FLORIDA INTERNATIONAL UNIVERSITY
Mechanical Engineering Department

Summer B 1987 | Dynamics | EGN 2321

Textbook: **Engineering Mechanics, Volume 2—Dynamics, 2nd Edition**
J.L. Meriam and L.G. Kraige

Session #			
1	Chapter 2, Section	1,2	
2		3,4	
3		5,6	
4		8-10	
5	Review Chapter 2		quiz*****
6	Chapter 3, Section	1-4	
7		5	
8		6,7	quiz*****
9		8,9	
10		10,11	
11		13,14 and Review Chapter 3	
12	Chapter 4, Section	1-3	quiz*****
13		4,5	
14	Review Chapters 3,4		
15	Chapter 5, Section	1-3	
16		4,5	quiz*****
17		6,7	
18	Review Chapter 5		
19	Chapter 6, Section	1,2	
20		3-5	quiz*****
21	Review Chapters 5,6		
22		6	
23		7-9	quiz*****
24	Review		
25	FINAL EXAM		

Grade will be determined on the basis of 6 Quizzes 11 % each
FINAL 34 %

quizzes will be at intervals of approximately EACH week

Grading Scheme:	90 and above	A	77 - 79	B-	60 - 62	D
	87 - 89	A-	74 - 76	C+	Below 60	F
	83 - 86	B+	70 - 73	C		
	80 - 82	B	67 - 69	C-		
			63 - 66	D+		

This is a preliminary syllabus subject to change

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6	Chapter 3, Section	1-4
7		5
8	EXAM #1 CHAPTER 2	
9		6,7
10		8,9
11		10,11
12		13,14 and Review Chapter 3
13	Chapter 4, Section	1-3
14		4,5
15	Review Chapters 3,4	
16	Chapter 5, Section	1-3
17	EXAM #2 CHAPTER 3,4	
18		4,5
19		6,7
20	Review Chapter 5	
21	Chapter 6, Section	1,2
22		3-5
23	Review Chapters 5,6	
24	EXAM #3 CHAPTER 5,6	
25		6
26		7-9
27	Review	
28		
29	FINAL EXAM	

Grade will be determined on the basis of

HW	10 %
Exam #1	15 %
Exam #2	20 %
Exam #3	20 %
FINAL	35 %

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90 and above	A	77 - 79	B-	60 - 62	D
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Session #				
1	Chapter 2, Section	1,2		29 Jun
2		3,4		30 Jun
3		5,6		5 Jul
4		8-10	4	6
5	Review Chapter 2		quiz*****	7
6	Chapter 3, Section	1-4		11
7		5	3	12
8		6,7	quiz*****	13
9		8,9		14
10	Ch. 4 Sect 1-3	~~10,11~~	4	18
11	Sect 4-5	~~13,14 and Review Chapter 3~~		19
12	Chapter 3, Section	~~1-3~~ 10,11	quiz*****	20
13		~~4,5~~ 13,14		21
14	Review Chapters 3,4			25
15	Chapter 5, Section	1-3	4	26
16		4,5	quiz*****	27
17		6,7		28
18	Review Chapter 5			1 Aug
19	Chapter 6, Section	1,2	4	2
20		3-5	quiz*****	3
21	Review Chapters 5,6			4
22		6	3	8
23		7-9	quiz*****	9
24	Review			10
25	FINAL EXAM			11

Grade will be determined on the basis of 6 Quizzes 11 % each
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		63 - 66	D+		

is is a preliminary syllabus subject to change

PURPOSE OF STUDYING DYNAMICS

to develop the capacity to predict the effects of force & motion while carrying out the design fn of engineering

Dynamics
- kinematics - study of motion of body irrespective to forces acting on body
- kinetics - study of forces on body due to changes in its motion

- PARTICLE DYNAMICS first Then rigid body motion

  ASSUMPTIONS
  - flat earth
  - char length << radius of curvature
  - measured wrt fixed coord system
  -

- SEVERAL WAYS TO MEASURE — spherical, cartesian, cylindr
  - motion in space
  - motion in plane
  - rectilinear motion

2/3 — $v_f^2 = v_i^2 + 2as.$

$= (3)^2 + 2 \cdot \frac{1}{6} 9.81 (6)$

$= 9 + 3.27(6) = 9 + 19.62 = 28.62$

$v_f = \sqrt{28.62} = 5.35$ m/s

2/32 $a_x - 4 = \frac{2 m/s}{80 mm} (x - 120)^{mm}$

$a_x = 25x + 1000$ mm/s

$v\,dv = a\,dx$

$\frac{v^2}{2} = 25\frac{x^2}{2} + 1000x + C$

$v^2 = 25x^2 + 2000x + C$

$(400)^2 = 25(40)^2 + 2000(40) + C$

$16 \times 10^4 = 4 \times 10^4 + 8 \times 10^4 + C \qquad C = 4 \times 10^4$

$v^2 = 25(120)^2 + 2000(120) + 4 \times 10^4 = 64 \times 10^4$

$v = 800$ mm/s

2/15

$$\frac{da}{dt} = \frac{-4g}{4\text{ sec}} = -g = -9.81$$

$$a = 0 = -9.81 \cdot 2 + 9.81 \cdot 2$$

$$a = -9.81t + C \quad \text{for } t > 2\text{ sec.}$$

$$v\Big|_{2s} = 100\text{ m/s}$$

$$a = -9.81(t-2)$$

$$v = \frac{-9.81(t-2)^2}{2} + C$$

$$v = \frac{-9.81(2)^2}{2} + 100$$

$$= -19.62 + 100 = 80.38\text{ m/s}$$

2/34 $a = -[c_1 + c_2 v^{-2}]$

$\sqrt{c_2}\ \frac{dv}{c_1 + c_2 v^2} = -dt$

$\sqrt{c_2}\sqrt{c_1}\ \sqrt{c_1}\left(1 + \frac{c_2}{c_1}v^2\right) = -t$

$\sqrt{\frac{1}{c_1 c_2}}\ \frac{dx}{1 + x^2}$ $x = \sqrt{\frac{c_2}{c_1}}v$

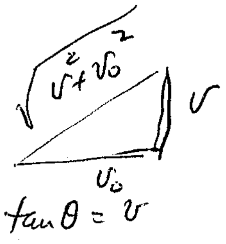
$dx = \sqrt{\frac{c_2}{c_1}}\,dv$

$\tan^{-1} x$

@ $t = 0$ $v = v_0$

$\sqrt{\frac{1}{c_1 c_2}}\tan^{-1} x = -t + C.$

$\sqrt{\frac{c_2}{c_1}}v_0$



$\sqrt{\frac{1}{c_1 c_2}}\ \underline{\tan^{-1} x - \overset{t^{-1}}{x_0}} = -t.$

$\tan^{-1}\frac{v}{v_0} = -\sqrt{c_1 c_2}\,t$

$\frac{v}{v_0} = -{}^{1}\tan\sqrt{c_1 c_2}\,t$

$v = -v_0 \tan$

$ds = -v_0 \tan \sqrt{\quad} t\,dt.$

$s =$

$v\,dv = dx\,a = K/(L-x)^2 . dx$

$v^2 = \frac{2K}{(L-x)} + C$ $C = -\frac{2K}{L.}$

$\frac{+1}{(L-x)^2}$

$= \sqrt{2K\left[\frac{1}{l-x} - \frac{1}{l}\right]}$ $\underline{v \rightarrow \infty.}$

@ $x \rightarrow l$

Soln to HW # 2

George

- Bring artwork 2983 CALL in 30 min.
  3025
- Requisition

$V_{x_0} = u\cos\theta$

$V_{y_0} = u\sin\theta$

$s_y = -g\frac{t^2}{2} + V_{y_0}t + \cancel{s_{y_0}}$

$s_x = \cancel{s_{x_0}} + V_{x_0}t$

$90 = u\overset{\cos}{\cancel{\sin}}\theta\, t$,

$10 = -g\frac{t^2}{2} + u\overset{\sin}{\cancel{\cos}}\theta\, t$,

$10 = -g\frac{t^2}{2} + u\cancel{\sin}\theta\, t$

$= -\frac{g}{2}\frac{(90)^2}{u^2\cancel{\cos^2}\theta} + u\cancel{\sin}\theta \cdot \frac{90}{u\cancel{\cos\theta}}$

$\left(10 + g\frac{t^2}{2}\right)^2 = u^2\cos^2\theta\, t^2$

1 $10 = -\frac{g}{2}\frac{(90)^2}{u^2\overset{}{\underset{\cos}{\cancel{\sin}}}{}^2\theta} + \frac{90}{\cancel{\tan\theta}}\tan\theta$

$90^2 = u^2\sin^2\theta\, t^2$

$90^2 + \left(10 + g\frac{t^2}{2}\right)^2 \neq u^2t^2$

$(90)^2 + 10^2 + 10gt^2 + g^2\frac{t^4}{4} - u^2t^2$

$(90^2 + 10^2) \neq (10g - u^2)t^2 + g^2\frac{t^4}{4} = 0$

100^2

find

$8200 \quad -9678\,t^2 + 259.21\,t^4 = 0$

$\frac{9678 \pm \sqrt{(9678)^2 - 4(8200)(259.21)}}{2(259.21)}$ $\quad 9678 \pm 9228.304$

~~820~~ $\quad 90 = 100\overset{\cos\theta}{\cancel{\sin\theta}}\, t$ $\quad 6.039 \quad 36.4691 = t^2$

1 $\quad 81.429 \quad t = 6.039$ 1 $\quad .931 \quad .8674 = t^2$

1 $\quad 14.83 \quad t = .931$ 1

$\theta = 45°$ $\quad 90 = 100(.707$
$\quad\quad 70.7\ t$

$t = 1.29 \sim 1.3$

$10 = -16.1(1.6) + 70.7(1.3)$

$-26 + 91$

Soln to HW #1

$v^2 = k/s$ @ $t=0$ $v = 2\,in/s$ $x = 9\,in$ find v when $t = 3s$.

$\frac{ds}{dt} = v = \sqrt{k/s}$ $\sqrt{s}\,ds = \sqrt{k}\,dt$ 1

$\frac{2}{3} s^{3/2} = \sqrt{k}t + C$ @ $t=0$ $s=9$.

$\frac{2}{\cancel{3}}\cdot\overset{9}{\cancel{27}} = 18 = C$ 1

$\frac{2}{3} s^{3/2} = \sqrt{k}\,t + 18$

$s^{3/2} = \left[\frac{3}{2}\sqrt{k}\;t + \overset{27}{\cancel{18}}\right]$ 1 $4\,in^2/s^2 = v^2 = \frac{k}{9\,in}$ $k = 36\,in^3/s^2$. 1

$s^{3/2} = \frac{3}{2}\cdot 6t + 27 = 9t + 27$

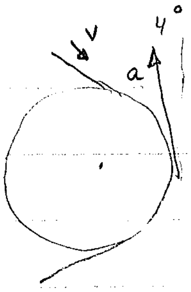
$t = 3s$ $s^{3/2} = 54$ $s = 14.2866\,in$

$v^2 = \frac{36}{14.2866} = 2.5183$

$v = 1.5869\,in/s.$ 1

Sol to #3

2/113



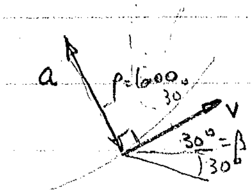
$V = 4\ m/s$ $\qquad a_n = V^2/\rho = \frac{4.4}{.12} = 133.33\ m/s^2$ 1

$\tan 4° = \frac{a_n}{a_t} = .0699 \quad \therefore \quad a_t = \frac{a_n}{.0699} = -1907.49\ m/s^2$ 1

∴ for constant deceleration $\quad v_f = v_i + a_t t$

$0 = 4 + (-1907.49)t \qquad t = .002097\ s$ 2

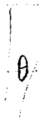
2/148



$V = 400\ km/hr$ const. $\Rightarrow \dot{v} = 0 = a_t$

$a_n = V^2/\rho = \frac{(400\ km/hr)^2}{.6\ km} = 2.667 \times 10^5 \frac{km}{hr^2} = 20.576 \frac{m}{s^2}$

$a_n = a$



1 $\quad a_r = a\cos 60° = 10.288\ m/s^2 \qquad a_\theta = -a\sin 60° = -17.82\ m/s^2$

$V = 400 \frac{km}{hr} = 111.11\ m/s^2 \qquad V_r = V\cos 30° = 96.22\ m/s^2 \qquad V_\theta = V\sin 30° = 55.56 \frac{m}{s^2}$ 1

1 1 1

How $\quad V_r = \dot{r} = 96.22 \frac{m}{s^2} \qquad V_\theta = r\dot{\theta} \qquad \therefore \dot{\theta} = \frac{V_\theta}{r} = \frac{55.56}{800} = +.06945\ rad/s$

1

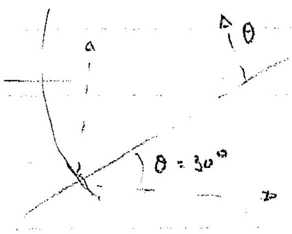
$-17.82 = a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta} = 2(96.22)(.06945) + 800(\ddot{\theta}) \Rightarrow \ddot{\theta} = -.039\ rad/s^2$

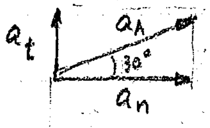
Soln to #4

2/188 Given $\vec{a}_{A/B} = \vec{0}$ find a_B and a_{A_t} 1

$\vec{a}_A - \vec{a}_B = \vec{a}_{A/B} = \vec{0} \Rightarrow \vec{a}_A = \vec{a}_B \qquad V_A = 30 \frac{mi}{hr} = 44\ ft/s$

$a_n = \frac{V^2}{\rho} = \frac{(44)^2}{500} = 3.872\ ft/s^2$ 1





Direction of a_A = direction of a_B 1

$\therefore |\vec{a}_A| = \frac{a_n}{\cos 30°} = 4.471\ ft/s^2 = |\vec{a}_B|$ 1

$a_t = a_n \tan 30° = 2.236\ ft/s^2$ 1

2/211 $\dot{V}_A = 0 \Rightarrow a_{t_A} = 0$ 1 $V_A = 50\,km/hr = 13.89$ $a_{n_A} = 3.215\,\frac{m}{s^2} = V_A^2/\rho$

$\therefore\ \vec{a}_A = 3.215\ \overset{60}{\nwarrow} = -3.215\cos 60^\circ \bar{i} + 3.215\sin 60^\circ \bar{j}$ 2

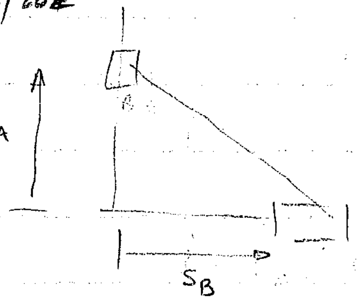
$\vec{a}_B = -1.5\bar{j}$

$\bar{a}_{A/B} = \bar{a}_A - \bar{a}_B = -1.608\,\bar{i} + 2.784\bar{j} - (-1.5\bar{j}) = -1.608\bar{i} + 4.284\bar{j}$ m/s^2 1

$a_{A/B} = 4.58\,\frac{m}{s^2}$

$\tan\varphi = \frac{a_{A/B}\,x}{a_{A/B}}$ $\varphi = 69.44^\circ$ $\varphi\ 69.44^\circ$ 1

2/202



$S_A^2 + S_B^2 = L^2$

$2S_A V_A + 2S_B V_B = 0$ $S_A V_A = -V_B S_B$ $S_A^2 V_A^2 = S_B^2 V_B^2$

$2V_A^2 + \cancel{2S_A \dot{V}_A} + 2V_B^2 + 2S_B a_B = 0$

$\cancel{2}V_A^2 + \cancel{2}V_B^2 + \cancel{2}S_B a_B = 0$

$a_B(S_A)$ $a_B = -\left(\frac{V_A^2 + V_B^2}{S_B}\right) = -\frac{\left(V_A^2 + S_A^2 V_A^2 / S_B^2\right)}{S_B}$

$= -V_A^2\left[\frac{L^2}{S_B^3}\right]$

$S_B^3 = \left(\sqrt{L^2 - y^2}\right)^3$

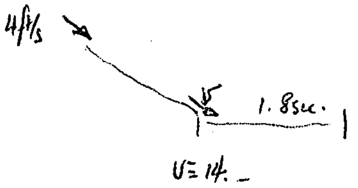
$a_B = -V_A^2\,\frac{L^2}{(L^2 - y^2)^{3/2}}$

$S = S_0 + v_0 (t-2) + \frac{1}{2} a (t-2)^2$

$v = \underset{0}{\cancel{v_0}} + a(t-2)$ $t = 27 sec.$ $v = 150$

150

$9.66 \times 2 = 19.32 \times 10$



$v^2 = 16 + 2(.3)(32.2)10$

$v = \sqrt{209.2} \sim 14.\_$

$v_f = v_i + at$

$14 = 4 + 9.66t$ $t \cong 1 sec.$

$v = 14 + a(1.8sec).$

$a = -$

33.33 $\frac{\overset{2}{\cancel{120{,}000}}}{6\,\cancel{3600}} \frac{m/hr}{s/hr}$ $\frac{\overset{250}{\cancel{150000}}}{6\,\cancel{3600}} = 41.67\ m/s.$

$6t = 41.67$ $t \cong 7 sec$

$a_x = \frac{\Delta a_x}{\Delta x} = \frac{2}{80} =$

$\left(a_x - \overset{4000}{\cancel{120}}\right) = \frac{2000}{80}(s - 120)$

$\frac{dv}{d}$

$a_x = \frac{2s}{80} + 4 - 3$ m/s.

$a_s = \frac{2s}{80} + 1^{000}\ mm/s:$

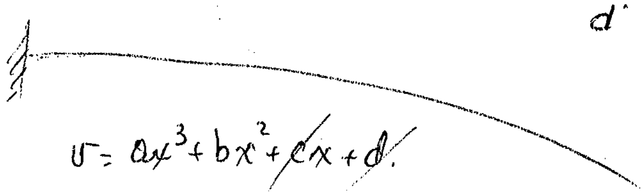
dv

$v\,dv = a\,ds = \frac{\overset{\cancel{2000}}{2s}}{80} ds + \overset{\cancel{1000}}{ds}$

$v^2 = \frac{s^2}{80} + s + C$

$(400)^2 = \frac{(40)^2}{80} + 40 + C.$

$= 20 + 40.$



$v = ax^3 + bx^2 + \underset{0}{\cancel{cx}} + \underset{0}{\cancel{d}}.$

$\left(a_x - \cancel{4 m/s}\right) = \frac{2 m/s^2}{80 mm} \cdot s + 1\ m/s$

a_x $v\,dv = 1000 \frac{s^2}{80} \frac{mm}{s^2} mm \overset{ds}{+} \cancel{1}000 m/s \cdot ds.$

$\frac{v^2}{2} = 1000 \frac{s^2}{80} + 1000 s + C.$

$\frac{(400)^2}{2} = 1000 \frac{40.40}{80} + 1000.40 + C$

$\frac{16 \times 10^4}{2} = 2 \times 10^4 + 4 \times 10^4 + C$

8×10^4 6×10^4 2×10^4 $C = 0$

$2000 \frac{(120)^2}{80} + 2000\ 120 + \cancel{1} \times 10^5$

$2000(180 + 120) = 3000\ 2000$

$\cancel{600 \times}$

6×10^5 $\sqrt{64 \times 10^4}$

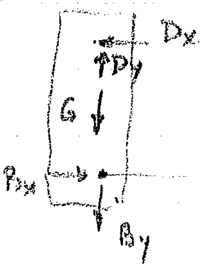
$8 \times 10^2\ mm/s = .8\ m/s$

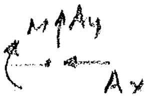
6/1 , 6/3 , 6/5 , 6/11 , 6/20 , 6/24 , 6/26 , 6/37 , 6/46 , 6/48 , 6/61

B10, B12 , B19 , B26 , B24

Curvilinear Travel

6/24





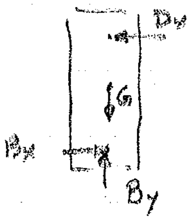
Dy $\Sigma F = ma = 0$ — $D_y = C_y$ $D_x = C_x$

$\Sigma M = I\alpha = 0$ — $D_y = C_y = 0$

$\Sigma F = ma = 0$ $A_x = B_x$

$\Sigma M = I\alpha = 0$ $A_y = B_y$

$B_y .600 = M$



$\Sigma \bar{F} = m\bar{a}$

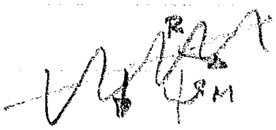
$\Sigma \bar{F}_n = m\frac{v^2}{\rho} = m\omega^2 r = B_x - D_x$

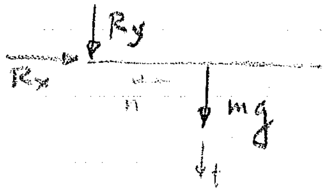
$\Sigma \bar{F}_t = m\alpha r = -B_y + mg$ ⇐ given α

$\Sigma M_G = 0$ since no rotation $(D_x)(.3) + B_x(.5) = 0$

6/38

$I_o = I_G + md^2$





$\Sigma F_x = 0$ $R_x = 0$

$\Sigma F_y = R_y + mg = ma_t = mr\alpha$.

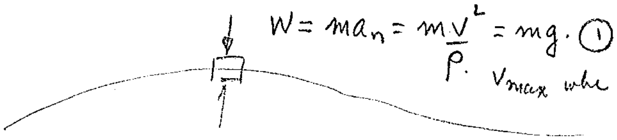
$\Sigma M_o = I_o\alpha = mg(.8)$

$I_o = I_G + md^2$
$(.8)$ ⟹ $\alpha = 9.20$ rad/s.

Give

① $v = \sqrt{\rho g}$.   4 + 11

3-76



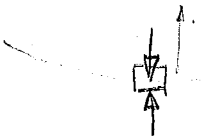
$W = ma_n = m\frac{v^2}{\rho} = mg$. ① $\frac{1}{\rho} = \frac{y''}{[1+y'^2]^{3/2}}$. ①

v_{max} whe ρ is max. or $\frac{1}{\rho}$ is min. $y' = 0 \Rightarrow x = \frac{L}{4}$.

$y = b\sin\frac{2\pi x}{L}$ $y' = \frac{b \cdot 2\pi}{L}\cos\frac{2\pi x}{L}$.

$y'' = -b\cdot\frac{4\pi^2}{L^2}\sin\frac{2\pi x}{L} = -b\cdot\frac{4\pi^2}{L^2}$ $\therefore$ $\frac{1}{\rho} = \frac{4\pi^2 b}{L^2}$

$v = \sqrt{\frac{L^2 g}{4\pi^2 b}} = \frac{L}{2\pi}\sqrt{\frac{g}{b}}$

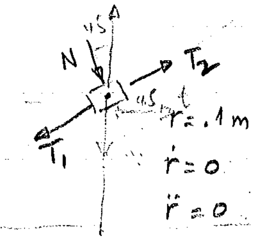


at $\frac{3L}{4}$ $y' = 0$ $y'' = \frac{4\pi^2}{L^2}$.

$N - W = \frac{mv^2}{\rho} = mg$. $N = W + mg = 2mg$. ①

3-69. given $\ddot{\theta} = .5\ \text{rad}/s^2 \curvearrowright = \text{const}$ $\therefore$ $\omega = \omega_0 + \alpha t$ $\omega_0 = 0$ $\therefore$ $\omega = .5t \curvearrowright$

③



$r = .1\,m$

$\dot{r} = 0$

$\ddot{r} = 0$

① $a_r = \cancel{\ddot{r}} - r\dot{\theta}^2 = -r\dot{\theta}^2 = -.1(.5t)^2 = -.025t^2 = -a_n$

① $a_\theta = \cancel{2\dot{r}\dot{\theta}} + r\ddot{\theta} = .1(.5) = .05 = a_t$

~~①~~ $-N\cos 45 + T_2\sin\theta + T_1\sin\theta = ma_r = -ma_n$

~~①~~ $+N\sin 45 + (T_2 - T_1)\cos\theta = ma_\theta = ma_t$

accel. in the direction of the wire T_2 is $a_\theta\cos 45^\circ + a_r\sin 45^\circ$; when this is zero or greater than zero T_2 is in tension. If < 0 then T_1 is in tension.

$\therefore$ $.05(.707) - .025t^2(.707) = 0$ $t = 1.414\,s$ ①

$\therefore$ $t \le 1.414\,s$ $T_1 = 0$

$t > 1.414\,s$ $T_2 = 0$.

$t \le 1.414\,s$. $(-N + T_2)\cos 45^\circ = 2(-.025t^2)$.

$(N + T_2)\cos 45^\circ = 2(.05)$

①/2 $T_2 = \frac{1}{2(.707)} 2\cdot[.05 - .025t^2] = .0707 - .0354t^2$

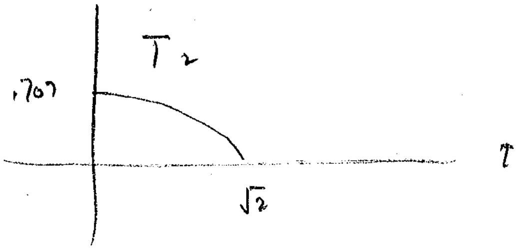
①/2 $N = \frac{1}{2(.707)} 2\cdot[.05 + .025t^2] = .0707 + .0354t^2$

$t \ge 1.414$ ①/2 $-(N + T_1)\sin 45^\circ = 2(-.025t^2)$

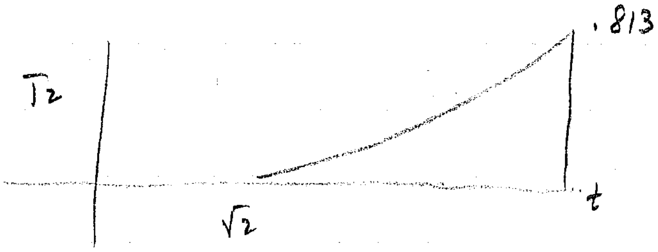
①/2 $+(N - T_1)\sin 45^\circ = 2(.05)$

$\Big\}$ $N = \frac{1}{2(.707)} 2[.05 + .025t^2]$

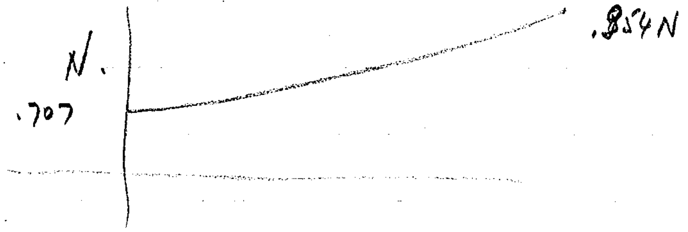
$T_1 = \frac{1}{} 2[.05 - .025t^2]$



①



①



①

HW # 1 2/10 , 2/19, 2/28, 2/38 TURN IN 2/33

2/57, 2/58, $2/\overset{63}{\cancel{\text{[illegible]}}}$, 2/77 TURN IN 2/82

$\underbrace{2/101,\ 2/115,}_{n,t}$ 2/118, $\underbrace{2/126,\ 2/143,}_{r,\theta}$ 2/149 2/113, 2/148

n-t r-θ

4 2/174, 2/178, 2/184, 2/197, 2/203, 2/214 2/188 2/211

5 3/5, 9, 24, 27 3/10

3/51, 55, 66 3/55, 3/68, 3/84 3/76, 3/69

N = 323 N $v = \frac{L}{2\pi}\sqrt{g/b}$ N = 2W

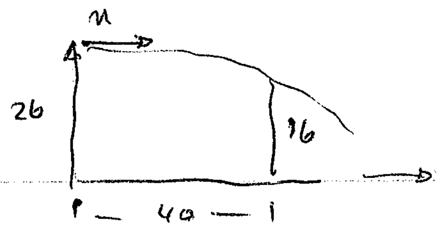
S = 160 m v = 4.28

#6 3/94, 98, 114, 118 k = 8.79 kN/m 3/129, 132, 141, 144

#7 3/164, 171, 184

#8 3/194, 3/199, 3/208, 3/210, 3/228

2/67



$s_x = \cancel{s_{x_0}} + V_{0_x} t$ ~~xxx~~

$s_y = s_{y_0} + \cancel{V_{0_y} t} + \frac{1}{2} g t^2.$

$s_y = 26 + \frac{1}{2} g t^2$

$s_x = 40 = V_{0_y} t$

$16 - 26 = +10 = +4.9\underset{\wedge}{\overset{0}{5}} t^2$

$\sim 2 \quad t =$ ~~1.414.~~ 1.4278 s $\quad 1.43$s.

$\frac{40}{\sqrt{2}} = 20\sqrt{2} = 28.28$ m/s $\quad 28.07 \frac{m}{s}$

if $V_{0_x} > 28.28 \rightarrow t < 1.414$

$V_0 \qquad s_y = 26 + \frac{1}{2} g t^2$

let $\quad V_0 = 30 \qquad s_x = V_{0_x} t$

$40 = 30t \qquad t = 1.33$s.

$s_y = s_{y_0} + \frac{1}{2} g t^2$

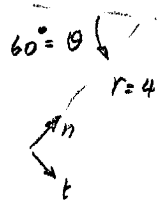
$= 26 - \underset{.55}{4.905} \cdot \frac{16}{9} = 26 - \overset{8.72}{\cancel{8.8}} = 17.28$ m

2/93 $V = \dot{s} = \text{const} \Rightarrow \ddot{s} = a_t = 0$

$a = a_n = \frac{V^2}{\rho}$ $\rho = 100\,m - .6\,m = 99.4\,m.$

$a = .5g = (.5)(9.81) \Rightarrow V = \sqrt{a_n \rho} = 22.08 \frac{m}{s} = 79.490 \frac{km}{h}.$

2/98.



2/103. $a = 3g$

$V = \dot{s} = 800\ km/h.$

$\ddot{s} = (20\ km/hr)/s$

2/105 $a_n = 40 \frac{m}{s^2} = \frac{v^2}{\rho.} = \frac{v^2}{.1m}$ $v = 2\,m/s.$

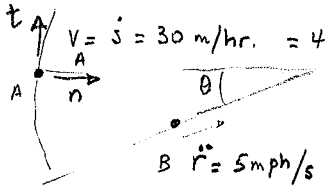
$a_t = \ddot{S} = \dot{v} = 30 \frac{m}{s^2}$

@ P_1 since length between P_1 & P_2 is fixed a_t must be same for both.
V must be same for both.

$\therefore a_{P_1} = \sqrt{a_t^2 + a_n^2} = 50 \frac{m}{s^2}$

@ $P_2 \Rightarrow a_{P_2} = \sqrt{a_t^2 + a_n^2} = \sqrt{80^2 + 30^2} = 85.44\ m/s^2.$

2/187



$V = \dot{s} = 30\ m/hr. = 44\ ft/s.$

$\ddot{r} = 5\,mph/s$

$V_A = 44 \vec{e}_t = 44 \vec{j}$

$V_B = +\dot{r}\vec{e}_r + \cancel{r\dot{\theta}\vec{e}_\theta}$

$a_A = \cancel{\dot{v}\vec{e}_t} + \frac{v^2}{\rho}\vec{e}_n = \frac{44^2}{500}\vec{i}$

$3.872\ ft/s^2\ \vec{e}_n$
$\vec{i}$

$a_B = (\ddot{r} - \cancel{r\dot{\theta}^2})\bar{e}_r + (\cancel{r\ddot{\theta}} + \cancel{2\dot{r}\dot{\theta}})\vec{e}_\theta$

$= +7.333\,\vec{e}_r$

$(-\cos 30\vec{i} - \sin 30\vec{j})\ \vec{e}_r$

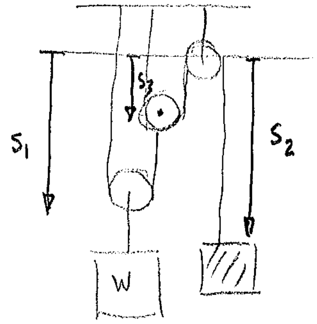
$= -(6.3509\,\vec{i} + 3.6667\,\vec{j})$

$\bar{a}_A = \bar{a}_B + \bar{a}_{A/B}$

$3.872\vec{i} = (6.3509\hat{i} + 3.667\bar{j}) + \bar{a}_{A/B}$

$10.223\,\bar{i} + 3.667\bar{j} = \bar{a}_{A/B}$

2/195



$s_1 + (s_1 - s_3) = C_1$

$2s_3 + s_2 = C_2$

$v_1 + (v_1 - v_3) = 0$

$2v_3 + v_2 = 0$.

$2v_1 = v_3 = -v_2/2$.

$v_1 = -v_2/4 = -\frac{320 mm/s}{4} = -80 mm/s$.

$\therefore \quad \Delta s_1 = v_1 \Delta t = -80(5) = -400 mm /$

$= .4 m$

2/116

$$x$$

$$S_x = \cancel{S_{x_0}} + V_{x_0} t + \cancel{\tfrac{a}{2}}$$

$$y. \quad S_y = \cancel{S_{y_0}} + V_{y_0} t + \frac{1}{2} a t^2$$

$$\frac{dy}{dt} = V_{y_0} + at \qquad\qquad \frac{dy}{dx} = \frac{y'}{x'} = \frac{V_{y_0} + at}{V_{x_0}}$$

$$\frac{dx}{dt} = V_{\cancel{x}_0}$$

$$y = V_{y_0} \frac{x}{V_{x_0}} + \frac{1}{2} a \left(\frac{x}{V_{x_0}}\right)^2$$

$$\frac{dy}{dx} = \frac{V_{y_0}}{V_{x_0}} + \frac{1}{2} a \cdot 2 \left(\frac{x}{V_{x_0}}\right) \cdot \frac{1}{V_{x_0}}$$

$$= \frac{V_{y_0}}{V_{x_0}} + a \left(\frac{x}{V_{x_0}^2}\right)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{y'}{x'}\right) = \cancel{\frac{dx}{dt}} \frac{d}{dt}\left(\frac{y'}{x'}\right) = V_{x_0} \cdot \left[-\frac{y'x''}{(x')^2} + \frac{y''x'}{x'^2}\right]$$

$$\frac{dt}{dx} \qquad\qquad = \frac{1}{V_{x_0}} \left[\frac{x'y'' - y'x''}{x'^2}\right]$$

$$x'' = 0 \qquad y'' = a \qquad\qquad = \frac{\cancel{V_{x_0}}}{V_{x_0}^3} \left[V_{x_0} a - (V_{y_0} + at) \cdot 0\right]$$

$$\frac{d^2y}{dx^2} = \frac{a}{V_{x_0}^2}$$

$$\frac{d^2y}{dx^2} = \frac{a}{V_{x_0}^2} \qquad\qquad \rho = \frac{\left[1 + \left(\frac{y'}{x'}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

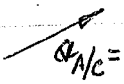
$$\rho = \frac{\left[1 + \left(\frac{V_{y_0} + at}{V_{x_0}}\right)^2\right]^{3/2}}{\left|\frac{a}{V_{x_0}^2}\right|}$$

@ max height $V_{y_0} + at = 0 \quad \therefore \rho = \frac{V_{x_0}^2}{a}$

@ " " $V = V_{x_0} \Rightarrow a = V_{x_0}^2/\rho = g \qquad a = a_n \qquad a_t = \dot{V} = 0$

relative accel.

2/210



$a_{A/c} = 50\,m/s^2$

$a_{A/c} = \frac{d V_{A/c}}{dt}$

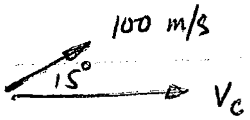
$V_f^2 = V_i^2 + 2 a_{A/c} (s_f - s_i)$

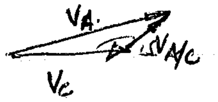
$= 0 + 2 \cdot 50 \cdot 100$

$V_{f\,A/c} = 100\,m/s.$

~~$a_{A/c} \cdot t + const = V_{A/c}$~~

~~and $\frac{1}{2} a_{A/c} t^2 + V_i t + s_i = s_f$~~





$V_A - V_c = V_{A/c}.$

$V_c = 30\,knots = 30(1.852 \frac{km}{hr}) \times \frac{1000}{3600}$

$= 15.43\,m/s.$

~~$\frac{V_A}{\sin}$~~

$V_A = \sqrt{V_c^2 + V_{A/c}^2 - 2 V_c V_{A/c} \cos 165^\circ}$

$= 114.98\,m/s$

$\cos(165) = \cos(180 - 15) = -\cos 15^\circ$

$V_A = \sqrt{V_c^2 + V_{AC}^2 + 2 V_c V_{A/c} \cos 15^\circ}$

r, θ

2/219 $x = r \sin\theta$ $\dot{x} = \cancel{\dot{r}\sin\theta} + r\dot{\theta}\cos\theta$ $\dot{r} = 0,\ \ddot{r} = 0$

$r = 4''$ $x = 2\,in$ $\Rightarrow \theta = 30^\circ$

$4 = 0 \cdot 0.5 + \frac{4}{12}\dot{\theta}(.866)$ ~~$\dot{\theta} = \frac{1}{.866} = \frac{2}{\sqrt{3}}$~~ $\dot{\theta} = \frac{4 \cdot 3}{.866} = \frac{4 \cdot 3 \cdot 2}{\sqrt{3}} = 8\sqrt{3} = 13.86\ \frac{rad}{s}$

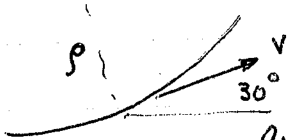
$\ddot{x} = \cancel{\ddot{r}\sin\theta} + \cancel{2\dot{r}\dot{\theta}\cos\theta} + r\ddot{\theta}\cos\theta - r\dot{\theta}^2\sin\theta$

$30 = \frac{4}{12}\ddot{\theta}(.866) - \frac{4}{12}(8\sqrt{3})^2\, 0.5$

$= \frac{1}{3}\ddot{\theta}\frac{\sqrt{3}}{2} - \frac{1}{3}(64 \cdot 3)(0.5) \implies 62 = \frac{1}{6}\sqrt{3}\,\ddot{\theta}$ $\ddot{\theta} = \frac{372}{\sqrt{3}} = 124\sqrt{3} = 214.77\ \frac{r}{s^2}$

n, t prob.

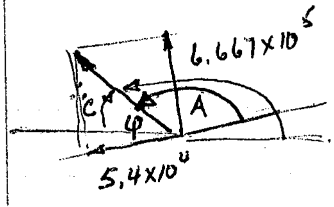
2/213 ~~$v = const \Rightarrow a = 0$ on straight part~~ ~~& $a_t = 0$~~



$v = 1000\,km/hr$ w/ $\dot{v} = -15\,\frac{km/hr}{s}$ if $\rho = 1.5\,km$

$a_t = \dot{v} = -15\,\frac{km/hr}{s} \times \frac{3600\,s}{hr} = 54000\,\frac{km}{hr^2}$

$a_n = v^2/\rho = \frac{1000 \times 1000}{1.5} = .6667 \times 10^6\,km/hr^2$



$$a = \sqrt{(6.667\times10^5)^2 + (5.4\times10^4)^2} = 6.68850\times10^5 \frac{km}{hr^2}$$

$$\varphi = \tan^{-1}\left(\frac{6.667\times10^5}{5.4\times10^4}\right) = 85.369$$

$$180 - \varphi = A = 94.63$$

$$A + 30 = B = 124.63$$

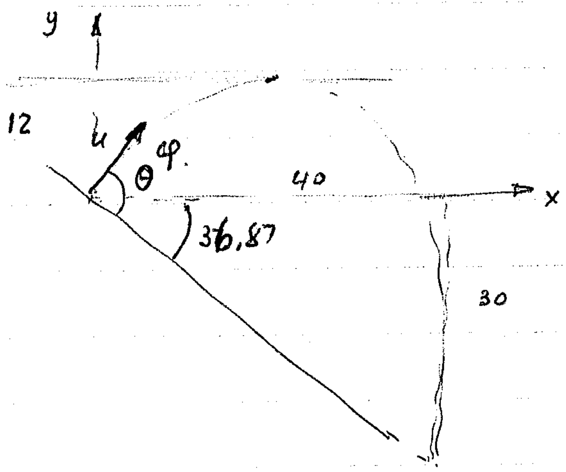
$$180 - B = C = 55.369^\circ$$

$$a \sin C = \ddot{y} = 5.5035\times10^5 \frac{km}{hr^2} = 42.47 \frac{m}{s^2}$$

$$a \cos C = -\ddot{x} \rightarrow \ddot{x} = -3.80\cancel{4}\times10^5 \frac{km}{hr^2} = -29.33 \frac{m}{s^2}$$

traj

2/215



let $u \sin\varphi = V_{y_i}$

$$V_{y_f}^2 = V_{y_i}^2 + 2g(\Delta s)$$

$$0 = V_{y_i}^2 - 2(9.81)(12)$$

$$V_{y_i} = \sqrt{2(9.81)(12)} = 15.344 \frac{m}{s}$$

$V_{x_i} = u\cos\varphi$ $\qquad$ $s_x = V_{x_i} t = 40$

$$s_f = s_i + V_{y_i} t + \frac{1}{2} g t^2$$

$$-30 = 0 + 15.344t + \frac{1}{2}(-9.81)t^2$$

$$0 = 30 + 15.344t - 4.905t^2$$

$$t = \frac{\overset{-15.344}{-V_{y_i}} \pm \overset{28.706}{\sqrt{V_{y_i}^2 + 4(30)(4.905)}}}{-\cancel{10}\ 9.81} = 4.49s \quad (-1.36s)$$

take +

$$V_{x_i} = \frac{40}{t} = 8.908m \qquad \sqrt{V_{x_i}^2 + V_{y_i}^2} = u = 17.74 m/s.$$

$$\varphi = \tan^{-1}\left(\frac{V_{y_i}}{V_{x_i}}\right) = 59.86^\circ \qquad \text{now} \qquad \varphi + 36.87 = \theta = \overset{96.73^\circ}{\cancel{96.86}}$$

$P - \frac{4}{3}T = m\ddot{x}_A = \Sigma F_x$

$N_A + \frac{3}{5}T - W = 0 = \Sigma F_y \quad T =$

5 eq

$T, \ddot{y}_B, \ddot{x}_A, N_B, N_A$

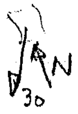
$T = 46.6\ N \qquad a_A = 1.364\ m/s^2 \qquad a_A = -9.32\ m/s^2$

N_A & N_B will be −

$N_B = -37.28\ N$

$N_A = -8.34\ N$

3/46

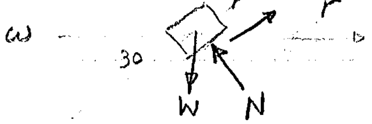


$W\cos 30° - N = ma_n = \cancel{m}\frac{v^2}{\rho} \qquad N = W\cos 30° - m\frac{v^2}{\rho} < 0$

$W\sin 30° = ma_t \qquad g\sin 30° = a_t \qquad \Rightarrow N \downarrow$

n, t

3/61 $\quad a_n = \frac{r^2\omega^2}{r} = \frac{v^2}{r}$



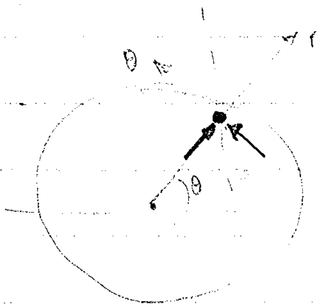
$\Sigma F_n = N\sin\theta \pm F_s\cos\theta = ma_n$

n, t $\quad \Sigma F_b = \pm F_s\sin\theta + N\cos\theta - W = 0$

$\dot{\Sigma} F_t = 0 \Rightarrow a_t = 0$

∴ from $\Sigma F_n \quad F = \mu_s N \quad$ solve for ω

3/71



$F_\theta = \quad \Sigma F_\theta = m(2\dot{r}\dot{\theta} + r\ddot{\theta}) = ma_\theta$

$F_r = \quad \Sigma F_r = m(\ddot{r} + r\dot{\theta}^2) = ma_r$

r, θ $\quad \dot{r} = 0,\ \ddot{r} = 0 \qquad \ddot{\theta} = \text{const} \therefore$

$\dot{\theta}_i = 0 \qquad \dot{\theta}_f = \sqrt{2\alpha\theta}$

for slipping total force $\quad F_{max} = \mu_s N = \sqrt{F_r^2 + F_\theta^2} = \sqrt{(mr\alpha)^2 + (mr\dot{\theta}_f^2)^2}$

$= mr\sqrt{\alpha^2 + \dot{\theta}_f^4} = mr\sqrt{\alpha^2 + 4\alpha^2\theta^2}$

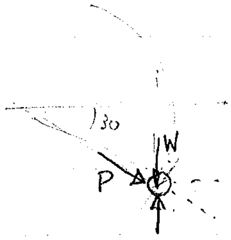
$\mu_s \cancel{m}g = mr\alpha\sqrt{1 + 4\theta^2}$

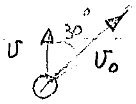
$\therefore \sqrt{\frac{\mu^2 g^2 - r^2\alpha^2}{4r^2\alpha^2}} = \theta =$

$2\pi\ \text{rad/rev} \times N = \theta$

$N = \frac{\theta}{2\pi}$

3/83





$$U_o = \frac{U}{\cos 30^\circ} = \frac{2}{.866} = 2.31 \text{ m/s.}$$

$$\Sigma F_r = P - F\cos 60^\circ + W\cos 60^\circ = m(\ddot r - r\dot\theta^2)$$

$\dot r = \ddot r = 0$

$$\Sigma F_\theta = -W\cos 30^\circ + F\cos 30^\circ = m(2\dot r\dot\theta + r\ddot\theta)$$

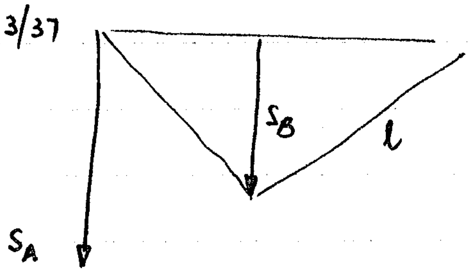
$$U_o = U_\theta = r\dot\theta \quad \therefore \quad \dot\theta = \frac{2.31}{.25} = 9.24 \text{ rad/s}$$

$$U = \text{const} \quad U_o = \text{const} \Rightarrow \dot\theta = \text{const} \Rightarrow \ddot\theta = 0$$

$$F = \frac{m(2\dot r\dot\theta + r\ddot\theta) = 0}{\cos 30^\circ} + W \qquad F = W$$

$$P = m(-r\dot\theta^2) = \frac{W}{g}\left(\frac{U_o^2}{r}\right).$$

3/37



$$v_B = -\frac{v_A\,l}{2y}$$

$$2l + s_A = \text{const.}$$

$$l = \sqrt{s_B^2 + b^2}$$

$$\frac{d}{dt}: \quad v_A + \cancel{2}\frac{2 s_B v_B}{\cancel{2}\sqrt{s_B^2+b^2}} = 0$$

$$a_A + \frac{2v_B^2 + 2s_B a_B}{\sqrt{s_B^2+b^2}} + \frac{1}{\cancel{2}}\frac{\cancel{2}s_B v_B}{(s_B^2+b^2)^{3/2}} = 0$$

$\omega = \text{const}$

$v_A = \text{const} = \omega \cdot \frac{d}{2}$

$\therefore \quad \dot{v}_A = a_{t_A} = 0$

$$a_{n_A} = \frac{v^2}{\rho} = \frac{\omega^2 d^2}{4 \cdot d/2} = \frac{\omega^2 d}{2} = a_A$$

$$v_A + \frac{2y\dot{y}}{s} = 0 \qquad y\dot{y} = s\left(-\frac{v_A}{2}\right)$$

$$a_A + \frac{2\left[\frac{v_A^2 l^2}{4y^2} + y a_B\right]}{l} - \frac{y\left(-\frac{v_A l}{2y}\right)}{l^3}$$

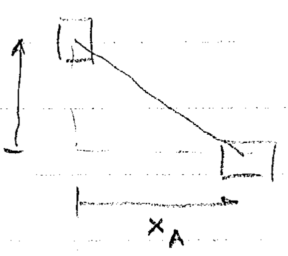
$$\frac{\omega^2 d}{2} + \frac{v_A^2 l}{2y^2} + \frac{2y a_B}{l} + \frac{v_A}{2l^2} = 0$$

$$\frac{\omega^2 d}{2} + \frac{\omega^2 d^2}{8y^2}l + \frac{2y}{l}a_B + \frac{\omega d}{4l^2} = 0$$

$$\frac{2y}{l}a_B = -\left[\frac{4\omega^2 d l^2}{y^2 8 l^2} + \frac{\omega^2 d^2 l^3}{8y^2 l^2} - \frac{2\omega d y^2}{8 l^2 y^2}\right]$$

$$a_B = -\left[\frac{4y^2\omega^2 d l^2 + \omega^2 d^2 l^3 - 2\omega d y^2}{16 y^3 l}\right]$$

3/33



$x_A^2 + y_B^2 = l^2 \qquad x_A = .4 \quad l = .5 \quad y_B = .5$

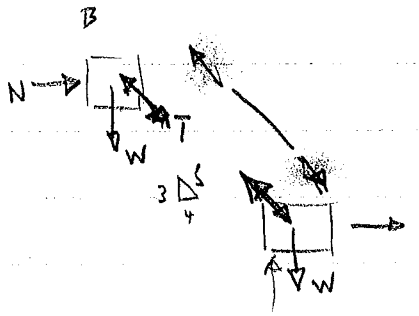
$x_A\dot{x}_A + y_B\dot{y}_B = 0 \qquad .4(.9) + \dot{y}_B(.3) = 0 \qquad \dot{y}_B = -1.2\ \text{m/s}$

$\dot{x}_A^2 + x_A\ddot{x}_A + \dot{y}_B^2 + y_B\ddot{y}_B = 0$

$(.9)^2 + (.4)\ddot{x}_A + (-1.2)^2 + (.3)(\ddot{y}_B) = 0$

$.81 + 1.44 + .4\ddot{x}_A + .3\ddot{y}_B = 0$

2.25

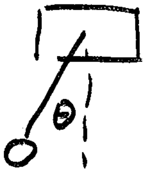


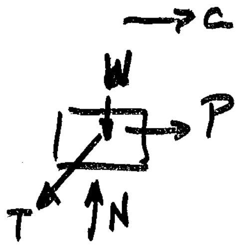
$-\frac{3}{5}T - W = m\ddot{y}_B \quad = \Sigma F_y$

$\frac{4}{5}T + N = 0 \qquad \Sigma F_x$

B

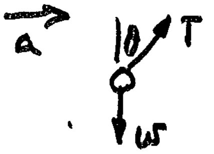
3/30





$W + T\cos\theta = N$

$P - T\sin\theta = \frac{W}{g} a$

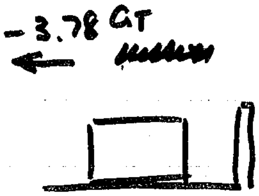


$T\cos\theta = w$ $\qquad P = (M+m)a$

$T\sin\theta = \frac{w}{g} a$

$\tan\theta = \frac{a}{g}$

3/10



$70 \frac{km}{hr} = 70 \cdot \frac{1}{3.6} \sim \overset{19.44}{20} \, m/s.$

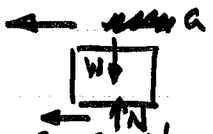
min stop distance when $f = \mu N$

$0 = (20)^2 + 2a\,\Delta s$

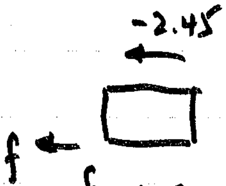
$a = (.3)g$

$400 = .6(9.81)\Delta s$

$= 6\,\Delta s \qquad \Delta s \sim 67$ m.



$f < \mu N = \mu W = \mu g \cdot m$



$\Delta s = 50 \qquad a_{Tru} = \frac{-400}{2(50)} = -\overset{4}{\cancel{8}} m/s^2$

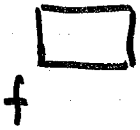
-3.78

$f = ma = 8m \overset{?}{\leq} \mu N = \mu mg$ no.

$f > \mu N$

slip

$\overset{-3.78}{a_T} = \overset{-2.45}{a_p} + \overset{1.33 m/s^2}{a_{p/T}}$



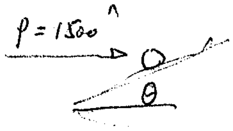
$-4 = \frac{\mu_k W}{W/g} + a_{p/T}$ $\qquad \overset{-2.5}{-.25g} + a_{p/T} = +1.5$

$3 = \frac{\overset{a_{p/T}}{1.33}}{2}(t^2) \qquad t \simeq \overset{2.13}{\cancel{2}}$ sec

$\therefore (v_{p/T})_f^2 - \underset{0}{\cancel{(v_{p/T})_i^2}} = 2\overset{1.33}{a_{p/T}} \overset{3}{\Delta s_{p/T}}$

$v_{p/T} = 2.82$

3/58



$$N\cos\theta - W = m a_b = 0.$$

$$N\sin\theta = \frac{mv^2}{\rho}$$

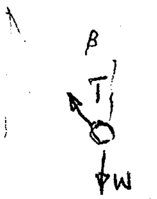
120 mi/hr ≈ 176 ft/s

$$N\cos\theta = W = mg$$

$$N\sin\theta = \frac{mv^2}{\rho}$$

$$\tan\theta = \frac{v^2}{\rho g} = \frac{(176)^2}{1500 \cdot 32.2}$$

3/52



$$T\cos\beta = W$$

$$T\sin\beta = \frac{mv^2}{\rho} = m\rho\dot\theta^2$$

$$\cancel{m}l\omega^2\cos\beta = \cancel{m}g$$

$$h = g/\omega^2$$

$$\rho\dot\theta = v \qquad \rho = l\sin\beta\,\dot\theta$$

$$T = ml\omega^2$$

$$a \approx \tan\beta = \frac{\rho\dot\theta^2}{g} = \frac{r}{h} = \frac{\sqrt{l^2-h^2}}{h}$$

$$ah = \sqrt{l^2-h^2} \qquad a^2h^2 = l^2-h^2 \qquad (a^2+1)h^2 = l^2$$

$$h = \frac{l}{\sqrt{\left(\rho\dot\theta^2/g\right)^2+1}}$$

$$T^2 = W^2 + m^2\rho^2\dot\theta^4 \qquad T = m\sqrt{g^2+\rho^2\dot\theta^4}$$

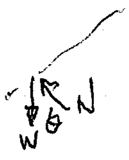
3/86 any pt on the tube of radius r from center $\bar V_T = r\omega_0 e_\theta$

particle move wrt tube $\dot r e_r = \bar V_{P/T}$

$$\bar V_P = \bar V_T + \bar V_{P/T} = \dot r e_r + r\omega_0 e_\theta$$

$$v = \sqrt{\dot r^2 + r^2\omega_0^2}$$

$$\bar a_{\cancel{r}} = (\ddot r - \dot r\dot\theta^2)\hat e_r + (2\dot r\dot\theta + \cancel{r\ddot\theta})e_\theta$$



cond. @ $\theta = 0$, $r = 0$

$$0 = C_1 + C_2$$

$$W\cos\theta - N = m a_\theta = m(2\dot r\dot\theta)$$

$$W\sin\theta = m a_r = m(\ddot r - r\dot\theta^2)$$

$$W\sin\omega_0 t = m(\ddot r - r\omega_0^2)$$

$$g\sin\omega_0 t = \ddot r - r\omega_0^2$$

$$r = C_1 e^{\omega_0 t} + C_2 e^{-\omega_0 t} - \frac{g}{2\omega_0^2}\sin\omega_0 t$$

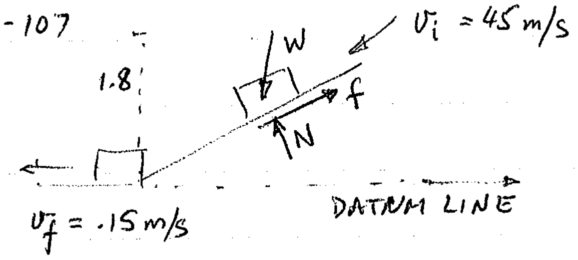
let $r = A\sin\omega_0 t$

$\ddot r = -A\omega_0 \sin\omega_0 t$

$$-2A\omega_0^2 = g$$

1. Do impact ~~problems~~
2. Relative Motion

3-107



Problem in work & energy

$T_1 = \frac{1}{2} m v_1^2$ $\quad v_1 = .45 m/s$

$T_2 = \frac{1}{2} m V_2^2$ $\quad v_2 = .15 m/s$

$V_1 = mgh$ $\quad h = 1.8m$

$V_2 = 0$

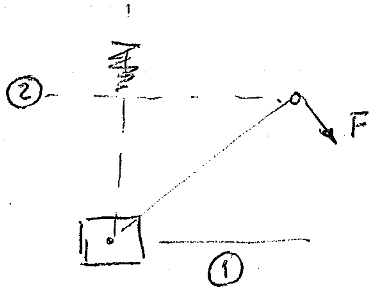
$U_{1-2} = \int f \Delta s = \mu N \Delta s$ $\quad \Delta s = \frac{1.8}{\sin\theta}$

$N = W \cos\theta \; \Delta s$

or $U_{1-2} = (W \sin\theta - f) \Delta s$

X

3 - 118



$F \cdot \Delta s = U_{1-2}$ of force

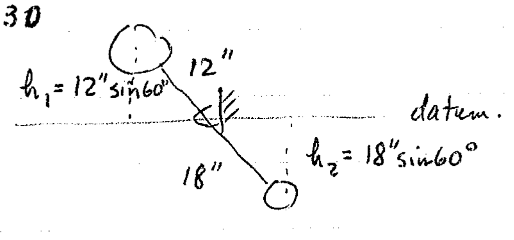
$\Delta s = \sqrt{(450)^2 + (225)^2} - 225 \text{ mm}$

$F = 700 N$

$T_1 = 0$ $\quad V_1 = 0 + 0$

$T_2 = 0$ $\quad V_2 = mg(\underbrace{450 - [illegible]}_{450-75+75}) + \frac{1}{2} k (75)^2$

3/130



$V_1 = \overset{10 lb}{m_1 g} \cdot h_1 + \overset{5 lb}{\overline{m_2 g}} h_2$

$\frac{12'' \sin 60°}{12''/ft} \quad \left(\frac{-18'' \sin 60°}{12''/ft.}\right)$

$V_2 = 0$

$T_1 = 0$ $\quad T_2 = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2$

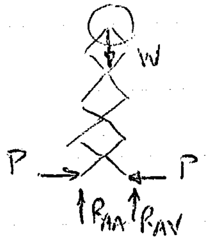
$V_1 = \omega \cdot \frac{12''}{12''/ft}$ $\quad V_2 = \omega \cdot \frac{18''}{12''/ft}$

solve for $\omega \Rightarrow V_2 = \omega \cdot 1.5$

for spring compression V_1 & $V_2 = 0$

$V_1 = W_1 h_1 + W_2 h_2$ $\quad T_1 = 0$ $\quad V_2 = \frac{1}{2} k \Delta x^2$ $\quad T_2 = 0$

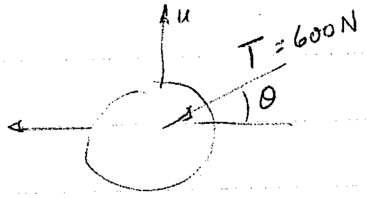
3/167 note $P\,dt$ for each will cancel $\therefore\ mV_{1_x} = mV_{2_x}$



$$\overset{3\cdot(2)}{mV_{1_y}} + 2R_{AV}\overset{2}{\Delta t} - \overset{3\cdot 9.81}{W}\overset{2}{\Delta t} = \overset{3\cdot(4)}{mV_{2_y}}$$

$$R_{AV} = 16.22\,N$$

3/184



$$mV_{y_1} = 0$$

$$\int T\cos\theta \cdot d\cancel{\theta}t = \int T\cos\theta \cdot \frac{dt}{d\theta}\cdot d\theta$$

$$= T\cdot\frac{10}{\pi}\int\cos\theta\,d\theta$$

$$= T\cdot\frac{10}{\pi}\sin\theta\Big|_0^{\pi/2}$$

$$= mV_{y_2} = T\cdot\frac{10}{\pi} = \frac{6000}{\pi}$$

3/197 $T_1 = 0 \quad V_1 = mg\cdot 6'$

$T_2 = \frac{1}{2}mv_1^2 \quad V_2 = 0 \qquad V_{1_{plug}} = \sqrt{2gh} = 19.66$ ft/s.

initially $m_1 V_1 + m_B V_B = (m_1 + m_B)V$

$$\frac{2}{32.2}19.66 + \frac{4}{32.2}\cdot 0 = \frac{6}{32.2}v \qquad v = 6.55 \text{ ft/s}$$

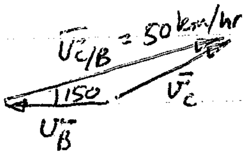
$$T_1 = \frac{m_1+m_B}{2}v^2 \qquad V_1 = 0+0$$

$$T_2 = 0 \qquad V_2 = \frac{1}{2}k\Delta s^2 \qquad \Delta s = .316\text{ ft}$$

originally $\frac{1}{2}m_1 v_1^2$ finally $\frac{1}{2}(m_1+m_B)v^2$

$$\frac{\Delta KE}{KE_{orig}} = .667$$

3/209 $\bar{v}_c = \bar{v}_B + \bar{v}_{c/B}$



$$v_{c_x} = v_{c/B}\cos\overset{15}{\cancel{60}}{}^\circ - v_B$$

$$m_c v_{c_i} + m_B v_B = 0 = m_c v_{c_x} + m_B v_B$$

$$0 = 1500(50\cdot\overset{15^\circ}{\cancel{\%}} - v_B) + 500\times 10^3 v_B$$

$$v_{cx} = v_{c/B}\cos 60^\circ - v_B = \overset{48.296}{\cancel{23}} - .144 = \overset{48.152}{\cancel{24.856}}\ \text{km/hr.}$$

Proj motion

$$\cancel{m_B v_B} + m_c v_{cx} = (m_B + m_c)\,v$$

$$0 \qquad 1500\,(48.152) = 501500\,v \qquad\qquad v = .144\ \text{km/hr} = .04\ \text{m/s}$$

4/10 $\quad m_B\bar{v}_B + m_1\bar{v}_1 + m_2\bar{v}_2 = (m_B + m_1 + m_2)\,\bar{v}$

$\bar{v}_B = 4\cos 30^\circ \qquad v_1 = 2 \qquad v_2 = -1 \qquad\qquad v = .68$ ft/s

~~to find time of fall $m_B v_{B_{y_i}} + W\Delta t = 0$~~

$$m_B\bar{v}_B + m_1\bar{v}_1 = (m_B + m_1)\,\tilde{v}$$

$$(m_B + m_1)\tilde{v} + m_2\bar{v}_2 = (m_B + m_1 + m_2)\,\bar{v}$$

$$m_B\bar{v}_B + m_1\bar{v}_1 + m_2\bar{v}_2$$

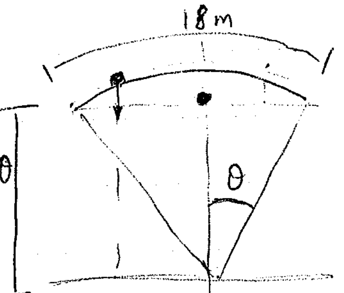
velocity same.

4/20 $\quad T_1 = \frac{1}{2}Mv_{c_1}^2 \qquad V_1 = \overset{Mg}{\cancel{mg}}h \qquad\qquad M = 6m.$

$$T_2 = \frac{1}{2}Mv_{c_2}^2 \qquad V_2 = 0$$

$$\frac{1}{2}v_{c_2}^2 = \frac{1}{2}v_{c_1}^2 + g\tilde{h}$$

$$v_{c_2}^2 = \left(\frac{30}{3.6}\right)^2 + 2g\tilde{h}$$



centroidal

$= \tilde{h} = \dfrac{r\sin\theta}{\theta}$

$= 17.26$m

~~$s = 18m = 2r\theta \qquad \theta = \frac{1}{2}\text{rad} = 28.65^\circ$~~

~~$18\cos\theta\ d\theta$~~

~~$r\cos\theta \qquad \rho\,dl \qquad dl = r\,d\theta$~~

~~$\Delta y = r\sin\theta$~~

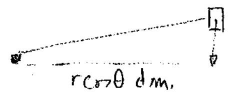
~~$mgr\sin\theta\ d\theta$~~

~~$dW = dmg\,l$~~

$\underline{\underline{dm}} \cdot r\sin\theta =$

$\rho\,ds = \rho r\,d\theta$

r

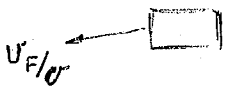


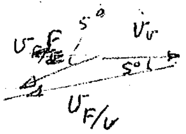
$$\int r\cos\theta\,dm \qquad \rho\,d\theta$$

$$\rho\theta\bar{r} = \rho r\sin\theta$$

$$\bar{r} = \frac{r\sin\theta}{\theta}$$

4/24

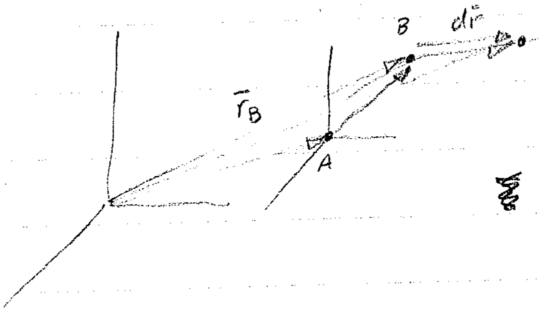




$$\bar{v}_F = \bar{v}_v + \bar{v}_{F/v}$$

3/266 relative motion

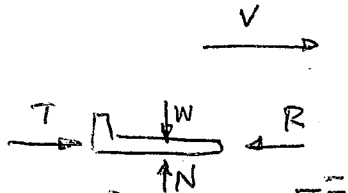
$$\bar{a}_B = \overset{0}{\cancel{\bar{a}_A}} + \bar{a}_{B/A} \qquad d\bar{r}_{ab} \neq d\bar{r}_{rel.}$$



$$\frac{1}{2}m(v_{B/A})_2^2 = U_{1-2_{Rel}} + \frac{1}{2}m(v_{B/A})_1^2$$

$$\int \bar{F}\, dt = m\bar{v}_{B/A_2} - m v_{B/A_1}$$

Chapter 3 – Kraige & Meriam



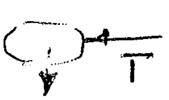
157 $m\bar{v}_i + \int \Sigma F\,dt = m\bar{v}_f$ $\qquad \bar{v}_f = 250\,\frac{km}{hr}\cdot\frac{1}{3.6} \cong 70\,m/s$

$\bar{v}_i = 0 \qquad \Sigma\bar{F} = (T-R)\bar{i} + (W-N)\bar{j} = (T-R)\bar{i}$

$$0 + \left(\int \Sigma F\,dt = T\Delta t - \bar{R}\Delta t\right) = m\bar{v}_f$$

$$0 + (48\times10^4 - \bar{R}\times10) = 6450\,v_f = 6450\times70$$

171 $m\bar{v}_i + \int \Sigma F\,dt = m\bar{v}_f$ $\qquad \Sigma F = -T$

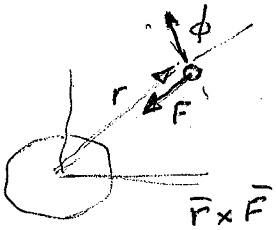


$$\int \Sigma F\,dt = -\int T\,dt = -\pi\frac{T_n\Delta t}{2}$$

$.2(9.81)(.08)$ $(.19)$ $\qquad .2(6) - \pi\overset{6\pi(.2)}{\frac{(30)(.08)}{2}} = .2\,V_{f_x}$ $\qquad 6 - 6\pi = V_{f_x}$ $\qquad V_{f_x} = 6(-2.14) = -12.$

$.2(0) - \underset{.1961(.08)}{W\Delta t} = .2\,V_{f_y}$ $\qquad V_{f_y} \cong .08\,m/s$ $\qquad 12.85\left(1+\frac{.04}{12.85}\right)$

177 $H_{o_i} + \int \Sigma M_o\,dt = H_{o_f}$



$\bar{r}\times\bar{F} = \bar{0}$

$\therefore \quad \bar{H}_{o_i} = \bar{H}_{o_f} \Rightarrow H_o$ is const.

~~$\dot{H}_o$~~ $= \quad \bar{r}\times m\bar{v} \qquad |r||v|\,m\sin\phi$ = const.

~~$\dot{H}_o = 0 \Rightarrow \bar{r}\times m\bar{a}$~~

~~$a_\theta = 0 = 2\dot{r}\dot{\theta} + r\ddot{\theta} = 2\frac{d}{dt}(r\dot{\theta}) = 0 \quad r\ddot{\theta}$~~

~~$= \frac{1}{r}\frac{d}{dt}[r^2\dot{\theta}]$~~

~~$\bar{r}\times m\bar{v}$~~ ~~$m\bar{v} = m(v_r\bar{e}_r$~~

$\bar{r}\times m\bar{v}$ = const. $\qquad \bar{r} = r\bar{e}_r$

$$m\bar{v} = m[v_r\bar{e}_r + v_\theta\bar{e}_\theta]$$

$\bar{r}\times m\bar{v} = mrv_\theta\,\underbrace{\bar{e}_r\times\bar{e}_\theta}_{\bar{e}_z}$ $\qquad$ but $v_\theta = r\dot{\theta}$

$= mr^2\dot{\theta}\,\bar{e}_z = \underline{\text{const.}}\,e_z \quad \Rightarrow \quad r^2\dot{\theta}$ = const

3/184

$\dot\theta = \pi/10$ $\quad \Delta\theta = \dot\theta\Delta t$ $\quad \Delta t = 5\text{sec.}$

$\int T\cos\theta\, dt = \int T\cos\theta \frac{dt}{d\theta} d\dot\theta = \frac{1}{\dot\theta}\int T\cos\theta d\theta$

$= \frac{T}{\dot\theta}\sin\theta$

$\int T\cos\theta dt$

$$mv_{y_i} + \int \cancel{T\cos\theta\, d\theta}^{\Delta t} = mv_{y_f}$$

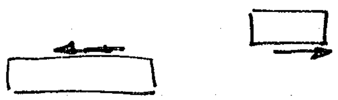
$$\cancel{\Delta t}\cdot\frac{600}{\dot\theta}\sin\theta\Big|_0^{\pi/2} = 260\, V_{y_f}$$

$\frac{600\cdot 10}{\pi}$ $\cancel{3000}$ $1 = 260$ $\quad V_{y_f} =$

$\cong 2000$ $\quad V_{y_f} = \frac{2000}{260} \cong 8$

angular

3/188



Equal & opposite.

$$\Sigma(mv)_i = \Sigma(mv_f)$$

$$12\,Mg\,(20) + m_b\cdot 0 = \Sigma(12+m_b)(x)$$

$\frac{240}{362} \cong \frac{2}{3}.$ $\quad$ 362 Mg.

3/189

$$m_p V_{p_i} + mw\cdot \cancel{V_{w_i}}^{0} = (m_p + \Sigma m_w)\, V_{com.}$$

$140(600) = 440(x)$ $\quad x = \frac{140}{440}(600) \cong \frac{1}{3}(600) \cong 200$

$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \Delta E$

$\frac{1}{2}140(600^2 - 200^2) = .07\text{kg}\,(32\times10^4) =$

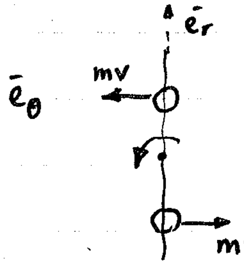
KE before $\quad$ KE after. $\quad$ PE befoe = PE after.

$\frac{1}{2}(140)(600)^2 - \frac{1}{2}440(200)^2$

$\frac{1}{2}(50.4)10^6 - \frac{1}{2}(17.6)\times10^6 = \frac{32.8}{2} = 16\times10^6$

3/195

Consew. of mom.
$\Sigma\bar M = 0$



$$2\bar r\times m\bar v_1 + \Sigma M_o = 2\bar r_n \times m\bar v_n$$

$\bar r_1 = re_r$ $\quad \bar v_1 = \omega_o r\bar e_\theta$ $\Big\}$ $\bar r_1\times m_1\bar v_1 = m\omega_o r^2\bar k$

$\bar F_1 = -mg\bar k$ $\quad \bar r_1\times\bar F_1 = -rmg\,\bar e_r\times\bar k = -rmg\,\bar e_\theta$

$\bar r_2 = -re_r$ $\quad \bar v_2 = -\omega_o r\bar e_\theta$ $\Big\}$ $\bar r_2\times m_2\bar v_2 = m\omega_o r^2\bar k$

$\bar F_2 = -mg\bar k$ $\quad \bar r_2\times\bar F_2 = -r\bar e_r\times -mg\bar k = rmg$

$\bar r_1\times m_1\bar v_1 + \bar r_2\times \bar m_2\bar v_2 + \bar r_h\times \cancel{m_h}\bar v_h + \Sigma M$

$\therefore \quad \Sigma M_o = 0 \qquad \therefore \qquad \bar{r}_f = 2r e_r \qquad v = 2r\omega$

$\therefore \quad 8r^2\omega m = 2\bar{r} \times m\hat{v}$

$2m\omega_o r^2 \bar{k} = 8r^2 m\omega\bar{k} \qquad \omega = \frac{\omega_o}{4.}$ ✓

now $\frac{1}{2}\Sigma m v_i^2 - \frac{1}{2}\Sigma m v_f^2 = \frac{1}{2}\left[2m(\omega_o r)^2\right] - \frac{1}{2}\left[\Sigma 2m\left(\frac{\omega_o}{4}\cdot 2r\right)^2\right]$

$= m\omega_o^2 r^2 - \frac{1}{4}m\omega_o^2 r^2 = \frac{3}{4}m\omega_o^2 r^2 = \Delta E_{lost.}$

$\therefore \Delta E_{lost} = \frac{3}{4}\Delta E_{init}$

1. Angular Momentum
2. Problem in Linear & Angular Momentum

$\bar{H}_o = \bar{r} \times m\bar{v} = |r||mv| \sin\theta \; \bar{u}$ ū is ⊥ to plane formed by $\bar{r}$ & $\bar{v}$

$|m\bar{v}||d|$

$$\begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ mv_x & mv_y & mv_z \end{vmatrix}$$

$$\sum \bar{F} = m\bar{a} = m\dot{\bar{v}}$$

$$\sum \bar{M}_o = \bar{r} \times \sum \bar{F} = \bar{r} \times m\dot{\bar{v}}$$

$$\frac{d\bar{H}_o}{dt} = \frac{d\bar{r}}{dt} \times m\bar{v} + \bar{r} \times m\frac{d\bar{v}}{dt}$$

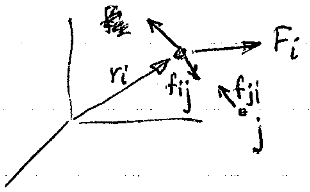
$$\underset{0}{\cancel{\bar{v} \times m\bar{v}}} + \bar{r} \times m\dot{\bar{v}} = \sum \bar{M}_o$$

- Sum of mom about a pt O due to forces acting on particle = $\frac{d}{dt}$ angular momentum.

$$(H_o)_2 = (H_o)_1 + \int M_o \, dt$$

lin mom $\sum \bar{F} = \frac{d}{dt}(m\bar{v})$

system of particles



$$\bar{r}_j = \bar{r}_i + \bar{r}_{j/i}$$

$$\sum \bar{r}_i \times (F_i + \bar{f}_{ij}) = \sum (M_o)_i = \sum \dot{\bar{H}}_{o_i} = \sum \bar{r}_i \times m\dot{\bar{v}}_i$$

$$\sum \bar{r}_i \times \bar{F}_i + \sum \bar{r}_i \times f_{ij}$$

$$\bar{r}_i \times f_{ij} + \bar{r}_j \times f_{ji}$$

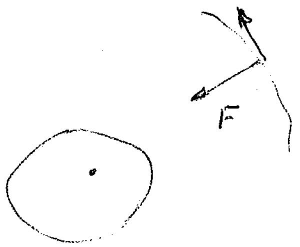
$$- \bar{r}_j \times f_{ij} = (\bar{r}_i + \bar{r}_{j/i}) \times \bar{f}_{ij}$$

$\bar{r}_{j/i}$ is // $\bar{f}_{ij}$ = 0 $\therefore$ $\sum \bar{r}_i \times \bar{f}_{ij} = 0$

for a system of particles $\sum \bar{r}_i \times \bar{F}_i = \sum \bar{M}_o = \frac{d}{dt}(\bar{H}_o)_{TOTAL.}$

- $(\bar{H}_o)_{2_{TOT}} = (\bar{H}_o)_{TOT} + \int \sum \bar{M}_o \, dt$

Do 177



$\bar{r} \times \bar{F} = 0$

$\therefore H_o$

$(\bar{H}_o)_2 = (\bar{H}_o)_1 = \bar{r} \times m\bar{v}$

$= r\bar{e}_r \times m(\dot{r}\bar{e}_r + r\dot{\theta}e_\theta)$

$= mr^2\dot{\theta}\,\bar{e}_r \times \bar{e}_\theta$

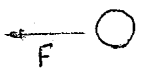
$mr^2\dot{\theta}\,\bar{e}_z$

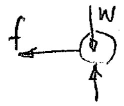
const const.

$r^2\dot{\theta} = \text{const}$

Do 2/195

3/206





$H_{o_1} = H_{o_2} \Rightarrow r_1^2 m\omega_1 = r_2^2 m\omega_2$

$\dot{H} = \frac{d}{dt}(mr^2\omega) = 0$

~~$\frac{d}{dt}$~~ $m \cdot 2r\frac{dr}{dt}\omega + mr^2\frac{d\omega}{dt} = 0$

$\frac{d\omega}{dr} = -\frac{2\omega}{r}$

$F = mv^2/r = mr^2\omega^2/r = mr\omega^2$

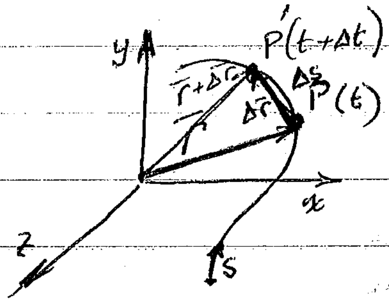
$dU = -F\,dr = dT$

$KE = \frac{1}{2}mv^2 = \frac{1}{2}mr^2\omega^2$

$dKE = \frac{1}{2}m \cdot 2r\,dr\,\omega^2 + \frac{1}{2}mr^2 \cdot 2\omega\,d\omega$

4/8

Curvilinear Motion (Particle)

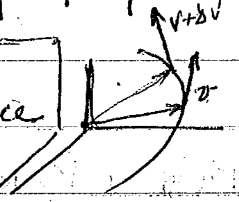


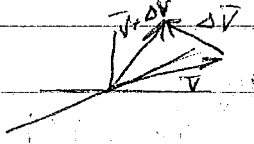
Definition $\bar v = \lim_{\Delta t\to 0} \frac{\Delta \bar r}{\Delta t}$

$\Delta \bar r = \Delta \bar r(t)$ $\boxed{\bar v = \frac{d\bar r}{dt}}$ velocity tangent to path

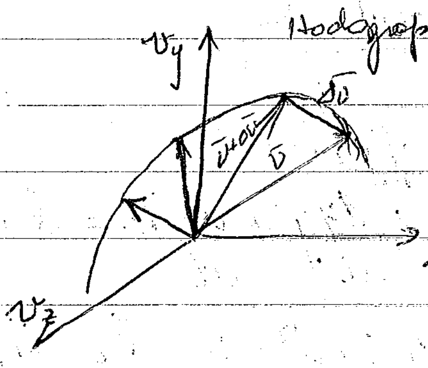
$\lim_{\Delta t\to 0} |\Delta \bar r| = |\Delta s|$ Speed $|\bar v| = \frac{ds}{dt}$

$$\boxed{\begin{array}{l} \bar a = \lim_{\Delta t\to 0} \frac{\Delta \bar v}{\Delta t} \text{ acce} \\ \bar a = \frac{d\bar v}{dt} = \frac{d^2\bar r}{dt^2} \end{array}}$$

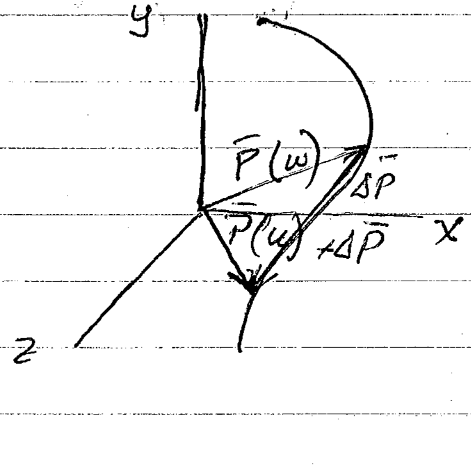




Hodograph Plane (Not responsible)



Derivative of Vector Function



$$\bar P(u) + \Delta \bar P = \bar P(u+\Delta u)$$

$$\Delta \bar P = \bar P(u+\Delta u) - \bar P(u)$$

$$\frac{d\bar P}{du} = \lim_{\Delta u\to 0} \frac{\bar P(u+\Delta u) - \bar P(u}{\Delta u}$$

$$= \lim_{\Delta u\to 0} \frac{\Delta \bar P}{\Delta u}$$

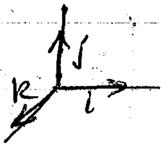
$$\frac{d(\bar{P}+\bar{Q})}{du} = \frac{d\bar{P}}{du} + \frac{d\bar{Q}}{du}$$

$f = f(u)$ $$\frac{d f\bar{P}}{du} = \frac{df}{du}\bar{P} + f\frac{d\bar{P}}{du}$$

$$\frac{d(\bar{P}\cdot\bar{Q})}{du} = \frac{d\bar{P}}{du}\cdot\bar{Q} + \bar{P}\cdot\frac{d\bar{Q}}{du}$$

$$\frac{d(\bar{P}\times\bar{Q})}{du} = \frac{d\bar{P}}{du}\times\bar{Q} + \bar{P}\times\frac{d\bar{Q}}{du}$$

eg



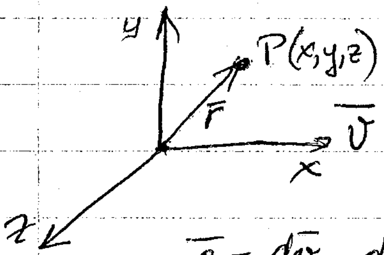
$$\frac{di}{du} = \frac{dj}{du} = \frac{dk}{du} = 0$$

i is fixed, constant

$$\bar{P} = P_x\bar{i} + P_y\bar{j} + P_z\bar{k}$$

$$\frac{d\bar{P}}{du} = \frac{dP_x}{du}\bar{i} + \frac{dP_y}{du}\bar{j} + \frac{dP_z}{du}\bar{k}$$

velocity and Accel in Rect coordinate.



$$\bar{r} = x\bar{i} + y\bar{j} + \bar{k}z$$

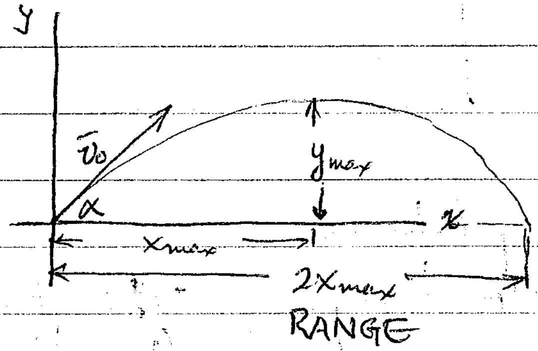
$$\bar{v} = \frac{d\bar{r}}{dt} = \frac{dx}{dt}\bar{i} + \frac{dy}{dt}\bar{j} + \frac{dz}{dt}\bar{k} = \dot{x}\bar{i} + \dot{y}\bar{j} + \dot{z}\bar{k}$$

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d^2\bar{r}}{dt^2} = \frac{d^2x}{dt^2}\bar{i} + \frac{d^2y}{dt^2}\bar{j} + \frac{d^2z}{dt^2}\bar{k} = \ddot{x}\bar{i} + \ddot{y}\bar{j} + \ddot{z}\bar{k}$$

$$|\bar{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2}$$

$v_x = \dot{x}$ $a_x = \ddot{x}$
$v_y = \dot{y}$ $a_y = \ddot{y}$
$v_z = \dot{z}$ $a_z = \ddot{z}$

Example - Projectile



$\ddot{x}=0 \qquad \dot{x}=v_0\cos\alpha$

$\ddot{y}=-g \qquad \dot{y}=v_0\sin\alpha$

$\dot{x}=C_1=v_0\cos\alpha$

$x=(v_0\cos\alpha)t+C_2$

$x(0)=0 \quad C_2=(-v_0\cos\alpha)t=0$

$$\boxed{x=(v_0\cos\alpha)t} \quad ②$$

$\dot{y}=-gt+C_3=v_0\sin\alpha$ at $t=0$,

$C_3=v_0\sin\alpha$

$y=-\frac{gt^2}{2}+(v_0\sin\alpha)t+C_4 \qquad C_4(\text{at } t=0)=0$

$$\boxed{y=-\frac{gt^2}{2}+(v_0\sin\alpha)t \quad ①}$$

$$y=-\frac{g}{2}\left[\frac{x^2}{v_0^2\cos^2\alpha}\right]+v_0(\sin\alpha)\left[\frac{x}{v_0\cos\alpha}\right]=x\left[\tan\alpha-\frac{g}{2}\frac{x}{v_0^2\cos^2\alpha}\right] \quad ③$$

$$\boxed{y=x\left[\tan\alpha-\frac{gx}{2v_0^2\cos^2\alpha}\right] \quad ③}$$

to find y_{max}

$\frac{d^2y}{dx^2}=0 \quad = \tan\alpha-\frac{gx}{v_0^2\cos^2\alpha}=0 \qquad x_{max}=\frac{\tan\alpha\cos^2\alpha\, v_0^2}{g}=\frac{\sin 2\alpha\, v}{2g}$

$$\boxed{x_{max}=\frac{v_0^2}{g}\frac{\sin 2\alpha}{2}}$$

$$\boxed{\text{Range}=\frac{v_0^2}{g}\sin 2\alpha=2x_{max}}$$

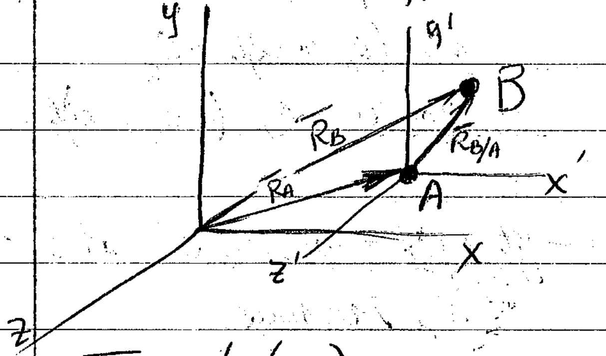
$$y_{max}=\frac{v_0^2}{g}\frac{\sin 2\alpha}{2}\left[\tan\alpha-\frac{g}{2v_0^2\cos^2\alpha}\,v_0^2\sin\alpha\cos\alpha\right]$$

$$=\frac{v_0^2}{g}\sin\alpha\cos\alpha\left[\frac{1}{2}\frac{\sin\alpha}{\cos\alpha}\right]=\frac{v_0^2}{2g}(\sin\alpha)^2$$

$$\boxed{y_{max} = \frac{v_0^2}{2g}(\sin\alpha)^2}$$

Moving Frames of Reference

Translation Only



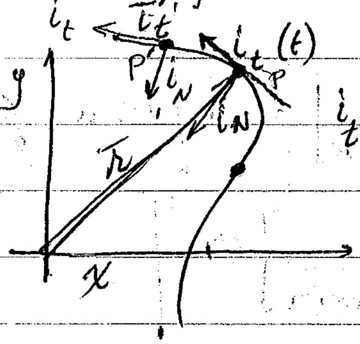
x, y, z "fixed" frame
(Newton's law holds - frame is when)
x'y'z' moving frame
x'//x

$$\bar{r}_B = \bar{r}_A + \bar{r}_{B/A}$$
$$\bar{v}_B = \bar{v}_A + \bar{v}_{B/A}$$
$$\bar{a}_B = \bar{a}_A + \bar{a}_{B/A}$$

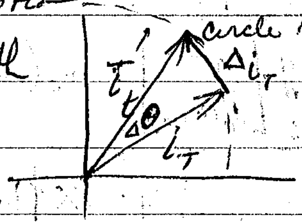
$$\bar{v}_{B/A} = \frac{d}{dt}(\bar{r}_{B/A})$$

Velocity and Acceleration - Normal & Tangential Components

Plane Motion



i_t tangent to path



$$\bar{i}_T + \Delta\bar{i}_T = \bar{i}_t'$$

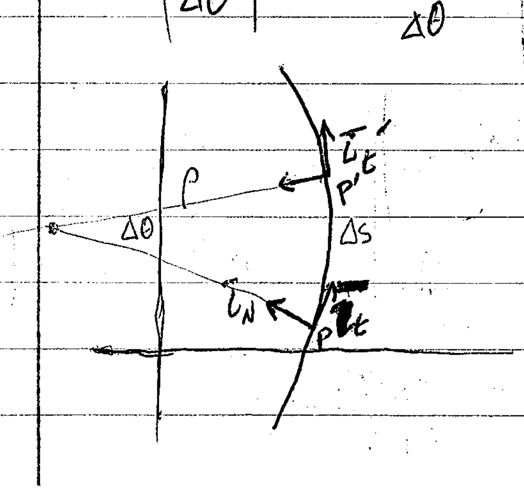
$$|\Delta\bar{i}_T| = 2\sin\frac{\Delta\theta}{2}$$

$$\lim_{\Delta\theta\to0}\left|\frac{\Delta\bar{i}_T}{\Delta\theta}\right| = \lim_{\Delta\theta\to0}\frac{2\sin\frac{\Delta\theta}{2}}{\Delta\theta} = 1$$

$$\lim_{\Delta\theta\to0}\left|\frac{\Delta\bar{i}_T}{\Delta\theta}\right| = \left|\frac{d\bar{i}_T}{d\theta}\right|$$ unit vector ⊥ to $\bar{i}_T$

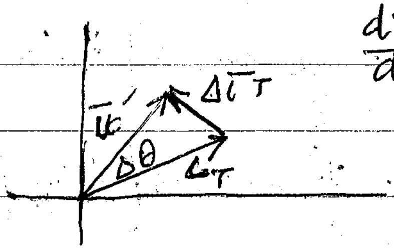
$$= \bar{i}_n$$

116, 136
Chapt 11



$$\lim_{\Delta\theta\to0}\frac{\Delta\bar{i}_T}{\Delta\theta} = \frac{d\bar{i}_t}{d\theta} = \lim_{\Delta\theta\to0}\left|\frac{\Delta\bar{i}_T}{\Delta\theta}\right| = 1$$

$$\frac{d\bar{i}_t}{d\theta} = \bar{i}_n$$



Intrinsic or Path Coordinate System

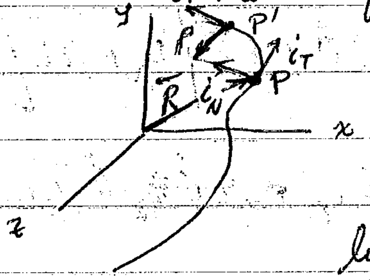
$\boxed{\bar v = v\,\bar i_t}$ $\bar a = \frac{d\bar v}{dt} = \dot v\,\bar i_T + v\,\frac{d\bar i_T}{d\theta}\cdot\frac{d\theta}{ds}\cdot\frac{ds}{dt}$

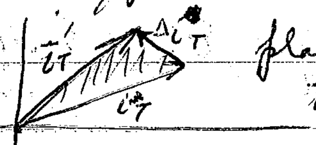
$\bar a = \dot v\,\bar i_T + v\,\bar i_N\,\rho\, v$ since $\Delta s = \rho\Delta\theta$ then $\rho = \frac{ds}{d\theta}$

or $\frac{1}{\rho} = \frac{d\theta}{ds}$ ρ radius of curvature

$$\boxed{\bar a = \underset{\overset{\uparrow}{a_T}}{\dot v\,\bar i_T} + \underset{\overset{\uparrow}{a_N}}{\frac{v^2}{\rho}\bar i_N}}$$

Motion of a Particle in Space





plane formed by $\bar i_T, \bar i_T', \Delta\bar i_T$

formed by

$\lim_{\Delta\theta\to 0}$ Plane $[\bar i_T, \bar i_T', \Delta\bar i_T] \longrightarrow$ Osculating plane

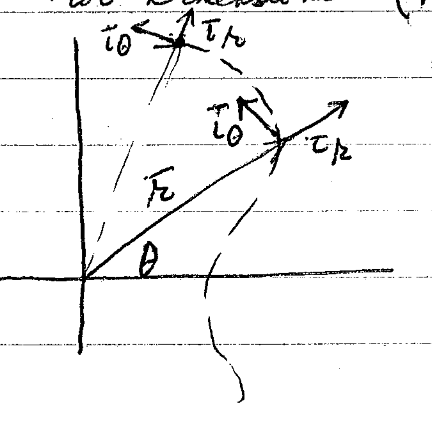
$\bar i_N$ is in the Osculating plane "principal normal vector".

A third vector: $\bar i_b = \bar i_T \times \bar i_N$ $\bar i_b$ "binormal" vector

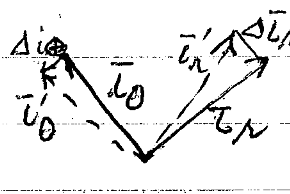
accelerations acting on a particle must act in osculating Plane, otherwise, acceleration of particle must also act in direction of that acceleration not in osculating plane but this is not ~~the~~ true since $\bar a$ has components in osculating plane $[\dot v\,\bar i_T, \frac{v^2}{\rho}\bar i_N]$.

Velocity & Acceleration in Radial & Transverse Coordinates

Two Dimensional (Polar Coordinate)



$\bar i_r$ in radial direction $\bar i_\theta \perp \bar i_r$



$\frac{d\bar i_r}{d\theta} = \bar i_\theta$; $\frac{d\bar i_\theta}{d\theta} = -\bar i_r$

$\bar{r} = r\,\bar{i}_r$

$$\bar{v} = \frac{d\bar{r}}{dt} = \dot{r}\,\bar{i}_r + r\,\frac{d\bar{i}_r}{d\theta}\cdot\frac{d\theta}{dt}$$

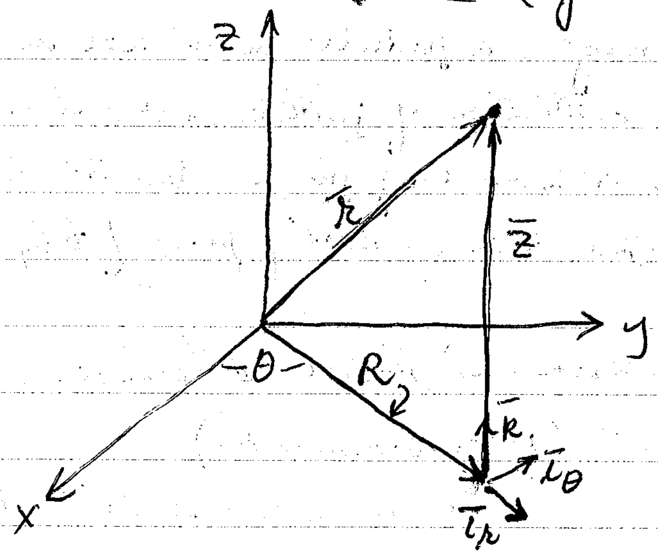
$$= \underbrace{(\dot{r})}_{v_r}\bar{i}_r + \underbrace{(r\,\dot{\theta})}_{v_\theta}\bar{i}_\theta \qquad \bar{v} = \dot{r}\,\bar{i}_r + r\theta\,\bar{i}_\theta$$

$$\bar{a} = \frac{d\bar{v}}{dt} = \ddot{r}\,\bar{i}_r + \dot{r}\,\frac{d\bar{i}_r}{d\theta}\cdot\frac{d\theta}{dt} + \left[\dot{r}\dot{\theta} + r\ddot{\theta}\right]\bar{i}_\theta + r\dot{\theta}\left[\frac{d\bar{i}_\theta}{d\theta}\cdot\frac{d\theta}{dt}\right]$$

$$\ddot{r}\,\bar{i}_r + \dot{r}\dot{\theta}\,\bar{i}_\theta + \dot{r}\theta\,\bar{i}_\theta + r\ddot{\theta}\,\bar{i}_\theta - r\dot{\theta}\,\dot{\theta}\,\bar{i}_r$$

$$\bar{a} = \underbrace{(\dot{r} - r\dot{\theta}^2)}_{a_r}\bar{i}_r + \underbrace{\left[2\dot{r}\dot{\theta} + r\ddot{\theta}\right]}_{a_\theta}\bar{i}_\theta$$

Extension to 3 dimensions (Cylindrical Coordinates)

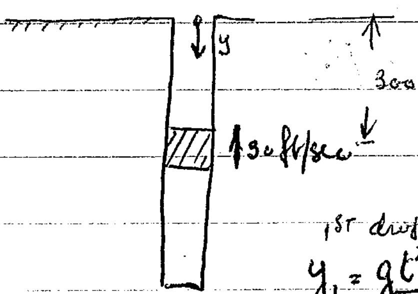


$\bar{r} = R\,\bar{i}_r + z\bar{k}$

$\bar{v} = (\dot{R})\bar{i}_r + R\dot{\theta}\,\bar{i}_\theta + \dot{z}\,\bar{k}$

$\bar{a} = (\ddot{R} - R\dot{\theta}^2)\bar{i}_r + (R\ddot{\theta}\,2\dot{R}\dot{\theta})\bar{i}_\theta$
$+ \ddot{z}\,\bar{k}$

11.32



Motion of 1ST PART.

$\ddot{y} = +g$

$\dot{y} = gt + C_1 \quad C_1 = 0 \quad \dot{y}(0) = 0$

$y = g\frac{t^2}{2} + C_2 \quad C_2 = 0 \quad y(0) = 0$

1ST drop.

$y_1 = g\frac{t^2}{2} \qquad y_2 = g\frac{(t-1)^2}{2}$ motion of second drop

Motion of elevator $\quad \dot{y}_E = -30. \qquad y_E = -30t + C_3$

1ST drop hits elevator $\quad t^* = t \qquad y_1 = \frac{g(t^*)^2}{2} \qquad 300 = \frac{32(t^*)^2}{2} = 16.1(t^*)^2$

$t^{*2} = 18.61 \qquad t^* = 4.32$ sec.

$y_E = 300 = -30(4.32) + C_3 \qquad C_3 = 429.6$ ft.

$y_E = -30t + 429.6$ ft

$y_2 = \frac{g(t-1)^2}{2}$ equate $y_2 = y_E$

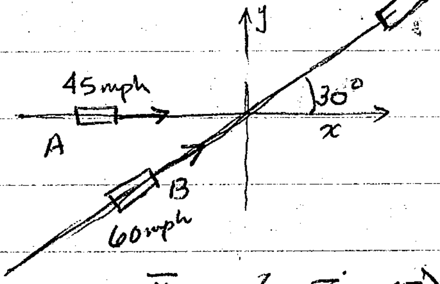
$\frac{g(t-1)^2}{2} = -30t + 429.6 \qquad (16.1)[t^2 - 2t + 1] = -30t + 429.6$

$t^2 - .1367t - 25.6 = 0 \qquad t = 5.13$ sec

$y_E = -30(5.13) + 429.6 = 275.5$ ft time 5.13 sec after first drop is dropped or .81 sec after 1st drop hit.

$\bar{V}_{B/A} = \bar{V}_B - V_A$

11.104



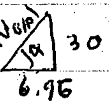
$\bar{V}_B = 60\cos 30°\,\bar{i} + 60\sin 30°\,\bar{j}$

$\bar{V}_B = 30\sqrt{3}\,\bar{i} + 30\bar{j}$

$\bar{V}_A = 45\bar{i}$

$\bar{V}_{B/A} = (30\sqrt{3} - 45)\bar{i} + 30\,\bar{j}$

$\bar{V}_{B/A} =$

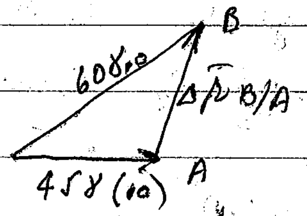


$V_{B/A} = 30.85$ mph $\quad \alpha = 76.9°$

Chapt #10, 30, 50, √4 + [redacted] 63

Part B

$v = 1.467$



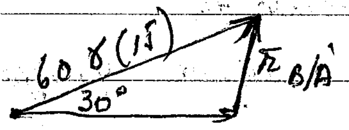
$$\bar{v}_{B/A} = \frac{d}{dt}(\bar{r}\,B/A)$$

$$\bar{r}_{B/A} = \bar{v}_{B/A}\,dt + (\bar{r}_{B/A})_0$$

$$(\bar{r}_{B/A})_{10} - (\bar{r}_{B/A})_0 = [7\bar{i} + 30\bar{j}]\,8.10$$

$$|\Delta\bar{r}_{B/A}| = 45\gamma \text{ feet}$$

Part (c)

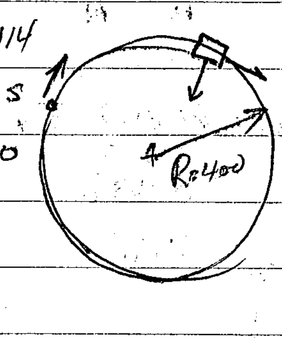


$$r^2_{B/A} = [60(15)\gamma]^2 + [450\,8]^2 - 2[900\gamma][450\gamma]\cos 30^\circ$$

$$r_{B/A} = 818 \text{ ft.}$$

11.114

s(0) = v(0) = 0



accelerates along track at $a = 3$ ft/sec²

Find $S \Rightarrow a$ total $= 6$ ft/sec²

$$a_t = \frac{dv}{dt}, \quad a_n = \frac{v^2}{R}$$

$a_t = 3$ $\qquad a_t^2 + a_n^2 = 6^2$ $\qquad a_n^2 = 6^2 - 3^2$

$\frac{v^4}{R^2} = 27$ $v^4 = 27(400)^2$ $v^2 = 400(3)\sqrt{3}$

$a_t = \frac{dv}{dt} = v\frac{dv}{ds}$ $3ds = v\,dv$

$3s + S_0 = \frac{v^2}{2} + v_0$

$3s = \frac{v^2}{2}$ $6s = 400(3)\sqrt{3}$

$S = 200\sqrt{3} = \underline{\underline{347 ft.}}$

KINETICS

Newton's Second Law $\overline{F} = m\overline{a}$

Cartesian -

$$F_x = m\ddot{x} \; ; \; F_y = m\ddot{y} \; ; \; F_z = m\ddot{z}$$

Plane

Polar Coordinary

$$F_r = m[\ddot{r} - r\dot{\theta}^2] \; ; \; F_\theta = m[2\dot{r}\dot{\theta} + r\ddot{\theta}]$$

Tang - Normal

$$F_T = m\frac{dv}{dt} \; , \; F_N = \frac{mv^2}{\rho}$$

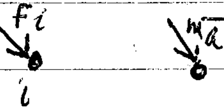
Fixed frame - Newtonian frame Newton's law hold.

British Gravitational units

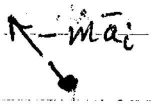
$$\left.\begin{array}{ll} \text{Force} & \text{lb} \\ \text{Length} & \text{ft} \\ \text{Time} & \text{sec} \end{array}\right\} \text{independent} \quad m \text{ slug} \quad \frac{\text{lb-sec}^2}{\text{ft}}$$

Dallmbert's Principle

Consider a particle



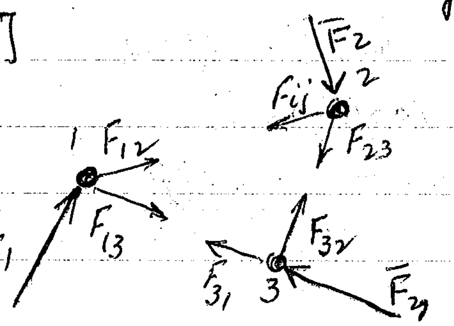
effective force particle $m\bar{a}_i$



$= [-m\bar{a}_i]$ reverse effective force

D'alembert force.

[S]

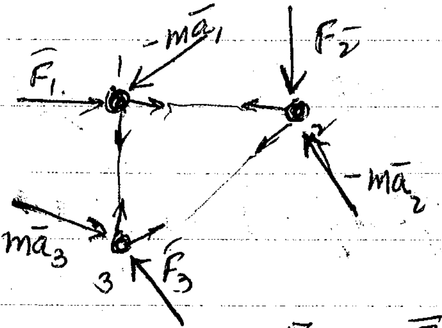


Dynamical

$F_{12} = -F_{21}$

Same configuration but at rest and a set of forces equal to reversed effective forces

[S']



In system [S'] $\bar{F}_i = m\bar{a}_i$

$\therefore \sum \bar{F}_i = \sum m_i a_i$

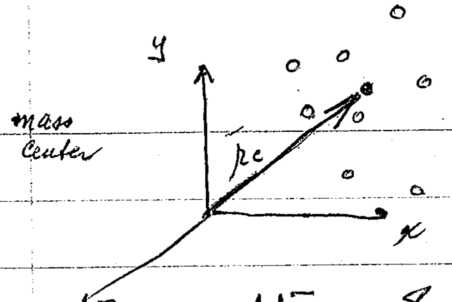
$$\boxed{\sum \bar{F}_i - \sum_i m_i a_i = 0}$$

D'Alembert The reversed effective forces and the real forces give static equilibrium

$$\bar{F}_i = \bar{F}_i^{(e)} + \sum_{j=1}^{j} \bar{F}_{ij}$$

$$\sum_i F_i = \sum_i \bar{F}_i^{(e)} + \sum_i \sum_j \bar{F}_{ij} = \sum_i \bar{F}_i^{(e)}$$ since $\bar{F}_{ij} = -\bar{F}_{ji}$

Motion of Mass Center



$$\bar{r}_c = \frac{\sum m_i \bar{r}_i}{\sum m_i}$$

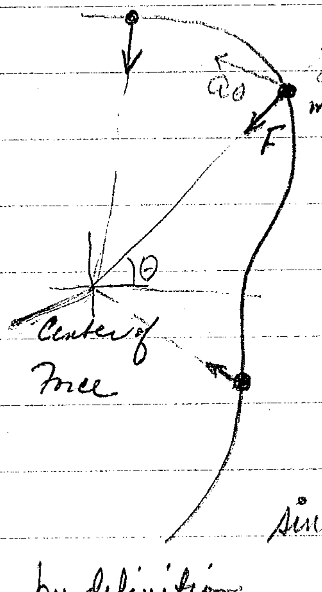
$$M\frac{d\bar{r}_c}{dt} = \quad M\bar{v}_c = \sum_i m_i \bar{v}_i$$

$$M\frac{d\bar{v}_c}{dt} \quad M\bar{a}_c = \sum_i m_i \bar{a}_i \quad ①$$

but $\sum \bar{F}_i = \sum M_i \bar{a}_i = \sum_i \bar{F}_i^{(e)}$ ②

$$\sum \bar{F}_i^{(e)} = M\bar{a}_c$$

Central Force Motion – F always acts along line connecting particle & center of Force which is fixed.



Polar Coordinates $\bar{F} = F_R + F_\theta$

$\bar{F}_\theta = 0$ in central force field

$$F_r = m\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right] = m\left[\ddot{r} - r\dot{\theta}^2\right] \quad ①$$

$$\overset{0}{\cancel{F_\theta}} = m\left[\frac{rd^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right] = m\left[r\ddot{\theta} + 2\dot{r}\dot{\theta}\right] ②$$

since only F_r is only F acting on particle

by definition

from ② $\quad r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \Longrightarrow \frac{1}{r}\frac{d(r^2\dot{\theta})}{dt} = 0$

$\frac{d}{dt}(r^2\dot{\theta}) = 0$ or

$$r^2\dot{\theta} = \text{Constant} = h \quad ③$$

Equation ①

let $r = \frac{1}{u}$, note: $\dot{\theta} = hu^2$ from ③

$$dr/dt = \frac{d(1/u)}{d\theta} \cdot \frac{d\theta}{dt} = \dot{\theta}\left(-u^{-2}\frac{du}{d\theta}\right) = hu^2 - \left(\frac{du/d}{u^2}\right.$$

$= -h\frac{du}{d\theta}$

$$\ddot{r} = \frac{d\left(-h\frac{du}{d\theta}\right)}{dt} = \frac{d}{d\theta}\left(-h\frac{du}{d\theta}\right)\cdot\dot{\theta} = hu^2\left(-h\frac{d^2u}{d\theta^2}\right) \quad ④$$

Take 4 and 3 into 1

$$F_r = m\left[-h^2u^2\frac{d^2u}{d\theta^2} - \frac{1}{u}h^2u^4\right]$$

$$\frac{F_r}{mh^2u^2} = -\frac{d^2u}{d\theta^2} - u \quad ⑤$$

Newton's Law

$$F_r = -GMmu^2 = -GMmr^2 \quad ⑥$$ since F_r acts towards center

$$\frac{-GMmu^2}{mh^2u^2} = -\frac{d^2u}{d\theta^2} - u \qquad \frac{-GM}{h^2} = -\frac{d^2u}{d\theta^2} - u$$

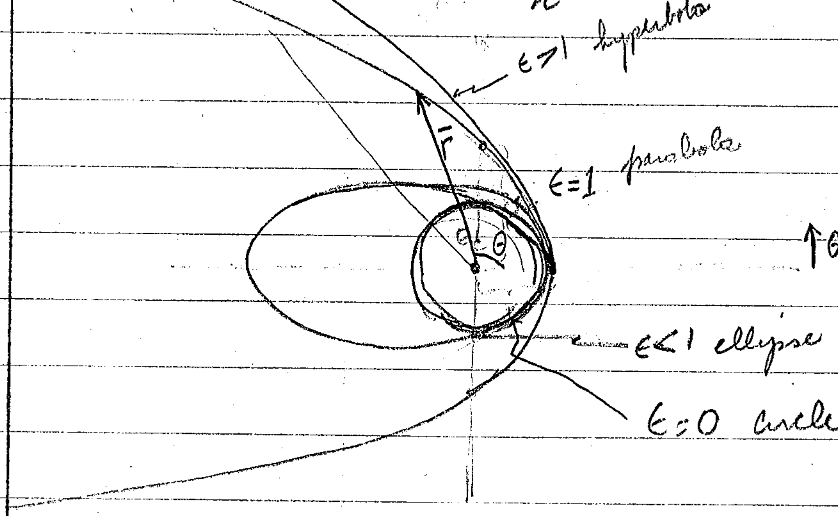
$$\frac{GM}{h^2} = \frac{d^2u}{d\theta^2} + u$$

origin at focus
conic section

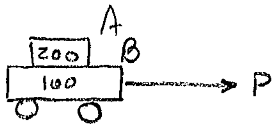
ⓐ $$u = \underbrace{A\sin\theta + B\cos\theta}_{u_c} + \underbrace{\frac{GM}{h^2}}_{u_p}$$

ⓒ $$u = \frac{GM}{h^2}\left[1 + \epsilon\cos\theta\right] = \frac{1}{r}$$

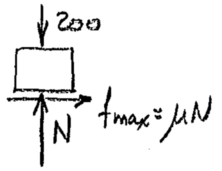
ⓑ $$u = C\cos(\theta - \theta_0) + \frac{GM}{h^2}$$



12.32



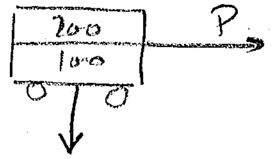
$\mu = 1/4$



$-200 + N = Ma(0) \quad N = 200^{\#}$

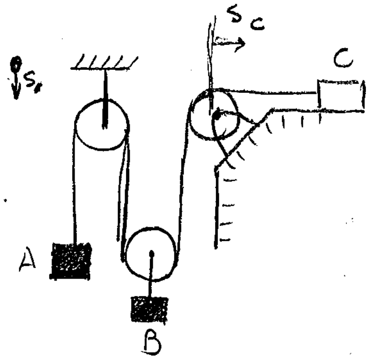
$f_{max} = m_A(a) \quad \frac{1}{4} N = \frac{200}{g}[a_A]_x$

$\frac{1}{4}(200) = \frac{200}{g}(a_A)_x = [a_A]_x = g/4$



$P_{max} = \frac{300}{g}\left(g/4\right) = 75^{\#}$

12.44



$\mu_c = .3 \quad W_c = 10 \quad a_A = a_B = g/5 \downarrow$

find W_A, W_B

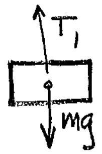
$s_A + 2s_B + s_c = $ Constant

$v_A + 2v_B + v_c = 0$

$a_A + 2a_B + a_c = 0$ ①

$\frac{3g}{5} + a_c = 0 \quad a_c = -\frac{3g}{5}$

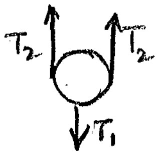
Isolate B



$W_b - T_1 = \frac{W_b}{g}(\ddot{s}) = g/5 \left(\frac{W_b}{g}\right) = \frac{W_b}{5}$

$T_1 = +\frac{4}{5} W_b$ ②

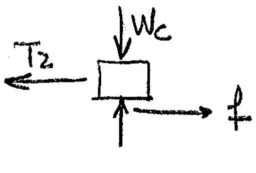
Isolate B pulley



$T_1 - 2T_2 = ma$ but $m = 0$

$\frac{T_1}{2} = T_2 = \frac{2}{5} W_b$

Isolate body C

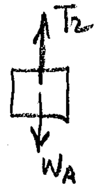


⑤ $W_c = N$ ④ $-T_2 + f = \frac{W_c}{g}\left(-\frac{3}{5}g\right) = \frac{W_c}{g} \cdot a_c$

$-\frac{2}{5} W_b + .3W_c = -\frac{W_c(3)}{5}$

$-\frac{2}{5} W_b = -W_c(.9) = -9 \quad \boxed{W_b = 22\frac{1}{2}^{\#}}$

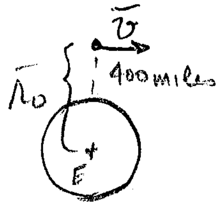
Isolate A



$$W_A - T_2 = \frac{W_A}{g}(a_g)$$

$$\therefore \; -T_2 = -\frac{4}{5}W_A = -\frac{2}{5}W_B \qquad \boxed{W_A = \frac{1}{2}W_B = 11\tfrac{1}{4}}$$

12,84



find $|\bar{v}| = ?$ $\quad \Rightarrow \quad \epsilon = 0$

$$\frac{1}{r} = u = \frac{GM}{h^2}\left[1 + \epsilon\cos\theta\right]$$

$$\frac{1}{r} = \frac{GM}{h^2} \qquad GM = gR^2 = 140.6 \times 10^{14} \text{ ft}^3/\text{sec}^2$$

$$\frac{GMm}{R^2} = mg \qquad R_0 = (3960 \text{ miles} + 400 \text{ miles}) = 23 \times 10^6 \text{ feet}$$

$$\frac{1}{r_0} = \frac{gR^2}{(r^2\dot{\theta})^2} \qquad \text{recall} \quad \bar{v} = \underbrace{\dot{r}}_{v_r}\bar{i}_r + \underbrace{r\dot{\theta}}_{v_\theta}\bar{i}_\theta$$

$$\frac{1}{r_0} = \frac{140.6 \times 10^{14}}{(r_0 V_\theta)^2} = \frac{140,6 \times 10^{14}}{r_0^2 v^2}$$

$$v^2 = \frac{140,6 \times 10^{14}}{23,0 \times 10^6} \qquad v = 24,750 \text{ ft/sec}$$

400 miles apogee

200 mile perigee

for apogee

$$\frac{1}{r_0} = \frac{GM}{h^2}\left(1 + \epsilon\cos\theta\right) = \frac{GM}{h^2}\left(1 + \epsilon\right) \quad ①$$

$$\frac{1}{r_1} = \frac{GM}{h^2}\left(1 - \epsilon\right) \quad ②$$

$$r_0(1+\epsilon) = r_1(1-\epsilon)$$
$$\epsilon(r_0 + r_1) = r_1 - r_0$$
$$\epsilon = \left(\frac{r_1 - r_0}{r_0 + r_1}\right)$$
$$\equiv ,024$$

$r_1 = 3960 + 200$

$r_2 = 3960 + 400$

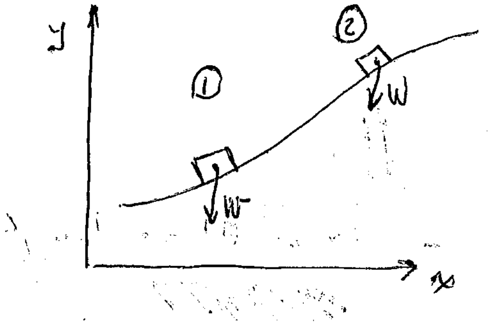
$$\frac{1}{r_0} = \quad \frac{1}{23 \times 10^6} = \frac{140,6 \times 10^{14}}{r_0^2 v^2}(0,976)$$

$$v_0^2 = \frac{140,6 \times 10^{14}}{23 \times 10^6}(,976)$$

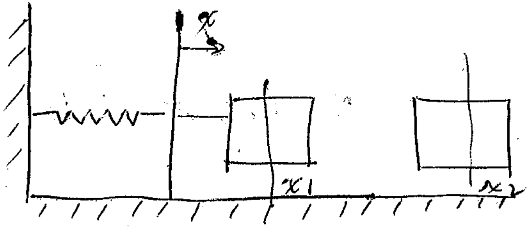
$$V_\theta = \sqrt{\downarrow \quad (,976)} = \sqrt{,976}\,(24,750$$

Work and energy Methods

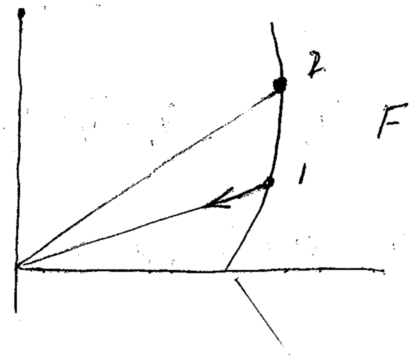
1. $\bar{F} = m\bar{a}$
2. Work and Energy Method. $F, m, v, s,$
3. Impulse & Momentum F, m, v, t



$$U_{1-2} = -Wy_1 - Wy_2$$

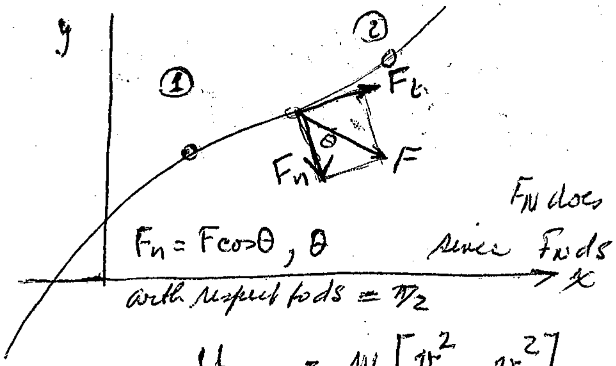


$$U_{1-2} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$



$$F = \frac{GMm}{r^2} \qquad U_{1-2} = \frac{GMm}{r_2} - \frac{GMm}{r_1}$$

Kinetic Energy of a particle



$F_T = ma_T$

$F_N = ma_N$

$F_t = m\frac{dv}{dt}\cdot\frac{ds}{ds}$

$= m v\frac{dv}{ds}$

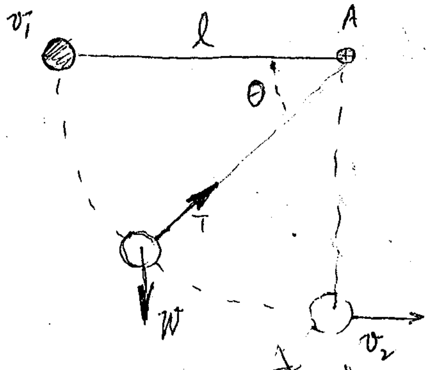
F_N does no work since $F_N ds = 0$ $\uparrow^{F_n}_{ds}$

$F_n = F\cos\theta$, θ with respect to $ds = \pi/2$

$$\int_1^2 F_t\,ds = \int_1^2 m v\,dv$$

$$U_{1-2} = m\left[\frac{v_2^2}{2} - \frac{v_1^2}{2}\right] \qquad \frac{mv^2}{2} = T$$

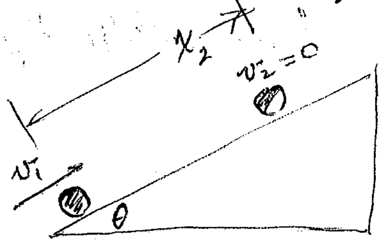
and $U_{1-2} = T_2 - T_1$ principle of Work and Energy



$\mathcal{U}_{1-2} = T_2 - T_1$

$W(l) = \frac{1}{2}mv_2^2 - 0$

$v_2 = \sqrt{2gl}$



$-W(x_2\sin\theta) = -\frac{1}{2}mv_1^2$

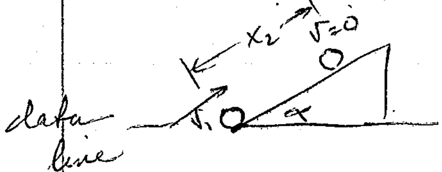
$x_2 = \frac{v_1^2 g}{2\sin\theta}$

$v_1 = \sqrt{2g\,x_2\sin\theta}$

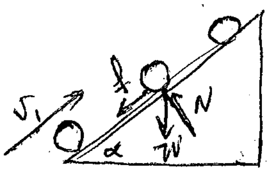
If field is conservative $\quad d\mathcal{U} = -dV$ or $\mathcal{U}_{1-2} = V_1 - V_2$ ①

$\mathcal{U}_{1-2} = T_2 - T_1$ ②

$V_1 - V_2 = T_2 - T_1 \qquad V_1 + T_1 = V_2 + T_2$ ③ Principle of Conservation of Energy



$\frac{1}{2}mv_1^2 + 0 = 0 + Wx_2\sin\alpha$



$\mathcal{U}_{1-2} = T_2 - T_1$

$Wx_2\sin\alpha + \mu x_2 W\cos\alpha = \frac{1}{2}mv_1^2$

$v_1 = \sqrt{2g(x_2[\sin\alpha + \mu\cos\alpha])}$

2 hr exam closed book

Questions in statics & dynamics 5 quest.; 3 quest. in dynamics no work & energy methods & impulse in momentum.

2 quest in statics Choice

Chapt 2. - scalar vectors, equal, free, sliding vectors equilibrium of particle

$$\Sigma \overline{F} = 0 \quad \Sigma F_x, \Sigma F_y, \Sigma F_y = 0$$

Newton's 3rd law. Draw free body diagram.

Chapt 3 - scalar product Work = F · ds, vector product Moment $\overline{r} \times \overline{F}$, moment about an axis

$\overline{M}_o = \overline{r} \times F$.

Chapt 3 - Moment of Couple

Reduce system to 1 force & 1 couple (forget Wrench)

Chapt 4 - Equilibrium of Rigid Body

$$\Sigma M = 0 \qquad \Sigma M_x = \Sigma M_y = \Sigma M_z = 0$$

2 force bodies & 3 force bodies moment about convenient pt.

Chapt 5 - Skip.

Chapt 7 - Internal forces 3 first pages.

Chapt 6 - Definition of Trusses - 2 forces members Method of Joints & Sections. Frames - resist bend Machines use loadings

Chapt 8 - Coulomb's law, write down equations of equil & impending motion

Chapter 10 - Omit

Chapter 11 - Position vector, velocity vector, acceleration vector $a = a(t)$, $a = (s) \Rightarrow a = v\,dv/ds$

$a = a(v)$

look at uniform acceleration don't memorize form

Curvilinear motion ~~as~~ $\bar{r} = \bar{r}(t)$, ~~radial coordinates~~
path coordinates, cylindrical, cartesian know derivat
Projectile problems, relative motion.

Chapter 12 – Particle dynamics

$$\bar{p} = m\bar{v}$$

$$F = d\bar{p}/dt = m\bar{a}$$

Know problems in all three types of coordinates

D'Alembert Principle can be used.

Center of Mass – mass moves as one particle having ~~the~~ total mass & all external forces acting on it.

integral problems & algebraic

→ Central force problem – Kepler's 2nd law
areal velocity, newton's inverse square law

$F = \frac{GMm}{r^2}$ conic section depending on eccentricity