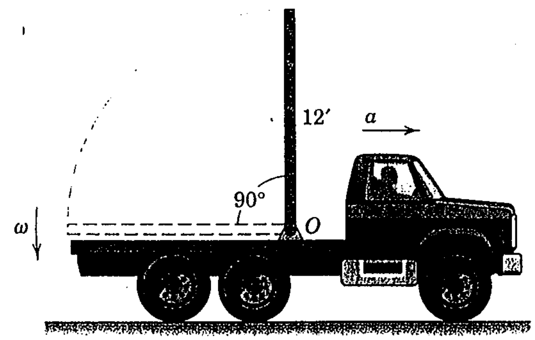
EGN 3321

Dynamics

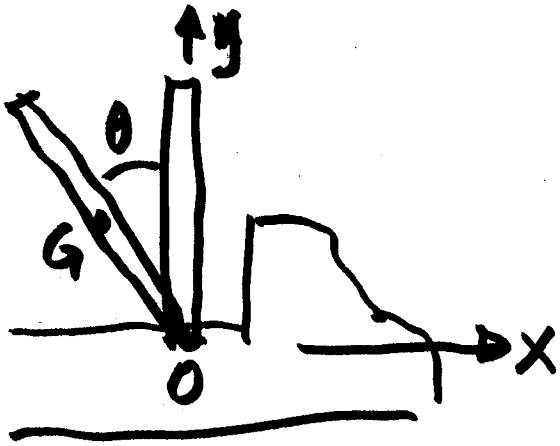
4/11/05

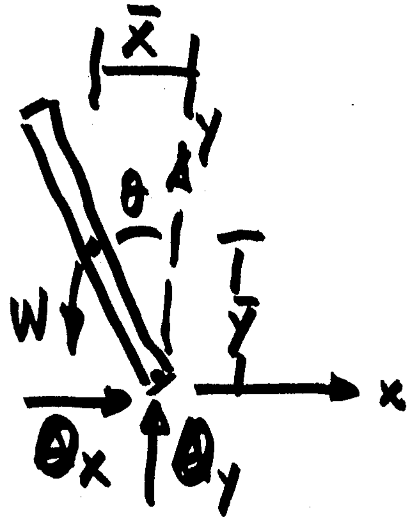
6/103 The uniform 12-ft pole is hinged to the truck bed and released from the vertical position as the truck starts from rest with an acceleration of 3 ft/sec^2. If the acceleration remains constant during the motion of the pole, calculate the angular velocity ω of the pole as it reaches the horizontal position.

Ans. $\omega = 2.97$ rad/sec



Problem 6/103





$$\Sigma F_x = m a_{G_x} = O_x = m a_{G_x}$$

$$\Sigma F_y = m a_{G_y} = O_y - W = m a_{G_y}$$

$$+\circlearrowleft \Sigma M_G = I_G \alpha \quad \text{or} \quad +\circlearrowleft \Sigma M_O = \bar{x}\, m a_{O_y} + \bar{y}\, m a_{O_x} + I_O \alpha$$

$$\bar{x} = -6\sin\theta \qquad a_{O_x} = 3\ \text{ft/s}^2$$

$$\bar{y} = 6\cos\theta \qquad a_{O_y} = 0\ \text{ft/s}^2$$

$$W \cdot 6\sin\theta = \cancel{-6\sin\theta\, m \cdot 0} - \cancel{6\cos\theta\, m\, 3} + \frac{1}{3} m l^2 \alpha$$

$$6g\sin\theta + 18\cos\theta = \frac{144}{3}\alpha = 48\alpha$$

$$\alpha = \frac{1}{8}\left(g\sin\theta + 3\cos\theta\right)$$

$$\alpha\, d\theta = \omega\, d\omega \qquad (a\, ds = v\, dv)$$

$$\int_0^{\pi/2} \frac{6g\sin\theta + 18\cos\theta}{48}\, d\theta = \int_0^{\omega} \bar{\omega}\, d\bar{\omega} = \frac{\omega^2}{2} - 0$$

$$-\frac{1}{8} g\cos\theta \Big|_0^{\pi/2} + \frac{3}{8}\sin\theta \Big|_0^{\pi/2} = \frac{1}{8}g + \frac{3}{8} = \frac{\omega^2}{2} \qquad \omega = 2.97\ \frac{\text{rad}}{\text{s}}$$

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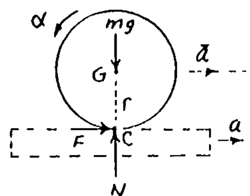
$\alpha = \frac{a_{c/G}}{r} = \frac{a - \bar{a}}{r}$



$\Sigma F_x = m a_x$; $F = m\bar{a}$

$\Sigma M_G = \bar{I}\alpha$; $Fr = \frac{1}{2} m r^2 \frac{a - \bar{a}}{r}$

solve & get $\bar{a} = \frac{1}{3} a$

$a_{G/c} = a - \bar{a} = \frac{2}{3} a$ to the left

Rel. to truck, $s = \frac{1}{2} a_{rel} t^2$, $d = \frac{1}{2}\left(\frac{2}{3} a\right) t^2$, $t^2 = \frac{3d}{a}$

Truck $s = \frac{1}{2} a t^2$, $s = \frac{1}{2} a \frac{3d}{a} = \underline{\frac{3d}{2}}$

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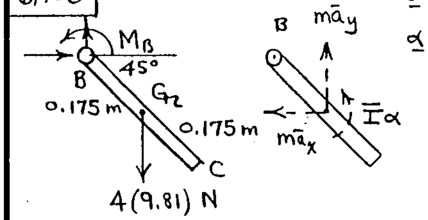
$\underline{\omega}_{BC} = \underline{\omega}_{AB} = 2\underline{k}$ rad/s

$\underline{\alpha}_{BC} = \underline{\alpha}_{AB} = 4\underline{k}$ rad/s²



$\underline{a}_{G_2} = \underline{\alpha} \times \underline{r}_{AG_2} - \omega^2 \underline{r}_{AG_2} = 4\underline{k} \times [(0.7 + 0.175)\cos 45° \underline{i} + (0.7 - 0.175)\sin 45° \underline{j}] - 2^2 [(0.7 + 0.175)\cos 45° \underline{i} + (0.7 - 0.175)\sin 45° \underline{j}] = -3.96\underline{i} + 0.990\underline{j}$ m/s²

$\Sigma M_B = \bar{I}\alpha + \Sigma m\bar{a}d$: $M_B - 4(9.81)(0.175 \sin 45°) = \frac{1}{12}(4)(0.35)^2(4) + 4(0.990)(0.175 \cos 45°) - 4(3.96)(0.175 \sin 45°)$, $\underline{M_B = 3.55 \text{ N·m}}$ (CCW)

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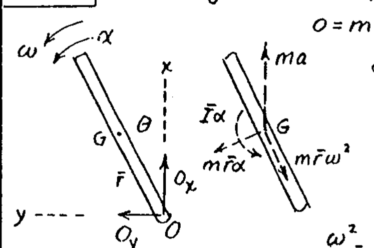
$\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d$; $\bar{I} = m(k_O^2 - \bar{r}^2)$

$0 = m(k_O^2 - \bar{r}^2)\alpha + m\bar{r}^2\alpha - m a \bar{r} \sin\theta$



$\alpha = \frac{a \bar{r} \sin\theta}{k_O^2}$

$\int_0^{\omega} \dot{\theta}\, d\dot{\theta} = \int_0^{\pi/2} \alpha\, d\theta$

$\frac{\omega^2}{2} = \frac{a\bar{r}}{k_O^2}(-\cos\theta)\Big|_0^{\pi/2} = \frac{a\bar{r}}{k_O^2}$

$\underline{\omega = \frac{1}{k_O}\sqrt{2a\bar{r}}}$

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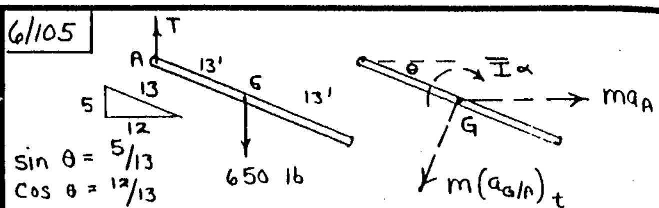

$\sin\theta = 5/13$

$\cos\theta = 12/13$

$(a_{G/A})_n = 0$ because $\omega = 0$.

Point A has no velocity, so a_A is horizontal.

$m(a_{G/A})_t = m \frac{L}{2}\alpha = \frac{650}{32.2}(13)\alpha = 262\alpha$

$\Sigma F_x = m\bar{a}_x$: $0 = m(13\alpha)\sin\theta - m a_A$, $a_A = 5\alpha$

$\Sigma F_y = m\bar{a}_y$: $650 - T = (262\alpha)\cos\theta$, $T = 650 - 242\alpha$

$\Sigma M_G = \bar{I}\alpha$: $12T = \frac{1}{12}\frac{650}{32.2}(26)^2\alpha$, $T = 94.8\alpha$

Solve for T & α: $\underline{T = 182.8 \text{ lb}}$

$\alpha = 1.929$ rad/sec²

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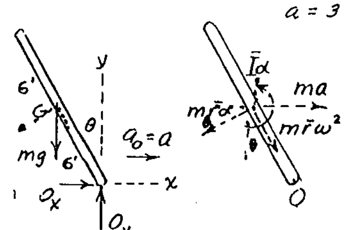
$m\underline{\bar{a}} = m\underline{a}_O + m\underline{a}_{G/O} = m\underline{a} + m\bar{r}\omega^2 + m\bar{r}\alpha$

$a = 3$ ft/sec², $\bar{r} = 6$ ft



$a_G = a_O + a_{G/O}$

$= 3\underline{i} - \cancel{r}\omega^2 + \alpha r + r\omega^2(\sin\theta \underline{i} - \cos\theta \underline{j}) + \alpha r(-\cos\theta \underline{i} + \sin\theta \underline{j})$

$a_{Gx} = 3 + r\omega^2 \sin\theta - \alpha r \cos\theta$

$\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d$;

$mg(6\sin\theta) = \frac{1}{12} m(12^2)\alpha + m(6\alpha)(6) - m(3)6\cos\theta$

$\alpha = \frac{1}{8}(g \sin\theta + 3\cos\theta)$

$\int \omega\, d\omega = \int \alpha\, d\theta$; $\int_0^{\omega} \omega\, d\omega = \frac{1}{8}\int_0^{\pi/2}(g \sin\theta + 3\cos\theta)d\theta$

$\omega^2 = \frac{1}{4}\left[-32.2\cos\theta + 3\sin\theta\right]_0^{\pi/2} = \frac{1}{4}[32.2 + 3] = \frac{35.2}{4}$

$\omega = \frac{1}{2}\sqrt{35.2} = \underline{2.97 \text{ rad/sec}}$

$r_{G/O} \times m a_G$

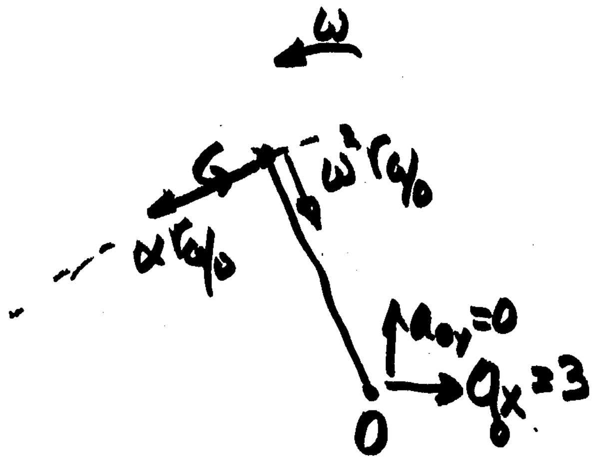
$\Sigma M_O = \bar{x} m a_{Gy} - \bar{y} m a_{Gx} + I_G \alpha$

$= \frac{1}{3} m l^2 \alpha$

$\cancel{m} g 6 \sin\theta = 0 - 6\cos\theta \cdot \cancel{m} \cdot 3 + \frac{1}{3} m 12^2 \alpha$

$g \frac{6 \sin\theta}{48} + \frac{18\cos\theta}{48} = \alpha$

$\frac{1}{8}(g \cos\theta + 3\cos\theta) = \alpha$

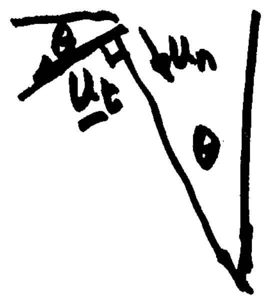


$$\underline{a}_G = \underline{a}_O + \underline{a}_{G/O}$$

$$a_{Gx}\underline{i} + a_{Gy}\underline{j} = 3\underline{i} + \alpha r_{G/O}\underline{u}_t + \omega^2 r_{G/O}\underline{u}_n$$

$$\underline{u}_t = -\cos\theta\,\underline{i} - \sin\theta\,\underline{j}$$

$$\underline{u}_n = \sin\theta\,\underline{i} - \cos\theta\,\underline{j}$$



$$a_{Gx} = 3 - \alpha r_{G/O}\cos\theta + \omega^2 r_{G/O}\sin\theta$$

$$a_{Gy} = -\alpha r_{G/O}\sin\theta + \omega^2 r_{G/O}(-\cos\theta)$$

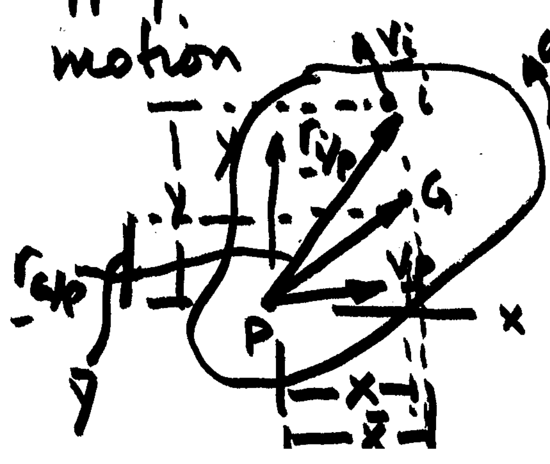
$$r_{G/O} = 6 \qquad \alpha = \frac{1}{8}(g\sin\theta + 3\cos\theta)$$

$$\omega^2 = -\frac{1}{4}g[\cos\theta - 1] + \frac{6}{8}\sin\theta$$

Chapter 18 – work & energy for a rigid body

Need to know $\bar{V}$, Δs, $\underline{F}$

Apply same ideas to a rigid body in planar motion



$$dT_i = \frac{1}{2}dm_i V_i^2$$

$$\int dT_i = T_{TOT}$$

$$V_i^2 = \underline{V}_i \cdot \underline{V}_i = (\underline{V}_P + \omega \times \underline{r}_{i/P})(\underline{V}_P + \omega \times \underline{r}_{i/P})$$

$$V_i^2 = V_p^2 - 2\omega\,(y\,V_{P_x} - x\,V_{P_y}) + \omega^2 r_{i/p}^2$$

$$\int \tfrac{1}{2} dm_i\, V_i^2 = \int \tfrac{1}{2} dm_i\, V_p^2 - \int \tfrac{1}{2} dm_i\, \omega (y\,V_{P_x} - x\,V_{P_y}) + \int \tfrac{1}{2} dm_i\, \omega^2 r_{i/p}^2$$

$$= \frac{V_p^2}{2} m - \tfrac{1}{2}\omega V_{P_x} m \bar{y} + \tfrac{1}{2}\omega V_{P_y} m \bar{x} + \tfrac{1}{2}\omega^2 I_p$$

$$T_{TOT} = \underline{\frac{m}{2} V_p^2} + \tfrac{1}{2} m\omega \left[\bar{x} V_{P_y} - \bar{y} V_{P_x} \right] + \tfrac{1}{2} I_p \omega^2$$

IF PT "p" IS THE "G"

$$T_{TOT} = \frac{m V_G^2}{2} + \tfrac{1}{2} I_G \omega^2$$

IF "p" IS FIXED

$$T_{TOT} = \tfrac{1}{2} I_p \omega^2$$

IF BODY HAS NO ROTATION

$$T_{TOT} = \frac{m V_p^2}{2} = \frac{m V_G^2}{2}$$

IF ONLY ROTATION ABOUT "p"

$$T_{TOT} = \frac{1}{2} I_p \omega^2 = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega^2$$

IN GENERAL MOTION THEN

$$T_{TOT} = \frac{m V_G^2}{2} + \frac{I_G \omega^2}{2}$$

$$= \frac{m V_p^2}{2} + \frac{I_p \omega^2}{2} + m\omega \left[\bar{x} V_{P_y} - \bar{y} V_{P_x} \right]$$

Example 18–1

PG 440 in your book

S_E

S_B

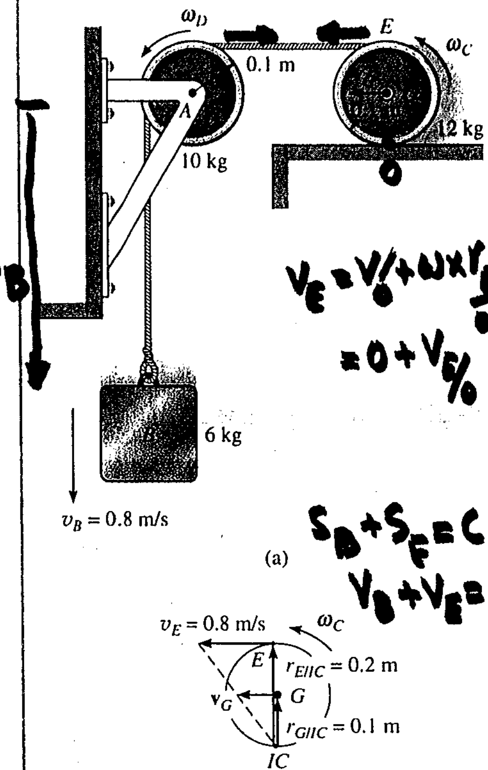


Fig. 18–5

$V_E = V_{/} + \omega \times r_{s}$

$= 0 + V_{E/O}$

$S_B + S_E = C$

$V_B + V_E = 0$

The system of three elements shown in Fig. 18–5*a* consists of a 6-kg block *B*, a 10-kg disk *D*, and a 12-kg cylinder *C*. A continuous cord of negligible mass is wrapped around the cylinder, passes over the disk, and is then attached to the block. If the block is moving downward with a speed of 0.8 m/s and the cylinder rolls without slipping, determine the total kinetic energy of the system at this instant.

SOLUTION

By inspection, the block is translating, the disk rotates about a fixed axis, and the cylinder has general plane motion. Hence, in order to compute the kinetic energy of the disk and cylinder, it is first necessary to determine ω_D, ω_C, and v_G, Fig. 18–5*a*. From the *kinematics* of the disk,

$$v_B = r_D\omega_D \qquad 0.8 \text{ m/s} = (0.1 \text{ m})\omega_D \qquad \omega_D = 8 \text{ rad/s}$$

Since the cylinder rolls without slipping, the instantaneous center of zero velocity is at the point of contact with the ground, Fig. 18–5*b*, hence,

$$v_E = r_{E/IC}\omega_C \qquad 0.8 \text{ m/s} = (0.2 \text{ m})\omega_C \qquad \omega_C = 4 \text{ rad/s}$$

$$v_G = r_{G/IC}\omega_C \qquad v_G = (0.1 \text{ m})(4 \text{ rad/s}) = 0.4 \text{ m/s}$$

The kinetic energy of the block is

$$T_B = \tfrac{1}{2}m_B v_B^2 = \tfrac{1}{2}(6 \text{ kg})(0.8 \text{ m/s})^2 = 1.92 \text{ J}$$

The kinetic energy of the disk is

$$T_D = \tfrac{1}{2} I_D\omega_D^2 = \tfrac{1}{2}(\tfrac{1}{2}m_D r_D^2)\omega_D^2$$
$$= \tfrac{1}{2}[\tfrac{1}{2}(10 \text{ kg})(0.1 \text{ m})^2](8 \text{ rad/s})^2 = 1.60 \text{ J}$$

Finally, the kinetic energy of the cylinder is

$$T_C = \tfrac{1}{2}mv_G^2 + \tfrac{1}{2} I_G\omega_C^2 = \tfrac{1}{2}mv_G^2 + \tfrac{1}{2}(\tfrac{1}{2}m_C r_C^2)\omega_C^2$$
$$= \tfrac{1}{2}(12 \text{ kg})(0.4 \text{ m/s})^2 + \tfrac{1}{2}[\tfrac{1}{2}(12 \text{ kg})(0.1 \text{ m})^2](4 \text{ rad/s})^2 = 1.44 \text{ J}$$

The total kinetic energy of the system is therefore

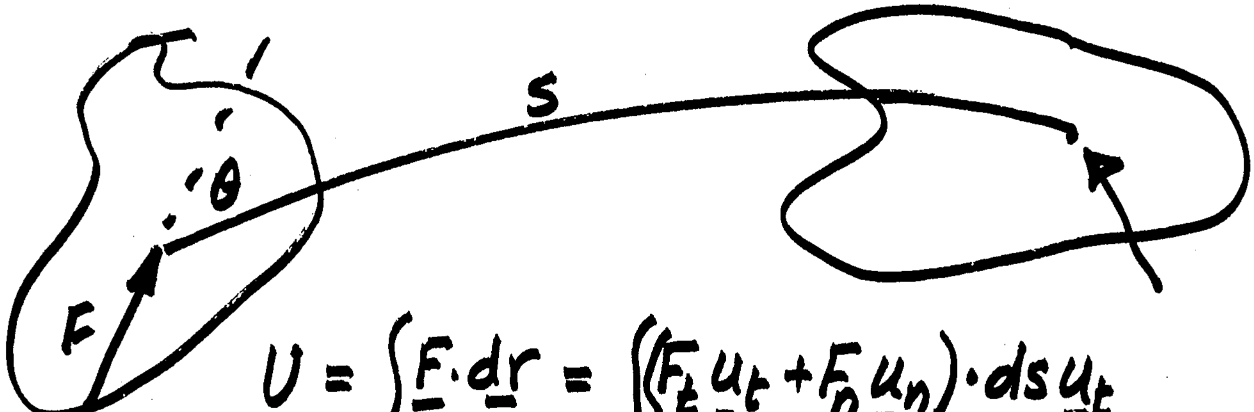
$$T = T_B + T_D + T_C$$
$$= 1.92 \text{ J} + 1.60 \text{ J} + 1.44 \text{ J} = 4.96 \text{ J} \qquad \textit{Ans.}$$

FOR A SYSTEM OF RIGID BODIES

SINCE KE IS A SCALAR : FOR A SYSTEM OF CONNECTED RIGID BODIES, TOTAL $KE = \sum_i KE_i$

$$\int \underline{F} \cdot d\underline{r} = \int \underline{F}_{external} \cdot d\underline{r} + \int \underline{F}_{internal} \cdot d\underline{r}$$

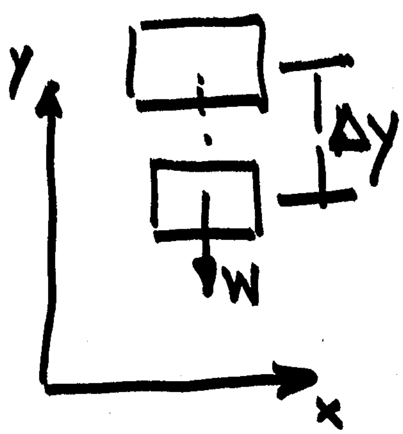
WORK & ENERGY FOR RIGID BODIES



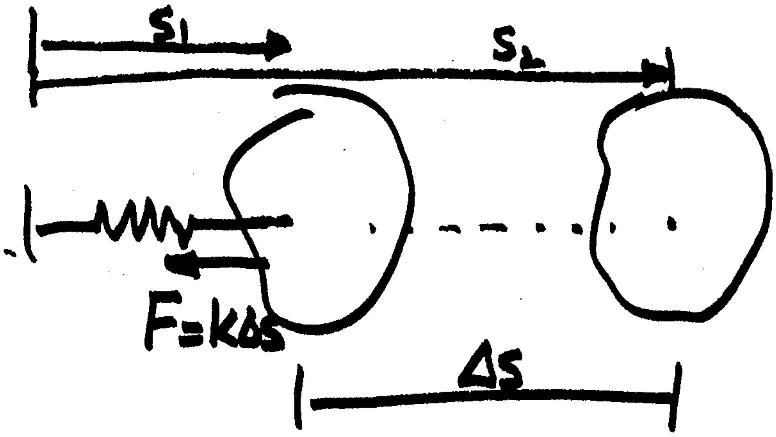
$$U = \int \underline{F} \cdot d\underline{r} = \int (F_t \underline{u}_t + F_n \underline{u}_n) \cdot ds\, \underline{u}_t$$

$$= \int F_t\, ds = \int F \cos\theta\, ds$$

WEIGHT

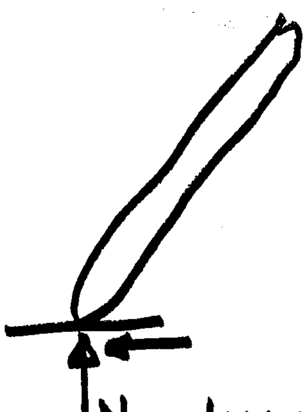


$$U = -W\Delta y$$

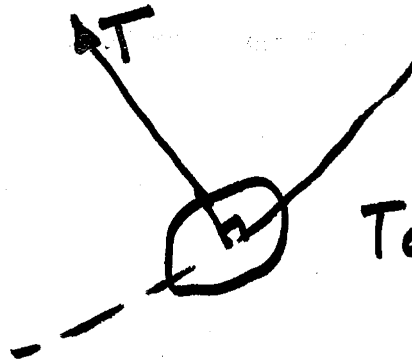


$$U = \frac{1}{2}\left(ks_1^2 - ks_2^2\right)$$
$$= -\frac{1}{2}\left(ks_2^2 - ks_1^2\right)$$

- Forces acting $\perp$ to path do no work

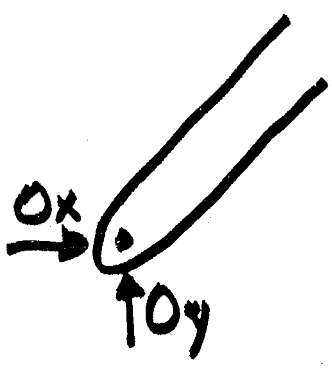


N - does no work

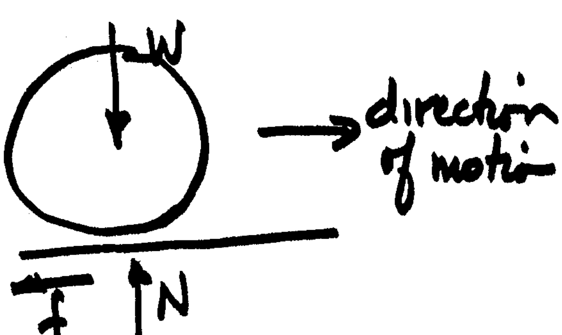


T does no work.

- F acts at a fixed pt. does no work ($\Delta s = 0$)



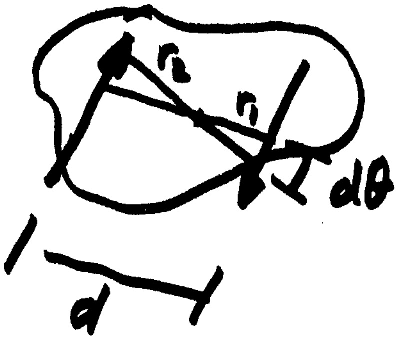
Ox, Oy at pinned pt. O does no work



if rolling & no slip f does no work

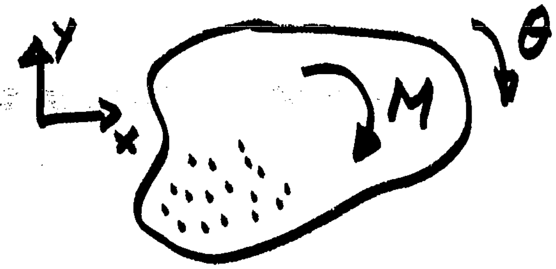
if body slips f does work.

work of a couple



- no work in translation
- under rotation

$$dU = M\,d\theta \qquad (\underline{F}\cdot d\underline{r})$$
$$= P\cdot r_1 d\theta + P\cdot r_2 d\theta$$
$$= P(r_1+r_2)\,d\theta = Pd\,d\theta$$
$$= M d\theta$$



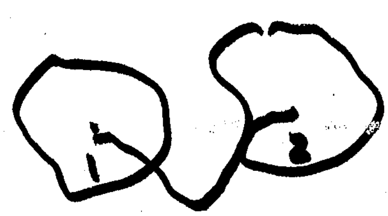
$$dU > 0$$
$$\underline{M} = -M\underline{k}$$
$$\underline{\theta} = -\theta\underline{k}$$
$$\underline{M}\cdot d\underline{\theta}$$

IF $M = \text{const}$ $\qquad U = M\Delta\theta = M(\theta_2 - \theta_1)$

Principle of work & energy still holds

$$T_i + \sum U_{i-f} = T_f \qquad (T_1 + \sum U_{1-2} = T_2)$$

$$\sum U_{1-2} = \int \underline{F}_{external}\cdot d\underline{r} + \cancel{\int \underline{F}_{internal}\cdot d\underline{r}}$$



Total ~~internal~~ initial translational + rotational KE + work done by external forces & moments = total final translational + Rotational KE.

$$T_1 = \left(\sum \tfrac{1}{2} m_i \underset{2}{\underline{v}_{G_i}^2} + \sum \tfrac{1}{2} I_{G_i} \omega_i^2\right)_1$$

$$T_2 = \left(\sum \tfrac{1}{2} m_i \underset{2}{\underline{v}_{G_i}^2} + \sum \tfrac{1}{2} I_{G_i} \omega_i^2\right)_2$$

$$\sum U_{1-2} = \sum \int \left(\underline{F}_{external} \cdot d\underline{r}\right)_i$$