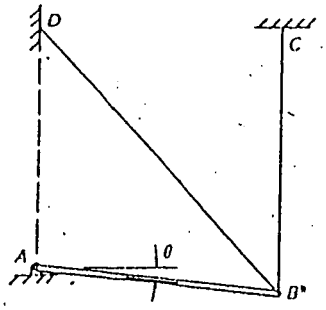

Fig. P2.17

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**2.21.** The bar $AB$ is rigid and is supported by two wires $DB$ and $CB$ as shown in Fig. P2.21. If the bar rotates through a small angle $\theta$, determine the average extensional strain of $DB$ and $CB$.

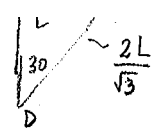



Before

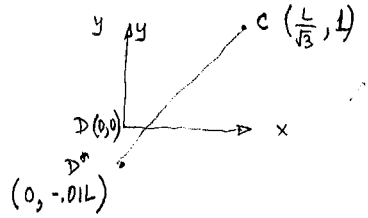



After

Fig. P2.21




$$\sin 30^\circ = \frac{\bar{BC}}{\bar{CD}} \Rightarrow \bar{BC} = \bar{CD}\sin 30^\circ = \frac{2L}{\sqrt{3}}\cdot\frac{1}{2} = L/\sqrt{3}$$




$$\epsilon_{BD} = \frac{\bar{BD}^* - BD}{BD} = \frac{1.01L - L}{L} = .01$$

$$\epsilon_{CD} = \frac{\bar{CD}^* - CD}{CD} = \frac{1.163L - 1.155L}{1.155L} = .0075$$

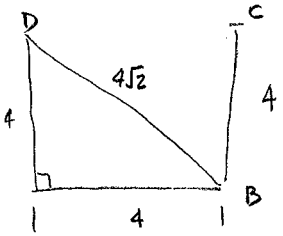
$$CD = \frac{L}{\sqrt{3}}\hat{i} + L(1.01)\hat{j} \Rightarrow L\sqrt{\tfrac{1}{3} + 1.02} = 1.163L$$

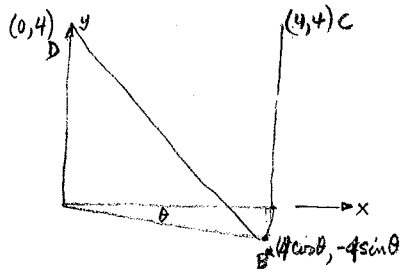
BY SYMMETRY

מכאן שמהנדס טעה

$$0.0075 = \epsilon_{AD} = \epsilon_{CD}$$

2.21




$$\bar{BD} = \left[(0 - 4\cos\theta)^2 + (4 + 4\sin\theta)^2\right]^{1/2}$$
$$= \left[16\cos^2\theta + 16 + 32\sin\theta + 16\sin^2\theta\right]^{1/2}$$
$$= \left[32 + 32\sin\theta\right]^{1/2} = 4\sqrt{2}\,(1+\sin\theta)^{1/2}$$

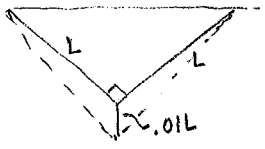
$$\epsilon_{BD} = \frac{\bar{B^*D} - BD}{BD} = \frac{4\sqrt{2}\,(1+\sin\theta)^{1/2} - 4\sqrt{2}}{4\sqrt{2}} = (1+\sin\theta)^{1/2}$$

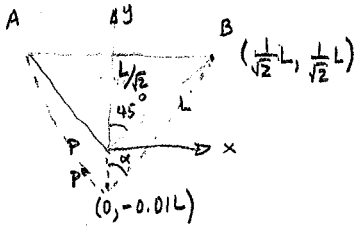
$$\epsilon_{BD} \cong \left(1 + \frac{\sin\theta}{2} + \ldots\right) - 1 = \frac{\sin\theta}{2} \approx \theta/2 \text{ rad}$$

$$\bar{CB}^* = \left[(4\cos\theta - 4)^2 + (-4\sin\theta - 4)^2\right]^{1/2}$$
$$= \left[16\cos^2\theta - 32\cos\theta + 16 + 16\sin^2\theta + 32\sin\theta + 16\right]^{1/2}$$
$$= \left[48 + 32\sin\theta - 32\cos\theta\right]^{1/2} \cong \left[16 + 32\sin\theta\right]^{1/2}$$
$$= 4\left[1 + 2\sin\theta\right]^{1/2}$$

$$\epsilon_{CB} = \frac{\bar{CB}^* - CB}{CB} = \frac{4(1+2\sin\theta)^{1/2} - 4}{4} = (1+2\sin\theta)^{1/2} - 1$$
$$= \left(1 + \frac{2\sin\theta}{2} + \ldots\right) - 1 = \sin\theta \cong \theta$$

2.32



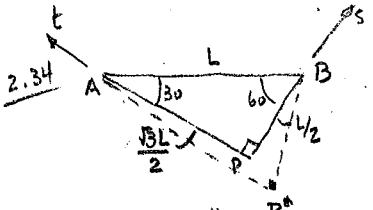




$$\tan\alpha = \frac{\frac{1}{\sqrt{2}}L}{\frac{L}{\sqrt{2}} + .01L} = .986 \Rightarrow \alpha = 44.598^\circ = .7784 \text{ rad}$$

$$45^\circ = .7854 \text{ rad}$$

$$\gamma_{st} = 90^\circ - 2(44.598^\circ) = .8046^\circ = .0140 \text{ rad.}$$

2.34



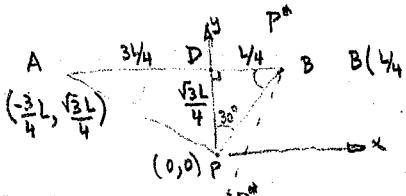

$\epsilon_s = .004$

$\epsilon_t = .003$

ASSUME AB IS OF LENGTH L

יוצא בדרך, אלו ואלו מתבדרים שווה ל-1.

נניח ש- $\bar{AB}$ שווה ל-L $\Leftarrow$ $\bar{AP} = \frac{\sqrt{3}L}{2}$, $\bar{BP} = L/2$, $\bar{PD} = \frac{\sqrt{3}L}{4}$




$$\epsilon_s = \frac{\bar{BP}^* - BP}{BP} = \frac{\bar{BP}^* - L/2}{L/2} \Rightarrow \bar{BP}^* = (\epsilon_s + 1)L/2 = (1.004)L/2$$

$$\epsilon_t = \frac{\bar{AP}^* - AP}{AP} = \frac{\bar{AP}^* - \sqrt{3}L/2}{\sqrt{3}L/2} \Rightarrow \bar{AP}^* = (\epsilon_t + 1)\frac{\sqrt{3}L}{2} = (1.003)\frac{\sqrt{3}L}{2}$$

כעת נמצא את $\theta = \angle AP^*B$ $\Leftarrow$




$$L^2 = \bar{AP}^{*2} + \bar{BP}^{*2} - 2\bar{AP}^*\bar{BP}^*\cos\theta \Rightarrow \cos\theta = \frac{L^2 - \bar{AP}^{*2} - \bar{BP}^{*2}}{-2\bar{AP}^*\bar{BP}^*}$$
$$= \frac{(1.003)^2\frac{3}{4}L^2 + (1.004)^2\frac{L^2}{4} - L^2}{2(1.003)\frac{\sqrt{3}}{2}(1.004)\frac{L^2}{2}} = .00745$$

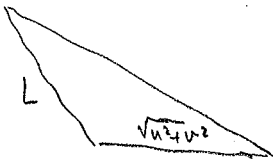
$$\gamma_{st} = (90 - 89.57)\pi/180 = .00747 \Leftarrow \theta = 89.57^\circ$$

כאשר $\theta$ קרובה ל- $90^\circ$, $90-\theta$ קרובה ל- $0^\circ$.

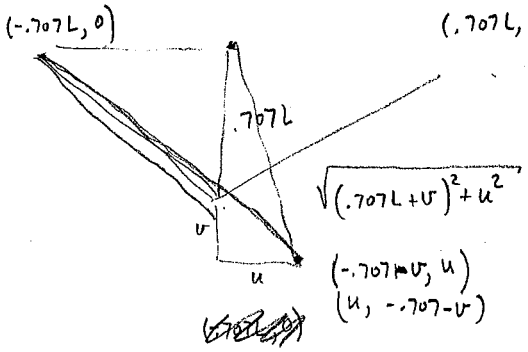
$$\cos\theta = \cos(90^\circ - \theta - 90^\circ) = \cos(90-\theta)\cos 90^\circ + \sin(90-\theta)\sin 90^\circ$$
$$= \sin(90-\theta) \cong 90^\circ - \theta$$

$\cos\theta \cong (90-\theta)\pi/180$ RADIANים $\Leftarrow$

$\simeq x$




$\sqrt{u^2+v^2}$ =




$(u+.707L)^2 + (.707L+v)^2$

$u^2 + \frac{2}{\sqrt{2}}uL + \frac{1}{2}L^2 + .\frac{1}{2}L^2 + \frac{2}{\sqrt{2}}vL + v^2$

$$\epsilon_{AC} = \frac{\sqrt{u^2+v^2+\frac{2L}{\sqrt{2}}(u+v)+L^2} - L}{L} = \frac{L\left(1+\frac{\sqrt{2}}{L}(u+v)+\frac{u^2+v^2}{L^2}\right)-L}{L} \cong \frac{\sqrt{2}}{2}\left(\frac{u+v}{L}\right)$$

$$\epsilon_{BC} = \sqrt{(u-.707L)^2+(-.707L+v)^2}$$

$$= \frac{\sqrt{u^2+v^2-\frac{2}{\sqrt{2}}L(u-v)+L^2} - L}{L} = \frac{L\left(1-\frac{\sqrt{2}}{L}(u-v)+\frac{u^2+v^2}{L^2}\right)}{L}$$

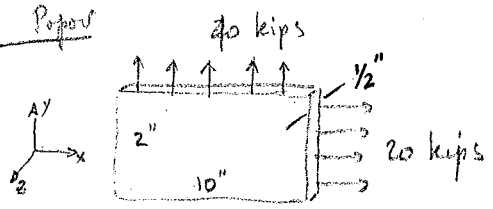
$$= -\frac{\sqrt{2}}{2}\left(\frac{u-v}{L}\right)$$

$$\epsilon_x = 2x^2+5y^2$$
$$\epsilon_y = -3x^2+2y^2$$
$$\gamma_{xy} = 5x^2+9y^2$$

$$\frac{\partial^2\epsilon_x}{\partial y^2} + \frac{\partial^2\epsilon_y}{\partial x^2} - \frac{\partial^2\gamma_{xy}}{\partial x\partial y} \neq 0$$

$$10 + (-6) - 0 \neq 0$$

4-5 Popov




$\sigma_y = \frac{P_y}{A} = \frac{40{,}000}{10\times\frac{1}{2}} = 8{,}000$ psi

$\sigma_x = \frac{P_x}{A} = \frac{20{,}000}{2\times\frac{1}{2}} = 20{,}000$ psi

$\sigma_z \cong 0$ { אפשר להניח שמאמצים בכיוון Z קטן מאוד בהשוואה לשני הכיוונים האחרים WIDTH HEIGHT THICKNESS

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_y+\sigma_x)$$

$$= 0 - \frac{0.25}{30\times10^6\text{psi}}(8{,}000+20{,}000) = -.25\left(\frac{28\times10^3}{30\times10^6}\right) = -.25(.933\times10^{-3}) = -.233\times10^{-3}$$

$$\epsilon_z = \frac{t_{final}-t_{init}}{t_{init}} \Rightarrow \Delta t = t_{final}-t_{init} = \epsilon_z t_{init} = -.233\times10^{-3}\left(\tfrac{1}{2}''\right) = -.1167\times10^{-3}\text{ in}$$

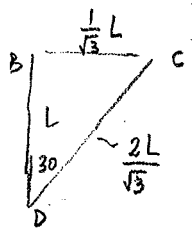
$$\frac{-.1167\times10^{-3}}{\frac{1}{2}} = \frac{\Delta t}{t_{init}} = \epsilon_z = -.233\times10^{-3} = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_y+\sigma_x) = -\frac{\nu}{E}\sigma_x \Rightarrow \sigma_x = -\frac{\epsilon_z E}{\nu} = -\frac{(-.233\times10^{-3})(30\times10^6)}{.25}$$

$$= (.933\times10^{-3})(30\times10^6) = 28000\text{ p}$$

$$P_x = \sigma_x \cdot A = 28000\times(2\times\tfrac{1}{2}) = 28\text{ kips}$$

A piece of steel 2in x 10in x ½in is loaded by a distributed load along its edges. If $P_x$ = 20 kips $P_y$ = 40 kips ... in thickness due to these forces?

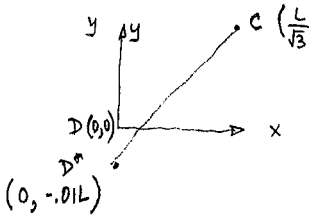
2.17




$\cos 30° = \frac{\overline{BD}}{\overline{CD}} = \frac{L}{\overline{CD}} = .866 = \frac{\sqrt{3}}{2} \Rightarrow \overline{CD} = \frac{2L}{\sqrt{3}}$ ✓

$\sin 30° = \frac{\overline{BC}}{\overline{CD}} \Rightarrow \overline{BC} = \overline{CD}\sin 30° = \frac{2L}{\sqrt{3}} \cdot \frac{1}{2} = L/\sqrt{3}$ ✓

$\epsilon_{BD} = \frac{\overline{BD}^* - BD}{BD} = \frac{1.01L - L}{L} = .01$ ✓




$\epsilon_{CD} = \frac{\overline{CD}^* - \overline{CD}}{\overline{CD}} = \frac{1.163L - 1.155L}{1.155L} = .0075$ ✓

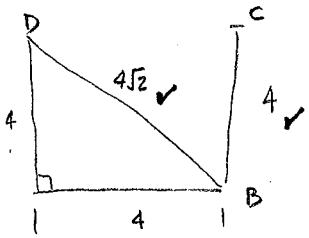
$\overline{CD}^* = \frac{L}{\sqrt{3}}\underline{i} + L(1.01)\underline{j} \Rightarrow L\sqrt{\frac{1}{3} + 1.02} = 1.163L$ ✓

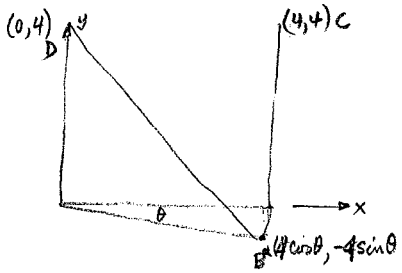
BY SYMMETRY

מכאן שמהנה מאלכי

$0.0075 = \epsilon_{AD} = \epsilon_{CD}$ ✓

2.21




$\overline{BD}^* = \left[(0 - 4\cos\theta)^2 + (4 + 4\sin\theta)^2\right]^{1/2}$

$= \left[16\cos^2\theta + 16 + 32\sin\theta + 16\sin^2\theta\right]^{1/2}$

$= \left[32 + 32\sin\theta\right]^{1/2} = 4\sqrt{2}\,(1+\sin\theta)^{1/2}$ ✓

$\epsilon_{BD} = \frac{\overline{B^*D} - BD}{BD} = \frac{4\sqrt{2}(1+\sin\theta)^{1/2} - 4\sqrt{2}}{4\sqrt{2}} = (1+\sin\theta)^{1/2} - 1$ ✓

$\epsilon_{BD} \cong \left(1 + \frac{\sin\theta}{2} + \ldots\right) - 1 = \frac{\sin\theta}{2} \approx \theta/2$ rad ✓

$\overline{CB}^* = \left[(4\cos\theta - 4)^2 + (-4\sin\theta - 4)^2\right]^{1/2}$

$= \left[16\cos^2\theta - 32\cos\theta + 16 + 16\sin^2\theta + 32\sin\theta + 16\right]^{1/2}$

$= \left[48 + 32\sin\theta - 32\cos\theta\right]^{1/2} \cong \left[16 + 32\sin\theta\right]^{1/2}$

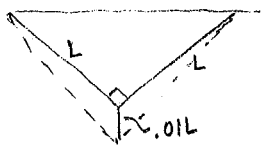
$= 4\left[1 + 2\sin\theta\right]^{1/2}$ ✓

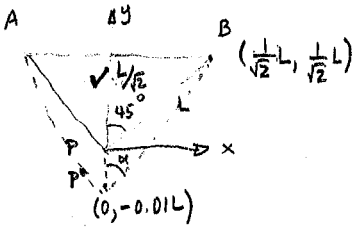
$\epsilon_{CB} = \frac{\overline{CB}^* - CB}{CB} = \frac{4(1+2\sin\theta)^{1/2} - 4}{4} = (1+2\sin\theta)^{1/2} - 1$ ✓

$= \left(1 + \frac{2\sin\theta}{2} + \ldots\right) - 1 = \sin\theta \cong \theta$ ✓

8/8

2.32



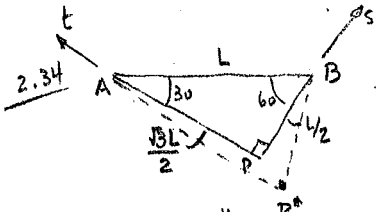

$\tan\alpha = \frac{\frac{1}{\sqrt{2}}L}{\frac{L}{\sqrt{2}} + .01L} = .986$ ✓ $\Rightarrow \alpha = 44.598° = .7784$ rad

$45° = .7854$ rad

$\gamma_{st} = 90° - 2(44.598°) = .8046° = .0140$ rad. ✓

3/3

2.34



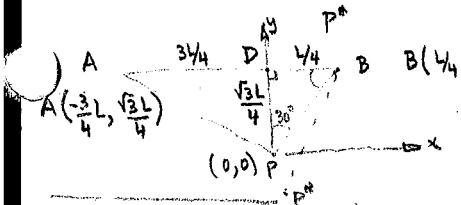

$\epsilon_s = .004$

$\epsilon_t = .003$

ASSUME AB IS OF LENGTH L

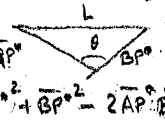
יוצר בראש, אנו יודעים שאורך $\overline{AB}$ שווה ל-1.

נניח ש- $\overline{AB}$ שווה ל-1 $\Leftarrow \overline{BP} = 1/2$, $\overline{AP} = \frac{\sqrt{3}L}{2}$, $\overline{PD} = \frac{\sqrt{3}L}{4}$




$\epsilon_s = \frac{\overline{BP}^* - BP}{BP} = \frac{\overline{BP}^* - 1/2}{1/2} \Rightarrow \overline{BP}^* = (\epsilon_s + 1)\frac{1}{2} = (1.004)\frac{1}{2}$

$\epsilon_t = \frac{\overline{AP}^* - AP}{AP} = \frac{\overline{AP}^* - \sqrt{3}L/2}{\sqrt{3}L/2} \Rightarrow \overline{AP}^* = (\epsilon_t + 1)\frac{\sqrt{3}L}{2} = (1.003)\frac{\sqrt{3}L}{2}$

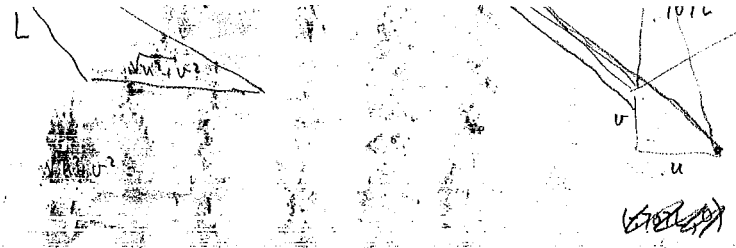



$\Leftarrow \theta = \angle AP^*B$ כלא תכונות

$L^2 = \overline{AP}^{*2} + \overline{BP}^{*2} - 2\overline{AP}^*\overline{BP}^*\cos\theta \Rightarrow \cos\theta = \frac{L^2 - \overline{AP}^{*2} - \overline{BP}^{*2}}{}$

כאשר $\theta$ קרובה ל- $90°$, $90°-\theta$ קרובה ל- $0°$. לכן

$\cos A = \cot\theta \cos(90° - A - 90°) = \cos(90 - A)$ כאשר $90°$




$$(u+.707L)^2 + (.707L+v)^2$$

$$u^2 + \frac{2}{\sqrt{2}}uL + \frac{1}{2}L^2 + \frac{1}{2}L^2 + \frac{2}{\sqrt{2}}vL + v^2$$

$$\epsilon_{AC} = \frac{\sqrt{u^2+v^2+\frac{2L}{\sqrt{2}}(u+v)+L^2} - L}{L} = \frac{L\left(1+\frac{\sqrt{2}}{L}(u+v)+\frac{u^2+v^2}{L^2}\right)-L}{L} \approx \frac{\sqrt{2}}{2}\left(\frac{u+v}{L}\right)$$

$$\epsilon_{BC} = \frac{\sqrt{(u-.707L)^2+(-.707L+v)^2}}{L} = \frac{\sqrt{u^2+v^2-\frac{2}{\sqrt{2}}L(u-v)+L^2} - L}{L} = \frac{L\left(1-\frac{\sqrt{2}}{L}(u-v)+\frac{u^2+v^2}{L^2}\right)-L}{L} = -\frac{\sqrt{2}}{2}\left(\frac{u-v}{L}\right)$$

$$\epsilon_x = 2x^2+5y^2$$

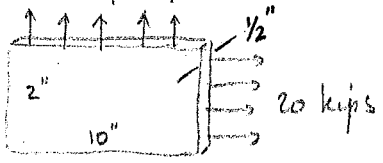
$$\epsilon_y = -3x^2+2y^2$$

$$\gamma_{xy} = 5x^2+9y^2$$

$$\frac{\partial^2\epsilon_x}{\partial y^2} + \frac{\partial^2\epsilon_y}{\partial x^2} - \frac{\partial^2\gamma_{xy}}{\partial x\partial y} \neq 0$$

$$10 + (-6) - 0 \neq 0$$

4-5 Popov




$$\sigma_y = \frac{P_y}{A} = \frac{40{,}000}{10\times\frac{1}{2}} = 8{,}000 \text{ psi}$$

$$\sigma_x = \frac{P_x}{A} = \frac{20{,}000}{2\times\frac{1}{2}} = 20{,}000 \text{ psi}$$

$\sigma_z \cong 0$ { שטח פנים קטן, מעגופה והרוחב WIDTH HEIGHT / עובי קטן THICKNESS

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_y+\sigma_x)$$

$$= 0 - \frac{0.25}{30\times10^6 \text{psi}}(8{,}000+20{,}000) = -.25\left(\frac{28\times10^3}{30\times10^6}\right) = -.25(.933\times10^{-3}) = -.233\times10^{-3}$$

$$\epsilon_z = \frac{t_{final}-t_{init}}{t_{init}} \Rightarrow \Delta t = t_{final}-t_{init} = \epsilon_z t_{init} = -.233\times10^{-3}\left(\tfrac{1}{2}''\right) = -.1167\times10^{-3} \text{ in}$$

$$\frac{-.1167\times10^{-3}}{\frac{1}{2}} = \frac{\Delta t}{t_{init}} = \epsilon_z = -.233\times10^{-3} = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_y+\sigma_x) = -\frac{\nu}{E}\sigma_x \Rightarrow \sigma_x = -\frac{\epsilon_z E}{\nu} = -\frac{(-.233\times10^{-3})(30\times10^6)}{.25}$$

$$= (.933\times10^{-3})(30\times10^6) = 28000 \text{ psi}$$

$$P_x = \sigma_x \cdot A = 28000\times(2\times\tfrac{1}{2}) = 28 \text{ kips}$$

A piece of steel $2\text{in}\times10\text{in}\times\frac{1}{2}\text{in}$ is loaded by a distributed load along its edges. If $P_x = 20$ kips $P_y = 40$ kips
what is the change in thickness due to these forces?
What is the value of $p_x$ alone to cause the same change in thickness $E = 30\times10^6$ psi $\nu = 0.25$

2.32. A thin triangular plate is deformed as indicated in Fig. P2.32. Compute approximately the shear strain $\gamma_{st}(P)$.

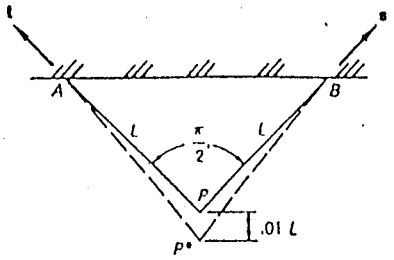



Fig. P2.32

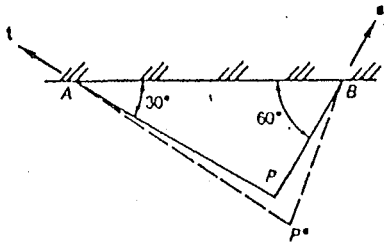



Fig. P2.34

2.34. A thin plate is in the form of a 30°–60°–90° triangle and is supported along its hypotenuse as shown in Fig. P2.34. It is subjected to the uniform strains $\epsilon_s = .004$, $\epsilon_t = .003$. Compute the approximate value of $\gamma_{st}(P)$.

2.37. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.37. It elongates 10 per cent of its original length and contracts 3 per cent of its original width. Compute approximately the shear strain $\gamma_{st}$.

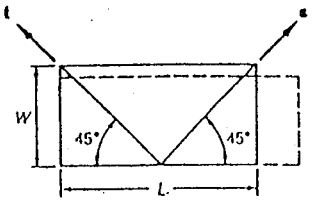



Fig. P2.37

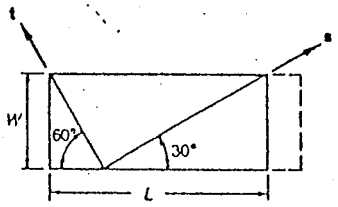



Fig. P2.38

2.38. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.38. The extensional strain along its length is .02 and the width remains unchanged. Compute approximately the shear strain $\gamma_{st}$.

**2.7.** A straight piece of wire $PQ$ lies along the line $y = mx$ as shown in Fig. P2.7. The wire is strained and displaced to the line $y = nx$ in such a way that a point originally at $x$ is displaced to $x^2/2$. Show that the extensional strain at any point along the wire is given by

$$\epsilon = \sqrt{\frac{1 + n^2}{1 + m^2}}\, x - 1$$

(where $x$ is the original coordinate of the point).

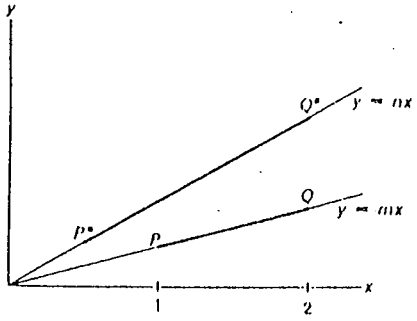



Fig. P2.7

**2.16.** The point $C$ is displaced to $C^*$ as shown in Fig. P2.16. The horizontal and vertical components of this displacement are $u$ and $v$. Express the average extensional strains of $AC$ and $BC$ in terms of $u$, $v$ and $L$. If $u$ and $v$ are small, what are the approximate expressions for these average extensional strains?

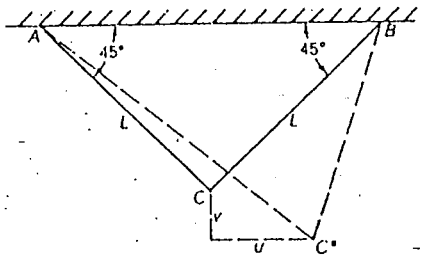



Fig. P2.16

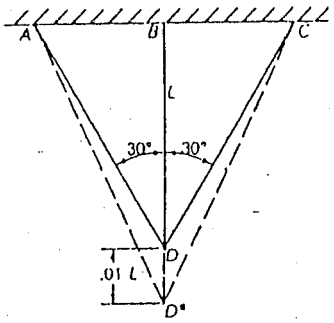



Fig. P2.17

**2.17.** The point $D$ in Fig. P2.17 is displaced vertically an amount $.01L$. Obtain the approximate (first-order) average extensional strain of $AD$, $BD$, and $CD$.

**2.21.** The bar $AB$ is rigid and is supported by two wires $DB$ and $CB$ as shown in Fig. P2.21. If the bar rotates through a small angle $\theta$, determine the average extensional strain of $DB$ and $CB$.

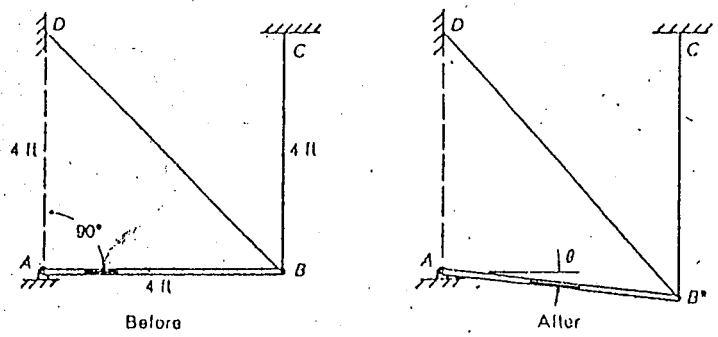
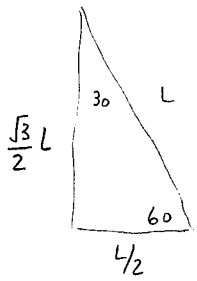


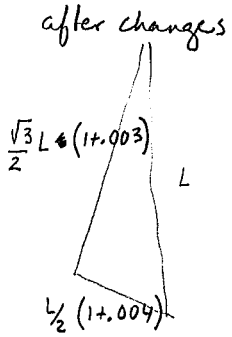

Fig. P2.21




$\epsilon_x = .004$

$\epsilon_y = .003$

remember $\epsilon = \frac{\Delta L}{L}$




$$\therefore \quad L^2 = \frac{3}{4}L^2(1+.003)^2 + \frac{L^2}{4}(1+.004)^2 - 2\cdot\frac{\sqrt{3}}{2}L(1+.003)\cdot\frac{L}{2}(1+.004)\cos\theta$$

$$0 = \frac{3}{4}(.006) + \frac{1}{4}(.008) - \frac{\sqrt{3}}{2}(1+.007)\cos\theta$$

$$\cos\theta = \frac{.0065}{\frac{\sqrt{3}}{2}(1.007)} \Rightarrow 89.573^\circ = \theta$$

7

$$\gamma_{xy} = 90^\circ - 89.573^\circ = .427^\circ \quad \text{or} \quad .427\times\frac{\pi}{180} = \underline{.00745}$$

2

also $\gamma_{xy} = \left(\frac{\Delta u}{\Delta y} + \frac{\Delta v}{\Delta y}\right) = \left(\frac{\frac{L}{2}\cdot.004}{\frac{L}{2}\sqrt{3}} + \frac{\frac{\sqrt{3}L}{2}(.003)}{L/2}\right) = .0075$

$\Delta u = \frac{L}{2}(.004) \qquad \Delta y = L\cdot\frac{\sqrt{3}}{2}$

$\Delta v = \frac{\sqrt{3}L}{2}(.003) \qquad \Delta x = L/2$

now

$$\left.\begin{aligned}\epsilon_x &= \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_z + \sigma_y)\\ \epsilon_y &= \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z)\\ \epsilon_z &= \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y)\end{aligned}\right\} \quad I_{1\epsilon} = \frac{1-2\nu}{E} I_{1\sigma} \quad \text{or} \quad I_{1\sigma} = I_{1\epsilon}\cdot\frac{E}{1-2\nu}$$

since thin plate $\sigma_z = 0$ ; since $\epsilon_z = 0$ then $\sigma_x + \sigma_y = 0 \quad \therefore \sigma_y = -\sigma_x$

4

$$\therefore \quad \epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\cancel{\sigma_z}^{\,0} + \sigma_y) = \sigma_x\frac{(1+\nu)}{E} \quad \therefore \sigma_x = \frac{E\epsilon_x}{1+\nu} = \frac{200\,GPa\,(.004)}{1.3}$$

$$\underline{\sigma_x = 615.39\,MPa}$$

2

$E(.004) = \sigma_x - .3\sigma_y$ $\qquad .3E(.003) = .3\sigma_y - .09\sigma_x$

$E(.003) = \sigma_y - .3\sigma_x$ $\qquad E(.004) = -.3\sigma_y + \sigma_x$

$$= \sigma_x(1-.09)$$

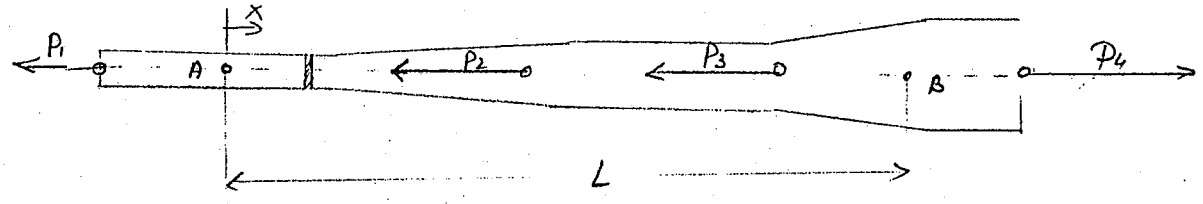
$\sigma_x = 1.0769$ GPa if not taking into account $\epsilon_z = 0$

15

לעתים מופיעים התומכים והתנועות בלתי נקודה בקודה שבמציאות ומוכנסים ומונחים שהם בטעויות. ישנם תומכים שכשר ראיתם שונה מאוד בין לתיאוריה למציאות כגון בטון ומבטן יבוקה.

## 6-8 התארכות מוטות מהירות בגורות

בנוסחאות 1.8 ו- 2.8 התוספה למבנה והמטרה ניתן של המומנט מהמטרי תשובה ההצבה (התארכות) שמתקבל מוקם המיוחס וחלוקת. נסתכל במוקם כלשהו כמשורטט




המיוחס במחוות בגורם $P_1, P_2, P_3, P_4$. בנוסף הוראם הוא מיתוך $AB=L$ במקום שמתנה המיתוך יקבל ערכים שה. התארכות לאות הוא סיו התארכותם של כל הוולמנטים בין A ל- B.

נקח וולמנט וחתך במרחק x
על וולמנט זה מופעל כוח $P_x$
על שטח חתך $A_x$ לכן

$$\sigma = \frac{P_x}{A_x} \qquad \varepsilon = \frac{\sigma_x}{E}$$

$$\varepsilon = \frac{P_x}{EA_x}$$

הוא ורוך המוט ההתחל הוא $dx$ ולכן אורכו הסופי והוא

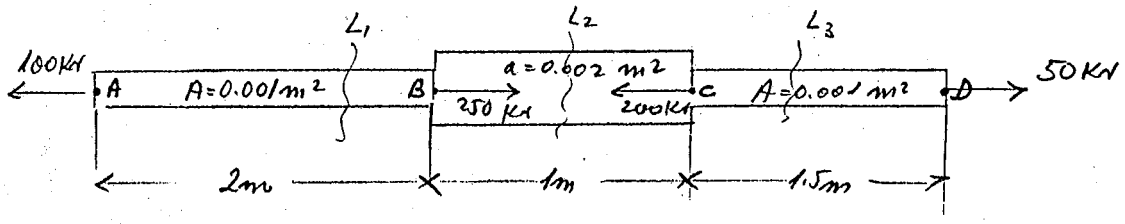
$$dx + \varepsilon dx$$

ושינוי אורכו הוא $\varepsilon dx$

$$d\Delta = \varepsilon dx = \frac{P_x}{A_x E} dx$$

פתרון (2.2 עמ' 39)

חשב את שינוי האורך AD עבור מוט בעל $E = 200\ GPa$ המתואר בשרטוט




כיון ששטח החתך משתנה יש לחשב כל חלק בנפרד.

קטע A-B

חותכים בתחום $L_1$ ומקבלים כי $P_1 = 100\ KN$

$A_1 = 0.001\ m^2$

קטע B-C

חותכים ב $L_2$ ומקבלים $P_2 = -150\ KN$

$A_2 = 0.002\ m^2$

קטע C-D

חותכים $L_3$ ומקבלים $P_3 = 50\ KN$

$A_3 = 0.0001\ KN$

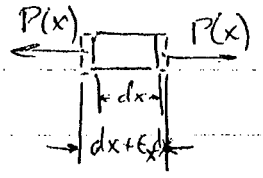
לכן ההתארכות הכללית

$$\Delta = \int_A^D \frac{P_x}{EA_x}dx = \frac{1}{E}\left\{\int_A^B \frac{P_1}{A_1}dx + \int_B^C \frac{P_2}{A_2}dx + \int_C^D \frac{P_3}{A_3}dx\right\} =$$

$$\frac{1}{E}\left\{\frac{P_1}{A_1}L_1 - \frac{P_2}{A_2}L_2 + \frac{P_3}{A_3}L_3\right\} = \sum\frac{PL}{EA} = 1\ mm.$$

## 2.7 מקדם פוסון

מכפלת ההתארכות בשטח החתך המקורי של חומר נתון נקראת נפח המשנה ואת שינוי הנפח היחסי לכיון כזה למשל מוט בעל חתך מלבני המתואר בשרטוט ה



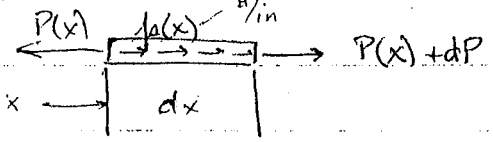

displacement or measure of elongations.

$du = \epsilon_x dx$

$$u = \int du = \int \epsilon_x dx = \frac{1}{E}\int \sigma_x dx = \frac{1}{E}\int \frac{P(x)}{A(x)} dx$$

$$\boxed{\frac{du}{dx} = \frac{1}{E}\frac{P(x)}{A(x)}}$$

distributed load



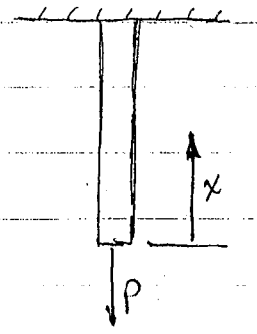

$$\sum F_x = P(x) + dP - P(x) + p_x dx = 0$$

$$\frac{dP}{dx} = -p_x$$

$$\frac{d}{dx}\left(\frac{du}{dx}\right) = \frac{1}{E}\frac{d\left(\frac{P(x)}{A(x)}\right)}{dx}$$ if A is a constant

$$\frac{d^2u}{dx^2} = \frac{1}{EA}\frac{dP}{dx} = -\frac{p_x}{EA}$$

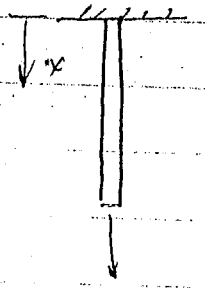
$$\boxed{\frac{d^2u}{dx^2} = -\frac{p_x}{EA}}$$




$$\epsilon_x = \frac{\Delta l}{l} \qquad \sigma_x = P/A \qquad \frac{\sigma_x}{E} = \frac{P}{AE} \qquad \Delta l = \frac{Pl}{AE}$$

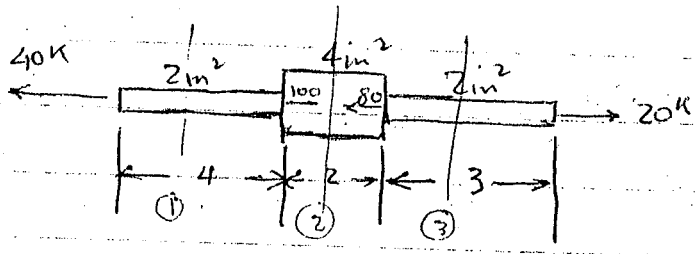
$$u = \frac{1}{E}\int \frac{P}{A} dx = \frac{Px}{AE} + C_1 \qquad u=0 \;\; x=0 \Rightarrow C_1 = 0$$

$$u = \frac{Px}{AE} \qquad u_L = \frac{PL}{AE}$$




$$u(L) = \frac{PL}{AE}$$

Problem 2



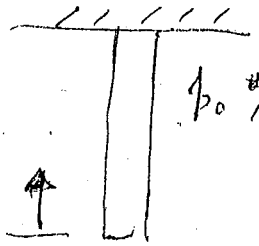

$$u = \sum \frac{PL}{AE}$$

$$u_1 = \frac{40K \times 48in}{2in^2 \times 30\times10^3 K} \qquad u_2 = -\frac{60K \times 24in}{4in^2 \times 30\times10^3}$$

$$u_3 = \frac{20\times36}{2in^2\times30\times10^3 K}$$

$$u_{T.} = 32\times10^{-3} - 12\times10^{-3} + 12\times10^{-3} = 32\times10^{-3}.$$

Problem 3




$$P_x = -P_0 \qquad \frac{du}{dx} = -\int \frac{P_x dx}{AE} dx = \frac{P_0 x}{AE} + C_1$$

$$\frac{du}{dx} = \frac{P(x)}{AE} \qquad \frac{du}{dx} = \frac{P_0 x}{AE} \quad \text{since } C_1 = 0$$

$\epsilon = \frac{\sigma}{E} - \nu\frac{\sigma}{E} - \frac{\nu\sigma}{E}$ $\epsilon = \frac{\sigma}{E}(1-2\nu) = \frac{30000}{30\times10^6} \cdot \frac{1}{2} = \frac{1}{2}(1000)$

$l = l_0 + \epsilon l_0$ $2'' + (.0005)2 = 1.999''$

## Thermal Strains

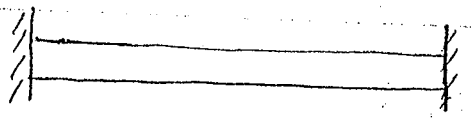
do not affect shear strains in isotropic material

$\epsilon_x = \epsilon_y = \epsilon_z$

$\alpha$ - coefficient of thermal expansion

$\alpha = \frac{\epsilon}{\Delta T}$ $\alpha_{steel} = 6.5\times10^{-6}/°F$

$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} + \alpha\Delta T$



$A = 4in^2$
$L = 20\ ft$

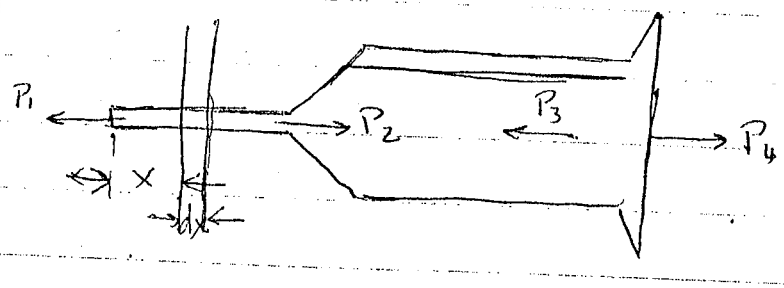
$\Delta T = 100°F$ $\nu =$ $E =$

$\epsilon_x = 0 = \frac{\sigma_x}{E} + \alpha\Delta T$ since it is not restrained in y, z direction $\sigma_y, \sigma_z = 0$

$\sigma_x = -E\alpha\Delta T$ .....

$\epsilon_y = -\frac{\nu\sigma_x}{E} + \alpha\Delta T$

$\epsilon_z = -\frac{\nu\sigma_x}{E} + \alpha\Delta T$



## 8.13 מאמצים טרמיים

במבנים מסוימים סטטית ניתן להזניח את הדפורמציות הנובעות מהתחממות המבנה שכן ניתן לו להתפשט חופשית. במבנים בלתי מסוימים לא כך הדבר. כיון שהגוף מוגבל מלהתפשט נוצרים בו מאמצים עקב רק העיוות הנוצר כתוצאה מהתפשטות חופשית הוא:

$$\Delta = \alpha L(\delta_T) \qquad 8.11$$

כאשר $\alpha$ מקדם ההתפשטות התרמית ו-$\delta_T$ שינוי הטמפ'.

### דוגמא 12.5 (411)

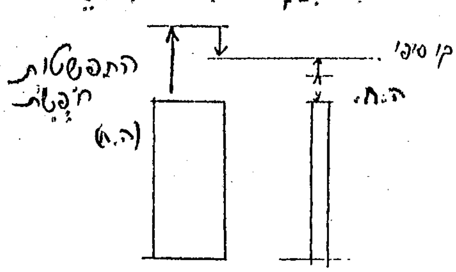
צינור נחושת שאורכו 300 מ"מ ובעל חתך של $2000\,mm^2$ מוכנס בין שתי פלטות קשיחות של Invar (מתכת שלמעשה בעלת $\alpha \cong 0$) המחוברות ביניהן ב-4 ברגי חבור עשויים פלדה ובעלי רדיוס של $4\,mm = r$. מחממים את המערכת מ-15° ל-$70^\circ C$. כמו כן נתון:

$$E_{cu} = 120\ GPa \ ; \ E_{st} = 200\ GPa$$

$$\alpha_{cu} = 16\times10^{-6}/^\circ C \qquad \alpha_{st} = 12\times10^{-6}/^\circ C$$

**פתרון:**

כתוצאה מהחימום נוטה הנחושת להתפשט יותר מהפלדה. ברגי הפלדה מעכבים זאת.




במצב שיווי המשקל יהיו הברגים מתוחים והגליל לחוץ שניהם בכוח P

$$P_{cu} = P_{st} = P$$

$$\Delta_{cu} = \Delta_s = \Delta$$

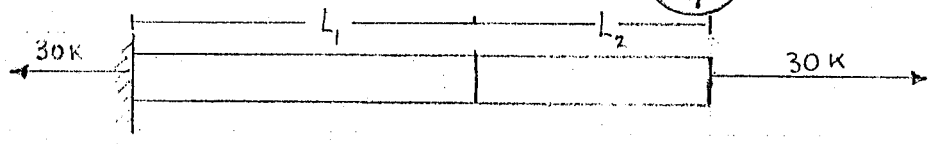
$$\alpha_{cu} L_{cu} \Delta_T - \frac{P_{cu} L_{cu}}{E_{cu} A_{cu}} = \alpha_s L_s \delta_{\Delta_s} + \frac{P_s L_s}{A_s E_s}$$

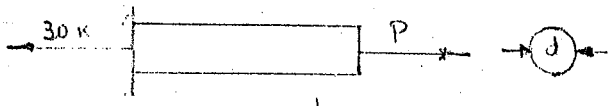
$$\Delta_T = 55^\circ$$

20

(9)

Cesar Levy
CE 201 D
Mr Blumenthal




$L_1 + L_2 = 36in$

$u_s = u_A$

$$u_s = \frac{1}{E_s}\int_0^L \frac{P_s}{A_s}dx = \frac{1}{E_s}\frac{P}{A}L_1 \qquad u_A = \frac{1}{E_A}\int \frac{P_A}{A_A}dx = \frac{1}{E_A}\frac{P}{A}L_2$$

since $u_s = u_A$ $\quad \frac{L_1}{L_2} = \frac{E_s}{E_A} = .333$

$L_1 + L_2 = 36in$ $\quad L_2 + 3L_2 = 36in$ $\quad L_2 = 9in$ $\quad L_1 = 27in$

$$u_A = \frac{30{,}000}{10\times10^6(\pi)}(9) = \frac{27}{\pi\times10^{-3}} = .859\times10^{-2} \qquad u_T = 2u_A = 1.718\times10^{-2}\,in$$

23 $$u_T = \frac{1}{E_A}\int\frac{P}{A_A}dx_1 + \frac{1}{E_s}\int\frac{P}{A_s}dx_2$$

$$u_T = \frac{P}{E_A A_A}(15) + \frac{P}{E_s A_s}(12)$$

$E_A = 10\times10^6$ psi $\quad A_A = 16\,in^2$ $\quad E_s = 30\times10^6$ psi $\quad A_s = 4\,in^2$

$$u_T = .101 = \frac{3P + 3.2P}{32\times10^6} \quad ; \quad P = 51.6\ kips$$

21. f is given in units of force/length $\therefore F = \int f\,dy = \frac{Ky^3}{3} + C_1$ $\quad @\ y=0,\ C_1 = 0$

$$u = \int\frac{F\,dy}{EA} = \frac{1}{EA}\int\frac{Ky^3}{3}dy = \frac{1}{EA}\left[\frac{Ky^4}{12}\right]_0^L = \frac{KL^4}{12EA}$$

$F = \frac{KL^3}{3} \Rightarrow K = \frac{3F}{L^3}$

but $K = \frac{3F}{L^3}$ $\longrightarrow$ $u = \frac{3F}{L^3}\cdot\frac{L^4}{12EA} = \frac{FL}{4EA}$

$F = 96000\,\#$ $\quad L = 40\times12\,in$ $\quad A = 100\,in^2$ $\quad E = 1.5\times10^6$ psi

$$\frac{(96\times10^3\,\#)(40\times12\,in)}{4\left(1.5\times10^6\frac{\#}{in^2}\right)(100\,in^2)} = \frac{16(40\times12)}{10^5} = 7.682\times10^{-2}\,in$$

# המכללה האקדמית יהודה ושומרון

## תורת החוזק 1

## תרגיל מס' 6

**2.9** The 36-mm-diameter steel rod $ABC$ and a brass rod $CD$ of the same diameter are joined at point $C$ to form the 7.5-m rod $ABCD$. For the loading shown, and neglecting the weight of the rod, determine the deflection ($a$) of point $C$, ($b$) of point $D$.

Answer: $\delta_C = 2.95\ mm\downarrow$, $\delta_D = 5.29\ mm\downarrow$.

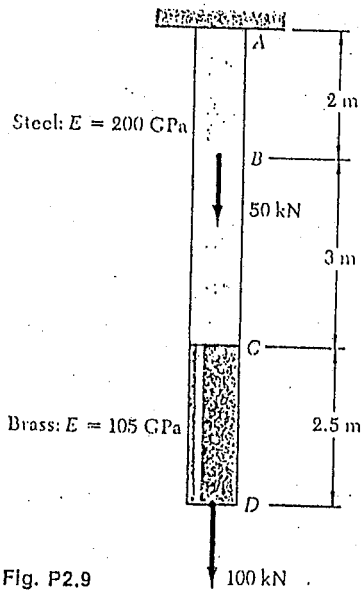



Fig. P2.9

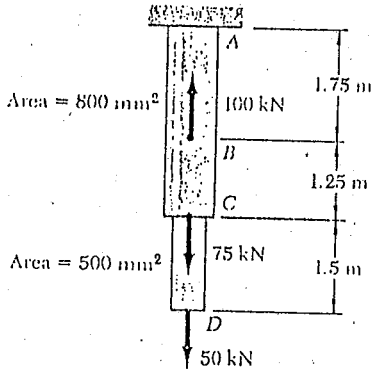



Fig. P2.10

**2.10** The rod $ABCD$ is made of an aluminum alloy for which $E = 70$ GPa. For the loading shown, and neglecting the weight of the rod, determine the deflection ($a$) of point $B$, ($b$) of point $D$.

Answer: $\delta_B = 0.781\ mm\downarrow$, $\delta_D = 5.71\ mm\downarrow$.

**2.26** An axial centric force of magnitude $P = 385$ kN is applied to the composite block shown by means of a rigid end plate. Determine the normal stress in ($a$) the steel core, ($b$) the aluminum plates.

Answer: $\sigma_S = -175\ MPa$, $\sigma_{Al} = -61.3\ MPa$.

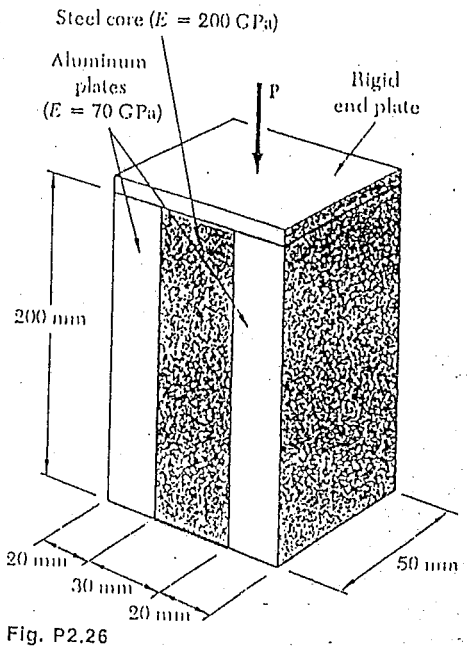



Fig. P2.26

**2.37** The rigid rod $ABC$ is suspended by three identical wires. Knowing that $x = \frac{2}{3}L$, determine the tension in each wire due to the force P.

Answer: $T_B = P/3$, $T_C = P/6$, $T_A = P/2$.

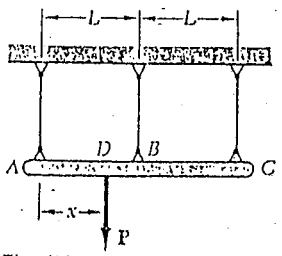



Fig. P2.37 and P2.38

**2.39** The rigid bar *ABCD* is suspended from four identical wires. Determine the tension in each wire caused by the load P shown.

Answer: $T_B = P/5, \quad T_D = 2P/5, \quad T_A = P/10, \quad T_C = 3P/10.$

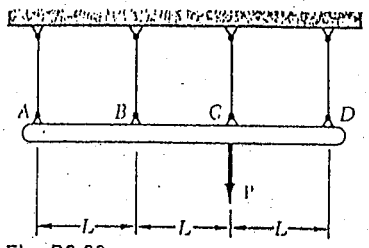
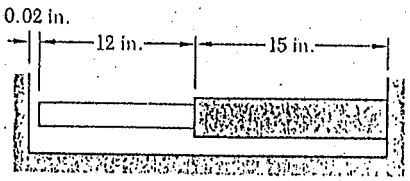


Fig. P2.39

**2.45** Determine (*a*) the compressive force in the bars shown after a temperature rise of 200°F, (*b*) the corresponding change in length of the aluminum bar.

Answer: $P = -52\ kips, \quad \delta_{Al} = 0.0124\ in.$




| Bronze | Aluminum |
|---|---|
| $A = 2.5\ \text{in}^2$ | $A = 3\ \text{in}^2$ |
| $E = 15 \times 10^6\ \text{psi}$ | $E = 10 \times 10^6\ \text{psi}$ |
| $\alpha = 10.1 \times 10^{-6}/°\text{F}$ | $\alpha = 12.8 \times 10^{-6}/°\text{F}$ |

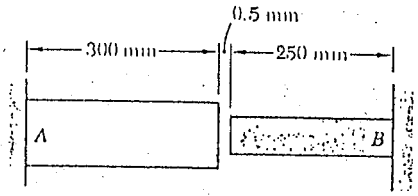
Fig. P2.45

**2.46** At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature reaches 140°C, determine (*a*) the normal stress in the aluminum, (*b*) the exact length of the aluminum rod.

Answer: $\sigma_{Al} = -114.6\ MPa, \quad L = 300.34\ mm.$

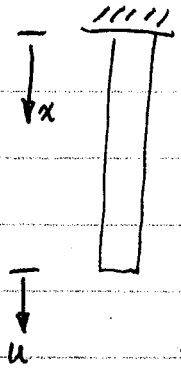
**2.47** Knowing that a 0.5-mm gap exists between the rods shown when the temperature is 20°C, determine (*a*) the temperature at which the normal stress in the stainless steel rod will be $\sigma = -150$ MPa, (*b*) the corresponding exact length of the stainless steel rod.

Answer: $T = 103.7°C, \quad L = 250.18\ mm.$

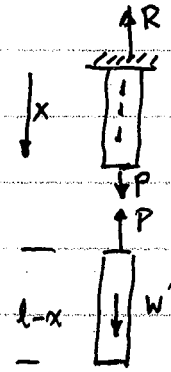



| Aluminum | Stainless steel |
|---|---|
| $A = 2000\ \text{mm}^2$ | $A = 800\ \text{mm}^2$ |
| $E = 70\ \text{GPa}$ | $E = 190\ \text{GPa}$ |
| $\alpha = 23 \times 10^{-6}/°\text{C}$ | $\alpha = 18 \times 10^{-6}/°\text{C}$ |

Fig. P2.46 and P2.47



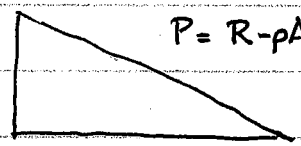

a bar $A, E, \ell$ find $u$




$R = \rho A g \ell$

$P = R - \rho A g x = \rho A g (\ell - x)$

$W' = \rho A g (\ell - x) = P$




$P = R - \rho A g x$

$$u = \int_0^x \frac{P\,dx'}{AE} = \int \frac{\rho A g (\ell - x')\,dx'}{AE}$$

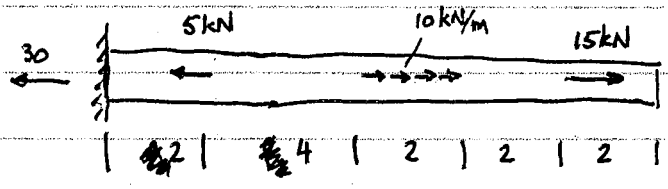
$$= \frac{\rho A g}{AE} \left(\ell - \frac{x'}{2}\right)^2 \Big|_0^x$$

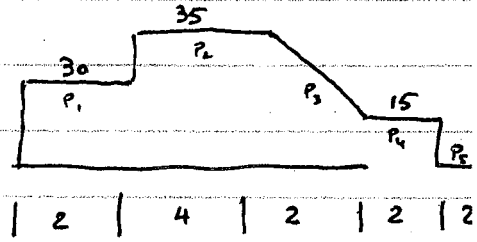
$$u = -\frac{\rho A g}{2AE}(\ell - x)^2 + \frac{\rho A g}{AE}\frac{\ell^2}{2}$$

when $x = 0$ $\quad u = 0$ ✓

$x = \ell \quad u = \frac{\rho A g}{AE}\frac{\ell^2}{2} = \frac{W\ell}{2AE}$

DO THIS →




$$u = \int_0^{12} \frac{P\,dx}{AE} = \int_0^2 \frac{P_1\,dx}{AE} + \int_2^6 \frac{P_2\,dx}{AE} + \int_6^8 \frac{P_3\,dx}{AE} + \int_8^{10} \frac{P_4\,dx}{AE} + \int_{10}^{12} \frac{P_5\,dx}{AE}$$

$$u = \frac{30 \cdot 2}{AE} + \frac{35 \cdot 4}{AE} + \int_6^8 \frac{(35 - 10(x-4))}{AE}dx + \frac{15 \cdot 2}{AE} + \frac{0 \cdot 2}{AE}$$

$6 \le x \le 8$

$$\frac{35 \cdot 2}{AE} - \frac{10(x-6)^2}{2AE}\Big|_6^8$$

**2.9** The 36-mm-diameter steel rod $ABC$ and a brass rod $CD$ of the same diameter are joined at point $C$ to form the 7.5-m rod $ABCD$. For the loading shown, and neglecting the weight of the rod, determine the deflection $(a)$ of point $C$, $(b)$ of point $D$.

Answer: $\delta_C = 2.95\ mm\downarrow$, $\delta_D = 5.29\ mm\downarrow$.

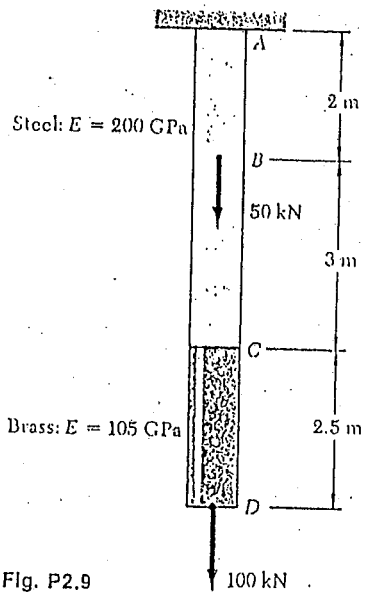


Fig. P2.9

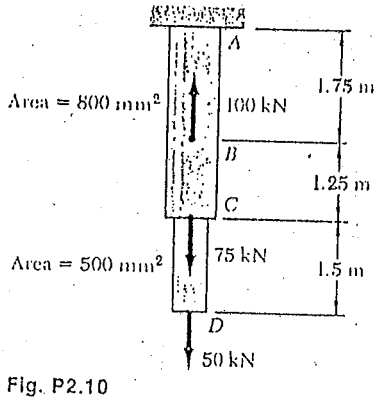


Fig. P2.10

**2.10** The rod $ABCD$ is made of an aluminum alloy for which $E = 70$ GPa. For the loading shown, and neglecting the weight of the rod, determine the deflection $(a)$ of point $B$, $(b)$ of point $D$.

Answer: $\delta_B = 0.781\ mm\downarrow$, $\delta_D = 5.71\ mm\downarrow$.

**2.26** An axial centric force of magnitude $P = 385$ kN is applied to the composite block shown by means of a rigid end plate. Determine the normal stress in $(a)$ the steel core, $(b)$ the aluminum plates.

Answer: $\sigma_S = -175\ MPa$, $\sigma_{Al} = -61.3\ MPa$.

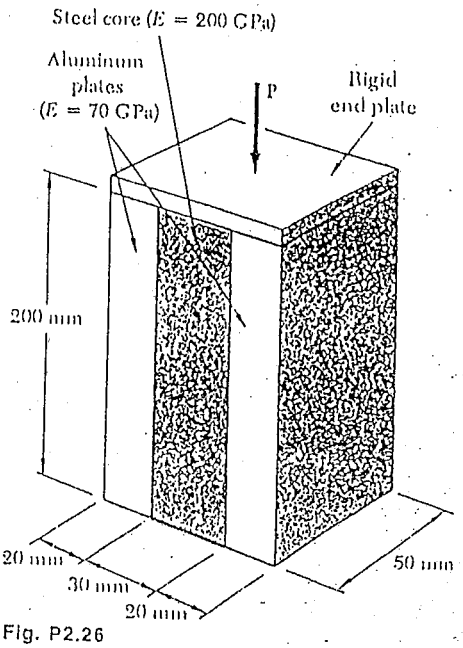


Fig. P2.26

**2.37** The rigid rod $ABC$ is suspended by three identical wires. Knowing that $x = \frac{2}{3}L$, determine the tension in each wire due to the force P.

Answer: $T_B = P/3$, $T_C = P/6$, $T_A = P/2$.

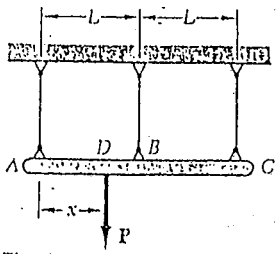


Fig. P2.37 and P2.38

**2.39** The rigid bar *ABCD* is suspended from four identical wires. Determine the tension in each wire caused by the load P shown.

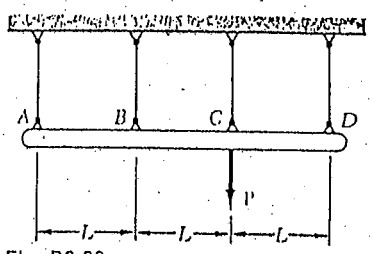
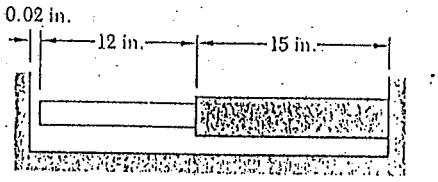


Fig. P2.39

Answer: $T_B = P/5, \quad T_D = 2P/5, \quad T_A = P/10, \quad T_C = 3P/10.$

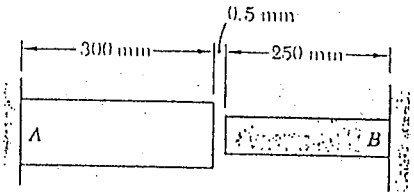
**2.45** Determine (*a*) the compressive force in the bars shown after a temperature rise of 200°F, (*b*) the corresponding change in length of the aluminum bar.

Answer: $P = -52\ kips, \quad \delta_{Al} = 0.0124\ in.$




| Bronze | Aluminum |
|---|---|
| $A = 2.5\ in^2$ | $A = 3\ in^2$ |
| $E = 15 \times 10^6$ psi | $E = 10 \times 10^6$ psi |
| $\alpha = 10.1 \times 10^{-6}/°F$ | $\alpha = 12.8 \times 10^{-6}/°F$ |

Fig. P2.45




| Aluminum | Stainless steel |
|---|---|
| $A = 2000\ mm^2$ | $A = 800\ mm^2$ |
| $E = 70$ GPa | $E = 190$ GPa |
| $\alpha = 23 \times 10^{-6}/°C$ | $\alpha = 18 \times 10^{-6}/°C$ |

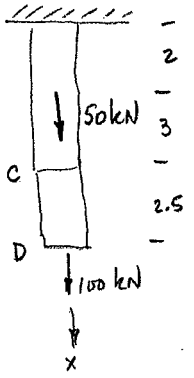
Fig. P2.46 and P2.47

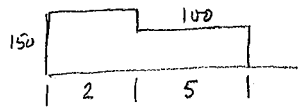
**2.46** At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature reaches 140°C, determine (*a*) the normal stress in the aluminum, (*b*) the exact length of the aluminum rod.

Answer: $\sigma_{Al} = -114.6\ MPa, \quad L = 300.34\ mm.$

**2.47** Knowing that a 0.5-mm gap exists between the rods shown when the temperature is 20°C, determine (*a*) the temperature at which the normal stress in the stainless steel rod will be $\sigma = -150$ MPa, (*b*) the corresponding exact length of the stainless steel rod.

Answer: $T = 103.7°C, \quad L = 250.18\ mm.$




$A = \frac{\pi d}{4} = \frac{\pi}{4}(.036) = 1.018\times10^{-3}\ m$

$E = 200\ GPa$ פלדה $\qquad E = 105\ GPa$ פלדה

$$\downarrow \delta_C = \int_0^5 \frac{P}{AE}dx = \int_0^2 \frac{150000\,dx}{(1.018\times10^{-3})(200\times10^9)} + \int_2^5 \frac{100000\,dx}{(1.018\times10^{-3})(200\times10^9)}$$

$$= 7.367\times10^{-4}\,x\Big|_0^2 + 4.912\times10^{-4}\,x\Big|_2^5 =$$

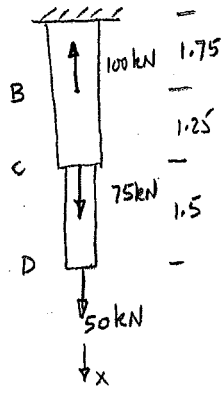
$$= 1.473\times10^{-3} + 1.473\times10^{-3} = \underline{2.947\times10^{-3}\ m}\ \downarrow$$

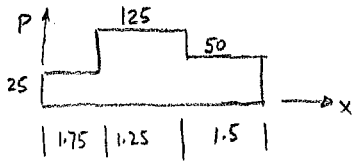
$$\delta_D = \int_0^{7.5} \frac{P}{AE}dx = \int_0^2 \frac{150000\,dx}{(1.018\times10^{-3})(200\times10^9)} + \int_2^5 \frac{100000\,dx}{(1.018\times10^{-3})(200\times10^9)} + \int_5^{7.5} \frac{100000\,dx}{(1.018\times10^{-3})(105\times10^9)}$$

$$= \delta_C + 9.355\times10^{-4}\,x\Big|_5^{7.5} = \delta_C + 2.339\times10^{-3} = \underline{5.286\times10^{-3}\ m}\ \downarrow$$

---

2.10




$A_{AC} = 8\times10^{-4}\ m^2 \qquad A_{CD} = 5\times10^{-4}\ m^2 \qquad E = 70\ GPa$

$$\downarrow \delta_B = \int_0^{1.75} \frac{P}{AE}dx = \int_0^{1.75} \frac{25000\,dx}{(8\times10^{-4})(70\times10^9)} = 4.464\times10^{-4}\,x\Big|_0^{1.75}$$

$$= 7.813\times10^{-4}\ m = .781\ mm\ \downarrow$$

$$\downarrow \delta_D = \int_0^{4.5} \frac{P}{AE}dx = \int_0^{1.75} \frac{25000}{(8\times10^{-4})(70\times10^9)}dx + \int_{1.75}^{3} \frac{125000\,dx}{(8\times10^{-4})(70\times10^9)} + \int_3^{4.5} \frac{50,000\,dx}{(5\times10^{-4})(70\times10^9)}$$

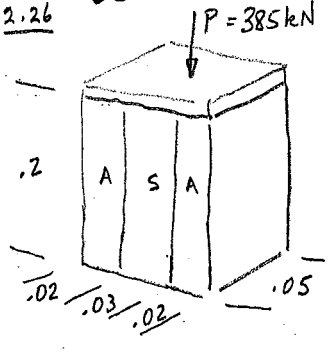
$$= \delta_B + 2.232\times10^{-3}\,x\Big|_{1.75}^{3} + 1.4286\times10^{-3}\,x\Big|_3^{4.5} = 7.813\times10^{-4} + 2.790\times10^{-3} + 2.1429\times10^{-3}$$

$$= 5.714\times10^{-3}\ m = \underline{5.71\ mm}$$

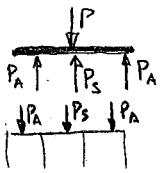
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**DO**

2.26




$E_A = 70\ GPa$
$E_S = 200\ GPa$




$2P_A + P_S = P = 385\times10^3\ N \qquad \Delta u_A = \Delta u_S = \epsilon_A L_A = \epsilon_S L_S\ ;\ L_S = L_A$

$$\epsilon_S = \epsilon_A = \frac{\sigma_A}{E_A} = \frac{\sigma_S}{E_S} \Rightarrow \frac{P_A}{E_A A_A} = \frac{P_S}{E_S A_S}$$

$A_S = .03\times.05 = 1.5\times10^{-3}\ m^2$

$A_A = .02\times.05 = 1.0\times10^{-3}\ m^2$

$$\frac{P_A}{(70\times10^9)(1\times10^{-3})} = \frac{P_S}{(200\times10^9)(1.5\times10^{-3})}$$

$$P_S = \frac{300\times10^6}{70\times10^6}P_A = 4.286\,P_A$$

$$P = 2P_A + P_S = 2P_A + 4.286P_A = 6.286\,P_A \Rightarrow P/6.286 = P_A$$

$$\sigma_A = P_A/A_A = \frac{6.125\times10^4}{1.0\times10^{-3}} = 61.25\times10^6\ Pa \Leftarrow P_A = 61247.22\ N$$

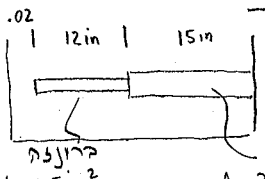
לכן,

$$\sigma_S = P_S/A_S = \frac{2.625\times10^5}{1.5\times10^{-3}} = 175\times10^6 \Leftarrow P_S = 262505.66\ N$$

הטוף $\sigma_A = 61.25\ MPa$ הפלדה $\sigma_S = 175\ MPa$ -ו

---

2.45




$A = 2.5\ in^2$
$E_B = 15\times10^6\ psi$
$\alpha_B = 10.1\times10^{-6}/°F$

$A_A = 3\ in^2$
$E_A = 10\times10^6\ psi$
$\alpha_A = 12.8\times10^{-6}/°F$

ההתארכות של המוט המורכב היא $\Delta u = L\epsilon_x$ אבל מפני שהמוט מחומם בדיוק למאזן את הדפורמציה שגורמת למוצר לקיר. לכן, מפני ש $\epsilon_x = \alpha\Delta T$

$$\Delta u = .02\ in = L_B\alpha_B\Delta T + L_A\alpha_A\Delta T = \Delta T\left[(12\times10.1\times10^{-6}) + (15\times12.8\times10^{-6})\right]$$

$$= \Delta T\,(3.132\times10^{-4}) \Rightarrow 63.857\,°F = \Delta T$$

לכן, יש עוד $\Delta T^* = 136.14°$ שהמוט נותן בקיר. כדי לפשוט פיתרון, נעבוד בקיר את התארכות של האלומיניום והברונזה עד שהמוט מתנתק

$\Delta u_A = L_A\alpha_A(63.86) = .01226in$, $\Delta u_B = .00774in$. לכן, אורך האלומיניום בהתחלת הנעיצה 15.01226 ושל הברונזה 12.00774

כאשר המוטות מתחילים להתמתח יותר מאשר המוצר יש כוח שפועל על המוט

$P$ $\qquad$ $L_B$ $\qquad$ $L_B+L_A$

$$0 = \Delta u = \Delta u_B + \Delta u_A$$

$$\begin{cases} \Delta u_B = \int_0^{l_B} \epsilon_{x_B} dx = \int_0^{l_B} \left(\frac{\sigma_{x_B}}{E_B} + \alpha_B \Delta T^*\right) dx = \left(\frac{\sigma_{x_B}}{E_B} + \alpha_B \Delta T^*\right) x \Big|_0^{l_B} & \text{1- התארכות הברונזה} \\ \Delta u_A = \int_{l_B}^{l_B+l_A} \epsilon_{x_A} dx = \int_{l_B}^{l_B+l_A} \left(\frac{\sigma_{x_A}}{E_A} + \alpha_A \Delta T^*\right) dx = \left(\frac{\sigma_{x_A}}{E_A} + \alpha_A \Delta T^*\right) x \Big|_{l_B}^{l_B+l_A} & \text{1- התקצרות האלומיניום} \end{cases}$$

כאן, העיבור הוא מהתימות והלחץ מאזן עם הקיר המשמאלי. $\sigma_{x_A} = \frac{P_A}{A_A}$, $\sigma_{x_B} = \frac{P_B}{A_B}$. אבל $P_A = P_B$ ; לכן

$$0 = \Delta u_B + \Delta u_A = \left(\frac{P}{E_B A_B} + \alpha_B \Delta T^*\right) l_B + \left(\frac{P}{E_A A_A} + \alpha_A \Delta T^*\right) l_A$$

או

$$P\left[\frac{l_B}{E_B A_B} + \frac{l_A}{E_A A_A}\right] = -\left(\alpha_B l_B + \alpha_A l_A\right) \Delta T^*$$

$$P\left[\frac{12.00774 \text{ in}}{15\times10^6 \frac{\text{lb}}{\text{in}^2} \cdot 2.5 \text{in}^2} + \frac{15.01226 \text{ in}}{10\times10^6 \frac{\text{lb}}{\text{in}^2} \cdot 3 \text{in}^2}\right] = -\left[10.1\times10^{-6}/^\circ\text{F} \cdot 12.00774 + 12.8\times10^{-6}/^\circ\text{F} \cdot 15.01226 \text{ in}\right](136.14^\circ\text{F})$$

$$P\left(3.2021\times10^{-7} \frac{\text{in}}{\text{lb}} + 5.0041\times10^{-7} \frac{\text{in}}{\text{lb}}\right) = -\left(1.21278\times10^{-4} \frac{\text{in}}{^\circ\text{F}} + 1.92157\times10^{-4}\right)(136.14^\circ\text{F})$$

$$P\left(8.2062\times10^{-7} \frac{\text{in}}{\text{lb}}\right) = -\left(3.13435\times10^{-4} \frac{\text{in}}{^\circ\text{F}}\right)(136.14^\circ\text{F}) = 4.2671\times10^{-2} \text{ in}$$

$$\underline{P = -5.20\times10^4 \text{ lb} = -52000 \text{ lb}}$$

$$\underline{\sigma_{x_A} = \frac{P}{A_A} = -17333 \text{ lb/in}^2} \qquad \sigma_{x_B} = \frac{P}{A_B} = -20800 \text{ lb/in}^2$$

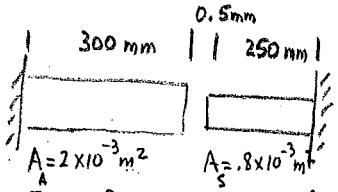
למצוא את התארכות של האלומיניום $\delta_{Al}$

$$\delta_{Al} = \int_{l_A}^{l_B+l_A} \left(\frac{\sigma_{x_A}}{E_A} + \alpha_A\left[\Delta T + \Delta T^*\right]\right) dx$$

$$\underline{\delta_{Al} = \left\{\frac{\sigma_{x_A}}{E_A} + \alpha_A \cdot (\Delta T + \Delta T^*)\right\} l_A = \left\{\frac{-17333}{10\times10^6} + 12.8\times10^{-6}(200)\right\}(15) = 1.24\times10^{-2} \text{ in}}$$

---

2.46




לפי הגישה הנ"ל

$$\Delta u = 0.5\times10^{-3} = L_A \alpha_A \Delta T + L_S \alpha_S \Delta T$$

$$= \left(.3 \cdot 23\times10^{-6} + .25 \cdot 18\times10^{-6}\right)\Delta T$$

$$0.5\times10^{-3} = 1.14\times10^{-5} \Delta T$$

$$43.86^\circ\text{C} = \Delta T$$

$$76.14^\circ = (140-20) - 43.86^\circ = \underline{\Delta T^*}$$

אחרי כת

$$0 = \Delta u_A + \Delta u_S = \left(\frac{P}{E_A A_A} + \alpha_A \Delta T^*\right) l_A + \left(\frac{P}{E_S A_S} + \alpha_S \Delta T^*\right) l_S$$

$$P\left(\frac{l_A}{E_A A_A} + \frac{l_S}{E_S A_S}\right) = -\left(\alpha_A l_A + \alpha_S l_S\right) \Delta T^*$$

$$P\left[\frac{0.3}{(70\times10^9)(2\times10^{-3})} + \frac{0.250}{(190\times10^9)(0.8\times10^{-3})}\right] = -\left((23\times10^{-6})(0.3) + (18\times10^{-6})(0.25)\right)(76.14^\circ)$$

$$P\left(3.7876\times10^{-9}\right) = -\left(1.14\times10^{-5}\right)(76.14^\circ) = -8.6800\times10^{-4} \text{ m}$$

$$\underline{P = -2.29169\times10^5 \text{ N} = -229169 \text{ N}}$$

$$\sigma_A = \frac{P}{A_A} = \frac{-229169 \text{ N}}{2\times10^{-3} m^2} = -114.58 \text{ MPa}$$

$$\delta_{Al} = \int_0^{l_A} \left(\frac{\sigma_{x_A}}{E_A} + \alpha_A\left(\Delta T + \Delta T^*\right)\right) dx = \left\{\frac{\sigma_{x_A}}{E_A} + \alpha_A\left(\Delta T + \Delta T^*\right)\right\} l_A = \left\{\frac{-1.1458\times10^8}{70\times10^9} + 23\times10^{-6}(120^\circ\text{C})\right\}$$

$R_A \qquad R_B$

$$0 = \Delta u_B + \Delta u_A = \left(\frac{P}{E_B A_B} + \alpha_B \Delta T^*\right) l_B + \left(\frac{P}{E_A A_A} + \alpha_A \Delta T^*\right) l_A$$

$$P\left[\frac{l_B}{E_B A_B} + \frac{l_B}{E_A A_A}\right] = -\left(\alpha_B l_B + \alpha_A l_A\right)\Delta T^*$$

$$P\left[\frac{12.00774\ in}{15\times10^6 \frac{lb}{in^2}\cdot 2.5 in^2} + \frac{15.01226\ in}{10\times10^6 \frac{lb}{in^2}\cdot 3 in^2}\right] = -\left[10.1\times10^{-6}/{}^\circ F \cdot 12.00774 + 12.8\times10^{-6}/{}^\circ F \cdot 15.01226\ in\right](136.14\,^\circ F)$$

$$P\left(3.2021\times10^{-7}\frac{in}{lb} + 5.0041\times10^{-7}\frac{in}{lb}\right) = -\left(1.21278\times10^{-4}\frac{in}{{}^\circ F} + 1.92157\times10^{-4}\right)(136.14\,^\circ F)$$

$$P\left(8.2062\times10^{-7}\frac{in}{lb}\right) = -\left(3.13435\times10^{-4}\frac{in}{{}^\circ F}\right)(136.14\,^\circ F) = 4.2671\times10^{-2} in$$

$$P = -5.20\times10^4 lb = -52000\ lb$$

$$\sigma_{x_A} = \frac{P}{A_A} = -17333\ lb/in^2 \qquad \sigma_{x_B} = \frac{P}{A_B} = -20800\ lb/in^2$$

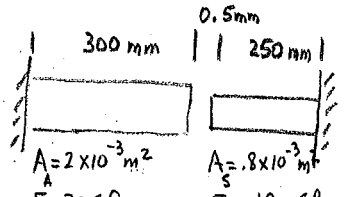
$$\delta_{Al} = \int_{l_A}^{l_B+l_A}\left(\frac{\sigma_{x_A}}{E_A} + \alpha_A\left[\Delta T + \Delta T^*\right]\right)dx$$

למצוא את התארכות של האלומיניום

$$\delta_{Al} = \left\{\frac{\sigma_{xA}}{E_A} + \alpha_A\cdot\left(\Delta T + \Delta T^*\right)\right\} l_A = \left\{\frac{-17333}{10\times10^6} + 12.8\times10^{-6}(200)\right\}(15) = 1.24\times10^{-2} in$$

---

2.46




לפי הנוסחה הנ"ל

$$\Delta u = 0.5\times10^{-3} = L_A\alpha_A\Delta T + L_S\alpha_S\Delta T$$
$$= \left(.3\cdot23\times10^{-6} + .25\cdot18\times10^{-6}\right)\Delta T$$
$$0.5\times10^{-3} = 7.14\times10^{-5}\Delta T$$
$$43.86°C = \Delta T$$
$$76.14° = (140-20) - 43.86° = \Delta T^*$$

אחרי כח

$$0 = \Delta u_A + \Delta u_S = \left(\frac{P}{E_A A_A} + \alpha_A\Delta T^*\right) l_A + \left(\frac{P}{E_S A_S} + \alpha_S\Delta T^*\right) l_S$$

$$P\left(\frac{l_A}{E_A A_A} + \frac{l_S}{E_S A_S}\right) = -\left(\alpha_A l_A + \alpha_S l_S\right)\Delta T^*$$

$$P\left[\frac{0.3}{(70\times10^9)(2\times10^{-3})} + \frac{0.250}{(190\times10^9)(0.8\times10^{-3})}\right] = -\left((23\times10^{-6})(0.3) + (18\times10^{-6})(0.25)\right)(76.14°)$$

$$P\left(3.7876\times10^{-9}\right) = -\left(1.14\times10^{-5}\right)(76.14°) = -8.6800\times10^{-4}\ m$$

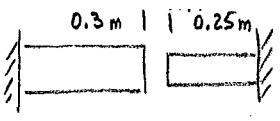
$$P = -2.29169\times10^5\ N = -229169\ N$$

$$\sigma_A = \frac{P}{A_A} = \frac{-229169\ N}{2\times10^{-3} m^2} = -114.58\ MPa$$

$$\delta_{Al} = \int_0^{l_A}\left(\frac{\sigma_{x_A}}{E_A} + \alpha_A\left(\Delta T + \Delta T^*\right)\right)dx = \left\{\frac{\sigma_{x_A}}{E_A} + \alpha_A\left(\Delta T + \Delta T^*\right)\right\} l_A = \left\{\frac{-1.1458\times10^8}{70\times10^9} + 23\times10^{-6}(120°C)\right\}$$
$$= 3.37\times10^{-4} m$$

$$L = L_0 + \delta_{Al} = 0.3m + 0.00034m = 0.30034m = 300.34mm$$

2.47




$\sigma_s = -150\times10^6\ N/m^2 = P/A_s = \dfrac{P}{.8\times10^{-3}\ m^2}$

$P = -1.2\times10^5\ N$

$$0 = \Delta u_A + \Delta u_s = \left(\frac{P}{E_A A_A} + \alpha_A \Delta T^*\right) l_A + \left(\frac{P}{E_s A_s} + \alpha_s \Delta T^*\right) l_s$$

$$= P\left[\frac{l_A}{E_A A_A} + \frac{l_s}{E_s A_s}\right] + (\alpha_A l_A + \alpha_s l_s)\Delta T^*$$

$$0 = -1.2\times10^5\left(3.7876\times10^{-9}\right) + \left(1.14\times10^{-5}\right)\Delta T^* = -4.545\times10^{-4} + (1.14\times10^{-5})\Delta T^*$$

2.46 מהמשוואות $\left[\frac{l_A}{E_A A_A} + \frac{l_s}{E_s A_s}\right]$ ו $(\alpha_A l_A + \alpha_s l_s)$ המספרים

לכן,

$$\Delta T^* = \frac{4.545\times10^{-4}}{1.14\times10^{-5}} = 39.87^\circ C$$

כדי להוסיף את הטמפרטורה לסיבוב של המרחק של 0.5mm, צריך לדעת את $\Delta T = 43.86^\circ C$ כמו שמצאנו בתרגיל הקודם. לכן, $\Delta T + \Delta T^* = 43.86 + 39.87 = 83.73^\circ C$. אפשר לחשב את שינוי הטמפרטורה T, $T = 20^\circ C$ הוסיף כדי להיות $83.73 + 20^\circ = 103.73^\circ C$

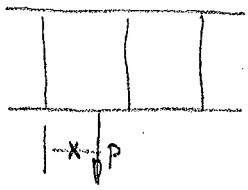
$$\delta_s = \left[\frac{\sigma_{xs}}{E_s} + \alpha_s(\Delta T + \Delta T^*)\right] l_s = \left[\frac{-150\times10^6}{190\times10^9} + (18\times10^{-6})(43.86 + 39.87)\right](250\ mm)$$

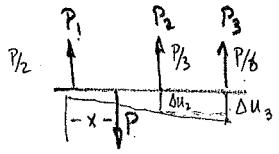
$$= (7.1767\times10^{-4})(250\ mm) = 1.794\times10^{-1}\ mm$$

$$L = L_0 + \delta_s = 250 + 0.1794\ mm = 250.1794\ mm$$

2.45 לפי השיטה של תרגיל

# Do




$P_1+P_2+P_3=P$

$\Sigma M_{P_1} = P\cdot\frac{2}{3}L - P_2\cdot L - P_3\cdot 2L = 0$

$\Delta u_2 = \frac{P_2 L}{AE}$

$\Delta u_3 = \frac{P_3 L}{AE}$

$\Delta u_1 = \frac{P_1 L}{AE}$

$\frac{2Pl}{3} = P_2 + 2P_3 \qquad P_2 = \frac{2P}{3} - 2P_3$

$P_1 + \left(\frac{2P}{3} - 2P_3\right) + P_3 = P$

$(P_1 - P_3) = P/3$

$P_3 - P_2 = P_2 - P_1$

$\frac{\Delta u_3 - \Delta u_2}{L} = \frac{u_3 - u_2}{L} = \frac{u_2 - u_1}{L}$

$\frac{1}{L}\left(\frac{P_3 L}{AE} - \frac{P_2 L}{AE}\right) = \frac{1}{L}\left(\frac{P_2 - P_1}{AE}\right)$

$P_3 - P_2 = P_2 - P_1$

$P_3 - 2P_2 + P_1 = 0$ ①

$P_3 + P_2 + P_1 = P$ ②

② − ① ⇒ $3P_2 = P \quad P_2 = P/3$

$P/3 = 2P/3 - 2P_3 \Rightarrow 2P_3 = P/3$

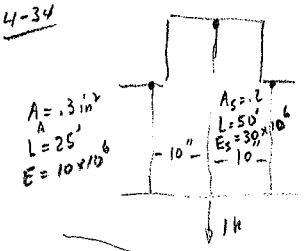
$P_3 = P/6$

$P_1+P_2+P_3=P \Longrightarrow P_1 = P/2$

$\epsilon_1 =$

את הכוח מכוון מטה.

**4-34**




$\Sigma M_{P=1k} = 0 \Rightarrow P_{AL} = P_{AR}$

$2P_A + P_S = P \Leftarrow \Sigma F_y = 0$

By symmetry bar moves vertical

$\Delta u_A = \Delta u_S \Rightarrow \epsilon_A L_A = \epsilon_S L_S$

$\frac{\epsilon_A}{\epsilon_S} = \frac{L_S}{L_A} = \frac{2}{1}$

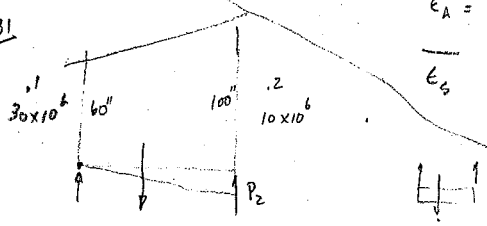
$\frac{\epsilon_A}{\epsilon_S} = \frac{\sigma_A/E_A}{\sigma_S/E_S} = \frac{P_A}{E_A A_A}\cdot\frac{E_S A_S}{P_S} = 2$

$\frac{P_A}{P_S}\cdot\frac{E_S A_S}{E_A A_A} = \frac{P_A}{P_S}\cdot\frac{30\times10^6}{1\times10^7}\cdot\frac{.2}{.3} = 2$

$\frac{P_A}{P_S}\cdot 2 = 8$

$P_A = \frac{4}{5}P_S$

**4-31**




$\Sigma M \quad P\cdot 10 - P_2\cdot 20 = 0 \quad P_2 = \frac{1}{2}P \quad P_1 = \frac{1}{2}P$

$P_A = \frac{E_A A_A}{E_S A_S} P_S \cdot 2 = \frac{10\times10^6 (.3)}{30\times10^6 (.2)}\cdot 2P_S$

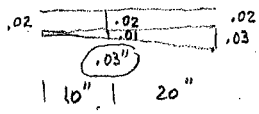
$P_A = \frac{1}{3}\left(\frac{3}{2}\right)\cdot 2P_S \qquad P_A = P_S \Longrightarrow 3P_A = P \qquad P_A = \frac{1}{3}P = P_S$

$\Delta u$

$\Delta u = \frac{P_1 L_1}{A_1 E_1} \qquad \Delta u_2 = \frac{P_2 L_2}{A_2 E_2}$

$\frac{\frac{P}{2}\cdot 60}{.1\times 30\times10^6} \qquad \frac{\frac{P}{2}\cdot 100}{.2\times10\times10^6}$

$\frac{1000\,(60)}{30\times10^5} = \frac{6\times10^4}{3\times10^6} = 2\times10^{-2} = .02\text{in} \qquad \frac{100\,000}{2\times10^6} = \frac{1\times10^5}{2\times10^6} = 0{,}5\times10^{-1} = .05$




שני חוטים נקטרים לאותו קשת במשורטט חוט השמאלי מפלדה בעל חתך של $0{,}1''^2$ ו $E=30\times10^6$ חוט הימני נעשה מסגסוגת אלומיניום בעל חתך $0{,}2''^2$ ו $E=10\times10^6$. את מקום מיושב במשורטט מה יהיה בהנחה שמני הקצוות התחתונים

26. $u_A = u_B$

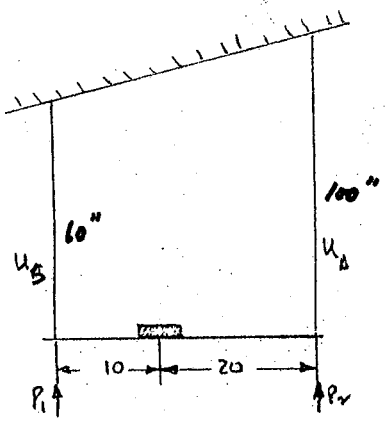
$$u_A = \int_0^{L_A} \frac{1}{E}\frac{P}{A}\,dx = \frac{P}{EX2}L_A$$

$$u_B = \int_0^{L_B} \frac{1}{E}\frac{P}{A}\,dx = \frac{P}{E}\int \frac{dx}{A_T} = \frac{PL_B}{E}\int_C^{L_B} \frac{dx}{2x + L_B} = \frac{PL_B}{E}\left\{\frac{1}{2}\ln\left(3L_B/L_B\right)\right\} = \frac{P_{L_B}}{E}[.55]$$

$u_A = u_B \quad \therefore \quad \frac{P}{EX2}L_A = \frac{P_{L_B}}{E}(.55)$

$L_A/L_B = 1.1$ ✓

31.




$P_1 + P_2 = 2000$ $\qquad 3P_2 = 2000$

$P_1(10) = P_2(20)$ $\qquad P_2 = \frac{2000}{3}$ $\qquad P_1 = \frac{4000}{3}$

$$u_B = \int_0^{60''} \frac{1}{E_S}\frac{P_1}{A_S}\,dx = \frac{\frac{4000}{3} \times 60}{3 \times 30\times10^6 \times .1} = \frac{12(2)}{9\times10^2} = .6667\times10^{-2}\text{ in} = 2.667$$

$$u_A = \int_0^{100''} \frac{1}{E_A}\frac{P_2}{A_A}\,dx = \frac{2000}{3\times10\times10^6\times.2} \times 100 = 3.33\times10^{-2}$$

find $P_1, P_2$

$\sigma_1, \sigma_2, \epsilon_1, \epsilon_2$ $\qquad u_1 = \epsilon_1 \Delta L_1$ $\qquad u_2 = \epsilon_2 \Delta L_2$

$2.66\times10^{-2}$ ..... deflection $\qquad 3.77\times10^{-2}$ $\qquad 4(.667\times10^{-2}\text{ in})$

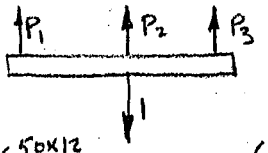
deflection = $.667\times10^{-2}\text{ in} + \frac{1}{3}4(.667\times10^{-2}\text{ in})$

$1.555\times10^{-2}\text{ in}$

$\frac{30}{.667} = \frac{10}{x}$ $\quad \therefore$

2.667 — — — — .667

34




$u_1 = u_2 = u_3$

$P_1 + P_2 + P_3 = 1\text{ K}$

$P_1 = P_3$

$$u_1 = \int_0^{25\times12} \frac{P_1}{E_1A_1}dx_1 = \int_0^{50\times12} \frac{P_2}{E_2A_2}dx_2 = \int_0^{25\times12} \frac{P_3}{E_3A_3}dx_3$$

$$u_1 = \frac{P_1}{E_1A_1}(25\times12) = \frac{P_2}{E_2A_2}(50\times12)$$

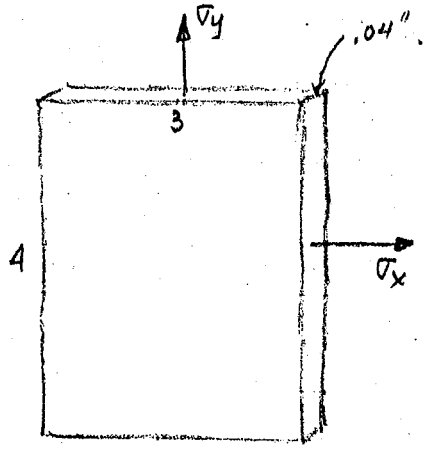
$$\frac{P_1}{E_1A_1} = \frac{2P_2}{E_2A_2}$$

$E_1A_1 = (10\times10^6)(.3) = 3\times10^6$

$E_2A_2 = (30\times10^6)(.2) = 6\times10^6$

$\therefore E_1A_1 = \frac{1}{2}E_2A_2 \quad \therefore 2P_1 = 2P_2$

$\therefore P_1 = P_2 = P_3 \implies P_1 = P_2 = P_3 = 1/3\text{ K}$




סרגפת 3" × 4" מלבנית בין מסגרת קשיחה
של סגסוגת INVAR, שלא יש התקדמות על 200°F. מ'ג'ו
מתכונות בסרגפת

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y + \alpha\Delta T \qquad 16\times10^6\ psi = E_{brass}$$

$$\epsilon_y = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E} + \alpha\Delta T \qquad 6\times10^6 = G$$

$$11\times10^{-6}\ \frac{in}{in/°F} = \alpha$$

since it is constrained to a rigid invar frame then

$$\epsilon_x = \epsilon_y = 0$$

$$-\alpha\Delta T\, E = \sigma_x - \nu\sigma_y$$
$$-\alpha\Delta T\, E = -\nu\sigma_x + \sigma_y$$

$$G = \frac{E}{2(1+\nu)}$$

$$\nu = \tfrac{1}{2}\left(\tfrac{E}{G}\right) - 1 = \tfrac{1}{3}$$

then $-\alpha\Delta T E\nu = \nu\sigma_x - \nu^2\sigma_y$

$-\alpha\Delta T E = -\nu\sigma_x + \sigma_y$

$$-\alpha\Delta T E(1+\nu) = (1-\nu^2)\sigma_y = (1-\nu)(1+\nu)\sigma_y$$

$$\sigma_y = \frac{-\alpha\Delta T E}{1-\nu} = -\left(11\times10^{-6}\ \frac{in}{in/deg\,F}\right)(-200\,°F)\left(16\times10^6\ psi\right)\Big/\left(2/3\right)$$

$$\sigma_y = 52.8\ KSI$$

Substituting back we get $\sigma_x = 26.4$ KSI  I made a mistake of compu

×

$-\alpha\Delta T E = +35.2$  $35.2 - 52.8 = -17.6 = -\tfrac{1}{3}\sigma_x$

$\sigma_x = 52.8$ KSI

A 3×4 in 0.040 in thick rectangular brass diaphragm is stretched between a rigid frame made of invar. Take $E_{br} = 16\times10^6$ psi $G_{br} = 6\times10^6$ psi & $\alpha = 11\times10^{-6}\ \frac{in}{in\,°F}$ and $\alpha_{invar} = 0$ for range of temp. involved

If a drop in temp of 200°F occurs what are the resulting ~~the~~ normal stresses in the diaphragm

$$\epsilon_{x'} = l_1^2 \epsilon_x + m_1^2 \epsilon_y + n_1^2 \epsilon_z + l_1 m_1 \gamma_{xy} + m_1 n_1 \gamma_{yz} + n_1 l_1 \gamma_{zx}$$

$$\vdots = \dots$$

$$\epsilon_{z'} = \dots$$

$$\gamma_{x'y'} = -2 l_1 m_1 (\epsilon_x - \epsilon_y) + (l_1 m_2 + m_1 l_2) \gamma_{xy}$$

$$\vdots = \dots$$

$$\gamma_{z'x'} = \dots$$

$\epsilon_1, \epsilon_2$ 2-D max, min $\quad \gamma_{12} = 0$

$\epsilon_1, \epsilon_2, \epsilon_3$ 3-D max, min, other $\quad \gamma_{12} = \gamma_{23} = \gamma_{31} = 0$

9-24.1

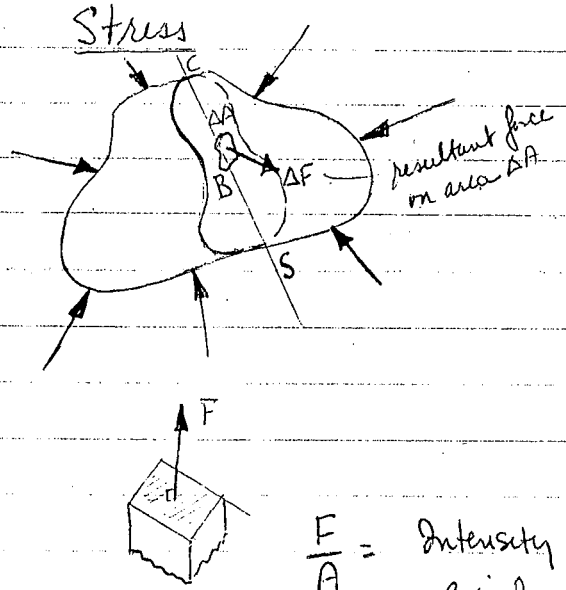
2-D
$$\begin{cases} \epsilon_x + \epsilon_y = I_{1\epsilon} & \text{invariant} \\ \epsilon_x \epsilon_y - \frac{1}{4}\gamma_{xy}^2 = I_{2\epsilon} & \text{invariant} \end{cases}$$

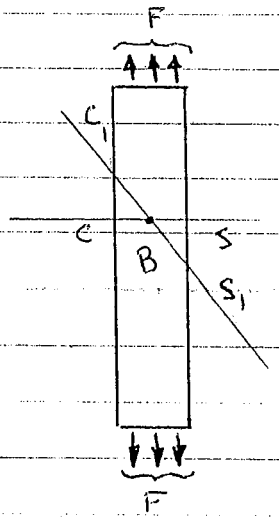
3-D
$$\begin{cases} \epsilon_x + \epsilon_y + \epsilon_z = I_{1\epsilon} & \text{invariant} \\ \epsilon_x \epsilon_y + \epsilon_y \epsilon_z + \epsilon_z \epsilon_x - \frac{1}{4}\left(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2\right) = I_{2\epsilon} \\ \epsilon_x \epsilon_y \epsilon_z + \frac{1}{4}\left(\gamma_{xy}\gamma_{yz}\gamma_{zx}\right) - \frac{1}{4}\left(\epsilon_x \gamma_{yz}^2 + \epsilon_y \gamma_{zx}^2 + \epsilon_z \gamma_{xy}^2\right) = I_{3\epsilon} \end{cases}$$

1.139

Stress

FIG 1
9-24




$\frac{F}{A}$ = Intensity of internal forces in the axial direction.

10-6-70

from FIG. 1 10-5 we can obtain

$$(\sigma_x')_{\substack{max \\ min}} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

$$\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

In 2-D the invariants are

10-6.1

$$I_{1\sigma} = \sigma_x + \sigma_y \quad ; \quad I_{2\sigma} = \sigma_x\sigma_y - \tau_{xy}^2$$

In 3-D the invariants become

$$I_{1\sigma} = \sigma_x + \sigma_y + \sigma_z \quad ; \quad I_{2\sigma} = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)$$

$$I_{3\sigma} = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{zx}\tau_{yz} - \left(\sigma_x\tau_{yz}^2 + \sigma_y\tau_{zx}^2 + \sigma_z\tau_{xy}^2\right)$$

from $\frac{\partial \sigma_x'}{\partial \ell}$ we had obtained $\frac{\ell\sigma_x + m\tau_{xy}}{\ell} = \frac{\ell\tau_{xy} + m\sigma_y}{m} = \frac{\bar{X}}{\ell} = \frac{\bar{Y}}{m}$

or $m/\ell = \bar{Y}/\bar{X} = \tan\alpha$

if ratio holds then we have only $\sigma_1$ and no shear

FIG. 1 10-6




From strain displacements we have 6 equations

$$\epsilon_x = \frac{\partial u}{\partial x} \quad ; \quad \epsilon_y = \frac{\partial v}{\partial y} \quad ; \quad \epsilon_z = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad ; \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \quad ; \quad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

From Stress Equilibrium Equations we got

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + \bar{X} = 0$$

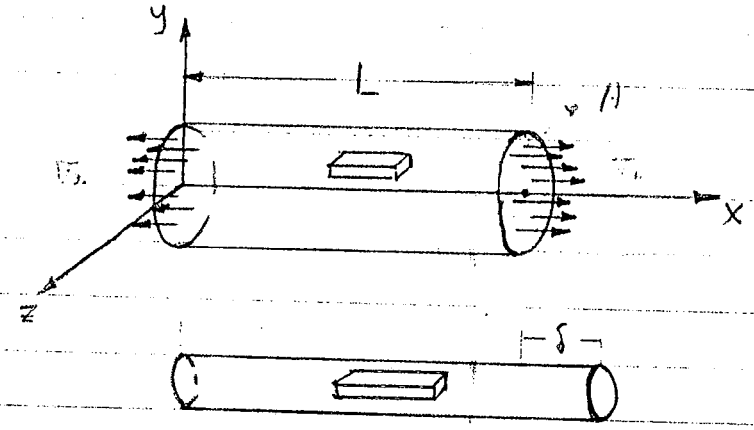
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \bar{Y} = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + \bar{Z} = 0$$

Total :- 9 equations 15 unknowns (stresses, strain, displ

## Stress - Strain Relationships

FIG. 2 10-6




cross-section must remain the same - isotropic properties

$$\delta = \frac{FL}{AE}$$ deformation of bar

$$\frac{\delta}{L} = \epsilon_x = \frac{F}{AE} = \frac{\sigma_x}{E}$$ ; E - modulus of elasticity

$$\epsilon_y = \epsilon_z = -\nu \epsilon_x$$ ; $\nu$ - Poisson's ratio

if $\epsilon_y = \frac{\sigma_y}{E}$ then the effect of this is $\epsilon_x = \epsilon_z = -\nu \epsilon_y$

" $\epsilon_z = \frac{\sigma_z}{E}$ " " " " " " " $\epsilon_x = \epsilon_y = -\nu \epsilon_z$

true if we are below Elastic or Proportional limit (FIG. 1(c) 9-15)

Within elastic limit we can use the principle of superposition

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z)$$

10-6.2

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y)$$

If load is axial then $\gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 0$ (no change in the angles of the parallelopiped of FIG. 2 10-6)

If the load is at some angle $\alpha$ to the axes then the strain relationships are

$$\epsilon x' = l_1^2 \epsilon_x + m_1^2 \epsilon_y + n_1^2 \epsilon_z + l_1 m_1 \gamma_{xy} + m_1 n_1 \gamma_{yz} + n_1 l_1 \gamma_{zx}$$

$$\gamma_{x'y'} = 2(l_1 l_2 \epsilon_x + m_1 m_2 \epsilon_y + n_1 n_2 \epsilon_z) + (l_1 m_2 + l_2 m_1)\gamma_{xy} + (m_1 n_2 + n_1 m_2)\gamma_{yz} + (n_1 l_2 + l_1 n_2)\gamma_{zx}$$

these 2 equations boil down to

$$\left.\begin{aligned} \epsilon x' &= l_1^2 \epsilon_x + m_1^2 \epsilon_y + n_1^2 \epsilon_z \\ \gamma_{x'y'} &= 2(l_1 l_2 \epsilon_x + m_1 m_2 \epsilon_y + n_1 n_2 \epsilon_z) \end{aligned}\right\} \text{Set 1}$$

iff* $\gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 0$

Similarly the Stress relationship is

$$\left.\begin{aligned} \sigma_{x'} &= l_1^2 \sigma_x + m_1^2 \sigma_y + n_1^2 \sigma_z \\ \tau_{x'y'} &= l_1 l_2 \sigma_x + m_1 m_2 \sigma_y + n_1 n_2 \sigma_z \end{aligned}\right\} \text{Set 2}$$

from 10-6.2 we obtain the following

$$\epsilon_x = \frac{(1+\nu)\sigma_x - \nu(\sigma_x + \sigma_y + \sigma_z)}{E}$$

10-6.3

$$\epsilon_y = \frac{(1+\nu)\sigma_y - \nu(\sigma_x + \sigma_y + \sigma_z)}{E}$$

$$\epsilon_z = \frac{(1+\nu)\sigma_z - \nu(\sigma_x + \sigma_y + \sigma_z)}{E}$$

from Set 1 and 10-6.3 we get

$$\epsilon_{x'} = \frac{(1+\nu)}{E}\left[l_1^2\sigma_x + m_1^2\sigma_y + n_1^2\sigma_z\right] - \frac{\nu}{E}\left(\sigma_x+\sigma_y+\sigma_z\right)\left[l_1^2+m_1^2+n_1^2\right]$$

$$= \frac{(1+\nu)}{E}\sigma_{x'} - \frac{\nu}{E}\left(\sigma_{x'}+\sigma_{y'}+\sigma_{z'}\right) = \frac{\sigma_{x'} - \nu\sigma_{y'} - \nu\sigma_{z'}}{E}$$

$$\gamma_{x'y'} = \frac{2(1+\nu)}{E}\left[l_1l_2\sigma_x + m_1m_2\sigma_y + n_1n_2\sigma_z\right] - \frac{2\nu}{E}\left(\sigma_x+\sigma_y+\sigma_z\right)\left[l_1l_2+m_1m_2+n_1n_2\right]$$

$$= \frac{2(1+\nu)}{E}\tau_{x'y'}$$

or

$$\boxed{\gamma_{x'y'} = \frac{2(1+\nu)}{E}\tau_{x'y'} = \frac{\tau_{x'y'}}{G}}$$

10-6.4 $$\boxed{G = E/[2(1+\nu)]}$$

If the loading is not axial and the angles change they are related in the following manner to the shear stress,

10-6.5 $$\boxed{\gamma_{xy} = \frac{\tau_{xy}}{G}\ ;\ \gamma_{yz} = \frac{\tau_{yz}}{G}\ ;\ \gamma_{xz} = \frac{\tau_{zx}}{G}}$$

we now have 15 unknowns & 15 equations.

10-8-70

$$\epsilon_x = \frac{(1+\nu)\sigma_x - \nu(\sigma_x+\sigma_y+\sigma_z)}{E}$$

$$\epsilon_y = \frac{(1+\nu)\sigma_y - \nu(\sigma_x+\sigma_y+\sigma_z)}{E}$$

$$\epsilon_z = \frac{(1+\nu)\sigma_z - \nu(\sigma_x+\sigma_y+\sigma_z)}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G},\quad \gamma_{yz} = \frac{\tau_{yz}}{G},\quad \gamma_{zx} = \frac{\tau_{zx}}{G}\ ;\quad G = \frac{E}{2(1+\nu)}$$

$$\sigma_{ij} = \lambda\delta_{ij}\epsilon_{kk} + 2\mu\epsilon_{ij}$$

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \qquad \mu = G$$



$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y - \frac{\nu}{E}\sigma_z$$

$$\varepsilon_y = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_z$$

$$\varepsilon_z = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \frac{\sigma_z}{E}$$

$$\varepsilon_{xy} = \frac{1}{2G}\tau_{xy}$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}$$

$$\gamma_{xz} = \frac{\tau_{xz}}{G}, \quad \gamma_{yz} = \frac{\tau_{yz}}{G}$$

E – modulus of elasticity – Young's modulus.

Mild steel $E \sim 29 \times 10^6$ psi $= 29 \times 10^3$ ksi

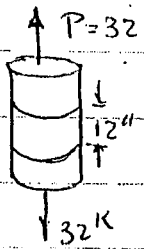
G – shearing modulus of elasticity –

Mild steel $G \sim 10\text{-}12 \times 10^6$ psi $= 10\text{-}12 \times 10^3$ ksi

$\nu$ – Poisson's ratio

0.1 – 0.5, 0.25 – 0.33 More practical

Problem:




$\Delta l = .00938$ Find E & $\nu$

$\Delta d = -.000585$

$E = \frac{\sigma_x}{\varepsilon_x}$

$\sigma_x = \frac{32}{\pi/4 (2.25)^2}$

$\varepsilon_x = \frac{.00938}{12"}$

$\nu = -\varepsilon_y/\varepsilon_x$ ; $\varepsilon_y = \frac{-.000585}{2.25}$

$$\nu = \frac{+\frac{.000585}{2.25}}{\frac{.00938}{12}} =$$

2" cube $\sigma_x = \sigma_y = \sigma_z = 30{,}000$ psi $E = 30 \times 10^6$ psi

$\nu = 1/4$

נהוג מהקדה היה שטח חתכו קבוע. לגופים בה קהויים וגופים פוואסון. מסתבר כי עבור חומרי מבין הומס בין בעבור מהירכי לעבור הרוחבי קבוע. אחרונה הנקרא היחס בין האורך מאחס בשטוש הרוחבית (x) ולזה. עבורם הטווחים $y$ ו- $z$ שווים:

$$\frac{\varepsilon_{zz}}{\varepsilon_{xx}} = \frac{\varepsilon_{yy}}{\varepsilon_{xx}} = -\nu \qquad (8.5)$$

$\nu$ נקרא מקדם פוואסון. ערכו תמיד $0.5$ מספרים ואפונולה: מהלויים $\nu = 0$, פלדה $0.25-0.3$, ואלומיניום $0.33$, פלסטיקים $0.4$ לעומי $\nu = 0.5$.






<u>דוגמא</u>

חשב את שינוי קוטר וגובה של מוט פלדה שהוטעון:
$E = 200\,GPa$, $\nu = 0.25$

$$\varepsilon_{xx} = \frac{P}{EA} = \frac{P}{E\cdot\pi r^2}$$

$$\varepsilon_{yy} = \varepsilon_{zz} = -\nu\varepsilon_{xx} = -\frac{\nu P}{E\pi r^2}$$

$$\varepsilon_{yy} = \frac{\Delta r}{r} \qquad \Delta r = r\cdot\varepsilon_{yy} = -\frac{\nu P}{E\pi r}$$

$$\varepsilon_{xx} = \frac{\Delta L}{L} \qquad \Delta L = L\varepsilon_{xx} = \frac{PL}{E\pi r^2}$$

ולכן

$$\frac{\pi(r+\Delta r)^2(L+\Delta L) - \pi r^2 L}{\pi r^2 L} = \frac{\pi(r^2+2r\Delta r+\Delta r^2)(L+\Delta L) - \pi r^2 L}{\pi r^2 L} =$$

$$= \frac{\pi r^2\Delta L + 2\pi r L\Delta r + \pi r^2 L + \text{מכפלות קטנות}}{\pi r^2 L} = \frac{\Delta V}{V} = \varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz} = \frac{P}{E\pi r^2}\left[1-2\nu\right] =$$

מה קורה כאשר ?

<u>קיים חומר הנקרא אידיאלי</u>

אוטפקט פוואסון נאמנה על חומרי ההוחבי הנידר כתוצאה מהעמסה ג זניחת. הנה ומתוך אחרים בשטוש העמסות הג קטנות כך ותת אהן ניתן סוץ על העבורם שם בטוחים ניתבומלה וסה"כ וקבל תוכ הוק מוכלל:

reasonably away from load solution is different in the area near the load.

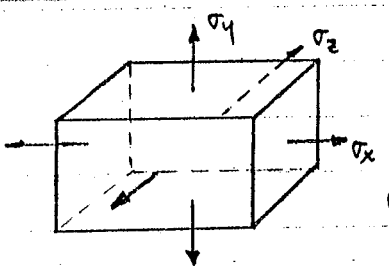
10/13/70

$$\epsilon_x = \frac{\sigma_x - \nu(\sigma_y + \sigma_z)}{E}$$

$$\epsilon_y = \frac{\sigma_y - \nu(\sigma_x + \sigma_z)}{E} \qquad 10\text{-}6.2$$

$$\epsilon_z = \frac{\sigma_z - \nu(\sigma_y + \sigma_x)}{E}$$

FIG. 1
10-13




התגלות הקוביה

$$\frac{\text{new volume} - \text{old vol}}{\text{old vol.}} = \text{cubical dilatation}$$

$$\frac{(1+\epsilon_x)dx(1+\epsilon_y)dy(1+\epsilon_z)dz - dx\,dy\,dz}{dx\,dy\,dz}$$

$$1 + \epsilon_x\epsilon_y + \epsilon_y\epsilon_z + \epsilon_z\epsilon_x + \epsilon_x\epsilon_y\epsilon_z + \epsilon_x + \epsilon_y + \epsilon_z - 1 \approx \epsilon_x + \epsilon_y + \epsilon_z$$

since $\epsilon_i\epsilon_j$ where $i \neq j \ll \epsilon_i, \epsilon_j$

$$\therefore \text{cubic dilatation} = \frac{\Delta V}{V} \approx \epsilon_x + \epsilon_y + \epsilon_z = \epsilon_x' + \epsilon_y' + \epsilon_z' = e$$

$$e = \frac{(1-2\nu)(\sigma_x + \sigma_y + \sigma_z)}{E} \qquad 10\text{-}13.1$$

if $\sigma_x = \sigma_y = \sigma_z = -p$ $\qquad e = -3p\frac{(1-2\nu)}{E} = -\frac{p}{E/3(1-2\nu)} = -\frac{p}{K}$

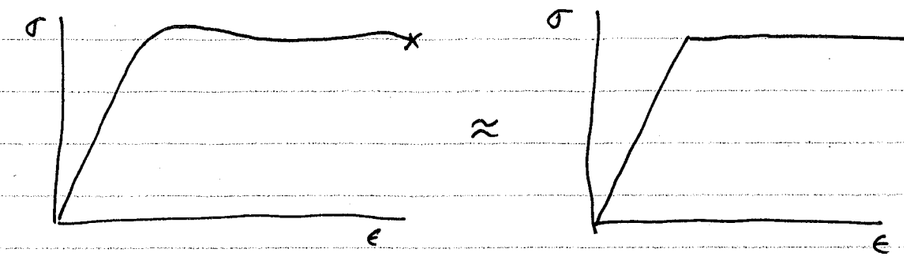
מודול אלסטיות נפחי

$K$ = bulk modulus $= E/3(1-2\nu)$

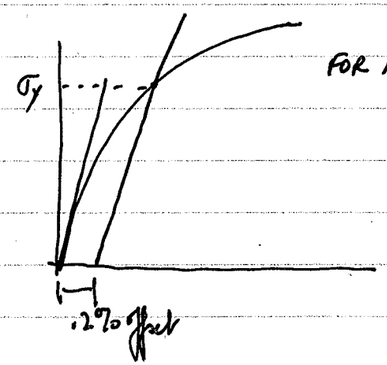
$$K = E/[3(1-2\nu)] \qquad 10\text{-}13.2$$

Strain Energy – consider problems only in statics $KE = 0$

TALK ABOUT ELASTIC - PERFECTLY PLASTIC




TALK ABOUT APPROX TO LINEAR VS .2% OFFSET



**SOLUTION**

In this case $P(x)$ is variable. It is conveniently expressed as $p_o x$ if the origin is taken at $A$. Here again Eq. 4-29 can be applied:

$$u = \int_0^x \frac{P(x)\,dx}{A(x)E} + C_1 = \frac{1}{AE}\int_0^x p_o x\,dx + C_1 = \frac{p_o x^2}{2AE} + C_1$$

At the boundary $B$, where $x = L$, the displacement is zero, i.e., $u(L) = 0$. This condition must be used to evaluate the constant of integration: $C_1 = -p_o L^2/2AE$. Thus $u = -p_o(L^2 - x^2)/2AE$ and $u(0) = -p_o L^2/2AE$. The negative sign indicates that the displacement $u$ is in the opposite direction to that of positive $x$. If $W$ designates the total weight of the rod, the absolute maximum deflection is $WL/2AE$. Compare this expression with Eq. 4-33.

In this problem Eq. 4-32 instead of Eq. 4-29 could be applied. With the gravity load acting downward and with the positive $x$ axis directed upward, the sign of the load in Eq. 4-32 must be negative, i.e., $AE\,d^2u/dx^2 = -(-p_o)$. As in the previous solution, one of the boundary conditions is $u(L) = 0$. The second one is $u'(0) = 0$, where $u' = du/dx$; this follows from the fact that at the free end $P = 0$. (See Eq. 4-30.)

If a concentrated force $P$, in addition to the bar's own weight, were acting on the bar $AB$ at the end $A$, the total end deflection due to the two causes by superposition would be

$$|u| = \frac{PL}{AE} + \frac{WL}{2AE} = \frac{[P + (W/2)]L}{AE}$$

**EXAMPLE 4-10**

A 30-in.-long aluminum rod is enclosed within a steel-alloy tube, Figs. 4-29(a) and (b). The two materials are bonded together. If the stress-strain diagrams for the two materials can be idealized as shown, respectively, in Fig. 4-29(d), what end deflection will occur for $P_1 = 80$ kips and for $P_2 = 125$ kips? The cross-sectional areas of steel $A_s$ and of aluminum $A_a$ are the same and equal to 0.5 in.$^2$.

**SOLUTION**

By applying the method of sections, one can easily determine the axial force at an arbitrary section, Fig. 4-29(c). However, unlike the case in any problem considered so far, the manner in which the resistance to the force $P$ is distributed between the two materials is not known. Thus, the problem is internally statically indeterminate. The requirements of equilibrium (statics) remain valid, but additional conditions are necessary to solve the problem. One of the auxiliary conditions comes from the requirements of the compatibility of deformations. However, since the requirements of statics involve forces and deformations involve displacements, a connecting condition based on the property of materials must be added.

Let subscripts $a$ and $s$ on $P$, $\varepsilon$, and $\sigma$ identify these quantities as being for aluminum and steel, respectively. Then, noting that the applied force is supported by a force developed in steel and aluminum and that

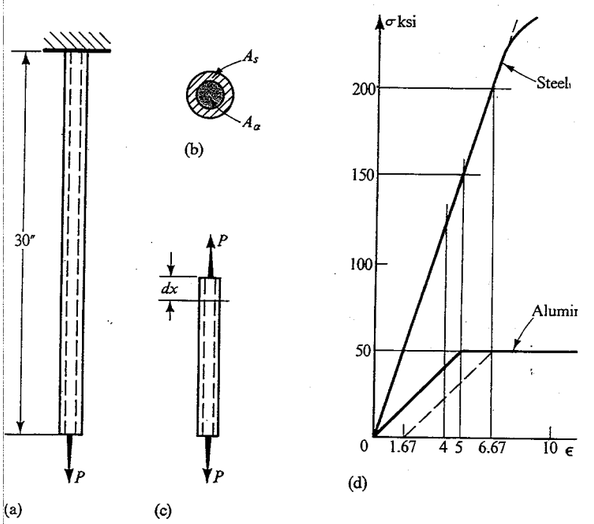



**Fig. 4-29**

at every section the displacement or the strain of the two materials is the same, and tentatively assuming elastic response of both materials, one has

Equilibrium: $P_a + P_s = P_1$ or $P_2$

Deformation: $u_a = u_s$ or $\varepsilon_a = \varepsilon_s$

Material properties: $\varepsilon_a = \sigma_a/E_a$ and $\varepsilon_s = \sigma_s/E_s$

By noting that $\sigma_a = P_a/A_a$ and $\sigma_s = P_s/A_s$, one can solve the three equations. From the diagram the elastic moduli are $E_s = 30 \times 10^6$ psi and $E_a = 10 \times 10^6$ psi. Thus

$$\varepsilon_a = \varepsilon_s = \frac{\sigma_a}{E_a} = \frac{\sigma_s}{E_s} = \frac{P_a}{A_a E_a} = \frac{P_s}{A_s E_s}$$

Hence $P_s = [A_s E_s/(A_a E_a)]P_a = 3P_a$, and $P_a + 3P_a = P_1 = 80$ k; therefore, $P_a = 20$ k, and $P_s = 60$ k.

Applying Eq. 4-33 to either material, the tip deflection for 80 kips will be

$$u = \frac{P_s L}{A_s E_s} = \frac{P_a L}{A_a E_a} = \frac{20(10^3)30}{0.5(10)10^6} = 0.120 \text{ in.}$$

This corresponds to a strain of $0.120/30 = 4 \times 10^{-3}$ in. per inch. In this range both materials respond elastically, which satisfies the material-property assumption made at the beginning of this solution. In fact, as may be seen from Fig. 4-29(d), since for the linearly elastic response the strain can reach $5 \times 10^{-3}$ in. per inch for both materials, by direct proportion the applied force $P$ can be as large as 100 kips.

At $P = 100$ kips the stress in aluminum reaches 50 ksi. According to the idealized stress-strain diagram no higher stress can be resisted by this material, although the strains may continue to increase. Therefore, beyond $P = 100$ kips, the aluminum rod can be counted upon to resist only $P_a = A_a\sigma_{yp} = 0.5 \times 50 = 25$ kips. The remainder of the applied load must be carried by the steel tube. For $P_2 = 125$ kips, 100 kips must be carried by the steel tube. Hence $\sigma_s = 100/0.5 = 200$ ksi. At this stress level $\varepsilon_s = 200/(30 \times 10^3) = 6.67 \times 10^{-3}$ in. per inch. Therefore, the tip deflection

$$u = \varepsilon_s L = 6.67 \times 10^{-3} \times 30 = 0.200 \text{ in.}$$

Note that it is not possible to determine $u$ from the strain in aluminum since no unique strain corresponds to the stress of 50 ksi, which is all that the aluminum rod can carry. However, in this case the elastic steel tube contains the plastic flow. Thus, the strains in both materials are the same, i.e., $\varepsilon_s = \varepsilon_a = 6.67 \times 10^{-3}$ in. per inch, see Fig. 4-29(d).

If the applied load $P_2 = 125$ kips were removed, both materials in the rod would rebound elastically. Thus if one imagines the bond between the two materials broken, the steel tube would return to its initial shape. But a permanent set (stretch) of $(6.67 - 5) \times 10^{-3} = 1.67 \times 10^{-3}$ in. per inch would occur in the aluminum rod. This incompatibility of strain cannot develop if the two materials are bonded together. Instead, residual stresses develop, which maintain the same axial deformations in both materials. In this case, the aluminum rod remains slightly compressed, and the steel tube is slightly stretched. The solution of such statically indeterminate problems is considered in greater detail in Chapter 12. The small effect due to Poisson's ratio is neglected in the above discussion.

## 4-18. STRESS CONCENTRATIONS

From the preceding articles of this chapter it is seen that stresses are accompanied by deformations. If such deformations take place at the same uniform rate in adjoining elements, no additional stresses, other than

the cross-sectional area of a member is interrupted or if the force is actually applied over a very small area, a perturbation in stresses takes place because the adjoining elements must be physically continuous in a deformed state. They must stretch or contract equal amounts at the adjoining sides of all particles. These deformations result from linear and shearing deformations involving the properties of materials $E$, $G$, and $\nu$ and the applied forces. Methods of obtaining this disturbed-stress distri-

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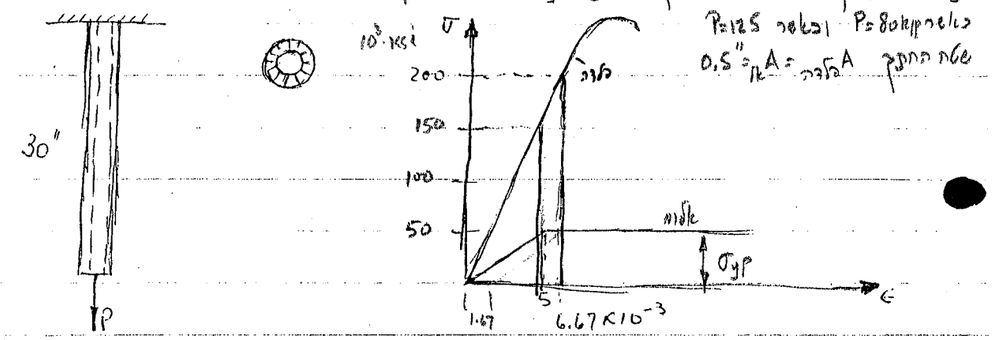
the cross-sectional area of a member is interrupted or if the force is actually applied over a very small area, a perturbation in stresses takes place because the adjoining elements must be physically continuous in a deformed state. They must stretch or contract equal amounts at the adjoining sides of all particles. These deformations result from linear and shearing deformations involving the properties of materials $E$, $G$, and $\nu$ and the applied forces. Methods of obtaining this disturbed-stress distri-

$$u = \frac{P_0 x^2}{2AE} + C_2 \qquad u=0 \quad x=0 \quad C_2 = 0$$

$$u = \frac{P_0 x^2}{2AE} \qquad u(L) = \frac{P_0 L^2}{2AE}$$

Problem 4-10 Page 128

"סם מט אלומינום מוצמד בינור פלדה
שני המוטות קשורים יחד. מצדיק לדיות המתאכזיות המוצרים
כוח מקסימלי $P=80$ kips וכוח $P=125$
שטח החתך $A_{פלדה} = A_{אל} = 0.5$"




$$E_s = \frac{\sigma}{\varepsilon} = \frac{200}{6.67 \times 10^{-3}} = 30 \times 10^3 \text{ KSI}$$ אלומיניום

$$E_a = \frac{\sigma}{\varepsilon} = \frac{50}{5 \times 10^{-3}} = 10 \times 10^3 \text{ KSI}$$

① $P_a + P_s = P$ ← equil $\qquad \varepsilon_s = \varepsilon_a$ ← compatability since $u_s = u_a$ ②

$$\varepsilon_s = \frac{\sigma_s}{E_s} \quad ; \quad \varepsilon_a = \frac{\sigma_a}{E_a}$$ ← elastic ③

$$\varepsilon_s = \frac{\sigma_s}{30 \times 10^3} = \varepsilon_a = \frac{\sigma_a}{10 \times 10^3} \qquad \sigma_s = 3\sigma_a$$ ④

$P_a = \sigma_a A_a \qquad P_s = \sigma_s A_s \qquad P_a = .5\sigma_a \qquad P_s = .5(3\sigma_a) = .5\sigma_s$

$.5\sigma_A + 1.5\sigma_A = P \qquad \sigma_a = P/2.0$

$\sigma_s = \frac{3P}{2.0}$

$$\Delta l = \varepsilon_s l = \varepsilon_a l = \frac{\sigma_a}{10 \times 10^3} .30 = 3\sigma_a \times 10^{-3} = \sigma_s \times 10^{-3}$$

for $P_1 = 80K$ $\qquad \frac{P}{AE_a}$

$\sigma_a = \frac{80}{2.0} = 40$ KSI  $\sigma_s = \frac{3.80}{2.0} = 120$ KSI

$\Delta l = 120 \times 10^{-3} = .12$ in

suppose $P_2 = 125$ K  $\sigma_a = \frac{125}{2} = 62.5 > 50$

$\sigma_s = \frac{3(125)}{2} = 187.5 < 200$  $\sigma_a \equiv 50$ KSI

$P_a \equiv 25$ K  $P_a + P_s = P$

$P_s \equiv 100$ K

$\therefore \sigma_s = \frac{100}{.5} = 200$ KSI

$\epsilon_s = \frac{200}{30 \times 10^3} = 6.67 \times 10^{-3}$

$\Delta l = \epsilon_s \times 30 = 200 \times 10^{-3} = .200$ in

4-3  $\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$

$\epsilon_y = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_z$

$\epsilon_z = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \frac{\sigma_z}{E}$

$\sigma_x = \lambda e + 2\mu\epsilon_x$

$\sigma_x = \frac{(1-\nu^2)E\,\epsilon_x + \nu E(1+\nu)[\epsilon_y + \epsilon_z]}{1-3\nu^2-2\nu^3}$

$= \frac{(1-\nu)E\,\epsilon_x + \nu E[\epsilon_z + \epsilon_y]}{1-\nu-2\nu^2} \quad \frac{}{(1-2\nu)(1+\nu)} = \frac{\nu E}{(1-2\nu)(1+\nu)}(\epsilon_x+\epsilon_y+\epsilon_z) + \frac{2E}{2(1+\nu)}\epsilon_x$

$= \lambda e + 2\mu\epsilon_x$

4-8  $\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E} + \alpha\Delta T$

$\epsilon_y = \frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_z + \alpha\Delta T$

Could contract in 3 directions

$\sigma_z = \epsilon_x = \epsilon_y = 0$

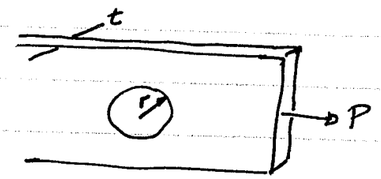
they are constrained

$\sigma_x = \sigma_y$

$\therefore 0 = \left(\frac{1-\nu}{E}\right)\sigma + \alpha\Delta T$

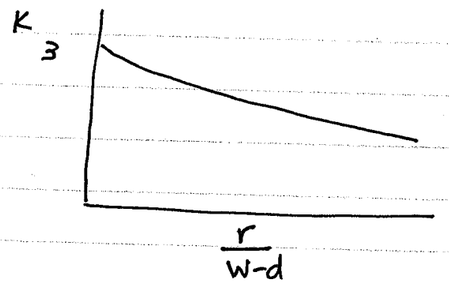
$\sigma = \frac{\alpha E \Delta T}{\nu - 1}$

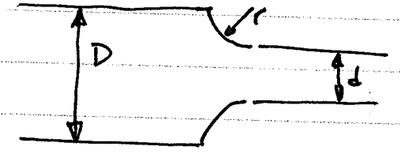
TALK ABOUT STRESS CONCENTRATION FACTORS




$$\sigma_{ave} = \frac{P}{A} = \frac{P}{t(W-d)}$$

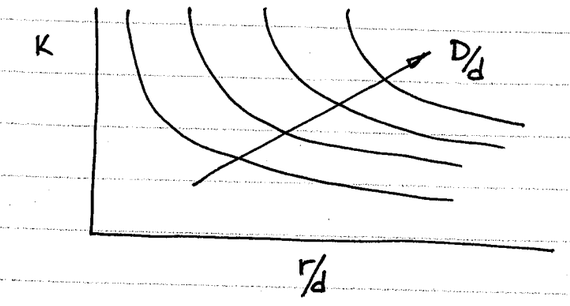
$$\sigma_{actual} = K\sigma_{ave}$$



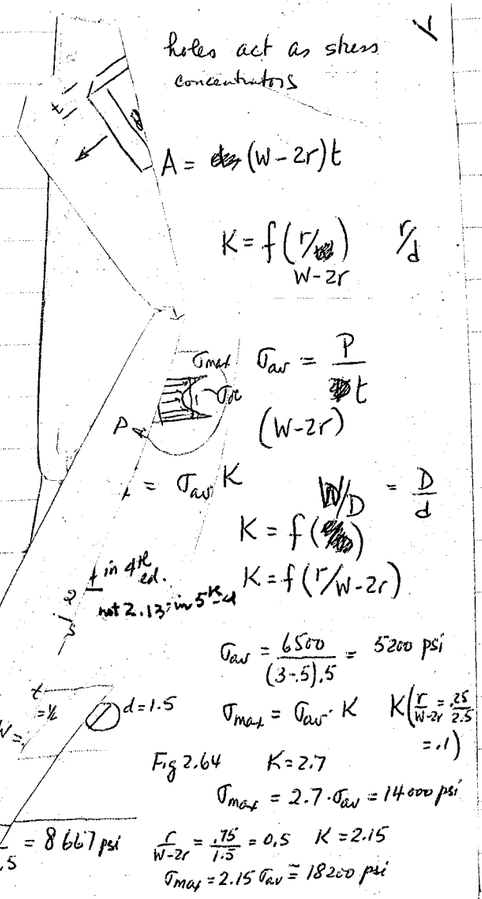




$$\sigma_{ave} = \frac{P}{A} = \frac{P}{td}$$

$$\sigma_{act} = K\sigma_{ave}$$




see other side




**2.94**

$\sigma_{av} = \frac{32000}{.015(.1-.02)} = 26.67$ MPa

$K\left(\frac{r}{W-d} = \frac{10}{80} = .125\right) = 2.67$

$\sigma = K\sigma_{av} =$ 71.2 MPa

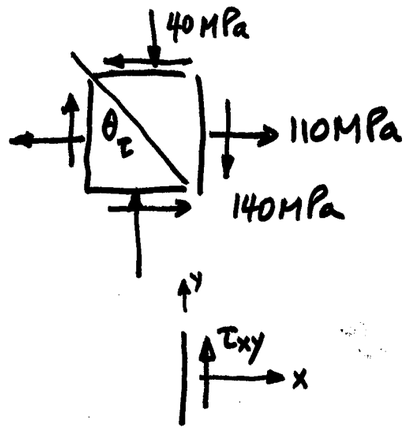
$\sigma_{av} = \frac{32000}{.015(.1-.05)} = 42.67$ MPa

$K\left(\frac{r}{W-d} = \frac{25}{050} = .5\right) = 2.17$

$\sigma_{max} = K\,\sigma_{av} = 92.6$ MPa

# EMA 3702 PART I

$\theta_t$ ?, $\tau_{max}$, $\sigma_x'$ ?




$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_x = 110\,MPa$

$\sigma_y = -40\,MPa$

$\tau_{xy} = -140\,MPa$

$$\tau_{max} = \sqrt{\left(\frac{110-(-40)}{2}\right)^2 + (-140)^2}$$

$$= 158.82\ MPa$$

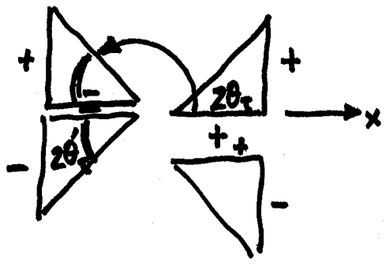
$$\tan 2\theta_t = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(110-(-40))/2}{-140}$$

$$= \frac{-150}{-280}$$

$$2\theta_t' = 28.18^\circ$$

$$2\theta_t = 180 + 28.18^\circ = 208.18^\circ$$

$$\theta_t = 104.09^\circ$$



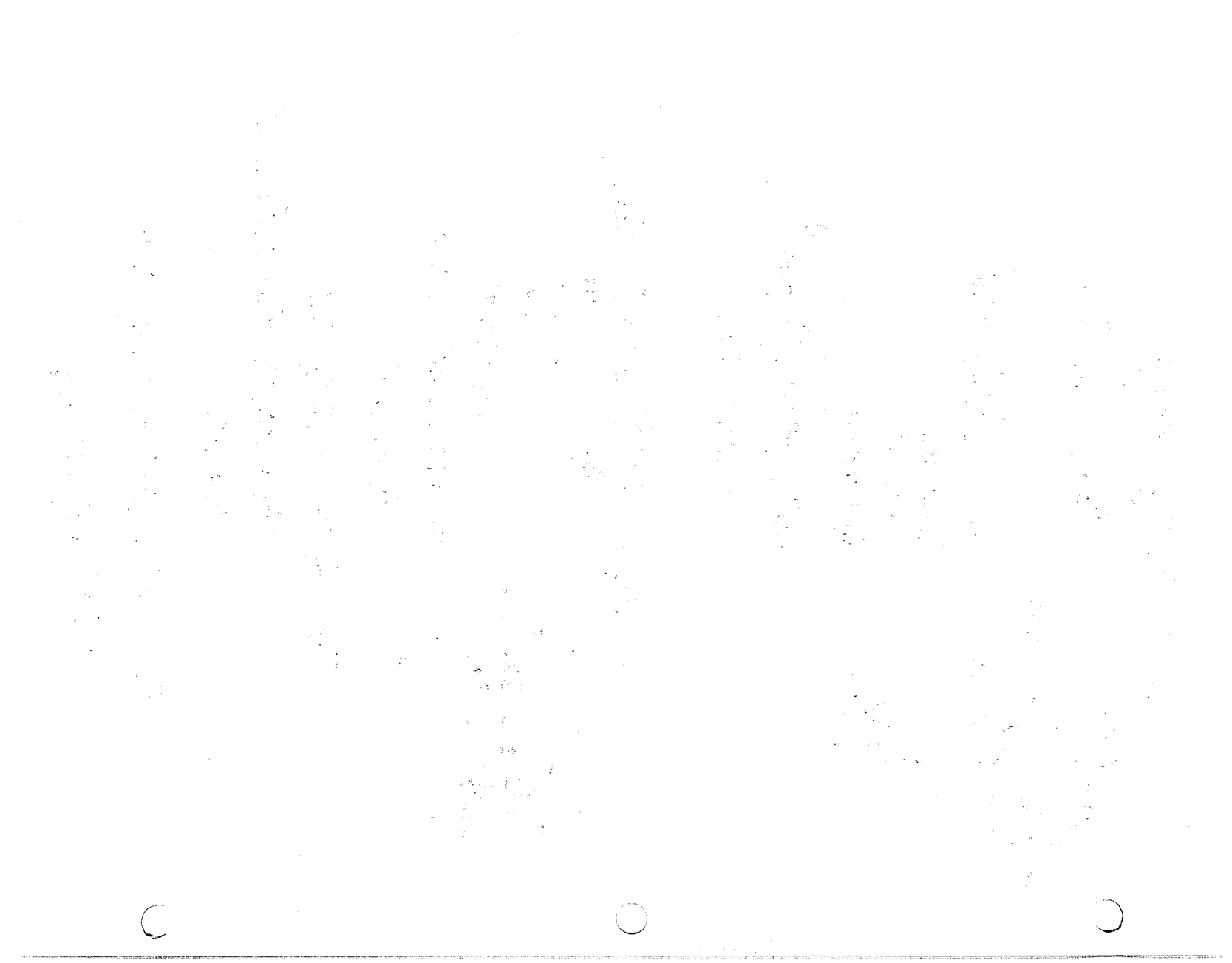

$$\tau_{x'y'} = \tau_{xy}\cos 2\theta - \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta$$

$$2\theta_t = 208.18^\circ + 180^\circ = 388.18^\circ \text{ or } 28.18^\circ$$

$$\theta_t = 14.09^\circ$$

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} = \frac{70}{2} = \underline{\underline{35\,MPa}}$$



1. The stress distribution on the rectangular cross-section shown in Fig. 1 is given by $\sigma_{xx}=1000y-500z+800$ kPa, $\sigma_{xy}=200z$ kPa, $\sigma_{xz}=0$.
What is the net internal force system on this cross-section ?

Answer : $F=1920$ N, $V_y=V_z=0$
$M_y=-1600$ N-cm., $M_z=-7200$ N-cm., $T=-640$ N-cm.

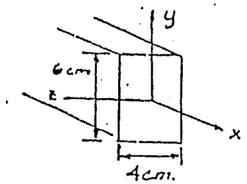


Fig. 1

2. Suppose the stress distribution on a cross-section of the circular cylinder of Fig. 2 is given by $\sigma_{xx}=\sigma_{xr}=0$, $\sigma_{x\theta}=k\sqrt{r}$, where k is unknown. What is the value of k in this case?

Answer : $k=7T_0/4\pi R^{7/2}$

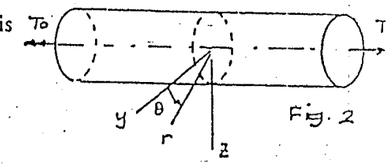


Fig. 2

**6.4** For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium equations of that element, as was done in the derivations presented in Sec. 6.2.

**6.6** For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

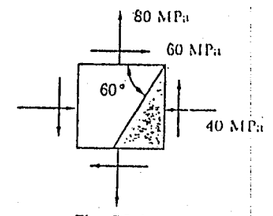


Fig. P6.4

**6.12** For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

**6.14** For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 40° counterclockwise, (*b*) 15° clockwise.

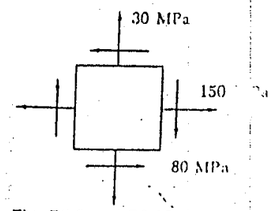
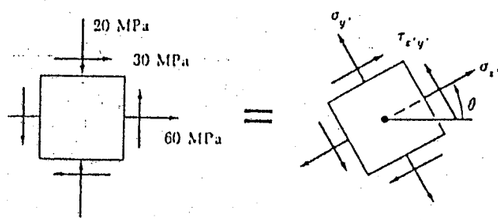


Fig. P6.6 and P6.10

**6.56** For the state of plane stress shown, determine the range of values of $\theta$ for which the normal stress $\sigma_{x'}$ is equal to or less than +65 MPa.



P6.55 and P6.56

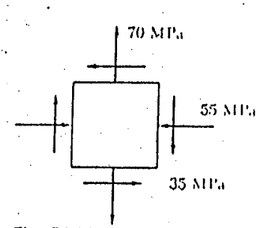


Fig. P6.14

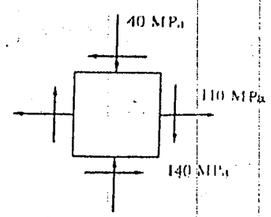


Fig. P6.8 and P6.12

25
12

Suppose $\epsilon_x = -800 \times 10^{-6}$ m/m

$\epsilon_y = -200 \times 10^{-6}$ m/m

$\gamma_{xy} = 800 \times 10^{-6}$

WHAT ARE THE MAX & MIN DIRECT STRAINS $(\epsilon_1, \epsilon_2)$ ✓

WHAT ARE THE PRINCIPAL DIRECTIONS FOR $\epsilon_1$ & $\epsilon_2$ ✓

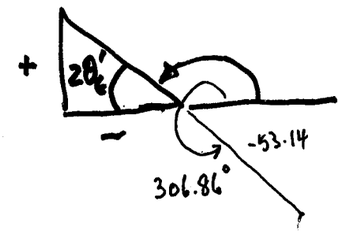
SUPPOSE THESE STRAINS OCCUR ON A PLATE OF STEEL W/ $E = 29.5 \times 10^6$ psi & $\nu = 0.3$, WHAT ARE THE MAX STRESSES $(\sigma_1, \sigma_2)$ & WHAT ARE PRINCIPAL DIRECTIONS $\theta_\sigma$

$$\epsilon_{max,min} = \frac{\epsilon_x + \epsilon_y}{2} \pm \frac{1}{2}\sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}$$

$$= \left\{ \frac{-800 + (-200)}{2} \pm \frac{1}{2}\sqrt{(-800 - (-200))^2 + (800)^2} \right\} \times 10^{-6}$$

$$= \{ -500 \pm 500 \} \times 10^{-6} = \begin{cases} 0 & \sim \epsilon_1 \\ -1000 \times 10^{-6} & \sim \epsilon_2 \end{cases}$$

$$\tan 2\theta_\epsilon = \frac{2\epsilon_{xy}}{\epsilon_x - \epsilon_y} = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{800}{-800 - (-200)} = \frac{800}{-600}$$




$2\theta_\epsilon' = -53.14°$

$2\theta_\epsilon'' = 2\theta_\epsilon' + 180 = 126.86°$

$\theta_{\epsilon_1} = 63.43°$

$\theta_{\epsilon_2} = 153.43°$

$$\epsilon_x' = \frac{\epsilon_x + \epsilon_y}{2} + \left(\frac{\epsilon_x - \epsilon_y}{2}\right) \cos 2\theta + \epsilon_{xy} \sin 2\theta$$

$\epsilon_{xy} \rightarrow \frac{\gamma_{xy}}{2}$

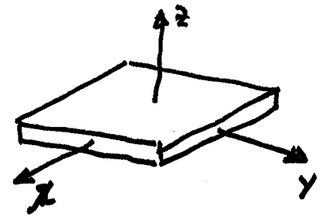
TO FIND $\sigma_1$ & $\sigma_2$ NEED $\sigma_x, \sigma_y, \tau_{xy}, \lambda, \mu$

$\sigma_x = \lambda e + 2\mu\epsilon_x = \lambda e + 2G\epsilon_x$ $\quad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$ $\quad \mu = G = \frac{E}{2(1+\nu)}$

$\sigma_y = \lambda e + 2\mu\epsilon_y$

$= 17019231$ psi $= 11346154$ psi

$e = \epsilon_x + \epsilon_y + \epsilon_z$




$\sigma_z = 0$

$\sigma_z = \lambda e + 2\mu\epsilon_z = 0$ $\quad \lambda(\epsilon_x + \epsilon_y) + (2\mu + \lambda)\epsilon_z = 0$

$\epsilon_z = \frac{-\lambda}{\lambda + 2\mu}(\epsilon_x + \epsilon_y) = 428.57 \times 10^{-6}$

$\sigma_x = \lambda e + 2\mu\epsilon_x = 17.02\times10^{\#6}(-571.43\times10^{-6}) + 2\cdot 11.35\times10^{\#6}(-800\times10^{-6})$

$= -27879$ psi

$\sigma_y = \lambda e + 2\mu\epsilon_y = 17.02\times10^{\#6}(-571.43\times10^{-6}) + 2\cdot 11.35\times10^{\#6}(-200\times10^{-6})$

$= -14264$ psi

$\tau_{xy} = G\gamma_{xy} = 11.35\times10^{\#6}(800\times10^{-6}) = 9077$ psi

$\sigma_{max,min} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \left(\frac{-27879 - 14264}{2}\right) \pm \sqrt{\left(\frac{-27879 + 14264}{2}\right)^2 + (9077)^2}$

$= -21071.5 \pm 11346.1 = \begin{cases} -9725 \text{ psi} - \sigma_1 \\ -32418 \text{ psi} - \sigma_2 \end{cases}$

$\tan 2\theta_\sigma = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$ $\quad 2\theta_\sigma = 126.86°$

$$\sigma_x = \lambda(\epsilon_x + \epsilon_y + \epsilon_z) + 2\mu\epsilon_x$$

$$I_{1\epsilon}$$

$$\epsilon_x = \lambda(\epsilon_1 + \epsilon_2 + \epsilon_3) + 2\mu\epsilon_x$$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z) \qquad \sigma_z = 0$$

$$\epsilon_x = \frac{\sigma_x}{E} + \frac{\nu}{E}\sigma_y \qquad \epsilon_1 = \frac{\sigma_1}{E} - \frac{\nu}{E}\sigma_2$$

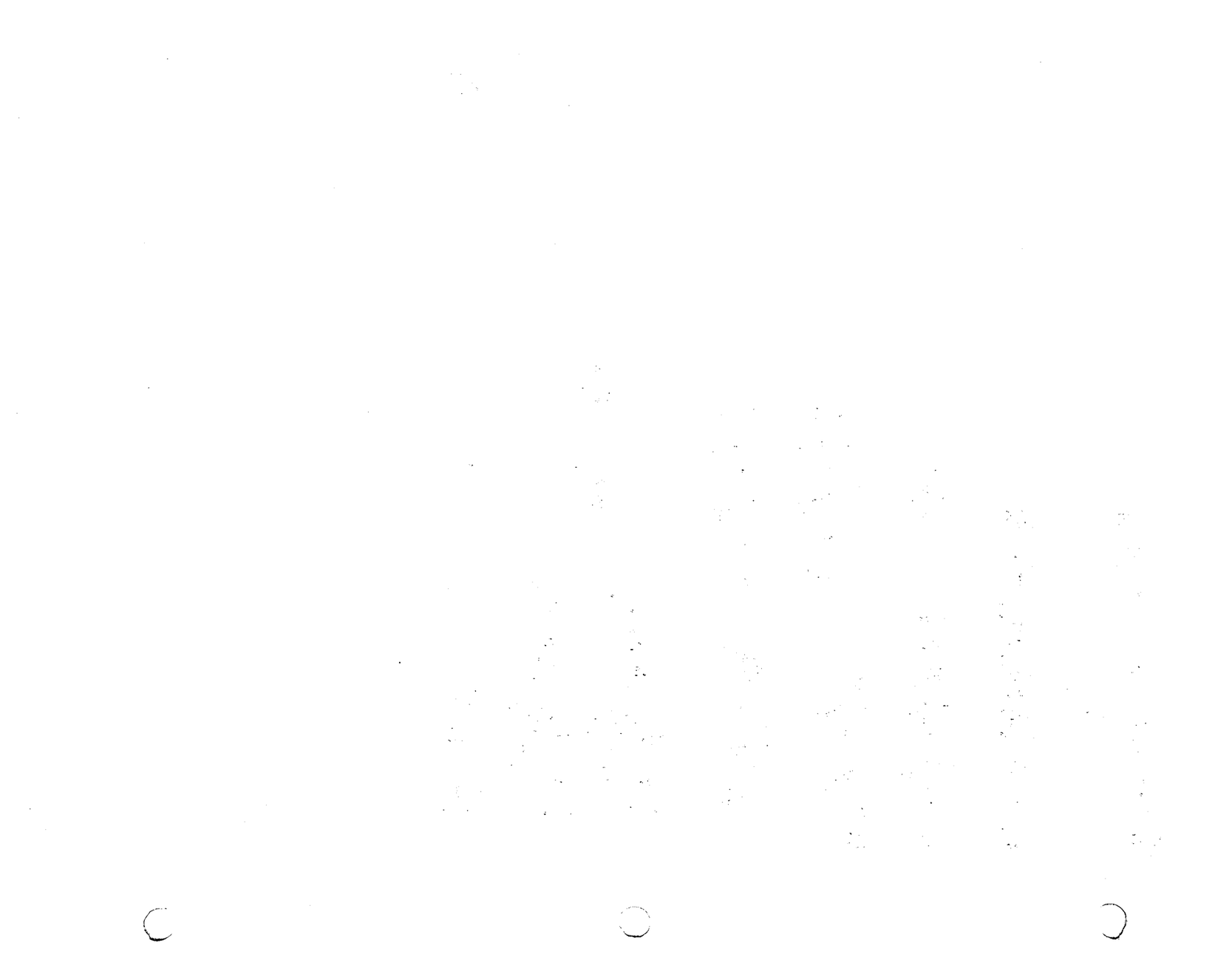
$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z) \qquad \epsilon_2 = \frac{\sigma_2}{E} - \frac{\nu}{E}\sigma_1$$

$$\sigma_2 = \frac{E}{(1-\nu^2)}(\epsilon_2 + \nu\epsilon_1)$$

only for $\sigma_2 = 0$

$$\sigma_1 = \frac{E}{(1-\nu^2)}(\epsilon_1 + \nu\epsilon_2)$$

$\sigma_1 = -9725$ $\sigma_2 = -32418$



**PROBLEM 2.1**

**2.1** A steel rod is 2.2 m long and must not stretch more than 1.2 mm when a 8.5 kN load is applied to it. Knowing that $E = 200$ GPa, determine (*a*) the smallest diameter rod which should be used, (*b*) the corresponding normal stress caused by the load.

**SOLUTION**

(a) $\delta = \frac{PL}{AE} \quad \therefore \quad A = \frac{PL}{E\delta} = \frac{(8.5\times10^{3})(2.2)}{(200\times10^{9})(1.2\times10^{-3})} = 77.92\times10^{-6}\ m^{2}$ 4

$A = \frac{\pi}{4}d^{2} \quad \therefore \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(77.92\times10^{-6})}{\pi}} = 9.96\times10^{-3}\ m$

$= 9.96\ mm$ ◄ 2

(b) $\sigma = \frac{P}{A} = \frac{8.5\times10^{3}}{77.92\times10^{-6}} = 109.1\times10^{6}\ Pa = 109.1\ MPa$ ◄ 2

**PROBLEM 2.8** like 2.6 HW Spot

**2.8** A cast-iron tube is used to support a compressive load. Knowing that $E = 10 \times 10^{6}$ psi and that the maximum allowable change in length is 0.025 percent, determine (*a*) the maximum normal stress in the tube, (*b*) the minimum wall thickness for a load of 1600 lb if the outside diameter of the tube is 2.0 in.

**SOLUTION**

(a) $\frac{\delta}{L} = \frac{0.025}{100} = 0.00025$ 2

$\sigma = \frac{E\delta}{L} = (10\times10^{6})(0.00025) = 2.5\times10^{3}\ psi = 2.5\ ksi$ ◄ 2

(b) $\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma} = \frac{1600}{2.5\times10^{3}} = 0.640\ in^{2}$

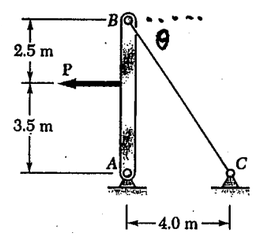
$A = \frac{\pi}{4}(d_o^{2} - d_i^{2})$

$d_i^{2} = d_o^{2} - \frac{4A}{\pi} = 2.0^{2} - \frac{(4)(0.64)}{\pi} = 3.1851\ in^{2} \quad \therefore \quad d_i = 1.7847\ in.$

$t = \frac{1}{2}(d_o - d_i) = \frac{1}{2}(2.0 - 1.7847) = 0.1077\ in.$ ◄ 1

**PROBLEM 2.11** 2.13

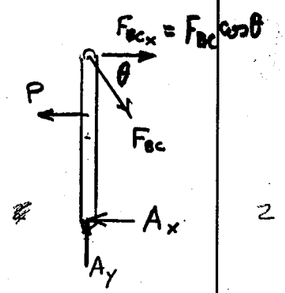
**2.11** The 4-mm-diameter cable *BC* is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load P that can be applied as shown.




**SOLUTION**

$L_{BC} = \sqrt{6^{2} + 4^{2}} = 7.2111\ m$

Use bar AB as a free body




$+\circlearrowleft \sum M_A = 0 \qquad 3.5P - (6)\left(\frac{4}{7.2111}F_{BC}\right) = 0$

$P = 0.9509\ F_{BC}$ 2

Considering allowable stress $\sigma = 190\times10^{6}$ Pa

$A = \frac{\pi}{4}d^{2} = \frac{\pi}{4}(0.004)^{2} = 12.566\times10^{-6}\ m^{2}$

$\sigma = \frac{F_{BC}}{A} \quad \therefore \quad F_{BC} = \sigma A = (190\times10^{6})(12.566\times10^{-6}) = 2.388\times10^{3}\ N$ 2

Considering allowable elongation $\delta = 6\times10^{-3}\ m$

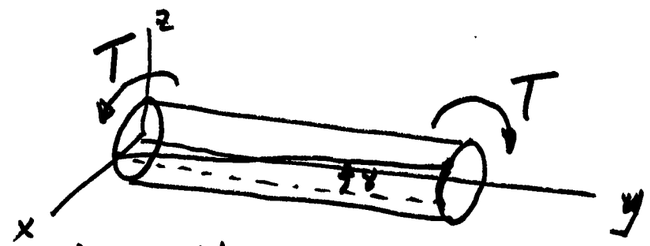
$\delta = \frac{F_{BC}L_{BC}}{AE} \quad \therefore \quad F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566\times10^{-6})(200\times10^{9})(6\times10^{-3})}{7.2111} = 2.091\times10^{3}\ N$ 2

Smaller value governs $F_{BC} = 2.091\times10^{3}\ N$

$P = 0.9509\ F_{BC} = (0.9509)(2.091\times10^{3}) = 1.988\times10^{3}\ N = 1.988\ kN$ ◄ 2

# EMA 3702 2ND TAPE

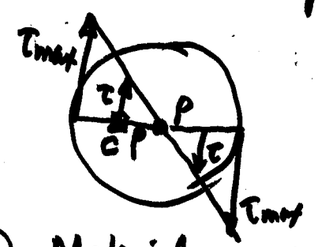
## TORSION




Assumptions

(1) Plane sections remain plane after deformation & no warping or distortion of the plane section normal to the long axis

(2) For cylindrical members, shear stresses are not constant but vary linearly from the center of the member to a maximum on the outer surface




$$\frac{\tau_{max}}{c} = \frac{\tau}{\rho}$$

(3) Material remains elastic & Hooke's Law applies

$\tau \sim \gamma \qquad \tau = G\gamma$

$dF = \tau dA$

$dT = (\tau dA)\rho$

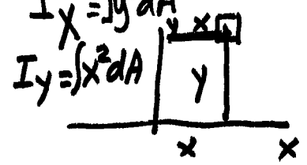
$$T = \int dT = \int \tau \rho \, dA = \int \rho \frac{\tau_{max}}{c} \cdot \rho \, dA$$

$$= \frac{\tau_{max}}{c} \int \rho^2 dA$$

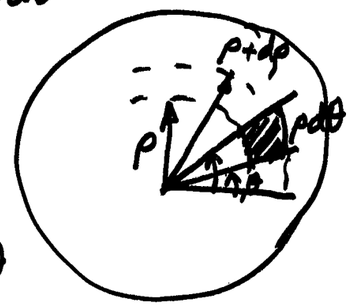
$J = \int \rho^2 dA$ Polar moment of inertia

$I_x = \int y^2 dA$

$I_y = \int x^2 dA$




$dA = \rho d\rho d\theta$




$J = \iint \rho^2 \cdot \rho d\rho \cdot d\theta$

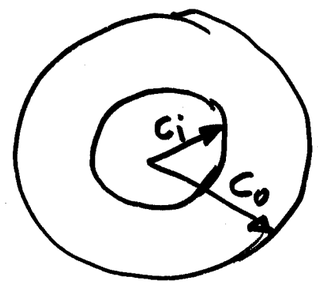
SOLID CYLINDER

$= \int_0^{2\pi} d\theta \int_0^c \rho^3 d\rho = 2\pi \cdot \frac{\rho^4}{4}\Big|_0^c = \frac{\pi}{2} c^4 = J$

$T = \frac{\tau_{max}}{c} \cdot J \Rightarrow \quad T = \frac{\tau}{\rho} J \quad \boxed{\tau = \frac{T\rho}{J}}$

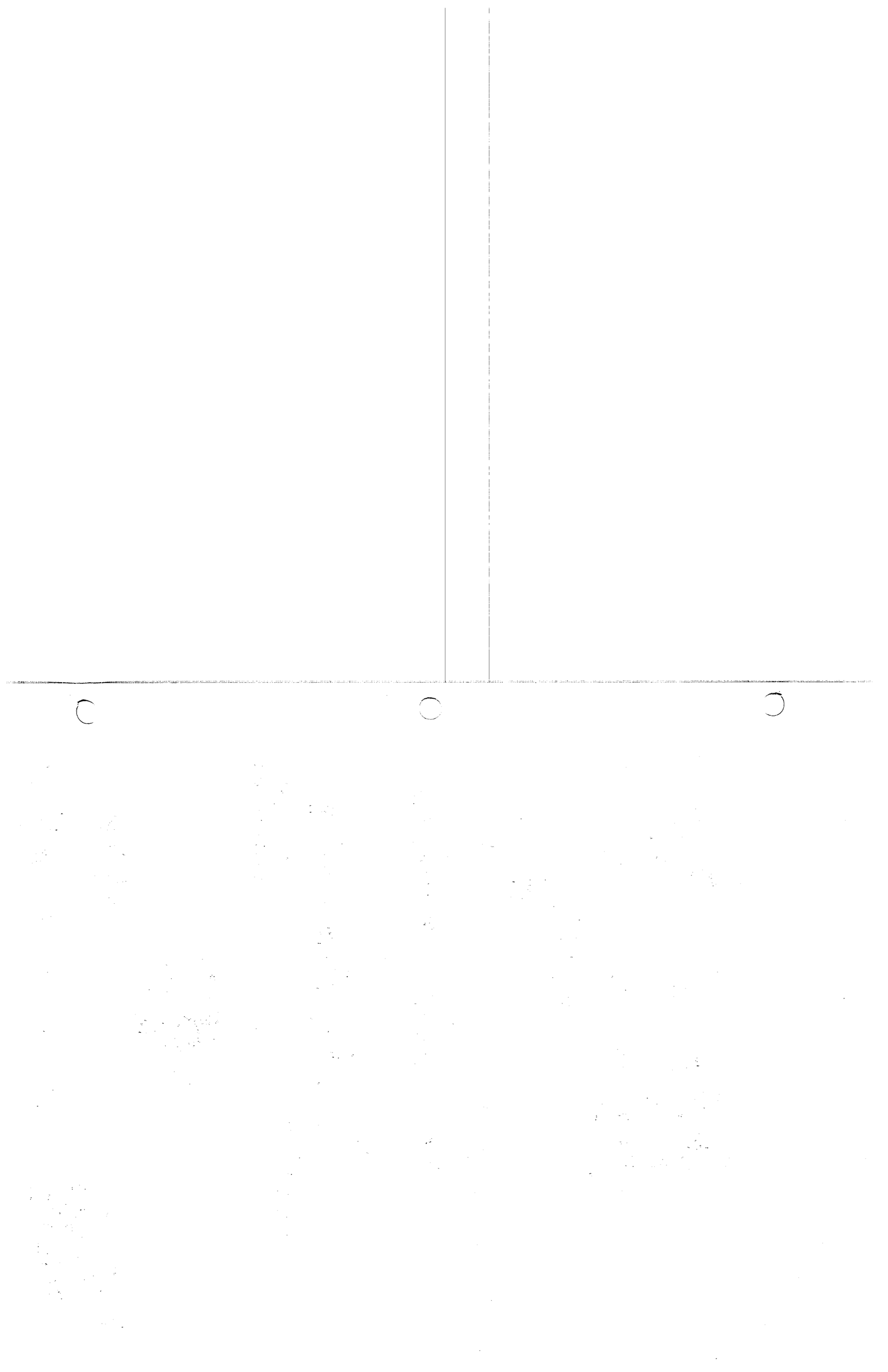
$\tau_{max} = \frac{Tc}{J}$

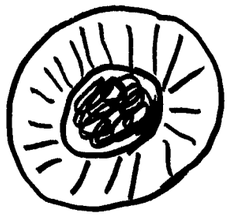
Thick Tube




$J = \int \rho^2 dA = \int_0^{2\pi} d\theta \int_{c_i}^{c_o} \rho^3 d\rho$

$J = 2\pi \left[ \frac{c_o^4}{4} - \frac{c_i^4}{4} \right]$





$\rho = .448 \Rightarrow$ carries only 4% of torque

$$T = \frac{\tau_{max}}{c} J = \frac{\tau}{\rho} J = \frac{\tau_{max}}{c} \cdot \frac{\pi c^4}{2}$$

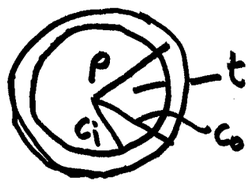
$$.04T = \frac{\tau_{max}}{c} \cdot \frac{2\pi r^4}{4}$$

$$.04 = \left(\frac{r}{c}\right)^4$$

$$.448 = \frac{r}{c}$$

$.06T \Rightarrow \frac{r}{c} = .495$

$.07T \Rightarrow \frac{r}{c} = .515$




Thin tube

$$c = \frac{c_i + c_o}{2}$$

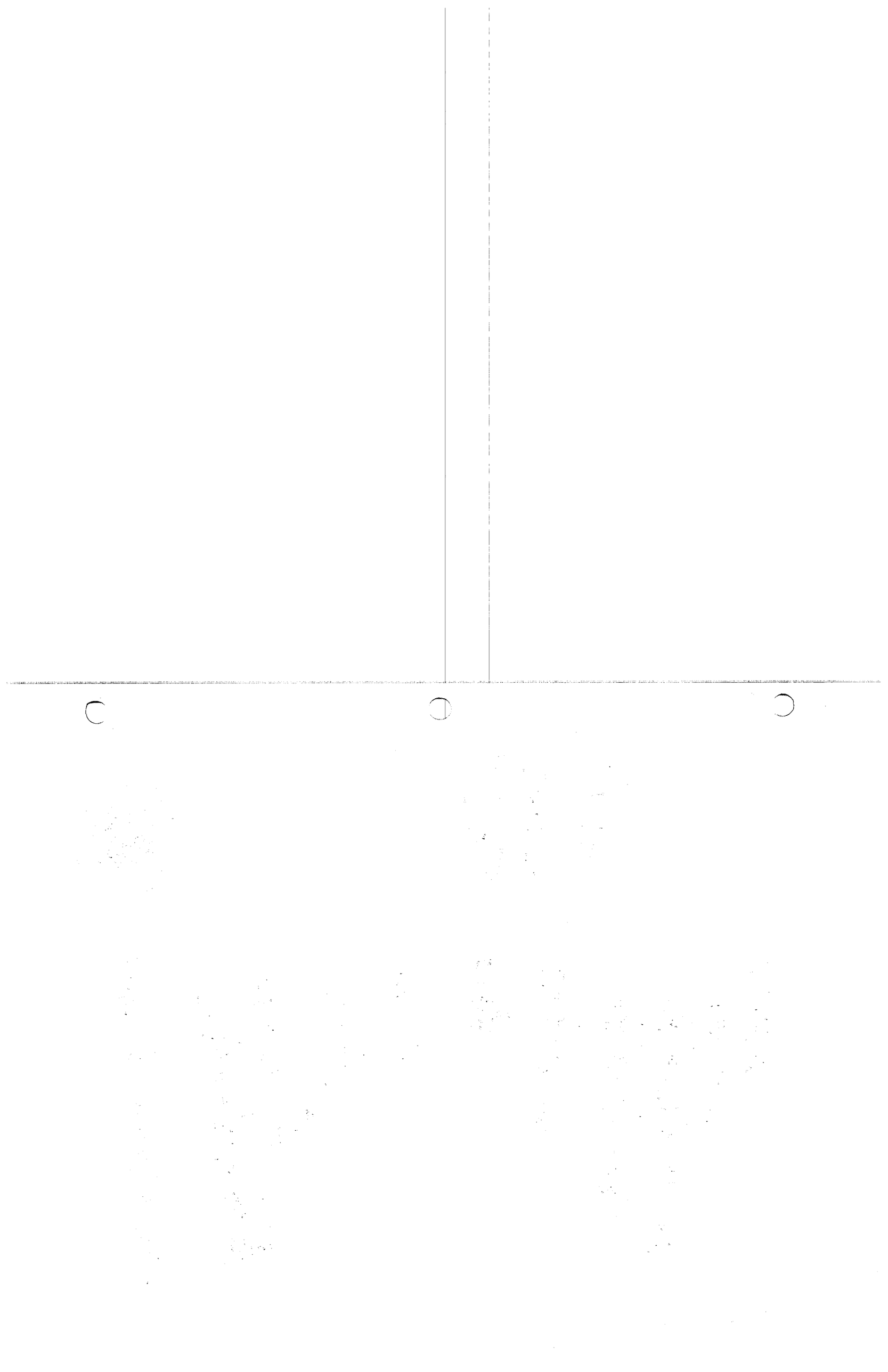
$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right)$$

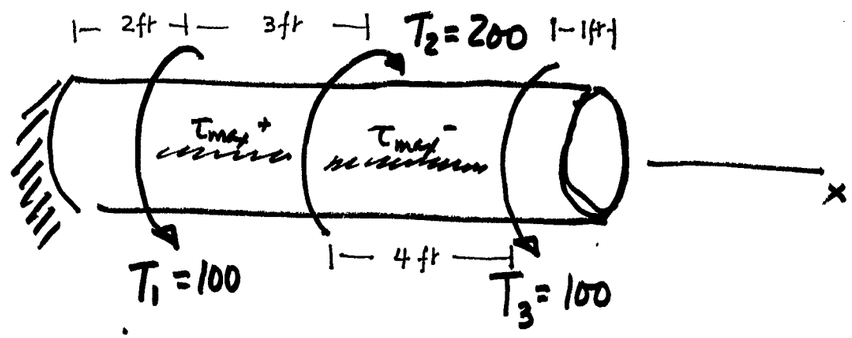
$$= \frac{\pi}{2}\left(c_o^2 - c_i^2\right)\left(c_o^2 + c_i^2\right)$$

$$= \frac{\pi}{2}\left(c_o - c_i\right)\left(c_o + c_i\right)\left(c_o^2 + c_i^2\right)$$

$$= \pi \cdot tc \cdot 2c^2$$

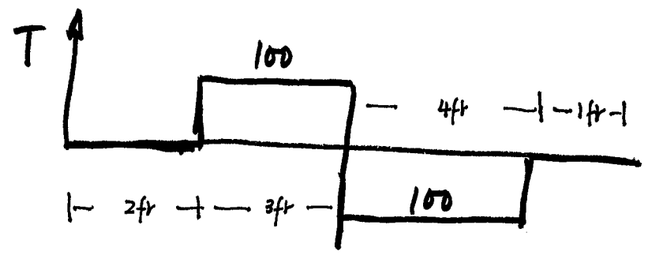
$$\underline{J = 2\pi t c^3}$$

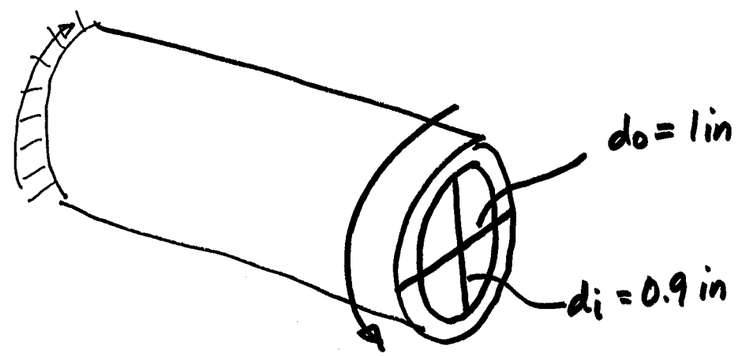





WHERE IS $\tau_{max}$

$$\tau_{max} = \frac{Tc}{J}$$



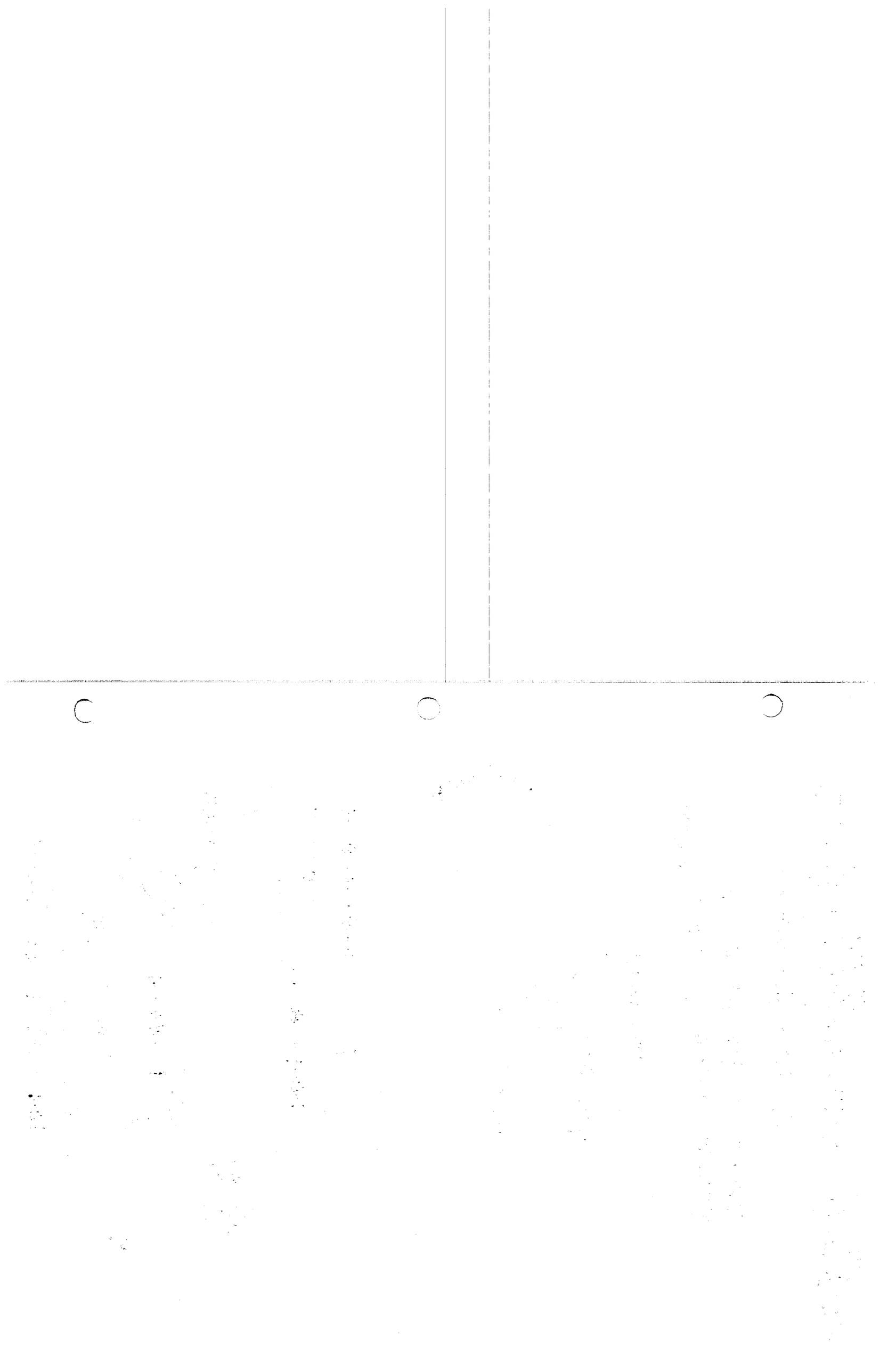




what is $\tau$ at the inner surface $c_i = 0.45$in

$\tau$ at the outer surface $c_o = 0.5$in

$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right) = 0.0337\ in^4$$

$$\tau_{inner} = \frac{Tc_i}{J} = \frac{400(.45)}{.0337\ in^4} \cong 5330\ psi\ ;\ \tau_{outer} = \frac{Tc_o}{J} = \frac{400(.5)}{.0337} \cong 5930$$



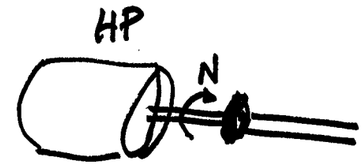
$$\text{Power} = F \cdot v \quad \text{or} \quad T \cdot \omega = T \cdot 2\pi N$$

($N$: revs/min)

$$[\text{lb-in}]\, T = 63025 \cdot \frac{HP}{N\ (\text{revs/min})}$$

$$[\text{lb-ft}]\, T = 5252 \cdot \frac{HP}{N\ (\text{revs/min})}$$

$$[\text{N-m}]\, T = 9540 \cdot \frac{P\ (\text{kw})}{N\ (\text{revs/min})}$$



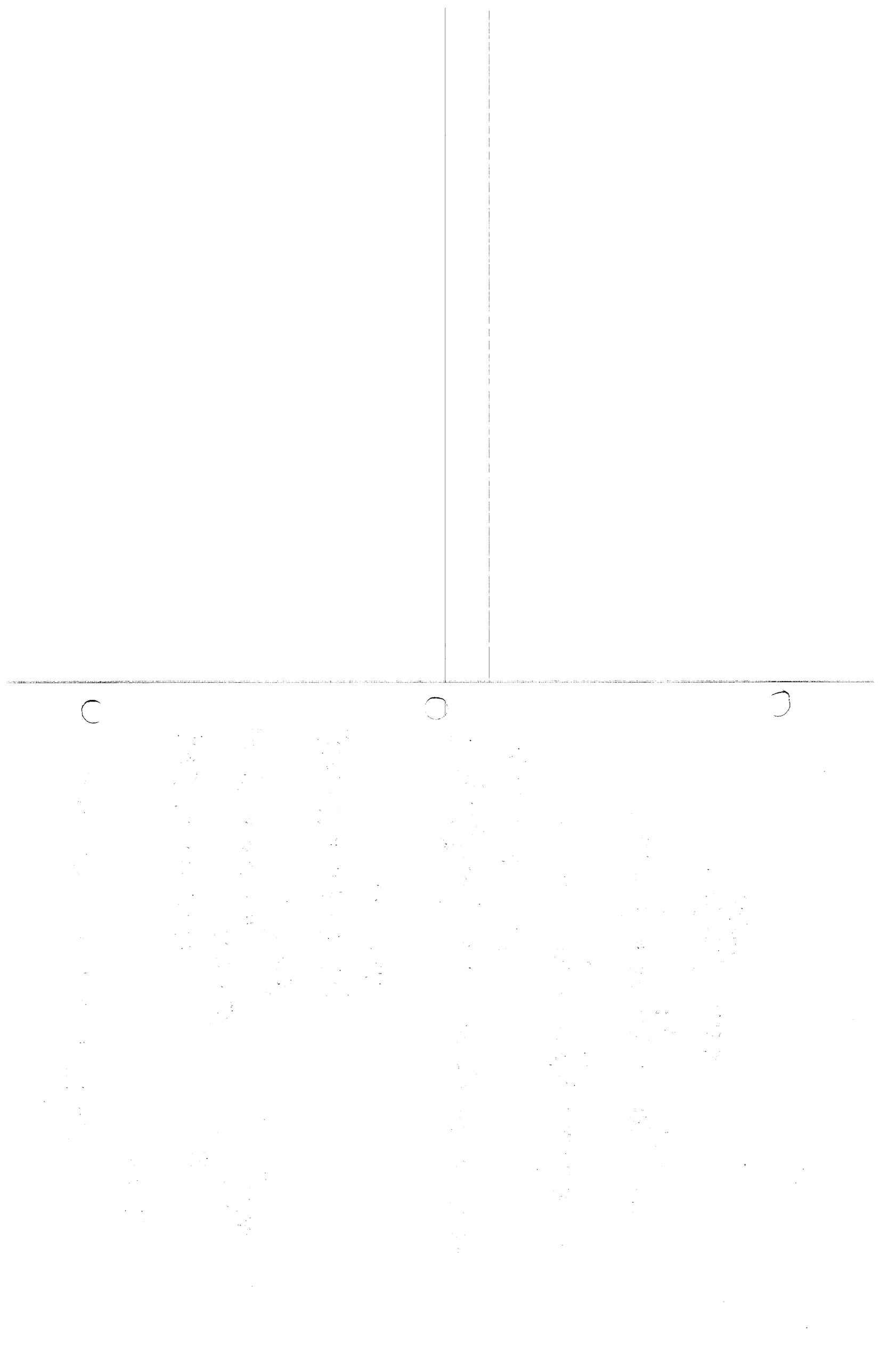

Design a steel shaft for 10 Hp motor operating at 1800 rpm

Max. shear is 8000 psi

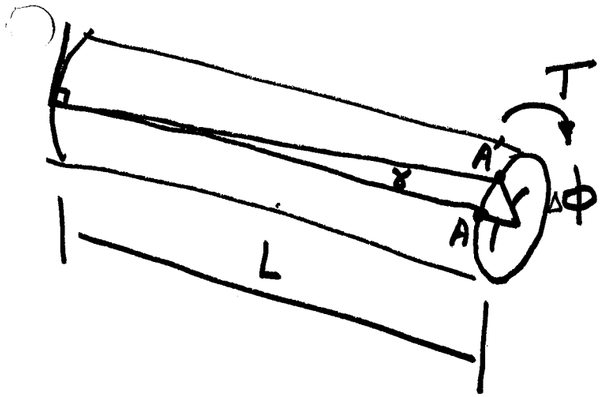
$$T = 63025 \frac{HP}{N} = \frac{63025 \cdot 10}{1800} = 350 \text{ lb-in}$$

$$\tau_{max} = \frac{Tc}{J} = 350 \frac{c}{\pi c^4/2} = \frac{700}{\pi c^3} = 8000$$

$$c = \sqrt[3]{\frac{700}{\pi(8000)}} = .303 \text{ in}$$



angle of twist.




$\overline{AA'} = L \cdot \gamma = c \Delta\phi$

$\gamma = \frac{\tau_{max}}{G}$

$L \frac{\tau_{max}}{G} = c \Delta\phi$

**Angle of Twist**

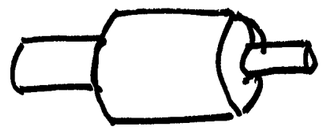
$$\Delta\phi = \frac{L}{G} \frac{\tau_{max}}{c} = \frac{L}{G} \cdot \frac{T}{J}$$



$\frac{\Delta\phi}{L}$ - angle of twist $= \frac{T}{GJ}$ (unit)

$$\phi_{TOT} = \sum \Delta\phi_i = \sum \frac{T_i L_i}{G_i J_i}$$

$$= \int_0^L \frac{T\,dx}{GJ}$$



$$u_{TOT} = \sum \frac{P_i L_i}{A_i E_i}$$

$$u = \int_0^L \frac{P\,dx}{AE}$$

6.96. Shafts AB and BC are made of different materials and unstressed when welded together at $B$ as shown in Fig. P6.96. The twisting couples at $A$ and $C$ are equal in magnitude when the couple $T_0$ is applied at $B$. Determine the relation between the elastic moduli of the two materials.

Answer: $G_A = 0.592 G_C$

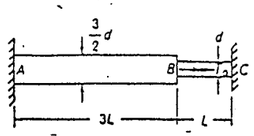



Fig. P6.96

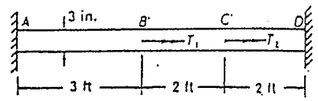


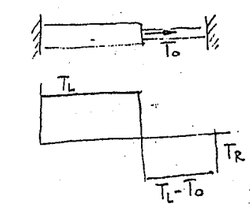

Fig. P6.97

6.97. An aluminum shaft is rigidly restrained at each end and loaded by couples as shown in Fig. P6.97. If $T_1 = 12{,}000$ in.-lb and $T_2 = 10{,}000$ in.-lb, compute the maximum shear stress in each section and the rotations of cross sections at $B$ and $C$.

Answer: $\tau_{CD} = -2300\ psi$, $\tau_{BC} = -433\ psi$, $\tau_{AB} = 1840\ psi$,

A.

6.96 Dint




$T_L - T_R = T_o$

$T_R = T_L - T_o$

since $|T_R| = |T_L|$ ●

$T_L = T_o/2 \quad T_R = -T_o/2$

$$\varphi_{AB} = \frac{T_L L_{AB}}{J_{AB} G_{AB}} = |\varphi_{BC}| = \left|\frac{T_R L_{BC}}{J_{BC} G_{BC}}\right|$$

$J_{AB} = \frac{\pi c^4}{2} = \frac{\pi d^4}{32} = \frac{\pi \cdot 81 d^4}{512}$; $L_{AB} = 3L$;

$J_{BC} = \frac{\pi c^4}{2} = \frac{\pi d^4}{32}$; $L_{BC} = L$

$$\varphi_{AB} = \frac{T_o/2 \cdot 3L}{\pi 81 d^4/512 \, G_{AB}} = \frac{T_o/2 \cdot L}{\frac{\pi d^4}{32} G_{BC}}$$

$$\therefore \frac{G_{AB} \cdot 81}{1536} = \frac{G_{BC}}{32} \quad \text{or} \quad G_{AB} = \frac{1536}{81 \cdot 32} G_{BC} = .5926 G_{BC}$$

Fig. P3.108 and P3.109

**3.132** A thin-walled tube has been fabricated by bending a metal plate of thickness $t$ into a cylinder of radius $c$ and bonding together the edges of the plate. A torque **T** is then applied to the tube, producing a shearing stress $\tau_1$ and an angle of twist $\phi_1$. Denoting by $\tau_2$ and $\phi_2$, respectively, the shearing stress and the angle of twist which will develop if the bond suddenly fails, express the ratios $\tau_2/\tau_1$ and $\phi_2/\phi_1$ in terms of the ratio $c/t$.

$\phi_2/\phi_1 = 3\left(\frac{c}{t}\right)^2$

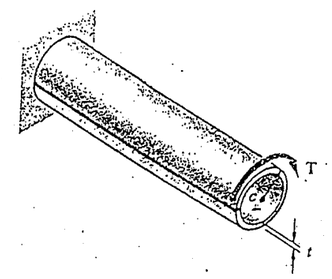



Fig. P3.132

**3.122** A 3.5-m-long steel member with a W310 × 143 cross section is subjected to a 4.5 kN · m torque. Knowing that $G = 77$ GPa and referring to Appendix C for the dimensions of the cross section, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the maximum shearing stress along line *b-b*, (*c*) the angle of twist. (*Hint*: Consider the web and the flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

39.7 MPa 24.2 MPa 4.72°

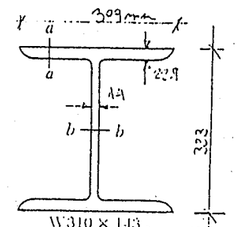
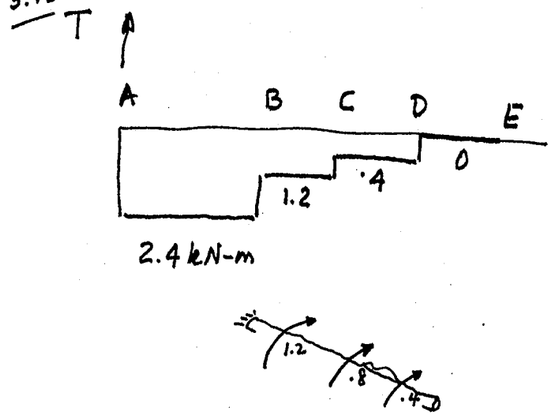



Fig. P3.122

# EMA 3702

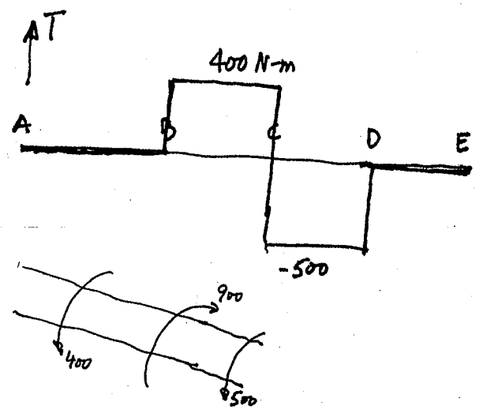
3.13




$$\tau = \frac{Tc}{J} \qquad \frac{2T}{\pi c^3}$$

$$\varphi = \frac{TL}{JG} \qquad \varphi = \int \frac{T\,dx}{JG}$$

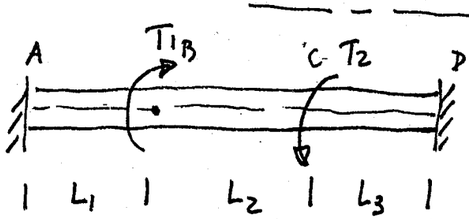
3.36

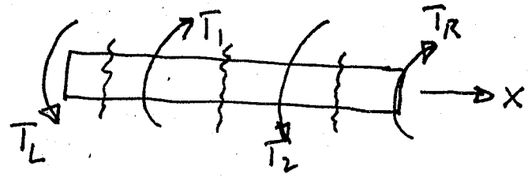



$\varphi_{BC}$?

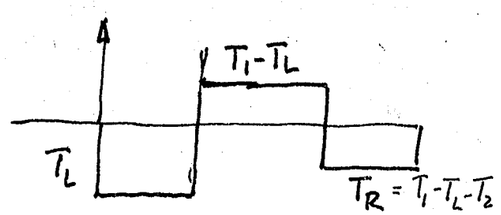
$\varphi_{CD}$?

$$\varphi_{AE} = \varphi_{AB} + \varphi_{BC} + \varphi_{CD} + \varphi_{DE}$$




$$\Sigma T = 0 = T_L - T_1 + T_2 - T_R = 0$$

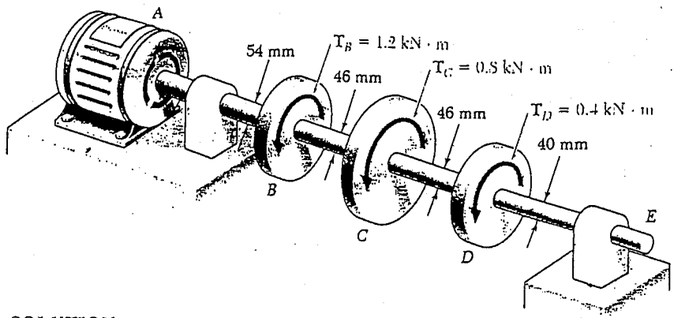



$$\Sigma\varphi = \varphi_{AB} + \varphi_{BC} + \varphi_{CD} = 0$$

$$= \frac{T_{AB} L_1}{J_{AB} \cdot G} + \frac{T_{BC} L_2}{J_{BC} \cdot G} + \frac{T_{CD} L_3}{J_{CD} \cdot G} = 0 \Rightarrow \quad T_{AB} \cdot L_1 + T_{BC} \cdot L_2 + T_{CD} \cdot L_3 = 0$$

PROBLEM 3.13

**3.13** Under normal operating conditions, the electric motor exerts a torque of 2.4 kN·m at $A$. Knowing that each shaft is solid, determine the maximum shearing stress (*a*) in shaft $AB$, (*b*) in shaft $BC$, (*c*) in shaft $CD$.




$$J = \frac{\pi c^4}{2}$$

SOLUTION

Shaft AB: $T_{AB} = 2.4\times10^3$ N-m, $c = \frac{1}{2}d = 0.027$ m

$$\tau_{AB} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2.4\times10^3)}{\pi(0.027)^3} = 77.625\times10^6 \text{ Pa} \qquad 77.6 \text{ MPa}$$

Shaft BC: $T_{BC} = 2.4$ kN-m $- 1.2$ kN-m $= 1.2$ kN-m, $c = \frac{1}{2}d = 0.023$ m

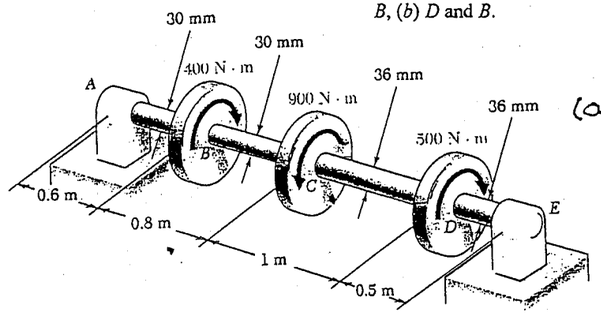
$$\tau_{BC} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1.2\times10^3)}{\pi(0.023)^3} = 62.788\times10^6 \text{ Pa} \qquad 62.8 \text{ MPa}$$

Shaft CD: $T_{CD} = 0.4\times10^3$ N-m $\quad c = \frac{1}{2}d = 0.023$ m

$$\tau_{CD} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(0.4\times10^3)}{\pi(0.023)^3} = 20.929\times10^6 \text{ Pa} \qquad 20.9 \text{ MPa}$$

PROBLEM 3.36

*3.36* The torques shown are exerted on pulleys $B$, $C$ and $D$. Knowing that the entire shaft is made of steel ($G = 27$ GPa), determine the angle of twist between (*a*) $C$ and $B$, (*b*) $D$ and $B$.




SOLUTION

(a) Shaft BC: $c = \frac{1}{2}d = 0.015$ m

$J_{BC} = \frac{\pi}{2}c^4 = 79.522\times10^{-9}$ m$^4$

$L_{BC} = 0.8$ m, $G = 27\times10^9$ Pa

$$\varphi_{BC} = \frac{TL}{GJ} = \frac{(400)(0.8)}{(27\times10^9)(79.522\times10^{-9})}$$

$$= 0.149904 \text{ rad} = 8.54^\circ$$

(b) Shaft CD: $c = \frac{1}{2}d = 0.018$ m $\quad J_{CD} = \frac{\pi}{2}c^4 = 164.896\times10^{-9}$ m$^4$

$L_{CD} = 1.0$ m $\quad T_{CD} = 400 - 900 = -500$ N·m

$$\varphi_{CD} = \frac{TL}{GJ} = \frac{(-500)(1.0)}{(27\times10^9)(164.896\times10^{-9})} = -0.11230 \text{ rad}$$

$\varphi_{AE} = \varphi_{BD} = \varphi_{BC} + \varphi_{CD} = 0.14904 - 0.11230 = 0.03674$ rad $= 2.11^\circ$

$$T_L + T_2 - T_1 - T_R = 0$$

$$-T_L \cdot L_1' + (T_1 - T_L')L_2 - T_R \cdot L_3 = 0 \Rightarrow -T_L(L_1+L_2) + T_1 L_2 - T_R L_3 = 0$$

$$T_L - T_R = T_1 - T_2$$

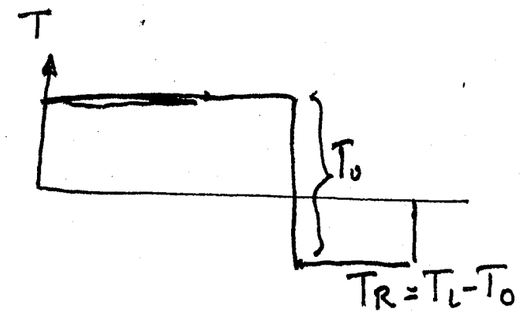
$$-T_L(L_1+L_2) - T_R \cdot L_3 = T_1 L_2$$

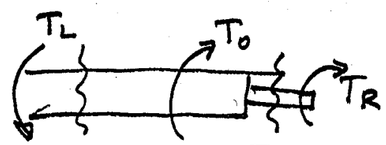
$$T_L = \frac{\begin{vmatrix} T_1 - T_2 & -1 \\ T_1 L_2 & -L_3 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -(L_1+L_2) & -L_3 \end{vmatrix}} = \frac{T_1\left(\frac{L_2+L_3}{L}\right) - T_2\left(\frac{L_3}{L}\right)}{L = L_1 + L_2 + L_3}$$

$$T_R = -T_1\left(\frac{L_1}{L}\right) + T_2\left(\frac{L_1+L_2}{L}\right)$$

$$\varphi_{AB} = \frac{T_{AB} L_1}{J_{AB} G} = \frac{-T_L \cdot L_1}{J_{AB} \cdot G} = [\text{rad.}] \qquad \text{rad} \cdot \frac{180^\circ}{\pi} = (\ )^\circ$$

6.96




$$T_R + T_0 - T_L = 0$$

$$T_0 = T_L - T_R$$

welded together at B as shown in Fig. P6.96. The twisting couples at A and C are equal in magnitude when the couple $T_0$ is applied at $B$. Determine the relation between the elastic moduli of the two materials.

Answer: $G_A = 0.592G_C$

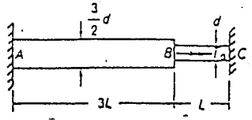


Fig. P6.96

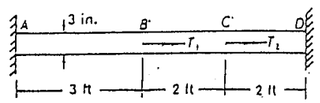


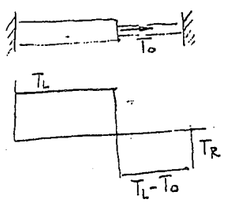
Fig. P6.97

6.97. An aluminum shaft is rigidly restrained at each end and loaded by couples as shown in Fig. P6.97. If $T_1 = 12{,}000$ in.-lb and $T_2 = 10{,}000$ in.-lb; compute the maximum shear stress in each section and the rotations of cross sections at $B$ and $C$.

Answer: $\tau_{CD} = -2300\ psi$, $\tau_{BC} = -433\ psi$, $\tau_{AB} = 1840\ psi$,

A

6.96




$T_L - T_R = T_0$

$T_R = T_L - T_0$

$|T_R| = |T_L|$ ו $T_0$ נתון

$T_L = T_0/2 \quad T_R = -T_0/2$

$$\varphi_{AB} = \frac{T_L L_{AB}}{J_{AB} G_{AB}} = |\varphi_{BC}| = \left|\frac{T_R L_{BC}}{J_{BC} G_{BC}}\right|$$

$$J_{AB} = \frac{\pi c^4}{2} = \frac{\pi d^4}{32} = \frac{\pi \cdot 81 d^4}{512}; \quad L_{AB} = 3L;$$

$$J_{BC} = \frac{\pi c^4}{2} = \frac{\pi d^4}{32}; \quad L_{BC} = L$$

$$\varphi_{AB} = \frac{T_0/2 \cdot 3L}{\pi 81 d^4/512 \, G_{AB}} = \frac{T_0/2 \cdot L}{\frac{\pi d^4}{32} G_{BC}}$$

$$\therefore \quad \frac{G_{AB} \cdot 81}{1536} = \frac{G_{BC}}{32} \quad \text{or} \quad G_{AB} = \frac{1536}{81 \cdot 32} G_{BC} = .5926 G_{BC}$$

Fig. P3.108 and P3.109

**3.132** A thin-walled tube has been fabricated by bending a metal plate of thickness $t$ into a cylinder of radius $c$ and bonding together the edges of the plate. A torque T is then applied to the tube, producing a shearing stress $\tau_1$ and an angle of twist $\phi_1$. Denoting by $\tau_2$ and $\phi_2$, respectively, the shearing stress and the angle of twist which will develop if the bond suddenly fails, express the ratios $\tau_2/\tau_1$ and $\phi_2/\phi_1$ in terms of the ratio $c/t$.

$\phi_2/\phi_1 = 3\left(\frac{c}{t}\right)^2$

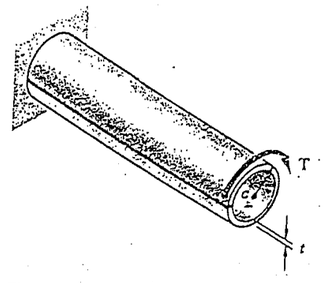


Fig. P3.132

**3.122** A 3.5-m-long steel member with a W310 × 143 cross section is subjected to a 4.5 kN · m torque. Knowing that $G = 77$ GPa and referring to Appendix C for the dimensions of the cross section, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the maximum shearing stress along line *b-b*, (*c*) the angle of twist. (*Hint*: Consider the web and the flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

39.7 MPa 24.2 MPa 4.72°

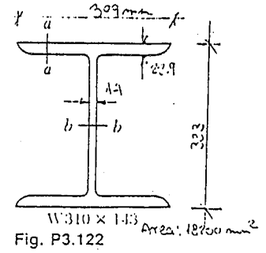
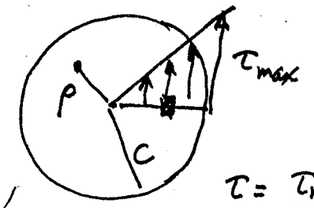
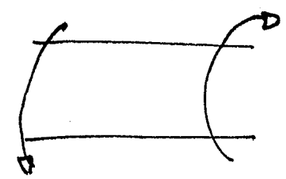
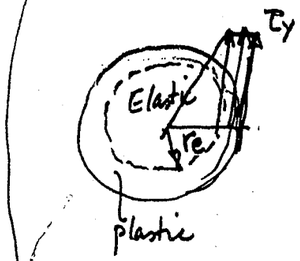


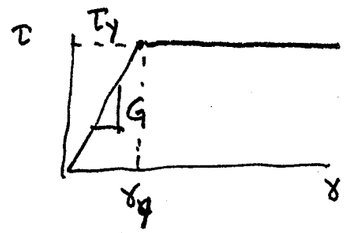
Fig. P3.122



$$\tau = \frac{\tau_{max}}{c} \cdot \rho$$




$\tau = G\gamma$

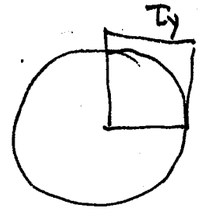
elasto plastic model

$$T = \int \tau \, dA \cdot \rho = \int \frac{\tau_{max}\,\rho}{c} \cdot \rho \, d\rho \, d\theta \cdot \rho = \frac{\tau_{max}}{c} \int_0^c \rho^3 d\rho \cdot \int_0^{2\pi} d\theta$$

$$= \frac{\tau_{max}}{c} \cdot \frac{c^4}{4} \cdot 2\pi$$

$$T = \tau_{max} \frac{c^3 \pi}{2}$$

$$T = \int_0^{r_e} \frac{\tau_{max}}{c} \cdot \rho \cdot \rho \, d\rho \, d\theta \cdot \rho + \int_{r_e}^{c} \tau_y \cdot \rho \, d\rho \, d\theta \cdot \rho$$




$$T_p = \int_0^c \tau_y \cdot \rho \, d\rho \, d\theta \cdot \rho$$

$$= \tau_y \int_0^c \rho^2 d\rho \int_0^{2\pi} d\theta$$

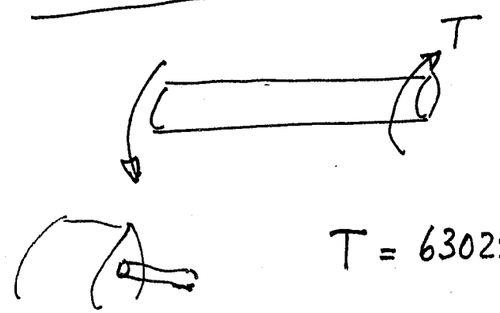
$$= \tau_y \frac{\rho^3}{3} \Big|_0^c \cdot 2\pi = \tau_y \cdot \frac{2\pi c^3}{3}$$

$$T_y \left(\text{when } \tau_{max} = \tau_y\right) = \tau_{max} \frac{c^3 \pi}{2} = \tau_y \cdot \frac{c^3 \pi}{2}$$

$$T_p = 4T$$

# EMA 3702 - PT2 (2/14/02)

## TORSION




$P = \underline{F} \cdot \underline{v}$

$= \underline{T} \cdot \underline{\omega} = \underline{T} \cdot 2\pi N$

$$T = 63025 \cdot \frac{P}{N}$$

T [lb-in]
P [hp]
N [rpm]

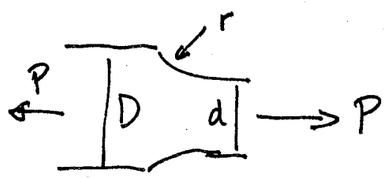
$$T = \frac{9540\ P}{N}$$

T = [N-m]
P [kw]
N [rev per min]

Hz [cyc/sec] $\times \frac{60 sec}{min}$ = cyc/min $\times$ ~~$\frac{2\pi\ rad}{cyc.}$~~

$$\tau = \frac{Tc}{J}$$

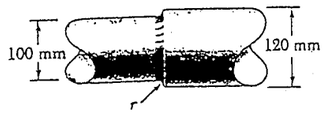
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$\sigma_{max} = K \sigma_{ave}$

$K = f(D/d, r/d)$

PROBLEM 3.87




**3.87** The stepped shaft shown rotates at 450 rpm. Knowing that $r = 10$ mm, determine the maximum power that can be transmitted without exceeding an allowable shearing stress of 45 MPa.

SOLUTION

$d = 100$ mm, $D = 120$ mm, $r = 10$ mm

$\frac{D}{d} = \frac{120}{100} = 1.2$, $\frac{r}{d} = \frac{10}{100} = 0.10$, From Fig. 3.32 $K = 1.33$

For smaller shaft $c = \frac{1}{2}d = 0.050$ m $\qquad \tau = \frac{Tc}{J} = \frac{2KT}{\pi c^3}$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.050)^3 (45 \times 10^6)}{(2)(1.33)} = 6.643 \times 10^3 \text{ N-m}$$

$f = 450$ rpm $= 7.5$ Hz $= 450/60$

Power $P = 2\pi f T = 2\pi (7.5)(6.643 \times 10^3) = 313 \times 10^3$ W $= 313$ kW

$$T = \frac{9540\,P}{N} \Rightarrow \quad P = \frac{T \cdot N}{9540} = 313.35 \text{ kW}$$

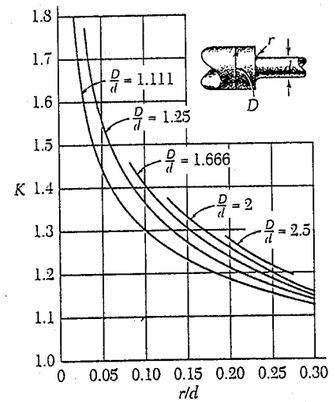


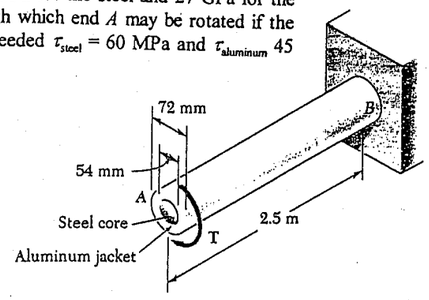

Fig. 3.32 Stress-concentration factors for fillets in circular shafts.†

TABLE 3.1. Coefficients for Rectangular Bars in Torsion

| $a/b$ | $c_1$ | $c_2$ |
|---|---|---|
| 1.0 | 0.208 | 0.1406 |
| 1.2 | 0.219 | 0.1661 |
| 1.5 | 0.231 | 0.1958 |
| 2.0 | 0.246 | 0.229 |
| 2.5 | 0.258 | 0.249 |
| 3.0 | 0.267 | 0.263 |
| 4.0 | 0.282 | 0.281 |
| 5.0 | 0.291 | 0.291 |
| 10.0 | 0.312 | 0.312 |
| $\infty$ | 0.333 | 0.333 |

**PROBLEM 3.53**

**3.53** The composite shaft shown is to be twisted by applying a torque **T** at end *A*. Knowing that the modulus of rigidity is 77 GPa for the steel and 27 GPa for the aluminum, determine the largest angle through which end *A* may be rotated if the following allowable stresses are not to be exceeded $\tau_{steel} = 60$ MPa and $\tau_{aluminum}$ 45 MPa.




SOLUTION

$\tau_{max} = G\gamma_{max} = G c_{max}\frac{\varphi}{L}$

$\frac{\varphi_{all}}{L} = \frac{\tau_{all}}{G c_{max}}$ for each material

Steel core: $\tau_{all} = 60\times10^{6}$ Pa, $c_{max} = \frac{1}{2}d = 0.027$ m, $G = 77\times10^{9}$ Pa

$$\frac{\varphi_{all}}{L} = \frac{60\times10^{6}}{(77\times10^{9})(0.027)} = 28.860\times10^{-3} \text{ rad/m}$$

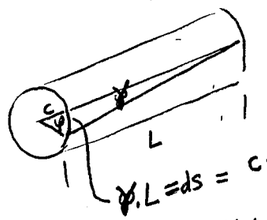
Aluminum jacket: $\tau_{all} = 45\times10^{6}$ Pa, $c_{max} = \frac{1}{2}d = 0.036$ m, $G = 27\times10^{9}$ Pa

$$\frac{\varphi_{all}}{L} = \frac{45\times10^{6}}{(27\times10^{9})(0.036)} = 46.296\times10^{-3} \text{ rad/m}$$

Smaller value governs $\frac{\varphi_{all}}{L} = 28.860\times10^{-3}$ rad/m

Allowable angle of twist $\varphi_{all} = L\frac{\varphi_{all}}{L} = (2.5)(28.860\times10^{-3})$

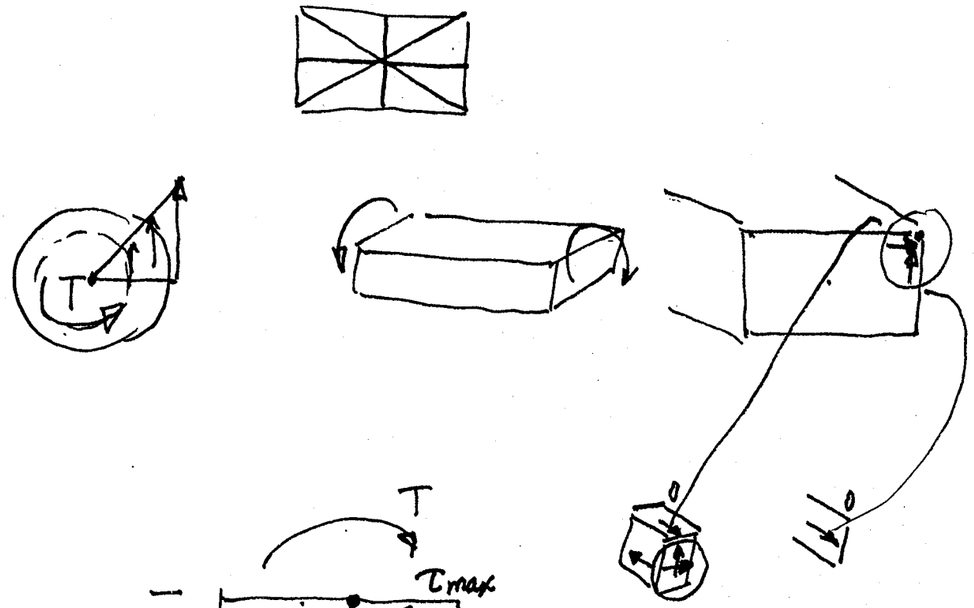
$= 72.15\times10^{-3}$ rad $= 4.13°$




$\gamma \cdot L = ds = c\cdot\varphi$

$\varphi = \frac{\gamma\cdot L}{c} = \frac{\tau_{max}\cdot L}{G\cdot c}$

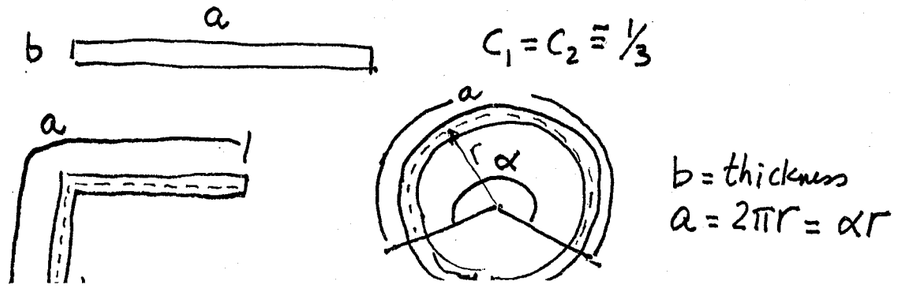
$\therefore \frac{\varphi}{L}$ must be same for both




$$\tau_{max} = \frac{T}{c_1 a b^2} \qquad c_1 = f_1(a/b)$$

$$\varphi = \frac{TL}{c_2 a b^3 G} \qquad c_2 = f_2(a/b)$$

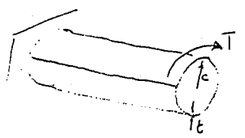
Table 2.1 pg 187




$c_1 = c_2 \cong 1/3$

b = thickness
$a = 2\pi r = \alpha r$

3.132




closed tube $J = \frac{\pi}{2}(a^2-b^2)(a^2+b^2)$ ; $\tau_1 = \frac{Tc}{J}$ ; $\phi_1 = \frac{TL}{JG}$

$= \frac{\pi}{2}(a-b)(a+b)(a^2+b^2)$

$J = \frac{\pi}{2} \cdot t \cdot 2c \cdot 2c^2 = 2\pi c^3 t$ $\quad \tau_1 = \frac{T}{2\pi c^2 t}$ ; $\phi_1 = \frac{TL}{2\pi c^3 t G}$

Open tube $J_R = \frac{bt^3}{3} = \frac{2\pi c t^3}{3}$ ; $\tau_2 = \frac{Tt}{J_R}$ ; $\beta = \frac{\phi_2}{L} = \frac{T}{GJ_R}$

$\tau_2 = \frac{Tt}{2\pi c t^3/3} = \frac{3Tt}{2\pi c t^3} = \frac{3T}{2\pi c t^2}$ ; $\phi_2 = \frac{TL}{\frac{2\pi c t^3 G}{3}} = \frac{3TL}{2\pi c t^3 G}$

$\frac{\tau_1}{\tau_2} = \frac{\frac{T}{2\pi c^2 t} \cdot t}{\frac{3T}{2\pi c t^2} \cdot c} = \frac{t}{3c}$ ; $\frac{\phi_1}{\phi_2} = \frac{\frac{TL}{2\pi c^3 t G} \cdot \frac{t^2}{t^2}}{\frac{3TL}{2\pi c t^3 G} \cdot \frac{c^2}{c^2}} = \frac{t^2}{3c^2}$

$\left(\frac{\tau_1}{\tau_2}\right)^2 = \frac{t^2}{9c^2} = \frac{1}{3}\left(\frac{\phi_1}{\phi_2}\right)$

so: $\frac{\tau_2}{\tau_1} = 3\left(\frac{c}{t}\right)$ and $\frac{\phi_2}{\phi_1} = 3\left(\frac{c}{t}\right)^2$

Fig. P3.108 and P3.109

**3.132** A thin-walled tube has been fabricated by bending a metal plate of thickness $t$ into a cylinder of radius $c$ and bonding together the edges of the plate. A torque T is then applied to the tube, producing a shearing stress $\tau_1$ and an angle of twist $\phi_1$. Denoting by $\tau_2$ and $\phi_2$, respectively, the shearing stress and the angle of twist which will develop if the bond suddenly fails, express the ratios $\tau_2/\tau_1$ and $\phi_2/\phi_1$ in terms of the ratio $c/t$.

$\phi_2/\phi_1 = 3\left(\frac{c}{t}\right)^2$

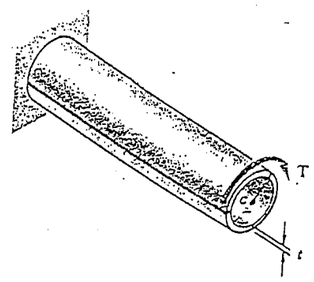



Fig. P3.132

**3.122** A 3.5-m-long steel member with a W310 × 143 cross section is subjected to a 4.5 kN · m torque. Knowing that $G = 77$ GPa and referring to Appendix C for the dimensions of the cross section, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the maximum shearing stress along line *b-b*, (*c*) the angle of twist. (*Hint*: Consider the web and the flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

39.7 MPa 24.2 MPa 4.72°

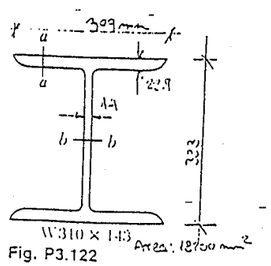


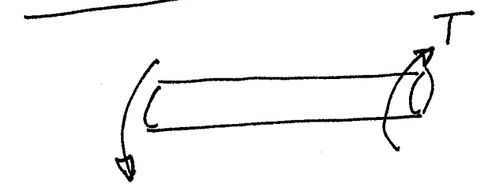

Fig. P3.122

7p

HW: DO 3.21, 3.47 & 3.53

# EMA 3702 - PT2 (2/14/02)

## TORSION




$P = \underline{F} \cdot \underline{v}$

$= \underline{T} \cdot \underline{\omega} = \underline{T} \cdot 2\pi N$

$$T = 63025 \cdot \frac{P}{N}$$

T [lb-in]
P [hp]
N [rpm]

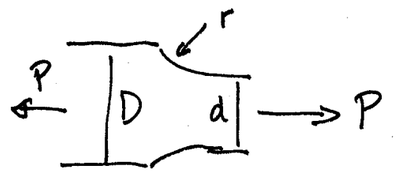
$$T = \frac{9540\ P}{N}$$

T = [N-m]
P [kw]
N [rev per min]

Hz [cyc/sec] $\times \frac{60 sec}{min}$ = cyc/min $\times$ ~~$\frac{2\pi\ rad}{cyc.}$~~

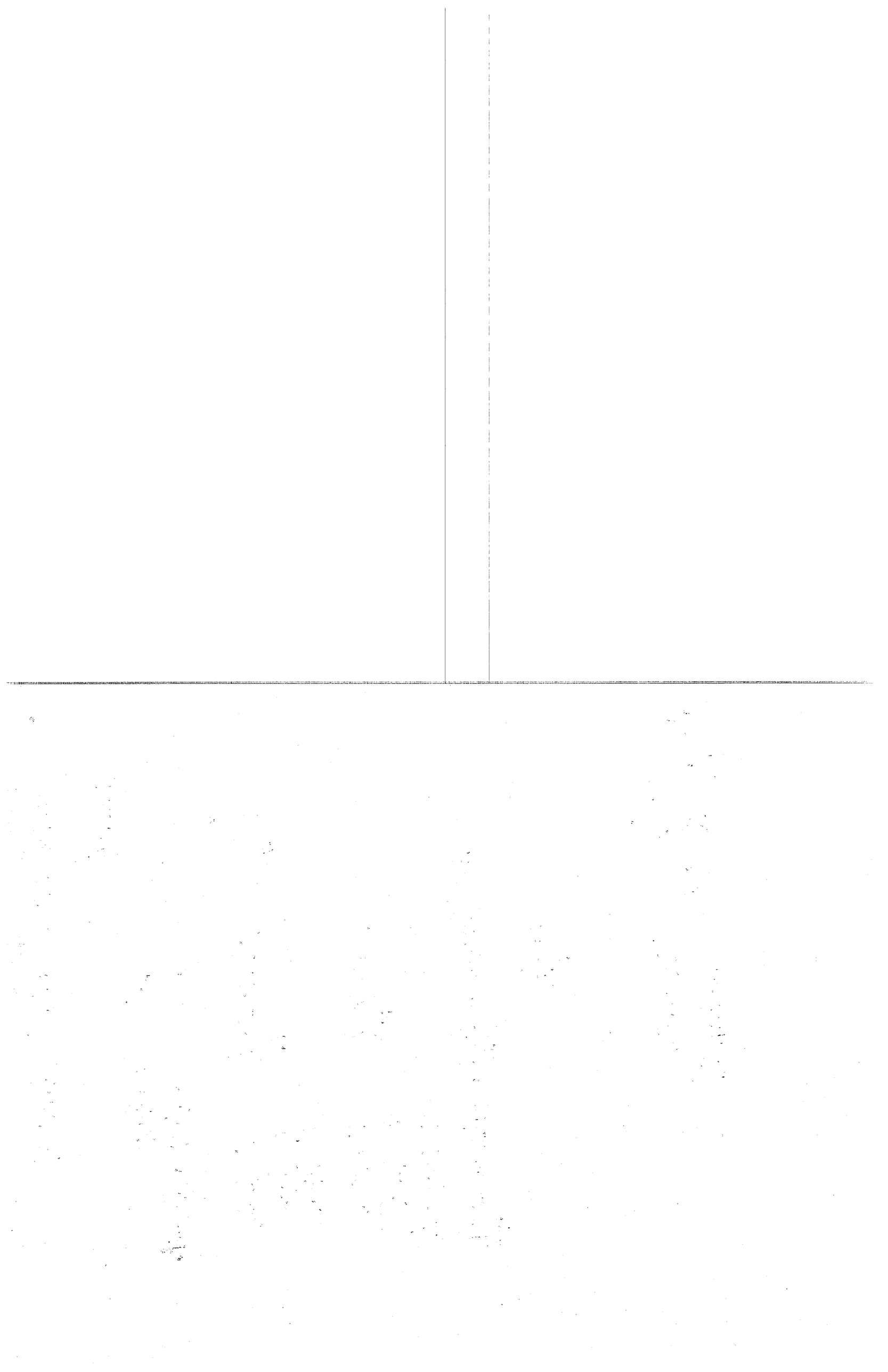
$$\tau = \frac{Tc}{J}$$

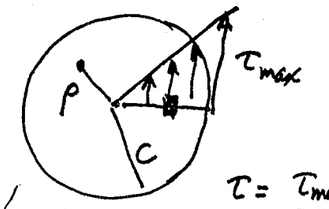
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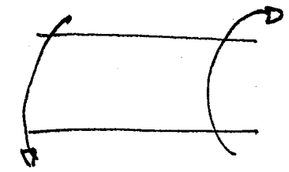



$\sigma_{max} = K \sigma_{ave}$

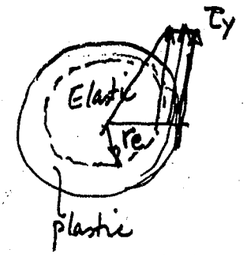
$K = f(D/d, r/d)$

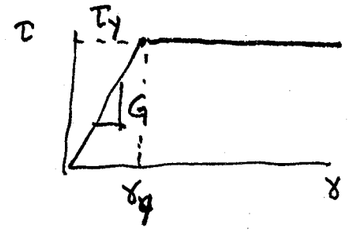




$$\tau = \frac{\tau_{max}}{c} \cdot \rho$$




$\tau = G\gamma$

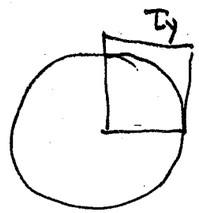
elasto plastic model

$$T = \int \tau \, dA \cdot \rho = \int \frac{\tau_{max}\rho}{c} \cdot \rho d\rho d\theta \cdot \rho = \frac{\tau_{max}}{c} \int_0^c \rho^3 d\rho \cdot \int_0^{2\pi} d\theta$$

$$= \frac{\tau_{max}}{c} \frac{c^4}{4} \cdot 2\pi$$

$$T = \tau_{max} \frac{c^3 \pi}{2}$$

$$T = \int_0^{r_e} \frac{\tau_{max}\rho}{r_e} \cdot \rho \cdot \rho d\rho d\theta \cdot \rho + \int_{r_e}^{c} \tau_y \cdot \rho d\rho d\theta \cdot \rho$$




$$T_p = \int_0^c \tau_y \cdot \rho d\rho d\theta \cdot \rho$$

$$= \tau_y \int_0^c \rho^2 d\rho \int_0^{2\pi} d\theta$$

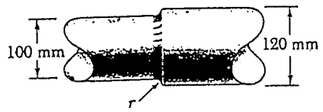
$$= \tau_y \frac{\rho^3}{3} \Big|_0^c \cdot 2\pi = \tau_y \cdot \frac{2\pi c^3}{3}$$

$$T_y \left(\text{when } \tau_{max} = \tau_y\right) = \tau_{max} \frac{c^3 \pi}{2} = \tau_y \cdot \frac{c^3 \pi}{2}$$

$$T_p = \frac{4}{3} T_y$$

HW Done on tape

PROBLEM 3.87




**3.87** The stepped shaft shown rotates at 450 rpm. Knowing that $r = 10$ mm, determine the maximum power that can be transmitted without exceeding an allowable shearing stress of 45 MPa.

SOLUTION

$d = 100$ mm, $D = 120$ mm, $r = 10$ mm

$\frac{D}{d} = \frac{120}{100} = 1.2$, $\frac{r}{d} = \frac{10}{100} = 0.10$, From Fig. 3.32 $K = 1.33$

For smaller shaft $c = \frac{1}{2}d = 0.050$ m $\qquad \tau = \frac{Tc}{J} = \frac{2KT}{\pi c^3}$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.050)^3 (45 \times 10^6)}{(2)(1.33)} = 6.643 \times 10^3 \text{ N-m}$$

$f = 450$ rpm $= 7.5$ Hz $= 450/60$

Power $P = 2\pi f T = 2\pi (7.5)(6.643 \times 10^3) = 313 \times 10^3$ W $= 313$ kW

$T = \frac{9540 P}{N} \Rightarrow P = \frac{T \cdot N}{9540} = 313.35$ kW

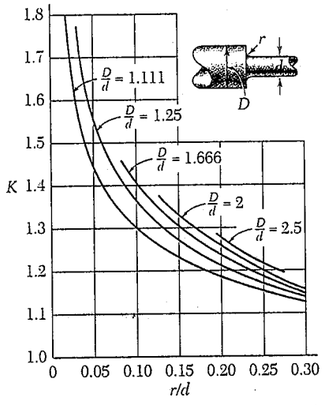



**Fig. 3.32** Stress-concentration factors for fillets in circular shafts.†

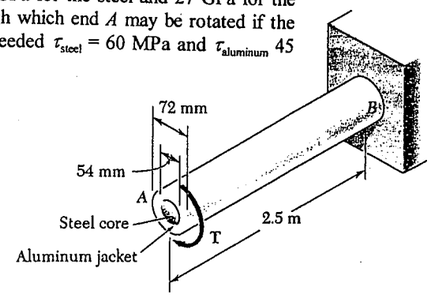
TABLE 3.1. **Coefficients for Rectangular Bars in Torsion**

| $a/b$ | $c_1$ | $c_2$ |
|---|---|---|
| 1.0 | 0.208 | 0.1406 |
| 1.2 | 0.219 | 0.1661 |
| 1.5 | 0.231 | 0.1958 |
| 2.0 | 0.246 | 0.229 |
| 2.5 | 0.258 | 0.249 |
| 3.0 | 0.267 | 0.263 |
| 4.0 | 0.282 | 0.281 |
| 5.0 | 0.291 | 0.291 |
| 10.0 | 0.312 | 0.312 |
| $\infty$ | 0.333 | 0.333 |

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**PROBLEM 3.53** **3.53** The composite shaft shown is to be twisted by applying a torque T at end *A*. Knowing that the modulus of rigidity is 77 GPa for the steel and 27 GPa for the aluminum, determine the largest angle through which end *A* may be rotated if the following allowable stresses are not to be exceeded $\tau_{steel}$ = 60 MPa and $\tau_{aluminum}$ 45 MPa.

**SOLUTION**




$\tau_{max} = G\gamma_{max} = G c_{max}\frac{\phi}{L}$

$\frac{\phi_{all}}{L} = \frac{\tau_{all}}{G c_{max}}$ for each material

Steel core: $\tau_{all} = 60\times10^{6}$ Pa, $c_{max} = \frac{1}{2}d = 0.027$ m, $G = 77\times10^{9}$ Pa

$$\frac{\phi_{all}}{L} = \frac{60\times10^{6}}{(77\times10^{9})(0.027)} = 28.860\times10^{-3} \text{ rad/m}$$

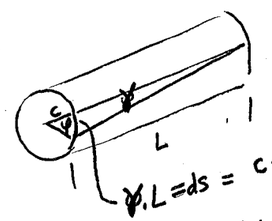
Aluminum jacket: $\tau_{all} = 45\times10^{6}$ Pa, $c_{max} = \frac{1}{2}d = 0.036$ m, $G = 27\times10^{9}$ Pa

$$\frac{\phi_{all}}{L} = \frac{45\times10^{6}}{(27\times10^{9})(0.036)} = 46.296\times10^{-3} \text{ rad/m}$$

Smaller value governs $\frac{\phi_{all}}{L} = 28.860\times10^{-3}$ rad/m

Allowable angle of twist $\phi_{all} = L\frac{\phi_{all}}{L} = (2.5)(28.860\times10^{-3})$

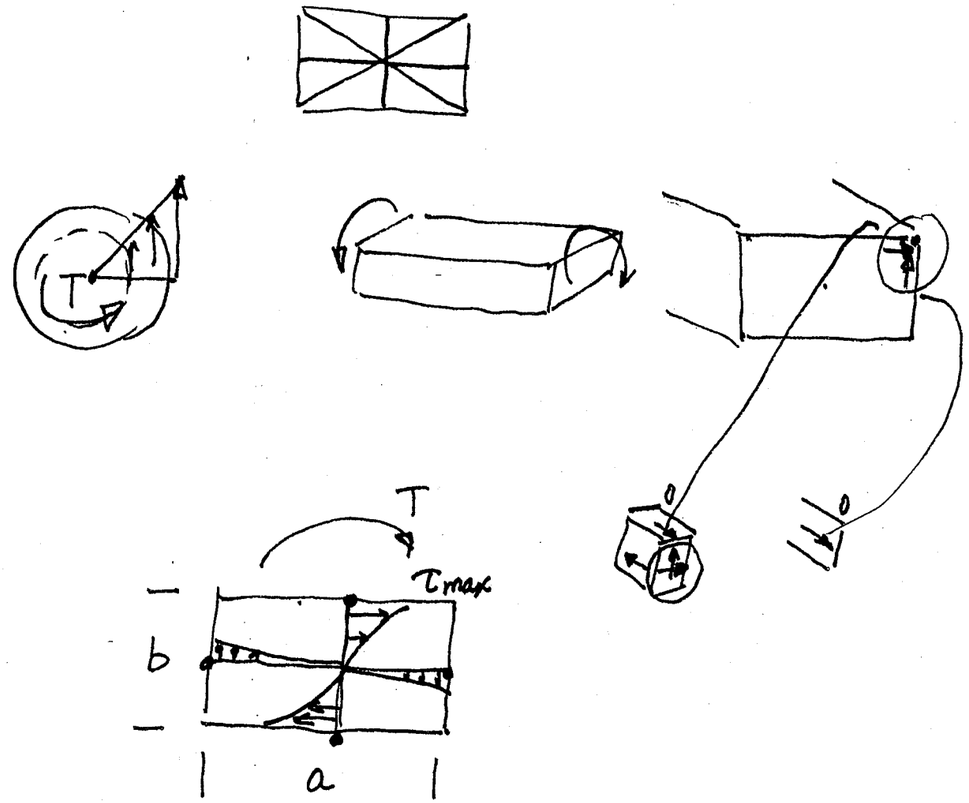
$= 72.15\times10^{-3}$ rad $= 4.13°$




$\gamma \cdot L = ds = c\cdot\varphi$

$\varphi = \frac{\gamma\cdot L}{c} = \frac{\tau_{max}\cdot L}{G\cdot c}$

$\therefore \frac{\varphi}{L}$ must be same for both



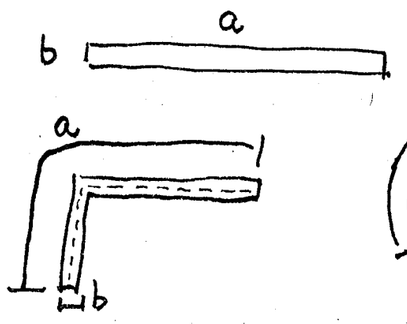




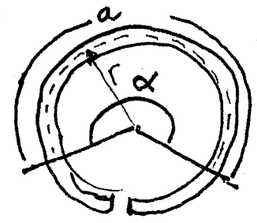
$$\tau_{max} = \frac{T}{c_1 a b^2} \qquad c_1 = f_1(a/b)$$

$$\varphi = \frac{TL}{c_2 a b^3 G} \qquad c_2 = f_2(a/b)$$

Table 3.1 pg 187




$c_1 = c_2 \cong 1/3$




b = thickness

$a = 2\pi r = \alpha r$

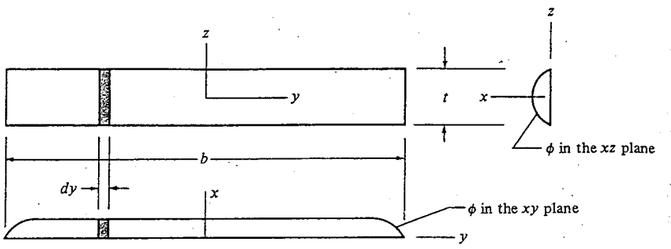



**FIGURE 8.4.2.** Narrow rectangular cross section ($b >> t$), with views of the stress function (or inflated membrane).

where $C$ is a constant. Substituting into Eq. 8.3.5, we find

$$-2C = -2G\beta \quad \text{so} \quad C = G\beta \tag{8.4.4}$$

The torque is

$$T = 2\int_{\text{area}} \phi \, dA = 2\int_{-t/2}^{t/2} \phi b \, dz = G\beta \frac{bt^3}{3} \tag{8.4.5}$$

The maximum shear stress, found along the edges $z = \pm t/2$, is

$$\tau = \left|\frac{\partial \phi}{\partial z}\right|_{z=\pm t/2} = 2C\frac{t}{2} = G\beta t \tag{8.4.6}$$

Substituting for $\beta$ from Eq. 8.4.5, we find

$$\tau = \frac{Tt}{bt^3/3} = \frac{3T}{bt^2} \quad \text{at } z = \pm\frac{t}{2} \tag{8.4.7}$$

Equations 8.4.5 and 8.4.7 can be written in the forms

$$\frac{d\theta}{dx} = \beta = \frac{T}{GJ_R} \qquad \tau = \frac{Tt}{J_R} \tag{8.4.8}$$

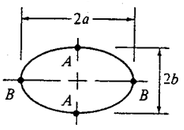
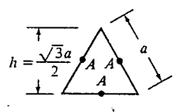
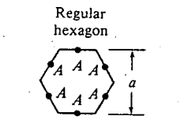
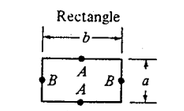
where

$$J_R = \frac{bt^3}{3} \quad (\text{for } b >> t) \tag{8.4.9}$$

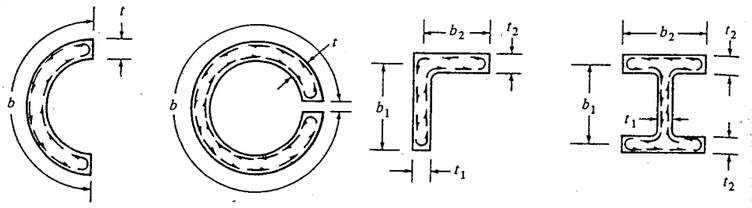
These expressions for $\beta$ and $\tau$ are similar in form to the corresponding expressions for a circular cross section, Eqs. 8.1.2 and 8.1.3. However, $J_R$ is *emphatically NOT* the polar moment of the cross-sectional area about the centroidal $x$ axis.

If (say) $b/t = 10$, Eqs. 8.4.8 give $\beta$ and $\tau$ values that are approximately 6.5 percent low. Accuracy improves as $b/t$ increases. Aspect ratios in the range $1 < b/t < 10$ can be analyzed with tabulated data obtained by other analytical or numerical methods (Table 8.4.1). If the centerline of the bar is curved, as

**TABLE 8.4.1** Expressions for Maximum Shear Stress and Rate of Twist in Selected Solid Sections [8.1, 8.4]

| Cross Section and Area | Maximum Shear Stress | Rate of Twist |
|---|---|---|
| Ellipse<br>  <br>Area = $\pi ab$ | $\tau_A = \dfrac{2T}{\pi ab^2}$ $(a > b)$<br>($\tau_{max}$ at $B$ if $b > a$) | $\beta = \dfrac{a^2 + b^2}{\pi a^3 b^3}\dfrac{T}{G} = \dfrac{d\theta}{dx}$ |
| Equilateral triangle<br>  <br>Area = $0.433\,a^2$ | $\tau_A = \dfrac{20T}{a^3}$ | $\beta = \dfrac{46.2}{a^4}\dfrac{T}{G} = \dfrac{d\theta}{dx}$ |
| Regular hexagon<br>  <br>Area = $0.866a^2$ | $\tau_A = \dfrac{5.7T}{a^3}$ | $\beta = \dfrac{8.8}{a^4}\dfrac{T}{G} = \dfrac{d\theta}{dx}$ |
| Rectangle<br>  <br>Area = $ab$ | $\tau_A = \dfrac{T}{C_\tau ba^2}$ | $\beta = \dfrac{1}{C_\beta ba^3}\dfrac{T}{G} = \dfrac{d\theta}{dx}$ |

| $b/a$ | $C_\tau$ | $\tau_B/\tau_A$ | $C_\beta$ |
|---|---|---|---|
| 1.0 | 0.208 | 1.000 | 0.1406 |
| 1.2 | 0.219 | 0.935 | 0.166 |
| 1.5 | 0.231 | 0.859 | 0.196 |
| 2.0 | 0.246 | 0.795 | 0.229 |
| 2.5 | 0.258 | 0.766 | 0.249 |
| 3.0 | 0.267 | 0.753 | 0.263 |
| 4.0 | 0.282 | 0.745 | 0.281 |
| 6.0 | 0.299 | 0.743 | 0.299 |
| 10.0 | 0.312 | 0.742 | 0.312 |
| $\infty$ | 0.333 | 0.742 | 0.333 |




$J_R = \frac{bt^3}{3}$ $J_R = \frac{bt^3}{3}$ $J_R = \frac{b_1 t_1^3 + b_2 t_2^3}{3}$ $J_R = \frac{b_1 t_1^3 + 2b_2 t_2^3}{3}$

(a) (b) (c) (d)

**FIGURE 8.4.3.** Thin-walled open sections, showing their shear stresses and torsional constants $J_R$. In (a) and (b), dimension $b$ is the length of the midline of the cross section.

when a bar is bent to form a helical spring, some corrections to these formulas may be necessary [8.3].

The membrane analogy permits a very useful generalization of the foregoing results to other thin-walled open cross sections. Imagine that the narrow rectangular cross section of Fig. 8.4.2 is distorted into a C or an L shape, or attached to another narrow rectangle to make a T or an I section. By visualizing the inflated membranes for these shapes, we decide that $\phi$ surfaces of all of them remain parabolic (excepting near ends and near reentrant corners, which we discuss later). The total torque is the sum of torques carried by each part of the cross section. Since $G\beta$ is the same for each part, Eqs. 8.4.5 and 8.4.8 still apply but with the $bt^3/3$ contributions summed to yield

$$J_R = \frac{1}{3}\sum_{i=1}^{n} b_i t_i^3 \tag{8.4.10}$$

where $n$ is the number of parts into which the cross section is divided for purposes of calculation. Examples of this calculation appear in Fig. 8.4.3. In angle and I sections, the part of greater thickness displays greater shear stress. Where there is taper, as in the cross section of a turbine blade or in flanges of a rolled section, we can use

$$J_R = \frac{1}{3}\int t^3\, ds \tag{8.4.11}$$

where $ds$ is an increment of length along the medial line of the cross section. More exact formulas are available [8.3]. These formulas, experimental results [8.5], and coefficient $C_\beta$ in Table 8.4.1 suggest that, for standard rolled structural shapes, $J_R$ may actually be some 10 percent higher than predicted by Eqs. 8.4.10 and 8.4.11. Omission of this adjustment yields a conservative design under either a stress limit or a deflection limit.

## 8.5 A HYDRODYNAMIC ANALOGY

There are several hydrodynamic analogies to the torsion problem [8.1]. In this section we outline one, without proof, and use it to draw useful conclusions about shear stresses in twisted bars.

If a fluid without viscosity executes motion in the $yz$ plane with constant vorticity, its motion is described by the equation

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = -2\omega \tag{8.5.1}$$

where $\phi$ is the stream function and $\omega$ is the (constant) vorticity. Fluid velocities $v$ and $w$ in $y$ and $z$ directions, respectively, are

$$v = \frac{\partial \phi}{\partial z} \qquad w = -\frac{\partial \phi}{\partial y} \tag{8.5.2}$$

Clearly, Eqs. 8.5.1 and 8.5.2 are analogous to Eqs. 8.3.5 and 8.3.1, respectively. Thus fluid velocities are proportional to shear stresses. Lines $\phi$ = constant are streamlines. The fluid boundary must be a streamline, therefore the boundary condition $d\phi = 0$ of Eq. 8.3.6 is met. Thus the analogy is complete.

Experiments have been done in the following way. Imagine that a square cross section is to be studied. A shallow square tank is painted black and placed on a turntable. A camera, looking down onto the tank, is also attached to the turntable. The tank is filled with water (whose viscosity is low, if not quite zero) and aluminum powder is sprinkled on the water. When all is quiet, the camera shutter is opened while the turntable is rotated about 10°. From the viewpoint of the photograph, the tank is stationary while the fluid rotates with nearly constant vorticity. The aluminum particles show as streaks on the film. Each streak is in the direction of a shear stress. The length of the streak is proportional to the magnitude of the shear stress.

Experiments aside, the hydrodynamic analogy has other uses. One is in aiding visualization of torsion problems. Another is that known solutions for fluid flow can be applied to the torsion problem. For example, consider an elliptical obstacle (Fig. 8.5.1a). Far from the obstacle the flow is uniform and horizontal, so $w = 0$ and $v = v_o$, a constant. Points $A$ are stagnation points, where $v = w = 0$. Fluid theory shows that at points $B$, where $z = \pm b$, fluid velocities are $v = v_o(1 + b/a)$ and $w = 0$. Applying these results to a twisted bar, Fig. 8.5.1b, we conclude that stress is zero at the sharp external corners $C$. At the root of the small elliptical notch, shear stress is approximately

$$\tau = \tau_o\left(1 + \frac{b}{a}\right) \tag{8.5.3}$$

where $\tau_o$ is the stress that would prevail near the boundary if the notch were not there. The calculation is approximate because the undisturbed stress is not the uniform value $\tau_o$ unless the notch is very small. The quantity in parentheses

welded together at $B$ as shown in Fig. P6.96. The twisting couples at $A$ and $C$ are equal in magnitude when the couple $T_0$ is applied at $B$. Determine the relation between the elastic moduli of the two materials.

Answer: $G_A = 0.592G_C$

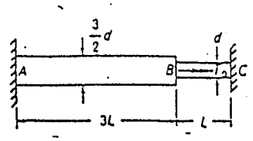


Fig. P6.96

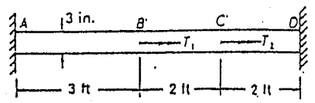


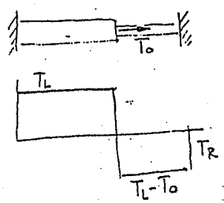
Fig. P6.97

6.97. An aluminum shaft is rigidly restrained at each end and loaded by couples as shown in Fig. P6.97. If $T_1 = 12{,}000$ in.-lb and $T_2 = 10{,}000$ in.-lb; compute the maximum shear stress in each section and the rotations of cross sections at $B$ and $C$.

Answer: $\tau_{CD} = -2300\ psi$, $\tau_{BC} = -433\ psi$, $\tau_{AB} = 1840\ psi$,

A

6.96




$T_L - T_R = T_0$

$T_R = T_L - T_0$

since $|T_R| = |T_L|$

$T_L = T_0/2 \quad T_R = -T_0/2$

$$\varphi_{AB} = \frac{T_L L_{AB}}{J_{AB} G_{AB}} = |\varphi_{BC}| = \left|\frac{T_R L_{BC}}{J_{BC} G_{BC}}\right|$$

$J_{AB} = \frac{\pi c^4}{2} = \frac{\pi d^4}{32} = \frac{\pi \cdot 81 d^4}{512}$; $L_{AB} = 3L$;

$J_{BC} = \frac{\pi c^4}{2} = \frac{\pi d^4}{32}$; $L_{BC} = L$

$$\varphi_{AB} = \frac{T_0/2 \cdot 3L}{\pi 81 d^4/512 \, G_{AB}} = \frac{T_0/2 \cdot L}{\frac{\pi d^4}{32} G_{BC}}$$

$$\therefore \frac{G_{AB} \cdot 81}{1536} = \frac{G_{BC}}{32} \quad \text{or} \quad G_{AB} = \frac{1536}{81 \cdot 32} G_{BC} = .5926 G_{BC}$$

Fig. P3.108 and P3.109

**3.132** A thin-walled tube has been fabricated by bending a metal plate of thickness $t$ into a cylinder of radius $c$ and bonding together the edges of the plate. A torque T is then applied to the tube, producing a shearing stress $\tau_1$ and an angle of twist $\phi_1$. Denoting by $\tau_2$ and $\phi_2$, respectively, the shearing stress and the angle of twist which will develop if the bond suddenly fails, express the ratios $\tau_2/\tau_1$ and $\phi_2/\phi_1$ in terms of the ratio $c/t$.

$\phi_2/\phi_1 = 3\left(\frac{c}{t}\right)^2$

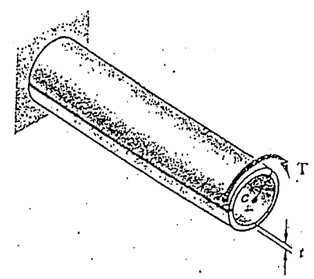


Fig. P3.132

**3.122** A 3.5-m-long steel member with a W310 × 143 cross section is subjected to a 4.5 kN · m torque. Knowing that $G = 77$ GPa and referring to Appendix C for the dimensions of the cross section, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the maximum shearing stress along line *b-b*, (*c*) the angle of twist. (*Hint*: Consider the web and the flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

39.7 MPa 24.2 MPa 4.12°

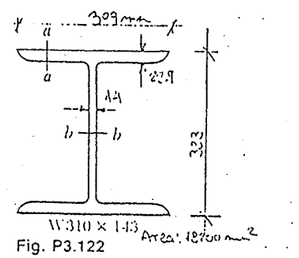
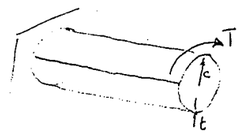


Fig. P3.122

3.132




closed tube $J = \frac{\pi}{2}(a^2-b^2)(a^2+b^2)$ ; $\tau_1 = \frac{Tc}{J}$ ; $\phi_1 = \frac{TL}{JG}$

$= \frac{\pi}{2}(a-b)(a+b)(a^2+b^2)$

$J = \frac{\pi}{2} \cdot t \cdot 2c \cdot 2c^2 = 2\pi c^3 t$ $\quad \tau_1 = \frac{T}{2\pi c^2 t}$ ; $\phi_1 = \frac{TL}{2\pi c^3 t G}$

Open tube $J_R = \frac{bt^3}{3} = \frac{2\pi c t^3}{3}$ ; $\tau_2 = \frac{Tt}{J_R}$ ; $\beta = \frac{\phi_2}{L} = \frac{T}{GJ_R}$

$\tau_2 = \frac{Tt}{2\pi c t^3/3} = \frac{3Tt}{2\pi c t^3} = \frac{3T}{2\pi c t^2}$ ; $\phi_2 = \frac{TL}{\frac{2\pi c t^3 G}{3}} = \frac{3TL}{2\pi c t^3 G}$

$\frac{\tau_1}{\tau_2} = \frac{\frac{T}{2\pi c^2 t}\frac{t}{t}}{\frac{3T}{2\pi c t^2}\frac{c}{c}} = \frac{t}{3c}$ ; $\frac{\phi_1}{\phi_2} = \frac{\frac{TL}{2\pi c^3 t G}\frac{t^2}{t^2}}{\frac{3TL}{2\pi c t^3 G}\frac{c^2}{c^2}} = \frac{t^2}{3c^2}$

$\left(\frac{\tau_1}{\tau_2}\right)^2 = \frac{t^2}{9c^2} = \frac{1}{3}\left(\frac{\phi_1}{\phi_2}\right)$

So: $\frac{\tau_2}{\tau_1} = 3\left(\frac{c}{t}\right)$ and $\phi_2/\phi_1 = 3\left(\frac{c}{t}\right)^2$

Fig. P3.108 and P3.109

**3.132** A thin-walled tube has been fabricated by bending a metal plate of thickness $t$ into a cylinder of radius $c$ and bonding together the edges of the plate. A torque T is then applied to the tube, producing a shearing stress $\tau_1$ and an angle of twist $\phi_1$. Denoting by $\tau_2$ and $\phi_2$, respectively, the shearing stress and the angle of twist which will develop if the bond suddenly fails, express the ratios $\tau_2/\tau_1$ and $\phi_2/\phi_1$ in terms of the ratio $c/t$.

$\phi_2/\phi_1 = 3\left(\frac{c}{t}\right)^2$

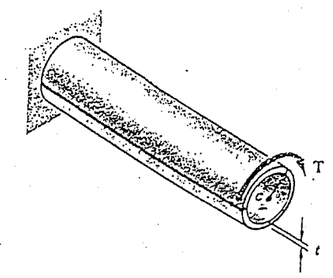



Fig. P3.132

**3.122** A 3.5-m-long steel member with a W310 × 143 cross section is subjected to a 4.5 kN · m torque. Knowing that $G = 77$ GPa and referring to Appendix C for the dimensions of the cross section, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the maximum shearing stress along line *b-b*, (*c*) the angle of twist. (*Hint*: Consider the web and the flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

39.7 MPa 24.2 MPa 4.72°

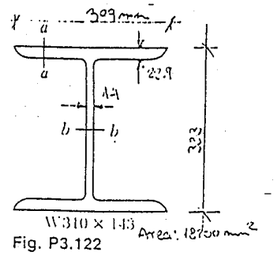
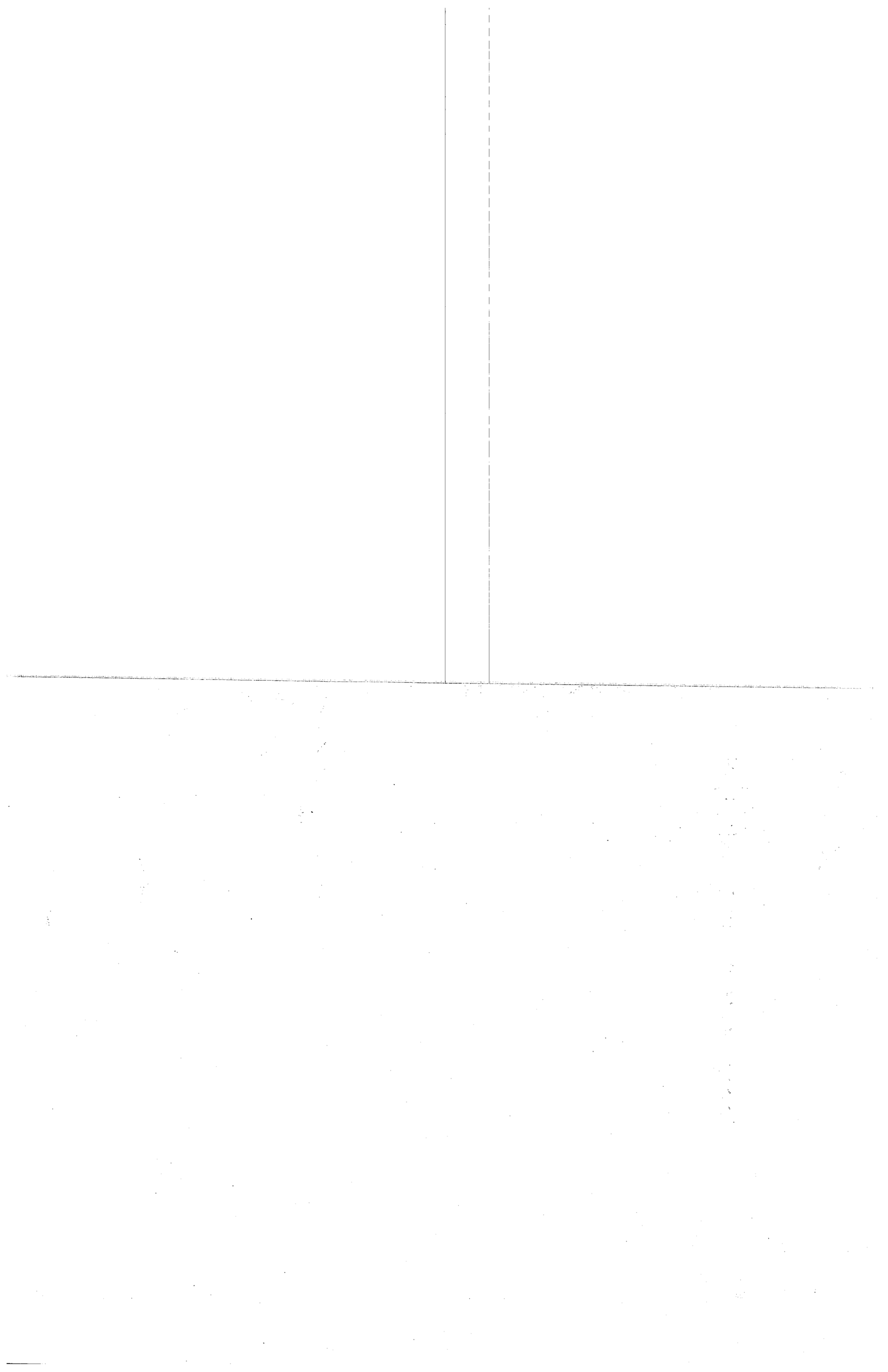
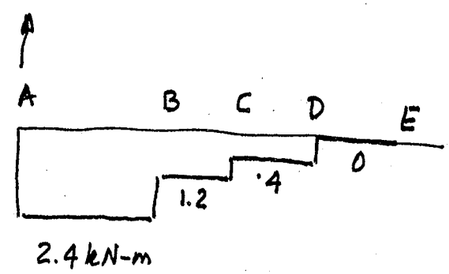



Fig. P3.122

HW: DO 3.21, 3.47 & 3.53



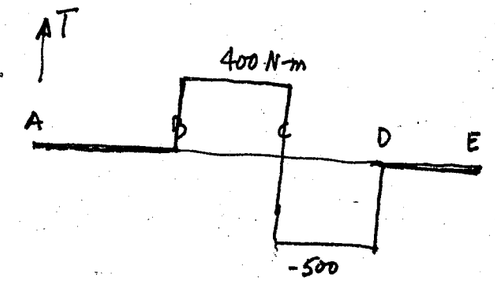
3.13




$$\tau = \frac{Tc}{J} \quad \frac{2T}{\pi c^3}$$

$$\varphi = \frac{TL}{JG} \quad \varphi = \int \frac{T dx}{JG}$$

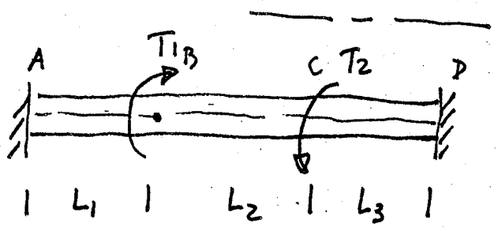
3.36

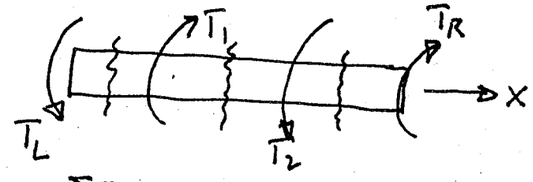



$\varphi_{BC}$?

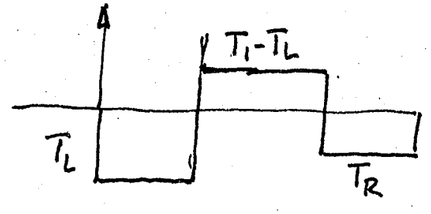
$\varphi_{CD}$?

$\varphi_{AE} = \varphi_{AB} + \varphi_{BC} + \varphi_{CD} + \varphi_{DE}$




$$\Sigma T = 0 = T_L - T_1 + T_2 - T_R = 0$$



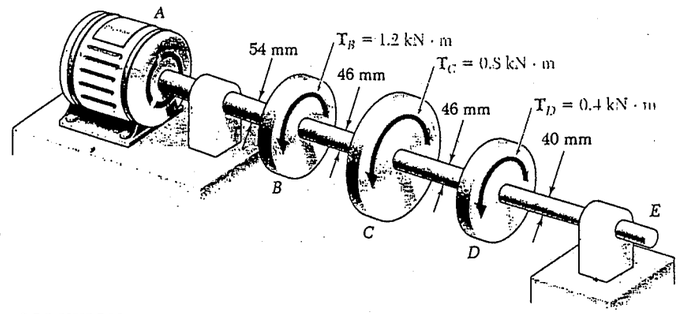

$$\Sigma \varphi = \varphi_{AB} + \varphi_{BC} + \varphi_{CD} = 0$$

$$= \frac{T_{AB} L_1}{J_{AB} \cdot G} + \frac{T_{BC} L_2}{J_{BC} \cdot G} + \frac{T_{CD} L_3}{J_{CD} \cdot G} = 0 \Rightarrow \quad T_{AB} \cdot L_1 + T_{BC} \cdot L_2 + T_{CD} \cdot L_3 = 0$$

$$-T_L \cdot L_1 + (T_1 - T_L) L_2 + -T_R \cdot L_3 = 0$$

**PROBLEM 3.13**

**3.13** Under normal operating conditions, the electric motor exerts a torque of 2.4 kN·m at *A*. Knowing that each shaft is solid, determine the maximum shearing stress (*a*) in shaft *AB*, (*b*) in shaft *BC*, (*c*) in shaft *CD*.




$$J = \frac{\pi c^4}{2}$$

**SOLUTION**

Shaft AB: $T_{AB} = 2.4\times10^3$ N·m, $c = \frac{1}{2}d = 0.027$ m

$$\tau_{AB} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2.4\times10^3)}{\pi(0.027)^3} = 77.625\times10^6 \text{ Pa}$$ 77.6 MPa

Shaft BC: $T_{BC} = 2.4$ kN·m $- 1.2$ kN·m $= 1.2$ kN·m, $c = \frac{1}{2}d = 0.023$ m

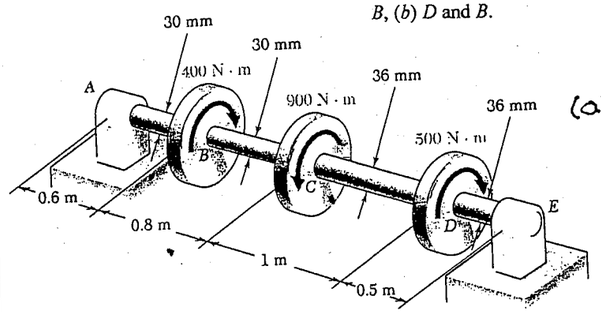
$$\tau_{BC} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1.2\times10^3)}{\pi(0.023)^3} = 62.788\times10^6 \text{ Pa}$$ 62.8 MPa

Shaft CD: $T_{CD} = 0.4\times10^3$ N·m $c = \frac{1}{2}d = 0.023$ m

$$\tau_{CD} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(0.4\times10^3)}{\pi(0.023)^3} = 20.929\times10^6 \text{ Pa}$$ 20.9 MPa

**PROBLEM 3.36**

*3.36* The torques shown are exerted on pulleys *B*, *C* and *D*. Knowing that the entire shaft is made of steel ($G = 27$ GPa), determine the angle of twist between (*a*) *C* and *B*, (*b*) *D* and *B*.




**SOLUTION**

(a) Shaft BC: $c = \frac{1}{2}d = 0.015$ m

$J_{BC} = \frac{\pi}{2}c^4 = 79.522\times10^{-9}$ m$^4$

$L_{BC} = 0.8$ m, $G = 27\times10^9$ Pa

$$\varphi_{BC} = \frac{TL}{GJ} = \frac{(400)(0.8)}{(27\times10^9)(79.522\times10^{-9})}$$

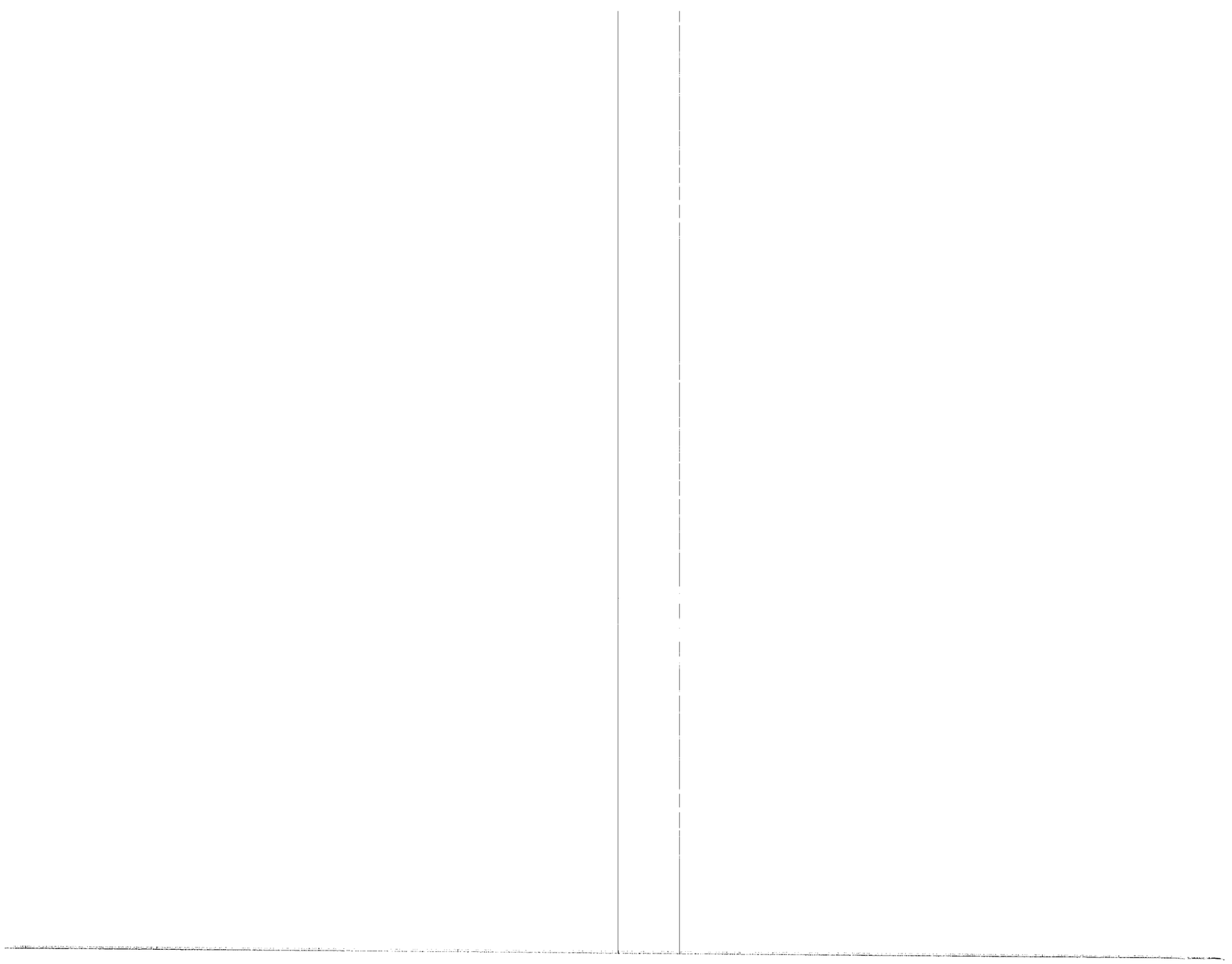
$= 0.149904$ rad $= 8.54°$

(b) Shaft CD: $c = \frac{1}{2}d = 0.018$ m $J_{CD} = \frac{\pi}{2}c^4 = 164.896\times10^{-9}$ m$^4$

$L_{CD} = 1.0$ m $T_{CD} = 400 - 900 = -500$ N·m

$$\varphi_{CD} = \frac{TL}{GJ} = \frac{(-500)(1.0)}{(27\times10^9)(164.896\times10^{-9})} = -0.11230 \text{ rad}$$

$\varphi_{AE} = \varphi_{BD} = \varphi_{BC} + \varphi_{CD} = 0.14904 - 0.11230 = 0.03674$ rad $= 2.11°$



$$T_L + T_2 - T_1 - T_R = 0$$

$$-T_L \cdot L_1' + (T_1 - T_L')L_2 - T_R \cdot L_3 = 0 \Rightarrow -T_L(L_1+L_2) + T_1 L_2 - T_R L_3 = 0$$

$$T_L - T_R = T_1 - T_2$$

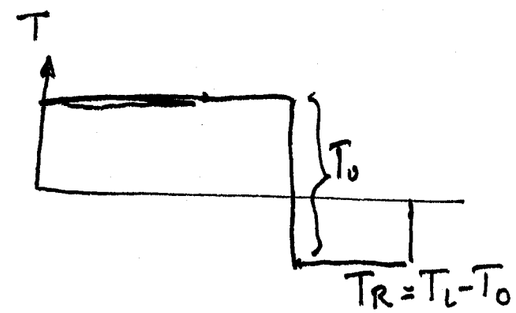
$$-T_L(L_1+L_2) - T_R \cdot L_3 = T_1 L_2$$

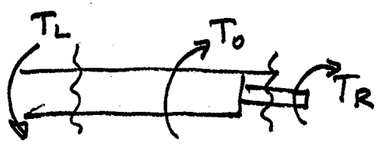
$$T_L = \frac{\begin{vmatrix} T_1 - T_2 & -1 \\ T_1 L_2 & -L_3 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -(L_1+L_2) & -L_3 \end{vmatrix}} = \frac{T_1\left(\frac{L_2+L_3}{L}\right) - T_2\left(\frac{L_3}{L}\right)}{L = L_1 + L_2 + L_3}$$

$$T_R = -T_1\left(\frac{L_1}{L}\right) + T_2\left(\frac{L_1+L_2}{L}\right)$$

$$\varphi_{AB} = \frac{T_{AB} L_1}{J_{AB} G} = \frac{-T_L \cdot L_1}{J_{AB} \cdot G} = [\text{rad.}] \qquad \text{rad} \cdot \frac{180^\circ}{\pi} = (\ )^\circ$$

6.96




$$T_R + T_0 - T_L = 0$$

$$T_0 = T_L - T_R$$





$$A_{incl} = \frac{\pi r^2}{2} + b2r + \frac{2r \cdot l}{2} = \pi \cdot 8 + 3 \cdot 8 + 4.12 = 97.133\ in^2$$ 3·2+1=7

$$\tau = \frac{T}{2tA_{inc}} = \frac{1000\ lb\text{-}in}{2(.04in)(97.133in^2)} = 128.69\ \frac{lb}{in^2}$$ 2

$$\frac{\phi}{L} = \frac{T}{4A_{inc}^2 G} \oint \frac{ds}{t}$$

$ds = \pi r + 2b + l + h$ $\quad h = \sqrt{(2r)^2 + l^2}$

$= \pi \cdot 4 + 2.3 + 12 + 14.42 = 44.99$ $\quad = 14.42$ 4

$= 18.566\ in$

$$= \frac{1000\ lb\text{-}in}{4(97.133)^2 G} \cdot \frac{18.566}{.040} = \frac{29.8}{G}\ \frac{rad}{in}$$ 2

15

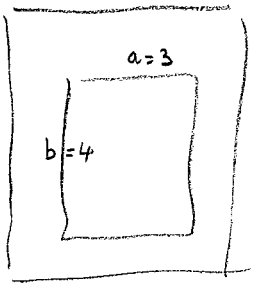
Thin plate $\sigma_z = 0$ 1

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y + \alpha\Delta T$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_x + \alpha\Delta T$$

2

$\epsilon_x = 0$, $\epsilon_y = 0$ since mirror doesn't expand.




$$0 = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y + \alpha(-200) = \frac{\sigma_x}{E} - \frac{1}{3E}\sigma_y - .0022$$

$$0 = \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_x + \alpha(-200) = \frac{\sigma_y}{E} - \frac{1}{3}\frac{\sigma_x}{E} - .0022$$

$$G = \frac{E}{2(1+\nu)}$$

$$\therefore \nu = \frac{E}{2G} - 1 = .\overline{333}$$ 2

$$\therefore .0022 = A - \tfrac{1}{3}B$$

$$.0022 = B - \tfrac{1}{3}A$$

$$\frac{.0022}{3} = \frac{A}{3} - \frac{1}{9}B$$

$$.0022 = -\tfrac{1}{3}A + B$$

$$B = \frac{.0022\left(1+\frac{1}{3}\right)}{1-\frac{1}{9}} = .0033$$

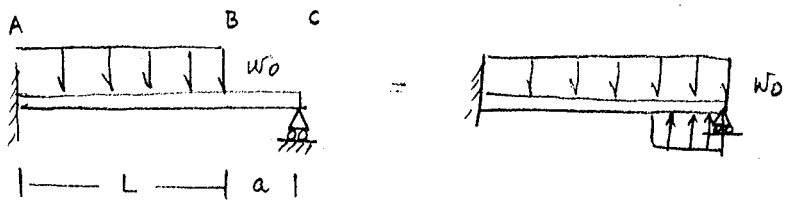
$\sigma_y = 52800\ psi = \sigma_x$

by symmetry of eqns. 10

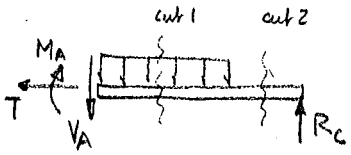
$$\begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} \\ -\frac{\nu}{E} & \frac{1}{E} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \end{bmatrix} = \begin{bmatrix} -\alpha\Delta T \\ -\alpha\Delta T \end{bmatrix}$$

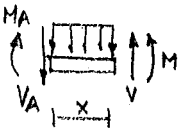
$$\sigma_x = \frac{\begin{vmatrix} -\alpha\Delta T & -\nu/E \\ -\alpha\Delta T & 1/E \end{vmatrix}}{\begin{vmatrix} \frac{1}{E} & -\nu/E \\ -\nu/E & \frac{1}{E} \end{vmatrix}} \qquad \sigma_y = \frac{\begin{vmatrix} 1/E & -\alpha\Delta T \\ -\nu/E & -\alpha\Delta T \end{vmatrix}}{\text{Same.}}$$

5 5



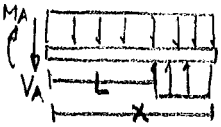

Indeterminate Problem: solve using shear form of equation




at cut 1: 

$V = w_0 x + V_A$

4 unknowns & 3 eqns : 1 extra unknown → choose $V_A$

at cut 2: 

$V = w_0 x - w_0 (x-L) + V_A$

$\therefore$ V can be written as $V = w_0 x - w_0 \langle x-L \rangle^1 + V_A$ where $V_A$ here is the unknown

Now $EI\,y''' = -V = -w_0 x + w_0 \langle x-L \rangle^1 - V_A$ .

Solving from here requires 3 BCs to solve for $C_2, C_3, C_4$ & 1 more BC for the unknown $V_A$.

$$EI\,y'' = -w_0 \frac{x^2}{2} + \frac{w_0}{2}\langle x-L\rangle^2 - V_A x + C_2 = M$$

$$EI\,y' = -w_0 \frac{x^3}{6} + \frac{w_0}{6}\langle x-L\rangle^3 - V_A \frac{x^2}{2} + C_2 x + C_3$$

$$EI\,y = -w_0 \frac{x^4}{24} + \frac{w_0}{24}\langle x-L\rangle^4 - V_A \frac{x^3}{6} + C_2 \frac{x^2}{2} + C_3 x + C_4$$

BCs: $y=0$ @ $x=0 \Rightarrow C_4 = 0$

$y'=0$ @ $x=0 \Rightarrow C_3 = 0$

$y=0$ @ $x=L+a \Rightarrow 0 = -w_0 \frac{(L+a)^4}{24} + \frac{w_0 a^4}{24} - V_A \frac{(L+a)^3}{6} + C_2 \frac{(L+a)^2}{2}$ ①

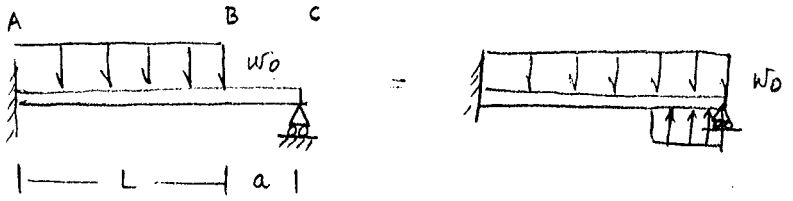
$M=0$ $(EIy''=0)$ @ $x=L+a \Rightarrow 0 = -w_0 \frac{(L+a)^2}{2} + \frac{w_0}{2}(a)^2 - V_A (L+a) + C_2$ ②

so
$$\begin{bmatrix} -(L+a)^3/6 & (L+a)^2/2 \\ -(L+a) & 1 \end{bmatrix} \begin{bmatrix} V_A \\ C_2 \end{bmatrix} = \begin{bmatrix} \frac{w_0}{24}(L+a)^4 - \frac{w_0 a^4}{24} \\ \frac{w_0}{2}(L+a)^2 - \frac{w_0 a^2}{2} \end{bmatrix}$$

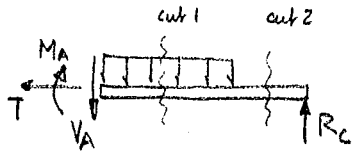
$$V_A = \frac{\begin{bmatrix} \frac{w_0}{24}\left[(L+a)^4 - a^4\right] & (L+a)^2/2 \\ \frac{w_0}{2}\left[(L+a)^2 - a^2\right] & 1 \end{bmatrix}}{\begin{bmatrix} -(L+a)^3/6 & (L+a)^2/2 \\ -(L+a) & 1 \end{bmatrix}} = \frac{\frac{w_0}{24}\left[(L+a)^4 - a^4\right] - \frac{w_0}{4}\left[(L+a)^4 - a^2(L+a)^2\right]}{(L+a)^3/3}$$

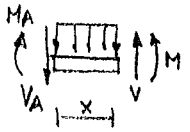
$$C_2 = \frac{\begin{bmatrix} -(L+a)^3/6 & \frac{w_0}{24}\left[(L+a)^4 - a^4\right] \\ -(L+a) & \frac{w_0}{2}\left[(L+a)^2 - a^2\right] \end{bmatrix}}{(L+a)^3/3} = \frac{\frac{-w_0}{12}\left[(L+a)^5 - a^2(L+a)^3\right] + \frac{w_0}{24}\left[(L+a)^5 - a^4(L+a)\right]}{(L+a)^3/3}$$

Note that @ $x=0$ $EIy'' = C_2 = M_A$ . Also $R_c = V_A + w_0 L$



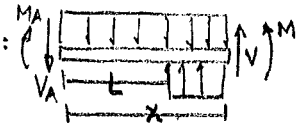

Indeterminate Problem: solve using shear form of equation




at cut 1: 


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4 unknowns & 3 eqns : 1 extra unknown → choose $V_A$

at cut 2: 


$V = w_0 x - w_0\langle x-L\rangle + V_A$

$\therefore$ V can be written as $V = w_0 x - w_0\langle x-L\rangle^1 + V_A$ where $V_A$ here is the unknown

Solving from here requires 3 BCs to solve for $C_2, C_3, C_4$ & 1 more BC for the unknown $V_A$.

Now $EI\,y''' = -V = -w_0 x + w_0\langle x-L\rangle^1 - V_A$ .

$$EI\,y'' = -w_0\frac{x^2}{2} + \frac{w_0}{2}\langle x-L\rangle^2 - V_A x + C_2 = M$$

$$EI\,y' = -w_0\frac{x^3}{6} + \frac{w_0}{6}\langle x-L\rangle^3 - V_A\frac{x^2}{2} + C_2 x + C_3$$

$$EI\,y = -w_0\frac{x^4}{24} + \frac{w_0}{24}\langle x-L\rangle^4 - V_A\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4$$

BCs: $y=0$ @ $x=0 \Rightarrow C_4 = 0$

$y'=0$ @ $x=0 \Rightarrow C_3 = 0$

$y=0$ @ $x=L+a \Rightarrow 0 = -w_0\frac{(L+a)^4}{24} + \frac{w_0 a^4}{24} - V_A\frac{(L+a)^3}{6} + C_2\frac{(L+a)^2}{2}$ ①

$M=0$ $(EIy''=0)$ @ $x=L+a \Rightarrow 0 = -w_0\frac{(L+a)^2}{2} + \frac{w_0}{2}(a)^2 - V_A(L+a) + C_2$ ②

so

$$\begin{bmatrix} -\frac{(L+a)^3}{6} & \frac{(L+a)^2}{2} \\ -(L+a) & 1 \end{bmatrix}\begin{bmatrix} V_A \\ C_2 \end{bmatrix} = \begin{bmatrix} \frac{w_0(L+a)^4}{24} - \frac{w_0 a^4}{24} \\ \frac{w_0(L+a)^2}{2} - \frac{w_0 a^2}{2} \end{bmatrix}$$

$$V_A = \frac{\begin{bmatrix} \frac{w_0}{24}\left[(L+a)^4 - a^4\right] & \frac{(L+a)^2}{2} \\ \frac{w_0}{2}\left[(L+a)^2 - a^2\right] & 1 \end{bmatrix}}{\begin{bmatrix} -\frac{(L+a)^3}{6} & \frac{(L+a)^2}{2} \\ -(L+a) & 1 \end{bmatrix}} = \frac{\frac{w_0}{24}\left[(L+a)^4 - a^4\right] - \frac{w_0}{4}\left[(L+a)^4 - a^2(L+a)^2\right]}{\frac{(L+a)^3}{3}}$$

$$C_2 = \frac{\begin{bmatrix} -\frac{(L+a)^3}{6} & \frac{w_0}{24}\left[(L+a)^4 - a^4\right] \\ -(L+a) & \frac{w_0}{2}\left[(L+a)^2 - a^2\right] \end{bmatrix}}{\frac{(L+a)^3}{3}} = \frac{-\frac{w_0}{12}\left[(L+a)^5 - a^2(L+a)^3\right] + \frac{w_0}{24}\left[(L+a)^5 - a^4(L+a)\right]}{\frac{(L+a)^3}{3}}$$

Note that @ $x=0$ $EIy'' = C_2 = M_A$ . Also $R_c = V_A + w_0 L$

## תורת החוזק 1
## תרגיל מס 9

**5.16 and 5.17** For the wide-flange beam and loading shown, determine in a section located halfway between points $D$ and $E$, ($a$) the largest normal stress, ($b$) the largest shearing stress.

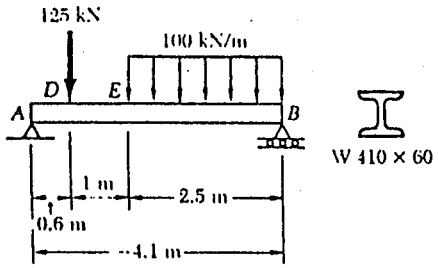



Fig. P5.16

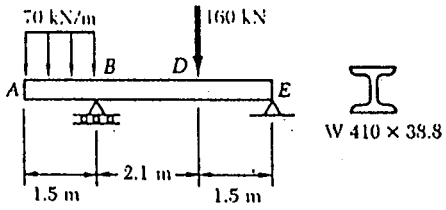



Fig. P5.17

**5.18** Two rectangular plates are welded to the 310-mm-wide-flange beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 90 MPa.

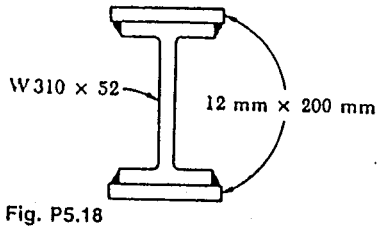
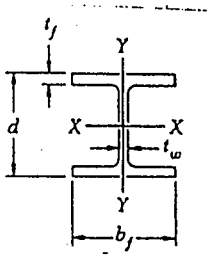



Fig. P5.18

| Designation† | Area $A$, mm² | Depth $d$, mm | Flange Width $b_f$, mm | Flange Thickness $t_f$, mm | Web Thickness $t_w$, mm | Axis X-X $I_x$ 10⁶ mm⁴ | Axis X-X $S_x$ 10³ mm³ | Axis X-X $r_x$ mm |
|---|---|---|---|---|---|---|---|---|
| W310 × 52 | 6650 | 317 | 167 | 13.2 | 7.6 | 118.6 | | |
| W410 × 114 | 14600 | 420 | 261 | 19.3 | 11.6 | 462 | 2200 | 177.8 |
| 85 | 10800 | 417 | 181 | 18.2 | 10.9 | 316 | 1516 | 170.7 |
| 60 | 7610 | 407 | 178 | 12.8 | 7.7 | 216 | 1061 | 168.4 |
| 46.1 | 5880 | 403 | 140 | 11.2 | 7.0 | 156.1 | 775 | 162.8 |
| 38.8 | 4950 | 399 | 140 | 8.8 | 6.4 | 125.3 | 628 | 159.0 |




**3. For a timber beam having the cross section shown, determine the dimension w if the maximum allowable vertical shear force is 7.67kN, and the shearing stress is not to exceed 1.MPa.**

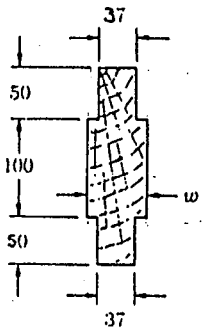
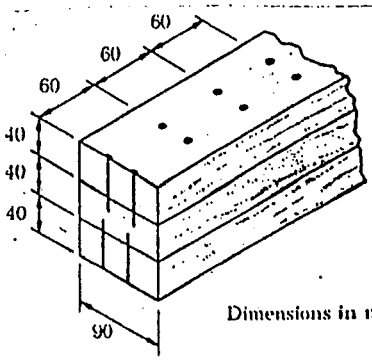


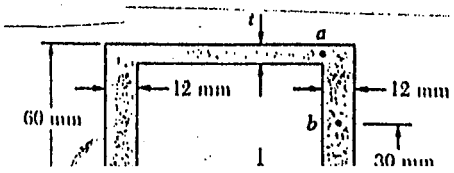

Fig. P5.20 Dimensions in mm

**5.1** Three boards, each of 40 mm × 90 mm rectangular cross section, are nailed together to form a beam which is subjected to a vertical shear of 1 kN. Knowing that the spacing between each pair of nails is 60 mm, determine the shearing force in each nail.




Dimensions in mm

**5.32** An extruded beam has the cross section shown and is subjected to a vertical shear of 50 kN. For $t$ = 6 mm, determine the shearing stress at ($a$) point $a$, ($b$) point $b$.



occur in the plastic zones, no unbalance in longitudinal forces occurs and no shearing stresses are developed.

This elementary solution has been refined by using a more carefully formulated criterion of yielding caused by the simultaneous action of normal and shearing stresses.* Some fundamental aspects of the interaction of such stresses will be considered in Chapter 9.

### EXAMPLE 7-6

An $I$ beam is loaded as in Fig. 7-14(a). If it has the cross section shown in Fig. 7-14(c), determine the shearing stresses at the levels indicated. Neglect the weight of the beam.

### SOLUTION

A free-body diagram of a segment of the beam is in Fig. 7-14(b). It is seen from this diagram that the vertical shear at every section is 50 kips. Bending moments do not enter directly into the present problem. The shear flow at the various levels of the beam is computed in the table below using Eq. 7-5. Since $\tau = q/t$ (Eq. 7-6), the shearing stresses are obtained by dividing the shear flows by the respective widths of the beam.

$$I = \frac{6(12)^3}{12} - \frac{(5.5)(11)^3}{12} = 254 \text{ in.}^4$$

For use in Eq. 7-5 the ratio $V/I = -50{,}000/254 = -197$ lb/in.$^4$

| Level | $A_{obj}$* | $\bar{y}$** | $Q = A_{obj}\bar{y}$ | $q = VQ/I$ | $t$ | $\tau$, psi |
|---|---|---|---|---|---|---|
| 1-1 | 0 | 6 | 0 | 0 | 6.0 | 0 |
| 2-2 | (0.5)6 = 3.00 | 5.75 | 17.25 | −3,400 | 6.0<br>0.5 | −570<br>−6,800 |
| 3-3 | (0.5)6 = 3.00<br>(0.5)(0.5) = 0.25 | 5.75<br>5.25 | 17.25<br>1.31 \| 18.56 | −3,650 | 0.5 | −7,300 |
| 4-4 | (0.5)6 = 3.00<br>(0.5)(5.5) = 2.75 | 5.75<br>2.75 | 17.25<br>7.56 \| 24.81 | −4,890 | 0.5 | −9,780 |

* $A_{obj}$ is the partial area of the cross section above a given level in square inches.
** $\bar{y}$ is the distance from the neutral axis to the centroid of the partial area in inches.

The negative signs of $\tau$ show that, for the section considered, the stresses act downward on the right face of the elements. The sense of the shearing stresses acting on the section coincides with the sense of the shearing force $V$. For this reason a strict adherence to the sign convention is often unnecessary. It is always true that $\int_A \tau\, dA$ is equal to $V$ and has the same sense.

* D. C. Drucker, "The Effect of Shear on the Plastic Bending of Beams," *Journal of Applied Mechanics*, 23 (1956), pp. 509–14.



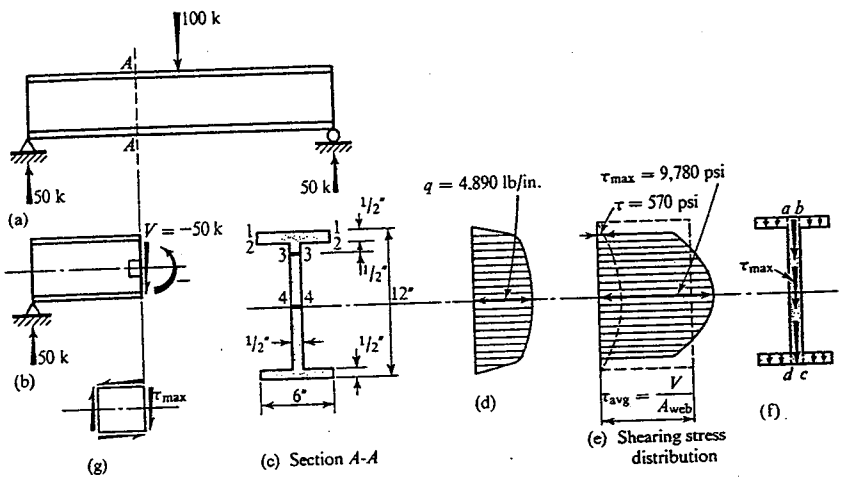



Fig. 7-14

Note that at the level 2-2 two widths are used to determine the shearing stress—one just above the line 2-2, and one just below. A width of 6 in. corresponds to the first case, and 0.5 in. to the second. This transition point will be discussed in the next article. The results obtained, which by virtue of symmetry are also applicable to the lower half of the section, are plotted in Fig. 7-14(d) and (e). By a method similar to the one used in the preceding example, it may be shown that the curves in Fig. 7-14(e) are parts of a second-degree parabola.

The variation of the shearing stress indicated by Fig. 7-14(e) may be interpreted as is shown in Fig. 7-14(f). The maximum shearing stress occurs at the neutral axis; the vertical shearing stresses throughout the web of the beam are nearly of the same magnitude. The shearing stresses occurring in the flanges are very small. For this reason the maximum shearing stress in an $I$ beam is often approximated by dividing the total shear $V$ by the cross-sectional area of the web (area *abcd* in Fig. 7-11(f)). Hence

$$(\tau_{max})_{approx} = V/A_{web} \tag{7-9}$$

In the example considered this gives

$$(\tau_{max})_{approx} = \frac{50{,}000}{(0.5)12} = 8{,}330 \text{ psi}$$

This stress differs by about 15 per cent from the one found by the accurate formula. For most cross sections a much closer approximation

תורת החוזק 1

תרגיל מס 9

**5.16 and 5.17** For the wide-flange beam and loading shown, determine in a section located halfway between points $D$ and $E$, ($a$) the largest normal stress, ($b$) the largest shearing stress.

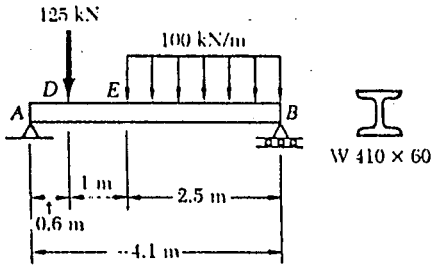



Fig. P5.16

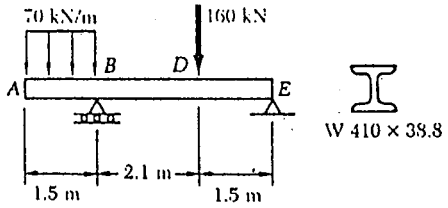



Fig. P5.17

**5.18** Two rectangular plates are welded to the 310-mm-wide-flange beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 90 MPa.

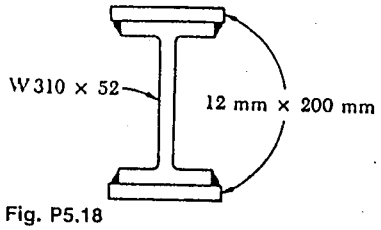
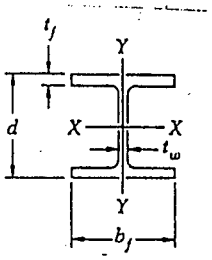



Fig. P5.18

| Designation† | Area $A$, mm² | Depth $d$, mm | Flange Width $b_f$, mm | Flange Thickness $t_f$, mm | Web Thickness $t_w$, mm | Axis X-X $I_x$ 10⁶ mm⁴ | Axis X-X $S_x$ 10³ mm³ | Axis X-X $r_x$ mm |
|---|---|---|---|---|---|---|---|---|
| W310 × 52 | 6650 | 317 | 167 | 13.2 | 7.6 | 118.6 | | |
| W410 × 114 | 14600 | 420 | 261 | 19.3 | 11.6 | 462 | 2200 | 177.8 |
| 85 | 10800 | 417 | 181 | 18.2 | 10.9 | 316 | 1516 | 170.7 |
| 60 | 7610 | 407 | 178 | 12.8 | 7.7 | 216 | 1061 | 168.4 |
| 46.1 | 5880 | 403 | 140 | 11.2 | 7.0 | 156.1 | 775 | 162.8 |
| 38.8 | 4950 | 399 | 140 | 8.8 | 6.4 | 125.3 | 628 | 159.0 |




**3. For a timber beam having the cross section shown, determine the dimension w if the maximum allowable vertical shear force is 7.67kN, and the shearing stress is not to exceed 1.MPa.**

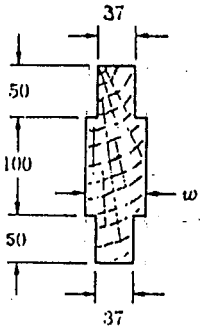
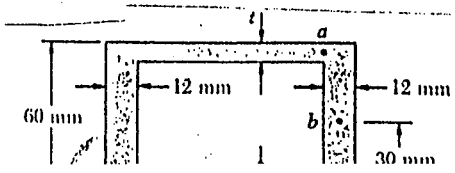
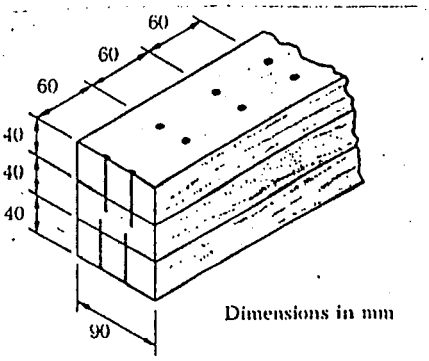



Fig. P5.20 Dimensions in mm

**5.1** Three boards, each of 40 mm × 90 mm rectangular cross section, are nailed together to form a beam which is subjected to a vertical shear of 1 kN. Knowing that the spacing between each pair of nails is 60 mm, determine the shearing force in each nail.

**5.32** An extruded beam has the cross section shown and is subjected to a vertical shear of 50 kN. For $t$ = 6 mm, determine the shearing stress at ($a$) point $a$, ($b$) point $b$.




Dimensions in mm

occur in the plastic zones, no unbalance in longitudinal forces occurs and no shearing stresses are developed.

This elementary solution has been refined by using a more carefully formulated criterion of yielding caused by the simultaneous action of normal and shearing stresses.* Some fundamental aspects of the interaction of such stresses will be considered in Chapter 9.

### EXAMPLE 7-6

An $I$ beam is loaded as in Fig. 7-14(a). If it has the cross section shown in Fig. 7-14(c), determine the shearing stresses at the levels indicated. Neglect the weight of the beam.

### SOLUTION

A free-body diagram of a segment of the beam is in Fig. 7-14(b). It is seen from this diagram that the vertical shear at every section is 50 kips. Bending moments do not enter directly into the present problem. The shear flow at the various levels of the beam is computed in the table below using Eq. 7-5. Since $\tau = q/t$ (Eq. 7-6), the shearing stresses are obtained by dividing the shear flows by the respective widths of the beam.

$$I = \frac{6(12)^3}{12} - \frac{(5.5)(11)^3}{12} = 254 \text{ in.}^4$$

For use in Eq. 7-5 the ratio $V/I = -50{,}000/254 = -197$ lb/in.$^4$

| Level | $A_{\text{part}}$* | $\bar{y}$** | $Q = A_{\text{part}}\bar{y}$ | $q = VQ/I$ | $t$ | $\tau$, psi |
|---|---|---|---|---|---|---|
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* $A_{\text{part}}$ is the partial area of the cross section above a given level in square inches.
** $\bar{y}$ is the distance from the neutral axis to the centroid of the partial area in inches.

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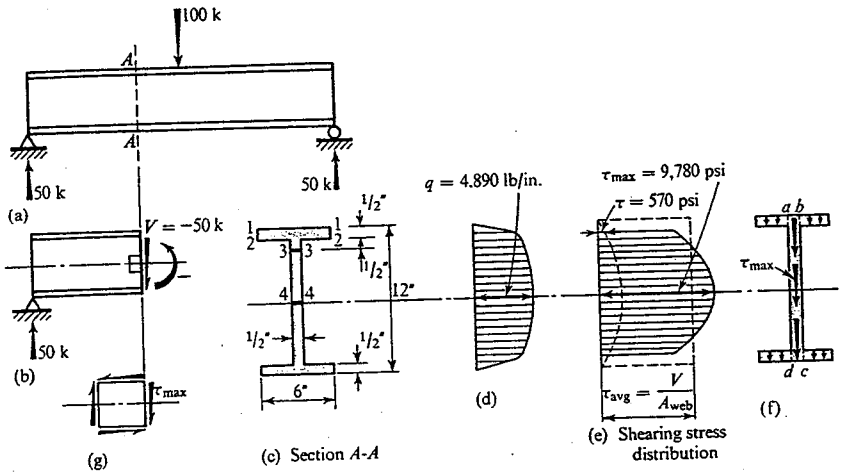



Fig. 7-14

Note that at the level 2-2 two widths are used to determine the shearing stress—one just above the line 2-2, and one just below. A width of 6 in. corresponds to the first case, and 0.5 in. to the second. This transition point will be discussed in the next article. The results obtained, which by virtue of symmetry are also applicable to the lower half of the section, are plotted in Fig. 7-14(d) and (e). By a method similar to the one used in the preceding example, it may be shown that the curves in Fig. 7-14(e) are parts of a second-degree parabola.

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In the example considered this gives

$$(\tau_{\max})_{\text{approx}} = \frac{50{,}000}{(0.5)12} = 8{,}330 \text{ psi}$$

This stress differs by about 15 per cent from the one found by the accurate formula. For most cross sections a much closer approximation

Stress in clevis of bar $AB$, Fig. 3-12(e):

$$(\sigma_{AB})_{\text{clevis}} = \frac{F_A}{A_{\text{net}}} = \frac{2.23}{2(0.20)(0.875 - 0.375)} = 11.2 \text{ ksi} \quad \text{(tension)}$$

Stress in main bar $BC$:

$$\sigma_{BC} = \frac{F_C}{A} = \frac{2.83}{(0.875)(0.25)} = 12.9 \text{ ksi} \quad \text{(compression)}$$

In the compression member the net section at the clevis need not be investigated; see Fig. 3-12(f) for the transfer of forces. The bearing stress at the pin is more critical. Bearing between pin $C$ and clevis:

$$\sigma_b = \frac{F_C}{A_{\text{bearing}}} = \frac{2.83}{(0.375)(0.20)2} = 18.8 \text{ ksi}$$

Bearing between the pin $C$ and the main plate:

$$\sigma_b = \frac{F_C}{A} = \frac{2.83}{(0.375)(0.25)} = 30.1 \text{ ksi}$$

Double shear in the pin $C$:

$$\tau = \frac{F_C}{A} = \frac{2.83}{2\pi(0.375/2)^2} = 12.9 \text{ ksi}^*$$

For a complete analysis of this bracket, other pins should be investigated. However, it may be seen by inspection that the other pins in this case are stressed the same amount as computed above, or less.

The advantages of the method used in the above example for finding forces in members should now be apparent. It can also be applied with success in a problem such as the one shown in Fig. 3-13. The force $F_A$ transmitted by the curved member $AB$ acts through points $A$ and $B$ since the forces applied at $A$ and $B$ must be collinear. By resolving this force at $A'$, the same procedure may be followed. Wavy lines through $F_A$ and $F_C$ indicate that these forces are replaced by the two components shown. Alternatively, the force $F_A$ may be resolved at $A$, and since $F_{Ay} = (x/y)F_{Ax}$, the application of $\Sigma M_C = 0$ yields $F_{Ax}$.

In frames where the applied forces do not act through a joint, proceed as above as far as possible. Then isolate an individual member, and using its free-body diagram, complete the determination of forces.

* Considering the pin in a two-dimensional state of stress $\tau_{xy} = \tau_{yx}$, the tensor representation of the results becomes $\begin{pmatrix} 0 & 12.9 \\ 12.9 & 0 \end{pmatrix}$ ksi



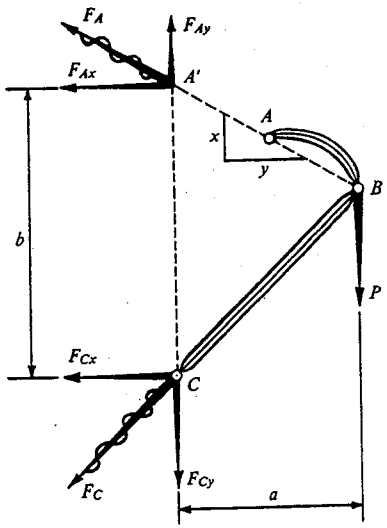



Fig. 3-13

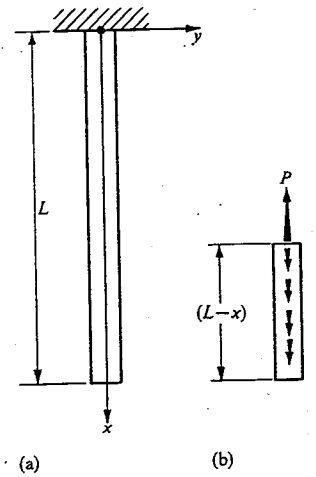



Fig. 3-14

If inclined forces are acting on the structure, resolve them into convenient components.

EXAMPLE 3-4

A 1-in.$^2$ rod $L$ in. long is suspended vertically as shown in Fig. 3-14(a). The unit weight of the material is $\gamma$. Determine the normal stress in this rod using differential equations of equilibrium.

SOLUTION

With the axes shown in the figure, $\tau_{xy} = 0$, and only the first part of Eq. 3-3 has relevance. The body force $X = \gamma$. By virtue of the boundary condition at the free end of the rod $\sigma_x(L) = 0$. On this basis, setting up a differential equation, intergating it, and determining the constant of integration from the boundary conditions, one has

$$\frac{d\sigma_x}{dx} + \gamma = 0 \quad \text{and} \quad \sigma_x = -\gamma x + C_1$$

$$\sigma_x(L) = -\gamma L + C_1 = 0 \quad \text{and} \quad \sigma_x = (L - x)\gamma$$

This result can be easily checked by cutting the rod $(L - x)$ above the free end, Fig. 3-14(b), and applying Eq. 3-5. Only very few problems can be analyzed using Eq. 3-3 alone. In more general problems deformations must be considered simultaneously in the analysis.

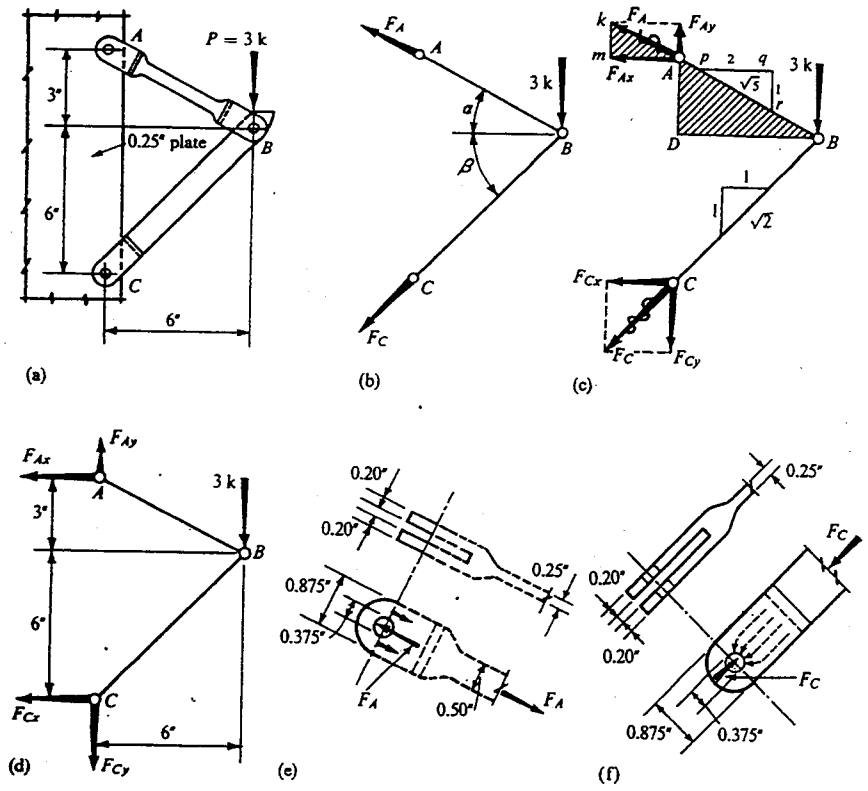



**Fig. 3-12**

**SOLUTION**

First an idealized free-body diagram consisting of the two bars pinned at the ends is prepared, Fig. 3-12(b). As there are no intermediate forces acting on the bars and the applied force acts through the joint at $B$, the forces in the bars are directed along the lines $AB$ and $BC$, and the bars $AB$ and $BC$ are loaded axially. The magnitudes of the forces are unknown and are labeled $F_A$ and $F_C$ in the diagram.* These forces may be determined graphically by completing a triangle of forces $F_A$, $F_B$, and $P$. These forces may also be found analytically from two simultaneous

* In frameworks it is convenient to assume all unknown forces are tensile. A negative answer in the solution then indicates that the bar is in compression.





equations $\Sigma F_y = 0$ and $\Sigma F_x = 0$, written in terms of the unknowns $F_A$ and $F_C$, a known force $P$, and two known angles $\alpha$ and $\beta$. Both these procedures are possible. However, in this course usually it will be found advantageous to proceed in a different way. Instead of treating forces $F_A$ and $F_C$ directly, their components are used; and instead of $\Sigma F = 0$, $\Sigma M = 0$ becomes the main tool.

Any force may be resolved into components. For example, $F_A$ may be resolved into $F_{Ax}$ and $F_{Ay}$ as in Fig. 3-12(c). Conversely, if any one of the components of a directed force is known, the force itself may be determined. This follows from similarity of dimension and force triangles. In Fig. 3-12(c) the triangles $Akm$ and $BAD$ are similar triangles (both are shaded in the diagram). Hence, if $F_{Ax}$ is known,

$$F_A = (AB/DB)F_{Ax}$$

Similarly, $F_{Ay} = (AD/DB)F_{Ax}$. However, note further that $AB/DB$ or $AD/DB$ are ratios, hence relative dimensions of members may be used. Such relative dimensions are shown by a little triangle on the member $AB$ and again on $BC$. In the problem at hand

$$F_A = (\sqrt{5}/2)F_{Ax} \quad \text{and} \quad F_{Ay} = F_{Ax}/2$$

Adopting the above procedure of resolving forces, the revised free-body diagram, Fig. 3-12(d), is prepared. Two components of force are necessary at the pin joints. After the forces are determined by statics, Eq. 3-5 is applied several times, thinking in terms of a free body of an individual member:

$$\sum M_C = 0 \circlearrowleft +, \quad +F_{Ax}(3 + 6) - 3(6) = 0, \quad F_{Ax} = +2 \text{ kips}$$

$$F_{Ay} = F_{Ax}/2 = 2/2 = 1 \text{ kip},$$

$$F_A = 2(\sqrt{5}/2) = +2.23 \text{ kips}$$

$$\sum M_A = 0 \circlearrowright +, \quad +3(6) + F_{Cx}(9) = 0,$$

$$F_{Cx} = -2 \text{ kips} \quad \text{(compression)}$$

$$F_{Cy} = F_{Cx} = -2 \text{ kips}$$

$$F_C = \sqrt{2}(-2) = -2.83 \text{ kips}$$

*Check:* $\sum F_x = 0, \quad F_{Ax} + F_{Cx} = 2 - 2 = 0$

$$\sum F_y = 0, \quad F_{Ay} - F_{Cy} - P = 1 - (-2) - 3 = 0$$

Stress in main bar $AB$:

$$\sigma_{AB} = \frac{F_A}{A} = \frac{2.23}{(0.25)(0.50)} = 17.8 \text{ ksi} \quad \text{(tension)}$$



possible.

**EXAMPLE 3-3**

A bracket of negligible weight shown in Fig. 3-12(a) is loaded with a force $P$ of 3 kips. For interconnection purposes the bar ends are clevised (forked). Pertinent dimensions are shown in the figure. Find the normal stresses in the members $AB$ and $BC$ and the bearing and shearing stresses for the pin $C$. All pins are 0.375 in. in diameter.

Stress in clevis of bar $AB$, Fig. 3-12(e):

$$(\sigma_{AB})_{\text{clevis}} = \frac{F_A}{A_{\text{net}}} = \frac{2.23}{2(0.20)(0.875 - 0.375)} = 11.2 \text{ ksi} \quad \text{(tension)}$$

Stress in main bar $BC$:

$$\sigma_{BC} = \frac{F_C}{A} = \frac{2.83}{(0.875)(0.25)} = 12.9 \text{ ksi} \quad \text{(compression)}$$

In the compression member the net section at the clevis need not be investigated; see Fig. 3-12(f) for the transfer of forces. The bearing stress at the pin is more critical. Bearing between pin $C$ and clevis:

$$\sigma_b = \frac{F_C}{A_{\text{bearing}}} = \frac{2.83}{(0.375)(0.20)2} = 18.8 \text{ ksi}$$

Bearing between the pin $C$ and the main plate:

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Double shear in the pin $C$:

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For a complete analysis of this bracket, other pins should be investigated. However, it may be seen by inspection that the other pins in this case are stressed the same amount as computed above, or less.

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In frames where the applied forces do not act through a joint, proceed as above as far as possible. Then isolate an individual member, and using its free-body diagram, complete the determination of forces.

* Considering the pin in a two-dimensional state of stress $\tau_{xy} = \tau_{yx}$, the tensor representation of the results becomes $\begin{pmatrix} 0 & 12.9 \\ 12.9 & 0 \end{pmatrix}$ ksi



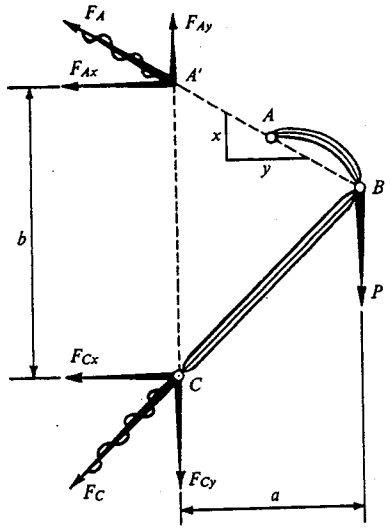



Fig. 3-13

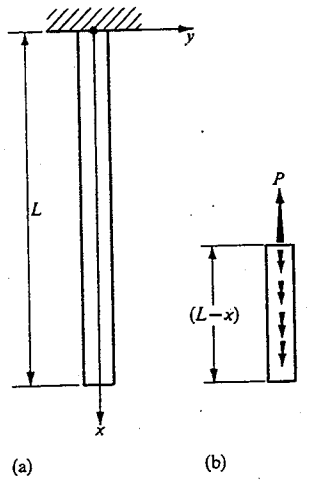



Fig. 3-14

If inclined forces are acting on the structure, resolve them into convenient components.

EXAMPLE 3-4

A 1-in.$^2$ rod $L$ in. long is suspended vertically as shown in Fig. 3-14(a). The unit weight of the material is $\gamma$. Determine the normal stress in this rod using differential equations of equilibrium.

SOLUTION

With the axes shown in the figure, $\tau_{xy} = 0$, and only the first part of Eq. 3-3 has relevance. The body force $X = \gamma$. By virtue of the boundary condition at the free end of the rod $\sigma_x(L) = 0$. On this basis, setting up a differential equation, intergating it, and determining the constant of integration from the boundary conditions, one has

$$\frac{d\sigma_x}{dx} + \gamma = 0 \quad \text{and} \quad \sigma_x = -\gamma x + C_1$$

$$\sigma_x(L) = -\gamma L + C_1 = 0 \quad \text{and} \quad \sigma_x = (L - x)\gamma$$

This result can be easily checked by cutting the rod $(L - x)$ above the free end, Fig. 3-14(b), and applying Eq. 3-5. Only very few problems can be analyzed using Eq. 3-3 alone. In more general problems deformations must be considered simultaneously in the analysis.

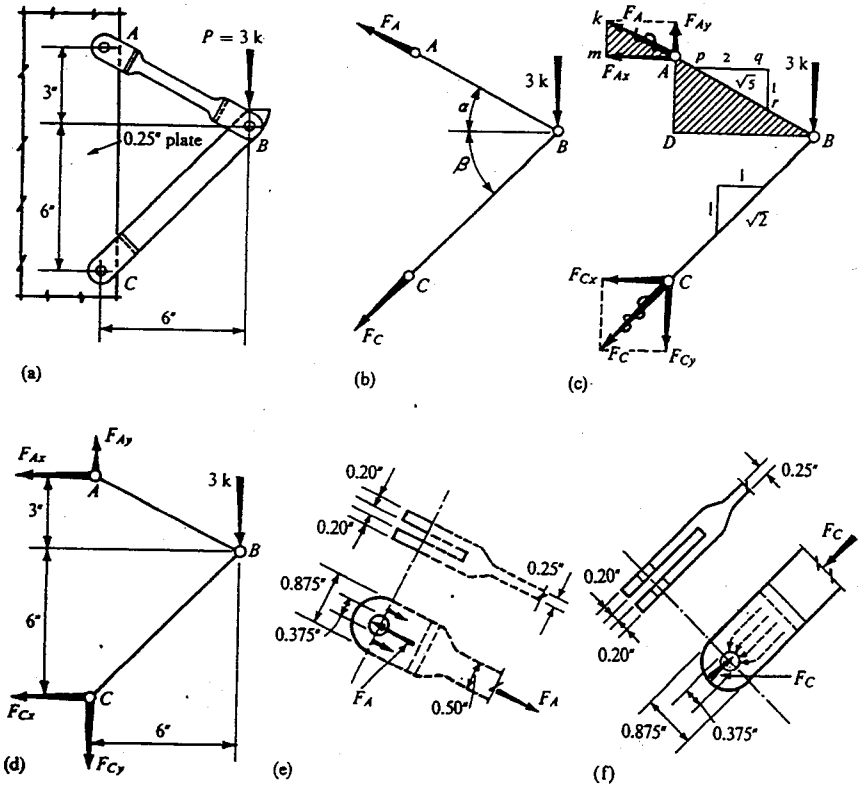



**Fig. 3-12**

SOLUTION

First an idealized free-body diagram consisting of the two bars pinned at the ends is prepared, Fig. 3-12(b). As there are no intermediate forces acting on the bars and the applied force acts through the joint at $B$, the forces in the bars are directed along the lines $AB$ and $BC$, and the bars $AB$ and $BC$ are loaded axially. The magnitudes of the forces are unknown and are labeled $F_A$ and $F_C$ in the diagram.* These forces may be determined graphically by completing a triangle of forces $F_A$, $F_B$, and $P$. These forces may also be found analytically from two simultaneous

* In frameworks it is convenient to assume all unknown forces are tensile. A negative answer in the solution then indicates that the bar is in compression.



*Section 3-7*
*Average shearing stress*

equations $\Sigma F_y = 0$ and $\Sigma F_x = 0$, written in terms of the unknowns $F_A$ and $F_C$, a known force $P$, and two known angles $\alpha$ and $\beta$. Both these procedures are possible. However, in this course usually it will be found advantageous to proceed in a different way. Instead of treating forces $F_A$ and $F_C$ directly, their components are used; and instead of $\Sigma F = 0$, $\Sigma M = 0$ becomes the main tool.

Any force may be resolved into components. For example, $F_A$ may be resolved into $F_{Ax}$ and $F_{Ay}$ as in Fig. 3-12(c). Conversely, if any one of the components of a directed force is known, the force itself may be determined. This follows from similarity of dimension and force triangles. In Fig. 3-12(c) the triangles $Akm$ and $BAD$ are similar triangles (both are shaded in the diagram). Hence, if $F_{Ax}$ is known,

$$F_A = (AB/DB)F_{Ax}$$

Similarly, $F_{Ay} = (AD/DB)F_{Ax}$. However, note further that $AB/DB$ or $AD/DB$ are ratios, hence relative dimensions of members may be used. Such relative dimensions are shown by a little triangle on the member $AB$ and again on $BC$. In the problem at hand

$$F_A = (\sqrt{5}/2)F_{Ax} \quad \text{and} \quad F_{Ay} = F_{Ax}/2$$

Adopting the above procedure of resolving forces, the revised free-body diagram, Fig. 3-12(d), is prepared. Two components of force are necessary at the pin joints. After the forces are determined by statics, Eq. 3-5 is applied several times, thinking in terms of a free body of an individual member:

$$\sum M_C = 0 \circlearrowleft +, \quad +F_{Ax}(3 + 6) - 3(6) = 0, \quad F_{Ax} = +2 \text{ kips}$$

$$F_{Ay} = F_{Ax}/2 = 2/2 = 1 \text{ kip},$$

$$F_A = 2(\sqrt{5}/2) = +2.23 \text{ kips}$$

$$\sum M_A = 0 \circlearrowleft +, \quad +3(6) + F_{Cx}(9) = 0,$$

$$F_{Cx} = -2 \text{ kips} \quad \text{(compression)}$$

$$F_{Cy} = F_{Cx} = -2 \text{ kips}$$

$$F_C = \sqrt{2}(-2) = -2.83 \text{ kips}$$

*Check:* $\sum F_x = 0, \quad F_{Ax} + F_{Cx} = 2 - 2 = 0$

$$\sum F_y = 0, \quad F_{Ay} - F_{Cy} - P = 1 - (-2) - 3 = 0$$

Stress in main bar $AB$:

$$\sigma_{AB} = \frac{F_A}{A} = \frac{2.23}{(0.25)(0.50)} = 17.8 \text{ ksi} \quad \text{(tension)}$$



possible.

EXAMPLE 3-3

A bracket of negligible weight shown in Fig. 3-12(a) is loaded with a force $P$ of 3 kips. For interconnection purposes the bar ends are clevised (forked). Pertinent dimensions are shown in the figure. Find the normal stresses in the members $AB$ and $BC$ and the bearing and shearing stresses for the pin $C$. All pins are 0.375 in. in diameter.

**2.9** The 36-mm-diameter steel rod $ABC$ and a brass rod $CD$ of the same diameter are joined at point $C$ to form the 7.5-m rod $ABCD$. For the loading shown, and neglecting the weight of the rod, determine the deflection ($a$) of point $C$, ($b$) of point $D$.

Answer: $\delta_C = 2.95\ mm\downarrow$, $\delta_D = 5.29\ mm\downarrow$.

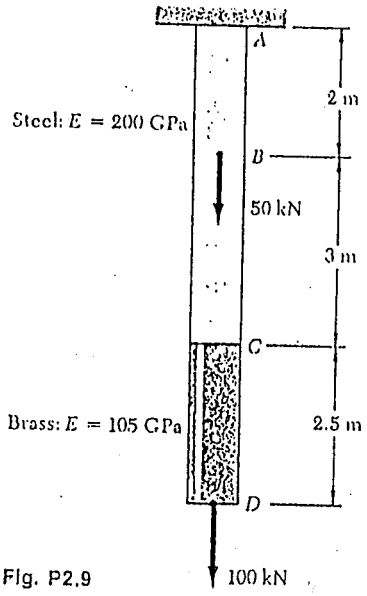


Fig. P2.9

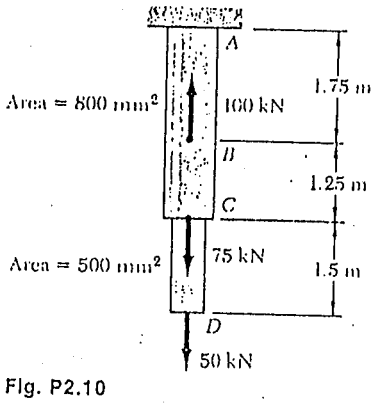


Fig. P2.10

**2.10** The rod $ABCD$ is made of an aluminum alloy for which $E = 70$ GPa. For the loading shown, and neglecting the weight of the rod, determine the deflection ($a$) of point $B$, ($b$) of point $D$.

Answer: $\delta_B = 0.781\ mm\downarrow$, $\delta_D = 5.71\ mm\downarrow$.

**2.26** An axial centric force of magnitude $P = 385$ kN is applied to the composite block shown by means of a rigid end plate. Determine the normal stress in ($a$) the steel core, ($b$) the aluminum plates.

Answer: $\sigma_S = -175\ MPa$, $\sigma_{Al} = -61.3\ MPa$.

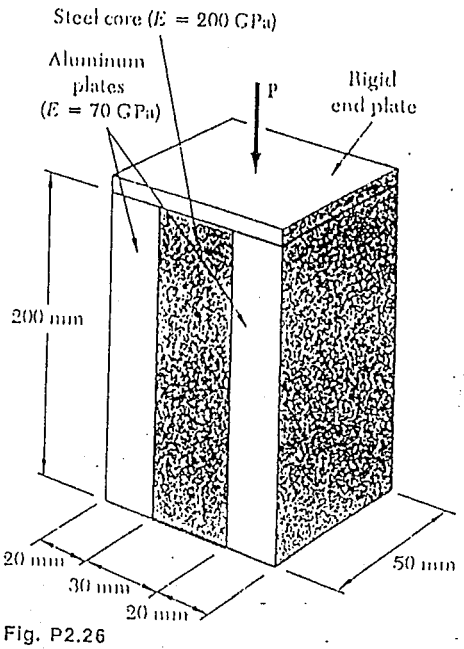


Fig. P2.26

**2.37** The rigid rod $ABC$ is suspended by three identical wires. Knowing that $x = \frac{3}{4}L$, determine the tension in each wire due to the force P.

Answer: $T_B = P/3$, $T_C = P/6$, $T_A = P/2$.

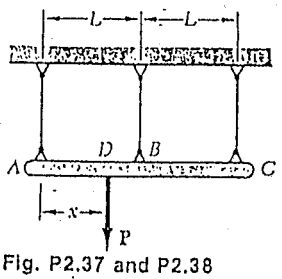


Fig. P2.37 and P2.38

**2.39** The rigid bar *ABCD* is suspended from four identical wires. Determine the tension in each wire caused by the load P shown.

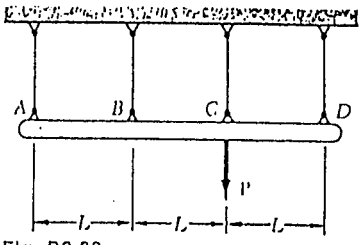
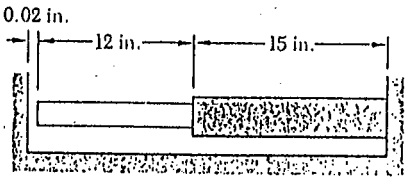


Fig. P2.39

Answer: $T_B = P/5, \quad T_D = 2P/5, \quad T_A = P/10, \quad T_C = 3P/10.$

**2.45** Determine (*a*) the compressive force in the bars shown after a temperature rise of 200°F, (*b*) the corresponding change in length of the aluminum bar.

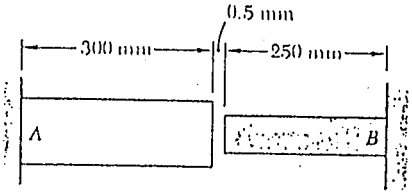
Answer: $P = -52\ kips, \quad \delta_{Al} = 0.0124\ in.$




| Bronze | Aluminum |
|---|---|
| $A = 2.5\ in^2$ | $A = 3\ in^2$ |
| $E = 15 \times 10^6$ psi | $E = 10 \times 10^6$ psi |
| $\alpha = 10.1 \times 10^{-6}/°F$ | $\alpha = 12.8 \times 10^{-6}/°F$ |

Fig. P2.45

**2.46** At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature reaches 140°C, determine (*a*) the normal stress in the aluminum, (*b*) the exact length of the aluminum rod.




| Aluminum | Stainless steel |
|---|---|
| $A = 2000\ mm^2$ | $A = 800\ mm^2$ |
| $E = 70$ GPa | $E = 190$ GPa |
| $\alpha = 23 \times 10^{-6}/°C$ | $\alpha = 18 \times 10^{-6}/°C$ |

Fig. P2.46 and P2.47

Answer: $\sigma_{Al} = -114.6\ MPa, \quad L = 300.34\ mm.$

**2.47** Knowing that a 0.5-mm gap exists between the rods shown when the temperature is 20°C, determine (*a*) the temperature at which the normal stress in the stainless steel rod will be $\sigma = -150$ MPa, (*b*) the corresponding exact length of the stainless steel rod.

Answer: $T = 103.7°C, \quad L = 250.18\ mm.$

2/2

Florida International University

Department of Mechanical and Materials Engineering

EMA 3702 QUIZ 1B 5/15/03

You are allowed one sheet of 8 ½ x 11 inch paper with whatever you wish on the sheet

Print your name and sign the following statement:

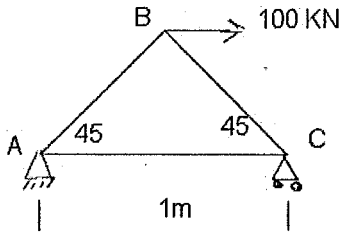
I will not give nor take any unpermitted aid during this quiz. I understand that violation of this statement will lead to automatic failure of the quiz.

_____ _____

PRINT NAME SIGN NAME

Problem 1.

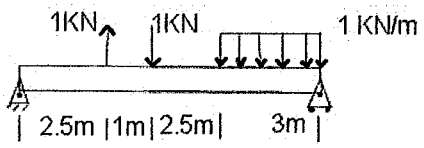
a) For the truss shown, find the maximum shear stress in pin C, assuming double shear conditions. Assume the pin has a diameter of 20 cm.
b) Find the maximum stress (normal or shear) in AB assuming that A and B are double and single shear type pins, respectively, of diameter 20 cm



Problem 2.

a) Given the following beam loaded as shown, find the location of the maximum moment.

b) What is the shear stress due to the shear load, 4 m from the left end, if the beam cross sectional area is 100 $cm^2$.



Florida International University
Department of Mechanical and Materials Engineering

EMA 3702 QUIZ 1A 5/15/03

You are allowed one sheet of 8 ½ x 11 inch paper with whatever you wish on the sheet

Print your name and sign the following statement:

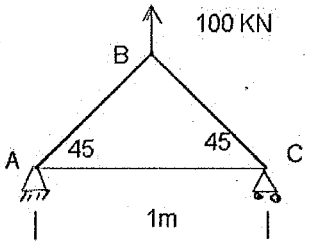
I will not give nor take any unpermitted aid during this quiz. I understand that violation of this statement will lead to automatic failure of the quiz.

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PRINT NAME SIGN NAME

Problem 1.

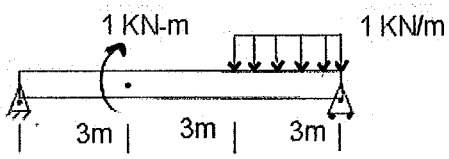
a) For the truss shown, find the maximum shear stress in pin C, assuming double shear conditions. Assume the pin has a diameter of 20 cm.
b) Find the maximum stress (normal or shear) in AB assuming that A and B are double and single shear type pins, respectively, of diameter 20 cm

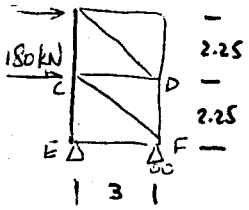


Problem 2.

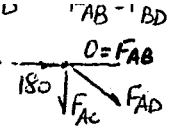
a) Given the following beam loaded as shown, find the location of the maximum moment.

b) What is the shear stress 4 m from the left end, if the beam cross sectional area is 100 $cm^2$.





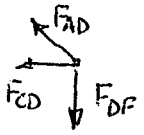

Using method of joints @ A




$180 + F_{AD}\cdot\frac{3}{\sqrt{2.25^2+3^2}} = 0 \Rightarrow F_{AD} = -\frac{180}{.8} = -225\,kN$

$F_{AD}\cdot\frac{2.25}{\sqrt{2.25^2+3^2}} + F_{AC} = 0 \qquad F_{AC} = 135\,kN$

@ D



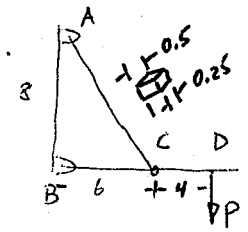

$F_{AD}\cdot\frac{2.25}{\sqrt{2.25^2+3^2}} - F_{DF} = 0 \qquad F_{DF} = -135\,kN$

$F_{AD}\cdot\frac{3}{\sqrt{2.25^2+3^2}} + F_{CD} = 0$

$\sigma = \frac{F_{DF}}{A} = \frac{-135\times10^3\,N}{2.5\times10^{-3}\,m^2} = -54\times10^6\,\frac{N}{m^2} = -54\,MPa$

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1.68

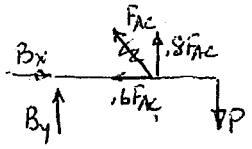



$\Sigma M_B = 0 \quad 6(.8F_{AC}) - 10P = 0 \quad P = .48\,F_{AC} \quad F_{AC} = 2.083P$

$\Sigma F_x = 0 \quad B_x - .6F_{AC} = 0 \quad B_x = .6F_{AC} = 1.25P$

$\Sigma F_y = 0 \quad B_y + .8F_{AC} - P = 0 \quad B_y = -2/3P$

$R_B = \sqrt{B_x^2 + B_y^2} = 1.417P$




- using Link AC $\sigma_{AC} = \frac{F_{AC}}{A} = \frac{2.083P}{\frac{1}{2}\cdot\frac{1}{4}} = 60ksi \Rightarrow P = 3.6\,kip$
- using net area at Pin C $\sigma_{net} = \frac{F_{AC}}{\frac{1}{4}\cdot(\frac{1}{2}-\frac{3}{8})} = 60ksi \Rightarrow P = .9\,kip$
- using shear at pin C (single shear) $\tau\cdot A = F_{AC}$

  $25ksi\cdot\frac{\pi}{4}d^2 = 25ksi\cdot\frac{\pi}{4}\left(\frac{3}{8}\right)^2 = 2.761\,kip$

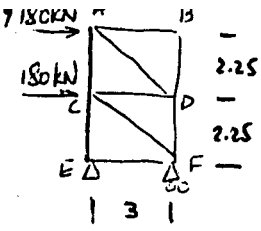
  $P = F_{AC}/2.083 = 1.325\,kip$
- using pin at B in shear (single shear) $\tau\cdot A = R_B \qquad A = \pi/4\,d^2$

  $25ksi\cdot\pi/4\,(5/16)^2 = R_B = 1.9175\,kip$

  $P = R_B/1.417 = 1.3535\,kip$

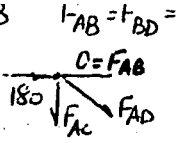
lowest $P = 0.9\,kip$ at net section of C

Now ALLOWABLE $P = \frac{0.9\,kip}{S.F} = \frac{0.9\,kip}{3.25} = .277\,kip$ or $277\,lb$




Using method of joints @ B $F_{AB} = F_{BD} = 0$

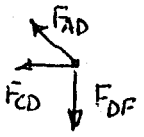
Using method of joints @ A




$180 + F_{AD}\cdot\frac{3}{\sqrt{2.25^2+3^2}} = 0 \Rightarrow F_{AD} = -\frac{180}{.8} = -225\,kN$

$F_{AD}\cdot\frac{2.25}{\sqrt{2.25^2+3^2}} + F_{AC} = 0 \qquad F_{AC} = 135\,kN$

@ D



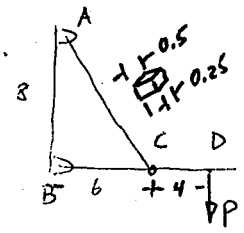

$F_{AD}\cdot\frac{2.25}{\sqrt{2.25^2+3^2}} - F_{DF} = 0 \qquad F_{DF} = -135\,kN$

$F_{AD}\cdot\frac{3}{\sqrt{2.25^2+3^2}} + F_{CD} = 0$

$\sigma = \frac{F_{DF}}{A} = \frac{-135\times10^3\,N}{2.5\times10^{-3}\,m^2} = -54\times10^6\,\frac{N}{m^2} = -54\,MPa$

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1.68

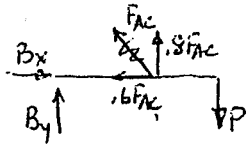



$\Sigma M_B = 0 \quad 6(.8F_{AC}) - 10P = 0 \quad P = .48\,F_{AC} \quad F_{AC} = 2.083P$

$\Sigma F_x = 0 \quad B_x - .6F_{AC} = 0 \quad B_x = .6F_{AC} = 1.25P$

$\Sigma F_y = 0 \quad B_y + .8F_{AC} - P = 0 \quad B_y = -\frac{2}{3}P$

$R_B = \sqrt{B_x^2 + B_y^2} = 1.417P$




- using Link AC $\sigma_{AC} = \frac{F_{AC}}{A} = \frac{2.083P}{\frac{1}{2}\cdot\frac{1}{4}} = 60ksi \Rightarrow P = 3.6\,kip$

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$25ksi\cdot\frac{\pi}{4}d^2 = 25ksi\cdot\frac{\pi}{4}\left(\frac{3}{8}\right)^2 = 2.761\,kip$

$P = F_{AC}/2.083 = 1.325\,kip$

- using pin at B in shear (single shear) $\tau\cdot A = R_B \qquad A = \pi/4\,d^2$

$25ksi\cdot\pi/4\,(5/16)^2 = R_B = 1.9175\,kip$

$P = R_B/1.417 = 1.3535\,kip$

lowest $P = 0.9\,kip$ at net section of C

Now ALLOWABLE $P = \frac{0.9\,kip}{S.F} = \frac{0.9\,kip}{3.25} = .277\,kip$ or $277\,lb$

References

EGOR P. POPOV - Introduction to Mechanics of Solids

E.P. POPOV - Mechanics of Materials

BYARS & SNYDER - Engineering Mechanics of Deformable Bodies

TIMOSHENKO & YOUNG - Elements of Strength of Materials

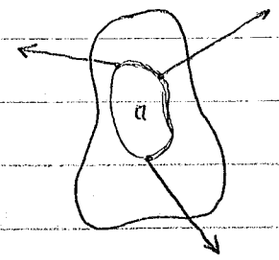
SINGER - Strength of Materials

GRINTER - Engineering Mechanics

BEER & JOHNSTON - AE 100 Book

SEELY & SMITH - Advanced Mechanics of Materials.

H.W. Read Pages 1-53 Prob 21, 2, 3, 5, 7, 9, 11




BASIC CONCEPTS - STATICS

Full Knowledge (Practical) must be had in:

a) Resolution & Vectoral Manipulation of Forces

b) Equilibrium Problems, equilibrium conditions

c) Free-Body diagram.

Look at structure through strength, stiffness, stability

Strength - ability to withstand load without breaking

stiffness - the ability to withstand change in dimensions (elasticity)

stability - depends on structure & dimensions of body
support given load w/o experiencing sudden change in its configuration

We will use Method of Sections - If body in equilibrium, any portion of body in equilibrium.

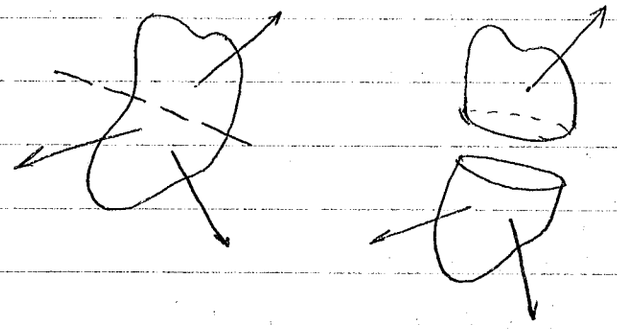
## Method of Sections

Approach No.

1. Free-Body diagram showing all forces - body must be in

equilibrium.

2. Pass arbitrary plane through body separating two pieces.



MAIN Problem: Approach

1) Isolate Member to be analyzed or designed.

2) Find reactions by

a) Statics $\Sigma F_x = \Sigma F_y = \Sigma F_z = \Sigma M_x = \Sigma M_y = \Sigma M_z = 0$

b) Differential Equations & Boundary Conditions.

c) If Indeterminate, By Kinematic Considerations – displacements are defined, restraints put on member.

3) Apply Method of Sections

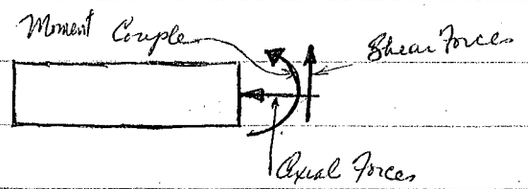
4) At Section find internal forces (INTERNAL REACTIONS)

5) Use appropriate formulas to find stresses

6) In Design, select appropriate material & member.

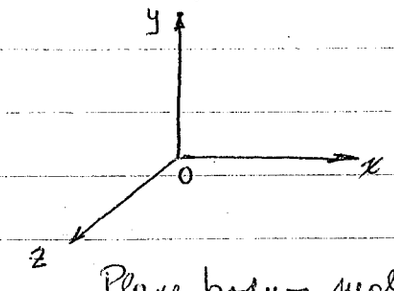
7) Predict deformations of the body

Internal Forces



Equilibrium

Solid body - $\Sigma F_x = \Sigma F_y = \Sigma F_z = \Sigma M_x = \Sigma M_y = \Sigma M_z = 0$




Plane body - neglect all distances & dimensions in Z direction

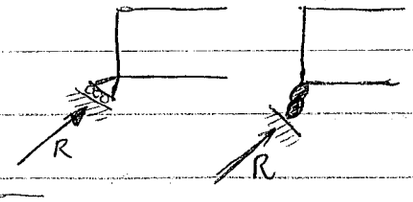
Forces in plane of body.

$\Sigma F_x = \Sigma F_y = \Sigma M_z = 0$

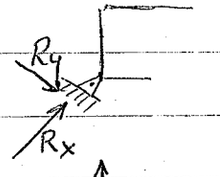
Dimensions will be taken as if before deformation

Supports

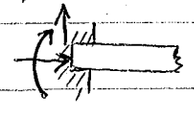
a) Roller or Link

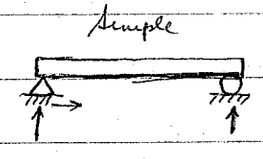


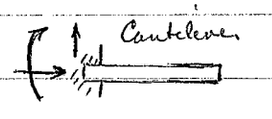

b) Pin

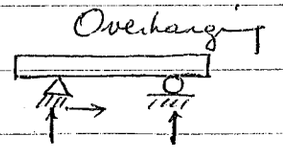


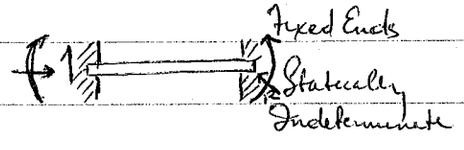

c) Fixed

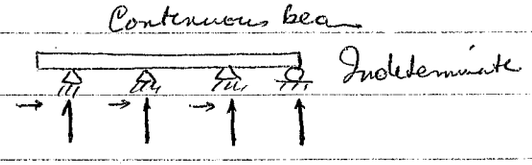


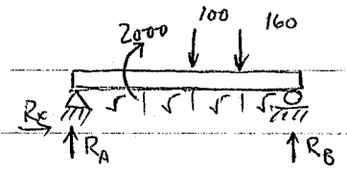


① 


$R_x = 0$

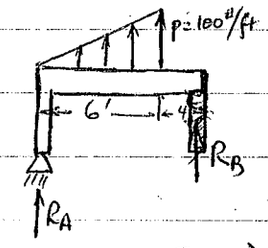
$\sum F_y = R_A + R_B - 260 = 0$

$R_A + R_B = 260$

$\sum H_B = 0$ $\quad R_A \times 20 + 2000 - 100 \times 10 - 160 \times 5 = 0$

$R_A \times 20 + 100 - 50 - 40 = 0 \quad R_A = -10^{\#} = 10^{\#} \downarrow$

$R_B = 270^{\#} \uparrow$

② 

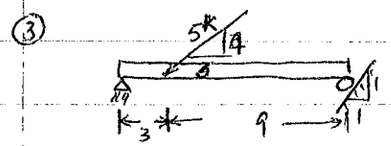

$\sum F_y = 0$

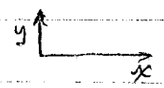
$\sum F_y = R_A + R_B + \frac{1}{2} \times 6 \times 100 = 0$

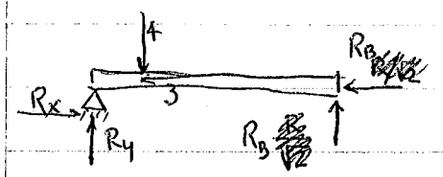
$R_A + R_B = -300$

$\sum H_B = 0 \quad R_A \times 10 + 300 \times 6 = 0; \; R_A = -180^{\#} \quad 180^{\#} \downarrow$

$R_B = 120^{\#} \downarrow$

③ 


$R_x - 3^K - R_B = 0$

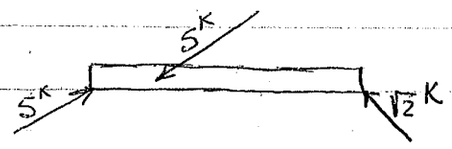
$\boxed{R_x - R_B = 3^K}$

$R_y - 4 + R_B = 0$

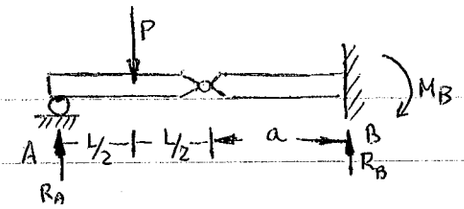
$\boxed{R_y + R_B = 4^K}$

$\sum H_B = 0 \quad R_y(12) - 4(9) = 0 \quad R_y = 3^K \quad R_B = 1^K$

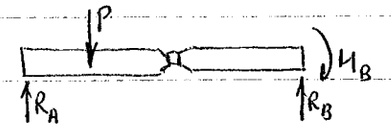
$R_x = 4^K$



④



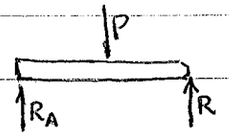

Find $R_A, R_B, M_B$




$\sum F_y = R_A + R_B - P = 0$

$\boxed{R_A + R_B = P}$

$\sum M_B = R_A(L+a) - P(L/2+a) + M_B = 0$ $\quad \boxed{R_A(L+a) + M_B = P(L/2+a)}$




$R_A - P + R = 0$ $\quad \boxed{R = P/2}$

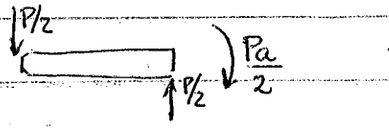
$\sum M_C = 0$ $\quad R_A(L) - PL/2 = 0$ $\quad \boxed{R_A = P/2}$

$\boxed{R_B = P/2}$

$\frac{P}{2}(L+a) + M_B = \frac{P}{2}(L+2a)$

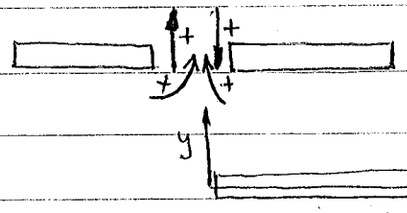
$M_B = P/2\,(L + 2a - L - a) = \frac{Pa}{2}$

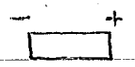
$\boxed{M_B = \frac{Pa}{2}}$




$\sum F_y = 0$ ✓ $\quad \sum M_B = 0$ ✓

Sign Convention for problems

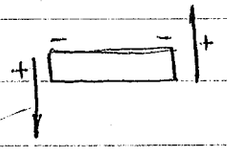


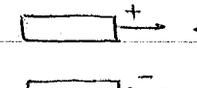

relative to the other end

negative face  positive face

depends on direction & face.

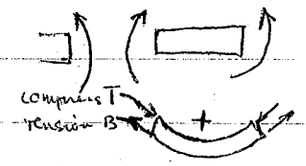


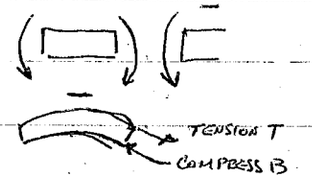

Shear

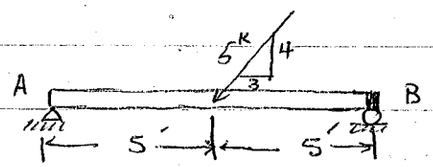








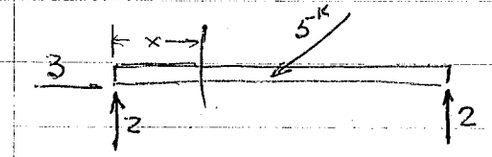



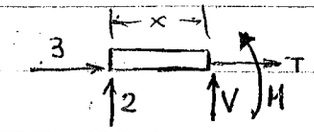

① 


$\boxed{R_{Ax} = 3^K \rightarrow}$

$R_{Ay} - 4 + R_B = 0$

$\circlearrowright \Sigma M_A = 4(5) - 10R_B = 0$ $\quad \boxed{R_B = 2^K \uparrow}$ $\quad \boxed{R_{Ay} = 2^K \uparrow}$



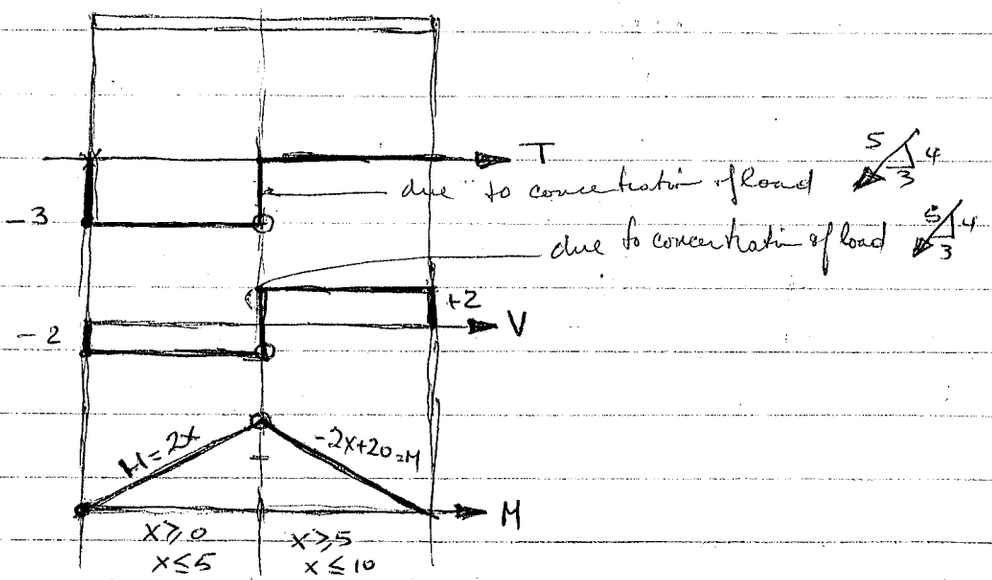




$0 \leq x < 5'$

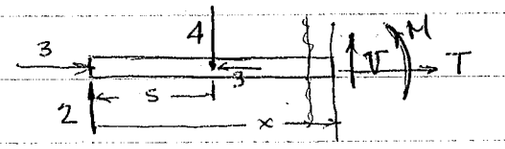
$T = 3^K \leftarrow$ $\quad V = 2^K \downarrow$

Always take M about cut

$2x - M = 0$ $\quad M = 2x$



$5 \leq x \leq 10$



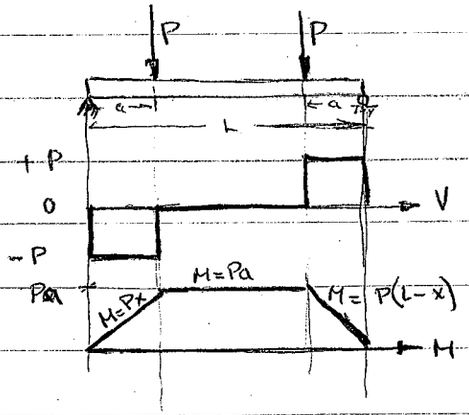

$\Sigma F_x = 0$ $\quad 3 - 3 + T = 0$ $\quad T = 0$

$\Sigma F_y = 0$ $\quad 2 - 4 + V = 0$ $\quad V = +2$ kips

$\Sigma M_C \circlearrowright = 2x - 4(x-5) - M = 0$ $\quad -2x + 20 = M$

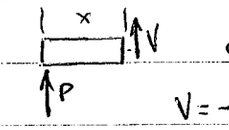
Shear curve displaces the curve an equal amount in the opposite direction to the direction of the concentrated load

Axial curve is displaced an equal amount in the same direction as the concentrated load.


P
P
a
L
a
+P
0
−P
V
M=Pa
M=P(L−x)
M=Px
M
R_A + R_B = 2P ; ΣF_H = 0

$\Sigma M_B = 0 \circlearrowright$ ; $R_A(L) - P(L-a) - P(a) = 0$

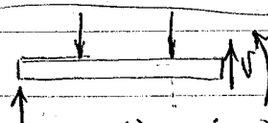
$R_A(L) - PL = 0$ $\quad \boxed{R_A = P}$ $\quad \boxed{R_B = P}$




$0 \leq x \leq a$

$V = -P$

$M = P(x)$ $\quad x \geq 0$ $\quad x \leq a$



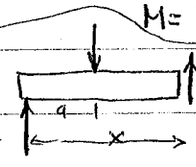

$V = P$

$P(x) - P(x-a) - P(x-(L-a)) - M = 0$

$P(x) - P(x) + P(a) - Px + P(L) - Pa - M = 0$

$\boxed{M = P(L) - P(x)}$ $\quad x \geq L-a$ $\quad x \leq L$

when $x = (L-a)$



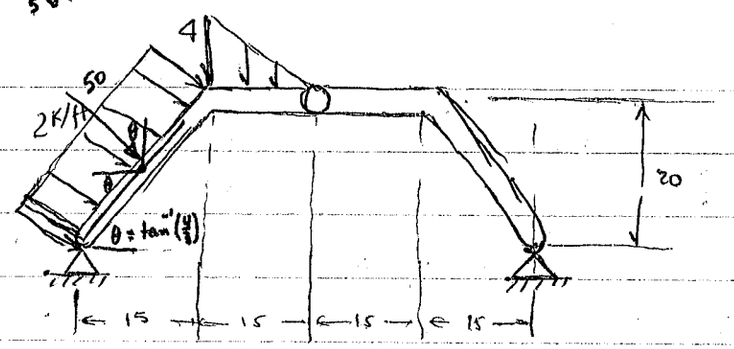

$a \leq x \leq L-a$

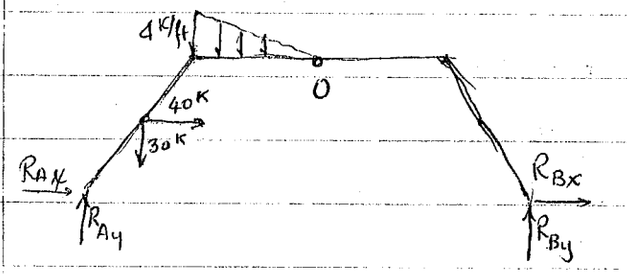
$V = 0$

$P(x) - P(x-a) - M = 0$

$+Pa = M$ $\quad x \geq a$ $\quad x \leq L-a$

#2-11




$\Sigma F_x = 0$

$R_{Ax} + R_{Bx} + 40 = 0$

$R_{Ax} + R_{Bx} = -40$

$\Sigma F_y = 0$

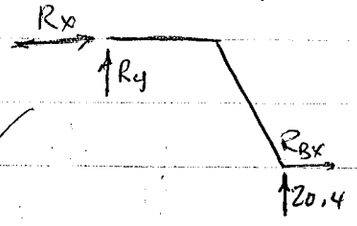
$R_{Ay} - 30 - \frac{1}{2} \times 15 \times 4 + R_{By} = 0$

$R_{Ay} + R_{By} = 60^K$

$\Sigma M_A = 0 \circlearrowright$ $\quad 50 \times 12.5 + \frac{1}{2} \times 15 \times 4 \times 20 - R_{By} \times 60 = 0$

$R_{By} = \frac{5}{6} \times 12.5 + 10 = 20.4^K$

$R_{Ay} = 39.6^K$




$\Sigma M_O \circlearrowright = 0$ $\quad -R_{Ax} \times 20 + 39.6 \times 30 - 30 \times 22.5 - 40 \times 10 - \frac{1}{2} \times 4 \times 15 \times 10 - R_{Bx} \times 20 + 20.4 \times 30 = 0$

$R_{Ax} + R_{Bx} = (39.6 - 20.4) \times \frac{30}{20} - \frac{30}{20} \times 22.5 - 35$

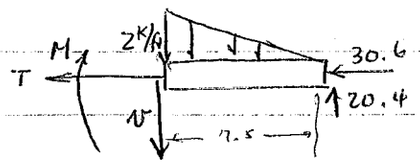
$(-3.2) \times \frac{30}{20} - 35$

$-35 - 5 = -40^K$

$-20.4(30) - R_{Bx}(20) = 0$ $\quad R_{Bx} = +30.6 \leftarrow$

$R_{Ax} = +9.4 \leftarrow$

Since we know everything about right member we can isolate the smaller member between pin & cut therefore



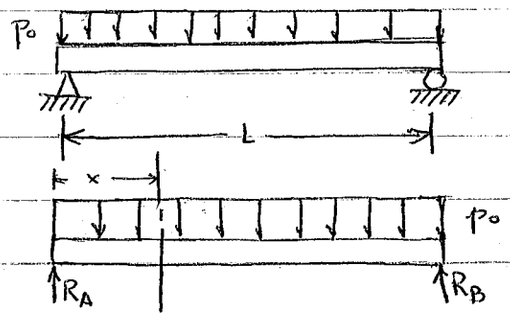

$\Sigma F_x = 0 \quad -T - 30.6 = 0 \quad T = -30.6^K$

$\Sigma F_y = 0 \quad -V + 20.4 - \frac{1}{2}(2)(7.5) = 0 \quad V = 12.9^K$

$\Sigma M_{\text{atcut}} = 0 \quad M + \frac{1}{2}(2)(7.5)(2.5) - 20.4(7.5) = 0$

$M = (17.9)(7.5) \cong 135\ K\text{-ft}.$

Consider



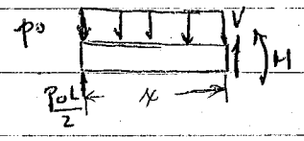

$\Sigma F_y = R_A - P_0L + R_B = 0$

$\boxed{R_A + R_B = P_0L}$

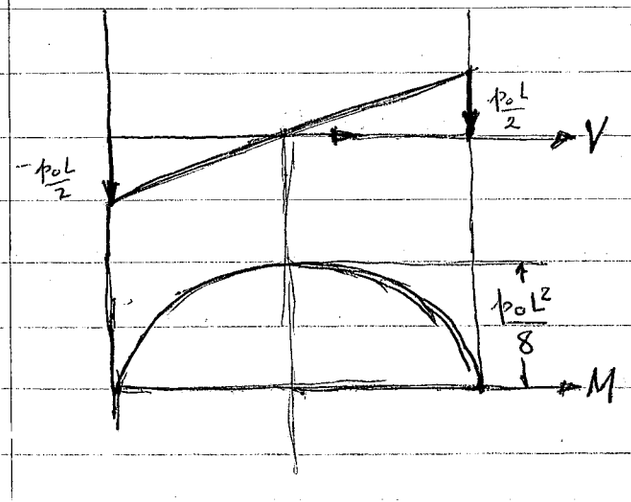
$\Sigma M_B = R_A L - \frac{P_0 L L}{2} = 0$

$\boxed{R_A = \frac{P_0L}{2}}$

$\boxed{R_B = \frac{P_0L}{2}}$




$\Sigma F_y = 0 \quad \frac{P_0L}{2} - P_0x + V = 0 \quad V = P_0\left(x - L/2\right)$

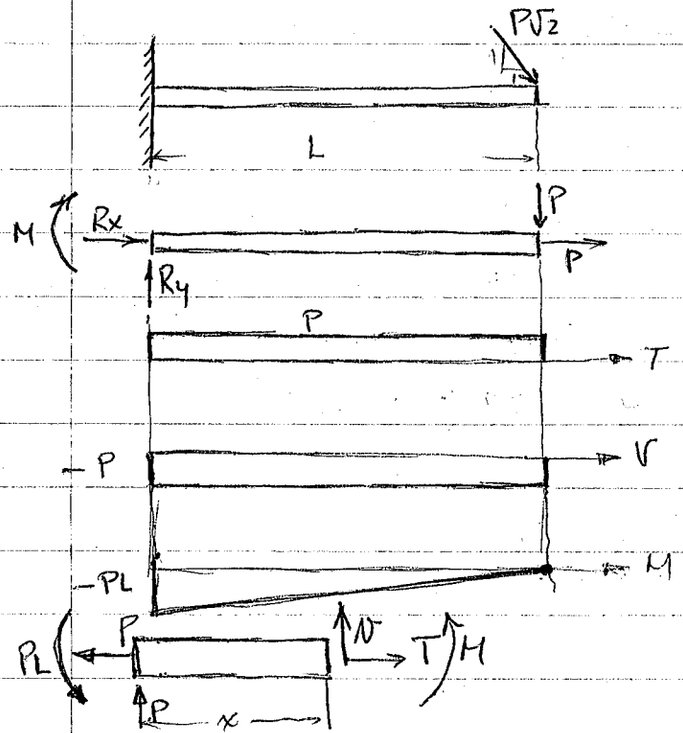



$\Sigma M_c = 0 \quad \frac{P_0L}{2}x - P_0\frac{x^2}{2} - M = 0$

$M = \frac{P_0x}{2}(L - x)$

$\frac{dM}{dx} = \frac{P_0L}{2} - P_0x = P_0(1/2 - x) = 0$

$x = L/2$




$\Sigma F_x = 0$

$R_x + P = 0$ $\boxed{R_x = -P}$

$\Sigma F_y = 0$

$R_y - P = 0$ $\boxed{R_y = P}$

$\Sigma M_A = 0$ $M + P(L) = 0$

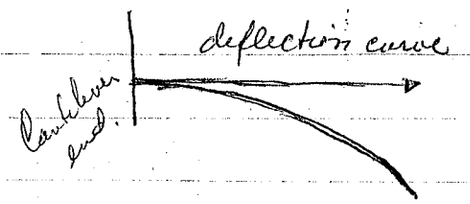
$\boxed{M = -PL}$

$T = P$

$V = -P$

$\Sigma M_C = 0$ $-PL + Px - M = 0$

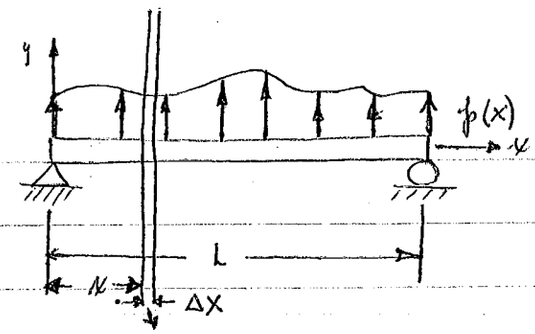
$M = P(x - L)$

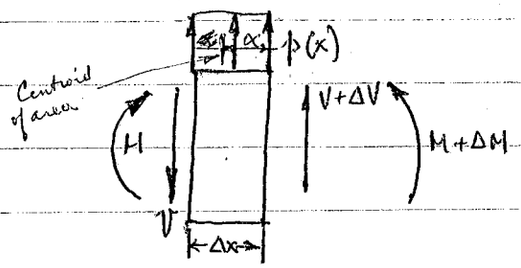



Rd Page 30-31 Problem 2-8 2-15, 17, 19, 21, 29

Steps to solve

1) Draw Sketch of Member - which shows all forces & dimensions
2) Unknown reactions are indicates
3) Resolve inclined forces into components
4) Find reactions
5) Cut arbitrary sections (depending on range that is good for that section)
6) Isolate segment to either side of cut. Show forces acting
7) Indicate internal reactions.
8) Find internal unknown reactions.




$$\Sigma F_y = 0$$

$$-V + p(x)\Delta x + V + \Delta V = 0$$

$$\frac{\Delta V}{\Delta x} = -p(x)$$

$$\Sigma M_R = 0 \qquad H + p(x)\Delta x \cdot \frac{\alpha \Delta x}{2} - V\Delta x - M - \Delta M = 0$$

$$\frac{\Delta M}{\Delta M} = -V + p(x)\alpha\Delta x$$

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = \boxed{\frac{dV}{dx} = -p(x)} \qquad \lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = \boxed{\frac{dM}{dx} = -V}$$

$$\boxed{-\frac{dV}{dx} = p = \frac{d^2M}{dx^2}}$$

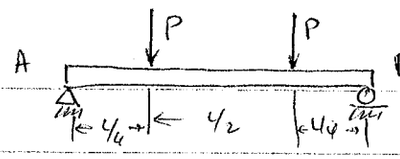
$$dV = -p\,dx$$

$$\boxed{V = -\int p(x)\,dx + C_1} \quad (1)$$

$$\boxed{M = -\int V(x)\,dx + C_2} \quad (2)$$

$$V_b - V_a = -\int_a^b p(x)\,dx$$

$$M_b - M_a = -\int_a^b V(x)\,dx$$

① 

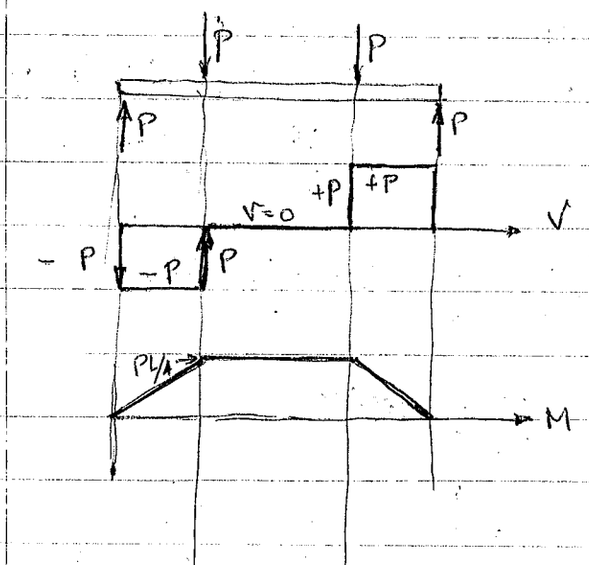

$\sum F_y = 0 \uparrow$ Inspectional

$R_A + R_B - 2P = 0$

$\boxed{R_A + R_B = 2P}$

$\sum M_B = 0 \circlearrowleft$

$R_A L - PL = 0 \qquad R_A = P = R_B$




Analytically

$p = 0 \qquad 0 \le x \le L/4 \qquad V_1 = C_{11} \qquad M_1 = -C_{11}x + C_{21}$

$C_{21} = 0$ ; when $x = 0, M = 0$.

$M_1 = -C_{11}x$ at $x = L/4 \quad V_1 = C_{11} \quad M_1 = -C_{11}L/4$

$L/4 \le x \le 3L/4 \qquad V_2 = C_{12} \qquad M_2 = -C_{12}x + C_{22}$

at $x = L/4 \quad C_{11} + p = C_{12} \Rightarrow C_{11} = C_{12} - p$

at $x = L/4 \quad -C_{11}L/4 = -C_{12}L/4 + C_{22} \Rightarrow (-C_{12} + p)L/4 = -C_{12}L/4 + C_{22}$

$\boxed{C_{22} = pL/4}$

at $x = 3L/4 \qquad V_2 = C_{12} \qquad M_2 = -C_{12}\left(\frac{3L}{4}\right) + \frac{pL}{4}$

$3L/4 \le x \le L \qquad V_3 = C_{13} \qquad M_3 = -C_{13}x + C_{23}$

$C_{12} + p = C_{13} \Rightarrow C_{12} = C_{13} - p$

$-C_{12}\frac{3L}{4} + \frac{PL}{4} = -C_{13}\frac{3L}{4} + C_{23}$

$$-C_{13}\frac{3L}{4} + P_1\frac{3L}{4} + \frac{PL}{4} = -C_{13}\frac{3L}{4} + C_{23}$$

$$\boxed{PL = C_{23}}$$

$M_3 = -C_{13}X + PL$ at $X = L$ $M_3 = 0$ $\therefore$ $\boxed{+C_{13} = P}$

$\boxed{C_{12} = 0}$ $\boxed{C_{11} = -P}$

$V_1 = -P$ $V_2 = 0$ $V_3 = P$

$M_1 = +PX$ $M_2 = PL/4$ $M_3 = P(L-X)$

(2)



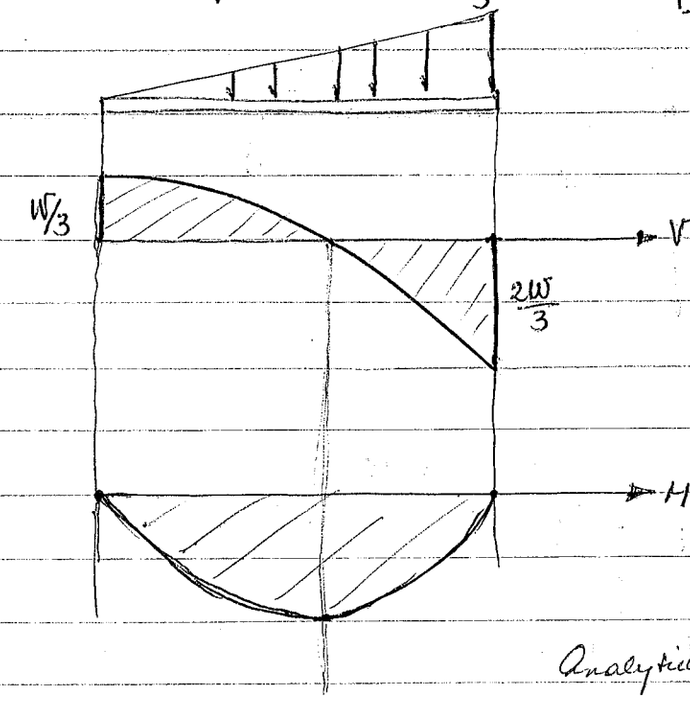

Inspectorily

$\Sigma F_y = 0\uparrow$

$R_A + R_B + W = 0$

$\boxed{R_A + R_B = -W}$

$\Sigma M_B = 0$ $R_A L + \frac{WL}{3} = 0$ $\boxed{R_A = -\frac{W}{3}}$ $\boxed{R_B = -\frac{2W}{3}}$




Analytically

$p = kx$ at $h$ $p = kL$ $W = \frac{1}{2}KL \cdot h$ $k = \frac{2W}{L^2}$

$p = \frac{2W}{L^2}x$

$$V = -\frac{2w}{L^2}x^2 + C_1$$

$$M = \frac{+w}{L^2}\frac{x^3}{3} + C_1 x + C_2 \qquad \text{at } x=0\ M=0 \therefore \boxed{C_2 = 0}$$

$$\text{at } x = L\ M = 0$$

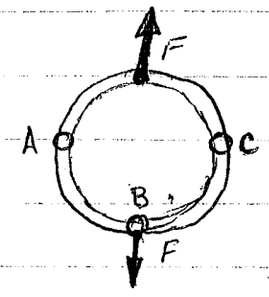
$$M = \frac{wL}{3} - C_1 L = 0 \qquad \boxed{C_1 = \frac{w}{3}}$$

$$\boxed{V = -\frac{w}{L^2}x^2 + \frac{w}{3}} \qquad M = \frac{w}{3}\left[\frac{x^3}{L^2} - x\right]$$

at $V=0$ $x = L/\sqrt{3}$ $M_{max} =$

problem: 2-38, 42, 47, 50, 54, 56, 61, 63 Study example 2-10
P. 37-39 Rel. Chapter 3.

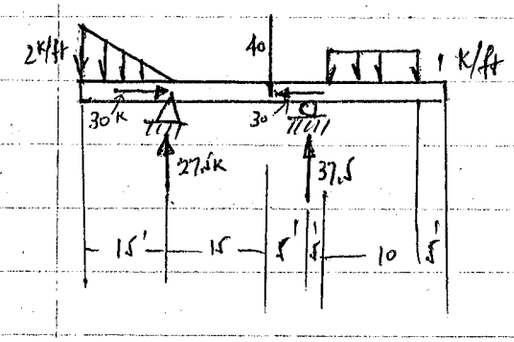
2-29

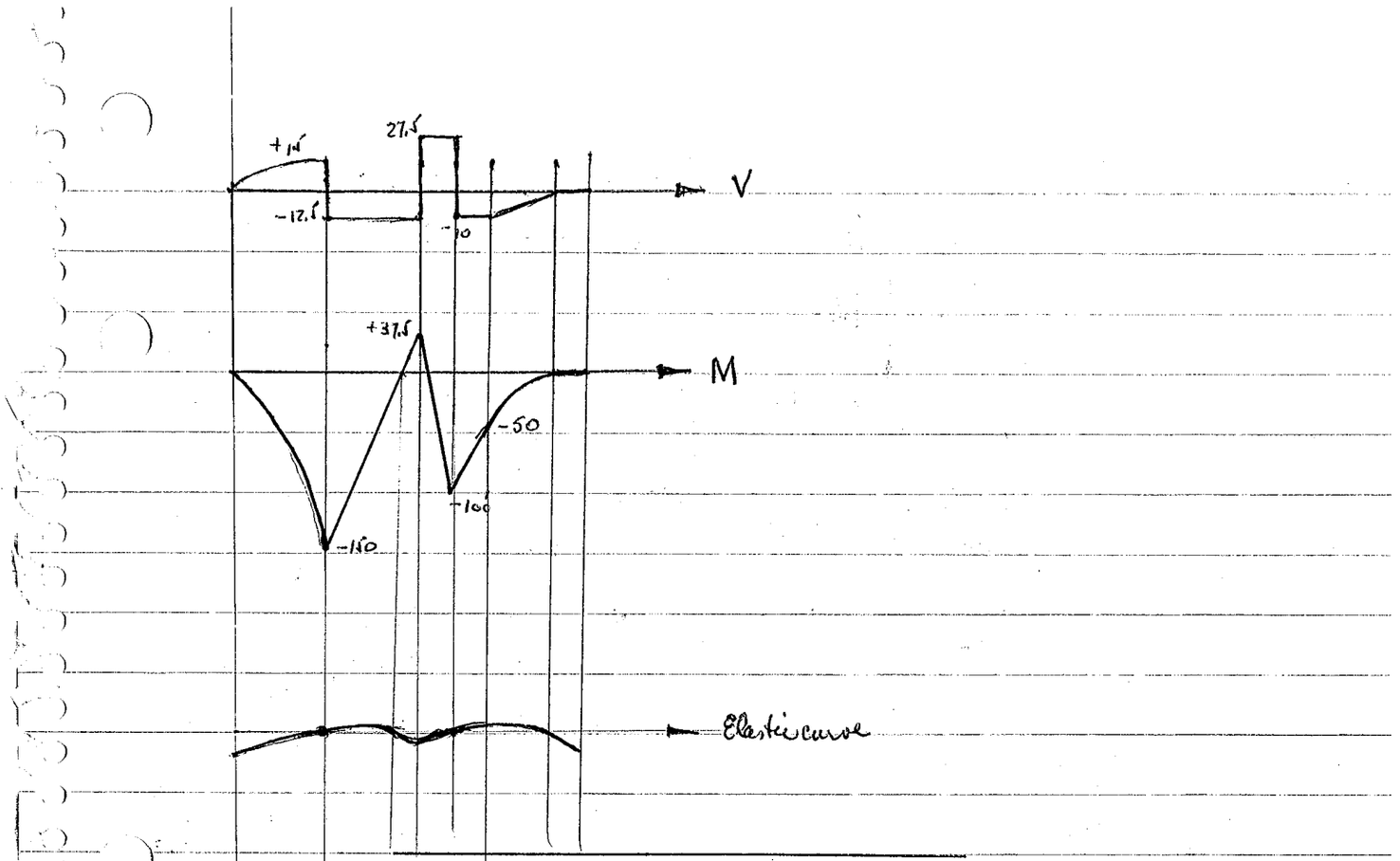


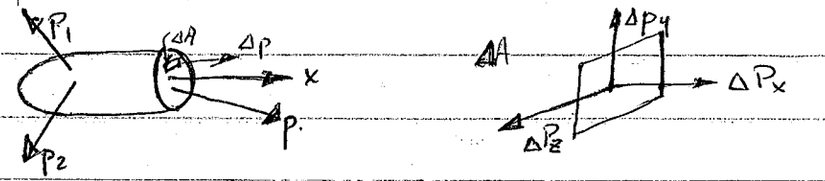




P. 38-9 Example 2-10






$\Delta P_x$ - axial (norma
$\Delta P_y$ & $\Delta P_z$ - shear.

Stress - force intensity per unit area. $\#/in^2 = psi$

$K/in^2 = Ksi$

$$\tau_{xx} = \lim_{\Delta A \to 0} \frac{\Delta P_x}{\Delta A} \rightarrow \text{compression tension}$$

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta P_y}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta P_y}{\Delta A}$$

$\tau_{a,b}$ a - tells the normal to the plane lies in the a direction

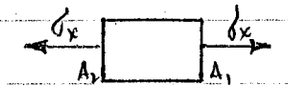
b - direction of force.

when $a = b$ we have an axial type force

$a \neq b$ we have shear-type forces

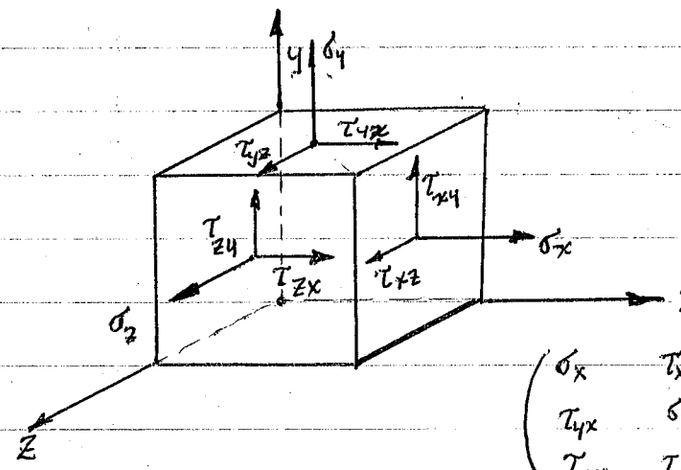
$\tau_{xx} \equiv \sigma_x$ , $\tau_{yy} \equiv \sigma_y$ , $\tau_{zz} \equiv \sigma_z$

Force = Stress by area




if $\sigma_x A_2 = \sigma_x A_1$ we have equilib.

if $\sigma_x A_2 \neq \sigma_x A_1$ we have motion




$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} \Rightarrow \text{Stress Tensor.}$$

Scalar 0-Rank

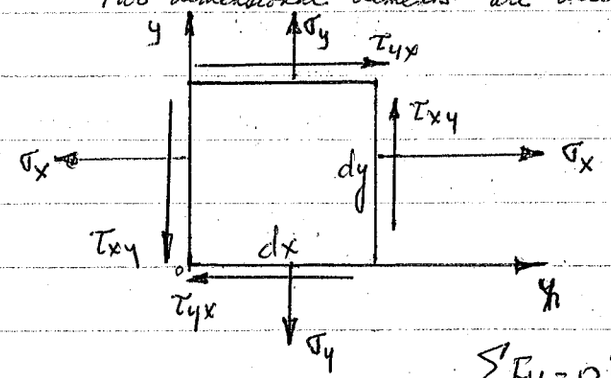
Vector 1st-Rank  Stress-Tensor 2nd rank

$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$$

tensors { Scalar — 0 rank — t — $2^0$ — $3^0$
vector — 1 rank — $V_x, V_y, V_z$ — $2^1$ — $3^1$
tensor — 2 rank — $2^2$ — $3^2$

# of components to describe it in ↓ Plane, ↓ Free Space

Two dimensional elements are used for the derivation.




uniform thickness $dz$

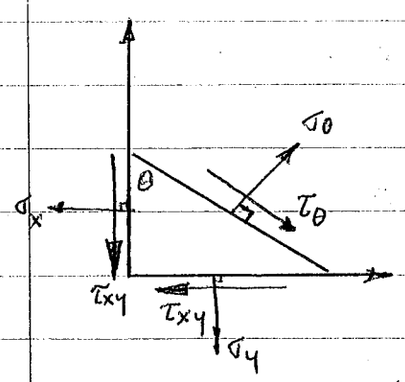
$$\sum F_x = 0$$

$$\sigma_x\, dy\, dz - \sigma_x\, dy\, dz + \tau_{yx}\, dx\, dz - \tau_{yx}\, dy\, dz = 0$$

$$\sum F_y = 0 \uparrow \quad \sigma_y\, dx\, dz - \sigma_y\, dx\, dz + \tau_{xy}\, dy\, dz - \tau_{xy}\, dy\, dz = 0$$

$\Sigma M_o \circlearrowright = 0 \qquad \tau_{yx}\,(dx\,dz)\,dy - \tau_{xy}\,(dy\,dz)\,dx = 0$

$$\boxed{\tau_{yx} = \tau_{xy}} \iff \tau_{xz} = \tau_{zx} \iff \tau_{yz} = \tau_{zy}$$



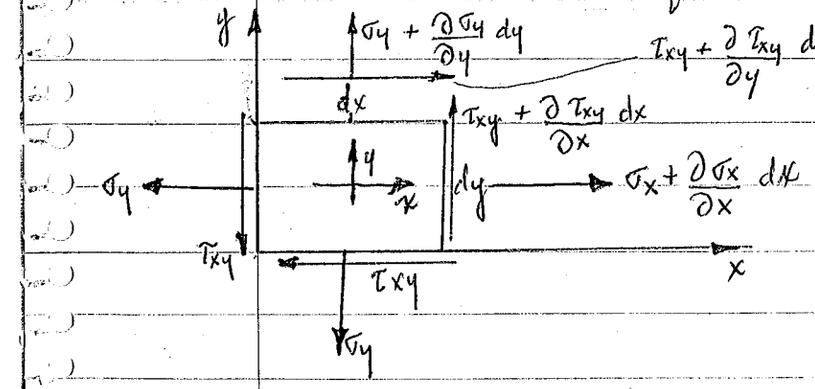

$\sigma_\theta = f(\theta)$ since $\sigma_\theta$ depends on $\sigma_x, \sigma_y, \tau_{xy}, \theta$; but $\sigma_x, \sigma_y, \tau_{xy}$ have been already calculated & they are constant; thus $\sigma_\theta$, and similarly $\tau_\theta$, are functions of $\theta$.

$\tau_\theta = f(\theta)$

we can find some $\theta \Rightarrow \tau_\theta, \sigma_\theta$ are 0 in the xy, yz, xz planes so that we can produce

$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} \Longrightarrow \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$$

We will derive the equations of unequal stresses




$\Sigma F_x = 0$;

$$\left(\sigma_x + \frac{\partial \sigma_x}{\partial x} dx\right) dy\,dz - \sigma_x\,dy\,dz + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y} dy\right) dx\,dz - \tau_{xy}\,dx\,dz + X\,dy\,dz\,dx = 0$$

$\Sigma F_y = 0$;

$$\left(\sigma_x + \frac{\partial \sigma_y}{\partial y} dy\right) dx\,dz - \sigma_y\,dx\,dz + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x} dx\right) dy\,dz - \tau_{xy}\,dy\,dz + Y\,dx\,dy\,dz = 0$$

$$\boxed{\begin{aligned} &(1)\quad \frac{\partial}{\partial x}(\sigma_x) + \frac{\partial}{\partial y}(\tau_{xy}) + X = 0 \\ &(2)\quad \frac{\partial}{\partial y}(\sigma_y) + \frac{\partial}{\partial x}(\tau_{xy}) + Y = 0 \end{aligned}}$$

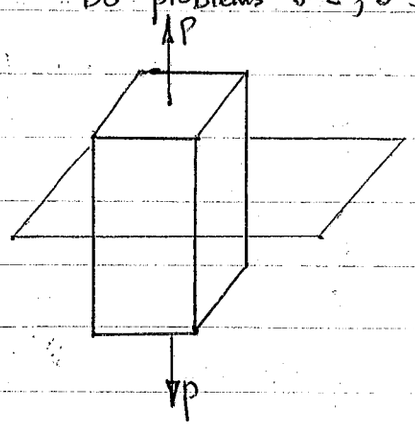
Three dimensionalization

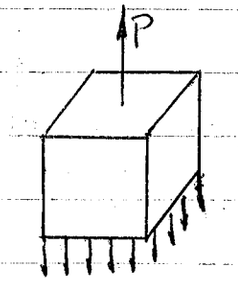
$$\frac{\partial}{\partial x}(\sigma_x) + \frac{\partial}{\partial y}(\tau_{xy}) + \frac{\partial}{\partial z}(\tau_{xz}) + X = 0$$

$$\frac{\partial}{\partial y}(\sigma_y) + \frac{\partial}{\partial x}(\tau_{xy}) + \frac{\partial}{\partial z}(\tau_{yz}) + Y = 0$$

$$\frac{\partial}{\partial z}(\sigma_z) + \frac{\partial}{\partial x}(\tau_{xz}) + \frac{\partial}{\partial y}(\tau_{yz}) + Z = 0$$

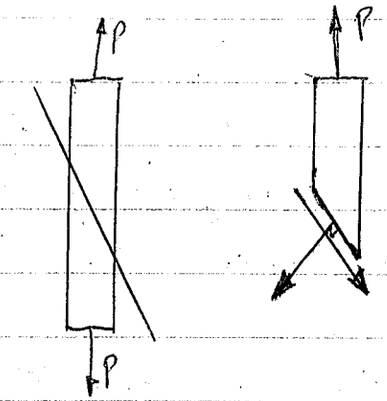
Do problems 3-2, 3-3



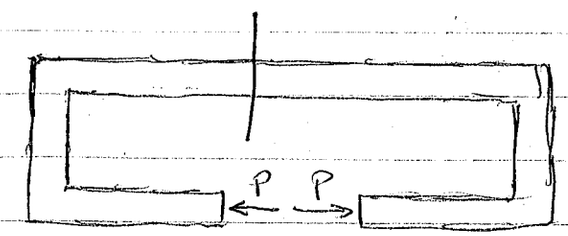

$$\sigma = \frac{P}{A}$$

centric load

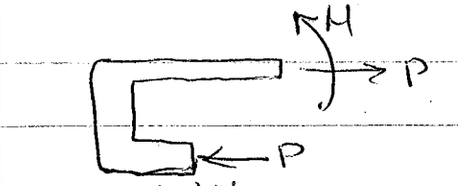




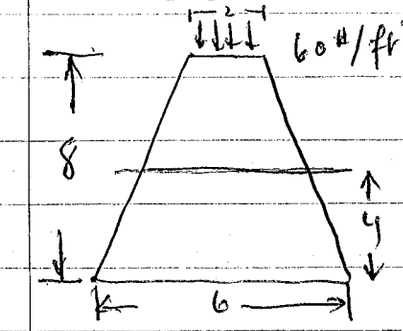




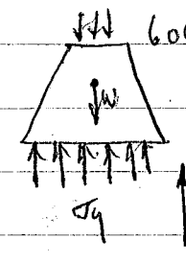
eccentric load




distribution is not uniform




wt of concrete = 120 #/ft³




Since we have resultants passing thru centroid we can say $\sigma_y$ must be uniform.

$$b_y = 6 - \tfrac{y}{8}(6-2) = (12-y)/2$$

$$w_y = \frac{\left(2 + \frac{12-y}{2}\right)}{2}(8-y)\,2 \cdot 150 = 75(8-y)(16-y)$$

$$P_y = W_y + 600(4) = 2400 + 75(8-y)(16-y)$$

$$\sigma_y = \frac{2400 + 75(8-y)(16-y)}{2\left(\frac{12-y}{2}\right)} \qquad \text{if } P_y = \sigma_y \cdot A$$

2nd way

There's a mistake

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + Y = 0$$ since F's are 0 we can

then say $\frac{\partial \sigma_y}{\partial y} + Y = 0$ $Y$ #/ft³ $\quad \frac{\partial \sigma_y}{\partial y} = -150$ $\quad \int d\sigma_y = \int -150\,dy$

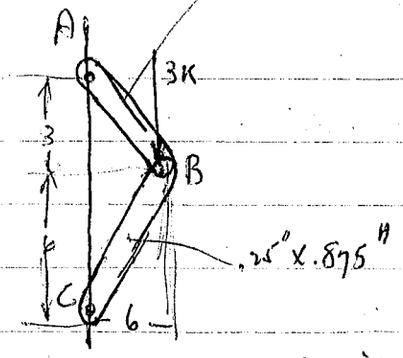
$$\boxed{\sigma_y = -150y + f(x)}$$

at $x=8$ $\sigma = 600$ #/ft²

$600 = -1200 + f(x)$ $\quad f(x) = 1800$ $\quad (\sigma_y = -150y + 1800)$

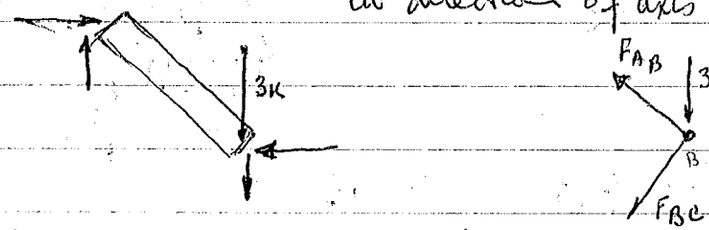
.25"X.50" - cross sectional area.

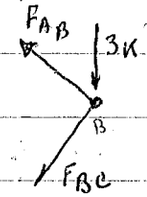
.375" Φ diameter




Neglect Weight of Members

Consider AB these members must have result in direction of axis of the members.




$$\sum F_x = 0 \qquad -\frac{2}{\sqrt{5}} F_{AB} + \frac{1}{\sqrt{2}} F_{BC} = 0$$

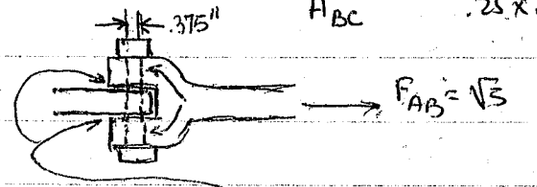
$$\sum F_y = 0 \qquad \frac{1}{\sqrt{5}} F_{AB} - \frac{1}{\sqrt{2}} F_{BC} = 3$$

$$\frac{3}{\sqrt{5}} F_{AB} = 3 \qquad F_{AB} = \sqrt{5}\,K\,T \qquad F_{BC} = +2\sqrt{2}\,K\,C$$

2.828 KN

Shesses $\qquad \sigma_{AB} = \dfrac{F_{AB}}{A_{AB}} = \dfrac{\sqrt{5}}{.025 \times .50} \quad T$

$$\sigma_{BC} = \frac{F_{CB}}{A_{BC}} = \frac{2\sqrt{2}}{.25 \times .875} \quad C$$




$\tau_{BOLT\,A} = \dfrac{\sqrt{5}}{\frac{\pi}{4} \times .375^2 \times 2}$ — 2 surface shear

Ultimate Strength.

strength at which member "failed". failure can be total rupture or excessive deflection.

Factor of Safety - Factor of Ignorance

those factors that can't calculated and those factors which we have doubt.

Allowable Stress -

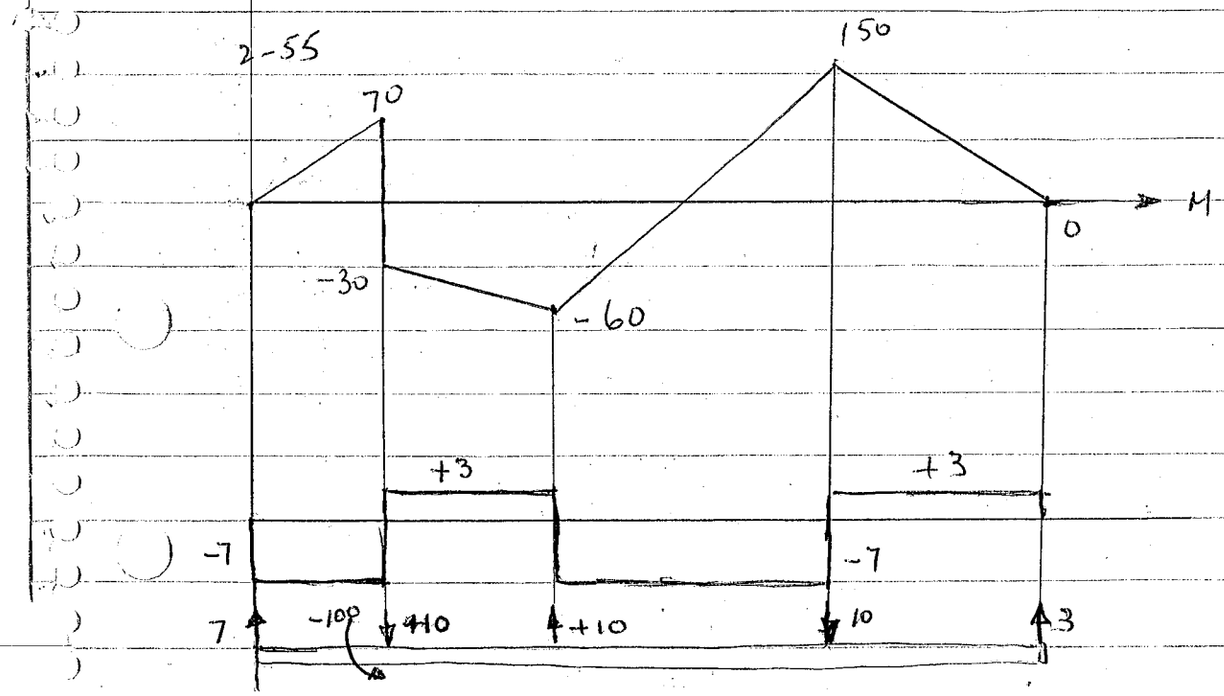
Maximum stress allowed to occur on job and that aren't exceeded

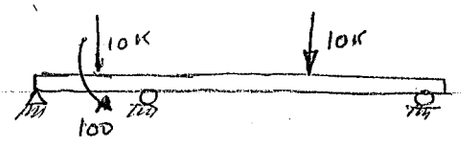
$$\text{Allowable Stress} = \frac{\text{Ult. Strength}}{\text{Factor of Safety}}$$

Creep - tendency of material to flow plastically under sustained load (load should produce stress almost = ultimate stress)

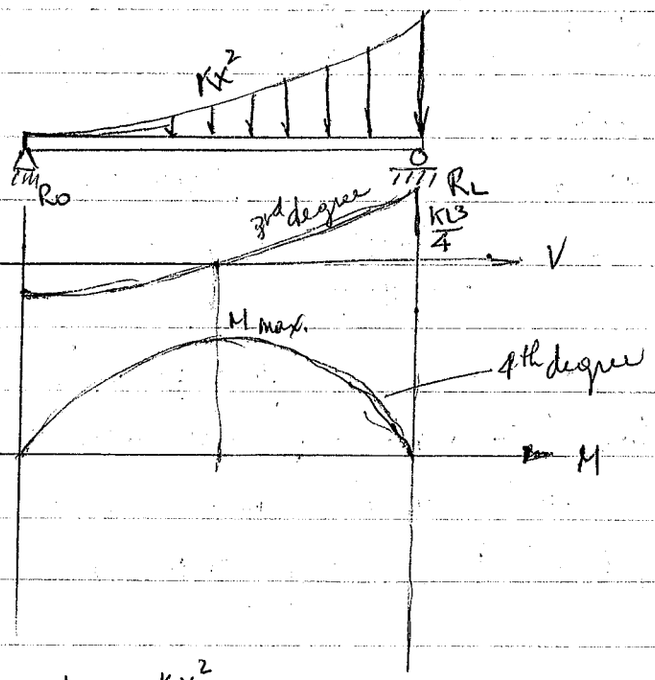
Fatigue - if a material carries a high stress & these stresses are constantly loading & unloading or change of state (T to C to T to C)

Residual stresses -






2-62




$p = -Kx^2$

$V = -\int Kx^2 dx = \frac{Kx^3}{3} + C_1$

$M = -\int \left(\frac{Kx^3}{3} + C_1\right) = -\frac{Kx^4}{12} - C_1 x + C_2$

at $X=0$ $M=0$ $\quad C_2 = 0$

at $X=L$ $M=0$ $\quad C_1 = -\frac{KL^3}{12}$

$\therefore \quad V = \frac{Kx^3}{3} - \frac{KL^3}{12} \qquad M = -\frac{Kx^4}{12} + \frac{KL^3}{12}x$

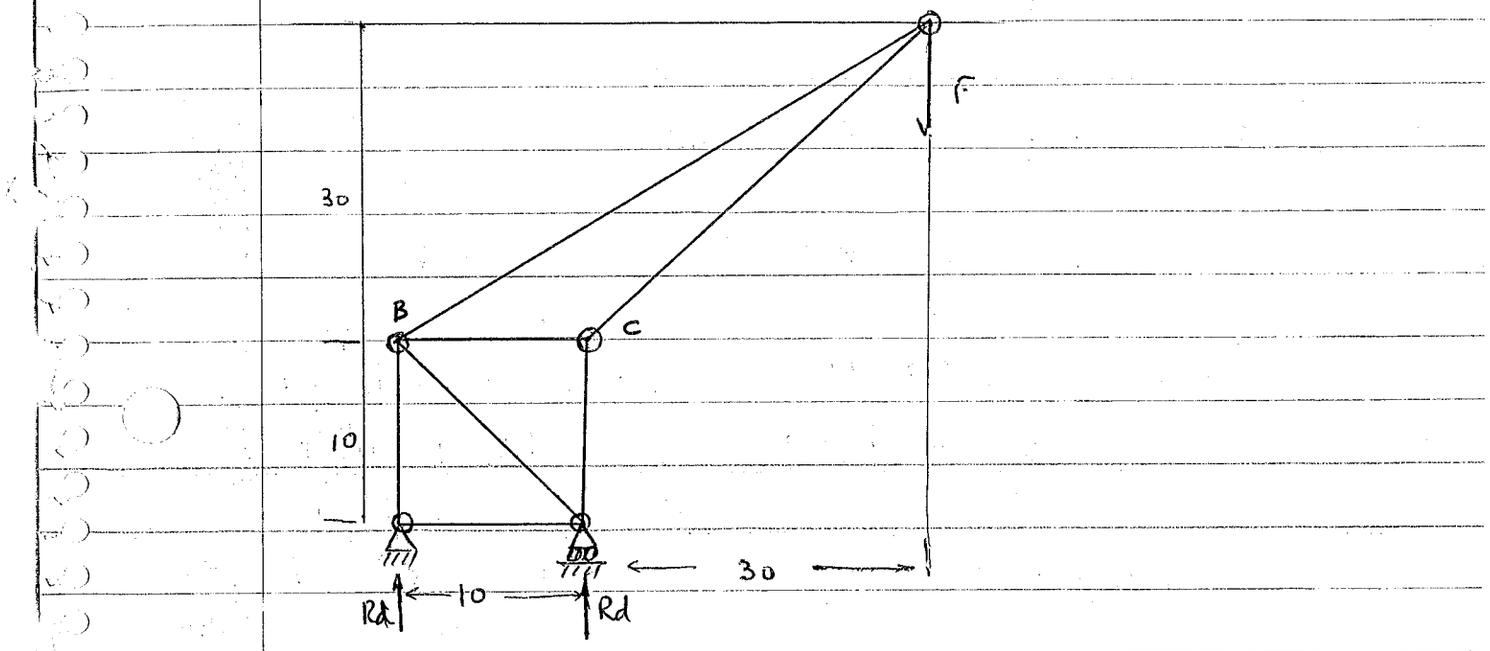
at $V=0$ $\quad x = \frac{L}{\sqrt[3]{4}}$ $\quad \therefore$ at $V=0$ $M = max.$

thus $M_{max} = \frac{KL}{12\sqrt[3]{4}}\left[\frac{3L}{4}^3\right] = \frac{KL^4}{16\sqrt[3]{4}}$

$R_0 = -V_0 = +\frac{KL^3}{12}$

$R_L = V_L = \frac{KL^3}{4}$

3-12

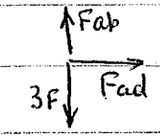



$Ra + Rd = F$

$\Sigma M_d = Ra(10) + F(30) = 0 \qquad Ra = -3F \qquad Rd = 4F$

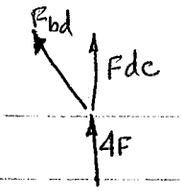
| Member | Force | Stress | Member | Force | Stress |
|---|---|---|---|---|---|
| AD | 0 | 0 | BC | 4F (C) | ½F (C) |
| AB | 3F (T) | ⅜F (T) | CE | $\frac{2\sqrt{2}F}{\sqrt{2}}$ (C) | $\frac{\sqrt{2}}{2}$F (C) |
| DC | 4F (C) | ½F (C) | | | |
| BD | 0 | 0 | | | |
| BE | 5F (T) | ⅝F (T) | | | |

Start at a




$Fad = 0 \qquad Fab = 3F$

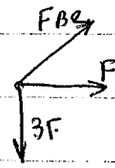
at d




$F_{bd} = 0$

$F_{dc}$ 4F

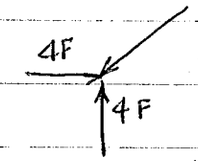
at B




$\frac{3}{5} F_{BE} = 3F$ $\quad F_{BG} = 5F$

$F_{bc} = -4F$ C

at C




$F_{CE} = \frac{2}{\sqrt{2}}$ C

Since a = 8" square

$\frac{5}{8} F \leq 20{,}000$ $\quad F \leq 32{,}000 = 16$ TONS

$4\sqrt{2} \cdot \frac{1}{2} F \leq 15{,}000$ $\quad F \leq \frac{30{,}000}{\sqrt{2}} = 15{,}000\sqrt{2}$

This is assumed to be reached first.

15000
1.414
15000
6000
150
60
21,210

Deformation - change in dimensions or shape due to stress, temperature



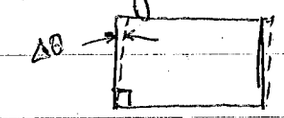
$\Delta l = l - l_0$ $\qquad \frac{\Delta l}{l_0} = \frac{l}{l_0} - 1 = \varepsilon$ strain

if uniform strain

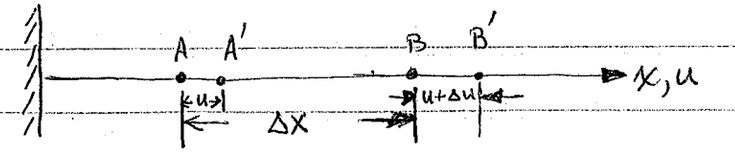
if non-uniform strain $\varepsilon = \frac{\Delta l}{l_0} = \int_0^{l_0} \frac{d(\Delta l)}{l_0}$

Order of Magnitude of $\varepsilon$: we will take $\varepsilon \leq .001$ $\qquad \varepsilon \leq .1\%$

Shearing strain - change in $\Delta\theta$ of a right angle of a rectangle

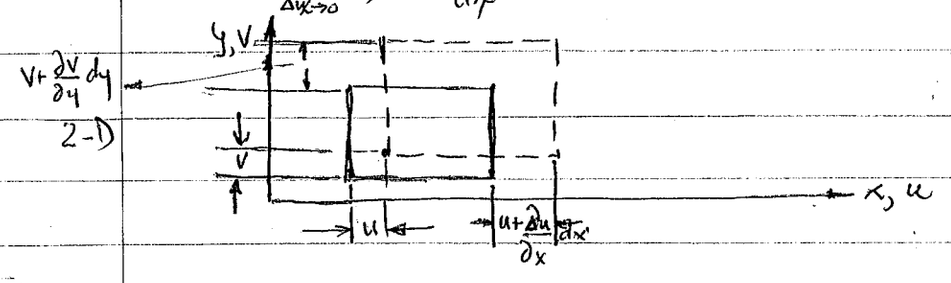



1-D

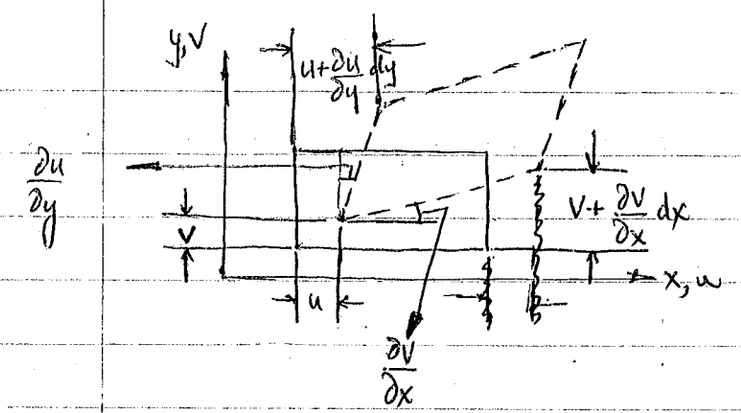



$\varepsilon = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = \frac{du}{dx}$

2-D




$\varepsilon_x = \frac{\partial u}{\partial x}$ $\qquad \varepsilon_y = \frac{\partial v}{\partial y}$ $\qquad \varepsilon_z = \frac{\partial w}{\partial z}$

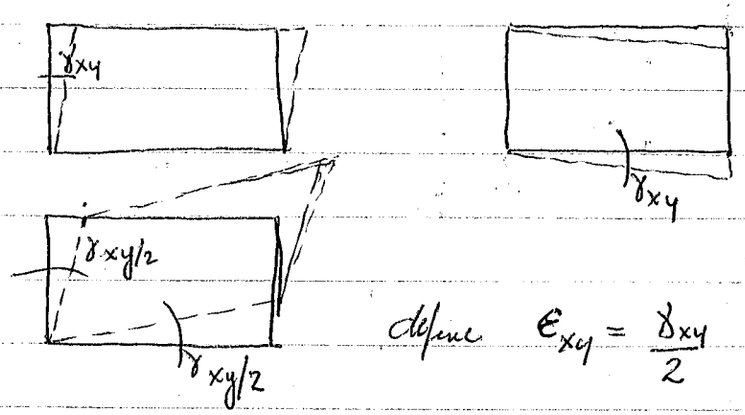



$$\gamma_{xy}\ (\text{shearing strain}) = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \gamma_{yx}$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \gamma_{zx}$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \gamma_{zy}$$

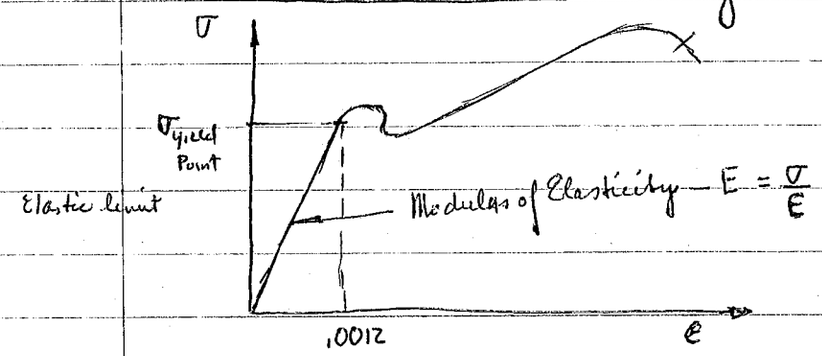
deformation must be such that compatability occurs between each deformation of a body with dimensions $dx, dy$

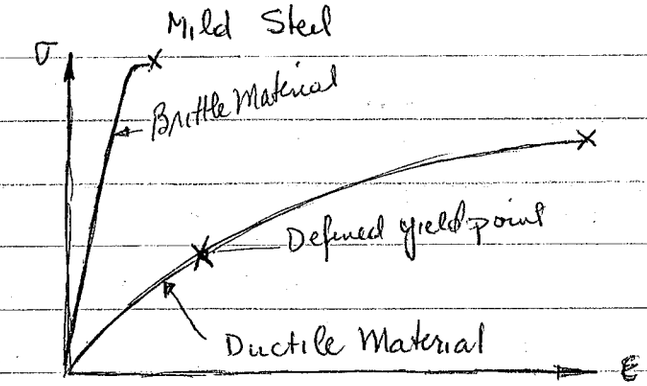



define $\epsilon_{xy} = \frac{\gamma_{xy}}{2}$

$$\begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix} = \begin{pmatrix} \varepsilon_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{yx}}{2} & \varepsilon_y & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{zx}}{2} & \frac{\gamma_{zy}}{2} & \varepsilon_z \end{pmatrix}$$

Constitutive laws – mostly empirical laws




Hookes Law

$\tau = C\,\epsilon$ $\qquad$ $\epsilon = A\tau$ — depends on material

Stress.

Principle of Superposition

1) each strain must be directly and linearly related to the stress causing the strain (Hookes law)

2) Strains due to one stress have negligible effect on another stress.

$$\epsilon_{xx} = A_{11}\tau_{xx} + A_{12}\tau_{yy} + A_{13}\tau_{zz} + A_{14}\tau_{xy} + A_{15}\tau_{yz} + A_{16}\tau_{zx}$$

$$\epsilon_{yy} = A_{21}\tau_{xx} + A_{22}\tau_{yy} + A_{23}\tau_{zz} + A_{24}\tau_{xy} + A_{25}\tau_{yz} + A_{26}\tau_{zx}$$

$\epsilon_{zz} =$ ...

$$\epsilon_{xy} = A_{41}\tau_{xx} + A_{42}\tau_{yy} + A_{43}\tau_{zz} + A_{44}\tau_{xy} + A_{45}\tau_{yz} + A_{46}\tau_{zx}$$

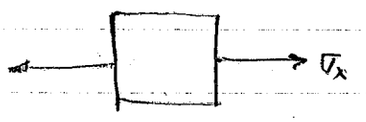
$\epsilon_{yz} =$ ...

$\epsilon_{zx} =$ ...

A homogeneous material - every infinitesimal element of the material has the mechanical properties of the material

Isotropic - Macroscopic - has same mechanical properties in all directions

Homogeneous Isotropic Material




$$\frac{\epsilon_{xx}}{\tau_{xx}} = \frac{\epsilon_{yy}}{\tau_{yy}} = \frac{\epsilon_{zz}}{\tau_{zz}}$$

$$A_{11} = A_{22} = A_{33} = 1/E$$

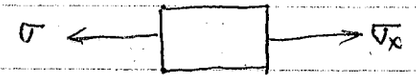
$$\frac{\epsilon_{yy}}{\tau_{xx}} = \frac{\epsilon_{xx}}{\tau_{yy}} = \quad A_{12} = A_{21} = A_{31} = A_{13} = A_{23} = A_{32} = \frac{-\nu}{E}$$

$$\frac{\epsilon_{xy}}{\tau_{xy}} = \frac{\epsilon_{yz}}{\tau_{yz}} = \frac{\epsilon_{xz}}{\epsilon_{xz}} = A_{44} = A_{55} = A_{66} = \frac{1}{2G}$$

All others are not non-zero.

$\epsilon_x = \sigma_x \frac{\nu}{E} + \sigma_y \frac{\nu}{E} + \sigma_z \frac{\nu}{E}$

$\epsilon_x = \frac{1}{E}\sigma_x \quad \sigma_x = \nu\sigma_x$




$$\epsilon_x = 1/E\,\sigma_x$$

$$\epsilon_y = -\nu\,\epsilon_x = -\frac{\nu}{E}\sigma_x$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y - \frac{\nu}{E}\sigma_z$$

$$\varepsilon_x = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_z$$

$$\varepsilon_z = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \frac{\sigma_z}{E}$$

$$\varepsilon_{xy} = \frac{1}{2G}\tau_{xy} \qquad \gamma_{xy} = \frac{1}{G}\tau_{xy} \qquad \gamma_{xz} = \frac{\tau_{xz}}{G},\ \gamma_{yz} = \frac{\tau_{yz}}{G}$$

E – modulus of elasticity – Young's modulus.

Mild steel $E \sim 29\times10^6$ psi $= 29\times10^3$ ksi

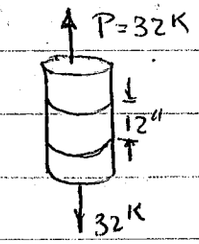
G – shearing modulus of elasticity –

Mild steel $G \sim 10\text{-}12\times10^6$ psi $= 10\text{-}12\times10^3$ psi

$\nu$ – Poisson's ratio

$0.1 - 0.5$, $0.25 - 0.33$ More practical

Problem:




$\Delta l = .00938$ Find E & $\nu$

$\Delta d = -.000585$

$E = \frac{\sigma_x}{\varepsilon_x}$ $\quad \sigma_x = \frac{32}{\pi/4(2.25)^2}$ $\quad \varepsilon_x = \frac{.00938}{12"}$

$\nu = -\varepsilon_y/\varepsilon_x$ ; $\quad \varepsilon_y = \frac{-.000585}{2.25}$

$$\nu = \frac{+\frac{.000585}{2.25}}{\frac{.00938}{12}} =$$

2in cube $\quad \sigma_x = \sigma_y = \sigma_z = 30{,}000$ psi $\quad E = 30\times10^6$ psi

$\nu = 1/4$

$\epsilon = \frac{\sigma}{E} - \nu\frac{\sigma}{E} - \nu\frac{\sigma}{E}$ $\quad \epsilon = \frac{\sigma}{E}(1-2\nu) = \frac{30000}{30\times10^6} \cdot \frac{1}{2} = \frac{1}{2}(1000)$

$l = l_0 + \epsilon l_0 \quad 2" + (.0005)2 = 1.999"$

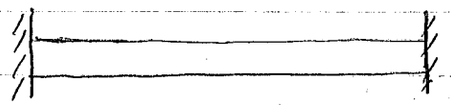
## Thermal Strains

$\epsilon_x = \epsilon_y = \epsilon_z$

$\alpha$ - coefficient of thermal expansion

$\alpha = \frac{\epsilon}{\Delta T}$ $\quad \alpha_{steel} = 6.5\times10^{-6}/°F$

$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} + \alpha\Delta T$



$A = 4in^2$
$L = 20\ ft$

$\Delta T = 100°F \quad \nu = \quad E =$

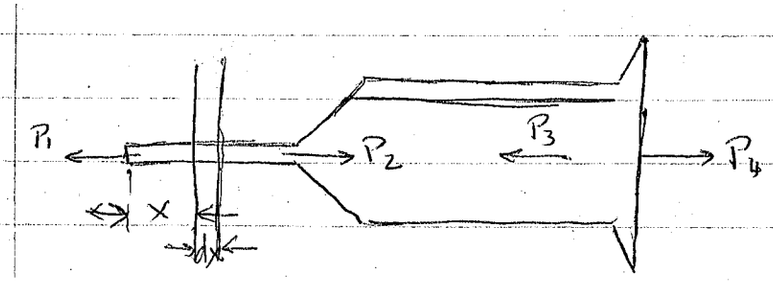
$\epsilon_x = 0 = \frac{\sigma_x}{E} + \alpha\Delta T$ since it is not restrained in y, z

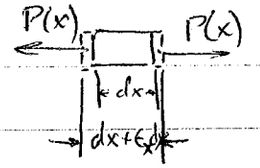
direction $\sigma_y, \sigma_z = 0$

$\sigma_x = -E\alpha\Delta T$ .....

$\epsilon_y = -\frac{\nu\sigma_x}{E} + \alpha\Delta T$

$\epsilon_z = -\frac{\nu\sigma_x}{E} + \alpha\Delta T$





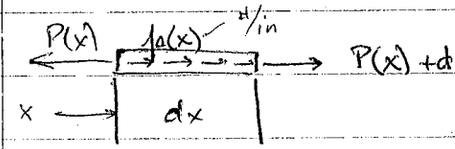

displacement or measure of elongations.

$du = \epsilon_x dx$

$$u = \int du = \int \epsilon_x dx = \frac{1}{E}\int \sigma_x dx = \frac{1}{E}\int \frac{P(x)}{A(x)} dx$$

$$\boxed{\frac{du}{dx} = \frac{1}{E}\frac{P(x)}{A(x)}}$$

distributed load



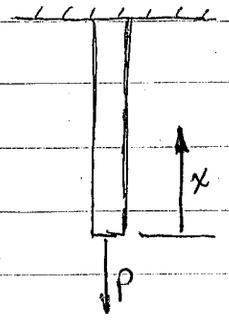

$$\sum F_x = P(x) + dP - P(x) + p_x dx = 0$$

$$\frac{dP}{dx} = -p_x$$

$$\frac{d}{dx}\left(\frac{du}{dx}\right) = \frac{1}{E}\frac{d\left(\frac{P(x)}{A(x)}\right)}{dx}$$ if A is a constant

$$\frac{d^2u}{dx^2} = \frac{1}{EA}\frac{dP}{dx} = -\frac{p_x}{EA}$$

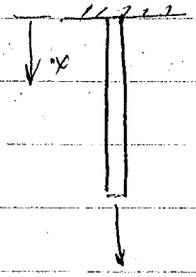
$$\boxed{\frac{d^2u}{dx^2} = -\frac{p_x}{EA}}$$




$$\epsilon_x = \frac{\Delta l}{l} \qquad \sigma_x = P/A \qquad \frac{\sigma_x}{E} = \frac{P}{AE} \qquad \Delta l = \frac{Pl}{AE}$$

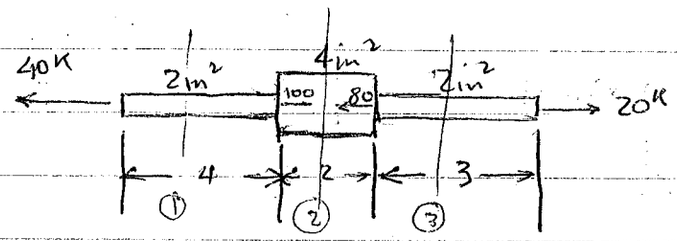
$$u = \frac{1}{E}\int \frac{P}{A} dx = \frac{Px}{AE} + C_1 \qquad u=0 \;\; x=0 \Rightarrow C_1 = 0$$

$$u = \frac{Px}{AE} \qquad u_L = \frac{PL}{AE}$$



$$u(L) = \frac{PL}{AE}$$

Problem 2



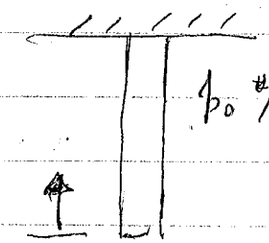

$$u = \sum \frac{PL}{AE}$$

$$u_1 = \frac{40^K \times 48in}{2in^2 \times 30\times10^3 K} \qquad u_2 = -\frac{60^K \times 24in}{4in^2 \times 30\times10^3}$$

$$u_3 = \frac{20\times36}{2in^2\times30\times10^3 K}$$

$$u_T = 32\times10^{-3} - 12\times10^{-3} + 12\times10^{-3} = 32\times10^{-3}.$$

Problem 3




$$p_x = -p_0 \qquad \frac{du}{dx} = -\int \frac{p_x dx}{AE} dx = \left\{ \right.$$

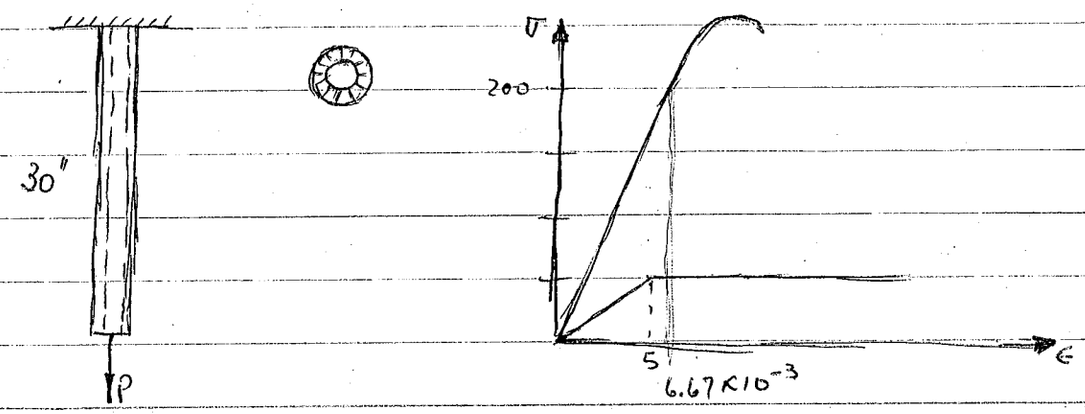
$$\frac{P_0 x}{AE} + C_1$$

$$\frac{du}{dx} = \frac{P(x)}{AE} \qquad \frac{du}{dx} = \frac{P_0 x}{AE} \quad \text{since } C_1 = 0$$

$$u = \frac{p_0 x^2}{2AE} + C_2 \qquad u=0 \quad x=0 \quad C_2 = 0$$

$$u = \frac{p_0 x^2}{2AE} \qquad u(L) = \frac{p_0 L^2}{2AE}$$

Problem 4-10 Page 128




$$E_s = \frac{\sigma}{\epsilon} = \frac{200}{6.67\times10^{-3}} = 30\times10^3 \text{ KSI}$$

$$E_a = \frac{\sigma}{\epsilon} = \frac{50}{5\times10^{-3}} = 10\times10^3 \text{ KSI}$$

$P_a + P_s = P$ ← equil. $\qquad \epsilon_s = \epsilon_a$ ← compatibility $\qquad$ hence $u_s = u_a$

$$\epsilon_s = \frac{\sigma_s}{E_s} \; ; \quad \epsilon_a = \frac{\sigma_a}{E_a} \leftarrow \text{elastic}$$

$$\epsilon_s = \frac{\sigma_s}{30\times10^3} \qquad \epsilon_a = \frac{\sigma_a}{10\times10^3} \qquad \sigma_s = 9/3\,\sigma_a \qquad \sigma_s = 3\sigma_a$$

$$P_a = \sigma_a A_a \qquad P_s = \sigma_s A_s \qquad \sigma_a = .5\sigma_a \qquad \sigma_s = .5(3\sigma_a)$$

$$.5\sigma_A + 1.5\sigma_A = P \qquad \sigma_a = P/2.0$$

$$\sigma_s = \frac{3P}{2.0}$$

$$\Delta l = \epsilon_s l = \epsilon_a l = \frac{\sigma_a}{10\times10^3}\,.30 = 3\sigma_a\times10^3 = \sigma_s\times10^3$$

for $P_1 = 80K$

$\sigma_a = \frac{80}{2.0} = 40$ KSI  $\sigma_s = \frac{3.80}{2.0} = 120$ KSI

$\Delta l = 120 \times 10^{-3} = .12$ in

suppose $P_2 = 125$ K  $\sigma_a = \frac{125}{2} = 62.5 > 50$

$\sigma_s = \frac{3(125)}{2} = 187.5 < 200$  $\sigma_a = 50$ KSI

$P_a = 25$ K  $P_a + P_s = P$

$\therefore \sigma_s = \frac{100}{.5} = 200$  $P_s = 100$ K

$\epsilon_s = \frac{200}{30 \times 10^3} = 6.67 \times 10^{-3}$

$\Delta l = \epsilon_s \times 30 = 200 \times 10^{-3} = .200$ in

4-3 $\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$

$\epsilon_y = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_z$

$\epsilon_z = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \frac{\sigma_z}{E}$

$\sigma_x = \lambda\theta + 2\mu\epsilon_x$

$$\sigma_x = \frac{(1-\nu^2)E\,\epsilon_x + \nu E(1+\nu)[\epsilon_y + \epsilon_z]}{1-3\nu^2-2\nu^3}$$

$$= \frac{(1-\nu)E\,\epsilon_x + \nu E[\epsilon_z + \epsilon_y]}{1-\nu-2\nu^2}$$

4-8 $\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E} + \alpha\Delta T$

$\epsilon_y = \frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_z + \alpha\Delta T$

$\therefore 0 = \left(\frac{1-\nu}{E}\right)\sigma + \alpha\Delta T$

$\sigma = \frac{\alpha E \Delta T}{\nu - 1}$

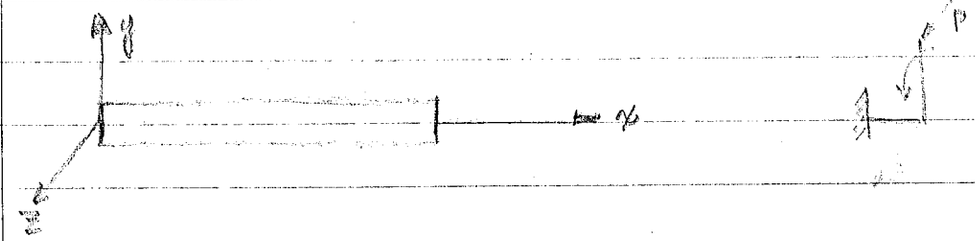
could extract in 3 directions

they are constrained

$\sigma_z = \epsilon_x = \epsilon_y = 0$  $\sigma_x = \sigma_y$

Chapter 5 and Chapter 6 - Deal with internal reactions

Torsion




1 hp. → 550 ft-lb/sec = 550×12×60 in-lb/min

if crankshaft rotates at N rpm Torque = T

power = $2\pi NT$ = 550×12×60 × # hp developed

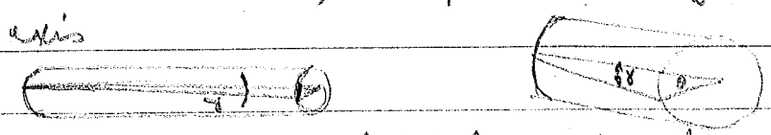
$$T = \frac{63000(hp)}{N} \qquad T \text{ (in lb-in)}$$

Assumptions

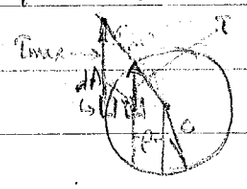
1) Plane sections normal to axis remain plane after deformation

No warping or distortion

2) In circular members, shearing strains, vary linearly from central axis




3) Material remains elastic, Hooke's law applies, the shear is proportional to strain.




$$\frac{\tau}{\rho} = \frac{\tau_{max}}{c}$$

$$\tau = \frac{\tau_{max}}{c}\rho$$

$dT = \tau\, dA\, \rho$ (moment force arm)

$$dT = \frac{\tau_{max}}{c}\rho\, dA\,\rho$$

$$T = \int_A \frac{\tau_{max}}{c}\rho^2 dA = \frac{\tau_{max}}{c}\int_A \rho^2 dA$$




$J$ (polar moment of inertia) $= \int_A \rho^2 dA$

$T = \frac{\tau_{max} J}{c}$ $\quad \left[ \tau_{max} = Tc/J \right.$ torsion formula for shear stress. $\left.$ circular shafts.$\right]$

$J = \int_A \rho^2 dA$ $\quad$  $\quad dA = 2\pi\rho\, d\rho$

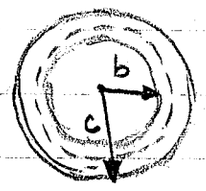
for solid circle

$$J = \int_0^c \rho^2 \times 2\pi\rho\, d\rho = 2\pi \left.\frac{\rho^4}{4}\right|_0^c = 2\pi \frac{c^4}{4} = \frac{\pi c^4}{2} = \frac{\pi d^4}{32}$$

$\tau = \frac{\tau_{max}\rho}{c}$ $\quad \frac{\tau_{max}}{c} = \frac{\tau\rho}{\rho^2} = \frac{T}{J}$ $\quad \boxed{\tau = T\rho/J}$

Also called Jpot

For circular tube :—




$$J = \int_b^c 2\pi\rho^3 d\rho = \frac{2\pi\rho^4}{4} = \frac{\pi c^4}{2} - \frac{\pi b^4}{2}$$

for a thin tube $b \cong c$ $\quad c - b = t$ is assumed

$$J = \frac{\pi}{2}(c^4 - b^4) = \frac{\pi}{2}(c^2 - b^2)(c^2 + b^2) = \frac{\pi}{2}(c^2 + b^2)(c - b)(c + b)$$

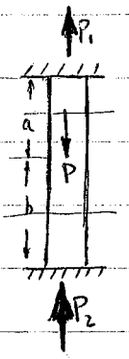
$$\cong \frac{\pi}{2} \cdot 2c^2 t \cdot 2c \cong 2c^3 t\pi$$

$$J \cong 2\pi c^3 t$$

$c = 10''$ $\quad t = 1''$ $\quad J = \frac{\pi}{2}(10^4 - 9^4) = \frac{\pi}{2}(10000 - 6561)$

$J \cong \pi\left(\frac{34 \cdot 39}{2}\right) = 1719.5\pi$ $\quad J \cong 2000\pi$

4-35




$P = P_1 + P_2$ - equil $\quad \Sigma F_y = 0$

$u = \sum \frac{PL}{AE} = \frac{P_1 a}{AE} - \frac{P_2 b}{AE} = 0$

$P_1 = P - P_2 \quad a = L - b \qquad P_1 = \frac{bP}{L} \quad P_2 = \frac{aP}{L}$

4-31

$\tau_{max} = \frac{Tc}{J}$ solid shaft $J = \frac{\pi d^4}{32}$

hollow shaft $J = \frac{\pi d_o^4}{32} - \frac{\pi d_i^4}{32}$

thin tube $J = 2\pi c^3 t = \frac{\pi d_o^3 t}{4}$

solid shaft $\frac{J}{\frac{Vol.}{unit length}} = \frac{\pi c^4/2}{\pi c^2} = \frac{c^2}{2}$

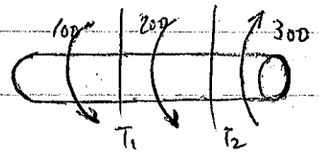
tube: $\frac{J}{V} = \frac{\pi}{2}\left(c_o^4 - c_i^4\right) \Big/ \pi\left(c_o^2 - c_i^2\right) = \frac{c_o^2 + c_i^2}{2}$

thin tube: $\frac{J}{V} = \frac{2\pi c^3 t}{2\pi c t} = c^2$

1. Equilibrium - Method of Sections
   Internal Torques
2. Assume shearing strain linear from axis

3. Apply the Material properties + perhaps Hookes Law.

Problem




$\phi = 1/2"$

$T_1 = 100$

$T_2 = 300$ $\quad \tau_{max} = \frac{TC}{J}$ $\quad J = \frac{\pi}{516}$

$$\tau_{max} = \frac{512(300)(.25)}{\pi} = 128\left(\frac{300}{\pi}\right) \approx 12000 \text{ psi}$$

Pblm 1" outside dia $\quad T = 400 \quad$ find $\tau_p \; \tau_c$

.9" inside dia $\quad J = \frac{\pi}{32}(1^4 - .9^4) = \frac{\pi}{32}(.6561) = \frac{\pi}{32}(.3439)$

$J \geq .01\pi$ $\quad$ approx $\rightarrow J_{\text{thin tube}} = \frac{\pi}{4} 1^3 (.05) = .0125\pi$

$$\tau_{max} = \frac{400(0.5)}{.01\pi} \sim 5930 \text{ psi} \qquad \tau_p = \frac{400(0.45)}{.01\pi} \sim 5330 \text{ psi}$$

Design

$\tau_{all} = 8000$ psi Steel (annual)

$\frac{J}{c} = \frac{T}{\tau_{max}}$ $\quad$ for solid shaft $\frac{\pi c^3}{2} = \frac{J}{c}$

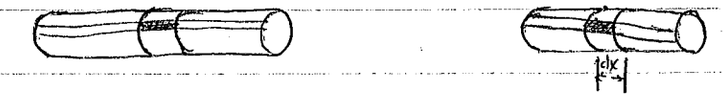
Select a shaft for a ten hp motor at 1800 rpm.

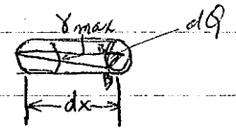
$\tau_{all} = 8000$ $\quad T = \frac{63000 \text{ Hp}}{N} = \frac{630000}{1800} = 350 \text{ in-lb}$

$$J/c = \frac{350\rho}{8000} = \frac{\pi c^3}{2} \qquad c^3 = \frac{700}{8000\pi} = (.303)^3$$

$c = .303$

## Angle of twist





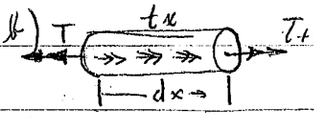

$\widehat{BD} = c\,d\theta$ $\qquad \gamma_{max}\,dx = \widehat{BD}$ $\qquad \frac{d\phi}{dx} = \frac{\gamma_{max}}{c}$

if hookes' law applies $\quad \gamma_{max} = \frac{\tau_{max}}{G}$ $\quad \tau_{max} = \frac{Tc}{J}$ ; $\gamma_{max} = \frac{Tc}{JG}$

or $\frac{d\theta}{dx} = \frac{T}{JG}$ $\qquad \boxed{\varphi = \int_0^L \frac{T}{JG}\,dx}$

1) Suppose $\frac{T}{GJ}$ = constant $\qquad \varphi = \frac{TL}{GJ}$ $\qquad \varphi_T = \sum \frac{TL}{GJ}$

2) Suppose $\frac{T}{JG}$ not constant $\qquad \varphi = \int_0^L \frac{T(x)}{J(x)G}\,dx$




$dT = -t_x\,dx$

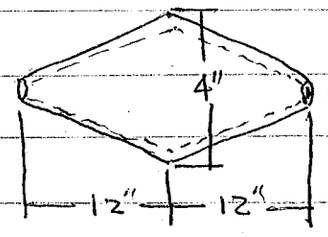
$\frac{dT}{dx} = -t_x$ $\qquad \frac{d\theta}{dx} = \frac{T}{JG}$

$\frac{d^2\theta}{dx^2} = \frac{1}{JG}\frac{dT}{dx} + T\frac{d}{dx}\left(\frac{1}{JG}\right)$

if $1/JG$ = const $\quad T\frac{d}{dx}\left(1/JG\right) = 0$ $\qquad \boxed{\frac{d^2\theta}{dx^2} = \frac{-t_x}{JG}}$

$\frac{d\theta}{dx} = -\int \frac{t_x}{JG}\,dx = f(x) + C_1 = \frac{T}{JG}$ $\qquad$ JG - torsional stiffness of rod.

5-14




$\frac{d\theta}{dx} = \frac{T}{JG}$ $\qquad t = C_o - C_i = \frac{1}{2\pi}$

$C_i = C_o - t$

$d_o = ax + b$ $\quad 0 \le x \le 12''$ $\quad$ @ $x = 0$ $d_o = 1$

$1 = b$ $\quad$ @ $x = 12$ $d_o = 4$

$4 = 12a + 1$ $\quad a = 1/4$ $\quad d_o = \frac{x}{4} + 4$

$x = 12$ $\quad d_o = 4$ $\quad 4 = 12a' + b'$ $\quad 3 = -12a'$ $\quad a' = -1/4$

$x = 24$ $\quad d_o = 1$ $\quad 1 = 24a' + b'$ $\quad b' = 28/4$

$d_o = \frac{-x + 28}{4}$

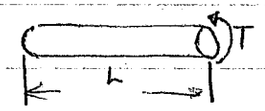
$0 \le x \le 12''$ $\quad d_o = \frac{x+4}{4}$ $\quad c_i = c_o - t$

$12'' + 0 \le x \le 12 + 12''$ $\quad d_o = \frac{-x+28}{4}$ $\quad J_x = \frac{\pi c_o^4}{2} - \frac{\pi c_i^4}{2}$

$$\frac{\pi c_o^4}{2} - \frac{\pi}{2}(c_o - t)^4 = -\frac{\pi}{2}(-4c_o^3 t + 6c_o^2 t^2 - 4c_o t^3 + t^4).$$

$$J_x = \frac{1}{4}(4c_o^3 - 6c_o^2 t + 4c_o t^2 - t^3) = c_o^3 - \frac{3}{2}c_o^2 t + c_o t^2 - \frac{1}{4}t^3$$

5-6 $\quad .1\text{in} = c$




$\tau_{max} = 6000$

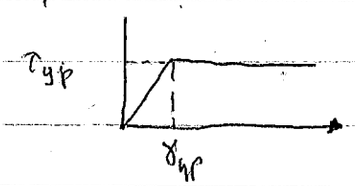
$G = 3.84 \times 10^6$

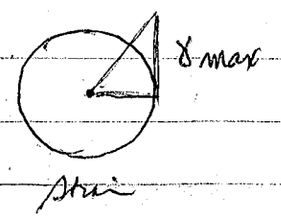
$\phi = \frac{TL}{JG}$ $\quad \tau_{max} = \frac{Tc}{J}$ $\quad \frac{\tau_{max} L}{cG} = \phi = 2\pi$

$L = \frac{2\pi cG}{\tau_{max}} = \frac{2\pi(.1)(3.84 \times 10^6 \text{ psi})}{6000 \text{ psi}}$

Relax some of the assumptions

1. relax hookes law



Case I $\gamma_{max} \leq \gamma_{yp}$ $\tau_{max} \leq \tau_{yp}$

already discussed.



$\tau_{max} \rightarrow \tau_{yp}$

Telastic ; max. stress reaches yield pt $T_{el} = \frac{\tau_{yp} J}{c}$

Case II $\gamma_{max} > \tau_{yp}$. $\tau_{max} = \tau_{yp}$.

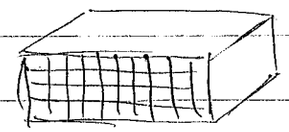


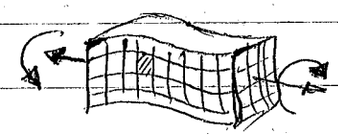
$$T = \int_A \tau dA\rho = \int_r^c \tau_{yp} 2\pi\rho^2 d\rho + \int_0^r \frac{\tau_{yp}\rho^2}{r} 2\pi\rho d\rho$$

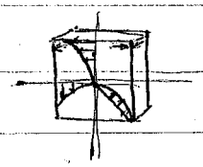
limiting case $r=0$ $T_{plastic} = \int_0^c \tau_{yp} 2\pi\rho^2 d\rho = \tau_{yp} \frac{2\pi c^3}{3} = \tau_{yp} \frac{2}{3} \frac{\pi c^4}{2} \frac{2}{c}$

$$T_{plastic} = \tau_{yp} \frac{4}{3} \frac{J}{c}$$

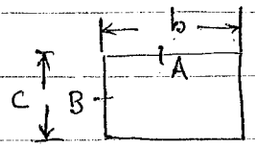
Solid non-circular member







$$\tau_{max} = \frac{T}{\alpha b c^2} \qquad \varphi = \frac{TL}{\beta b c^3 G}$$




$\alpha = f(b/c)$

$$T_A = \frac{T}{\alpha b c^2} \qquad T_B = \frac{T}{\alpha b^2 c}$$

$\int \frac{du}{u^4 - a^4}$ a contains x & constant ten.

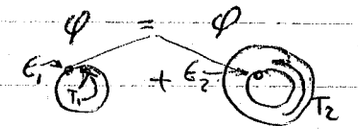
$$\frac{dx}{(ax+b)^4-c^4} = \frac{1}{a}\int \frac{a\,dx}{(ax+b)^4-c^4} = \frac{1}{a}\int \frac{du}{u^4-c^4}$$

let $u = ax-b$

$du = a\,dx$

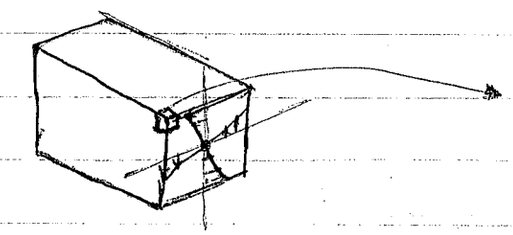
19 

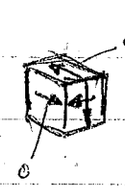
$T_1 + T_2 = T$




$\epsilon_1 = \epsilon_2$

## Solid Noncircular Members

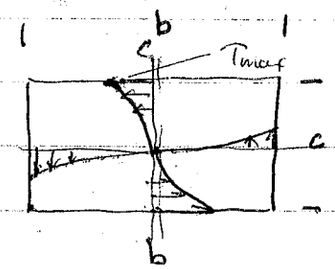





Stress at corner = 0

Strain at corner = 0

All assumptions are not valid.

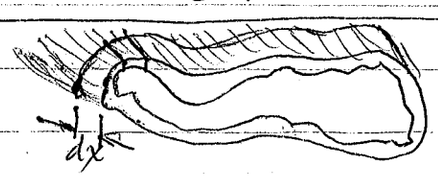


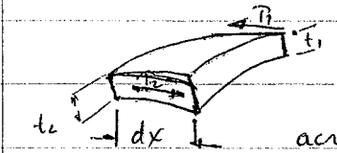

$\tau_{max} = \frac{T}{\alpha b c^2}$ $\quad \alpha = f(b/c)$

$\varphi = \frac{TL}{G(\beta b c^3)}$ $\quad \beta = f(b/c)$

Table on page 167.

## Thin-walled hollow members

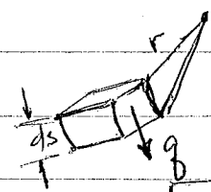





since $t$ is small assume that $\tau$ is constant across thickness

$\Sigma F_x = 0 \Rightarrow \tau_2 (t_2 dx) - \tau_1 (t_1 dx) = 0$

$\tau_1 t_1 = \tau_2 t_2 = q$ constant about perimeter. (shear flow) lb/in



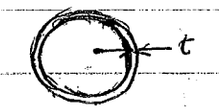

$\Sigma M_o = \oint q\, ds\, r = T \qquad q \oint r\, ds = T = q\, 2A_{enclosed}$

$$\tau = \frac{T}{2A_{enc}t}$$

$J = 2\pi c^3 t \qquad T = Tc/J$

$T/2\pi c^2 t = \tau \; \checkmark$

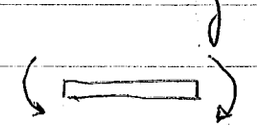
$$\tau = \frac{T}{2A_{enc}t} = \frac{T}{2(\pi c^2)t} \; \checkmark\checkmark$$

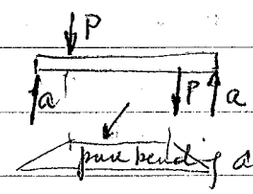


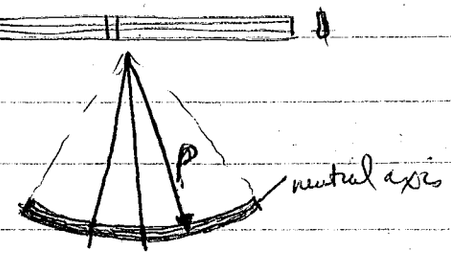

## Chapt 6 Flexural (Bending) Stresses in Beams

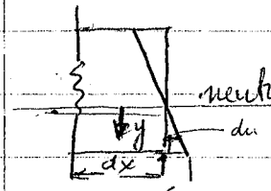
Straight beams. (homogeneous)

Pure bending = $V = 0 \Rightarrow M$ = constant







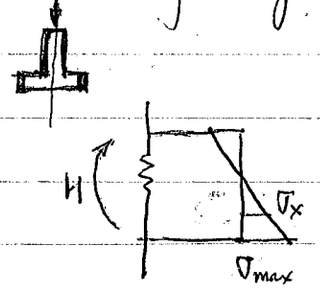

$$\epsilon_x = \frac{du}{dx} \qquad \epsilon_{max} = \frac{du_{max}}{dx}$$

$\epsilon_x = by$ b depends on geometry, moment

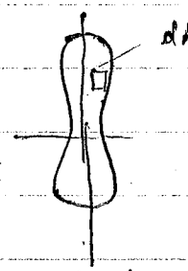
Suppose

1) Member has axis of symmetry

2) loading along axis of symmetry.




$\sigma_x = Eby$




$\Sigma F_x = 0$

$$\int_A \sigma_x dA = 0 \qquad Eb\int_A y dA$$

$$\int_A y dA = 0 \Rightarrow \text{Neutral axis} \equiv \text{centroid}$$

$$\frac{\int_A y dA}{\int_A dA} = \bar{y}$$

but since $\int_A y dA = 0$ $\quad \bar{y} = 0$

$\therefore$ neutral axis $\equiv$ centroid

$\Sigma M_L = 0$

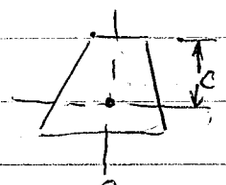
$$M + \int_A \sigma_x dA y = 0$$

$$M + Eb \int_A y^2 dA = 0 \qquad M = -Eb \int_A y^2 dA$$

$I_z = \int_A y^2 dA$ moment of inertia $\qquad I_y = \int_A z^2 dA$

$$\boxed{M = -EbI} \quad \text{since } \sigma_x = Eby \quad \boxed{\frac{\sigma_x I}{y} = -M} \quad \boxed{\sigma_x = \frac{-My}{I}}$$

call c largest distance from one of the free surfaces to NA.

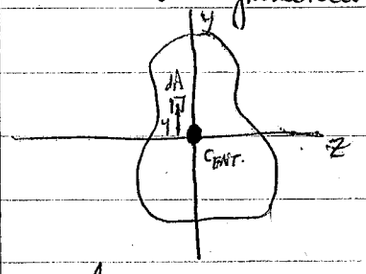



$\sigma_{max} = Mc/I \qquad \boxed{\sigma_{max} = -Mc/I}$

Basic Assumption

1) Deformation bases on linear distribution of strain,
2) Material properties relate stress – strain. (Hooke's law)
3) Equilibrium

Unsymmetrical Bending | either member doesn't have axis of symmetry
Load is not along axis of symmetry.

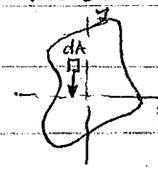



$M_z$

$$\int dA \sigma_x = \int Eby \, dA = F$$

$$My = 0$$

Submit on Tuesday we come back → Exam




$M_z \qquad \sigma_x dA \longrightarrow$ force

$$\int_{M_z} dM_z = 0 \Rightarrow M_z = -\int_A \sigma_x dA y$$

$\Sigma M_y = 0 \qquad 0 = \int_A \sigma_x \, dA z$

Suppose $\epsilon_x = by \qquad \sigma_x = Eby$ Define: $\int_A yz\,dA = I_{yz}$ product of inertia

$0 \stackrel{?}{=} Eb \int_A yz\,dA \qquad E \neq b \neq 0$ if $\int_A yz\,dA \neq 0$ the assumption is false

Define If $I_{yz} = 0$ y & z are principle axes.

If $I_{yz} \neq 0$ assumptions are false

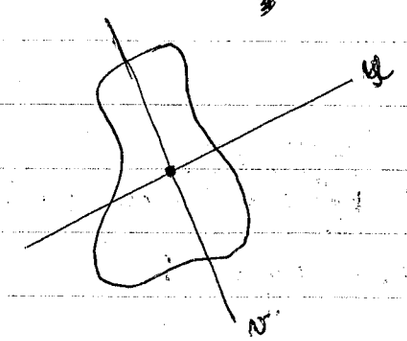
Cor: Axes of symmetry is a principle axis.

Centroidal Axis normal to a principle axis is itself principle

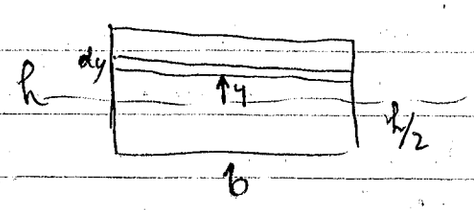
Suppose $I_{yz} \neq 0 \qquad \sigma_x = E(by + dz)$

$\Sigma M_y = 0 = \int_A \sigma_x dA z = E\int (by + dz) z\,dA = E\left[b\int yz\,dA + d\int z^2 dA\right]$

$E\left[bI_{yz} + dI_{yy}\right] = 0 \qquad E \neq 0$




principle axes for unsymmetric beams

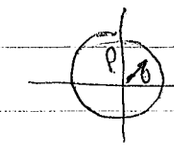



$dA = b\,dy$

$I_{zz} = b\int_A y^2 dy$

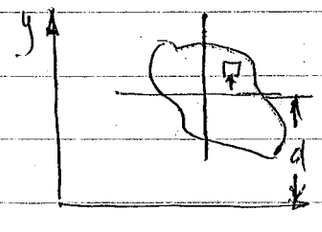
$I_{zz} = b/12\, h^3$

$I_{yy} = b^3h/12$



$I_{zz} = \int_A y^2 dA \quad ; \quad I_{yy} = \int_A z^2 dA$

$2I_{zz} = \int_A (y^2+z^2)\,dA = \int_A \rho^2 dA = J \qquad I_{zz} = I_{yy} = J/2$




$$I_{zz} = \int_A (y+d)^2 dA$$

$$\int_A y^2 dA + \int 2yd\,dA + \int d^2 dA$$

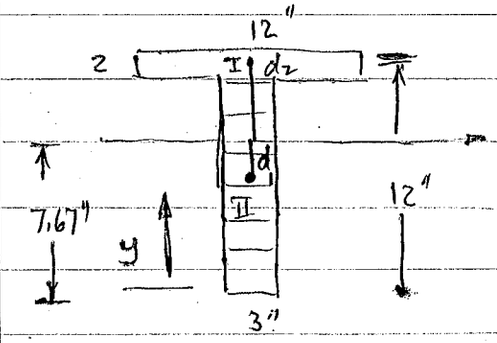
$$\int_A y^2 dA + 2d\int_A y\,dA + d^2\int_A dA$$

if y is measured from centroid $y=0 \Rightarrow \int y\,dA = 0$

$I_{zz} = I_0 + 2d(0) + d^2A$

The Parallel Axis Theorem:

$I_{zz} = I_0 + Ad^2$ ⟶ the fake area × $d^2$ from global axis
⟶ moment about its own centroid




$$I_{zz}_{total} = \sum I_{o_i} + \sum A_i d_i^2$$

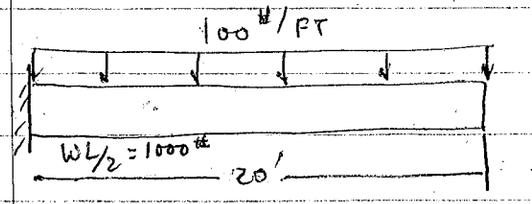
$$\bar{y} = \frac{\sum Ay}{\sum A}$$

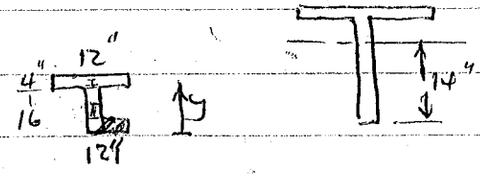
| | A | y | Ay | d | $Ad^2$ | $I_o$ |
|---|---|---|---|---|---|---|
| I | 24 | 11 | 264 | 3.33 | | |
| II | 30 | 5 | 150 | 2.67 | | |
| | $\sum 4$ | | 414 | | | |

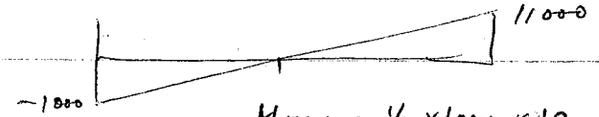
$\frac{69}{9} = \frac{414}{54} = \frac{23}{3}$

$= 7.67"$

| | $Ad^2$ | $I_o$ |
|---|---|---|
| I | $24\times3.3^2$ | $12\times2^3/12$ |
| II | $30\times$ | $3\times10^3/12$ |
| | $2.67^2$ | |






$M_{max} = \frac{1}{2} \times 1000 \times 10 = 5000$

$= \frac{1}{2} \times \frac{wL}{2} \times \frac{L}{2}$

| | A | y | Ay | d | $Ad^2$ | $I_o$ ($\frac{bh^3}{12}$) |
|---|---|---|---|---|---|---|
| I | 48 | 18 | 864 | 4" | 768 | 64 |
| II | $32$ | 8 | 256 | -6" | 1152 | $682\frac{2}{3}$ |
| | 80 | | 1120 | | 1920 | $746\frac{2}{3}$ |

$\bar{y} = \frac{\Sigma Ay}{\Sigma A}$

$I_{zz} = \Sigma (I_o + Ad^2)$

$\bar{y} = \frac{112}{8} = 14$

$I_{zz} = 2666\frac{2}{3}$

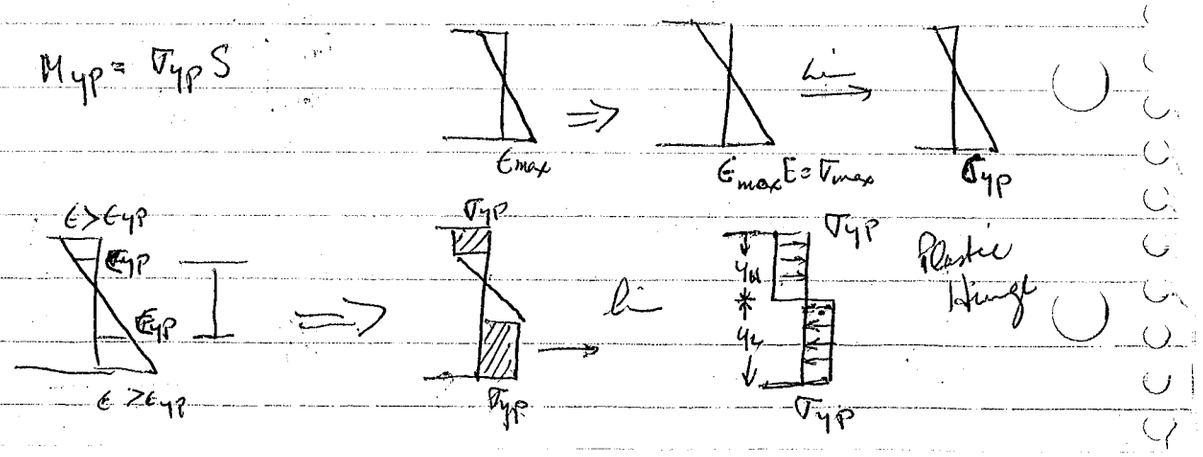
$\sigma_{max} = \frac{5000(12)14}{2666.67}$

$\sigma_{max} = \frac{5000(12)(6)}{2666.67}$

$\sigma_{max} = \frac{Mc}{I}$ let $\frac{I}{c} = S$ - section modulus

$\sigma_{max} = \frac{M}{S}$

$M_{yp} = \sigma_{yp} S$



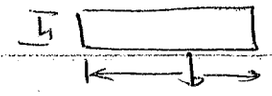
We must now find a new neutral axis since Hooke's law doesn't hold any more

$$\Sigma F_x = 0 \qquad \int_A \sigma_x \, dA = 0$$

$$\Sigma M_{\text{Neutral axis}} = 0 \qquad M_{\text{plast.}} + \int_A \sigma_x \, dA \, y = 0$$

$$\Sigma F_x = 0 = \int_A \sigma_x dA = \sigma_{yp} \int_{\text{lower area}} dA - \sigma_{yp} \int_{\text{upper area}} dA = \sigma_{yp} \left( \int_{y_L}^{0} dA - \int_{0}^{y_u} dA \right)$$

$$\int_{y_L}^{0} dA = \int_{0}^{y_u} dA$$ areas about & below the neutral axis are equal.



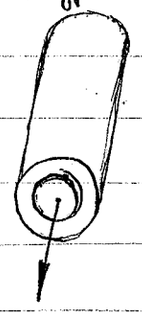
In rectangle

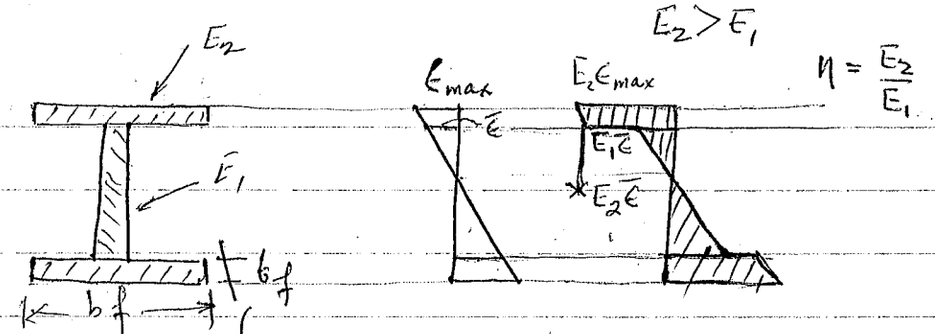
$$M_{plastic} = -\sigma_{yp} b \int_{-h/2}^{0} y \, dy + \sigma_{yp} b \int_{0}^{h/2} y \, dy$$

$$M_{plastic} = \frac{b h^2}{4} (\sigma_{yp})$$

$M_{plastic} = \sigma_{yp} Z$ $Z$ is the plastic modulus.

$\frac{M_{plastic}}{M_{yp}} \cdot \frac{Z}{S}$ = Shape factor.






$E_2 > E_1$

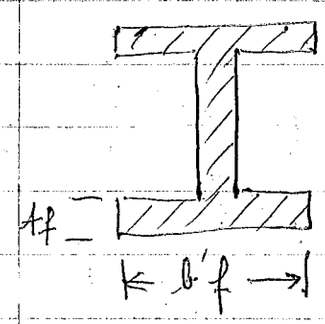
$\eta = \frac{E_2}{E_1}$

$$M = -\int_A \sigma_x \, dA(y) \qquad \sigma = \frac{My}{I}$$

Equivalent beam

web same.

$$F_{in\ flange} = \int_{A_{flange}} \sigma_x \, dA = \frac{E_2(\epsilon_{max} + \bar{\epsilon})}{2} t_f \, b_f$$




$$F'_{flange} = t_f \, b'_f \, \frac{E_1(\epsilon_{max} + \bar{\epsilon})}{2}$$

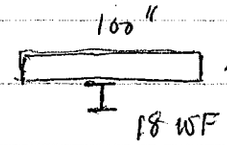
$$b'_f E_1 = b_f E_2$$

$$\boxed{b'_f = \eta \, b_f}$$

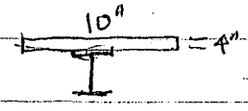
$\sigma_{max}$ in orginal flange = $\eta \, \sigma_{max}$ of equiv material

1)






$$\frac{E_s}{E_c} = 10 = \eta$$



| | A | y | $A_y$ | d | $Ad^2$ | $I_o$ |
|---|---|---|---|---|---|---|
| PL | 40 | 20 | 800 | 3 | 360 | 13.33 |
| WF | 14.71 | 9 | 132.39 | -8 | 942 | 800.6 |
| | 54.71 | | 932.39 | | 1302 | 853.93 |

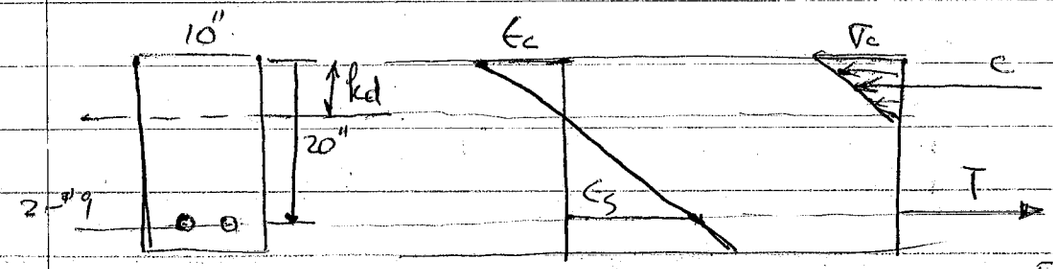
$$\bar{y} = \frac{932.39}{54.71} = 17$$

1) $\sigma^t_{max} = \frac{M}{I}\,17''$

2) $\sigma^c_{max} = \frac{M}{I}\,1''$

} WF

$$\sigma^c_{max_{conc}} = \frac{M}{I}\,\frac{5''}{n} = \frac{M}{2I} \quad \text{Concrete}$$




A = 1" sq. $\qquad n = \frac{E_s}{E_c} = 15$

$T = C \qquad \epsilon_s = \epsilon_c \frac{(20 - kd)}{kd}$

$$\epsilon_s = \frac{\sigma_s}{E_s} = \frac{T}{A_s E_s}\,; \quad \epsilon_c = \frac{\sigma_c}{E_c}\,; \quad \frac{T}{A_s E_s} = \frac{\sigma_c}{E_c}\,\frac{(20 - kd)}{kd}$$

$$T = \sigma_c\, n A_s\, \frac{(20 - kd)}{kd} \qquad T = \tfrac{1}{2}\sigma_c\, kd \cdot 10$$

$$T = 5\sigma_c\, kd$$

$nA_s = 30$

$$I = \sum\left(Ad^2 + I_o\right) \qquad \frac{bh^3}{12} = \frac{Ah^2}{12}$$

$$= \frac{10\,kd^3}{12} + \frac{10\,kd^3}{4} + \frac{30\cdot(1\tfrac{1}{8})^2}{12} \leftarrow \text{not needed.}$$

$$+ 30(20 - kd)^2 \qquad \sigma_c = \frac{M}{I}\,kd$$

$$5kd = \frac{30(20-kd)}{kd}$$

$$\nabla_s = \frac{H}{I}(20-kd)\eta = \frac{15H}{I}(20-kd)$$

$$T = A_s \nabla_s$$

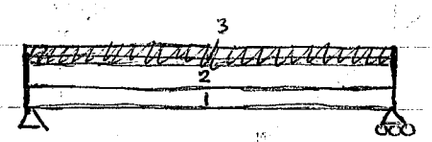
18.28
-2.94
-11.5
18.28
14.44
3.84

## Shearing Stresses in Beams.

Stress tensor $T = \begin{pmatrix} \nabla_x & \tau_{xy} \\ \tau_{yx} & 0 \end{pmatrix}$

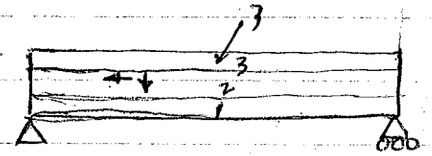
$$-\frac{dM}{dx} = V$$

$$dM = -V dx$$



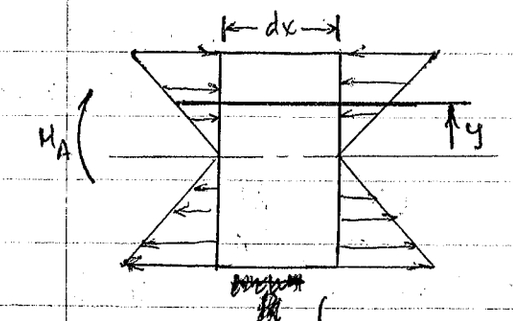


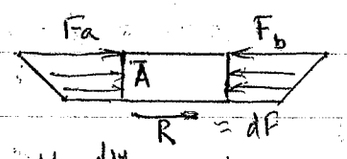

if bonded adequately
3 same laminar layer




if not bonded






$$F_a = \int_A \nabla_a dA \qquad \nabla_a = -\frac{M_A y}{I}$$

$$F_a = \int_A -\frac{M_A}{I}(y)dA = -\frac{M_A}{I}\int_{\bar{A}} y dA = -\frac{M_A}{I} Q_y$$

$Q_y = \int_{\bar{A}} y\,dA$ - statical moment of $\bar{A}$ about the neutral axis

$F_b = \frac{-M_B Q_y}{I}$ if $\bar{A}$ is constant across beam & $M_a = M_B$

$F_a \equiv F_b \quad R = 0$

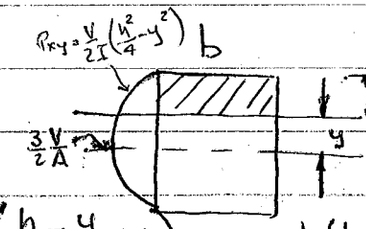
if $\bar{A}$ is constant across beam & $M_a \neq M_b$

$F_a \neq F_b \quad R \neq 0 \quad R = F_b - F_a \neq 0$

$dF = F_b - F_a = -\frac{(M_A + dM) Q_y}{I} + \frac{M_A Q_y}{I} = -\frac{dM\, Q_y}{I}$

$\frac{dF}{dx} = -\frac{dM}{dx}\frac{Q_y}{I}$ ; $\boxed{q = \frac{V Q_y}{I}}$ $q = \tau t$ Shear Flow

$\tau_{xy} = \frac{V Q_y}{I t}$




$Q = \int y\,dA$

$Q = b\left(\frac{h}{2} - y\right)\left(\frac{h}{4} - \frac{y}{2} + y\right) = b\left(\frac{h}{2} - y\right)\left(\frac{2y+h}{4}\right) = \frac{b}{2}\left(\frac{h-2y}{2}\right)\left(\frac{h+2y}{2}\right)$

$Q = \frac{b}{2}\left(\frac{h^2 - 4y^2}{4}\right)$ $\tau_{xy} = \frac{VQ}{Ib} = \frac{V}{2I}\left(\frac{h^2}{4} - y^2\right)$

$\tau_{xy} = 0 \Rightarrow y = \pm h/2$ $\tau_{max}$ $\partial y = 0$

$\tau_{max} = \frac{V}{2I}\frac{h^2}{4} = \frac{12Vh^2}{8bh^3} = \frac{3}{2}\frac{V}{A}$

$\tau = \begin{pmatrix} \tau_x & \tau_{xy} \\ \tau_{yx} & 0 \end{pmatrix} = \begin{pmatrix} My & \frac{V}{2I}\left(\frac{h^2}{4} - y^2\right) \\ \frac{V}{2I}\left(\frac{h^2}{4} - y^2\right) & 0 \end{pmatrix}$

Prove $\int_A \tau_{xy}\, dA = V = b\int_{-h/2}^{h/2} \frac{V}{2I}\left(\frac{h^2}{4} - y^2\right) dy = \frac{Vb}{2I}\left[\frac{h^2}{4}y - \frac{y^3}{3}\right]_{-h/2}^{h/2}$

$\frac{Vb}{2I}\left[\frac{h^3}{4} - \frac{h^3}{12}\right]$

$$\frac{Vb}{24I}\left[2h^3\right] = \frac{2V}{2I}\cdot\frac{bh^3}{12} = V$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \qquad \sigma_x = -\frac{My}{I}$$

$$\frac{\partial \sigma_x}{\partial x} = -\frac{y}{I}\frac{\partial M}{\partial x} = \frac{Vy}{I}$$

$$\frac{\partial \tau_{xy}}{\partial y} = -\frac{Vy}{I}$$

$$\tau_{xy} = -\frac{V}{I}\int y\,dy \quad \text{if} \quad \tau_{xy} \text{ is } f(y) \text{ only}$$

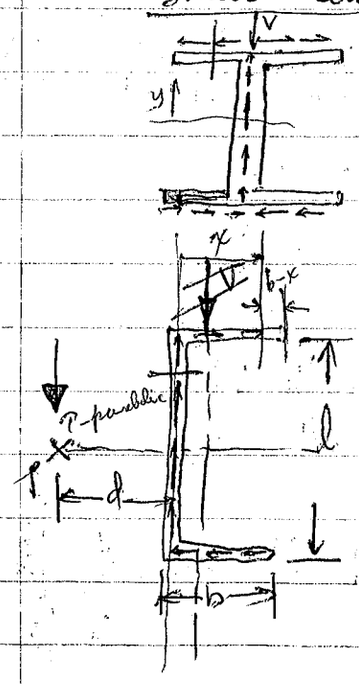
$$\tau_{xy} = -\frac{V}{I}\int y\,dy = -\frac{V}{I}\frac{y^2}{2} + C_1 \qquad @\ y = \pm\frac{h}{2} \text{ since } \tau_{xy} \text{ at free surfaces are } = 0$$

(if symmetry about neutral axis)

$$C_1 = \frac{Vh^2}{8I} \quad \text{if} \quad \text{body is rectangular in nature}$$

$$\tau_{xy} = \frac{V}{2I}\left(\frac{h^2}{4} - y^2\right) \qquad V = f(x) \quad \tau_{xy}$$

## Shear Center




since y is constant throughout the flange the only variable in Q is dA. ∴ Q is linear

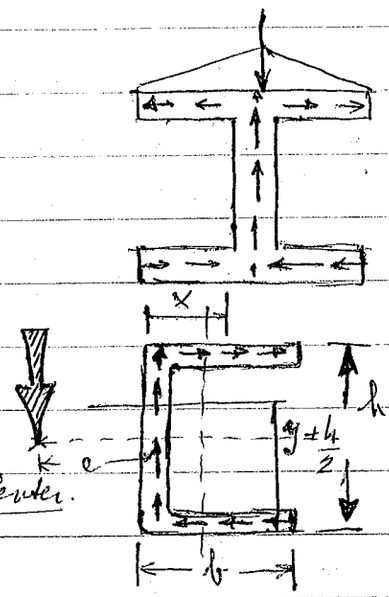
$\tau_{xy}$ is thus linear. Vertical, Horizontal & Rotational equil are satisfied.

torsion will occur by unbalanced shears in flange & shear $\int V$ of web

to remove torsion there exists a pt ∋

$Vd = \tau \cancel{A} l$ p is a shear center.

Assume thin walled member




T is linear in flanges, T parabolic in web

thin member. $t \ll h$, $t \ll b$

in flange: $Q = (b-x)\,t\,y\left(\frac{h}{2}\right)$

$q = \frac{V}{I}\,\frac{th}{2}(b-x)$

$Q_a = bht/2$

$q_a = \frac{V}{I}\,\frac{thb}{2}$



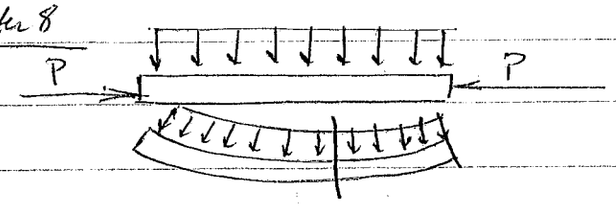

in web: $Q = Q_a + \left(\frac{h}{2}-y\right)t\left(y+\frac{\frac{h}{2}-y}{2}\right)$

$q = q_a + \frac{V}{I}\left(\frac{t}{2}\right)\left(\frac{h}{2}-y\right)\left(\frac{h}{2}+y\right) = q_a + \frac{V}{I}\left(\frac{t}{2}\right)\left[\frac{h^2}{4}-y^2\right]$

$V = \int_A q\,dy$ $\qquad F_{flanges} = \frac{1}{2}q_a b$ $\qquad T = \frac{1}{2}q_a b\,h = Ve$

$e = \frac{q_a b h}{2V} = \frac{th^2b^2}{4I}$

Chapter 8

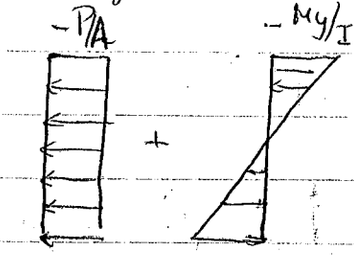



this must change to $P_y + H$ to account for superposition of bending



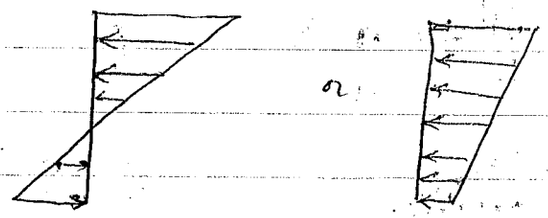
if $\bar{y}$ is small $\Rightarrow P y \ll M$ then we can superimpose

If bar is allowed to bend elastically, $\bar{y}$ will be small, P must be small enough $\Rightarrow$ buckling doesn't occur.

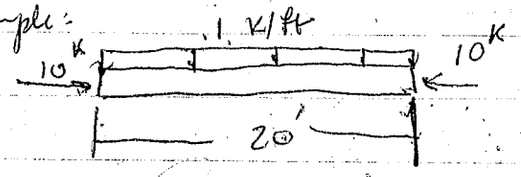



$$\sigma_x = -P/A - \frac{My}{I}$$

=

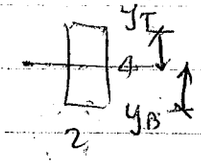



Example:




$$M_{max} = \frac{wL^2}{8} = \frac{1 \times 400}{8} = 50 \text{ k-ft}$$

$$\sigma_x = -\frac{10}{8} - \frac{60}{10,67} y$$



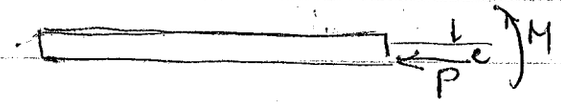

$$I = \frac{2 \cdot 4^3}{12} = 10,67$$

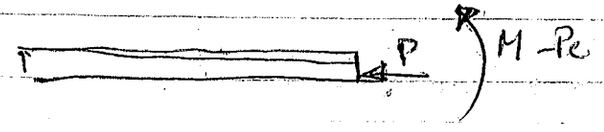
$M =$

$$\sigma_T = \frac{-10}{8} \quad \frac{-120}{10.67} =$$

$$\sigma_B = -\frac{10}{8} + \frac{120}{10.67} =$$

$$\boxed{\sigma_i = -F/A - \frac{Mc}{I}}$$

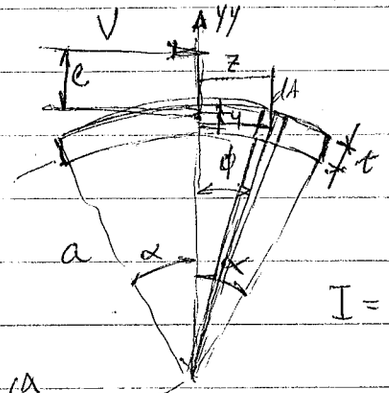


$\sigma = -\frac{P}{A} \pm \frac{[M^*c]}{I}$ where $M^* = M - Pe$

In concrete $\sigma \begin{cases} \leq 0 \\ \geq f_c \end{cases}$

7-28




$dA = a\,t\,d\phi$, $z = a\sin\phi$

$q = \frac{VQ}{I}$ $\quad y = a(1-\cos\phi)$

$I = \int_A z^2 dA$ $\quad Q = \int_\phi^\alpha z\,dA$

$Q = \int_\phi^\alpha a\sin\phi\; a\,t\,d\phi = a^2 t \int \sin\phi\,d\phi = -a^2 t\cos(\alpha-\phi)$

$I = \int_{-\alpha}^{\alpha} a^2\sin^2\phi\; a\,t\,d\phi = a^3 t \int_{-\alpha}^{\alpha} \left(\frac{1-\cos 2\phi}{2}\right) d\phi$

$\frac{a^3 t}{2}\left(\alpha - \frac{1}{2}\sin 2\phi\right)_{-\alpha}^{\alpha} = \frac{a^3 t}{2}(2\alpha - \sin 2\alpha) = a^3 t(\alpha - \sin\alpha$

$q = \frac{V\left(-a^2 t\cos(\alpha-\phi)\right)}{\frac{a^3 t}{2}(2\alpha - \sin 2\alpha)} = -\frac{2V}{a}\frac{(\cos(\alpha-\phi))}{2\alpha - \sin 2\alpha}$

$T = \int q\,a\,d\phi\; y = \int_{-\alpha}^{\alpha} -\frac{2V\cos(\alpha-\phi)}{2\alpha-\sin 2\alpha}\, a(1-\cos\phi)\,d\phi$

$= \frac{-2Va}{2\alpha-\sin 2\alpha}\int_{-\alpha}^{\alpha} \cos(\alpha-\phi)(1-\cos\phi)\,d\phi =$

$\int_{-\alpha}^{\alpha}(\cos\alpha\cos\phi + \sin\alpha\sin\phi)(1-\cos\phi)\,d\phi$

$$\cos\alpha\cos\phi + \sin\alpha\sin\phi - \cos\alpha\left(\frac{1+\cos 2\phi}{2}\right) - \sin\alpha\frac{\sin 2\phi}{2}$$

$$T + Ve = 0 \qquad C = \frac{-T}{V}$$

$$\left(+\cos\alpha\sin\phi + \sin\alpha\cos\phi - \cos\alpha\left[\frac{\phi}{2} + \frac{\sin 2\phi}{4}\right] + \sin\alpha\left\{\frac{\cos 2\phi}{4}\right)\right|_{-\alpha}^{\alpha}$$

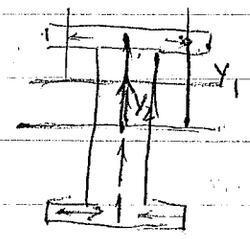
7-21

Capacity - that load at which the max allowable shear is reached.

$q_y \neq q_z$ but $\tau_y = \tau_z$

$I_z = 166$

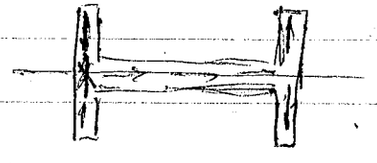
$I_y = 36.5$




$$\tau_{max_1} = \frac{V_z}{I_z}\frac{Q_z}{t_z} \qquad Q_z = 6\cdot1\cdot\left(4-\tfrac{1}{2}\right) + 3\cdot1\cdot1.5 = 25.5$$

$$t_z = 1$$

$$\frac{V_z}{166}\ \frac{25.5}{1.}$$



$$\tau_{max} = \frac{V_y}{I_y}\frac{Q_y}{t_y}$$

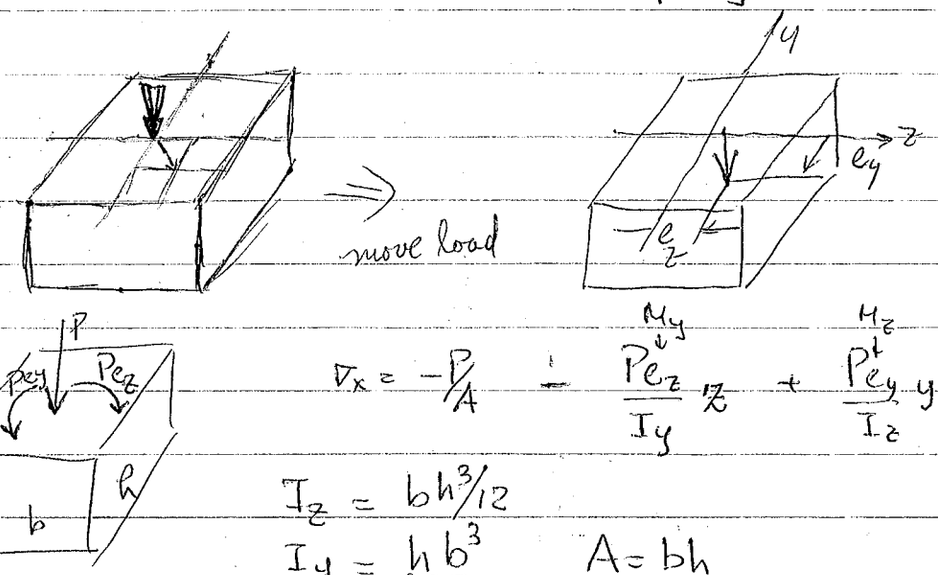
$$Q_y = 3\cdot1\cdot1.5 + 3\cdot1\cdot1.5 + 6\cdot.5\cdot.25$$

$$9.75$$

$$\tau_{max_2} = \frac{V_y}{I_y}\frac{9.75}{2} \qquad t_z = \text{thickness of each flange}$$

$\tau_{max_1} = \tau_{max_2}$

$$\frac{V_y}{I_y}\ \frac{9.75}{2} = \frac{V_z}{I_z}\ \frac{25.1}{1} \qquad V_y = \frac{I_y}{I_z}\frac{Q_z}{Q_y}\frac{t_z}{t_y}V_z$$




$$\sigma_x = -\frac{P}{A} - \frac{Pe_z}{I_y}z + \frac{Pe_y}{I_z}y \leq 0$$

$$I_z = bh^3/12$$

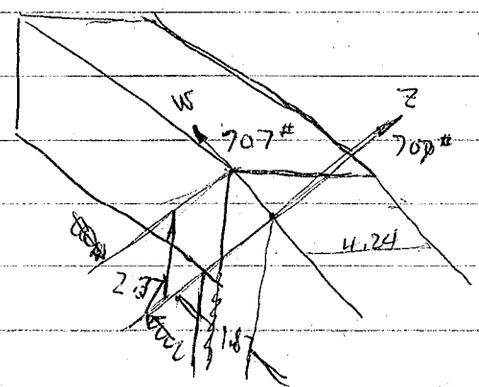
$$I_y = \frac{hb^3}{12} \qquad A = bh$$

if $\sigma_x = 0$ then $-1 - \frac{e_z z}{b^2/12} + \frac{e_y y}{h^2/12} = 0$

$$\pm \frac{e_z\, b/2}{b^2/12} \pm \frac{e_y\, h/2}{h^2/12} = 1 \qquad \pm \frac{e_z}{b/6} \pm \frac{e_y}{h/6} = 1$$

defines delineated area called here

8-21




$I_{zz} = 8.0 = Ar_z^2$

$I_{ww} = 31.8$

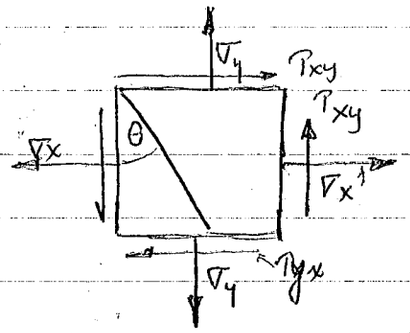
$M_z = M_w = .707 \times 70.7 \text{ or } 50^k$

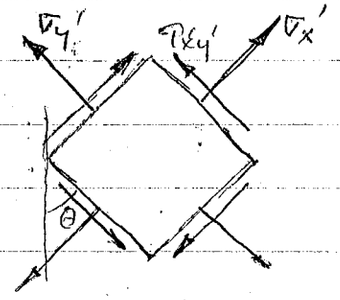
$$\sigma_A = \frac{50 \times 4.24}{31.8} + \frac{50 \times 1.87}{8.0}$$

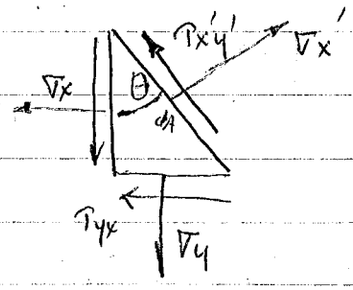
$$-\sigma_B = \frac{50 \times 2.37}{8.0} =$$

$$\sigma_C = \frac{50 \times 1.87}{8.0} - \frac{50 \times 4.24}{31.8}$$

# Transformation of Plane Stress




$\Sigma F_{x'} = 0$

$\sigma_x' dA - \sigma_x dA\cos^2\theta - \sigma_y dA\sin^2\theta + \tau_{xy}\left(-2dA\sin\theta\cos\theta\right) = 0$

$\sigma_x' = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + \tau_{xy}\sin 2\theta$

$\sigma_x' = \sigma_x\left(\frac{1+\cos 2\theta}{2}\right) + \sigma_y\left(\frac{1-\cos 2\theta}{2}\right) + \tau_{xy}\sin 2\theta$

$$\boxed{\sigma_x' = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta}$$

$\frac{\partial \sigma_x'}{\partial \theta} = -2\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + 2\tau_{xy}\cos 2\theta$

$(\sigma_y-\sigma_x)\sin 2\theta + 2\tau_{xy}\cos 2\theta = 0$

$\tan 2\theta_{1,2} = 2\tau_{xy}/(\sigma_x-\sigma_y)$ ∴ (A)

$\Sigma F_{y'} = 0$ $\quad \tau_{x'y'}dA + \sigma_x dA\cos\theta\sin\theta - \sigma_y dA\sin\theta\cos\theta + \tau_{xy}\left(dA\cos^2\theta - dA\sin^2\theta\right) = 0$

$\tau_{x'y'} = -\sigma_x\frac{\sin 2\theta}{2} + \sigma_y\frac{\sin 2\theta}{2} + \tau_{xy}\cos 2\theta$

$$\boxed{\tau_{x'y'} = -\frac{(\sigma_x-\sigma_y)}{2}\sin 2\theta + \tau_{xy}\cos 2\theta}$$

$\frac{\partial \tau_{x'y'}}{\partial \theta} = -(\sigma_x-\sigma_y)\cos 2\theta - 2\tau_{xy}\sin 2\theta = 0$

$\therefore \left(\frac{2\tau_{xy}}{\sigma_x-\sigma_y}\right)^{-1} = \tan 2\theta_s$ (B)

$$\sigma_{y'} = \frac{\sigma_x+\sigma_y}{2} - \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$\sigma_{x'}+\sigma_{y'} = \sigma_x+\sigma_y$ if $\sigma_{x'} = \max$ $\sigma_{y'} = \min$

$\sigma_{x'_{max}}$ $\perp$ $\sigma_{y'_{min}}$

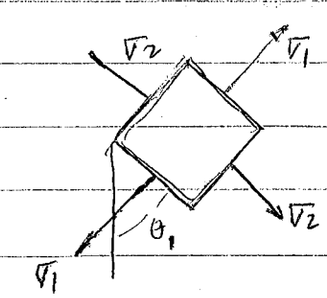
if $\sigma_{x_{\substack{max\\min}}}$ $\tau_{xy} = 0$

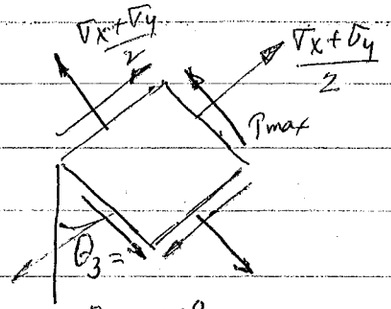
(A) $\sigma_{1,2}$ (principal stresses) $= \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$

$\sigma_1+\sigma_2 = \sigma_x+\sigma_y$

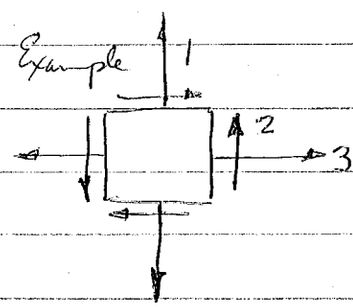
(B) $\tau_{\substack{max\\min}} = \pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = \pm\frac{\sigma_1-\sigma_2}{2}$

when $\tau_{max} = \frac{\sigma_x+\sigma_y}{2}$

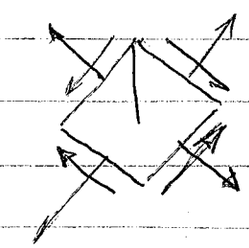



Example




Suppose $\theta = -22.5°$



$\sigma_{x'} = 2 + .707 - 1.414$

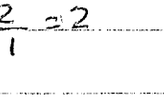
$= 1.293$

$\sigma_{y'} = 2.707$

$\tau_{x'y'} = +.707 + 2 \times .707 = 2.12$

at $-22.5°$

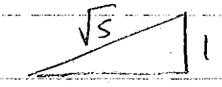
Principal shearers $\tan 2\theta_1 = \frac{2}{1} = 2$




$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 2 \pm \sqrt{1+4} = 2 \pm \sqrt{5}$$

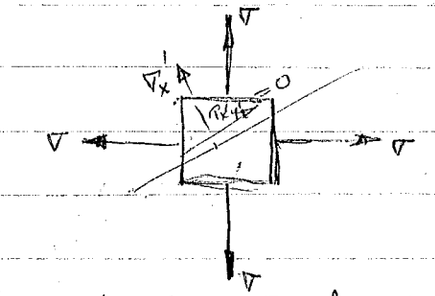
$$\tau_{1,2} = -1\ \frac{2}{\sqrt{5}} + 2 \times \frac{1}{\sqrt{5}} = 0$$

$\tan 2\theta_3 = -\frac{1}{2}$




$$\sigma_3 = 2 + 1 \cdot \frac{2}{\sqrt{5}} - 2 \cdot \frac{1}{\sqrt{5}} = 2$$

$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \sqrt{5}$



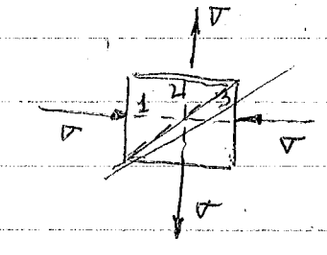

principal stresses no shear

$\sigma_x' = \frac{\sigma+\sigma}{2} + \frac{\sigma-\sigma}{2}(\quad) + \tau_{xy}'(\quad)$

$\sigma_x' = \sigma$ irregardless of angle

$\tau_{x'y'} = 0$

degenerate case $\sigma$ stays constant at any theta, fluid cannot sustain shear $\therefore$ must be in a state $\Rightarrow \sigma_{x'} = \sigma_x = \sigma_y$

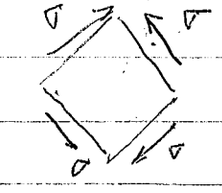



pure shear $\sigma_x' = \underbrace{\frac{\sigma_x+\sigma_z}{2}}_{0} + \underbrace{\frac{\sigma_x-\sigma_y}{2}}_{-\sigma} \cos 2\theta + \tau_{xy} \sin 2\theta$

$\sigma_y' = -\sigma_x' + \tau_{xy}\sin 2\theta$

$\tau_{x'y'} = \sigma \sin 2\theta + \tau_{xy} \cos 2\theta$

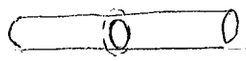
$\tau_{max} = \sigma$



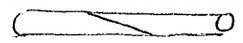

if weak in tension breaks at face 1

if weak in compression breaks at plane 2

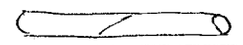
if " " shear breaks at phase 3



if weak in torque shear 2 planes shear



if weak in tension buckling at 45° angle



if weak in compression buckling at 45° angle

## Mohr's circle of Stress

$$\sigma_{x'} - \left(\frac{\sigma_x+\sigma_y}{2}\right) = \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

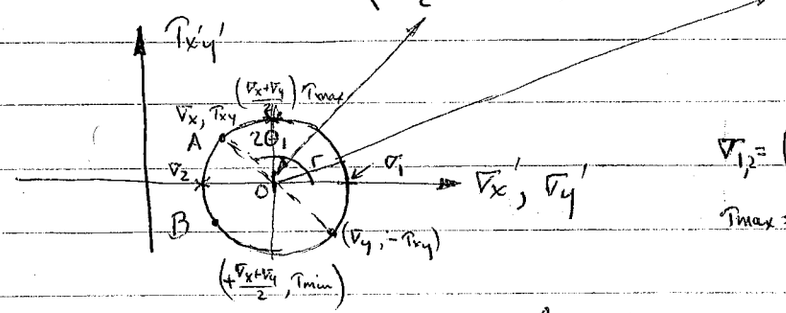
$$\tau_{x'y'} = \frac{\sigma_y-\sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\left(\sigma_{x'} - \frac{\sigma_x+\sigma_y}{2}\right)^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2\cos^2 2\theta + 2\tau_{xy}\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta \sin 2\theta + \tau_{xy}^2\sin^2 2\theta$$

$$\tau_{x'y'}^2 = \left(\frac{\sigma_y-\sigma_x}{2}\right)^2\sin^2 2\theta + 2\tau_{xy}\cos 2\theta\left(\frac{\sigma_y-\sigma_x}{2}\right)\sin 2\theta + \tau_{xy}^2\cos^2 2\theta$$

$$\left(\sigma_{x'} - \left(\frac{\sigma_x+\sigma_y}{2}\right)\right)^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_y-\sigma_x}{2}\right)^2 + \tau_{xy}^2$$

center of circle is at $\left(\frac{\sigma_x+\sigma_y}{2}, 0\right)$ Radius $= \sqrt{\left(\frac{\sigma_y-\sigma_x}{2}\right)^2+\tau_{xy}^2} = \tau_{max}$



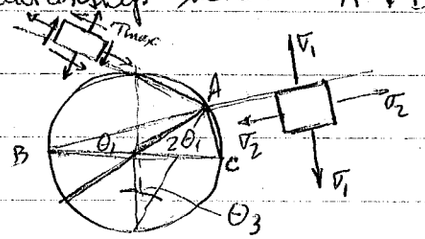

$$\sigma_{1,2} = \left(\frac{\sigma_x+\sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

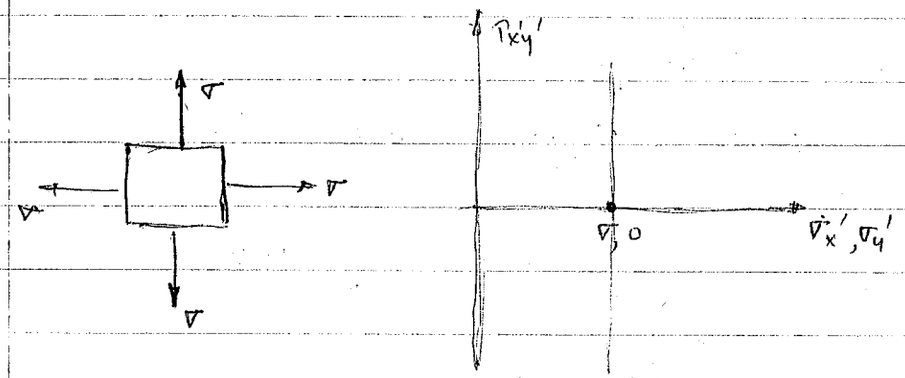
$$\tau_{max} = \sqrt{\left(\frac{\sigma_y-\sigma_x}{2}\right)^2+\tau_{xy}^2}$$

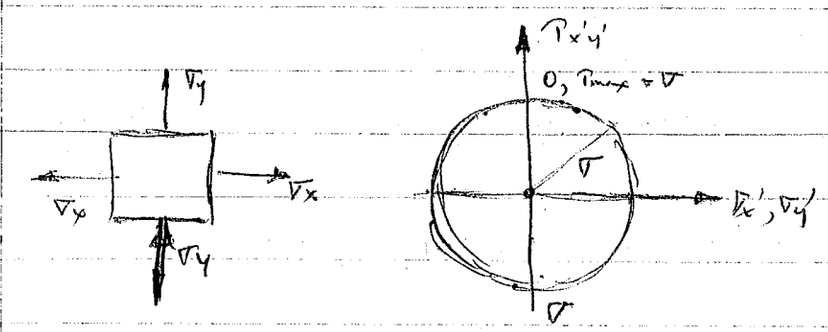
knowing $\sigma_x$, $\sigma_y$ & $\tau_{xy}$: locate center of circle, O @ $\left(\frac{\sigma_x+\sigma_y}{2}, 0\right)$; locate A @ $(\sigma_x, \tau_{xy})$

Construct Mohrs' circle, with center @ O & radius $\overline{OA}$

Find a relationship between A & B



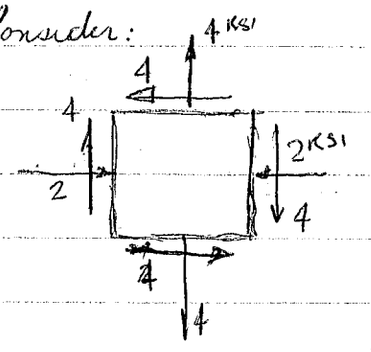







Consider:

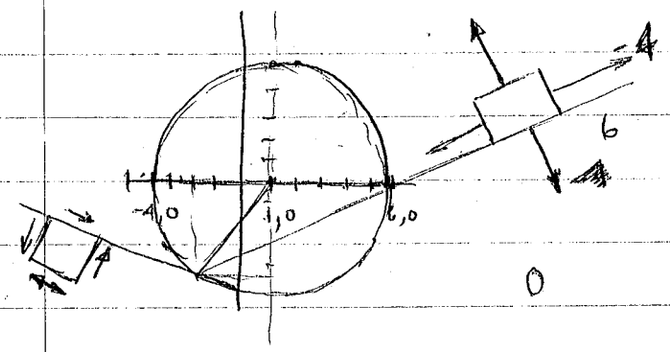



$$\tan 2\theta_{1,2} = \frac{-4(2)}{-2-4} = \frac{4}{3}$$




$$\sigma_{1,2} = \frac{4-2}{2} \pm \sqrt{\left(\frac{4-(-2)}{2}\right)^2 + (-4)^2} = \begin{cases} 6 \\ -4 \end{cases}$$

$$\tau_{max} = \pm 5 \qquad \sigma_{\tau_{max}} = \frac{\sigma_1 + \sigma_2}{2} = 1$$



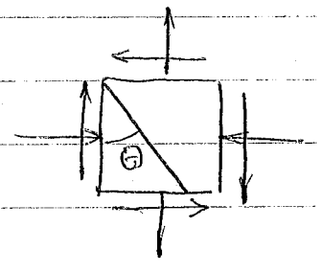

$\sigma_1 = 1+5 = 6$

$\sigma_2 = 1-5 = -4$

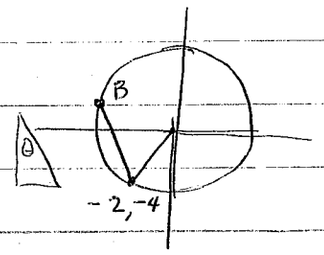
$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 5$

$\tau_{min} = -5$

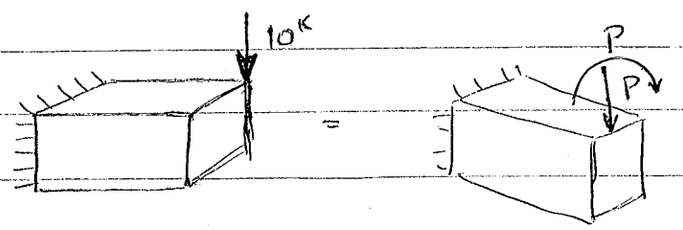
0

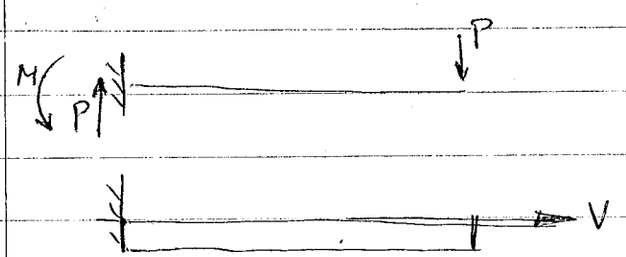



to find stresses at angle θ from vertical face draw a line ∥ to cut and where it intersects circle is it.

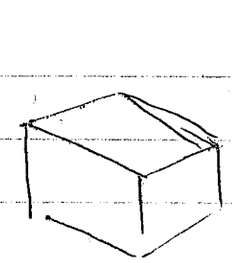


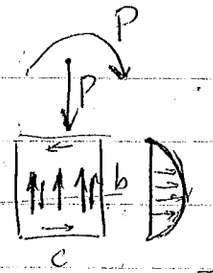

8-29




$P = 10^K$ $T = 10^K\text{-in}$



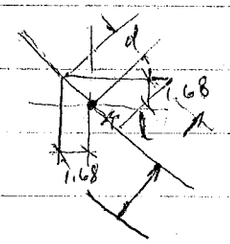



$$\tau = \frac{3}{2}\frac{V}{A}$$

$$\tau_T = \frac{T}{\alpha b c^2}$$

$$\tau_{max} = \frac{3}{2}\frac{V}{A} + \frac{T}{\alpha b c^2} = \frac{3}{2}\frac{10}{4 \cdot 2} + \frac{10}{(.246)(4)(2)^2}$$

8-21.



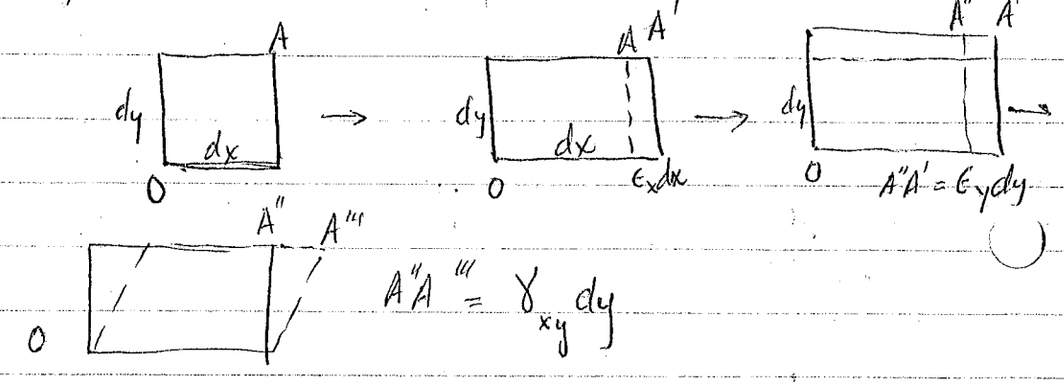

$$d = \sqrt{2 \times 1.68^2} = 2.37$$

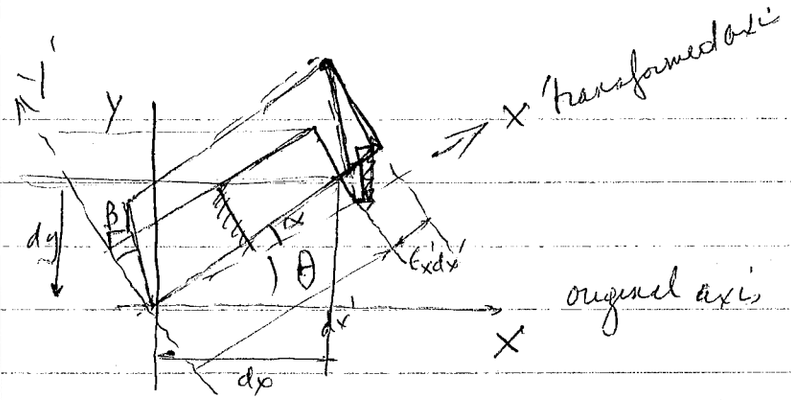
$$K = \sqrt{18} - 2.37 = 1.87$$

$$l = 4.24$$

Transformation of Strain

$$\begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} \\ \epsilon_{xy} & \epsilon_{yy} \end{pmatrix}$$






$$\epsilon_x' dx' = AA' \cos\theta + A'A'' \sin\theta + A''A''' \cos\theta$$

$$\epsilon_x' = \epsilon_x \frac{dx}{dx'} \cos\theta + \epsilon_y \frac{dy}{dx'} \sin\theta + \gamma_{xy} \frac{dy}{dx'} \cos\theta$$

$$= \epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + \gamma_{xy} \frac{\sin 2\theta}{2}$$

$$\left(\frac{1+\cos 2\theta}{2}\right) \quad \left(\frac{1-\cos 2\theta}{2}\right)$$

$$\epsilon_x' = \frac{\epsilon_x + \epsilon_y}{2} + \left(\frac{\epsilon_x - \epsilon_y}{2}\right)\cos 2\theta + \gamma_{xy} \frac{\sin 2\theta}{2} \; ; \quad \text{but } \epsilon_{xy} = \frac{\gamma_{xy}}{2}$$

therefore $$\boxed{\epsilon_x' = \frac{\epsilon_x + \epsilon_y}{2} + \left(\epsilon_x - \epsilon_y\right)\frac{\cos 2\theta}{2} + \epsilon_{xy} \sin 2\theta}$$

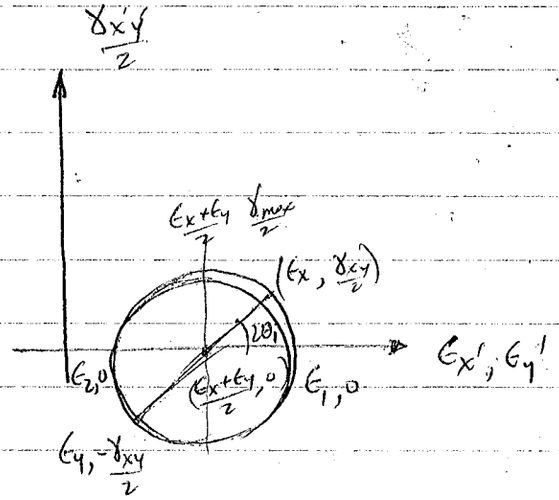
$\gamma_{xy'} = \alpha + \beta$ $\qquad \alpha \approx \tan\alpha = -\dfrac{AA' \sin\theta + A'A'' \cos\theta - A''A''' \sin\theta}{dx'}$

$$\alpha = -\epsilon_x \frac{\sin 2\theta}{2} + \epsilon_y \frac{\cos 2\theta}{2} - \gamma_{xy} \sin^2\theta$$

$$\beta = -\epsilon_x \frac{\sin 2\theta}{2} + \epsilon_y \frac{\cos 2\theta}{2} + \gamma_{xy} \cos^2\theta$$

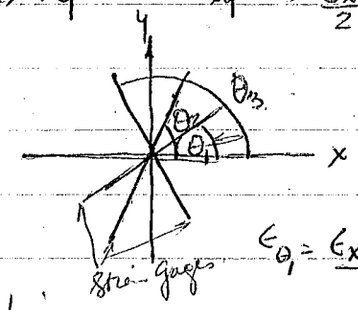
$$\gamma_{x'y'} = \alpha + \beta \quad -\left(\epsilon_x - \epsilon_y\right) \sin 2\theta + \gamma_{xy} \cos 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = \tau_{x'y'} = -\frac{\left(\epsilon_x - \epsilon_y\right)}{2} \sin 2\theta + \frac{\gamma_{xy}}{2} \cos 2\theta$$




$\sigma_x \Longleftrightarrow \epsilon_x \qquad \sigma_y \Longleftrightarrow \epsilon_y \qquad \tau_{xy} \Longleftrightarrow \frac{\gamma_{xy}}{2}$

$$\begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{pmatrix}$$




$$\epsilon_\theta = \frac{\epsilon_x+\epsilon_y}{2} + \left(\frac{\epsilon_x-\epsilon_y}{2}\right)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

Start with

$$\epsilon_{\theta_2} = \ldots + \ldots \cos 2\theta_2 + \ldots \sin 2\theta_2$$

$$\epsilon_{\theta_3} = \ldots + \ldots \cos 2\theta_3 + \ldots \sin 2\theta_3$$

$\sigma_z = 0$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu}{E}\sigma_z \quad (\sigma_z \to 0)$$

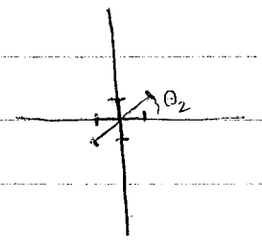
$$\epsilon_y = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E}$$

$$\epsilon_x + \nu\epsilon_y = \frac{(1-\nu^2)}{E}\sigma_x$$

$$\boxed{\sigma_x = \frac{(\epsilon_x + \nu\epsilon_y)E}{1-\nu^2}}$$

$$\boxed{\tau_{xy} = G\,\gamma_{xy}}$$

If $E, G$ are known

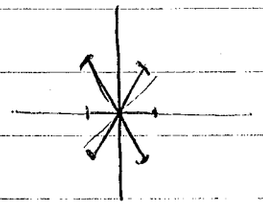



$\theta_1 = 0^\circ \quad \theta_2 = 45^\circ \quad \theta_3 = 90^\circ$

$\epsilon_{\theta_1} = \epsilon_x$

$\epsilon_{\theta_3} = \epsilon_y \qquad \epsilon_{45^\circ} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\gamma_{xy}}{2}$

$\gamma_{xy} = 2\epsilon_{45^\circ} - (\epsilon_{0^\circ} + \epsilon_{90^\circ})$



$\theta_1 = 0^\circ \quad \theta_2 = 60^\circ \quad \theta_3 = 120^\circ$

$\epsilon_{0^\circ} = \epsilon_x \qquad \epsilon_{60^\circ} = \frac{\epsilon_x + \epsilon_y}{2} - \left(\frac{\epsilon_x - \epsilon_y}{4}\right) + \frac{\gamma_{xy}\sqrt{3}}{4}$

$\epsilon_{120^\circ} = \frac{\epsilon_x + \epsilon_y}{2} - \left(\frac{\epsilon_x - \epsilon_y}{4}\right) - \frac{\gamma_{xy}\sqrt{3}}{4}$

$\epsilon_0 = -.0005 \qquad \nu = .3$

$\epsilon_{45^\circ} = .0002 \qquad E = 30 \times 10^6 \text{ psi}$

$\epsilon_{90^\circ} = .0003$

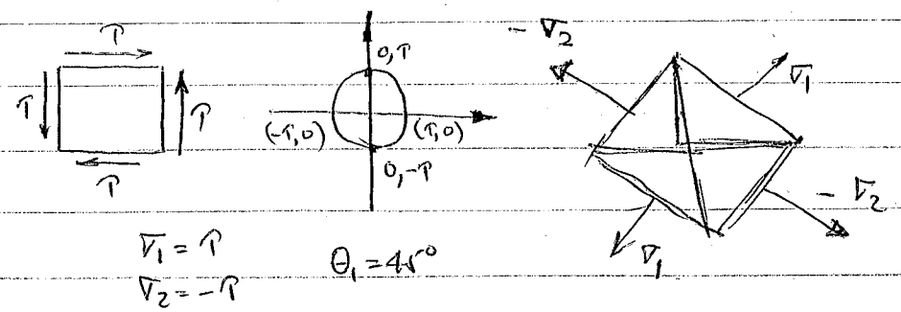
$\epsilon_x = -.0005 \qquad \sigma_x = \frac{-.0005 + (.3)(.0003)}{.91} 30 \times 10^6$

$\epsilon_y = .0003$

$\sigma_y = \frac{.0003 - .0005}{1.91} 30 \times 10^6$

$\gamma_{xy} = 2(.0002) - (-.0005 + .0003)$

$\tau_{xy} = G(\gamma_{xy})$




$\sigma_1 = \tau$

$\sigma_2 = -\tau$

$\theta_1 = 45^\circ$

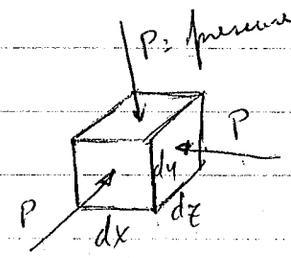
$\sigma_x = \sigma_y = 0$

$\gamma_{xy} \neq 0$

$$\epsilon_{45^\circ} = \frac{\epsilon_x + \epsilon_y}{2}^{0} + \frac{\epsilon_x - \epsilon_y}{2}^{0} \cos 2\theta_1 + \frac{\gamma_{xy}}{2}^{1} \sin 2\theta_1$$

$$\epsilon_{45^\circ} = \frac{\gamma_{xy}}{2} = \frac{\tau_{xy}}{2G} = \frac{\sigma_1}{E} - \nu\frac{\sigma_2}{E} = \left(\frac{1+\nu}{E}\right)\tau_{xy}$$

$$\frac{\tau_{xy}}{2G} = \frac{1+\nu}{E}\tau_{xy} \qquad G = \frac{E}{2(1+\nu)}$$




$$\text{Bulk Modulus} = K = \frac{\text{compressive stress}}{\text{decrease in volume}}$$

$$E = \frac{\sigma}{\epsilon}$$

$$\text{Change in volume} = (1+\epsilon_x)dx \cdot (1+\epsilon_y)dy \cdot (1-\epsilon_z)dz$$

$$- dx\,dy\,dz = \left[(1+\epsilon_x)(1+\epsilon_y)(1+\epsilon_z) - 1\right] dx\,dy\,dz$$

Assume: second order changes are 0.

$$\approx (\epsilon_x + \epsilon_y + \epsilon_z)\,dx\,dy\,dz$$

However,

$$\text{dilatation} = e = \frac{\text{change in volume}}{\text{original vol}} = \epsilon_x + \epsilon_y + \epsilon_z$$

dilatation is an invariant since $\Delta V$, $V_{orig}$ do not change with change in coordinate system.

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$e = \epsilon_x + \epsilon_y + \epsilon_z = \frac{(1-2\nu)}{E}\sigma_x + \left(\frac{1-2\nu}{E}\right)\sigma_y + \left(\frac{1-2\nu}{E}\right)\sigma_z$$

$\sigma_x + \sigma_y + \sigma_z$ is also an invariant

Suppose $\sigma_x = \sigma_y = \sigma_z = -p$

$$e = \epsilon_x + \epsilon_y + \epsilon_z = \left(\frac{1-2\nu}{E}\right)(-3p)$$

Compressive stress $= -p$

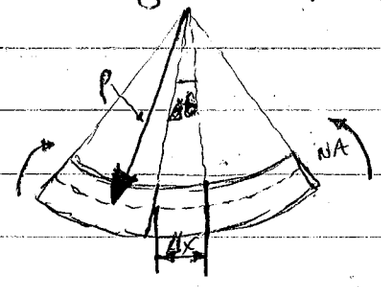
decrease in volume $= e$ $\qquad K = \dfrac{-p}{e} = \dfrac{E}{3(1-2\nu)}$ $\qquad \boxed{K = \dfrac{E}{3(1-2\nu)}}$

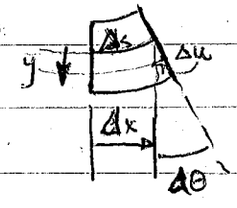
$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E} = \frac{1-2\nu}{E}(-p)$$

if $\epsilon_x = 0 \Rightarrow \dfrac{1-2\nu}{E} = 0$ $\qquad \nu = .5$ $\qquad K = \infty$ you need an infinite compressive stress to change volume

## Deflections of Beams

1) Plane Sections Remain Plane
2) Shear deformation are neglected
3) bending about principal axis




$$\frac{\Delta u}{\Delta s} = -y\frac{\Delta\theta}{\Delta s}$$

$$\frac{d\theta}{ds} = \frac{1}{\rho}$$

$$\lim_{\Delta s \to 0} \frac{\Delta u}{\Delta s} = \frac{du}{ds} \qquad \lim_{\Delta s \to 0} -y\frac{\Delta\theta}{\Delta s} = -y\frac{d\theta}{ds}$$

$$\epsilon_x = \lim_{\Delta s \to 0} \frac{\Delta u}{\Delta s} = -y \lim_{\Delta s \to 0} \frac{\Delta\theta}{\Delta s}$$

$$\epsilon_x = -y\frac{1}{\rho} = -y\kappa$$

$$\boxed{\kappa = \frac{1}{\rho} = -\frac{\epsilon_x}{y}}$$ Universal (General Equations)

Assume that the problem satisfies three assumptions made & it occurs in elastic range.

If elastic $\quad \epsilon_x = \frac{\sigma_x}{E} = \frac{-My}{IE} \qquad \frac{1}{\rho} = \frac{-\sigma_x}{Ey} = \frac{M}{IE}$

$v = f(x)$ deflection of beam

$$\frac{1}{\rho} = \frac{\frac{d^2v}{dx^2}}{\left[1+\left(\frac{dv}{dx}\right)^2\right]^{3/2}} \approx 1 \qquad \frac{dv}{dx} \ll 1$$

$$\frac{d^2v}{dx^2} \approx \frac{M}{EI}$$

EI - flexural stiffness

$v(x)$ elastic curve

$\theta(x) = \frac{dv}{dx}$ = slope of elastic curve

$M(x) = EI\frac{d^2v}{dx^2}$ = moment curve.

$V(x) = -\frac{dM}{dx} = \frac{d}{dx}\left(-EI\frac{d^2v}{dx^2}\right)$ - shear curve

$p(x) = \frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right)$ - load curve.

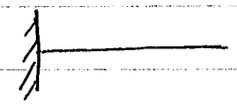
If EI is constant

$$p(x) = EI\frac{d^4v}{dx^4}$$

$$V(x) = -EI\frac{d^3v}{dx^3}$$

## Boundary Conditions

Fixed Support



$v=0$ $\quad$ $\theta=0$ $\quad$ at free end $V=0$, $M=0$

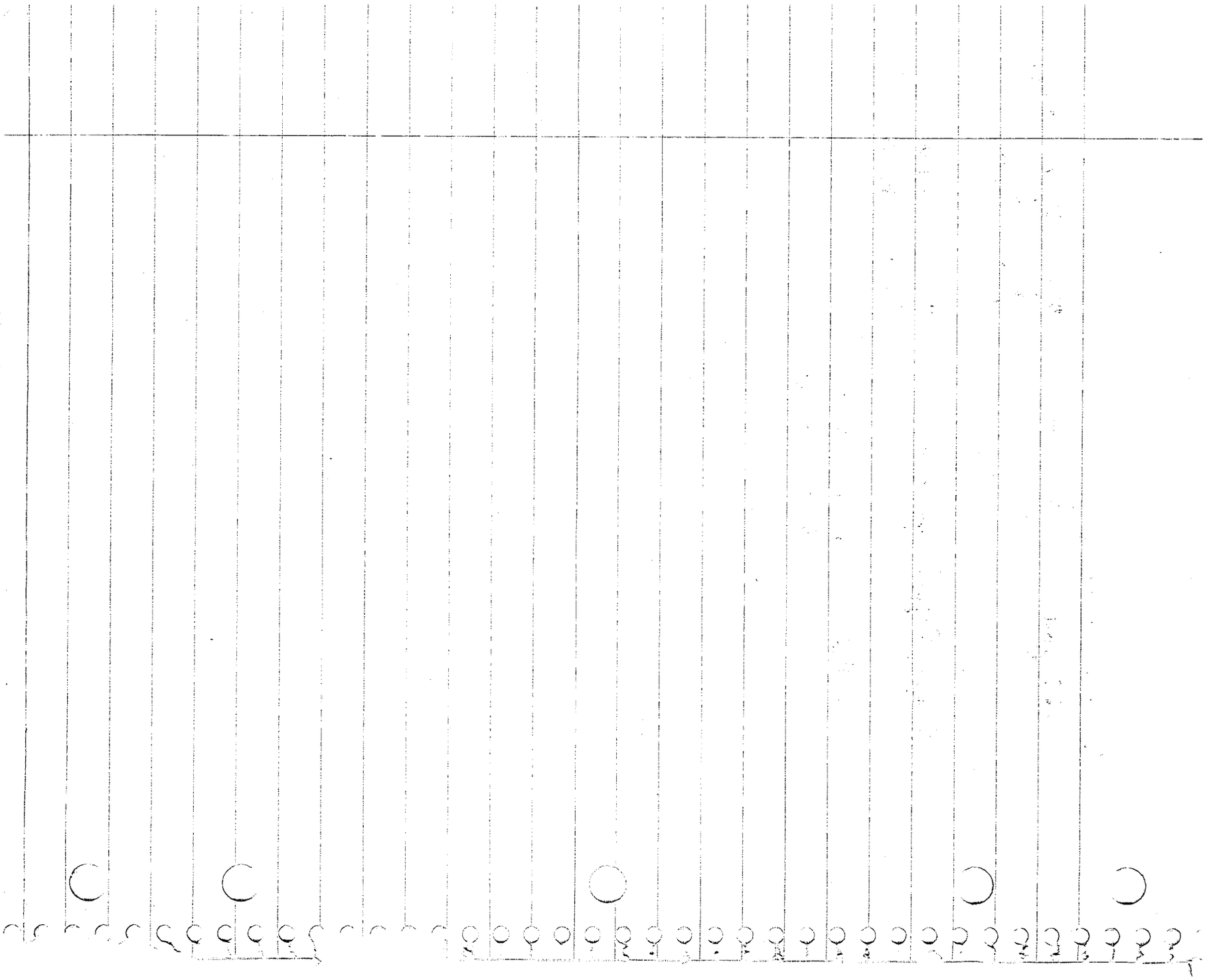
$V\neq 0$ $\quad$ $M\neq 0$

Simply Support (pin or roller) $\quad$ $v=0 \rightarrow V\neq 0$

$\theta\neq 0 \rightarrow M=0$

Free End $\quad$ $v\neq 0 \rightarrow V=0$

$\theta=0 \rightarrow M=0$



at e

| | A | y | Q | q | T |
|---|---|---|---|---|---|
| 1. | .5 | 2.0 | 1.0 | (7.38)(196) | 2900 |
| 2. | 1.5 | 2.75 | 2.75 | 1450 | |
| 3. | 1.5 | 1.5 | 2.25 | | |

at e $T = 2900 psi$

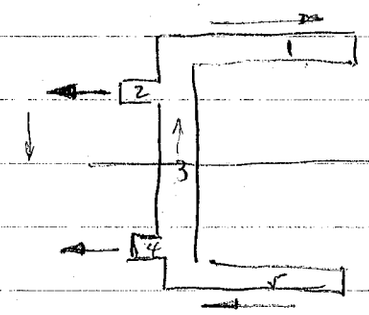
8-21, 22, 24, 29 9-1, 2, 6, 7

On May 26 Exam 6-9

9-14, 16, 17, 23



26

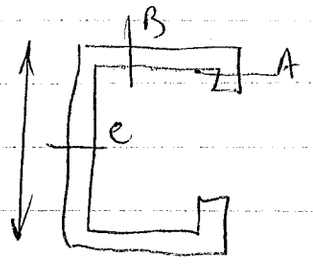



| | A | d | $Ad^2$ | $I_o$ |
|---|---|---|---|---|
| 1 | .075 | 2.0 | .3 | |
| 2 | .025 | 1.5 | .056 | Small |
| 3 | .2 | 0 | 0 | |
| 4 | .025 | -1.5 | .056 | |
| 5 | .075 | -2.0 | .3 | |

$Ve = F_2 h$

$$e = \frac{b^2 h^2 t}{4I} = \frac{(2)^2 (4)^2 (.05)}{4 (.712\ in)}$$

$e = 1.12\ in - .05\ in = 1.07\ in$




$V = 7000\ lb$

$I = 35.7\ in$

$t = .5\ in$

$V/I = 196$

| at A | A | y | Q | $q = VQ/I$ | $\tau = q/t$ |
|---|---|---|---|---|---|
| | 1.5 | 2.0 | 1.0 | 196 | 392 |

$\tau = 392$

| at B | A | y | Q | q | $\tau$ |
|---|---|---|---|---|---|
| | .5 | 2.0 | 1.0 | (5.13)(196) | 2000 |
| | 1.5 | 2.75 | 4.13 | 1070 | |

at B $\tau = 2000\ psi$

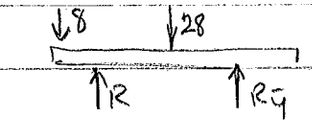
$$\frac{3\cancel{6}\sec^2\theta d\theta \cdot \cos^2\theta}{9}$$

$$d\theta$$

$$\cancel{\text{Arc}}\arccos\frac{3}{\sqrt{9+(x/12)^2}}$$

$$\frac{4}{18}\arccos\frac{3}{\sqrt{9+(x/12)^2}}\Bigg|_0^{24}$$

$$-\frac{4}{18}\ln\left|\frac{36-x}{\sqrt{9(144)-x^2}}\right|$$



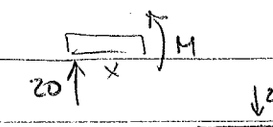

$R+R_y=36$

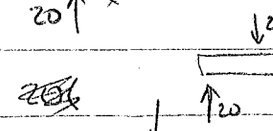
$32(-28(14))+R_y(20)=0$

$R_y(20)=392-32=360$

$R_y=18$

$R_y=18$




$-20(x)+20(x-4)+M=0$

$M=-8x$

$20x-240+144$

$-20(x)+20(x-4)$

$+20(x-8)+M$

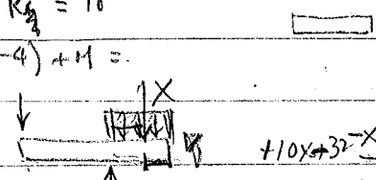
$M=-8x+18x-36-\frac{x^2}{2}+x$

$M=240-20x$

$11x-36-\frac{x^2}{2}$

$x=4$

$44=36-8$




$+10x+32-\frac{x^2}{2}+8x+16$

$14x-40-\frac{x^2}{2}$

$x-4\ \ x-4$

$56-40-8$

$8x-18(x-4)+8(\quad)+M=0$

$-10x+36+\frac{x^2}{2}-2x+2+M=0$

$8x-18(x-2)+\frac{1}{2}(x-2)^2+M=0$

$M=-36+12x-\frac{x^2}{2}$

$-36+48-16$

Feb 26 - Exam Ch 1 & 2 and all part of chapter 3

Read Chapt 4 Prob 4-1

Prob. 4 3, 5, 6, 7, 8

4-20, 23, 24, 26, 31, 34, 35

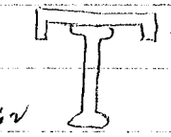
Prob 5-1, 2, 3, 4

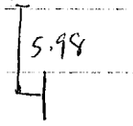
Proble 6-10, 6-9, 6-11, 6-17

Proble 6-1, 3, 7, 12

TA 405 P68

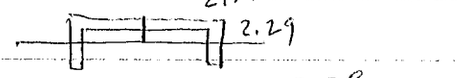
TA 405 T485 1962

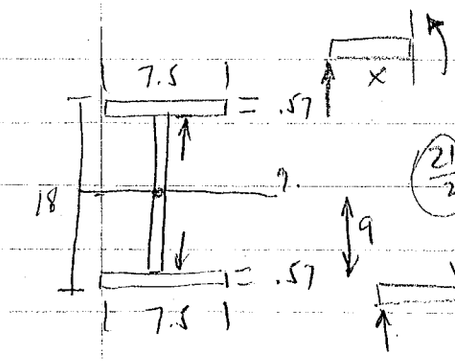





2. $\frac{I}{\frac{I}{c}}$

$\frac{128.1}{21.4} = c$ .598

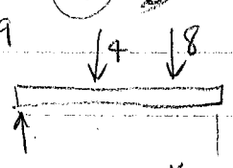



$-\frac{15}{2}x + (x-2)4 + M = 0$

$\frac{21}{2}\frac{15}{2}x = M$

$M = +\frac{24}{2}x + (2-x)4$




$\frac{24}{2}x + 8 - 4x$

$M = \frac{13}{2}x + 8$

8422-1250

1.14 (7.5)

16.86 (.358)

8.5 + 6.4

14.9

$-\frac{10\frac{1}{2}}{2}x + 4(x-2) + 8(x-6) + M = 0$ 3948

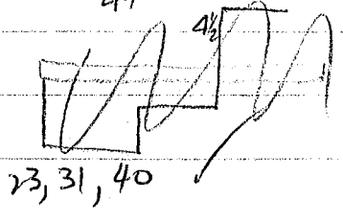
$M = \frac{24}{2}x - 4x + 8 - 8x + 48$

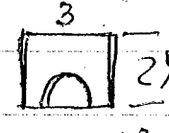
$-\frac{3}{2}x + 56 = M$ $\frac{24}{2}x + 4 - 8 = M$

$x = 8$ $x = 6$ $-27 + 56 = 29$

$-12 + 56$ $x = 8$ $-(36) + 56 + 20$

44



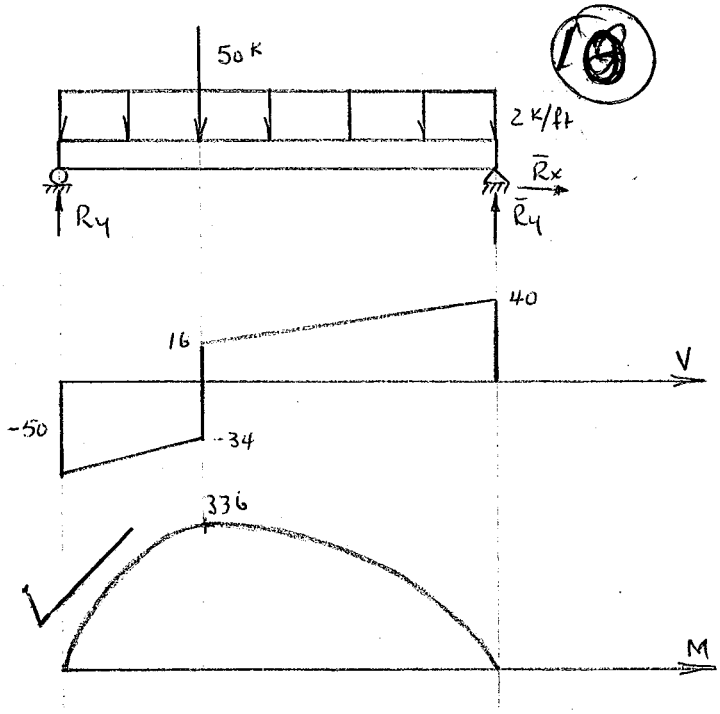

$\frac{3 \cdot \left(\frac{5}{2}\right)^3 - .11(1)}{12}$

HW 6-19, 22, 23, 31, 40

7-2, 7-4, 7-9, 7-10, 7-16 7-21, 22, 25, 28

C. Levy
MA 153A

(19)

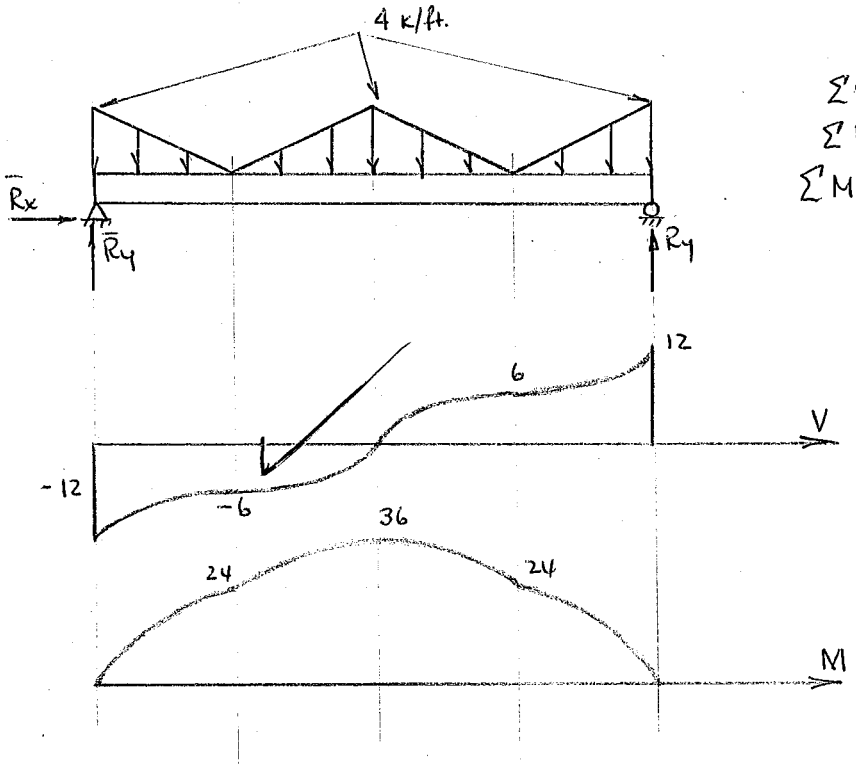



$\Sigma F_x = 0 \qquad \bar{R}_x = 0$

$\Sigma F_y = 0 \qquad 2(20) + 50 = R_y + \bar{R}_y$

$\Sigma M_{y\uparrow} = 0 \qquad \bar{R}_y(20) - 50(8) - 40(10) = 0$

$\bar{R}_y = 40\text{K} \quad R_y = 50\text{K}$

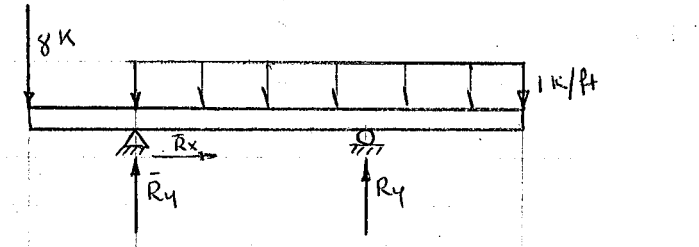



$\Sigma F_x = 0 \qquad \bar{R}_x = 0$

$\Sigma F_y = 0 \qquad R_y + \bar{R}_y = 4[4 \cdot 3 \cdot \tfrac{1}{2}]$

$\Sigma M_{\bar{y}\downarrow} = 0 \qquad 6 \cdot 1 + 6 \cdot 5 + 6 \cdot 7 + 6 \cdot 11 = R_y(12)$

$R_y = 12\text{K} \quad \bar{R}_y = 12\text{K}$

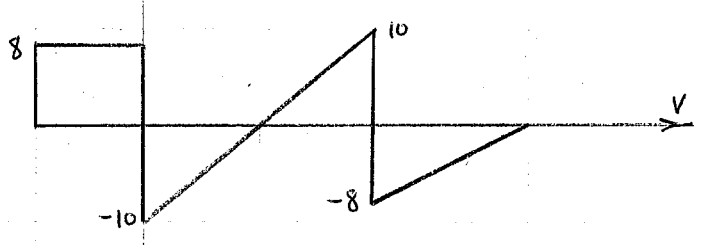


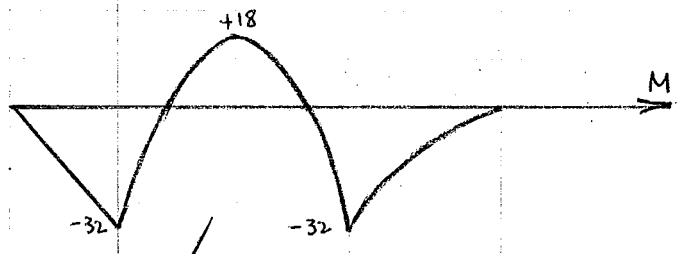

$\Sigma F_x = 0 \quad \bar{R}_x = 0$

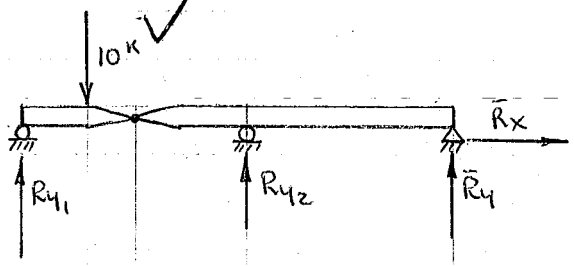
$\Sigma F_y = 0 \quad R_y + \bar{R}_y = 36$

$\Sigma M_{\bar{y}} \circlearrowleft = 0 \quad 32 + R_y(20) - 28(14) = 0$

$R_y = 18\text{ K} \quad \bar{R}_y = 18\text{K}$




$\Sigma F_x = 0 \quad \bar{R}_x = 0$

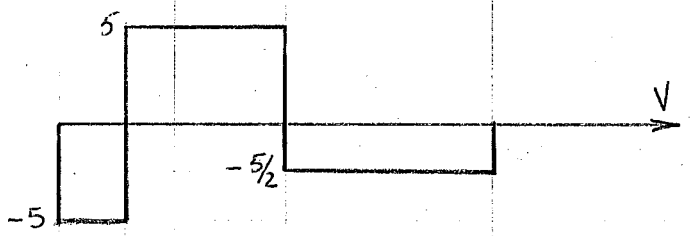
① $\Sigma F_y = 0 \quad R_{y_1} + V = 10$

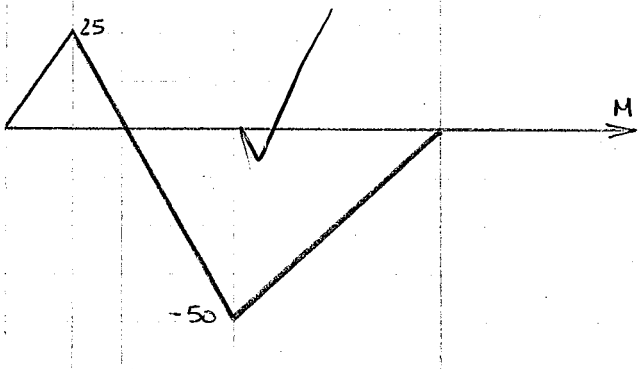
② $\Sigma F_y = 0 \quad R_{y_2} + \bar{R}_y - V = 0$

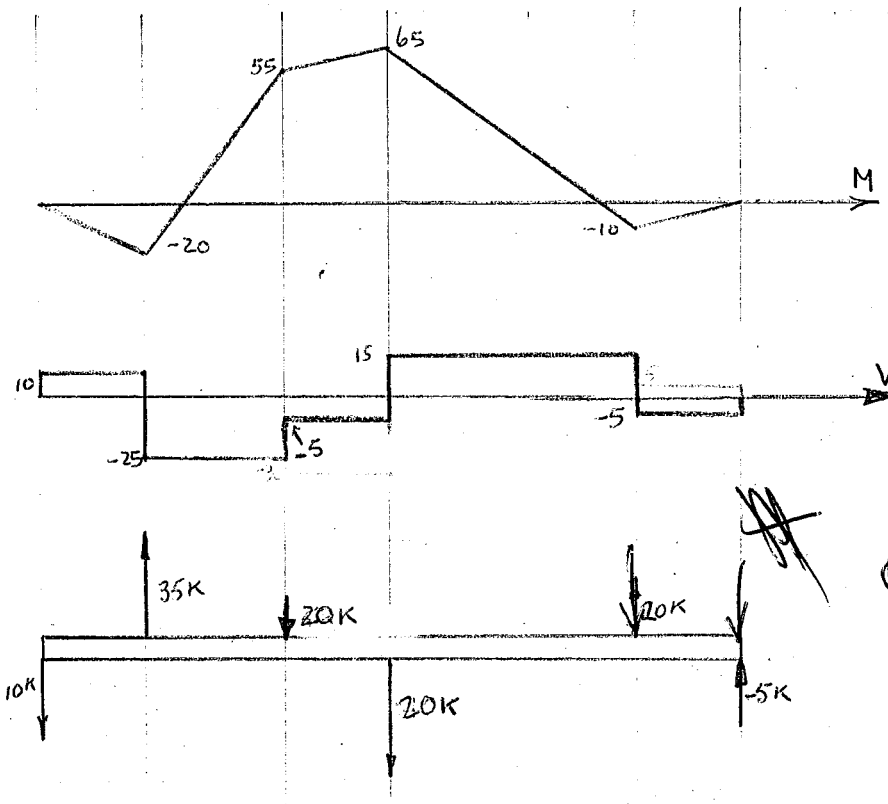
① $\Sigma M_{y_1} = 0 \quad V(10) = 5(10) \quad V = 5\text{K} \quad \boxed{R_{y_1} = 5\text{K}}$

② $\Sigma M_{y_2} = 0 \quad V(10) + \bar{R}_y(20) = 0 \quad \boxed{\bar{R}_y = -2\tfrac{1}{2}\text{K}}$

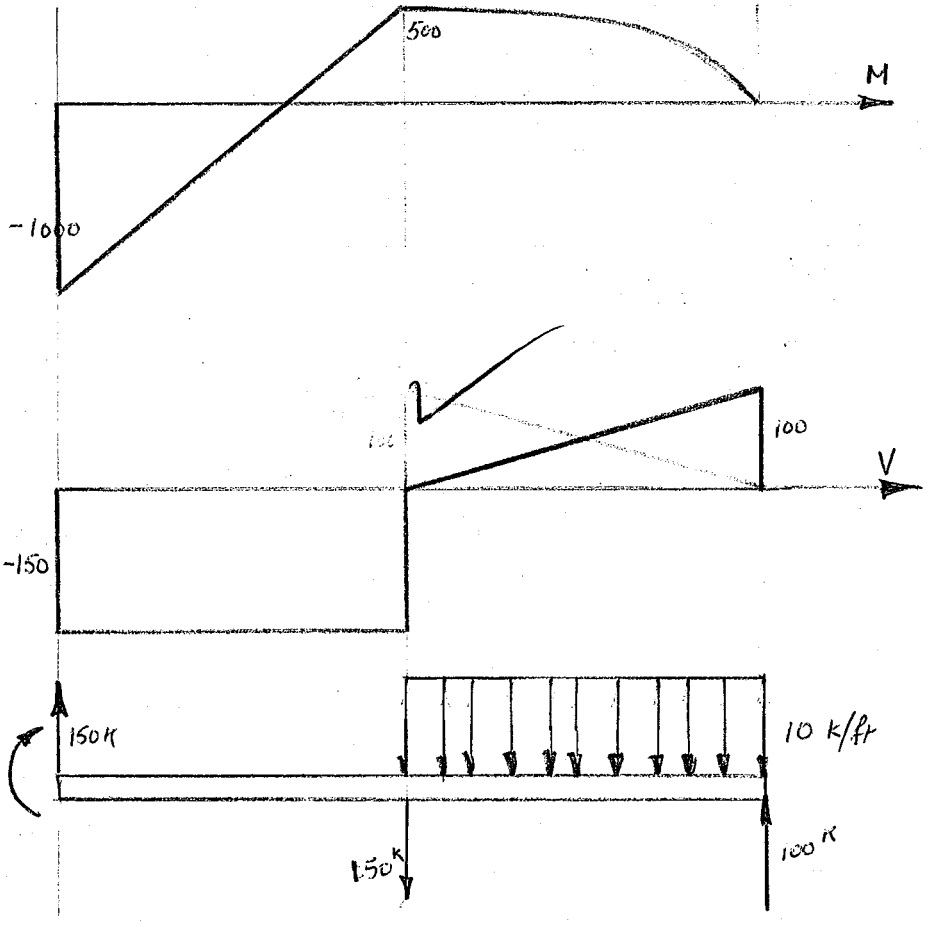
$\boxed{R_{y_2} = 7\tfrac{1}{2}\text{K}}$




65
55
M
-20
-10
15
10
V
-25
-5
-5
35K
20K
20K
10K
20K
-5K

OK


500
M
-1000
100
V
-150
150K
1000
K-ft
10 k/ft
150K
100K

10

2-61 $p(x) = -k(x)$ at $L$ $p(L) = -KL$ $\bar{L} = -\frac{KL^2}{2}$ $K = -\frac{2\bar{L}}{L^2}$

$\therefore p = +\frac{2\bar{L}}{L^2}x \Rightarrow V = -\frac{2\bar{L}x^2}{2L^2} + C_1 \Rightarrow M = +\frac{\bar{L}}{3L^2}x^3 + C_1x + C_2$

at $x=L$ $M=0$ $\therefore M = +\frac{L}{3}\cdot\bar{L} - \bar{L}L + C_2 = 0$ $C_2 = +\frac{2}{3}L(\bar{L})$

at $x=L$ $V=0$ $\therefore C_1 = +\bar{L}$

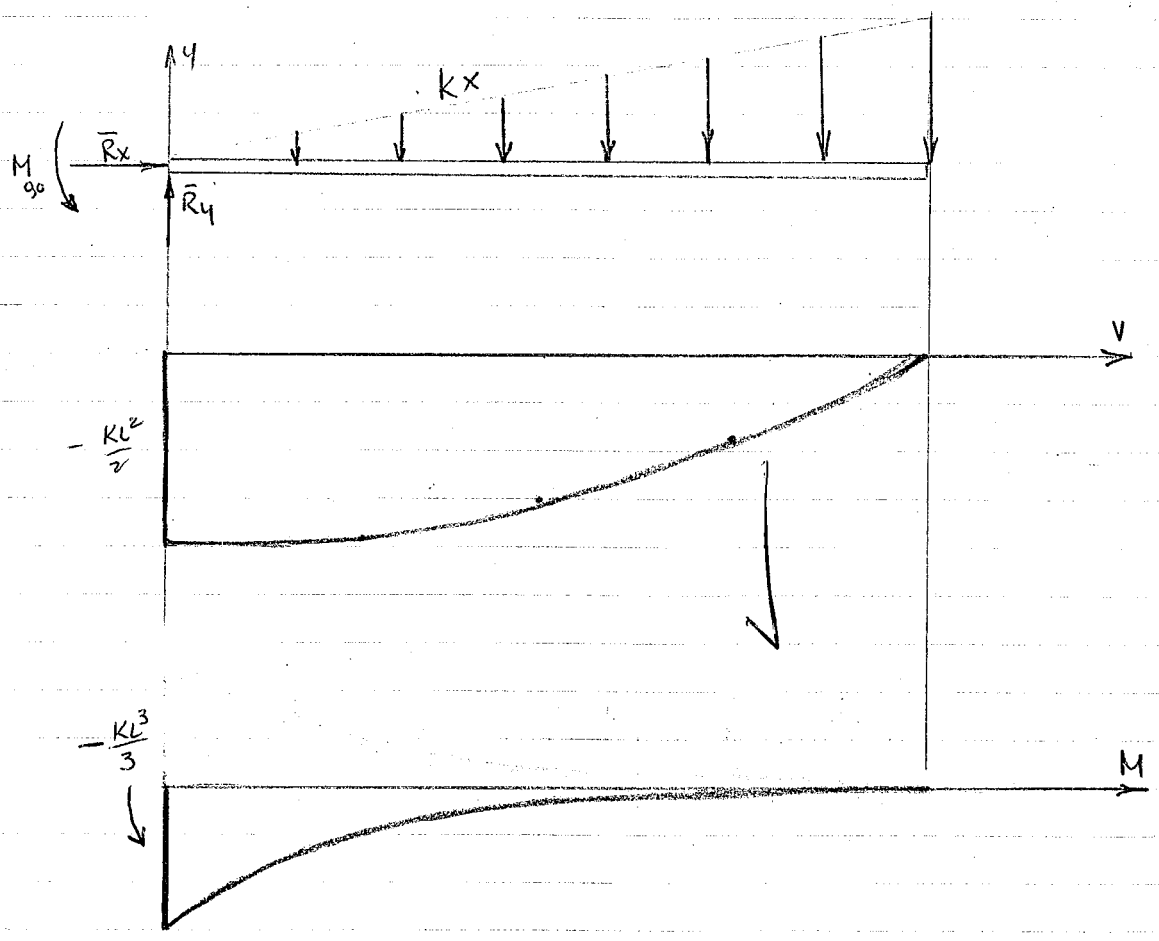
$V = -\bar{L}(x^2/L^2 - 1)$ $M = +\bar{L}\left(\frac{x^3}{3L^2} - x + \frac{2}{3}L\right)$

$\Sigma F_x = 0$ $\bar{R}_x = 0$ at $x=0$

$\Sigma F_y = 0$ $\bar{R}_y = \frac{KL^2}{2}$ $\Sigma M_{0,0} = 0$ $M = \frac{KL^3}{3}$

formulas give at $x=0$ $V = \bar{L} = -\frac{KL^2}{2}$ but the graph value is the negative of the reaction $\therefore$ it checks

for the moment it is in same direction as if we were to take $M$ + the formulas give at $x=0$ $M = \frac{KL^3}{3}$ which checks



2-63 $p(x) = K\sin(2\pi x/L)$ $\qquad -V = -\frac{KL}{2\pi}\cos\frac{2\pi x}{L} + C_1$

$$M = -\frac{KL^2}{4\pi^2}\sin\frac{2\pi x}{L} + C_1 x + C_2$$

at $x=0;L$ $\quad M=0 \quad \therefore \quad C_2 = C_1 = 0$

$$V = \frac{KL}{2\pi}\cos\frac{2\pi x}{L} \qquad M = -\frac{KL^2}{4\pi^2}\sin\frac{2\pi x}{L}$$

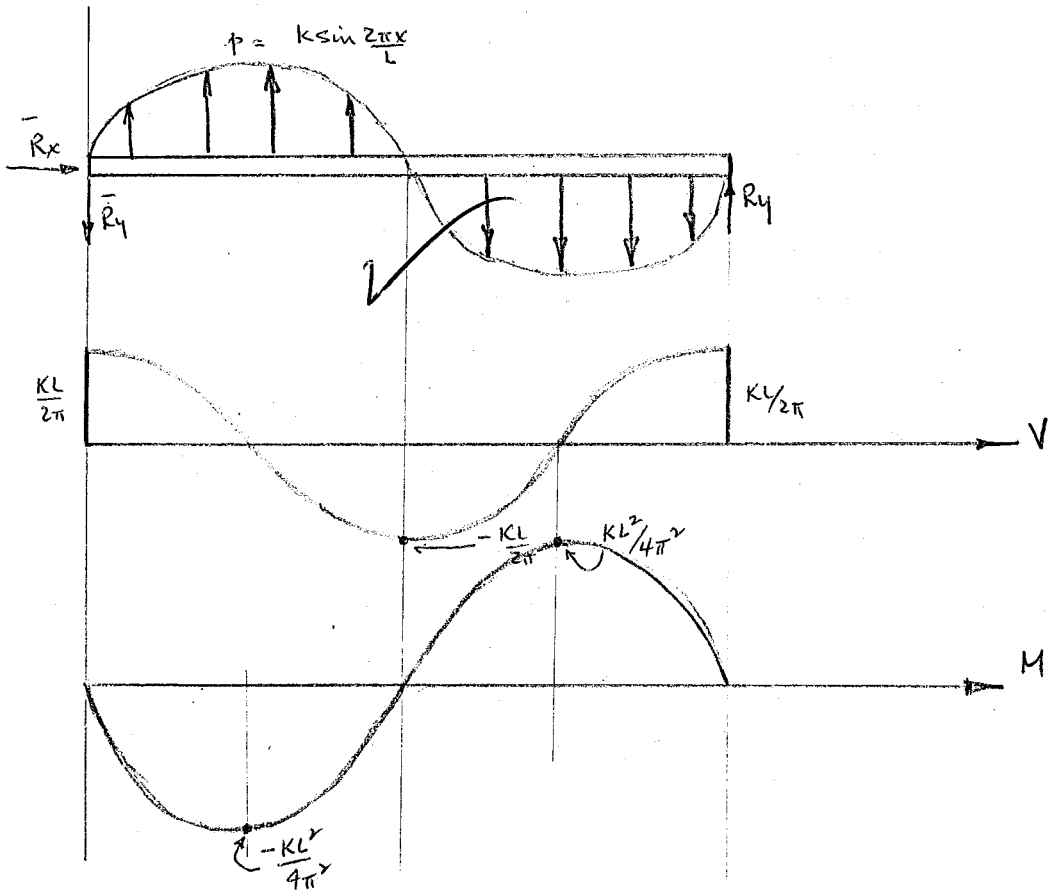
at $x=0$ $\quad V = \frac{KL}{2\pi}$ but $V_{graph} = -V_{reaction}$ $\therefore$ at $x=0$ there must be a downward reaction $= \frac{KL}{2\pi}$; at $x=L$ $\quad V$

① $\displaystyle\int_0^{L/2} K\sin\frac{2\pi x}{L}\,dx = \frac{KL}{\pi}$

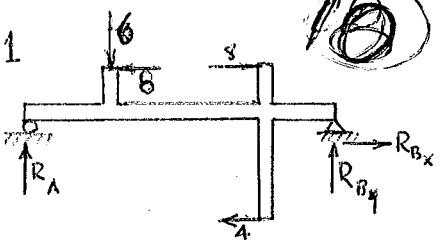
$\therefore \quad \Sigma F_x = 0 \quad \bar{R}_x = 0 \qquad \Sigma F_y = 0 \quad -\bar{R}_y + R_y = \frac{KL}{\pi} - \frac{KL}{\pi} = 0$

$$\Sigma M_{0,0} = \bar{R}_y \cdot 0 + \frac{KL}{\pi}(L/4) - \frac{KL}{\pi}(3L/4) + R_y L$$

$R_y = \frac{KL}{2\pi}$ $\qquad \bar{R}_y = \frac{KL}{2\pi}$ which checks results.



1




$\Sigma F_x = 0 \quad -4+8-8+R_{B_x}=0 \quad R_{B_x} = 4 \text{ kips} \rightarrow$

$\Sigma F_y = 0 \quad R_A - 6 + R_{B_y} = 0$

$\Sigma M_B = 0 \circlearrowleft + \quad -8\times3 - 4\times11 - R_A\times20 + 6\times14 + 8\times3 = 0$

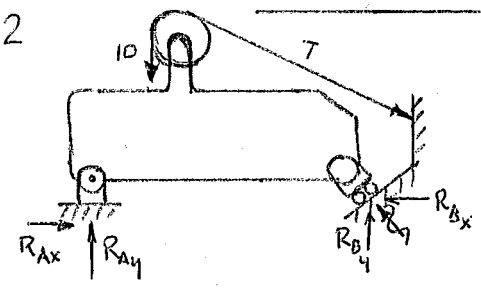
$-24-44-20R_A+108=0$

$R_A = 2 \text{ kip} \uparrow$

$R_{B_y} = 4 \text{ kip} \uparrow$

---

2




$\Sigma F_x = 0 \quad R_{A_x} - R_{B_x} + T_x = 0$

$\Sigma F_y = 0 \quad R_{A_y} + R_{B_y} - 10 - T_y = 0$

$R_{B_x} = R_B/\sqrt{2}$

$T_x = 2T/\sqrt{5}$

$R_{B_y} = R_B/\sqrt{2}$

$T_y = T/\sqrt{5}$

$T = 10 \text{ kips}$

$R_{A_x} - R_B/\sqrt{2} + 20/\sqrt{5} = 0$

$R_{A_y} + R_B/\sqrt{2} - 10 - 10/\sqrt{5} = 0$

$\Sigma M_A = 0 \circlearrowleft + \quad -10(3) - 20/\sqrt{5}(5) - \frac{20}{\sqrt{5}}(4) + \frac{R_B}{\sqrt{2}}(10) = 0$

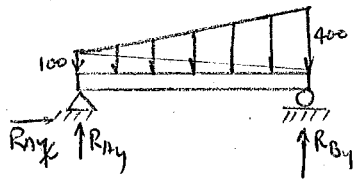
$R_B = 13.1 \text{ kips}$ (1 : 1)

$R_{A_x} = 35.5 \text{ kips} \leftarrow$

$R_{A_y} = 23.125 \text{ kips} \rightarrow$

---

3

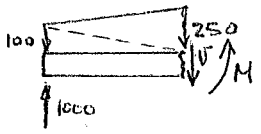



$\Sigma F_x = 0 \quad R_{A_x} = 0$

$\Sigma F_y = 0 \quad R_{A_y} + R_{B_y} - \frac{1}{2}(100)(10) - \frac{1}{2}(400)(10) = 0$

$\Sigma M_A = 0 \circlearrowleft \quad R_{B_y}(10) - 500\left(\frac{10}{3}\right) - 2000\left(\frac{20}{3}\right) = 0$

$R_{B_y} = 4500(1/3) = 1500 \text{ \#} \uparrow \quad R_{A_y} = 1000 \text{ \#} \uparrow$



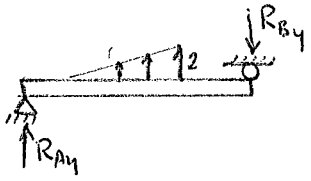

Axial Force = 0

$\Sigma F_y = 1000 - V - \frac{1}{2}(100)(5) - \frac{1}{2}(250)(5) = 0 \quad V = +125 \text{ \#} \downarrow$

$\Sigma M_A = \frac{5}{3}(-250) - 125(5) - \frac{10}{3}(625) + M = 0 \quad M = 3125 \text{ kip-ft} \circlearrowleft$

---

5

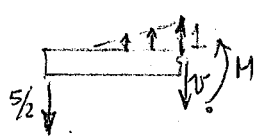



$\Sigma F_y = 0 \quad R_{A_y} + 6 - R_{B_y} = 0$

$\Sigma M_A = 0 \quad +6(7) - R_{B_y}(12) = 0$

$R_{B_y} = 7/2 \text{ kip} \downarrow$

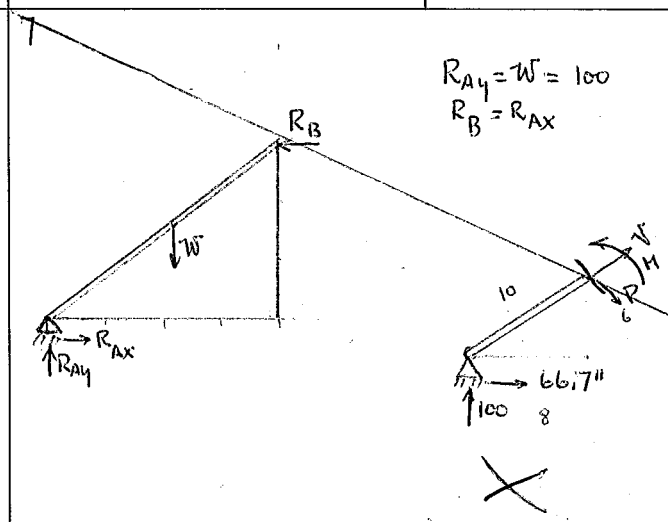
$R_{A_y} = 5/2 \text{ kip} \downarrow$




$\Sigma F_y = 0 \quad -5/2 - V + 3/2 = 0 \quad V = +1 \text{ kip} \uparrow$

$\Sigma M_A = 0 \quad 5(3/2) + 6 + M = 0 \quad M = 13.5 \text{ kip-ft} \circlearrowleft$

7




$R_{AY} = W = 100$ ; $R_B = R_{AX}$

$\Sigma M_A = 0 \quad -100(8) + R_B(12) = 0$

$R_B = \frac{100(8)}{12} = 66.7^{\#} = R_{AX}$

$100 + V\frac{3}{5} - P\frac{4}{5} = 0$ ; $3V - 4P = -500$

$66.7^{\#} + \frac{3}{5}P + \frac{4}{5}V = 0$ ; $3P + 4V = -\frac{1000}{3}$

$\Sigma M_A = 0$ ; $-P(10) + M = 0$

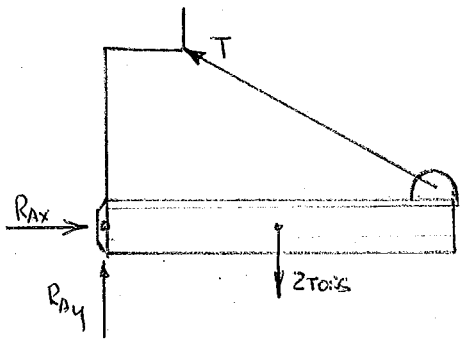
$V = +113\frac{1}{3}$ $\quad P = 210^{\#}$

$M = 2100^{\#\text{-ft}}$

Problem 7 is found as last problem done

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9



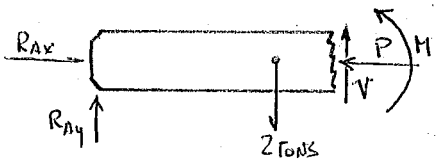

$R_{AX} = \frac{2}{\sqrt{5}}T$ ; $R_{AY} + \frac{T}{\sqrt{5}} = 2\left(\frac{2000}{1000}\right) = 4k$

$\Sigma M_A = 0 \quad -4000(3) + \frac{T}{\sqrt{5}}\left(\frac{35}{6}\right) + \frac{2}{\sqrt{5}}T\left(\frac{1}{2}\right) = 0$

$T = 3.93$ kips

$R_{AX} = 3.5$ kips

$R_{AY} = 2.25$ kips



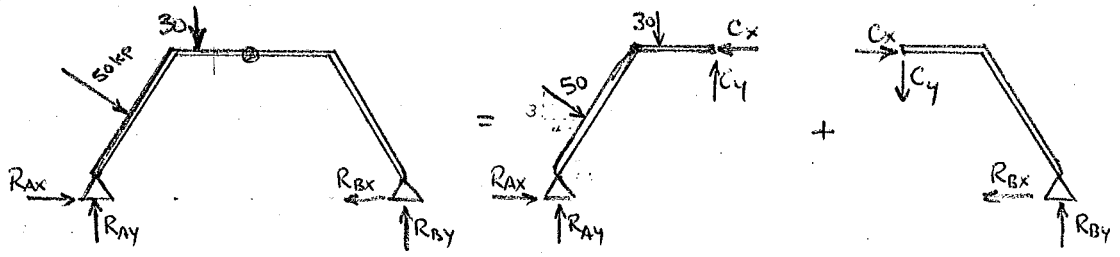

$R_{AX} = P$ ; $P = 3.5$ kips ←

$R_{AY} - 2000(2) + V = 0$ ; $V = 1.75$ kips ↑

$\Sigma M_A = -4000(3) + V(4) + M = 0$ ; $M = 5$ k-ft $= 60$ kip-in.

---

11




$\rightarrow R_{AX} - C_x + 40 = 0$

$C_y + R_{AY} - 30 - 30 = 0$ $\quad R_{AY} = 100$

$\Sigma M_A = -30(15\frac{1}{2}) - 40(10) - 30(20) + C_x(20) + C_y(30) = 0$

$-225 - 400 - 600 + 40C_x = 0$

$C_x = R_{BX}$ ; $C_y = R_{BY}$

$\Sigma M_B = 0$ ; $C_y(30) - C_x(20) = 0$

$C_y = \frac{2}{3}C_x$

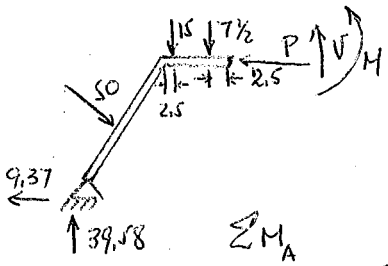
$C_x = \frac{1225}{40} = 30.63$ kp ; $C_y = 20.42$ kp ;

$R_{AX} = 9.37$ kp ← $\quad R_{AY} = 39.58$ kp ↑

11 CONT.

$$-9.37+40-P=0$$

$$P=30.63 \text{ kp} \leftarrow$$




$$39.58-30-22.5+V=0$$

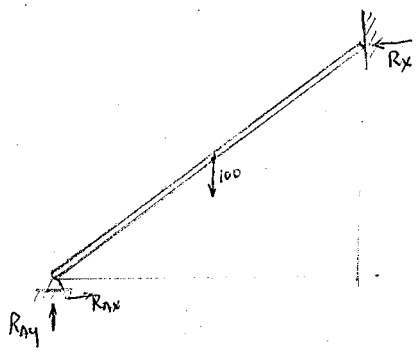
$$V=12.92 \text{ kp} \uparrow$$

$$\Sigma M_A \qquad 0=-30(15/2)-40(10)-15(17.5)-15/2(20)+P(20)+V(22.5)+M$$

$$-225-400-262.5-150+612.6+290.7+M=0$$

$$M=134.2 \text{ kp-ft} \curvearrowleft$$

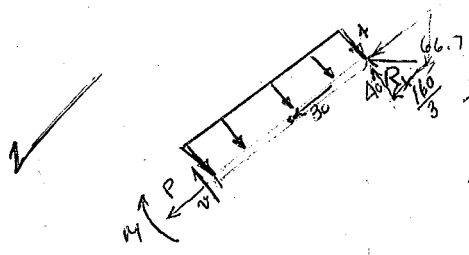
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$R_{Ay}=100^{\#}$

$R_{AX}=R_X$

$\Sigma M=0;\ 100(8)=R_x(12);\quad R_x=R_{AX}=\frac{100(2)}{3}$



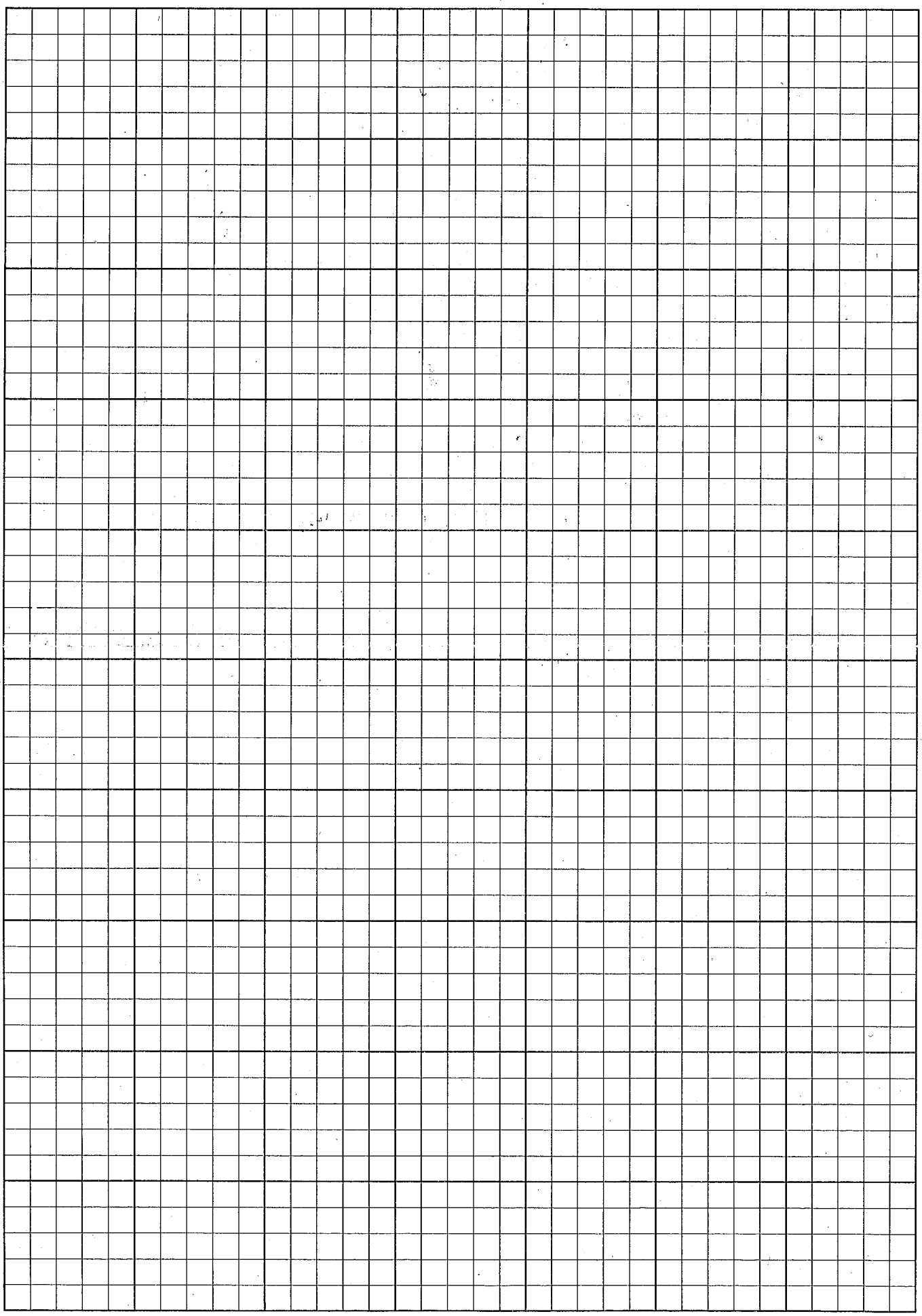

$P=30^{\#}$ (4, 3)

$V=40^{\#}$ (4, 3)

$M=200^{\#}\text{-inches}$ ↺

$P+30+53\frac{1}{3}=0 \qquad P=83\frac{1}{3}^{\#}$ (4, 3)

$V-40+40=0 \qquad V=0$



Clark Kelly
CE 201 D
Exam

100

1. $\tau_{max} = \frac{T}{\alpha b c^2}$ $\quad \varphi = \frac{TL}{\beta c^3 b G} = \frac{L}{\beta c G}\left(T/bc^2\right)$

$\tau_{max}\alpha = T/bc^2$ $\quad \varphi = \frac{L}{\beta c G}\tau_{max}\alpha$

a) $\tau_{max} = \frac{\varphi \beta c G}{\alpha L}$

$\tau_{max} = 8000$ psi $\quad \varphi = .1$ $\quad b/c = 2 \Rightarrow \alpha = .246,\ \beta = .229$

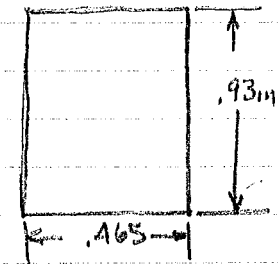
if $\tau_{max} = 8000$ $\varphi = .1$ $\alpha = .246$, $\beta = .229$ then $\frac{\tau_{max}\alpha}{\beta\varphi} = \frac{cG}{L} = \frac{8000(.246)}{.229(.1)} =$

$85900 = cG/L_{min}$

$\varphi = \frac{TL}{\beta c^3 b G} = \frac{400}{(.229)(c^2b)cG} = \frac{400}{.229(2c^3)85900} = .1$

$\frac{400}{2(2.29)(859)} = c^3 = .1018$ $\quad c = .465$ $\quad b = .93$

$T/\alpha bc^2 = \tau_{max}$ $\quad \frac{400}{2(.246)c^3} = \tau_{max}$ $\quad \tau_{max} = \frac{400}{2(.246)(.1018)} = 8000$ ✓

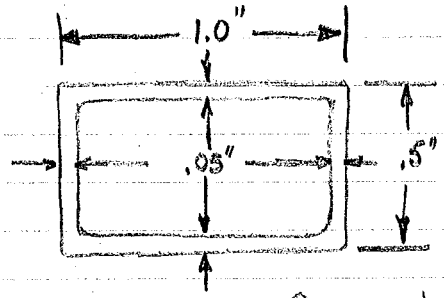



b) $b = .93$in $\quad c = .465$in

$L \le \frac{\varphi\beta c G}{\tau\alpha} \le .1(.229)(.465)(G)/(8000)(.246)$

$L \le 5.45 \times 10^{-6} G$

Part 2 c)




$\tau_{max} = \frac{T}{2A_{encl}t}$ $\quad A_{encl} = \frac{T}{2\tau_{max}t}$

$A_{encl} = \frac{400}{800} = .5 = bc = 2c^2$ $\quad c^2 = .5/2 = .25$

$c = .5"$ $\quad b = 1"$

Ans. $b = 1.0"$ $\quad c = .5"$ $\quad t = .05"$ $\quad L \le 4.16 \times 10^{-6} G$ from eq in Part I

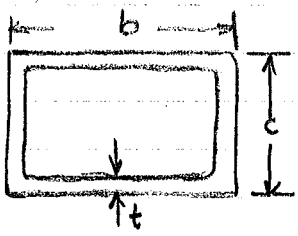
Part 1 $\tau_{max} = \frac{T}{2A_m t}$ $\quad \theta = \frac{d\varphi}{dx} = \frac{T}{4A^2G}\oint\frac{ds}{t} = \frac{T}{4A^2Gt}\oint ds$ $t$ is uniform

$\oint ds = 2(b+c)$

$$\frac{\tau_{max}^2 t^2}{T^2} = \frac{1}{4A_m^2} \qquad \theta = \frac{T}{4A^2 G t}(2b+2c) = \frac{d\varphi}{dx}$$

$$\theta = \frac{\tau_{max}^2 t^2}{T^2} \cdot \frac{T}{Gt}(2b+2c) = \frac{(\tau_{max})^2 t}{TG}(2b+2c)$$

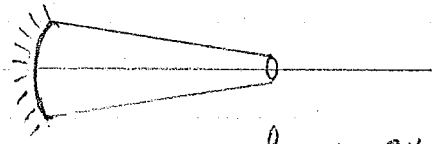
$$\varphi = \int_0^L \frac{\tau_{max}^2 t}{TG}\left[2b+2c\right]dx = \frac{\tau_{max}^2 t}{TG}\left[2b+2c\right]\int_0^L dx$$

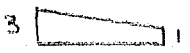



if t is uniform and material is homogeneous ✓

$$\boxed{\varphi = \frac{\tau_{max}^2 t}{TG}\left[2b+2c\right]L \quad \text{or} \quad \varphi = \frac{\tau_{max} L\left[b+c\right]}{AG} \qquad A = bc}$$

2) Example 5-12





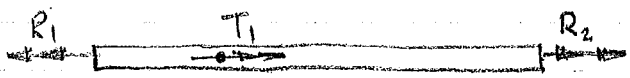

at $x=0$ $y=3$
$x=20$ $y=1$

if $y = ax+b$ $\quad y = 3 - x/10 = c_0$

$$\varphi = \int_0^{20} \frac{T\,dx}{\frac{\pi}{2}G(3-x/10)^4} = \frac{2T}{\pi G}\int_0^{20} \frac{dx}{(3-x/10)^4} = \frac{2T(10)}{3\pi G}\left[3-x/10\right]^{-3}\Big|_0^{20}$$

$$\varphi_{\text{in degrees}} = \frac{2T(10)(26)(180)}{27(3\pi G)} = \frac{2(27000)(10)(26)(180)}{27(3\pi^2)(12\times10^6)} = \frac{260}{987} = .263^\circ$$

3)

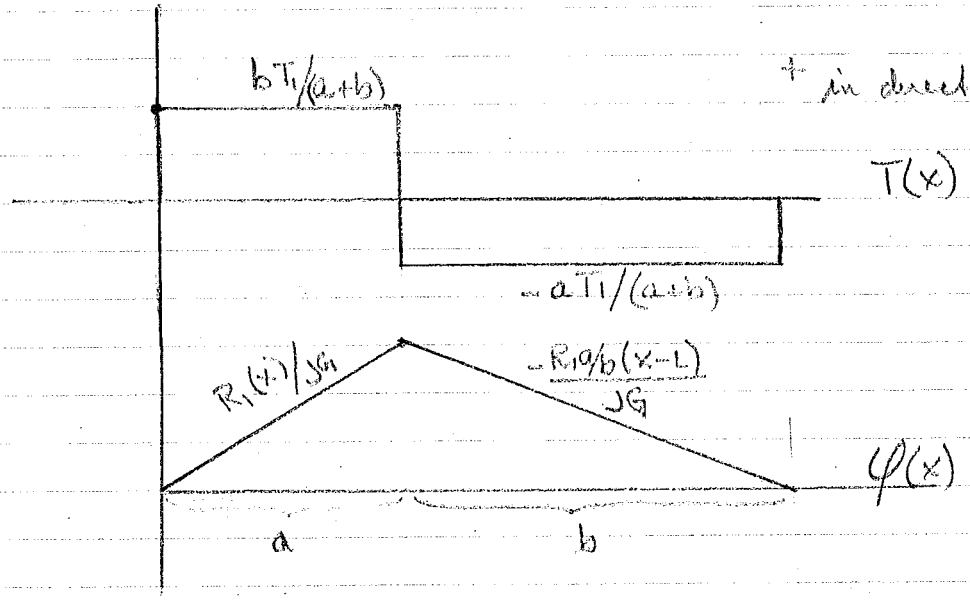



$T_1 + R_2 = R_1$

$$\varphi = 0 = \int_0^L \frac{T\,dx}{JG} = \int_0^a \frac{R_1\,dx}{JG} + \int_a^L \frac{R_2\,dx}{JG}$$

$$\frac{R_1 a}{JG} + \frac{R_2 b}{JG} = 0 \qquad R_2 = -\frac{R_1 a}{b}$$

$$T_1 - \frac{R_1 a}{b} - R_1 = 0 \quad \Rightarrow \quad \boxed{\frac{bT_1}{a+b} = R_1} \qquad \text{but} \quad \boxed{R_2 = -\frac{R_1 a}{b} = \frac{-aT_1}{a+b}}$$




+ in direction as shown in diagram. ⟵ +Torque

i) $\tau_{max} = Tc/J \qquad J = \frac{\pi}{2}(2)^4 \quad c = 2 \quad T = \frac{bT_1}{a+b} = \frac{2}{3}(10000)$

$$\tau_{max} = \frac{\frac{2}{3}(10000)\,2}{\frac{\pi}{2}\,2^4} = \frac{10000}{6\pi} = 531 \text{ lb/in}^2$$

ii) $$\varphi_{max} = \int_0^a \frac{R_1\,dx}{JG} = \frac{\frac{2}{3}(10000)(3(12))}{\pi/2\,(2^4)\,(10\times10^6)} = \frac{5}{\pi}\,10^{-3} = 1.593\times10^{-3} \text{ rad.}$$

iii) at larger torque material is fully plastic – $\tau_{max} = \tau_{yp}$

$$T = \int_A \tau_{max}\,dA\,\rho = \tau_{max}\int \rho\,dA = \tau_{max}\int_0^c 2\pi\rho\cdot\rho\,d\rho = \tau_{max}(2\pi)\frac{\rho^3}{3}\Big|_0^c$$

$$T = \frac{2}{3}\pi c^3 \tau_{max}\,; \quad J = \frac{\pi}{2}c^4 \quad \Rightarrow \quad \frac{2J}{c} = \pi c^3$$

$$\therefore \tau_{plastic} = \frac{4}{3}\tau_{max}\,J/c \qquad \text{but} \quad \tau_{plastic} = R_1 = \frac{2}{3}(10000)$$

$$\tau_{max} = \frac{3T_p c}{4J} = \frac{3}{4}\cdot\frac{2}{3}(19{,}000)(2)\cdot\frac{2}{\pi\,2^4} = \frac{10000}{8\pi} = 398 \text{ lb/in}^2$$

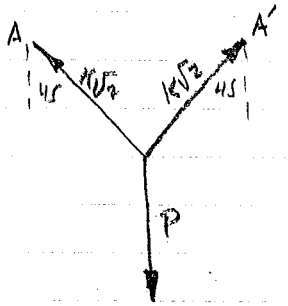
**iff** $\tau_{plastic} = R_1$ otherwise not solvable since $\tau_{yp}$ wasn't given.

iv) Part IV is at the end of the exam

25

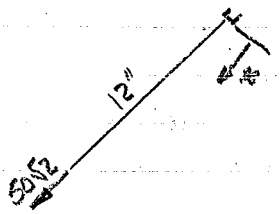
PART IV

4.




$\Sigma F_y\ (+\uparrow) = 0 \quad 2K - P = 0 \quad K = P/2 = 50$

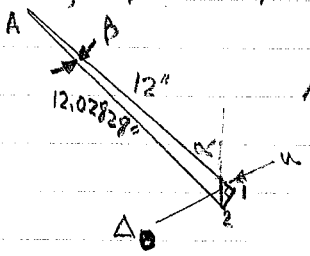
$\Sigma F_x\ (+\rightarrow) = 0 \quad K - K = 0$ ✓




$$u = \int_0^L \frac{P(x)\,dx}{E\,A(x)} = \frac{K\sqrt{2}}{EA}\,h + C_1 \qquad \text{at } h = 0,\ u = 0,\ C_1 = 0$$

$$\therefore u = \frac{50\sqrt{2}\,(12)}{(30\times10^6)(1\times10^{-3})} = \frac{600\sqrt{2}}{30\times10^3} = 20\sqrt{2}\times10^{-3} = 2\sqrt{2}\times10^{-2} = 2.828\times10^{-2}\ \text{in.}$$

using $(h+u)$ as radius strike an arc from A such that it intersects perpendicular

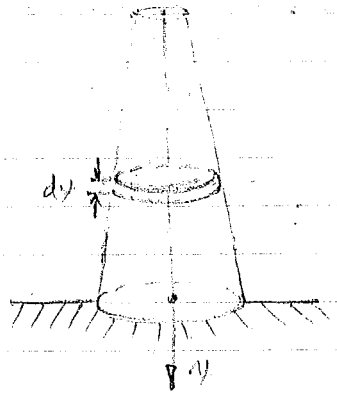



Since deflection is small $\beta$ will be small therefore arc 1-2 will be small and can be approximated as a line. Therefore from trigonometry $\cos\alpha = \frac{u}{\Delta_0}$ $\quad \Delta_0 = \frac{u}{\cos\alpha}$ ; $\alpha = 45°$

$$\Delta_0 = \frac{2.828\times10^{-2}\ \text{in}}{.707} = 4\times10^{-2}\ \text{in}$$

$pt_0$ deflection $\approx 4\times10^{-2}$ in.

5.




@ $x=0$ $r=1$

@ $x=24$ $r=3$ $\quad r = x/12 + 1$

$dV = A\,dx$

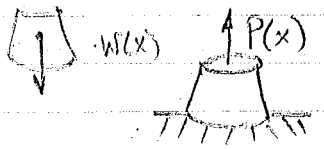
$dW = \gamma\, dV = \gamma A\,dx = \gamma\pi r^2 dx$

$$W = \int \gamma \pi r^2 dx = \gamma\pi \int \left(1 + x/12\right)^2 dx$$

$$W(x) = 48\gamma\pi \left(1 + x/12\right)^3 + C \qquad @\ x=0\ \ W(0)=0$$

$\therefore \quad C = -48\gamma\pi$

$$W(x) = 48\gamma\pi\left[\left(1 + x/12\right)^3 - 1\right]$$




$W(x) = P(x)$

$$A(x) = \pi r^2 = \pi\left[1 + x/12\right]^2$$

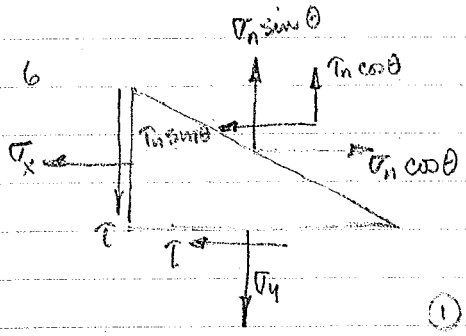
$$\therefore \quad du/dx = \frac{P}{AE} = \frac{48\gamma\pi\left[\left(1+x/12\right)^3 - 1\right]}{\pi\left[1 + x/12\right]^2 E}$$

$$u = \int_0^{24} \frac{48\gamma\left[\left(1+x/12\right)^3 - 1\right]}{\left[1 + x/12\right]^2 E}\,dx = \frac{48\gamma}{E}\int_0^{24}\left[\left(1 + x/12\right) - \frac{1}{\left(1+x/12\right)^2}\right]dx$$

$$= \frac{48\gamma}{E}\left[6\left(1 + x/12\right)^2 + 12\left(1 + x/12\right)^{-1}\right]_0^{24}$$

$$= \frac{48\gamma}{E}\left(54 + 4 - 6 - 12\right) = \frac{1608}{E}$$

6




$\Sigma F_y (\uparrow +) = 0$

(1) $-\tau \cos\theta ds + \sigma_n \sin\theta ds + \tau_n \cos\theta ds - \sigma_y \sin\theta ds = 0$

$\Sigma F_x (\rightarrow +) = 0$

(2) $-\sigma_x \cos\theta ds - \tau_n \sin\theta ds + \sigma_n \cos\theta ds - \tau \sin\theta ds = 0$

(1) $-\tau \cos\theta + \sigma_n \sin\theta + \tau_n \cos\theta - \sigma_y \sin\theta = 0$

(2) $-\sigma_x \cos\theta - \tau_n \sin\theta + \sigma_n \cos\theta - \tau \sin\theta = 0$

$-\tau \cos\theta \sin\theta + \sigma_n \sin^2\theta + \tau_n \cos\theta \sin\theta - \sigma_y \sin^2\theta = 0$

$-\sigma_x \cos^2\theta - \tau_n \sin\theta \cos\theta + \sigma_n \cos^2\theta - \tau \sin\theta \cos\theta = 0$

(i) $-2\tau \cos\theta \sin\theta + \sigma_n - \sigma_y \sin^2\theta - \sigma_x \cos^2\theta = 0$

$$\sigma_n = \sigma_y \sin^2\theta + \sigma_x \cos^2\theta + \tau \sin 2\theta$$

(1) $-\tau \cos\theta + \sigma_n \sin\theta + \tau_n \cos\theta - \sigma_y \sin\theta = 0$

(2) $-\sigma_x \cos\theta - \tau_n \sin\theta + \sigma_n \cos\theta - \tau \sin\theta = 0$

$-\tau \cos^2\theta + \sigma_n \sin\theta \cos\theta + \tau_n \cos^2\theta - \sigma_y \sin\theta \cos\theta = 0$

$+\sigma_x \cos\theta \sin\theta + \tau_n \sin^2\theta - \sigma_n \cos\theta \sin\theta + \tau \sin^2\theta = 0$

(ii) $\tau_n + (\sigma_x - \sigma_y)\frac{\sin 2\theta}{2} + \tau(\sin^2\theta - \cos^2\theta) = 0$

$$\tau_n = (\sigma_y - \sigma_x)\frac{\sin 2\theta}{2} + \tau \cos 2\theta$$

$\frac{d\sigma_n}{d\theta} = 0$

$0 = -(\sigma_x - \sigma_y)\sin 2\theta + 2\tau \cos 2\theta \qquad \tan 2\theta_p = \frac{2\tau}{(\sigma_x - \sigma_y)}$ (iii)

Substituting this equation (iii) into (i) we get

$$\sigma_{n_{max}} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

(work is done on last sheet)

$\frac{d\tau_n}{d\theta} = 0$

$0 = (\sigma_y - \sigma_x)\cos 2\theta - 2\tau \sin 2\theta \qquad \tan 2\theta_s = \frac{(\sigma_y - \sigma_x)}{2\tau}$ (iv)

Substituting (iv) into (ii) yields

$$\tau_{n_{max}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

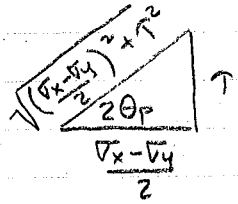
(work is done on last sheet)

$$\tan 2\theta_p = 2\tau/(\sigma_x - \sigma_y)$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2} \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sigma_n = \sigma_y\left(\frac{1-\cos 2\theta}{2}\right) + \sigma_x\left(\frac{1+\cos 2\theta}{2}\right) + \tau \sin 2\theta$$

$$\sigma_n = \frac{\sigma_y+\sigma_x}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau \sin 2\theta$$




$$\cos 2\theta = (\sigma_x-\sigma_y)/2\left[1/\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2}\right]$$

$$\sin 2\theta = \tau\left[/\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2}\right]$$

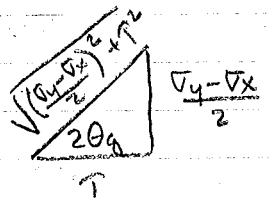
$$\sigma_n = \frac{\sigma_y+\sigma_x}{2} + \left[\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2\right]\Big/\left[\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2}\right]$$

$$\sigma_{n_1} = \frac{\sigma_y+\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2}$$

$\tan 2\theta_p = 2\tau/(\sigma_x-\sigma_y)$ is positive in 1ST & 3rd quadrants but both $\cos 2\theta_p$, $\sin 2\theta_p$ are negative in 3rd quadrant. For $\sigma_{n\,max}$ we must take $\tan 2\theta_p$ in 1ST quadrant $\therefore \sigma_{n_1} = \sigma_{n\,max}$

$$\tan 2\theta_q = (\sigma_y-\sigma_x)/2\tau$$

$$\tau_n = (\sigma_y-\sigma_x)\frac{\sin 2\theta}{2} + \tau\cos 2\theta$$




$$\sin 2\theta = \left[\frac{\sigma_y-\sigma_x}{2}\right]\Big/\left[\sqrt{\left(\frac{\sigma_y-\sigma_x}{2}\right)^2+\tau^2}\right]$$

$$\cos 2\theta = [\tau]\Big/\left[\sqrt{\left(\frac{\sigma_y-\sigma_x}{2}\right)^2+\tau^2}\right.$$

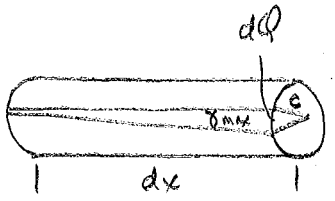
$$\tau_n = \left[\left(\frac{\sigma_y-\sigma_x}{2}\right)^2+\tau^2\right]\Big/\left[\sqrt{\left(\frac{\sigma_y-\sigma_x}{2}\right)^2+\tau^2}\right]$$

$$\tau_{n_1} = \sqrt{\left(\frac{\sigma_y-\sigma_x}{2}\right)^2+\tau^2}$$

Same argument as before. Thus $\tau_{n_1} = \tau_{n\,max}$

3)

iv)




$\gamma_{max}\, dx = dQ\, c$

also $\gamma\, dx = dQ\, \rho$

since $\tau^n = k\gamma$ since distribution of $\gamma_{max}$ is still linear

$$\frac{\gamma}{\gamma_{max}} = \rho/c = \frac{\tau^n}{\tau_{max}^n}$$

$$\therefore \quad \tau = (\rho/c)^{1/n}\, \tau_{max}$$

$$T = \int_A \tau\, dA\, \rho = \tau_{max} \int_A (\rho/c)^{1/n}\, dA(\rho) \quad ; \quad dA = 2\pi\rho\, d\rho$$

$$\therefore \quad 2\pi\tau_{max}\left(\frac{1}{c}\right)^{1/n} \int_A \rho^{\frac{2n+1}{n}}\, d\rho = T$$

$$\therefore \quad T = 2\pi\rho^3\tau_{max}\,(\rho/c)^{1/n}\,\frac{n}{3n+1}$$

and $$\tau_{max} = \frac{3n+1}{n}\left(\frac{\rho}{c}\right)^n \frac{T}{2\pi\rho^3} \qquad T = \frac{10000}{2.585}$$

$T_a = T_1/2.585 \qquad T_b = T_1/3.828 \qquad T_a > T_b$

$$\tau_{max} = \frac{10000}{2.585}\,\frac{3.667}{\cdot 16\pi} = 282 \text{ lbs/in}^2$$

Cesar Levy
CE201 D Mr Blumert.

7-21

$\tau_y = \tau_z$

$I_z = 166 \quad I_y = 36.5$

$\tau_{max_z} = \frac{VQ}{It} = \frac{V_z Q_z}{I_z t_z}$ $\quad t_z = 1in$ ; $Q_z = (6.1)3.5 + 3\cdot1\cdot1.5 = 25.5$

$I_z = 166$

$\tau_{max_z} = V_z(.1536)$

$\tau_{max_y} = \frac{V_y Q_y}{I_y t_y}$ $\quad t_y = 2in \quad I_y = 36.5 \quad Q_y = 3\cdot1\cdot1.5 + 3\cdot1\cdot1.5 + 6\cdot1.5\cdot2 = 9.75$

$\tau_{max_y} = V_y(.1335)$ $\quad V_y\frac{(.1335)}{(.1536)} = V_z$ ; $V_z = .869\,V_y$

why was this marked wrong Answer: →

7-22 $\quad V = 7000\#$ $\quad I = 35.7\,in^4$ $\quad t = .5$

at A $\quad A = (1\cdot.5) = .5 \quad \bar{y} = 2.0 \quad Q = A\bar{y} = 1.0\,in^3$

$\tau = \frac{q}{t} = \frac{VQ}{It} = \frac{7000(1.0)}{35.7(.5)} = 393\,psi$

at B $\quad A_T = (2.5)(.5) + (1.5)(.5) = 2\,in^4 \quad \bar{y}_1 = 2.25 \quad \bar{y}_2 = 2.75$

$A_1\bar{y}_1 + A_2\bar{y}_2 = (2.5)(.5)(2.75) + (1.5)(.5)(2.25) = Q$

$\tau = q/t = \frac{VQ}{It} = \frac{7000(5.12)}{35.7\ .5} = 2005\,psi$

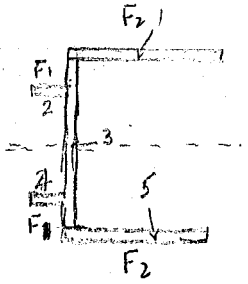
at C $\quad A_1 = (2.5)(.5) = 1.25 \quad A_2 = (3.5)(.5) = 1.75 \quad A_3 = (1.0)(.5) = .5$

$\bar{y}_1 = 1.25 \quad \bar{y}_2 = 2.75 \quad \bar{y}_3 = 2.00$

$A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 = (1.563 + 4.820 + 1.000) = 7.383$

$\tau = \frac{VQ}{It} = \frac{7000(7.383)}{35.7(.5)} = 2900\,psi$

7-25

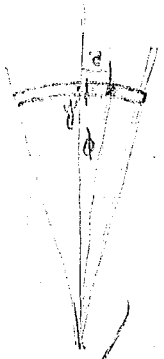



| | A | d | $Ad^2$ | $I_o$ |
|---|---|---|---|---|
| 1. | .075 | 2.0 | .3 | $\frac{(1.5)(.05)^3}{12}$ = small |
| 2. | .025 | 1.5 | .056 | $\frac{(.5)(.05)^3}{12}$ |
| 3. | .2 | 0 | 0 | $\frac{.05(4)^3}{12} = .267$ |
| 4. | .025 | -1.5 | .056 | $\frac{(.5)(.05)^3}{12}$ |
| 5 | .075 | -2.0 | .3 | $\frac{(1.5)(.05)^3}{12}$ |

$Ad^2 + I_o \approx .712 + .267 \approx .990$

$$e = \frac{b^2h^2t}{4I} = \frac{(2)^2(4)^2(.05)}{4(.99)} = .81 \text{ inches from center}$$

7-28




$dA = at\,d\phi \qquad z = a\sin\phi \qquad y = a(1-\cos\phi)$

$$I = \int_A z^2\,dA$$

$$I = \int_{-\alpha}^{\alpha} a^2\sin^2\phi\, t\,d\phi = a^3t(\alpha + \sin\alpha\cos\alpha)$$

$$Q = \int_{\phi}^{\alpha} z\,dA = \int_{\phi}^{\alpha} a^2\sin\phi\, t\,d\phi = -a^2t\cos(\alpha-\phi)$$

$$q = \frac{VQ}{I} = \frac{V(-a^2t\cos(\alpha-\phi))}{a^3t(\alpha - \sin\alpha\cos\alpha)}$$

$$T = \int \tau\,dA\,y = \int_{-\alpha}^{\alpha} \frac{q}{t}\,at\,d\phi\; a(1-\cos\phi) = -\int_{-\alpha}^{\alpha} \frac{V\cos(\alpha-\phi)\,a(1-\cos\phi)\,d\phi}{\alpha - \sin\alpha\cos\alpha}$$

$$\frac{-Va}{\alpha-\sin\alpha\cos\alpha}\int_{-\alpha}^{\alpha}\cos(\alpha-\phi)(1-\cos\phi)\,d\phi = \frac{-Va}{\alpha-\sin\alpha\cos\alpha}\int_{-\alpha}^{\alpha}\left(\cos\alpha\cos\phi+\sin\alpha\sin\phi-\cos\alpha\cos^2\phi-\frac{\sin\alpha\sin 2\phi}{2}\right)d\phi$$

$$\frac{-Va}{\alpha-\sin\alpha\cos\alpha}\left[\cos\alpha\sin\phi\Big|_{-\alpha}^{\alpha}+\sin\alpha\cos\phi\Big|_{-\alpha}^{\alpha}-\cos\alpha\left(\frac{\phi}{2}+\frac{\sin 2\phi}{4}\right)\Big|_{-\alpha}^{\alpha}+\frac{\sin\alpha\cos 2\phi}{4}\Big|_{-\alpha}^{\alpha}\right]$$

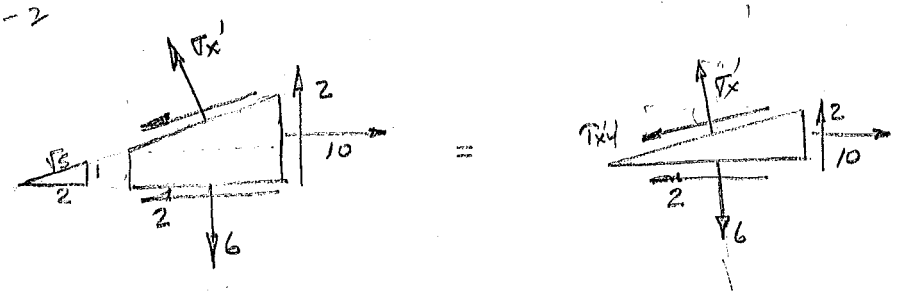
$$\frac{-Va}{\alpha-\sin\alpha\cos\alpha}\left(2\cos\alpha\sin\alpha-\cos\alpha\left(\alpha+\frac{\sin 2\alpha}{2}\right)\right)=\frac{-Va\,2}{\alpha-\sin\alpha\cos\alpha}\left(\sin\alpha-\alpha\cos\alpha\right)=T$$

$$e=-\frac{T}{V}=\frac{2a\left(\sin\alpha-\alpha\cos\alpha\right)}{\alpha-\sin\alpha\cos\alpha}$$

7-1 —

Clear new corner
Mr Blumert.

9-2




$$\sigma_x' \frac{2}{\sqrt{5}} dA + 2 \cdot \frac{1}{\sqrt{5}} dA + 6 \cdot \frac{2}{\sqrt{5}} dA - \tau_{x'y'} dA \frac{1}{\sqrt{5}} = 0$$

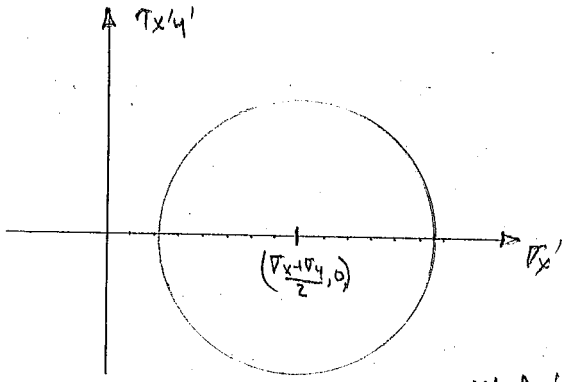
$$10 \cdot \frac{1}{\sqrt{5}} - 2 \cdot \frac{2}{\sqrt{5}} - \tau_{x'y'} \frac{2}{\sqrt{5}} - \sigma_x' \frac{1}{\sqrt{5}} = 0$$

$$\frac{2\sigma_x'}{\sqrt{5}} - \frac{\tau_{x'y'}}{\sqrt{5}} = \frac{10}{\sqrt{5}} \longrightarrow 2\sigma_x' - \tau_{x'y'} = 10$$

$$-\frac{\sigma_x'}{\sqrt{5}} - \frac{2\tau_{x'y'}}{\sqrt{5}} = -\frac{6}{\sqrt{5}} \longrightarrow +\sigma_x' + 2\tau_{x'y'} = 6$$

$5\sigma_x' = 26$

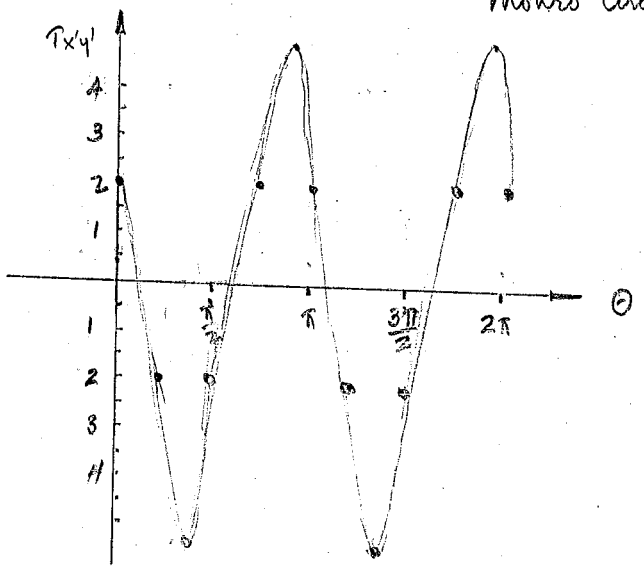
$$\boxed{\begin{aligned} \sigma_x' &= 5.2 \text{ KSI} \\ \tau_{x'y'} &= .4 \text{ KSI} \end{aligned}}$$




Mohr's Circle

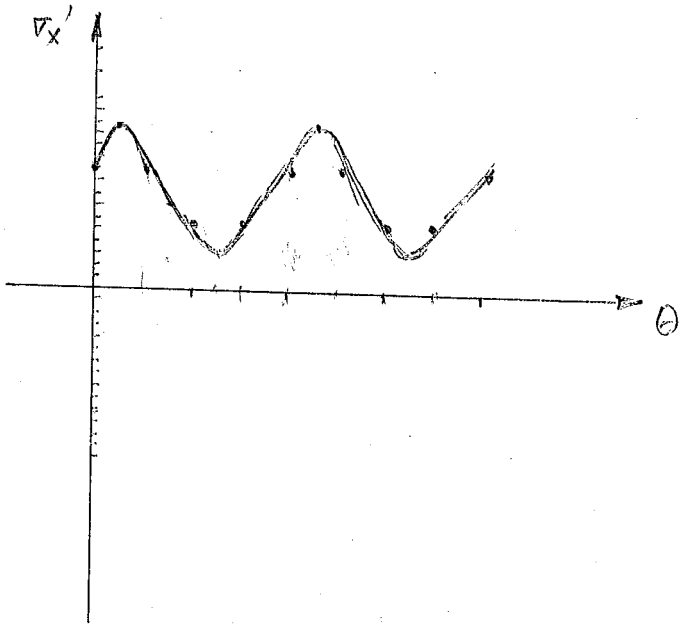
$$\sigma_{1,2} = \left(\frac{\sigma_x+\sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$8 \pm \sqrt{4+4} = 8 \pm 4\sqrt{2}$$




$$\tau_{x'y'} = \frac{\sigma_y-\sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

| | |
|---|---|
| $\theta = 0$ | $\tau_{x'y'} = 2$ |
| $\theta = \frac{\pi}{4}$ | $= -2$ |
| $\theta = \frac{\pi}{2}$ | $= -2$ |
| $\theta = \pi$ | $= 2$ |
| $\theta = \frac{3\pi}{2}$ | $= +2$ |
| $\theta = 2\pi$ | $= 2$ |




$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

| | |
|---|---|
| $\theta = 0$ | $\sigma_{x'} = 8+2 = 10$ |
| $\theta = \frac{\pi}{2}$ | $\sigma_{x'} = 8-2 = 6$ |
| $\theta = \pi$ | $\sigma_{x'} = 8+2 = 10$ |
| $\theta = \frac{3\pi}{2}$ | $\sigma_{x'} = 8-2 = 6$ |
| $\theta = 2\pi$ | $\sigma_{x'} = 10$ |
| $\theta = \frac{\pi}{4}$ | $\sigma_{x'} = 8+2 = 10$ |
| $\theta = \frac{3\pi}{4}$ | $\sigma_{x'} = 8-2 = 6$ |
| $\theta = \frac{\pi}{8}$ | $\sigma_{x'} = 8+4\sqrt{2}$ |
| $\theta = \frac{5\pi}{8}$ | $\sigma_{x'} = 8+2\sqrt{2}-2\sqrt{2}$ |

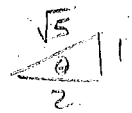
7-1

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$\theta = \arctan .5$

$2\theta = (\arctan .5)2$

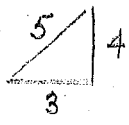
$$\tau_{x'y'} = +\frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$




$\theta = 26°34'$

$2\theta = 53°8'$

$$\sigma_{x'} = 8 - 2\left(\frac{3}{5}\right) - 2\left(\frac{4}{5}\right) = 8 - 2.8 = 5.2 \text{ KSI} \checkmark$$



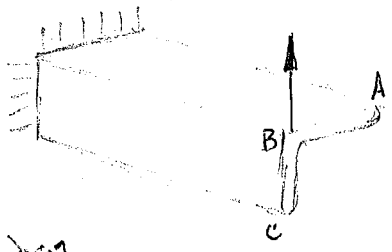


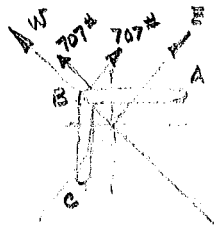


$$\tau_{x'y'} = \frac{6 - 10}{2}\left(-\frac{4}{5}\right) + 2\left(\frac{3}{5}\right) = \frac{2}{5} \text{ KSI} = .4 \text{ KSI} \checkmark$$

8-21

Mr. Blemmer



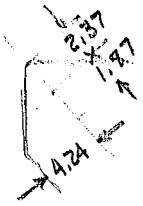




$I_x + I_y = 39.8$

$I_{min} = Ar^2_{min} = 8.0 = I_z$

$I_{max} = Ar^2_{max} = 31.8 = I_w$

$M_z = M_w = .707^k \times 70.7 = 50^{k\text{-}in}$




$$\sigma = \frac{M_z w}{I_z} + \frac{M_w z}{I_w}$$

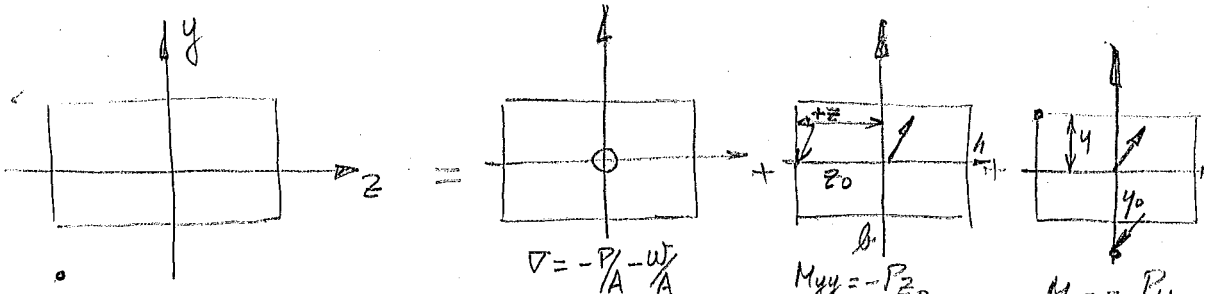
$$\sigma_A = \frac{-50(-1.87)}{8.0} - \frac{50(4.24)}{31.8} = 11.67 - 6.67 = 5.0 \text{ KSI}$$

$$\sigma_B = \frac{-50(2.37)}{8.0} - \frac{50(0)}{31.8} = -14.84 \text{ KSI}$$

$$\sigma_C = \frac{-50(-1.87)}{8.0} - \frac{50(-4.24)}{31.8} = 18.34 \text{ KSI}$$

$\sigma_A$ at wall $= +5.0$ KSI  $\sigma_B$ at wall $= -14.84$ KSI  $\sigma_C$ at wall $= +18.34$ KSI

8-22




$w = .283 \times V = 6 \times 12 \times 20 \times .283 = 407.5$

$$\sigma = -\frac{(P+w)}{A} - \frac{M_{yy} z}{I_{yy}} + \frac{M_{zz} y}{I_{zz}}$$

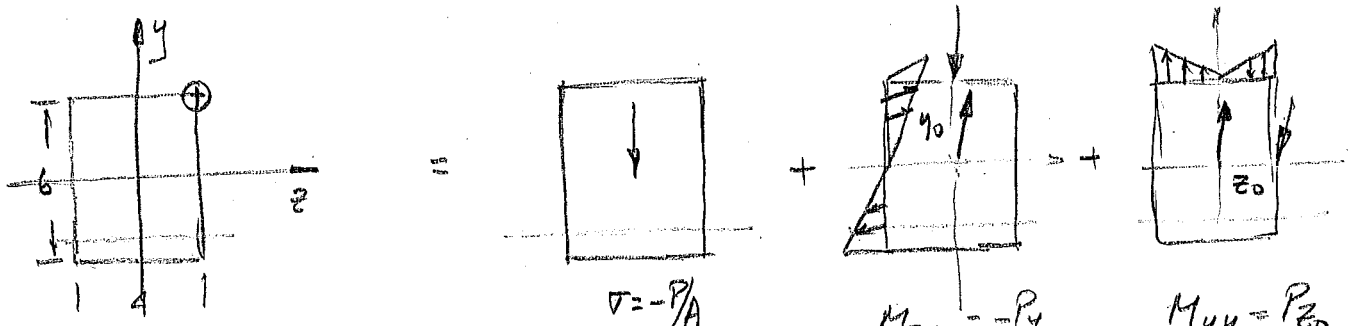
$$0 = -\frac{(P+407.5)}{72} - \frac{P\,6\cdot6}{\frac{6\cdot12\cdot12\cdot12}{12}} + \frac{P\,6\cdot3}{\frac{12\cdot6\cdot6\cdot6}{12}}$$

$$0 = -\frac{(P+407.5)}{72} + \frac{P}{24} = \frac{2P - 407.5}{72}$$

$P = 203.75^{\#}$

24

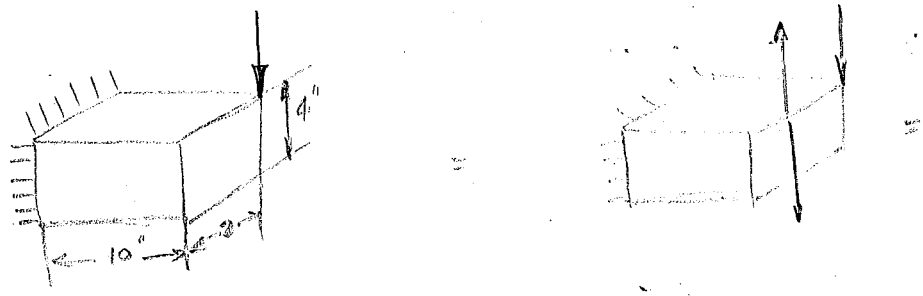
$\sigma = E\epsilon = (500\times10^{-6})(10\times10^{6}) = 5000$ psi

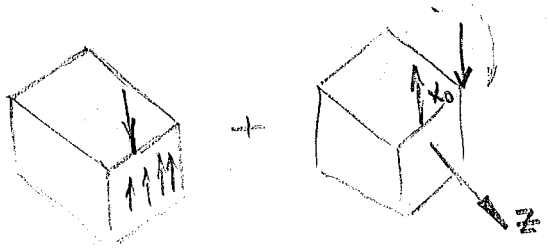



$$\sigma = -\frac{P}{A} - \frac{M_{yy}z}{I_{yy}} + \frac{M_{zz}y}{I_{zz}}$$

$$= \frac{-P}{24} + \frac{3P}{24} + \frac{2P}{24} \Rightarrow 6\sigma = P \quad \checkmark \quad P = 30{,}000\ \#$$

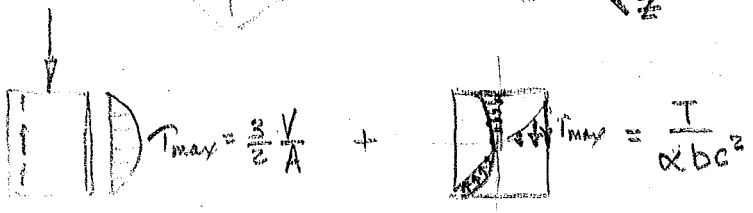
8-29




$M_z = -Px_0$

at $x \geq 0$ $\sigma \geq 0$




$\sum M_z = 0 \quad -T = M_z$

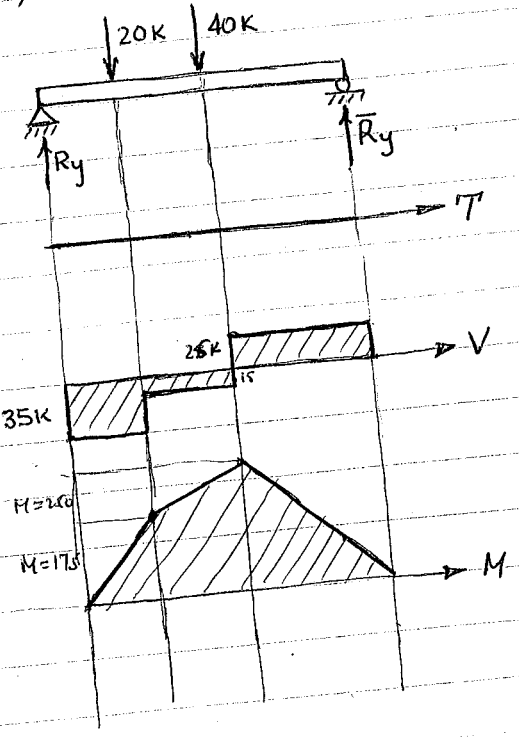
$$\tau_{max} = \frac{1.5V}{A} + \frac{Px_0}{\alpha bc^2} = \frac{1.5(10)}{4.2} + \frac{10(1)}{(.246)4.4} = \frac{15}{8} + 2.54 = 4.415 \text{ ksi}$$

To

Cesar Levy

Cesar Levy

2-15, 17, 19, 21, 29

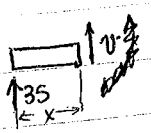



$R_y + \bar{R}_y - 60 = 0$

$R_y \cdot 0 - 100 - 400 + \bar{R}_y(20) = 0$

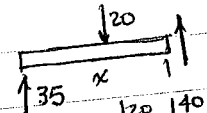
$\bar{R}_y = 25K$

$R_y = 35K$



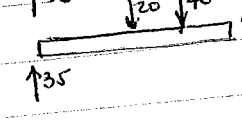

$35 + V = 0$

$V = -35K$




$35 + V - 20 = 0$

$V = -15K$




$V = 25K$

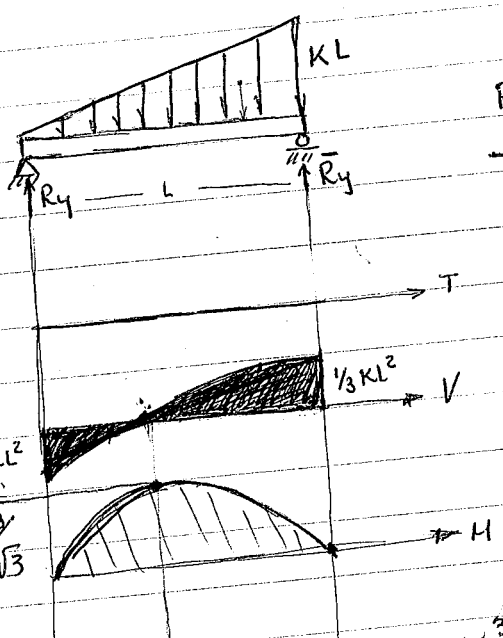
$\sum M_c = 0 \quad M = 35x \quad x \geq 0, \; x \leq 5$

$35x - 20(x-5) = M$

$15x + 100 = M$

$35x - 20(x-5) - 40(x-10) = M$

$M = -25x + 500$




$R_y + \bar{R}_y - \frac{1}{2}KL^2 = 0$

$-\frac{1}{2}KL^2 \cdot \frac{2}{3}L + \bar{R}_y L = 0$

$\bar{R}_y = \frac{1}{3}KL^2$

$R_y = \frac{1}{6}KL^2$




$-R_y(x) + \frac{1}{2}Kx^2 \cdot \frac{x}{3} + M = 0$

$M = R_y x - \frac{Kx^3}{6}$

$M = \frac{1}{6}K\left[L^2x - x^3\right]$

$R_y - \frac{1}{2}K(x)^2 = V$

$\frac{1}{6}KL^2 - \frac{1}{2}K(x)^2 = V$

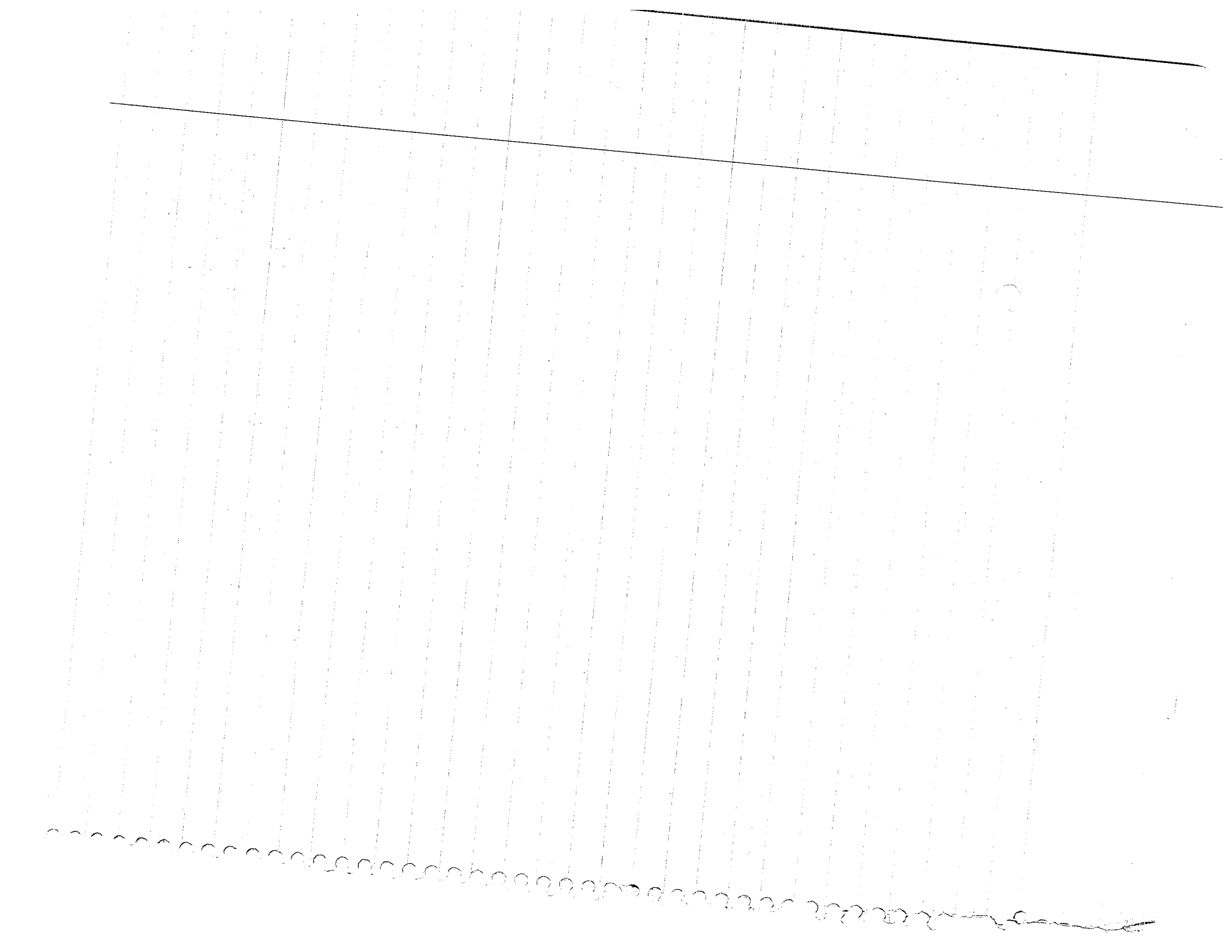
$-\frac{K}{6}\left[L^2 - 3x^2\right] = V$

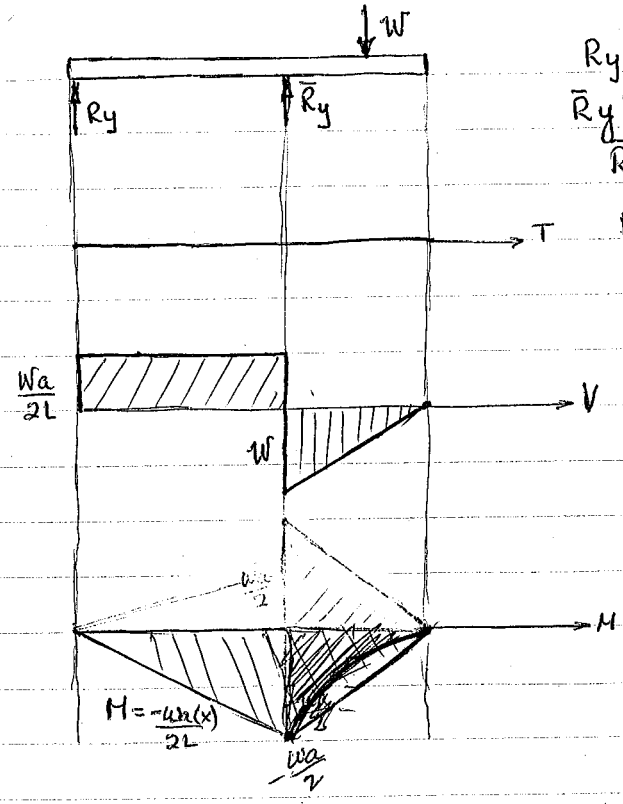
$\frac{K}{6}\left[3x^2 - L^2\right] = V$

$\frac{dy}{dx} = 0 \; ; \; x = L\sqrt{3}/3$

$\frac{1}{6}KL^2(x) - \frac{1}{6}K(x)^3 = M$

$R_y x - \frac{1}{6}K(x)^3 = M$



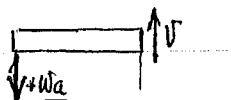



$R_y + \bar{R}_y = W$

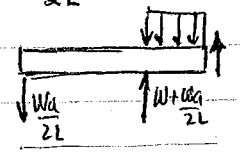
$\bar{R}_y L - W(L + a/2) = 0$

$\bar{R}_y = \frac{W}{L}(L + a/2) = W + \frac{Wa}{2L}$

$R_y = W\left(1 - \frac{L - a/2}{L}\right) = -\frac{Wa}{2L}$



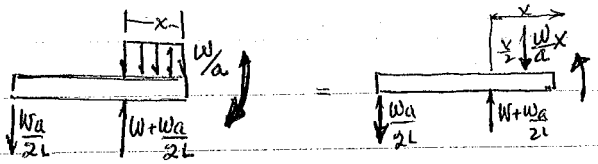

$V = +\frac{Wa}{2L}$




$-\frac{Wa}{2L} + W + \frac{Wa}{2L} - \frac{Wx}{a} + V = 0$

$V = \frac{Wx}{a} - W$

$M = -\frac{Wa}{2L}(x)$



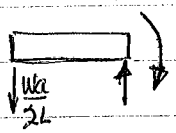

$-\frac{Wa}{2L}(L+x) + \left(W + \frac{Wa}{2L}\right)x - \frac{W}{2a}x^2 + M = 0$

$-\frac{Wa}{2L}(L+x) + \left(\frac{W + Wa}{2L}\right)x - \frac{Wx^2}{a} + M = 0$

$M = \frac{Wa}{2} + \frac{Wax}{2L} - Wx - \frac{Wax}{2L} + \frac{Wx^2}{2a}$

$M = \frac{Wa}{2} + \frac{Wax}{2L} - Wx + \frac{Wax}{2L} + \frac{Wx^2}{a} =$

$M = \frac{W}{2}(a - 2x) + \frac{W}{2a}x^2$




$\frac{Wa}{2} = M$

$M = \frac{W}{2}(a - 2x) + \frac{Wx^2}{2a} - \frac{Wa}{2} + \frac{Wa}{2}$

$+\frac{Wa}{2} - \frac{Wax}{2L} + Wx + \frac{Wax}{2L} + \frac{Wx^2}{2a} = +M$

$+\frac{Wa}{2L}(L+x) - \left(W + \frac{Wa}{2L}\right)x + \frac{Wx^2}{2a} + M = 0$

$M = -\frac{Wa}{2L}(x)$

$-M = \frac{Wa}{2} + \frac{Wax}{2L} - Wx - \frac{Wax}{2L} + \frac{Wx^2}{2a}$

$M = \frac{W}{2}(-a + 2x) + \frac{Wx^2}{2a} = \frac{W}{2}\left[2x - a - \frac{x^2}{a}\right]$

$2 - a - \frac{a}{4}$

$-\frac{W}{8}$

3.04
3.04
1216
912
9.2416

$18.33 - 2x^2 = 0$

$9.17 = x^2 \quad = 3.03$

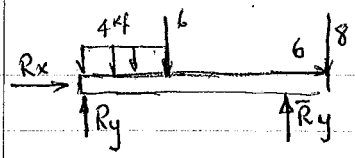
$x = 3.$

12.

$4x - 18.33 = 0$

$x = 4.6$

Problem 2.21



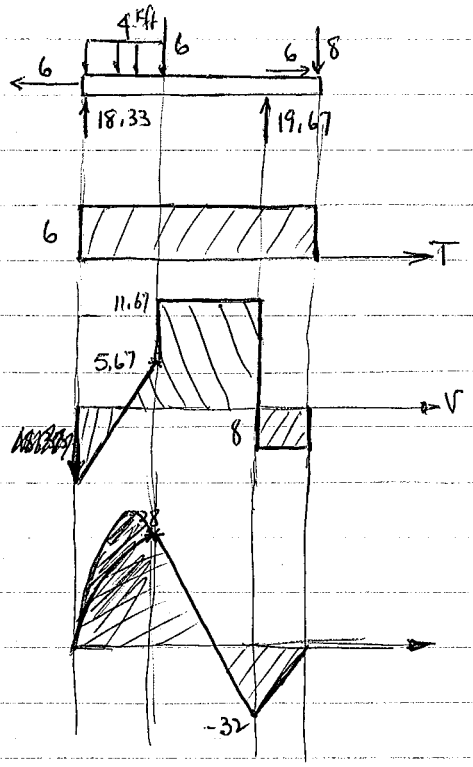

$R_x + 6 = 0 \quad R_x = -6$

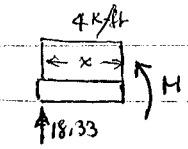
$R_y + \bar{R}y - 8 - 6 - 24 = 0$

$-72 - 36 + \bar{R}y(12) - 8(16) = 0$

$\frac{+108+128}{12} = \bar{R}y \quad \bar{R}y = 19.67$

$R_y = 18.33$

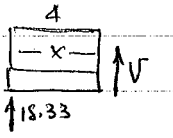



$x \geq 0 \quad x \leq 6$

$-18.33x + \frac{4x^2}{2} + M = 0$

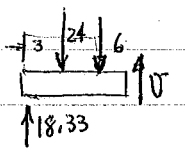
$M = 18.33x - 2x^2$




$18.33 - 4x + V = 0$

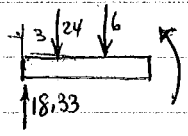
$4x - 18.33 = V$

$5.67 \quad x \geq 0 \quad x \leq 6$




$V = 30 - 18.33$

$= 11.67 \quad x \geq 6 \quad x \leq 12$




$-18.33x + 24(x-3) + 6(x-6) + M = 0$

$M = -11.67x + 108 \quad x \geq 6 \quad x \leq 12$

$-\frac{35(4)}{3} + 108 = -32$

$55 - 18.37$

60
48
110

$-18.33x + 24x - 72 + 6x - 36$

$-19.67x + 236 + M = 0$

$-7x = 38$

$18.33(5) - 50$

91.67

50

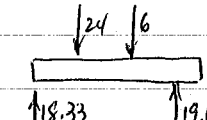
$-38x + 30x + 128 + M = 0$

$+8x - 128 = M$

80
48

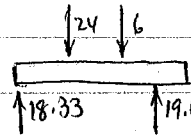
$18.33 - 4x = 0$

$x = \frac{18.33}{4} = 4.58$




$V \geq 12 \quad V \leq 16$

$V + 38 - 30 = 0 \quad V = -8$




$-18.33(x) + 24(x-3) + 6(x-6) - 19.67(x-12) + M = 0$

$V(x) = 4X - 18.33K$ $\quad 0 \le x \le 6$

$= 11.67\ K$ $\quad 6 \le x \le 12$

$= -8\ K$ $\quad 12 \le x \le 16$

$= 0\ K$ $\quad x = 16$

$M(x) = 18.33x - 2x^2$ $\quad 0 \le x \le 6$

$= -11.67x + 108$ $\quad 6 \le x \le 12$

$= 8x - 128$ $\quad 12 \le x \le 16$

$= 0$ $\quad x = 16$

$\frac{\partial M}{\partial x} = -V$

$\frac{\partial M}{\partial x^2} = -\frac{\partial V}{\partial x} = +P$

$18.33x - 2x^2 = M$

$\frac{\partial M}{\partial x} = 18.33 - 4x = 0$

$4.58 = x$

$\frac{55}{3}(4.58) - 2(4.58)^2 =$

4.6

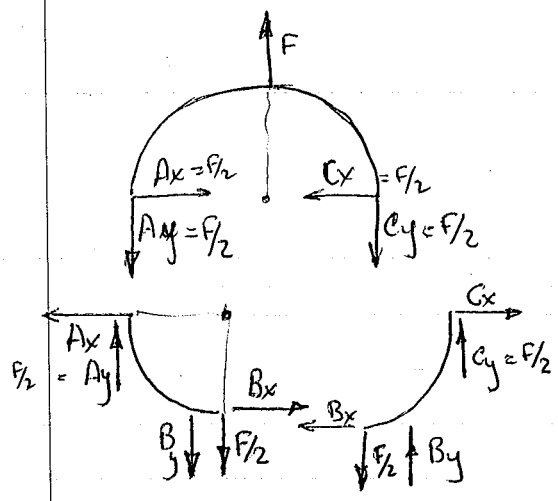
$\frac{230}{23}$

$\frac{253}{3} = 84$

4.6
4.6
276
184
21.16

$84 - 42.3x$

41.

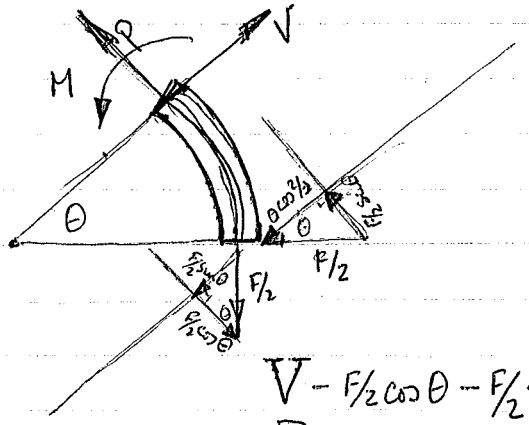


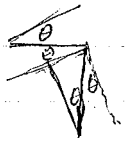

$$A_x - C_x = 0$$
$$F - A_y - C_y = 0$$
$$A_y R - C_y R = 0 \qquad A_y = C_y$$
$$F - 2C_y = 0 \qquad F/2 = C_y = A_y$$




$$-A_x + B_x = 0$$
$$A_y - F/2 - B_y = 0 \qquad B_y = 0$$
$$B_x(R) - A_y(R) = 0 \qquad B_x = A_y = F/2 = A_x$$
$$C_x = F/2$$

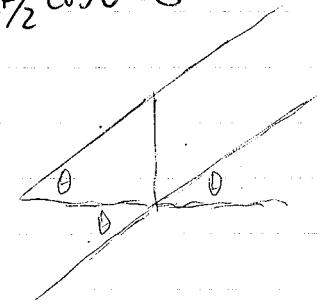
$$C_x - B_x = 0 \qquad F/2 - F/2 = 0$$
$$C_y + B_y - F/2 = 0 \qquad F/2 + 0 - F/2 = 0$$
$$-B_x(R) + C_y R = 0 \qquad B_x = C_y = F/2 \checkmark$$



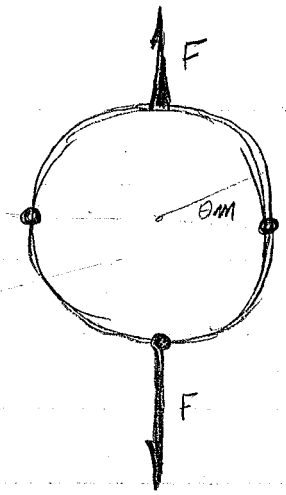



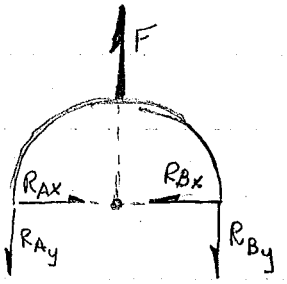

$$V - F/2\cos\theta - F/2\sin\theta = 0$$
$$P + F/2\sin\theta - F/2\cos\theta = 0$$

$$P = F/2\,(\cos\theta - \sin\theta)$$
$$V = F/2\,(\cos\theta + \sin\theta)$$
$$P_R - F/2\,R + M = 0$$
$$M = F/2\,R - R\left[F/2\sin\theta - F/2\cos\theta\right]$$
$$M = \frac{FR}{2}\left[1 + \cos\theta - \sin\theta\right]$$



$0 < \theta < \pi/2$



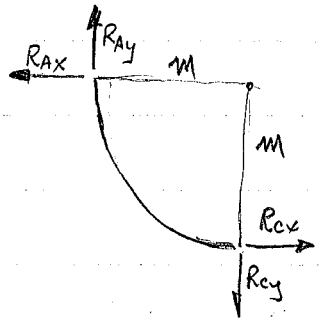

$R_{Ax} = R_{Bx}$

$R_{Ay} + R_{By} = F$

$R_{Ay}[m] = R_{By}[m]$

$R_{Ay} = R_{By}$

$\therefore R_{Ay} = R_{By} = F/2$



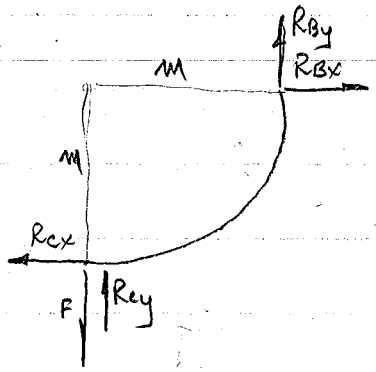

$R_{Ay} = R_{cy} = F/2$

$R_{Ax} = R_{cx}$

$R_{cx}[m] = R_{Ay}[m]$

$R_{cx} = R_{Ay} = R_{Ax} = F/2$

since $R_{Ax} = F/2$ ; $R_{Bx} = F/2$

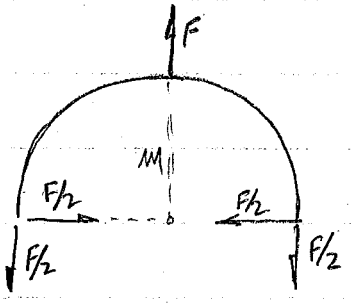


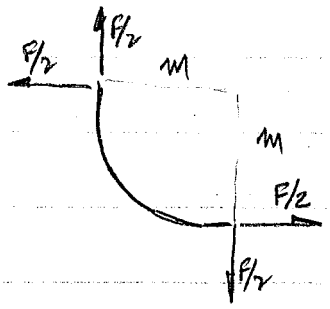

$R_{By} + R_{cy} = F$

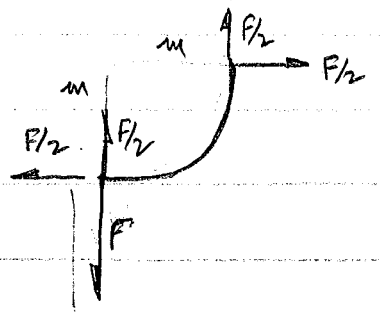
$\therefore R_{By} = F/2$ ✓

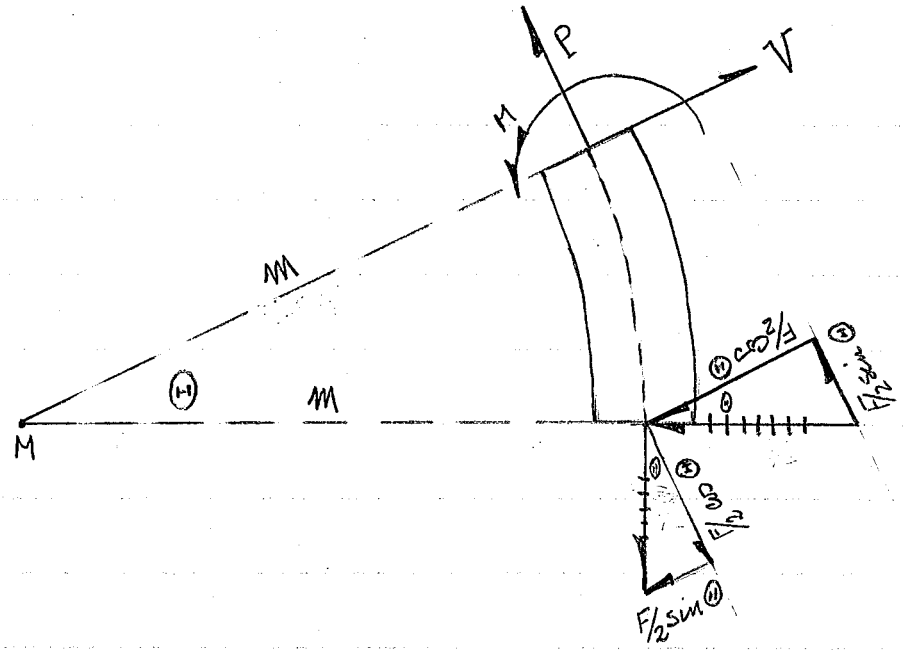
$R_{Bx} = R_{cx} = F/2$

Final Shear-Stress Force position






$$\sum F_P = 0 \qquad P + \frac{F}{2}\sin\theta - \frac{F}{2}\cos\theta = 0$$

$$\sum F_V = 0 \qquad V - \frac{F}{2}\cos\theta - \frac{F}{2}\sin\theta = 0$$

$$\sum M_M = 0 \qquad P[m] + M - \frac{F}{2}[m] = 0$$

$$P(\theta) = \frac{F}{2}\left[\cos\theta - \sin\theta\right]$$

$$V(\theta) = \frac{F}{2}\left[\sin\theta + \cos\theta\right]$$

$$M(\theta) = \frac{Fm}{2}\left[1 + \sin\theta - \cos\theta\right]$$

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$$\varepsilon_x = \frac{\partial u}{\partial x}, \qquad \varepsilon_y = \frac{\partial v}{\partial y}, \qquad \varepsilon_z = \frac{\partial w}{\partial z}$$

① $$\gamma_{xy} = \gamma_{yx} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

$$\frac{\partial(\gamma_{xy})}{\partial y} = \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial^2 \gamma_{xy}}{\partial y \partial x} = \frac{\partial^3 v}{\partial x \partial y \partial x} + \frac{\partial^3 u}{\partial y^2 \partial x}$$

but $\frac{\partial^2 F}{\partial y \partial x} = \frac{\partial^2 F}{\partial x \partial y}$ therefore

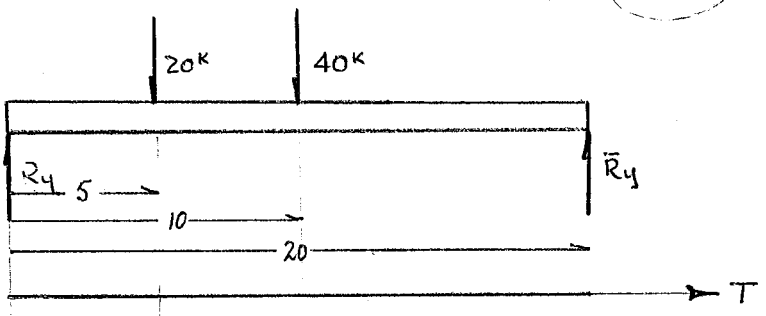
② $$\frac{\partial^2 \gamma_{xy}}{\partial y \partial x} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

③ $$\frac{\partial^3 v}{\partial x \partial y \partial x} = \frac{\partial^3 v}{\partial x^2 \partial y} = \frac{\partial^2}{\partial x^2}\left[\frac{\partial v}{\partial y}\right] = \frac{\partial^2 \epsilon_y}{\partial x^2}$$

④ $$\frac{\partial^3 u}{\partial y^2 \partial x} = \frac{\partial^2}{\partial y^2}\left[\frac{\partial u}{\partial x}\right] = \frac{\partial^2 \epsilon_x}{\partial y^2}$$

$$\therefore \quad \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2}$$

QED

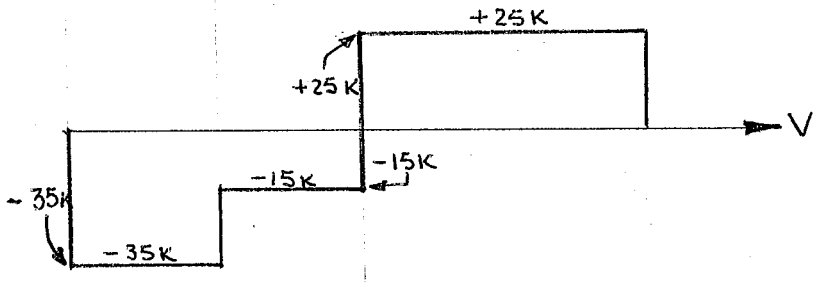


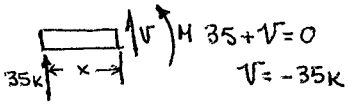

$R_y + \bar{R}_y - 60 = 0$

$-100 - 400 + \bar{R}_y(20) = 0$

$\bar{R}_y = 25K$

$R_y = 35K$

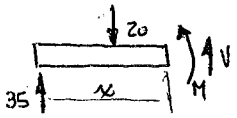



$35 + V = 0$

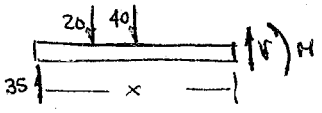
$V = -35K$

$35x = M$; $V = -35K$ $\quad 0 \le x \le 5$

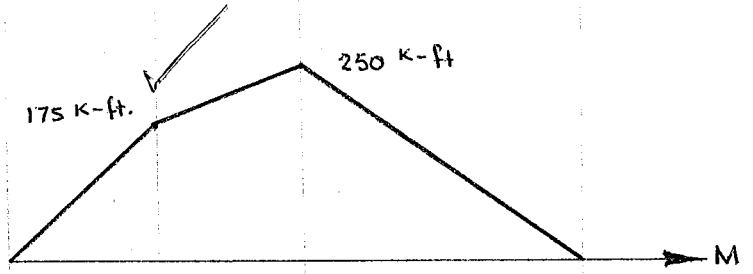


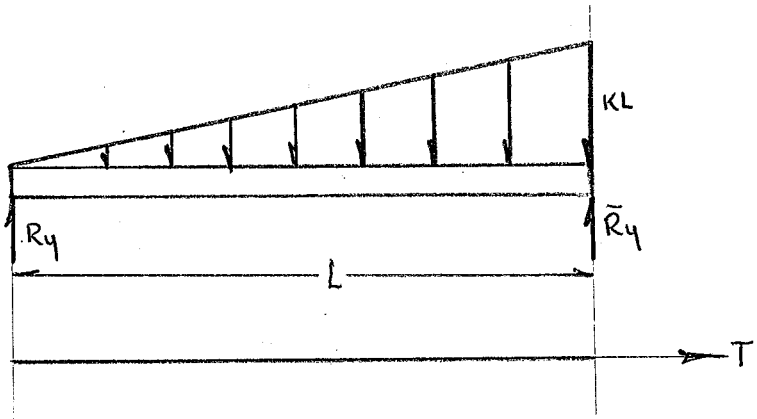

$V = -15K$

$M = 15x + 100$ $\quad 5 \le x \le 10$




$V = 25K$ $\quad M = -25x + 500$ $\quad 10 \le x \le 20$



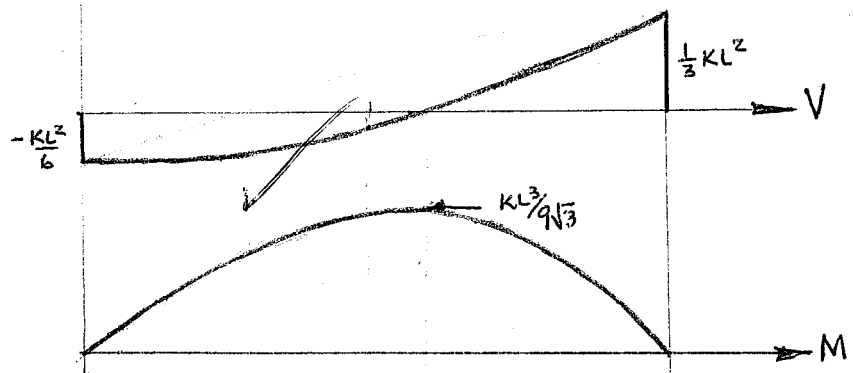



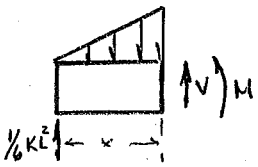

$R_y + \bar{R}_y - \frac{1}{2}KL^2 = 0$

$-\frac{KL^3}{3} + \bar{R}_y L = 0$

$\bar{R}_y = \frac{1}{3}KL^2$

$R_y = \frac{1}{6}KL^2$



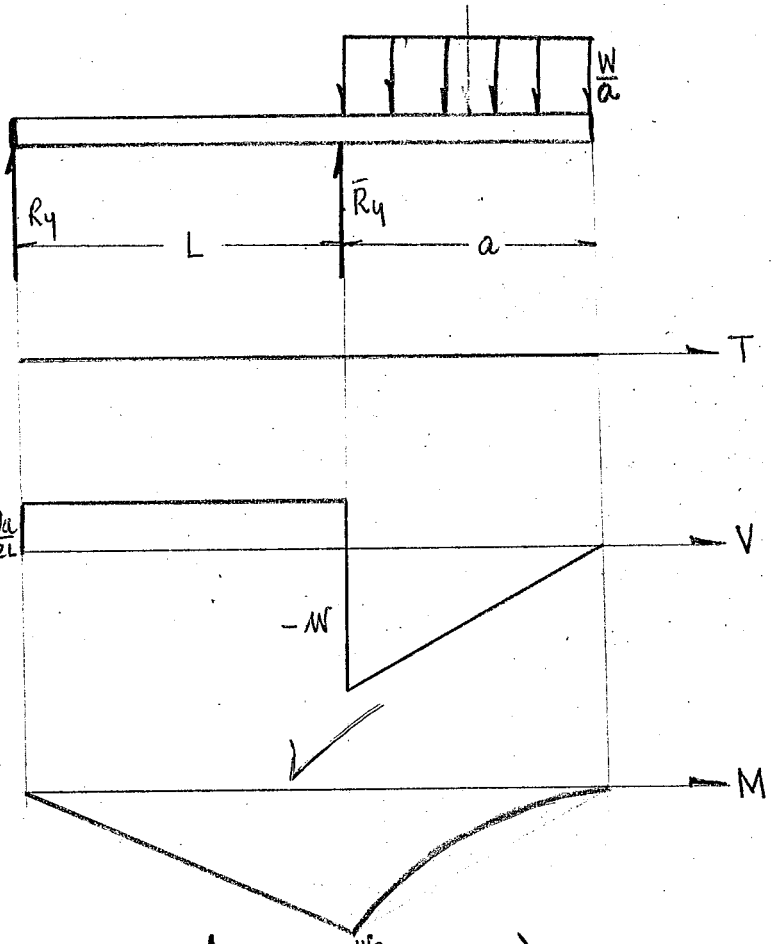




$R_y - \frac{1}{2}Kx^2 + V = 0$

$V = [3x^2 - L^2]K/6$

$-\frac{1}{6}KL^2x + \frac{1}{6}Kx^3 + M = 0$

$M = \frac{1}{6}K[L^2x - x^3]$

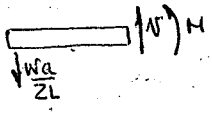



$R_y + \bar{R}_y = W$

$\bar{R}_y L - \frac{W}{a}\left(L + \frac{a}{2}\right) = 0$

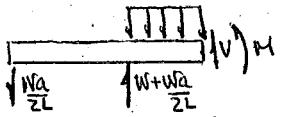
$\bar{R}_y = W + \frac{Wa}{2L}$

$R_y = -\frac{Wa}{2L}$



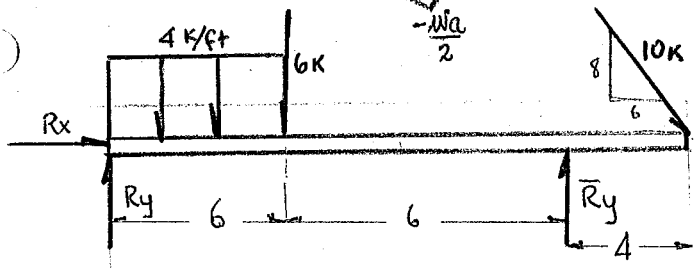

$V = \frac{Wa}{2L}$ ; $M = -\frac{Wa}{2L}(x)$

$0 \le x \le L$




$V = \frac{Wx}{a} - W$ $\quad L \le x \le L+a$

$M = \frac{W}{2}\left[2x - a - \frac{x^2}{a}\right]$



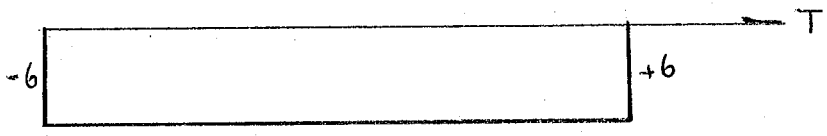

$R_x + 6 = 0$

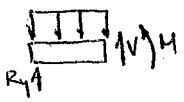
$R_y + \bar{R}_y - 8 - 6 - 24 = 0$

$-72 - 36 + \bar{R}_y(12) - 8(16) = 0$

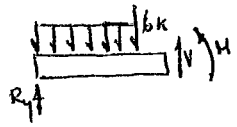
$\bar{R}_y = 19.67$

$R_y = 18.33$

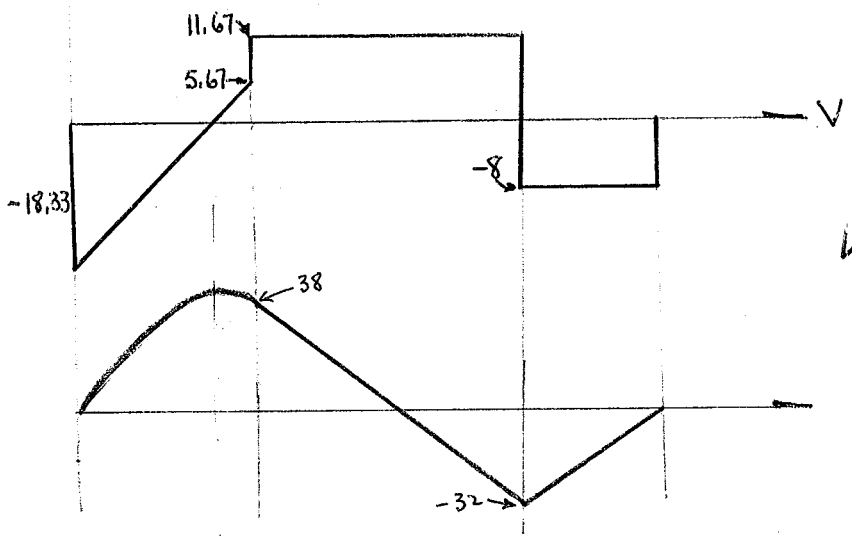


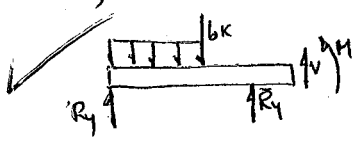




$T = -6$ ; $V = 4x - 18.33$ ; $M = 18.33x - 2x^2$ ; $0 < x \le 6$

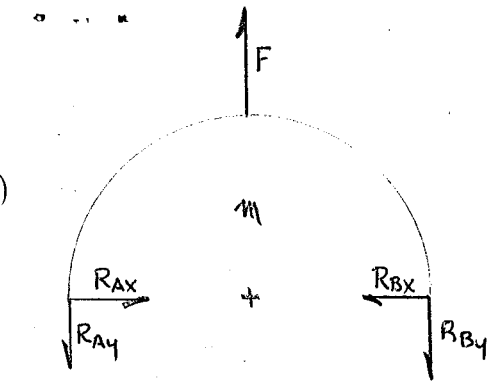



$T = -6$ ; $V = 11.67$ ; $M = -11.67x + 108$ ; $6 \le x \le 12$




$T = -6$ ; $V = -8$ ; $M = 8x - 128$ ; $12 \le x \le 16$



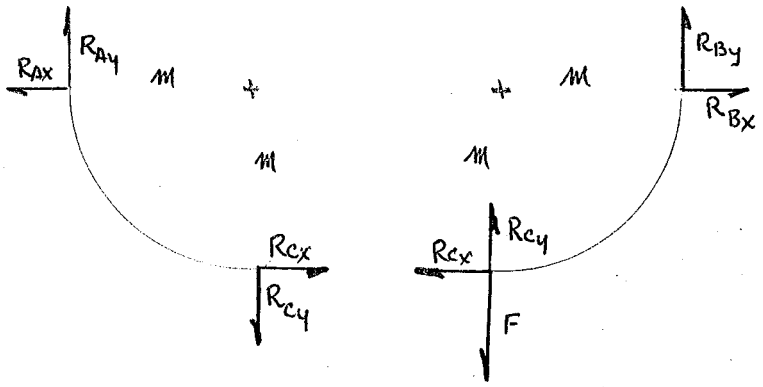

$R_{AX} = R_{BX}$

$R_{AY} + R_{BY} = F$

$R_{AY}[M] = R_{BY}[M]$

$R_{AY} = R_{BY}$

$\therefore R_{AY} = R_{BY} = F/2$



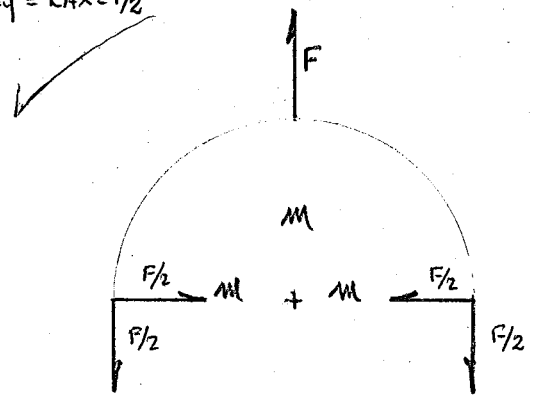

$R_{AY} = R_{CY} = F/2$

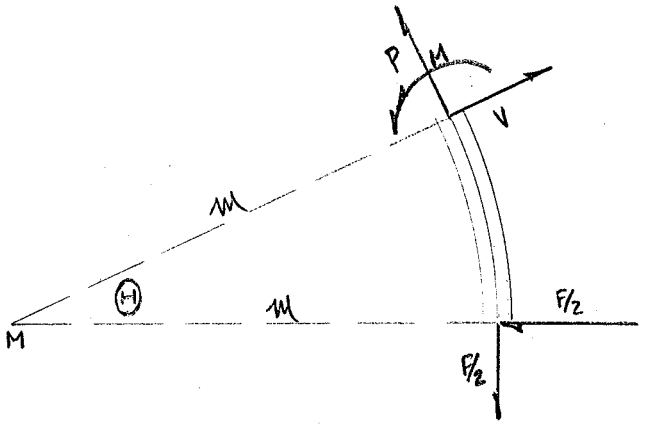
$R_{AX} = R_{CX}$

$R_{CX}[M] = R_{AY}[M]$

$\therefore R_{CX} = R_{AY} = R_{AX} = F/2$

Dado $R_{AX} = F/2$ ; $R_{BX} = F/2$




$\sum F_P = 0 \quad P + F/2 \sin\theta - F/2 \cos\theta = 0$

$\sum F_V = 0 \quad V - F/2 \cos\theta - F/2 \sin\theta = 0$

$\sum M_M = 0 \quad P[M] + M - F/2[M] = 0$

$P(\theta) = F/2 \left[\cos\theta - \sin\theta\right]$

$V(\theta) = F/2 \left[\sin\theta + \cos\theta\right]$

$M(\theta) = \frac{FM}{2} \left[1 + \sin\theta - \cos\theta\right]$

$$d\varphi = \frac{T\,dx}{JG} \qquad \varphi = \int_0^x \frac{T(x)}{J(x)G}\,dx + C_1$$

$$J_{AB} = \frac{\pi c^4}{2} = \frac{\pi}{2} = J_{BK} \qquad J_{KC} = J_{CD} = \frac{\pi}{2}\left(c_o^4 - c_i^4\right) = \frac{\pi}{2}\left(\frac{15}{16}\right)$$

$L_{AB} = 10$ in $\quad L_{BK} = 6$ in $\quad L_{KC} = 6$ in $\quad L_{CD} = 10$ in

$T_{AB} = 450\pi$ ↷ $\quad T_{BK} = 240\pi$ ↶ $\quad T_{KC} = 240\pi$ ↷ $\quad T_{CD} = 375\pi$ ↶

$$\varphi = \int_A^D \frac{T(x)}{G(JK)}\,dx = \int_A^B \frac{T_{AB}}{G\,J_{AB}}\,dx_{AB} + \int_B^K \frac{T_{BK}}{G\,J_{BK}}\,dx_{BK} + \int_K^R \frac{T_{KC}}{G\,J_{KC}}\,dx_{KC} + \int_C^D \frac{T_{CD}\,dx_{CD}}{G\,J_{CD}}$$

$$= \frac{T_{AB}L_{AB}}{G\,J_{AB}} + \frac{T_{BK}L_{BK}}{G\,J_{BK}} + \frac{T_{KC}L_{KC}}{G\,J_{KC}} + \frac{T_{CD}L_{CD}}{G\,J_{CD}}$$

$$= \frac{(450\pi)(10)}{(12\times10^6)(\pi/2)} + \frac{240\pi(6)}{(12\times10^6)(\pi/2)} + \frac{240\pi(6)}{(12\times10^6)\left(\frac{\pi(15)}{32}\right)} + \frac{375\pi(10)}{(12\times10^6)\left(\frac{\pi}{32}(15)\right)}$$

$$= \frac{9000}{12\times10^6} + \frac{480}{2\times10^6} + \frac{240(32)}{(2\times10^6)(15)} + \frac{375(16)}{(6\times10^6)(15)}$$

$$= .75\times10^{-3} + .24\times10^{-3} + .128\times10^{-3} + .667\times10^{-3} = 1.785\times10^{-3}\ \text{rad.}$$

$\varphi$ in degrees $= .001785\left(\frac{180}{\pi}\right) = .123^\circ$

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$\tau_{max} = \frac{Tc}{J}$ $\quad J = \frac{\pi c^4}{2}$ for solid shaft ; $J = \frac{\pi c^4}{2} - \frac{\pi b^4}{2}$ for hollow shaft

$\tau_{max_1} = \tau_{max_2}$ ; $T_1 = T_2$ ; $\therefore \quad c_1 J_2 = c_2 J_1$

$c_1 = 1$ in , $c_2 = 1.5$ in , $\quad J_1 = \frac{\pi}{2}$ $\quad J_2 = \frac{\pi}{2}\left(\frac{81}{16} - b^4\right)$

$1.5\left(\frac{\pi}{2}\right) = 1\left(\frac{\pi}{2}\right)\left(\frac{81}{16} - b^4\right)$ ; $\quad 1.5 = \left(\frac{81}{16} - b^4\right)$

$b = \sqrt[4]{5.0625 - 1.5} = \sqrt[4]{3.5625} = 1.375$ in

thickness must be $1.5 - 1.375 = .125$ in.

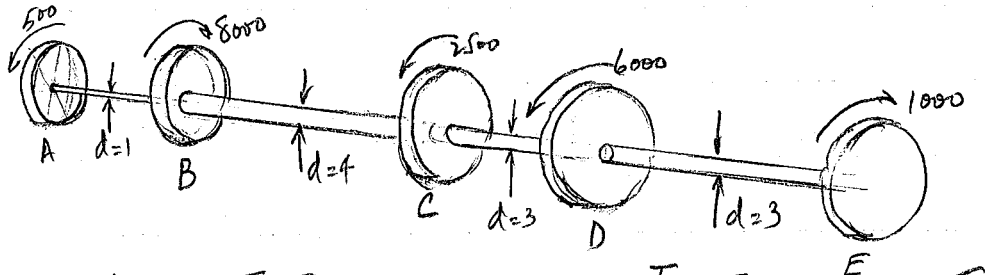
b)

weight of solid / weight of hollow = $\rho$ Volume solid / $\rho$ Volume hollow.

Volume of solid $= \pi r^2 h$ $\quad$ volume of hollow $= \pi R^2 h - \pi r^2 h$

$\frac{W_s}{W_h} = \frac{r^2}{R^2 - r^2}$ $\quad r = 1.375$; $R = 1.5$; $r = 1$; $\quad \frac{W_s}{W_h} = \frac{1}{.365} = \underline{\underline{2.74}}$

5-2




| | | | | |
|---|---|---|---|---|
| between AB | T = 500 | c = 1/2 | $J = \frac{\pi}{32}$ | $\tau_{max} \stackrel{?}{=}$ 2545 psi |
| between BC | T = 7500 | c = 2 | $J = 8\pi$ | $\tau_{max} \stackrel{?}{=}$ 596 psi |
| between CD | T = 5000 | c = 1.5 | $J = \frac{\pi}{2}\left(\frac{81}{16}\right)$ | $\tau_{max} \stackrel{?}{=}$ 944 psi |
| between DE | T = 1000 | c = 1.5 | $J = \frac{\pi}{2}\left(\frac{81}{16}\right)$ | $\tau_{max} \stackrel{?}{=}$ 188.5 psi |

$\tau_{max} = 2545$ psi between A & B

5-3 $T_l = \frac{63000(90)}{63000} = 9000$ in-lb $\qquad T_r = \frac{63000}{630,00}(30) = 3000$ in-lb

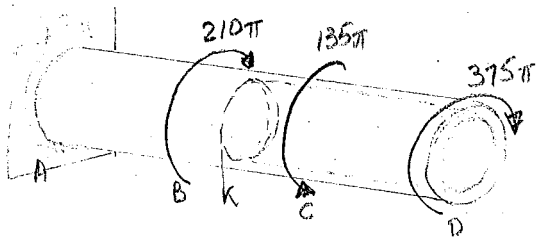
Since $kT = T_{max}$, we must find a rod that will withstand $T_{max}$. We can therefore say that we must overcome the torque produced by the left machine since it has the higher torque.

$$\therefore T_{max} = \frac{Tc}{J} \; ; \quad \frac{c}{J} = \frac{c}{\frac{\pi}{2}c^4} = \frac{2}{\pi}\frac{1}{c^3} \; ; \quad \frac{T_{max}}{T} = \frac{2}{\pi}\frac{1}{c^3}$$

$$\frac{\pi}{2}c^3 = \frac{T}{T_{max}} \qquad c = \sqrt[3]{\frac{2T}{\pi T_{max}}} = \sqrt[3]{\frac{18000}{18090}} \approx \sqrt[3]{1.0} = 1.0$$

$$\therefore c = 1.0 \qquad \underline{d = 2 \text{ in}}$$

5-4




1. between AB $\quad T = 450\pi \quad c = 1'' \quad J = \frac{\pi c^4}{2}$

$$T_{max_1} = \frac{Tc}{J} = \frac{2(450)\pi}{\pi c^3} = \frac{900\pi}{\pi} = 900 \text{ psi}$$

2. between BK $\quad T = 240\pi \quad c = 1 \quad J = \frac{\pi c^4}{2}$

$$T_{max_2} = \frac{Tc}{J} = \frac{2(240)\pi}{\pi c^3} = 480 \text{ psi}$$

3. between KC $\quad T = 240\pi \quad c = 1 \quad b = \frac{1}{2} \quad J = \frac{\pi b^4}{2} + \frac{\pi c^4}{2}$

$$J = \frac{\pi}{2}\left(1 - \frac{1}{16}\right) = \frac{\pi}{2}\left(\frac{15}{16}\right) \; ; \quad T_{max_3} = \frac{Tc}{J} = \frac{240\pi(1)2(16)}{\pi(15)} = 480\frac{(16)}{15} = 512 \text{ psi}$$

4 between CD $\quad T = 375\pi \quad c = 1 \quad b = \frac{1}{2} \quad J = \frac{\pi}{2}\left(\frac{15}{16}\right)$

$$T_{max_4} = \frac{Tc}{J} = \frac{375\pi(1)2(16)}{\pi(15)} = \frac{750(16)}{15} = 800 \text{ psi}$$

$\boxed{T_{max} = 900 \text{ psi} \text{ between AB}}$

Mechanics of Solids

CE201D



Problems.

4-3, 5, 6, 7, 8

C. LEVY

4/9/70

4-3, 5, 6, 7, 8

C. LEVY CE201 D

Mr. Blumenf

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\epsilon_y = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_z$$

$$\epsilon_z = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \frac{\sigma_z}{E}$$

$$\therefore \begin{bmatrix} 1/E & -\nu/E & -\nu/E \\ -\nu/E & +1/E & -\nu/E \\ -\nu/E & -\nu/E & 1/E \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{bmatrix} = \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \end{bmatrix}$$

TAKE THE AUGMENTED MATRIX AND REDUCE IT

$$\begin{bmatrix} 1/E & -\nu/E & -\nu/E & \epsilon_x \\ -\nu/E & +1/E & -\nu/E & \epsilon_y \\ -\nu/E & -\nu/E & 1/E & \epsilon_z \end{bmatrix} \longrightarrow \begin{bmatrix} 1/E+\nu/E & 0 & -\nu/E-1/E & \epsilon_x-\epsilon_z \\ 0 & 1/E+\nu/E & -\nu/E-1/E & \epsilon_y-\epsilon_z \\ -\nu/E & -\nu/E & 1/E & \epsilon_z \end{bmatrix} \longrightarrow$$

$$\begin{bmatrix} 1 & 0 & -1 & (\epsilon_x-\epsilon_z)/(1/E+\nu/E) \\ 0 & 1 & -1 & (\epsilon_y-\epsilon_z)/(1/E+\nu/E) \\ -\nu/E & -\nu/E & 1/E & \epsilon_z \end{bmatrix} \longrightarrow$$

$$\begin{bmatrix} 1 & 0 & -1 & (\epsilon_x-\epsilon_z)/(1/E+\nu/E) \\ 0 & 1 & -1 & (\epsilon_y-\epsilon_z)/(1/E+\nu/E) \\ 0 & -\nu/E & 1/E-\nu/E & \epsilon_z+\nu/E\left[(\epsilon_x-\epsilon_z)/(1/E+\nu/E)\right] \end{bmatrix} \longrightarrow$$

$$\begin{bmatrix} 1 & 0 & -1 & (\epsilon_x-\epsilon_z)/(1/E+\nu/E) \\ 0 & 1 & -1 & (\epsilon_y-\epsilon_z)/(1/E+\nu/E) \\ 0 & 0 & -2\nu/E+1/E & \epsilon_z+\nu/E\left[(\epsilon_x-\epsilon_z)/(1/E+\nu/E)\right]+\frac{\nu}{E}\left[(\epsilon_y-\epsilon_z)/(1/E+\nu/E)\right] \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & (\epsilon_x-\epsilon_y)/(1+\nu/E) \\ 0 & 1 & -1 & (\epsilon_y-\epsilon_z)/(1+\nu/E) \\ 0 & 0 & 1 & \frac{E}{1-2\nu}\left[\epsilon_z+\nu/E\left[(\epsilon_x-\epsilon_z)/\left(\frac{1+\nu}{E}\right)\right]+\nu/E\left[(\epsilon_y-\epsilon_z)/\left(\frac{1+\nu}{E}\right)\right]\right] \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & (\epsilon_x-\epsilon_z)/\left(\frac{1+\nu}{E}\right)+K \\ 0 & 1 & 0 & (\epsilon_y-\epsilon_z)/\left(\frac{1+\nu}{E}\right)+K \\ 0 & 0 & 1 & K \end{bmatrix} \Rightarrow \begin{aligned} \sigma_x &= (\epsilon_x-\epsilon_z)/\left(\tfrac{1+\nu}{E}\right)+K \\ \sigma_y &= (\epsilon_y-\epsilon_z)/\left(\tfrac{1+\nu}{E}\right)+K \\ \sigma_z &= K \end{aligned}$$

$$\sigma_x = (\epsilon_x-\epsilon_z)/\left(\tfrac{1+\nu}{E}\right) + E/(1-2\nu)\left[\epsilon_z+\nu/E\left[(\epsilon_x+\epsilon_y-2\epsilon_z)/\left(\tfrac{1+\nu}{E}\right)\right]\right]$$

$$\sigma_y = (\epsilon_y-\epsilon_z)/\left(\tfrac{1+\nu}{E}\right) + E/(1-2\nu)\left[\epsilon_z+\nu/E\left[(\epsilon_x+\epsilon_y-2\epsilon_z)/\left(\tfrac{1+\nu}{E}\right)\right]\right]$$

$$\sigma_z = E/(1-2\nu)\left[\epsilon_z+\nu/E\left[(\epsilon_x+\epsilon_y-2\epsilon_z)/\left(\tfrac{1+\nu}{E}\right)\right]\right]$$

from $\sigma_z = \frac{EV}{(1+V)(1-2V)}\left[\frac{\epsilon_z}{V} + \epsilon_x + \epsilon_y - \epsilon_z\right]$ add to both sides an $\lambda\epsilon_z$

so that $\sigma_z = \lambda\left[\frac{\epsilon_z}{V} + \epsilon_x + \epsilon_y + \epsilon_z - \epsilon_z\right] - \lambda\epsilon_z$

and that $\sigma_z = \lambda e + \lambda\epsilon_z\left[\frac{1}{V} - 2\right] = \lambda e + \lambda\epsilon_z\left[\frac{1-2V}{V}\right]$

$$\lambda\left[\frac{1-2V}{V}\right] = \frac{E}{(1+V)} = 2G$$

$\therefore \quad \sigma_z = \lambda e + 2G\epsilon_z$ if $G = \mu$ then $\sigma_z = \lambda e + 2\mu\epsilon_z$ (same as book)

therefore $\lambda = \frac{EV}{(1+V)(1-2V)}$ $\qquad \mu = G = \frac{E}{2(1+V)}$

for $\sigma_y$ proving that $\sigma_y = \sigma_z - 2\mu\epsilon_z + 2\mu\epsilon_y$ is simple

since $\sigma_y = E(\epsilon_y - \epsilon_z)/(1+V) + \sigma_z$

$\therefore \quad E(\epsilon_y - \epsilon_z)/(1+V) = 2\mu(\epsilon_y - \epsilon_z) = 2G(\epsilon_y - \epsilon_z) = E/(1+V)[\epsilon_y - \epsilon_z]$

since $\sigma_x$ just is $E(\epsilon_x - \epsilon_z)/(1+V) + \sigma_z$ by the same way $2\mu\epsilon_x - 2\mu\epsilon_z = \frac{E}{(1+V)}(\epsilon_x - \epsilon_z)$

5. $P_x = 20$ KIPS $\qquad P_y = 40$ KIPS $\qquad \sigma_z = 0$

$\epsilon_z = -\frac{V}{E}(\sigma_x + \sigma_y)$ $\qquad \sigma_x = \frac{20}{1\text{in}^2} = 20\ \text{K/in}^2$ $\qquad \sigma_y = \frac{40}{5} = 8\ \text{K/in}^2$

$= -\frac{1}{120\times10^3\text{KSI}}(28\ \text{KSI})$ $\qquad V = 1/4 \qquad E = 30\times10^3\ \text{KSI}$

$= -\frac{7}{30\times10^3\text{KSI}} = -.2335\times10^{-3}$

$\epsilon_z = \frac{\Delta}{z} = \frac{\Delta}{.5\text{in}} = -.2335\times10^{-3}$; $\quad -.1168\times10^{-3}\text{in} = \Delta$ depth decreases by $.1168\times10^{-3}$ in.

b. $\sigma_x = \frac{P_x}{1\text{in}^2} = P_x/\text{in}^2$ $\qquad \sigma_z = \sigma_y = 0$

$\epsilon_z = -.2335\times10^{-3}$ $\qquad \epsilon_z = -\frac{V}{E}\sigma_x$

$-.2335\times10^{-3} = -\frac{7}{30\times10^3} = -\frac{1}{4(30\times10^3)}\sigma_x \quad \therefore \quad \sigma_x = 28\ \text{KSI}$

$\boxed{P_x = 28\ \text{K}}$

6. $\sigma_x = \frac{P_x}{A_x}$ $\sigma_y = \frac{P_y}{A_y}$ ;

$\epsilon_x = \frac{\Delta_x}{L_x} = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y$ $E = 30\times10^3$ KSI $\nu = \frac{E}{2G} - 1$

$\epsilon_y = \frac{\Delta_y}{L_y} = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E}$ $G = 12\times10^3$ KSI $\nu = \frac{30}{24} - 1 = 1/4$

$\epsilon_x = \frac{\Delta_x}{L_x} = \frac{.0768}{12(8)} = .0008$ ; $\epsilon_y = \frac{\Delta_y}{L_y} = \frac{.0864}{12(12)} = .0006$

$.0008 = \frac{P_x}{EA_x} - \frac{1}{4E}\frac{P_y}{A_y}$ $A_x = (12\times12)(1/4) = 36\,in^2$

$A_y = (8\times12)(1/4) = 24\,in^2$

$.0006 = -\frac{1}{4E}\frac{P_x}{A_x} + \frac{P_y}{EA_y}$

$.0008 = \frac{P_x}{30\times10^3(36)} - \frac{P_y}{(120\times10^3)(24)}$ ; $.0006 = -\frac{P_x}{120\times10^3(36)} + \frac{P_y}{(30\times10^3)(24)}$

$P_x = \frac{28.5(576)}{15}$ and $P_x$ in KIPS $= \frac{(28.5)(576)}{15(12)} = 91.2$ KIPS

$P_y$ by substitution $= 76.8$ KIPS

7. $\epsilon_x = \frac{\nu\sigma_x}{\nu E} - \frac{\nu}{E}\sigma_y - \frac{\nu}{E}\sigma_z$ ;

$\sigma_x = \frac{40\text{ KIPS}}{8\,in^2}$ ; $\sigma_y = \frac{48\text{ KIPS}}{6\,in^2}$ ; $\sigma_z = \frac{36\text{ KIPS}}{12\,in^2}$

$\epsilon_x = \frac{20}{40\times10^3} - \frac{8}{40\times10^3} - \frac{3}{40\times10^3}$

$\sigma_x = 5$ KSI ; $\sigma_y = 8$ KSI ; $\sigma_z = 3$ KSI

$E = 10\times10^3$ KSI $\nu = 1/4$

$= \frac{9}{40\times10^3} = .225\times10^{-3}$

$\Delta l_x = \epsilon_x l_{0x}$ $\Delta l_x = (.225\times10^{-3})(3) = .675\times10^{-3}$ in

find $P_x \Rightarrow \Delta l$ is same $\therefore \epsilon_x$ must be same

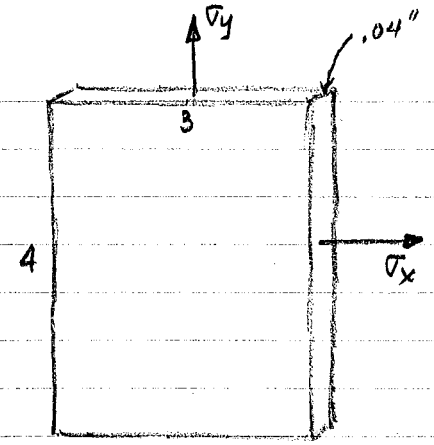
$.225\times10^{-3} = \frac{\sigma_x}{E}$ since $\sigma_y = \sigma_z = 0$

$\frac{9}{40\times10^3}(10\times10^3) = \sigma_x$ ; $\sigma_x = 2.25$ KSI $P_x = \sigma_x A_x$ $A_x = 8\,in^2$

$\boxed{P_x = 8(2.25) = 18\text{ KIPS}}$

8. $\Delta T = -200^\circ F$ $E_B = 16\times10^6$ PSI , $G_B = 6\times10^6$ PSI , $\alpha_B = 11\times10^{-6}/^\circ F$

$\alpha_{INVAR} = 0$




$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y + \alpha\Delta T$$

$$\epsilon_y = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E} + \alpha\Delta T$$

since it is constrained to a rigid unwarp frame then

$$\epsilon_x = \epsilon_y = 0$$

$$-\alpha\Delta T\, E = \sigma_x - \nu\sigma_y$$

$$-\alpha\Delta T\, E = -\nu\sigma_x + \sigma_y$$

then $-\alpha\Delta T E\nu = \nu\sigma_x - \nu^2\sigma_y$

$$-\alpha\Delta T E = -\nu\sigma_x + \sigma_y$$

$$-\alpha\Delta T E(1+\nu) = (1-\nu^2)\sigma_y = (1-\nu)(1+\nu)\sigma_y$$

$$\sigma_y = \frac{-\alpha\Delta T E}{1-\nu} = -\left(11\times10^{-6}\,\frac{\text{in}}{\text{in/deg F}}\right)(-200\,^\circ\text{F})\left(16\times10^6\,\text{psi}\right)\Big/\left(\tfrac{2}{3}\right)$$

$\sigma_y = 52.8$ KSI

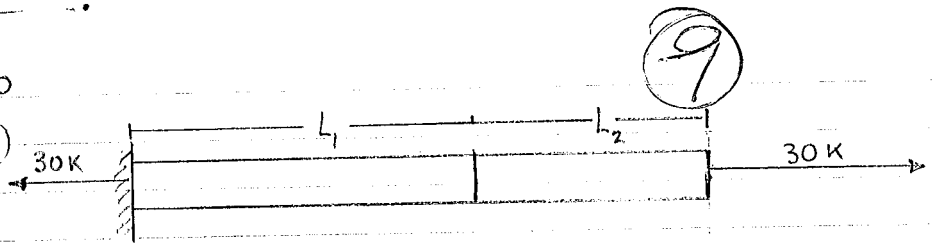
Substituting back we get $\sigma_x = 26.4$ KSI I made a mistake of computation

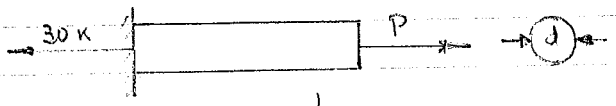
$$-\alpha\Delta T E = +35.2 \qquad 35.2 - 52.8 = -17.6 = -\tfrac{1}{3}\sigma_x$$

$\sigma_x = 52.8$ KSI

Cesar Levy
CE 201 D
Mr. Blumenthal

20




$L_1 + L_2 = 36in$

$u_S = u_A$

$$u_S = \frac{1}{E_S}\int_0^L \frac{P_S}{A_S}dx = \frac{1}{E_S}\frac{P}{A}L_1 \qquad u_A = \frac{1}{E_A}\int \frac{P_A}{A_A}dx = \frac{1}{E_A}\frac{P}{A}L_2$$

since $u_S = u_A$ $\quad \frac{L_1}{L_2} = \frac{E_S}{E_A} = .333$

$L_1 + L_2 = 36in \quad L_2 + 3L_2 = 36in \quad L_2 = 9in \quad L_1 = 27in$

$$u_A = \frac{30{,}000\,(9)}{10\times10^6\,(\pi)} = \frac{27}{\pi\times10^{-3}} = .859\times10^{-2} \qquad u_T = 2u_A = 1.718\times10^{-2}\,in$$

23 $$u_T = \frac{1}{E_A}\int \frac{P}{A_A}dx_1 + \frac{1}{E_S}\int \frac{P}{A_S}dx_2$$

$$u_T = \frac{P}{E_A A_A}(15) + \frac{P}{E_S A_S}(12) \qquad E_A = 10\times10^6\,psi \quad E_S = 30\times10^6\,psi \quad A_A = 16\,in^2 \quad A_S = 4\,in^2$$

$$u_T = .01 = \frac{3P + 3.2P}{32\times10^6} \;; \quad P = 51.6\,kips$$

21. f is given in units of force/length $\therefore F = \int f\,dy = \frac{Ky^3}{3} + C_1 \quad C_1 = 0 \quad y = L$

$$u = \int \frac{F\,dy}{EA} = \frac{1}{EA}\int \frac{Ky^3}{3}dy = \frac{1}{EA}\left[\frac{Ky^4}{12}\right]_0^L = \frac{KL^4}{12EA}$$

$\therefore$ but $K = \frac{3F}{L^3}$ $\longrightarrow$ $u = \frac{3F}{L^3}\cdot\frac{L^4}{12EA} = \frac{FL}{4EA}$

$P = 96000\,\#$ $\quad L = 40\times12\,in$ $\quad A = 100\,in^2$ $\quad E = 1.5\times10^6\,psi$

$$\frac{(96\times10^3\,\#)(40\times12\,in)}{4\left(1.5\times10^6\,\frac{\#}{in^2}\right)(100\,in^2)} = \frac{16(40\times12)}{10^5} = 7.682\times10^{-2}\,in$$

26. $u_A = u_B$

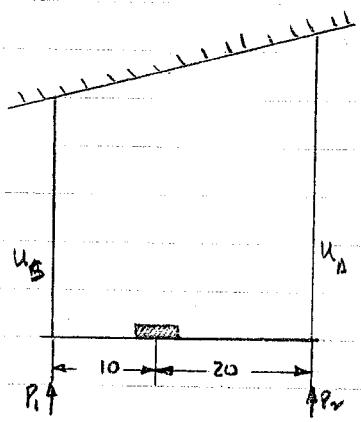
$$u_A = \int_0^{L_A} \frac{1}{E}\frac{P}{A}\,dx = \frac{P}{EX2}L_A$$

$$u_B = \int_0^{L_B} \frac{1}{E}\frac{P}{A}\,dx = \frac{P}{E}\int \frac{dx}{A_T} = \frac{PL_B}{E}\int_0^{L_B}\frac{dx}{2x+L_B} = \frac{P_{L_B}}{E}\left\{\frac{1}{2}\ln\left(3L_B/L_B\right)\right\} = \frac{P_{L_B}}{E}[.55]$$

$u_A = u_B \quad \therefore \quad \frac{P}{EX2}L_A = \frac{P_{L_B}}{E}(.55)$

$L_A/L_B = 1.1$ ✓

31.




$P_1 + P_2 = 2000$ $\qquad 3P_2 = 2000$

$P_1(10) = P_2(20)$ $\qquad P_2 = \frac{2000}{3}$ $\qquad P_1 = \frac{4000}{3}$

$$u_B = \int_0^{60''} \frac{1}{E_S}\frac{P_1}{A_S}\,dx = \frac{4000}{3}\times\frac{60}{30\times10^6\times.1} = \frac{12(2)}{9\times10^2} = .6667\times10^{-2}\text{ in} \downarrow$$
$$= 2.667$$

$$u_A = \int_0^{100''} \frac{1}{E_A}\frac{P_2}{A_A}\,dx = \frac{2000}{3}\times100 = 3.33\times10^{-2}$$
$$3\times10\times10^6\times.2$$

find $P_1, P_2$ $\qquad u_1 = \epsilon_1 \Delta L_1$

$\sigma_1, \sigma_2, \epsilon_1, \epsilon_2$ $\qquad u_2 = \epsilon_2 \Delta L_2$

$2.66\times10^{-2}$ deflection $\qquad 3.77\times10^{-2}$ $\quad 4(.667\times10^{-2}\text{ in})$

deflection $= .667\times10^{-2}\text{ in} + \frac{1}{3}4(.667\times10^{-2}\text{ in})$

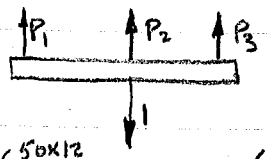
$1.555\times10^{-2}\text{ in}$

$\frac{30}{.667} = \frac{10}{x}$ $\quad .22$

2.667

.667

34.




$u_1 = u_2 = u_3$

$P_1 + P_2 + P_3 = 1\text{ K}$

$P_1 = P_3$

$$u_1 = \int_0^{25\times12}\frac{P_1}{E_1A_1}dx_1 = \int_0^{50\times12}\frac{P_2}{E_2A_2}dx_2 = \int_0^{25\times12}\frac{P_3}{E_3A_3}dx_3$$

$$u_1 = \frac{P_1}{E_1A_1}(25\times12) = \frac{P_2}{E_2A_2}(50\times12)$$

$$\frac{P_1}{E_1A_1} = \frac{2P_2}{E_2A_2}$$

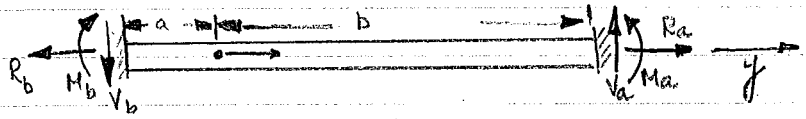
$E_1A_1 = (10\times10^6)(.3) = 3\times10^6$

$E_2A_2 = (30\times10^6)(.2) = 6\times10^6$

$\therefore E_1A_1 = \frac{1}{2}E_2A_2 \quad \therefore 2P_1 = 2P_2$

$\therefore P_1 = P_2 = P_3 \implies P_1 = P_2 = P_3 = 1/3\text{ K}$

$$AE\frac{d^2u}{dx^2} = -p_x$$




$V_a = V_b$ $\quad R_a + P = R_b$

$u(0)=0 \quad u(L)=0$

$$AE\frac{d^2u}{dx^2} = -p_x = -P\langle x-a\rangle^{-1}$$

$$AE\frac{du}{dx} = -P\langle x-a\rangle^{0} + C_1$$

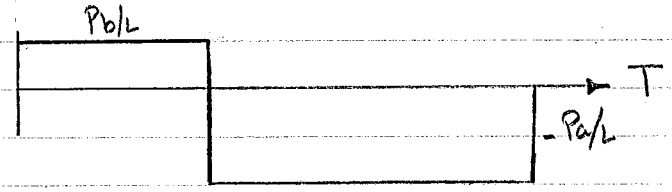
$$AEu = -P\langle x-a\rangle^{1} + C_1x + C_2$$

$AEu(0) = 0 = C_2$

$AEu(L) = 0 = -Pb + C_1L \qquad C_1 = Pb/L$

$$R_b = P(0) = AE\left(\frac{du}{dx}\right)_{x=0} = Pb/L$$

$$R_a = P(L) = AE\left(du/dx\right)_{x=L} = -P + C_1 = -Pa/L$$




for $x<a$ axial force distrib $= Pb/AL$

for $x=a$ " " " $= P/A$

for $x>a$ axial force dist $= Pa/AL$

# EXAMINATION BOOK

Name: Cesar Lang

Subject: C 201

Instructor: Mr Stewart

Exam Start No. _____ Section D

Date: 4/18/19 Grade _____

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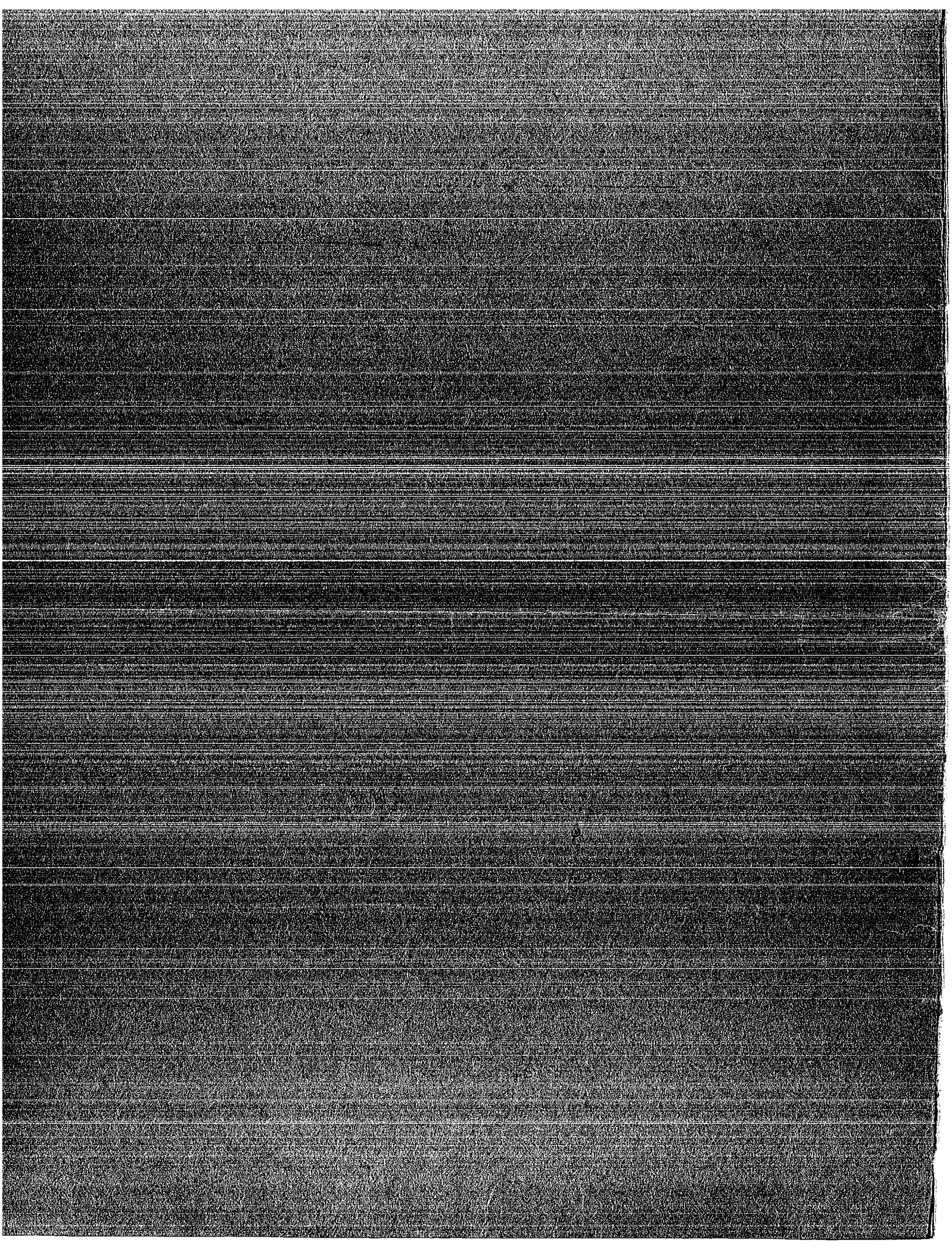
1- 30/30

2- 30/35

3- 25/35

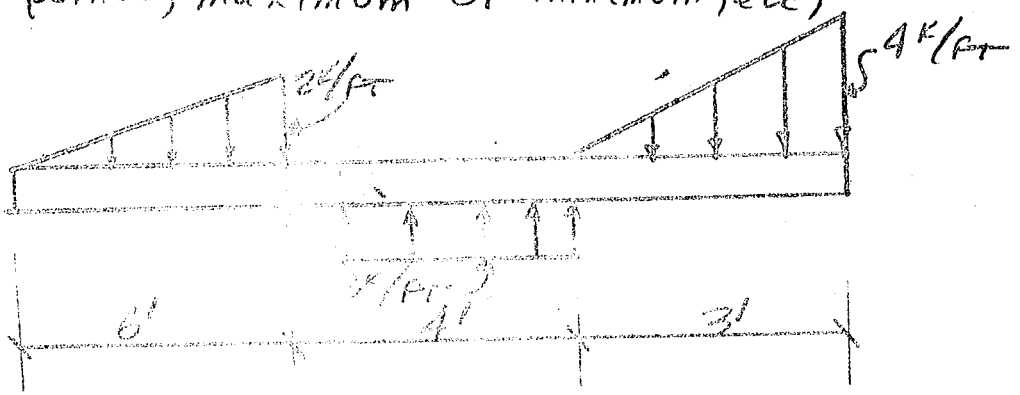
85

50-12 PAGES

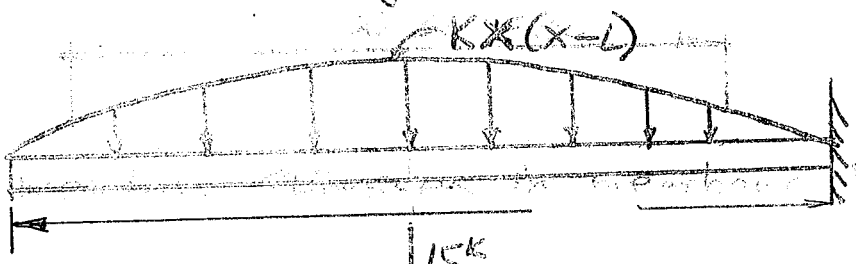


# Exam I – February 26, 1970

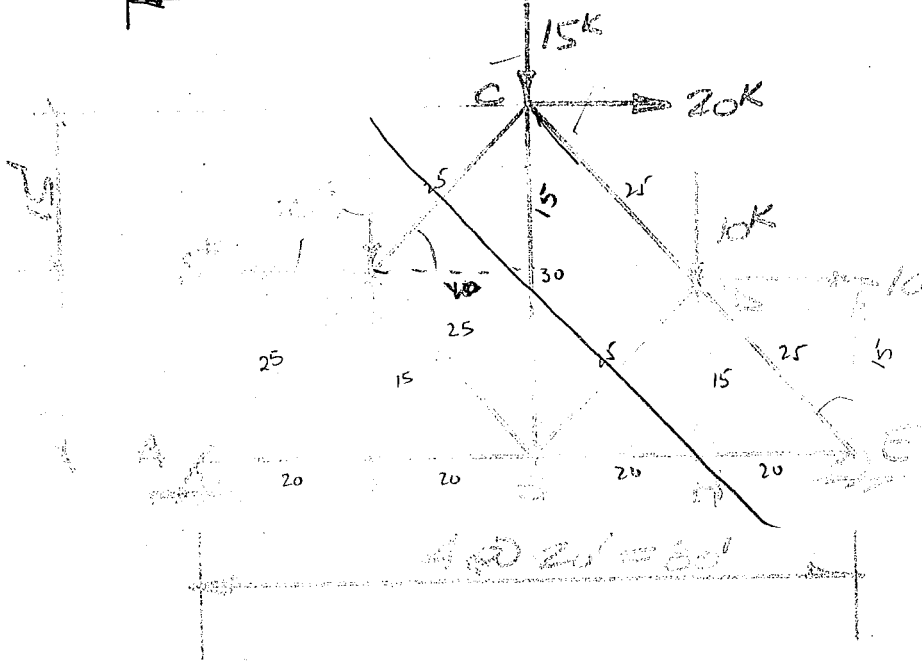
1) Draw a shear & bending moment diagram, using any method. Show values at critical points. (transition points, end points, maximum or minimum, etc)




2) Find expressions for shear & bending moment and draw diagrams.




3)



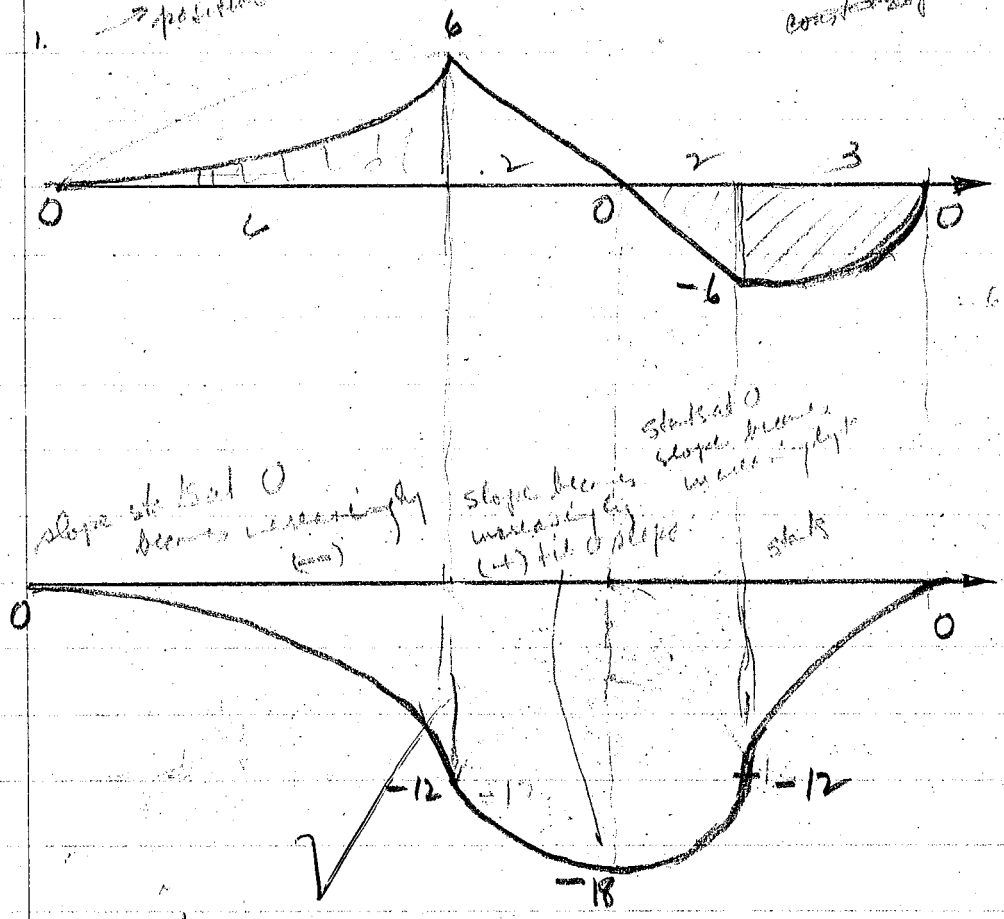

$\sigma_{GH} = 13.34$ KSI

$\sigma_{BC} = 8.45$ KSI

$\sigma_{CG} = 1$ KSI

Find member stresses in members BC, GC, & GH.
All members pin-connected & 2.5 in² area.

1.




$\frac{6(3)}{3}$ $\frac{2}{3}(12)(3)$

$6(3)$

$\frac{6(2)}{2}$ $6(3) =$

$\downarrow 6$

$-6 + 3(x-6) + V = 0$

$3(x-6) =$ $V = -3x + 24$ $M = \frac{3x^2}{2} - 24x + C$

$x = 6$

$\frac{1}{3}6(6) = 12$

$M =$

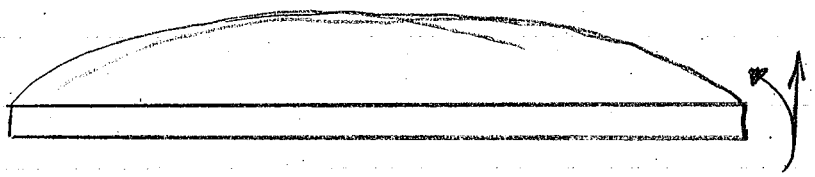
$\downarrow 6(4)$ $\downarrow 6(12)$

$\uparrow 8(12)$

$24 + 72 = 96$

All done by inspection starting from right end

2.



$$p = +Kx(x-L) = +Kx^2 - KLx$$

$$-V = -\frac{KLx^2}{2} + \frac{Kx^3}{3} + C_1$$

$$M = -\frac{Kx^3 L}{6} + \frac{Kx^4}{12} + C_1 x + C_2$$

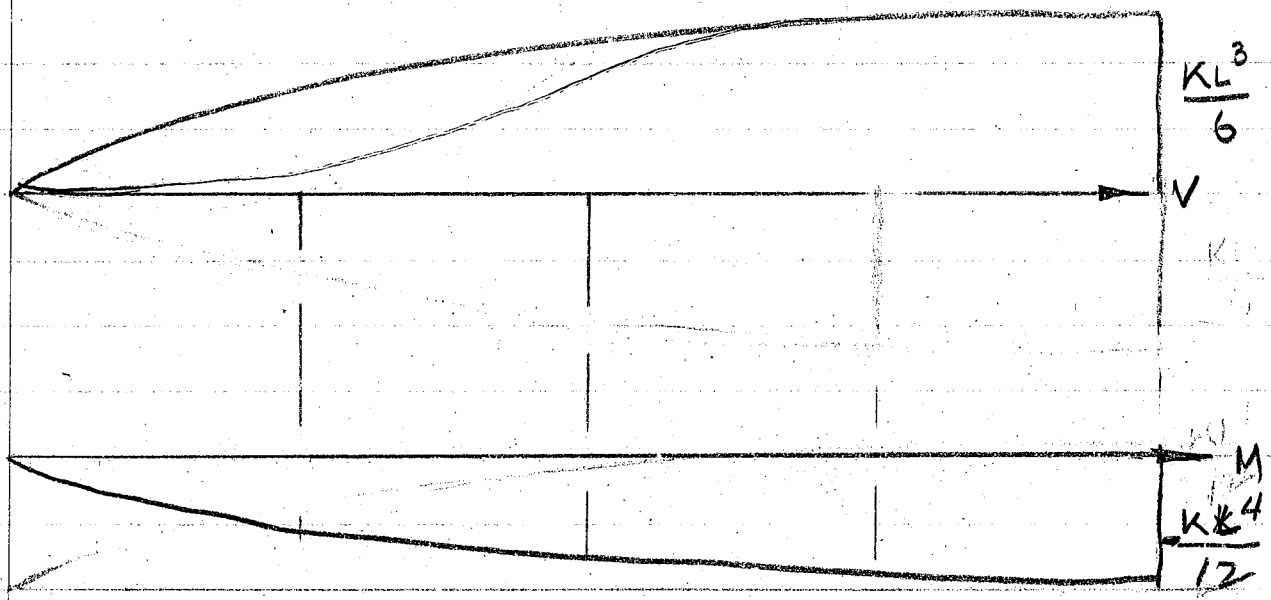
at $x=0$ $M=0$

at $x=0$ $V=0$

$M = 0$ $\quad 0 - 0 + 0 + C_2$ $\quad C_2 = 0$

$V = 0$ $\quad 0 - 0 + C_1 = 0$ $\quad C_1 = 0$

$$V = -\frac{Kx^3}{3} + \frac{KLx^2}{2} \qquad M = -\frac{Kx^3}{6}L + \frac{Kx^4}{12}$$




at $x = L/4$ $\quad V = -\frac{KL^3}{192} + \frac{6KL^3}{6(32)} = \frac{15KL^3}{192}$

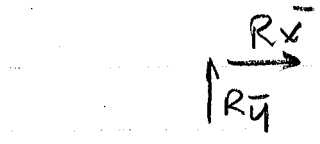
$\frac{Kx^3}{6} = \frac{Kx^4}{12} \times$

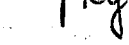
at $x = L$ $\quad -\frac{KL^3}{3} + \frac{KL^3}{2} = +\frac{KL^3}{6} = V$

$\frac{2L}{12} = \frac{x}{12}$

at $X = L$ $\quad M = -\frac{KL^4}{6} + \frac{KL^4}{12} = \frac{KL^4}{12} = M$ $\quad 2L = x$

3.




$\Sigma F_x = 0 \qquad R\bar{x} - 10^K + 20^K + 10^K = 0$

$R\bar{x} = +20^K \leftarrow$

$\Sigma F_y = 0 \qquad R\bar{y} - 10^K - 15^K - 10^K + R_y = 0$

$R\bar{y} + R_y = 35^K$

$\Sigma M_A = 0 \qquad 10(15) - 10(20) + 15(40) - 20(30) - 10(60)$

$- 10(15) + R_y(80) = 0$

$-5(15) - 10(20) - 15(40) - 20(30) - 10(60) = -R_y$

80

$15(+45) + 10(+80) + 20(+30) = R_y$

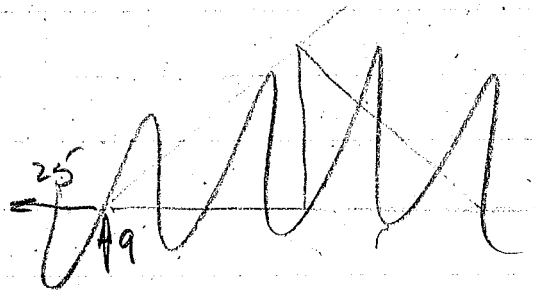
80

450
225
675

$\frac{675 + 800 + 600}{80} = R_y$

2075
8

450
225
675

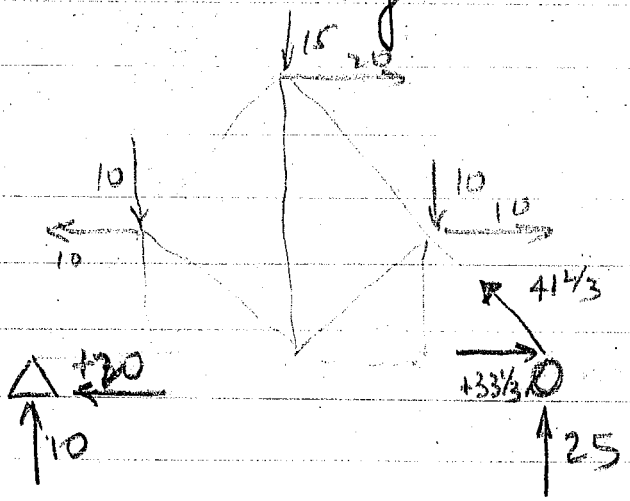


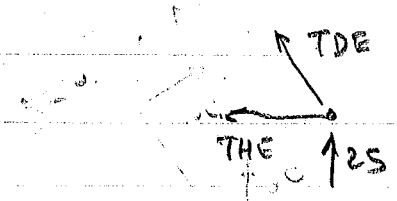
$\Sigma M_A \circlearrowleft = 0$ $\quad 10(15) - 10(20) - 15(40) - 20(30)$
$- 10(60) - 10(15) + R_y(80) = 0$

$+200 + 600 + 600 + 600 = R_y$
$80$

$\frac{2000}{80} = 25^K = R_y$

$10^K = R_y$



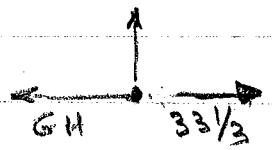




$25 + 3/5\ T_{DE} = 0$

$T_{DE} = \frac{-5(25)}{3} = -41\frac{2}{3}$

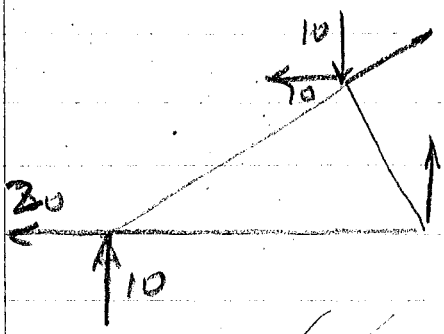
$-HE - \left(-4/5\left(\frac{5(25)}{3}\right)\right) = 0$

$+33\frac{1}{3}K$




$GH = 33\frac{1}{3}$ in ~~compression~~ tension

$\sigma_{GH} = \frac{33.3^K}{2.5\ in^2} = 13.34\ KSI \quad (T)$




$10 - 10 + GC + \frac{3}{5} BC = 0$

$-30 + \frac{4}{5} GC = 0$ $\qquad GC = \frac{5}{4}(30) = \frac{150}{4} = 37\frac{1}{2}$

$GC = 37.5\ K$

$BC = -37.5\left(\frac{5}{3}\right) = -(12.5)5 = -62.5\ K$

$\sigma_{GC} = \frac{37\frac{1}{2}}{2.5\ in^2} = 15\ KSI \quad (T)$

$\sigma_{BC} = \frac{62.5}{2.5} = 25\ KSI \quad (C)$

$37.5\left(\frac{5}{3}\right) = 12.5$

$\frac{1}{4}\left(\frac{1}{12}\right) = \frac{1}{48}$

$\sigma_x = \frac{P_x}{12 \times \frac{1}{48}} \qquad \sigma_y = \frac{P_y}{8 \times \frac{1}{48}}$

$E = \frac{\sigma}{\epsilon} = \frac{4P_x}{\epsilon}$ K/ft²

144 KSI = KSF

$\frac{30 \times 10^3 \text{ KSI}}{144 \text{ KSI/KSF}} = \frac{300}{144} \times 10^5 = 2.083 \times 10^5 \text{ KSF}$

$2.083 \times 10^{\cancel{3}\,5} \text{ KSF} = \frac{4R_x}{\epsilon_x}$

$\frac{2.083 \times 10^2}{4} \epsilon_x$

$\epsilon_x = \frac{\Delta x}{L_x} = \frac{.0768}{12.8'} = .0008 = 8 \times 10^{-4}$

$\frac{2.083 \times 10^2 (8 \times 10^{-4})}{4} = 4.163 \times 10^{-2}$

$\frac{1}{4}\left[.0006(30 \times 10^3)\right] = \frac{1}{4}\left(\frac{-P_x}{144}\right) + \frac{1}{4}\left(\frac{-P_y}{24}\right)$

$.0008(30 \times 10^3) = \frac{P_x}{36} - \frac{P_y}{4(24)}$

$(.00085 + .00015)(.0009)(30 \times 10^3) = \frac{-P_x}{576} + \frac{P_x}{16(36)}$

$28.5 = \frac{-R_x}{576} + \frac{16R_x}{576}$

$24 - 2.535$

$\frac{24}{4}(21.465)$

$24 - 2.535$

$(24)(21.5)$

$\frac{24(21.5)}{4.8}$

$24 - \frac{P_x}{36} = -\frac{P_y}{4(24)}$

$24 - \frac{28.5(576)}{15(36)}$

$24 - 30.4$

$-6.4$

$\frac{24}{4}(-6.4)$

$36$

$16$

$360$

$216$

$P_x = \frac{28.5(576)}{15(12)}$

$$\sigma_y = \sigma_z - 2\mu\epsilon_z + 2\mu\epsilon_y$$

$$\sigma_y = (\epsilon_y - \epsilon_z)\left[(1+\nu)/E\right) + \sigma_z$$

$$\frac{E}{(1+\nu)}(\epsilon_y - \epsilon_z) = 2\mu(\epsilon_y - \epsilon_z) \quad QED$$

$$G = \frac{E}{2+2\nu}$$

$$1+\nu = \frac{E}{2G}$$

$$\nu = \frac{E}{2G} - 1$$

$$\frac{30}{24} - 1 = \nu = 1/4$$

$$\epsilon_x = \frac{\Delta x}{L_x} \qquad \sigma_x = \frac{P_x}{12 \times 1/48} = \sigma_x = P_x$$

$$\epsilon_y = \frac{\Delta y}{L_y} \qquad \frac{.0096}{.0768} = .0008 \qquad \sigma_y = \frac{P_y}{8 \times 1/48}$$

$$\frac{.0096}{8(12)} = .0008 \qquad 8\times10^{-4} = \epsilon_x$$

$$\frac{.0864}{12(12)} = \frac{.0072}{12} = .0006 = 6\times10^{-4} = \epsilon_y$$

$$6\times10^{-4} = \epsilon_y = -\frac{\nu}{E}\sigma_x + \frac{\sigma_y}{E} = -\frac{\nu 4P_x}{E\,4} + \frac{6P_y}{E}$$

$$8\times10^{-4} = \epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y = \frac{4P_x}{E} - \frac{\nu}{E}6P_y$$

$$E(6\times10^{-4}) = -P_x + 6P_y$$

$$E(8\times10^{-4}) = 4P_x - 6/4\,P_y$$

$$180\times10^{-2} = -P_x + 6P_y$$

$$4(240\times10^{-2}) = 16P_x - 6P_y$$

$$15P_x = 1140\times10^{-2}$$

$$P_x =$$

$$\begin{array}{r|l} & 76 \\ 15 & 1140 \\ & 105 \\ \hline & 90 \end{array}$$

$$\frac{1140}{15} \quad 90$$

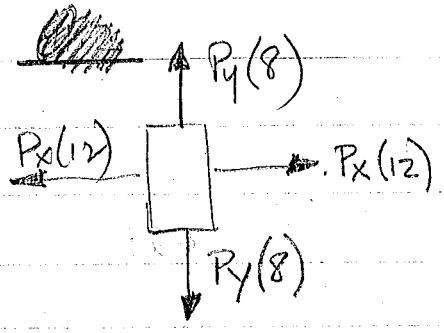
$$180\times10^{-2} = -90 + 6P_y$$

$$6P_y = 270$$

$$P_y = 45$$

$P_x = 91.2\ (12) = \frac{30\times10^3\ KSI}{144\ \frac{KSI}{KSF}}$

$\sigma_x =$ 




$\frac{P_x(12)\ FT.}{12 \times 1/48} \quad 48\,P_x = \sigma_x\ _{KSF} \quad \frac{P_y(8)}{8 \times 1/48} = 48P_y = \sigma_y$

$K/FT.$

$48\ K/F^2$

$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y \qquad \frac{24}{30\times10^3\,(.0008)} \qquad E = \frac{\sigma_x}{\epsilon_x} = \frac{P_x}{\frac{E\epsilon_x}{48} = \sigma_x}$

$144(48)$

$E\epsilon_x = \sigma_x - \nu\sigma_y$

$\frac{30\times10^3\ KSI}{144\ \frac{KSI}{KSF}}[\epsilon_x] = 48P_x - \frac{1}{4}(48)P_y \qquad \frac{30\times10^3}{144}\epsilon_y = -12P_x + 48P_y$

$\frac{30\times10^3}{144}(.0008) = 48P_x - 12P_y \qquad \frac{(18)4}{144} = -48P_x + 192P_y$

$\frac{24}{144} = 48P_x - 12P_y \qquad \frac{24}{144} = 48P_x - 12P_y$

$\frac{4(18)+24}{144} = 180P_y$

$E = 30\times10^6\ \#/in^2$

$P_x$

$24$

$96$

$144$

$180$

$91.2(12) = P_x$

$76.8\ (8) = P_y$

$\sigma_x = \frac{P_x}{A_x} = \frac{91.2(12)}{12 \times 1/48}$

$\sigma_y = \frac{P_y}{A_y} = \frac{76.8(8)}{8 \times 1/48}$

$\epsilon_x = \frac{192}{4(30\times10^3)}(91.2)144 - \frac{48}{120\times10^3}(76.8)144$

$\sigma_x = 48\,(91.2)$

$\sigma_y = 48\,(76.8)$

$\frac{144(48)}{120\times10^3}\left(\frac{4}{48}(91.2) - 48(76.8)\right) \qquad 364.8 - 76.8 = 288.$

$\frac{144(48)}{120\times10^3}(288) \qquad \frac{288(288)}{5000}$

5

$$\frac{\epsilon_z\left(\frac{\nu}{E}\right)\left(\frac{1+\nu}{E}\right)}{\left(\frac{\nu}{E}\right)\left(\frac{1+\nu}{E}\right)} + \frac{\nu/E}{\left(\frac{1+\nu}{E}\right)}$$

$\frac{\nu}{1+\nu}$ eq. $\left[\epsilon_z\left(\frac{1+\nu}{\nu}\right) + \epsilon_x + \epsilon_y - \epsilon_z\right] \qquad \left(\epsilon_y - \epsilon_z\right)\left(\frac{1+\nu}{E}\right.$

$$\epsilon_z\left(\frac{1+\nu}{\nu}\right) + \frac{\epsilon_z\nu + \epsilon_x\nu + \epsilon_y\nu}{\nu}$$

$$\epsilon_z +$$

$$\sigma_z = \frac{\nu}{1+\nu}\left[\frac{\epsilon_z}{\nu} + \epsilon_x + \epsilon_y\right]\frac{E}{(1-2\nu)}$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

$$\sigma_y = \left(\epsilon_y - \epsilon_z\right)\left(\frac{1+\nu}{E}\right)$$

~~$\sigma_x = (\epsilon_x + \epsilon_y + \epsilon_z)\nu$~~ ~~$\lambda\epsilon_z\nu$~~

$$\sigma_z = \lambda\left[\frac{\epsilon_z}{\nu} + \epsilon_x + \epsilon_y\right]$$

$$\lambda\epsilon_z + \lambda\sigma_z = \lambda e + \frac{\lambda\epsilon_z}{\nu} = \lambda(\epsilon_x + \epsilon_y + \epsilon_z)$$

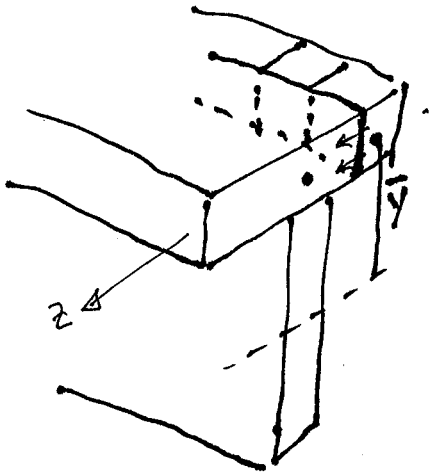
$$\sigma_z = \lambda e + \lambda\epsilon_z\left(\frac{1-\nu}{\nu}\right) = \lambda e + \frac{\lambda\epsilon_z}{\nu} - \frac{\lambda\epsilon_z\nu}{\nu}$$

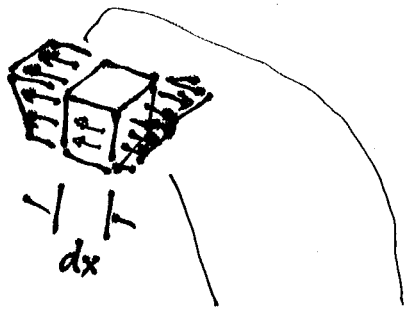
$$\lambda\epsilon_z\left(\frac{1-\nu}{\nu}\right)$$

$\frac{E}{1+\nu} = 2G$ $\qquad \frac{E\nu}{(1+\nu)(1-2\nu)}\frac{1-\nu}{\nu}$ $\qquad \frac{E\nu}{(1+\nu)(1-2\nu)}\left(\frac{1-2\nu}{\nu}\right)\epsilon_z$

$$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

$$2G\frac{(1-\nu)}{(1-2\nu)}$$




$$\Sigma F_x = 0 = \int(\sigma_x + d\sigma_x)dA - \int \sigma_x dA - \tau_{zx} dx t = 0$$

$$\sigma_x = \frac{-M_z \bar{y}}{I_z}$$

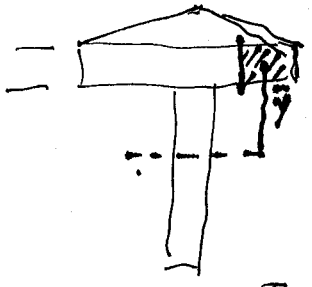
$$d\sigma_x = \frac{-dM_z \cdot y}{I_z}$$

$$\Sigma F_x = 0 \quad \int \frac{-dM_z}{I_z} \cdot y \, dA - \tau_{xz}\, dx \cdot t = 0$$

$$\int \frac{-dM_z}{dx} \cdot \frac{y\,dA}{I_z} = \tau_{xz} \cdot t = q$$

$$\int_{-b/2}^{z} V \cdot \frac{\bar{y}\,dA}{I_z} = \frac{V}{I_z}\bar{y}\,t\left(z + \frac{b}{2}\right) = \tau_{zx} \cdot t$$

$$dA = t \cdot dz$$

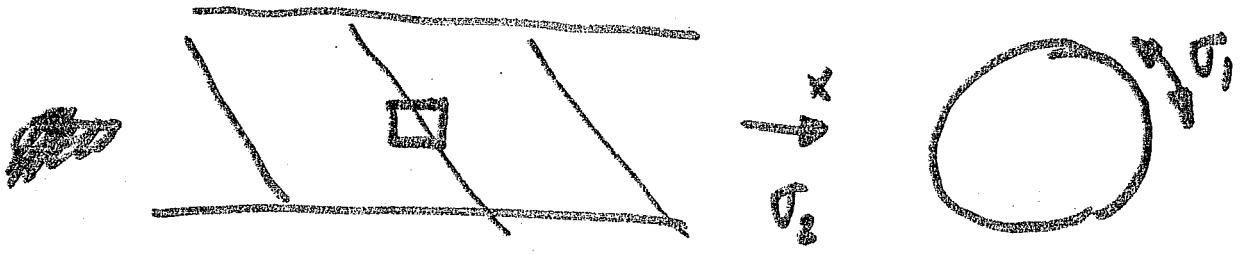


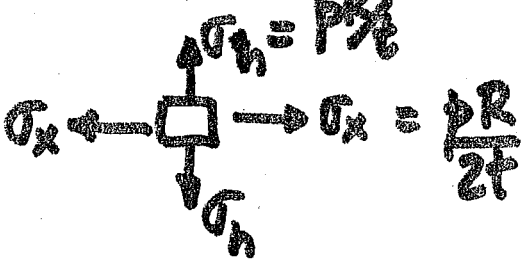

$$\tau_{zx} = \frac{VQ}{I_z \cdot t}$$

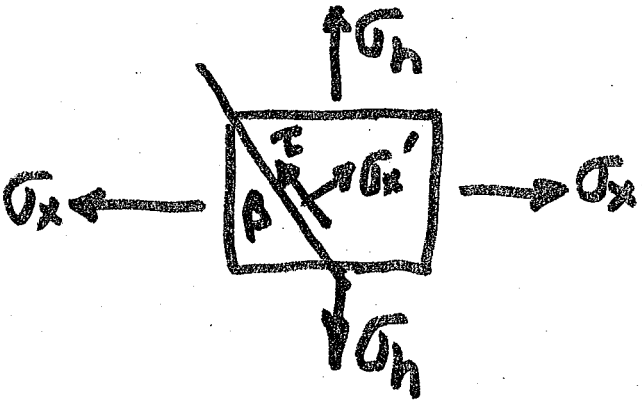
$$Q = \bar{y} \cdot t\left(z + \frac{b}{2}\right)$$

$$\tau_{zx} = \frac{VQ}{It}$$

7.117



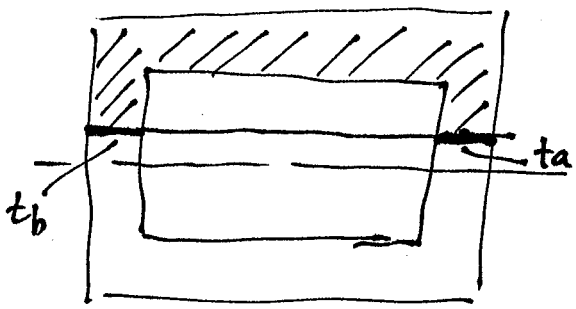




$$\sigma_x' = \left(\frac{\sigma_x + \sigma_h}{2}\right) + \left(\frac{\sigma_x - \sigma_h}{2}\right)\cos 2\beta + \tau_{xy}\sin 2\beta$$

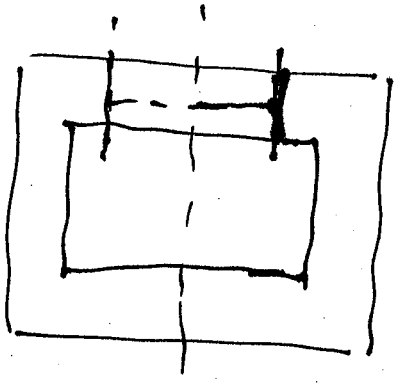
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_h}{2}\right)^2 + \tau_{xy}^2} \Rightarrow \frac{\sigma_x - \sigma_h}{2}$$

$$\tau_{max} = \tau_{xy}\cos 2\beta - \left(\frac{\sigma_x - \sigma_h}{2}\right)\sin 2\beta$$



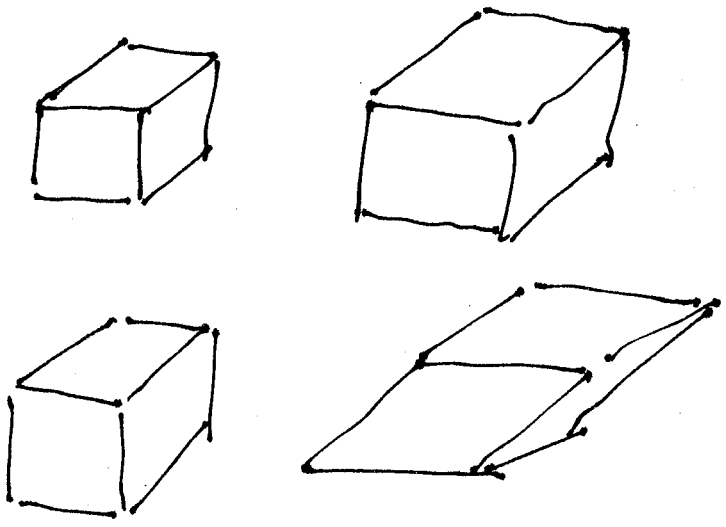

$$\tau = \frac{VQ}{It}$$

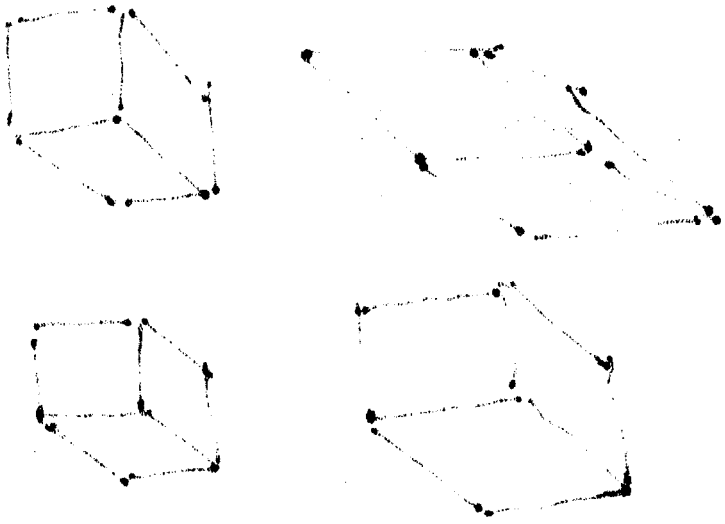
$$t = t_a + t_b$$

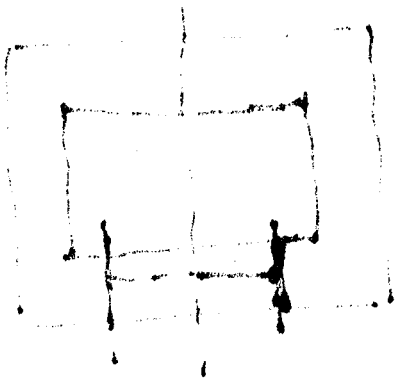


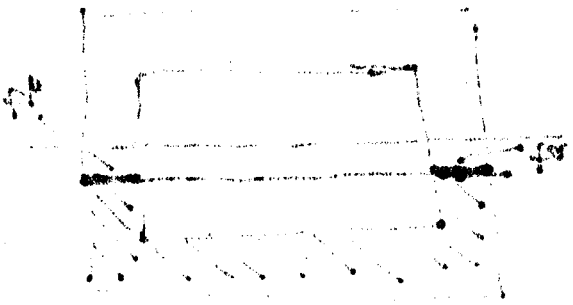
$$\tau = \frac{VQ}{It}$$

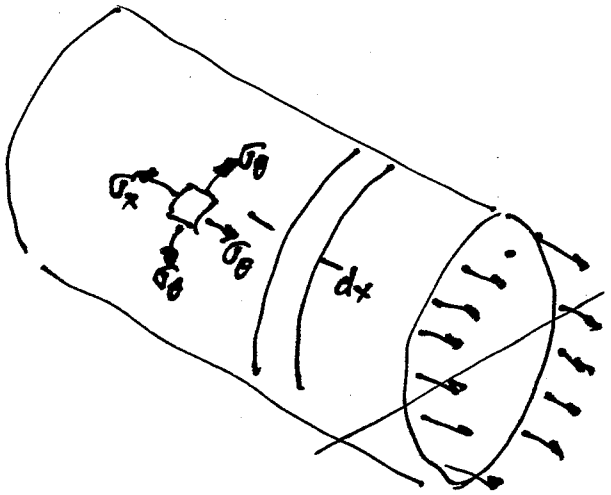
$$Q = \bar{y}\int_0 t\,dz$$



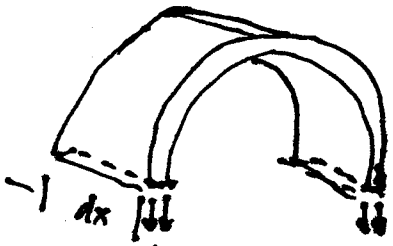




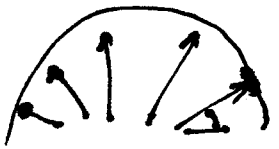





$\sigma_x \cdot 2\pi Rt$




$2\sigma \cdot t\,dx$

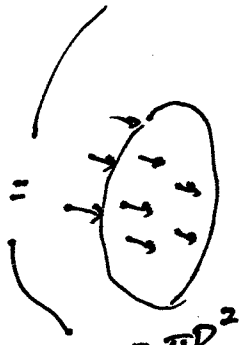


$= \int_0^{\pi} p \sin\theta \cdot R\,d\theta\,dx$

$= pR[-\cos\theta]_0^{\pi} dx = 2pR\,dx$

$\sigma_\theta = \sigma_{circum} = \sigma_1 = \frac{pR}{t}$

pg 463



$= p \cdot \pi R^2$

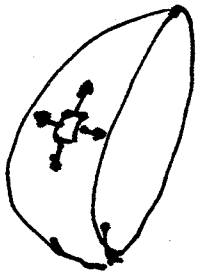
$\sigma_x \cdot 2\pi Rt = p\pi R^2$

$\sigma_x = \frac{pR}{2t}$

$R$ = internal radius

$R' = R + t/2$

pg 463

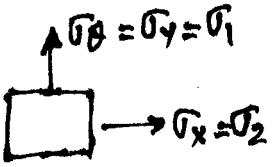


$\sigma_\theta = \sigma_\varphi = \frac{pR}{2t}$

pg 464

For Problem 7.5 # 465



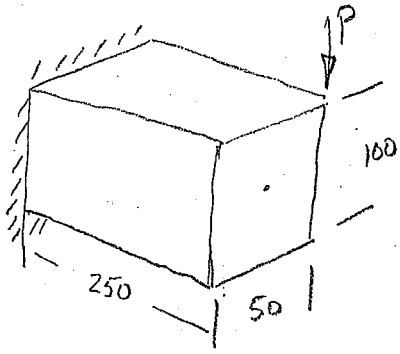




$$\sigma_x' = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \cancel{\tau_{xy}}^{\,0}\cos 2\theta$$

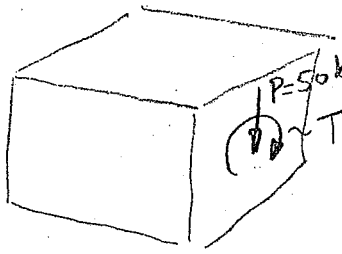
$$= \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\sin 50°$$

$$\tau_{x'y'} = \cancel{\tau_{xy}}\cos 2\theta - \frac{(\sigma_x-\sigma_y)}{2}\sin 2\theta$$

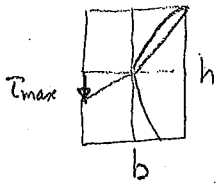
8-28




=

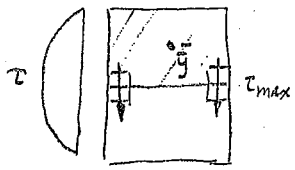



| h/b | 1.00 | 1.50 | 2.00 | 3.00 | ∞ |
|---|---|---|---|---|---|
| α | 0.208 | 0.231 | 0.246 | 0.267 | 0.333 |




$\frac{h}{b} = \frac{100}{50} = 2 \qquad \alpha = 0.246$

$$\tau_{max} = \frac{T}{\alpha h b^2} = \frac{1250\ N\text{-}m}{0.246(0.10)(0.05)^2} = 20.3252\ MPa$$




$$\tau_{max} = \frac{VQ}{It} = \frac{V\ \frac{bh^2}{8}}{\frac{bh^3}{12}\cdot b}$$

$$Q = \int y\,dA = \bar{y}\cdot A = \frac{h}{4}\cdot\frac{hb}{2} = \frac{h^2 b}{8}$$

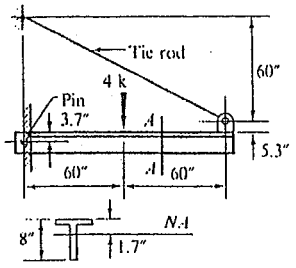
$$\tau_{max} = \frac{3V}{2A} = \frac{3\cdot 50000N}{2(0.10)(0.05)} = 15\ MPa$$

$$\tau_{max} = 20.33\ MPa + 15\ MPa = 35.33\ MPa$$

$$\sigma_{max} = +\frac{Pl\cdot y}{I_{zz}} = \frac{50000\cdot(0.25)(0.050)}{4.167\times10^{-6}} = 150\ MPa \qquad I_{zz} = \frac{bh^3}{12} = \frac{(0.05)(0.1)^3}{12} = 4.166\times10^{-6}\ m^4$$

$\sigma_{min} = 0 \qquad 0 = y$

**10-50.** Plot shear, moment, and axial force diagrams for the jib-crane loaded as shown in the figure. Neglect the weight of the beam. *Ans:* 128.6 k-in.

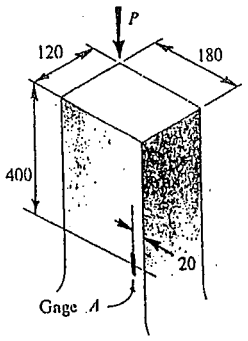



Section *A-A*

PROB. 10 - 50

8-24

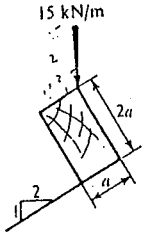
**7-35.** An aluminum alloy block is loaded as shown in the figure. The application of this load produces a tensile strain of $500 \times 10^{-6}$ mm. per mm at $A$ as measured by means of an electrical strain gage. Compute the magnitude of the applied force $P$. Let $E = 10 \times 10^6$ kN/m². All dimensions given in the figure are in mm.




PROB. 7 - 35

8-19 1&2

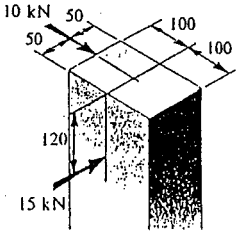
**7-44.** A tilted, simply supported beam with a depth to width ratio of 2 to 1 is to span 4 m and is to carry a uniformly distributed load of 15 kN per linear meter, including its own weight, applied as shown in the figure. (a) Determine the required dimensions of the beam so that the maximum stress due to bending does not exceed 10 MN/m². (b) Locate the neutral axis of the beam and show its position on the sketch.



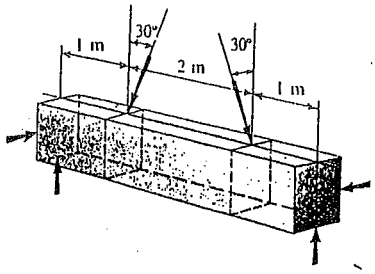

PROB. 7 - 44

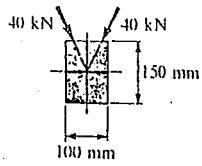
8-23

**7-34.** A cast iron block is loaded as shown in the figure. Neglecting the weight of the block, determine the stresses acting normal to a section taken 0.5 m below the top and locate the line of zero stress. All dimensions given in the figure are in mm. *Ans:* ±24.6 MPa, ±9.6 MPa.




**10-91.** A 4 m long beam is loaded as shown in the figure. The two applied forces act perpendicularly to the long axis of the beam and are inclined 30° with the vertical. If these forces act through the centroid of the cross-sectional area find the location and magnitude of the maximum bending stress. Neglect the weight of the beam. *Ans:* 132 MN/m².

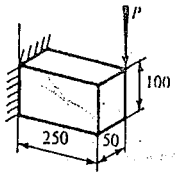



PROB. 10 - 91

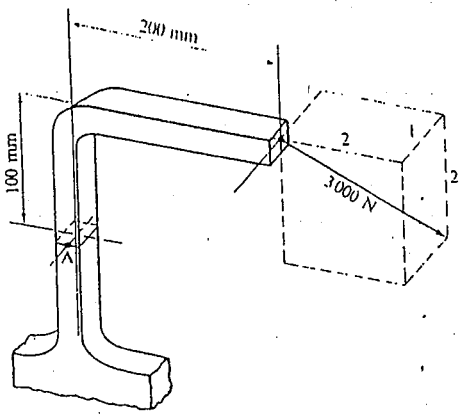
8-28

**7-45.** A rectangular cantilever 250 mm long is loaded with $P = 50$ kN at the free end as shown in the figure. Determine the maximum shearing stress at the built-in end due to the direct shear and the torque. Show the result on a sketch analogous to Fig. 7-17(e). All dimensions shown in the figure are in mm. *Ans:* 35.3 MPa.

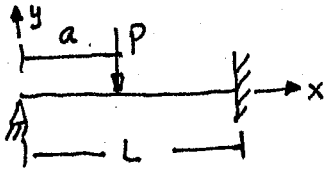



PROB. 7 - 45

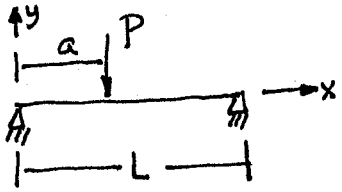
**9-26.** A bent rectangular bar is subjected to an inclined force of 3 000 N as shown in the figure. The cross-section of the bar is 12 mm by 12 mm. (a) Determine the state of stress at point $A$ caused by the applied force and show the results on an element. (b) Find the maximum principal stress. *Ans:* (a) 1·333 MPa, 578 MPa



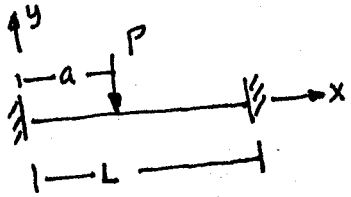
Use these formulas as they are clearer to understand




$$y(x) = \frac{1}{EI}\left[-\frac{P\langle x-a\rangle^3}{6} + \left(\frac{x}{L}\right)^3 \cdot \frac{P}{24}(L-a)^2(5L+a) - \left(\frac{x}{L}\right)\frac{P}{24}(L-a)^2(11L+4a)\right]$$




$$y(x) = \frac{1}{EI}\left[-\frac{P\langle x-a\rangle^3}{6} + \frac{Px^3}{6L}(L-a) - \frac{P}{6L}x\cdot a\cdot(L-a)(2L-a)\right]$$



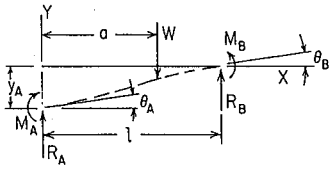

$$y(x) = \frac{1}{EI}\left[-\frac{P\langle x-a\rangle^3}{6} + \frac{P(L-a)^2(L+2a)}{6}\left(\frac{x}{L}\right)^3 - \left(\frac{x}{L}\right)^2\cdot\frac{P}{2}a(L-a)^2\right]$$

Note $\langle x-a\rangle^n = \begin{cases} 0 & x \le a \\ (x-a)^n & x \ge a \end{cases}$ singularity function

**TABLE 3 Shear, moment, slope, and deflection formulas for elastic straight beams**

NOTATION: $W$ = load (force); $w$ = unit load (force per unit length); $M_o$ = applied couple (force-length); $\theta_o$ = externally created concentrated angular displacement (radians); $\Delta_o$ = externally created concentrated lateral displacement; $T_1$ and $T_2$ = temperatures on the top and bottom surfaces, respectively (degrees). $R_A$ and $R_B$ are the vertical end reactions at the left and right, respectively, and are positive upward. $M_A$ and $M_B$ are the reaction end moments at the left and right, respectively. All moments are positive when producing compression on the upper portion of the beam cross section. The transverse shear force $V$ is positive when acting upward on the left end of a portion of the beam. All applied loads, couples, and displacements are positive as shown. All deflections are positive upward, and all slopes are positive when up and to the right. $E$ is the modulus of elasticity of the beam material, and $I$ is the area moment of inertia about the centroidal axis of the beam cross section. $\gamma$ is the temperature coefficient of expansion (unit strain per degree)

1. Concentrated intermediate load



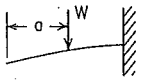
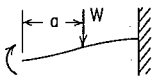
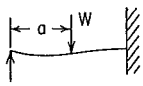
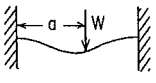
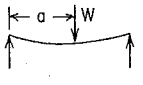
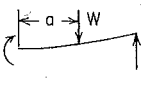

Transverse shear $= V = R_A - W\langle x - a\rangle^0$

Bending moment $= M = M_A + R_A x - W\langle x - a\rangle$

$$\text{Slope} = \theta = \theta_A + \frac{M_A x}{EI} + \frac{R_A x^2}{2EI} - \frac{W}{2EI}\langle x - a\rangle^2$$

$$\text{Deflection} = y = y_A + \theta_A x + \frac{M_A x^2}{2EI} + \frac{R_A x^3}{6EI} - \frac{W}{6EI}\langle x - a\rangle^3$$

(*Note:* see page 98 for a definition of the term $\langle x - a\rangle^n$.)

| End restraints, reference no. | Boundary values | Selected maximum values of moments and deformations |
|---|---|---|
| 1a. Left end free, right end fixed (cantilever)   | $R_A = 0 \quad M_A = 0 \quad \theta_A = \frac{W(l-a)^2}{2EI}$<br>$y_A = \frac{-W}{6EI}(2l^3 - 3l^2a + a^3)$<br>$R_B = W \quad M_B = -W(l-a)$<br>$\theta_B = 0 \quad y_B = 0$ | Max $M = M_B$; max possible value $= -Wl$ when $a = 0$<br>Max $\theta = \theta_A$; max possible value $= \frac{Wl^2}{2EI}$ when $a = 0$<br>Max $y = y_A$; max possible value $= \frac{-Wl^3}{3EI}$ when $a = 0$ |
| 1b. Left end guided, right end fixed   | $R_A = 0 \quad M_A = \frac{W(l-a)^2}{2l} \quad \theta_A = 0$<br>$y_A = \frac{-W}{12EI}(l-a)^2(l+2a)$<br>$R_B = W \quad M_B = \frac{-W(l^2-a^2)}{2l}$<br>$\theta_B = 0 \quad y_B = 0$ | Max $+M = M_A$; max possible value $= \frac{Wl}{2}$ when $a = 0$<br>Max $-M = M_B$; max possible value $= \frac{-Wl}{2}$ when $a = 0$<br>Max $y = y_A$; max possible value $= \frac{-Wl^3}{12EI}$ when $a = 0$ |
| 1c. Left end simply supported, right end fixed   | $R_A = \frac{W}{2l^3}(l-a)^2(2l+a) \quad M_A = 0$<br>$\theta_A = \frac{-Wa}{4EIl}(l-a)^2 \quad y_A = 0$<br>$R_B = \frac{Wa}{2l^3}(3l^2 - a^2) \quad \theta_B = 0$<br>$M_B = \frac{-Wa}{2l^2}(l^2 - a^2) \quad y_B = 0$ | Max $+M = \frac{Wa}{2l^3}(l-a)^2(2l+a)$ at $x = a$; max possible value $= 0.174Wl$ when $a = 0.366l$<br>Max $-M = M_B$; max possible value $= -0.1924Wl$ when $a = 0.5773l$<br>Max $y = \frac{-Wa}{6EI}(l-a)^2\left(\frac{a}{2l+a}\right)^{1/2}$ at $x = l\left(\frac{a}{2l+a}\right)^{1/2}$ when $a > 0.414l$<br>Max $y = \frac{-Wa(l^2-a^2)^3}{3EI(3l^2-a^2)^2}$ at $x = \frac{l(l^2+a^2)}{3l^2-a^2}$ when $a < 0.414l$; max possible $y = -0.0098\frac{Wl^3}{EI}$ when $x = a = 0.414l$ |
| 1d. Left end fixed, right end fixed   | $R_A = \frac{W}{l^3}(l-a)^2(l+2a)$<br>$M_A = \frac{-Wa}{l^2}(l-a)^2$<br>$\theta_A = 0 \quad y_A = 0$<br>$R_B = \frac{Wa^2}{l^3}(3l - 2a)$<br>$M_B = \frac{-Wa^2}{l^2}(l-a)$<br>$\theta_B = 0 \quad y_B = 0$ | Max $+M = \frac{2Wa^2}{l^3}(l-a)^2$ at $x = a$; max possible value $= \frac{Wl}{8}$ when $a = \frac{l}{2}$<br>Max $-M = M_A$ if $a < \frac{l}{2}$; max possible value $= -0.1481Wl$ when $a = \frac{l}{3}$<br>Max $y = \frac{-2W(l-a)^2a^3}{3EI(l+2a)^2}$ at $x = \frac{2al}{l+2a}$ if $a > \frac{l}{2}$; max possible value $= \frac{-Wl^3}{192EI}$ when $x = a = \frac{l}{2}$ |
| 1e. Left end simply supported, right end simply supported   | $R_A = \frac{W}{l}(l-a) \quad M_A = 0$<br>$\theta_A = \frac{-Wa}{6EIl}(2l-a)(l-a) \quad y_A = 0$<br>$R_B = \frac{Wa}{l} \quad M_B = 0$<br>$\theta_B = \frac{Wa}{6EIl}(l^2 - a^2) \quad y_B = 0$ | Max $M = R_A a$ at $x = a$; max possible value $= \frac{Wl}{4}$ when $a = \frac{l}{2}$<br>Max $y = \frac{-Wa}{3EIl}\left(\frac{l^2-a^2}{3}\right)^{3/2}$ at $x = l - \left(\frac{l^2-a^2}{3}\right)^{1/2}$ when $a < \frac{l}{2}$; max possible value $= \frac{-Wl^3}{48EI}$ at $x = \frac{l}{2}$ when $a = \frac{l}{2}$<br>Max $\theta = \theta_A$ when $a < \frac{l}{2}$; max possible value $= -0.0642\frac{Wl^2}{EI}$ when $a = 0.423l$ |
| 1f. Left end guided, right end simply supported   | $R_A = 0 \quad M_A = W(l-a) \quad \theta_A = 0$<br>$y_A = \frac{-W(l-a)}{6EI}(2l^2 + 2al - a^2)$<br>$R_B = W \quad M_B = 0$<br>$\theta_B = \frac{W}{2EI}(l^2 - a^2) \quad y_B = 0$ | Max $M = M_A$ for $0 < x < a$; max possible value $= Wl$ when $a = 0$<br>Max $\theta = \theta_B$; max possible value $= \frac{Wl^2}{2EI}$ when $a = 0$<br>Max $y = y_A$; max possible value $= \frac{-Wl^3}{3EI}$ when $a = 0$ |

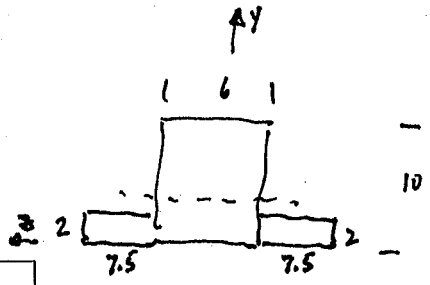
Probl 1 Q2A

A incl=pi*0.75$^2$ + 2*1.5 5

$\tau$ =T/2*Aincl*t=4 MPa= T/2*Aincl*0.010 to get T 10

$\delta\phi/\cancel{\Phi}$ = $\left\{T/(4*Ainc l^2*G)*[2*2+2*\pi*0.75]/0.010\right\}$ .4 m 10

$\oint \frac{ds}{t}$




Probl 2 Q2A

From steel to wood n=20

| | y | A | yA |
|---|---|---|---|
| ~~S~~ S | ~~1~~ 1 | 2*3/8*20*2=30 | ~~1200/8=150~~ 30 |
| W | ~~5~~ 5 | 6x10 | ~~60~~ 300 |
| | | 90 | ~~210~~ 330 |

Y=~~210~~ 330/90=~~1~~ 11/3=~~3.33~~ 3.67 9

| | Iz | A | d | Iz' =Iz+Ad$^2$ |
|---|---|---|---|---|
| S | 2*(1/12)*(3/8*20)*2$^3$ | 30 | ~~1-3.33~~ 3.67 | 223.87 |
| w | (1/12)*6*10$^3$ | 60 | ~~5-3.33~~ 3.67 | 606.13 |
| | | | | 830 |

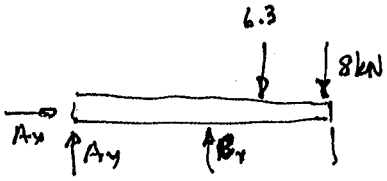
9

$\sigma$ = -(200000 lb-in)(10-~~3.33~~ 3.67)/Iz' at top in wood — 4 — $\sigma$ = 1525.3 psi = 1.525 ksi

$\sigma$ = -(200000 lb-in)(-~~3.33~~ 3.67)/Iz' at bottom in wood — 4 — $\sigma$ = 884.34 psi = 0.884 ksi

$\sigma$ = -20 (200000 lb-in)(-~~3.33~~ 3.67)/Iz' at bottom in steel — 4 — $\sigma$ = 17,686.7 psi = 17.69 ksi

Probl 3 Q2A




5

$\Sigma$Fx=0 = Ax=0 (1)

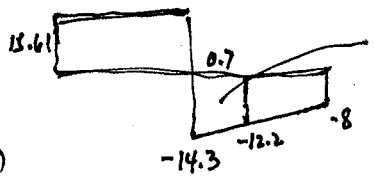
$\Sigma$Fy=0 = Ay+By-3x2.1-8=0 Ay=-15.61 (5)

$\Sigma$Ma=0 = By*1.5-3x2.1*2.55-8*3.6=0 By=29.91 (5)

Mom =-17.64KN-m ~~-14.145~~ kN·m

Iz=128 (10)$^{-6}$ m$^4$ and y=.151m (2+2)

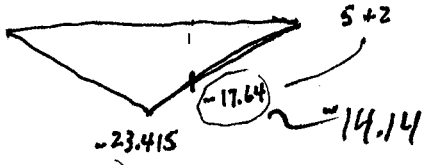
$\sigma$ =-(-17640 N-m)(0.151)/Iz at top of beam) (5)




$\frac{5+2}{2}(14.3+12.2)(0.7) = 9.27$

$\sigma$ = 16686679.7 psi

= 16686.7 ksi




-23.415+9.27 = -14.145

$\ell$ = 15.61(1.5)

Probl 1 Q2B

The centroid of the composite is at the center of the beam, i.e., y=5in

Convert steel to wood n=15

| | y | A | yA |
|---|---|---|---|
| S | 5 | 10*0.5*15=75 | 375 |
| W | 5 | 2*3*10=60 | 300 |
| | | 135 | 675 |

$\bar{y} = \frac{675}{135} = 5$ 9

| | Iz $= \frac{1}{12} bh^3$ | A | d | Iz' |
|---|---|---|---|---|
| S | $(1/12)*(1/2*15)*10^3 = 625$ | 75 | 5-5 | SAME as Iz |
| w | $2*(1/12)*3*10^3 = 500$ | 60 | 5-5 | 1125 |
| | 1125 | | | |

9

$M_{wood} = 450$ kips

$\sigma = -(M$ lb-in)(5)/Iz' at top in wood=2000 psi gives M due to wood 5

$\sigma = -15(M$ lb-in)(5)/Iz' at top in steel = 22000 psi gives M due to steel 5

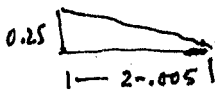
$M_{steel} = 330$ kips

$\sigma = \frac{-My}{Iz}$ $\quad M = \frac{\sigma Iz}{y}$

Use smaller value 2

$M_{steel} = 330$ kips

Probl 2 Q2B




Aincl = Pi*0.5²/2 + (1+0.5)*(2-0.005)/2 5

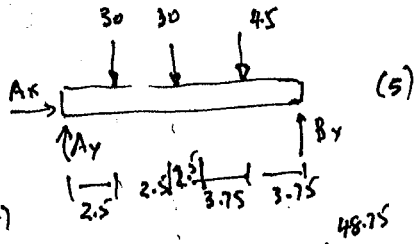
$\tau = T/2*Aincl*t = 4.5\ (10)^6/2*Aincl*0.010$ to get $\tau$ 10

$\lambda = Sqrt[0.25^2 + (2-0.005)^2]$ 3

$\delta\phi/L = 4.5\ (10)^6/(4*Aincl^2*G)*[2*\lambda+\pi*0.5+0.5]/0.010$ $= \frac{T}{4A_{inc}^2 G} \oint \frac{ds}{t}$

10

Probl 3 Q2B




(5)

$\Sigma Fx=0 = Ax=0$ (1)

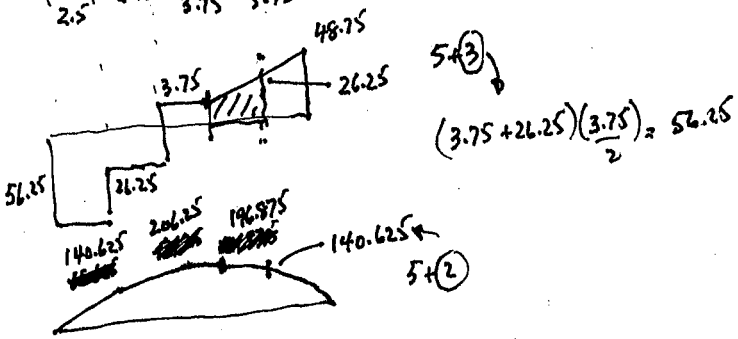
$\Sigma Fy=0 = Ay+By-30-30-6*7.5=0$ Ay=56.25 kip (5)

$\Sigma Ma=0 = By*15-30x2.5-30*5-6*7.5*11.25=0$ By=48.75 kip (5)

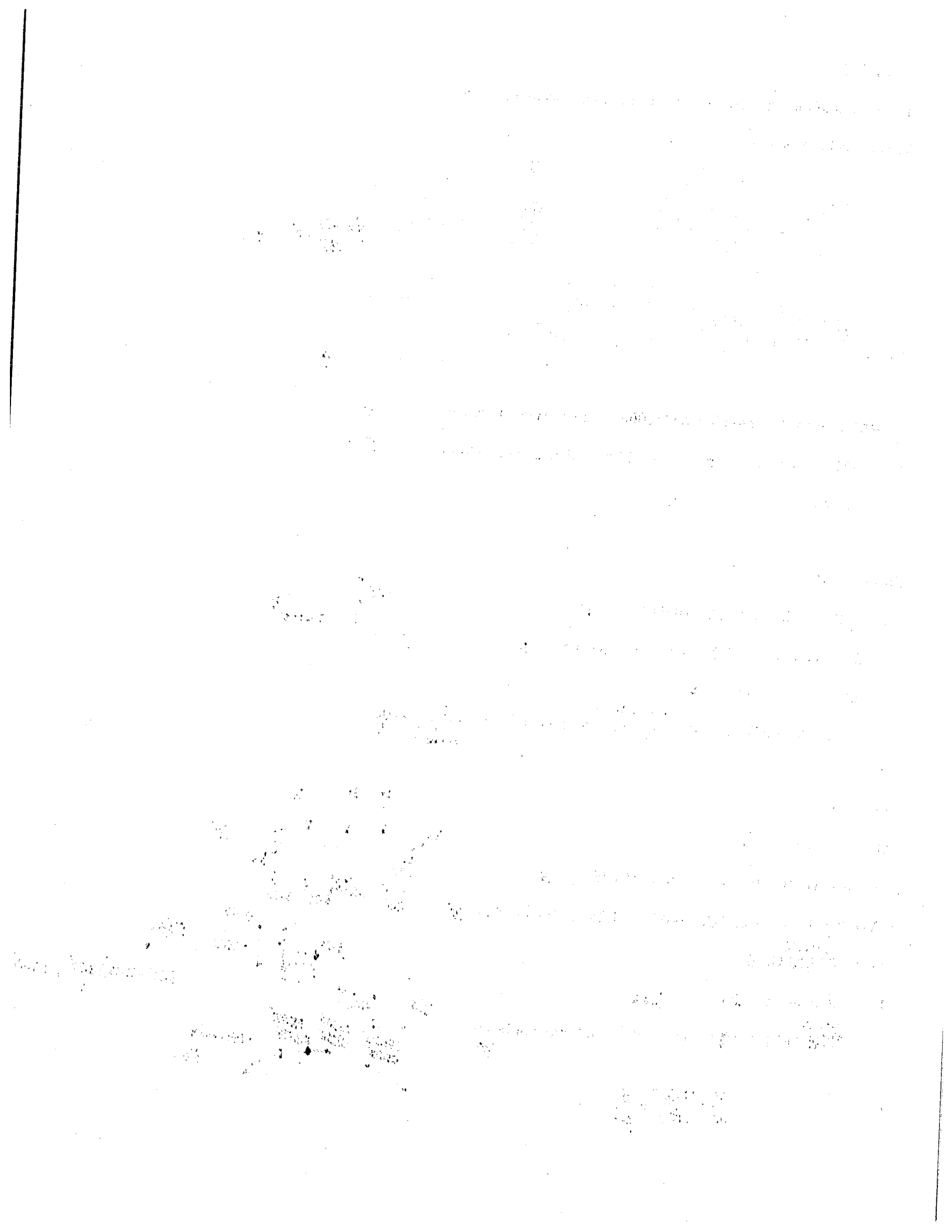
Mom = 140.625 kip-ft

Iz=1330 in⁴ and y=-9in (2+2)

$\sigma = -(140625$ lb-ft)(-0.75ft) x $(12$ in/ft$)^2$/Iz at bottom of beam (5)




$\frac{\#}{in^2} \cdot \left(\frac{12\,in}{ft}\right)^2 = \frac{\#}{ft^2}$



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EMA 3702 EXAM 3B April 16, 2013

You are allowed seven sheets of 8 ½ x 11 inch paper with information to help you solve problems

Print your name and sign the following statement:

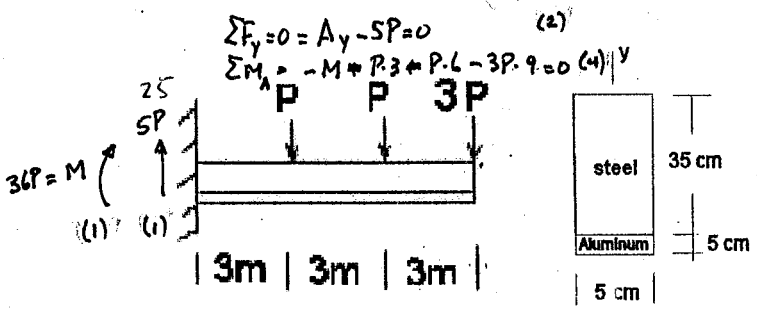
I will not give nor take any unpermitted aid during this quiz. I understand that violation of this statement will lead to automatic failure of the quiz.

PRINT NAME SIGN NAME

Problem 1.

a) Given the following beam loaded as shown, find the shear and moment diagraph, given P=5 KN.

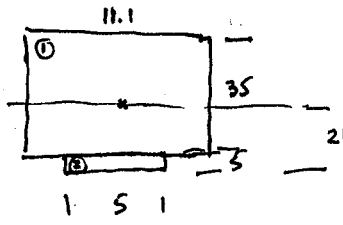
b) What is the maximum shear stress $\tau_{xy}$ and where is it located, given the following information: $E_{steel} = 206$ GPa, $E_{aluminum} = 92.7$ GPa, for the cross-section below. The poisson ratio in both materials is 0.3

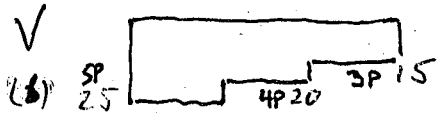






steel to al $\frac{206}{92.7} = 2.22$ (1)

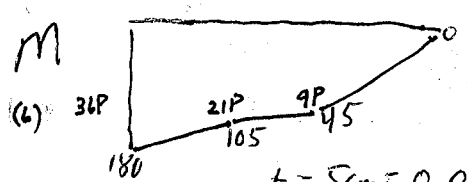


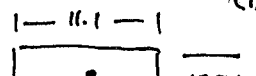

| | $\bar{y}$ | A | $\bar{y}A$ |
|---|---|---|---|
| ① | 22.5 | 388.5 | 8741.25 |
| ② | 2.5 | 25 | 62.5 |
| | | 413.5 | 8803.75 |

$\bar{y} = \frac{8803.75}{413.5} = 21.29$ cm (9)

| | $I_z$ | A | d | $I_z'$ |
|---|---|---|---|---|
| ① | $\frac{1}{12}(11.1)(35)^3$ | 388.5 | $(22.5-21.29)^2$ | 40222 |
| ② | $\frac{1}{12}5.5^3 = 52.08$ cm⁴ | 25 | $(21.29-2.5)^2$ | 8878 |
| | | | 18.71 | 49107 cm⁴ (9) |




$\tau_{max} = \frac{VQ}{I_z t}$




$= \frac{[5(5000)]\left(11.1 \times 18.71^2/2\right)(10^{-6}\ m^3/cm^3)}{10^{-8}\ cm^4 \cdot 5cm\ (10^{-2}\ m/cm)}$

Q = (4)
V = (1)
t = (1)
τ = (5)

$t = 5cm = 0.05m$

$V = 25000\ N$

$I_z = 49107 cm^4 = 4.9107 E^{-4}\ m^4$

$Q = 11.1 \times \frac{18.71^2}{2} = 1942.86 cm^3 = 0.001943 m^3$

$\tau_{max} = \frac{25000N(0.001943m^3)}{(4.9107 \times 10^{-4} m^4)(0.05m)}$

$= 1.978 MPa$

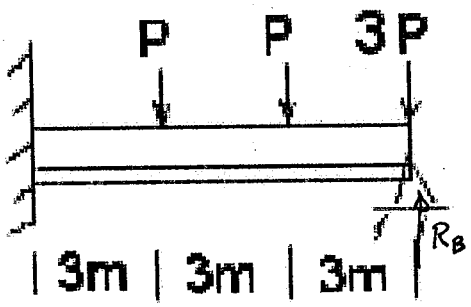
12-15 - nothing
20
25 - eqns
30
35 - c values
40
45 - slope wrong
50

QUIZ 3B

Problem 2.

a) Given the following beam loaded as shown, find the deflection y as a function of x.

b) What is the slope dy/dx at x=6m from the deflection equation y(x) ?




$y=0$ $\quad$ $y=0$

$y'=0$ $\quad$ $y''=0$

$$EIy''' = -P\langle x-3\rangle^0 - P\langle x-6\rangle^0 - 3P\langle x-9\rangle^0 + C_1$$

$$EIy'' = -P\langle x-3\rangle^1 - P\langle x-6\rangle^1 - 3P\langle x-9\rangle^1 + C_1 x + C_2$$

$$EIy' = -\frac{P}{2}\langle x-3\rangle^2 - \frac{P}{2}\langle x-6\rangle^2 - \frac{3P}{2}\langle x-9\rangle^2 + C_1\frac{x^2}{2} + C_2 x + C_3$$

$$EIy = -\frac{P}{6}\langle x-3\rangle^3 - \frac{P}{6}\langle x-6\rangle^3 - \frac{3P}{6}\langle x-9\rangle^3 + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4$$

$y=0$ @ $x=0$ $\Rightarrow$ $\boxed{C_4 = 0}$

$y'=0$ @ $x=0$ $\Rightarrow$ $\boxed{C_3 = 0}$

$y=0$ @ $x=9$ $\Rightarrow$ $EI\cdot 0 = -\frac{P}{6}(6)^3 - \frac{P}{6}(3)^3 + 0 + C_1\frac{9^3}{6} + C_2\frac{9^2}{2}$ (1)

$y''=0$ @ $x=9$ $\Rightarrow$ $EI\cdot 0 = -P\cdot 6 - P\cdot 3 + 0 + C_1\cdot 9 + C_2$ (2)

$(1) - (2)\cdot\frac{9^2}{2} \Rightarrow C_1$

$9P - 9C_1 = C_2$

$$EIy'\Big|_{x=6} = -\frac{P}{2}\langle 3\rangle^2 + 0 + 0 + C_1\frac{6^2}{2} + C_2 6$$

$-36P - 4.5P + 121.5C_1 + 40.5C_2 + 243P + 121.5P - 364.5C_1 - 40.5C_2 = 0$

$\Rightarrow 324P = 243C_1 \Rightarrow \boxed{C_1 = \frac{4}{3}P}$

$C_2 = 9P - 12P = \boxed{-3P}$

$$\text{Slope} = EIy'\Big|_{x=6} = -4.5P + \frac{4}{3}P(18) + (-3P)(6) = -4.5P + 24P - 18P = \boxed{1.5P}$$

$$\left.\frac{dy}{dx}\right|_{x=6} = \frac{1.5P}{EI}$$

Florida International University
Department of Mechanical and Materials Engineering

EMA 3702 EXAM 3C April 16, 2013

You are allowed seven sheets of 8 ½ x 11 inch paper with information to help you solve problems

Print your name and sign the following statement:

I will not give nor take any unpermitted aid during this quiz. I understand that violation of this statement will lead to automatic failure of the quiz.

PRINT NAME SIGN NAME

Problem 1.

$$\tau_{max} = \frac{(1500\,N)(0.006695\,m^3)}{(0.003366\,m^4)(0.05\,m)} = 59670\,Pa$$

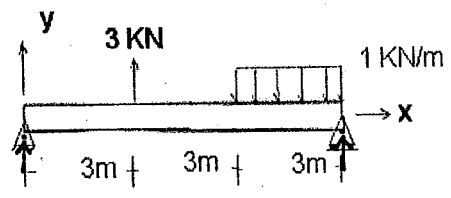
| | $I_z$ |
|---|---|
| ① | 233355.9 cm⁴ |
| ② | 103199.1 cm⁴ |
| | 336555 cm⁴ |

$= 0.003366\,m^4$

a) Given the following beam loaded as shown with the cross-section as shown, find the location of the maximum moment by obtaining the shear and moment diagram.

b) What is the maximum shear stress $\tau_{xy}$ and where can it be found, given the following information:
$E_{aluminum} = 92.7$ GPa, $E_{wood} = 10.3$ GPa, for the cross-section below and both materials have poisson ratios of 0.3.

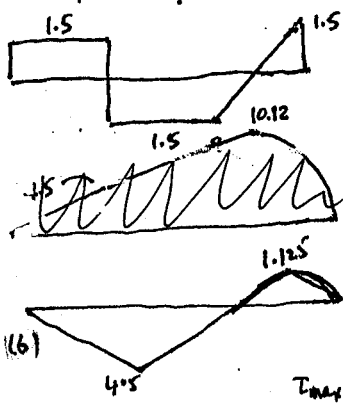
Al to wood $\Rightarrow n = 9$

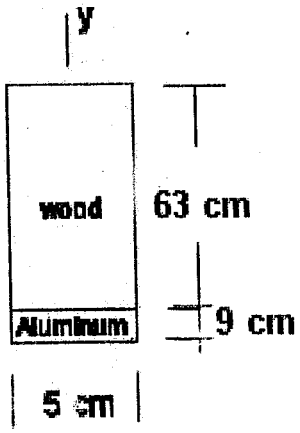


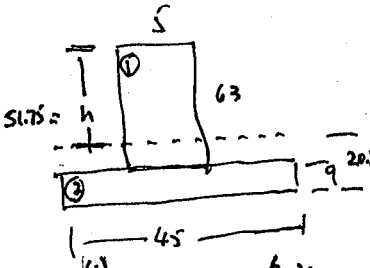

$\Sigma F_y = A_y + 3K - 3K + B_y = 0$ (4)

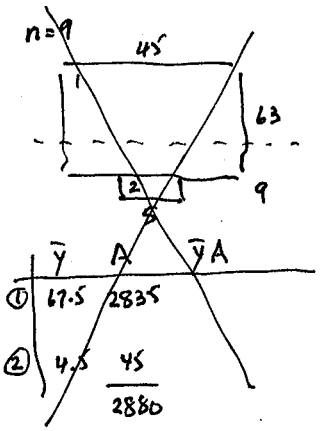
$\Sigma M_A = +3K \cdot 3 - 3K \cdot 7.5 + B_y \cdot 9 = 0$ (3)

$B_y = 1.5$ $A_y = -1.5$ (2)




| | $\bar{y}$ | A | $\bar{y}A$ |
|---|---|---|---|
| ① | 67.5 | 2835 | |
| ② | 4.5 | 45 | |
| | | 2880 | |

| | $\bar{y}$ | A | $\bar{y}A$ |
|---|---|---|---|
| ① | 40.5 | 315 | 12757.5 |
| ② | 4.5 | 40.5 | 1822.5 |
| | | 720 | 14580 |

$\bar{y} = 20.25$ (9)

| | $I_z$ | A | d |
|---|---|---|---|
| ① | $\frac{1}{12}5 \cdot 63^3$ 104186 | 315 | 40.5 − 20.25 = 20.25 |
| ② | $\frac{1}{12}45 \cdot 9^3$ 2733.75 | 40.5 | 20.25 − 4.5 = 15.75 |

(9)

$Q = 5 \cdot h^2/2$

$= 5(51.75)^2/2$ (4)

$= 6695\,cm^3$

$= 0.006695\,m^3$

$\tau_{max} = \frac{VQ}{I_z t} = \frac{(1500N)(6695.16 \times 10^{-6}\,m^3)}{5 \times 10^{-2}\,m^2}$ (1)

(5)



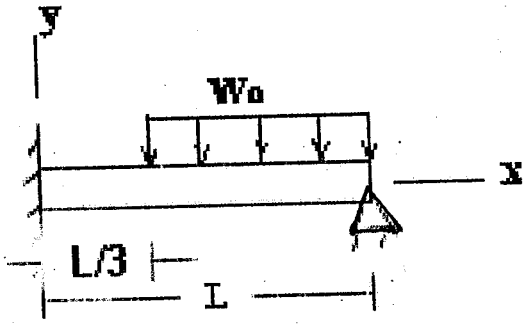
12-15 - nothing
20
25 eqns
30
35 C values
40
45 moment wrong
50

QUIZ 3C

Problem 2.

a) Given the following beam loaded as shown, find the deflection y as a function of x.

a) Determine the moment at x=0 from the deflection equation y(x)?




$y=0$, $y'=0$ $\qquad$ $y=0$, $y''=0$

$$EIy^{IV} = -w_0\langle x-L/3\rangle^0$$

$$EIy''' = -w_0\langle x-L/3\rangle^1 + C_1$$

$$EIy'' = -\frac{w_0}{2}\langle x-L/3\rangle^2 + C_1x + C_2$$

$$EIy' = -\frac{w_0}{6}\langle x-L/3\rangle^3 + C_1\frac{x^2}{2} + C_2x + C_3$$

a) $$\boxed{EIy = -\frac{w_0}{24}\langle x-L/3\rangle^4 + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3x + C_4}$$

$y=0$ @ $x=0$ $\Rightarrow$ $\boxed{C_4=0}$

$y'=0$ @ $x=0$ $\Rightarrow$ $\boxed{C_3=0}$

$y=0$ @ $x=L$ $\Rightarrow$ $EI\cdot 0 = -\frac{w_0}{24}\left(\frac{16}{81}L^4\right) + C_1\frac{L^3}{6} + C_2\frac{L^2}{2}$ (1)

$y''=0$ @ $x=L$ $\Rightarrow$ $EI\cdot 0 = -\frac{w_0}{2}\left(\frac{4}{9}L^2\right) + C_1L + C_2$ (2)

(1) $-$ (2)$\cdot L^2/2$ $\Rightarrow$ $C_1$

$$C_2 = \frac{4w_0}{18}L^2 - C_1L$$

$M = EIy'' = -\frac{w_0}{2}\langle\ \ \rangle + C_1\cdot 0 + C_2 = C_2$

$$-\frac{w_0}{24}\left(\frac{16}{81}L^4\right) + C_1\frac{L^3}{6} + C_2\frac{L^2}{2} + \frac{w_0}{2}\left(\frac{2}{9}L^4\right) - C_1\frac{L^3}{2} - C_2\frac{L^2}{2} = 0$$

$$\Rightarrow -w_0\left(\frac{2}{243}L^4\right) + w_0\left(\frac{1}{9}L^4\right) = C_1\frac{L^3}{2} - C_1\frac{L^3}{6} = C_1\frac{L^3}{3}$$

$$\Rightarrow w_0\left(\frac{25}{243}\right)L^4 = C_1\frac{L^3}{3} \Rightarrow \boxed{C_1 = \frac{25}{81}w_0L}$$

$$C_2 = \frac{4}{18}w_0L^2 - \frac{25}{81}w_0L^2 = \boxed{-\frac{7}{81}w_0L^2}$$

$$M\Big|_{x=0} = EIy'' = C_2 = -\frac{7}{81}w_0L^2$$



Solution

Florida International University
Department of Mechanical and Materials Engineering

EMA 3702 QUIZ 2A April 4, 2013

This exam is 75 minutes long. You are allowed five sheets of 8 ½ x 11 inch paper with whatever you wish on the sheets

Print your name and sign the following statement:

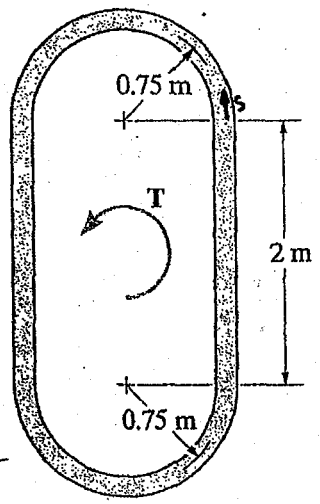
I will not give nor take any unpermitted aid during this quiz. I understand that violation of this statement will lead to automatic failure of the quiz.

PRINT NAME _____ SIGN NAME _____

Problem 1.

A portion of an airplane fuselage can be approximated by the cross section shown. If the thickness of its 2014-T6-aluminum skin is 10 mm, determine the maximum wing torque **T** that can be applied if $\tau_{allow} = 4$ MPa. Also, in a 4-m long section, determine the angle of twist.

**Take G=27 GPa**




Given: $t = 10\,mm = 0.010\,m$

$\tau_{allow} = 4\,MPa$

$L = 4\,m$

$G = 27\,GPa$

$A_{included} = A_{circle} + A_{rectangle}$

$= \pi(0.75)^2 + 2(1.5)$

$= 4.767\,m^2$ 5

$\tau = \frac{T}{2 \cdot A_{incl} \cdot t} \Rightarrow T = (4\,MPa)(2)(4.767)(0.010)$ $\rightarrow = 4{,}000{,}000\ \frac{N}{m^2}$

$T = 381{,}371.68\ N \cdot m$

$= 381.37\ kN \cdot m$

$= 0.38137\ MN \cdot m$ 10

$J = 4 \cdot A_{incl}^2 = 4(4.767\,m^2)^2$

$= 90.9\,m^4$

$\oint \frac{ds}{t} = \frac{(2 \cdot 2) + (2 \cdot \pi \cdot 0.75)}{0.010}$ 5

$= \frac{8.712}{0.01} = 871.24$

$\frac{\delta\phi}{L} = \frac{T}{JG} \Rightarrow \delta\phi = \frac{TL}{JG}$

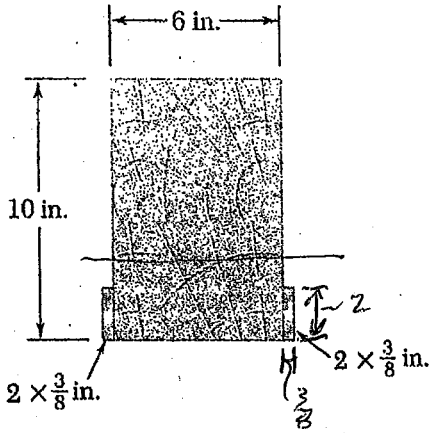
$\phi = \frac{TL}{4 \cdot A_{incl}^2 \cdot G} \oint \frac{ds}{t}$

$\phi = \frac{(381{,}371.68\ N \cdot m)(4\,m)}{(90.9\,m^4)(27 \times 10^9\,Pa)\ \cdot \frac{N}{m^2}} (871.2) = 5.41 \times 10^{-4}\,rad = 3.1 \times 10^{-2}\,deg$

$= 0.031\,deg$ 10

Problem 2.

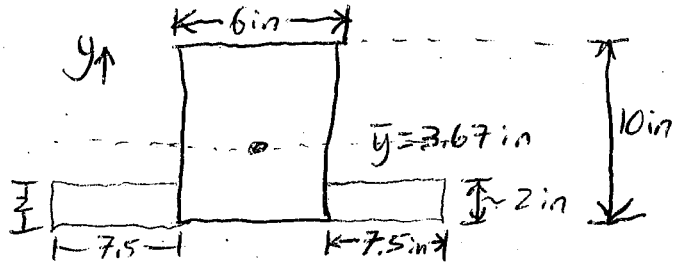
A 6 × 10-in. timber beam has been strengthened by bolting to it the steel straps shown. The modulus of elasticity is $E = 1.5 \times 10^6$ psi for the wood and $E = 30 \times 10^6$ psi for the steel. Knowing that the beam is bent about a horizontal axis by a couple of moment 200 kip · in., determine the maximum stress in (*a*) the wood, (*b*) the steel.




converting steel to wood:

$n_{wood} = 1$

$n_{steel} = \frac{E_s}{E_w} = \frac{30}{1.5} = 20$




| | y in. | $A = n \cdot b \cdot h$ $in^2$ | $yA$ $in^3$ |
|---|---|---|---|
| S | 1 | $2 \cdot 20 \cdot \frac{3}{8} \cdot 2 = 30$ | 30 |
| W | 5 | $6 \cdot 10 = 60$ | 300 |
| Σ | | 90 | 330 |

$\bar{y} = \frac{\Sigma yA}{\Sigma A} = \frac{330}{90} = 3.67$ in

9

| | $I_z = \frac{n}{12} bh^3$ | A | d | $I_z' = I_z + Ad^2$ |
|---|---|---|---|---|
| S | $2 \cdot \frac{20}{12} \cdot \frac{3}{8} \cdot 2^3 = 10$ | 30 | 1-3.67 | 223.87 |
| W | $\frac{1}{12} \cdot 6 \cdot 10^3 = 500$ | 60 | 5-3.67 | 606.13 |
| Σ | | | | 830 $in^4$ |

9

$\sigma = \frac{nMc}{I_z'}$

a) For Wood: Top: $\sigma = \frac{(1)(200{,}000\ lb \cdot in)(10-3.67)in}{830\ in^4} = 1525.3 psi = 1.525 ksi$ 4

Bottom: $\sigma = \frac{(1)(200{,}000\ lb \cdot in)(3.67)in}{830\ in^4} = 884.34 psi = 0.884 ksi$ 3

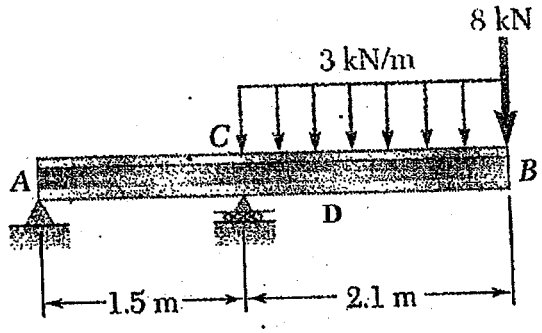
b) For Steel: Bottom: $\sigma = \frac{(20)(200{,}000\ lb \cdot in)(3.67)in}{830\ in^4} = 17{,}686.7 psi = 17.69 ksi$ 4

Top: $\sigma = \frac{(20)(200{,}000)(-3.67)}{830} = -17{,}867.5 psi = 12.87 ksi$

Problem 3

For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at **D**.

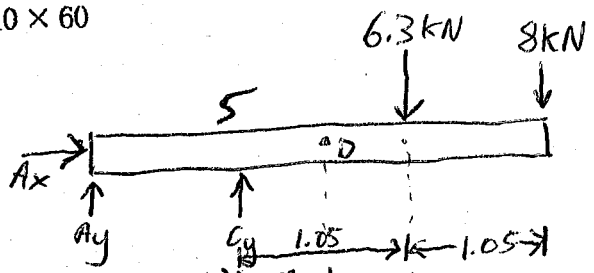
**D is one-third the way between C and B**







Check the last page for pertinent information related to this problem.



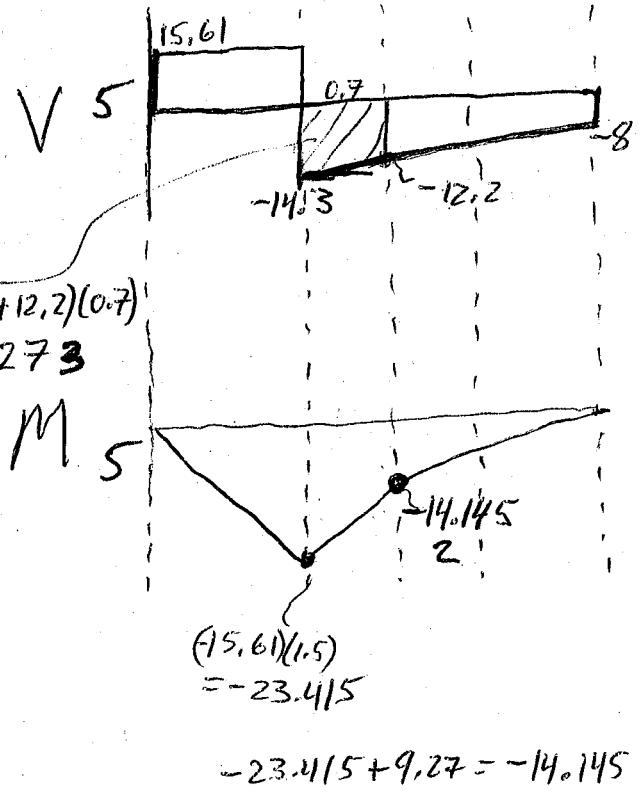

1 $\Sigma F_x = 0 \Rightarrow A_x = 0$

$\Sigma F_y = 0 = A_y + C_y - 6.3 - 8 \Rightarrow A_y = 14.3 - C_y$

$\Sigma M_a = 0 = C_y(1.5) - 6.3(2.55) - 8(3.6)$

$\Rightarrow C_y = 29.91 \text{ kN}$ 5

$\therefore A_y = -15.61 \text{ kN}$ 5




$\frac{1}{2}(14.3+12.2)(0.7) = 9.273$

$-23.415 + 9.27 = -14.145$

$M_D = -14.145 \text{ kN·m}$

$I_z = 128 \times 10^6 \text{ mm}^4 = 128 \times 10^{-6} \text{ m}^4$ 2

$\bar{y} = 151 \text{ mm} = 0.151 \text{ m}$ 2

$$\sigma = \frac{-M\bar{y}}{I_z} = \frac{-(-14145 \text{ N·m})(0.151 \text{ m})}{128 \times 10^{-6} \text{ m}^4}$$

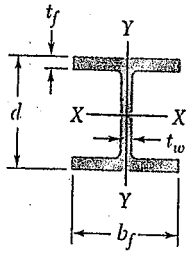
$= 16686679.7 \text{ psi}$ 5

$= 16686.7 \text{ ksi}$

# APPENDIX B Properties of Rolled-Steel Shapes

(SI Units)

Continued from page 681




## W Shapes

(Wide-Flange Shapes)

| Designation† | Area $A$, mm$^2$ | Depth $d$, mm | Flange Width $b_f$, mm | Flange Thickness $t_f$, mm | Web Thickness $t_w$, mm | Axis X-X $I_x$, $10^6$ mm$^4$ | Axis X-X $S_x$, $10^3$ mm$^3$ | Axis X-X $r_x$, mm | Axis Y-Y $I_y$, $10^6$ mm$^4$ | Axis Y-Y $S_y$, $10^3$ mm$^3$ | Axis Y-Y $r_y$, mm |
|---|---|---|---|---|---|---|---|---|---|---|---|
| W310 × 143 | 18200 | 323 | 310 | 22.9 | 14.0 | 347 | 2150 | 138 | 112 | 728 | 78.5 |
| 107 | 13600 | 312 | 305 | 17.0 | 10.9 | 248 | 1600 | 135 | 81.2 | 531 | 77.2 |
| 74 | 9420 | 310 | 205 | 16.3 | 9.40 | 163 | 1050 | 132 | 23.4 | 228 | 49.8 |
| 60 | 7550 | 302 | 203 | 13.1 | 7.49 | 128 | 844 | 130 | 18.4 | 180 | 49.3 |
| 52 | 6650 | 318 | 167 | 13.2 | 7.62 | 119 | 747 | 133 | 10.2 | 122 | 39.1 |
| 44.5 | 5670 | 312 | 166 | 11.2 | 6.60 | 99.1 | 633 | 132 | 8.45 | 102 | 38.6 |
| 38.7 | 4940 | 310 | 165 | 9.65 | 5.84 | 84.9 | 547 | 131 | 7.20 | 87.5 | 38.4 |
| 32.7 | 4180 | 312 | 102 | 10.8 | 6.60 | 64.9 | 416 | 125 | 1.94 | 37.9 | 21.5 |
| 23.8 | 3040 | 305 | 101 | 6.73 | 5.59 | 42.9 | 280 | 119 | 1.17 | 23.1 | 19.6 |
| W250 × 167 | 21200 | 290 | 264 | 31.8 | 19.2 | 298 | 2060 | 118 | 98.2 | 742 | 68.1 |
| 101 | 12900 | 264 | 257 | 19.6 | 11.9 | 164 | 1240 | 113 | 55.8 | 433 | 65.8 |
| 80 | 10200 | 257 | 254 | 15.6 | 9.4 | 126 | 983 | 111 | 42.9 | 338 | 65.0 |
| 67 | 8580 | 257 | 204 | 15.7 | 8.89 | 103 | 805 | 110 | 22.2 | 218 | 51.1 |
| 58 | 7420 | 252 | 203 | 13.5 | 8.00 | 87.0 | 690 | 108 | 18.7 | 185 | 50.3 |
| 49.1 | 6260 | 247 | 202 | 11.0 | 7.37 | 71.2 | 574 | 106 | 15.2 | 151 | 49.3 |
| 44.8 | 5700 | 267 | 148 | 13.0 | 7.62 | 70.8 | 531 | 111 | 6.95 | 94.2 | 34.8 |
| 32.7 | 4190 | 259 | 146 | 9.14 | 6.10 | 49.1 | 380 | 108 | 4.75 | 65.1 | 33.8 |
| 28.4 | 3630 | 259 | 102 | 10.0 | 6.35 | 40.1 | 308 | 105 | 1.79 | 35.1 | 22.2 |
| 22.3 | 2850 | 254 | 102 | 6.86 | 5.84 | 28.7 | 226 | 100 | 1.20 | 23.5 | 20.6 |
| W200 × 86 | 11000 | 222 | 209 | 20.6 | 13.0 | 94.9 | 852 | 92.7 | 31.3 | 300 | 53.3 |
| 71 | 9100 | 216 | 206 | 17.4 | 10.2 | 76.6 | 708 | 91.7 | 25.3 | 246 | 52.8 |
| 59 | 7550 | 210 | 205 | 14.2 | 9.14 | 60.8 | 582 | 89.7 | 20.4 | 200 | 51.8 |
| 52 | 6650 | 206 | 204 | 12.6 | 7.87 | 52.9 | 511 | 89.2 | 17.7 | 174 | 51.6 |
| 46.1 | 5880 | 203 | 203 | 11.0 | 7.24 | 45.8 | 451 | 88.1 | 15.4 | 152 | 51.3 |
| 41.7 | 5320 | 205 | 166 | 11.8 | 7.24 | 40.8 | 398 | 87.6 | 9.03 | 109 | 41.1 |
| 35.9 | 4570 | 201 | 165 | 10.2 | 6.22 | 34.4 | 342 | 86.9 | 7.62 | 92.3 | 40.9 |
| 31.3 | 3970 | 210 | 134 | 10.2 | 6.35 | 31.3 | 298 | 88.6 | 4.07 | 60.8 | 32.0 |
| 26.6 | 3390 | 207 | 133 | 8.38 | 5.84 | 25.8 | 249 | 87.1 | 3.32 | 49.8 | 31.2 |
| 22.5 | 2860 | 206 | 102 | 8.00 | 6.22 | 20.0 | 193 | 83.6 | 1.42 | 27.9 | 22.3 |
| 19.3 | 2480 | 203 | 102 | 6.48 | 5.84 | 16.5 | 162 | 81.5 | 1.14 | 22.5 | 21.4 |
| W150 × 37.1 | 4740 | 162 | 154 | 11.6 | 8.13 | 22.2 | 274 | 68.6 | 7.12 | 91.9 | 38.6 |
| 29.8 | 3790 | 157 | 153 | 9.27 | 6.60 | 17.2 | 220 | 67.6 | 5.54 | 72.3 | 38.1 |
| 24 | 3060 | 160 | 102 | 10.3 | 6.60 | 13.4 | 167 | 66.0 | 1.84 | 36.1 | 24.6 |
| 18 | 2290 | 153 | 102 | 7.11 | 5.84 | 9.20 | 120 | 63.2 | 1.24 | 24.6 | 23.3 |
| 13.5 | 1730 | 150 | 100 | 5.46 | 4.32 | 6.83 | 91.1 | 62.7 | 0.916 | 18.2 | 23.0 |
| W130 × 28.1 | 3590 | 131 | 128 | 10.9 | 6.86 | 10.9 | 167 | 55.1 | 3.80 | 59.5 | 32.5 |
| 23.8 | 3040 | 127 | 127 | 9.14 | 6.10 | 8.91 | 140 | 54.1 | 3.13 | 49.2 | 32.0 |
| W100 × 19.3 | 2470 | 106 | 103 | 8.76 | 7.11 | 4.70 | 89.5 | 43.7 | 1.61 | 31.1 | 25.4 |

†A wide-flange shape is designated by the letter W followed by the nominal depth in millimeters and the mass in kilograms per meter.

Solution

Florida International University
Department of Mechanical and Materials Engineering

EMA 3702 QUIZ 2B April 4, 2013

This exam is 75 minutes long. You are allowed five sheets of 8 ½ x 11 inch paper with whatever you wish on the sheets

Print your name and sign the following statement:

I will not give nor take any unpermitted aid during this quiz. I understand that violation of this statement will lead to automatic failure of the quiz.

PRINT NAME _____ SIGN NAME _____

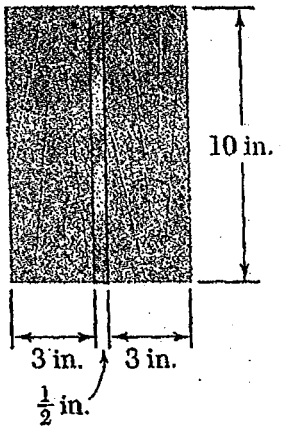
Problem 1.

Wooden beams and steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the composite member is bent about a horizontal axis.

| | Wood | Steel |
|---|---|---|
| Modulus of elasticity | $2 \times 10^6$ psi | $30 \times 10^6$ psi |
| Allowable stress | 2000 psi | 22 ksi |

converting steel to wood: $n_{wood} = 1$

$n_{steel} = \frac{E_s}{E_w} = \frac{30}{2} = 15$




| | $y$ | $A$ | $yA$ |
|---|---|---|---|
| S | 5 | $n \cdot b \cdot h = 15 \cdot \frac{1}{2} \cdot 10 = 75$ | 375 |
| W | 5 | $2 \cdot b \cdot h = 2 \cdot 3 \cdot 10 = 60$ | 300 |
| | | 135 | 675 |

$\bar{y} = \frac{\Sigma yA}{\Sigma A} = \frac{675}{135} = 5$ in.

| | $I_z = \frac{1}{12} b h^3 \cdot n$ | $A$ | $d$ | $I_z'$ |
|---|---|---|---|---|
| S | $\frac{1}{12} \cdot \frac{1}{2} \cdot 10^3 \cdot 15 = 625$ | 75 | 5-5 | $I_z$ |
| W | $2 \cdot \frac{1}{12} \cdot 3 \cdot 10^3 \cdot 1 = 500$ | 60 | 5-5 | $I_z$ |
| | 1125 | | | 1125 |

$|\sigma| = \left|\frac{nMc}{I_z}\right| \Rightarrow M = \frac{\sigma I_z'}{nc}$, $c = 5$

For Wood: $M = \frac{(2000 \text{ psi})(1125)}{(1)(5)} = 450{,}000 \text{ lb·in} = 450 \text{ kips·in}$

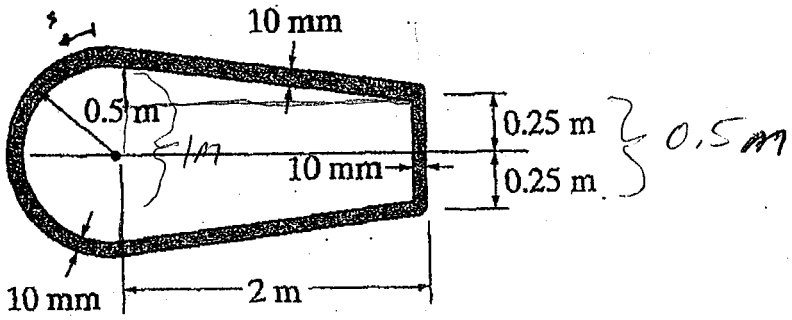
For steel: $M = \frac{(22000 \text{ psi})(1125)}{(15)(5)} = 330{,}000 \text{ lb·in} = 330 \text{ kips}$

Problem 2.

The mean dimensions of the cross section of the leading edge and torsion box of an airplane wing can be approximated as shown. If the wing is subjected to a torque of 4.5 MN·m and the wall thickness is 10 mm, determine the average shear stress developed in the wing and the angle of twist per meter length of the wing. The wing is made of 2014-T6 aluminum alloy.

**Take G = 27 GPa**

Given:
$T = 4.5\,MN\cdot m$
$t = 10mm = 0.01m$
$G = 27GPa$




$$A_{incl} = \frac{\pi}{2}(0.5)^2 + \frac{1}{2}(1+0.5)(2-0.005) = 1.89m^2$$

$$\tau = \frac{T}{2\cdot A_{incl}\cdot t} = \frac{(4500000\ N\cdot m)}{2(1.89m^2)(0.01m)} = 119.047619\,Pa = 119MPa$$

$$\phi = \frac{TL}{4\cdot A_{incl}^2\cdot G}\oint \frac{ds}{t} \Rightarrow \frac{\phi}{L} = \frac{T}{4\cdot A_{inc}^2\cdot G}\cdot\frac{1}{t}\cdot\oint ds$$

$$\oint ds = 2\sqrt{0.25^2+(2-0.005)^2} + \pi(0.5) + 0.5 = 6.09m$$

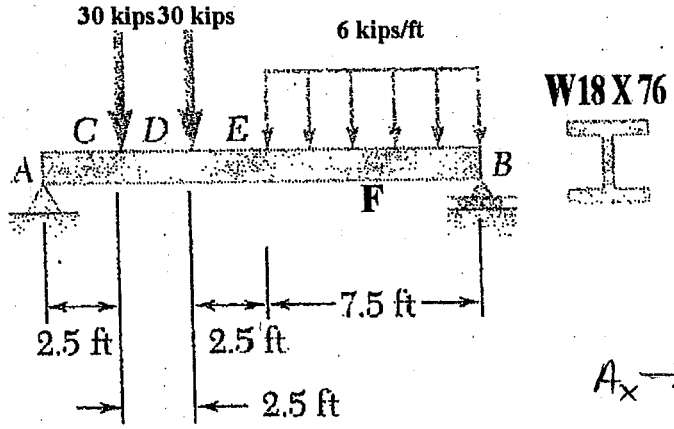
$$\frac{\phi}{L} = \frac{(4500000\ N\cdot m)}{4(1.89m^2)^2\cdot(27\times10^9\ \frac{N}{m^2})}\cdot\frac{1}{0.01m}\cdot 6.09m = 0.0071\frac{rad}{m} = 0.407\frac{deg}{m}$$

$= 14.288m^4$

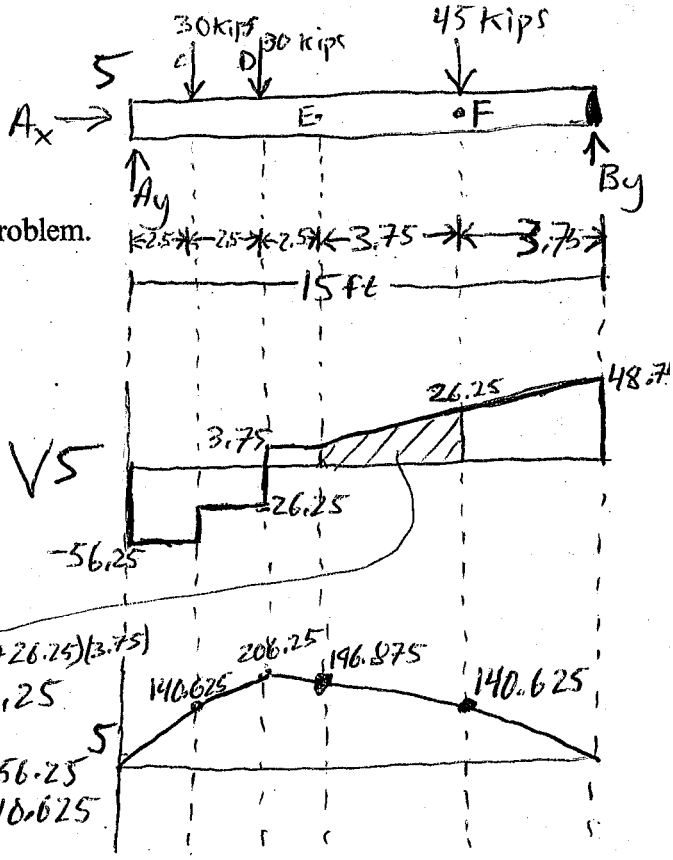
Problem 3

For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at **F**.

**F is midway between E and B**




Check the last page for pertinent information related to this problem.




$\Sigma F_x = 0 \Rightarrow A_x = 0$ 1

$\Sigma F_y = 0 = A_y + B_y - 30 - 30 - 45 \Rightarrow A_y = 105 - B_y$

$\Sigma M_A = 0 = -30(2.5) - 30(5) - 45(11.25) + B_y(15)$

$\Rightarrow B_y = 48.75$ kips 5

$\therefore A_y = 56.25$ kips 5

$M_F = 140.625$ kip·ft

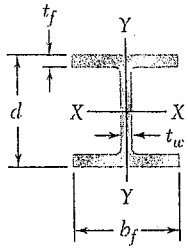
$I_z = 1330\ in^4 \left(\frac{1\ ft}{12\ in}\right)^4 = 0.064\ ft^4$ 2

$\bar{y} = -9.1\ in = 0.758333\ ft$ 2

$$\sigma = \frac{-M\bar{y}}{I_z} = \frac{-(140.625\ kip\cdot ft)(-0.758333\ ft)}{0.064\ ft^4} = 1666.259\ \frac{kip}{ft^2}$$ 5

# APPENDIX B Properties of Rolled-Steel Shapes
(U.S. Customary Units)

## W Shapes
(Wide-Flange Shapes)




| Designation† | Area $A$, in$^2$ | Depth $d$, in. | Flange Width $b_f$, in. | Flange Thickness $t_f$, in. | Web Thickness $t_w$, in. | Axis X-X $I_x$, in$^4$ | Axis X-X $S_x$, in$^3$ | Axis X-X $r_x$, in. | Axis Y-Y $I_y$, in$^4$ | Axis Y-Y $S_y$, in$^3$ | Axis Y-Y $r_y$, in. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| W36 × 302 | 88.8 | 37.3 | 16.7 | 1.68 | 0.945 | 21100 | 1130 | 15.4 | 1300 | 156 | 3.82 |
| 135 | 39.7 | 35.6 | 12.0 | 0.790 | 0.600 | 7800 | 439 | 14.0 | 225 | 37.7 | 2.38 |
| W33 × 201 | 59.2 | 33.7 | 15.7 | 1.15 | 0.715 | 11600 | 686 | 14.0 | 749 | 95.2 | 3.56 |
| 118 | 34.7 | 32.9 | 11.5 | 0.740 | 0.550 | 5900 | 359 | 13.0 | 187 | 32.6 | 2.32 |
| W30 × 173 | 51.0 | 30.4 | 15.0 | 1.07 | 0.655 | 8230 | 541 | 12.7 | 598 | 79.8 | 3.42 |
| 99 | 29.1 | 29.7 | 10.50 | 0.670 | 0.520 | 3990 | 269 | 11.7 | 128 | 24.5 | 2.10 |
| W27 × 146 | 43.1 | 27.4 | 14.0 | 0.975 | 0.605 | 5660 | 414 | 11.5 | 443 | 63.5 | 3.20 |
| 84 | 24.8 | 26.70 | 10.0 | 0.640 | 0.460 | 2850 | 213 | 10.7 | 106 | 21.2 | 2.07 |
| W24 × 104 | 30.6 | 24.1 | 12.8 | 0.750 | 0.500 | 3100 | 258 | 10.1 | 259 | 40.7 | 2.91 |
| 68 | 20.1 | 23.7 | 8.97 | 0.585 | 0.415 | 1830 | 154 | 9.55 | 70.4 | 15.7 | 1.87 |
| W21 × 101 | 29.8 | 21.4 | 12.3 | 0.800 | 0.500 | 2420 | 227 | 9.02 | 248 | 40.3 | 2.89 |
| 62 | 18.3 | 21.0 | 8.24 | 0.615 | 0.400 | 1330 | 127 | 8.54 | 57.5 | 14.0 | 1.77 |
| 44 | 13.0 | 20.7 | 6.50 | 0.450 | 0.350 | 843 | 81.6 | 8.06 | 20.7 | 6.37 | 1.26 |
| W18 × 106 | 31.1 | 18.7 | 11.2 | 0.940 | 0.590 | 1910 | 204 | 7.84 | 220 | 39.4 | 2.66 |
| 76 | 22.3 | 18.2 | 11.0 | 0.680 | 0.425 | 1330 | 146 | 7.73 | 152 | 27.6 | 2.61 |
| 50 | 14.7 | 18.0 | 7.50 | 0.570 | 0.355 | 800 | 88.9 | 7.38 | 40.1 | 10.7 | 1.65 |
| 35 | 10.3 | 17.7 | 6.00 | 0.425 | 0.300 | 510 | 57.6 | 7.04 | 15.3 | 5.12 | 1.22 |
| W16 × 77 | 22.6 | 16.5 | 10.3 | 0.76 | 0.455 | 1110 | 134 | 7.00 | 138 | 26.9 | 2.47 |
| 57 | 16.8 | 16.4 | 7.12 | 0.715 | 0.430 | 758 | 92.2 | 6.72 | 43.1 | 12.1 | 1.60 |
| 40 | 11.8 | 16.0 | 7.00 | 0.505 | 0.305 | 518 | 64.7 | 6.63 | 28.9 | 8.25 | 1.57 |
| 31 | 9.13 | 15.9 | 5.53 | 0.440 | 0.275 | 375 | 47.2 | 6.41 | 12.4 | 4.49 | 1.17 |
| 26 | 7.68 | 15.7 | 5.50 | 0.345 | 0.250 | 301 | 38.4 | 6.26 | 9.59 | 3.49 | 1.12 |
| W14 × 370 | 109 | 17.9 | 16.5 | 2.66 | 1.66 | 5440 | 607 | 7.07 | 1990 | 241 | 4.27 |
| 145 | 42.7 | 14.8 | 15.5 | 1.09 | 0.680 | 1710 | 232 | 6.33 | 677 | 87.3 | 3.98 |
| 82 | 24.0 | 14.3 | 10.1 | 0.855 | 0.510 | 881 | 123 | 6.05 | 148 | 29.3 | 2.48 |
| 68 | 20.0 | 14.0 | 10.0 | 0.720 | 0.415 | 722 | 103 | 6.01 | 121 | 24.2 | 2.46 |
| 53 | 15.6 | 13.9 | 8.06 | 0.660 | 0.370 | 541 | 77.8 | 5.89 | 57.7 | 14.3 | 1.92 |
| 43 | 12.6 | 13.7 | 8.00 | 0.530 | 0.305 | 428 | 62.6 | 5.82 | 45.2 | 11.3 | 1.89 |
| 38 | 11.2 | 14.1 | 6.77 | 0.515 | 0.310 | 385 | 54.6 | 5.87 | 26.7 | 7.88 | 1.55 |
| 30 | 8.85 | 13.8 | 6.73 | 0.385 | 0.270 | 291 | 42.0 | 5.73 | 19.6 | 5.82 | 1.49 |
| 26 | 7.69 | 13.9 | 5.03 | 0.420 | 0.255 | 245 | 35.3 | 5.65 | 8.91 | 3.55 | 1.08 |
| 22 | 6.49 | 13.7 | 5.00 | 0.335 | 0.230 | 199 | 29.0 | 5.54 | 7.00 | 2.80 | 1.04 |

†A wide-flange shape is designated by the letter W followed by the nominal depth in inches and the weight in pounds per foot.

*(Table continued on page 682)*

$v = A\cos\lambda x + B\sin\lambda x + Cx + D$

לפי תנאי השפה על הקורה.

כפי שקבלה שלת הדרך לפתור את הבעיה נצכיר ש-

$$EI\frac{d^2v}{dx^2} + Pv = M_t$$

כאשר $P=0$

$$EI\frac{d^2v_t}{dx^2} = M_t$$

פה $M_t$ הוא המומנט שבא מעומסים אנכיים בלבד, ו- $v_t$ הוא התצוגה הכוון ע שבאה מהעומס האנכיים. מניחים שצורת עיקום התצוגה יהיה שווה בערך עם או בלי הכוח האופקי P.

לכן, לפי תנאי השפה, כאן קורה על סמך נייד ומסך נייד, מניחים ש- $v = \bar{v}\sin\frac{\pi x}{L}$

ו- $v_t = \bar{v}_t\sin\frac{\pi x}{L}$. $\bar{v}$, ו- $\bar{v}_t$ הם ה- AMPLITUDE של v ו- $v_t$. לפי הנוסחאות למעלה

$$-EI\frac{\pi^2}{L^2}\bar{v}_t = -EI\frac{\pi^2}{L^2}\bar{v} + P\bar{v}$$

מפני ש- $P_E = \frac{\pi^2EI}{L^2}$, או $\bar{v}(P-P_E) = -P_E\bar{v}_t$

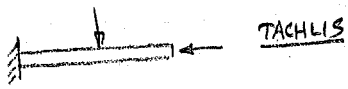
$$\bar{v} = \frac{\bar{v}_t}{1 - P/P_E}$$

מומנט גם כן עוקב אחרי נוסחא דומה לזה

$$M = \frac{M_t}{1 - P/P_E}$$

אם P חיובי, $\bar{v} > \bar{v}_t$ ו- $M > M_t$. אם P שלילי, $\bar{v} < \bar{v}_t$ ו- $M < M_t$

TACHLIS



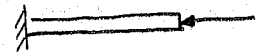
כדי לקבל את $v$ מתוך הערכים שנמצאו בכל מקום אחר. קודם לכה $\bar{v}_t$.



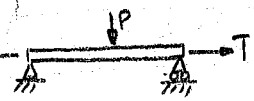
קבל $P_E$ לקורה עמוסה כמו בקורה

$v$ יהיה

$$v = \frac{\bar{v}_t}{1 - P/P_E}$$



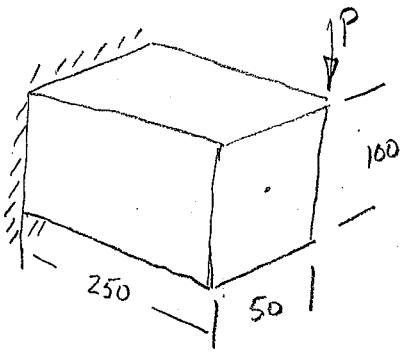
בתרגיל בית 12.10



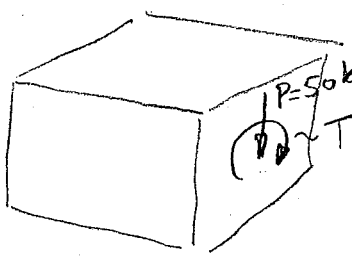
כי P היה שלילי.

$$T_E = \frac{\pi^2EI}{L^2}, \quad \bar{v}_t\Big|_{x=L/2} = \frac{PL^3}{48EI}, \quad v = \frac{\bar{v}_t}{1 + T/T_E}$$

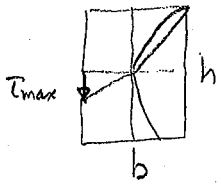
8-28




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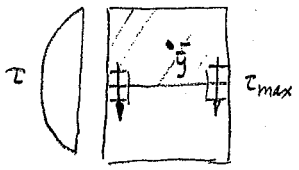



| h/b | 1.00 | 1.50 | 2.00 | 3.00 | ∞ |
|---|---|---|---|---|---|
| α | 0.208 | 0.231 | 0.246 | 0.267 | 0.333 |




$\frac{h}{b} = \frac{100}{50} = 2 \qquad \alpha = 0.246$

$$\tau_{max} = \frac{T}{\alpha h b^2} = \frac{1250\ N\text{-}m}{0.246(0.10)(0.05)^2} = 20.3252\ MPa$$




$$\tau_{max} = \frac{VQ}{It} = \frac{V \frac{bh^2}{8}}{\frac{bh^3}{12} \cdot b}$$

$$Q = \int y\,dA = \bar{y} \cdot A = \frac{h}{4} \cdot \frac{hb}{2} = \frac{h^2 b}{8}$$

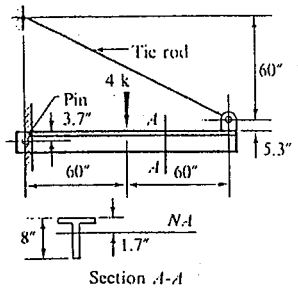
$$\tau_{max} = \frac{3V}{2A} = \frac{3 \cdot 50000N}{2(0.10)(0.05)} = 15\ MPa$$

$$\tau_{max} = 20.33\ MPa + 15\ MPa = 35.33\ MPa$$

$$\sigma_{max} = +\frac{Pl \cdot y}{I_{zz}} = \frac{50000 \cdot (0.25)(0.050)}{4.167 \times 10^{-6}} = 150\ MPa \qquad I_{zz} = \frac{bh^3}{12} = \frac{(0.05)(0.1)^3}{12} = 4.166 \times 10^{-6}\ m^4$$

$\sigma_{min} = 0 \qquad 0 = y$ [redacted]

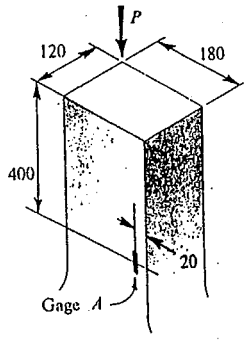
**10-50.** Plot shear, moment, and axial force diagrams for the jib-crane loaded as shown in the figure. Neglect the weight of the beam. *Ans:* 128.6 k-in.




PROB. 10 – 50

8-29

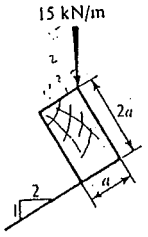
**7-35.** An aluminum-alloy block is loaded as shown in the figure. The application of this load produces a tensile strain of $500 \times 10^{-6}$ mm. per mm at $A$ as measured by means of an electrical strain gage. Compute the magnitude of the applied force $P$. Let $E = 10 \times 10^6$ kN/m². All dimensions given in the figure are in mm.




PROB. 7 – 35

8-19

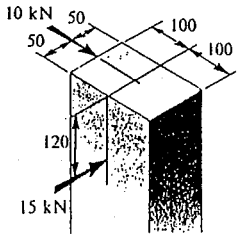
**7-44.** A tilted, simply supported beam with a depth to width ratio of 2 to 1 is to span 4 m and is to carry a uniformly distributed load of 15 kN per linear meter, including its own weight, applied as shown in the figure. (a) Determine the required dimensions of the beam so that the maximum stress due to bending does not exceed 10 MN/m². (b) Locate the neutral axis of the beam and show its position on the sketch.



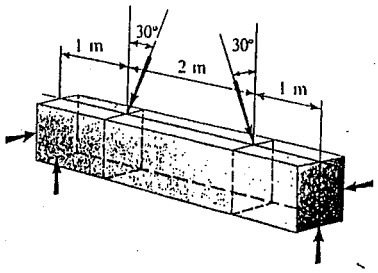

PROB. 7 – 44

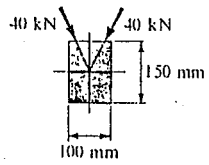
8-23

**7-34.** A cast iron block is loaded as shown in the figure. Neglecting the weight of the block, determine the stresses acting normal to a section taken 0.5 m below the top and locate the line of zero stress. All dimensions given in the figure are in mm. *Ans:* ±24.6 MPa, ±9.6 MPa.




**10-91.** A 4 m long beam is loaded as shown in the figure. The two applied forces act perpendicularly to the long axis of the beam and are inclined 30° with the vertical. If these forces act through the centroid of the cross-sectional area find the location and magnitude of the maximum bending stress. Neglect the weight of the beam. *Ans:* 132 MN/m².

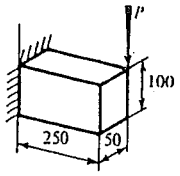



PROB. 10 – 91

8-28

**7-45.** A rectangular cantilever 250 mm long is loaded with $P = 50$ kN at the free end as shown in the figure. Determine the maximum shearing stress at the built-in end due to the direct shear and the torque. Show the result on a sketch analogous to Fig. 7-17(e). All dimensions shown in the figure are in mm. *Ans:* 35.3 MPa.




PROB. 7 – 45

**9-26.** A bent rectangular bar is subjected to an inclined force of 3 000 N as shown in the figure. The cross-section of the bar is 12 mm by 12 mm. (a) Determine the state of stress at point $A$ caused by the applied force and show the results on an element. (b) Find the maximum principal stress. *Ans:* (a) -1·333 MPa, 578 MPa

