LESSON #1

- STATICS - EQUILIB OF BODIES AT REST OR MOVING W/CONST. VEL
- DYNAMICS - BODIES UNDERGO ACCELERATION
  - KINEMATICS LOOK AT GEOMETRICAL ASPECTS OF MOTION ONLY
    - RECTILINEAR MOTION
    - CURVILINEAR MOTION
  - KINETICS LOOKS AT FORCES CAUSING MOTION

- LOOK AT PARTICLE DYNAMICS FIRST.
- NEXT - RIGID BODY MOTION

FIRST TOPIC - KINEMATICS OF A PARTICLE

PARTICLE HAS MASS BUT SHAPE IS NEGLIGIBLE

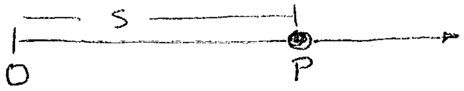
- BY FOLLOWING MASS CENTER OF A FINITE DIMENSION BODY WE CAN TREAT BODIES OF FINITE SIZE AS PARTICLES
- TO DO THIS MUST NEGLECT ANY ROTATIONS OF THE BODY.

→ FIRST STUDY MOTION OF PARTICLE MEASURED WRT FIXED COORD. SYSTEM

RECTILINEAR MOTION - MOTION OF PARTICLE ALONG STRAIGHT PATH.

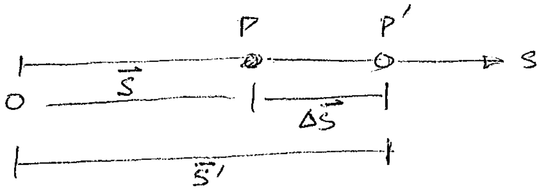
KINEMATICS ARE PARTICLE'S POSITION, VELOC & ACCEL

POSITION - THE LOCATION OF A PARTICLE AT ANY INSTANT MEASURED RELATIVE TO A FIXED PT O, IT IS A VECTOR < MAGNITUDE, DIRECTION



DISTANCE TRAVELED IS THE SCALAR MAGNITUDE OF THE POSITION VECTOR

DISPLACEMENT - IS A CHANGE IN POSITION, IT IS A VECTOR



IF $P' > P \quad \Delta\vec{S} > 0$

IF $P' < P \quad \Delta\vec{S} < 0$

VELOCITY - AVERAGE VELOCITY = DISPLACEMENT DIVIDED BY TIME TAKEN FOR PARTICLE TO MOVE THRU DISPL.

$$\vec{V}_{ave} = \frac{\Delta\vec{S}}{\Delta t} = \frac{\vec{S}' - \vec{S}}{\Delta t}$$

$\underline{S}' = 4\underline{i} + 3\underline{j} + 2\underline{k}$

$\underline{S} = 4\underline{i} - 2j + 7\underline{k}$

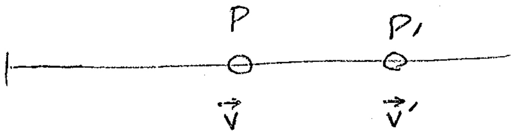
$\Delta\underline{s} = 0\underline{i} + 5\underline{j}$

$\Delta t = 1 sec$

$\longrightarrow$ INSTANTANEOUS VELOCITY AT POSITION P $\quad \vec{v}\Big|_P = \frac{d\vec{s}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\vec{s}}{\Delta t}$

VELOCITY IS A VECTOR MAGNITUDE OF VELOCITY = SPEED

IF WE CAN FIND THE VELOCITIES A 2 PTS P, P'



$$\Delta\bar{V} = \bar{V}' - \bar{V}$$

- DEFINE ACCELERATION

AVERAGE ACCEL = VELOC. CHANGE DIVIDED BY TIME TAKEN TO CAUSE THIS CHANGE

$$\vec{a}_{ave} = \frac{\bar{V}' - \bar{V}}{\Delta t} = \frac{\Delta\bar{V}}{\Delta t}$$

INSTANTANEOUS ~~[illegible]~~ ACCELERATION AT POSITION P $\quad a\Big|_P = \frac{d\bar{v}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\bar{v}}{\Delta t}$

$$\vec{a}\Big|_P = \frac{d\bar{v}}{dt}\Big|_P = \frac{d}{dt}\left(\frac{d\vec{s}}{dt}\right) = \frac{d^2}{dt^2}\vec{s}\Big|_P$$

IT IS ALSO A ~~SCALAR~~ VECTOR. IF $\Delta\vec{v} > 0 \quad \vec{a} > 0$ ACCELERATION

$\Delta\vec{v} < 0 \quad \vec{a} < 0$ DECELERATION

CAN ELIMINATE dt. Multiply $a = \frac{dv}{dt}$ by $v = \frac{ds}{dt}$

$$\frac{dv}{\cancel{dt}} v = a \frac{ds}{\cancel{dt}} \Rightarrow \underline{a\,ds = v\,dv}$$

IF $\boxed{a = \text{const}}$ $\quad \int a\,ds = a \int ds = \int v\,dv = \frac{v^2}{2} + \text{constant}$

IF when $V = V_0$ $\quad S = S_0 \Rightarrow 2a(S - S_0) = V^2 - V_0^2 \Rightarrow \underline{\underline{V^2 = V_0^2 + 2a(S - S_0)}}$

$a = \text{constant}$ $\quad a = \frac{dv}{dt} \Rightarrow \int a\,dt = \int dv \Rightarrow a \int dt = V + \text{const}$

IF at $t = t_0$ $\quad V = V_0 \Rightarrow a(t - t_0) = V - V_0$ or $\underline{V = V_0 + a(t - t_0)}$

IF $t_0 = 0 \Rightarrow \boxed{V = V_0 + at}$

IF at $t = t_0$ $\quad V = V_0$ $\quad S = S_0$ $\quad V = \frac{ds}{dt} \Rightarrow \int V\,dt = \int ds$

$$\int [V_0 + a(t - t_0)]\,dt = \int ds \Rightarrow V_0 t + \frac{a}{2}(t - t_0)^2 + \text{const} = S$$

$$V_0(t - t_0) + \frac{a}{2}(t - t_0)^2 = S - S_0 \Rightarrow \underline{S = S_0 + V_0(t - t_0) + \frac{a}{2}(t - t_0)^2}$$

IF $t_0 = 0$ $\quad \underline{\underline{S = S_0 + V_0 t + \frac{at^2}{2}}}$

IN ORDER TO SOLVE PROBLEMS MUST ASK WHAT AM I GIVEN
WHAT EQUATIONS CAN I USE
HOW CAN I GET ANSWER

EXAMPLE 12-10

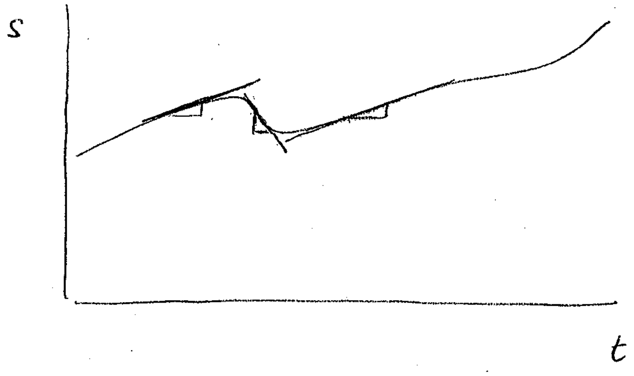
$v = \frac{4}{a} \Rightarrow v \frac{dv}{dt} = 4$ and $\frac{v^2}{2} = 4t + \text{Const}$

$v = 6$ @ $t = 2$ $\quad 18 = 8 + \text{Const}$ $\quad \text{Const} = 10 \Rightarrow V^2 = 8t + 20$

find V at $t = 3$ sec $\quad V = \sqrt{8t + 20}$ $\quad dV/dt = \frac{1}{2} \frac{8}{\sqrt{8t + 20}} = \frac{4}{\sqrt{8t + 20}} = \frac{4}{\sqrt{44}} = \frac{2}{\sqrt{11}}$

GRAPHICAL REPRESENTATION

- Suppose data given in graphical form ie s as a fn of t



- TAKE SLOPE AT A PT THIS GIVES VELOCITY AT PT

IF V is given as a fn of t

- SLOPE AT A PT GIVES ACCELERATION

$V = \lim_{\delta t \to 0} \frac{\Delta s}{\Delta t}$ = slope of curve (s, t curve)

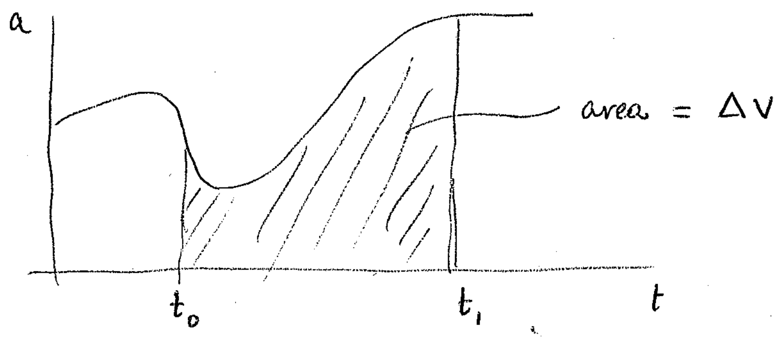
LOOK AT
EXAMPLE 12-7
p. 15

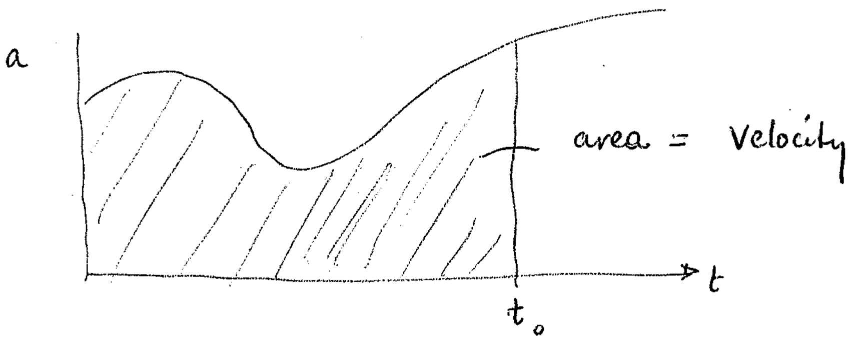
Similarly $a = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t}$ = slope of (v-t curve)

- suppose given a as a fn of t & want velocity

$$a = \frac{dv}{dt} \quad \Rightarrow \quad \int_{t_0}^{t_1} a\,dt = \int_{V_0}^{V_1} dV = V_1 - V_0 = V\Big|_{V_0}^{V_1}$$

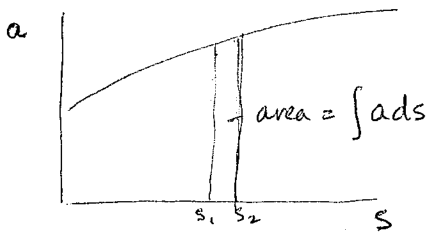
Area under curve represents relative velocity change or change in particle speed





LOOK AT
EXAMPLE 12-8
p. 17

Suppose you are given a vs. s curve ~~& you want v vs. s curve~~



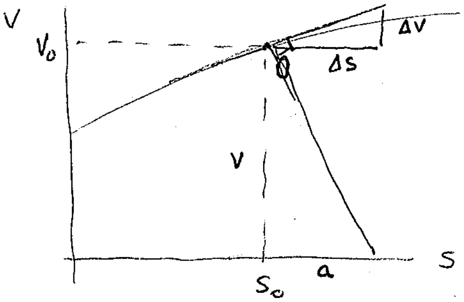
$$a = \frac{dv}{dt} \qquad v = \frac{ds}{dt} \Rightarrow v\,dt = ds$$

$$a \cdot ds = v\,dt \cdot \frac{dv}{dt} = v\,dv.$$

$$\int_{s_1}^{s_2} a\,ds = \int_{v_1}^{v_2} v\,dv = \frac{v^2}{2}\Big|_{v_1}^{v_2} = (v_2^2 - v_1^2)/2.$$

- area under the graph gives one half the difference between the square of the velocities

Suppose you are given V. vs. S curve & you want a vs. s curve.

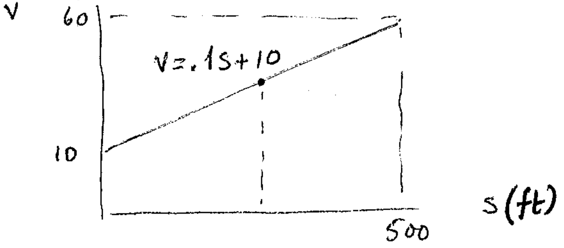


$$\text{slope at a pt. } 0 = \frac{dV}{ds}\Big|_{s=S_0} = \frac{a}{V}$$

since at 0 you can get $V = V_0$

then $a = V_0 \cdot \left(\frac{dV}{ds}\right)\Big|_{s=S_0}$

LOOK AT problem 12-33 p. 21
12-60 p29 in 10th ed.



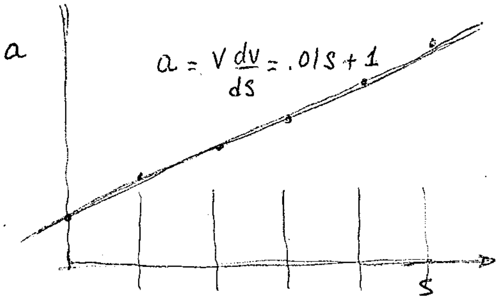
@ pt 0 slope $= \frac{dV}{ds} = .1 = \text{const} = \frac{a}{V}$

when $s = 0$ $V = 10$ $\quad .1 = \frac{a}{10} \Rightarrow a = 1$

when $s = 100$ $V = .1s + 10 \;\; .1 = \frac{a}{20} \Rightarrow a = 2$

when $s = 200$ $V = 30$ $\quad .1 = \frac{a}{30} \Rightarrow a = 3$

when $s = 500$ $V = 60$ $\quad a = 6$



linear

$$V = \frac{ds}{dt} = .1s + 10 \Rightarrow \int_{t=0}^{t=t_f} dt = \int_{s=0}^{s=500} \frac{ds}{.1s + 10}$$

$$t = 10\ln(.1s + 10)\Big|_{s=0}^{s=500} = 10(\ln 60 - \ln 10) = 10\ln 6$$

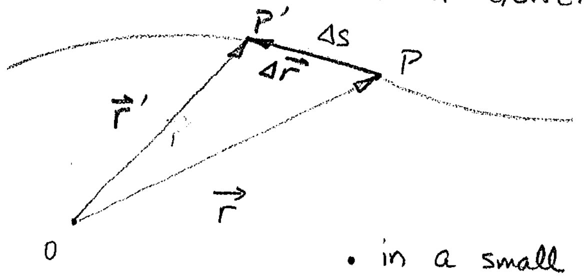
$= 17.92$ se

H 12-3

LESSON #2

- Particle moves in a general curved path MOTION IS CURVILINEAR
- Path is usually 3-D : MUST USE VECTOR ANALYSIS TO FORMULATE $\vec{r}, \vec{v}, \vec{a}$
- GENERAL CURVILINEAR MOTION

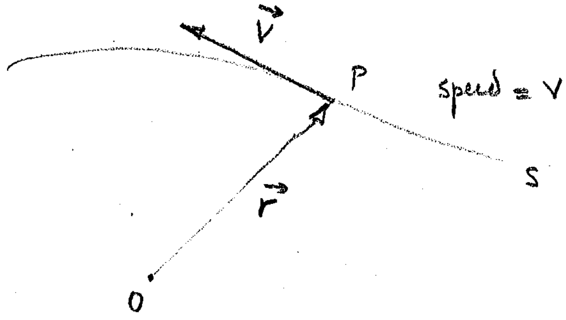
SUPPOSE WE HAVE A GENERAL SPACE CURVE. POSITION WRT FIXED PT O.



- $\vec{r} = \vec{r}(t)$ since position & direction change with time
- in a small increment in time Δt, particle moves from P to P' along S.
- magnitude is ΔS and $\vec{r}' = \vec{r} + \Delta\vec{r}$
- $\Delta\vec{r}$ is the displacement & $|\Delta\vec{r}| = \Delta s$

- Velocity $\vec{V}_{ave} = \dfrac{\Delta\vec{r}}{\Delta t}$ (average velocity)

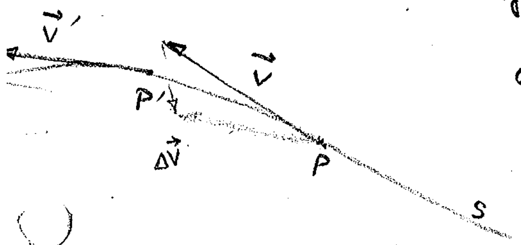
if $\lim\limits_{\Delta t \to 0}$ then $\vec{V} = \dfrac{d\vec{r}}{dt}$ at the point t



since $d\vec{r}$ is tangent to path so will $\vec{v}$

speed = v = magnitude of $\vec{v} = |\vec{v}| = \left|\dfrac{d\vec{r}}{dt}\right| = \dfrac{|d\vec{r}|}{dt} = \dfrac{ds}{dt}$

- acceleration



if at P velocity is $\vec{v}$ at time t

and velocity is $\vec{v}' = \vec{v} + \Delta\vec{v}$ at time $t + \Delta t$

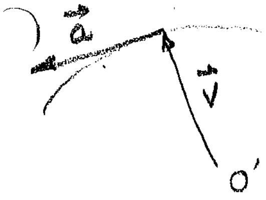
$$\vec{a}_{average} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}' - \vec{v}}{\Delta t}$$

Plot $\vec{v}$ & $\vec{v}'$ so that the tails are at the same pt O' and the relative angles between $\vec{v}$ & $\vec{v}'$ are preserved. THIS IS HODOGRAPH

it serves the same fn. as the path S describes the locus of pts. of the position vector.

if we let $\Delta t \to 0$

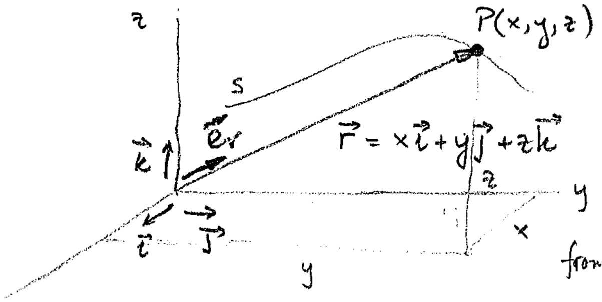
$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$



- $\vec{a}$ is tangent to the hodograph
- $\vec{a}$ in general is not tangent to the path of motion

12-4 CURVILINEAR MOTION : RECTANGULAR COMPONENTS (CARTESIAN REPRESENTATION)

- in some cases the motion can be expressed in terms of the x, y, z components
- $\vec{r}$ - measured from the origin



since $\vec{r} = \vec{r}(t)$

- fixed reference frame
- $\vec{i}, \vec{j}, \vec{k}$ are fixed & do not change w/time
- x, y, z are fns of time

distance from origin $r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

$$\vec{e}_r = \vec{u}_r = \vec{r}/r$$

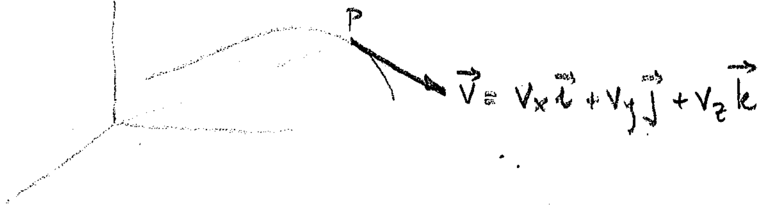
- $\vec{V} = \frac{d}{dt}\vec{r} = \frac{d}{dt}(x\vec{i} + y\vec{j} + z\vec{k}) = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}$
    - $\frac{d}{dt}(x\vec{i}) = \frac{dx}{dt}\vec{i} + x\cancelto{0}{\frac{d\vec{i}}{dt}} = \dot{x}\vec{i}$

      since mag & direction of $\vec{i}$ do not change w/time
- $\vec{V} = V_x\vec{i} + V_y\vec{j} + V_z\vec{k} = \underline{\dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}}$

$$\left.\begin{aligned} V_x &= \dot{x} \\ V_y &= \dot{y} \\ V_z &= \dot{z} \end{aligned}\right\} \quad V = |\vec{V}| = \sqrt{V_x^2 + V_y^2 + V_z^2} = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2}$$

$$\vec{e}_v = \vec{u}_v = \vec{V}/V$$

Direction of $\vec{V}$ is always tangent to the path



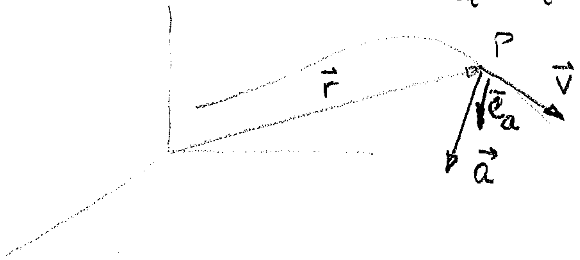
- ACCELERATION

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(v_x\vec{i}+v_y\vec{j}+v_z\vec{k}\right) = \frac{d}{dt}\left(\dot{x}\vec{i}+\dot{y}\vec{j}+\dot{z}\vec{k}\right)$$

$$a_x\vec{i}+a_y\vec{j}+a_z\vec{k} = \dot{v}_x\vec{i}+\dot{v}_y\vec{j}+\dot{v}_z\vec{k} = \ddot{x}\vec{i}+\ddot{y}\vec{j}+\ddot{z}\vec{k}$$

$$\left.\begin{array}{l} a_x = \dot{v}_x = \ddot{x} \\ a_y = \dot{v}_y = \ddot{y} \\ a_z = \dot{v}_z = \ddot{z} \end{array}\right\} \quad a = |\vec{a}| = \sqrt{a_x^2+a_y^2+a_z^2}$$

$$\vec{u}_a = \vec{a}/a = \vec{e}_a$$



- $\vec{a}$ will not be in general tangent to path

PROBLEM 12-43

$$\vec{V} = (2t^2\vec{i} + 10t^{1/2}\vec{j} + 4\vec{k}) \text{ ft/sec} \qquad \vec{a} = \frac{d\vec{v}}{dt} = (4t\vec{i} + 5t^{-1/2}\vec{j} + 0\vec{k}) \text{ ft/sec}^2$$

a) t=3sec

$$\vec{a} = (12\vec{i} + 2.89\vec{j}) \text{ ft/sec}^2$$

$$a = \sqrt{a_x^2+a_y^2} = \sqrt{144+\frac{25}{3}} = 12.34 \text{ ft/sec}^2$$

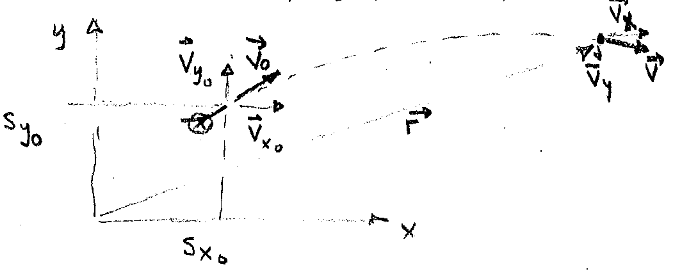
$$\vec{u}_a = \frac{\vec{a}}{a} = \frac{12\vec{i}}{12.34} + \frac{2.89}{12.34}\vec{j} = .972\vec{i} + .234\vec{j} + 0\vec{k}$$

$$= \cos\alpha\,\vec{i} + \cos\beta\,\vec{j} + \cos\gamma\,\vec{k} \qquad \alpha = 13.53^\circ \quad \beta = 76.46^\circ \quad \gamma = 90^\circ$$

12-5 MOTION OF A PROJECTILE - PLANAR MOTION

RECTANGULAR components can be used to study the free flight motion of a projectile

- consider a gun firing a projectile. The gun is located at some point P(S_{x_0}, S_{y_0}) projectile is assumed to move only in the x,y plane



- velocity out of the muzzle is $\vec{V}_0$ having components $\vec{V}_{x_0}$ & $\vec{V}_{y_0}$
- earth is flat
- acceleration of gravity is constant

IF AIR RESISTANCE IS NEGLECTED the only component of force acting on the particle is THE weight which creates a downward component of acceleration

$$a_y = g = 9.81\ m/sec^2 \quad \text{or} \quad 32.2\ ft/sec^2 \qquad \text{THIS IS constant.}$$

$$\therefore\ \vec{a} = -g\vec{j}$$

GIVEN HORIZONTAL

$\overset{+}{\rightarrow}\ \dot{V}_x = \ddot{X} = a_x = 0$

$\overset{+}{\rightarrow}\ V_x = \dot{X} = a_x t + \text{constant}\ ;\ @\ t=0\ \ V_x = 0 + \text{constant} = V_{x_0}\ ;\ \text{constant} = V_{x_0} \Rightarrow \underline{V_x = V_{x_0}} = \dot{S}_x$

$$\frac{a_x = 0}{V_x = V_{x_0}}$$

$\overset{+}{\rightarrow}\ S_x = X = V_{x_0} t + \text{constant}\ ;\ @\ t=0\ \ S_x = 0 + \text{constant} = S_{x_0}\ ;\ \text{constant} = S_{x_0} \Rightarrow \underline{S_x = V_{x_0} t + S_{x_0}}$

V_x is a constant throughout the flight of the projectile

GIVEN VERTICAL

$+\uparrow\ \dot{V}_y = \ddot{y} = a_y = -g$

$+\uparrow\ V_y = \dot{y} = a_y t + \text{const} = -gt + \text{constant}\ ;\ @\ t=0\ \ V_y = 0 + \text{const} = V_{y_0}\ ;\ \text{const} = V_{y_0} \Rightarrow V_y = -gt + V_{y_0}$

$$\frac{a_y = -g}{V_y = -gt + V_{y_0}}$$

$+\uparrow\ S_y = y = -g\frac{t^2}{2} + V_{y_0} t + \text{constant}\ ;\ @\ t=0\ \ S_y = 0 + 0 + \text{const} = S_{y_0}\ ;\ \text{const} = S_{y_0} \Rightarrow \underline{S_y = -g\frac{t^2}{2} + V_{y_0} t + S}$

$\vec{r} = x\vec{i} + y\vec{j} = (V_{x_0} t + S_{x_0})\vec{i} + (-g\frac{t^2}{2} + V_{y_0} t + S_{y_0})\vec{j}$

$\vec{r}_0$ = position of the gun is $S_{x_0}\vec{i} + S_{y_0}\vec{j}$

$\vec{r} - \vec{r}_0$ relative position of the particle wrt the gun muzzle

$\vec{V} = V_{x_0}\vec{i} + (-gt + V_{y_0})\vec{j}$

$\vec{V}_0 = V_{x_0}\vec{i} + V_{y_0}\vec{j}$

TO ANALYZE A PROBLEM

- DRAW THE TRAJECTORY & ESTABLISH YOUR COORDINATE AXES x, y
- FOR EACH PT PUT DOWN ALL KNOWN INFO ; ACCEL OF GRAVITY IS DOWNWARD
- CHOOSE WHICH EQUATIONS TO USE
- HORIZONTAL MOTION $V_x = (V_x)_0 = \text{constant}$

  $S_x = S_{x_0} + V_{x_0} t$
- VERTICAL MOTION

$$a_y = -g$$
$$V_y = -gt + V_{y_0}$$
$$S_y = -g\frac{t^2}{2} + V_{y_0} t + S_{y_0}$$
$$V_y^2 = V_{y_0}^2 - 2g(S_y - S_{y_0})$$

$V_{0_x} = V_0 \cos\theta$

$V_{0_y} = V_0 \sin\theta$

for motion starting at $\underline{\underline{t=0}}$

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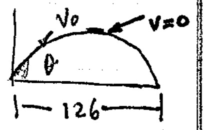
10^{th} ed

12-85 Pro-player kicked a football 126 ft in 3.6 sec find V_0 & θ



$S = S_{x_0} + V_{x_0} t$ $\qquad S_{x_0} = 0$

$126 = 0 + V_{x_0} \cdot 3.6$ $\qquad V_{x_0} = \frac{126}{3.6} = 35$ ft/s

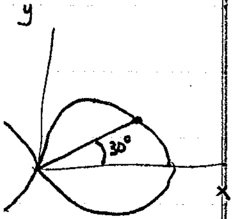
$S = -g\frac{t^2}{2} + V_{y_0} t + \cancel{S_{y_0}}$ $\qquad 0 = t\left(V_{y_0} - g\frac{t}{2}\right)$ where $t = 3.8$ sec

or $V = -gt + V_{y_0} = 0$ $\qquad$ or $V_{y_0} = gt = \underline{32.2}\,(1.8\text{ sec}) = \overset{57.96}{}$ ft/s $\qquad$ here $t = 1.8$ sec

$V_0 = \sqrt{V_{x_0}^2 + V_{y_0}^2} = 67.71$ ft/s

$V_0 \cos\theta = V_{x_0}$ $\qquad \cos\theta = \frac{V_{x_0}}{V_0} = \frac{35}{67.71}$ $\qquad \theta = \overset{58.88}{\cancel{60}}{}^\circ$

similar to 12-164



Given that a mechanism follows the path prescribed by $r = 200\cos 2\theta$ m

If the angular veloc $\dot\theta = 0.008t^2$ rad/s and $\theta = 0$ when $t = 0$

determine $v_r, v_\theta, a_r, a_\theta$ when $\theta = 30^\circ$ as well as v & a

since $\dot\theta = .008t^2 \Rightarrow \theta = .008\frac{t^3}{3} + C$ rad when $t=0$ $\theta = 0 \Rightarrow C = 0$

when $\theta = 30^\circ$ ie $\frac{30}{180}\cdot\pi = \frac{\pi}{6}$ rad time to reach that is $t = \sqrt[3]{\frac{3\theta}{.008}} = \sqrt[3]{\frac{3(\pi/6)}{.008}} = 5.8122$ s

$v_r = \dot r = \frac{dr}{d\theta}\cdot\frac{d\theta}{dt} = -200\sin 2\theta \cdot 2\dot\theta = -400\sin(60^\circ)\cdot\dot\theta$ $\qquad \dot\theta\Big|_{5.8122} = .008(5.8122)^2 = .2703$ rad/s

$= -93.635$ m/s

$v_\theta = r\dot\theta = 200\cos 60 \cdot (.2703) = 27.03$ m/s

$v = \sqrt{v_r^2 + v_\theta^2} = 97.46$ m/s

$a_r = \ddot r - r\dot\theta^2$

$\ddot r = \dot v_r = \frac{d}{dt}\left[-400\sin 2\theta\,\dot\theta\right] = -800\cos 2\theta\,\dot\theta^2 - 400\sin 2\theta\,\ddot\theta = -61.35$ m/s²

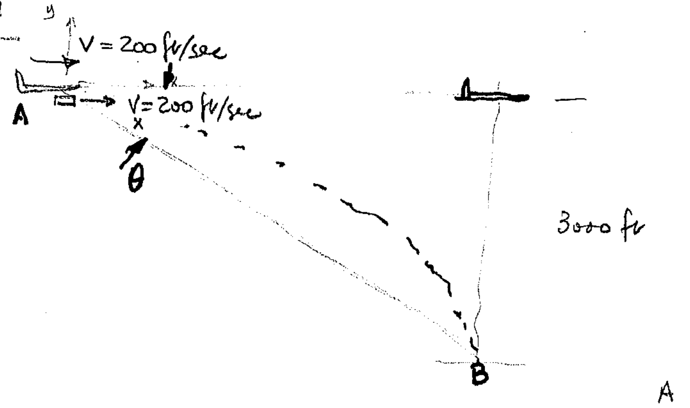
$\ddot\theta = \frac{d}{dt}\dot\theta = +.016t = .093$ rad/s²

$a_r = -61.35 - 200\cos 60\,(.2703)^2 = -68.65$ m/s²

$a_\theta = 2\dot r\dot\theta + r\ddot\theta = 2(-93.635)(.2703) + 200\cos 60 \cdot .093 = -41.32$ m/s²

$a = \sqrt{a_r^2 + a_\theta^2} = 80.13$ m/s²

Problem 12-54



Do 2/67
in Meriam / Kraige

The plane is flying horizontally with a constant speed of 200 ft/s at an altride of 3000 ft. If the pilot drops a package with the same horiz speed of 200 $\frac{ft}{s}$ determine the angle θ at which he must sight the target B so that when the package is released it falls & strikes the target. Neglect air resistance.

B

Package is given a horiz velocity $V_{x_0} = 200$ ft/sec

$a_x = 0$ ft/sec²
$S_{x_0} = 0$ ft

$V_{y_0} = 0$ ft/sec
$a_y = -g = -32.2$ ft/sec²
$S_{y_0} = 0$ ft

$S_{y_f} = -3000$ ft

$V_x = V_{x_0}$
$S_x = V_{x_0} t + S_{x_0} = V_{x_0} t$

$a_y = -g$
$V_y = -gt + V_{y_0} = -gt$
$S_y = -gt^2/2 + V_{y_0} t + S_{y_0} = -gt^2/2$

$$S_{y_f} = -3000\text{ ft} = -g\frac{t_f^2}{2} = -\frac{32.2}{2}(t_f)^2 \quad \therefore \quad t_f = \sqrt{-2 \cdot S_{y_f}/g} = \sqrt{\frac{6000\text{ ft}}{32.2\text{ ft/sec}^2}} = 13.65\text{ sec}$$

$S_x = V_{x_0} \cdot t = 200$ ft/sec · 13.65 sec = 2730.01 ft.

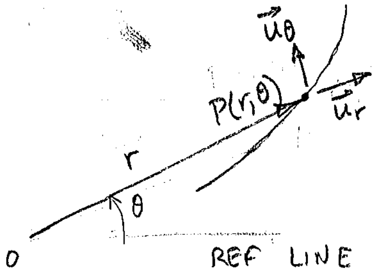
$$\tan\theta = \frac{-3000\text{ ft}}{2730.01\text{ ft}} = -1.099 \qquad \underline{\theta = -47.7°}$$

since $V_x = V_{x_0}$ = constant, the package will be traveling w/ the same horizontal speed as the plane. When one looks down one will always see the package below the plane.

SECTION 12-6

LESSON #3

CYLINDRICAL COORDINATES (r, θ, z)
if planar motion use r, θ only.



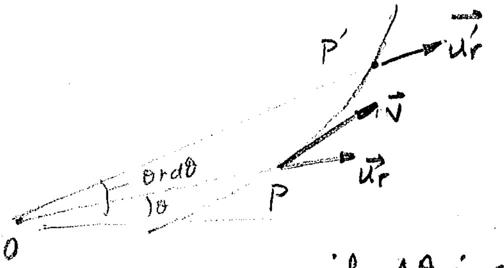
in Polar coordinates a pt P can be specified by magnitude & direction (r, θ)
r - position from a fixed pt O & θ an angle measured from some fixed reference line to the r axis

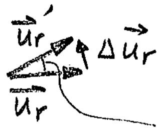
- can define unit vectors $\vec{u}_r$ & $\vec{u}_\theta$ where $\vec{u}_r$ line of action is along the r axis and $\vec{u}_\theta$ is ⊥ to $\vec{u}_r$

Position: $\vec{r} = r\vec{u}_r$

- PARTICULARLY USEFUL WHEN MOTION IS CONSTRAINED THROUGH CONTROL OF RADIAL DISTANCE AND ANGULAR POSITION
- UNCONSTRAINED MOTION IS OBSERVED BY MEASUREMENTS OF A RADIAL DISTANCE AND AN ANGULAR POSITION

velocity $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d(r\vec{u}_r)}{dt} = \dot{r}\vec{u}_r + r\frac{d\vec{u}_r}{dt}$

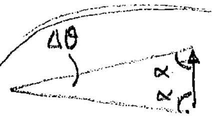




- $\vec{u}_r'$ & $\vec{u}_r$ have same magnitude (1) only different direction
- angle is $\Delta\theta$ over a time Δt

- if $\Delta\theta$ is small (ie Δt is small) $\Delta u_r = \Delta\theta \cdot$ length of $\vec{u}_r = \Delta\theta \cdot 1$
- thus $\lim_{\Delta t \to 0} \frac{\Delta u_r}{\Delta t} \equiv \frac{du_r}{dt} = \lim_{\Delta t \to 0} \frac{1 \cdot \Delta\theta}{\Delta t} = \dot{\theta}$
- the direction of $\Delta\vec{u}_r$ is in the $\vec{u}_\theta$ direction

$$\text{thus } \lim_{\Delta t \to 0} \frac{\Delta\vec{u}_r}{\Delta t} = \frac{d\vec{u}_r}{dt} = \dot{\theta}\vec{u}_\theta$$



angles of Δ are = 180°

$\Delta\theta \to 0 \quad \therefore \quad 2\alpha = 180° \; \alpha = 90°$

$\therefore \; \Delta\vec{u}_r$ is $\perp$ $\vec{u}_r$

or $\Delta\vec{u}_r$ is in direction of $\vec{u}_\theta$

thus $\vec{v} = \dot{r}\vec{u}_r + r\dot{\theta}\vec{u}_\theta = v_r\vec{u}_r + v_\theta\vec{u}_\theta$

$$\boxed{\begin{aligned} v_r &= \dot{r} \\ v_\theta &= r\dot{\theta} \end{aligned}}$$

$\dot{\theta}$ is known as the angular velocity

speed $v = |\vec{v}| = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$

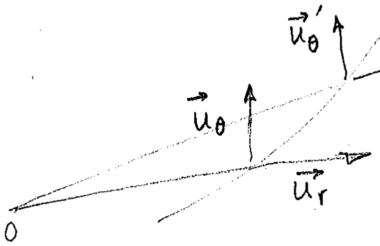
the direction of $\vec{v}$ is tangent to the path at pt P

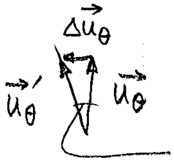
do you want to see the module again

Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\dot{r}\vec{u}_r + r\theta\vec{u}_\theta\right) = \ddot{r}\vec{u}_r + 2\dot{r}\,\underbrace{\dot{\theta}\vec{u}_\theta}_{\frac{d}{dt}\vec{u}_r} + r\ddot{\theta}\vec{u}_\theta + r\dot{\theta}\frac{d}{dt}\vec{u}_\theta$$

do you want to review





$\Delta\theta$ change over the length of time Δt

- $\vec{u}_\theta'$ & $\vec{u}_\theta$ are of same magnitude only change is in direction

- if $\Delta\theta$ is small $\Delta u_\theta = \Delta\theta \cdot$ length of $\vec{u}_\theta = \Delta\theta \cdot 1$
- thus $\lim_{\Delta t \to 0} \frac{\Delta u_\theta}{\Delta t} = \frac{du_\theta}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\theta \cdot 1}{\Delta t} = \frac{d\theta}{dt} = \dot{\theta}$
- the direction of $\Delta\vec{u}_\theta$ is in the $-\vec{u}_r$ $\quad \Delta\vec{u}_\theta$ is $\perp$ to $\vec{u}_\theta$ pointing

towards O.

$$\therefore \quad \vec{a} = \frac{d\vec{v}}{dt} = \ddot{r}\vec{u_r} + 2\dot{r}\theta\,\vec{u_\theta} + r\ddot{\theta}\,\vec{u_\theta} - r\dot{\theta}\dot{\theta}\,\vec{u_r}$$

$$= (\ddot{r} - r\dot{\theta}^2)\vec{u_r} + (2\dot{r}\theta + r\ddot{\theta})\vec{u_\theta}$$

$$\vec{a} = a_r\vec{u_r} + a_\theta\vec{u_\theta} \quad \Longrightarrow \quad a_r = (\ddot{r} - r\dot{\theta}^2)$$

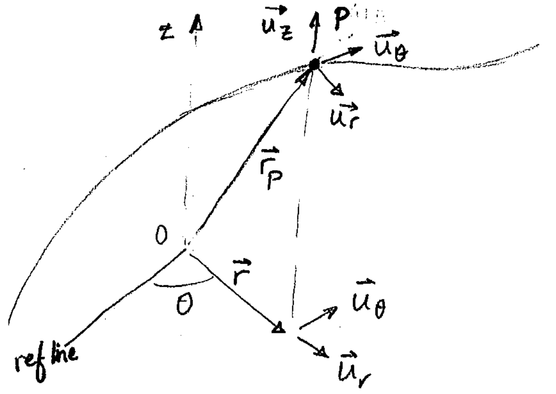
$$a_\theta = 2\dot{r}\ddot{\theta} + r\ddot{\theta}$$

$\ddot{\theta}$ is the angular acceleration

$$a = |\vec{a}| = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (2\dot{r}\dot{\theta} + r\ddot{\theta})^2}$$

Note that $\vec{a}$ in general will not be tangent to the path of the particle

FOR A SPACE CURVE - z component is like that for rectangular coordinates



$$\vec{r_P} = r\vec{u_r} + z\vec{k_z}$$

$$\vec{v_P} = \dot{r}\vec{u_r} + r\dot{\theta}\vec{u_\theta} + \dot{z}\vec{k_z}$$

$$\vec{a_P} = (\ddot{r} - r\dot{\theta}^2)\vec{u_r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{u_\theta} + \ddot{z}\vec{k_z}$$

$$V = \sqrt{\dot{r}^2 + (r\dot{\theta})^2 + \dot{z}^2}$$

$$a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (2\dot{r}\dot{\theta} + r\ddot{\theta})^2 + \ddot{z}^2}$$

IF $\vec{r} = \vec{r}(t) = x\vec{i} + y\vec{j} + z\vec{k} \Longrightarrow x(t), y(t), z(t)$ can be differentiated to obtain $\vec{V}, \vec{a}$

IF $\vec{r} = \vec{r}(\theta)$ and $\theta = \theta(t)$ ie $\vec{r} = r\vec{u_r}$ then to find $\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt}$

use chain rule for differentiation

$$\ddot{r} = \frac{d}{dt}(\dot{r}) = \frac{d}{dt}\left(\frac{dr}{d\theta}\right)\dot{\theta} + \frac{dr}{d\theta}\ddot{\theta} = \frac{d^2r}{d\theta^2}\frac{d\theta}{dt}\frac{d\theta}{dt} + \frac{dr}{d\theta}\frac{d^2\theta}{dt^2}$$

$$\ddot{r} = r''\dot{\theta}^2 + r'\ddot{\theta}$$

PROBLEM 12-68

Car drives around a circular curve ($r = 50$m) at constant speed $v = 15$ m/s find $\dot{\theta}$ and acceleration. $r = 50$m $=$ const $\dot{r} = 0$ $\ddot{r} = 0$

$$v = \sqrt{\dot{r}^2 + (r\dot{\theta})^2} = \sqrt{(r\dot{\theta})^2} = r\dot{\theta} \qquad \dot{\theta} = v/r = \frac{15\text{ m/sec}}{50\text{ m}} = .3\text{ rad/sec} = \text{const}$$

$\ddot{\theta} = 0$ $\quad a = \sqrt{(\cancel{\ddot{r}} - r\dot{\theta}^2)^2 + (r\cancel{\ddot{\theta}} + 2\cancel{\dot{r}}\dot{\theta})^2} = -r\dot{\theta}^2 = -50\text{m} \cdot (.3\text{ rad/sec})^2 = -4.5\text{ m/sec}^2$ toward the center

12-114
simular to
12-164
10th ed.

$r = 200\cos 2\theta$

$\dot{\theta} = .008t^2 \ \frac{rad}{s}$

~~$\theta = 30$~~

what v_r, v_θ when $\theta = 30°$

a_r, a_θ

a portion of path defined
by $r = (200\cos 2\theta)$ m
If the angular velocity of the
radial coord. line is $\dot{\theta} = .008t^2$
find $v_r, v_\theta, a_r, a_\theta$ when $\theta = 30°$,
given $\theta = 0$
when $t = 0$

whe $t = 0$ $\theta = 0$

$$\theta = \frac{.008t^3}{3} + \cancel{C} \longrightarrow \left(\frac{30°\cdot\pi}{180}\cdot\frac{3}{.008}\right)^{1/3} = t = 5.812 \text{ sec}$$

$$\dot{\theta} = .008t^2 = .2703$$

$$\dot{r} = v_r = 200(-\sin 2\theta)\cdot 2\dot{\theta} = -400\sin(2\cdot 30°)\cdot\dot{\theta} = -93.6197 \text{ ft/s}$$

$$r\dot{\theta} = v_\theta = 200\cos 60°(.2703) = 27.03 \text{ ft/s}$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta}$$

need $\ddot{r}$ $\ddot{\theta}$

$$\ddot{r} = -800\cos(2\theta)\dot{\theta}^2 - 400\sin(2\theta)\ddot{\theta} = -61.44 \text{ ft/s}^2$$

$$\ddot{\theta} = -.016t = .0930 \text{ rad/s}^2$$

$$a_r = -68.7 \text{ ft/s}^2$$

$$a_\theta = -41.3 \text{ ft/s}^2$$

$$\bar{u}_r = \cos\theta\,\bar{i} + \sin\theta\,\bar{J} \qquad \dot{\bar{u}}_r = (-\sin\theta\,\bar{i} + \cos\theta\,\bar{J})\dot{\theta} = \bar{u}_\theta\,\dot{\theta}$$

$$\bar{u}_\theta = -\sin\theta\,\bar{i} + \cos\theta\,\bar{J} \qquad \dot{\bar{u}}_\theta = (-\cos\theta\,\bar{i} - \sin\theta\,\bar{J})\dot{\theta} = -\bar{u}_r\,\dot{\theta}$$

EDUCATION

9/78-1/83	Ph.D., Mechanical Engineering, Stanford University
6/72-8/74	M.S., Applied Mathematics, Courant Institute of Mathematical Sciences, New York University
9/68-6/72	B.S., Aerospace Engineering, Polytechnic Institute of Brooklyn

EXPERIENCE

8/85 - Present	FLORIDA INTERNATIONAL UNIVERSITY, Miami, Florida, Department of Mechanical Engineering. Responsible for teaching, advising and developing research proposals in fracture mechanics and applied mechanics in general. Continue work with Biology Department colleague on the applied mechanics' implications of hypergravity on mouse femora.
8/83-7/85	TECHNION-Israel Institute of Technology, Faculty of Mechanical Engineering at Haifa, Israel. Lady Davis Post-Doctoral Fellow.
4/83-7/83	APPLE COMPUTERS,INC. at Cupertino, California. Consultant. Provided engineering analysis, suggested improvements and provided new designs for existing products.

PUBLICATIONS

One submitted for publication; three in preparation.

1. Bone, in press
2. Eng. Fracture Mech., 29, pp 263-274 (1988)
3. Int J Solids and Structures, 22, pp 1525-1539 (1986)
4. ASM Proceedings, 4, pp 359-365 (1985)
5. Eng. Fracture Mech., 18, pp 39-48 (1983)
6. Eng. Fracture Mech., 18, pp 49-58 (1983)
7. Eng. Fracture Mech., 18, pp 295-305 (1983)
8. J. Applied Mech., 49, pp 773-778 (1982)
9. J. Applied Mech., 49, pp 656-658 (1982)
10. Eng. Fracture Mech., 17, pp 125-131 (1982)
11. Int. J. Fracture, 14, pp 221-239 (1982)
12. Bull. Seism. Soc. Amer., 64, pp 1789-1808 (1974)

Other Publications--two proceedings papers and three abstracts

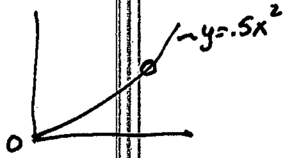
RESEARCH AREAS OF INTEREST: fracture mechanics, biomechanics, large scale numerical methods in applied mechanics

GRANTS AWARDED

- IBM Unsponsored Research Grant--March 1986 to January 1988
- FIU Provost's Summer Grant--Summer "A" 1988

OTHER DATA

- Lady Davis Post-Doctoral Fellowship for academic years 1983/1984 and 1984/1985.
- Alexander von Humboldt Fellowship (West Germany) 1983/1984.
- Member: Tau Beta Pi and The American Society of Mechanical Engineers.



The particle travels along the path defined by the parabola $y = 0.5x^2$. If the compon of velocity along the x axis is $v_x = 5t$ ft/s, where t is in seconds, determine the particle's distance from the origin O & the magnitude of its accel. when $t = 1$. When $t = 0$, $x = y = 0$.

12-78 pg 44

Given: $y = 0.5x^2$

$v_x = 5t$

find @ $t=1$ find d & a given $t=0, x=0, y=0$

$$v_x = \dot{x} = 5t \Rightarrow x = \int \dot{x}\,dt = \frac{5t^2}{2} + C \ ; \quad \text{since } x=0 \text{ @ } t=0 \Rightarrow C=0$$

$$\therefore \ y = 0.5x^2 = .5\left[6.25t^4\right] = 3.125\,t^4 \qquad \underline{r} = x\underline{i} + y\underline{j} = \frac{5t^2}{2}\underline{i} + 3.125t^4\,\underline{j} \text{ ft}$$

$$d\Big|_{t=1} = \sqrt{x^2+y^2} = \sqrt{(2.5)^2+(3.125)^2} = 4 \text{ ft}$$

$$\underline{a} = \ddot{x}\underline{i} + \ddot{y}\underline{j} = 5\underline{i} + 37.5t^2\,\underline{j} \ \text{ft/s}^2 \quad \& \quad a = \sqrt{\ddot{x}^2+\ddot{y}^2} = 37.83 \text{ ft/s}^2$$

direction of $\underline{a}$

$$\underline{u}_a = \frac{\underline{a}}{a} = .1322\underline{i} + .9912\underline{j} \quad \therefore \ \cos\alpha = .1322 \quad \alpha = 82.41°$$

$$= \cos\alpha\,\underline{i} + \cos\beta\,\underline{j} + \cos\gamma\,\underline{k} \quad \cos\beta = .9912 \quad \beta = 7.59°$$

$$\gamma = 90°$$

$$\underline{u}_d = \frac{\underline{r}}{d} = .625\underline{i} + .7809\underline{j} \qquad \cos\alpha = .625 \quad \alpha = 51.34°$$

$$\cos\beta = .7809 \quad \beta = 38.66°$$

$$\cos\gamma = 0 \quad \gamma = 90°$$

since $y = 3.125t^4 \Rightarrow \dot{y} = v_y = 12.5t^3$ or $\dot{y} = 0.5 \cdot 2x \cdot \dot{x} = 0.5 \cdot 2 \cdot \frac{5t^2}{2} \cdot 5t = \frac{25}{2}t^3 = 12.5t^3$

SEC 12-7 Normal and Tangential Components in planar motion

USED: When path of the particle is known use n, t coordinates

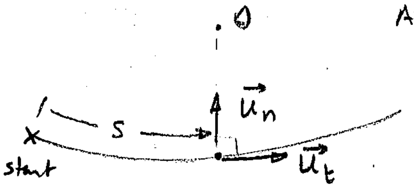
n is normal to the path t is tangent to the path

CONSIDER FIRST

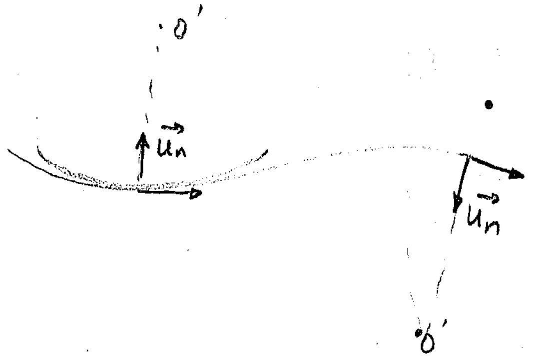
Planar Motion

LOOK AT PARTICLE MOVING IN A PLANE $\ni$.

AT SOME INSTANT IN TIME IT IS LOCATED AT A PT S ON THE CURVE



- $\vec{u}_t$ POINTS IN DIRECTION OF INCREASING S
- PICK A $\vec{u}_n \perp \vec{u}_t$ SO THAT IT ALWAYS POINTS TOWARDS OF THE CENTER OF CURVATURE O'
- CURVE WILL BE CONCAVE ON THAT SIDE



- THE PLANE CONTAINING $\vec{u}_n$ & $\vec{u}_t$ IS THE OSCULATING PLANE

$\vec{u}_b = \vec{u}_t \times \vec{u}_n$ binormal.

POSITION - always known along S

VELOCITY - since it is tangent to the path

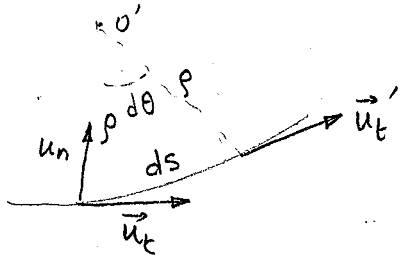
$$\vec{V} = V\vec{u}_t$$

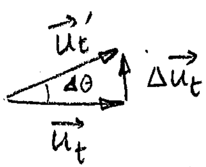
$$V = ds/dt = \dot{s}$$

ACCELERATION

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(V\vec{u}_t) = \dot{V}\vec{u}_t + V\frac{d\vec{u}_t}{dt}$$

$\frac{d\vec{u}_t}{dt}$ can be described as $\lim_{\Delta t \to 0} \frac{\Delta\vec{u}_t}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{u}_t{}' - \vec{u}_t}{\Delta t}$





direction of $\Delta\vec{u}_t$; as $\Delta t \to 0$ $\Delta\theta \to 0$ $\therefore$ $\Delta\vec{u}_t \to \perp$ to $\vec{u}_t$ or $\vec{u}_n$

magnitude of $\vec{u}_t$; is just the arc length formed or $\Delta\theta$. length of vector $\vec{u}_t = \Delta\theta$

- but $\rho\Delta\theta = \Delta s$ ρ being the radius of curvature ;
- for small Δt ρ is assumed not to change very much

thus $\Delta\theta = \frac{\Delta s}{\rho}$ thus $\frac{\Delta\vec{u}_t}{\Delta t} = \frac{\Delta s}{\rho\Delta t}\vec{u}_n$ or $\frac{d\vec{u}_t}{dt} = \frac{1}{\rho}\frac{ds}{dt}\vec{u}_n$

since $a_t = 2\,\text{ft/s}^2 = r\ddot{\theta}$ $\quad \ddot{\theta} = \frac{1}{4}\,\text{rad/s}$

$\dot{\theta} = \frac{1}{4}t + \mathcal{C}$ $\quad$ but where $t=0$ $\quad \underline{V} = 0 = \cancel{\dot{r}\underline{u}_r} + r\dot{\theta}\underline{u}_\theta \Rightarrow \dot{\theta} = 0$

$\Rightarrow \mathcal{C} = 0$

$\theta = \frac{t^2}{8} \quad \therefore \quad \theta = \frac{(2.63)^2}{8} = .866\,\text{rad}$ or 49.62°

and $\frac{ds}{dt} = \dot{s} = V$ we also have that $\dot{\theta} = \frac{1}{\rho} V$ ρ is the instantaneous radius of curvatur

thus

$$\vec{a} = \dot{V}\,\vec{u}_t + V\frac{V}{\rho}\,\vec{u}_n = \dot{V}\,\vec{u}_t + V^2/\rho\,\vec{u}_n = a_t\,\vec{u}_t + a_n\,\vec{u}_n$$

IN <u>3-D</u> DEFINE A 3rd unit vector $\vec{u}_b$ (BINORMAL) in such a manner that $\vec{u}_b = \vec{u}_t \times \vec{u}_n$

- $\vec{u}_b$ WILL BE A VECTOR ⊥ TO THE OSCULATIN PLANE (ie ⊥ to $\vec{u}_t$ & $\vec{u}_n$)
- $\vec{u}_b$ WILL SATISFY THE RH SCREW RULE

- EVERYTHING WE'VE SAID ABOUT $\vec{u}_t$ & $\vec{u}_n$ STILL HOLD

IF a_t is a constant $\dot{V} = a_t = \text{constant}$

$V = a_t t + \text{constant}$ @ $t=0$ assume $\overset{\text{known}}{\overline{V = V_0}}$

$\dot{s} = V = a_t t + V_0$

$s = a_t t^2/2 + V_0 t + \text{const}$ @ $t=0$ assume $s = s_0$

$s = a_t t^2/2 + V_0 t + s_0$

FOR NORMAL acceleration

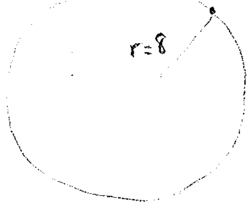
$$a_n = V^2/\rho$$

- direction of a_n is towards the center
- V is known from above
- ρ must be obtained if $y=f(x)$ is known

FROM CALCULUS :

$$\frac{1}{\rho} = \left| \frac{d^2y/dx^2}{\left[1 + (dy/dx)^2\right]^{3/2}} \right|$$

<u>12-78</u> A particle is traveling along a circular path having a radius of 8 ft. If its speed at $t=0$ is zero ft/s, determine the time it takes for it to reach and acceleration $a = 4\,ft/s^2$, assuming that its speed is increasing $a_t = 2$ ft/sec^2 at a rate ~~of~~ 2 ft/s



$V = a_t t + \text{const} = a_t t$

$a_n = V^2/\rho = \frac{a_t^2 t^2}{8} = \frac{1}{2} t^2$

$a = \sqrt{a_t^2 + a_n^2} = 4 = \sqrt{4 + a_n^2}$ $\therefore a_n^2 = 12 = \frac{1}{4} t^4$

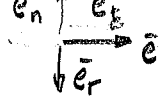
$48 = t^4$ $t = 2.63$

FOR CIRCULAR MOTION $r = \text{const}$ $\dot{r} = \ddot{r} = 0$

$\Rightarrow \bar{V} = V_r \bar{e}_r + V_\theta \bar{e}_\theta$

$= 0 + r\dot{\theta}\,\bar{e}_\theta$

$\bar{V} = V\bar{e}_t$



$\bar{e}_n = -\bar{e}_r$

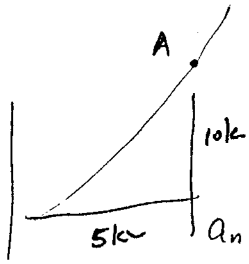
$\bar{e}_t = \bar{e}_\theta$

$\bar{a} = a_r\bar{e}_r + a_\theta\bar{e}_\theta = -r\dot{\theta}^2\bar{e}_r + r\ddot{\theta}\,\bar{e}_\theta = a_n\bar{e}_n + a_t\bar{e}_t$

$s = r\theta \Rightarrow V = \frac{ds}{dt} = r\dot{\theta}$

$a_t = \dot{V} = d^2s/dt^2 = r\ddot{\theta}$ $a_n = V^2/\rho = \frac{r^2\dot{\theta}^2}{r} = r\dot{\theta}^2$

12-106 10^{th} ed.



$y = .4x^2$ km

$\left.\begin{aligned} v &= 200\ m/s \\ \dot{v} &= .8\ m/s^2 \end{aligned}\right]$ at $x=5$

$$a_n = \frac{v^2}{\rho} = \frac{(200)^2}{87616} = .4565\ \frac{m}{s^2}$$

$$a_t = .8\ m/s^2$$

$$a = \sqrt{a_t^2 + a_n^2}$$

$$= \sqrt{.8^2 + .4565^2} = \overset{.9211}{\cancel{.8313}}\ m/s^2$$

$$\frac{1}{\rho} = \frac{d^2y/dx^2}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}} = \frac{.8}{\left[1+(.8x)^2\right]^{3/2}} \quad \frac{1}{km}$$

$$= \frac{.8}{\left[1+16\right]^{3/2}} = \frac{.8}{\cancel{64}\ 70.093} = \overset{.0114}{\cancel{.0125}}\ \frac{1}{km} \cdot \cancel{\frac{km}{1000\,m}}$$

$$= \cancel{1.28 \times 10^{-5}\ \frac{1}{m}}$$

$$\rho = \frac{\overset{87.616\ km}{\cancel{1000}}}{\cancel{.0125}}\ \cancel{m} = \cancel{\frac{1}{.0125}\ km}$$

$$= \cancel{80\ km}$$

$$\rho = \cancel{80000\ m}\ 87616\ m$$

A jet plane travels along the vertical parabolic path. When it is at A it has a speed of 200 m/s which is increasing at a rate of 0.8 m/s². Determine |a| at A

CURRICULUM VITA

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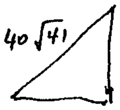
12-89 $\bar{a} = 16\underline{i} + 4tj$

$v = 16t\bar{i} + 2t^2\bar{j}$

$\bar{r} = 8t^2\bar{i} + \frac{2t^3}{3}\bar{j}$

$\underline{v} = \overset{40\cdot4}{160\bar{i}} + \overset{40\cdot5}{200j}$



$V = \underline{256.125}$ ft/s^2

$\bar{a} = \underset{4\cdot4}{16i} + \underset{4\cdot10}{40j}$

$4 \cdot \sqrt{116} = 43.081$ ft/s^2

$x = 8t^2$

$y = 2t^3/3$

$t = \sqrt{X/8}$

$y = \frac{2}{3}\left(\frac{X}{8}\right)^{3/2}$

$y' = \frac{1}{8}\left(\frac{X}{8}\right)^{1/2}$

$y'' = \frac{1}{2}\left(\frac{X}{8}\right)^{-1/2} \cdot \frac{1}{64}$

$= \frac{1}{20} \cdot \frac{1}{64}$

$t = 10$ $x = 800$

$y = \frac{2}{3} \times 10^3$

$$\frac{1}{\rho} = \frac{1/20 \cdot \frac{1}{64}}{\left[1 + \frac{100}{64}\right]^{3/2}}$$

$\rho = 1/{}_{1/\rho}$ = ~~20300.7~~ 5250.56 ft

<u>12-92</u>

since $v = $ const $\dot{v} = a_t = 0$

$a = a_n = v^2/\rho$

$$\frac{1}{\rho} = \frac{d^2y/dx^2}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

$y = .4x^2$

$y' = .8x$

$y'' = .8$

12-171

$r = .5z$

$z = 100 - 0.1t^2$

$\dot{\theta} = 0.04\pi t$

find V + a when $z = 10$ ft

$V = \sqrt{V_r^2 + V_\theta^2 + V_z^2}$ $\quad V_r = \dot{r} \cdot \quad V_\theta = r\dot{\theta} \quad V_z = \dot{z}$

since $r = .5z \quad \dot{r} = .5\dot{z}$ or $\frac{dr}{dt} = \frac{dr}{dz} \cdot \frac{dz}{dt}$

$\dot{z} = -0.2t \quad \therefore \dot{r} = -0.1t$

Now when $z = 10 = 100 - 0.1t^2 \quad \therefore t = 30 \text{ sec}$

$\dot{z} = -6 \text{ ft/s} \quad \dot{r} = -3 \text{ ft/s} \quad r = .5z = 5 \text{ ft} \quad \dot{\theta} = 0.04\pi t = 1.2\pi = 3.77 \frac{\text{rad}}{\text{s}}$

$\therefore \quad V = \sqrt{(-3)^2 + (18.85)^2 + (-6)^2} = 20 \text{ ft/s} \quad \underline{(20.01 \text{ ft/s})}$

now $\quad a = \sqrt{a_r^2 + a_\theta^2 + a_z^2} \quad a_r = \ddot{r} - r\dot{\theta}^2 \quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \quad a_z = \ddot{z}$

$\ddot{r} = \frac{d}{dt}(\dot{r}) = \frac{d}{dt}(.5\dot{z}) = .5\ddot{z} = \frac{d}{dt}(-0.1t) = -0.1$

$\ddot{\theta} = 0.04\pi \quad \ddot{z} = -0.2$

$\therefore \quad a_r = \ddot{r} - r\dot{\theta}^2 = -0.1 - (5)(3.77)^2 = -71.165 \text{ ft/s}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (5)(0.04\pi) + 2(-3)(3.77) = -21.992 \text{ ft/s}$

$a_z = \ddot{z} = -0.2$

$a = \sqrt{(-71.165)^2 + (-21.992)^2 + (-0.2)^2} = \underline{74.485 \text{ ft/s}}$

12-131 $\quad V_{A_0} = V_{B_0} = 8 \text{ m/s} \quad \dot{v}_A = 4S_A = a_{t_A}$ $\quad$ now $\quad v_A \dot{v}_A = 4S_A \cdot v_A = 4s_A \frac{ds_A}{dt}$

or $\quad v_A dv_A = 4S_A dS_A$

or $\quad v_A^2/2 = 2S_A^2 - 2S_{A_0}^2 + v_{A_0}^2/2$

$v_A = \sqrt{4S_A^2 - 4S_{A_0}^2 + v_{A_0}^2}$

$$\frac{1}{2}\int_{S_{A_0}}^{S_A} \frac{ds_A}{\sqrt{S_A^2 - \left[S_{A_0}^2 - \frac{v_{A_0}^2}{4}\right]}} = \int_0^1 dt = 1$$

since $a_{t_B} = 0 \Rightarrow S_B = V_{B_0} \cdot t + \cancel{S_{B_0}} = 8 \text{m}$

$S_{A_0} = r \cdot \frac{2\pi}{3} = 10\frac{\pi}{3} = 10.472 \text{ m}$ $\quad$ let $S_{A_0}^2 - \frac{v_{A_0}^2}{4} = (10.472)^2 - \frac{8^2}{4} = 93.6623$

$$\frac{1}{2}\int_{10.472}^{S_A}\frac{ds}{\sqrt{S^2-9.678^2}} = \frac{1}{2}\left\{\ln\left[S_A+\sqrt{S_A^2-9.678^2}\right]-\ln\left[10.472+\sqrt{10.472^2-9.678^2}\right]\right\} = \text{[redacted]}\ 1$$

$$\frac{1}{2}\left\{\ln\left[S_A+\sqrt{S_A^2-9.678^2}\right]-2.672\right\} = \text{[redacted]}\ 1$$

$$S_A = \text{[redacted]}\ 55\,m$$

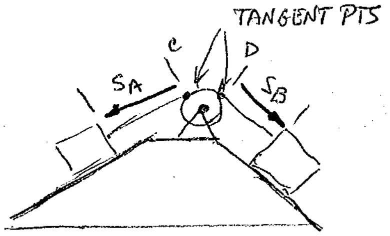
$$S_B = 8m \qquad \therefore \quad S_A - S_B = \overset{47}{\text{[redacted]}}\,m$$

LESSON #4

12.8 ABSOLUTE DEPENDENT MOTION

SOME CASES WHERE ONE PARTICLE'S MOTION IS DEPENDENT ON THE MOTION OF ANOTHER

EXAMPLES : 2 masses connected OVER PULLEY BY INEXTENSIBLE CORDS



REF WRT FIXEDI PT

TO DO PROBLEMS : USE PATH COORDINATES

(1) MEASURE THEM FROM A FIXED PT OR DATUM LINE

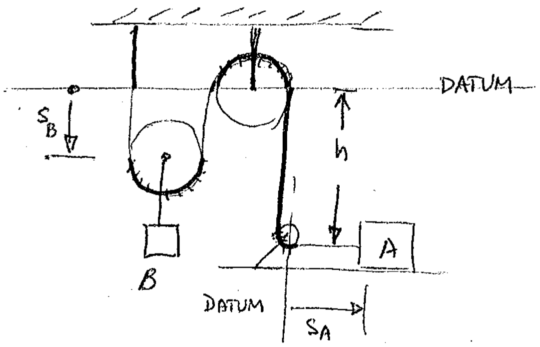
(2) MEASURE THEM IN DIRECTION OF MOTION

(3) MEASURE POSITIVE SENSE

$$S_A + S_B + l_{CD} = \text{constant}$$

$$\Delta S_A + \Delta S_B + \overset{0}{\cancel{\Delta l_{CD}}} = 0$$

take $\frac{d}{dt}$ to get $v_A + v_B = 0 \Rightarrow v_A = -v_B$



REF WRT FIXED DATUM LINE

H IS FIXED

FOR EACH ~~WEIGHT~~ PARTICLE DEFINE A DATUM

- MOTION OF A DEPENDS ON B SINCE BOTH CONNECTED BY ONE CORD

$$2S_B + h + S_A = \text{constant}$$

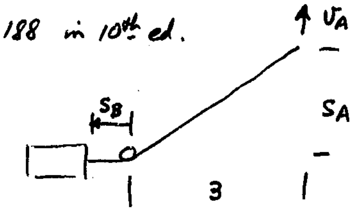
$$2\Delta S_B + \overset{0}{\cancel{\Delta h}} + \Delta S_A = 0$$

take $\frac{d}{dt}$ $2v_B + v_A = 0$ $v_B = -\frac{1}{2} v_A$

METHOD : FOR MASSES WHOSE MOTION DEPEND ON SAME CORD

- DEFINE DATUM OR FIXED PT
- DEFINE POSITION COORDS FROM THIS PT TO EACH PARTICLE
- POS. COORDS MUST LIE ALONG PATH OF MOTION
- USE GEOMETRY OR TRIG TO RELATE COORDINATES TO THE TOTAL LENGTH OF ~~CORD~~ CORD
- EXCLUDE THOSE PORTIONS THAT DON'T CHANGE LENGTH WHEN PARTICLES MOVE.

12-188 in 10th ed. $\uparrow v_A = 3 \quad a_A = 4$ when $s_A = 4$ find v_B, a_B



$$s_B + l = \text{const} \qquad l = \sqrt{9+s_A^2}$$

$$v_B + \frac{1}{2\sqrt{9+s_A^2}} \cdot 2s_A \cdot v_A = 0$$

$$\frac{1}{\cancel{2}} \frac{\cancel{2} \cdot 4 \cdot 3}{5} \qquad v_B = -\frac{12}{5} \text{ ft/s} = -2.4 \text{ ft/s}$$

$$a_B + \frac{1}{2\sqrt{\quad}}\left[2v_A^2 + 2s_A a_A\right] - \frac{1}{4} \cdot \frac{2s_A v_A \cdot 2s_A v_A}{(9+s_A^2)^{3/2}} = 0$$

$$a_B + \frac{1}{2 \cdot 5}\left[2 \cdot 3^2 + 2 \cdot 4 \cdot 4\right] - \frac{1}{4} \cdot \frac{4 \cdot 4^2 \cdot 3^2}{125}$$

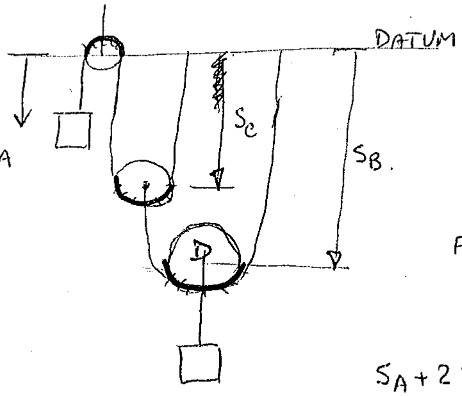
$$a_B + 5 - \frac{144}{125} = 0 \qquad a_B = - \text{[illegible]}$$

-3.85

- TAKE $\frac{d}{dt}$ remember $\frac{d}{dt}(\text{constant}) = 0$
- FIND RELATIONSHIP BETWEEN COORDS, VELOCITIES, ACCEL.

IF MORE THAN 1 CORD INVOLVED – MUST RELATE MOTION OF A PT ON ONE CORD TO MOTION OF A PT. ON ANOTHER

PG 65.



since cord BD is inextensible

speed of pulley D = speed of mass B

FROM DATUM $r_B = r_D + \underbrace{BD}_{\text{const}} \Rightarrow \underline{v_B = v_D.}$

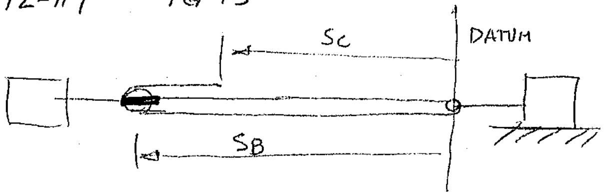
$S_A + 2S_C = \text{Const} \Rightarrow v_A = -2v_C$

$\uparrow v_B = 6 \text{ ft/sec}$ $S_B + (S_B - S_C) = \text{const}$ $2v_B = v_C.$

$v_C = 12 \text{ ft/sec}$ $v_A = -24 \text{ ft/sec.}$ or $24 \text{ ft/sec} \downarrow.$

PROBLEM 12-119 PG 73

like problem 12-176 in 10th ed



Determine the displacement of the box if the man at C pulls the cable 4 ft to the right

$2S_B + (S_B - S_C) = \text{Const.}$

$3S_B - S_C = \text{const.}$

IF $S_C \rightarrow S_C - 4 \Rightarrow 3S_B' - S_C' = \text{const}$

$3S_B' - (S_C - 4) = \text{const.} = 3S_B - S_C$

$3S_B' - \cancel{S_C} + 4 = \text{const} = 3S_B - \cancel{S_C}$

$S_B' + \frac{4}{3} = S_B$

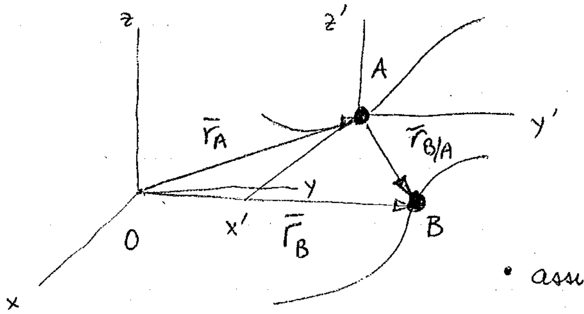
$\Rightarrow S_B' = S_B - \frac{4}{3}$

OR ~~bq~~ moves 4/3 ft to right

12.9 Relative Motion

- Up to now use fixed reference frame to measure motion
- Some situations are complicated and cannot be analyzed in simple means
  - must use 2 or more frames of reference
- not necessary that the same coord system be used in these frames

- we will use only translating frames of reference



Consider 2 particles A & B moving

- we can get the absolute position vectors $\bar{r}_A, \bar{r}_B$
- if we attach a moving reference frame with A
- assume moving reference only translates wrt fixed frame

FROM PARALLELOGRAM LAW OF ADDITION $\bar{r}_B = \bar{r}_A + \bar{r}_{B/A}$

$\bar{r}_{B/A}$ is relative position vector

Velocity vector take $\frac{d}{dt}$: $\bar{V}_B = \bar{V}_A + \bar{V}_{B/A}$ $\bar{V}_i = \frac{d\bar{r}_i}{dt}$

- $\bar{V}_{B|A}$ is the relative velocity of particle B wrt particle A
  is relative velocity observed from moving frame of reference

- since x', y', z' only translates – components of $\bar{r}_{B/A}$ change only in magnitude ~~and~~ but not direction | x', y', z' are still // to x, y, z

acceleration : $\bar{a}_B = \bar{a}_A + \bar{a}_{B/A}$ $\bar{a}_i = \frac{d\bar{V}_i}{dt}$

- $\bar{a}_{B/A}$ – accel of B as seen by observer located at A and translating

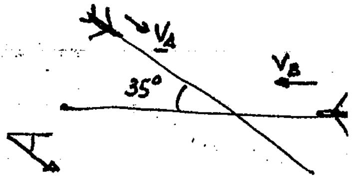
iF translating frame moves at constant velocity then $\frac{d\bar{V}_A}{dt} = \bar{0}$ & $\bar{a}_{B/A} = \bar{a}_B$

12-146

like 12-196 tenth ed.

$\bar{V}_{A/B} = \bar{V}_A - \bar{V}_B$

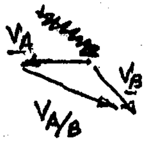
Two planes are flying at the same altitude
If the velocity of A is 300 km/h & B is 250 km/hr
so that the angle between them is 35°
find velocity of A wrt B



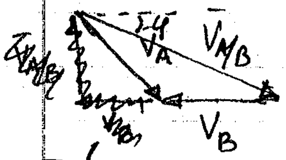
$V_A = 300\cos 35\,\bar{i} - 300\sin 35\,\bar{j}$

$V_B = -250\,\bar{i}$

$$\bar{V}_{A/B} = \bar{V}_A - \bar{V}_B = -(300\cos 35 + 250)\,\bar{i} \overset{+}{} - 300\sin 35\,\bar{j}$$



$$|\bar{V}_{A/B}| = \sqrt{(300\cos 35 + 250)^2 + (300\sin 35)^2} = 524.7 \text{ km/hr}$$



$$\tan\varphi = \frac{-300\sin 35}{300\cos 35 + 250} \Rightarrow \theta = -19.1^{\circ}$$

$\varphi =$

12-154 Cont.

~~$s_A = r_A\theta_A \quad v_A = r_A\dot{\theta}_A \quad \dot{v}_A = r_A\ddot{\theta}_A \quad \dot{r}_A = 0 \quad \ddot{r}_A = 0$~~

~~$\bar{a}_A = (\ddot{r} - r\dot{\theta}^2)\bar{u}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\bar{u}_\theta = (0 - 300(\frac{90}{300})^2)\bar{u}_r + (0 + 300(\frac{15}{300})^2)\bar{u}$~~

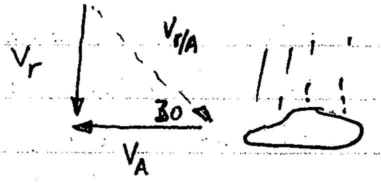
$\underline{a}_A = -15i - 27j \qquad \underline{a}_B = -25\left(-\frac{1}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}\right) + 44.1\left(-\frac{\sqrt{3}}{2}\underline{i} - \frac{1}{2}\underline{j}\right)$

$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = \{-15 - [(12.5 - 22.05\sqrt{3})\}\underline{i} + \{(-12.5\sqrt{3} \qquad \}j = 10.69\underline{i} + 16.70\underline{j} \qquad a_{A/B} = 19.8$

$\theta = 57.4^{\circ}$

152

12-206 in 10th ed



~~problem~~ A passenger in a car observes that raindrops make an angle of 30° w/ horizontal as the auto travels forward w/ speed of 60 km/hr. Find velocity of ra

v_r if it is assumed to fall vertically

$$V_{r/A} = V_r - V_A$$

$$\frac{V_r}{V_A} = \tan 30°$$

$$V_r = \frac{1}{\sqrt{3}}\, 60$$

$V_r = V_A + V_{r/A}$

$-V_A + V_{r/A}\cos 30° = 0$
$-V_r - V_{r/A}\sin 30° = 0$

or $-V_r\underline{j} = V_A + V_{r/A} = -60\underline{i} + V_{r/A}\cos 30\underline{i} - V_{r/A}\sin 30\underline{j}$

$0 = -60 + V_{r/A}\cos 30$
$V_r = +V_{r/A}\sin 30$

$V_{r/A} = \frac{120}{\sqrt{3}} = 69.28$
$V_r = 60/\sqrt{3} = 34.64$

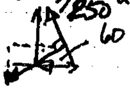
12-203 in 10th ed.

<u>12.-154</u> $V_A = 90\,ft/s$ $\quad \dot{V}_A = 15\,ft/s^2$ $\quad \rho_A = 300$ $\quad \bar{V}_A = V_A\,\bar{u}_t = 90(-\bar{i})$

$$a_A = 15\,u_t + \frac{(90)^2}{300}\,u_n = 15u_t + 27u_n = -15\underline{i} - 27\underline{j}$$

$\rho_B = 250$ $\quad V_B = 105$ & $\quad \dot{V}_B = -25\,ft/sec^2$ $\qquad \bar{V}_B = V_B\,\bar{u}_t$

$$a_B = -25\overset{u_t}{+} (105)^2/250\, u_n = -25u_t + 44.1u_n$$



$u_n = -\cos 30\, i - \sin 30\, j^n$

$$\bar{u}_t = -\bar{i}\cos 60° + \bar{j}\sin 60°$$

$$\bar{V}_B = 105\left(-\tfrac{1}{2}\bar{i} + \tfrac{\sqrt{3}}{2}\bar{j}\right)$$
$$= -52.5\,\bar{i} + 52.5\sqrt{3}\,\bar{j}$$

$$\bar{V}_{A/B} = \bar{V}_A - \bar{V}_B = -90\bar{i} - \left[-52.5\bar{i} + 52.5\sqrt{3}\bar{j}\right]$$
$$= -37.5\,\bar{i} - 52.5\sqrt{3}\,\bar{j}$$

$$|\bar{V}_{A/B}| = \sqrt{(37.5)^2 + (52.5)^2 \cdot 3} = 98.4\ ft/s$$

over

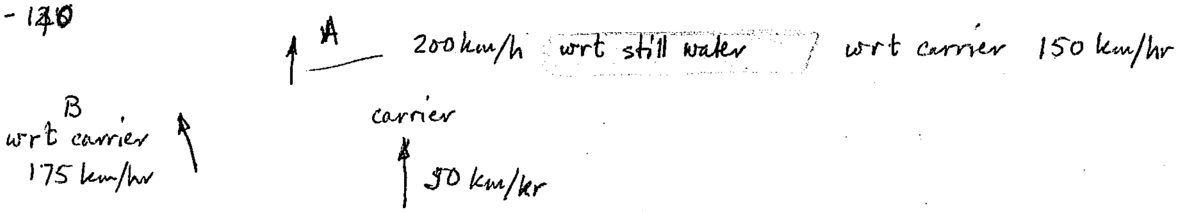
$$\tan\varphi = \frac{52.5\sqrt{3}}{37.5} \Rightarrow 180° + 67.6 = 247.6°$$

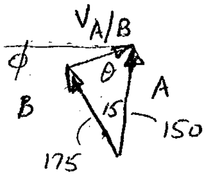
METHOD OF ATTACK for $\bar{r}_B = \bar{r}_A + \bar{r}_{B/A}$

1. Define fixed & moving reference frames
2. pick origin of moving frame as a known pt. wrt inertial frame
3. Can be no more than 2 unknowns (magnitude / direction)

for relative veloc ~~&~~ or accel : origin of fixed frame need not be specified

PROB 12-140





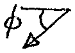
$$\bar{V}_{A/B} = \bar{V}_A - \bar{V}_B$$

$$V_{A/B} = \sqrt{(175)^2 + (150)^2 - 2(175)(150)\cos 15^\circ}$$
$$= 49.13 \text{ km/hr.}$$

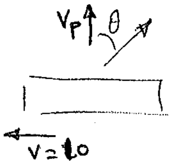
$$\frac{49.13}{\sin 15^\circ} = \frac{175}{\sin\theta}$$

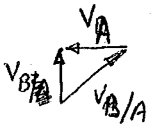
$$\sin\theta = \frac{175}{49.13}\sin 15^\circ \Rightarrow \theta = 67.21^\circ$$

$$\phi = 22.79^\circ$$



PROB 12-135





$$\bar{V}_B = \bar{V}_A + \bar{V}_{B/A}$$

$$\therefore \sin\theta = \frac{V_A}{V_{B/A}} = \frac{1}{2} \qquad \theta = 30^\circ$$

$$\bar{V}_B = \bar{V}_P = 17.32 \text{ mi/hr.}$$

LESSON # 5

- WE HAVE DEVELOPED METHOD TO FIND ACCEL OF PARTICLE
- WILL USE NEWTON'S 2nd LAW TO STUDY MOTION OF PARTICLE
  - METHOD OF SOLUTION, COORD. SYSTEM IS PROBLEM-DEPENDENT

LAW$\overset{3}{\text{S}}$ OF MOTION – REVIEW

- PARTICLE AT REST OR MOVING IN A STRAIGHT LINE w/CONST VELOCITY, REMAINS IN THIS STATE UNTIL ACTED UPON BY AN UNBALANCING FORCE

⊛ - A PARTICLE ACTED UPON BY AN UNBALANCED FORCE $\vec{F}$ EXPERIENCES AN ACCELERATION $\vec{a}$ IN THE SAME DIRECTION AS $\vec{F}$ & DIRECTLY PROPORTIONAL IN MAGNITUDE TO F.

⊛ - FOR EVERY ACTION ON A PARTICLE THERE IS AN EQUAL BUT OPPOSITE REACTION THAT THE PARTICLE EXERTS.

2nd LAW IS BASIC : FORCE ACTING ON A PARTICLE IS PROPORTIONAL TO TIME RATE OF CHANGE OF PARTICLE'S LINEAR MOMENTUM

MOMENTUM $= m\vec{v} \Rightarrow \vec{F} = \frac{d}{dt}(m\vec{v}) = m\vec{a}$ IF m=constant

- IF $\vec{F}$ is unknown and $\vec{a}$ is known then $m\vec{a} = \vec{F}$
  m is inertia or resistance to motion.
- $\vec{F} = m\vec{a}$ equation of motion experimentally observed.
- EINSTEIN DEVELOPED LIMITATIONS TO THIS EQN BASED ON THEORY OF RELATIVITY.
  - NEWTON ASSUMED TIME TO BE ABSOLUTE QUANTITY – NOT TRUE

- NEWTON'S GRAVITATIONAL LAW $F = G\dfrac{m_1 m_2}{r^2}$

③ - on surface of earth $\dfrac{G m_1}{r^2} = g \Rightarrow F = m_2 g =$ weight

② - NEAR SURFACE ATTRACTIVE FORCE IS GRAVITION

① - LAW STATES ∃ LAW OF MUTUAL ATTRACTION BET. PARTICLE

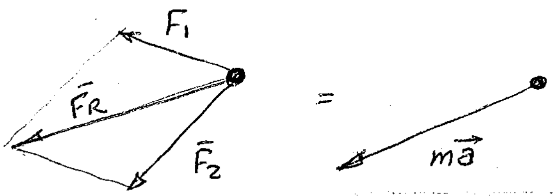
- mass IS AN ABSOLUTE QUANTITY - NOT LOCATION DEPENDENT
- weight is <u>not</u> - is LOCATION DEPENDENT

$g = 9.81\ m/sec^2$ or	$32.2\ ft/sec^2$
F = newtons	F = lbs
m = kg	m = slugs

- EQUATION OF MOTION OF PARTICLE ACTED ON BY SEVERAL FORCES

TOTAL RESULTANT FORCE $\bar{F}_R = \Sigma \bar{F} = m\vec{a}$

<u>DRAW FBD</u>

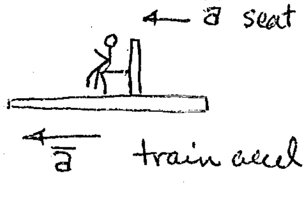


KINETIC DIAGRAM.

EQUIVALENCE BETWEEN FBD AND KINETIC DIAGRAM

- IF $\Sigma \bar{F} = \bar{0} \Rightarrow \vec{a} = 0$ either body at rest, or moving w/constant velocity

- CAN ALSO WRITE $\Sigma \bar{F} - m\bar{a} = \vec{O}$ THIS IS DYNAMIC EQUIL EQN
  - $-m\vec{a}$ IS TREATED AS A VECTOR - INERTIA FORCE VECTOR
  - D'ALEMBERT'S PRINCIPLE : $\Sigma \bar{F} - m\bar{a} = \bar{0}$



wrt train person has 0 accel.

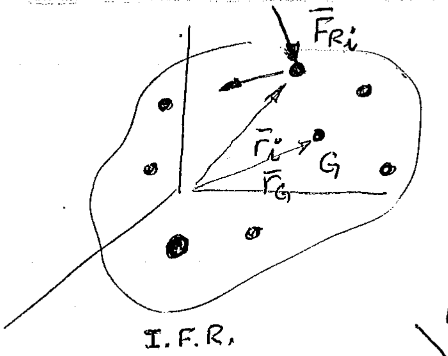
chair exerts $\vec{F}$ on person $= m\vec{a}$

IF rel accel $= \bar{O} \Rightarrow$ ∃ a force $-m\bar{a}$ inertia force

∋ $\bar{a}$ of person relative to train $= \bar{O}$

- WHEN EQS OF MOTION ARE APPLIED MUST BE MEASURED FROM INERTIAL REFERENCE FRAME (NEWTONIAN FRAME) I.F.R.
    - DOESN'T ROTATE
    - FIXED OR ~~MOVES~~ TRANSLATES AT CONSTANT VELOCITY
    - USED TO INSURE THAT ACCELERATION MEASURED IN 2 I.F.R. ARE SAME.
    - "EARTH BOUND" PROBLEMS - EARTH IS I.F.R.
    - "SPACE MOTION" PROBLEMS - STARS IS I.F.R.

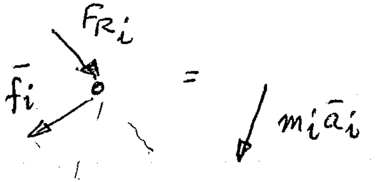
- EQN OF MOTION FOR SYSTEM OF PARTICLES



LOOK AT set of particles in a fixed region

For each particle -

1. external forces acting $\Sigma \bar F = \bar F_{R_i}$
2. internal forces $\Sigma \bar f_{ij} = \bar f_i$



$$\bar f_i + \bar F_{R_i} = m_i \bar a_i$$

1. by 3rd law all internal forces cancel ∴ $\Sigma \bar f_i = 0$
2. Take sum of $\Sigma \bar F_{R_i} = \bar F_R = \sum_{i=} m_i \bar a_i = M \bar a_G$ $M = \Sigma m_i$
   ~~&~~ $\bar a_G = d^2 \bar r_G / dt^2$ and $\bar a_i = d^2/dt^2 \bar r_i$

- EXTERNAL FORCES - Gravitational, Electric, Magnetic, Contact other particles outside the syste

$\Sigma \bar F = M \bar a_G$ replace system by one particle having mass $= \Sigma m_i$ located at center of mass of all particles

- THIS JUSTIFIES REPLACEMENT OF BODY BY ONE PARTICLE IF WE TRACK center of mass of body.

IN RECTANGULAR COORD. FOR A PARTICLE

$$\Sigma \bar{F} = m\bar{a}$$

$$\Sigma F_x \bar{i} + \Sigma F_y \bar{j} + \Sigma F_z \bar{k} = m(a_x \bar{i} + a_y \bar{j} + a_z \bar{k})$$

SCALAR EQNS $\Sigma F_l = m a_l$ $\quad l = x, y, z$

IF MOTION IS PLANAR e.g. x,y plane $\Rightarrow a_z = 0$

$$\left.\begin{array}{l} \Sigma F_x = ma_x \\ \Sigma F_y = ma_y \end{array}\right\} \text{eqns of motion.}$$

PROCEDURE

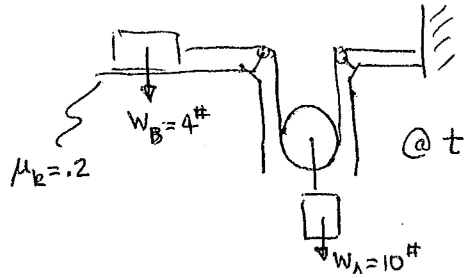
1. DEFINE I.F.R. Inertial Frame of Ref.
2. DRAW F.B.D. showing all forces
3. Assume a_x, a_y to be + in direction of positive coord
4. Resolve all forces into components along x, y, z axes
5. Write eqns of motion
   - if frictional force $F = \mu_k N$
   - spring force $F = k\Delta$ $\quad \Delta$ is stretch or compression
6. if needed: use kinematics to give you $a = \frac{dv}{dt}$ $\quad v = \frac{ds}{dt}$

   or $vdv = ads$

   - if a=constant $\Rightarrow$ $v = v_0 + at$ & $s = s_0 + v_0 t + \frac{1}{2}at^2$

     $v^2 = v_0^2 + 2a(s - s_0)$
7. if dependent motion: use ideas derived in section 12.8 to find velocity & acceleration

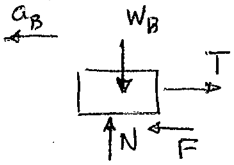
EXAMPLE 13-14 **13-24 in 10th ed pg 117**

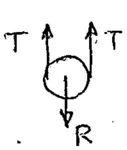


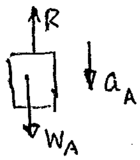
@ t=0 $V_A = 6$ ft/sec

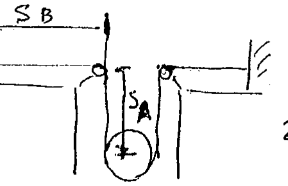
find V_A @ t=2 sec.

FBD of each









$2S_A + S_B = \text{const}$

$2V_A + V_B = 0$

$\boxed{2a_A + a_B = 0}$

$\sum F_y = m\cancel{a_y}^{\,0} = W_B - N = 0$

$N = W_B$

$\overset{+}{\leftarrow} \sum F_x = m_B a_x = \boxed{\frac{W_B}{g} a_B = F - T = \mu_k W_B - T}$

$+\!\downarrow \sum F_y = W_A - R = m_A a_y = \frac{W_A}{g} a_A$

$\boxed{W_A - 2T = \frac{W_A}{g} a_A}$

PULLEY $\sum F_y = R - 2T = \cancel{m}\overset{0}{a_y} = 0 \Rightarrow \boxed{R = 2T}$

$$\left.\begin{aligned} \mu_k W_B &= T + \frac{W_B}{g} a_B = T - \frac{2W_B}{g} a_A \\ W_A &= \frac{W_A}{g} a_A + 2T \end{aligned}\right\} \qquad a_A = \frac{2\mu_k W_B - W_A}{-\left[4\frac{W_B}{g} + \frac{W_A}{g}\right]} = \underline{\underline{10.40\ \text{ft/sec}^2}}$$

Note $a_A = \text{const} \Longrightarrow$ $V_A = V_{0_A} + a_A t$ where when $t = 0$ $V_{0_A} = 6$ ft/sec.

now @ $t = 2\text{sec}$ $\Rightarrow$ $V_A = 6 + 10.40(2) = \underline{\underline{26.80\ \text{ft/sec}}}$

FOR MY OWN EDIFICATION $\Rightarrow$

$a_B = -2a_A = -20.80$ ft/sec $=$ const.

$V_B = V_{0_B} + a_B t$: when $V_{0_B} = -2V_{0_A} = -12$ ft/sec

$V_B = -12 - 20.80(2) = -53.6$ ft/sec 2 seconds later

or $V_B = -2V_A = -2(26.80) = -53.6$ ft/sec. ✓✓

—— ∘ —— ∘ —— ∘ ——

LESSON # 6 REVIEW

⊛ Problem 12-56

Given $y = x^2$ in meters. $V_y = 3$ m/s always (ie const) $a_y \equiv 0 = \frac{dV_y}{dt}$

find V at t = 2sec given that @ $t = 0$ $(x, y) = (1, 1)$; and at $t = 2$ s

find S

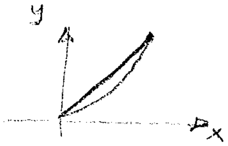
$y = x^2 \Rightarrow \dot{y} = 2x\dot{x} \Rightarrow 3 = 2x\frac{dx}{dt}$ or $3dt = 2x\,dx \Rightarrow 3t = x^2 + C$

when $t = 0$ $x = 1$ $\Rightarrow$ $C = -1$ or $\underline{\underline{x^2 = 3t + 1 = y}}$

@ $t = 2$s $\dot{y} = 3$ $x = \sqrt{3t+1}$ $\dot{x} = \frac{3}{2}\frac{1}{\sqrt{3t+1}} = \frac{3}{2\sqrt{7}} = .567\ \frac{m}{s}$

$V = \sqrt{\dot{y}^2 + (.567)^2} = 3.053$ m/s

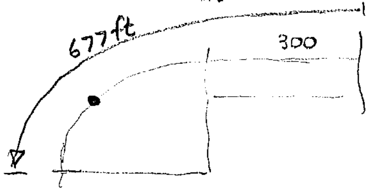
$@t = 2\,\text{sec} \quad y = 3t+1 = 7\,m \quad x = \sqrt{7}\,m \quad y = x^2$



$\therefore \quad s = \sqrt{x^2+y^2} = \sqrt{49+7} = 7.483\,m$

$\underline{12\text{-}88}$ $\quad \dot{v} = .05t^2 \quad v = .05\,t^3/3 + C \Rightarrow$ @ rest $v=0, s=0$ @ $t=0 \Rightarrow C=0$

$\frac{ds}{dt} = v = .05\,t^3/3 \Rightarrow s = \frac{.05t^4}{12} + C \Rightarrow s = \frac{.05t^4}{12}$ @ $t = 18\,\text{sec} = 437.4f$



since @ $t = 18$ sec you are in the curve

$a_t = \dot{v} = .05\,t^2 \quad a_n = \frac{v^2}{\rho} \quad \underline{\underline{v = .05t^3/3}} = \underline{\underline{97.2\,ft/s}}$

$a_t = 16.2\,ft/s^2 \quad a_n = \frac{(97.2)^2}{240} = 39.37\,ft/s^2$

$\underline{\underline{a = \sqrt{a_t^2+a_n^2} = 42.57\,ft/s^2}}$

$\underline{12\text{-}86}$ $\quad V_A$ initial $= 15\,m/s \quad \rho = 150\,m \quad a_t = \dot{v} = .4s \Rightarrow vdv = ads$ or

$v^2/2 = .2s^2 + C$ if when $s=0 \quad v=15 \Rightarrow C = \frac{225}{2}\,m^2/s^2$

$\boxed{v^2 = .4s^2 + 225}$

when $s = 100\,m \Rightarrow v = 65\,m/s$; when $s = 100\,m \quad \dot{v} = .4s = 40\,m/s^2 = a_t$

$a_n = v^2/\rho = \frac{(65)^2}{150} = 28.17\,m/s^2$

$\underline{12\text{-}101}$ $\quad \dot{\theta} = 20$ rad/sec $\quad r = 400\,mm = .4\,m \quad z = (15\sin\theta)\,mm \Rightarrow \dot{z} = 15\cos\theta\,\dot{\theta}$ mm/s

$\ddot{\theta} = 0$ and $\dot{r} = \ddot{r} = 0$ also $\ddot{z} = -15\sin\theta\,\dot{\theta}^2 + 15\cos\theta\,\cancel{\ddot{\theta}} = -15\sin\theta\,\dot{\theta}$

@ $\theta = \pi/2 \quad \ddot{z} = -15(1)(20)^2 = -6\,m/s^2$

$V_r = \dot{r} = 0 \quad a_r = \ddot{r} - r\dot{\theta}^2 = -160\,m/s^2$

$V_\theta = r\dot{\theta} = 8\,m/s \quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

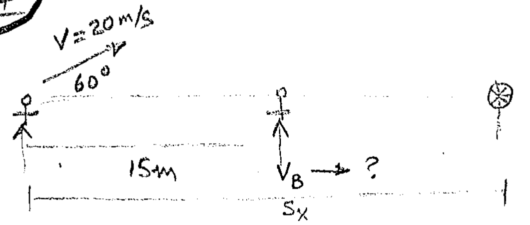
$V_z = \dot{z} = 15\cos\theta\,\dot{\theta}\Big|_{\theta=\pi/2} = 0 \quad a_z = \ddot{z} = -6\,m/s^2$

$a = \sqrt{(-160)^2 + (-6)^2} = 160.11\,m/s^2$

$V = 8\,m/s$

Same as 12-207 in 10th ed

12-144



a) find V_B

b) find $V_{F/B}$ $a_{F/B}$

$V_{oy} = 20 \sin 60° = 17.321$ m/s.

$V_{ox} = 20 \cos 60° = 10$ m/sec.

$$S_x = S_{ox} + V_{ox} t = 10t = 15 + V_B t \qquad (1)$$

$S_y = S_{oy} + V_{oy} t + \frac{1}{2} g t^2$ where $S_{oy} = S_y$ since same elev. $\Rightarrow t = 0$ or

$$t = -\frac{2V_{oy}}{g} = -\frac{2\ (17.321)}{-9.81} = 3.53 \text{ sec.}$$

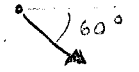
put this into (1) to get $\underline{V_B = 5.752 \text{ m/s}}$.

By symmetry

$\underline{V}_F - \underline{V}_B = \underline{V}_{F/B}$

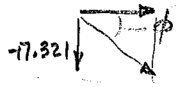
$\bullet \rightarrow V_{F_x} = V_{ox} = 10$ m/sec =

$V_{F_y} \downarrow -17.321$ m/sec.



$\bullet \rightarrow V_B = 5.752$ m/s

10 − 5.752 = 4.248 m/s



$V = \sqrt{(4.248)^2 + (17.321)^2} = 17.834$ m/s

$\phi = \tan^{-1} \frac{17.321}{4.248} = 76.22°$

For accel. $\underline{a}_F - \underline{a}_B = \underline{a}_{F/B}$

since $V_B = \text{const} \Rightarrow \vec{a}_B = \vec{0}$

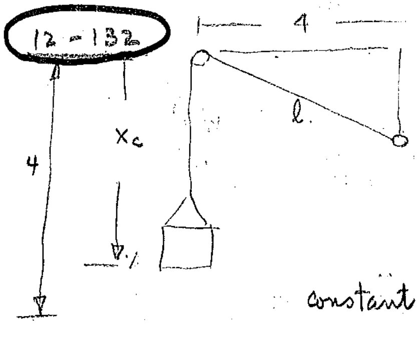
$\vec{a}_F = -9.81 \text{ m/s}^2 \downarrow.$

$\downarrow a_{F/B} = -9.81 \text{ m/s}^2$

$\bar{a}_{F/B} = \bar{a}_F - \bar{a}_B = \downarrow$

same as 12-189 in 10th ed.

12-132



x_A $\downarrow S_A$ $\downarrow v_A, a_A$

$x_c + l = \text{const.} \Rightarrow x_c + \sqrt{4^2 + x_A^2} = \text{const.}$

$$\dot{x}_c + \frac{1}{2} \frac{2x_A}{\sqrt{4^2 + x_A^2}} \dot{x}_A = 0$$

constant = 8m when roller is at B crate is on ground.

when s = 1 $x_c = 3$ m & $l = 5$ m $\dot{x}_c + \frac{1}{2} \frac{2x_A}{5} \dot{x}_A = 0$ (2) also $x_A = 3$ m.

$$\dot{x}_c = -\frac{1}{2} \frac{2 \cdot 3}{5} \cdot 2 = -\frac{6}{5} = -1.2 \text{ m/s or } 1.2 \text{m} \uparrow$$

also
$$\ddot{x}_c - \frac{1}{4} \frac{4x_A^2}{\sqrt{(4^2 + x_A^2)^3}} \dot{x}_A^2 + \frac{\dot{x}_A^2}{\sqrt{}} + \frac{\cancel{x_A}}{\sqrt{}} \cancel{\ddot{x}_A} = 0$$

$$\ddot{x}_c - \frac{1}{\cancel{4}} \frac{4(9)}{125} \cdot \cancel{4} + \frac{4}{5} = 0 \qquad \ddot{x}_c = \frac{36}{125} - \frac{100}{125} = \frac{-64}{125} = -.512 \text{ m/s}^2$$

LESSON #7

EQNS OF MOTION IN n, t COORD.

since $\Sigma \bar{F} = m\bar{a}$

in n, t coord. $\bar{a} = a_t \bar{u}_t + a_n \bar{u}_n + 0\,\bar{u}_b$ $\qquad \bar{u}_t \times \bar{u}_n = \bar{u}_b$

IF $\Sigma \bar{F} = \Sigma F_t \bar{u}_t + \Sigma F_n \bar{u}_n + \Sigma F_b \bar{u}_b \quad \Rightarrow$

$$\Sigma F_i = m a_i \qquad \text{where } i = t, n, b$$

ΣF_t - forces in tangential dir $= m a_t$ $\qquad a_t = \dot{v} = v\frac{dv}{ds}$

$\Sigma F_n =$ normal dir $= m a_n$ $\qquad a_n = v^2/\rho$

$\Sigma F_b =$ binormal direction $= m \cdot 0 = 0$

- SINCE a_n TOWARDS CENTER OF CURVATURE SO MUST ΣF_n
- ΣF_n FOR MOTION ABOUT CIRCLE W/CONST SPEED $\Rightarrow$ ΣF_n CENTRIPETAL FORCE

PROCEDURE

1. Establish Coordinate system
2. assume positive directions & a_t, a_n act in those directions
3. FBD showing all forces
4. write eqns of motion
5. Use kinematics to find a_t, a_n

Show Example 13-6 P. 103 } one or the other
Example 13-8 P. 105
Example 13-10 P. 107

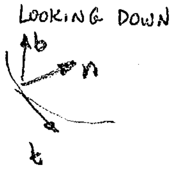
like 13-55/13-56

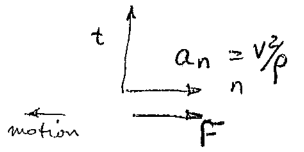
Problem 13-44 / 13-45

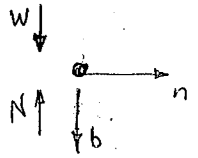
$m = 25$ kg. $\rho = 1.5$ m.   in Problem 13-43 $a_t \sim 0$

15 kg   5 m

find V







$\Sigma F_b = 0 = -N + W = 0$

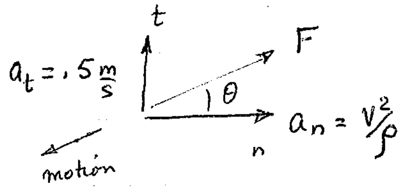
$\underline{N = W =}$

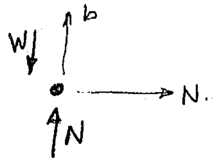
$$\Sigma F_n = ma_n = F = ma_n$$

$$\mu N = ma_n = \frac{W}{g} a_n \Rightarrow \mu W = \frac{W}{g} a_n = \frac{W}{g} \frac{V^2}{\rho}$$

$$\therefore \quad \mu g \rho = V^2 \quad \text{or} \quad V = \sqrt{\mu g \rho} = \sqrt{\underbrace{.3}_{.2}(9.81)\underbrace{(1.5)}_{5}} = 2.10 \text{ m/s.}$$

3.13 m/s





$\Sigma F_b = 0 \Rightarrow N = W.$

$$\Sigma F_n = F\cos\theta = \mu N \cos\theta = m a_n = \frac{W}{g} \frac{V^2}{\rho} \qquad V = \sqrt{\mu g \rho \cos\theta}$$

$$\Sigma F_t = F\sin\theta = m a_t \Rightarrow \mu N \sin\theta = \frac{W}{g}(.5) \Rightarrow \sin\theta = \frac{a_t}{\mu g}$$

$$\text{or} \quad \theta = \sin^{-1}\left(\frac{.5}{\underbrace{.3}_{.2}(9.81)}\right) = 9.78^\circ \quad 14.76^\circ$$

$$\& \quad \sqrt{\underset{.2}{\mu}\, g\, \underset{5}{\rho} \cos \underset{14.76^\circ}{\theta}} = V = 2.09 \text{ m/s}$$

3.08 m/s

Cylindrical coordinates

$$\Sigma \bar{F} = m\bar{a} = m(a_r \bar{u}_r + a_\theta \bar{u}_\theta + a_z \bar{u}_z)$$

$$\Sigma F_r \bar{u}_r + \Sigma F_\theta \bar{u}_\theta + \Sigma F_z \bar{u}_z = m(\ddot{r} - r\dot{\theta}^2)\bar{u}_r + m(r\ddot{\theta} + 2\dot{r}\dot{\theta})\bar{u}_\theta + m\ddot{z}\bar{u}_z$$

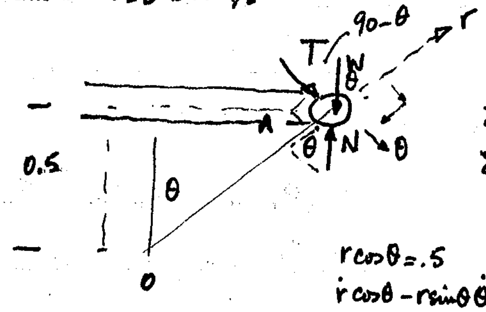
ΣF_r – forces in radial dir $= m(\ddot{r} - r\dot{\theta}^2)$

ΣF_θ – angular dir $= m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

ΣF_z $= m\ddot{z}$

1. Establish coordinates r, θ, z
2. assume $\bar{a}_r, \bar{a}_\theta, \bar{a}_z$ act in + direction of coord.
3. Draw FBD showing forces in r, θ, z dir
4. Use eqn of motion

The particle has a mass of .5 kg & is confined to move along the smooth horizontal slot due to rotation of arm OA. Find force rod applies to particle & normal force on slot when $\theta = 30°$
The rod rotates at constant $\omega = \dot{\theta} = 2$ rad/s



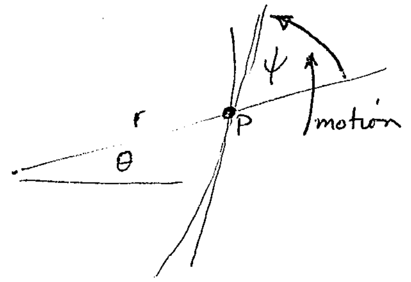
$$\Sigma F_r = m(\ddot{r} - r\dot{\theta}^2) = -W\cos\theta + N\cos\theta = m(\ddot{r} - r\dot{\theta}^2)$$
$$\Sigma F_\theta = m(2\dot{r}\dot{\theta} + r\cancel{\ddot{\theta}}) = T - N\sin\theta + W\sin\theta$$

$$r\cos\theta = .5$$
$$\dot{r}\cos\theta - r\sin\theta\,\dot{\theta} = 0 \qquad \dot{r} = r\tan\theta\,\dot{\theta} = .67\text{ m/s}$$
$$\ddot{r}\cos\theta - 2\dot{r}\sin\theta\,\dot{\theta} - r\cos\theta\,\dot{\theta}^2 - r\sin\theta\,\cancel{\ddot{\theta}}$$
$$\ddot{r} = 2\dot{r}\dot{\theta}\tan\theta + r\dot{\theta}^2 = 3.85\text{ m/s}^2$$

$$-.5(9.81)\cos 30 + N\cos 30 = .5\left[3.85 - .577\cdot 2^2\right] = m(\ddot{r} - r\dot{\theta}^2)$$
$$N = 5.79\text{ N}$$

$$T - 5.79\sin 30 + (.5)(9.81)\sin 30 = .5\left[2(.67)2\right] = m(2\dot{r}\dot{\theta} + \cancel{r\ddot{\theta}})$$
$$T = 1.78\text{ N}$$

5. Use kinematics to find a_r, a_θ, a_z



$\tan\psi = r / \frac{dr}{d\theta}$

if $r = f(\theta)$ $\quad \frac{dr}{d\theta} = f'$

NEED TO DETERMINE $\Sigma F_r, \Sigma F_\theta, \Sigma F_z$:

IF accel is not known completely - must know something about direction or magnitude of forces acting on particle

A normal force act ⊥ to tangent of path.

A friction force acts ∥ to tangent of path opposing motion

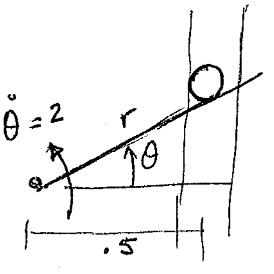
DIRECTION OF F & N CAN BE FOUND IF ψ is known.

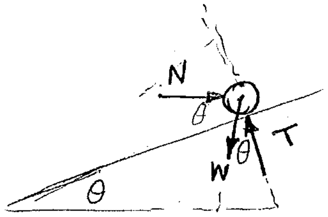
ψ is always measured counterclockwise

EXAMPLE 13-13 PG 117

Same as 13-91 in 10th ed. 13-114/115 in 12th ed

PROBLEM 13-77





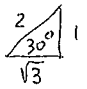
T acts in the θ dir

$W\sin\theta$ acts in - r dir

$W\cos\theta$ acts in - θ dir

$N\cos\theta$ acts in r dir

$N\sin\theta$ acts in - θ dir



$$\Sigma F_r = N\cos\theta - W\sin\theta = m(\ddot r - r\dot\theta^2)$$

$$\Sigma F_\theta = T - W\cos\theta - N\sin\theta = m(r\ddot\theta + 2\dot r\dot\theta)$$

$$\Sigma F_z = 0 \qquad = m a_z \Rightarrow \ddot z = 0 \quad \dot z = C = C$$

$$z = C = C$$

Given : $r\cos\theta = .5\,m \Rightarrow$ $\quad \dot r\cos\theta - r\sin\theta\,\dot\theta = 0$

$$\ddot r\cos\theta - 2\dot r\sin\theta\,\dot\theta - r\cos\theta\,\dot\theta^2 - r\sin\theta\,\ddot\theta = 0$$

since $\dot\theta$ = const $\quad \ddot\theta = 0$; @ $\theta = 30°$ $\quad r\cos(30°) = .5\,m \quad r = \frac{\sqrt3}{3} = .577\,m$

$\dot r = r\tan\theta\,\dot\theta = \frac{\sqrt3}{3}\frac{1}{\sqrt3}.2 = .667$ m/s

$\ddot r = 2\dot r\dot\theta\tan\theta + r\dot\theta^2 = 2(\frac{2}{3})(2)\frac{1}{\sqrt3} + \frac{\sqrt3}{3}(2)^2 = \frac{8}{9}\sqrt3 + \frac{12}{9}\sqrt3 = \frac{20}{9}\sqrt3 =$

3.46400
.34640
.03464
3.84504
~3.849

@ $\theta = 30°$ $N(.866) - .5(9.81)(½) = .5[3.849 - .5774(4)]$

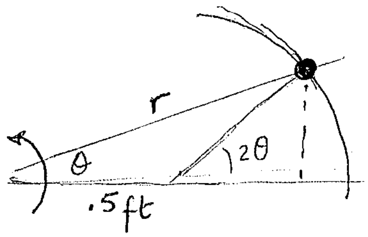
$T - .5(9.81)(.866) - N(.5) = .5[.5774(0) + 2(.667)(2)]$

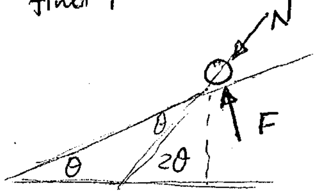
from ① $N = 3.72\,N$

from ② $\underline{T = 7.44\,N}$.

<u>Problem 13-73</u> $W = .5\,lb$ $\dot{\theta} = 4\ rad/s$ $\ddot{\theta} = 8\ rad/s^2$ when $\theta = 30°$

find F





N cos θ acts in − r dir
N sin θ acts in − θ dir
F acts in θ dir

$r\sin\theta = .5\sin 2\theta \Rightarrow \dot{r}\sin\theta + r\cos\theta\,\dot{\theta} = .5\cos 2\theta\,(2\dot{\theta})$.

$r = 1\cos\theta$ or $\dot{r} = -\sin\theta\,\dot{\theta}$ $\ddot{r} = -\cos\theta\,\ddot{\theta}^2 - \sin\theta\,\ddot{\theta}$

when $\theta = 30°$ $r = .866\ ft$ $\dot{r} = -\frac{1}{2}.4 = -2\frac{ft}{s}$ $\ddot{r} = -.866(16) - \frac{1}{2}(8) = -17.8$ $\frac{ft}{s}$

① $\Sigma F_r = -N\cos\theta = \frac{W}{g}(\ddot{r} - r\dot{\theta}^2) \Rightarrow \frac{.5}{32.2}(-17.856 - .866(16)) = -N(.866)$

$\Sigma F_\theta = F - N\sin\theta = \frac{W}{g}(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \Rightarrow F - \frac{1}{2}N = \frac{.5}{32.2}[.866(8) + 2(-2)(4)]$

① $N = .569\ lb$ ② $F = .143\ lb$

— • — • — • — • —

SESSION # 8 EXAM

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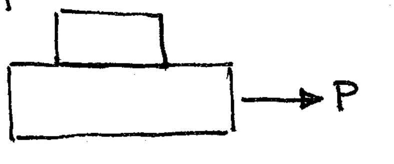
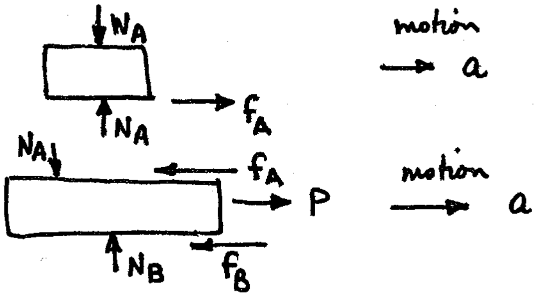
SESSION # 9

1. DETERMINING FORCES ON BODY

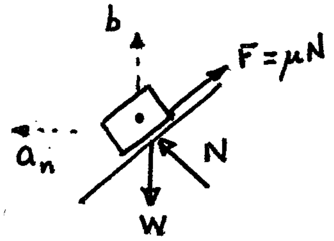
$\boxed{13\text{-}34}$

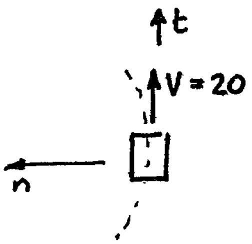




13-40

13-54

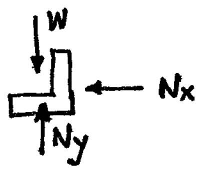




b is out of the paper.

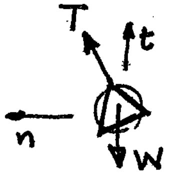
13-41

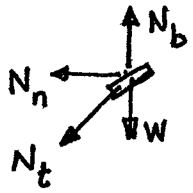
13-42



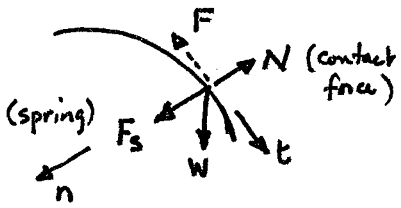
13-46

13-67





13-54



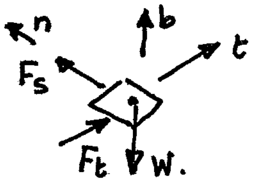
when it leaves surface: N = 0

if friction: F

b is into paper.

13-56

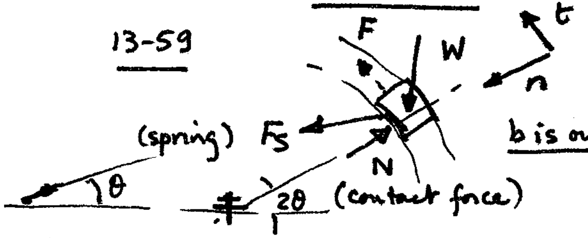
13-80



13-82

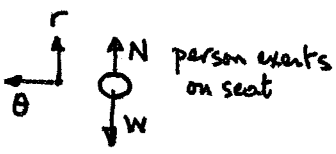
13-59

if Friction: add F



b is out of paper

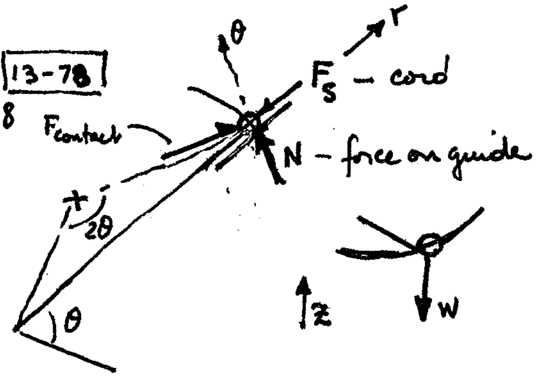
13-75



person exerts on seat

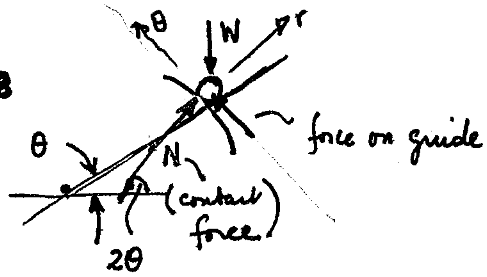
13-78

13-98



13-90

13-73

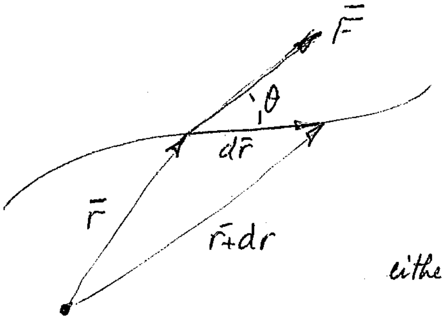


WORK & ENERGY

- USE EQNS OF MOTION TO GET PRINCIPLE OF WORK & ENERGY
- USED TO RELATE FORCE, VELOC & DISP.

FROM STATICS - VIRTUAL WORK.

- WORK OF A FORCE - WHEN FORCE UNDERGOES DISPLACEMENT IN DIRECTION OF FORCE



$\bar{F}\cdot d\bar{r} = F ds \cos\theta = dU$ SCALAR

$|d\bar{r}| = ds$ θ between $\bar{F}$ & $d\bar{r}$

F is $|\bar{F}|$

either $dU = F \underbrace{ds\cos\theta}_{\text{component of ds in dir. of F}} = \underbrace{F\cos\theta}_{\text{component of F in dir. of ds}} ds$

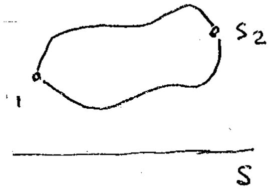
UNIT OF WORK is Joule, ft-lb

$$U_2 - U_1 = U_{1\to 2} = \int_{s_1}^{s_2} F\cos\theta\, ds = \int F_t\, ds \quad \text{if} \quad \bar{F} = \bar{F}(s)$$

skip

~~if F is fn of s work is dependent on path~~

~~F is not fn of s work is independent on path but only on end pts.~~



- work is area under curve

IT FORCE MOVING ALONG STRAIGHT LINE ⇒ θ = constant

$= F\cos\theta \int ds = F\cos\theta\,(s_2 - s_1)$

$\bar{\imath} + dy\bar{\jmath} + dz\bar{k}$ $\quad \bar{F} = F_x\bar{\imath} + F_y\bar{\jmath} + F_z\bar{k}$

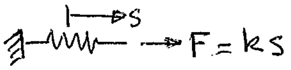
$$= \int_{\bar{r}_1}^{\bar{r}_2} \bar{F}\cdot d\bar{r} = \int_{x_1}^{x_2} F_x\, dx + \int_{y_1}^{y_2} F_y\, dy + \int_{z_1}^{z_2} F_z\, dz$$

NOTE F_x does no work in y dir
F_y does no work in x dir

- iF F = weight $= -W\bar{\jmath}$ $\Rightarrow U_{1-2} = \int -W\,dy = -W(y_2 - y_1) = -W\Delta y$

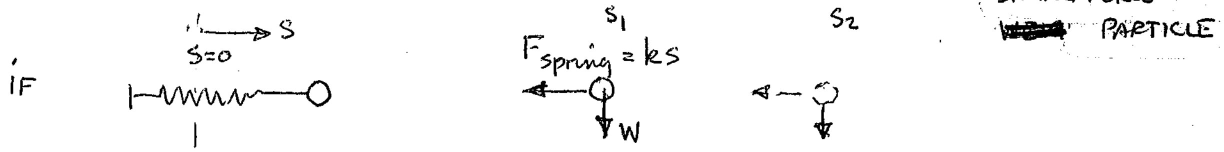
- negative work since ds is + ↑ & force is ↓

WORK OF A SPRING FOR DISTANCE MEASURED FROM <u>UNSTRETCHED</u> POSITION



$$U_{1-2} = \int \bar{F}\cdot d\bar{r} = \int_{s_1}^{s_2} ks\,\bar{i}\cdot ds\,\bar{i} = \int_{s_1}^{s_2} ks\,ds = \left.\frac{ks^2}{2}\right|_{s_1}^{s_2} = \frac{ks_2^2}{2} - \frac{ks_1^2}{2}$$

- in stretching since $\bar{F}$ & $d\bar{r}$ are in same dir U is +



- weight does no work since s is ⊥ to W

Thus $\int \bar{F}\cdot d\bar{r}$ on particle $-\int F s\,ds = -k\left(\frac{s_2^2}{2} - \frac{s_1^2}{2}\right) = U_{1-2}$

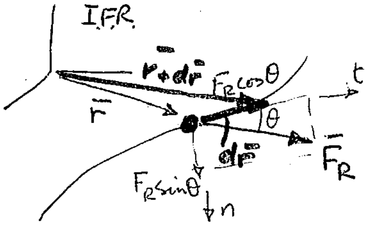
- note sign : $\bar{F}$ & $d\bar{r}$ are in opposite dir

<u>NOW</u> <u>LOOK AT particle along path.</u>

$$\bar{F}_R = \Sigma\bar{F} = m\bar{a} = \Sigma F_t\,\bar{u}_t + \Sigma F_n\,\bar{u}_n = m a_t \overset{\bar{u}_t}{} + m a_n \bar{u}_n$$

along path: $d\bar{s} = ds\,\bar{u}_t$

NORMAL COMPONENTS
<u>DO NO WORK</u>



$$\Sigma F_t\,ds\;\bar{u}_t\cdot\bar{u}_t \qquad \Sigma F_n\,ds\;\cancel{\bar{u}_n\cdot\bar{u}_t}_{0}$$

$$\int \bar{F}_R\cdot d\bar{r} = \int \Sigma F_t\,\bar{u}_t\cdot ds\,\bar{u}_t + \cancel{\int \Sigma F_n\,\bar{u}_n\cdot ds\,\bar{u}_t}_{0}$$

$$= \int \Sigma F_t\,ds$$

$$\bar{F}_R = F_R\cos\theta\,\bar{u}_t + F_R\sin\theta\,\bar{u}_n$$

$$= \int (m a_t\,\bar{u}_t + m a_n\,\bar{u}_n)\cdot ds\,\bar{u}_t$$

$$= m\int a_t\,ds + m\int a_n\,ds\,\cancel{\bar{u}_n}_{0}$$

$$\int \Sigma F_t\,ds \qquad = \int m\,a_t\,ds$$

$$\Sigma\int F_i\cos\theta_i\,ds = \int F_R\cos\theta\,ds = \Sigma\underbrace{\int F_t\,ds}_{} = \int m\,\dot{v}\,ds = \int m v\frac{dv}{ds}\,ds$$

work done by the tangential forces in moving thru distance ds.

$$= m\int v\,dv.$$

$$\Sigma\int F_i\cos\theta_i\,ds = \int F_R\cos\theta\,ds = \Sigma\,U_{1-2} = m\left[\frac{v_2^2}{2} - \frac{v_1^2}{2}\right]$$

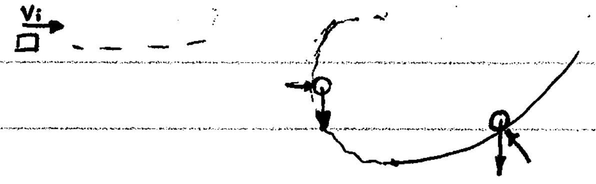
$\theta_i \neq \theta$

work of a force $\bar{F}_R$ = change of kinetic energy of the particle

14-18 in 10th
in 13th 14-19

$$W = \frac{mv^2}{\rho}$$

$$g = \frac{v^2}{\rho} \qquad \sqrt{9.81(25)} = V \cong 15.\_ \ m/s$$



work of mass is mg Δy

$$\frac{1}{2}mv_B^2 + \sum U_{BC} = \frac{1}{2}mv_C^2$$

$$\frac{1}{2}mv_B^2 \quad - mg\,\Delta y \quad = \frac{1}{2}\,mg\rho_C$$

$$v_B = \sqrt{g\rho_C + 2g\,\Delta y} = \sqrt{(9.81)\left[25+2\cdot 35\right]} = 30.53\ m/s$$

now from B to D $\quad \frac{1}{2}mv_B^2 + \sum U_{B\text{-}C} = \frac{1}{2}m\cancel{v_D^2}\,0$

$$- mg\,h$$

$$\therefore \quad h = \frac{v_B^2}{2g} = \frac{95\cdot g}{2} = 47.5\ m$$

14-17 in 10th / 14-18 in 13th

$v_i = 0$ Spring one is stretched +.5m
spring two is compressed −.5m

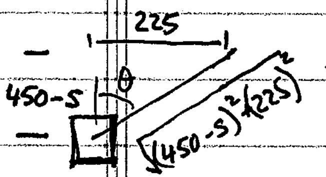
$$\frac{1}{2}\cancel{mv_i^2} + \frac{1}{2}k_1\,\Delta s_1^2 + \frac{1}{2}k_2\,\Delta s^2 = \frac{1}{2}m\,v_f^2$$

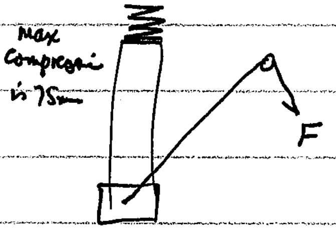
$$\Delta s_1 = .5 \qquad \Delta s_2 = -.5$$

$$v_f^2 = \frac{k_1\Delta s_1^2 + k_2\Delta s_2^2}{m} = \frac{50(.5)^2 + 100(.5)^2}{20} \Rightarrow \frac{150(.25)}{20} = 1.875\ m^2/s^2$$

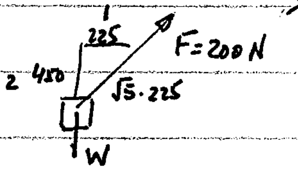
$$v_f = 1.37\ m/s$$

7kg





$$\cancel{0 + F\cdot\frac{2}{\sqrt{5}}\cdot\left(\frac{450-75+75}{1000}\right) - \frac{1}{2}k\left(\frac{75}{1000}\right)^2} = 0$$



$$\frac{2F}{(.075)^2}\cdot\frac{2(.45)}{\sqrt{5}} = k$$

$$200\left[\sqrt{5}\cdot 225 - 225\right]$$
$$200(\sqrt{5}-1)\cdot\frac{225}{1000}$$

let $x = 450 - s$

$$\int_{450}^{0}\frac{-x\,dx}{\sqrt{x^2+a^2}} = \int_{0}^{450}\frac{x\,dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}\,F\cos\theta \quad \begin{matrix}\sqrt{5}\cdot 225\\ 225\end{matrix}$$

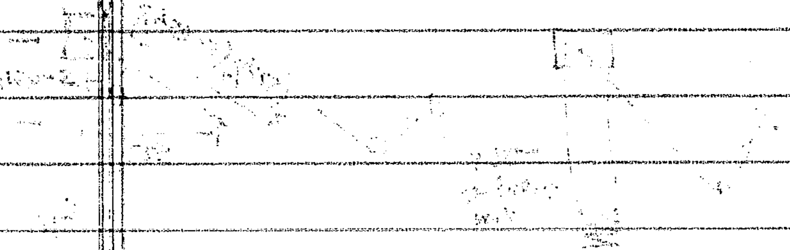
$$+\int_0^{450} 200\cdot\frac{450-s\ \ ds}{\sqrt{(450-s)^2+(225)^2}} - mg\cdot\frac{450}{\cancel{4}} - \frac{1}{2}k\,(0.075)^2 = 0$$

55.623 −30.902

$450^2 - 2\cdot 450 s + s^2 + 225^2$

$$k = 8789.7\ N/m$$

[illegible]



[illegible]

[illegible]

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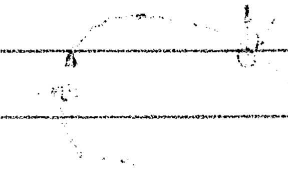


[illegible]

[illegible]

if we define $T = \frac{1}{2}mv^2$ kinetic energy of a particle

$$\frac{1}{2}mv_1^2 + \Sigma U_{1-2} = \frac{1}{2}mv_2^2$$
$$T_1 + \Sigma U_{1-2} = T_2$$
} principle of work & energy

$$\Sigma U_{1-2} = \int F_R \cos\theta \, ds \quad \text{or} \quad \Sigma \int F_i \cos\theta_i \, ds$$
$$\theta = \measuredangle(F_R, ds) \qquad \theta_i = \measuredangle(F_i, ds)$$

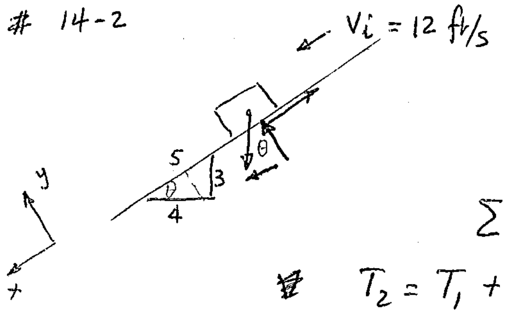
- since this equation arises from the tangential component of eqns of motion
  - CAN USE THIS EQN IN PLACE OF $\Sigma F_t = ma_t$
  - MUST STILL USE $\Sigma F_n = ma_n = mv^2/\rho$
- CAN USE THIS EQN TO OBTAIN V_2 DIRECTLY
- PUT INTO $\Sigma F_n = mv^2/\rho$ TO FIND ΣF_n ie the normal component of F_R.

To apply principle
- DRAW FBD of particle showing all forces acting on particle
- velocities are always measured wrt I.F. Reference
- get algebraic sum of work due to forces - sign depends on direction of $\bar{F}$ & $d\bar{r}$

SESSION #10

Pg 147
\# 14-2

a package whose initial veloc = 12 ft/s travels down an inclined ramp which has a coeff of fric of 0.2 what is its velocity after it travels 6 ft.



along surface $W\sin\theta = W \cdot \frac{3}{5} \leftarrow$ & $F = \mu N = \mu W\cos$
$= \frac{4}{5}\mu W$

here we take $\int \Sigma F\cos\theta \cdot ds = \Sigma F\cos\theta \cdot s$

$$\Sigma U_{1-2} = W\sin\theta \cdot s - \mu W\cos\theta \cdot s = Ws\left[\frac{3}{5} - .2\,\frac{4}{5}\right] = Ws(.4$$

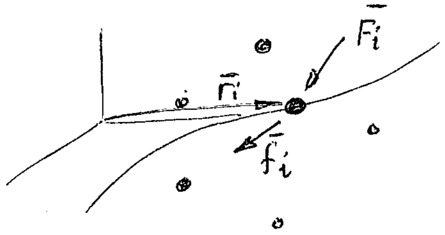
$$T_2 = T_1 + \Sigma U_{1-2} = \frac{1}{2}\frac{W}{g}V_2^2 = \frac{1}{2}\frac{W}{g}V_1^2 + Ws[\sin\theta - \mu\cos\theta]$$

CONSTANT FORCE ALONG STRAIGHT PATH. -

$$V_2^2 = V_1^2 + 2sg[\sin\theta - \mu\cos\theta]$$
$$= 144 + 2(6)[.44](32.2) = 314.02$$

$$V_2 = 17.72 \text{ ft/s}$$

FOR A SYSTEM OF PARTICLES $\overset{\text{IN A REGION}}{\vee}$ EACH MOVING ALONG ITS PATH



we can write for particle i

$$\frac{1}{2} m_i V_{i_1}^2 + \int_{\bar{r}_{i_1}}^{\bar{r}_{i_2}} \bar{F}_i \cdot d\bar{r}_i + \int_{\bar{r}_{i_1}}^{\bar{r}_{i_2}} \bar{f}_i \cdot d\bar{r}_i = \frac{1}{2} m_i V_{i_2}^2$$

I.F.R.

take $\sum_i \left[\frac{1}{2} m_i V_{i_1}^2 + \int \quad + \int \quad = \frac{1}{2} m_i V_{i_2}^2 \right]$

- but since each particle moves over its own path $\Rightarrow d\bar{r}_i$ are not equal
  thus even though $\Sigma \bar{f}_i = 0$ $\quad \int \bar{f}_i \cdot d\bar{r}_i \neq 0$

Hence $\Sigma T_1 + \Sigma U_{1\to2} = \Sigma T_2$ where $\Sigma T = \Sigma \frac{1}{2} m_i V_i^2$

$$\Sigma U_{1\to2} = \Sigma \int (\bar{F}_i + \bar{f}_i) \cdot d\bar{r}_i$$

- exceptions: for a translating body all $d\bar{r}_i$ are the same $\Rightarrow \Sigma \int \bar{f}_i \cdot d\bar{r}_i = 0$

- PARTICLES CONNECTED BY INEXTENSIBLE CABLES $\overset{\text{OR RODS}}{\wedge}$: FORM SYSTEM ~~###~~ FOR WHICH INTERNAL FORCES HAVE COMPONENTS THAT UNDERGO SAME DISPL. & THUS $\Sigma \int \bar{f}_i \cdot d\bar{r}_i = 0$.

YOU GET JUST 1 eqn for the entire system

pg. 146

KINEMATICS TO RELATE DISTANCES, ASSUME

IN POSITIVE COORDINATE DIRECTION

KINEMATIC & PRINCIPLE OF WORK & ENERGY)

$T_1 = \frac{1}{2} m_A V_{A_1}^2 + \frac{1}{2} m_B V_{B_1}^2 = 0$

$2S_A + S_B = \text{const}$

$U_{1-2} = W_A \Delta S_A + W_B \Delta S_B$

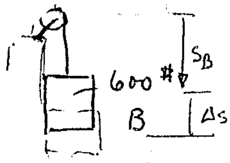
$= W_A \Delta S_A + W_B(-2\Delta S_A)$

$= (W_A - 2W_B)\Delta S_A = 40.3$

$T_2 = \frac{1}{2} m_A V_{A_2}^2 + \frac{1}{2} m_B V_{B_2}^2 =$

$= \frac{1}{2g}[W_A V_{A_2}^2 + W_B(-2V_{A_2})^2]$

" CO1



SYSTEM STARTS FROM REST
FIND VELOC OF A AFTER A HAS MOVED 5 FT.

$S_A + S_B = 0$

$\Delta S_A = -\Delta S_B$

-5 ft $\quad \Delta S_B = 5$ ft ↓

$V_A = -V_B$

13.9 ft/s

lly

$T_1 = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2 = 0$

$\Sigma U_{1-2} = -W_B \Delta y = W_B \Delta s = 600(5)$

Read Section 14.4   CAN SKIP & ASK THEM TO READ

POWER - AMOUNT OF WORK DONE PER UNIT TIME

$$P = \frac{dU}{dt} \quad \text{or} \quad P_{AVE} = \frac{\Delta U}{\Delta t}$$

since $dU = \bar{F}\cdot d\bar{r} \Rightarrow \frac{dU}{dt} = \underline{\underline{\bar{F}\cdot\bar{v} = P}}$ scalar

Units are horsepower (british)   watts (si)

1 watt = 1 Joule/s = 1 N·m/s

1 hp = 550 ft·lb/s = 746 watts

POWER - USEFUL TO DETERMINE TYPE OF MACHINE TO DO REQUIRED AMOUNT OF WORK

MECHANICAL EFFICIENCY OF A MACHINE IS RATIO OF $\frac{\text{POWER OUTPUT}}{\text{POWER INPUT}} = \varepsilon$

ALSO $\frac{\text{ENERGY OUTPUT}}{\text{ENERGY INPUT}} = \varepsilon$ ENERGY BEING POWER · TIME

TO ANALYZE:

- ALWAYS DETERMINE FORCE ACTING ON BODY
- DRAW FBD & APPLY EQS OF MOTION
- DETERMINE ~~FORCE~~ VELOCITY & DIRECTION BETWEEN $\bar{F}$ & $\bar{v}$
- OR USE $P = \frac{dU}{dt}$

- A FORCE IS CONSERVATIVE
  - IF WORK DONE IS INDEPENDENT OF PATH TAKEN (ONLY DEPENDS ON END PTS)
  - IF FORCE IS FUNCTION OF PARTICLE'S POSITION $F = F(s)$
  - NOT A FN OF VELOCITY OR ACCEL. FRICTION IS NOT CONSERVATIVE

WEIGHT IS CONSERVATIVE - WORK DEPENDS ON Δy $U = -W\Delta y$

SPRING FORCE ON A PARTICLE IS CONSERVATIVE : $F = ks$ $U = -\frac{1}{2}k(s_2^2 - s_1^2)$

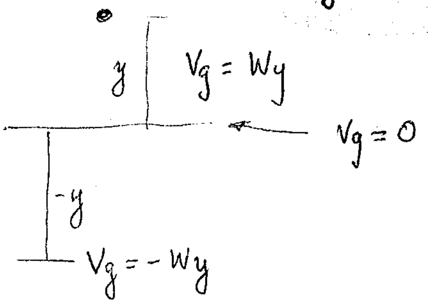
- FORCE ACTING ON PARTICLE HAS CAPACITY TO DO WORK.
  - CAPACITY IS POTENTIAL ENERGY V
  - V DEPENDS ON LOCATION OF PARTICLE WHEN ACTED ON BY F

- GRAVITATIONAL POTENTIAL ENERGY: POTENTIAL ENERGY DUE TO GRAVITY
  - DEPENDENT ON DEF'N OF A FIXED DATUM LINE

$$V_g = Wy$$

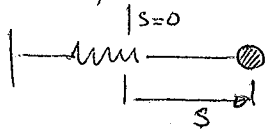
W - weight y - distance above datum

$$V_{g_2} - V_{g_1} = W(y_2 - y_1)$$



ASSUMPTIONS: FOR SMALL CHANGES IN y W = constant.

- ELASTIC POTENTIAL ENERGY: POTENTIAL ENERGY STORED IN SPRING IMPARTED TO A PARTICLE.
  - ALWAYS MEASURED FROM UNDEFORMED POSITION OF SPRING



$$V_e = \tfrac{1}{2}ks^2$$

always + for stretched or compressed spring

$$V_{e_2} - V_{e_1} = \tfrac{1}{2}(ks_2^2 - ks_1^2)$$

WORK DONE BY CONSERVATIVE FORCE

$$U_{1-2} = V_1 - V_2$$

by any force & specifically a conservative force is U_{1-2}

- body affected by 2 or more conservative forces

$$V_{TOTAL} = \sum V_i \Rightarrow V = V_g + V_e$$

- $dU = -dV$ change in work = negative of change in potentia

$$dU = \bar{F}\cdot d\bar{r} = (F_x \cancel{d}\hat{\imath} + F_y \bar{j} + F_z \bar{k})\cdot(dx\,\hat{\imath} + dy\,\bar{j} + dz\,\bar{k})$$

$$= F_x\,dx + F_y\,dy + F_z\,dz$$

since V is a fn of position $V = V(x, y, z)$

dV is a total differential $dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$

$$\Rightarrow \qquad F_i = -\frac{\partial V}{\partial x_i} \quad \Rightarrow \quad \bar{F} = -\left(\frac{\partial V}{\partial x} \bar{i} + \frac{\partial V}{\partial y} \bar{j} + \frac{\partial V}{\partial z} \bar{j}\right)$$

$$\bar{F} = -\nabla V$$

FOR CONSERVATIVE FORCE $\bar{F} = -\nabla V$

if $V = Wy$ $\quad -\nabla V = -W\bar{j} = \bar{F}$ weight acts downward.

if $V = \frac{1}{2} kx^2$ $\quad -\nabla V = -\frac{\partial V}{\partial x} i = kxi = \bar{F}$ restoring force on mass by spring

---

SESSION #11

NOW DEFINE PRINCIPLE OF CONSERVATION OF ENERGY

RECALL $T_1 + \sum U_{1-2} = T_2$: PRINCIPLE OF WORK + ENERGY

$$\sum U_{1-2} = \left(\sum U_{1-2}\right)_{consew} + \left(\sum U_{1-2}\right)_{nonconsew} = V_1 - V_2 + \left(\sum U_{1-2}\right)_{n.c.}$$

$$T_1 + V_1 - V_2 + \left(\sum U_{1-2}\right)_{n.c.} = T_2 \quad \text{or}$$

$$\underline{\underline{T_1 + V_1 + \left(\sum U_{1-2}\right)_{n.c.} = T_2 + V_2}}$$

IF NO nonconservative forces $\Rightarrow$ $T_1 + V_1 = T_2 + V_2$ $= \text{constant}$ CONSERVATION OF ENERGY EQ.

P.E. converts to K.E. & vise versa.

FOR A SYSTEM OF PARTICLES UNDER CONSERVATIVE FORCES

$$\sum T_1 + \sum U_{1-2} = \sum T_2 \quad \text{where} \quad \sum U_{1-2} = \sum V_1 - \sum V_2$$

$$\sum T_1 + \sum V_1 = \sum T_2 + \sum V_2$$

kinetic + potential energies caused by internal & external forces is a constant

REMEMBER $\sum U_{1-2} = \int \bar{F}_i \cdot d\bar{r}_i + \int \bar{f}_i \cdot d\bar{r}_i = \sum V_1 - \sum V_2$

- CAN USE CONSERVATION OF ENERGY EQN W/ PROBLEMS THAT INVOLVE VELOCITY, DISPL. & CONSERVATIVE FORCE SYSTEMS
- EASIER THAN PRINCIPLE OF WORK & ENERGY

TO USE EQN

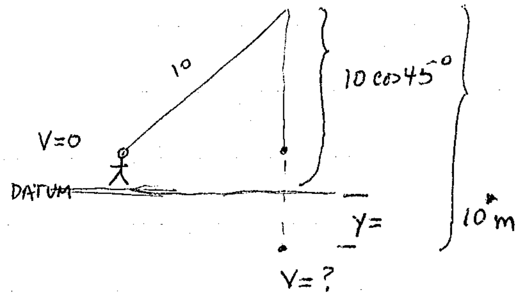
- DRAW PARTICLE AT INITIAL & FINAL PT.
- ESTABLISH FIXED DATUM IS PARTICLE UNDERGOES y displ.
- EITHER PICK DATUM AT FINAL OR INITIAL PT. (one of $V_g = 0$)
- DETERMINE STRETCHES IN ANY SPRINGS
- APPLY $T_1 + V_1 = T_2 + V_2$ where $T = \frac{1}{2} m v^2$ v measured in I.F.R.

- REMEMBER CAN ONLY BE USED FOR CONSERVATIVE FORCES SYSTEMS

→ • USE PRINCIPLE OF WORK & ENERGY FOR PROBLEMS INVOLVING FRICTIO

- REMEMBER THIS EQN IS OBTAINED FROM TANGENTIAL EQN OF MOTION
  - MUST USE NORMAL EQN OF MOTION FOR normal components of force or acceleration.

14-49 **14-81 in 10th ed** **14-86 in 12th ed.**



$$T_1 + V_1 = T_2 + V_2$$

@ 1 $v_1 = 0 \Rightarrow T_1 = 0$

@ 1 $V_1 = 0 \Rightarrow$ $T_1 + V_1 = 0$

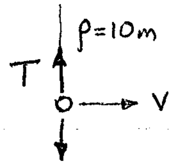
@ 2 $v_2 = ?$

2 $V_2 = -W\left[10 - 10\cos 45^\circ\right]$

$$0 = T_1 + V_1 = T_2 + V_2 = \frac{1}{2}\frac{W}{g}v_2^2 - W\left[10\,(1 - .7071)\right]$$

thus $v_2 = \sqrt{10\,(1 - \cos 45^\circ)\cdot 2g} = 7.581$ m/s

Just before



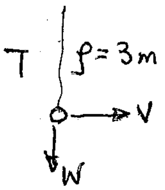
$$a_n = \frac{v^2}{\rho} = \frac{(7.581)^2}{10} = 5.747 \text{ m/s}^2$$

$$T - W = m\frac{v^2}{\rho} = m a_n \Rightarrow T = \frac{W}{g}a_n + W$$

$$= W\left[\frac{a_n}{g} + 1\right] =$$

$$\boxed{T = 1555.72\text{ N}}$$

Just after



$$a_n = \frac{v^2}{\rho} = \frac{(7.581)^2}{3} = 19.157 \text{ m/s}^2$$

$$T - W = m\frac{v^2}{\rho} = m a_n \Rightarrow T = W\left[\frac{a_n}{g} + 1\right]$$

$$\boxed{T = 2896.72\text{ N}}$$

HW. READ CH. 14.5, 6 **HW** 14-50, 55, 51, 61

SO FAR HAVE LEARNED

FOR A <u>PARTICLE</u>

$$T_1 + \Sigma U_{1-2} = T_2$$

ΣU_{1-2} : work done by external forces

AND $\Sigma U_{1-2} = V_1 - V_2$ FOR CONSERVATIVE FORCE ACTING ON PARTICLE

SUBSTITUTE → $T_1 + (V_1 - V_2) = T_2$ or $T_1 + V_1 = T_2 + V_2 = \text{CONST.}$

THIS IN PRINCIPLE OF CONSERVATION OF ENERGY

i.e. P.E. CONVERTS TO K.E. & VICE VERSA

- IF FORCES ACTING ON A PARTICLE ARE MADE UP OF CONSERVATIVE & NON-CONSERVATIVE FORCES

$$\Rightarrow \Sigma U_{1-2} = (\Sigma U_{1-2})_{\text{cons.}} + (\Sigma U_{1-2})_{n.c.}$$

$$\therefore \quad T_1 + \Sigma U_{1-2} = T_2 \Rightarrow T_1 + (\Sigma U_{1-2})_{\text{cons}} + (\Sigma U_{1-2})_{n.c} = T_2$$

OR $$T_1 + V_1 + (\Sigma U_{1-2})_{n.c.} = T_2 + V_2$$

$V = V_{\text{gravitational}} + V_{\text{elastic}} +$ any other potential energy due to conservative forces acting on particle

FOR A SYSTEM OF PARTICLES

TAKE [redacted] $T_1 + \Sigma U_{1-2} = T_2$ & SUM OVER NO. OF PARTICLES

$$\therefore \quad \Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2$$

where $\Sigma T_1 = \sum_i \frac{1}{2} m_i (V_1)_i^2$; $\Sigma T_2 = \sum_i \frac{1}{2} m_i (V_2)_i^2$

$$\Sigma U_{1-2} = \Sigma \left(\int \bar{F}_i \cdot d\bar{r}_i + \int \bar{f}_i \cdot d\bar{r}_i \right)$$

WORK DUE TO EXTERNAL + INTERNAL FORCE

FOR CONSERVATIVE FORCES $\sum U_{1-2} = \sum V_1 - \sum V_2$

and $\sum T_1 + \sum V_1 = \sum T_2 + \sum V_2$ = constant

$\Rightarrow$ system's initial & K.E. + P.E. = system's FINAL K.E. + P.E. = constant.
HERE KE + PE is caused by <u>INTERNAL</u> + <u>EXTERNAL</u> FORCES

PROCEDURE FOR ANALYSIS

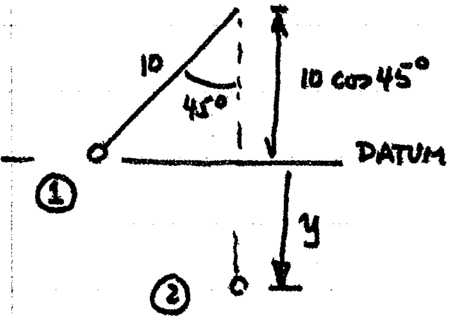
- DRAW PARTICLE AT INITIAL & FINAL PT
- ESTABLISH DATUM LINE FOR PARTICLES HAVING y DISPL.
- DETERMINE STRETCHES IN SPRINGS IF NECESSARY
- APPLY CONSERVATION OF ENERGY FOR CONSERVATIVE FORCE SYSTEMS
- USE PRINCIPLE OF WORK & ENERGY FOR NON CONSERVATIVE FORCES
- REMEMBER: THIS EQN IS FOUND USING TANGENTIAL EQN OF MOTION; MUST STILL USE NORMAL EQN OF MOTION FOR INFORMATION ⊥ TO PATH OF PARTICLE

~~FOR A SET OF SPRINGS AS IN PROB 14-54~~
~~NOTE EACH SPRINGS FEEL SAME CHANGE IN DISPLACEMENT~~

~~FOR A SET OF SPRINGS AS IN PROB 14-62, 14-64~~
~~EACH SPRING FEELS SAME FORCE ACTING ON IT~~
~~BUT NOT SAME DISPLACEMENT~~

PROB. 14-49

14-74



$$T_1+V_1 = T_2+V_2$$

at ① $V=0 \Rightarrow T_1=0$

① $V_1 = Wy = 0$ since $y=0$ $\Big\}$ $T_1+V_1=0$

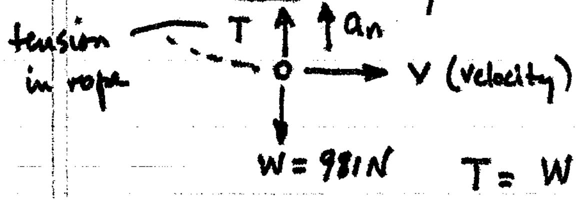
at ② $V_2 = Wy = -W\left[10-10\cos 45°\right]$

② $T_2 = \frac{1}{2}mV_2^2$

$$\therefore\ 0 = T_1+V_1 = T_2+V_2 = \tfrac{1}{2}mV_2^2 - W\left[10(1-\cos 45°)\right]$$

$$\therefore\ V_2 = \sqrt{2g\left[10(1-\cos 45°)\right]} = 7.581 \text{ m/s}$$

Just <u>before</u> hitting branch @ B   radius of curvature $\rho = 10$m.

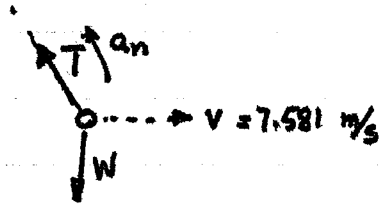


Use normal eqn of motion

$$\Sigma F_n = T - W = ma_n = \frac{W}{g}\frac{V^2}{\rho}$$

$$T = W\left[1+\frac{V^2}{\rho g}\right] = 1555.72\text{ N}$$

Just <u>after</u> hitting branch @ B $\rho = 3$m (radius of curvature)



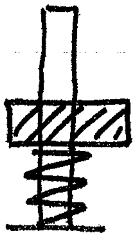
$$\Sigma F_n = T - W = ma_n = \frac{W}{g}\frac{V^2}{\rho}$$

$$T = W\left[1+\frac{V^2}{\rho g}\right] = 2896.72\text{ N.}$$

NOTE: CONSERVATION OF ENERGY EQN REPLACES TANGENTIAL EQN OF MOTION **BUT** **NOT** NORMAL EQN OF MOTION

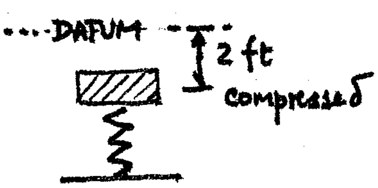
14.73 in 10th ed.

PROBLEM ~~14-48~~

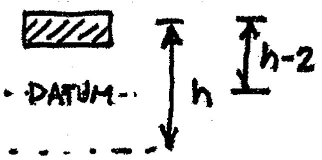


Collar weight

BEFORE



AFTER



BEFORE $v_1 = 0 \Rightarrow T_1 = 0$ PICK DATUM AT SPRING'S UNCOMPRESSED HEIGHT

$$V_1 = V_g + V_e = \frac{1}{2}ks^2 - Ws$$ s - 2 ft spring is compressed

$$= \frac{1}{2}(30)(2)^2 - 8(2) = 44 \text{ ft-lb}$$

AFTER $V_2 = ?$ we measure distance weight has moved from datum

$$V_2 = W(h-2) = W(2.5\text{ft}) = 20 \text{ ft-lb}$$

$$T_1 + V_1 = T_2 + V_2 = 0 + 44 = \frac{1}{2}mv_2^2 + 20 \Rightarrow 24 = \frac{1}{2}mv_2^2$$

$$v_2 = \sqrt{2 \cdot 24 \cdot \frac{32.2}{8}} = 13.9 \text{ ft/s}.$$

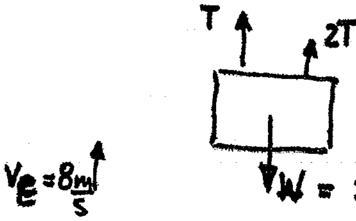
COULD HAVE PICKED DATUM AT LOCATION WHERE WEIGHT WAS RELEASED $\Rightarrow$ $V_g = Wy = 0$ $V_e = \frac{1}{2}ks^2 = \frac{1}{2}(30)(2)^2 = 60$ ft-lb

$\therefore$ AT PT ② $V_g = Wy = W(h) = 8(4.5) = 36$ ft-lb.

GIVES SAME RESULTS.

14-36

14-52



- forces on the elevator T, 2T are due to pulley
  - since elevator velocity = constant $\Rightarrow$ tension in rope = constant

$W = 500(9.81)$
$= 4905$ N

$+\uparrow \Sigma F_y = ma_y = 0$ since elevator doesn't accelerate $\Rightarrow$ $3T - W = 0$ or $T = \frac{W}{3}$

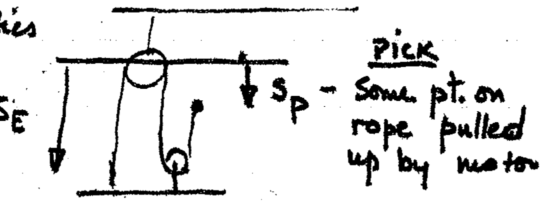
$\therefore$ T= 1635N

Using kinematics

$2s_E + (s_E - s_p) = C$

$3s_E - s_p = \text{const}$

$3v_E - v_p = 0$



PICK s_p - Some pt. on rope pulled up by motor

$v_p = 3v_E = 3(-8)$

$v_p = -24$ m/s or $24\frac{m}{s}\uparrow$

minus sign $\Rightarrow$ motion upward since s_p is $\downarrow +$

now Motor pulls rope up $\Longrightarrow$

Power ~~input~~ output = Force in rope being pulled up · velocity of rope

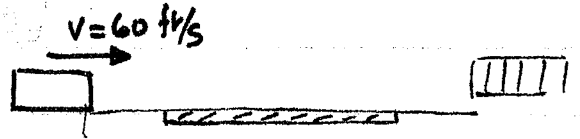
$= 1635(24) = 39,240$ Watts

Power input $= 60$ kW $= 60,000$ W

$\therefore \quad \varepsilon = \frac{\text{power output}}{\text{power input}} = \frac{39240}{60000} = .65$ efficiency

14-20 in 10th ed.

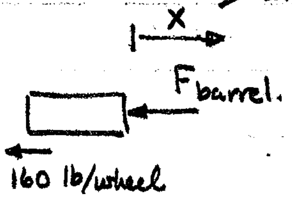
14-15



initially: $T_1 = \frac{1}{2}mv_1^2$

when truck hits

— distance barrels collapses



ALSO $T_2 = \frac{1}{2}mV_2^2 = 0$

since truck comes to rest.

$$\therefore \quad T_1 + \Sigma U_{1-2} = T_2 = 0$$

$$\frac{1}{2}mv_1^2 + \int \bar{F}_{barrel} \cdot d\bar{r} + \int \bar{F}_{wheels} \cdot d\bar{r}_{wheels} = 0$$

$d\bar{r} = +dx\,\bar{i}$

$\bar{F}_{barrel} = -10^3x^3\,\bar{i}$ lb

$\bar{F}_{wheels} = -4\times160\,\bar{i}$ lb.

this force works through a distance of 50ft $\bar{i} = \Delta s$

$$+\left[\int_0^x -10^3x^3dx + -640\cdot\Delta s\right] = 0$$

$$\frac{1}{2}mv_1^2 + \left[-10^3\frac{x^4}{4} - 640(50)\right] = 0$$

thus $\quad x = \left\{\left[\frac{1}{2}mv_1^2 - 640(50)\right]\frac{4}{1000}\right\}^{1/4}$

$= 5.44$ ft

for $\quad m = \frac{W}{g} \cong 139.75$ slugs $\quad v_1 = 60$ ft/s

——— • ——— • ——— • ———

12 - 127

Given $\theta = 30°$ $\dot\theta = .6\ \text{rad/s}$ $\ddot\theta = .2\ \text{rad/s}^2$ find V & a

from diag $r\cos\theta = 800\ \text{m} \Rightarrow r = 800/\cos\theta \approx 923.76\ \text{m}$

$\dot r\cos\theta + r(-\sin\theta)\dot\theta = 0$ this gives $\dot r$

$\dot r = r\dot\theta\tan\theta = .\overset{320}{\text{[redacted]}}\ \text{m/s}$

$\ddot r = \ddot\theta\tan\theta + \dot\theta(\sec^2\theta)\dot\theta \underset{+\dot r\dot\theta\tan\theta}{,} \Rightarrow$ this gives $\ddot r$

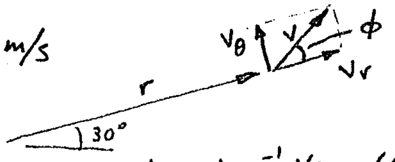
knowing $r, \dot r, \ddot r, \dot\theta, \theta, \ddot\theta$ we can find

$V_r, V_\theta, a_r, a_\theta$

for example $V_r = \dot r = .\overset{320}{\text{[redacted]}}\ \text{m/s}$

$V_\theta = r\dot\theta = (923.76)(.6) = 554.256\ \text{m/s}$

$V = \sqrt{V_r^2 + V_\theta^2} = \overset{640\ \text{m/s}}{\text{[redacted]}}$ direction



$\phi = \tan^{-1}\frac{V_\theta}{V_r} = \underline{60°}$

V vector is ↑

what are the V components in cartesian coordinates?

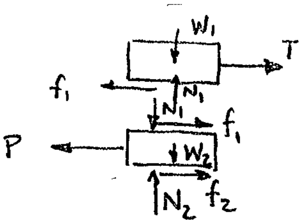
since $r\cos\theta = x = \text{const}$

$\dot x = \dot r\cos\theta + r(-\sin\theta)\dot\theta = 0\ \text{m/s}$

$\dot y = V = 640.\ \text{m/s}$

13-34

13-14 in tenth edition



$W_1 = N_1$

$f_1 = T$

$P - f_1 - f_2 = m_2 a_{x_2}$

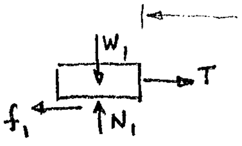
$-(N_1 + W_2) + N_2 = 0 \Rightarrow N_2 = W_1 + W_2$
$= W_2 + N_1$

$f_2 = \mu(W_1 + W_2)$

$f_1 = \mu W_1 = T$

$P - [\mu(2W_1 + W_2)] = m_2 a_{x_2}$

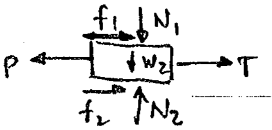
$a_{x_2} = \frac{P}{m} - 3\mu g$



$W_1 = N_1$

$f_1 - T = m_1 a_{x_1}$

Kinematics $a_{x_1} = -a_{x_2}$



$-(N_1 + W_2) + N_2 = 0$

$P - f_1 - f_2 - T = m_2 a_{x_2}$

$N_2 = N_1 + W_2 = W_1 + W_2$

$P - \mu W_1 - \mu(W_1 + W_2) - f_1 + m_1 a_{x_1} = m_2 a_{x_2}$
$-\mu W_1 + m_1 a_{x_2}$

$P - 3\mu W_1 - \mu W_2 = (m_1 + m_2) a_{x_2}$

$\frac{P}{2m} - 2g = a_{x_2}$

SESSION # 12

- 3rd method of soln ⇒ EQNS OF MOTION, PRINC OF WORK & ENERGY (CONSERV OF ENERGY)
- DERIVE PRINCIPLE OF LINEAR IMPULSE & MOMENTUM FOR PARTICLE
- FOR PROBLEMS INVOLVING FORCE, VELOCITY & TIME
- EXAMPLES: BALL HITTING BAT, GOLF CLUB HITTING BALL, EXPLOSION

  SYSTEMS THAT GAIN OR LOSE MASS, STEADY FLOW PROB.

START w/ EQNS OF MOTION & INTEGRATE

$$\Sigma \bar{F} = m\bar{a} = m\frac{d}{dt}\bar{v} \qquad \text{PARTICLE}$$

or

$$\int_{t_1}^{t_2} \Sigma \bar{F}\, dt = \int_{v_1}^{v_2} m\, d\bar{v} \Rightarrow \Sigma \int_{t_1}^{t_2} \bar{F} dt = m \int_{v_1}^{v_2} d\bar{v} = m(\bar{v}_2 - \bar{v}_1)$$

or

$$\boxed{m\bar{v}_1 + \underbrace{\Sigma \int_{t_1}^{t_2} \bar{F} dt}_{\text{forces acting on particle}} = m\bar{v}_2} \qquad \boxed{\text{VECTOR EQN}}$$

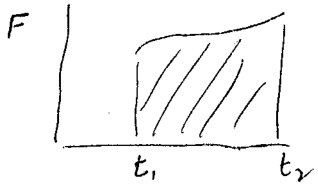
PRINCIPLE OF LINEAR IMPULSE & MOMENTUM:

- ALLOWS YOU TO FIND $\bar{V}_2$ KNOWING Δt, $\Sigma \bar{F}$, $\bar{V}_1$

- TO USE THIS MUST FIRST KNOW ALL FORCES ON BODY & USE $\Sigma \bar{F} = m\bar{a}$ THEN INTEGRATE

- DEFINE $\int_{t_1}^{t_2} \bar{F} dt = \bar{I}$ (LINEAR IMPULSE VECTOR) IN SAME DIRECTION AS $\bar{F}$

  - IF $\bar{F}$ IS IN A CONSTANT DIRECTION ⇒ $\int \bar{F} dt$ IS AREA UNDER GRAPH



$$\boxed{I = \int F dt}$$

IE $\bar{F}$ only changes magnitude

- IF $\bar{F}$ IS $\boxed{\text{CONSTANT MAGNITUDE \& DIRECTION}}$ ⇒ $\boxed{\bar{I} = = \bar{F}(t_2 - t_1)}$

- Linear momentum $m\bar{V} = \bar{L}$

CAN TAKE THE VECTOR EQUATION & WRITE AS SCALAR

$$m(V_1)_x + \Sigma \int F_x\,dt = m(V_2)_x$$
$$m(V_{\cancel{1}})_y + \Sigma \int F_y\,dt = m(V_2)_y$$
$$m(V_1)_z + \Sigma \int F_z\,dt = m(V_2)_z$$

} FOR EQN IN CARTESIAN COORD

REMEMBER IN WRITING THESE $\bar{a}$ & $\bar{V}$ in IFR

IN CERTAIN PROB. F acts over small Δt to produce I large. CAN CAUSE LARGE CHANGE IN MOMENTUM OF OBJECT

IN CERTAIN PROB. IMPULSE DUE TO AN EXTERNAL FORCE WHICH ISN'T IMPULSIVE IN NATURE CAN BE NEGLECTED WRT IMPULSIVE FORCE.
EXAMPLE: W OF BALL WRT I OF BATS - BALL INTERACTION

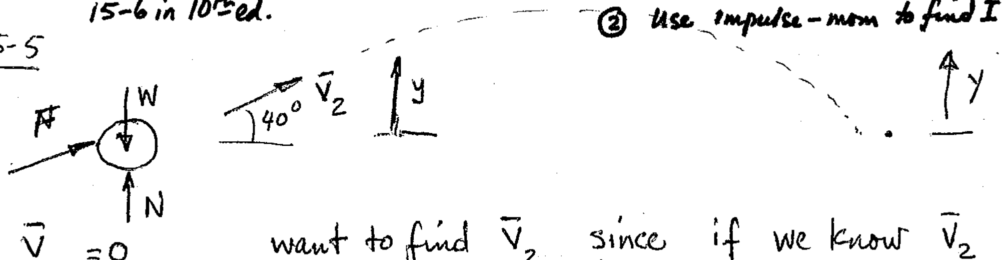
TO DO PROBLEMS

- DRAW FBD SHOWING ALL FORCES
- ESTABLISH IFR
- SHOW INITIAL & FINAL VELOCITY DIRECTIONS IF KNOWN
- IF NOT KNOWN ~~TAKE~~ ASSUME + is + COORDINATE DIRECTION
- APPLY PRINCIPLE OF IMPULSE & MOMENTUM
- IF IT INVOLVES SEVERAL PARTICLES IN DEPENDENT MOTION
  USE KINEMATICS OF DEPENDENT MOTION
- LOOK AT EXAMPLES 15-1 , 15-2 ON YOUR OWN
- GO THROUGH 15-3 WITH CLASS

15-6 in 10th ed.

① Find $\bar{V}_2$
② Use impulse-mom to find I

Problem 15-5



initially $\bar{V}_{Ball} = 0$

want to find $\bar{V}_2$ since if we know $\bar{V}_2$ then $m\bar{V}_1 + \int \bar{F}dt = m\bar{V}_2$

since ball lands 20 m away

$$V_2 \cos 40^\circ t = R = 20\,m$$

$$\cancel{s_{y_f}} = \cancel{s_{y_i}} + V_2 \sin 40^\circ t + \frac{1}{2} g t^2 = 0 \Rightarrow t = 0$$

or $t = \dfrac{-2V_2 \sin 40^\circ}{g}$

$$V_2 \cos 40^\circ \left[\frac{-2V_2 \sin 40^\circ}{g}\right] = -\frac{V_2^2}{g} \sin 80^\circ = R \Rightarrow V_2 = \sqrt{\frac{-Rg}{\sin 80^\circ}}$$

$V_2 = 14.11$ m/s $\quad t = 1.85$ s

$\bar{u} = \cos 40° \bar{i} + \sin 40° \bar{j}$

now $m(\bar{V}_1) + \int \bar{F} dt = m(\bar{V}_2) = m(V_2 \cos 40° \bar{i} + V_2 \sin 40° \bar{j}) = mV_2 \bar{u}$

$0 + \int \bar{F} dt = (.05 \text{ kg})(14.1 \text{ m/s}) \bar{u}$

$\int \bar{F} dt = (.706 \text{ N}\cdot\text{s}) \bar{u}$

⇒ impulse has magnitude .706 N.S at an angle of 40° same as velocity.

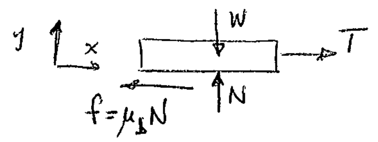
impulse due to weight is small in comparison to I due to F.

since we are told to neglect impklse of weight ⇒ neglect impulse of N also.

① Find time to impending motion

② Use impulse-momentum to get V_{final}

15-19 $\quad$ 15-31 in 10th ed.

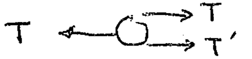


For movement must look at IMPENDING MOTION

$\sum F_y = N - W = 0 \quad N = W$

$\sum F_x = T - f = T - \mu_s N = 0$ or $T = \mu_s W$

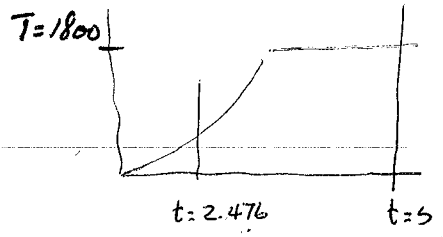
∴ T (impending motion) $= .5(500)(9.81) = 2452.5$ N

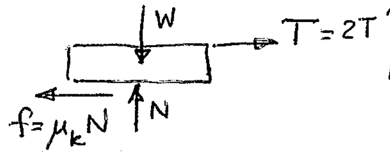


$\Rightarrow T' = T/2 = .1226,25$

since $T' < 1800$ N $\Rightarrow T' = 200t^2$ or $t = \sqrt{\frac{1226.25}{200}}$

$t = 2.476$ s has passed to impending motion stage





Note since no movement in y dir W=N

$m\bar{V}_1 + \sum \int \bar{F} dt = m\bar{V}_2$

since at impending motion $\bar{V}_1 = 0 \quad \bar{L}_1 = 0$

$\sum \int_{2.476}^{5} \bar{F} dt = m\bar{V}_2 \Rightarrow -\int_{2.476}^{5} \mu_k N dt + 2\int_{2.476}^{5} \bar{T} dt = m\bar{V}_2$

$-\mu_k \underset{4905}{W} [5 - 2.476] + 2\int_{2.476}^{3} 200t^2 dt + 2\int_{3}^{5} 1800 dt = m\bar{V}_2$

$-(.4)(\overset{4905}{\quad})[2.524] + \frac{400t^3}{3}\Big|_{2.476}^{3} + 3600(2) = 500 \bar{V}_2$

$-4952.088 + 1576.0925 + 7200 = 3824.0045 = 500 \bar{V}_2$

$\bar{V}_2 = 7.648$ m/s @ $t = 5$s

FOR A SET OF PARTICLES WE CAN TAKE

$$\sum \bar{L}_{1_i} + \sum \bar{I}_{1-2_i} = \sum \bar{L}_{2_i}$$ FOR EACH PARTICLE & SUM

$$\text{where } \sum \bar{I}_{1-2\,i} = \sum \int \bar{F}_i \, dt + \sum \int \bar{f}_i \, dt$$

$$\text{but } \sum \int \bar{f}_i \, dt = \int \left(\sum \bar{f}_i\right) dt = 0 \quad \text{since } \sum \bar{f}_i = 0$$

FOR A SYSTEM OF PARTICLES $\sum m_i (\bar{V}_i)_1 + \underbrace{\sum \int_{t_1}^{t_2} \bar{F}_i \, dt}_{\text{impulse due to external force}} = \sum m_i (\bar{V}_i)_2$

FOR AGGREGATE WE CAN LOOK AT MASS CENTER

FOR $\sum m_i (\bar{V}_i)_1 = \sum m_i \frac{d(\bar{r}_i)_1}{dt} = \frac{d}{dt} \sum m_i (\bar{r}_i)_1 = \frac{d}{dt} M \bar{r}_{G_1} = M \bar{V}_{G_1}$

$$M \bar{V}_{G_1} + \sum \int_{t_1}^{t_2} \bar{F}_i \, dt = M \bar{V}_{G_2}$$ where $M = \sum m_i$

$\bar{V}_{G_i}$ is veloc of mass center at time t_i

SESSION #13

CONSERVATION OF LINEAR MOMENTUM

WHEN $\sum \int \bar{F}_i \, dt = 0$ (SUM OF EXTERNAL IMPULSES = 0)

1. NO EXTERNAL - FORCES BALANCED
2. Δt IS << & IMPULSE CAN BE NEGLECTED

$$\sum m_i (\bar{V}_i)_1 = \sum m_i (\bar{V}_i)_2$$ CONSERVATION OF LINEAR MOMENTUM

FOR A SYSTEM OF PARTICLES $\Rightarrow M \bar{V}_{G_1} = M \bar{V}_{G_2}$ or $\bar{V}_{G_1} = \bar{V}_{G_2}$

VELOC OF MASS CENTER DOESN'T CHANGE WHEN NO EXTERNAL IMPULS

- WHEN ~~[illegible]~~ EXTERNAL FORCES CAUSE NEGLIGIBLE IMPULSES CAN BE NEGLECTED - FORCES ARE NONIMPULSIVE

EXAMPLE : IN CASE OF GOLF BALL IF IMPULSE = .706 N·S ACTS OVER .1 SEC

$\Rightarrow F_{AV} = \frac{I}{\Delta t} = 7.06\,N$ & DUE TO WEIGHT $= W = .05(9.81) = .49$

$F_{AV} \sim 14.40\, F_W$

15-26

$$T_1 = 0$$

Princ of work & energy

$$\frac{1}{2}k(\Delta s)^2 + T_1 = \frac{1}{2}k\cdot 0 + T_2$$

$$T_2 = \frac{1}{2}m_B v_B^2 + \frac{1}{2}m_C v_C^2 = \frac{1}{2}\frac{m_{\cancel{B}}^2 v_{\cancel{C}_2}^2}{m_B} + \frac{1}{2}m_C v_C^2$$

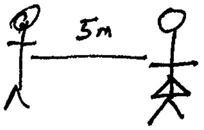
$$= \frac{1}{2}\left(\frac{m_C^2}{m_B} + m_C\right) v_C^2$$

princ of linear mom.

$$0 = \sum m_{\cancel{B}} v_B + m_C v_C = m_B v_{B_2} + m_C v_{C_2}$$

$$v_{B_2} = \frac{m_C}{m_B} v_{\cancel{C}_2}$$

15-24



A boy & girl are each standing on ice skates & holding on to a rope. If they begin to pull on rope such that their relative speed is 2m/s, determine their velocities when they meet if rope is 5 m long. How long before they reach each other! $m_b = 60$ $m_g = 50$ kg

$$m_B v_B + m_G v_G = \text{const.} = 0$$

$$v_{B/G} = 2.$$

$$v_B = v_{B/G} + v_G$$

$$m_B\left[v_{B/G} + v_G\right] + m_G v_G = 0$$

$$\underset{v_G}{\cancel{(m_B+m_G)}} = \frac{-m_B v_{B/G}}{m_B + m_G} = \frac{-60(2)}{110} = -1.09 \text{ m/s}$$

$$v_B = v_{B/G} + v_G = +2 - 1.09 = .91 \text{ m/s}$$

$$T = m_B a_B \qquad T = m_G a_G.$$

$$m_B \underset{0}{\cancel{v_{B_1}}} + \int T\,dt = m_B v_{B_2} \qquad \int T\,dt = m_G v_G.$$

takes $\frac{5m}{2 m/s} = \overset{2.5}{\cancel{5}}$ sec. $\quad \frac{\Delta s}{v_{rel}} = \Delta t$

$$v_{B_2} = v_{B/G} + v_G$$

$$+m_b v_b - m_G v_G = 0$$

$$m_b\left(v_{B/C} + \cancel{b}v_G\right) - m_G v_G = 0$$

or

$$v_G = \frac{m_b v_{B/G}}{m_b + m_G} = 1.09 \text{ m/s}$$

$$v_B = .91 \text{ m/s}$$

February 16, 1990

Graduate Admissions Committee
Department of Mechanical Engineering

To whom it may concern:

I am writing this letter in support of the application of **Mr. Narendra Kota, 480-15-8511**, into the PhD program in Mechanical Engineering and for financial assistance at your university.

I have known Mr. Kota since his arrival at Florida International University (FIU) and I have had the pleasure of having him as a student in my Advanced Analysis of Mechanica Systems course. His course grade was an A. **My impressions of Narendra are that he grasps the material presented very quickly, he is meticulous in his work and he is very motivated.** My course required knowledge of the FORTRAN programming language. Because of his weakness in that area, he taught himself FORTRAN, while still producing the required projects at their due dates (normally ten days from assignment). **is areas of interest are in CAD/CAM and robotics. He presently has a GPA of 3.7 or better in our MSME program.** He is a conscientious individual who, I believe, will do well in graduate work. If we had a PhD program here, I would be want to write this letter, as I would try to pursuade him to stay at FIU. I believe that he would be a welcome addition to your corps of graduate students and **unequivocally support his application into the PhD program in Mechanical Engineering.**

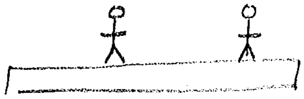
Mr. Kota has been supported by various grants to cover his living expenses and tuition while at FIU. Being a non resident of your state, he would have to also receive financial assistance in order to meet his expenses and tuition at your university.

Respectfully,

Cesar Levy
Assistant Professor
Mechanical Engineering

~38

15-36 in 10th ed



$$v_{c_1} m_{c_1} + m_A v_{A_1} + m_B v_{B_1} = v_{c_2} m_c + m_A v_{A_2} + m_B v_{B_2}$$

$$v_A = v_{A/c} + v_c$$

$$v_B = v_{B/c} + v_c$$

a) $v_{c_1} = v_{A_1} = v_{B_1} = 0$ $v_{B_2} = v_{c_2}$ $v_{A_2} = v_{A/c} + v_{c_2}$

$$0 = (m_c + m_B) v_{c_2} + m_A (v_{A/c} + v_{c_2})$$

$$v_{c_2} = -\frac{m_A v_{A/c}}{m_c + m_A + m_B} = \frac{-3[160]}{[200 + 2(160)]} = -\frac{3 \cdot 160}{520} = \frac{-480}{520} = \frac{-12}{13}$$

@ pt where A jumps first & B stays $v_{c_2} = -.923$ ft/s

$$(m_B + m_c) v_{c_2} = m_B (v_{B_3}) + m_c v_{c_3} \qquad v_{B_3} = v_{B/c} + v_{c_3}$$

$$(m_B + m_c) v_{c_2} = m_B (v_{B/c} + v_{c_3}) + m_c v_{c_3} = (m_B + m_c) v_{c_3} + m_B v_{B/c}$$

at pt where B jumps next $v_{c_3} = v_{c_2} + \frac{m_B v_{B/c}}{m_B + m_c} = -.923 - \frac{160 \cdot 3}{360} = -2.256$ ft/s

b) both jump $v_{c_1} = v_{A_1} = v_{B_1}$

$$0 = m_A (v_{A/c} + v_{c_2}) + m_B (v_{B/c} + v_{c_2}) + m_c v_{c_2}$$

$$v_{c_2} = -\frac{(m_A + m_B) v_{A/c}}{m_A + m_B + m_c} = -\frac{3(320)}{520} = -2(.923) = -1.8$$ ft/s

January 10, 1990

Dr. T.O. Kvalseth, Chairman
Graduate Fellowship and Assistantship
Committee
Department of Mechanical Engineering
University of Minnesota
111 Church Street, SE
Minneapolis, Minnesota 55455

Dear Dr. Kvalseth:

I am writing this letter in support of the application of **Mr. Narendra Kota, 480-15-8511**, into the PhD program in Mechanical Engineering and for financial assistance at the University of Minnesota.

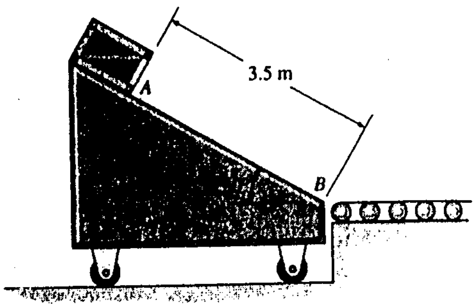
I have known Mr. Kota since his arrival at Florida International University (FIU) and I have had the pleasure of having him as a student in my Advanced Analysis of Mechanica Systems course. His course grade was an A. **y impressions of Narendra are that he grasps the material presented very quickly, he is meticulous in his work and he is very motivated.** My course required knowledge of the FORTRAN programming language. Because of his weakness in that area, he taught himself FORTRAN, while still producing the required projects at their due dates (normally ten days from assignment). **is areas of interest are in CAD/CAM and robotics. He presently has a GPA of 3.7 or better in our MSME program.** He is a conscientious individual who, I believe, will do well in graduate work. If we had a PhD program here, I would be want to write this letter, as I would try to pursuade him to stay at FIU. I believe that he would be a welcome addition to your corps of graduate students and **unequivocally support his application into the PhD program in Mechanical Engineering.**

Mr. Kota has been supported by various grants to cover his living expenses and tuition while at FIU. Being a non resident of Minnesota, he would have to also receive financial assistance in order to meet his expenses and tuition at your university.

Respectfully,

Cesar Levy
Assistant Professor
Mechanical Engineering

15-51. The free-rolling ramp has a mass of 40 kg. A 10-kg crate is released from rest at A and slides down 3.5 m to point B. If the surface of the ramp is smooth, determine the ramp's speed when the crate reaches B. Also, what is the velocity of the crate?



Conservation of Energy : The datum is set at lowest point B. When the crate is at point A, it is $3.5\sin 30° = 1.75$ m *above* the datum. Its gravitational potential energy is $10(9.81)(1.75) = 171.675$ N · m. Applying Eq. 14 – 21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 171.675 = \frac{1}{2}(10)\,v_C^2 + \frac{1}{2}(40)\,v_R^2$$

$$171.675 = 5v_C^2 + 20v_R^2 \qquad [1]$$

Relative Velocity : The velocity of the crate is given by

$$\mathbf{v}_C = \mathbf{v}_R + \mathbf{v}_{C/R}$$

$$= -v_R\mathbf{i} + (v_{C/R}\cos 30°\mathbf{i} - v_{C/R}\sin 30°\mathbf{j})$$

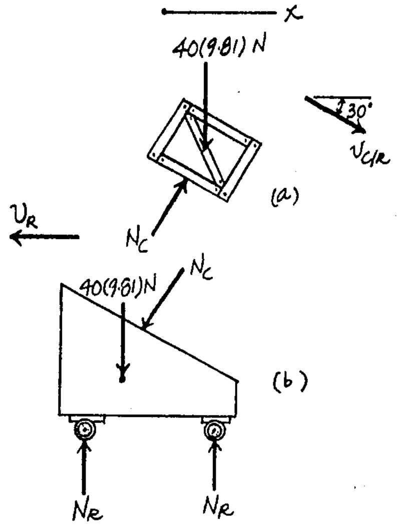
$$= (0.8660v_{C/R} - v_R)\mathbf{i} - 0.5v_{C/R}\mathbf{j} \qquad [2]$$

The magnitude of $\mathbf{v}_C$ is

$$v_C = \sqrt{(0.8660v_{C/R} - v_R)^2 + (-0.5v_{C/R})^2}$$

$$= \sqrt{v_{C/R}^2 + v_R^2 - 1.732v_R\,v_{C/R}} \qquad [3]$$

Conservation of Linear Momentum : If we consider the crate and the ramp as a system, from the FBD, one realizes that the normal reaction N_C (*impulsive force*) is *internal* to the system and will cancel each other. As the result, the linear momentum is conserved along the x axis.



$$0 = m_C(v_C)_x + m_R v_R$$

$$(\overset{+}{\rightarrow}) \qquad 0 = 10(0.8660v_{C/R} - v_R) + 40(-v_R)$$

$$0 = 8.660v_{C/B} - 50v_B \qquad [4]$$

Solving Eqs. [1], [3] and [4] yields

$$v_R = 1.101 \text{ m/s} = 1.10 \text{ m/s} \qquad v_C = 5.43 \text{ m/s} \qquad \textbf{Ans}$$

$$v_{C/R} = 6.356 \text{ m/s}$$

From Eq. [1]

$$\mathbf{v}_C = [0.8660(6.356) - 1.101]\mathbf{i} - 0.5(6.356)\mathbf{j} = \{4.403\mathbf{i} - 3.178\mathbf{j}\} \text{ m/s}$$

Thus, the directional angle ϕ of v_C is

$$\phi = \tan^{-1}\frac{3.178}{4.403} = 35.8° \qquad \textbf{Ans}$$

- NORMALLY APPLY PRINCIPLE TO A SYSTEM OF PARTICLES TO FIND VELOCITY JUST AFTER COLLISION
- THIS ALLOWS INTERNAL FORCES TO BE ELIMINATED FROM CONSIDERATION
  - DRAW FBD SHOWING ALL FORCES
  - APPLY CONSERVATION OF LINEAR MOMENTUM IF NO IMPULSIVE FORCES
  - ESTABLISH DIRECTION OF INITIAL & FINAL VELOCITIES
  - APPLY PRINCIPLE OF IMPULSE & MOMENTUM IF IMPULSIVE FORCES

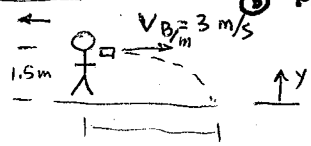
EXAMPLES OF NON IMPULSIVE FORCES WEIGHT, LIGHT SPRING (K small) W/ SMALL ΔS

like 15-37

PROBLEM 15-30

a 70 kg man releases an 8 kg block horiz so that it has a relative velocity of 3 m/s determine how far apart the man and the block are when the block hits the ground. The block is 1.5 m off the ground when released.

① Find absolute veloc of man & block
② Find time of fall
③ Find distance



Just before Just after

$$m_m \bar{V}_{m_1} + m_B \bar{V}_{B_1} = m_m \bar{V}_{m_2} + m_B \bar{V}_{B_2}$$

$$0 + 0 = -70 V_{m_2} + 8 V_{B_2}$$ **ASSUME + →**

Block follows proj motion

$$S_f = S_i + V_{y_i} t + \frac{1}{2} g t^2$$

$$0 = 1.5 + 0 + \frac{1}{2}(-9.81)t^2 \qquad t = \sqrt{\frac{2(1.5)}{9.81}} = .553 s$$

$$\bar{V}_{B_2} = \bar{V}_{M_2} + \bar{V}_{B/m}$$

$$V_{B_2} = -V_{m_2} + 3 \qquad \xrightarrow{+}$$

$$-70 V_{m_2} + 8(-V_{m_2} + 3) = 0 \quad \text{or}$$

$$24 = 78 V_{m_2} \Rightarrow V_{m_2} = .308 \text{ m/s}$$

$$\underline{\underline{V_{B_2} = 2.692 \text{ m/s}}}$$

Now distance travelled by block is

$$V_{B_2} t = (2.692)(.553) = 1.49 \overset{m}{\rightarrow}$$

Distance traveled by man

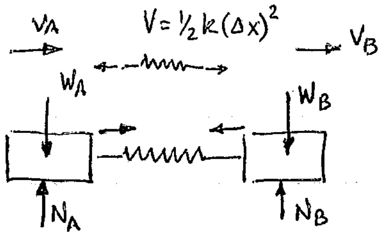
$$V_{m_2} t = (.308)(.553) = .17 \overset{m}{\leftarrow}$$

$$\text{TOTAL DIS} = \underset{\rightarrow}{V_{B_2} t} + \underset{\leftarrow}{V_{m_2} t} = 3(.553) = 1.66 \text{ m}$$

15-36

①

15-54 in 10th ed.



impulses due to W_A & N_A cancel
W_B & N_B cancel

① Get relation for cons. of momentum F internal ∴ cancel
② Get relation for cons. of energy.

$\Sigma m_i(v_i)_1 = \Sigma m_i(v_i)_2$ initially $V_A = V_B = 0$ $\Rightarrow \Sigma m_i(v_i)_1 = 0$

$\Sigma m_i(v_i)_2 = 40V_A - 60V_B = 0$ from ① $\Rightarrow V_A = \frac{3}{2}V_B$

All the forces are conservative ∴ use $T_1 + V_1 = T_2 + V_2$

$T_1 = \Sigma \frac{1}{2}m_i(v_i)_1^2 = 0$ $V_1 = \frac{1}{2}ks^2$ since spring is stretched 2m

$T_2 = \Sigma \frac{1}{2}m_i(v_i)_2^2$ $V_2 = 0$ since spring is unstretched

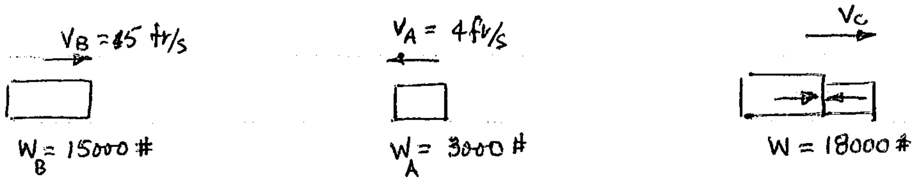
$\frac{1}{2}(180)(2)^2 = \frac{1}{2}40(V_A)^2 + \frac{1}{2}60(V_B)^2$ ALSO $V_A = \frac{3}{2}V_B$

$360 = 45V_B^2 + 30V_B^2 = 75V_B^2$ ∴ $V_B = \sqrt{\frac{360}{75}} = 2.191$ m/s ←

$V_A = 3.286$ m/s →

15-37 in 10th ed.

15-25



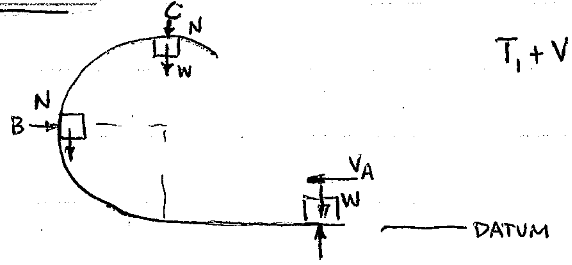
$\Sigma m_i(v_i)_1 = \Sigma m_i(v_i)_2 \Rightarrow W_BV_B - W_AV_A = (W_B + W_A)V_C$

~~213000 #~~ 63000 ft/s = (18000 #)(V_C)

$V_C = 3.5$ ft/s →

③

R 1-45 P. 242



① use cons. of energy
② normal eqn of motion

$T_1 + V_1 = T_2 + V_2$

@ A $V_1 = 0$ $T_1 = \frac{1}{2}mV_A^2$

@ B $V_2 = Wr$ $T_2 = T_1 + V_1 - V_2$

$\frac{1}{2}\frac{W}{g}V_B^2 = 0 + \frac{1}{2}\frac{W}{g}V_A^2 - Wr$

$V_B^2 = V_A^2 - 2gr$

$= (900) - 2(32.2)(5$

$V_B = 24.04$ ft/s

$\Sigma F_n = N = \frac{mV_B^2}{r} = \frac{W}{g}\frac{V_B^2}{r} = 7.18$ lb.

Note N is always ⊥ to path ∴ no work done

@ C $V_2 = W(2r)$ $T_2 = T_1 + V_1 - V_2$

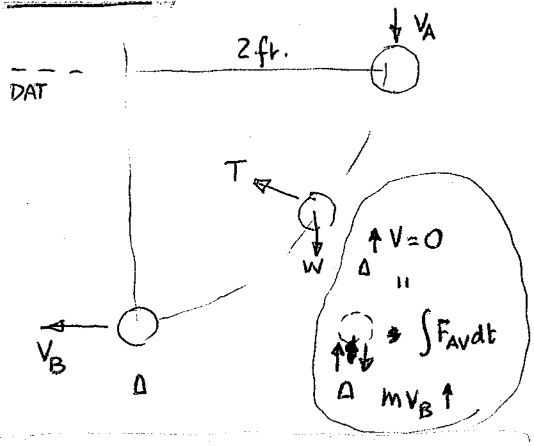
$$\frac{1}{2}\frac{W}{g}v_c^2 = 0 + \frac{1}{2}\frac{W}{g}v_A^2 - W(2r)$$

$$v_c^2 = v_A^2 - 4gr \qquad v_c = 16 \text{ ft/s}$$

$$\Sigma F_n = N + W = \frac{mv_c^2}{r} \quad \text{or} \quad N = \frac{mv_c^2}{r} - W = 1.18 \text{ lb.}$$

②

R 1-38 PG 241



① use conv of energy ③ use normal
② use impulse & moment eqn.

$$\frac{1}{2}mv_A^2 + W(2) = \frac{1}{2}mv_B^2$$

T always normal to path does no work
W does work

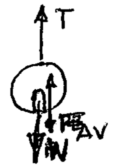
BULLET STOPS HAVING COMPONENT OF VELOC IN y DIRECTION - Weight impulse ≈ 0

$$\uparrow + \quad m_B(v_B)_1 + \int F_{AV}\,dt = m_B\cancel{(v_B)_2}^{\,0}$$

$$\frac{.1}{32.2}(1600) - F_{AV}(.2\text{sec}) = 0 \quad \text{or}$$

$$F_{AV} = \frac{1600}{32.2}\left(\frac{.1}{.2}\right) = 24.84 \text{ lb}$$

WHEN IMPACT OCCURS : IE BULLET IS NOT IMBEDDED JUST TOUCHES.



$$T + F_{AV} - W = \frac{mv_B^2}{\rho}$$

$$24.8 - 2 = \frac{2}{32.2}\,\frac{1}{2}\,(v_B)^2$$

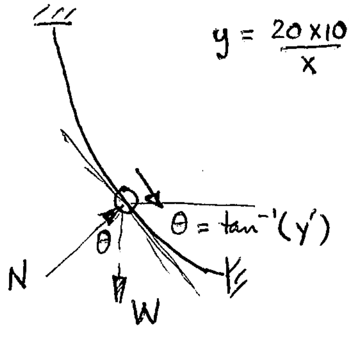
T=0
NO SLACK AT MIN VELOCITY

$$v_B = 27.12 \text{ ft/s}$$

$$v_A = \sqrt{v_B^2 + \frac{W(2)\cdot 2}{m}} = 24.63\text{f}$$

④

R 1-17 (R-129 in 10th ed)



$$y = \frac{20\times10^3}{x} \qquad \frac{dy}{dx} = -\frac{20(10^3)}{x^2} \qquad \frac{d^2y}{dx^2} = \frac{2(20)(10^3)}{x^3}$$

$$@\ x = .2m \quad y' = \frac{-20(10^3)}{4\times10^4} = -.5 \qquad y'' = \frac{40\times10^3}{8\times10^6} = .005$$

$$\rho = \frac{[1 + y'^2]^{3/2}}{y''} = \frac{[1 + .25]^{3/2}}{.005} = 279.5 \text{ mm}$$

$$a_n = \frac{v^2}{\rho} = \frac{(300)^2}{279.5} = 322 \text{ mm/s}^2$$

$a_t = 0$ since $V = \text{constant}$

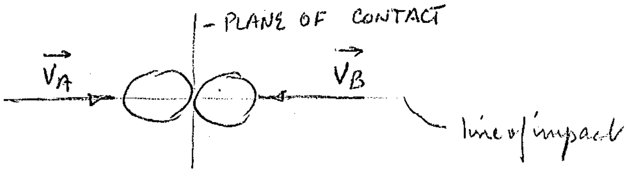
$$a = a_n = 322 \text{ m/s}^2$$

SESSION #14

~~DM 144 9:25-10:40~~
~~163 10:50 12:05~~

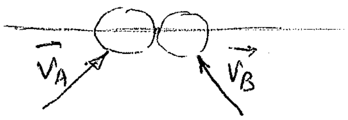
- HAVE LEARNED THAT $\vec{F} = \frac{d}{dt}(m\vec{v})$ or $m\vec{v}_2 = m\vec{v}_1 + \int \vec{F}\,dt$
- HAVE LEARNED ABOUT CONSERVATION OF LINEAR MOMENTUM
- APPLICATION IN CASE OF IMPACT
  - COLLISION OF TWO BODIES DURING SHORT TIME
  - CAUSE LARGE (IMPULSIVE) FORCES BETWEEN BODIES

- TWO TYPES
  - CENTRAL IMPACT - DIRECTION OF MOTION OF MASS CENTER IS ALONG LINE OF IMPACT



centers lie on line ⊥ to plane of contact.

  - OBLIQUE IMPACT - ONE OR BOTH OF MASSES HAVE MOTION AT AN ANGLE TO LINE OF IMPACT



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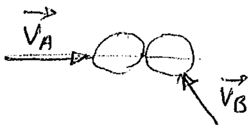
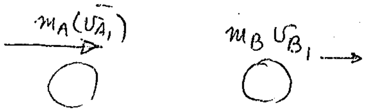
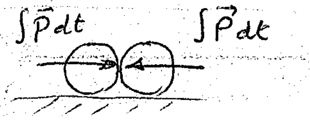


- ANALYSIS OF CENTRAL IMPACT
  - ASSUME BEFORE IMPACT $v_{A_1} > v_{B_1}$



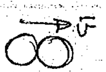
$\Rightarrow$ IMPACT WILL OCCUR

  - FOR A SHORT DURATION EQUAL & OPPOSITE IMPULSES EXIST



BODIES DEFORM

  - AT MAXIMUM DEFORMATION both bodies move with velocity $\vec{v}$

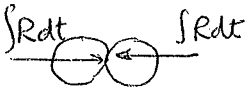


DURING PERIOD OF RESTITUTION

- BODIES WILL EITHER REMAIN DEFORMED OR RETURN TO THE ORIGINAL SHAPE (MATERIAL DEPENDENT & IMPULSE FORCE DEPENDENT)

- EQUAL BUT OPPOSITE RESTITUTION IMPULSE PUSH THE BODIES AWAY

NORMALLY $\int P dt > \int R dt$



- FINALLY $v_{B_2} > v_{A_2}$

- NORMALLY v_{A_1}, v_{B_1} are known want v_{B_2}, v_{A_2}
- BY CONSIDERING THE SYSTEM OF PARTICLES $\int Pdt$ & $\int Rdt$ ARE <u>INTERNAL</u>
- THUS MOMENTUM IS CONSERVED

$$m_A(\vec{v}_A)_1 + m_B(\vec{v}_B)_1 = m_A \vec{v}_{A_2} + m_B \vec{v}_{B_2}$$

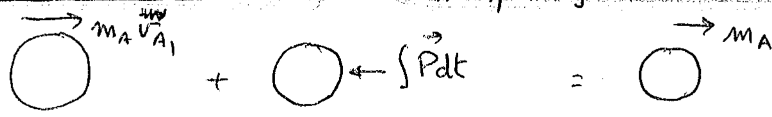
- SINCE ALL VELOCITY COMPONENTS ACT ALONG LINE OF IMPACT USE SCALA

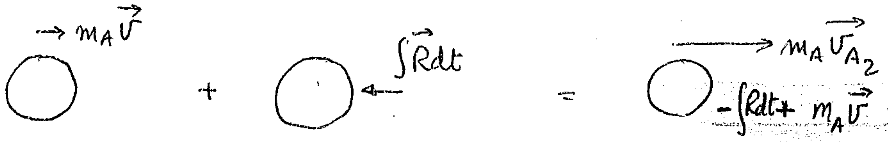
$$m_A v_{A_1} + m_B v_{B_1} = m_A v_{A_2} + m_B v_{B_2}$$

- NEED ANOTHER EQ.

CONSIDER PARTICLE A $\quad m_A \vec{v}_{A_1} - \int \vec{P} dt = m_A \vec{v}$

DURING DEFORMATION



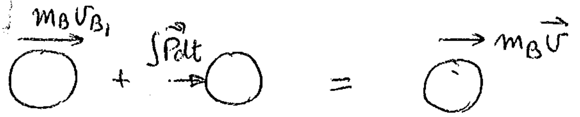


$$-\int Rdt + m_A \vec{v} = m_A \vec{v}_{A_2}$$

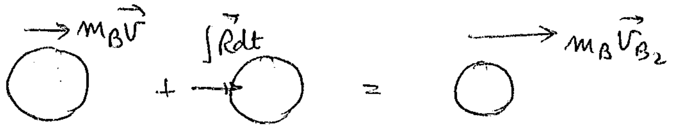
SINCE ONLY IMPULSIVE FORCES ARE DUE TO B

$$\text{define} \quad e = \frac{\int Rdt}{\int Pdt} = \frac{m_A[\vec{v} - \vec{v}_{A_2}]}{m_A[\vec{v}_{A_1} - \vec{v}]} \quad (i) = \frac{m_A[v - v_{A_2}]\vec{i}}{m_A[v_{A_2} - v]\vec{i}}$$

CONSIDER B



$$m_B \bar{v}_{B_1} + \int \bar{P}dt = m_B \bar{v}$$



$$m_B\bar{v} + \int \bar{R}dt = m_B \bar{v}_{B_2}$$

$$e = \frac{\int\vec{R}dt}{\int\vec{P}dt} = \frac{m_B\;[\vec{v}_{B_2} - \vec{v}_B]}{m_B\;[\vec{v} - \vec{v}_{B_1}]} \qquad (2) \quad = \frac{m_B\;[v_{B_2} - v]\vec{i}}{m_B\;[\vec{v} - v_{B_1}]\vec{i}}$$

DON'T NEED TO DO THIS

Now (1) $(e v_{A_1} - e v) = v - v_{A_2}$ or $e v_{A_1} + v_{A_2} = v(1+e)$

(2) $e v - e v_{B_1} = v_{B_2} - v$ or $v_{B_2} + e v_{B_1} = (e+1)v$

$$v = (e v_{A_1} + v_{A_2})/e+1 = (v_{B_2} + e v_{B_1})/e+1$$

or $e v_{A_1} + v_{A_2} = v_{B_2} + e v_{B_1} \Rightarrow e[v_{A_1} - v_{B_1}] = v_{B_2} - v_{A_2}$

$$\boxed{e = \frac{v_{B_2} - v_{A_2}}{v_{A_1} - v_{B_1}}} = \frac{\text{relative veloc after impact}}{\text{relative veloc before impact}}$$

Given $e, v_{A_1}, v_{B_1} \Rightarrow v_{B_2} - v_{A_2}$

- ELASTIC IMPACT $e = 1 \Rightarrow$ relative veloc after = relative veloc before

RESTITUTION IMPULSE $\int\vec{R}dt = \int\vec{P}dt$ DEFORMATION IMPULSE

- PLASTIC IMPACT $e=0 \Rightarrow V_{B_2} = V_{A_2} = \vec{V}$ also $\int\vec{R}(dt) = 0$

$$m_A\vec{V}_{A_1} + m_B\vec{V}_{B_1} = m_A\vec{V}_{A_2} + m_B\vec{V}_{B_2} = (m_A + m_B)\vec{V}$$

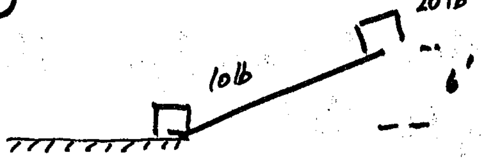
- $e = 1$ MOMENTUM CONSERVED $\Rightarrow$ ENERGY IS CONSERVED

$$\left.\begin{array}{l} m_A(V_{A_1} - V_{A_2}) = m_B(V_{B_2} - V_{B_1}) \\ e=1 \Rightarrow \quad V_{A_1} - V_{B_1} = V_{B_2} - V_{A_2} \Rightarrow V_{A_1} + V_{A_2} = V_{B_2} + V_{B_1} \end{array}\right\}\text{multiply}$$

$$m_A(V_{A_1}^2 - V_{A_2}^2) = m_B(V_{B_2}^2 - V_{B_1}^2)$$

①

15-49



$T_1 + V_1 = T_2 + V_2$

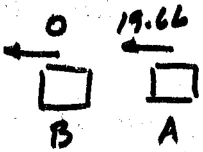
$T_1 = 0 \quad V_1 = 20.6$

$T_2 = \frac{1}{2} \cdot \frac{20}{32.2} V^2 \quad V_2 = 0$

$20.6 = \frac{20}{64.4} V^2$

A package^{A} (20lb) slides down an incline and strikes another package^{B} (10lb). The 10 lb package slides along a surface having a coeff of friction $\mu = 0.4$. How far does it slide before stopping

$V_{A_1} = \sqrt{6(64.4)} = 19.66$ ft/s



$.3 = e = \frac{\cancel{19.66 - 0}}{\cancel{V_B - V_A}}$ $V_A \to V_B$ inverse

$20(19.66) + 10(0) = 20V_{A_2} + 10V_{B_2}$

$V_{A_2} = 19.66 - \frac{1}{2} V_{B_2}$

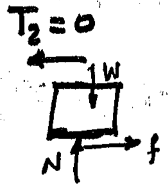
~~$.3 = \frac{19.66}{V_B - 19.66 + .5V_B} \Rightarrow 3 = \frac{19.66}{1.5V_B - 19.66}$~~

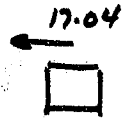
~~$.45V_B = 1.3(19.66)$ $V_B =$~~

$.3 = \frac{V_{B_2} - V_{A_2}}{19.66 - 0} = 1.5V_{B_2} - 19.66$

$\frac{1.3(19.66)}{1.5} = V_{B_2} = 17.04$ ft/s

$V_{A_2} = 19.66 - 8.52 = 11.14$ ft/s





$\Sigma F_y = 0 \quad W = N$

$f = \mu_k N$

$T_1 + \Sigma U_{1-2} = T_2$

$\frac{1}{2} m v_{B_2}^2 - \mu m g s = 0$

$\frac{1}{2} \cdot \frac{10}{32.2} v^2 - \mu W s = 0$

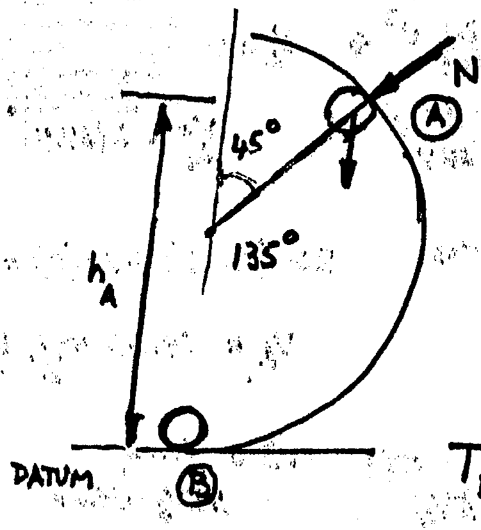
$\mu_k = 0.4$

$s = \frac{v^2}{2\mu g} = \cancel{4.82}\ 11.27$ ft

HW 15-51, 53, 56, 62

14-15

Since it leaves the track at 135°, then the normal forces N=0



That implies that

$$\Sigma F_n = W\cos 45° = \frac{m v^2}{\rho}$$

or $v_A = \sqrt{g\rho\cos 45°}$ when N = 0

since the only forces acting on this system are conservative, then

$$T_B + V_B = T_A + V_A$$

$$T_B = \frac{1}{2} m v_B^2 = 0$$

$$T_A = \frac{1}{2} m v_A^2 = \frac{1}{2} m g \rho \cos 45°$$

$$V_{A_{weight}} = mgh_A = mg[1.5\sin 45° + 1.5]$$

$$V_{A_{spring}} = \frac{1}{2} k\Delta^2 = \frac{1}{2}(500 N/m)(0.08 m)^2$$

$V_{B_{weight}} = 0$ since datum is set there

$$V_{B_{spring}} = \frac{1}{2} k(s+\Delta)^2 = \frac{1}{2} k(s+.08)^2$$

∴ $T_A + V_A = T_B + V_B$

$$\frac{1}{2}(500)(s+.08)^2 = \frac{1}{2}(.5)(9.81)(1.5)(.707) + (.5)(9.81)[1.5(1.707)] + \frac{1}{2}(500)(.08)^2$$

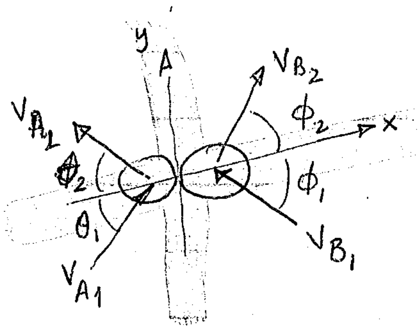
s = .1789 m = 178.9 mm

Suggestion to you: do problem 14-16

$$\frac{1}{2} m_A V_{A_1}^2 + \frac{1}{2} m_B V_{B_1}^2 = \frac{1}{2} m_A V_{A_2}^2 + \frac{1}{2} m_A V_{B_2}^2$$

WHEN $e \neq 1$ energy is not conserved.

• OBLIQUE CENTRAL IMPACT

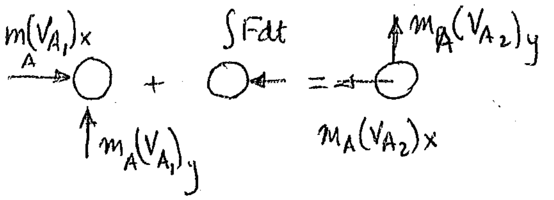


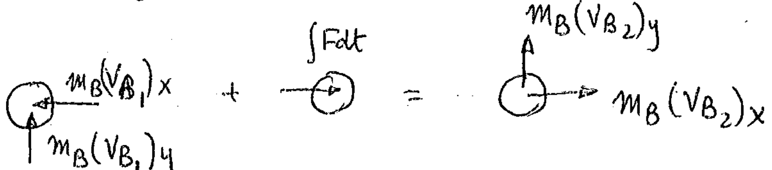
4 unknowns: $V_{A_2}, V_{B_2}, \theta_2, \phi_2$

FINAL VELOCITIES & DIRECTIONS ARE UNKNOWN

- ESTABLISH COORD AXES
- IMPULSIVE FORCES ARE ALONG LINE OF IMPACT (x-DIRECTION)
- MOMENTUM IN Y dir is conserved for A & B

$$m_A (V_{A_1})_y = m_A (V_{A_2})_y$$
$$m_B (V_{B_1})_y = m_B (V_{B_2})_y$$





- total momentum in x dir is conserved

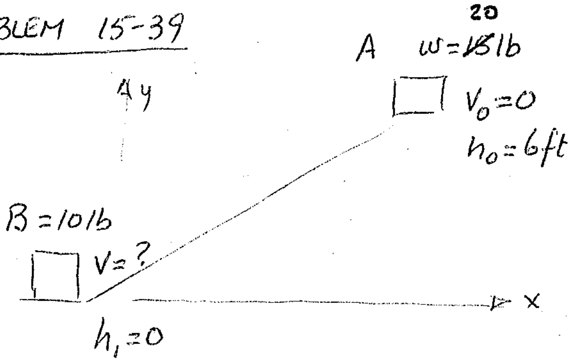
$$m_A(V_{A_1})_x + m_B(V_{B_1})_x = m_A(V_{A_2})_x + m_B(V_{B_2})_x$$

along the line of impact

- $$e = \frac{(V_{B_2})_x - (V_{A_2})_x}{(V_{A_1})_x - (V_{B_1})_x}$$

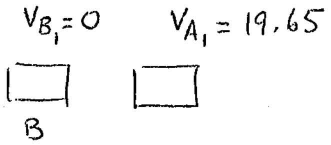
PROBLEM 15-39

4y



$$T_0 + V_0 = T_1 + V_1$$
$$0 + Wh = T_1 + 0$$
$$(\cancel{15}\overset{}{\underset{20}{}}lb)(6ft) = T_1 = \frac{1}{2} m_1 V_1^2$$
$$\frac{2(\overset{20}{\cancel{15}})(6)}{\underset{20}{\cancel{15}}/(32.2)} = V_1^2 = 2(32.2)(6)$$
$$V_1 = 19.66 \text{ ft/sec}$$

at impact



$$m_{B}\cancel{V_{B_1}}^{\,0} + m_A V_{A_1} = m_B V_{B_2} + m_A V_{A_2} \quad (1)$$

$$e = \frac{V_{B_2}-V_{A_2}}{V_{A_1}-\cancel{V_{B_1}}^{\,0}} = .3 = \frac{V_{B_2}-V_{A_2}}{19.66} \quad (2)$$

$$\frac{\overset{20}{\cancel{15}}}{\cancel{32.2}}\cdot 19.66 = \frac{10}{\cancel{32.2}}\cdot V_{B_2} + \frac{\overset{20}{\cancel{15}}}{\cancel{32.2}}\cdot V_{A_2} \quad (1')$$

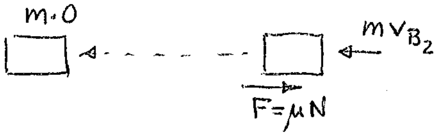
$$.3(19.66) = V_{B_2}-V_{A_2} \quad \text{or} \quad V_{B_2} = .3(19.66)+V_{A_2} \quad (2')$$

$$\overset{20}{\cancel{15}}(19.66) = 10(.3)(19.66) + 10V_{A_2} + \overset{20}{\cancel{15}}V_{A_2}$$

$$\overset{17}{\cancel{12}}(19.66) = \qquad + \overset{30}{\cancel{25}}V_{A_2}$$

$$V_{A_2} = \frac{\overset{17}{\cancel{12}}(19.66)}{30\,\cancel{25}} = \overset{11.14}{\cancel{9.44}}\ \text{ft/sec}$$

$$V_{B_2} = \underset{17.04}{\cancel{15.34}}\ \text{ft/sec}$$



Principle of impulse & mom.

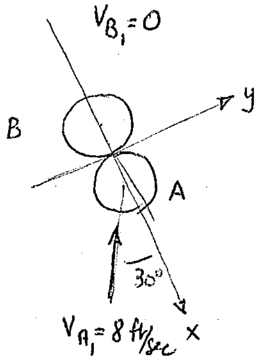
$N=W \qquad F=\mu N=\mu W$

$$mV_{B_2} - \mu W t = m\cdot 0 \quad \text{thus} \quad t = \frac{mV_{B_2}}{\mu W} = \frac{V_{B_2}}{\mu g} = \overset{1.32}{\cancel{1.19}}\,\text{sec} \quad \text{time to slide to a stop}$$

Energy needed to be dissipated = initial energy ie $\int F ds = \frac{1}{2}mV_{B_2}^2 = Fs = \mu N s = \mu W s$

$$\therefore \quad s = \frac{V_B^{\,2}}{2\mu g} = \overset{11.87}{\cancel{9.13}}\ \text{ft.}$$

PROBLEM 15-45 15-88 in 10th ed but e is different



$W_A = W_B = 47$ lb

$e=.7$

$$m_A(V_{A_1})_y = m_A(V_{A_2})_y$$

$$m_B(V_{B_1})_y = m_B(V_{B_2})_y$$

$$(V_{A_1})_x = -(V_{A_1})\cos 30^\circ = -8(.866) = -6.93\ \text{ft/sec}$$

$$(V_{A_1})_y = (V_{A_1})\sin 30^\circ = 8(.5) = 4\ \text{ft/sec} = (V_{A_{2y}})$$

$$(V_{B_1})_y = 0 = (V_{B_2})_y \qquad (V_{B_{1x}}) = 0$$

$$\overset{W_A}{\cancel{m_A}}(V_{A_1})_x + \overset{W_B}{\cancel{m_B}}\cancel{(V_{B_1})_x}^{\,0} = \overset{W_A}{\cancel{m_A}}(V_{A_2})_x + \overset{W_B}{\cancel{m_B}}(V_{B_2})_x$$

$$e = .7 = \frac{(V_{B_2})_x - (V_{A_2})_x}{(V_{A_1})_x - \cancel{(V_{B_1})_x}^{\,0}} \quad \text{or} \quad .7(-6.93) = V_{B_{2x}} - (V_{A_2})_x$$

$$(V_{B_2})_x = (V_{A_2})_x - 4.85$$

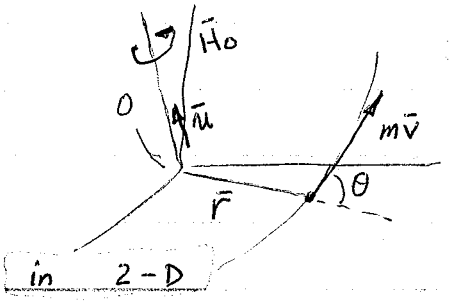
$$-6.93 = (V_{A_2})_x + (V_{A_2})_x - 4.85 \Rightarrow (V_{A_2})_x = -1.04\ \text{ft/sec} \qquad (V_{A_2}) = \sqrt{(-1.04)^2+4^2} = 4.133\ \text{ft/s}$$

$$V_{B_{2x}} = -5.89\ \text{ft/sec} \qquad V_{B_2} = \sqrt{(-5.89)^2+0^2} = 5.89\ \text{ft/sec}$$

LESSON #15

- CAN DEFINE ANGULAR MOMENTUM OF A PARTICLE
    - MOMENT OF LINEAR MOMENTUM OR MOMENT OF MOMENTUM

- DEFINE INERTIAL FRAME & FIXED PT O

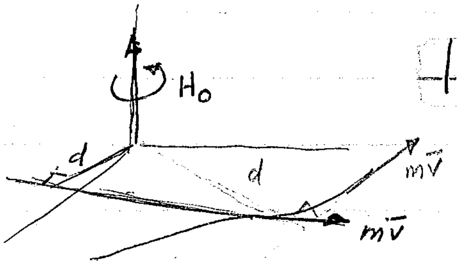


$$\boxed{\overrightarrow{\bar{H}_o} = \bar{r} \times m\bar{v}}$$

where $\bar{r}$ is from O to moment vector. Axis of H_o is ⊥ to $\bar{r}$ & $\bar{v}$

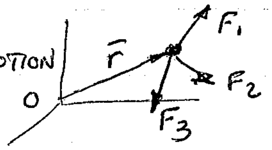
$$= |\bar{r}||m\bar{v}|\sin\theta\,\bar{u}$$

$$= \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ mv_x & mv_y & mv_z \end{vmatrix}$$



$$\boxed{|\bar{H}_o| = |m\bar{v}||d|}$$ & if $m\bar{v}$ & $\bar{d}$ in x-y plane $\bar{H}_o$ in z dir

- Relationship between moments & angular Momentum
    - FOR A PARTICLE LOOK AT EQN OF MOTION



Given $\quad \sum \bar{F} = m\bar{a} = m\dot{\bar{v}}$ $\quad$. Define

$\boxed{\text{Take } \bar{r} \times \sum \bar{F}} = \underline{\underline{\bar{r} \times m\dot{\bar{v}} = \sum \bar{M}_o}}$

- NOW LOOK AT ANGULAR MOMENTUM

Given $\quad \bar{H}_o = \bar{r} \times m\bar{v}$

$\boxed{\text{Take } \frac{d}{dt}\bar{H}_o} = \frac{d}{dt}(\bar{r} \times m\bar{v}) = \dot{\bar{r}} \times m\bar{v} + \bar{r} \times m\dot{\bar{v}}$

$$\dot{\bar{r}} = \bar{v} \quad \& \quad \bar{v} \times \bar{v} = 0 \quad \Rightarrow \quad \boxed{\dot{\bar{H}}_o} = \bar{r} \times m\dot{\bar{v}} = \boxed{\sum \bar{M}_o}$$

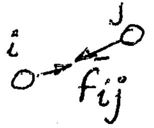
SUM OF MOMENTS ABOUT A PT O DUE TO FORCES ACTING ON PARTICLE = TIME RATE OF CHANGE OF ANGULAR MOMENTUM ABOUT PT O.

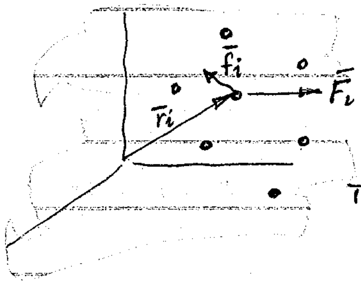
- FROM EQS OF MOTION: $\Sigma \bar{F} = m\bar{a} = m\dot{\bar{v}} = \dot{\bar{L}}$

SUM OF FORCES ACTING ON PARTICLE = TIME RATE OF CHANGE OF LINEAR MOMENTUM.

- LOOK AT A SYSTEM OF PARTICLES WITH EACH PARTICLE EXPERIENCING EXTERNAL FORCE $\bar{F}_i$ & INTERNAL forces

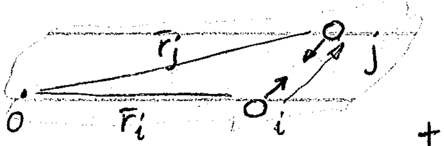
~~Σ~~ $\bar{f}_i = \sum_{i \neq j} \bar{f}_{ij}$





TAKE $\bar{r}_i \times (\bar{F}_i + \bar{f}_i) = (\bar{M}_o)_i = (\dot{\bar{H}}_o)_i = \bar{r}_i \times m_i \dot{\bar{v}}_i$

- TAKE $\sum_i \bar{r}_i \times \bar{F}_i + \sum_i \bar{r}_i \times \underset{\times \sum_{i \neq j} \bar{f}_{ij}}{\bar{f}_i} = \sum_i (\bar{M}_o)_i = \sum_i (\dot{\bar{H}}_o)_i$



$\bar{r}_j = \bar{r}_i + \bar{r}_{j/i}$

$\bar{r}_j \times \bar{f}_{ji} = \bar{r}_i \times \bar{f}_{ji} + \bar{r}_{j/i} \times \bar{f}_{ji}$

+

$\bar{r}_i \times \bar{f}_{ij} = -\bar{r}_i \times \bar{f}_{ji}$

$\bar{f}_{ji} = -\bar{f}_{ij}$ & $\bar{r}_{j/i}$ is ∥ $\bar{f}_{ji}$

$$\therefore \quad \sum_i \bar{r}_i \times \bar{F}_i = \sum_i (\bar{M}_o)_i = \sum_i (\dot{\bar{H}}_o)_i = \frac{d}{dt}(\Sigma \bar{H}_o)_i$$

Σ moments about point O of external forces acting on system = $\frac{d}{dt}$ TOTAL ANGULAR MOMENTUM OF SYSTEM ABOUT O.

$$\Sigma \bar{r}_i \times \bar{F}_i = \Sigma \bar{M}_o = \frac{d}{dt}(\bar{H}_{o\,TOTAL}) = \Sigma \bar{r}_i \times m_i \dot{\bar{v}}_i$$

FOR A PARTICLE

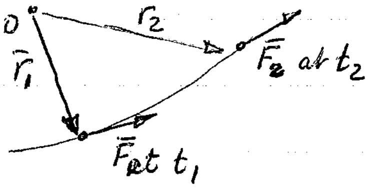
- INTEGRATE

$$\int_{t_1}^{t_2} \Sigma \bar{M}_o \, dt = \int_{H_{o_1}}^{H_{o_2}} d\bar{H}_o \Rightarrow \bar{H}_{o_2} - \bar{H}_{o_1} = \int_{t_1}^{t_2} \Sigma \bar{M}_o \, dt$$

or $\bar{H}_{o_2} = \bar{H}_{o_1} + \underbrace{\int_{t_1}^{t_2} \Sigma \bar{M}_o \, dt}_{\text{ANGULAR IMPULSE}}$ PRINCIPLE OF ANGULAR IMPULSE & MOMENTUM

$\bar{H}_{o_{\substack{2\\1}}} = \bar{H}_o$ @ time $t_{\substack{2\\1}}$

$$\int_{t_1}^{t_2} \bar{M}_0 \, dt = \int_{t_1}^{t_2} \bar{r} \times \bar{F} \, dt$$



- FOR A SYSTEM we had $\Sigma \bar{M}_0 = \frac{d}{dt}(\Sigma \bar{H}_0)$

INTEGRATE

$$(\Sigma \bar{H}_0)_2 = (\Sigma \bar{H}_0)_1 + \Sigma \int_{t_1}^{t_2} \bar{M}_0 \, dt \longrightarrow \text{moments due to external forces only.}$$

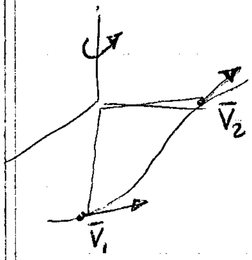
$$(\Sigma \bar{H}_0)_{\substack{2\\1}} = \Sigma (\bar{r}_i \times m_i \bar{v}_i) \ @ \ t_2, t_1$$

BASED ON ABOVE FOR A PARTICLE - MOMENTA BEFORE & AFTER

VECTOR FORM

$$\left.\begin{array}{l} m\bar{v}_1 + \Sigma \int \bar{F} dt = m\bar{v}_2 \\ (\bar{H}_0)_1 + \Sigma \int \bar{M}_0 dt = (\bar{H}_0)_2 \end{array}\right\} \quad \text{6 eqns - scaler}$$

FOR MOTION IN x-y plane only. ($\bar{v}$ has only v_x, v_y components)



$$\left.\begin{array}{lll} mV_{1x} + \Sigma \int F_x \, dt = mV_{2x} & x \text{ dir} \\ mV_{1y} + \Sigma \int F_y \, dt = mV_{2y} & y \text{ dir} \end{array}\right\} \text{LINEAR IMPULSE \& MOM.}$$

$$(H_0)_1 + \Sigma \int M_0 \, dt = (H_0)_2 \qquad z \text{ dir} \qquad \text{ANGULAR IMP. \& MOM.}$$

$$\bar{H}_0 = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \cancel{mv_x} r_x & r_y & 0 \\ mv_x & mv_y & 0 \end{vmatrix} = \bar{k}\,(r_x m v_y - r_y m v_x)$$

$$\bar{M}_0 = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ r_x & r_y & 0 \\ F_x & F_y & 0 \end{vmatrix} = \bar{k}\,(r_x F_y - r_y F_x)$$

- WHEN ANGULAR IMPULSES = 0 $\Sigma \int \bar{M}_0 dt = 0 \Rightarrow (\bar{H}_0)_1 = (\bar{H}_0)_2$

conservation of angular momentum

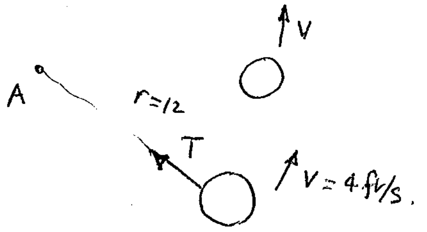
- FOR A SYSTEM $\Sigma(\bar{H}_0)_1 = \Sigma(\bar{H}_0)_2$ $\quad \Sigma H_0 = \Sigma \bar{r}_i \times m_i \bar{v}_i$

DRAW FBD SHOWING ALL FORCES; DETERMINE AXIS ABOUT WHEN TO TAKE MOMENTS. IF ANGULAR MOMENTUM IS CONSERVED USE $(\bar{H}_0)_1 = (\bar{H}_0)_2$ IF NOT USE PRINCIPLE OF ANGULAR IMPULSE & MOMENTUM.

LOOK AT PROB. 15-12 pg. 208

15-107 in 10th ed.

15-61



$\overset{\circ}{r} = .5$ ft/s. in 3 seconds $r = 12-(1.5)$
$= 10.5$ ft

T passes through A & ∴ $\bar{M}_0 = \bar{r} \times \bar{T} = 0$

W, is ⊥ to axis passing through 0 ∴ $\bar{M}_0 = \bar{r} \times \bar{W} = 0$ in z plane
($\bar{M}_0$ is in x-y plane)

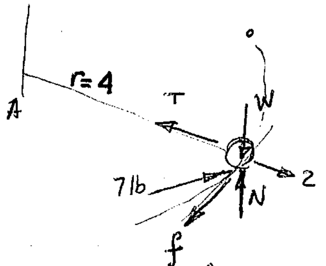
so in z dir ∴ $H_{0_1} = H_{0_2}$

$$r_1 \cdot \cancel{m} V_1 = r_2 \cdot \cancel{m} V_{2_T} \qquad V_{2_T} = \frac{r_1 V_1}{r_2} = \frac{12(4)}{10.5} = 4.57 \text{ ft/s.}$$

$$V_{2n} = \overset{\circ}{r} = .5 \text{ ft/s.} \qquad \therefore \qquad V_2 = \sqrt{(4.57)^2 + (.5)^2} = 4.60 \text{ ft/s.}$$

15-67

① find V when T = 20 lb next use principle of imp + mom



When T = 20 lb

$$T - 7\sin 38 - 2 = \frac{mV^2}{\rho} = \frac{10}{32.2}\,\frac{V^2}{4}$$

$$\therefore \quad V = 13.67 \text{ ft/s}$$

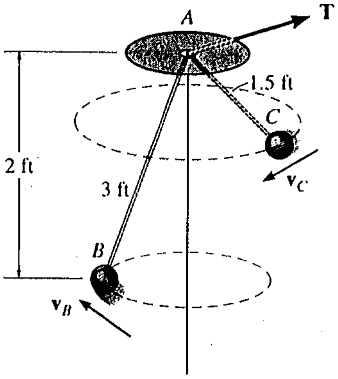
$$H_{A_1} + \Sigma \int M_A\,dt = H_{A2} \qquad \text{where } H_{A_1} = 4 \cdot \frac{10}{32.2} \cdot 2 = (rmV)_1$$

now T, 7 sin 30° & Z produce no $\int M_A dt$, W & N produce no Mdt
but 7 cos 30° & $f = \mu N = \mu W$ does => $+ 7\cos 30° \cdot 4\Delta t - .5(10) 4 \Delta t$

$$H_{A_2} = (rmV)_1 + (7\cos 30°)(4\,\Delta t) - .5(10 \cdot 4)\,\Delta t = 4 \cdot \frac{10}{32.2}(V = 13.67)$$

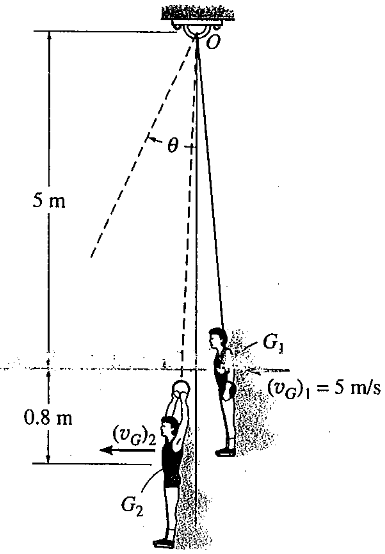
Thus $\Delta t = 3.41$s

15–77. The ball B has a weight of 5 lb and is originally rotating in a circle. As shown, the cord AB has a length of 3 ft and passes through the hole at A, which is 2 ft above the plane of motion. If 1.5 ft of cord is pulled through the hole, determine the speed of the ball when it moves in a circular path at C.



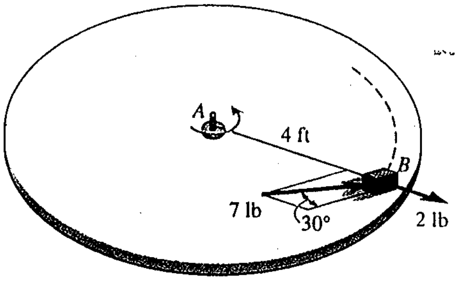
Prob. 15–77

15–78. A gymnast having a mass of 80 kg holds the two rings with his arms down in the position shown as he swings downward. His center of mass is located at point G_1. When he is at the bottom position of his swing, his velocity is $(v_G)_1 = 5$ m/s. At this lower position he *suddenly* lets his arms come up, shifting his center of mass to position G_2. Determine his new velocity in the upswing due to this sudden movement, and the angle θ to which he swings before coming to rest. Treat his body as a particle.



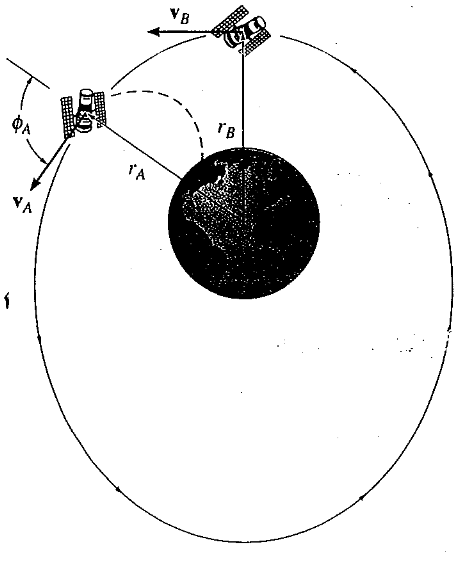
Prob. 15–78

15–79. The 10-lb block rests on a surface for which the coefficient of kinetic friction is $\mu_k = 0.5$. The block is acted upon by a radial force of 2 lb and a horizontal force of 7 lb, always directed at 30° from the tangent to the path as shown. If the block is initially moving in a circular path with a speed $v_1 = 2$ ft/s at the instant the forces are applied, determine the time required before the tension in cord AB becomes 20 lb. Neglect the size of the block for the calculation.



Prob. 15–79

***15–80.** An earth satellite of mass 800 kg is launched into a free-flight trajectory about the earth with an initial velocity of $v_A =$ 12 km/s when the distance from the center of the earth is $r_A =$ 15 Mm. If the launch angle at this position is $\phi_A = 60°$, determine the velocity v_B of the satellite and its closest distance r_B from the center of the earth. The earth has a mass of $M_e = 5.976(10^{24})$ kg. *Hint:* Under these conditions, the satellite is subjected only to the earth's gravitational force, $F = G\, M_e m_s / r^2$ (Eq. 13–2). For part of the solution, use the conservation of energy (see Prob. 14–63).



Prob. 15–80

LESSON # 16

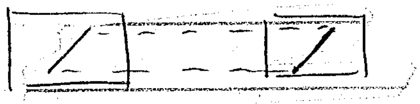
- LOOK AT ~~RIGID BODY~~ PLANAR MOTION OF A RIGID BODY
  - USED IN DESIGN OF PARTS ~~WHOSE MOTION~~ DEPENDS ON GEOMETRY OF THEIR MOTIONS
    NEED TO TRANSMIT CERTAIN MOTIONS USING CAMS, GEARS & LINKAGE ⇒ NEED TO KNOW $s, \bar{v}, \bar{a}$
    NEED TO KNOW $\bar{F}$ & $\bar{M}$ due to these. ALSO WHAT IS MOTION DUE TO FORCES ~~[illegible]~~ GENERATED
- MOTION : TRANSLATION - BOX MOVING IN STRAIGHT LINE

  ROTATION ABOUT AN AXIS FIXED PENDULUM

  GENERAL MOTION - COMBINATION ; ROLLING WHEEL
- IF 2 PTS ARE KNOWN & SO IS THEIR MOTION - SO IS WHOLE BODY

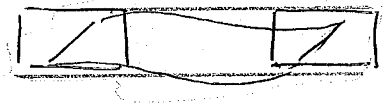
PLANAR MOTION - EACH OF PARTICLES OF BODY MOVE ALONG A PATH A FIXED DISTANCE FROM A FIXED PLANE

- TRANSLATION - IF A FIXED LINE IN A BODY KEEPS THE SAME DIRECTION DURING MOTION



IF PATH OF THIS LINE IS STRAIGHT

RECTILINEAR TRANSLATION



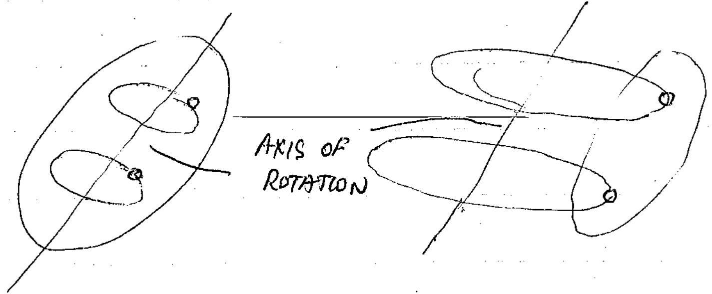
IF NOT RECTILINEAR

THEN CURVILINEAR MOTION

- ROTATION ABOUT FIXED AXIS

  - PARTICLES MOVE IN ~~[illegible]~~ CIRCULAR PATHS IN PARALLEL PLANES

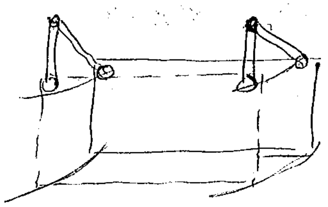
  - ALL CIRCLES CENTERED ON SAME AXIS.

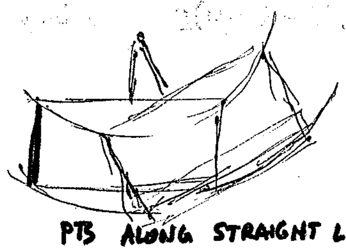


PTS ALONG AXIS HAVE
ZERO VELOCITY & ACCEL.

ROTATION WRT CURVILINEAR MOTION

- CURV. MOTION — PARTICLES UNDERGO MOVEMENT WHICH FORM PARALLEL CIRCLES
- ROTATION — PARTICLES MAY UNDERGO CONCENTRIC CIRCLES



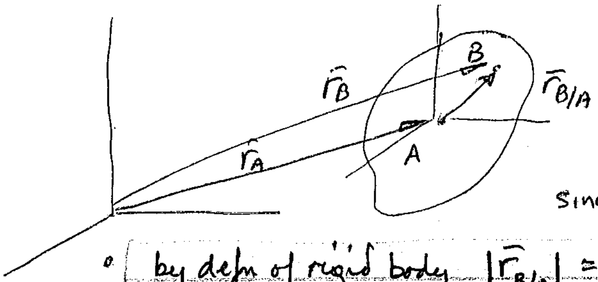


PTS ALONG STRAIGHT LINE TRACE OUT CONCENTRIC CIRCLES

- GENERAL MOTION — COMBO OF THE TWO

ANALYSIS OF TRANSLATION

- LOOK AT BODY IN RECTILINEAR OR CURVILINEAR TRANSLATION



- LOOK AT 2 PTS IN BODY

$$\bar r_B = \bar r_A + \bar r_{B/A}$$

- VELOCITIES $\frac{d}{dt}\bar r_B = \frac{d}{dt}\bar r_A + \frac{d\bar r_{B/A}}{dt}$

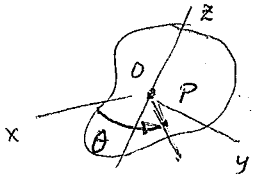
since only translating $\frac{d}{dt}\bar r_{B/A} = \bar 0$

- by defn of rigid body $|\bar r_{B/A}| = \text{const.}$
- by defn of translation dir = const. ∴ $\frac{d}{dt} = 0 \Rightarrow \bar V_A = \bar V_B$
- ACCEL $\bar a_A = \bar a_B$ $\quad \frac{d}{dt}(\bar V_A = \bar V_B)$

- WHEN A BODY TRANSLATES ALL PTS OF BODY HAVE SAME VELOC & ACCEL
- ⊙ MAY APPLY KINEMATICS OF PARTICLE MOTION TO A RIGID BODY
-

ROTATION ABOUT AN AXIS

- CONSIDER A RIGID BODY WHICH ROTATES ABOUT AN AXIS

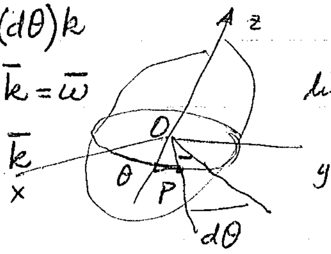


PICK A PT P & LET $\overline{OP}$ define the ANGULAR POSITION OF PT. P wrt X AXIS (X AXIS IS OUR FIXED REFERENCE LINE)

$\underline{a} \times (\underline{b} \times \underline{c}) = (\underline{a} \cdot \underline{c})\underline{b} - (\underline{b} \cdot \underline{a})\underline{c}$

$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = (\underline{\omega} \cdot \underline{r})\underline{\omega} - (\underline{\omega} \cdot \underline{\omega})\underline{r}$ since $\underline{\omega} \perp \underline{r}$ $\underline{\omega} \cdot \underline{r} = 0$ $\underline{\omega} \cdot \underline{\omega} = \omega^2$

- MAGNITUDE IS θ & DIRECTION IS DEFINED BY RIGHT HAND RULE
- CHANGE IN ANGULAR POSITION IS $d\bar{\theta} = (d\theta)\bar{k}$
- ANGULAR VELOCITY IS $\frac{d}{dt}\bar{\theta} = \dot{\theta}\bar{k} = \omega\bar{k} = \bar{\omega}$
- ANGULAR ACCEL IS $\frac{d^2}{dt^2}\bar{\theta} = \frac{d}{dt}\bar{\omega} = \bar{\alpha} = \alpha\bar{k}$



like $\frac{d\bar{r}}{dt} = \bar{v}$

$\frac{d\bar{v}}{dt} = \bar{a}$

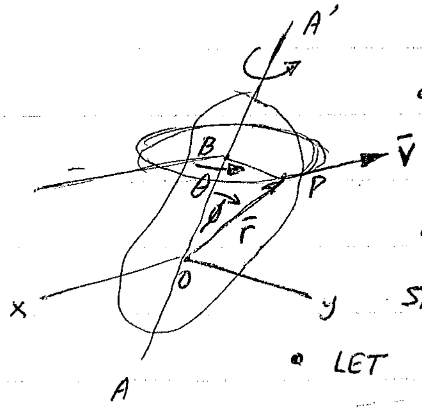
- since $\frac{d\omega}{dt} = \alpha \Rightarrow d\omega = \alpha dt$ also $\omega = \frac{d\theta}{dt}$ multiply $\omega d\omega = \alpha d\theta$

similar to $\underline{\underline{ads = vdv}}$

- θ, ω, α – rad, rad/s, rad/s^2
- IF α = const. $\Rightarrow C + \alpha t = \dot{\theta} = \omega \Rightarrow \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

  also $2\alpha(\theta - \theta_0) + \omega_0^2 = \omega^2$

Just like rectilinear particle motion



- CONSIDER P A PT OF BODY

  $r\sin\phi = \overline{BP} = \text{const.}$

- let B be a proj of P on AA'

  Since $\bar{v} = \frac{d\bar{r}}{dt}$ and $|\bar{v}| = \frac{ds}{dt}$. $\bar{v}$ is // to path

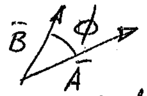
- LET P MOVE ALONG CIRCLE SO THAT $\exists$ A $\Delta\theta$

  now $\Delta s = \overline{BP}(\Delta\theta) = r\sin\phi\Delta\theta \Rightarrow v = r\dot{\theta}\sin\phi$

- $\bar{v}$ is $\perp$ to AA' axis & $\bar{v}$ is $\perp$ to $\overline{BP}$

  $\therefore$ $\bar{v}$ is $\perp$ plane formed by $\overline{BO}$ & $\overline{BP}$ ie $\bar{v}$ is $\perp$ TO ANY VECTOR in that plane & $\Rightarrow \bar{v}$ is $\perp$ to $\bar{r}$

- $\bar{C} = \bar{A} \times \bar{B} \Rightarrow AB\sin\phi = C$



  when $\bar{C}$ is $\perp \bar{A}$ & $\bar{C}$ is $\perp \bar{B}$

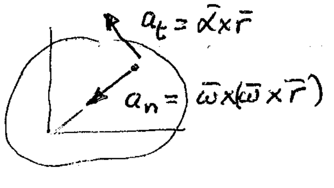
  - NOTE that $\bar{\omega} = \dot{\theta}\bar{u}_{A'A}$ and $\bar{r} = r\bar{u}_r$

- IF $v = r\dot{\theta}\sin\phi \Rightarrow \bar{v} = \bar{\omega} \times \bar{r} \Rightarrow \dot{\theta} r \sin\phi = v$

- $$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d\bar{\omega}}{dt} \times \bar{r} + \bar{\omega} \times \frac{d\bar{r}}{dt} = \underbrace{\bar{\alpha} \times \bar{r}}_{\text{TANGENT TO PATH}} + \underbrace{\bar{\omega} \times \bar{v}}_{\text{NORMAL TO PATH}} = \bar{\alpha} \times \bar{r} + \bar{\omega} \times (\bar{\omega} \times \bar{r})$$

now $\bar{\alpha} \times \bar{r}$ has magnitude $r\ddot{\theta} = a_t$

$\bar{\omega} \times (\bar{\omega} \times \bar{r})$ has magnitude $\omega^2 r = a_n$



$$\bar{\omega}\times(\bar{\omega}\times\bar{r}) = \underbrace{(\bar{\omega}\cdot\bar{r})}_{0}\bar{\omega} - (\bar{\omega}\cdot\bar{\omega})\bar{r} = -\omega^2\bar{r}$$

since $\bar{\omega}$ is $\perp$ $\bar{r}$

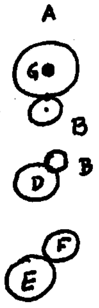
$$\bar{a} = \bar{\alpha}\times\bar{r} - \omega^2\bar{r} = \underbrace{\bar{a}_t}_{\text{time rate of change of magnitude of veloc}} + \underbrace{\bar{a}_n}_{\text{time rate of change of direction of velocity}}$$

EXAMPLE 16-2 — GEARS IN CONTACT : $\bar{V}$ are same at contact pt & $\bar{a}_t = \bar{\alpha}\times\bar{r}$ are same at contact pt.

16-27 in 10th edition

PROBLEM 16-5 ① PTS IN CONTACT HAVE SAME VELOCITY

② GEARS ALONG SAME SHAFT HAVE SAME ANGULAR VELOCITIES



① $V_A = \omega_G r_A = (60\text{ rad/s})(90\text{ mm}) = V_B = \omega_B r_B = \omega_B(30\text{ mm}) \Rightarrow \omega_B = 180\text{ rad/s}$

② $\omega_B = \omega_C = 180\text{ rad/s}$ ↺

① $V_D = \omega_D r_D = V_C = \omega_C r_C \Rightarrow \omega_D(50) = 180(30) \Rightarrow \omega_D = 108\text{ rad/s}$ ↻

② $\omega_D = \omega_E = 108\text{ rad/s}$ ↻

① $V_E = \omega_E r_E = V_F = \omega_F r_F \Rightarrow 108(70) = \omega_f(60) \Rightarrow \omega_f = 126\text{ rad/s}$ ↺ $= \omega_H$

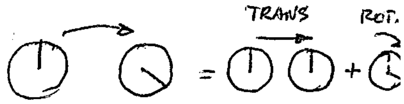
PROBLEM 16-4

VELOCITY OF A' & B' must be same $\bar{r}_{A'} + \bar{r}_{B'/A'} = \bar{r}_{B'}$ $r_{B'/A'} = \text{Const}$

$\therefore \bar{V}_{B'/A'} = \bar{0}$

LESSON #18

GENERAL PLANE MOTION - **SCALAR APPROACH**



BODY UNDERGOES TRANSLATION & ROTATION

TRANSLATES IN PLANE & ROTATES ABOUT AXIS $\perp$ TO PLANE

- RELATE POSITION OF A POINT ON BODY TO THE ANGULAR POSITION OF A LINE IN THE BODY
- NEXT APPLY $V = \frac{ds}{dt}$ $a = \frac{dv}{dt}$ $\alpha = \frac{d\omega}{dt}$ $\omega = \frac{d\theta}{dt}$

TO ANALYZE MOTION, RELATE:

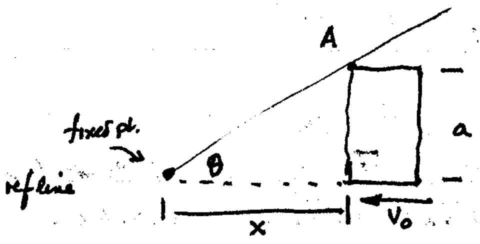
- POSITION OF A PT IN BODY wrt FIXED PT
- ANGULAR POSITION wrt fixed REFERENCE LINE

pick ref line to pass through fixed pt.

EXAMPLE 16-4 ↗ same as EX. 16-3 in 10th ed.

Block moves to left w/ a constant velocity V_0. Find $\dot{\theta}$ & $\ddot{\theta}$ of the bar as a fn. of θ

16-36 in 10th ed



$\frac{d}{dt}\tan = \left(\frac{c}{c} + 1\cdot\frac{s}{c^2}s\right)\dot{\theta} = \frac{\dot{\theta}}{c^2} = \sec^2\theta\,\dot{\theta}$

$\tan\theta = \frac{a}{x}$

$x\tan\theta = a$

$\dot{x}\tan\theta + x\sec^2\theta\,\dot{\theta} = 0$

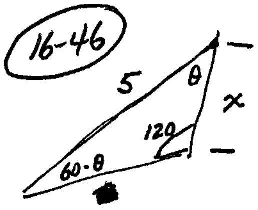
$\dot{x} = -V_0$

$$\therefore\ \dot{\theta} = \frac{-\dot{x}\tan\theta}{x\sec^2\theta} = \frac{V_0\tan^2\theta}{a\sec^2\theta} = \frac{V_0\sin^2\theta}{a}$$

$$\ddot{\theta} = V_0\cdot\frac{2\sin\theta\cos\theta}{a}\,\dot{\theta} = \frac{V_0^2}{a^2}\cdot 2\sin^3\theta\cos\theta$$

15-102, 109 ; 16-12, 18, 38, 48
due Wed 3/16

16-46



$$\frac{x}{\sin 60-\theta} = \frac{5}{\sin 120} = \frac{y}{\sin\theta}$$

$\dot{x} = \frac{5}{\sin 120}\,\overset{\bullet}{\sin(60-\theta)}$

$-6 = \frac{5}{\sin 120}\cos(60-\theta)(-\dot{\theta})$

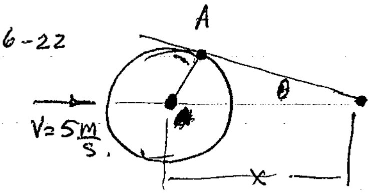
$\dot{\theta} = 1.076\ r/s$

$\dot{y} = \frac{5}{\sin 120}\cos\theta\,\dot{\theta}$

$= 4.393\ ft/s$

A PT IN BODY

16-22



FIXEDREF

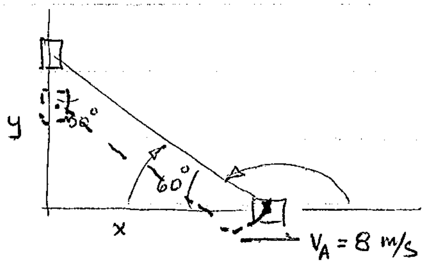
$\sin\theta = \frac{100}{x} \Rightarrow x\sin\theta = 100$

$\frac{dx}{dt}\sin\theta + \underline{x\cos\theta}\,\overset{\circ}{\theta} = 0$

BUT $\frac{dx}{dt} = -V = -5\ m/s$. when $\theta = 30^\circ$ $x = 200$ mm

$\therefore\ -5(\tfrac{1}{2}) + .2(.866)\overset{\circ}{\theta} = 0$ or $\overset{\circ}{\theta} = \frac{2.5}{.1732} = 14.44$ rad/s ↻

16-29



FIXED REF

$x^2 + y^2 = 4$ when $\theta = 60^\circ$ $x = 1$ m

$2x\dot{x} + 2y\dot{y} = 0$ $y = 1.732$ m

$2(1)8 + 2(1.732)V_B = 0$

IF $V_A=6$ $V_B \overset{=}{-} 3.46$ m/s

$V_B = -\frac{8}{1.732} = -4.62$ m/s 4.62 m/s ↓

$x = 2\cos\theta$ $\overset{\circ}{x} = -2\sin\theta\,\overset{\circ}{\theta}$

$+8 = -2(.866)\overset{\circ}{\theta} \Rightarrow \overset{\circ}{\theta} = \frac{8}{-1.732} = -4.62$

IF $V_A = 6$ THEN $\dot{\theta} = -3.46$

$\overset{\circ}{\theta} = -4.62$ rad/s or 4.62 rad/s ↻ .

EXAM: 4 PROBLEMS

CALCULATOR + 1 8½ x 11 SHEET
75min long
come 10 min early.

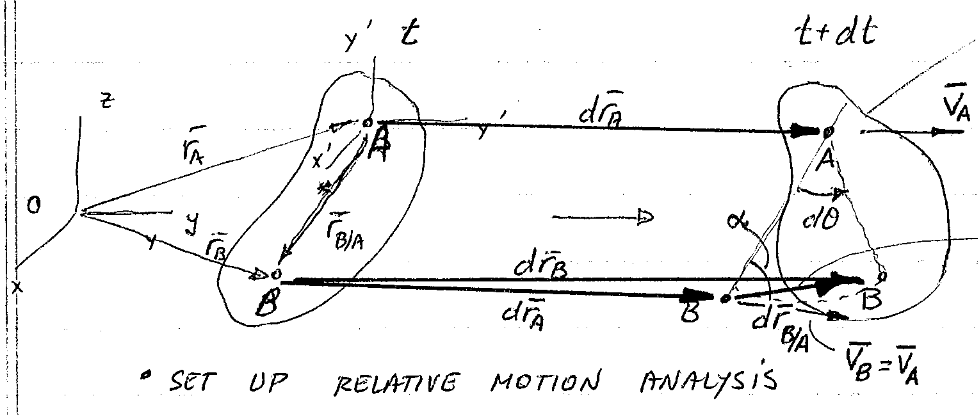
LESSON #17 EXAM

LESSON #18

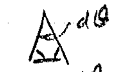
- RE-EXPLAIN ABOUT $\bar{\omega} \times \bar{r} = \bar{V}$
- GO BACK TO GENERAL MOTION

- GENERAL MOTION IS A COMBINATION OF TRANS + ROT ABOUT FIXED AXIS - GEOMETRIC-VECTOR

TO SEE THIS



isos triangle



$2\alpha + d\theta = 180°$

as $dt \to 0$ $d\theta \to 0$
$\Rightarrow \alpha \to 90°$

FIXED IN MAG.

$$\left|\frac{d\bar{r}}{dt}\right| = \frac{ds}{dt} = \bar{r}_{B/A}\frac{d\theta}{dt}$$
$$= \omega r_{B/A}$$

- SET UP RELATIVE MOTION ANALYSIS
- SET UP FIXED REF. FRAME TO MEASURE ABSOLUTE POSITIONS
- SET UP TRANSLATING REF FRAME WRT FIXED FRAME. - ORIGIN AT A PT IN BODY. PICK A FOR CONV. NOT FIXED IN BODY but translate

IF TRANSLATING $\bar{r}_{B/A}$ would have kept dir fixed

$$\bar{r}_B + d\bar{r}_B = \bar{r}_A + d\bar{r}_A + \bar{r}_{B/A} + d\bar{r}_{B/A}$$

- A HAS MOVED $d\bar{r}_A$ in dt $\Rightarrow$ $\bar{V}_A$

$$\bar{r}_B = \bar{r}_A + \bar{r}_{B/A}$$

- B HAS MOVED $d\bar{r}_B$ in dt $\Rightarrow$ $\bar{V}_B$

$$d\bar{r}_B = d\bar{r}_A + d\bar{r}_{B/A} \quad \Rightarrow \quad \bar{V}_B = \bar{V}_A + \frac{d\bar{r}_{B/A}}{dt}$$

- $d\bar{r}_{B/A}$ ACCOUNTS ONLY FOR CHANGE IN DIR. $|d\bar{r}_{B/A}| = |\bar{r}_{B/A}| d\theta$

  $$\left|\frac{d\bar{r}_{B/A}}{dt}\right| = |\bar{r}_{B/A}|\,\dot{\theta} = \omega r_{B/A}$$

  $\bar{r}_{B/A}$ is fixed in magnitude
- ANGULAR VELOC OF BODY AT INSTANT CONSIDERED
- THIS ROTATION IS ABSOLUTE - SAME IF MEASURED WRT X,Y,Z OR x'y'z'

- DEFINE $\frac{d\bar{r}_{B/A}}{dt} = \bar{V}_{B/A}$ relative velocity

- $|\bar{V}_{B/A}| = \omega r_{B/A}$

observer at A sees pt B moving in circular arc with angular velocity $\bar{\omega}$ about fixed axis through $\bar{A}$

- CAN STUDY MOTION OF A RIGID BODY WHICH IS PIN CONNECTED OR IN CONTACT w/ OTHER MOVING BODIES

⊛
- CHOOSE PTS A & B AT JOINTS OR CONTACT PTS WHICH HAVE KNOWN MOTIONS
- SEE PG 265 FIG 16-11
- $\bar{V}_B = \bar{V}_A + \bar{V}_{B/A}$ APPLIED TO 2 PTS ON SAME BODY

ANALYSIS

- DRAW DIAGRAM SHOWING ABSOLUTE VELOCITIES $\bar{V}_A$ & $\bar{V}_B$ & ANGULAR VELOC OF BODY
- DRAW BODY INDICATING RELATIVE VELOC

  $\bar{V}_{B/A}$ magn is $\omega r_{B/A}$

  dir is $\perp$ to $\bar{r}_{B/A}$
- WRITE $\bar{V}_B = \bar{V}_A + \bar{V}_{B/A}$ showing magnitude & dir

  This is vector eqn.

- EXAMPLE 16-5 P. 266

  EXAMPLE 16-6

MATHEMATICAL APPROACH.

SINCE $\bar{V}_{B/A}$ REPRESENTS CIRCULAR MOTION OBSERVED FROM $x'y'z'$

$$\bar{V}_{B/A} = \bar{\omega} \times \bar{r}_{B/A}$$

$$\bar{V}_{B/A} + \bar{V}_A = \bar{V}_A + \bar{\omega} \times \bar{r}_{B/A} = \bar{V}_B$$

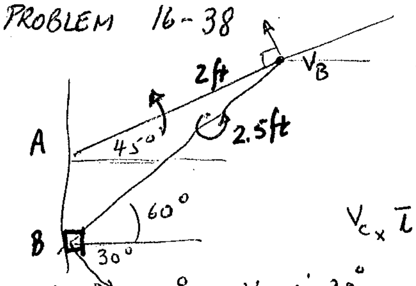
LOOK AT ~~PROBLEM~~ EXAMPLE 16-8 PG 270

Rod AB is rotating with angular velocity $\omega_{AB} = 5$ rad/s
Find veloc of collar C at the instant shown

LESSON # 19

See fig 16-120
10th ed.

PROBLEM 16-38



$$\bar{V}_B = \bar{V}_A + \bar{V}_{B/A} \qquad \bar{V}_A = 0$$

$$\bar{V}_B = V_{B/A} = \omega r = 5(2) = 10 \text{ ft/s} \quad \nwarrow 135°$$

$$\bar{V}_c = \bar{V}_B + \bar{V}_{c/B}$$

$$V_{c_x}\bar{i} = 10\cos 135\,\bar{i} + 10\sin 135°\,j + \bar{V}_{c/B_x}\bar{i} + V_{c/B_y}\bar{j}$$

$$\bar{V}_{c/B} = V_{c/B}\cos 30°\,i - V_{c/B}\sin 30°\,j$$

$$\Rightarrow 10\sin 135° = -(V_{c/B})_y = +V_{c/B}\overset{\sin}{\cancel{\cos}}30°$$

$$V_{c/B} = \frac{10 \sin 135^\circ}{\sin 30^\circ} = \frac{10 \cos 45^\circ}{\sin 30^\circ} = \frac{10(.7071)}{.5} = 14.41 \text{ ft/s}$$

$$V_{cx} = 10 \cos 135^\circ + V_{c/B} \cos 30^\circ = -10 \sin 45^\circ + 14.41(.866) = 5.176 \text{ ft/s}$$

$$V_{B} = \omega r_{B} = \omega(2.5) \implies \omega = \frac{14.41}{2.5} = 5.657 \text{ rad/s.}$$

READ 16.7 ON your own

LESSON 19

RELATIVE MOTION ANALYSIS: ACCEL.

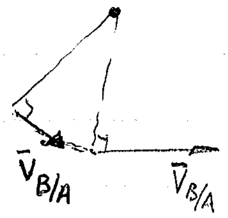
$$\bar{r}_B = \bar{r}_A + \bar{r}_{B/A}$$

$$\frac{d}{dt}: \quad \bar{V}_B = \bar{V}_A + \frac{d}{dt}(\bar{r}_{B/A})$$

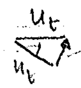
magnitude $\omega r_{B/A}$
$\perp$ to $r_{B/A}$

$$\frac{d}{dt}: \quad \bar{a}_B \neq \bar{a}_A + \frac{d}{dt}(\bar{V}_{B/A})$$

- $\bar{a}_B, \bar{a}_A$ are absolute ~~[illegible]~~ accelerations wrt xyz fixed inertia frame
- THINK IN TANGENTIAL - NORMAL COMP.



$$\frac{d}{dt}(V_{B/A}\bar{u}_t) = \dot{V}_{B/A}\bar{u}_t + V_{B/A}\frac{d\bar{u}_t}{dt}$$



$$\frac{d\theta}{dt}\bar{u}_n = \frac{d\bar{u}_t}{dt}$$

$$\frac{V_{B/A}}{\rho}\bar{u}_n = \dot{\theta}\bar{u}_n$$

$\omega r_{B/A} \Rightarrow \omega$ changes — changes magnitude of $V_{B/A}$

$$\dot{V}_{B/A}\bar{u}_t + \frac{V_{B/A}^2}{\rho}\bar{u}_n$$

$$V_{B/A} = \omega r_{B/A} \qquad \dot{V}_{B/A} = \dot{\omega} r_{B/A} = \ddot{\theta} r_{B/A} = \alpha r_{B/A}$$

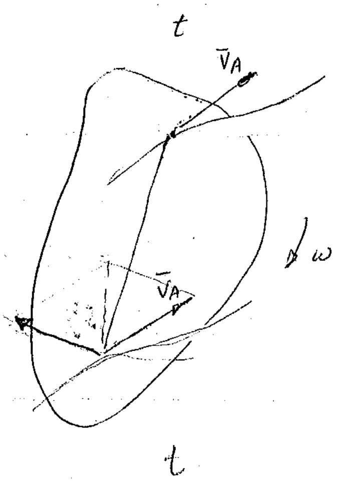
$$\rho = r_{B/A} \implies \frac{V_{B/A}^2}{r_{B/A}} = \frac{V_{B/A}^2}{\rho} = \frac{\omega^2 r_{B/A}^2}{r_{B/A}} = \omega^2 r_{B/A}$$

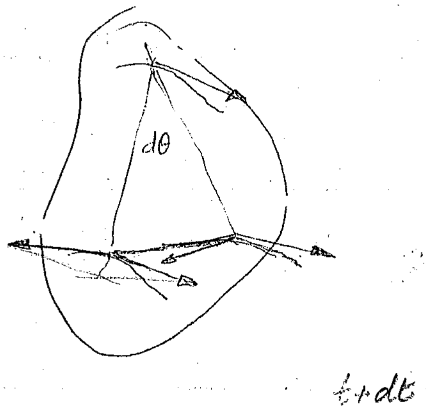
$$\therefore \quad \bar{a}_B = \bar{a}_A + \ddot{\theta} r_{B/A}\bar{u}_t + \omega^2 r_{B/A}\bar{u}_n = \bar{a}_A + \bar{a}_{B/A}$$

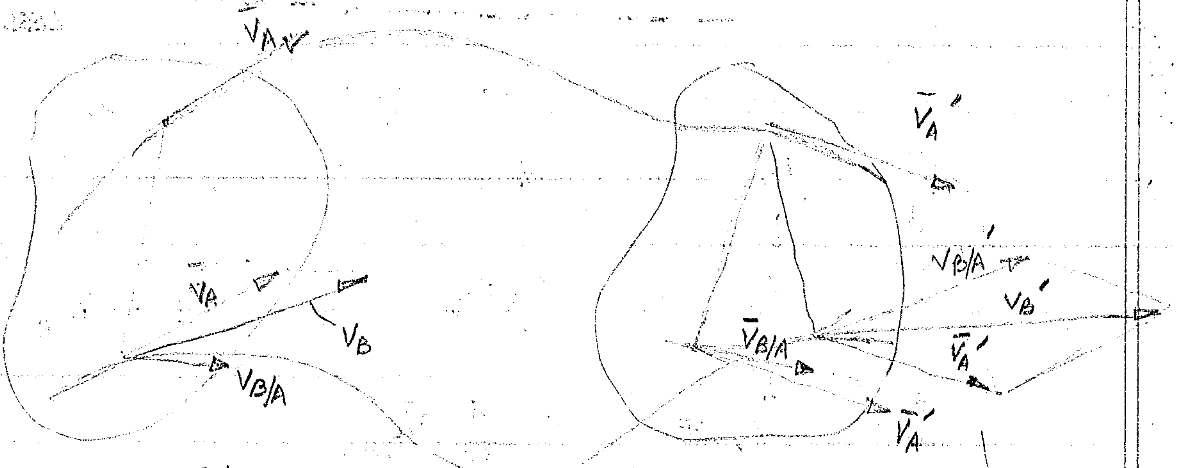
$\alpha r_{B/A}$

RELATIVE TANGENTIAL COMP.

RELATIVE NORMAL COMP.



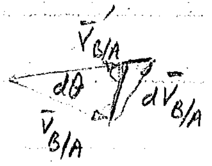




GENERAL @t

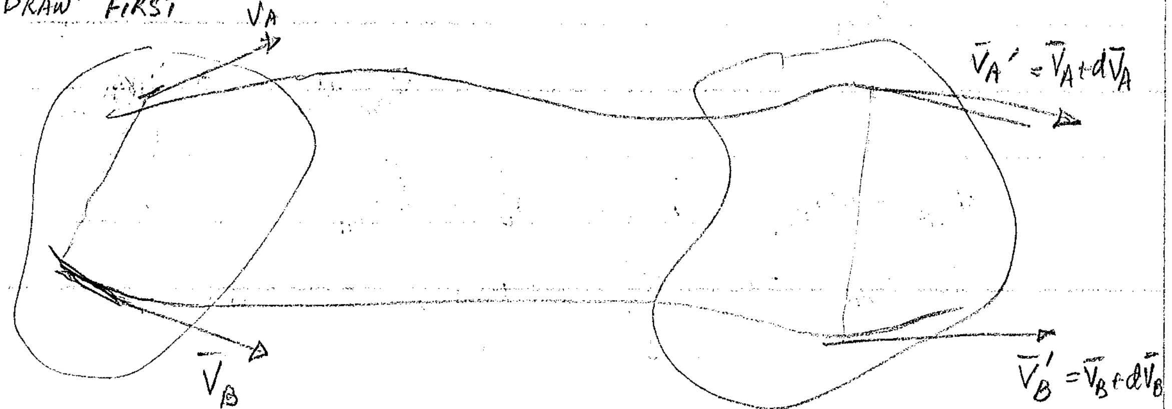
ADDED TRANS, @dt+t

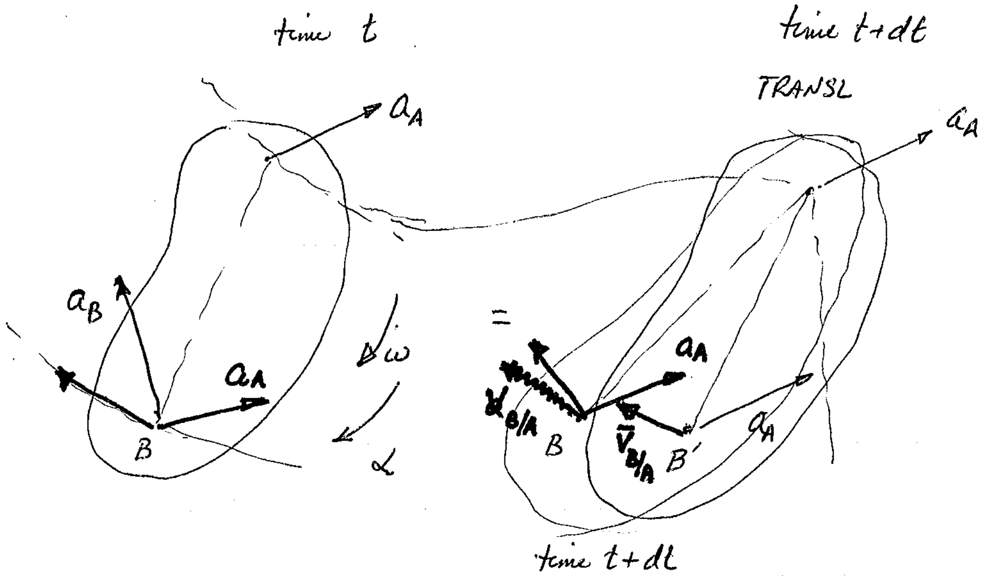
$\bar{V}_i' = \bar{V}_i + d\bar{V}_i$

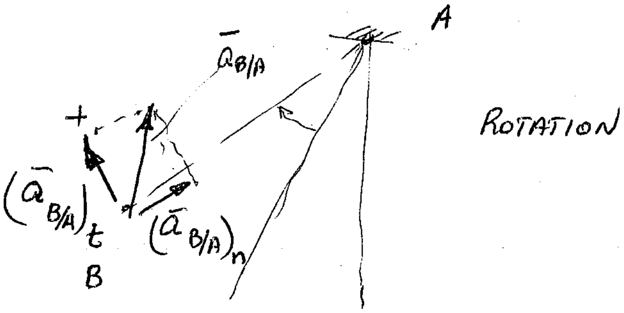


made up of rot (change in dir of $r_{B/A}$
+ change in mag. due to change in ω

DRAW FIRST







$\bar{\alpha}$ & $\bar{\omega}$ are absolute angular & accel + velocity.

- BODY TRANSLATES with an instantaneous accel $\bar{a}_A$ & undergoes instantaneous rotation.

- A & B MOVE ALONG CURVED PATHS SINCE BOTH PTS HAVE tangential & normal components of velocity
- IF PATHS ARE RECTILINEAR - NORMAL COMPONENT = 0
  - WHEN USING FOR PROBLEMS WITH PIN CONNECTIONS & PTS IN CONTACT
    - PIN CONNECTION FOR 2 BODIES ACCELS ARE SAME SINCE PATH MUST BE SAME

SEE PAGE 286 -

- CONTACT PTS w/o SLIPPING TANGENTIAL ACCEL are equal BUT MOVE ALONG DIFFERING PATHS

GEARS

ANALYSIS $\bar{a}_B = \bar{a}_A + (\bar{a}_{B/A})_t + (\bar{a}_{B/A})_n$ VECTOR EQS.

- MAY NEED TO FIND $\bar{V}_A$ & $\bar{V}_B$ IF $\bar{\omega}$ IS UNKNOWN

- REMEMBER A & B MOVE ALONG CURVED PATHS ∴

$$\bar{a}_{\substack{A\\B}} = (\bar{a}_{\substack{A\\B}})_t + (\bar{a}_{\substack{A\\B}})_n$$

- PICK BASE PT HAVING KNOWN ACCEL.

- DRAW KINEMATIC DIAG FOR $\bar{a}_{B/A}$ & SHOW $(\bar{a}_{B/A})_t$ $(\bar{a}_{B/A})_n$

remember $|(\bar{a}_{B/A})_t| = \ddot{\theta} r_{B/A} = \dot{\omega} r_{B/A} = \alpha r_{B/A}$

$$|(\bar{a}_{B/A})_n| = \omega^2 r_{B/A} = \frac{v_{B/A}^2 \, r_{B/A}^2}{r_{B/A}}$$

directions $(\bar{a}_{B/A})_t$ is tangent to path, in dir of $\bar{v}_{B/A}$

$(\bar{a}_{B/A})_n$ is ⊥ to path pointed toward A.

LOOK AT EXAMPLE 16-14 p. 289

16-15 p. 290

↓ MON/WED START HERE LESSON #20

- since relative acceleration components represent the effect of circular motion observed from translating axes

$$\bar{V}_B = \bar{V}_A + \bar{\omega} \times \bar{r}_{B/A}$$

$\frac{d}{dt}$:

$$\bar{a}_B = \bar{a}_A + \dot{\bar{\omega}} \times \bar{r}_{B/A} + \bar{\omega} \times \frac{d}{dt}(\bar{r}_{B/A})$$

$$\bar{\alpha} \times \bar{r}_{B/A} + \bar{\omega} \times \bar{v}_{B/A}$$

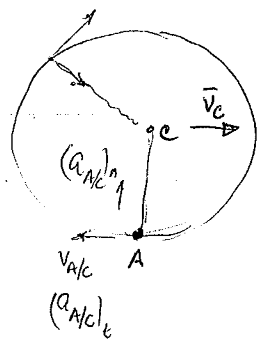
$$+ \bar{\omega} \times (\bar{\omega} \times \bar{r}_{B/A})$$

$$- \omega^2 \bar{r}_{B/A}$$

$$\bar{\alpha} \times \bar{r}_{B/A} = (\bar{a}_{B/A})_t \qquad -\omega^2 \bar{r}_{B/A} = (\bar{a}_{B/A})_n$$

OLD versions

Problem 16-77



Given $\alpha = 4\ rad/s^2$

$\omega = 2\ rad/s.$

no slip $\Rightarrow \bar V_A = \bar 0$ & ~~$(\bar a_A)_t = \bar 0$~~ $\bar a_A = \bar 0$

$\bar 0\ \cancel{\bar V_A} = \cancel{\bar V_C} + \bar V_{A/B}$

$= \bar V_c + \omega r_{A/c} \bar i \Rightarrow \overrightarrow{\bar V_c = 2(1.5) = 3 ft/s}$

$\overset{0}{\cancel{\bar a_A}} = \bar a_c + (\bar a_{A/c})_t + (\overline{a_{A/c}})_n$

FOR NO SLIPPING $(\bar a_{A/c})_n = \bar 0$

$(\bar a_{A/c})_t = -\alpha r_{A/c} \bar i = -4(1.45)\hat i$

$\therefore\ \bar a_c = +5.8\ \overset{ft}{\cancel{rad}}/s^2\ \bar i$

$\bar a_B = \bar a_c + (\bar a_{B/c})_t + (\bar a_{B/c})_n \qquad (a_{B/c})_t = \alpha r_{B/c} = 4(1.45)$

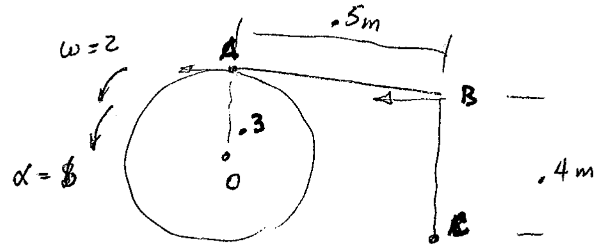
$= 5.8\bar i + (5.8 \cos 30° \bar i - 5.8 \sin 30° \bar j) + \qquad (a_{B/c})_n = \omega^2 r_{B/c} = 4(1.45$

$(5.8 \cos 60° \bar i + 5.8 \sin 60° \bar j)$

$\bar a_B = 13.72\ ft/s^2\ \bar i + 2.12\ ft/s^2\ \bar j$

$a_B = \sqrt{(13.72)^2 + (2.12)^2} = 13.9\ ft/s \qquad \angle\theta = \tan^{-1}\frac{2.12}{13.72} = 8.8°$

Problem 16-80 **Problem 6-111 in 10th ed.**



at A pin joint $\therefore (\bar V_A)_F = (\bar V_A)_{AB}$

$(\bar a_A)_F = (\bar a_A)_{AB}$

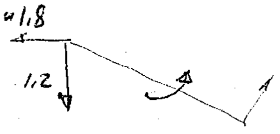
at B pin joint $\therefore (\bar V_B)_{AB} = (\bar V_B)_{BC}$

$(\bar a_B)_{AB} = (\bar a_B)_{BC}$

$\bar V_A = \cancel{\bar V_O} + \bar V_{A/O} = \omega_F r_{A/O} \leftarrow \qquad (2)(.3) = .6\ m/s$

$\bar V_B = \cancel{\bar V_C} + \bar V_{B/C} = \omega_{BC} r_{B/C} \leftarrow$

$\bar V_A = \bar V_B + \bar V_{A/B} \longrightarrow \bar V_{A/B} = \bar\omega_{AB} \times \bar r_{A/B}$



$\omega \bar k \times (\bar i + \bar j)$

$= (\quad)\bar j + (\quad)$

$\omega r_x \bar j - \omega r_y \bar i$

$\omega r_x = 0 \Rightarrow \omega_{AB} = 0$

& $\bar V_A = \bar V_B = .6\ m/s$

$V_B = \omega_{BC} r_{B/C} \Rightarrow$

$\underline{\omega_{BC} = 1.5\ rad/s}$ ↻

$\bar a_A = (\bar a_{A/O})_t + (\bar a_{A/O})_n + \cancel{\bar a_O}$

$= -\alpha r_{A/O} \bar i - \omega^2 r_{A/O} \bar j = -1.8\ m/s^2\ i - 1.2\ m/s^2\ \bar j$

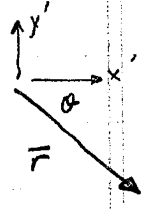
$\bar a_B = \cancel{\bar a_C} + (\bar a_{B/C})_t + (\bar a_{B/C})_n$

$(a_{B/C})_n = \omega^2 r_{B/C} = (2.25)(.4) = .9\ m/s^2 \downarrow$

$(a_{B/C})_t = \alpha_{BC} r_{B/C} \leftarrow$

Remind Students $\bar{a} = \bar{a}_t + \bar{a}_n$ in general at A & B

Look @ Example ~~16-15~~ 16-17 in 10th ed.



$\bar{r} = r\cos\theta\,\bar{i} - r\sin\theta\,\bar{j}$

$\circlearrowleft$

$\bar{V}_B = V_A + \bar{\omega}_{BA} \times \bar{r}_{B/A}$ $\quad \bar{r}_{B/A} = -.2\bar{j}$ $\quad \bar{\omega} = \omega_{BA}\bar{k}$ $\quad \bar{\omega}\times\bar{r} = .2\omega_{BA}\bar{i}$

$\bar{V}_A = \bar{0}$

$\bar{V}_B = .2\omega_{BA}\bar{i}$

$\bar{V}_c = -2\bar{j}$

$\bar{V}_B = \bar{V}_c + \bar{\omega}_{BC} \times \bar{r}_{B/c}$ $\quad \bar{r}_{B/c}$ $\quad \omega_{BC}\bar{k} \times [.2\bar{i} - .2\bar{j})$

$.2\omega_{BA}\bar{i} = -2\bar{j} + [.2\omega_{BC}\bar{j} + .2\omega_{BC}\bar{i}]$

$\omega_{BC} = 10 \text{ rad/s} \quad \omega_{BA} = 10 \text{ rad/s}$

$\bar{a}_B = \cancel{\bar{a}_A}^{0} + \bar{\alpha}_{BA} \times \bar{r}_{B/A} - \omega_{BA}^2 \bar{r}_{B/A}$ $\quad \bar{\alpha}_{BA} = \alpha_{BA}\bar{k}$ $\quad \bar{\alpha}\times\bar{r}_{B/A} = .2\alpha_{BA}\bar{i}$

$-\omega_{BA}^2\,\bar{r}_{B/A} = -.2\omega_{BA}^2\bar{j}$

$\bar{a}_B = .2\alpha_{BA}\bar{i} + .2\omega_{BA}^2\bar{j}$

$\bar{a}_c = -1\bar{j}$

$\bar{a}_B = \bar{a}_C + \alpha_{BC} \times \bar{r}_{B/C} - \omega_{B/C}^2\,\bar{r}_{B/c}$ $\quad \bar{\alpha}_{BC} = \alpha_{BC}\bar{k}$

$.2\alpha_{BA}\bar{i} + .2\omega_{BA}^2\bar{j} = -1\bar{j} + .2\alpha_{BC}\bar{i} + .2\alpha_{BC}\bar{j} - \omega_{BC}^2[.2\bar{i} - .2\bar{j}]$

$.2\alpha_{BA}\bar{i} + .2(100)\bar{j} = -1\bar{j} + .2\alpha_{BC}\bar{i} + .2\alpha_{BC}\bar{j} - 100(.2\bar{i} - .2\bar{j})$

$\bar{j}: \quad +20 = .2\alpha_{BC} + 20 - 1 \implies \alpha_{BC} = 5 \text{ rad/s}^2$

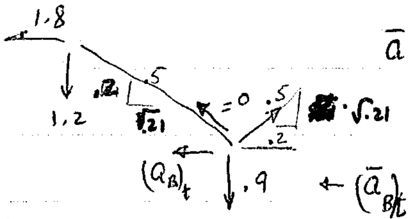
$\bar{i}: \quad .2(\alpha_{BA}) = .2\alpha_{BC} - 20 \implies \alpha_{BA} = -19/.2 = -95 \text{ rad/s}^2 \circlearrowright$

$\bar{a}_B = \bar{a}_A + \bar{\alpha}_{BA} \times \bar{r}_{B/A} - \omega_{BA}^2\,\bar{r}_{B/A}$ $\quad \bar{\alpha}_{BA} = \alpha_{BA}\bar{k}$ $\quad \bar{r}_{B/A} = -.2\bar{j}$

$= +.2\alpha_{BA}\bar{i} - 100[-.2\bar{j}]$

$\bar{a}_B = (.2\alpha_{BA}\bar{i} + 20\bar{j})\ \text{m/s}^2$

$a_c = -1\bar{j} \ \text{m/s}^2$

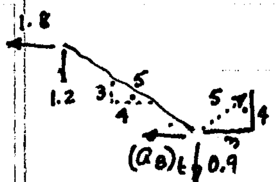


$$\overset{\alpha_{AB} r_{B/A}}{} \qquad \cancel{-\omega^2_{AB} r_{B/A}} \to 0$$

$$\bar{a}_B = \bar{a}_A + (\bar{a}_{B/A})_t + (\bar{a}_{B/A})_n$$

$$\alpha_{AB} r_{B/A} + \cancel{(\omega^2_{AB} r_{B/A})}$$

$$\leftarrow (\bar{a}_B)_t + .9\downarrow = \underset{\leftarrow}{1.8} + \underset{\downarrow}{1.2} + \frac{.2}{.5}\alpha_{AB} r_{B/A} \rightarrow + \frac{\sqrt{.21}}{.5}\alpha_{AB} r_{B/A}$$



all x

$$-a_{B/t} = -1.8 + .4\alpha_{AB}(.4)$$

y

$$-.9 = -1.2 + \frac{\sqrt{.21}}{.5}\alpha_{AB}(.4) \Rightarrow \alpha_{AB} = .8183 \text{ rad/s}$$

gives $(\bar{a}_B)_t$

$$\leftarrow (a_B)_t + 0.9\downarrow = \overleftarrow{1.8} + 1.2\downarrow + \alpha_{AB} r_{B/A}\cdot\frac{3}{5}\rightarrow + \alpha_{AB} r_{B/A}\cdot 4/5\uparrow$$

$$-a_{B_t} = -1.8 + \alpha_{AB}\cdot(0.5)\cdot\frac{3}{5}$$

$$-0.9 = -1.2 + \alpha_{AB}(0.5)\cdot\frac{4}{5}$$

$$\alpha_{AB} = \frac{.3}{.4} = 0.75$$

$$(a_B)_t = 1.575 \text{ m/s}^2$$

1.669 m/s²

Now $(\bar{a}_B)_t = (\bar{a}_{B/A})_t = \alpha_{BA} r_{B/A} \Rightarrow \alpha_{BA} = (\bar{a}_B)_t/.4 = .4173$ rad/s²

Now $\underline{a}_B = \cancel{a_C} + \underline{a}_{B/C} = \underline{a}_{B/C}$

$\underline{a}_{B_t} = \underline{a}_{B/C_t} = \alpha_{BC} r_{B/C}$ or $\alpha_{BC} = a_{Bt}/r_{B/C} = \frac{1.575}{.4} = 3.938$

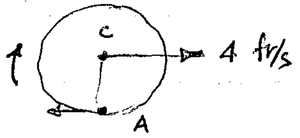
LESSON #20 REVIEW

PROBLEM 16-42 Pg 273 TAKES 3 min TOPS

NOTE PT C HAS ONLY TRANSLATIONAL MOTION

$\therefore V_C = V_{BICYCLE} = 4$ ft/s $\longrightarrow$

DON'T DO

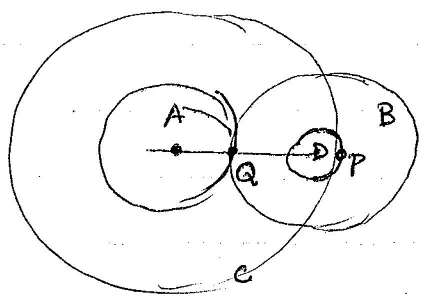


$\bar{V}_A = \bar{V}_C + \bar{V}_{A/C}$ $\qquad \bar{V}_{A/C} = \omega r_{A/C} = 3(2.167 \text{ ft}) = 6.5$ ft/s

$= 4\hat{i} - 6.5\hat{i} = -2.5$ ft/s

PROBLEM 16-54

find ω_B & ω_A



$V_A = V_B$ & $V_D = V_C$ & $\omega_D = \omega_B$

Given $\omega_{DE} = 18$ rad/s

(1) use $\bar{V}_D = \cancel{\bar{V}_E}^{0} + \bar{V}_{D/E}$ to find $\bar{V}_D$ (known)

(2) $\bar{V}_D$ of DE $\doteq$ $\bar{V}_D$ of hub C $= \bar{V}_D$ of hub B

(3) use $\bar{V}_P = \bar{V}_D + \bar{V}_{P/D}$ $\qquad \bar{V}_P$ will depend on ω_D

(4) but $\bar{V}_P = \bar{0}$ since R is fixed $\Rightarrow \omega_D$

(5) $\bar{V}_D$ of B $= \bar{V}_Q + \bar{V}_{D/Q}$ $\Rightarrow \bar{V}_Q$ will be known

(6) $\bar{V}_Q = \cancel{\bar{V}_E \text{ of hub}}^{0}$ A $+ \bar{V}_{Q/E}$ $\Rightarrow \omega_A$

(1) $\bar{V}_D = \bar{V}_{D/E} = \omega_{DE} r_{D/E}\uparrow = 18(.5\text{m}) = 9$ m/s $\uparrow$

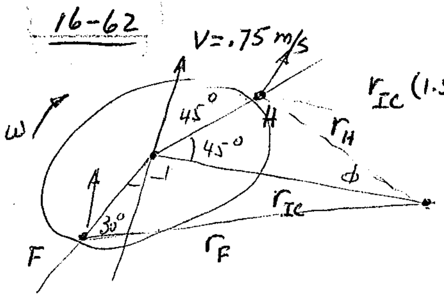
(3) $\bar{V}_P = 0 = 9 \text{ m/s}\uparrow + \bar{V}_{P/D}$ $\qquad \bar{V}_{P/D} = \omega_D r_{P/D}\downarrow = \omega_D(.1)\downarrow$ $\qquad \underline{\omega_D = 90 \text{ rad/}}$

(5) $\bar{V}_D = \bar{V}_Q + \bar{V}_{D/Q}$

$9 \text{ m/s}\uparrow = \bar{V}_Q + \omega_B r_{D/Q}\downarrow = \bar{V}_Q + 90(.3)\downarrow \Rightarrow \bar{V}_Q = 36$ m/s $\uparrow$

(6) $\bar{V}_Q \uparrow = \bar{V}_{Q/E} = \omega_A r_{Q/E} \uparrow = \omega_A (.2) = 36\ m/s \uparrow$ $\qquad \omega_A = 180\ rad/s$ ↻

16-62



$r_{IC}(1.5) = r_{IC}\,\omega = V = .75\ m/s \qquad r_{IC} = .5 m$

using law of cosines

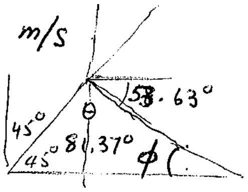
$$r_H = \sqrt{r_{IC}^2 + r_{H/G}^2 - 2 r_{IC} r_{H/G} \cos 45^\circ}$$
$$= .3576\, m.$$

$$\therefore \quad V_H = \omega r_H = 1.5\,(.3576) = .536\ m/s$$

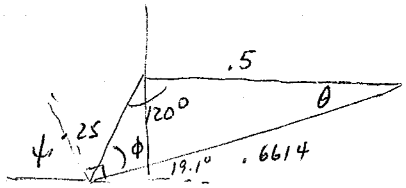
using law of cosines

$$r_F = \sqrt{r_{IC}^2 + r_{F/G}^2 - 2 r_{IC} r_{F/G} \cos 120^\circ} = .6614\ m$$
$$V_F = \omega r_F = 1.5\,(.6614) = .992\ m/s$$

$$\frac{\sin\phi}{.3} = \frac{\sin\theta}{r_{IC}} = \frac{\sin 45}{r_H} \Rightarrow \sin\theta = \frac{r_{IC}}{r_H}\sin 45^\circ$$
$$\Rightarrow \theta = 81.37^\circ$$



$\phi = 180^\circ - \theta - 45^\circ = 53.63^\circ$ $\qquad V_H$ is at an angle of $90 - 53.63^\circ = 36.37^\circ$



$$\frac{\sin\phi}{.5} = \frac{\sin\theta}{.25} = \frac{\sin 120^\circ}{.664} \qquad \theta = 19.1^\circ \ \measuredangle$$

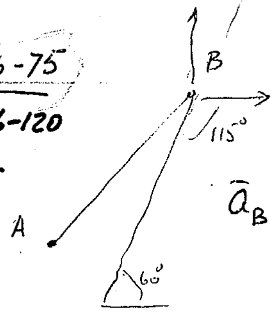
$\measuredangle\ \psi = 180 - 90 - \theta = 70.89^\circ$

USING VECTOR CROSSPRODUCT

DO

16-75

Same as 16-120 in 10th ed.



$\bar{r}_{AB} = (2\cos 45^\circ \bar{i} + 2\sin 45^\circ \bar{j})$ ft

$\bar{\omega}_{AB} = 5\bar{k}\ rad/s \qquad \bar{\alpha} = 3\bar{k}\ rad/s^2$

$$\bar{a}_B = \bar{a}_A + \bar{\alpha} \times \bar{r}_{A/B} - \omega^2 \bar{r}_{AB}$$
$$= \bar{0} + 3\bar{k} \times (2\cos 45^\circ \bar{i} + 2\sin 45^\circ \bar{j}) - 25\,(2\cos 45^\circ \bar{i} + 2\sin 45^\circ \bar{j}$$
$$= +6\cos 45^\circ \bar{j} - 6\sin 45^\circ \bar{i} - 50\cos 45^\circ \bar{i} - 50\sin 45^\circ \bar{j}$$
$$\bar{a}_B = -56\cos 45^\circ \bar{i} - 44\cos 45^\circ \bar{j} = -39.6\,\bar{i} - 31.1\,\bar{j}\ ft/s^2$$

$$\bar{a}_C = \bar{a}_B + \bar{\alpha}_{BC} \times \bar{r}_{B/B} - \omega_{BC}^2 \bar{r}_{C/B}$$

FROM PROB 16-38 we did in class we found $\omega_{BC} = 5.66\ rad/s$ ↻

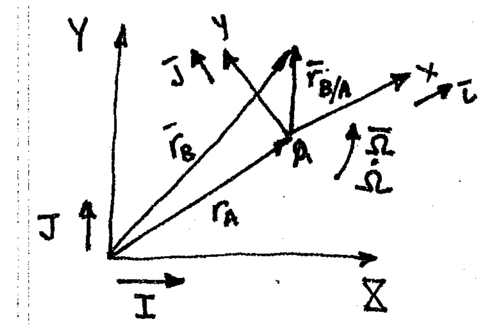
$\bar{\omega}_{BC} = \omega_{BC}\bar{k} \qquad$ let $\bar{\alpha}_{BC} = \alpha_{BC}\bar{k}$

$$\bar{r}_{C/B} = (2.5)\cos 120^\circ \bar{i} - \overset{2.5}{2}\sin 120^\circ \bar{j}\ \text{ft}$$

$$\bar{a}_C = -39.6\,\bar{i} - 31.1\,\bar{j} + \alpha_{BC}\bar{k} \times (2.5\cos 120^\circ \bar{i} - 2.5\sin 120^\circ \bar{j}) - (5.66)^2 [2.5\cos 120^\circ \bar{i} - \overset{2.5}{2}\sin 12$$

USED WHEN SLIDING OCCURS AT CONNECTION

ANALYSIS: USED FRAME THAT ROTATES & TRANSLATES



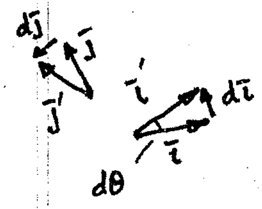
$$\bar{r}_B = \bar{r}_A + \bar{r}_{B/A}$$

$\bar{r}_{B/A}$ relative pos. vector whether measured from fixed or rotating frame

$$\bar{r}_{B/A} = x_{B/A}\bar{i} + y_{B/A}\bar{J}$$

$$\frac{d}{dt}: \quad \bar{V}_B = \bar{V}_A + \frac{d}{dt}(\bar{r}_{B/A})$$

$$\dot{x}_{B/A}\bar{i} + \dot{y}_{B/A}\bar{J} + x_{B/A}\frac{d\bar{i}}{dt} + y_{B/A}\frac{d\bar{J}}{dt}$$



$|d\bar{i}| = |\bar{i}|d\theta$ direction is $\bar{J}$ $\therefore$ $d\bar{i} = d\theta\bar{J}$ ∗

$d\bar{j} = -d\theta\,\bar{i}$

$$\bar{V}_B = \bar{V}_A + \underbrace{\dot{x}_{B/A}\bar{i} + \dot{y}_{B/A}\bar{J}}_{(\bar{V}_{B/A})_{rel}} + x_{B/A}\dot{\theta}\bar{J} - y_{B/A}\dot{\theta}\bar{i}$$

$$\underbrace{x_{B/A}\bar{\Omega}\times\bar{i} + y_{B/A}\bar{\Omega}\times\bar{J}}_{\bar{\Omega}\times\bar{r}_{B/A}}$$

$$\bar{V}_B = \bar{V}_A + \bar{\Omega}\times\bar{r}_{B/A} + (\bar{V}_{B/A})_{rel}$$

absolute veloc of origin of x,y,z frame

angular veloc effect of frame rotation

motion of x,y,z frame relative to X,Y,Z frame

relative velocity of B/A measured wrt x,y,z frame

first two terms equivalent to translating frame last term is rotating frame

$$\frac{d}{dt}: \quad \bar{a}_B = \bar{a}_A + \dot{\bar{\Omega}}\times\bar{r}_{B/A} + \bar{\Omega}\times\frac{d\bar{r}_{B/A}}{dt} + \frac{d}{dt}(\bar{V}_{B/A})_{rel}.$$

$$\bar{\Omega}\times\left[(\bar{V}_{B/A})_{rel} + \bar{\Omega}\times\bar{r}_{B/A}\right] +$$

$$\frac{d}{dt}(V_{B/A})_{rel} = \frac{d}{dt}(\dot{x}_{B/A}\bar{i} + \dot{y}_{B/A}\bar{J}) = \underbrace{\ddot{x}_{B/A}\bar{i} + \ddot{y}_{B/A}\bar{J}}_{(\bar{a}_{B/A})_{rel}} + \underbrace{\dot{x}_{B/A}(\bar{\Omega}\times\bar{i}) + \dot{y}_{B/A}(\bar{\Omega}\times\bar{J})}_{\bar{\Omega}\times(\bar{V}_{B/A})_{rel}}$$

$$* \ \bar{a}_B = \underbrace{\bar{a}_A + \dot{\bar{\Omega}}\times\bar{r}_{B/A} + \bar{\Omega}\times(\bar{\Omega}\times\bar{r}_{B/A})}_{\text{motion of x,y,z frame wrt X,Y,Z frame}} + \underbrace{2\bar{\Omega}\times(\bar{V}_{B/A})_{rel}}_{\text{interaction between rotation \& trans.}} + \underbrace{(\bar{a}_{B/A})_{rel}}_{\text{relative motion of B wrt A measured wrt x,y,z frame}}$$

TRANS. ROT + TRANS. ROTATION

$\bar{a}_A$ - accel of rotating frame's origin measured from $\bar{X}, Y, Z$ ref

~~$\dot{\bar{\Omega}}, \bar{\Omega}$ are~~

$\dot{\bar{\Omega}} \times (\bar{r}_{B/A})$ angular accel effect of frame's rotation

$\bar{\Omega} \times (\bar{\Omega} \times \bar{r}_{B/A})$ angular velocity effect due to frame's rotation

$2\bar{\Omega} \times (\bar{v}_{B/A})_{rel}$ combined effect of B moving rel to rotating frame & rotation of frame

$(\bar{a}_{B/A})_{rel}$ relative accel of B wrt A measured wrt x, y, z frame

To solve: set the inertial frame

set the rotating frame — define $\bar{\Omega}$, $\dot{\bar{\Omega}}$, $\bar{v}_A$, $\bar{a}_A$

determine motions of particle wrt rot. frame $(\bar{a}_{B/A})_{rel}$, $(\bar{v}_{B/A})_{rel}$, $\bar{r}_{B/A}$

relate I & J to $\bar{i}$ & $\bar{j}$ note: K & k are normally same

<u>pick</u> one frame to do the calculations & stick to it

16-133
in 10th ed

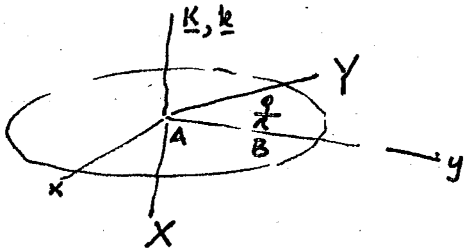
$\underline{\Omega} = 0.5\underline{k}$

$\underline{\alpha} = 0.2\underline{k}$

$\underline{v}_A = 0$ $\underline{r}_{B/A} = 5\underline{j}$

$(\underline{v}_{B/A})_{rel} = 2\underline{j}$

$(\underline{a}_{B/A})_{rel} = 3\underline{j}$



$$\underline{v}_B = \cancel{\underline{v}_A} + \underline{\Omega} \times \underline{r}_{B/A} + (\underline{v}_{B/A})_{rel}$$
$$= 0 + 0.5\underline{k} \times 5\underline{j} + 2\underline{j} = -2.5\underline{i} + 2\underline{j}$$

$$\underline{a}_B = \cancel{\underline{a}_A} + \underline{\alpha} \times \underline{r}_{B/A} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{B/A}) + 2\underline{\Omega} \times (\underline{v}_{B/A})_{rel} + (\underline{a}_{B/A})_{rel}$$
$$= (0.2\underline{k} \times 5\underline{j}) + (0.5\underline{k}) \times (-2.5\underline{i}) + 2(0.5\underline{k}) \times 2\underline{j} + 3\underline{j}$$
$$= -\underline{i} + 1.25\underline{j} + 2\underline{i} + 3\underline{j} = -3\underline{i} + 1.75\underline{j}$$

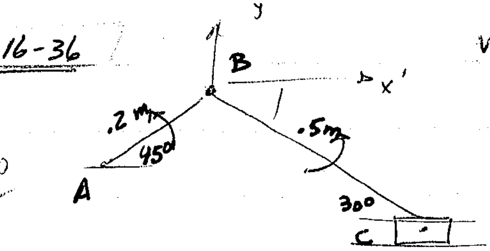
$$a_c\bar{i} = \bar{a}_c = -39.6\bar{i} - 31.1\bar{j} - 1.25\,\alpha_{BC}\bar{j} + 2.1651\alpha_{BC}\bar{i} + 40.0445\,\bar{i} + 69.3591\bar{j}$$

$$a_c = -39.6 + 2.1651\,\alpha_{BC} + 40.0445 \qquad a_B = 66.7\ \text{ft}/s^2$$

$$0 = -31.1 - 1.25\,\alpha_{BC} + 69.3591 \qquad \alpha_{BC} = 30.\cancel{6}\ \text{rad}/s^2$$

USING VECTOR CROSSPROD.

16-36



DO

$$\bar{V}_B = \overset{0}{\cancel{\bar{V}_A}} + \bar{\omega}_{BA} \times \bar{r}_{B/A} = (6\bar{k}) \times (.2\cos 45^\circ\bar{i} + .2\sin 45^\circ\bar{j})$$

$$\bar{V}_B = (.8485\,\bar{J} - .8485\,\bar{k})\ \text{m/s}$$

$$\bar{V}_c = V_c\bar{i} = \bar{V}_B + \bar{V}_{c/B} = \bar{V}_B + \bar{\omega}_{BC} \times \bar{r}_{c/B}$$

$$\bar{\omega}_{BC} = \omega_{BC}\bar{k} \qquad \bar{r}_{c/B} = .5\cos 30^\circ\bar{i} - .5\sin 30^\circ\bar{j}$$

$$\bar{V}_{c/B} = \omega_{BC}(.433\,\bar{j}) + \omega_{BC}(.25\,\hat{i})$$

$$\therefore \quad V_c\,\hat{i} = .8485\bar{j} - .8485\,\hat{i} + \omega_{BC}(.433\bar{j}) + \omega_{BC}(.25)\hat{i}$$

$$\curvearrowleft \omega_{BC} \quad \Rightarrow \quad 0 = .8485 + \omega_{BC}(.433) \Rightarrow \omega_{BC} = -1.96\ \text{rad/s}$$

$$\xleftarrow{\ } V_c \qquad V_c = -.8485 - (1.9596)(.25) = -1.3384\ \text{m/s}$$

—— • —— • —— • ——

LESSON # 21

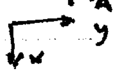
CORIOUS EFFECT

SKETCH PROB. 16-88 P. 306

$$V_A = \omega \times r_A$$

$$V_m = V_A + V_{m/A}$$

$\uparrow V_A \rightarrow V_{m/A}$



$$a_A = \bar{\alpha} \times \bar{r} - \omega^2\bar{r}$$

$$a_m \neq a_A + a_{m/A}$$

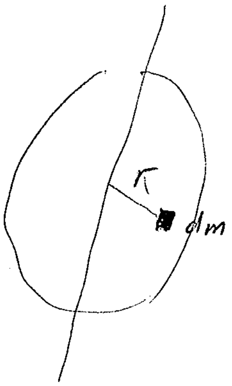
- SO FAR WE HAVE LOOKED AT KINEMATICS OF PLANAR RIGID BODY MOTION
- AS WITH CHAPTER 13 WE WILL DERIVE THE EQS OF MOTION FOR RIGID BODY
- TO DO THIS WE MUST REVIEW THE QUANTITY OF MOMENT OF INERTI
- IN THE PAST CHAPTERS – MOST FORCES WERE CONCURRENT
  - IN GENERAL BODIES ARE NOT POINTS & FORCES ARE NOT CONCURRENT
- WE HAVE FOUND THAT TRANSLATION IS GOVERNED BY

$$\bar{F} = m\bar{a}$$

- ROTATION IS GOVERNED BY $\bar{M} = I\bar{\alpha}$ (TO BE SHOWN

I - is mass moment of inertia

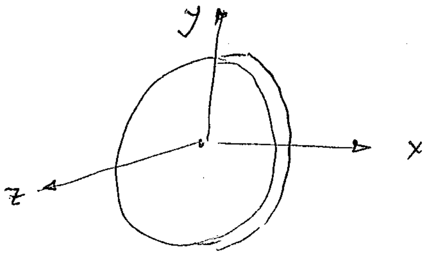
m - is a resistance to acceleration I is a measure of resistance to angular accel.



$$I = \int_m r^2 dm \qquad I = \int_A r^2 dA$$

$$dm = \rho dV$$

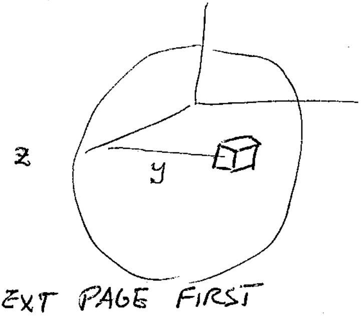
- IN PLANAR KINETICS AXIS PASSES THROUGH MASS CENTER
- $\perp$ TO PLANE OF MOTION



I_G = moment of inertia about mass center.

TO FIND VOLUME ELEMENT

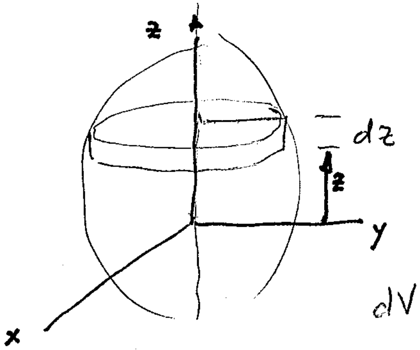
$$I = \int r^2 \rho dV \qquad \text{if } \rho = \text{const} \qquad \rho \int r^2 dV = I$$



$$dV = dx\,dy\,dz$$

$$\rho \iiint r^2 dV = I_z$$

GO TO NEXT PAGE FIRST

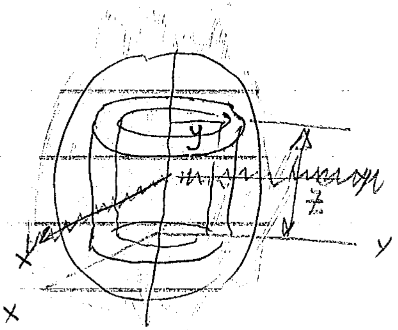


$r = y \qquad dV = \pi y^2 dz$

$$dI_z = \frac{1}{2} dm\, y^2$$

$$= \frac{1}{2} \rho \left[\pi y^2 dz \right] y^2$$

$$dV = \int_0^z \int_0^{2\pi} \int_0^{r(z)} r\,dr\,d\theta\,dz$$



VOLUME OF CYL. SHELL.

$$dV = (2\pi y)\, dy \cdot z$$

RIGHT CIRCULAR CYLINDER

$$\rho\, dV = dm$$

$$r = y$$

SINCE ALL POINTS AT SAME DISTANCE FROM Z AXIS

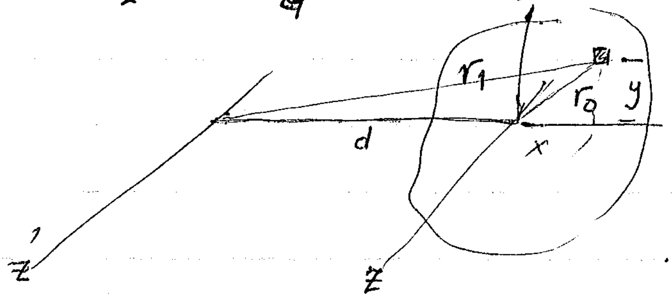
$$\int \rho (2\pi y\, dy) z \cdot y^2$$

EXAMPLE 17-1 P. 312 THEN GO BACK TO SLAB METHOD

- PARALLEL AXIS THEOREM - IF Z PASSES THROUGH MASS CENTER

$$I_{z'} = I_G + (z'-z)^2 m$$

$z'-z$ is ⊥ distance



$$r_1^2 = (d+x)^2 + y^2 = d^2 + 2dx + x^2 + y^2 = d^2 + 2xd + r_0^2$$

$$\int d^2 (dm) = md^2$$

$$\int r_0^2 (dm) = I_z = I_G$$

$$\int r_1^2 (dm) = I_{z'}$$

$$\int d\, x\, dm = d \underbrace{\int x\, dm}_{d\bar{x}m} = 0$$ since axis passes through centroid of mass

$\bar{x}$ distance ~~from G to center of mass~~

$$\therefore \quad I_z' = I_G + md^2$$

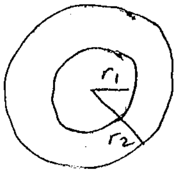
- RADIUS OF GYRATION

NOTICE $[I] = [\text{slugs}][\text{length}]^2$ + $I = \int r^2 dm \doteq k^2 m$

$$k = \sqrt{\frac{I}{m}}$$ RADIUS OF GYRATION

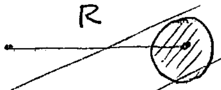
- COMPOSITE BODIES
  - SIMPLE SHAPES FIND I_G for each shape
  - USE $I_i = (I_G)_i + m_i d_i^2$ TO FIND I ABOUT THE COMMON AXIS
  - TAKE $\sum I_i = I_{TOT}$

- FOR SYSTEMS WITH "HOLES" TREAT AS $-(I_G + md^2)$



find I for full plate − I for smaller plate
EXAMPLE 17-3.

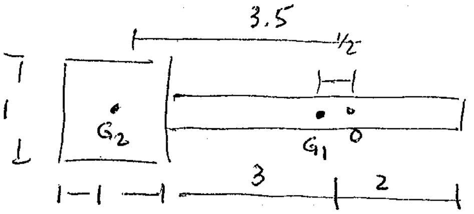
PROBLEM 17-82



~~$dI = R^2 dm$~~
~~$dm = \rho dV$~~
~~$= \rho dA \cdot l = \rho\pi a^2 \cdot 2\pi$~~

~~$dI = \rho\pi a^2 \cdot 2\pi R \cdot$~~

Problem 17-19 in 10th ed.



back cover

$$I_{G_2} = \frac{1}{12} m_P (a^2 + b^2) = \frac{1}{12} m_P [2] = \frac{1}{6} m_P$$

$$I_{G_1} = \frac{1}{12} m_R l^2 = \frac{1}{12} m_R (5)^2 = 2.0833 m_R$$

$$I_0 = I_{G_2} + m_P [3.5]^2 + I_{G_1} + m_R [.5]^2$$

$$m_P = \frac{W_P}{g} \qquad m_R = \frac{W_R}{g}$$

$$I_0 = \frac{1}{6}\frac{W_P}{g} + \frac{W_P}{g}[3.5]^2 + \overset{2.0833 \;"\; \frac{l^2}{12}}{2.0833}\frac{W_R}{g} + \frac{W_R}{g}[.5]^2$$

$$\frac{12}{6(32.2)} + \frac{12}{32.2}[12.25] + [\frac{4}{32.2}] + \frac{4}{32.2}[.25] = \overset{4.918}{\cancel{5.663}} \text{ slug-ft}^2$$

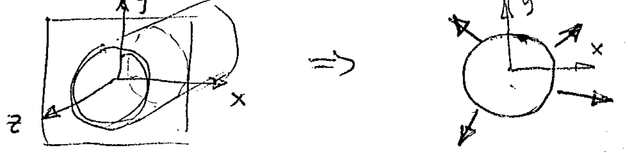
$$I_0 = 4.918 = mk^2 \qquad m = \frac{\Sigma W}{g} = .497 \text{ slugs}$$

$$k = 3.146 \text{ ft.}$$

- Planar Kinetics
- remember for a system of particles we found $\Sigma\bar{F} = m\bar{a}_G$
  when $\Sigma\bar{F}$ ARE EXTERNAL FORCES TRANSLATIONAL

- LOOK AT SYSTEM OF PARTICLES (RIGID BODY) & LOADINGS

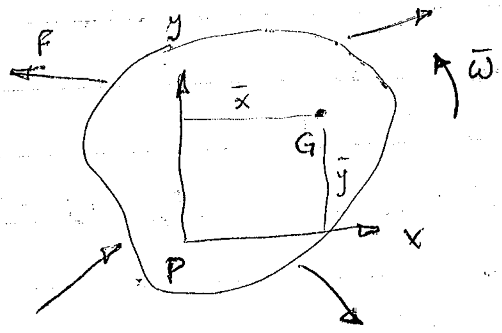
And Support $\Sigma F_z = 0$



~~DISTRIB LOADING IN Z DIR~~

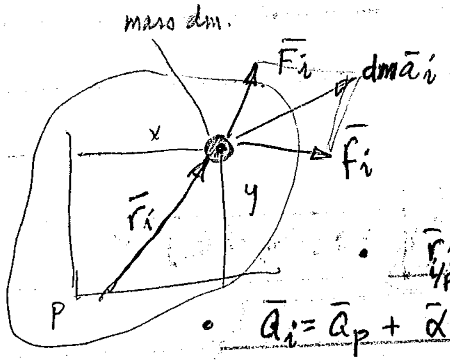
PLANAR –
$\bar{F}_i$ are in plane
motion in plane

- LOOK AT SLAB WITH TOTAL MASS & EXTERIOR SHAPE OF BODY ONLY
- FORCES CONSIDERED IN PLANE OF SLAB — FORCE ~~COMPONENTS~~ IN PLANE
- PICK A PT P ON BODY : IFR AT P or TRANSLATING AXES (FRAME) W/ CONST VELOCITY



- LOOK AT ~~ROTATIONAL~~ MOMENTS DUE TO $\bar{F}_s'$

--- --- --- --- LESSON # 22



- LOOK AT TYPICAL PT
- HAS FORCES $\bar{F}_i$ & $\bar{f}_i$

MON/WED

- $\bar{r}_{i/p} \times \bar{F}_i + \bar{r}_{i/p} \times \bar{f}_i = \bar{r}_{i/p} \times dm\,\bar{a}_i = d\bar{M}_p$
- $\bar{a}_i = \bar{a}_p + \bar{\alpha} \times \bar{r}_{i/p} - \omega^2 \bar{r}_{i/p}$

$$d\bar{M}_p = dm\left[\bar{r}_{i/p} \times \bar{a}_p + \bar{r}_{i/p} \times (\bar{\alpha} \times r_{i/p}) - \bar{r}_{i/p} \times \omega^2 r_{i/p}\right]$$

$\bar{r}\times(\alpha \times r) - \bar{r}\times(-\bar{r}\times\alpha) = -(-r^2\bar{\alpha})$

$\bar{r}\times(\omega^2\bar{r}) = \omega^2(\bar{r}\times\bar{r}) = \bar{0}$

$\bar{a}_p = [a_{px}\bar{i} + a_{py}\bar{j}]$

planar motion

$\bar{r} = x\bar{i} + y\bar{j}$

$\bar{\alpha} = \alpha\bar{k}$

$$= dm\left[\bar{r}_{i/p} \times \bar{a}_p + r_{i/p}^2 \bar{\alpha}\right]$$

$$dM_p\bar{k} = dm\left[-y a_{px}\bar{k} + x a_{py}\bar{k} + \alpha\left[x^2 + y^2\right]\bar{k}\right]$$

$$dM_p = dm\left[-y a_{px} + x a_{py} + \alpha\left[r_{i/p}^2\right]\right]$$

TAKE $\sum dM_p = \sum\left(\bar{r}_{i/p} \times \bar{F}_i + \bar{r}_{i/p} \times \bar{f}_i\right) = \sum \bar{r}_{i/p} \times \bar{F}_i = \sum \bar{M}_p$

DISCUSSION OF ANGULAR MOM

$r_{1/2}$ OVER

r_1

r_2

$r_1 = r_2 + r_{1/2}$

$$= \left(\int -y\,dm\right)a_{px} + \left(\int x\,dm\right)a_{py} + \alpha\left(\int r^2 dm\right)$$

$$\circlearrowleft + \sum M_p \dot{=} -\bar{y}\,m a_{px} + \bar{x}\,m a_{py} + \alpha I_p$$

$$\sum \bar{M}_p = \bar{r}_{G/p} \times m\bar{a}_p + \bar{\alpha} I_p$$

ASSUME RH rule

$$\begin{matrix} i & j & k \\ \bar{x} & \bar{y} & 0 \\ a_{px} & a_{py} & 0 \end{matrix}$$

$\bar{y}$ & $\bar{x}$ are coordinates of mass center measured from P

$\bar{y} = \int \frac{y\,dm}{m}$

$\bar{x} = \int \frac{x\,dm}{m}$

if P is the mass center $\sum M_G = \alpha I_G$ since $\bar{y} = \bar{x} = 0$.

if P is fixed: $a_{px} = a_{py} = 0 \Rightarrow \sum M_p = \alpha I_p$

$$\Sigma \bar{M}_P = \bar{r}_{G/P} \times m\bar{a}_P + I_P \bar{\alpha}$$

$$\bar{a}_P = \bar{a}_G + \bar{\alpha} \times \bar{r}_{P/G} - \omega^2 \underline{r}_{P/G}$$

$$\bar{r}_{G/P} \times m\bar{a}_G + \bar{r}_{G/P} \times (m\bar{\alpha} \times \bar{r}_{P/G}) - r_{P/G} \times (m\omega^2 \underline{r}_{P/G}) + I_P \bar{\alpha}$$

$$\bar{r}_{P/G} = -\bar{r}_{G/P} \qquad -m\omega^2 r_{P/G} \times (-\bar{r}_{G/P})$$

$$m[-\bar{r}_{G/P} \times (\bar{\alpha} \times \bar{r}_{G/P})] \qquad \overset{''}{0}$$

$$m[+\bar{r}_{G/P} \times (\bar{r}_{G/P} \times \bar{\alpha})]$$

$$-m r_{G/P}^2 \bar{\alpha} + I_P \bar{\alpha} \qquad I_P - m r_{G/P}^2 = I_G$$

$$\Sigma \bar{M}_P = \bar{r}_{G/P} \times m\bar{a}_G + I_G \bar{\alpha}$$

$$+\circlearrowleft \Sigma M_P = +\bar{x}\, m a_{G_y} - \bar{y}\, m a_{G_x} + I_G \alpha$$

$$\bar{r}_{G/P} = \bar{x}\hat{\imath} + \bar{y}\hat{\jmath}$$

$$\begin{vmatrix} i & j & k \\ \bar{x} & \bar{y} & 0 \\ a_{G_x} & a_{G_y} & 0 \end{vmatrix} = \underline{k}\left[\bar{x} a_{G_y} - \bar{y} a_{G_x}\right] \checkmark$$

- SUM OF MOMENTS OF EXTERNAL FORCES ABOUT MASS CENTER G

  = MASS MOM. OF INERTIA ~~THROUGH~~ ABOUT AN AXIS THROUGH G TIMES

  ANGULAR ACCEL of body

$$\Sigma \bar{M}_G = I_G \bar{\alpha} \quad . \quad \Sigma \bar{M}_P = I_P \bar{\alpha}$$

- SUM OF MOMENTS OF EXTERNAL FORCES ABOUT A FIXED PT IN BODY

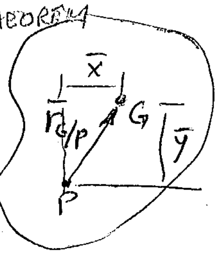
  P = mass mom of inertia about an axis through P times

  absolute angular accel of body

- RETURN TO $\Sigma M_P = -\bar{y} m a_{px} + \bar{x} m a_{py} + I_P \alpha$

- LET $I_P = I_G + m(\bar{x}^2 + \bar{y}^2)$ PARALLEL AXIS THEOREM

- Let $\bar{a}_G = \bar{a}_P + \bar{\alpha} \times \bar{r}_{G/P} - \omega^2 \bar{r}_{G/P}$



PLANAR MOTION

$$a_{Gx}\bar{i} + a_{Gy}\bar{j} = a_{Px}\bar{i} + a_{Py}\bar{j} + \alpha\bar{x}\bar{j} - \alpha\bar{y}\bar{i} - \omega^2\bar{x}\bar{i} - \omega^2\bar{y}\bar{j}$$

$\bar{\alpha} = \alpha\bar{k}$
$\bar{a} = a_x\bar{i} + a_y\bar{j}$
$\bar{r} = \bar{x}\bar{i} + \bar{y}\bar{j}$

$$a_{Gx} = a_{Px} - \alpha\bar{y} - \omega^2\bar{x} \quad \Rightarrow -a_{Gx} - \omega^2\bar{x} = \alpha\bar{y} - a_{Px}$$

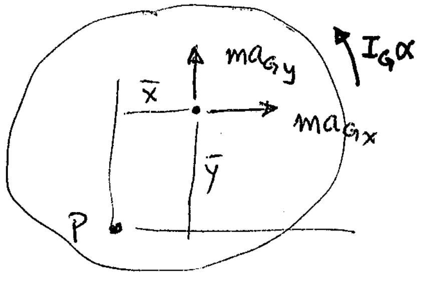
$$a_{Gy} = a_{Py} + \alpha\bar{x} - \omega^2\bar{y} \quad \Rightarrow +a_{Gy} + \omega^2\bar{y} = \alpha\bar{x} + a_{Py}$$

ERROR IN BOOK! p 324

$$\circlearrowleft + \Sigma M_P = -\bar{y} m a_{px} + \underline{\bar{x} m_p a_{py}} + \alpha I_G + \underline{m\bar{x}^2\alpha} + m\bar{y}^2\alpha$$

$$= m\bar{y}\left[-a_{Gx} - \omega^2\bar{x}\right] + m\bar{x}\left[+a_{Gy} + \omega^2\bar{y}\right] + \alpha I_G$$

$$\Sigma M_P = -m\left[a_{Gx}\bar{y}\right] + m\left[a_{Gy}\bar{x}\right] + \alpha I_G$$



$\Sigma \bar{M}_P$ = "kinetic moments" due to $m\bar{a}_G$ & $I\bar{\alpha}$

ma_{Gx} produces $\circlearrowright M$

ma_{Gy} produces $\circlearrowleft M$

αI_G produces $\circlearrowleft M$

3 eqs are $\Sigma \bar{F} = m\bar{a}_G$

if $P = G \Rightarrow \bar{r}_{G/P} = \bar{0}$

$$\Sigma \bar{M}_P = \bar{r}_{G/P} \times m\bar{a}_G + I_G\bar{\alpha} \quad \text{or} \quad \Sigma \bar{M}_G = I_G\bar{\alpha}$$

$$= \bar{r}_{G/P} \times (\Sigma \bar{F})_{EXT} + I_G\bar{\alpha}$$

applied at G.

$$\bar{r}_{G/P} = \bar{x}\bar{i} + \bar{y}\bar{j}$$

$$\bar{a}_G = a_{Gx}\bar{i} + a_{Gy}\bar{j}$$

$$\bar{\alpha} = \alpha\bar{k}$$

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Textbook: **Engineering Mechanics--Dynamics, 10th Edition**
R. C. Hibbeler

Session #			
1	Chapter 12, Section 1,2,3	1/10	
2	4,5	1/12	
3	7,8	1/19	
4	9,10	1/24	
5	Review Chapter 12	1/26	
6	Chapter 13, Section 1-4	1/31	x,y
7	5,6	2/2	n-t, r-θ
8	**Exam #1--Chapter 12**	2/7	
9	Chapter 14, Section 1-3	2/9	
10	4-6	2/14	
11	Review Chapters 13 and 14	2/16	
12	Chapter 15, Section 1,2	2/21	
13	3,4 and review	2/23	
14	4,5	2/28	
15	6,7	3/2	
16	**Exam #2--Chapters 13, 14 and 15.1-4**	3/7	
17	Chapter 16, Section 1-4	3/9	
18	5,6	3/14	
19	7,8	3/16	
20	Review	3/28	
21	Chapter 17, Section 1,2	3/30	
22	3-5	4/4	
23	Review Chapters 15.5-7, 16, 17	4/6	
24	Chapter 18, Section 1-3	4/11	
25	**Exam #3--Chapters 15.4-6, 16, 17**	4/13	
26	4,5	4/18	
27	Chapter 19, Section 1-3	4/20	
28	Review Chapters 18, 19	4/[illegible] 25	
29	**FINAL EXAM -- During Exam Week**		

Grade will be determined on the basis of

Exam #1	15 %
Exam #2	20 %
Exam #3	25 %
Homework	10 %
FINAL	30 %

Grading Scheme:	90 and above	A	77 - 79	B-	60 - 64	D
	87 - 89	A-	74 - 76	C+	Below 60	F
	83 - 86	B+	70 - 73	C		
	80 - 82	B	65 - 69	C-		

Homeworks are due a week after being assigned. These problems should be neatly worked out, preferably on engineering paper. Use the "Given, Required, Solution" format and completely draw appropriate diagrams and coordinate systems. All numerical answers should have the appropriate units. Note in each exam there will be one problem quite similar to the assigned problems. Problems submitted after the class hour the due day or the next day will be penalized with 15% of the total grade for that assignment. No homework will be accepted after two days without a medical or any other documented excuse. If you realize you will miss the due day and you have a valid excuse, please contact me **before** the due day and not after. You must keep up with the homework in order to do well in class

Cellphones are to be put on silent mode before the start of and during class and turned off before and during examinations. Thanks.

If you come in late, do it quietly. Find a seat in a manner that does not disturb the class or your neighbors. Thanks.

Cheating will not be tolerated. Cheating is defined as the giving or taking of unpermitted aid of any kind. Cheating affects all students. The cheater gets an undeserved grade and the rest of the class is affected by being lower on the merit order list. Cheating on an exam will be cause to fail the exam. Cheating on the final exam is cause to fail the course. So, please don't cheat nor give aid to those cheating.

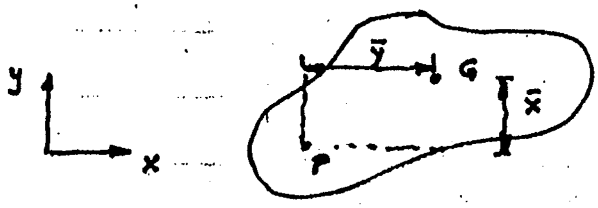
Our class meets M 540-655pm and W 540-820pm in room EAS 1105.

Office hours are M and W 1pm-4pm and by appointment.

I will be out for a few days in the beginning of April. More on that as we get close to the date.

This is a preliminary syllabus subject to change. All changes will be announced in class.

THESE EQNS ASSUME COORDINATE AXES ARE FIXED OR <u>TRANSLATE ONLY</u>



TRANSLATION ONLY (RECTILINEAR)

$$\xrightarrow{+} \sum F_x = m a_{G_x}$$
$$+\uparrow \sum F_y = m a_{G_y}$$
$$+\circlearrowleft \sum M_G = 0 \qquad \text{or } \circlearrowleft \sum M_P = -\bar{y} m a_{G_x} + \bar{x} m a_{G_y}$$

TRANSLATION - CURVILINEAR

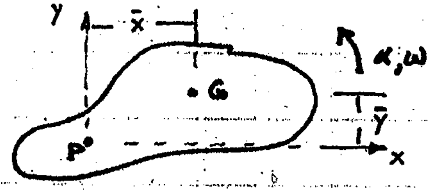
$$\sum F_t = m a_{G_t}$$
$$\sum F_n = m a_{G_n}$$

$$+\circlearrowleft \sum M_G = 0 \quad \text{or } \circlearrowleft \sum M_P = -h m a_{G_n} + e m a_{G_t}$$

ROTATION ABOUT AN AXIS O

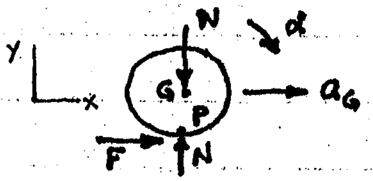
$$\sum F_t = m a_{G_t} = m\alpha|\bar{r}_{G/O}|$$
$$\sum F_n = m a_{G_n} = m\omega^2|\bar{r}_{G/O}|$$
$$+\circlearrowleft \sum M_G = I_G \alpha \qquad \text{or } \circlearrowleft \sum M_O = I_O \alpha$$

GENERAL MOTION



$$\sum F_x = m a_{G_x}$$
$$\sum F_y = m a_{G_y}$$
$$+\circlearrowleft \sum M_G = I_G \alpha \qquad +\circlearrowleft \sum M_P = -\bar{y} m a_{G_x} + \bar{x} m a_{G_y} + I_G \alpha$$
$$= -\bar{y} m a_{P_x} + \bar{x} m a_{P_y} + I_P \alpha$$

For problems involving rolling friction



IF <u>NO SLIP</u> $a_G = \alpha r$ $\quad a_{P_x} = 0$ $\quad V_P = 0$

IF <u>SLIP</u> $F = \mu N$

particles move along // straight lines

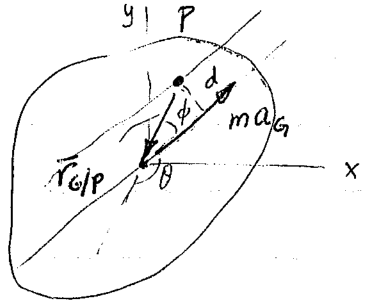
- FOR BODIES IN PURE TRANSLATION $\Rightarrow \bar{\alpha} = 0$ & $\Sigma \bar{M}_G = \bar{0}$

or $\Sigma \bar{M}_P = \bar{r}_{G/P} \times (\Sigma \bar{F}) = (-\bar{y}\, m a_{G_x} + \bar{x}\, m a_{G_y}) \bar{k}$

- AND $\Sigma \bar{F} = m \bar{a}_G$

$$\Sigma F_x = m a_{G_x} \quad , \Sigma F_y = m a_{G_y} \quad , \quad \Sigma M_G = 0$$

AXIS THROUGH G.



$\Sigma \bar{M}_P = \bar{r}_{G/P} \times m\bar{a}_G = r_{G/P}(ma_G) \sin\theta$

$\phi = 180 - \theta \quad \sin\phi = \sin\theta \Rightarrow$

$r_{G/P} \sin\phi = d.$

$\Sigma \bar{M}_P = m a_G d.$ ↶ about z axis

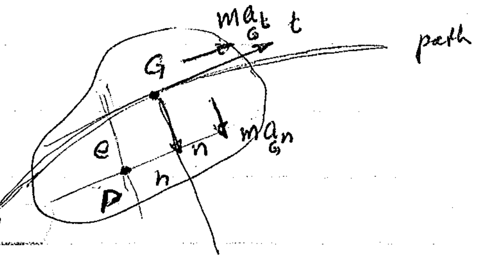
particles move along // curved paths.

For curvilinear translation

$\Sigma \bar{F} = m\bar{a}_G \quad \Rightarrow \quad \Sigma F_t = m a_{G_t} = m \dot{v}_G$

$\Sigma F_n = m a_{G_n} = m v_G^2/\rho$ instantaneous

$\Sigma M_G = 0 \quad$ or $\overset{+}{\circlearrowleft} \Sigma M_P = -(m a_{G_n}) h - (m a_{G_t}) e$



FOR TRANSLATION

1. Establish IFR
2. Draw FBD & show all forces acting on body
3. apply either $\Sigma M_G = 0$ or $\overset{+}{\circlearrowleft} \Sigma M_P = -y\, m a_{G_x} + x\, m a_{G_y}$: if CARTESIAN

   $\overset{+}{\circlearrowleft} \Sigma M_P = -e\, m a_n - h\, m a_{t_o}$: IF CURVILINEAR
4. Remember $a_{G_n} = \frac{v_G^2}{\rho} \qquad a_{G_t} = \dot{v}_G = \alpha\rho$

FOR RECTILINEAR $\quad a = \frac{dv}{dt}$ & $a\,ds = v\,dv$ & $\frac{ds}{dt} = v$

FOR center of mass [CURVILIN $\quad a_t = \frac{dv}{dt}$ & $a_t\,ds = v\,dv$ & $a_n = \frac{v^2}{\rho} = \omega^2\rho$

397 in 10 $= \alpha\rho$

EXAMPLE 17-5 p. 329 Rectilinear Trans. ALSO 17-7 p. 330

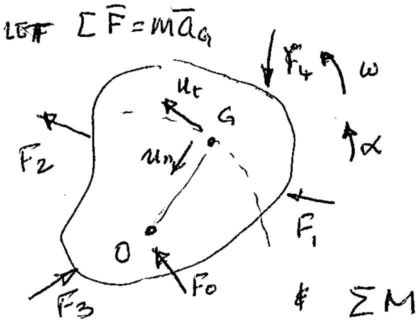
EXAMPLE 17-8 p. 331 Curvilinear

ROTATIONAL MOTION THROUGH O — — — — — LESSON # 23

Motion about a fixed axis : $\Rightarrow$ CENTER OF MASS MOVES IN CIRCLE

LET $\Sigma \bar{F} = m\bar{a}_G$

$$\Rightarrow \bar{a}_G = \dot{v}_G u_t + \frac{v_G^2}{r_{G/O}} \bar{u}_n$$



$$\therefore \Sigma\bar{F} = m\bar{a}_G \Rightarrow \Sigma F_t = m\dot{v}_G = m\alpha r_{G/O}$$

$$\Sigma F_n = m\frac{v_G^2}{r_{G/O}} = m\omega^2 r_{G/O}$$

& $\Sigma M_G = I_G\alpha$: USE // AXIS THEOREM I

- or $\Sigma M_O = I_O\alpha$ since $(\Sigma M_O = I_G\alpha + r_{G/O}\, m a_t = I_G\alpha + m r_{G/O}^2\alpha$
  $= I_O\alpha$

TAKE MOMENT ABOUT O TO ELIMINATE UNKNOWNS THROUGH O

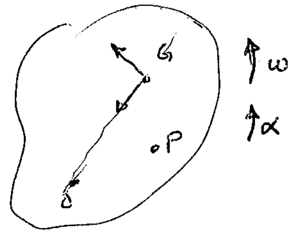
- ~~BY // AXIS THEOREM~~

REMEMBER HERE
- DRAW FBD SHOWING ALL FORCES
- SHOW $(\bar{a}_G)_n, (\bar{a}_G)_t, \bar{\alpha}$ INCLUDE DIRECTIONS
- FIND I_G OR I_O or I_P
- FOR MOMENTS
  - SHOW ACCELERATION COMPONENTS
- USE KINEMATICS $\alpha = \frac{d\omega}{dt}$ $\omega = \frac{d\theta}{dt}$ $\alpha\, d\theta = \omega\, d\omega$

$\Sigma M_G = I_G\alpha$

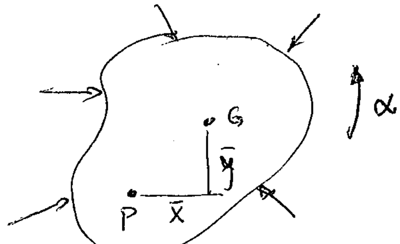
$\Sigma M_O = I_O\alpha$

$\Sigma M_P = \bar{r}_{G/P} \times m\bar{a}_G + I_P$



EXAMPLE 17-9 P. 339 also 17-12 P. 342
17-13 in 10th ed.

GENERAL PLANE MOTION



$$\Sigma\bar{F} = m\bar{a}_G \quad — \quad \Sigma F_x = m a_{Gx}, \quad \Sigma F_y = m a_{Gy}$$

$$+\circlearrowleft \Sigma M_G = I_G\alpha$$

$$\circlearrowleft + \text{ or } \Sigma M_P = -\bar{y}\, m a_{Gx} + \bar{x}\, m a_{Gy} + I_P\alpha$$

$$\Sigma \bar{M}_P = (\bar{r} \times m\bar{a}_G) + I_P\bar{\alpha}$$

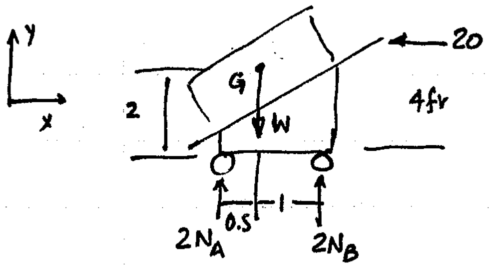
- CHOOSE IFR
- DRAW FBD showing direction of $\bar{a}_G$ & $\bar{\alpha}$.
- find I_G or I_P (to eliminate moments due to unknown forces) THROUGH P.
- WRITE EQS OF MOTIONS
- KINEMATICS LOOK AT P. 350

NO SLIP P. 259
~~Skipping~~ $a_G = \alpha r$
$v_{gn} = 0$
$a_{gt} = 0$
$a_{gn} \neq 0$

EXAMPLE 17-14 p. 352
17-16 p. 354 now 17-17 in 10th

17-41 The drum truck supports the 600 lb drum that has a CG at G. If the operator pushes forward

Take 2 together



$$\Sigma F_x = m a_{G_x} \Rightarrow \quad -20 = \frac{600}{32.2} a_{G_x} \qquad a_{G_x} = -1.07 \text{ ft/s}^2$$

$$\Sigma F_y = m a_{G_y} = 0 \Rightarrow -600 + 2N_A + 2N_B = 0$$

$$+\circlearrowleft \Sigma M_A = -W(0.5) + 20\cdot 4 + 2N_B(1.5) = -\bar{y} m a_{G_x} + \bar{x}\, \cancel{m a_{G_y}} + \cancel{I_G \alpha}$$

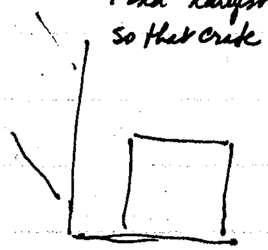
$$= -2\cdot\frac{600}{32.2}(-1.07)$$

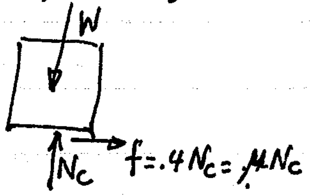
$$= -600(.5) + 80 + 3N_B = 2.15\cdot\frac{600}{32.2}$$

$N_B = 86.7$ lb.

$N_A = 213$ lb.

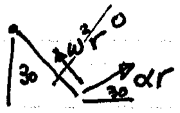
17-51 The crate C has a weight of 150 lb and rests of the truck elevator ($\mu_s = 0.4$) Find largest α, starting from rest, that AB, DE can have so that crate will not slip

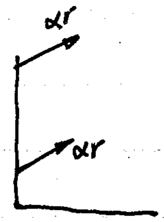




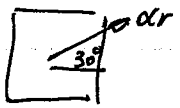
$$\Sigma F_x = m a_{G_x} = f = m a_{G_x}$$

$$\Sigma F_y = m a_{G_y} = N_c - W = m a_{G_y}$$





so the crate has



$a_{G_x} = \alpha r \cos 30$

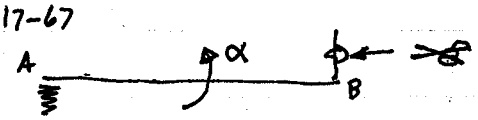
$a_{G_y} = \alpha r \sin 30$

$$a_D = \underset{0}{\cancel{a_E}} + \alpha \times r_{D/E} - \overset{0}{\cancel{\omega^2 r_{D/E}}} = \alpha \times r_{D/E} = \alpha r_{D/E}$$

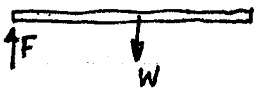
So $0.4N_c = f = \frac{150}{32.2}\cdot 2\alpha\ (0.866)$

$N_c - 150 = \frac{150}{32.2}\cdot 2\alpha\cdot\frac{1}{2}$

17-67



The 4 kg slender rod is supported horiz by a spring & cord find α. & a_G if cord is cut.



$F=\frac{W}{2}=\frac{4\cdot9.81}{2}=19.6\,N$

$$\Sigma F_x = m a_{G_x} \Rightarrow 0 = m a_{G_x} = 0 \qquad a_{G_x} = 0$$

$$+\uparrow \Sigma F_y = m a_{G_y} \Rightarrow F - W = m a_{G_y}$$

$$+\circlearrowleft \Sigma M_A = +\bar{x}\, m a_{G_y} + \cancel{\bar{y}\, m a_{G_x}} + I_A \alpha$$

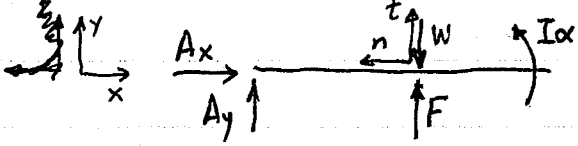
$$-4\cdot g\cdot 1 = -W(1) = 1\cdot 4\cdot a_{G_y} + \frac{1}{12} m (2)^2 \alpha$$

$$-g = a_{G_y} + \frac{\alpha}{3}$$

$$a_{G_y} = \frac{F-W}{m} = \frac{g}{2} - g = -g/2 = -4.91\ m/s^2$$

$$\therefore \quad \alpha = 3(-g - a_{G_y}) = 3(-g + g/2) = -\frac{3}{2} g = -14.7\ rad/s^2$$

17-72 Determine α of the 25kg diving board & A_x & A_y when man jump off. Assume that the board is uniform & rigid & $\Delta x = 0.2m$ $\omega = 0$ & board is horizontal. Take $k = 7kN/m$.



since $\omega = 0$

$V_G = \omega \cdot r_{G/A} = 0$

$a_{G_n} = \frac{V_G^2}{r_{G/A}} = 0$

$a_{G_t} = \alpha r_{G/A}$

$$\Sigma F_t = \Sigma F_y = A_y + F - W = m a_{G_t} = m \alpha r_{G/A}$$

$$\Sigma F_n = -\Sigma F_x = -[A_x] = m a_{G_n} = 0 \Rightarrow \underline{A_x = 0}$$

$$+\circlearrowleft \Sigma M_A = (F - W) r_{G/A} = I_A \alpha = \frac{m l^2}{3} \alpha \qquad I_A = I_G + m\left(\frac{l}{2}\right)^2 = \frac{m l^2}{3}$$

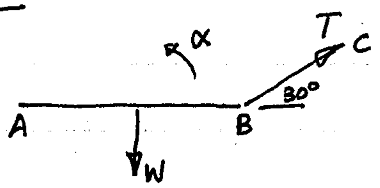
$$= [1400 - 25(9.81)]\,1.5 = 25 \cdot \frac{3^2}{3} \alpha$$

$$\alpha = 23.1\ rad/s^2$$

so $\underline{A_y} = W - F + m\alpha r_{G/A} = 25(9.81) - 1400 + 25(23.1)\cdot 1.5 = \underline{-288.9\ N}$

A slender 2kg rod is supported by a cord BC & released from rest at A. Find α & T

17-99



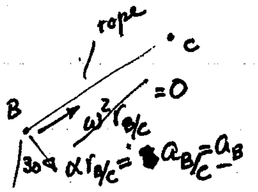
$+\circlearrowleft \sum M_G = T\sin 30\,(0.15) = I_G\,\alpha = \frac{1}{12}m(0.3)^2\alpha$ (3)

$\sum F_x = T\cos 30^\circ = m a_{G_x}$ (1)

$\sum F_y = T\sin 30 - W = m a_{G_y}$ (2)

$\underline{a}_{G} = \underline{a}_B + \alpha \times \underline{r}_{G/B} - \cancel{\omega^2 \underline{r}_{G/B}}\ 0$

$a_{G_x}\underline{i} + a_{G_y}\underline{j} = -a_B\cos 30\,\underline{j} + a_B\sin 30\,\underline{i} + \alpha(0.15)\,\underline{j}$



$\underline{a}_B = \cancel{\underline{a}_C} + \underline{a}_{B/C} = |_{30}\searrow a_B$

So $a_{G_x} = a_B\sin 30$ (2) (1) (3)

$a_{G_y} = -a_B\cos 30 + \alpha(0.15)$ & $a_{G_y} = -a_{G_x}\sqrt{3} + \alpha(0.15)$

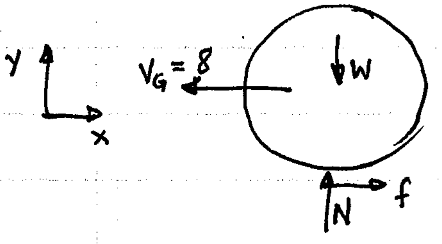
& solve for T = 5.61 N

$a_{G_x} = 2.43\ m/s^2$

$a_{G_y} = -8.41\ m/s^2$

$\alpha = 28.0\ rad/s^2$

17-107



initially $\sum F_y = N - W = 0$ $N = W$

$\sum F_x = f = m a_{G_x} = 0.12N = 0.12W = \frac{W}{g}a_{G_x}$

$a_{G_x} = 0.12g = 3.864\ ft/s^2$

$\therefore$ ~~$\frac{V_f^2}{2} - \frac{V_i^2}{2} = 2a_{G_x}\Delta S$~~

~~$0 - \frac{8^2}{2} = -32 = 2(0.12)(32.2)\Delta S$~~

~~$\Delta S = -\frac{32}{32.2}\frac{1}{2(0.12)}$~~

~~$= -4.14\ ft$~~

$\sum M_G = I_G\,\alpha$

$f(0.375) = \frac{2}{5}mr^2\alpha = \frac{2}{5}m(0.375)^2\alpha$

$0.12mg(0.375)$

$\alpha = .12g \cdot \frac{5}{2(.375)} = 25.76\ rad/s^2$

when rolls w/o slipping

$V = \omega(0.375)$

$\omega = \cancel{\omega_0}^{0} + \alpha t$

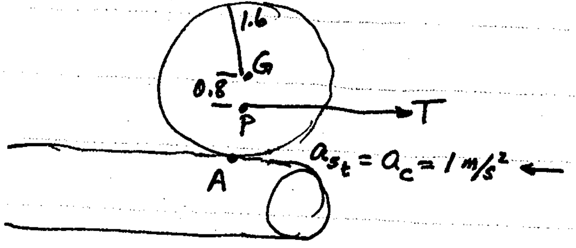
$\omega = V/.375 = 25.76t$ or $9.66t = V$

$V = -9.66t = V_0 + a_c t = -8 + 3.864t$ $t = 0.592s$

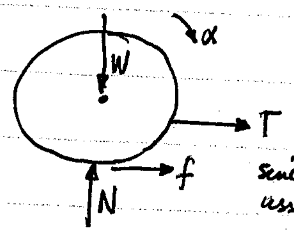
and $s = s_0 + V_0 t + \frac{1}{2}a_c t^2$

$= 0 - 8t + \frac{1}{2}3.64t^2 = -4.06ft$

17-93 Spool of mass 500 kg & $k_G = 1.30$ m. Sits on conveyor belt where $\mu_s = 0.5$ & $\mu_k = 0.4$ If conveyor moves w/ accel of $1\,m/s^2$ find T in rope, α of spool spool originally at rest.



initially at rest $V=0$ $\quad V_G = \omega\, r_{G/A} = 0 \Rightarrow \omega$
initi



since a_c to left assume f to right

$$\Sigma F_x = f + T = m a_{G_x}$$
$$\Sigma F_y = N - W = m a_{G_y} = 0 \quad N = W$$
$$\underline{a}_G = \underline{a}_c + \underline{a}_{G/A}$$
$$a_{G_x}\underline{i} + \cancel{a_{G_y}\underline{j}} = -1\underline{i} + \alpha r_{G/A}\underline{i} \cancel{- \omega^2 r \underline{j}} \Rightarrow -1 + \alpha r_{G/A} = a_{G_x}$$

$$\therefore\ f + T = m[-1 + \alpha 1.6]$$

$$+\circlearrowleft \Sigma M_G = I_G\, \alpha$$
$$T(0.8) + f(1.6) = \underset{= -mk^2\alpha}{\alpha} \quad .\quad \therefore\ f = \overset{-500(1.3)^2\alpha/1.6}{\cancel{0.8\ 500\alpha}} - \frac{T}{2}$$

Now $\underline{a}_P = a_G + \underline{a}_{P/G}$ $\quad a_{P_x} = 0$ since no slip at the contact between rope & spool

$$a_{P_y}\underline{j} = \underline{a}_P = a_{G_x}\underline{i} + \cancel{a_{G_y}\underline{j}} + \alpha r_{P/G}\underline{i} \qquad \therefore\ \underline{a_{G_x} = \alpha(0.8)} \ \ \&\ a_{P_y} = 0$$

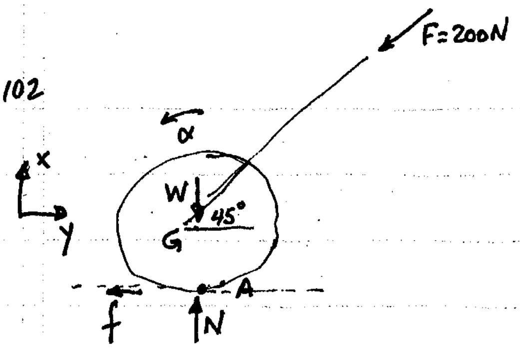
$$\therefore\ f + T = m a_{G_x} = \overset{-1.69(500\alpha) - T/2}{\cancel{-0.8(500\alpha)}} + T = m\alpha(0.8) \quad \text{or} \quad \frac{T}{2} = 1.85\, m\alpha \quad \text{or} \quad T = 3.7\, m\alpha$$

also since $a_{G_x} = -1 + \alpha r_{G/C} = 0.8\alpha$ $\qquad \alpha = \frac{-1}{-0.8} = 1.25\ rad/s^2$

$$a_{G_x} = 1$$
$$T = 3.7(500)(1.25) = 2312.5\,N$$
$$f = m a_{G_x} - T = 500(1) - 2312.5\,N = -1812.5\,N$$ means direction is opposite

& $f_{max} = 0.5\,N = 0.5W = 4905(.5) = 2403N$ $\quad$ Since $f < f_{max}$ not at impending motion

17-102



$$\Sigma F_x = m a_{G_x}$$

$$-F\cos 45 - f = m a_{G_x}$$

$$-F\sin 45 - W + N = m a_{G_y} = 0 \qquad \overset{80(9.81)+200(.707)}{N = W + F\sin 45} = 926.2N$$

$$+\circlearrowleft \Sigma M_G = I_G \alpha \quad \text{or} \quad -f \cdot 0.2 = 80 \cdot (.175)^2 \alpha$$

$$a_{A_t} = 0 \qquad a_{A_n} \neq 0$$

$$\underline{a}_G = \underline{a}_A + \underline{a}_{G/A}$$

$$a_{G_x}\underline{i} = \cancel{a_{A_t}\underline{i}} + (-\alpha r \underline{i}) \quad \therefore \quad a_{G_x} = \alpha(0.2) = -\alpha r$$

$$0 = a_{G_y}\underline{j} = a_{A_n}\underline{j} + \omega^2 r \underline{j} \quad \therefore \quad a_{A_N} = \omega^2 r$$

$$\text{Now } f = -\frac{80(.175)^2}{0.2}\alpha \quad \&$$

$$-F\cos 45 + \frac{80(.175)^2}{.2}\alpha = -80 \cdot \alpha(.2)$$

$$-F\cos 45^\circ = \left[80(.2) - \frac{80(.175)^2}{.2}\right]\alpha$$

$$-141.42 = \overset{-28.25}{} \alpha$$

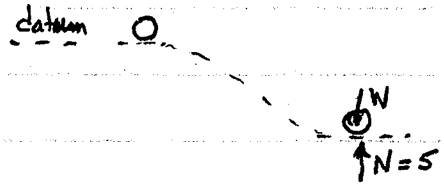
$$\alpha = 5.006 \text{ rad/s}^2$$

so $f = -61.32$ N meaning in opposite direction

$\mu N = \overset{111.14N}{} > f$ no slipping

Angular Momentum

15-100:



$$\cancel{\tfrac{1}{2}mv_i^2} + \overset{0}{\cancel{P_i}} = \tfrac{1}{2}mv_f^2 + P_f$$

$$\tfrac{1}{2}mv_f^2 = mgh$$

$$\therefore \quad v_f^2 = 2gh \qquad v_f = 25.38 \text{ ft/s}$$

$$\text{Now } \Sigma F_n = N - W = ma_n = \frac{mv_f^2}{\rho}$$

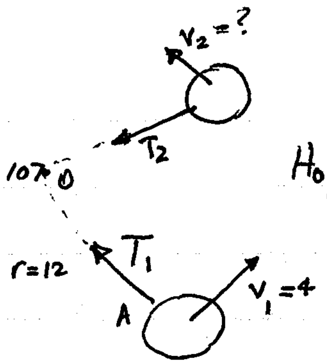
$$5 - 3 = 2 = \frac{3}{\cancel{32.2}} \cdot \frac{2(\cancel{32.2}) \cdot 10}{\rho}$$

$$2 = \frac{20 \cdot 3}{\rho} = \frac{60}{\rho} \qquad \rho = 30 \text{ ft}$$

$$\text{Angular mom} = \rho m v_f = 30 \cdot \frac{3}{32.2} \cdot \sqrt{2 \cdot 32.2 \cdot 10} = 70.93 \ \frac{\text{lb-ft}}{\text{s}}$$

An amusement park ride consists of a car attached to a cable OA. Car rotates in horizontal circular path & has a speed $v_1 = 4$ ft/s when $r = 12$ ft. Now if cable is pulled in at speed of 0.5 ft/s determine the car speed after 3 sec.

15-107



$$H_{O_1} = \Sigma \underline{r}_1 \times m\underline{v}_1 = \Sigma \underline{r}_2 \times m v_2 = H_{O_2}$$

Conservation of angular momentum about pt O.

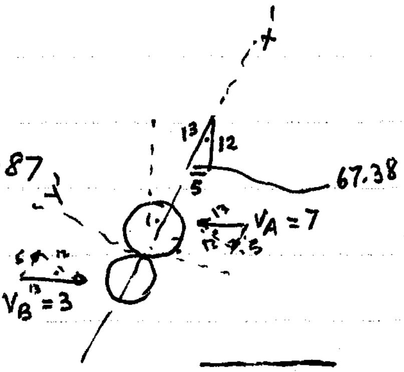
$$12 \cdot m \cdot 4 = (12 - 0.5 \times 3) \cdot m \cdot V_{2_t}$$

$$\frac{12 \cdot 4}{10.5} = V_{2_t} = 4.57 \text{ ft/s} \qquad V_{2_n} = 0.5 \text{ ft/s}$$

$$V_2 = \sqrt{V_{2t}^2 + V_{2n}^2} = 4.599 \text{ ft/s}$$

Two smooth disks collide at a $\underline{V}_A = -7i$ $\underline{V}_B = +3\underline{i}$
$m_A = 8$ kg $m_B = 6$ kg. find V_A & V_B after impact if $e = 0.5$

15-87



$$\Sigma m v_{i_1} = \Sigma m v_{i_2}$$

$$6\left(3 \cdot \frac{5}{13}\right) + 8\left(-7 \cdot \frac{5}{13}\right) = 6 V_{B_{2x'}} + 8 V_{A_{2x'}}$$

$$V_{B_{1y'}} = V_{B_{2y'}} = -3\left(\frac{12}{13}\right) = -2.769 \text{ m/s}$$

$$V_{A_{1y'}} = V_{A_{2y'}} = 7 \cdot \frac{12}{13} = 6.462 \text{ m/s}$$

$$\frac{V_{B_{2x}} - V_{A_{2x}}}{V_{A_{1x}} - V_{B_{1x}}} = e$$

$$= \frac{(V_{B_2})_{x'} - (V_{A_2})_{x'}}{-7 \cdot \frac{5}{13} - 3 \cdot \frac{5}{13}} = 0.5$$

~~$m_A \underline{V}_{A_1} + m_B \underline{V}_{B_1} = m_A \underline{V}_{A_2} + m_B \underline{V}_{B_2}$~~

~~$8(-7) + 6(3) = 8 \underline{V}_{A_2} + 6 \underline{V}_{B_2}$~~

~~$8 V_{A_{2x}} + 6 V_{B_{2x}}$~~

$$V_{B_{2x'}} = -2.14 \text{ m/s}$$

$$V_{A_{2x'}} = -0.220 \text{ m/s}$$

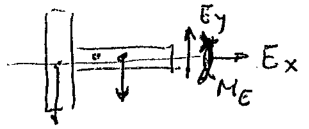
Now $V_{B_2} = \sqrt{V_{B_{2x'}}^2 + V_{B_{2y}}^2} = 3.50$

$$V_{A_2} = \sqrt{V_{A_{2x'}}^2 + V_{A_{2y'}}^2} = 6.47$$

17-34

Curvilinear

Find a_E AC has $\omega_{AC}=0$



$\bar{y}=0$

$\bar{x}=\frac{\Sigma \bar{x} m}{\Sigma m}$

$=\frac{-1(3)-2(3)}{6}$

$=-\frac{9}{6}=-1.5ft$

$$\bar{a}_{G} = a_E = a_A = \cancel{a_B}_{0} + \alpha_{AB} \times r_{A/B} - \omega_{AB}^2 r_{A/B}.$$

$$= 8k \times (-3\cos 30^\circ \bar{i} + 3\sin 30^\circ \bar{j}) - (5^2)[-3\cos 30^\circ \bar{i} + 3\sin 30^\circ \bar{j}]$$

$$\Sigma \bar{F} = m\bar{a}_G = \frac{6 lb}{32.2}[a_G] = \frac{6}{32.2}[a_{G_x}\bar{i} + a_{G_y}\bar{j}]$$

$$\Sigma \bar{F} = E_x \bar{i} + 6\bar{j} + E_y \bar{j}$$

$$E_x = \frac{6a_{G_x}}{32.2}.$$

$$E_y = 6 + \frac{6}{32.2} a_{G_y}$$

~~ΣM_G~~

$$\bar{a}_G = -24(.866)\bar{j} - 24(.5)\bar{i} + 75(.866)\bar{i} - 75(.5)\bar{j}$$

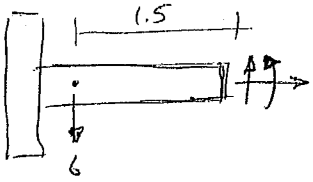
$$= (-58.284\bar{j} + 52.95\bar{i}) \ ft/s^2$$

$$E_x = \frac{6(52.95)}{32.2} = 9.866 \ lb$$

DONE

$$E_y = 6 - \frac{6(58.284)}{32.2} = -4.86 \ lb$$

↓



$$\Sigma M_G = M_E + E_y(1.5) = 0 \qquad M_E = 7.291 \ lb\text{-}ft$$

1. Find center of mass
2. Find a_G of system using kinematics
3. Use $\Sigma \bar{F} = m\bar{a}_G$ to find unknown forces
4. Use $\Sigma M_G = 0$ to find unknown moments.

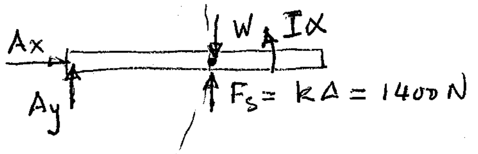
DONE

↙

Rotation about fixed axis

17-72 in 10th

17-60



at instant he jumps off $V_G = 0$

$$\Rightarrow a_{G_n} = \frac{V_G^2}{\rho} = 0 = \frac{V_G^2}{r}$$

$$a_{G_t} = \alpha r = \alpha(1.5)$$

$$\Sigma F_t = \Sigma F_y = A_y - W + F_s = m\alpha(1.5)$$

$$\Sigma F_x = -\Sigma F_n = -m a_{G_n} = 0 = \underline{\underline{A_x = 0}}$$

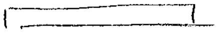
$$+\circlearrowleft \Sigma M_A = I_A \alpha \quad \text{since } a \text{ is fixed}$$

$$I_A = I_G + mr^2$$

$$= \frac{1}{12}ml^2 + m\frac{l^2}{4} = \frac{ml^2}{3}$$

$$= \frac{ml^2}{3}\alpha = -W\frac{l}{2} + F_s\frac{l}{2}$$

$$\alpha = (F_s - W)\frac{l}{2} \cdot \frac{3}{ml^2} = \frac{(F_s - W)}{m}\frac{1.5}{l} = 23.1 \ rad/s^2 \circlearrowleft$$



~~$+\circlearrowleft \sum M_G = I_G\alpha$~~ put into $\sum F_t$ to get $A_y = -288.94\,N \downarrow$

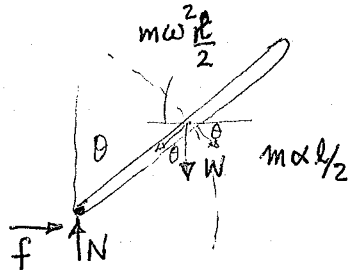
~~$-A_y \frac{l}{2}$~~

1. Define all forces
2. Find a_G interms of system. a_{G_t}, a_{G_n} (instantaneous values
3. Use $\sum \bar{F} = m\bar{a}_G$

Use $\circlearrowleft \sum M_G = I_G \alpha$ or $\circlearrowleft \sum M_A = I_A \alpha$

$\sum M_G = -A_y l/2 = +\left(\frac{1}{12} m l^2\right)\alpha$

17-67



$\sum F$

$\sum M_A = -W\frac{l}{2}\sin\theta = -I_A\alpha$ $I_A = I_G + m\frac{l^2}{2} = \frac{ml^2}{3}$

$= -\frac{ml^2}{3}\alpha$

$$\boxed{\alpha = \frac{3g}{2l}\sin\theta}$$

$\sum F_x = f = m\alpha\frac{l}{2}\cos\theta - m\omega^2\frac{l}{2}\sin\theta$

$\sum F_y = N - W = -m\alpha\frac{l}{2}\sin\theta - m\omega^2\frac{l}{2}\cos\theta$

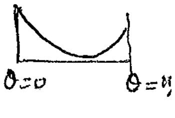
$\alpha\, d\theta = \omega\, d\omega = \frac{3g}{2l}\sin\theta\, d\theta$ or $\frac{\omega^2}{2}\Big|_0^\theta = -\frac{3g}{2l}\cos\theta\Big|_0^\theta$

$\frac{\omega^2}{2} = -\frac{3g}{2l}\left[\cos\theta - 1\right]$ or $\omega^2 = \frac{3g}{l}\left[1-\cos\theta\right]$

$f = m\frac{3g}{2l}\sin\theta\left(\frac{l}{2}\cos\theta\right) - m\frac{3g}{l}(1-\cos\theta)\frac{l}{2}\sin\theta = \frac{3}{2}mg\sin\theta\left(\frac{3}{2}\cos\theta - 1\right)$

$N = mg - m\frac{3g}{2l}\sin\theta\cdot\frac{l}{2}\sin\theta - m\frac{3g}{l}(1-\cos\theta)\frac{l}{2}\cos\theta = \frac{mg}{4}(1-3\cos\theta)^2$

for slipping for $\theta \in [0, 90°]$ $N \in \left[\frac{mg}{4}(4)^2, \frac{mg}{4}\right]$

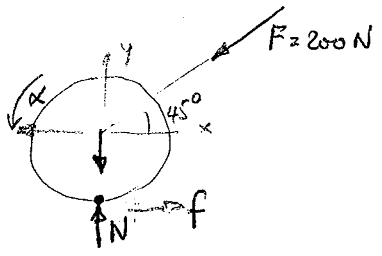


$\theta \in [0, 90°]$ $f \in \left[0, -\frac{3}{2}mg\right]$

~~since $f = \mu_k N$ for slip $\Rightarrow f, N > 0 \Rightarrow \theta = \cos^{-1}\frac{2}{3} =$~~ as long as $N > 0$ we have slip

17-77. Given m & k_G $I_G = mk_G^2 = 2.45\ m^2\text{-}kg$

17-102 in 10th



Assume No slip $a_G = \alpha r$

$\Sigma F_x = -F\cos 45° + f = -ma_{G_x} = -m\alpha r$

$\Sigma F_y = -F\sin 45° + N - W = 0 \Rightarrow N = W + F\sin 45°$ $\underline{N = 926.2\ N}$

$+\circlearrowleft \Sigma M_G = +F(.2) = mk_G^2\alpha$

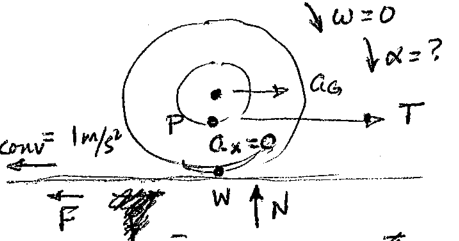
Solve for $f = 61.32N$ $\alpha = 5.01\ rad/s^2$

Check $\mu N = .12(926.2) = 111.1N > f$ no slip ✓ $f \le \mu_s N$

17-75

Same as 17-93 in 10th

Need to find a_G



Assume no slipping of Rope at P

since $\bar{a}_{W_t} = \bar{a}_{conv.} = 1\ m/s$ initially $\bar{a}_{W_n} = \bar{0}$

$a_G = \cancel{\ } a_W + a_{G/W} = -1 + 1.6\alpha \rightarrow$

$a=0$

$\Sigma F_x = T - F = ma_{G_x} = -m(-1 + 1.6\alpha)$

$\Sigma F_y = -W + N = ma_{G_y} = 0$ $N = W = 4905N$

$\bar{a}_{G/W} = \bar{\alpha} \times \bar{r}_{G/W} = +\alpha r \bar{i}$

$+\circlearrowleft \Sigma M_G = I_G\alpha = \frac{1}{2}m(1.6)^2\alpha$

$= -F(1.6) + T(.8) = \cancel{\frac{1}{2}m(1.6)^2}\alpha \quad mk_G^2\alpha$

$a_P = a_G + a_{P/G}$

$a_{Py} = a_G + \alpha r \quad \alpha \times r_{P/G} - \omega^2 r_{P/G} \quad \bar{\alpha} \times \bar{r}_{P/G} = -\alpha\bar{k} \times -.8\bar{j} = -.8\alpha\bar{i}$

let $\bar{\alpha} = -\alpha\bar{k}$

$a_{Py} = 0 \Rightarrow a_G = \alpha r \rightarrow$ $=0$

no slip at initially: at P $\bar{a}_P = \bar{a}_G + \alpha \times r_{P/G} - \cancel{\omega^2 r_{P/G}}^{\,0} =$ $\bar{a}_P = (-1 + 1.6\alpha)\bar{i} - .8\alpha\bar{i} = 0\bar{i}$

$\alpha = 1.25\ rad/s^2$ $a_G = 1\ m/s^2 = -1 + 1.6\alpha$

$T - F = 500\ N$

$-F(1.6) + T(.8) = \cancel{250(1.6)^2(1.25)} = \cancel{800}\ -500(1.3)^2(1.25) = -1056.25$

$-F(1.6) + F(.8) + 500(.8) = -1056.25$

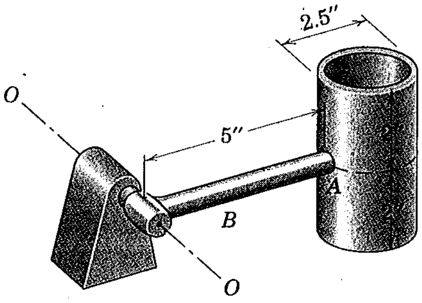
$-.8F = -1456.25$ $F = 1820.31\ N$

$T = 2320.31\ N$

FOR SLIP $F_s = \mu N = .4(4905) = 1962 > F$ ∴ no slip

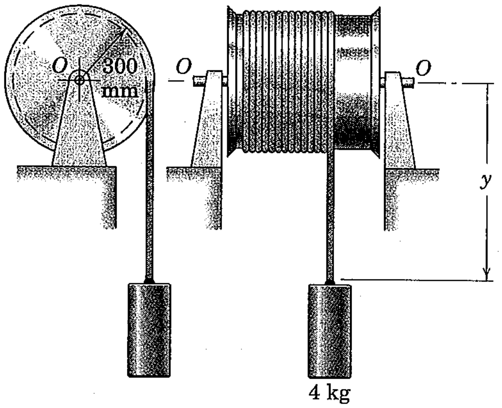
6/63 The link B weighs 0.80 lb with center of mass 2.20 in. from O-O and has a radius of gyration about O-O of 2.76 in. The link is welded to the steel tube and is free to rotate about the fixed horizontal shaft at O-O. The tube weighs 1.84 lb. If the tube is released from rest with the link in the horizontal position, calculate the initial angular acceleration α of the assembly and the corresponding reaction O exerted by the shaft on the link.

Ans. $\alpha = 62.6$ rad/sec^2, $O = 0.492$ lb



Problem 6/63

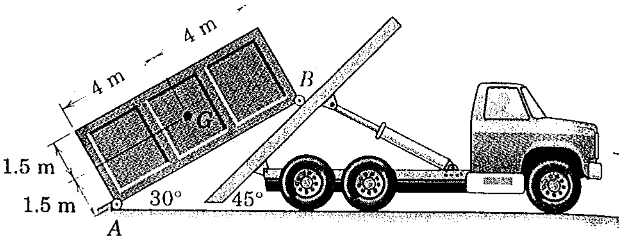
6/64 A flexible cable 60 meters long with a mass of 0.160 kg per meter of length is wound around the reel. With $y = 0$, the weight of the 4-kg cylinder is required to start turning the reel to overcome friction in its bearings. Determine the downward acceleration a in meters per second squared of the cylinder as a function of y in meters. The empty reel has a mass of 16 kg with a radius of gyration about its bearing of 200 mm.



Problem 6/64

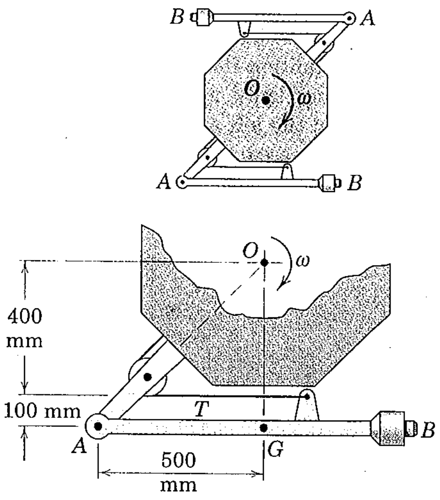
6/65 The figure shows a roll-off truck ramp for discharging loaded containers. The loaded 120-Mg container may be treated as a homogeneous solid rectangular block with mass center at G. If the supporting wheel A is restrained from movement, calculate the force F_B exerted by the ramp on the supporting wheel B when the truck starts from rest with a forward acceleration of 3 m/s^2. Neglect friction at B.

Ans. $F_B = 310$ kN



Problem 6/65

6/66 Prior to deployment of its two instrument arms AB, the spacecraft shown in the upper view is spinning at the constant rate of 1 revolution per second. Each instrument arm, shown in the lower view, has a mass of 10 kg with mass center at G. Calculate the tension T in the deployment cable prior to its release. Also find the magnitude of the force on the pin at A. Neglect any acceleration of the center O of the spacecraft.

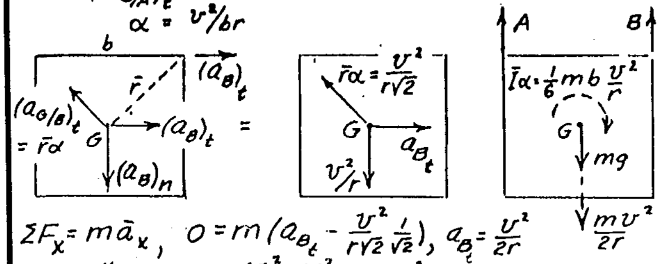


Problem 6/66

6/119 $v_{B/A} = 0$ so $\omega = 0$ $\quad \underline{a}_G = (\underline{a}_B)_n + (\underline{a}_B)_t + (\underline{a}_{G/B})_t$

$(a_{B/A})_t = b\alpha = v^2/r$

$\alpha = v^2/br$



$\Sigma F_x = m\bar{a}_x, \quad 0 = m\left(a_{B_t} - \frac{v^2}{r\sqrt{2}}\frac{1}{\sqrt{2}}\right), \; a_{B_t} = \frac{v^2}{2r}$

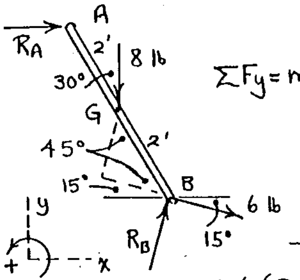
$a_G = \bar{a}_y = \frac{v^2}{r} - \frac{v^2}{r\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{v^2}{2r}$

$\bar{I}\alpha = \frac{1}{6}mb^2\frac{v^2}{br} = \frac{1}{6}mb\frac{v^2}{r}$

$$\left.\begin{aligned}\Sigma M_G = \bar{I}\alpha;\; (A-B)\frac{b}{2} &= \frac{1}{6}mb\frac{v^2}{r}\\ \Sigma F_y = m\bar{a}_y;\; mg - (A+B) &= \frac{mv^2}{2r}\end{aligned}\right\}\text{solve \& get}\quad \underline{A = \frac{m}{2}\left(g - \frac{v^2}{6r}\right)},\quad \underline{B = \frac{m}{2}\left(g - \frac{5v^2}{6r}\right)}$$

▶6/120

$\Sigma F_x = m\bar{a}_x:\; R_A + 6\cos 15° + R_B \sin 15° = \frac{8}{32.2}\bar{a}_x \quad (1)$



$\Sigma F_y = m\bar{a}_y:\; R_B\cos 15° - 6\sin 15° - 8 = \frac{8}{32.2}\bar{a}_y \quad (2)$

$\Sigma M_G = \bar{I}\alpha:$

$-R_A(2\cos 30°) + R_B(2\cos 45°) + 6(2\sin 45°) = \frac{1}{12}\frac{8}{32.2}(4)^2\alpha \quad (3)$

Kinematics:

$\underline{a}_A = \underline{a}_G + \underline{a}_{A/G} = \bar{a}_x\underline{i} + \bar{a}_y\underline{j} + \underline{\alpha} \times \underline{r}_{A/G} - \omega^2\underline{r}_{A/G}$

With $\underline{r}_{A/G} = 2[-\sin 30°\underline{i} + \cos 30°\underline{j}]$, we have

$a_A\underline{j} = [\bar{a}_x - 2\cos 30°\alpha - 2^2\cdot(-2\sin 30°)]\,\underline{i} + [\bar{a}_y - 2\sin 30°\alpha - 2^2\cdot(2\cos 30°)]\,\underline{j}$

$$\Rightarrow \begin{cases} 0 = \bar{a}_x - \sqrt{3}\alpha + 4 & (4)\\ a_A = \bar{a}_y - \alpha - 4\sqrt{3} & (5)\end{cases}$$

$\underline{a}_B = \underline{a}_G + \underline{a}_{B/G} = \bar{a}_x\underline{i} + \bar{a}_y\underline{j} + \alpha\underline{k} \times \underline{r}_{B/G} - \omega^2\underline{r}_{B/G}$

With $\underline{r}_{B/G} = 2[\sin 30°\underline{i} - \cos 30°\underline{j}]$, we have

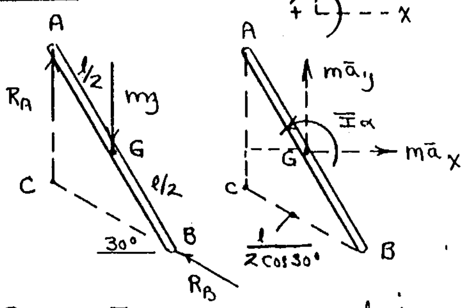
$a_B[\cos 15°\underline{i} - \sin 15°\underline{j}] = [\bar{a}_x + 2\cos 30°\alpha - 2^2\cdot(2\sin 30°)]\,\underline{i} + [\bar{a}_y + 2\sin 30°\alpha - 2^2\cdot(-2\cos 30°)]\,\underline{j}$

$$\Rightarrow \begin{cases} a_B\cos 15° = \bar{a}_x + \sqrt{3}\alpha - 4 & (6)\\ -a_B \sin 15° = \bar{a}_y + \alpha + 4\sqrt{3} & (7)\end{cases}$$

Solution of Eqs. (1)–(7):

$\underline{R_A = 1.128 \text{ lb}}$	$\underline{\alpha = 18.18 \text{ rad/sec}^2}$
$\underline{R_B = -0.359 \text{ lb}}$	$\underline{a_A = -65.0 \text{ ft/sec}^2}$
$\bar{a}_x = 27.5 \text{ ft/sec}^2$	$\underline{a_B = 56.9 \text{ ft/sec}^2}$
$\bar{a}_y = -39.8 \text{ ft/sec}^2$	

▶6/121



$\curvearrowleft \Sigma M_C = \bar{I}\alpha + \Sigma m\bar{a}d:\; -mg\frac{l}{2}\sin 30° = \frac{1}{12}ml^2\alpha + m\bar{a}_y\left(\frac{l}{2}\sin 30°\right) - m\bar{a}_x\left(\frac{l}{2\cos 30°} - \frac{l}{2}\cos 30°\right) \quad (1)$

Kinematics: $\underline{a}_A = \underline{a}_G + \underline{a}_{A/G}$

$a_A\underline{i} = \bar{a}_x\underline{i} + \bar{a}_y\underline{j} + \alpha\underline{k} \times \frac{l}{2}[-\sin 30°\underline{i} + \cos 30°\underline{j}]$

$\underline{j}:\; 0 = \bar{a}_y - \alpha\frac{l}{2}\sin 30° \quad (2)$

$\underline{a}_B = \underline{a}_G + \underline{a}_{B/G}$

$a_B(\sin 30°\underline{i} + \cos 30°\underline{j}) = \bar{a}_x\underline{i} + \bar{a}_y\underline{j} + \alpha\underline{k} \times \frac{l}{2}[\sin 30°\underline{i} - \cos 30°\underline{j}]$

$$\Rightarrow \begin{cases} a_B \sin 30° = \bar{a}_x + \alpha\frac{l}{2}\cos 30° & (3)\\ a_B\cos 30° = \bar{a}_y + \alpha\frac{l}{2}\sin 30° & (4)\end{cases}$$

Solution strategy: Eliminate a_B from (3) & (4):

(3): $a_B = 2\bar{a}_x + \alpha l\frac{\sqrt{3}}{2}$

(4): $\sqrt{3}\bar{a}_x + \alpha l\frac{3}{4} = \bar{a}_y + \alpha\frac{l}{4}$

With (2): $\sqrt{3}\bar{a}_x + \frac{3}{4}\alpha l = \alpha\frac{l}{4} + \alpha\frac{l}{4}$

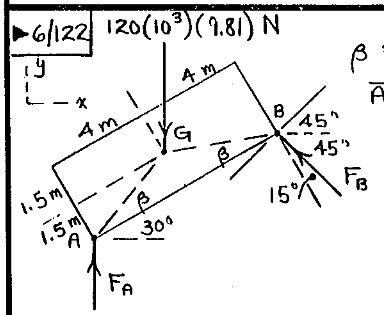
$\bar{a}_x = -\frac{1}{4\sqrt{3}}\alpha l \quad (5)$

(2) & (5) into (1):

$-\frac{g}{4} = \frac{1}{12}l\alpha + \frac{1}{4}\alpha\left(\frac{l}{4}\right) - \alpha\left(\frac{1}{-4\sqrt{3}}\right)\left(\frac{l}{4\sqrt{3}}\right)$

$\underline{\alpha = -\frac{3}{2}\frac{g}{l}\;(\text{CW})}$

▶6/122



$\beta = \tan^{-1}\frac{1.5}{4} = 20.6°$

$\overline{AG} = \overline{BG} = (4^2 + 1.5^2)^{1/2} = 4.27 \text{ m}$

$\bar{I} = \frac{120(10^3)}{12}[8^2 + 3^2] = 73(10^4) \text{ kg}\cdot\text{m}^2$

$\Sigma F_x = ma_{G_x}:\; -F_B\cos 45° = 120(10^3)a_{G_x} \quad (1)$

$\Sigma F_y = ma_{G_y}:\; F_A + F_B\sin 45° - 120(10^3)(9.81) = 120(10^3)a_{G_y} \quad (2)$

$\curvearrowleft \Sigma M_G = \bar{I}\alpha:\; -F_A[\overline{AG}\cos(30° + \beta)] + F_B\cos 15°(4) - F_B\sin 15°(1.5) = 73(10^4)\alpha \quad (3)$

(CONT.)

6/122, CONT.

$\underline{a}_A = \underline{a}_G + \underline{a}_{A/G}$

$a_A \underline{i} = a_{Gx}\underline{i} + a_{Gy}\underline{j} + \alpha \underline{k} \times \left[-\cos(30°+\beta)\underline{i} - \sin(30°+\beta)\underline{j}\right]\overline{AG}$

$\underline{i}: \quad a_A = a_{Gx} + \alpha \overline{AG} \sin(30°+\beta) \qquad (4)$

$\underline{j}: \quad 0 = a_{Gy} - \alpha \overline{AG} \cos(30°+\beta) \qquad (5)$

$\underline{a}_B = \underline{a}_G + \underline{a}_{B/G}$

$a_B\left[\cos 45° \underline{i} + \sin 45° \underline{j}\right] = a_{Gx}\underline{i} + a_{Gy}\underline{j}$
$\qquad + \alpha \underline{k} \times \left[\cos(30°-\beta)\underline{i} + \sin(30°-\beta) j\right] \overline{BG}$

$\underline{i}: \quad a_B \cos 45° = a_{Gx} - \alpha \overline{BG} \sin(30°-\beta) \qquad (6)$

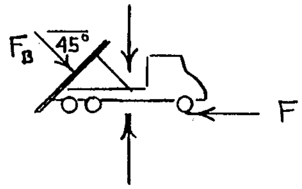
$\underline{j}: \quad a_B \sin 45° = a_{Gy} + \alpha \overline{BG} \cos(30°-\beta) \qquad (7)$

Now 7 eqs. in $F_A, F_B, a_{Gx}, a_{Gy}, \alpha, a_A, a_B$

Solution:

$F_A = 716\,000$ N $\qquad \alpha = -0.372$ rad/s²
$F_B = 481\,000$ N $\qquad a_A = -4.06$ m/s²
$a_{Gx} = -2.83$ m/s² $\qquad a_B = -3.64$ m/s²
$a_{Gy} = -1.009$ m/s²

FBD of truck



$\Sigma F_x = 0: \quad F_B \cos 45° - F = 0$

$F = F_B \cos 45° = 481 \frac{\sqrt{2}}{2} = \underline{340 \text{ kN}}$

6/123

$T_1 + U_{1-2} = T_2$ { ①: horizontol ②: vertical

$0 + mg\frac{l}{4} = \frac{1}{2}\left[\frac{1}{12}ml^2 + m\left(\frac{l}{4}\right)^2\right]\omega^2$

$\underline{\omega = \sqrt{\frac{24g}{7l}}} \qquad \left(1.852\sqrt{\frac{g}{l}}\right)$

6/124

$T_1 + U_{1-2} = T_2$

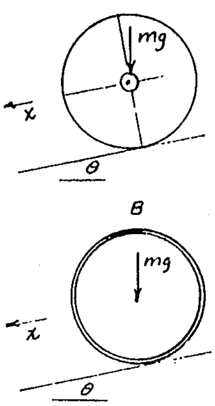
$T_1 = \frac{1}{2} 8(0.3)^2 + \frac{1}{2} 12(0.210)^2\left(\frac{0.3}{0.2}\right)^2 = 0.955$ J

$U_{1-2} = 8(9.81)(1.5) - 3\left(\frac{1.5}{0.2}\right) = 95.2$ J

$T_2 = \frac{1}{2} 8 v^2 + \frac{1}{2} 12(0.210)^2\left(\frac{v}{0.2}\right)^2 = 10.62 v^2$

So $0.955 + 95.2 = 10.62 v^2$, $\underline{v = 3.01 \text{ m/s}}$

6/125



$U = \Delta T$

$U = mgx \sin\theta$

$\Delta T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2$

Case A: $\Delta T = \frac{1}{2} m v^2 + 0$

case B: $\Delta T = \frac{1}{2} m v^2 + \frac{1}{2} m r^2 \left(\frac{v}{r}\right)^2$
$\qquad = m v^2$

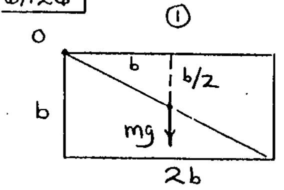
Case A: $mgx \sin\theta = \frac{1}{2} m v^2$

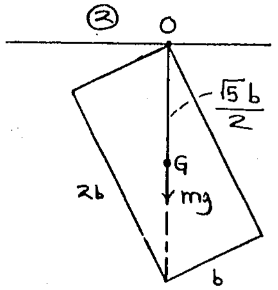
$\underline{v_A = \sqrt{2gx \sin\theta}}$

Case B: $mgx \sin\theta = m v^2$

$\underline{v_B = \sqrt{gx \sin\theta}}$

6/126





$I_O = \bar{I} + md^2 = \frac{1}{12} m\left[b^2 + (2b)^2\right] + m\left[b^2 + \left(\frac{b}{2}\right)^2\right]$
$\quad = \frac{5}{3} mb^2$

$T_1 + U_{1-2} = T_2$:

$0 + mgb\left[\frac{\sqrt{5}}{2} - \frac{1}{2}\right] = \frac{1}{2}\left[\frac{5}{3} mb^2\right]\omega^2$

$\omega^2 = \frac{3g}{5b}\left(\sqrt{5} - 1\right)$, $\quad \underline{\omega = 0.861\sqrt{\frac{g}{b}}}$

6/127

Power $P = \frac{d(\text{Energy})}{dt} = \frac{\Delta E}{t}$

$\Delta E = \frac{1}{2}\bar{I}(\omega_2^2 - \omega_1^2) = \frac{1}{2}(1200)(0.4)^2([5000]^2 - [3000]^2)\left(\frac{2\pi}{60}\right)^2$
$\quad = 16.84(10^6)$ J

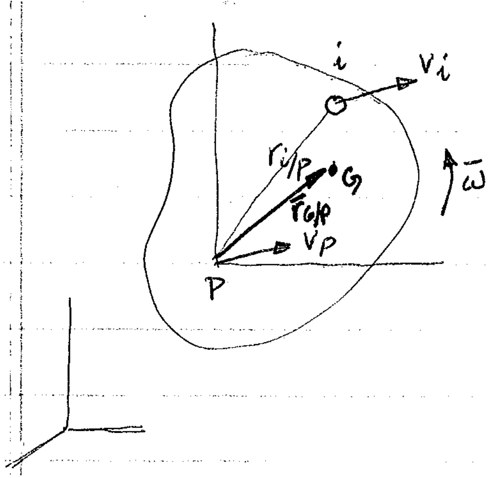
$P = \frac{16.84(10^6)}{2(60)} = 140.4(10^3)$ J/s or W

So $\underline{P = 140.4 \text{ kW}}$ or $P = \frac{140.4(10^3)}{7.457(10^2)} = \underline{188 \text{ hp}}$

LESSON #24

- CHAPTER 14 - USED WORK & ENERGY TO SOLVE PROBLEMS FOR SYSTEM OF PARTICLES OR A PARTICLE.
- NEED TO KNOW $\bar{V}$, s, F
- WILL APPLY SAME IDEAS TO RIGID BODY IN PLANAR MOTION

- FIRST WE WILL FIND KE OF BODY IN TRANSLATION, ROTATION & GENERAL MOTION



- CONSIDER SLAB MOVING WRT IFR.
- LET PARTICLE HAVE VELOC. $\bar{V}_i$
- LET P BE AN ARBITRARY PT W/ VELOC $\bar{V}_p$
- LET P BE A DISTANCE $r_{i/p}$ FROM i
- dT_i (ELEMENTAL KE) $= \frac{1}{2} dm_i V_i^2$
- $\int_m \frac{1}{2} dm_i V_i^2 = T_{TOT}$

$$V_i^2 = \bar{V}_i \cdot \bar{V}_i = (\bar{V}_p + \bar{\omega} \times \bar{r}_{i/p}) \cdot (\bar{V}_p + \bar{\omega} \times \bar{r}_{i/p}) = V_p^2 + 2\bar{V}_p \cdot (\bar{\omega} \times \bar{r}_{i/p}) + (\bar{\omega} \times \bar{r}_{i/p}) \cdot (\bar{\omega} \times \bar{r}_{i/p})$$

$\bar{u} \perp \bar{\omega}$ & $\bar{u} \perp \bar{r}_{i/p}$

- Now $\bar{\omega} \times \bar{r}_{i/p} = \omega r_{i/p} \bar{u} \Rightarrow$ LAST TERM $= \omega^2 r_{i/p}^2$
- $$\bar{V}_p \cdot (\bar{\omega} \times \bar{r}_{i/p}) \begin{vmatrix} V_{Px} & V_{Py} & 0 \\ 0 & 0 & \omega \\ x & y & 0 \end{vmatrix} = -\omega (V_{Px} y - x V_{Py})$$

$$V_i^2 = V_p^2 - 2\omega (y V_{Px} - x V_{Py}) + \omega^2 r_{i/p}^2$$

Now $\int \frac{1}{2} dm_i V_i^2 = V_p^2 \frac{M}{2} - \frac{2\omega}{2} V_{Px} \int y \, dm_i + \frac{2\omega}{2} V_{Py} \int x \, dm_i + \frac{\omega^2}{2} \int r_{i/p}^2 \, dm_i$

$$T = \frac{m V_p^2}{2} - \omega V_{Px} \bar{y} m + \omega V_{Py} \bar{x} m + \frac{\omega^2}{2} I_P$$

if P is G: $\bar{y} = \bar{x} = 0 \Rightarrow T = \frac{m V_G^2}{2} + \frac{\omega^2}{2} I_G$

if P is fixed: $\bar{V}_P = 0 \Rightarrow T = \frac{m V_P^2}{2} + \frac{\omega^2}{2} I_P$

$T \geq 0$ since V_G & ω are squared

- TRANSLATION ONLY $\bar{\omega} = \bar{0}$ $\Rightarrow$ $T = \frac{mV_G^2}{2}$

  or if fixed pt. $T = \cancel{\frac{mV_G^2}{2}}\; I_P\frac{\omega^2}{2}$

RECTILINEAR OR CURVILINEAR TRANSLATION

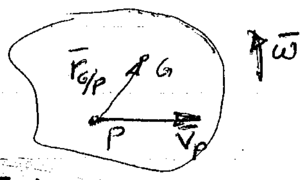
ROTATION ABOUT A FIXED AXIS

- ROTATION ABOUT A FIXED AXIS $\bar{V}_P = 0 \Rightarrow T = \frac{\omega^2}{2} I_P$

  IF FIXED AXIS IS CENTER OF MASS $T = \frac{\omega^2}{2} I_G$

  IF FIXED AXIS IS NOT CENTER OF MASS $T = \frac{mV_G^2}{2} + I_G\frac{\omega^2}{2}$

- GENERAL PLANAR MOTION



$$\boxed{T = \frac{mV_G^2}{2} + I_G\frac{\omega^2}{2}}$$

or

$$T = \frac{mV_P^2}{2} + I_P\frac{\omega^2}{2} + m\bar{V}_P \cdot (\bar{\omega} \times \bar{r}_{G/P})$$

$$-m\omega\left[V_{Px}\bar{y} - V_{Py}\bar{x}\right]$$

$\bar{\omega} = \omega\bar{k}$, $\bar{r}_{G/P} = \bar{x}\bar{i} + \bar{y}\bar{j}$

$\bar{V}_P = V_{Px}\bar{i} + V_{Py}\bar{j}$

$\frac{mV_G^2}{2}$ – TRANSLATIONAL KE

$I_G\frac{\omega^2}{2}$ – ROTATIONAL KE

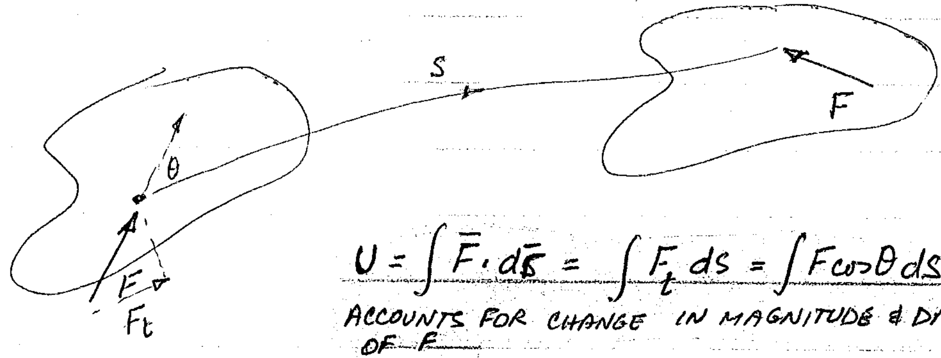
SYSTEM OF CONNECTED RIGID BODIES

- SINCE KE IS A SCALAR QUANTITY: FOR A SYSTEM OF CONNECTED RIGID BODIES TOTAL KE OF SYSTEM = Σ OF KE OF ALL MOVING PARTS, WHICH IS DEPENDENT ON THAT PART.

EXAMPLE 18-1 SYSTEM

- AS A REVIEW LOOK AT THE WORK OF A FORCE

- WORK OF A VARIABLE EXTERNAL FORCE



$$U = \int \bar{F}\cdot d\bar{s} = \int F_t\, ds = \int F\cos\theta\, ds$$

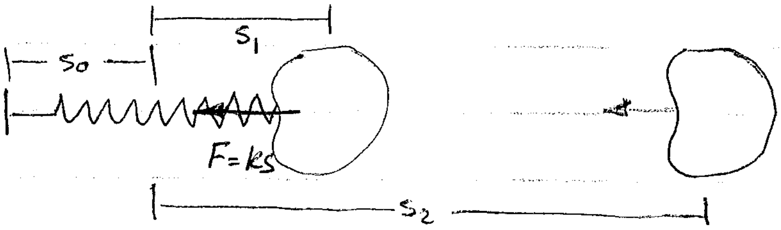
ACCOUNTS FOR CHANGE IN MAGNITUDE & DIRECTION OF F

- if F is constant & θ = constant $U = \int F\cos\theta\, ds = F\cos\theta \int ds = F\cos\theta\, \Delta s$

- WORK OF WEIGHT $U = -W\Delta y$ Δy is vertical displ +↑

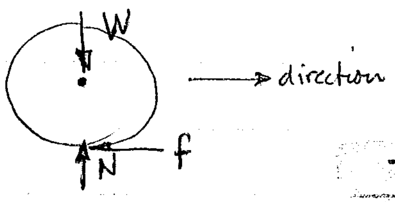
- WORK OF A SPRING FORCE : MEASURED FROM UNSTRETCHED SPRING LENGTH

$$U = -\left(\frac{1}{2}k s_2^2 - \frac{1}{2}k s_1^2\right)$$



- FORCES DOING NO WORK F ⊥ to S &

F acts at fixed pt.



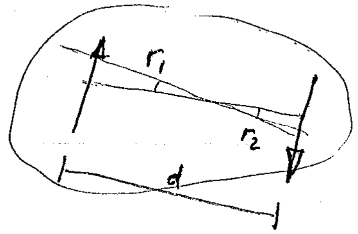
W, N do no work

f does no work when body rolls without slipping ($\bar{V}=0$)

f does work if body slips

A_x, A_y do no work

- WORK OF A COUPLE



- DOES NO WORK UNDER TRANSLATION
- UNDER ROTATION

$$dU_M = Fr_1 d\theta + Fr_2 d\theta = (Fd)d\theta = M d\theta$$

$$dU_M = \bar{M} \cdot d\bar{\theta} = -(Fd)\bar{k} \cdot -d\theta \bar{k}$$

- IF $\bar{M}$ & $d\bar{\theta}$ are in same direction $dU_M > 0$
  opposite < 0

- LINE OF ACTION OF $\bar{M}$ & $d\bar{\theta}$ are //

- CONSTANT MAGNITUDE $dU_m = \int_{\theta_1}^{\theta_2} M d\theta = M(\theta_2 - \theta_1)$ IF M is constant in magnitude

THE PRINCIPLE OF WORK & ENERGY STILL HOLDS

$$T_1 + \Sigma U_{1-2} = T_2$$

WHEN T_1 & T_2 ARE OBTAINED USING $T = \frac{mv_G^2}{2} + \frac{I_G\omega^2}{2}$

- APPLY $T_1 + \Sigma U_{1-2} = T_2$ TO each particle $\Rightarrow \Sigma T_i = \Sigma \frac{dm}{2} v_i^2$

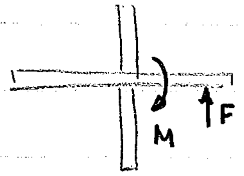
  $\Sigma U_{1-2} = \int F_{ext} \cdot d\bar{r} + \int F_{int} \cdot d\bar{r}$

  - internal forces come in equal but opposite pairs
  - body is rigid $\Rightarrow$ no relative motion $\therefore \int F_{int} \cdot d\bar{r} = 0$

- TOTAL INITIAL TRANSLATIONAL + ROTATIONAL KE ENERGY + WORK DONE BY EXTERNAL FORCES & MOMENTS = TOTAL FINAL TRANS. + ROTATIONAL KE.

- PIN CONNECTED, ~~[illegible] CONNECTED~~ in contact, INEXTENSIBLY CONNECTED BODIES MAY BE LOOKED AT AS A SYSTEM TO WHICH THIS EQN IS APPLIED
  - CALCULATE T_1 & T_2
  - FBD : CALCULATE WORK DONE
  - USE PRINCIPLE OF WORK & ENERGY

PROB
~~EXAMPLE~~ 18-4



$T_1 = 0$

$Fs - M\theta = \sum U_{1-2}$

$T_2 = T_A = \frac{1}{2} I_A \omega^2$

since $V_A = 0$

$I_A = \frac{1}{12} m l^2$ of 1 plate

$I_{A_{TOT}} = 2(I_A$ for 1 plate$)$

LESSON #25

EXAM

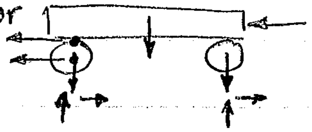
LESSON #26

PROB
~~EXAMPLE~~ 18-11

$T_1 = 0$ for system $V_1 = 0, \omega_1 =$

BLOCK TRANSLATES
CYL - GEN. MOTION $\omega r = V_G$

$V_B = 2\omega r$



$T_2 = \left(\frac{I\omega^2}{2} + \frac{m V_G^2}{2}\right) 2 + m_B \frac{V_B^2}{2}$

$= \left[\left(\frac{1}{2} m r^2\right)\left[\frac{\omega^2}{2}\right] + \frac{m\omega^2 r^2}{2}\right] 2 + m_B \frac{(4\omega}{2}$

frictional forces - internal
weight forces & normals - ⊥ to dir of motion

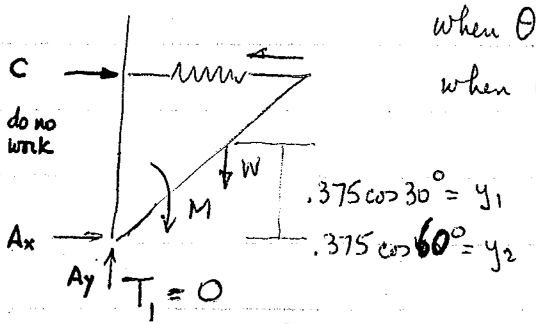
$P(2) = \sum U_{1-2} = 50$ lb-ft

$0 + P(2) = \left[\frac{1}{4} m_{cyl} r^2 \omega^2 + m_{cy} \frac{\omega^2 r^2}{2}\right] 2 + \frac{4 m_B}{2} \omega^2 r^2$

$50 = 252.5 V_G^2 = \left\{\left[\frac{m_{cyl}}{2} \cancel{r^2} + m_{cyl} \cancel{r^2}\right] + 2 m_B \cancel{r^2}\right\} V_G^2$ $V_B = 2V_G$

$V_G = \frac{\cancel{2.234}}{2.525}$ ft/s $V_B = \frac{\cancel{4.469}}{5.05}$ ft/s

PROB
~~EXAMPLE~~ 18-18



Ax, Ay, C do no work

when $\theta = 30°$ x of spring $= .375$ m

when $\theta = 60°$ x of spring $= .\frac{\cancel{707}}{866}(\cancel{.375}\ .75)$

$.375 \cos 30° = y_1$

$.375 \cos 60° = y_2$

$T_1 = 0$

$T_2 = \cancel{\frac{I_G \omega^2}{2} = \frac{1}{12} m l^2 \cdot \frac{\omega^2}{2}}$ $V_G = \omega r$

$I_G \omega^2/2 + m V_G^2/2 = I_A \omega^2/2$

$\sum U_{1-2} = W(.375)(\cos 60° - \cos 30°)$

$+ M\left[\frac{60° \cdot \pi}{180°} - \frac{30° \cdot \pi}{180°}\right]$

$- \frac{1}{2} k \left[(.866)(.75) - .375\right]^2$

$$-\overset{10(9.81)}{W}[.375](\cos 60^\circ - \cos 30^\circ] + \overset{(15)}{M}\left[\frac{30^\circ \cdot \pi}{180}\right] + \frac{1}{2}\overset{(40)}{k}\left[(\overset{(.866)}{\cancel{\;\;\;}})(.75) - .375\right]^2$$

$$= \frac{1}{6} m_{ROD} l^2 \omega^2 = \frac{1}{6}(10)(.75)^2 \omega^2$$

and $\omega = 4.597$ rad/s

Conservation of Energy

- FOR A RIGID BODY TO WHICH ARE APPLIED CONSERVATIVE FORCES
  - CAN USE CONSERVATION OF ENERGY
  - REMEMBER WORK OF CONSERVATIVE FORCE IS INDEPENDENT OF PATH

- REVIEW POTENTIAL ENERGY FOR A CONSERVATIVE FORCE
  - WEIGHT FORCE HAS A GRAVITATIONAL POTENTIAL ENERGY

    $V_g = W y_G$ $\quad$ y_G is that of CENTER OF MASS
  - ELASTIC POTENTIAL ENERGY $\quad V_e = \frac{1}{2} k s^2$

- since $T_1 + \sum U_{1-2} = T_2$
  - $\sum U_{1-2} = \sum U_{1-2_{cons}} + \sum U_{1-2_{noncon.}}$
  - $\sum U_{1-2_{cons}} = V_1 - V_2$

$$\therefore \quad T_1 + V_1 + \sum U_{1-2_{noncons}} = T_2 + V_2$$

- if $\sum U_{1-2_{noncons}} = 0 \Rightarrow T_1 + V_1 = \text{Cons.}$

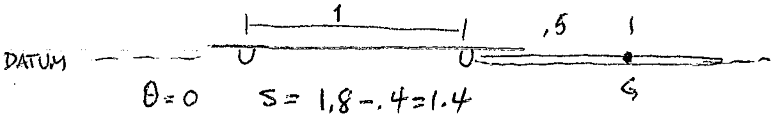
$T_1 + V_1 = T_2 + V_2$ applies to a system of smooth, pin connected rigid bodies

system connected by an inextensible cord

bodies in mesh with other bodies or in contact

USE CONSERVATION OF ENERGY FOR VELOCITY, DISPL + CONSERVATIVE FORCE PROBLEMS

PROBLEM 18-35



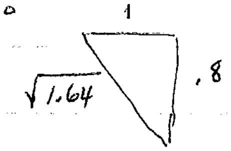
since from rest $V_G, \omega = 0 \Rightarrow T_1 =$

$\theta = 0$ $s = 1.8 - .4 = 1.4$ $V_{1g} = 0$ $V_{1e} = \frac{1}{2}ks^2 = \frac{1}{2}(50)(1.4)$

$I_G = mk_G^2 = 10(.2)^2$; $V_G = \omega r_G = \omega(.5)$

at $\theta = 90°$ $T_2 = \frac{1}{2}mV_G^2 + I_G\frac{\omega^2}{2} = \frac{1}{2}10\underbrace{\omega^2(.5)^2}_{V_G^2} + \frac{10(.2)^2\omega^2}{2} = 1.45\omega^2$

$\theta = 90°$



$s = \sqrt{1.64} - .4$ $V_{1g} = \frac{1}{2}k\left[\sqrt{1.64} - .4\right]^2$

$V_e = -mgh_G = -10(9.81)(.5)$

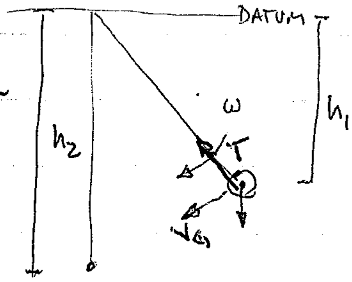
$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(50)(1.4)^2 = 1.45\omega^2 + \frac{1}{2}k\left[\sqrt{1.64} - .4\right]^2 - 10(9.81)(.5)$$

$$\Rightarrow \omega = 7.37 \text{ rad/s}$$

PROBLEM 18-29 Note tension in rope is ⊥ to path & forces on seat are intern

T does no work



$T_1 = 0$ since $\omega r = V_G$ $r \neq 0 \Rightarrow V_G, \omega = 0$

$V_1 = -mgh_1 = -\frac{110}{32.2}(32.2)(15\cos 30°)$

$T_2 = \frac{1}{2}mV_G^2 + \frac{1}{2}I_G\omega^2$ $I_G = mk_G^2 = \frac{110}{32.2}(1.8)^2$

$= \frac{1}{2}m\omega^2 r_G^2 + \frac{1}{2}I_G\omega^2$

$= \left[\frac{1}{2}m(15)^2 + \frac{1}{2}(mk_G^2)\right]\omega^2$

$V_2 = -mgh_2 = -mg(15)$

$\therefore T_1 + V_1 = T_2 + V_2$

$0 - mgh_1 = \frac{1}{2}m\left[r_G^2 + k_G^2\right]\omega^2 - mg(15)$ $\omega = .753 \text{ rad/s}$

LESSON #27

- PROBLEMS INVOLVING FORCES, VELOCITY & TIME CAN BE SOLVED BY LINEAR & ANGULAR MOMENTUM

- EXTEND THESE PRINCIPLES TO RIGID BODY

- remember from CHAPTER 15 $\sum m_i \bar{v}_i = \bar{L}_{tot}$ for a body $\bar{L}_{Body} = m\bar{v}_G$

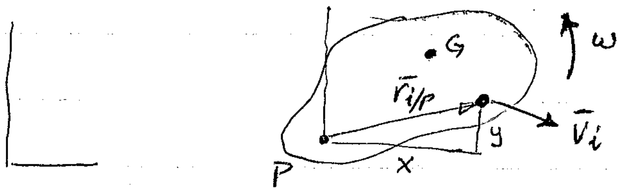
THUS

- $$m\bar{v}_{G_1} + \int \sum \bar{F}\, dt = m\bar{v}_{G_2}$$ where $\sum \bar{F}$ = EXTERNAL FORCES ACTING ON BODY

- $\int \sum \bar{f}\, dt = 0$ FOR INTERNAL FORCES

MOMENT OF LINEAR MOMENTUM

- ANGULAR MOMENTUM – PICK INERTIAL FRAME



- DEFINE A FRAME TRANSL. WITH BODY
- LET BODY BE IN GENERAL MOTION

$$d\bar{H}_{P_i} = \bar{r}_{i/p} \times dm_i \bar{v}_i \qquad \text{where } \bar{v}_i = \bar{v}_p + \bar{\omega} \times \bar{r}_{i/p}$$

$$= \bar{r}_{i/p} \times dm_i (\bar{v}_p + \bar{\omega} \times r_{i/p}) = \bar{r}_{i/p} \times dm_i \bar{v}_p + dm_i \bar{r}_{i/p} \times [\bar{\omega} \times r_{i/p}]$$

$$dm_i [\bar{r}_{i/p} \times \bar{v}_p] \quad + dm_i \bar{\omega} r_{i/p}^2$$

$$\bar{r} \times \bar{v}_p = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x_i & y_i & 0 \\ v_{px} & v_{py} & 0 \end{vmatrix} = \bar{k}(x_i v_{py} - y_i v_{px})$$

$$\bar{\omega} = \omega \bar{k}$$

- $$d\bar{H}_{pi} = \bar{k}\left\{dm_i x_i v_{py} - dm_i y_i v_{px} + dm_i \omega r_{i/p}^2\right\}$$

- TAKE $\sum_i$

$$\bar{H}_p = \int d\bar{H}_{P_i} = \bar{k}\left[v_{py} \int x_i dm_i - v_{px} \int y_i dm_i + \omega \int r_{i/p}^2 dm_i\right]$$

$$= \bar{k}\left[\bar{x} v_{py} m - \bar{y} v_{px} m + \omega I_p\right] = \bar{r}_{G/p} \times m\bar{v}_p + I_p \bar{\omega}$$

$$\vec{r}_{G/p} = \bar{x}\vec{i} + \bar{y}\vec{j} \qquad \bar{\omega} = \omega\bar{k} \qquad \bar{v}_p = v_{px}\bar{i} + v_{py}\bar{j}$$

- IF $P = G$ $\quad \bar{x} = \bar{y} = 0 \Rightarrow \bar{H}_G = I_G \bar{\omega}$ or $\overset{+}{\circlearrowleft} H_G = I_G \omega$

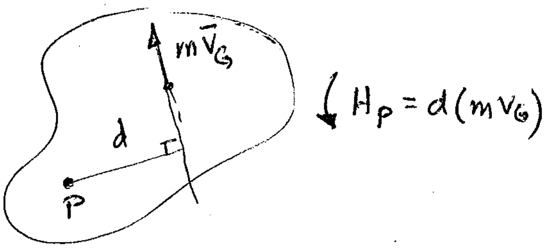
IF P is fixed $\quad \bar{H}_p = I_p \bar{\omega}$ or $\overset{+}{\circlearrowleft} H_p = I_p \omega$

IF $\bar{r}_{G/p}$ is $\parallel \bar{v}_p$ $\quad \bar{H}_p = I_p \bar{\omega}$

- FOR TRANSLATION $\bar{\omega} = 0$ $\therefore$ $\bar{L} = m\bar{v}_G$ & $\bar{H}_G = \bar{0}$

if $\bar{H}_P = \bar{r}_{G/P} \times m\bar{v}_P$ and for translation $\bar{v}_P = \bar{v}_G + \overset{0}{\cancel{\bar{\omega}}} \times r_{P/G} = \bar{v}_G$

$\bar{H}_P = \bar{r}_{G/P} \times m\bar{v}_G$ or



$\circlearrowleft H_P = d(mv_G)$

- FOR ROTATION ABOUT A FIXED AXIS THROUGH P

$\bar{L} = m\bar{v}_G$ & $\bar{H}_G = I_G\bar{\omega}$ or $\circlearrowleft H_G = I_G\omega$

or $\bar{H}_P = I_P\bar{\omega}$ or $\circlearrowleft H_P = I_P\omega$

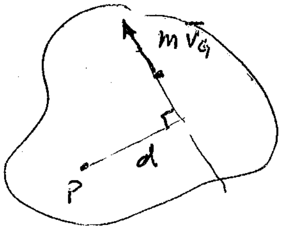
$\omega I_P = (I_G + m r_{G/P}^2)\omega = I_G\omega + r_{G/P}(mv_G)$ where $V_G = \omega r_{G/P}$

- GENERAL PLANE MOTION

$\bar{L} = m\bar{v}_G$ or $L = mv_G$

$H_G = I_G\omega$

for a point P other than G



$+\circlearrowleft H_P = I_G\omega + d(mv_G)$

$\bar{H}_P = I_G\bar{\omega} + \bar{r}_{G/P} \times m\bar{v}_G$

FOR MOST GENERAL CASE

$$H_{P_2} = H_{P_1} + \int \Sigma M_P \, dt$$

where $$H_P = I_P \omega + m[\bar{x} v_{Py} - \bar{y} v_{Px}]$$

$$H_P = I_G \omega + m[\bar{x} v_{G_y} - \bar{y} v_{G_x}]$$

- FROM CHAPTER 14 FOR PROBLEMS DEALING W/ F, V, t

$$\Sigma \bar{F} = \frac{d}{dt}(\bar{L}) = \frac{d}{dt}(m V_G)$$

thus $$m\bar{V}_{G_2} = m\bar{V}_{G_1} + \int_{t_1}^{t_2} \Sigma \bar{F}\, dt$$ $\bar{F}$ are external forces

$$\Sigma M_o = \dot{H}_o$$

- ALSO $\Sigma M_G = I_G \alpha = \frac{d}{dt}(I_G \omega)$ FOR GENERAL MOTION

$$\therefore \quad I_G \omega_2 = I_G \omega_1 + \int_{t_1}^{t_2} \Sigma M_G\, dt$$ M_G = MOM OF EXTERNAL FORCES ABOUT G

FOR ROTATION ABOUT FIXED AXIS $\Sigma M_o = I_o \alpha = \frac{d}{dt}(I_o \omega)$

or $$(I_o \omega)_2 = (I_o \omega)_1 + \int_{t_1}^{t_2} \Sigma M_o\, dt$$

FOR PLANAR MOTION

$$m V_{G_{2x}} = m V_{G_{1y}} + \int \Sigma F_x\, dt$$ x dir

$$m V_{G_{2y}} = m V_{G_{1y}} + \int \Sigma F_y\, dt$$ y dir

$$\{I_G \omega\}_2 = \{I_G \omega\}_1 + \int \Sigma M_G\, dt$$ about axis ⊥ to x, y plane

- FOR A SYSTEM OF CONNECTED BODIES
  - WE CAN USE THIS
  - ELIMINATES NEED TO COMPUTE INTERNAL FORCES

- IN ALL THESE EQUATIONS ANGULAR MOMENTUM & IMPULSE MUST BE COMPUTED ABOUT THE SAME AXIS.

DRAW FBD SHOWING FORCES & VELOCITIES ~~&~~ V, ω

- USE IMPULSE & MOM EQ.
- USE KINEMATICS TO FIND V, ω

for: 1 body $\bar{L}_1 = \bar{L}_2 \Rightarrow \bar{v}_{G_1} = \bar{v}_{G_2}$

for: 1 body $H_{O_1} = H_{O_2} \Rightarrow \omega_1 = \omega_2$

- IF THE LINEAR IMPULSES ARE ZERO – LINEAR MOM IS CONSERVED
  - CONTACT PROBLEMS – WEIGHT FORCES ARE NON IMPULSIVE WRT COLLISION'S DEFORMATION & RESTITUTION IMPULSE

$$\Sigma \bar{L}_1 = \Sigma \bar{L}_2 \quad \text{FOR A SYSTEM}$$

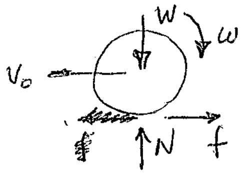
- IF THE ANGULAR IMPULSES ARE ZERO – ANGULAR MOM IS CONSERVED MEASURED ABOUT SAME PT

$$H_{G_1} = H_{G_2} \quad \text{or} \quad (I_G\omega)_1 = (I_G\omega)_2$$

PROBLEM 19-29

LESSON ~~#~~28

PROBLEM 19-18 (3 min)



Lin mom x: $-mV_0 + \int f dt = m(0)$ (1) mult by r & subtract from (2).

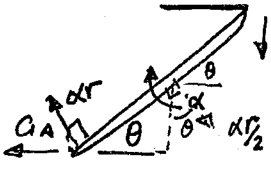
Angular mom: $-I_G\omega + r\int f dt = I_G \cdot 0$. (2)

$$mV_0 r = I_G\omega = \frac{2mr^2}{5}\omega$$

$$\Rightarrow \boxed{\frac{5}{2}\frac{V_0}{r} = \omega} = 2.5\frac{\omega}{r}$$

Lin mom y: $\Rightarrow$ $\underline{\underline{W=N}}$

DO LAST PROBLEM ON EXAM.



$$\bar{a}_A = \bar{a}_B + \bar{a}_{A/B}$$

$$\omega_{AB} = 0 \Rightarrow (a_{A/B})_n = 0$$

$$a_{A/B} = \alpha r\cos\theta\bar{j} - \alpha r\sin\theta\bar{i} \qquad \cos\theta = \frac{1.5}{2.5} = .6$$

$$= .6\alpha r\bar{j} - .8\alpha r\bar{i} \qquad \sin\theta = \frac{2}{2.5} = .8$$

$\bar{a}_A = a_A\bar{i}$ $\quad \bar{a}_B = +a_B\bar{j}$ $\quad a_A = -.8\alpha r$ $\quad a_B = .6\alpha r$

$a_A = -2\alpha$ $\quad a_B = -1.5\alpha$

$a_G = \bar{a}_A + \bar{a}_{G/A}$ $\quad (a_{G/A})_n = 0$ since $\omega_{AB} = 0$ $\quad (a_{G/A})_t = -\frac{\alpha r}{2}\cos\theta\bar{j} + \frac{\alpha r}{2}\bar{i}$

$$= -2\alpha\bar{i} + \underbrace{\alpha(1.25)(.8)\bar{i}}_{\alpha\bar{i}} - \underbrace{\alpha(1.25)(.6)\bar{j}}_{-.75j} = -\alpha\bar{i} - .75\alpha\bar{j}$$

Meriam & Kraige 3rd Ed

for a rigid body $m\underline{v}_{G_1} + \int \Sigma \underline{F}_{ext}\,dt = m\underline{v}_{G_2}$

for rigid body $H_p = I_p \underset{\sim}{\omega} + \underline{\tilde{r}}_{G/p} \times m\underline{\tilde{v}}_p$

if p is center of mass $H_G = I_G \underline{\underline{\omega}}$

p is fixed $\underline{v}_p = 0$ $H_p = I_p \underline{\underline{\omega}}$

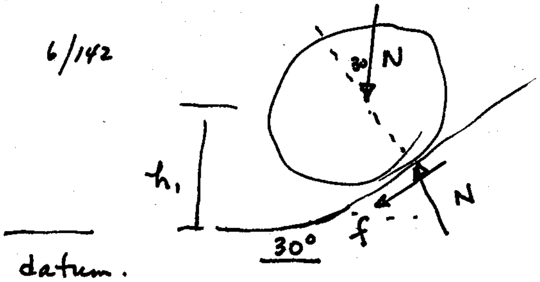
if v_p is $\parallel \underline{r}_{G/p}$ $H_p = I_p \underline{\underline{\omega}}$

conservation of angular momentum $H_{G_1} + \int \Sigma M_G\,dt = H_{G_2}$

or $I_G \underline{\omega}_1 + \int \Sigma M_G\,dt = I_G \underline{\omega}_2$

6-129 in 4th ed M&K

6/142



$-W\cos 30 + N = m a_{G_y}$

$f + N\sin 30° = m a_{G_x}$

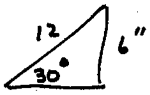
$\Sigma M_G = -f \cdot \frac{1}{2} = I_G \alpha$

$V_G = 2$ ft/s

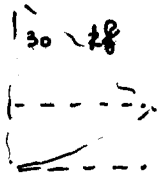
$\omega = 4$ rad/s²

$I_G \frac{\omega_1^2}{2} + m\frac{V_G^2}{2} = I_p \frac{\omega_1^2}{2} + mgh_1 = I_p \frac{\omega_2^2}{2}$

$I_G = mk^2$ $I_p = mk^2 + mr^2 = m(k^2+r^2)$ $I_p = 2.24$ lb-ft sec²



$h_1 =$



$18 - 28\cos 30° = 28(1-\cos 30°)$ in

$h_1 = \frac{28}{12}(1-\cos 30°) + \frac{6''}{12}$

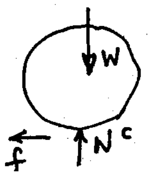
$V_1 = mgh_1 = 140.2$ ft·lb

$I_p \frac{\omega_2^2}{2} = 2.24\frac{(4^2)}{2} + 140.2$

$\frac{2.24}{2}\omega_2^2$

$\omega_2^2 =$ ~~[illegible]~~ 141.2

$\omega = 11.86$ rad/s $v = 5.94$ ft/s



$\therefore N - W = m a_{G_y}$

$a_{c_y} = 0$ a_{c_t}

$\underset{a_{G_x}\underline{i} + a_{G_y}\underline{j}}{\underline{a}_G} = \cancel{a_c} + \alpha r_{G/c} - \omega^2 r_{G/c}$

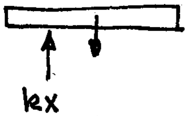
$\omega^2 r_{G/c} = \frac{v_G^2}{r}$

$= \underline{5.94}$

$\therefore \quad \underline{N = 346 \text{ lb}}$

6/128 initially

finally



$I_p \frac{\omega_1^2}{2} = 0 \quad mgh = (15)\, 9.81 \cdot (.3)$

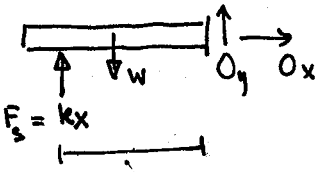
$I_p = \frac{1}{3} m (.6)^2$

$I_p \frac{\omega_2^2}{2}$

$\frac{1}{3} m (.6)^2 \frac{\omega_2^2}{2} + \frac{k}{2} x^2$

$$mgh = \frac{k}{2} x^2 + \frac{1}{3} m (.6) \left(\frac{4}{2}\right)^2 \qquad x = .0545 \text{ m}$$

has no effect.



need

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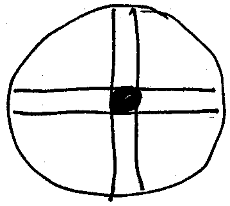


case A since all mass at center

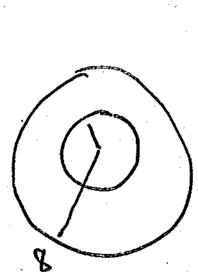
$T = \frac{1}{2} m V_{center}^2$

case B since mass around rim

$T = \frac{1}{2} m V_{center}^2 + I_G \frac{\omega^2}{2}$

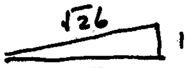
$I_G = \frac{1}{2} m r^2 \quad \omega = \frac{V}{r}$

6/



$T_1 = 0$

$V_1 = \frac{1}{2} k x_1^2 + mgh = 0$



$\frac{\sqrt{26}}{9} = \frac{1}{hi} \qquad h = \frac{9}{\sqrt{26}}$

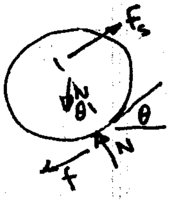
$T_2 = I_G \omega^2/2 + \frac{1}{2} m v^2 = I_p \omega^2/2$

$V_2 = \frac{1}{2} k x_2^2 - mg(9/\sqrt{26})$

$\uparrow_9$

get $\omega^2 = v^2/.667^2$

V



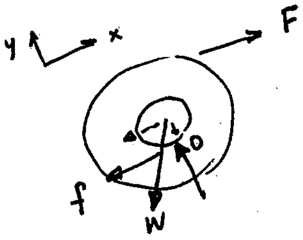
$-F_s + f + W \cdot \frac{1}{\sqrt{26}} = 0$

f

$F_s \cdot 3 = f \cdot 8 \qquad f = -\frac{3}{8} F_s$

$\therefore -F_s - \frac{3}{8} F_s + W \frac{1}{\sqrt{26}} = 0$

6/19 0



$$\int \Sigma M_O dt = H_{O_2} - H_{O_1}$$

$$\int (F \cdot 9 - W \sin\theta \cdot 6)dt = I_O \omega$$

$$I_O = mk^2 + mr^2$$

$\omega = 23.4$ rad/s ↻

6/200 slender bar in ① & a sphere in ②

conserv of mom. $H_1 = H_2$

$$\frac{1}{12} m l^2 \omega_1 = \frac{2}{5} m r^2 N_2 \qquad N_2 = 2.04 \text{ rev/s}$$

$\omega_1 = .3$ rev/s.

$$a_G = a_O + \alpha \times r - \omega^2 r$$

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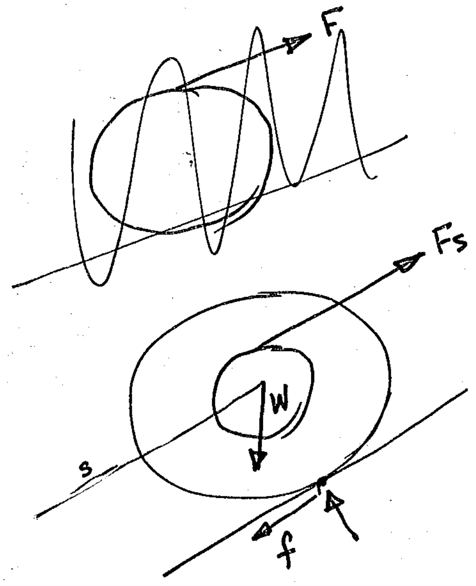
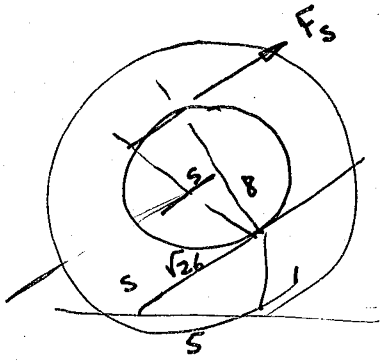
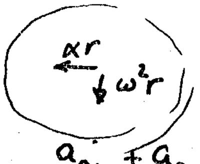






$a_{O_t} + a_{O_n}$

$a_{O_t} = 0$

$\therefore a_{O_n} = \omega^2 r$

$a_G = \alpha r$

$f = \mu N$

Scenter = 8θ

Srope = 11θ

Scenter = Srope · $\frac{8}{11}$

Srope = Scenter · $\frac{11}{8}$

$s = r\theta = 8\theta$ at the center

s at the rope is 11θ

Srope = Scenter · $\cancel{\frac{8}{11}}\ \frac{11}{8}$

∴ Scenter = $\frac{8}{11}$ Srope

$T_1 = 0$

$$V_1 = \frac{1}{2}k\left(\frac{9}{12}\right)^2 + 0$$

$$T_2 = \frac{1}{2} I_O \omega^2 = \frac{1}{2} I_O/r^2\, v^2$$

$$V_2 = \underbrace{\frac{1}{2}k\left[\left(\frac{11}{8}x + \frac{9}{12}\right)^2\right]}_{\text{spring}} - \underbrace{mg\frac{x}{\sqrt{26}}}_{mgh}$$

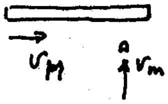
$$\frac{s}{y} = \frac{\sqrt{26}}{1} \qquad x = \frac{s}{\sqrt{26}}$$

$$\Longrightarrow I_O = \frac{1}{2}m(8/12)^2 + m(8/12)^2$$

$\therefore\ 0.4139 v^2 + 3.922x + 37.81x^2 - 41.25x = 0$ or $.4139v^2 + 37.81x^2 - 37.33x$

$\frac{d}{dx} v^2 = 0 \implies x = .494$ ft $\quad\therefore\ v = 4.62$ ft/s

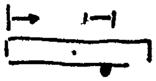
6/195



$$M v_M \underline{i} + m v_m \underline{j} = (M+m) \underset{\sim}{v_f}$$ Conserv of L. mom.

$$\therefore \qquad v_{fx}\underline{i} + v_{fy}\underline{j}$$

$$\therefore \quad v_{fx} = \frac{M v_M}{M+m} \quad \& \quad v_{fy} = \frac{m v_m}{M+m}$$



mass center is at $M \cdot \frac{L}{2} + m \cdot \frac{3L}{4} = \frac{4(M+m)}{4} x_G$

$$\frac{2M+3m}{4(M+m)} L = x_G.$$

$$\frac{3L}{4} - \frac{ML}{4(M+m)}$$

~~if A is $v_M \underline{i}$~~

~~B is $v_{fx} i + v_{fy} \underline{j}$~~ ~~then $\omega \times r =$~~

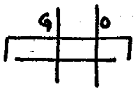
~~$v_B = v_A + \omega \times \underline{r}$~~ ~~$v_{B/A} = (v_{fx} - v_M)\underline{i} + \omega_{fy}\underline{j}$~~

~~$= -\frac{m v_M}{M+m}\underline{i} + \frac{m v_m}{M+m}\underline{j}$~~

~~$\therefore \frac{m}{M+m}\sqrt{v_M^2 + v_m^2} = |v_{B/A}|$~~

~~$= \omega \cdot \frac{3L}{4} \qquad \omega = \frac{4m}{3L(M+m)}(v_M^2 + v_m^2)$~~

look at m $\qquad H_G = \cancel{I_G \omega} + r_{o/G} \times m \underline{V}_o$



find H_G about mass center of sys just before impact.

for particle $+\left(\frac{ML}{4(m+M)}\right) i \times m V_m \underline{j} = + m v_M \cdot \frac{ML}{4(m+M)} \underline{k}$

* ~~$\int \Sigma M_o dt$~~

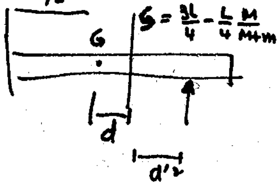
look at M $\qquad H_o = I_o \omega + r_{G/o} \times m \underline{V}_o \qquad r_{G/o}$ is // $V_o \qquad r_{G/o} = -\frac{ML}{4(M+M)}\underline{i} \quad V_o = V_M \underline{i}$

also $\omega = 0$

$\therefore$ H_o for syste is $m v_M \cdot \frac{ML}{4(m+M)} \underline{k}$ $\qquad \int \Sigma M_G dt = 0$ equal & opposite forces acting at = distance from G

$$H_{G\,final} = I_G \omega + \underline{r_{G/o} \times m \overset{0}{\underline{v}_G}} \quad \text{of each}$$

$$= \frac{1}{12} M l^2 \omega + \underset{d^2}{M\left[\frac{L}{4} - \frac{ML}{4(M+m)}\right]^2} \omega^2 + \underset{d'^2}{\underline{m\left(\frac{L}{4}\frac{M}{m+M}\right)^2}} \omega^2$$



solve to get $\omega = \frac{12 v_m}{L}\left(\frac{m}{4M+7m}\right)$ ↺

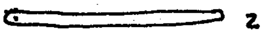
6-183 in 4th Ed

6/198 H

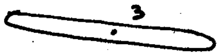
$T_1 = 0$

$V_1 = mg \cdot l/2$

$T_2 = \frac{1}{2} I_o \omega^2 \quad V_1 = 0$



$\therefore \ \omega = \sqrt{\frac{mgl}{I_o}} \qquad \omega = \sqrt{\frac{mgl}{1/3\, m l^2}} = \sqrt{\frac{3g}{l}}$



$H_{G_1} = I_{G_1} \omega -$

$H_{O_i} = I_G \omega_2$

$\int \Sigma M_G \, dt = 0$ since forces at A are equal & opposite

$H_{O_f} = I_G \omega_3$

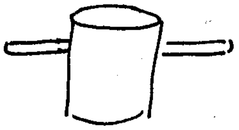
$H_{G_i} = I_{G_1} \omega_2$

just before

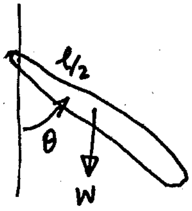
$\omega_3 = \frac{I_G \omega_2}{I_G} = \frac{\cancel{\frac{1}{3} m l^2} \, \omega_2}{\cancel{\frac{1}{12} m l^2}}$

$= 4\omega_2$

6/205



6/233



$\Sigma M_o = W \, l/2 \sin\theta = I_o \alpha = \frac{1}{3} m l^2 \alpha$

$\therefore \quad I_o \omega_f = I_o \omega_i + \int \Sigma M_o \, dt$

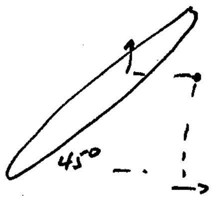
$0 = \frac{1}{3} m l^2 \omega_i + \int \left(W \frac{l}{2} \sin\theta \right) dt$

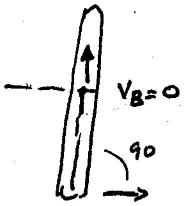
$=$

or $\frac{1}{2} I_o \cancel{\omega_f^2} = \frac{1}{2} I_o \omega_i^2 - mg \left(l/2 \right) \left[1 - \cos\beta \right]$

or $\omega_i^2 = \frac{mg(l/2)}{1/2 (1/3 m l^2)} \left[1 - \cos\beta \right] \qquad \therefore \ \overset{\cancel{\theta}}{\frac{dt}{d\theta}} = \frac{1}{\omega_i} = \sqrt{\frac{3g}{l}} \ \overline{\sqrt{1-\cos\beta}}$

6/135





$T_1 = 0$

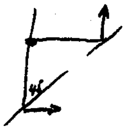
$V_1 = mg\cdot 4\sqrt{2} + M_1\cdot\frac{\pi}{4} +$

$T_2 = \frac{I_B}{2}\omega^2 + \cancel{m\frac{v_B^2}{2}}\,0 \qquad M_1\cdot \pi/2$

$\omega = v_A/r$

$V_2 = mg\cdot 8 \qquad M_1\cdot\pi/2$

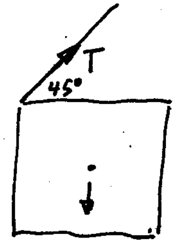
$8/\sqrt{2} = 4\sqrt{2}$



$T_1 + V_1 =$

$T_2 + V_2 =$

$\Delta E = .435$ ft-lb



$\Sigma M_G = I_G \alpha$

$T\cos 45° \cdot b/2 + T\sin 45° \cdot b/2 = I_G \alpha$

also

$a_A = \alpha \cdot r_{A/C} - \omega^2 \overset{0}{\cancel{r_{A/C}}}$

$a_G = a_A + \alpha r_{A/G} - \overset{0}{\cancel{\omega^2}} \cdot r_{A/G}$

$I_G = \frac{1}{12} m(a^2+b^2)$

$= \frac{1}{6} mb^2$

$\therefore \quad T \cdot \frac{2}{\sqrt{2}} \cdot \frac{b}{2} = \frac{1}{6} mb^2 \cdot \alpha$

$m\, b/\sqrt{2} \cdot \alpha \cdot \underline{r}_{A/C} \frac{b}{\sqrt{2}}$

also [illegible] $\Sigma M_A = I_A \alpha +$ [illegible] $= I_A \alpha + r_{G/A} \times m \underline{a}_P$

$W \cdot b/2 = \left[\left(\frac{1}{6} mb^2\right) + m \cdot 2\left(\frac{b}{2}\right)^2\right] \alpha +$ [illegible] $\times m \alpha\, r_{A/c}$

$\alpha = \frac{3g}{4b}$

$\Sigma M_G = I_G \alpha \qquad T\cos 45 \cdot b/2 + T\sin 45 \cdot b/2 = T \cdot 2\frac{b}{2} \cdot \frac{1}{\sqrt{2}} = I_G \alpha = \frac{1}{6} mb^2 \alpha$

$\Rightarrow T = 20.8N$

DO 6/128

6/125

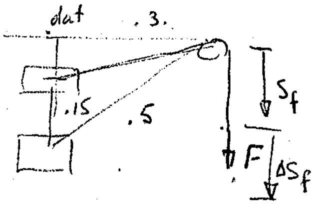
6/142

6/154

[illegible] Do

6/191 $m_1 v_1 \cdot 15'' = (I_0 + mb^2)\omega =$ ~~$I_G\omega + m_{bar} \cdot \frac{v_{bar}}{\omega} + m_{bul} v_{bull}$~~

14-23



~~.4(.4~~ $.4-.15=.25$

$\sqrt{(.25)^2+(.3)^2}=\sqrt{.0625+.09}=\sqrt{.1525}\cong .39$

$F\Delta s = F(.5-.39) = F(.11)$

$T_1=0$

$V_{1e}=0$

$V_{2e}=\frac{1}{2}ks^2 = \frac{1}{2}k(.15)^2$

$T_2 = \frac{1}{2} m_A v_B^2$

$V_1 g = -mg(.4)$

$V_2 g = -mg(.25)$

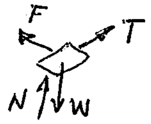
$V_{1e} + \cancel{T_1} + \cancel{V_{1e}} + \Sigma U_{1-2} = T_2 + V_{2e} + V_{2g}$

$F(.11) = \frac{1}{2}mv_B^2 + \frac{1}{2}(100)(.15)^2 + (-.8)(9.81)($

$F(.11) = \overset{4.805}{\cancel{3.1635}} = \frac{1}{2}(.8)(2.5)^2 + \frac{1}{2}(100)(.15)^2 + 1.177$

$F = \overset{43.68}{\cancel{31.95}}$ lb

13.56



$\cancel{F}F = \frac{mv^2}{\rho}$

$T = ma_t = m\dot{v} = 0 \Rightarrow T=0$

$F = k(\rho - 1.25) = \frac{mv^2}{\rho}$

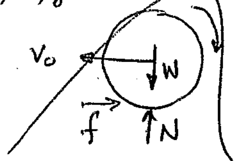
$k\rho^2 - (2.5)(1.25)\rho - mv^2 = 0$

$2.5\rho^2 - 3.125\rho - \frac{2}{32.2}(12)^2 = 0$

$\rho = \frac{3.125 \pm 9.96}{5} = 2.617$ ft

$F = k(\rho-1.25) = 3.418$ lb

19-18



$-I_0\omega_1 + \int fr\,dt = -I_0\cancel{\omega_2} = 0$

$-mv_{0_1} + \int f\,dt = mv_{0_2} = 0$

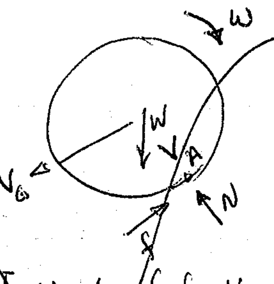
$-I_0\omega_1 + mv_{0_1}r = 0$

$I_0 = \frac{2}{5}mr^2$

$\frac{2}{5}\cancel{m}r^{\cancel{2}}\omega_1 = \cancel{m}v_{0_1}\cancel{r}$

$\omega_1 = \frac{5}{2}\frac{v_{0_1}}{r}$

~~19~~-11



$V_A = \bar{V}_G + \bar{\omega} \times r_{A/G}$

$= -3\bar{i} - \omega\bar{k} \times (-.5\bar{j}) = -3\bar{i} - \omega .5\bar{i}$

$V_A = -7.$

for no slip $-3 - \frac{\omega}{2} = 0$ or $\omega = -6$ rad/s.

$W\cos 30^\circ = N$ ①

② $f = \mu N = .6(W\cos 30^\circ)$

$-I_0\omega_1 + \int fr\,dt - \cancel{\int W\sin 30^\circ dt \cdot r} = -I_0\omega_2$

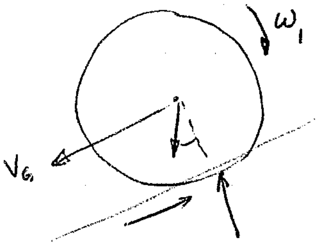
6

$-1.25(8) + .6(\cancel{5})(9.81)\cos 30^\circ t(.5) = -1.25(6)$

$.6W\cos 30^\circ (.5)t$

$I_0 = \cancel{2m}\, mR^2 = 5(.25) = 1.25$ kg m^2

19-11

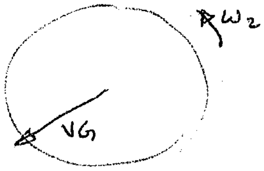


$$-mV_{G_1} + f\Delta t - W\sin 30^\circ \Delta t = -mV_{G_2}$$

$$W\cos 30^\circ - N = 0 \qquad f = \mu N = \mu W\cos 30^\circ.$$

$$-I_o\omega_1 + \int f r\,dt = I_o\omega_2$$

$$-I_o\omega_1 + f r\Delta t = I_o\omega_2 = I_o\frac{V_{G2}}{r}$$



$$I_o = mr^2 = 1.25\,kg\,m^2$$

$$(-W\sin 30^\circ + f)\Delta t = mV_{G_1} - mV_{G_2}$$

$$\Delta t = \left(I_o\frac{V_{G_2}}{r} + I_o\omega_1\right)/fr$$

$$\overset{.0962}{\cancel{50.012}}\,\Delta t = 15 - 5V_{G_2}$$

$$(2.5V_{G_2} + 10)/_{12.744} = \Delta t \qquad .7847 + .1962\,V_{G_2} = \Delta t$$

$$\cancel{39.2448} + \cancel{9.818\,V_{G_2}} = 15 - 5V_{G_2}$$

$$.7549 + .1887\,V_{G_2}$$

$$\boxed{V_{G_2} = 2.745\ m/s. \qquad \Delta t = 1.323\,s.}$$

$$V_A = V_G + \omega\times r = \boxed{-2.745 + \frac{V_G}{r}\cdot r = 0}$$

$$mR^2 = I_G \qquad H_G = I_G\,\omega$$

$$H_{G_1} + \int \Sigma M_G\,dt = H_{G_2}$$

$$+\circlearrowleft \quad -mR^2\omega_1 + f r\Delta t = I_G\,\omega_2^{*} \qquad \omega_2 = \frac{V_G}{r}$$

$$\text{also} \quad mV_{G_1} + \int \Sigma F\,dt = mV_{G_2}$$

				Wu
HW #1	12-2, 12-5, 12-9, 12-13		#1	12-5, 7, 19, 21
HW #2	12-35, [illegible], 12-36, 12-41		←	—
HW #3	12-58, 12-63, 12-68, 12-70		2	12-51, 57, 66, 70
HW #4	12-79, 12-86, 12-96, 12-98, 12-114		3	12-85, 90, 97, 99 ✓
HW #5	12-118, 12-128, 12-146, 12-148		4	✓
HW #6	13-11, 13-18, 13-23, 13-26		5	
HW #7	13-46, 13-55, 13-59, 13-75		6	
8	14-10, 14-19, 14-20, 14-36,	} m. HW.	7	
9	14-50, 14-55, 14-51, 14-61		8	
	15-17, 15-12, 15-6, 15-11	wed HW	9	
	15-26, 15-28, 15-32, 15-37	Monday.	10	
	15-47, 15-48, 15-54, 15-63, 15-65		11	
Rotation	16-7, 10, 13, 30		12	
Veloc.	16-40, 47, 16-50, [illegible] 70		13	
Acc	16-74, 75, 78, 79		14.	

~~17-2, 7, 13, 14~~ ~~17-2, 17, 21, 31, 49, 53, 115~~

17-2, 17, 21, 31, 49, 53, 78, 91

18-6, 10, 17, 15

18-27, 38

12-11, 15, 25, 24 ⊛

HW #1

12-5. Sober driver has reaction distance of 44 ft/sec × reaction time = 33 ft.

Drunk driver " " " " " × reaction time = 132 ft.

Both have breaking distance of $(s_2 - s_1) = \frac{v_2^2 - v_1^2}{2a} = \frac{0-(44)^2}{2(-2)} = 484\ ft$

Total distance sober = 33 ft + 484 ft = 517 ft

Total distance drunk = 132 ft + 484 ft = 616 ft

12-7 To find position at t = 6 sec $s\Big|_{t=6\,sec} = (t^3 - 9t^2 + 15t)\Big|_{t=6\,sec} = -18\ ft.$

To find total distance :

$$\begin{aligned} @\ t&=0 & s&=0 \\ t&=1\,sec & s&=7\,ft \\ t&=5\,sec & s&=-25\,ft \\ t&=6\,sec & s&=-18\,ft \end{aligned} \quad \begin{aligned} &\}\ \Delta s = 7\,ft \\ &\}\ \Delta s = 32\,ft \\ &\}\ \Delta s = 7\,ft \\ \hline &\Delta s = 46\,ft \end{aligned}$$

@ t = 1 and t = 5 sec V = 0 ft/sec

i.e. particle changes direction of motion

12-19 a) $V = (100 - s)\ \frac{mm}{sec} \Rightarrow \frac{d}{dt}V = a = \frac{d}{dt}(100-s) = -\frac{ds}{dt} = -V = -(100-s).$

@ s = 50 mm $\underline{a = -50\ mm/sec}$

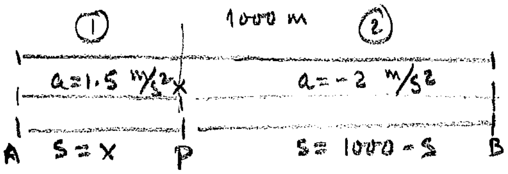
b) when particle stops V = 0 $\Rightarrow 0 = (100 - s)$ or $\underline{s = 100\ mm.}$

c) since $V = \frac{ds}{dt} = 100 - s \Rightarrow \frac{ds}{100-s} = dt \Rightarrow -\ln(100-s)\Big|_{s=0}^{s=100} = t\Big|_{t=0}^{t=t_{final}}$

upper limit is distance traveled when particle comes to a stop. Evaluate integral to get

$-\ln(0) + \ln 100 = \underline{t_{final} = -(-\infty) + \ln 100 = \infty}$

12-21



region ① $V_P^2 = \cancel{V_A^2} + 2a(s_P - s_A)$

$\quad\quad\quad =0$

$V_P^2 = 2(1.5)X = 3X$

region ② $\cancel{V_B^2} = V_P^2 + 2a(s_B - s_A)$

$=0 \quad = V_P^2 + (2)(-2)(1000 - X)$

thus $V_P^2 = 3X = 4(1000 - X) \Rightarrow X = \frac{4000}{7} = 571.43\ m.$

now region ① $s_P = \overset{0}{\cancel{s_A}} + \overset{0}{\cancel{V_A}}t + \frac{1}{2}at_1^2 = 571.43\ m \quad t_1 = \sqrt{\frac{571.43}{.75}} = \underline{27.603\ sec}$

region ② $s_B = s_P + V_P t_2 + \frac{1}{2}at_2^2 \Rightarrow 1000 = X + (1.5t_1)t_2 + \frac{1}{2}(-2)t_2^2$ where $V_P = \underset{=0}{\cancel{V_A}} + at_1 = 1.5t_1$

solving with X = 571.43 m $t_1 = 27.603$ s gives $\underline{t_2 = 20.702\ sec}$

thus $t_{TOT} = t_1 + t_2 = 45.305$ sec. <u>Note</u> pt P is unknown and <u>is not</u> halfway between A & B

12-57, 12-66, 12-75, 12-81

HW #2

12-51 When collision occurs $\bar{r}_A = \bar{r}_B \Rightarrow \overset{①}{3t = 3(t^2-2t+2)}$ and $\overset{②}{9t(2-t) = 3(t-2)}$

solution for ① is $t=2$ and $t=1$; solution for ② is $t=2$ and $t=-1/3$. Common solution to both is $\boxed{t = 2\text{sec}}$ and this is time of collision

at $t=2 \Rightarrow \bar{r}_A = \bar{r}_B = 6\bar{i}$ m. $\bar{V}_A = [3\bar{i} + (18-18t)\bar{j}]\frac{m}{s}$ @ $t=2$sec $\bar{V}_A = (3\bar{i} - 18\bar{j})\frac{m}{s} = 18.25$ m/s

$\bar{V}_B = [(6t-6)\bar{i} + 3\bar{j}]\frac{m}{s}$ @ $t=2$sec $\bar{V}_B = (6\bar{i}+3\bar{j})\frac{m}{s} \Rightarrow V_B = \sqrt{6^2+3^2} = 6.71$ m/sec

12-57 since $(y-40)^2 = 160x$ take $\frac{d}{dt}$ of the eqn $\Rightarrow 2(y-40)\frac{dy}{dt} = 160\frac{dx}{dt} \Rightarrow 2(y-40)V_y = 160\,V_x$

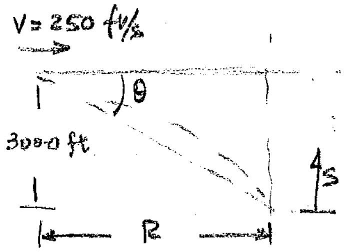
@ $y = 80$ m w/ $V_y = 180$ m/sec $\Rightarrow V_x = \frac{2\cdot 40\cdot 180}{160} = 90\,\frac{m}{sec}$; now $\underline{V = \sqrt{V_x^2+V_y^2} = 201.25 \text{ m/s}}$

To find acceleration: take $\frac{d}{dt}$ again $\Rightarrow 2\left(\frac{dy}{dt}\right)\left(\frac{dy}{dt}\right) + 2(y-40)\frac{d^2y}{dt^2} = 160\frac{d^2x}{dt^2}$

or $2V_y^2 + 2(y-40)a_y = 160\,a_x$; but $\underline{a_y = 0}$ since $V_y = $ const. $\Rightarrow 2V_y^2 = 160\,a_x$ or

$\underline{a_x} = \frac{2V_y^2}{160} = \frac{2(180)^2}{160} = \underline{405 \text{ m/s}^2}$ now $a = \sqrt{a_x^2 + a_y^2} = 405$ m/s²

12-66



$V_{Ax} = 250$ ft/sec ; $V_{Ay} = 0$; $a_y = -32.2$ ft/s ; $a_x \equiv 0$

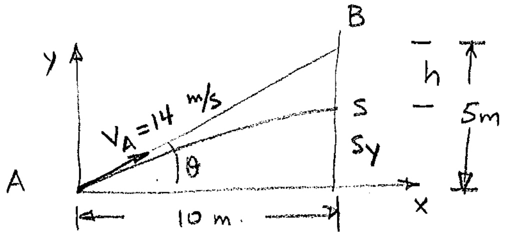
now $S_{final} = S_{initial} + V_A t + \frac{1}{2}gt^2 \Rightarrow 0 = 3000\text{ft} + 0\cdot t + \frac{1}{2}(-32.2)t^2$

$\Rightarrow t = \sqrt{\frac{2(3000)}{32.2}} = 13.65$ sec

$R = V_{Ax}t = 250\cdot(13.65) = 3412.5$ ft $\theta = \tan^{-1}\left(\frac{3000'}{3412.5'}\right) = 41.3°$

Package & plane move with same horiz velocity and also $a_x \equiv 0 \Rightarrow$ package appears below plane.

12-70



place origin wrt girl's hand as she lets go of acorn

$\theta = \tan^{-1} 5/10 = 26.6°$

$V_{Ax} = V_A\cos\theta = 14\cos 26.6° = 12.52$ m/s

$V_{Ay} = V_A\sin\theta = 14\sin 26.6° = 6.26$ m/s

Now distance traveled in x dir $= 10\text{ m} = V_{Ax}t \Rightarrow t = \frac{10\text{m}}{V_{Ax}} = .8$sec time of flight for acorn

" " in y dir by acorn $= S_y = \cancel{S_{y_0}} + V_{A_y}t + \frac{1}{2}gt^2 = (6.26)(.8) + \frac{1}{2}(-9.81)(.8)^2 = \underline{1.869\text{m}}$

distance traveled by squirrel $S_y = S_{y_0} + V_A t^0 + \frac{1}{2}gt^2 = 5\text{m} + 0(.8) + \frac{1}{2}(-9.81)(.8)^2 = 1.869$ m

thus squirrel can reach acorn $\Rightarrow h = 5 - S_y = 3.13$ m

12-88, 12-85, 1294, 12-108, 12-117, 12-126

HW #3

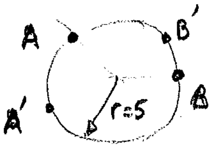
12-85 $\dot{V} = 4t \Rightarrow V = 2t^2 + C$. Since wheel starts from rest $V = 0$ @ $t = 0 \Rightarrow C = 0$

$\dot{S} = V = 2t^2 \Rightarrow S = 2t^3/3 + C$. From rest $S = 0$ @ $t = 0 \Rightarrow C = 0$. Since $\rho\dot{\theta} = V$ and $\rho = $ const.

then $\dot{\theta} = \frac{V}{\rho} = \frac{2t^2}{\rho} \Rightarrow \theta = \frac{2t^3}{3\rho} \Rightarrow$ @ $\theta = \frac{\pi}{6} = (30°) \Rightarrow t = \left[\frac{3(\pi/6)(40')}{2}\right]^{1/3} = 3.156$ sec

$\Rightarrow 4t = \dot{V} = 12.624$ ft/sec & $\underline{V = 19.92 \text{ ft/sec}}$ $\underline{a = \sqrt{(\dot{V})^2 + (V^2/\rho)^2} = 16.06 \text{ ft/sec}^2}$

12-90 Particles A & B are at a distance of $\frac{2\pi r}{3} = \frac{10\pi}{3}$ m @ $t = 0$. Particle B moves a distance $vt = 8 \text{ m/sec}(1) = 8\text{m}$ in 1 sec (since $V = \dot{S} \Rightarrow Vt = S$ since B moves at constant speed ie zero tangential acceleration). Particle A $\dot{V} = 4S_A \Rightarrow VdV = 4S_A dS$ or $V^2/2 = 2S^2 + C$, @ $t = 0$ $V = 8$ m/s and measure S when $t = 0 \Rightarrow C = 8^2/2 = 32 \text{ ft}^2 \Rightarrow V^2 = 4S^2 + 64 \text{ ft}^2 \Rightarrow V = 2\sqrt{S^2 + 16}$



now $V = \frac{dS}{dt} = 2\sqrt{S^2 + 16}$ or $\left.\frac{dS}{\sqrt{S^2 + 16}}\right|_{S=0}^{S} = \left.2dt\right|_{t=0}^{t}$ or $\underline{\frac{1}{2}\ln\left(\frac{S_A + \sqrt{S_A^2 + 16}}{4}\right) = t}$

by trial & error $t = 1$ sec occurs when $\underline{\underline{S_A = 14.5 \text{ m}}}$. Thus $\overset{\frown}{A'B'} = \overset{\frown}{AB} + \overset{\frown}{AA'} - \overset{\frown}{BB'} = \frac{10\pi}{3} + 14.5 - 8$ $= 16.97$ m this is arc $\overset{\frown}{A'AB'}$; now arc $\overset{\frown}{A'BB'} = 2\pi(5) - \overset{\frown}{A'AB'} = \underline{\underline{14.45 \text{ m}}}$.

$a_B = \frac{V^2}{r} = \frac{8 \cdot 8}{5} = \underline{\underline{12.8 \text{ m/sec}^2}}$; $\bar{a}_A = \dot{V}\bar{u}_t + V^2/r\,\bar{u}_n = 4S_A\,\bar{u}_t + \frac{4S_A^2 + 64}{r}\bar{u}_n$

thus $\bar{a}_A = (58\bar{u}_t + 181\ \bar{u}_n)$ m/sec² $\underline{a_A = \sqrt{\dot{V}^2 + (V^2/r)^2} = 190.07 \text{ m/sec}^2}$

$\underline{12\text{-}97}$ $r = (1 + .5\cos\theta)\text{m} \Rightarrow \dot{r} = -.5\sin\theta\,\dot{\theta} \Rightarrow \ddot{r} = -.5\cos\theta\,\dot{\theta}^2 - .5\sin\theta\,\ddot{\theta}$

when $\theta = \pi/4$ $\dot{\theta} = .6$ rad/s $\dot{\theta}' = .25$ rad/sec² $\Rightarrow r = 1 + .5(.7071) = 1.354$ m ; $\dot{r} = -.5(.7071)(.6) = -.212$

$\ddot{r} = -.5[.7071][.36 + .25] = -.2157 \text{ m/s}^2$

$V_r = \dot{r} = -.212$ m/s $V_\theta = r\dot{\theta} = (1.354)(.6) = .8124$ m/s $a_r = \ddot{r} - r\dot{\theta}^2 = -.7031 \text{ m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = .08398 \text{ m/s}^2$

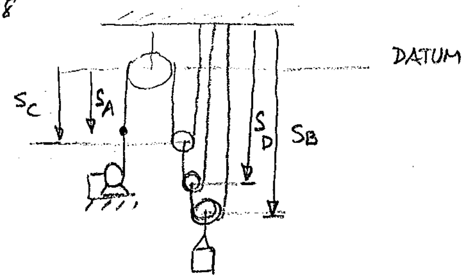
12-99: if $\dot{\theta} = 3$ rad/sec, $r = (4\theta)$m find $V_r, V_\theta, a_r, a_\theta$ when $\theta = \pi/3$ rad $r = 4.1875$ m

$\dot{r} = 4\dot{\theta}$, $\ddot{r} = 4\ddot{\theta} \Rightarrow \dot{r} = 1.2$ m/s $\ddot{r} = 0 \Rightarrow V_r = \dot{r} = 12$ m/s ; $V_\theta = r\dot{\theta} = 4 \cdot \frac{\pi}{3} \cdot (3) = 4\pi =$ 12.57 m/s ; $a_r = \ddot{r} - r\dot{\theta}^2 = -37.7 \text{ m/s}^2$; $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 8\dot{\theta}^2 = 72 \text{ m/s}^2$

12-135 , 12-131 , 12-140 , 12-155 , 12-149 , 12-~~[illegible]~~ 150 ~~[illegible]~~

HW # 4

12-118



Given $\Delta S = 6m$ $\Delta t = 1.5 s$ at B $\Rightarrow$ $V_B = 4 m/s \uparrow$

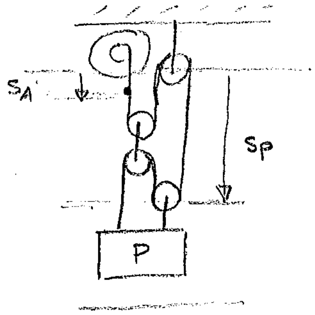
$\boxed{V_B = -4 m/s}$

Using the path coord. for each wire:

$$S_B + (S_B - S_D) = l_1 \Rightarrow 2V_B - V_D = 0$$
$$S_D + (S_D - S_C) = l_2 \Rightarrow 2V_D - V_C = 0 \Big\} \Rightarrow 4V_B = V_C \Big\} \Rightarrow V_A = -8V$$
$$2S_C + S_A = l_3 \qquad 2V_C + V_A = 0$$

$\therefore$ $V_A = 32 m/s \downarrow$

12-121

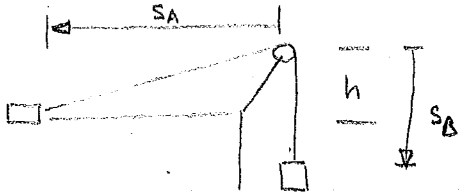


Given $V_A = 2 m/s \uparrow$

Using path coordinate for the wire

$$2S_P + (S_P - S_A) = \text{const} \Rightarrow 3V_P = V_A \Rightarrow$$
$$V_P = V_A/3 = .667 \text{ m/s} \uparrow$$

12-126



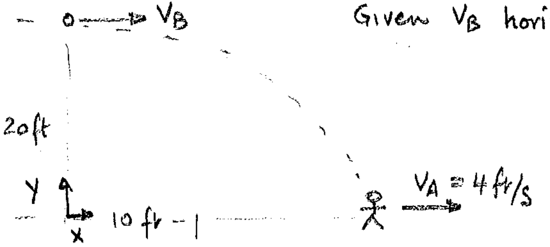
length of rope $= S_B + \sqrt{S_A^2 + h^2}$

$$(1) \quad 0 = V_B + \frac{S_A}{\sqrt{S_A^2 + h^2}} V_A \Rightarrow V_A = -\frac{V_B\sqrt{S_A^2+h^2}}{S_A}$$

take $\frac{d}{dt}$(1) : $$a_B + \frac{\dot{S}_A}{\sqrt{S_A^2+h^2}} V_A + \frac{S_A \dot{V}_A}{\sqrt{S_A^2+h^2}} - \frac{S_A^2 V_A^2}{(S_A^2+h^2)^{3/2}} = 0 \quad \text{or} \quad a_B + \frac{V_A^2 h^2}{(S_A^2+h^2)^{3/2}} + \frac{S_A a_A}{(S_A^2+h^2)^{1/2}} = 0$$

$$\Rightarrow \quad a_A = -\left[\frac{a_B}{S_A}\sqrt{S_A^2+h^2} + \frac{V_A^2 h^2}{(S_A^2+h^2)S_A}\right]$$

12-143



Given $\bar{V}_B$ horizontal when A is 10ft from wall. find V_B & $V_{B/A}$

Define coordinate system at wall. Need to find time of flight = time for ball to drop 20ft.

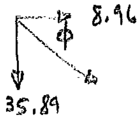
$$\therefore \quad S_y = S_{y_0} + V_{0_y} t_f + \frac{1}{2} a t_f^2 = 20 + 0t_f - \frac{1}{2}(32.2) t_f^2 = 0 ft$$

$$\therefore \quad t_f = \sqrt{\frac{2 S_{y_0}}{g}} = 1.114 s$$

Ball must travel in x direction : $10 ft + V_A t_f = 14.456 ft = V_B t_f \Rightarrow \underline{V_B} = \frac{14.456 ft}{1.114 s} = \underline{12.96 ft.}$

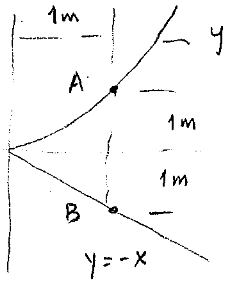
Components of velocity for ball : $V_y = V_{y_0} + a t_f = 0 - 32.2(1.114) = -35.89 ft/s$ and $V_x = V_B = 12.96 ft.$

$\bar{V}_B = (12.96\,\bar{i} - 35.89\,\bar{j}) ft/s$; $\bar{V}_A = (4\,\bar{i}) ft/s$ $\quad \bar{V}_{B/A} = \bar{V}_B - \bar{V}_A = (8.96\,\bar{i} - 35.89\,\bar{j}) ft/s \Rightarrow \underline{V_{B/A} = 37.0 ft/s}$



$\phi = \tan^{-1}\left(\frac{35.89}{8.96}\right)$

12-146



NOTE!

Given V_A & $V_B = 8 \, m/s$ $\quad \dot{V}_A = 5 \, m/s^2 \quad a_B = -6 \, m/s^2$

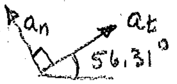
find: $a_{A/B}$

Since V_A & $\dot{V}_A$ is given for particle A use n, t coordinates to find $\bar{a}_A \Rightarrow$ need to find $\rho = \left| \frac{[1+(y')^2]^{3/2}}{y''} \right|$

y' from eqn $\Rightarrow 2yy' = 3x^2$ or

$$y' = \left.\frac{3x^2}{2y}\right|_{\substack{x=1\\y=1}} = 3/2 \qquad y'' = \left.\frac{3}{4\sqrt{x}}\right|_{x=1} = 3/4 \qquad \therefore \rho = \frac{[1+(1.5)^2]^{3/2}}{3/4} = 7.812 \, m \therefore a_n = \frac{V^2}{\rho} = \frac{8^2}{7.812} = 8.1925 \frac{m}{s}$$

$a_t = \dot{V}_A = 5 \, m/s$; direction of a_t is tangent to curve at $x = y = 1$ $\therefore$ direction is $\tan^{-1}\left(\frac{dy}{dx}\right) = 56.31°$



$$\bar{a}_A = \bar{a}_n + \bar{a}_t = (-6.817\bar{i} + 4.544\bar{j}) + (2.774\bar{i} + 4.1603\bar{j}) = (-4.043\bar{i} + 8.714\bar{j}) \frac{m}{s}$$

For particle B: take $\frac{d}{dt}(y = -x) \Rightarrow \dot{y} = -\dot{x}$ & $V_B = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{2\dot{x}^2} \Rightarrow \dot{x} = 5.657 \frac{m}{s} \quad \dot{y} = -\dot{x}$

from the slope of the curve, change in + x cause - change in y.

Now take $\frac{d}{dt}$ of equation to get $\ddot{y} = -\ddot{x}$ or $6\frac{m}{s^2} = |\bar{a}_B| = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{2\ddot{x}^2} \Rightarrow \ddot{x} = -4.243 \frac{m}{s^2} \quad \ddot{y} = -\ddot{x}$

The signs of $\ddot{x}$ and $\ddot{y}$ were chosen as follows: since B decelerates along $y = -x$, its x component of deceleration must also be −. The y component also decelerates but along the −y axis $\Rightarrow$ a_{By} is a positive quantity. Then $\bar{a}_B = (-4.243\bar{i} + 4.243\bar{j}) \, m/s^2$

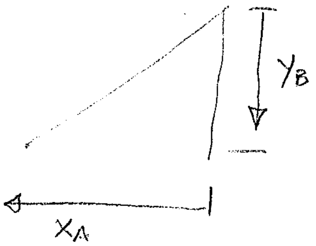
$$\text{Now} \quad \bar{a}_{A/B} = \bar{a}_A - \bar{a}_B = [-4.043 - (-4.243)]\bar{i} + [8.714 - 4.243]\bar{j} \quad \frac{m}{s^2}$$
$$= (+.2\bar{i} \quad + 4.471\bar{j}) \, m/s$$

$$a_{A/B} = |\bar{a}_{A/B}| = 4.475 \, m/s^2$$

$\bar{a}_{A/B}$

$$\theta = \tan^{-1}\left(\frac{4.475}{.2}\right) = 87.44°$$

12-129



$y_B + \sqrt{x_A^2 + 64} = $ Const **@ $y_B = 8$, $x_A = 0$ $\Rightarrow$ Const = 16 m**

$$v_B + \frac{x_A \, v_A}{\sqrt{x_A^2 + 64}} = 0$$

Given $v_A = 1.5 \, m/s \leftarrow \quad x_A = 4m$

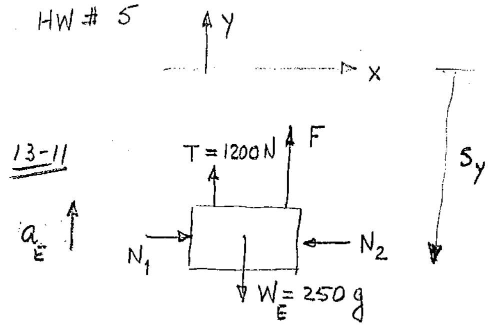
$$v_B = -\frac{(1.5 \, m/s)(4m)}{\sqrt{80}} = -.671 \, m/s$$

$\Rightarrow v_B = .671 \, m/s \uparrow$

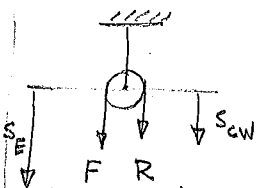
13-19, 25, 32, 38

HW # 5

Prob 13-11



Elevator



F=R due to massless pulley

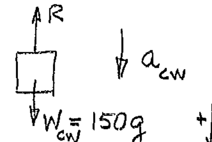
From kinematics

$S_E + S_{cw} = \text{const}$

$V_E + V_{cw} = 0$

$a_E + a_{cw} = 0$

or $\boxed{a_{cw} = -a_E = a}$



Counterweight

$+\downarrow \Sigma F_y = m a_{cw}$

$$W_{cw} - R = \frac{W_{cw}}{g} a_{cw}$$

$$\boxed{F = R = W_{cw}\left[1 - \frac{a}{g}\right]} \quad (1)$$

N_1, N_2 are contact forces due wheel-track contact.

$\therefore$ $N_1 = N_2$ since a_x of elevator $= 0$

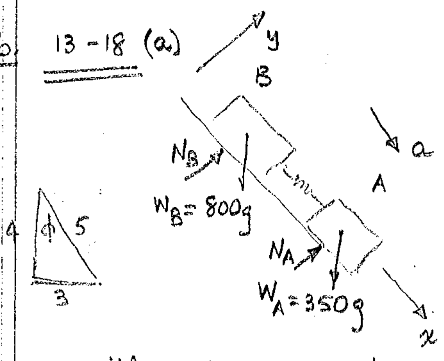
$$\boxed{\Sigma F_y = F + 1200 - W_E = \frac{W_E}{g} a} \quad (2)$$

thus put (1) into (2) & solve for a $\Rightarrow$ $a = .5475\ m/s^2$ and then solve for $F = R = 1389.4\ N$

$v_{f_E}^2 = v_{i_E}^2 + 2a_E(s_{f_E} - s_{i_E})$; but $s_{f_E} - s_{i_E} = 10\ m$ $a_E = .5475\ m/s^2$ and $v_{i_E} = 0 \Rightarrow v_{f_E} = \sqrt{2a_E \Delta s_E}$

$v_{f_E} = \sqrt{2(.5475)(10)} = 3.31\ m/s^2$ ↑

Prob 13-18 (a)

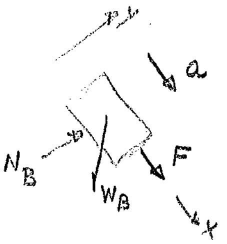


without brakes applied

$$\Sigma F_x = W_B \cos\phi + W_A \cos\phi = \frac{W_B + W_A}{g} a \quad \text{and } \cos\phi = 4/5$$

$$\underline{a = 7.848\ m/s^2}$$

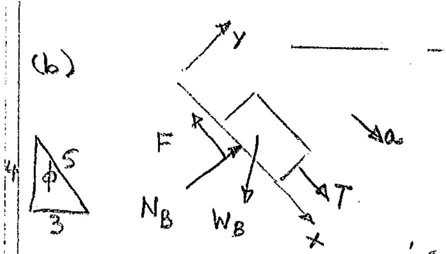
Now look at boxcar B.



$$\Sigma F_x = W_B \cos\phi + F = \frac{W_B}{g} a$$

$$F = W_B\left[\frac{a}{g} - \cos\phi\right] = 0 \quad \text{and } F = k\Delta x \Rightarrow \underline{\Delta x = 0}$$

(b)

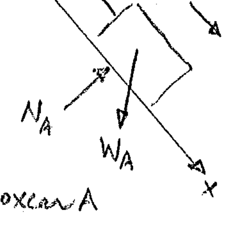


with brakes applied at B

$$\Sigma F_y = N_B - W_B \sin\phi = 0 \quad \& \sin\phi = 3/5$$

$$N_B = W_B \sin\phi = 800 \cdot g \cdot 3/5 = 4708.8\ N$$

$$\underline{F = \mu N_B = 1883.52\ N.}$$



For boxcar A

$$\Sigma F_x = T - F + W_B \cos\phi = \frac{W_B}{g} a$$

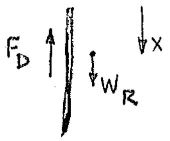
$$\Sigma F_x = -T + W_A \cos\phi = \frac{W_A}{g} a$$

} solve simultaneously to get T and a

$\underline{a = 6.21\ m/s^2}$ $\underline{T = 573.25\ N}$

but $T = k\Delta s$ and $\underline{\Delta s = \frac{T}{k} = .955 m}$

Prob. 13-23



$$\Sigma F_x = W_R - F_D = \frac{W_R}{g} a \Rightarrow 20 - .25v = \frac{20}{g} a = \frac{20}{g} v \frac{dv}{ds}.$$

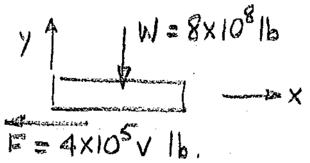
$$\text{thus} \int_{s=0}^{s_f} \frac{g}{20} ds = \int_{v=0}^{v_f} \frac{v\,dv}{20-.25v} = \frac{1}{.25}\int \frac{.25v\,dv}{20-.25v} = \frac{1}{.25}\int \frac{(-20+.25v+20)dv}{20-.25v}$$

$$= \frac{1}{.25}\int \left[-dv + \frac{20\,dv}{20-.25v}\right] = \frac{1}{.25}\left[-v - \frac{20}{.25}\ln(20-.25v)\right]_{v=0}^{v_f}$$

$$\frac{g}{20} s_f = -\frac{v_f}{.25} - \frac{20}{(.25)^2}\left\{\ln\left[\frac{20-.25v_f}{20}\right]\right\}$$

when $v_f = 12$ ft/s $\quad s_f = -\frac{20}{g}\left[\frac{v_f}{.25} + \frac{20}{(.25)^2}\ln\left(\frac{20-.25v_f}{20}\right)\right] = \underline{\underline{2.49 \text{ ft}}}$

Problem 13-26



$$\Sigma F_x = -F = \frac{W}{g} a \Rightarrow -4\times10^5 v = \frac{8\times10^8}{g} a$$

$$\text{or } -v = \frac{2\times10^3}{g} v\frac{dv}{ds} \quad \text{or } -1 = \frac{2\times10^3}{g}\frac{dv}{ds}$$

then $-g\,ds = 2\times10^3 dv \Rightarrow -gs = 2\times10^3 v + C$

if $v = 3$ ft/s measure s from when engine shuts off, ie $v = 3$ ft/s, $s = 0$ ft at $t = 0$.

$\Rightarrow C = -6\times10^3$ ft/s $\Rightarrow -gs = 2\times10^3(v-3)$

when $v = 1$ ft/s $\quad -32.2s = -4\times10^3 \Rightarrow \underline{\underline{s = 124.224 \text{ ft.}}}$

also since

$$-4\times10^5 v = \frac{8\times10^8}{g}\frac{dv}{dt} \Rightarrow -g\,dt = 2\times10^3 \frac{dv}{v} \Rightarrow -gt = 2\times10^3 \ln v + C$$

if $t=0$ when $v = 3$ ft/s $\Rightarrow C = -2\times10^3 \ln 3$ or $-gt = 2\times10^3 \ln(v/3)$

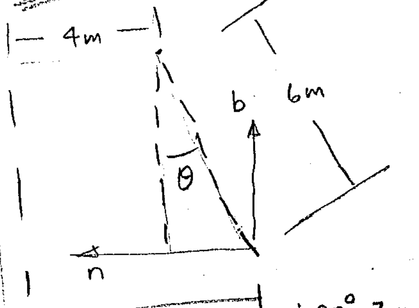
when $v = 1$ ft/s $\Rightarrow t = -\frac{2\times10^3}{g}\ln(1/3) = \underline{\underline{68.24 \text{ s}}}$.

Note: since $-gt = 2\times10^3 \ln v/3 \Rightarrow 3e^{-\frac{gt}{2000}} = v$; thus $s = \frac{2\times10^3}{g}(3-v)$

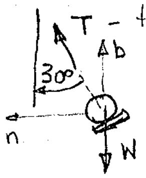
$$s = \frac{6\times10^3}{g}\left[1 - e^{-gt/2000}\right] \text{ ft}$$

HW #6

13-46



$\rho = 4m + 6m \sin 30^\circ = 7m$



T - tension in cable

$$\Sigma \bar{F} = T\sin 30^\circ \, \bar{u}_n + T\cos 30^\circ \bar{u}_b - W \bar{u}_b$$

Thus $T\sin 30^\circ = \frac{W}{g}\frac{v^2}{\rho} = ma_n \quad (1)$

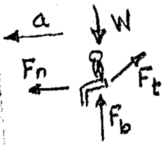
$T\cos 30^\circ - W = m\cancel{a_b} = 0$

Thus $\boxed{\frac{W}{\cos 30^\circ} = T}$ $\Rightarrow$ from (1) $V^2 = \frac{\rho g}{W} T\sin 30^\circ = \rho g \tan 30^\circ$

$$V = \sqrt{\rho g \tan 30^\circ} = \sqrt{7(9.81)(.5773)}$$
$$= \underline{6.296 \ m/s}$$

Note: since constant speed $\Rightarrow a_t = 0$

since $T = \frac{W}{\cos 30^\circ} = \frac{80(9.81)}{.866} = \underline{\underline{906.24 N}}$

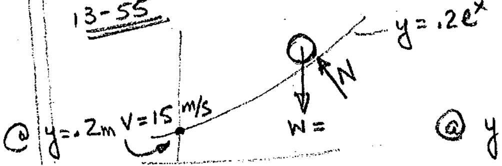


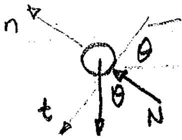
This is FBD of forces on man due to seat. What man exerts on seat is of equal magnitude but opposite direction. From this $\Sigma F_b = F_b - W = 0$ or $F_b = W = 50(9.81) = \underline{490.5 \text{ Newtons}}$

Also there are no forces or accelerations in the t direction $\Rightarrow \Sigma F_t = \underline{F_t = 0}$

And $\Sigma F_n = F_n = m\frac{v^2}{\rho} = 50\frac{(6.296)^2}{7m} = 283.14 \ N.$

13-55





tangent to path $= \frac{dy}{dx} = \tan\theta$

put into this to get $\theta = 11.31^\circ$

@ $y = .2m \quad x = 0m \Rightarrow y' = .2e^x \Rightarrow y'\big|_{x=0} = .2$ & $y''\big|_{x=0} = .2e^x\big|_{x=0} = .2$

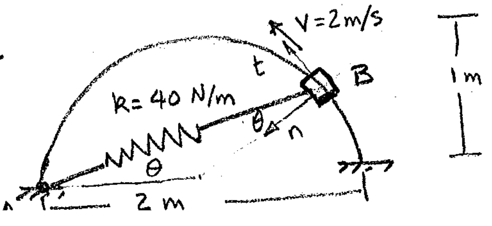
now $\underline{\underline{\rho}} = (1 + y'^2)^{3/2}/y'' = \frac{(1 + .04)^{3/2}}{.2} = \underline{\underline{5.303 \ m}}$

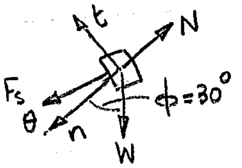
Given also: $V = 15$ m/s when $x = 0$m (ie when $y = .2$m). Now use eqns of motion

$\Sigma F_t = W\sin\theta = ma_t \quad \therefore \quad g\sin\theta = a_t = \underline{\underline{1.924 \ m/s^2 = \dot{V}}}$

$\Sigma F_n = N - W\cos\theta = mv^2/\rho \quad \underline{\underline{N}} = W\cos\theta + \frac{mv^2}{\rho} = m\left[g\cos\theta + \frac{v^2}{\rho}\right] = \underline{\underline{4163.86 N}}$

13-59





N is contact force of collar to rod (in

F_s is spring force

W is weight

when $\theta = 30°$, length of spring is $2(1\cos\theta) = 2\cos 30° = 1.732\text{m} = \overline{AB}$

$\Delta s = \overline{AB}$ − unstretched length $= 1.732 - .2 = 1.532\text{m}$ and $F_s = k\Delta s = \underline{\underline{61.28\text{N}}}$ (1)

The eqns of motion lead to $\Sigma F_t = -W\sin 30° + F_s \sin 30° = m a_t$ (2)

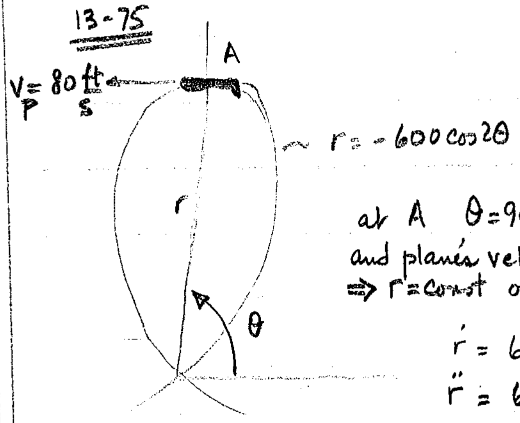
$\Sigma F_n = W\cos\phi + F_s\cos\theta - N = \frac{mv^2}{\rho}$ (3) where $V = 2\text{m/s}$ $\rho = 1\text{m}$ = radius of circle

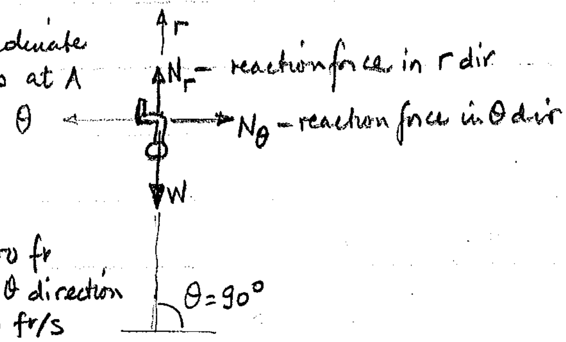
Put (1) into (2) and with $m = 5\text{kg}$ $\Rightarrow$ $\underline{\underline{a_t}} = \left(\frac{F_s}{m} - g\right)\sin 30° = \underline{\underline{1.223\text{ m/s}^2}}$

Put (1) into (3) with the values of v, ρ $\Rightarrow$ $\underline{\underline{N}} = (F_s + W)\cos 30° - \frac{mv^2}{\rho} = \underline{\underline{75.55\text{N}}}$

$a_n = \frac{v^2}{\rho} = 4\text{ m/s}^2$ $\therefore$ $\underline{\underline{a}} = \sqrt{a_t^2 + a_n^2} = \underline{\underline{4.183\text{ m/s}^2}}$

13-75





at A $\theta = 90°$, $r = 600$ ft
and plane's veloc. is in θ direction
$\Rightarrow$ r = const or $\underline{\underline{\dot{r} = 0\text{ ft/s}}}$

$$\dot{r} = 600(2\sin 2\theta\ \dot{\theta}) \quad (1)$$

$$\ddot{r} = 600(\cos 2\theta)(4\dot{\theta}^2) + 600(\sin 2\theta)(2\ddot{\theta}) \quad (2)$$

$V_p^2 = \dot{r}^2 + (r\dot{\theta})^2 = 0 + (600\dot{\theta})^2 = (80)^2$ or $\underline{\underline{\dot{\theta} = .133\text{ rad/s.}}}$ = constant since V_p & r are const.

$\Rightarrow$ $\underline{\underline{\ddot{\theta} = 0}}$ $\Rightarrow$ $\underline{\underline{\ddot{r}}} = 2400\cos(2 \cdot 90°)(\dot{\theta}^2) = \underline{\underline{-42.67\text{ ft/s}^2}}$

$a_r = \ddot{r} - r\dot{\theta}^2 = -42.67 - 600(.133)^2 = -53.33\text{ ft/s}^2$

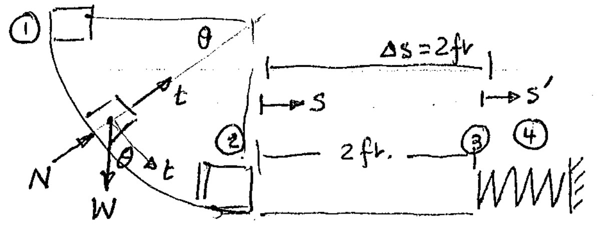
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = r(0) + 2(0)\dot{\theta} = 0$ $\Rightarrow$ $\Sigma F_\theta = -N_\theta = m\cancelto{0}{a_\theta} = 0$ or $\underline{\underline{N_\theta = 0}}$

$+\uparrow \Sigma F_r = N_r - W = m a_r = N_r - 150 = \frac{150}{32.2}(-53.33)$ or $\underline{\underline{N_r = -98.45\text{ lb}}}$

The negative sign means he will have to be strapped into his seat or fall out of the seat at the top of the loop.

HW # 8

14-10



when the block slides along the arc the forces acting are the normal force N and the weight W. The only force that does work is the component of W along the t axis ∴ at the bottom of the slide

$T_1 + \Sigma U_{1-2} = T_2$, Since $V_1 = 0$ then $T_1 = 0$.

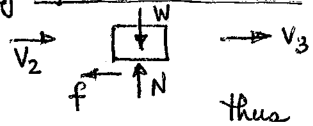
$$\Sigma U_{1-2} = \int_{\theta=0}^{\theta=\pi/2} (W\cos\theta)\, r\, d\theta = +Wr\sin\theta \Big|_{\theta=0}^{\theta=\pi/2} = +Wr$$

where r is the radius (3ft)

Thus $Wr = T_2 = \frac{mV_2^2}{2} \Rightarrow \sqrt{2rg} = V_2 = 13.9$ ft/s

To go from ② to ③

FBD:



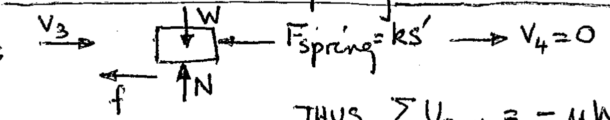
$\Rightarrow T_2 + \Sigma U_{2-3} = T_3$ where $\Sigma U_{2-3} = \int_{S=0}^{S=2} -f\,ds = -\mu W \Delta s$

thus $\frac{1}{2}mV_2^2 - \mu W \Delta s = \frac{1}{2}mV_3^2$ or $Wr - \mu W\Delta s = \frac{1}{2}\frac{W}{g}V_3^2$

thus $V_3 = \sqrt{(r - \mu\Delta s)2g} = 12.94$ ft/s.

When it strikes the spring and comes to rest (max compression)

FBD:



$T_3 = \frac{1}{2}mV_3^2$; $\Sigma U_{3-4} = \int_{s'=0}^{S_{stop}} -f\,ds' - \int_{s'=0}^{S_{stop}} F_{Spring} \cdot ds'$

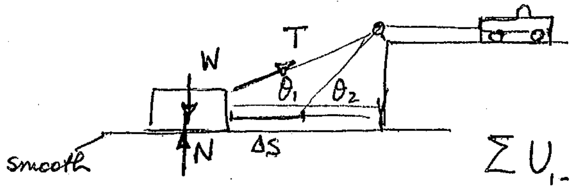
THUS $\Sigma U_{3-4} = -\mu W S_{STOP} - \frac{1}{2}kS_{STOP}^2$

thus $T_3 + \Sigma U_{3-4} = T_4 = \frac{1}{2}mV_4^2 = 0 \Rightarrow \frac{1}{2}mV_3^2 - \mu W S_{STOP} - \frac{1}{2}kS_{STOP}^2 = 0$

or $13 - .2(5)S_{STOP} - \frac{1}{2}(40)S_{STOP}^2 = 0$, Solve for $S_{STOP} = +.7816$ft or $-.8316$ft

THUS $\underline{S_{STOP} = .7816 \text{ ft}}$

14-17 This is the case of an inextensible cord connecting 2 particles. Thus work done by cable is internal and $\Sigma\int \bar{f}_i \cdot d\bar{r}_i = 0$. But to find work done by the truck, must isolate the block with all forces acting on it.



$$\Sigma U_{1-2} = \int_{S_1}^{S_2} T\cos\theta\, ds$$

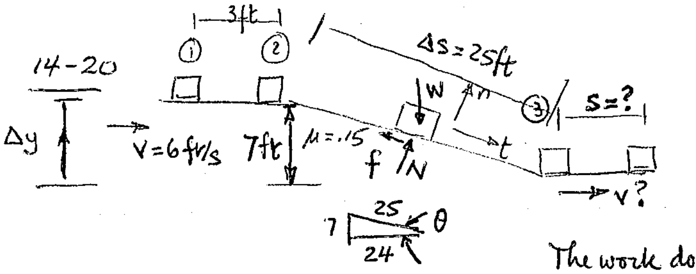
now $\tan\theta = \frac{8}{S}$ and take $\frac{d}{ds}$ of both sides $\frac{d}{ds}\tan\theta = \sec^2\theta \frac{d\theta}{ds}$; $d/ds\,(8/S) = -\frac{8}{S^2} = -\frac{1}{8}\left(\frac{8}{S}\right)^2 = -\frac{1}{8}\tan^2\theta$

thus $\sec^2\theta \frac{d\theta}{ds} = -\frac{1}{8}\tan^2\theta$ or $\boxed{ds = -8\, d\theta/\sin^2\theta}$

and $$\Sigma U_{1-2} = -8\int_{\theta=30^\circ}^{\theta=45^\circ} T\frac{\cos\theta}{\sin^2\theta}\, d\theta = 8T/\sin\theta \Big|_{\theta=30^\circ}^{\theta=45^\circ} = 8T\left[\frac{1}{.7071} - \frac{1}{.5}\right] = -2343.1 J$$

this is work done in moving the block. Work done by the truck is negative of this. Here we assume that when $S=S_1$ $\theta=30°$, $S=S_2$ $\theta=45°$.

14-20



look at package on the ramp.

in ⊥ dir to ramp (n dir): $\Sigma F_n = 0$

or $\underline{N} = W\cos\theta = 15 \cdot \frac{24}{25} = \underline{\underline{14.4 lb}}$.

The work done to move package from pt ② to ③:

$$\Sigma U_{2-3} = -W\,\Delta y - f\Delta s = -15(-7ft) - \mu N\,(25ft) = (105-54)\,lbft = 51\ lbft$$

$$\therefore\quad T_2 + \Sigma U_{2-3} = T_3 \quad\text{or}\quad \frac{1}{2}\left(\frac{15}{32.2}\right)(6)^2 + 51\ lb\text{-}ft = \frac{1}{2}\left(\frac{15}{32.2}\right)(V_3)^2$$

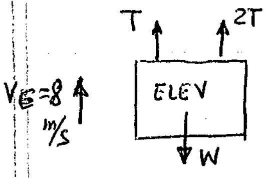
$\Rightarrow V_3 = 15.97$ ft/s. This is speed of package at pt ③. If it is not to slide speed of conveyor ≡ speed of package at pt ③ $\therefore$ $\underline{V_{conveyor} = 15.97\ ft/s}$

Note that weight moves downward 7ft ∴ work contrib is +.

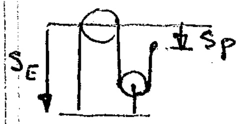
Since conveyor spacing at top is 3ft. ⇒ time between packages is $\frac{3ft}{6ft/s} = .5sec$.

This is ~~speed~~ time interval packages are delivered to ramp. This time interval is also that at lower conveyor ∴ $\underline{\underline{S}} = V_3 \cdot (.5sec) = \underline{\underline{7.98\ ft}}$.

Problem 14-36 was done in class last Thursday.



Forces on elevator T, 2T are due to pulley; since elevator veloc = const, tension in rope = const. By $\Sigma F_y = ma_y = 0 \Rightarrow T = \frac{W}{3} = 1635N$



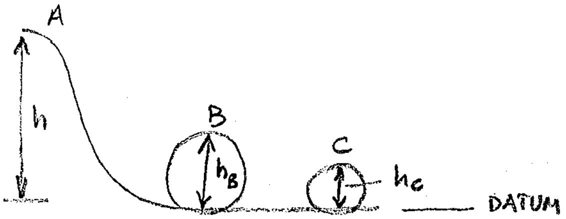
By kinematics $3S_E - S_p = \text{const}$ where S_p is a pt on the rope to motor thus $3V_E = V_p$ or $V_p = 24$ m/s ↑ V_E here is -8 m/s since S_E is ↓+

Since motor pulls rope power output = force of rope · velocity of rope or $(1635N)(24\ m/s) = 39{,}240$ watts. Power input is 60 kW

thus $\varepsilon = \frac{\text{power output}}{\text{power input}} = \frac{39240\ W}{60000\ W} = .65$ or 65% efficiency

HW # 8b

14-50



If the car can make it up to B, it will make it to C. Since, the only forces acting on the car are conservative forces $T_A + V_A = T_B + V_B = T_C + V_C$. But since $V_A > V_B > V_C$ then $T_A < T_B < T_C$ ie $V_B < V_C$. $\Rightarrow T_A = \frac{1}{2} m V_A^2 = \frac{1}{2}(800)(3)^2$; $V_A = (800)(9.81)(h) = Wh$

$V_B = W h_B = W(2)(10)$; $T_B = \frac{1}{2} m V_B^2$. From min. height condition:

$\Rightarrow \Sigma F_n = W + N_B = \frac{m V_B^2}{\rho_B}$ where $\underline{N_B}$(contact force) $= \underline{0}$ $\Rightarrow V_B = \sqrt{\frac{W \rho_B}{m}} = \sqrt{\frac{W h_B}{2m}}$

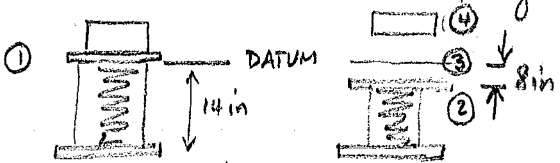
$\therefore$ $\underline{V_B = 9.90 \text{ m/s}}$ and from $T_A + V_A = T_B + V_A \Rightarrow \underline{h = 24.5 m}$

at C: $T_A + V_A = T_C + V_C = \frac{1}{2} m V_C^2 + W(14) \Rightarrow V_C = 14.69$ m/s, also at C

$\Sigma F_n = W + N_C = \frac{m V_C^2}{\rho_C}$ or $\underline{N_C} = \frac{m V_C^2}{\rho_C} - W = \frac{800(14.69)^2}{7} - 800(9.81) = \underline{16.8 kN}$

14-51

Pick the datum when the spring is 14 in long. The potential $V = V_g + V_e$



@ ① $V_g = W(0) = 0$ $V_e = \frac{1}{2} k (\frac{1}{2}')^2 = 2.5$ lb-ft

$T = 0$ since we start from rest

Pt ① when weight is put on

Pt ② is after weight & spring are pushed 8 in

Pt ③ is when plate & weight lose contact

Pt ④ is max height of block

@ ② $V_g = -W(\frac{2}{3})'$ $V_e = \frac{1}{2} k (\frac{1}{2}' + \frac{2}{3}')^2$

$T = 0$ since we start from rest

@ ③ $V_g = W(0)$ $V_e = \frac{1}{2} k (\frac{1}{2})^2$

$T = \frac{1}{2} m_{Block} V_{Block}^2$

@ ④ $V_g = Wh$ $V_e = \frac{1}{2} k (\frac{1}{2})^2$

$T = \frac{1}{2} m_{Block} \cdot 0^2$

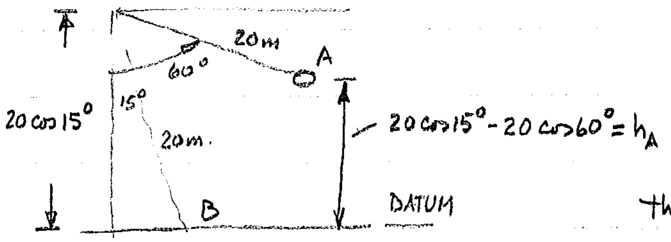
$T_2 + V_2 = T_3 + V_3 \Rightarrow T_3 = T_2 + V_2 - V_3$ also $T_3 + V_3 = T_4 + V_4$ where $T_4 = 0$

Thus $V_4 = Wh = T_2 + V_2 = 0 + \left[-W \cdot \frac{2}{3} + \frac{1}{2} k (\frac{1}{2} + \frac{2}{3})^2 \right]$

or $\underline{h = 1.59 \text{ ft}}$

Caution: must convert all distances to feet first. Only forces acting are conservative forces (and internal forces that travel same distance & whose $\Sigma U = 0$)

14-55



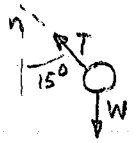
@ A : $T_1 = 0$ since we start from rest

$V_1 = W h_A$

@ B : $T_2 = \frac{1}{2} m V_B^2$ and $V_2 = 0$

thus $V_B = \sqrt{\frac{2Wh_A}{m}} = 13.52$ m/s.

Note only forces acting ~~one~~ particle is tension in cable and weight. Tension produces

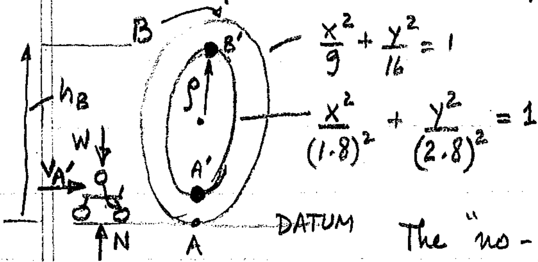


no work since T is ⊥ to direction of motion & W is conservative ∴ work done by is is $W\Delta y$. Just before crash $T - W\cos 15° = \frac{mV_B^2}{\rho}$

$T = W\cos 15° + \frac{mV_B^2}{\rho} = 148\,937.6$ N. This is maximum since at any intermediate position $T = W\cos\theta + \frac{mV^2}{\rho}$. As θ↓ cos θ↑ and V↑; thus max is at min θ.

14-61

Since the man's center of mass is 1.2 m above ground he will trace out an ellipsoid within the loop given by $\frac{x^2}{(1.8)^2} + \frac{y^2}{(2.8)^2} = 1$. Since the only forces acting

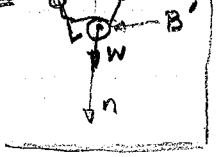


on him are the weight force & the normal force, Just as with the previous problems, the normal force will do no work, whereas the weight force is conservative. The "no-falling condition" is critical at the top of the loop; thus at B' we must relate the values of T & V to those at A'. $T_{A'} + V_{A'} = T_{B'} + V_{B'}$

@ A' $T_{A'} = \frac{1}{2} m V_{A'}^2$, $V_{A'} = W(1.2m)$. $T_{B'} = \frac{1}{2} m V_{B'}^2$ & , $V_{B'} = W(8-1.2) = W(16.8\ m)$

Note $V_{A'}$ & $V_{B'}$ are to the center of mass of the system. But at B' we also know that $\Sigma F_n = \frac{mV_{B'}^2}{\rho} = W + N_B = \frac{mV_{B'}^2}{\rho}$. To find ρ use eqn for center of mass & take $\frac{d}{dx}$ of it $\Longrightarrow \frac{2x}{(1.8)^2} + \frac{2yy'}{(2.8)^2} = 0$ ∴ $y' = -\frac{(2.8)^2}{(1.8)^2}\frac{x}{y}\Big|_{\substack{x=0,\,y=2.8m\\(\text{pt } B')}} = 0$

at B



also $\frac{2}{(1.8)^2} + \frac{\cancelto{0}{2y'^2} + 2yy''}{(2.8)^2}\Big|_{\substack{x=0,\,y=2.8m\\(\text{pt } B')}} = 0$ ∴ $y'' = -\frac{(2.8)^2}{(1.8)^2}\frac{1}{y}\Big|_{\substack{x=0,\,y=2.8m\\(\text{pt } B')}} = -.864$

Thus $\underline{\underline{\rho}} = \left|\frac{(1+y'^2)^{3/2}}{y''}\right| = \left|-\frac{1}{.864}\right| = \underline{\underline{1.157\ m}}$. Critical condition at B is $N_B = 0$ thus

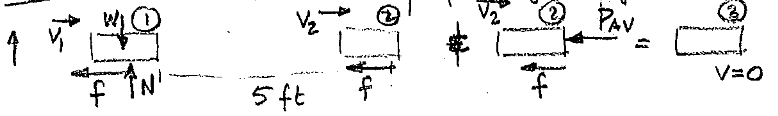
$W = \frac{mV_{B'}^2}{\rho}$ or $\underline{V_{B'}} = \sqrt{\rho g} = \underline{\underline{3.37\ m/s}}$. Since $\frac{1}{2}\cancel{m}V_{A'}^2 + \overset{g}{\cancel{W}}(1.2) = \frac{1}{2}\cancel{m}V_{B'}^2 + \overset{g}{\cancel{W}}(16.8)$

$\Longrightarrow V_{A'} = \sqrt{V_{B'}^2 + 2g[6.8 - 1.2]} = 11.01$ m/s. $V_{A'}$ is veloc. of center of mass of man + bike.

<u>Remember</u> <u>must</u> <u>take</u> <u>potentials</u> & <u>kinetic energies</u> <u>wrt</u> <u>center of mass</u> of system.

HW # 9

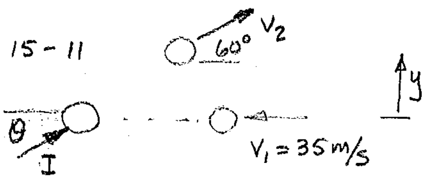
15-6 ① Must find velocity of car just before crash using work & energy. Then use impulse and momentum.



From $\Sigma F_y = 0$ W=N . From work & energy $T_1 + \Sigma U_{1-2} = T_2$ or $\frac{1}{2}mV_1^2 - \mu W s = \frac{1}{2}mV_2^2$

finally $mV_2 - \mu W \Delta t - P_{AV}\Delta t = mV_3 = 0 \Rightarrow V_2 = 11.71$ m/s & $P_{AV} = 78956$ N.

15-11



Use $V_{yf}^2 = V_{yi}^2 + 2a_c(s_f - s_i)$ to get $V_{yi} = V_2 \sin 60°$

then use impulse & momentum to find $\bar{I}$. $a_c = -9.81\ m/s^2$

$V_{yf} = 0$ $V_{yi} = ?$ $s_f - s_i = 50m$ $\therefore V_{yi} = \sqrt{-2a_c(50)} = 31.32$ m/s

$\therefore \bar{V}_2 = \frac{V_{yi}}{\sin 60°} = 36.17$ m/s $\measuredangle 60°$

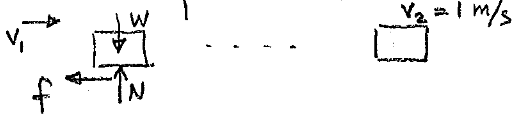
Now assume +x is to right $\Rightarrow -mV_{1_x} + I_x = mV_{2_x} \Rightarrow -.4(35) + I_x = .4(36.17\cos 60°)$

$I_x = 21.233$ N·s

also $mV_{1_y} + I_y = mV_{2_y} \Rightarrow .4(0) + I_y = .4(36.17 \sin 60) = 12.528$ N·s

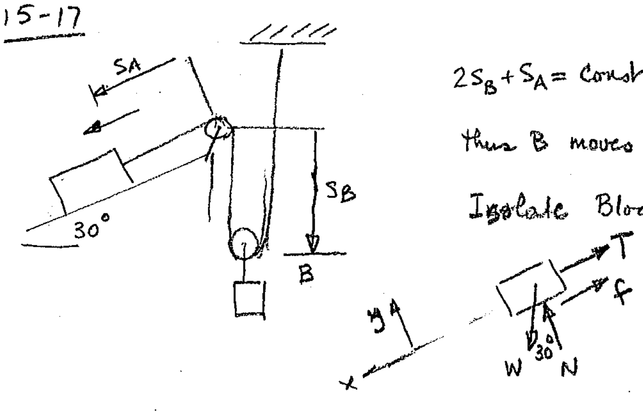
$I = \sqrt{I_x^2 + I_y^2} = 24.654$ N·s $\theta = \tan^{-1}\left(\frac{I_y}{I_x}\right) = 30.54° \measuredangle \theta$

15-12 Use impulse & momentum where the impulse is due to the friction force only (weight & normal force produce opposite but equal impulses). Final velocity = 1 m/s initial velocity is 3 m/s. Assume both velocities + to right, impulse is to left



$mV_1 - \mu W \Delta t = mV_2 \Rightarrow \frac{m(V_1 - V_2)}{\mu W} = \Delta t = 1.36s$

15-17



$2S_B + S_A = \text{Const} \Rightarrow 2V_B + V_A = 0$. Since $(V_A)_1 = +2$ m/s $\Rightarrow (V_B)_1 = -$

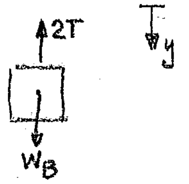
thus B moves up initially w/ velocity of 1 m/s

Isolate Block A : From impulse & momentum in y dir $W_A \cos 30°$:

From impulse & momentum in x dir for block

$$m_A V_{A_1} \underbrace{- T\Delta t}_{\text{Impulse due to T}} + \underbrace{W_A \sin 30 \Delta t}_{\text{Impulse due to comp. W in x dir}} - \mu \underbrace{W_A \cos 30° \Delta t}_{\text{Impulse due to f } f=\mu N} = m_A$$

Isolate block B :



$$m_B V_{B_1} + \overbrace{2T\Delta t}^{\text{Impulse due to 2T}} + \overbrace{W_B \Delta t}^{\text{Impulse due to weight}} = m_B V_{B_2}$$ here $V_{B_1} = -1$ m/s

by the kinematics eqn $V_{B_2} = -V_{A_2}/2$

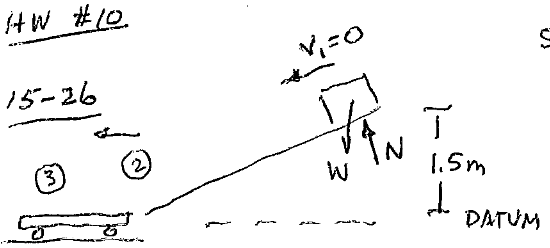
With these 3 eqns we can solve for T and V_{A_2} and V_{B_2} when $\Delta t = 2$sec

$\therefore$ @ t=2sec $V_{A_2} = 6.02$ m/s $V_{B_2} = -3.01$ m/s or 3.01 m/s ↑ T = 16.2 N

Here we needed to use impulse & momentum to relate the velocities of the block to the tension in the cord and kinematics to relate the velocities of the block. Remember since by kinematics we picked the directions for $+s_A$ & $+s_B$, we must use same notation for the definition of the directions for the impulse & momentum equations.

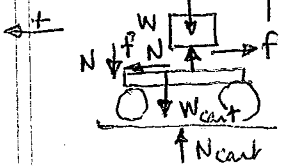
HW #10

15-26



Since forces on body as it moves down ramp are conservative Use conservation of energy to find V_2. $T_1+V_1=T_2+V_2$; $T_1=0$ $V_1=W(1.5)$
$T_2=\frac{1}{2}mV_2^2$ $V_2=0$ $\Rightarrow$ $V_2=\sqrt{2(1.5)g}$
or $V_2=\underline{5.42}$ m/s ←

This is velocity when package starts slide on cart. Since f & N are internal and

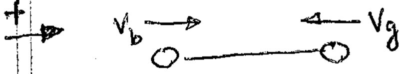


$N_{cart}=W_{cart}+W_{block}$ the impulses due to them add up to zero thus momentum is conserved. Thus $m_{Block}V_2+m_{cart}\cdot V_{cart}=(m_{Block}+m_{cart})\tilde{V}$

Where $V_{cart}=0$ initial & $\tilde{V}$ is velocity of syste when block stops sliding. This gives $\tilde{V}=\underline{1.55}$ m/s ←

To find the time use $m_{block}V_{block}+\Sigma\int Fdt=m_{block}\tilde{V}$ where $V_{block}=V_2$ and
$\Sigma\int Fdt=-f\Delta t=\mu W\Delta t$. This gives $\Delta t=\underline{.493}$ seconds

15-28 Use relative velocity relation the relate V_{boy} & V_{girl}



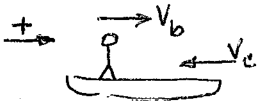
$V_b-(-V_g)=V_{rel}$ $\Rightarrow$ $V_b=-V_g+2$ m/s

Next use conservation of momentum since weights & contact force balance & force in rope produce internal impulses (equal & opposite on each body).

$\Sigma mv_{initial}=\Sigma mv_{final}$ initially both at rest ∴ $0=m_bV_b-m_gV_g$

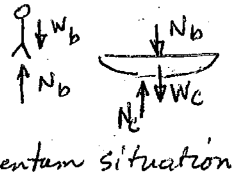
with $m_b=60$ kg $m_g=50$ kg $\Rightarrow$ $\underline{\underline{V_b=.909\text{ m/s}}}$ → $\underline{\underline{V_g=1.09\text{ m/s}}}$ ←

Now $V_b\cdot t+V_gt=5m$ but $V_b+V_g=V_{rel}$ $\Rightarrow$ $t=\frac{5m}{V_{rel}}=2.5$ seconds



15-32 Must find absolute velocity of boy : $V_b-(-V_c)=V_{rel}$

Also the only forces are the weight of the boy & canoe and their normal reactions.



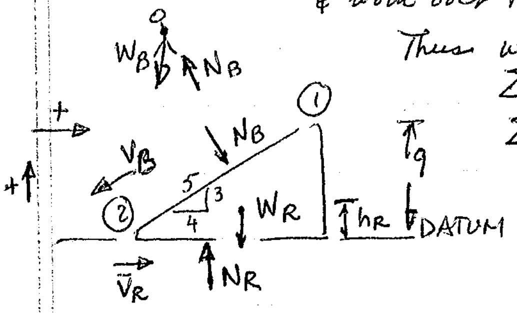
N_b produces internal impulses and $N_c=W_c+W_b$ and $\int N_cdt-\int(W_c+W_b)dt=0$. Thus we have an conservation of momentum situation $\Rightarrow$ $(m_bV_b-m_cV_c)=(\Sigma mV)_{final}=(\Sigma mV)_{initial}=0$.

with $m_b=\frac{75}{32.2}$ slugs ; $m_c=\frac{50}{32.2}$ slugs ; $V_{rel}=4$ ft/s $\Rightarrow$ $V_b=1.6$ ft/s ⇒ &
$V_c=2.4$ ft/s ←.

Boy takes 2 seconds to travel length of canoe $=\frac{8\text{ ft}}{4\text{ ft/s}}=\frac{\text{length of canoe}}{\text{relative velocity}}$

thus canoe travels 2 seconds × V_c = 4.8 ft ←

15-37 Treat the ramp & boy as a system. Since the forces N_B are internal & work over the slides length, the work done by them gives 0. Thus we can use conservation of energy



$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2 \qquad ⑤$$

$$\Sigma T_1 = \tfrac{1}{2} m_B V_{B_1}^2 + \tfrac{1}{2} m_R V_{R_1}^2 = 0 \text{ initially}$$

$$\Sigma V_1 = W_B(9) + W_R h_R$$

$$\Sigma T_2 = \tfrac{1}{2} m_B V_{B_2}^2 + \tfrac{1}{2} V_{R_2}^2 m_R$$

$$\Sigma V_2 = W_R h_R$$

By kinematics $\bar{V}_B - \bar{V}_R = \bar{V}_{B/R}$

$$-(V_B)_x - (V_R)_x = (V_{B/R})_x \qquad ①$$

$$-(V_B)_y - \underset{0}{\cancel{(V_R)_y}} = (V_{B/R})_y \qquad ②$$

$V_{R_y} = 0$ since ramp has no y velocity

Also wrt ramp $\bar{V}_{B/R}$ is ↙ down the ramp. ∴ $(V_{B/R})_x = -\frac{4}{5} V_{B/R}$ ③

$$(V_{B/R})_y = -3/5\, V_{B/R} \qquad ④$$

Also since forces are either internal or balance each other ($N_R = W_R + W_B$), we have no impulses & we can use conservation of momentum

$$\left(\Sigma m\bar{V}\right)_{initial} = \bar{0} = \left(\Sigma m\bar{V}\right)_{final} = m_B \bar{V}_{B_2} + m_R \bar{V}_{R_2}$$

or $m_B(-V_{B_2})_x + m_R (V_{R_2})_x = 0$ and with ①-④

and $W_B = 80 lb$ $W_R = 120 lb$ we find $(V_{B_2})_x = 1.5 (V_{R_2})_x = 13.4$ ft/s ←

$$(V_{B_2})_y = 1.875 (V_{R_2}) = 16.74 \text{ ft/s} \rightarrow$$

Now $V_{B_2}^2 = (V_{B_2})_x^2 + (V_{B_2})_y^2 = 5.7656\, V_{R_2}^2$. Put into ⑤ to find

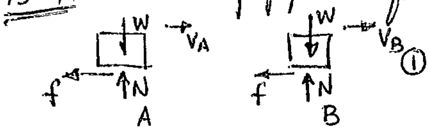
$$(V_{R_2}) = 8.93 \text{ ft/s} \rightarrow$$

$$V_{B_2} = 21.44 \text{ ft/s} \swarrow$$

$$\left.\begin{aligned} (V_{B/R})_{x_2} &= -13.4 - 8.93 = -22.33 \text{ ft/s} \\ (V_{B/R})_{y_2} &= -16.74 \text{ ft/s} \end{aligned}\right\} (V_{B/R}) = 27.91 \text{ ft/s} \swarrow$$

HW #10 ① use work & energy to get velocity of box at B. ② use collision to find

15-47 velocity of plate after collision ③ use work & energy to find spring compression.

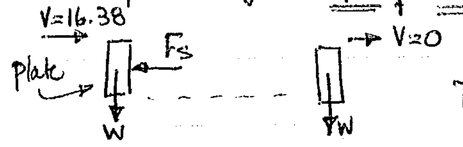


① $\frac{1}{2} m_{box} V_A^2 - \mu_k W_{box} \Delta S = \frac{1}{2} m_{box} V_B^2$ w/ $V_A = 15$ ft/s, $W_{box} = 20$ lb, $\Delta S = 2$ ft, $\mu_k = .3$

$\Rightarrow V_B = 13.65$ ft/s.

② before collision $(m_{box} \overset{V_B}{\cancel{V_{box}}} + m_{plate} \overset{0}{\cancel{V_{plate}}}) = (m_{box} V_{box} + m_{plate} V_{plate})$ after collision and $e = .8 = \frac{V_{P_f} - V_{box\,f}}{V_{B_i} - \overset{0}{\cancel{V_{plate_i}}}}$

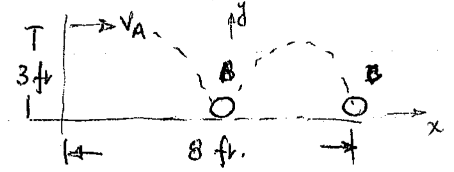
these 2 equations give $\underline{V_{plate_f}} = \underline{\underline{16.38 \text{ ft/s}}}$ $\underline{V_{box_f}} = \underline{\underline{5.46 \text{ ft/s}}}$

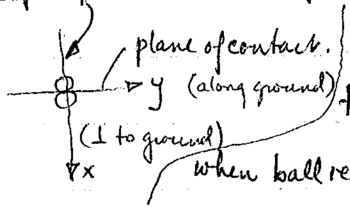
③


Thus $\frac{1}{2} m_{plate} V^2 - \overbrace{\frac{1}{2} k s^2}^{\text{work due to spring}} = \frac{1}{2} m_{plate} \overset{0}{\cancel{V^2}} = 0$

w/ $W_{plate} = 10$ lb, $k = 400$ lb/ft we get $\underline{\underline{S = .456 \text{ ft}}}$

15-48 plane of contact is surface ; line of impact is ⊥ to surface





① Use $V_f^2 = V_i^2 + 2a(\Delta S)$ to find y component of velocity when ball reaches ground. Ground is the second object having $\bar{V}_i = \bar{V}_f = \bar{0}$ ② Use collision to find $(V_{ball})_y$ after collision ③ Use fact that ball starts & ends at same altitude to say that $V_{B_y} = -V_{C_y}$ & $V_{B_x} = V_{C_x} = V_A$ since momentum along ~~line~~ plane of contact is conserved.

① Thus $V_{A_y}^2 = V_{B_y}^2 + 2a_y(\overbrace{S_A - S_B}^{3\text{ ft}}) \Rightarrow 0 = V_{B_y}^2 - 2(32.2)(3)$ or $V_{B_y} = 13.90$ ft/s ↓ $= -13.90$

② Collision $e = \frac{\overset{0}{\cancel{V_{Ground}}} - V_{Ball\,after}}{V_{Ball\,before} - \overset{0}{\cancel{V_{ground}}}} \Rightarrow .6 = \frac{-V_{Ball\,after}}{-13.90}$ or $V_{Ball\,after} = 8.34$ ft/s ↑

③ $V_{C_y}^2 = V_{ball\,after}^2 + 2a\overset{0}{\cancel{(S_B - S_C)}} \Rightarrow |V_{C_y}| = |V_{ball\,after}|$ but in opposite direction.

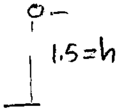
also from ① $\overset{(-13.90)}{V_{f_y}} = \overset{0}{\cancel{V_{i_y}}} + \overset{(-32.2)}{a_y} t \Rightarrow t = \frac{-13.90}{-32.2} = .432$ s time from A to B

also from ③ $\overset{(-8.34)}{V_{f_y}} = \overset{(8.34)}{V_{i_y}} + \overset{(-32.2)}{a_y} t \Rightarrow t = \frac{-16.68}{-32.2} = .518$ s time from B to C

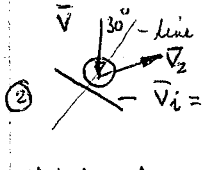
thus $t_{TOTAL} = .950$ s to go from A to C. Now $S_{TOT} = 8$ ft $= V_A t_{TOT} \Rightarrow \underline{\underline{V_A = 8.42 \text{ ft/s}}}$

15-54 ① find velocity of ball before contact ② Use oblique impact to find components of ball's velocity. ③ Use S vs. t relationship to get relationships involving d

① Use conservation of energy.



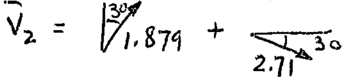
$\overset{0}{\cancel{T_1}} + V_1 = T_2 + \overset{0}{\cancel{V_2}} \Rightarrow Wh = \frac{1}{2} m V^2 \Rightarrow V = \sqrt{2gh} = 5.42$



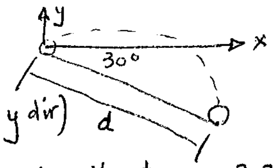
② $\bar{V}_i = \bar{V}_f = \bar{0}$ for ground also $e = \frac{\cancel{V_{ground\,f}} - (V_{ball})}{V\cos 30° - \cancel{V_{ground_i}}} \Rightarrow .4 = \frac{-V_{ball}}{5.42 \cos 30°} \Rightarrow V_{Ball} = -1.879 \frac{m}{s}$

This is rebound velocity of ball <u>but</u> this is component along line of impact. Along plane of contact

momentum is conserved; hence $V \sin 30^\circ = (V_{ball})_2$ along plane of contact. $\therefore (V_{ball})_2 = 2.71 \frac{m}{s}$

$\bar{V}_2 =$  $\Rightarrow$ $V_{2x} = 1.879 \sin 30^\circ + 2.71 \cos 30^\circ = 3.286$ m/s

$V_{2y} = 1.879 \cos 30^\circ - 2.71 \sin 30^\circ = .272$ m/s

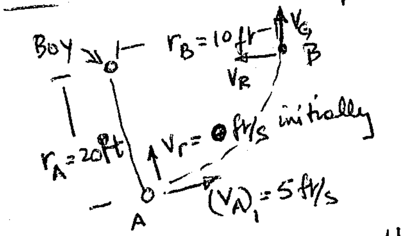


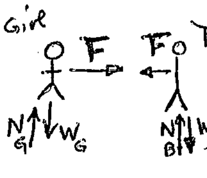
③ $d \cos 30^\circ$ (distance traveled in x dir) $= V_{2x} t$

$- d \sin 30^\circ = 0 + .272t - \frac{1}{2}(9.81)t^2 =$ (distance traveled in y dir)

$$-\tan 30^\circ = \frac{V_{2y} + \frac{1}{2} a_y t}{V_{2x}} \Rightarrow \underline{\underline{t = .442s}} \quad \text{and} \quad \underline{\underline{d}} = \frac{V_{2x} t}{\cos 30^\circ} = \frac{3.286(.442)}{.866} = \underline{\underline{1.68m}}$$

15-63 Since the rope changes 10 ft between A & B at a rate of 4ft/s the girl reaches B in 2.5s.





This is a conservation of angular, momentum system since F points towards Boy & N & W are ⊥ to axis through Boy's position.

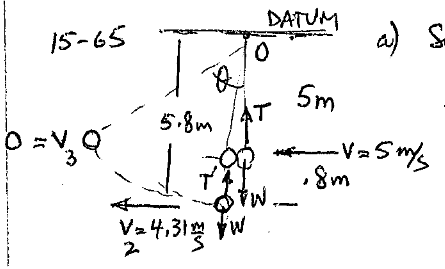
$$\therefore H_{o_1} = H_{o_2} \Rightarrow (20)\left(\frac{80}{32.2}\right)\left[5 \text{ ft/s}\right] = 10\left(\frac{80}{32.2}\right)(V_G) \Rightarrow V_G = 10 \text{ ft/s}$$

→ but to this must add V_R component $\therefore V_{TOT_B} = (10^2 + 4^2) = 10.77$ ft/s.

Only work done is by F which undergoes a change of distance. Thus $T_B - T_A = \int \bar{F} \cdot ds$

$T_B = \frac{1}{2} m_G V_{TOT_B}^2 \quad T_A = \frac{1}{2} m_G (V_A)_1^2 \quad \therefore T_B - T_A = \underline{\underline{113 \text{ ft-lb}}} = \underline{\underline{\sum U_{A \to B}}}$

15-65

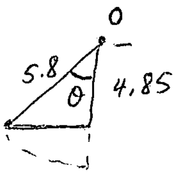


a) Suddenly means instantaneously. Only forces acting at these 2 events at along line of action through O. Thus we have conservation of momentum (angular) about O.

$$H_{o_1} = H_{o_2} \Rightarrow 5(80)5 = 5.8(80)(V_2) \Rightarrow V_2 = \frac{5.5}{5.8} = 4.31 \frac{m}{s}$$

b) This is instantaneous velocity. Assume this is velocity initially. Only forces acting are conservative. Thus use conservation of energy $T_2 + V_2 = T_3 + V_3 \quad T_2 = \frac{1}{2} m V_2^2, V_2 = -W(5$

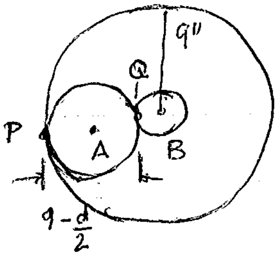
$V_3 = ? \quad T_3 = 0 \quad \therefore \quad W h_3 = \frac{1}{2}(80)(4.31)^2 - 80(9.81)(5.8) \quad h_3 = -4.85 m$



$\therefore \cos\theta = \frac{4.85}{5.8} = 33.2^\circ$

HW #12

16-7



Velocity of common contact pts are the same. V_Q & V_P are the same for their respective bodies. Also $V_P = V_Q$ for A

$$(V_Q)_A = (V_Q)_B = \frac{d}{2}\omega_B = \frac{1}{2}\left(9 - \frac{d}{2}\right)\omega_A$$

$$(V_P)_A = (V_P)_T = \frac{1}{2}\left(9 - \frac{d}{2}\right)\omega_A = 9 \cdot \omega_T$$

$$\Rightarrow \frac{d}{2}\omega_B = 9\omega_T \Rightarrow \left(\frac{\omega_B}{18\omega_T}\right)^{-1} = \underline{\underline{d = 2.49 \text{ in}}}$$

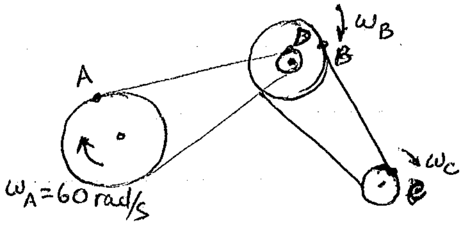
$\omega_B = 25$ rad/s

$\omega_T = 33 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{rev}} \cdot \frac{1 \text{mi}}{60 \text{s}}$

$= 3.456$ rad/s

$$\omega_A = \frac{18\omega_T}{9 - d/2} = 8.02 \text{ rad/s.}$$

16-10.



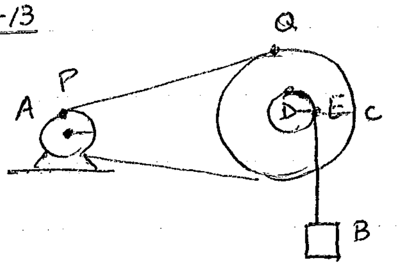
$V_A = V_B$: only undergo translation ; also $\omega_D = \omega_B$ rigid connection requirement

$V_C = V_B$: only undergo translation

$$V_A = \omega_A r_A = \omega_D r_D \Rightarrow \omega_D = \frac{\omega_A r_A}{r_D} = 180 \text{ rad/s}$$

$$V_C = \omega_C r_C = \omega_B r_B = \omega_D r_B \Rightarrow \omega_C = \omega_D \frac{r_B}{r_C} = 360 \text{ rad/s}$$

16-13



$\alpha_A = 6\theta_A$ & $\alpha = \frac{d\omega}{dt}$ $\omega = \frac{d\theta}{dt}$ $\Rightarrow \alpha d\theta = \omega d\omega$

$\Rightarrow 6\theta_A d\theta = \omega d\omega$ or $3\theta_A^2 + C = \frac{\omega_A^2}{2}$

but it starts from rest $\Rightarrow$ $\omega = 0$ @ $t = 0$ & $\theta = 0$

$\Rightarrow 6\theta_A^2 = \omega_A^2$ or $\omega_A = \sqrt{6}\,\theta_A$ rad/s.

As in 16-10 $V_P = V_Q$ & $\omega_C = \omega_D$

$$V_P = \omega_A r_P = \sqrt{6}\,\theta_A (50 \text{mm}) = \omega_C (150 \text{mm}) \Rightarrow \omega_C = \frac{\sqrt{6}}{3}\theta_A \text{ rad/s} = \omega_D$$

$$\Rightarrow V_E = \omega_D r_D = \frac{\sqrt{6}\,\theta_A}{3}(75 \text{mm}) = \sqrt{6}\,\theta_A (25 \text{mm})$$

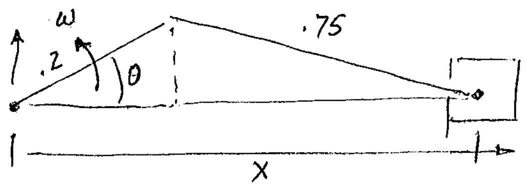
when block B has risen 6 m., 6 m of rope are taken up by pulley D

$\therefore$ 6 meter $= r_D \Delta\theta_D \Rightarrow \Delta\theta_D = \frac{6 \text{ m}}{.075 \text{m}} = 80$ rad. We must find a relation between $\Delta\theta_D$ & $\Delta\theta_A$. Since D rotates through 80 rad so must C (they are attached). $\therefore \Delta\theta_C = \Delta\theta_D = 80$ rad. $\Rightarrow$ a pt on rim of pulley C has travelled $\Delta s = \Delta\theta_C r_C$ meters. Since PQ is a fixed distance $\Rightarrow$ pulley A must move through the same distance $\Delta s = \Delta\theta_A r_A = \Delta\theta_C r_C \Rightarrow \Delta\theta_A \cdot 50 = 80 \cdot 150$

$\therefore \Delta\theta_A = 240$ rad. $\Rightarrow V_E = \sqrt{6}\,(240 \text{ rad})(.025) = 14.697$ m/s

Here we assume that we start w/ $\theta = 0$ @ $t = 0$ ie $\Delta\theta_A \equiv \theta_A$ $\Delta\theta_C = \theta_C$ since $\Delta\theta = \theta_{final} - \cancelto{0}{\theta_{initial}}$.

16-30



using law of cosines

$(.75)^2 = (.2)^2 + x^2 - 2(.2)(x)\cos\theta$ (1)

take $\frac{d}{dt}$ of eq $\Rightarrow 0 = 2x\dot{x} - 2(.2)\dot{x}\cos\theta + 2(.2)x\cdot\sin\theta\,\dot{\theta}$ (2)

when $\theta = 30^\circ$ $\dot{\theta} = \omega = 150$ rad/s from the eqn (1) $.5225 \neq x^2 - .3464x$

$$x = \frac{.3464 \pm \sqrt{(.3464)^2 - 4(-.5225)}}{2} = .9165 \text{ ft}$$

from (2) $0 = \dot{x}(2x - .4\cos\theta) + .4x\sin\theta\,\dot{\theta} \Longrightarrow \dot{x} = \frac{-.4x\sin\theta\,\dot{\theta}}{2x - .4\cos\theta} = -18.495 \frac{ft}{s}$

or 18.495 ft/s $\leftarrow$

16-20 $\alpha = 20e^{-.6t}$ $\omega = \int_0^t \alpha\,dt' = \frac{-20}{.6} e^{-.6t}\Big|_0^3 = -33.33e^{-.6t}\Big|_0^3$ rad/s

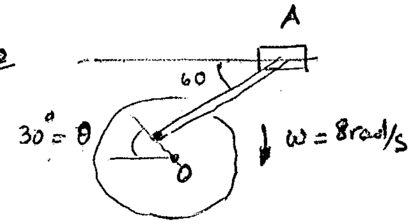
$V = \omega r = -33.33(1.75ft)e^{-.6t}\Big|_0^3$; @ $t = 3s$ $V = -58.33[e^{-1.8} - e^0] = 48.7\frac{ft}{s}$

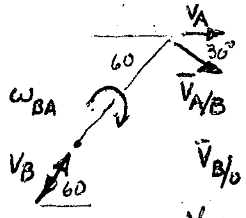
$\theta = \int_0^3 \omega\,dt = \int_0^3 (-33.33)[e^{-.6t} - 1]dt = 55.55[e^{-1.8} - e^0] + 33.33t\Big|_0^3 = 53.63$ rad.

$53.63 \times \frac{1}{2\pi \text{ rad/rev}} = 8.54$ rev.

HW #13

16-40





$\bar{V}_B = \bar{V}_O + \bar{V}_{B/O}$

$V_{B/O} = \omega r_{B/O} = 8(.15) = 1.2$ m/s

$\bar{V}_{B/O} = 1.2\cos 60\bar{i} + 1.2\sin 60\bar{j}$

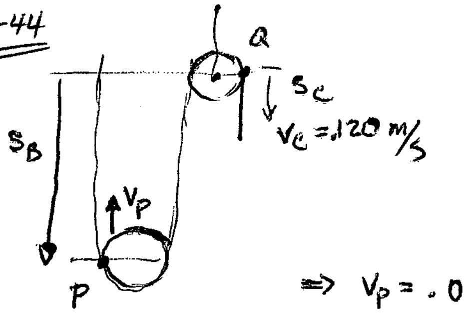
$V_{A/B} = \omega_{BA} r_{A/B} = \omega_{BA}(.5)$

$\bar{V}_A = \bar{V}_B + \bar{V}_{A/B} \Rightarrow V_A\bar{i} = 1.2\cos 60\bar{i} + 1.2\sin 60^\circ\bar{j} + .5\omega_{BA}\cos 30\bar{i} - .5\omega_{BA}\sin 30^\circ\bar{j}$

$\Rightarrow \quad 1.2\sin 60^\circ - .5\omega_{BA}\sin 30^\circ = 0 \quad$ or $\quad \omega_{BA} = \dfrac{1.2\sin 60^\circ}{.5\sin 30^\circ} = \dfrac{1.2(.866)}{(.5)(.5)} = 4.16\ \dfrac{\text{rad}}{\text{s}}$

$V_A = 1.2\cos 60^\circ + .5(4.16)\cos 30^\circ$
$= (1.2)(.5) + (.5)(4.16)(.866) = 2.4$ m/s

16-44



$2S_B + S_c = \text{const.} \Rightarrow 2V_B + V_c = 0$

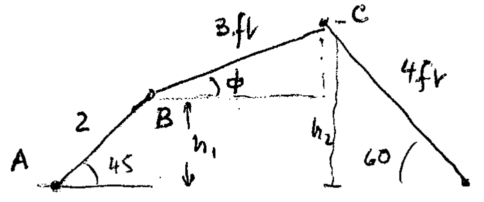
$\Rightarrow$ for $V_c = .12$ m/s $\quad V_B = -\dfrac{V_c}{2} = -.06\ \dfrac{\text{m}}{\text{s}}$

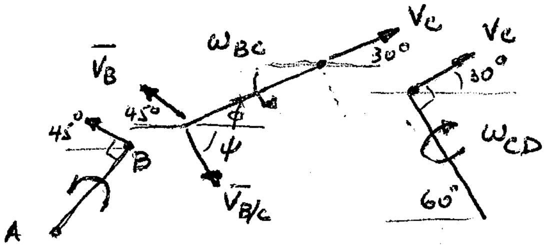
or $.06\ \dfrac{\text{m}}{\text{s}} \uparrow = V_B = V_D$

$\Rightarrow V_P = .06$ m/s $= \omega_B r_B \Rightarrow \underline{\underline{\omega_B}} = \dfrac{V_P}{r_B} = \dfrac{60 \text{ mm/s}}{60 \text{ mm/s}} = 1$ rad/s ↻

$\Rightarrow V_A = V_c$ since A & C only translate $\qquad \underline{\underline{V_A}} = 120$ mm/s $= \omega_A r_c \Rightarrow \omega_A = \dfrac{V_A}{r_c}$
$= \dfrac{120 \text{ mm/s}}{30 \text{ m/s}} = 4$ rad/s ↻

16-50





$\bar{V}_B = \bar{V}_A + \bar{V}_{B/A} \qquad \bar{V}_A = \bar{0} \qquad V_{B/A} = \omega_{AB} r_{B/A} = 3(2) = 6$ ft/s $\Rightarrow \bar{V}_{B/A} = -6\cos 45^\circ\bar{i} + 6\sin 45^\circ\bar{j}$

$\therefore \quad \bar{V}_B = \bar{V}_{B/A} = -6\cos 45^\circ\bar{i} + 6\sin 45^\circ\bar{j}$

$\bar{V}_c = \bar{V}_D + \bar{V}_{c/D} \qquad \bar{V}_D = \bar{0} \qquad V_{c/D} = \omega_{CD} r_{c/D} = \omega_{CD}(4) = 4\omega_{CD} \Rightarrow \bar{V}_{c/D} = 4\omega_{CD}\cos 30^\circ\bar{i} + 4\omega_{CD}\sin 30^\circ\bar{j}$

Now $\quad h_1 = 2\sin 45^\circ = \sqrt{2}$ ft $\qquad h_2 = 4\sin 60^\circ = 3.464$ ft. $\qquad h_2 - h_1 = 2.05$ ft

and $\phi = \sin^{-1}\left(\dfrac{h_2 - h_1}{3}\right) = 43.1^\circ \qquad \psi = 90^\circ - \phi = 46.9^\circ$

$\Longrightarrow V_{B/c} = \omega_{BC} r_{B/c} = 3\omega_{BC} \quad$ & $\quad \bar{V}_{B/c} = 3\omega_{BC}\cos\psi\bar{i} - 3\omega_{BC}\sin\psi\bar{j}$

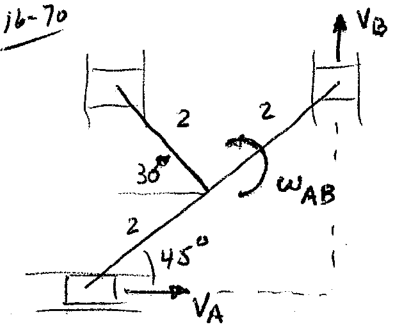
thus $\quad \bar{V}_B = \bar{V}_c + \bar{V}_{B/c} \Rightarrow -6\cos 45^\circ\bar{i} + 6\sin 45^\circ\bar{j} = 4\omega_{CD}\cos 30^\circ\bar{i} + 4\omega_{CD}\sin 30^\circ\bar{j} + 3\omega_{BC}\cos\psi\bar{i} - 3\omega_{BC}\sin\psi\bar{j}$

$\underline{\underline{\text{or}}} \quad \left.\begin{array}{l} -6\cos 45^\circ = 4\omega_{CD}\cos 30^\circ + 3\omega_{BC}\cos 46.9^\circ \\ 6\sin 45^\circ = 4\omega_{CD}\sin 30^\circ - 3\omega_{BC}\sin 46.9^\circ \end{array}\right\}$

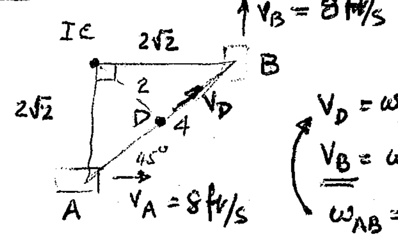
$\omega_{CD} = -.0508$ rad/s
$\Rightarrow \omega_{CD}$ ↺

$\omega_{BC} = -1.984$ rad/s
$\Rightarrow \omega_{BC}$ ↺

16-70



<u>using instantaneous center</u>



$V_D = \omega_{AB}\, r_{D/IC} = 5.656$ ft/s

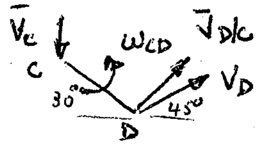
$\underline{\underline{V_B}} = \omega_{AB}\, r_{B/IC} = \omega\,(2\sqrt{2}) = \underline{\underline{8\,\text{ft}}}$

$\underline{\omega_{AB} = 2\sqrt{2} = 2.828 \text{ rad/s}}$ ↻

Now $\bar{V}_D$ is at an angle of 45° $\Rightarrow$ ↗

$$\bar{V}_D = 5.656 \cos 45^\circ \bar{i} + 5.656 \sin 45^\circ \bar{j} = 4\bar{i} + 4\bar{j} \text{ ft/s}$$

Ⓐ



$\bar{V}_{D/C}$ is $\perp$ to $r_{D/C}$ $\Rightarrow$ $\bar{V}_{D/C} = V_{D/C}(\cos 60^\circ \bar{i} + \sin 60^\circ \bar{j})$

where $\bar{V}_{D/C} = \omega_{DC}\, r_{D/C} = 2\omega_{DC}$

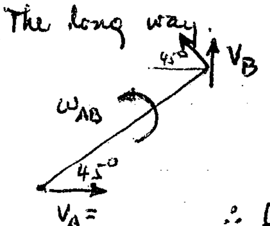
$\bar{V}_C = -V_C \bar{j}$ $\quad\therefore\quad$ $\bar{V}_D = \bar{V}_C + \bar{V}_{D/C}$

$$(4\bar{i} + 4\bar{j}) = -V_C \bar{j} + V_{D/C} \cos 60^\circ \bar{i} + V_{D/C} \sin 60^\circ \bar{j}$$

$\Rightarrow \quad 4 = -V_C + V_{D/C} \sin 60^\circ$ and $4 = V_{D/C} \cos 60^\circ$ $\quad V_{D/C} = 8$ ft/s $\quad \underline{\underline{V_C = 2.928 \text{ ft/s}}}\downarrow$

since $V_{D/C} = \omega_{DC}\, r_{D/C} = 2\omega_{DC}$ $\Rightarrow$ $\underline{\omega_{DC} = 4 \text{ rad/s}}.$ ↺

The long way.

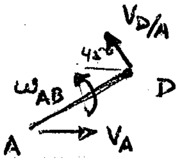


(1) $\bar{V}_B = \bar{V}_A + \bar{V}_{B/A}$ $\qquad V_{B/A} = \omega_{AB}\, r_{B/A} = 4\omega_{AB}$; $\bar{V}_B = V_B \bar{j}$, $\bar{V}_A = 8\bar{i}$

$$\bar{V}_{B/A} = -V_{B/A} \cos 45^\circ \bar{i} + V_{B/A} \sin 45^\circ \bar{j}$$

$\therefore$ from (1) $\quad V_B \bar{j} = 8\bar{i} - V_{B/A} \cos 45^\circ \bar{i} + V_{B/A} \sin 45^\circ \bar{j} \Rightarrow V_{B/A} = +8\sqrt{2}$ ft/s

and $\omega_{AB} = V_{B/A}/r_{B/A} = 2\sqrt{2}$ rad/s. $\quad \underline{\underline{V_B}} = V_{B/A} \sin 45^\circ = 8\sqrt{2} \cdot \frac{1}{\sqrt{2}} = \underline{\underline{8 \text{ ft/s}}}.$



$\bar{V}_D = \bar{V}_A + \bar{V}_{D/A} = 8\bar{i} + \bar{V}_{D/A}$ $\qquad V_{D/A} = \omega_{AB}\, r_{D/A} = 2(2\sqrt{2})$ rad/s $= 5.656$ ft/s

and $\bar{V}_{D/A}$ is ~~ok to~~ $\bar{r}_{D/A}$ $\therefore$ $\bar{V}_{D/A} = -V_{D/A} \cos 45^\circ \bar{i} + V_{D/A} \sin 45^\circ \bar{j}$

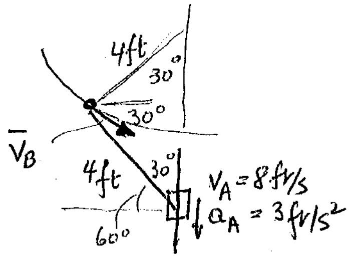
$= -4\bar{i} + 4\bar{j}$ ft/s.

Thus $\bar{V}_D = 8\bar{i} + (-4\bar{i} + 4\bar{j}) = 4\bar{i} + 4\bar{j}$ ft/s

Now go back to the 1st method – to section Ⓐ to find $\bar{V}_C$ etc.

HW #14

16-74



$\bar{V}_B = V_B\cos 30°\bar{i} - V_B\sin 30°\bar{j}$ (3)

$\bar{V}_A = -8\bar{j}$

$\bar{V}_{B/A} = \bar{\omega}_{BA} \times \bar{r}_{B/A}$ (2)

$= -4\omega_{BA}\cos 60°\bar{j} - 4\omega_{BA}\sin 60°\bar{i}$

$\bar{V}_B = \bar{V}_A + \bar{V}_{B/A}$ (1)

$\bar{r}_{B/A} = -4\cos 60°\bar{i} + 4\sin 60°\bar{j}$

let $\bar{\omega}_{BA} = \omega\bar{k}$, $\bar{\alpha}_{BA} = \alpha\bar{k}$

$\overset{k}{+}$ i j

$\therefore V_B\cos 30°\bar{i} - V_B\sin 30°\bar{j} = -8\bar{j} - 4\omega_{BA}\cos 60°\bar{j} - 4\omega_{BA}\sin 60°\bar{i}$ from (1),(2),(3)

Collect all i terms to get $V_B\cos 30° = -4\omega_{BA}\sin 60°$ $\Rightarrow \underline{V_B = -4\omega_{BA}}$

$-V_B\sin 30° = -8 - 4\omega_{BA}\cos 60°$

$\underline{\omega_{BA}} = -2$ rad/s or $\underline{\underline{2\ \text{rad/s}}}$ ↻

$\underline{V_B} = 8$ ft/s

$\bar{a}_B = (a_B)_t\left[\cos 30°\bar{i} - \sin 30°\bar{j}\right] + \frac{V_B^2}{\rho}\left[\cos 60°\bar{i} + \sin 60°\bar{j}\right]$; $\bar{a}_A = -3\bar{j}$ (4,5)

$\bar{a}_{B/A} = \bar{\alpha}_{AB} \times \bar{r}_{B/A} - \omega^2\bar{r}_{B/A} = \alpha_{AB}\bar{k} \times (-4\cos 60°\bar{i} + 4\sin 60°\bar{j}) - (4\ \text{rad}^2/\text{s}^2)\left[-4\cos 60°\bar{i} + 4\sin 60°\bar{j}\right]$

$= -4\alpha_{AB}\cos 60°\bar{j} - 4\alpha_{AB}\sin 60°\bar{i} + 16\cos 60°\bar{i} - 16\sin 60°\bar{j}$ (6)

$\bar{a}_B = \bar{a}_A + \bar{\alpha} \times \bar{r}_{B/A} - \omega^2\bar{r}_{B/A}$ (7) ; $V_B^2/\rho = \frac{64}{4} = 16$ ft/s² . From (4-7)

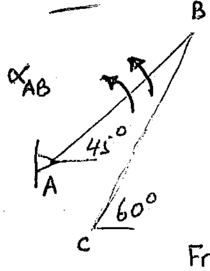
$a_{B_t}\cos 30° + 16\cos 60° = -4\alpha_{AB}\sin 60° + 16\cos 60°$ $\Rightarrow \underline{a_{B_t} = -4\alpha_{AB}}$

$-a_{B_t}\sin 30° + 16\sin 60° = -4\alpha_{AB}\cos 60° - 16\sin 60° - 3$

$\underline{\underline{\alpha_{AB}}} = -7.678\ \frac{\text{rad}}{\text{s}^2}$ or $\underline{\underline{7.68\ \frac{\text{rad}}{\text{s}^2}}}$ ↻

$a_{B_t} = 30.713$ ft/s²

16-75



$\bar{r}_{AB} = (2\cos 45°\bar{i} + 2\sin 45°\bar{j})$; $\bar{\omega}_{AB} = 5\bar{k}$ rad/s ; $\bar{\alpha} = 3\bar{k}$ rad/s²

$\bar{a}_B = \bar{a}_A + \bar{\alpha}_{AB} \times \bar{r}_{B/A} - \omega_{AB}^2\bar{r}_{B/A}$ with $\bar{a}_A = \bar{0}$

$\bar{a}_B = \bar{a}_{B/A} = 5\bar{k} \times \left[2\cos 45°\bar{i} + 2\sin 45°\bar{j}\right] - (25)\left[2\cos 45°\bar{i} + 2\sin 45°\bar{j}\right] = -39.6\bar{i} - 31.1\bar{j}$ ft/s²

$\bar{a}_c = \bar{a}_B + \bar{\alpha}_{BC} \times \bar{r}_{C/B} - \omega_{BC}^2\bar{r}_{C/B}$ with $\bar{r}_{C/B} = -(2.5\cos 60°\bar{i} + 2.5\sin 60°\bar{j})$ ft.

From Prob. 16-38 we did in class we found $\bar{\omega}_{BC} = 5.66$ rad/s $\bar{k}$ ↺; let $\bar{\alpha}_{BC} = \alpha_{BC}\bar{k}$

from above $\bar{a}_c = \bar{a}_B + \alpha_{BC}\bar{k} \times \left[-2.5\cos 60°\bar{i} - 2.5\sin 60°\bar{j}\right] - (5.66)^2\left[-2.5\cos 60°\bar{i} - 2.5\sin 60°\bar{j}\right]$

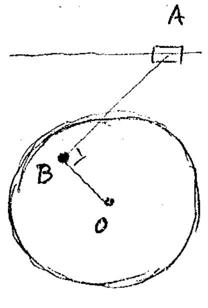
since C moves in a horizontal line let $\bar{a}_c = a_c\bar{i}$

Collecting all $\bar{i}$ terms & all $\bar{j}$ terms leads to $a_c = 66.7$ ft/s² & $\alpha_{BC} = 30.6$ rad/s² ↺

i.e. $a_c = -39.6 + \alpha_{BC}(2.5\sin 60°) + (5.66)^2(2.5)\cos 60°$ ← to get a_c

$0 = -31.1 - \alpha_{BC}(2.5\cos 60°) + (5.66)^2(2.5)\sin 60°$ → this leads to α_{BC}

16-78



$\bar{V}_B = \cancel{\bar{V}_O} + \bar{V}_{B/O} = \bar{\omega}_{BO} \times \bar{r}_{B/O}$ $\Rightarrow$ let $\bar{\omega}_{BO} = \omega_{BO}\bar{k}$ $\bar{r}_{B/O} = -.15\cos 30°\bar{i} + .15\sin 3$

$\bar{V}_B = -.15\omega_{BO}\cos 30°\bar{j} - .15\omega_{BO}\sin 30°\bar{i}$; $\bar{V}_A = V_A\bar{i}$; $\omega_{BO} = -8$ rad/s

$\bar{V}_{A/B} = \bar{\omega}_{AB} \times \bar{r}_{A/B} = \omega_{BA}\bar{k} \times \left[.5\cos 60°\bar{i} + .5\sin 60°\bar{j}\right]$

$= \omega_{BA}(.5\cos 60°)\bar{j} - .5\omega_{BA}\sin 60°\bar{i}$

$\bar{V}_A = \bar{V}_B + \bar{V}_{A/B} \Rightarrow V_A\bar{i} = -.15\omega_{BO}\sin 30°\bar{i} - .15\omega_{BO}\cos 30°\bar{j} - .5\omega_{BA}\sin 60°\bar{i} + .5\omega_{BA}\cos 60°\bar{j}$

FROM PROBLEM 16-40 $\omega_{BA} = -4.16$ rad/s or $\omega_{BA} = 4.16$ rad/s ↻ $\bar{V}_A = 2.4$ m/s →

Now $\bar{a}_A = \bar{a}_B + \bar{\alpha}_{AB} \times \bar{r}_{A/B} - \omega_{AB}^2 \bar{r}_{A/B}$; $\bar{a}_B = \bar{a}_o + \bar{\alpha}_{BO} \times \bar{r}_{B/o} - \omega_{BO}^2 \bar{r}_{B/o}$, with $\bar{a}_o = \bar{0}, \bar{\alpha}_{BO} = \alpha_{BO} k$ when $\alpha_{BO} = -16 \text{ rad/s}^2$. $\bar{a}_B = -16\bar{k} \times [-.15\cos 30° \bar{i} + .15 \sin 30° \bar{j}] - (-8)^2 [-.15 \cos 30° \bar{i} + .15 \sin 30° \bar{j}]$.

let $\bar{\alpha}_{AB} = \alpha_{AB} \bar{k}$. Thus $\alpha_{AB} \bar{k} \times [.5\cos 60° \bar{i} + .5 \sin 60° \bar{j}] - (-4.16)^2 [.5 \cos 60° \bar{i} + .5 \sin 60° \bar{j}] = \bar{a}_{A/B}$

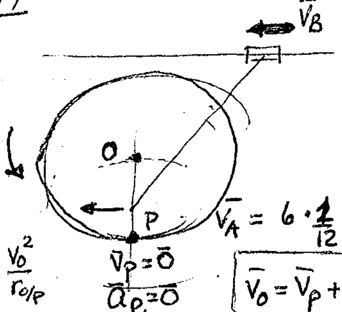
Also $\bar{a}_A = a_A \bar{i}$. Putting all the above together yields

$$\left.\begin{aligned} a_A &= 8.314 + 1.200 - .433\alpha_{AB} - 4.326 \\ 0 &= -4.800 + 2.0785 + .25\alpha_{AB} - 7.4935 \end{aligned}\right\} \text{ this gives } \alpha = 40.9 \text{ rad/s}^2 \circlearrowright$$

$a_A = 12.5 \text{ m/s}^2 \longleftrightarrow$

16-79

Let units be in ft.



$(a_P)_n = \frac{V_o^2}{r_{o/p}}$

$\bar{V}_p = \bar{0}$

$\bar{a}_{P_t} = \bar{0}$

$\bar{a}_o = \bar{a}_P + \bar{\alpha} \times \bar{r}_{o/p} - \omega^2 \bar{r}_{o/p}$
$= \bar{\alpha}_{cp} \times \bar{r}_{o/p} = 12\bar{k} \times (\frac{1}{4}\bar{j}) = -3\bar{i}$

Note that $\bar{a}_p = +\omega^2 \bar{r}_{o/p} = +9\bar{j} \text{ ft/s}^2$

$\bar{V}_A = 6 \cdot \frac{1}{12} = \frac{1}{2} \text{ ft/s} = \bar{\omega}_{pA} \times \bar{r}_{A/p}$

$\bar{V}_o = \bar{V}_p + \bar{V}_{o/p} = 6 \cdot \frac{3}{12} = 1.5 \text{ ft/s}$

here $\bar{V}_{o/p} = \bar{\omega}_{op} \times \bar{r}_{o/p}$
$= -1.5\bar{i}$ ft/s

Since $\bar{V}_B$ & $\bar{V}_A$ are in same direction $\Rightarrow \bar{\omega}_{AB} = \bar{0} \Rightarrow \bar{V}_A = \bar{V}_B$

$\bar{V}_A = \bar{V}_B = .5 \text{ ft/s } \bar{i}$; $\bar{\omega}_{pA} = 6 \text{ rad/s } \bar{k}$; $\bar{\omega}_{AB} = \bar{0}$; $\bar{\alpha}_{pA} = 12\bar{k} \frac{\text{rad}}{\text{s}^2}$

let $\bar{\alpha}_{BA} = \alpha_{BA}\bar{k} \Rightarrow \bar{a}_A = \bar{a}_p + \bar{\alpha}_{AP} \times \bar{r}_{A/p} - \omega_{AP}^2 \bar{r}_{A/p}$

$\bar{a}_{A/p} = \alpha_{AP}\bar{k} \times [\frac{1}{12}\bar{j}] - [6]^2(\frac{1}{12}\bar{j}) = -\frac{\alpha_{AP}}{12}\bar{i} - 3\bar{j}$

$\boxed{\bar{a}_A = 9\bar{j} - \bar{i} - 3\bar{j} = 6\bar{j} - \bar{i} \text{ ft/s}^2} = \bar{a}_p + \bar{a}_{A/p}$

To find $\bar{a}_B$ we need $\bar{r}_{B/A} = \frac{2}{3}\cos 60° \bar{i} + \frac{2}{3} \sin 60° \bar{j}$ & let $\bar{\alpha}_{BA} = \alpha_{BA}\bar{k}$

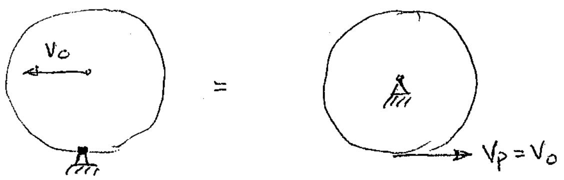
Now. $\bar{a}_B = \bar{a}_A + \omega_{BA}^2 \bar{r}_{B/A} + \bar{\alpha}_{BA} \times \bar{r}_{B/A} = (6\bar{j} - \bar{i}) - 0^2 \cdot \bar{r}_{B/A} + \alpha_{BA}\bar{k} \times [\frac{2}{3}\cos 60° \bar{i} + \frac{2}{3}\sin 60° \bar{j}]$

since B moves horizontally $\Rightarrow \bar{a}_B = a_B \bar{i}$

$\underline{\underline{a_B \bar{i}}} = \underline{6\bar{j}} - \underline{\underline{\bar{i}}} + \underline{\frac{2}{3}\alpha_{AB} \cos 60° \bar{j}} - \underline{\underline{\frac{2}{3}\alpha_{AB} \sin 60° \bar{i}}} \Rightarrow \alpha_{AB} = -18 \text{ rad/s}^2 \text{ or } 18 \circlearrowright$

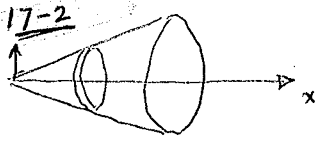
$\Rightarrow a_B = +9.392 \text{ ft/s}^2$

In this problem at the gear teeth interface $V = 0$ & $a_t = 0$. The $a_n = \frac{V^2}{\rho}$ for the path of the point P. P Travels a circular path whose radius is the radius of the gear & whose velocity is viewed as if the point O is fixed & P is moving along a circular path but in the opposite direction to V_o.



HW #15

17-2



Using the disk method $dI_x = \frac{1}{2}dm\,y^2 \quad dm = \rho\pi y^2 dx$

$$I_x = \int_0^h \frac{1}{2}\rho\pi y^4 dx \quad ; \quad y^4 = \frac{r^4}{h^4}x^4$$

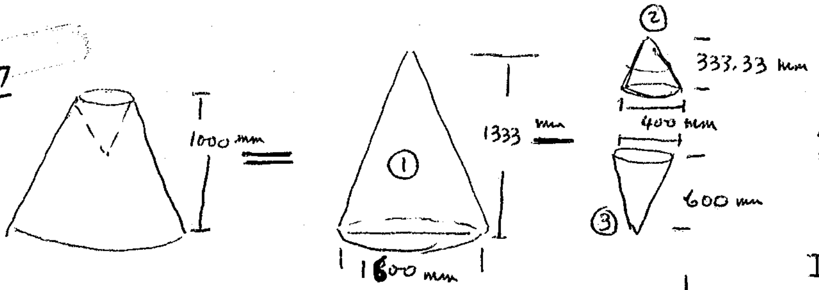
$$I_x = \int_0^h \frac{\rho\pi}{2}\cdot\frac{r^4}{h^4}x^4 dx = \frac{\rho\pi}{2}\frac{r^4}{h^4}\frac{x^5}{5}\Big|_0^h = \rho\pi\frac{r^4h}{10}$$

$$m = \int\rho\pi y^2 dx = \int_0^h \rho\pi\frac{r^2}{h^2}x^2 dx = \rho\pi\frac{r^2}{h^2}\frac{x^3}{3}\Big|_0^h = \rho\pi\frac{r^2h}{3}$$

Thus

$$I_x = \rho\pi\frac{r^2h}{3}\cdot\frac{3r^2}{10} = \frac{3}{10}mr^2$$

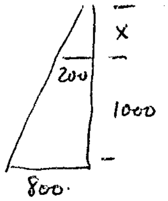
17-17



$$I_{zz} = \frac{3}{10}mR^2 = \rho\pi\frac{R^4h}{10}$$

$$I_{zz_{TOT}} = I_{zz_1} - I_{zz_2} - I_{zz_3}$$

Using similar triangles



$$\frac{800}{200} = \frac{1000+x}{x} \Rightarrow x = 333.33\text{ mm}$$

$$I_{zz_1} = \frac{200(\pi)(.8)^4(1.333)}{10}$$

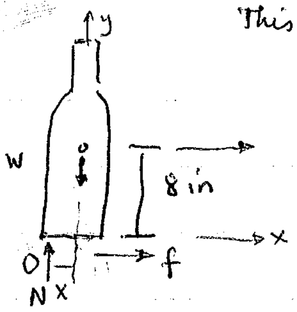
$$I_{zz_2} = \frac{200(\pi)(.2)^4(.333)}{10}$$

$$I_{zz_3} = \frac{200\pi(.2)^4(.6)}{10}$$

$$I_{zz_{TOT}} = 34.315 - .0335 - .0603 = 34.221\text{ kg-m}^2$$

here we used to results shown above $I_{zz} = \rho\pi\frac{R^4h}{10} = \frac{3}{10}mR^2$

17-21



This is a rectilinear accel problem. IF BOTTLE TIPS Bottle will tip about O ⇒ f to the righ

$$\Sigma F_x = +f = ma_{G_x}$$

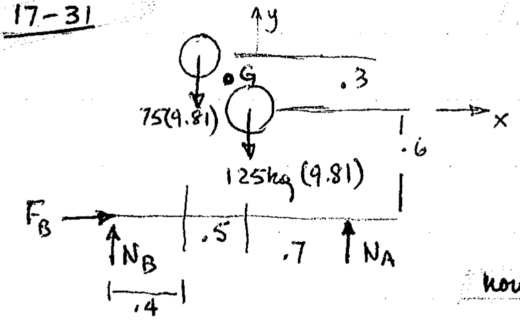
$$\Sigma F_y = N - W = ma_{G_y} = 0 \qquad N = W = 2\text{lb}$$

if bottle tips first N is at O $\Rightarrow \circlearrowleft_{+}\Sigma M_G = -N(1.5) + f(8) = 0$

$$\Rightarrow f_{tip} = \frac{2(1.5)}{8} = .375\text{ lb} \quad \text{and} \quad a_{G_x} = \frac{.375}{2/32.2} = 6.0375\text{ ft/s}^2$$

if bottle slips first: $\mu N = f_{slip} = 2(.2) = .4\text{ lb}$. Since $f_{slip} > f_{tip}$ bottle tips first.

17-31



For a wheelie $N_A = 0$. Find center of mass + eqns of motion

$$\Sigma F_x = F_B = ma_{G_x\,sys.}$$

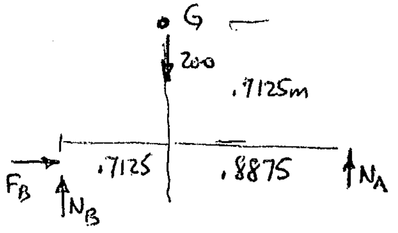
$$\Sigma F_y = -W_{BIKE} - W_{man} + N_B + N_A = ma_{G_y\,SYSTEM} = 0$$

$$N_B = W_{BIKE} + W_{man} = 200(9.81) \approx 1962\text{ N}$$

now the location of G_{SYSTEM}: $\bar{x} = \frac{\Sigma x_i m_i}{\Sigma m_i} = \frac{-.5(75)+0(125)}{200}$

$$= -.1875\text{m}$$

$$\bar{y} = \frac{\Sigma y_i m_i}{\Sigma m_i} = \frac{75(.3)+0(125)}{200} = .1125\text{ m}$$



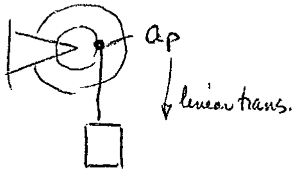
$+\circlearrowright \sum M_G = F_B(.7125) - N_B(.7125) = 0$ for $N_A = 0$

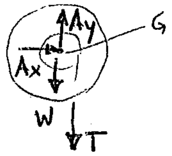
$F_B = N_B = 1962N.$

since $F_B > \mu N_B \Rightarrow$ rear wheel will slip & wheelie cannot be done. Since $F_B = m a_{G_{x\,sys}} \Rightarrow a_{G_x} = 9.81\ m/s^2$

17-49

$I_A = m k_A^2 = \frac{180}{32.2}(1.25)^2 = 8.7345$ slug-ft^2





(B) Write eqns of motion of wheel

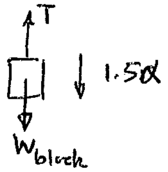
$\sum F_x = A_x = m a_{G_x} = 0 \quad \Rightarrow A_x = 0$

$\sum F_y = A_y - W_{Spool} - T = m a_{G_y} = 0 \quad (3)$

$+\circlearrowright \sum M_G = \cancel{-}I_G\alpha = -T(1.5) \Rightarrow I_G\alpha = T(1.5) \quad (2)$

(B) $a_{P_t} = \alpha r = 1.5\alpha = a_{block}$

(B) Write eqn of motion for block



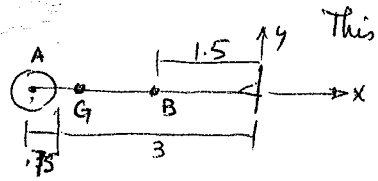
$W_{block} - T = \frac{W_{block}}{g} 1.5\alpha \quad (1)$

Using (1), (2) we find $\alpha = .826\ rad/s^2 \circlearrowright$, $T = 4.808 lb$

and $A_y = W_{spool} + T = 184.81\ lb. \quad (3)$ $\qquad a = 1.5(.826) = 1.239\ ft/s^2$

Note center of mass of wheel is fixed. Outer edge of spool has $a_{P_t} = \alpha r = a_{block}$

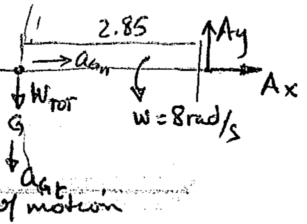
17-53



This is symmetric about x axis $\therefore \bar{y} = \frac{\sum y_i m_i}{\sum m_i} = \frac{\sum y_i W_i}{\sum W_i}$

$\bar{y} = \frac{-1.5(10) - 3.75(15)}{25} = -2.85 ft$

(A) FIND CENTER OF MASS + I_G



$I_A(\text{circle}) = \frac{1}{2}mR^2 = \frac{1}{2}\frac{15}{32.2}(.75)^2 = .131$ slug-ft^2

$I_B(\text{rod}) = \frac{1}{12}ml^2 = \frac{1}{12}\frac{10}{32.2}(3)^2 = .233$ slug-ft^2

$I_G = I_A + m_{circ}(.9)^2 + I_B + m_{rod}(1.35)^2 = 1.307$ slug-ft^2

$I_o = I_G + md^2 = 1.307 + \frac{25}{32.2}(2.85)^2 = 7.613$

(B) Write eqns of motion

$\sum F_t = m_{TOT}\alpha r = \frac{25}{32.2}\alpha(2.85) = W - A_y$

$\sum F_n = m_{TOT}\omega^2 r = \frac{25}{32.2}(64)(2.85) = A_x = 141.615 lb$

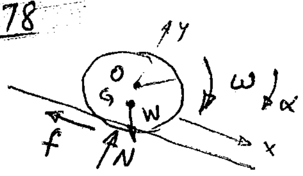
$+\circlearrowright \sum M_G = I_G\alpha \Rightarrow A_y(2.85) = 1.307\alpha$

solving $\alpha = 9.359\ rad/s \circlearrowright$

$A_y = 4.292\ lb$;

17-78

$I_G = m_{TOT} k_G^2 = \frac{185}{32.2}(1.65)^2 = 15.642$ slug-ft^2



since no slip $a_o = \alpha r = \alpha(2)$. Next find $\bar{a}_G$

Eqs of motion

$\sum F_x = W\sin 20° - f = m a_{G_x} \quad (1)$

$\sum F_y = -W\cos 20° + N = m a_{G_y} \quad (2)$

$+\circlearrowright \sum M_G = -I_G\alpha \Rightarrow -f(2-.75) = -I_G\alpha \quad (3)$

$\bar{a}_G = \bar{a}_o + \bar{\alpha}\times\bar{r}_{G/o} - \omega^2\bar{r}_{G/o}$

$= \alpha(2)\bar{i} + (-\alpha\bar{k})\times(-.75\bar{j}) - 36(-.75\bar{j})$

$\bar{a}_G = \alpha(2)\bar{i} - .75\alpha\bar{i} + 27\bar{j} = 1.25\alpha\bar{i} + 27\bar{j}$ ft/s^2

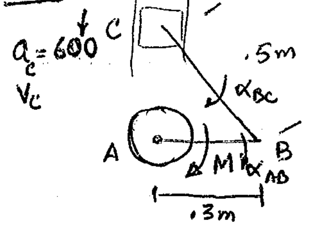
from (1) $f = W\sin 20^\circ - m a_{G_x} = 185\sin 20^\circ - \frac{185}{32.2}(1.25\alpha)$
(3) $f(1.25) = 15.642\alpha$
$\Big\}$ $\alpha = 3.213\ \text{rad}/s^2$ ↻ $f = 40.202\ lb$

from (2) $N = W\cos 20^\circ + m a_{G_y} = 185\cos 20^\circ + \frac{185}{32.2}(27) = 328.97\ lb.$

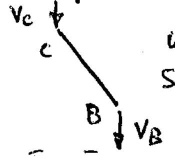
Note here that since the tire rolls w/no slip the center's accel is αr but not its center of mass.

METHOD

17-91



$m_{CB} = 1\ kg$ $\quad m_{AB} = .6\ kg$



$\omega_{CB} = 0$
Since C & B move vertically

. First we need to find $\bar{a}_G$ of AB
Next we need to find forces on AB
Thus find $\bar{a}_B$ & THEN a_G

$$\bar{a}_B = \bar{a}_C + \bar{\alpha}_{BC} \times \bar{r}_{B/C} - \omega_{BC}^2 r_{B/C}$$
$$= -600\bar{j} + (-\alpha_{BC}\bar{k}) \times (+.3\bar{i} - .4\bar{j}) = -600\bar{j} - .3\alpha_{BC}\bar{j} - .4\alpha_{BC}\bar{i} \quad (1)$$

also $\bar{a}_B = \bar{a}_A + \bar{\alpha}_{AB} \times \bar{r}_{B/A} - \omega_{AB}^2 \bar{r}_{B/A}$. Since system was at rest $\omega_{AB} = 0$ & $\bar{a}_A = 0 \Rightarrow \bar{a}_B = \bar{\alpha}_{AB} \times \bar{r}_{B/A}$

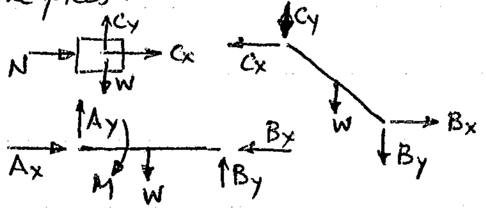
$\bar{a}_B = (-\alpha_{AB}\bar{k}) \times .3\bar{i} = -.3\alpha_{AB}\bar{j}$ (2) . From (1) & (2) $\Rightarrow$ $\underline{\underline{\alpha_{BC} = 0}}$ & $\underline{\underline{\alpha_{AB} = 2000\ \text{rad}/s^2}}$

Now I_G of AB $= \frac{1}{12} m_{AB} l^2 = \frac{1}{12}(.6)(.3)^2 = \underline{\underline{.0045\ kg\text{-}m^2 = I_{G_{AB}}}}$

Also $\bar{a}_G$ of AB $= \bar{\alpha}_{AB} \times \bar{r}_{B/A/2} = \underline{\underline{-300\bar{j}\ m/s^2 = \bar{a}_G}}$

To find the forces:

These are the FBD's



For the piston
$\overset{+}{\rightarrow} \sum F_x = C_x + N = 0 = m a_{G_x}$
$+\downarrow \sum F_y = W - C_y = m a_{G_y} = 3(+600)$
$C_y = 3(9.81) - 3(600) = -1770.57\ N$
↓

For BC

Note that BC only has accel which is rectilinear $(\bar{a}_B = \bar{a}_c)$ $\Rightarrow \sum F_x = -C_x + B_x = m a_{G_x} = 0$
Therefore $\bar{a}_G$ of BC $= \bar{a}_B$
$+\downarrow \sum F_y = W + B_y + C_y = m a_{G_y} = 1(600)$
$B_y = -C_y - W + m a_{G_y} = \underline{\underline{2360.76N}}$

For AB $+\circlearrowleft \sum M_G = -I_G \alpha_{AB}$ since α_{AB} is clockwise $\Rightarrow$ $B_y(.15) - A_y(.15) - M = -I_G \alpha_{AB}$

also $+\downarrow \sum F_y = -B_y - A_y + W_{AB} = m a_G = .6(300)$ $\Rightarrow A_y = -B_y + W_{AB} - .6(300) = -2534.874N$

thus $M = B_y(.15) - A_y(.15) + I_G \alpha_{AB} = (2360.76)(.15) + 2534.874(.15) + .0045(2000)$
$= 743.345\ Nm$ ↻

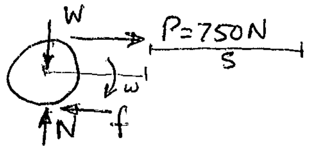
Since $\sum F_x = A_x - B_x = 0 \Rightarrow A_x = B_x$ since $\bar{a}_G$ was only in y dir for AB

Since is in rectilinear motion $^+\circlearrowleft \sum M_G = 0 = B_x(.4) + C_y(.15) - B_y(.15)$ gives B_x

thus knowing $B_x = 1549.25\ lb = C_x = A_x$ $\Rightarrow$ $N = -C_x = -1549.25\ lb.$

HW #16

18-6



Originally at rest $V_G = \omega = 0 \Rightarrow T_1 = 0$

f, N, W do no work. $I_G = \frac{1}{2} mR^2$

$\Sigma U_{1-2} = P \cdot s = 750s$ N-m

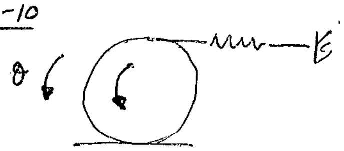
No slip $\Rightarrow V_G = \omega r$

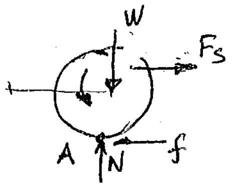
$$T_2 = I_G \frac{\omega^2}{2} + \frac{m V_G^2}{2} = \frac{1}{4} mR^2\omega^2 + \frac{m}{2}\omega^2 R^2 = \frac{3m}{4}\omega^2 R^2$$

since center moves 2 m $\Rightarrow$ cylinder rolls $\frac{2m}{.75m} = \frac{8}{3}$ rad. Thus load moves through a distance of $\frac{8}{3}$ rad (1.5 m) = 4 m. The $\Sigma U_{1-2} = 3000$ N-m

thus $3000 \text{ N-m} = \frac{3}{4}(80)(.75)^2\omega^2$ or $\omega = 9.428$ rad/s ↻

18-10





Originally $V_G = \omega = 0 \Rightarrow T_1 = 0$

f, N, W do no work $I_G = mk^2$

As in 18-6 since at mesh pt $V_A = 0$ then for $S_G = 2$ ft

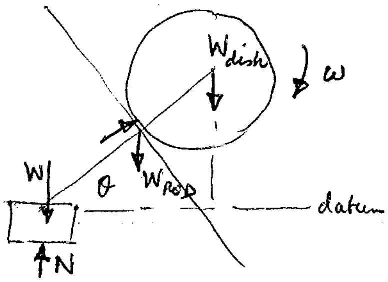
$\theta_{gear} = \frac{s}{.5 \text{ ft}} = 4$ rad $\Rightarrow$ F_s moves through a distance of $4(.9) = 3.6$ ft ←

thus $\Sigma U_{1-2} = -\frac{1}{2}k(3.6)^2 + M\theta = -\frac{1}{2}(3)(3.6)^2 + 6(4) = 4.56$ lb-ft

$$T_2 = \frac{1}{2} m V_G^2 + I_G \frac{\omega^2}{2} = \frac{1}{2} m_{gear}\omega^2 r^2 + .06551\frac{\omega^2}{2} = .09098\omega^2$$

$\omega = 7.0795 \sim 7.08$ rad/s ↺

18-15



when $\theta = 45°$ weight of disk is $.6\sin 45°$ above datum

when $\theta = 0$ weight is at 0 m above datum

$\therefore$ U of disk = $W_{DISK}(.6\sin 45°)$

No work is done by forces on the block

U of rod = $W_{rod}(.3\sin 45° - .3\sin 0°) = W_{rod}(.3\sin 45°)$

now $\bar{V}_{G_{disk}} = \omega(.2m) \searrow 45° = 1.6$ m/s $\qquad \bar{V}_{Block} = -V_B\bar{i}$ thus $\bar{V}_B = \bar{V}_{G_{disk}} + \bar{\omega} \times \bar{r}_{B/G}$

@θ=0: $V_{G\,disk} = \omega_D(.2)$ m/s $\searrow 45°$ here ω_D is disk angular velocity at θ = 0

Now: $\bar{r}_{B/G} = -.6\cos\theta\,\bar{i} - .6\sin\theta\,\bar{j}$ and let $\bar{\omega}_{BA} = \omega\bar{k}$ $\therefore$ $\bar{\omega} \times \bar{r} = -\omega .6\cos\theta\,\bar{j} + \omega .6\sin\theta\,\bar{i}$

thus $-\bar{V}_B\bar{i} = 1.6\cos 45°\bar{i} - 1.6\sin 45°\bar{j} + \omega(.6)\sin\theta\,\bar{i} - .6\omega\cos\theta\,\bar{j}$.

$$\Rightarrow \omega_{AB} = \frac{-1.6\cos 45°}{.6\cos\theta} \quad \text{and} \quad V_B = -1.6\cos 45° - .6\omega\sin\theta$$

when $\theta = 45°$ $\omega_{AB} = -2.67$ rad/s $\quad V_B = 0\,\frac{m}{s}$; when $\theta = 0°$ $\omega_{AB} = \frac{-(.2\omega_D)(.7071)}{.6} = -.2357\omega_D$

and $V_B = -(.2\omega_D)(.7071) = -.1414\omega_D$ m/s

also $\bar{V}_{G_{rod}} = \bar{V}_{G_{disk}} + \bar{\omega}_{AB} \times \bar{r}/2 = +1.6\cos 45°\bar{i} - 1.6\sin 45°\bar{j} + .3\omega_{AB}\sin\theta\,\bar{i} - .3\omega_{AB}\cos\theta\,\bar{j}$

@ θ=45° $\bar{V}_{G_{rod}} = .565\bar{i} - .565\bar{j} = [1.6\cos 45° + .3(-2.67)(.7071)]\bar{i} + [-1.6\sin 45° - .3(-2.67)(.707$

$$\theta=0:\ \vec{V}_{G_{rod}} = +.2\omega_D\cos 45^\circ\,\bar{i} - .2\omega_D \sin 45^\circ\,\bar{j} - .3\omega_{AB}\bar{j} = .1414\omega_D\,\bar{i} - \overset{-3(-.2357\omega_D)}{.0707\,\omega_D}\,\bar{j}$$

$$\theta=45^\circ:\ T_{\cancel{1}_{TOT}} = \tfrac{1}{2} m_B V_{BLOCK,}^2 + \tfrac{1}{2} m_{ROD} V_{ROD}^2 + \tfrac{1}{2} I_G \omega_{ROD}^2 + \frac{\omega^2}{2} m_{DISK} k^2 + \tfrac{1}{2} m V_{G_{disk}}^2$$

$$= \tfrac{1}{2}(\#)(0) + \tfrac{1}{2}(4)(.565^2 + .565^2) + \tfrac{1}{2}\left[\tfrac{1}{12}\right](4)[.6]^2[2.67]^2 + \frac{2}{2}(.15)^2(8)^2 + \frac{(2)(1.6)^2}{2}$$

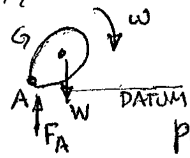
$$\theta=0:\ T_{2_{TOT}} = \tfrac{1}{2}\,\#\,(\cdot1414\omega_D)^2 + \tfrac{1}{2}(4)[.15809\,\omega_D]^2 + \tfrac{1}{2}\left[\tfrac{1}{12}(4)[.6]^2[-.2357\omega_D]^2\right] + \frac{2(.15)^2(\omega_D)^2}{2} + \frac{2(.2)^2\omega_D^2}{2}$$

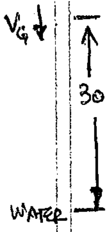
here $I_{DISK} = mk^2$ $\quad V_{G_{rod}}\Big|_{\theta=0} = \sqrt{(.1414\,\omega_D)^2 + (.0707\,\omega_D)^2} = .15809\,\omega_D$

$$V_{G_{rod}}\Big|_{\theta=45^\circ} = \sqrt{.565^2 + .565^2} = .8 \text{ m/s}$$

Thus $T_{1_{TOT}} + W_{DISK}(.6\sin 45^\circ) + W_{ROD}(.3\sin 45^\circ) = T_{2_{TOT}} \Rightarrow \omega_D = 13.3$ rad/s ↻

18-17 Look at the problem as a rigid body problem from $\theta=0^\circ$ to 90°, rotating about pt. A. At $\theta=90^\circ$ to the water level this is a free fall problem. During this time the body also rotates.





when $\theta=0^\circ$ $\quad V_G = \omega = 0 \Rightarrow T_1 = 0$, $\quad V_1 = mgh_{G_1} = 150(1.5)$ $\qquad I_G = mk_G^2$

when $\theta=90^\circ$ $\quad V_G = \omega(1.5)$ and $T_2 = \tfrac{1}{2}mV_G^2 + \frac{I_G}{2}\omega^2 = \tfrac{1}{2}m[1.5^2 + 1.2^2]\omega^2$

and $V_2 = mgh_{G_2} = 0$. Thus $T_1 + V_1 = T_2 + V_2 \Rightarrow \underline{\omega = 5.12 \text{ rad/s}}$ ↻

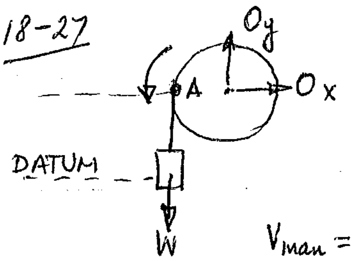
The time required to fall the 30 feet $\Rightarrow 0 = 30 \cancel{+} V_G t + \tfrac{1}{2}a_G t^2 = 30 \cancel{+} \underbrace{(5.12)(1.5)}_{V_G} t - 16.1$

Here $V_G = -7.68$ ft/s . Take the + roots ($t = -1.624$ or 1.147 s) $\Rightarrow \underline{\underline{t = 1.147s.}}$

Using $T_1 + V_1 = T_2 + V_2$ between $\theta = 90^\circ$ position and water we will find that ω of body at impact is same as that at $\theta = 90^\circ \Longrightarrow \alpha = 0$ rad/s². This means that $\theta = \theta_{initial} + \omega_{initial}\,t + \frac{\overset{0}{\alpha} t^2}{2}$. $t = 1.147s$; $\theta_{initial}$ assume to be zero, $\Rightarrow \theta = (5.12)(1.147) \cong 5.87$ rad or ~.935 revolutions,

HW #17

18-27



since man starts from rest $V_{man} = V_{spool} = \omega = 0$ $\therefore T_1 = 0$. Also the weight moves through a distance of 4m. Thus $V_1 = 0$ and $V_2 = -W(4)$ Nm

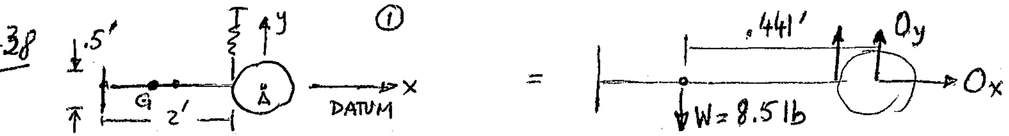
$$T_2 = \frac{1}{2} m_{man} V_{man}^2 + \frac{1}{2} m_{spool} \cancel{V_{spool}^2} + \frac{1}{2} I_G \omega^2$$

$V_{man} = V_A = \omega(.8)$ m/s also $I_G = k_G^2 m = 200(.525)^2$ kg m^2

Thus $T_1 + V_1 = 0 = T_2 + V_2 = -70(9.81)(4) + \frac{1}{2}(70)[.8\omega]^2 + \frac{1}{2}[200(.525)^2]\omega^2$

and $V_{man} = .8\omega = 5.93$ m/s ↓

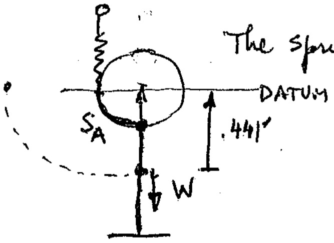
18-38



$\bar{x} = \frac{\Sigma \bar{x}_i m_i}{\Sigma m_i} = \frac{\Sigma \bar{x}_i W_i}{\Sigma W_i} = \frac{-(1.3)(2)-(2.3)(.5)}{8.5} = -.441'$

With this datum $V_1 = \frac{1}{2}k(.3)^2$ and $T_1 = 0$ since $V_G = \omega = 0$

When system rotates it will look like this. The spring stretches $.3 + .\frac{3\pi}{2} = .3 + S_A$



The spring will stretch .77ft and the weight will move through a distance of .441 ft $\therefore V_2 = \frac{1}{2}k(.77)^2 + (8.5 \text{ lb})(.441 \text{ft})$

$V_G = \omega(.441')$. Also we need $I_G = (I_{G_1} + m_1 d_1^2) + (I_{G_2} + m_2 d_2^2) + (I_{G_3} + m_3 d_3^2)$

disk: $I_{G_1} = \frac{1}{2} m_{disk} r^2$; $m_1 d_1^2 = m_{disk}(.441-0)^2$; rod AB: $I_{G_2} = \frac{1}{12} m_{AB} l_{AB}^2$; $m_2 d_2^2 = m_{AB}(1.8$

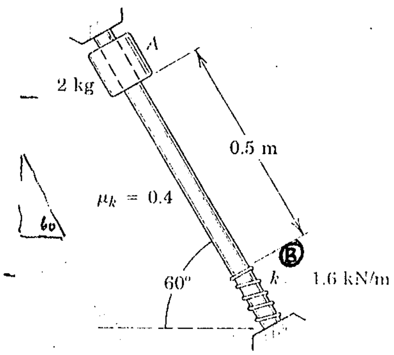
rod$_{CD}$: $I_{G_3} = \frac{1}{12} m_{CD} l_{CD}^2$; $m_3 d_3^2 = m_{CD}(1.3 - .441)^2$

Finally $T_2 = \frac{1}{2} m_{TOT} V_G^2 + I_G \omega^2/2$; Thus

$T_1 + V_1 = T_2 + V_2 \Rightarrow 0 + \frac{1}{2}(8)(.3)^2 = \frac{1}{2}\left(\frac{8.5}{32.2}\right)(.441\omega)^2 + I_G \omega^2/2 + \frac{1}{2}8(.77)^2 - (8.5)(.441)$

Solving: $\omega = 4.00$ rad/s ↻

The 2-kg collar is released from rest at A and slides down the inclined fixed rod in the vertical plane. The coefficient of kinetic friction is 0.4. Calculate (*a*) the velocity v of the collar as it strikes the spring and (*b*) the maximum deflection x of the spring.



③

$m a_n = N - W\cos 60^\circ$

$N = W\cos 60^\circ$ ①

$= 2 \cdot 9.81 \left(\frac{1}{2}\right)$

$= 9.81\,N$

$f = 0.4(9.81)$

$= 3.924\,N$

$T_1 = 0$

$U_{1-2} = m_1 g\,[0.5 \sin 60^\circ] - f\,[0.5\,m]$

$T_2 = \frac{1}{2} m v_2^2$

$\overset{①}{0} + m_1 g\,\overset{①}{[0.5 \sin 60^\circ]} - \overset{①}{\mu m_1 g \cos 60^\circ (.5)} = \overset{①}{\tfrac{1}{2} m_1 v_2^2}$

$2g\,\{(0.5)(.866) - 0.4(.5)\overset{(.5)}{}\} = v_2^2$

$= v_2^2$

$\sqrt{2(9.81)(.333)} = 2.56\ m/s$ ① /9 for v

at Ⓑ $T_2 = \frac{1}{2} m v_2^2 = 6.534$ ①

$U_{1-2} = \overset{①}{m_1 g\,x \sin 60^\circ} - \overset{①}{\mu m_1 g \cos 60^\circ x} - \overset{①}{\tfrac{1}{2} k x^2}$

$T_3 = 0$

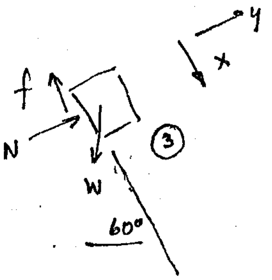
$\frac{1}{2} m v_2^2 + x\,[m_1 g \sin 60^\circ - \mu m_1 g \cos 60^\circ] - \frac{1}{2} k x^2 = 0$

$\frac{1}{2} \cdot 2\ (2)(9.81)(.333) + x\,[2(9.81)(.866) - .4(2)(9.81)(.5)] - \frac{1}{2}(1600)x^2 = 0$

$-6.534 - x(13.067) + 800x^2 = 0$

$x = \dfrac{13.067 \pm \sqrt{(13.067)^2 + 4(6.534)(800)}}{1600} = \underline{\underline{.099\ m}}$ ② /15

other way



③

$W \sin 60^\circ - f = m a_x$ ①

$N = W\cos 60^\circ$

$f = \mu N$ ① $\therefore$ ①

$m a_x = m g \sin 60^\circ - \mu m g \cos 60^\circ$

$a_x = 9.81(.866) - .4(9.81)(.5)$

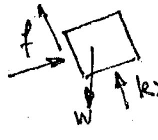
$= 9.81(.666)$ ①

$v_f^2 = v_i^2 + 2as$ ①

$v_f = 0 + \sqrt{2(\quad)(.5)}$ at bot

$= 2.56\ m/s$ ① /⑨

also



$\Sigma F_x = -f + W \sin 60^\circ - kx = m a_x$

$\Sigma F_y = 0 \quad N = W\cos 60^\circ$

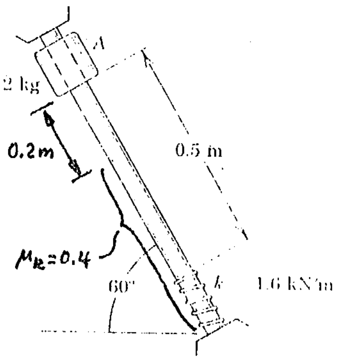
$C - kx = -\mu W\cos 60^\circ + W \sin 60^\circ - kx = m v \frac{dv}{dx}$; now integrate

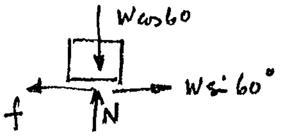
$Cx - kx^2/2 = \int [C - kx]\,dx = m\left(v^2/2_f - v^2/2_i\right) = -m v_i^2/2$ or /⑥

Hopler 12/15

The 2-kg collar is released from rest at A and slides down the inclined fixed rod in the vertical plane. The coefficient of kinetic friction is 0.4. Calculate (*a*) the velocity v of the collar as it strikes the spring and (*b*) the maximum deflection x of the spring.

Note $\mu_k = 0.4$ over the distance shown





$T_1 = 0$

$U_{1-2} = m_1 g\ (0.5\sin 60) - f\ (0.3)$

$T_2 = \frac{1}{2} m v_2^2$

$$0 + m_1 g\left[0.5 \sin 60°\right] - \mu m_1 g \cos 60° \ (0.3) = \frac{1}{2} m_1 v_2^2$$

$$2g\left[0.5(.866) - .04(.5)(.3)\right] = v_2^2$$

$$.433 - .006$$

$$\sqrt{2(9.81)(.427)} = v_2 = 2.895 \text{ m/s}$$

$T_2 = \frac{1}{2} m v_2^2$

$U_{1-2} = \left[m_1 g \sin 60 - \mu m_1 g \cos 60\right] x - \frac{1}{2} k x^2$

$T_3 = 0$

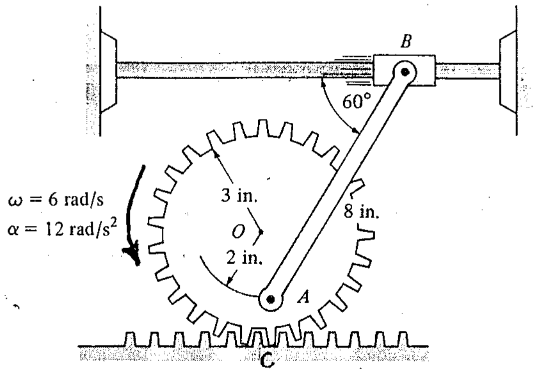
$$\frac{1}{2} \cdot 2 \cdot \left[2 \cdot 9.81 \cdot (.427)\right]^2 + 2(9.81)\left[\overset{(.666)}{.866 - 0.4(.5)}\right] x - \frac{1}{2} k x^2 = 0$$

$$(.666)$$

$$-8.378 - 13.067x + 800x^2$$

$x_1 = .094$ m or 94 mm

At a given instant the gear has the angular motions shown. Determine the accelerations of points *A* and *B* on the link and the link's angular acceleration at this instant.



No slip at C $\therefore$ $V_o = \omega r \leftarrow$ or $6(3/12) = 1.5\, ft/s \leftarrow$

$a_o = \alpha r \leftarrow$ or $12(3/12) = 3\, ft/s^2 \leftarrow$

$$\underline{V_A} = \underline{V_o} + \underline{\omega_{AO}} \times \underline{r_{A/o}}$$

$$= -1.5\underline{i} + 6\underline{k} \times \underline{r_{A/o}} = -0.5\,\underline{i}\,\tfrac{ft}{s} \text{ since } r_{A/o} = -\tfrac{2}{12}\underline{j} \text{ ft.}$$

$$\underline{V_A} = \underline{V_B} + \underline{\omega_{AB}} \times \underline{r_{A/B}}$$

$$-0.5\underline{i} = V_B\underline{i} + \omega_{AB}\underline{k} \times \left(\tfrac{8}{12}\right)\left[-\cos 60\,\underline{i} - \sin 60\,\underline{j}\right]$$

$$\Longrightarrow \underline{\omega_{AB} = 0} \quad \text{and} \quad \underline{V_B = -0.5\, ft/s}$$

$$\underline{a_A} = \underline{a_o} + \underline{\alpha_{AO}} \times \underline{r_{A/o}} - \omega_{AO}^2\, \underline{r_{A/o}}$$

$$= -3\underline{i} + 12\underline{k} \times \left(\tfrac{-2}{12}\underline{j}\right) - 6^2\left(-\tfrac{2}{12}\underline{j}\right)$$

$$= -3\underline{i} + 2\underline{i} + 6\underline{j}$$

$$= -\underline{i} + 6\underline{j} \quad ft/sec^2$$

$$\underline{a_B} = \underline{a_A} + \underline{\alpha_{AB}} \times \underline{r_{B/A}} - \cancelto{0}{\omega_{AB}^2}\, \underline{r_{B/A}}$$

$$= -\underline{i} + 6\underline{j} + \alpha\underline{k} \times \tfrac{8}{12}(\cos 60°\,\underline{i} + \sin 60°\,\underline{j})$$

$$a_B\underline{i} = \left(-1 - \tfrac{8\alpha}{12}\sin 60°\right)\underline{i} + \left(6 + \tfrac{8\alpha}{12}\cos 60°\right)\underline{j}$$

$$\Rightarrow \alpha = \frac{-72}{8\cos 60°} = -18 \text{ rad/sec}^2 \text{ or } \underline{\alpha = 18 \text{ rad/sec}^2} \text{ CW}$$

$$\Rightarrow a_B = -1 - \frac{8(-18)}{12}\sin 60° = \underline{9.39\, ft/sec^2} \rightarrow$$

LET'S LOOK AT $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$ one dim heat equation

EXPLICIT SCHEME $u_{i,j+1} = u_{i,j} + \overset{C}{\boxed{\frac{\alpha \Delta t}{\Delta x^2}}} \left[u_{i+1,j} - 2u_{i,j} + u_{i-1,j} \right]$

U^j is the amplitude at t_j,

NEUMANN STABILITY ANALYSIS let $u_{ij} = U^j e^{Ik\Delta x}$

$I = \sqrt{-1}$

$$e^{Ik\Delta x} \left\{ U^{j+1} = U^j + U^j C \left[e^{Ik\Delta x} - 2 + e^{-Ik\Delta x} \right] \right\}$$

$$U^{j+1} = U^j \{ \underbrace{1 + 2C[\cos k\Delta x - 1]}_{A} \} = U^0 A^{j+1}$$

k is sep var cn

$i\Delta x = x_i$

$e^{I\xi} + e^{-I\xi} = 2\cos\xi$

if $|A|$ is > 1 $\Rightarrow$ solution becomes unbounded

FOR BOUNDEDNESS let $\{ 1 + 2C[\cos k\Delta x - 1] \}^2 \leq 1$

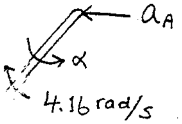
$$4C[\quad] + (2C)^2 [\quad]^2 \leq 0$$

$$4C[\quad] \{ 1 + C[\quad] \} \leq 0$$

$$C \leq \frac{1}{1 - \cos k\Delta x}$$

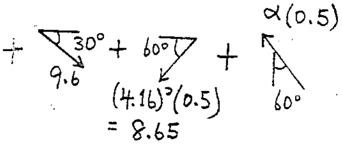
6-40,

ad/s ↻



$(a_B)_n = (8)^2(0.15) = 9.6$

$\vec{a}_{A/B}$



$60° + 9.6\cos 30° -$

$60° - \alpha(0.5)\sin 60°$

$-9.6 \sin 30° -$

$+ \alpha(0.5)\cos 60°$

ad/s² ↻

$\underline{\underline{m/s^2 \leftarrow}}$

$\times \vec{r}_{A/B} - \omega^2 \vec{r}_{A/B}$

$s 30° \hat{i} - (8)^2(0.15)\sin 30° \hat{j}$

$30° \hat{i} + 16(0.15)\cos 30° \hat{j}$

$60° \hat{i} + 0.5 \sin 60° \hat{j})$

$60° \hat{i} + 0.5 \sin 60° \hat{j})$

$- 0.433\alpha - 4.326$

$785 + 0.25\alpha - 7.4935$

$a_B = -36 + 24 - 6.93\alpha_{AB}$

$0 = 72 + 4\alpha_{AB}$

$v_B = \underline{v}_A + \omega \times r_{B/A}$ (4)

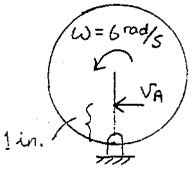
16-79

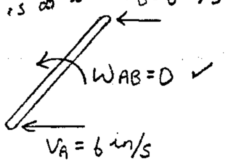
$v_B \underline{i} = -6\underline{i} + \omega \underline{k} \times (8)(\cos 60 i + \sin 60 j)$

$j \quad 0 = \omega \cdot 8\cos 60 \quad \omega = 0$ (1)

$\underline{i} \quad -6 = v_B$ (1)

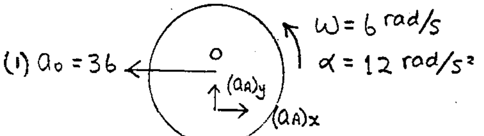
Since instantaneous is ∞ away $v_B = 6$ in/s





7 to find v_B & ω

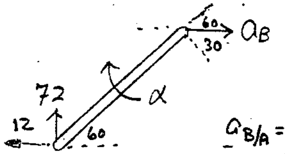
$v_A = 1(6) = 6$ in/s ✓ (1)



$\vec{a}_A = \vec{a}_0 + \vec{a}_{A/0}$

$\vec{a}_A = \underset{\leftarrow}{36} + (6)^2(2)^{(1)} \uparrow + \underset{\rightarrow}{12(2)}^{(1)}$

$\vec{a}_A = \underset{\leftarrow}{12} + 72\uparrow$ (2) 5 for a_A



IF $\underline{\alpha} = -\alpha \underline{k}$

$\underline{a}_{B/A} = \underline{\alpha} \times \underline{r}_{B/A} - \omega^2 \underline{r}_{B/A} \nearrow 0$

$= -|\alpha| 8 \; \searrow 30$ (2)

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$\underset{\rightarrow}{a_B} = 72\uparrow + \underset{\leftarrow}{12} + \underset{30°\searrow}{\alpha(8)}$

$\overset{+}{\rightarrow} \quad a_B = -12 + \alpha(8)\cos 30°$ (2)

$+\uparrow \quad 0 = 72 - \alpha(8)\sin 30°$ (2)

$\alpha = 18 \text{ rad/s}^{2\,(1)}$, $\underline{\underline{a_B = 113 \text{ in/s}^2 \rightarrow}}$ (1)

$a_A = \sqrt{(15)^2 + (72)^2} = \underline{\underline{73.5 \text{ in/s}^2}}$ 8 for α, a_B

$\theta = \tan^{-1}\left(\frac{72}{15}\right) = \underline{\underline{78.2°}}$ ∠θ $\frac{\quad}{20}$

Also,

$\vec{a}_A = \vec{a}_0 + \vec{\alpha} \times \vec{r}_{A/0} - \omega^2 \vec{r}_{A/0}$

$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$

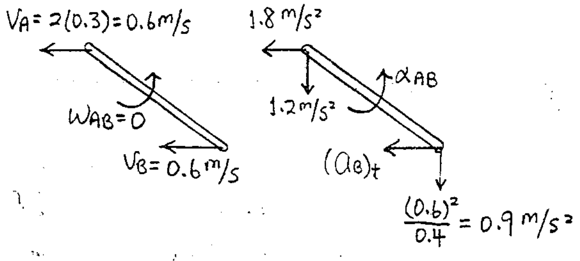
$a_B \hat{i} = -(12)(3)\hat{i} + (12\hat{k}) \times (-2\hat{j}) - (6)^2(-2\hat{j}) +$

$(\alpha_{AB}\hat{k}) \times (8\cos 60° \hat{i} + 8\sin 60° \hat{j}) - \vec{0}$

16-80

$\alpha_{AB} = 18 \text{ rad/s}^2$ ↻

$a_B = 113 \text{ in/s}^2$



$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$\underset{\leftarrow}{(a_B)_t} + 0.9\downarrow = \underset{\leftarrow}{1.8} + 1.2\downarrow + \overset{5 \, 4 \, 3}{\nearrow} \alpha_{AB}(0.5)$

$\overset{+}{\leftarrow} \quad (a_B)_t = 1.8 - \frac{3}{5}(\alpha_{AB})(0.5)$

$+\downarrow \quad 0.9 = 1.2 - \frac{4}{5}(\alpha_{AB})(0.5)$

$\underline{\underline{\alpha_{AB} = 0.75 \text{ rad/s}^2}}$ ↻

$(a_B)_t = 1.575 \text{ m/s}^2$

$\alpha_{BC} = \frac{1.575}{0.4} = \underline{\underline{3.94 \text{ rad/s}^2}}$ ↻

Also,

$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB}\vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$

$-(a_B)_t \hat{i} - \frac{(0.6)^2}{0.4}\hat{j} = -6(0.3)\hat{i} - (2)^2(0.3)\hat{j}$

$+ (\alpha_{AB}\hat{k}) \times (0.4\hat{i} - 0.3\hat{j}) - \vec{0}$

$-(a_B)_t = -1.8 + 0.3\alpha_{AB}$

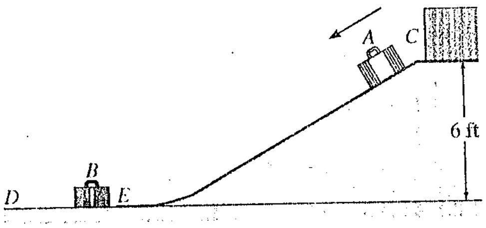
$-0.9 = -1.2 + 0.4\alpha_{AB}$

$\underline{\underline{\alpha_{AB} = 0.75 \text{ rad/s}^2}}$ ↻

$(a_B)_t = 1.575 \text{ m/s}^2$

$\alpha_{BC} = \frac{1.575}{0.4} = \underline{\underline{3.94 \text{ rad/s}^2}}$ ↻

The 15-lb suitcase *A* is released from rest at *C*. After it slides down the smooth ramp, it strikes the 10-lb suitcase *B*, which is originally at rest. If the coefficient of restitution between the suitcases is $e = 0.3$ and the coefficient of kinetic friction between the floor *DE* and each suitcase is $\mu_k = 0.4$, determine *(a)* the velocity of *A* just before impact, *(b)* the velocities of *A* and *B* just after impact, and *(c)* the distance *B* slides before coming to rest.



$(T_A+V_A)_i = (T_A+V_A)_F$ ④ $T_{A_i}=0$ $V_{A_i} = 15.6$ $T_{A_F} = \frac{1}{2}\frac{W}{g}V_f^2$ $T_{A_F}=0$

$\therefore \quad mgh = \frac{1}{2}mv_f^2 \qquad V_f = \sqrt{2gh}$ (a)

$= \sqrt{2(32.2)(6)} = 19.7$ ft/s ① (19.66 ft/s)

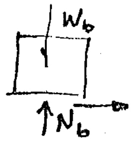
$$\overset{\frac{15}{32.2}}{\cancel{m_{A_i}}}\overset{\sqrt{2gh}}{\cancel{V_{A_i}}} + \cancel{m_B \overset{0}{V_{B_i}}} = \overset{\frac{15}{32.2}}{m_A} V_{A_f} + \overset{\frac{10}{32.2}}{m_B} V_{B_f}$$ ④

$$e = \frac{V_{B_f} - V_{A_f}}{V_{A_i} - 0}$$ ② $\therefore \quad V_{B_f} - V_{A_f} = eV_{A_i} = 0.3(19.66) = 5.9$

gives $V_{B_f} = 15.3$ ft/s $V_{A_f} = 9.43$ ft/s (b) ②

(c) $T_{B_i} - f\Delta s = T_{B_f}$ ③

$T_{B_f} = 0 \qquad T_{B_i} = \frac{1}{2}m_b V_{B_f}^2 = \frac{1}{2}\frac{10}{32.2}(15.3)^2 \qquad f = \mu_k W_B$



③ $f = \mu_k N_b = .4(10) = 4$ lb

$\therefore \quad \frac{1}{2}m_b V_{B_f}^2 - \mu_k W_B \Delta s \qquad \Delta s = \frac{V_{B_f}^2}{2\mu_k g}$ (c)

① $= \frac{15.3^2}{2(.4)(32.2)} = \underline{9.13 \text{ ft.}}$

Fundamental Equations of Dynamics

KINEMATICS

Particle Rectilinear Motion

Variable a	*Constant $a = a_c$*
$a = \dfrac{dv}{dt}$	$v = v_0 + a_c t$
$v = \dfrac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$
$v\,dv = a\,ds$	$v^2 = v_0^2 + 2a_c(s - s_0)$

Particle Curvilinear Motion

x, y, z Coordinates		*r, θ, z Coordinates*	
$v_x = \dot{x}$	$a_x = \ddot{x}$	$v_r = \dot{r}$	$a_r = \ddot{r} - r\dot{\theta}^2$
$v_y = \dot{y}$	$a_y = \ddot{y}$	$v_\theta = r\dot{\theta}$	$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
$v_z = \dot{z}$	$a_z = \ddot{z}$	$v_z = \dot{z}$	$a_z = \ddot{z}$

n, t, Coordinates

$v = \dot{s}$	$a_t = \dot{v} = v\dfrac{dv}{ds}$ $a_n = \dfrac{v^2}{\rho} \qquad \rho = \left\lvert \dfrac{[1 + (dy/dx)^2]^{3/2}}{d^2y/dx^2} \right\rvert$

Relative Motion

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \qquad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Rigid Body Motion About a Fixed Axis

Variable α	*Constant $\alpha = \alpha_c$*
$\alpha = \dfrac{d\omega}{dt}$	$\omega = \omega_0 + \alpha_c t$
$\omega = \dfrac{d\theta}{dt}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$
$\omega\,d\omega = \alpha\,d\theta$	$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

For Point P

$s = \theta r \qquad v = \omega r \qquad a_t = \alpha r \qquad a_n = \omega^2 r$

Relative General Plane Motion-Translating Axes

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})} \qquad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$

Relative General Plane Motion-Trans. and Rot. Axis

$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{\text{rel}}$

$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{B/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/A}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{B/A})_{\text{rel}} + (\mathbf{a}_{B/A})_{\text{rel}}$

KINETICS

Mass Moment of Inertia	$I = \int r^2\,dm$
Parallel-Axis Theorem	$I = I_G + md^2$
Radius of Gyration	$k = \sqrt{\dfrac{I}{m}}$

Equations of Motion

Particle	$\Sigma\mathbf{F} = m\mathbf{a}$
Rigid Body (Plane Motion)	$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G\alpha \,/\, \Sigma M_P = \Sigma(\mathcal{M}_k)_P$

Principle of Work and Energy

$T_1 + U_{1-2} = T_2$

Kinetic Energy

Particle	$T = \frac{1}{2}mv^2$
Rigid Body (Plane Motion)	$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

Work

Variable force	$U_F = \int F\cos\theta\,ds$
Constant force	$U_{F_c} = (F_c\cos\theta)\,\Delta s$
Weight	$U_W = -W\,\Delta y$
Spring	$U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$
Couple moment	$U_M = M\,\Delta\theta$

Power and Efficiency

$P = \dfrac{dU}{dt} = \mathbf{F} \cdot \mathbf{v} \qquad \varepsilon = \dfrac{P_{\text{out}}}{P_{\text{in}}} = \dfrac{U_{\text{out}}}{U_{\text{in}}}$

Conservation of Energy Theorem

$T_1 + V_1 = T_2 + V_2$

Potential Energy

$V = V_g + V_e$, *where* $V_g = \pm Wy,\ V_e = +\frac{1}{2}ks^2$

Principle of Linear Impulse and Momentum

Particle	$m\mathbf{v}_1 + \Sigma\int \mathbf{F}\,dt = m\mathbf{v}_2$
Rigid Body	$m(\mathbf{v}_G)_1 + \Sigma\int \mathbf{F}\,dt = m(\mathbf{v}_G)_2$

Conservation of Linear Momentum

$\Sigma(\text{syst. } m\mathbf{v})_1 = \Sigma(\text{syst. } m\mathbf{v})_2$

Coefficient of Restitution $\quad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

Principle of Angular Impulse and Momentum

Particle	$(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2,$ *where* $H_O = (d)(mv)$
Rigid Body (Plane Motion)	$(\mathbf{H}_G)_1 + \Sigma\int \mathbf{M}_G\,dt = (\mathbf{H}_G)_2,$ *where* $H_G = I_G\omega$ $(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2,$ *where* $H_O = I_G\omega + (d)(mv_G)$

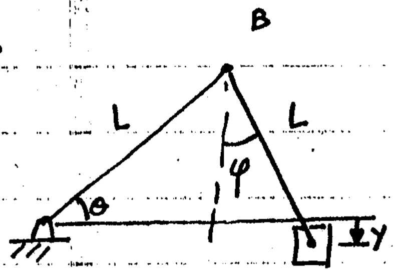
Conservation of Angular Momentum

$\Sigma(\text{syst. } \mathbf{H})_1 = \Sigma(\text{syst. } \mathbf{H})_2$

also to convert from mph to ft/sec $\qquad$ mph × $\frac{44}{30}$ = ft/sec

km/h to m/sec $\qquad$ km/h × $\frac{1}{3.6}$ = m/sec

16-36



$$L\cos\theta + L\sin\varphi = L \qquad (1)$$

and $$y = L\cos\varphi - L\sin\theta \qquad (2)$$

take $\frac{d}{dt}(1) \Rightarrow \dot{\varphi} = \dot{\theta}\,\frac{\sin\theta}{\cos\varphi}$

and $\ddot{\varphi} = \frac{\cos\theta\,\dot{\theta}^2 + \sin\varphi\,\dot{\varphi}^2}{\cos\varphi}$

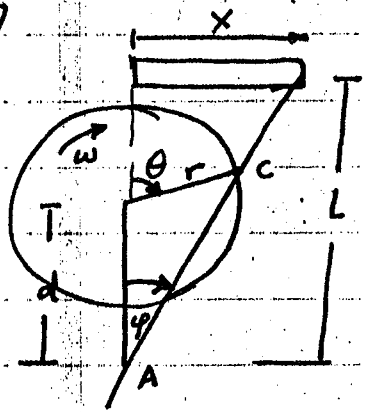
for $\theta = 60° \Rightarrow \varphi = 30° \Rightarrow \boxed{\dot{\varphi} = \dot{\theta} = \omega}$

take $\frac{d}{dt}(2) \Rightarrow \quad V = \frac{dy}{dt} = -L\left[\sin\varphi\,\dot{\varphi} + \cos\theta\,\dot{\theta}\right] = -L\omega$

take $\frac{d}{dt}V = a = -L\left[\cos\varphi\,\dot{\varphi}^2 + \sin\varphi\,\ddot{\varphi} - \sin\theta\,\dot{\theta}^2 + \cos\theta\,\cancel{\ddot{\theta}}\,\right] = \frac{-1}{\sqrt{3}}L\omega^2$

this means $V = L\omega\uparrow$ and $a = \frac{1}{\sqrt{3}}L\omega^2\uparrow$

16-37



$\therefore \quad x = L\tan\varphi$

$\overline{AC}\cos\varphi = d + r\cos\theta$

$\overline{AC}\sin\varphi = r\sin\theta$

$\therefore \tan\varphi = \frac{r\sin\theta}{d + r\cos\theta}$

now $\dot{x} = L\frac{d}{dt}\tan\varphi = L\frac{d}{dt}\left(\frac{r\sin\theta}{d + r\cos\theta}\right)$

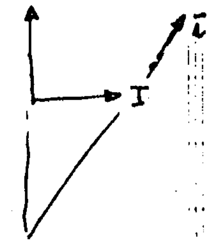
$$V = \dot{x} = l\dot{\theta}r\,\frac{(r + d\cos\theta)}{(d + r\cos\theta)^2}$$

and $$a = \dot{V} = \ddot{x} = lr\dot{\theta}^2\sin\theta\left[\frac{2r^2 - d^2 + rd\cos\theta}{(d + r\cos\theta)^3}\right]$$

$\dot{\phi} = \frac{r\dot{\theta}(r + d\cos\theta)}{d^2 + 2rd\cos\theta + r^2} \Rightarrow$ this is the angular velocity of bar AC

note that it is <u>not</u> the same as that of the disk (which is ω)

16-37



$$\bar{V}_c = -\omega\bar{k} \times (r\sin\theta\, I + r\cos\theta\, J) = -\omega r\sin\theta\, J + \omega r\cos\theta\, I$$

$$\bar{V}_c = \underset{0}{\cancel{\bar{V}_A}} + \bar{\Omega} \times \bar{r}_{c/A} + (\bar{V}_{c/A})_{rel} \qquad \text{let } \bar{\Omega} = -\dot{\Phi}\bar{k} \quad \bar{r}_{c/A} = (d+r\cos\theta)\bar{J} + r\sin\theta I$$

$$\text{let } (\bar{V}_{c/A})_{rel} = -V_{rel}\, \bar{i} = -V_{rel} \left[\frac{(d+r\cos\theta)\bar{J} + r\sin\theta I}{\sqrt{d^2+2dr\cos\theta+r^2}}\right.$$

$$\bar{V}_c = \dot{\Phi}(d+r\cos\theta)I - \dot{\Phi}r\sin\theta J - \frac{V_{rel}(d+r\cos\theta)}{\sqrt{\ }}J - \frac{V_{rel}\, r\sin\theta}{\sqrt{\ }} I$$

$$(J) \qquad -\omega r\sin\theta = -\dot{\Phi}r\sin\theta - \frac{V_{rel}(d+r\cos\theta)}{\sqrt{\ }} \qquad (*)$$

$$(I) \qquad \omega r\cos\theta = \dot{\Phi}(d+r\cos\theta) - \frac{V_{rel}\, r\sin\theta}{\sqrt{\ }}$$

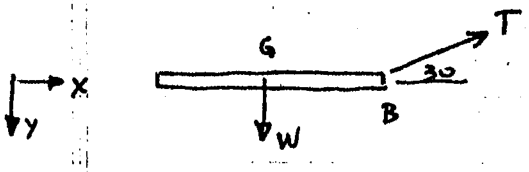
$$\text{from } (*) \quad \frac{(\dot{\Phi}-\omega)r\sin\theta}{d+r\cos\theta} = -\frac{V_{rel}}{\sqrt{\ }} \Rightarrow \omega r\cos\theta = \dot{\Phi}(d+r\cos\theta) + \frac{(\dot{\Phi}-\omega)r^2\sin^2\theta}{d+r\cos\theta}$$

$$\Longrightarrow \frac{\omega(dr\cos\theta + r^2)}{d+r\cos\theta} = \dot{\Phi}\frac{(d^2+2dr\cos\theta+r^2)}{d+r\cos\theta}$$

$$\text{or} \quad \dot{\Phi} = \frac{r+d\cos\theta}{d^2+2dr\cos\theta+r^2}\,\omega r$$

$$\text{and} \quad V_{rel} = -\frac{(\dot{\Phi}-\omega)r\sin\theta}{d+r\cos\theta}\sqrt{d^2+2dr\cos\theta+r^2}$$

17-88



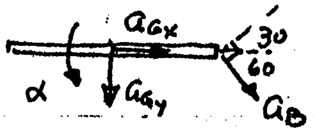
FBD

$$\overset{+}{\rightarrow} \sum F_x = m a_{G_x} = T\cos 30° \qquad (1)$$

$$+\downarrow \sum F_y = m a_{G_y} = W - T\sin 30° \qquad (2)$$

$$\circlearrowright + \sum M_G = I_G \alpha = T\sin 30°(.15) \qquad (3)$$

From kinematics



$$\bar{a}_G = \bar{a}_B + \bar{a}_{G/B}$$

$$a_{G_x}\bar{i} + a_{G_y}\bar{j} = a_{B}\cos 60°\bar{i} + a_{B}\sin 60°\bar{j} + (+\alpha)(.15)\bar{j}$$

$\bar{a}_{G/B} = \alpha r \downarrow$ only since $\omega = 0$

$I_G = \frac{1}{2} m l^2 = \frac{1}{2} m (.3)^2$

$$\therefore \left.\begin{aligned} a_{G_x} &= a_{B}\cos 60° \\ a_{G_y} &= a_{B}\sin 60° + 0.15\alpha \end{aligned}\right\}$$

$$a_{G_y} = a_{G_x}\tan 60° + 0.15\alpha \qquad (4)$$

we have unknowns: T, a_{G_x}, a_{G_y}, α & 4 eqns

we can solve $(3)/(1) \Rightarrow \tan 30°(0.15) = \frac{I_G \alpha}{m a_{G_x}} = \frac{1}{12} \frac{l^2 \alpha}{a_{G_x}}$

$$\therefore \quad a_{G_x} \tan 30°(0.15) \cdot \frac{12}{l^2} = \alpha \qquad (*)$$

$\therefore$ from (4) $\quad a_{G_y} = a_{G_x}\left[\tan 60° + \frac{12(0.15)^2}{l^2}\tan 30°\right] \qquad (5)$

from (1) & (2)

$$m a_{G_y} = W - T\sin 30° = W - m a_{G_x}\tan 30°$$

$$a_{G_y} = g - a_{G_x}\tan 30° \qquad (6)$$

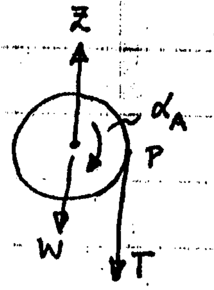
$\therefore$ Put (6) into (5) to get that $g = a_{G_x}\left[\tan 60° + \frac{12(0.15)^2}{l^2}\tan 30° + \tan 30\right.$

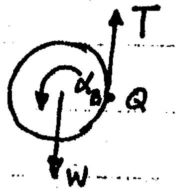
$a_{G_x} = 2.427 \text{ m/s}^2$

from (5) $\quad a_{G_y} = 8.409 \text{ m/s}^2 \qquad \alpha = 28.03 \text{ rad/s}^2$ from (*)

from (1) $\quad T = 5.605 \text{ N}$

17-84





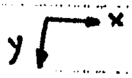
for A

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$+\curvearrowright \Sigma M_G = I_{A_G}\alpha_A = TR \quad (1)$

$+\downarrow \Sigma F_y = m_A a_{G_A} = W + T - Z = 0$

$I_{A_G} = I_{B_G} = \frac{1}{2} mR^2$



for B

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$+\downarrow \Sigma F_y = m a_{G_B} = W - T \quad (2)$

$+\curvearrowleft \Sigma M_G = I_{B_G}\alpha_B = TR \quad (3)$

Kinematics : since $\omega_A = \omega_B = 0$

$a_P = \alpha_A R = a_Q \downarrow$

$\bar{a}_{G_B} = \bar{a}_Q + \bar{a}_{G/Q} = a_Q \bar{j} + \alpha_B R \bar{j} = (\alpha_A R + \alpha_B R)\bar{j}$

Thus $a_{G_B} = (\alpha_A + \alpha_B)R \quad (4)$

We have 4 eqns 4 unknowns : $\alpha_A, T, a_{G_B}, \alpha_B$

$(1)+(3) \Rightarrow (I_{A_G})(\alpha_A + \alpha_B) = 2TR \Rightarrow T = \frac{I_{A_G}(\alpha_A+\alpha_B)}{2R} \quad (*)$

also (4) $\Rightarrow T = I_{AG}\frac{(\alpha_A+\alpha_B)}{2R} = \frac{I_{A_G} a_{G_B}}{2R^2}$

from (2) $m a_{G_B} = W - T = W - \frac{I_{AG} a_{G_B}}{2R^2}$ $\quad \therefore \; a_{G_B} = W/\left[m + \frac{I_{AG}}{2R^2}\right]$

$= \frac{4}{5} g = 7.85 \, m/s^2$

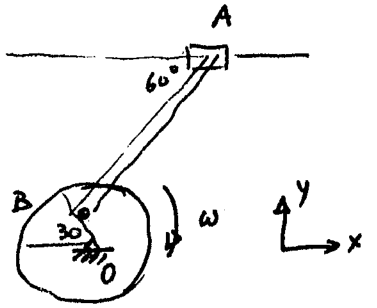
$\alpha_A = \frac{TR}{I_{AG}} = 43.6 \text{ rad/s}^2$

$T = \frac{I_{AG} a_{G_B}}{2R^2} = \frac{m}{4} \cdot a_{G_B} = 19.62 \text{ N}$

$\alpha_B = a_{G_B}/R - \alpha_B = 43.6 \text{ rad/s}^2$

12

$|\bar{\omega}| = 8$ rad/s



$\bar{\omega} = -8\bar{k}$ rad/s since clockwise

$\bar{V}_B = \bar{V}_o + \bar{\omega} \times \bar{r}_{B/o}$

$\overset{i}{k \; + \; j}$

$\bar{r}_{B/o} = 150[-\cos 30^\circ \bar{i} + \sin 30^\circ \bar{j}]$

$\bar{\omega} \times \bar{r}_{B/o} = [8 \cdot 30 \cdot \cos 30^\circ \bar{j} + 8 \cdot 30 \sin 30^\circ \bar{i}] \cdot 5$

$\bar{V}_o = \bar{0}$

$\bar{V}_B = [207.85\,\bar{j} + 120\,\bar{i}] \cdot 5$ mm/s

$\bar{V}_A = \bar{V}_B + \bar{\omega}_{BA} \times \bar{r}_{A/B}$ $\qquad \bar{V}_A = V_A \bar{i}$

let $\bar{\omega}_{BA} = \omega_{BA} \bar{k}$ $\qquad$ THIS ASSUMES ω ↺+

$\bar{r}_{A/B} = 500[\cos 60^\circ \bar{i} + \sin 60^\circ \bar{j}]$

$\therefore \quad V_A \bar{i} = 5[120\bar{i} + 207.85\bar{j}] + \omega_{BA} \cdot 500 \cos 60^\circ \bar{j} - \omega_{BA} \cdot 500 \sin 60^\circ \bar{i}$

since $\bar{V}_A$ has no $\bar{i}$ component. $\Rightarrow$ $5(207.85) + \omega_{BA} \cdot 500 \cos 60^\circ = 0$

or $\omega_{BA} = -\frac{5(207.85)}{250} = -4.16$ rad/sec

$\omega_{BA} = 4.16$ rad/s ↻

- SIGN IMPLIES ↻ SINCE WE ASSUMED ω_{BA} ↺+

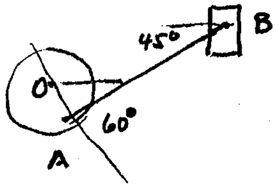
$V_A = 600 - \omega_{BA} \cdot 500 \sin 60^\circ$

$= 2400$ mm/sec ⟶

$= 2.4$ m/sec

-80 $\qquad$ let $\bar{\omega} = -2\bar{k}$ rad/s

$\bar{\alpha} = -6$ rad/sec^2 $\bar{k}$



$\bar{r}_{B/A} = .5[\cos 45^\circ \bar{i} + \sin 45^\circ \bar{j}]$

let $\bar{\omega}_{BA} = \omega_{BA} \bar{k}$ $\qquad \bar{V}_B = V_B \bar{j}$

$\bar{V}_B = \bar{V}_A + \bar{\omega}_{BA} \times \bar{r}_{B/A}$

$\bar{V}_B \bar{j} = (-.52\bar{i} - .3\bar{j}) + .5\omega_{BA} \cos 45^\circ \bar{j} - \omega_{BA}(.5) \sin 45^\circ \bar{i}$

$\Rightarrow (-.52 - .5\,\omega_{BA} \sin 45^\circ) = 0$ $\quad \underline{\omega_{BA} = +1.471 \text{ rad/s}}$ ↻

$\underline{V_B} = .5\omega_{BA} \cos 45^\circ - .3 = -.82$ m/s or $\underline{.82 \text{ m/s}}$ ↓

$\bar{V}_A = \bar{V}_o + \bar{\omega} \times \bar{r}_{A/o}$

$\overset{i}{k \; + \; j}$

$\bar{V}_o = \bar{0} \qquad \bar{a}_o = 0$

$\bar{r}_{A/o} = .3[\cos 60^\circ \bar{i} - \sin 60^\circ \bar{j}]$

$\bar{\omega} \times \bar{r}_{A/o} = -.6 \cos 60^\circ \bar{j} - .6 \sin 60^\circ \bar{i}$

$\underline{\bar{V}_A = (-.52\bar{i} \; - .3\bar{j}) \text{ m/s}}$

$\bar{a}_A = \bar{a}_o + \bar{\alpha} \times \bar{r}_{A/o} - \omega^2 \bar{r}_{A/o}$

$= \bar{0} + (-6\bar{k}) \times \bar{r}_{A/o} - 4(.3)[-.866\bar{j} + .5\bar{i}]$

$= -1.56\bar{i} - .9\bar{j} + 1.04\bar{j} + .6\bar{i}$

$= -2.16\bar{i} + .14\bar{j}$ m/s^2

$$\bar{a}_B = \bar{a}_A + \bar{\alpha}_{BA} \times \bar{r}_{B/A} - \omega_{BA}^2 \bar{r}_{B/A}$$

$$\bar{j} = -2.16\,\bar{i} + .14\,\bar{j} + \left[-.5\alpha_{BA}\sin 45^\circ\,\bar{i} + .5\alpha_{BA}\cos 45^\circ\,\bar{j}\right]$$

$$- (1.471)^2 \left[.5\cos 45^\circ\,\bar{i} + .5\sin 45^\circ\,\bar{j}\right]$$

let $\bar{\alpha}_{BA} = \alpha_{BA}\bar{k}$

$$\bar{\alpha}_{BA} \times \bar{r}_{B/A} = \alpha_{BA} \cdot .5\cos 45^\circ\,\bar{j} - .5\alpha_{BA}\sin 45^\circ\,\bar{i}$$

let $\bar{a}_B = a_B\bar{J}$

$$\therefore\ -2.16 - .5\alpha_{BA}\sin 45^\circ - (1.471)^2(.5\cos 45^\circ) = 0$$

$$\alpha_{BA} = 8.27 \text{ rad/sec}^2 \ \downarrow$$

$$_B = .14 + .5\alpha_{BA}\cos 45^\circ - (1.471)^2(.5)\sin 45^\circ$$

$$= 3.55 \text{ m/s} \ \downarrow$$

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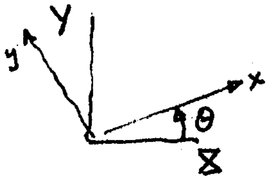
Given $(\bar{V}_{B/o})_{rel} = (5 \text{ ft/s})\,\bar{i}$ $\quad \bar{\Omega} = 3\bar{k}$ rad/sec

$(\bar{a}_{B/o})_{rel} = (3 \text{ ft/s}^2)\,\bar{i}$ $\quad \dot{\bar{\Omega}} = 5\bar{k}$ rad/sec²

$$\bar{V}_B = \bar{V}_o + \bar{\Omega} \times \bar{r}_{B/o} + (\bar{V}_{B/o})_{rel}$$

$$= \bar{0} + (3\bar{k}) \times (2\bar{i}) + 5\bar{i}$$

$$\bar{V}_B = (6\bar{j} + 5\bar{i}) \text{ ft/s}$$



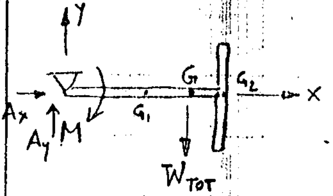
center of rotating frame (origin of x, y, z) in the same as origin of fixed X, Y, Z

$$\bar{a}_B = \bar{a}_o + \dot{\bar{\Omega}} \times \bar{r}_{B/o} + \bar{\Omega} \times (\bar{\Omega} \times \bar{r}_{B/o}) + 2\bar{\Omega} \times (\bar{V}_{B/o})_{rel} + (\bar{a}_{B/o})_{rel}$$

$$= \bar{0} + (5\bar{k}) \times (2\bar{i}) + 3\bar{k} \times (3\bar{k} \times 2\bar{i}) + 2(3\bar{k}) \times (5\bar{i}) + 3\bar{i}$$

$$= 10\bar{j} + 3\bar{k} \times 6\bar{j} + 30\bar{j} + 3\bar{i} = 10\bar{j} - 18\bar{i} + 30\bar{j} + 3\bar{i} = 40\bar{j} - 15\bar{i} \text{ ft/s}^2$$

18-8



$$\bar{x} = \frac{\Sigma x_i m_i}{\Sigma m_i} = \frac{1(4)(2) + 2(4)(1)}{4 \cdot 2 + 4 \cdot 1} = \frac{16}{12} = 1.33\,m$$

$$T = \tfrac{1}{2} m V_G^2 + \tfrac{1}{2} I_G \omega^2 \quad V_G = \omega \bar{x} \Rightarrow T = \tfrac{1}{2}(m\bar{x}^2 + I_G)\omega^2$$

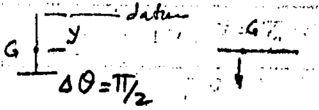
$$T_1 + \Sigma U_{1-2} = T_2$$

assume $\omega_1 = 0$

Bar	1	2
I_{G_i}	$\frac{1}{12} \cdot 4 \cdot 2 \cdot (2)^2$	$\frac{1}{12} \cdot 4 \cdot 1 \cdot (1)^2$
md^2	$4 \cdot 2 \cdot (\frac{1}{3})^2$	$4 \cdot 1\,(\frac{2}{3})^2$
TOT =	$3.5\bar{5}$	$2.1\bar{1}$

TOTAL $I_G = 5.67$ kg-m²

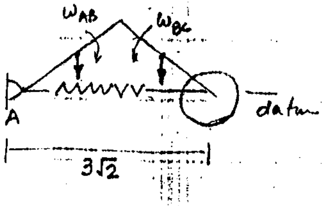
$$\Sigma U_{1-2} = M\Delta\theta - Wy$$



if $\omega_1 = 0$ & $\Delta\theta = \pi/2$
$$\omega = \sqrt{\frac{2(M\Delta\theta + mg\bar{x})}{m\bar{x}^2 + I_G}} = 4.18 \text{ rad/s}$$

$\Delta\theta = \pi$
$$\omega = \sqrt{\frac{2M\Delta\theta}{m\bar{x}^2 + I_G}} = 3.41 \text{ rad/s}$$

18-20



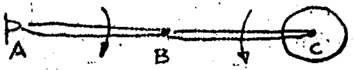
$$T = \tfrac{1}{2} I_{AB_G}\omega_{AB}^2 + \tfrac{1}{2} m_{AB} V_{AB_G}^2 + \tfrac{1}{2} I_{BC_G}\omega_{BC}^2 + \tfrac{1}{2} m_{BC} V_{BC_G}^2 + \tfrac{1}{2} M_c V_c^2 + \tfrac{1}{2} I_c \omega_c^2$$

initially all were zero ∴ $T_1 = 0$

$\underline{V_1} = W_{AB}\frac{3\sin 45°}{2} + W_{BC}\frac{3\sin 45°}{2} = mg \cdot 3/\sqrt{2} = \underline{31.82 \text{ lb-ft}}$

$\underline{V_2} = \frac{1}{2}k(6 - 3\sqrt{2})^2 = \frac{1}{2} \cdot 4(1.76)^2 = \underline{6.18 \text{ lb-ft}}$

In the final position



$\bar{V}_B = 3\omega_{AB} \downarrow \quad \bar{V}_c = |V_c| \rightarrow$

and $\bar{V}_{C/B} = 3\omega_{BC} \uparrow$ ∴ $V_c\,\bar{i} = -3\omega_{AB}\bar{j} + 3\omega_{BC}\bar{j}$ using $\bar{V}_C = \bar{V}_B + \bar{V}_{C/B}$

$\Rightarrow \underline{V_c = 0}$ $\underline{\omega_{BC} = W_{AB} = \omega}$ $\Rightarrow$ $\underline{V_{AB_G} = 1.5\,\omega}$ & $\bar{V}_{BC_G} = \bar{V}_B + \omega_{AB}(1.5)\bar{j} = -1.5\omega\bar{j}$

$\underline{V_{BC_G} = 1.5\,\omega} \downarrow$ also $\underline{V_{AB_G} = 1.5\,\omega} \downarrow$

∴ since $V_c = 0$ & disk doesn't slip velocity at contact $= 0 \Rightarrow \omega_c = 0$

$$\therefore T_2 = \left[\tfrac{1}{2}(0.3494)\omega^2 + \tfrac{1}{2}(0.4658)(-1.5\omega)^2\right]2 + \left[\tfrac{1}{2}(.6211)\cdot 0^2 + \tfrac{1}{2} I_c (0)^2\right] = 1.3975\,\omega^2$$

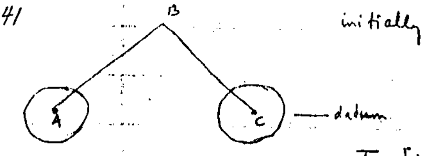
also: $I_{AB_G} = \frac{1}{12} m l^2 = \frac{1}{12}\left(\frac{15}{32.2}\right)(3)^2 = I_{CB_G} = .3494$ slug ft²

$m_{AB} = \frac{15}{32.2} = 0.4658$ slug $\quad m_c = \frac{20}{32.2} = .6211$ slug

THUS $T_1 + V_1 = T_2 + V_2$

$$0 + 31.82 = 1.3975\,\omega^2 + 6.18 \Rightarrow \omega = 4.283 \text{ rad/s}$$

18-41



initially $T_1 = 0$

$V_1 = 2\left[8 \cdot \frac{3}{2} \sin 60^\circ\right]$

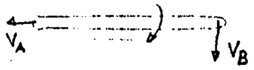
$V_2 = 0$

$T_2 = \left[\frac{1}{2} m_A V_A^2 + \frac{1}{2} I_{A_G}\omega_A^2 + \frac{1}{2} m_{AB} V_{AB_G}^2 + \frac{1}{2} I_{AB_G}\omega_{AB}^2\right]2$

By symmetry

$V_A = \omega_A(.5)$ $\quad I_{AB_G} = \frac{1}{12} m_{AB} l^2 = \frac{1}{12}\left(\frac{8}{32.2}\right)(9) = .1863 \text{ slug ft}^2$

$I_A = \frac{1}{2} m_A R^2 = \frac{1}{2}\left(\frac{10}{32.2}\right)(.5)^2 = .0388 \text{ slug ft}^2$



$-V_A \vec{i} = -V_B \vec{j} - \omega_{AB} \cdot 3\vec{j} \Rightarrow V_A = 0 \qquad V_B = -\omega_{AB} \cdot 3$

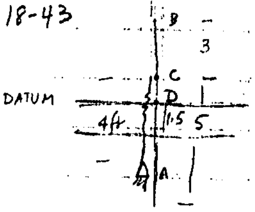
$V_{AB_G} = -1.5\,\omega_{AB}$

$T_1 + V_1 = T_2 + V_2$

$0 + 20.7846 = 2\left[0 + 0 + \frac{1}{2}\left(\frac{8}{32.2}\right)(1.5\omega)^2 + \frac{1}{2}(.1863)\omega^2\right] = .7453\omega^2$

$\omega = 5.281 \text{ rad/s}$

18-43



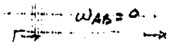
$\bar{V}_{AB_G} = \bar{V}_A + \bar{V}_{G/A} = -3\bar{i} + 4(2)\bar{i} = 5\bar{i} \qquad V_{AB_G} = 5 \text{ ft/s} \qquad \omega_{AB} = 2 \text{ rad/s}$

$V_A = 3 \text{ ft/s} \leftarrow \Rightarrow \omega_{AC} = .75 \text{ rad/s} \Rightarrow V_{AC_G} = 9/8 \text{ ft/s}$

$\Delta s_1 = 3 \text{ ft}$

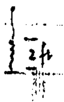
$V_1 = -m_{AC} g(1.5) \cdot 2 + \left(\frac{1}{2} k \Delta s_1^2\right)2$

in final position



$V_{AB_G} = V_A$

$V_{AC_G} = V_A/2 \qquad \omega_{AC} = V_{AC_G}/1.5 = V_A/3$



$V_2 = m_{AB} g(4) + 2 m_{AC} g(1.5) + \left(\frac{1}{2} k \Delta s_2^2\right)2$

$\Delta s_2 = 1 \text{ ft}$

$T_1 = \frac{1}{2} m_{AB} V_{AB_G}^2 + \frac{1}{2} I_{AB_G}\omega_{AB}^2 + 2\left[\frac{1}{2} m_{AC} V_{AC_G}^2 + \frac{1}{2} I_{AC_G}\omega_{AC}^2\right]$

$V_1 = 2\frac{1}{2} k \Delta s_1^2 - m_{AC} g \cdot 2(1.5)$

$T_2 = \frac{1}{2} m_{AB} V_A^2 + 2\left[\frac{1}{2} m_{AC}\left(V_A/2\right)^2 + \frac{1}{2} I_{AC_G}\left(V_A/3\right)^2\right]$

$V_1 = 2\frac{1}{2} k \Delta s_2^2 + 4 m_{AB} g + 2 m_{AC} g(1.5)$

But we were told that $m_{AC} = 0 \Rightarrow I_{AC_G} = 0$

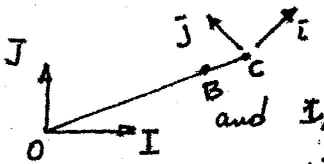
$\therefore \quad T_1 = \frac{1}{2} m_{AB}(5)^2 + \frac{1}{2}\left(\frac{1}{12} m_{AB} \cdot 8^2\right)(2)^2$

$V_1 = 2 \cdot \frac{1}{2} k \Delta s_1^2$

$T_2 = \frac{1}{2} m_{AB} V_A^2 + 2 \cdot \frac{1}{2} k \Delta s_2^2 + 4 m_{AB} g$

Solving $\quad V_A = 10.5 \text{ ft/s}$

15.145 fix

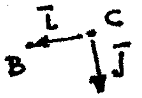


and I, J at the center of the circular slot. Given $\bar{a}_B = \bar{0}$

also $(\bar{V}_{B/C})_{rel} = u\bar{j}$ @ B

$$\bar{V}_C = -\omega r_1 \bar{J} = \bar{V}_0 + \bar{\omega} \times \bar{r}_{C/0}$$

$$\bar{V}_B = \bar{V}_C + \bar{\Omega} \times \bar{r}_{B/C} + (V_{B/C})_{rel} = -\omega r_1 \bar{j} + \Omega r_2 \bar{j} + u\bar{j}$$



$$\bar{a}_C = \bar{a}_0 + \bar{\alpha} \times \bar{r}_{C/0} - \omega^2 \bar{r}_{C/0} = +\omega^2 r_1 \bar{i}$$

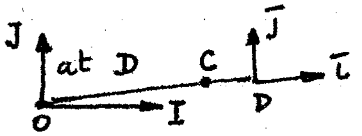
$$\bar{a}_B = \bar{a}_C + \dot{\bar{\Omega}} \times \bar{r}_{B/C} - \Omega^2 \bar{r}_{B/C} + 2\bar{\Omega} \times (\bar{V}_{B/C})_{rel} + (\bar{a}_{B/C})_{rel}$$

$$\bar{0} = \omega^2 r_1 \bar{i} + \dot{\Omega} r_2 \bar{j} - \Omega^2 r_2 \bar{i} - 2\Omega u \bar{i} - \Omega^2 r_2 \bar{i}$$

$$\left\{\begin{array}{l} (\bar{a}_{B/C})_{rel} = -u^2/r_2\, \bar{i} \\ u = \Omega r_2 \end{array}\right.$$

$\Rightarrow \quad \dot{\Omega} = 0 \qquad$ also $\quad \omega^2 r_1 - 4\Omega^2 r_2 = 0$

$$\Omega = \frac{\omega}{2}\left(r_1/r_2\right)^{1/2} \quad \& \quad \underline{u = \Omega r_2 = \frac{\omega}{2}\sqrt{r_1 r_2}}$$



$$\bar{a}_D = \bar{a}_c + \dot{\bar{\Omega}} \times \bar{r}_{D/C} - \Omega^2 \bar{r}_{D/C} + 2\bar{\Omega} \times (\bar{V}_{D/C})_{rel} + (\bar{a}_{D/C})_{rel}$$

$$= -\omega^2 r_1 \bar{i} + 0 - \Omega^2 r_2 \bar{i} + 2\Omega \bar{k} \times (u\bar{j}) - \frac{u^2}{r_2} \bar{i}$$

$$= -\omega^2 r_1 \bar{i} - \Omega^2 r_2 \bar{i} - 2\Omega u \bar{i} - \Omega^2 r_2 \bar{i} = (-\omega^2 r_1 - 4\Omega^2 r_2)\bar{i}$$

$$= -2\omega^2 r_1 \bar{i} \qquad \underline{a_D = 2\omega^2 r_1}$$

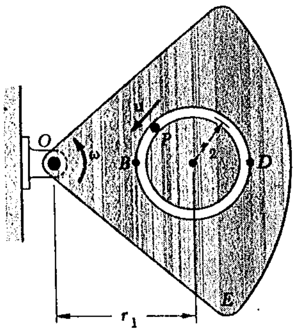
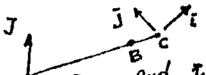


FIG. P 15.145

15.145. A pin P slides in a circular slot of radius r_2 which is cut in the plate OE. The velocity of P relative to the plate is of constant magnitude u and is directed as shown. Knowing that the plate rotates counterclockwise with a constant angular velocity ω, derive an expression for (a) the magnitude u for which the acceleration of the pin is zero as it passes through point B, (b) the corresponding magnitude of the acceleration of the pin as it passes through point D.

5.145 fix

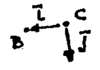


and I, J at the center of the circular slot. Given $\bar{a}_B = \bar{0}$

also $(\bar{V}_{B/C})_{rel} = u\bar{j}$ @ B

$\bar{V}_C = -\omega r_1 \bar{J} = \bar{V}_O + \bar{\omega} \times \bar{r}_{C/O}$

$\bar{V}_B = \bar{V}_C + \bar{\Omega} \times \bar{r}_{B/C} + (V_{B/C})_{rel} = -\omega r_1 \bar{j} + \Omega r_2 \bar{j} + u\bar{j}$



$\bar{a}_C = \bar{a}_O + \bar{\alpha} \times \bar{r}_{C/O} - \omega^2 \bar{r}_{C/O} = +\omega^2 r_1 \bar{i}$

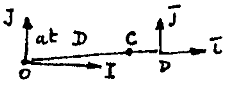
$\bar{a}_B = \bar{a}_C + \dot{\bar{\Omega}} \times \bar{r}_{B/C} - \Omega^2 \bar{r}_{B/C} + 2\bar{\Omega} \times (\bar{V}_{B/C})_{rel} + (\bar{a}_{B/C})_{rel}$

$\bar{0} = \omega^2 r_1 \bar{i} + \dot{\Omega} r_2 \bar{J} - \Omega^2 r_2 \bar{i} - 2\Omega u \bar{i} - \Omega^2 r_2 \bar{i}$

$\left\{ \begin{array}{l} (\bar{a}_{B/C})_{rel} = -u^2/r_2 \bar{i} \\ u = \Omega r_2 \end{array} \right.$

$\Rightarrow \dot{\Omega} = 0$ also $\omega^2 r_1 - 4\Omega^2 r_2 = 0$

$\Omega = \frac{\omega}{2}\left(\frac{r_1}{r_2}\right)^{1/2}$ ∢ $\underline{u = \Omega r_2 = \frac{\omega}{2}\sqrt{r_1 r_2}}$



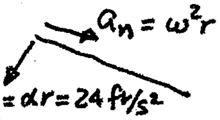
$\bar{a}_D = \bar{a}_C + \dot{\bar{\Omega}} \times \bar{r}_{D/C} - \Omega^2 \bar{r}_{D/C} + 2\bar{\Omega} \times (\bar{V}_{D/C})_{rel} + (\bar{a}_{D/C})_{rel}$

$= -\omega^2 r_1 \bar{i} + 0 - \Omega^2 r_2 \bar{i} + 2\Omega \bar{k} \times (u\bar{j}) - \frac{u^2}{r_2} \bar{i}$

$= -\omega^2 r_1 \bar{i} - \Omega^2 r_2 \bar{i} - 2\Omega u \bar{i} - \Omega^2 r_2 \bar{i} = (-\omega^2 r_1 - 4\Omega^2 r_2)\bar{i}$

$= -2\omega^2 r_1 \bar{i}$ $\underline{a_D = 2\omega^2 r_1}$

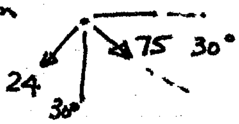
17-36 at A



$a_n = \omega^2 r = 75$ ft/s²

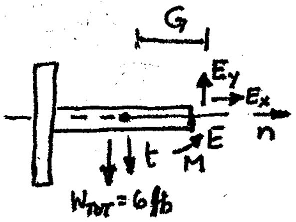
$a_t = \alpha r = 24$ ft/s²

at E since AC involves curvilinear motion



$a_E = a_A$

Now for T



the centroid G is at $\frac{3\cdot 0 + 3\cdot 2}{6} = \frac{3}{2}$ ft

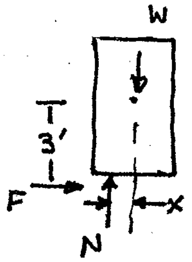
$\Sigma F_n = E_x = m a_n = m[-24 \sin 30° + 75 \cos 30°]$

$\Sigma F_t = W_{TOT} - E_y = m a_t = m[24 \cos 30° + 24 \sin 30°]$

$E_x = 9.87$ lb $E_y = -4.86$ lb

$+\circlearrowleft \Sigma M_G = M - E_y(1.5) = 0$ $M = 7.29$ lb-ft ↻

17-27



Rect. Translational Problem

for no slip or tip $F = m a_{G_x}$

$\Sigma F_y = 0 \Rightarrow N - W = 0$

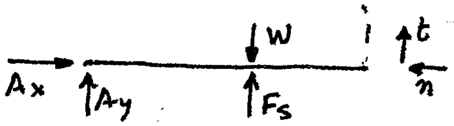
$+\circlearrowleft \Sigma M_G = 0 \Rightarrow F \cdot 3 - N x = 0$

Assume slip before tip : $F = \mu N \longrightarrow \Rightarrow 3\mu = x = 1.8$ ft

but max $x = 1.5$ ft $\quad\therefore$ tips before slips and $x = 1.5$ ft

$\therefore \quad F = \frac{Nx}{3} = \frac{W(1.5)}{3} = \frac{W}{2}$ but $F = m a_{G_x}$ $\quad \therefore a_{G_x} = g/_2 = 16.1$ ft/s^2

17-61 Rotational Problem about A



$\Sigma F_n = m a_{G_n}$

$\Sigma F_t = m a_{G_t}$

$+\circlearrowleft \Sigma M_A = I_A \alpha$

for a slender bar $I_A = \frac{1}{3} m l^2$

$\Sigma M_A = F_s - W = \frac{1}{3} m l^2 \alpha \qquad F_s = 7(.2) = 1.4\,kN \qquad W = 25(9.81) = 245.25\,N$

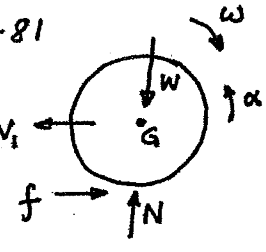
$\therefore \quad \alpha = 23.1\ rad/s^2 \circlearrowleft$

$\Sigma F_t = A_y + F_s - W = m a_{G_t} = m l \alpha \qquad \therefore A_y = m l \alpha + W - F_s = -288.69$

$\Sigma F_n = -A_x = m\omega^2 l$ but $\omega = 0 \Rightarrow A_x = 0$

General Motion

17-81



slip $\Rightarrow f = \mu N$

$\overset{+}{\rightarrow} \Sigma F_x = m a_{G_x}$

$+\uparrow \Sigma F_y = m a_{G_y}$

$+\circlearrowleft \Sigma M_G = I_G \alpha$

$I_G = \int r^2 dm = 2\pi \rho r^3$

$m = 2\pi\rho r \quad \therefore I_G = m r^2$

$\Sigma F_x = f = \mu N = m a_{G_x} \qquad \Sigma F_y = N - W = 0 \qquad N = W \qquad \Rightarrow \mu m g = m a_x \ \boxed{a_x = \mu g} \rightarrow$

$+\circlearrowleft \Sigma M_G = f r = m r^2 \alpha \quad \Rightarrow \quad \alpha = \mu g/_r \circlearrowleft$

t to no slip $\quad \omega_f = 0 = \omega_i + \alpha t = -\omega_1 + \mu g/_r\, t \qquad t = \frac{\omega_1 r}{\mu g}$

now $\underset{+}{\overset{}{\rightarrow}}\ s = s_i + v_i t + \frac{1}{2} a t^2 = 0$ [illegible] $V_1 \left[\frac{\omega_1 r}{\mu g}\right] + \frac{1}{2}\mu g \left[\frac{\omega_1 r}{\mu g}\right]^2$

$$\underset{\rightarrow}{s} = \frac{\omega_1 r}{\mu g}\left[\frac{\omega_1 r}{2} - V_1\right]$$

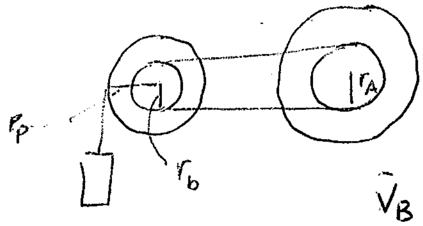
16-13 Given $\bar{\alpha}_A = 6\ rad/s^2$ ↻ find $\bar{V}_C$ and rise of C after 3 sec

Given $\bar{\alpha}_A = const$ $\bar{\omega}_A = 6t\ rad/s$ ↻ since we start from rest

∴ find $\bar{V}_A$ where pulley & belt meet. This equals $\bar{V}_B$ when pulley/belt meet

Knowing $\bar{V}_B = \bar{\omega}_B \times \bar{r}_B$ find $\bar{\omega}_B$ now find $\bar{V}_P = \bar{\omega}_B \times \bar{r}_P = \bar{V}_C$

integrate $\bar{V}_C$ to find height



$$\bar{V}_A = \bar{\omega}_A \times \bar{r}_A = 6t(0.3) = 1.8t \leftarrow \text{rad/s}$$

$$\bar{V}_B = \bar{V}_A = 1.8t \leftarrow = \bar{\omega}_B \times \bar{r}_B = \omega_B(0.2m)$$

$$\therefore \quad \bar{\omega}_B = 9t\ rad/s \ ↻$$

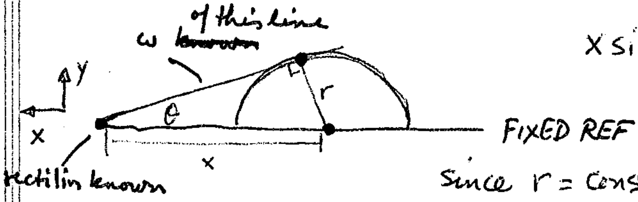
$$\bar{V}_P = \bar{\omega}_B \times \bar{r}_P = 9t(0.4m) = 3.6t\ m/s \uparrow$$

after 3 sec $\bar{V}_P = 10.8\ m/s \uparrow$

integrate $S = \int_0^3 \bar{V}_P dt = \int_0^3 3.6t\,dt = 1.8t^2 \Big|_0^3 = 16.2\ m$

HW DO 16-16, 16-19, 16-36, 16-37

16-33 General Motion



$x \sin\theta = r$

Given $V_A = const \leftarrow$

find $\bar{\omega}$ & $\bar{\alpha}$ of rod

since r = const $\frac{dr}{dt} = \dot{x}\sin\theta + x\cos\theta\,\dot{\theta} = 0$

but $\dot{x} = V_A$ ∴

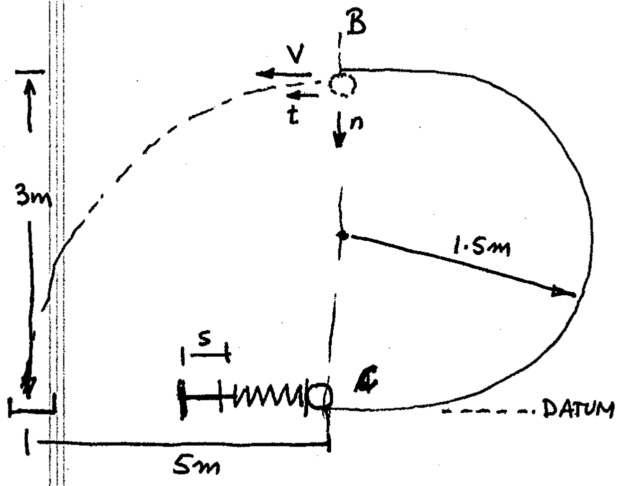
$$\bar{\omega} \curvearrowright \quad \dot{\theta} = -\frac{\dot{x}}{x}\frac{\sin\theta}{\cos\theta} = -\frac{V_A}{x}\tan\theta$$

$$\text{Now } \frac{d^2r}{dt^2} = \cancel{\ddot{x}\sin\theta} + 2\dot{x}\cos\theta\,\dot{\theta} - x\sin\theta\,\dot{\theta}^2 + x\cos\theta\,\ddot{\theta} = 0$$

$\swarrow 0$ since $\ddot{x} = \dot{V}_A = 0$

$$\therefore \quad \ddot{\theta} = \frac{x\sin\theta\,\dot{\theta}^2 - 2\dot{x}\cos\theta\,\dot{\theta}}{x\cos\theta} = \dot{\theta}^2\tan\theta - \frac{2V_A}{x}\dot{\theta}$$

$$\bar{\alpha} \curvearrowright \quad \ddot{\theta} = \frac{V_A^2}{x^2}\tan^2\theta\cdot\tan\theta + \frac{2V_A^2}{x^2}\tan\theta = \frac{V_A^2}{x^2}\tan\theta\,(2+\tan^2\theta)$$



Since ball reaches the top it must have a velocity ← call it V

∴ $Vt = 5m$ t is time of fall

to find t

$$s_y = s_{y_0} + V_{y_0}t + \frac{1}{2}at^2$$

$$3 = 0 + 0\cdot t + \frac{1}{2}(-9.81)t^2$$

note there is <u>no</u> $V_{y_\bullet}$ at B only V_x

∴ solving for t = .782 sec.

Now from $Vt = 5m$ $\underline{\underline{V = 6.39\ m/s}} = V_B$

<u>Since there is no friction</u>, there is no acceleration in the t direction ∴ <u>projectile motion is OK to use</u>

To find S $T_c + V_c = T_B + V_B$ (*)

$T_c = 0$ since $V_c = 0$ (given)

$V_c = \frac{1}{2}k(s+.08)^2$ since when $S=0$ spring is already compressed by .08m
(s+.08) is total amount spring is compressed

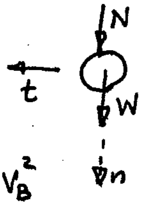
$T_B = \frac{1}{2}mV_B^2$

$V_B = mg(3\ \text{meters}) + \frac{1}{2}k(.08)^2$ since at $S=0$ there is an initial .08m compression

Putting the results into (*) and solving leads to $S = .246\ m$

Note that $\underline{S + .08 = .326\ m}$

To find N at B draw FBD



N is force exerted by track on ball

<u>THERE'S NO TANGENTIAL ACCEL.</u>: $\Sigma F_t = 0 = ma_t$ ∴ $\underline{\underline{a_t = 0}}$

$$\Sigma F_n = N + W = ma_n = \frac{mV_B^2}{\rho}$$

$$N = \frac{mV_B^2}{\rho} - W = \underline{8.72\ N}$$

note $N \neq 0$

<u>NOTE</u>: in fact if <u>N=0</u> the ball will <u>never</u> reach the dish since

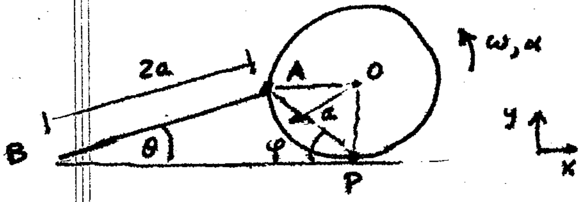
$$\Sigma F_n = W = \frac{mV_B^2}{\rho} \Rightarrow V_B = 3.836\ m/s$$

and the time to fall the 3 meters <u>is</u> <u>still</u> .782 sec ∴ $V_B t = 3$ <u>meters</u>

<u>3 m</u> is the horizontal distance travelled by the ball when N=0

HW : 16-66, 16-71, 16-96, 16-95, 16-93

16-84



- based on diagram sin $\theta = \frac{a}{2a} = .5 \Rightarrow \theta = 30^\circ$
- since no slip at P $V_p = 0$
- also $\varphi = 45^\circ$ since $\overline{AO} = \overline{OP}$ & $\measuredangle AOP = 90^\circ$
- $\bar{V}_A = \bar{V}_p + \bar{\omega} \times \bar{r}_{A/P} = \bar{O} + \omega \bar{k} \times [a\sqrt{2}] \cdot (-\cos 45^\circ \bar{i} + \sin 45^\circ \bar{j})$

$$\bar{V}_A = -a\omega\sqrt{2}\cos 45^\circ \bar{j} - a\omega\sqrt{2}\sin 45^\circ \bar{i} = -a\omega(\bar{i}+\bar{j})$$

$\bar{V}_B = \bar{V}_A + \bar{\omega}_{AB} \times \bar{r}_{B/A}$ let $\bar{\omega}_{AB} = \omega_{AB}\bar{k}$ $\quad \bar{r}_{B/A} = 2a[-\cos 30^\circ \bar{i} - \sin 30^\circ \bar{j}]$

$$= -a\omega(\bar{i}+\bar{j}) + \omega_{AB}\bar{k} \times -2a[.866\bar{i} + .5\bar{j}]$$

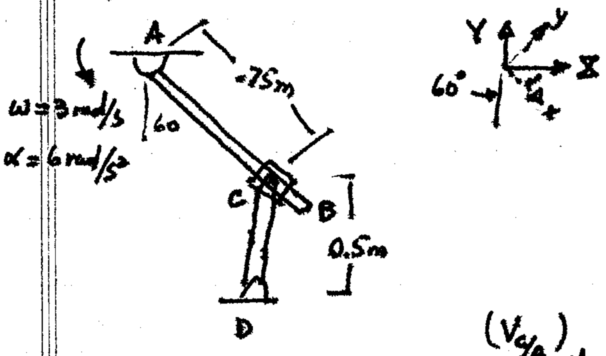
$$V_B\bar{i} = -a\omega\bar{i} + \omega_{AB} \cdot 2a(.5)\bar{i} - a\omega\bar{j} - \omega_{AB} \cdot 2a(.866\bar{j})$$

$\Rightarrow -a\omega - 1.732a\,\omega_{AB} = 0$ or $\omega_{AB} = -\frac{\omega}{\sqrt{3}} = \omega(.577) \circlearrowright$ rad/s

$\Rightarrow V_B = -a\omega + (.577\,\omega a) = -.423\,\omega a = .423\,\omega a \leftarrow$

YOU DO THE ACCELERATION PART AT HOME

16-98



Will use rotating system attached to AB

$\Omega = 3$ rad/s $\circlearrowleft \Rightarrow \bar{\Omega} = 3\bar{k} = 3K$

$$\bar{V}_c = \bar{V}_A + \bar{\Omega} \times (\bar{r}_{c/A}) + (\bar{V}_{c/A})_{rel} \qquad (1)$$

$\bar{V}_A = \bar{O} \quad \bar{r}_{c/A} = .75[\cos 30^\circ I - \sin 30^\circ J]$

$(V_{c/A})_{rel}$ represents the speed of the collar relative to bar AB in the rotating frame of ref $= V\bar{i} = V[\cos 30^\circ I - \sin 30^\circ J]$

now $\bar{V}_c = \underset{0}{\cancel{\bar{V}_D}} + \bar{\omega}_{CD} \times \bar{r}_{c/D}$ (2) with $\bar{\omega}_{CD} = \omega_{CD} K$

$\bar{r}_{c/D} = r_{c/D} J = 0.5 J$

From (1) $\bar{V}_c = (3K) \times .75[\cos 30^\circ I - \sin 30^\circ J] + V[\cos 30^\circ I - \sin 30^\circ J]$

$$= 3(.75)\sin 30^\circ I + 3(.75)\cos 30^\circ J + V\cos 30^\circ I - V\sin 30^\circ J$$

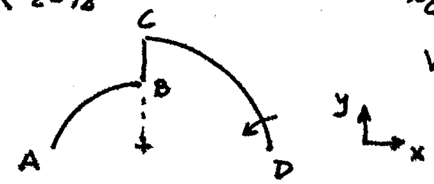
(2) $\bar{V}_c = \omega_{CD} K \times r_{c/D} J = -\omega_{CD}(0.5) I$

equate the 2 eqns $\Rightarrow -\omega_{CD}(0.5) = 3(.75)(.5) + V\cos 30^\circ \quad$ (I)

$0 = 3(.75)(.866) - V(.5)$

$V = 3.897$ m/s $\overset{30^\circ}{\searrow}$ and $\omega = -9$ rad/s $\Rightarrow \omega = 9$ rad/s $\circlearrowright$

R 2-18

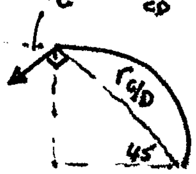


$\omega_{CD} = 3\ rad/s$ ↺

$V_D = V_A = 0$

for CD

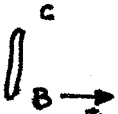
$\vec{V}_C = \omega_{CD} \cdot \underbrace{6 \cdot \sqrt{2}}_{}$ ↙ 45° $r_{C/D}$



assume ω_{AB} is ↺

$\vec{V}_B = \omega_{AB} \cdot 3\sqrt{2}$ ↖ 45°

now $\bar{V}_B = \bar{V}_C + \bar{V}_{B/C}$



if ω_{BC} ↺

$\vec{V}_{B/C} = \omega_{BC} \cdot 3 \rightarrow$

$$\therefore \quad \omega_{AB} \cdot 3\sqrt{2}\left[-\cos 45^\circ \bar{i} + \sin 45^\circ \bar{j}\right] = \omega_{CD} \cdot 6\sqrt{2}\left[-\cos 45^\circ \bar{i} - \sin 45^\circ \bar{j}\right] + \omega_{BC} \cdot 3\,\bar{i}$$

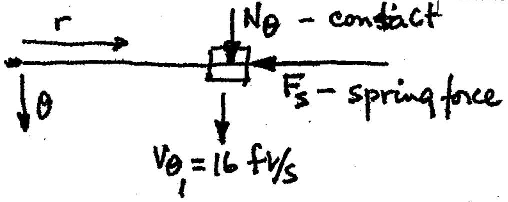
$\bar{j}$ comp: $\omega_{AB} \cdot 3\sqrt{2} \cdot \frac{1}{\sqrt{2}} = -\omega_{CD} \cdot 6\sqrt{2} \frac{1}{\sqrt{2}}$ $\omega_{AB} = -\omega_{CD} \cdot 2 = -6\ rad/s = 6\ rad/s$ ↻

$\bar{i}$ comp: $\omega_{AB} \cdot 3\sqrt{2}\left(-\frac{1}{\sqrt{2}}\right) = \omega_{CD} \cdot 6\sqrt{2}\left(-\frac{1}{\sqrt{2}}\right) + \omega_{BC} \cdot 3 \Rightarrow -3\omega_{AB} = -6\omega_{CD} + 3\omega_{BC}$

$$\Rightarrow \omega_{BC} = \frac{1}{3}\left[-3\omega_{AB} + 6\omega_{CD}\right] = \frac{1}{3}\left[-3(-6) + 6 \cdot 3\right] = 12\ rad/s$$ ↺

REVIEW OF ANGULAR M

14.58 Collar B weighs 10 lb and is attached to a spring of constant 50 lb/ft and of undeformed length equal to 18 in. The system is set in motion with $r = 12$ in., $v_\theta = 16$ ft/s, and $v_r = 0$. Neglecting the mass of the rod and the effect of friction, determine the radial and transverse components of the velocity of the collar when $r = 21$ in.

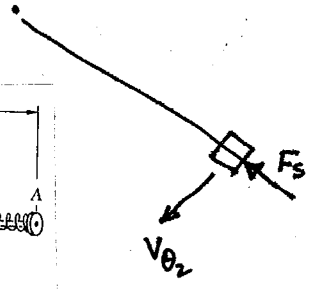


into plane $W = 10 lb$

out of plane N_z $W = N_z$

since rod is massless

$N_\theta = \cancel{m}_0 a_\theta \Rightarrow N_\theta = 0$



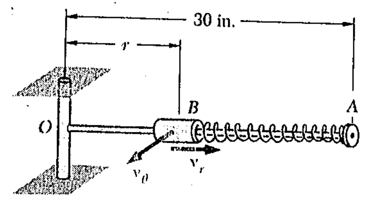


Fig. P14.58

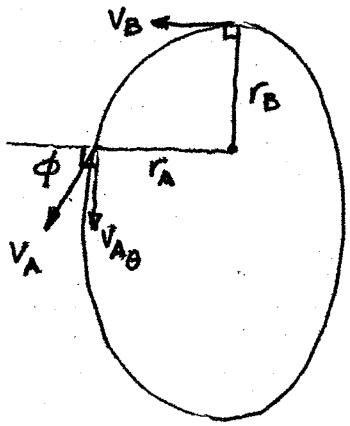
$r_1 m V_{\theta_1} = r_2 m V_{\theta_2}$ $V_{\theta_2} = \frac{r_1}{r_2} V_{\theta_1} = 9.14$ ft/s

$T_1 + V_1 = T_2 + V_2$

$\frac{1}{2} m V_1^2 + 0 = \frac{1}{2} m V_2^2 + \frac{1}{2} k \Delta s^2$ $V_1 = V_{\theta_1}$ $\Delta s = .75$ ft

$\therefore \quad V_2 = 12.86$ ft/s $V_r = \sqrt{V_2^2 - V_{\theta_2}^2} = 9.05$ ft/s

Problem 15-73



on gravitational force $F = \frac{GMm}{r^2}$ directed toward earth center ∴ $H_B = H_A$

$r_B m V_B = r_A m V_{A_\theta}$ note $r \perp V$ here

$V_{A_\theta} = V_A \sin\phi$

also $T_B + V_B = T_A + V_A$

$$\frac{1}{2} m V_B^2 - \frac{GMem}{r_B} = \frac{1}{2} m V_A^2 - \frac{GMem}{r_A}$$

here V_B, V_A are <u>total</u> velocities

also $F = -\frac{\partial V}{\partial r}$ ∴ $V = \int F dr = -\frac{GMem}{r}$

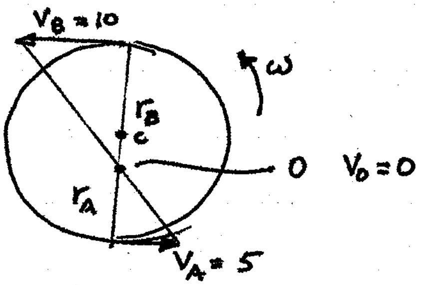
For a satellite outside normal gravity field $V = -\frac{GMem}{r}$

using $\phi = 70^\circ$ $M_e = 5.976(10^{24})$ kg $m = 700$ kg $r_A = 15\times10^6$ m $r_B = ?$

$V_A = 1\times10^4$ m/s $V_{B_\theta} = ?$ we find $V_{B_\theta} = 10.2$ km/s $r_B = 13.8\times10^6$ m

INSTANTANEOUS CENTER OF ZERO VELOCITY

16-65



$V_B = \omega r_B$ $V_A = \omega r_A$ $V_B = 2V_A$ ∴ $r_B = 2r_A$

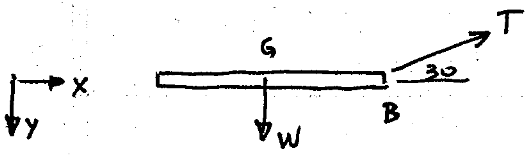
$r_B + r_A = 1.6$ ft

∴ $r_A = .533$ ft $r_B = 1.067$ ft ∴ $\omega = 9.372 \curvearrowright$

$V_c = \omega r_{c/o}$ $r_{c/o} = .8 - .533 = .267$ ft

$V_c = 9.372(.267) = 2.50$ ft ←

17-88



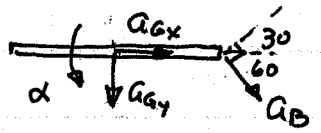
FBD

$$\overset{+}{\rightarrow} \sum F_x = m a_{G_x} = T\cos 30° \qquad (1)$$

$$+\downarrow \sum F_y = m a_{G_y} = W - T\sin 30° \qquad (2)$$

$$\circlearrowright + \sum M_G = I_G \alpha = T\sin 30° (.15) \qquad (3)$$

From kinematics



$$\bar{a}_G = \bar{a}_B + \bar{a}_{G/B}$$

$$a_{G_x}\bar{i} + a_{G_y}\bar{j} = a_B \cos 60° \bar{i} + a_B \sin 60° \bar{j} + (+\alpha)(.15)\bar{j}$$

$\bar{a}_{G/B} = \alpha r \downarrow$ only since $\omega = 0$

$I_G = \frac{1}{2} m l^2 = \frac{1}{2} m (.3)^2$

$$\therefore \quad \left.\begin{aligned} a_{G_x} &= a_B \cos 60° \\ a_{G_y} &= a_B \sin 60° + 0.15\alpha \end{aligned}\right\}$$

$$a_{G_y} = a_{G_x} \tan 60° + 0.15\alpha \qquad (4)$$

we have unknowns: T, a_{G_x}, a_{G_y}, α & 4 eqns

we can solve (3)/(1) $\Rightarrow$ $\tan 30° (0.15) = \frac{I_G \alpha}{m a_{G_x}} = \frac{1}{12} \frac{l^2 \alpha}{a_{G_x}}$

$$\therefore \quad a_{G_x} \tan 30° (0.15) \frac{12}{l^2} = \alpha \qquad (*)$$

$$\therefore \text{ from (4)} \quad a_{G_y} = a_{G_x}\left[\tan 60° + \frac{12(0.15)^2}{l^2} \tan 30°\right] \qquad (5)$$

from (1) & (2)

$$m a_{G_y} = W - T\sin 30° = W - m a_{G_x} \tan 30°$$

$$a_{G_y} = g - a_{G_x} \tan 30° \qquad (6)$$

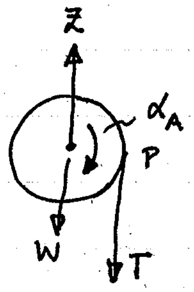
$\therefore$ Put (6) into (5) to get that $g = a_{G_x}\left[\tan 60° + \frac{12(0.15)^2}{l^2} \tan 30° + \tan 30\right.$

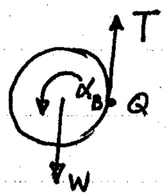
$a_{G_x} = 2.427 \text{ m/s}^2$

from (5) $a_{G_y} = 8.409 \text{ m/s}^2$ $\qquad \alpha = 28.03 \text{ rad/s}^2$ from (*)

from (1) $T = 5.605 \text{ N}$

17-84





for A

$\xrightarrow{+} \Sigma F_x = 0$

$+\circlearrowright \Sigma M_G = I_{A_G}\alpha_A = TR \quad (1)$

$+\downarrow \Sigma F_y = m_A a_{G_A} = W + T - Z = 0$

$I_{A_G} = I_{B_G} = \frac{1}{2} mR^2$

for B

$\xrightarrow{+} \Sigma F_x = 0$

$+\downarrow \Sigma F_y = m a_{G_B} = W - T \quad (2)$

$+\circlearrowleft \Sigma M_G = I_{B_G}\alpha_B = TR \quad (3)$

Kinematics : since $\omega_A = \omega_B = 0$

$a_P = \alpha_A R = a_Q \downarrow$

$\bar{a}_{G_B} = \bar{a}_Q + \bar{a}_{G/Q} = a_Q \bar{j} + \alpha_B R \bar{j} = (\alpha_A R + \alpha_B R)\bar{j}$

Thus $a_{G_B} = (\alpha_A + \alpha_B)R \quad (4)$

We have 4 equs 4 unknowns : α_A, T, a_{G_B}, α_B

$(1)+(3) \Rightarrow (I_{A_G})(\alpha_A + \alpha_B) = 2TR \Rightarrow T = \frac{I_{A_G}(\alpha_A + \alpha_B)}{2R} \quad (*)$

also (2) $\Rightarrow T = I_{A_G}\frac{(\alpha_A + \alpha_B)}{2R} = \frac{I_{A_G} a_{G_B}}{2R^2}$

from (2) $m a_{G_B} = W - T = W - \frac{I_{A_G} a_{G_B}}{2R^2}$ $\quad \therefore \quad a_{G_B} = W\Big/\left[m + \frac{I_{A_G}}{2R^2}\right]$

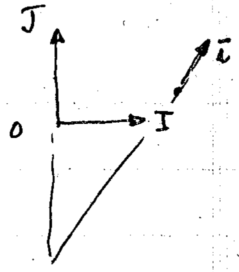
$= \frac{4}{5} g = 7.85 \, m/s^2$

$\alpha_A = \frac{TR}{I_{A_G}} = 43.6 \text{ rad/s}^2$

$T = \frac{I_{A_G} a_{G_B}}{2R^2} = \frac{m}{4} \cdot a_{G_B} = 19.6$ N

$\alpha_B = a_{G_B}/R - \alpha_B = 43.6 \text{ rad/s}^2$

16-37

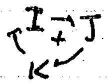


$$\bar{V}_c = -\omega\bar{k}\times(r\sin\theta\,I + r\cos\theta\,J) = -\omega r\sin\theta\,J + \omega r\cos\theta\,I$$

$$\bar{V}_c = \cancel{\bar{V}_A}_{0} + \bar{\Omega}\times\bar{r}_{c/A} + (\bar{V}_{c/A})_{rel}$$

let $\bar{\Omega} = -\dot{\Phi}\bar{k}$ $\quad \bar{r}_{c/A} = (d+r\cos\theta)\bar{J} + r\sin\theta I$

let $(\bar{V}_{c/A})_{rel} = -V_{rel}\,\bar{i} = -V_{rel}\left[\dfrac{(d+r\cos\theta)\bar{J}+r\sin\theta I}{\sqrt{d^2+2dr\cos\theta+r^2}}\right.$



$$\bar{V}_c = \dot{\Phi}(d+r\cos\theta)I - \dot{\Phi}r\sin\theta J - \frac{V_{rel}(d+r\cos\theta)\bar{J}}{\sqrt{}} - \frac{V_{rel}\,r\sin\theta I}{\sqrt{}}$$

$$(J)\qquad -\omega r\sin\theta = -\dot{\Phi}r\sin\theta - \frac{V_{rel}(d+r\cos\theta)}{\sqrt{}}\qquad (*)$$

$$(I)\qquad \omega r\cos\theta = \dot{\Phi}(d+r\cos\theta) - \frac{V_{rel}\,r\sin\theta}{\sqrt{}}$$

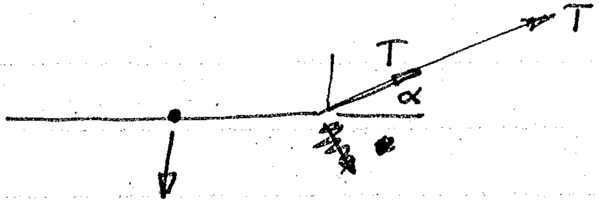
from $(*)$ $\dfrac{(\dot{\Phi}-\omega)r\sin\theta}{d+r\cos\theta} = -\dfrac{V_{rel}}{\sqrt{}} \Rightarrow \omega r\cos\theta \neq \dot{\Phi}(d+r\cos\theta) + \dfrac{(\dot{\Phi}-\omega)r^2\sin^2\theta}{d+r\cos\theta}$

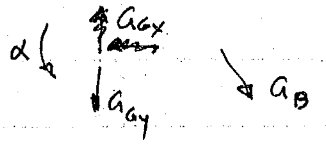
$$\Rightarrow \frac{\omega(dr\cos\theta+r^2)}{d+r\cos\theta} = \dot{\Phi}\cdot\frac{(d^2+2dr\cos\theta+r^2)}{d+r\cos\theta}$$

or $\dot{\Phi} = \dfrac{r+d\cos\theta}{d^2+2dr\cos\theta+r^2}\,\omega r$

and $V_{rel} = -\dfrac{(\dot{\Phi}-\omega)r\sin\theta}{d+r\cos\theta}\sqrt{d^2+2dr\cos\theta+r^2}$

17-88





$+\downarrow\ \Sigma F_y = -T\sin 30^\circ + W = m a_{y_G}$

$\overset{+}{\rightarrow}\ \Sigma F_x = T\cos 30^\circ = m a_{G_x}$

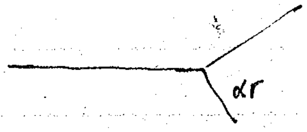
$+\circlearrowleft\ \Sigma M_G = T\sin 30^\circ \cdot (.15) = I\alpha$

$a_G = a_B + a_{G/B}$

$-a_{G_y}\bar{j} + a_{G_x}\bar{i} = \searrow 60$

also since we start from rest.

$\underline{a_{y_G} = .15\alpha}$



$W(.15) = m a_{y_G}(.15) + I\alpha$

$W(.15) = (m(.15)^2 + I)\alpha$

$g(.15) = \cancel{m}\left[.15^2 + \frac{1}{12}(.3)^2\right]\alpha$

$9.81(.15) = [.03]\alpha$

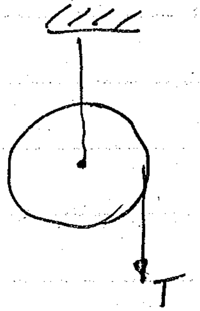
19
5

$49.05\ \text{rad/s}^2\ \circlearrowleft$

$a_{G_y} = \begin{array}{r} 4.905 \\ 2.45 \\ \hline 7.36 \end{array}\ \text{m/s}^2$

$\frac{1}{12}\cdot 2 \cdot \frac{49.05}{.15} \cdot .2 = \frac{49.05}{.45} = 109\ N.$

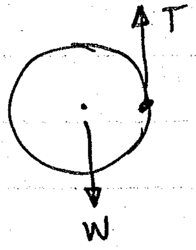
17-84



$\Sigma F_y = W + T \overset{+\,T_y}{=} m a_G$

$+\circlearrowright\ \Sigma M_{G_1} = I\alpha = Tr$

$a_{G_y} = \alpha r$



$\downarrow\ \Sigma F_y = W \bullet T = m a_{G_y}$

$+\circlearrowright\ \Sigma M_{G_2} = Tr = I\alpha$

$T = \frac{I\alpha}{r}$

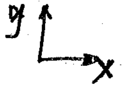
$W - T = m\alpha r$

$W - T = m\frac{Tr^2}{I}$

$\therefore\ T = \frac{W}{1 + \frac{mr^2}{I}} = \frac{10\cdot 9.81}{1+2} = \frac{98.1}{3} = 32.7\ N$

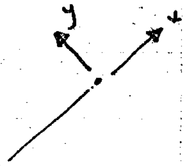
$I_G = \frac{1}{2}mr^2$

$\underline{\dot{\alpha} = \frac{Tr}{I} = \frac{32.7\cancel{(.09)}}{\frac{10}{2}\cdot(.09)^{\cancel{2}}} = \frac{\overset{6.54}{65.4}}{.9} = 72\ \text{rad/s}^2}$



$$\bar{V}_c = -\omega\bar{k}\times(r\sin\theta I + r\cos\theta J) = -\omega r\sin\theta J + \omega r\cos\theta I$$

$$\bar{V}_c = \underset{0}{\cancel{\bar{V}_A}} + \bar{\Omega}\times\bar{r}_{c/A} + (\bar{V}_{c/A})_{rel} \qquad \text{let} \quad \bar{\Omega} = \dot{\Phi}\bar{k}\times\left[(d+r\cos\theta)J + r\sin$$



$$= -\dot{\Phi}(d+r\cos\theta)I + \dot{\Phi}r\sin\theta J + -V_{rel}\,\bar{i}$$

$$\bar{i} = \frac{(d+r\cos\theta)J + r\sin\theta I}{\sqrt{d^2+2rd\cos\theta+r^2}}$$

$$-\omega r\sin\theta J + \omega r\cos\theta I = \left\{-\dot{\Phi}(d+r\cos\theta) - \frac{V_{rel}(d+r\cos\theta)}{\sqrt{\ }}\right\} I + \left\{-\frac{V_{rel}\, r\sin\theta}{\sqrt{\ }} + \dot{\Phi}r\sin\right.$$

$$\Rightarrow \omega r\cos\theta = -\dot{\Phi}(d+r\cos\theta) - \frac{V_{rel}(d+r\cos\theta)}{\sqrt{\ }}$$

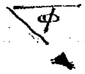
$$+\omega r\sin\theta = -\dot{\Phi}r\sin\theta + \frac{V_{rel}\ r\sin\theta}{\sqrt{\ }}$$

$$\omega = -\dot{\Phi} + \frac{V_{rel}}{\sqrt{\ }} \qquad \Rightarrow \frac{V_{rel}}{\sqrt{\ }} = \omega + \dot{\Phi}$$

$$\omega r\cos\theta = -\dot{\Phi}(d+r\cos\theta) - (d+r\cos\theta)(\omega+\dot{\Phi})$$

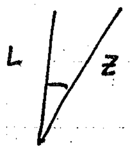
$$\omega[r\cos\theta + d + r\cos\theta] = -2\dot{\Phi}(d+r\cos\theta)$$

$$\omega[d+2r\cos\theta] = -2\dot{\Phi}(d+r\cos\theta)$$

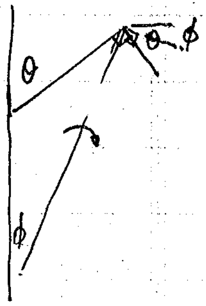


$$-\frac{\omega}{2}\,\frac{d+2r\cos\theta}{d+r\cos\theta} = \dot{\Phi}$$

now



$$\frac{L}{Z} = \cos\varphi \qquad \text{and} \quad Z\dot{\Phi}\cos\phi = \dot{X} =$$



$$\omega r\cos\theta\, I - \omega r\sin\theta\, J = V_c$$

$$\dot{\Phi}\,\overline{CA}\cos\phi\, I - \dot{\Phi}\,\overline{CA}\sin\phi\, J$$

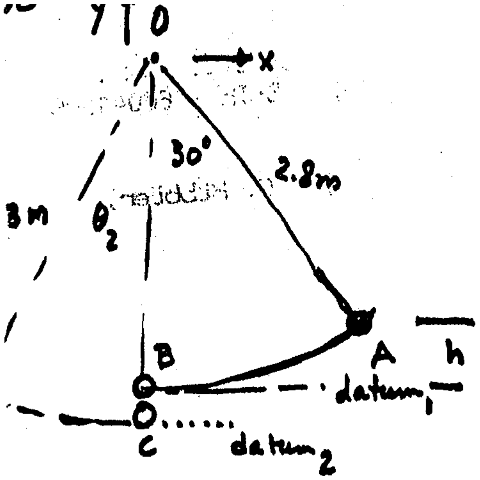
$$\dot{\Phi}(d+r\cos\theta)I - \dot{\Phi}r\sin\theta J + V_{rel}\,\bar{i} \qquad V_{rel}\left(\frac{d+r\cos\theta}{\overline{CA}}J + \frac{r\sin\theta\, I}{\overline{CA}}\right)$$

$$\omega r\cos\theta = \dot{\Phi}(d+r\cos\theta) + V_{rel}\frac{r\sin\theta}{\overline{CA}}$$

$$-\omega r\sin\theta = -\dot{\Phi}r\sin\theta + V_{rel}\frac{d+r\cos\theta}{\overline{CA}} \qquad \frac{V_{rel}}{\overline{CA}} = \frac{(\dot{\Phi}-\omega)r\sin\theta}{d+r\cos\theta}$$

$$\omega r\cos\theta = \frac{V_{rel}}{\overline{CA}} = \frac{(\dot{\Phi}-\omega)r^2\sin^2\theta}{d+r\cos\theta} + \frac{\dot{\Phi}(d+r\cos\theta)^2}{d+r\cos\theta} = \dot{\Phi}\,\frac{(r^2+2rd\cos\theta+d^2)}{d+r\cos\theta}$$

$$\omega\frac{rd\cos\theta + \omega r^2\cancel{\cos^2\theta}}{d+r\cos\theta} \qquad \omega r(r+d\cos\theta)/(\qquad) = \dot{\Phi} \ ✓$$



since only conservative forces from A to B using datum$_1$

$$T_1+V_1=T_2+V_2$$

$$T_1=\frac{1}{2}mV_A^2=0$$

$$V_1=mgh=mg\left[2.8\right](1-\cos 30^\circ)$$

$$T_2=\frac{1}{2}mV_B^2$$

$$V_2=0$$

nt V_c & θ_2

can solve for $V_B=\sqrt{2g(2.8)(1-\cos 30^\circ)}=2.713\text{ m/s}$

when girl drops legs so that CG shifts from B to C the forces are still conservative. We now use datum$_2$ for motion between C & D

$$T_1+V_1=T_2+V_2$$

$T_1=\frac{1}{2}mV_c^2$ V_c unknown

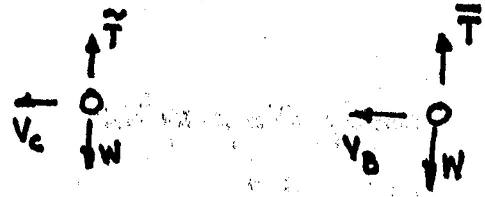
$$V_1=0$$

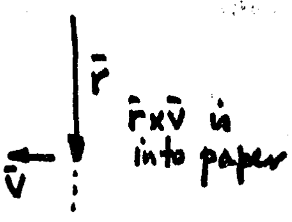
$$T_2=\frac{1}{2}mV_D^2=0$$

$V_2=mgy=mg[3](1-\cos\theta_2)$ θ_2 unknown

we have 1 eqn 2 unknowns. Use of angular momentum & impulse

@ C (after move) @ B (before move)





only forces acting at $\theta=0^\circ$ are tension & weight. They lie along line of action of position vector producing zero moments over the time interval in which the girl shifts her legs. Thus

$$(\bar{H}_O)_c=(\bar{H}_O)_B$$

$$-(r_c mV_c)\bar{k}=-(r_B mV_B)\bar{k}$$

$$(3)m\,V_c=(2.8)m(2.713)$$

$$V_c=2.532\text{ m/s}$$

put this into $T_1+V_1=T_2+V_2$

$$\frac{1}{2}mV_c^2=mg(3)(1-\cos\theta_2)\implies\theta_2=27^\circ$$

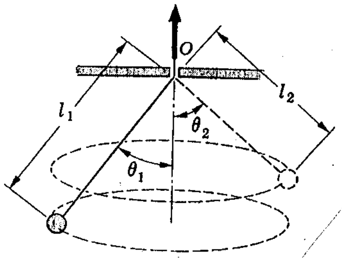


FIG. P 14.83

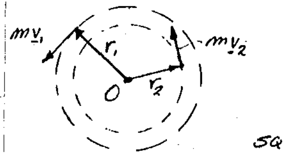
14.83. A small ball swings in a horizontal circle at the end of a cord of length l_1 which forms an angle θ_1 with the vertical. If the cord is slowly drawn through the support at O until the free length is l_2, derive a relation between l_1, l_2, θ_1, and θ_2.

Angular Momentum problem.

Note: slowly means $a_y = 0$

14.83 SINCE FORCE EXERTED BY CORD ON BALL PASSES THROUGH O, AND SINCE WEIGHT IS VERTICAL, THE ANGULAR MOMENTUM IS CONSTANT ABOUT VERTICAL AXIS THROUGH O.

LOOKING DOWNWARD: $r_1 = l_1 \sin\theta_1 \quad r_2 = l_2 \sin\theta_2$



CONS. OF ANG. MOMENTUM

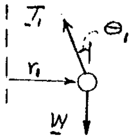
↺ ABOUT O: $\cancel{m}v_1 r_1 = \cancel{m}v_2 r_2$

$$v_1 l_1 \sin\theta_1 = v_2 l_2 \sin\theta_2 \qquad (1)$$

SQUARING BOTH SIDES OF EQ(1), WE HAVE

$$v_1^2 l_1^2 \sin^2\theta_1 = v_2^2 l_2^2 \sin^2\theta_2 \qquad (2)$$

NOW, CONSIDER THE MOTION IN POSITION 1.



SINCE BALL MOVES IN CIRCLE OF RADIUS r_1

$$\underline{a}_n = v_1^2/r_1 = \frac{v_1^2}{l_1 \sin\theta_1} \leftarrow$$

$$\Sigma F_y = 0: \quad T_1 \cos\theta_1 - W = 0$$

$$T_1 = W/\cos\theta_1 \qquad (3)$$

$$\overset{+}{\leftarrow}\Sigma F_n = ma_n: \quad T_1 \sin\theta_1 = \frac{W}{g}\frac{v_1^2}{l \sin\theta_1}$$

$$v_1^2 = T_1 \sin^2\theta_1 \frac{gl}{W} = \frac{W}{\cos\theta_1}\sin^2\theta_1 \frac{g\,l}{W}$$

$$v_1^2 = g\,l_1 \frac{\sin^2\theta_1}{\cos\theta_1} \qquad (4)$$

IN A SIMILAR FASHION WE FIND FOR POSITION 2:

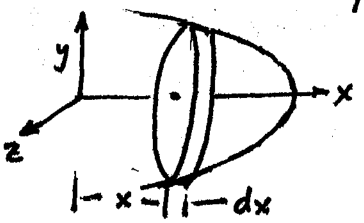
$$v_2^2 = g\,l_2 \frac{\sin^2\theta_2}{\cos\theta_1} \qquad (5)$$

SUBSTITUTING FOR v_1^2 AND v_2^2 IN EQ (2), WE HAVE

$$\left[g l_1 \frac{\sin^2\theta_1}{\cos\theta_1}\right] l_1^2 \sin^2\theta_1 = \left[g l_2 \frac{\sin^2\theta_2}{\cos\theta_2}\right] l_2 \sin^2\theta_2$$

$$l_1^3 \frac{\sin^4\theta_1}{\cos\theta_1} = l_2^3 \frac{\sin^4\theta_2}{\cos\theta_2} \blacktriangleleft$$

17-5 P 302 Using slab method $\quad dI_x = \frac{1}{2}y^2 dm \quad dm = \rho\, dV = \rho \cdot \pi y^2 dx$



$$dI_x = \frac{\rho\pi}{2} y^4 dx \quad \text{but } y^2 = b^2[1 - x^2/a^2]$$

and $$I_x = \int_0^a \frac{\rho\pi}{2} b^4 (1 - x^2/a^2)^2 dx$$

and $$m = \int_0^a \rho\pi b^2 (1 - x^2/a^2) dx$$