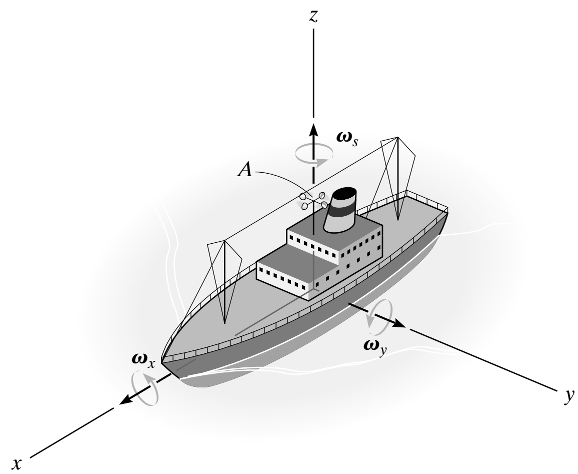
•20–1. The anemometer located on the ship at A spins about its own axis at a rate ω_s, while the ship rolls about the x axis at the rate ω_x and about the y axis at the rate ω_y. Determine the angular velocity and angular acceleration of the anemometer at the instant the ship is level as shown. Assume that the magnitudes of all components of angular velocity are constant and that the rolling motion caused by the sea is independent in the x and y directions.



$$\omega = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$$ **Ans.**

Let $\Omega = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j}$.

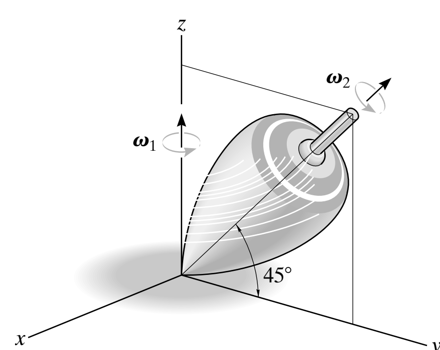
Since ω_x and ω_y are independent of one another, they do not change their direction or magnitude. Thus,

$$\alpha = \dot{\omega} = \left(\ddot{\omega}\right)_{xyz} + (\omega_x + \omega_y) \times \omega_z$$

$$\alpha = \mathbf{0} + (\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j}) \times (\omega_y\,\mathbf{k})$$

$$\alpha = \omega_y\,\omega_x\,\mathbf{i} - \omega_x\,\omega_z\,\mathbf{j}$$ **Ans.**

20–2. The motion of the top is such that at the instant shown it rotates about the z axis at $\omega_1 = 0.6$ rad/s, while it spins at $\omega_2 = 8$ rad/s. Determine the angular velocity and angular acceleration of the top at this instant. Express the result as a Cartesian vector.



$$\omega = \omega_1 + \omega_2$$

$$\omega = 0.6\mathbf{k} + 8\cos 45°\,\mathbf{j} + 8\sin 45°\mathbf{k}$$

$$\omega = \{5.66\mathbf{j} + 6.26\mathbf{k}\}\ \text{rad/s}$$ **Ans.**

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$$

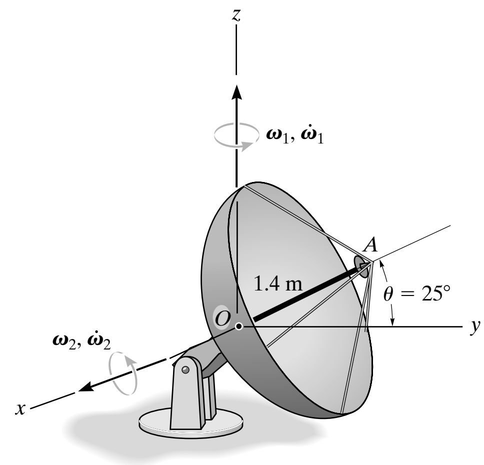
Let x, y, z axes have angular velocity of $\Omega = \omega_1$, thus

$$\dot{\omega}_1 = \mathbf{0}$$

$$\omega_2 = \left(\omega_2\right)_{xyz} + (\omega_1 \times \omega_2) = \mathbf{0} + (0.6\mathbf{k}) \times (8\cos 45°\mathbf{j} + 8\sin 45°\mathbf{k}) = -3.394\mathbf{i}$$

$$\alpha = \omega = \{-3.39\mathbf{i}\}\ \text{rad/s}^2$$ **Ans.**

20–3. At a given instant, the satellite dish has an angular motion $\omega_1 = 6$ rad/s and $\dot{\omega}_1 = 3$ rad/s^2 about the z axis. At this same instant $\theta = 25°$, the angular motion about the x axis is $\omega_2 = 2$ rad/s, and $\dot{\omega}_2 = 1.5$ rad/s^2. Determine the velocity and acceleration of the signal horn A at this instant.



Angular Velocity: The coordinate axes for the fixed frame (X, Y, Z) and rotating frame (x, y, z) at the instant shown are set to be coincident. Thus, the angular velocity of the satellite at this instant (with reference to X, Y, Z) can be expressed in terms of **i, j, k** components.

$$\omega = \omega_1 + \omega_2 = \{2\mathbf{i} + 6\mathbf{k}\} \text{ rad/s}$$

Angular Acceleration: The angular acceleration α will be determined by investigating separately the time rate of change of *each angular velocity component* with respect to the fixed XYZ frame. ω_2 is observed to have a *constant direction* from the rotating xyz frame if this frame is rotating at $\Omega = \omega_1 = \{6\mathbf{k}\}$ rad/s. Applying Eq. 20–6 with $(\dot{\omega}_2)_{xyz} = \{1.5\mathbf{i}\}$ rad/s^2. we have

$$\dot{\omega}_2 = (\dot{\omega}_2)_{xyz} + \omega_1 \times \omega_2 = 1.5\mathbf{i} + 6\mathbf{k} \times 2\mathbf{i} = \{1.5\mathbf{i} + 12\mathbf{j}\} \text{ rad/s}^2$$

Since ω_1 is always directed along the Z axis ($\Omega = \mathbf{0}$), then

$$\dot{\omega}_1 = (\dot{\omega}_1)_{xyz} + \mathbf{0} \times \omega_1 = \{3\mathbf{k}\} \text{ rad/s}^2$$

Thus, the angular acceleration of the satellite is

$$\alpha = \dot{\omega}_1 + \dot{\omega}_2 = \{1.5\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}\} \text{ rad/s}^2$$

Velocity and Acceleration: Applying Eqs. 20–3 and 20–4 with the ω and α obtained above and $\mathbf{r}_A = \{1.4 \cos 25°\mathbf{j} + 1.4 \sin 25°\mathbf{k}\}$ m $= \{1.2688\mathbf{j} + 0.5917\mathbf{k}\}$ m, we have

$$\mathbf{v}_A = \omega \times \mathbf{r}_A = (2\mathbf{i} + 6\mathbf{k}) \times (1.2688\mathbf{j} + 0.5917\mathbf{k})$$

$$= \{-7.61\mathbf{i} - 1.18\mathbf{j} + 2.54\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans.}$$

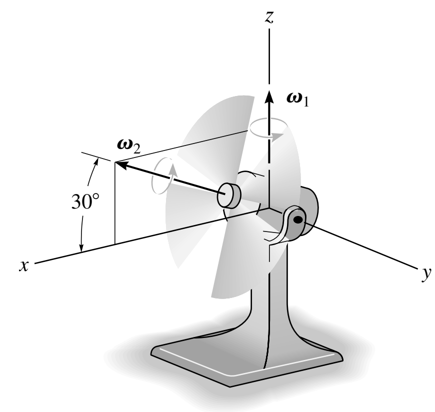
$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times (\omega \times \mathbf{r}_A)$$

$$= (1.3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}) \times (1.2688\mathbf{j} + 0.5917\mathbf{k})$$

$$+ (2\mathbf{i} + 6\mathbf{k}) \times [(2\mathbf{i} + 6\mathbf{k}) \times (1.2688\mathbf{j} + 0.5917\mathbf{k})]$$

$$= \{10.4\mathbf{i} - 51.6\mathbf{j} - 0.463\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

***20–4.** The fan is mounted on a swivel support such that at the instant shown it is rotating about the z axis at $\omega_1 = 0.8$ rad/s, which is increasing at 12 rad/s^2. The blade is spinning at $\omega_2 = 16$ rad/s, which is decreasing at 2 rad/s^2. Determine the angular velocity and angular acceleration of the blade at this instant.



$$\omega = \omega_1 + \omega_2$$

$$= 0.8\mathbf{k} + (16\cos 30°\mathbf{i} + 16\sin 30°\mathbf{k})$$

$$= \{13.9\mathbf{i} + 8.80\mathbf{k}\}\ \text{rad/s}$$ **Ans.**

For ω_2, $\Omega = \omega_1 = \{0.8\mathbf{k}\}$ rad/s.

$$(\dot{\omega}_2)_{XYZ} = (\dot{\omega}_2)_{xyz} + \Omega \times \omega_2$$

$$= (-2\cos 30°\mathbf{i} - 2\sin 30°\mathbf{k}) + (0.8\mathbf{k}) \times (16\cos 30°\mathbf{i} + 16\sin 30°\mathbf{k})$$

$$= \{-1.7320\mathbf{i} + 11.0851\mathbf{j} - 1\mathbf{k}\}\ \text{rad/s}^2$$

For ω_1, $\Omega = \mathbf{0}$.

$$(\omega_1)_{XYZ} = (\omega_1)_{xyz} + \Omega \times \omega_1$$

$$= (12\mathbf{k}) + 0$$

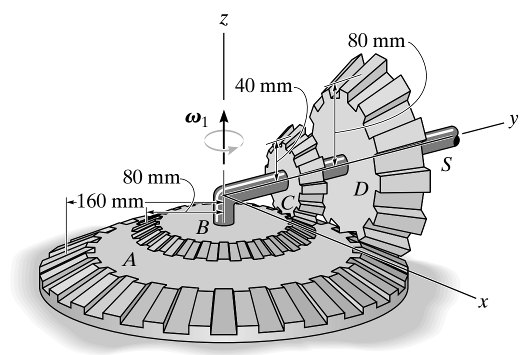
$$= \{12\mathbf{k}\}\ \text{rad/s}^2$$

$$\alpha = \dot{\omega} = (\dot{\omega}_1)_{XYZ} + (\dot{\omega}_2)_{XYZ}$$

$$\alpha = 12\mathbf{k} + (-1.7320\mathbf{i} + 11.0851\mathbf{j} - 1\mathbf{k})$$

$$= \{-1.73\mathbf{i} + 11.1\mathbf{j} + 11.0\mathbf{k}\}\ \text{rad/s}^2$$ **Ans.**

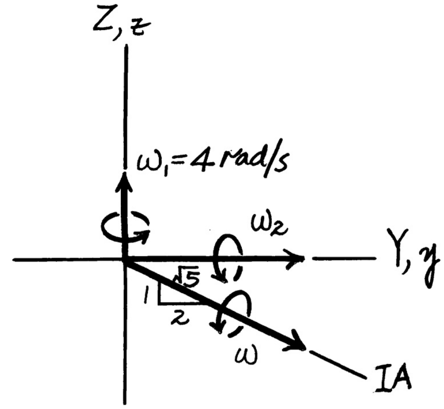
•**20–5.** Gears A and B are fixed, while gears C and D are free to rotate about the shaft S. If the shaft turns about the z axis at a constant rate of $\omega_1 = 4$ rad/s, determine the angular velocity and angular acceleration of gear C.



The resultant angular velocity $\omega = \omega_1 + \omega_2$ is always directed along the instantaneous axis of zero velocity IA.

$$\omega = \omega_1 + \omega_2$$

$$\frac{2}{\sqrt{5}}\omega\mathbf{j} - \frac{1}{\sqrt{5}}\omega\mathbf{k} = 4\mathbf{k} + \omega_2\mathbf{j}$$



Equating **j** and **k** components

$$-\frac{1}{\sqrt{5}}\omega = 4 \qquad \omega = -8.944 \text{ rad/s}$$

$$\omega_2 = \frac{2}{\sqrt{5}}(-8.944) = -8.0 \text{ rad/s}$$

Hence $\quad \omega = \frac{2}{\sqrt{5}}(-8.944)\mathbf{j} - \frac{1}{\sqrt{5}}(-8.944)\mathbf{k} = \{-8.0\mathbf{j} + 4.0\mathbf{k}\}$ rad/s **Ans.**

For ω_2, $\Omega = \omega_1 = \{4\mathbf{k}\}$ rad/s.

$$(\dot{\omega}_2)_{XYZ} = (\dot{\omega}_2)_{xyz} + \Omega \times \omega_2$$

$$= 0 + (4\mathbf{k}) \times (-8\mathbf{j})$$

$$= \{32\mathbf{i}\} \text{ rad/s}^2$$

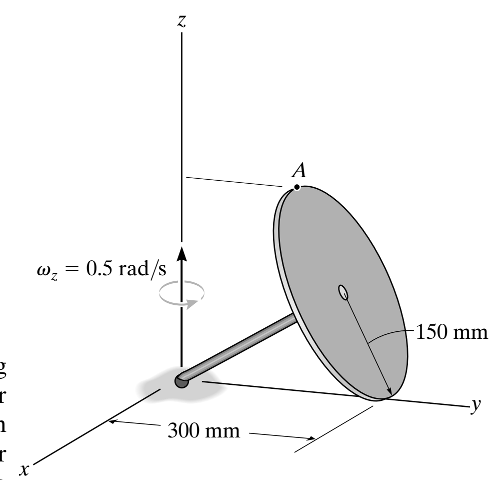
For ω_1, $\Omega = \mathbf{0}$.

$$(\dot{\omega}_1)_{XYZ} = (\dot{\omega}_1)_{xyz} + \Omega \times \omega_1 = 0 + 0 = 0$$

$$\alpha = \dot{\omega} = (\dot{\omega}_1)_{XYZ} + (\dot{\omega}_2)_{XYZ}$$

$$\alpha = \mathbf{0} + (32\mathbf{i}) = \{32\mathbf{i}\} \text{ rad/s}^2$$ **Ans.**

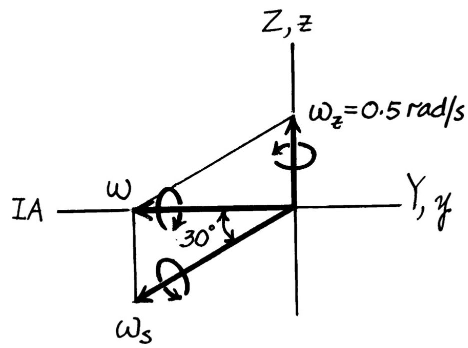
20–6. The disk rotates about the z axis $\omega_z = 0.5$ rad/s without slipping on the horizontal plane. If at this same instant ω_z is increasing at $\dot{\omega}_z = 0.3$ rad/s^2, determine the velocity and acceleration of point A on the disk.



Angular Velocity: The coordinate axes for the fixed frame (X, Y, Z) and rotating frame (x, y, z) at the instant shown are set to be coincident. Thus, the angular velocity of the disk at this instant (with reference to X, Y, Z) can be expressed in terms of **i, j, k** components. Since the disk rolls without slipping, then its angular velocity $\omega = \omega_s + \omega_z$ is always directed along the instantaneous axis of zero velocity (y axis). Thus,

$$\omega = \omega_s + \omega_z$$

$$-\omega\mathbf{j} = -\omega_s \cos 30°\mathbf{j} - \omega_s \sin 30°\mathbf{k} + 0.5\mathbf{k}$$



Equating **k** and **j** components, we have

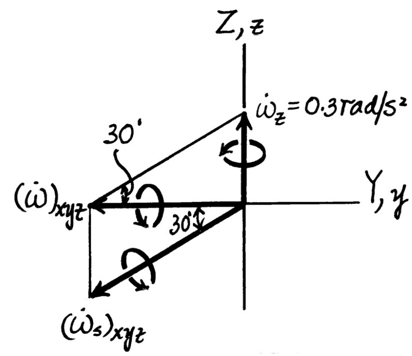
$$0 = -\omega_s \sin 30° + 0.5 \qquad \omega_s = 1.00 \text{ rad/s}$$

$$-\omega = -1.00 \cos 30° \qquad \omega = 0.8660 \text{ rad/s}$$

Angular Acceleration: The angular acceleration α will be determined by investigating the time rate of change of *angular velocity* with respect to the fixed XYZ frame. Since ω always lies in the fixed X–Y plane, then $\omega = \{-0.8660\mathbf{j}\}$ rad/s is observed to have a *constant direction* from the rotating xyz frame if this frame is rotating at $\Omega = \omega_z = \{0.5\mathbf{k}\}$ rad/s. $(\dot{\omega}_s)_{xyz}$

$$= \left\{-\frac{0.3}{\sin 30°}(\cos 30°)\,\mathbf{j} - \frac{0.3}{\sin 30°}(\sin 30°)\,\mathbf{k}\right\} \text{rad/s}^2 = \{-0.5196\mathbf{j} - 0.3\mathbf{k}\} \text{ rad/s}^2.$$

Thus, $(\dot{\omega})_{xyz} = \dot{\omega}_z + (\dot{\omega}_x)_{xyz} = \{-0.5196\mathbf{j}\}$ rad/s^2. Applying Eq. 20–6, we have



$$\alpha = \dot{\omega} = (\dot{\omega})_{xyz} + \omega_z \times \omega$$

$$= -0.5196\mathbf{j} + 0.5\mathbf{k} \times (-0.8660\mathbf{j})$$

$$= \{0.4330\mathbf{i} - 0.5196\mathbf{j}\} \text{ rad/s}^2$$

Velocity and Acceleration: Applying Eqs. 20–3 and 20–4 with the ω and α obtained above and $\mathbf{r}_A = \{(0.3 - 0.3\cos 60°)\mathbf{j} + 0.3 \sin 60°\mathbf{k}\}$ m $= \{0.15\mathbf{j} + 0.2598\mathbf{k}\}$ m, we have

$$\mathbf{v}_A = \omega \times \mathbf{r}_A = (-0.8660\mathbf{j}) \times (0.15\mathbf{j} + 0.2598\mathbf{k}) = \{-0.225\mathbf{i}\} \text{ m/s} \qquad \textbf{Ans.}$$

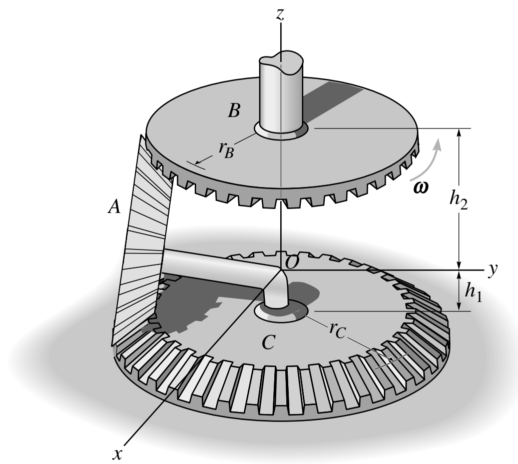
$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times (\omega \times \mathbf{r}_A)$$

$$= (0.4330\mathbf{i} - 0.5196\mathbf{j}) \times (0.15\mathbf{j} + 0.2598\mathbf{k})$$

$$+ (-0.8660\mathbf{j}) \times [(-0.8660\mathbf{j}) \times (0.15\mathbf{j} + 0.2598\mathbf{k})]$$

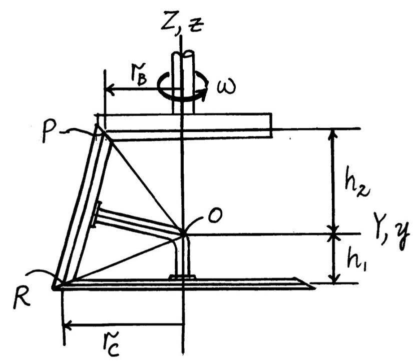
$$= \{-0.135\mathbf{i} - 0.1125\mathbf{j} - 0.130\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

20–7. If the top gear B rotates at a constant rate of $\boldsymbol{\omega}$, determine the angular velocity of gear A, which is free to rotate about the shaft and rolls on the bottom fixed gear C.



$$\mathbf{v}_P = \omega\mathbf{k} \times (-r_B\,\mathbf{j}) = \omega r_B\,\mathbf{i}$$

Also,



$$\mathbf{v}_P = \boldsymbol{\omega}_A \times (-r_B\,\mathbf{j} + h_2\mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_{Ax} & \omega_{Ay} & \omega_{Az} \\ 0 & -r_B & h_2 \end{vmatrix}$$

$$= (\omega_{Ay}\,h_2 + \omega_{Az}\,r_B)\mathbf{i} - (\omega_{Ax}\,h_2)\mathbf{j} - \omega_{Ax}\,r_B\,\mathbf{k}$$

Thus,

$$\omega r_B = \omega_{Ay}\,h_2 + \omega_{Az}\,r_B \qquad \textbf{(1)}$$

$$0 = \omega_{Ax}\,h_2$$

$$0 = \omega_{Ax}\,r_B$$

$$\omega_{Ax} = 0$$

$$\mathbf{v}_R = \mathbf{0} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \omega_{Ay} & \omega_{Az} \\ 0 & -r_C & -h_1 \end{vmatrix} = (-\omega_{Ay}\,h_1 + \omega_{Az}\,r_C)\mathbf{i}$$

$$\omega_{Ay} = \omega_{Az}\left(\frac{r_C}{h_1}\right)$$

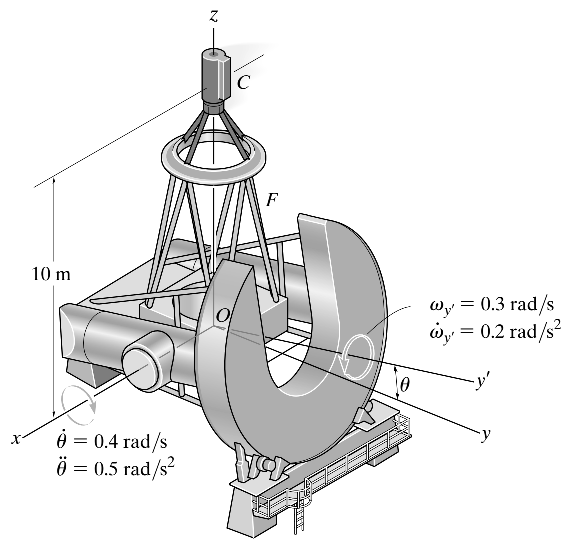
From Eq. (1)

$$\omega r_B = \omega_{Az}\left[\left(\frac{r_C\,h_2}{h_1}\right) + r_B\right]$$

$$\omega_{Az} = \frac{r_B\,h_1\,\omega}{r_C\,h_2 + r_B\,h_1}; \qquad \omega_{Ay} = \left(\frac{r_C}{h_1}\right)\left(\frac{r_B\,h_1\omega}{r_C\,h_2 + r_B\,h_1}\right)$$

$$\boldsymbol{\omega}_A = \left(\frac{r_C}{h_1}\right)\left(\frac{r_B\,h_1\,\omega}{r_C\,h_2 + r_B\,h_1}\right)\mathbf{j} + \left(\frac{r_B\,h_1\omega}{r_C\,h_2 + r_B\,h_1}\right)\mathbf{k} \qquad \textbf{Ans.}$$

***20–8.** The telescope is mounted on the frame F that allows it to be directed to any point in the sky. At the instant $\theta = 30°$, the frame has an angular acceleration of $\alpha_{y'} = 0.2\ \text{rad/s}^2$ and an angular velocity of $\omega_{y'} = 0.3$ rad/s about the y' axis, and $\ddot{\theta} = 0.5\ \text{rad/s}^2$ while $\dot{\theta} = 0.4$ rad/s. Determine the velocity and acceleration of the observing capsule at C at this instant.



Angular Velocity: The coordinate axes for the fixed frame (X, Y, Z) and rotating frame (x, y, z) at the instant shown are coincident. Thus, the angular velocity of the frame at this instant is

$$\omega = \dot{\theta} + \omega_{y'} = -0.4\mathbf{i} + (0.3\cos 30°\mathbf{j} + 0.3\sin 30°\mathbf{k})$$

$$= [-0.4\mathbf{i} + 0.2598\mathbf{j} + 0.15\mathbf{k}]\ \text{rad/s}$$

Angular Acceleration: $\omega_{y'}$ is observed to have a constant direction relative to the rotating xyz frame which rotates at $\Omega = \dot{\theta} = [-0.4\mathbf{i}]$ rad/s. With $\left(\dot{\omega}_{y'}\right)_{xyz} = \alpha_{y'} = 0.2\cos 30°\mathbf{j} + 0.2\sin 30°\mathbf{k} = [0.1732\mathbf{j} + 0.1\mathbf{k}]\ \text{rad/s}^2$, we obtain

$$\dot{\omega}_{y'} = \left(\dot{\omega}_{y'}\right)_{xyz} + \Omega \times \omega_{y'}$$

$$= (0.1732\mathbf{j} + 0.1\mathbf{k}) + (-0.4\mathbf{i}) \times (0.3\cos 30°\mathbf{j} + 0.3\sin 30°\mathbf{k})$$

$$= [0.2332\mathbf{j} - 0.003923\mathbf{k}]\ \text{rad/s}^2$$

Since $\dot{\theta}$ is always directed along the X axis ($\Omega = 0$), then

$$\ddot{\theta} = (\ddot{\theta})_{xyz} + 0 \times \dot{\theta} = [-0.5\mathbf{i}]\ \text{rad/s}^2$$

Thus, the angular acceleration of the frame is

$$\alpha = \dot{\omega}_{y'} + \ddot{\theta} = [-0.5\mathbf{i} + 0.2332\mathbf{j} - 0.003923\mathbf{k}]\ \text{rad/s}^2$$

Velocity and Acceleration:

$$\mathbf{v}_c = \omega \times \mathbf{r}_{oc} = (-0.4\mathbf{i} + 0.2598\mathbf{j} + 0.15\mathbf{k}) \times (10\mathbf{k})$$

$$= [2.598\mathbf{i} + 4.00\mathbf{j}]\ \text{m/s} = [2.60\mathbf{i} + 4.00\mathbf{j}]\ \text{m/s} \qquad \textbf{Ans.}$$

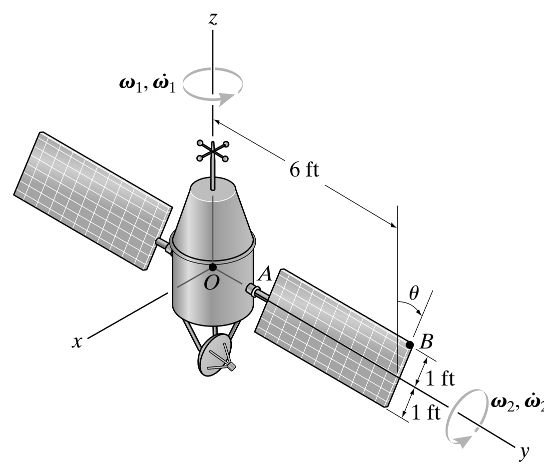
$$\mathbf{a}_c = \alpha \times \mathbf{r}_{oc} + \omega \times (\omega \times \mathbf{r}_{oc})$$

$$= (-0.5\mathbf{i} + 0.2332\mathbf{j} - 0.003923\mathbf{k}) \times (10\mathbf{k}) + (-0.4\mathbf{i} + 0.2598\mathbf{j} + 0.15\mathbf{k}) \times [(-0.4\mathbf{i} + 0.2598\mathbf{j} + 0.15\mathbf{k}) \times (10\mathbf{k})]$$

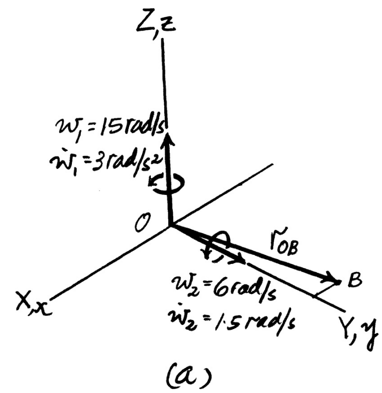
$$= [1.732\mathbf{i} + 5.390\mathbf{j} - 2.275\mathbf{k}]\ \text{m/s}^2$$

$$= [1.73\mathbf{i} + 5.39\mathbf{j} - 2.275\mathbf{k}]\ \text{m/s}^2 \qquad \textbf{Ans.}$$

•20–9. At the instant when $\theta = 90°$, the satellite's body is rotating with an angular velocity of $\omega_1 = 15$ rad/s and angular acceleration of $\dot{\omega}_1 = 3$ rad/s^2. Simultaneously, the solar panels rotate with an angular velocity of $\omega_2 = 6$ rad/s and angular acceleration of $\dot{\omega}_2 = 1.5$ rad/s^2. Determine the velocity and acceleration of point B on the solar panel at this instant.



Here, the solar panel rotates about a fixed point O. The XYZ fixed reference frame is set to coincide with the xyz rotating frame at the instant considered. Thus, the angular velocity of the solar panel can be obtained by vector addition of ω_1 and ω_2.



$$\omega = \omega_1 + \omega_2 = [6\mathbf{j} + 15\mathbf{k}] \text{ rad/s} \qquad \textbf{Ans.}$$

The angular acceleration of the solar panel can be determined from

$$\alpha = \dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$$

If we set the xyz frame to have an angular velocity of $\Omega = \omega_1 = [15\mathbf{k}]$ rad/s, then the direction of ω_2 will remain constant with respect to the xyz frame, which is along the y axis. Thus,

$$\dot{\omega}_2 = (\dot{\omega}_2)_{xyz} + \omega_1 \times \omega_2 = 1.5\mathbf{j} + (15\mathbf{k} \times 6\mathbf{j}) = [-90\mathbf{i} + 1.5\mathbf{j}] \text{ rad/s}^2$$

Since ω_1 is always directed along the Z axis when $\Omega = \omega_1$, then

$$\dot{\omega}_1 = (\dot{\omega}_1)_{xyz} + \omega_1 \times \omega_1 = [3\mathbf{k}] \text{ rad/s}^2$$

Thus,

$$\alpha = 3\mathbf{k} + (-90\mathbf{i} + 1.5\mathbf{j})$$

$$= [-90\mathbf{i} + 1.5\mathbf{j} + 3\mathbf{k}] \text{ rad/s}^2$$

When $\theta = 90°$, $\mathbf{r}_{OB} = [-1\mathbf{i} + 6\mathbf{j}]$ ft. Thus,

$$\mathbf{v}_B = \omega \times r_{OB} = (6\mathbf{j} + 15\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j})$$

$$= [-90\mathbf{i} - 15\mathbf{j} + 6\mathbf{k}] \text{ ft/s} \qquad \textbf{Ans.}$$

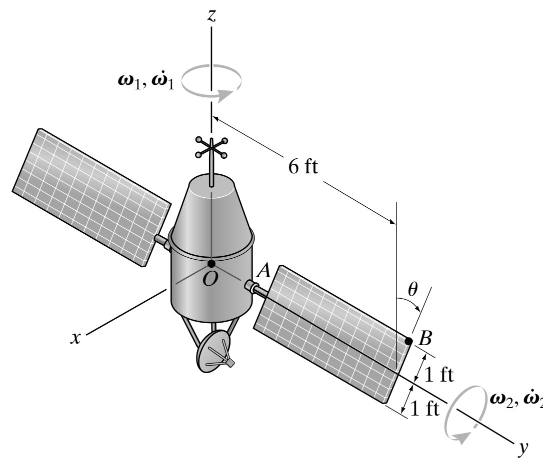
and

$$\mathbf{a}_B = \alpha \times \mathbf{r}_{OB} + \omega \times (\omega \times \mathbf{r}_{OB})$$

$$= (-90\mathbf{i} + 15\mathbf{j} + 3\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j}) + (6\mathbf{j} + 15\mathbf{k}) \times [(6\mathbf{j} + 15\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j})]$$

$$= [243\mathbf{i} - 1353\mathbf{j} + 1.5\mathbf{k}] \text{ ft/s}^2 \qquad \textbf{Ans.}$$

20–10. At the instant when $\theta = 90°$, the satellite's body travels in the x direction with a velocity of $\mathbf{v}_O = \{500\mathbf{i}\}$ m/s and acceleration of $\mathbf{a}_O = \{50\mathbf{i}\}$ m/s^2. Simultaneously, the body also rotates with an angular velocity of $\omega_1 = 15$ rad/s and angular acceleration of $\dot{\omega}_1 = 3$ rad/s^2. At the same time, the solar panels rotate with an angular velocity of $\omega_2 = 6$ rad/s and angular acceleration of $\dot{\omega}_2 = 1.5$ rad/s^2 Determine the velocity and acceleration of point B on the solar panel.



The XYZ translating reference frame is set to coincide with the xyz rotating frame at the instant considered. Thus, the angular velocity of the solar panel at this instant can be obtained by vector addition of ω_1 and ω_2.

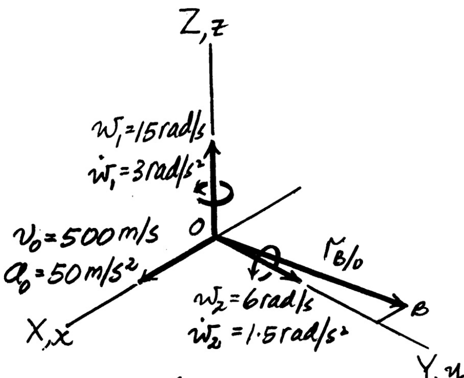
$$\omega = \omega_1 + \omega_2 = [6\mathbf{j} + 15\mathbf{k}] \text{ rad/s}$$

The angular acceleration of the solar panel can be determined from

$$\alpha = \dot{\omega} = \omega_1 + \omega_2$$

If we set the xyz frame to have an angular velocity of $\Omega = \omega_1 = [15\mathbf{k}]$ rad/s, then the direction of ω_2 will remain constant with respect to the xyz frame, which is along the y axis. Thus,

$$\dot{\omega}_2 = (\dot{\omega}_2)_{xyz} + \omega_1 \times \omega_2 = 1.5\mathbf{j} + (15\mathbf{k} \times 6\mathbf{j}) = [-90\mathbf{i} + 15\mathbf{j}] \text{ rad/s}^2$$



(a)

Since ω_1 is always directed along the Z axis when $\Omega = \omega_1$, then

$$\dot{\omega}_1 = (\dot{\omega}_1)_{xyz} + \omega_1 \times \omega_1 = [3\mathbf{k}] \text{ rad/s}^2$$

Thus,

$$\alpha = 3\mathbf{k} + (-90\mathbf{i} + 1.5\mathbf{j})$$

$$= [-90\mathbf{i} + 1.5\mathbf{j} + 3\mathbf{k}] \text{ rad/s}^2$$

When $\theta = 90°$, $\mathbf{r}_{B/O} = [-1\mathbf{i} + 6\mathbf{j}]$ ft. Since the satellite undergoes general motion, then

$$\mathbf{v}_B = \mathbf{v}_O + \omega \times r_{B/O} = (500\mathbf{i}) + (6\mathbf{j} + 15\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j})$$

$$= [410\mathbf{i} - 15\mathbf{j} + 6\mathbf{k}] \text{ ft/s} \qquad \textbf{Ans.}$$

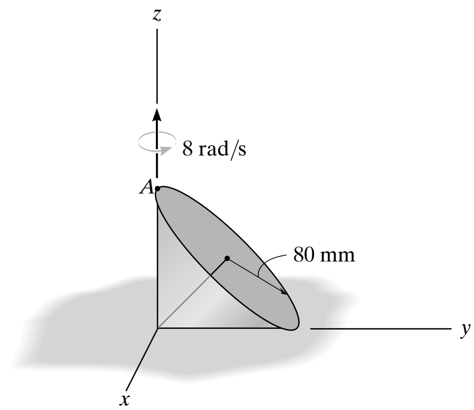
and

$$\mathbf{a}_B = \mathbf{a}_O + \alpha \times \mathbf{r}_{B/O} + \omega \times (\omega \times \mathbf{r}_{B/O})$$

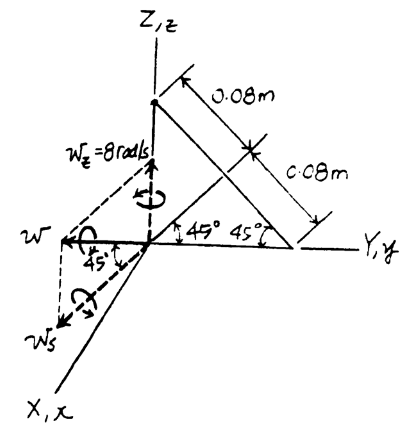
$$= 50\mathbf{i} + (-90\mathbf{i} + 1.5\mathbf{j} + 3\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j}) + (6\mathbf{j} + 15\mathbf{k}) \times [(6\mathbf{j} + 15\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j})]$$

$$= [293\mathbf{i} - 1353\mathbf{j} + 1.5\mathbf{k}] \text{ ft/s}^2 \qquad \textbf{Ans.}$$

20–11. The cone rolls in a circle and rotates about the z axis at a constant rate $\omega_z = 8$ rad/s. Determine the angular velocity and angular acceleration of the cone if it rolls without slipping. Also, what are the velocity and acceleration of point A?



Angular Velocity: The coordinate axes for the fixed frame (X, Y, Z) and rotating frame (x, y, z) at the instant shown are set to be coincident. Thus, the angular velocity of the disk at this instant (with reference to X, Y, Z) can be expressed in terms of **i, j, k** components. Since the disk rolls without slipping, then its angular velocity $\omega = \omega_s + \omega_z$ is always directed along the instantaneuos axis of zero velocity (y axis). Thus,



$$\omega = \omega_s + \omega_z$$

$$-\omega\mathbf{j} = -\omega_s \cos 45°\mathbf{j} - \omega_s \sin 45°\mathbf{k} + 8\mathbf{k}$$

Equating **k** and **j** components, we have

$$0 = -\omega_s \sin 45° + 8 \qquad \omega_s = 11.31 \text{ rad/s}$$

$$-\omega = -11.13 \cos 45° \qquad \omega = 8.00 \text{ rad/s}$$

Thus, $\omega = \{-8.00\mathbf{j}\}$ rad/s **Ans.**

Angular Acceleration: The angular acceleration α will be determined by investigating the time rate of change of *angular velocity* with respect to the fixed XYZ frame. Since ω always lies in the fixed X–Y plane, then $\omega = \{-8.00\mathbf{j}\}$ rad/s is observed to have a *constant direction* from the rotating xyz frame if this frame is rotating at $\Omega = \omega_z = \{8\mathbf{k}\}$ rad/s. Applying Eq. 20–6 with $(\dot{\omega})_{xyz} = \mathbf{0}$, we have

$$\alpha = \dot{\omega} = (\dot{\omega})_{xyz} + \omega_z \times \omega = \mathbf{0} + 8\mathbf{k} \times (-8.00\mathbf{j}) = \{64.0\mathbf{i}\} \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Velocity and Acceleration: Applying Eqs. 20–3 and 20–4 with the ω and α obtained above and $\mathbf{r}_A = \{0.16 \cos 45°\mathbf{k}\}$ m $= \{0.1131\mathbf{k}\}$ m, we have

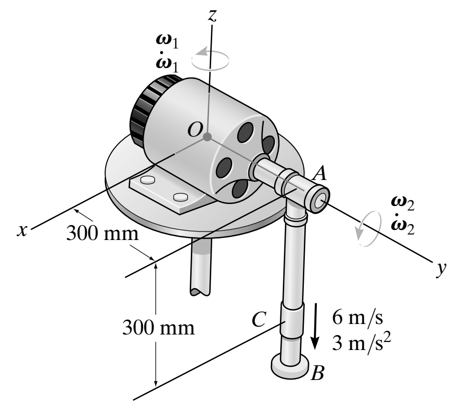
$$\mathbf{v}_A = \omega \times \mathbf{r}_A = (-8.00\mathbf{j}) \times (0.1131\mathbf{k}) = \{-0.905\mathbf{i}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times (\omega \times \mathbf{r}_A)$$

$$= (64.0\mathbf{i}) \times (0.1131\mathbf{k}) + (-8.00\mathbf{j}) \times [(-8.00\mathbf{j}) \times (0.1131\mathbf{k})]$$

$$= \{-7.24\mathbf{j} - 7.24\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

***20–12.** At the instant shown, the motor rotates about the z axis with an angular velocity of $\omega_1 = 3$ rad/s and angular acceleration of $\dot{\omega}_1 = 1.5$ rad/s^2. Simultaneously, shaft OA rotates with an angular velocity of $\omega_2 = 6$ rad/s and angular acceleration of $\dot{\omega}_2 = 3$ rad/s^2, and collar C slides along rod AB with a velocity and acceleration of 6 m/s and 3 m/s^2. Determine the velocity and acceleration of collar C at this instant.



The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [3\mathbf{k}] \text{ rad/s} \qquad \dot{\omega} = [1.5\mathbf{k}] \text{ rad/s}^2$$

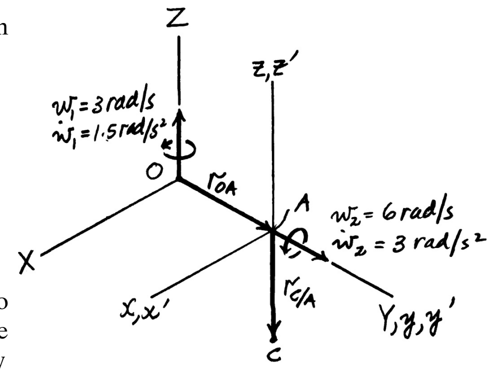
Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (3\mathbf{k}) \times (0.3\mathbf{j}) = [-0.9\mathbf{i}] \text{ m/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega \times \mathbf{r}_{OA})$$

$$= (1.5\mathbf{k}) \times (0.3\mathbf{j}) + (3\mathbf{k}) \times (3\mathbf{k} \times 0.3\mathbf{j})$$

$$= [-0.45\mathbf{i} - 2.7\mathbf{j}] \text{ m/s}^2$$



(a)

In order to determine the motion of point C relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [6\mathbf{j}]$ rad/s, the direction of $\left(\mathbf{r}_{C/A}\right)_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $\left(\mathbf{r}_{C/A}\right)_{xyz}$,

$$(\mathbf{v}_{C/A})_{xyz} = (\dot{\mathbf{r}}_{C/A})_{xyz} = \left[(\dot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{C/A})_{xyz}\right]$$

$$= (-6\mathbf{k}) + 6\mathbf{j} \times (-0.3\mathbf{k})$$

$$= [-1.8\mathbf{i} - 6\mathbf{k}] \text{ m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = [3\mathbf{j}]$ rad/s^2. Taking the time derivative of $(\dot{\mathbf{r}}_{C/A})_{xyz}$,

$$(\mathbf{a}_{C/A})_{xyz} = (\ddot{\mathbf{r}}_{C/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{C/A})_{x'y'z'}\right] + \dot{\omega}_2 \times (\mathbf{r}_{C/A})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{C/A})_{xyz}$$

$$= [(-3\mathbf{k}) + 6\mathbf{j} \times (-6\mathbf{k})] + (3\mathbf{j}) \times (-0.3\mathbf{k}) + 6\mathbf{j} \times (-1.8\mathbf{i} - 6\mathbf{k})$$

$$= [-72.9\mathbf{i} + 7.8\mathbf{k}] \text{ m/s}$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$

$$= (-0.9\mathbf{i}) + 3\mathbf{k} \times (-0.3\mathbf{k}) + (-1.8\mathbf{i} - 6\mathbf{k})$$

$$= [-2.7\mathbf{i} - 6\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

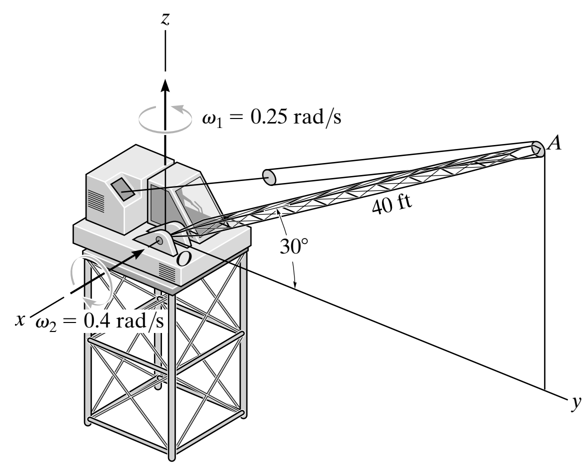
and

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$

$$= (-0.45\mathbf{i} - 2.7\mathbf{j}) + 1.5\mathbf{k} \times (-0.3\mathbf{k}) + (3\mathbf{k}) \times [(3\mathbf{k}) \times (-0.3\mathbf{k})] + 2(3\mathbf{k}) \times (-1.8\mathbf{i} - 6\mathbf{k}) + (-72.9\mathbf{i} + 7.8\mathbf{k})$$

$$= [-73.35\mathbf{i} - 13.5\mathbf{j} + 7.8\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

•**20–13.** At the instant shown, the tower crane rotates about the z axis with an angular velocity $\omega_1 = 0.25$ rad/s, which is increasing at 0.6 rad/s^2. The boom OA rotates downward with an angular velocity $\omega_2 = 0.4$ rad/s, which is increasing at 0.8 rad/s^2. Determine the velocity and acceleration of point A located at the end of the boom at this instant.



$\omega = \omega_1 \cdot \omega_2 = \{-0.4\,\mathbf{i} + 0.25\mathbf{k}\}$ rad/s

$\Omega = \{0.25\,\mathbf{k}\}$ rad/s

$\omega = \omega_{1\cdot 2} + \Omega \times \omega = (-0.8\,\mathbf{i} + 0.6\mathbf{k}) + (0.25\mathbf{k}) \times (-0.4\,\mathbf{i} + 0.25\mathbf{k})$

$= \{-0.8\mathbf{i} - 0.1\mathbf{j} + 0.6\mathbf{k}\}$ rad/s^2

$\mathbf{r}_A = 40 \cos 30°\mathbf{j} + 40 \sin 30°\mathbf{k} = \{34.64\mathbf{j} + 20\mathbf{k}\}$ ft

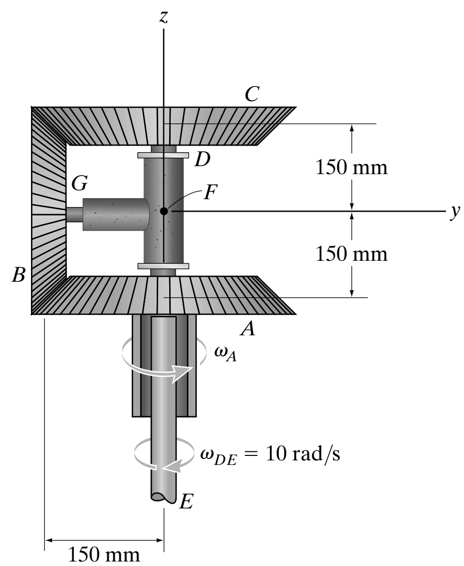
$\mathbf{v}_A = \omega \times \mathbf{r}_A = 1 - 0.4\,\mathbf{i} + 0.25\,\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k})$

$\mathbf{v}_A = \{-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k}\}$ft/s **Ans.**

$\mathbf{a}_A = \alpha \cdot \mathbf{r}_A + \omega \times \mathbf{v}_A = (-0.8\mathbf{i} - 0.1\mathbf{j} + 0.6\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k}) + (-0.4\mathbf{i} + 0.25\mathbf{k}) \times (-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k})$

$\mathbf{a}_A = \{-24.8\mathbf{i} + 8.29\mathbf{j} - 30.9\mathbf{k}\}$ft/s^2 **Ans.**

20–14. Gear C is driven by shaft DE, while gear B spins freely about its axle GF, which precesses freely about shaft DE. If gear A is held fixed ($\omega_A = 0$), and shaft DE rotates with a constant angular velocity of $\omega_{DE} = 10$ rad/s, determine the angular velocity of gear B.



Since gear C rotates about the fixed axis ($z$axis), the velocity of the contact point P between gears B and C is

$$\mathbf{v}_P = \omega_{DE} \times \mathbf{r}_C = (10\mathbf{k}) \times (-0.15\mathbf{j}) = [1.5\mathbf{i}] \text{ m/s}$$

Here, gear B spins about its axle with an angular velocity of $(\omega_B)_y$ and precesses about shaft DE with an angular velocity of $(\omega_B)_z$. Thus, the angular velocity of gear B is

$$\omega_B = (\omega_B)_y \mathbf{j} + (\omega_B)_z \mathbf{k}$$

Here, $\mathbf{r}_{FP} = [-0.15\mathbf{j} + 0.15\mathbf{k}]$ m. Thus,

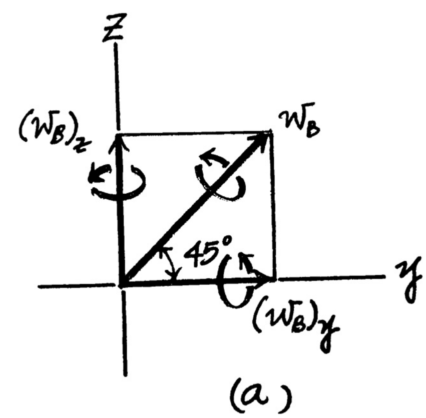
$$\mathbf{v}_P = \omega_B \times \mathbf{r}_{FP}$$

$$1.5\mathbf{i} = \left[(\omega_B)_y \mathbf{j} + (\omega_B)_z \mathbf{k}\right] \times (-0.15\mathbf{j} + 0.15\mathbf{k})$$

$$1.5\mathbf{i} = \left[0.15(\omega_B)_y - (-0.15)(\omega_B)_z\right]\mathbf{i}$$

$$1.5 = 0.15(\omega_B)_y + 0.15(\omega_B)_z$$

$$(\omega_B)_y + (\omega_B)_z = 10 \qquad \textbf{(1)}$$



Since gear A is held fixed, ω_B will be directed along the instantaneous axis of zero velocity, which is along the line where gears A and B mesh. From the geometry of Fig. a,

$$\frac{(\omega_B)_z}{(\omega_B)_y} = \tan 45° \qquad (\omega_B)_z = (\omega_B)_y \qquad \textbf{(2)}$$

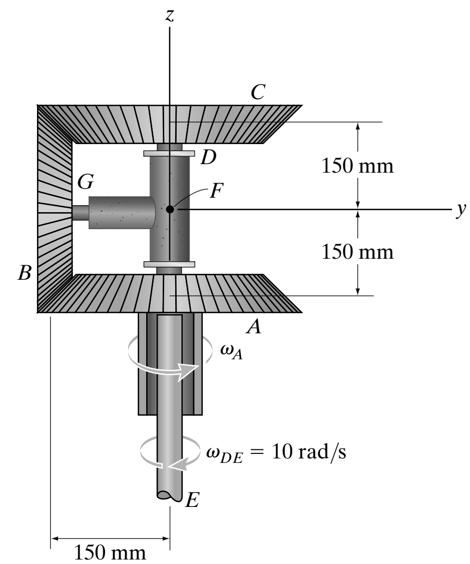
Solving Eqs. (1) and (2),

$$(\omega_B)_y = (\omega_B)_z = 5 \text{ rad/s}$$

Thus,

$$\omega_B = [5\mathbf{j} + 5\mathbf{k}] \text{ rad/s} \qquad \textbf{Ans.}$$

20–15. Gear C is driven by shaft DE, while gear B spins freely about its axle GF, which precesses freely about shaft DE. If gear A is driven with a constant angular velocity of $\omega_A = 5$ rad/s and shaft DE rotates with a constant angular velocity of $\omega_{DE} = 10$ rad/s, determine the angular velocity of gear B.



Since gears A and C rotate about the fixed axis (z axis), the velocity of the contact point P between gears B and C and point P' between gears A and B are

$$\mathbf{v}_P = \omega_{DE} \times \mathbf{r}_C = (10\mathbf{k}) \times (-0.15\mathbf{j}) = [1.5\mathbf{i}] \text{ m/s}$$

and

$$\mathbf{v}_{P'} = \omega_A \times \mathbf{r}_A = (-5\mathbf{k}) \times (-0.15\mathbf{j}) = [-0.75\mathbf{i}] \text{ m/s}$$

Gear B spins about its axle with an angular velocity of $(\omega_B)_y$ and precesses about shaft DE with an angular velocity of $(\omega_B)_z$. Thus, the angular velocity of gear B is

$$\omega_B = (\omega_B)_y\mathbf{j} + (\omega_B)_z\,\mathbf{k}$$

Here, $r_{FP} = [-0.15\mathbf{j} + 0.15\mathbf{k}]$ m and $r_{FP'} = [-0.15\mathbf{j} - 0.15\mathbf{k}]$. Thus,

$$\mathbf{v}_P = \omega_B \times \mathbf{r}_{FP}$$

$$1.5\mathbf{i} = \left[(\omega_B)_y\mathbf{j} + (\omega_B)_z\,\mathbf{k}\right] \times (-0.15\mathbf{j} + 0.15\mathbf{k})$$

$$1.5\mathbf{i} = \left[0.15(\omega_B)_y + 0.15(\omega_B)_z\right]\mathbf{i}$$

so that

$$1.5 = 0.15(\omega_B)_y + 0.15(\omega_B)_z$$

$$(\omega_B)_y + (\omega_B)_z = 10 \qquad \textbf{(1)}$$

and

$$\mathbf{v}_{P'} = \omega_B \times \mathbf{r}_{FP'}$$

$$-0.75\mathbf{i} = \left[(\omega_B)_y\mathbf{j} + (\omega_B)_z\mathbf{k}\right] \times (-0.15\mathbf{j} - 0.15\mathbf{k})$$

$$-0.75\mathbf{i} = \left[0.15(\omega_B)_z - 0.15(\omega_B)_y\right]\mathbf{i}$$

Thus,

$$-0.75 = 0.15(\omega_B)_z - 0.15(\omega_B)_y$$

$$(\omega_B)_y - (\omega_B)_z = 5 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), we obtain

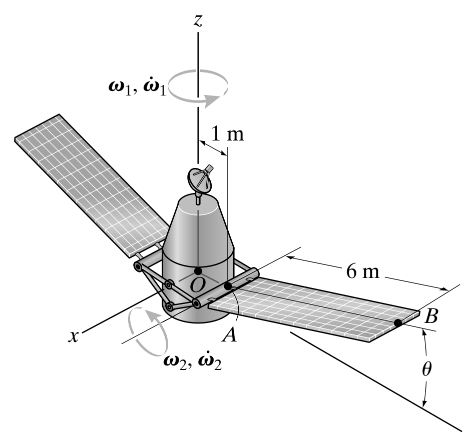
$$(\omega_B)_y = 7.5 \text{ rad/s}$$

$$(\omega_B)_z = 2.5 \text{ rad/s}$$

Thus,

$$\omega_B = [7.5\mathbf{j} + 2.5\mathbf{k}] \text{ rad/s} \qquad \textbf{Ans.}$$

***20–16.** At the instant $\theta = 0°$, the satellite's body is rotating with an angular velocity of $\omega_1 = 20$ rad/s, and it has an angular acceleration of $\dot{\omega}_1 = 5$ rad/s^2. Simultaneously, the solar panels rotate with an angular velocity of $\omega_2 = 5$ rad/s and angular acceleration of $\dot{\omega}_2 = 3$ rad/s^2. Determine the velocity and acceleration of point B located at the end of one of the solar panels at this instant.



The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

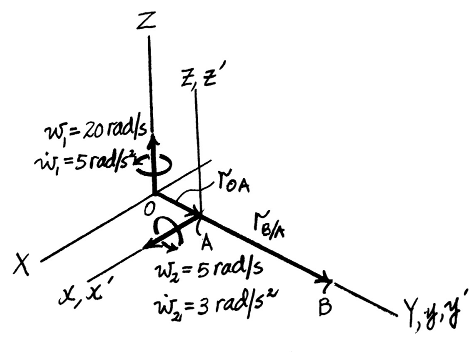
$$\Omega = \omega_1 = [20\mathbf{k}] \text{ rad/s} \qquad \dot{\omega} = \dot{\omega}_1 = [5\mathbf{k}] \text{ rad/s}^2$$

Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (20\mathbf{k}) \times (1\mathbf{j}) = [-20\mathbf{i}] \text{ m/s}$$

and

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$
$$= (5\mathbf{k}) \times (1\mathbf{j}) + (20\mathbf{i}) \times \left[(20\mathbf{i}) \times (1\mathbf{j})\right]$$
$$= [-5\mathbf{i} - 400\mathbf{j}] \text{ m/s}^2$$



(a)

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [5\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{B/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{B/A})_{xyz}$,

$$(\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{B/A})_{xyz}\right]$$
$$= 0 + (5\mathbf{i}) \times (6\mathbf{j})$$
$$= [30\mathbf{k}] \text{ m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = [3\mathbf{i}]$ rad/s^2. Taking the time derivative of $(\dot{\mathbf{r}}_{B/A})_{xyz}$,

$$(\mathbf{a}_{B/A})_{xyz} = (\ddot{\mathbf{r}}_{B/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{x'y'z'}\right] + \dot{\omega}_2 \times (\mathbf{r}_{B/A})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{xyz}$$
$$= [\mathbf{0} + \mathbf{0}] + (3\mathbf{i}) \times (6\mathbf{j}) + (5\mathbf{i}) \times (30\mathbf{k})$$
$$= [-150\mathbf{j} + 18\mathbf{k}] \text{ m/s}^2$$

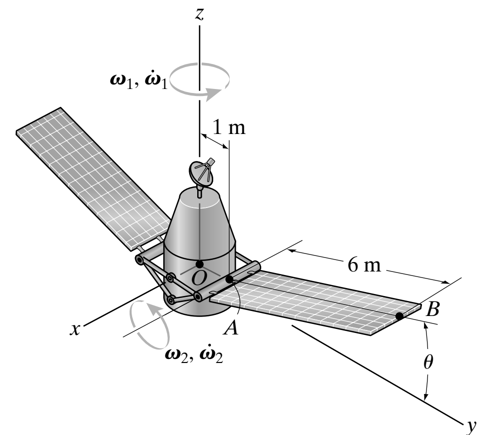
Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$
$$= (-20\mathbf{i}) + (20\mathbf{k}) \times (6\mathbf{j}) + (30\mathbf{k})$$
$$= [-140\mathbf{i} + 30\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times r_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$
$$= (-5\mathbf{i} - 400\mathbf{j}) + (5\mathbf{k}) \times (6\mathbf{j}) + (20\mathbf{k}) \times \left[(20\mathbf{k}) \times (6\mathbf{j})\right] + 2(20\mathbf{k}) \times 30\mathbf{k} + (-150\mathbf{j} + 18\mathbf{k})$$
$$= [-35\mathbf{i} - 2950\mathbf{j} + 18\mathbf{k}] \text{ m/s}^2 \qquad \textbf{Ans.}$$

•20–17. At the instant $\theta = 30°$, the satellite's body is rotating with an angular velocity of $\omega_1 = 20$ rad/s, and it has an angular acceleration of $\dot{\omega}_1 = 5$ rad/s^2. Simultaneously, the solar panels rotate with a constant angular velocity of $\omega_2 = 5$ rad/s. Determine the velocity and acceleration of point B located at the end of one of the solar panels at this instant.

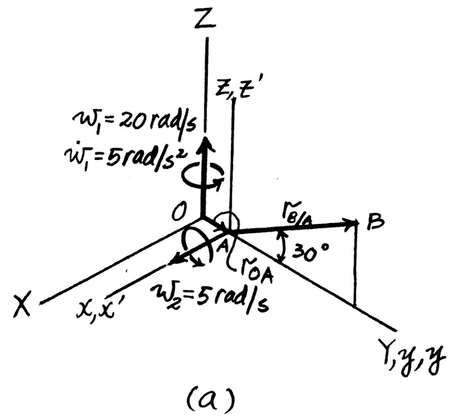


The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [20\mathbf{k}] \text{ rad/s} \qquad \dot{\omega} = \dot{\omega}_1 = [5\mathbf{k}] \text{ rad/s}^2$$

Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (20\mathbf{k}) \times (1\mathbf{j}) = [-20\mathbf{i}] \text{ m/s}$$



and

$$a_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$
$$= (5\mathbf{k}) \times (1\mathbf{j}) + (20\mathbf{k}) \times [(20\mathbf{k}) \times (1\mathbf{j})]$$
$$= [-5\mathbf{i} - 400\mathbf{j}] \text{ m/s}^2$$

In order to determined the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [5\mathbf{k}]$ rad/s, the direction of $(\mathbf{r}_{B/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{B/A})_{xyz}$,

$$(\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{B/A})_{xyz}\right]$$
$$= \mathbf{0} + (5\mathbf{i}) \times (6 \cos 30° \,\mathbf{j} + 6 \sin 30° \,\mathbf{k})$$
$$= [-15\mathbf{j} + 25.98\mathbf{k}] \text{ m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = \mathbf{0}$. Taking the time derivative of $(\dot{\mathbf{r}}_{B/A})_{xyz}$,

$$(\mathbf{a}_{B/A})_{xyz} = \left(\ddot{\mathbf{r}}_{B/A}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{B/A}\right)_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{x'y'z'}\right] + \dot{\omega}_2 \times (\mathbf{r}_{B/A})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{xyz}$$
$$= [\mathbf{0} + \mathbf{0}] + \mathbf{0} + (5\mathbf{i}) \times (-15\mathbf{j} + 25.98\mathbf{k})$$
$$= [-129.90\mathbf{j} - 75\mathbf{k}] \text{ m/s}^2$$

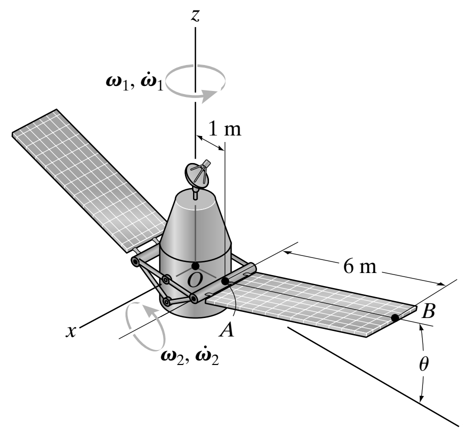
Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$
$$= (-20\mathbf{i}) + (20\mathbf{k}) \times (6 \cos 30° \,\mathbf{j} + 6 \sin 30° \mathbf{k}) + (-15\mathbf{j} + 25.98\mathbf{k})$$
$$= [-124\mathbf{i} - 15\mathbf{j} + 26.0\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$
$$= (-5\mathbf{i} - 400\mathbf{j}) + (5\mathbf{k}) \times (6 \cos 30° \mathbf{j} + 6 \sin 30° \,\mathbf{k}) + (20\mathbf{k}) \times \left[(20\mathbf{k}) \times (6 \cos 30° \,\mathbf{j} + 6 \sin 30° \mathbf{k})\right]$$
$$+ 2(20\mathbf{k}) \times (-15\mathbf{j} + 25.98\mathbf{k}) + (-129.90\mathbf{j} - 75\mathbf{k})$$
$$= [569\mathbf{i} - 2608\mathbf{j} - 75\mathbf{k}] \text{m/s}^2 \qquad \textbf{Ans.}$$

20–18. At the instant $\theta = 30°$, the satellite's body is rotating with an angular velocity of $\omega_1 = 20$ rad/s, and it has an angular acceleration of $\dot{\omega}_1 = 5$ rad/s^2. At the same instant, the satellite travels in the x direction with a velocity of $\mathbf{v}_O = \{5000\mathbf{i}\}$ m/s, and it has an acceleration of $\mathbf{a}_O = \{500\mathbf{i}\}$ m/s^2. Simultaneously, the solar panels rotate with a constant angular speed of $\omega_2 = 5$ rad/s. Determine the velocity and acceleration of point B located at the end of one of the solar panels at this instant.



The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [20\mathbf{k}] \text{ rad/s} \qquad \dot{\omega} = \dot{\omega}_1 = [5\mathbf{k}] \text{ rad/s}^2$$

Since the body of the satellite undergoes general motion, the motion of points O and A can be related using

$$\mathbf{v}_A = \mathbf{v}_O + \omega_1 \times \mathbf{r}_{A/O} = 5000\mathbf{i} + (20\mathbf{k}) \times (1\mathbf{j}) = [4980\mathbf{i}] \text{ m/s}$$

and

$$\mathbf{a}_A = \mathbf{a}_O + \dot{\omega}_1 \times \mathbf{r}_{A/O} + \omega_1 \times (\omega_1 \times \mathbf{r}_{O/A})$$

$$= (500\mathbf{i}) + (5\mathbf{k}) \times (1\mathbf{j}) + (20\mathbf{k}) \times [(20\mathbf{k}) \times (1\mathbf{j})]$$

$$= [495\mathbf{i} - 400\mathbf{j}] \text{ m/s}^2$$

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity of $\Omega' = \omega_2 = [5\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{B/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{B/A})_{xyz}$,

$$(\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{B/A})_{xyz}\right]$$

$$= \mathbf{0} + (5\mathbf{i}) \times (6 \cos 30° \,\mathbf{j} + 6 \sin 30° \,\mathbf{k})$$

$$= [-15\mathbf{j} + 25.98\mathbf{k}] \text{ m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega} = \dot{\omega}_2 = \mathbf{0}$. Taking the time derivative of $(\dot{\mathbf{r}}_{B/A})_{xyz}$,

20–18. Continued

$$(\mathbf{a}_{B/A})_{xyz} = \left(\ddot{\mathbf{r}}_{B/A}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{B/A}\right)_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{x'y'z'}\right] + \dot{\omega}_2 \times (\mathbf{r}_{B/A})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{xyz}$$

$$= [0 + 0] + 0 + (5\mathbf{i}) \times (-15\mathbf{j} + 25.98\mathbf{k})$$

$$= [-129.90\mathbf{j} - 75\mathbf{k}]\ \text{m/s}^2$$

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= (4980\mathbf{i}) + (20\mathbf{k}) \times (6\cos 30^\circ\,\mathbf{j} + 6\sin 30^\circ\mathbf{k}) + (-15\mathbf{j} + 25.98\mathbf{k})$$

$$= [4876\mathbf{i} - 15\mathbf{j} + 26.0\mathbf{k}]\text{m/s}$$ **Ans.**

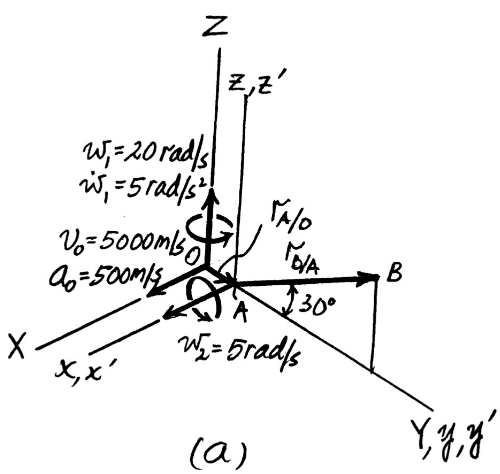
and

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= (495\mathbf{i} - 400\mathbf{j}) + (5\mathbf{k})\times(6\cos 30^\circ\mathbf{j} + 6\sin 30^\circ\,\mathbf{k}) + (20\mathbf{k})\times\left[(20\mathbf{k})\times(6\cos 30^\circ\,\mathbf{j} + 6\sin 30^\circ\mathbf{k})\right]$$

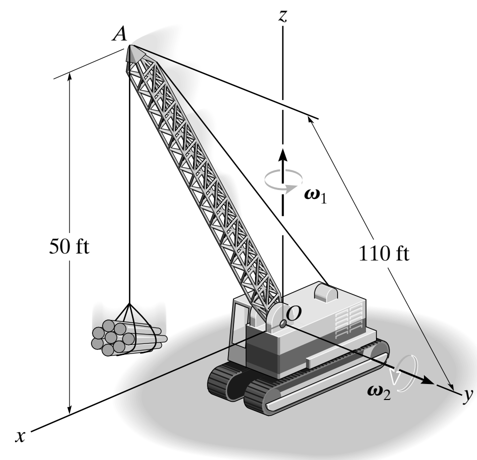
$$+ 2(20\mathbf{k})\times(-15\mathbf{j} + 25.98\mathbf{k}) + (-129.90\mathbf{j} - 75\mathbf{k})$$

$$= [1069\mathbf{i} - 2608\mathbf{j} - 75\mathbf{k}]\ \text{m/s}^2$$ **Ans.**



(a)

20–19. The crane boom OA rotates about the z axis with a constant angular velocity of $\omega_1 = 0.15$ rad/s, while it is rotating downward with a constant angular velocity of $\omega_2 = 0.2$ rad/s. Determine the velocity and acceleration of point A located at the end of the boom at the instant shown.



$\dot{\omega} = \omega_1 + \omega_2 = \{0.2\mathbf{j} + 0.15\mathbf{k}\}$ rad/s

$\omega = \omega_1 - \omega_2$

Let the x, y, z axes rotate at $\Omega = \omega_1$, then

$\omega = \omega = |\omega| + \omega_1 \times \omega_2$

$\omega = \mathbf{0} + 0.15\mathbf{k} \times \mathbf{0.2j} = \{-0.03\mathbf{i}\}$ rad/s^2

$\mathbf{r}_A = \left[\sqrt{(110)^2 - (50)^2}\right]\mathbf{i} + 50\mathbf{k} = \{97.98\mathbf{i} + 50\mathbf{k}\}$ ft

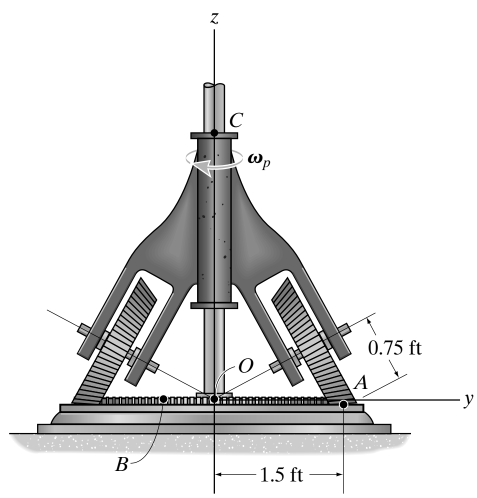
$$\mathbf{v}_A = \omega_A\,\mathbf{r}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.2 & 0.15 \\ 97.98 & 0 & 50 \end{vmatrix}$$

$\mathbf{v}_A = \{10\mathbf{i} + 117\mathbf{j} - 19.6\mathbf{k}\}$ ft/s **Ans.**

$$\mathbf{a}_A = \alpha + \mathbf{r}_A + \omega + \mathbf{v}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.03 & 0 & 0 \\ 97.98 & 0 & 50 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.2 & 0.15 \\ 10 & 14.7 & -19.6 \end{vmatrix}$$

$\mathbf{a}_A = \{-6.12\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}$ ft/s^2 **Ans.**

***20–20.** If the frame rotates with a constant angular velocity of $\omega_p = \{-10\mathbf{k}\}$ rad/s and the horizontal gear B rotates with a constant angular velocity of $\omega_B = \{5\mathbf{k}\}$ rad/s, determine the angular velocity and angular acceleration of the bevel gear A.



If the bevel gear A spins about its axle with an angular velocity of ω_S, then its angular velocity is

$$\omega = \omega_s + \omega_p$$

$$= (\omega_s \cos 30^\circ\, \mathbf{j} + \omega_s \sin 30^\circ\, \mathbf{k}) - 10\,\mathbf{k}$$

$$= 0.8660\omega_s \mathbf{j} + (0.5\omega_s - 10)\mathbf{k}$$

Since gear B rotates about the fixed axis ($z$axis), the velocity of the contact point P between gears A and B is

$$\mathbf{v}_p = \omega_B \times \mathbf{r}_B = (5\mathbf{k}) \times (1.5\mathbf{j}) = [-7.5\mathbf{i}]\text{ft/s}$$

Since gear A rotates about a fixed point O then $r_{OP} = [1.5\mathbf{j}]$ ft. Also,

$$\mathbf{v}_p = \omega \times \mathbf{r}_{OP}$$

$$-7.5\mathbf{i} = [0.8660\omega_s \mathbf{j} + (0.5\omega_s - 10)\mathbf{k}] \times (1.5\mathbf{j})$$

$$-7.5\mathbf{i} = -1.5(0.5\omega_s - 10)\mathbf{i}$$

$$-7.5 = -1.5(0.5\omega_s - 10)$$

$$\omega_s = 30 \text{ rad/s}$$

Thus,

$$\omega_s = 30\cos 30^\circ\, \mathbf{j} + 30\sin 30^\circ \mathbf{k} = [25.98\mathbf{j} + 15\mathbf{k}] \text{ rad/s}$$

$$\omega = 0.8660(30)\mathbf{j} + [0.5(30) - 10]\mathbf{k} = [26.0\mathbf{j} + 5\mathbf{k}] \text{ rad/s} \qquad \textbf{Ans.}$$

We will set the XYZ fixed reference frame to coincide with the xyz rotating reference frame at the instant considered. If the xyz frame rotates with an angular velocity of $\Omega = \omega_p = [-10\mathbf{k}]$ rad/s, the direction of ω_s will remain constant with respect to the xyz frame. Thus,

$$\dot{\omega}_s = (\dot{\omega}_S)_{xyz} + \omega_p \times \omega_s$$

$$= \mathbf{0} + (-10\mathbf{k}) \times (25.98\mathbf{j} + 15\mathbf{k})$$

$$= [259.81\mathbf{i}] \text{ rad/s}^2$$

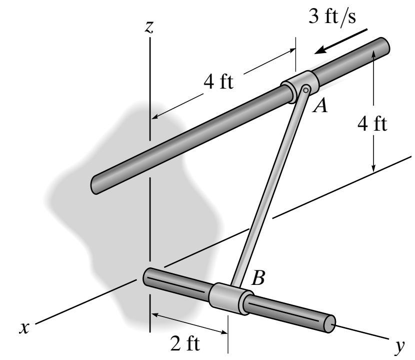
If $\Omega = \omega_p$, then ω_p is always directed along the z axis. Thus,

$$\dot{\omega}_p = \left(\dot{\omega}_p\right)_{xyz} + \omega_p \times \omega_p = \mathbf{0} + \mathbf{0} = 0$$

Thus,

$$\alpha = \dot{\omega} = \dot{\omega}_s + \dot{\omega}_p = (259.81\mathbf{i}) + \mathbf{0} = [260\mathbf{i}] \text{ rad/s}^2 \qquad \textbf{Ans.}$$

•20–21. Rod AB is attached to collars at its ends by ball-and-socket joints. If the collar A has a velocity of $v_A = 3$ ft/s, determine the angular velocity of the rod and the velocity of collar B at the instant shown. Assume the angular velocity of the rod is directed perpendicular to the rod.



Velocity Equation: Here, $\mathbf{r}_{B/A} = \{[0 - (-4)]\,\mathbf{i} + (2 - 0)\,\mathbf{j} + (0 - 4)\mathbf{k}\}$ ft $= \{4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\}$ ft, $\mathbf{v}_A = \{3\mathbf{i}\}$ ft/s, $\mathbf{v}_B = v_B\,\mathbf{j}$ and $\omega = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$. Applying Eq. 20–7, we have

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$$

$$v_B\,\mathbf{j} = 3\mathbf{i} + \left(\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\mathbf{k}\right) \times (4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$$

$$v_B\,\mathbf{j} = \left(3 - 4\omega_y - 2\omega_z\right)\mathbf{i} + (4\omega_x + 4\omega_z)\,\mathbf{j} + \left(2\omega_x - 4\omega_y\right)\mathbf{k}$$

Equating **i**, **j** and **k** components, we have

$$3 - 4\omega_y - 2\omega_z = 0 \qquad \textbf{[1]}$$

$$v_B = 4\omega_x + 4\omega_z \qquad \textbf{[2]}$$

$$2\omega_x - 4\omega_y = 0 \qquad \textbf{[3]}$$

If ω is specified acting *perpendicular* to the axis of the rod AB. then

$$\omega \cdot \mathbf{r}_{B/A} = 0$$

$$\left(\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}\right)\cdot(4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}) = 0$$

$$4\omega_x + 2\omega_y - 4\omega_z = 0 \qquad \textbf{[4]}$$

Solving Eqs. [1], [2], [3] and [4] yields

$$v_B = 6.00 \text{ ft/s} \qquad \omega_x = 0.6667 \text{ rad/s}$$

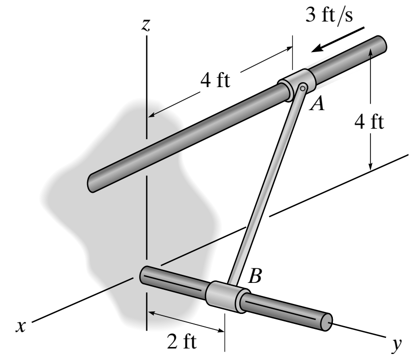
$$\omega_y = 0.3333 \text{ rad/s} \qquad \omega_z = 0.8333 \text{ rad/s}$$

Thus,

$$\mathbf{v}_B = \{6.00\mathbf{j}\} \text{ ft/s} \qquad \textbf{Ans.}$$

$$\omega = \{0.667\mathbf{i} + 0.333\mathbf{j} + 0.833\mathbf{k}\} \text{ rad/s} \qquad \textbf{Ans.}$$

20–22. The rod AB is attached to collars at its ends by ball-and-socket joints. If collar A has an acceleration of $\mathbf{a}_A = \{8\mathbf{i}\}$ ft/s^2 and a velocity $\mathbf{v}_A = \{3\mathbf{i}\}$ ft/s, determine the angular acceleration of the rod and the acceleration of collar B at the instant shown. Assume the angular acceleration of the rod is directed perpendicular to the rod.



Velocity Equation: Here, $\mathbf{r}_{B/A} = \{[0 - (-4)]\,\mathbf{i} + (2 - 0)\,\mathbf{j} + (0 - 4)\mathbf{k}\}$ ft $= \{4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\}$ ft, $\mathbf{v}_A = \{3\mathbf{i}\}$ ft/s, $\mathbf{v}_B = v_B\,\mathbf{j}$ and $\omega = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$. Applying Eq. 20–7. we have

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$$

$$v_B\,\mathbf{j} = 3\mathbf{i} + \left(\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\mathbf{k}\right) \times (4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$$

$$v_B\,\mathbf{j} = \left(3 - 4\omega_y - 2\omega_z\right)\mathbf{i} + (4\omega_x + 4\omega_z)\,\mathbf{j} + \left(2\omega_x - 4\omega_y\right)\mathbf{k}$$

Equating **i**, **j**, **k** components, we have

$$3 - 4\omega_y - 2\omega_z = 0 \qquad \textbf{[1]}$$

$$v_B = 4\omega_x + 4\omega_z \qquad \textbf{[2]}$$

$$2\omega_x - 4\omega_y = 0 \qquad \textbf{[3]}$$

If ω is specified acting *perpendicular* to the axis of rod AB, then

$$\omega \cdot \mathbf{r}_{B/A} = 0$$

$$\left(\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}\right) \cdot (4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}) = 0$$

$$4\omega_x + 2\omega_y - 4\omega_z = 0 \qquad \textbf{[4]}$$

Solving Eqs. [1], [2], [3] and [4] yields

$$v_B = 6.00 \text{ ft/s} \qquad \omega_x = 0.6667 \text{ rad/s}$$

$$\omega_y = 0.3333 \text{ rad/s} \qquad \omega_z = 0.8333 \text{ rad/s}$$

Thus, $\omega = \{0.6667\mathbf{i} + 0.3333\mathbf{j} + 0.8333\mathbf{k}\}$ rad/s

Acceleration Equation: With $\alpha = \alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\,\mathbf{k}$ and the result obtained above, applying Eq. 20–8, we have

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} + \omega \times (\omega \times \mathbf{r}_{B/A})$$

$$a_B\,\mathbf{j} = 8\mathbf{i} + \left(\alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\mathbf{k}\right) \times (4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$$

20–22. Continued

$$+(0.6667\mathbf{i} + 0.3333\mathbf{j} + 0.8333\mathbf{k}) \times [(0.6667\mathbf{i} + 0.3333\mathbf{j} + 0.8333\mathbf{k}) \times (4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$$

$$a_B\,\mathbf{j} = \left(3 - 4\alpha_y - 2\alpha_z\right)\mathbf{i} + (-2.50 + 4\alpha_x + 4\alpha_z)\mathbf{j} + \left(5 + 2\alpha_x - 4\alpha_y\right)\mathbf{k}$$

Equating **i**, **j**, **k** components, we have

$$3 - 4\alpha_y - 2\alpha_z = 0 \qquad \textbf{[5]}$$

$$a_B = -2.50 + 4\alpha_x + 4\alpha_z \qquad \textbf{[6]}$$

$$5 + 2\alpha_x - 4\alpha_y = 0 \qquad \textbf{[7]}$$

If α is specified acting *perpendicular* to the axis of rod AB, then

$$\alpha \cdot \mathbf{r}_{B/A} = 0$$

$$\left(\alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_2\,\mathbf{k}\right) \cdot (4\mathbf{i} + 2 - 4\mathbf{k}) = 0$$

$$4\alpha_x + 2\alpha_y - 4\alpha_z = 0 \qquad \textbf{[8]}$$

Solving Eqs. [5], [6], [7] and [8] yields

$$a_B = -6.50 \text{ ft/s}^2 \qquad \alpha_x = -0.7222 \text{ rad/s}^2$$

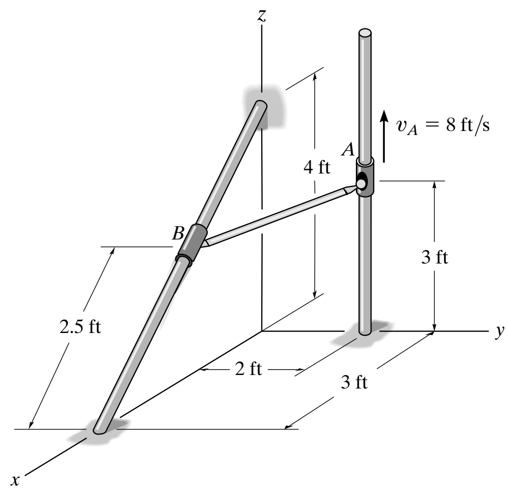
$$\alpha_y = 0.8889 \text{ rad/s} \qquad \alpha_z = -0.2778 \text{ rad/s}^2$$

Thus,

$$\mathbf{a}_B = \{-6.50\mathbf{j}\} \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$\alpha = \{-0.722\mathbf{i} + 0.889\mathbf{j} - 0.278\mathbf{k}\} \text{ rad/s}^2 \qquad \textbf{Ans.}$$

20–23. Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A moves upward with a velocity of $\mathbf{v}_A = \{8\mathbf{k}\}$ ft/s, determine the angular velocity of the rod and the speed of collar B at the instant shown. Assume that the rod's angular velocity is directed perpendicular to the rod.



$\mathbf{v}_B = \{8\mathbf{k}\}$ ft/s $\qquad \mathbf{v}_B = -\frac{3}{5}v_B\mathbf{i} + \frac{4}{5}v_B\,\mathbf{k} \qquad \omega_{AB} = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$

$$\mathbf{r}_{B/A} = \{1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}\}\text{ ft}$$

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$$

$$-\frac{3}{5}v_B\,\mathbf{i} + \frac{4}{5}v_B\mathbf{k} = 8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 1.5 & -2 & -1 \end{vmatrix}$$

Equating $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$

$$-\omega_z + 2\omega_z = -\frac{3}{5}v_B \qquad \textbf{(1)}$$

$$\omega_z + 1.5\omega_z = 0 \qquad \textbf{(2)}$$

$$\omega - 2\omega_x - 1.5\omega_x = \frac{4}{5}v_B \qquad \textbf{(3)}$$

Since ω_{AB} is perpendicular to the axis of the rod,

$\omega_{AB}\cdot\mathbf{r}_{B/A} = (\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\mathbf{k})\cdot(1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}) = 0$

$$1.5\omega_x - 2\omega_y - \omega_z = 0 \qquad \textbf{(4)}$$

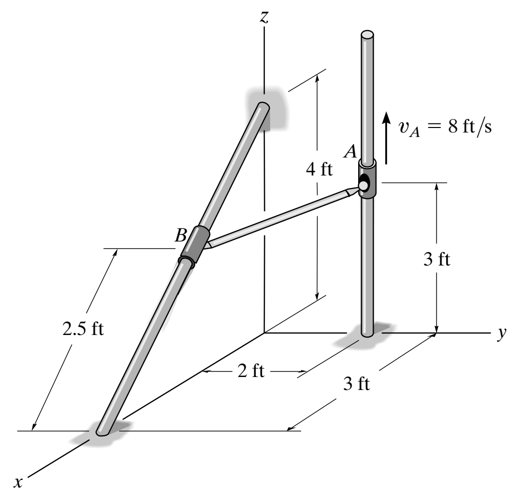
Solving Eqs.(1) to (4) yields:

$\omega_x = 1.1684$ rad/s $\qquad \omega_y = 1.2657$rad/s $\qquad \omega_z = -0.7789$ rad/s

$$v_B = 4.71\text{ ft/s} \qquad \textbf{Ans.}$$

Then $\omega_{AB} = \{1.17\mathbf{i} + 1.27\mathbf{j} - 0.779\mathbf{k}\}$ rad/s **Ans.**

***20–24.** Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A moves upward with an acceleration of $\mathbf{a}_A = \{4\mathbf{k}\}\text{ ft/s}^2$, determine the angular acceleration of rod AB and the magnitude of acceleration of collar B. Assume that the rod's angular acceleration is directed perpendicular to the rod.



From Prob. 20–23

$\omega_{AB} = \{1.1684\mathbf{i} + 1.2657\mathbf{j} - 0.7789\mathbf{k}\}$ rad/s

$\mathbf{r}_{B \cdot A} = \{1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}\}$ ft

$\alpha_{AB} = \alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\,\mathbf{k}$

$\mathbf{a}_A = \{4\mathbf{k}\}\text{ ft/s}^2 \qquad \mathbf{a}_B = -\frac{3}{5}a_B\,\mathbf{i} + \frac{4}{5}a_B\,\mathbf{k}$

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A})$$

$$-\frac{3}{5}a_B\,\mathbf{i} + \frac{4}{5}a_B\,\mathbf{k} = 4\mathbf{k} + (\alpha_x\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\,\mathbf{k}) \times (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k})$$

$$+(1.1684\mathbf{i} + 1.2657\mathbf{j} - 0.7789\mathbf{k})$$

$$\times\left[(1.1684\mathbf{i} + 1.2657\mathbf{j} - 0.7789\mathbf{k}) \times (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k})\right]$$

Equating **i**, **j**, and **k** components

$$-\alpha_y + 2\alpha_z - 5.3607 = -\frac{3}{5}a_B \qquad \textbf{(1)}$$

$$\alpha_x + 1.5\alpha_z + 7.1479 = 0 \qquad \textbf{(2)}$$

$$7.5737 - 2\alpha_x - 1.5\alpha_y = \frac{4}{5}a \qquad \textbf{(3)}$$

Since α_{AB} is perpendicular to the axis of the rod,

$\alpha_{AB} \cdot \mathbf{r}_{B/A} = (\alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\mathbf{k}) \cdot (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}) = 0$

$$1.5\alpha_x - 2\alpha_y - \alpha_z = 0 \qquad \textbf{(4)}$$

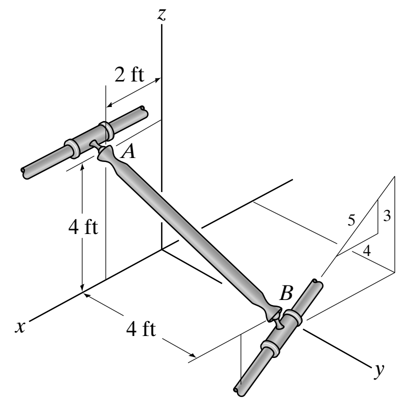
Solving Eqs.(1) to (4) yields:

$\alpha_x = -2.7794\text{ rad/s}^2 \qquad \alpha_y = -0.6285\text{rad/s}^2 \qquad \alpha_z = -2.91213\text{ rad/s}^2$

$a_B = 17.6\text{ ft/s}^2$ **Ans.**

Then $\alpha_{AB} = \{-2.78\mathbf{i} - 0.628\mathbf{j} - 2.91\mathbf{k}\}\text{ rad/s}^2$ **Ans.**

•20–25. If collar A moves with a constant velocity of $\mathbf{v}_A = \{10\mathbf{i}\}$ ft/s, determine the velocity of collar B when rod AB is in the position shown. Assume the angular velocity of AB is perpendicular to the rod.



Since rod AB undergoes general motion $\mathbf{v}_A$ and $\mathbf{v}_B$ can be related using the relative velocity equation.

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

Assume $\mathbf{v}_B = \frac{4}{5}v_B\,\mathbf{i} - \frac{3}{5}v_B\,\mathbf{k}$ and $\boldsymbol{\omega}_{AB} = (\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\,\mathbf{j} + (\omega_{AB})_z\,\mathbf{k}$. Also, $\mathbf{r}_{B/A} = [-2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}]$ ft. Thus,

$$\frac{4}{5}v_B\,\mathbf{i} - \frac{3}{5}v_B\mathbf{k} = 10\mathbf{i} + \left[(\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\mathbf{j} + (\omega_{AB})_z\mathbf{k}\right] \times (-2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})$$

$$\frac{4}{5}v_B\mathbf{i} - \frac{3}{5}v_B\,\mathbf{k} = \left[10 - 4(\omega_{AB})_y - 4(\omega_{AB})_z\right]\mathbf{i} + \left[4(\omega_{AB})_x - 2(\omega_{AB})_z\right]\mathbf{j} + \left[4(\omega_{AB})_x + 2(\omega_{AB})_y\right]\mathbf{k}$$

Equating the **i**, **j**, and **k** components

$$\frac{4}{5}v_B = 10 - 4(\omega_{AB})_y - 4(\omega_{AB})_z \qquad \textbf{(1)}$$

$$0 = 4(\omega_{AB})_x - 2(\omega_{AB})_z \qquad \textbf{(2)}$$

$$-\frac{3}{5}v_B = 4(\omega_{AB})_x + 2(\omega_{AB})_y \qquad \textbf{(3)}$$

The fourth equation can be obtained from the dot product of

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$\left[(\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\,\mathbf{j} + (\omega_{AB})_z\mathbf{k}\right] \cdot (-2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}) = 0$$

$$-2(\omega_{AB})_x + 4(\omega_{AB})_y - 4(\omega_{AB})_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) through (4),

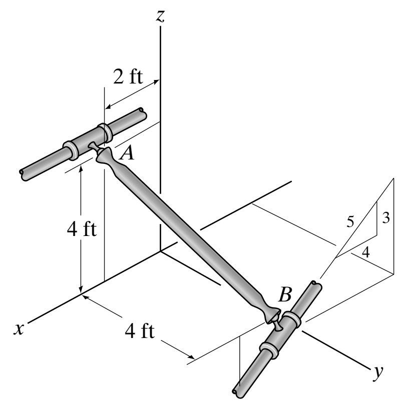
$(\omega_{AB})_x = 1.667$ rad/s $\qquad (\omega_{AB})_y = 4.167$ rad/s $\qquad (\omega_{AB})_z = 3.333$ rad/s

$$v_B = -25 \text{ ft/s}$$

Then,

$$\mathbf{v}_B = \frac{4}{5}(-25)\mathbf{i} - \frac{3}{5}(-25)\mathbf{k} = [-20\mathbf{i} + 15\mathbf{k}] \text{ ft/s} \qquad \textbf{Ans.}$$

20–26. When rod AB is in the position shown, collar A moves with a velocity of $\mathbf{v}_A = \{10\mathbf{i}\}$ ft/s and acceleration of $\mathbf{a}_A = \{2\mathbf{i}\}$ ft/s^2. Determine the acceleration of collar B at this instant. Assume the angular velocity and angular acceleration of AB are perpendicular to the rod.



For general motion, $\mathbf{a}_A$ and $\mathbf{a}_B$ can be related using the relative acceleration equation.

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A})$$

Using the result of Prob. 19-17, $\omega_{AB} = [1.667\mathbf{i} + 4.166\mathbf{j} + 3.333\mathbf{k}]$ rad/s.

Assume $\mathbf{a}_B = \frac{4}{5} a_B \mathbf{i} - \frac{3}{5} a_B \mathbf{k}$ and $\alpha_{AB} = (\alpha_{AB})_x \mathbf{i} + (\alpha_{AB})_y \mathbf{j} + (\alpha_{AB})_z \mathbf{k}$. Also,

$\mathbf{r}_{B/A} = [-2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}]$ ft. Thus,

$$\frac{4}{5} a_B \mathbf{i} - \frac{3}{5} a_B \mathbf{k} = 2\mathbf{i} + \left[(\alpha_{AB})_x \mathbf{i} + (\alpha_{AB})_y \mathbf{j} + (\alpha_{AB})_z \mathbf{k}\right] \times (-2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})$$

$$+(1.667\mathbf{i} + 4.166\mathbf{j} + 3.333\mathbf{k}) \times [(1.667\mathbf{i} + 4.166\mathbf{j} + 3.333\mathbf{k}) \times (-2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})]$$

$$\frac{4}{5} a_B \mathbf{i} - \frac{3}{5} a_B \mathbf{k} = \left[64.5 - 4(\alpha_{AB})_y - 4(\alpha_{AB})_z\right]\mathbf{i} + \left[4(\alpha_{AB})_x - 2(\alpha_{AB})_z - 125\right]\mathbf{j} + \left[4(\alpha_{AB})_x + 2(\alpha_{AB})_y + 125\right]\mathbf{k}$$

Equating the **i**, **j**, and **k** components

$$\frac{4}{5} a_B = 64.5 - 4(\alpha_{AB})_y - 4(\alpha_{AB})_z \qquad \textbf{(1)}$$

$$0 = 4(\alpha_{AB})_x - 2(\alpha_{AB})_z - 125 \qquad \textbf{(2)}$$

$$-\frac{3}{5} a_B = 4(\alpha_{AB})_x + 2(\alpha_{AB})_y + 125 \qquad \textbf{(3)}$$

The fourth equation can be obtained from the dot product of

$$\alpha_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$\left[(\alpha_{AB})_x \mathbf{i} + (\alpha_{AB})_y \mathbf{j} + (\alpha_{AB})_z \mathbf{k}\right] \cdot (-2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}) = 0$$

$$-2(\alpha_{AB})_x + 4(\alpha_{AB})_y - 4(\alpha_{AB})_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) through (4),

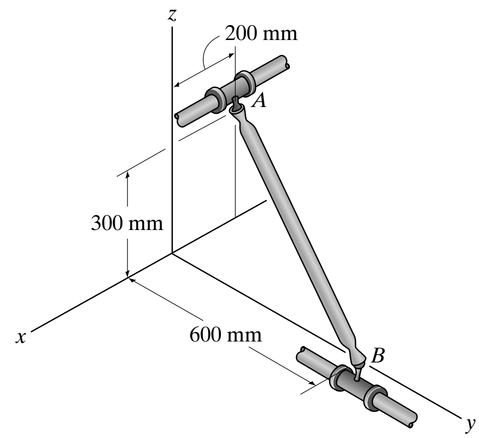
$(\alpha_{AB})_x = 94.08$ rad/s^2 $\qquad$ $(\alpha_{AB})_y = 172.71$ rad/s^2 $\qquad$ $(\alpha_{AB})_z = 125.67$ rad/s^2

$$a_B = -1411.25 \text{ ft/s}^2$$

Then,

$$\mathbf{a}_B = \frac{4}{5}(-1411.25)\mathbf{i} - \frac{3}{5}(-1411.25)\mathbf{k} = [-1129\mathbf{i} + 846.75\mathbf{k}] \text{ ft/s} \qquad \textbf{Ans.}$$

20–27. If collar A moves with a constant velocity of $\mathbf{v}_A = \{3\mathbf{i}\}$ m/s, determine the velocity of collar B when rod AB is in the position shown. Assume the angular velocity of AB is perpendicular to the rod.



Since rod AB undergoes general motion $\mathbf{v}_A$ and $\mathbf{v}_B$ can be related using the relative velocity equation.

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

Assume $\mathbf{v}_B = v_B\,\mathbf{j}, \omega_{AB} = \left[(\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\,\mathbf{j} + (\omega_{AB})_z\,\mathbf{k}\right]$, and

$\mathbf{r}_{B/A} = [0.2\mathbf{i} + 0.6\mathbf{j} - 0.3\mathbf{k}]$ ft. Thus,

$$v_B\,\mathbf{j} = 3\mathbf{i} + \left[(\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\mathbf{j} + (\omega_{AB})_z\,\mathbf{k}\right] \times (0.2\mathbf{i} + 0.6\mathbf{j} - 0.3\mathbf{k})$$

$$v_B\,\mathbf{j} = \left[3 - 0.3(\omega_{AB})_y - 0.6(\omega_{AB})_z\right]\mathbf{i} + \left[0.3(\omega_{AB})_x + 0.2(\omega_{AB})_z\right]\mathbf{j} + \left[0.6(\omega_{AB})_x - 0.2(\omega_{AB})_y\right]\mathbf{k}$$

Equating the **i**, **j**, and **k** components

$$0 = 3 - 0.3(\omega_{AB})_y - 0.6(\omega_{AB})_z \quad \textbf{(1)}$$

$$v_B = 0.3(\omega_{AB})_x + 0.2(\omega_{AB})_z \quad \textbf{(2)}$$

$$0 = 0.6(\omega_{AB})_x - 0.2(\omega_{AB})_y \quad \textbf{(3)}$$

The fourth equation can be obtained from the dot product of

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$\left[(\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\mathbf{j} + (\omega_{AB})_z\,\mathbf{k}\right] \cdot (0.2\mathbf{i} + 0.6\mathbf{j} - 0.3\mathbf{k}) = 0$$

$$0.2(\omega_{AB})_x + 0.6(\omega_{AB})_y - 0.3(\omega_{AB})_z = 0 \quad \textbf{(4)}$$

Solving Eqs. (1) through (4),

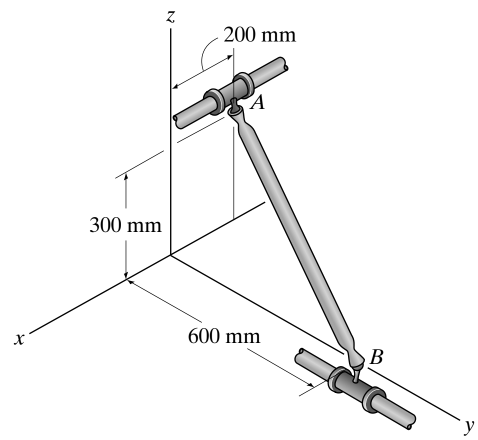
$(\omega_{AB})_x = 0.6122$ rad/s $\quad (\omega_{AB})_y = 1.837$ rad/s $\quad (\omega_{AB})_z = 4.082$ rad/s

$$v_B = 1 \text{ m/s}$$

Then,

$$\mathbf{v}_B = [1\mathbf{j}]\text{m/s} \quad \textbf{Ans.}$$

***20–28.** When rod AB is in the position shown, collar A moves with a velocity of $\mathbf{v}_A = \{3\mathbf{i}\}$ m/s and acceleration of $\mathbf{a}_A = \{0.5\mathbf{i}\}$ m/s^2. Determine the acceleration of collar B at this instant. Assume the angular velocity and angular acceleration of AB are perpendicular to the rod.



For general motion, $\mathbf{a}_A$ and $\mathbf{a}_B$ can be related using the relative acceleration equation.

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A})$$

Using the result of Prob. 19-22, $\omega_{AB} = [0.6122\mathbf{i} + 1837\mathbf{j} + 4.082\mathbf{k}]$ rad/s

Also, $\mathbf{a}_B = a_B\,\mathbf{j}$, $\alpha_{AB} = (\alpha_{AB})_x\,\mathbf{i} + (\alpha_{AB})_y\,\mathbf{j} + (\alpha_{AB})_z\,\mathbf{k}$, and $\mathbf{r}_{B/A} = [0.2\mathbf{i} + 0.6\mathbf{j} - 0.3\mathbf{k}]$ m. Thus,

$$\mathbf{a}_B\,\mathbf{j} = (0.5\mathbf{i}) + \left[(\alpha_{AB})_x\,\mathbf{i} + (\alpha_{AB})_y\,\mathbf{j} + (\alpha_{AB})_z\,\mathbf{k}\right] \times (0.2\mathbf{i} + 0.6\mathbf{j} - 0.3\mathbf{k})$$
$$+(0.6122\mathbf{i} + 1837\mathbf{j} + 4.082\mathbf{k}) \times [(0.6122\mathbf{i} + 1837\mathbf{j} + 4.082\mathbf{k}) \times (0.2\mathbf{i} + 0.6\mathbf{j} - 0.3\mathbf{k})]$$

$$a_B\mathbf{j} = -\left[0.3(\alpha_{AB})_y + 0.6(\alpha_{AB})_z + 3.5816\right]\mathbf{i} + \left[0.3(\alpha_{AB})_x + 0.2(\alpha_{AB})_z - 12.2449\right]\mathbf{j} + \left[0.6(\alpha_{AB})_x - 0.2(\alpha_{AB})_y + 6.1224\right]\mathbf{k}$$

Equating the **i**, **j**, and **k** components

$$0 = -\left[0.3(\alpha_{AB})_y + 0.6(\alpha_{AB})_z + 3.5816\right] \qquad \textbf{(1)}$$

$$a_B = 0.3(\alpha_{AB})_x + 0.2(\alpha_{AB})_z - 12.2449 \qquad \textbf{(2)}$$

$$0 = 0.6(\alpha_{AB})_x - 0.2(\alpha_{AB})_y + 6.1224 \qquad \textbf{(3)}$$

The fourth equation can be obtained from the dot product of

$$\alpha_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$\left[(\alpha_{AB})_x\,\mathbf{i} + (\alpha_{AB})_y\,\mathbf{j} + (\alpha_{AB})_z\,\mathbf{k}\right] \cdot (0.2\mathbf{i} + 0.6\mathbf{j} - 0.3\mathbf{k}) = 0$$

$$0.2(\alpha_{AB})_x + 0.6(\alpha_{AB})_y - 0.3(\alpha_{AB})_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) through (4),

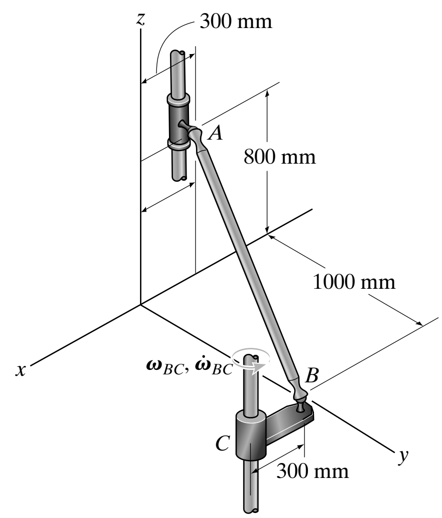
$(\alpha_{AB})_x = -10.1020$ rad/s^2 $\qquad (\alpha_{AB})_y = 0.3061$rad/s^2 $\qquad (\alpha_{AB})_z = -6.1224$ rad/s^2

$$a_B = -16.5 \text{ m/s}^2$$

Then,

$$\mathbf{a}_B = [-16.5\mathbf{j}]\text{m/s}^2 \qquad \textbf{Ans.}$$

•20–29. If crank BC rotates with a constant angular velocity of $\omega_{BC} = 6$ rad/s, determine the velocity of the collar at A. Assume the angular velocity of AB is perpendicular to the rod.



Here, $\mathbf{r}_{C/B} = [-0.3\mathbf{i}]$ m and $\omega_{BC} = [6\mathbf{k}]$ rad/s. Since crank BC rotates about a fixed axis, then

$$\mathbf{v}_B = \omega_{AB} \times r_{B/C} = \left(6\mathbf{k}\right) \times \left(-0.3\mathbf{i}\right) = [-1.8\mathbf{j}] \text{ m/s}$$

Since rod AB undergoes general motion $\mathbf{v}_A$ and $\mathbf{v}_B$ can be related using the relative velocity equation.

$$\mathbf{v}_A = \mathbf{v}_B + \omega_{AB} \times \mathbf{r}_{A/B}$$

Here, $\mathbf{v}_B = v_A\,\mathbf{k}$, $\omega_{AB} = \left[(\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\,\mathbf{j} + (\omega_{AB})_z\,\mathbf{k}\right]$, and

$$\mathbf{r}_{A/B} = [-0.3\mathbf{i} - 1\mathbf{j} + 0.8\mathbf{k}] \text{ m. Thus,}$$

$$v_A\,\mathbf{k} = -1.8\mathbf{j} + \left[(\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\mathbf{j} + (\omega_{AB})_z\,\mathbf{k}\right] \times (-0.3\mathbf{i} - 1\mathbf{j} + 0.8\mathbf{k})$$

$$v_A\,\mathbf{k} = \left[0.8(\omega_{AB})_y + (\omega_{AB})_z\right]\mathbf{i} - \left[0.8(\omega_{AB})_x + 0.3(\omega_{AB})_z + 1.8\right]\mathbf{j} + \left[0.3(\omega_{AB})_y - (\omega_{AB})_x\right]\mathbf{k}$$

Equating the **i**, **j**, and **k** components

$$0 = 0.8(\omega_{AB})_y + (\omega_{AB})_z \qquad \textbf{(1)}$$

$$0 = -\left[0.8(\omega_{AB})_x + 0.3(\omega_{AB})_z + 1.8\right] \qquad \textbf{(2)}$$

$$v_A = 0.3(\omega_{AB})_y - (\omega_{AB})_x \qquad \textbf{(3)}$$

The fourth equation can be obtained from the dot product of

$$\omega_{AB} \cdot \mathbf{r}_{A/B} = 0$$

$$\left[(\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\,\mathbf{j} + (\omega_{AB})_z\,\mathbf{k}\right] \cdot (-0.3\mathbf{i} - 1\mathbf{j} + 0.8\mathbf{k}) = 0$$

$$-0.3(\omega_{AB})_x - (\omega_{AB})_y + 0.8(\omega_{AB})_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) through (4),

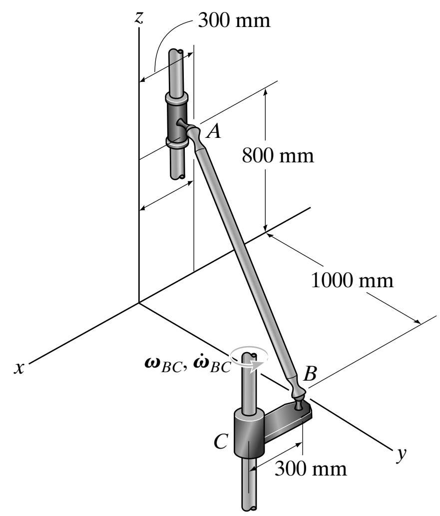
$(\omega_{AB})_x = -2.133$ rad/s $\qquad (\omega_{AB})_y = 0.3902$rad/s $\qquad (\omega_{AB})_z = -0.3121$ rad/s^2

$v_A = 2.25$ m/s

Then,

$$\mathbf{v}_A = [2.25\mathbf{k}]\text{m/s} \qquad \textbf{Ans.}$$

20–30. If crank BC is rotating with an angular velocity of $\omega_{BC} = 6$ rad/s and an angular acceleration of $\dot{\omega}_{BC} = 1.5$ rad/s^2, determine the acceleration of collar A at this instant. Assume the angular velocity and angular acceleration of AB are perpendicular to the rod.



Here, $\mathbf{r}_{CB} = [-0.3\mathbf{i}]$ m and $\alpha_{BC} = [1.5\mathbf{k}]$ rad/s^2. Since crank BC rotates about a fixed axis, then

$$\mathbf{a}_B = \alpha_{BC} \times \mathbf{r}_{CB} + \omega_{BC} \times (\omega_{BC} \times r_{CB}) = (1.5\mathbf{k}) \times (-0.3\mathbf{i}) + 6\mathbf{k} \times [(6\mathbf{k}) \times (-0.3\mathbf{i})]$$

$$= [10.8\mathbf{i} - 0.45\mathbf{j}] \text{ m/s}^2$$

For general motion, $\mathbf{a}_A$ and $\mathbf{a}_B$ can be related using the acceleration equation.

$$\mathbf{a}_A = \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{A/B} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{A/B})$$

Using the result of Prob. 20-29, $\omega_{AB} = [-2.133\mathbf{i} + 0.3902\mathbf{j} - 0.3121\mathbf{k}]$ rad/s.

Also, $\mathbf{a}_A = a_A\,\mathbf{k}$, $\alpha_{AB} = (\alpha_{AB})_x\,\mathbf{i} + (\alpha_{AB})_y\,\mathbf{j} + (\alpha_{AB})_z\,\mathbf{k}$, and

$\mathbf{r}_{A/B} = [-0.3\mathbf{i} - 1\mathbf{j} + 0.8\mathbf{k}]$ m. Thus,

$$a_A\mathbf{k} = (10.8\mathbf{i} - 0.45\mathbf{j}) + \left[(\alpha_{AB})_x\,\mathbf{i} + (\alpha_{AB})_y\mathbf{j} + (\alpha_{AB})_z\,\mathbf{k}\right] \times (-0.3\mathbf{i} - 1\mathbf{j} + 0.8\mathbf{k})$$

$$+(-2.133\mathbf{i} + 0.3902\mathbf{j} - 0.3121\mathbf{k}) \times [(-2.133\mathbf{i} + 0.3902\mathbf{j} - 0.3121\mathbf{k}) \times (-0.3\mathbf{i} - 1\mathbf{j} + 0.8\mathbf{k})]$$

$$a_A\mathbf{k} = \left[0.8(\alpha_{AB})_y + (\alpha_{AB})_z + 12.24\right]\mathbf{i} + \left[4.349 - 0.8(\alpha_{AB})_x - 0.3(\alpha_{AB})_z\right]\mathbf{j} + \left[0.3(\alpha_{AB})_y - (\alpha_{AB})_x - 3.839\right]\mathbf{k}$$

Equating the **i**, **j**, and **k** components

$$0 = 0.8(\alpha_{AB})_y + (\alpha_{AB})_z + 12.24 \quad \textbf{(1)}$$

$$0 = 4.349 - 0.8(\alpha_{AB})_x - 0.3(\alpha_{AB})_z \quad \textbf{(2)}$$

$$a_A = 0.3(\alpha_{AB})_y - (\alpha_{AB})_x - 3.839 \quad \textbf{(3)}$$

The fourth equation can be obtained from the dot product of

$$\alpha_{AB} \cdot \mathbf{r}_{A/B} = 0$$

$$\left[(\alpha_{AB})_x\,\mathbf{i} + (\alpha_{AB})_y\,\mathbf{j} + (\alpha_{AB})_z\,\mathbf{k}\right] \cdot (-0.3\mathbf{i} - 1\mathbf{j} + 0.8\mathbf{k}) = 0$$

$$-0.3(\alpha_{AB})_x - (\alpha_{AB})_y + 0.8(\alpha_{AB})_z = 0 \quad \textbf{(4)}$$

Solving Eqs. (1) through (4).

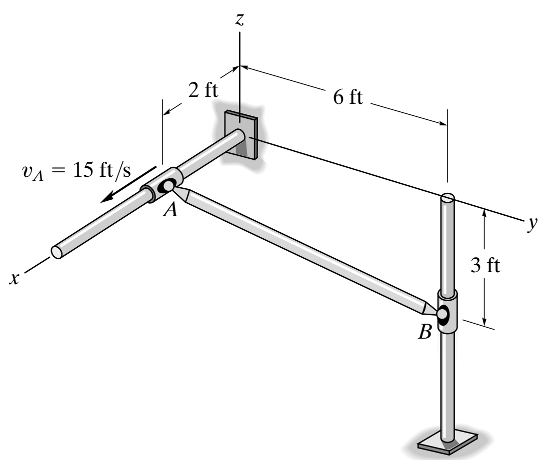
$(\alpha_{AB})_x = 7.807$ rad/s^2 $\quad (\alpha_{AB})_y = -7.399$rad/s^2 $\quad (\alpha_{AB})_z = -6.321$ rad/s^2

$$a_A = -13.9 \text{ m/s}^2$$

Then,

$$\mathbf{a}_A = [-13.9\mathbf{k}]\text{m/s}^2 \qquad \textbf{Ans.}$$

20–31. Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A has a velocity $v_A = 15$ ft/s at the instant shown, determine the velocity of collar B. Assume the angular velocity is perpendicular to the rod.



$\mathbf{v}_A = \{15\mathbf{i}\}$ ft/s $\qquad \mathbf{v}_B = v_B\mathbf{k} \qquad \boldsymbol{\omega}_{AB} = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$

$$\mathbf{r}_{B/A} = \{-2\,\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\}\text{ ft}$$

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$v_B\mathbf{k} = 15\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -2 & 6 & -3 \end{vmatrix}$$

Equating **i**, **j**, and **k** components yields:

$$15 - 3\,\omega_y - 6\omega_z = 0 \qquad \textbf{(1)}$$

$$3\,\omega_x - 2\,\omega_z = 0 \qquad \textbf{(2)}$$

$$6\omega_x + 2\omega_y = v_B \qquad \textbf{(3)}$$

If ω_{AB} is perpendicular to the axis of the rod,

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = (\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}) \cdot (-2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) = 0$$

$$-2\omega_x + 6\omega_y - 3\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4) yields:

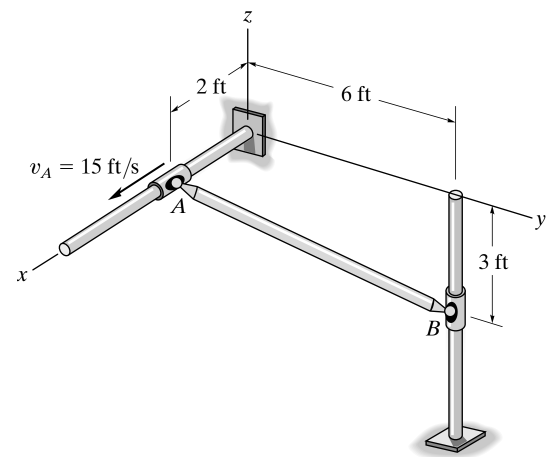
$\omega_x = 1.2245$ rad/s $\qquad \omega_y = 1.3265$ rad/s $\qquad \omega_z = 1.8367$ rad/s $\qquad v_B = 10$ ft/s

Note: v_B can be obtained by solving Eqs. (1)-(3) without knowing the direction of ω

Hence $\boldsymbol{\omega}_{AB} = \{1.2245\mathbf{i} + 1.3265\mathbf{j} + 1.8367\mathbf{k}\}$ rad/s

$$\mathbf{v}_B = \{10\mathbf{k}\}\text{ ft/s} \qquad \textbf{Ans.}$$

***20–32.** Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A has a velocity of $\mathbf{v}_A = \{15\mathbf{i}\}$ ft/s and an acceleration of $\mathbf{a}_A = \{2\mathbf{i}\}$ ft/s^2 at the instant shown, determine the acceleration of collar B. Assume the angular velocity and angular acceleration are perpendicular to the rod.



From Prob. 20–31

$$\omega_{AB} = \{1.2245\mathbf{i} + 1.3265\mathbf{j} + 1.8367\mathbf{k}\}\text{ rad/s}$$

$$\mathbf{r}_{B/A} = \{-2\,\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\}\text{ ft}$$

$$\alpha_{AB} = \alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\,\mathbf{k}$$

$$\mathbf{a}_A = \{\,2\mathbf{i}\,\}\quad \text{ft/s}^2 \qquad\qquad \mathbf{a}_B = a_B\,\mathbf{k}$$

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A})$$

$$a_B\,\mathbf{k} = 2\mathbf{i} + (\alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\mathbf{k}) \times (-2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k})$$
$$+(1.2245\mathbf{i} + 1.3265\mathbf{j} + 1.8367\mathbf{k})$$
$$\times\left[(1.2245\mathbf{i} + 1.3265\mathbf{j} + 1.8367\mathbf{k}) \times (-2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k})\right]$$

Equating **i**, **j**, and **k** components yields:

$$15.2653 - 3\,\alpha_y - 6\alpha_z = 0 \qquad \textbf{(1)}$$

$$3\alpha_x - 2\alpha_z - 39.7955 = 0 \qquad \textbf{(2)}$$

$$6\alpha_x + 2\alpha_y + 19.8975 = a_B \qquad \textbf{(3)}$$

If α_{AB} is perpendicular to the axis of the rod,

$$\alpha_{AB}\cdot\mathbf{r}_{B/A} = (\alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\mathbf{k})\cdot(-2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) = 0$$

$$-2\alpha_x + 6\alpha_y - 3\alpha_z = 0 \qquad \textbf{(4)}$$

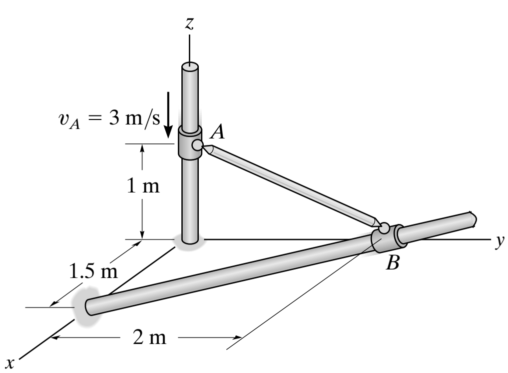
Solving Eqs. (1) to (4) yields:

$\alpha_x = 13.43\text{ rad/s}^2 \qquad \alpha_y = 4.599\text{ rad/s}^2 \qquad \alpha_z = 0.2449\text{ rad/s}^2 \qquad a_B = 109.7\text{ ft/s}^2$

Note: $\mathbf{a}_B$ can be obtained by solving Eqs. (1)-(3) without knowing the direction of α

Hence $\qquad \mathbf{a}_B = \{\,110\mathbf{k}\,\}\ \text{ft/s}^2 \qquad$ **Ans.**

•20–33. Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A has a speed $v_A = 3$ m/s, determine the speed of collar B at the instant shown. Assume the angular velocity is perpendicular to the rod.



Velocity Equation: Here, $\mathbf{r}_{B/A} = \{(2-0)\,\mathbf{j} + (0-1\,\mathbf{k})\text{ m} = \{2\mathbf{j} - 1\mathbf{k}\}$ m,

$\mathbf{v}_A = \{-3\mathbf{k}\}$ m/s, $\mathbf{v}_B = v_B\left[\dfrac{(0-1.5)\,\mathbf{i} + (2-0)\,\mathbf{j}}{\sqrt{(0-1.5)^2 + (2-0)^2}}\right] = -0.6\,v_B\,\mathbf{i} + 0.8\,v_B\,\mathbf{j}$ and

$\omega = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$. Applying Eq. 20–7, we have

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$$

$$-0.6\,v_B\,\mathbf{i} + 0.8\,v_B\,\mathbf{j} = -3\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})$$

$$-0.6\,v_B\,\mathbf{i} + 0.8\,v_B\,\mathbf{j} = (-\omega_y - 2\omega_z)\mathbf{i} + \omega_x\,\mathbf{j} + (2\omega_x - 3)\,\mathbf{k}$$

Equating **i**, **j** and **k** components, we have.

$$-0.6\,v_B = -\omega_y - 2\omega_z \qquad \textbf{[1]}$$

$$0.8\,v_B = \omega_x \qquad \textbf{[2]}$$

$$0 = 2\,\omega_x - 3 \qquad \textbf{[3]}$$

If ω is specified acting *perpendicular* to the axis of the rod AB, then

$$\omega \cdot \mathbf{r}_{B/A} = 0$$

$$(\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}) \cdot (2\mathbf{j} - 1\mathbf{k}) = 0$$

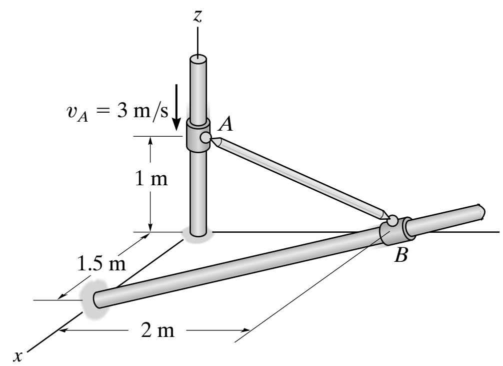
$$2\omega_y - \omega_z = 0 \qquad \textbf{[4]}$$

Solving Eqs. [1], [2], [3] and [4] yields

$$v_B = 1.875 \text{ m/s} \qquad \textbf{Ans.}$$

$$\omega_x = 1.50 \text{ rad/s} \qquad \omega_y = 0.225 \text{ rad/s} \qquad \omega_z = 0.450 \text{ rad/s}$$

20–34. If the collar at A in Prob 20–33 has an acceleration of $\mathbf{a}_A = \{-2\mathbf{k}\}\ \text{m/s}^2$ at the instant its velocity is $\mathbf{v}_A = \{-3\mathbf{k}\}\ \text{m/s}$, determine the magnitude of the acceleration of the collar at B at this instant. Assume the angular velocity and angular acceleration are perpendicular to the rod.



Velocity Equation: Here, $\mathbf{r}_{B/A} = \{(2-0)\mathbf{j} + (0-1)\,\mathbf{k}\}\ \text{m} = \{2\mathbf{j} - 1\mathbf{k}\}\ \text{m}$,

$\mathbf{v}_A = \{-3\mathbf{k}\}\ \text{m/s}, \mathbf{v}_B = v_B\left[\dfrac{(0-1.5)\mathbf{i} + (2-0)\mathbf{j}}{\sqrt{(0-1.5)^2 + (2-0)^2}}\right] = -0.6\,v_B\,\mathbf{i} + 0.8\,v_B\,\mathbf{j}$ and

$\omega = \omega\,\mathbf{j} + \omega_z\,\mathbf{k}$. Applying Eq. 20–7, we have

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$$

$$-0.6v_B\,\mathbf{i} + 0.8\,v_B\,\mathbf{j} = -3\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})$$

$$-0.6\,v_B\,\mathbf{i} + 0.8\,v_B\,\mathbf{j} = (-3-\omega_y - 2\omega_z)\mathbf{i} + \omega_x\,\mathbf{j} + (2\,\omega_x - 3)\,\mathbf{k}$$

Equating **i**, **j**, and **k** components, we have

$$-0.6\,v_B = -3-\omega_y - 2\omega_z \qquad \textbf{[1]}$$

$$0.8\,v_B = \omega_x \qquad \textbf{[2]}$$

$$0 = 2\omega_x - 3 \qquad \textbf{[3]}$$

If ω is specified acting *perpendicular* to the axis of the rod AB, then

$$\omega \cdot \mathbf{r}_{B/A} = 0$$

$$\left(\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}\right)\cdot(2\mathbf{j} - 1\mathbf{k}) = 0$$

$$2\omega_y - \omega_z = 0 \qquad \textbf{[4]}$$

Solving Eqs. [1], [2], [3] and [4] yields

$$v_B = 1.875\ \text{m/s} \qquad \omega_x = 1.50\ \text{rad/s}$$

$$\omega_y = 0.225\ \text{rad/s} \qquad \omega_z = 0.450\ \text{rad/s}$$

Thus,

$$\omega = \{1.50\mathbf{i} + 0.225\mathbf{j} + 0.450\mathbf{k}\}\ \text{rad/s}$$

20–34. Continued

Acceleration Equation: With $\alpha = \alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\,\mathbf{k}$ and the result obtained above, Applying Eq. 20–8, we have

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} + \omega \times (\omega \times \mathbf{r}_{B/A})$$

$$-0.6a_B\,\mathbf{i} + 0.8a_B\,\mathbf{j} = -2\mathbf{k} + \left(\alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\,\mathbf{k}\right) \times (2\mathbf{j} - 1\mathbf{k})$$

$$+ (1.50\mathbf{i} + 0.225\mathbf{j} + 0.450\mathbf{k}) \times [(1.50\mathbf{i} + 0.225\mathbf{j} + 0.450\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})]$$

$$-0.6a_B\,\mathbf{i} + 0.8a_B\,\mathbf{j} = \left(-\alpha_y - 2\alpha_z\right)\mathbf{i} + (\alpha_x - 5.00625)\,\mathbf{j} + (2\alpha_x + 0.503125)\,\mathbf{k}$$

Equating $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components. we have

$$-0.6a_B = -\alpha_y - 2\alpha_z \qquad \textbf{[5]}$$

$$0.8a_B = \alpha_y - 5.00625 \qquad \textbf{[6]}$$

$$0 = 2\alpha_y + 0.503125 \qquad \textbf{[7]}$$

Solving Eqs. [6] and [7] yields

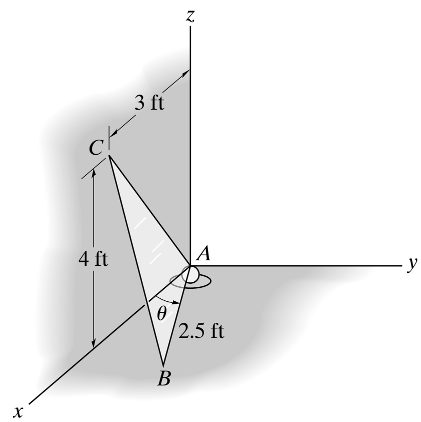
$$\alpha_x = -0.2515 \text{ rad/s}^2$$

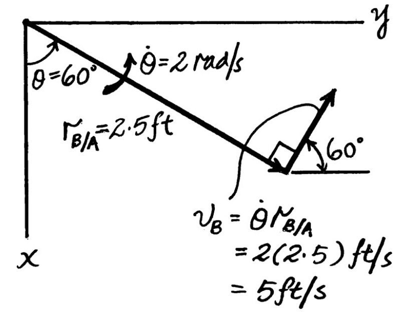
$$a_B = -6.57 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Negative sign indicates that $\mathbf{a}_B$ is directed in the opposite direction to that of the above assumed direction

Note: In order to determine α_y and α_z. one should obtain another equation by pacifying the direction of α which acts *perpendicular* to the axis of rod AB.

20–35. The triangular plate ABC is supported at A by a ball-and-socket joint and at C by the $x-z$ plane. The side AB lies in the $x-y$ plane. At the instant $\theta = 60°$, $\dot{\theta} = 2$ rad/s and point C has the coordinates shown. Determine the angular velocity of the plate and the velocity of point C at this instant.





$$\mathbf{v}_B = -5 \sin 60°\mathbf{i} + 5 \cos 60°\mathbf{j}$$

$$= \{-4.33\mathbf{i} + 2.5\mathbf{j}\} \text{ ft/s}$$

$$\mathbf{v}_C = (v_C)_x\,\mathbf{i} + (v_C)_z\,\mathbf{k}$$

$$\mathbf{r}_{C/A} = \{3\mathbf{i} + 4\mathbf{k}\} \text{ ft}$$

$$\mathbf{r}_{B/A} = \{1.25\mathbf{i} + 2.165\mathbf{j}\} \text{ ft}$$

$$\mathbf{v}_B = \omega \times \mathbf{r}_{B/A}$$

$$-4.33\mathbf{i} + 2.5\mathbf{j} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 1.25 & 2.165 & 0 \end{vmatrix}$$

$$-2.165\omega_z = -4.33\,; \qquad \omega_z = 2 \text{ rad/s}$$

$$2.165\omega_x - 1.25\omega_y = 0; \qquad \omega_y = 1.732\omega_x$$

$$\mathbf{v}_C = \omega \times \mathbf{r}_{C/A}$$

$$(v_C)_x\,\mathbf{i} + (v_C)_z\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & 2 \\ 3 & 0 & 4 \end{vmatrix}$$

$$(v_C)_x = 4\omega_y$$

$$0 = 4\omega_x - 6; \qquad \omega_x = 1.5 \text{ rad/s}$$

$$(v_C)_z = -3\omega_y$$

Solving,

$$\omega_y = 2.5981 \text{ rad/s}$$

$$(v_C)_x = 10.392 \text{ ft/s}$$

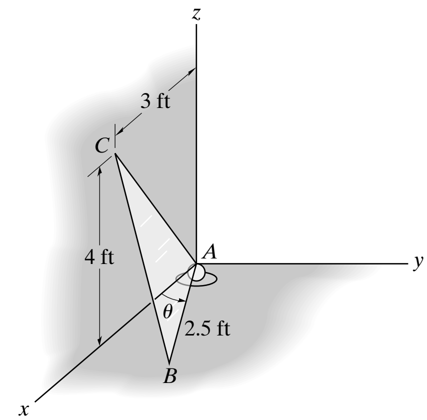
$$(v_C)_z = -7.7942 \text{ ft/s}$$

Thus,

$$\omega = \{1.50\mathbf{i} + 2.60\mathbf{j} + 2.00\mathbf{k}\} \text{ rad/s}$$ **Ans.**

$$\mathbf{v}_C = \{10.4\mathbf{i} - 7.79\mathbf{k}\} \text{ ft/s}$$ **Ans.**

***20–36.** The triangular plate ABC is supported at A by a ball-and-socket joint and at C by the $x-z$ plane. The side AB lies in the $x-y$ plane. At the instant $\theta = 60°$, $\dot{\theta} = 2$ rad/s, $\ddot{\theta} = 3$ rad/s^2 and point C has the coordinates shown. Determine the angular acceleration of the plate and the acceleration of point C at this instant.



From Prob. 20–35.

$$\omega = 1.5\mathbf{i} + 2.5981\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{r}_{B/A} = 1.25\mathbf{i} + 2.165\mathbf{j}$$

$$\mathbf{v}_B = -4.33\mathbf{i} + 2.5\mathbf{j}$$

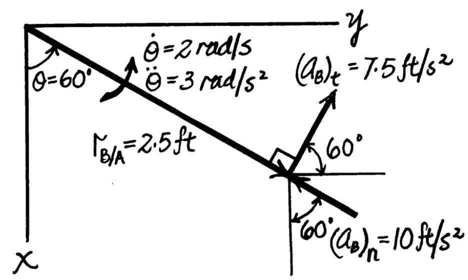
$$(a_B)_t = 3(2.5) = 7.5 \text{ ft/s}^2$$

$$(a_B)_n = (2)^2(2.5) = 10 \text{ ft/s}^2$$

$$\mathbf{a}_B = -7.5 \sin 60°\mathbf{i} + 7.5 \cos 60°\mathbf{j} - 10 \cos 60°\mathbf{i} - 10 \sin 60°\mathbf{j}$$

$$\mathbf{a}_B = -11.4952\mathbf{i} - 4.91025\mathbf{j}$$

$$\mathbf{a}_B = \alpha \times \mathbf{r}_{B/A} + \omega \times \mathbf{v}_{B/A}$$



$$-11.4952\mathbf{i} - 4.91025\mathbf{j} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \alpha_x & \alpha_y & \alpha_z \\ 1.25 & 2.165 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 2.5981 & 2 \\ -4.33 & 2.5 & 0 \end{vmatrix}$$

$$-11.4952 = -2.165\alpha_z - 5$$

$$-4.91025 = 1.25\alpha_z - 8.66$$

$$\alpha_z = 3 \text{ rad/s}^2$$

$$0 = 2.165\alpha_x - 1.25\alpha_y + 15 \qquad \textbf{(1)}$$

$$\mathbf{a}_C = \alpha \times \mathbf{r}_{C/A} + \omega \times \mathbf{v}_{C/A}$$

$$\mathbf{v}_{C/A} = 10.39\mathbf{i} - 7.794\mathbf{k}$$

$$\mathbf{a}_C = (a_C)_x\,\mathbf{i} + (a_C)_z\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \alpha_x & \alpha_y & \alpha_z \\ 3 & 0 & 4 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 2.5981 & 2 \\ 10.39 & 0 & -7.794 \end{vmatrix}$$

$$(a_C)_x = 4\alpha_y - 20.25 \qquad \textbf{(2)}$$

$$0 = 3\alpha_z - 4\alpha_x + 32.4760 \qquad \textbf{(3)}$$

$$(a_C)_z = -3\alpha_y - 27 \qquad \textbf{(4)}$$

Solving Eqs. (1)-(4),

$$\alpha_x = 10.369 \text{ rad/s}^2$$

$$\alpha_y = 29.96 \text{ rad/s}^2$$

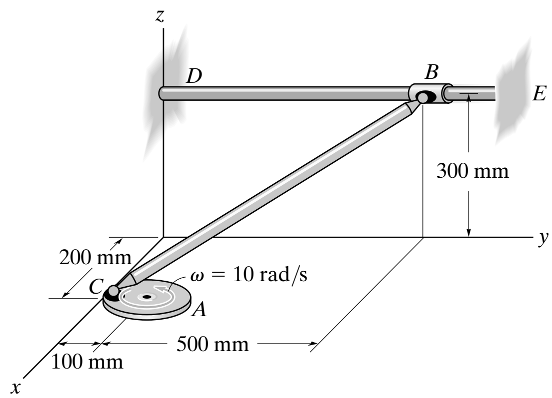
$$(a_C)_x = 99.6 \text{ ft/s}^2$$

$$(a_C)_z = -117 \text{ ft/s}^2$$

$$\mathbf{a}_C = \{99.6\mathbf{i} - 117\mathbf{k}\}\text{ft/s}^2 \qquad \textbf{Ans.}$$

$$\alpha = \{10.4\mathbf{i} + 30.0\mathbf{j} + 3\mathbf{k}\} \text{ rad/s}^2 \qquad \textbf{Ans.}$$

•20–37. Disk A rotates at a constant angular velocity of 10 rad/s. If rod BC is joined to the disk and a collar by ball-and-socket joints, determine the velocity of collar B at the instant shown. Also, what is the rod's angular velocity $\boldsymbol{\omega}_{BC}$ if it is directed perpendicular to the axis of the rod?

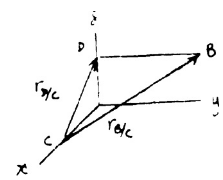


$\mathbf{v}_C = \{1\mathbf{i}\}$ m/s $\quad \mathbf{v}_B = -v_B\mathbf{j} \quad \omega_{BC} = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$

$$r_{B/C} = \{-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}\}\text{ m}$$

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$-\mathbf{v}_B = 1\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -0.2 & 0.6 & 0.3 \end{vmatrix}$$



Equating **i**, **j**, and **k** components

$$1 - 0.3\omega_y - 0.6\omega_z = 0 \qquad \textbf{(1)}$$

$$0.3\omega_x + 0.2\omega_z = v_B \qquad \textbf{(2)}$$

$$0.6\omega_x + 0.2\omega_y = 0 \qquad \textbf{(3)}$$

Since ω_{BC} is perpendicular to the axis of the rod,

$$\omega_{BC} \quad \mathbf{r}_{B/C} = (\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\mathbf{k})\cdot(-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}) = 0$$

$$-0.2\omega_x + 0.6\omega_y + 0.3\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4) yields:

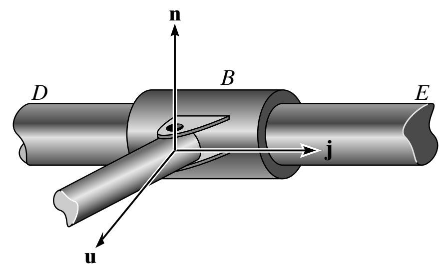
$\omega_x = 0.204$ rad/s $\quad \omega_y = -0.612$ rad/s $\quad \omega_z = 1.36$ rad/s $\quad v_B = 0.333$ m/s

Then

$$\omega_{BC} = \{0.204\mathbf{i} - 0.612\mathbf{j} + 1.36\mathbf{k}\}\text{ rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_B = \{-0.333\mathbf{j}\}\text{ m/s} \qquad \textbf{Ans.}$$

20–38. Solve Prob. 20–37 if the connection at B consists of a pin as shown in the figure below, rather than a ball-and-socket joint. *Hint:* The constraint allows rotation of the rod both about bar DE ($\mathbf{j}$ direction) and about the axis of the pin ($\mathbf{n}$ direction). Since there is no rotational component in the $\mathbf{u}$ direction, i.e., perpendicular to $\mathbf{n}$ and $\mathbf{j}$ where $\mathbf{u} = \mathbf{j} \times \mathbf{n}$, an additional equation for solution can be obtained from $\boldsymbol{\omega} \cdot \mathbf{u} = 0$. The vector $\mathbf{n}$ is in the same direction as $\mathbf{r}_{B/C} \times \mathbf{r}_{D/C}$.



$\mathbf{v}_C = \{1\mathbf{i}\}$ m/s $\qquad \mathbf{v}_B = -v_B\mathbf{j} \qquad \omega_{BC} = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$

$$\mathbf{r}_{B/C} = \{-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}\}\text{ m}$$

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$-v_B\mathbf{j} = 1\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -0.2 & 0.6 & 0.3 \end{vmatrix}$$

Equating $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components

$$1 + 0.3\omega_x - 0.6\omega_z = 0 \qquad \textbf{(1)}$$

$$0.3\omega_x + 0.2\omega_z = v_B \qquad \textbf{(2)}$$

$$0.6\omega_x + 0.2\omega_z = 0 \qquad \textbf{(3)}$$

Also,

$$\mathbf{r}_{B/C} = \{-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}\}\text{ m}$$

$$\mathbf{r}_{D/C} = \{-0.2\mathbf{i} + 0.3\mathbf{k}\}\text{ m}$$

$$\mathbf{r}_{B/C} \times \mathbf{r}_{D/C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.2 & 0.6 & 0.3 \\ -0.2 & 0 & 0.3 \end{vmatrix} = \{0.18\mathbf{i} + 0.12\mathbf{k}\}\text{ m}^2$$

$$\mathbf{n} = \frac{0.18\mathbf{i} + 0.12\mathbf{k}}{\sqrt{0.18^2 + 0.12^2}} = 0.8321\mathbf{i} + 0.5547\mathbf{k}$$

$$\mathbf{u} = \mathbf{j} \times \mathbf{n} = \mathbf{j} \times (0.8321\mathbf{i} + 0.5547\mathbf{k}) = 0.5547\mathbf{i} - 0.8321\mathbf{k}$$

$$\omega_{BC} \cdot \mathbf{u} = (\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\mathbf{k}) \cdot (0.5547\mathbf{i} - 0.8321\mathbf{k}) = 0$$

$$0.5547\omega_x - 0.8321\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4) yields:

$\omega_x = 0.769$ rad/s $\qquad \omega_y = -2.31$ rad/s $\qquad \omega_z = 0.513$ rad/s $\qquad v_B = 0.333$ m/s

Then

$$\omega_{BC} = \{0.769\mathbf{i} - 2.31\mathbf{j} + 0.513\mathbf{k}\}\text{ rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_B = \{-0.333\mathbf{j}\}\text{ m/s} \qquad \textbf{Ans.}$$

20–39. Solve Example 20–5 such that the x, y, z axes move with curvilinear translation, $\mathbf{\Omega} = \mathbf{0}$ in which case the collar appears to have both an angular velocity $\mathbf{\Omega}_{xyz} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$ and radial motion.

Relative to XYZ, let xyz have

$$\Omega = 0 \qquad \dot{\Omega} = 0$$

$$\mathbf{r}_B = \{-0.5\mathbf{k}\} \text{ m}$$

$$\mathbf{v}_B = \{2\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_B = \{0.75\mathbf{j} + 8\mathbf{k}\} \text{ m/s}^2$$

Relative to xyz, let $x'\ y'\ z'$ be coincident with xyz and be fixed to BD. Then

$$\Omega_{xyz} = \omega_1 + \omega_2 = \{4\mathbf{i} + 5\mathbf{k}\} \text{ rad/s} \qquad \dot{\omega}_{xyz} = \dot{\omega}_1 + \dot{\omega}_2 = \{1.5\mathbf{i} - 6\mathbf{k}\} \text{ rad/s}^2$$

$$(\mathbf{r}_{C/B})_{xyz} = \{0.2\mathbf{j}\} \text{ m}$$

$$(\mathbf{v}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{x'y'z'} + (\omega_1 + \omega_2) \times (\mathbf{r}_{C/B})_{xyz}$$

$$= 3\mathbf{j} + (4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})$$

$$= \{-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = (\ddot{\mathbf{r}}_{C/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/B})_{x'y'z'} + (\omega_1 + \omega_2) \times (\dot{\mathbf{r}}_{C/B})_{x'y'z'}\right]$$

$$+\left[(\dot{\omega}_1 + \dot{\omega}_2) \times (\mathbf{r}_{C/B})_{xyz}\right] + \left[(\omega_1 + \omega_2) \times (\dot{\mathbf{r}}_{C/B})_{xyz}\right]$$

$$(\mathbf{a}_{C/B})_{xyz} = \left[2\mathbf{j} + (4\mathbf{i} + 5\mathbf{k}) \times 3\mathbf{j}\right] + \left[(1.5\mathbf{i} - 6\mathbf{k}) \times 0.2\mathbf{j}\right] + \left[(4\mathbf{i} + 5\mathbf{k}) \times (-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k})\right]$$

$$= \{-28.8\mathbf{i} - 6.2\mathbf{j} + 24.3\mathbf{k}\} \text{ m/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= 2\mathbf{j} + 0 + (-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k})$$

$$= \{-1.00\mathbf{i} + 5.00\mathbf{j} + 0.800\mathbf{k}\} \text{ m/s}$$ **Ans.**

$$\mathbf{a}_C = \mathbf{a}_B + \Omega \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= (0.75\mathbf{j} + 8\mathbf{k}) + \mathbf{0} + \mathbf{0} + \mathbf{0} + (-28.8\mathbf{i} - 6.2\mathbf{j} + 24.3\mathbf{k})$$

$$= \{-28.8\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\} \text{ m/s}^2$$ **Ans.**

***20–40.** Solve Example 20–5 by fixing x, y, z axes to rod BD so that $\mathbf{\Omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$. In this case the collar appears only to move radially outward along BD; hence $\mathbf{\Omega}_{xyz} = \mathbf{0}$.

Relative to XYZ, let $x'\ y'\ z'$ be concident with XYZ and have $\Omega' = \omega_1$ and $\dot{\Omega}' = \dot{\omega}_1$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2 = \{4\mathbf{i} + 5\mathbf{k}\}\text{ rad/s}$$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2 = \left[\left(\dot{\omega}_1\right)_{x'y'z'} + \omega_1 \times \omega_1\right] + \left[\left(\omega_2\right)_{x'y'z'} + \omega_1 \times \omega_2\right]$$

$$= (1.5\mathbf{i} + \mathbf{0}) + \left[-6\mathbf{k} + (4\mathbf{i}) \times (5\mathbf{k})\right] = \{1.5\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}\}\text{ rad/s}^2$$

$$\mathbf{r}_B = \{-0.5\mathbf{k}\}\text{ m}$$

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = \left(\dot{\mathbf{r}}_B\right)_{x'y'z'} + \omega_1 \times \mathbf{r}_B = \mathbf{0} + (4\mathbf{i}) \times (-0.5\mathbf{k}) = \{2\mathbf{j}\}\text{ m/s}$$

$$\mathbf{a}_B = \dot{\mathbf{r}}_B = \left[\left(\ddot{\mathbf{r}}_B\right)_{x'y'z'} + \omega_1 \times \left(\dot{\mathbf{r}}_\mathbf{B}\right)_{x'y'z'}\right] + \dot{\omega}_1 \times r_B + \omega_1 \times \dot{\mathbf{r}}_B$$

$$= \mathbf{0} + \mathbf{0} + \left[(1.5\mathbf{i}) \times (-0.5\mathbf{k})\right] + (4\mathbf{i} \times 2\mathbf{j}) = \{0.75\mathbf{j} + 8\mathbf{k}\}\text{ m/s}^2$$

Relative to $x'y'z'$, let xyz have

$$\Omega_{x'y'z'} = \mathbf{0}; \qquad \dot{\Omega}_{x'y'z'} = \mathbf{0};$$

$$\left(r_{C/B}\right)_{xyz} = \{0.2\mathbf{j}\}\text{ m}$$

$$(\mathbf{v}_{C/B})_{xyz} = \{3\mathbf{j}\}\text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \{2\mathbf{j}\}\text{ m/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= 2\mathbf{j} + \left[(4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})\right] + 3\mathbf{j}$$

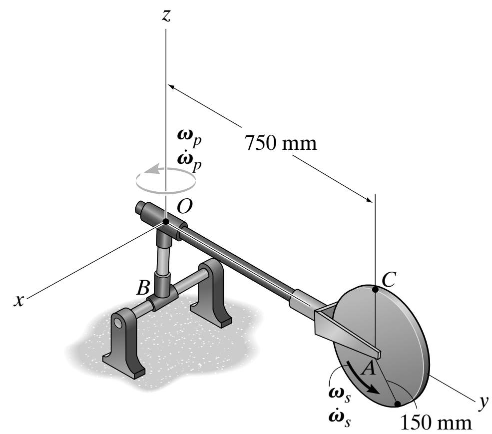
$$= \{-1\mathbf{i} + 5\mathbf{j} + 0.8\mathbf{k}\}\text{m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = \mathbf{a}_B + \Omega \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= (0.75\mathbf{j} + 8\mathbf{k}) + \left[(1.5\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}) \times (0.2\mathbf{j})\right] + (4\mathbf{i} + 5\mathbf{k}) \times \left[(4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})\right] + 2\left[(4\mathbf{i} + 5\mathbf{k}) \times (3\mathbf{j})\right] + 2\mathbf{j}$$

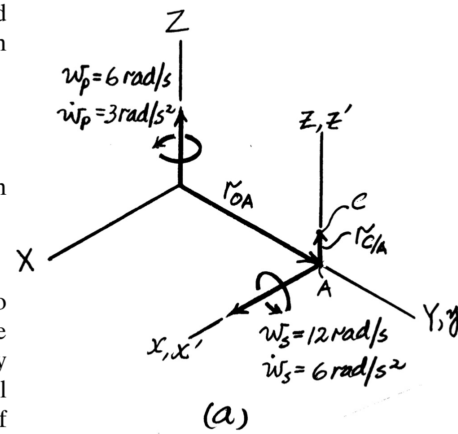
$$\mathbf{a}_C = \{-28.2\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\}\text{ m/s}^2 \qquad \textbf{Ans.}$$

•20–41. At the instant shown, the shaft rotates with an angular velocity of $\omega_p = 6$ rad/s and has an angular acceleration of $\dot{\omega}_p = 3$ rad/s^2. At the same instant, the disk spins about its axle with an angular velocity of $\omega_s = 12$ rad/s, increasing at a constant rate of $\dot{\omega}_s = 6$ rad/s^2. Determine the velocity of point C located on the rim of the disk at this instant.



The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. The angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_p = [6\mathbf{k}]\text{ rad/s} \qquad \dot{\omega} = \dot{\omega}_p = [3\mathbf{k}]\text{ rad/s}^2$$



Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_P \times \mathbf{r}_{OA} = (6\mathbf{k}) \times (0.75\mathbf{j}) = [-4.5\mathbf{i}]\text{ m/s}$$

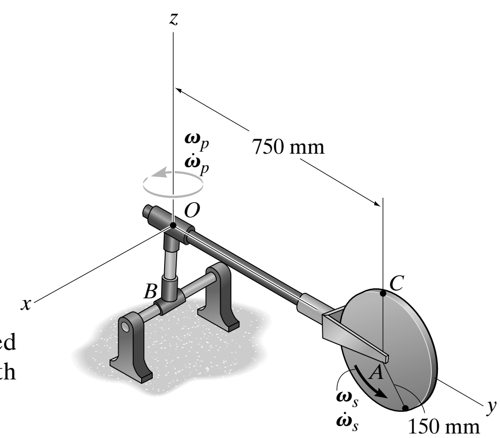
In order to determine the motion of point C relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_S = [12\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{C/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{C/A})_{xyz}$,

$$\begin{aligned}(\mathbf{v}_{C/A})_{xyz} = (\dot{\mathbf{r}}_{C/A})_{xyz} &= \left[(\dot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_s \times (\mathbf{r}_{C/A})_{xyz}\right] \\ &= 0 + (12\mathbf{i}) \times (0.15\mathbf{k}) \\ &= [-1.8\mathbf{j}]\text{ m/s}\end{aligned}$$

Thus,

$$\begin{aligned}\mathbf{v}_C &= \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz} \\ &= (-4.5\mathbf{i}) + 6\mathbf{k} \times 0.15\mathbf{k} + (-1.8\mathbf{j}) \\ &= [-4.5\mathbf{i} - 1.8\mathbf{j}]\text{ m/s} \qquad \textbf{Ans.}\end{aligned}$$

20–42. At the instant shown, the shaft rotates with an angular velocity of $\omega_p = 6$ rad/s and has an angular acceleration of $\dot{\omega}_p = 3$ rad/s^2. At the same instant, the disk spins about its axle with an angular velocity of $\omega_s = 12$ rad/s, increasing at a constant rate of $\dot{\omega}_s = 6$ rad/s^2. Determine the acceleration of point C located on the rim of the disk at this instant.



The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. The angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_p = [6\mathbf{k}]\text{ rad/s} \qquad \dot{\omega} = \dot{\Omega}_p = [3\mathbf{k}]\text{ rad/s}^2$$

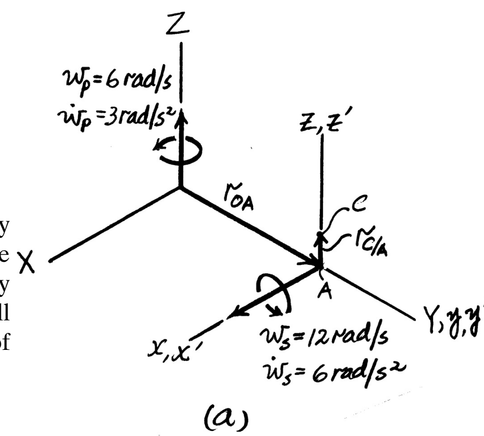
Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_p \times \mathbf{r}_{OA} = (6\mathbf{k}) \times (0.75\mathbf{j}) = [-4.5\mathbf{i}]\text{ m/s}$$

$$a_A = \dot{\omega}_p \times \mathbf{r}_{OA} + \omega_p \times \left(\omega_p \times \mathbf{r}_{OA}\right)$$

$$= (3\mathbf{k}) \times (0.75\mathbf{j}) + 6\mathbf{k} \times [6\mathbf{k} \times 0.75\mathbf{j}]$$

$$= [-2.25\mathbf{i} - 27\mathbf{j}]\text{ m/s}^2$$



(a)

In order to determine the motion of the point C relative to point A, it is neccessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_s = [12\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{C/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{C/A})_{xyz}$,

$$(\mathbf{v}_{C/A})_{xyz} = (\dot{\mathbf{r}}_{C/A})_{xyz} = \left[(\dot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_S \times (\mathbf{r}_{C/A})_{xyz}\right]$$

$$= 0 + (12\mathbf{i}) \times (0.15\mathbf{k})$$

$$= [-1.8\mathbf{j}]\text{ m/s}$$

Since $\Omega' = \omega_s$ has a constant direction with respect to the xyz frame, then $\dot{\Omega} = \dot{\omega}_s = [6\mathbf{i}]$ rad/s^2. Taking the time derivative of $(\dot{\mathbf{r}}_{C/A})_{xyz}$,

$$(\mathbf{a}_{C/A}) = (\ddot{\mathbf{r}}_{C/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_s \times (\dot{\mathbf{r}}_{C/A})_{x'y'z'}\right] + \dot{\omega}_s \times (\mathbf{r}_{C/A})_{xyz} + \omega_s \times (\dot{r}_{C/A})_{xyz}$$

$$= [0 + 0] + (6\mathbf{i}) \times (0.15\mathbf{k}) + (12\mathbf{i}) \times (-1.8\mathbf{j})$$

$$= [-0.9\mathbf{j} - 21.6\mathbf{k}]\text{ m/s}^2$$

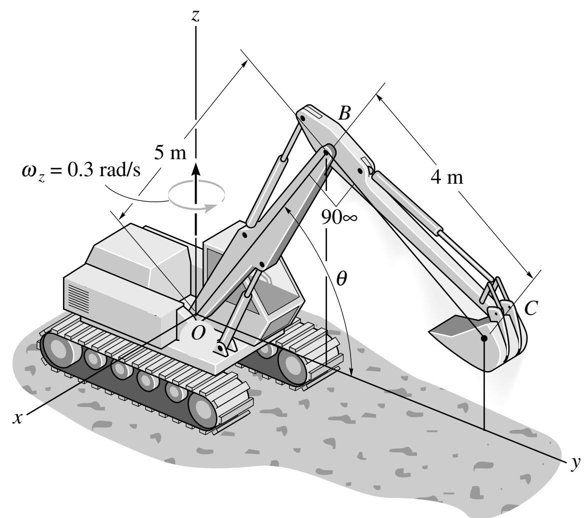
Thus,

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$

$$= (-2.25\mathbf{i} - 27\mathbf{j}) + 3\mathbf{k} \times 0.15\mathbf{k} + 6\mathbf{k} \times (6\mathbf{k} \times 0.15\mathbf{k}) + 2(6\mathbf{k}) \times (-1.8\mathbf{j}) + (-0.9\mathbf{j} - 21.6\mathbf{k})$$

$$= [19.35\mathbf{i} - 27.9\mathbf{j} - 21.6\mathbf{k}]\text{m/s}^2 \qquad \textbf{Ans.}$$

20–43. At the instant shown, the cab of the excavator rotates about the z axis with a constant angular velocity of $\omega_z = 0.3$ rad/s. At the same instant $\theta = 60°$, and the boom OBC has an angular velocity of $\dot{\theta} = 0.6$ rad/s, which is increasing at $\ddot{\theta} = 0.2$ rad/s^2, both measured relative to the cab. Determine the velocity and acceleration of point C on the grapple at this instant.



Relative to XYZ, let xyz have

$\Omega = \{0.3\mathbf{k}\}$ rad/s, $\quad \dot{\omega} = \mathbf{0}$ (Ω does not change direction relative to XYZ.)

$$\mathbf{r}_O = \mathbf{0}, \quad \mathbf{v}_O = \mathbf{0}, \quad \mathbf{a}_O = \mathbf{0}$$

Relative to xyz, let $x'y'z'$ be coincident with xyz at O and have

$\Omega_{xyz} = \{0.6\mathbf{i}\}$ rad/s, $\quad \dot{\Omega}_{xyz} = \{0.2\mathbf{i}\}$ rad/s^2 (Ω_{xyz} does not change direction relative to XYZ.)

$$(\mathbf{r}_{C/O})_{xyz} = (5\cos 60° + 4\cos 30°)\mathbf{j} + (5\sin 60° - 4\sin 30°)\mathbf{k} = \{5.9641\mathbf{j} + 2.3301\mathbf{k}\}\text{ m}$$

$(\mathbf{r}_{C/O})_{xyz}$ change direction relative to XYZ.)

$$(\mathbf{v}_{C/O})_{xyz} = \left(\dot{\mathbf{r}}_{C/O}\right)_{xyz} = \left(\dot{\mathbf{r}}_{C/O}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\mathbf{r}_{C/O}\right)_{xyz}$$

$$= \mathbf{0} + (0.6\mathbf{i}) \times (5.9641\mathbf{j} + 2.3301\mathbf{k}) = \{-1.3981\mathbf{j} + 3.5785\mathbf{k}\}\text{ m/s}$$

$$(\mathbf{a}_{C/O})_{xyz} = \left(\ddot{\mathbf{r}}_{C/O}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{C/O}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/O}\right)_{x'y'z'}\right] + \dot{\Omega}_{xyz} \times \mathbf{r}_{C/O} + \Omega_{xyz} \times \dot{\mathbf{r}}_{C/O}$$

$$= \left[\mathbf{0} + \mathbf{0}\right] + (0.2\mathbf{i}) \times (5.9641\mathbf{j} + 2.3301\mathbf{k}) + (0.6\mathbf{i}) \times (-1.3981\mathbf{j} + 3.5785\mathbf{k})$$

$$= \{-2.61310\mathbf{j} + 0.35397\mathbf{k}\}\text{ m/s}^2$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_O + \Omega \times \mathbf{r}_{C/O} + (\mathbf{v}_{C/O})_{xyz} = \mathbf{0} + (0.3\mathbf{k}) \times (5.9641\mathbf{j} + 2.3301\mathbf{k}) - 1.3981\mathbf{j} + 3.5785\mathbf{k}$$

$$= \{-1.79\mathbf{i} - 1.40\mathbf{j} + 3.58\mathbf{k}\}\text{ m/s} \qquad \textbf{Ans.}$$

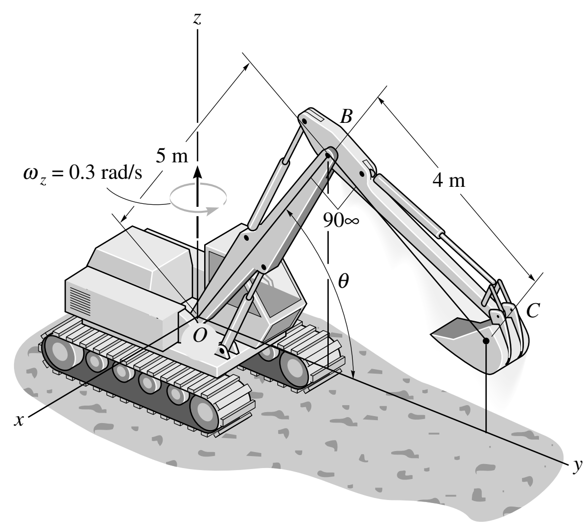
$$\mathbf{a}_C = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{C/O} + \Omega \times \left(\Omega \times \mathbf{r}_{C/O}\right) + 2\Omega \times (\mathbf{v}_{C/O})_{xyz} + (\mathbf{a}_{C/O})_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + (0.3\mathbf{k}) \times \left[(0.3\mathbf{k}) \times (5.9641\mathbf{j} + 2.3301\mathbf{k})\right]$$

$$+ 2(0.3k) \times (-1.3981\mathbf{j} + 3.5785\mathbf{k}) - 2.61310\mathbf{j} + 0.35397\mathbf{k}$$

$$= \{0.839\mathbf{i} - 3.15\mathbf{j} + 0.354\mathbf{k}\}\text{ m/s}^2 \qquad \textbf{Ans.}$$

***20–44.** At the instant shown, the frame of the excavator travels forward in the y direction with a velocity of 2 m/s and an acceleration of 1 m/s^2, while the cab rotates about the z axis with an angular velocity of $\omega_z = 0.3$ rad/s, which is increasing at $\alpha_z = 0.4$ rad/s^2. At the same instant $\theta = 60°$, and the boom OBC has an angular velocity of $\dot{\theta} = 0.6$ rad/s, which is increasing at $\ddot{\theta} = 0.2$ rad/s^2, both measured relative to the cab. Determine the velocity and acceleration of point C on the grapple at this instant.



Relative to XYZ, let xyz have

$\Omega = \{0.3\mathbf{k}\}$ rad/s, $\dot{\omega} = \{0.4\mathbf{k}\}$ rad/s^2 (Ω does not change direction relative to XYZ.)

$\mathbf{r}_O = \mathbf{0}$ (r_o does not change direction relative to XYZ.)

$$\mathbf{v}_O = \{2\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_O = \{1\mathbf{j}\} \text{ m/s}^2$$

Relative to xyz, let $x'y'z'$ have

$\Omega_{xyz} = \{0.6\mathbf{i}\}$ rad/s, $\dot{\omega}_{xyz} = \{0.2\mathbf{i}\}$ rad/s^2 (Ω_{xyz} does not change direction relative to xyz.)

$$(\mathbf{r}_{C/O})_{xyz} = (5\cos 60° + 4\cos 30°)\mathbf{j} + (5\sin 60° - 4\sin 30°)\mathbf{k} = \{5.9641\mathbf{j} + 2.3301\mathbf{k}\} \text{ m}$$

($(\mathbf{r}_{C/O})_{xyz}$ change direction relative to xyz.)

$$(\mathbf{v}_{C/O})_{xyz} = \left(\dot{\mathbf{r}}_{C/O}\right)_{xyz} = \left(\dot{\mathbf{r}}_{C/O}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\mathbf{r}_{C/O}\right)_{xyz}$$

$$= 0 + (0.6\mathbf{i}) \times (5.9641\mathbf{j} + 2.3301\mathbf{k}) = \{-1.3981\mathbf{j} + 3.5785\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{C/O})_{xyz} = \left(\ddot{\mathbf{r}}_{C/O}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{C/O}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/O}\right)_{x'y'z'}\right] + \dot{\Omega}_{xyz} \times \left(\mathbf{r}_{C/O}\right)_{xyz} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/O}\right)_{xyz}$$

$$= \left[\mathbf{0} + \mathbf{0}\right] + (0.2\mathbf{i}) \times (5.9641\mathbf{j} + 2.3301\mathbf{k}) + (0.6\mathbf{i}) \times (-1.3981\mathbf{j} + 3.5785\mathbf{k})$$

$$= \{-2.61310\mathbf{j} + 0.35397\mathbf{k}\} \text{ m/s}^2$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_O + \Omega \times \mathbf{r}_{C/O} + (\mathbf{v}_{C/O})_{xyz} = 2\mathbf{j} + (0.3\mathbf{k}) \times (5.9641\mathbf{j} + 2.3301\mathbf{k}) - 1.3981\mathbf{j} + 3.5785\mathbf{k}$$

$$= \{-1.79\mathbf{i} + 0.602\mathbf{j} + 3.58\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans.}$$

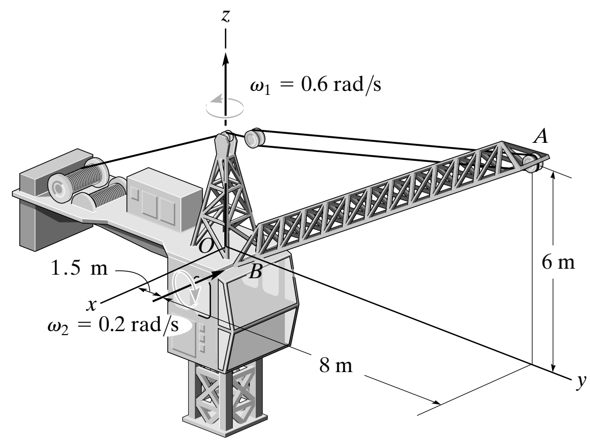
$$\mathbf{a}_C = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{C/O} + \Omega \times \left(\Omega \times \mathbf{r}_{C/O}\right) + 2\Omega \times (\mathbf{v}_{C/O})_{xyz} + (\mathbf{a}_{C/O})_{xyz}$$

$$= 1\mathbf{j} + 0.4\mathbf{k} \times (5.9641\mathbf{j} + 2.3301\mathbf{k}) + (0.3\mathbf{k}) \times \left[(0.3\mathbf{k}) \times (5.9641\mathbf{j} + 2.3301\mathbf{k})\right]$$

$$+ 2(0.3\mathbf{k}) \times (-1.3981\mathbf{j} + 3.5785\mathbf{k}) - 2.61310\mathbf{j} + 0.35397\mathbf{k}$$

$$= \{-1.55\mathbf{i} - 2.15\mathbf{j} + 0.354\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

•20–45. The crane rotates about the z axis with a constant rate $\omega_1 = 0.6$ rad/s, while the boom rotates downward with a constant rate $\omega_2 = 0.2$ rad/s. Determine the velocity and acceleration of point A located at the end of the boom at the instant shown.



Coordinate Axes: The rotating x, y, z frame and fixed X, Y, Z frame are set with the origin at point B and O respectively.

Motion of B: Here, $\mathbf{r}_B$ changes direction with respect to X, Y, Z frame. The time derivatives of $\mathbf{r}_B$ can be found by setting another set of coordinate axis x', y', z', coincident with X, Y, Z rotating at $\Omega = \omega_1 = \{0.6\mathbf{k}\}$ rad/s and $\Omega' = \dot{\omega}_1 = \mathbf{0}$. Here, $\mathbf{r}_B = \{1.5\mathbf{j}\}$ m

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = (\dot{\mathbf{r}}_B)_{x'y'z'} + \Omega' \times \mathbf{r}_B = \mathbf{0} + 0.6\mathbf{k} \times 1.5\mathbf{j} = \{-0.9\mathbf{i}\} \text{ m/s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{x'y'z'} + \Omega' \times (\dot{\mathbf{r}}_B)_{x'y'z'}\right] + \dot{\Omega}' \times \mathbf{r}_B + \Omega' \times \dot{\mathbf{r}}_B$$

$$= (\mathbf{0} + \mathbf{0}) + \mathbf{0} + 0.6\mathbf{k} \times (-0.9\mathbf{i}) = \{-0.540\mathbf{j}\} \text{ m/s}^2$$

Motion of A with Respect to B: Let xyz axis rotate at $\Omega_{xyz} = \omega_2 = \{-0.2\mathbf{i}\}$ rad/s and $\dot{\Omega}_{xyz} = \dot{\omega}_2 = \mathbf{0}$. Here, $\mathbf{r}_{A/B} = \{8\mathbf{j} + 6\mathbf{k}\}$ m.

$$(\mathbf{v}_{A/B})_{xyz} = \dot{\mathbf{r}}_{A/B} = (\dot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{A/B} = \mathbf{0} + (-0.2\mathbf{i}) \times (8\mathbf{j} + 6\mathbf{k}) = \{1.20\mathbf{j} - 1.60\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{A/B})_{xyz} = \ddot{\mathbf{r}}_{A/B} = \left[(\ddot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{A/B})_{xyz}\right] + \dot{\Omega}_{xyz} \times \mathbf{r}_{A/B} + \Omega_{xyz} \times \dot{\mathbf{r}}_{A/B}$$

$$= \mathbf{0} + \mathbf{0} + \mathbf{0} + (-0.2\mathbf{i}) \times (1.20\mathbf{j} - 1.60\mathbf{k})$$

$$= \{-0.320\mathbf{j} - 0.240\mathbf{k}\} \text{ m/s}^2$$

Motion of Point A: Here, $\Omega = \omega_1 = \{0.6\mathbf{k}\}$ rad/s and $\dot{\omega} = \dot{\omega}_1 = \mathbf{0}$. Applying Eqs. 20–11 and 20-12, we have

$$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz} = (-0.9\mathbf{i}) + 0.6\mathbf{k} \times (8\mathbf{j} + 6\mathbf{k}) + (1.20\mathbf{j} - 1.60\mathbf{k})$$

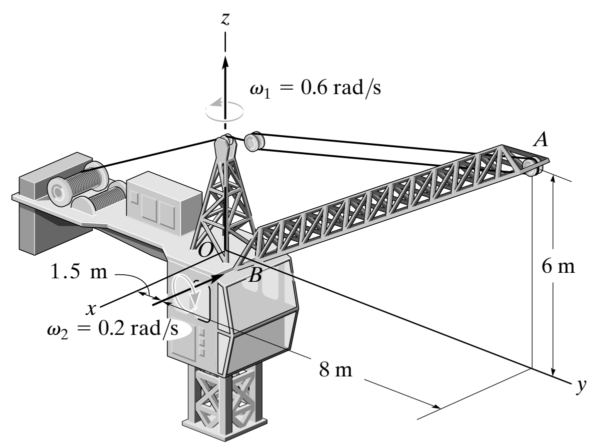
$$= \{-5.70\mathbf{i} + 1.20\mathbf{j} - 1.60\mathbf{k}\}\text{m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{AB} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$$

$$= (-0.540\mathbf{j}) + \mathbf{0} + 0.6\mathbf{k} \times [0.6\mathbf{k} \times (8\mathbf{j} + 6\mathbf{k})] + 2(0.6\mathbf{k}) \times (1.20\mathbf{j} - 1.60\mathbf{k}) + (-0.320\mathbf{j} - 0.240\mathbf{k})$$

$$= \{-1.44\mathbf{i} - 3.74\mathbf{j} - 0.240\mathbf{k}\}\text{m/s}^2 \qquad \textbf{Ans.}$$

20–46. The crane rotates about the z axis with a rate of $\omega_1 = 0.6$ rad/s, which is increasing at $\dot{\omega}_1 = 0.6$ rad/s^2. Also, the boom rotates downward at $\omega_2 = 0.2$ rad/s, which is increasing at $\dot{\omega}_2 = 0.3$ rad/s^2. Determine the velocity and acceleration of point A located at the end of the boom at the instant shown.



Coordinate Axes: The rotating x, y, z frame and fixed and fixed X, Y, Z frame are set with the origin at point B and O respectively.

Motion of B: Here, $\mathbf{r}_B$ change direction with respect to X, Y, Z frame. The time derivatives of $\mathbf{r}_B$ can be found by seeting another set of coordinate axis x', y', z' coincident with X, Y, Z rotating at $\Omega' = \omega_1 = \{0.6\mathbf{k}\}$ rad/s and $\dot{\Omega} = \dot{\omega}_1 = \{0.6\mathbf{k}\}$ rad/s^2. Here, $\mathbf{r}_B = \{1.5\mathbf{j}\}$ m

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = (\dot{\mathbf{r}}_B)_{'y'z'} + \Omega' \times \mathbf{r}_B = \mathbf{0} + 0.6\mathbf{k} \times 1.5\mathbf{j} = \{-0.9\mathbf{i}\} \text{ m/s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{x'y'z'} + \Omega' \times (\dot{\mathbf{r}}_B)_{x'y'z'}\right] + \dot{\Omega}' \times \mathbf{r}_B + \Omega' \times \dot{\mathbf{r}}_B$$

$$= (\mathbf{0} + \mathbf{0}) + 0.6\mathbf{k} \times 1.5\mathbf{j} + 0.6\mathbf{k} \times (-0.9\mathbf{i}) = \{-0.9\mathbf{i} - 0.540\mathbf{j}\} \text{ m/s}^2$$

Motion of A with Respect to B: Let xyz axis rotate at $\Omega_{xyz} = \omega_2 = \{-0.2\mathbf{i}\}$ rad/s and $\Omega_{xyz} = \dot{\omega}_2 = \{-0.3\mathbf{i}\}$ rad/s^2. Here, $\mathbf{r}_{A/B} = \{8\mathbf{j} + 6\mathbf{k}\}$ m

$$(\mathbf{v}_{A/B})_{xyz} = \dot{\mathbf{r}}_{A/B} = (\dot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{A/B} = \mathbf{0} + (-0.2\mathbf{i}) \times (8\mathbf{j} + 6\mathbf{k}) = \{1.20\mathbf{j} - 1.60\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{A/B})_{xyz} = \ddot{\mathbf{r}}_{A/B} = \left[(\ddot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{A/B})_{xyz}\right] + \dot{\Omega}_{xyz} \times \mathbf{r}_{A/B} + \Omega_{xyz} \times \dot{\mathbf{r}}_{A/B}$$

$$= \mathbf{0} + \mathbf{0} + (-0.3\mathbf{i}) \times (8\mathbf{i} + 6\mathbf{k}) + (-0.2\mathbf{i}) \times (1.20\mathbf{j} - 1.60\mathbf{k})$$

$$= \{1.48\mathbf{j} - 2.64\mathbf{k}\} \text{ m/s}^2$$

Motion of Point A: Here, $\Omega = \omega_1 = \{0.6\mathbf{k}\}$ rad/s and $\dot{\Omega} = \dot{\omega}_1 = \{0.6\mathbf{k}\}$ rad/s^2. Applying Eqs. 20–11 and 20-12, we have

$$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz} = (-0.9\mathbf{i}) + 0.6\mathbf{k} \times (8\mathbf{j} + 6\mathbf{k}) + (1.20\mathbf{j} - 1.60\mathbf{k})$$

$$= \{-5.70\mathbf{i} + 1.20\mathbf{j} - 1.60\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans.}$$

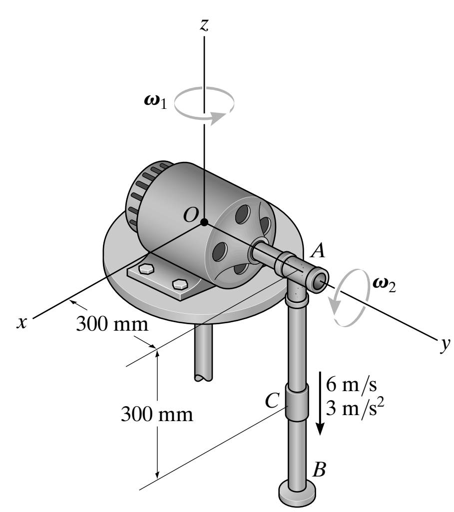
$$\mathbf{a}_A = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{A/B} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$$

$$= (-0.9\mathbf{i} - 0.540\mathbf{j}) + 0.6\mathbf{k} \times (8\mathbf{j} + 6\mathbf{k}) + 0.6\mathbf{k} \times [0.6\mathbf{k} \times (8\mathbf{j} + 6\mathbf{k})]$$

$$+ 2(0.6\mathbf{k}) \times (1.20\mathbf{j} - 1.60\mathbf{k}) + (1.48\mathbf{j} - 2.64\mathbf{k})$$

$$= \{-7.14\mathbf{i} - 1.94\mathbf{j} - 2.64\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

20–47. The motor rotates about the z axis with a constant angular velocity of $\omega_1 = 3$ rad/s. Simultaneously, shaft OA rotates with a constant angular velocity of $\omega_2 = 6$ rad/s. Also, collar C slides along rod AB with a velocity and acceleration of 6 m/s and 3 m/s^2. Determine the velocity and acceleration of collar C at the instant shown.



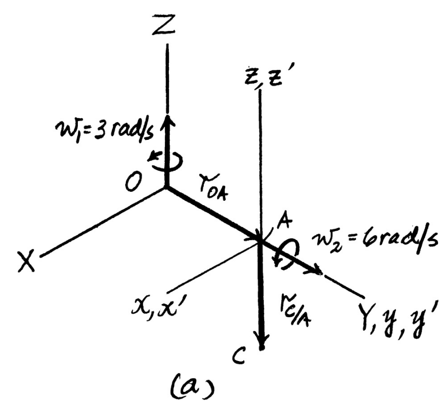
The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [3\mathbf{k}] \text{ rad/s} \qquad \dot{\Omega} = \dot{\omega}_1 = \mathbf{0}$$

Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (3\mathbf{k}) \times (0.3\mathbf{j}) = [-0.9\mathbf{i}] \text{ m/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$
$$= \mathbf{0} + (3\mathbf{k}) \times \left[(3\mathbf{k}) \times (0.3\mathbf{j})\right]$$
$$= [-2.7\mathbf{j}] \text{ m/s}^2$$



In order to determine the motion of the point C relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [6\mathbf{j}]$ rad/s, the direction of $(\mathbf{r}_{C/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{C/A})_{xyz}$,

$$(\mathbf{v}_{C/A})_{xyz} = (\dot{\mathbf{r}}_{C/A})_{xyz} = \left[(\dot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{C/A})_{xyz}\right]$$
$$= (-6\mathbf{k}) + 6\mathbf{j} \times (-0.3\mathbf{k})$$
$$= [-1.8\mathbf{i} - 6\mathbf{k}] \text{ m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = \mathbf{0}$. Taking the time derivative of $(\dot{\mathbf{r}}_{C/A})_{xyz}$,

$$(\mathbf{a}_{C/A})_{xyz} = (\ddot{\mathbf{r}}_{C/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{C/A})_{x'y'z'}\right] + \dot{\omega}_2 \times (\mathbf{r}_{C/A})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{C/A})_{xyz}$$
$$= [(-3\mathbf{k}) + 6\mathbf{j} \times (-6\mathbf{k})] + \mathbf{0} + [6\mathbf{j} \times (-18\mathbf{j} - 6\mathbf{k})]$$
$$= [-72\mathbf{i} + 7.8\mathbf{k}] \text{ m/s}^2$$

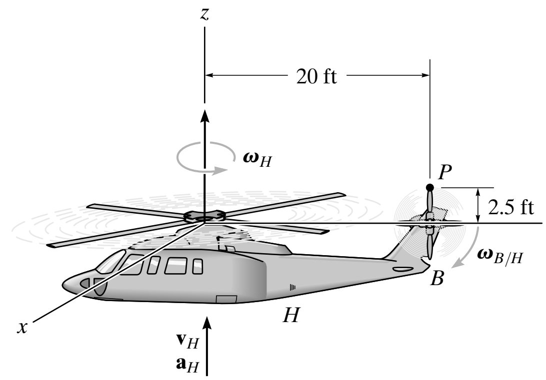
Thus,

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$
$$= (-0.9\mathbf{i}) + 3\mathbf{k} \times (-0.3\mathbf{k}) + (-1.8\mathbf{i} - 6\mathbf{k})$$
$$= [-2.7\mathbf{i} - 6\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$
$$= (-2.7\mathbf{j}) + \mathbf{0} + 3\mathbf{k} \times [(3\mathbf{k}) \times (-0.3\mathbf{k})] + 2(3\mathbf{k}) \times (-1.8\mathbf{i} - 6\mathbf{k}) + (-72\mathbf{i} + 7.8\mathbf{k})$$
$$= [-72\mathbf{i} - 13.5\mathbf{j} + 7.8\mathbf{k}] \text{ m/s}^2 \qquad \textbf{Ans.}$$

***20–48.** At the instant shown, the helicopter is moving upwards with a velocity $v_H = 4$ ft/s and has an acceleration $a_H = 2$ ft/s^2. At the same instant the frame H, *not* the horizontal blade, rotates about a vertical axis with a constant angular velocity $\omega_H = 0.9$ rad/s. If the tail blade B rotates with a constant angular velocity $\omega_{B/H} = 180$ rad/s, measured relative to H, determine the velocity and acceleration of point P, located on the end of the blade, at the instant the blade is in the vertical position.



Relative to XYZ, let xyz have

$\Omega = \{0.9\mathbf{k}\}$ rad/s $\quad \dot{\omega} = \mathbf{0}$ (Ω does note change direction relative to XYZ.)

$\mathbf{r}_B = \{20\mathbf{j}\}$ ft ($\mathbf{r}_B$ changes direction relative to XYZ.)

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = (\dot{\mathbf{r}}_B)_{xyz} + \Omega \times \mathbf{r}_B = 4\mathbf{k} + (0.9\mathbf{k}) \times (20\mathbf{j}) = \{-18\mathbf{i} + 4\mathbf{k}\} \text{ ft/s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{xyz} + \Omega \times \left(\dot{\mathbf{r}}_B\right)_{xyz}\right] + \dot{\Omega} \times \mathbf{r}_B + \Omega \times \dot{\mathbf{r}}_B$$
$$= \left[2\mathbf{k} + \mathbf{0}\right] + \mathbf{0} + \left[(0.9\mathbf{k}) \times (-18\mathbf{i} + 4\mathbf{k})\right]$$
$$= \{-16.2\mathbf{j} + 2\mathbf{k}\} \text{ ft/s}^2$$

Relative to xyz, let $x'y'z'$ have

$\Omega_{xyz} = \{-180\mathbf{i}\}$ rad/s $\quad \dot{\Omega}_{xyz} = \mathbf{0}$ (Ω_{xyz} does not change direction relative to xyz.)

$(\mathbf{r}_{P/B})_{xyz} = \{2.5\mathbf{k}\}$ ft ($(\mathbf{r}_{P/B})_{xyz}$ change direction relative to xyz.)

$$(\mathbf{v}_{P/B})_{xyz} = (\dot{\mathbf{r}}_{P/B})_{xyz} = (\dot{\mathbf{r}}_{P/B})_{x'y'z'} + \Omega_{xyz} \times (\mathbf{r}_{P/B})_{xyz} = \mathbf{0} + (-180\mathbf{i}) \times (2.5\mathbf{k}) = \{450\mathbf{j}\} \text{ ft/s}$$

$$(\mathbf{a}_{P/B})_{xyz} = (\ddot{\mathbf{r}}_{P/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{P/B})_{x'y'z'} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{P/B})_{x'y'z'}\right] + \dot{\Omega}_{xyz} \times (\mathbf{r}_{P/B})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{P/B})_{xyz}$$

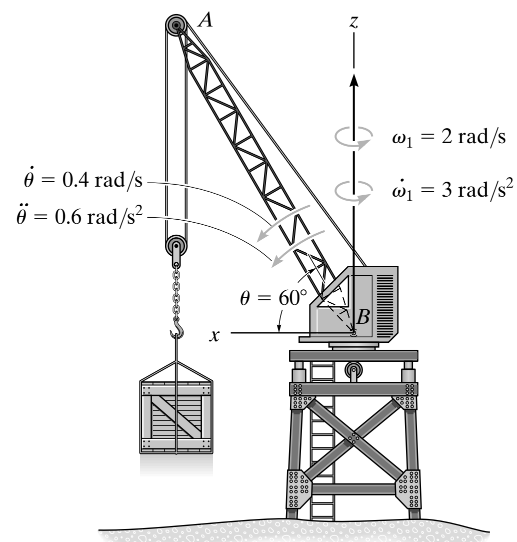
$$(\mathbf{a}_{P/B})_{xyz} = \left[\mathbf{0} + \mathbf{0}\right] + \mathbf{0} + (-180\mathbf{i}) \times (450\mathbf{j}) = \{-81\,000\mathbf{k}\} \text{ ft/s}^2$$

Thus,

$$\mathbf{v}_F = \mathbf{v}_B + \Omega \times \mathbf{r}_{P/B} + (\mathbf{v}_{P/B})_{xyz}$$
$$= (-18\mathbf{i} + 4\mathbf{k}) + \left[(0.9\mathbf{k}) \times (2.5\mathbf{k})\right] + (450\mathbf{j})$$
$$= \{-18\mathbf{i} + 450\mathbf{j} + 4\mathbf{k}\} \text{ft/s}$$ **Ans.**

$$\mathbf{a}_P = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{P/B} + \Omega \times (\Omega \times \mathbf{r}_{P/B}) + 2\Omega \times (\mathbf{v}_{P/B})_{xyz} + (\mathbf{a}_{P/B})_{xyz}$$
$$= (-16.2\mathbf{j} + 2\mathbf{k}) + \mathbf{0} + (0.9\mathbf{k}) \times \left[(0.9\mathbf{k}) \times (2.5\mathbf{k})\right] + \left[2(0.9\mathbf{k}) \times (450\mathbf{j})\right] + (-81000\mathbf{k})$$
$$= \{-810\mathbf{i} - 16.2\mathbf{j} - 81\,000\mathbf{k}\} \text{ ft/s}^2$$ **Ans.**

•20–49. At a given instant the boom AB of the tower crane rotates about the z axis with the motion shown. At this same instant, $\theta = 60°$ and the boom is rotating downward such that $\dot{\theta} = 0.4$ rad/s and $\ddot{\theta} = 0.6$ rad/s^2. Determine the velocity and acceleration of the end of the boom A at this instant. The boom has a length of $l_{AB} = 40$ m.



Coordinate Axis: The rotating x, y, z frame is set to be coincident with the fixed X, Y, Z frame with origin at point B.

Motion of B: Since point B does not move, then

$$\mathbf{a}_B = \mathbf{v}_B = \mathbf{0}$$

Motion of A with Respect to B: Let xyz axis rotate at $\Omega_{xyz} = \dot{\theta} = \{0.4\mathbf{j}\}$ rad/s and $\dot{\Omega}_{xyz} = \ddot{\theta} = \{0.6\mathbf{j}\}$ rad/s^2. Here.
$\mathbf{r}_{A/B} = \{40 \cos 60°\mathbf{i} + 40 \sin 60°\mathbf{k}\}$ m $= \{20.0\mathbf{i} + 34.64\mathbf{k}\}$ m.

$$(\mathbf{v}_{A/B})_{xyz} = \dot{\mathbf{r}}_{A/B} = (\dot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{A/B} = \mathbf{0} + 0.4\mathbf{j} \times (20.0\mathbf{i} + 34.640\mathbf{k}) = \{13.86\mathbf{i} - 8.00\mathbf{k}\}\text{ m/s}$$

$$(\mathbf{a}_{A/B})_{xyz} = \ddot{\mathbf{r}}_{A/B} = \left[(\ddot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{A/B})_{xyz}\right] + \dot{\Omega}_{xyz} \times \mathbf{r}_{A/B} + \Omega_{xyz} \times \dot{\mathbf{r}}_{A/B}$$

$$= \mathbf{0} + \mathbf{0} + 0.6\mathbf{j} \times (20.0\mathbf{i} + 34.64\mathbf{k}) + 0.4\mathbf{j} \times (13.86\mathbf{i} - 8.00\mathbf{k})$$

$$= \{17.58\mathbf{i} - 17.54\mathbf{k}\}\text{ m/s}^2$$

Motion of Point A: Here, $\Omega = \omega_1 = \{2\mathbf{k}\}$ rad/s and $\dot{\Omega} = \dot{\omega}_1 = \{3\mathbf{k}\}$ rad/s^2. Applying Eqs. 20–11 and 20-12. we have

$$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz} = \mathbf{0} + 2\mathbf{k} \times (20.0\mathbf{i} + 34.64\mathbf{k}) + (13.86\mathbf{i} - 8.00\mathbf{k})$$

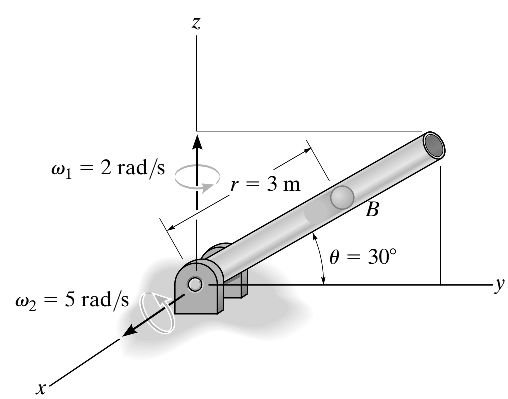
$$= \{13.9\mathbf{i} - 40.0\mathbf{j} - 8.00\mathbf{k}\}\text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{A/B} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$$

$$= \mathbf{0} + 3\mathbf{k} \times (20.0\mathbf{i} + 34.64\mathbf{k}) + 2\mathbf{k} \times [2\mathbf{k} \times (20.0\mathbf{i} + 34.64\mathbf{k})] + 2(2\mathbf{k}) \times (13.86\mathbf{i} - 8.00\mathbf{k}) + 17.58\mathbf{i} - 17.54\mathbf{k}$$

$$= \{-62.4\mathbf{i} + 115\mathbf{j} - 17.5\mathbf{k}\}\text{ m/s}^2 \qquad \textbf{Ans.}$$

20–50. At the instant shown, the tube rotates about the z axis with a constant angular velocity $\omega_1 = 2$ rad/s, while at the same instant the tube rotates upward at a constant rate $\omega_2 = 5$ rad/s. If the ball B is blown through the tube at a rate $\dot{r} = 7$ m/s, which is increasing at $\ddot{r} = 2\text{ m/s}^2$, determine the velocity and acceleration of the ball at the instant shown. Neglect the size of the ball.



Coordinate Axis: The rotating x, y, z frame is set to be coincident with the fixed X, Y, Z frame with origin at point A.

Motion of A: Since point A does not move, then

$$\mathbf{a}_A = \mathbf{v}_A = \mathbf{0}$$

Motion of B with Respect to A: Let xyz axis rotate at $\Omega_{xyz} = \omega_2 = \{5\mathbf{i}\}$ rad/s and $\dot{\Omega}_{xyz} = \dot{\omega}_2 = \mathbf{0}$. Here, $\mathbf{r}_{B/A} = \{3\cos 30°\mathbf{j} + 3\sin 30°\mathbf{k}\}\text{ m} = \{2.5981\mathbf{j} + 1.50\mathbf{k}\}$ m, $(\dot{\mathbf{r}}_{B/A})_{xyz} = \{7\cos 30°\mathbf{j} + 7\sin 30°\mathbf{k}\}\text{ m/s} = \{6.0621\mathbf{j} + 3.50\mathbf{k}\}$ m/s and $(\ddot{\mathbf{r}}_{B/A})_{xyz} = \{2\cos 30\mathbf{j} + 2\sin 30°\mathbf{k}\}\text{ m/s}^2 = \{1.7321\mathbf{j} + 1\mathbf{k}\}\text{ m/s}^2$.

$$(\mathbf{v}_{B/A})_{xyz} = \dot{\mathbf{r}}_{B/A} = (\dot{\mathbf{r}}_{B/A})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{B/A}$$
$$= 6.0621\mathbf{j} + 3.50\mathbf{k} + 5\mathbf{i} \times (2.5981\mathbf{j} + 1.50\mathbf{k}) = \{-1.4378\mathbf{j} + 16.4903\mathbf{k}\}\text{ m/s}$$

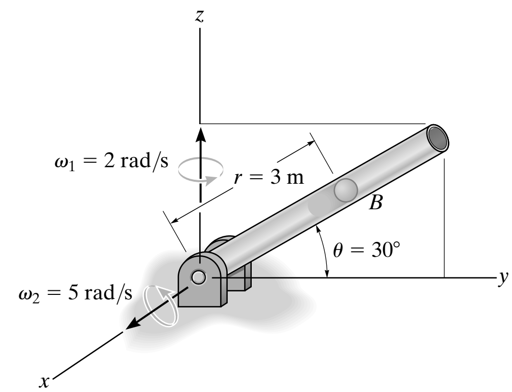
$$(\mathbf{a}_{B/A})_{xyz} = \ddot{\mathbf{r}}_{B/A} = \left[(\ddot{\mathbf{r}}_{B/A})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{B/A})_{xyz}\right] + \dot{\Omega}_{xyz} \times \mathbf{r}_{B/A} + \Omega_{xyz} \times \dot{\mathbf{r}}_{B/A}$$
$$= 1.7321\mathbf{j} + 1\mathbf{k} + 5\mathbf{i} \times (6.0621\mathbf{j} + 3.50\mathbf{k}) + \mathbf{0} + 5\mathbf{i} \times (-1.4378\mathbf{j} + 16.4903\mathbf{k})$$
$$= \{-98.2199\mathbf{j} - 24.1218\mathbf{k}\}\text{ m/s}^2$$

Motion of Point B: Here, $\Omega = \omega_1 = \{2\mathbf{k}\}$ rad/s and $\dot{\Omega} = \dot{\omega}_1 = \mathbf{0}$. Applying Eqs. 20–11 and 20–12, we have

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz} = \mathbf{0} + 2\mathbf{k} \times (2.5981\mathbf{j} + 1.50\mathbf{k}) + (-1.4378\mathbf{j} + 16.4903\mathbf{k})$$
$$= \{-5.20\mathbf{i} - 1.44\mathbf{j} + 16.5\mathbf{k}\}\text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_B = \mathbf{a}_A + \;\; + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$
$$= \mathbf{0} + \mathbf{0} + 2\mathbf{k} \times [2\mathbf{k} \times (2.5981\mathbf{j} + 1.50\mathbf{k})] + 2(2\mathbf{k}) \times (-1.4378\mathbf{j} + 16.4903\mathbf{k}) + (-98.2199\mathbf{j} + 24.1218\mathbf{k})$$
$$= \{5.75\mathbf{i} - 109\mathbf{j} + 24.1\mathbf{k}\}\text{ m/s}^2 \qquad \textbf{Ans.}$$

20–51. At the instant shown, the tube rotates about the z axis with a constant angular velocity $\omega_1 = 2$ rad/s, while at the same instant the tube rotates upward at a constant rate $\omega_2 = 5$ rad/s. If the ball B is blown through the tube at a constant rate $\dot{r} = 7$ m/s, determine the velocity and acceleration of the ball at the instant shown. Neglect the size of the ball.



Coordinate Axis: The rotating x, y, z frame is set to be coincident with the fixed X, Y, Z frame with origin at point A.

Motion of A: Since point A does not move, then

$$\mathbf{a}_A = \mathbf{v}_A = \mathbf{0}$$

Motion of B with Respect to A: Let xyz axis rotate at $\Omega_{xyz} = \omega_2 = \{5\mathbf{i}\}$ rad/s and $\dot{\Omega}_{xyz} = \dot{\omega}_2 = \mathbf{0}$. Here, $\mathbf{r}_{B/A} = \{3\cos 30°\mathbf{j} + 3\sin 30°\mathbf{k}\}$ m $= \{2.5981\mathbf{j} + 1.50\mathbf{k}\}$ m and $(\dot{\mathbf{r}}_{B/A})_{xyz} = \{7\cos 30\mathbf{j} + 7\sin 30°\mathbf{k}\}$ m/s $= \{6.0621\mathbf{j} + 3.50\mathbf{k}\}$ m/s.

$$(\mathbf{v}_{B/A})_{xyz} = \dot{\mathbf{r}}_{B/A} = (\dot{\mathbf{r}}_{B/A})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{B/A}$$

$$= 6.0621\mathbf{j} + 3.50\mathbf{k} + 5\mathbf{i} \times (2.5981\mathbf{j} + 1.50\mathbf{k}) = \{-1.4378\mathbf{j} + 16.4903\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{B/A})_{xyz} = \ddot{\mathbf{r}}_{B/A} = \left[(\ddot{\mathbf{r}}_{B/A})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{B/A})_{xyz}\right] + \dot{\Omega}_{xyz} \times \mathbf{r}_{B/A} + \Omega_{xyz} \times \dot{\mathbf{r}}_{B/A}$$

$$= \mathbf{0} + 5\mathbf{i} \times (6.0621\mathbf{j} + 3.50\mathbf{k}) + \mathbf{0} + 5\mathbf{i} \times (-1.4378\mathbf{j} + 16.4903\mathbf{k})$$

$$= \{-99.9519\mathbf{j} + 23.1218\mathbf{k}\} \text{ m/s}^2$$

Motion of Point B: Here, $\Omega = \omega_1 = \{2\mathbf{k}\}$ rad/s and $\dot{\Omega} = \dot{\omega}_1 = \mathbf{0}$. Applying Eqs. 20–11 and 20–12, we have

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz} = \mathbf{0} + 2\mathbf{k} \times (2.5981\mathbf{j} + 1.50\mathbf{k}) + (-1.4378\mathbf{j} + 16.4903\mathbf{k})$$

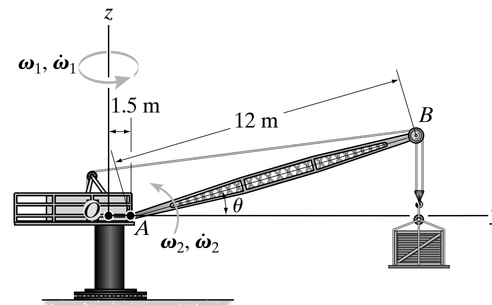
$$= \{-5.20\mathbf{i} - 1.44\mathbf{j} + 16.5\mathbf{k}\} \text{m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_B = \mathbf{a}_A + \ + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + 2\mathbf{k} \times [2\mathbf{k} \times (2.5981\mathbf{j} + 1.50\mathbf{k})] + 2(2\mathbf{k}) \times (-1.4378\mathbf{j} + 16.4903\mathbf{k}) + (-99.9519\mathbf{j} + 23.1218\mathbf{k})$$

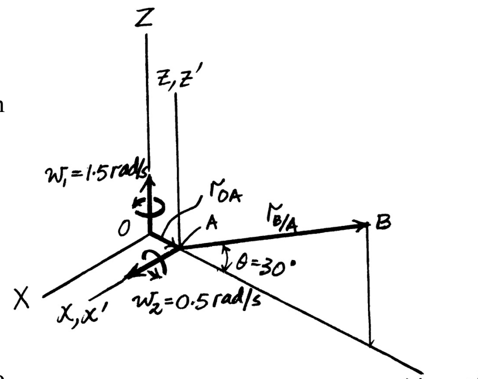
$$= \{5.75\mathbf{i} - 110\mathbf{j} + 23.1\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

***20–52.** At the instant $\theta = 30°$, the frame of the crane and the boom AB rotate with a constant angular velocity of $\omega_1 = 1.5$ rad/s and $\omega_2 = 0.5$ rad/s, respectively. Determine the velocity and acceleration of point B at this instant.



The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. The angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [1.5\mathbf{k}]\ \text{rad/s} \qquad \dot{\Omega} = \dot{\omega}_1 = \mathbf{0}$$



(a)

Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (1.5\mathbf{k}) \times (1.5\mathbf{j}) = [-2.25\mathbf{i}]\ \text{m/s}$$

$$\begin{aligned}\mathbf{a}_A &= \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})\\ &= \mathbf{0} + (1.5\mathbf{k}) \times [(1.5\mathbf{k}) \times (1.5\mathbf{j})]\\ &= [-3.375\mathbf{j}]\ \text{m/s}^2\end{aligned}$$

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [0.5\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{B/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{B/A})_{xyz}$,

$$\begin{aligned}(\mathbf{v}_{B/A})_{xyz} &= (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{B/A})_{xyz}\right]\\ &= \mathbf{0} + (0.5\mathbf{i}) \times (12\cos 30°\,\mathbf{j} + 12\sin 30°\mathbf{k})\\ &= [-3\mathbf{j} + 5.196\mathbf{k}]\ \text{m/s}\end{aligned}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = \mathbf{0}$. Taking the time derivative of $(\dot{\mathbf{r}}_{A/B})_{xyz}$,

$$\begin{aligned}(\mathbf{a}_{A/B})_{xyz} &= (\ddot{\mathbf{r}}_{A/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{A/B})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{A/B})_{x'y'z'}\right] + \dot{\omega}_2 \times (\mathbf{r}_{A/B})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{A/B})_{xyz}\\ &= [0 + 0] + 0 + (0.5\mathbf{i}) \times (-3\mathbf{j} + 5.196\mathbf{k})\\ &= [-2.598\mathbf{j} - 1.5\mathbf{k}]\ \text{m/s}^2\end{aligned}$$

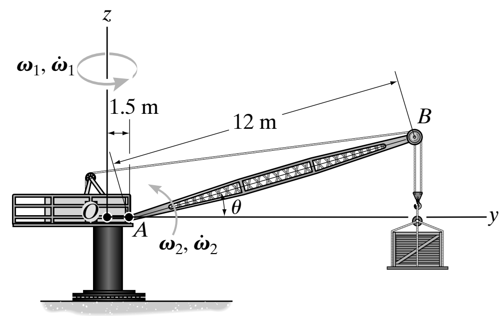
Thus,

$$\begin{aligned}\mathbf{v}_B &= \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}\\ &= (-2.25\mathbf{i}) + 1.5\mathbf{k} \times (12\cos 30°\,\mathbf{j} + 12\sin 30°\,\mathbf{k}) + (-3\mathbf{j} + 5.196\mathbf{k})\\ &= [-17.8\mathbf{i} - 3\mathbf{j} + 5.20\mathbf{k}]\ \text{m/s} \qquad \textbf{Ans.}\end{aligned}$$

and

$$\begin{aligned}\mathbf{a}_B &= \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{AB})_{xyz} + (\mathbf{a}_{AB})_{xyz}\\ &= (-3.375\mathbf{j}) + 0 + 1.5\mathbf{k} \times \left[(1.5\mathbf{k}) \times (12\cos 30°\,\mathbf{j} + 12\sin 30°\,\mathbf{k})\right] + 2(1.5\mathbf{k}) \times (-3\mathbf{j} + 5.196\mathbf{k}) + (-2.598\mathbf{j} - 1.5\mathbf{k})\\ &= [9\mathbf{i} - 29.4\mathbf{j} - 1.5\mathbf{k}]\ \text{m/s}^2 \qquad \textbf{Ans.}\end{aligned}$$

•20–53. At the instant $\theta = 30°$, the frame of the crane is rotating with an angular velocity of $\omega_1 = 1.5$ rad/s and angular acceleration of $\dot{\omega}_1 = 0.5$ rad/s^2, while the boom AB rotates with an angular velocity of $\omega_2 = 0.5$ rad/s and angular acceleration of $\dot{\omega}_2 = 0.25$ rad/s^2. Determine the velocity and acceleration of point B at this instant.



The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [1.5\mathbf{k}] \text{ rad/s} \qquad \dot{\Omega} = [0.5\mathbf{k}] \text{ rad/s}^2$$

Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (1.5\mathbf{k}) \times (1.5\mathbf{j}) = [-2.25\mathbf{i}] \text{ m/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$

$$= (0.5\mathbf{k}) \times (1.5\mathbf{j}) + (1.5\mathbf{k}) \times \left[(1.5\mathbf{k}) \times (1.5\mathbf{j})\right]$$

$$= [-0.75\mathbf{i} - 3.375\mathbf{j}] \text{ m/s}^2$$

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [0.5\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{B/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{B/A})_{xyz}$,

$$(\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{B/A})_{xyz}\right]$$

$$= \mathbf{0} + (0.5\mathbf{i}) \times (12 \cos 30° \,\mathbf{j} + 12 \sin 30° \,\mathbf{k})$$

$$= [-3\mathbf{j} + 5.196\mathbf{k}] \text{ m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = [0.25\mathbf{i}]$ m/s^2. Taking the time derivative of $(\dot{\mathbf{r}}_{B/A})_{xyz}$,

•20–53. Continued

$$(\mathbf{a}_{B/A}) = (\ddot{\mathbf{r}}_{B/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{x'y'z'}\right] + \dot{\Omega}_2 \times (\mathbf{r}_{B/A})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{xyz}$$

$$= [0 + 0] + (0.25\mathbf{i}) \times (12 \cos 30° \,\mathbf{j} + 12 \sin 30° \mathbf{k}) + 0.5\mathbf{i} \times (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-4.098\mathbf{j} + 1.098\mathbf{k}] \text{ m/s}^2$$

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= (-2.25\mathbf{i}) + 1.5\mathbf{k} \times (12 \cos 30° \,\mathbf{j} + 12 \sin 30° \mathbf{k}) + (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-17.8\mathbf{i} - 3\mathbf{j} + 5.20\mathbf{k}] \text{ m/s}$$ **Ans.**

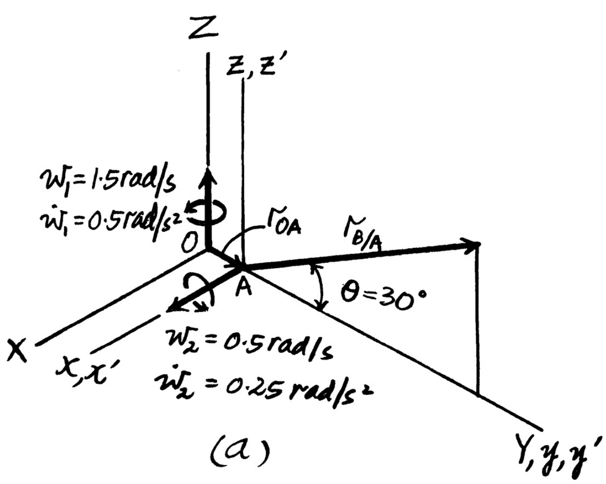
and

$$\mathbf{a}_B = \mathbf{a}_A = \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= (-0.75\mathbf{i} - 3.375\mathbf{j}) + 0.5\mathbf{k} \times (12 \cos 30° \mathbf{j} + 12 \sin 30° \,\mathbf{k}) + (1.5\mathbf{k}) \times \left[(1.5\mathbf{k}) \times (12 \cos 30° \,\mathbf{j} + 12 \sin 30° \,\mathbf{k})\right]$$

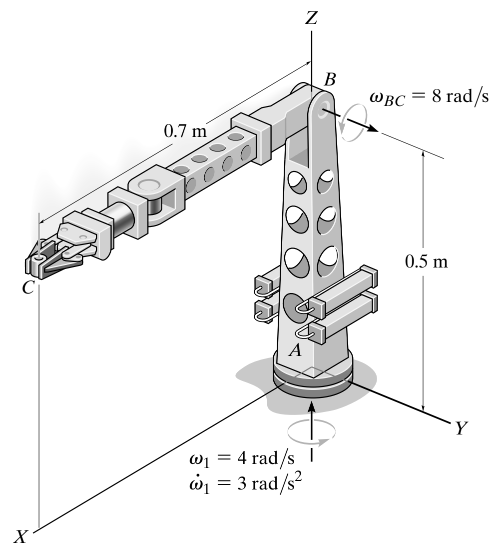
$$+2(1.5\mathbf{k}) \times (-3\mathbf{j} + 5.196\mathbf{k}) + (-4.098\mathbf{j} + 1.098\mathbf{k})$$

$$= [3.05\mathbf{i} - 30.9\mathbf{j} + 1.10\mathbf{k}] \text{ m/s}^2$$ **Ans.**



(a)

20–54. At the instant shown, the base of the robotic arm rotates about the z axis with an angular velocity of $\omega_1 = 4$ rad/s, which is increasing at $\dot{\omega}_1 = 3$ rad/s^2. Also, the boom BC rotates at a constant rate of $\omega_{BC} = 8$ rad/s. Determine the velocity and acceleration of the part C held in its grip at this instant.



Relative to XYZ, let xyz have origin at B and have

$\Omega = \{4\mathbf{k}\}$ rad/s, $\quad \Omega = \{3\mathbf{k}\}$ rad/s^2 (Ω does not change direction relative to XYZ.)

$\mathbf{r}_B = \{0.5\mathbf{k}\}$ m ($\mathbf{r}_B$ does not change direction relative to XYZ.)

$\mathbf{v}_B = 0$

$\mathbf{a}_B = 0$

Relative to xyz, let coincident $x'y'z'$ have origin at B and have

$\Omega_{xyz} = \{8\mathbf{j}\}$ rad/s, $\quad \dot{\Omega}_{xyz} = \mathbf{0}$ (Ω_{xyz} does not change direction relative to xyz.)

$(\mathbf{r}_{C/B})_{xyz} = \{0.7\mathbf{i}\}$ m ($(\mathbf{r}_{C/B})_{xyz}$ changes direction relative to xyz.)

$$(\mathbf{v}_{C/B})_{xyz} = \left(\mathbf{r}_{C/B}\right)_{xyz} = \left(\dot{\mathbf{r}}_{C/B}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz} = \mathbf{0} + (8\mathbf{j}) \times (0.7\mathbf{i}) = \{-5.6\mathbf{k}\}\text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \left(\mathbf{r}_{C/B}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{C/B}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/B}\right)_{x'y'z'}\right] + \dot{\Omega}_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/B}\right)_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + \mathbf{0} + (8\mathbf{j}) \times (-5.6\mathbf{k}) = \{-44.8\mathbf{i}\}\text{ m/s}^2$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz} = \mathbf{0} + (4\mathbf{k}) \times (0.7\mathbf{i}) + (-5.6\mathbf{k})$$

$$= \{2.80\mathbf{j} - 5.60\mathbf{k}\}\text{ m/s} \qquad \textbf{Ans.}$$

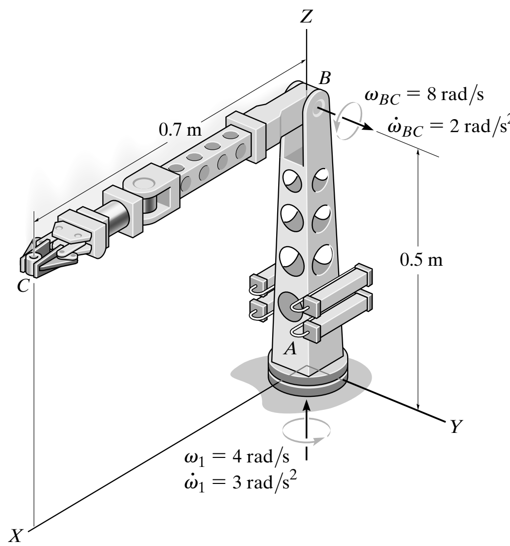
$$\mathbf{a}_C = \mathbf{a}_B + \Omega \times \mathbf{r}_{C/B} + \Omega \times \left(\Omega \times \mathbf{r}_{C/B}\right) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= \mathbf{0} + (3\mathbf{k}) \times (0.7\mathbf{i}) + (4\mathbf{k}) \times \left[(4\mathbf{k}) \times (0.7\mathbf{i})\right]$$

$$= 2(4\mathbf{k}) \times (-5.6\mathbf{k}) - 44.8\mathbf{i}$$

$$= \{-56\mathbf{i} + 2.1\mathbf{j}\}\text{ m/s}^2 \qquad \textbf{Ans.}$$

20–55. At the instant shown, the base of the robotic arm rotates about the z axis with an angular velocity of $\omega_1 = 4$ rad/s, which is increasing at $\dot{\omega}_1 = 3$ rad/s^2. Also, the boom BC rotates at $\omega_{BC} = 8$ rad/s, which is increasing at $\dot{\omega}_{BC} = 2$ rad/s^2. Determine the velocity and acceleration of the part C held in its grip at this instant.



Relative to XYZ, let xyz with origin at B have

$\Omega = \{4\mathbf{k}\}$ rad/s, $\quad \Omega = \{3\mathbf{k}\}$ rad/s^2 (Ω does not change direction relative to XYZ.)

$\mathbf{r}_B = \{0.5\mathbf{k}\}$ m (r_B does not change direction relative to XYZ.)

$\mathbf{v}_B = \mathbf{0}$

$\mathbf{a}_B = \mathbf{0}$

Relative to xyz, let coincident $x'y'z'$ have origin at B and have

$\Omega_{xyz} = \{8\mathbf{j}\}$ rad/s, $\quad \Omega_{xyz} = \{2\mathbf{j}\}$ rad/s^2 (Ω does not change direction relative to xyz.)

$(\mathbf{r}_{C/B})_{xyz} = \{0.7\mathbf{i}\}$ m (Ω does not change direction relative to xyz.)

$$(\mathbf{v}_{C/B})_{xyz} = \left(\mathbf{r}_{C/B}\right)_{xyz} = \left(\mathbf{r}_{C/B}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz} = \mathbf{0} + (8\mathbf{j}) \times (0.7\mathbf{i}) = \{-5.6\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \left(\mathbf{r}_{C/B}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{C/B}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{C/B}\right)_{x'y'z'}\right] + \dot{\Omega}_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz} + \Omega_{xyz} \times \left(\mathbf{r}_{C/B}\right)_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + (2\mathbf{j}) \times (0.7\mathbf{i}) + (8\mathbf{j}) \times (-5.6\mathbf{k}) = \{-44.8\mathbf{i} - 1.40\mathbf{k}\} \text{ m/s}^2$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz} = \mathbf{0} + (4\mathbf{k}) \times (0.7\mathbf{i}) + (-5.6\mathbf{k})$$

$$= \{2.80\mathbf{j} - 5.60\mathbf{k}\} \text{ m/s}$$ **Ans.**

$$\mathbf{a}_C = \mathbf{a}_B + \Omega \times \mathbf{r}_{C/B} + \Omega \times \left(\Omega \times \mathbf{r}_{C/B}\right) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= \mathbf{0} + (3\mathbf{k}) \times (0.7\mathbf{i}) + (4\mathbf{k}) \times \left[(4\mathbf{k}) \times (0.7\mathbf{i})\right]$$

$$= 2(4\mathbf{k}) \times (-5.6\mathbf{k}) - 44.8\mathbf{i} - 1.40\mathbf{k}$$

$$= \{-56\mathbf{i} + 2.1\mathbf{j} - 1.40\mathbf{k}\} \text{ m/s}^2$$ **Ans.**