•19–1.   The rigid body (slab) has a mass m and rotates with an angular velocity $\boldsymbol{\omega}$ about an axis passing through the fixed point O. Show that the momenta of all the particles composing the body can be represented by a single vector having a magnitude mv_G and acting through point P, called the *center of percussion,* which lies at a distance $r_{P/G} = k_G^2/r_{G/O}$ from the mass center G. Here k_G is the radius of gyration of the body, computed about an axis perpendicular to the plane of motion and passing through G.
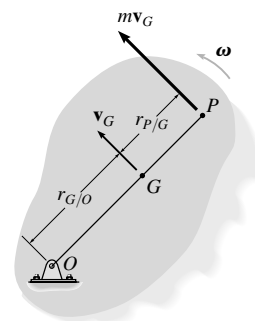
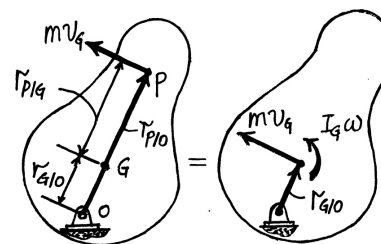$$H_O = (r_{G/O} + r_{P/G})\, mv_G = r_{G/O}\,(mv_G) + I_G\omega, \qquad \text{where} \quad I_G = mk_G^2$$

$$r_{G/O}\,(mv_G) + r_{P/G}\,(mv_G) = r_{G/O}\,(mv_G) + (mk_G^2)\,\omega$$

$$r_{P/G} = \frac{k_G^2}{v_G/\omega}$$

However, $v_G = \omega r_{G/O}$ or $r_{G/O} = \dfrac{v_G}{\omega}$

$$r_{P/G} = \frac{k_G^2}{r_{G/O}} \qquad\qquad \textbf{Q.E.D.}$$

19–2.   At a given instant, the body has a linear momentum $\mathbf{L} = m\mathbf{v}_G$ and an angular momentum $\mathbf{H}_G = I_G\boldsymbol{\omega}$ computed about its mass center. Show that the angular momentum of the body computed about the instantaneous center of zero velocity IC can be expressed as $\mathbf{H}_{IC} = I_{IC}\boldsymbol{\omega}$, where I_{IC} represents the body's moment of inertia computed about the instantaneous axis of zero velocity. As shown, the IC is located at a distance $r_{G/IC}$ away from the mass center G.
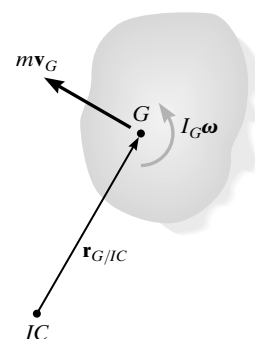
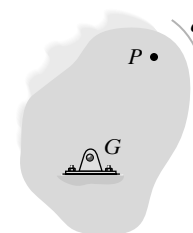$$H_{IC} = r_{G/IC}\,(mv_G) + I_G\,\omega, \qquad \text{where} \quad v_G = \omega r_{G/IC}$$

$$= r_{G/IC}\,(m\omega r_{G/IC}) + I_G\,\omega$$

$$= (I_G + mr_{G/IC}^2)\,\omega$$

$$= I_{IC}\,\omega \qquad\qquad \textbf{Q.E.D.}$$

19–3.   Show that if a slab is rotating about a fixed axis perpendicular to the slab and passing through its mass center G, the angular momentum is the same when computed about any other point P.


Since $v_G = 0$, the linear momentum $L = mv_G = 0$. Hence the angular momentum about any point P is

$$H_P = I_G\,\omega$$

Since ω is a free vector, so is $\mathbf{H}_P$.   **Q.E.D.**

*19–4. The pilot of a crippled jet was able to control his plane by throttling the two engines. If the plane has a weight of 17 000 lb and a radius of gyration of $k_G = 4.7$ ft about the mass center G, determine the angular velocity of the plane and the velocity of its mass center G in $t = 5$ s if the thrust in each engine is altered to $T_1 = 5000$ lb and $T_2 = 800$ lb as shown. Originally the plane is flying straight at 1200 ft/s. Neglect the effects of drag and the loss of fuel.
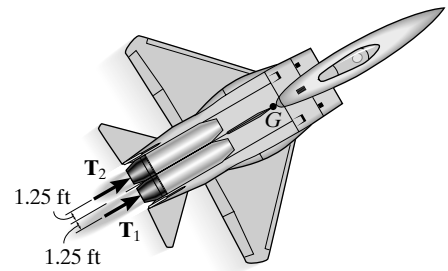
$$(\zeta+) \qquad (H_G)_1 + \Sigma \int M_G \, dt = (H_G)_2$$

$$0 + 5000(5)(1.25) - 800(5)(1.25) = \left[\left(\frac{17\,000}{32.2}\right)(4.7)^2\right]\omega$$
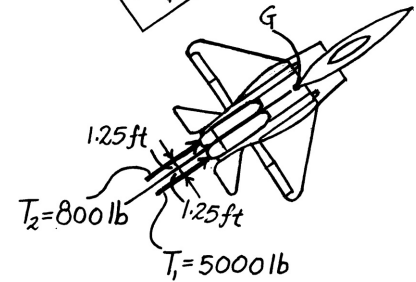
$$\omega = 2.25 \text{ rad/s} \qquad \textbf{Ans.}$$

$$\left(\overset{+}{\rightarrow}\right) \qquad m(v_{Gx})_1 + \Sigma \int F_x \, dt = m(v_{Gx})_2$$

$$\left(\frac{17\,000}{32.2}\right)(1200) + 5800(5) = \left(\frac{17\,000}{32.2}\right)(v_G)_2$$

$$(v_G)_2 = 1.25\left(10^3\right) \text{ ft/s} \qquad \textbf{Ans.}$$

•19–5. The assembly weighs 10 lb and has a radius of gyration $k_G = 0.6$ ft about its center of mass G. The kinetic energy of the assembly is 31 ft·lb when it is in the position shown. If it rolls counterclockwise on the surface without slipping, determine its linear momentum at this instant.
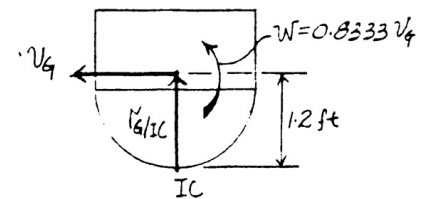
Kinetic Energy: Since the assembly rolls without slipping, then $\omega = \dfrac{v_G}{r_{G/IC}} = \dfrac{v_G}{1.2} = 0.8333 v_G$.
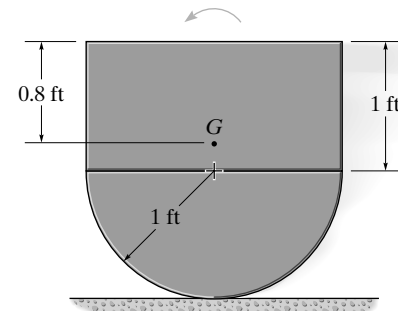
$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

$$31 = \frac{1}{2}\left(\frac{10}{32.2}\right) v_G^2 + \frac{1}{2}\left[\frac{10}{32.2}(0.6^2)\right](0.8333 v_G)^2$$

$$v_G = 12.64 \text{ ft/s}$$

Linear Momentum: Applying Eq. 19–7, we have

$$L = m v_G = \frac{10}{32.2}(12.64) = 3.92 \text{ slug·ft/s} \qquad \textbf{Ans.}$$

19–6. The impact wrench consists of a slender 1-kg rod AB which is 580 mm long, and cylindrical end weights at A and B that each have a diameter of 20 mm and a mass of 1 kg. This assembly is free to rotate about the handle and socket, which are attached to the lug nut on the wheel of a car. If the rod AB is given an angular velocity of 4 rad/s and it strikes the bracket C on the handle without rebounding, determine the angular impulse imparted to the lug nut.
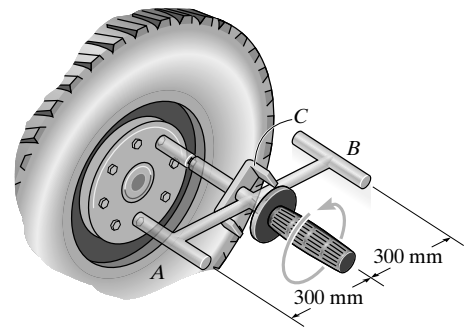
$$I_{\text{axle}} = \frac{1}{12}(1)(0.6 - 0.02)^2 + 2\left[\frac{1}{2}(1)(0.01)^2 + 1(0.3)^2\right] = 0.2081 \text{ kg·m}^2$$

$$\int M \, dt = I_{\text{axle}} \, \omega = 0.2081(4) = 0.833 \text{ kg·m}^2/\text{s} \qquad \textbf{Ans.}$$

19–7. The space shuttle is located in "deep space," where the effects of gravity can be neglected. It has a mass of 120 Mg, a center of mass at G, and a radius of gyration $(k_G)_x = 14$ m about the x axis. It is originally traveling forward at $v = 3$ km/s when the pilot turns on the engine at A, creating a thrust $T = 600(1 - e^{-0.3t})$ kN, where t is in seconds. Determine the shuttle's angular velocity 2 s later.

$(\zeta +)$ $\qquad (H_G)_1 + \Sigma \int M_G \, dt = (H_G)_2$

$$0 + \int_0^2 600(10^3)(1 - e^{-0.3\,t})(2) \, dt = \left[120(10^3)(14)^2 \right]\omega$$

$$1200(10^3)\left[t + \frac{1}{0.3}e^{-0.3\,t} \right]_0^2 = 120(10^3)(14)^2 \, \omega$$

$$\omega = 0.0253 \text{ rad/s} \qquad \qquad \textbf{Ans.}$$
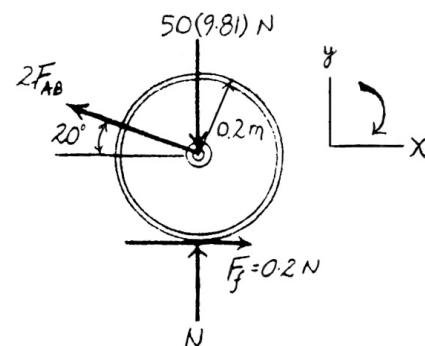
$T = 600(10^3)(1 - e^{-0.3t})$ N

---
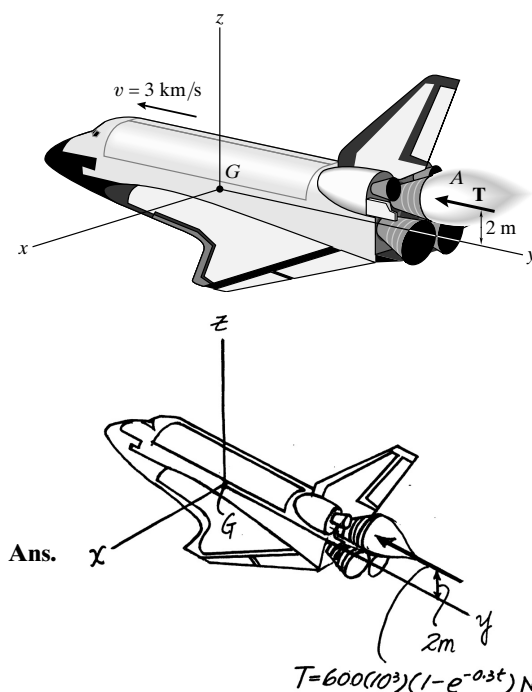
*19–8. The 50-kg cylinder has an angular velocity of 30 rad/s when it is brought into contact with the horizontal surface at C. If the coefficient of kinetic friction is $\mu_C = 0.2$, determine how long it will take for the cylinder to stop spinning. What force is developed in link AB during this time? The axle through the cylinder is connected to two symmetrical links. (Only AB is shown.) For the computation, neglect the weight of the links.
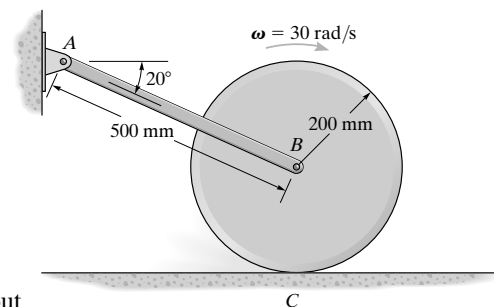
$\omega = 30$ rad/s

Principle of Impulse and Momentum: The mass moment inertia of the cylinder about its mass center is $I_G = \dfrac{1}{2}(50)(0.2^2) = 1.00$ kg·m². Applying Eq. 19–14, we have

$$m(v_{G_y})_1 + \Sigma \int_{t_1}^{t_2} F_y \, dt = m(v_{G_y})_2$$

$(+\uparrow) \qquad 0 + N(t) + 2F_{AB}\sin 20°\,(t) - 50(9.81)(t) = 0 \qquad \textbf{(1)}$

$$m(v_{G_x})_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_{G_x})_2$$

$(\xrightarrow{+}) \qquad 0 + 0.2N(t) - 2F_{AB}\cos 20°(t) = 0 \qquad \textbf{(2)}$

$$I_G\omega_1 + \Sigma \int_{t_1}^{t_2} M_G \, dt = I_G \, \omega_2$$

$(\zeta +) \qquad\qquad -1.00(30) + [0.2N(t)](0.2) = 0 \qquad \textbf{(3)}$

Solving Eqs. (1), (2), and (3) yields

$$F_{AB} = 48.7 \text{ N} \qquad t = 1.64 \text{ s} \qquad\qquad \textbf{Ans.}$$

$$N = 457.22 \text{ N}$$

$50(9.81)$ N

$2F_{AB}$

$0.2m$

$F_f = 0.2 N$

N

•**19–9.** If the cord is subjected to a horizontal force of $P = 150$ N, and the gear rack is fixed to the horizontal plane, determine the angular velocity of the gear in 4 s, starting from rest. The mass of the gear is 50 kg, and it has a radius of gyration about its center of mass O of $k_O = 125$ mm.
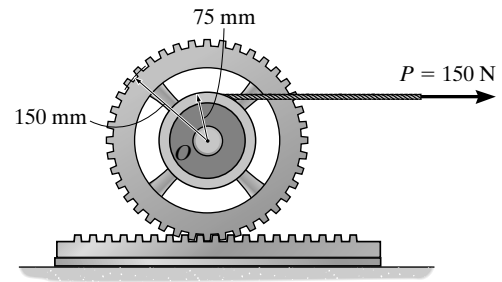
Kinematics: Referring to Fig. *a*,

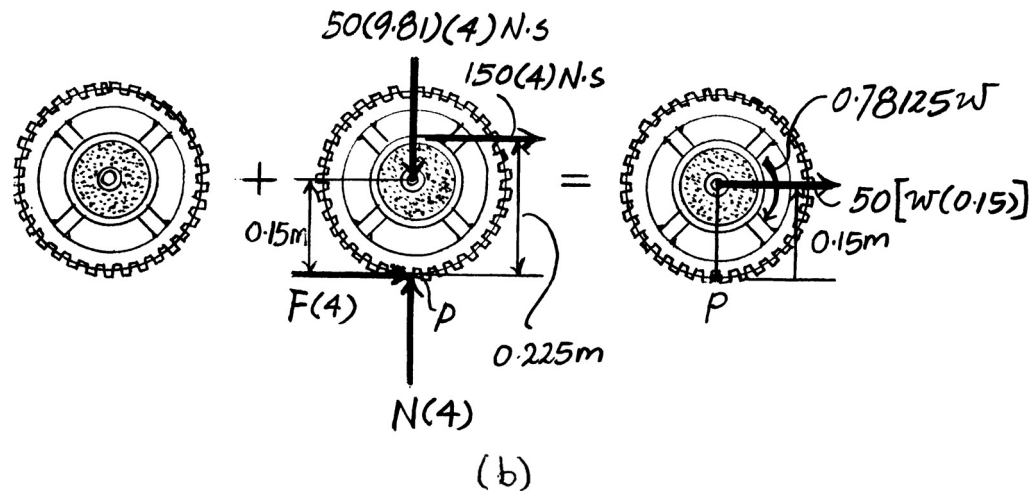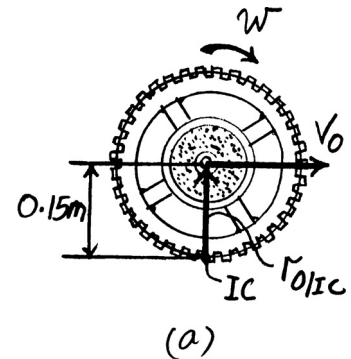$$v_O = \omega r_{O/IC} = \omega(0.15)$$

Principle of Angular Impulse and Momentum: The mass moment of inertia of the gear about its mass center is $I_O = mk_O{}^2 = 50(0.125^2) = 0.78125$ kg $\cdot$ m². From Fig. *b*,

$$\zeta + I_P\omega_1 + \Sigma \int_{t_1}^{t_2} M_P \, dt = I_P\omega_2$$

$$0 + 150(4)(0.225) = 0.78125\omega + 50[\omega(0.15)](0.15)$$

$$\omega = 70.8 \text{ rad/s} \qquad\qquad \textbf{Ans.}$$

(a)

(b)

19–10. If the cord is subjected to a horizontal force of $P = 150$ N, and gear is supported by a fixed pin at O, determine the angular velocity of the gear and the velocity of the 20-kg gear rack in 4 s, starting from rest. The mass of the gear is 50 kg and it has a radius of gyration of $k_O = 125$ mm. Assume that the contact surface between the gear rack and the horizontal plane is smooth.
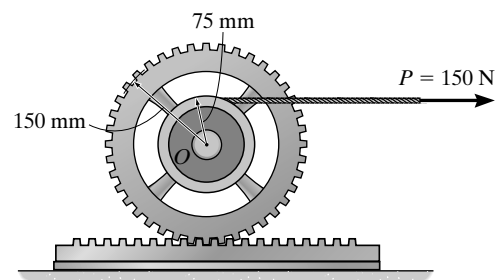
75 mm

$P = 150$ N

150 mm

O

Principle of Impulse and Momentum: The mass moment of inertia of the gear about its mass center is $I_O = mk_O{}^2 = 50(0.125^2) = 0.78125$ kg·m². Referring to the free-body diagram of the gear shown in Fig. a,

$$\zeta + I_O\omega_1 + \Sigma \int_{t_1}^{t_2} M_O\,dt = I_O\omega_2$$

$$0 + F(4)(0.15) - 150(4)(0.075) = -0.78125\omega_A$$

$$F = 75 - 1.302\omega_A \tag{1}$$

Since the gear rotates about the fixed axis, $v_P = \omega_A r_P = \omega_A(0.15)$. Referring to the free-body diagram of the gear rack shown in Fig. b,

$$(\pm) \qquad mv_1 + \Sigma \int_{t_1}^{t_2} F_x\,dt = mv_2$$

$$0 + F(4) = 20[\omega_A(0.15)]$$
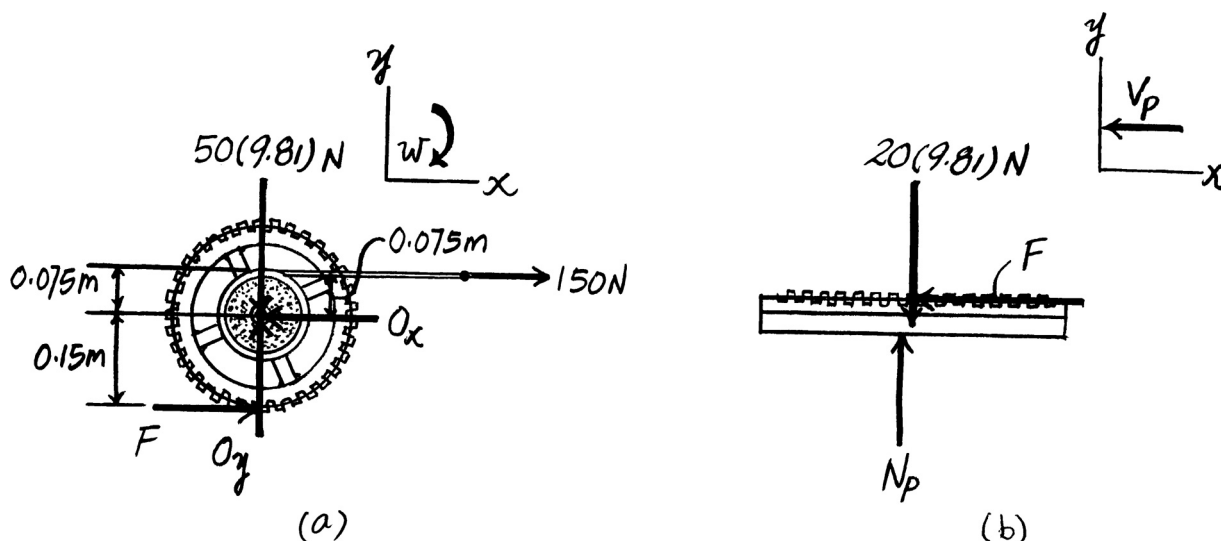
$$F = 0.75\omega_A \tag{2}$$

Equating Eqs. (1) and (2),

$$0.75\omega_A = 75 - 1.302\omega_A$$

$$\omega_A = 36.548 \text{ rad/s} = 36.5 \text{ rad/s} \qquad \textbf{Ans.}$$

Then,

$$v = 36.548(0.15) = 5.48 \text{ m/s} \qquad \textbf{Ans.}$$

(a)

(b)

19–11. A motor transmits a torque of $M = 0.05$ N·m to the center of gear A. Determine the angular velocity of each of the three (equal) smaller gears in 2 s starting from rest. The smaller gears (B) are pinned at their centers, and the masses and centroidal radii of gyration of the gears are given in the figure.

$m_A = 0.8$ kg
$M = 0.05$ N · m
$k_A = 31$ mm

40 mm
A

B
$m_B = 0.3$ kg
20 mm
$k_B = 15$ mm
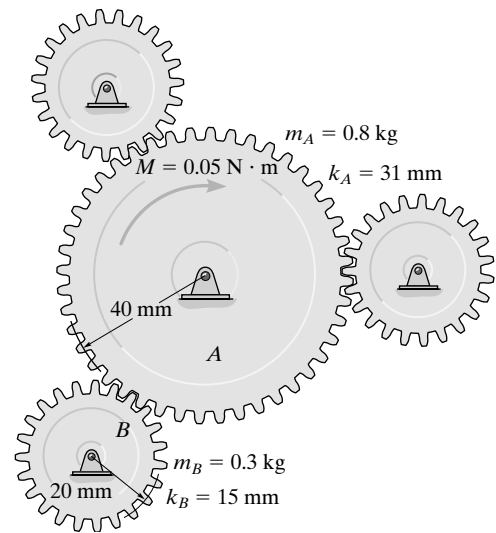
Gear A:

$(\zeta +)$ $\qquad (H_A)_1 + \Sigma \int M_A \, dt = (H_A)_2$

$\qquad 0 - 3(F)(2)(0.04) + 0.05(2) = [0.8(0.031)^2]\omega_A$

Gear B:

$(\zeta +)$ $\qquad (H_B)_1 + \Sigma \int M_B \, dt = (H_B)_2$

$\qquad 0 + (F)(2)(0.02) = [0.3(0.015)^2]\omega_B$
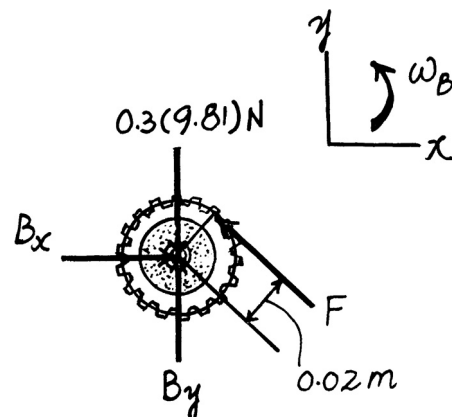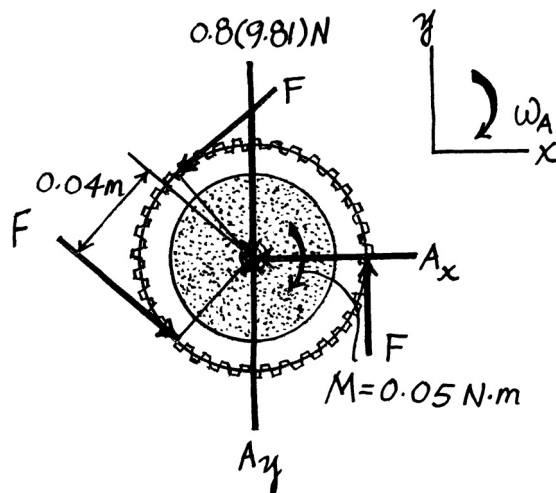
Since $0.04\omega_A = 0.02\omega_B$, or $\omega_B = 2\omega_A$, then solving,

$F = 0.214$ N

$\omega_A = 63.3$ rad/s

$\omega_B = 127$ rad/s $\qquad\qquad\qquad\qquad\qquad\qquad$ **Ans.**

0.8(9.81)N
F
0.04m
F
ω_A
x
A_x
F
$M = 0.05$ N·m
A_y

0.3(9.81)N
B_x
F
ω_B
x
B_y
0.02m

*19–12. The 200-lb flywheel has a radius of gyration about its center of gravity O of $k_O = 0.75$ ft. If it rotates counterclockwise with an angular velocity of 1200 rev/min before the brake is applied, determine the time required for the wheel to come to rest when a force of $P = 200$ lb is applied to the handle. The coefficient of kinetic friction between the belt and the wheel rim is $\mu_k = 0.3$. (*Hint:* Recall from the statics text that the relation of the tension in the belt is given by $T_B = T_C e^{\mu\beta}$, where β is the angle of contact in radians.)
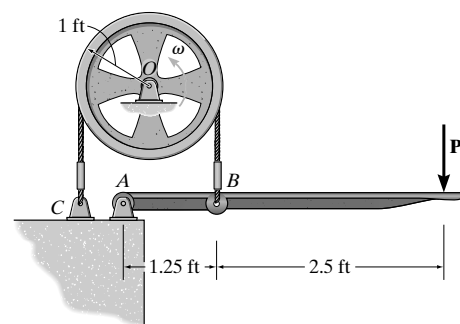
Equilibrium: Writing the moment equation of equilibrium about point A and referring to the free-body diagram of the arm brake shown in Fig. *a*,

$\zeta + \Sigma M_A = 0;$ $T_B(1.25) - 200(3.75) = 0$ $T_B = 600$ lb

Using the belt friction formula,

$T_B = T_C e^{\mu\beta}$

$600 = T_C e^{0.3(\pi)}$
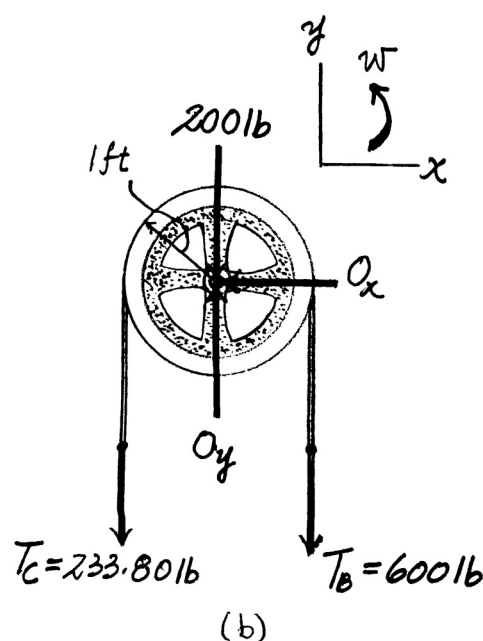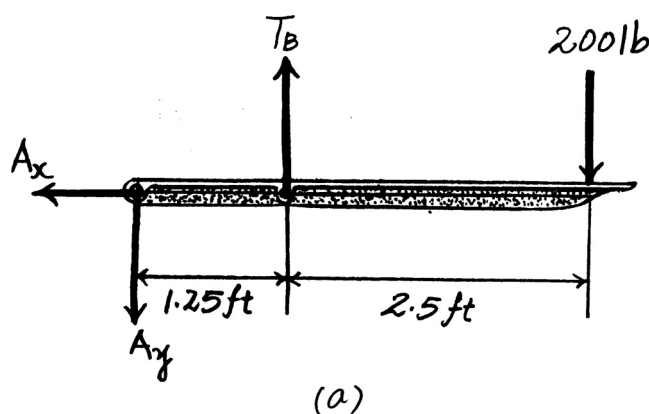
$T_C = 233.80$ lb

Principle of Angular Impulse and Momentum: The mass moment of inertia of the wheel about its mass center is $I_O = mk_O^2 = \left(\dfrac{200}{32.2}\right)(0.75^2) = 3.494$ slug·ft², and the initial angular velocity of the wheel is $\omega_1 = \left(1200\,\dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi\,\text{rad}}{1\,\text{rev}}\right)\left(\dfrac{1\,\text{min}}{60\,\text{s}}\right)$ $= 40\pi$ rad/s. Applying the angular impulse and momentum equation about point O using the free-body diagram of the wheel shown in Fig. *b*,

$\zeta + \quad I_O\omega_1 + \Sigma \displaystyle\int_{t_1}^{t_2} M_O\, dt = I_O\omega_2$

$3.494(40\pi) + 233.80(t)(1) - 600(t)(1) = 0$

$t = 1.20$ s   **Ans.**

(a)

(b)

•**19–13.** The 200-lb flywheel has a radius of gyration about its center of gravity O of $k_O = 0.75$ ft. If it rotates counterclockwise with a constant angular velocity of 1200 rev/min before the brake is applied, determine the required force **P** that must be applied to the handle to stop the wheel in 2 s. The coefficient of kinetic friction between the belt and the wheel rim is $\mu_k = 0.3$. (*Hint:* Recall from the statics text that the relation of the tension in the belt is given by $T_B = T_C e^{\mu\beta}$, where β is the angle of contact in radians.)
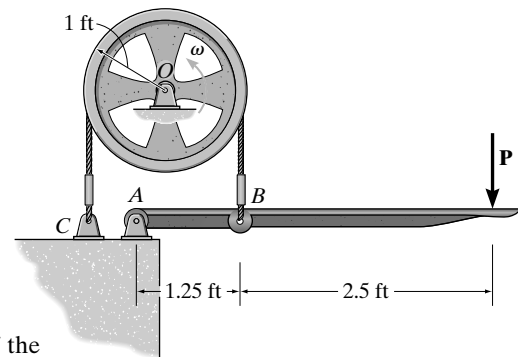
Principle of Angular Impulse and Momentum: The mass moment of inertia of the wheel about its mass center is $I_O = mk_O{}^2 = \left(\dfrac{200}{32.2}\right)(0.75^2) = 3.494$ slug · ft^2, and

the initial angular velocity of the wheel is $\omega_1 = \left(1200 \dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)$

$= 40\pi$ rad/s. Applying the angular impulse and momentum equation about point O using the free-body diagram shown in Fig. *a*,

$$\zeta + \quad I_O\omega_1 + \Sigma \int_{t_1}^{t_2} M_O \, dt = I_O\omega_2$$

$$3.494(40\pi) + T_C(2)(1) - T_B(2)(1) = 0$$

$$T_B - T_C = 219.52 \qquad\qquad \textbf{(1)}$$

Using the belt friction formula,

$$T_B = T_C e^{\mu\beta}$$

$$T_B = T_C e^{0.3(\pi)} \qquad\qquad \textbf{(2)}$$
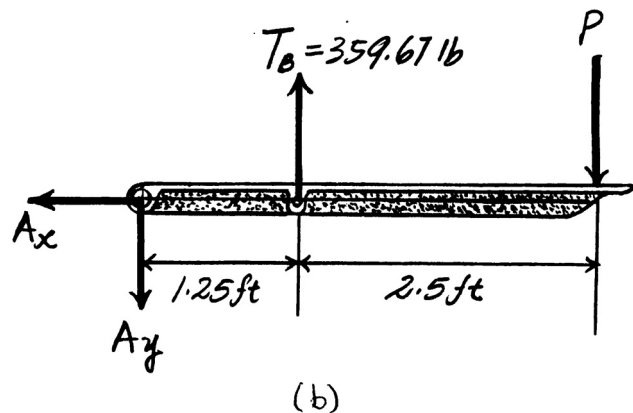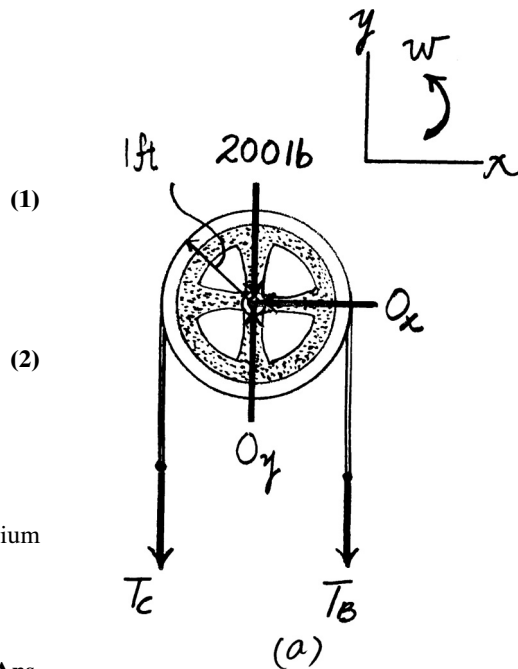
Solving Eqs. (1) and (2),

$$T_C = 140.15 \text{ lb} \qquad T_B = 359.67 \text{ lb}$$

Equilibrium: Using this result and writing the moment equation of equilibrium about point A using the free-body diagram of the brake arm shown in Fig. *b*,
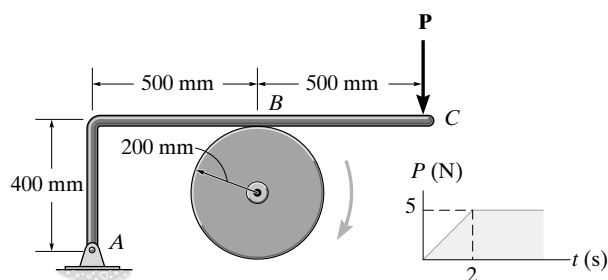
$$\zeta + \Sigma M_A = 0; \qquad 359.67(1.25) - P(3.75) = 0$$

$$P = 120 \text{ lb} \qquad\qquad\qquad \textbf{Ans.}$$

(a)

(b)

19–14. The 12-kg disk has an angular velocity of $\omega = 20$ rad/s. If the brake ABC is applied such that the magnitude of force **P** varies with time as shown, determine the time needed to stop the disk. The coefficient of kinetic friction at B is $\mu_k = 0.4$. Neglect the thickness of the brake.

Equation of Equilibrium: Since slipping occurs at B, the friction $F_f = \mu_k N_B = 0.4N_B$. From FBD(a), the normal reaction N_B can be obtained directed by summing moments about point A.

$$\zeta + \Sigma M_A = 0; \qquad N_B(0.5) - 0.4N_B(0.4) - P(1) = 0$$

$$N_B = 2.941P$$

Thus, the friction $F_f = 0.4N_B = 0.4(2.941P) = 1.176P$.

Principle of Impulse and Momentum: The mass moment inertia of the cylinder about its mass center is $I_O = \dfrac{1}{2}(12)(0.2^2) = 0.240$ kg·m^2. Applying Eq. 19–14, we have

$$I_O \omega_1 + \Sigma \int_{t_1}^{t_2} M_O \, dt = I_O \omega_2$$

$$(\zeta +) \qquad -0.240(20) + \left[-\left(1.176 \int_0^t P dt\right)(0.2)\right] = 0 \qquad \textbf{(1)}$$

However, $\int_0^t P dt$ is the area under the $P-t$ graph. Assuming $t > 2$ s, then

$$\int_0^t P dt = \frac{1}{2}(5)(2) + 5(t-2) = (5t - 5) \text{ N·s}$$

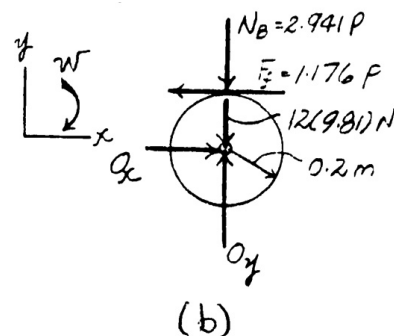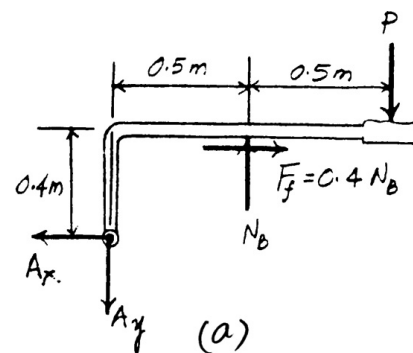Substitute into Eq. (1) yields

$$-0.240(20) + [-1.176(5t - 5)(0.2)] = 0$$

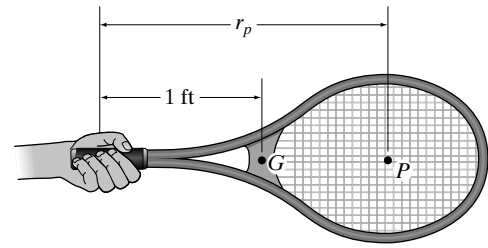$$t = 5.08 \text{ s} \qquad \textbf{Ans.}$$

Since $t = 5.08$ s > 2 s, the above assumption is correct.

19–15. The 1.25-lb tennis racket has a center of gravity at G and a radius of gyration about G of $k_G = 0.625$ ft. Determine the position P where the ball must be hit so that 'no sting' is felt by the hand holding the racket, i.e., the horizontal force exerted by the racket on the hand is zero.

Principle of Impulse and Momentum: Here, we will assume that the tennis racket is initially at rest and rotates about point A with an angular velocity of ω immediately after it is hit by the ball, which exerts an impulse of $\int F \, dt$ on the racket, Fig. a. The mass moment of inertia of the racket about its mass center is $I_G = \left(\dfrac{1.25}{32.2}\right)(0.625^2)$ $= 0.01516$ slug $\cdot$ ft^2. Since the racket about point A,

$(v_G) = \omega r_G = \omega(1)$. Referring to Fig. b,

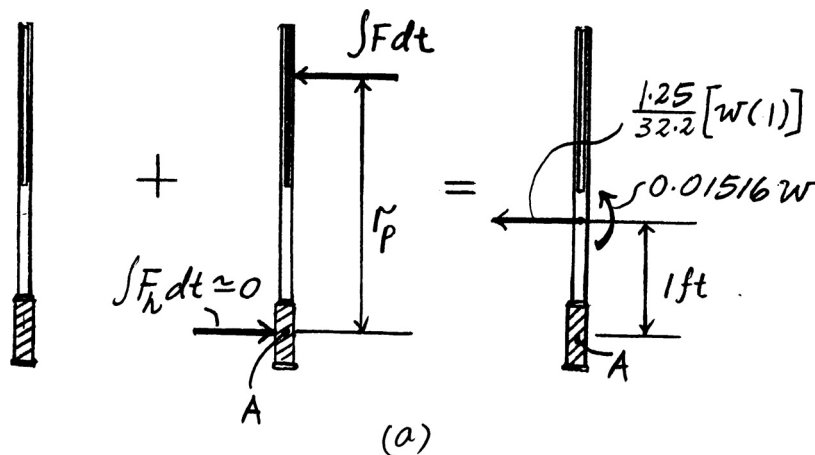$\xleftarrow{+}$ $\qquad m(v_G)_1 + \Sigma \displaystyle\int_{t_1}^{t_2} F_x \, dt = m(v_G)_2$

$\qquad 0 + \displaystyle\int F \, dt = \left(\dfrac{1.25}{32.2}\right)\left[\omega(1)\right]$

$\qquad \displaystyle\int F \, dt = 0.03882\omega \qquad\qquad\qquad\qquad\qquad (1)$

and

$\zeta+ \quad (H_A)_1 + \Sigma \displaystyle\int_{t_1}^{t_2} M_A \, dt = (H_A)_2$

$\qquad 0 + \left(\displaystyle\int F \, dt\right) r_P = 0.01516\omega + \dfrac{1.25}{32.2}\left[\omega(1)\right](1)$

$\qquad \displaystyle\int F \, dt = \dfrac{0.05398\omega}{r_P} \qquad\qquad\qquad\qquad (2)$

Equating Eqs. (1) and (2) yields

$\qquad\qquad 0.03882\omega = \dfrac{0.05398\omega}{r_P} \qquad\qquad r_P = 1.39 \text{ ft} \qquad\qquad$ **Ans.**

(a)

*19–16. If the boxer hits the 75-kg punching bag with an impulse of $I = 20 \text{ N} \cdot \text{s}$, determine the angular velocity of the bag immediately after it has been hit. Also, find the location d of point B, about which the bag appears to rotate. Treat the bag as a uniform cylinder.
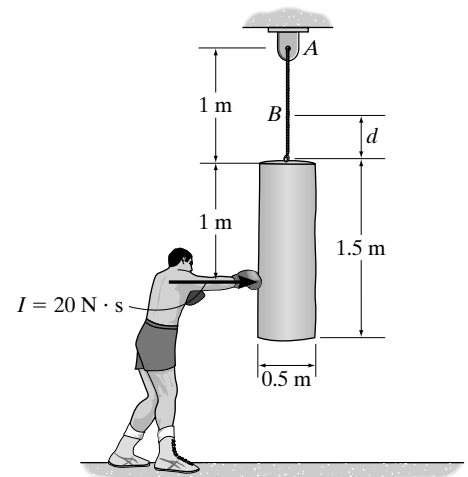
Principle of Impulse and Momentum: The mass moment of inertia of the bag about its mass center is $I_G = \dfrac{1}{12}m(3r^2 + h^2) = \dfrac{1}{12}(75)\left[3(0.25^2) + 1.5^2\right] = 15.23 \text{ kg} \cdot \text{m}^2$.

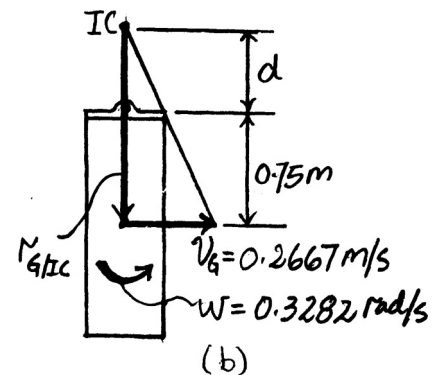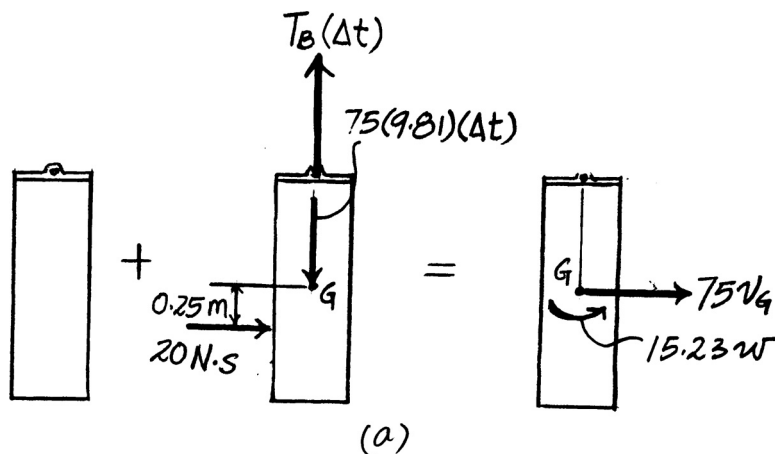Referring to the impulse and momentum diagrams of the bag shown in Fig. a,

$$(\,\xrightarrow{+}\,) \qquad m(v_G)_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m\,(v_G)_2$$

$$0 + 20 = 75v_G \qquad\qquad\qquad v_G = 0.2667 \text{ m/s}$$

and

$$\zeta + I_G\omega_1 + \Sigma \int_{t_1}^{t_2} M_G \, dt = I_G\omega_2$$

$$0 + 20(0.25) = 15.23\omega$$

$$\omega = 0.3282 \text{ rad/s} = 0.328 \text{ rad/s} \qquad\qquad \textbf{Ans.}$$

Kinematics: Referring to Fig. b,

$$v_G = \omega r_{G/IC}$$

$$0.2667 = 0.3282(0.75 + d)$$

$$d = 0.0625 \text{ m} \qquad\qquad \textbf{Ans.}$$

(a)

(b)

•19–17. The 5-kg ball is cast on the alley with a backspin of $\omega_0 = 10$ rad/s, and the velocity of its center of mass O is $v_0 = 5$ m/s. Determine the time for the ball to stop back spinning, and the velocity of its center of mass at this instant. The coefficient of kinetic friction between the ball and the alley is $\mu_k = 0.08$.

$\omega_0 = 10$ rad/s

100 mm

$v_0 = 5$ m/s

O
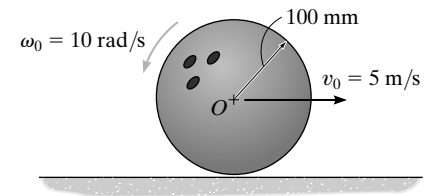
Principle of Impulse and Momentum: Since the ball slips, $F_f = \mu_k N = 0.08N$. The mass moment of inertia of the ball about its mass center is

$$I_O = \frac{2}{5} mr^2 = \frac{2}{5}(5)(0.1^2) = 0.02 \text{ kg} \cdot \text{m}^2$$
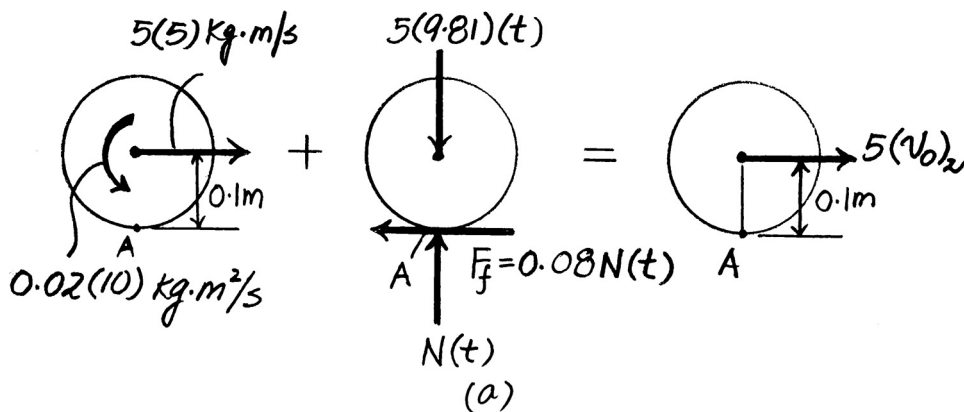
Referring to Fig. a,

$$(+\uparrow) \quad m\left[(v_O)_y\right]_1 + \Sigma \int_{t_1}^{t_2} F_y \, dt = m\left[(v_O)_y\right]_2$$

$$0 + N(t) - 5(9.81)t = 0 \qquad\qquad N = 49.05 \text{ N}$$

$$\zeta + (H_A)_1 + \Sigma \int_{t_1}^{t_2} M_A \, dt = (H_A)_2$$

$$0.02(10) - 5(5)(0.1) + 0 = -5(v_O)_2(0.1)$$

$$(v_O)_2 = 4.6 \text{ m/s} \qquad\qquad \textbf{Ans.}$$

$$(\overset{+}{\rightarrow}) \quad m\left[(v_O)_x\right]_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m\left[(v_O)_x\right]_2$$

$$5(5) - 0.08(49.05)(t) = 5(4.6)$$

$$t = 0.510 \text{ s} \qquad\qquad \textbf{Ans.}$$

$5(5)\,kg\cdot m/s \qquad 5(9.81)(t)$

$+ \qquad = \qquad 5(V_O)_2$

$0.1m$

$0.1m$

A

$0.02(10)\,kg\cdot m^2/s$

$A \quad F_f = 0.08N(t) \quad A$

$N(t)$

(a)

19–18. The smooth rod assembly shown is at rest when it is struck by a hammer at A with an impulse of 10 N·s. Determine the angular velocity of the assembly and the magnitude of velocity of its mass center immediately after it has been struck. The rods have a mass per unit length of 6 kg/m.
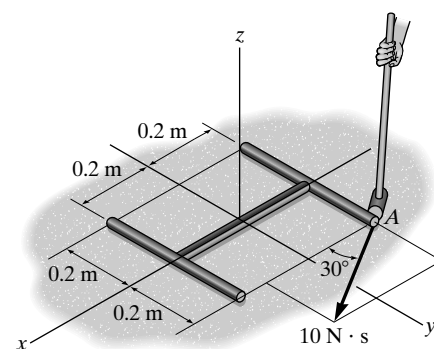
Principle of Impulse and Momentum: The total mass of the assembly is $m = 3[6(0.4)] = 7.2$ kg. The mass moment of inertia of the assembly about its mass center is

$$I_G = \frac{1}{12}[6(0.4)](0.4^2) + 2\left[\frac{1}{12}[6(0.4)](0.4^2) + 6(0.4)(0.2^2)\right] = 0.288 \text{ kg} \cdot \text{m}^2$$

Referring to Fig. b,

$(+\downarrow)$ $m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$

$\qquad 0 + 10 \cos 30° = 7.2(v_G)_x$ $\qquad\qquad\qquad (v_G)_x = 1.203$ m/s

$(\overset{+}{\rightarrow})$ $m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$

$\qquad 0 + 10 \sin 30° = 7.2(v_G)_y$ $\qquad\qquad\qquad (v_G)_y = 0.6944$ m/s

Thus, the magnitude of $\mathbf{v}_G$ is

$v_G = \sqrt{(v_G)_x^2 + (v_G)_y^2} = \sqrt{1.203^2 + 0.6944^2} = 1.39$ m/s $\qquad$ **Ans.**

Also

$\zeta+$ $I_G\omega_1 + \Sigma \int_{t_1}^{t_2} M_G\, dt = I_G\omega_2$

$\qquad 0 + [-10 \cos 30°(0.2) - 10 \sin 30°(0.2)] = -0.288\omega$

$\qquad \omega = 9.49$ rad/s $\qquad\qquad\qquad\qquad\qquad$ **Ans.**

(a)

791
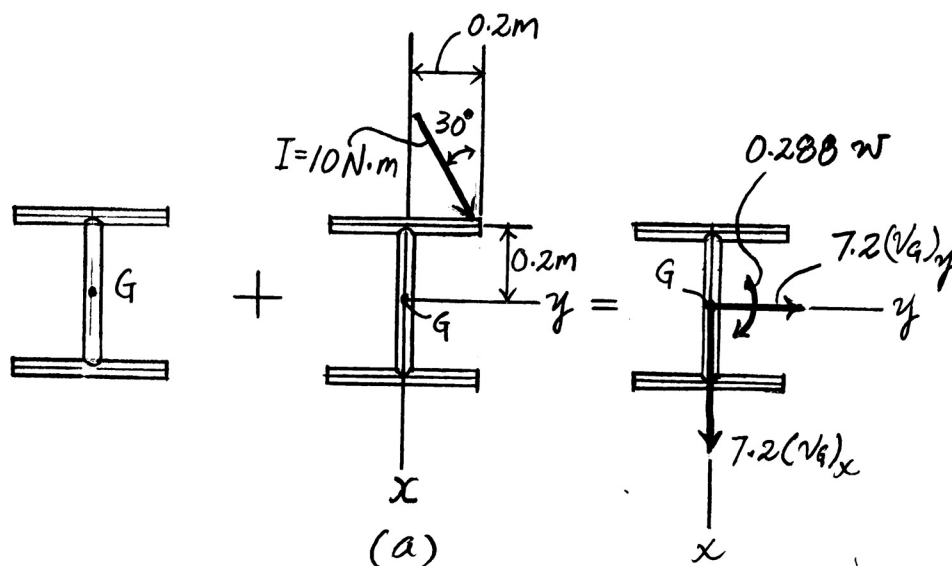
19–19. The flywheel A has a mass of 30 kg and a radius of gyration of $k_C = 95$ mm. Disk B has a mass of 25 kg, is pinned at D, and is coupled to the flywheel using a belt which is subjected to a tension such that it does not slip at its contacting surfaces. If a motor supplies a counterclockwise torque or twist to the flywheel, having a magnitude of $M = (12t)$ N $\cdot$ m, where t is in seconds, determine the angular velocity of the disk 3 s after the motor is turned on. Initially, the flywheel is at rest.
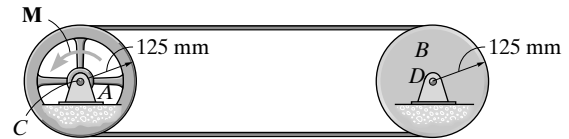
Principle of Impulse and Momentum: The mass moment inertia of the flywheel about point C is $I_C = 30(0.095^2) = 0.27075$ kg $\cdot$ m^2. Applying Eq. 19–14 to the flywheel [FBD(a)], we have

$$I_C \omega_1 + \Sigma \int_{t_1}^{t_2} M_C \, dt = I_C \omega_2$$

$(\zeta +) \quad 0 + \int_0^{3\,s} 12t \, dt + [T_2(3)](0.125) - T_1(3)](0.125) = 0.27075\omega$

$$54.0 + 0.375T_2 - 0.375T_1 = 0.27075\omega \qquad \textbf{(1)}$$

(a)

The mass moment inertia of the disk about point D is $I_D = \dfrac{1}{2}(25)(0.125^2)$ = 0.1953125 kg $\cdot$ m^2. Applying Eq. 19–14 to the disk [FBD(b)], we have

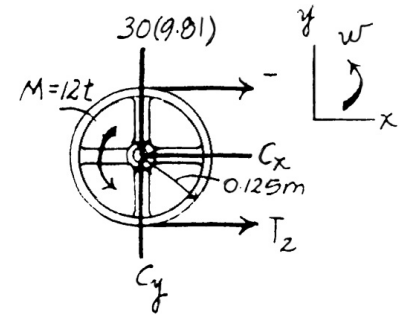$$I_D \omega_1 + \Sigma \int_{t_1}^{t_2} M_D \, dt = I_D \omega_2$$

$(\zeta +) \quad 0 + \left[T_1(3)\right](0.125) - \left[T_2(3)\right](0.125) = 0.1953125\omega$

$$0.375T_2 - 0.375T_1 = -0.1953125\omega \qquad \textbf{(2)}$$
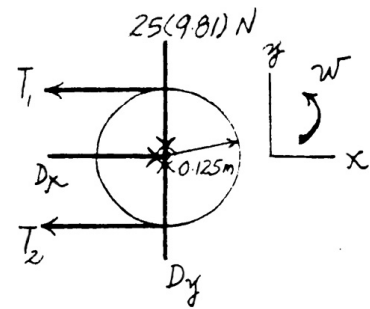
Substitute Eq. (2) into Eq. (1) and solving yields

$$\omega = 116 \text{ rad/s} \qquad \textbf{Ans.}$$


(b)

*19–20. The 30-lb flywheel A has a radius of gyration about its center of 4 in. Disk B weighs 50 lb and is coupled to the flywheel by means of a belt which does not slip at its contacting surfaces. If a motor supplies a counterclockwise torque to the flywheel of $M = (50t)$ lb · ft, where t is in seconds, determine the time required for the disk to attain an angular velocity of 60 rad/s starting from rest.
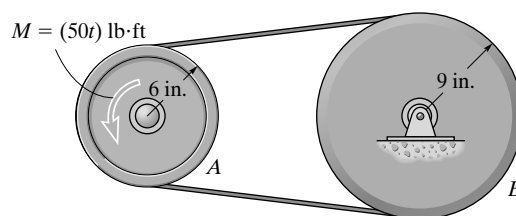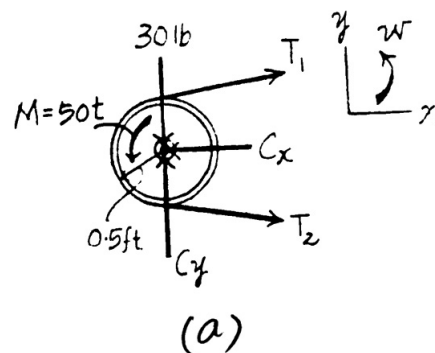
Principle of Impulse and Momentum: The mass moment inertia of the flywheel about point C is $I_C = \dfrac{30}{32.2}\left(\dfrac{4}{12}\right)^2 = 0.1035$ slug · ft^2. The angular velocity of the flywheel is $\omega_A = \dfrac{r_B}{r_A}\omega_B = \dfrac{0.75}{0.5}(60) = 90.0$ rad/s. Applying Eq. 19–14 to the flywheel [FBD(a)], we have

$$I_C\,\omega_1 + \Sigma \int_{t_1}^{t_2} M_C\,dt = I_C\,\omega_2$$

$(\circlearrowright+)\quad 0 + \displaystyle\int_0^t 50t\,dt + \left[\int T_2\,(dt)\right](0.5) - \left[\int T_1(dt)\right](0.5) = 0.1035(90)$

$$25t^2 + 0.5\int (T_2 - T_1)dt = 9.317 \tag{1}$$

The mass moment inertia of the disk about point D is $I_D = \dfrac{1}{2}\left(\dfrac{50}{32.2}\right)(0.75^2)$ $= 0.4367$ slug · ft^2. Applying Eq. 19–14 to the disk [FBD(b)], we have

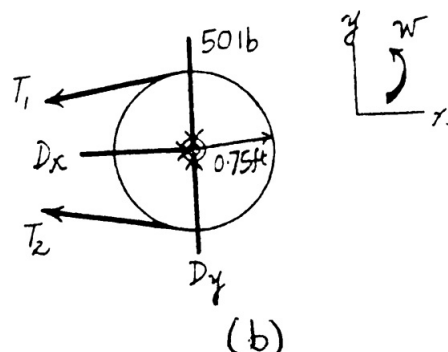$$I_D\,\omega_1 + \Sigma \int_{t_1}^{t_2} M_D\,dt = I_D\,\omega_2$$

$(\circlearrowright+)\quad 0 + \left[\int T_1\,(dt)\right](0.75) - \left[\int T_2\,(dt)\right](0.75) = 0.4367(60)$

$$\int (T_2 - T_1)dt = -34.94 \tag{2}$$

Substitute Eq. (2) into Eq. (1) and solving yields

$$t = 1.04 \text{ s} \qquad\qquad \textbf{Ans.}$$

793

•19–21. For safety reasons, the 20-kg supporting leg of a sign is designed to break away with negligible resistance at B when the leg is subjected to the impact of a car. Assuming that the leg is pinned at A and approximates a thin rod, determine the impulse the car bumper exerts on it, if after the impact the leg appears to rotate clockwise to a maximum angle of $\theta_{max} = 150°$.
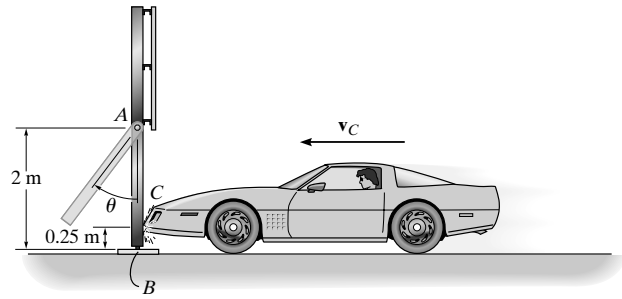
$(+\circlearrowleft)$ $\qquad I_A \omega_1 + \Sigma \int_{t_1}^{t_2} M_A\, dt = I_A \omega_2$

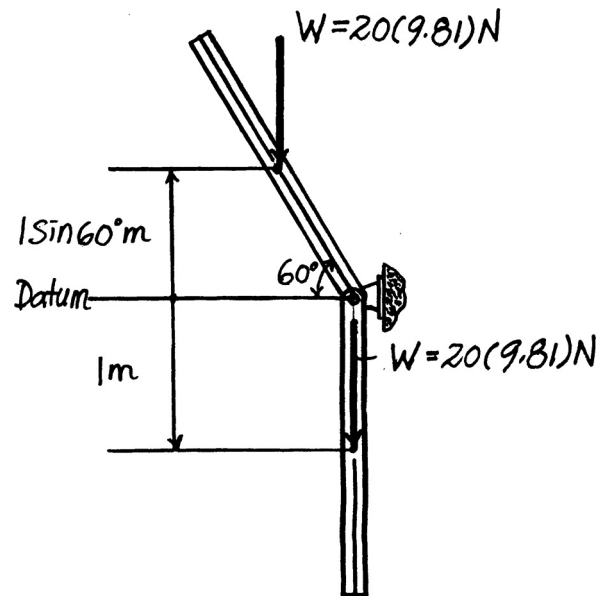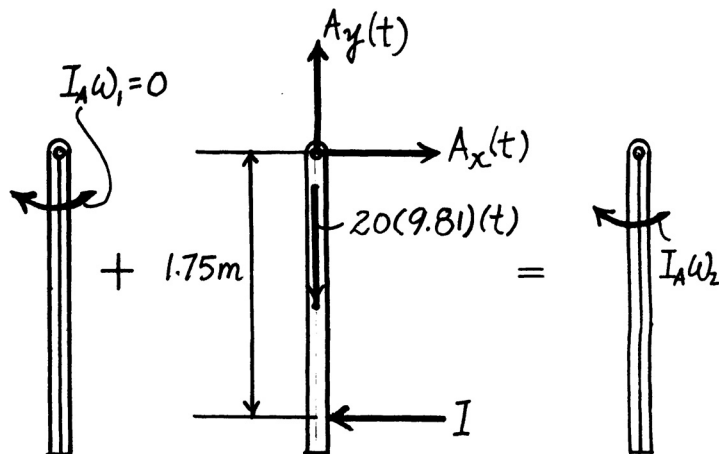$$0 + I(1.75) = \left[\frac{1}{3}(20)(2)^2\right]\omega_2$$
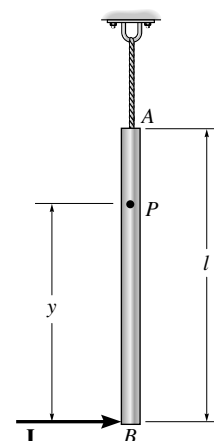
$$\omega_2 = 0.065625 I$$

$T_2 + V_2 = T_3 + V_3$

$$\frac{1}{2}\left[\frac{1}{3}(20)(2)^2\right](0.065625 I)^2 + 20(9.81)(-1) = 0 + 20(9.81)(1 \sin 60°)$$

$I = 79.8 \text{ N} \cdot \text{s}$ $\qquad\qquad\qquad\qquad\qquad\qquad$ **Ans.**

19–22. The slender rod has a mass m and is suspended at its end A by a cord. If the rod receives a horizontal blow giving it an impulse $\mathbf{I}$ at its bottom B, determine the location y of the point P about which the rod appears to rotate during the impact.

Principle of Impulse and Momentum:

$(\zeta +)$
$$I_G \omega_1 + \Sigma \int_{t_1}^{t_2} M_G \, dt = I_G \, \omega_2$$

$$0 + I\left(\frac{l}{2}\right) = \left[\frac{1}{12} ml^2\right]\omega \qquad I = \frac{1}{6}ml\omega$$

$\left(\stackrel{+}{\rightarrow}\right)$
$$m(v_{Ax})_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_{Ax})_2$$

$$0 + \frac{1}{6}ml\omega = mv_G \qquad\qquad v_G = \frac{l}{6}\omega$$
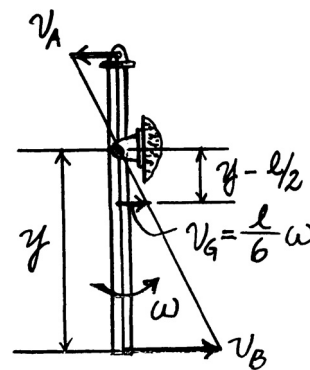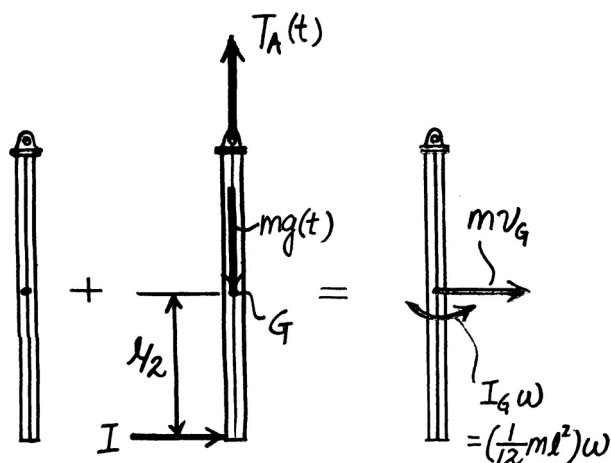
Kinematics: Point P is the IC.

$$v_B = \omega \, y$$

Using similar triangles,

$$\frac{\omega y}{y} = \frac{\frac{l}{6}\omega}{y - \frac{l}{2}} \qquad y = \frac{2}{3}l \qquad\qquad\qquad \text{Ans.}$$

795

19–23. The 25-kg circular disk is attached to the yoke by means of a smooth axle A. Screw C is used to lock the disk to the yoke. If the yoke is subjected to a torque of $M = (5t^2)\,\text{N} \cdot \text{m}$, where t is in seconds, and the disk is unlocked, determine the angular velocity of the yoke when $t = 3\,\text{s}$, starting from rest. Neglect the mass of the yoke.
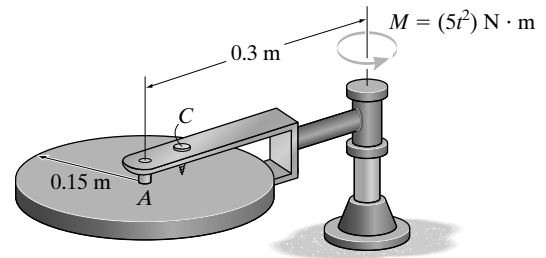
Principle of Angular Momentum: Since the disk is not rigidly attached to the yoke, only the linear momentum of its mass center contributes to the angular momentum about point O. Here, the yoke rotates about the fixed axis, thus $v_A = \omega r_{OA} = \omega(0.3)$. Referring to Fig. a,

$$\zeta + \qquad (H_O)_1 + \Sigma \int_{t_1}^{t_2} M_O dt = (H_O)_2$$

$$0 + \int_0^{3\,\text{s}} 5t^2 dt = 25\big[\omega(0.3)\big](0.3)$$

$$\frac{5t^3}{3}\bigg|_0^{3\,\text{s}} = 2.25\omega$$

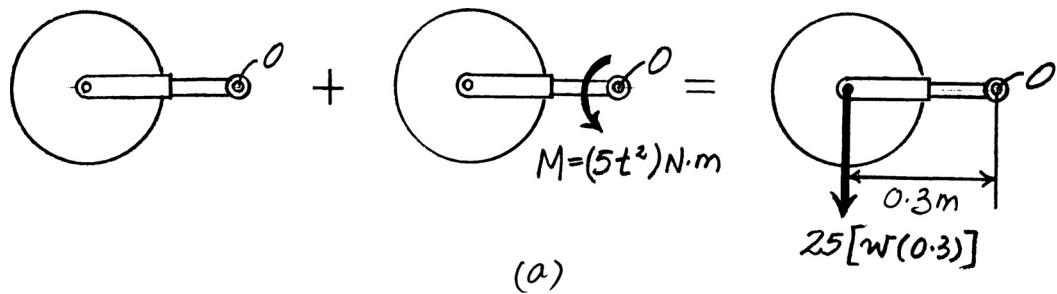$$\omega = 20\,\text{rad/s} \qquad\qquad \textbf{Ans.}$$

(a)

*19–24. The 25-kg circular disk is attached to the yoke by means of a smooth axle A. Screw C is used to lock the disk to the yoke. If the yoke is subjected to a torque of $M = (5t^2)$ N·m, where t is in seconds, and the disk is locked, determine the angular velocity of the yoke when $t = 3$ s, starting from rest. Neglect the mass of the yoke.
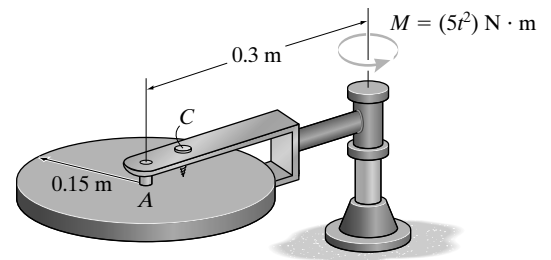
Principle of Angular Momentum: The mass moment of inertia of the disk about its mass center is $I_A = \dfrac{1}{2}mr^2 = \dfrac{1}{2}(25)(0.15^2) = 0.28125$ kg·m^2. Since the yoke rotates about a fixed axis, $v_A = \omega r_{OA} = \omega(0.3)$. Referring to Fig. a,

$\zeta +$ $(H_O)_1 + \Sigma \displaystyle\int_{t_1}^{t_2} M_O \, dt = (H_O)_2$

$0 + \displaystyle\int_0^{3\,s} 5t^2 dt = 0.28125\omega + 25\big[\omega(0.3)\big](0.3)$
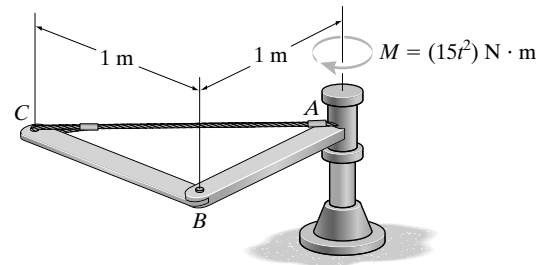
$\left.\dfrac{5t^3}{3}\right|_0^{3\,s} = 2.53125\omega$

$\omega = 17.8$ rad/s   **Ans.**

(a)
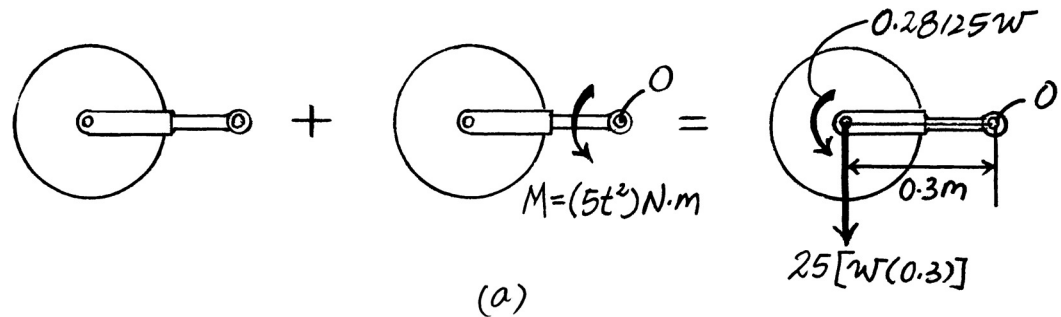
•19–25. If the shaft is subjected to a torque of $M = (15t^2)\,\text{N} \cdot \text{m}$, where t is in seconds, determine the angular velocity of the assembly when $t = 3$ s, starting from rest. Rods AB and BC each have a mass of 9 kg.

1 m 1 m $M = (15t^2)\,\text{N} \cdot \text{m}$

C

A

B

Principle of Impulse and Momentum: The mass moment of inertia of the rods about their mass center is $I_G = \dfrac{1}{12}\,ml^2 = \dfrac{1}{12}\,(9)(1^2) = 0.75\text{ kg} \cdot \text{m}^2$. Since the assembly rotates about the fixed axis, $(v_G)_{AB} = \omega(r_G)_{AB} = \omega(0.5)$ and $(v_G)_{BC} = \omega(r_G)_{BC} = \omega\left(\sqrt{1^2 + (0.5)^2}\right) = \omega(1.118)$. Referring to Fig. a,

$\zeta +$ $(H_z)_1 + \Sigma \displaystyle\int_{t_1}^{t_2} M_z\,dt = (H_z)_2$

$0 + \displaystyle\int_0^{3\text{ s}} 15t^2 dt = 9\big[\omega(0.5)\big](0.5) + 0.75\omega + 9\big[\omega(1.118)\big](1.118) + 0.75\omega$

$5t^3 \Big|_0^{3\text{ s}} = 15\omega$

$\omega = 9\text{ rad/s}$ **Ans.**

(a)
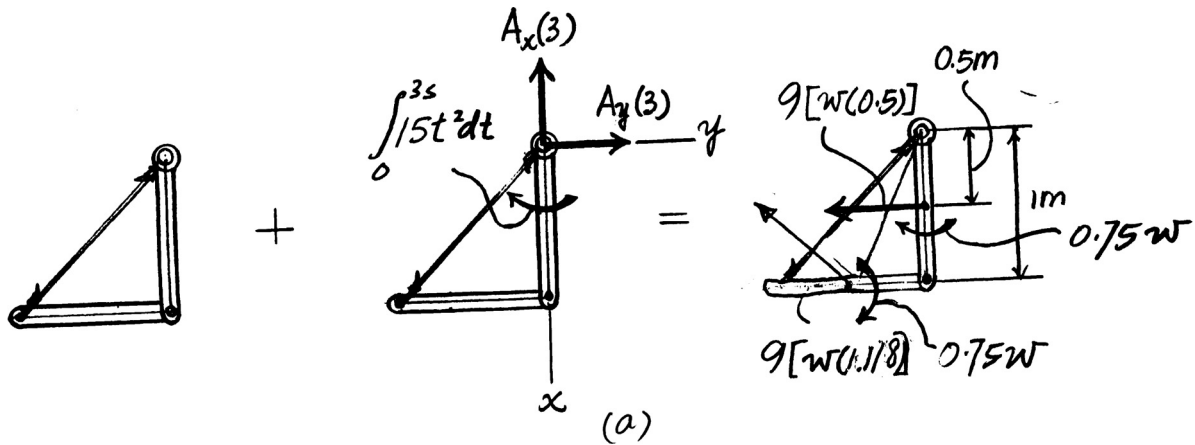
19–26. The body and bucket of a skid steer loader has a weight of 2000 lb, and its center of gravity is located at G. Each of the four wheels has a weight of 100 lb and a radius of gyration about its center of gravity of 1 ft. If the engine supplies a torque of $M = 100$ lb·ft to each of the rear drive wheels, determine the speed of the loader in $t = 10$ s, starting from rest. The wheels roll without slipping.
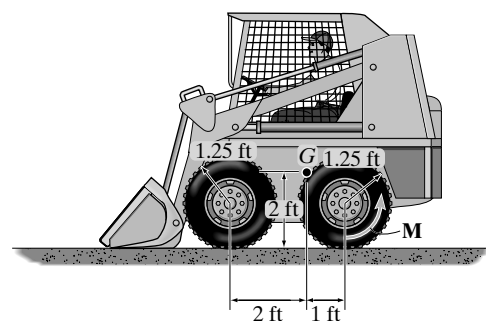
Principle of Impulse and Momentum. The mass moment of inertia of the wheels about their mass center are $I_A = I_B = 2mk^2 = 2\left(\dfrac{100}{32.2}\right)(1^2) = 6.211$ slug·ft^2.

Since the wheels roll without slipping, $\omega = \dfrac{v}{r} = \dfrac{v}{1.25} = 0.8v$. From Figs. a, b, and c,

$\zeta +$ $\qquad (H_C)_1 + \Sigma \displaystyle\int_{t_1}^{t_2} M_C\, dt = (H_C)_2$

$\qquad\qquad 0 + 2(100)(10) - A_x(10)(1.25) = 6.211(0.8v) + 2\left[\left(\dfrac{100}{32.2}\right)v\right](1.25)$

$\qquad\qquad A_x = 160 - 1.019v$ $\qquad\qquad\qquad\qquad\qquad$ **(1)**

and

$\zeta +$ $\qquad (H_D)_1 + \Sigma \displaystyle\int_{t_1}^{t_2} M_D\, dt = (H_D)_2$

$\qquad\qquad 0 + B_x(10)(1.25) = 6.211(0.8v) + 2\left[\left(\dfrac{100}{32.2}\right)v\right](1.25)$

$\qquad\qquad B_x = 1.019v$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ **(2)**

From Fig. d,

$\left(\overset{+}{\underset{}{\rightarrow}}\right)$ $\qquad m\left[(v_G)_x\right]_1 + \Sigma \displaystyle\int_{t_1}^{t_2} F_x\, dt = m\left[(v_G)_x\right]_2$
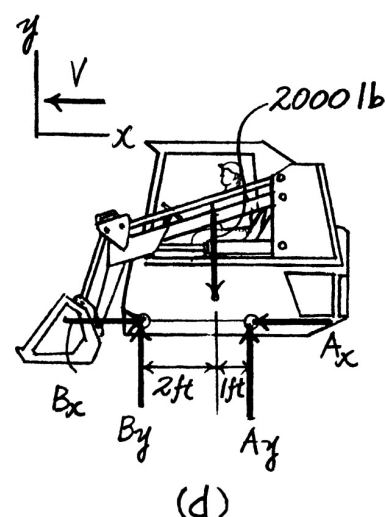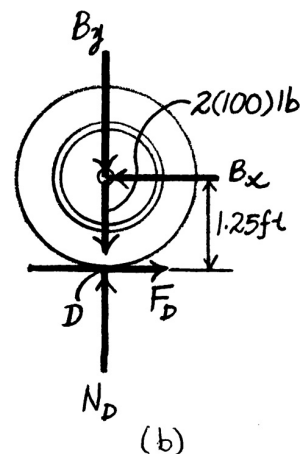
$\qquad\qquad 0 + A_x(10) - B_x(10) = \left(\dfrac{2000}{32.2}\right)v$ $\qquad\qquad$ **(3)**

Substituting Eqs. (1) and (2) into Eq. (3),

$\qquad\qquad (160 - 1.019v)(10) - 1.019v(10) = \left(\dfrac{2000}{32.2}\right)v$

$\qquad\qquad v = 19.4$ ft/s $\qquad\qquad\qquad\qquad\qquad\qquad$ **Ans.**

(a)

(b)

(c)

(d)
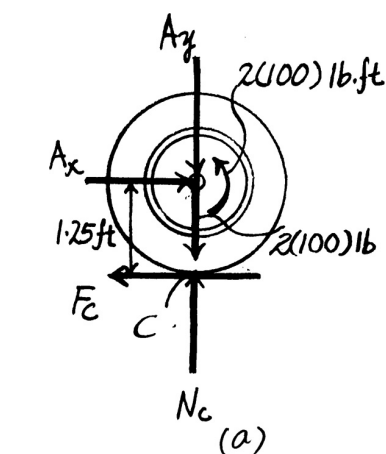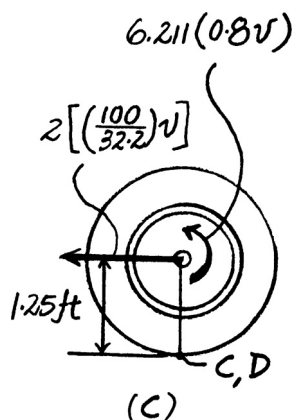
19–27. The body and bucket of a skid steer loader has a weight of 2000 lb, and its center of gravity is located at G. Each of the four wheels has a weight of 100 lb and a radius of gyration about its center of gravity of 1 ft. If the loader attains a speed of 20 ft/s in 10 s, starting from rest, determine the torque $\mathbf{M}$ supplied to each of the rear drive wheels. The wheels roll without slipping.
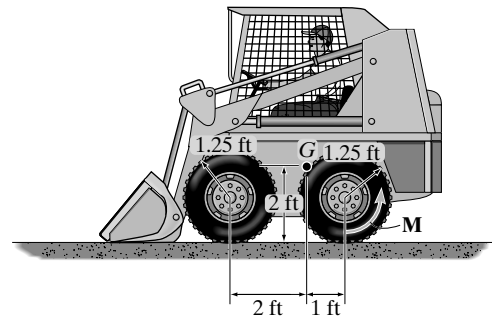
Principle of Impulse and Momentum: The mass momentum of inertia of the wheels about their mass centers are $I_A = I_B = 2mk^2 = 2\left(\dfrac{100}{32.2}\right)(1^2) = 6.211 \text{ slug} \cdot \text{ft}^2$.

Since the wheels roll without slipping, $\omega = \dfrac{v}{r} = \dfrac{20}{1.25} = 16 \text{ rad/s}$. From Figs. a, b, and c,

$\zeta + \qquad (H_C)_1 + \Sigma \displaystyle\int_{t_1}^{t_2} M_C\, dt = (H_C)_2$

$\qquad 0 + 2M(10) - A_x(10)(1.25) = 6.211(16) + 2\left[\dfrac{100}{32.2}(20)\right](1.25)$

$\qquad A_x = 1.6M - 20.37 \qquad\qquad\qquad\qquad (1)$

and

$\zeta + \qquad (H_D)_1 + \Sigma \displaystyle\int_{t_1}^{t_2} M_D\, dt = (H_D)_2$

$\qquad 0 + B_x(10)(1.25) = 6.211(16) + 2\left[\dfrac{100}{32.2}(20)\right](1.25)$

$\qquad B_x = 20.37 \text{ lb} \qquad\qquad\qquad\qquad (2)$

From Fig. d,

$\left(\overset{+}{\leftarrow}\right) \qquad m\left[(v_G)_x\right]_1 + \Sigma \displaystyle\int_{t_1}^{t_2} F_x\, dt = m\left[(v_G)_x\right]_2$
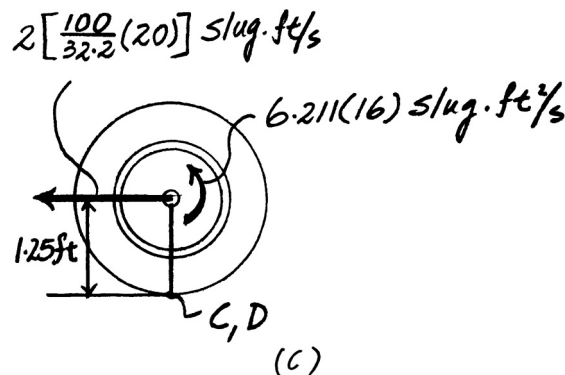
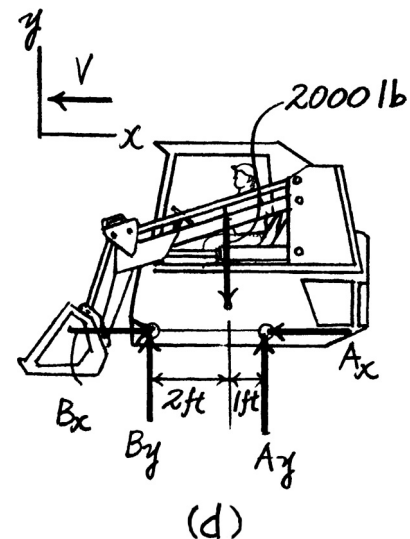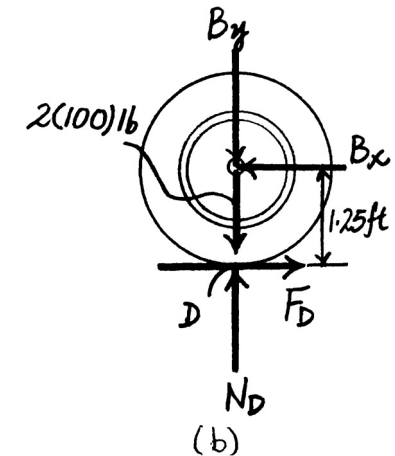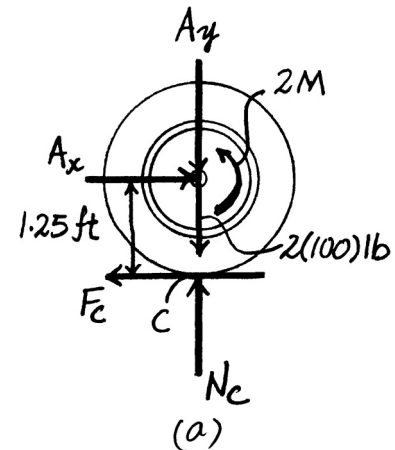$\qquad 0 + A_x(10) - B_x(10) = \dfrac{2000}{32.2}(20) \qquad\qquad (3)$

Substituting Eqs. (1) and (2) into Eq. (3),

$\qquad (1.6M - 20.37)(10) - 20.37(10) = \dfrac{2000}{32.2}(20)$

$\qquad M = 103 \text{ lb} \cdot \text{ft} \qquad\qquad\qquad\qquad$ **Ans.**

(a)

(b)

$2\left[\dfrac{100}{32.2}(20)\right]$ slug·ft/s

$6.211(16)$ slug·ft²/s

1.25 ft

C, D

(c)

(d)

*19–28. The two rods each have a mass m and a length l, and lie on the smooth horizontal plane. If an impulse **I** is applied at an angle of 45° to one of the rods at midlength as shown, determine the angular velocity of each rod just after the impact. The rods are pin connected at B.
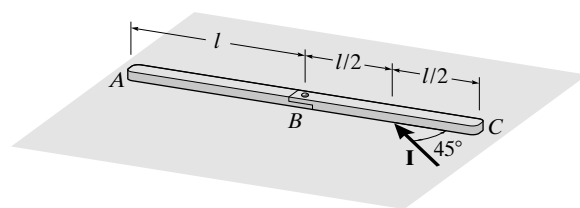
Bar BC:

$$(\zeta+) \qquad (H_G)_1 + \Sigma \int M_G \, dt = (H_G)_2$$

$$0 + \int B_y \, dt \left(\frac{l}{2}\right) = I_G \, \omega_{BC} \qquad\qquad \textbf{(1)}$$

$$(+\uparrow) \qquad m(v_{Gy})_1 + \Sigma \int F_y \, dt = m(v_{Gy})_2$$

$$0 - \int B_y \, dt + I \sin 45° = m(v_G)_y \qquad\qquad \textbf{(2)}$$

Bar AB:

$$(\zeta+) \qquad (H_{G'})_1 + \Sigma \int M_{G'} \, dt = (H_{G'})_2$$

$$0 + \int B_y \, dt \left(\frac{l}{2}\right) = I_G \, \omega_{AB} \qquad\qquad \textbf{(3)}$$

$$\left(+\uparrow\right) \qquad m(v_{Gy})_1 + \Sigma \int F_y \, dt = m(v_{Gy})_2$$

$$0 + \int B_y \, dt = m(v_{G'})_y \qquad\qquad \textbf{(4)}$$

$$\mathbf{v}_B = \mathbf{v}_{G'} + \mathbf{v}_{B/G'} = \mathbf{v}_G + \mathbf{v}_{B/G}$$

$$\left(+\uparrow\right) \qquad v_{By} = (v_{G'})_y + \omega_{AB}\left(\frac{l}{2}\right) = (v_G)_y - \omega_{BC}\left(\frac{l}{2}\right) \qquad\qquad \textbf{(5)}$$
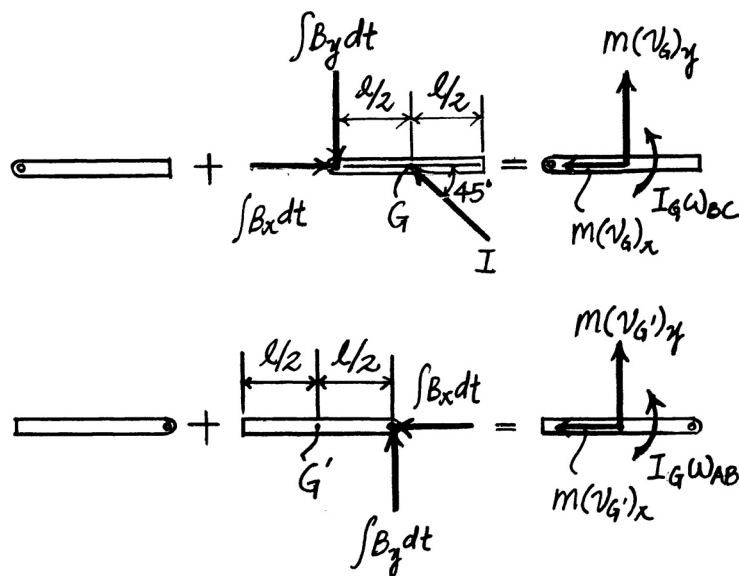
*19–28.   Continued

Eliminate $\int B_y \, dt$ from Eqs. (1) and (2), from Eqs. (3) and (4), and between Eqs. (1) and (3). This yields

$$I_G \, \omega_{BC} = \frac{l}{2}(I \sin 45° - m(v_G)_y)$$

$$m(v_G)_y \left(\frac{l}{2}\right) = I_G \, \omega_{AB}$$

$$\omega_{BC} = \omega_{AB}$$

Substituting into Eq. (5),

$$\frac{1}{m}\left(\frac{2}{l}\right)I_G \, \omega_{AB} + \omega_{AB}\left(\frac{l}{2}\right) = -\left[I_G\left(\frac{\omega_{AB}}{m}\right)\left(\frac{2}{l}\right)\right] + \frac{I}{m}\sin 45° - \omega_{AB}\left(\frac{l}{2}\right)$$
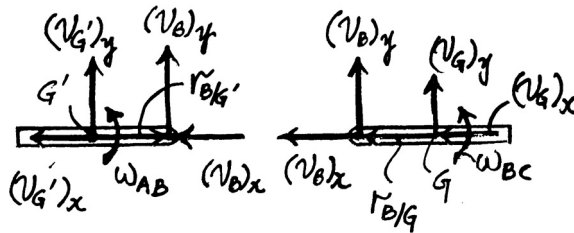
$$\left(\frac{4}{ml}\right)I_G\omega_{AB} + \omega_{AB}\, l = \frac{I}{m}\sin 45°$$

$$\left(\frac{4}{ml}\right)\left(\frac{1}{12}ml^2\right)\omega_{AB} + \omega_{AB}\, l = \frac{I}{m}\sin 45°$$

$$\frac{4}{3}\omega_{AB}\, I = \frac{I}{m}\sin 45°$$

$$\omega_{AB} = \omega_{BC} = \frac{3}{4\sqrt{2}}\left(\frac{I}{ml}\right) \qquad\qquad\qquad \textbf{Ans.}$$

•19–29. The car strikes the side of a light pole, which is designed to break away from its base with negligible resistance. From a video taken of the collision it is observed that the pole was given an angular velocity of 60 rad/s when AC was vertical. The pole has a mass of 175 kg, a center of mass at G, and a radius of gyration about an axis perpendicular to the plane of the pole assembly and passing through G of $k_G = 2.25$ m. Determine the horizontal impulse which the car exerts on the pole at the instant AC is essentially vertical.
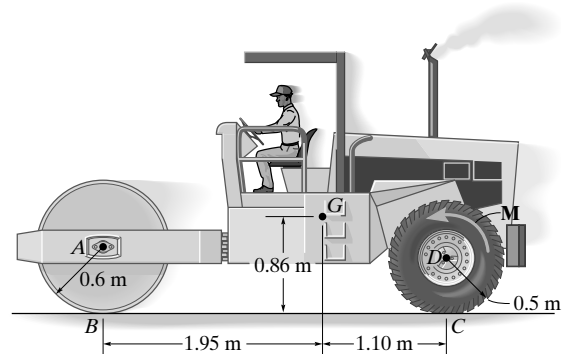
$$(\zeta +) \qquad (H_G)_1 + \Sigma \int M_G \, dt = (H_G)_2$$

$$0 + \left[\int F \, dt \right](3.5) = 175(2.25)^2 (60)$$

$$\int F \, dt = 15.2 \text{ kN} \cdot \text{s}$$
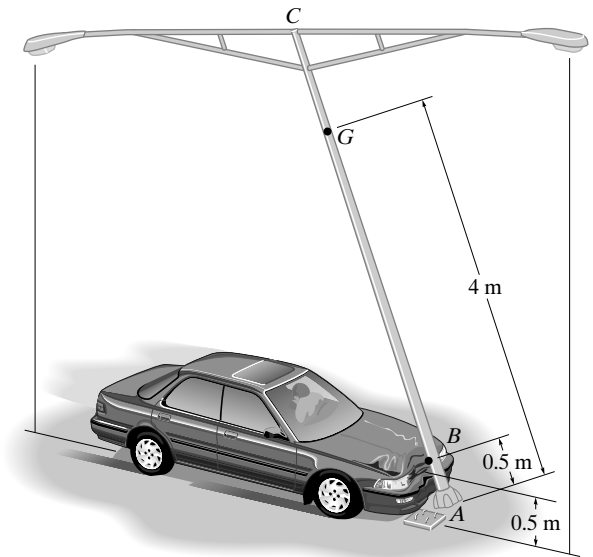
Ans.
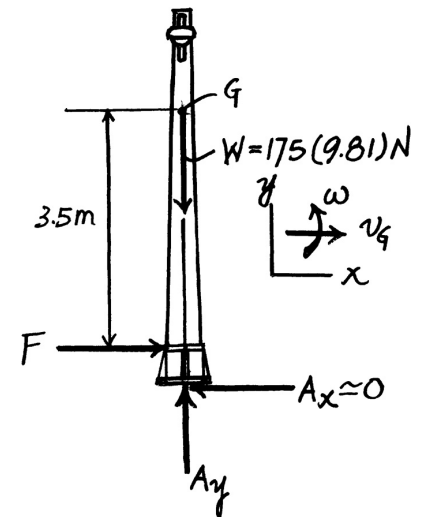
19–30. The frame of the roller has a mass of 5.5 Mg and a center of mass at G. The roller has a mass of 2 Mg and a radius of gyration about its mass center of $k_A = 0.45$ m. If a torque of $M = 600$ N·m is applied to the rear wheels, determine the speed of the compactor in $t = 4$ s, starting from rest. No slipping occurs. Neglect the mass of the driving wheels.

Driving Wheels: (mass is neglected)

$$\zeta + \Sigma M_D = 0; \qquad 600 - F_C(0.5) = 0$$

$$F_C = 1200 \text{ N}$$

Frame and driving wheels:

$$\left(\xleftrightarrow{} \right) \qquad m(v_{Gx})_1 + \Sigma \int F_x \, dt = m(v_{Gx})_2$$

$$0 + 1200(4) - A_x(4) = 5500v_G$$

$$A_x = 12\,00 - 1375v_G \qquad\qquad (1)$$

Roller:

$$v_G = v_A = 0.6\omega$$

$$(\zeta +) \qquad (H_B)_1 + \Sigma \int M_B \, dt = (H_B)_2$$

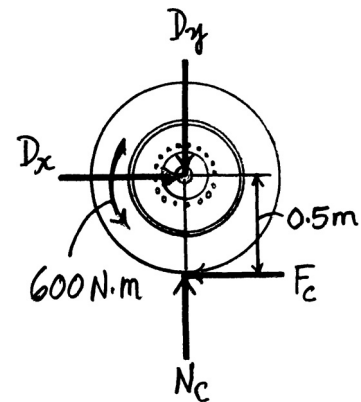$$0 + A_x(4)(0.6) = \left[2000(0.45)^2\right]\left(\frac{v_G}{0.6}\right) + \left[2000(v_G)\right](0.6)$$

$$A_x = 781.25v_G \qquad\qquad (2)$$

Solving Eqs. (1) and (2):

$$A_x = 435 \text{ N}$$

$$v_G = 0.557 \text{ m/s} \qquad\qquad \textbf{Ans.}$$
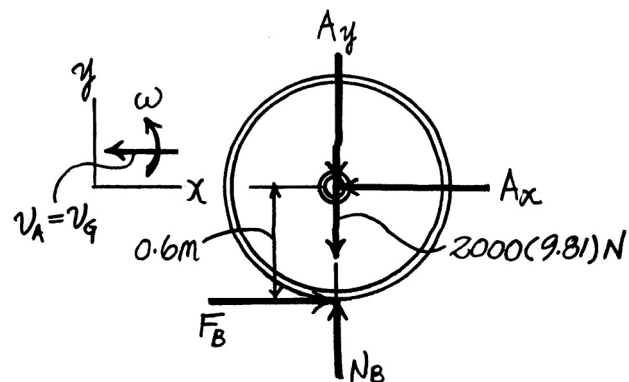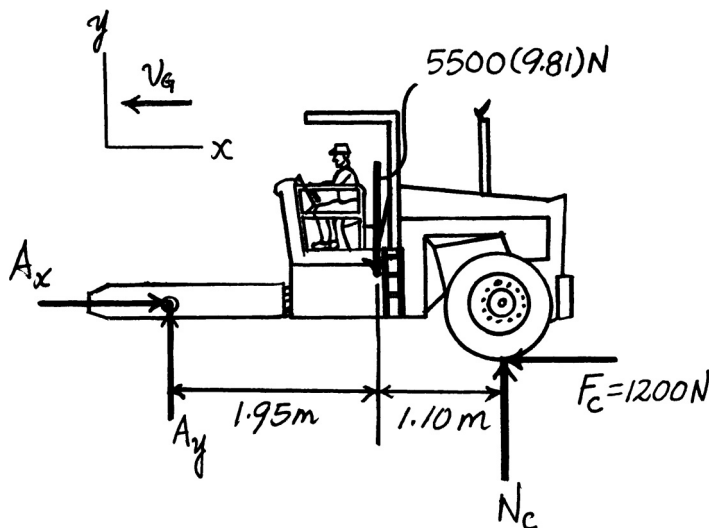
19–31. The 200-kg satellite has a radius of gyration about the centroidal z axis of $k_z = 1.25$ m. Initially it is rotating with a constant angular velocity of $\boldsymbol{\omega}_0 = \{1500\,\mathbf{k}\}$ rev/min. If the two jets A and B are fired simultaneously and produce a thrust of $T = (5e^{-0.1t})$ kN, where t is in seconds, determine the angular velocity of the satellite, five seconds after firing.

$T = (5e^{(-t/10)})$ kN

1.5 m

B

A

1.5 m y

$T = (5e^{(-t/10)})$ kN
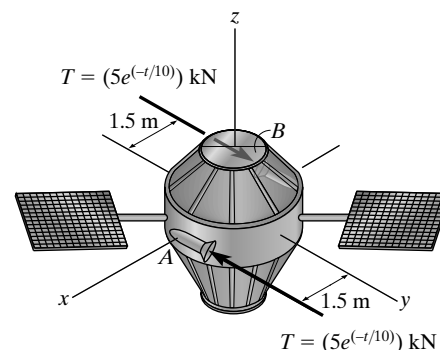
Principle of Angular Impulse and Momentum: The mass moment of inertia of the satellite about its centroidal z axis is $I_z = mk_z^2 = 200(1.25^2) = 312.5$ kg·m². The

initial angular velocity of the satellite is $\omega_1 = \left(1500\,\dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi\,\text{rad}}{1\,\text{rev}}\right)\left(\dfrac{1\,\text{min}}{60\,\text{s}}\right)$

$= 50\pi$ rad/s. Applying the angular impulse and momentum equation about the z axis,

$$I_z\omega_1 + \Sigma \int_{t_1}^{t_2} M_z\,dt = I_z\omega_2$$

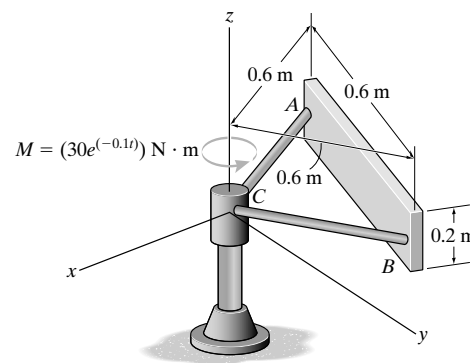$$312.5(50\pi) - \left[2\int_0^{5\,\text{s}} 5000e^{-0.1t}(1.5)dt\right] = 312.5\omega_2$$

$$15625\pi + \left(150\,000e^{-0.1t}\right)\Big|_0^{5\,\text{s}} = 312.5\omega_2$$

Thus,

$$\omega_2 = [-31.8\mathbf{k}]\,\text{rad/s} \qquad\qquad \textbf{Ans.}$$

---

*19–32. If the shaft is subjected to a torque of $M = (30e^{-0.1t})\,\text{N·m}$, where t is in seconds, determine the angular velocity of the assembly when $t = 5$ s, starting from rest. The rectangular plate has a mass of 25 kg. Rods AC and BC have the same mass of 5 kg.

0.6 m
0.6 m
A

$M = (30e^{(-0.1t)})\,\text{N·m}$

0.6 m

C

B

0.2 m

Principle of Angular Impulse and Momentum: The mass moment of inertia of the

assembly about the z axis is $I_z = 2\left[\dfrac{1}{3}(5)\left(0.6^2\right)\right] + \left[\dfrac{1}{12}(25)\left(0.6^2\right) + 25(0.6\sin 60°)^2\right]$

$= 8.70$ kg·m². Using the free-body diagram of the assembly shown in Fig. a,

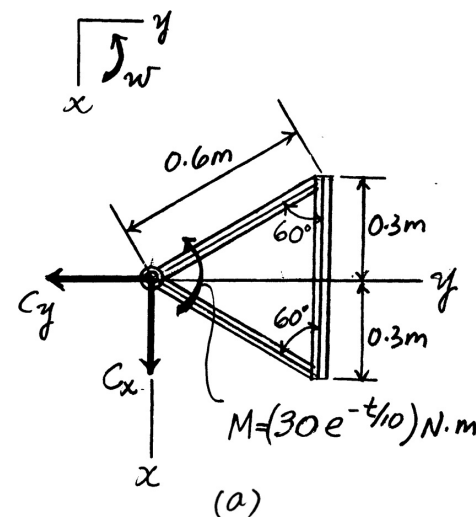$$\zeta + I_z\omega_1 + \Sigma \int_{t_1}^{t_2} M_z\,dt = I_z\omega_2$$

$$0 + \int_0^{5\,\text{s}} 30e^{-0.1t}\,dt = 8.70\omega_2$$

$$\left(-300e^{-0.1t}\right)\Big|_0^{5\,\text{s}} = 8.70\omega_2$$

Thus,

$$\omega_2 = 13.6\,\text{rad/s} \qquad\qquad \textbf{Ans.}$$

$\mathbf{J}_w$

y

x

0.6m

60°
0.3m

C_y

60°
0.3m

y

C_x

$M = (30e^{-t/10})\,\text{N·m}$

x

(a)

•19–33. The 75-kg gymnast lets go of the horizontal bar in a fully stretched position A, rotating with an angular velocity of $\omega_A = 3$ rad/s. Estimate his angular velocity when he assumes a tucked position B. Assume the gymnast at positions A and B as a uniform slender rod and a uniform circular disk, respectively.
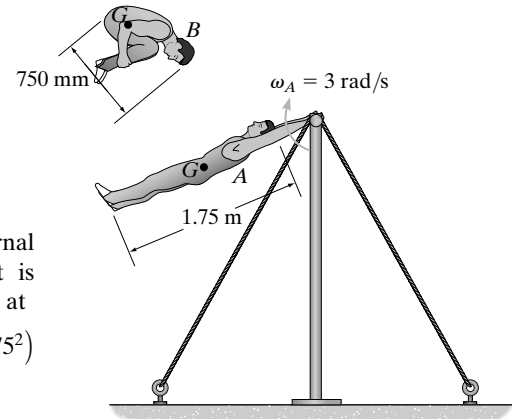
Conservation of Angular Momentum: Other than the weight, there is no external impulse during the motion. Thus, the angular momentum of the gymnast is conserved about his mass center G. The mass moments of inertia of the gymnast at the fully-stretched and tucked positions are $(I_A)_G = \dfrac{1}{12}ml^2 = \dfrac{1}{12}(75)(1.75^2)$

$= 19.14 \text{ kg} \cdot \text{m}^2$ and $(I_B)_G = \dfrac{1}{2}mr^2 = \dfrac{1}{2}(75)(0.375^2) = 5.273 \text{ kg} \cdot \text{m}^2$. Thus,

$$(H_A)_G = (H_B)_G$$

$$19.14(3) = 5.273\omega_B$$

$$\omega_B = 10.9 \text{ rad/s} \qquad\qquad\qquad \textbf{Ans.}$$

19–34. A 75-kg man stands on the turntable A and rotates a 6-kg slender rod over his head. If the angular velocity of the rod is $\omega_r = 5$ rad/s measured relative to the man and the turntable is observed to be rotating in the opposite direction with an angular velocity of $\omega_t = 3$ rad/s, determine the radius of gyration of the man about the z axis. Consider the turntable as a thin circular disk of 300-mm radius and 5-kg mass.
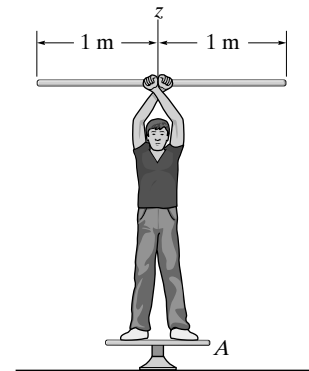
Conservation of Angular Momentum: The mass moment of inertia of the rod about the z axis is $(I_r)_z = \dfrac{1}{12}ml^2 = \dfrac{1}{12}(6)(2^2) = 2 \text{ kg} \cdot \text{m}^2$ and the mass moment of inertia of the man and the turntable about the z axis is $(I_m)_z = \dfrac{1}{2}(5)(0.3^2) + 75k_z^2$

$= 0.225 + 75k_z^2$. Since no external angular impulse acts on the system, the angular momentum of the system is conserverved about the z axis.

$$(H_z)_1 = (H_z)_2$$

$$0 = 2(\omega_r) - (0.225 + 75k_z^2)(3)$$

$$\omega_r = \omega_m + lm$$

$$\omega_r = -3 + 5 = 2 \text{ rad/s}$$

$$2(2) = (0.225 + 75k_z^2)3$$

$$k_z = 0.122 \text{ m} \qquad\qquad\qquad \textbf{Ans.}$$

19–35. A horizontal circular platform has a weight of 300 lb and a radius of gyration $k_z = 8$ ft about the z axis passing through its center O. The platform is free to rotate about the z axis and is initially at rest. A man having a weight of 150 lb begins to run along the edge in a circular path of radius 10 ft. If he maintains a speed of 4 ft/s relative to the platform, determine the angular velocity of the platform. Neglect friction.
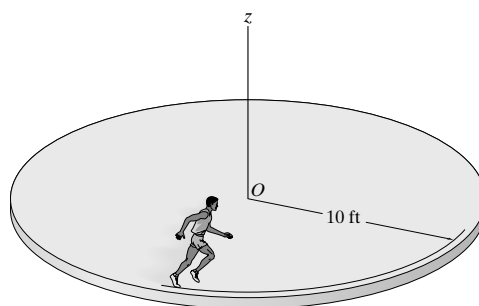
$$\mathbf{v}_m = \mathbf{v}_p + \mathbf{v}_{m/p}$$

$(\overset{+}{\rightarrow})$ $v_m = -10\omega + 4$
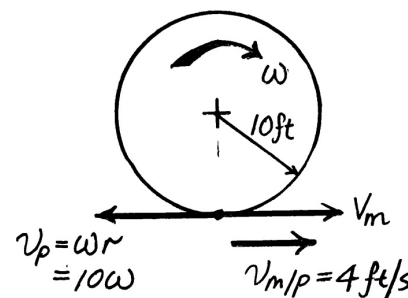
$(\zeta +)$ $(H_z)_1 = (H_z)_2$

$$0 = -\left(\frac{300}{32.2}\right)(8)^2\,\omega + \left(\frac{150}{32.2}\right)(-10\omega + 4)(10)$$

$\omega = 0.175$ rad/s   **Ans.**

*19–36. A horizontal circular platform has a weight of 300 lb and a radius of gyration $k_z = 8$ ft about the z axis passing through its center O. The platform is free to rotate about the z axis and is initially at rest. A man having a weight of 150 lb throws a 15-lb block off the edge of the platform with a horizontal velocity of 5 ft/s, *measured relative to the platform*. Determine the angular velocity of the platform if the block is thrown (a) tangent to the platform, along the $+t$ axis, and (b) outward along a radial line, or $+n$ axis. Neglect the size of the man.

a) $(H_z)_1 = (H_z)_2$

$$0 + 0 = \left(\frac{15}{32.2}\right)(v_b)(10) - \left(\frac{300}{32.2}\right)(8)^2\,\omega - \left(\frac{150}{32.2}\right)(10\omega)(10)$$

$v_b = 228\omega$

$$\mathbf{v}_b = \mathbf{v}_m + \mathbf{v}_{b/m}$$

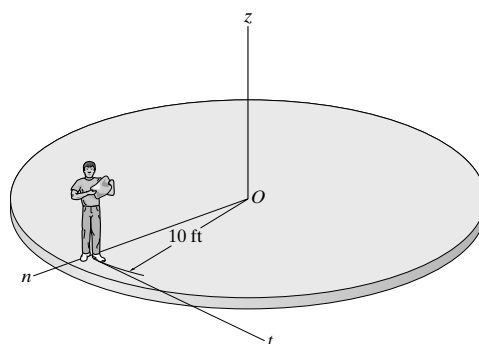$(\overset{+}{\rightarrow})$ $v_b = -10\omega + 5$

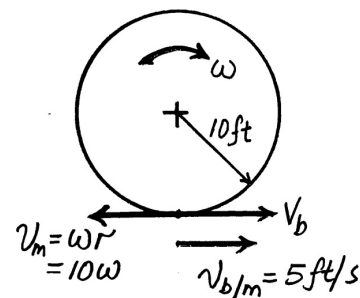$228\omega = -10\omega + 5$

$\omega = 0.0210$ rad/s   **Ans.**

b) $(H_z)_1 = (H_z)_2$

$$0 + 0 = 0 - \left(\frac{300}{32.2}\right)(8)^2\,\omega - \left(\frac{150}{32.2}\right)(10\omega)(10)$$

$\omega = 0$ **Ans.**

•**19–37.** The man sits on the swivel chair holding two 5-lb weights with his arms outstretched. If he is rotating at 3 rad/s in this position, determine his angular velocity when the weights are drawn in and held 0.3 ft from the axis of rotation. Assume he weighs 160 lb and has a radius of gyration $k_z = 0.55$ ft about the z axis. Neglect the mass of his arms and the size of the weights for the calculation.
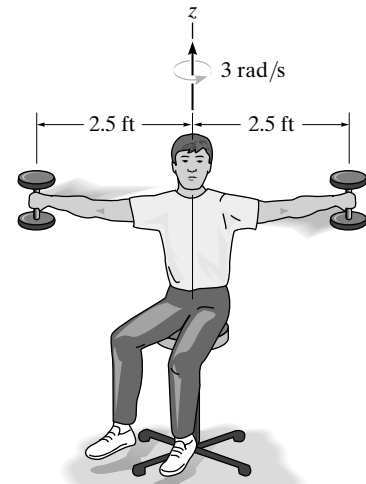
Mass Moment of Inertia: The mass moment inertia of the man and the weights about z axis when the man arms are fully stretched is

$$(I_z)_1 = \left(\frac{160}{32.2}\right)(0.55^2) + 2\left[\frac{5}{32.2}(2.5^2)\right] = 3.444 \text{ slug} \cdot \text{ft}^2$$

The mass moment inertia of the man and the weights about z axis when the weights are drawn in to a distance 0.3 ft from z axis

$$(I_z)_2 = \left(\frac{160}{32.2}\right)(0.55^2) + 2\left[\frac{5}{32.2}(0.3^2)\right] = 1.531 \text{ slug} \cdot \text{ft}^2$$

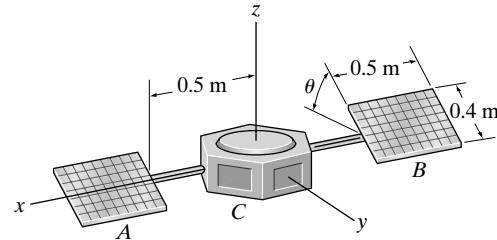**Conservation of Angular Momentum:** Applying Eq. 19–17, we have

$$(H_z)_1 = (H_z)_2$$

$$3.444(3) = 1.531(\omega_z)_2$$

$$(\omega_z)_2 = 6.75 \text{ rad/s} \qquad \textbf{Ans.}$$

19–38. The satellite's body C has a mass of 200 kg and a radius of gyration about the z axis of $k_z = 0.2$ m. If the satellite rotates about the z axis with an angular velocity of 5 rev/s, when the solar panels A and B are in a position of $\theta = 0°$, determine the angular velocity of the satellite when the solar panels are rotated to a position of $\theta = 90°$. Consider each solar panel to be a thin plate having a mass of 30 kg. Neglect the mass of the rods.

Conservation of Angular Momentum: When $\theta = 0°$ and $\theta = 90°$, the mass momentum of inertia of the satellite are

$$(I_z)_1 = 200(0.2^2) + 2\left[\frac{1}{12}(30)(0.5^2 + 0.4^2) + 30(0.75^2)\right]$$

$$= 43.8 \text{ kg} \cdot \text{m}^2$$

$$(I_z)_2 = 200(0.2^2) + 2\left[\frac{1}{12}(30)(0.5^2) + 30(0.75^2)\right]$$

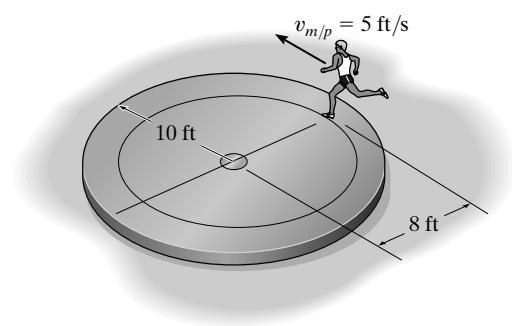$$= 43 \text{ kg} \cdot \text{m}^2$$

Thus,

$$(H_z)_1 = (H_z)_2$$

$$(I_z)_1 \omega_1 = (I_z)_2 \omega_2$$

$$43.8(5) = 43\omega_2$$

$$\omega_2 = 5.09 \text{ rev/s} \qquad \textbf{Ans.}$$

19–39. A 150-lb man leaps off the circular platform with a velocity of $v_{m/p} = 5$ ft/s, relative to the platform. Determine the angular velocity of the platform afterwards. Initially the man and platform are at rest. The platform weighs 300 lb and can be treated as a uniform circular disk.

Kinematics: Since the platform rotates about a fixed axis, the speed of point P on the platform to which the man leaps is $v_P = \omega r = \omega(8)$. Applying the relative velocity equation,

$$v_m = v_P + v_{m/P}$$

$$(+\uparrow) \qquad v_m = -\omega(8) + 5 \qquad \qquad \textbf{(1)}$$

Conservation of Angular Momentum: As shown in Fig. b, the impulse $\int F dt$ generated during the leap is internal to the system. Thus, angular momentum of the system is conserved about the axis perpendicular to the page passing through point O. The mass moment of inertia of the platform about this axis is

$$I_O = \frac{1}{2} mr^2 = \frac{1}{2}\left(\frac{300}{32.2}\right)(10^2) = 465.84 \text{ slug} \cdot \text{ft}^2$$
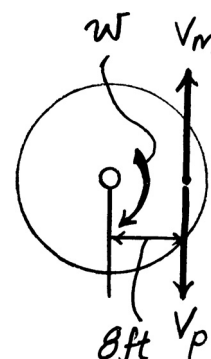
Then

$$(H_O)_1 = (H_O)_2$$

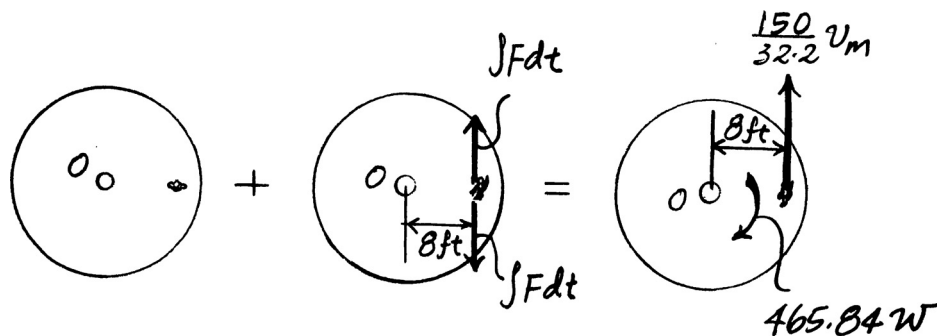$$0 = \left(\frac{150}{32.2} v_m\right)(8) - 465.84\omega$$

$$v_m = 12.5\omega \qquad \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields
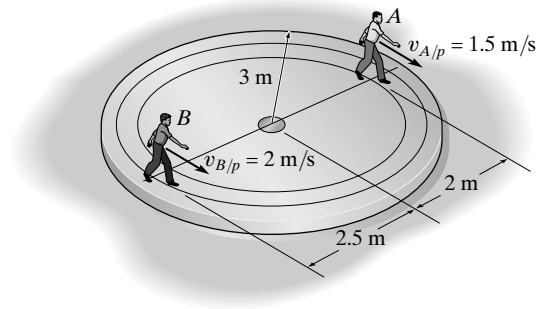
$$\omega = 0.244 \text{ rad/s} \qquad \qquad \textbf{Ans.}$$

$$v_m = 3.05 \text{ ft/s}$$

*19–40. The 150-kg platform can be considered as a circular disk. Two men, A and B, of 60-kg and 75-kg mass, respectively, stand on the platform when it is at rest. If they start to walk around the circular paths with speeds of $v_{A/p} = 1.5$ m/s and $v_{B/p} = 2$ m/s, measured relative to the platform, determine the angular velocity of the platform.

Kinematics: Since the platform rotates about a fixed axis, the speed of points P and P' on the platform at which men B and A are located is $v_P = \omega r_P = \omega(2.5)$ and $v_{P'} = \omega r_{P'} = \omega(2)$. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_P + \mathbf{v}_{B/P}$$

$(+\downarrow) \qquad v_B = -\omega(2.5) + 2 \qquad\qquad (1)$

and

$$\mathbf{v}_A = \mathbf{v}_{P'} + \mathbf{v}_{A/P'}$$

$(+\downarrow) \qquad v_B = \omega(2) + 1.5 \qquad\qquad (2)$

Conservation of Angular Momentum: As shown in Fig. b, the impulses $\int F_A\, dt$ and $\int F_B\, dt$ are internal to the system. Thus, angular momentum of the system is conserved about the axis perpendicular to the page passing through point O. The mass moment of inertia of the platform about this axis is $I_O = \frac{1}{2} mr^2 = \frac{1}{2}(150)(3^2)$ $= 675$ kg·m². Then
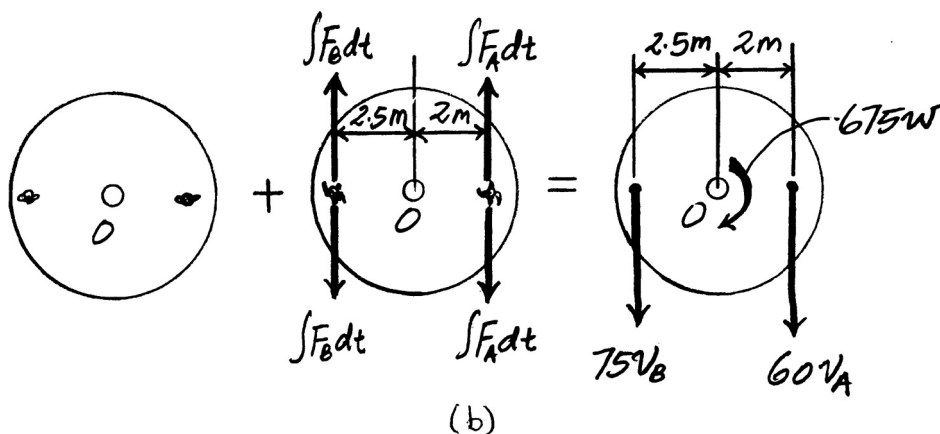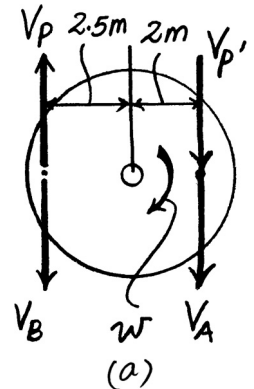
$$(H_O)_1 = (H_O)_2$$

$$0 = 75v_B\,(2.5) - 60v_A\,(2) - 675\omega \qquad\qquad (3)$$

Substituting Eqs. (1) and (2) into Eq. (3),

$$0 = 75(-2.5\omega + 2)(2.5) - 60(2\omega + 1.5)(2) - 675\omega$$

$$\omega = 0.141\ \text{rad/s} \qquad\qquad\qquad\qquad \textbf{Ans.}$$

(a)

(b)

•**19–41.**   Two children A and B, each having a mass of 30 kg, sit at the edge of the merry-go-round which rotates at $\omega = 2$ rad/s. Excluding the children, the merry-go-round has a mass of 180 kg and a radius of gyration $k_z = 0.6$ m. Determine the angular velocity of the merry-go-round if A jumps off horizontally in the $-n$ direction with a speed of 2 m/s, measured relative to the merry-go-round. What is the merry-go-round's angular velocity if B then jumps off horizontally in the $-t$ direction with a speed of 2 m/s, measured relative to the merry-go-round? Neglect friction and the size of each child.
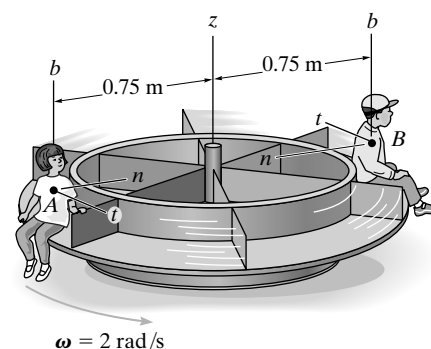
Mass Moment of Inertia: The mass moment inertia of the merry-go-round about the z axis when both children are still on it is

$$(I_z)_1 = 180(0.6^2) + 2[30(0.75^2)] = 98.55 \text{ kg} \cdot \text{m}^2$$

The mass moment inertia of the merry-go-round about z axis when child A jumps off

$$(I_z)_2 = 180(0.6^2) + 30(0.75^2) = 81.675 \text{ kg} \cdot \text{m}^2$$

The mass moment inertia of the merry-go-round about z axis when both children jump off

$$(I_z)_3 = 180(0.6^2) + 0 = 64.80 \text{ kg} \cdot \text{m}^2$$

Conservation of Angular Momentum: When child A jumps off in the $-n$ direction, applying Eq. 19–17, we have

$$(H_z)_1 = (H_z)_2$$

$$(I_z)_1 \, \omega_1 = (I_z)_2 \, \omega_2$$

$$98.55(2) = 81.675\omega_2$$

$$\omega_2 = 2.413 \text{ rad/s} = 2.41 \text{ rad/s} \qquad \textbf{Ans.}$$

Subsequently, when child B jumps off from the merry-go-round in the $-t$ direction, applying Eq. 19–17, we have

$$(H_z)_2 = (H_z)_3$$

$$(I_z)_2 \, \omega_2 = (I_z)_3 \, \omega_3 - (m_B \, v_B)(0.75)$$

$$81.675(2.413) = 64.80\omega_3 - 30v_B \, (0.75) \qquad \textbf{(1)}$$

Relative Velocity: The speed of a point located on the edge of the merry-go-round at the instant child B jumps off is $v_M = \omega_3 \, (0.75)$.
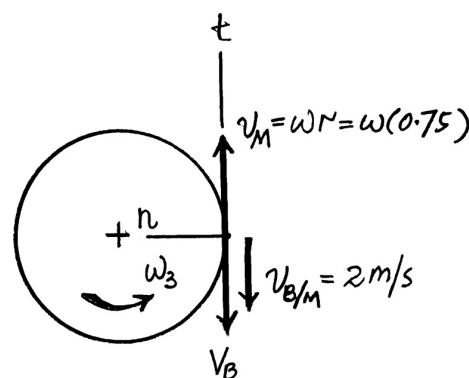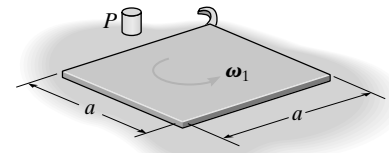
$$v_B = -v_M + v_{B/M} = -\omega_3 \, (0.75) + 2 \qquad \textbf{(2)}$$

Substituting Eq. (2) into Eq. (1) and solving yields

$$\omega_3 = 2.96 \text{ rad/s} \qquad \textbf{Ans.}$$

19–42. A thin square plate of mass m rotates on the smooth surface with an angular velocity ω_1. Determine its new angular velocity just after the hook at its corner strikes the peg P and the plate starts to rotate about P without rebounding.

Mass Moment of Inertia: The mass moment inertia of the thin plate about the z axis passing through its mass center is

$$(I_z)_G = \frac{1}{12}(m)(a^2 + a^2) = \frac{1}{6}ma^2$$

The mass moment inertia of the thin plate about z axis passing through peg P is

$$(I_z)_P = \frac{1}{12}(m)(a^2 + a^2) + m\left[\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2}\right]^2 = \frac{2}{3}ma^2$$

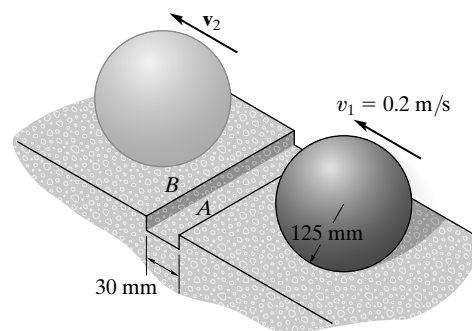**Conservation of Angular Momentum:** Applying Eq. 19–17, we have

$$H_G = H_P$$

$$\left(\frac{1}{6}ma^2\right)\omega_1 = \left(\frac{2}{3}ma^2\right)\omega_2$$

$$\omega_2 = \frac{1}{4}\omega_1 \qquad \text{Ans.}$$

19–43. A ball having a mass of 8 kg and initial speed of $v_1 = 0.2$ m/s rolls over a 30-mm-long depression. Assuming that the ball rolls off the edges of contact first A, then B, without slipping, determine its final velocity $\mathbf{v}_2$ when it reaches the other side.

$\omega_1 = \dfrac{0.2}{0.125} = 1.6$ rad/s $\omega_2 = \dfrac{v_2}{0.125} = 8v_2$

$\theta = \sin^{-1}\left(\dfrac{15}{125}\right) = 6.8921°$

$h = 125 - 125 \cos 6.8921° = 0.90326$ mm

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(8)(0.2)^2 + \frac{1}{2}\left[\frac{2}{5}(8)(0.125)^2\right](1.6)^2 + 0$$

$$= -(0.90326)(10^{-3})8(9.81) + \frac{1}{2}(8)\omega^2(0.125)^2 + \frac{1}{2}\left[\frac{2}{5}(8)(0.125)^2\right](\omega)^2$$

$$\omega = 1.836 \text{ rad/s}$$

$$(H_B)_2 = (H_B)_3$$

$$\left[\frac{2}{5}(8)(0.125)^2\right](1.836) + 8(1.836)(0.125) \cos 6.892°(0.125 \cos 6.892°)$$

$$- 8(0.22948 \sin 6.892°)(0.125 \sin 6.892°)$$

$$= \left[\frac{2}{5}(8)(0.125)^2\right]\omega_3 + 8(0.125)\omega_3 (0.125)$$

$$\omega_3 = 1.7980 \text{ rad/s}$$

$$T_3 + V_3 = T_4 + V_4$$

$$\frac{1}{2}\left[\frac{2}{5}(8)(0.125)^2\right](1.7980)^2 + \frac{1}{2}(8)(1.7980)^2(0.125)^2 + 0$$
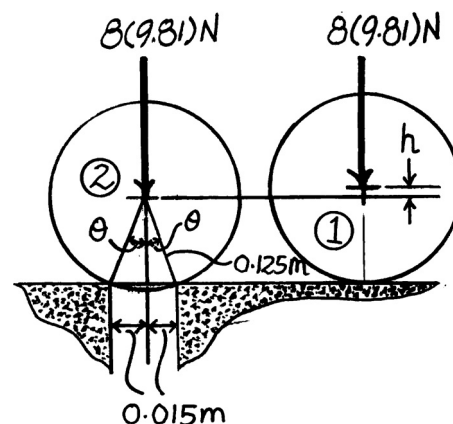
$$= 8(9.81)(0.90326(10^{-3})) + \frac{1}{2}\left[\frac{2}{5}(8)(0.125)^2\right](\omega_4)^2$$
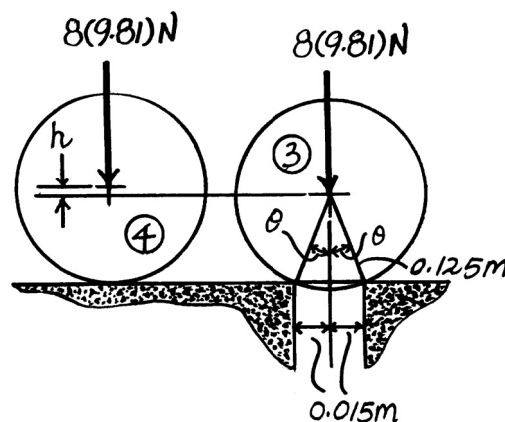
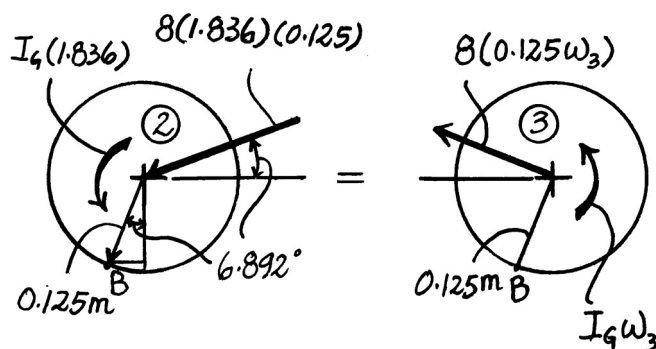$$+ \frac{1}{2}(8)(\omega_4)^2(0.125)^2$$
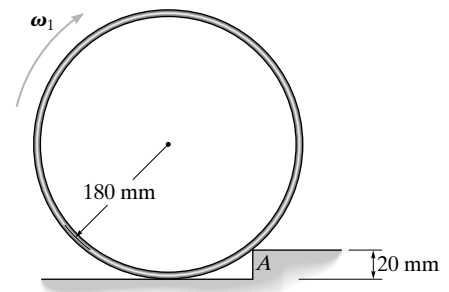
$$\omega_4 = 1.56 \text{ rad/s}$$

So that

$$v_2 = 1.56(0.125) = 0.195 \text{ m/s} \qquad\qquad \textbf{Ans.}$$

813

*19–44. The 15-kg thin ring strikes the 20-mm-high step. Determine the smallest angular velocity ω_1 the ring can have so that it will just roll over the step at A without slipping

The weight is non-impulsive.

$(H_A)_1 = (H_A)_2$

$15(\omega_1)(0.18)(0.18 - 0.02) + \left[15(0.18)^2\right](\omega_1) = \left[15(0.18)^2 + 15(0.18)^2\right]\omega_2$
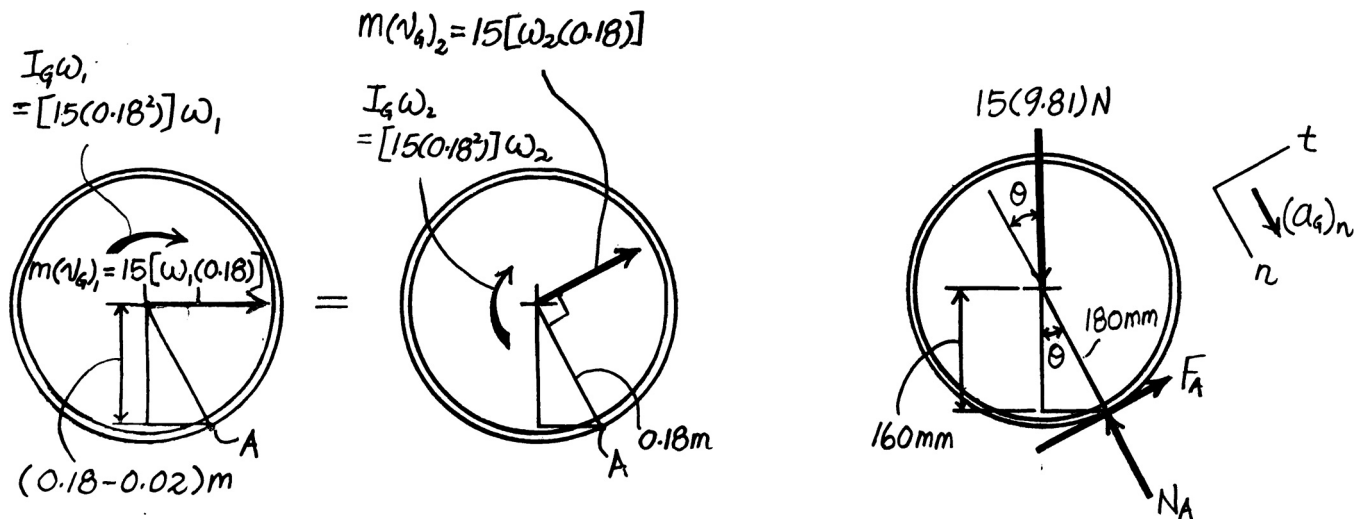
$\omega_2 = 0.9444\omega_1$

$_+\searrow\Sigma F_n = m(a_G)_n;$ $(15)(9.81)\cos\theta - N_A = 15\omega_2^2(0.18)$

When hoop is about to rebound, $N_A - 0$. Also, $\cos\theta = \dfrac{160}{180}$, and so

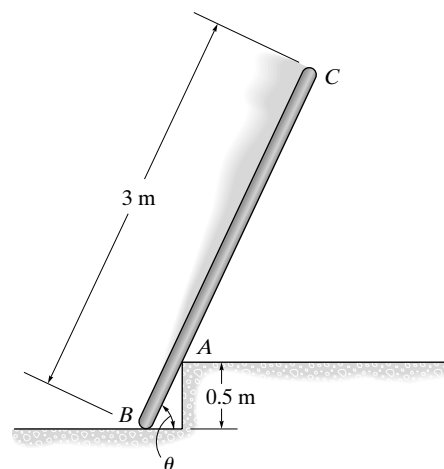$\qquad\qquad \omega_2 = 6.9602 \text{ rad/s}$

$\qquad\qquad \omega_1 = \dfrac{6.9602}{0.9444} = 7.37 \text{ rad/s}$ **Ans.**

•19–45. The uniform pole has a mass of 15 kg and falls from rest when $\theta = 90°$. It strikes the edge at A when $\theta = 60°$. If the pole then begins to pivot about this point after contact, determine the pole's angular velocity just after the impact. Assume that the pole does not slip at B as it falls until it strikes A.

Conservation of Energy: Datum is set at point B. When the pole is at its initial and final position, its center of gravity is located 1.5 m and 1.5 sin 60° m = 1.299 m *above* the datum. Its initial and final potential energy are 15(9.81)(1.5) = 220.725 N·m and 15(9.81)(1.299) = 191.15 N·m. The mass moment of inertia about point B is $I_B = \dfrac{1}{12}(15)(3^2) + 15(1.5^2) = 45.0$ kg·m². The kinetic energy of the pole before the impact is $\dfrac{1}{2} I_B \omega_1^2 = \dfrac{1}{2}(45.0)\omega_1^2 = 22.5\omega_1^2$. Applying Eq. 18–18, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 220.725 = 22.5\omega_1^2 + 191.15$$

$$\omega_1 = 1.146 \text{ rad/s}$$

Conservation of Angular Momentum: Since the weight of the pole is *nonimpulsive* force, the angular momentum is conserved about point A. The velocity of its mass center before impact is $v_G = \omega_1 r_{GB} = 1.146(1.5) = 1.720$ m/s. The mass moment of inertia of the pole about its mass center and point A are

$$I_G = \frac{1}{12}(15)(3^2) = 11.25 \text{ kg·m}^2$$

and

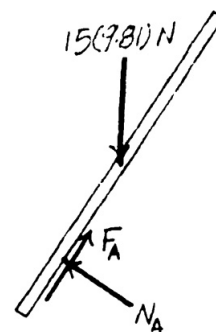$$I_A = \frac{1}{12}(15)(3^2) + 15\left(1.5 - \frac{0.5}{\sin 60°}\right)^2 = 24.02 \text{ kg·m}^2$$

Applying Eq. 19–17, we have

$$(H_A)_1 = (H_A)_2$$

$$(mv_G)(r_{GA}) + I_G\omega_1 = I_A\omega_2$$

$$[15(1.720)]\left(1.5 - \frac{0.5}{\sin 60°}\right) + 11.25(1.146) = 24.02\omega_2$$

$$\omega_2 = 1.53 \text{ rad/s} \qquad\qquad \textbf{Ans.}$$

19–46. The 10-lb block slides on the smooth surface when the corner D hits a stop block S. Determine the minimum velocity $\mathbf{v}$ the block should have which would allow it to tip over on its side and land in the position shown. Neglect the size of S. *Hint:* During impact consider the weight of the block to be nonimpulsive.
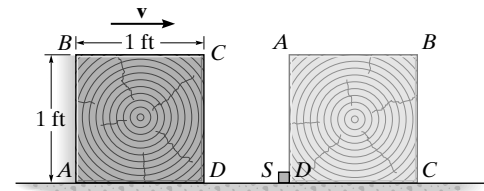
Conservation of Energy: If the block tips over about point D, it must at least achieve the dash position shown. Datum is set at point D. When the block is at its initial and final position, its center of gravity is located 0.5 ft and 0.7071 ft *above* the datum. Its initial and final potential energy are $10(0.5) = 5.00$ ft · lb and $10(0.7071) = 7.071$ ft · lb. The mass moment of inertia of the block about point D is

$$I_D = \frac{1}{12}\left(\frac{10}{32.2}\right)(1^2 + 1^2) + \left(\frac{10}{32.2}\right)(\sqrt{0.5^2 + 0.5^2})^2 = 0.2070 \text{ slug} \cdot \text{ft}^2$$

The initial kinetic energy of the block (after the impact) is $\frac{1}{2}I_D\,\omega_2^2 = \frac{1}{2}(0.2070)\,\omega_2^2$. Applying Eq. 18–18, we have

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}(0.2070)\,\omega_2^2 + 5.00 = 0 + 7.071$$
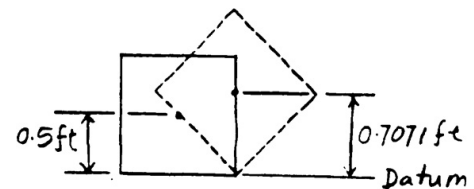
$$\omega_2 = 4.472 \text{ rad/s}$$
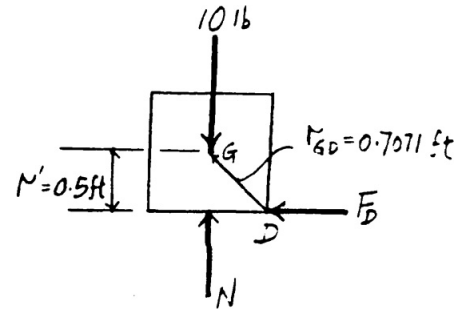
Conservation of Angular Momentum: Since the weight of the block and the normal reaction N are *nonimpulsive* forces, the angular momentum is conserves about point D. Applying Eq. 19–17, we have

$$(H_D)_1 = (H_D)_2$$
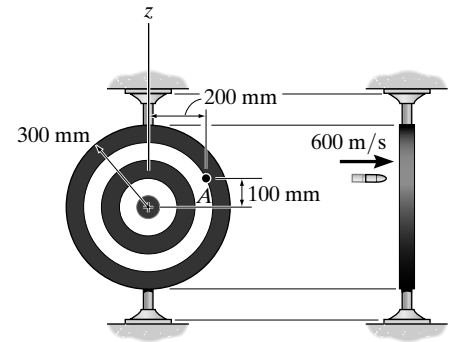
$$(mv_G)(r') = I_D\,\omega_2$$

$$\left[\left(\frac{10}{32.2}\right)v\right](0.5) = 0.2070(4.472)$$

$$v = 5.96 \text{ ft/s} \qquad\qquad \textbf{Ans.}$$

19–47. The target is a thin 5-kg circular disk that can rotate freely about the z axis. A 25-g bullet, traveling at 600 m/s, strikes the target at A and becomes embedded in it. Determine the angular velocity of the target after the impact. Initially, it is at rest.
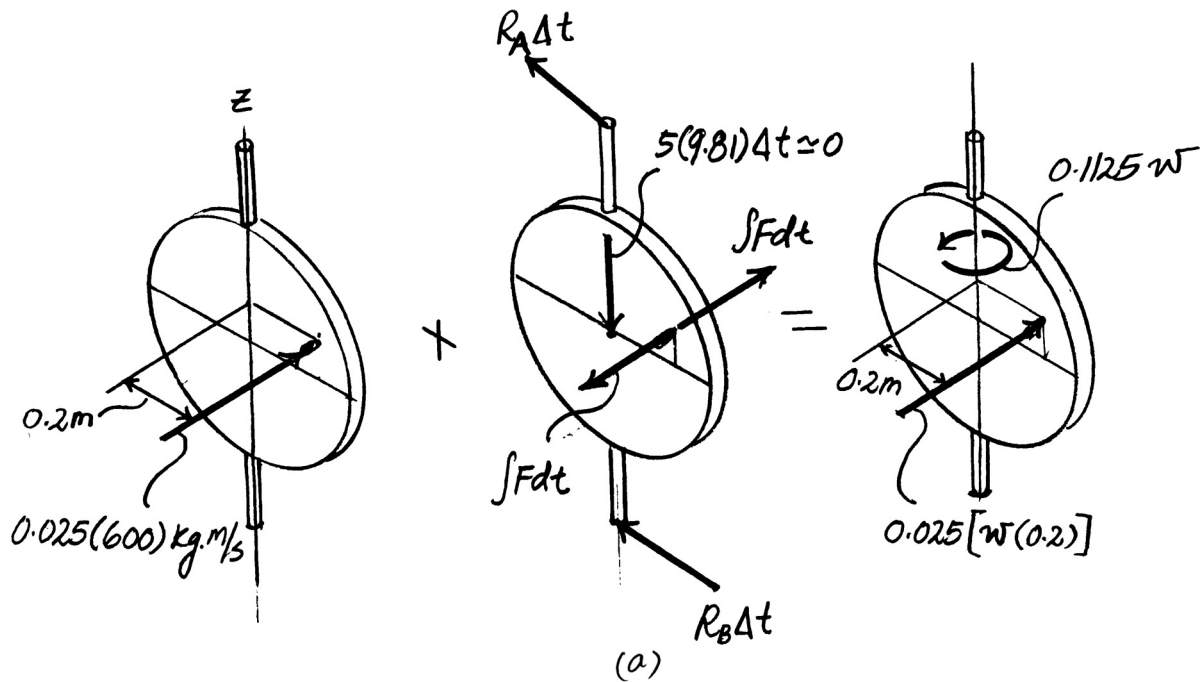
Conservation of Angular Momentum: Referring to Fig. a, the sum of the angular impulse of the system about the z axis is zero. Thus, the angular impulse of the system is conserved about the z axis. The mass moment of inertia of the target about the z axis is $I_z = \dfrac{1}{4}mr^2 = \dfrac{1}{4}(5)(0.3^2) = 0.1125 \text{ kg} \cdot \text{m}^2$. Since the target rotates about the z axis when the bullet is embedded in the target, the bullet's velocity is $(v_b)_2 = \omega(0.2)$. Then,
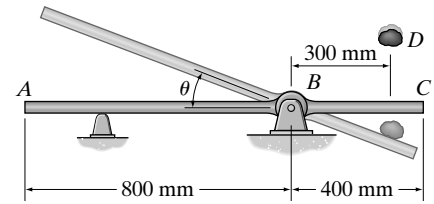
$$(H_z)_1 = (H_z)_2$$

$$0.025(600)(0.2) = 0.1125\omega + 0.025\big[\omega(0.2)\big](0.2)$$

$$\omega = 26.4 \text{ rad/s} \qquad\qquad \textbf{Ans.}$$

(a)

*19–48. A 2-kg mass of putty D strikes the uniform 10-kg plank ABC with a velocity of 10 m/s. If the putty remains attached to the plank, determine the maximum angle θ of swing before the plank momentarily stops. Neglect the size of the putty.

Conservation of Angular Momentum: Referring to Fig. a, the sum of the angular impulses about point B is zero. Thus, angular impulse of the system is conserved about this point. Since rod AC rotates about point B, $(v_{GAC})_2 = \omega_2 r_{GAC} = \omega_2(0.2)$ and $(v_D)_2 = \omega_2 r_{GD} = \omega_2(0.3)$. The mass moment of inertia of rod AC about its mass center is $I_{GAC} = \dfrac{1}{12} ml^2 = \dfrac{1}{12}(10)(1.2^2) = 1.2$ kg·m². Then,

$$(H_B)_1 = (H_B)_2$$

$$2(10)(0.3) = 1.2\omega_2 + 10\big[\omega_2(0.2)\big](0.2) + 2\big[\omega_2(0.3)\big](0.3)$$

$$\omega_2 = 3.371 \text{ rad/s} \qquad\qquad \textbf{Ans.}$$

Conservation of Energy: With reference to the datum in Fig. a,

$$V_2 = \big(V_g\big)_2 = W_{AC}(y_{GAC})_2 + W_D(y_{GD})_2 = 0$$

and

$$V_3 = \big(V_g\big)_3 = W_{AC}(y_{GAC})_3 - W_D(y_{GD})_3$$

$$= 10(9.81)(0.2 \sin\theta) - 2(9.81)(0.3 \sin\theta) = 13.734 \sin\theta$$

The initial kinetic energy of the system is

$$T_2 = \frac{1}{2} I_{GAC}\,\omega_2{}^2 + \frac{1}{2} m_{AC}(v_{GAC})_2{}^2 + \frac{1}{2} m_D(v_{GD})_2{}^2$$

$$= \frac{1}{2}(1.2)(3.371^2) + \frac{1}{2}(10)\big[3.371(0.2)\big]^2 + \frac{1}{2}(2)\big[3.371(0.3)\big]^2 = 10.11 \text{ J}$$
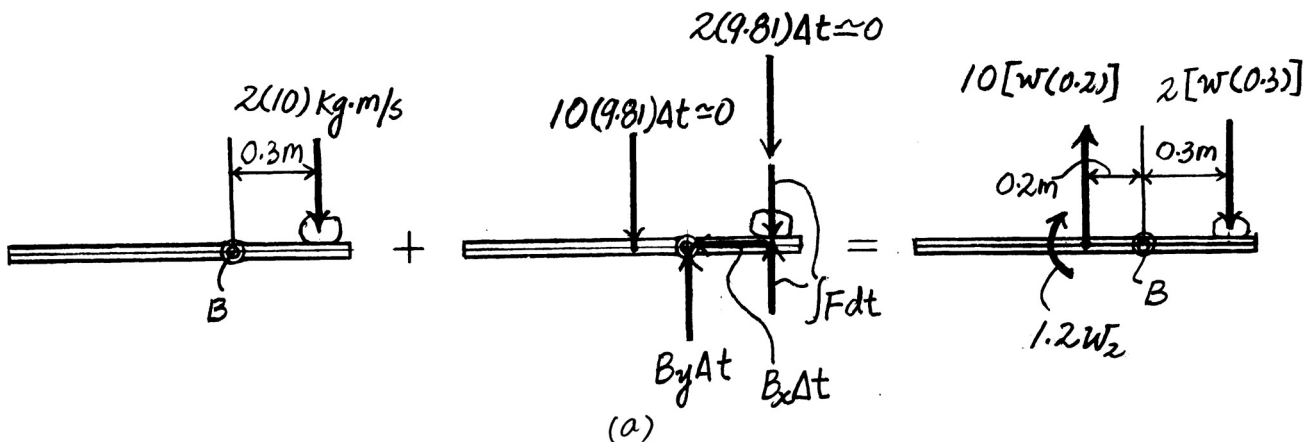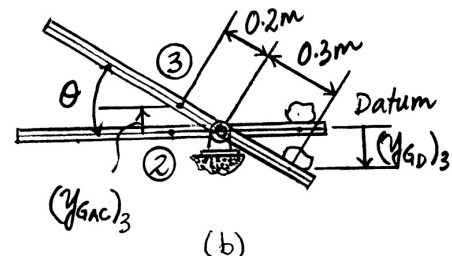
Since the system is required to be at rest in the final position, $T_3 = 0$. Then,

$$T_2 + V_2 = T_3 + V_3$$

$$10.11 + 0 = 0 + 13.734 \sin\theta$$

$$\theta = 47.4° \qquad\qquad \textbf{Ans.}$$

(b)

(a)

•**19–49.** The uniform 6-kg slender rod AB is given a slight horizontal disturbance when it is in the vertical position and rotates about B without slipping. Subsequently, it strikes the step at C. The impact is perfectly plastic and so the rod rotates about C without slipping after the impact. Determine the angular velocity of the rod when it is in the horizontal position shown.
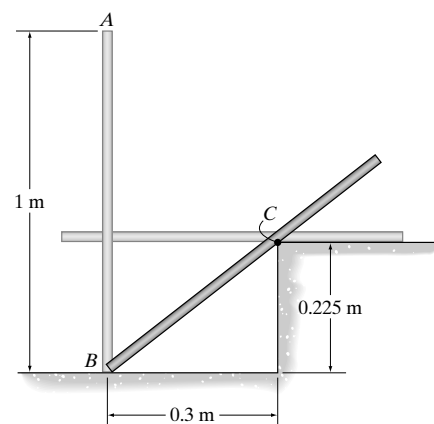
Conservation of Energy: From the geometry of Fig. a, $\theta = \tan^{-1}\left(\dfrac{0.225}{0.3}\right) = 36.87°$

and $BC = \sqrt{0.3^2 + 0.225^2} = 0.375$ m. Thus, $r_{CG} = 0.5 - 0.375 = 0.125$ m. With reference to the datum, $V_1 = W(y_G)_1 = 6(9.81)(0.5) = 29.43$ J, $V_2 = V_3 = W(y_G)_3 = 6(9.81)(0.5 \sin 36.87°) = 17.658$ J, and $V_4 = W(y_G)_4 = 6(9.81)(0.225) = 13.2435$ J. Since the rod is initially at rest, $T_1 = 0$. The rod rotates about point B before impact. Thus, $(v_G)_2 = \omega_2 r_{BG} = \omega_2 (0.5)$. The mass moment of inertia of the rod about its mass center is $I_G = \dfrac{1}{12} ml^2 = \dfrac{1}{12}(6)(1^2) = 0.5$ kg · m². Then, $T_2 = \dfrac{1}{2}m(v_G)_2^2 + \dfrac{1}{2}I_G\omega^2$

$= \dfrac{1}{2}(6)\left[\omega_2(0.5)\right]^2 + \dfrac{1}{2}(0.5)\omega_2^2 = 1\omega_2^2$. Therefore,

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 29.43 = 1\omega_2^2 + 17.658$$

$$\omega_2 = 3.431 \text{ rad/s}$$

The rod rotates about point C after impact. Thus, $v_G = \omega r_{CG} = \omega(0.125)$. Then,

$$T = \dfrac{1}{2}m(v_G)^2 + \dfrac{1}{2}I_G\omega^2 = \dfrac{1}{2}(6)\left[\omega(0.125)\right]^2 + \dfrac{1}{2}(0.5)\omega^2 = 0.296875\omega^2$$

so that

$$T_3 = 0.296875\omega_3^2 \text{ and } T_4 = 0.296875\omega_4^2$$

$$T_3 + V_3 = T_4 + V_4$$

$$0.296875\omega_3^2 + 17.658 = 0.296875\omega_4^2 + 13.2435$$

$$\omega_4^2 - \omega_3^2 = 14.87 \qquad \qquad \textbf{(1)}$$

Conservation of Angular Momentum: Referring to Fig. b, the sum of the angular impulses about point C is zero. Thus, angular momentum of the rod is conserved about this point during the impact. Then,

$$(H_C)_1 = (H_C)_2$$

$$6\left[3.431(0.5)\right](0.125) + 0.5(3.431) = 6\left[\omega_3(0.125)\right](0.125) + 0.5\omega_3$$
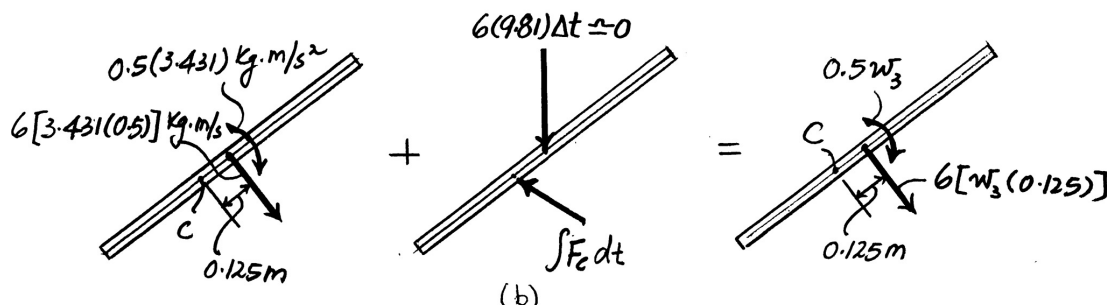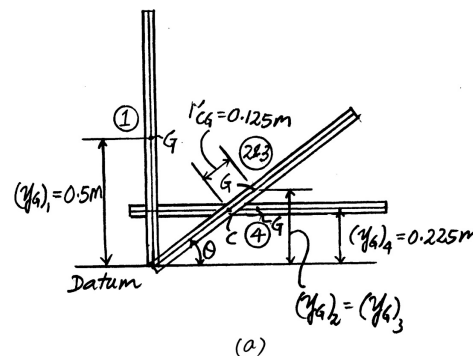
$$\omega_3 = 5.056 \text{ rad/s}$$

Substituting this result into Eq. (1), we obtain

$$\omega_4^2 - (5.056)^2 = 14.87$$

$$\omega_4 = 6.36 \text{ rad/s} \qquad \qquad \textbf{Ans.}$$

(a)

(b)

19–50. The rigid 30-lb plank is struck by the 15-lb hammer head H. Just before the impact the hammer is gripped loosely and has a vertical velocity of 75 ft/s. If the coefficient of restitution between the hammer head and the plank is $e = 0.5$, determine the maximum height attained by the 50-lb block D. The block can slide freely along the two vertical guide rods. The plank is initially in a horizontal position.
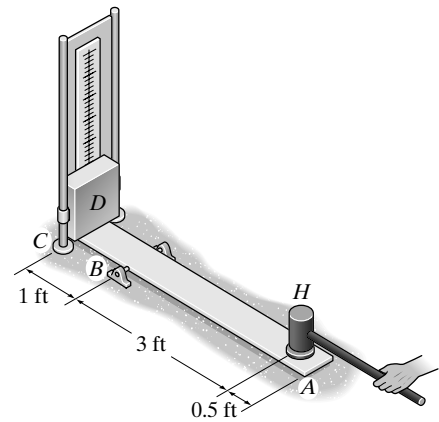
Conservation of Angular Momentum: Referring to Fig. a, the sum of the angular impulses about point B is zero. Thus, angular momentum of the system is conserved about this point during the impact. Since the plank rotates about point B, $(v_D)_2 = \omega_2(1)$ and $(v_G)_2 = \omega_2(1.25)$. The mass moment of inertia of the plank about its mass center is $I_G = \dfrac{1}{12} ml^2 = \dfrac{1}{12}\left(\dfrac{30}{32.2}\right)(4.5^2) = 1.572$ slug · ft². Thus,

$$(H_B)_1 = (H_B)_2$$

$$\frac{15}{32.2}(75)(3) = \frac{50}{32.2}\big[\omega_2(1)\big](1) + \frac{30}{32.2}\big[\omega_2(1.25)\big](1.25) + 1.572\omega_2 - \frac{15}{32.2}(v_H)_2(3)$$

$$4.581\omega_2 - 1398(v_H)_2 = 104.81 \qquad\qquad \textbf{(1)}$$

Coefficient of Restitution: Here, $(v_A)_2 = \omega_2(3)\downarrow$. Thus,

$$(+\uparrow) \qquad e = \frac{(v_A)_2 - (v_H)_2}{(v_H)_1 - (v_A)_1}$$

$$0.5 = \frac{-\omega_2(3) - (v_H)_2}{-75 - 0}$$

$$3\omega_2 + (v_H)_2 = 37.5 \qquad\qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$\omega_2 = 17.92 \text{ rad/s} \qquad (v_H)_2 = -16.26 \text{ ft/s} = 16.26 \text{ ft/s}\downarrow$$

Conservation of Energy: With reference to the datum in Fig. b, $V_2 = \left(V_g\right)_2 = W_D(y_G)_2 = 0$ and $V_3 = \left(V_g\right)_3 = W_D(y_G)_3 = 50h$.

$$\left(v_D\right)_2 = \omega_2(1) = 17.92(1) = 17.92 \text{ ft/s and } (v_D)_3 = 0$$

Thus,

$$T_2 = \frac{1}{2} m_D(v_D)_2{}^2 = \frac{1}{2}\left(\frac{50}{32.2}\right)(17.92^2) = 249.33 \text{ ft · lb and } T_3 = 0$$
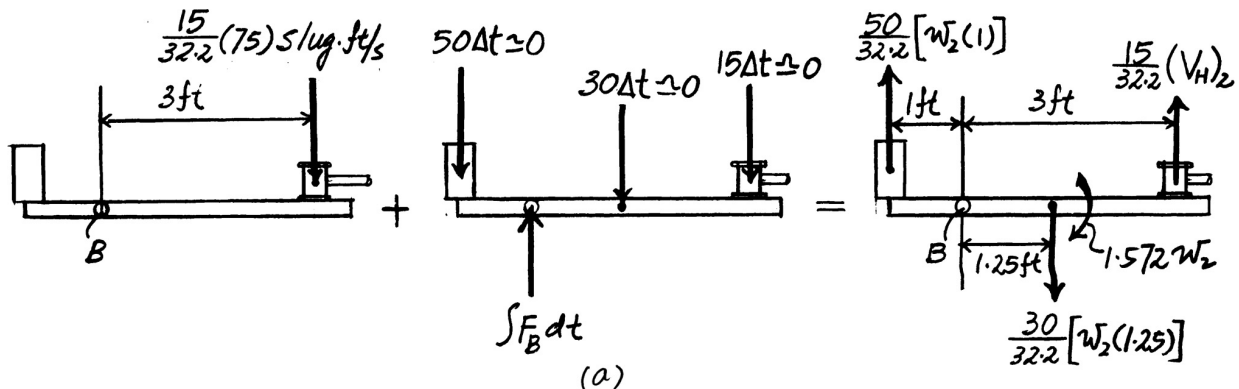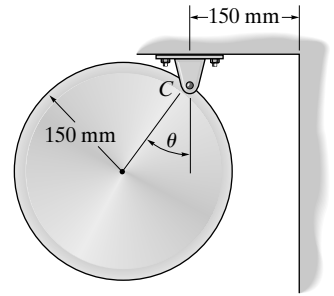
Then

$$T_2 + V_2 = T_3 + V_3$$

$$249.33 + 0 = 0 + 50h$$

$$h = 4.99 \text{ ft} \qquad\qquad\qquad\qquad \textbf{Ans.}$$

(a)

19–51. The disk has a mass of 15 kg. If it is released from rest when $\theta = 30°$, determine the maximum angle θ of rebound after it collides with the wall. The coefficient of restitution between the disk and the wall is $e = 0.6$. When $\theta = 0°$, the disk hangs such that it just touches the wall. Neglect friction at the pin C.

Datum at lower position of G.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (15)(9.81)(0.15)(1 - \cos 30°) = \frac{1}{2}\left[\frac{3}{2}(15)(0.15)^2\right]\omega^2 + 0$$

$$\omega = 3.418 \text{ rad/s}$$

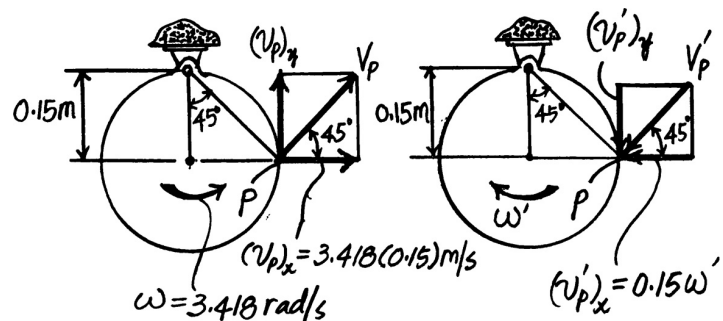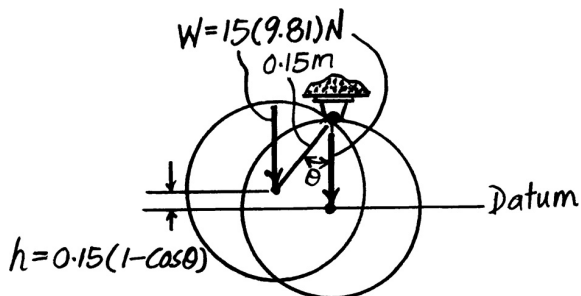$$\left(\xrightarrow{+}\right) \quad e = 0.6 = \frac{0 - (-0.15\omega')}{3.418(0.15) - 0}$$

$$\omega' = 2.0508 \text{ rad/s}$$
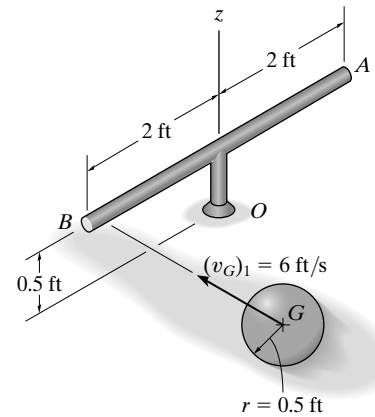
$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}\left[\frac{3}{2}(15)(0.15)^2\right](2.0508)^2 + 0 = 0 + 15(9.81)(0.15)(1 - \cos\theta)$$

$$\theta = 17.9° \qquad\qquad\qquad\qquad \textbf{Ans.}$$

821

*19–52. The mass center of the 3-lb ball has a velocity of $(v_G)_1 = 6$ ft/s when it strikes the end of the smooth 5-lb slender bar which is at rest. Determine the angular velocity of the bar about the z axis just after impact if $e = 0.8$.

Conservation of Angular Momentum: Since force F due to the impact is *internal* to the system consisting of the slender bar and the ball, it will cancel out. Thus, angular momentum is conserved about the z axis. The mass moment of inertia of the slender bar about the z axis is $I_z = \dfrac{1}{12}\left(\dfrac{5}{32.2}\right)(4^2) = 0.2070$ slug · ft². Here, $\omega_2 = \dfrac{(v_B)_2}{2}$. Applying Eq. 19–17, we have

$$(H_z)_1 = (H_z)_2$$

$$\left[m_b\,(v_G)_1\right](r_b) = I_z\,\omega_2 + \left[m_b\,(v_G)_2\right](r_b)$$

$$\left(\frac{3}{32.2}\right)(6)(2) = 0.2070\left[\frac{(v_B)_2}{2}\right] + \left(\frac{3}{32.2}\right)(v_G)_2(2) \qquad\qquad \textbf{(1)}$$

Coefficient of Restitution: Applying Eq. 19–20, we have

$$e = \frac{(v_B)_2 - (v_G)_2}{(v_G)_1 - (v_B)_1}$$

$$0.8 = \frac{(v_B)_2 - (v_G)_2}{6 - 0} \qquad\qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$(v_G)_2 = 2.143 \text{ ft/s} \qquad (v_B)_2 = 6.943 \text{ ft/s}$$

Thus, the angular velocity of the slender rod is given by

$$\omega_2 = \frac{(v_B)_2}{2} = \frac{6.943}{2} = 3.47 \text{ rad/s} \qquad\qquad \textbf{Ans.}$$

•19–53. The 300-lb bell is at rest in the vertical position before it is struck by a 75-lb wooden post suspended from two equal-length ropes. If the post is released from rest at $\theta = 45°$, determine the angular velocity of the bell and the velocity of the post immediately after the impact. The coefficient of restitution between the bell and the post is $e = 0.6$. The center of gravity of the bell is located at point G and its radius of gyration about G is $k_G = 1.5$ ft.
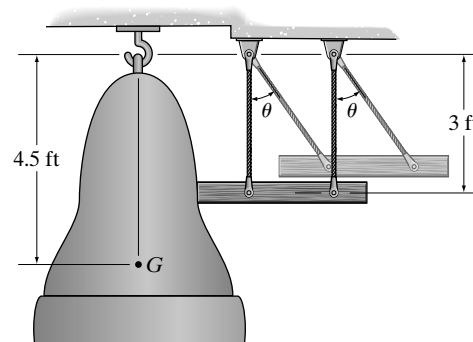
Conservation of Energy: With reference to the datum in Fig. *a*, $V_1 = (V_g)_1$
$= -W(y_G)_1 = -75(3 \cos 45°) = -159.10$ ft · lb and $V_2 = (V_g)_2 = -W(y_G)_2$
$= -75(3) = -225$ ft · lb. Since the post is initially at rest, $T_1 = 0$. The post undergoes curvilinear translation, $T_2 = \frac{1}{2} m(v_P)_2{}^2 = \frac{1}{2}\left[\frac{75}{32.2}\right](v_P)_2{}^2$. Thus,

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (-159.10) = \frac{1}{2}\left[\frac{75}{32.2}\right](v_G)_2{}^2 + (-225)$$

$$(v_P)_2 = 7.522 \text{ ft/s}$$

Conservation of Angular Momentum: The sum of the angular impulses about point O is zero. Thus, angular momentum of the system is conserved about this point during the impact. Since the bell rotates about point O, $(v_G)_3 = \omega_3 r_{OG} = \omega_3(4.5)$. The mass moment of inertia of the bell about its mass center is $I_G = \frac{1}{12} m k_G{}^2$
$= \frac{300}{32.2}(1.5^2) = 20.96$ slug · ft². Thus,

$$(H_O)_2 = (H_O)_3$$

$$\frac{75}{32.2}(7.522)(3) = \frac{300}{32.2}\left[\omega_3(4.5)\right](4.5) + 20.96\omega_3 - \frac{75}{32.2}(v_P)_3(3)$$

$$209.63\omega_3 - 6.988(v_P)_3 = 52.56 \tag{1}$$

Coefficient of Restitution: The impact point A on the bell along the line of impact (x axis) is $\left[(v_A)_3\right]_x = \omega_3(3)$. Thus,

$$e = \frac{\left[(v_A)_3\right]_x - (v_P)_3}{(v_P)_2 - \left[(v_A)_2\right]_x}$$

$$(\stackrel{+}{\rightarrow}) \qquad 0.6 = \frac{-\omega_3(3) - (v_P)_3}{-7.522 - 0}$$

$$3\omega_3 + (v_P)_3 = 4.513 \tag{2}$$

Solving Eqs. (1) and (2),

$$\omega_3 = 0.365 \text{ rad/s} \qquad (v_P)_3 = 3.42 \text{ ft/s} \qquad \textbf{Ans.}$$

19–54. The 4-lb rod AB hangs in the vertical position. A 2-lb block, sliding on a smooth horizontal surface with a velocity of 12 ft/s, strikes the rod at its end B. Determine the velocity of the block immediately after the collision. The coefficient of restitution between the block and the rod at B is $e = 0.8$.
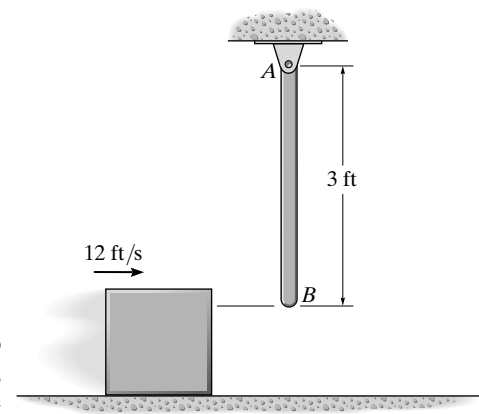
A

3 ft

12 ft/s

B

Conservation of Angular Momentum: Since force F due to the impact is *internal* to the system consisting of the slender rod and the block, it will cancel out. Thus, angular momentum is conserved about point A. The mass moment of inertia of the slender rod about point A is $I_A = \dfrac{1}{12}\left(\dfrac{4}{32.2}\right)(3^2) + \dfrac{4}{32.2}(1.5^2) = 0.3727$ slug·ft².

Here, $\omega_2 = \dfrac{(v_B)_2}{3}$. Applying Eq. 19–17, we have

$$(H_A)_1 = (H_A)_2$$

$$\left[m_b\,(v_b)_1\right](r_b) = I_A\,\omega_2 + \left[m_b\,(v_b)_2\right](r_b)$$

$$\left(\dfrac{2}{32.2}\right)(12)(3) = 0.3727\left[\dfrac{(v_B)_2}{3}\right] + \left(\dfrac{2}{32.2}\right)(v_b)_2(3) \tag{1}$$

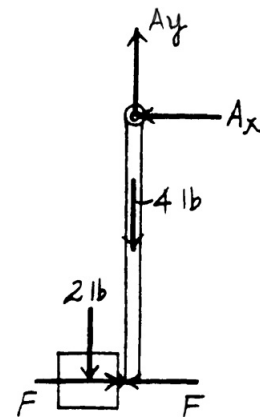**Coefficient of Restitution:** Applying Eq. 19–20, we have

$$e = \dfrac{(v_B)_2 - (v_b)_2}{(v_b)_1 - (v_B)_1}$$

$$(\pm)\qquad 0.8 = \dfrac{(v_B)_2 - (v_b)_2}{12 - 0} \tag{2}$$

Solving Eqs. (1) and (2) yields

$$(v_b)_2 = 3.36 \text{ ft/s} \rightarrow \qquad\qquad \textbf{Ans.}$$

$$(v_B)_2 = 12.96 \text{ ft/s} \rightarrow$$

A_y

A_x

4 lb

2 lb

F

F

19–55. The pendulum consists of a 10-lb sphere and 4-lb rod. If it is released from rest when $\theta = 90°$, determine the angle θ of rebound after the sphere strikes the floor. Take $e = 0.8$.
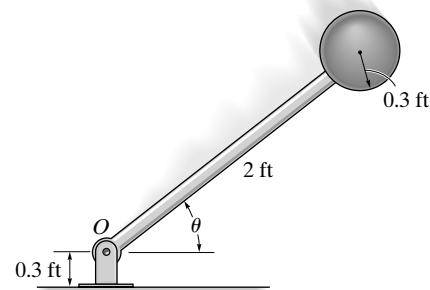
$$I_A = \frac{1}{3}\left(\frac{4}{32.2}\right)(2)^2 + \frac{2}{5}\left(\frac{10}{32.2}\right)(0.3)^2 + \left(\frac{10}{32.2}\right)(2.3)^2 = 1.8197 \text{ slug} \cdot \text{ft}^2$$

Just before impact:

Datum through O.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 4(1) + 10(2.3) = \frac{1}{2}(1.8197)\omega^2 + 0$$

$$\omega_2 = 5.4475 \text{ rad/s}$$

$$v = 2.3(5.4475) = 12.529 \text{ ft/s}$$

Since the floor does not move,

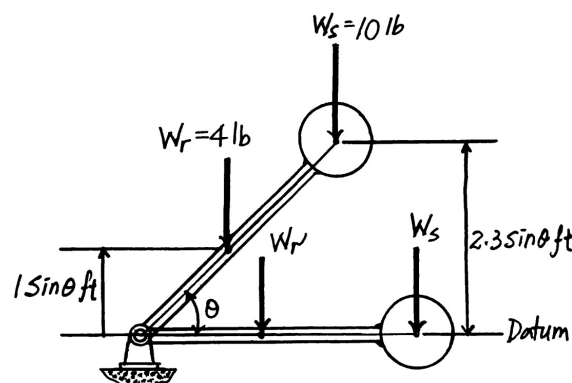$$(+\uparrow) \qquad e = 0.8 = \frac{(v_P) - 0}{0 - (-12.529)}$$

$$(v_P)_3 = 10.023 \text{ ft/s}$$

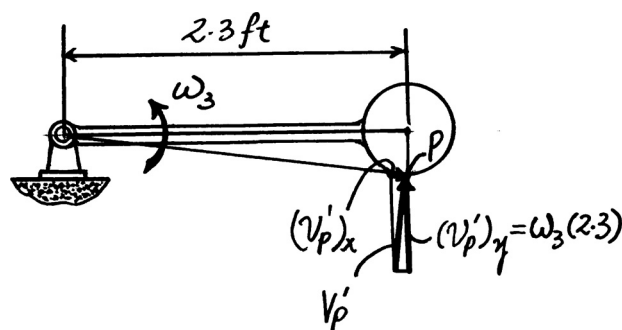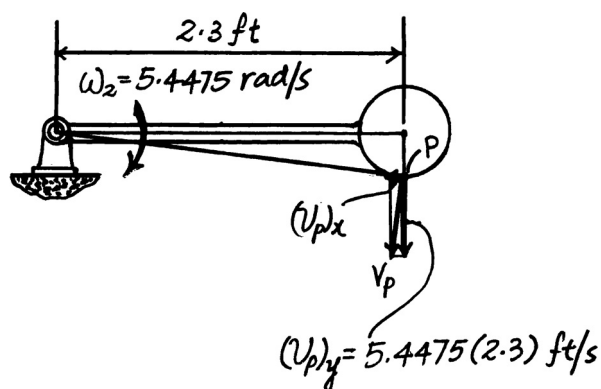$$\omega_3 = \frac{10.023}{2.3} = 4.358 \text{ rad/s}$$

$$T_3 + V_3 = T_4 + V_4$$

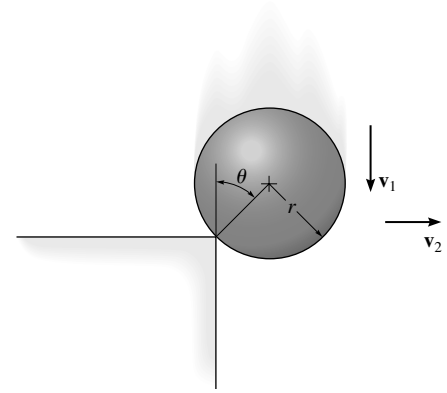$$\frac{1}{2}(1.8197)(4.358)^2 + 0 = 4(1 \sin \theta_1) + 10(2.3 \sin \theta_1)$$

$$\theta_1 = 39.8° \qquad\qquad \textbf{Ans.}$$

*19–56. The solid ball of mass m is dropped with a velocity $\mathbf{v}_1$ onto the edge of the rough step. If it rebounds horizontally off the step with a velocity $\mathbf{v}_2$, determine the angle θ at which contact occurs. Assume no slipping when the ball strikes the step. The coefficient of restitution is e.

Conservation of Angular Momentum: Since the weight of the solid ball is a *nonimpulsive force*, then angular momentum is conserved about point A. The mass moment of inertia of the solid ball about its mass center is $I_G = \dfrac{2}{5} mr^2$. Here, $\omega_2 = \dfrac{v_2 \cos \theta}{r}$. Applying Eq. 19–17, we have

$$(H_A)_1 = (H_A)_2$$

$$\left[m_b\, (v_b)_1 \right](r') = I_G\, \omega_2 + \left[m_b\, (v_b)_2 \right](r'')$$

$$(mv_1)(r \sin \theta) = \left(\frac{2}{5} mr^2 \right)\left(\frac{v_2 \cos \theta}{r} \right) + (mv_2)(r \cos \theta)$$

$$\frac{v_2}{v_1} = \frac{5}{7} \tan \theta \qquad \qquad (1)$$

Coefficient of Restitution: Applying Eq. 19–20, we have

$$e = \frac{0 - (v_b)_2}{(v_b)_1 - 0}$$

$$e = \frac{-(v_2 \sin \theta)}{-v_1 \cos \theta}$$

$$\frac{v_2}{v_1} = \frac{e \cos \theta}{\sin \theta} \qquad \qquad (2)$$

Equating Eqs. (1) and (2) yields

$$\frac{5}{7} \tan \theta = \frac{e \cos \theta}{\sin \theta}$$

$$\tan^2 \theta = \frac{7}{5} e$$

$$\theta = \tan^{-1}\left(\sqrt{\frac{7}{5} e} \right) \qquad \qquad \textbf{Ans.}$$