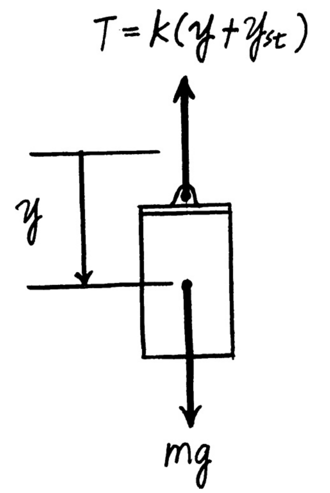
•22–1. A spring is stretched 175 mm by an 8-kg block. If the block is displaced 100 mm downward from its equilibrium position and given a downward velocity of 1.50 m/s, determine the differential equation which describes the motion. Assume that positive displacement is downward. Also, determine the position of the block when $t = 0.22$ s.



$+\downarrow \Sigma F_y = ma_y; \qquad mg - k(y + y_{st}) = m\ddot{y} \qquad \text{where } ky_{st} = mg$

$$\ddot{y} + \frac{k}{m}y = 0$$

Hence $p = \sqrt{\dfrac{k}{m}}$ Where $k = \dfrac{8(9.81)}{0.175} = 448.46$ N/m

$$= \sqrt{\frac{448.46}{8}} = 7.487$$

$\therefore \qquad \ddot{y} + (7.487)^2 y = 0 \qquad \ddot{y} + 56.1y = 0$ **Ans.**

The solution of the above differential equation is of the form:

$$y = A \sin pt + B \cos pt \qquad \textbf{(1)}$$

$$v = \dot{y} = Ap \cos pt - Bp \sin pt \qquad \textbf{(2)}$$

At $t = 0$, $y = 0.1$ m and $v = v_0 = 1.50$ m/s

From Eq. (1) $0.1 = A \sin 0 + B \cos 0 \qquad B = 0.1$ m

From Eq. (2) $v_0 = Ap \cos 0 - 0 \qquad A - \dfrac{v_0}{p} = \dfrac{1.50}{7.487} = 0.2003$ m

Hence $\quad y = 0.2003 \sin 7.487t + 0.1 \cos 7.487t$

At $t = 0.22$ s, $\quad y = 0.2003 \sin \left[7.487(0.22)\right] + 0.1 \cos \left[7.487(0.22)\right]$

$= 0.192$ m **Ans.**

22–2. When a 2-kg block is suspended from a spring, the spring is stretched a distance of 40 mm. Determine the frequency and the period of vibration for a 0.5-kg block attached to the same spring.

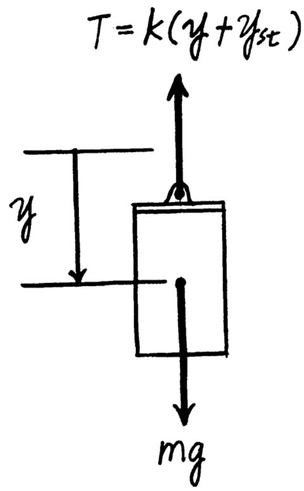
$$k = \frac{F}{y} = \frac{2(9.81)}{0.040} = 490.5 \text{ N/m}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{490.5}{0.5}} = 31.321$$

$$f = \frac{p}{2\pi} = \frac{31.321}{2\pi} = 4.985 \text{ Hz}$$ **Ans.**

$$\tau = \frac{1}{f} = \frac{1}{4.985} = 0.201 \text{ s}$$ **Ans.**

22–3. A block having a weight of 8 lb is suspended from a spring having a stiffness $k = 40$ lb/ft. If the block is pushed $y = 0.2$ ft upward from its equilibrium position and then released from rest, determine the equation which describes the motion. What are the amplitude and the natural frequency of the vibration? Assume that positive displacement is downward.



$+\downarrow \Sigma F_y = ma_y;\qquad mg - k(y + y_{st}) = m\ddot{y}\qquad \text{where } ky_{st} = mg$

$$\ddot{y} + \frac{k}{m}y = 0$$

Hence $$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{40}{8/32.2}} = 12.689$$

$$f = \frac{p}{2\pi} = \frac{12.689}{2\pi} = 2.02 \text{ Hz}\qquad \textbf{Ans.}$$

The solution of the above differential equation is of the form:

$$v = A \sin pt + B \cos pt \qquad \textbf{(1)}$$

$$v = \dot{y} = Ap \cos pt - Bp \sin pt \qquad \textbf{(2)}$$

At $t = 0$, $y = -0.2$ ft and $v = v_0 = 0$

From Eq. (1) $-0.2 = A \sin 0° + B \cos 0° \qquad B = -0.2$ ft

From Eq. (2) $\quad v_0 = Ap \cos 0° - 0 \qquad A = \dfrac{v_0}{p} = \dfrac{0}{12.689} = 0$

Hence $\qquad y = -0.2 \cos 12.7t \qquad$ **Ans.**

Amplitude $\qquad C = 0.2$ ft $\qquad$ **Ans.**

***22–4.** A spring has a stiffness of 800 N/m. If a 2-kg block is attached to the spring, pushed 50 mm above its equilibrium position, and released from rest, determine the equation that describes the block's motion. Assume that positive displacement is downward.

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{800}{2}} = 20$$

$$y = A \sin pt + B \cos pt$$

$$y = -0.05 \text{ m when } t = 0,$$

$$-0.05 = 0 + B; \qquad B = -0.05$$

$$v = Ap \cos pt - Bp \sin pt$$

$$v = 0 \text{ when } t = 0,$$

$$0 = A(20) - 0; \qquad A = 0$$

Thus,

$$y = -0.05 \cos (20t) \qquad \textbf{Ans.}$$

•22–5. A 2-kg block is suspended from a spring having a stiffness of 800 N/m. If the block is given an upward velocity of 2 m/s when it is displaced downward a distance of 150 mm from its equilibrium position, determine the equation which describes the motion. What is the amplitude of the motion? Assume that positive displacement is downward.

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{800}{2}} = 20$$

$$x = A \sin pt + B \cos pt$$

$$x = 0.150 \text{ m when } t = 0,$$

$$0.150 = 0 + B; \qquad B = 0.150$$

$$v = Ap \cos pt - Bp \sin pt$$

$$v = -2 \text{ m/s when } t = 0,$$

$$-2 = A(20) - 0; \qquad A = -0.1$$

Thus,

$$x = 0.1 \sin (20t) + 0.150 \cos (20t) \qquad \textbf{Ans.}$$

$$C = \sqrt{A^2 + B^2} = \sqrt{(0.1)^2 + (0.150)^2} = 0.180 \text{ m} \qquad \textbf{Ans.}$$

22–6. A spring is stretched 200 mm by a 15-kg block. If the block is displaced 100 mm downward from its equilibrium position and given a downward velocity of 0.75 m/s, determine the equation which describes the motion. What is the phase angle? Assume that positive displacement is downward.

$$k = \frac{F}{y} = \frac{15(9.81)}{0.2} = 735.75 \text{ N/m}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{735.75}{15}} = 7.00$$

$$y = A \sin pt + B \cos pt$$

$$y = 0.1 \text{ m when } t = 0,$$

$$0.1 = 0 + B; \qquad B = 0.1$$

$$v = Ap \cos pt - Bp \sin pt$$

$$v = 0.75 \text{ m/s when } t = 0,$$

$$0.75 = A(7.00)$$

$$A = 0.107$$

$$y = 0.107 \sin (7.00t) + 0.100 \cos (7.00t) \qquad \textbf{Ans.}$$

$$\phi = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{0.100}{0.107}\right) = 43.0^\circ \qquad \textbf{Ans.}$$

22–7. A 6-kg block is suspended from a spring having a stiffness of $k = 200$ N/m. If the block is given an upward velocity of 0.4 m/s when it is 75 mm above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the block measured from the equilibrium position. Assume that positive displacement is downward.

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{6}} = 5.774$$

$$x = A \sin pt + B \cos pt$$

$$x = 0.075 \text{ m when } t = 0.$$

$$-0.075 = 0 + B; \qquad B = -0.075$$

$$v = Ap \cos pt - Bp \sin pt$$

$$v = -0.4 \text{ m/s when } t = 0,$$

$$-0.4 = A(5.774) - 0; \qquad A = -0.0693$$

Thus,

$$x = -0.0693 \sin (5.77t) - 0.075 \cos (5.77t)$$ **Ans.**

$$C = \sqrt{A^2 + B^2} = \sqrt{(-0.0693)^2 + (-0.075)^2} = 0.102 \text{ m}$$ **Ans.**

***22–8.** A 3-kg block is suspended from a spring having a stiffness of $k = 200$ N/m. If the block is pushed 50 mm upward from its equilibrium position and then released from rest, determine the equation that describes the motion. What are the amplitude and the frequency of the vibration? Assume that positive displacement is downward.

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{3}} = 8.165$$

$$f = \frac{p}{2\pi} = \frac{8.165}{2\pi} = 1.299 = 1.30 \text{ Hz}$$ **Ans.**

$$x = A \sin pt + B \cos pt$$

$$x = -0.05 \text{ m when } t = 0,$$

$$-0.05 = 0 + B; \qquad B = -0.05$$

$$v = Ap \cos pt - Bp \sin pt$$

$$v = 0 \text{ when } t = 0,$$

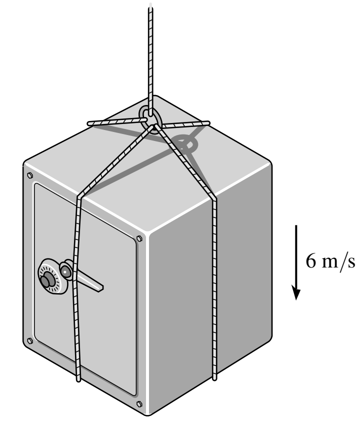
$$0 = A(8.165) - 0; \qquad A = 0$$

Hence,

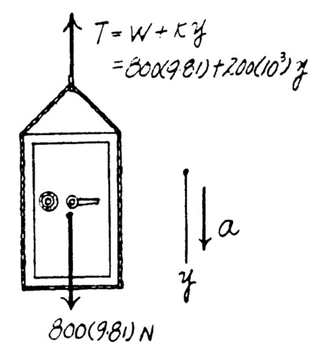
$$x = -0.05 \cos (8.16t)$$ **Ans.**

$$C = \sqrt{A^2 + B^2} = \sqrt{(0)^2 + (-0.05)} = 0.05 \text{ m} = 50 \text{ mm}$$ **Ans.**

•22–9. A cable is used to suspend the 800-kg safe. If the safe is being lowered at 6 m/s when the motor controlling the cable suddenly jams (stops), determine the maximum tension in the cable and the frequency of vibration of the safe. Neglect the mass of the cable and assume it is elastic such that it stretches 20 mm when subjected to a tension of 4 kN.



Free-body Diagram: Here the stiffness of the cable is $k = \dfrac{4000}{0.02} = 200\left(10^3\right)$ N/m. When the safe is being displaced by an amount y downward vertically from its equilibrium position, the *restoring force* that developed in the cable $T = W + ky = 800(9.81) + 200\left(10^3\right) y$.



Equation of Motion:

$$+\uparrow \Sigma F_x = 0; \qquad 800(9.81) + 200\left(10^3\right) y - 800(9.81) = -800a \qquad \textbf{[1]}$$

Kinematics: Since $a = \dfrac{d^2y}{dt^2} = \ddot{y}$, then substituting this value into Eq. [1], we have

$$200\left(10^3\right) y = -800\ddot{y}$$

$$\ddot{y} + 250x = 0 \qquad \textbf{[2]}$$

From Eq. [2], $p^2 = 250$, thus, $p = 15.81$ rad/s. Applying Eq. 22–14, we have

$$f = \frac{p}{2\pi} = \frac{15.81}{2\pi} = 2.52 \text{ Hz} \qquad \textbf{Ans.}$$

The solution of the above differential equation (Eq. [2]) is in the form of

$$y = C \sin (15.81t + \phi) \qquad \textbf{[3]}$$

Taking the time derivative of Eq. [3], we have

$$\dot{y} = 15.81\, C \cos (15.81t + \phi) \qquad \textbf{[4]}$$

Applying the initial condition of $y = 0$ and $\dot{y} = 6$ m/s at $t = 0$ to Eqs. [3] and [4] yields

$$0 = C \sin \phi \qquad \textbf{[5]}$$

$$6 = 15.81\, C \cos \phi \qquad \textbf{[6]}$$

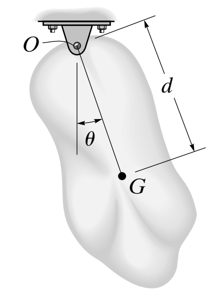
Solving Eqs. [5] and [6] yields

$$\phi = 0^\circ \qquad C = 0.3795 \text{ m}$$

Since $v_{max} = C = 0.3795$ m, the maximum cable tension is given by

$$T_{max} = W + ky_{max} = 800(9.81) + 200\left(10^3\right)(0.3795) = 83.7 \text{ kN} \qquad \textbf{Ans.}$$

22–10. The body of arbitrary shape has a mass m, mass center at G, and a radius of gyration about G of k_G. If it is displaced a slight amount θ from its equilibrium position and released, determine the natural period of vibration.



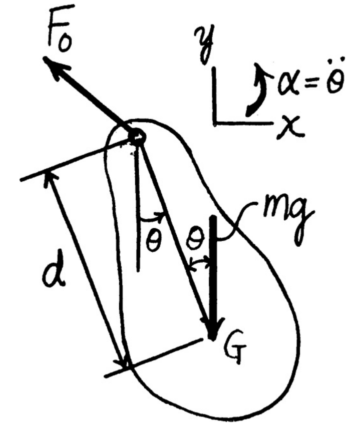
$$\zeta +\Sigma M_O = I_O\,\alpha; \qquad -mgd\sin\theta = \left[mk_G^2 + md^2\right]\ddot{\theta}$$

$$\ddot{\theta} + \frac{gd}{k_G^2 + d^2}\sin\theta = 0$$

However, for small rotation $\sin\theta \approx \theta$. Hence

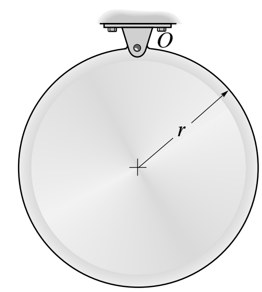
$$\ddot{\theta} + \frac{gd}{k^2 + d^2}\theta = 0$$

From the above differential equation, $p = \sqrt{\dfrac{gd}{k_G^2 + d^2}}$.



$$\tau = \frac{2\pi}{p} = \frac{2\pi}{\sqrt{\dfrac{gd}{k_G^2 + d^2}}} = 2\pi\sqrt{\frac{k_G^2 + d^2}{gd}}$$ **Ans.**

22–11. The circular disk has a mass m and is pinned at O. Determine the natural period of vibration if it is displaced a small amount and released.

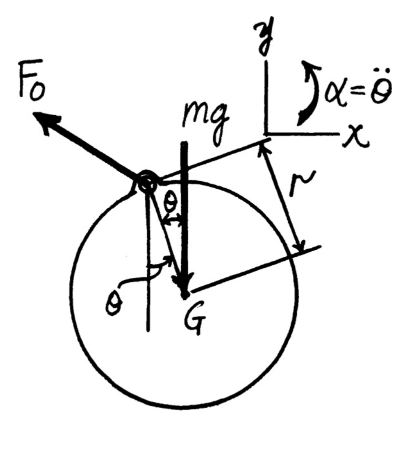


$$\zeta +\Sigma M_O = I_O\,\alpha; \qquad -mgr\theta = \left(\frac{3}{2}mr^2\right)\ddot{\theta}$$

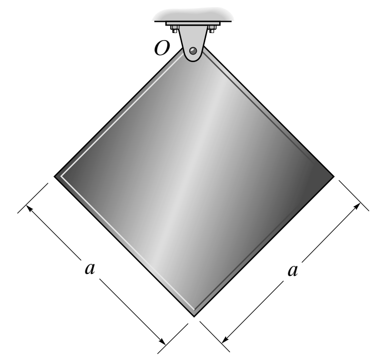
$$\ddot{\theta} + \left(\frac{2g}{3r}\right)\theta = 0$$

$$p = \sqrt{\frac{2g}{3r}}$$

$$\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{3r}{2g}}$$ **Ans.**



***22–12.** The square plate has a mass m and is suspended at its corner from a pin O. Determine the natural period of vibration if it is displaced a small amount and released.



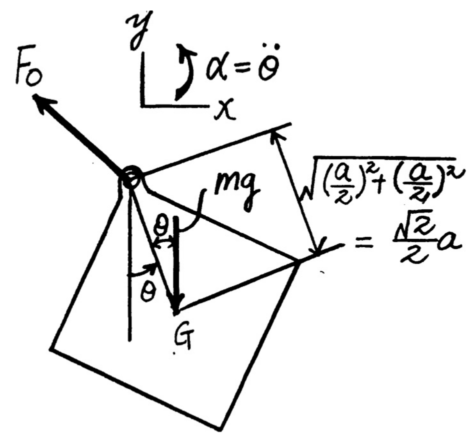
$$I_O = \frac{1}{12}m(a^2 + a^2) + m\left(\frac{\sqrt{2}}{2}a\right)^2 = \frac{1}{6}ma^2 + \frac{1}{2}ma^2 = \frac{2}{3}ma^2$$

$$\circlearrowleft +\Sigma M_O = I_O\,\alpha; \qquad -mg\left(\frac{\sqrt{2}}{2}a\right)\theta = \left(\frac{2}{3}ma^2\right)\ddot{\theta}$$

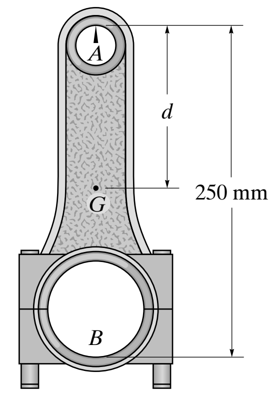
$$\dot{\theta} + \left(\frac{3\sqrt{2}g}{4a}\right)\theta = 0$$

$$p = \sqrt{\frac{3\sqrt{2}g}{4a}}$$

$$\tau = \frac{2\pi}{p} = 6.10\sqrt{\frac{a}{g}}$$ **Ans.**



•22–13. The connecting rod is supported by a knife edge at A and the period of vibration is measured as $\tau_A = 3.38$ s. It is then removed and rotated 180° so that it is supported by the knife edge at B. In this case the perod of vibration is measured as $\tau_B = 3.96$ s. Determine the location d of the center of gravity G, and compute the radius of gyration k_G.



Free-body Diagram: When an object of arbitary shape having a mass m is pinned at O and is displaced by an angular displacement of θ, the tangential component of its weight will create the *restoring moment* about point O.

Equation of Motion: Sum monent about point O to eliminate O_x and O_y.

$$\zeta + \Sigma M_O = I_O\alpha; \qquad -mg \sin\theta(l) = I_O\,\alpha \qquad \textbf{[1]}$$

Kinematics: Since $\alpha = \dfrac{d^2\theta}{dt^2} = \ddot{\theta}$ and $\sin\theta = \theta$ if θ is small, then substitute these values into Eq. [1], we have

$$-mgl\theta = I_O\ddot{\theta} \qquad \text{or} \qquad \ddot{\theta} + \frac{mgl}{I_O}\theta = 0 \qquad \textbf{[2]}$$

From Eq. [2], $p^2 = \dfrac{mgl}{I_O}$, thus, $p = \sqrt{\dfrac{mgl}{I_O}}$. Applying Eq. 22–12, we have

$$\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{I_O}{mgl}} \qquad \textbf{[3]}$$

When the rod is rotating about B, $\tau = \tau_A = 3.38$ s and $l = d$. Substitute these values into Eq. [3], we have

$$3.38 = 2\pi\sqrt{\frac{I_A}{mgd}} \qquad I_A = 0.2894mgd$$

When the rod is rotating about B, $\tau = \tau_B = 3.96$ s and $l = 0.25 - d$. Substitute these values into Eq. [3], we have

$$3.96 = 2\pi\sqrt{\frac{I_B}{mg\,(0.25 - d)}} \qquad I_B = 0.3972mg\,(0.25 - d)$$

However, the mass moment inertia of the rod about its mass center is

$$I_G = I_A - mg^2 = I_B - m(0.25 - d)^2$$

Then,

$$0.2894mgd - md^2 = 0.3972mg\,(0.25 - d) - m\,(0.25 - d)^2$$

$$d = 0.1462 \text{ m} = 146 \text{ mm} \qquad \textbf{Ans.}$$

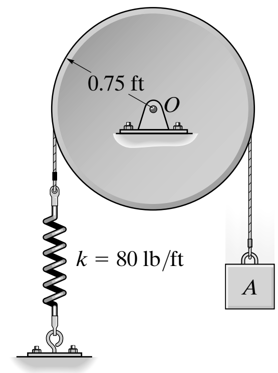
Thus, the mass moment inertia of the rod about its mass center is

$$I_G = I_A - md^2 = 0.2894m\,(9.81)(0.1462) - m\left(0.1462^2\right) = 0.3937\,m$$

The radius of gyration is

$$k_G = \sqrt{\frac{I_G}{m}} = \sqrt{\frac{0.3937m}{m}} = 0.627 \text{ m} \qquad \textbf{Ans.}$$

22–14. The disk, having a weight of 15 lb, is pinned at its center O and supports the block A that has a weight of 3 lb. If the belt which passes over the disk does not slip at its contacting surface, determine the natural period of vibration of the system.



For equilibrium:

$T_{st} = 3 \text{ lb}$

$\zeta\, \Sigma M_O = I_O\,\alpha + ma(0.75)$

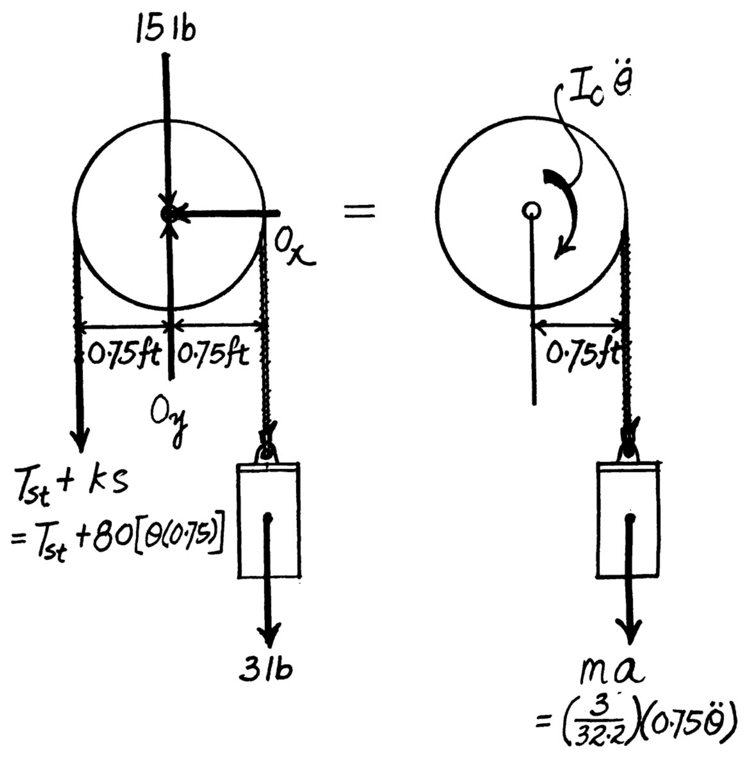
$a = 0.75\alpha$

$$-T_{st}(0.75) - (80)(\theta)(0.75)(0.75) + (3)(0.75) = \left[\frac{1}{2}\left(\frac{15}{32.2}\right)(0.75)^2\right]\ddot{\theta} + \left(\frac{3}{32.2}\right)(0.75)\ddot{\theta}(0.75)$$

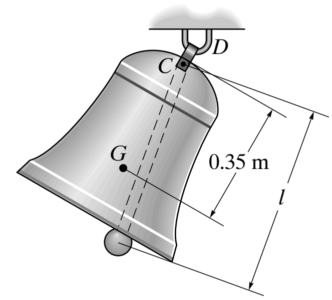
$$-2.25 - 45\theta + 2.25 = 0.131\ddot{\theta} + 0.05241\ddot{\theta}$$

$$\ddot{\theta} + 245.3\theta = 0$$

$$\tau = \frac{2\pi}{p} = \frac{2\pi}{\sqrt{245.3}} = 0.401 \text{ s}$$ **Ans.**



22–15. The bell has a mass of 375 kg, a center of mass at G, and a radius of gyration about point D of $k_D = 0.4$ m. The tongue consists of a slender rod attached to the inside of the bell at C. If an 8-kg mass is attached to the end of the rod, determine the length l of the rod so that the bell will "ring silent," i.e., so that the natural period of vibration of the tongue is the same as that of the bell. For the calculation, neglect the small distance between C and D and neglect the mass of the rod.



For an arbitrarily shaped body which rotates about a fixed point.

$\zeta + \Sigma M_O = I_O\,\alpha; \qquad mgd\sin\theta = -I_O\ddot{\theta}$

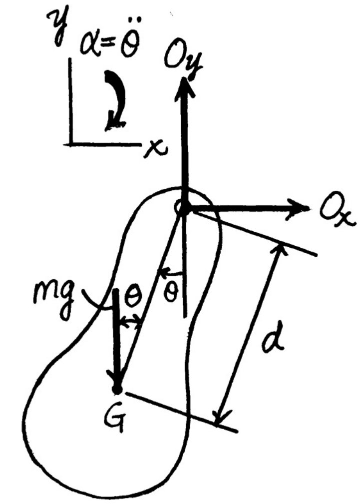
$$\ddot{\theta} + \frac{mgd}{I_O}\sin\theta = 0$$

However, for small rotation $\sin\theta \approx \theta$. Hence

$$\ddot{\theta} + \frac{mgd}{I_O}\theta = 0$$

From the above differential equation, $p = \sqrt{\dfrac{mgd}{I_O}}$.

$$\tau = \frac{2\pi}{p} = \frac{2\pi}{\sqrt{\dfrac{mgd}{I_O}}} = 2\pi\sqrt{\frac{I_O}{mgd}}$$



In order to have an equal period

$$\tau = 2\pi\sqrt{\frac{(I_O)_T}{m_T g d_T}} = 2\pi\sqrt{\frac{(I_O)_B}{m_B\,g d_B}}$$

$(I_O)_T$ = moment of inertia of tongue about O.

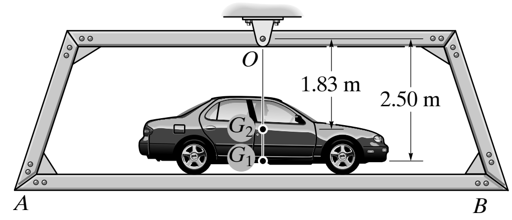
$(I_O)_B$ = moment of inertia of bell about O.

$$\frac{(I_O)_T}{m_T g d_T} = \frac{(I_O)_B}{m_B\,g d_B}$$

$$\frac{8(l^2)}{8gl} = \frac{375(0.4)^2}{375g(0.35)}$$

$$l = 0.457 \text{ m} \qquad \textbf{Ans.}$$

***22–16.** The platform AB when empty has a mass of 400 kg, center of mass at G_1, and natural period of oscillation $\tau_1 = 2.38$ s. If a car, having a mass of 1.2 Mg and center of mass at G_2, is placed on the platform, the natural period of oscillation becomes $\tau_2 = 3.16$ s. Determine the moment of inertia of the car about an axis passing through G_2.



Free-body Diagram: When an object of arbitrary shape having a mass m is pinned at O and being displaced by and angular displacement of θ, the tangential component of its weight will create the *restoring moment* about point O.

Equation of Motion: Sum moment about point O to eliminate O_x and O_y.

$$\zeta + \Sigma M_O = I_O\,\alpha; \qquad -mg\sin\theta(l) = I_O\,\alpha \qquad \textbf{[1]}$$

Kinematics: Since $\alpha = \dfrac{d^2\theta}{dt^2} = \ddot{\theta}$ and $\sin\theta \approx \theta$ if θ is small, then substitute these values into Eq. [1], we have

$$-mgI\theta = I_O\,\ddot{\theta} \qquad \text{or} \qquad \ddot{\theta} + \frac{mgl}{I_O}\theta = 0 \qquad \textbf{[2]}$$

From Eq. [2], $p^2 = \dfrac{mgl}{I_O}$, thus, $p = \sqrt{\dfrac{mgl}{I_O}}$. Applying Eq. 22–12, we have

$$\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{I_O}{mgl}} \qquad \textbf{[3]}$$

When the platform is empty, $\tau = \tau_1 = 2.38$ s. $m = 400$ kg and $l = 250$ m. Substitute these values into Eq. [3], we have

$$2.38 = 2\pi\sqrt{\frac{(I_O)_p}{400(9.81)(2.50)}} \qquad (I_O)_p = 1407.55\ \text{kg}\cdot\text{m}^2$$

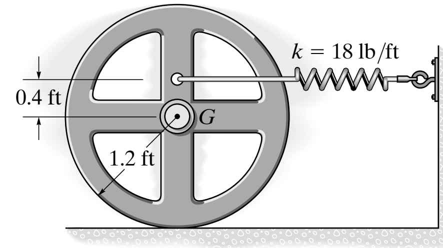
When the car is on the platform, $\tau = \tau_2 = 3.16$ s, $m = 400\ \text{kg} + 1200\ \text{kg} = 1600\ kg$, $l = \dfrac{2.50(400) + 1.83(1200)}{1600} = 1.9975$ m and $I_O = (I_O)_C + (I_O)_p = (I_O)_C + 1407.55$. Substitute these values into Eq. [3], we have

$$3.16 = 2\pi\sqrt{\frac{(I_O)_C + 1407.55}{1600(9.81)(1.9975)}} \qquad (I_O)_C = 6522.76\ \text{kg}\cdot\text{m}^2$$

Thus, the mass moment inertia of the car about its mass center is

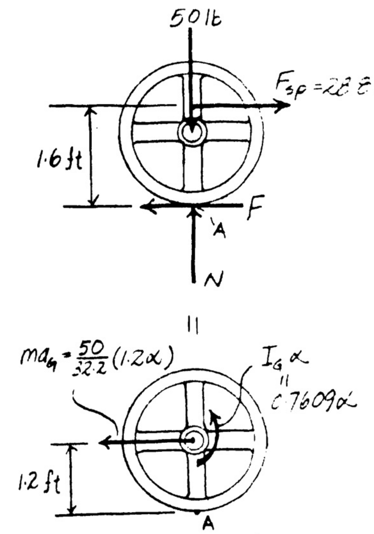
$$(I_G)_C = (I_O)_C - m_C d^2$$
$$= 6522.76 - 1200\left(1.83^2\right) = 2.50\left(10^3\right)\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

•**22–17.** The 50-lb wheel has a radius of gyration about its mass center G of $k_G = 0.7$ ft. Determine the frequency of vibration if it is displaced slightly from the equilibrium position and released. Assume no slipping.



Kinematics: Since the wheel rolls without slipping, then $a_G = \alpha r = 1.2\alpha$. Also when the wheel undergoes a small angular displacement θ about point A, the spring is stretched by $x = 1.6 \sin\theta\, \theta$. Since θ us small, then $\sin\theta - \theta$. Thus, $x = 1.6\,\theta$.

Free-body Diagram: The spring force $F_{sp} = kx = 18(1.6\theta) = 28.8\theta$ will create the *restoring moment* about point A.



Equation of Motion: The mass moment inertia of the wheel about its mass center is $I_G = mk_G^2 = \frac{50}{32.2}\left(0.7^2\right) = 0.7609 \text{ slug} \cdot \text{ft}^2$.

$$\zeta + \Sigma M_A = (M_k)_A; \qquad -28.8\theta(1.6) = \frac{50}{32.2}(1.2\alpha)(1.2) + 0.7609\alpha$$

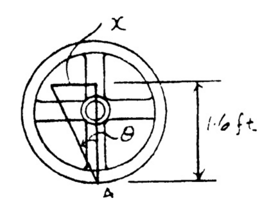
$$\alpha + 15.376\theta = 0 \qquad \textbf{[1]}$$

Since $\alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}$, then substitute this values into Eq. [1], we have

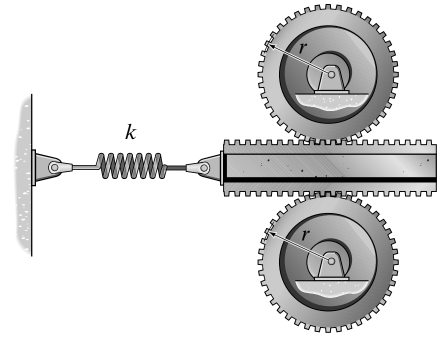
$$\ddot{\theta} + 15.376\theta = 0 \qquad \textbf{[2]}$$

From Eq. [2], $p^2 = 15.376$, thus, $p = 3.921$ rad/s. Applying Eq. 22–14, we have

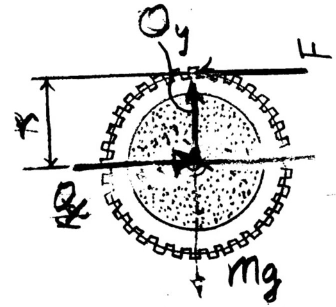
$$f = \frac{p}{2\pi} = \frac{3.921}{2\pi} = 0.624 \text{ Hz} \qquad \textbf{Ans.}$$



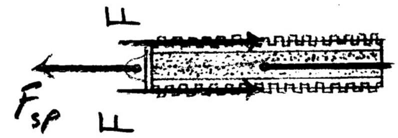
22–18. The two identical gears each have a mass of m and a radius of gyration about their center of mass of k_0. They are in mesh with the gear rack, which has a mass of M and is attached to a spring having a stiffness k. If the gear rack is displaced slightly horizontally, determine the natural period of oscillation.



Equation of Motion: When the gear rack is displaced horizontally downward by a small distance x, the spring is stretched by $s_1 = x$ Thus, $F_{sp} = kx$. Since the gears rotate about fixed axes, $\overline{x} = \dot{\theta} r$ or $\dot{\theta} = \dfrac{\overline{x}}{r}$. The mass moment of inertia of a gear about its mass center is $I_O = mk_O{}^2$. Referring to the free-body diagrams of the rack and gear in Figs. *a* and *b*,



(a)



(b)

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 2F - (kx) = M\overline{x}$$

$$2F - kx = M\overline{x} \qquad \textbf{(1)}$$

and

$$\circlearrowleft +\Sigma M_O = I_O\,\alpha; \qquad -F(r) = mk_O{}^2\left(\frac{\overline{x}}{r}\right)$$

$$F = -\frac{mk_O{}^2}{r^2}\overline{x} \qquad \textbf{(2)}$$

Eliminating **F** from Eqs. (1) and (2),

$$M\overline{x} + \frac{2mk_O{}^2}{r^2}\overline{x} + kx = 0$$

$$\overline{x} + \left(\frac{kr^2}{Mr^2 + 2mk_O{}^2}\right)x = 0$$

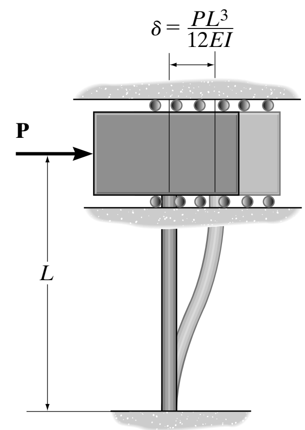
Comparing this equation to that of the standard from, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{kr^2}{Mr^2 + 2mk_O{}^2}}$$

Thus, the natural period of the oscillation is

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{Mr^2 + 2mk_O{}^2}{kr^2}} \qquad \textbf{Ans.}$$

22–19. In the "lump mass theory", a single-story building can be modeled in such a way that the whole mass of the building is lumped at the top of the building, which is supported by a cantilever column of negligible mass as shown. When a horizontal force **P** is applied to the model, the column deflects an amount of $\delta = PL^3/12EI$, where L is the effective length of the column, E is Young's modulus of elasticity for the material, and I is the moment of inertia of the cross section of the column. If the lump mass is m, determine the frequency of vibration in terms of these parameters.



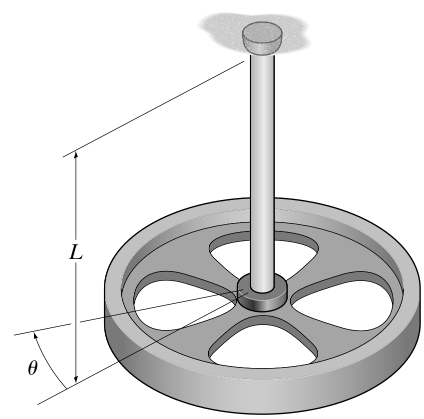
Since δ very small, the vibration can be assumed to occur along the horizontal. Here, the equivalent spring stiffness of the cantilever column is $k_{eq} = \frac{P}{\delta} = \frac{P}{PL^3/12EI} = \frac{12EI}{I^3}$. Thus, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{\frac{12EI}{I^3}}{m}} = \sqrt{\frac{12EI}{mL^3}}$$

Then the natural frequency of the system is

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{12EI}{mL^3}} \qquad \textbf{Ans.}$$

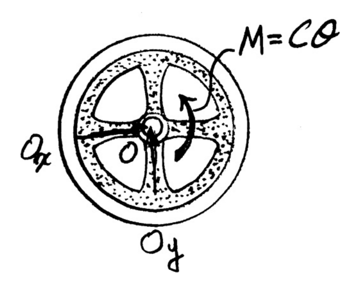
***22–20.** A flywheel of mass m, which has a radius of gyration about its center of mass of k_O, is suspended from a circular shaft that has a torsional resistance of $M = C\theta$. If the flywheel is given a small angular displacement of θ and released, determine the natural period of oscillation.



Equation of Motion: The mass moment of inertia of the wheel about point O is $I_O = mk_O{}^2$. Referring to Fig. a,

$$\circlearrowleft + \qquad \Sigma M_O = I_O\,\alpha; \qquad -C\theta = mk_O{}^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{C}{mk_O{}^2}\theta = 0$$



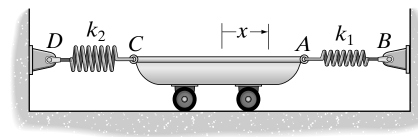
Comparing this equation to the standard equation, the natural circular frequency of the wheel is

$$\omega_n = \sqrt{\frac{C}{mk_O{}^2}} = \frac{1}{k_O}\sqrt{\frac{C}{m}}$$

Thus, the natural period of the oscillation is

$$\tau = \frac{2\pi}{\omega_n} = 2\pi k_O\sqrt{\frac{m}{C}} \qquad \textbf{Ans.}$$

•22–21. The cart has a mass of m and is attached to two springs, each having a stiffness of $k_1 = k_2 = k$, unstretched length of l_0, and a stretched length of l when the cart is in the equilibrium position. If the cart is displaced a distance of $x = x_0$ such that both springs remain in tension $(x_0 < l - l_0)$, determine the natural frequency of oscillation.



Equation of Motion: When the cart is displaced x to the right, the stretch of springs AB and CD are $s_{AB} = (l - l_0) - x_0$ and $s_{AC} = (l - l_0) + x$. Thus, $F_{AB} = ks_{AB} = k[(l - l_0) - x]$ and $F_{AC} = ks_{AC} = k[(l - l_0) + x]$. Referring to the free-body diagram of the cart shown in Fig. *a*,

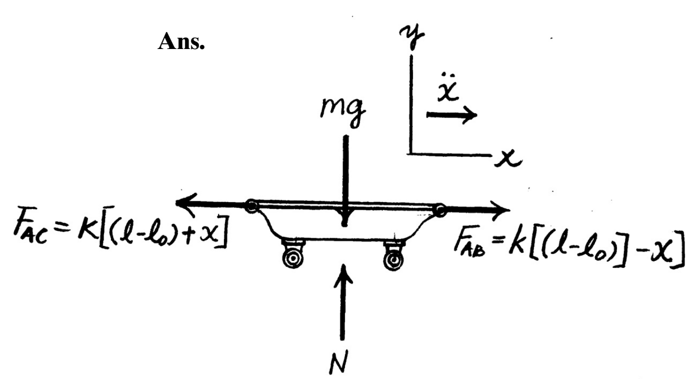
$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad k[(l - l_0) - x] - k[(l - l_0) + x] = m\bar{x}$$

$$-2kx = m\bar{x}$$

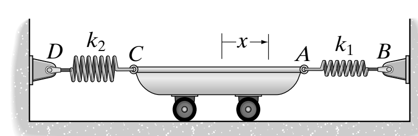
$$\bar{x} + \frac{2k}{m}x = 0$$

Simple Harmonic Motion: Comparing this equation with that of the standard form, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{2k}{m}} \qquad \textbf{Ans.}$$



22–22. The cart has a mass of m and is attached to two springs, each having a stiffness of k_1 and k_2, respectively. If both springs are unstretched when the cart is in the equilibrium position shown, determine the natural frequency of oscillation.

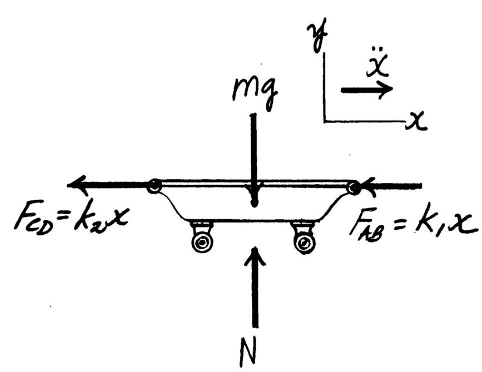


Equation of Motion: When the cart is displaced x to the right, spring CD stretches $s_{CD} = x$ and spring AB compresses $s_{AB} = x$. Thus, $F_{CD} = k_2 s_{CD} = k_2 x$ and $F_{AB} = k_1 s_{AB} = k_1 x$. Referring to the free-body diagram of the cart shown in Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -k_1 x - k_2 x = m\bar{x}$$

$$-(k_1 + k_2)x = m\bar{x}$$

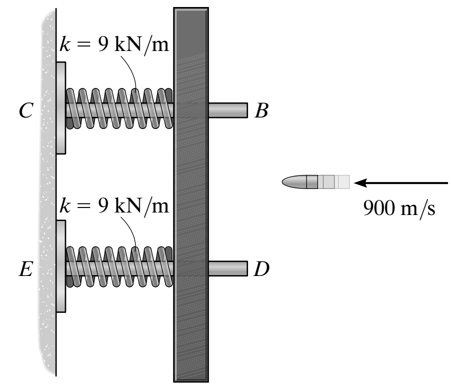
$$\bar{x} + \left(\frac{k_1 + k_2}{m}\right)x = 0$$



Simple Harmonic Motion: Comparing this equation with that of the standard equation, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{k_1 + k_2}{m}} \qquad \textbf{Ans.}$$

22–23. The 3-kg target slides freely along the smooth horizontal guides BC and DE, which are 'nested' in springs that each have a stiffness of $k = 9$ kN/m. If a 60-g bullet is fired with a velocity of 900 m/s and embeds into the target, determine the amplitude and frequency of oscillation of the target.



Conservation of Linear Momentum: The velocity of the target after impact can be determined from

$$m_b(v_b)_1 = (m_b + m_A)v$$

$$0.06(900) = (0.06 + 3)v$$

$$v = 17.65 \text{ m/s}$$

Since the springs are arranged in parallel, the equivalent stiffness of a single spring is $k_{eq} = 2k = 2(9000 \text{ N/m}) = 18000 \text{ N/m}$. Thus, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{18000}{3.06}} = 76.70 \text{ rad/s} = 76.7 \text{ rad/s} \qquad \textbf{Ans.}$$

The equation that describes the oscillation of the system is

$$y = C \sin(76.70t + \phi) \text{ m} \qquad \textbf{(1)}$$

Since $y = 0$ when $t = 0$,

$$0 = C \sin \phi$$

Since $C \neq 0$, $\sin \phi = 0$. Then $\phi = 0°$. Thus, Eq. (1) becomes

$$y = C \sin(76.70t) \qquad \textbf{(2)}$$

Taking the time derivative of Eq. (2),

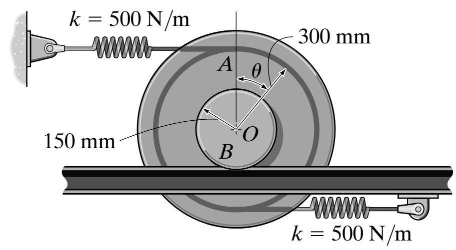
$$\dot{y} = v = 76.70C \cos(76.70t) \text{ m/s} \qquad \textbf{(3)}$$

Here, $v = 17.65$ m/s when $t = 0$. Thus, Eq. (3) gives

$$17.65 = 76.70C \cos 0$$

$$C = 0.2301 \text{ m} = 230 \text{ mm} \qquad \textbf{Ans.}$$

***22–24.** If the spool undergoes a small angular displacement of θ and is then released, determine the frequency of oscillation. The spool has a mass of 50 kg and a radius of gyration about its center of mass O of $k_O = 250$ mm. The spool rolls without slipping.



Equation of Motion: Referring to the kinematic diagram of the spool, Fig. *a*, the stretch of the spring at A and$B$ when the spool rotates through a small angle θ is $s_A = \theta r_{A/IC} = \theta(0.45)$ and $s_B = \theta r_{B/IC} = \theta(0.15)$. Thus, $\left(F_{sp}\right)_A = ks_A = 500[\theta(0.45)] = 225\theta$ and $\left(F_{sp}\right)_B = ks_B = 500[\theta(0.15)] = 75\theta$. Also, $a_O = \ddot{\theta} r_{O/IC} = \ddot{\theta}(0.15)$. The mass moment of inertia of the spool about its mass center is $I_O = mk_O{}^2 = 50\left(0.25^2\right) = 3.125\ \text{kg}\cdot\text{m}^2$. Referring the free-body and kinetic diagrams of the spool, Fig. *b*,

$$+\qquad \Sigma M_{IC} = \Sigma (M_k)_{IC}; \quad -225\theta(0.045) - 75\theta(0.15) = 50\left[\ddot{\theta}(0.15)\right](0.15) + 3.125\ddot{\theta}$$

$$-112.5\theta = 4.25\ddot{\theta}$$

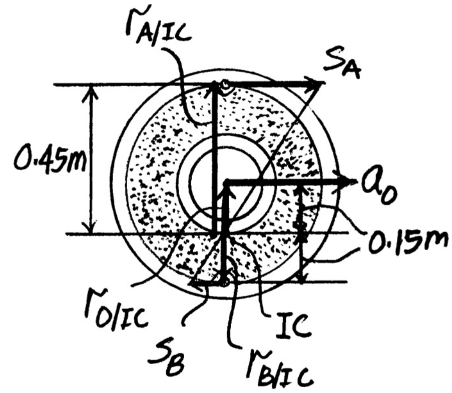
$$\ddot{\theta} + 26.47\theta = 0$$

Comparing this equation to that of the standard equation, the natural circular frequency of the spool is

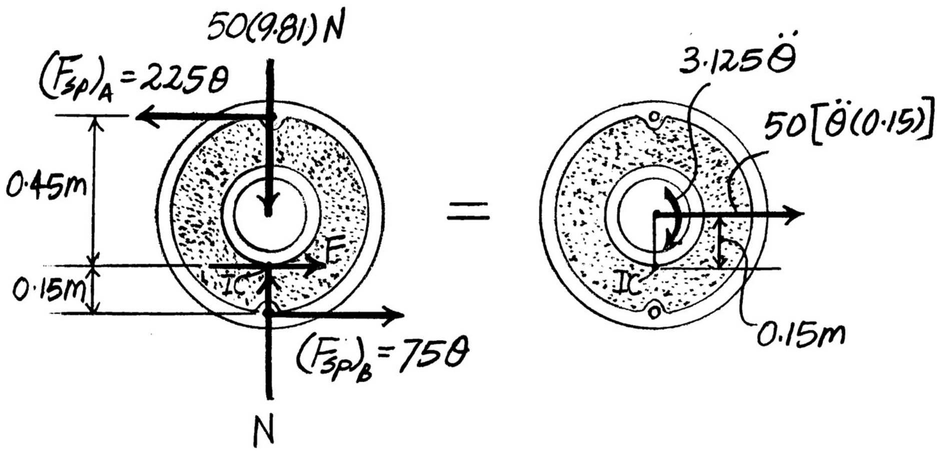
$$\omega_n = \sqrt{26.47}\ \text{rad/s} = 5.145\ \text{rad/s}$$

Thus, the natural frequency of the oscillation is

$$f_n = \frac{\omega_n}{2\pi} = \frac{5.145}{2\pi} = 0.819\ \text{Hz} \qquad \textbf{Ans.}$$

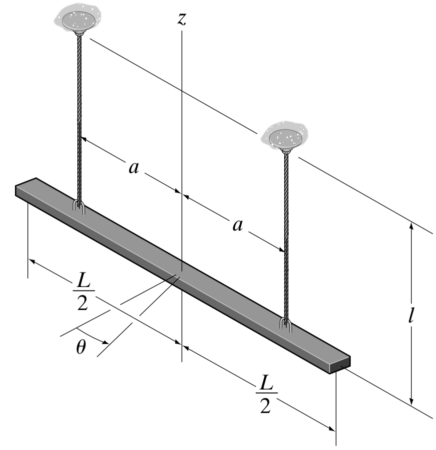


(a)



(b)

•22–25. The slender bar of mass m is supported by two equal-length cords. If it is given a small angular displacement of θ about the vertical axis and released, determine the natural period of oscillation.



Equation of Motion: The mass moment of inertia of the bar about the z axis is $I_z = \frac{1}{12}mL^2$. Referring to the free-body diagram of the bar shown in Fig. a,

$$+\uparrow \Sigma F_z = ma_z; \qquad 2T\cos\phi - mg = 0 \qquad\qquad T = \frac{mg}{2\cos\phi}$$

Then,

$$\uparrow + \Sigma M_z = I_z\alpha; \qquad -2\left(\frac{mg}{2\cos\phi}\right)\sin\phi(a) = \left(\frac{1}{12}mL^2\right)\ddot{\theta}$$

$$\ddot{\theta} + \frac{12ga}{L^2}\tan\phi = 0 \qquad \textbf{(1)}$$

Since θ is very small, from the geometry of Fig. b,

$$l\phi = a\theta$$

$$\phi = \frac{a}{l}\theta$$

Substituting this result into Eq. (1)

$$\ddot{\theta} + \frac{12ga}{L^2}\tan\left(\frac{a}{l}\theta\right) = 0$$

Since θ is very small, $\tan\left(\frac{g}{l}\theta\right) \cong \frac{g}{l}\theta$. Thus,

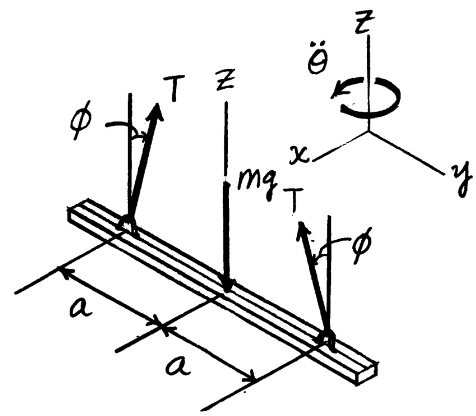
$$\ddot{\theta} + \frac{12ga^2}{IL^2}\theta = 0$$

Comparing this equation to that of the standard form, the natural circular frequency of the bar is

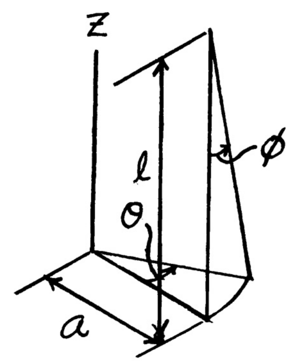
$$\omega_n = \sqrt{\frac{12ga^2}{IL^2}} = \frac{a}{L}\sqrt{\frac{12g}{l}}$$

Thus, the natural period of oscillation is

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi L}{a}\sqrt{\frac{l}{12g}} \qquad \textbf{Ans.}$$

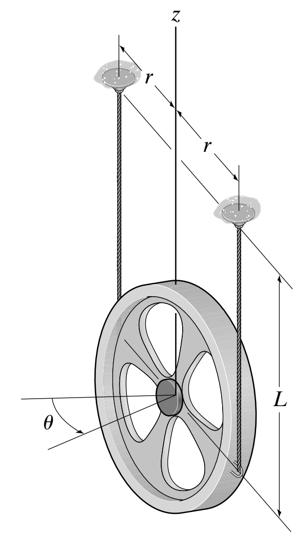


(a)



(b)

22–26. A wheel of mass m is suspended from two equal-length cords as shown. When it is given a small angular displacement of θ about the z axis and released, it is observed that the period of oscillation is τ. Determine the radius of gyration of the wheel about the z axis.



Equation of Motion: The mass moment of inertia of the wheel about is z axis is $I_z = mk_z^{\,2}$. Referring to the free-body diagram of the wheel shown in Fig. a,

$$+\uparrow \Sigma F_z = ma_z; \qquad 2T\cos\phi - mg = 0 \qquad T = \frac{mg}{2\cos\phi}$$

Then,

$$\uparrow + \Sigma M_z = I_z\,\alpha; \qquad -2\left(\frac{mg}{2\cos\phi}\right)\sin\phi(\mathrm{r}) = \left(\mathrm{mk_z}^2\right)\ddot{\theta}$$

$$\ddot{\theta} + \frac{gr}{k_z^{\,2}}\tan\phi = 0 \qquad \textbf{(1)}$$

Since θ is very small, from the geometry of Fig. b,

$$L\phi = r\theta$$

$$\phi = \frac{r}{L}\theta$$

Substituting this result into Eq. (1)

$$\ddot{\theta} + \frac{gr}{k_z^{\,2}}\tan\left(\frac{r}{L}\theta\right) = 0$$

Since θ is very small, $\tan\left(\frac{r}{L}\theta\right) \cong \frac{r}{L}\theta$. Thus,

$$\ddot{\theta} + \frac{gr^2}{k_z^{\,2}L}\theta = 0$$

Comparing this equation to that of the standard form, the natural circular frequency of the wheel is

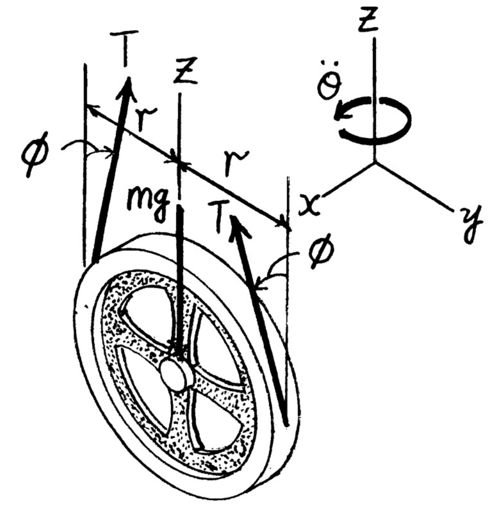
$$\omega_n = \sqrt{\frac{gr^2}{k_z^{\,2}L}} = \frac{r}{k_z}\sqrt{\frac{g}{L}}$$

Thus, the natural period of oscillation is

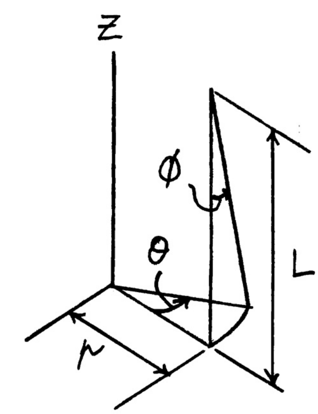
$$\tau = \frac{2\pi}{\omega_n}$$

$$\tau = 2\pi\left(\frac{k_z}{r}\sqrt{\frac{L}{g}}\right)$$

$$k_z = \frac{\tau r}{2\pi}\sqrt{\frac{g}{L}} \qquad \textbf{Ans.}$$

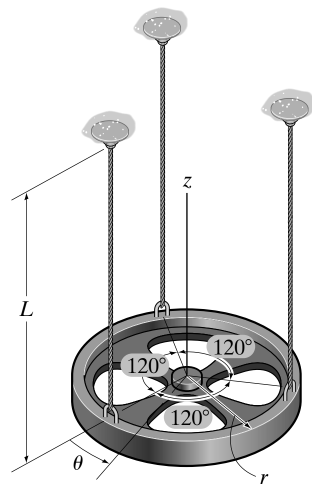


(a)



(b)

22–27. A wheel of mass m is suspended from three equal-length cords. When it is given a small angular displacement of θ about the z axis and released, it is observed that the period of oscillation is τ. Determine the radius of gyration of the wheel about the z axis.



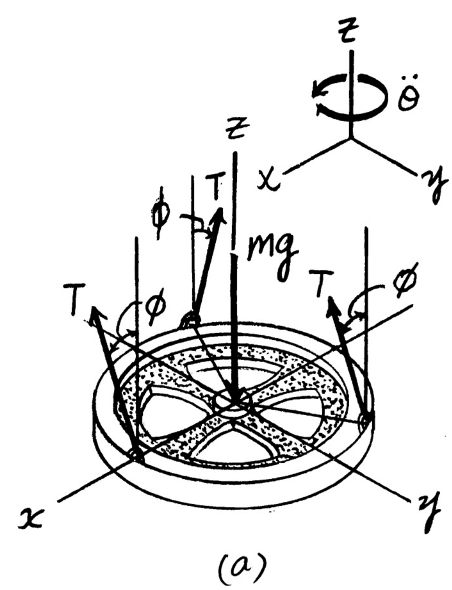
Equation of Motion: Due to symmetry, the force in each cord is the same. The mass moment of inertia of the wheel about is z axis is $I_z = mk_z{}^2$. Referring to the free-body diagram of the wheel shown in Fig. a,

$$+\uparrow \Sigma F_z = ma_z; \qquad 3T\cos\phi - mg = 0 \qquad\qquad T = \frac{mg}{3\cos\phi}$$

Then,

$$\uparrow + \Sigma M_z = I_z\,\alpha; \qquad -3\left(\frac{mg}{3\cos\phi}\right)\sin\phi(\mathrm{r}) = mk_z{}^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{gr}{k_z{}^2}\tan\phi = 0 \qquad \textbf{(1)}$$



Since θ is very small, from the geometry of Fig. b,

$$r\theta = L\phi$$

$$\phi = \frac{r}{L}\theta$$

Substituting this result into Eq. (1)

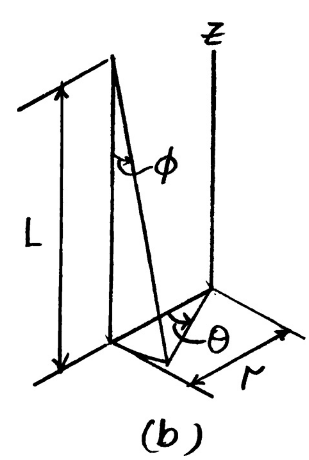
$$\ddot{\theta} + \frac{gr}{k_z{}^2}\tan\left(\frac{r}{L}\theta\right) = 0 \qquad \textbf{(2)}$$

Since θ is very small, $\tan\left(\frac{r}{L}\theta\right) \cong \frac{r}{L}\theta$. Thus,

$$\ddot{\theta} + \frac{gr^2}{kz^2L}\theta = 0$$

Comparing this equation to that of the standard form, the natural circular frequency of the wheel is

$$\omega_n = \sqrt{\frac{gr^2}{k_z{}^2L}} = \frac{r}{k_z}\sqrt{\frac{g}{L}}$$



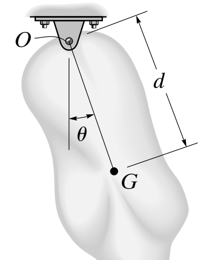
Thus, the natural period of oscillation is

$$\tau = \frac{2\pi}{\omega_n}$$

$$\tau = 2\pi\left(\frac{k_z}{r}\sqrt{\frac{L}{g}}\right)$$

$$k_z = \frac{\tau r}{2\pi}\sqrt{\frac{g}{L}} \qquad \textbf{Ans.}$$

***22–28.** Solve Prob. 22–10 using energy methods.



$$T + V = \frac{1}{2}\left[mk_G^2 + md^2\right]\theta^2 + mg(d)(1 - \cos\theta)$$

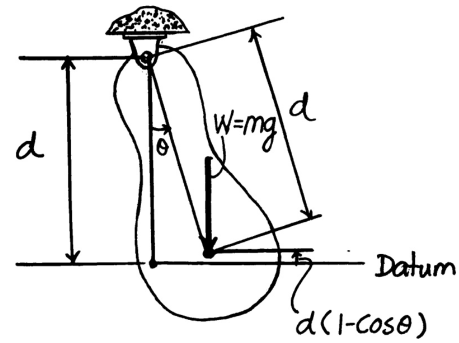
$$\left(k_G^2 + d^2\right)\dot{\theta}\,\ddot{\theta} + gd(\sin\theta)\dot{\theta} = 0$$

$$\sin\theta = \theta$$

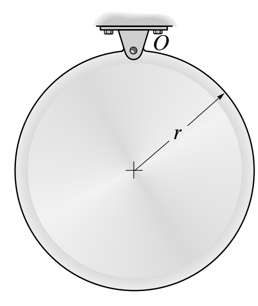
$$\ddot{\theta} + \frac{gd}{\left(k_G^2 + d^2\right)}\theta = 0$$

$$\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{\left(k_G^2 + d^2\right)}{gd}}$$

Ans.



•22–29. Solve Prob. 22–11 using energy methods.



$$T + V = \frac{1}{2}\left[\frac{3}{2}mr^2\right]\dot{\theta}^2 + mg(r)(1 - \cos\theta)$$

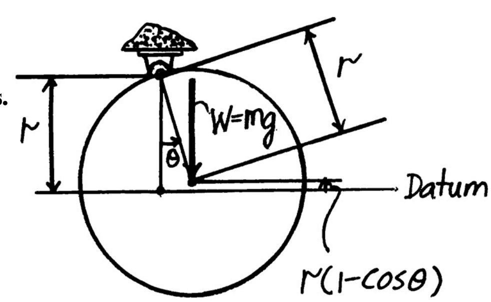
$$\frac{3}{2}mr^2\,\dot{\theta}\ddot{\theta} + mg(r)(\sin\theta)\dot{\theta} = 0$$

$$\sin\theta = \theta$$

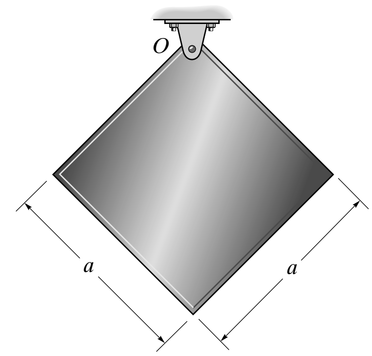
$$\theta + \left(\frac{2}{3}\right)\left(\frac{g}{r}\right)\theta = 0$$

$$\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{3r}{2g}}$$

Ans.



22–30. Solve Prob. 22–12 using energy methods.



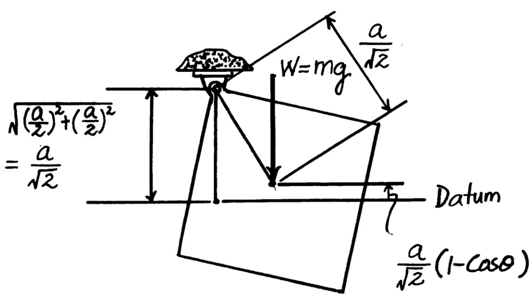
$$T + V = \frac{1}{2}\left[\frac{1}{12}m\left(a^2 + a^2\right) + m\left(\frac{a}{\sqrt{2}}\right)^2\right]\dot{\theta}^2 + mg\left(\frac{a}{\sqrt{2}}\right)(1 - \cos\theta)$$

$$\frac{2}{3}ma^2\,\dot{\theta}\,\ddot{\theta} + mg\left(\frac{a}{\sqrt{2}}\right)(\sin\theta)\dot{\theta} = 0$$

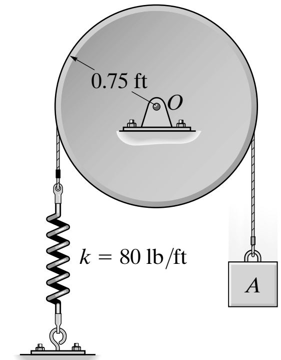
$$\sin\theta = \theta$$

$$\theta + \frac{3g}{2\sqrt{2}a}\theta = 0$$

$$\tau = \frac{2\pi}{p} = \frac{2\pi}{1.0299}\left(\sqrt{\frac{a}{g}}\right) = 6.10\sqrt{\frac{a}{g}}$$ **Ans.**



22–31. Solve Prob. 22–14 using energy methods.



$$s = 0.75\theta, \qquad \dot{s} = 0.75\dot{\theta}$$

$$T + V = \frac{1}{2}\left[\frac{1}{2}\left(\frac{15}{32.2}\right)(0.75)^2\right]\dot{\theta}^2 + \frac{1}{2}\left(\frac{3}{32.2}\right)\left(0.75\,\dot{\theta}\right)^2$$

$$+\frac{1}{2}(80)\,(s_{eq} + 0.75\theta)^2 - 3(0.75\theta)$$

$$0 = 0.1834\dot{\theta}\,\ddot{\theta} + 80(s_{eq} + 0.75\theta)\left(0.75\,\dot{\theta}\right) - 2.25\dot{\theta}$$

$$F_{eq} = 80s_{eq} = 3$$

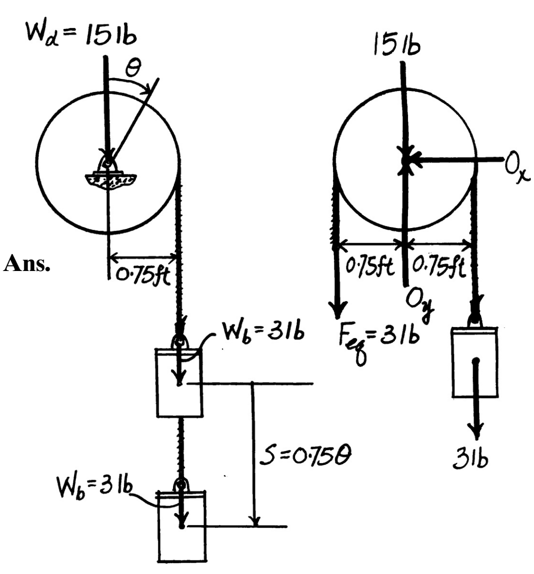
$$s_{eq} = 0.0375 \text{ ft}$$

Thus,

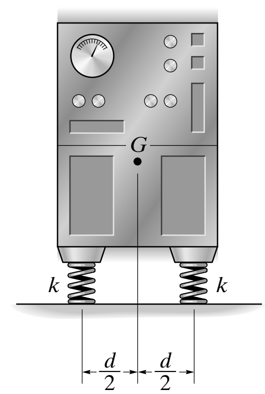
$$0.1834\ddot{\theta} + 45\theta = 0$$

$$\ddot{\theta} + 245.36 = 0$$

$$\tau = \frac{2\,\pi}{p} = \frac{2\pi}{\sqrt{245.3}} = 0.401 \text{ s}$$ **Ans.**



***22–32.** The machine has a mass m and is uniformly supported by *four* springs, each having a stiffness k. Determine the natural period of vertical vibration.



$$T + V = \text{const.}$$

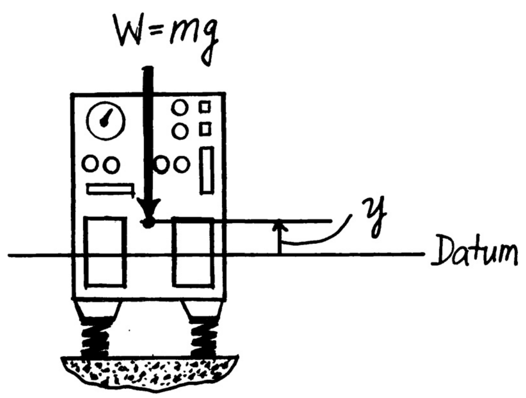
$$T = \frac{1}{2}m(\dot{y})^2$$

$$V = m\,g\,y + \frac{1}{2}(4k)(\Delta s - y)^2$$

$$T + V = \frac{1}{2}m(\dot{y})^2 + m\,g\,y + \frac{1}{2}(4k)(\Delta s - y)^2$$

$$m\,\dot{y}\,\ddot{y} + m\,g\,\dot{y} - 4k(\Delta s y)\dot{y} = 0$$

$$m\,\ddot{y} + m\,g + 4ky - 4k\Delta s = 0$$



Since $\Delta s = \dfrac{mg}{4k}$

Then

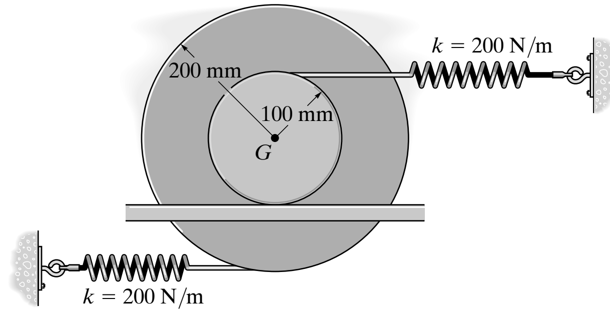
$$m\ddot{y} + 4ky = 0$$

$$y + \frac{4k}{m}y = 0$$

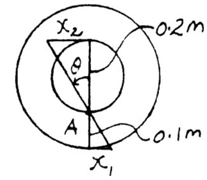
$$p = \sqrt{\frac{4k}{m}}$$

$$\tau = \frac{2\pi}{p} = \pi\sqrt{\frac{m}{k}} \qquad \textbf{Ans.}$$

•22–33. Determine the differential equation of motion of the 15-kg spool. Assume that it does not slip at the surface of contact as it oscillates. The radius of gyration of the spool about its center of mass is $k_G = 125$ mm. The springs are originally unstretched.



Energy Equation: Since the spool rolls without slipping, the stretching of both springs can be approximated as $x_1 = 0.1\theta$ and $x_2 = 0.2\theta$ when the spool is being displaced by a small angular displacement θ. Thus, the elastic potential energy is $V_p = \frac{1}{2}k\,x_1^2 + \frac{1}{2}k\,x_2^2 = \frac{1}{2}(200)(0.1\theta)^2 + \frac{1}{2}(200)\,(0.2\theta)^2 = 5\theta^2$. Thus,



$$V = V_e = 5\,\theta^2$$

The mass moment inertia of the spool about point A is $I_A = 15\left(0.125^2\right) + 15\left(0.1^2\right) = 0.384375\ \text{kg}\cdot\text{m}^2$. The kinetic energy is

$$T = \frac{1}{2}I_A\,\omega^2 = \frac{1}{2}(0.384375)\,\dot{\theta}^2 = 0.1921875\dot{\theta}^2$$

The total energy of the system is

$$U = T + V = 0.1921875\dot{\theta}^2 + 5\theta^2 \qquad \textbf{[1]}$$

Time Derivative: Taking the time derivative of Eq. [1], we have

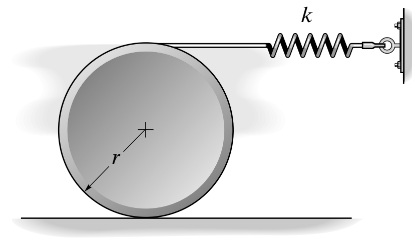
$$0.384375\dot{\theta}\,\ddot{\theta} + 10\,\theta\,\dot{\theta} = 0$$

$$\dot{\theta}\,(0.384375\,\ddot{\theta} + 10\,\theta) = 0$$

Since $\dot{\theta} \neq 0$, then $0.384375\,\ddot{\theta} + 10\,\theta = 0$

$$\ddot{\theta} + 26.0\,\theta = 0 \qquad \textbf{Ans.}$$

22–34. Determine the natural period of vibration of the disk having a mass m and radius r. Assume the disk does not slip on the surface of contact as it oscillates.



$$T + V = \text{const.}$$

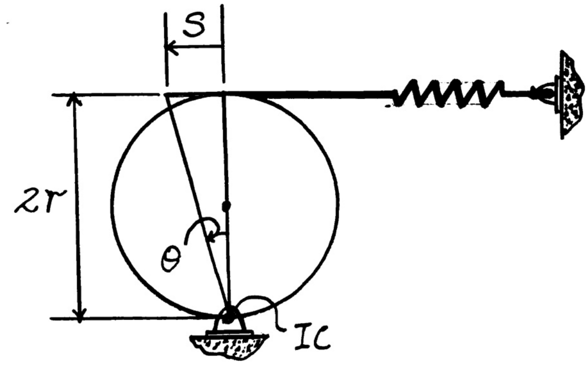
$$s = (2r)\,\theta$$

$$T + V = \frac{1}{2}\left[\frac{1}{2}mr^2 + mr^2\right]\dot{\theta}^2 + \frac{1}{2}k(2r\,\theta)^2$$

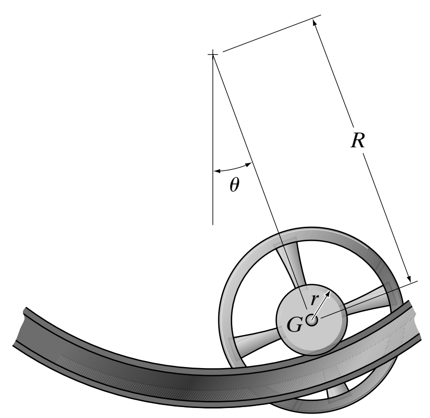
$$0 = \frac{3}{2}mr^2\,\theta\dot{\theta} + 4\,kr^2\,\theta\dot{\theta}$$

$$\theta + \frac{8k}{3m}\theta = 0$$

$$\tau = \frac{2\,\pi}{p} = \frac{2\pi}{\sqrt{\frac{8k}{3m}}} = 3.85\sqrt{\frac{m}{k}}$$ **Ans.**



22–35. If the wheel is given a small angular displacement of θ and released from rest, it is observed that it oscillates with a natural period of τ. Determine the wheel's radius of gyration about its center of mass G. The wheel has a mass of m and rolls on the rails without slipping.



Potential and Kinetic Energy: With reference to the datum established in Fig. a, the gravitational potential energy of the wheel is

$$V = V_g = -Wy_G = -mgR\cos\theta$$

As shown in Fig. b, $v_G = \dot{\theta}R$. Also, $v_G = \omega r_{G/IC} = \omega r$. Then, $\omega r = \dot{\theta}R$ or $\omega = \left(\frac{R}{r}\right)\dot{\theta}$. The mass moment of inertia of the wheel about its mass center is $I_G = mk_G{}^2$. Thus, the kinetic energy of the wheel is

$$T = \frac{1}{2}mv_G{}^2 + \frac{1}{2}I_G\,\omega^2$$

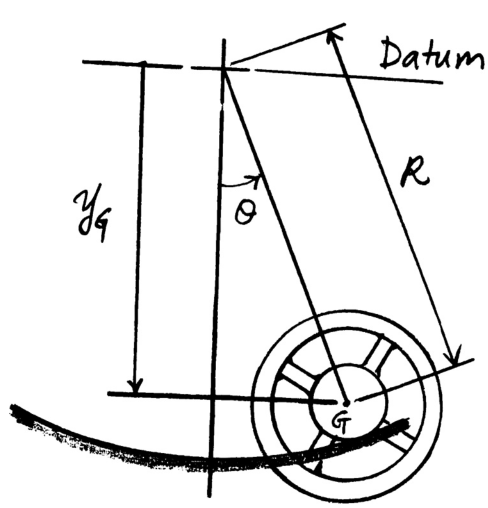
$$= \frac{1}{2}m\left(\dot{\theta}R\right)^2 + \frac{1}{2}(mk_G^2)\left[\left(\frac{R}{r}\right)\dot{\theta}\right]^2$$

$$= \frac{1}{2}mR^2\left(\frac{r^2 + k_G{}^2}{r^2}\right)\dot{\theta}^2$$

The energy function of the wheel is

$$T + V = \text{constant}$$

$$\frac{1}{2}mR^2\left(\frac{r^2 + k_G{}^2}{r^2}\right)\dot{\theta}^2 - mgR\cos\theta = \text{constant}$$



(a)

Taking the time derivative of this equation,

$$mR^2\left(\frac{r^2 + k_G{}^2}{r^2}\right)\dot{\theta}\,\ddot{\theta} + mgR\sin\theta\dot{\theta} = 0$$

$$\dot{\theta}\left[mR^2\left(\frac{r^2 + k_G{}^2}{r^2}\right)\ddot{\theta} + mgR\sin\theta\right] = 0$$

Since $\dot{\theta}$ is not always equal to zero, then

$$mR^2\left(\frac{r^2 + k_G{}^2}{r^2}\right)\ddot{\theta} + mgR\sin\theta = 0$$

$$\ddot{\theta} + \frac{g}{R}\left(\frac{r^2}{r^2 + k_G{}^2}\right)\sin\theta = 0$$

Since θ is small, $\sin\theta \cong \theta$. This equation becomes

$$\ddot{\theta} + \frac{g}{R}\left(\frac{r^2}{r^2 + k_G{}^2}\right)\theta = 0$$

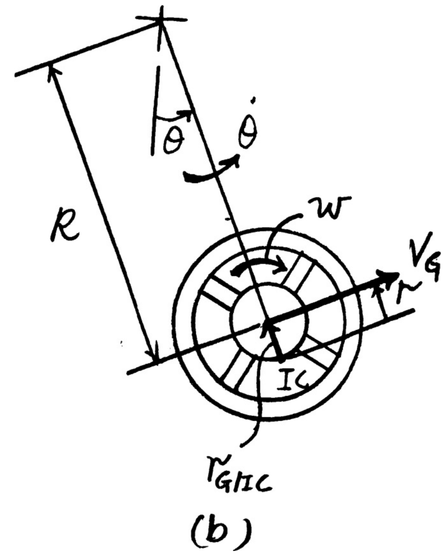
Comparing this equation to that of the standard form, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{g}{R}\left(\frac{r^2}{r^2 + k_G{}^2}\right)}$$

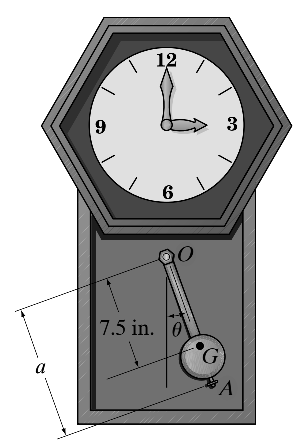
The natural period of the oscillation is therefore

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{R}{g}\left(\frac{r^2 + k_G{}^2}{r^2}\right)}$$

$$k_G = \frac{r}{2\pi}\sqrt{\frac{\tau^2 g - 4\pi^2 R}{R}}$$ **Ans.**



***22–36.** Without an adjustable screw, A, the 1.5-lb pendulum has a center of gravity at G. If it is required that it oscillates with a period of 1 s, determine the distance a from pin O to the screw. The pendulum's radius of gyration about O is $k_O = 8.5$ in. and the screw has a weight of 0.05 lb.



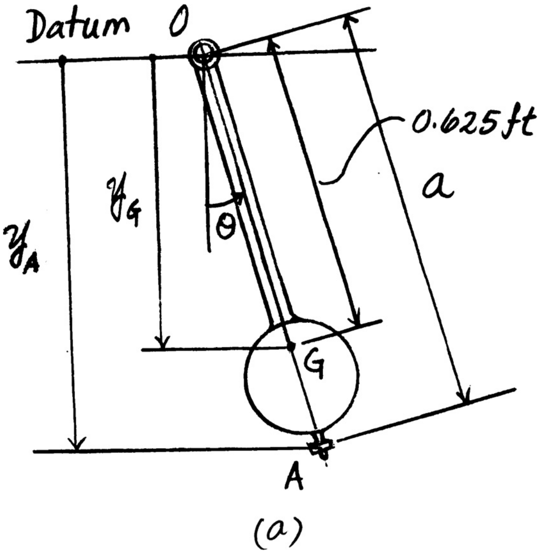
Potential and Kinetic Energy: With reference to the datum established in Fig. a, the gravitational potential energy of the system is

$$\begin{aligned} V = V_g &= -W_P\, y_G - W_A\, y_A \\ &= -1.5(0.625\cos\theta) - 0.05(a\cos\theta) \\ &= -(0.9375 + 0.05\,a)\cos\theta \end{aligned}$$

The mass moment of inertia of the pendulum about point O is $I_O = mk_O{}^2 = \frac{1.5}{32.2}\left(\frac{8.5}{12}\right)^2 = 0.02337\ \text{slug}\cdot\text{ft}^2$. Since the pendulum rotates about point O, $v_A = \dot{\theta}r_{OA} = \dot{\theta}a$. Thus, the kinetic energy of the system is

$$T = \frac{1}{2}I_O\dot{\theta}^2 + \frac{1}{2}m_A v_A{}^2$$

***22–36. Continued**



$$= \frac{1}{2}(0.02337)\dot{\theta}^2 + \frac{1}{2}\left(\frac{0.05}{32.2}\right)\left(\dot{\theta}a\right)^2$$

$$= \left(0.01169 + 0.0007764\,a^2\right)\dot{\theta}^2$$

Thus, the energy function of the system is

$$T + V = \text{constant}$$

$$\left(0.01169 + 0.0007764a^2\right)\dot{\theta}^2 - (0.9375 + 0.05a)\cos\theta = \text{constant}$$

Taking the time derivative of this equation,

$$\left(0.02337 + 0.001553a^2\right)\dot{\theta}\ddot{\theta} + (0.9375 + 0.05a)\sin\theta\dot{\theta} = 0$$

$$\dot{\theta}\left[\left(0.02337 + 0.001553\,a^2\right)\ddot{\theta} + (0.9375 + 0.05a)\sin\theta\right] = 0$$

Since θ is not always equal to zero, then

$$\left(0.02337 + 0.001553a^2\right)\ddot{\theta} + (0.9375 + 0.05a)\sin\theta = 0$$

$$\ddot{\theta} + \left(\frac{0.9375 + 0.05a}{0.02337 + 0.001553a^2}\right)\sin\theta = 0$$

Since θ is small, $\sin\theta \cong 0$. This equation becomes

$$\ddot{\theta} + \left(\frac{0.9375 + 0.05a}{0.02337 + 0.001553a^2}\right)\theta = 0$$

Comparing this equation to that of the standard form, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{0.9375 + 0.05a}{0.02337 + 0.001553a^2}}$$

The natural period of the oscillation is therefore

$$\tau = \frac{2\pi}{\omega_n}$$

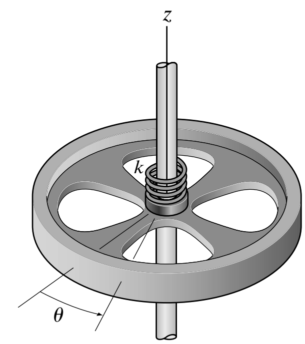
$$1 = 2\pi\sqrt{\frac{0.02337 + 0.001553a^2}{0.9375 + 0.05a}}$$

$$0.06130a^2 - 0.05a - 0.01478 = 0$$

Solving for the positive root,

$$a = 1.05 \text{ ft}$$ **Ans.**

•22–37. A torsional spring of stiffness k is attached to a wheel that has a mass of M. If the wheel is given a small angular displacement of θ about the z axis determine the natural period of oscillation. The wheel has a radius of gyration about the z axis of k_z.



Potential and Kinetic Energy: The elastic potential energy of the system is

$$V = V_e = \frac{1}{2}k\theta^2$$

The mass moment of inertia of the wheel about the z axis is $I_z = Mk_z{}^2$. Thus, the kinetic energy of the wheel is

$$T_1 = \frac{1}{2}I_z\dot{\theta}^2 = \frac{1}{2}Mk_z{}^2\dot{\theta}^2$$

The energy function of the wheel is

$$T_1 + V = \text{constant}$$

$$\frac{1}{2}Mk_z{}^2\dot{\theta}^2 + \frac{1}{2}k\theta^2 = \text{constant}$$

Taking the time derivative of this equation,

$$Mk_z{}^2\dot{\theta}\ddot{\theta} + k\theta\dot{\theta} = 0$$

$$\dot{\theta}\left(Mk_z{}^2\dot{\theta} + k\theta\right) = 0$$

Since $\dot{\theta}$ is not always equal to zero, then

$$Mk_z{}^2\ddot{\theta} + k\theta = 0$$

$$\dot{\theta} + \frac{k}{Mk_z{}^2}\theta = 0$$

Comparing this equation to that of the standard form, the natural circular frequency of the system is

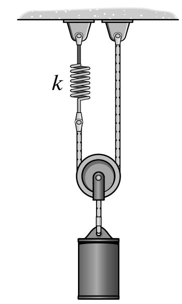
$$(\omega_n) = \sqrt{\frac{k}{Mk_z{}^2}}$$

The natural period of the oscillation is therefore

$$\tau = \frac{2\pi}{(\omega_n)_1}$$

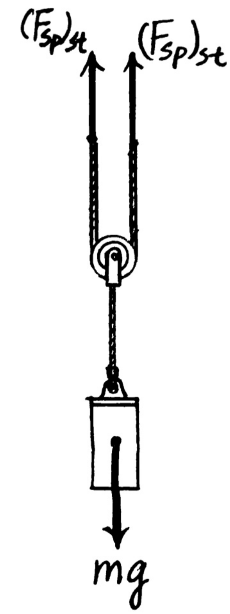
$$\tau = 2\pi\sqrt{\frac{Mk_z{}^2}{k}}$$ **Ans.**

22–38. Determine the frequency of oscillation of the cylinder of mass m when it is pulled down slightly and released. Neglect the mass of the small pulley.



Potential and Kinetic Energy: Referring to the free-body diagram of the system at its equilibrium position, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad 2(F_{sp})_{st} - mg = 0 \qquad (F_{sp})_{st} = \frac{mg}{2}$$



(a)

Thus, the initial stretch of the spring is $s_0 = \dfrac{(F_{sp})_{st}}{k} = \dfrac{mg}{2k}$. Referring to Fig. *b*,

$$(+\downarrow) \qquad s_C + (s_C - s_P) = l$$

$$2s_C - s_P = l$$

$$2\Delta s_C - \Delta s_P = 0$$

$$\Delta s_P = 2\Delta s_C$$

When the cylinder is displaced vertically downward a distance $\Delta s_C = y$, the spring is stretched further by $s_1 = \Delta s_P = 2y$.

Thus, the elastic potential energy of the spring is

$$V_e = \frac{1}{2}k(s_0 + s_1)^2 = \frac{1}{2}k\left(\frac{mg}{2k} + 2y\right)^2$$

With reference to the datum established in Fig. *b*, the gravitational potential energy of the cylinder is

$$V_g = -Wy = -mgy$$

The kinetic energy of the system is $T = \frac{1}{2}m\dot{y}^2$. Thus, the energy function of the system is

$$T + V = \text{constant}$$

$$\frac{1}{2}m\dot{y}^2 + \frac{1}{2}k\left(\frac{mg}{2k} + 2y\right)^2 - mgy = 0$$

Taking the time derivative of this equation,

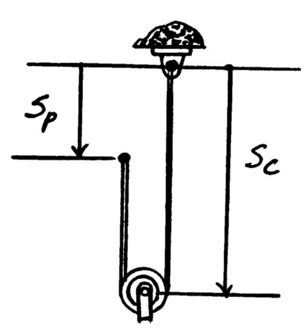
$$m\dot{y}\,\ddot{y} + k\left(\frac{mg}{2k} + 2y\right)(2\dot{y}) - mg\dot{y} = 0$$

$$\dot{y}(m\ddot{y} + 4ky) = 0$$

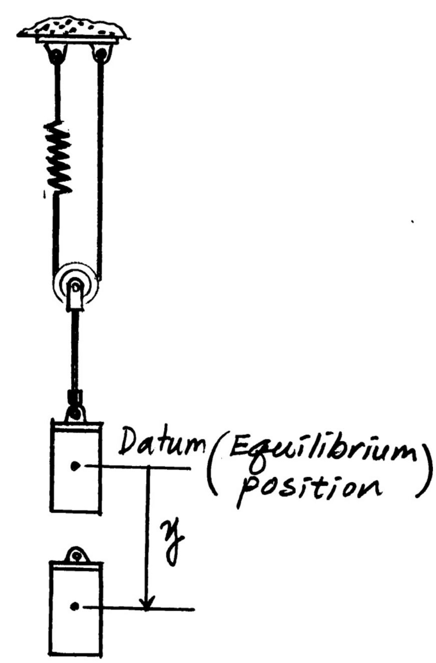
Since $\dot{y}$ is not equal to zero,

$$m\ddot{y} + 4\,ky = 0$$

$$\ddot{y} + \frac{4k}{m}y = 0$$



(b)



(C)

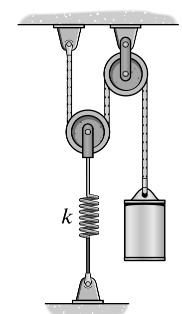
Comparing this equation to that of the standard form, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{4k}{m}} = 2\sqrt{\frac{k}{m}}$$

Thus, the frequency of the oscillation is

$$f = \frac{\omega_n}{2\pi} = \frac{1}{\pi}\sqrt{\frac{k}{m}} \qquad \textbf{Ans.}$$

22–39. Determine the frequency of oscillation of the cylinder of mass m when it is pulled down slightly and released. Neglect the mass of the small pulleys.



Potential and Kinetic Energy: Referring to the free-body diagram of the system at its equilibrium position, Fig. *a*,

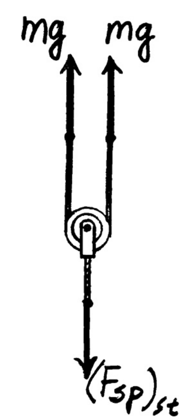
$$+\uparrow\Sigma F_y = 0; \qquad 2mg - (F_{sp})_{st} = 0 \qquad (F_{sp})_{st} = 2mg$$

Thus, the initial stretch of the spring is $s_0 = \frac{(F_{sp})_{st}}{k} = \frac{2mg}{k}$. Referring to Fig. *b*,

$$(+\downarrow) \qquad 2s_P + s_C = l$$

$$2\Delta s_P - \Delta s_C = 0$$

$$\Delta s_P = -\frac{\Delta s_C}{2} = \frac{\Delta s_C}{2} \quad \uparrow$$



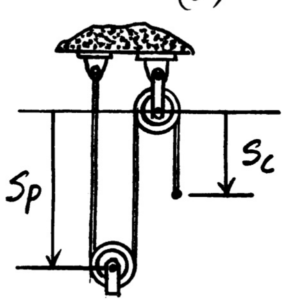
(a)

When the cylinder is displaced vertically downward a distance $\Delta s_C = y$, the spring stretches further by $s_1 = \Delta s_P = \frac{y}{2}$. Thus, the elastic potential energy of the spring is

$$V_e = \frac{1}{2}k(s_0 + s_1)^2 = \frac{1}{2}k\left(\frac{2mg}{k} + \frac{y}{2}\right)^2$$

With reference to the datum established in Fig. *c*, the gravitational potential energy of the cylinder is

$$V_g = -Wy = -mgy$$



(b)

The kinetic energy of the cylinder is $T = \frac{1}{2}m\dot{y}^2$. Thus, the energy function of the system is

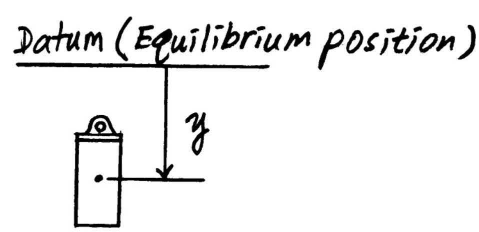
$$T + V = \text{constant}$$

$$\frac{1}{2}m\dot{y}^2 + k\left(\frac{2mg}{2k} + \frac{y}{2}\right)^2 - mgy = \text{constant}$$

Taking the time derivative of this equation,

$$m\dot{y}\,\ddot{y} + k\left(\frac{2mg}{k} + \frac{y}{2}\right)\left(\frac{\dot{y}}{2}\right) - mg\,\dot{y} = 0$$

$$\dot{y}\left(m\ddot{y} + \frac{k}{4}y\right) = 0$$



(c)

Since $\dot{y}$ is not always equal to zero,

$$m\ddot{y} + \frac{k}{4}y = 0$$

$$\ddot{y} + \frac{1}{4}\left(\frac{k}{m}\right)y = 0$$

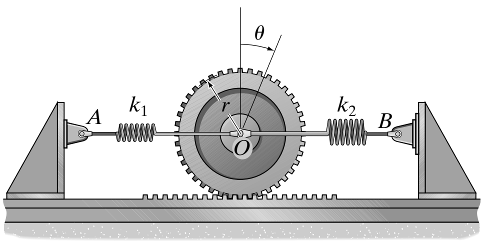
Comparing this equation to that of the standard form, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{1}{4}\left(\frac{k}{m}\right)} = \frac{1}{2}\sqrt{\frac{k}{m}}$$

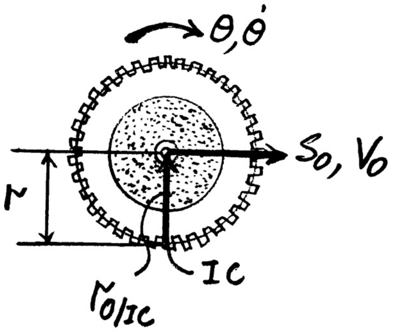
Thus, the natural frequency of oscillation is

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{4\pi}\sqrt{\frac{k}{m}} \qquad \textbf{Ans.}$$

***22–40.** The gear of mass m has a radius of gyration about its center of mass O of k_O. The springs have stiffnesses of k_1 and k_2, respectively, and both springs are unstretched when the gear is in an equilibrium position. If the gear is given a small angular displacement of θ and released, determine its natural period of oscillation.



Potential and Kinetic Energy: Since the gear rolls on the gear rack, springs AO and BO stretch and compress $s_O = r_{O/IC}\theta = r\theta$. When the gear rotates a small angle θ, Fig. a, the elastic potential energy of the system is



(a)

$$V = V_e = \frac{1}{2}k_1 s_O{}^2 + \frac{1}{2}k_2 s_O{}^2$$

$$= \frac{1}{2}k_1(r\theta)^2 + \frac{1}{2}k_2(r\theta)^2$$

$$= \frac{1}{2}r^2(k_1 + k_2)\theta^2$$

Also, from Fig. a, $v_O = \dot{\theta}r_{O/IC} = \dot{\theta}r$. The mass moment of inertia of the gear about its mass center is $I_O = mk_O{}^2$.

Thus, the kinetic energy of the system is

$$T = \frac{1}{2}mv_O{}^2 + \frac{1}{2}I_O\,\omega^2$$

$$= \frac{1}{2}m(\dot{\theta}r)^2 + \frac{1}{2}\left(mk_O{}^2\right)\dot{\theta}^2$$

$$= \frac{1}{2}m(r^2 + k_O{}^2)\dot{\theta}^2$$

The energy function of the system is therefore

$$T + V = \text{constant}$$

$$\frac{1}{2}m\left(r^2 + k_O{}^2\right)\dot{\theta}^2 + \frac{1}{2}r^2(k_1 + k_2)\theta^2 = \text{constant}$$

Taking the time derivative of this equation,

$$m\left(r^2 + k_O{}^2\right)\dot{\theta}\,\ddot{\theta} + r^2(k_1 + k_2)\theta\dot{\theta} = 0$$

$$\dot{\theta}\left[m\left(r^2 + k_O{}^2\right)\ddot{\theta} + r^2(k_1 + k_2)\theta\right] = 0$$

Since $\dot{\theta}$ is not always equal to zero, then

$$m\left(r^2 + k_O{}^2\right)\ddot{\theta} + r^2(k_1 + k_2)\theta = 0$$

$$\ddot{\theta} + \frac{r^2(k_1 + k_2)}{m\left(r^2 + k_O{}^2\right)}\theta = 0$$

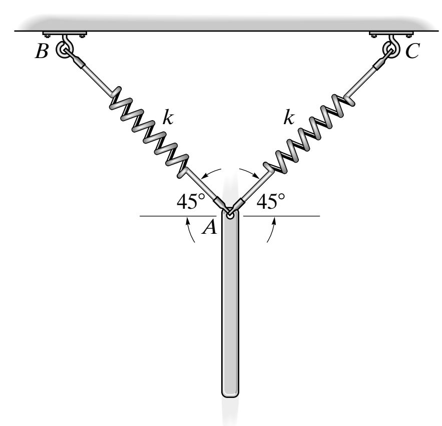
Comparing this equation to that of the standard form, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{r^2(k_1 + k_2)}{m\left(r^2 + k_O{}^2\right)}}$$

Thus, the natural period of the oscillation is

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m\left(r^2 + k_O{}^2\right)}{r^2(k_1 + k_2)}} \qquad \textbf{Ans.}$$

22–41. The bar has a mass of 8 kg and is suspended from two springs such that when it is in equilibrium, the springs make an angle of 45° with the horizontal as shown. Determine the natural period of vibration if the bar is pulled down a short distance and released. Each spring has a stiffness of $k = 40$ N/m.



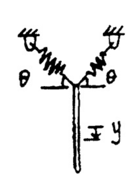
$$E = 2\left(\frac{1}{2}\right)k(s_{eq} + y\sin\theta)^2 - mgy + \frac{1}{2}m\dot{y}^2$$

$$\dot{E} = 2k(s_{eq} + y\sin\theta)\dot{y}\sin\theta - mg\dot{y} + m\dot{y}\,\ddot{y} = 0$$

$$2k\left(\frac{mg}{2k\sin\theta} + y\sin\theta\right)\sin\theta - mg + m\ddot{y} = 0$$

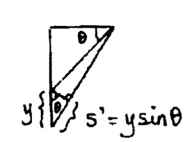
$$2ky\sin^2\theta + m\ddot{y} = 0$$

$$\ddot{y} + \frac{2k\sin^2\theta}{m}y = 0$$

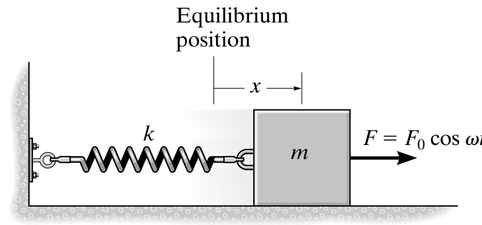


$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sin\theta}\sqrt{\frac{m}{2k}} = \frac{2\pi}{\sin 45°}\sqrt{\frac{8}{2(40)}}$$

$$\tau = 2.81\text{ s} \qquad \textbf{Ans.}$$



22–42. If the block-and-spring model is subjected to the periodic force $F = F_0 \cos \omega t$, show that the differential equation of motion is $\ddot{x} + (k/m)x = (F_0/m)\cos \omega t$, where x is measured from the equilibrium position of the block. What is the general solution of this equation?



$\xrightarrow{+} \Sigma F_x = ma_x;\qquad F_0 \cos \omega t - kx = m\ddot{x}$

$$\ddot{x} + \frac{k}{m}x = \frac{F_0}{m}\cos \omega t \qquad \textbf{(Q.E.D.)}$$

$$\ddot{x} + p^2 x = \frac{F_0}{m}\cos \omega t \qquad \text{Where } p = \sqrt{\frac{k}{m}} \qquad \textbf{(1)}$$

The general solution of the above differential equation is of the form of $x = x_c + x_p$.

The complementary solution:

$x_c = A \sin pt + B \cos pt$

The particular solution:

$$s_p = .C \cos \omega t \qquad \textbf{(2)}$$

$$\ddot{x}_P = -C\omega^2 \cos \omega t \qquad \textbf{(3)}$$

Substitute Eqs. (2) and (3) into (1) yields:

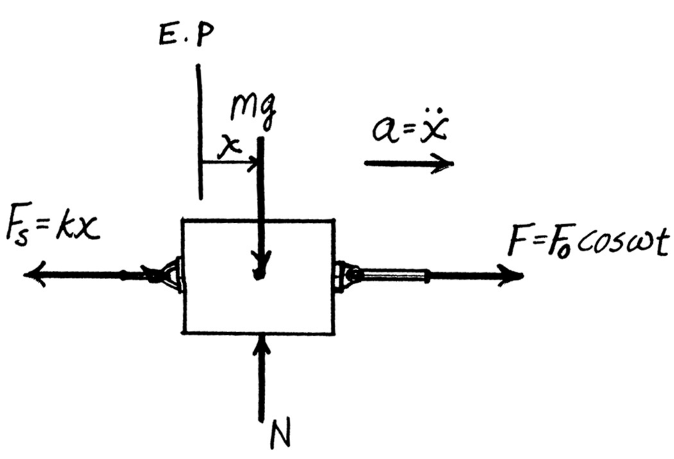
$$-C\omega^2 \cos \omega t + p^2 (C \cos \omega t) = \frac{F_0}{m}\cos \omega t$$

$$C = \frac{\frac{F_0}{m}}{p^2 - \omega^2} = \frac{F_0/k}{1 - \left(\frac{\omega}{p}\right)^2}$$

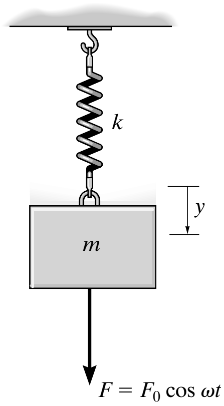
The general solution is therefore

$$s = A \sin pt + B \cos pt + \frac{F_0/k}{1 - \left(\frac{\omega}{p}\right)^2}\cos \omega t \qquad \textbf{Ans.}$$

The constants A and B can be found from the initial conditions.



22–43. If the block is subjected to the periodic force $F = F_0 \cos \omega t$, show that the differential equation of motion is $\ddot{y} + (k/m)y = (F_0/m) \cos \omega t$, where y is measured from the equilibrium position of the block. What is the general solution of this equation?



$$+\downarrow \Sigma F_y = ma_y; \qquad F_0 \cos \omega t + W - k\delta_{st} - ky = m\ddot{y}$$

Since $W = k\delta_{st}$,

$$\ddot{y} + \left(\frac{k}{m}\right)y = \frac{F_0}{m} \cos \omega t \qquad \textbf{(1) (Q.E.D.)}$$

$y_c = A \sin py + B \cos py$ (complementary solution)

$y_p = C \cos \omega t$ (particular solution)

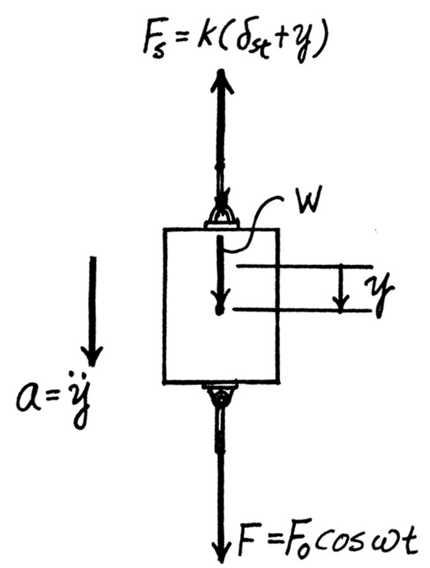
Substitute y_p into Eq. (1).

$$C\left(-\omega^2 + \frac{k}{m}\right)\cos \omega t = \frac{F_0}{m} \cos \omega t$$

$$C = \frac{\frac{F_0}{m}}{\left(\frac{k}{m} - \omega^2\right)}$$

$$y = y_c + y_p$$

$$y = A \sin pt + B \cos pt + \left(\frac{F_0}{(k - m\omega^2)}\right)\cos \omega t \qquad \textbf{Ans.}$$



***22–44.** A block having a mass of 0.8 kg is suspended from a spring having a stiffness of 120 N/m. If a dashpot provides a damping force of 2.5 N when the speed of the block is 0.2 m/s, determine the period of free vibration.

$$F = cv \qquad c = \frac{F}{v} = \frac{2.5}{0.2} = 12.5 \text{ N} \cdot \text{s/m}$$

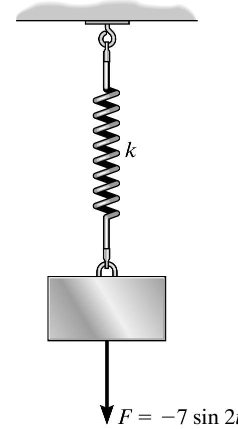
$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{120}{0.8}} = 12.247 \text{ rad/s}$$

$$C_c = 2mp = 2(0.8)(12.247) = 19.60 \text{ N} \cdot \text{s/m}$$

$$p_d = p\sqrt{1 - \left(\frac{c}{c_c}\right)^2} = 12.247\sqrt{1 - \left(\frac{12.5}{19.6}\right)^2} = 9.432 \text{ rad/s}$$

$$\tau_d = \frac{2\pi}{p_d} = \frac{2\pi}{9.432} = 0.666 \text{ s} \qquad \textbf{Ans.}$$

•22–45. The spring shown stretches 6 in. when it is loaded with a 50-lb weight. Determine the equation which describes the position of the weight as a function of time if the weight is pulled 4 in. below its equilibrium position and released from rest at $t = 0$. The weight is subjected to the periodic force of $F = (-7 \sin 2t)$ lb, where t is in seconds.



$$+\uparrow \Sigma F_y = ma_y; \qquad k(y_{st} + y) - mg - F_O \sin \omega t = -m\ddot{y}$$

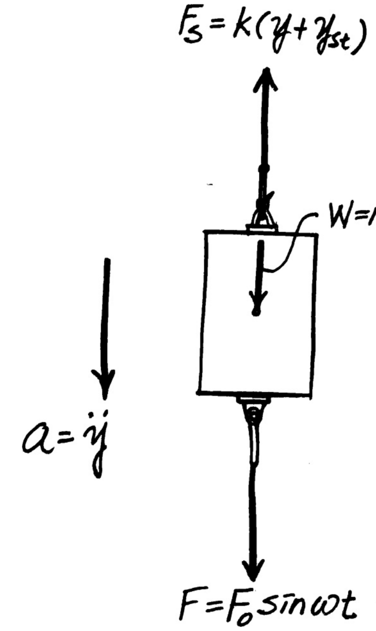
$$m\ddot{y} + ky + ky_{st} - mg = F_O \sin \omega t$$

However, from equilibrium $ky_{st} - mg = 0$, therefore

$$m\ddot{y} + ky = F_O \sin \omega t$$

$$\ddot{y} + \frac{k}{m}y = \frac{F_O}{m}\sin \omega t \qquad \text{where } p = \sqrt{\frac{k}{m}}$$

$$\ddot{y} + p^2 y = \frac{F_O}{m}\sin \omega t$$



From the text, the general solution of the above differential equation is

$$y = A \sin pt + B \cos pt + \frac{F_O/k}{1 - \left(\frac{\omega}{p}\right)^2}\sin \omega t$$

$$v = \dot{y} = Ap \cos pt - Bp \sin pt + \frac{(F_O/k)\omega}{1 - \left(\frac{\omega}{p}\right)^2}\cos \omega t$$

The initial condition when $t = 0$, $y = y_0$ and $v = v_0$.

$$y_0 = 0 + B + 0 \qquad B = y_0$$

$$v_0 = Ap - 0 + \frac{(F_O/k)\omega}{1 - \left(\frac{\omega}{p}\right)^2} \qquad A = \frac{v_0}{p} - \frac{(F_O/k)\omega}{p - \frac{\omega^2}{p}}$$

The solution is therefore

$$y = \left(\frac{v_0}{p} - \frac{(F_O/k)\omega}{p - \frac{\omega^2}{p}}\right)\sin pt + y_0 \cos pt + \frac{F_O/k}{1 - \left(\frac{\omega}{p}\right)^2}\sin \omega t$$

For this problem:

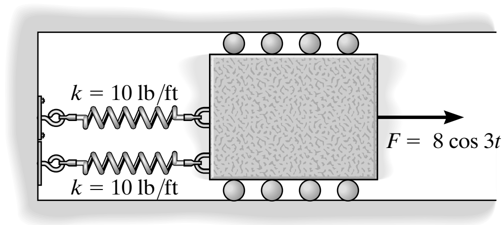
$$k = \frac{50}{6/12} = 100 \text{ lb/ft} \qquad p = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{50/32.2}} = 8.025 \text{ rad/s}$$

$$\frac{F_O/k}{1 - \left(\frac{\omega}{p}\right)^2} = \frac{-7/100}{1 - \left(\frac{2}{8.025}\right)^2} = -0.0746 \qquad y_0 = 0.333$$

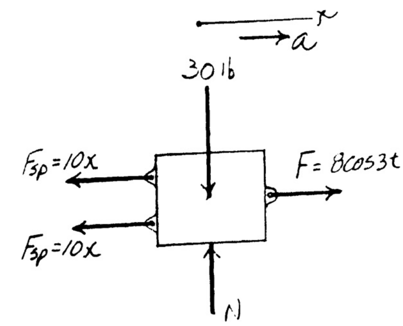
$$\frac{v_0}{p} - \frac{(F_O/k)\omega}{p - \frac{\omega^2}{p}} = 0 - \frac{(-7/100)2}{8.025 - \frac{2^2}{8.025}} = 0.0186$$

$$y = (0.0186 \sin 8.02t + 0.333 \cos 8.02t - 0.0746 \sin 2t) \text{ ft} \qquad \textbf{Ans.}$$

22–46. The 30-lb block is attached to two springs having a stiffness of 10 lb/ft. A periodic force $F = (8 \cos 3t)$ lb, where t is in seconds, is applied to the block. Determine the maximum speed of the block after frictional forces cause the free vibrations to dampen out.



Free-body Diagram: When the block is being displaced by amount x to the right, the *restoring force* that develops in both springs is $F_{sp} = kx = 10x$.



Equation of Motion:

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -2(10x) + 8\cos 3t = \frac{30}{32.2}a$$

$$a + 21.47x = 8.587\cos 3t \qquad \textbf{[1]}$$

Kinematics: Since $a = \dfrac{d^2x}{dt^2} = \ddot{x}$, then substituting this value into Eq. [1], we have

$$\ddot{x} + 21.47x = 8.587\cos 3t \qquad \textbf{[2]}$$

Since the friction will eventually dampen out the free vibration, we are only interested in the *particular solution* of the above differential equation which is in the form of

$$x_p = C\cos 3t$$

Taking second time derivative and substituting into Eq. [2], we have

$$-9C\cos 3t + 21.47C\cos 3t = 8.587\cos 3t$$

$$C = 0.6888 \text{ ft}$$

Thus,

$$x_p = 0.6888\cos 3t \qquad \textbf{[3]}$$

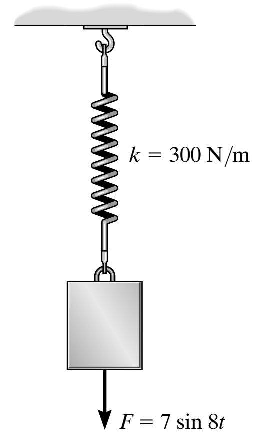
Taking the time derivative of Eq. [3], we have

$$v_p = \dot{x}_p = -2.0663\sin 3t$$

Thus,

$$\left(v_p\right)_{max} = 2.07 \text{ ft/s} \qquad \textbf{Ans.}$$

22–47. A 5-kg block is suspended from a spring having a stiffness of 300 N/m. If the block is acted upon by a vertical periodic force $F = (7 \sin 8t)$ N, where t is in seconds, determine the equation which describes the motion of the block when it is pulled down 100 mm from the equilibrium position and released from rest at $t = 0$. Consider positive displacement to be downward.



The general solution is defined by:

$$v = A \sin pt + B \cos pt + \left(\frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{p}\right)^2} \right) \sin \omega t$$

Since

$$F = 7 \sin 8t, \qquad F_0 = 7 \text{ N} \qquad \omega = 8 \text{ rad/s}, \qquad k = 300 \text{ N/m}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{300}{5}} = 7.746 \text{ rad/s}$$

Thus,

$$y = A \sin 7.746t + B \cos 7.746t + \left(\frac{\frac{7}{300}}{1 - \left(\frac{8}{7.746}\right)^2} \right) \sin 8t$$

$$y = 0.1 \text{ m when } t = 0,$$

$$0.1 = 0 + B - 0; \qquad B = 0.1 \text{ m}$$

$$v = A(7.746) \cos 7.746 - B(7.746) \sin 7.746t - (0.35)(8) \cos 8t$$

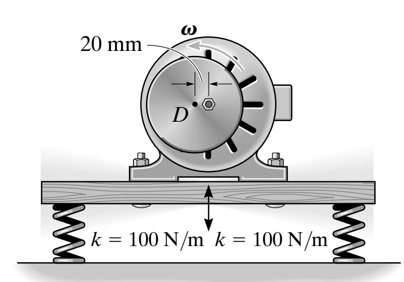
$$v = y = 0 \text{ when } t = 0,$$

$$v = A(7.746) - 2.8 = 0; \qquad A = 0.361$$

Expressing the results in mm, we have

$$y = (361 \sin 7.75t + 100 \cos 7.75t - 350 \sin 8t) \text{ mm}$$ **Ans.**

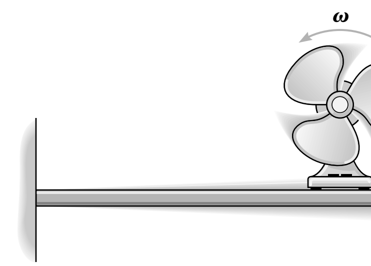
***22–48.** The electric motor has a mass of 50 kg and is supported by *four springs,* each spring having a stiffness of 100 N/m. If the motor turns a disk D which is mounted eccentrically, 20 mm from the disk's center, determine the angular velocity ω at which resonance occurs. Assume that the motor only vibrates in the vertical direction.



$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4(100)}{50}} = 2.83 \text{ rad/s}$$

$$\omega_n = \omega = 2.83 \text{ rad/s}$$ **Ans.**

•22–49. The fan has a mass of 25 kg and is fixed to the end of a horizontal beam that has a negligible mass. The fan blade is mounted eccentrically on the shaft such that it is equivalent to an unbalanced 3.5-kg mass located 100 mm from the axis of rotation. If the static deflection of the beam is 50 mm as a result of the weight of the fan, determine the angular velocity of the fan blade at which resonance will occur. *Hint:* See the first part of Example 22.8.



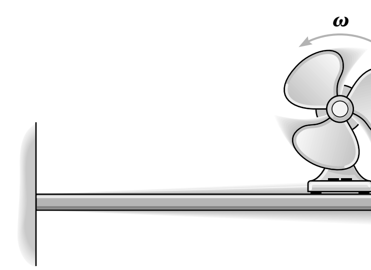
$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905 \text{ N/m}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01 \text{ rad/s}$$

Resonance occurs when

$$\omega = p = 14.0 \text{ rad/s}$$ **Ans.**

22–50. The fan has a mass of 25 kg and is fixed to the end of a horizontal beam that has a negligible mass. The fan blade is mounted eccentrically on the shaft such that it is equivalent to an unbalanced 3.5-kg mass located 100 mm from the axis of rotation. If the static deflection of the beam is 50 mm as a result of the weight of the fan, determine the amplitude of steady-state vibration of the fan if the angular velocity of the fan blade is 10 rad/s. *Hint:* See the first part of Example 22.8.



$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905 \text{ N/m}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01 \text{ rad/s}$$

The force caused by the unbalanced rotor is

$$F_0 = mr\,\omega^2 = 3.5(0.1)(10)^2 = 35 \text{ N}$$

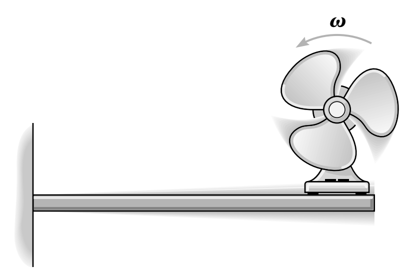
Using Eq. 22–22, the amplitude is

$$(x_p)_{\max} = \left| \frac{\frac{F_0}{k}}{1 - (\frac{\omega}{p})^2} \right|$$

$$(x_p)_{\max} = \left| \frac{\frac{35}{4905}}{1 - (\frac{10}{14.01})^2} \right| = 0.0146 \text{ m}$$

$$(x_p)_{\max} = 14.6 \text{ mm}$$ **Ans.**

22–51. What will be the amplitude of steady-state vibration of the fan in Prob. 22–50 if the angular velocity of the fan blade is 18 rad/s? *Hint:* See the first part of Example 22.8.



$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905 \text{ N/m}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01 \text{ rad/s}$$

The force caused by the unbalanced rotor is

$$F_0 = mr\omega^2 = 3.5(0.1)(18)^2 = 113.4 \text{ N}$$

Using Eq. 22–22, the amplitude is

$$(x_p)_{\max} = \left| \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{p}\right)^2} \right|$$

$$(x_p)_{\max} = \left| \frac{\frac{113.4}{4905}}{1 - \left(\frac{18}{14.01}\right)^2} \right| = 0.0355 \text{ m}$$

$$(x_p)_{\max} = 35.5 \text{ mm}$$ **Ans.**

***22–52.** A 7-lb block is suspended from a spring having a stiffness of $k = 75$ lb/ft. The support to which the spring is attached is given simple harmonic motion which can be expressed as $\delta = (0.15 \sin 2t)$ ft, where t is in seconds. If the damping factor is $c/c_c = 0.8$, determine the phase angle ϕ of forced vibration.

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{75}{\left(\frac{7}{32.2}\right)}} = 18.57$$

$$\delta = 0.15 \sin 2t$$

$$\delta_0 = 0.15, \omega = 2$$

$$\phi' = \tan^{-1}\left(\frac{2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{p}\right)}{1 - \left(\frac{\omega}{p}\right)^2}\right) = \tan^{-1}\left(\frac{2(0.8)\left(\frac{2}{18.57}\right)}{1 - \left(\frac{2}{18.57}\right)^2}\right)$$

$$\phi' = 9.89^\circ$$ **Ans.**

•22–53. Determine the magnification factor of the block, spring, and dashpot combination in Prob. 22–52.

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{75}{\left(\frac{7}{32.2}\right)}} = 18.57$$

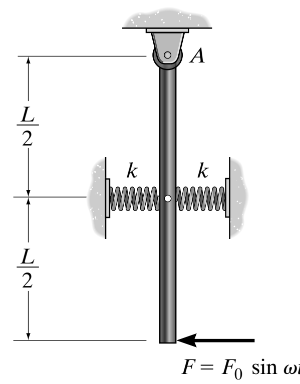
$$\delta = 0.15 \sin 2t$$

$$\delta_0 = 0.15, \omega = 2$$

$$\text{MF} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{p}\right)\right]^2}} = \frac{1}{\sqrt{\left[1 - \left(\frac{2}{18.57}\right)^2\right]^2 + \left[2(0.8)\left(\frac{2}{18.57}\right)\right]^2}}$$

$$\text{MF} = 0.997$$ **Ans.**

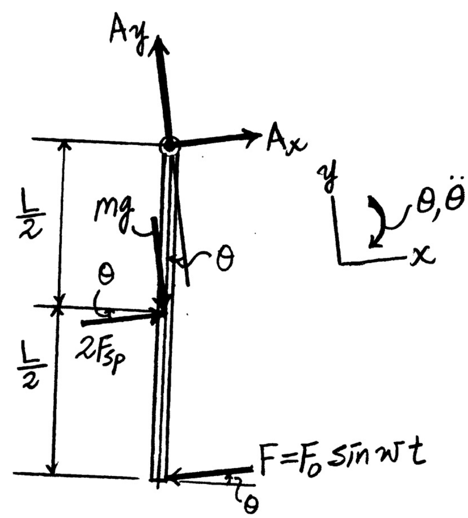
22–54. The uniform rod has a mass of m. If it is acted upon by a periodic force of $F = F_0 \sin \omega t$, determine the amplitude of the steady-state vibration.



Equation of Motion: When the rod rotates through a small angle θ, the springs compress and stretch $s = r_{AG}\theta = \frac{L}{2}\theta$. Thus, the force in each spring is $F_{sp} = ks = \frac{kL}{2}\theta$. The mass moment of inertia of the rod about point A is $I_A = \frac{1}{3}mL^2$. Referring to the free-body diagram of the rod shown in Fig. a,

$$+\Sigma M_A = I_A\alpha; \qquad F_O \sin \omega t \cos \theta(L) - mg \sin \theta\left(\frac{L}{2}\right) - 2\left(\frac{kL}{2}\theta\right)\cos \theta\left(\frac{L}{2}\right)$$

$$= \frac{1}{3}mL^2\ddot{\theta}$$



(a)

Since θ is small, $\sin \theta \cong 0$ and $\cos \theta \cong 1$. Thus, this equation becomes

$$\frac{1}{3}mL\ddot{\theta} + \frac{1}{2}(mg + kL)\theta = F_O \sin \omega t$$

$$\ddot{\theta} + \frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right)\theta = \frac{3F_O}{mL}\sin \omega t \qquad \textbf{(1)}$$

The particular solution of this differential equation is assumed to be in the form of

$$\theta_p = C \sin \omega t \qquad \textbf{(2)}$$

Taking the time derivative of Eq. (2) twice,

$$\ddot{\theta}_p = -C\omega^2 \sin \omega t \qquad \textbf{(3)}$$

Substituting Eqs. (2) and (3) into Eq. (1),

$$-C\omega^2 \sin \omega t + \frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right)(C \sin \omega t) = \frac{3F_O}{mL}\sin \omega t$$

$$C\left[\frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right) - \omega^2\right]\sin \omega t = \frac{3F_O}{mL}\sin \omega t$$

$$C = \frac{3F_O/mL}{\frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right) - \omega^2}$$

$$C = \frac{3F_O}{\frac{3}{2}(mg + Lk) - mL\omega^2} \qquad \textbf{Ans.}$$

22–55. The motion of an underdamped system can be described by the graph in Fig. 20–16. Show that the relation between two successive peaks of vibration is given by $\ln(x_n/x_{n+1}) = 2\pi(c/c_c)/\sqrt{1-(c/c_c)^2}$, where c/c_c is the *damping factor* and $\ln(x_n/x_{n+1})$ is called the *logarithmic decrement*.

If the first peak occurs when $t_n = t$, then the successive peaks occur when $t_{n+1} = l + \tau_d = t + \dfrac{2\pi}{\omega_d} = t + \dfrac{2\pi}{\omega_n\sqrt{1-(c/c_c)^2}}$.

Thus, the two successive peaks are

$$y_n = De^{-((c/c_c)\omega_n)}$$

and

$$y_{n+1} = De^{-\left[(c/c_c)\omega_n\left(t+\frac{2\pi}{\omega_n\sqrt{1-(c/c_c)^2}}\right)\right]}$$

$$= De^{-((c/c_c)\omega_n)t}\, e^{-\frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}}$$

Thus,

$$\frac{y_n}{y_n + 1} = e^{\left(\frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}\right)}$$

$$\ln\left(\frac{y_n}{y_n + 1}\right) = \frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}} \qquad \textbf{(Q.E.D.)}$$

***22–56.** Two successive amplitudes of a spring-block underdamped vibrating system are observed to be 100 mm and 75 mm. Determine the damping coefficient of the system. The block has a mass of 10 kg and the spring has a stiffness of $k = 1000$ N/m. Use the result of Prob. 22–55.

Using the result of Prob. 22-55,

$$\ln\left(\frac{y_n}{y_n + 1}\right) = \frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}$$

$$\ln\left(\frac{100}{75}\right) = \frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}$$

$$\frac{c}{c_c} = 0.04574$$

However,

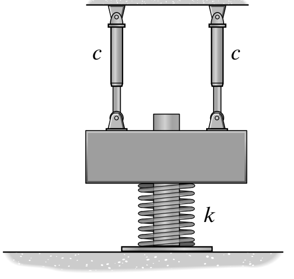
$$c_c = 2m\omega_n = 2(10)\sqrt{\frac{1000}{10}} = 200 \text{ N}\cdot\text{s/m}$$

Thus,

$$\frac{c}{200} = 0.04574$$

$$c = 9.15 \text{ N}\cdot\text{s/m}$$ **Ans.**

•22–57. Two identical dashpots are arranged parallel to each other, as shown. Show that if the damping coefficient $c < \sqrt{mk}$, then the block of mass m will vibrate as an underdamped system.



When the two dash pots are arranged in parallel, the piston of the dashpots have the same velocity. Thus, the force produced is

$$F = c\dot{y} + c\dot{y} = 2c\dot{y}$$

The equivalent damping coefficient c_{eq} of a single dashpot is

$$c_{eq} = \frac{F}{\dot{y}} = \frac{2c\dot{y}}{\dot{y}} = 2c$$

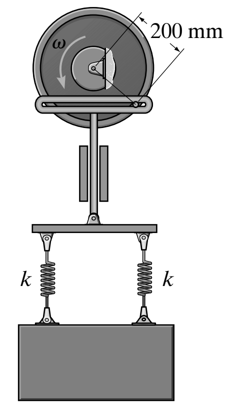
For the vibration to occur (underdamped system), $c_{eq} < c_c$. However, $c_c = 2m\omega_n = 2m\sqrt{\frac{k}{m}}$. Thus,

$$c_{eq} < c_c$$

$$2c < 2m\sqrt{\frac{k}{m}}$$

$$c < \sqrt{mk}$$ **Ans.**

22–58. The spring system is connected to a crosshead that oscillates vertically when the wheel rotates with a constant angular velocity of $\boldsymbol{\omega}$. If the amplitude of the steady-state vibration is observed to be 400 mm, and the springs each have a stiffness of $k = 2500$ N/m, determine the two possible values of $\boldsymbol{\omega}$ at which the wheel must rotate. The block has a mass of 50 kg.



In this case, $k_{eq} = 2k = 2(2500) = 5000$ N/m Thus, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{5000}{50}} = 10 \text{ rad/s}$$

Here, $\delta_O = 0.2$ m and $(Y_P)_{\max} = \pm 0.4$ m, so that

$$(Y_P)_{\max} = \frac{\delta_O}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2}$$

$$\pm 0.4 = \frac{0.2}{1 - \left(\dfrac{\omega}{10}\right)^2}$$

$$\frac{\omega^2}{100} = 1 \pm 0.5$$

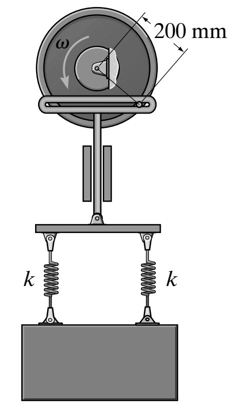
Thus,

$$\frac{\omega^2}{100} = 1.5 \qquad \omega = 12.2 \text{ rad/s} \qquad \textbf{Ans.}$$

or

$$\frac{\omega^2}{100} = 0.5 \qquad \omega = 7.07 \text{ rad/s} \qquad \textbf{Ans.}$$

22–59. The spring system is connected to a crosshead that oscillates vertically when the wheel rotates with a constant angular velocity of $\omega = 5$ rad/s. If the amplitude of the steady-state vibration is observed to be 400 mm, determine the two possible values of the stiffness k of the springs. The block has a mass of 50 kg.



In this case, $k_{eq} = 2k$ Thus, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{2k}{50}} = \sqrt{0.04k}$$

Here, $\delta_O = 0.2$ m and $(Y_P)_{\max} = \pm 0.4$ m, so that

$$(Y_P)_{\max} = \frac{\delta_O}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$$\pm 0.4 = \frac{0.2}{1 - \left(\frac{5}{\sqrt{0.04k}}\right)^2}$$

$$\frac{625}{k} = 1 \pm 0.5$$

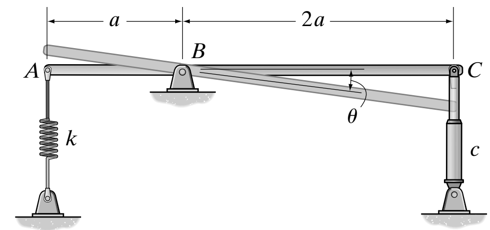
Thus,

$$\frac{625}{k} = 1.5 \qquad k = 417 \text{ N/m} \qquad \textbf{Ans.}$$

or

$$\frac{625}{k} = 0.5 \qquad k = 1250 \text{ N/m} \qquad \textbf{Ans.}$$

***22–60.** Find the differential equation for small oscillations in terms of θ for the uniform rod of mass m. Also show that if $c < \sqrt{mk}/2$, then the system remains underdamped. The rod is in a horizontal position when it is in equilibrium.



Equation of Motion: When the rod is in equilibrium, $\theta = 0°$, $F_c = c\dot{y}_c = 0$ and $\ddot{\theta} = 0$. writing the moment equation of motion about point B by referring to the free-body diagram of the rod, Fig. a,

$$+\Sigma M_B = 0; \qquad -F_A(a) - mg\left(\frac{a}{2}\right) = 0 \qquad F_A = \frac{mg}{2}$$

Thus, the initial stretch of the spring is $s_O = \frac{F_A}{k} = \frac{mg}{2k}$. When the rod rotates about point B through a small angle θ, the spring stretches further by $s_1 = a\theta$. Thus, the force in the spring is $F_A = k(s_0 + s_1) = k\left(\frac{mg}{2k} + a\theta\right)$. Also, the velocity of end C of the rod is $v_c = \dot{y}_c = 2a\dot{\theta}$. Thus, $F_c = c\dot{y}_c = c(2a\dot{\theta})$. The mass moment of inertia of the rod about B is $I_B = \frac{1}{12}m(3a)^2 + m\left(\frac{a}{2}\right)^2 = ma^2$. Again, referring to Fig. a and writing the moment equation of motion about B,

$$\Sigma M_B = I_B\,\alpha; \qquad k\left(\frac{mg}{2k} + a\theta\right)\cos\theta(a) + \left(2a\dot{\theta}\right)\cos\theta(2a) - mg\cos\theta\left(\frac{a}{2}\right)$$

$$= -ma^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{4c}{m}\cos\theta\dot{\theta} + \frac{k}{m}(\cos\theta)\theta = 0$$

Since θ is small, $\cos\theta \cong 1$. Thus, this equation becomes

$$\ddot{\theta} + \frac{4c}{m}\dot{\theta} + \frac{k}{m}\theta = 0 \qquad \textbf{Ans.}$$

Comparing this equation to that of the standard form,

$$\omega_n = \sqrt{\frac{k}{m}} \qquad c_{eq} = 4c$$

Thus,

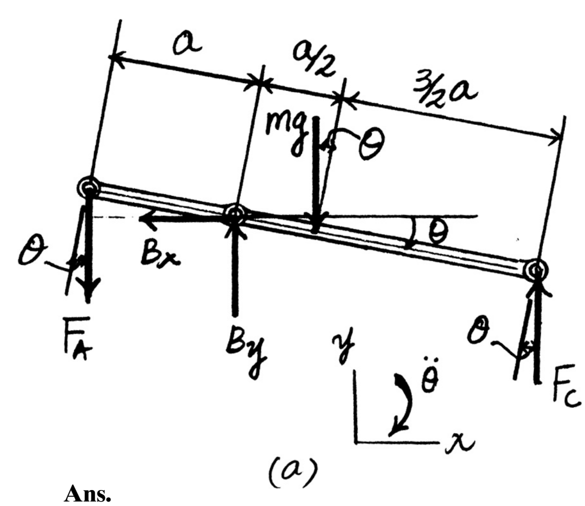
$$c_c = 2m\omega_n = 2m\sqrt{\frac{k}{m}} = 2\sqrt{mk}$$

For the system to be underdamped,

$$c_{eq} < c_c$$

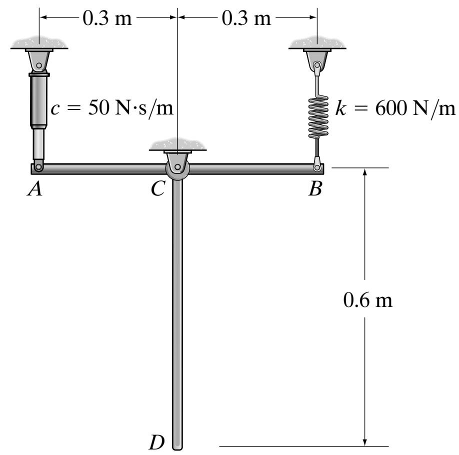
$$4c < 2\sqrt{mk}$$

$$c < \frac{1}{2}\sqrt{mk} \qquad \textbf{Ans.}$$

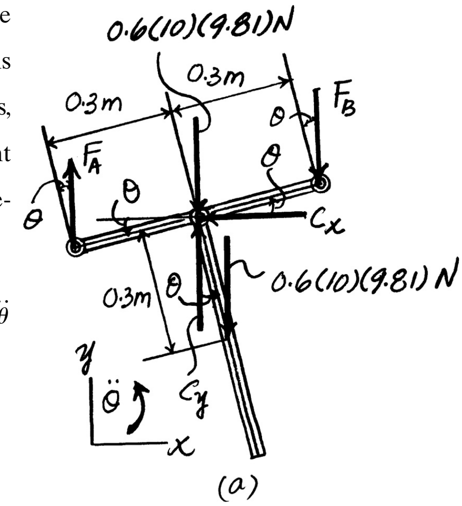


(a)

•22–61. If the dashpot has a damping coefficient of $c = 50\ \text{N}\cdot\text{s/m}$, and the spring has a stiffness of $k = 600\ \text{N/m}$, show that the system is underdamped, and then find the pendulum's period of oscillation. The uniform rods have a mass per unit length of 10 kg/m.



Equation of Motion: When the pendulum rotates point C through a small angle θ, the spring compresses $s = 0.3\theta$. Thus, the force in the spring is $F_B = ks = 600(0.3\theta) = 180\theta$. Also, the velocity of end A is $v_A = \dot{y}_A = 0.3\dot{\theta}$. Thus, $F_A = c\dot{y}_A = 50(0.3\dot{\theta}) = 15\dot{\theta}$. The mass moment of inertia of the pendulum about point C is $I_C = \frac{1}{12}\left[0.6(10)\left(0.6^2\right)\right] + \frac{1}{3}\left[0.6(10)\left(0.6^2\right)\right] = 0.9\ \text{kg}\cdot\text{m}^2$. Referring to the free-body diagram of the pendulum, Fig. *a*,



$$\Sigma M_C = I_C\,\alpha; \qquad -180\theta\cos\theta(0.3) - 15\dot{\theta}\cos\theta(0.3) - 0.6(10)(9.81)\sin\theta(0.3) = 0.9\ddot{\theta}$$

$$\ddot{\theta} + 5\cos\theta\dot{\theta} + 60\cos\theta\theta + 16.62\sin\theta = 0$$

Since θ is small, $\sin\theta \cong \theta$ and $\cos\theta \cong 1$. Thus, this equation becomes

$$\ddot{\theta} + 5\dot{\theta} + 79.62 = 0$$

Comparing this equation to that of the standard form,

$$c_{eq} = 5 \qquad \omega_n = 8.923\ \text{rad/s}$$

Here, $m = 2\left[(10)(0.6)\right] = 12\ \text{kg}$. Thus, $c_{eq} = 5m = 5(12) = 60\ \text{N}\cdot\text{m/s}$. Also, $c_c = 2m\omega_n = 2(12)8.923 = 214.15\ \text{N}\cdot\text{m/s}$. Since $c_{eq} < c_c$, the system is *underdamped* (Q.E.D.). Thus,

$$\omega_d = \omega_n\sqrt{1 - \left(\frac{c_{eq}}{c_c}\right)^2}$$

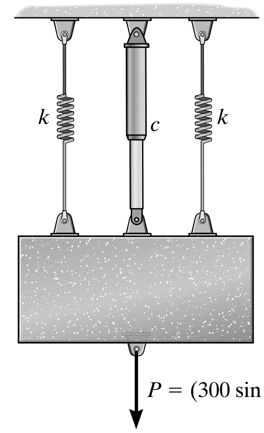
$$= 8.923\left(\sqrt{1 - \left(\frac{60}{214.15}\right)^2}\right)$$

$$= 8.566\ \text{rad/s}$$

Thus, the period of under-damped oscillation of the pendulum is

$$\tau_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{8.566} = 0.734\ \text{s} \qquad \textbf{Ans.}$$

22–62. If the 30-kg block is subjected to a periodic force of $P = (300 \sin 5t)$ N, $k = 1500$ N/m, and $c = 300$ N·s/m, determine the equation that describes the steady-state vibration as a function of time.



Here, $k_{eq} = 2k = 2(1500) = 3000$ N/m. Thus, the circular frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{3000}{30}} = 10 \text{ rad/s}$$

The critical damping coefficient is

$$c_c = 2m\omega_n = 2(30)(10) = 600 \text{ N}\cdot\text{s/m}$$

Then, the damping factor is

$$\frac{c}{c_c} = \frac{300}{600} = 0.5$$

Here, $F_O = 300$ N and $\omega = 5$ rad/s.

$$Y = \frac{F_O / k_{eq}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[\left(2\frac{c}{c_c}\right)\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$$= \frac{300/3000}{\sqrt{\left[1 - \left(\frac{5}{10}\right)^2\right]^2 + \left[\frac{2(0.5)(5)}{10}\right]^2}}$$

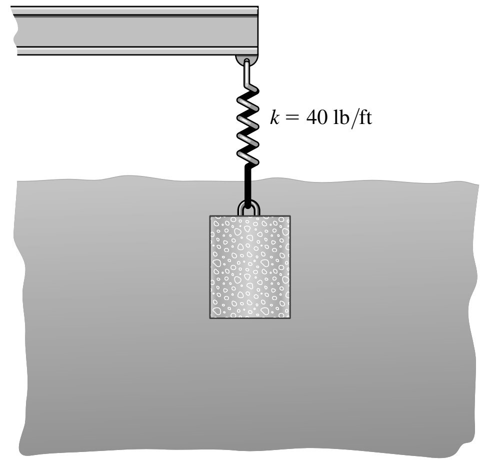
$$= 0.1109 \text{ m}$$

$$\phi' = \tan^{-1}\left[\frac{2\frac{c}{c_c}\left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right] = \tan^{-1}\left[\frac{2(0.5)\left(\frac{5}{10}\right)}{1 - \left(\frac{5}{10}\right)^2}\right] = 33.69° = 0.588 \text{ rad}$$

Thus,

$$y_P = 0.111 \sin(5t - 0.588) \text{ m}$$ **Ans.**

22–63. The block, having a weight of 15 lb, is immersed in a liquid such that the damping force acting on the block has a magnitude of $F = (0.8|v|)$ lb, where v is the velocity of the block in ft/s. If the block is pulled down 0.8 ft and released from rest, determine the position of the block as a function of time. The spring has a stiffness of $k = 40$ lb/ft. Consider positive displacement to be downward.



Viscous Damped Free Vibration: Here $c = 0.8$ lb·s/ft, $p = \sqrt{\frac{k}{m}} = \sqrt{\frac{40}{15/32.2}}$ $= 9.266$ rad/s and $c_c = 2mp = 2\left(\frac{15}{32.2}\right)(9.266) = 8.633$ lb·s/ft. Since $c < c_c$, the system is underdamped and the solution of the differential equation is in the form of

$$y = D\left[e^{-(c/2m)t}\sin(p_d t + \phi)\right] \qquad \textbf{[1]}$$

Taking the time derivative of Eq. [1], we have

$$v = \dot{y} = D\left[-\left(\frac{c}{2m}\right)e^{-(c/2m)t}\sin(p_d t + \phi) + p_d e^{-(c/2m)t}\cos(p_d t + \phi)\right]$$

$$= De^{-(c/2m)t}\left[-\left(\frac{c}{2m}\right)\sin(p_d t + \phi) + p_d\cos(p_d t + \phi)\right] \qquad \textbf{[2]}$$

Applying the initial condition $v = 0$ at $t = 0$ to Eq. [2], we have

$$0 = De^{-0}\left[-\left(\frac{c}{2m}\right)\sin(0 + \phi) + p_d\cos(0 + \phi)\right]$$

$$0 = D\left[-\left(\frac{c}{2m}\right)\sin\phi + p_d\cos\phi\right] \qquad \textbf{[3]}$$

Here, $\frac{c}{2m} = \frac{0.8}{2(15/32.2)} = 0.8587$ and $p_d = p\sqrt{1 - \left(\frac{c}{c_c}\right)^2} = 9.266\sqrt{1 - \left(\frac{0.8}{8.633}\right)^2}$ $= 9.227$ rad/s. Substituting these values into Eq. [3] yields

$$0 = D[-0.8587\sin\phi + 9.227\cos\phi] \qquad \textbf{[4]}$$

Applying the initial condition $y = 0.8$ ft at $t = 0$ to Eq. [1], we have

$$0.8 = D\left[e^{-0}\sin(0 + \phi)\right]$$

$$0.8 = D\sin\phi \qquad \textbf{[5]}$$

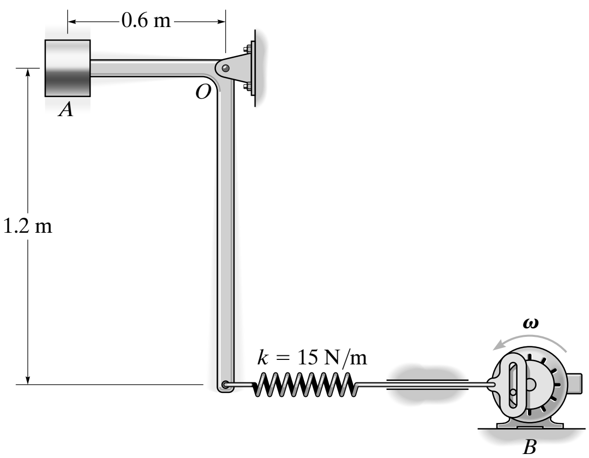
Solving Eqs. [4] and [5] yields

$$\phi = 84.68° = 1.50 \text{ rad} \qquad D = 0.8035 \text{ ft}$$

Substituting these values into Eq. [1] yields

$$y = 0.803\left[e^{-0.8597}\sin(9.23t + 1.48)\right] \qquad \textbf{Ans.}$$

***22–64.** The small block at A has a mass of 4 kg and is mounted on the bent rod having negligible mass. If the rotor at B causes a harmonic movement $\delta_B = (0.1 \cos 15t)$ m, where t is in seconds, determine the steady-state amplitude of vibration of the block.



$+\Sigma M_O = I_O\,\alpha; \qquad 4(9.81)(0.6) - F_s(1.2) = 4(0.6)^2\ddot{\theta}$

$$F_s = kx = 15(x + x_{st} - 0.1 \cos 15t)$$

$$x_{st} = \frac{4(9.81)(0.6)}{1.2(15)}$$

Thus,

$$-15(x - 0.1 \cos 15t)(1.2) = 4(0.6)^2\ddot{\theta}$$

$$x = 1.2\theta$$

$$\theta + 15\theta = 1.25 \cos 15t$$

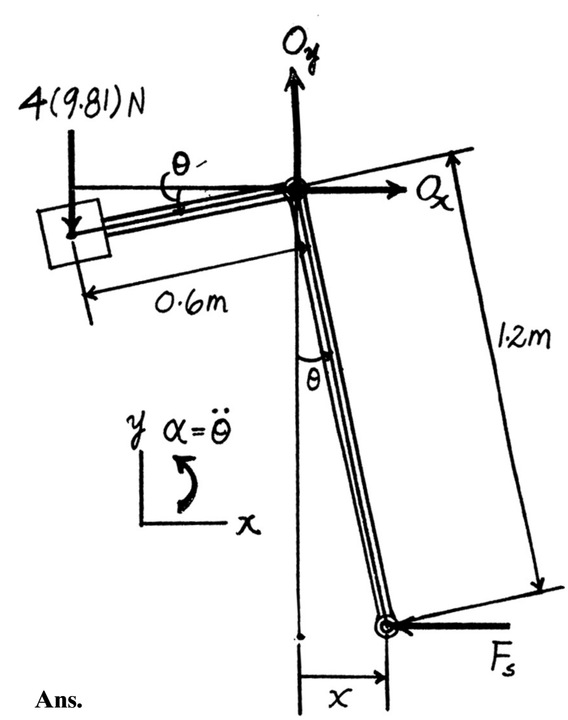
Set $x_p = C \cos 15t$

$$-C(15)^2 \cos 15t + 15(C \cos 15t) = 1.25 \cos 15t$$

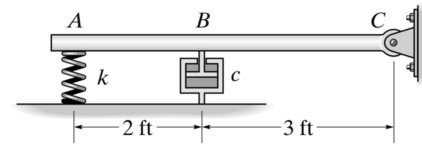
$$C = \frac{1.25}{15 - (15)^2} = -0.00595 \text{ m}$$

$$\theta_{\max} = C = 0.00595 \text{ rad}$$

$$y_{\max} = (0.6 \text{ m})(0.00595 \text{ rad}) = 3.57 \text{ rad} \qquad \textbf{Ans.}$$



•22–65. The bar has a weight of 6 lb. If the stiffness of the spring is $k = 8\ \text{lb/ft}$ and the dashpot has a damping coefficient $c = 60\ \text{lb} \cdot \text{s/ft}$, determine the differential equation which describes the motion in terms of the angle θ of the bar's rotation. Also, what should be the damping coefficient of the dashpot if the bar is to be critically damped?



$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad 6(2.5) - (60\dot{y}_2)(3) - 8(y_1 + y_{st})(5) = \left[\frac{1}{3}\left(\frac{6}{32.2}\right)(5)^2\right]\ddot{\theta}$$

$$1.5528\ddot{\theta} + 180\dot{y}_2 + 40y_1 + 40y_{st} - 15 = 0 \qquad \textbf{[1]}$$

From equilibrium $40y_{st} - 15 = 0$. Also, for small θ, $y_1 = 5\theta$ and $y_2 = 3\theta$ hence $\dot{y}_2 = 3\theta$.

From Eq. [1] $\qquad 1.5528\ddot{\theta} + 180(3\dot{\theta}) + 40(5\theta) = 0$

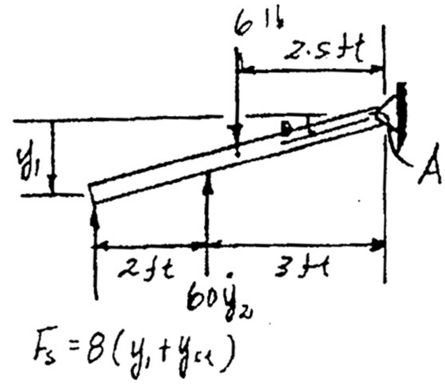
$$1.55\dot{\theta} + 540\dot{\theta} + 200\theta = 0 \qquad \textbf{Ans.}$$

By comparing the above differential equation to Eq. 22-27

$$m = 1.55 \qquad k = 200 \qquad \omega_n = \sqrt{\frac{200}{1.55}} = 11.35\ \text{rad/s} \qquad c = 9c_{d \cdot p}$$

$$\left(\frac{9\left(c_{d \cdot p}\right)_c}{2m}\right)^2 - \frac{k}{m} = 0$$

$$\left(c_{d \cdot p}\right)_c = \frac{2}{9}\sqrt{km} = \frac{2}{9}\sqrt{200(1.55)} = 3.92\ \text{lb} \cdot \text{s/ft} \qquad \textbf{Ans.}$$



22–66. A block having a mass of 7 kg is suspended from a spring that has a stiffness $k = 600$ N/m. If the block is given an upward velocity of 0.6 m/s from its equilibrium position at $t = 0$, determine its position as a function of time. Assume that positive displacement of the block is downward and that motion takes place in a medium which furnishes a damping force $F = (50|v|)$ N, where v is the velocity of the block in m/s.

$$c = 50\text{ N s/m} \quad k = 600\text{ N/m} \quad m = 7\text{ kg}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{7}} = 9.258\text{ rad/s}$$

$$c_c = 2m\omega_n = 2(7)(9.258) = 129.6\text{ N}\cdot\text{s/m}$$

Since $c < c_z$, the system is underdamped,

$$\omega_d = \omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)^2} = 9.258\sqrt{1 - \left(\frac{50}{129.6}\right)^2} = 8.542\text{ rad/s}$$

$$\frac{c}{2m} = \frac{50}{2(7)} = 3.751$$

From Eq. 22-32

$$y = D\left[e^{-\left(\frac{c}{2m}\right)t}\sin(\omega_d t + \phi)\right]$$

$$v = \dot{y} = D\left[e^{-\left(\frac{c}{2m}\right)t}\,\omega_d\,\cos(\omega_d t + \phi) + \left(-\frac{c}{2m}\right)e^{-\left(\frac{c}{2m}\right)t}\sin(\omega_d t + \phi)\right]$$

$$v = De^{-\left(\frac{c}{2m}\right)t}\left[\omega_d\cos(\omega_d t + \phi) - \frac{c}{2m}\sin(\omega_d t + \phi)\right]$$

Applying the initial condition at $t = 0$, $y = 0$ and $v = -0.6$ m/s.

$$0 = D[e^{-0}\sin(0 + \phi)] \qquad \text{since} \quad D \neq 0$$

$$\sin\phi = 0 \qquad \phi = 0°$$

$$-0.6 = De^{-0}[8.542\cos 0° - 0]$$

$$D = -0.0702\text{ m}$$

$$y = \{-0.0702[e^{-3.57t}\sin(8.540)]\}\text{ m}$$ **Ans.**

22–67. A 4-lb weight is attached to a spring having a stiffness $k = 10\ \text{lb/ft}$. The weight is drawn downward a distance of 4 in. and released from rest. If the support moves with a vertical displacement $\delta = (0.5 \sin 4t)$ in., where t is in seconds, determine the equation which describes the position of the weight as a function of time.

From Prob. 22-46

$$y = A \sin \omega_n t + B \cos \omega_n t + \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_0}\right)^2} \sin \omega_0 t$$

$$v = \dot{y} = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t + \frac{\delta_0\, \omega_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \cos \omega_0 t$$

The initial condition when $t = 0$, $y = y_0$ and $v = v_0$

$$y_0 = 0 + B + 0 \qquad B = y_0$$

$$v_0 = A\omega_n - 0 + \frac{\delta_0\, \omega_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \qquad A = \frac{v_0}{\omega_n} - \frac{\delta_0\, \omega_0}{\omega_n - \frac{\omega_0^2}{\omega_n}}$$

Thus,

$$y = \left(\frac{v_0}{\omega_n} - \frac{\delta_0\, \omega_0}{\omega_n - \frac{\omega_0^2}{\omega_n}}\right) \sin \omega_n t + y_0 \cos \omega_n t + \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \sin \omega_0 t$$

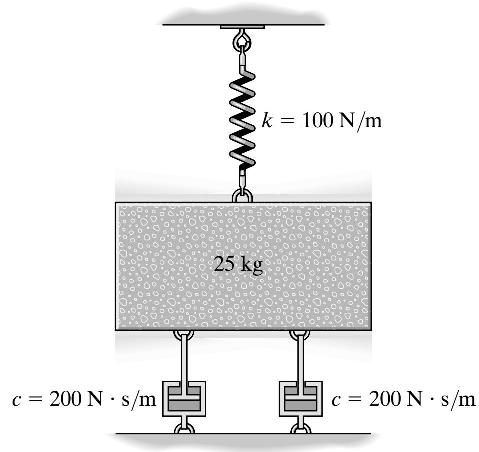
$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10}{4/32.2}} = 8.972$$

$$\frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} = \frac{0.5/12}{1 - \left(\frac{4}{8.972}\right)^2} = 0.0520$$

$$\frac{v_0}{\omega_n} - \frac{\delta_0\, \omega_0}{\omega_n - \frac{\omega_0^2}{\omega_n}} = 0 - \frac{(0.5/12)4}{8.972 - \frac{4^2}{8.972}} = -0.0232$$

$$y = \{-0.0232 \sin 8.97t + 0.333 \cos 8.97t + 0.0520 \sin 4t\}\ \text{ft} \qquad \textbf{Ans.}$$

***22–68.** Determine the differential equation of motion for the damped vibratory system shown. What type of motion occurs?



$+\downarrow \Sigma F_y = ma_y;\qquad mg - k(y + y_{st}) - 2c\dot{y} = m\ddot{y}$

$$m\ddot{y} + ky + 2c\dot{y} + ky_{st} - mg = 0$$

Equilibrium $\qquad ky_{st} - mg = 0$

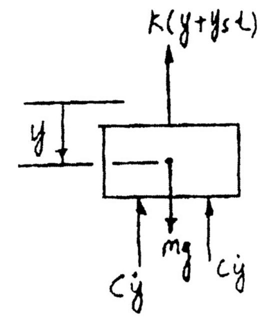
$m\ddot{y} + 2c\dot{y} + ky = 0$ Here $m = 25$ kg $\quad k = 100$ N/m

$c = 200$ N · s/m

$$25\ddot{y} + 400\dot{y} + 100y = 0 \qquad \textbf{(1)}$$

$$\ddot{y} + 16\dot{y} + 4y = 0 \qquad \textbf{Ans.}$$

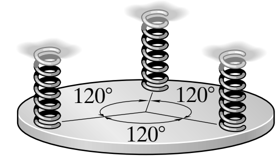
By comparing Eq. (1) to Eq. 22–27



$$m = 25 \qquad k = 100 \qquad c = 400 \qquad \omega_n = \sqrt{\frac{4}{1}} = 2\text{ rad/s}$$

$$c_c = 2m\omega_n = 2(25)(2) = 100\text{ N} \cdot \text{s/m}$$

Since $c > c_c$, the system will not vibrate. Therefore, it is **overdamped**. **Ans.**



•**22–69.** The 4-kg circular disk is attached to three springs, each spring having a stiffness $k = 180$ N/m. If the disk is immersed in a fluid and given a downward velocity of 0.3 m/s at the equilibrium position, determine the equation which describes the motion. Consider positive displacement to be measured downward, and that fluid resistance acting on the disk furnishes a damping force having a magnitude $F = (60|v|)$ N, where v is the velocity of the block in m/s.

$$k = 540 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{540}{4}} = 11.62 \text{ rad/s}$$

$$c_c = 2m\omega_n = 2(4)(11.62) = 92.95$$

$$F = 60v, \text{ so that } c = 60$$

Since $c < c_c$, system is underdamped.

$$\omega_d = \omega_n\sqrt{1 - (\frac{c}{c_c})^2}$$

$$= 11.62\sqrt{1 - (\frac{60}{92.95})^2}$$

$$= 8.87 \text{ rad/s}$$

$$y = A[e^{-(\frac{c}{2m})t} \sin(\omega_d t + \phi)] \qquad \textbf{(1)}$$

$$y = 0, v = 0.3 \text{ at } t = 0$$

$$0 = A \sin \phi$$

$$A \neq 0 \text{ (trivial solution) so that } \phi = 0$$

$$v = y = A[-\frac{c}{2m} e^{-(\frac{c}{2m})t} \sin(\omega_d t + \phi) + e^{-(\frac{c}{2m})t} \cos(\omega_d t + \phi)(\omega_d)]$$

Since $\phi = 0$

$$0.3 = A[0 + 1(8.87)]$$

$$A = 0.0338$$

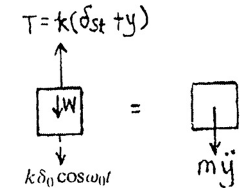
Substituting into Eq. (1)

$$y = 0.0338[e^{-(\frac{60}{2(4)})t} \sin(8.87)t]$$

Expressing the result in mm

$$y = 33.8[e^{-7.5t} \sin(8.87t)] \text{ mm} \qquad \textbf{Ans.}$$

22–70. Using a block-and-spring model, like that shown in Fig. 22–13*a*, but suspended from a vertical position and subjected to a periodic support displacement of $\delta = \delta_0 \cos \omega_0 t$, determine the equation of motion for the system, and obtain its general solution. Define the displacement y measured from the static equilibrium position of the block when $t = 0$.



$+\downarrow \Sigma F_y = ma_y; \qquad k\delta_0 \cos \omega_0 t + W - k\delta_{st} - ky = m\ddot{y}$

Since $W = k\delta_{st}$,

$$\ddot{y} + \frac{k}{m} y = \frac{k\delta_0}{m} \cos \omega_0 t \qquad \textbf{(1)}$$

$$y_C = A \sin \omega_n y + B \cos \omega_n y \text{ (General sol.)}$$

$$y_p = C \cos \omega_0 t \text{ (Particular sol.)}$$

Substitute y_p into Eq. (1)

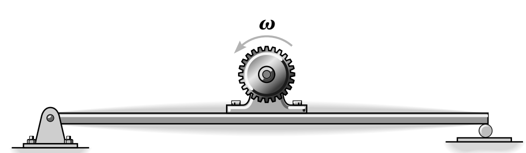
$$C(-\omega_0{}^2 + \frac{k}{m}) \cos \omega_0 t = \frac{k\delta_0}{m} \cos \omega_0 t$$

$$C = \frac{\frac{k\delta_0}{m}}{(\frac{k}{m} - \omega_0{}^2)}$$

Thus, $y = y_C + y_P$

$$y = A \sin \omega_n t + B \cos \omega_n t + \frac{\frac{k\delta_0}{m}}{(\frac{k}{m} - \omega_0{}^2)} \cos \omega_0 t \qquad \textbf{Ans.}$$

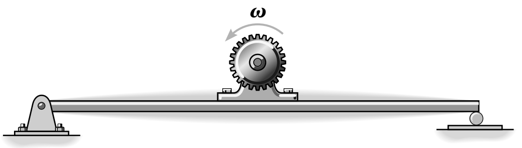
22–71. The electric motor turns an eccentric flywheel which is equivalent to an unbalanced 0.25-lb weight located 10 in. from the axis of rotation. If the static deflection of the beam is 1 in. due to the weight of the motor, determine the angular velocity of the flywheel at which resonance will occur. The motor weights 150 lb. Neglect the mass of the beam.



$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.66$$

Resonance occurs when $\qquad \omega_0 = \omega_n = 19.7 \text{ rad/s} \qquad$ **Ans.**

***22–72.** What will be the amplitude of steady-state vibration of the motor in Prob. 22–71 if the angular velocity of the flywheel is 20 rad/s?



The constant value F_0 of the periodic force is due to the centrifugal force of the unbalanced mass.

$$F_0 = ma_n = mr\omega_0{}^2 = \left(\frac{0.25}{32.2}\right)\left(\frac{10}{12}\right)(20)^2 = 2.588 \text{ lb}$$

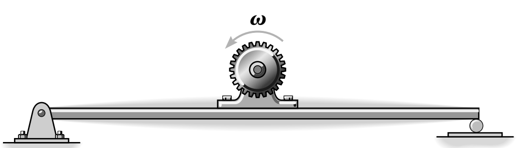
Hence $F = 2.588 \sin 20t$

$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.657$$

From Eq. 22–21, the amplitude of the steady state motion is

$$C = \left|\frac{F_0/k}{1-\left(\frac{\omega_0}{\omega_n}\right)^2}\right| = \left|\frac{2.588/1800}{1-\left(\frac{20}{19.657}\right)^2}\right| = 0.04085 \text{ ft} = 0.490 \text{ in.} \qquad \textbf{Ans.}$$

•22–73. Determine the angular velocity of the flywheel in Prob. 22–71 which will produce an amplitude of vibration of 0.25 in.



The constant value F_0 of the periodic force is due to the centrifugal force of the unbalanced mass.

$$F_O = ma_n = mr\omega^2 = \left(\frac{0.25}{32.2}\right)\left(\frac{10}{12}\right)\omega^2 = 0.006470\omega^2$$

$$F = 0.006470\omega^2 \sin \omega t$$

$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad p = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.657$$

From Eq. 22.21, the amplitude of the steady state motion is

$$C = \left|\frac{F_O/k}{1-\left(\frac{\omega}{p}\right)^2}\right|$$

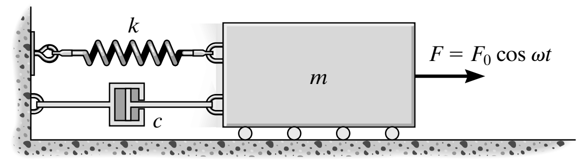
$$\frac{0.25}{12} = \left|\frac{0.006470\left(\frac{\omega^2}{1800}\right)}{1-\left(\frac{\omega}{19.657}\right)^2}\right|$$

$$\omega = 19.0 \text{ rad/s} \qquad \textbf{Ans.}$$

Or,

$$\theta_0 = 20.3 \text{ rad/} \qquad \textbf{Ans.}$$

22–74. Draw the electrical circuit that is equivalent to the mechanical system shown. Determine the differential equation which describes the charge q in the circuit.

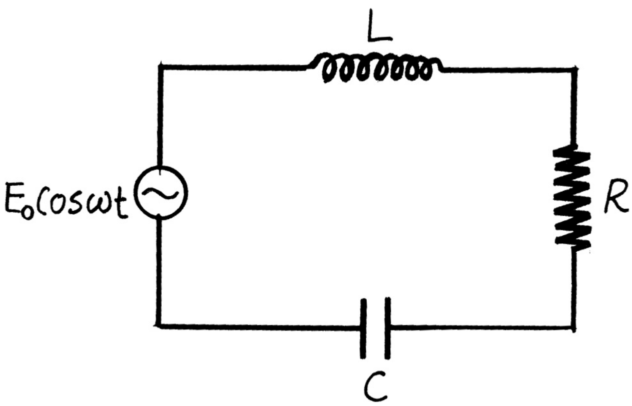


For the block,

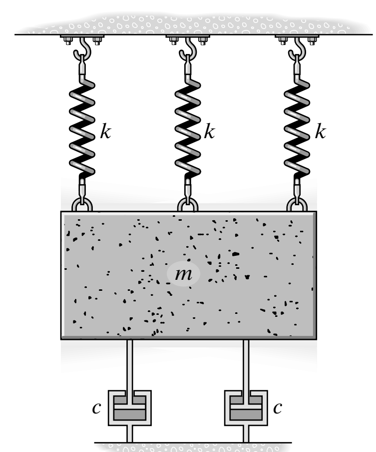
$$mx + cx + kx = F_0 \cos \omega t$$

Using Table 22–1,

$$Lq + Rq + \left(\frac{1}{C}\right)q = E_0 \cos \omega t \qquad \textbf{Ans.}$$



22–75. Determine the differential equation of motion for the damped vibratory system shown. What type of motion occurs? Take $k = 100$ N/m, $c = 200$ N·s/m, $m = 25$ kg.



Free-body Diagram: When the block is being displaced by an amount y vertically downward, the *restoring force* is developed by the three springs attached the block.

Equation of Motion:

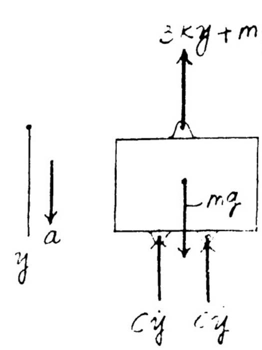
$$+\uparrow \Sigma F_x = 0; \qquad 3ky + mg + 2c\dot{y} - mg = -m\ddot{y}$$

$$m\ddot{y} + 2c\dot{y} + 3ky = 0 \qquad \textbf{[1]}$$

Here, $m = 25$ kg, $c = 200$ N·s/m and $k = 100$ N/m. Substituting these values into Eq. [1] yields

$$25\ddot{y} + 400\dot{y} + 300y = 0$$

$$\ddot{y} + 16\dot{y} + 12y = 0 \qquad \textbf{Ans.}$$

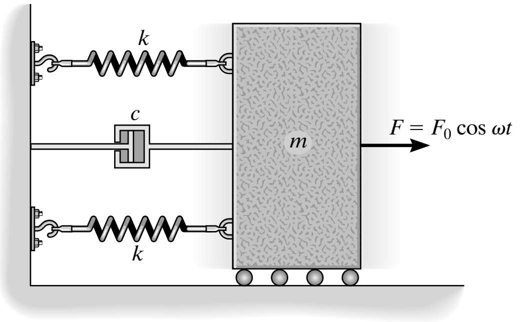


Comparing the above differential equation with Eq. 22–27, we have $m = 1$kg, $c = 16$ N·s/m and $k = 12$ N/m. Thus, $p = \sqrt{\frac{k}{m}} = \sqrt{\frac{12}{1}} = 3.464$ rad/s.

$$c_c = 2mp = 2(1)(3.464) = 6928 \text{ N}\cdot\text{s/m}$$

Since $c > c_c$, the system will not vibrate. Therefore it is **overdamped.** **Ans.**

***22–76.** Draw the electrical circuit that is equivalent to the mechanical system shown. What is the differential equation which describes the charge q in the circuit?

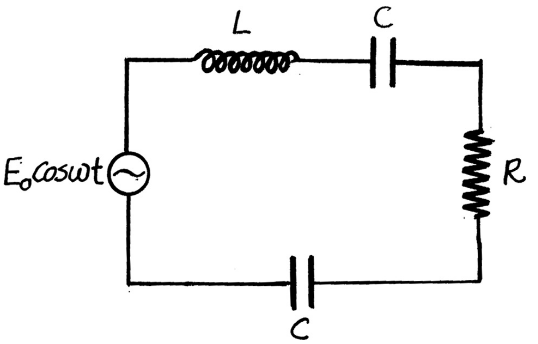


Electrical Circuit Analogs: The differential equation that deseribes the motion of the given mechanical system is

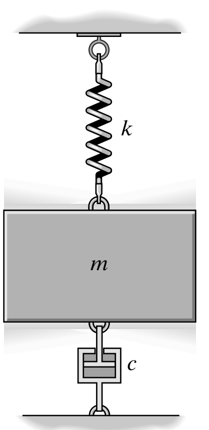
$$m\ddot{x} + c\dot{x} + 2kx = F_0 \cos \omega t$$

From Table 22–1 of the text, the differential equation of the analog electrical circuit is

$$L\ddot{q} + R\dot{q} + \left(\frac{2}{C}\right)q = E_0 \cos \omega t \qquad \textbf{Ans.}$$



•22–77. Draw the electrical circuit that is equivalent to the mechanical system shown. Determine the differential equation which describes the charge q in the circuit.



For the block

$$m\ddot{y} + c\dot{y} + ky = 0$$

Using Table 22–1

$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = 0 \qquad \textbf{Ans.}$$