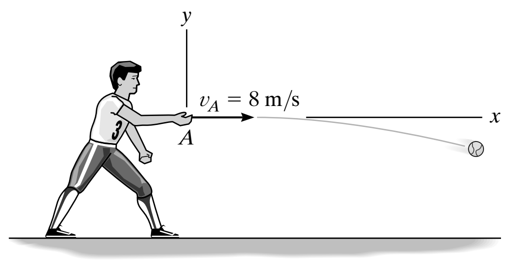
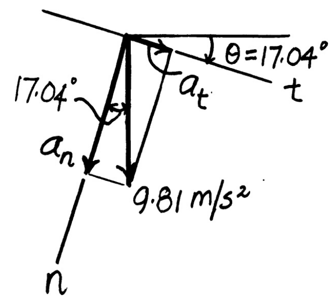
R1–1. The ball is thrown horizontally with a speed of 8 m/s. Find the equation of the path, $y = f(x)$, and then determine the ball's velocity and the normal and tangential components of acceleration when $t = 0.25$ s.



Horizontal Motion: The horizontal component of velocity is $v_x = 8$ m/s and the initial horizontal position is $(s_0)_x = 0$.



$(\overset{+}{\rightarrow})$ $$s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + 8t \qquad \textbf{[1]}$$

Vertical Motion: The vertical component of initial velocity $(v_0)_y = 0$ and the initial vertical position are $(s_0)_y = 0$.

$(+\uparrow)$ $$s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + 0 + \frac{1}{2}(-9.81)t^2 \qquad \textbf{[2]}$$

Eliminate t from Eqs. [1] and [2] yields

$$y = -0.0766x^2 \qquad \textbf{Ans.}$$

The vertical component of velocity when $t = 0.25$ s is given by

$(+\uparrow)$ $$v_y = (v_0)_y + (a_c)_y t$$

$$v_y = 0 + (-9.81)(0.25) = -2.4525 \text{ m/s} = 2.4525 \text{ m/s} \downarrow$$

The magnitude and direction angle when $t = 0.25$ s are

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{8^2 + 2.4525^2} = 8.37 \text{ m/s} \qquad \textbf{Ans.}$$

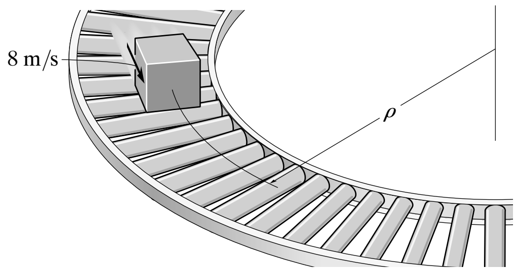
$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{2.4525}{8} = 17.04° = 17.0° \searrow \qquad \textbf{Ans.}$$

Since the velocity is always directed along the tangent of the path and the acceleration $a = 9.81$ m/s^2 is directed downward, then tangential and normal components of acceleration are

$$a_t = 9.81 \sin 17.04° = 2.88 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_n = 9.81 \cos 17.04° = 9.38 \text{ m/s}^2 \qquad \textbf{Ans.}$$

R1–2. Cartons having a mass of 5 kg are required to move along the assembly line with a constant speed of 8 m/s. Determine the smallest radius of curvature, ρ, for the conveyor so the cartons do not slip. The coefficients of static and kinetic friction between a carton and the conveyor are $\mu_s = 0.7$ and $\mu_k = 0.5$, respectively.



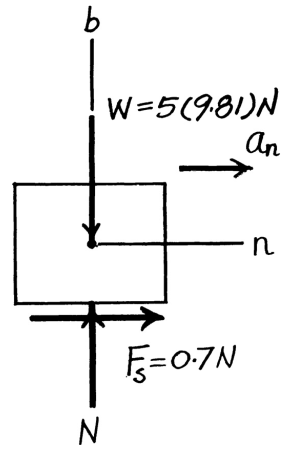
$+\uparrow \Sigma F_b = m\,a_b;\qquad N - W = 0$

$N = W$

$F_s = 0.7W$

$\xrightarrow{+} \Sigma F_n = m\,a_n;\qquad 0.7W = \dfrac{W}{9.81}\left(\dfrac{8^2}{\rho}\right)$

$\rho = 9.32\text{ m}$ **Ans.**



R1–3. A small metal particle travels downward through a fluid medium while being subjected to the attraction of a magnetic field such that its position is $s = (15t^3 - 3t)$ mm, where t is in seconds. Determine (a) the particle's displacement from $t = 2$ s to $t = 4$ s, and (b) the velocity and acceleration of the particle when $t = 5$ s.

a) $s = 15t^3 - 3t$

At $t = 2$ s, $s_1 = 114$ mm

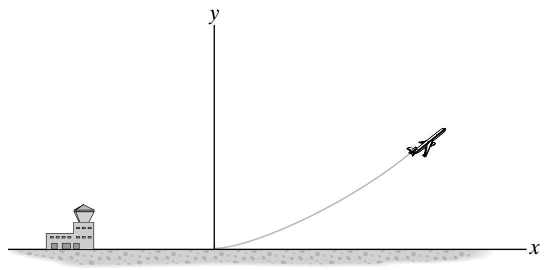
At $t = 4$ s, $s_3 = 948$ mm

$\Delta s = 948 - 114 = 834$ mm **Ans.**

b) $v = \dfrac{ds}{dt} = 45t^2 - 3\Big|_{t=5} = 1122\text{ mm/s} = 1.12\text{ m/s}$ **Ans.**

$a = \dfrac{dv}{dt} = 90t\Big|_{t=5} = 450\text{ mm/s}^2 = 0.450\text{ m/s}^2$ **Ans.**

***R1–4.** The flight path of a jet aircraft as it takes off is defined by the parametric equations $x = 1.25t^2$ and $y = 0.03t^3$, where t is the time after take-off, measured in seconds, and x and y are given in meters. If the plane starts to level off at $t = 40$ s, determine at this instant (a) the horizontal distance it is from the airport, (b) its altitude, (c) its speed, and (d) the magnitude of its acceleration.



Position: When $t = 40\ s$, its horizontal position is given by

$$x = 1.25\left(40^2\right) = 2000\text{ m} = 2.00\text{ km} \qquad \textbf{Ans.}$$

and its altitude is given by

$$y = 0.03\left(40^3\right) = 1920\text{ m} = 1.92\text{ km} \qquad \textbf{Ans.}$$

Velocity: When $t = 40$ s, the horizontal component of velocity is given by

$$v_x = \dot{x} = 2.50t\big|_{t=40\text{ s}} = 100\text{ m/s}$$

The vertical component of velocity is

$$v_y = \dot{y} = 0.09t^2\big|_{t=40\text{ s}} = 144\text{ m/s}$$

Thus, the plane's speed at $t = 40$ s is

$$v_y = \sqrt{v_x^2 + v_y^2} = \sqrt{100^2 + 144^2} = 175\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The horizontal component of acceleration is

$$a_x = \ddot{x} = 2.50\text{ m/s}^2$$

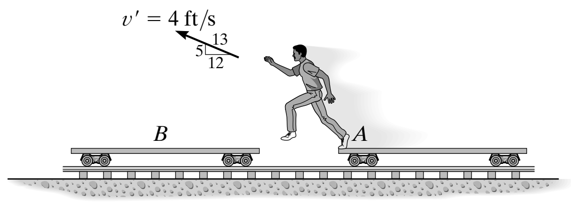
and the vertical component of acceleration is

$$a_y = \ddot{y} = 0.18t\big|_{t=40\text{ s}} = 7.20\text{ m/s}^2$$

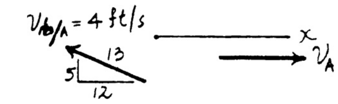
Thus, the magnitude of the plane's acceleration at $t = 40$ s is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{2.50^2 + 7.20^2} = 7.62\text{ m/s}^2 \qquad \textbf{Ans.}$$

R1–5. The boy jumps off the flat car at A with a velocity of $v' = 4$ ft/s relative to the car as shown. If he lands on the second flat car B, determine the final speed of both cars after the motion. Each car has a weight of 80 lb. The boy's weight is 60 lb. Both cars are originally at rest. Neglect the mass of the car's wheels.

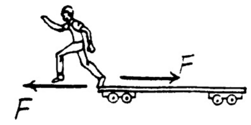


Relative Velocity: The horizontal component of the relative velocity of the boy with respect to the car A is $(v_{b/A})_x = 4\left(\frac{12}{13}\right) = 3.692$ ft/s. Thus, the horizontal component of the velocity of the boy is

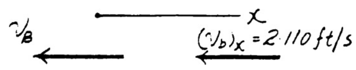


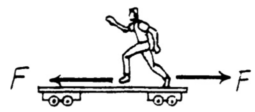
$$(v_b)_x = v_A + (v_{b/A})_x$$

$$(\overset{+}{\leftarrow}) \qquad (v_b)_x = -v_A + 3.692 \qquad \textbf{[1]}$$



Conservation of Linear Momentum: If we consider the boy and the car as a system, then the impulsive force caused by traction of the shoes is *internal* to the system. Therefore, they will cancel out. As the result, the linear momentum is conserved along x axis. For car A





$$0 = m_b(v_b)_x + m_A v_A$$

$$(\overset{+}{\leftarrow}) \qquad 0 = \left(\frac{60}{32.2}\right)(v_b)_x + \left(\frac{80}{32.2}\right)(-v_A) \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$v_A = 1.58 \text{ ft/s} \qquad \textbf{Ans.}$$

$$(v_b)_x = 2.110 \text{ ft/s}$$

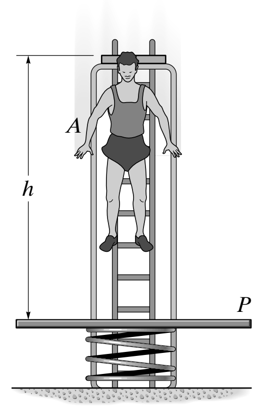
For car B

$$m_b\,(v_b)_x = (m_b + m_B)v_B$$

$$(\overset{+}{\leftarrow}) \qquad \left(\frac{60}{32.2}\right)(2.110) = \left(\frac{60+80}{32.2}\right)v_B$$

$$v_B = 0.904 \text{ ft/s} \qquad \textbf{Ans.}$$

R1-6. The man A has a weight of 175 lb and jumps from rest at a height $h = 8$ ft onto a platform P that has a weight of 60 lb. The platform is mounted on a spring, which has a stiffness $k = 200$ lb/ft. Determine (a) the velocities of A and P just after impact and (b) the maximum compression imparted to the spring by the impact. Assume the coefficient of restitution between the man and the platform is $e = 0.6$, and the man holds himself rigid during the motion.



Conservation of Energy: The datum is set at the initial position of platform P. When the man falls from a height of 8 ft *above* the datum, his initial gravitational potential energy is $175(8) = 1400$ ft · lb. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 1400 = \frac{1}{2}\left(\frac{175}{32.2}\right)(v_M)_1^2 + 0$$

$$(v_H)_1 = 22.70 \text{ ft/s}$$

Conservation of Momentum:

$$m_M\,(v_M)_1 + m_P(v_P)_1 = m_M(v_M)_2 + m_p(v_p)_2$$

$$(+\downarrow) \qquad \left(\frac{175}{32.2}\right)(22.70) + 0 = \left(\frac{175}{32.2}\right)(v_M)_2 + \left(\frac{60}{32.2}\right)(v_p)_2 \qquad \textbf{[1]}$$

Coefficient of Restitution:

$$e = \frac{(v_P)_2 - (v_M)_2}{(v_M)_1 - (v_p)_1}$$

$$(+\downarrow) \qquad 0.6 = \frac{(v_P)_2 - (v_P)_2}{22.70 - 0} \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$(v_p)_2 = 27.04 \text{ ft/s} \downarrow = 27.0 \text{ ft/s} \downarrow \qquad (v_M)_2 = 13.4 \text{ ft/s} \downarrow \qquad \textbf{Ans.}$$

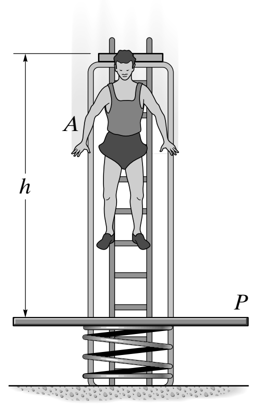
Conservation of Energy: The datum is set at the spring's compressed position. Initially, the spring has been compressed $\frac{60}{200} = 0.3$ ft and the elastic potential energy is $\frac{1}{2}(200)\left(0.3^2\right) = 9.00$ ft · lb. When platform P is at a height of s *above* the datum, its initial gravitational potential energy is $60s$. When platform P stops momentary, the spring has been compressed to its maximum and the elastic potential energy at this instant is $\frac{1}{2}(200)(s + 0.3)^2 = 100s^2 + 60s + 9$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{60}{32.2}\right)\left(27.04^2\right) + 60s + 9.00 = 100s^2 + 60s + 9$$

$$s = 2.61 \text{ ft} \qquad \textbf{Ans.}$$

R1–7. The man A has a weight of 100 lb and jumps from rest onto the platform P that has a weight of 60 lb. The platform is mounted on a spring, which has a stiffness $k = 200$ lb/ft. If the coefficient of restitution between the man and the platform is $e = 0.6$, and the man holds himself rigid during the motion, determine the required height h of the jump if the maximum compression of the spring is 2 ft.



Conservation of Energy: The datum is set at the initial position of platform P. When the man falls from a height of h *above* the datum, his initial gravitational potential energy is $100h$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 100h = \frac{1}{2}\left(\frac{100}{32.2}\right)(v_M)_1^2 + 0$$

$$(v_H)_1 = \sqrt{64.4h}$$

Conservation of Momentum:

$$m_M\,(v_M)_1 + m_P\,(v_P)_1 = m_M\,(v_M)_2 + m_P\,(v_P)_2$$

$$(+\downarrow) \qquad \left(\frac{100}{32.2}\right)(\sqrt{64.4h}) + 0 = \left(\frac{100}{32.2}\right)(v_M)_2 + \left(\frac{60}{32.2}\right)(v_P)_2 \qquad \textbf{[1]}$$

Coefficient of Restitution:

$$e = \frac{(v_p)_2 - (v_M)_2}{(v_M)_1 - (v_p)_1}$$

$$(+\downarrow) \qquad 0.6 = \frac{(v_p)_2 - (v_M)_2}{\sqrt{64.4h} - 0} \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$(v_p)_2 = \sqrt{64.4h}\ \downarrow \qquad (v_M)_2 = 0.4\sqrt{64.4h}\ \downarrow$$

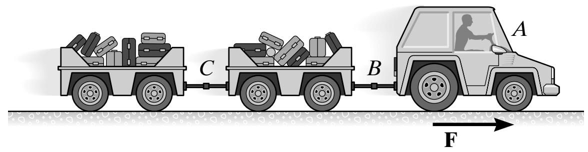
Conservation of Energy: The datum is set at the spring's compressed position. Initially, the spring has been compressed $\frac{60}{200} = 0.3$ ft and the elastic potential energy is $\frac{1}{2}(200)\left(0.3^2\right) = 9.00$ ft·lb. Here, the compression of the spring caused by impact is $(2 - 0.3)$ ft $= 1.7$ ft. When platform P is at a height of 1.7 ft *above* the datum, its initial gravitational potential energy is $60(1.7) = 102$ ft·lb. When platform P stops momentary, the spring has been compressed to its maximum and the elastic potential energy at this instant is $\frac{1}{2}(200)\left(2^2\right) = 400$ ft·lb. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{60}{32.2}\right)\left(\sqrt{64.4h}\right)^2 + 102 + 9.00 = 400$$

$$h = 4.82 \text{ ft} \qquad \textbf{Ans.}$$

***R1–8.** The baggage truck A has a mass of 800 kg and is used to pull each of the 300-kg cars. Determine the tension in the couplings at B and C if the tractive force $\mathbf{F}$ on the truck is $F = 480$ N. What is the speed of the truck when $t = 2$ s, starting from rest? The car wheels are free to roll. Neglect the mass of the wheels.



$\xrightarrow{+} \Sigma F_x = ma_x;\qquad 480 = [800 + 2(300)]a$

$a = 0.3429 \text{ m/s}^2$

$(\xrightarrow{+})\qquad v = v_0 + a_c t$

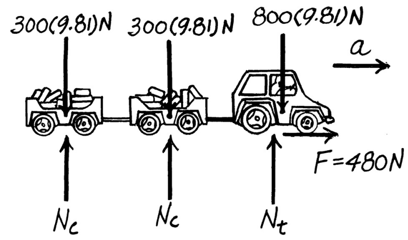
$v = 0 + 0.3429(2) = 0.686 \text{ m/s}$ **Ans.**

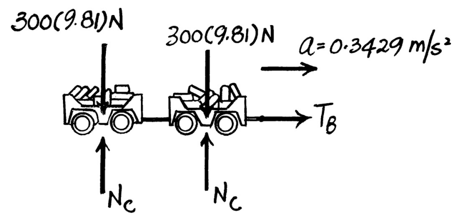
$\xrightarrow{+} \Sigma F_x = ma_x;\qquad T_B = 2(300)(0.3429)$

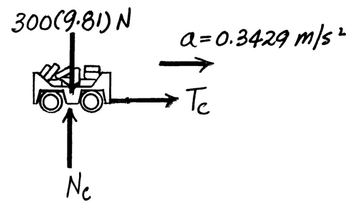
$T_B = 205.71 = 206 \text{ N}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = ma_x;\qquad T_C = (300)(0.3429)$

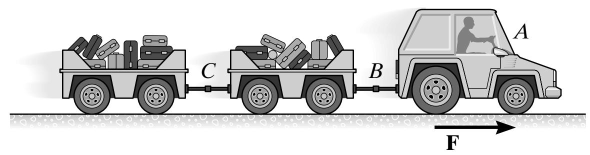
$T_C = 102.86 = 103 \text{ N}$ **Ans.**







R1–9. The baggage truck A has a mass of 800 kg and is used to pull each of the 300-kg cars. If the tractive force $\mathbf{F}$ on the truck is $F = 480$ N, determine the acceleration of the truck. What is the acceleration of the truck if the coupling at C suddenly fails? The car wheels are free to roll. Neglect the mass of the wheels.

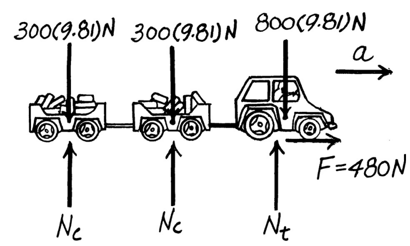


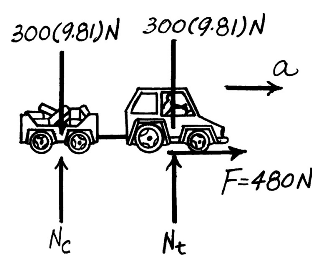
$\xrightarrow{+} \Sigma F_x = ma_x;\qquad 480 = [800 + 2(300)]a$

$a = 0.3429 = 0.343 \text{ m/s}^2$ **Ans.**

$\xrightarrow{+} \Sigma F_x = ma_x;\qquad 480 = (800 + 300)a$

$a = 0.436 \text{ m/s}^2$ **Ans.**





R1–10. A car travels at 80 ft/s when the brakes are suddenly applied, causing a constant deceleration of 10 ft/s². Determine the time required to stop the car and the distance traveled before stopping.

$(\overset{+}{\rightarrow})$ $\quad v = v_0 + a_c t$

$0 = 80 + (-10)t$

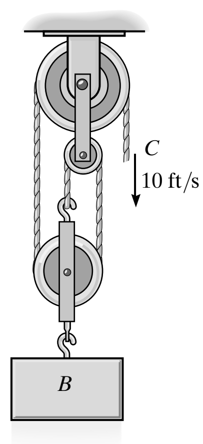
$t = 8 \text{ s}$ **Ans.**

$(\overset{+}{\rightarrow})$ $\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$0 = (80)^2 + 2(-10)(s - 0)$

$s = 320 \text{ ft}$ **Ans.**

R1–11. Determine the speed of block B if the end of the cable at C is pulled downward with a speed of 10 ft/s. What is the relative velocity of the block with respect to C?



$3s_B + s_C = l$

$3v_B = -v_C$

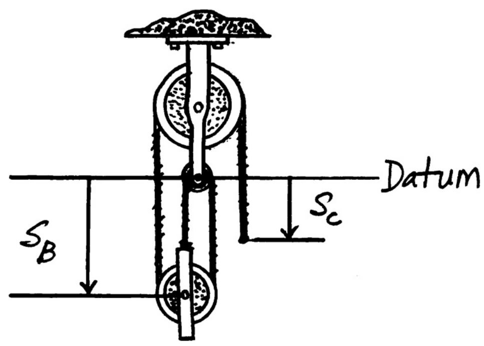
$3v_B = -(10)$

$v_B = -3.33 \text{ ft/s} = 3.33 \text{ ft/s} \uparrow$ **Ans.**

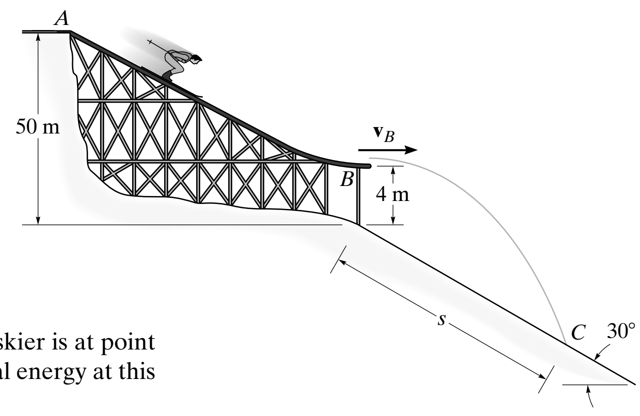
$(+\downarrow)$ $\quad v_B = v_C + v_{B/C}$

$-3.33 = 10 + v_{B/C}$

$v_{B/C} = -13.3 \text{ ft/s} = 13.3 \text{ ft/s} \uparrow$ **Ans.**



***R1–12.** The skier starts fom rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, compute the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass of 70 kg.



Potential Energy: The datum is set at the lowest point B. When the skier is at point A, he is $(50 - 4) = 46$ m *above* the datum. His gravitational potential energy at this position is $70(9.81)(46) = 31588.2$ J.

Conservation of Energy: Applying Eq. 14–21, we have

$$T_A + V_A = T_B + V_B$$

$$0 + 31588.2 = \frac{1}{2}(70)\,v_B^2$$

$$v_B = 30.04 \text{ m/s} = 30.0 \text{ m/s} \qquad \textbf{Ans.}$$

Kinematics: By considering the vertical motion of the skier, we have

$$(+\downarrow) \qquad s_y = (s_0)_y + (v_0)_y\,t + \frac{1}{2}(a_c)_y\,t^2$$

$$4 + s\sin 30^\circ = 0 + 0 + \frac{1}{2}(9.81)\,t^2 \qquad \textbf{[1]}$$

By considering the horizontal motion of the skier, we have

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + v_x\,t$$

$$s\cos 30^\circ = 0 + 30.04t \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$s = 130 \text{ m} \qquad \textbf{Ans.}$$

$$t = 3.753 \text{ s}$$

R1–13. The position of a particle is defined by $r = \{5(\cos 2t)\mathbf{i} + 4(\sin 2t)\mathbf{j}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. Determine the magnitudes of the velocity and acceleration of the particle when $t = 1$ s. Also, prove that the path of the particle is elliptical.

Velocity: The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{-10 \sin 2r\mathbf{i} + 8 \cos 2r\mathbf{j}\} \text{ m/s}$$

When $t = 1$ s, $\mathbf{v} = -10 \sin 2(1)\mathbf{i} + 8 \cos 2(1)\,\mathbf{j} = (-9.093\mathbf{i} - 3.329\mathbf{j}\}$ m/s. Thus, the magnitude of the velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-9.093)^2 + (-3.329)^2} = 9.68 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The acceleration express in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{-20 \cos 2r\mathbf{i} - 16 \sin 2r\mathbf{j}\} \text{ m/s}^2$$

When $t = 1$ s, $\mathbf{a} = -20 \cos 2(1)\,\mathbf{i} - 16 \sin 2(1)\,\mathbf{j} = \{8.323\mathbf{i} - 14.549\mathbf{j}\}$ m/s^2. Thus, the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8.323^2 + (-14.549)^2} = 16.8 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Travelling Path: Here, $x = 5 \cos 2t$ and $y = 4 \sin 2t$. Then,

$$\frac{x^2}{25} = \cos^2 2t \qquad \textbf{[1]}$$

$$\frac{y^2}{16} = \sin^2 2t \qquad \textbf{[2]}$$

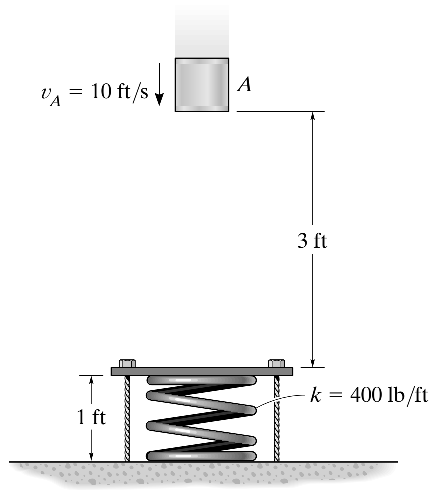
Adding Eqs [1] and [2] yields

$$\frac{x^2}{25} + \frac{y^2}{16} = \cos^2 2r + \sin^2 2t$$

However, $\cos^2 2r + \sin^2 2t = 1$. Thus,

$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \quad (\textbf{Equation of an Ellipse}) \qquad \textbf{(Q.E.D.)}$$

R1–14. The 5-lb cylinder falls past A with a speed $v_A = 10$ ft/s onto the platform. Determine the maximum displacement of the platform, caused by the collision. The spring has an unstretched length of 1.75 ft and is originally kept in compression by the 1-ft-long cables attached to the platform. Neglect the mass of the platform and spring and any energy lost during the collision.



Potential Energy: Datum is set at the final position of the platform. When the cylinder is at point A, its position is $(3 + s)$ *above* the datum where s is the maximum displacement of the platform when the cylinder stops momentary. Thus, its gravitational potential energy at this position is $5(3 + s) = (15 + 5s)$ ft·lb. The initial and final elastic potential energy are $\frac{1}{2}(400)(1.75 - 1)^2 = 112.5$ ft·lb and $\frac{1}{2}(400)(s + 0.75)^2 = 200s^2 + 300s + 112.5$, respectively.

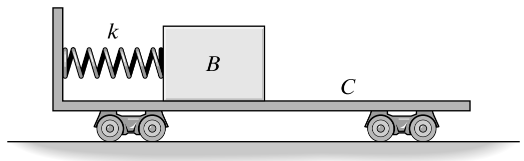
Conservation of Energy: Applying Eq. 14–22, we have

$$\Sigma T_A + \Sigma V_A = \Sigma T_B + \Sigma V_B$$

$$\frac{1}{2}\left(\frac{5}{32.2}\right)\left(10^2\right) + (15 + 5s) + 112.5 = 0 + 200s^2 + 300s + 112.5$$

$$s = 0.0735 \text{ ft}$$ **Ans.**

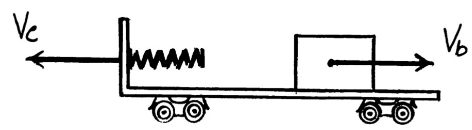
R1–15. The block has a mass of 50 kg and rests on the surface of the cart having a mass of 75 kg. If the spring which is attached to the cart and not the block is compressed 0.2 m and the system is released from rest, determine the speed of the block after the spring becomes undeformed. Neglect the mass of the cart's wheels and the spring in the calculation. Also neglect friction. Take $k = 300$ N/m.



$$T_1 + V_1 = T_2 + V_2$$

$$[0 + 0] + \frac{1}{2}(300)(0.2)^2 = \frac{1}{2}(50)\,v_b^2 + \frac{1}{2}(75)\,v_e^2$$

$$12 = 50\,v_b^2 + 75\,v_e^2$$



$(\overset{+}{\rightarrow})\qquad \Sigma m v_1 = \Sigma m v_2$

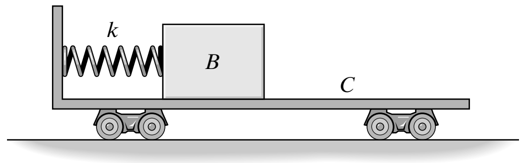
$$0 + 0 = 50\,v_b - 75\,v_e$$

$$v_b = 1.5 v_e$$

$$v_c = 0.253 \text{ m/s} \leftarrow$$

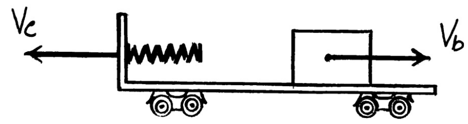
$$v_b = 0.379 \text{ m/s} \rightarrow$$ **Ans.**

***R1–16.** The block has a mass of 50 kg and rests on the surface of the cart having a mass of 75 kg. If the spring which is attached to the cart and not the block is compressed 0.2 m and the system is released from rest, determine the speed of the block with respect to the cart after the spring becomes undeformed. Neglect the mass of the cart's wheels and the spring in the calculation. Also neglect friction. Take $k = 300$ N/m.



$$T_1 + V_1 = T_2 + V_2$$

$$[0 + 0] + \frac{1}{2}(300)(0.2)^2 = \frac{1}{2}(50)v_b^2 + \frac{1}{2}(75)\,v_e^2$$



$$12 = 50\,v_b^2 + 75\,v_e^2$$

$(\xrightarrow{+})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0 + 0 = 50\,v_b - 75\,v_e$$

$$v_b = 1.5\,v_e$$

$$v_c = 0.253 \text{ m/s} \leftarrow$$

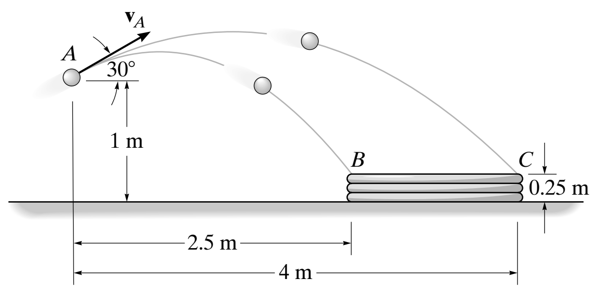
$$v_b = 0.379 \text{ m/s} \rightarrow$$

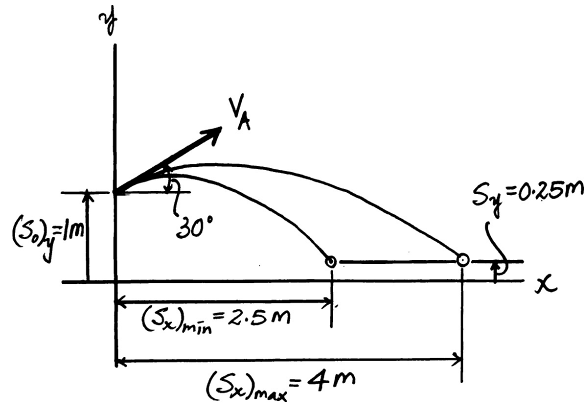
$$\mathbf{v}_b = \mathbf{v}_c + \mathbf{v}_{b/c}$$

$(\xrightarrow{+})$ $\quad 0.379 = -0.253 + \mathbf{v}_{b/c}$

$v_{b/c} = 0.632 \text{ m/s} \rightarrow$ **Ans.**

R1–17. A ball is launched from point A at an angle of 30°. Determine the maximum and minimum speed v_A it can have so that it lands in the container.





Min. speed:

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$2.5 = 0 + v_A \cos 30° t$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$0.25 = 1 + v_A \sin 30° t - \frac{1}{2}(9.81)t^2$$

Solving

$t = 0.669$ s

$v_A = (v_A)_{min} = 4.32$ m/s **Ans.**

Max. speed:

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$4 = 0 + v_A \cos 30° t$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

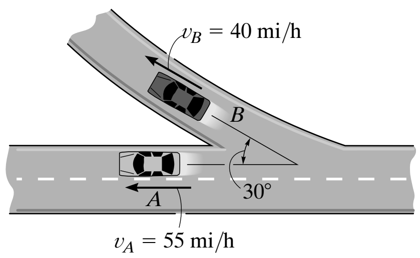
$$0.25 = 1 + v_A \sin 30° t - \frac{1}{2}(9.81)t^2$$

Solving:

$t = 0.790$ s

$v_A = (v_A)_{max} = 5.85$ m/s **Ans.**

R1–18. At the instant shown, cars A and B travel at speeds of 55 mi/h and 40 mi/h, respectively. If B is increasing its speed by 1200 mi/h^2, while A maintains its constant speed, determine the velocity and acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.5 mi.

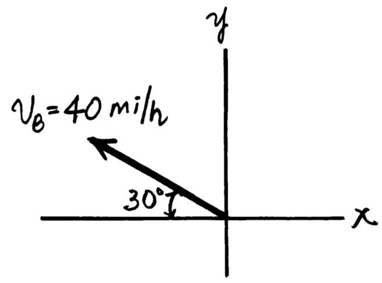


$$\mathbf{v}_B = -40\cos 30°\mathbf{i} + 40\sin 30°\mathbf{j} = \{-34.64\mathbf{i} + 20\mathbf{j}\}\text{ mi/h}$$

$$\mathbf{v}_A = \{-55\mathbf{i}\}\text{ mi/h}$$

$$\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A$$

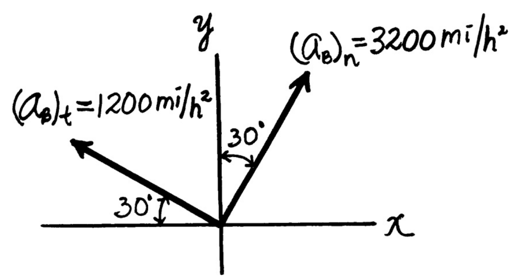
$$= (-34.64\mathbf{i} + 20\mathbf{j}) - (-55\mathbf{i}) = \{20.36\mathbf{i} + 20\mathbf{j}\}\text{ mi/h}$$



$$v_{B/A} = \sqrt{20.36^2 + 20^2} = 28.5\text{ mi/h}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{20}{20.36}\right) = 44.5° \measuredangle$$ **Ans.**

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{40^2}{0.5} = 3200\text{ mi/h}^2 \qquad (a_B)_t = 1200\text{ mi/h}^2$$



$$\mathbf{a}_B = (3200\sin 30° - 1200\cos 30°)\mathbf{i} + (3200\cos 30° + 1200\sin 30°)\mathbf{j}$$

$$= \{560.77\mathbf{i} + 3371.28\mathbf{j}\}\text{ mi/h}^2$$

$$\mathbf{a}_A = \mathbf{0}$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

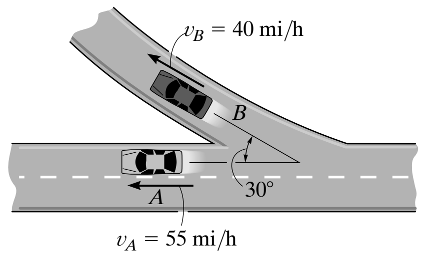
$$560.77\mathbf{i} + 3371.28\mathbf{j} = \mathbf{0} + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{560.77\mathbf{i} + 3371.28\mathbf{j}\}\text{ mi/h}^2$$

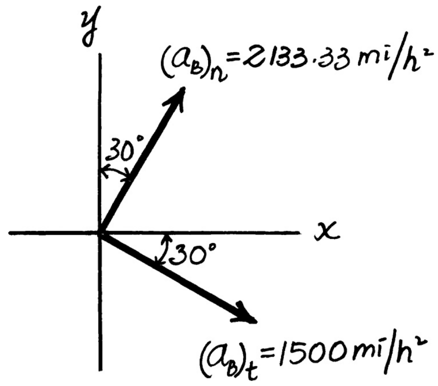
$$a_{B/A} = \sqrt{(560.77)^2 + (3371.28)^2} = 3418\text{ mi/h}^2 = 3.42\left(10^3\right)\text{ mi/h}^2$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{3371.28}{560.77}\right) = 80.6° \measuredangle$$ **Ans.**

R1–19. At the instant shown, cars A and B travel at speeds of 55 mi/h and 40 mi/h, respectively. If B is decreasing its speed at 1500 mi/h^2 while A is increasing its speed at 800 mi/h^2, determine the acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.75 mi.



$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{(40)^2}{0.75} = 2133.33 \text{ mi/h}^2$$



$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$2133.33 \sin 30°\mathbf{i} + 2133.33 \cos 30°\mathbf{j} + 1500 \cos 30°\mathbf{i} - 1500 \sin 30°\mathbf{j}$$

$$= -800\mathbf{i} + (a_{B/A})_x\,\mathbf{i} + (a_{B/A})_y\,\mathbf{j}$$

$(\overset{+}{\rightarrow})$ $\quad 2133.33 \sin 30° + 1500 \cos 30° = -800 + (a_{B/A})_x$

$$(a_{B/A})_x = 3165.705 \rightarrow$$

$(+\uparrow)$ $\quad 2133.33 \cos 30° - 1500 \sin 30° = (a_{B/A})_y$

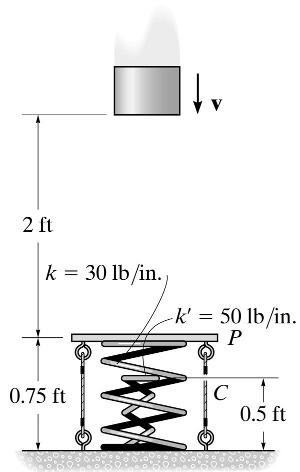
$$(a_{B/A})_y = 1097.521 \uparrow$$

$$(a_{B/A}) = \sqrt{(1097.521)^2 + (3165.705)^2}$$

$$a_{B/A} = 3351 \text{ mi/h}^2 = 3.35\left(10^3\right) \text{ mi/h}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{1097.521}{3165.705}\right) = 19.1° \measuredangle \qquad \textbf{Ans.}$$

***R1–20.** Four inelastic cables C are attached to a plate P and hold the 1-ft-long spring 0.25 ft in compression when *no weight* is on the plate. There is also an undeformed spring nested within this compressed spring. If the block, having a weight of 10 lb, is moving downward at $v = 4$ ft/s, when it is 2 ft above the plate, determine the maximum compression in each spring after it strikes the plate. Neglect the mass of the plate and springs and any energy lost in the collision.



$k = 30(12) = 360$ lb/ft

$k' = 50(12) = 600$ lb/ft

Assume both springs compress;

$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(4)^2 + 0 + \frac{1}{2}(360)(0.25)^2 = 0 + \frac{1}{2}(360)(s + 0.25)^2 + \frac{1}{2}(600)(s - 0.25)^2 - 10(s + 2)$$

$$13.73 = 180(s + 0.25)^2 + 300(s - 0.25)^2 - 10s - 20 \qquad \textbf{(1)}$$

$$33.73 = 180(s + 0.25)^2 + 300(s - 0.25)^2 - 10s$$

$$480s^2 - 70s - 3.73 = 0$$

Choose the positive root;

$$s = 0.1874 \text{ ft} < 0.25 \text{ ft} \qquad \textbf{NG!}$$

The nested spring does not deform.

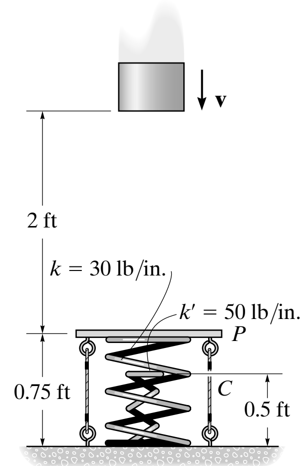
Thus Eq. (1) becomes

$$13.73 = 180(s + 0.25)^2 - 10s - 20$$

$$180\,s^2 + 80\,s - 22.48 = 0$$

$$s = 0.195 \text{ ft} \qquad \textbf{Ans.}$$

R1–21. Four inelastic cables C are attached to plate P and hold the 1-ft-long spring 0.25 ft in compression when *no weight* is on the plate. There is also a 0.5-ft-long undeformed spring nested within this compressed spring. Determine the speed v of the 10-lb block when it is 2 ft above the plate, so that after it strikes the plate, it compresses the nested spring, having a stiffness of 50 lb/in., an amount of 0.20 ft. Neglect the mass of the plate and springs and any energy lost in the collision.



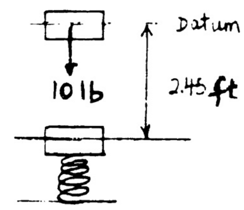
$$k = 30(12) = 360 \text{ lb/ft}$$

$$k' = 50(12) = 600 \text{ lb/ft}$$

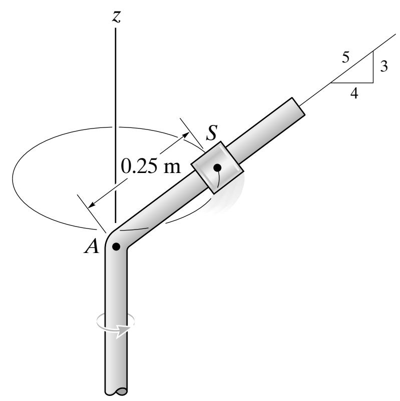
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)v^2 + \frac{1}{2}(360)(0.25)^2 = \frac{1}{2}(360)(0.25 + 0.25 + 0.20)^2 + \frac{1}{2}(600)(0.20)^2 - 10(2 + 0.25 + 0.20)$$

$v = 20.4$ ft/s **Ans.**



R1–22. The 2-kg spool S fits loosely on the rotating inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the minimum constant speed the spool can have so that it does not slip down the rod.



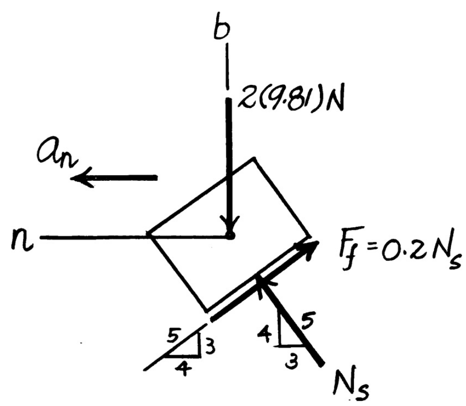
$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2 \text{ m}$$

$\overset{+}{\leftarrow} \Sigma F_n = ma_n;$ $\quad N_s\left(\frac{3}{5}\right) - 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$

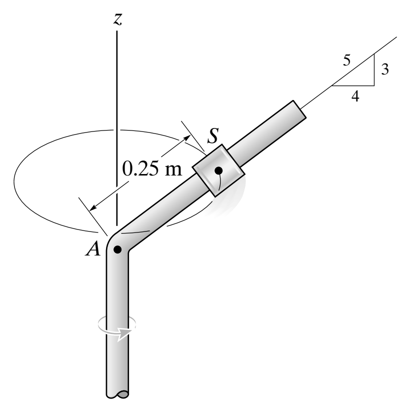
$+\uparrow \Sigma F_b = m\,a_b;$ $\quad N_s\left(\frac{4}{5}\right) + 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$

$N_s = 21.3$ N

$v = 0.969$ m/s **Ans.**



R1–23. The 2-kg spool S fits loosely on the rotating inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the maximum constant speed the spool can have so that it does not slip up the rod.



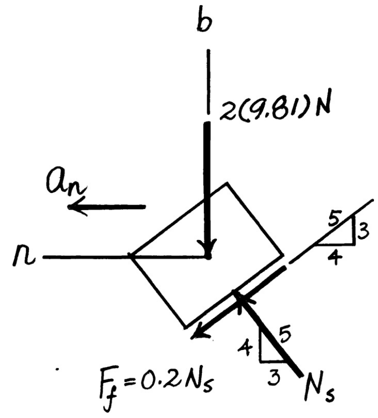
$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2 \text{ m}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad N_s\left(\frac{3}{5}\right) + 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$$

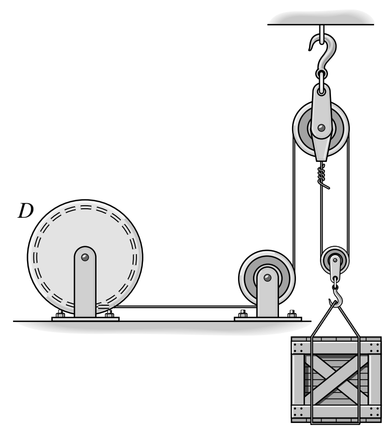
$$+\uparrow \Sigma F_b = m\,a_b; \qquad N_s\left(\frac{4}{5}\right) - 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$$

$$N_s = 28.85 \text{ N}$$

$$v = 1.48 \text{ m/s} \qquad \textbf{Ans.}$$



***R1–24.** The winding drum D draws in the cable at an accelerated rate of 5 m/s^2. Determine the cable tension if the suspended crate has a mass of 800 kg.



$$s_A + 2\,s_B = l$$

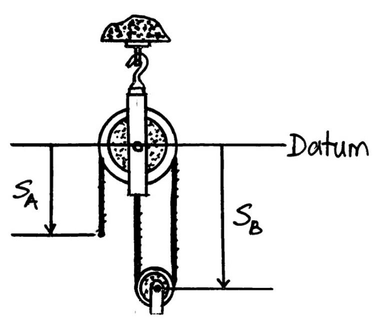
$$a_A = -2\,a_B$$

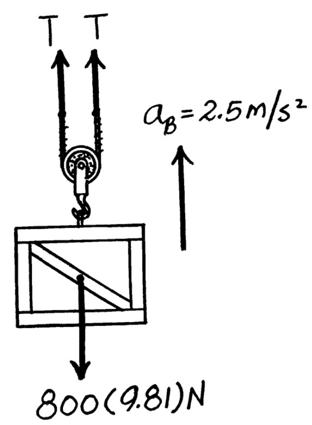
$$5 = -2\,a_B$$

$$a_B = -2.5 \text{ m/s}^2 = 2.5 \text{ m/s}^2 \uparrow$$

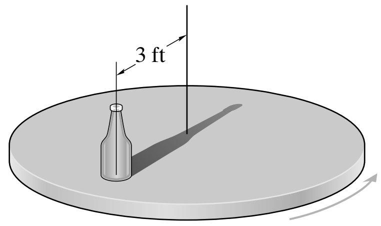
$$+\uparrow \Sigma F_y = ma_y; \qquad 2T - 800(9.81) = 800(2.5)$$

$$T = 4924 \text{ N} = 4.92 \text{ kN} \qquad \textbf{Ans.}$$





R1–25. The bottle rests at a distance of 3 ft from the center of the horizontal platform. If the coefficient of static friction between the bottle and the platform is $\mu_s = 0.3$, determine the maximum speed that the bottle can attain before slipping. Assume the angular motion of the platform is slowly increasing.

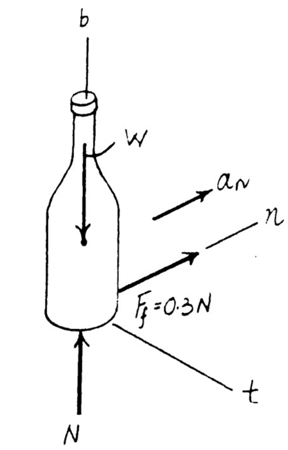


$\Sigma F_b = 0;$ $\quad N - W = 0 \qquad N = W$

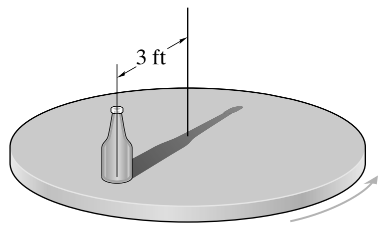
Since the bottle is on the verge of slipping, then $F_f = \mu_s N = 0.3W$.

$\Sigma F_n = ma_n;$ $\quad 0.3W = \left(\dfrac{W}{32.2}\right)\left(\dfrac{v^2}{3}\right)$

$v = 5.38 \text{ ft/s}$ **Ans.**



R1–26. Work Prob. R1–25 assuming that the platform starts rotating from rest so that the speed of the bottle is increased at 2 ft/s^2.



Applying Eq. 13–8, we have

$\Sigma F_b = 0;$ $\quad N - W = 0 \qquad N = W$

Since the bottle is on the verge of slipping, then $F_f = \mu_s N = 0.3W$.

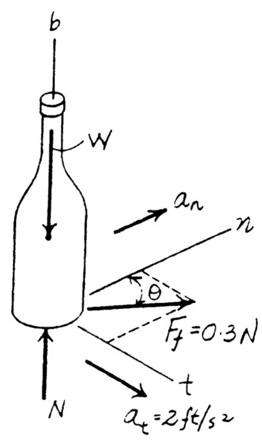
$\Sigma F_t = ma_t;$ $\quad 0.3W \sin\theta = \left(\dfrac{W}{32.2}\right)(2)$ **[1]**

$\Sigma F_n = ma_n;$ $\quad 0.3W \cos\theta = \left(\dfrac{W}{32.2}\right)\left(\dfrac{v^2}{3}\right)$ **[2]**

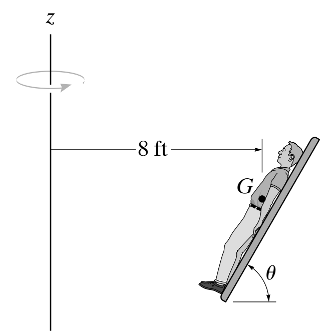
Solving Eqs. [1] and [2] yields

$v = 5.32 \text{ ft/s}$ **Ans.**

$\theta = 11.95°$

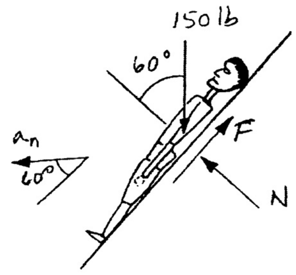


R1–27. The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. Determine the resultant normal and frictional forces the cushion exerts on him if, due to rotation about the z axis, he has a constant speed $v = 20$ ft/s. Neglect the size of the man. Take $\theta = 60°$.



$+\nwarrow \Sigma F_y = m(a_n)_y;\qquad N - 150\cos 60° = \dfrac{150}{32.2}\left(\dfrac{20^2}{8}\right)\sin 60°$

$N = 277$ lb **Ans.**



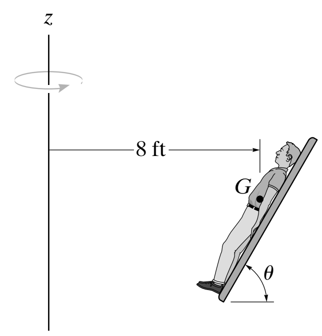
$+\swarrow \Sigma F_x = m(a_n)_x;\qquad -F + 150\sin 60° = \dfrac{150}{32.2}\left(\dfrac{20^2}{8}\right)\cos 60°$

$F = 13.4$ lb **Ans.**

Note: No slipping occurs

Since $\mu_s N = 138.4 \text{ lb} > 13.4 \text{ lb}$

*__R1–28.__ The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. If he rotates about the z axis with a constant speed $v = 30$ ft/s, determine the smallest angle θ of the cushion at which he will begin to slip up the cushion.



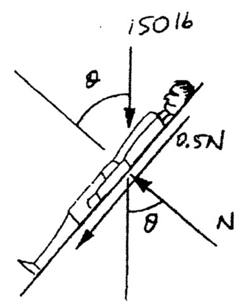
$\overset{+}{\leftarrow} \Sigma F_n = ma_n;\qquad 0.5N\cos\theta + N\sin\theta = \dfrac{150}{32.2}\left(\dfrac{(30)^2}{8}\right)$

$+\uparrow \Sigma F_b = 0;\qquad -150 + N\cos\theta - 0.5\,N\sin\theta = 0$

$$N = \frac{150}{\cos\theta - 0.5\sin\theta}$$

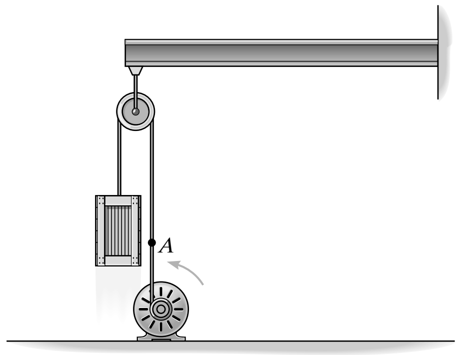
$$\frac{(0.5\cos\theta + \sin\theta)150}{(\cos\theta - 0.5\sin\theta)} = \frac{150}{32.2}\left(\frac{(30)^2}{8}\right)$$

$$0.5\cos\theta + \sin\theta = 3.493\,79\cos\theta - 1.746\,89\sin\theta$$



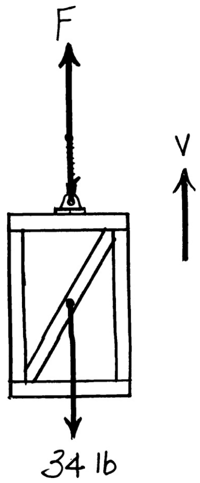
$\theta = 47.5°$ **Ans.**

R1–29. The motor pulls on the cable at A with a force $F = (30 + t^2)$ lb, where t is in seconds. If the 34-lb crate is originally at rest on the ground when $t = 0$, determine its speed when $t = 4$ s. Neglect the mass of the cable and pulleys. *Hint:* First find the time needed to begin lifting the crate.



$$30 + t^2 = 34$$

$t = 2$ s for crate to start moving



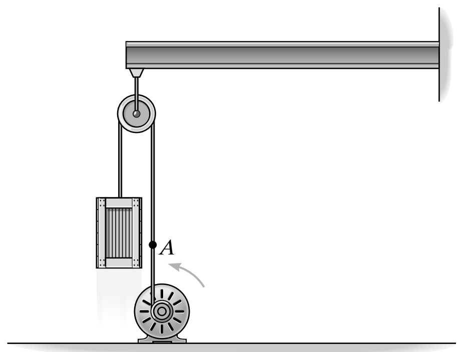
$$(+\uparrow) \qquad mv_1 + \Sigma \int F dt = mv_2$$

$$0 + \int_2^4 (30 + t^2)dt - 34(4 - 2) = \frac{34}{32.2} v_2$$

$$[30t + \frac{1}{3} t^3]_2^4 - 68 = \frac{34}{32.2} v_2$$

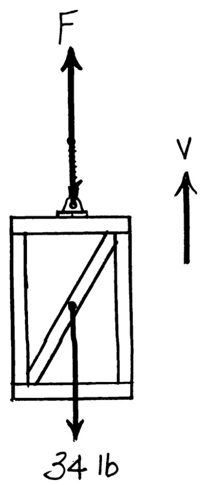
$$v_2 = 10.1 \text{ ft/s} \qquad \textbf{Ans.}$$

R1–30. The motor pulls on the cable at A with a force $F = (e^{2t})$ lb, where t is in seconds. If the 34-lb crate is originally at rest on the ground when $t = 0$, determine the crate's velocity when $t = 2$ s. Neglect the mass of the cable and pulleys. *Hint:* First find the time needed to begin lifting the crate.



$$F = e^{2t} = 34$$

$t = 1.7632$ s for crate to start moving



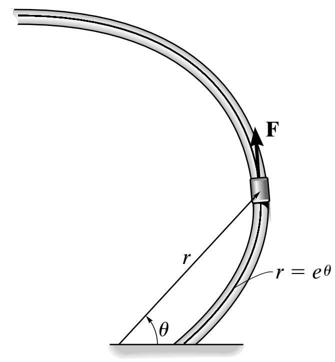
$$(+\uparrow) \qquad mv_1 + \Sigma \int F\, dt = mv_2$$

$$0 + \int_{1.7632}^{2} e^{2t}\, dt - 34(2 - 1.7632) = \frac{34}{32.2} v_2$$

$$\frac{1}{2} e^{2t} \Big|_{1.7632}^{2} - 8.0519 = 1.0559\, v_2$$

$$v_2 = 2.13 \text{ ft/s} \qquad \textbf{Ans.}$$

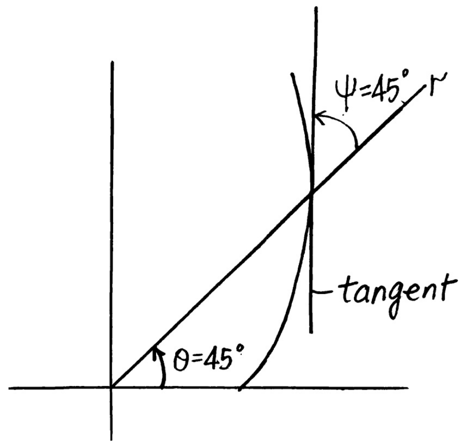
R1–31. The collar has a mass of 2 kg and travels along the smooth *horizontal* rod defined by the equiangular spiral $r = (e^{\theta})$ m, where θ is in radians. Determine the tangential force **F** and the normal force **N** acting on the collar when $\theta = 45°$, if force **F** maintains a constant angular motion $\dot{\theta} = 2$ rad/s.



$$r = e^{\theta}$$

$$\dot{r} = e^{\theta}\,\dot{\theta}$$

$$\ddot{r} = e^{\theta}(\dot{\theta})^2 + e^{\theta}\dot{\theta}$$



At $\theta = 45°$

$$\dot{\theta} = 2 \text{ rad/s}$$

$$\theta = 0$$

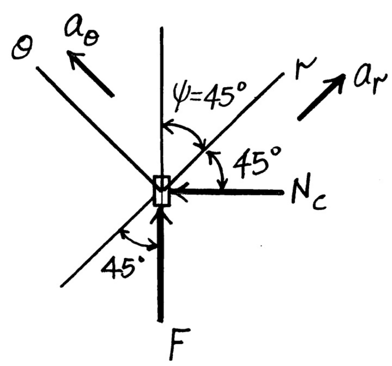
$$r = 2.1933 \text{ m}$$

$$r = 4.38656 \text{ m/s}$$

$$\ddot{r} = 8.7731 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\ (\dot{\theta})^2 = 8.7731 - 2.1933(2)^2 = 0$$

$$a_\theta = r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta} = 0 + 2(4.38656)(2) = 17.5462 \text{ m/s}^2$$



$$\tan\psi = \frac{r}{\left(\frac{dr}{d\theta}\right)} = e^{\theta}/e^{\theta} = 1$$

$$\psi = \theta = 45°$$

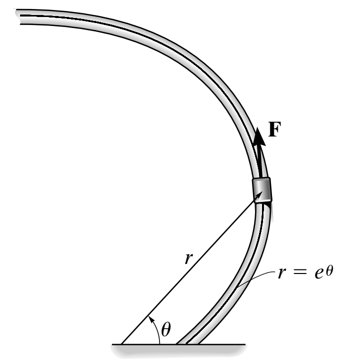
$\Sigma F_r = m\,a_r;\qquad -N_r\cos 45° + F\cos 45° = 2(0)$

$\Sigma F_\theta = m\,a_\theta;\qquad F\sin 45° + N_\theta \sin 45° = 2(17.5462)$

$N = 24.8$ N **Ans.**

$F = 24.8$ N **Ans.**

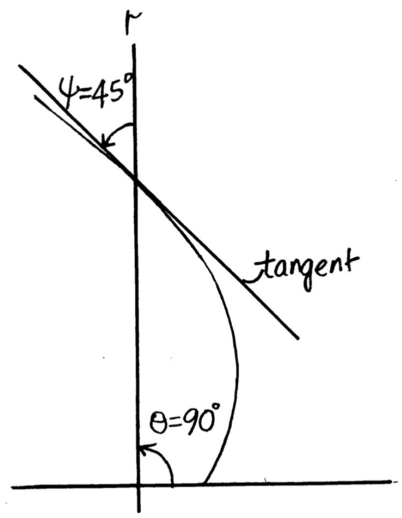
***R1–32.** The collar has a mass of 2 kg and travels along the smooth *horizontal* rod defined by the equiangular spiral $r = (e^{\theta})$ m, where θ is in radians. Determine the tangential force **F** and the normal force **N** acting on the collar when $\theta = 90°$, if force **F** maintains a constant angular motion $\dot{\theta} = 2$ rad/s.



$$r = e^{\theta}$$

$$\dot{r} = e^{\theta}\,\dot{\theta}$$

$$\ddot{r} = e^{\theta}\,(\dot{\theta})^2 + e^{\theta}\,\ddot{\theta}$$



At $\theta = 90°$

$$\dot{\theta} = 2 \text{ rad/s}$$

$$\dot{\theta} = 0$$

$$r = 4.8105 \text{ m}$$

$$\dot{r} = 9.6210 \text{ m/s}$$

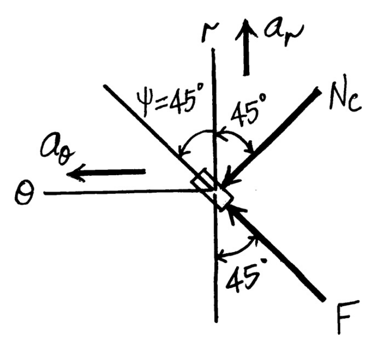
$$\ddot{r} = 19.242 \text{ m/s}^2$$

$$a_r = \ddot{r} - r(\dot{\theta})^2 = 19.242 - 4.8105(2)^2 = 0$$

$$a_{\theta} = r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta} = 0 + 2(9.6210)(2) = 38.4838 \text{ m/s}^2$$

$$\tan\psi = \frac{r}{\left(\frac{dr}{d\theta}\right)} = e^{\theta}/e^{\theta} = 1$$

$$\psi = \theta = 45°$$



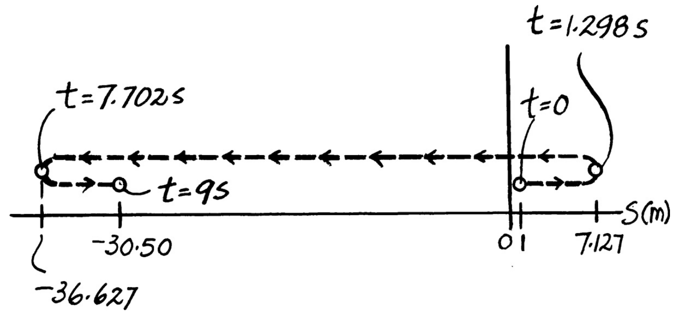
$+\uparrow \Sigma F_t = m\,a_t;\qquad -N\cos 45° + F\cos 45° = 2(0)$

$\overset{+}{\leftarrow} \Sigma F_{\theta} = m\,a_{\theta};\qquad F\sin 45° + N\sin 45° = 2(38.4838)$

$N_t = 54.4$ N **Ans.**

$F = 54.4$ N **Ans.**

R1–33. The acceleration of a particle along a straight line is defined by $a = (2t - 9)\ \text{m/s}^2$, where t is in seconds. When $t = 0$, $s = 1\ \text{m}$ and $v = 10\ \text{m/s}$. When $t = 9\ \text{s}$, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity. Assume the positive direction is to the right.



$$a = (2t - 9)$$

$$dv = a\,dt$$

$$\int_{10}^{v} dv = \int_0^t (2t - 9)\,dt$$

$$v - 10 = t^2 - 9t$$

$$v = t^2 - 9t + 10$$

$$ds = v\,dt$$

$$\int_1^s ds = \int_0^t \left(t^2 - 9t + 10\right)dt$$

$$s - 1 = \frac{1}{3}t^3 - 4.5t^2 + 10t$$

$$s = \frac{1}{3}t^3 - 4.5t^2 + 10t + 1$$

Note $v = 0$ at $t^2 - 9t + 10 = 0$

$$t = 1.298\ \text{s and } t = 7.702\ \text{s}$$

At $t = 1.298$ s, $s = 7.127$ m

At $t = 7.702$ s, $s = -36.627$ m

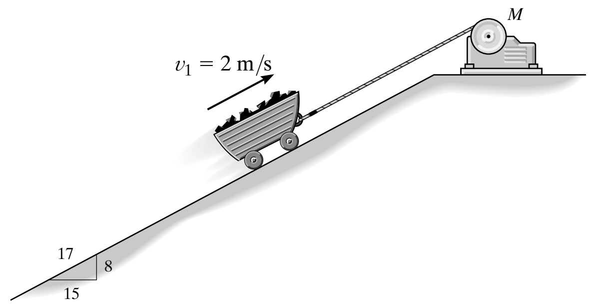
At $t = 9$ s, $s = -30.50$ m

a) $s = -30.5\ \text{m}$ **Ans.**

b) $s_{\text{tot}} = (7.127 - 1) + 7.127 + 36.627 + (36.627 - 30.50) = 56.0\ \text{m}$ **Ans.**

c) $v|_{t=9} = (9)^2 - 9(9) + 10 = 10\ \text{m/s}$ **Ans.**

R1–34. The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s when $t = 0$, determine its velocity when $t = 2$ s.



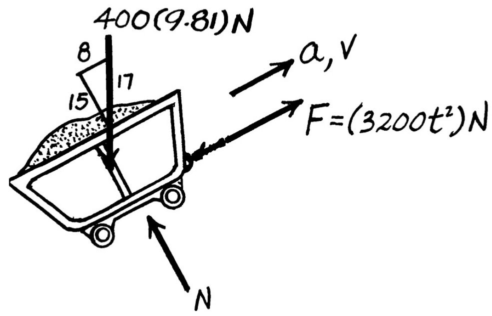
$$+\nearrow\Sigma F_{x'} = ma_{x'}; \qquad 3200t^2 - 400(9.81)\left(\frac{8}{17}\right) = 400a \qquad a = 8t^2 - 4.616$$

$$dv = a\,dt$$

$$\int_2^v dv = \int_0^2 \left(8t^2 - 4.616\right)dt$$

$$v = 14.1 \text{ m/s}$$ **Ans.**

Also,



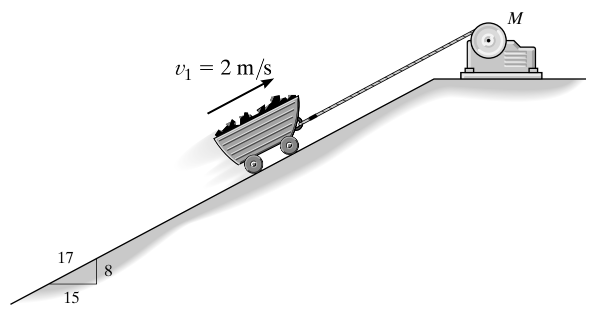
$$m\mathbf{v}_1 + \Sigma\int \mathbf{F}\,dt = m\mathbf{v}_2$$

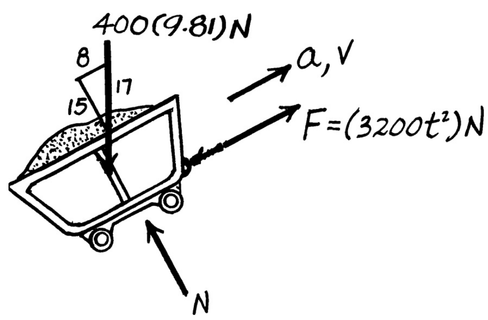
$$+\nearrow \qquad 400(2) + \int_0^2 3200\,t^2\,dt - 400(9.81)(2-0)\left(\frac{8}{17}\right) = 400v_2$$

$$800 + 8533.33 - 3693.18 = 400v_2$$

$$v_2 = 14.1 \text{ m/s}$$

R1–35. The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s at $s = 0$ and $t = 0$, determine the distance it moves up the plane when $t = 2$ s.





$$\Sigma F_{x'} = ma_{x'}; \qquad 3200t^2 - 400(9.81)\left(\frac{8}{17}\right) = 400a \qquad a = 8t^2 - 4.616$$

$$dv = a\,dt$$

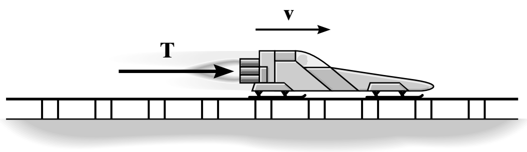
$$\int_2^v dv = \int_0^t \left(8t^2 - 4.616\right)dt$$

$$v = \frac{ds}{dt} = 2.667t^3 - 4.616t + 2$$

$$\int_2^s ds = \int_0^2 \left(2.667t^3 - 4.616t + 2\right)dt$$

$$s = 5.43 \text{ m}$$ **Ans.**

***R1–36.** The rocket sled has a mass of 4 Mg and travels along the smooth horizontal track such that it maintains a constant power output of 450 kW. Neglect the loss of fuel mass and air resistance, and determine how far the sled must travel to reach a speed of $v = 60$ m/s starting from rest.



$$\xrightarrow{+} \Sigma F_x = m\,a_x; \qquad F = m\,a = m\left(\frac{v\,dv}{ds}\right)$$

$$P = F\,v = m\left(\frac{v^2\,dv}{ds}\right)$$

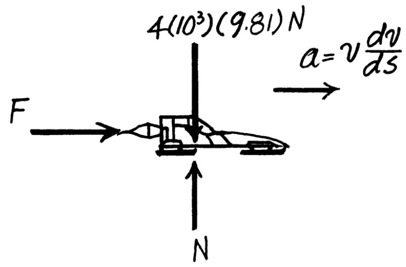
$$\int P\,ds = m\int v^2\,dv$$

$$P\int_0^s ds = m\int_0^v v^2\,dv$$

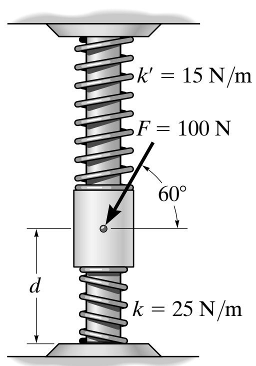
$$Ps = \frac{m\,v^3}{3}$$

$$s = \frac{m\,v^3}{3\,P}$$

$$s = \frac{4(10^3)(60)^3}{3(450)(10^3)} = 640 \text{ m}$$ **Ans.**



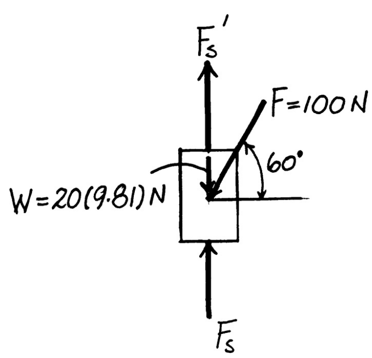
R1–37. The collar has a mass of 20 kg and can slide freely on the smooth rod. The attached springs are undeformed when $d = 0.5$ m. Determine the speed of the collar after the applied force $F = 100$ N causes it to be displaced so that $d = 0.3$ m. When $d = 0.5$ m the collar is at rest.



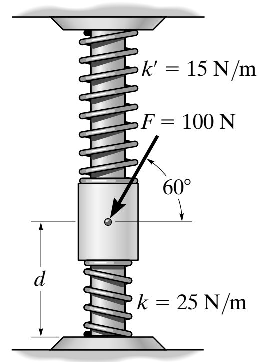
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 100 \sin 60°(0.5 - 0.3) + 20(9.81)(0.5 - 0.3) - \frac{1}{2}(15)(0.5 - 0.3)^2 - \frac{1}{2}(25)(0.5 - 0.3)^2 = \frac{1}{2}(20)v_C^2$$

$v_C = 2.36$ m/s **Ans.**



R1–38. The collar has a mass of 20 kg and can slide freely on the smooth rod. The attached springs are both compressed 0.4 m when $d = 0.5$ m. Determine the speed of the collar after the applied force $F = 100$ N causes it to be displaced so that $d = 0.3$ m. When $d = 0.5$ m the collar is at rest.

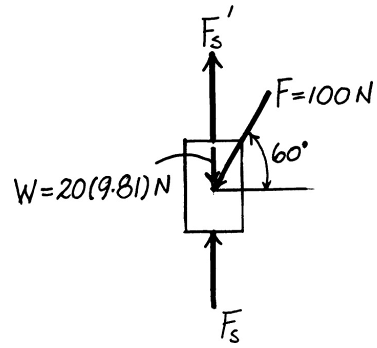


$$T_1 + \Sigma U_{1-2} = T_2$$

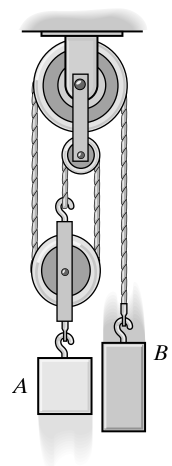
$$0 + 100 \sin 60°(0.5 - 0.3) + 196.2(0.5 - 0.3) - [\frac{1}{2}(25)[0.4 + 0.2]^2 - \frac{1}{2}(25)(0.4)^2]$$

$$- [\frac{1}{2}(15)[0.4 - 0.2]^2 - \frac{1}{2}(15)(0.4)^2] = \frac{1}{2}(20)v_C^2$$

$v_C = 2.34$ m/s **Ans.**



R1–39. The assembly consists of two blocks A and B which have masses of 20 kg and 30 kg, respectively. Determine the speed of each block when B descends 1.5 m. The blocks are released from rest. Neglect the mass of the pulleys and cords.



$$3\,s_A + s_B = l$$

$$3\,\Delta s_A = -\Delta s_B$$

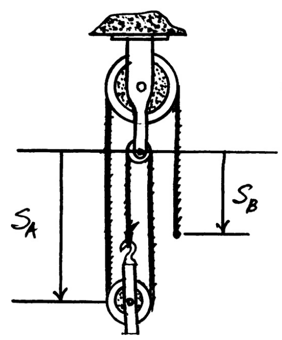
$$3\,v_A = -v_B$$

$$T_1 + V_1 = T_2 + V_2$$

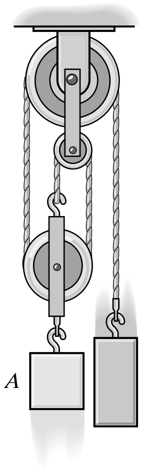
$$(0 + 0) + (0 + 0) = \frac{1}{2}(20)(v_A)^2 + \frac{1}{2}(30)(-3v_A)^2 + 20(9.81)\left(\frac{1.5}{3}\right) - 30(9.81)(1.5)$$

$v_A = 1.54$ m/s **Ans.**

$v_B = 4.62$ m/s **Ans.**



***R1–40.** The assembly consists of two blocks A and B, which have masses of 20 kg and 30 kg, respectively. Determine the distance B must descend in order for A to achieve a speed of 3 m/s starting from rest.



$$3\,s_A + s_B - l$$

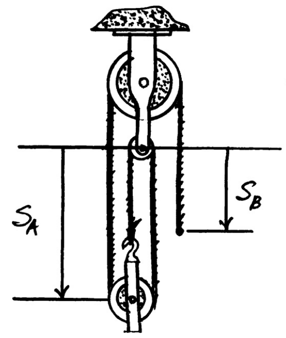
$$3\,\Delta s_A = -\,\Delta s_B$$

$$3\,v_A = -\,v_B$$

$$v_B = -9 \text{ m/s}$$

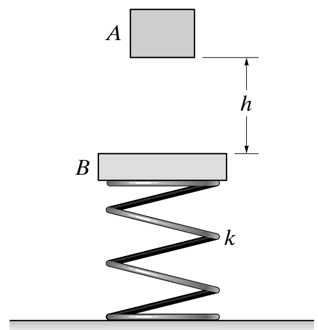
$$T_1 + V_1 = T_2 + V_2$$

$$(0 + 0) + (0 + 0) = \frac{1}{2}(20)(3)^2 + \frac{1}{2}(30)(-9)^2 + 20(9.81)\left(\frac{s_B}{3}\right) - 30(9.81)(s_B)$$



$s_B = 5.70$ m **Ans.**

R1–41. Block A, having a mass m, is released from rest, falls a distance h and strikes the plate B having a mass $2m$. If the coefficient of restitution between A and B is e, determine the velocity of the plate just after collision. The spring has a stiffness k.



Just before impact, the velocity of A is

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}mv_A^2 - mgh$$

$$v_A = \sqrt{2gh}$$

$$(+\downarrow) \qquad e = \frac{(v_B)_2 - (v_A)_2}{\sqrt{2gh}}$$

$$e\sqrt{2gh} = (v_B)_2 - (v_A)_2 \qquad \textbf{(1)}$$

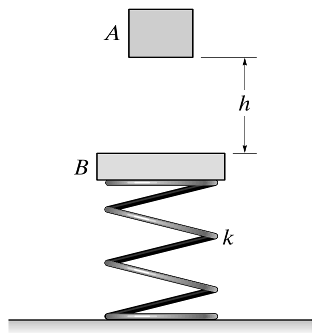
$$(+\downarrow) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$m(v_A) + 0 = m(v_A)_2 + 2m(v_B)_2 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) for $(v_B)_2$ yields

$$(v_B)_2 = \frac{1}{3}\sqrt{2gh}\,(1 + e)$$ **Ans.**

R1–42. Block A, having a mass of 2 kg, is released from rest, falls a distance $h = 0.5$ m, and strikes the plate B having a mass of 3 kg. If the coefficient of restitution between A and B is $e = 0.6$, determine the velocity of the block just after collision. The spring has a stiffness $k = 30$ N/m.



Just before impact, the velocity of A is

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}(2)(v_A)_2^2 - 2(9.81)(0.5)$$

$$(v_A)_2 = \sqrt{2(9.81)(0.5)} = 3.132 \text{ m/s}$$

$$(+\downarrow) \qquad e = \frac{(v_B)_3 - (v_A)_3}{3.132 - 0}$$

$$0.6(3.132) = (v_B)_3 - (v_A)_3$$

$$1.879 = (v_B)_3 - (v_A)_3 \qquad \textbf{(1)}$$

$$(+\downarrow) \qquad \Sigma m v_2 = \Sigma m v_3$$

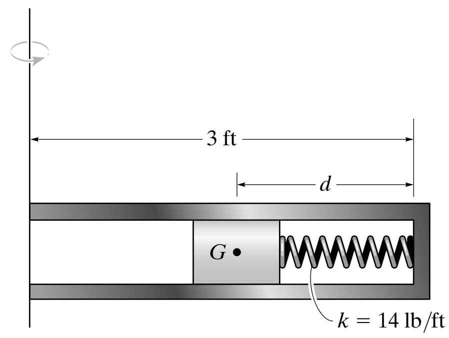
$$2(3.132) + 0 = 2(v_A)_3 + 3(v_B)_3 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) for $(v_A)_3$ yields

$$(v_B)_3 = 2.00 \text{ m/s}$$

$$(v_A)_3 = 0.125 \text{ m/s} \qquad \textbf{Ans.}$$

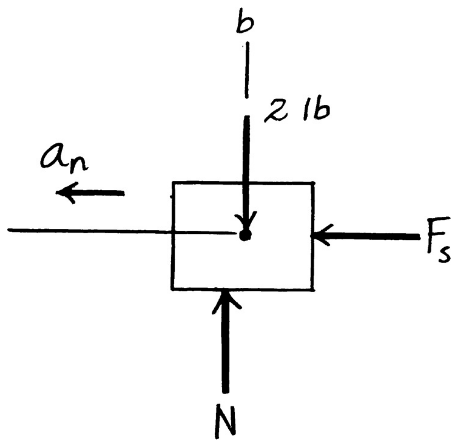
R1–43. The cylindrical plug has a weight of 2 lb and it is free to move within the confines of the smooth pipe. The spring has a stiffness $k = 14$ lb/ft and when no motion occurs the distance $d = 0.5$ ft. Determine the force of the spring on the plug when the plug is at rest with respect to the pipe. The plug travels in a circle with a constant speed of 15 ft/s, which is caused by the rotation of the pipe about the vertical axis. Neglect the size of the plug.



$$\overset{+}{\leftarrow} \Sigma F_n = m\, a_n; \qquad F_s = \frac{2}{32.2}\left[\frac{(15)^2}{3 - d}\right]$$

$$F_s = ks; \qquad F_s = 14(0.5 - d)$$

Thus,



$$14(0.5 - d) = \frac{2}{32.2}\left[\frac{(15)^2}{3 - d}\right]$$

$$(0.5 - d)(3 - d) = 0.9982$$

$$1.5 - 3.5d + d^2 = 0.9982$$

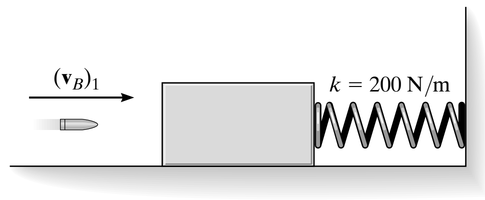
$$d^2 - 3.5d + 0.5018 = 0$$

Choosing the root < 0.5 ft

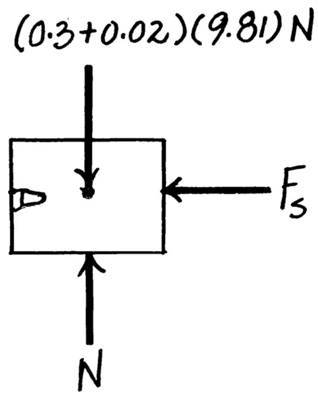
$$d = 0.1498 \text{ ft}$$

$$F_s = 14(0.5 - 0.1498) = 4.90 \text{ lb} \qquad \textbf{Ans.}$$

***R1–44.** A 20-g bullet is fired horizontally into the 300-g block which rests on the smooth surface. After the bullet becomes embedded into the block, the block moves to the right 0.3 m before momentarily coming to rest. Determine the speed $(v_B)_1$ of the bullet. The spring has a stiffness $k = 200$ N/m and is originally unstretched.



After collision



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(0.320)(v_2)^2 - \frac{1}{2}(200)(0.3)^2 = 0$$

$$v_2 = 7.50 \text{ m/s}$$

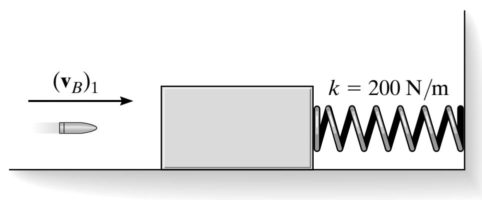
Impact

$$\Sigma m v_1 = \Sigma m v_2$$

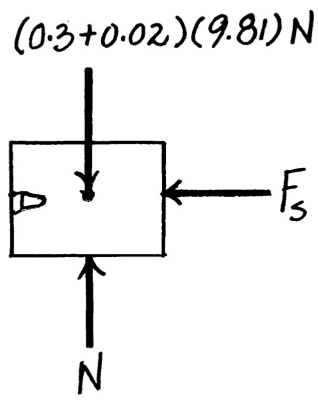
$$0.02(v_B)_1 + 0 = 0.320(7.50)$$

$$(v_B)_1 = 120 \text{ m/s} \qquad \textbf{Ans.}$$

R1–45. The 20-g bullet is fired horizontally at $(v_B)_1 = 1200$ m/s into the 300-g block which rests on the smooth surface. Determine the distance the block moves to the right before momentarily coming to rest. The spring has a stiffness $k = 200$ N/m and is originally unstretched.



Impact



$$\Sigma m v_1 = \Sigma m v_2$$

$$0.02(1200) + 0 = 0.320(v_2)$$

$$v_2 = 75 \text{ m/s}$$

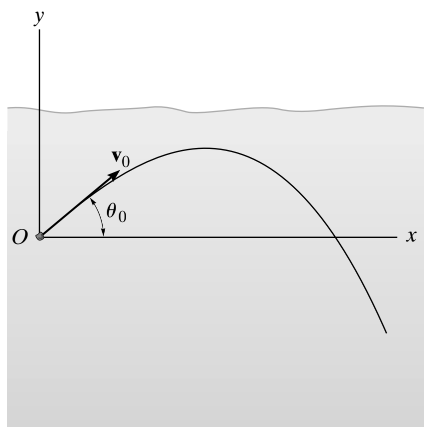
After collision;

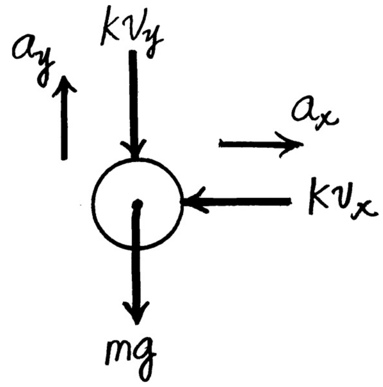
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(0.320)(75)^2 - \frac{1}{2}(200)(x^2) = 0$$

$$x = 3 \text{ m} \qquad \textbf{Ans.}$$

R1–46. A particle of mass m is fired at an angle θ_0 with a velocity $\mathbf{v}_0$ in a liquid that develops a drag resistance $F = -kv$, where k is a constant. Determine the maximum or terminal speed reached by the particle.





Equation of Motion: Applying Eq. 13–7, we have

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -kv_x = ma_x \qquad a_x = -\frac{k}{m}v_x \qquad \textbf{[1]}$$

$$+\uparrow \Sigma F_y = ma_y; \qquad -mg - kv_y = ma_y \qquad a_y = -g - \frac{k}{m}v_y \qquad \textbf{[2]}$$

However, $v_x = \dfrac{dx}{dt}$, $a_x = \dfrac{d^2x}{dt^2}$, $v_y = \dfrac{dy}{dt}$ and $a_y = \dfrac{d^2y}{dt^2}$. Substitute these values into Eqs. [1] and [2], we have

$$\frac{d^2x}{dt^2} + \frac{k}{m}\frac{dx}{dt} = 0 \qquad \textbf{[3]}$$

$$\frac{d^2y}{dt^2} + \frac{k}{m}\frac{dy}{dt} = -g \qquad \textbf{[4]}$$

The solution for the differential equation, Eq. [3], is in the form of

$$x = C_1 e^{-\frac{k}{m}t} + C_2 \qquad \textbf{[5]}$$

Thus,

$$\dot{x} = -\frac{C_1 k}{m} e^{-\frac{k}{m}t} \qquad \textbf{[6]}$$

However, at $t = 0$, $x = 0$ and $\dot{x} = v_0 \cos\theta_0$. Substitute these values into Eqs. [5] and [6], one obtains $C_1 = -\dfrac{m}{k} v_0 \cos\theta_0$ and $C_2 = \dfrac{m}{k} v_0 \cos\theta_0$. Substitute C_1 into Eq. [6] and rearrange. This yields

$$\dot{x} = e^{-\frac{k}{m}t}(v_0 \cos\theta_0) \qquad \textbf{[7]}$$

The solution for the differential equation, Eq. [4], is in the form of

$$y = C_3 e^{-\frac{k}{m}t} + C_4 - \frac{mg}{k}t \qquad \textbf{[8]}$$

Thus,

$$\dot{y} = -\frac{C_3 k}{m} e^{-\frac{k}{m}t} - \frac{mg}{k} \qquad \textbf{[9]}$$

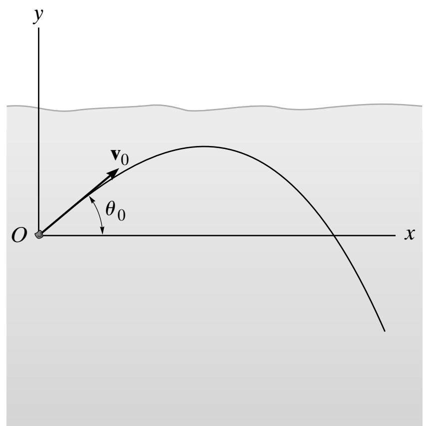
However, at $t = 0$, $y = 0$ and $\dot{y} = v_0 \sin\theta_0$. Substitute these values into Eq. [8] and [9], one obtains $C_3 = -\dfrac{m}{k}\left(v_0 \sin\theta_0 + \dfrac{mg}{k}\right)$ and $C_4 = \dfrac{m}{k}\left(v_0 \sin\theta_0 + \dfrac{mg}{k}\right)$. Substitute C_3 into Eq. [9] and rearrange. This yields

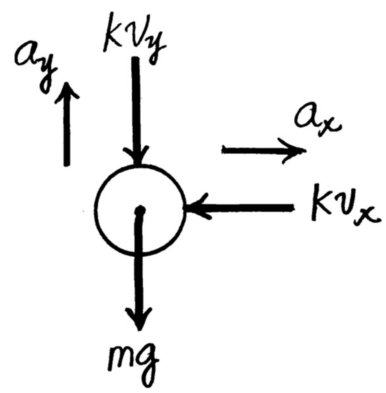
$$\dot{y} = e^{-\frac{k}{m}t}\left(v_0 \sin\theta_0 + \frac{mg}{k}\right) - \frac{mg}{k} \qquad \textbf{[10]}$$

For the particle to achieve terminal speed, $t \to \infty$ and $e^{-\frac{k}{m}t} \to 0$. When this happen, from Eqs. [7] and [10], $v_x = \dot{x} = 0$ and $v_y = \dot{y} = -\dfrac{mg}{k}$. Thus,

$$v_{\max} = \sqrt{v_x^2 + v_y^2} = \sqrt{0^2 + \left(-\frac{mg}{k}\right)^2} = \frac{mg}{k} \qquad \textbf{Ans.}$$

R1–47. A projectile of mass m is fired into a liquid at an angle θ_0 with an initial velocity $\mathbf{v}_0$ as shown. If the liquid develops a friction or drag resistance on the projectile which is proportional to its velocity, i.e., $F = -kv$, where k is a constant, determine the x and y components of its position at any instant. Also, what is the maximum distance x_{max} that it travels?





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However, $v_x = \dfrac{dx}{dt}$, $a_x = \dfrac{d^2x}{dt^2}$, $v_y = \dfrac{dy}{dt}$ and $a_y = \dfrac{d^2y}{dt^2}$. Substituting these values into Eqs. [1] and [2], we have

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The solution for the differential equation, Eq. [3], is in the form of

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Thus,

$$\dot{x} = -\frac{C_1 k}{m} e^{-\frac{k}{m}t} \qquad \textbf{[6]}$$

However, at $t = 0$, $x = 0$ and $\dot{x} = v_0 \cos\theta_0$. Substituting these values into Eq. [5] and [6], one can obtain $C_1 = -\dfrac{m}{k}v_0\cos\theta_0$ and $C_2 = \dfrac{m}{k}v_0\cos\theta_0$. Substituting C_1 and C_2 into Eq. [5] and rearrange yields

$$x = \frac{m}{k}v_0\cos\theta_0\left(1 - e^{-\frac{k}{m}t}\right) \qquad \textbf{Ans.}$$

When $t \to \infty$, $e^{-\frac{k}{m}t} \to 0$ and $x = x_{max}$. Then,

$$x_{max} = \frac{m}{k}v_0\cos\theta_0 \qquad \textbf{Ans.}$$

The solution for the differential equation. Eq. [4], is in the form of

$$y = C_3 e^{-\frac{k}{m}t} + C_4 - \frac{mg}{k}t \qquad \textbf{[7]}$$

Thus,

$$\dot{y} = -\frac{C_3 k}{m} e^{-\frac{k}{m}t} - \frac{mg}{k} \qquad \textbf{[8]}$$

However, at $t = 0$, $y = 0$ and $\dot{y} = v_0\sin\theta_0$. Substitute these values into Eq. [7] and [8], one can obtain $C_3 = -\dfrac{m}{k}\left(v_0\sin\theta_0 + \dfrac{mg}{k}\right)$ and $C_4 = \dfrac{m}{k}\left(v_0\sin\theta_0 + \dfrac{mg}{k}\right)$. Substitute C_3 and C_4 into Eq. [7] and rearrange yields

$$y = \frac{m}{k}\left(v_0\sin\theta_0 + \frac{mg}{k}\right)\left(1 - e^{-\frac{k}{m}t}\right) - \frac{mg}{k}t \qquad \textbf{Ans.}$$

***R1–48.** The position of particles A and B are $\mathbf{r}_A = \{3t\mathbf{i} + 9t(2 - t)\mathbf{j}\}$ m and $\mathbf{r}_B = \{3(t^2 - 2t + 2)\mathbf{i} + 3(t - 2)\mathbf{j}\}$ m, respectively, where t is in seconds. Determine the point where the particles collide and their speeds just before the collision. How long does it take before the collision occurs?

When collision occurs, $\mathbf{r}_A = \mathbf{r}_B$.

$$3t = 3(t^2 - 2t + 2)$$

$$t^2 - 3t + 2 = 0$$

$$t = 1\text{ s}, \qquad t = 2\text{ s}$$

Also,

$$9t(2 - t) = 3(t - 2)$$

$$3t^2 - 5t - 2 = 0$$

The positive root is $t = 2$ s

Thus,

$$t = 2\text{ s} \qquad \textbf{Ans.}$$

$$x = 3(2) = 6\text{ m} \qquad y = 9(2)(2 - 2) = 0$$

Hence, (6 m, 0) **Ans.**

$$\mathbf{v}_A = \frac{d\mathbf{r}_A}{dt} = 3\mathbf{i} + (18 - 18t)\mathbf{j}$$

$$\mathbf{v}_A\big|_{t=2} = \{3\mathbf{i} - 18\mathbf{j}\}\text{ m/s}$$

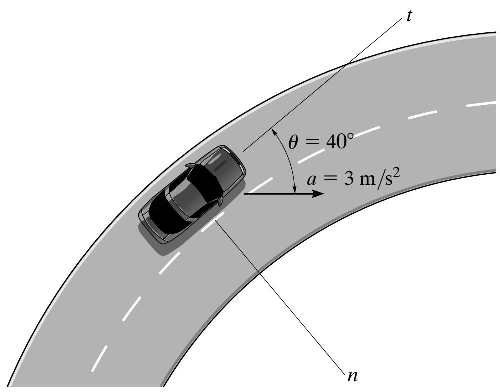
$$v_A = \sqrt{(3)^2 + (-18)^2} = 18.2\text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_B = \frac{d\mathbf{r}_B}{dt} = 3(2t - 2)\mathbf{i} + 3\mathbf{j}$$

$$\mathbf{v}_B\big|_{t=2} = \{6\mathbf{i} + 3\mathbf{j}\}\text{ m/s}$$

$$v_B = \sqrt{(6)^2 + (3)^2} = 6.71\text{ m/s} \qquad \textbf{Ans.}$$

R1–49. Determine the speed of the automobile if it has the acceleration shown and is traveling on a road which has a radius of curvature of $\rho = 50$ m. Also, what is the automobile's rate of increase in speed?



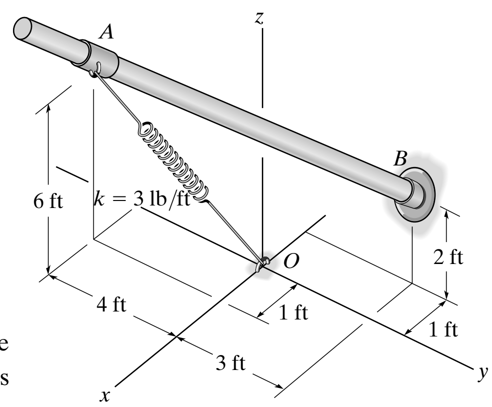
$$a_n = \frac{v^2}{\rho}$$

$$3 \sin 40° = \frac{v^2}{50}$$

$v = 9.82$ m/s **Ans.**

$a_t = 3 \cos 40° = 2.30\ \text{m/s}^2$ **Ans.**

R1–50. The spring has a stiffness $k = 3$ lb/ft and an unstretched length of 2 ft. If it is attached to the 5-lb smooth collar and the collar is released from rest at A, determine the speed of the collar just before it strikes the end of the rod at B. Neglect the size of the collar.



Potential Energy: Datum is set at point B. The collar is $(6 - 2) = 4$ ft *above* the datum when it is at A. Thus, its gravitational potential energy at this point is $5(4) = 20.0$ ft · lb. The length of the spring when the collar is at points A and B are calculated as $l_{OA} = \sqrt{1^2 + 4^2 + 6^2} = \sqrt{53}$ ft and $l_{OB} = \sqrt{1^2 + 3^2 + 2^2} = \sqrt{14}$ ft, respectively. The initial and final elastic potential energy are $\frac{1}{2}(3)\left(\sqrt{53} - 2\right)^2 = 41.82$ ft · lb and $\frac{1}{2}(3)\left(\sqrt{14} - 2\right)^2 = 4.550$ ft · lb, respectively.

Conservation of Energy: Applying Eq. 14–22, we have

$$\Sigma T_A + \Sigma V_A = \Sigma T_B + \Sigma V_B$$

$$0 + 20.0 + 41.82 = \frac{1}{2}\left(\frac{5}{32.2}\right)v_B^2 + 4.550$$

$v_B = 27.2$ ft/s **Ans.**