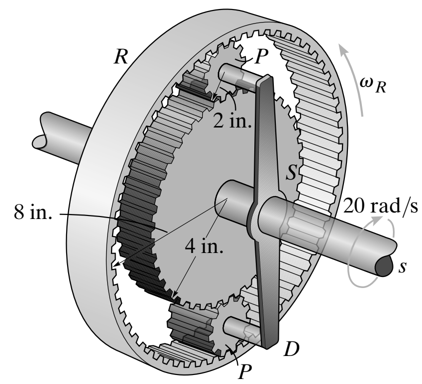
**R2–1.** An automobile transmission consists of the planetary gear system shown. If the ring gear $R$ is held fixed so that $\omega_R = 0$, and the shaft $s$ and sun gear $S$, rotates at 20 rad/s, determine the angular velocity of each planet gear $P$ and the angular velocity of the connecting rack $D$, which is free to rotate about the center shaft $s$.




For planet gear $P$: The velocity of point $A$ is $v_A = \omega_s r_s = 20\left(\frac{4}{12}\right) = 6.667$ ft/s.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$0 = \left[\underset{\rightarrow}{6.667}\right] + \left[\underset{\leftarrow}{\omega_P\left(\frac{4}{12}\right)}\right]$$

$$(\overset{+}{\rightarrow}) \qquad 0 = 6.667 - \omega_P\left(\frac{4}{12}\right) \qquad \omega_P = 20 \text{ rad/s} \qquad \textbf{Ans.}$$

For connecting rack $D$:

$$\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$$

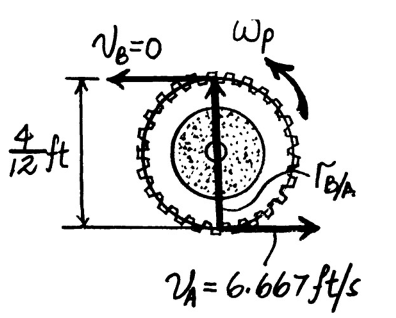
$$\left[\underset{\rightarrow}{v_C}\right] = \left[\underset{\rightarrow}{6.667}\right] + \left[\underset{\leftarrow}{20\left(\frac{2}{12}\right)}\right]$$

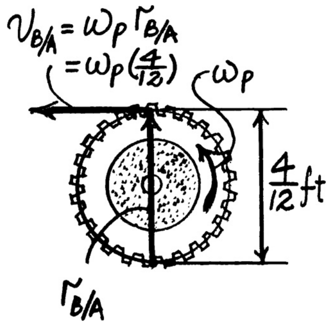
$$(\overset{+}{\rightarrow}) \qquad v_C = 6.667 - 20\left(\frac{2}{12}\right) \qquad v_C = 3.333 \text{ ft/s}$$

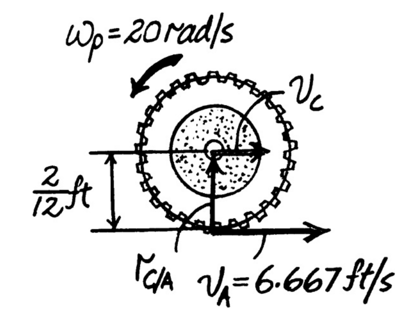
The rack is rotating about a fixed axis (shaft $s$). Hence,

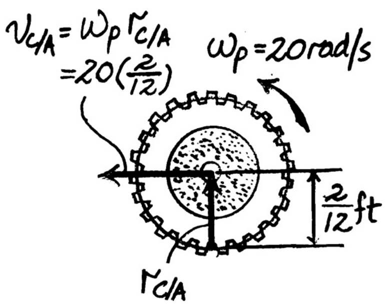
$$v_C = \omega_D r_D$$

$$3.333 = \omega_D\left(\frac{6}{12}\right) \qquad \omega_D = 6.67 \text{ rad/s} \qquad \textbf{Ans.}$$

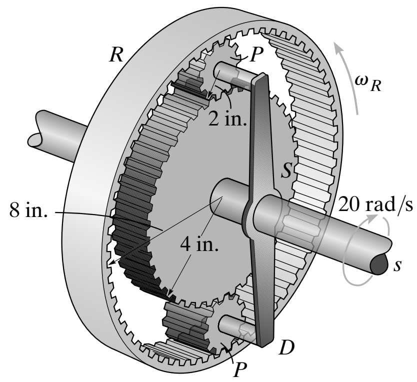


**R2–2.** An automobile transmission consists of the planetary gear system shown. If the ring gear $R$ rotates at $\omega_R = 2$ rad/s, and the shaft $s$ and sun gear $S$, rotates at 20 rad/s, determine the angular velocity of each planet gear $P$ and the angular velocity of the connecting rack $D$, which is free to rotate about the center shaft $s$.




For planet gear $P$: The velocity of points $A$ and $B$ are $v_A = \omega_S r_S = 20\left(\frac{4}{12}\right) = 6.667$ ft/s and $v_B = \omega_B r_B = 2\left(\frac{8}{12}\right) = 1.333$ ft/s.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$\left[\underset{\leftarrow}{1.333}\right] = \left[\underset{\rightarrow}{6.667}\right] + \left[\underset{\leftarrow}{\omega_P\left(\frac{4}{12}\right)}\right]$$

$$(\overset{+}{\rightarrow}) \qquad -1.333 = 6.667 - \omega_P\left(\frac{4}{12}\right) \qquad \omega_P = 24 \text{ rad/s} \qquad \textbf{Ans.}$$

For connecting rack $D$:

$$\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$$

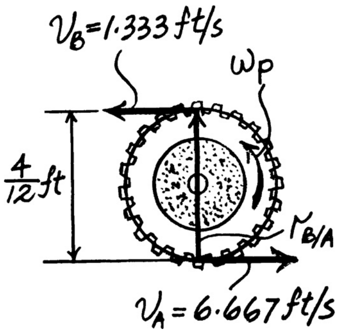
$$\left[\underset{\rightarrow}{v_C}\right] = \left[\underset{\rightarrow}{6.667}\right] + \left[\underset{\leftarrow}{24\left(\frac{2}{12}\right)}\right]$$

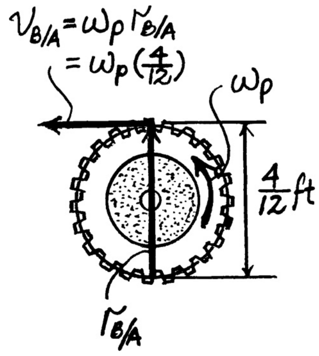
$$(\overset{+}{\rightarrow}) \qquad v_C = 6.667 - 24\left(\frac{2}{12}\right) \qquad v_C = 2.667 \text{ ft/s}$$

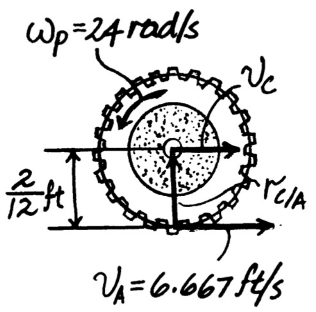
The rack is rotating about a fixed axis (shaft $s$). Hence,

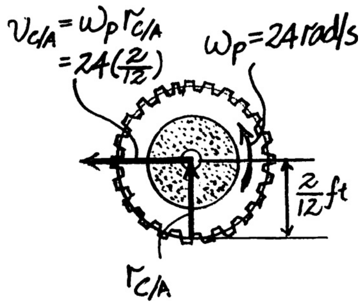
$$v_C = \omega_D r_D$$

$$2.667 = \omega_D\left(\frac{6}{12}\right) \qquad \omega_D = 5.33 \text{ rad/s} \qquad \textbf{Ans.}$$

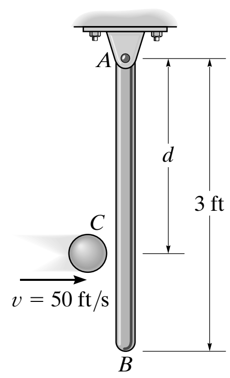


**R2–3.** The 6-lb slender rod $AB$ is released from rest when it is in the *horizontal position* so that it begins to rotate clockwise. A 1-lb ball is thrown at the rod with a velocity $v = 50$ ft/s. The ball strikes the rod at $C$ at the instant the rod is in the vertical position as shown. Determine the angular velocity of the rod just after the impact. Take $e = 0.7$ and $d = 2$ ft.




Datum at $A$:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{6}{32.2}\right)(3)^2\right]\omega^2 - 6(1.5)$$

$$\omega = 5.675 \text{ rad/s}$$

$$\circlearrowleft + \qquad (H_A)_1 = (H_A)_2$$

$$\frac{1}{32.2}(50)(2) - \left[\frac{1}{3}\left(\frac{6}{32.2}\right)(3)^2\right](5.675) = \left[\frac{1}{3}\left(\frac{6}{32.2}\right)(3)^2\right]\omega_2 + \frac{1}{32.2}(v_{BL})(2)$$

$$e = 0.7 = \frac{v_C - v_{BL}}{50 - [-5.675(2)]}$$

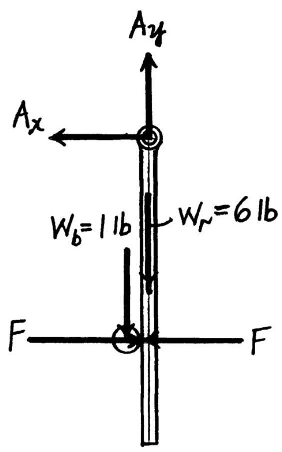
$$v_C = 2\omega_2$$

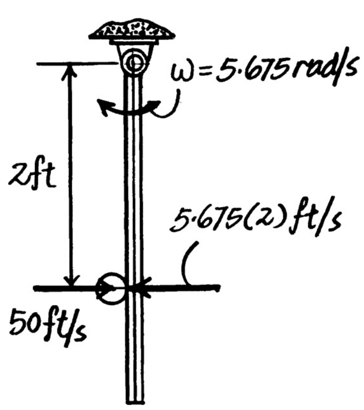
Solving,

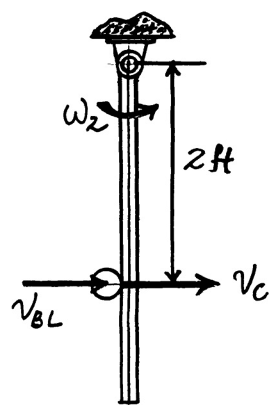
$$\omega_2 = 3.81 \text{ rad/s} \qquad \textbf{Ans.}$$

$$v_{BL} = -35.3 \text{ ft/s}$$

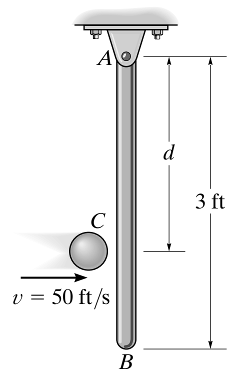
$$v_C = 7.61 \text{ ft/s}$$



***R2–4.** The 6-lb slender rod $AB$ is originally at rest, suspended in the vertical position. A 1-lb ball is thrown at the rod with a velocity $v = 50$ ft/s and strikes the rod at $C$. Determine the angular velocity of the rod just after the impact. Take $e = 0.7$ and $d = 2$ ft.




$\circlearrowleft + \ (H_A)_1 = (H_A)_2$

$$\left(\frac{1}{32.2}\right)(50)(2) = \left[\frac{1}{3}\left(\frac{6}{32.2}\right)(3)^2\right]\omega_2 + \frac{1}{32.2}(v_{BL})(2)$$

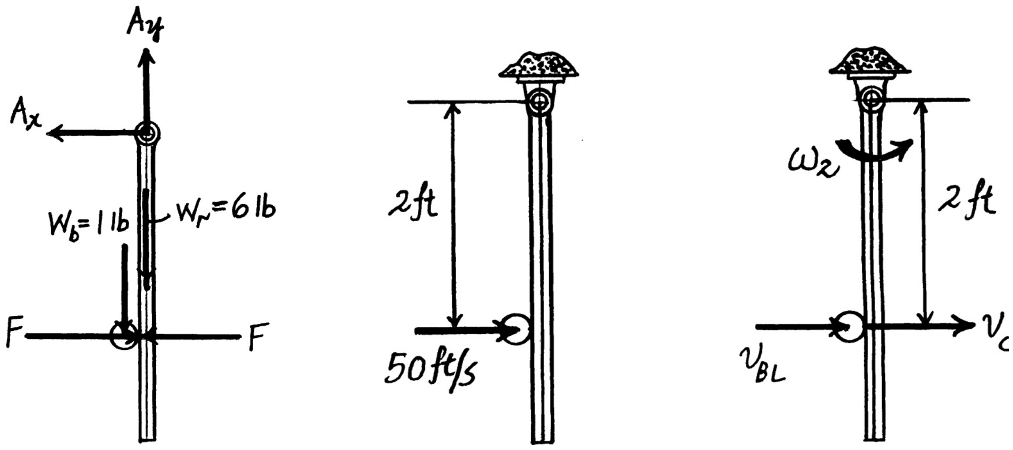
$$e = 0.7 = \frac{v_C - v_{BL}}{50 - 0}$$

$$v_C = 2\omega_2$$

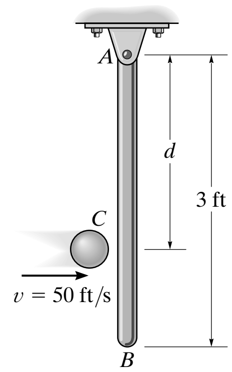
Thus,

$$\omega_2 = 7.73 \text{ rad/s} \qquad \textbf{Ans.}$$

$$v_{BL} = -19.5 \text{ ft/s}$$



**R2–5.** The 6-lb slender rod is originally at rest, suspended in the vertical position. Determine the distance $d$ where the 1-lb ball, traveling at $v = 50$ ft/s, should strike the rod so that it does not create a horizontal impulse at $A$. What is the rod's angular velocity just after the impact? Take $e = 0.5$.




Rod:

$$\circlearrowleft + \quad (H_G)_1 + \Sigma \int M_G\, dt = (H_G)_2$$

$$0 + \int F\, dt\,(d - 1.5) = \left(\frac{1}{12}(m)(3)^2\right)\omega$$

$$m(v_G)_1 + \Sigma \int F\, dt = m(v_G)_2$$

$$0 + \int F\, dt = m(1.5\omega)$$

Thus,

$$m(1.5\omega)(d - 1.5) = \frac{1}{12}(m)(3)^2\,\omega$$

$$d = 2 \text{ ft} \qquad \textbf{Ans.}$$

This is called the center of percussion. See Example 19–5.

$$\circlearrowleft + \qquad (H_A)_1 = (H_A)_2$$

$$\frac{1}{32.2}(50)(2) = \left[\frac{1}{3}\left(\frac{6}{32.2}\right)(3)^2\right]\omega_2 + \frac{1}{32.2}(v_{BL})(2)$$

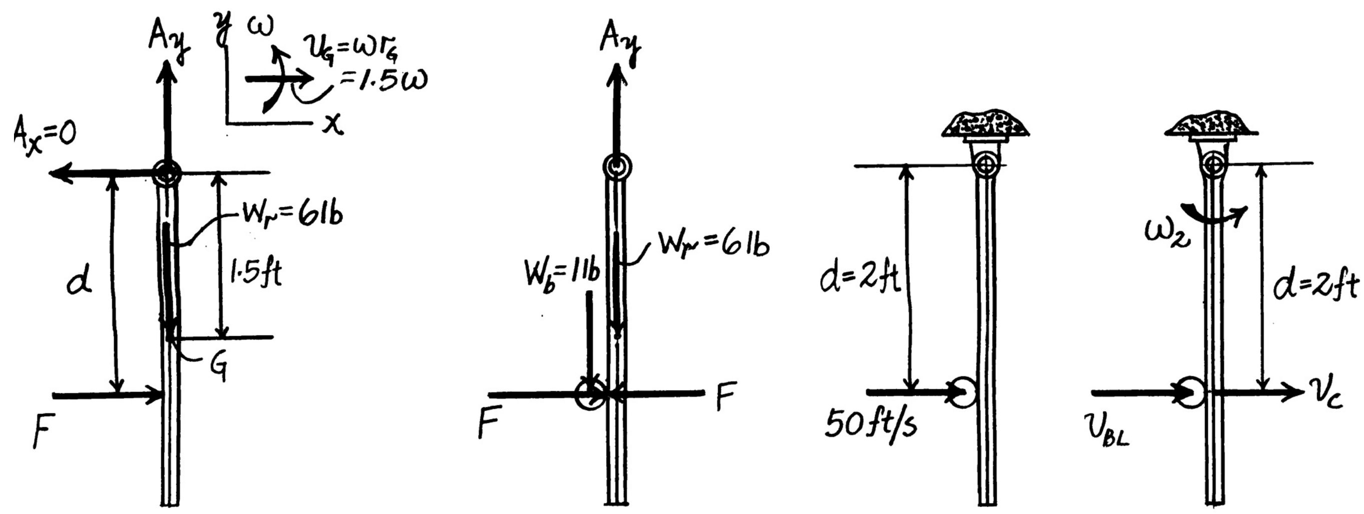
$$e = 0.5 = \frac{v_C - v_{BL}}{50 - 0}$$

$$v_C = 2\omega_2$$

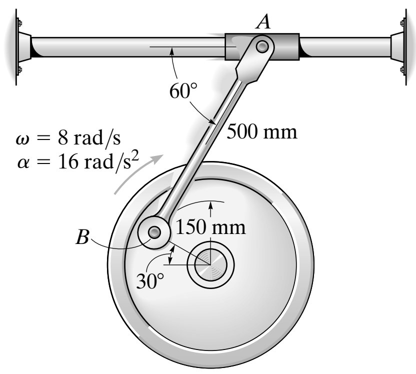
Thus,

$$\omega_2 = 6.82 \text{ rad/s} \qquad \textbf{Ans.}$$

$$v_{BL} = -11.4 \text{ ft/s}$$



**R2–6.** At a given instant, the wheel rotates with the angular motions shown. Determine the acceleration of the collar at $A$ at this instant.




Using instantaneous center method:

$$\omega_{AB} = \frac{v_B}{r_{B/IC}} = \frac{8(0.15)}{0.5 \tan 30^\circ} = 4.157 \text{ rad/s}$$

$$\mathbf{a}_B = 16(0.15)\mathbf{i} - 8^2\,(0.15)\mathbf{j} = \{2.4\mathbf{i} - 9.6\mathbf{j}\}\ \text{m/s}^2$$

$$\mathbf{a}_A = -a_A \cos 60°\mathbf{i} + a_A \sin 60°\mathbf{j} \qquad \alpha = \alpha\mathbf{k} \qquad \mathbf{r}_{B/A} = \{-0.5\mathbf{i}\}\ \text{m}$$

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2\,\mathbf{r}_{B/A}$$

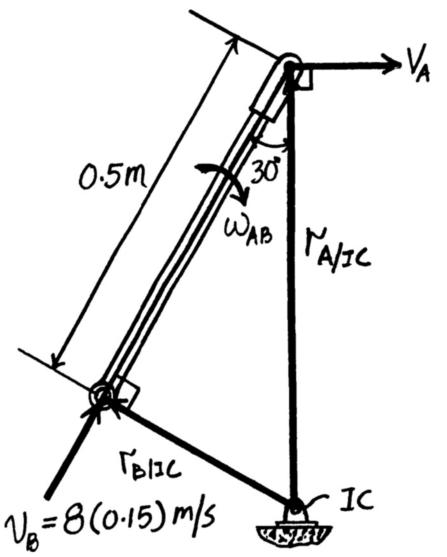
$$2.4\mathbf{i} - 9.6\mathbf{j} = (-a_A \cos 60°\mathbf{i} + a_A \sin 60°\mathbf{j}) + (\alpha\mathbf{k}) \times (-0.5\mathbf{i}) - (4.157)^2(-0.5\mathbf{i})$$

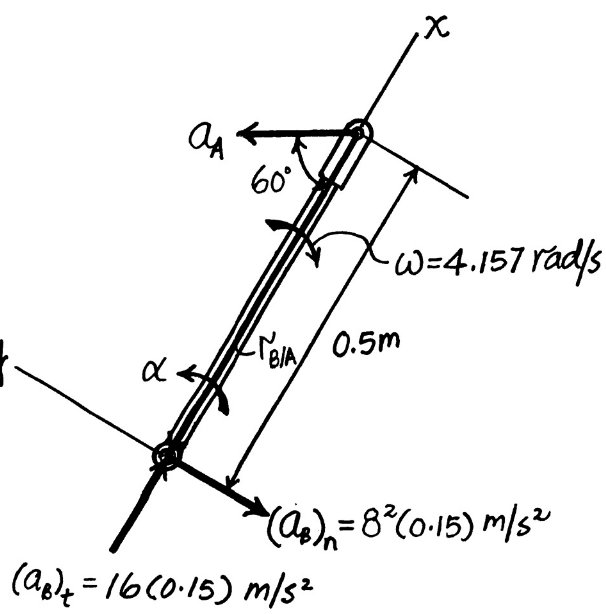
$$2.4\mathbf{i} - 9.6\mathbf{j} = (-a_A \cos 60° + 8.64)\mathbf{i} + (-0.5\alpha + a_A \sin 60°)\mathbf{j}$$

Equating the **i** and **j** components yields:

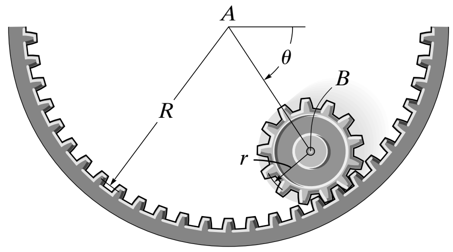
$$2.4 = -a_A \cos 60° + 8.64 \qquad a_A = 12.5 \text{ m/s}^2 \leftarrow \qquad \textbf{Ans.}$$

$$-9.6 = -0.5\alpha + (12.5)\sin 60° \qquad \alpha = 40.8 \text{ rad/s}^2 \circlearrowleft$$



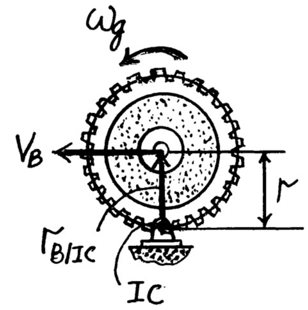



**R2–7.** The small gear which has a mass $m$ can be treated as a uniform disk. If it is released from rest at $\theta = 0°$, and rolls along the fixed circular gear rack, determine the angular velocity of the radial line $AB$ at the instant $\theta = 90°$.




**Potential Energy:** Datum is set at point $A$. When the gear is at its final position ($\theta = 90°$), its center of gravity is located $(R - r)$ *below* the datum. Its gravitational potential energy at this position is $-mg(R - r)$. Thus, the initial and final potential energies are

$$V_1 = 0 \qquad V_2 = -mg(R - r)$$




**Kinetic Energy:** When gear $B$ is at its final position ($\theta = 90°$), the velocity of its mass center is $v_B = \omega_g r$ or $\omega_g = \dfrac{v_B}{r}$ since the gear rolls without slipping on the fixed circular gear track. The mass moment of inertia of the gear about its mass center is $I_B = \dfrac{1}{2}mr^2$. Since the gear is at rest initially, the initial kinetic energy is $T_1 = 0$. The final kinetic energy is given by

$$T_2 = \frac{1}{2}mv_B^2 + \frac{1}{2}I_B\,\omega_g^2 = \frac{1}{2}mv_B^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v_B}{r}\right)^2 = \frac{3}{4}mv_B^2$$

**Conservation of Energy:** Applying Eq. 18–18, we have

$$T_1 + V_1 = T_2 + V_2$$

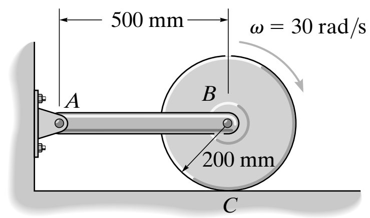
$$0 + 0 = \frac{3}{4}mv_B^2 + [-mg(R - r)]$$

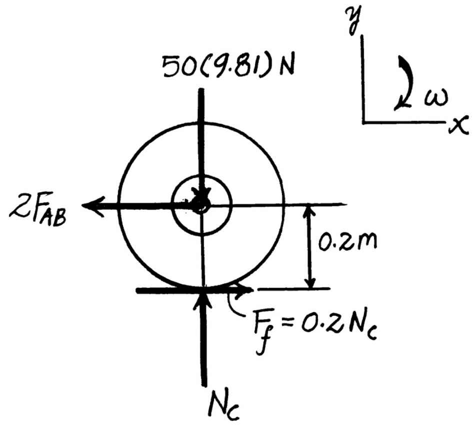
$$v_B = \sqrt{\frac{4g(R - r)}{3}}$$

Thus, the angular velocity of the radical line $AB$ is given by

$$\omega_{AB} = \frac{v_B}{R - r} = \sqrt{\frac{4g}{3(R - r)}} \qquad \textbf{Ans.}$$

***R2–8.** The 50-kg cylinder has an angular velocity of 30 rad/s when it is brought into contact with the surface at $C$. If the coefficient of kinetic friction is $\mu_k = 0.2$, determine how long it will take for the cylinder to stop spinning. What force is developed in link $AB$ during this time? The axis of the cylinder is connected to *two* symmetrical links. (Only $AB$ is shown.) For the computation, neglect the weight of the links.




$$(+\uparrow) \qquad m(v_{Ay})_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_{Ay})_2$$

$$0 + N_C(t) - 50(9.81)(t) = 0 \qquad N_C = 490.5\text{ N}$$

$$\left(\xrightarrow{+}\right) \qquad m(v_{Ax})_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_{Ax})_2$$

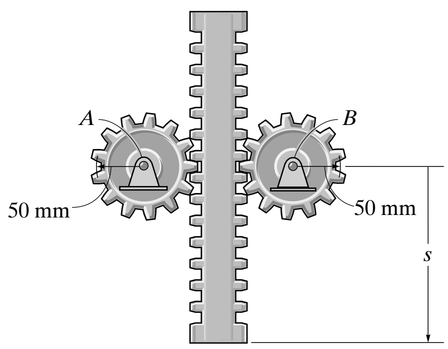
$$0 + 0.2(490.5)(t) - 2F_{AB}(t) = 0 \qquad F_{AB} = 49.0\text{ N}$$ **Ans.**

$$(\circlearrowleft +) \qquad I_B\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_B\,dt = I_B\,\omega_2$$

$$-\left[\frac{1}{2}(50)(0.2)^2\right](30) + 0.2(490.5)(0.2)(t) = 0$$

$$t = 1.53\text{ s}$$ **Ans.**

**R2–9.** The gear rack has a mass of 6 kg, and the gears each have a mass of 4 kg and a radius of gyration of $k = 30$ mm about their center. If the rack is originally moving downward at 2 m/s, when $s = 0$, determine the speed of the rack when $s = 600$ mm. The gears are free to rotate about their centers, $A$ and $B$.




Originally, both gears rotate with an angular velocity of $\omega_t = \dfrac{2}{0.05} = 40$ rad/s. After the rack has traveled $s = 600$ mm, both gears rotate with an angular velocity of $\omega_2 = \dfrac{v_2}{0.05}$, where $v_2$ is the speed of the rack at that moment.

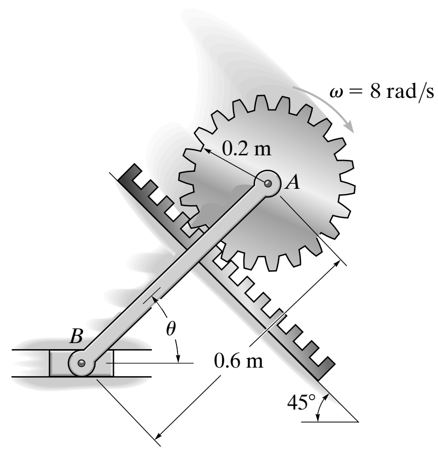
Put datum through points $A$ and $B$.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(6)(2)^2 + \left\{\frac{1}{2}\left[4(0.03)^2\right](40)^2\right\} + 0 = \frac{1}{2}(6)v_2^2 + 2\left\{\frac{1}{2}\left[4(0.03)^2\right]\left(\frac{v_2}{0.05}\right)\right\} - 6(9.81)(0.6)$$

$$v_2 = 3.46\text{ m/s}$$ **Ans.**

**R2–10.** The gear has a mass of 2 kg and a radius of gyration $k_A = 0.15$ m. The connecting link $AB$ (slender rod) and slider block at $B$ have a mass of 4 kg and 1 kg, respectively. If the gear has an angular velocity $\omega = 8$ rad/s at the instant $\theta = 45°$, determine the gear's angular velocity when $\theta = 0°$.




At position 1:

$$(\omega_{AB})_1 = \frac{(v_A)_1}{r_{A/IC}} = \frac{1.6}{0.6} = 2.6667 \text{ rad/s} \qquad (v_B)_1 = 0$$

$$(v_{AB})_1 = (\omega_{AB})_1\, r_{G/IC} = 2.6667(0.3) = 0.8 \text{ m/s}$$

At position 2:

$$(\omega_{AB})_2 = \frac{(v_A)_2}{r_{A/IC}} = \frac{\omega_2\,(0.2)}{\dfrac{0.6}{\cos 45°}} = 0.2357\omega_2$$

$$(v_B)_2 = (\omega_{AB})_2\, r_{B/IC} = 0.2357\,\omega_2(0.6) = 0.1414\omega_2$$

$$(v_{AB})_2 = (\omega_{AB})_2\, r_{G/IC} = 0.2357\,\omega_2(0.6708) = 0.1581\omega_2$$

$$T_1 = \frac{1}{2}\left[(2)(0.15)^2\right](8)^2 + \frac{1}{2}(2)(1.6)^2 + \frac{1}{2}(4)(0.8)^2 + \frac{1}{2}\left[\frac{1}{12}(4)(0.6)^2\right](2.6667)^2$$

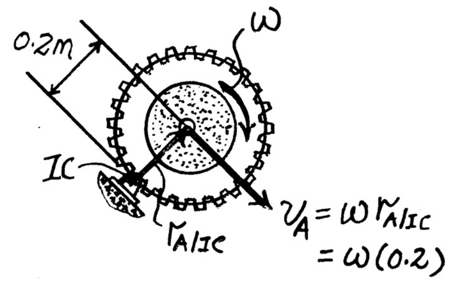
$$= 5.7067 \text{ J}$$

$$T_2 = \frac{1}{2}\left[(2)(0.15)^2\right](\omega_2)^2 + \frac{1}{2}(2)(0.2\,\omega_2)^2 + \frac{1}{2}(4)(0.1581\omega_2)^2$$

$$+ \frac{1}{2}\left[\frac{1}{12}(4)(0.6)^2\right](0.2357\omega_2)^2 + \frac{1}{2}(1)(0.1414\omega_2)^2$$

$$T_2 = 0.1258\,\omega_2^2$$

Put datum through bar in position 2.

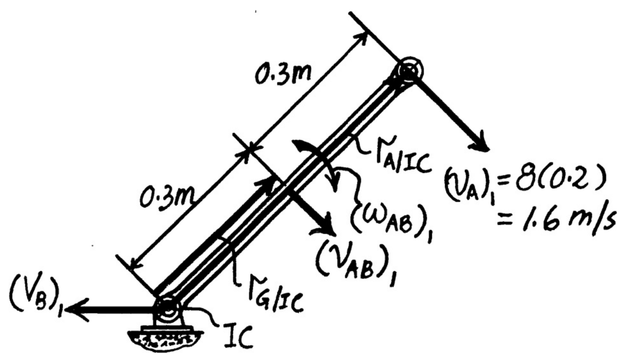


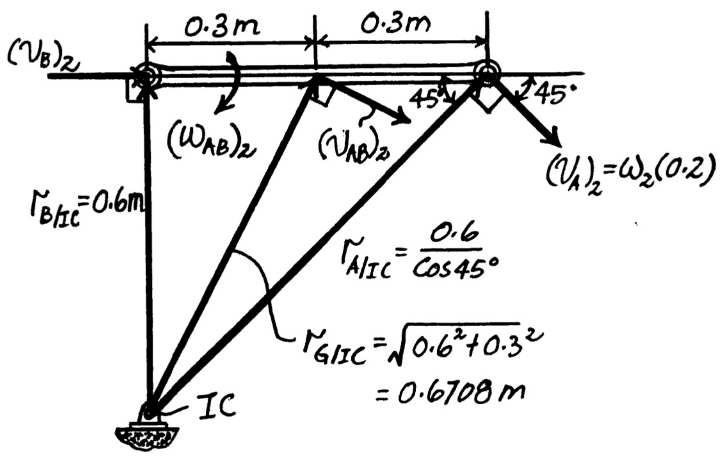

$$V_1 = 2(9.81)(0.6 \sin 45°) + 4(9.81)(0.3 \sin 45°) = 16.6481 \text{ J} \qquad V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2$$

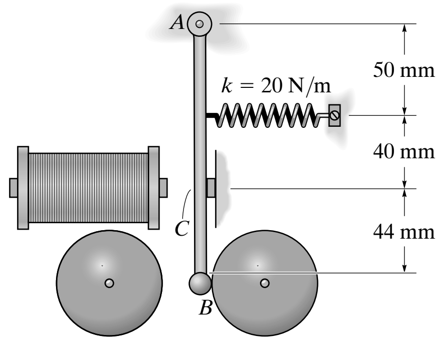
$$5.7067 + 16.6481 = 0.1258\omega_2^2 + 0$$

$$\omega_2 = 13.3 \text{ rad/s} \qquad \textbf{Ans.}$$

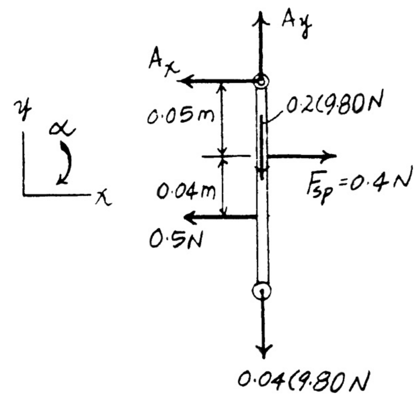


***R2–11.** The operation of a doorbell requires the use of an electromagnet, that attracts the iron clapper $AB$ that is pinned at end $A$ and consists of a 0.2-kg slender rod to which is attached a 0.04-kg steel ball having a radius of 6 mm. If the attractive force of the magnet at $C$ is 0.5 N when the switch is on, determine the initial angular acceleration of the clapper. The spring is originally stretched 20 mm.



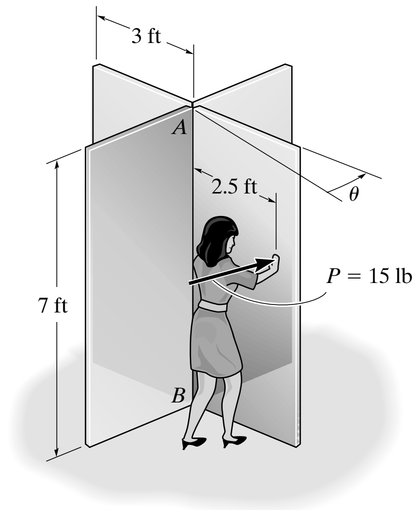

**Equation of Motion:** The spring force is given by $F_{sp} = kx = 20(0.02) = 0.4$ N. The mass moment of inertia for the clapper $AB$ is $(I_{AB})_A = \frac{1}{12}(0.2)\left(0.134^2\right) + 0.2\left(0.067^2\right) + \frac{2}{5}(0.04)\left(0.006^2\right) + 0.04\left(0.14^2\right) = 1.9816\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$. Applying Eq. 17–12, we have




$$+\Sigma M_A = I_A\,\alpha; \qquad 0.4(0.05) - 0.5(0.09) = -1.9816\left(10^{-3}\right)\alpha$$

$$\alpha = 12.6\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

***R2–12.** The revolving door consists of four doors which are attached to an axle $AB$. Each door can be assumed to be a 50-lb thin plate. Friction at the axle contributes a moment of 2 lb · ft which resists the rotation of the doors. If a woman passes through one door by always pushing with a force $P = 15$ lb perpendicular to the plane of the door as shown, determine the door's angular velocity after it has rotated 90°. The doors are originally at rest.



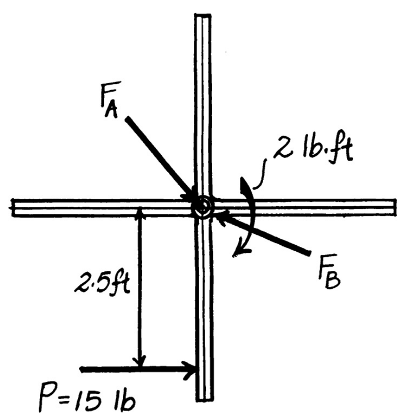

Moment of inertia of the door about axle $AB$:

$$I_{AB} = 2\left[\frac{1}{12}\left(\frac{100}{32.2}\right)(6)^2\right] = 18.6335\ \text{slug}\cdot\text{ft}^2$$

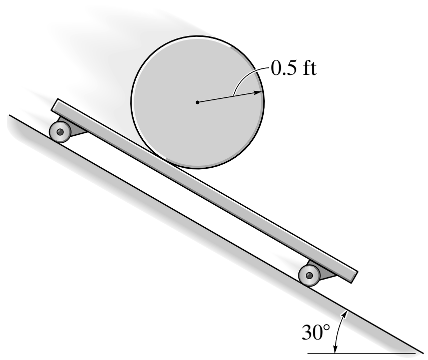
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left\{15(2.5)\left(\frac{\pi}{2}\right) - 2\left(\frac{\pi}{2}\right)\right\} = \frac{1}{2}(18.6335)\,\omega^2$$

$$\omega = 2.45\ \text{rad/s} \qquad \textbf{Ans.}$$



**R2–13.** The 10-lb cylinder rests on the 20-lb dolly. If the system is released from rest, determine the angular velocity of the cylinder in 2 s. The cylinder does not slip on the dolly. Neglect the mass of the wheels on the dolly.




For the cylinder,

$$(+\searrow) \quad m(v_{Cx'})_1 + \Sigma\int_{t_1}^{t_2} F_{x'}\,dt = m(v_{Cx'})_2$$

$$0 + 10\sin 30°(2) - F(2) = \left(\frac{10}{32.2}\right)v_C \qquad \textbf{(1)}$$

$$(\circlearrowleft +) \quad I_C\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_C\,dt = I_C\,\omega_2$$

$$0 + F(0.5)(2) = \left[\frac{1}{2}\left(\frac{10}{32.2}\right)(0.5)^2\right]\omega \qquad \textbf{(2)}$$

For the dolly,

$$(+\searrow) \quad m(v_{Dx'})_1 + \Sigma\int_{t_1}^{t_2} F_{x'}\,dt = m(v_{Dx'})_2$$

$$0 + F(2) + 20\sin 30°(2) = \left(\frac{20}{32.2}\right)v_D \qquad \textbf{(3)}$$

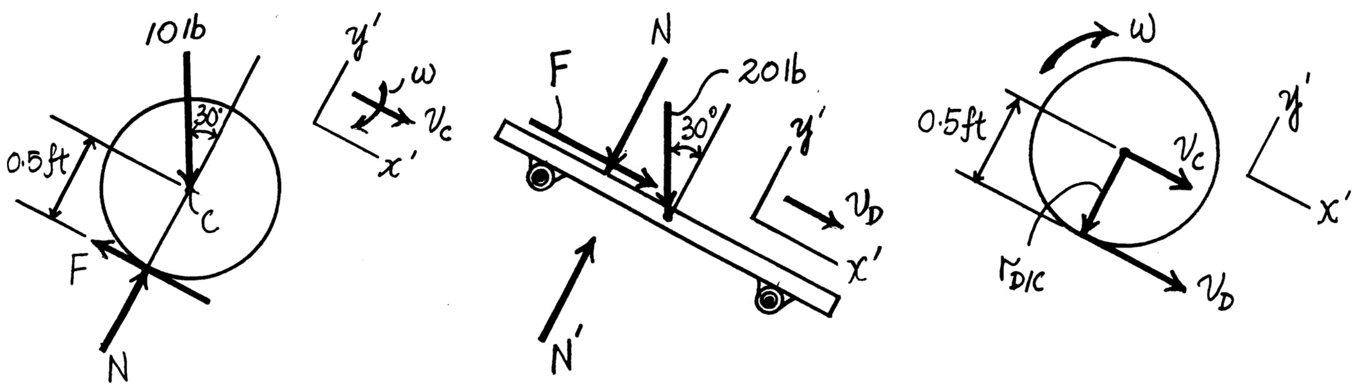
$$(+\searrow) \quad \mathbf{v}_D = \mathbf{v}_C + \mathbf{v}_{D/C}$$

$$v_D = v_C - 0.5\omega \qquad \textbf{(4)}$$

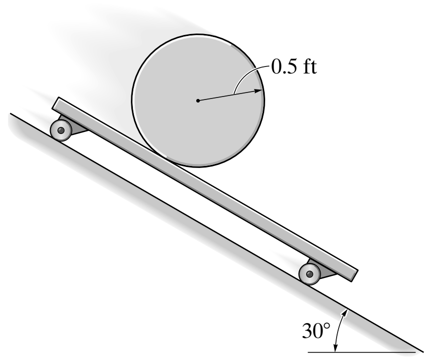
Solving Eqs. (1) to (4) yields:

$$\omega = 0 \qquad \textbf{Ans.}$$

$$v_C = 32.2 \text{ ft/s} \qquad v_D = 32.2 \text{ ft/s} \qquad F = 0$$



**R2–14.** Solve Prob. R2–13 if the coefficients of static and kinetic friction between the cylinder and the dolly are $\mu_s = 0.3$ and $\mu = 0.2$, respectively.




For the cylinder,

$$(+\searrow)\quad m(v_{Cx'})_1 + \Sigma\int_{t_1}^{t_2} F_{x'}\,dt = m(v_{Cx'})_2$$

$$0 + 10\sin 30°(2) - F(2) = \left(\frac{10}{32.2}\right)v_C \qquad \textbf{(1)}$$

$$(\circlearrowleft +)\quad I_C\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_C\,dt = I_C\,\omega_2$$

$$0 + F(0.5)(2) = \left[\frac{1}{2}\left(\frac{10}{32.2}\right)(0.5)^2\right]\omega \qquad \textbf{(2)}$$

For the dolly,

$$(+\searrow)\quad m(v_{Dx'})_1 + \Sigma\int_{t_1}^{t_2} F_{x'}\,dt = m(v_{Dx'})_2$$

$$0 + F(2) + 20\sin 30°(2) = \left(\frac{20}{32.2}\right)v_D \qquad \textbf{(3)}$$

$$(+\searrow)\quad \mathbf{v}_D = \mathbf{v}_C + \mathbf{v}_{D/C}$$

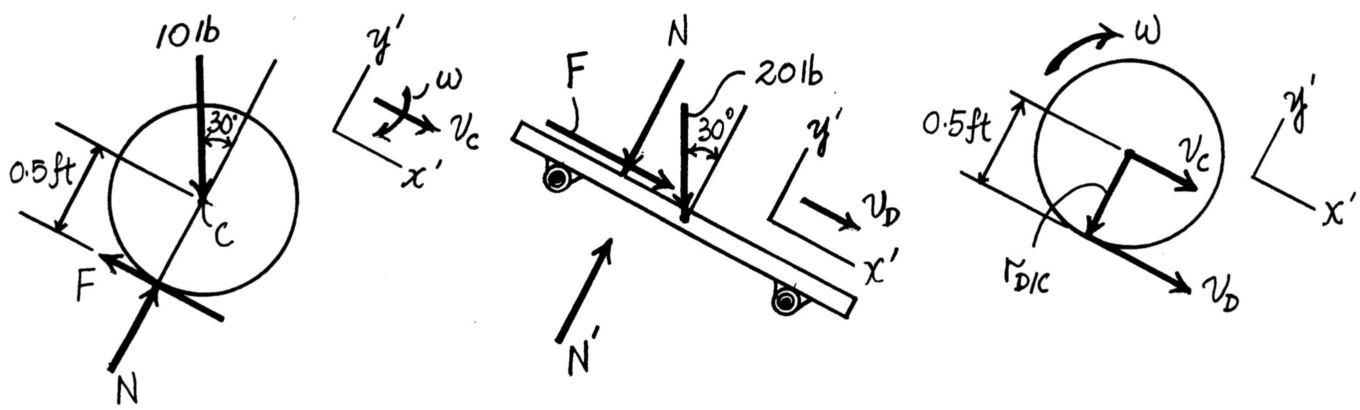
$$v_D = v_C - 0.5\omega \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4) yields:

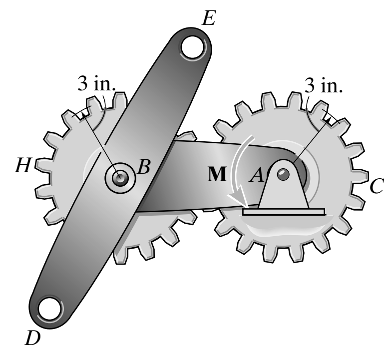
$$\omega = 0 \qquad \textbf{Ans.}$$

$$v_C = 32.2\text{ ft/s} \qquad v_D = 32.2\text{ ft/s} \qquad F = 0$$

**Note:** No friction force develops.

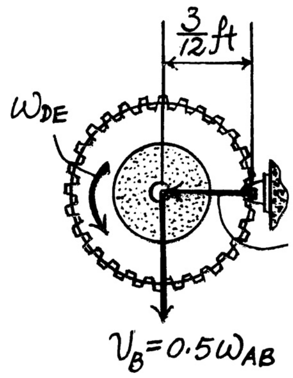


**R2–15.** Gears $H$ and $C$ each have a weight of 0.4 lb and a radius of gyration about their mass center of $(k_H)_B = (k_C)_A = 2$ in. Link $AB$ has a weight of 0.2 lb and a radius of gyration of $(k_{AB})_A = 3$ in., whereas link $DE$ has a weight of 0.15 lb and a radius of gyration of $(k_{DE})_B = 4.5$ in. If a couple moment of $M = 3\text{ lb}\cdot\text{ft}$ is applied to link $AB$ and the assembly is originally at rest, determine the angular velocity of link $DE$ when link $AB$ has rotated 360°. Gear $C$ is prevented from rotating, and motion occurs in the horizontal plane. Also, gear $H$ and link $DE$ rotate together about the same axle at $B$.




For link $AB$,

$$v_B = \omega_{AB}\, r_{AB} = \omega_{AB}\left(\frac{6}{12}\right) = 0.5\omega_{AB}$$

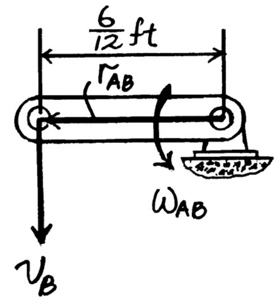



For gear $H$,

$$\omega_{DE} = \frac{v_B}{r_{B/IC}} = \frac{0.5\omega_{AB}}{3/12} = 2\omega_{AB}$$

$$\omega_{AB} = \frac{1}{2}\omega_{DE}$$

$$v_B = \left(\frac{1}{2}\omega_{DE}\right)\frac{6}{12} = 0.25\omega_{DE}$$




**Principle of Work and Energy:** For the system,

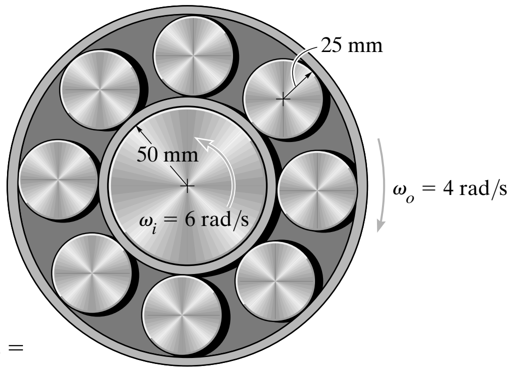
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 3(2\pi) = \frac{1}{2}\left[\left(\frac{0.2}{32.2}\right)\left(\frac{3}{12}\right)^2\right]\left(\frac{1}{2}\omega_{DE}\right)^2 + \frac{1}{2}\left[\left(\frac{0.4}{32.2}\right)\left(\frac{2}{12}\right)^2\right]\omega_{DE}^2$$

$$+ \frac{1}{2}\left(\frac{0.4}{32.2}\right)(0.25\omega_{DE})^2 + \frac{1}{2}\left[\left(\frac{0.15}{32.2}\right)\left(\frac{4.5}{12}\right)^2\right]\omega_{DE}^2 + \frac{1}{2}\left(\frac{0.15}{32.2}\right)(0.25\omega_{DE})^2$$

$$\omega_{DE} = 132 \text{ rad/s} \qquad \textbf{Ans.}$$

***R2–16.** The inner hub of the roller bearing rotates with an angular velocity of $\omega_i = 6$ rad/s, while the outer hub rotates in the opposite direction at $\omega_o = 4$ rad/s. Determine the angular velocity of each of the rollers if they roll on the hubs without slipping.

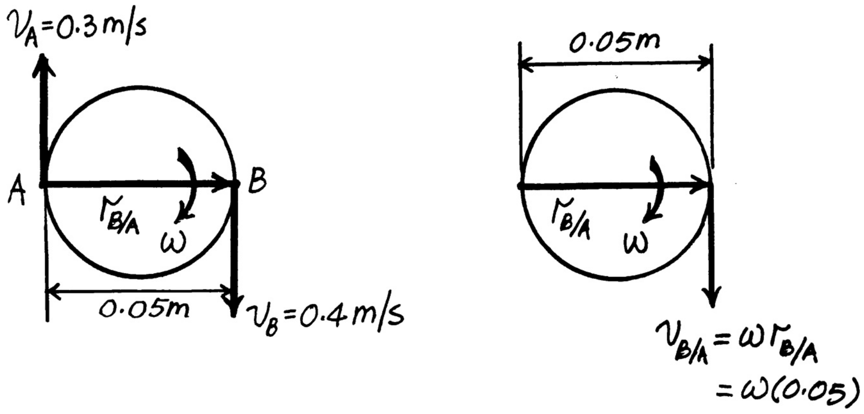



Since the hub does not slip, $v_A = \omega_i r_i = 6(0.05) = 0.3$ m/s and $v_B = \omega_O r_O = 4(0.1) = 0.4$ m/s.

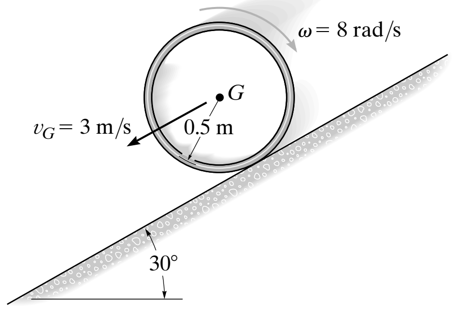
$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$\begin{bmatrix} 0.4 \\ \downarrow \end{bmatrix} = \begin{bmatrix} 0.3 \\ \uparrow \end{bmatrix} + \begin{bmatrix} \omega(0.05) \\ \downarrow \end{bmatrix}$$

$(+\downarrow)$ $\quad 0.4 = -0.3 + 0.05\omega \qquad \omega = 14 \text{ rad/s} \curvearrowright$ **Ans.**



**R2–17.** The hoop (thin ring) has a mass of 5 kg and is released down the inclined plane such that it has a backspin $\omega = 8$ rad/s and its center has a velocity $v_G = 3$ m/s as shown. If the coefficient of kinetic friction between the hoop and the plane is $\mu_k = 0.6$, determine how long the hoop rolls before it stops slipping.




$(+\nwarrow) \qquad mv_{y1} + \Sigma \int F_y\, dt = mv_{y2}$

$$0 + N_h(1) - 5(9.81)t \cos 30° = 0$$

$$N_h = 42.479 \text{ N}$$

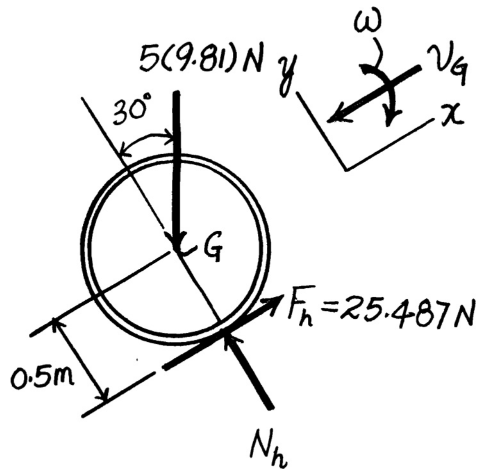
$$F_h = 0.6N_h = 0.6(42.479 \text{ N}) = 25.487 \text{ N}$$

$(\swarrow +) \qquad mv_{x1} + \Sigma \int F_x\, dt = mv_{x2}$

$$5(3) + 5(9.181) \sin 30°(t) - 25.487t = 5v_G$$

$(\circlearrowleft +) \qquad (H_G)_1 + \Sigma \int M_G\, dt = (H_G)_2$

$$-5(0.5)^2(8) + 25.487(0.5)(t) = 5(0.5)^2\left(\frac{v_G}{0.5}\right)$$

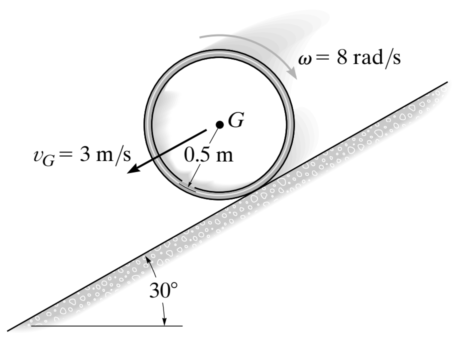



Solving,

$$v_G = 2.75 \text{ m/s}$$

$$t = 1.32 \text{ s} \qquad \textbf{Ans.}$$

**R2–18.** The hoop (thin ring) has a mass of 5 kg and is released down the inclined plane such that it has a backspin $\omega = 8$ rad/s and its center has a velocity $v_G = 3$ m/s as shown. If the coefficient of kinetic friction between the hoop and the plane is $\mu_k = 0.6$, determine the hoop's angular velocity 1 s after it is released.




See solution to Prob. R2–17. Since backspin will not stop in $t = 1 \text{ s} < 1.32$ s, then

$(+\nwarrow) \qquad mv_{y1} + \Sigma \int F_y\, dt = mv_{y2}$

$$0 + N_h(t) - 5(9.81)t \cos 30° = 0$$

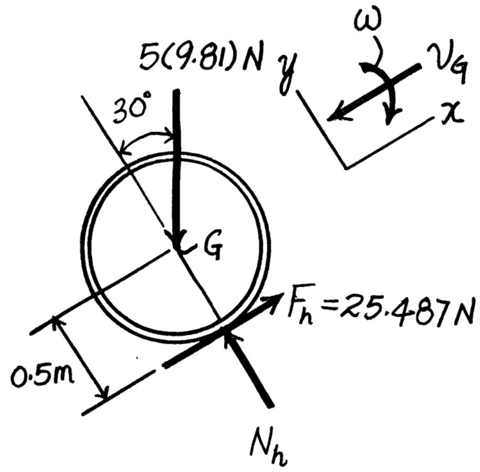
$$N_h = 42.479 \text{ N}$$

$$F_h = 0.6N_h = 0.6(42.479 \text{ N}) = 25.487 \text{ N}$$

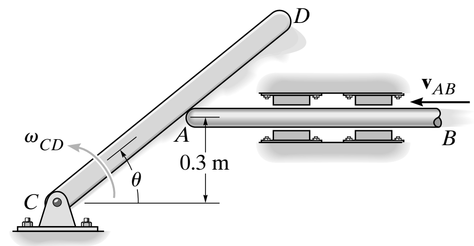
$\circlearrowleft + \qquad (H_G)_1 + \Sigma \int M\, dt = (H_G)_2$

$$-5(0.5)^2(8) + 25.487(0.5)(1) = -5(0.5)^2\,\omega$$

$$\omega = 2.19 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$



**R2–19.** Determine the angular velocity of rod $CD$ at the instant $\theta = 30°$. Rod $AB$ moves to the left at a constant speed of $v_{AB} = 5$ m/s.

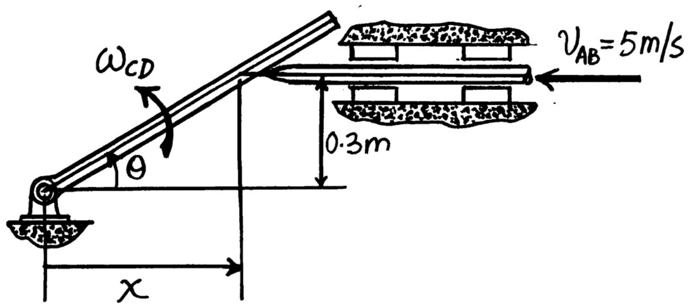



$$x = \frac{0.3}{\tan\theta} = 0.3\cot\theta$$

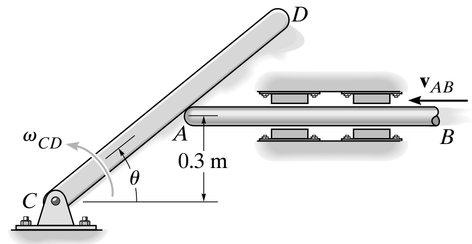
$$\dot{x} = v_{AB} = -0.3\csc^2\theta\dot{\theta}$$

Here $\dot{\theta} = \omega_{CD}, v_{AB} = -5$ m/s and $\theta = 30°$.

$-5 = -0.03\csc^2 30°(\omega_{CD})$ $\qquad$ $\omega_{CD} = 4.17$ rad/s $\qquad$ **Ans.**



***R2–20.** Determine the angular acceleration of rod $CD$ at the instant $\theta = 30°$. Rod $AB$ has zero velocity, i.e., $v_{AB} = 0$, and an acceleration of $a_{AB} = 2\text{ m/s}^2$ to the right when $\theta = 30°$.




$$x = \frac{0.3}{\tan\theta} = 0.3\cot\theta$$

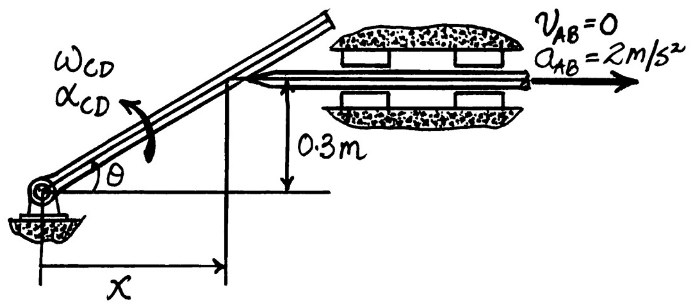
$$\dot{x} = v_{AB} = -0.3\csc^2\theta\dot{\theta}$$

$$\ddot{x} = a_{AB} = -0.3\left[\csc^2\theta\ddot{\theta} - 2\csc^2\theta\cot\theta\dot{\theta}^2\right] = 0.3\csc^2\theta\left(2\cot\theta\dot{\theta}^2 - \ddot{\theta}\right)$$

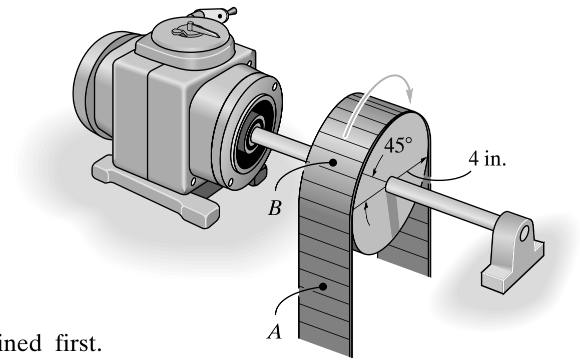
Here $\dot{\theta} = \omega_{CD}, v_{AB} = 0, a_{AB} = 2\text{ m/s}^2, \ddot{\theta} = \alpha_{CD}$, and $\theta = 30°$.

$$0 = -0.3\csc^2 30°(\omega_{CD}) \qquad \omega_{CD} = 0$$

$$2 = 0.3\csc^2 30°\left[2\cot 30°(0)^2 - \alpha_{CD}\right] \qquad \alpha_{CD} = -1.67\text{ rad/s}^2 \qquad \textbf{Ans.}$$



**R2–21.** If the angular velocity of the drum is increased uniformly from 6 rad/s when $t = 0$ to 12 rad/s when $t = 5$ s, determine the magnitudes of the velocity and acceleration of points $A$ and $B$ on the belt when $t = 1$ s. At this instant the points are located as shown.




**Angular Motion:** The angular acceleration of drum must be determined first. Applying Eq. 16–5, we have

$$\omega = \omega_0 + \alpha_c t$$

$$12 = 6 + \alpha_c (5)$$

$$\alpha_c = 1.20 \text{ rad/s}^2$$

The angular velocity of the drum at $t = 1$ s is given by

$$\omega = \omega_0 + \alpha_c t = 6 + 1.20(1) = 7.20 \text{ rad/s}$$

**Motion of *P*:** The magnitude of the velocity of points $A$ and $B$ can be determined using Eq. 16–8.

$$v_A = v_B = \omega r = 7.20\left(\frac{4}{12}\right) = 2.40 \text{ ft/s} \qquad \textbf{Ans.}$$

Also,

$$a_A = (a_t)_A = \alpha_c r = 1.20\left(\frac{4}{12}\right) = 0.400 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

The tangential and normal components of the acceleration of points $B$ can be determined using Eqs. 16–11 and 16–12, respectively.

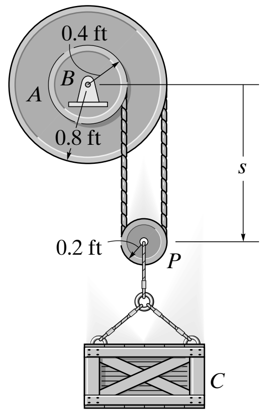
$$(a_t)_B = \alpha_c r = 1.20\left(\frac{4}{12}\right) = 0.400 \text{ ft/s}^2$$

$$(a_n)_B = \omega^2 r = \left(7.20^2\right)\left(\frac{4}{12}\right) = 17.28 \text{ ft/s}^2$$

The magnitude of the acceleration of points $B$ is

$$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{0.400^2 + 17.28^2} = 17.3 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

**R2–22.** Pulley $A$ and the attached drum $B$ have a weight of 20 lb and a radius of gyration of $k_B = 0.6$ ft. If pulley $P$ "rolls" downward on the cord without slipping, determine the speed of the 20-lb crate $C$ at the instant $s = 10$ ft. Initially, the crate is released from rest when $s = 5$ ft. For the calculation, neglect the mass of pulley $P$ and the cord.

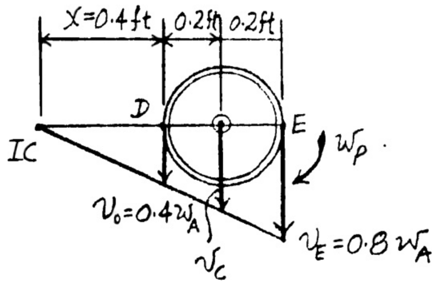



**Kinematics:** Since pulley $A$ is rotating about a fixed point $B$ and pulley $P$ rolls down without slipping, the velocity of points $D$ and $E$ on the pulley $P$ are given by $v_D = 0.4\omega_A$ and $v_E = 0.8\omega_A$ where $\omega_A$ is the angular velocity of pulley $A$. Thus, the instantaneous center of zero velocity can be located using similar triangles.

$$\frac{x}{0.4\omega_A} = \frac{x + 0.4}{0.8\omega_A} \qquad x = 0.4 \text{ ft}$$

Thus, the velocity of block $C$ is given by

$$\frac{v_C}{0.6} = \frac{0.4\omega_A}{0.4} \qquad v_C = 0.6\omega_A$$




**Potential Energy:** Datumn is set at point $B$. When block $C$ is at its initial and final position, its locations are 5 ft and 10 ft *below* the datum. Its initial and final gravitational potential energies are $20(-5) = -100$ ft·lb and $20(-10) = -200$ ft·lb, respectively. Thus, the initial and final potential energy are

$$V_1 = -100 \text{ ft}\cdot\text{lb} \qquad V_2 = -200 \text{ ft}\cdot\text{lb}$$

**Kinetic Energy:** The mass moment of inertia of pulley $A$ about point $B$ is $I_B = mk_B^2 = \frac{20}{32.2}\left(0.6^2\right) = 0.2236$ slug·ft². Since the system is initially at rest, the initial kinetic energy is $T_1 = 0$. The final kinetic energy is given by

$$\begin{aligned} T_2 &= \frac{1}{2} m_C v_C^2 + \frac{1}{2} I_B \omega_A^2 \\ &= \frac{1}{2}\left(\frac{20}{32.2}\right)(0.6\omega_A)^2 + \frac{1}{2}(0.2236)\,\omega_A^2 \\ &= 0.2236\omega_A^2 \end{aligned}$$

**Conservation of Energy:** Applying Eq. 18–19, we have

$$T_1 + V_1 = T_2 + V_2$$

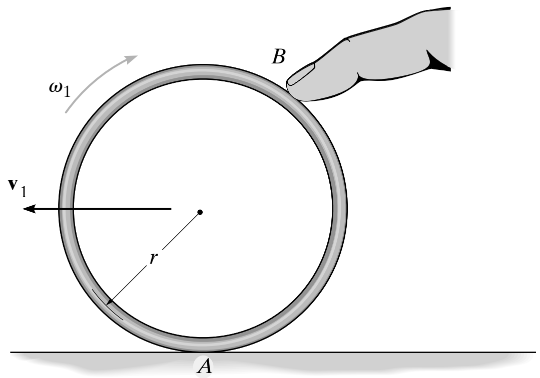
$$0 + -100 = 0.2236\omega_A^2 + (-200)$$

$$\omega_A = 21.15 \text{ rad/s}$$

Thus, the speed of block $C$ at the instant $s = 10$ ft is

$$v_C = 0.6\omega_A = 0.6(21.15) = 12.7 \text{ ft/s} \qquad \textbf{Ans.}$$

**R2–23.** By pressing down with the finger at $B$, a thin ring having a mass $m$ is given an initial velocity $v_1$ and a backspin $\omega_1$ when the finger is released. If the coefficient of kinetic friction between the table and the ring is $\mu$, determine the distance the ring travels forward before the backspin stops.

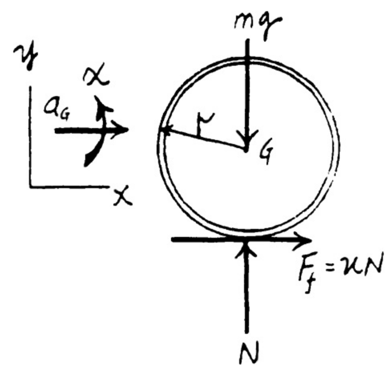



**Equations of Motion:** The mass moment of inertia of the ring about its mass center is given by $I_G = mr^2$. Applying Eq. 17–16, we have

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - mg = 0 \qquad N = mg$$

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad \mu mg = ma_G \qquad a_G = \mu g$$

$$\circlearrowleft + \Sigma M_G = I_G \alpha; \qquad \mu mgr = mr^2 \alpha \qquad \alpha = \frac{\mu g}{r}$$




**Kinematics:** The time required for the ring to stop back spinning can be determined by applying Eq. 16–5.

$$\omega = \omega_0 + \alpha_c t$$

$$(\circlearrowright +) \qquad 0 = \omega_1 + \left(-\frac{\mu g}{r}\right)t$$

$$t = \frac{\omega_1 r}{\mu g}$$

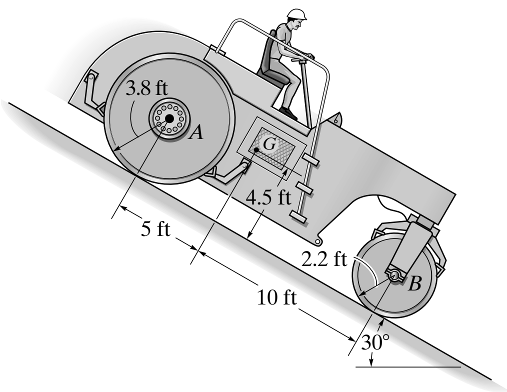
The distance traveled by the ring just before back spinning stops can be determine by applying Eq. 12–5.

$$(\overset{+}{\leftarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$= 0 + v_1 \left(\frac{\omega_1 r}{\mu g}\right) + \frac{1}{2}(-\mu g)\left(\frac{\omega_1 r}{\mu g}\right)^2$$

$$= \frac{\omega_1 r}{2\mu g}(2v_1 - \omega_1 r) \qquad \textbf{Ans.}$$

***R2–24.** The pavement roller is traveling down the incline at $v_1 = 5$ ft/s when the motor is disengaged. Determine the speed of the roller when it has traveled 20 ft down the plane. The body of the roller, excluding the rollers, has a weight of 8000 lb and a center of gravity at $G$. Each of the two rear rollers weighs 400 lb and has a radius of gyration of $k_A = 3.3$ ft. The front roller has a weight of 800 lb and a radius of gyration of $k_B = 1.8$ ft. The rollers do not slip as they turn.




The wheels roll without slipping, hence $\omega = \dfrac{v_G}{r}$.

$$T_1 = \frac{1}{2}\left(\frac{8000 + 800 + 800}{32.2}\right)(5)^2 + \frac{1}{2}\left[\left(\frac{800}{32.2}\right)(3.3)^2\right]\left(\frac{5}{3.8}\right)^2 + \frac{1}{2}\left[\left(\frac{800}{32.2}\right)(1.8)^2\right]\left(\frac{5}{2.2}\right)^2$$

$$= 4168.81 \text{ ft} \cdot \text{lb}$$

$$T_2 = \frac{1}{2}\left(\frac{8000 + 800 + 800}{32.2}\right)v^2 + \frac{1}{2}\left[\left(\frac{800}{32.2}\right)(3.3)^2\right]\left(\frac{v}{3.8}\right)^2 + \frac{1}{2}\left[\left(\frac{800}{32.2}\right)(1.8)^2\right]\left(\frac{v}{2.2}\right)^2$$

$$= 166.753\, v^2$$

Put datum through the mass center of the wheels and body of the roller when it is in the initial position.

$$V_1 = 0$$

$$V_2 = -800(20 \sin 30°) - 8000(20 \sin 30°) - 800(20 \sin 30°)$$

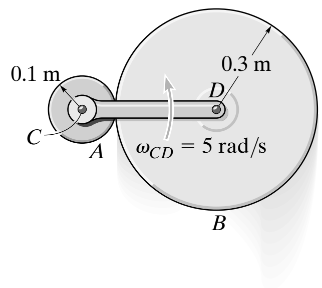
$$= -96000 \text{ ft} \cdot \text{lb}$$

$$T_1 + V_1 = T_2 + V_2$$

$$4168.81 + 0 = 166.753v^2 - 96000$$

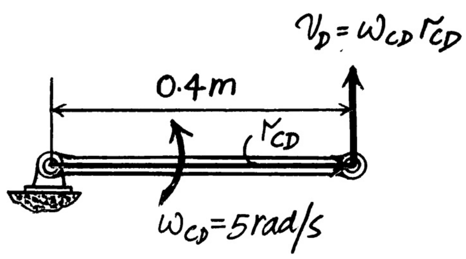
$$v = 24.5 \text{ ft/s}$$ **Ans.**

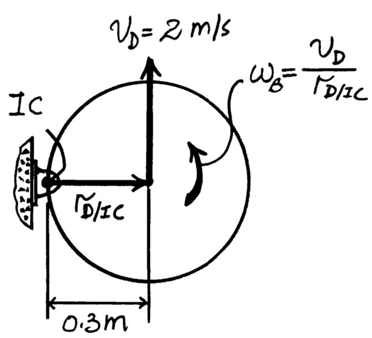
**R2–25.** The cylinder $B$ rolls on the fixed cylinder $A$ without slipping. If bar $CD$ rotates with an angular velocity $\omega_{CD} = 5$ rad/s, determine the angular velocity of cylinder $B$. Point $C$ is a fixed point.



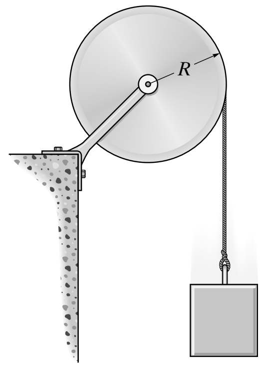

$$v_D = 5(0.4) = 2 \text{ m/s}$$

$$\omega_B = \frac{2}{0.3} = 6.67 \text{ rad/s}$$ **Ans.**



**R2–26.** The disk has a mass $M$ and a radius $R$. If a block of mass $m$ is attached to the cord, determine the angular acceleration of the disk when the block is released from rest. Also, what is the distance the block falls from rest in the time $t$?




$$I_0 = \frac{1}{2}MR^2$$

$\circlearrowleft +\Sigma M_O = \Sigma tM_k)_0;$ $\quad mgR = \frac{1}{2}MR^2(\alpha) + m(\alpha R)R$

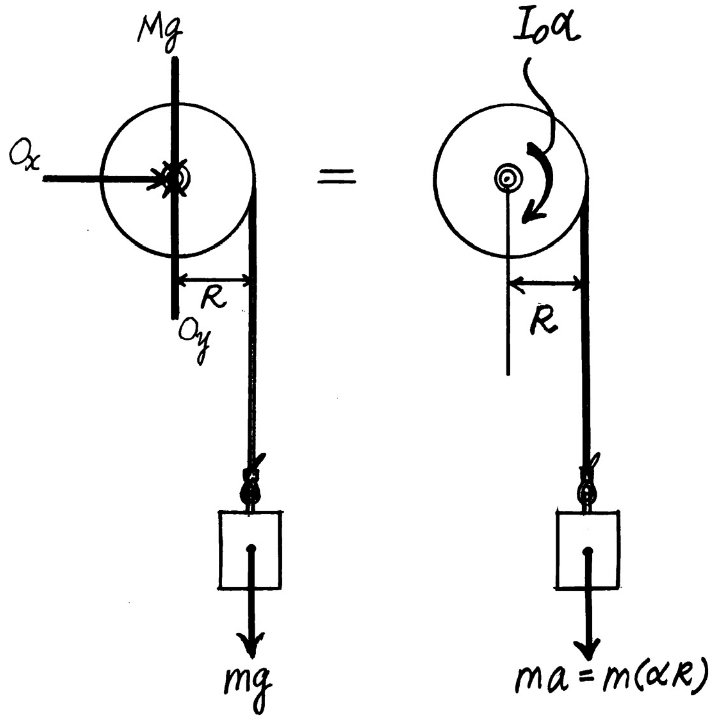
$$\alpha = \frac{2mg}{R(M + 2m)} \qquad \textbf{Ans.}$$

The displacement $h = R\theta$, hence $\theta = \dfrac{h}{R}$

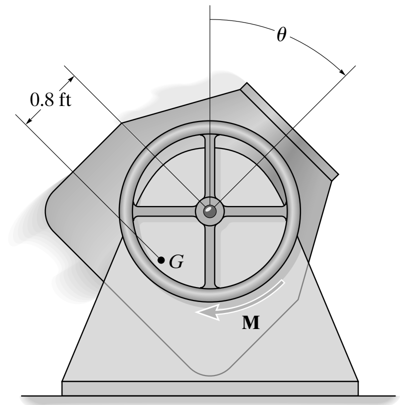
$$\theta - \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$$

$$\frac{h}{R} = 0 + 0 + \frac{1}{2}\left(\frac{2mg}{R(M + 2m)}\right)t^2$$

$$h = \frac{mg}{M + 2m}t^2 \qquad \textbf{Ans.}$$



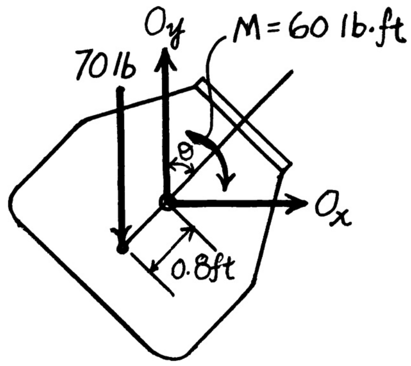
**R2–27.** The tub of the mixer has a weight of 70 lb and a radius of gyration $k_G = 1.3$ ft about its center of gravity $G$. If a constant torque $M = 60$ lb · ft is applied to the dumping wheel, determine the angular velocity of the tub when it has rotated $\theta = 90°$. Originally the tub is at rest when $\theta = 0°$. Neglect the mass of the wheel.



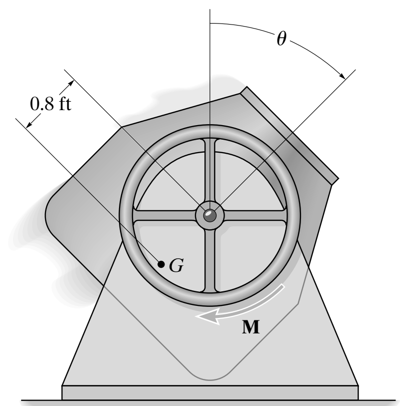

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 60\left(\frac{\pi}{2}\right) - 70(0.8) = \frac{1}{2}\left[\left(\frac{70}{32.2}\right)(1.3)^2\right](\omega)^2 + \frac{1}{2}\left[\frac{70}{32.2}\right](0.8\omega)^2$$

$$\omega = 3.89 \text{ rad/s}$$ **Ans.**



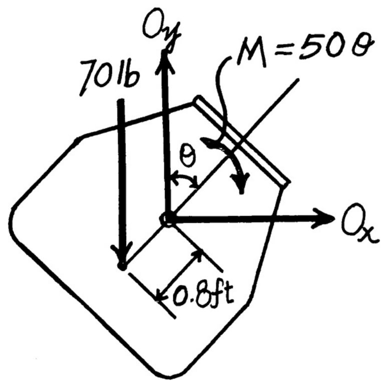

***R2–28.** Solve Prob. R2–27 if the applied torque is $M = (50\theta)$ lb · ft, where $\theta$ is in radians.



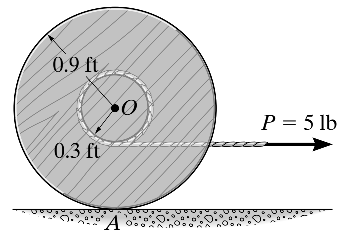

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \int_0^{\pi/2} 50\theta \, d\theta - 70(0.8) = \frac{1}{2}\left[\left(\frac{70}{32.2}\right)(1.3)^2\right]\omega^2 + \frac{1}{2}\left[\frac{70}{32.2}\right](0.8\omega)^2$$

$$\omega = 1.50 \text{ rad/s}$$ **Ans.**



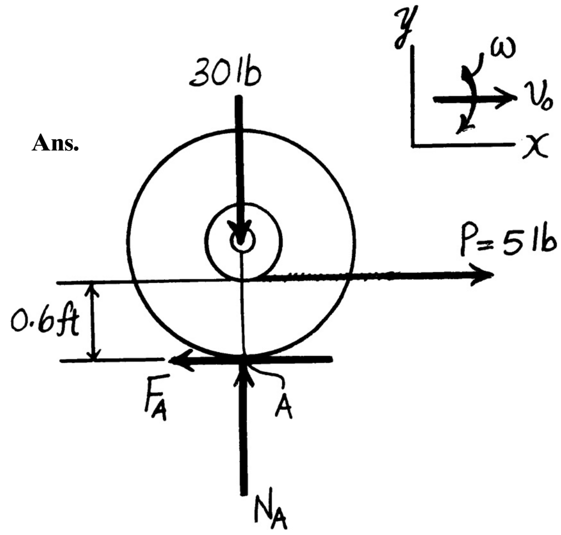
**R2–29.** The spool has a weight of 30 lb and a radius of gyration $k_O = 0.45$ ft. A cord is wrapped around the spool's inner hub and its end subjected to a horizontal force $P = 5$ lb. Determine the spool's angular velocity in 4 s starting from rest. Assume the spool rolls without slipping.



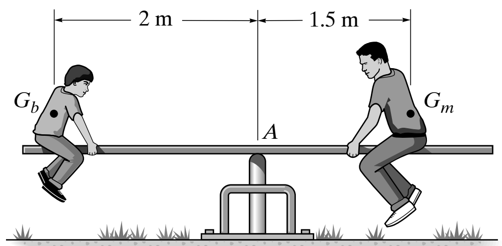

$(+\circlearrowright)$ $$I_A\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = I_A\omega_2$$

$$0 + 5(0.6)(4) = \left[\left(\frac{30}{32.2}\right)(0.45)^2 + \left(\frac{30}{32.2}\right)(0.9)^2\right]\omega_2$$

$$\omega_2 = 12.7 \text{ rad/s}$$ **Ans.**




**R2–30.** The 75-kg man and 40-kg boy sit on the horizontal seesaw, which has negligible mass. At the instant the man lifts his feet from the ground, determine their accelerations if each sits upright, i.e., they do not rotate. The centers of mass of the man and boy are at $G_m$ and $G_b$, respectively.




$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A;$ $\quad 40(9.81)(2) - 75(9.81)(1.5)$

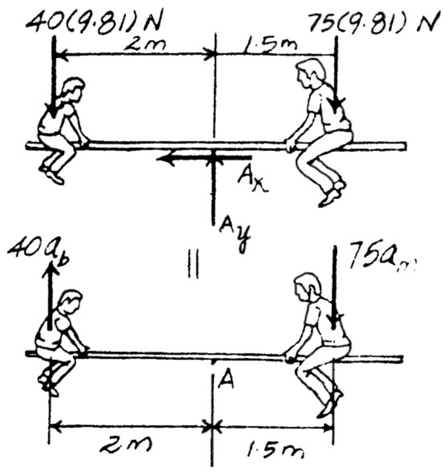
$$= -40a_b\,(2) - 75a_m\,(1.5) \qquad [1]$$

Since the seesaw is rotating about point $A$, then

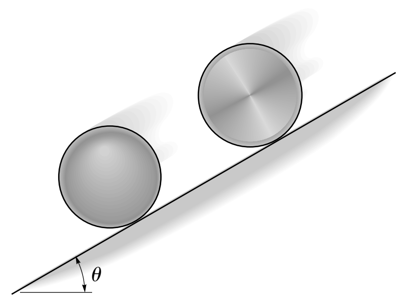
$$\alpha = \frac{a_b}{2} = \frac{a_m}{1.5} \qquad \text{or} \qquad a_m = 0.75a_b \qquad [2]$$

Solving Eqs. (1) and (2) yields:

$$a_m = 1.45 \text{ m/s}^2 \qquad a_b = 1.94 \text{ m/s}^2$$ **Ans.**



**R2–31.** A sphere and cylinder are released from rest on the ramp at $t = 0$. If each has a mass $m$ and a radius $r$, determine their angular velocities at time $t$. Assume no slipping occurs.




**Principle of Impulse and Momentum:** For the sphere,

$$(+\circlearrowleft) \qquad I_A\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = I_A\omega_2$$

$$0 + mg\sin\theta(r)(t) = \left[\frac{2}{5}mr^2 + mr^2\right](\omega_S)_2$$

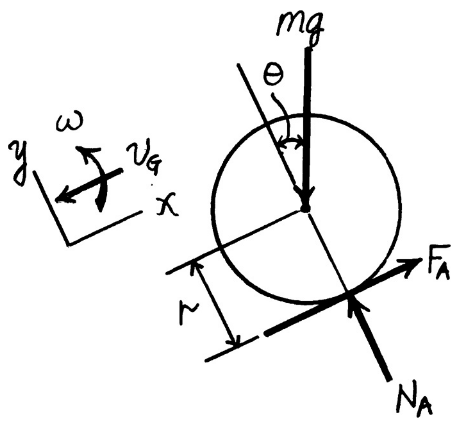
$$(\omega_S)_2 = \frac{5g\sin\theta}{7r}t \qquad \textbf{Ans.}$$

**Principle of Impulse and Momentum:** For the cyclinder,

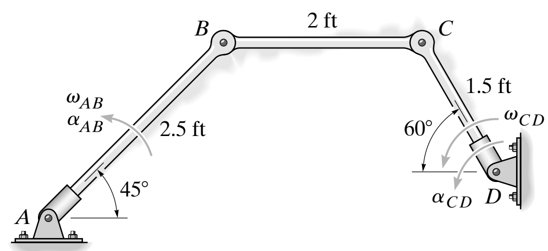
$$(+\circlearrowleft) \qquad I_A\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = I_A\omega_2$$

$$0 + mg\sin\theta(r)(t) = \left[\frac{1}{2}mr^2 + mr^2\right](\omega_C)_2$$

$$(\omega_C)_2 = \frac{2g\sin\theta}{3r}t \qquad \textbf{Ans.}$$



***R2–32.** At a given instant, link $AB$ has an angular acceleration $\alpha_{AB} = 12\ \text{rad/s}^2$ and an angular velocity $\omega_{AB} = 4\ \text{rad/s}$. Determine the angular velocity and angular acceleration of link $CD$ at this instant.




$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$$\begin{bmatrix} v_C \\ 30°\nearrow \end{bmatrix} = \begin{bmatrix} 10 \\ 45°\nwarrow \end{bmatrix} + \begin{bmatrix} 2\omega_{BC} \\ \downarrow \end{bmatrix}$$

$(\overset{+}{\leftarrow})$ $\quad v_C \cos 30° = 10 \cos 45° + 0$

$(+\downarrow)$ $\quad v_C \sin 30° = -10 \sin 45° + 2\omega_{BC}$

$$\omega_{BC} = 5.58\ \text{rad/s}$$

$$v_C = 8.16\ \text{ft/s}$$

$$\omega_{CD} = \frac{8.16}{1.5} = 5.44\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C/B}$$

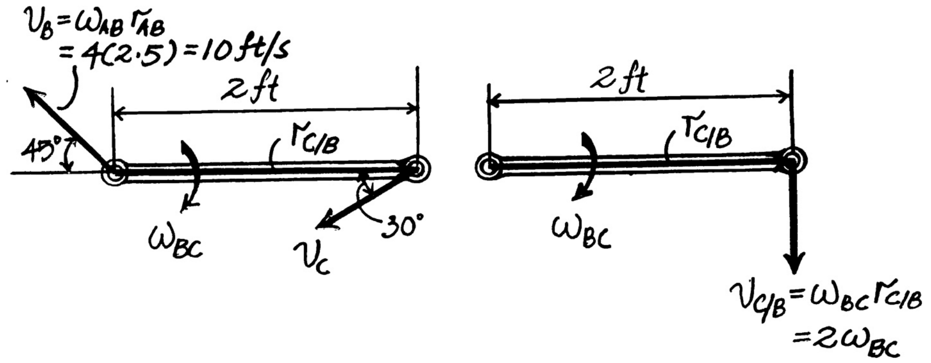
$$\begin{bmatrix} 44.44 \\ \swarrow 60° \end{bmatrix} + \begin{bmatrix} (a_C)_t \\ 30°\nearrow \end{bmatrix} = \begin{bmatrix} 30 \\ 45°\nwarrow \end{bmatrix} + \begin{bmatrix} 40 \\ 45°\nearrow \end{bmatrix} + \begin{bmatrix} 2(5.58)^2 \\ \leftarrow \end{bmatrix} + \begin{bmatrix} 2\alpha_{BC} \\ \downarrow \end{bmatrix}$$

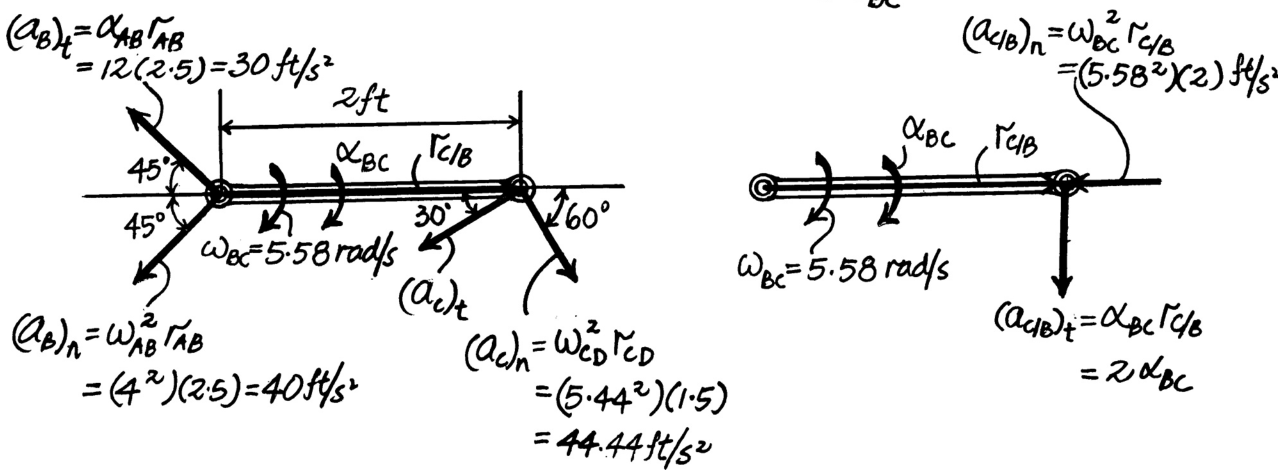
$(\overset{+}{\leftarrow})$ $\quad -44.44 \cos 60° + (a_C)_t \cos 30° = 30 \cos 45° + 40 \cos 45° + 62.21$

$(+\downarrow)$ $\quad 44.44 \sin 60° + (a_C)_t \sin 30° = -30 \sin 45° + 40 \sin 45° + 2\alpha_{BC}$

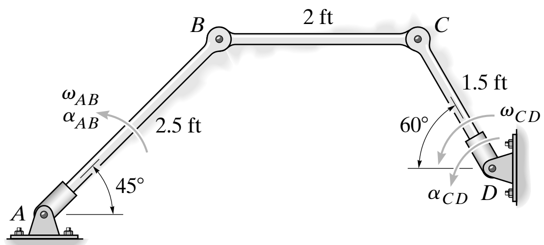
$$(a_C)_t = 155\ \text{ft/s}^2 \qquad \alpha_{BC} = 54.4\ \text{rad/s}^2$$

$$\alpha_{CD} = \frac{155}{1.5} = 103\ \text{rad/s}^2 \circlearrowleft \qquad \textbf{Ans.}$$



**R2–33.** At a given instant, link $CD$ has an angular acceleration $\alpha_{CD} = 5\ \text{rad/s}^2$ and an angular velocity $\omega_{CD} = 2\ \text{rad/s}$. Determine the angular velocity and angular acceleration of link $AB$ at this instant.




$$\frac{r_{IC-C}}{\sin 45^\circ} = \frac{2}{\sin 75^\circ} \qquad r_{IC-C} = 1.464\ \text{ft}$$

$$\frac{r_{IC-B}}{\sin 60^\circ} = \frac{2}{\sin 75^\circ} \qquad r_{IC-B} = 1.793\ \text{ft}$$

$$\omega_{BC} = \frac{3}{1.464} = 2.0490\ \text{rad/s}$$

$$v_B = 2.0490(1.793) = 3.6742\ \text{ft/s}$$

$$\omega_{AB} = \frac{3.6742}{2.5} = 1.47\ \text{rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

$$(a_B)_n = \frac{v_B^2}{r_{BA}} = \frac{(3.6742)^2}{2.5} = 5.4000\ \text{ft/s}^2$$

$$(a_C)_n = \frac{v_C^2}{r_{CD}} = \frac{(3)^2}{1.5} = 6\ \text{ft/s}^2$$

$$(a_C)_t = \alpha_{CD}(r_{CD}) = 5(1.5) = 7.5\ \text{ft/s}^2$$

$$\mathbf{a}_B = \mathbf{a}_C + \alpha \times \mathbf{r}_{B/C} - \omega^2\,\mathbf{r}_{B/C}$$

$$-5.400\cos 45^\circ\mathbf{i} - 5.400\sin 45^\circ\mathbf{j} - (a_B)_t\cos 45^\circ\mathbf{i} + (a_B)_t\sin 45^\circ\mathbf{j} = 6\sin 30^\circ\mathbf{i} - 6\cos 30^\circ\mathbf{j}$$
$$-7.5\cos 30^\circ\mathbf{i} - 7.5\sin 30^\circ\mathbf{j} + (\alpha\mathbf{k}) \times (-2\mathbf{i}) - (2.0490)^2(-2\mathbf{i})$$

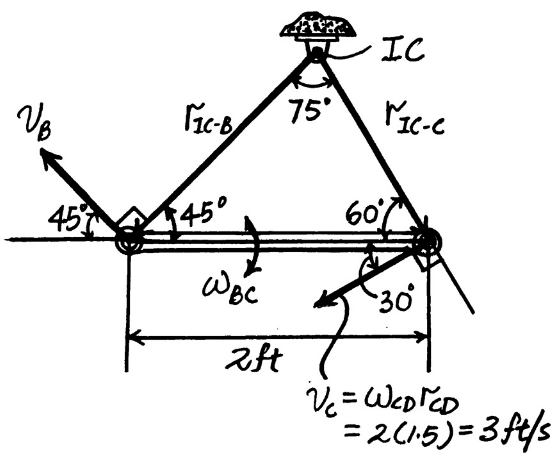
$$-3.818 - (a_B)_t(0.7071) = 3 - 6.495 + 8.3971$$

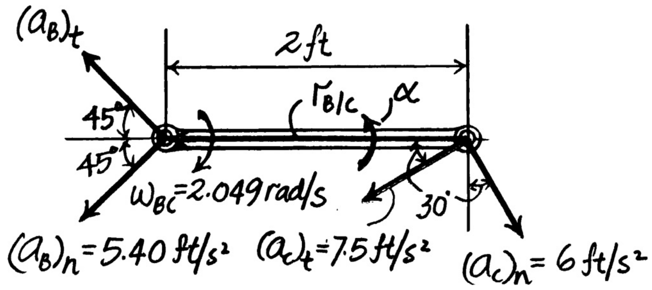
$$-3.818 + (a_B)_t(0.7071) = -5.1962 - 3.75 - 2\alpha$$

$$(a_B)_t = -12.332\ \text{ft/s}^2$$

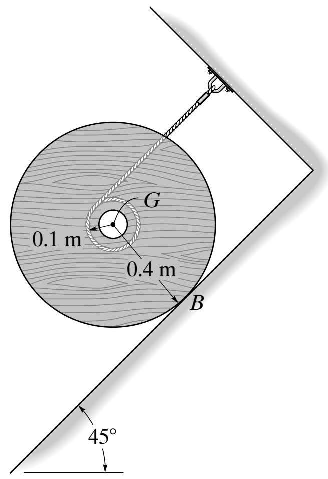
$$\alpha = 1.80\ \text{rad/s}^2$$

$$\alpha_{AB} = \frac{12.332}{2.5} = 4.93\ \text{rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$



**R2–34.** The spool and the wire wrapped around its core have a mass of 50 kg and a centroidal radius of gyration of $k_G = 235$ mm. If the coefficient of kinetic friction at the surface is $\mu_k = 0.15$, determine the angular acceleration of the spool after it is released from rest.




$$I_G = mk_G^2 = 500(0.235)^2 = 2.76125 \text{ kg}\cdot\text{m}^2$$

$+\swarrow \Sigma F_{x'} = m(a_G)_{x'}$;  $50(9.81)\sin 45° - T - 0.15N_B = 50a_G$  **(1)**

$+\nwarrow \Sigma F_{y'} = m(a_G)_{y'}$;  $N_B - 50(9.81)\cos 45° = 0$  **(2)**

$\circlearrowleft + \Sigma M_G = I_G\alpha$;  $T(0.1) - 0.15N_B(0.4) = 2.76125\alpha$  **(3)**

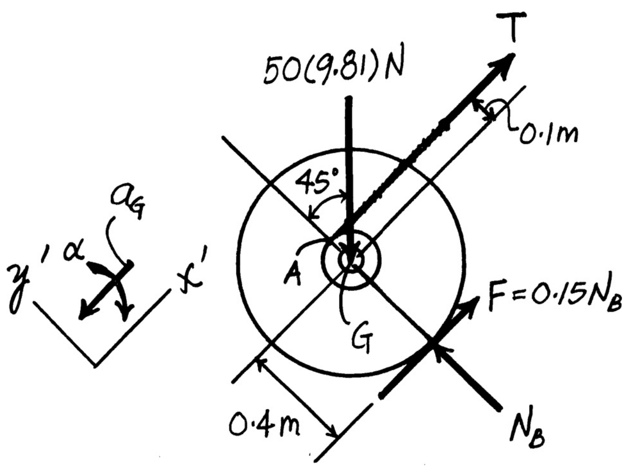
The spool does not slip at point $A$, therefore

$$a_G = 0.1\alpha \qquad \textbf{(4)}$$

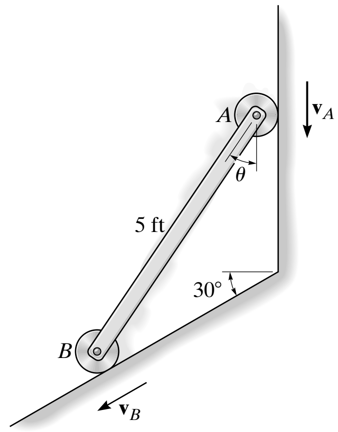
Solving Eqs. (1) to (4) yields:

$N_B = 346.8 \text{ N}$  $T = 281.5 \text{ N}$  $a_G = 0.2659 \text{ m/s}^2$

$\alpha = 2.66 \text{ rad/s}^2 \circlearrowright$  **Ans.**



**R2–35.** The bar is confined to move along the vertical and inclined planes. If the velocity of the roller at $A$ is $v_A = 6$ ft/s when $\theta = 45°$, determine the bar's angular velocity and the velocity of $B$ at this instant.




$$s_B \cos 30° = 5 \sin \theta$$

$$s_B = 5.774 \sin \theta$$

$$\dot{s}_B = 5.774 \cos \theta\dot{\theta} \qquad \textbf{(1)}$$

$$5 \cos \theta = s_A + s_B \sin 30°$$

$$-5 \sin \theta\, \ddot{\theta} = \dot{s}_A + \dot{s}_B \sin 30° \qquad \textbf{(2)}$$

Combine Eqs. (1) and (2):

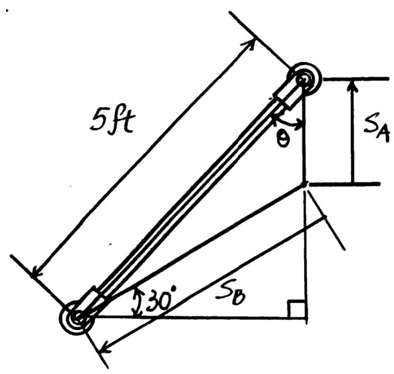
$$-5 \sin \theta\, \dot{\theta} = -6 + 5.774 \cos \theta(\dot{\theta})(\sin 30°)$$

$$-3.536\dot{\theta} = -6 + 2.041\dot{\theta}$$

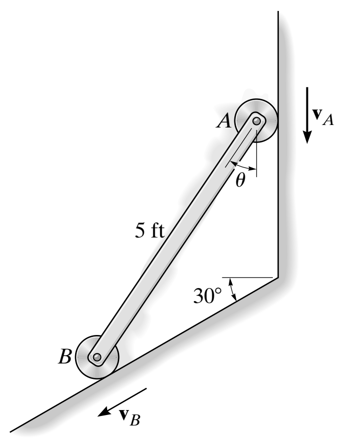
$$\omega = \theta = 1.08 \text{ rad/s} \qquad \textbf{Ans.}$$

From Eq. (1),

$$v_B = s_B = 5.774 \cos 45°(1.076) = 4.39 \text{ ft/s} \qquad \textbf{Ans.}$$




***R2–36.** The bar is confined to move along the vertical and inclined planes. If the roller at $A$ has a constant velocity of $v_A = 6$ ft/s, determine the bar's angular acceleration and the acceleration of $B$ when $\theta = 45°$.




See solution to Prob. R2–35.

Taking the time derivatives of Eqs. (1) and (2) yields:

$$a_B = \ddot{s}_B = -5.774 \sin \theta(\dot{\theta})^2 + 5.774 \cos \theta(\ddot{\theta})$$

$$-5 \cos \theta\, \dot{\theta}^2 - 5 \sin \theta(\ddot{\theta}) = \ddot{s}_A + \ddot{s}_B \sin 30°$$

Substitute the data:

$$a_B = -5.774 \sin 45°(1.076)^2 + 5.774 \cos 45°(\ddot{\theta})$$

$$-5 \cos 45°(1.076)^2 - 5 \sin 45°(\ddot{\theta}) = 0 + a_B \sin 30°$$

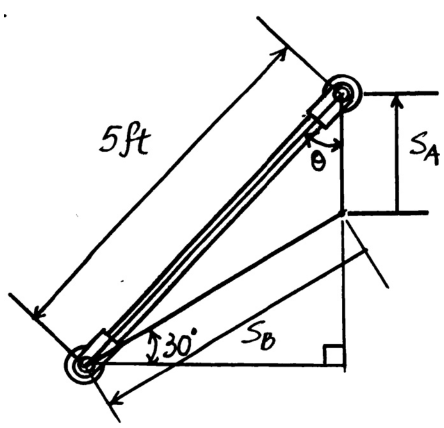
$$a_B = -4.726 + 4.083\, \ddot{\theta}$$

$$a_B = -8.185 - 7.071\, \ddot{\theta}$$

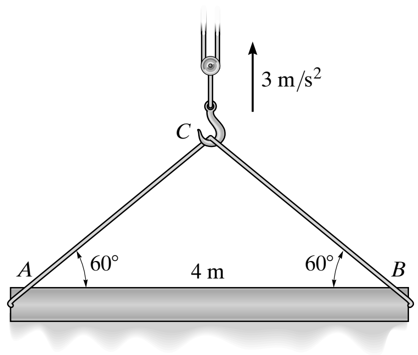
Solving:

$$\dot{\theta} = -0.310 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$a_B = -5.99 \text{ ft/s}^2 \qquad \textbf{Ans.}$$



**R2–37.** The uniform girder $AB$ has a mass of 8 Mg. Determine the internal axial force, shear, and bending moment at the center of the girder if a crane gives it an upward acceleration of 3 m/s$^2$.




Equations of Motion: By considering the entire beam [FBD(a)], we have

$+\uparrow \Sigma F_y = ma_y;\qquad 2T \sin 60° - 8000(9.81) = 8000(3)$

$$T = 59166.86 \text{ N}$$

From the FBD(b) (beam segment),

$\circlearrowleft + \Sigma M_O = \Sigma (M_k)_O;\qquad M + 4000(9.81)(1)$

$$-59166.86 \sin 60°(2) = -4000(3)(1)$$

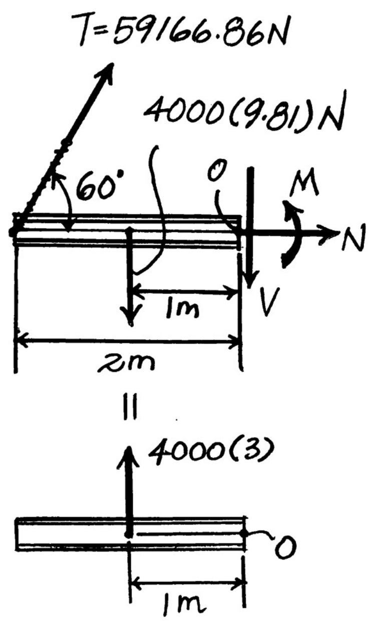
$$M = 51240 \text{ N}\cdot\text{m} = 51.2 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad 59166.86 \cos 60° + N = 0$

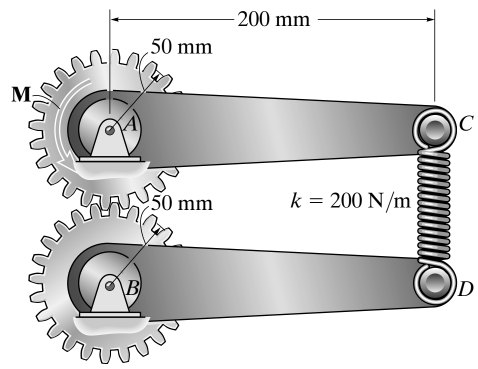
$$N = -29583.43 \text{ N} = -29.6 \text{ kN} \qquad \textbf{Ans.}$$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad 59166.86 \sin 60° - 4000(9.81) - V = 4000(3)$

$$V = 0 \qquad \textbf{Ans.}$$



**R2–38.** Each gear has a mass of 2 kg and a radius of gyration about its pinned mass centers $A$ and $B$ of $k_g = 40$ mm. Each link has a mass of 2 kg and a radius of gyration about its pinned ends $A$ and $B$ of $k_l = 50$ mm. If originally the spring is unstretched when the couple moment $M = 20\ \text{N}\cdot\text{m}$ is applied to link $AC$, determine the angular velocities of the links at the instant link $AC$ rotates $\theta = 45°$. Each gear and link is connected together and rotates in the horizontal plane about the fixed pins $A$ and $B$.



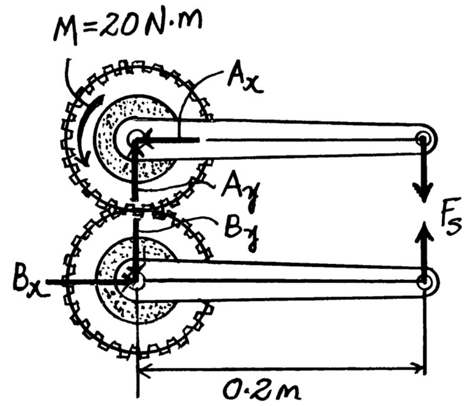

Consider the system of both gears and the links.

The spring stretches $s = 2(0.2 \sin 45°) = 0.2828$ m.

$$T_1 + \Sigma U_{1-2} = T_2$$

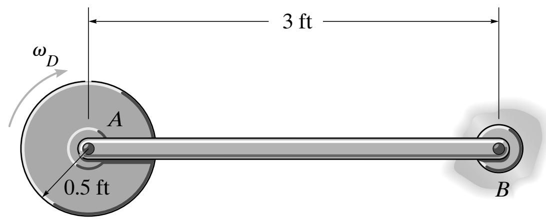
$$0 + \left\{20\left(\frac{\pi}{4}\right) - \frac{1}{2}(200)(0.2828)^2\right\} = 2\left\{\frac{1}{2}\left[(2)(0.05)^2 + (2)(0.04)^2\right]\omega^2\right\}$$

$$\omega = 30.7 \text{ rad/s} \qquad \textbf{Ans.}$$




Note that work is done by the tangential force between the gears since each move. For the system, though, this force is equal but opposite and the work cancels.

**R2–39.** The 5-lb rod $AB$ supports the 3-lb disk at its end $A$. If the disk is given an angular velocity $\omega_D = 8$ rad/s while the rod is held stationary and then released, determine the angular velocity of the rod after the disk has stopped spinning relative to the rod due to frictional resistance at the bearing $A$. Motion is in the *horizontal plane.* Neglect friction at the fixed bearing $B$.




**Conservation of Momentum:**

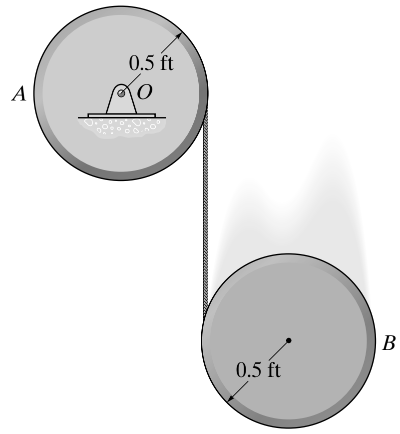
$$\circlearrowleft + \Sigma(H_B)_1 = \Sigma(H_B)_2$$

$$\left[\frac{1}{2}\left(\frac{3}{32.2}\right)(0.5)^2\right](8) + 0 = \left[\frac{1}{3}\left(\frac{5}{32.2}\right)(3)^2\right]\omega$$

$$+\left[\frac{1}{2}\left(\frac{3}{32.2}\right)(0.5)^2\right]\omega + \left(\frac{3}{32.2}\right)(3\omega)(3)$$

$$\omega = 0.0708 \text{ rad/s} \qquad \textbf{Ans.}$$

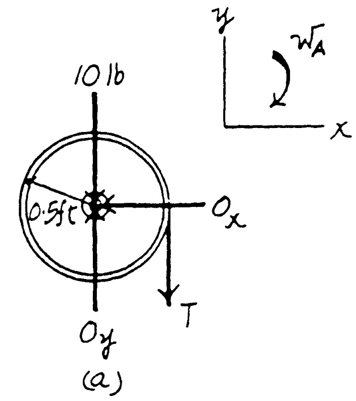
***R2–40.** A cord is wrapped around the rim of each 10-lb disk. If disk $B$ is released from rest, determine the angular velocity of disk $A$ in 2 s. Neglect the mass of the cord.




**Principle of Impulse and Momentum:** The mass moment inertia of disk $A$ about point $O$ is $I_O = \frac{1}{2}\left(\frac{10}{32.2}\right)\left(0.5^2\right) = 0.03882\ \text{slug}\cdot\text{ft}^2$. Applying Eq. 19–14 to disk $A$ [FBD(a)], we have

$$I_O\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_O\,dt = I_O\,\omega_2$$

$$(\curvearrowright +) \qquad 0 - [T(2)](0.5) = -0.03882\omega_A \qquad \textbf{(1)}$$



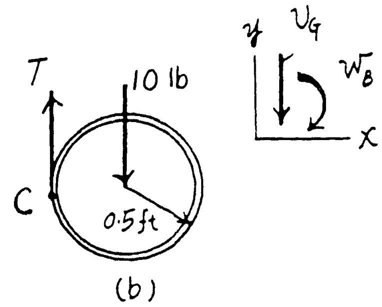

The mass moment inertia of disk $B$ about its mass center is $I_G = \frac{1}{2}\left(\frac{10}{32.2}\right)\left(0.5^2\right) = 0.03882\ \text{slug}\cdot\text{ft}^2$. Applying Eq. 19–14 to disk $B$ [FBD(b)], we have

$$m\left(v_{G_y}\right)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m\left(v_{G_y}\right)_1$$

$$(+\uparrow) \qquad 0 + T(2) - 10(2) = -\left(\frac{10}{32.2}\right)v_G \qquad \textbf{(2)}$$

$$I_G\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_G\,dt = I_G\,\omega_2$$

$$(\curvearrowright +) \qquad 0 - [T(2)](0.5) = -0.03882\omega_B \qquad \textbf{(3)}$$

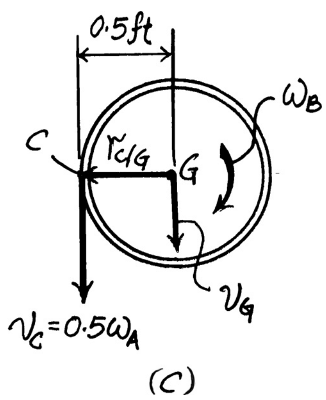



**Kinematics:** The speed of point $C$ on disk $B$ is $v_C = \omega_A r_A = 0.5\omega_A$. Here, $v_{C/G} = \omega_B r_{C/G} = 0.5\omega_B$ which is directed vertically upward. Applying Eq. 16–15, we have

$$\mathbf{v}_C = \mathbf{v}_G + \mathbf{v}_{C/G}$$

$$\begin{bmatrix} 0.5\omega_A \\ \downarrow \end{bmatrix} = \begin{bmatrix} v_G \\ \downarrow \end{bmatrix} + \begin{bmatrix} 0.5\omega_B \\ \uparrow \end{bmatrix}$$

$$(+\uparrow) \qquad -0.5\omega_A = -v_G + 0.5\omega_B \qquad \textbf{(4)}$$

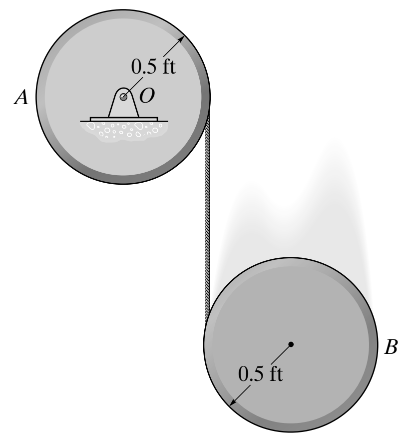



Solving Eqs. (1), (2), (3), and (4) yields:

$$\omega_A = 51.5\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\omega_B = 51.52\ \text{rad/s} \qquad v_G = 51.52\ \text{ft/s} \qquad T = 2.00\ \text{lb}$$

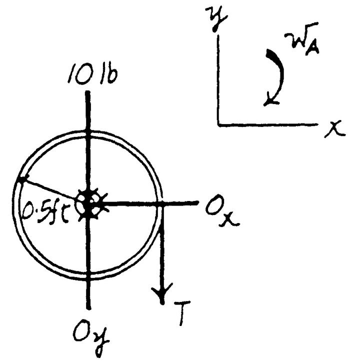
**R2–41.** A cord is wrapped around the rim of each 10-lb disk. If disk $B$ is released from rest, determine how much time $t$ is required before $A$ attains an angular velocity $\omega_A = 5$ rad/s.




**Principle of Impulse and Momentum:** The mass moment inertia of disk $A$ about point $O$ is $I_O = \frac{1}{2}\left(\frac{10}{32.2}\right)(0.5^2) = 0.03882 \text{ slug}\cdot\text{ft}^2$. Applying Eq. 19–14 to disk $A$ [FBD(a)], we have

$$I_O\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_O\,dt = I_O\,\omega_2$$

$$(+) \qquad 0 - [T(t)](0.5) = -0.03882(5) \qquad \textbf{(1)}$$



(a)

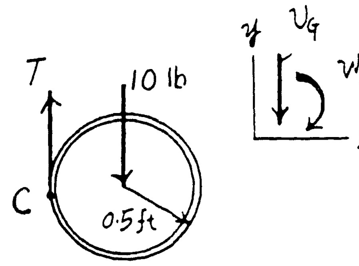
The mass moment inertia of disk $B$ about its mass center is $I_G = \frac{1}{2}\left(\frac{10}{32.2}\right)(0.5^2) = 0.03882 \text{ slug}\cdot\text{ft}^2$. Applying Eq. 19–14 to disk $B$ [FBD(b)], we have

$$m\left(v_{G_y}\right)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m\left(v_{G_y}\right)_1$$

$$(+\uparrow) \qquad 0 + T(t) - 10(t) = -\left(\frac{10}{32.2}\right)v_G \qquad \textbf{(2)}$$

$$I_G\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_G\,dt = I_G\,\omega_2$$

$$(\circlearrowleft +) \qquad 0 - [T(t)](0.5) = -0.03882\omega_B \qquad \textbf{(3)}$$



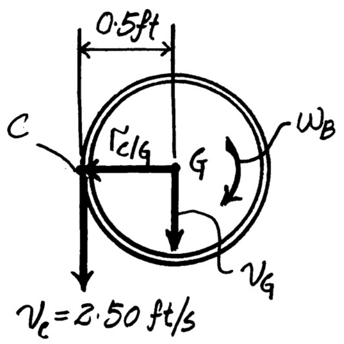
(b)

**Kinematics:** The speed of point $C$ on disk $B$ is $v_C = \omega_A\,r_A = 0.5(5) = 2.50$ ft/s. Here, $v_{C/G} = \omega_B\,r_{C/G} = 0.5\,\omega_B$ which is directed vertically upward. Applying Eq. 16–15, we have

$$\mathbf{v}_C = \mathbf{v}_G + \mathbf{v}_{C/G}$$

$$\begin{bmatrix} 2.50 \\ \downarrow \end{bmatrix} = \begin{bmatrix} v_G \\ \downarrow \end{bmatrix} + \begin{bmatrix} 0.5\,\omega_B \\ \uparrow \end{bmatrix}$$

$$(+\uparrow) \qquad -2.50 = -v_G + 0.5\,\omega_B \qquad \textbf{[4]}$$



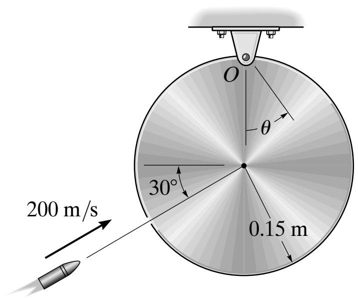
(C)

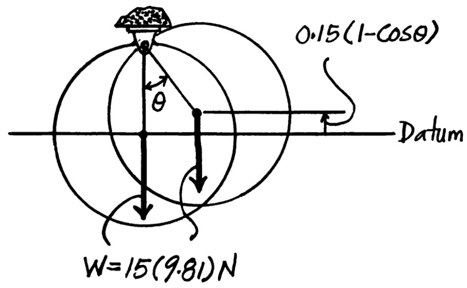
Solving Eqs. (1), (2), (3), and (4) yields:

$$t = 0.194 \text{ s} \qquad \textbf{Ans.}$$

$$\omega_B = 5.00 \text{ rad/s} \qquad v_G = 5.00 \text{ ft/s} \qquad T = 2.00 \text{ lb}$$

**R2–42.** The 15-kg disk is pinned at $O$ and is initially at rest. If a 10-g bullet is fired into the disk with a velocity of 200 m/s, as shown, determine the maximum angle $\theta$ to which the disk swings. The bullet becomes embedded in the disk.




$$\circlearrowleft+(H_O)_1 = (H_O)_2$$

$$0.01(200 \cos 30°)(0.15) = \left[\left[\frac{1}{2}(15)(0.15)^2 + 15(0.15)^2\right]\omega\right.$$

$$\omega = 0.5132 \text{ rad/s}$$

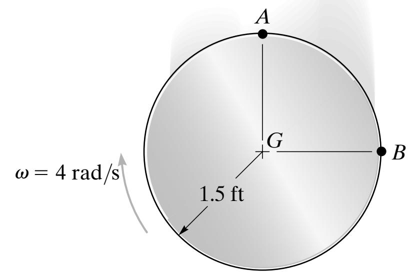
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{2}(15)(0.15)^2 + 15(0.15)^2\right](0.5132)^2 + 0 = 0 + 15(9.81)(0.15)(1 - \cos\theta)$$

$$\theta = 4.45° \qquad \textbf{Ans.}$$

Note that the calculation neglects the small mass of the bullet after it becomes embedded in the plate, since its position in the plate is not specified.

**R2–43.** The disk rotates at a constant rate of 4 rad/s as it falls freely so that its center $G$ has an acceleration of 32.2 ft/s$^2$. Determine the accelerations of points $A$ and $B$ on the rim of the disk at the instant shown.




$$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$$

$$\begin{bmatrix} (a_A)_x \\ \rightarrow \end{bmatrix} + \begin{bmatrix} (a_A)_y \\ \uparrow \end{bmatrix} = \begin{bmatrix} 32.2 \\ \downarrow \end{bmatrix} + 0 + \begin{bmatrix} (4)^2(1.5) \\ \downarrow \end{bmatrix}$$

$(\overset{+}{\rightarrow})$ $\quad (a_A)_x = 0$

$(+\uparrow)$ $\quad (a_A)_y = -32.2 - (4)^2\,(1.5) = -56.2 \text{ ft/s}^2 = 56.2 \text{ ft/s}^2 \downarrow$

$a_A = (a_A)_y = 56.2 \text{ ft/s}^2 \downarrow$ **Ans.**

$$\mathbf{a}_B = \mathbf{a}_G + (\mathbf{a}_{B/G})_t + (\mathbf{a}_{B/G})_n$$

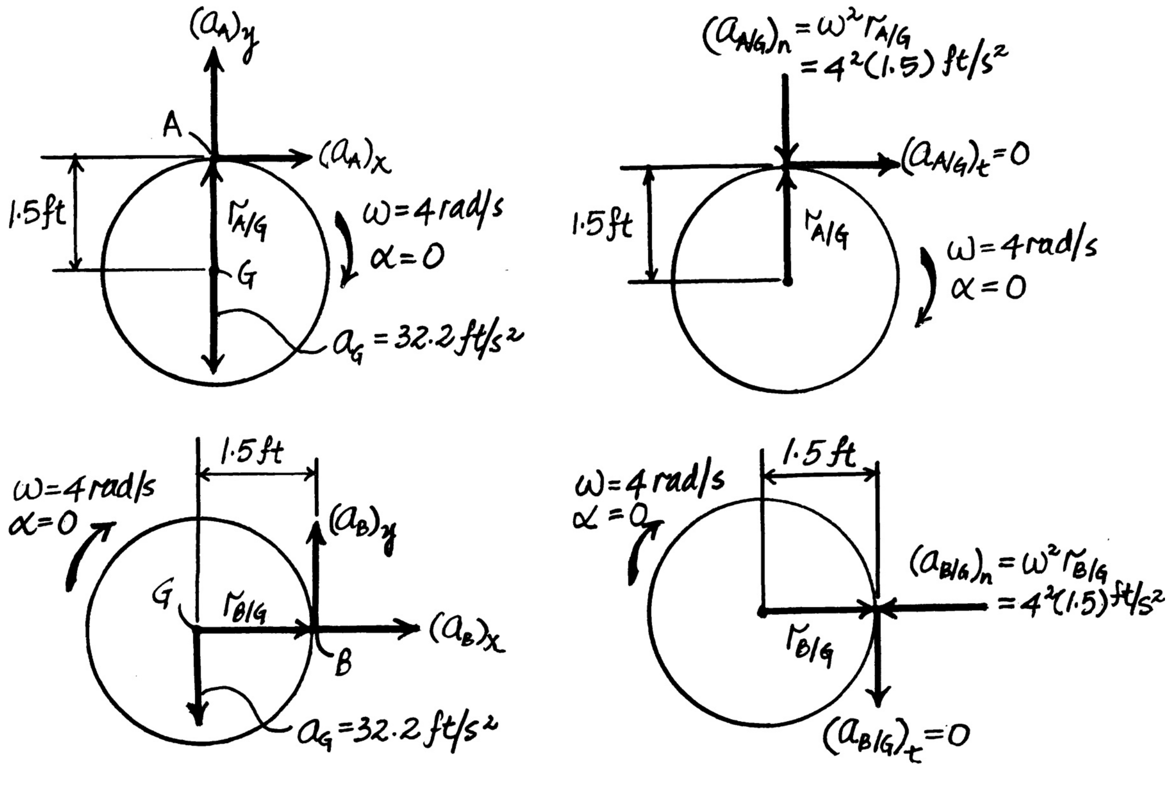
$$\begin{bmatrix} (a_B)_x \\ \rightarrow \end{bmatrix} + \begin{bmatrix} (a_B)_y \\ \uparrow \end{bmatrix} = \begin{bmatrix} 32.2 \\ \downarrow \end{bmatrix} + 0 + \begin{bmatrix} (4)^2\,(1.5) \\ \leftarrow \end{bmatrix}$$

$(\overset{+}{\rightarrow})$ $\quad (a_B)_x = -(4)^2(1.5) = -24 \text{ ft/s}^2 = 24 \text{ ft/s}^2 \leftarrow$

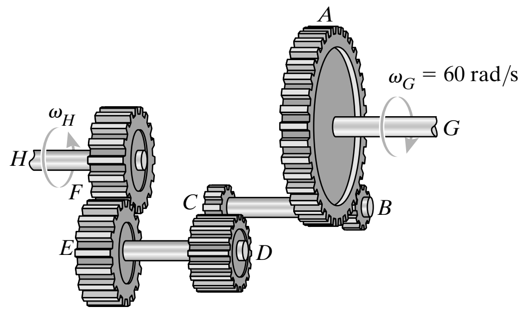
$(+\uparrow)$ $\quad (a_B)_y = -32.2 \text{ ft/s}^2 = 32.2 \text{ ft/s}^2 \downarrow$

$a_B = \sqrt{(a_B)_x^2 + (a_B)_y^2} = \sqrt{24^2 + 32.2^2} = 40.2 \text{ ft/s}^2$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{(a_B)_y}{(a_B)_x}\right) = \tan^{-1}\left(\dfrac{32.2}{24}\right) = 53.3° \measuredangle$ **Ans.**



***R2–44.** The operation of "reverse" for a three-speed automotive transmission is illustrated schematically in the figure. If the shaft $G$ is turning with an angular velocity of $\omega_G = 60$ rad/s, determine the angular velocity of the drive shaft $H$. Each of the gears rotates about a fixed axis. Note that gears $A$ and $B$, $C$ and $D$, $E$ and $F$ are in mesh. The radius of each of these gears is reported in the figure.



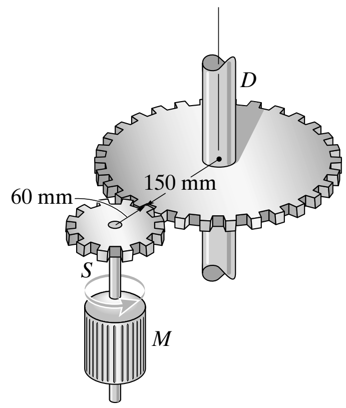

$r_A$ = 90 mm
$r_B = r_C$ = 30 mm
$r_D$ = 50 mm
$r_E$ = 70 mm
$r_F$ = 60 mm

$$\omega_C = \omega_B = \frac{r_A}{r_B}\,\omega_G = \frac{90}{30}(60) = 180 \text{ rad/s}$$

$$\omega_E = \omega_D = \frac{r_C}{r_D}\,\omega_C = \frac{30}{50}(180) = 108 \text{ rad/s}$$

$$\omega_H = \frac{r_E}{r_F}\,\omega_E = \frac{70}{60}(108) = 126 \text{ rad/s}$$ **Ans.**

**R2–45.** Shown is the internal gearing of a "spinner" used for drilling wells. With constant angular acceleration, the motor $M$ rotates the shaft $S$ to 100 rev/min in $t = 2$ s starting from rest. Determine the angular acceleration of the drill-pipe connection $D$ and the number of revolutions it makes during the 2-s startup.




For shaft $S$,

$$\omega = \omega_0 + \alpha_c t$$

$$\frac{100(2\pi)}{60} = 0 + \alpha_S(2) \qquad \alpha_S = 5.236 \text{ rad/s}^2$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$$

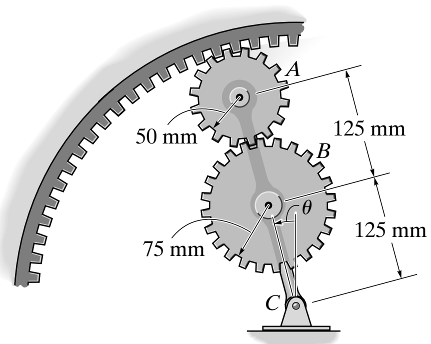
$$\theta_S = 0 + 0 + \frac{1}{2}(5.236)(2)^2 = 10.472 \text{ rad}$$

For connection $D$,

$$\alpha_D = \frac{r_S}{r_D}\,\alpha_S = \frac{60}{150}(5.236) = 2.09 \text{ rad/s}^2$$ **Ans.**

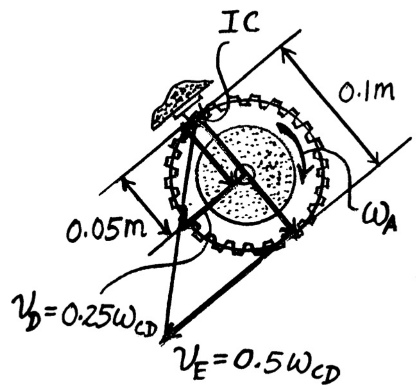
$$\theta_D = \frac{r_S}{r_D}\,\theta_S = \frac{60}{150}(10.472) = 4.19 \text{ rad} = 0.667 \text{ rev}$$ **Ans.**

**R2–46.** Gear $A$ has a mass of 0.5 kg and a radius of gyration of $k_A = 40$ mm, and gear $B$ has a mass of 0.8 kg and a radius of gyration of $k_B = 55$ mm. The link is pinned at $C$ and has a mass of 0.35 kg. If the link can be treated as a slender rod, determine the angular velocity of the link after the assembly is released from rest when $\theta = 0°$ and falls to $\theta = 90°$.




**Kinematics:** The velocity of the mass center of gear $A$ is $v_D = 0.25\,\omega_{CD}$, and since is rolls without slipping on the fixed circular gear track, the location of the instantaneous center of zero velocity is as shown. Thus,

$$\omega_A = \frac{v_D}{r_{D/IC}} = \frac{0.25\,\omega_{CD}}{0.05} = 5\omega_{CD} \qquad v_E = \omega_A\, r_{E/IC} = 5\omega_{CD}(0.1) = 0.5\,\omega_{CD}$$




The velocity of the mass center of gear $B$ is $v_F = 0.125\omega_{CD}$. The location of the instantaneous center of zero velocity is as shown. Thus,

$$\omega_B = \frac{v_E}{r_{E/(IC)1}} = \frac{0.5\,\omega_{CD}}{0.1} = 5\omega_{CD}$$

**Potential Energy:** Datum is set at point $C$. When gears $A$, $B$ and link $AC$ are at their initial position ($\theta = 0°$), their centers of gravity are located 0.25 m, 0.125 m, and 0.125 m *above* the datum, respectively. The total gravitational potential energy when they are at these positions is $0.5(9.81)(0.25) + 0.8(9.81)(0.125) + 0.35(9.81)(0.125) = 2.636\ \text{N}\cdot\text{m}$. Thus, the initial and final potential energy is

$$V_1 = 2.636\ \text{N}\cdot\text{m} \qquad V_2 = 0$$

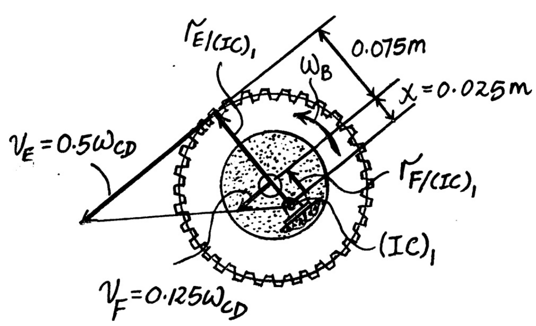
**Kinetic Energy:** The mass moment of inertia of gears $A$ and $B$ about their mass center is $I_D = 0.5\left(0.04^2\right) = 0.8\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$ and $I_F = 0.8\left(0.055^2\right) = 2.42\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$. The mass moment of inertia of link $CD$ about point $C$ is $(I_{CD})_C = \frac{1}{12}(0.35)\left(0.25^2\right) + 0.35\left(0.125^2\right) = 7.292\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2$. Since the system is at rest initially, the initial kinetic energy is $T_1 = 0$. The final kinetic energy is given by

$$T_2 = \frac{1}{2}m_A v_D^2 + \frac{1}{2}I_D\,\omega_A^2 + \frac{1}{2}m_B v_F^2 + \frac{1}{2}I_F\omega_B^2 + \frac{1}{2}(I_{CD})_C\,\omega_{CD}^2$$

$$= \frac{1}{2}(0.5)(0.25\,\omega_{CD})^2 + \frac{1}{2}\left[0.8\left(10^{-3}\right)\right](5\omega_{CD})^2 + \frac{1}{2}(0.8)(0.125\,\omega_{CD})^2$$

$$+ \frac{1}{2}\left[2.42\left(10^{-3}\right)\right](5\,\omega_{CD})^2 + \frac{1}{2}\left[7.292\left(10^{-3}\right)\right]\left(\omega_{CD}^2\right)$$

$$= 0.06577\,\omega_{CD}^2$$



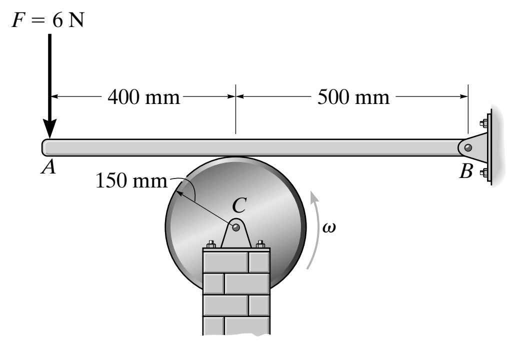

**Conservation of Energy:** Applying Eq. 18–19, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2.636 = 0.06577\,\omega_{CD}^2$$

$$\omega_{CD} = 6.33\ \text{rad/s}$$ **Ans.**

**R2–47.** The 15-kg cylinder rotates with an angular velocity of $\omega = 40$ rad/s. If a force $F = 6$ N is applied to bar $AB$, as shown, determine the time needed to stop the rotation. The coefficient of kinetic friction between $AB$ and the cylinder is $\mu_k = 0.4$. Neglect the thickness of the bar.




For link $AB$,

$\circlearrowleft + \Sigma M_B = 0; \quad 6(0.9) - N_E(0.5) = 0 \qquad N_E = 10.8\text{ N}$

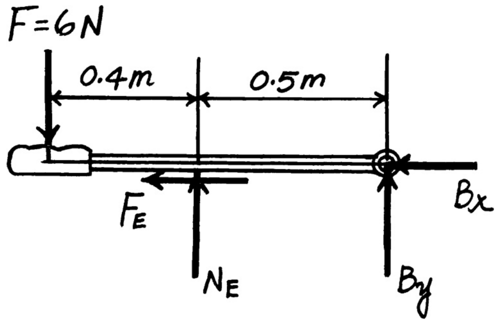
$$I_C = \frac{1}{2}mr^2 = \frac{1}{2}(15)(0.15)^2 = 0.16875\text{ kg}\cdot\text{m}^2$$

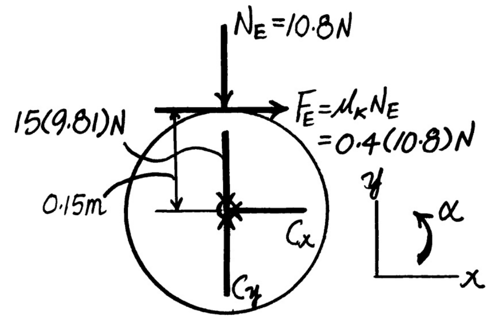
$\circlearrowleft + \Sigma M_C = I_C\,\alpha; \quad -0.4(10.8)(0.15) = 0.16875(\alpha) \qquad \alpha = -3.84\text{ rad/s}^2$

$\circlearrowleft + \omega = \omega_0 + \alpha t$

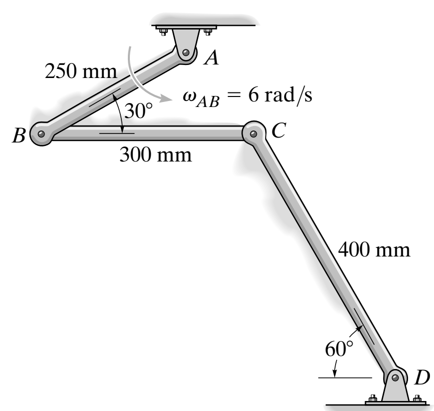
$0 = 40 + (-3.84)\,t$

$t = 10.4$ s **Ans.**




***R2–48.** If link $AB$ rotates at $\omega_{AB} = 6$ rad/s, determine the angular velocities of links $BC$ and $CD$ at the instant shown.




Link $AB$ rotates about the fixed point $A$. Hence,

$$v_B = \omega_{AB}\,r_{AB} = 6(0.25) = 1.5\text{ m/s}$$

For link $BC$,

$r_{B/IC} = 0.3\cos 30° = 0.2598$ m $\qquad r_{C/IC} = 0.3\cos 60° = 0.15$ m

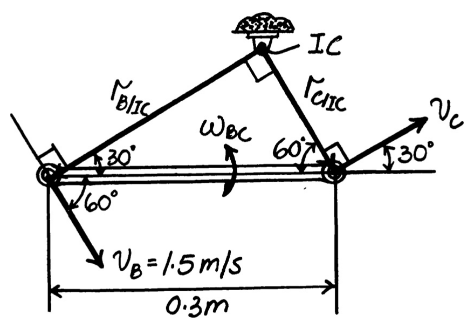
$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{1.5}{0.2598} = 5.77\text{ rad/s}$$ **Ans.**

$$v_C = \omega_{BC}\,r_{C/IC} = 5.77(0.15) = 0.8660\text{ m/s}$$

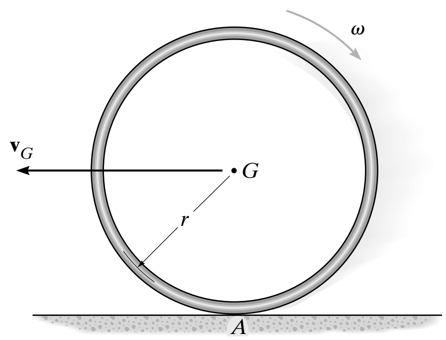
Link $CD$ rotates about the fixed point $D$. Hence,

$$v_C = \omega_{CD}\,r_{CD}$$

$0.8660 = \omega_{CD}\,(0.4) \qquad \omega_{CD} = 2.17$ rad/s **Ans.**



**R2–49.** If the thin hoop has a weight $W$ and radius $r$ and is thrown onto a *rough surface* with a velocity $\mathbf{v}_G$ parallel to the surface, determine the backspin, $\boldsymbol{\omega}$, it must be given so that it stops spinning at the same instant that its forward velocity is zero. It is not necessary to know the coefficient of kinetic friction at $A$ for the calculation.

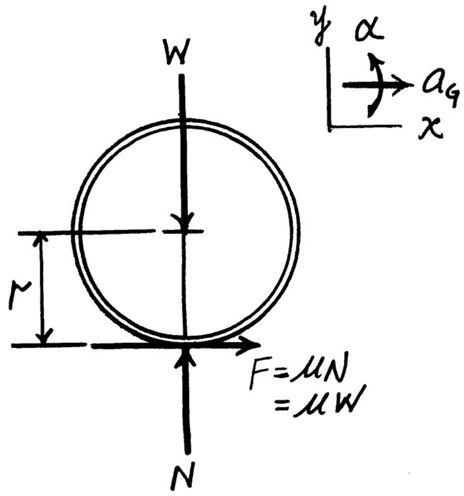



**Equations of Motion:** The mass moment of inertia of the hoop about its mass center is given by $I_G = mr^2 = \frac{W}{g} r^2$. Applying Eq. 17–16, we have

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - W = 0 \qquad N = W$$

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad \mu W = \frac{W}{g} a_G \qquad a_G = \mu g$$

$$\circlearrowleft + \Sigma M_G = I_G \alpha; \qquad \mu W r = \frac{W}{g} r^2 \alpha \qquad \alpha = \frac{\mu g}{r}$$




**Kinematics:** The time required for the hoop to stop back spinning can be determined by applying Eq. 16–5.

$$\omega = \omega_0 + \alpha t_1$$

$$(\circlearrowleft +) \qquad 0 = \omega + \left(-\frac{\mu g}{r}\right) t_1$$

$$t_1 = \frac{\omega r}{\mu g}$$

The time required for the hoop to stop can be determined by applying Eq. 12–4.

$$(\xleftarrow{+}) \qquad v = v_0 + a t_2$$

$$0 = v_G + (-\mu g)\, t_2$$

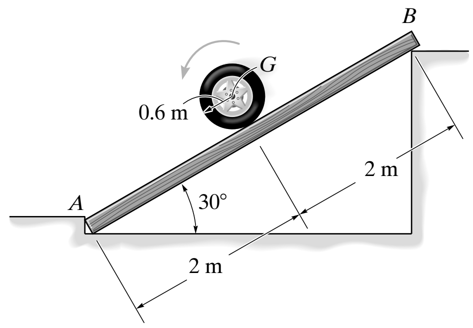
$$t_2 = \frac{v_G}{\mu g}$$

It is required that $t_1 = t_2$. Thus,

$$\frac{\omega r}{\mu g} = \frac{v_G}{\mu g}$$

$$\omega = \frac{v_G}{r} \qquad \textbf{Ans.}$$

**R2–50.** The wheel has a mass of 50 kg and a radius of gyration $k_G = 0.4$ m. If it rolls without slipping down the inclined plank, determine the horizontal and vertical components of reaction at $A$, and the normal reaction at the smooth support $B$ at the instant the wheel is located at the midpoint of the plank. The plank has negligible thickness and has a mass of 20 kg.

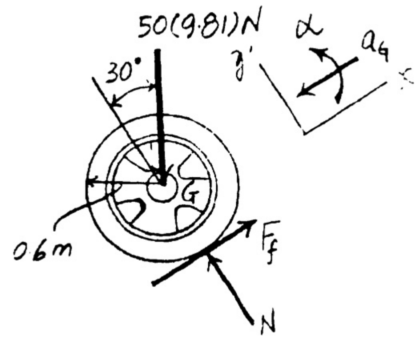



**Equations of Motion:** Since the tire rolls down the plane without slipping, then $a_G = \alpha r = 0.6\alpha$. The mass moment of inertia of the tire about its mass center is given by $I_G = mk_G^2 = 50(0.4^2) = 8.00\ \text{kg}\cdot\text{m}^2$. Applying Eq. 17–16 to [FBD(a)], we have

$\nwarrow + \Sigma F_{y'} = m(a_G)_{y'}$; $\quad N - 50(9.81)\cos 30° = 0 \qquad N = 424.79\ \text{N}$

$\swarrow + \Sigma F_{x'} = m(a_G)_{x'}$; $\quad 50(9.81)\sin 30° - Ff = 50(0.6\alpha)$ **(1)**

$\circlearrowleft + \Sigma M_G = I_G\alpha$; $\quad F_f(0.6) = 8.00\alpha$ **(2)**




Solving Eqs. [1] and [2] yields

$$F_f = 75.46\ \text{N} \qquad \alpha = 5.660\ \text{rad/s}^2$$

**Equations of Equilibrium:** From FBD(b).

$\circlearrowleft + \Sigma M_A = 0$; $\quad N_B(4) - 20(9.81)\cos 30°(2) - 424.79(2) = 0$

$$N_B = 297.35\ \text{N} = 297\ \text{N} \qquad \textbf{Ans.}$$

$+\uparrow \Sigma F_y = 0$; $\quad A_y + 297.35\cos 30° - 20(9.81)$

$$- 424.79\cos 30° - 75.46\sin 30° = 0$$

$$A_y = 344\ \text{N} \qquad \textbf{Ans.}$$

$\xrightarrow{+} \Sigma F_x = 0$; $\quad A_x + 424.79\sin 30° - 75.46\cos 30° - 297.35\sin 30° = 0$

$$A_x = 1.63\ \text{N} \qquad \textbf{Ans.}$$