HW #1

12-11, 15, 24, 25 in 6 pgs

Homework.

Ch 12

(11)

For Normal driver:

Initial velocity = 44 ft/s
acceleration = -2 ft/s^2

Using: $v^2 = u^2 + 2as$ where v = final velocity.
u = Initial velocity.
S = displacement.

$0 = 44^2 - 4S$

$S = \frac{44^2}{4} = 484$ ft

Now the driver also has a delay reaction time of 0.75 secs.
So Using: $S = ut$ where u = Initial velocity
$S = 44(0.75)$ t = time
$S = 33$ ft

$\therefore$ The shortest stopping distance needed = 484 + 33
= 517 ft

(15)

$$s = (t^3 - 9t^2 + 15t)\ \text{ft}$$

at $t = 6s$.

$$s = 6^3 - 9(6)^2 + 15(6) = 216 - 324 + 90$$
$$= \underline{-18\ \text{ft}}$$ ✓

$$s = t^3 - 9t^2 + 15t$$
$$\frac{ds}{dt} = v = 3t^2 - 18t + 15$$

Now particle stops when $3t^2 - 18t + 15 = 0$

$$(3t - 3)(t - 5) = 0$$

$t = 1s$, $t = 5s$. ✓

at $t = 1s$

$$s = 1 - 9 + 15 = 7\ \text{ft}$$

at $t = 5s$

$$s = 125 - 225 + 75 = -25\ \text{ft}$$

Distance travelled so far is $\underset{(7-(-25))}{7 + 32} = 39\ \text{ft}$ ✓

at $t = 6s$

$$s = -18\ \text{ft}$$

So total distance travelled $= \underset{(-18-(-25))}{39 + 7} = \underline{46\ \text{ft}}$ ✓

For the drunken driver :

Initial velocity, $u = 44\ ft/s$
acceleration, $a = -2\ ft/s^2$

Using $v^2 = u^2 + 2as$

$0 = 44^2 - 4s$

$s = 484\ ft$

But the delay time for the drunken driver = 3 secs.

Using $s = ut$ where u = initial velocity
t = time

$s = 44(3) = 132\ ft.$

$\therefore$ The shortest stopping distance $= 484 + 132$
$= \underline{616\ ft}$

(24) For Car A :

$u = 0$ ft/s

$a = 6$ ft/s^2

$V = 80$ ft/s

Using $V^2 = u^2 + 2as$

$6400 = 12s \Rightarrow s = 533\frac{1}{3}$ ft

Using $V = u + at \Rightarrow 80 = 6t \Rightarrow t = 13\frac{1}{3}$ sec.

(t during which car accelerates)

For Car B: (car B also is moving during this time)

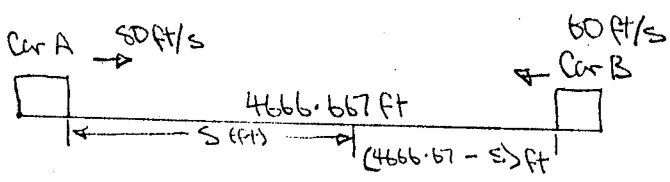
at $t = 13\frac{1}{3}$ sec.

Using $s = ut$

$s = 60(13\frac{1}{3}) = 800$ ft.

Now Car A is travelling at a constant velocity of 80 ft/s and Car B at 60 ft/s with a distance of

$6000 - 800 - 533\frac{1}{3} = 4666.667$ ft between them.



Using $s = ut$

For Car A:

$$S = 80t \quad --- \quad (1)$$

For Car B:

$$4666.667 - S = 60t \quad --- \quad (2)$$

Sub. $80t$ for S in (2).

$$4666.667 - 80t = 60t$$

$$140t = 4666.667$$

$$t = 33.33 \text{ secs.}$$

For Car A, $S = 80(33.33) = 2666\tfrac{2}{3}$ ft.

Hence Car A will travel a distance of <u>3200 ft</u> in a time of <u>$46\tfrac{2}{3}$</u> secs when it encounters Car B.

25

$a = (2 - v^2/500)\ ft/s^2$

For max. speed, $a = 0$

$2 - v^2/500 = 0$

$v^2 = 1000$

$v = \sqrt{1000} = \underline{31.6\ ft/s}$

$a = 2 - v^2/500$

$v\dfrac{dv}{ds} = 2 - v^2/500$

$v\,dv = (2 - v^2/500)\,ds$

$\dfrac{v}{2 - v^2/500}\,dv = ds$

$s = \displaystyle\int \frac{500v}{1000 - v^2}\,dv = -250\ln(1000 - v^2) + C$

when $s = 0$, $v = 0$

$0 = -250\ln(1000) + C$

$C = 250\ln(1000)$

$s = -250\ln(1000 - v^2) + 250\ln(1000)$

when $v = 8\ ft/s$

$s = -250\ln(936) + 250\ln(1000) = 1726.9 - 1710.4$

$s = \underline{16.5\ ft}$

RESF 246.06

SOLUTION HW #2

12-57, 66, 75, 81 in 8 PAGES

Homework Problems ( )

Ch 12

57) $\bar{r}_A = \{4.5t\hat{i} + 13.5t(2-t)\hat{j}\}$ m

$\bar{r}_B = \{4.5(t^2-2t+2)\hat{i} + (4.5t-9)\hat{j}\}$ m

For Particles to collide: $r_{A_x} = r_{B_x}$ $r_{A_y} = r_{B_y}$

$$4.5t = 4.5(t^2-2t+2)$$
$$4.5(t^2-3t+2) = 0$$
$$4.5(t-2)(t-1) = 0$$
$$(t-2)(t-1) = 0$$
$$t = 2,\ t = 1$$

$$13.5t(2-t) = 4.5t-9$$
$$-13.5t^2 + 27t = 4.5t - 9$$
$$13.5t^2 - 22.5t - 9 = 0$$
$$4.5(3t^2 - 5t - 2) = 0$$
$$4.5(3t+1)(t-2) = 0$$
$$t = 2,\ t = -1/3$$

Particles will collide at t = 2 secs.

The Position vector for the pt. of collision is

$\bar{r}_c = \{9\hat{i}\}$

The Collision will take place at (9 m, 0)

$$\bar{V}_a = \frac{d\bar{r}_a}{dt} = \{4.5\hat{i} + (-27t + 27)\hat{j}\}\ m/s$$

$$\bar{V}_b = \frac{d\bar{r}_b}{dt} = \{4.5(2t-2)\hat{i} + 4.5\hat{j}\}\ m/s$$

Just before collision:

$$\bar{V}_a = \{4.5\hat{i} + (-27)\hat{j}\}\ m/s$$

$$|\bar{V}_a| = \sqrt{(4.5)^2 + (-27)^2} = \sqrt{749.25} = 27.37\ m/s$$

$$\bar{V}_b = \{9\hat{i} + 4.5\hat{j}\}\ m/s$$

$$V_b = \sqrt{(9)^2 + (4.5)^2} = \sqrt{101.25} = 10.06\ m/s$$

(2) ÷ (1)

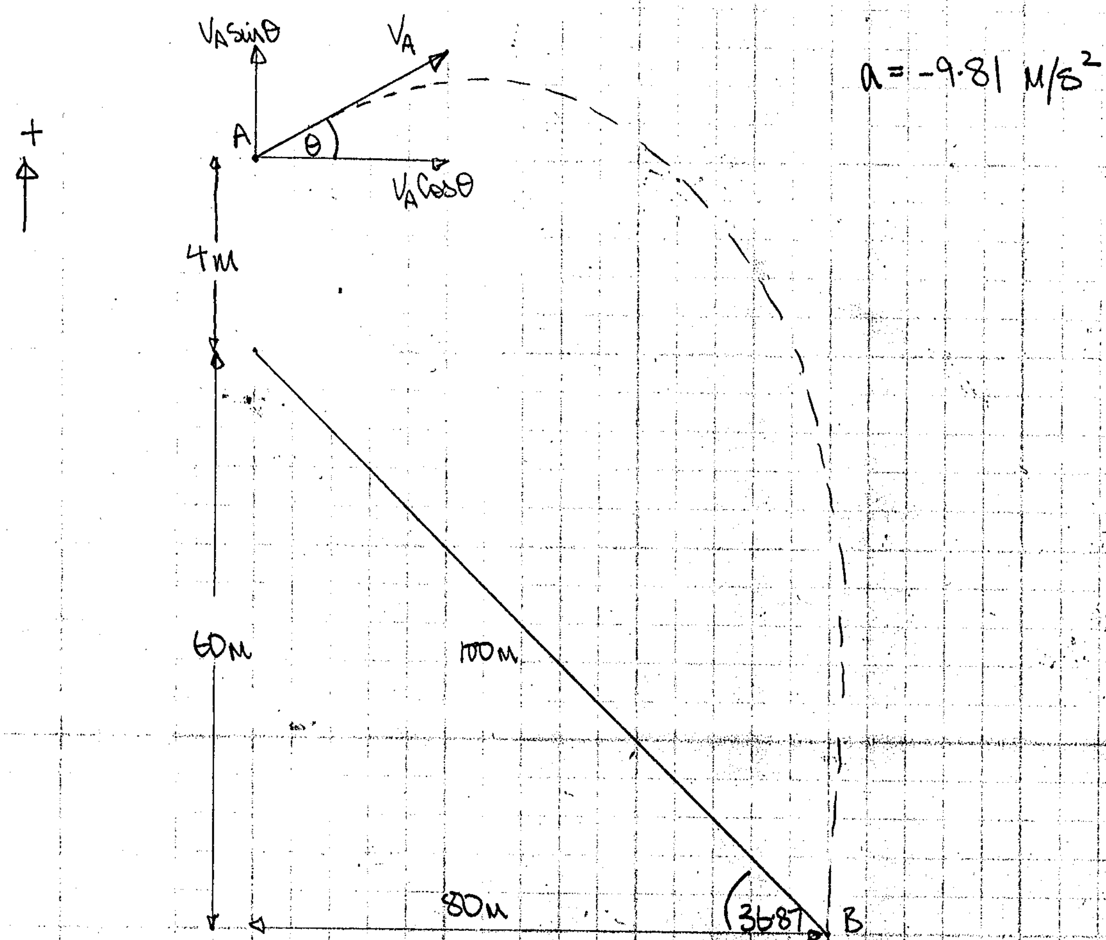
$$\frac{V_A \sin\theta}{V_A \cos\theta} = \frac{11.725}{16} \Rightarrow \tan\theta = 0.733$$

$$\theta = \tan^{-1} 0.733 = \underline{36.2^\circ}$$ ✓

$$V_A \cos\theta = 16.$$

$$V_A = \frac{16}{\cos\theta} = \frac{16}{0.807} = \underline{19.8 \text{ m/s}}$$ ✓

(66)



For Horizontal Motion:

$$V_A \cos\theta = \frac{80}{5} = 16\ \frac{m}{s} \quad --- (1)$$

For Vertical Motion:

Using: $s = V_{oy}t + \frac{1}{2}at^2$

$$-64 = 5V_A \sin\theta - \frac{1}{2}(25)(9.81)$$

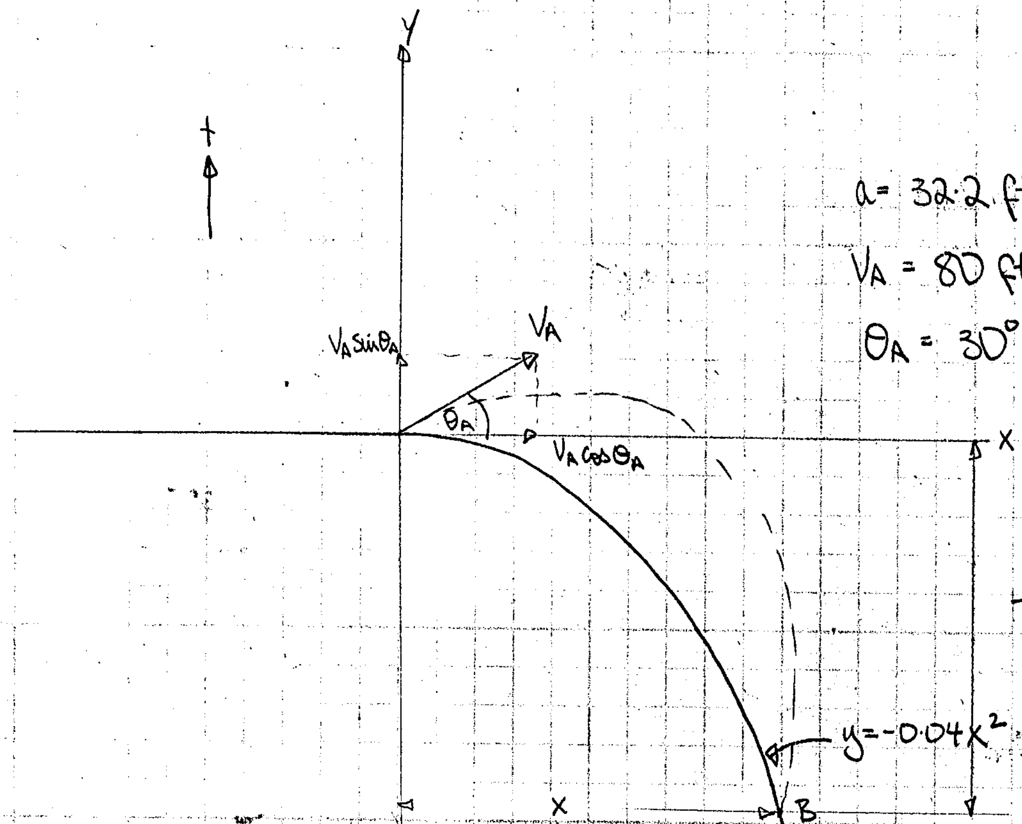
$$V_A \sin\theta = 11.725\ \frac{m}{s} \quad --- (2)$$

75

$a = 32.2\ ft/s^2$

$V_A = 80\ ft/s$

$\theta_A = 30^\circ$



For Vertical Motion: Using $S = v_y t + \frac{1}{2}at^2$

$$-Y = V_A \sin\theta_A t - \tfrac{1}{2}(32.2)t^2$$

$$-Y = 40t - 16.1t^2 \quad --- \quad (1)$$

For Horizontal Motion:

$$X = V_A \cos\theta_A t = 80(0.866)t$$

$$X = 69.28t$$

$$t = \frac{X}{69.28} \quad -- \quad (2)$$

Sub. (2) in (1):

$$-Y = \frac{40X}{69.28} - 16.1\left(\frac{X}{69.28}\right)^2$$

$$-y = 0.577x - 3.35\times10^{-3}x^2 \quad --- (3)$$

$$y = -0.04x^2 \quad (\text{eq}^n \text{ of parabola given})$$ ✓

$$0.04x^2 = 0.577x - 3.35\times10^{-3}x^2$$

$$4.335\times10^{-2}x^2 = 0.577x$$

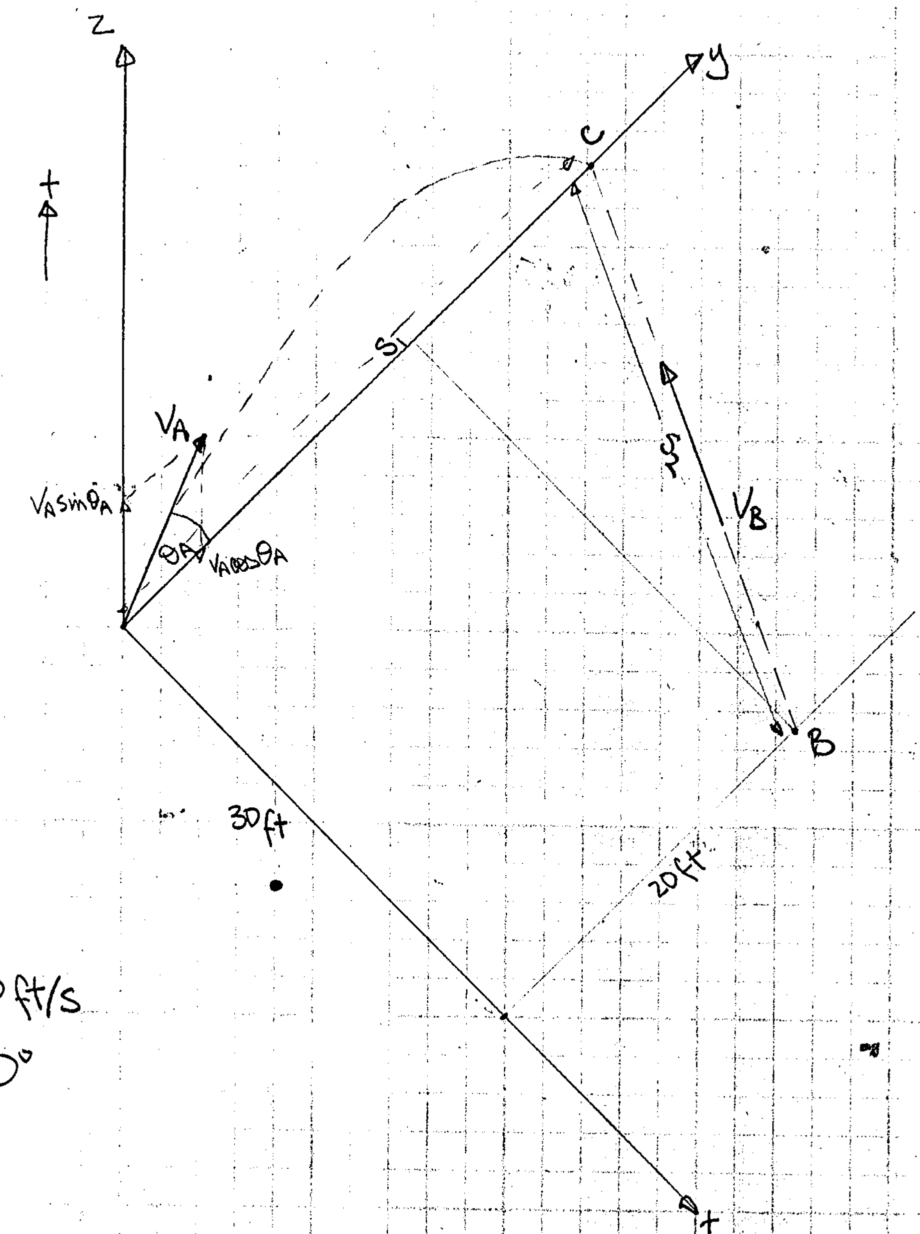
also the trivial solution $X=0$ also solves eqn.

$$4.335\times10^{-2}x = 0.577$$

$$x = \frac{0.577}{4.335\times10^{-2}} = \underline{13.3 \text{ ft}}$$ ✓

$$y = -0.04x^2 = -0.04(13.3)^2 = \underline{-7.1 \text{ ft}}$$ ✓

(81)



$V_A = 50$ ft/s

$\theta_A = 60°$

For Horizontal motion: $S_1 = 25\, t_1$

For Vertical motion: Using $S = V_0 t + \frac{1}{2} a t_1^2$

$0 = V_A \sin\theta_A\, t - \frac{1}{2}(32.2)t^2$

$0 = 43.3t - 16.1 t_1^2$

$16.1 t_1^2 = 43.3t$

$16.1 t_1 = 43.3$

$t_1 = \frac{43.3}{16.1} = 2.69$ secs. and $t=0$ solve the eqn.; $t = 2.69$ sec solves problem

$S_1 = 25(2.69) = 67.24$ ft

$S_2^2 = (67.24 - 20)^2 + 30^2$

$S_2 = \sqrt{2231.24 + 900} = \sqrt{3131.24} = 55.96$ ft

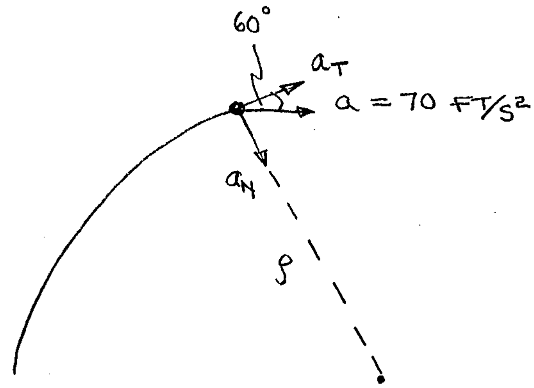
$V_B = \frac{S_2}{t_1} = \frac{55.96}{2.69} = \underline{20.8 \text{ ft/s}}$

HW # 3 in 6 pages

12 - 85, 88, 94, 108, 117, 126

DYNAMICS HOMEWORK #3

12-85



GIVEN

VELOCITY $(v) = 400$ FT/S

ACCELERATION $(a) = 70$ FT/S²

$\Rightarrow \quad a_T = 70 \cos 60°$

$\underline{a_T = 35 \text{ FT/S}^2}$

FROM

$$\frac{v^2}{\rho} = a_n$$

$$\Rightarrow \quad (70 \sin 60°) = \frac{400^2}{\rho}$$

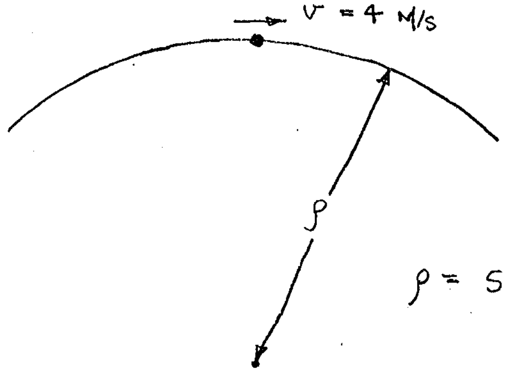
$$\rho = \frac{400^2}{(70 \sin 60°)}$$

$\underline{\rho = 2639 \text{ FT}}$

12-88

$\dot{v} = (0.05s)\ M/s^2$

$v = 4\ M/s$



$\rho = 50\ M$

GIVEN

$\dot{v} = a_T = 0.05s$

BUT $\quad a\,ds = v\,dv$

$0.05\,s\,ds = v\,dv$

$$\Rightarrow \int_4^v v\,dv = \int_0^s 0.05\,s\,ds$$

$$\tfrac{1}{2}v^2\Big|_4^v = \frac{0.05}{2}s^2$$

$$\tfrac{1}{2}v^2 - 8 = \frac{0.05}{2}s^2$$

$$v = \sqrt{0.05s^2 + 16}$$

WHEN $s = 10M$

$$\underline{v = 4.58\ M/s}$$

FROM $\quad a_n = \dfrac{v^2}{\rho}$

$$\Rightarrow a_n = \frac{0.05s^2 + 16}{50}$$

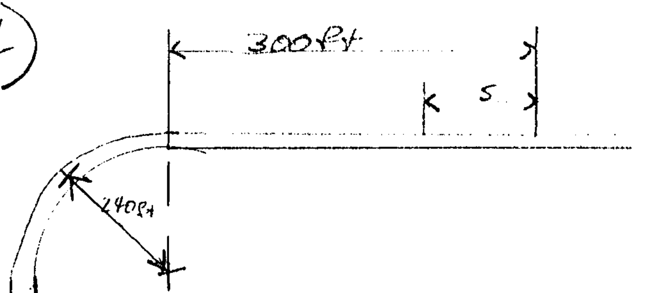
WHEN $s = 10M$ $\qquad a_n = 0.42\ M/s^2$

AND $\quad a_T = 0.05(10)$

$= 0.5$ units

$$a = \sqrt{0.5^2 + 0.42^2} \qquad a = \underline{0.653}\ M/s^2$$

94)



$V_0 = 0$ at $s = 0$

$\dot{V} = (0.05t^2)\,\text{ft/s}^2 \quad t > 0$

Determine magnitude of velocity and acceleration for $t = 18\,s$

$$a_t = \dot{v} = v\frac{dv}{ds} = \frac{\cancel{ds}}{dt}\frac{dv}{\cancel{ds}}$$

$$0.05t^2 = \frac{dv}{dt}$$

$$\int 0.05t^2\,dt = \int dv$$

$$\frac{0.05t^3}{3} = v \Big|_{t=18} \qquad |V| = 97.2\ \text{ft/s}$$

$$\frac{0.05t^3}{3} = \frac{ds}{dt}$$

$$\int \frac{0.05t^3}{3}\,dt = \int ds$$

$$\frac{0.05t^4}{12} = s \Big|_{t=18} \qquad s = 437.40\ \text{ft}$$

IMPORTANT!! MUST BE SHOWN

location of vehicle $\quad 437.4 - 300\,\text{ft} = 137.4$

length of curve $\quad 377\,\text{f}$

Vehicle is located on the curve

$$\vec{a} = \dot{V}\,\vec{U}_T + \frac{V^2}{\rho}\,\vec{U}_n$$

$$\vec{a} = (0.05t^2)\,\vec{U}_T + \frac{(97.2)^2}{240}\,\vec{U}_n = \left[16.2\,\vec{U}_T + 39.37\,\vec{U}_n\right.\ \text{ft/s}^2$$

$$|a| = 42.57\ \text{ft/s}^2 \qquad |V| = 97.2\ \text{ft/s}$$

excellent

12-108

GIVEN
$$\dot{\theta} = \frac{d\theta}{dt} = 0.8 \text{ RAD/S}$$

$$\ddot{\theta} = -0.2 \text{ RAD/S}^2$$

AND $r = 0.75\cos\theta + 2$ (FT.)

$$\Rightarrow \quad \dot{r} = -0.75\dot{\theta}\sin\theta$$

AND $\ddot{r} = -0.75\dot{\theta}\cos\theta \cdot \dot{\theta} - 0.75\ddot{\theta}\sin\theta$

$$\ddot{r} = -0.75\dot{\theta}^2\cos\theta - 0.75\ddot{\theta}\sin\theta$$

$\Rightarrow$ RADIAL COMP OF VELOCITY $v_r = \dot{r}$

WHEN $\theta = \pi/8$

$$v_r = -0.75(0.8)\sin(\pi/8)$$

$$\underline{v_r = -0.23 \text{ FT/S}} \quad \checkmark$$

TRANSVERSE COMP. OF VELOCITY v_θ

$$v_\theta = r\dot{\theta}$$

$$= (0.75\cos\theta + 2)0.8$$

WHEN $\theta = \pi/8$

$$v_\theta = (0.75\cos(\pi/8) + 2)0.8$$

$$\underline{v_\theta = 2.15 \text{ FT/S}} \quad \checkmark$$

AND

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$= (-0.75(0.8)^2\cos(\pi/8) - 0.75(-0.2)\sin(\pi/8))$$

$$- (0.75\cos(\pi/8) + 2)(0.8^2)$$

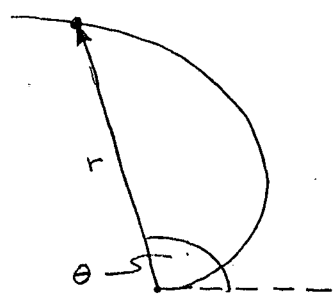
$$\underline{a_r = -2.11 \text{ FT/S}^2} \quad \checkmark$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$= (0.75\cos(\pi/8) + 2)(-0.2) + 2(-0.75(0.8)\sin(\pi/8))(0.8)$$

$\underline{a_\theta = -0.91 \text{ FT}}/s^2$

12-117



GIVEN

$$r = a\theta$$

RATE OF ANGULAR ROTATION $= \dot{\theta}$ (CONSTANT)

$\Rightarrow$ " " " ACCELERATION $= \ddot{\theta} = 0$

$$\dot{r} = a\dot{\theta}$$

AND $\ddot{r} = a\ddot{\theta}$

$\ddot{\theta} = 0$

$\Rightarrow \ddot{r} = 0$

$v_r = \dot{r}$

$\underline{v_r = a\dot{\theta}}$

$v_\theta = r\dot{\theta}$

$\underline{v_\theta = a\theta\dot{\theta}}$

$\Rightarrow \quad a_r = \cancel{\ddot{r}}^{\,0} - r\dot{\theta}^2$

$\underline{a_r = -r\dot{\theta}^2 = -a\theta\dot{\theta}^2}$

$a_\theta = \cancel{r\ddot{\theta}}^{\,0} + 2\dot{r}\dot{\theta}$

$\underline{a_\theta = 2a\dot{\theta}^2}$

12-126

ACCELERATION $a = (\ddot{r} - r\dot{\theta}^2)u_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})u_\theta + \ddot{z}u_z$

OR $a = \ddot{r}u_r - r\dot{\theta}^2u_r + r\ddot{\theta}u_\theta + 2\dot{r}\dot{\theta}u_\theta + \ddot{z}u_z$

$$\frac{da}{dt} = \frac{d}{dt}(\ddot{r}u_r) - \frac{d}{dt}(r\dot{\theta}^2u_r) + \frac{d}{dt}(r\ddot{\theta}u_\theta) + \frac{d}{dt}(2\dot{r}\dot{\theta}u_\theta) + \dddot{z}u_z$$

$$= \dddot{r}u_r + \ddot{r}\dot{u}_r - \left\{\left[\frac{d}{dt}(r\dot{\theta}^2)\right]u_r + r\dot{\theta}^2\dot{u}_r\right\} + \left[\frac{d}{dt}(r\ddot{\theta})\right]u_\theta + r\ddot{\theta}\dot{u}_\theta + \left[\frac{d}{dt}(2\dot{r}\dot{\theta})\right]u_\theta + 2\dot{r}\dot{\theta}\dot{u}_\theta + \dddot{z}u_z$$

$$= \dddot{r}u_r + \ddot{r}\dot{u}_r - (2r\dot{\theta}\ddot{\theta} + \dot{r}\dot{\theta}^2)u_r - r\dot{\theta}^2\dot{u}_r + (r\dddot{\theta} + \dot{r}\ddot{\theta})u_\theta + r\ddot{\theta}\dot{u}_\theta + (2\dot{r}\ddot{\theta} + 2\ddot{r}\dot{\theta})u_\theta - 2\dot{r}\dot{\theta}^2u_r + \dddot{z}u_z$$

BUT $\dot{u}_\theta = -\dot{\theta}u_r$

AND $\dot{u}_r = \dot{\theta}u_\theta$

SUBSTITUTING

$$\dot{a} = \dddot{r}u_r + \ddot{r}\dot{\theta}u_\theta - (2r\dot{\theta}\ddot{\theta} + \dot{r}\dot{\theta}^2)u_r + r\dot{\theta}^3u_\theta + (r\dddot{\theta} + \dot{r}\ddot{\theta})u_\theta - r\dot{\theta}\ddot{\theta}u_r + (2\dot{r}\ddot{\theta} + 2\ddot{r}\dot{\theta})u_\theta - 2\dot{r}\dot{\theta}^2u_r + \dddot{z}u_z$$

$$= (\dddot{r} - 2r\dot{\theta}\ddot{\theta} - \dot{r}\dot{\theta}^2 - r\dot{\theta}\ddot{\theta} - 2\dot{r}\dot{\theta}^2)u_r + (\ddot{r}\dot{\theta} + r\dot{\theta}^3 + r\dddot{\theta} + \dot{r}\ddot{\theta} + 2\dot{r}\ddot{\theta} + 2\ddot{r}\dot{\theta})u_\theta$$

$$\dot{a} = (\dddot{r} - 3r\dot{\theta}\ddot{\theta} - 3\dot{r}\dot{\theta}^2)u_r + (3\dot{r}\ddot{\theta} + 3\ddot{r}\dot{\theta} + r\dot{\theta}^3 + r\dddot{\theta})u_\theta$$

KESB 246.06

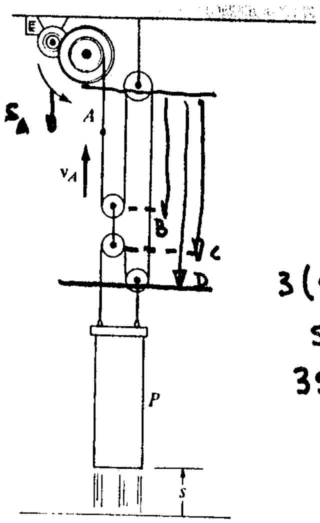
SOLN TO HW #4 in 2 pgs

12-131, 135, 140, 149, 150, 155

Ch 12 · Kinematics of a Particle

✓

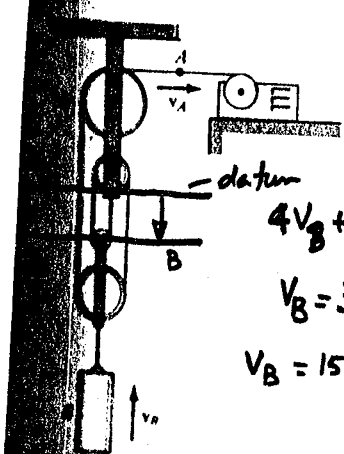
12–135. The pipe P is being lifted using the cable and pulley system shown. If point A on the cable is being drawn toward the drum with a speed of 2 m/s, determine the speed of the pipe.



Probs. 12–135/12–136

$$3(S_D - S_C) + S_C + S_B + S_B - S_A = C_1$$
$$S_C - S_B = C_2$$
$$3S_D - 2S_C + 2S_B - S_A = C_1$$
$$3S_D \cancel{+} - C_2 - S_A = C_1$$
$$3V_D - V_A = 0$$
$$3V_D - (-2)$$
$$V_D = -2/3 \text{ m/s}$$

...131. Determine the constant speed at which the cable at A ... be drawn in by the motor in order to hoist the load at B 15 ft ...s.



– datum

$$4V_B + V_A = 0$$

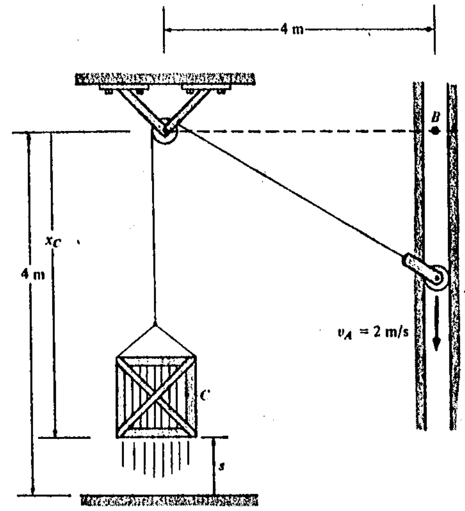
$$V_B = 3 \text{ ft/s} \qquad V_A = 12 \text{ ft/s}$$

$$V_B = 15 \text{ ft/sec}$$

...s. 12–131/12–132

or as done in class

*12–140. The crate C is being lifted by moving the roller at A downward with a constant speed of $v_A = 2$ m/s along the guide. Determine the velocity and acceleration of the crate at the instant $s = 1$ m. When the roller is at B, the crate rests on the ground. Neglect the size of the pulley in the calculation. *Hint:* Relate the coordinates x_C and x_A using the problem geometry, then take the first and second time derivatives.



Prob. 12–140

SOLN

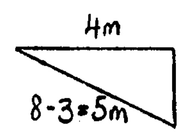
⇓

$$x_C + \sqrt{x_A^2 + 4^2} = l$$
$$\dot{x}_C + \tfrac{1}{2}(x_A^2 + 16)^{-1/2}(2x_A)\dot{x}_A = 0$$
$$\ddot{x}_C - \tfrac{1}{4}(x_A^2 + 16)^{-3/2}(4x_A^2)\dot{x}_A^2$$
$$+ (x_A^2 + 16)^{-1/2}(\dot{x}_A)^2 + (x_A^2 + 16)^{-1/2} x_A \ddot{x}_A = 0$$

When $s = 1$ m, $x_C = 3$ m

$v_A = \dot{x}_A = 2$ m/s

$a_A = \ddot{x}_A = 0$



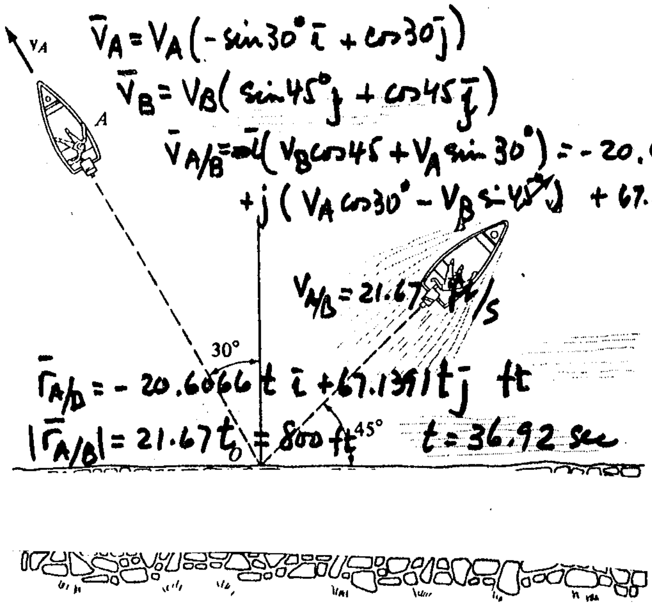
Thus,

$$v_C + (3^2 + 16)^{-1/2}(3)(2) = 0$$
$$v_C = -1.2 \text{ m/s} = \underline{1.2 \text{ m/s} \uparrow}$$

$$a_C - (3^2 + 16)^{-3/2}(3)^2(2)^2 + (3^2 + 16)^{-1/2}(2)^2$$
$$+ 0 = 0$$
$$a_C = -0.512 \text{ m/s}^2 = \underline{\underline{0.512 \text{ m/s}^2 \uparrow}}$$

✓

12–149. Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 20$ ft/s and $v_B = 15$ ft/s, determine the speed of boat A with respect to boat B. How long after leaving the shore will the boats be 800 ft apart?



$\bar{V}_A = V_A(-\sin 30° \bar{i} + \cos 30 \bar{j})$

$\bar{V}_B = V_B(\sin 45° \bar{j} + \cos 45 \bar{j})$

$\bar{V}_{A/B} = -\bar{i}(V_B \cos 45 + V_A \sin 30°) = -20.6066$

$+\bar{j}(V_A \cos 30° - V_B \sin 45°) + 67.1391 \bar{j}$

$V_{A/B} = 21.67$ ft/s

$\bar{r}_{A/B} = -20.6066 t \bar{i} + 67.1391 t \bar{j}$ ft

$|\bar{r}_{A/B}| = 21.67 t = 800$ ft $t = 36.92$ sec

Probs. 12–149/12–150

12–150. Each boat begins from rest at point O and heads in the direction shown at the same instant. If A has an acceleration of $a_A = 2$ ft/s², and B has an acceleration of $a_B = 3$ ft/s², determine the speed of boat A with respect to boat B at the instant they become 800 ft apart. How long will this take?

$\bar{r}_A = t^2(-\sin 30 \bar{i} + \cos 30 \bar{j})$

$\bar{r}_B = \frac{3}{2} t^2(\sin 45 \bar{i} + \cos 45 \bar{j})$

$\bar{r}_{A/B} = -1.56066 t^2 \bar{i} + .1946 t^2 \bar{j}$

800 ft $= |\bar{r}_{A/B}| = 1.57275 t^2 = |\bar{r}_A - \bar{r}_B|$

$\therefore \quad t = 22.5536$ sec

$\bar{V}_A = 2t(-\sin 30 \bar{i} + \cos 30 \bar{j})$

74

$\bar{V}_B = 3t(\sin 45 \bar{i} + \cos 45 \bar{j})$

$\therefore \quad |\bar{V}_{A/B}| = -3.121 t \bar{i} - .38927 t \bar{j} = |\bar{V}_A - \bar{V}_B|$

$-70.39 \bar{i} - 8.78 \bar{j} = 70.94$ ft/s

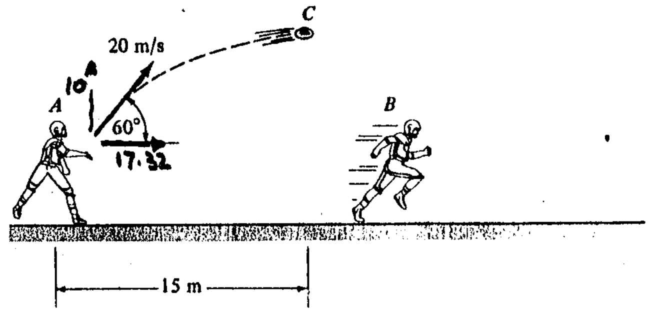
note here constant acceleration a_A & a_B & 0 initial veloc

$\therefore \quad \bar{V}_A = a_A t \cdot \bar{u}_A \quad \bar{V}_B = a_B t \, \bar{u}_B$

$\bar{r}_A = a_A \frac{t^2}{2} \bar{u}_A \quad \bar{r}_B = a_B \frac{t^2}{2} \bar{u}_B$

$\bar{u}_A$ & $\bar{u}_B$ are unit vectors in direction of motion: given in ( ) above.

12–155. At a given instant the football player at A throws a football C with a velocity of 20 m/s in the direction shown. Determine the constant speed at which the player at B must run so that he can catch the football at the same elevation at which it was thrown. Also calculate the relative velocity and relative acceleration of the football with respect to B at the instant the catch is made. Player B is 15 m away from A when A starts to throw the football.



⇓

Ball:

$\xrightarrow{+}$ $S_c = 20 \cos 60° t$

$+\uparrow$ $-20 \sin 60° = 20 \sin 60° - 9.81 t$

$t = 3.53$ s

$S_c = 35.31$ m

Player B:

$S_B = V_B t$

Require,

$35.31 = 15 + V_B(3.53)$

$V_B = \underline{5.75 \text{ m/s}}$

At the time of the catch

$(V_c)_x = 20 \cos 60° = 10$ m/s →

$(V_c)_y = 20 \sin 60° = 17.32$ m/s ↓

$\vec{V}_c = \vec{V}_B + \vec{V}_{c/B}$

$10\hat{i} - 17.32\hat{j} = 5.75\hat{i} + (V_{c/B})_x \hat{i} + (V_{c/B})_y \hat{j}$

$\xrightarrow{+}$ $10 = 5.75 + (V_{c/B})_x$

$+\uparrow$ $-17.32 = (V_{c/B})_y$

$(V_{c/B})_x = 4.25$ m/s →

$(V_{c/B})_y = 17.32$ m/s ↓

$V_{c/B} = \sqrt{(4.25)^2 + (17.32)^2} = \underline{17.8 \text{ m/s}}$

$\theta = \tan^{-1}\left(\frac{17.32}{4.25}\right) = \underline{76.2°}$

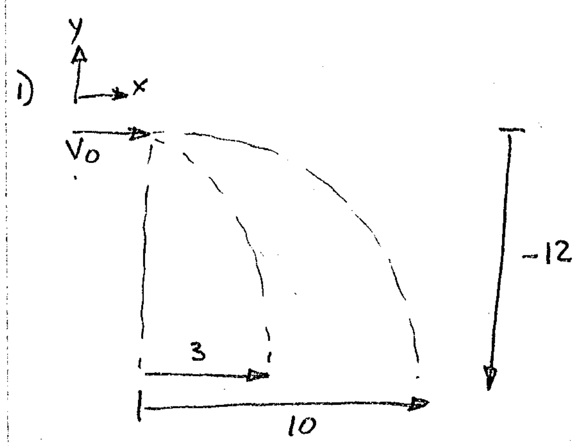
$\vec{a}_c = \vec{a}_B + \vec{a}_{c/B}$

$-9.81\hat{j} = 0 + \vec{a}_{c/B}$

$a_{c/B} = \underline{9.81 \text{ m/s}^2 \downarrow}$

EXAM #1

1)



$3 \leq R \leq 10$

$V_{0x} = V_0 \qquad V_{0y} = 0$

$V_{0x} t = R$

$s = -\frac{1}{2} g t^2 = -12 \text{ ft}$

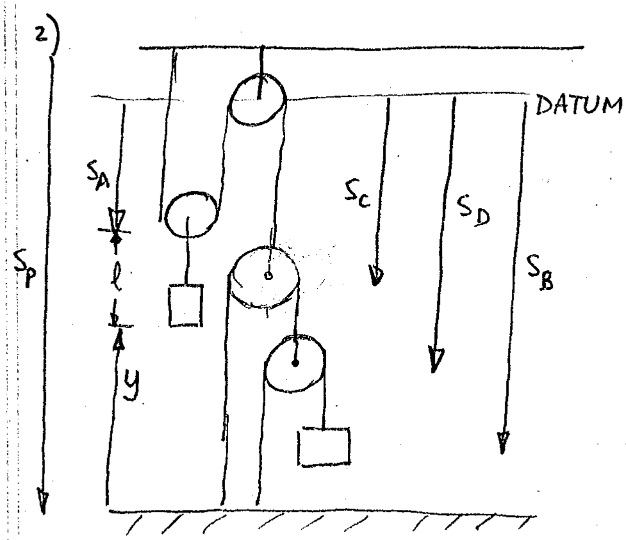
$t = \sqrt{\frac{2D}{g}} = .863 \text{ sec}$ to fall 12 ft

$V_{0x} = \frac{R}{t} = \frac{3}{.863} = 3.475 \text{ ft/s}$

$= \frac{10}{.863} = 11.58 \text{ ft/s}$

$3.47 \text{ ft/s} \leq V_{0x} \leq 11.58 \text{ ft/s}$

2)



$2S_A + S_C = C_1 \Rightarrow 2\dot{V}_A + \dot{V}_C = 0$

$(S_P - S_C) + (S_D - S_C) = C_2 \Rightarrow \dot{V}_D = 2\dot{V}_C$

$(S_P - S_D) + (S_B - S_D) = C_3 \Rightarrow \dot{V}_B = 2\dot{V}_D$

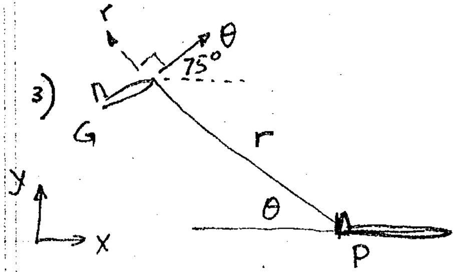
$S_A + l + y = S_P \Rightarrow \dot{V}_A = -\ddot{y}$

$y = t^2/4 \Rightarrow \ddot{y} = \frac{1}{2} \text{ m/s}^2 \qquad \dot{V}_A = a_A = -\frac{1}{2} \text{ m/s}^2$

$a_B = 2a_D = 2(2a_C) = 2(2(-2a_A)) = -8a_A$

$= -8\left(-\frac{1}{2}\right) = 4 \text{ m/s}^2$

3)



Given $\bar{V}_G = \bar{V}_P + \bar{V}_{G/P}$

$\bar{V}_P = 200 \text{ km/hr } \bar{i} =$

$r = 60 \text{ m} = \text{const} \Rightarrow \dot{r} = \ddot{r} = 0$

$\theta = 15° \qquad \dot{\theta} = 5°/\text{sec} = 0.0873 \text{ rad/sec}$

$\ddot{\theta} = 0$

$(V_{G/P})_r = \dot{r} = 0 \qquad (V_{G/P})_\theta = r\dot{\theta} = 60(.0873) = 5.24 \text{ m/s} \qquad \bar{V}_{G/P} = (\bar{V}_{G/P})_\theta = 5.24 \bar{u}_\theta$

θ direction is 75° to horizontal $\therefore \bar{u}_\theta = \bar{i} \cos 75° + \bar{j} \sin 75°$

$\bar{V}_G = 55.56 \bar{i} + 5.24 \cos 75° \bar{i} + 5.24 \sin 75° \bar{j} = 56.92 \bar{i} + 5.06 \bar{j} \text{ m/s}$

$|\bar{V}_G| = 57.14 \text{ m/s}$

$\bar{a}_G = \bar{a}_P + \bar{a}_{G/P}$ $\qquad \bar{a}_P = \bar{0}$ (constant vel)

$(a_{G/P})_r = \ddot{r} - r\dot{\theta}^2 = 0 - 60(.0873)^2 = -.457 \text{ m/s}^2$

$(a_{G/P})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 - 0 = 0$

$\bar{a}_G = (\bar{a}_{G/P})_r = -.457\,\bar{u}_r \qquad |\bar{a}_G| = .457 \text{ m/s}^2$

3b.) Given $\dot{r} = 2 \text{ m/s} \Rightarrow \ddot{r} = 0 \qquad \bar{a}_P = 5 \text{ km/hr/s}\ \bar{\imath} = 1.389 \text{ m/s}^2\ \bar{\imath}$

$\theta = \text{const} \Rightarrow \dot{\theta} = \ddot{\theta} = 0$

$\bar{a}_G = \bar{a}_P + \bar{a}_{G/P}$

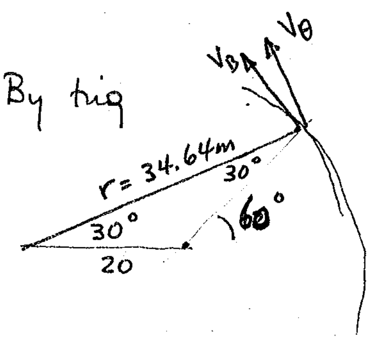
$(a_{G/P})_r = \ddot{r} - r\dot{\theta}^2 = 0 \qquad (a_{G/P})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$\therefore \quad \bar{a}_G = \bar{a}_P = 1.389 \text{ m/s}^2\ \bar{\imath} \qquad |\bar{a}_G| = 1.389 \text{ m/s}^2$

Note: the info given was about the relative change in r between the Glider & the plane

Note that 3a) could have also been solved using n,t coordinates where t is in θ direction n in $-r$ direction

4) By trig



$r = 2(20\cos 30°) = 40\cos\theta$

$\dot{r} = -40\sin\theta\,\dot{\theta}$

$\ddot{r} = -40\cos\theta\,\dot{\theta}^2 - 40\sin\theta\,\ddot{\theta}$

$V = 30 \text{ m/s} = \sqrt{V_r^2 + V_\theta^2} = \sqrt{\dot{r}^2 + (r\dot{\theta})^2}$

$= \sqrt{(-40\sin\theta)^2\dot{\theta}^2 + (40\cos\theta)^2\dot{\theta}^2}$

$30 = 40\dot{\theta} \Longrightarrow \qquad \dot{\theta} = .75 \text{ rad/s}$

also $V_B\cos 30° = V_\theta = r\dot{\theta}$

$30\cos 30° = 40\cos 30°\,\dot{\theta} \Rightarrow \dot{\theta} = .75 \text{ rad/s}$

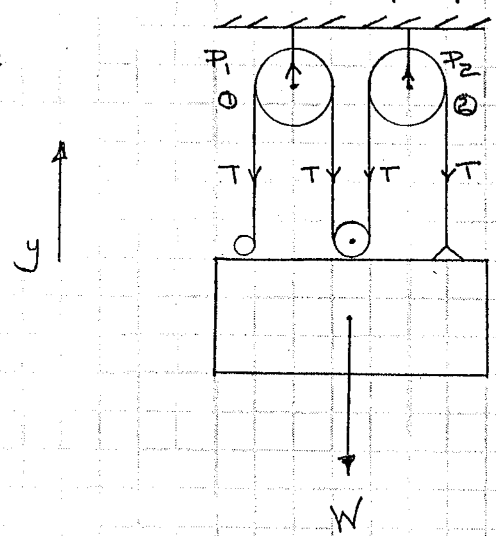
HW #5

13-19, 25, 32, 38 in 5 pages

Florida International University
The State University of Florida at Miami
College of Engineering & Applied Sciences

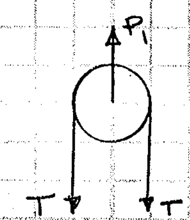
DYNAMICS HOME WORK # 5

13-19



TENSION IN STRING IS CONSTANT

FREE BODY OF PULLEY ①



$$\Sigma F_y = P_1 - T - T = 0$$

$$P_1 = 2T$$

FREE BODY OF ② REVEALS THE SAME I.E.

$$P_2 = 2T$$

$\Rightarrow$ NET FORCE ACTING ON ELEVATOR

$$F_N = 4T - W$$

BUT $F_N = M_E a_E$

WHERE M_E = MASS OF ELEVATOR
a_E = ACC. " "

$$\Rightarrow \quad a_E = \frac{4T - W}{M_E}$$

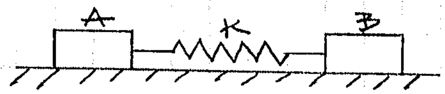
BUT $v_E^2 = v_{E_0}^2 + 2 a_E s_E$

GIVEN $v_{E_0} = 0$ M/S

$$\Rightarrow \quad v_E = \sqrt{\frac{2 s_E (4T - W)}{M_E}}$$ ✓

$$\underline{v_E = 3.62 \text{ M/S}}$$

13-25



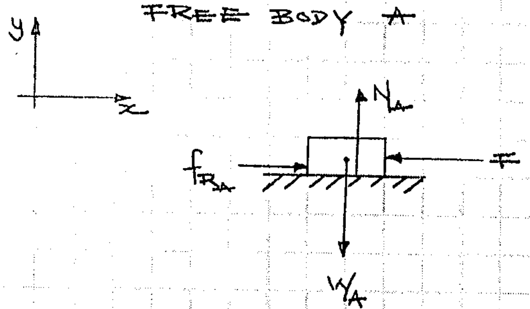
GIVEN $W_A = 8\ lb$

$W_B = 6\ lb$

SPRING CONSTANT $K = 20\ lb/FT$

COMPRESSED DISTANCE = 0.2 FT

FREE BODY A



F IS FORCE EXERTED BY SPRING

$F = Kx$

$\Rightarrow$ NET FORCE ACTING ON A

$F_{N_A} = f_{R_A} - F$

$f_{R_A} = \mu N$

BUT $F_{N_A} = m_A a_A$

$\Rightarrow \quad m_A a_A = \mu N_A - Kx$

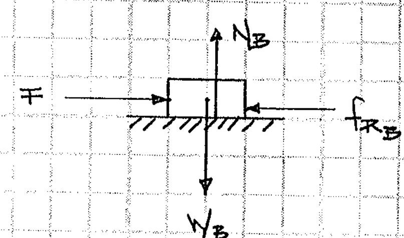
$$a_A = \frac{\mu N_A - Kx}{m_A}$$

$$m_A = \frac{8\ lb}{32.2\ FT/s^2}$$

$\mu = 0.2$

$\underline{a_A = -9.66\ FT/s^2}$ ✓

FREE BODY B



NET FORCE ACTING ON B

$F_{N_B} = F - f_{RB}$

$$a_B = \frac{Kx - \mu N_B}{m_B}$$

$\underline{a_B = 15.03\ FT/s^2}$ ✓

$$\int_0^t dt = \int_0^s \frac{ds}{(v_0 - Ks/m)}$$

~~LET~~ $u = v_0 - \frac{Ks}{m}$

$du = -\frac{K}{m} ds$

$-\frac{m}{K} du = ds$

$$\Rightarrow \quad t = -\frac{m}{K} \ln\left(v_0 - \frac{Ks}{m}\right)\Big|_0^s$$

BUT $s = \frac{1}{2K} m v_0$

$$\Rightarrow \quad t = -\frac{m}{K} \ln\left(v_0 - \frac{1}{2\cancel{K}} \cancel{m} v_0 \left(\frac{\cancel{K}}{\cancel{m}}\right)\right) + \frac{m}{K} \ln v_0$$

$$= -\frac{m}{K} \ln\left(\frac{1}{2} v_0\right) + \frac{m}{K} \ln v_0$$

$$= \frac{m}{K}\left(-\ln \frac{1}{2} v_0 + \ln v_0\right)$$

$$= \frac{m}{K}\left(\ln\left(\frac{\cancel{v_0}}{\frac{1}{2}\cancel{v_0}}\right)\right)$$

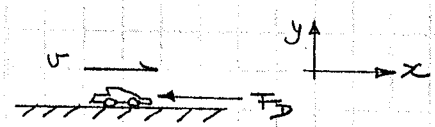
$$\boxed{t = \frac{m}{K} \ln 2 \ \text{secs}} \ \checkmark$$

also $m\frac{dv}{dt} = -kv \quad \therefore \quad \frac{m\,dv}{-kv} = dt$

$$\frac{m}{k} \left(-\ln v\right)\Big|_{v_0}^{\frac{1}{2}v_0} = t$$

$$\frac{m}{k} \ln 2 = t$$

13.32



F_D = DRAG RESISTANCE OF WIND

$F_D = -Kv$

INITIAL VELOCITY $= v_0$

FINAL VELOCITY $= \frac{1}{2}v_0$

MASS OF CAR $= m$

$$\Rightarrow \quad ma = -Kv$$

BUT $\quad a\,ds = v\,dv$

$$a = -\frac{Kv}{m}$$

$$\Rightarrow ds = -\frac{m\cancel{v}}{K\cancel{v}}\,dv$$

$$S = \int_{v_0}^{\frac{1}{2}v_0} -\frac{m}{K}\,dv$$

$$= -\frac{mv}{K}\Bigg|_{v_0}^{\frac{1}{2}v_0}$$

$$\boxed{S = \frac{1}{2K}mv_0} \quad (m, Ft, Km, ??)$$

AND $\quad S = -\frac{mv}{K} + C \qquad$ WHEN $\quad S = 0, \; v = v_0$

$$\Rightarrow 0 = -\frac{mv_0}{K} + C$$

$$C = \frac{mv_0}{K}$$

NOW $\quad S = -\frac{mv}{K} + \frac{mv_0}{K}$

BUT $\quad v = \frac{ds}{dt}$

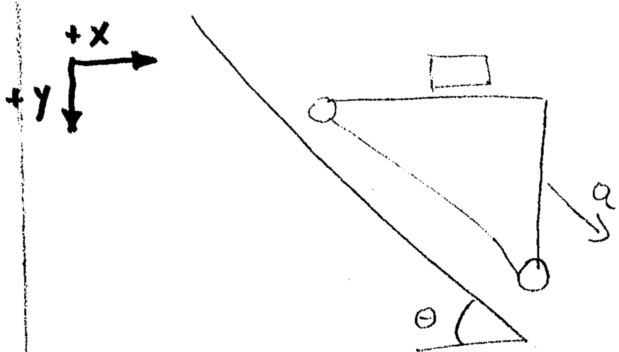
$$S = -\frac{m\,ds}{K\,dt} + \frac{mv_0}{K}$$

$$\frac{m}{K}\frac{ds}{dt} = \frac{mv_0}{K} - S$$

$$\frac{ds}{dt} = v_0 - \frac{Ks}{m}$$

$$dt = \frac{ds}{\left(v_0 - \frac{Ks}{m}\right)}$$

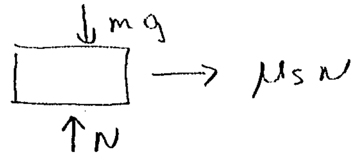
13-38



$m = 10\,kg$

$\mu_s = 0.3$

$a = 6\,m/s^2$



FBD for box

For no slip cart & box have same a

$\xrightarrow{+} \Sigma F_x = m a_x$

$+\downarrow \Sigma F_y = m a_y$

$\mu_s N = m a \cos\theta$

$mg - N = m a \sin\theta$

$$mg - \left(\frac{m a \cos\theta}{\mu_s}\right) = m a \sin\theta$$

$$g \mu_s = (\cos\theta + \mu_s \sin\theta)\, a$$

$$9.81\,(0.3) = \cos\theta + 0.3(\sin\theta)\,(6)$$

$$2.943 = \cos\theta + 1.8 \sin\theta$$

$$-2.943 + \cos\theta + 1.8 \sin\theta = 0$$

$$\boxed{\theta = 78.7^\circ}$$

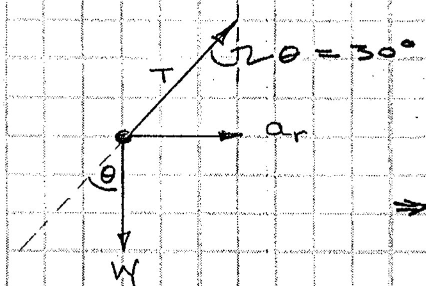
SOL'N TO HW # 6

13-48, 64, 66, 77, 90 in 6 pages

DYNAMICS HOME WORK # 6

13-48

FREE BODY OF PLANE

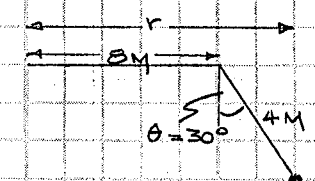


$\Rightarrow \tan\theta = \frac{a_r}{g}$

$a_r = g \tan 30°$

$= 5.66 \ M/S^2$

BUT $a_r = \frac{v^2}{r}$



$r = 8 + 4 \sin 30°$
$= 10 M$

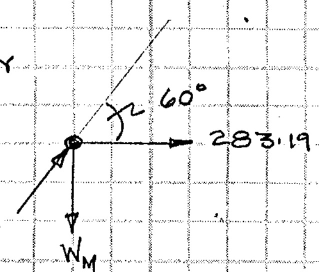
$\Rightarrow 5.66 = \frac{v^2}{10}$

$v = 7.52 \ M/S$

MAN WEIGHS 50 Kg

$\Rightarrow$ FORCE TOWARD CENTER $F_c = m a_r$
$= 50(5.66) = 283.19$

FREE BODY OF MAN

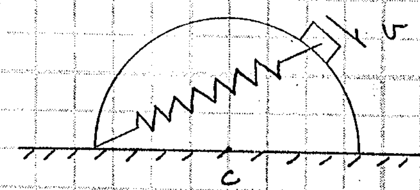


$\Rightarrow \cos 60° = \frac{283.19}{N_F}$

$N_F = 566.4 N$

13-64

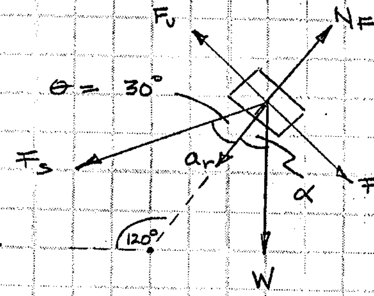
1349 in sixth ed



FREE BODY OF BLOCK

F_U = SPRING'S FORCE COMPONENT ACTING UP CURVE.

F_D = WEIGHT COMPONENT ACTING DOWN CURVE



F_s = FORCE DUE TO SPRING

a_r = ACCELERATION TOWARD CENTER

W = WEIGHT OF BLOCK

$$a_r = \frac{v^2}{r} = \frac{4^2}{3}$$

$$a_r = 5.33 \text{ FT/s}^2$$

AT $\theta = 30°$ LENGTH OF SPRING –

$$L = \sqrt{9 + 9 - 18\cos 120°}$$

$$L = 5.20$$

STRETCHED DISTANCE x GIVEN BY

$$x = 5.20 - x_0 \quad , \quad x_0 = 3 \text{ FT}$$

$$x = 2.20 \text{ FT}$$

$$\Rightarrow \quad F_s = kx$$
$$= 2(2.20)$$

$$F_s = 4.40 \text{ lb.}$$

$$\Rightarrow \quad \Sigma F_n = Ma_n = -N_F + W\cos\alpha + F_s\cos\theta$$

$$N_F = -\frac{5}{32.2}(5.33) + 4.4\cos 30° + 5\cos 30°$$

$$N_F = 7.31 \text{ lb}$$

NET FORCE ACTING DOWN CURVE

$$\Sigma F_t = F_D - F_U = ma_t$$
$$= W\sin\alpha - F_s\sin\theta$$
$$= 5\sin 30 - 4.4\sin 30°$$

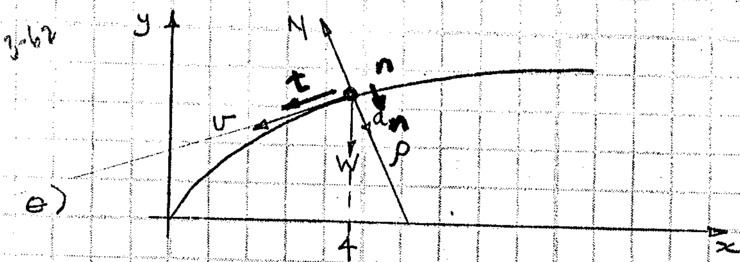
$\left(m = \frac{5}{32.2} \text{ SLUGS}\right)$ $\Rightarrow$ $ma_t = 0.3 \text{ lb}$ $\quad a_t = 1.93 \text{ FT/s}^2$

Now $|a| = \sqrt{a_N^2 + a_T^2} = \sqrt{1.93^2 + 5.33^2}$

$|a| = 5.67 \text{ FT}/s^2$

13-66

like 13-62



GIVEN $v = 4$ M/s

MASS (m) = 80kg

$y = 3(1 - e^{-x/2})$

$\Rightarrow \frac{dy}{dx} = \frac{3}{2} e^{-x/2}$ $\quad \frac{d^2y}{dx^2} = -\frac{3}{4} e^{-x/2}$

WHEN $x = 4$ $\quad \frac{dy}{dx} = \frac{3}{2} e^{-2}$

$= 0.203$

$\Rightarrow \tan\theta = 0.203$

$\theta = 11.48°$

$\Rightarrow$ ACCELERATION $a_t = g \sin\theta$ from $W\sin\theta = ma_t$

$= 9.81 \sin 11.48°$

$a_t = 1.95 \text{ M}/s^2$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$$

$$\rho = -2\frac{\left[1 + (0.203)^2\right]^{3/2}}{(0.203)}$$

$\rho = 10.47$ M

$\Rightarrow a_n = \frac{v^2}{\rho} = \frac{16}{10.47}$

$a_n = 1.53 \text{ M}/s^2$

$\Rightarrow$ FORCE TOWARD CENTER IS F_N

WHEN $\dot{\theta} = 2/30$ rad/s

$$\Rightarrow \quad \ddot{r} = -600\left(\ddot{\theta} + \frac{4}{900}\right)$$

(WHEN $\theta = 0 \Rightarrow \dot{r} = 0$)

$$\Rightarrow \quad \ddot{\theta} = \frac{-(0)(\ddot{r}) - 1200(0)(2/30)^2}{(1200)^2\,\dot{\theta}}$$

$$\ddot{\theta} = 0 \text{ RAD/s}^2$$

$$\Rightarrow \quad a_r = \ddot{r} - r(\dot{\theta})^2$$

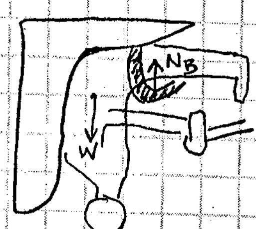
$$= -2.667 - 1200(2/30)^2$$

$$a_r = -8.00 \text{ FT/s}^2$$

F_p IS FORCE TOWARD CENTER

$$F_p = \frac{150}{32.2}(-8.00)$$

$$= -37.27 \text{ lb}$$



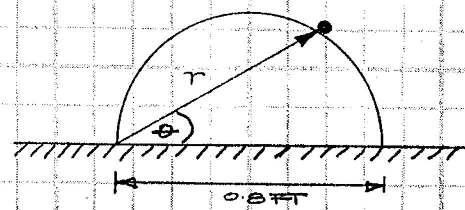
$$\Rightarrow \quad N_B - W = F_p = ma_r$$

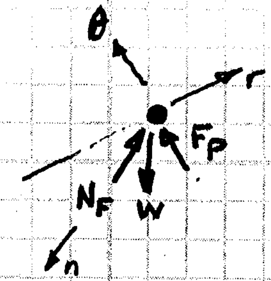
$$N_B = W + F_p$$

$$= 150 - 37.27$$

$$\underline{N_B = 113 \text{ lb}}$$

15 – 77





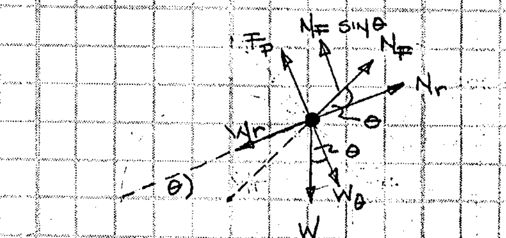
$r = 0.8\cos\theta$

$\dot{r} = -0.8\sin\theta\,.\,\dot{\theta}$

$\ddot{r} = -(0.8\sin\theta)\ddot{\theta} + (-0.8\cos\theta)\dot{\theta}^2$

$\ddot{r} = -0.8(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta)$

FREE BODY OF PARTICLE



N_r – comp of N_F in r dir
W_r – comp of W in r dir
W_θ – comp of W in θ dir
F_p is in θ dir

$\Rightarrow \quad \sum F_R = N_r - W_r = m a_r$

$N_F\cos 30^\circ - W\sin\theta = m a_r$

$0.866 N_F - 0.25 = \frac{0.5}{32.2} a_r \longrightarrow ①$

$\sum F_\theta = F_p + N_F\sin\theta - W_\theta = m a_\theta$

$F_p = m a_\theta + W\cos\theta - N_F\sin\theta \longrightarrow ②$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$= 0.8\cos(30)\,0.8 + 2(-0.8)(0.4)(\sin 30^\circ)(0.4)$

$a_\theta = 0.4263 \text{ FT/s}^2$

AND $a_r = \ddot{r} - r\dot{\theta}^2$

$= -0.8(0.8\sin 30^\circ + 0.4^2\cos 30^\circ)$
$- 0.8\cos 30^\circ (0.4)^2$

$a_r = -0.542 \text{ FT/s}^2$

$F_N = ma_n$

$= 80\,(1.53)$

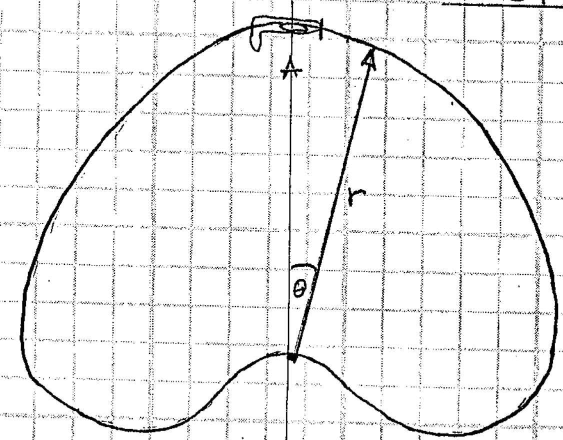
$F_N = 122.3\ N$ ✓

$\Sigma F_n = W\cos\theta - N = ma_n = F_N$

$\Rightarrow$ NORMAL FORCE $= W\cos 11.48° - 122.3$

$= \underline{647\,N}$ ✓

13-90



$r = 600(1+\cos\theta)$ ~~FT~~

AT A $v_P = 80$ ~~FT~~/S

$\dot r = -600\sin\theta\,\dot\theta$

$\ddot r = (-600\sin\theta)\ddot\theta - \dot\theta\,600\cos\theta\dot\theta$

$\ddot r = -600(\ddot\theta + \dot\theta^2\cos\theta)$

VELOCITY $v_P = r\dot\theta$

$\Rightarrow 80 = r(\theta=0)\dot\theta$

$\dot\theta = \frac{80}{1200}$

$\dot\theta = \frac{2}{30}\,\frac{rad}{s}$ ✓

SINCE $v = \sqrt{(\dot r)^2 + (r\dot\theta)^2}$

$\Rightarrow \dot v = \frac{1}{2}\left((\dot r)^2 + (r\dot\theta)^2\right)^{-1/2}\left(\cancel{2}\dot r\ddot r + \cancel{2}(r\dot\theta)(r\ddot\theta + \dot\theta\dot r)\right)$

BUT $\dot v = 0 \Rightarrow 0 = \dot r\ddot r + r\dot\theta(r\ddot\theta + \dot\theta\dot r)$

$0 = \dot r\ddot r + r^2\dot\theta\ddot\theta + r\dot r\dot\theta^2$

$r^2\dot\theta\ddot\theta = -\dot r\ddot r - r\dot r\dot\theta^2$

$\ddot\theta = \dfrac{-\dot r\ddot r - r\dot r\dot\theta^2}{r^2\dot\theta}$

PUT $a_r = -0.542$ IN ① TO GET N_F

$$0.866 N_F - 0.25 = \frac{0.5}{32.2}(-0.542)$$

$$N_F = 0.2790 \text{ lb}$$

FROM ② $\Rightarrow$ $F_p = \frac{0.5}{32.2}(0.4263) + 0.5\cos 30° - 0.2790 \sin 30°$

$$\underline{F_p = 0.30 \text{ lb}}$$

HW # 7 in 3 pages

14-5, 12, 17

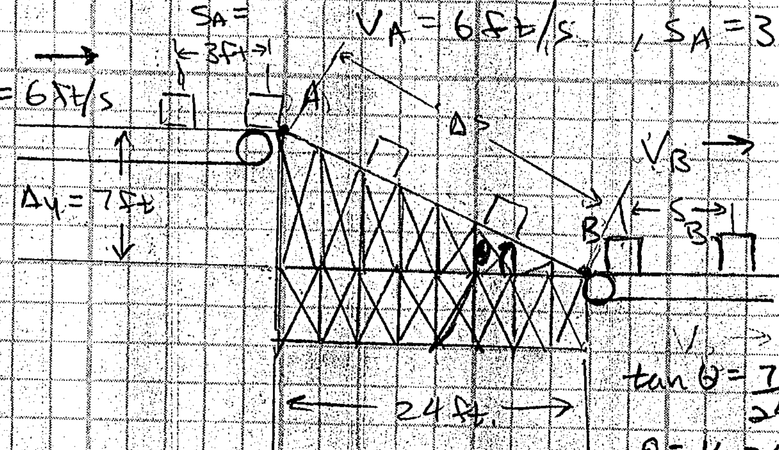
PROB 14-51 Given: W = 15 lb, $\mu_K = 0.15$

$V_B = ?$, $S_B = ?$ $V_A = 6\,ft/s$, $S_A = 3\,ft$

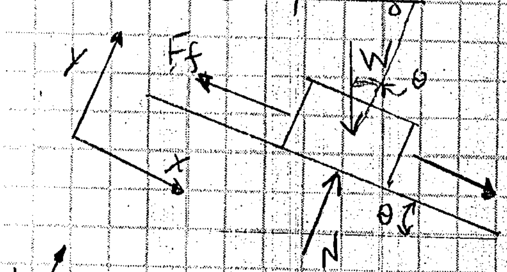
$\Delta y = 7\,ft$

$s = 25\,ft = \overline{AB}$

$\theta = 16.26^\circ$



Free-Body Dig.



$\tan\theta = \frac{7}{24}$

$\theta = 16.26^\circ$

$s = \overline{AB} = \sqrt{7^2 + 24^2}$

$s = \overline{AB} = 25\,ft$

$+\nearrow$

$\Sigma F_N = 0$, $N - W\cos 16.26 = 0 \Rightarrow N = 15\cos 16.26 \Rightarrow N = 14.4\,lbs$ ✓

$F_f = \mu_K N = 0.15(14.4) \Rightarrow F_f = 2.16\,lbs$ ✓

$T_A + \Sigma U_{A-B} = T_B$; $T_A = \frac{1}{2} m v_A^2 = \frac{1}{2}\left(\frac{15}{32.2}\right)(6)^2$,

$\frac{1}{2}\left(\frac{15}{32.2}\right)(6)^2 + (15\cdot 7 - 2.16\cdot 25)$

$= \frac{1}{2}\left(\frac{15}{32.2}\right) v_B^2$

, $U_{A-B} = W\Delta y - F_f \Delta s$

$= 15(7) - 2.16(25)$ ✓

$\boxed{v_B \approx 16.0\,ft/s}$ ✓

$T_B = \frac{1}{2} m v_B^2 = \frac{1}{2}\left(\frac{15}{32.2}\right) v_B^2$

If $v_A = 6\,ft/s$ and $S_A = 3\,ft$ then $t = \left(\frac{3\,ft}{6\,ft/s}\right) = 0.5\,s$, ✓
which means that the blocks are 0.5 s apart
at $v_B = 16\,ft/s$ the blocks will be $S_B = (0.5\,s)(16\,ft/s)$ ✓

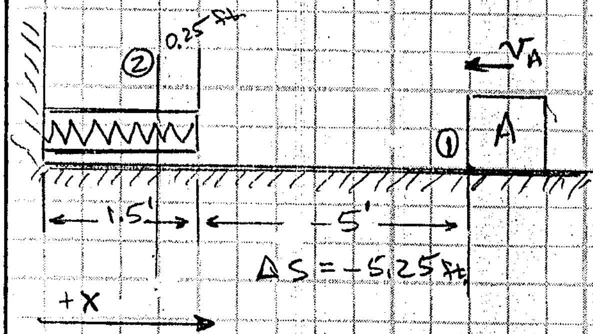
$\boxed{S_B \approx 8\,ft \text{ apart}}$

Dynamics, 2-13-90 | Professor Levy

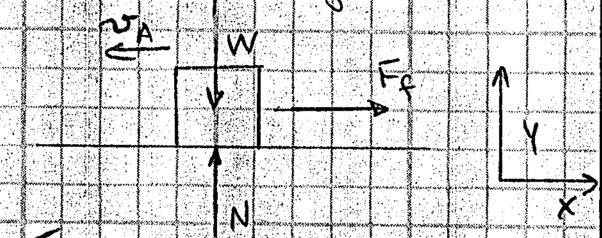
PROB. 14-12 GIVEN: $K = 50\ lb/ft.$, $S_o = 2\ ft.$, $W = 4\ lb$

① $S_f = 1.5\ ft.$, $\mu_K = 0.2$

② $S_f = 1.25\ ft.$



Free Body Diagram



$\Sigma F_y = 0 \quad N - W = 0$

$N = W = 4\ lb.$

Work due to friction

$= F_f\, \Delta S = \mu_K N(-5.25)$

Work due to spring

$= -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right) = -\frac{1}{2}ks_2^2 + \frac{1}{2}ks_1^2$

$T_1 + \Sigma U_{1-2} = \cancel{T_2}^{\,0}$, but $T_2 = 0$

$\Rightarrow T_1 = -\Sigma U_{1-2} = -\left[\mu_K N(-5.25) + \left(-\frac{1}{2}ks_2^2 + \frac{1}{2}s_1^2\right)\right]$

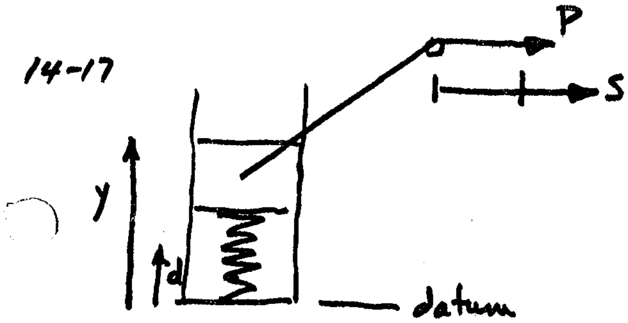
$\frac{1}{2}m_A v_A^2 = -\left[0.2(4)(-5.25) - \left(\frac{1}{2}(50)(0.75)^2 - \frac{1}{2}(50)(0.5)^2\right)\right]$

$s_2 = s_{f_2} - s_o$

$s_1 = s_{f_1} - s_o$

$\boxed{v_A \approx 13.91\ ft/s}$

14-17



since $T_1 + \sum U_{1-2} = T_2$

and $v|_y = 0$ & $v|_d = 0$

then $\sum U_{1-2} = 0$

forces that do work are found from FBD

U_{1-2} (of weight) $= -W(y-d)$

U_{1-2} (of spring) $= -\frac{1}{2}k(s_2^2 - s_1^2)$

$s_2 = y-d$
$s_1 = d-d=0$] measured from unstretched length

$U_{1-2}(\text{of } P) = \int_d^y P\cos\theta \, d\bar{y}$

note: cannot pull $P\cos\theta$ out of integral since θ changes as s changes and y changes

$$\therefore \quad \sum U_{1-2} = \int_d^y P\cos\theta \, d\bar{y} - W(y-d) - \frac{1}{2}k([y-d]^2 - 0)$$

$$\therefore \quad \int_d^y P\cos\theta \, d\bar{y} = U_{1-2}(\text{of } P) = \left[\frac{1}{2}k(y-d) + W\right](y-d)$$

note: cannot take $\sum F_y = 0$ & multiply by Δy to get this expression since work $= \int \bar{F} \cdot d\bar{r}$ and $\bar{F}$ changes with position

here $\bar{F} = P\cos\theta - W - F_s = P\cos\theta - W - k(y-d)$

$\bar{F}\Delta y = P\cos\theta(y-d) - W(y-d) - k(y-d)^2 \neq \sum U_{1-2}$

HW # 8

14-55, 62, 64 in 4 pages

(55)

21/30

The stretched length of the rubber band is :

$$l = \sqrt{(.240)^2 + (0.05)^2} = 0.245 \text{ M}$$

The energy of the band $= \frac{1}{2}(50)(0.245 - 0.200)^2$

$= 0.050625$ J.

Now 2 bands $= 2(0.050625) = 0.10125$ J

K.E of Pellet = stored energy of the two bands

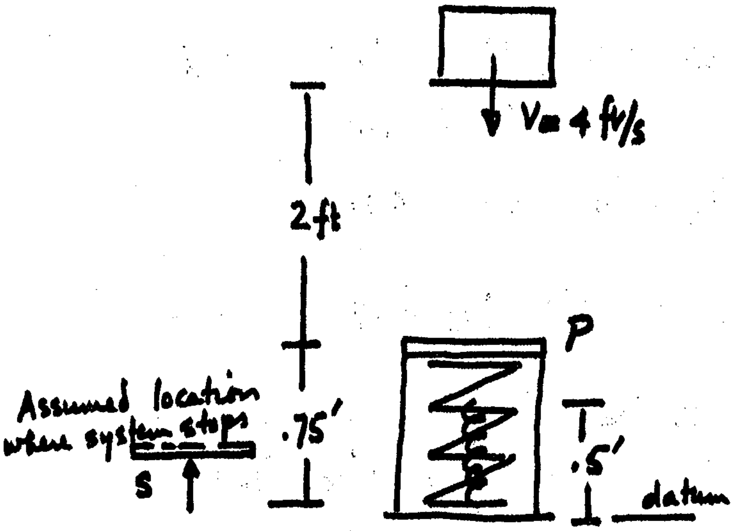
$$\tfrac{1}{2} M V^2 = 0.10125$$

$$\tfrac{1}{2}(0.025)(V^2) = 0.10125$$

$$V^2 = 8.1$$

$$V = 2.85 \text{ M/s}$$

62



All forces are conservative

$$T_1 = \tfrac{1}{2}mV^2 = \tfrac{1}{2}\tfrac{10}{32.2}(4)^2 = 2.49 \text{ lb-ft}$$

$$V_1 = W(2.75) + \tfrac{1}{2}k\Delta s^2$$
$$= 10(2.75) + \tfrac{1}{2}30\cdot 12\cdot(.25)^2 = 38.75 \text{ lb-ft}$$

$$T_2 = 0$$

$$V_2 = Ws + \tfrac{1}{2}k\Delta s_2^2 + \tfrac{1}{2}k'\Delta s_3^2$$
$$= 10s + \tfrac{1}{2}30\cdot 12(1-s)^2 + \tfrac{1}{2}50\cdot 12(.5-s)^2$$
$$= 10s + 180(1-2s+s^2) + 300(.25-s+s^2)$$
$$= 255 - 650s + 480s^2$$

$$\Delta s_2 = 1 - s$$
$$\Delta s_2 = .5 - s$$

$$T_1 + V_1 = 41.2345 = T_2 + V_2$$
$$480s^2 - 650s + 213.7655 = 0$$

$s = .7915$ ft
$s = .5626$ ft] in either case this implies that the inner spring is **not** compressed

Therefore $V_2 = Ws + \tfrac{1}{2}k\Delta s_2^2 = 180 - 350s + 180s^2$

$$\therefore \; T_1 + V_1 = T_2 + V_2$$
$$41.2345 = 180 - 350s + 180s^2$$
$$180s^2 - 350s + 138.7655 = 0$$

$s = 1.3897$ ft
$s = \underline{.5547 \text{ ft}}$

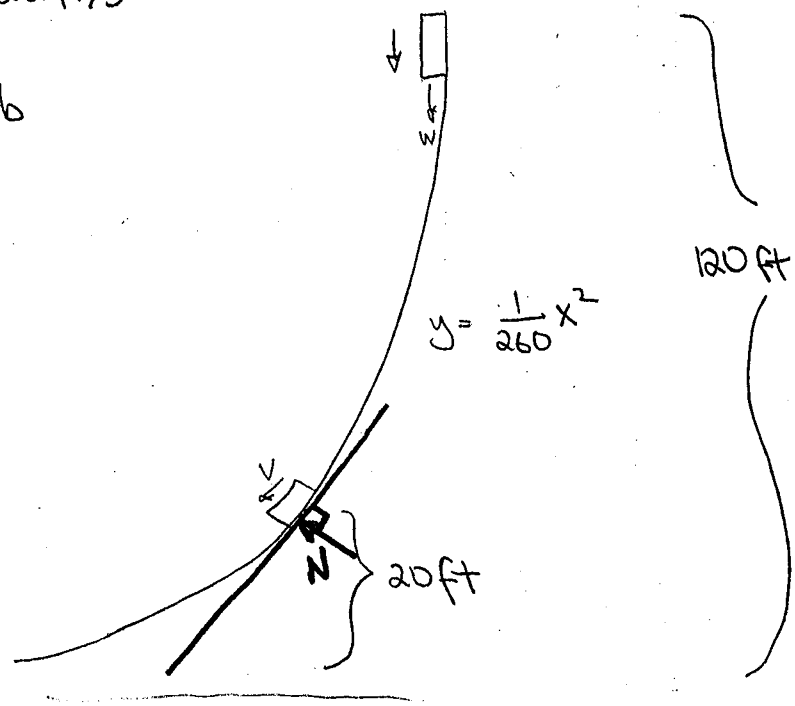
But we want to find how far the plate P depresses

$$= .75 - .5547 = \underline{\underline{.1953 \text{ ft}}}$$

(64)

$g = 32.2\ ft/s$

$W = 500 lb$



By the conservation of energy:

$$W h_1 = W h_2 + \tfrac{1}{2} M V^2$$

$$500(120) = 500(20) + \tfrac{1}{2}(500/32.2)V^2$$

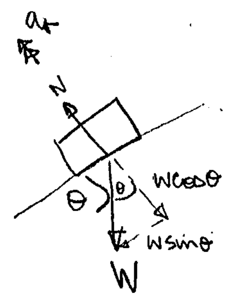
$$60000 = 10000 + 7.764 V^2$$

$$V^2 = 6439.98$$

$$V = \underline{80.25\ ft/s}$$ ✓

$y = \frac{1}{260}x^2 \qquad \frac{dy}{dx} = \frac{1}{130}x \quad , \frac{d^2y}{dx^2} = \frac{1}{130}$

at $y = 20$, $x = 72.11$



$\theta = \tan^{-1} \frac{72.11}{130} = 29.017°$

Now $\rho = \left| \frac{[1 + (dy/dx)^2]^{3/2}}{d^2y/dx^2} \right|$

$\rho = \left| \frac{[1 + (0.5547)^2]^{3/2}}{1/130} \right| = 194.4 \text{ ft}$

$a_r = \frac{V^2}{\rho} = \frac{6439.98}{194.4} = 33.13 \text{ ft/s}^2$

$Ma_r = N - W\cos\theta$

$N = Ma_r + W\cos\theta = (500/32.2)\,33.13 + 500\cos 29.017°$

$N = \underline{951.68 \text{ lb}}$

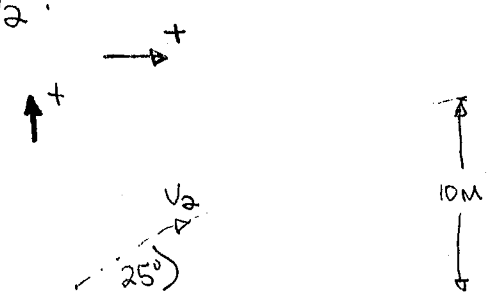
HW # 9 in 5 pages EGN 3321

15 - 8, 18, 23, 31 & 39

(8)

For V_2:

u = initial velocity



Using $V^2 = u^2 + 2as$

$$0 = +(V_2 \sin 25°)^2 - 2(9.81)10$$

$$V_2 \sin 25° = 14.007$$

$$V_2 = 33.144 \text{ M/s}$$

By conservation of linear momentum:

initial velocity is

$\leftarrow$ 15 $\quad \therefore \; V_{x_1} = -15$ m/s, $V_{y_1} = 0$ m/s

x-direction

$$M(V_x)_1 + I_x = M(V_x)_2$$

$$-M(15) + I_x = M(V_2 \cos 25°)$$

$$I_x = 0.18(30.04) + 0.18(15)$$

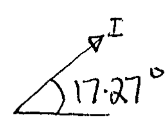
$$I_x = 8.107 \text{ N-s}$$

$$M(V_y)_1 + I_y = M(V_y)_2$$

$$0 + I_y = 0.18(V_2 \sin 25°)$$

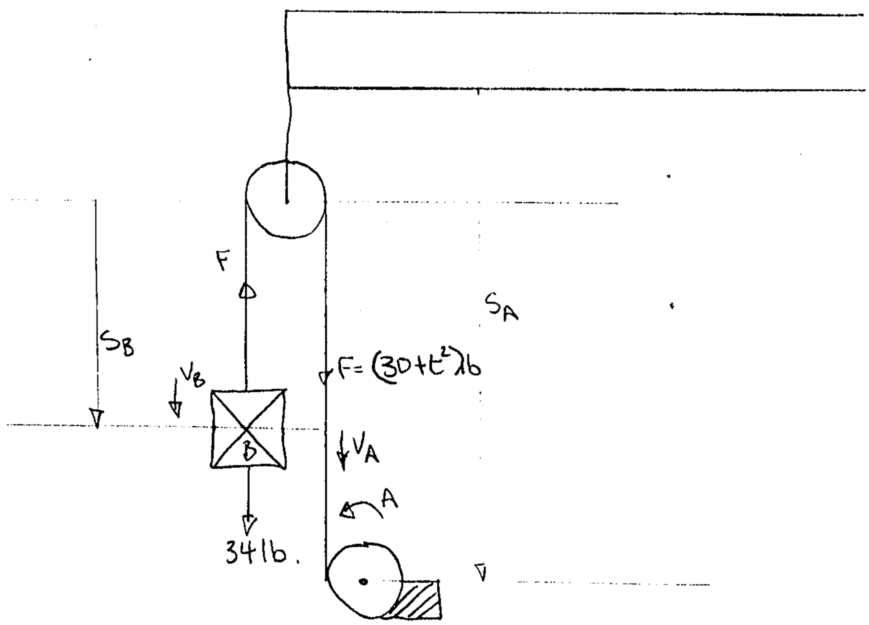
$$I_y = 0.18(14.007) = 2.52126 \text{ N}$$

$$I = \sqrt{(I_x)^2 + (I_y)^2} = \underline{8.5 \text{ N-s}}$$



Homework Problems

(18) Ch15

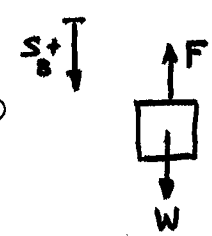


For block B to begin moving:

$$30 + t^2 = 34 \Rightarrow t^2 = 4 \Rightarrow t = \pm 2$$

$t = 2$ secs.

By Conservation of linear momentum: (after 4 secs)



$$34t - \int_2^4 (30 + t^2)\,dt = MV_2$$

$$34(2) - \left[30t + \tfrac{1}{3}t^3 \right]_2^4 = MV_2$$

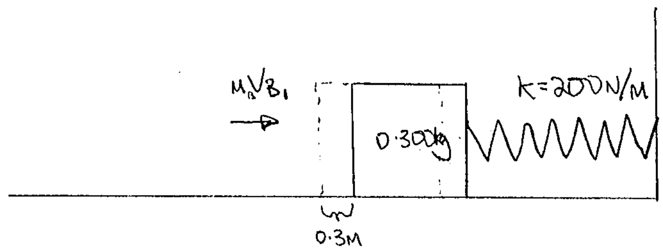
$$68 - (141\tfrac{1}{3} - 62\tfrac{2}{3}) = \tfrac{34}{32.2} V_2$$

$$68 - 78\tfrac{2}{3} = \tfrac{34}{32.2} V_2$$

$$-10\tfrac{2}{3} = \tfrac{34}{32.2} V_2 \Rightarrow V_2 = -10.102 \text{ ft/s}$$

indicates velocity is ↑

(23)



$M_B = 0.02$ kg

By wording of problem momentum of bullet becomes momentum of bullet + block **before** block starts moving

$$0.02\, V_{B1} = 0.320\, V_{B2}$$

$$V_{B2} = 0.0625\, V_{B1}$$

By Conservation of energy:

$$(1/2)(0.32)(0.0625\, V_{B1})^2 = 1/2(200)(0.3)^2$$

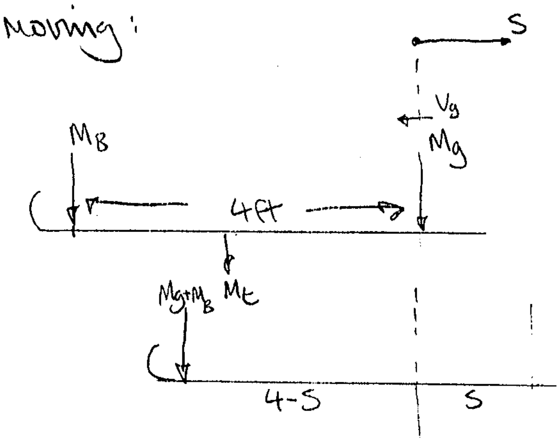
$$(1/2)\, 0.00125\, V_{B1}^2 = 9$$

$$V_{B1}^2 = 14400$$

$$V_{B1} = \underline{120 \text{ m/s}}$$

(31)

For girl moving:



$$-M_g V_g + (M_t + M_B) V_t = 0$$

integrate and use fact that initially there is no motion

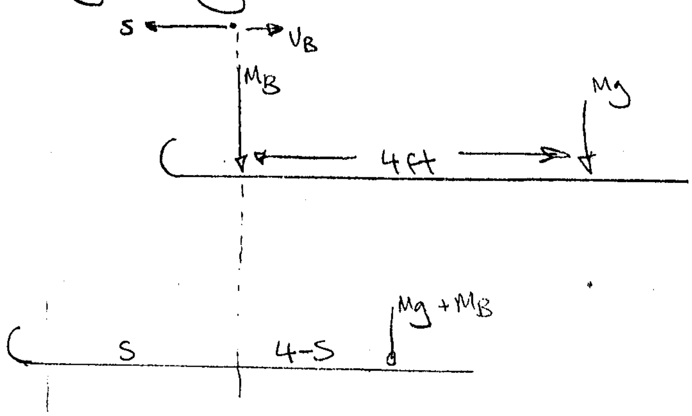
$$M_g S_g + \frac{100}{32.2} S_t = 0$$

$$-M_g(4-S) + \frac{100}{32.2} S = 0$$

$$\frac{65}{32.2} S + \frac{100}{32.2} S = \frac{65}{32.2}(4)$$

$$S = 1.575757 \text{ ft}$$

For boy moving:



$$-M_B V_B + (M_t + M_g) V_t = 0$$

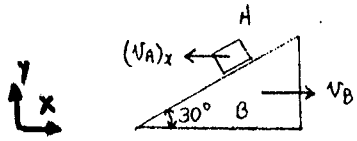
$$-M_B S_B + (M_t + M_g) S_t = 0$$

$$-M_B(4-S) + \frac{85}{32.2} S = 0$$

$$\frac{80}{32.2} S + \frac{85}{32.2} S = \frac{80}{32.2}(4) \Rightarrow S = 1.939394 \text{ ft}$$

Thus the net distance toboggan moves = 0.363636 ft ($\overset{S}{\leftarrow}$)

15-39



$\overset{+}{\rightarrow}\ \Sigma m(v_1) = \Sigma m(v_2)$

$0 = 30 v_B - 5(v_A)_x$

$(v_A)_x = 6 v_B$

$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

$\overset{+}{\rightarrow}\quad v_B = -(v_A)_x + (v_{B/A})_x$

$v_B = -6 v_B + (v_{B/A})_x$

Here conservation of linear momentum only in x direction

Motion down the block implies no conservation of linear momentum in y direction

Note here v_{A_x} is assumed to be to left ∴ − sign.

$(v_{B/A})_x = 7 v_B$

Integrate $(s_{B/A})_x = 7 s_B$

$(s_{B/A})_x = 0.5\text{m}$

thus $s_B = \dfrac{0.5}{7} = 0.0714\text{m} = \underline{\underline{71.4\ \text{mm}}} \rightarrow$

integration leads to

$(s_{B/A})_x = 7 s_B + \mathcal{C}$ ← integrating constant

initially no motion ∴ $\mathcal{C} = 0$

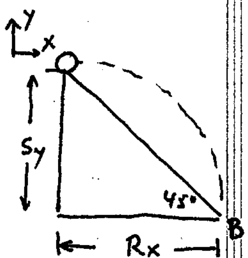
A moves a relative distance of ← 0.5 m horizontally to B

∴ B moves a relative distance of → 0.5 m horizontally to A

HW # 10

in 4 pages

15.51, 15.56, 15.62, 15.53



15-51 This must be looked at as 2 separate events: Projectile motion to just before ball hits, then oblique impact

$V_{x_0} = V_A = 3\text{ ft/s}$ $\quad V_{y_0} = 0$ $\quad R_x = V_A t$ $\quad S_y = \cancel{S_{y_0}}^{0} + \cancel{V_{y_0}}t - \frac{1}{2}gt^2$

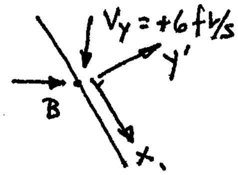
also $S_y = -R_x \tan 45°$ (minus since S_y is −)

$\therefore \quad S_y = -\frac{1}{2}g\left(\frac{R_x^2}{V_A^2}\right) = -R_x \tan 45° \qquad \therefore \quad R_x = \frac{2V_A^2}{g}\tan 45°$

$\therefore$ since $R_x = V_A t = \frac{2V_A^2}{g}\tan 45° \qquad \therefore \quad t = \frac{2V_A}{g}\tan 45°$

also V_y at B $= \cancel{V_{y_0}}^{0} - gt = -2V_A \tan 45° = -6\text{ ft/s}$

at B



} These are components of velocity of Ball. Must now convert them into components along x', y'

$V_{x'_{initial}} = V_x \cos 45° + V_y \cos 45° = 9\cos 45°$

$V_{y'_{initial}} = V_x \sin 45° - V_y \sin 45° = -3\sin 45°$

Remember along plane of contact (x' axis) velocity is unchanged $\therefore V_{x'_{final}} = 9\cos 45°$

Along line of impact (y' axis) $V_{bearing\ inital\ y'} = -3\sin 45° = -2.12\text{ ft/s}$ $\quad V_{plane\ init} = 0$

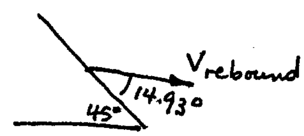
$V_{bearing\ final\ y'} = ?$ $\quad V_{plane\ final} = 0$

also $$e = \frac{V_{plane\ final} - V_{bearing\ final\ y'}}{V_{bearing\ initial\ y'} - V_{plane\ initial}} = 0.8 = \frac{0 - V_{bearing\ final\ y'}}{-2.12 - 0}$$

$V_{bearing\ final\ y'} = (0.8)(2.12) = 1.70\text{ ft/s}$ in $+y'$ direction

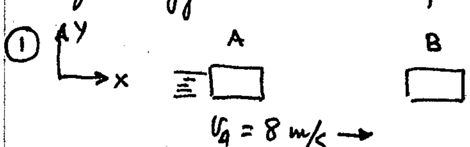
$$\therefore \quad V_{bearing} = \sqrt{V_{x'_{final}}^2 + V_{bearing\ final\ y'}^2} = \sqrt{(6.364)^2 + (1.697)^2} = 6.586\text{ ft/s}$$

at angle of $\tan^{-1}\left(\frac{1.697}{6.364}\right) = 14.93°$ to the x' axis



15-53 again we look at this as 2 separate events ① collision ② then using velocity of B after collision as the initial velocity for conservation of energy. B initially at rest A has velocity of 8 m/s →

①



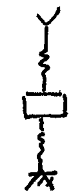
$$m_A v_{A_1} + m_B \cancel{v_{B_1}}^{0} = m_A v_{A_2} + m_B v_{B_2} \Rightarrow v_{A_1} = v_{A_2} + v_{B_2}$$

also

$$e = \frac{v_{B_2} - v_{A_2}}{v_{A_1} - \cancel{v_{B_1}}^{0}} \Rightarrow v_{B_2} - v_{A_2} = e\,v_{A_1}$$

∴ add the 2 eqns $2v_{B_2} = v_{A_1}(1+e)$ or $v_{B_2} = \frac{v_{A_1}}{2}(1+e) = .7(8) = 5.6$ m/s $\xrightarrow{+}$

also $v_{A_2} = v_{A_1} - v_{B_2} = 2.4$ m/s ; let $v_{B_i} = v_{B_2}$ $v_{A_i} = v_{A_2}$



② $T_1 + V_1 = T_2 + V_2$ system is A, B + 2 springs

$$T_1 = \tfrac{1}{2} m_A v_{A_i}^2 + \tfrac{1}{2} m_B v_{B_i}^2 = \tfrac{1}{2}(1.5)(2.4)^2 + \tfrac{1}{2}(1.5)(5.6)^2$$

$$V_1 = 2\left[\tfrac{1}{2} k s_1^2\right] = 2\left[\tfrac{1}{2}(30\,\text{N/m})(0.5)^2\right];$$ s_1 = distance spring is stretched initially

$$T_2 = \tfrac{1}{2} m_A v_{A_f}^2 + \tfrac{1}{2} m_B v_{B_f}^2 = \tfrac{1}{2}(1.5)(2.4)^2 + \tfrac{1}{2}(1.5)(0)^2$$

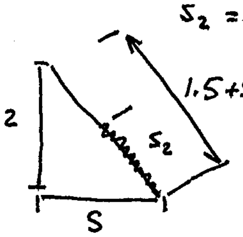
note here that block A continues moving unimpeded ; B comes to a stop

Note: we could have taken the system as B + 2 springs ONLY ; then first terms in T_1 & T_2 would be deleted

$$V_2 = 2\left[\tfrac{1}{2} k s_2^2\right] = 2\left[\tfrac{1}{2}(30)(s_2^2)\right];$$ s_2 = stretch in the spring when B comes to a stop

$$\therefore \tfrac{1}{2}(1.5)(2.4)^2 + \tfrac{1}{2}(1.5)(5.6)^2 + 2\left(\tfrac{1}{2}\right)(30)(.5)^2 = \tfrac{1}{2}(1.5)(2.4)^2 + \tfrac{1}{2}(1.5)(0)^2 + 2\left(\tfrac{1}{2}\right)(30)($$

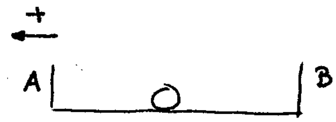
$s_2 = 1.01686$ m this represents the stretch as shown, along the direction of the spring, measured from its unstretched length (1.5 m)*

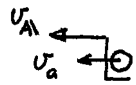


$$s = \sqrt{(1.5+s_2)^2 - (2)^2} = \underline{\underline{1.528\ \text{m}}}$$

* When s = 0 the spring was stretched 0.5 m. Looking at the diagram the spring was 2 m long when s = 0 ∴ initial length = 2 − 0.5 m = 1.5 m

15-56





By implulse-momentum from start to just before impact

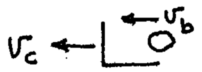
$+ \leftarrow \quad mV_o = mV_a + MV_A$

from that point to after the impact

$mV_a + MV_A = \underline{mV_b + MV_c = mV_o} \quad (*)$

V_b - v of ball after

V_c - v of cart after



Using e:

$$e = \frac{V_c - V_b}{V_o} \quad (**) \quad \text{if } V_b \text{ is } - \Rightarrow \text{ball moves} \rightarrow$$

Solving

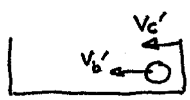
$eV_o + V_b = V_c$ put into $mV_b + M(eV_o + V_b) = mV_o$

$$\therefore \quad V_b = \frac{mV_o - MeV_o}{m+M} = \frac{m - Me}{m+M} V_o < c$$

if $Me > m \Rightarrow V_b < 0$

$$\therefore \quad V_c = eV_o + V_b = eV_o + \frac{m - Me}{m+M} V_o = \frac{m(1+e)v_o}{m+M} > 0$$

let's look at B: $mV_b + MV_c = mV_b' + MV_c' = mV_o$ $\quad V_b'$ & V_c' after collision

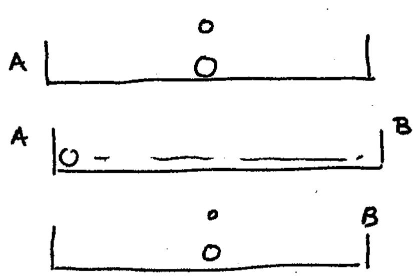


$$e = \frac{V_b' - V_c'}{V_c - V_b} = \frac{V_b' - V_c'}{eV_o} \quad \text{from } (**)$$

$$\therefore \quad e^2 V_o = V_b' - V_c' \quad \text{etc} \quad \therefore \quad V_b' = \frac{(m - Me^2)V_o}{m+M} \quad V_c' = \frac{m(1+e^2)v_o}{m+M}$$

Note: $V_c' < V_c$ since $e^2 < e$ also $V_b' > V_b$ for same reason

to find the time required $\quad$ travel time $= \dfrac{\text{distance}}{\text{relative velocity}}$



O-A : rel velocity is V_o

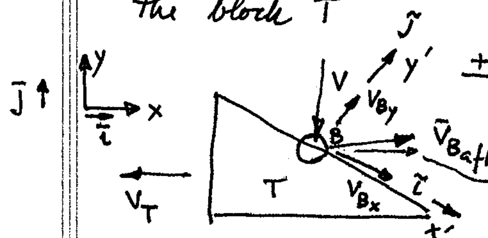
A-B : rel veloc is $V_c - V_b = eV_o$

B-O : rel veloc is $V_b' - V_c' = e^2 V_o$

$$\therefore \quad t = \frac{d}{V_o} + \frac{2d}{eV_o} + \frac{d}{e^2 V_o} = \frac{d[1 + 2e + e^2]}{e^2 V_o} = \frac{d(1+e)^2}{e^2 V_o}$$

15-62

Since all the forces at impact are internal we have conservation of momentum for the ball-triangle system. Also after impact the triangle will move but only in the horizontal direction $\leftarrow$. Since we want the velocity of the block T



$\overset{+}{\rightarrow}$ $\sum m_i \bar{v}_i \Big|_{before} = \sum m_i \bar{v}_i \Big|_{after}$ look at x momentum

$$\underbrace{m_T \bar{V}_{T\,Before}}_{0} + \underbrace{m_B \bar{V}_{B\,before}}_{\substack{0 \\ \text{since } \bar{V} \text{ in -y dir only}}} = m_T V_{T\,after_x} + m_B V_{B\,after\,x}$$

$\bar{V}_{T\,after} = -V_T$ $\qquad$ $\bar{V}_{B\,after} = V_{Bx}\tilde{i} + V_{By}\tilde{j}$ $\qquad$ $\tilde{i}, \tilde{j}$ unit vectors in x', y' dir

in terms of $\bar{i}, \bar{j}$

$$= V_{Bx}\cos 45^\circ \bar{i} - V_{Bx}\sin 45^\circ \bar{j} + V_{By}\cos 45^\circ \bar{i} + V_{By}\sin 45^\circ \bar{j}$$

$$= \underbrace{(V_{Bx} + V_{By})\frac{1}{\sqrt{2}}}_{V_{B\,after\,x}}\bar{i} + (-V_{Bx} + V_{By})\frac{1}{\sqrt{2}}\bar{j}$$

$$\therefore \quad -3m V_T + m(V_{Bx} + V_{By})\frac{1}{\sqrt{2}} = 0 \qquad (1)$$

for e:

y': along line of impact

$$e = \frac{V_{T_{y'after}} - V_{B_{y'after}}}{V_{B_{y'before}} - V_{T_{y'before}}} = \frac{-V_T\cos 45^\circ - V_{By}}{-V\cos 45^\circ - 0}$$

$$\therefore \quad eV\cos 45^\circ = V_T\cos 45^\circ + V_{By} \qquad (2)$$

x': along plane of contact: $\quad \underline{V\sin 45^\circ = V_{Bx}} \qquad (3)$

Put (3) & (2) into (1) and solve for V_T

$$-3mV_T + m(V\sin 45^\circ - V_T\cos 45^\circ + eV\cos 45^\circ)\frac{1}{\sqrt{2}} = 0$$

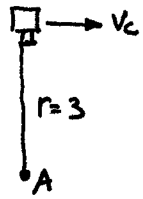
$$-V_T\left[3m + \frac{m}{2}\right] + \frac{mV}{2}(1+e) = 0 \quad \text{or}$$

$$V_T = \frac{V}{7}(1+e)$$

HW # 11 in 3 pages

15-71 , 72, 74

To find H_A : since V_c is $\perp$ to distance to pt A from C



$$H_A = r m V_c = 3 \cdot \left(\frac{2}{32.2}\right)(43.95) = 8.19 \text{ ft-lb-s} \quad (\text{slug-ft}^2/\text{s})$$

We can use conserv of energy to find V_D

$$T_D + V_D = T_c + V_c \qquad T_c = \tfrac{1}{2} m V_c^2 \qquad V_c = \tfrac{1}{2} k \Delta s_c^2$$

$$T_D = \tfrac{1}{2} m V_D^2 \qquad V_D = 0$$ (cord becomes unstretched & this is a horizontal plane)

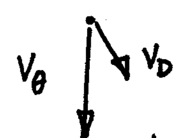
$$V_D = \sqrt{V_c^2 + \frac{k}{m}\Delta s_c^2} = 48.65 \text{ ft/s}$$

We can use the angular momentum since it is conserved to find the θ component of V_D ; thus $\sqrt{V_D^2 - (V_{D_\theta})^2} = V_{D_r}$

$$\therefore \quad H_c = H_D \qquad r_c \cdot m V_c = r_D \cdot m V_{D_\theta}$$

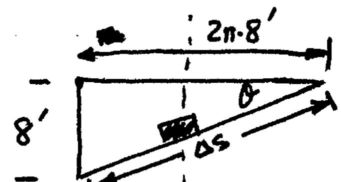
$$V_{D_\theta} = \frac{r_c V_c}{r_D} = \frac{3 \cdot 43.95}{1.5} = 87.9 \text{ ft/s}$$

now we have a problem $|V_D| > |V_{D_\theta}|$ which is impossible since $V_D^2 = V_{D_\theta}^2 + V_{D_r}^2 \Rightarrow V_D^2 > V_{D_\theta}^2$

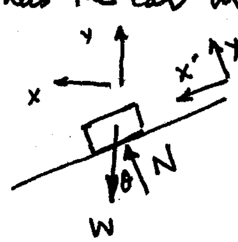


Thus the assumption of the cord becoming unstretched is <u>impossible</u>

15-74 If we look at one revolution of the helix "straightened out"



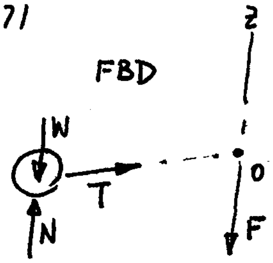
Thus the car drops through an angle $\theta = \tan^{-1}\left(\frac{8'}{2\pi \cdot 8'}\right) = 9.04°$



$$\Sigma F_{y'} = W \cos 9.04 - N = 0 \qquad N = 790 \text{ lbs}$$

in the x-y plane $N \sin 9.04$ produces a moment about the center of the helix $\therefore \int_0^4 r N \sin 9.04 \, dt$ = angular impulse

15-71



F, W, N, T produce no moments about the z axis

W, N do produce moments but in the x, y plane

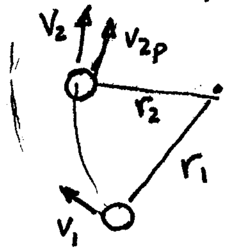
$\therefore$ in the z direction

$$H_{z_1} = H_{z_2} \qquad H_{z_1} = r_1 \cdot m v_1$$

since ball is moving in a circular path $\bar{r} \times m\bar{v} = rmv$ since $\bar{r} \perp \bar{v}$

$H_{z_2} = \bar{r}_2 \times m\bar{v}_2$ since $\bar{r}_2$ & the absolute velocity $\bar{V}_2$ are not $\perp$. However we can find a component of V_2 which is $\perp$ to r_2 call this V_{2p}

$$\therefore \quad r_1 m v_1 = r_2 m V_{2p} \qquad V_{2p} = r_1 V_1 / r_2 = 6 \cdot 3/2 = 9 \text{ ft/s}$$



V_{2p} is $\perp$ r_2 meaning it is in the θ direction

we are also told that the rope is pulled with $V_r = 2$ ft/s

$$\therefore \quad V_2 = \sqrt{V_\theta^2 + V_r^2} = \sqrt{V_{2p}^2 + V_r^2} = 9.22 \text{ ft/s}$$

Note that $\bar{F}$ has no moment about the z axis since $M = \bar{r} \times \bar{F}$ and $\bar{r} = \bar{0}$ always. $\bar{F}$ is the force used to pull the rope.

To find the work of $\bar{F}$: $\quad T_1 + \Sigma U_{1-2} = T_2 \qquad T_1 = \frac{1}{2} m v_1^2$

$$T_2 = \frac{1}{2} m V_2^2$$

Note that W & N do not produce work since they are $\perp$ to path of ball & T is internal $\therefore$ ΣU_{1-2} is the work of F only

$$\therefore \quad \Sigma U_{1-2} = \frac{1}{2} m \left[V_2^2 - V_1^2 \right] = \frac{1}{2} \left(\frac{4}{32.2} \right) \left[\overline{9.22}^2 - 6^2 \right] = 3.04 \text{ lb-ft}$$

15-72 To solve – 1) find V_c using conserv of energy

2) Determine V_D

Smooth guide – no friction ; note weight plays no part ; it is $\perp$ to plane of motion

$$T_B + V_B = T_c + V_c \qquad T_B = 0 \qquad V_B = \frac{1}{2} k \, \Delta s_B^2 \qquad \Delta s_B = 5 - 1.5 = 3.5'$$

$$T_c = \frac{1}{2} m v_c^2 \qquad V_c = \frac{1}{2} k \, \Delta s_c^2 \qquad \Delta s_c = 3 - 1.5 = 1.5'$$

$$\therefore \quad v_c = \sqrt{[V_B - V_c] \cdot \frac{2}{m}} = \sqrt{\frac{1}{2} k \left[\Delta s_B^2 - \Delta s_c^2 \right] \cdot \frac{2}{m}} = 43.95 \text{ ft/s}$$

and $H_{y_0} = 0$ $\quad H_{y_1} = rmV$ where V is the component of the velocity in the x,z plane

$$\therefore \int_0^4 r N \sin 9.04\, dt = \overset{8}{r}\underset{800/32.2}{m} V$$

thus $V = 20$ ft/s $\longleftarrow$

but V along the track $\swarrow$ is $\dfrac{20}{\cos 9.04^\circ} = 20.24$ ft/s

to find the drop: Let the datum be where the car started from rest since N is $\perp$ to the motion & $W\cos\theta$ is $\perp$ to motion; then $W\sin\theta$ produces work.

$$T_1 + \sum U_{1-2} = T_2 \qquad T_1 = 0 \quad T_2 = \tfrac{1}{2} m \left(20.24\,\tfrac{ft}{s}\right)^2$$

$$\sum U_{1-2} = W\sin\theta \cdot \Delta s = W \cdot \Delta h$$

$$\therefore \underline{\Delta h} = \tfrac{1}{2} m \left(20.24\,\text{ft/s}\right)^2 / mg = \underline{6.36'}$$

$\rightarrow$ could have also found v by using $\overset{\sum F_{x'} = ma_{x'}}{W\sin\theta = m a_{x'}} \Rightarrow a_{x'} = g\sin\theta$

then $v = v_0 + a_{x'} t = g\sin\theta\, t = 20.24$ ft/s

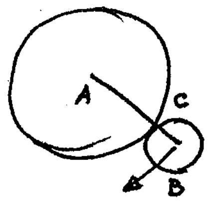
the ~~same~~ rest of problem is as above.

HW # 12

Problems 15.24, 26, 31, 37 in 2 pages

HW # 12

15-31



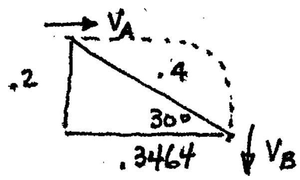
$\omega_{AB} = 120\text{ rpm} \times \frac{2\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ sec}} = 4\pi\text{ rad/s}$ ↻

$V_B = \omega_{AB} \cdot r_{B/A}$ ↙ $= 4\pi(.225) = .9\pi = 2.83$ m/s since $V_A = 0$

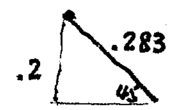
$V_C = V_B = r_{C/A}\,\omega_A$ since $V_A = 0 \Rightarrow$ $\underline{\omega_A = 18.85\text{ rad/s}}$ ↻

$\underline{V_B = 2.83\text{ m/s}}$ ↙

15.26



note that A is not in the circular part since



$.283 = r_{B/A}$ is distance that separates whether A is in circle or straight part. $r_{B/A} > .283$ m A in strait

$\bar{V}_B = -1.2\bar{j}$ $V_A = V_A\bar{i}$ $\bar{r}_{B/A} = .4\cos30°\bar{i} - .4\sin30°\bar{j}$

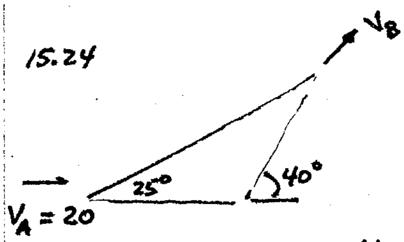
let $\bar{\omega} = \omega\bar{k}$

$$\bar{V}_B = \bar{V}_A + \bar{\omega} \times \bar{r}_{B/A} = V_A\bar{i} + .4\cos30°\omega\bar{j} + .4\sin30°\omega\bar{i} = -1.2\bar{j}$$

$\therefore$ $V_A + \omega(.4\sin30°) = 0$ & $-1.2 = .4\cos30°\omega \Rightarrow \omega = 3.464\text{ rad/s}$ ↻

$V_A = +.693$ m/s →

15.24



$\bar{V}_B = V_B(\cos40°\bar{i} + \sin40°\bar{j}) = 20\bar{i} + \bar{\omega} \times \bar{r}_{B/A}$

$\bar{r}_{B/A} = 6[\cos25°\bar{i} + \sin25°\bar{j}]$; let $\bar{\omega} = \bar{\omega}\bar{k}$

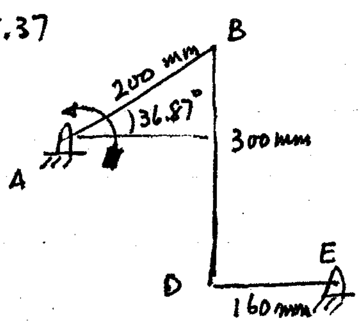
$$V_B\cos40°\bar{i} + V_B\sin40°\bar{j} = 20\bar{i} + \omega\cdot6\cos25°\bar{j} - 6\omega\sin25°\bar{i}$$

$\therefore$ $\left.\begin{array}{l} V_B\sin40° = +6\omega\cos25° \\ V_B\cos40° = 20 - 6\omega\sin25° \end{array}\right]$ $\tan40° = \dfrac{+6\omega\cos25°}{20 - 6\omega\sin25°}$

$\therefore$ $20\tan40° = +\omega\left[6\tan40°\sin25° + 6\cos25°\right]$

$\omega = 2.218$ rad/s ↺ $V_B = 18.77$ ft/s ∠40°

15.37



$V_B = \omega r_{B/A}$ ↖36.87° $= .6[-\sin36.87\bar{i} + \cos36.87\bar{j}]$

if $\bar{\omega}_{DE} = \omega_{DE}\bar{k} \Rightarrow \bar{V}_D = .16\,\omega_{DE}$ ↓

also $\bar{V}_{D/B} = \omega_{DB}(.3)$ → if $\bar{\omega}_{DB} = \omega_{DB}\bar{k}$

$\therefore$ $\bar{V}_D = \bar{V}_B + \bar{V}_{D/B}$

$$-.16\,\omega_{DE}\,\bar{j} = .6\left[-.6\bar{i} + .8\bar{j}\right] + .3\,\omega_{DB}\,\bar{i}$$

$$\Rightarrow \quad -.36 + .3\,\omega_{DB} = 0 \quad \text{or} \quad \omega_{DB} = 1.2 \text{ rad/sec}$$

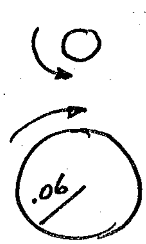
$$-.16\,\omega_{DE} = .48 \quad \text{or} \quad \omega_{DE} = 3 \text{ rad/sec}$$

EGN 3321

HW # 13

16 - 16, 19, 36, 37 in 2 pages

16-16



$\alpha_A = .06\theta_A^2$

$\alpha_B = \frac{1}{5}\alpha_A = .012\,\theta_A^2 = .3\theta_B^2$

since the gears are in contact over the time of motion they must travel the same distance

∴ $\Delta\theta_A \cdot r_A = \Delta\theta_B \cdot r_B$

now $\alpha_A \, d\theta_A = \omega_A \, d\omega_A$

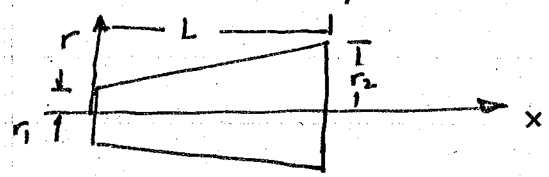
or $\frac{.06\,\theta_A^3}{3} = \frac{(\omega_{A_2}^2 - \omega_{A_1}^2)}{2}$ with $\theta_A = 10\,\text{rev} = 20\pi$ rad

$\omega_{A_1} = 50$ rad/s

$\Rightarrow \omega_{A_2} = 111.454$ rad/s

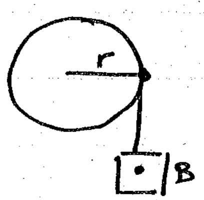
now at contact pt $\omega_A r_A = \omega_B r_B$ ∴ $\omega_{B_2} = \omega_{A_2} \, r_A/r_B = 22.29$ rad/s

16-19 The tapered drum's radius as a fn. of x



$r = r_1 + \frac{(r_2 - r_1)}{L}x$

also $\omega = \text{const.}$



at P $V = \omega r$ and since the rope is taught $V_P = V_B$

∴ $\frac{dV_B}{dt} = a_B = \frac{dV_P}{dt} = \cancel{\frac{d\omega}{dt}}\, r + \frac{dr}{dt}\,\omega$

∴ $a_P = \omega \frac{dr}{dt}$ now for one full revolution

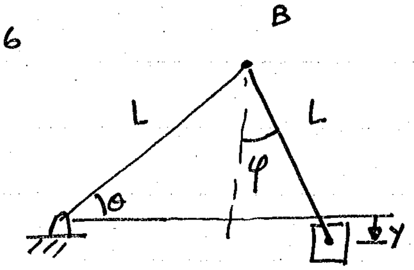
dr increases by $\frac{(r_2 - r_1)}{L}$

distance; also dx increases by an amount d

∴ $a_P = \omega \frac{dr}{dt} = \omega \frac{dr}{dx}\frac{d\theta}{dt}\frac{dx}{d\theta} = \omega \, \frac{(r_2 - r_1)/L}{} \cdot \omega \cdot \frac{d}{2\pi}$

$= \frac{\omega^2 d}{2\pi}\left(\frac{r_2 - r_1}{L}\right)$

16-36



$$L\cos\theta + L\sin\varphi = L \qquad (1)$$

and $$y = L\cos\varphi - L\sin\theta \qquad (2)$$

take $\frac{d}{dt}(1) \Rightarrow \dot\varphi = \dot\theta \frac{\sin\theta}{\cos\varphi}$

and $\ddot\varphi = \frac{\cos\theta\,\dot\theta^2 + \sin\varphi\,\dot\varphi^2}{\cos\varphi}$

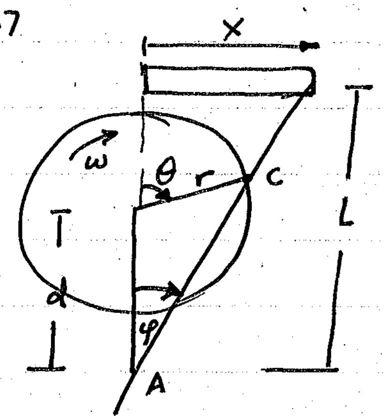
for $\theta = 60^\circ \Rightarrow \varphi = 30^\circ \Rightarrow \boxed{\dot\varphi = \dot\theta = \omega}$

take $\frac{d}{dt}(2) \Rightarrow V = \frac{dy}{dt} = -L\left[\sin\varphi\,\dot\varphi + \cos\theta\,\dot\theta\right] = -L\omega$

take $\frac{d}{dt}V = a = -L\left[\cos\varphi\,\dot\varphi^2 + \sin\varphi\,\ddot\varphi - \sin\theta\,\dot\theta^2 + \cos\theta\,\cancel{\ddot\theta}\right] = \frac{-1}{\sqrt3}L\omega^2$

this means $V = L\omega \uparrow$ and $a = \frac{1}{\sqrt3}L\omega^2 \uparrow$

16-37



$\therefore \quad x = L\tan\varphi$

$\bar{AC}\cos\varphi = d + r\cos\theta$

$\bar{AC}\sin\varphi = r\sin\theta$

$\therefore \quad \tan\varphi = \frac{r\sin\theta}{d + r\cos\theta}$

now $\dot x = L\frac{d}{dt}\tan\varphi = L\frac{d}{dt}\left(\frac{r\sin\theta}{d + r\cos\theta}\right)$

$$V = \dot x = l\dot\theta r\,\frac{(r + d\cos\theta)}{(d + r\cos\theta)^2}$$

and $$a = \dot v = \ddot x = lr\dot\theta^2\sin\theta\,\frac{\left[2r^2 - d^2 + rd\cos\theta\right]}{(d + r\cos\theta)^3}$$

$\dot\phi = \frac{r\dot\theta(r + d\cos\theta)}{d^2 + 2rd\cos\theta + r^2}$ $\Rightarrow$ this is the angular velocity of bar AC

note that it is <u>not</u> the same as that of the disk (which is ω)

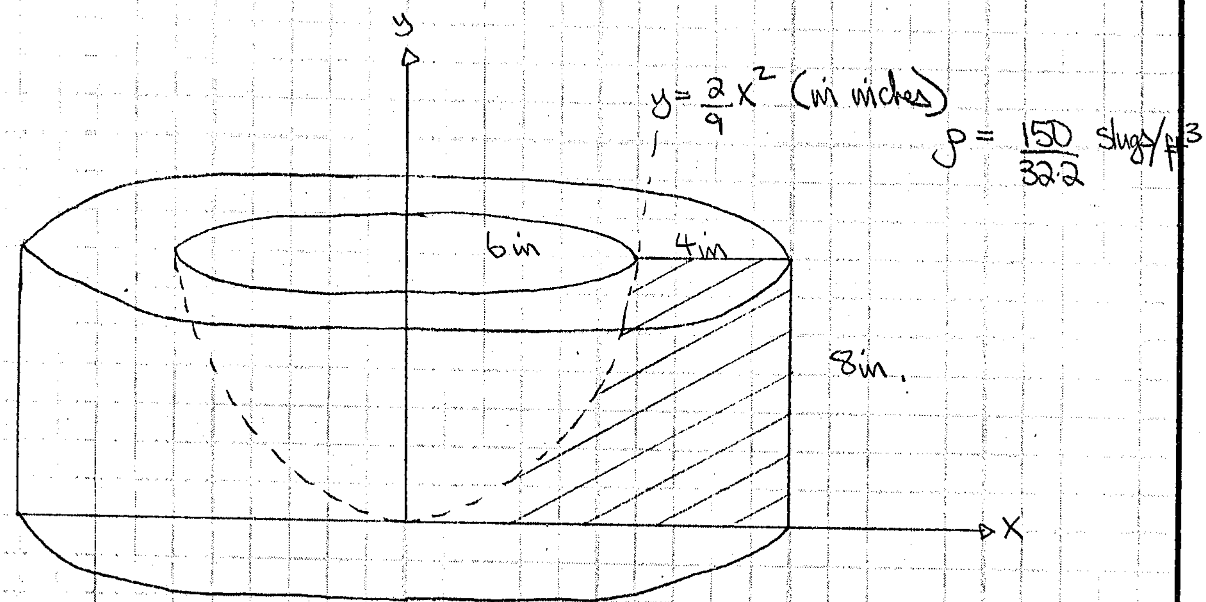
EGN 3321

HW # 14

17-9, 11, 14, 22, 37, 41 in 5 pages

Ch 17

(9)



For a solid cylinder of radius $^{10}/_{12}$ ft and height $^{8}/_{12}$ ft.

$$dV = (2\pi r)(h)\,dr$$

$$dm = \rho\,dV = (2\pi r h \rho)\,dr$$

$$dI_y = r^2 dm = 2\pi r^3 h\rho\,dr$$

$$I_y = \int_m r^2 dm = \rho 2\pi h \int_0^{10/12} r^3\,dr = \frac{\rho\pi}{2}\left(\frac{10}{12}\right)^4\left(\frac{8}{12}\right)$$

$$I_y = 2.3526 \text{ slugs/ft}^2$$

For a solid inner cup only: The equation becomes $y = {}^8/_3 x^2$ (in ft)

NOTE: SINCE X, Y have units of inches ⟹ constant 2/9 also must have units of $\frac{1}{in}$, THUS must also convert it.

$$dm = \rho\,dV = \rho(\pi x^2)\,dy$$

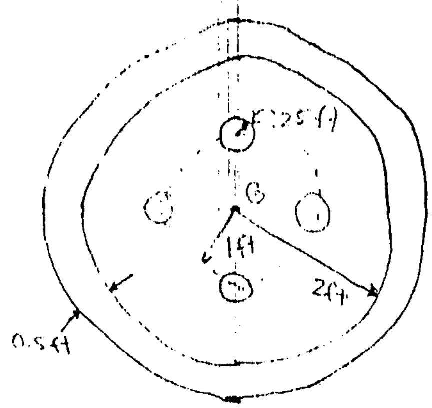
$$dI_y = \tfrac{1}{2}(dm)x^2 = \tfrac{1}{2}[\rho(\pi x^2)\,dy]x^2$$

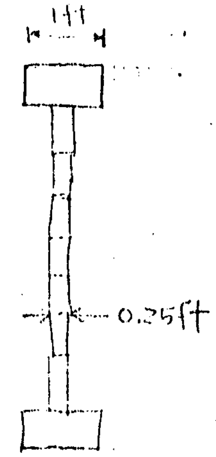
$$I_y = \frac{\pi\rho}{2}\int_0^{8/12} x^4\,dy = \frac{\pi\rho}{2}\int_0^{8/12}\frac{9}{64}y^2\,dy$$

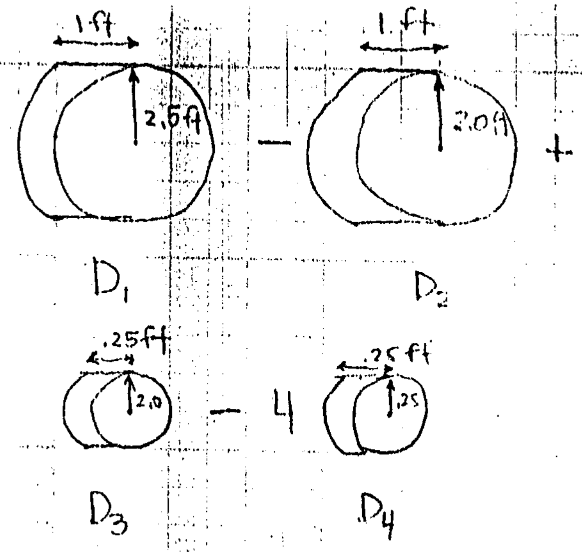
$$I_y = \frac{\pi\rho}{2}\left[\frac{9}{128}y^3\right]_0^{8/12} = 0.1016 \text{ slugs/ft}^2$$

Moment of Inertia of shaded region = 2.25 slugs/ft^2

17-14







$\gamma = 90\, lb/ft^3 \qquad t = \text{thickness}$

$\rho = \frac{\gamma}{G} = \frac{90}{32.2} = 4.66 \frac{slugs}{ft^3} \qquad V = \pi r^2 t$

$m = \rho V \qquad I_G = \frac{1}{2} m r^2$

$m_1 = (4.6)[\pi(2.5)^2(1)]$
$= 54.8$ slugs

$I_1 = \frac{1}{2}(54.8)(2.5)^2$
$= \underline{171.5 \text{ slugs}\cdot ft^2}$

$m_2 = (4.6)[\pi(2.0)^2 1]$
$= 35.12$ slugs

$I_2 = \frac{1}{2}(35.12)(2.0)^2$
$= \underline{70.24 \text{ slugs}\cdot ft^2}$ ✓

$m_3 = (4.6)[\pi(2.0)^2(.25)]$
$= 8.78$ slugs

$I_3 = \frac{1}{2}(8.78)(2.0)^2$
$= \underline{17.56 \text{ slugs}\cdot ft^2}$ ✓

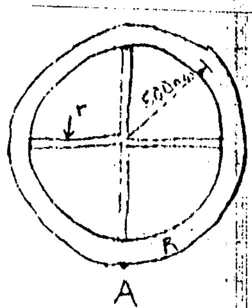
$4m_4 = (4.6)[\pi(.25)^2(.25)]4$
$= .5488$ slugs

$I_{G4} = \frac{1}{2}(.5488)(.25)^2$
$= 0.017 \text{ slugs}\cdot ft^2$

$I_4 = I_{G4} + 4m_4(d^2)$
$= .017 + (.5488)(1)^2$
$= \underline{.5659 \text{ slugs}\cdot ft^2}$

$I = I_1 - I_2 + I_3 - I_4$
$= \underline{118.2 \text{ slugs}\cdot ft^2}$ ✓

17-11



$M_R = 10 kg$
$M_r = 2 kg$

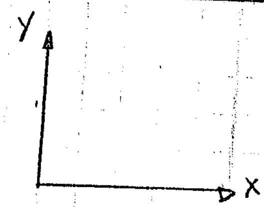
$R = r = 500 mm$
$= .5 m$

$d = r.$

$m = M_R + 4M_r$
$= 10 + 8 = 18 kg$

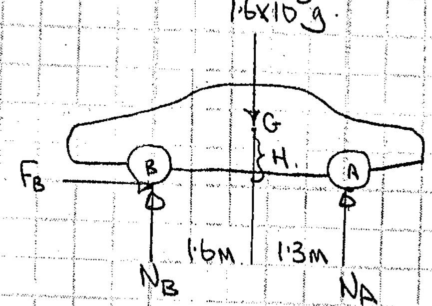
$I_G = I_R + I_r$
$= m_R R^2 + [\frac{1}{3} m_r l_r^2]4$ (four rods)
$= (10)(.5)^2 + \frac{4}{3}(2)(\frac{1}{2})^2$
$= 3.1\overline{6}$ kg-m² ✓

$I = I_G + md^2$
$= 3.1\overline{6} + (10)(.5)^2$
$I = 5.6\overline{6}$ kg-m²



(37)

$\mu_k = 0.3$



Eq^{ns} of Motion:

$\xrightarrow{+} \sum F_x = m(a_G)_x; \quad 0.3\,N_B = 1.6\times10^3\,a_G$ ✓

$+\uparrow \sum F_y = m(a_G)_y; \quad N_A + N_B - 1.6\times10^3(9.81) = 0$ ✓

$\circlearrowleft + \sum M_G = 0; \quad -N_B(1.6) + 0.3\,N_B(H) + N_A(1.3) = 0$ ✓

Now for regular car, $H = 0.4\,m$

$$N_A + N_B = 15696$$

$$-1.48\,N_B + 1.3\,N_A = 0$$

$N_A = 8356.14\ N \qquad N_B = 7339.86\ N \qquad a_G = \underline{1.376\ m/s^2}$ ✓

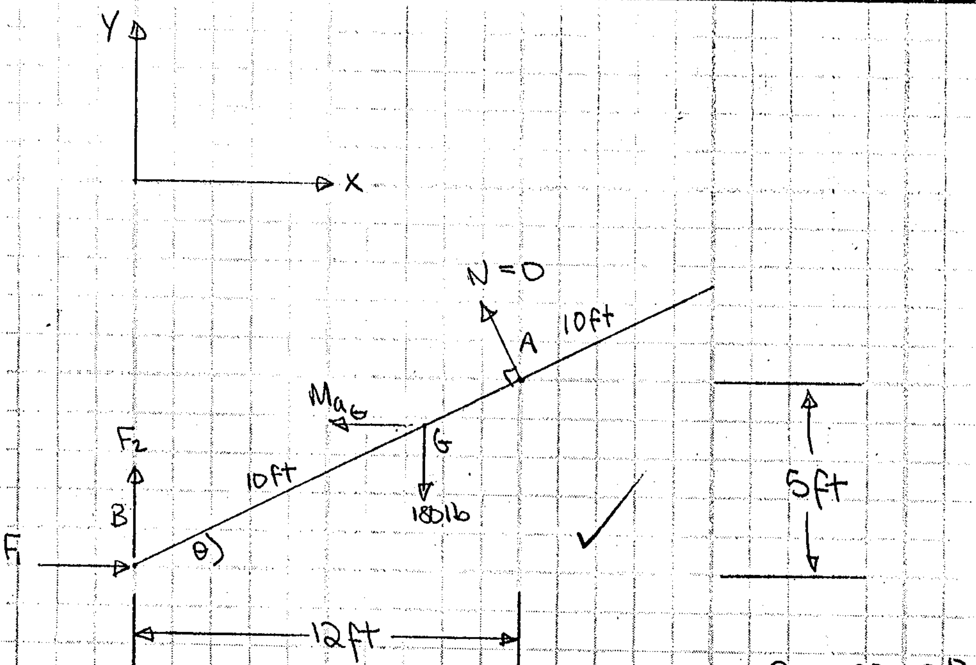
For "Raked" car; $H = 0.6\,m$

$$N_A + N_B = 15696$$

$$-1.42\,N_B + 1.3\,N_A = 0$$

$N_A = 8194.24\,N \qquad N_B = 7501.76\,N \qquad a_G = \underline{1.407\ m/s^2}$ ✓

(22)



$\theta = 22.62°$

Summing moments about B:

$$\cancel{180}\,(\cancel{10}\cos 22.62°) = \frac{\cancel{180}}{32.2}\,(\cancel{10}\sin 22.62°)\,a_G$$

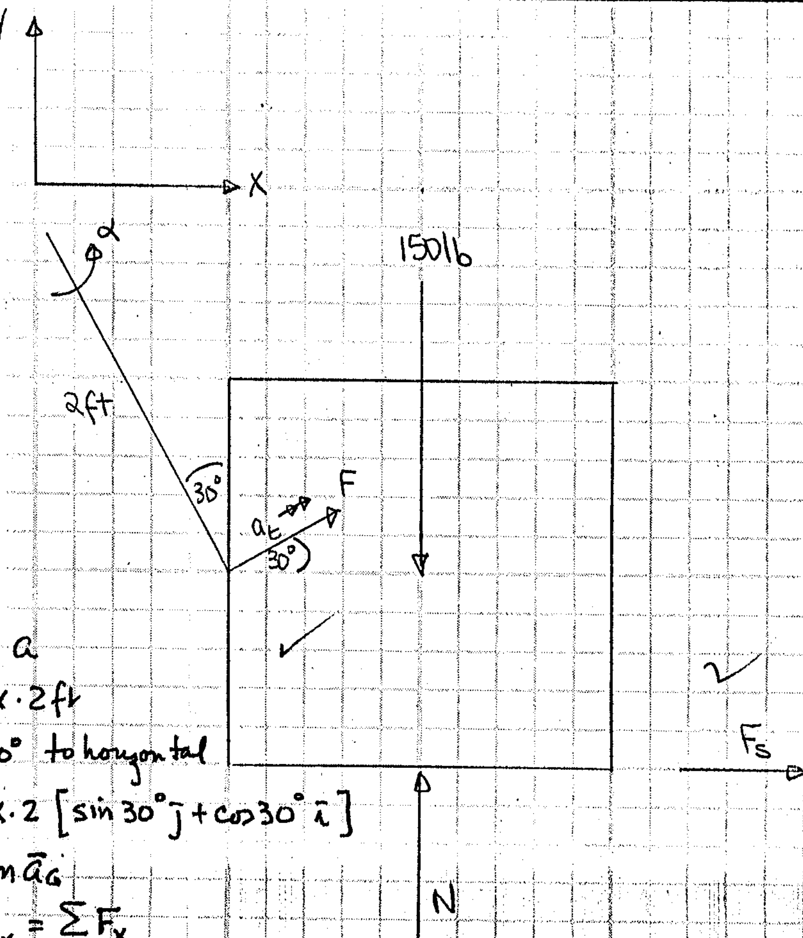
$$a_G = \frac{32.2\cos 22.62°}{\sin 22.62°} = \underline{77.3\ ft/s^2}$$

Resolving Forces in the x-y directions.

$F_2 = \underline{180\ lb}$

$F_1 = Ma_G = \underline{432\ lb}$

(41)



since the arms have an α then a of the lift is $= \alpha \cdot 2ft$ acting at an $\measuredangle 30°$ to horizontal

$$\therefore \; m\bar{a}_G = m\alpha \cdot 2\left[\sin 30° \, j + \cos 30° \, i\right]$$

call this $\bar{F} = m\bar{a}_G$

$$\therefore \; F_x = m\bar{a}_{G_x} = \Sigma F_x$$

$$F_y = m\bar{a}_{G_y} = \Sigma F_y$$

For limiting friction $F_S = \mu_s N = 0.4N$

Resolving forces in the $+Y$ direction :

$$F \sin 30° = N - 150 \quad \text{----} \quad (1)$$

Resolving forces in the $+X$ direction :

$$F \cos 30° = 0.4N \quad \text{-----} \quad (2)$$

$$F \cos 30° = 0.4(150 + F\sin 30°)$$

$$F \cos 30° - 0.4(F \sin 30°) = 60 \quad \Rightarrow \quad F = 90.0866 \text{ lb}$$

$$a_t = F/M = 19.338 \text{ ft/s}^2$$

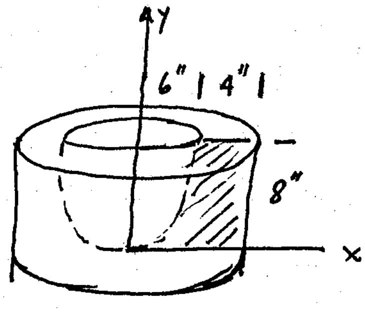
$$\alpha = \underline{9.67 \text{ rad/s}^2}$$

EGN 3321

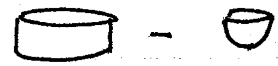
HW #15

Problems 17- 9, 11, 14, 22, 37, 41 in 3 pages

17-9



$y = \frac{2}{9}x^2$ units in inches $\quad \gamma = \frac{\text{weight}}{\text{volume}} = 150 \text{ lb/ft}^3$

solve this as 

for cylinder $I_y = \frac{1}{2} m R^2 = \frac{1}{2}\frac{W}{g} R^2 = \frac{1}{2}\frac{\gamma}{g} \text{Volume} \cdot R^2$

$= \frac{1}{2}\frac{\gamma}{g}(\pi R^2 h) R^2 = \frac{1}{2}\frac{\gamma}{g}\pi R^4 h = \frac{1}{2}\frac{150}{32.2}\cdot\pi\left(\frac{10}{12}\right)^4\left(\frac{8}{12}\right)$

$= \underline{2.3526 \text{ slug-ft}^2}$ Here inches are converted to ft.

for parabola x, y are in inches $\Rightarrow$ constant $2/9$ is in $(\text{inches})^{-1}$ $\therefore$ in feet the equation becomes $y = \frac{8}{3}x^2$. Now $dI_y = \frac{1}{2} dm R^2 = \frac{1}{2} dm\, x^2$ with $dm = \pi x^2 dy\rho$

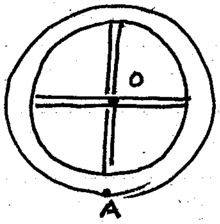
Thus $dI_y = \frac{1}{2}(\rho \pi x^2 dy) x^2 = \frac{1}{2}\rho\pi x^4 dy \qquad \rho = \frac{m}{V} = \frac{W}{Vg} = \frac{\gamma}{g}$

$\therefore \int dI_y = \int \frac{1}{2}\frac{\gamma}{g}\pi\left(\frac{3}{8}\right)^2 y^2 dy = \frac{9}{128}\frac{\gamma}{g}\pi\int_0^{8/12} y^2 dy$

$$I_y = \frac{9}{128}\frac{\gamma}{g}\pi\left.\frac{y^3}{3}\right|_0^{.667} = \underline{.1016 \text{ slug-ft}^2}$$

$$\therefore \quad I_y = 2.3526 - .1016 = \underline{\underline{2.251 \text{ slug ft}^2}}$$

17-11



Solve this by finding I_G first then $I_A = I_G + md^2$

for ring $I_G = I_o = mR^2 \quad I_A = mR^2 + mR^2 = 2mR^2$

$\underline{I_{A_{ring}}} = 2\cdot 10(.5)^2 = \underline{5 \text{ kg m}^2}$

I_G of horizontal spokes $= \frac{1}{12} m l^2 = \frac{1}{12}(4)(1)^2 = .333 \text{ kg m}^2 = I_o$

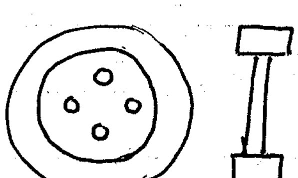
using the whole horizontal spoke $\quad I_A = I_G + mR^2 = I_G + 4(.5)^2 = 1.333 \text{ kg-m}^2$

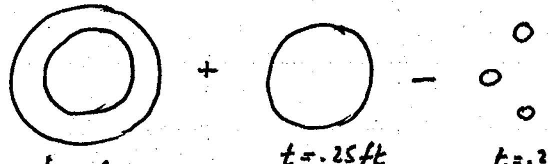
I_G for the vertical is same are for the ~~vertic~~ horizontal and I_A is also the same

$\therefore$ TOTAL $\underline{I_A \text{ spokes}} = 2(1.333) = \underline{2.667 \text{ kg-m}^2}$

Total $\underline{I_A} = 5 + 2.667 = \underline{7.667 \text{ kg-m}^2}$

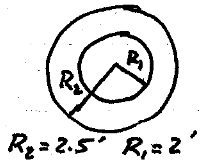
17-14



solve as 

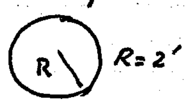
Given $\gamma = 90 \text{ lb/ft}^3 = \frac{\text{weight}}{\text{volume}} = \frac{W}{V}$

for the ring $\quad I_G = \left(\frac{1}{2} m R^2\right)_{\text{for outer}} - \left(\frac{1}{2} m R^2\right)_{\text{for inner}} \qquad m = \frac{\gamma V}{g} \qquad V = \pi R^2 t$



$$\underline{I_G} = \frac{1}{2}\frac{\gamma}{g}\pi R_2^2 t \cdot R_2^2 - \frac{1}{2}\frac{\gamma}{g}\pi R_1^2 t \cdot R_1^2 = \frac{1}{2}\frac{\gamma}{g}\pi t (R_2^4 - R_1^4) = \underline{101.254 \text{ slug ft}^2}$$

for inner plate $\quad I_G = \frac{1}{2} m R^2 = \frac{1}{2}\frac{\gamma}{g}\pi R^2 t \cdot R^2 = \frac{1}{2}\frac{\gamma}{g}\pi R^4 t = 17.562 \text{ slug ft}^2$



for the holes: each hole is equidistant from G $\therefore$ find $I_{\text{hole about its centroid}}$ then add $m(1)^2$

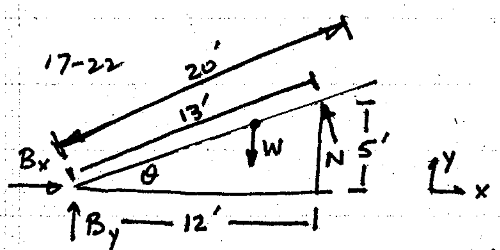
$$I_{\text{centroid}} = \frac{1}{2} m R^2 = \frac{1}{2}\pi R^2 t \cdot \frac{\gamma}{g} R^2 = \frac{1}{2}\pi \frac{\gamma}{g} R^4 t = \frac{1}{2}\pi\left(\frac{90}{32.2}\right)(.25)^4(.25) = .00136 \text{ slug ft}^2$$

add $m\ell^2 = \frac{\gamma}{g}\pi R^2 t\, \ell^2 = .1372 \text{ slug ft}^2 \quad \therefore$ total is $.13856 \text{ slug ft}^2$

$\underline{I_G \text{ for one hole is } .13856 \text{ slug ft}^2}$

$\therefore \quad \underline{\underline{I_{G_{TOT}}}} = 101.254 + 17.562 - 4(.13856) = \underline{118.26 \text{ slug ft}^2}$

17-22



$\cos\theta = 12/13 \qquad \sin\theta = 5/13$

$\overset{+}{\rightarrow} \Sigma F_x = B_x - N\sin\theta = m a_{G_x}$

$+\uparrow \Sigma F_y = -W + B_y + N\cos\theta = 0$

$+\circlearrowleft \Sigma M = I_G \alpha = 0 = N \cdot 3 - B_y \cdot 10\cos\theta + B_x \cdot 10\sin\theta$

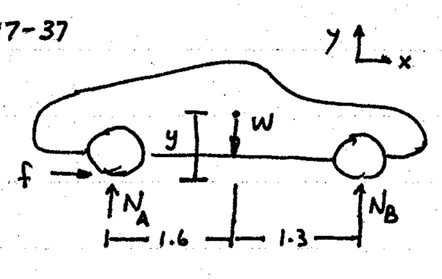
since $N = 0$ for max $a_{G_x} \Rightarrow \underline{W = B_y = 180 \text{ lb}}$

$\underline{B_y = B_x \tan\theta \Rightarrow B_x = 432 \text{ lb}}$

ALWAYS DRAW FBD ; contact forces are $\perp$ to contact surface

$\underline{a_{G_x}} = \frac{B_x}{m} = \left(\frac{W}{g}\right)^{-1} B_x = \underline{77.3 \text{ ft/s}^2}$

17-37



$\overset{+}{\rightarrow} \Sigma F_x = f = m a_{G_x}$

$+\uparrow \Sigma F_y = N_A + N_B - W = 0$

$+\circlearrowleft \Sigma M_G = f y - N_A(1.6) + N_B(1.3) = 0$

$= \mu N_A y - N_A(1.6) + N_B(1.3) = (\mu y - 1.6) N_A + N_B(1.3) = 0$

SEE EXAMPLE 17-5 AS TO THE FORCES ON THE BODY

$\therefore \quad N_A = \frac{N_B(1.3)}{1.6 - \mu y}$

unraked $y = 0.4 m \quad N_A = \frac{N_B(1.3)}{1.48}$

raked $y = .6 m \quad N_A = N_B(1.3)/1.42$

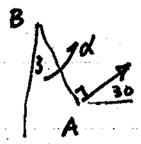
unraked $N_B = W\dfrac{(1.48)}{2.78} = 8356\ N$ $\quad N_A = 7339.9\ N$ $\quad a_G = 1.376\ m/s^2$

raked $N_B = W\dfrac{(1.42)}{2.72} = 8194 N$ $\quad N_A = 7501.8\ N$ $\quad a_G = 1.407\ m/s^2$

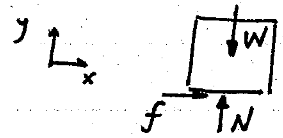
Thus raked gives a higher acceleration

17-41 The elevator is in curvilinear translation. Since it starts from rest $\omega = 0$

and $\bar{a}_A = \bar{a}_B = \bar{a}_{elevator} = \alpha r = \alpha(2') \angle 30°$ $\quad \therefore \quad \alpha \cdot 2 \left[\cos 30° \vec{i} + \sin 30° \vec{j}\right]$



This is also $\bar{a}_G$ of crate



$\stackrel{+}{\rightarrow} \Sigma F_x = f = m a_{G_x} = m 2\alpha \cos 30°$

$+\uparrow \Sigma F_y = N - W = m a_{G_y} = m \cdot 2\alpha \sin 30°$

$f = \mu N$ for slip to occur

$\therefore \quad \mu N = m a_{G_x}$ or $N = \dfrac{m}{\mu} a_{G_x}$ $\quad \therefore \quad \dfrac{m}{\mu} a_{G_x} - W = m a_{G_y}$

so $\dfrac{m}{\mu}\left[2 \cdot \alpha \cos 30°\right] - W = m\left[2\alpha \sin 30°\right]$ or

$\left[\dfrac{1}{\mu} \cdot 2\cos 30° - 2\sin 30°\right]\alpha = g$ $\qquad \alpha = 9.67\ rad/s^2$

HW #16

17-57, 60, 90, 94, 98 in 6 pages

Dynamics, 4-5-90 | Professor Levy

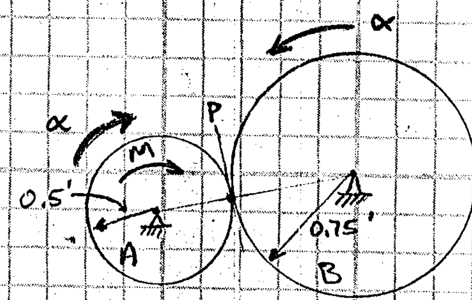
PROB 17-60

Given: $\alpha_A = 4\,rad/s^2$

$W_A = 5\,lb$, $W_B = 10\,lb$, no slipping

Det. couple M need for disk A

to have $\alpha = 4\,rad/s^2$

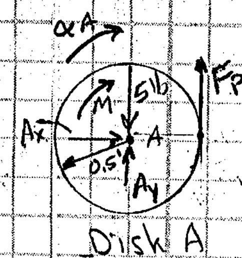


$(a_t)_A = (a_t)_B$ @ pt. P $\Rightarrow r\alpha_A = r\alpha_B$

$(0.5)4 = (0.75)\alpha_B \Rightarrow \alpha_B = 2.67\,rad/s^2$

FBD

Disk A

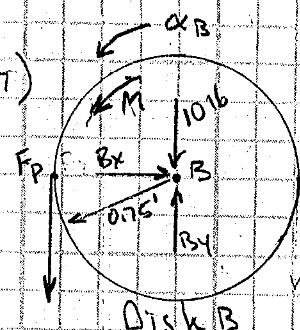


$\circlearrowleft + \Sigma M_A = I_A \alpha_A \Rightarrow M - F_P(0.5) = \frac{1}{2} m_A r_A^2 (4)$

$\Rightarrow M - F_P(0.5) = \frac{1}{2}\left(\frac{5}{32.2}\right)(0.5)^2(4)$

$\Rightarrow$ ① $M - F_P(0.5) = (77.64 \times 10^{-3})$ ft·lb

Disk B



$\circlearrowleft + \Sigma M_B = I_B \alpha_B \Rightarrow \cancel{M}^0 + F_P(0.75) = \frac{1}{2} m_B r_B^2 (2.67)$

$\Rightarrow F_P(0.75) = \frac{1}{2}\left(\frac{10}{32.2}\right)(0.75)^2(2.67)$

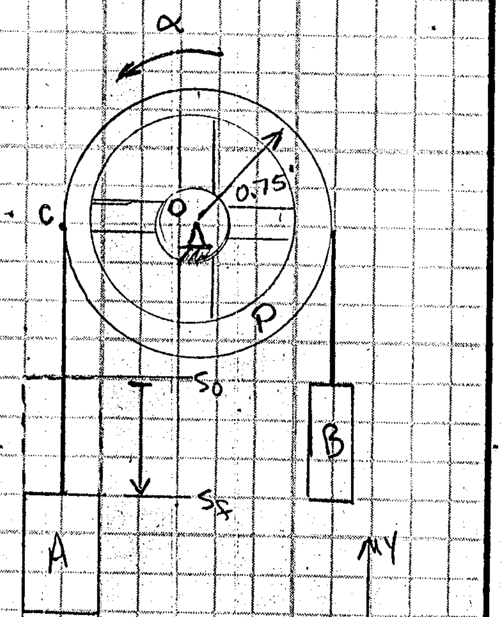
$F_P = 0.311$ lb

① $M - (0.311)(0.5) = (77.64 \times 10^{-3})$

$\boxed{M = 0.233\ \text{ft}\cdot\text{lb}}$

Dynamics, 4-5-90 | Professor Levy

PROB. 17-57



Given: $W_A = 10\text{ lb}, W_B = 5\text{ lb}$

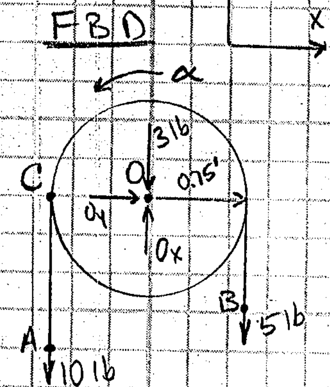
$W_P = 3\text{ lb}, k_o = 0.5\text{ ft}$,

If blocks released from rest, $(v_0 = 0)$ det. their speed just when A has descended 3 ft.

$$I_o = m_P k_o^2 = \left(\frac{3}{32.2}\right)(0.5)^2 = 23.3 \times 10^{-3} \text{ slugs} \cdot \text{ft}^2$$

$$a = \alpha r = \alpha(0.75) = a_c$$

Solution I - (system as a whole)



$$\circlearrowleft + \Sigma M_o = \Sigma (M_k)_o$$

$$10(0.75) - 5(0.75) = I_o \alpha + m_A a r + m_B a r$$

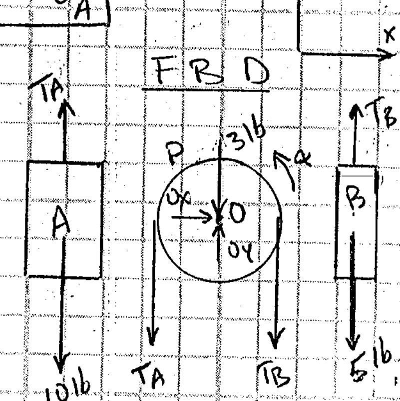
$$3.75 = (23.3 \times 10^{-3})\alpha + 0.175\alpha + 0.087\alpha$$

$$\alpha = 13.14 \text{ rad/s}^2 \circlearrowleft, \; a_c = \alpha(0.75) = 9.86 \text{ ft/s}^2 \downarrow,$$

$$v_{c_f}^2 = \cancel{v_{c_i}^2}^{\,0} + 2a_c(s_f - s_o) \qquad (v_c = v_A)$$

$$v_c^2 = 2(-9.86)(0-3) \Rightarrow \boxed{v_c = 7.69 \text{ ft/s} \downarrow = v_A}$$

Solution II - (system as seperate parts)



Block-A - n

$$\uparrow \Sigma F_y = m(a_G)_y \Rightarrow T_A - 10 = \left(\frac{10}{32.2}\right)\alpha(0.75)$$

$$T_A = 10 + 0.233\alpha$$

Pulley-P - take ΣM_o to eliminate O_x & O_y

$$\circlearrowleft + \Sigma M_o = I_o \alpha \Rightarrow T_A(0.75) - T_B(0.75) = (23.3 \times 10^{-3})\alpha$$

Block-B $\quad \Sigma F_y = m(a_G)_y \Rightarrow T_B - 5 = \left(\frac{5}{32.2}\right)\alpha(0.75) \Rightarrow T_B = 5 + 0.116\alpha$

$$\alpha = 13.14 \text{ rad/s}^2, \; a = 9.86 \text{ ft/s}^2$$

$$\therefore \boxed{v_A = 7.69 \text{ ft/s}}$$

Florida International University
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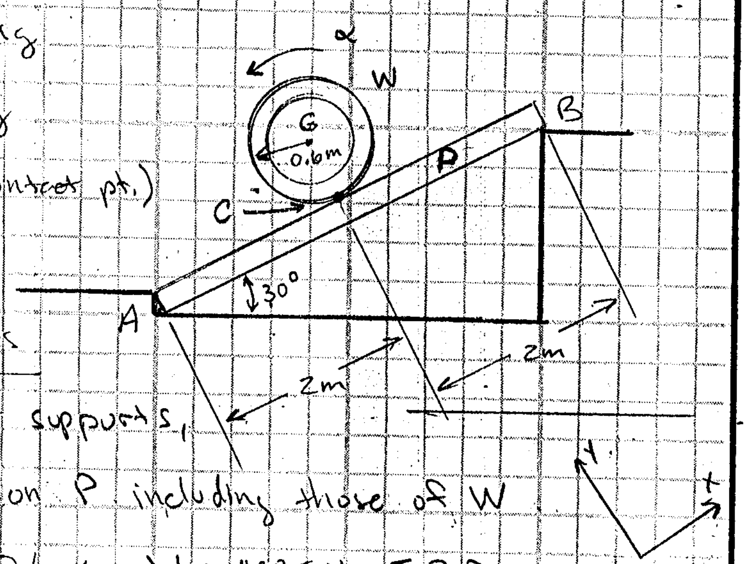
Dynamics, 4-5-90 | Professor Levy

Prob. 17-94



Given: $m_w = 50\,kg$, $m_p = 20\,kg$

$k_G = 0.4$, W rolls w/o slipping

(means some force exerted @ contact pt.)

Assume A_x, A_y, & N_B

Det. those reactions & forces

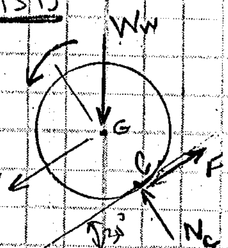
To find Rx's & $\vec{F}$'s exerted at supports,

1st need to find all $\vec{F}$'s acting on P including those of W

FOR Wheel

$\rightarrow \Sigma F_x = m(a_G)_x$, $a_G = -r\alpha = -0.6\alpha$, $W_w = 490.5\,N$, FBD

$F - W_w \sin 30 = -30\alpha \Rightarrow$ ① $F - 245.25 = 30\alpha$



$\uparrow \Sigma F_y = m(a_G)_y$, $(a_G)_y = 0$

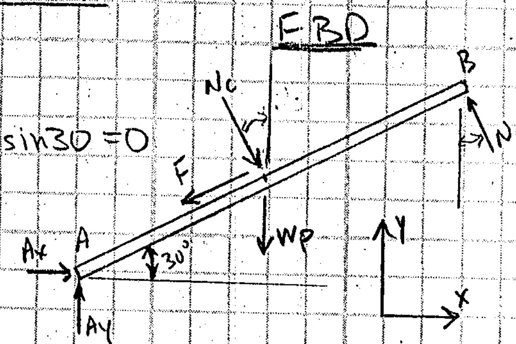
$N_c - W_w \cos 30 = 0 \Rightarrow \underline{N_c = 425\,N}$

$\Sigma M_c = \Sigma(m_k)_c \Rightarrow W_w \sin 30(0.6) = I_G\alpha + Fr$

$\Rightarrow 147.15 = mk^2\alpha + m(0.6\alpha)(0.6) \Rightarrow 147.15 = 26\alpha \Rightarrow \alpha = 5.66\,\frac{rad}{s^2}$

① $F - 245.25 = -30(5.66) \Rightarrow \underline{F = 75.45\,N}$

For Plank



$\rightarrow \Sigma F_x = 0 \Rightarrow A_x + N_c \sin 30 - F\cos 30 - N_B \sin 30 = 0$

② $A_x + 147.15 - N_B \sin 30 = 0$

$\uparrow \Sigma F_y = 0$

$\Rightarrow A_y + N_B \cos 30 - N_c \cos 30 - W_p - F\sin 30 = 0$

③ $A_y + N_B \cos 30 = 602$

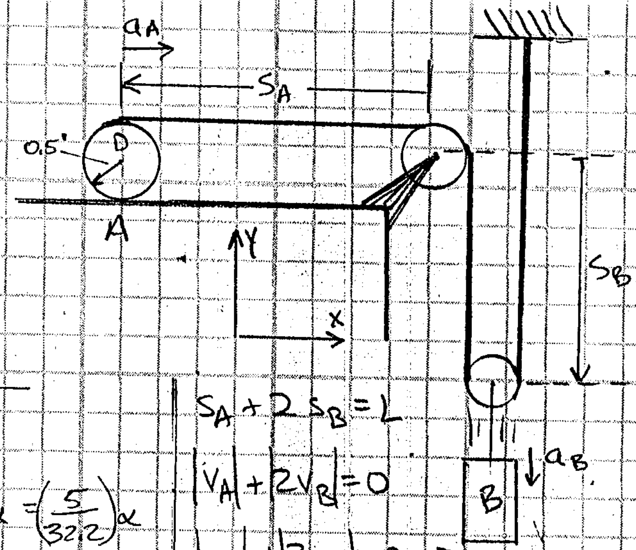
$\Sigma M_A = 0 \quad W_P(2\cos 30) + N_c(2) - N_B(4) = 0 \Rightarrow \boxed{N_B = 297.46\,N}$ 60°

③ $A_y + (297.46)\cos 30 = 602 \Rightarrow \boxed{A_y = 344.4\,N \uparrow}$

② $A_x + 147.15 - (297.46)\sin 30 = 0 \Rightarrow \boxed{A_x = 1.57\,N \rightarrow}$

Dynamics, 4-5-90 | Professor Levy

Prob. 17-90



Given $W_D = 10\,lb$, $W_B = 4\,lb$

$v_i = 0$

Det. v_B @ $t = 3\,s$

$\mu_s = 0.2$, $\mu_k = 0.15$ @ A

$S_A + 2S_B = L$

$|v_A| + |2v_B| = 0$

$|a_A| + |2a_B| = 0 \Rightarrow a_A = 2a_B$
(different directions)

For Disc $\quad a_G = r\alpha$

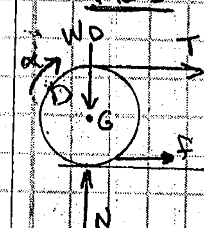
$\rightarrow \Sigma F_x = m(a_G)_x \Rightarrow$ ① $f + T = \left(\frac{10}{32.2}\right)(0.5)\alpha = \left(\frac{5}{32.2}\right)\alpha$

$\uparrow \Sigma F_y = m(a_G)_y \Rightarrow$ ② $N - W_D = 0 \Rightarrow N = 10\,lbs$

$\circlearrowleft \Sigma M_G = I_G\alpha \Rightarrow T(0.5) - f(0.5) = \frac{1}{2}\left(\frac{10}{32.2}\right)(0.5)^2\alpha$

$\Rightarrow 0.5(T - f) = \left(\frac{1.25}{32.2}\right)\alpha \Rightarrow$ ③ $T - f = \left(\frac{2.5}{32.2}\right)\alpha$

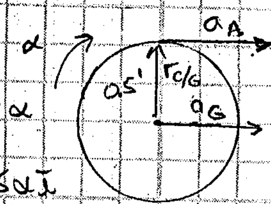
FBD



For Block

$\uparrow \Sigma F_y = m(a_B)_y \Rightarrow 2T - W_B = -\left(\frac{4}{32.2}\right)a_B$

④ $\Rightarrow 2T - 4 = \frac{-4}{32.2}a_B \Rightarrow 2T = 4 - \frac{4}{32.2}a_B$



$a_G = r\alpha$

$a_G = 0.5\alpha\bar{i}$

① & ② $\quad T + f = \frac{5}{32.2}\alpha + T - f = \frac{2.5}{32.2}\alpha$

⑤ $\Rightarrow 2T = \frac{7.5}{32.2}\alpha \Rightarrow 2T = \frac{15}{32.2}a_B$

$a_A = a_G + a_{c/G}$

$a_A\bar{i} = 0.5\alpha\bar{i} + (-\alpha\bar{k} \times .5\bar{j})$

$a_A\bar{i} = 0.5\alpha\bar{i} + 0.5\alpha\bar{i}$

$a_A = \alpha$; if $a_A = 2a_B$

$\Rightarrow \alpha = 2a_B$

④ & ⑤ $\quad 4 - \frac{4}{32.2}a_B = \frac{15}{32.2}a_B \Rightarrow 4 = \frac{19}{32.2}a_B$

$a_B = 6.78\ ft/s^2 \downarrow$

$v_f = \cancel{v_0}^{\,0} + at$

$v_f = 6.78(3)$

$\boxed{v_f = 20.34\ ft/s \downarrow}$

Dynamics, 4-5-90 | Professor Levy

Prob. 17-98 cont.

So far, we have:

① $6(g-(a_G)_y)\cos\theta - 6(a_G)_x\sin\theta = L\alpha$

2a $(a_G)_x = a_A - \frac{L}{2}\alpha\sin\theta$, 2b $(a_G)_y = \frac{L}{2}\alpha\cos\theta$

③ $a_A = L\alpha\sin\theta$

$3 \to 2_a = ④$ $(a_G)_x = L\alpha\sin\theta - \frac{L}{2}\alpha\sin\theta = \frac{L}{2}\alpha\sin\theta$

2b & ④ → ① $\Rightarrow 6\left(g-\frac{L}{2}\alpha\cos\theta\right)\cos\theta - 6\left(\frac{L}{2}\alpha\sin\theta\right)\sin\theta = L\alpha$

$6g\cos\theta - 3L\alpha\cos^2\theta - 3L\alpha\sin^2\theta = L\alpha$

$6g\cos\theta - L\alpha = 3L\alpha(\underbrace{\cos^2\theta + \sin^2\theta}_{1})$

$4L\alpha = 6g\cos\theta$

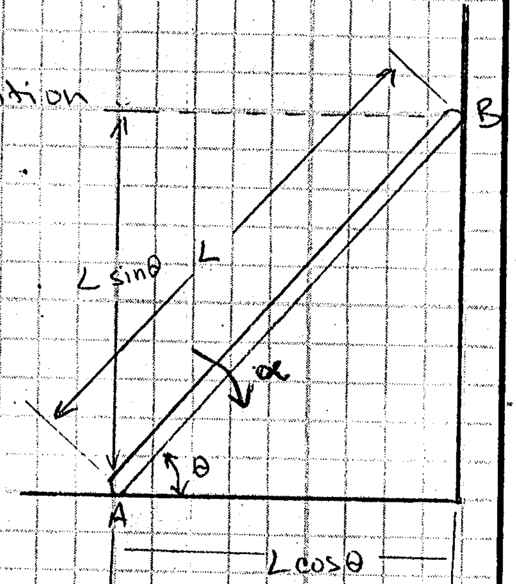
$$\boxed{\alpha = \frac{3}{2}\frac{g}{L}\cos\theta}$$

Dynamics, 4-5-90 | Professor Levy

PROB. 17-98

Given: mass = m, no friction

Det α: when release from rest from position θ and allowed to slide downward



from rest $\omega = 0$

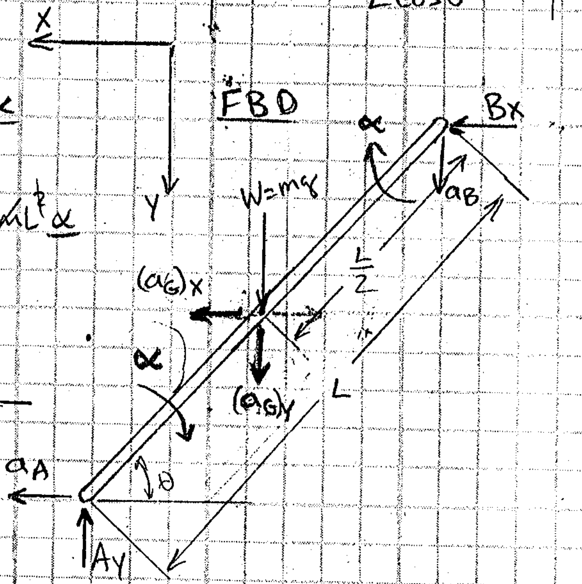
$\leftarrow \sum F_x = m(a_G)_x \Rightarrow B_x = m(a_G)_x$

$\downarrow \sum F_y = m(a_G)_y \Rightarrow W - A_y = m(a_G)_y$

$\circlearrowright \sum M_G = I_G \alpha$

$\Rightarrow A_y\left(\frac{L}{2}\right)\cos\theta - B_x\left(\frac{L}{2}\right)\sin\theta = \frac{1}{12} m L^2 \alpha$

$\Rightarrow (W - m(a_G)_y)\left(\frac{L}{2}\right)\cos\theta - m(a_G)_x\left(\frac{L}{2}\right)\sin\theta = \frac{1}{12} m L^2 \alpha$



$\Rightarrow 6(\cancel{m}g - \cancel{m}(a_G)_y)\cancel{L}\cos\theta - 6\cancel{m}(a_G)_x \cancel{L}\sin\theta = \cancel{m}L^{\cancel{2}}\alpha$

$\Rightarrow 6(g - (a_G)_y)\cos\theta - 6(a_G)_x \sin\theta = L\alpha$

$a_G = a_A + a_{G/A}$, $\omega = 0 \Rightarrow (a_{G/A})_n = 0$

$\alpha = -\alpha \bar{k}$, $r_{G/A} = -\frac{L}{2}\cos\theta\bar{i} - \frac{L}{2}\sin\theta\bar{j}$

$a_{Gx}\bar{i} + a_{Gy}\bar{j} = a_A\bar{i} + (\alpha \times r_{G/A})$

$(a_{Gx})\bar{i} + (a_G)_y\bar{j} = a_A\bar{i} + \frac{L}{2}\alpha\cos\theta\bar{j} - \frac{L}{2}\alpha\sin\theta\bar{i}$

Eq. Coeff.

$(a_G)_x = a_A - \frac{L}{2}\alpha\sin\theta$, $(a_G)_y = \frac{L}{2}\alpha\cos\theta$

$a_B = a_A + a_{B/A}$, $\omega = 0 \Rightarrow (a_{B/A})_n = 0$, $\alpha = -\alpha\bar{k}$, $r_{B/A} = -L\cos\theta\bar{i} - L\sin\theta\bar{j}$

$a_B\bar{j} = a_A\bar{i} + (\alpha \times r_{B/A}) = a_A\bar{i} + L\alpha\cos\theta\bar{j} - L\alpha\sin\theta\bar{i}$

Eq. Coeff.

$a_B = L\alpha\cos\theta$

$0 = a_A - L\alpha\sin\theta \Rightarrow a_A = L\alpha\sin\theta$