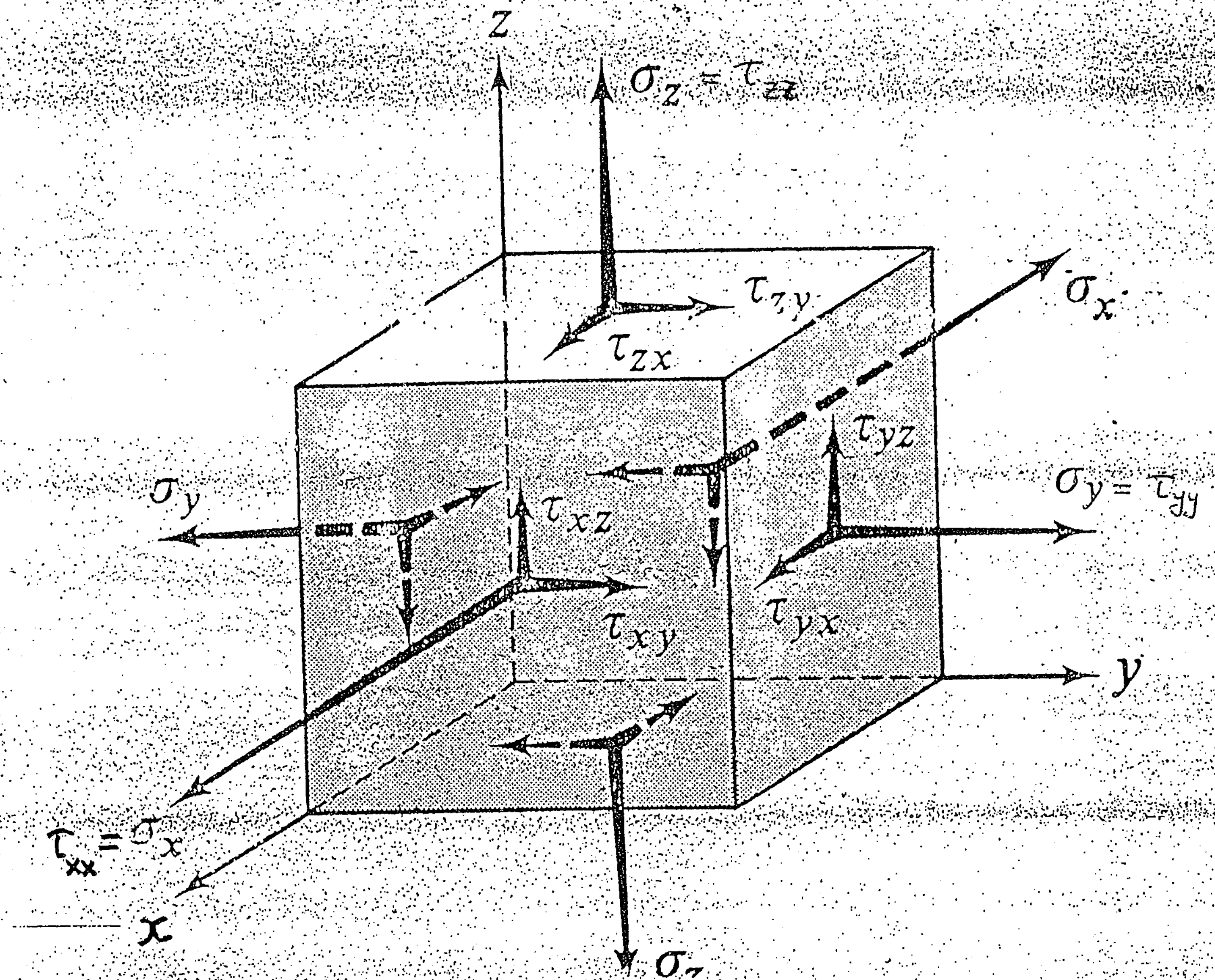



Fig. 1-3. The most general state of stress acting on an element

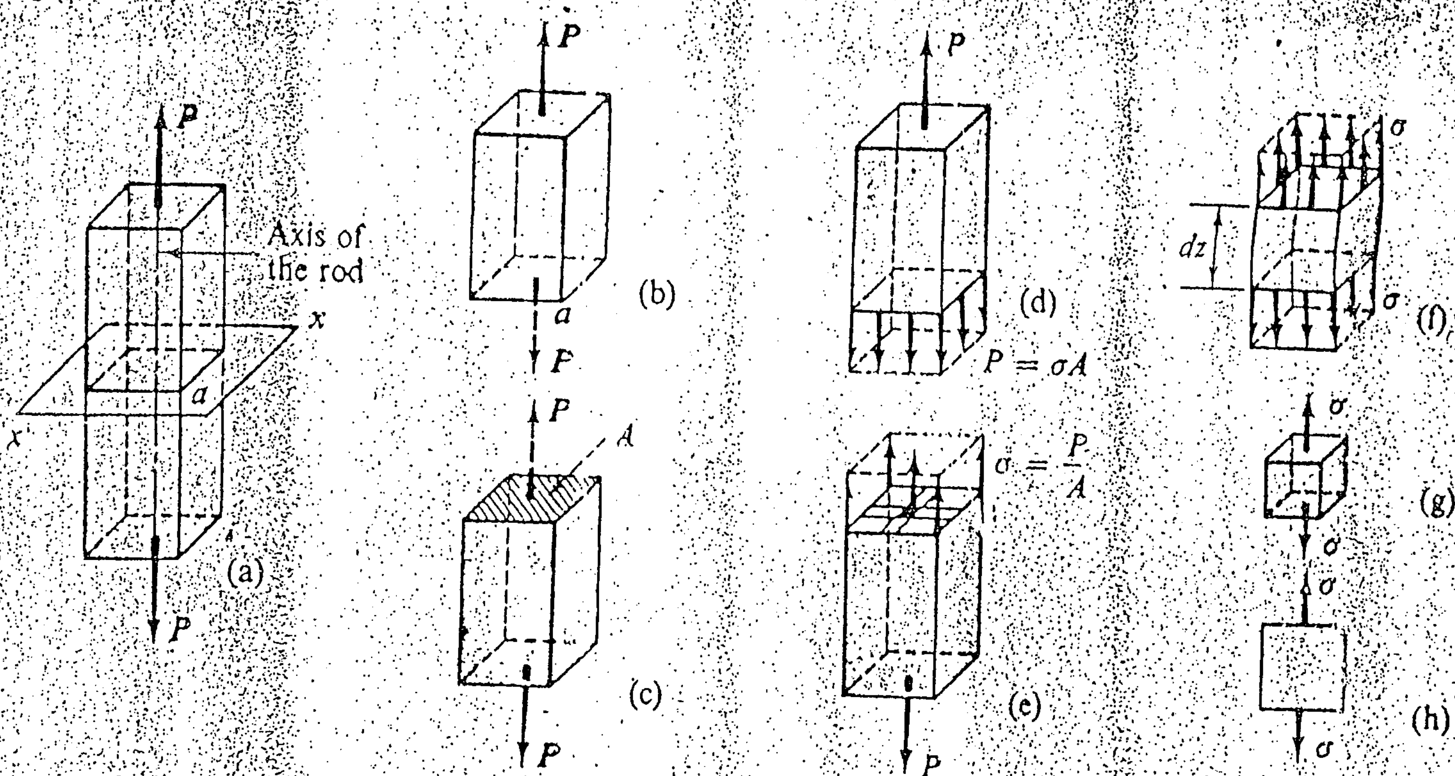



Fig. 1. Successive steps in the analysis of a body for stress

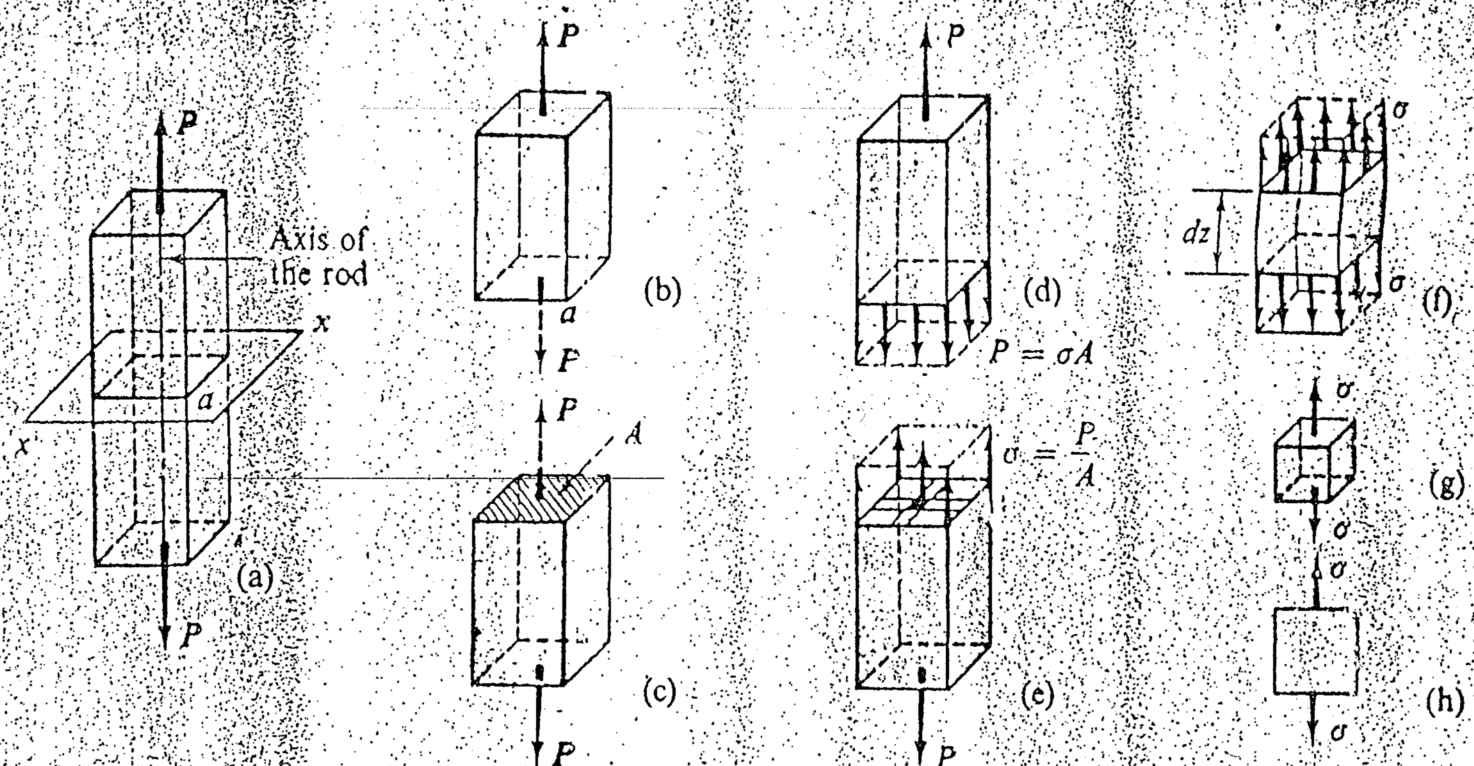



Fig. 1. Successive steps in the analysis of a body for stress

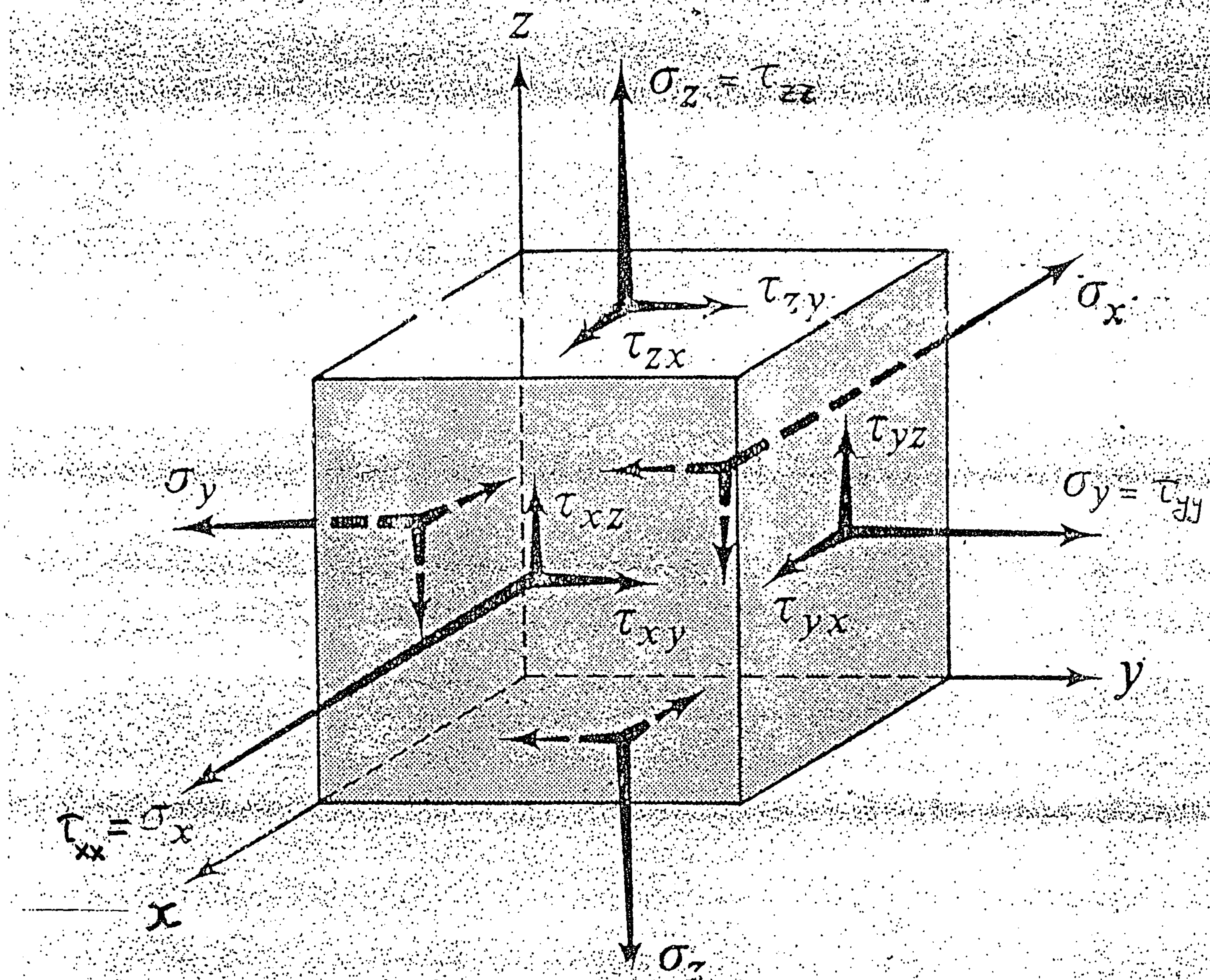



Fig. 1-3. The most general state of stress acting on an element

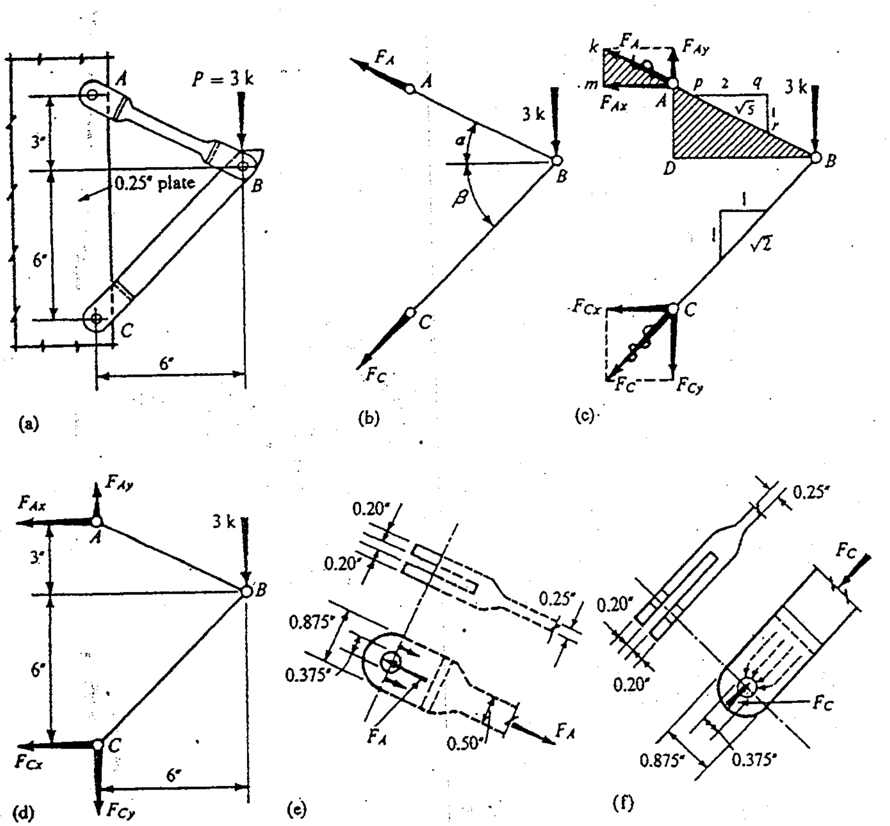



Fig. 3-12

SOLUTION

First an idealized free-body diagram consisting of the two bars pinned at the ends is prepared, Fig. 3-12(b). As there are no intermediate forces acting on the bars and the applied force acts through the joint at $B$, the forces in the bars are directed along the lines $AB$ and $BC$, and the bars $AB$ and $BC$ are loaded axially. The magnitudes of the forces are unknown and are labeled $F_A$ and $F_C$ in the diagram.* These forces may be determined graphically by completing a triangle of forces $F_A$, $F_B$, and $P$. These forces may also be found analytically from two simultaneous

* In frameworks it is convenient to assume all unknown forces are tensile. A negative answer in the solution then indicates that the bar is in compression.





equations $\Sigma F_y = 0$ and $\Sigma F_x = 0$, written in terms of the unknowns $F_A$ and $F_C$, a known force $P$, and two known angles $\alpha$ and $\beta$. Both these procedures are possible. However, in this course usually it will be found advantageous to proceed in a different way. Instead of treating forces $F_A$ and $F_C$ directly, their components are used; and instead of $\Sigma F = 0$, $\Sigma M = 0$ becomes the main tool.

Any force may be resolved into components. For example, $F_A$ may be resolved into $F_{Ax}$ and $F_{Ay}$ as in Fig. 3-12(c). Conversely, if any one of the components of a directed force is known, the force itself may be determined. This follows from similarity of dimension and force triangles. In Fig. 3-12(c) the triangles $Akm$ and $BAD$ are similar triangles (both are shaded in the diagram). Hence, if $F_{Ax}$ is known,

$$F_A = (AB/DB)F_{Ax}$$

Similarly, $F_{Ay} = (AD/DB)F_{Ax}$. However, note further that $AB/DB$ or $AD/DB$ are ratios, hence relative dimensions of members may be used. Such relative dimensions are shown by a little triangle on the member $AB$ and again on $BC$. In the problem at hand

$$F_A = (\sqrt{5}/2)F_{Ax} \quad \text{and} \quad F_{Ay} = F_{Ax}/2$$

Adopting the above procedure of resolving forces, the revised free-body diagram, Fig. 3-12(d), is prepared. Two components of force are necessary at the pin joints. After the forces are determined by statics, Eq. 3-5 is applied several times, thinking in terms of a free body of an individual member:

$$\sum M_C = 0 \circlearrowleft +, \quad +F_{Ax}(3+6) - 3(6) = 0, \qquad F_{Ax} = +2 \text{ kips}$$

$$F_{Ay} = F_{Ax}/2 = 2/2 = 1 \text{ kip},$$

$$F_A = 2(\sqrt{5}/2) = +2.23 \text{ kips}$$

$$\sum M_A = 0 \circlearrowright +, \quad +3(6) + F_{Cx}(9) = 0,$$

$$F_{Cx} = -2 \text{ kips} \quad \text{(compression)}$$

$$F_{Cy} = F_{Cx} = -2 \text{ kips},$$

$$F_C = \sqrt{2}(-2) = -2.83 \text{ kips}$$

*Check:* $\sum F_x = 0, \quad F_{Ax} + F_{Cx} = 2 - 2 = 0$

$\sum F_y = 0, \quad F_{Ay} - F_{Cy} - P = 1 - (-2) - 3 = 0$

Stress in main bar $AB$:

$$\sigma_{AB} = \frac{F_A}{A} = \frac{2.23}{(0.25)(0.50)} = 17.8 \text{ ksi} \quad \text{(tension)}$$



possible.

EXAMPLE 3-3

A bracket of negligible weight shown in Fig. 3-12(a) is loaded with a force $P$ of 3 kips. For interconnection purposes the bar ends are clevised (forked). Pertinent dimensions are shown in the figure. Find the normal stresses in the members $AB$ and $BC$ and the bearing and shearing stresses for the pin $C$. All pins are 0.375 in. in diameter.

Stress in clevis of bar $AB$, Fig. 3-12(e):

$$(\sigma_{AB})_{\text{clevis}} = \frac{F_A}{A_{\text{net}}} = \frac{2.23}{2(0.20)(0.875 - 0.375)} = 11.2 \text{ ksi} \qquad \text{(tension)}$$

Stress in main bar $BC$:

$$\sigma_{BC} = \frac{F_C}{A} = \frac{2.83}{(0.875)(0.25)} = 12.9 \text{ ksi} \qquad \text{(compression)}$$

In the compression member the net section at the clevis need not be investigated; see Fig. 3-12(f) for the transfer of forces. The bearing stress at the pin is more critical. Bearing between pin $C$ and clevis:

$$\sigma_b = \frac{F_C}{A_{\text{bearing}}} = \frac{2.83}{(0.375)(0.20)2} = 18.8 \text{ ksi}$$

Bearing between the pin $C$ and the main plate:

$$\sigma_b = \frac{F_C}{A} = \frac{2.83}{(0.375)(0.25)} = 30.1 \text{ ksi}$$

Double shear in the pin $C$:

$$\tau = \frac{F_C}{A} = \frac{2.83}{2\pi(0.375/2)^2} = 12.9 \text{ ksi*}$$

For a complete analysis of this bracket, other pins should be investigated. However, it may be seen by inspection that the other pins in this case are stressed the same amount as computed above, or less.

The advantages of the method used in the above example for finding forces in members should now be apparent. It can also be applied with success in a problem such as the one shown in Fig. 3-13. The force $F_A$ transmitted by the curved member $AB$ acts through points $A$ and $B$ since the forces applied at $A$ and $B$ must be collinear. By resolving this force at $A'$, the same procedure may be followed. Wavy lines through $F_A$ and $F_C$ indicate that these forces are replaced by the two components shown. Alternatively, the force $F_A$ may be resolved at $A$, and since $F_{Ay} = (x/y)F_{Ax}$, the application of $\Sigma M_C = 0$ yields $F_{Ax}$.

In frames where the applied forces do not act through a joint, proceed as above as far as possible. Then isolate an individual member, and using its free-body diagram, complete the determination of forces.

* Considering the pin in a two-dimensional state of stress $\tau_{xy} = \tau_{yx}$, the tensor representation of the results becomes $\begin{pmatrix} 0 & 12.9 \\ 12.9 & 0 \end{pmatrix}$ ksi



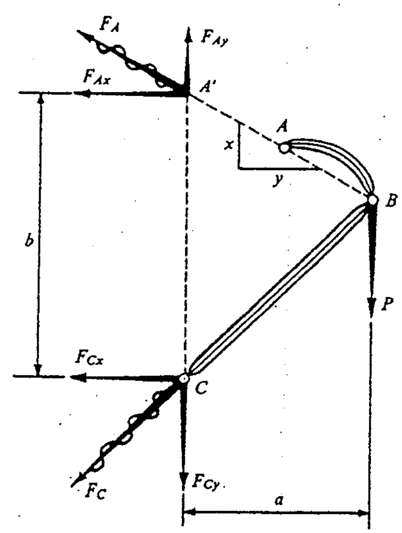



Fig. 3-13

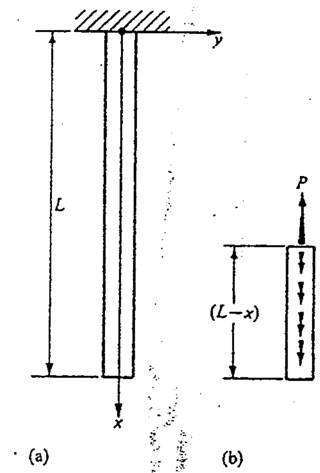



Fig. 3-14

If inclined forces are acting on the structure, resolve them into convenient components.

**EXAMPLE 3-4**

A 1-in.² rod $L$ in. long is suspended vertically as shown in Fig. 3-14(a). The unit weight of the material is $\gamma$. Determine the normal stress in this rod using differential equations of equilibrium.

**SOLUTION**

With the axes shown in the figure, $\tau_{xy} = 0$, and only the first part of Eq. 3-3 has relevance. The body force $X = \gamma$. By virtue of the boundary condition at the free end of the rod $\sigma_x(L) = 0$. On this basis, setting up a differential equation, intergating it, and determining the constant of integration from the boundary conditions, one has

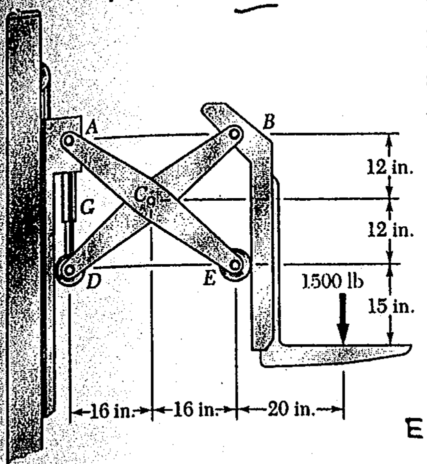
$$\frac{d\sigma_x}{dx} + \gamma = 0 \quad \text{and} \quad \sigma_x = -\gamma x + C_1$$

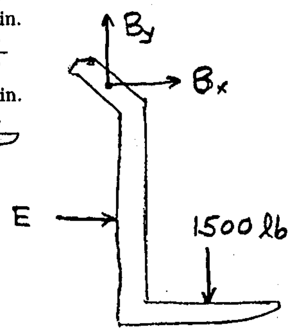
$$\sigma_x(L) = -\gamma L + C_1 = 0 \quad \text{and} \quad \sigma_x = (L - x)\gamma$$

This result can be easily checked by cutting the rod $(L - x)$ above the free end, Fig. 3-14(b), and applying Eq. 3-5. Only very few problems can be analyzed using Eq. 3-3 alone. In more general problems deformations must be considered simultaneously in the analysis.

## Problem 1.26

**1.26** Two identical linkage-and-hydraulic-cylinder systems control the position of the forks of a fork-lift truck. The load supported by the one system shown is 1500 lb. Knowing that the thickness of member $BD$ is $\frac{5}{8}$ in., determine (*a*) the average shearing stress in the $\frac{1}{2}$-in.-diameter pin at $B$, (*b*) the bearing stress at $B$ in member $BD$.




Use one fork as a free body.

$+\circlearrowleft \Sigma M_B = 0:$

$24E - (20)(1500) = 0$

$E = 1250 \text{ lb.} \rightarrow$ ✓

$\xrightarrow{+} \Sigma F_x = 0:$

$E + B_x = 0 \qquad B_x = -E$

$B_x = 1250 \text{ lb} \leftarrow$ ✓

$+\uparrow \Sigma F_y = 0: \quad B_y - 1500 = 0 \qquad B_y = 1500 \text{ lb}$ ✓

$B = \sqrt{B_x^2 + B_y^2} = \sqrt{1250^2 + 1500^2} = 1952.56 \text{ lb.}$ ✓

(a) <u>Shearing stress in pin at B.</u>

$A_{pin} = \frac{\pi}{4} d_{pin}^2 = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$ ✓

$\tau = \frac{B}{A_{pin}} = \frac{1952.56}{0.19635} = 9.94 \times 10^3 \text{ psi}$ ✓ $\qquad \tau = 9.94 \text{ ksi}$ ◀

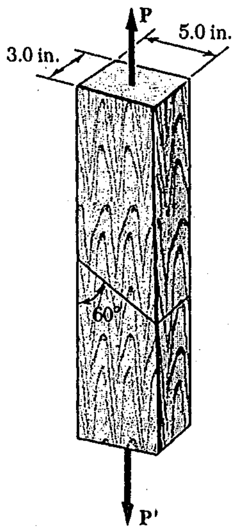
(b) <u>Bearing stress at B.</u>

$\sigma = \frac{B}{dt} = \frac{1952.56}{(\frac{1}{2})(\frac{5}{8})} = 6.25 \times 10^3 \text{ psi}$ ✓ $\qquad \sigma = 6.25 \text{ ksi}$ ◀

$A = \frac{5}{16} \text{ in}^2$ ✓

8/8

## Problem 1.29




**1.29** The 1.4-kip load **P** is supported by two wooden members of uniform cross section that are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

$P = 1400$ lb $\qquad \theta = 90° - 60° = 30°$

$A_o = (5.0)(3.0) = 15 \text{ in}^2$

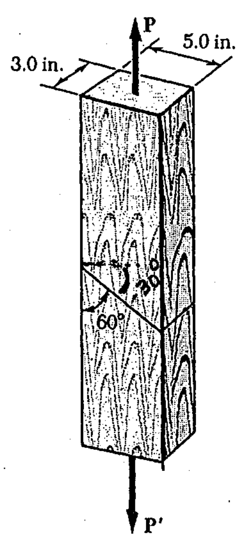
$\sigma = \dfrac{P\cos^2\theta}{A_o} = \dfrac{(1400)(\cos 30°)^2}{15}$

$\sigma = 70.0$ psi

$\tau = \dfrac{P\sin 2\theta}{2A_o} = \dfrac{(1400)\sin 60°}{(2)(15)}$

$\tau = 40.4$ psi

## Problem 1.30




***1.30*** Two wooden members of uniform cross section are joined by the simple scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 60 psi, determine (*a*) the largest load **P** that can be safely supported, (*b*) the corresponding tensile stress in the splice.

$A_o = (5.0)(3.0) = 15 \text{ in}^2$ ✓

$\theta = 90° - 60° = 30°$ ✓

$\tau = \dfrac{P\sin\theta\cos\theta}{A_o}$

✓

(a) $P = \dfrac{\tau A_o}{\sin\theta\cos\theta} = \dfrac{(60)(15)}{\sin 30° \cos 30°} = 2.0785$

✓

$P = 2.08$ kips

(b) $\sigma = \dfrac{P\cos^2\theta}{A_o} = \dfrac{2.0785 \cos^2 30°}{15}$

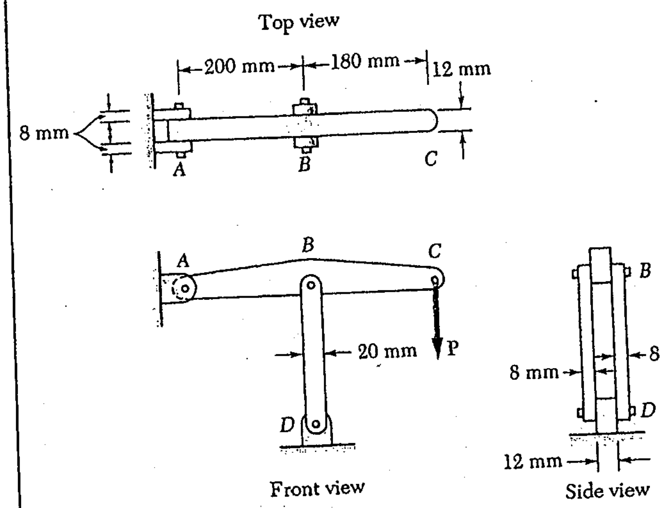
✓

$\sigma = 103.9$ psi

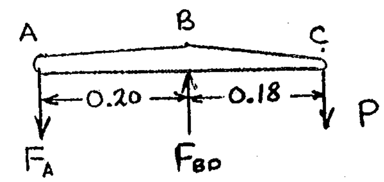
5/5

## Problem 1.55

**1.55** In the structure shown, an 8-mm-diameter pin is used at $A$, and 12-mm-diameter pins are used at $B$ and $D$. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining $B$ and $D$, determine the allowable load $\mathbf{P}$ if an overall factor of safety of 3.0 is desired.




Statics: Use ABC as free body.




$+\circlearrowleft \Sigma M_B = 0: \quad 0.20\,F_A - 0.18\,P = 0$

$P = \frac{10}{9} F_A$ ✓

$+\circlearrowleft \Sigma M_A = 0: \quad 0.20\,F_{BD} - 0.38\,P = 0$

$P = \frac{10}{19} F_{BD}$ ✓

Based on double shear in pin A.

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.008)^2 = 50.266 \times 10^{-6}\ m^2$ ✓

$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times10^6)(50.266\times10^{-6})}{3.0} = 3.351 \times 10^3\ N$ ✓

$\tau_{allow} = \frac{\tau_{ult}}{FS} = \frac{100 MPa}{3} = 33.3 MPa$

$P = \frac{10}{9} F_A = 3.72 \times 10^3\ N$ ✓

Based on double shear in pins at B and D.

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.012)^2 = 113.10 \times 10^{-6}\ m^2$ ✓

$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times10^6)(113.10\times10^{-6})}{3.0} = 7.54 \times 10^3\ N$ ✓

$P = \frac{10}{19} F_{BD} = 3.97 \times 10^3\ N$ ✓

Based on compression in links BD.

For one link $\quad A = (0.020)(0.008) = 160 \times 10^{-6}\ m^2$

$\sigma_{allow} = \frac{\sigma_{ult}}{FS} = \frac{250 MPa}{3} = 83.3 MPa$

$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250\times10^6)(160\times10^{-6})}{3.0} = 26.7 \times 10^3\ N$ ✓

$P = \frac{10}{19} F_{BD} = 14.04 \times 10^3\ N$ ✓

Allowable value of P is smallest. ∴ $\quad P = 3.72 \times 10^3\ N$ ✓

$P = 3.72\ kN$

**2.9** The 36-mm-diameter steel rod $ABC$ and a brass rod $CD$ of the same diameter are joined at point $C$ to form the 7.5-m rod $ABCD$. For the loading shown, and neglecting the weight of the rod, determine the deflection (*a*) of point $C$, (*b*) of point $D$.

Answer: $\delta_C = 2.95\ mm\downarrow$, $\delta_D = 5.29\ mm\downarrow$.

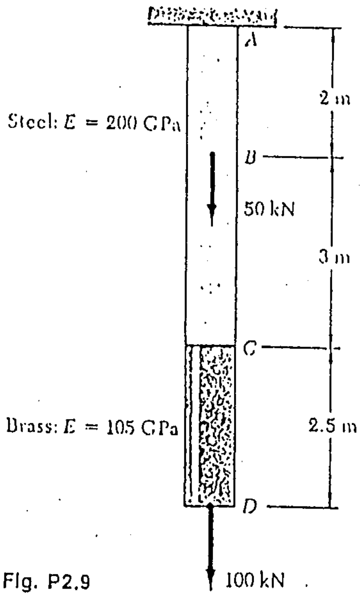



Fig. P2.9

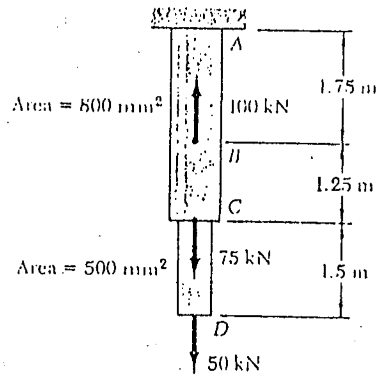



Fig. P2.10

**2.10** The rod $ABCD$ is made of an aluminum alloy for which $E = 70$ GPa. For the loading shown, and neglecting the weight of the rod, determine the deflection (*a*) of point $B$, (*b*) of point $D$.

Answer: $\delta_B = 0.781\ mm\downarrow$, $\delta_D = 5.71\ mm\downarrow$.

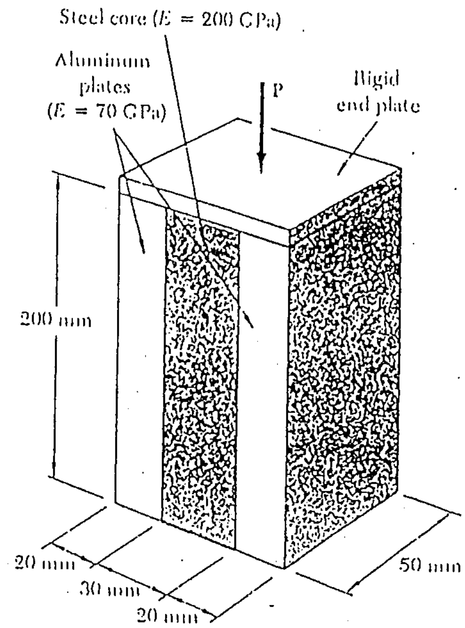



Fig. P2.26

**2.26** An axial centric force of magnitude $P = 385$ kN is applied to the composite block shown by means of a rigid end plate. Determine the normal stress in (*a*) the steel core, (*b*) the aluminum plates.

Answer: $\sigma_S = -175\ MPa$, $\sigma_{Al} = -61.3\ MPa$.

**2.37** The rigid rod $ABC$ is suspended by three identical wires. Knowing that $x = \frac{3}{4}L$, determine the tension in each wire due to the force **P**.

Answer: $T_B = P/3$, $T_C = P/6$, $T_A = P/2$.

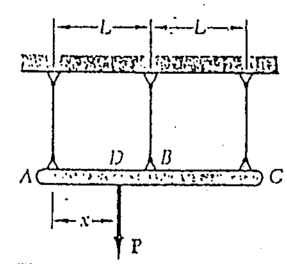



Fig. P2.37 and P2.38

**2.39** The rigid bar $ABCD$ is suspended from four identical wires. Determine the tension in each wire caused by the load P shown.

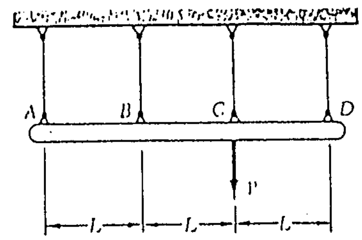
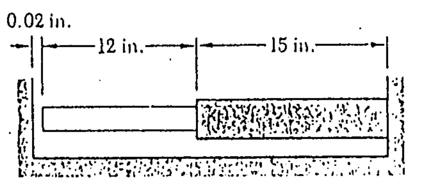


Fig. P2.39

Answer: $T_B = P/5, \quad T_D = 2P/5, \quad T_A = P/10, \quad T_C = 3P/10.$

**2.45** Determine ($a$) the compressive force in the bars shown after a temperature rise of 200°F, ($b$) the corresponding change in length of the aluminum bar.

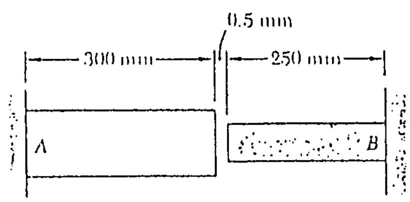
Answer: $P = -52\ kips, \quad \delta_{Al} = 0.0124\ in.$




Bronze
$A = 2.5\ \text{in}^2$
$E = 15 \times 10^6$ psi
$\alpha = 10.1 \times 10^{-6}/°F$

Aluminum
$A = 3\ \text{in}^2$
$E = 10 \times 10^6$ psi
$\alpha = 12.8 \times 10^{-6}/°F$

Fig. P2.45




Aluminum
$A = 2000\ \text{mm}^2$
$E = 70$ GPa
$\alpha = 23 \times 10^{-6}/°C$

Stainless steel
$A = 800\ \text{mm}^2$
$E = 190$ GPa
$\alpha = 18 \times 10^{-6}/°C$

Fig. P2.46 and P2.47

**2.46** At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature reaches 140°C, determine ($a$) the normal stress in the aluminum, ($b$) the exact length of the aluminum rod.

Answer: $\sigma_{Al} = -114.6\ MPa, \quad L = 300.34\ mm.$

**2.47** Knowing that a 0.5-mm gap exists between the rods shown when the temperature is 20°C, determine ($a$) the temperature at which the normal stress in the stainless steel rod will be $\sigma = -150$ MPa, ($b$) the corresponding exact length of the stainless steel rod.

Answer: $T = 103.7°C, \quad L = 250.18\ mm.$

2/2

**2.39** The rigid bar $ABCD$ is suspended from four identical wires. Determine the tension in each wire caused by the load P shown.

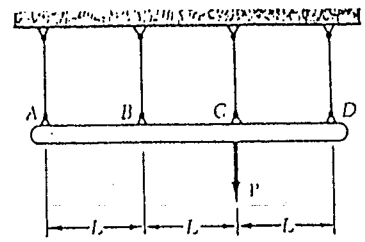
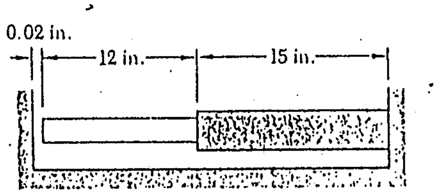


Fig. P2.39

Answer: $T_B = P/5, \quad T_D = 2P/5, \quad T_A = P/10, \quad T_C = 3P/10.$

**2.45** Determine (*a*) the compressive force in the bars shown after a temperature rise of 200°F, (*b*) the corresponding change in length of the aluminum bar.

Answer: $P = -52\ kips, \quad \delta_{Al} = 0.0124\ in.$

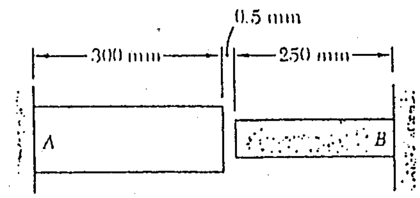



Bronze
$A = 2.5\ in^2$
$E = 15 \times 10^6$ psi
$\alpha = 10.1 \times 10^{-6}/°F$

Aluminum
$A = 3\ in^2$
$E = 10 \times 10^6$ psi
$\alpha = 12.8 \times 10^{-6}/°F$

Fig. P2.45

**2.46** At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature reaches 140°C, determine (*a*) the normal stress in the aluminum, (*b*) the exact length of the aluminum rod.




Aluminum
$A = 2000\ mm^2$
$E = 70$ GPa
$\alpha = 23 \times 10^{-6}/°C$

Stainless steel
$A = 800\ mm^2$
$E = 190$ GPa
$\alpha = 18 \times 10^{-6}/°C$

Fig. P2.46 and P2.47

Answer: $\sigma_{Al} = -114.6\ MPa, \quad L = 300.34\ mm.$

**2.47** Knowing that a 0.5-mm gap exists between the rods shown when the temperature is 20°C, determine (*a*) the temperature at which the normal stress in the stainless steel rod will be $\sigma = -150$ MPa, (*b*) the corresponding exact length of the stainless steel rod.

Answer: $T = 103.7°C, \quad L = 250.18\ mm.$

2/2

**2.9** The 36-mm-diameter steel rod $ABC$ and a brass rod $CD$ of the same diameter are joined at point $C$ to form the 7.5-m rod $ABCD$. For the loading shown, and neglecting the weight of the rod, determine the deflection (*a*) of point $C$, (*b*) of point $D$.

Answer: $\delta_C = 2.95\ mm\downarrow$, $\delta_D = 5.29\ mm\downarrow$.

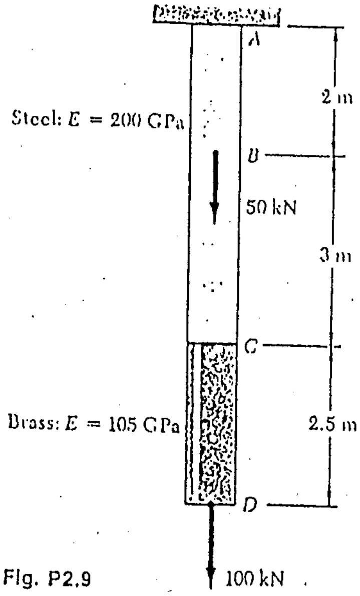



Fig. P2.9

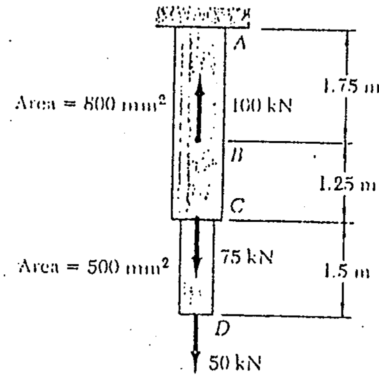



Fig. P2.10

**2.10** The rod $ABCD$ is made of an aluminum alloy for which $E = 70$ GPa. For the loading shown, and neglecting the weight of the rod, determine the deflection (*a*) of point $B$, (*b*) of point $D$.

Answer: $\delta_B = 0.781\ mm\downarrow$, $\delta_D = 5.71\ mm\downarrow$.

**2.26** An axial centric force of magnitude $P = 385$ kN is applied to the composite block shown by means of a rigid end plate. Determine the normal stress in (*a*) the steel core, (*b*) the aluminum plates.

Answer: $\sigma_S = -175\ MPa$, $\sigma_{Al} = -61.3\ MPa$.

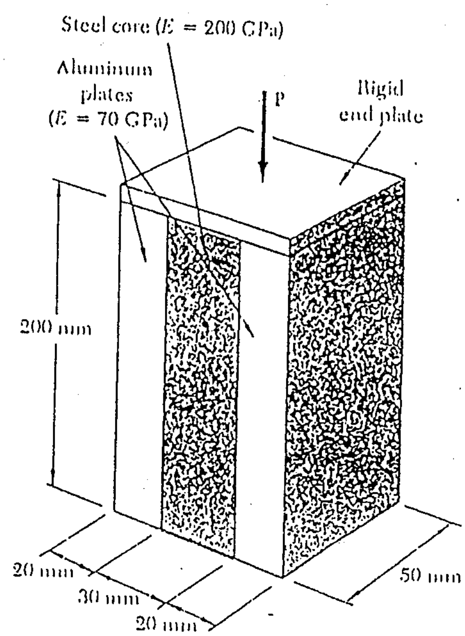



Fig. P2.26

**2.37** The rigid rod $ABC$ is suspended by three identical wires. Knowing that $x = \frac{3}{4}L$, determine the tension in each wire due to the force **P**.

Answer: $T_B = P/3$, $T_C = P/6$, $T_A = P/2$.

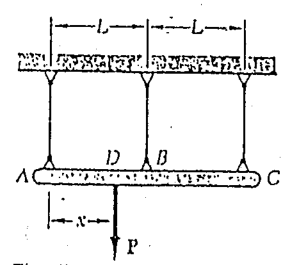
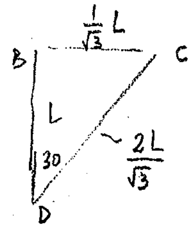



Fig. P2.37 and P2.38

2.17

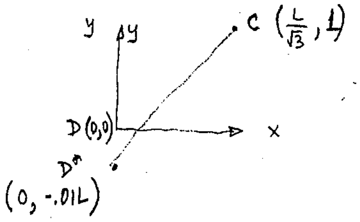


$$\cos 30^\circ = \frac{\overline{BD}}{\overline{CD}} = \frac{L}{\overline{CD}} = .866 = \frac{\sqrt{3}}{2} \Rightarrow \overline{CD} = \frac{2L}{\sqrt{3}}$$

$$\sin 30^\circ = \frac{\overline{BC}}{\overline{CD}} \Rightarrow \overline{BC} = \overline{CD}\sin 30^\circ = \frac{2L}{\sqrt{3}} \cdot \frac{1}{2} = L/\sqrt{3}$$

$$\epsilon_{BD} = \frac{\overline{BD}^* - BD}{BD} = \frac{1.01L - L}{L} = .01$$

6/6



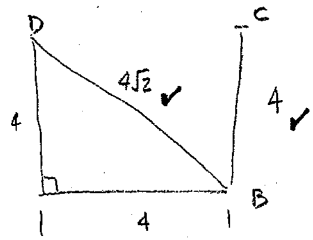
$$\epsilon_{CD} = \frac{\overline{CD}^* - CD}{CD} = \frac{1.163L - 1.155L}{1.155L} = .0075$$

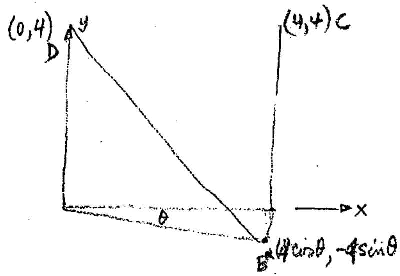
$$CD^* = \frac{L}{\sqrt{3}}\underline{i} + L(1.01)\underline{j} \Rightarrow L\sqrt{\tfrac{1}{3} + 1.02} = 1.163L$$

BY SYMMETRY

$0.0075 = \epsilon_{AD} = \epsilon_{CD}$ מפני שהמבנה סמטרי

2.21





$$\overline{BD}^* = \left[(0 - 4\cos\theta)^2 + (4 + 4\sin\theta)^2\right]^{1/2}$$
$$= \left[16\cos^2\theta + 16 + 32\sin\theta + 16\sin^2\theta\right]^{1/2}$$
$$= \left[32 + 32\sin\theta\right]^{1/2} = 4\sqrt{2}\,(1+\sin\theta)^{1/2}$$

$$\epsilon_{BD} = \frac{\overline{B^*D} - \overline{BD}}{BD} = \frac{4\sqrt{2}(1+\sin\theta)^{1/2} - 4\sqrt{2}}{4\sqrt{2}} = (1+\sin\theta)^{1/2} - 1$$

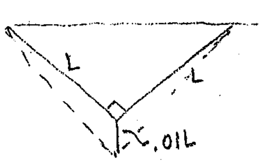
$$\epsilon_{BD} \cong \left(1 + \frac{\sin\theta}{2} + \dots\right) - 1 = \frac{\sin\theta}{2} \approx \theta/2$$

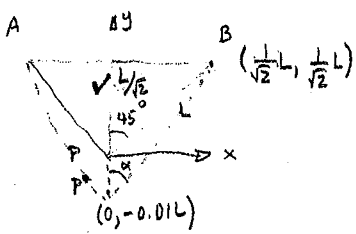
$$\overline{CB}^* = \left[(4\cos\theta - 4)^2 + (-4\sin\theta - 4)^2\right]^{1/2}$$
$$= \left[16\cos^2\theta - 32\cos\theta + 16 + 16\sin^2\theta + 32\sin\theta + 16\right]^{1/2}$$
$$= \left[48 + 32\sin\theta - 32\cos\theta\right]^{1/2} \cong \left[16 + 32\sin\theta\right]^{1/2}$$
$$= 4\left[1 + 2\sin\theta\right]^{1/2}$$

8/8

$$\epsilon_{CB} = \frac{\overline{CB}^* - CB}{CB} = \frac{4(1+2\sin\theta)^{1/2} - 4}{4} = (1+2\sin\theta)^{1/2} - 1,$$
$$= \left(1 + \frac{2\sin\theta}{2} + \dots\right) - 1 = \sin\theta \cong \theta$$

2.32





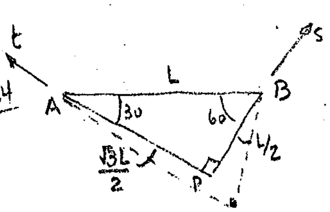
$$\tan\alpha = \frac{\frac{1}{\sqrt{2}}L}{\frac{L}{\sqrt{2}} + .01L} = .986 \Rightarrow \alpha = 44.598^\circ = .7784$$

$45^\circ = .7854$ rad

$$\gamma_{st} = 90^\circ - 2(44.598^\circ) = .8046^\circ = \underline{.0140 \text{ rad.}}$$

3/3

2.34



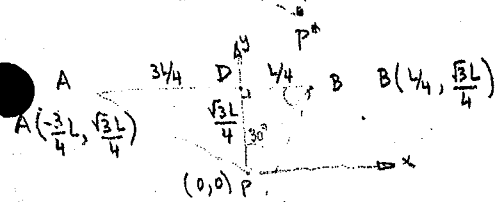
$\epsilon_s = .004$

$\epsilon_t = .003$

ASSUME AB IS OF LENGTH L

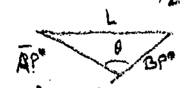
עוד פעם, מניחים שאחד מהדברים שווה ל-L.

$\frac{\sqrt{3}L}{4} = \overline{PD}$, $\frac{\sqrt{3}L}{2} = \overline{AP}$, $L/2 = \overline{BP}$ ⇐ נניח ש- $\overline{AB}$ שווה ל-L



$$\epsilon_s = \frac{\overline{BP}^* - BP}{BP} = \frac{\overline{BP}^* - L/2}{L/2} \Rightarrow \overline{BP}^* = (\epsilon_s + 1)L/2 = (1.004)L/2$$

$$\epsilon_t = \frac{\overline{AP}^* - AP}{AP} = \frac{\overline{AP}^* - \sqrt{3}L/2}{\sqrt{3}L/2} \Rightarrow \overline{AP}^* = (\epsilon_t + 1)\frac{\sqrt{3}L}{2} = (1.003)\frac{\sqrt{3}L}{2}$$



אם הזווית $\theta = \measuredangle AP^*B$ ⇐

כאשר $\theta$ קרובה ל-$90^\circ$, $90-\theta^\circ$ קרובה ל-$0^\circ$. לכן,

$L^2 - \overline{AP^*}^2 - \overline{BP^*}^2$

2.7. A straight piece of wire $PQ$ lies along the line $y = mx$ as shown in Fig. P2.7. The wire is strained and displaced to the line $y = nx$ in such a way that a point originally at $x$ is displaced to $x^2/2$. Show that the extensional strain at any point along the wire is given by

$$\epsilon = \sqrt{\frac{1+n^2}{1+m^2}}\,x - 1$$

(where $x$ is the original coordinate of the point).

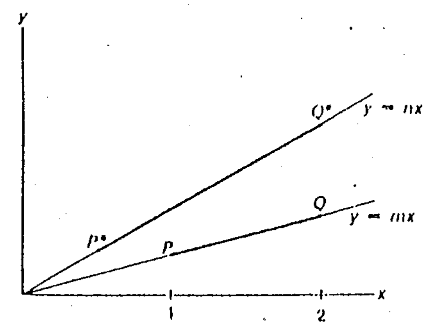



Fig. P2.7

2.16. The point $C$ is displaced to $C^*$ as shown in Fig. P2.16. The horizontal and vertical components of this displacement are $u$ and $v$. Express the average extensional strains of $AC$ and $BC$ in terms of $u$, $v$ and $L$. If $u$ and $v$ are small, what are the approximate expressions for these average extensional strains?

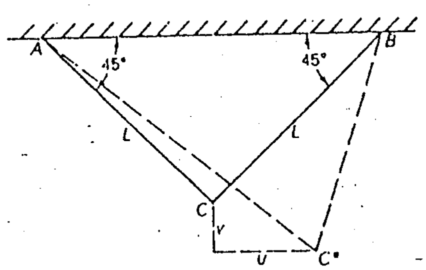



Fig. P2.16

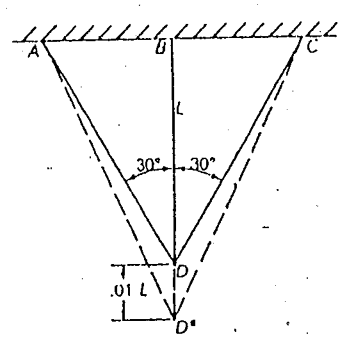



Fig. P2.17

2.17. The point $D$ in Fig. P2.17 is displaced vertically an amount $.01L$. Obtain the approximate (first-order) average extensional strain of $AD$, $BD$, and $CD$.

2.21. The bar $AB$ is rigid and is supported by two wires $DB$ and $CB$ as shown in Fig. P2.21. If the bar rotates through a small angle $\theta$, determine the average extensional strain of $DB$ and $CB$.

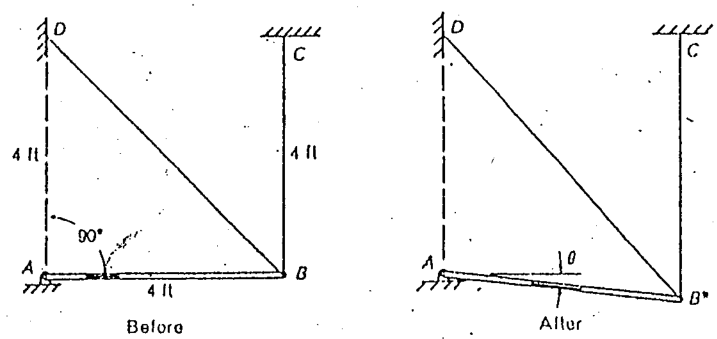



Fig. P2.21

3.5
1/2

2.32. A thin triangular plate is deformed as indicated in Fig. P2.32. Compute approximately the shear strain $\gamma_{st}(P)$.

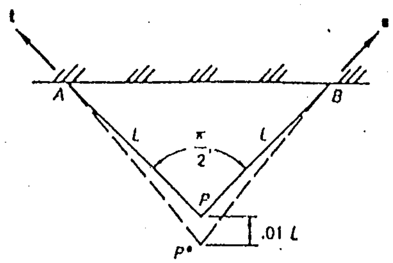


Fig. P2.32

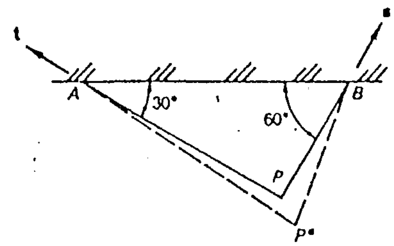


Fig. P2.34

2.34. A thin plate is in the form of a 30°–60°–90° triangle and is supported along its hypotenuse as shown in Fig. P2.34. It is subjected to the uniform strains $\epsilon_s = .004$, $\epsilon_t = .003$. Compute the approximate value of $\gamma_{st}(P)$.

2.37. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.37. It elongates 10 per cent of its original length and contracts 3 per cent of its original width. Compute approximately the shear strain $\gamma_{st}$.

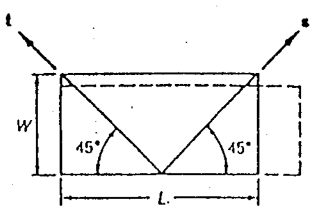


Fig. P2.37

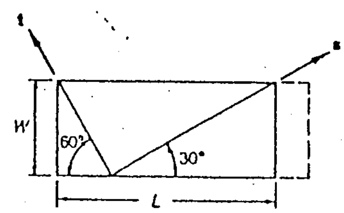


Fig. P2.38

2.38. A thin rectangular plate is deformed uniformly into another rectangle as shown in Fig. P2.38. The extensional strain along its length is .02 and the width remains unchanged. Compute approximately the shear strain $\gamma_{st}$.

**2.56** A homogeneous plate $ABCD$ is subjected to a biaxial loading which results in the normal stresses $\sigma_x = 150$ MPa and $\sigma_z = 100$ MPa. Knowing that the plate is made of steel for which $E = 200$ GPa and $\nu = 0.30$, determine the change in length of (a) edge $AB$, (b) edge $BC$, (c) diagonal $AC$.

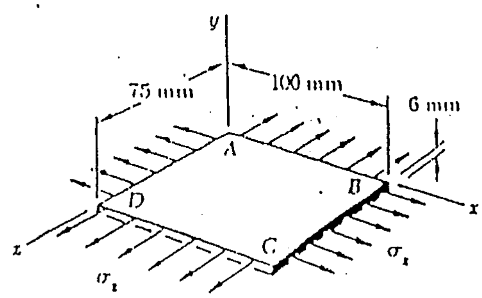


Fig. P2.56 and P2.57

Answer: $(a)\delta_{AB} = 60\mu m$, $(b)\delta_{BC} = 20.6\mu m$, $(c)\delta_{AC} = 60.4\mu m$

$1\mu m = 10^{-6} m$

**2.57** The homogeneous plate $ABCD$ is subjected to a biaxial loading as shown. It is known that $\sigma_z = \sigma_0$ and that the change in length of the plate in the $x$ direction must be zero, that is, $\varepsilon_x = 0$. Denoting by $E$ the modulus of elasticity and by $\nu$ Poisson's ration, determine (a) the required magnitude of $\sigma_x$, (b) the ratio $\sigma_0/\varepsilon_z$.

Answer: $(a)\sigma_x = \nu\sigma_0$, $(b)\frac{\sigma_0}{\varepsilon_z} = \frac{E}{1-\nu^2}$

**4.12.** A steel member ($E = 30 \times 10^6$ psi, $\nu = .3$) is subjected to the stresses

$$\sigma_x = 15{,}000 \text{ psi}, \quad \sigma_y = -5000 \text{ psi}, \quad \sigma_z = 0,$$

$$\tau_{xy} = -8000 \text{ psi}, \quad \tau_{xz} = 0, \quad \tau_{yz} = 0.$$

Determine the principal strains and the principal directions.

Answer: $\varepsilon_1 = 6.66\ 10^{-4}$, $\varepsilon_2 = -4.34\ 10^{-4}$, $\theta = 70.7°,\ 160.7°$

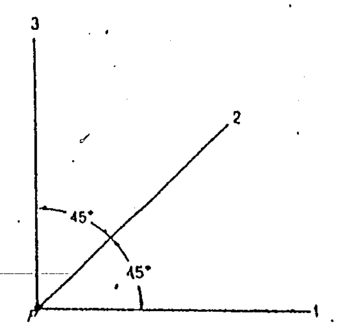
**4.13.** For a steel, $E = 30 \times 10^6$ psi, $\nu = \frac{1}{4}$. Determine the state of stress which corresponds to the following state of strain if the material obeys Hooke's law:

$$\epsilon_x = .001, \quad \epsilon_y = -.005, \quad \epsilon_z = 0,$$

$$\gamma_{xy} = -.0025, \quad \gamma_{yz} = -.0025, \quad \gamma_{xz} = 0.$$

Answer: $\sigma_{xy} = \sigma_{yz} = -2.81\ 10^4$, $\sigma_{xx} = -6.75\ 10^4$, $\sigma_{yy} = -20.25\ 10^4$, $\sigma_{zz} = -9\ 10^4$ psi

**4.17.** A state of plane stress (see Prob. 4.7) exists at the free surface of a body. Strain measurements are taken at a point $P$ on such a free surface by the 45° strain rosette shown in Fig. P4.17. If the material obeys Hooke's law, compute the principal stresses in terms of $\epsilon_1$, $\epsilon_2$, $\epsilon_3$, $E$, and $\nu$.



P 4.17

Answer: $\sigma_{1,2} = \frac{E}{2(1-\nu)}(\varepsilon_1 + \varepsilon_3) \pm \frac{E}{2(1+\nu)}\left[(\varepsilon_1 - \varepsilon_3)^2 + (2\varepsilon_2 - \varepsilon_1 - \varepsilon_3)^2\right]^{1/2}$

**4.18.** Repeat Prob. 4.17 for the 60° strain rosette shown in Fig. P4.18.

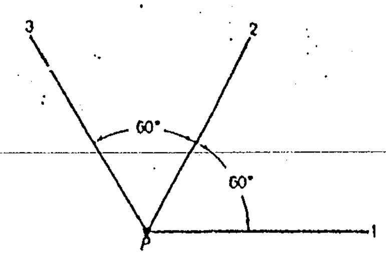


Fig. P4.18

Answer: $\sigma_{1,2} = \frac{E}{3}\left[\frac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{1-\nu} \pm \frac{2}{1+\nu}\sqrt{(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2) - (\varepsilon_1\varepsilon_2 + \varepsilon_1\varepsilon_3 + \varepsilon_2\varepsilon_3)}\right]$

## SOLUTION

In this case $P(x)$ is variable. It is conveniently expressed as $p_o x$ if the origin is taken at $A$. Here again Eq. 4-29 can be applied:

$$u = \int_0^x \frac{P(x)\,dx}{A(x)E} + C_1 = \frac{1}{AE}\int_0^x p_o x\,dx + C_1 = \frac{p_o x^2}{2AE} + C_1$$

At the boundary $B$, where $x = L$, the displacement is zero, i.e., $u(L) = 0$. This condition must be used to evaluate the constant of integration: $C_1 = -p_o L^2/2AE$. Thus $u = -p_o(L^2 - x^2)/2AE$ and $u(0) = -p_o L^2/2AE$. The negative sign indicates that the displacement $u$ is in the opposite direction to that of positive $x$. If $W$ designates the total weight of the rod, the absolute maximum deflection is $WL/2AE$. Compare this expression with Eq. 4-33.

In this problem Eq. 4-32 instead of Eq. 4-29 could be applied. With the gravity load acting downward and with the positive $x$ axis directed upward, the sign of the load in Eq. 4-32 must be negative, i.e., $AE\,d^2u/dx^2 = -(-p_o)$. As in the previous solution, one of the boundary conditions is $u(L) = 0$. The second one is $u'(0) = 0$, where $u' = du/dx$; this follows from the fact that at the free end $P = 0$. (See Eq. 4-30.)

If a concentrated force $P$, in addition to the bar's own weight, were acting on the bar $AB$ at the end $A$, the total end deflection due to the two causes by superposition would be

$$|u| = \frac{PL}{AE} + \frac{WL}{2AE} = \frac{[P + (W/2)]L}{AE}$$

## EXAMPLE 4-10

A 30-in.-long aluminum rod is enclosed within a steel-alloy tube, Figs. 4-29(a) and (b). The two materials are bonded together. If the stress-strain diagrams for the two materials can be idealized as shown, respectively, in Fig. 4-29(d), what end deflection will occur for $P_1 = 80$ kips and for $P_2 = 125$ kips? The cross-sectional areas of steel $A_s$ and of aluminum $A_a$ are the same and equal to 0.5 in.$^2$.

## SOLUTION

By applying the method of sections, one can easily determine the axial force at an arbitrary section, Fig. 4-29(c). However, unlike the case in any problem considered so far, the manner in which the resistance to the force $P$ is distributed between the two materials is not known. Thus, the problem is internally statically indeterminate. The requirements of equilibrium (statics) remain valid, but additional conditions are necessary to solve the problem. One of the auxiliary conditions comes from the requirements of the compatibility of deformations. However, since the requirements of statics involve forces and deformations involve displacements, a connecting condition based on the property of materials must be added.

Let subscripts $a$ and $s$ on $P$, $\varepsilon$, and $\sigma$ identify these quantities as being for aluminum and steel, respectively. Then, noting that the applied force is supported by a force developed in steel and aluminum and that

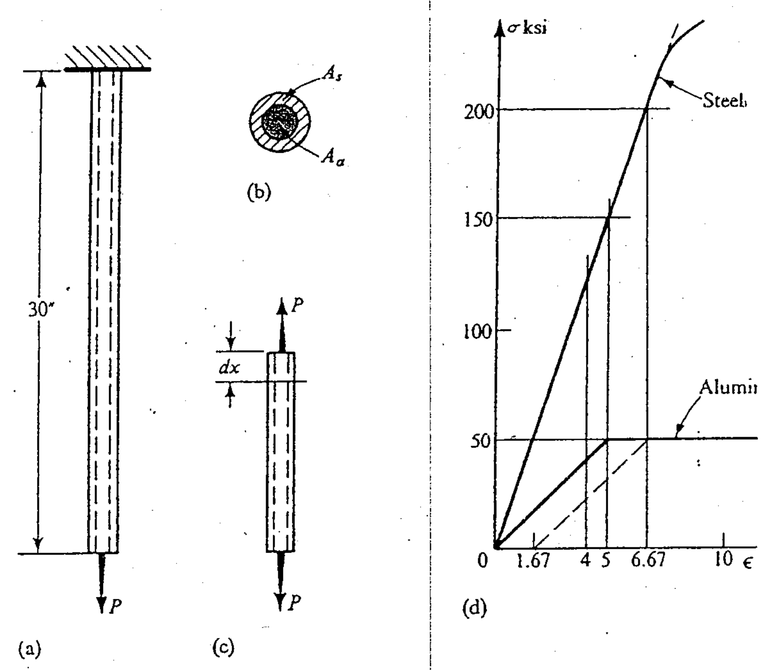



Fig. 4-29

at every section the displacement or the strain of the two materials is the same, and tentatively assuming elastic response of both materials, one has

Equilibrium: $\quad P_a + P_s = P_1 \quad$ or $\quad P_2$

Deformation: $\quad u_a = u_s \quad$ or $\quad \varepsilon_a = \varepsilon_s$

Material properties: $\quad \varepsilon_a = \sigma_a/E_a \quad$ and $\quad \varepsilon_s = \sigma_s/E_s$

By noting that $\sigma_a = P_a/A_a$ and $\sigma_s = P_s/A_s$, one can solve the three equations. From the diagram the elastic moduli are $E_s = 30 \times 10^6$ psi and $E_a = 10 \times 10^6$ psi. Thus

$$\varepsilon_a = \varepsilon_s = \frac{\sigma_a}{E_a} = \frac{\sigma_s}{E_s} = \frac{P_a}{A_a E_a} = \frac{P_s}{A_s E_s}$$

Hence $P_s = [A_s E_s/(A_a E_a)]P_a = 3P_a$, and $P_a + 3P_a = P_1 = 80$ k; therefore, $P_a = 20$ k, and $P_s = 60$ k.

Applying Eq. 4-33 to either material, the tip deflection for 80 kips will be

$$u = \frac{P_s L}{A_s E_s} = \frac{P_a L}{A_a E_a} = \frac{20(10^3)30}{0.5(10)10^6} = 0.120 \text{ in.}$$

This corresponds to a strain of $0.120/30 = 4 \times 10^{-3}$ in. per inch. In this range both materials respond elastically, which satisfies the material-property assumption made at the beginning of this solution. In fact, as may be seen from Fig. 4-29(d), since for the linearly elastic response the strain can reach $5 \times 10^{-3}$ in. per inch for both materials, by direct proportion the applied force $P$ can be as large as 100 kips.

At $P = 100$ kips the stress in aluminum reaches 50 ksi. According to the idealized stress-strain diagram no higher stress can be resisted by this material, although the strains may continue to increase. Therefore, beyond $P = 100$ kips, the aluminum rod can be counted upon to resist only $P_a = A_a\sigma_{yp} = 0.5 \times 50 = 25$ kips. The remainder of the applied load must be carried by the steel tube. For $P_2 = 125$ kips, 100 kips must be carried by the steel tube. Hence $\sigma_s = 100/0.5 = 200$ ksi. At this stress level $\varepsilon_s = 200/(30 \times 10^3) = 6.67 \times 10^{-3}$ in. per inch. Therefore, the tip deflection

$$u = \varepsilon_s L = 6.67 \times 10^{-3} \times 30 = 0.200 \text{ in.}$$

Note that it is not possible to determine $u$ from the strain in aluminum since no unique strain corresponds to the stress of 50 ksi, which is all that the aluminum rod can carry. However, in this case the elastic steel tube contains the plastic flow. Thus, the strains in both materials are the same, i.e., $\varepsilon_s = \varepsilon_a = 6.67 \times 10^{-3}$ in. per inch, see Fig. 4-29(d).

If the applied load $P_2 = 125$ kips were removed, both materials in the rod would rebound elastically. Thus if one imagines the bond between the two materials broken, the steel tube would return to its initial shape. But a permanent set (stretch) of $(6.67 - 5) \times 10^{-3} = 1.67 \times 10^{-3}$ in. per inch would occur in the aluminum rod. This incompatibility of strain cannot develop if the two materials are bonded together. Instead, residual stresses develop, which maintain the same axial deformations in both materials. In this case, the aluminum rod remains slightly compressed, and the steel tube is slightly stretched. The solution of such statically indeterminate problems is considered in greater detail in Chapter 12. The small effect due to Poisson's ratio is neglected in the above discussion.

## 4-18. STRESS CONCENTRATIONS

From the preceding articles of this chapter it is seen that stresses are accompanied by deformations. If such deformations take place at the same uniform rate in adjoining elements, no additional stresses, other than

the cross-sectional area of a member is interrupted or if the force is actually applied over a very small area, a perturbation in stresses takes place because the adjoining elements must be physically continuous in a deformed state. They must stretch or contract equal amounts at the adjoining sides of all particles. These deformations result from linear and shearing deformations involving the properties of materials $E$, $G$, and $\nu$ and the applied forces. Methods of obtaining this disturbed-stress distri-

## Problem 2.5

**2.5** A cast-iron tube is used to support a compressive load. Knowing that $E = 69$ GPa and that the maximum allowable change in length is 0.025 %, determine (*a*) the maximum normal stress in the tube, (*b*) the minimum wall thickness for a load of 7.2 kN if the outside diameter of the tube is 50 mm.

$E = 69\text{ GPa} = 69 \times 10^9\text{ Pa}$

$\varepsilon = \frac{\delta}{L} = \frac{0.00025 L}{L} = 0.00025 \quad (1)$

(a) $\sigma = E\varepsilon = (69 \times 10^9)(0.00025) = 17.25 \times 10^6\text{ Pa} \quad (1) \qquad \sigma = 17.25\text{ MPa}$

(b) $\sigma = \frac{P}{A} \qquad A = \frac{P}{\sigma} = \frac{7.2 \times 10^3}{17.25 \times 10^6} = 417.39 \times 10^{-6}\text{ m}^2 = 417.39\text{ mm}^2 \quad (1)$

$A = \frac{\pi}{4}(d_o^2 - d_i^2) \qquad d_i^2 = d_o^2 - \frac{4A}{\pi} = 50^2 - \frac{(4)(417.39)}{\pi} = 1968.56\text{ mm}^2$

$d_i = 44.368\text{ mm} \quad (1) \qquad t = \frac{1}{2}(d_o - d_i) = \frac{1}{2}(50 - 44.368)$

$(1)$

$t = 2.82\text{ mm}$

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## Problem 2.6

**2.6** A control rod made of yellow brass must not stretch more than 3 mm when the tension in the wire is 4 kN. Knowing that $E = 105$ GPa and that the maximum allowable normal stress is 180 MPa, determine (*a*) the smallest diameter that can be selected for the rod, (*b*) the corresponding maximum length of the rod.

(a) $\sigma = \frac{P}{A} \qquad A = \frac{P}{\sigma} = \frac{4 \times 10^3}{180 \times 10^6} = 22.222 \times 10^{-6}\text{ m}^2$

$A = \frac{\pi}{4}d^2 \qquad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(22.222 \times 10^{-6})}{\pi}} = 5.32 \times 10^{-3}\text{ m}$

$d = 5.32\text{ mm}$

(b) $\delta = \frac{PL}{AE} \qquad L = \frac{AE\delta}{P} = \frac{(22.222 \times 10^{-6})(105 \times 10^9)(3 \times 10^{-3})}{4 \times 10^3}$

$L = 1.750\text{ m}$

## Problem 2.7

**2.7** Two gage marks are placed exactly 10 in. apart on a $\frac{1}{2}$-in.-diameter aluminum rod with $E = 10.1 \times 10^6$ psi and an ultimate strength of 16 ksi. Knowing that the distance between the gage marks is 10.009 in. after a load is applied, determine (*a*) the stress in the rod, (*b*) the factor of safety.

(a) $\delta = 10.009 - 10.000 = 0.009\text{ in.}$

$\varepsilon = \frac{\delta}{L} = \frac{\sigma}{E} \qquad \sigma = \frac{E\delta}{L} = \frac{(10.1 \times 10^6)(0.009)}{10} = 9.09 \times 10^3\text{ psi}$

$\sigma = 9.09\text{ ksi}$

(b) $F.S. = \frac{\sigma_U}{\sigma} = \frac{16}{9.09} \qquad F.S. = 1.760$

## Problem 2.8

**2.8** An 80-m-long wire of 5-mm diameter is made of a steel with $E = 200$ GPa and an ultimate tensile strength of 400 MPa. If a factor of safety of 3.2 is desired, determine (*a*) the largest allowable tension in the wire, (*b*) the corresponding elongation of the wire.

(a) $\sigma_U = 400 \times 10^6$ Pa $\qquad A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(5)^2 = 19.635\ mm^2 = 19.635 \times 10^{-6}\ m^2$

$P_U = \sigma_U A = (400 \times 10^6)(19.635 \times 10^{-6}) = 7854$ N

$P_{all} = \frac{P_U}{F.S.} = \frac{7854}{3.2} = 2454$ N $\qquad P_{all} = 2.45$ kN

(b) $\delta = \frac{PL}{AE} = \frac{(2454)(80)}{(19.635 \times 10^{-6})(200 \times 10^9)} = 50.0 \times 10^{-3}$ m

$\delta = 50.0$ mm

## Problem 2.9

*2.9* A block of 250-mm length and 50 × 40 mm cross section is to support a centric compressive load **P**. The material to be used is a bronze for which $E = 95$ GPa. Determine the largest load which can be applied, knowing that the normal stress must not exceed 80 MPa and that the decrease in length of the block should be at most 0.12% of its original length.

$A = (50)(40) = 2000\ mm^2$
$= 2 \times 10^{-3}\ m^2$

$\sigma_U = 80$ MPa $= 80 \times 10^6$ Pa $\qquad E = 95 \times 10^9$ Pa

Considering allowable stress:

$\sigma = \frac{P}{A} \qquad P = A\sigma = (2 \times 10^{-3})(80 \times 10^6) = 160 \times 10^3$ N

Considering allowable deformation:

$\delta = \frac{PL}{AE} \qquad P = AE\left(\frac{\delta}{L}\right) = (2 \times 10^{-3})(95 \times 10^9)(0.0012) = 228 \times 10^3$ N

The smaller value governs. $\quad P = 160 \times 10^3$ N $\qquad P = 160.0$ kN

## Problem 2.10

**2.10** A 1.5-m-long aluminum rod must not stretch more than 1 mm and the normal stress must not exceed 40 MPa when the rod is subjected to a 3-kN axial load. Knowing that $E = 70$ GPa, determine the required diameter of the rod.

$L = 1.5$ m

$\delta = 1 \times 10^{-3}$ m, $\quad \sigma = 40 \times 10^6$ Pa, $\quad E = 70 \times 10^9$ Pa, $\quad P = 3 \times 10^3$ N

Stress: $\sigma = \frac{P}{A} \qquad A = \frac{P}{\sigma} = \frac{3 \times 10^3}{40 \times 10^6} = 75 \times 10^{-6}\ m^2 = 75\ mm^2$

Deformation: $\delta = \frac{PL}{AE}$

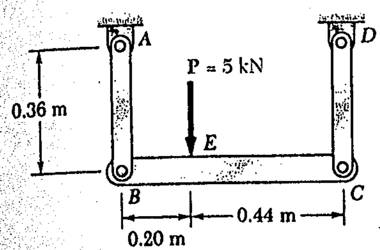
$A = \frac{PL}{E\delta} = \frac{(3 \times 10^3)(1.5)}{(70 \times 10^9)(1 \times 10^{-3})} = 64.29 \times 10^{-6}\ m^2 = 64.29\ mm^2$

Larger value of A governs. $\qquad A = 75\ mm^2$

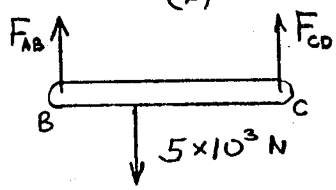
$A = \frac{\pi}{4} d^2 \qquad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(75)}{\pi}} \qquad d = 9.77$ mm

## Problem 2.27

**2.27** Each of the links $AB$ and $CD$ is made of aluminum ($E = 75$ GPa) and has a cross-sectional area of $125\ \text{mm}^2$. Knowing that they support the rigid member $BC$, determine the deflection of point $E$.




(2)




Use member BC as a free body.

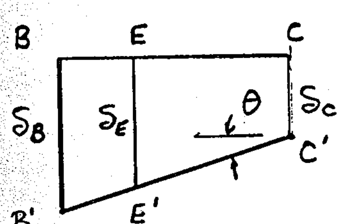
$\circlearrowleft \Sigma M_C = 0: \quad -(0.64)F_{AB} + (0.44)(5\times10^3) = 0 \qquad F_{AB} = 3.4375\times10^3\ \text{N}$ (2)

$\circlearrowleft \Sigma M_B = 0: \quad (0.64)F_{CD} - (0.20)(5\times10^3) = 0 \qquad F_{CD} = 1.5625\times10^3\ \text{N}$ (2)

For links AB and CD, $\quad A = 125\ \text{mm}^2 = 125\times10^{-6}\ \text{m}^2$

$$\delta_{AB} = \frac{F_{AB}L_{AB}}{EA} = \frac{(3.4375\times10^3)(0.36)}{(75\times10^9)(125\times10^{-6})} = 132.00\times10^{-6}\ \text{m} = \delta_B \quad (2)$$

$$\delta_{CD} = \frac{F_{CD}L_{CD}}{EA} = \frac{(1.5625\times10^3)(0.36)}{(75\times10^9)(125\times10^{-6})} = 60.00\times10^{-6}\ \text{m} = \delta_C \quad (2)$$




Deformation diagram.

$$\text{Slope } \theta = \frac{\delta_B - \delta_C}{l_{BC}} = \frac{72.00\times10^{-6}}{0.64} = 112.5\times10^{-6}\ \text{rad}$$

$$\delta_E = \delta_C + l_{EC}\,\theta$$

$$= 60.00\times10^{-6} + (0.44)(112.5\times10^{-6})$$

$$= 109.5\times10^{-6}\ \text{m} \qquad \delta_E = 0.1095\ \text{mm} \downarrow$$

$$\frac{\Delta u_b - \Delta u_e}{L_{BE}} = \frac{\Delta u_e - \Delta u_c}{L_{EC}} = \text{slope} =$$

(2)

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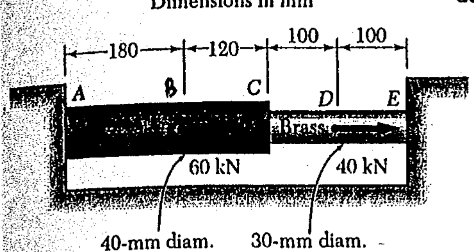
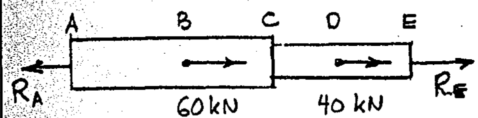
## Problem 2.41




**2.41** Two cylindrical rods, one of steel and the other of brass, are joined at $C$ and restrained by rigid supports at $A$ and $E$. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at $A$ and $E$, (*b*) the deflection of point $C$.

$$\Sigma F_x = R_E - R_A + 60 + 40 = 0$$

$$\Sigma \Delta u = 0$$




A to C: $E = 200 \times 10^9$ Pa

$A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3 \text{ mm}^2 = 1.25664 \times 10^{-3} \text{ m}^2$

$EA = 251.327 \times 10^6$ N

C to E: $E = 105 \times 10^9$ Pa

$A = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$

$EA = 74.220 \times 10^6$ N

A to B: $P = R_A$ $\quad L = 180 \text{ mm} = 0.180$ m

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{251.327 \times 10^6} = 716.20 \times 10^{-12} R_A \qquad (2)$$

B to C: $P = R_A - 60 \times 10^3$ $\quad L = 120 \text{ mm} = 0.120$ m

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{251.327 \times 10^6} = 447.47 \times 10^{-12} R_A - 26.848 \times 10^{-6} \qquad (2)$$

C to D: $P = R_A - 60 \times 10^3$ $\quad L = 100 \text{ mm} = 0.100$ m

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.100)}{74.220 \times 10^6} = 1.34735 \times 10^{-9} R_A - 80.841 \times 10^{-6} \qquad (2)$$

D to E: $P = R_A - 100 \times 10^3$ $\quad L = 100 \text{ mm} = 0.100$ m

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100 \times 10^3)(0.100)}{74.220 \times 10^6} = 1.34735 \times 10^{-9} R_A - 134.735 \times 10^{-6} \qquad (2)$$

A to E: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 3.85837 \times 10^{-9} R_A - 242.424 \times 10^{-6} = 0$

Since point $E$ cannot move relative to A, $\quad \delta_{AE} = 0$

(a) $3.85837 \times 10^{-9} R_A - 242.424 \times 10^{-6} = 0 \qquad R_A = 62.831 \times 10^3$ N $\qquad$ 62.8 kN ← (1)

$R_E = R_A - 100 \times 10^3 = 62.8 \times 10^3 - 100 \times 10^3 = -37.2 \times 10^3$ N $\qquad$ 37.2 kN ← (1)

(b) $\delta_C = \delta_{AB} + \delta_{BC} = 1.16367 \times 10^{-9} R_A - 26.848 \times 10^{-6}$

$= (1.16369 \times 10^{-9})(62.831 \times 10^3) - 26.848 \times 10^{-6}$

$= 46.3 \times 10^{-6}$ m $\quad (2) \qquad \delta_c = 46.3\ \mu$m →

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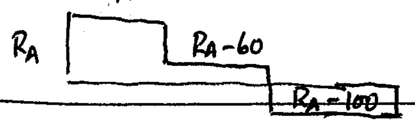
Loading diag

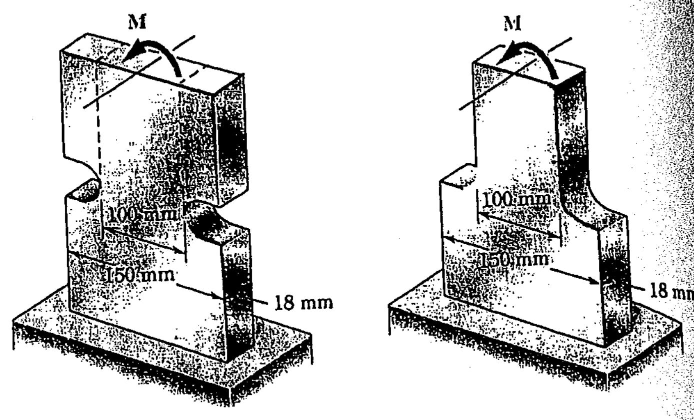



$$\Delta u = \frac{P_{AB} L_{AB}}{A_{AC} E_s} + \frac{P_{BC} L_{BC}}{A_{AC} E_s} + \frac{P_{CD} L_{CD}}{A_{CD} E_b} + \frac{P_{DE} L_{DE}}{A_{DE} E_b} = 0$$

$$\frac{R_A \times 10^3 \cdot 180 \times 10^{-3}}{A_{AC} E_s} + \frac{(R_A - 60) \times 10^3 \cdot 120 \times 10^{-3}}{A_{AC} E_s} + \frac{(R_A - 60) \times 10^3 \cdot 100 \times 10^{-3}}{A_{CD} \cdot E_b} + \frac{(R_A - 100) \times 10^3 \cdot 100 \times 10^{-3}}{A_{DE} E_b} = 0$$

## Problem 4.65

*4.65* A couple of moment $M = 2$ kN · m is to be applied to the end of a steel bar. Determine the maximum stress in the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = 10$ mm, as shown in Fig. 4.65*a*, (*b*) if the bar is redesigned by removing the material above the grooves as shown in Fig. 4.65*b*.




(*a*) (*b*)

For both configurations

$D = 150$ mm, $d = 100$ mm

$r = 10$ mm.

$$\frac{D}{d} = \frac{150}{100} = 1.50$$

$$\frac{r}{d} = \frac{10}{100} = 0.10$$

For configuration (a),
Fig 4.32 gives $K_a = 2.21$.

For configuration (b), Fig. 4.31 gives $K_b = 1.79$.

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2} d = 50 \text{ mm} = 0.05 \text{ m}$$

(a) $$\sigma = \frac{KMc}{I} = \frac{(2.21)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 147 \times 10^6 \text{ Pa} = 147 \text{ MPa}$$

(b) $$\sigma = \frac{KMc}{I} = \frac{(1.79)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 119 \times 10^6 \text{ Pa} = 119 \text{ MPa}$$

6

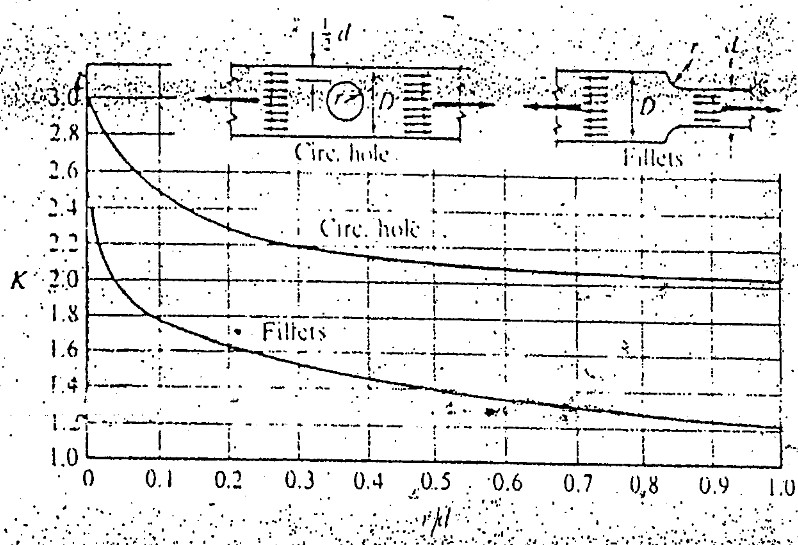



Fig. 2-17. Stress-concentration factors for flat bars in tension

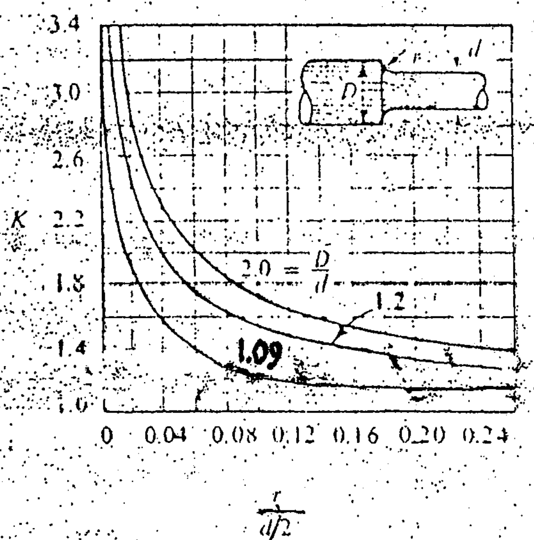



Fig. 3-15. Torsional stress-concentration factors in circular shafts of two diameters.

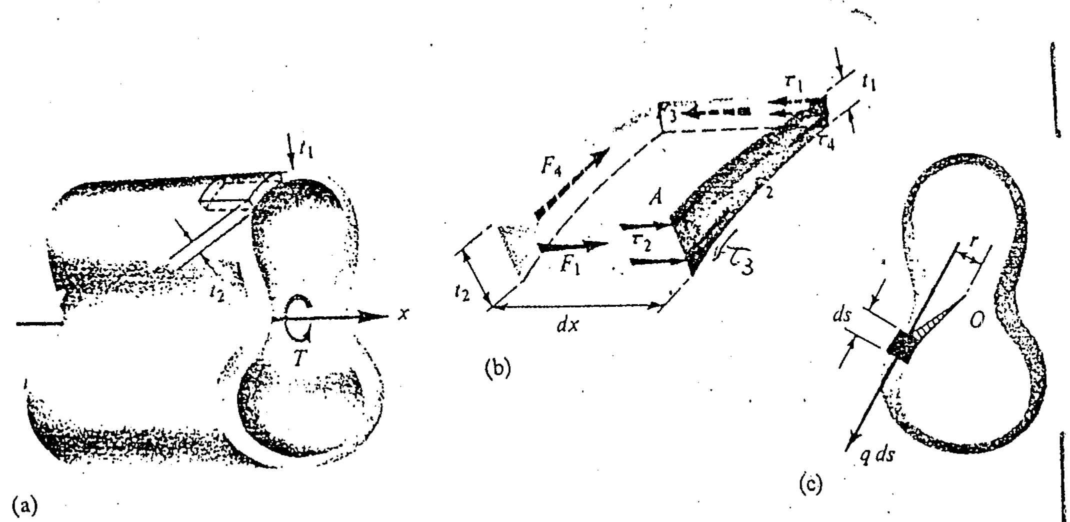
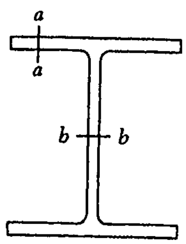



Fig. 3-22. Thin-walled member of variable thickness.

HW 7

## Problem 3.137 ×

**3.137** An 8-ft-long steel member with a W8 × 31 cross section is subjected to a kip · in. torque. The properties of the rolled-steel section are given in Appendix. Knowing that $G = 11.2 \times 10^6$ psi, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the maximum shearing stress along line *b-b*, (*c*) the angle of twist. (*Hint*: consider the web and flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)



W8 × 31

Flange: $a = 7.995$ in, $b = 0.435$, $\frac{a}{b} = \frac{7.995}{0.435} = 18.38$

$$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.3219 \qquad \varphi_F = \frac{T_F L}{c_2 a b^3 G}$$

$$T_F = c_2 a b^3 \frac{G\varphi_F}{L} = K_F \frac{G\varphi}{L} \quad \text{where } K_F = c_2 a b^3$$

$$K_F = (0.3219)(7.995)(0.435)^3 = 0.2138 \text{ in}^3$$

Web: $a = 8.0 - (2)(0.435) = 7.13$ in, $b = 0.285$ in, $\frac{a}{b} = \frac{7.13}{0.285} = 25.02$

$$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.3249 \qquad \varphi_W = \frac{T_W L}{C_2 a b^3 G}$$

$$T_W = c_2 a b^3 \frac{G\varphi_W}{L} = K_W \frac{G\varphi}{L} \quad \text{where } K_W = c_2 a b^3$$

$$K_W = (0.3249)(7.13)(0.285)^3 = 0.0563 \text{ in}^4$$

For matching twist angles $\quad \varphi_F = \varphi_W = \varphi$

Total torque. $\quad T = 2T_F + T_W = (2K_F + K_W)\frac{G\varphi}{L}$

$$\frac{G\varphi}{L} = \frac{T}{2K_F + K_W}, \quad T_F = \frac{K_F T}{2K_F + K_W}, \quad T_W = \frac{K_W T}{2K_W + K_W}$$

$$T_F = \frac{(0.2138)(5000)}{(2)(0.2138) + 0.0563} = 2221 \text{ lb·in}; \quad T_W = \frac{(0.0563)(5000)}{(2)(0.2138) + 0.0563} = 557 \text{ lb·in}$$

(a) $$\tau_F = \frac{T_F}{c_1 a b^2} = \frac{2221}{(0.3219)(7.995)(0.435)^2} = 4570 \text{ psi} = 4.57 \text{ ksi}$$

(b) $$\tau_W = \frac{T_W}{c_1 a b^2} = \frac{557}{(0.3249)(7.13)(0.285)^2} = 2960 \text{ psi} = 2.96 \text{ ksi}$$

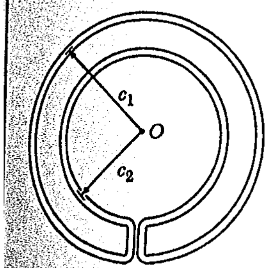
(c) $$\frac{G\varphi}{L} = \frac{T}{2K_F + K_W} \quad \therefore \varphi = \frac{TL}{G(2K_F + K_W)} \quad \text{where } L = 8 \text{ ft} = 96 \text{ in.}$$

$$\varphi = \frac{(5000)(96)}{(11.2 \times 10^6)[(2)(0.2138) + 0.563]} = 88.6 \times 10^{-3} \text{ rad} = 5.08°$$

3.137 $\quad$ V

3-148 $\quad$ 3

## Problem 3.148




**3.148** A cooling tube having the cross section shown is formed from a sheet of stainless steel of 3-mm thickness. The radii $c_1 = 150$ mm and $c_2 = 100$ mm are measured to the center line of the sheet metal. Knowing that a torque of magnitude $T = 3$ kN · m is applied to the tube, determine (*a*) the maximum shearing stress in the tube, (*b*) the magnitude of the torque carried by the outer circular shell. Neglect the dimension of the small opening where the outer and inner shells are connected.

Area bounded by centerline,

$$a = \pi(c_1^2 - c_2^2) = \pi(150^2 - 100^2) = 39.27\times10^3 \text{ mm}^2$$
$$= 39.27\times10^{-3} \text{ m}^2$$

$$t = 0.003 \text{ m}$$

(a) $$\tau = \frac{T}{2ta} = \frac{3\times10^3}{(2)(0.003)(39.27\times10^{-3})} = 12.73\times10^6 \text{ Pa}$$ $\tau = 12.76$ MPa

(b) $$T_1 = (2\pi c_1 t\,\tau\, c_1) = 2\pi c_1^2 t\,\tau$$
$$= 2\pi(0.150)^2(0.003)(12.73\times10^6) = 5.40\times10^3 \text{ N·m}$$
$T_1 = 5.40$ kN·m

3

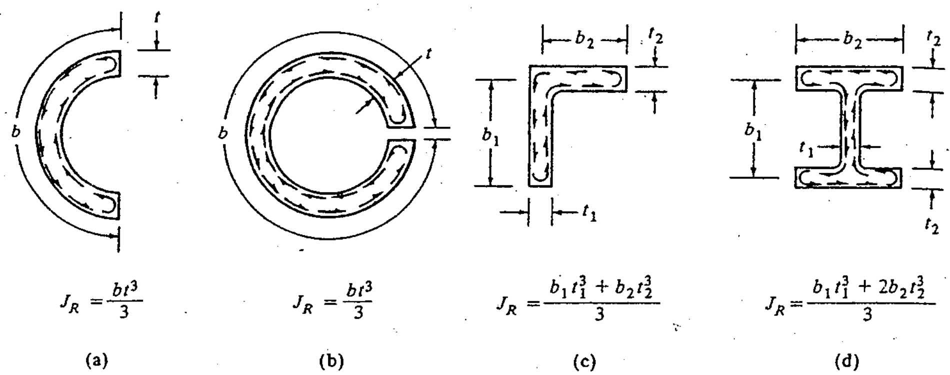



**FIGURE 8.4.3.** Thin-walled open sections, showing their shear stresses and torsional constants $J_R$. In (a) and (b), dimension $b$ is the length of the midline of the cross section.

when a bar is bent to form a helical spring, some corrections to these formulas may be necessary [8.3].

The membrane analogy permits a very useful generalization of the foregoing results to other thin-walled open cross sections. Imagine that the narrow rectangular cross section of Fig. 8.4.2 is distorted into a C or an L shape, or attached to another narrow rectangle to make a T or an I section. By visualizing the inflated membranes for these shapes, we decide that $\phi$ surfaces of all of them remain parabolic (excepting near ends and near reentrant corners, which we discuss later). The total torque is the sum of torques carried by each part of the cross section. Since $G\beta$ is the same for each part, Eqs. 8.4.5 and 8.4.8 still apply but with the $bt^3/3$ contributions summed to yield

$$J_R = \frac{1}{3}\sum_{i=1}^{n} b_i t_i^3 \tag{8.4.10}$$

where $n$ is the number of parts into which the cross section is divided for purposes of calculation. Examples of this calculation appear in Fig. 8.4.3. In angle and I sections, the part of greater thickness displays greater shear stress. Where there is taper, as in the cross section of a turbine blade or in flanges of a rolled section, we can use

$$J_R = \frac{1}{3}\int t^3\, ds \tag{8.4.11}$$

where $ds$ is an increment of length along the medial line of the cross section. More exact formulas are available [8.3]. These formulas, experimental results [8.5], and coefficient $C_\beta$ in Table 8.4.1 suggest that, for standard rolled structural shapes, $J_R$ may actually be some 10 percent higher than predicted by Eqs. 8.4.10 and 8.4.11. Omission of this adjustment yields a conservative design under either a stress limit or a deflection limit.

## 8.5 A HYDRODYNAMIC ANALOGY

There are several hydrodynamic analogies to the torsion problem [8.1]. In this section we outline one, without proof, and use it to draw useful conclusions about shear stresses in twisted bars.

If a fluid without viscosity executes motion in the $yz$ plane with constant vorticity, its motion is described by the equation

$$\frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = -2\omega \tag{8.5.1}$$

where $\phi$ is the stream function and $\omega$ is the (constant) vorticity. Fluid velocities $v$ and $w$ in $y$ and $z$ directions, respectively, are

$$v = \frac{\partial\phi}{\partial z} \qquad w = -\frac{\partial\phi}{\partial y} \tag{8.5.2}$$

Clearly, Eqs. 8.5.1 and 8.5.2 are analogous to Eqs. 8.3.5 and 8.3.1, respectively. Thus fluid velocities are proportional to shear stresses. Lines $\phi$ = constant are streamlines. The fluid boundary must be a streamline, therefore the boundary condition $d\phi = 0$ of Eq. 8.3.6 is met. Thus the analogy is complete.

Experiments have been done in the following way. Imagine that a square cross section is to be studied. A shallow square tank is painted black and placed on a turntable. A camera, looking down onto the tank, is also attached to the turntable. The tank is filled with water (whose viscosity is low, if not quite zero) and aluminum powder is sprinkled on the water. When all is quiet, the camera shutter is opened while the turntable is rotated about 10°. From the viewpoint of the photograph, the tank is stationary while the fluid rotates with nearly constant vorticity. The aluminum particles show as streaks on the film. Each streak is in the direction of a shear stress. The length of the streak is proportional to the magnitude of the shear stress.

Experiments aside, the hydrodynamic analogy has other uses. One is in aiding visualization of torsion problems. Another is that known solutions for fluid flow can be applied to the torsion problem. For example, consider an elliptical obstacle (Fig. 8.5.1a). Far from the obstacle the flow is uniform and horizontal, so $w = 0$ and $v = v_o$, a constant. Points $A$ are stagnation points, where $v = w = 0$. Fluid theory shows that at points $B$, where $z = \pm b$, fluid velocities are $v = v_o(1 + b/a)$ and $w = 0$. Applying these results to a twisted bar, Fig. 8.5.1b, we conclude that stress is zero at the sharp external corners $C$. At the root of the small elliptical notch, shear stress is approximately

$$\tau = \tau_o\left(1 + \frac{b}{a}\right) \tag{8.5.3}$$

where $\tau_o$ is the stress that would prevail near the boundary if the notch were not there. The calculation is approximate because the undisturbed stress is not the uniform value $\tau_o$ unless the notch is very small. The quantity in parentheses

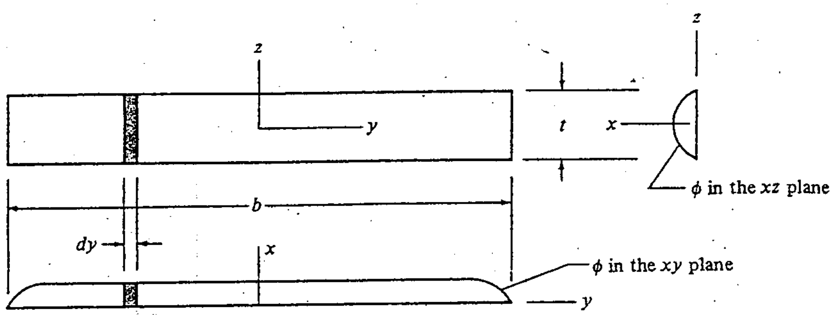



**FIGURE 8.4.2.** Narrow rectangular cross section ($b >> t$), with views of the stress function (or inflated membrane).

where $C$ is a constant. Substituting into Eq. 8.3.5, we find

$$-2C = -2G\beta \qquad \text{so} \qquad C = G\beta \tag{8.4.4}$$

The torque is

$$T = 2\int_{\text{area}} \phi \, dA = 2\int_{-t/2}^{t/2} \phi b \, dz = G\beta \frac{bt^3}{3} \tag{8.4.5}$$

The maximum shear stress, found along the edges $z = \pm t/2$, is

$$\tau = \left| \frac{\partial \phi}{\partial z} \right|_{z=\pm t/2} = 2C\frac{t}{2} = G\beta t \tag{8.4.6}$$

Substituting for $\beta$ from Eq. 8.4.5, we find

$$\tau = \frac{Tt}{bt^3/3} = \frac{3T}{bt^2} \qquad \text{at} \quad z = \pm\frac{t}{2} \tag{8.4.7}$$

Equations 8.4.5 and 8.4.7 can be written in the forms

$$\frac{d\theta}{dx} = \beta = \frac{T}{GJ_R} \qquad \tau = \frac{Tt}{J_R} \tag{8.4.8}$$

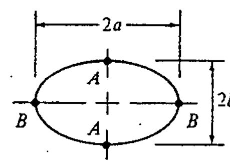
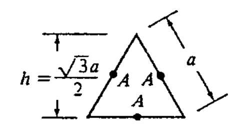
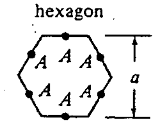
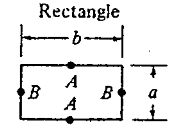
where

$$J_R = \frac{bt^3}{3} \qquad (\text{for } b >> t) \tag{8.4.9}$$

These expressions for $\beta$ and $\tau$ are similar in form to the corresponding expressions for a circular cross section, Eqs. 8.1.2 and 8.1.3. However, $J_R$ is *emphatically NOT* the polar moment of the cross-sectional area about the centroidal $x$ axis.

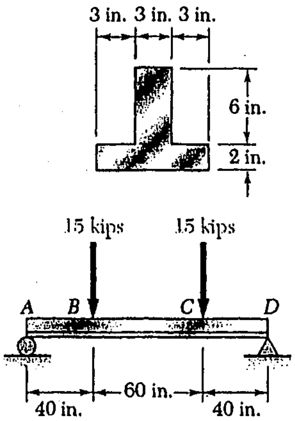
If (say) $b/t = 10$, Eqs. 8.4.8 give $\beta$ and $\tau$ values that are approximately 6.5 percent low. Accuracy improves as $b/t$ increases. Aspect ratios in the range $1 < b/t < 10$ can be analyzed with tabulated data obtained by other analytical or numerical methods (Table 8.4.1). If the centerline of the bar is curved, as

**TABLE 8.4.1** Expressions for Maximum Shear Stress and Rate of Twist in Selected Solid Sections [8.1, 8.4]

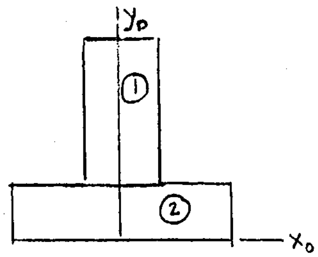
| Cross Section and Area | Maximum Shear Stress | Rate of Twist |
|---|---|---|
| Ellipse  Area = $\pi ab$ | $\tau_A = \frac{2T}{\pi ab^2}$ $(a > b)$ ($\tau_{max}$ at $B$ if $b > a$) | $\beta = \frac{a^2 + b^2}{\pi a^3 b^3}\frac{T}{G} = \frac{d\theta}{dx}$ |
| Equilateral triangle  Area = $0.433\,a^2$ | $\tau_A = \frac{20T}{a^3}$ | $\beta = \frac{46.2}{a^4}\frac{T}{G} = \frac{d\theta}{dx}$ |
| Regular hexagon  Area = $0.866a^2$ | $\tau_A = \frac{5.7T}{a^3}$ | $\beta = \frac{8.8}{a^4}\frac{T}{G} = \frac{d\theta}{dx}$ |
| Rectangle  Area = $ab$ | $\tau_A = \frac{T}{C_\tau ba^2}$ | $\beta = \frac{1}{C_\beta ba^3}\frac{T}{G} = \frac{d\theta}{dx}$ |

| $b/a$ | $C_\tau$ | $\tau_B/\tau_A$ | $C_\beta$ |
|---|---|---|---|
| 1.0 | 0.208 | 1.000 | 0.1406 |
| 1.2 | 0.219 | 0.935 | 0.166 |
| 1.5 | 0.231 | 0.859 | 0.196 |
| 2.0 | 0.246 | 0.795 | 0.229 |
| 2.5 | 0.258 | 0.766 | 0.249 |
| 3.0 | 0.267 | 0.753 | 0.263 |
| 4.0 | 0.282 | 0.745 | 0.281 |
| 6.0 | 0.299 | 0.743 | 0.299 |
| 10.0 | 0.312 | 0.742 | 0.312 |
| $\infty$ | 0.333 | 0.742 | 0.333 |

## Problem 4.7



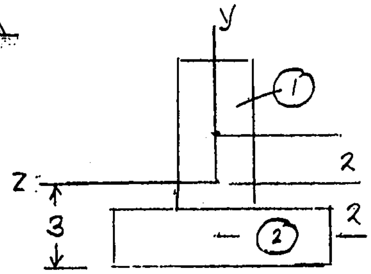

**4.7 through 4.9** Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.




| | $A$ | $\bar{y}_o$ | $A\bar{y}_o$ |
|---|---|---|---|
| 1 | 18 | 5 | 90 |
| 2 | 18 | 1 | 18 |
| $\Sigma$ | 36 | | 108 |

$$\bar{Y}_o = \frac{108}{36} = 3 \text{ in.}$$

Neutral axis lies 3 in. above the base.

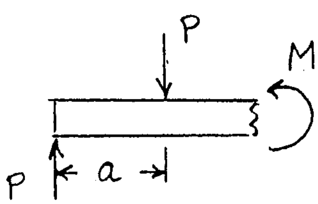



$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(3)(6)^3 + (18)(2)^2 = 126 \text{ in}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(9)(2)^3 + (18)(2)^2 = 78 \text{ in}^4$$

$$I = I_1 + I_2 = 126 + 78 = 204 \text{ in}^4$$

$y_{top} = 5$ in. $\quad y_{bot} = -3$ in.




$$M - Pa = 0$$

$$M = Pa = (15)(40) = 600 \text{ kip}\cdot\text{in.}$$

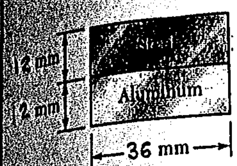
$$\sigma_{top} = -\frac{My_{top}}{I} = -\frac{(600)(5)}{204}$$

$\sigma_{top} = -14.71$ ksi (compression)

$$\sigma_{bot} = -\frac{My_{bot}}{I} = -\frac{(600)(-3)}{204}$$

$\sigma_{bot} = 8.82$ ksi (tension)

Problem 4.39




**4.39 and 4.40** A steel bar ($E_s = 210$ GPa) and an aluminum bar ($E_a = 70$ GPa) are bonded together to form the composite bar shown. Determine the maximum stress in (*a*) the aluminum, (b) the steel, when the bar is bent about a horizontal axis, with $M = 200$ N · m.

Use aluminum as the reference material.

For aluminum $n = 1$

For steel $n = E_s/E_a = 210/70 = 3$

Transformed section

| | ① n=3 |
|---|---|
| | ② n=1 |

| | $A, mm^2$ | $nA, mm^2$ | $\bar{y}_o, mm$ | $nA\bar{y}_o, mm^3$ |
|---|---|---|---|---|
| ① | 432 | 1296 | 18 | 23328 |
| ② | 432 | 432 | 6 | 2592 |
| Σ | | 1728 | | 25920 |

$\bar{Y}_o = \frac{25920}{1728} = 15$ mm  The neutral axis lies 15 mm above the bottom.

$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{3}{12}(36)(12)^3 + (1296)(3)^2 = 27.216 \times 10^3 \text{ mm}^4$

$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12}(36)(12)^3 + (432)(9)^2 = 40.176 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 = 67.392 \times 10^3 \text{ mm}^4 = 67.392 \times 10^{-9} \text{ m}^4$

$M = 200$ N·m

$\sigma = -\frac{nMy}{I}$

(a) Aluminum: $n = 1$, $y = -15 \text{ mm} = -0.015$ m

$\sigma_a = -\frac{(1)(200)(-0.015)}{67.392 \times 10^{-9}} = 44.516 \times 10^6$ Pa  $\sigma_a = 44.5$ MPa ◄

(b) Steel: $n = 3$, $y = 9 \text{ mm} = 0.009$ m

$\sigma_s = -\frac{(3)(200)(0.009)}{67.392 \times 10^{-9}} = -80.128 \times 10^6$ Pa  $\sigma_s = -80.1$ MPa ◄

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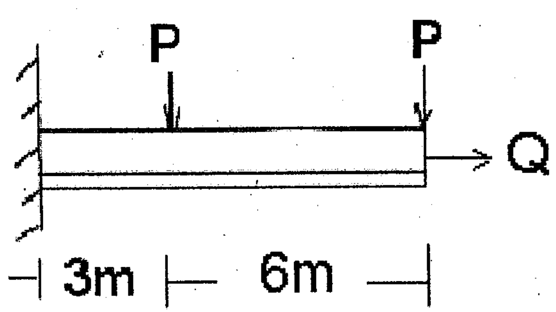
PRINT NAME SIGN NAME

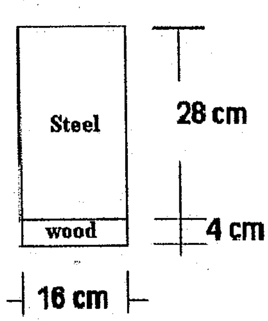
Problem 1.

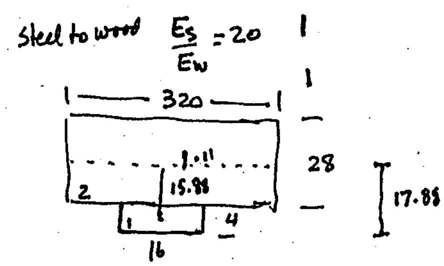
a) Given the following beam loaded as shown, find the location of the maximum moment, given P=10 KN. Take Q=0 KN initially. 2

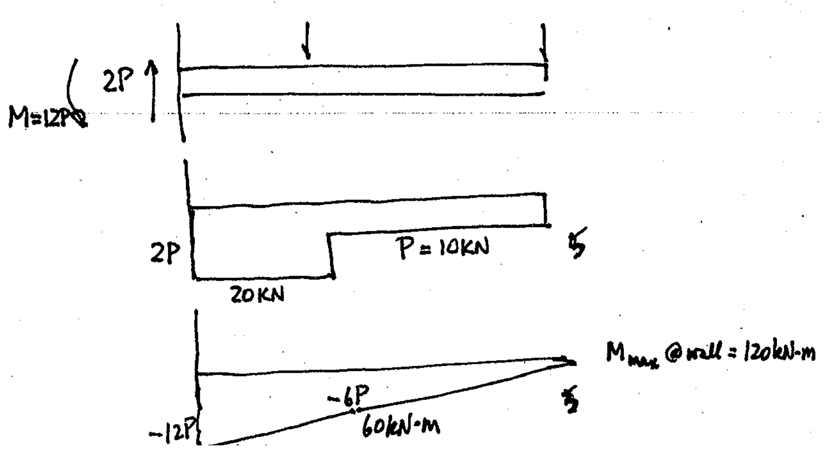
b) What is the maximum stress $\sigma_x$ and where can it be found, given the following information: $E_{steel} = 206$ GPa, $E_{wood} = 10.3$ GPa, for the cross-section below.

c) Now let Q=10 KN so that it acts at the centroid of the cross-section of the equivalent beam. Where is the neutral axis now? What is the maximum stress $\sigma_x$ and where can it be found?




$M_{max}$ @ wall = 120kN-m

|  | $\bar{y}$ | A | $\bar{y}A$ |
|---|---|---|---|
| 1 | 2 | 64 | 128 |
| 2 | 18 | 8960 | 161280 |
|  |  | 9024 | 161408 |

$\bar{y} = 17.89$ cm

9

|  | $I_z$ | A | d | $I_z'$ |
|---|---|---|---|---|
| 1 | $\frac{16 \cdot 4^3}{12} = 85.33$ | 64 | 15.89 | 16244.83 |
| 2 | $\frac{320 \cdot 28^3}{12} = 585386.67$ | 8960 | .11 | 585495.08 |
|  |  |  |  | 601739.91 cm$^4$ = .60174X10$^{-2}$ m$^4$ |

8

in wood $\frac{(120 \times 10^3 \text{ N-m})(.1789\text{ m})}{.60174 \times 10^{-2}} = 3.56$ MPa 4

in steel $\frac{(120 \times 10^3 \text{ N-m})(.1411\text{ m})}{}$ = 56.[illegible] MPa 5

$$\frac{Q}{A} + \frac{M_z y}{I_z} = 0$$

5

$$y = -\frac{Q}{A}\frac{I_z}{M_z} = -\frac{10000}{9024 \times 10^{-4}} \cdot \frac{.60174 \times 10^{-2}}{120000}$$

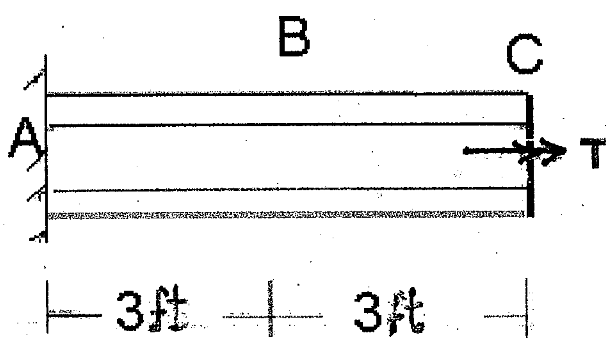
$-.000556$ m or $-.556$ mm

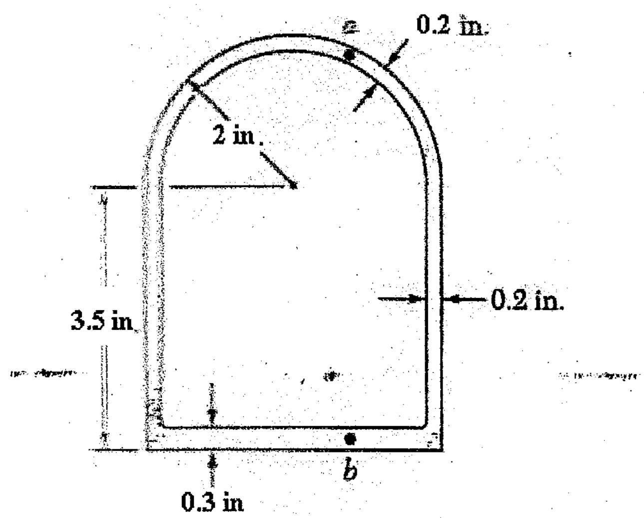
Problem 2.

A beam is loaded by means of a torque, T, equal to 20000 ft-lb at end C. The cross-section of the beam is given in the figure on the right.

a) What is the maximum shear stress in the cross-section and where can it be found? The beam is made of aluminum with $E_{aluminum} = 10 \times 10^6$ psi and the Poisson ratio, $\nu = 0.3$.

b) What is the angle of twist at the cross-section located at B.




torque = const 20000 ft-lb

$$\tau = \frac{T}{2Q_{line}t} = \frac{20000 \text{ ft-lb} \times 12 \text{in/ft}}{2 \cdot 18.4 \text{in}^2 \cdot 0.2 \text{in}}$$

$= 326$ ksi @ A 4

$$G = \frac{E}{2(1+\nu)} = 3.846 \times 10^6 \text{psi}$$ 2

$$\varphi = \frac{TL}{4Q_{line}^2 G} \oint \frac{ds}{t}$$

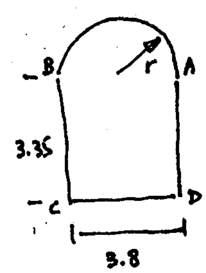
$$= \frac{20000\,(12)\,(36)}{4\,(18.4)^2\,(3.846 \times 10^6 \frac{lb}{in^2})} \quad 76.012$$

$= .00166 \times 76.012 = .1261$ rad

$= 7.225°$ 7

$$\oint \frac{ds}{t} = \int_A^B \frac{ds}{.2} + \int_B^C \frac{ds}{.2} + \int_C^D \frac{ds}{.3} + \int_D^A \frac{ds}{.2}$$

$$= \frac{\pi r}{.2} + \frac{3.35}{.2} + \frac{3.8}{.3} + \frac{3.35}{.2} = \frac{12.669}{.2} + \frac{3.8}{.3} = 76.012$$ 9

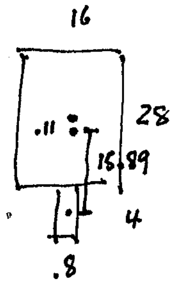



$r = 2 - .1 = 1.9$

$A = \pi r^2/2 = 5.67$

$A = 3.35 \times 3.8 = 12.73$

$18.4 \text{ in}^2$ .3




| | I | A | d | |
|---|---|---|---|---|
| 1 | $\frac{1}{12}(.8)(4)^3$ | 3.2 | 15.89 | 812.24 |
| 2 | $\frac{1}{12}(16)(28)^3$ | 448 | .11 | 29274.75 |
| | | | | 30086.99 |

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PRINT NAME SIGN NAME

Problem 1.

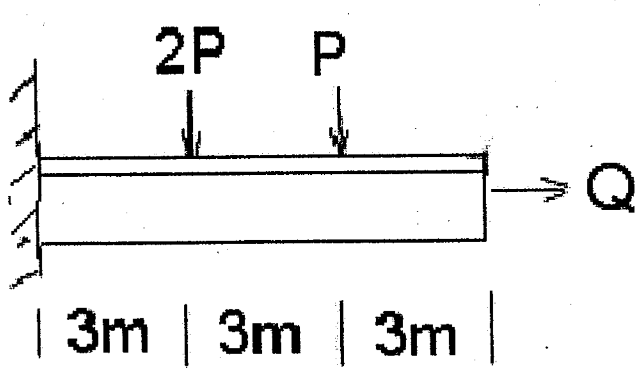
a) Given the following composite beam loaded as shown, find the location of the maximum moment, given P=10 KN. Take Q=0 KN initially. $M_{max}$ at wall = 120kN-m

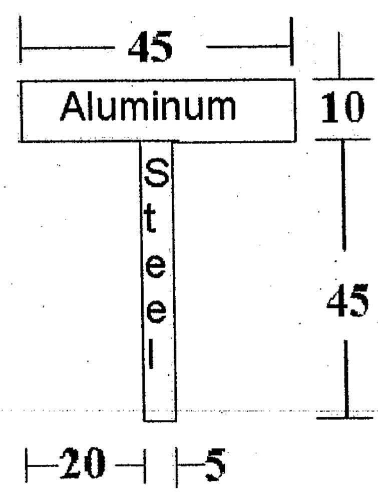
b) What is the maximum stress $\sigma_x$ and where can it be found, given the following information: $E_{steel}$ = 206 GPa, $E_{aluminum}$ = 72 GPa, for the cross-section below.

c) Now let Q=10 KN so that it acts at the centroid of the cross-section of the equivalent beam. Where is the neutral axis now? What is the maximum stress $\sigma_x$ and where can it be found?

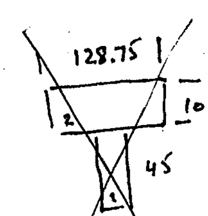
conv steel to al

$n = \frac{E_{steel}}{E_{al}} = \frac{206}{72} = 2.861$



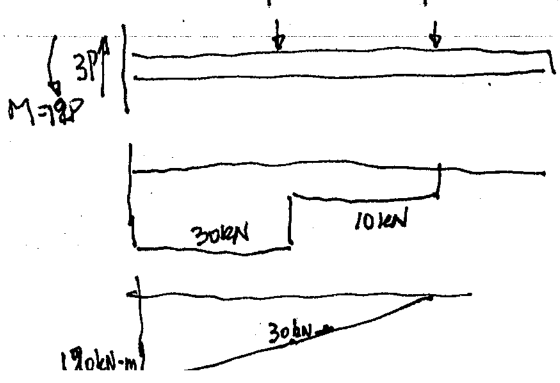

Units of length for the cross section are in cm.




| | $\bar{y}$ | A | $\bar{y}A$ |
|---|---|---|---|
| 1 | 22.5 | 225 | 5062.5 |
| 2 | 50 | 1287.5 | 64375.- |
| | | 1512.5 | 69437.5 |

$\bar{y} = 45.91$ cm

| | I | A | d | |
|---|---|---|---|---|
| 1 | $\frac{1}{12} 5 \cdot 45^3 = 37968.75$ | 225 | 23.41 | 161275.0 |
| 2 | $\frac{1}{12} 128.75 \cdot 10^3$ 10729.17 | 1287.5 | 4.09 | 3226660 |
| | | | | 193541.67 cm |




$\sigma_x$ all $= \frac{(120000)(.0909)}{193541.67 \times 10^{-8}} = 28.465$ MPa

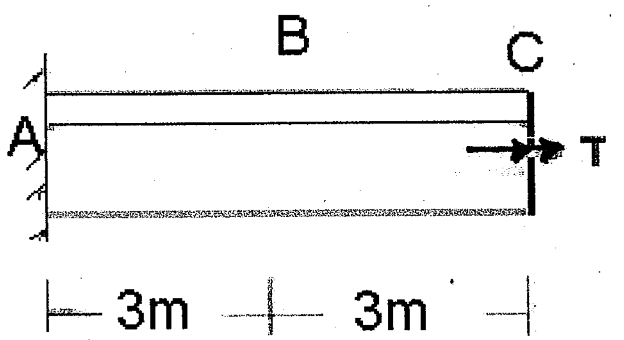
$\sigma$

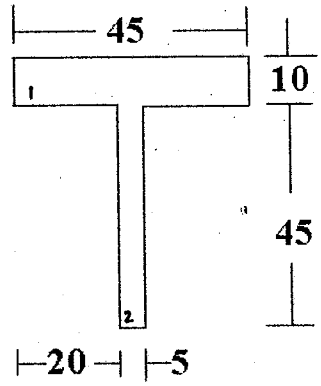
Problem 2.

A beam is loaded by means of a torque, T, equal to 5000 N-m at end C. The cross-section of the beam is given in the figure on the right.

a) What is the maximum shear stress in the cross-section and where can it be found? The beam is made of aluminum with $E_{aluminum} = 72$ GPa and the Poisson ratio, $\nu = 0.3$.

b) What is the angle of twist at the cross-section located at B.




**Units of length for the cross section are in cm.**

$T_1 + T_2 = T$

$\frac{T_1 L}{J_1 G} = \frac{T_2 L}{J_2 G}$ $\qquad T_1 = \frac{T_2 J_1}{J_2}$

for 1 $\quad a/b = 4.5 \quad C_2 = .286$

2 $\quad a/b = 9 \quad C_1 = .307$

$J_1 = C_2\, 45 (10^3) = .286(45)(10^3) = 12870$ $\qquad T_1 = T_2 (7.453)$

$J_2 = C_2\, 45 (5)^3 = .307 (45)(5^3) = 1726.875$ $\qquad T_1 + T_2 = T$

$J_1 + J_2 = J = 14596.875 \times 10^{-8}\ m^4$ $\quad \therefore\ 8.453 T_2 = T$

$\tau_1 = \frac{Tt}{J} = \frac{5000\,(.01)}{14596.875 \times 10^{-8}} = 342.54\ kPa$

$T_2 = 5000 / 8.453 = \cancel{670.89}\ 591.5$

$T_1 = 5000 - \cancel{670.89}\ 591.5 = \cancel{4329.105}\ 4408.49$

$\tau_2 = \frac{Tt}{J} = \frac{5000\,(.005)}{14596.875 \times 10^{-8}} = 171.27\ kPa$

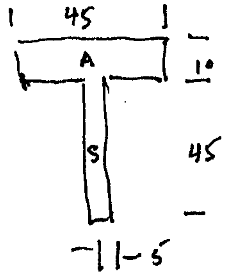
$\tau_1 = \frac{\cancel{4329.105}\ 4408.49\,(.01)}{12870 \times 10^{-8}} = \cancel{336.37 kPa}\ 342.54$

$\tau_2 = \frac{\cancel{670.89}\ 591.5\,(.005)}{1726.875 \times 10^{-8}} = \cancel{194.249 kPa}\ 171.263$

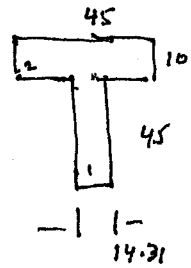
$G = \frac{E}{2(1+\nu)} = 27.69 \times 10^9 \frac{N}{m^2}$

$\varphi = \frac{TL}{J_{tot} G} = \frac{(5000 N)\, 3m}{(14596.875 \times 10^{-8})(27.69 \times 10^9 N/m^2)}$

$= .000371\ rad = .213°$




$\frac{E_s}{E_{al}} = 2.861$




| | $\bar{y}$ | A | $\bar{y}A$ |
|---|---|---|---|
| 1 | 22.5 | 643.75 | 14484.37 |
| 2 | 50 | 450 | 22500.– |
| | | 1093.75 | 36984.37 |

$\bar{y} = 33.8143$

| | I | A | d | |
|---|---|---|---|---|
| 1 | $\frac{1}{12}(14.31)(45)^3 = 108666.56$ | 643.75 | 11.314 | 191070.81 |
| 2 | $\frac{1}{12}(45)(10)^3 = 3750$ | 450 | 16.186 | 121643.97 |
| | | | | 312714.78 |

$$\sigma_{x_{al}} = \frac{120000\,(21.186)}{312784.78 \times 10^{-8}} \approx 8.13\ MPa$$

$$\sigma_{x_{st}} = 2.861\ \frac{120000\,(.33814)}{312784.78 \times 10^{-8}} = 37.115\ MPa$$

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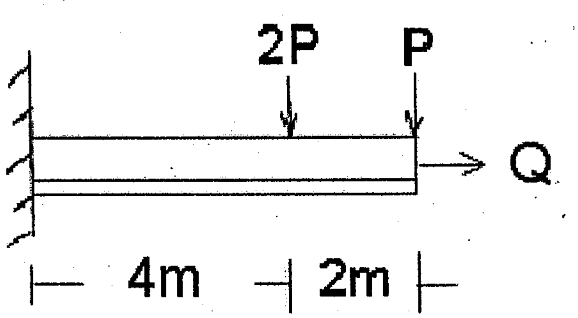
PRINT NAME SIGN NAME

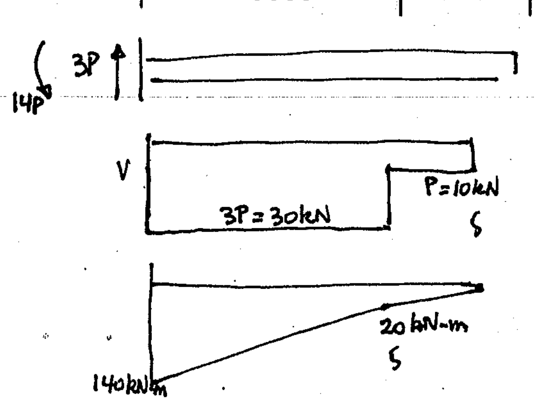
Problem 1.

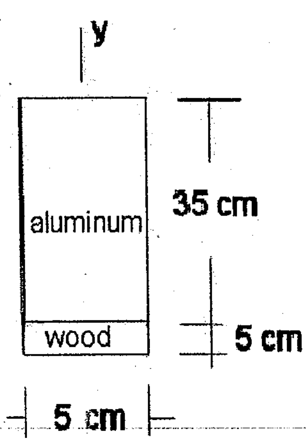
a) Given the following beam loaded as shown, find the location of the maximum moment, given that P=10KN. Take Q=0 KN initially. at the wall

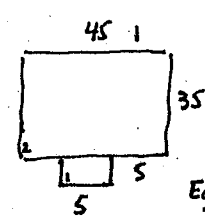
b) What is the maximum stress $\sigma_x$ and where can it be found, given the following information: $E_{aluminum}$ = 92. 7 GPa, $E_{wood}$ = 10.3 GPa, for the cross-section below.

c) Now let Q=20 KN so that it acts at the centroid of the cross-section of the equivalent beam. Where is the neutral axis now? What is the maximum stress $\sigma_x$ and where can it be found?




$E_a/E_w = 9$ 1

| $\bar{y}$ | $\bar{y}$ | A | $\bar{y}A$ |
|---|---|---|---|
| | 2.5 | 25 | 62.5 |
| | 22.5 | 1575 | 35437.5 |
| | | 1600 | 35500 |

$\bar{y} = 22.19$ 9

Units of length for the cross section are in cm.

| I | I | A | d | I' |
|---|---|---|---|---|
| 1 | $\frac{5\cdot5^3}{12} = 52.083$ | 25 | 19.69 | ~~10456.08~~ 9744.49 |
| 2 | $\frac{45\times35^3}{12} = 160781.25$ | 1575 | .31 | 160932.61 |

~~171388.69~~ $cm^4$
~~.001714~~ $m^4$ 8
$.17068\times10^{-4}$

$\sigma_{x_w} = \frac{(140000)(.2219)}{.0017068}$ 4
$= 18.202$ MPa

$\sigma_{x_A} = \frac{(140000)(17.81)}{.0017068}(9)$ 5

$$\frac{Q}{A} + \frac{My}{I} = 0 \qquad y = \frac{QI}{AM} \qquad y = .001$$

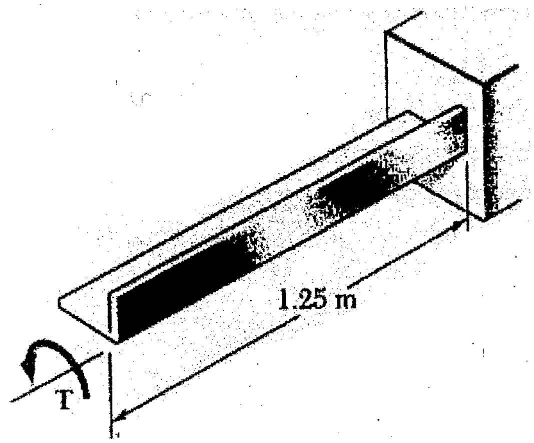
Problem 2.

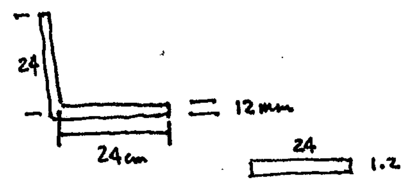
The angle bracket having the cross-section shown is to be formed from the sheet metal of thickness 12mm. Each leg of the angle bracket is 24.6 cm long to the centerline.

a) If the angle bracket has an applied torque, T, of 2500 N-m, find the maximum shear stress and where it can be found on the cross-section.

b) Also, find the angle of twist.

The Young's Modulus, E = 206 GPa and that the Poisson ratio, ν = 0.3.




20 = ratio c = .3228

$J = \frac{1}{3} 24 (1.2)^3 \cdot 2 \quad cm^4$

$\tau = \frac{Tt}{J} = \frac{2500 \text{ N-m } (.012) \text{ m}}{23.648 \times 10^{-8} \text{ m}^4}$

$= \cancel{217.014 \text{ GPa}}$ 108.51 MPa

halfway along each leg

$\varphi = \frac{T_1 L}{J_1 G} = \frac{T_2 L}{J_2 G}$

$T_1 + T_2 = T$

$\frac{T_1 L}{J_1 G} = \frac{(T - T_1) L}{J_2 G}$

$\frac{T_1 L}{G}\left[\frac{1}{J_1} + \frac{1}{J_2}\right] = \frac{T \cdot L}{J_2 G}$

$T_1 = T \frac{J_1 \cancel{J_2}}{\cancel{J_2}(J_1 + J_2)} = \frac{1}{2} T = T_2$

$G = \frac{E}{2(1+\nu)}$

$= 92.33$ GPa

$\varphi = \frac{TL}{JG}$

$= \frac{2 \cdot 1250 \, (1.25)}{(23.648 \times 10^{-8})(79.23 \times 10^9)}$

$= \cancel{.2853} \text{ rad} = \cancel{16.347^\circ}$

$.14266 \qquad 8.174^\circ$

$\varphi = \cancel{\frac{2500 / 1.25}{.4}}$

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EMA 3702 QUIZ 3A April 17, 2012

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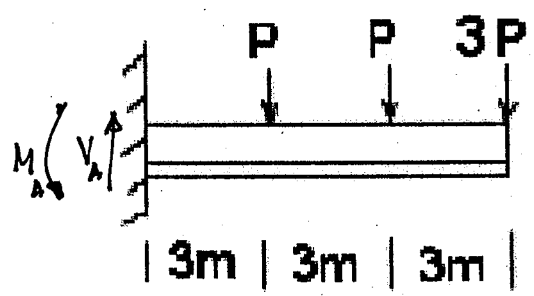
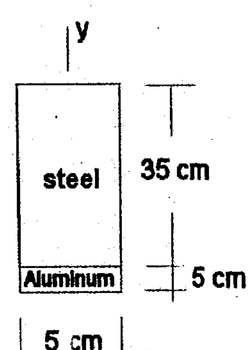
PRINT NAME SIGN NAME

Problem 1.

a) Given the following beam loaded as shown, find the location of the maximum shear, given P=10 KN, given the following information: $E_{steel} = 206$ GPa, $E_{aluminum} = 92.7$ GPa, for the cross-section below. Note that the maximum shear may not be at the centroid.

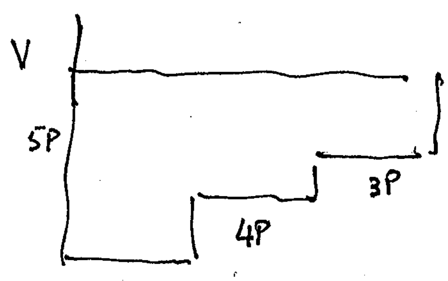
$n = \frac{206}{92.7} = 2.22$

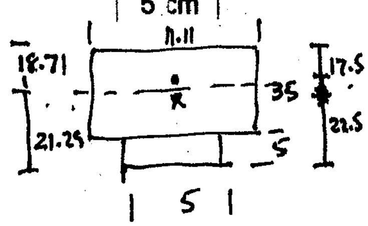



| | $A$ | $\bar{y}$ | $\bar{y}A$ |
|---|---|---|---|
| S | 11.11(35) 388.89 | 22.5 | ~~389·22.5~~ 8750 |
| A | 25 | 2.5 | 62.5 |
| | 414 | | 8812.5 |

$\bar{y} = 21.29$ cm




| | $I_o$ | $A$ | $d$ | $I$ |
|---|---|---|---|---|
| S | $\frac{1}{12}(11.11)(35)^3$ 39695.10 | 388.89 | (22.5−21.29) 1.21 | 40264.48 |
| A | $\frac{1}{12}(5)(5)^3$ 52.08 | 25 | 18.79 (21.29−2.5) | 8878.69 |
| | | | | 49143.17 cm |

$\sum F_y = V_A - 5P = 0 \quad \underline{V_A = 5P}$

$V_{max} = 50$ kN $\quad Q_{centr} = A\bar{y} = (11.11)(40-21.29)^2/2 = 1944.61$

$\tau_{c} = \frac{VQ}{It} = \frac{(50\times10^3 N)(1944.61\times10^{-6})}{(49143.17\times10^{-8})(5\times10^{-2})} = 3.957$ MPa

@ interface in steel $\quad Q = A\bar{y} = (11.11)(35)(18.71-17.5) = 470.51$

$\tau = .957$ MPa

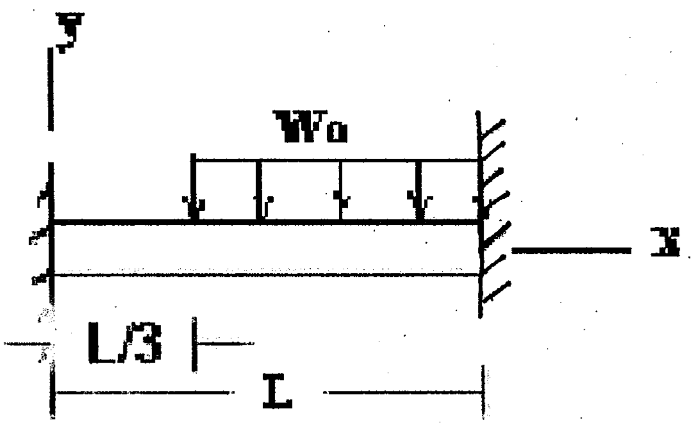
@ interface in al $\quad Q = A\bar{y} = 5 \cdot 5\,(21.29-2.5) = 469.75 \quad \tau$ about same

QUIZ 3A

Problem 2.

a) Given the following beam loaded as shown, find the deflection y as a function of x.

b) What is the moment at x=L/2 from the deflection equation y(x)?




$$EIy^{IV} = -w_0 \langle x - L/3 \rangle^0$$

$$EIy''' = -w_0 \langle x - L/3 \rangle^1 + C_1$$

$$EIy'' = -\frac{w_0}{2} \langle x - L/3 \rangle^2 + C_1 x + C_2 \quad (3)$$

$$EIy' = -\frac{w_0}{6} \langle x - L/3 \rangle^3 + C_1 x^2/2 + C_2 x + C_3 \qquad y'(0) = 0 \Rightarrow C_3 = 0$$

$$EIy = -\frac{w_0}{24} \langle x - L/3 \rangle^4 + C_1 x^3/6 + C_2 x^2/2 + C_4 \qquad y(0) = 0 \Rightarrow C_4 = 0$$

$$EIy(x=L) = 0 = \frac{-w_0}{24} \cdot \frac{16}{81} L^4 + C_1 L^3/6 + C_2 L^2/2 \qquad (1)$$

$$EIy'(x=L) = 0 = \left[ \frac{-w_0}{6} \langle \frac{8}{27} L^3 \rangle + C_1 L^2/2 + C_2 L \right] \qquad (2)$$

mult (2) × L/2

$$\frac{-w_0}{12} \cdot \frac{8}{27} L^4 + C_1 L^3/4 + C_2 L^2/2$$

$$(1) - (2) = \frac{-w_0 \cdot 2}{3 \cdot 81} L^4 + \frac{w_0 \cdot 3}{3 \cdot 27 \cdot 3} L^4 + C_1 \left[ L^3/6 - L^3/4 \right] = 0$$

$$+ w_0 \cdot \frac{1}{3 \cdot 81} L^4 + C_1 \left( -\frac{L^3}{12} \right) = 0 \Longrightarrow \boxed{C_1 = \frac{w_0}{3 \cdot 81} L \cdot 12 = \frac{4 w_0 L}{81}}$$ put into (2)

$$-\frac{w_0 \cdot 8}{6 \cdot 27} L^3 + \frac{2}{81} w_0 L^3 + C_2 L = 0$$

$$-\frac{2}{81} w_0 L^3 + C_2 L = 0 \qquad \boxed{C_2 = \frac{2}{81} w_0 L^2}$$

put $C_1$ & $C_2$ in (3)

$$M = EIy'' = \frac{w_0}{2} \left( \frac{L}{6} \right)^2 + \frac{4 w_0}{81} \cdot \frac{L}{2}^2 + \frac{2}{81} w_0 L^2$$

$$= \left( \frac{w_0}{72} + \frac{2 w_0}{81} + \frac{2}{81} w_0 \right) L^2 \qquad \boxed{\frac{41}{648} w_0 L^2 = M}$$

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EMA 3702 QUIZ 3B April 17, 2012

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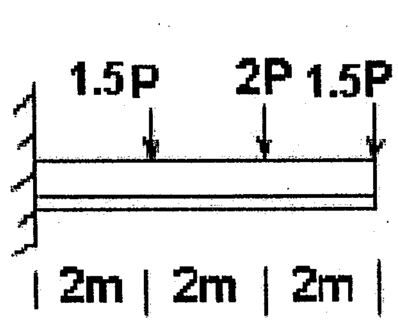
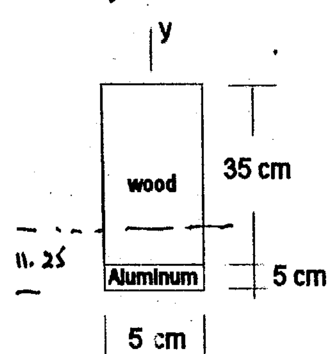
____________________ ____________________
PRINT NAME SIGN NAME

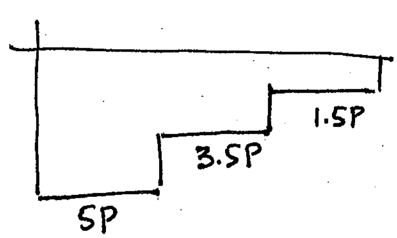
Problem 1.

a) Given the following beam loaded as shown, find the location of the maximum shear, given that P=20KN, given the following information: $E_{aluminum} = 92.7$ GPa, $E_{wood} = 10.3$ GPa, for the cross-section below. Note that the maximum shear may not be at the centroid.

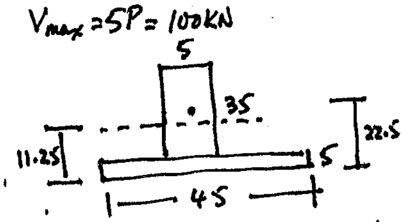



$n = \frac{92.7}{10.3} = 9$




$V_{max} = 5P = 100KN$




| | A | $\bar{y}$ | $A\bar{y}$ |
|---|---|---|---|
| W | $35 \cdot 5 = 175$ | 22.5 | 3937.5 |
| A | $45 \cdot 5 = 225$ | 2.5 | 562.5 |
| | 400 | | 4500 |

$\bar{y} = 11.25$

| | $I_0$ | A | d | I |
|---|---|---|---|---|
| W | $\frac{5 \cdot 35^3}{12} = 17864.58$ | 175 | $(22.5-11.25)$ | 40013.02 |
| A | $\frac{45 \cdot 5^3}{12}$ 468.75 | 225 | $(11.25-2.5)$ | 17695.31 |
| | | | | 57708.33 |

$Q_{c} = 5\left[40-11.25\right]^2 / 2 = 2066.41 cm^3$

$\tau = \frac{(100000)(2066.41 \times 10^{-6})}{(57708.33 \times 10^{-8})(.05)} = 7.162 MPa$

$Q_{int} = 5(35)(22.5-11.25) = 1968.75 cm^3$

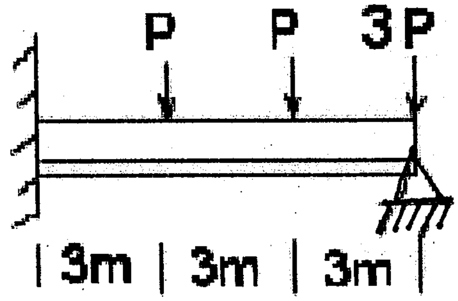
$\tau = 6.823 MPa$

QUIZ 3B

Problem 2.

a) Given the following beam loaded as shown, find the deflection y as a function of x.

b) What is the slope dy/dx at x=6m from the deflection equation y(x) ?




$$EIy^{IV} = -P\langle x-3\rangle^{-1} - P\langle x-6\rangle^{-1} - 3P\langle x-9\rangle^{-1}$$

$$EIy''' = -P\langle x-3\rangle^{0} - P\langle x-6\rangle^{0} - 3P\langle x-9\rangle^{0} + C_1$$

$$EIy'' = -P\langle x-3\rangle^{1} - P\langle x-6\rangle^{1} - 3P\langle x-9\rangle^{1} + C_1x + C_2$$

$$EIy' = -\frac{P}{2}\langle x-3\rangle^{2} - \frac{P}{2}\langle x-6\rangle^{2} - \frac{3P}{2}\langle x-9\rangle^{2} + C_1\frac{x^2}{2} + C_2x + C_3$$

@ $x=0$ $y'=0 \Rightarrow C_3=0$

$$EIy = -\frac{P}{6}\langle x-3\rangle^{3} - \frac{P}{6}\langle x-6\rangle^{3} - \frac{3P}{6}\langle x-9\rangle^{3} + \frac{C_1x^3}{6} + \frac{C_2x^2}{2} + C_4$$

@ $x=0$ $y=0 \Rightarrow C_4=0$

$-\frac{9P}{6}\cdot 27$

$$EIy\Big|_{x=9} = 0 = -\frac{P}{6}\cdot 6^3 - \frac{P}{6}\cdot 3^3 - \frac{3P}{6}\cdot 0 + C_1\cdot\frac{9^3}{6} + C_2\cdot\frac{9^2}{2} = 0$$

$$EIy''\Big|_{x=9} = -P\cdot 6 - P\cdot 3 - 3P\cdot 0 + C_1\cdot 9 + C_2$$

factor $9^2$

$$-P\cdot\frac{3}{6} + C_1\cdot\frac{9}{6} + \frac{C_2}{2} = 0$$

$$-9P + C_1\cdot 9 + C_2 = 0$$

$$\begin{pmatrix} 9/6 & 1/2 \\ 9 & 1 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} P/2 \\ 9P \end{pmatrix}$$

$-3 = \det$

$$\begin{pmatrix} P/2 & 1/2 \\ 9P & 1 \end{pmatrix} = \boxed{\frac{-4P}{-3} = C_1}$$

$$\begin{pmatrix} 9/2 & P/2 \\ 9 & 9P \end{pmatrix} = \boxed{\frac{-36P}{-3} = C_2}$$

$$\cancel{-\frac{3P}{2}} \quad \cancel{C_1\frac{9^3}{3} = 0} \quad \cancel{C_1 = -\frac{3P}{2\cdot 81}}$$

$$\cancel{-3P - \frac{P}{2\cdot 9} + C_2 = 0} \quad \cancel{C_2 = \frac{55P}{18}}$$

$$\cancel{-P\cdot 6 - P\cdot 3 - 3P\cdot 1 + C_1\cdot 9 + C_2 = 0}$$

$$\cancel{-\frac{P}{6}\cdot 6^3 - \frac{P}{6}\cdot 3^3 - \frac{3P}{2} + C_1\frac{9^3}{6} + C_2\cdot\frac{9^2}{2} = 0}$$

$$\cancel{-\frac{9\cdot P\cdot 3^2}{2} - \frac{3P}{2} + C_1\frac{9^3}{6} + C_2\frac{9^2}{2} = 0}$$

$$\cancel{-42P + C_1\cdot\frac{9^3}{6} + C_2\frac{9^2}{2} = 0}$$

$$\cancel{-12\cdot\frac{9^2}{2}P + C_1\cdot\frac{9^3}{2} + C_2\frac{9^2}{2} = 0}$$

$$\cancel{-489P}$$

$$\cancel{447P + C_1\cdot\frac{9^3}{3} = 0} \qquad \boxed{C_1 = \frac{447P\cdot 3}{729}}$$

$$\cancel{-12P + \frac{447P\cdot 3}{81} + C_2 = 0}$$

$C_2 =$

$$EIy = -\frac{P}{6}\langle x-3\rangle^3 - \frac{P}{6}\langle x-6\rangle^3 - \frac{3P}{6}\langle x-9\rangle^3 + \frac{4}{18}Px^3 + \frac{12}{2}Px^2$$

$$EIy' = -\frac{P}{2}\langle x-3\rangle^2 - \frac{P}{2}\langle x-6\rangle^2 - \frac{3P}{2}\langle x-9\rangle^2 + \frac{2}{3}Px^2 + 12Px$$

$$= -\frac{P}{2}\cdot 9 + \frac{2}{3}P\cdot 36 + 12P\cdot 6$$

$$+24P + 72P$$

$$+96P$$

$$\boxed{y'\Big|_{x=6} = \frac{91.5\,P}{EI}}$$

[illegible]

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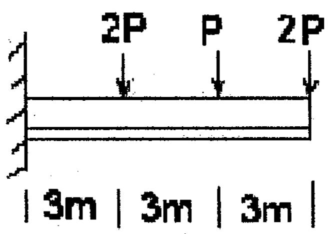
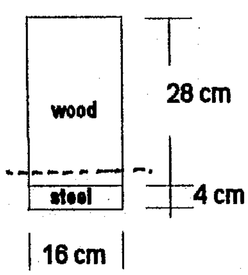
____________________ PRINT NAME ____________________ SIGN NAME

Problem 1.

a) Given the following beam loaded as shown, find the location of the maximum shear, given P=10 KN, given the following information: $E_{steel}$ = 206 GPa, $E_{wood}$ = 10.3 GPa, for the cross-section below. Note that the maximum shear may not be at the centroid.

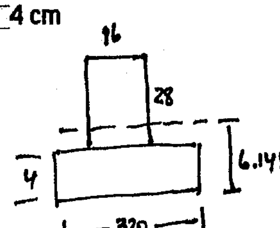



$\eta = 20$

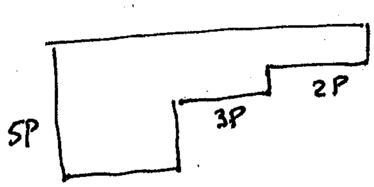
| | A | $\bar{y}$ | $A\bar{y}$ |
|---|---|---|---|
| w | 28·16 = 448 | 18 | 8064 |
| s | 16·20·4 = 1280 | 2 | 2560 |
| | 1728 | | 10624 |

$\bar{y} = 6.148$




| | $I_o$ | A | d | I |
|---|---|---|---|---|
| w | 29269.33 | 448 | (18−6.148) | 92199.8 |
| s | 1706.67 | 1280 | (6.148−2) | 23730.2 |
| | | | | 115930.07 |

n = .05
5796.5




$V_{max} = 5P = 50KN$

$Q = 16 \times (32-6.148)^2/2 = 5346.61\ cm^3$

$\tau_4 = \frac{VQ}{It}$

$= \frac{(50000)(5346.61 \times 10^{-6})}{(115930.07 \times 10^{-8})(.16)} = 1.441 MPa$

$Q = (16 \times 28)(18-6.148) = 5309.70$

$\tau_{interface} = 1.431\ MPa$

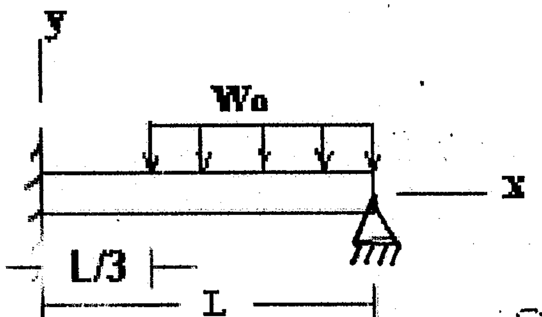
$Q = (320 \cdot 4)(6.148-2) + \cancel{\quad}$ 5309.44

QUIZ 3C

Problem 2.

a) Given the following beam loaded as shown, find the deflection y as a function of x.

a) What is the shear at x=L/2 from the deflection equation y(x)?




$EIy^{IV} = -w_0\langle x - L/3\rangle^0$

$EIy''' = -w_0\langle x - L/3\rangle^1 + C_1$

$EIy'' = -\frac{w_0}{2}\langle x - L/3\rangle^2 + C_1 x + C_2$

$EIy' = -\frac{w_0}{6}\langle x - L/3\rangle^3 + C_1 x^2/2 + C_2 x + C_3$ @ $x=0$ $y'=0 \Rightarrow C_3 = 0$

$EIy = -\frac{w_0}{24}\langle x - L/3\rangle^4 + C_1 x^3/6 + C_2 x^2/2 + C_4$ @ $x=0$ $y=0 \Rightarrow C_4 = 0$

$EIy(L) = 0$ $\quad \frac{-w_0}{3\cdot 24}\cdot\frac{16}{81}L^4 + C_1 L^3/6 + C_2 L^2/2 = 0$

$EIy''(L) = 0$ $\quad -\frac{w_0}{2}\cdot\frac{4}{9}L^2 + C_1\cdot L + C_2 = 0$

$+w_0\cdot\frac{2}{3}\cdot\frac{L^2}{81} = C_1\frac{L}{6} + \frac{C_2}{2} \Leftarrow$

$\frac{w_0}{2}\cdot\frac{4}{9}L^2 = C_1 L + C_2 \Leftarrow$

$\begin{pmatrix} 1/6 & \frac{1}{2} \\ L & 1 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{matrix} w_0L^2\cdot\frac{2}{243} \\ w_0L^2\cdot\frac{2}{9} \end{matrix}$

$\det = -L/3$

$\det\begin{pmatrix} w_0L^2\cdot\frac{2}{243} & \frac{L}{2} \\ 2w_0L^2/9 & 1 \end{pmatrix} \Big/ -L/3 = w_0L^2\cdot\frac{2}{243} - \frac{w_0L^2\cdot 27}{9\cdot 27}$

$= -\frac{25\, w_0L^2}{243}\Big/ -L/3$

$= +\frac{25\, w_0L}{81} = C_1$

$\det\begin{pmatrix} L/6 & w_0L^2\cdot\frac{2}{243} \\ L & 2w_0L^2/9 \end{pmatrix} \Big/ -L/3 = \frac{9\cdot 2w_0L^3/54}{9\cdot 9\cdot 3\cdot 2} - \frac{2w_0L^3}{243}\Big/ -L/3 = 7w_0L^3/243 \Big/ -L/3 = -\frac{7}{81}w_0L^2$

$\left(-w_0\cdot\frac{2}{3\cdot 81} + w_0\frac{3\cdot 1\cdot 9}{3\cdot 9\cdot 9}\right)L^4 + C_1\left(L^3/6 - L^3/2\right) = 0$

$\frac{25}{3}\cdot\frac{w_0}{81}\cdot L^4 \qquad - C_1\, L^3/3 = 0$ $\quad \boxed{C_1 = \frac{25 w_0 L}{81}}$ ✓

$C_2 = -C_1 L + w_0\cdot\frac{2}{9}L^2$

$= -\frac{25 w_0L^2}{81} + w_0\cdot\frac{2\cdot 9}{9\cdot 9}L^2 = \boxed{\frac{-7}{81}w_0L^2 = C_2 =} -\frac{7w_0L^2}{81}$

@ $x = L$ $\quad EIy''' = \cancel{-w_0\cdot\frac{2}{3}L + \frac{25 w_0 L}{81} = -\frac{53}{81}w_0L = -V}$

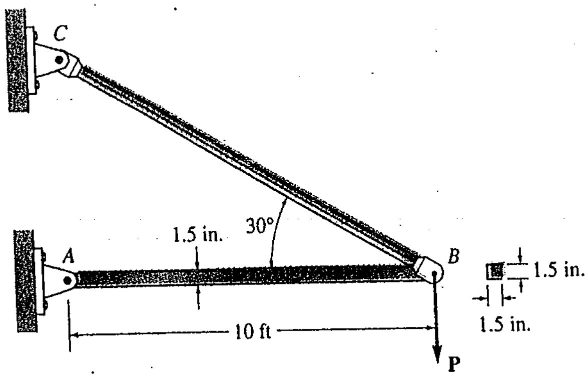
$\therefore \cancel{V@x=L = \frac{53}{81}w_0L}$

$EIy''' = -V = -w_0\langle L/2 - L/3\rangle^1 + C_1$

$= -w_0\, L/6 + \frac{25 w_0 L}{81} = -\frac{w_0L\cdot 27}{162} + \frac{50 w_0 L}{162} = \frac{23 w_0 L}{162}$

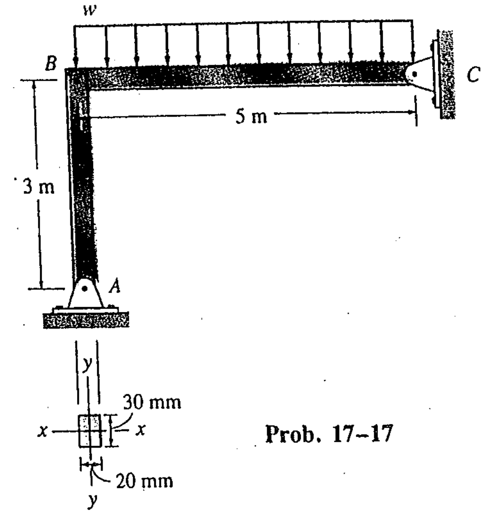
$V = -23 w_0L/162$

**17–15.** The steel bar $AB$ has a square cross section. If it is pin-connected at its ends, determine the maximum allowable load $P$ that can be applied to the frame. Use a factor of safety with respect to buckling of F.S. = 2. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 36$ ksi.



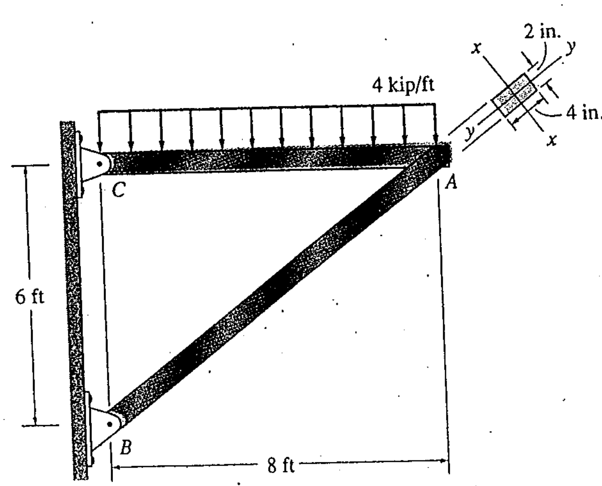

**Prob. 17–15**

**17–17.** The steel bar $AB$ has a rectangular cross section. If it is pin-connected at its ends, determine the maximum allowable intensity $w$ of the distributed load that can be applied to $BC$ without causing bar $AB$ to buckle. Use a factor of safety with respect to buckling of F.S. = 1.5. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



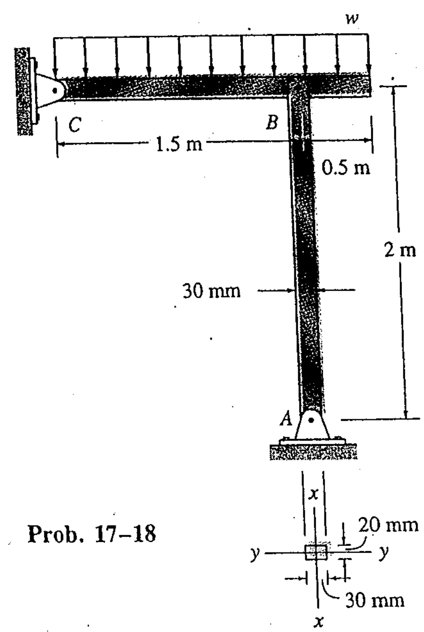

**Prob. 17–17**

***17–16.** The steel bar $AB$ of the frame is pin-connected at its ends. Determine the factor of safety with respect to buckling about the $y$–$y$ axis due to the applied loading. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 36$ ksi.




**Prob. 17–16**

**17–18.** Determine the maximum allowable intensity $w$ of the distributed load that can be applied to member $BC$ without causing member $AB$ to buckle. Assume that $AB$ is made of steel and is pinned at its ends for $x$–$x$ axis buckling and fixed at its ends for $y$–$y$ axis buckling. Use a factor of safety with respect to buckling of F.S. = 3. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.




**Prob. 17–18**

EMA 3702 SPRING 2012 DR. C. LEVY

FINAL EXAMINATION-A

April 26, 2012

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**Please sign the following:**

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________________________ Print your name

________________________ Sign your name

This examination consists of 3 problems <u>with several parts to each of the problems. You are to answer all the problems!</u>

<u>GOOD LUCK!</u>

| Problem # | Breakdown by Problem | Score |
|---|---|---|
| 1 | 30% | |
| 2 | 30% | |
| 3 | 40% | |

**TOTAL**

Problem 1A

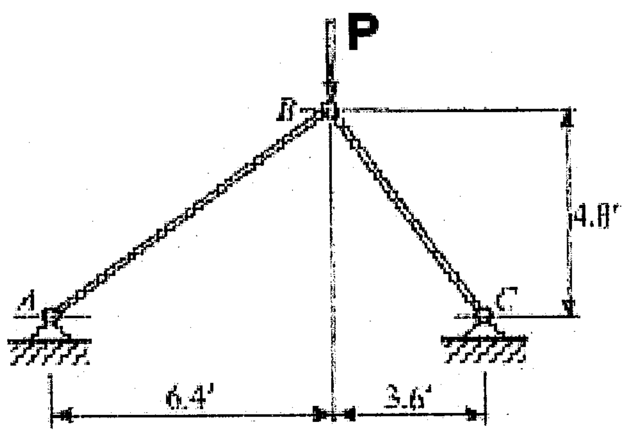
A bar truss, made of steel, is loaded by P as shown in the diagram on the left.

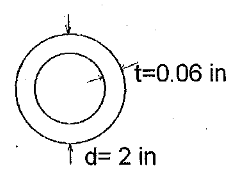
a. If each bar has a cross-section as shown below on the right
   - find the value of the load P that will cause buckling to occur and
   - find in which bar this buckling will happen first.

b. Suppose the dimensions of the bars' cross-section were not given and the bars were solid. What must be the minimum diameter of the solid bars if failure by buckling ($\sigma = P_{cr}/A$) and failure by yielding ($\sigma = \sigma_{yp}$) were to occur simultaneously. $P_{cr}$ is the Euler Buckling Load

For part a, calculate the load P according to the following conditions:
All the bars are pin connected at both ends for buckling in the plane of the page and are considered fixed at both ends for buckling out of the plane of the page.

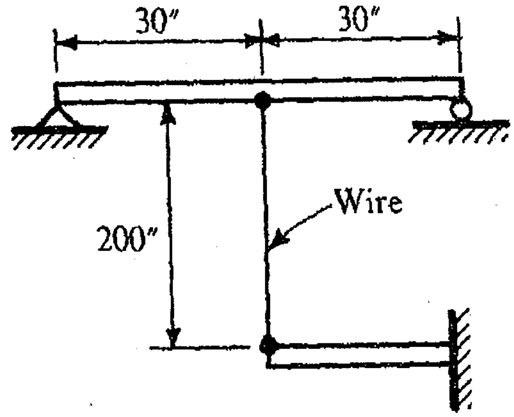
Take $E = 29 \times 10^6$ lb/in$^2$ and $\sigma_{yp}$=36000 lb/in$^2$. The dimensions of the bars can be determined from the picture and are given in FEET.



Problem 2A.

A steel wire 200 in. in length with cross-sectional area equal to 0.25 in.$^2$ is stretched tightly between the midpoint of the simple beam and the free end of the cantilever as shown in the figure. Determine the deflection of the end of the cantilever as a result of a temperature drop of 50°F. For steel wire: $E = 30 \times 10^6$ psi, $\alpha = 6.5 \times 10^{-6}$ per °F. For both beams: $I = 21.3$ in.$^4$ and $E = 1.5 \times 10^6$ psi.

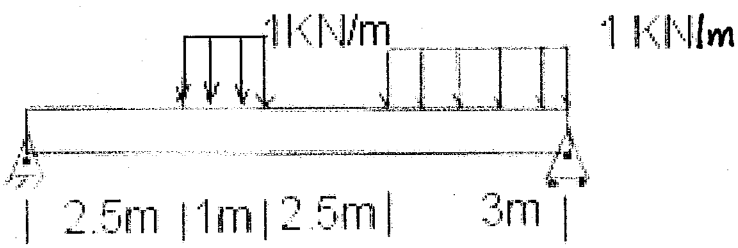


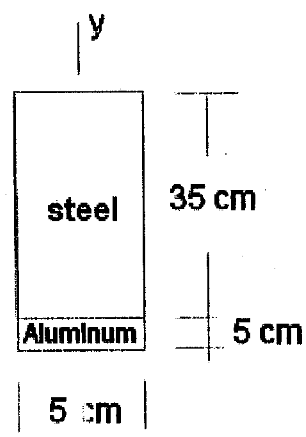
Problem 3A.

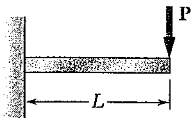
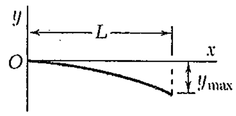
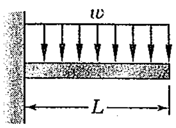
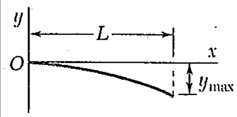
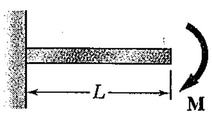
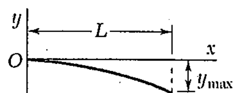
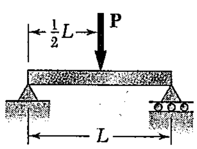
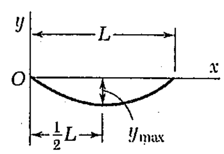
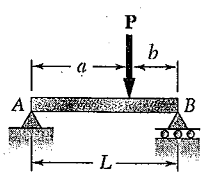
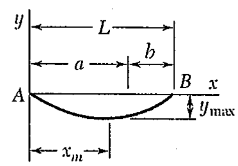
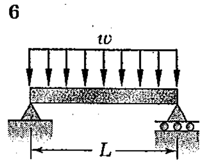
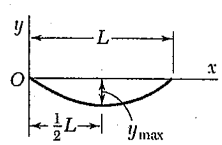
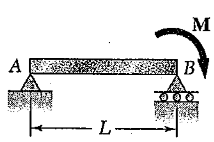
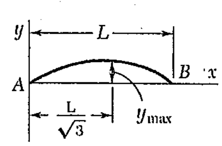
You are asked to solve the following problem by your boss. Given a beam made of wood and aluminum that is loaded as seen in the left figure and the cross-section as shown in the right figure. The Young's modulus for the steel is $E_s$=198 GPa and for the aluminum it is $E_{al}$=72 GPa.

The loading acts along the y axis in the downward direction. You decide that in your solution methodology <u>you will convert the cross-section to aluminum</u>.

a) Find the equation of the displacement function, y(x), in terms of EI.
b) Find the location and value of the maximum direct stress, $\sigma_x$, and the maximum shear stress, $\tau_{xy}$.
c) If the aluminum and steel are held together with a bolt that can handle a load of 200N, what would be the minimum spacing required of the bolts in the region x=6m to x=9m of the beam.
d) Find $\tau_{max}$ and $\sigma_{max}$ at a point x=3m, and 5cm below the top surface (hint: determine what stresses are acting at that point, first).



| Beam and Loading | Elastic Curve | Maximum Deflection | Slope at End | Equation of Elastic Curve |
|---|---|---|---|---|
| 1   |   | $-\frac{PL^3}{3EI}$ | $-\frac{PL^2}{2EI}$ | $y=\frac{P}{6EI}(x^3-3Lx^2)$ |
| 2   |   | $-\frac{wL^4}{8EI}$ | $-\frac{wL^3}{6EI}$ | $y=-\frac{w}{24EI}(x^4-4Lx^3+6L^2x^2)$ |
| 3   |   | $-\frac{ML^2}{2EI}$ | $-\frac{ML}{EI}$ | $y=-\frac{M}{2EI}x^2$ |
| 4   |   | $-\frac{PL^3}{48EI}$ | $\pm\frac{PL^2}{16EI}$ | For $x \le \frac{1}{2}L$: $y=\frac{P}{48EI}(4x^3-3L^2x)$ |
| 5   |   | For $a > b$: $-\frac{Pb(L^2-b^2)^{3/2}}{9\sqrt{3}EIL}$ at $x_m=\sqrt{\frac{L^2-b^2}{3}}$ | $\theta_A=-\frac{Pb(L^2-b^2)}{6EIL}$ $\theta_B=+\frac{Pa(L^2-a^2)}{6EIL}$ | For $x < a$: $y=\frac{Pb}{6EIL}[x^3-(L^2-b^2)x]$ For $x=a$: $y=-\frac{Pa^2b^2}{3EIL}$ |
| 6   |   | $-\frac{5wL^4}{384EI}$ | $\pm\frac{wL^3}{24EI}$ | $y=-\frac{w}{24EI}(x^4-2Lx^3+L^3x)$ |
| 7   |   | $\frac{ML^2}{9\sqrt{3}EI}$ | $\theta_A=+\frac{ML}{6EI}$ $\theta_B=-\frac{ML}{3EI}$ | $y=-\frac{M}{6EIL}(x^3-L^2x)$ |

EMA 3702 SPRING 2012 DR. C. LEVY

FINAL EXAMINATION-C

April 26, 2012

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**Please sign the following:**

I certify that I will neither receive nor give unpermitted aid on this examination. Violation of this will result in failure of the course and possibly other academic disciplinary actions.

______________________________ ______________________________

Print your name Sign your name

This examination consists of 3 problems with several parts to each of the problems. You are to answer all the problems!

GOOD LUCK!

| Problem # | Breakdown by Problem | Score |
|---|---|---|
| 1 | 30% | |
| 2 | 30% | |
| 3 | 40% | |

**TOTAL**

Problem 1C

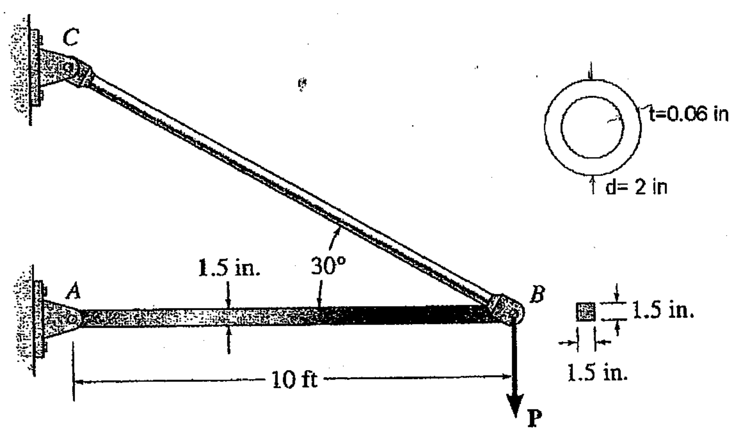
A bar truss, made of steel, is loaded by P as shown in the diagram on the left. This load can occur either pointing down along the –y direction or pointing up along the +y direction. <u>So you must solve for both cases.</u>

a. If each bar has a cross-section as shown below on the right of the structure,
   - find the value of the load P that will cause buckling to occur first, the direction of P and
   - find in which bar this buckling will happen first.

Assume a safety factor for buckling is 2.

Calculate the load P according to the following conditions:
All the bars are pin connected at both ends for buckling in the plane of the page and are considered fixed at both ends for buckling out of the plane of the page.

Take $E = 29 \times 10^6$ lb/in$^2$ and $\sigma_{yp}$=36000 lb/in$^2$. The dimensions of the bars can be determined from the picture and are given in FEET.



Problem 2C.

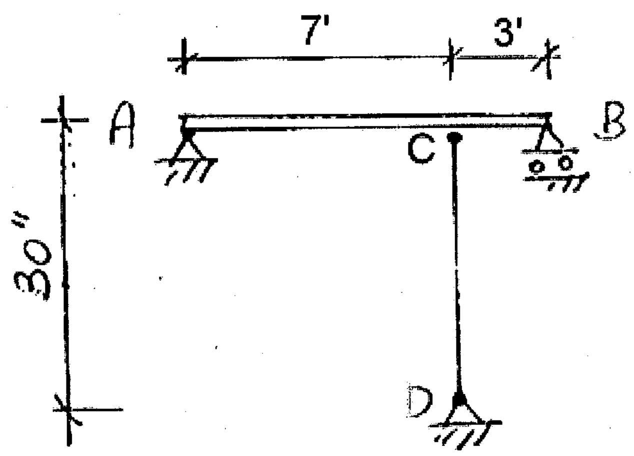
Given:
Beam AB having a bending rigidity EI=1040 lb-in$^2$
Wire CD having a cross-sectional area A=0.0001 in$^2$, an unstretched length of 30 inches, E=30 x10$^6$ psi, and $\alpha$=6.5 x 10$^{-6}$ in/in-deg F

Originally the system is at room temperature, no loads are applied to the system, and the unstretched wire CD is connected to beam AB at C.

a) Find the change in load in the wire CD if the temperature of the wire increases by 100 degrees F. The beam is unaffected by the change in temperature. To do this, you will need to determine the displacement of the beam at C, first. So,

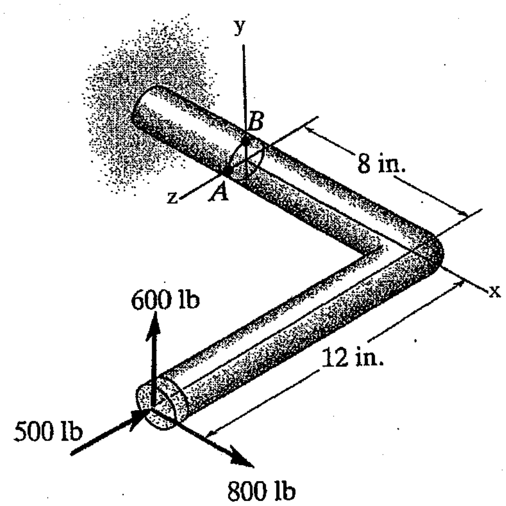
b) Derive the displacement function for the beam y(x) if a concentrated load is 7 feet from the left support. Use this function to find the displacement at C.

Note: 1'=1 foot=12"=12 inches



Problem 3.

The 2-in.-diameter rod is subjected to the loads shown. Determine the state of stress at point *A*, and show the results on a differential element located at this point.



EMA 3702 SPRING 2012 DR. C. LEVY

FINAL EXAMINATION-B

April 26, 2012

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__________________________ __________________________

Print your name Sign your name

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GOOD LUCK!

| Problem # | Breakdown by Problem | Score |
|---|---|---|
| 1 | 30% | |
| 2 | 40% | |
| 3 | 30% | |

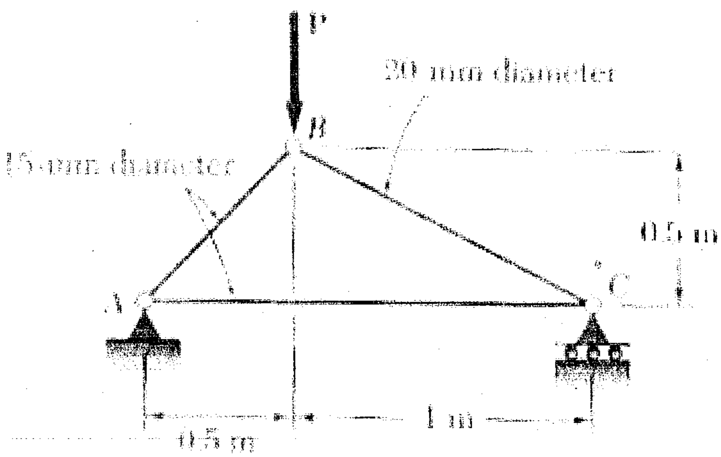
**TOTAL**

Problem 1B.

Knowing that a factor of safety of 2.0 is required,

a) Determine the largest load P that can be applied to the structure shown and which bar will fail first. Use E = 200 GPa

Calculate the load P under the following conditions:
Each rod is pin connected at both ends for buckling in the plane of the page and are considered fixed at both ends of each rod for buckling in the out of page direction.

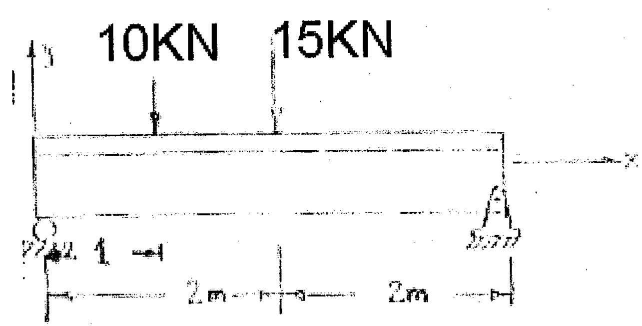


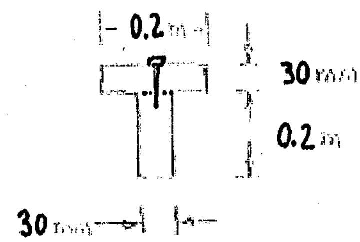

b) Assume that the diameters of the three members are <u>not given</u> and the safety factor is 1. For the load P found in (a), find the diameter in each of the three members so that the bar(s) that are in compression fail in buckling and yielding simultaneously and the bar(s) in tension fail in yielding.
Assume that $\sigma_{yp} = 360$ MPa

Problem 2B.

The T shaped beam is made of 2 steel plates 200 mm x 30 mm which are joined by bolts.

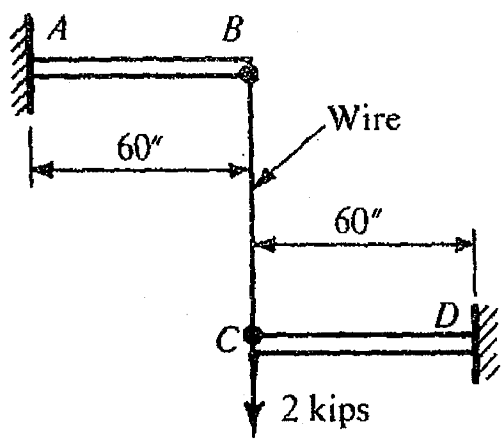
a) Find the equation of the displacement function, y(x)
b) If the allowable bending stress is 200 MPa, and the allowable shearing stress is 80 MPa determine if the beam is able to support safely the loads shown in the picture both in shear and in bending by determining the shear and moment diagram first, then finding the maximum bending and shear stresses.
c) Find the maximum spacing between the bolts if each nail is able to support safely 1500 N of shear force.
d) Find $\tau_{max}$ and $\sigma_{max}$ at a point x=3m, and 3.5cm below the top surface (hint: determine what stresses are acting at that point, first).




All dimensions in the left figure are in meters

Problem 3B.




Two steel cantilever beams $AB$ and $CD$ are connected by a taut steel wire $BC$ having a length equal to 150 in. under initial no-load conditions, see figure. Determine the stress in the wire produced by a 2-kip load applied at $C$ and a temperature drop, in the wire only, of 100°F. For beams $AB$ and $CD$: $E = 30 \times 10^6$ psi, and $I = 24$ in.$^4$ For wire $BC$: $E = 30 \times 10^6$ psi, $A = 0.1$ in.$^2$, and $\alpha = 6.0 \times 10^{-6}$ per °F.

to as the *shear flow*. Since force is usually measured in pounds, shear flow $q$ has units of pounds per inch. Then, recalling that $dM/dx = -V$, one obtains the following expression for the shear flow in beams:

$$q = \frac{dF}{dx} = -\frac{dM}{dx}\frac{1}{I}\int_{\substack{\text{area}\\ fghj}} y\,dA = \frac{V A_{fghj}\bar{y}}{I} = \frac{VQ}{I} \qquad (7\text{-}5)$$

In this equation $I$ stands for the moment of inertia of the entire cross-sectional area around the neutral axis, just as it does in the flexure formula from which it came. The total shearing force at the section investigated is represented by $V$, and the integral of $y\,dA$ for determining $Q$ extends only over the cross-sectional area of the beam to one side of this area at which $q$ is investigated.

In retrospect, note carefully that Eq. 7-5 was derived on the basis of the elastic flexure formula, but no term for a bending moment appears in the final expressions. This resulted from the fact that only the change in the bending moments at the adjoining sections had to be considered, and the latter quantity is linked with the shear $V$. The shear $V$ was substituted for $-dM/dx$, and this masks the origin of the established relations. Equation 7-5 is very useful in determining the necessary interconnection between the elements making up a beam. This will be illustrated by examples.

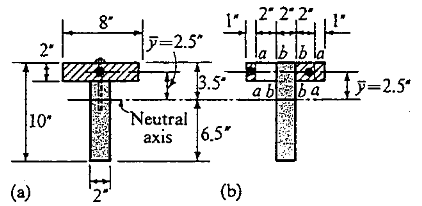



**Fig. 7-6**

EXAMPLE 7-1

Two long wooden planks form a $T$ section of a beam as shown in Fig. 7-6(a). If this beam transmits a constant vertical shear of 690 lb, find the necessary spacing of the nails between the two planks to make the beam act as a unit. Assume that the allowable shearing force per nail is 150 lb.

SOLUTION

In attacking such problems the analyst must ask: What part of a beam has a tendency to slide longitudinally from the remainder? Here it is the plane of contact of the two planks; Eq. 7-5 must be applied to determine the shear flow in this plane. To do this the neutral axis of the whole section and its moment of inertia around the neutral axis must be found. Then as $V$ is known and $Q$ is defined as the statical moment of the area of the upper plank around the neutral axis, $q$ may be determined. The distance $y_c$ from the top to the neutral axis is

$$y_c = \frac{2(8)1 + 2(8)6}{2(8) + 2(8)} = 3.5 \text{ in.}$$

$$I = \frac{8(2)^3}{12} + (2)8(2.5)^2 + \frac{2(8)^3}{12} + (2)8(2.5)^2 = 291 \text{ in.}^4$$





$$Q = A_{fghj}\bar{y} = (2)8(3.5 - 1) = 40 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{690(40)}{291} = 95 \text{ lb per in.}$$

Thus, a force of 95 lb must be transferred from one plank to the other in every linear inch along the length of the beam. However, from the data given, each nail is capable of resisting a force of 150 lb, hence one nail can take care of 150/95 = 1.59 linear inches along the length of the beam. As shear remains constant at the consecutive sections of the beam, the nails should be spaced throughout at 1.59-in. intervals. In a practical problem a 1.5-in. spacing would probably be used.

SOLUTION FOR AN ALTERNATE ARRANGEMENT OF PLANKS

If, instead of using the two planks as above, a beam of the same cross section were made from five pieces, Fig. 7-6(b), a different nailing schedule would be required.

To begin, the shear flow between one of the 1-in.-by-2-in. pieces and the remainder of the beam is found, and although the contact surface $a$-$a$ is vertical, the procedure is the same as before. The push or pull on an element is built up in the same manner as formerly:

$$Q = A_{fghj}\bar{y} = (1)2(2.5) = 5 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{690(5)}{291} = 11.8 \text{ lb per in.}$$

If the same nails as before are used to join the 1-in.-by-2-in. piece to the 2-in.-by-2-in. piece, they may be 150/11.8 = 12.7 in. apart. This nailing applies to both sections $a$-$a$.

To determine the shear flow between the 2-in.-by-10-in. vertical piece and either one of the 2-in.-by-2-in. pieces, the whole 3-in.-by-2-in. area must be used to determine $Q$. It is the difference of pushes (or pulls) on this whole area that causes the unbalanced force which must be transferred at the surface $b$-$b$:

$$Q = A_{fghj}\bar{y} = (3)2(2.5) = 15 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{690(15)}{291} = 35.6 \text{ lb per in.}$$

Space nails at 150/35.6 = 4.2 in. or, in practice, 4-in. intervals along the length of the beam in both sections $b$-$b$. These nails could be driven in first and then the 1-in.-by-2-in. pieces put on.

EXAMPLE 7-2

A simple beam on a 20-ft span carries a load of 200 lb per foot including its own weight. The beam cross section is to be made from several full-sized wooden pieces as in Fig. 7-7(a). Specify the spacing of the ½-in.

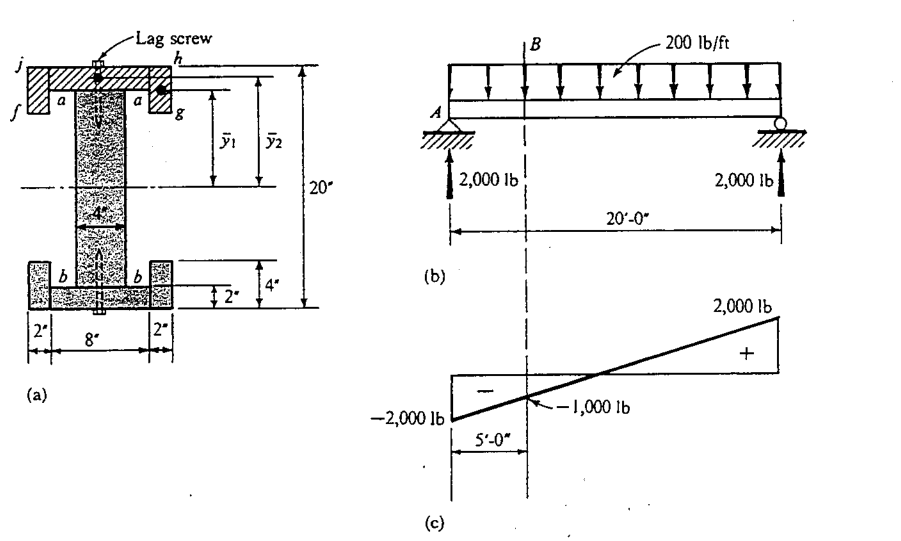



Fig. 7-7

lag screws shown which is necessary to fasten this beam together. Assume that one ½-in. lag screw, as determined by laboratory tests, is good for 500 lb when transmitting a lateral load parallel to the grain of the wood. For the entire section $I$ is equal to 6,060 in.⁴

SOLUTION

To find the spacing of the lag screws, the shear flow at section $a$-$a$ must be determined. The loading on the given beam is shown in Fig. 7-7(b); to show the variation of the shear along the beam, the shear diagram is constructed in Fig. 7-7(c). Then, to apply the shear flow formula, $Q$ must be determined. This is done by considering the shaded area to one side of the cut $a$-$a$ in Fig. 7-7(a). The statical moment of this area is most conveniently computed by multiplying the area of the two 2-in.-by-4-in. pieces by the distance from their centroid to the neutral axis of the beam and adding to this product a similar quantity for the 2-in.-by-8-in. piece. The largest shear flow occurs at the supports, as the largest vertical shears $V$ of 2,000 lb act there:

$$Q = A_{fghj}\bar{y} = \sum A_i\bar{y}_i = 2A_1\bar{y}_1 + A_2\bar{y}_2$$

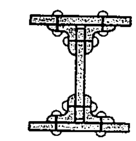
$$= 2(2)4(8) + 2(8)9 = 272 \text{ in.}^3$$

$$q = \frac{VQ}{I} = \frac{2{,}000(272)}{6{,}060} = 90 \text{ lb per in.}$$



At the supports the spacing of the lag screws must be 500/90 = 5.56 in. apart. This spacing of the lag screws applies only at a section where the shear $V$ is equal to 2,000 lb. Similar calculations for a section where $V = 1{,}000$ lb gives $q = 45$ lb per inch, and the spacing of the lag screws becomes 500/45 = 11.12 in. Thus it is proper to specify the use of ½-in. lag screws at 5½ in. on centers for a distance of 5 ft nearest both the supports and at 11 in. for the middle half of the beam. A greater refinement in making the transition from one spacing of fastenings to another may be desirable in some problems. The same spacing of lag screws should be used at the section $b$-$b$ as at the section $a$-$a$.

In a manner analogous to the above, the spacing of rivets or bolts in fabricated beams made from continuous angles and plates, Fig. 7-8, may be determined. Welding requirements are established similarly. The nominal shearing stress in a rivet is determined by dividing the total shearing force transmitted by the rivet (shear flow times spacing of the rivets) by the cross-sectional area of the rivet.



(a)

We

Fig. 7-8. Typical beam se of several components: (a) I beam reinforced with pla

## 7-4. THE SHEARING STRESS FORMULA FOR BEAMS

The shearing stress formula for beams may be obtained from the shear flow formula. Analogously to the earlier procedure, an element of a beam may be isolated between two adjoining sections taken perpendicular to the axis of the beam. Then by passing another imaginary section through this element parallel to the axis of the beam, a new element is obtained, which corresponds to the element of one "plank" used in the earlier derivations. A side view of such an element is shown in Fig. 7-9(a), where the imaginary longitudinal cut is made at a distance $y_1$ from the neutral axis.* The cross-sectional area of the beam is shown in Fig. 7-9(c).

If shearing forces exist at the sections through the beam, a different bending moment acts at section $A$ than at $B$. Hence more push or pull is developed on one side of the area $fghj$ than on the other, and, as before, the difference in the longitudinal forces in a distance $dx$ is

$$dF = -\frac{dM}{I}\int_{\substack{\text{area}\\ fghj}} y\, dA = -\frac{dM}{I} A_{fghj}\bar{y} = -\frac{dM}{I} Q$$

The force equilibrating $dF$ is developed in the plane of the longi-

* Since $dM/dx = -V$, for a positive $V$ the change in moment $dM = -V\,dx$. For this reason $M_A > M_B$ and the magnitudes of the normal stresses in Fig. 7-9(a) are shown accordingly.

occur in the plastic zones, no unbalance in longitudinal forces occurs and no shearing stresses are developed.

This elementary solution has been refined by using a more carefully formulated criterion of yielding caused by the simultaneous action of normal and shearing stresses.* Some fundamental aspects of the interaction of such stresses will be considered in Chapter 9.

**EXAMPLE 7-6**

An $I$ beam is loaded as in Fig. 7-14(a). If it has the cross section shown in Fig. 7-14(c), determine the shearing stresses at the levels indicated. Neglect the weight of the beam.

**SOLUTION**

A free-body diagram of a segment of the beam is in Fig. 7-14(b). It is seen from this diagram that the vertical shear at every section is 50 kips. Bending moments do not enter directly into the present problem. The shear flow at the various levels of the beam is computed in the table below using Eq. 7-5. Since $\tau = q/t$ (Eq. 7-6), the shearing stresses are obtained by dividing the shear flows by the respective widths of the beam.

$$I = \frac{6(12)^3}{12} - \frac{(5.5)(11)^3}{12} = 254 \text{ in.}^4$$

For use in Eq. 7-5 the ratio $V/I = -50{,}000/254 = -197$ lb/in.$^4$

| Level | $A_{fghj}$* | $\bar{y}$** | $Q = A_{fghj}\bar{y}$ | $q = VQ/I$ | $t$ | $\tau$, psi |
|---|---|---|---|---|---|---|
| 1-1 | 0 | 6 | 0 | 0 | 6.0 | 0 |
| 2-2 | $(0.5)6 = 3.00$ | 5.75 | 17.25 | −3,400 | 6.0<br>0.5 | −570<br>−6,800 |
| 3-3 | $(0.5)6 = 3.00$<br>$(0.5)(0.5) = 0.25$ | 5.75<br>5.25 | $\left.\begin{matrix}17.25\\1.31\end{matrix}\right\}18.56$ | −3,650 | 0.5 | −7,300 |
| 4-4 | $(0.5)6 = 3.00$<br>$(0.5)(5.5) = 2.75$ | 5.75<br>2.75 | $\left.\begin{matrix}17.25\\7.56\end{matrix}\right\}24.81$ | −4,890 | 0.5 | −9,780 |

* $A_{fghj}$ is the partial area of the cross section above a given level in square inches.
** $\bar{y}$ is the distance from the neutral axis to the centroid of the partial area in inches.

The negative signs of $\tau$ show that, for the section considered, the stresses act downward on the right face of the elements. The sense of the shearing stresses acting on the section coincides with the sense of the shearing force $V$. For this reason a strict adherence to the sign convention is often unnecessary. It is always true that $\int_A \tau\, dA$ is equal to $V$ and has the same sense.

* D. C. Drucker, "The Effect of Shear on the Plastic Bending of Beams," *Journal of Applied Mechanics*, **23** (1956), pp. 509–14.



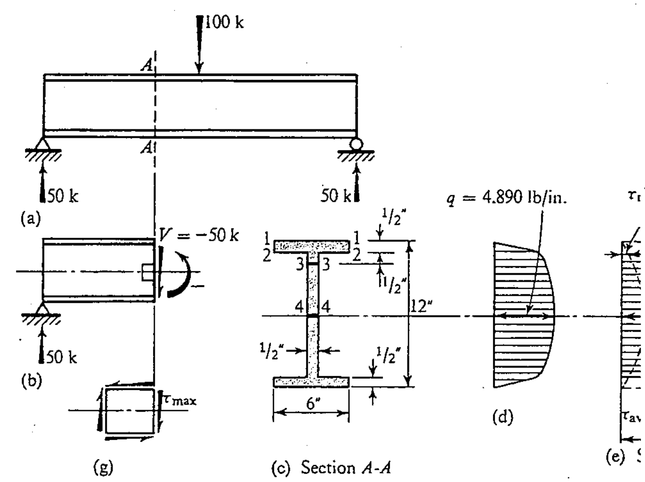



Fig. 7-14

Note that at the level 2-2 two widths are used to determine the shearing stress—one just above the line 2-2, and one just below. A width of 6 in. corresponds to the first case, and 0.5 in. to the second. This transition point will be discussed in the next article. The results obtained, which by virtue of symmetry are also applicable to the lower half of the section, are plotted in Fig. 7-14(d) and (e). By a method similar to the one used in the preceding example, it may be shown that the curves in Fig. 7-14(e) are parts of a second-degree parabola.

The variation of the shearing stress indicated by Fig. 7-14(e) may be interpreted as is shown in Fig. 7-14(f). The maximum shearing stress occurs at the neutral axis; the vertical shearing stresses throughout the web of the beam are nearly of the same magnitude. The shearing stresses occurring in the flanges are very small. For this reason the maximum shearing stress in an $I$ beam is often approximated by dividing the total shear $V$ by the cross-sectional area of the web (area *abcd* in Fig. 7-14(f)). Hence

$$(\tau_{max})_{approx} = V/A_{web} \tag{7-9}$$

In the example considered this gives

$$(\tau_{max})_{approx} = \frac{50{,}000}{(0.5)12} = 8{,}330 \text{ psi}$$

This stress differs by about 15 per cent from the one found by the accurate formula. For most cross sections a much closer approximation best approach. For example, if for the beam segment in Fig. 7-17(b) positive bending moments increase toward the reader, larger normal forces act on the nearer cross section. For the elements shown, $\tau t\,dx$ or $q\,dx$ must aid the smaller force acting on the partial area of the cross section. This determines the sense of the shearing stresses in the longitudinal cuts. Numerically equal shearing stresses act on the mutually perpendicular planes of an infinitesimal element, and the shearing stresses on such planes either meet or part with their directional arrowheads at a corner. In this manner the sense of the shearing stresses in the plane of the cross section becomes known.

The magnitude of the shearing stresses varies for the different vertical cuts. For example, if the cut $c$-$c$ in Fig. 7-17(a) is at the edge of the beam, the shaded area of the beam's cross section is zero. However, if the thickness of the flange is constant, and the cut $c$-$c$ is made progressively closer to the web, the shaded area increases from zero at a linear rate.

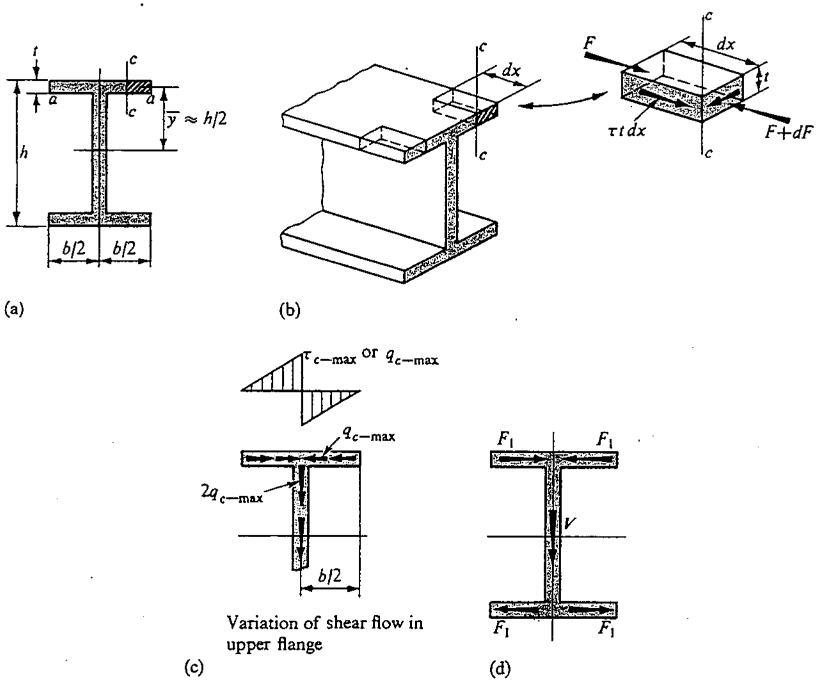



**Fig. 7-17.** Existence of shearing forces in the flange of an $I$ beam which act perpendicularly to the axis of symmetry.



Moreover, as $\bar{y}$ remains constant for any such area, $Q$ also increases linearly from zero toward the web. Therefore, since $V$ and $I$ are constant at any section through the beam, the shear flow $q_c = VQ/I$ follows the same variation. If the thickness of the flange remains the same, the shearing stress $\tau_c = VQ/It$ varies similarly. The same variation of $q_c$ and $\tau_c$ exists on either side of the axis of symmetry of the cross section. However, these quantities in the plane of the cross section act in opposite directions on the two sides as may be determined by isolating another flange element to the left side of the web in Fig. 7-17(b). The variation of these shearing stresses or shear flows is represented in Fig. 7-17(c), where it is assumed that the web is very thin.

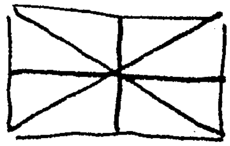
In common with all stresses, the shearing stresses in Fig. 7-17(c), when integrated over the area on which they act, are equivalent to a force. The magnitude of the horizontal force $F_1$ for one-half the flange, Fig. 7-17(d), is equal to the average shearing stress multiplied by one-half the flange area, i.e.,

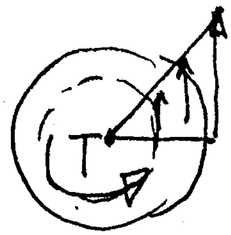
$$F_1 = (\tau_{c\text{-max}}/2)(bt/2) \qquad \text{or} \qquad F_1 = (q_{c\text{-max}}/2)(b/2)$$

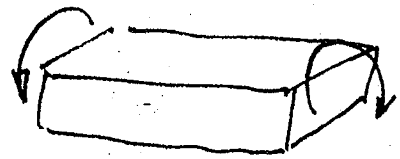
These horizontal forces act in the upper and lower flanges. Because of the symmetry of the cross section, these equal forces occur in pairs and oppose each other; thus they cause no apparent external effect.

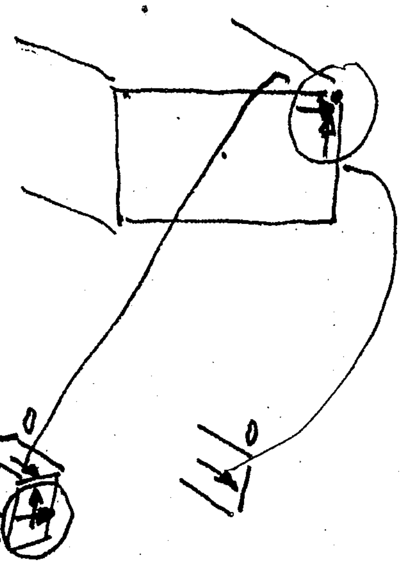
To determine the shear flow at the juncture of the flange and the web (cut $a$-$a$ in Fig. 7-17(a)), the whole area of the flange times $\bar{y}$ must be used in computing the value of $Q$. However, since in finding $q_{c\text{-max}}$ one-half the flange area times the same $\bar{y}$ has already been used, the sum of the two horizontal shear flows coming in from opposite sides gives the vertical shear flow at the cut $a$-$a$. Hence, figuratively speaking, the horizontal shear flows turn through 90° and merge to become the vertical shear flow. Then the shear flows at the various horizontal cuts through the web may be determined in the manner explained in the preceding articles. Moreover, as the resistance to the vertical shear $V$ in thin-walled $I$ beams is developed mainly in the web, it is so shown in Fig. 7-17(d). The sense of the shearing stresses and shear flows in the web coincides with the direction of the shear $V$. Note that the vertical shear flow "splits" upon reaching the lower flange. This is represented in Fig. 7-17(d) by the two forces $F_1$, which are the result of the horizontal shear flows in the flanges.

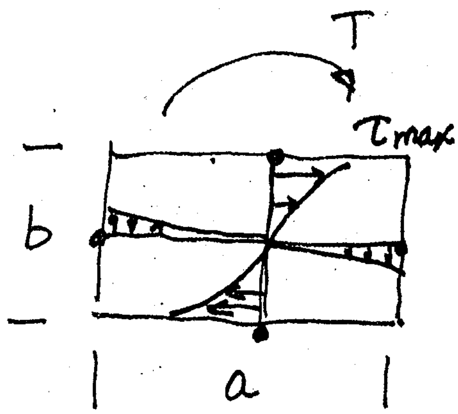
The shearing forces which act at a section of an $I$ beam are shown in Fig. 7-17(d), and, for equilibrium, the applied vertical forces must act through the centroid of the cross-sectional area to be coincident with $V$. If the forces are so applied, no torsion of the member will occur. This is true for all sections having cross-sectional areas with an axis of symmetry. Thus, to avoid torsion of such members, the applied forces must act in the plane of symmetry of the cross section and the axis of the beam. A beam with an unsymmetrical section will be discussed next.





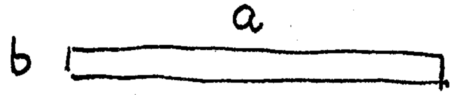




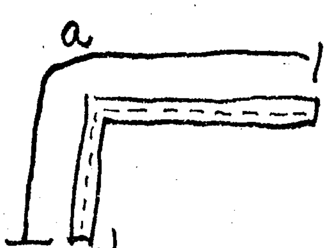

$$\tau_{max} = \frac{T}{c_1 a b^2} \qquad c_1 = f_1(a/b)$$

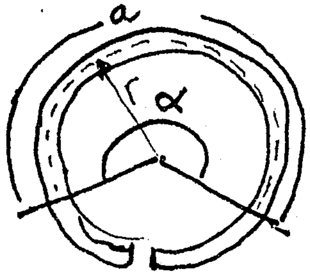
$$\varphi = \frac{TL}{c_2 a b^3 G} \qquad c_2 = f_2(a/b)$$

Table 3.1 pg 187




$c_1 = c_2 \cong 1/3$



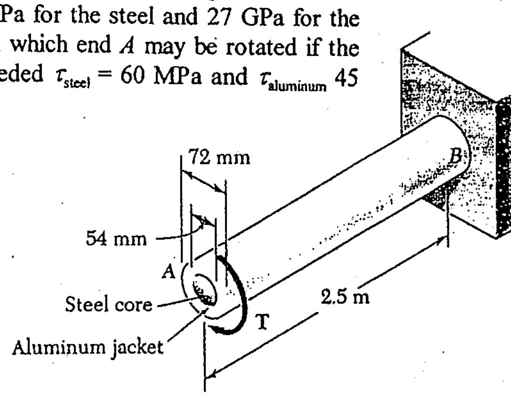




b = thickness
$a = 2\pi r = \alpha r$

Done on tape

**PROBLEM 3.53**

**3.53** The composite shaft shown is to be twisted by applying a torque **T** at end $A$. Knowing that the modulus of rigidity is 77 GPa for the steel and 27 GPa for the aluminum, determine the largest angle through which end $A$ may be rotated if the following allowable stresses are not to be exceeded $\tau_{steel} = 60$ MPa and $\tau_{aluminum}$ 45 MPa.




SOLUTION

$$\tau_{max} = G\gamma_{max} = G\, c_{max} \frac{\varphi}{L}$$

$$\frac{\varphi_{all}}{L} = \frac{\tau_{all}}{G\, c_{max}} \quad \text{for each material}$$

Steel core: $\tau_{all} = 60 \times 10^6$ Pa, $c_{max} = \frac{1}{2}d = 0.027$ m, $G = 77 \times 10^9$ Pa

$$\frac{\varphi_{all}}{L} = \frac{60 \times 10^6}{(77 \times 10^9)(0.027)} = 28.860 \times 10^{-3} \text{ rad/m}$$

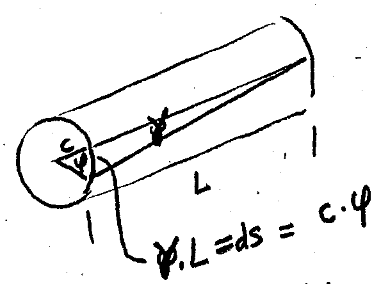
Aluminum jacket: $\tau_{all} = 45 \times 10^6$ Pa, $c_{max} = \frac{1}{2}d = 0.036$ m, $G = 27 \times 10^9$ Pa

$$\frac{\varphi_{all}}{L} = \frac{45 \times 10^6}{(27 \times 10^9)(0.036)} = 46.296 \times 10^{-3} \text{ rad/m}$$

Smaller value governs $\quad \frac{\varphi_{all}}{L} = 28.860 \times 10^{-3}$ rad/m

Allowable angle of twist $\quad \varphi_{all} = L\frac{\varphi_{all}}{L} = (2.5)(28.860 \times 10^{-3})$

$$= 72.15 \times 10^{-3} \text{ rad} = 4.13°$$




$\gamma \cdot L = ds = c \cdot \varphi$

$$\varphi = \frac{\gamma \cdot L}{c} = \frac{\tau_{max} \cdot L}{G \cdot c}$$

$\therefore \frac{\varphi}{L}$ must be same for both welded together at $B$ as shown in Fig. P6.96. The twisting couples at $A$ and $C$ are equal in magnitude when the couple $T_0$ is applied at $B$. Determine the relation between the elastic moduli of the two materials.

Answer: $G_A = 0.592G_C$

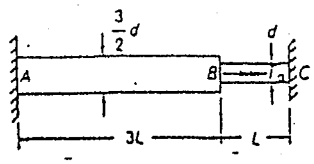



Fig. P6.96

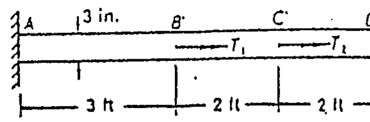
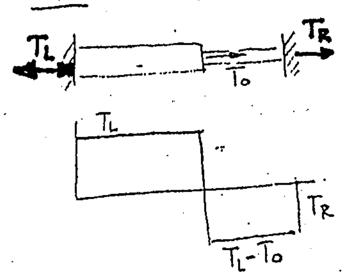



Fig. P6.97

6.97. An aluminum shaft is rigidly restrained at each end and loaded by couples as shown in Fig. P6.97. If $T_1 = 12{,}000$ in.-lb and $T_2 = 10{,}000$ in.-lb; compute the maximum shear stress in each section and the rotations of cross sections at $B$ and $C$.

Answer: $\tau_{CD} = -2300\ psi,\quad \tau_{BC} = -433\ psi,\quad \tau_{AB} = 1840\ psi,$

6:96 Done




$T_L - T_R = T_0$

$T_R = T_L - T_0$

since $|T_R| = |T_L|$

$T_L = T_0/2 \quad T_R = -T_0/2$

$$\varphi_{AB} = \frac{T_L L_{AB}}{J_{AB} G_{AB}} = |\varphi_{BC}| = \left|\frac{T_R L_{BC}}{J_{BC} G_{BC}}\right|$$

$$J_{AB} = \frac{\pi c^4}{2} = \frac{\pi d^4}{32} = \frac{\pi \cdot 81 d^4}{512};\quad L_{AB} = 3L;$$

$$J_{BC} = \frac{\pi c^4}{2} = \frac{\pi d^4}{32};\quad L_{BC} = L$$

$$\varphi_{AB} = \frac{T_0/2 \cdot \boxed{3L}}{\boxed{\pi}\, 81 d^4/512\, G_{AB}} = \frac{\boxed{T_0/2} \cdot \boxed{L}}{\boxed{\pi d^4}/32\, G_{BC}}$$

$$\therefore \frac{G_{AB} \cdot 81}{1536} = \frac{G_{BC}}{32} \quad \text{or} \quad G_{AB} = \frac{1536}{81 \cdot 32} G_{BC} = .5926\, G_{BC}$$

Fig. P3.108 and P3.109

3.132 A thin-walled tube has been fabricated by bending a metal plate of thickness $t$ into a cylinder of radius $c$ and bonding together the edges of the plate. A torque T is then applied to the tube, producing a shearing stress $\tau_1$ and an angle of twist $\phi_1$. Denoting by $\tau_2$ and $\phi_2$, respectively, the shearing stress and the angle of twist which will develop if the bond suddenly fails, express the ratios $\tau_2/\tau_1$ and $\phi_2/\phi_1$ in terms of the ratio $c/t$.

$\phi_2/\phi_1 = 3\left(\frac{c}{t}\right)^2$

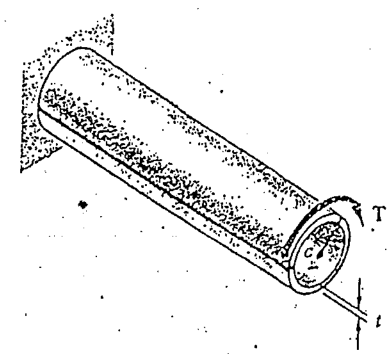



Fig. P3.132

3.122 A 3.5-m-long steel member with a W310 × 143 cross section is subjected to a 4.5 kN · m torque. Knowing that $G = 77$ GPa and referring to Appendix C for the dimensions of the cross section, determine (a) the maximum shearing stress along line $a$-$a$, (b) the maximum shearing stress along line $b$-$b$, (c) the angle of twist. (*Hint*: Consider the web and the flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

39.7 MPa 24.2 MPa 4.72°

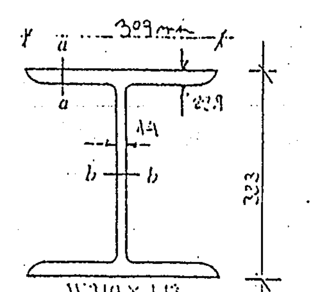
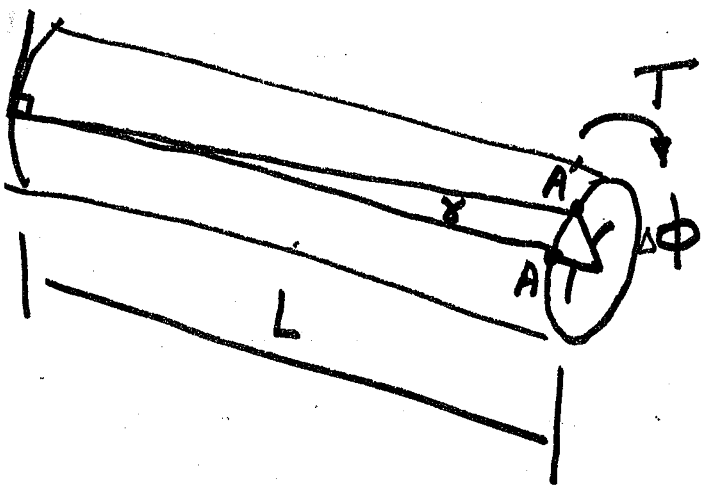



Fig. P3.122

angle of twist.



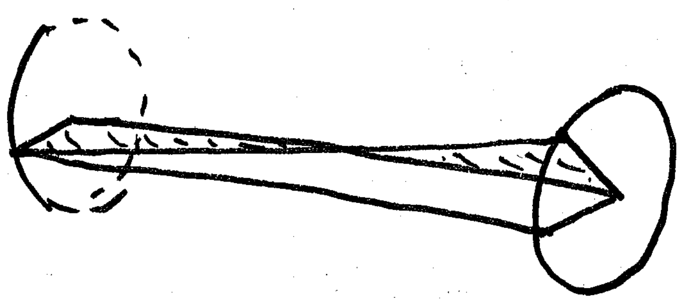

$$\overset{\frown}{AA'} = L \cdot \gamma = c\Delta\phi$$

$$\gamma = \frac{\tau_{max}}{G}$$

$$L\frac{\tau_{max}}{G} = c\Delta\phi$$

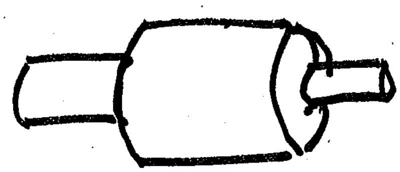
Angle of Twist

$$\Delta\phi = \frac{L}{G}\frac{\tau_{max}}{c} = \frac{L}{G}\cdot\frac{T}{J}$$



$$\frac{\Delta\phi}{L} - \text{unit angle of twist} = \frac{T}{GJ}$$

$$\phi_{TOT} = \sum \Delta\phi_i = \sum \frac{T_i L_i}{G_i J_i}$$

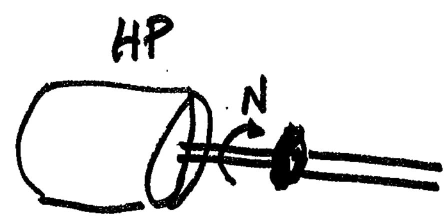


$$= \int_0^L \frac{T\,dx}{GJ}$$

$$u_{TOT} = \sum \frac{P_i L_i}{A_i E_i}$$

$$u = \int_0^L \frac{P\,dx}{AE}$$

$$\text{Power} = F \cdot V \quad \text{or} \quad T \cdot \omega = T \cdot 2\pi \underbrace{N}_{\text{revs/min}}$$




$$[\text{lb-in}]\; T = 63025 \cdot \frac{HP}{N\,(\text{revs/min})}$$

$$[\text{lb-ft}]\; T = 5252 \cdot \frac{HP}{N\,(\text{revs/min})}$$

$$[\text{N-m}]\; T = 9540 \cdot \frac{P\,(\text{kw})}{N\,(\text{revs/min})}$$

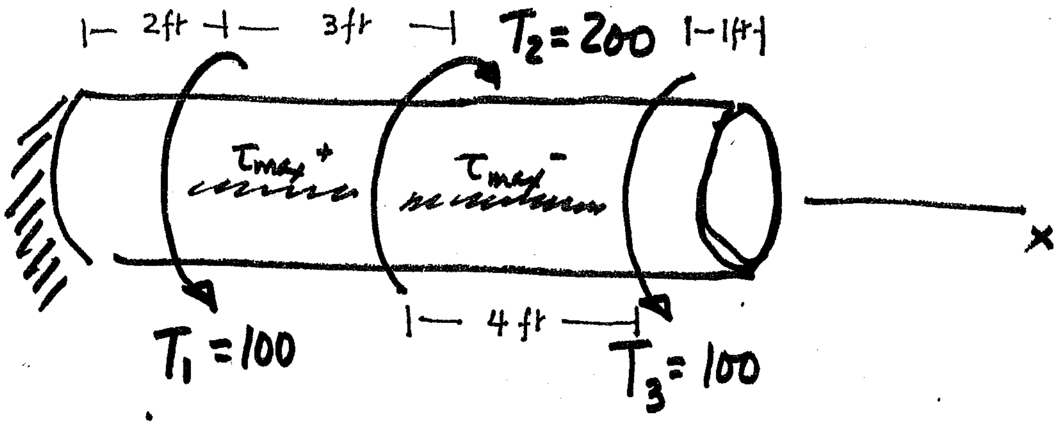
Design a steel shaft for 10 Hp motor operating at 1800 rpm

Max. shear is 8000 psi

$$T = 63025\,\frac{HP}{N} = \frac{63025 \cdot 10}{1800} = 350 \text{ lb-in}$$

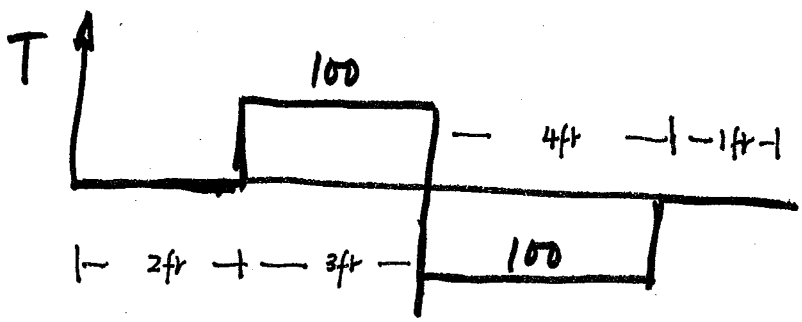
$$\tau_{max} = \frac{Tc}{J} = 350\,\frac{c}{\pi c^4/2} = \frac{700}{\pi c^3} = 8000$$

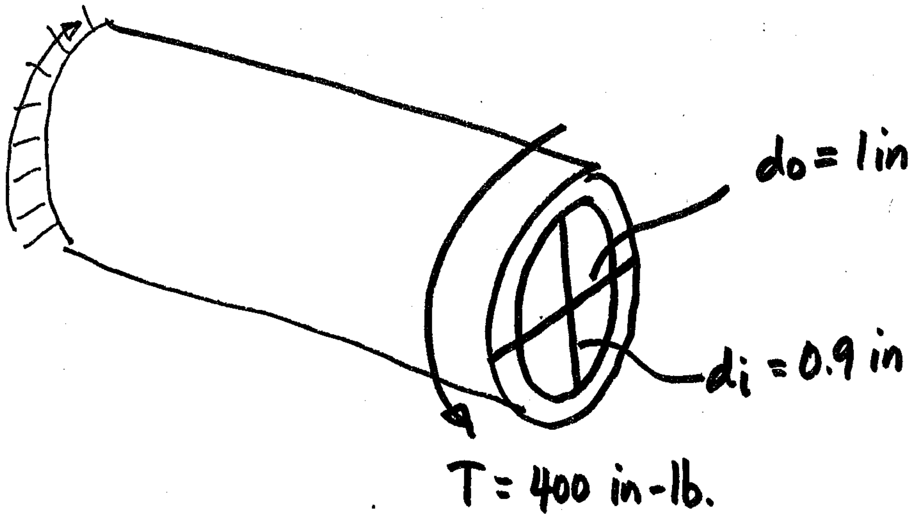
$$c = \sqrt[3]{\frac{700}{\pi(8000)}} = .303 \text{ in}$$




WHERE is $\tau_{max}$

$$\tau_{max} = \frac{Tc}{J}$$



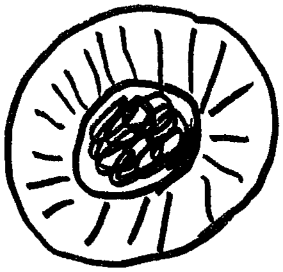




what is $\tau$ at the inner surface $c_i = 0.45 in$

$\tau$ at the outer surface $c_o = 0.5 in$

$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right) = 0.0337 \, in^4$$

$$\tau_{inner} = \frac{Tc_i}{J} = \frac{400(.45)}{.0337 \, in^4} \cong 5330 \, psi \, ; \quad \tau_{outer} = \frac{Tc_o}{J} = \frac{400(.5)}{.0337} \cong 5930$$



$\rho = .448 \Rightarrow$ carries only 4% of torque

$$T = \frac{\tau_{max}}{c} J = \frac{\tau}{\rho} J = \frac{\tau_{max}}{c} \cdot \frac{\pi c^4}{2}$$

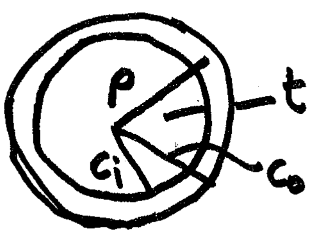
$.06T \Rightarrow \frac{r}{c} = .495$

$.07T \Rightarrow \frac{r}{c} = .515$

$$.04T = \frac{\tau_{max}}{c} \cdot \frac{2\pi r^4}{4}$$

$$.04 = \left(\frac{r}{c}\right)^4$$

$$.448 = \frac{r}{c}$$




Thin tube

$$c = \frac{c_i + c_o}{2}$$

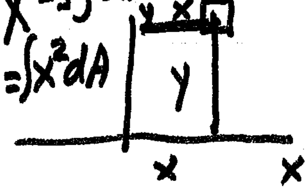
$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right)$$
$$= \frac{\pi}{2}\left(c_o^2 - c_i^2\right)\left(c_o^2 + c_i^2\right)$$
$$= \frac{\pi}{2}\left(c_o - c_i\right)\left(c_o + c_i\right)\left(c_o^2 + c_i^2\right)$$
$$= \pi \cdot tc \cdot 2c^2$$

$$\underline{J = 2\pi t c^3}$$

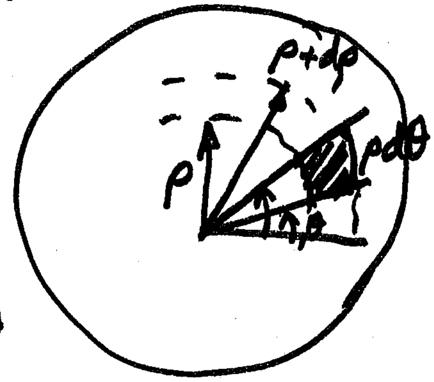
$J = \int \rho^2 dA$ Polar moment of inertia

$I_x = \int y^2 dA$

$I_y = \int x^2 dA$




$dA = \rho \, d\rho \, d\theta$




$J = \iint \underbrace{\rho^2 \cdot \rho \, d\rho}_{} \cdot d\theta$

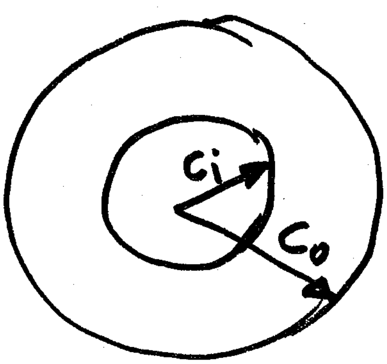
SOLID CYLINDER

$$= \int_0^{2\pi} d\theta \int_0^{c} \rho^3 d\rho = 2\pi \cdot \frac{\rho^4}{4}\Big|_0^c = \frac{\pi}{2} c^4 = J$$

$$T = \frac{\tau_{max}}{c} \cdot J \Rightarrow \quad T = \frac{\tau}{\rho} J \qquad \boxed{\tau = \frac{T\rho}{J}}$$

$$\tau_{max} = \frac{Tc}{J}$$

Thick Tube




$$J = \int \rho^2 dA = \int_0^{2\pi} d\theta \cdot \int_{c_i}^{c_o} \rho^3 d\rho$$

$$J = 2\pi \left[ \frac{c_o^4}{4} - \frac{c_i^4}{4} \right]$$

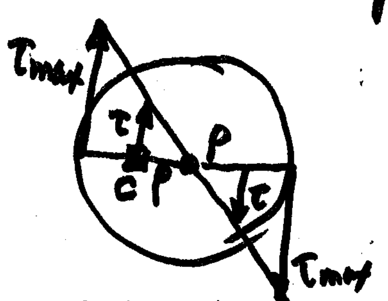
EMA 3702 $2^{nd}$ TAPE

TORSION


Assumptions

① Plane sections remain plane after deformation & no warping or distortion of the plane section normal to the long axis

② For cylindrical members, shear stresses are not constant but vary linearly from the center of the member to a maximum on the outer surface




$$\frac{\tau_{max}}{c} = \frac{\tau}{\rho}$$

③ Material remains elastic & Hooke's Law applies

$$\tau \sim \gamma \qquad \tau = G\gamma$$

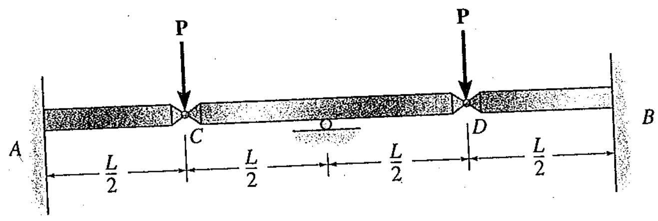
$$dF = \tau\, dA$$

$$dT = (\tau\, dA)\rho$$

$$T = \int dT = \int \tau \rho\, dA = \int \rho \frac{\tau_{max}}{c} \cdot \rho\, dA$$

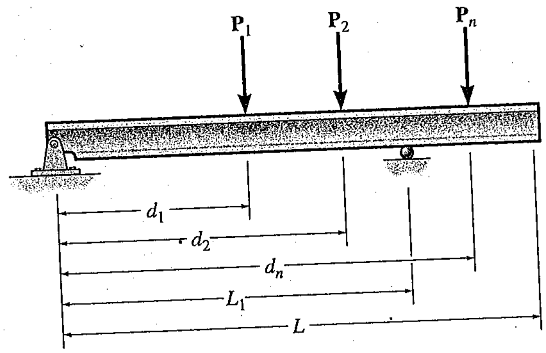
$$= \frac{\tau_{max}}{c} \int \rho^2 dA$$

**6-13.** The bars are connected by pins at $C$ and $D$. Draw the shear and moment diagrams for the assembly. Neglect the effect of axial load.



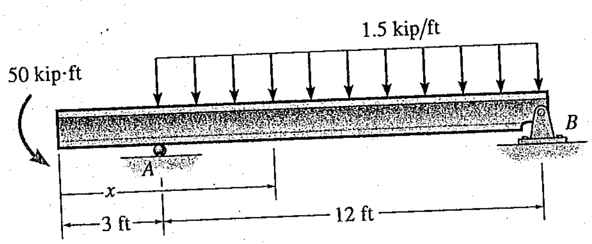

**Prob. 6–13**

■**6-14.** Consider the general problem of a simply supported beam subjected to $n$ concentrated loads. Write a computer program that can be used to determine the internal shear and moment at any specified location $x$ along the beam, and plot the shear and moment diagrams for the beam. Show an application of the program using the values $P_1 = 500$ lb, $d_1 = 5$ ft, $P_2 = 800$ lb, $d_2 = 15$ ft, $L_1 = 10$ ft, $L = 15$ ft.



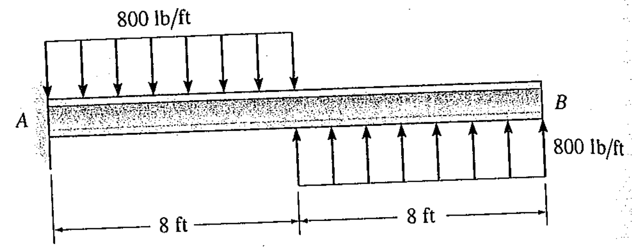

**Prob. 6–14**

**6-15.** Draw the shear and moment diagrams for the beam. Also, determine the shear and moment in the beam as functions of $x$, where 3 ft $< x \leq$ 15 ft.



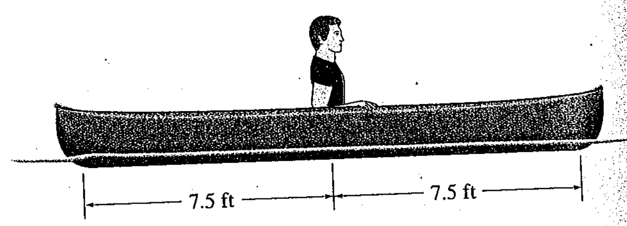

**Prob. 6–15**

***6-16.** Draw the shear and moment diagrams for the beam.



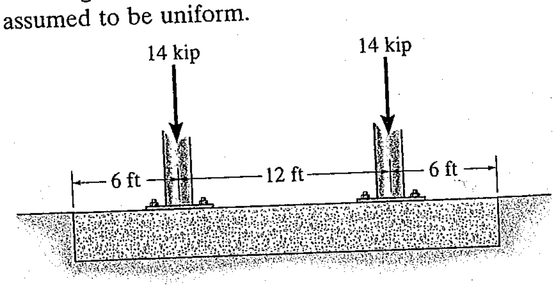

**Prob. 6–16**

**6-17.** The 150-lb man sits in the center of the boat, which has a uniform width and a weight per linear foot of 3 lb/ft. Determine the maximum bending moment exerted on the boat. Assume that the water exerts a uniform distributed load upward on the bottom of the boat.



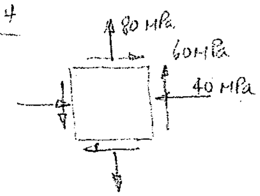

**Prob. 6–17**

**6-18.** The footing supports the load transmitted by the two columns. Draw the shear and moment diagrams for the footing if the reaction of soil pressure on the footing is assumed to be uniform.




**Prob. 6–18**

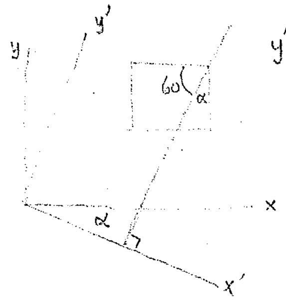
6.4




$\sigma_x = -40$ MPa
$\tau_{xy} = 60$ MPa
$\sigma_y = 80$ MPa

$$\sigma_x' = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\alpha + \tau_{xy}\cos 2\alpha$$



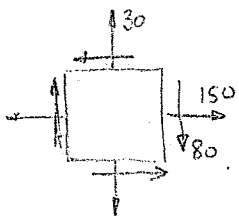

לפי הדיאגרמה $\alpha = -30°$ ; כי האלכסון הוא מקביל ל-$y'$

$$\sigma_x' = \left(\frac{-40+80}{2}\right) + \left(\frac{-40-80}{2}\right)\cos(-60) + 60\sin(-60)$$
$$= 20 + (-60)\cdot\tfrac{1}{2} + 60(-.866) = \underline{-61.96 \text{ MPa}}$$

$$\tau_{x'y'} = -\left(\frac{-40-80}{2}\right)\sin(-60) + 60\cos(-60)$$
$$= +60(-.866) + 60\left(\tfrac{1}{2}\right) = \underline{-21.96 \text{ MPa}}$$

---

6.6




$\sigma_x = 150$ MPa ✓
$\sigma_y = 30$ MPa ✓
$\tau_{xy} = -80$ MPa ✓

למצוא את הצירים הראשיים וגם המאמצים הראשיים

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \left[\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

לכן

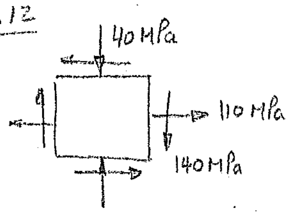
$$\sigma_{1,2} = \left(\frac{150+30}{2}\right) \pm \left[\left(\frac{150-30}{2}\right)^2 + (-80)^2\right]^{1/2}$$
$$= 90 \pm 100 \Rightarrow \sigma_1 = 190 \text{ MPa} ✓ \quad \sigma_2 = -10 \text{ MPa} ✓$$

$$\tan 2\theta = \frac{2(-80)}{150-30} = -1.333$$

7/7 $\quad 2\theta = -53.13 \quad \theta = \underline{-26.57°}$ ✓ $\quad \theta_2 = \theta + 90° = \underline{63.43°}$ ✓

---

6.12




$\sigma_x = 110$ MPa ✓
$\sigma_y = -40$ MPa ✓
$\tau_{xy} = -140$ MPa ✓

למצוא את הצירים של מאמץ הגזירה המקסימלי והמאמץ המקס'.

$$\tau_{\text{מקס}} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \left[\left(\frac{110-(-40)}{2}\right)^2 + (-140)^2\right]^{1/2} = \pm 158.82 \text{ MPa} ✓$$

$$\tan 2\alpha = \frac{-(\sigma_x-\sigma_y)}{2\tau_{xy}} = \frac{-[110-(-40)]}{2(-140)} = \frac{-150}{-280} \Rightarrow 208.18°$$

מפני שהיחס הוא שלילי חלקי שלילי

$\alpha = 104.09°$ ✓

$\alpha + 90° = 194.09° = 14.09°$ ✓

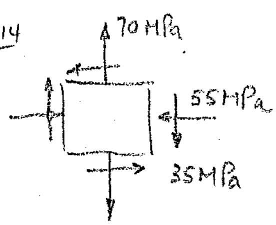
$$\sigma_x' = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha$$

7/7

$$= \frac{110-40}{2} + \left(\frac{110-(-40)}{2}\right)\cos 208.18 + (-140)\sin 208.18°$$
$$= 35 + 75\cos 208.18 - 140\sin 208.18 = 35 \text{ MPa} = (\sigma_x+\sigma_y)/2 ✓$$

---

6.14




$\sigma_x = -55$ MPa ✓
$\sigma_y = 70$ MPa ✓

בסעיף א) למצוא את המאמצים כאשר החלקיק הולך ומסתובב 40° נגד השעון (מאמצים נורמלי וגזירה).
בסעיף ב) אותו תהליך כאשר החלקיק הולך ומסובב 15° בכיוון השעון.

$$\sigma_x' = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha \qquad \alpha = 40° ✓$$
$$= \frac{-55+70}{2} + \left(\frac{-55-70}{2}\right)\cos 80 + (-35)\sin 80$$
$$= 7.5 - 62.5\cos 80 - 35\sin 80 = -37.82 \text{ MPa} ✓$$

$$I_{1\sigma} = \sigma_x' + \sigma_y' = (\sigma_x+\sigma_y) \Rightarrow \sigma_y' = (\sigma_x+\sigma_y) - \sigma_x' = (-55+70) - (-37.82) = 52.82\ MPa$$

✓ $\alpha = -15°$ (ב

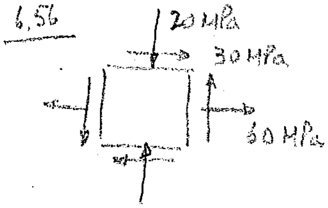
$$\sigma_x' = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha$$
$$= 7.5 - 62.5\cos(-30) + (-35)\sin(-30) = 7.5 - 62.5(.866) - 35(-\tfrac{1}{2}) = -29.13\ MPa$$ ✓

$$\sigma_y' = I_{1\sigma} - \sigma_x' = (\sigma_x+\sigma_y) - \sigma_x' = 15 - (-29.13\ MPa) = 44.13\ MPa$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\alpha + \tau_{xy}\cos 2\alpha$$
$$= 62.5\sin(-30) + (-35)\cos(-30) = 62.5(-\tfrac{1}{2}) - 35(.866) = -61.56\ MPa$$ ✓

9/9

6.56




$\sigma_x = 60\ MPa$
$\sigma_y = -20\ MPa$
$\tau_{xy} = 30\ MPa$

$$\sigma_x' = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\alpha + \tau_{xy}\sin 2\alpha$$
$$= \frac{60-20}{2} + \frac{60+20}{2}\cos 2\alpha + 30\sin 2\alpha$$
$$\sigma_x' = 20 + 40\cos 2\alpha + 30\sin 2\alpha$$

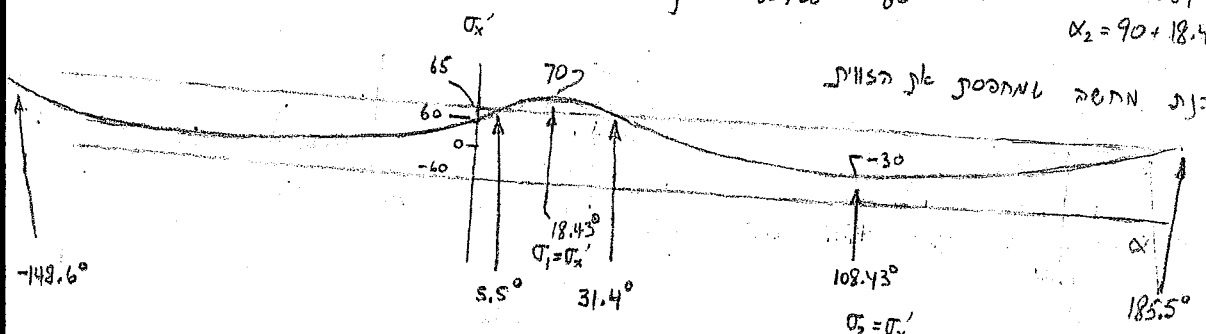
מקס, מין $\sigma_x' = \frac{\sigma_x+\sigma_y}{2} \pm \left[\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$

$$= \frac{60-20}{2} \pm \left[\left(\frac{60+20}{2}\right)^2 + 30^2\right]^{1/2} = 20 \pm \left[40^2+30^2\right]^{1/2} = 20 \pm 50 = \begin{cases} 70 \\ -30 \end{cases} MPa$$

$$\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(30)}{60+20} = \frac{60}{80} = 0.75$$

$2\alpha = 36.87°$ $\alpha = 18.43°$
$\alpha_2 = 90 + 18.43° = 108.43°$

אם נשתמש בתוכנת מחשב ונחפש את הזווית.




$$\cos 2\alpha = \cos(360+2\alpha) = \cos(-[360-2\alpha])$$

אם $\alpha = 5.5$
$371 = 360+2\alpha$
וחצי מזה $185.5°$. רואים ש- $180° = 185.5° - 5.5$

אם $\alpha = 31.4$
$-297.2 = -(360-2\alpha)$
וחצי מזה $-148.6°$. רואים ש- $-180° = -148.6° - 31.4$

הערות: הפכה (השיפוע של הנקודה) ב- $31.4°$ ו- $-148.6°$ זהות
הפכה של $5.5°$ ו- $185.5°$ זהות

לכן, בין $31.4° \le \alpha \le 185.5$ $\quad 65 > \sigma_x'$
ובין $-148.6° \le \alpha \le 5.5$ $\quad 65 > \sigma_x'$

Fig. 1 is given by $\sigma_{xx}=1000y-500z+800$ kPa, $\sigma_{xy}=200z$ kPa, $\sigma_{xz}=0$.
What is the net internal force system on this cross-section ?

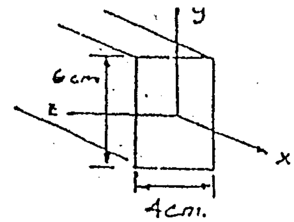


Fig. 1

Answer : $F=1920$ N, $V_y=V_z=0$
$M_y=-1600$ N-cm., $M_z=-7200$ N-cm., $T=-640$ N-cm.

2. Suppose the stress distribution on a cross-section of the circular cylinder of Fig. 2 is given by $\sigma_{xx}=\sigma_{xr}=0$, $\sigma_{x\theta}=k\sqrt{r}$, where k is unknown. What is the value of k in this case?

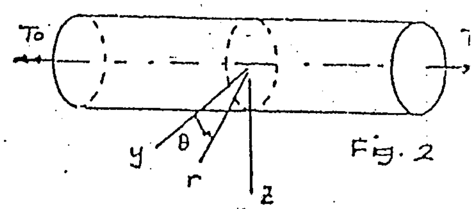


Fig. 2

Answer : $k=7T_0/4\pi R^{7/2}$

6.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium equations of that element, as was done in the derivations presented in Sec. 6.2.

6.6 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

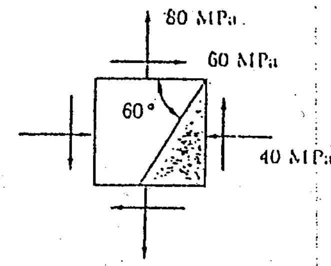


Fig. P6.4

*6.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

*6.14 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 40° counterclockwise, (*b*) 15° clockwise.

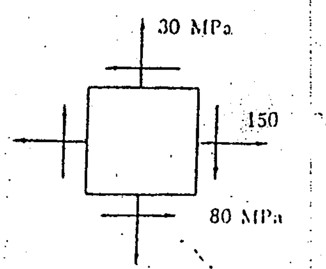
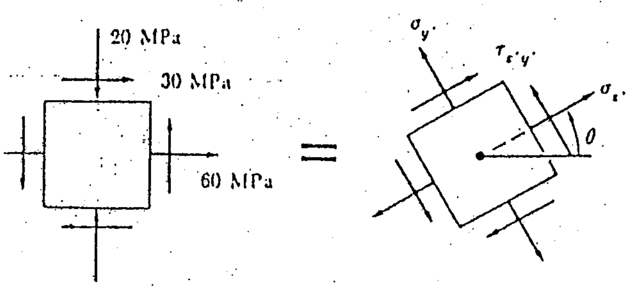


Fig. P6.6 and P6.10

*6.56 For the state of plane stress shown, determine the range of values of $\theta$ for which the normal stress $\sigma_{x'}$ is equal to or less than +65 MPa.



P6.55 and P6.56

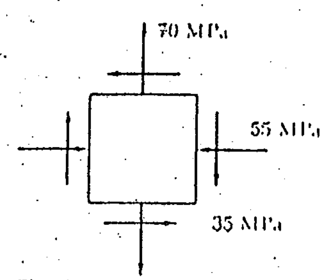


Fig. P6.14

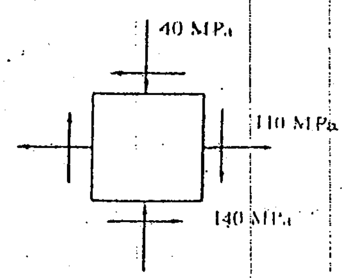


Fig. P6.8 and P6.12

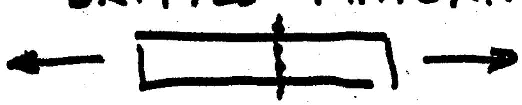
25
/12

# EMA 3702

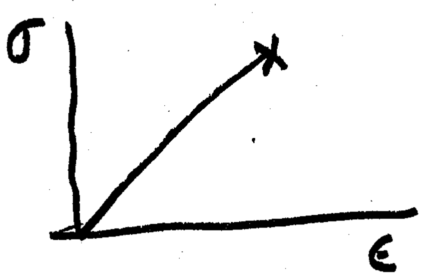
## EXTRA SESSIONS

1. FAILURE CRITERION - Chpt 7
2. BUCKLING - Chpt 10

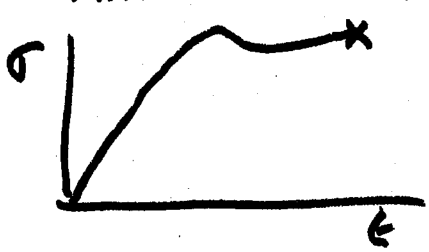
BRITTLE MATERIAL

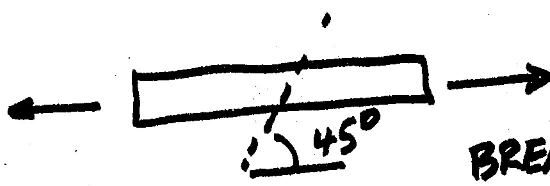


BREAK $\perp$ TO AXIS OF LOAD $\sigma_1 > \sigma_{max}$




MATERIALS W/ PLASTICITY




BREAK SURFACE IS 45° TO APPLIED LOAD.

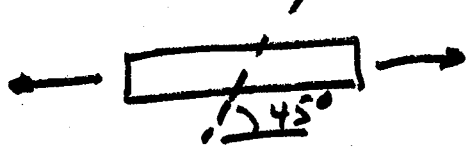
# TRESCA – MATERIALS W/ PLASTICITY

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\sigma_{1_{max}} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{2_{min}} = (\;\text{"}\;) - \sqrt{\;\text{"}\;}$$

$$\frac{\sigma_1 - \sigma_2}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \tau_{max}$$




$$\tau_{max} = \frac{\sigma_{yp}}{2} = \frac{(\sigma_1 - \sigma_2)}{2}$$

uniaxial test　　BIAXIAL

$\sigma_1 > \sigma_2 > \sigma_3$　MOST GEN　$\tau_{max} = \frac{\sigma_{yp}}{2} = \frac{(\sigma_1 - \sigma_3)}{2}$

$\sigma_1 > 0 > \sigma_2$　BIAXIAL PROB

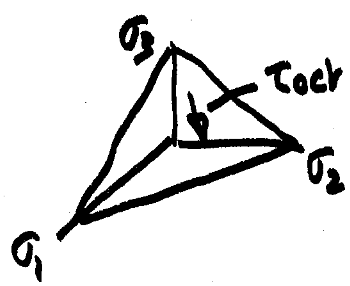
$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$$

# VON MISES – ENERGY CRITERION – MATERIALS W/ PLASTICITY

FAILURE: $\sigma_e$ – effective stress $\geq \sigma_{yp}$

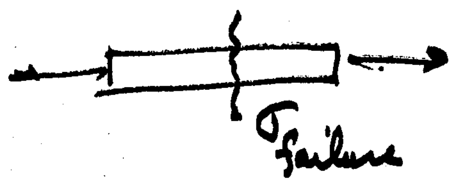
$$\sigma_{yp} \leq \sigma_e = \frac{3}{\sqrt{2}}\tau_{oct} = \frac{1}{\sqrt{2}}\sqrt{\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right]}$$

$$= \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$




FOR BIAXIAL all $\sigma_z, \tau_{yz}, \tau_{zx} = 0$
or $\sigma_z = 0$

## PRINCIPAL STRESS - BRITTLE $\sigma_1 > \sigma_{failure} \Rightarrow$ FAILED




IN 2-D

$$\sigma_{max} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} > \sigma_{fail}$$

FOR EXAMPLE $\sigma_1 \geq \sigma_{yp}$ is failure

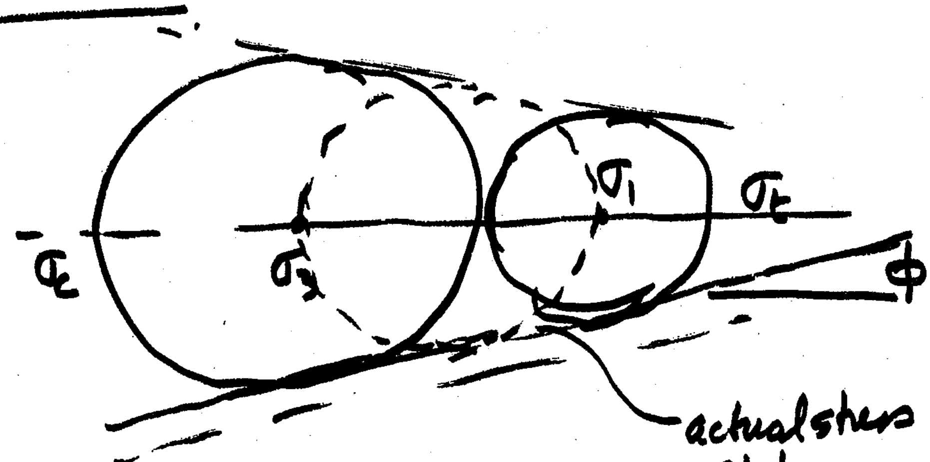
$$\sigma_{max} = \qquad \geq \sigma_{yp}$$

## PRINC. STRAIN

$$\epsilon_1 > \epsilon_{failure} \Rightarrow \frac{1}{2}\left[(\epsilon_x + \epsilon_y) + \sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}\right] > \epsilon_{fail}$$

FOR EXAMPE $\epsilon_{fail} = \dfrac{\sigma_{yp}}{E}$

## MOHR CRITERION - BRITTLE MATERIAL



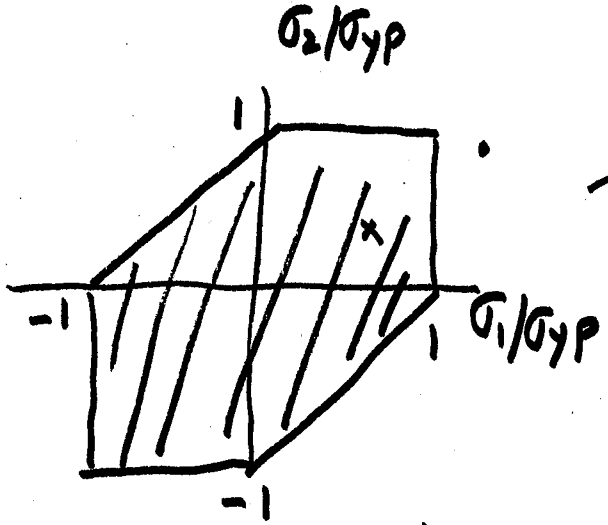

plane of failure $45° + \frac{1}{2}\phi$

FAILURE IF $\dfrac{\sigma_1}{\sigma_t} - \dfrac{\sigma_3}{|\sigma_c|} \geq 1$

$\sigma_t$ - max failure stress in tension

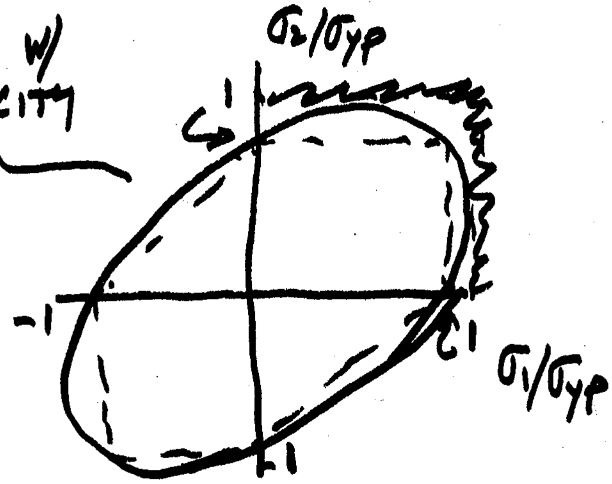
$\sigma_c$ - max failure stress in compression

MATERIALS W/ PLASTICITY




inside you are safe

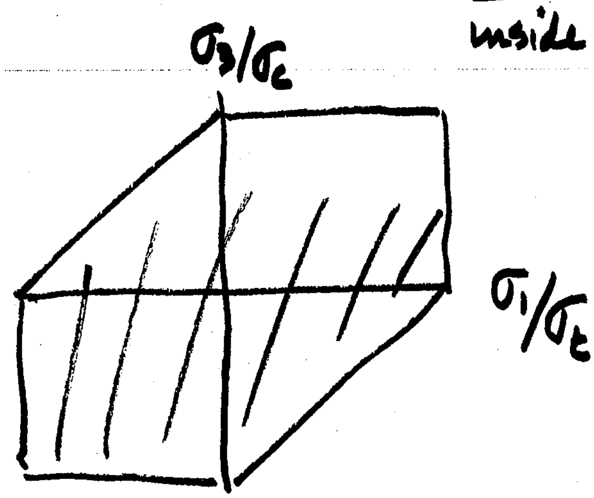
TRESCA




VON MISES

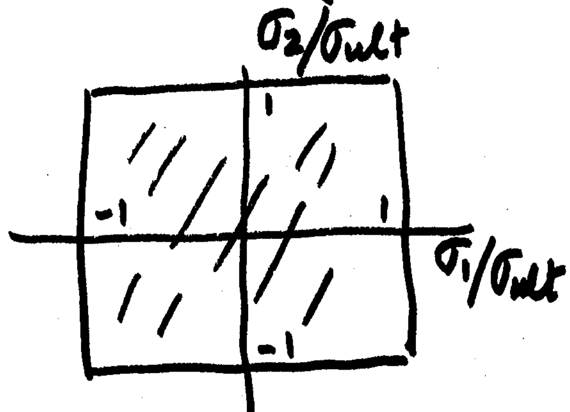
inside you are safe

$$\begin{pmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_y & 0 \\ 0 & 0 & 0 \end{pmatrix}$$




MOHR

inside you are safe




PRINCIPAL STRESS

inside safe

BRITTLE

TENSION & COMP. TESTS FOR BRITTLE MATERIAL

$\sigma_t = 14$ MPa $\qquad \sigma_c = 120$ MPa

UNDER A CERTAIN LOADING : $\sigma_x = 0$, $\sigma_y = -18$ MPa, $\tau_{xy} = 20$ MPa

ARE WE SAFE ? CONSIDER PLANE STRESS ($\sigma_z, \tau_{xz}, \tau_{yz} = 0$)

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0-18}{2} + \sqrt{\left(\frac{0-(-18)}{2}\right)^2 + (20)^2} = -9 + 21.9 = 12.9 \text{ MPa}$$

$$\sigma_2 = \left(\frac{\sigma_x - \sigma_y}{2}\right) - \sqrt{\phantom{xxxx}} = -9 - 21.9 = -30.9 \text{ MPa}$$

$$\sigma_1 > 0 > \sigma_2 \implies 12.9 > 0 > -30.9$$

$$\sigma_e = \frac{1}{\sqrt{2}}\left[(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right]^{1/2}$$

$$= \frac{1}{\sqrt{2}}\left[(0+18)^2 + (-18-0)^2 + (0-0)^2 + 6(20^2 + 0^2 + 0^2)\right]^{1/2}$$

$$= 39.04 \text{ MPa}$$

PRINC. STRESS CRIT : $\sigma_1 > \sigma_t \Rightarrow$ FAIL $\qquad 12.9 \overset{?}{>} 14$ NO ✓

TRESCA : $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{12.9 - (-30.9)}{2} = 21.9$ MPa

$\Rightarrow$ 1-D TEST $\quad \sigma_{yp} = \sigma_t = 14$ MPa , $\tau_{max} = \dfrac{\sigma_{yp}}{2} = 7$ MPa

$\tau_{max} \overset{?}{>} \dfrac{\sigma_{yp}}{2} \qquad 21.9 > 7$ MPa $\quad$ YES $\quad$ X

MISES : $\sigma_e = 39.04$ MPa $\qquad \sigma_e \geq \sigma_{yp} \Rightarrow$ FAIL $\qquad \sigma_{yp} = \sigma_t = 14$ MPa

$39.04 \overset{?}{>} 14$ $\quad$ YES $\quad$ X

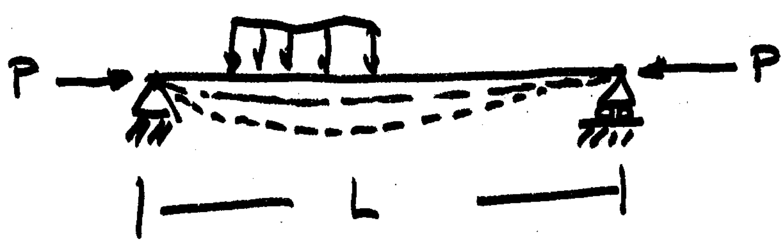
## MOHR CRITERIA

$$\frac{\sigma_1}{\sigma_t} - \frac{\sigma_3}{|\sigma_c|} \geq 1 \implies \text{FAIL}$$

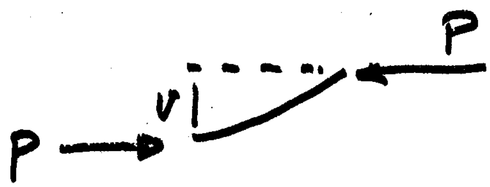
$$\frac{12.9}{14} - \frac{(-30.9)}{120} = 1.18 \overset{?}{\geq} 1 \quad \text{YES} \qquad \times$$

---

FAILURE — DUE TO EXCESSIVE DISPLACEMENT
BUCKLING



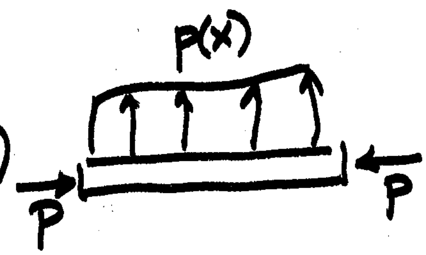

$$\frac{v}{L} << 1$$




Normally we have $EIv'' = M$

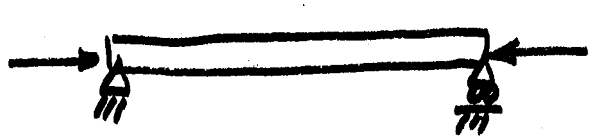
$$EIv'' + Pv = M$$

$$(EIv'')'' + (Pv)'' = +p(x)$$




IF $EI = \text{const.}$ $\quad v^{IV} + \frac{P}{EI} v'' = \frac{p(x)}{EI}$

$$\frac{P}{EI} = \lambda^2$$



$$v^{IV} + \lambda^2 v'' = 0$$

$$v''' + \lambda^2 v' = C_1$$

$$v'' + \lambda^2 v = C_1 x + C_2$$

homog soln $\quad v'' + \lambda^2 v = 0 \Rightarrow A\cos\lambda x + B\sin\lambda x = v_h$

particular $\quad v'' + \lambda^2 v = C_1 x + C_2$

$$\underline{C_1 x} \to v_p = Cx \qquad \underline{\underline{C_2}} \to v_p = D$$

$v_p = D$ : $v_p'' + \lambda^2 v_p = 0 + \lambda^2 D = C_2 \qquad D = C_2/\lambda^2$

$$v_T = v_h + v_p = A\cos\lambda x + B\sin\lambda x + Cx + D$$
$$v_T'' = -\lambda^2 A\cos\lambda x - \lambda^2 B\sin\lambda x$$

$v(x=0) = 0 \qquad v(x=L) = 0$

$v''(x=0) = 0 \qquad v''(x=L) = 0$

$$A + 0B + 0C + D = 0$$
$$-\lambda^2 A + 0\cdot B + 0C + 0\cdot D = 0$$
$$A\cos\lambda L + B\sin\lambda L + CL + D = 0$$
$$-\lambda^2 A\cos\lambda L - \lambda^2 B\sin\lambda L + 0\cdot C + 0\cdot D = 0$$

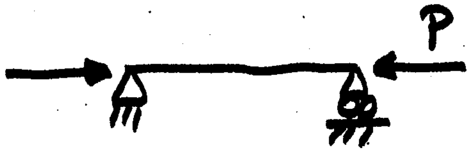
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -\lambda^2 & 0 & 0 & 0 \\ \cos\lambda L & \sin\lambda L & L & 1 \\ -\lambda^2\cos\lambda L & -\lambda^2\sin\lambda L & 0 & 0 \end{bmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

det matrix $= L\lambda^4 \sin\lambda L = 0 \Rightarrow \sin\lambda L = 0$

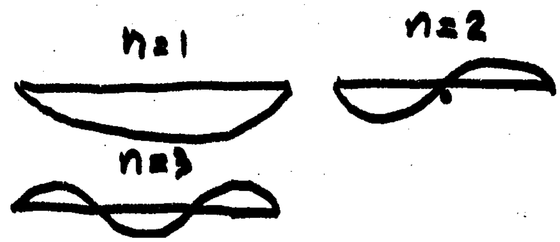
$\Rightarrow \lambda L = n\pi$

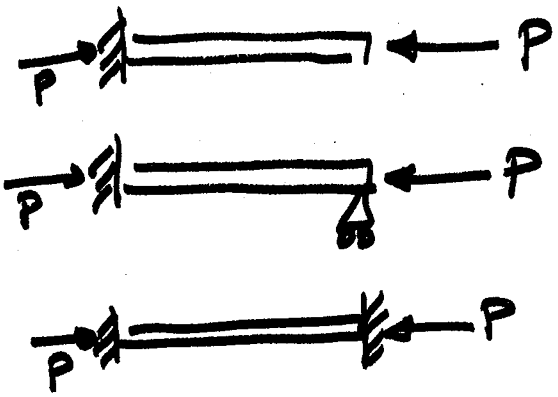
$$\sqrt{\frac{P}{EI}} = \lambda = \frac{n\pi}{L}$$

$$P = \frac{n^2\pi^2}{L^2}\cdot EI \qquad n=1 \quad P = \frac{\pi^2 EI}{L^2}$$ Euler Buckling Load.




$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

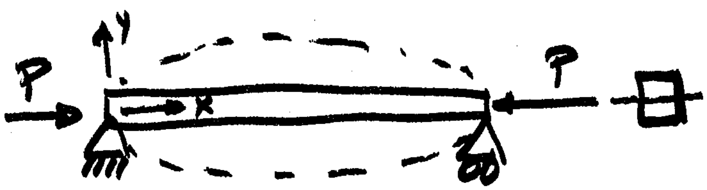




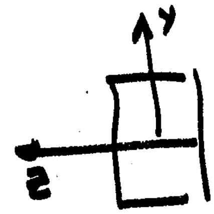

$P = \frac{1}{4} P_{cr} = \frac{1}{4} \frac{\pi^2 EI}{L^2}$

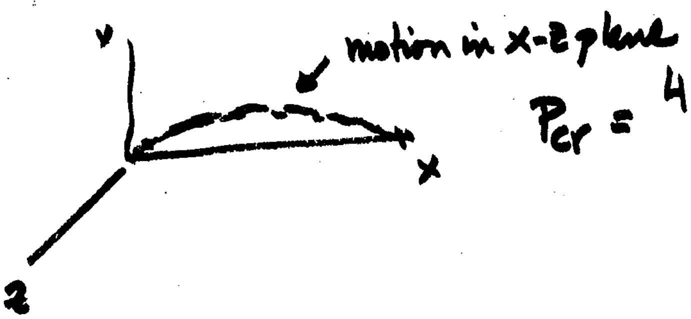
$P_{cr} = 2.05\, P_{cr} = 2.05 \frac{\pi^2 EI}{L^2}$

$P = 4 P_{cr} = \frac{4\pi^2 EI}{L^2}$

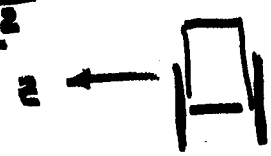



$P_{cr} = \frac{\pi^2 EI_z}{L^2}$

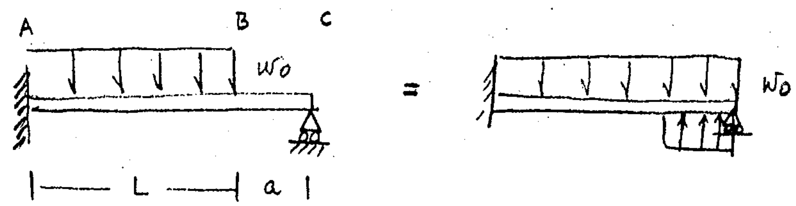


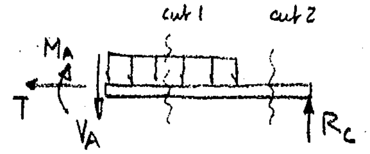

$P_{cr} = \frac{4\pi^2 EI_y}{L^2} \gg$ if $I_z = I_y$



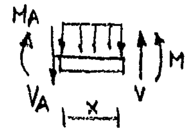

if $4I_y < I_z$ then buckling is out of the plane of the paper




Indeterminate Problem: solve using shear form of equation



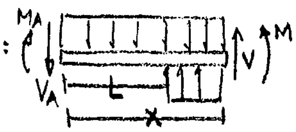

at cut 1:




$V = w_0 x + V_A$

$\boxed{\text{4 unknowns \& 3 eqns : 1 extra unknown} \rightarrow \text{choose } V_A}$

at cut 2:




$V = w_0 x - w_0 (x-L) + V_A$

---

$\therefore$ V can be written as $V = w_0 x - w_0 \langle x-L \rangle^1 + V_A$ where $V_A$ here is the unknown

$\boxed{\text{Solving from here requires 3 BCs to solve for } C_2, C_3, C_4 \text{ \& 1 more BC for the unknown } V_A}$

Now $EI y''' = -V = -w_0 x + w_0 \langle x-L \rangle^1 - V_A$.

$$EI y'' = -w_0 \frac{x^2}{2} + \frac{w_0}{2} \langle x-L \rangle^2 - V_A x + C_2 = M$$

$$EI y' = -w_0 \frac{x^3}{6} + \frac{w_0}{6} \langle x-L \rangle^3 - V_A \frac{x^2}{2} + C_2 x + C_3$$

$$EI y = -w_0 \frac{x^4}{24} + \frac{w_0}{24} \langle x-L \rangle^4 - V_A \frac{x^3}{6} + C_2 \frac{x^2}{2} + C_3 x + C_4$$

BC's: $y=0$ @ $x=0$ $\Rightarrow$ $C_4 = 0$

$y' = 0$ @ $x=0$ $\Rightarrow$ $C_3 = 0$

$y = 0$ @ $x = L+a$ $\Rightarrow$ $0 = -w_0 \frac{(L+a)^4}{24} + \frac{w_0 a^4}{24} - V_A \frac{(L+a)^3}{6} + C_2 \frac{(L+a)^2}{2}$ ①

$M=0$ $(EIy''=0)$ @ $x=L+a$ $\Rightarrow$ $0 = -w_0 \frac{(L+a)^2}{2} + \frac{w_0}{2}(a)^2 - V_A (L+a) + C_2$ ②

so
$$\begin{bmatrix} -(L+a)^3/6 & (L+a)^2/2 \\ -(L+a) & 1 \end{bmatrix} \begin{bmatrix} V_A \\ C_2 \end{bmatrix} = \begin{bmatrix} w_0 \frac{(L+a)^4}{24} - \frac{w_0 a^4}{24} \\ w_0 \frac{(L+a)^2}{2} - \frac{w_0 a^2}{2} \end{bmatrix}$$

$$V_A = \frac{\begin{vmatrix} \frac{w_0}{24}\left[(L+a)^4 - a^4\right] & (L+a)^2/2 \\ \frac{w_0}{2}\left[(L+a)^2 - a^2\right] & 1 \end{vmatrix}}{\begin{vmatrix} -(L+a)^3/6 & (L+a)^2/2 \\ -(L+a) & 1 \end{vmatrix}} = \frac{\frac{w_0}{24}\left[(L+a)^4 - a^4\right] - \frac{w_0}{4}\left[(L+a)^4 - a^2(L+a)^2\right]}{(L+a)^3/3}$$

$$C_2 = \frac{\begin{vmatrix} -(L+a)^3/6 & \frac{w_0}{24}\left[(L+a)^4 - a^4\right] \\ -(L+a) & \frac{w_0}{2}\left[(L+a)^2 - a^2\right] \end{vmatrix}}{(L+a)^3/3} = \frac{-\frac{w_0}{12}\left[(L+a)^5 - a^2(L+a)^3\right] + \frac{w_0}{24}\left[(L+a)^5 - a^4(L+a)\right]}{(L+a)^3/3}$$

Note that @ $x=0$ $EIy'' = C_2 = M_A$. Also $R_C = V_A + w_0 L$

$$EIy^{IV} = -w_0 \langle x - \tfrac{2L}{5} \rangle^0$$

$$EIy''' = -w_0 \langle x - \tfrac{2L}{5} \rangle^1 + C_1$$

$$EIy'' = -\frac{w_0}{2} \langle x - \tfrac{2L}{5} \rangle^2 + \overset{3/5 L w_0}{\cancel{C_1}} x + \overset{-21/50\, w_0 L^2}{\cancel{C_2}}$$

$$EIy' = -\frac{w_0}{6} \langle x - \tfrac{2L}{5} \rangle^3 + C_1 \frac{x^2}{2} + C_2 x + C_3$$

$$EIy = -\frac{w_0}{24} \langle x - \tfrac{2L}{5} \rangle^4 + C_1 \frac{x^3}{6} + C_2 \frac{x^2}{2} + C_3 x + C_4$$

$y(x=0)=0 \Rightarrow C_4 = 0$

$y'(x=0)=0 \Rightarrow C_3 = 0$

$V=0 \Rightarrow y'''(x=L)=0 \qquad -w_0 \langle 3L/5 \rangle^1 + C_1 = 0 \qquad C_1 = w_0 \cdot \frac{3L}{5}$

$M=0 \Rightarrow y''(x=L)=0 = -\frac{w_0}{2} \langle \frac{3L}{5} \rangle^2 + C_1 L + C_2 \qquad C_2 = \cancel{-\frac{w_0}{2}(3L/5)^2} \quad -\frac{21}{50} w_0 L^2$

$$-w_0 \cdot \frac{9L^2}{50} + w_0 \frac{30L^2}{50} + C_2$$

$$EIy'' = M = -\frac{w_0}{2}(.1L)^2 + w_0 \cdot \frac{3L}{5} \cdot \frac{L}{2} - \cancel{\frac{w_0}{2}\left(\frac{3L}{5}\right)^2} - \frac{21}{50} w_0 L^2$$

$$= w_0 L^2 \left[ -.005 + .3 - .42 \right]$$

$$= w_0 L^2 \left[ -.125 \right]$$

30pts 5/3pts/part.

---

3B

$y(0)=0 \Rightarrow C_4=0 \quad y(L)=0 \rightarrow EIy = 0 = -\frac{w_0}{24}\left(\frac{3L}{5}\right)^4 + C_1 \frac{L^3}{6} + C_2 \frac{L^2}{2}$

$y'(0)=0 \Rightarrow C_3=0 \quad y'(L)=0 \rightarrow EIy' = 0 = -\frac{w_0}{6}\left(\frac{3L}{5}\right)^3 + C_1 \frac{L^2}{2} + C_2 L$

$$-V = EIy'''(x=L/2) = -w_0(.1L) + C_1$$

$$V = w_0(.1L) - C_1$$

---

3C

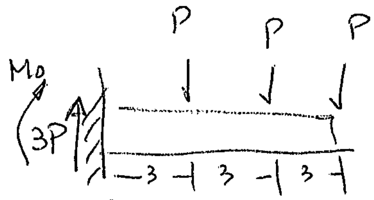
$EIy^{IV} = w_0 \langle x - \frac{2L}{5} \rangle^0 \qquad EIy''' = w_0 \langle x - \frac{2L}{5} \rangle^1 + C_1$

$EIy'' = \frac{w_0}{2} \langle \quad \rangle^2 + C_1 x + C_2$

$y(x=0)=0 \qquad y'''(x=L)=0 \Leftarrow V$

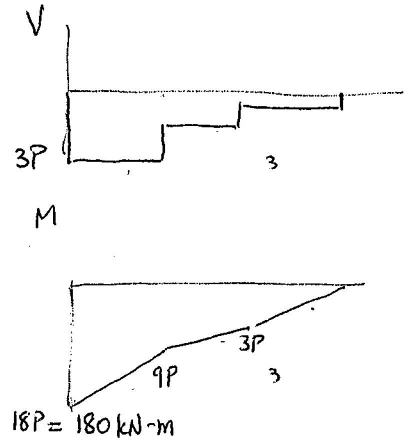
$y'(x=0)=0 \qquad y''(x=L)=0 \Leftarrow M$

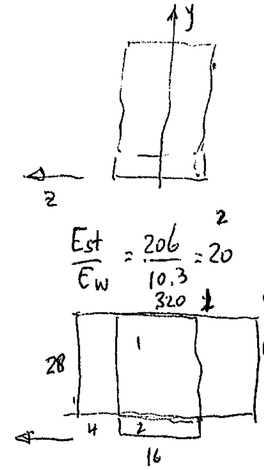
3A




$\Sigma M = 0 = -M_0 - P\cdot 3 - P\cdot 6 - P\cdot 9$

$M_0 = -18 \cdot P$




| | A | $\bar{y}$ | $A\bar{y}$ |
|---|---|---|---|
| ① | ~~28×320~~ | 18 | |
| ② | ~~4×16~~ | 2 | |

~~4~~ 7 pk for $\bar{y}$

2

$\frac{E_{st}}{E_w} = \frac{206}{10.3} = 20$

| | $I_0$ | A | d | I |
|---|---|---|---|---|
| ① | | | | |
| ② | | | | |

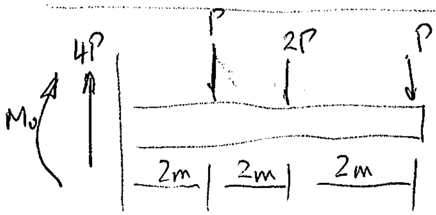
9 pk I

wood $-\frac{My}{I_z} = 2$

steel $-\frac{My}{I_z} \cdot n = 2$

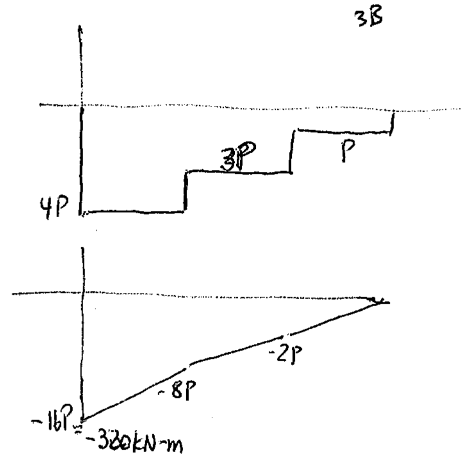
34 parts = 1.5 pts

3B

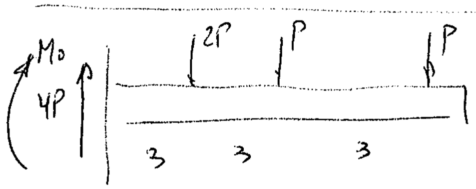



$\Sigma M = 0 = -M_0 - P\cdot 2 - 2P\cdot 4 - P\cdot 6$

$M_0 = -16P$

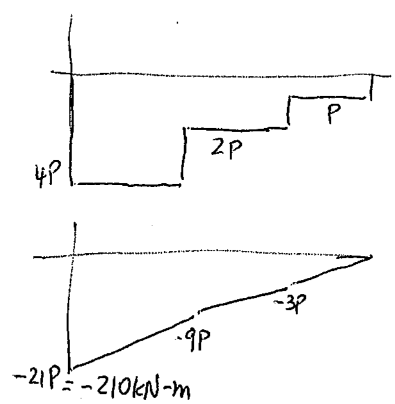



3C




$\Sigma M = -M_0 - 2P\cdot 3 - P\cdot 6 - P\cdot 9$

$M_0 = -21P$




$7.656 \times 10^{-3}$

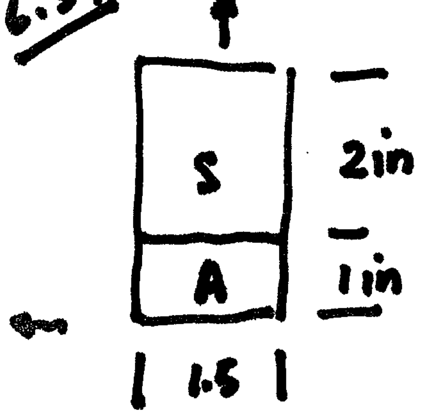
$\frac{320 \times 10^3 \times .2875}{5.769} \quad 15.94 \times 10^4$

7

EMA 3702

FOR CLASS ON 4/19/2011

6.57

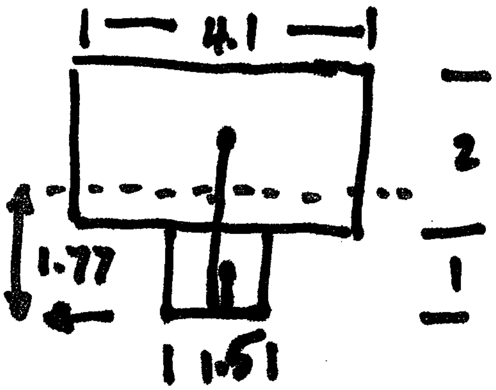



| | $\bar{y}$ | $A$ | $\bar{y}A$ |
|---|---|---|---|
| S | 2 | 8.21 | 16.42 |
| A | 0.5 | 1.5 | 0.75 |
| | | 9.71 | 17.17 |

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = 1.77\text{ in}$$

$$\frac{E_S}{E_A} = \frac{29\times10^6}{10.6\times10^6} = 2.736$$

0.736
1.363
4.1·




$$\tau = \frac{VQ}{It}$$

| | $I_{z'}$ | $A$ | $d$ | $I_z = I_z' + Ad^2$ |
|---|---|---|---|---|
| S | $\frac{1}{12}(4.1)(2)^3$ | 8.21 | -.23 | $.441 + 2.736 = 3.177$ |
| A | $\frac{1}{12}(1.5)(1)^3$ | 1.5 | 1.27 | $2.413 + .125 = 2.538$ |
| | | | | $5.715\text{ in}^4$ |


$$Q = \bar{y}A = (.23)(4.1)(2) = 1.902\text{ in}^3$$

$$Q = \bar{y}A = (1.27)(1.5)(1) = 1.902\text{ in}^3$$

$$\tau = \frac{VQ}{It} = \frac{(4000\,\text{lb})(1.902\,\text{in}^3)}{(5.715\,\text{in}^4)(1.5)} = 888\,\frac{\text{lb}}{\text{sq in}}$$

$$\tau = \frac{VQ}{It'} = \frac{(4000\,\text{lb})(1.902\,\text{in}^3)}{(5.715\,\text{in}^4)(4.1\,\text{in})}$$

$$\tau_{al} = G_{al}\gamma = G_{al}\gamma$$

$$\tau_s = G_{st}\gamma$$

$$\tau_s = \tau_{al}\cdot\frac{G_s}{G_{al}} = \tau_{al}\cdot\frac{\left[\frac{E_s}{2(1+\nu)}\right]}{\left[\frac{E_{al}}{2(1+\nu)}\right]}$$

$$\tau_s = \tau_{al}\cdot\frac{E_s}{E_{al}}$$

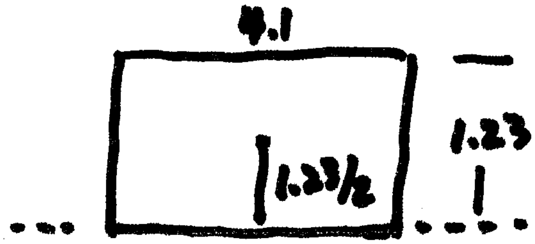
$$\frac{\tau_{al}}{G_{al}} = \gamma = \frac{\tau_s}{G_s}$$

$$\tau_s = \tau_{al}\cdot\frac{G_s}{G_{al}} \cong \tau_{al}\cdot\frac{E_s}{E_{al}}$$

$$\tau_s = \frac{VQ}{It'}\cdot\frac{E_s}{E_{al}} = \frac{(4000\,\text{lb})(1.902\,\text{in}^3)}{(5.715\,\text{in}^4)\underbrace{(4.1)}_{\parallel\; 2.736\times1.5}}\cdot\overbrace{\frac{29\times10^6}{10.6\times10^6}}^{2.736}$$

means that $\frac{1}{t'}\cdot\frac{E_s}{E_{al}} = \frac{1}{t}$

$$\tau_s = \frac{VQ}{It_s}$$

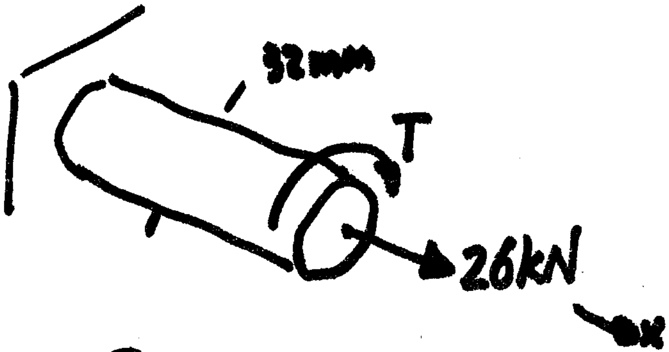



$$Q = \bar{y}A = \frac{1.23}{2}(4.1)(1.23) = 3.113\ in^3$$

$$\tau = \frac{VQ}{It'} \cdot \frac{E_s}{E_a} = \frac{VQ}{It} = \frac{(4000\ lb)(3.113\ in^3)}{(5.715\ in^4)(1.5\ in)}$$

$$= 1453\ lb/in^2$$

7.96




$$\frac{\sigma_1}{\sigma_t} - \frac{\sigma_3}{|\sigma_c|} = 1$$

$$\sigma_x = P/A = \frac{26000\,N}{\pi(32\times10^{-3})^2/4} = 32.33\ MPa = 32.33\times10^6\ Pa$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 16.16\ MPa$$

$$\sigma_1 = \sigma_{max} = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{ave} + \tau_{max}$$

$$\sigma_3 = \sigma_{min} = \quad " \quad - \quad " \quad = \sigma_{ave} - \tau_{max}$$

$$\frac{\sigma_{ave} + \tau_{max}}{\sigma_t} - \frac{(\sigma_{ave} - \tau_{max})}{|\sigma_c|} = 1$$

$$\tau_{max} \cdot \left[\frac{1}{\sigma_t} + \frac{1}{|\sigma_c|}\right] = 1 - \frac{\sigma_{ave}}{\sigma_t} + \frac{\sigma_{ave}}{|\sigma_c|}$$

$$\tau_{max} = \frac{1 - \sigma_{ave}\left[\frac{1}{\sigma_t} - \frac{1}{|\sigma_c|}\right]}{\left[\frac{1}{\sigma_t} + \frac{1}{|\sigma_c|}\right]}$$

$$= \frac{1 - 16.16\left[\frac{1}{75} - \frac{1}{120}\right]}{\left[1/75 + 1/120\right]} = 34.61\ MPa$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 34.61 \qquad \frac{\sigma_x - \sigma_y}{2} = 16.16 \text{ MPa}$$

$$\tau_{xy} = \sqrt{-\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{max}^2} = 30.61 \text{ MPa}$$

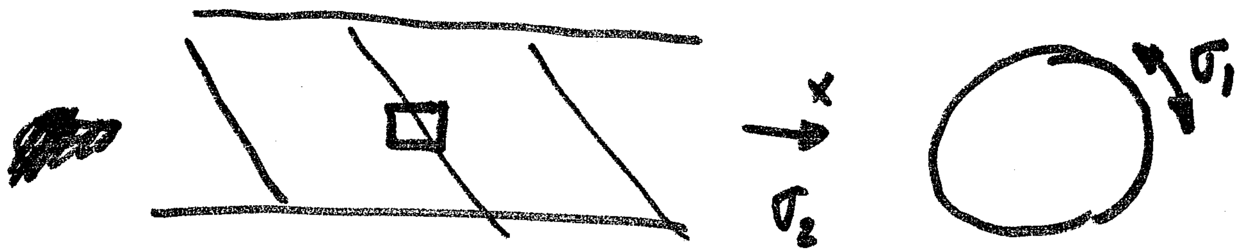
$$\tau = \frac{Tc}{J} \qquad J = \frac{\pi c^4}{2}$$

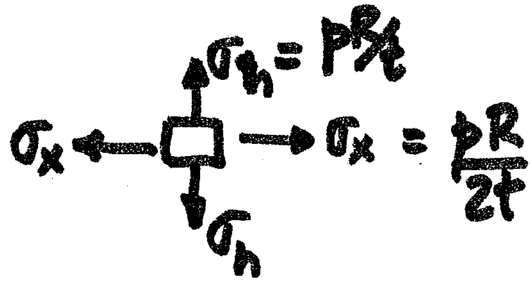
$$= \frac{2T}{\pi c^3} \qquad T = \frac{\pi c^3 \tau}{2}$$

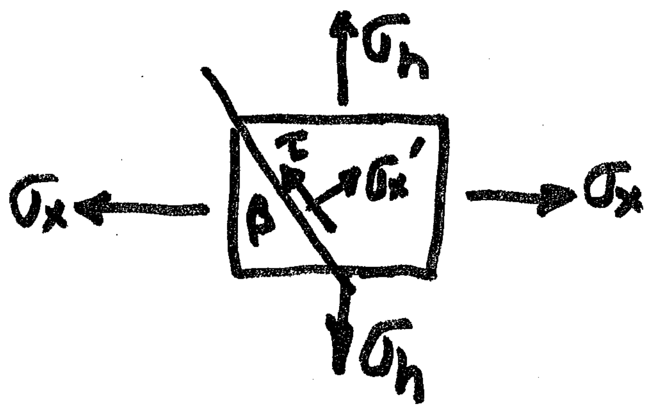
$$= \frac{\pi (16 \times 10^{-3})^3 \cdot 30.61 \times 10^6 \text{ Pa}}{2}$$

$$= 196.9 \text{ N-m}$$

7.117



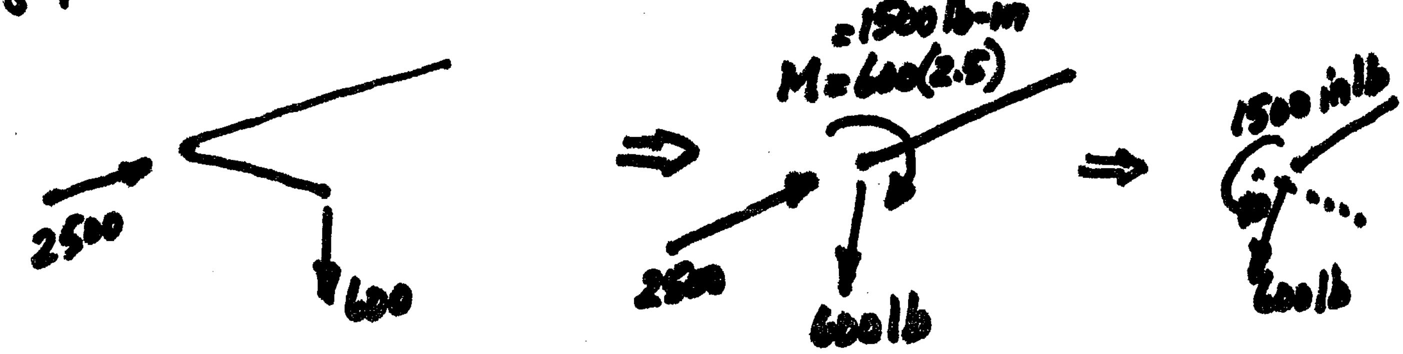



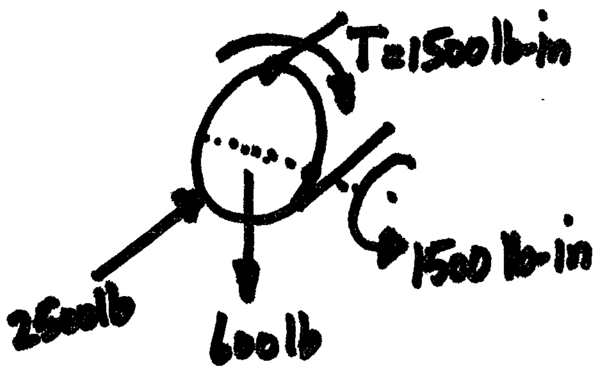

$$\sigma_x' = \left(\frac{\sigma_x+\sigma_h}{2}\right) + \left(\frac{\sigma_x-\sigma_h}{2}\right)\cos 2\beta + \cancel{\tau_{xy}} \sin 2\beta$$

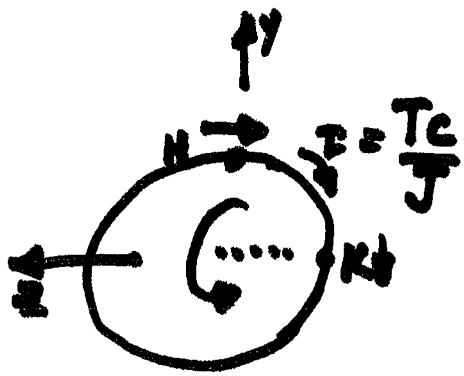
$$R = \sqrt{\left(\frac{\sigma_x-\sigma_h}{2}\right)^2 + \tau_{xy}^2} \Rightarrow \frac{\sigma_x-\sigma_h}{2}$$

$$\tau_{max} = \cancel{\tau_{xy}} \cos 2\beta - \left(\frac{\sigma_x-\sigma_h}{2}\right)\sin 2\beta$$

8.43




$\sigma_x = \frac{P}{A} = \frac{-2500\,lb}{\pi d^2/4}$ @ H & K

@ H $\tau$ due to 600 lb = 0

K " " " " $= \frac{VQ}{It}$

$\sigma_x = \frac{-My}{I} = \frac{-(-1500)(d/2)}{\frac{1}{4}\pi(d/2)^4}$ @ H

$= 0$ @ K

at H


$\sigma_x = +P/A - \frac{My}{I}$

at K


$\tau = \frac{Tc}{J} + \frac{VQ}{It}$

$\tau = Tc/J + \frac{VQ}{It}$


$Q = \bar{y}A$


$\bar{y} = \frac{4r}{3\pi}$ $A = \pi r^2/2$ $t = d$

$$\sigma_{\substack{max \\ min}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

@K $\sigma_x = P/A$

$\sigma_y = 0$

$\tau_{xy} = \frac{Tc}{J} + \frac{VQ}{It}$

@H $\sigma_x = P/A - \frac{My}{I}$

$\sigma_y = 0$

$\tau_{xy} = \frac{Tc}{J}$

## MOHR CRITERIA

$$\frac{\sigma_1}{\sigma_t} - \frac{\sigma_3}{|\sigma_c|} \geq 1 \implies \text{FAIL}$$

$$\frac{13.9}{14} - \frac{(-30.9)}{120} = 1.18 \overset{?}{\geq} 1 \quad \text{YES} \qquad \times$$

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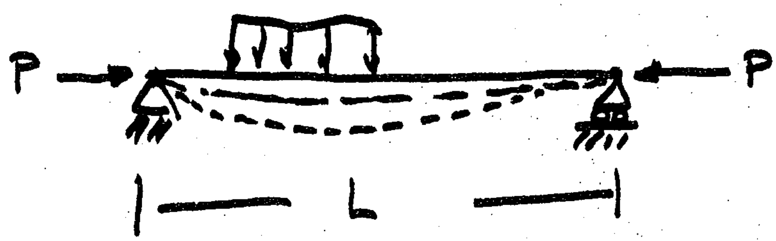
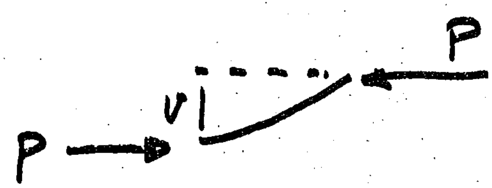
FAILURE – DUE TO EXCESSIVE DISPLACEMENT
BUCKLING



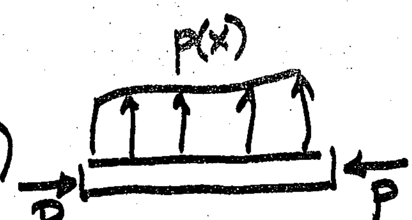

$$\frac{v}{L} \ll 1$$




Normally we have $EIv'' = M$

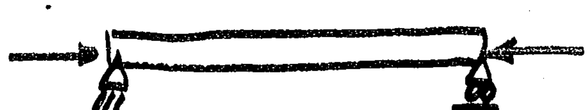
$$EIv'' + Pv = M$$

$$(EIv'')'' + (Pv)'' = +p(x)$$




IF $EI = \text{const.}$ $\quad v^{IV} + \frac{P}{EI}v'' = \frac{p(x)}{EI}$

$$\frac{P}{EI} = \lambda^2$$



$$v^{IV} + \lambda^2 v'' = 0$$

$$v''' + \lambda^2 v' = C_1$$

$$v'' + \lambda^2 v = C_1 x + C_2$$

homog soln $v'' + \lambda^2 v = 0 \Rightarrow A\cos\lambda x + B\sin\lambda x = v_h$

particular $v'' + \lambda^2 v = C_1 x + C_2$

$\rightarrow v_p = D$

$\rightarrow v_p = Cx$

$v_p = D: \; v_p'' + \lambda^2 v_p = 0 + \lambda^2 D = C_2 \qquad D = C_2/\lambda^2$

$$v_T = v_h + v_p = A\cos\lambda x + B\sin\lambda x + Cx + D$$
$$v_T'' = -\lambda^2 A\cos\lambda x - \lambda^2 B\sin\lambda x$$

$v(x=0)=0 \qquad v(x=L)=0$

$v''(x=0)=0 \qquad v''(x=L)=0$

$$A + 0B + 0C + D = 0$$
$$-\lambda^2 A + 0\cdot B + 0C + 0\cdot D = 0$$
$$A\cos\lambda L + B\sin\lambda L + CL + D = 0$$
$$-\lambda^2 A\cos\lambda L - \lambda^2 B\sin\lambda L + 0\cdot C + 0\cdot D = 0$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -\lambda^2 & 0 & 0 & 0 \\ \cos\lambda L & \sin\lambda L & L & 1 \\ -\lambda^2\cos\lambda L & -\lambda^2\sin\lambda L & 0 & 0 \end{bmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

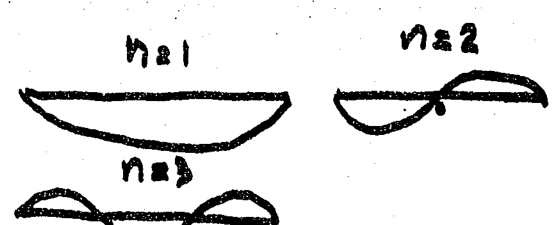
det matrix $= L\lambda^4 \sin\lambda L = 0 \Rightarrow \sin\lambda L = 0$

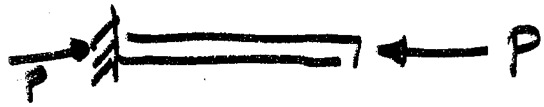
$\Rightarrow \lambda L = n\pi$

$$\sqrt{\frac{P}{EI}} = \lambda = \frac{n\pi}{L}$$

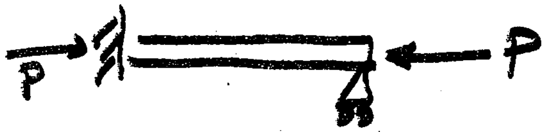
$$P = \frac{n^2\pi^2}{L^2} \cdot EI \qquad n=1 \quad P = \frac{\pi^2 EI}{L^2}$$ Euler Buckling Load.


$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

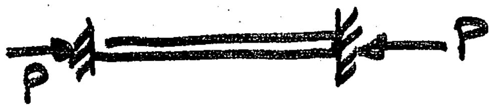




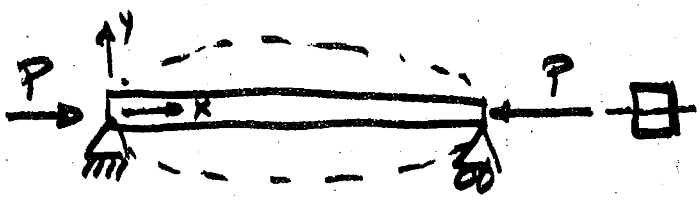

$$P = \frac{1}{4}P_{cr} = \frac{1}{4}\frac{\pi^2 EI}{L^2}$$



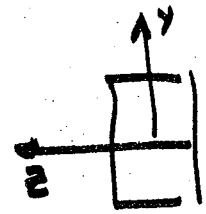

$$P_{cr} = 2.05\,P_{cr} = 2.05\,\frac{\pi^2 EI}{L^2}$$

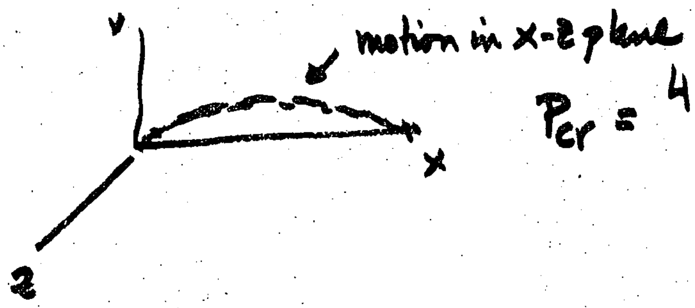



$$P = 4P_{cr} = \frac{4\pi^2 EI}{L^2}$$

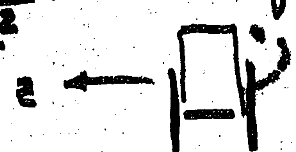



$$P_{cr} = \frac{\pi^2 EI_z}{L^2}$$

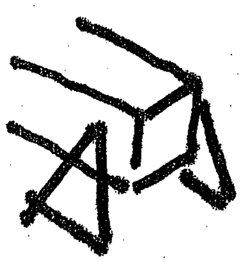


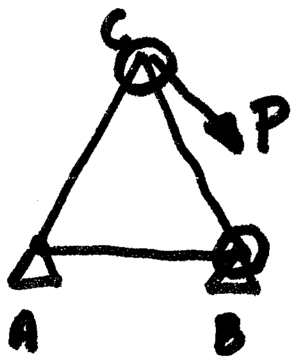



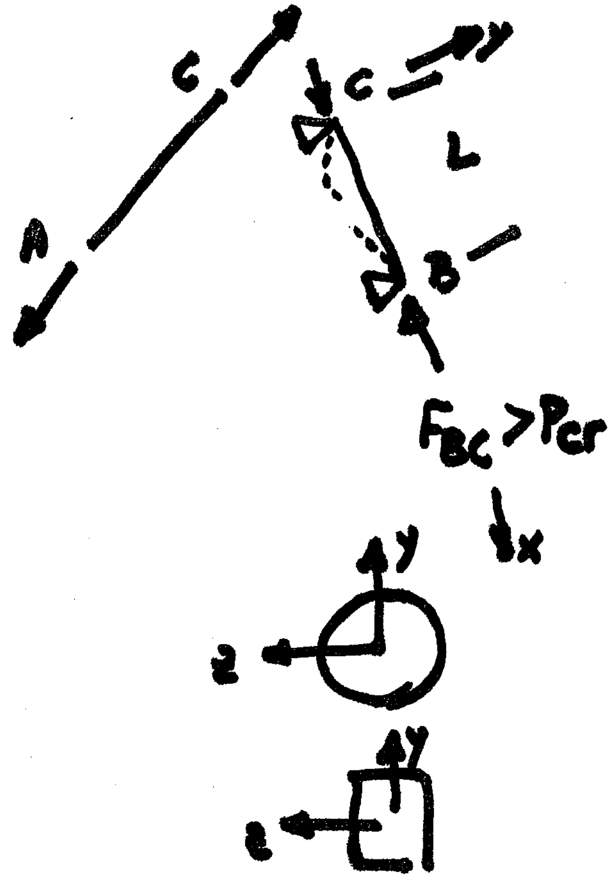

$$P_{cr} = \frac{4\pi^2 EI_y}{L^2} \gg \quad \text{if } I_z = I_y$$




if $4I_y < I_z$ then buckling is out of the plane of the paper






EXAM ON TUESDAY 4/19 $12^{30}-1^{45}$PM in RMS EC 1110/EC 1109. EMAIL WILL DETAIL WHO GOES WHERE

Florida International University
Department of Mechanical and Materials Engineering

EMA 3702 QUIZ 4B April 15, 2010

You are allowed nine sheets of 8 ½ x 11 inch paper with whatever you wish on the sheet

Print your name and sign the following statement:

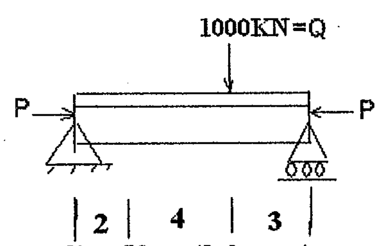
I will not give nor take any unpermitted aid during this quiz. I understand that violation of this statement will lead to automatic failure of the quiz.

PRINT NAME SIGN NAME

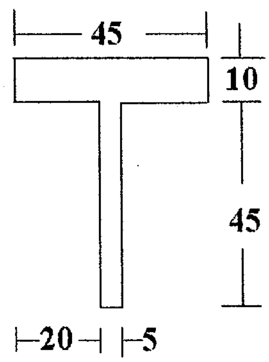
Problem.

The following beam is supported as shown and has axial loads P= 0.5 MN applied at the centroid of the cross-section.

1) Determine the buckling load for the system given E=25 GPa and the cross-section of the beam is given on the figure on the right and whose units are in cm.
2) Determine the displacement under the vertical load.
The formula for the displacement of the neutral axis of the beam without axial loads P is $y(x) = Q(L-x)^2x/(3EI)$ where x is the distance measured from the left end of the beam to the vertical load Q and L is the length of the beam.




units of length in meters



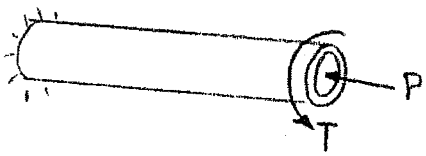
Problem 2.

Consider a hollow cylindrical tube of outer radius $R_0 = 140$ mm and inner radius $R_i = 125$ mm. The tube has a flat end cap. The tube is fixed at one end and subjected to a torque of T together with an axial compressive force of 68 kN as shown in the diagram. If the tube is also pressurized to a pressure of 2.1 MPa:

What must the torque be if the maximum shearing stress in yield is 340 MPa?

$R_o = .14 m \quad C = .14 m \quad R_i = .125 m$

$t = .015 M$




$P = 68000 N \qquad T = ?$

$p = 2.1 \times 10^6$ MPa $\qquad \tau_y = 340$ MPa

$A = 2\pi R_0 t = 2 \times 3.14 \times .125 \times .015 = .011775 \qquad J = 1.84 \times 10^{-4}$

$C = .125$

$$\tau = \frac{Tc}{J} = \frac{T \cdot .125}{1.84 \times 10^{-4}} = \frac{T\,1250}{1.84} = 680\,T$$

$\sigma x = 8.75 \times 10^6$

$\sigma y = 2\sigma x = 17.5 \times 10^6$

$$\tau_{max} = \sigma y/2 = \sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau^2}$$

$$\sigma x = 8.75 \times 10^6 - \frac{68000}{.011775}$$

$8.75 \times 10^6 - 5.77 \times 10^6$

$\cancel{14.52}\ 2.98 \times 10^6$

$\sigma y = 17.5 \times 10^6$

$$\frac{340 \times 10^6}{2} = \sqrt{\left(\frac{2.98 \times 10^6 - 17.5 \times 10^6}{2}\right)^2 + \tau^2}$$

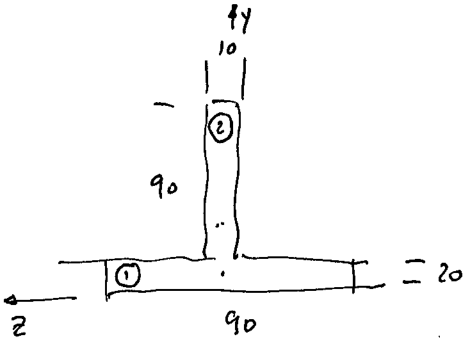
$170 \times 10^6 = \sqrt{52.70 \times 10^{12} + \tau^2}$

$28900 \times 10^{12} = 52.70 \times 10^{12} + \tau^2$

$28837.8 \times 10^{12}$

$169.84 \times 10^6 = 680\,T$

$.240 \times 10^6 = T \qquad .245\,MN\text{-}m$



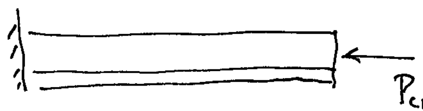

4A

| | $\bar{y}$ | $A$ | $\bar{y}A$ |
|---|---|---|---|
| 1 | 10 | 1800 | 18000 |
| 2 | 65 | 900 | 58500 |
| | | 2700 | 76500 |

$\bar{y} = \frac{76500}{2700} = 28.33$

$I_{zz}' = I_{zz} + Ad^2$

| | $I_{zz}$ | $A$ | $d$ | $I_{zz}'$ | $I_{yy}$ | $A$ | $e$ |
|---|---|---|---|---|---|---|---|
| 1 | $\frac{1}{12}(90)(20)^3$ | 1800 | 28.33-10 | | $\frac{1}{12}(20)(90)^3$ | | 0 |
| 2 | $\frac{1}{12}(10)(90)^3$ | 900 | $(65-28.33)$ | | $\frac{1}{12}(90)(10)^3$ | | 0 |
| | | | $cm^4 = (\quad)\times 10^{-8} m^4$ | | $I_{yy}'$ | | |



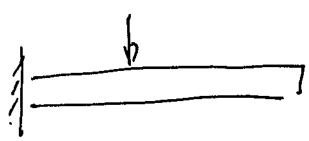
$$P_{cr_{in\,pl}} = \frac{1}{4}\pi^2 \frac{E I_z'}{L^2} \text{ in plane} = \frac{\frac{1}{4}\pi^2 (20\times 10^9)(I_z')}{9^2}$$

$$P_{cr_{out\,pl}} = \frac{1}{4}\pi^2 \frac{E I_y'}{L^2} \text{ out of p} = \frac{\frac{1}{4}\pi^2 (20\times 10^9)\, I_y'}{9^2}$$

smaller of two is $P_{cr}$

$$v = \frac{v_t}{1 - P/P_{cr}}$$

$$v_t = Qx^3/3EI_z' = \frac{1000000(4)^3}{3(20\times 10^9)(I_z')}$$



$$v = \frac{v_t}{1 - \frac{1\times 10^6}{P_{cr}}}$$

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4C
Problem 2

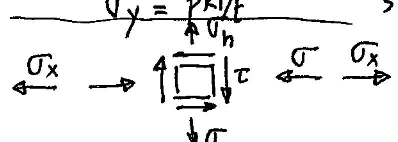
$\sigma = -P/A \qquad A = 2\pi R t = 2\pi(.125m)(.015m)$ 2 $\rightarrow \sigma = \underline{-68000}$ 2

$\tau = Tc/J$ 2 $\qquad J = 2\pi R_i^3 t$ 1 $\qquad c = R + t = .14$ 1

$J = 2\pi(.125)^3(.015)$

$\sigma_x = pR_i/2t =$ 3

$\sigma_y = pR_i/t$ 3




$$\tau_{max} = \frac{\sigma_y}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$ 3

$$\frac{340\times 10^6}{2} = \sqrt{\left(\frac{p\cdot .125}{2(.015)} - \quad\right)^2 + \tau^2}$$

$\sigma_x = \frac{pR_i}{2t} - \frac{P}{A}$

$\sigma_y = pR_i/t =$

$\tau = Tc/J = \underline{(35000\ N\text{-}m)(.14)}$

$$\sqrt{\frac{\sigma_y^2}{4} - \tau^2} = \left(p\frac{R_i}{2t} - \frac{P}{A}\right) - \frac{pR_i}{t}$$

$$= -pR_i/2t - P/A$$

$$\frac{pR_i}{2t} = \frac{|P|}{A} + \sqrt{\left(\frac{\sigma_y^2}{4} - \tau^2\right)}$$

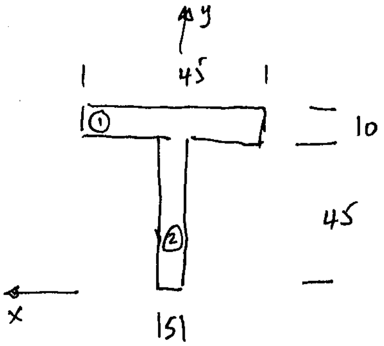
$$p = \frac{2t}{R_i}\left[ \qquad '' \qquad \right]$$

$$p = \frac{2(.015)}{.125}\left[\frac{68000}{A} + \sqrt{\left(\frac{340\times10^6}{2}\right)^2 - \tau^2}\right]$$

3

4C

$\bar{y} \to 9$ pts




| | $\bar{y}$ | $A$ | $\bar{y}A$ |
|---|---|---|---|
| ① | 50 | 450 | 22500 |
| ② | 22.5 | 225 | 5062.5 |
| | | 675 | 27562.5 |

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{27562.5}{675} = 40.83$$

$225 \times 22.5 = 112.5 + 450 + 4500 = 5062.5$

$675 \times .8 = 5400$

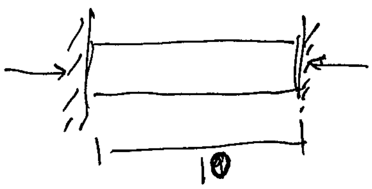
| | | $I_{zz}$ | $A$ | $d$ | $I_{zz}'$ |
|---|---|---|---|---|---|
| $I_{zz}'$ | ① | $\frac{1}{12}(45)(10)^3$ | 450 | $(50-\bar{y})$ | |
| | ② | $\frac{1}{12}(5)(45)^3$ | 225 | $(\bar{y}-22.5)$ | |
| $I_{yy}'$ | ① | $\frac{1}{12}(50)(45)^3$ | 450 | 0 | |
| | ② | $\frac{1}{12}(45)(5)^3$ | 225 | 0 | |

$I_{zz} + Ad^2 = I_{zz}'$

$I_{yy} + Ae^2 = I_{yy}'$

$I_{zz}' \to 9$ pts

$I_{yy}' \to 9$ pts




$$P_{cr} = \frac{4\pi^2 E I_z'}{L^2} = \frac{4\pi^2 (20\times10^9)(I_{zz}')}{10^2}$$ for in plane

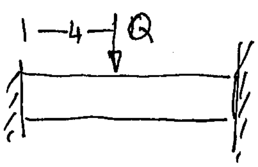
$$P_{cr} = \frac{4\pi^2 E I_y'}{L^2} = \frac{4\pi^2 (20\times10^9)(I_{yy}')}{10^2}$$ out of plane

$P_{cr}$ is smallest of two

$$v = \frac{v_t}{1 - P/P_{cr}}$$

$P_{cr} \to 4$ pts

choice 1 pts




$$v_t = Q\,6^3 \cdot 4^3 / 3EI_z' \cdot 10^3 = \frac{(1000000)\,4^3 \cdot 6^3}{3 \cdot 20\times10^9 \cdot I_{z'} \cdot 10^3}$$ ← 2 pts

$L = 10\,m$

$$v = v_t / 1 - P/P_{cr} = \frac{v_t}{1 - 0.75\times10^6/P_{cr}}$$ ← 2 pts

$v_t \to 2+2$

$v \to 2+2$

each part 1.25 pts

---

# 4A

$\sigma = P/A \qquad A = 2\pi R_i t = 2\pi(.125)(.015)$    A 2   + 2 $\sigma$

$\tau = Tc/J = \leftarrow \begin{cases} c = R_i + t = .14\,m \\ J = 2\pi R_i^3 t = 2\pi(.125)^3(.015)\ m^4 \\ T = 35000\ N\text{-}m \end{cases}$    c 1, J 1, $\tau$ 2

$$\sigma_x = \frac{pR_i}{2t} = \frac{(2.1\times10^6)(.125)}{2(.015)}$$    $\sigma_x$ 3, $\sigma_y$ 3

$\sigma_y = 2\sigma_x =$

3

$$\sigma_{max} = \sigma_{yp} = 300\ MPa = 300\times10^6 = \sqrt{\left(\frac{[\sigma_x - \sigma] - \sigma_y}{2}\right)^2 + \tau^2} + \frac{[\sigma_x - \sigma] + \sigma_y}{2}$$

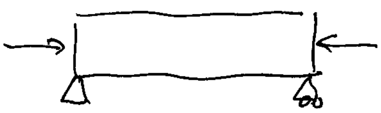
$$\left(\sigma_{yp} - \left[\frac{(\sigma_x - \sigma) + \sigma_y}{2}\right]\right)^2 = \left[\frac{(\sigma_x - \sigma) - \sigma_y}{4}\right]^2 + \tau^2$$ solution 3 pts

2.5 pts per part

same $\bar{y}$ calc as 4C

same $I_{zz}'$ calc as 4C

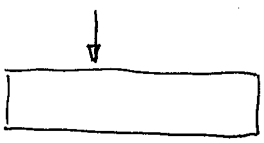
$I_{yy}'$ calc as 4C



$$P_{cr} = \frac{\pi^2 E I_z'}{L^2} = \frac{\pi^2 (25\times10^9)(\;\; I_z' \;\;)}{9^2}$$

$$P_{cr_{out}} = \frac{4\pi^2 E I_y'}{L^2} = \frac{4\pi^2 (25\times10^9)(\;\; I_y' \;\;)}{9^2}$$

} smallest of two is $P_{cr}$

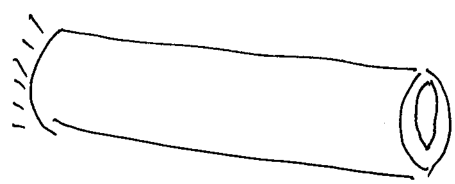


$$v_t = Q(9-6)^2 6/3EI_{z'} = \frac{9.6 \cdot 1{,}000{,}000}{3(25\times10^9)(I_z')}$$

$$\frac{v_t}{1 - P/P_{cr}} = \qquad v = \frac{v_t}{1 - \frac{0.5\times10^6}{P_{cr}}}$$

---

4B prob 2



$\sigma = -P/A = -\frac{68000}{A}$ 2 $\leftarrow$ $A = 2\pi R t = 2\pi(.125)(.015)$ 2

$\tau = \frac{Tc}{J}$ 2 $\quad J = 2\pi R_i^3 t = 2\pi(.125)^3(.015)$ 1

$c = R_o = R_i + t = .14$ 1

$$\sigma_x = \frac{pR_i}{2t} = \frac{(2.1\times10^6)(.125)}{2(.015)}$$ 3

$$\sigma_y = \frac{pR_i}{t} = \frac{(2.1\times10^6)(.125)}{.015}$$ 3

$$\frac{340\times10^6}{2} = \tau_{max} = \frac{\sigma_y}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$ 3

$$\sigma_x = \sigma_x - \sigma = \frac{pR_i}{2t} - \frac{68000}{A}$$

$$\sigma_y = \frac{pR_i}{t}$$

solve for $\tau \Rightarrow T = \frac{\tau J}{c} \Leftarrow \sqrt{\frac{\sigma_y^2}{4} - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \tau$ 3

Florida International University
Department of Mechanical and Materials Engineering

EMA 3702 QUIZ 1B February 14, 2013

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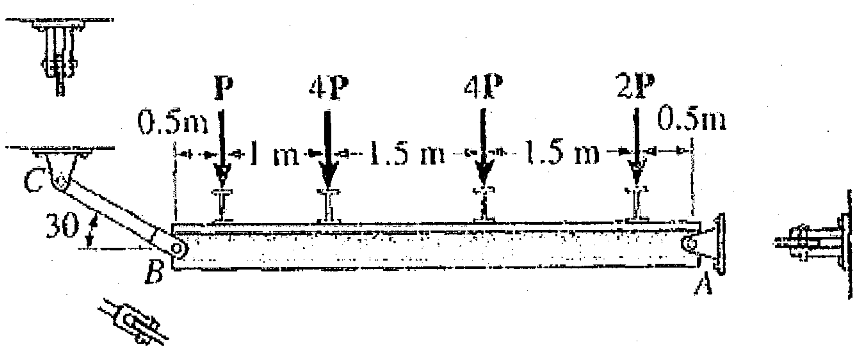
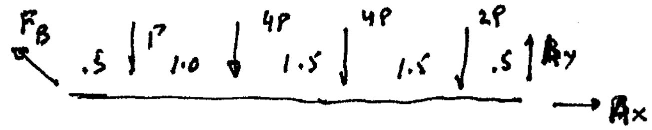
PRINT NAME SIGN NAME

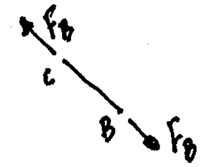
**P1** . The beam is supported by a pin at $A$ and a short link $BC$. Determine the maximum magnitude $P$ of the loads the beam will support if the average shear stress in each pin is not to exceed **100**MPa. All pins are in double shear as shown, and each has a diameter of 18 mm.
Solve for support forces first.




$n = 23$
$\bar{x} = 54.26$
$\sigma = 17.02$




10pts




$\Sigma M_B = .5P + 6P + 9P + 12P - A_y 5 = 0$ $\quad A_y = \dfrac{27.5P}{5} = 5.5P$ 3pts

$\Sigma F_y = F_B \cdot \frac{1}{2} - P - 4P - 4P - 2P + A_y = 0$ $\quad \underline{F_B = 11P}$ 3pts

$\Sigma F_x \quad - F_B \cos 30^\circ + A_x = 0$ $\quad A_x = 11P(.866) = 9.53P$ 3pts $\quad A = \sqrt{A_x^2 + A_y^2} = 11P$ 3pts

$A_{pin} = \dfrac{\pi d^2}{4} = \dfrac{\pi(.018)^2}{4} = \dfrac{\pi}{4}(3.24 \times 10^{-4}) = 2.545 \times 10^{-4}\, m^2$ 2pts

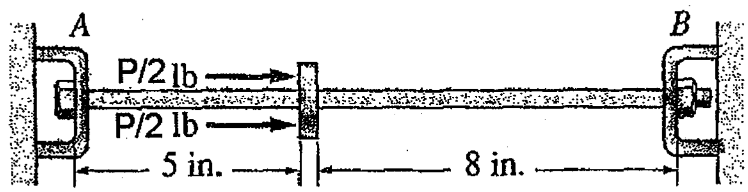
$\tau_B = \dfrac{F_B}{2A_{pin}}$ $\quad \tau_A = \dfrac{A}{2A_{pin}}$ $\quad \tau_C = \dfrac{F_B}{2A_{pin}} \Rightarrow P = 4626$ $\quad$ 3pts / 3pts / 3pts

$100 \times 10^6 = \dfrac{11P}{2 \cdot 2.545 \times 10^{-4}}$ $\quad = \dfrac{11P}{2 \cdot 2.545 \times 10^{-4}}$

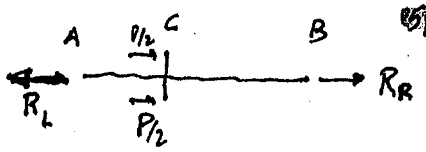
$P = 4626$ $\quad \Rightarrow \underline{P = 4626}$

$\overline{33}$

**P2 .** The aluminum bar has a diameter of 0.6 in. and is attached to the rigid supports at $A$ and $B$ when $T_1 = 80°F$. If the temperature becomes $T_2 = -10°F$, and an axial force of P lb is applied to the rigid collar as shown, find the value of P so that reaction at B is zero. $E_{al} = 10(10^3)$ ksi, $\alpha_{al} = 13(10^{-6})/°F$. Hint: find force diagram




Also, determine the final length of the 5 inch section of the bar.




5pts

$-R_L + R_R + 2P = 0$ 2pts

want $R_R = 0 \Rightarrow R_L = 2P$ 2pts

$\Delta u_{AC} + \Delta u_{AB} = 0$ 3pts

$$\Delta u_{AC} = \frac{+R_L L_{AC}}{A_{AC}E_{AC}} + \alpha_{AC}\Delta T \cdot L_{AC}$$ 5pts

$$\Delta u_{BC} = \frac{+(R_L - 2P)\cdot L_{BC}}{A_{BC}E_{AC}} + \alpha_{BC}\Delta T \cdot L_{BC}$$ 5pts

$$A_{AC} = A_{BC} = \frac{\pi d^2}{4} = .283\,in^2$$

$$\frac{R_L L_{AC}}{A_{AC}E_{AC}} + \alpha_{AC}L_{AC}\Delta T + \cancel{\frac{R_L - 2P}{A_{BC}E_{BC}}L_{BC}} + \alpha_{BC}L_{BC}\Delta T = 0$$

$$R_L = -(\alpha_{AC}L_{AC} + \alpha_{BC}L_{BC})\overset{(-90)}{\Delta T}\cdot\frac{A_{AC}E_{AC}}{L_{AC}}$$

$$= -(13\times10^{-6})(13)(-90)\cdot(.2827)\frac{(10\times10^{6}\,psi)}{5\,in}$$

$$\underline{= 8601.\,lb}$$ 5pts $\Rightarrow$ $\underline{P = 8602\,lb}$ 2pts

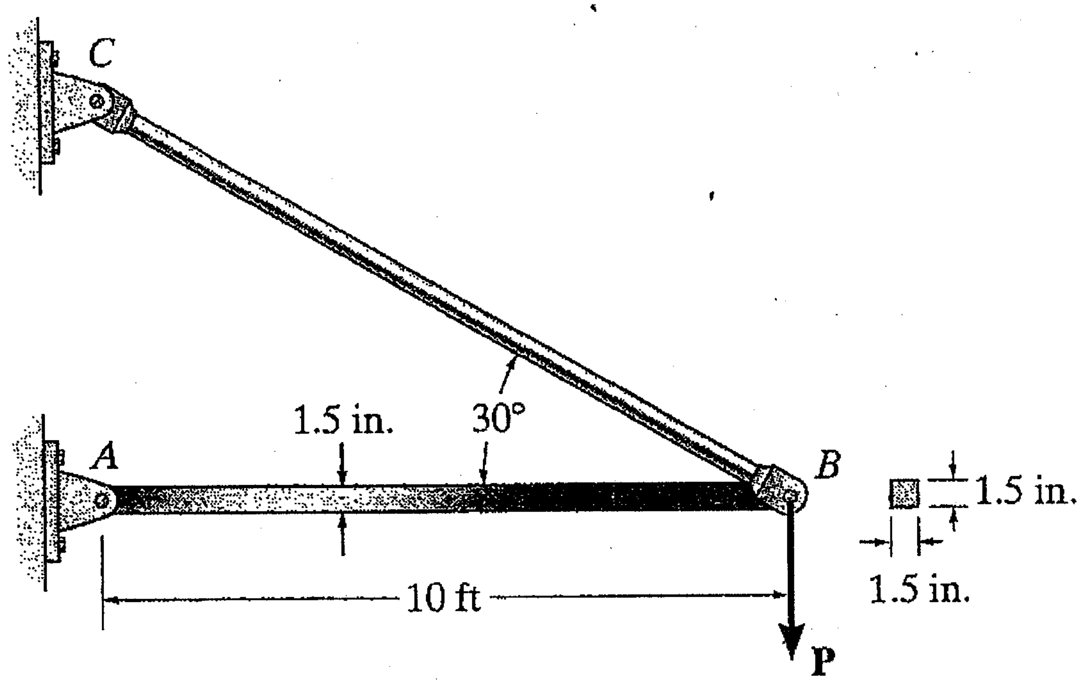
$$\Delta u_{AC} = \frac{R_L\cdot L_{AC}}{A_{AC}E_{AC}} + \alpha_{AC}L_{AC}\cdot\Delta T = \underline{.00936}$$ 3pts

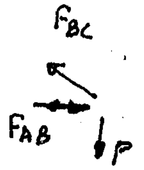
$$\ell_{AC} = 5\,in + \Delta u_{AC} = \underline{5.00936\,in.}$$ 1pts

$\overline{34}$

P3 . The A-36 steel bar *AB* has a square cross section. If it is pin connected at its ends, determine the maximum allowable load *P* that can be applied to the frame. Use a factor of safety of 2. $\sigma_Y$ for A-36 is 36 ksi. **All pins are single lap jointed.**

**Bar CB is circular with diameter d=1 in.**

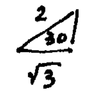





4pts

$\sum F_y = F_{BC}\sin 30 - P = 0$ $\quad \underline{F_{BC} = 2P}$ 5pts

$F_{AB} - F_{BC}\cos 30 = 0$ $\quad F_{AB} = 2P \cdot \frac{\sqrt{3}}{2}$




$\underline{F_{AB} = P\sqrt{3}}$ 5pts

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{P\sqrt{3}}{(1.5)^2} = .7698P$$ 5pts $\quad A_{AB} = 2.25$

6pts

$$\frac{\sigma_{allow}}{2} = \sigma_{act} = \frac{P\sqrt{3}}{(1.5)^2} = 18 \text{ KSI} \qquad P = \frac{18\,(2.25)}{\sqrt{3}} = 23383$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{2P}{\frac{\pi(1)^2}{4}} = \frac{8}{\pi}P = 2.5465P \qquad A_{BC} = \frac{\pi(1)^2}{4} = .7854$$

6pts

$$\frac{\sigma_{allow}}{2} = \sigma_{act} = \frac{8P}{\pi} \qquad \underline{P = \frac{18 \cdot \pi}{8} = \frac{28274}{4}\,lb} = 7069$$

P is smaller of 2 $\Rightarrow$ 7069 1pt

33

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EMA 3702 QUIZ 1A February 14, 2013

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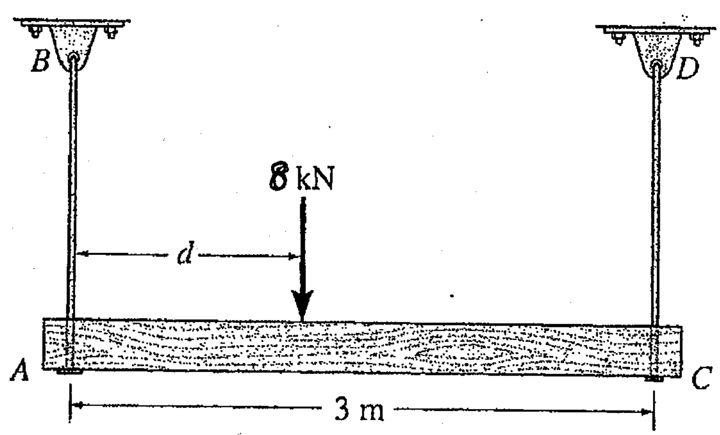
PRINT NAME SIGN NAME

**P1** The uniform beam is supported by two rods $AB$ and $CD$ that have cross-sectional areas of 12 $mm^2$ and 8 $mm^2$, respectively. Determine the position $d$ of the 8-kN load so that the average normal stress in each rod is the same.

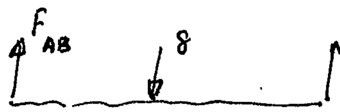



$n = 23$
$\bar{x} =$ 52.08
$\sigma =$ 16.06

Don't worry about the pins at B and D, here.

If both rods are made of steel with $E_{st}$=200 GPA, $\nu$=0.3, determine the strain in the horizontal direction of rod CD

#1




5pts

$\overset{+}{\curvearrowleft}\sum M_A = -8\cdot d + F_{CD}\cdot 3 = 0$ 2pts

$+\uparrow \sum F_y = 8 = F_{AB} + F_{CD} \Rightarrow 8 = 2.5F_{CD}$ 2pts

$3.2\text{ kN} = \frac{8}{2.5} = F_{CD}$ 1pt

$d = F_{CD}\cdot \frac{3}{8} = \frac{9.6}{8} = 1.2\text{ m}$ 1pt

$\sigma = \frac{F_{AB}}{A_{AB}} = \frac{F_{CD}}{A_{CD}}$

$\frac{F_{AB}}{12} = \frac{F_{CD}}{8}$

$F_{AB} = 1.5F_{CD}$ 2pts

$F_{CD} = 3.2\text{kN}$

$\sigma_{CD} = \frac{3.2\text{kN}}{8\times10^{-6}\text{m}^2} = 400\text{ MPa}$ 2pts

$\epsilon_{CD} = \frac{\sigma}{E} = \frac{400\times10^6}{200\times10^9} = 2\times10^{-3} = \epsilon_{CD_{vertical}}$ 2pts

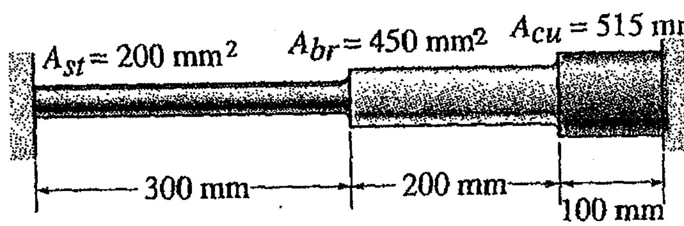
$\epsilon_{CD_{horiz}} = -\nu\,\epsilon_{CD_{vert}} = -.3\,(2\times10^{-3}) = -.6\times10^{-3}$ 2pts

x1.6

33pts

**P2** . Three bars each made of different materials are connected together and placed between two walls when the temperature is $T_1 = 12°C$. Determine the force exerted on the (rigid) supports when the temperature becomes $T_2 = 18°C$. The material properties and cross-sectional area of each bar are given in the figure.

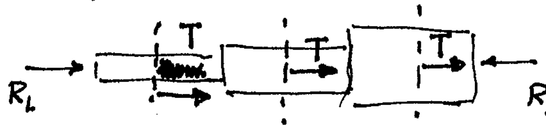
| Steel | Brass | Copper |
|---|---|---|
| $E_{st} = 200$ GPa | $E_{br} = 100$ GPa | $E_{cu} = 120$ GPa |
| $\alpha_{st} = 12(10^{-6})/°C$ | $\alpha_{br} = 21(10^{-6})/°C$ | $\alpha_{cu} = 17(10^{-6})/°C$ |




What is the overall change in diameter of the copper portion of the bar, assuming ν=0.3?

#2

reaction at cut T = $-R_L$




5pts

$$\Sigma F_x = 0 \Rightarrow R_L = R_R$$ 5pts

$$\Delta u_s + \Delta u_b + \Delta u_c = 0$$ 3pts

$$\Delta u_s = \frac{-R_L \cdot L_s}{A_s E_s} + \alpha_s \Delta T\, L_s$$ 3pts

$$\Delta u_b = \frac{-R_L \cdot L_b}{A_b E_b} + \alpha_b \Delta T\, L_b$$ 3pts

$$\Delta u_c = \frac{-R_L L_c}{A_c E_c} + \alpha_c \Delta T\, L_c$$ 3pts

$$+R_L \underbrace{\left[ +\frac{L_s}{A_s E_s} + \frac{L_b}{A_b E_b} + \frac{L_c}{A_c E_c} \right]}_{1.356\times10^{-8}} = +\left[ \alpha_s L_s + \alpha_b L_b + \alpha_c L_c \right] \Delta T$$

$$9.5\times10^{-6}\,(6) = 5.7\times10^{-5}$$

$$R_L = \frac{5.7\times10^{-5}}{1.356\times10^{-8}} = 4203.5\,N$$ 5pts

$$\epsilon_{c_{x_{TOT}}} = \frac{-R_L}{A_c E_c} + \alpha_c \Delta T \qquad \epsilon_y = -\nu\, \epsilon_{c_{x_{med}}} + \alpha_c \Delta T$$

$$\epsilon_{c_{x_{med}}} = \frac{-R_L}{A_c E_c} = \frac{-4203.5}{(515\times10^{-6})(120\times10^{9})}$$

$$\epsilon_{c_{x_{mech}}} = -6.802\times10^{-5}$$ 3pts

$$\Delta d = \epsilon_y \cdot d \qquad A = \pi d^2/4 \quad d = \sqrt{\frac{4A}{\pi}} = .0256\,m$$

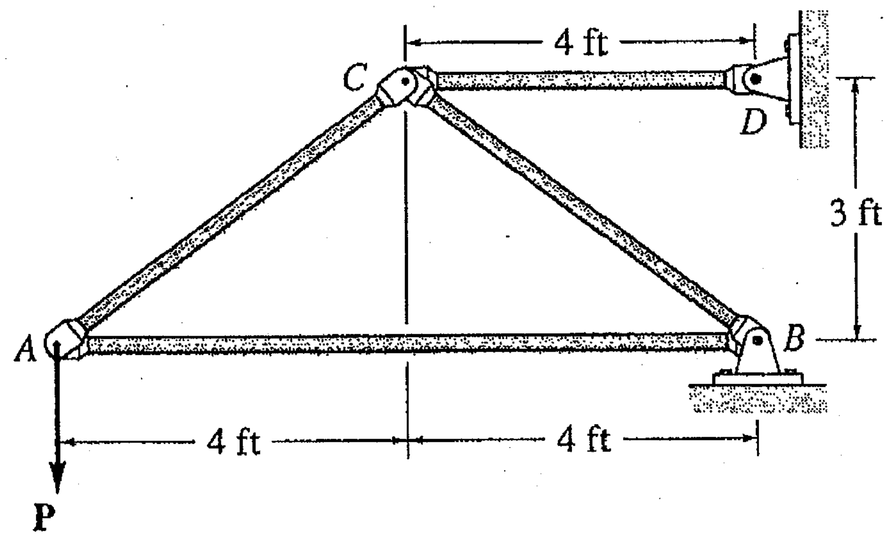
$$\epsilon_y = -(0.3)(-6.802\times10^{-5}) + (17\times10^{-6})(6)$$

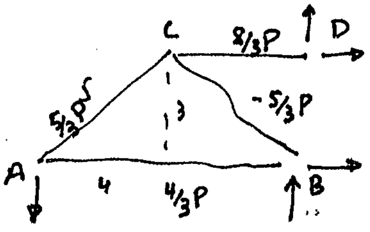
$$= 1.22\times10^{-4}$$ 3pts

$$\Delta d = (1.22\times10^{-4})\,d = 3.13\times10^{-6}\,m$$ 1pts.

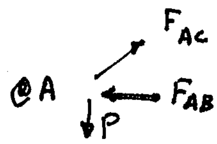
34

**P3** The truss is made from steel bars, each of which has a circular cross section with a diameter of 1.5 in. Determine the maximum force $P$ that can be applied without causing any of the members to fail . The members are pin-supported at their ends. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 36$ ksi. failure is when $\sigma > \sigma_Y$
All pins are single lap jointed at A and D; and double lap at B and C.





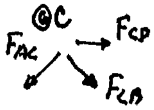

[redacted]



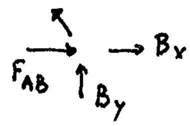

[redacted]

$\Sigma F_y \Rightarrow F_{AC} \cdot \frac{3}{5} = P \qquad F_{AC} = \frac{5P}{3}$ [redacted]

$\Sigma F_x \Rightarrow F_{AC} \cdot \frac{4}{5} = F_{AB} \qquad = \frac{4}{3} P$ [redacted]




[redacted]




4pts [redacted]

$\cancel{F_{CD} = F_{AC} \cdot \frac{4}{5} = \frac{4}{3} P}$ $\Sigma F_x = -F_{AC} \cdot \frac{4}{5} + F_{CD} + F_{CB} \cdot \frac{4}{5} = 0 \Rightarrow -\frac{4}{3} P + F_{CD} - \frac{4}{3} P = 0 \quad \boxed{F_{CD} = \frac{8}{3} P}$

$\Sigma F_y = F_{AC} \cdot \frac{3}{5} + F_{CB} \cdot \frac{3}{5} = 0 \Rightarrow P + F_{CB} \cdot \frac{3}{5} = 0 \quad F_{CB} = -\frac{5}{3} P$

[redacted] 4pts

since $F_{CD}$ is largest $F_{CD} = \frac{8P}{3}$ [redacted]

$\Rightarrow \sigma_{CD}$ largest

$$\sigma_{CD} = \frac{\frac{8P}{3}}{A_{CD}} = \frac{8P/3}{\frac{\pi (1.5)^2}{4}} = \frac{8P}{3} \cdot \frac{4}{\pi (1.5)^2} = 36K$$

$\frac{\pi(1.5)^2}{4} = 1.767$ [redacted]

$P = 23856.5$ lb [redacted]

33

Florida International University
Department of Mechanical and Materials Engineering

EMA 3702 QUIZ 2A March 4, 2010

You are allowed four sheets of 8 ½ x 11 inch paper with whatever you wish on the sheet

Print your name and sign the following statement:

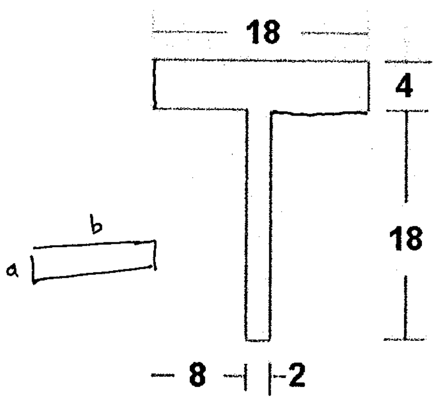
I will not give nor take any unpermitted aid during this quiz. I understand that violation of this statement will lead to automatic failure of the quiz.

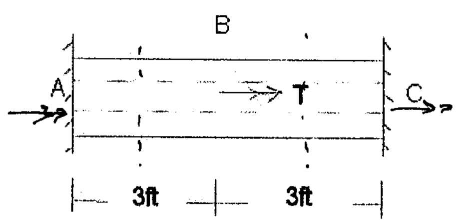
____________________ ____________________
PRINT NAME SIGN NAME

Problem.

a) The T shaped member, having the cross-section shown, is fixed between two walls and has an applied torque, T, of 2500 lb-in., determine the largest shearing stress and where it can be found. The Young's Modulus, $E = 30 \times 10^6$ psi and that the Poisson ratio, $\nu = 0.3$. NOTE: The dimensions in the left picture are given in inches.

b) Find the maximum angle of twist, $\phi$, and its location.




$$J_R = c_1 a_1^3 b_1 + c_2 a_2^3 b_2$$

$$b/a = 18/4 = 4.5 \rightarrow c_1 = .28$$

$$b/a = \frac{18}{2} = 9 \rightarrow c_2 = .31$$

$$\vec{T_A} + \vec{T} + \vec{T_c} = 0$$

$$T_A = -T - T_c$$

$$\Delta\phi = 0$$

$$\frac{-T_A \cdot L_{AB}}{J \cdot G} + \frac{(-T - T_A) L_{BC}}{J \cdot G} = 0$$

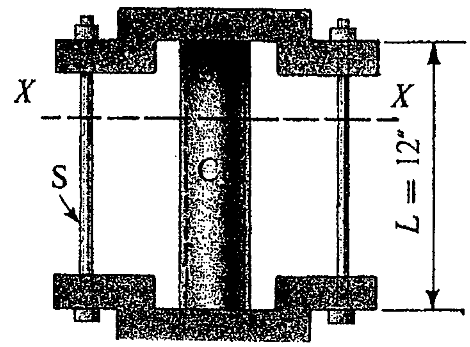
$$\Rightarrow -T_A L_{AB} - T L_{BC} - T_A L_{BC} = 0$$

$$T_A = \frac{-T L_{BC}}{L_{AB} + L_{BC}} \quad \frac{-2500 \cdot 36}{72} = -1250$$

$$T_c = \frac{-T L_{AB}}{L_{AB} + L_{BC}} = -1250$$

Problem.

A copper tube 12 in. long and having a cross-sectional area of 3 in.$^2$ is placed between two very rigid caps made of Invar,* Fig. 12-6(a). Four 1/2-in. steel bolts are symmetrically arranged parallel to the axis of the tube and are lightly tightened. Find the stress in the tube if the temperature of the assembly is raised from 60°F to 160°F. Let $E_{cu} = 17 \times 10^6$ psi, $E_s = 30 \times 10^6$ psi, $\alpha_{cu} = 0.0000091$ per °F, and $\alpha_s = 0.0000065$ per °F.



Florida International University
Department of Mechanical and Materials Engineering

EMA 3702 | QUIZ 2C | March 4, 2010

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Print your name and sign the following statement:

I will not give nor take any unpermitted aid during this quiz. I understand that violation of this statement will lead to automatic failure of the quiz.

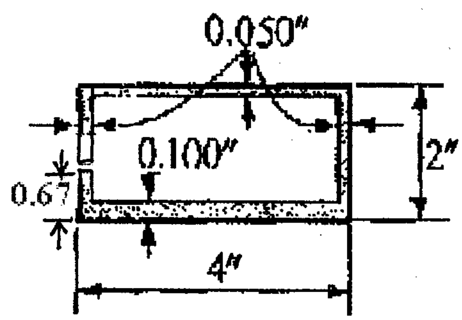
____________________ PRINT NAME

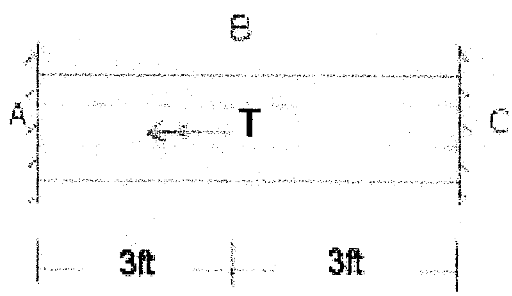
____________________ SIGN NAME

Problem.

a) The thin walled open member with a small horizontal slit in the left wall, having the cross-section shown, is fixed between two walls and has an applied torque, T, of 5000 lb-in. at B. Determine the largest shearing stress and where it is found. The Young's Modulus, E = 30 x $10^6$ psi and that the Poisson ratio, $\nu$ =0.3. NOTE: all the dimensions of the left side figure are in inches

b) Find the maximum angle of twist, $\phi$, and its location.




$$J \sum_{i=1}^{5} c_i a_i t_i^3$$

$c_i \Rightarrow \frac{4}{.05} \rightarrow .333$

$\frac{2}{.05} \rightarrow .333$

$\frac{1.33}{.05} \rightarrow .333$

$\frac{.67}{.05} \rightarrow .333$

$\frac{4}{.05} \rightarrow .333$

$$T_1+T_2+T_3+T_4+T_5 = 2500\#$$

$$\frac{T_1}{c_1a_1b_1^2} = \frac{T_2}{c_1a_2b_2^2} = \frac{T_3}{c_1a_3b_3^2} = \frac{T_4}{c_1a_4b_4^2} = \frac{T_5}{a_1a_5b_5^2}$$

Problem.

A copper tube 12 in. long and having a cross-sectional area of 3 in.$^2$ is placed between two very rigid caps made of Invar,* Fig. 12-6(a). Four $\frac{3}{4}$-in. steel bolts are symmetrically arranged parallel to the axis of the tube and are lightly tightened. Find the stress in the tube if the temperature of the assembly is raised from 60°F to 160°F. Let $E_{cu} = 17 \times 10^6$ psi, $E_s = 30 \times 10^6$ psi, $\alpha_{cu} = 0.0000091$ per °F, and $\alpha_s = 0.0000065$ per °F.

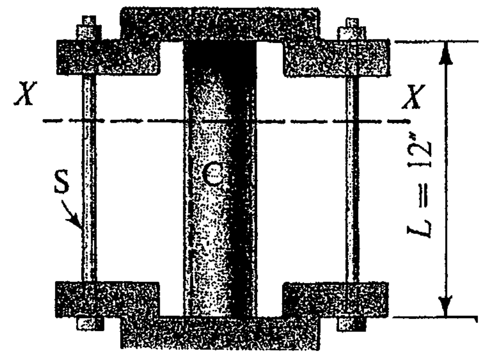
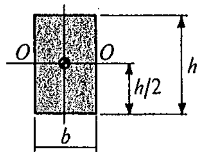


**TABLE 2**

## USEFUL PROPERTIES OF AREAS

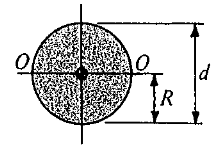
### AREAS AND MOMENTS OF INERTIA OF AREAS AROUND CENTROIDAL AXES

*RECTANGLE*



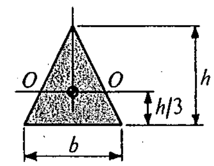

$A = bh$
$I_o = bh^3/12$

*CIRCLE*



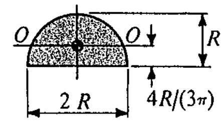

$A = \pi R^2$
$I_o = J/2 = \pi R^4/4$

*TRIANGLE*



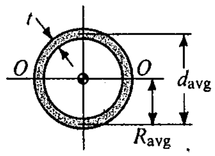

$A = bh/2$
$I_o = bh^3/36$

*SEMICIRCLE*



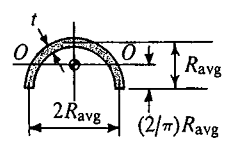

$A = \pi R^2/2$
$I_o = 0.110R^4$

*THIN TUBE*




$A = 2\pi R_{avg}t$
$I_o = J/2 \approx \pi R_{avg}^3 t$

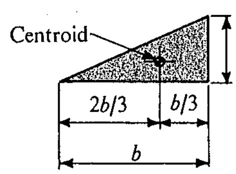
*HALF OF THIN TUBE*




$A = \pi R_{avg}t$
$I_o \approx 0.095\pi R_{avg}^3 t$

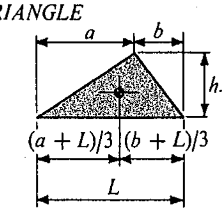
### AREAS AND CENTROIDS OF AREAS

*TRIANGLE*



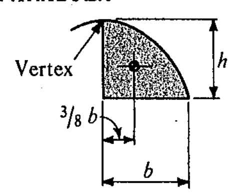

$A = bh/2$

*TRIANGLE*



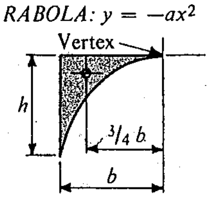

$A = hL/2$

*PARABOLA*



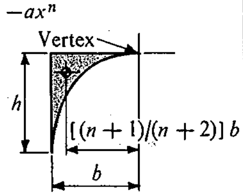

$A = {}^2/_3\, bh$

*PARABOLA:* $y = -ax^2$



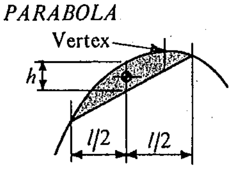

$A = bh/3$

$y = -ax^n$




$A = bh/(n+1)$

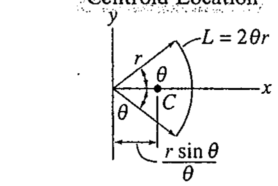
*PARABOLA*



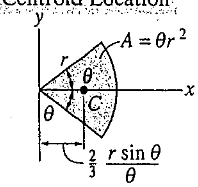

The area for any segment of a parabola is $A = {}^2/_3\, hl$

# Geometric Properties of Line and Area Elements

| Centroid Location | Centroid Location | Area Moment of Inertia |
|---|---|---|



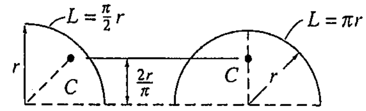
Circular arc segment



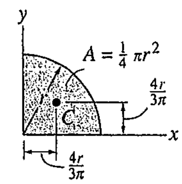
Circular sector area

$I_x = \frac{1}{4} r^4 (\theta - \frac{1}{2}\sin 2\theta)$

$I_y = \frac{1}{4} r^4 (\theta + \frac{1}{2}\sin 2\theta)$



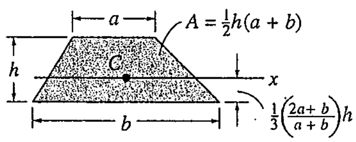
Quarter and semicircle arcs



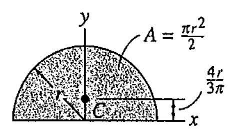
Quarter circle area

$I_x = \frac{1}{16}\pi r^4$

$I_y = \frac{1}{16}\pi r^4$



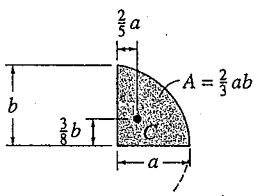
Trapezoidal area



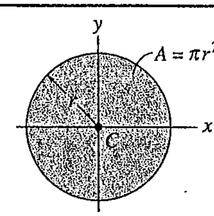
Semicircular area

$I_x = \frac{1}{8}\pi r^4$

$I_y = \frac{1}{8}\pi r^4$



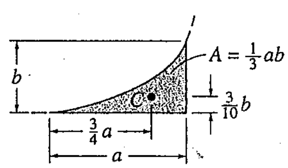
Semiparabolic area



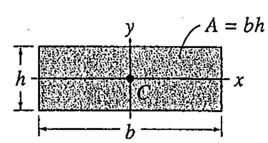
Circular area

$I_x = \frac{1}{4}\pi r^4$

$I_y = \frac{1}{4}\pi r^4$



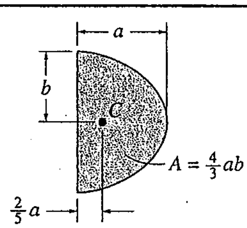
Exparabolic area



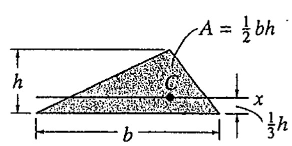
Rectangular area

$I_x = \frac{1}{12}bh^3$

$I_y = \frac{1}{12}hb^3$



Parabolic area



Triangular area

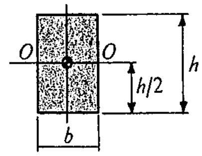
$I_x = \frac{1}{36}bh^3$

**TABLE 2**

## USEFUL PROPERTIES OF AREAS

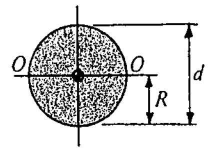
### AREAS AND MOMENTS OF INERTIA OF AREAS AROUND CENTROIDAL AXES

*RECTANGLE*



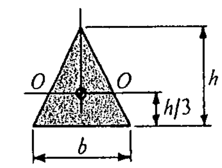

$A = bh$
$I_o = bh^3/12$

*CIRCLE*



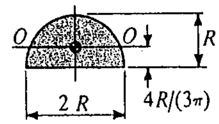

$A = \pi R^2$
$I_o = J/2 = \pi R^4/4$

*TRIANGLE*



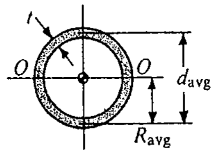

$A = bh/2$
$I_o = bh^3/36$

*SEMICIRCLE*



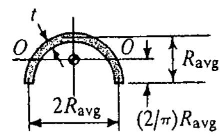

$A = \pi R^2/2$
$I_o = 0.110R^4$

*THIN TUBE*




$A = 2\pi R_{avg}t$
$I_o = J/2 \approx \pi R_{avg}^3 t$

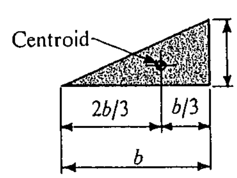
*HALF OF THIN TUBE*




$A = \pi R_{avg}t$
$I_o \approx 0.095\pi R_{avg}^3 t$

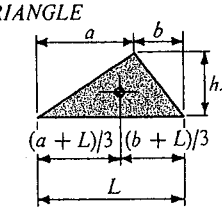
### AREAS AND CENTROIDS OF AREAS

*TRIANGLE*



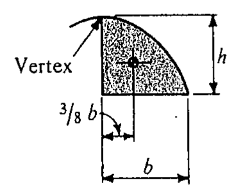

$A = bh/2$

*TRIANGLE*



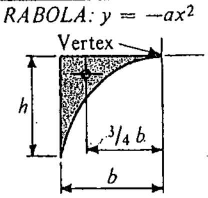

$A = hL/2$

*PARABOLA*



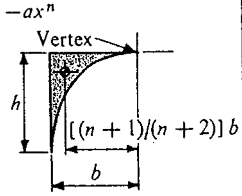

$A = {}^2/_3\, bh$

*PARABOLA:* $y = -ax^2$



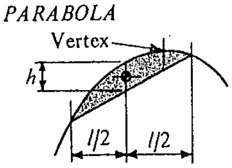

$A = bh/3$

$y = -ax^n$




$A = bh/(n + 1)$

*PARABOLA*




The area for any segment of a parabola is $A = {}^2/_3\, hl$

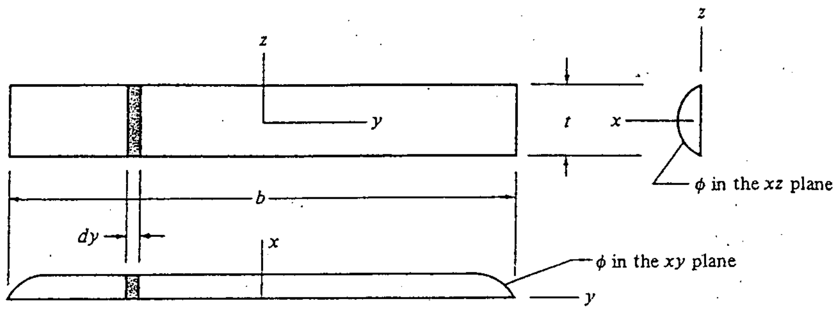



**FIGURE 8.4.2.** Narrow rectangular cross section ($b >> t$), with views of the stress function (or inflated membrane).

where $C$ is a constant. Substituting into Eq. 8.3.5, we find

$$-2C = -2G\beta \qquad \text{so} \qquad C = G\beta \tag{8.4.4}$$

The torque is

$$T = 2\int_{\text{area}} \phi\, dA = 2\int_{-t/2}^{t/2} \phi b\, dz = G\beta \frac{bt^3}{3} \tag{8.4.5}$$

The maximum shear stress, found along the edges $z = \pm t/2$, is

$$\tau = \left|\frac{\partial \phi}{\partial z}\right|_{z=\pm t/2} = 2C\frac{t}{2} = G\beta t \tag{8.4.6}$$

Substituting for $\beta$ from Eq. 8.4.5, we find

$$\tau = \frac{Tt}{bt^3/3} = \frac{3T}{bt^2} \qquad \text{at} \quad z = \pm\frac{t}{2} \tag{8.4.7}$$

Equations 8.4.5 and 8.4.7 can be written in the forms

$$\frac{d\theta}{dx} = \beta = \frac{T}{GJ_R} \qquad \tau = \frac{Tt}{J_R} \tag{8.4.8}$$

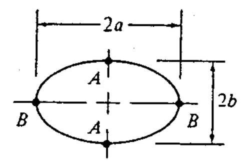
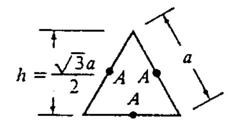
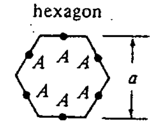
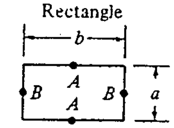
where

$$J_R = \frac{bt^3}{3} \qquad (\text{for } b >> t) \tag{8.4.9}$$

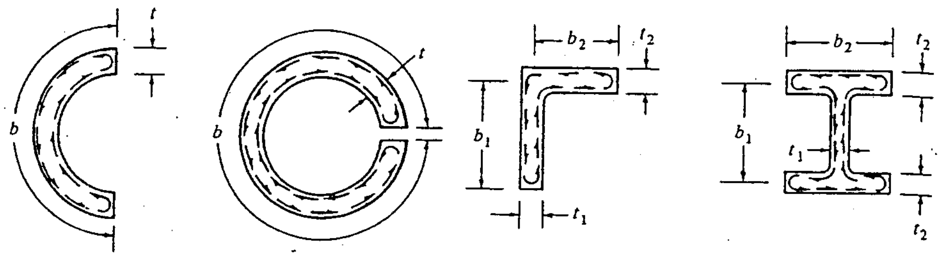
These expressions for $\beta$ and $\tau$ are similar in form to the corresponding expressions for a circular cross section, Eqs. 8.1.2 and 8.1.3. However, $J_R$ is *emphatically NOT* the polar moment of the cross-sectional area about the centroidal $x$ axis.

If (say) $b/t = 10$, Eqs. 8.4.8 give $\beta$ and $\tau$ values that are approximately 6.5 percent low. Accuracy improves as $b/t$ increases. Aspect ratios in the range $1 < b/t < 10$ can be analyzed with tabulated data obtained by other analytical or numerical methods (Table 8.4.1). If the centerline of the bar is curved, as

**TABLE 8.4.1** Expressions for Maximum Shear Stress and Rate of Twist in Selected Solid Sections [8.1, 8.4]

| Cross Section and Area | Maximum Shear Stress | Rate of Twist |
|---|---|---|
| Ellipse  Area $= \pi ab$ | $\tau_A = \dfrac{2T}{\pi ab^2}$ $(a > b)$ ($\tau_{max}$ at $B$ if $b > a$) | $\beta = \dfrac{a^2+b^2}{\pi a^3 b^3}\dfrac{T}{G} = \dfrac{d\theta}{dx}$ |
| Equilateral triangle  Area $= 0.433\,a^2$ | $\tau_A = \dfrac{20T}{a^3}$ | $\beta = \dfrac{46.2}{a^4}\dfrac{T}{G} = \dfrac{d\theta}{dx}$ |
| Regular hexagon  Area $= 0.866a^2$ | $\tau_A = \dfrac{5.7T}{a^3}$ | $\beta = \dfrac{8.8}{a^4}\dfrac{T}{G} = \dfrac{d\theta}{dx}$ |
| Rectangle  Area $= ab$ | $\tau_A = \dfrac{T}{C_\tau ba^2}$ | $\beta = \dfrac{1}{C_\beta ba^3}\dfrac{T}{G} = \dfrac{d\theta}{dx}$ |

| $b/a$ | $C_\tau$ | $\tau_B/\tau_A$ | $C_\beta$ |
|---|---|---|---|
| 1.0 | 0.208 | 1.000 | 0.1406 |
| 1.2 | 0.219 | 0.935 | 0.166 |
| 1.5 | 0.231 | 0.859 | 0.196 |
| 2.0 | 0.246 | 0.795 | 0.229 |
| 2.5 | 0.258 | 0.766 | 0.249 |
| 3.0 | 0.267 | 0.753 | 0.263 |
| 4.0 | 0.282 | 0.745 | 0.281 |
| 6.0 | 0.299 | 0.743 | 0.299 |
| 10.0 | 0.312 | 0.742 | 0.312 |
| $\infty$ | 0.333 | 0.742 | 0.333 |



$J_R = \frac{bt^3}{3}$ (a) $\quad J_R = \frac{bt^3}{3}$ (b) $\quad J_R = \frac{b_1t_1^3 + b_2t_2^3}{3}$ (c) $\quad J_R = \frac{b_1t_1^3 + 2b_2t_2^3}{3}$ (d)

**FIGURE 8.4.3.** Thin-walled open sections, showing their shear stresses and torsional constants $J_R$. In (a) and (b), dimension $b$ is the length of the midline of the cross section.

when a bar is bent to form a helical spring, some corrections to these formulas may be necessary [8.3].

The membrane analogy permits a very useful generalization of the foregoing results to other thin-walled open cross sections. Imagine that the narrow rectangular cross section of Fig. 8.4.2 is distorted into a C or an L shape, or attached to another narrow rectangle to make a T or an I section. By visualizing the inflated membranes for these shapes, we decide that $\phi$ surfaces of all of them remain parabolic (excepting near ends and near reentrant corners, which we discuss later). The total torque is the sum of torques carried by each part of the cross section. Since $G\beta$ is the same for each part, Eqs. 8.4.5 and 8.4.8 still apply but with the $bt^3/3$ contributions summed to yield

$$J_R = \frac{1}{3}\sum_{i=1}^{n} b_i t_i^3 \tag{8.4.10}$$

where $n$ is the number of parts into which the cross section is divided for purposes of calculation. Examples of this calculation appear in Fig. 8.4.3. In angle and I sections, the part of greater thickness displays greater shear stress. Where there is taper, as in the cross section of a turbine blade or in flanges of a rolled section, we can use

$$J_R = \frac{1}{3}\int t^3\, ds \tag{8.4.11}$$

where $ds$ is an increment of length along the medial line of the cross section. More exact formulas are available [8.3]. These formulas, experimental results [8.5], and coefficient $C_\beta$ in Table 8.4.1 suggest that, for standard rolled structural shapes, $J_R$ may actually be some 10 percent higher than predicted by Eqs. 8.4.10 and 8.4.11. Omission of this adjustment yields a conservative design under either a stress limit or a deflection limit.

## 8.5 A HYDRODYNAMIC ANALOGY

There are several hydrodynamic analogies to the torsion problem [8.1]. In this section we outline one, without proof, and use it to draw useful conclusions about shear stresses in twisted bars.

If a fluid without viscosity executes motion in the $yz$ plane with constant vorticity, its motion is described by the equation

$$\frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = -2\omega \tag{8.5.1}$$

where $\phi$ is the stream function and $\omega$ is the (constant) vorticity. Fluid velocities $v$ and $w$ in $y$ and $z$ directions, respectively, are

$$v = \frac{\partial\phi}{\partial z} \qquad w = -\frac{\partial\phi}{\partial y} \tag{8.5.2}$$

Clearly, Eqs. 8.5.1 and 8.5.2 are analogous to Eqs. 8.3.5 and 8.3.1, respectively. Thus fluid velocities are proportional to shear stresses. Lines $\phi$ = constant are streamlines. The fluid boundary must be a streamline, therefore the boundary condition $d\phi = 0$ of Eq. 8.3.6 is met. Thus the analogy is complete.

Experiments have been done in the following way. Imagine that a square cross section is to be studied. A shallow square tank is painted black and placed on a turntable. A camera, looking down onto the tank, is also attached to the turntable. The tank is filled with water (whose viscosity is low, if not quite zero) and aluminum powder is sprinkled on the water. When all is quiet, the camera shutter is opened while the turntable is rotated about 10°. From the viewpoint of the photograph, the tank is stationary while the fluid rotates with nearly constant vorticity. The aluminum particles show as streaks on the film. Each streak is in the direction of a shear stress. The length of the streak is proportional to the magnitude of the shear stress.

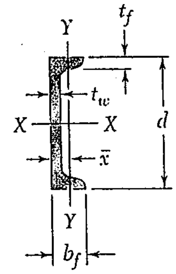
Experiments aside, the hydrodynamic analogy has other uses. One is in aiding visualization of torsion problems. Another is that known solutions for fluid flow can be applied to the torsion problem. For example, consider an elliptical obstacle (Fig. 8.5.1a). Far from the obstacle the flow is uniform and horizontal, so $w = 0$ and $v = v_o$, a constant. Points $A$ are stagnation points, where $v = w = 0$. Fluid theory shows that at points $B$, where $z = \pm b$, fluid velocities are $v = v_o(1 + b/a)$ and $w = 0$. Applying these results to a twisted bar, Fig. 8.5.1b, we conclude that stress is zero at the sharp external corners $C$. At the root of the small elliptical notch, shear stress is approximately

$$\tau = \tau_o\left(1 + \frac{b}{a}\right) \tag{8.5.3}$$

where $\tau_o$ is the stress that would prevail near the boundary if the notch were not there. The calculation is approximate because the undisturbed stress is not the uniform value $\tau_o$ unless the notch is very small. The

## Appendix C. Properties of Rolled-Steel Shapes
(SI Units)

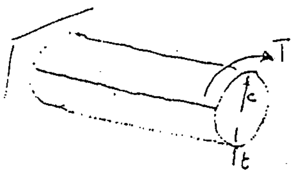
### C Shapes
(American Standard Channels)




| Designation† | Area $A$, mm² | Depth $d$, mm | Flange: Width $b_f$, mm | Flange: Thickness $t_f$, mm | Web Thickness $t_w$, mm | Axis X-X: $I_x$ $10^6$ mm⁴ | Axis X-X: $S_x$ $10^3$ mm³ | Axis X-X: $r_x$ mm | Axis Y-Y: $I_y$ $10^6$ mm⁴ | Axis Y-Y: $S_y$ $10^3$ mm³ | Axis Y-Y: $r_y$ mm | Axis Y-Y: $\bar{x}$ mm |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C380 × 74 | 9480 | 381 | 94 | 16.5 | 18.2 | 167 | 877 | 133 | 4.54 | 61.5 | 21.9 | 20.2 |
| 60 | 7570 | 381 | 89 | 16.5 | 13.2 | 144 | 756 | 138 | 3.79 | 54.7 | 22.4 | 19.7 |
| 50.4 | 6430 | 381 | 86 | 16.5 | 10.2 | 134 | 688 | 143 | 3.34 | 50.5 | 22.8 | 19.9 |
| C310 × 45 | 5690 | 305 | 80 | 12.7 | 13.0 | 67.2 | 441 | 109 | 2.09 | 33.2 | 19.2 | 17.0 |
| 37 | 4720 | 305 | 77 | 12.7 | 9.8 | 59.7 | 391 | 112 | 1.83 | 30.5 | 19.7 | 17.0 |
| 30.8 | 3920 | 305 | 74 | 12.7 | 7.2 | 53.4 | 350 | 117 | 1.57 | 27.7 | 20.0 | 17.4 |
| C250 × 45 | 5670 | 254 | 76 | 11.1 | 17.1 | 42.7 | 336 | 86.8 | 1.58 | 26.5 | 16.7 | 16.3 |
| 37 | 4750 | 254 | 73 | 11.1 | 13.4 | 37.9 | 298 | 89.3 | 1.38 | 24.0 | 17.0 | 15.6 |
| 30 | 3780 | 254 | 69 | 11.1 | 9.6 | 32.6 | 257 | 92.9 | 1.14 | 21.2 | 17.4 | 15.3 |
| 22.8 | 2880 | 254 | 65 | 11.1 | 6.1 | 27.7 | 218 | 98.1 | 0.912 | 18.5 | 17.8 | 15.8 |
| C230 × 30 | 3800 | 229 | 67 | 10.5 | 11.4 | 25.4 | 222 | 81.8 | 0.997 | 19.1 | 16.2 | 14.7 |
| 22 | 2840 | 229 | 63 | 10.5 | 7.2 | 21.2 | 185 | 86.4 | 0.796 | 16.5 | 16.7 | 14.9 |
| 19.9 | 2530 | 229 | 61 | 10.5 | 5.9 | 19.8 | 173 | 88.5 | 0.708 | 15.4 | 16.7 | 15.0 |
| C200 × 27.9 | 3560 | 203 | 64 | 9.9 | 12.4 | 18.2 | 179 | 71.5 | 0.817 | 16.4 | 15.1 | 14.3 |
| 20.5 | 2660 | 203 | 59 | 9.9 | 7.7 | 14.9 | 147 | 75.7 | 0.620 | 13.7 | 15.4 | 13.9 |
| 17.1 | 2170 | 203 | 57 | 9.9 | 5.6 | 13.4 | 132 | 78.6 | 0.538 | 12.6 | 15.7 | 14.4 |
| C180 × 18.2 | 2310 | 178 | 55 | 9.3 | 8.0 | 10.0 | 112 | 65.8 | 0.470 | 11.2 | 14.3 | 13.1 |
| 14.6 | 1850 | 178 | 53 | 9.3 | 5.3 | 8.83 | 99.2 | 69.1 | 0.400 | 10.2 | 14.7 | 13.7 |
| C150 × 19.3 | 2450 | 152 | 54 | 8.7 | 11.1 | 7.11 | 93.6 | 53.9 | 0.420 | 10.2 | 13.1 | 12.9 |
| 15.6 | 1980 | 152 | 51 | 8.7 | 8.0 | 6.21 | 81.7 | 56.0 | 0.347 | 9.01 | 13.2 | 12.5 |
| 12.2 | 1540 | 152 | 48 | 8.7 | 5.1 | 5.35 | 70.4 | 58.9 | 0.276 | 7.82 | 13.4 | 12.7 |
| C130 × 13 | 1710 | 127 | 48 | 8.1 | 8.3 | 3.70 | 58.3 | 46.5 | 0.264 | 7.37 | 12.4 | 12.2 |
| 10.4 | 1310 | 127 | 47 | 8.1 | 4.8 | 3.25 | 51.2 | 49.8 | 0.229 | 6.74 | 13.2 | 13.0 |
| C100 × 10.8 | 1370 | 102 | 43 | 7.5 | 8.2 | 1.90 | 37.3 | 37.2 | 0.172 | 5.44 | 11.2 | 11.4 |
| 8.0 | 1020 | 102 | 40 | 7.5 | 4.7 | 1.61 | 31.6 | 39.7 | 0.130 | 4.56 | 11.3 | 11.5 |
| C75 × 8.9 | 1130 | 76.2 | 40 | 6.9 | 9.0 | 0.850 | 22.3 | 27.4 | 0.122 | 4.25 | 10.4 | 11.3 |
| 7.4 | 936 | 76.2 | 37 | 6.9 | 6.6 | 0.751 | 19.7 | 28.3 | 0.0948 | 3.62 | 10.1 | 10.8 |
| 6.1 | 765 | 76.2 | 35 | 6.9 | 4.3 | 0.671 | 17.6 | 29.6 | 0.0765 | 3.16 | 10.0 | 10.8 |

†An American Standard Channel is designated by the letter C followed by the nominal depth in millimeters and the mass in kilograms per meter.

3.132




closed tube
$$J = \frac{\pi}{2}(a^4 - b^4) = \frac{\pi}{2}(a^2-b^2)(a^2+b^2) \; ; \; \tau_1 = \frac{Tc}{J} \; ; \; \phi_1 = \frac{TL}{JG}$$
$$= \frac{\pi}{2}(a-b)(a+b)(a^2+b^2)$$
$$\underline{J = \tfrac{\pi}{2} \cdot t \cdot 2c \cdot 2c^2 = 2\pi c^3 t} \qquad \underline{\tau_1 = \frac{T}{2\pi c^2 t}} \; ; \; \underline{\phi_1 = \frac{TL}{2\pi c^3 t G}}$$

Open tube
$$\underline{J_R = \frac{bt^3}{3} = \frac{2\pi c t^3}{3}} \qquad ; \; \tau_2 = \frac{Tt}{J_R} \; ; \; \beta = \frac{\phi_2}{L} = \frac{T}{GJ_R}$$
$$\tau_2 = \frac{Tt}{2\pi c t^3/3} = \frac{3Tt}{2\pi c t^3} = \frac{3T}{2\pi c t^2} \; ; \; \phi_2 = \frac{TL}{\frac{2\pi c t^3}{3} G} = \frac{3TL}{2\pi c t^3 G}$$
$$\frac{\tau_1}{\tau_2} = \frac{\frac{T}{2\pi c^2 t} \cdot \frac{t}{t}}{\frac{3T}{2\pi c t^2} \cdot \frac{c}{c}} = \frac{t}{3c} \; ; \qquad \frac{\phi_1}{\phi_2} = \frac{\frac{TL}{2\pi c^3 t G} \cdot \frac{t^2}{t^2}}{\frac{3TL}{2\pi c t^3 G} \cdot \frac{c^2}{c^2}} = \frac{t^2}{3c^2}$$
$$\left(\frac{\tau_1}{\tau_2}\right)^2 = \frac{t^2}{9c^2} = \frac{1}{3}\left(\frac{\phi_1}{\phi_2}\right)$$

so, $\underline{\frac{\tau_2}{\tau_1} = 3(c/t)}$ and $\underline{\phi_2/\phi_1 = 3(c/t)^2}$

Fig. P3.108 and P3.109

3.132 A thin-walled tube has been fabricated by bending a metal plate of thickness $t$ into a cylinder of radius $c$ and bonding together the edges of the plate. A torque T is then applied to the tube, producing a shearing stress $\tau_1$ and an angle of twist $\phi_1$. Denoting by $\tau_2$ and $\phi_2$, respectively, the shearing stress and the angle of twist which will develop if the bond suddenly fails, express the ratios $\tau_2/\tau_1$ and $\phi_2/\phi_1$ in terms of the ratio $c/t$.

$\phi_2/\phi_1 = 3\left(\frac{c}{t}\right)^2$

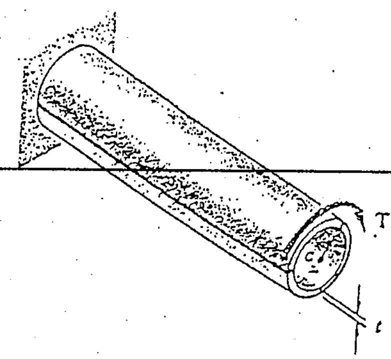



Fig. P3.132

3.122 A 3.5-m-long steel member with a W310 × 143 cross section is subjected to a 4.5 kN · m torque. Knowing that $G = 77$ GPa and referring to Appendix C for the dimensions of the cross section, determine (a) the maximum shearing stress along line a-a, (b) the maximum shearing stress along line b-b, (c) the angle of twist. (*Hint:* Consider the web and the flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

39.7 MPa 24.2 MPa 4.72°

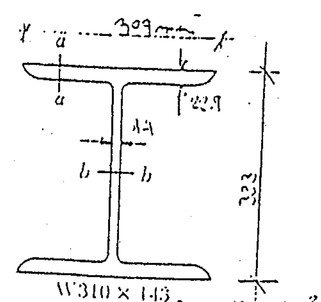
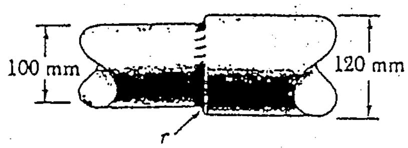



Fig. P3.122

7P

HW: DO 3.21, 3.47 & 3.53

PROBLEM 3.87




3.87 The stepped shaft shown rotates at 450 rpm. Knowing that $r = 10$ mm, determine the maximum power that can be transmitted without exceeding an allowable shearing stress of 45 MPa.

SOLUTION

$d = 100$ mm, $D = 120$ mm, $r = 10$ mm

$\frac{D}{d} = \frac{120}{100} = 1.2$, $\frac{r}{d} = \frac{10}{100} = 0.10$, From Fig. 3.32 $K = 1.33$

For smaller shaft $c = \frac{1}{2}d = 0.050$ m $\qquad \tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.050)^3 (45 \times 10^6)}{(2)(1.33)} = 6.643 \times 10^3 \text{ N-m}$$

$f = 450$ rpm $= 7.5$ Hz $= 450/60$

Power $P = 2\pi f T = 2\pi (7.5)(6.643 \times 10^3) = 313 \times 10^3$ W $= 313$ kW

$T = \frac{9540 P}{N} \Rightarrow P = \frac{T \cdot N}{9540} = 313.35$ kW

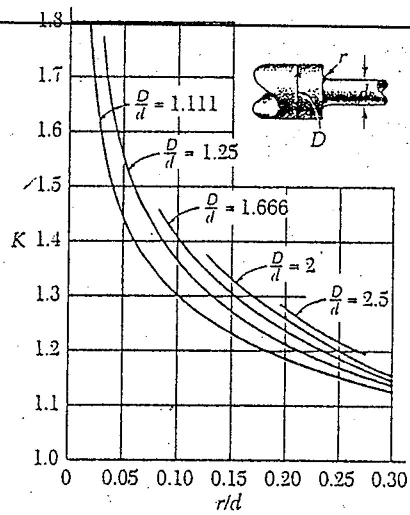


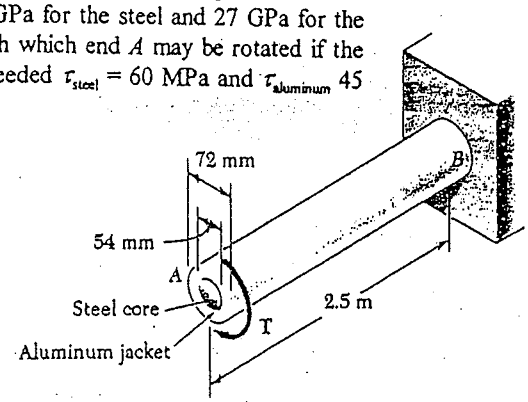

Fig. 3.32 Stress-concentration factors for fillets in circular shafts.†

TABLE 3.1. Coefficients for Rectangular Bars in Torsion

| $a/b$ | $c_1$ | $c_2$ |
|---|---|---|
| 1.0 | 0.208 | 0.1406 |
| 1.2 | 0.219 | 0.1661 |
| 1.5 | 0.231 | 0.1958 |
| 2.0 | 0.246 | 0.229 |
| 2.5 | 0.258 | 0.249 |
| 3.0 | 0.267 | 0.263 |
| 4.0 | 0.282 | 0.281 |
| 5.0 | 0.291 | 0.291 |
| 10.0 | 0.312 | 0.312 |
| ∞ | 0.333 | 0.333 |

PROBLEM 3.53

3.53 The composite shaft shown is to be twisted by applying a torque **T** at end $A$. Knowing that the modulus of rigidity is 77 GPa for the steel and 27 GPa for the aluminum, determine the largest angle through which end $A$ may be rotated if the following allowable stresses are not to be exceeded $\tau_{steel} = 60$ MPa and $\tau_{aluminum}$ 45 MPa.




SOLUTION

$$\tau_{max} = G\gamma_{max} = G\, c_{max} \frac{\varphi}{L}$$

$$\frac{\varphi_{all}}{L} = \frac{\tau_{all}}{G\, c_{max}} \quad \text{for each material}$$

Steel core: $\tau_{all} = 60\times10^6$ Pa, $c_{max} = \frac{1}{2}d = 0.027$ m, $G = 77\times10^9$ Pa

$$\frac{\varphi_{all}}{L} = \frac{60\times10^6}{(77\times10^9)(0.027)} = 28.860\times10^{-3} \text{ rad/m}$$

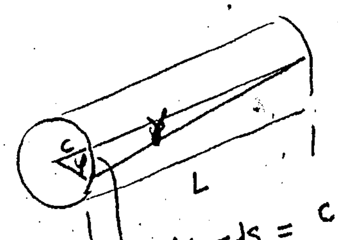
Aluminum jacket: $\tau_{all} = 45\times10^6$ Pa, $c_{max} = \frac{1}{2}d = 0.036$ m, $G = 27\times10^9$ Pa

$$\frac{\varphi_{all}}{L} = \frac{45\times10^6}{(27\times10^9)(0.036)} = 46.296\times10^{-3} \text{ rad/m}$$

Smaller value governs $\quad \frac{\varphi_{all}}{L} = 28.860\times10^{-3}$ rad/m

Allowable angle of twist $\quad \varphi_{all} = L\frac{\varphi_{all}}{L} = (2.5)(28.860\times10^{-3})$

$$= 72.15\times10^{-3} \text{ rad} = 4.13°$$




$\gamma . L = ds = c \cdot \varphi$

$$\varphi = \frac{\gamma \cdot L}{c} = \frac{\tau_{max} \cdot L}{G \cdot c}$$

$\therefore \frac{\varphi}{L}$ must be same for both

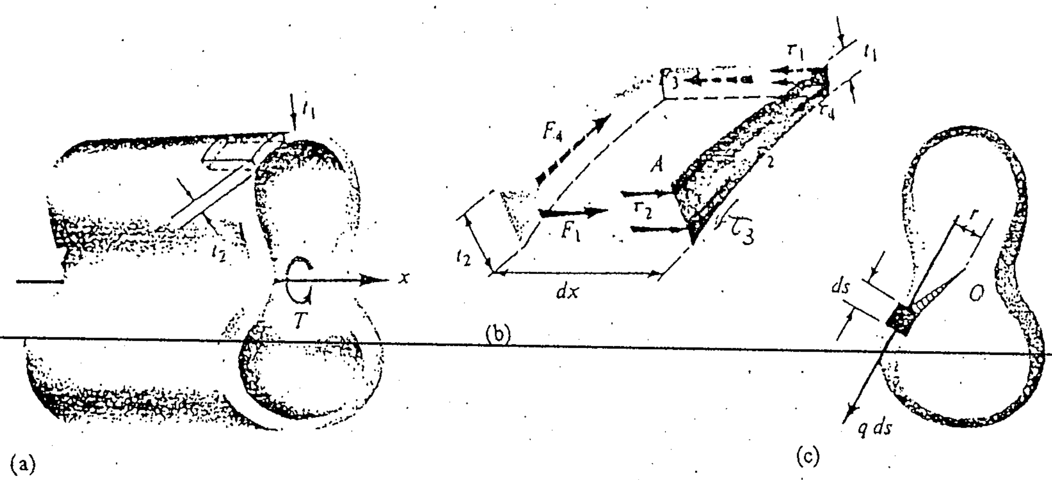
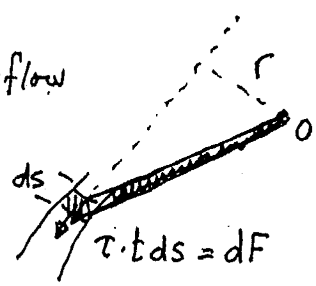
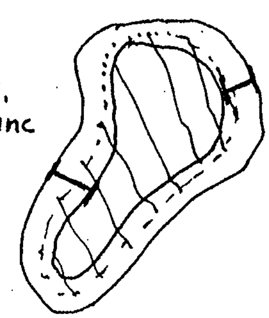



Fig. 3-22. Thin-walled member of variable thickness.

$$F_1 = F_3$$
$$\tau_1 t_1 dx = \tau_2 t_2 dx$$
$$\tau_1 t_1 = \tau_2 t_2 = q = \text{shearflow}$$




$$T = \oint dT = \oint r \cdot q ds = q \oint r ds$$
$$T = q \cdot 2A_{inc}$$
$$\tau t = q = \frac{T}{2A_{inc}}$$
$$\tau = \frac{T}{2A_{inc} t}$$



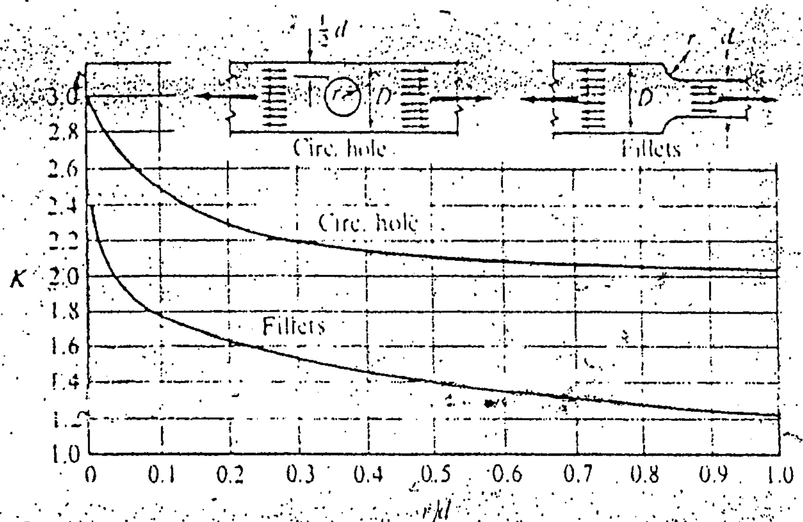



Fig. 2-17. Stress-concentration factors for flat bars in tension

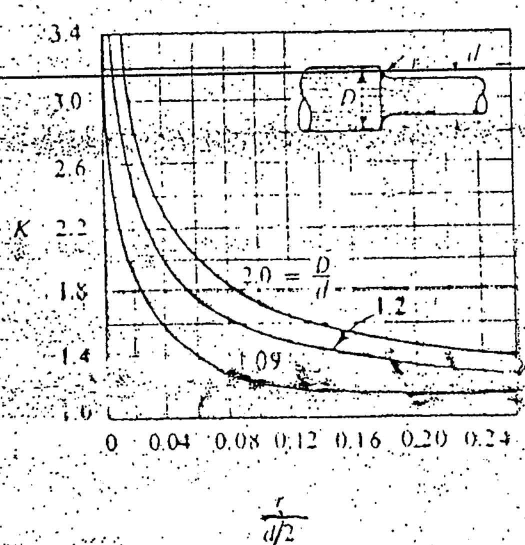



Fig. 3-15. Torsional stress-concentration factors in circular shafts of two diameters.

Florida International University
Department of Mechanical and Materials Engineering

EMA 3702 QUIZ 4A April 19, 2011

You are allowed eight sheets of 8 ½ x 11 inch paper. None are to include solutions

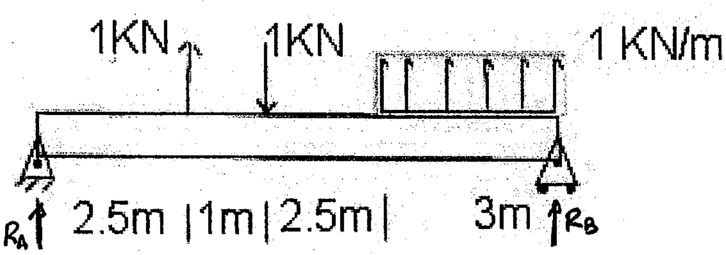
Print your name and sign the following statement:

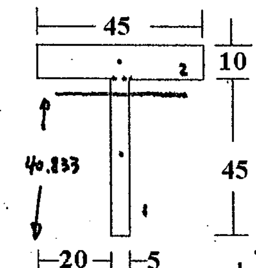
I will not give nor take any unpermitted aid during this quiz. I understand that violation of this statement will lead to automatic failure of the quiz.

__________________ PRINT NAME __________________ SIGN NAME

Problem 1.

The figure gives the beam with the loads applied to it, as well as its cross-section. Find the largest shear stress, where is it located along the length of the beam, and its location on the cross section.




| | $\bar{y}$ | A | $\bar{y}A$ |
|---|---|---|---|
| 1 | 22.5 | 225 | 5062.5 |
| 2 | 50 | 450 | 22500.- |
| | | 675 | 27562.5 |

$\bar{y} = 40.833$ (9)

| | I | A | d | $I_z$ |
|---|---|---|---|---|
| 1 | $\frac{1}{12}\cdot 5\cdot 45^3$ | 225 | $(22.5 - 40.83)$ -18.33 | 113566.3 |
| 2 | $\frac{1}{12}\cdot 45\cdot 10^3$ | 450 | $(40.83 - 50)$ 9.17 | 41590.0 |

$\times 10^{-8}$
155156.31 $m^4$
(9)

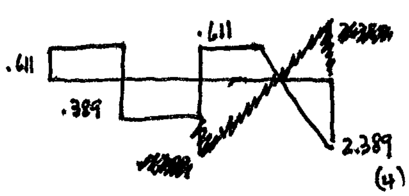
Units of length for the cross section are in cm.

$+\uparrow \Sigma F_y = R_A + 1 - 1 + 3 + R_B = 0 \qquad R_A + R_B + 3 = 0 \quad (3)$

$+\circlearrowleft \Sigma M_A = 1\cdot 2.5 - 1\cdot 3.5 + 1\cdot 3\cdot 7.5 + R_B\cdot 9 = 0$

$21.5 + R_B\cdot 9 = 0 \qquad R_B = -21.5/9 = -2.389\ kN \quad (5)$

$R_A = -.611\ kN \quad (1)$




$V = 2.389$ @ 9m at N/A (1)

$Q = A\bar{y} = (40.833\times 5)\left(\frac{40.833}{2}\right) = 4168.335\ cm^3$

$= 4168.335\times 10^{-6}\ m^4 \quad (3)$

$\tau = \frac{VQ}{It} = \frac{(2389)(4168.335\times 10^{-6})}{(155156.31\times 10^{-8})(5\times 10^{-2} m)}$

$=$ Pa

128.363 kPa (5)

$\frac{5}{4}$ pts/pt $\times$ 40 points

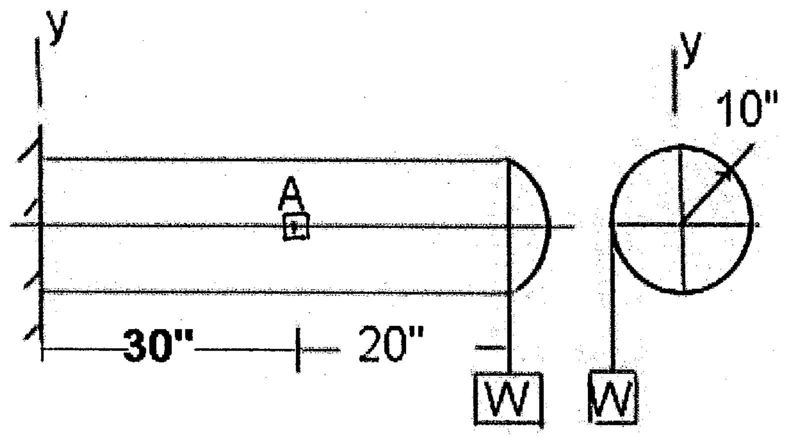
9 for Equil
4 for Shear diag + 1 for loc
9 for $\bar{y}$
9 for I
3 for Q
5 for $\tau$/40

Problem 2.

A steel pressure vessel has a 20 inch outer diameter and a 0.25 inch wall thickness. It acts also as an eccentrically loaded cantilever as in the figure. If the internal pressure is 250 psi and the applied weight W=31.4 kips,

- Determine the state of stress at the point A.
- Show the results on an infinitesimal element.
- Find the principal stresses and state whether the cylinder has failed using maximum stress criterion.

Take E=30 x $10^6$ psi and $\sigma_{yp}$ = 60 ksi if needed




$r = 10 - t = 9.75\text{in}$ (1)

$\sigma_1 = \frac{pR}{t}$ $\sigma_2 = \frac{\sigma_1}{2}$

$\sigma_1 = \frac{(250)(9.75)}{.25}$ $\sigma_2 = 4875\text{psi}$ (1)

$= 9750\text{psi}$ (4)

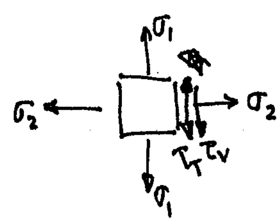
$\tau_T = \frac{Tc}{J}$ $T = W \cdot 10\text{in} = 31400(10)$ (1)

$= 314000\text{ lb-in}$

$= \frac{3.14\times10^5(10)}{1512.867}$ $c = 10\text{in} = c_o$ $c_i = 9.75\text{in}$

$J = \frac{\pi}{2}(c_o^4 - c_i^4) = 1512.867\text{in}^4$ (2)

$= 2075.53\text{psi}$ (4)

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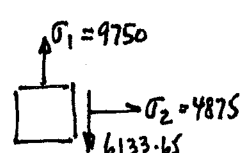



$\cancel{1972.58}$

(4) for numbers + 3 for directions

$\tau_v = \frac{VQ}{It}$

$Q = \frac{4c_o}{3\pi}\cdot\frac{\pi c_o^2}{2} - \frac{4c_i}{3\pi}\cdot\frac{\pi c_i^2}{2} = \Sigma\bar{y}A$

$Q = \frac{2}{3}(c_o^3 - c_i^3) = 48.76\text{in}^3$ (4)


$\tau_T$

$\tau_v = \frac{(31400)(48.76)}{I\ (2)(.25)}$ $I = J/2 = 756.433\text{in}^4$ (1)

$\sigma_{max} = \frac{\sigma_1+\sigma_2}{2} \pm \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau_{xy}^2}$

$= 7312.5 \pm \sqrt{(2437.5)^2 + (6133.65)^2}$ $= 4048.115\text{ psi}$ (5)

$\pm \cancel{3135.68}\ 6590.94$

$= \frac{13903.4}{\cancel{10448.2}}\text{psi}$ (3) did not failure since $\sigma_{max} < \sigma_{yp}$.

$\tau_{max} = 6590.94 < \frac{\sigma_{yp}}{2} = 30\text{ksi}$

$\cancel{30}$ 33 pnts

$\int y\,dA = \int_0^R\int_0^\pi r\sin\theta \cdot r\,dr\,d\theta = r^3/3 \cdot -\cos\theta\Big|_0^\pi = -\frac{R^3}{3}\cdot(-1-1) = \frac{2R^3}{3}$

1.5pts/part

$\int dA = \pi R^2/2$

$\bar{y} = \frac{\int y\,dA}{\int dA} = \frac{2R^3/3}{\pi R^2/2} = \frac{4R}{3\pi}$ ✓

if you ~~double~~ ~~quadruple~~ double p & W then $\sigma_{max} = \cancel{27.8}\text{psi}$ 27.8 ksi

Safety factor is 2.16

Florida International University
Department of Mechanical and Materials Engineering

EMA 3702 QUIZ 4C April 19, 2011

You are allowed eight sheets of 8 ½ x 11 inch paper. None are to include solutions

Print your name and sign the following statement:

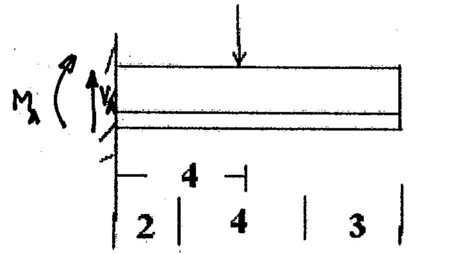
I will not give nor take any unpermitted aid during this quiz. I understand that violation of this statement will lead to automatic failure of the quiz.

PRINT NAME SIGN NAME

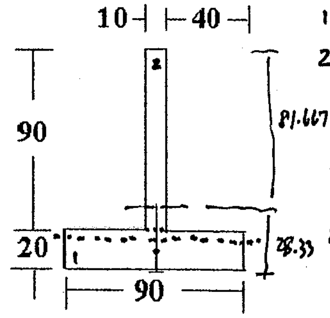
Problem 1.

- The figure gives the beam with the loads applied to it. Also given is the cross-section made up of a horizontal plank 20 x90 cm and a vertical plank 10 x 90 cm joined together by nails that can support loads of 700 N according to lab tests. Find the nail spacing required.

- What is the maximum shear stress and where is it located?




**units of length in meters**




**Units of length for the cross section are in cm.**

| | $\bar{y}$ | A | $\bar{y}A$ |
|---|---|---|---|
| 1 | 10 | 1800 | 18000 |
| 2 | 65 | 900 | 58500 |
| | | 2700 | ~~23850~~ 76500 |

$\bar{y} = 28.333$ (9)

| | I | A | d | I |
|---|---|---|---|---|
| 1 | $\frac{1}{12} \cdot 90 \cdot 20^3$ | 1800 | $28.333 - 10$ | ~~64451.4~~ 664978 |
| 2 | $\frac{1}{12} \cdot 10 \cdot 90^3$ | 900 | $65 - 28.333$ | ~~344675.87~~ 1917522 |

~~3509210.1~~ $\times 10^{-8}\ m^4$
2482500 (9)

$+\uparrow \Sigma F \Rightarrow V_A - 1000\ kN = 0$ (2) $V_A = 1000\ kN$

$+\circlearrowleft \Sigma M_A \Rightarrow M_A = -1000kN(4) = -4000\ kN\text{-}m$ (2)


$V = 1000\ kN$ bet $x = 0 \,\&\, 4m$
(2)

$Q = \bar{y}A = \cancel{(10-8.833)(20)}(90)$ $(28.33-10)(20\times90) + \cancel{10(8.33)^2/2}$

$= \cancel{2100.6} \times 10^{-6}\ m^3$ (4)
~~33318.18~~ 33000

$[N/m]$ $q = \frac{VQ}{I} = \frac{1000\times10^3\,(\cancel{2100.6}\times10^{-6})}{24825^{00}\,\cancel{3509210.1}\times10^{-8}}$ 33000 ~~33318.18~~

$= \cancel{59859.625}\ N/m = S/d$ (4)
1329305 ~~1343047.33~~

$d = \frac{700\ N}{\cancel{59859.625\ N/m}} = \cancel{.01169m} = \cancel{11.69mm}$ .000527m 0.527mm
1343047.33 (2)

$\cancel{\tau = q/t = \frac{59859.625\ N/m}{90\times10^{-2}\ m}} = \cancel{66510.7\ \frac{N}{m^2}}$ ~~13.43MPa~~ (2)

$Q_{@NA} = 33000 + 10\left(\frac{8.33}{2}\right)^2 = 33341.15 \times 10^{-6}\ m^3$

$\tau = \frac{VQ}{It} = \frac{(1000\times10^3)(33346.15\times10^{-6})}{(2482500\times10^{-8})(\ldots)}$

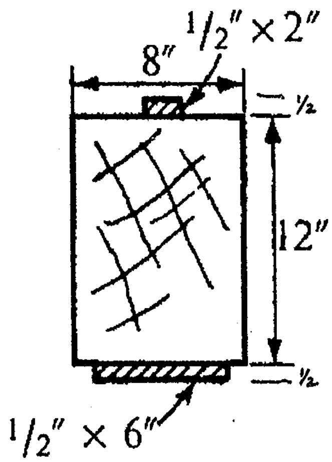
47
~~33~~ pts = [illegible]

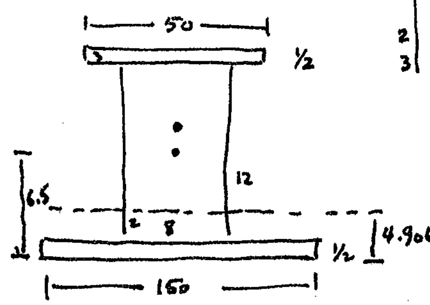
Problem 2.

Determine the allowable shear force for the composite beams of wood and two steel plates having the cross-sectional dimensions shown in the figures. Materials are fastened together so that they act as a unit.

Given:
$E_S = 30 \times 10^6$ psi; $E_W = 1.2 \times 10^6$ psi. The allowable bending stresses are $\tau_S = 20$ ksi and $\tau_W = 1.2$ ksi

$\frac{E_S}{E_W} = \frac{30}{1.2} = 25$ (1) converts steel to wood.




| | $\bar{y}$ | A | $\bar{y}A$ |
|---|---|---|---|
| 1 | 1/4 | 75 | 18.75 |
| 2 | 6.5 | 96 | 624 |
| 3 | 12.75 | 25 | 318.75 |
| | | 196 | 961.5 |

(12) $\bar{y} = 4.906$

| | I | A | d | $I_z$ |
|---|---|---|---|---|
| 1 | $\frac{1}{12}(150)(.5)^3$ | 75 | (4.906 − .25) | 1627.44 |
| 2 | $\frac{1}{12}(8)(12)^3$ | 96 | (6.5 − 4.906) | 1395.92 |
| 3 | $\frac{1}{12}(50)(.5)^3$ | 25 | (12.75 − 4.906) | 1538.73 |
| | | | | 4562.09 $in^4$ |

(13)

$\tau_S = \frac{VQ}{It} = \frac{VQ_1}{It_1} \rightarrow Q_1 = \bar{y} \cdot A = (13 - 4.906 - .25)(50)(.5) = 196.1\ in^3$ (4)

$t_1 = 2\ in$

$20000 = \frac{V(196.1)}{(4562.09)(2)}$ $V = 930{,}564$ lb (5) 930.5 kip

or

$20000 = \frac{VQ_2}{It_2} \rightarrow Q_2 = \bar{y}A = (4.9603 - .25)(150)(.5) = 349.2\ in^3$ (4)

$t_2 = 6\ in$

$= \frac{V(349.2)}{(4562.09)(6)}$ $V = 1567728.22$ lb (5) 1567.7 kip

$\tau_W = 1200 = \frac{VQ_3}{It_3} \rightarrow Q_3 = \bar{y}_a A_a + \bar{y}_b A_b = (13 - 4.906 - .25)(50)(.5) + \frac{8(12.5 - 4.906)^2}{2}$ (2)

$= 426.776$ (7)

$t_3 = 8\ in$

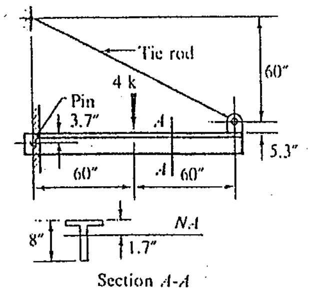
$1200 = \frac{V(426.775)}{(4562.09)(8)}$ $V = 102620.97$ lb (5)

$= 102.62$ kip

53 points

[redacted]

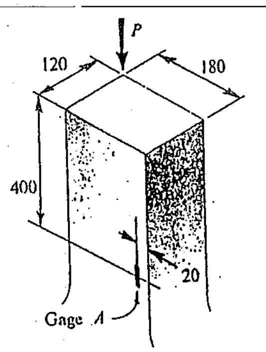
**10-50.** Plot shear, moment, and axial force diagrams for the jib-crane loaded as shown in the figure. Neglect the weight of the beam. *Ans:* 128.6 k-in.




PROB. 10 – 50

8-24

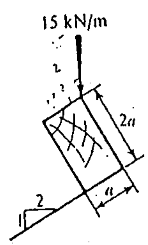
~~7-35.~~ An aluminum-alloy block is loaded as shown in the figure. The application of this load produces a tensile strain of $500 \times 10^{-6}$ mm. per mm at $A$ as measured by means of an electrical strain gage. Compute the magnitude of the applied force $P$. Let $E = 10 \times 10^{6}$ kN/m². All dimensions given in the figure are in mm.




PROB. 7 – 35

8-19 ind

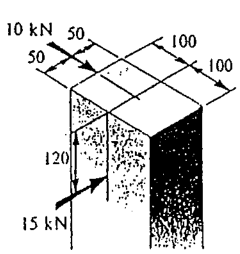
**7-44.** A tilted, simply supported beam with a depth to width ratio of 2 to 1 is to span 4 m and is to carry a uniformly distributed load of 15 kN per linear meter, including its own weight, applied as shown in the figure. (a) Determine the required dimensions of the beam so that the maximum stress due to bending does not exceed 10 MN/m². (b) Locate the neutral axis of the beam and show its position on the sketch.



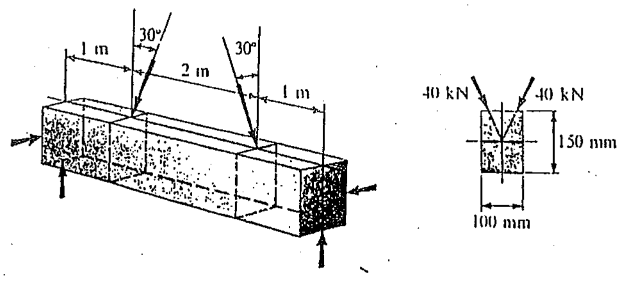

PROB. 7 – 44

8-23

~~7-34.~~ A cast iron block is loaded as shown in the figure. Neglecting the weight of the block, determine the stresses acting normal to a section taken 0.5 m below the top and locate the line of zero stress. All dimensions given in the figure are in mm. *Ans:* ±24.6 MPa, ±9.6 MPa.



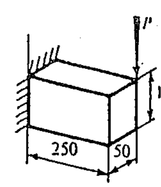

**10-91.** A 4 m long beam is loaded as shown in the figure. The two applied forces act perpendicularly to the long axis of the beam and are inclined 30° with the vertical. If these forces act through the centroid of the cross-sectional area find the location and magnitude of the maximum bending stress. Neglect the weight of the beam. *Ans:* ±132 MN/m².




PROB. 10 – 91

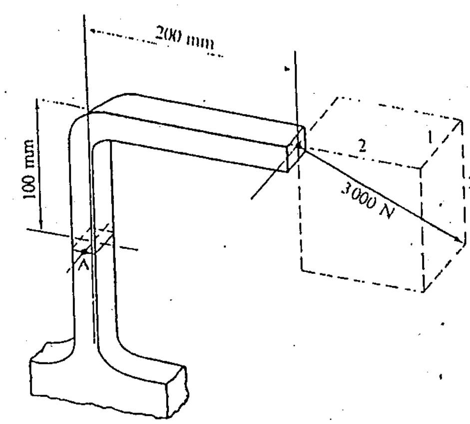
8-28

~~7-45.~~ A rectangular cantilever 250 mm long is loaded with $P = 50$ kN at the free end as shown in the figure. Determine the maximum shearing stress at the built-in end due to the direct shear and the torque. Show the result on a sketch analogous to Fig. 7-17(e). All dimensions shown in the figure are in mm. *Ans:* 35.3 MPa.



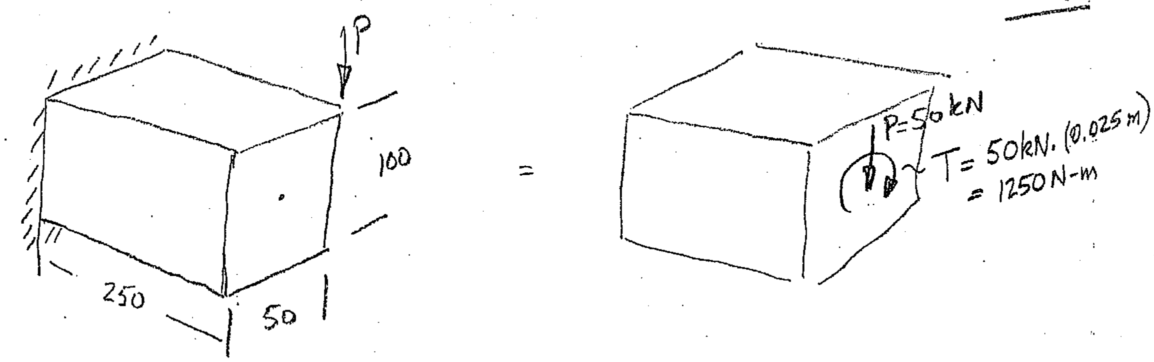

PROB. 7 – 45

**9-26.** A bent rectangular bar is subjected to an inclined force of 3 000 N as shown in the figure. The cross-section of the bar is 12 mm by 12 mm. (a) Determine the state of stress at point $A$ caused by the applied force and show the results on an element. (b) Find the maximum principal stress. *Ans:* (a) +333 MPa, 578 MPa

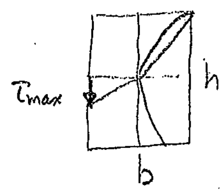


[redacted]



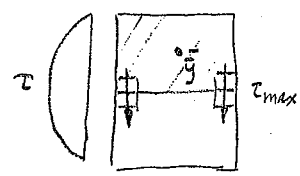



| h/b | 1.00 | 1.50 | 2.00 | 3.00 | ∞ |
|---|---|---|---|---|---|
| α | 0.208 | 0.231 | 0.246 | 0.267 | 0.333 |




$\frac{h}{b} = \frac{100}{50} = 2$ $\qquad \alpha = 0.246$

[redacted] $\tau_{max} = \frac{T}{\alpha h b^2} = \frac{1250\ N\text{-}m}{0.246(0.10)(0.05)^2} = 20.3252\ MPa$




$\tau_{max} = \frac{VQ}{It} = \frac{V}{\frac{bh^3}{12}\cdot b}\ \frac{bh^2}{8}$

$Q = \int y\,dA = \bar{y}\cdot A = \frac{h}{4}\cdot\frac{hb}{2} = \frac{h^2 b}{8}$

[redacted] $\tau_{max} = \frac{3V}{2A} = \frac{3\cdot 50000N}{2(0.10)(0.05)} = 15\ MPa$

$\tau_{max} = 20.33\,MPa + 15\,MPa = 35.33\ MPa$

$\sigma_{max} = \frac{+P\ell\cdot y}{I_{zz}} = \frac{50000\cdot(0.25)(0.050)}{4.167\times10^{-6}} = 150\ MPa$ $\qquad I_{zz} = \frac{bh^3}{12} = \frac{(0.05)(0.1)^3}{12} = 4.166\times10^{-6}\ m^4$

$\sigma_{min} = 0$ $\qquad 0 = y$ [redacted]