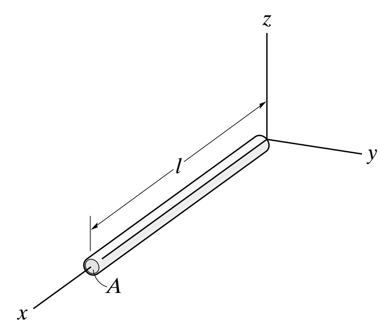
•17–1. Determine the moment of inertia I_y for the slender rod. The rod's density ρ and cross-sectional area A are constant. Express the result in terms of the rod's total mass m.

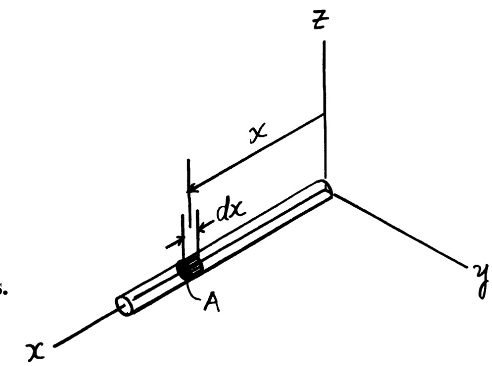


$$I_y = \int_M x^2\,dm$$

$$= \int_0^l x^2\,(\rho\,A\,dx)$$

$$= \frac{1}{3}\rho\,A\,l^3$$

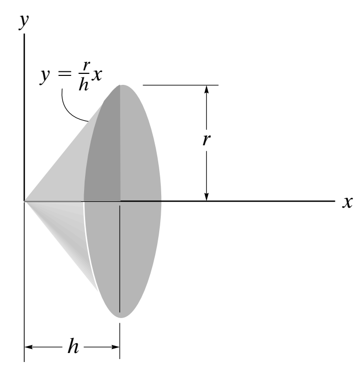
$$m = \rho\,A\,l$$



Thus,

$$I_y = \frac{1}{3}m\,l^2$$ **Ans.**

17–2. The right circular cone is formed by revolving the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the cone. The cone has a constant density ρ.



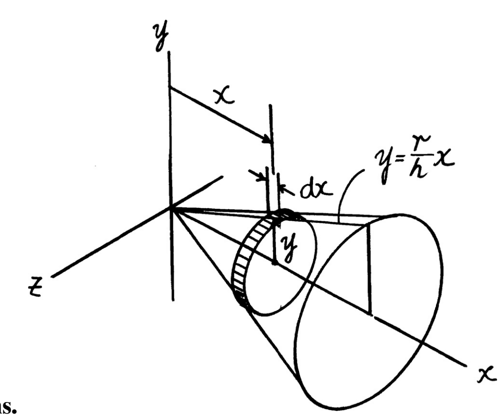
$$dm = \rho\,dV = \rho(\pi\,y^2\,dx)$$

$$m = \int_0^h \rho(\pi)\left(\frac{r^2}{h^2}\right)x^2\,dx = \rho\pi\left(\frac{r^2}{h^2}\right)\left(\frac{1}{3}\right)h^3 = \frac{1}{3}\rho\pi\,r^2h$$

$$dI_x = \frac{1}{2}\,y^2\,dm$$

$$= \frac{1}{2}\,y^2\,(\rho\pi\,y^2\,dx)$$

$$= \frac{1}{2}\,\rho(\pi)\left(\frac{r^4}{h^4}\right)x^4\,dx$$

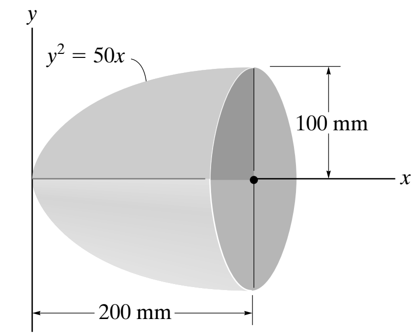


$$I_x = \int_0^h \frac{1}{2}\rho(\pi)\left(\frac{r^4}{h^4}\right)x^4\,dx = \frac{1}{10}\rho\pi\,r^4\,h$$

Thus,

$$I_x = \frac{3}{10}m\,r^2$$ **Ans.**

17–3. The paraboloid is formed by revolving the shaded area around the x axis. Determine the radius of gyration k_x. The density of the material is $\rho = 5\ \text{Mg/m}^3$.

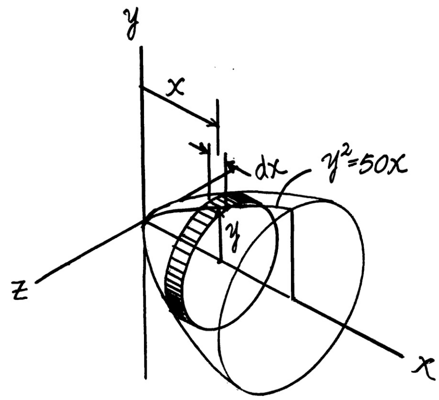


$$dm = \rho\,\pi\,y^2\,dx = \rho\,\pi\,(50x)\,dx$$

$$I_x = \int \frac{1}{2} y^2\,dm = \frac{1}{2}\int_0^{200} 50\,x\,\{\pi\,\rho\,(50x)\}\,dx$$

$$= \rho\,\pi\left(\frac{50^2}{2}\right)\left[\frac{1}{3}x^3\right]_0^{200}$$

$$= \rho\,\pi\left(\frac{50^2}{6}\right)(200)^3$$



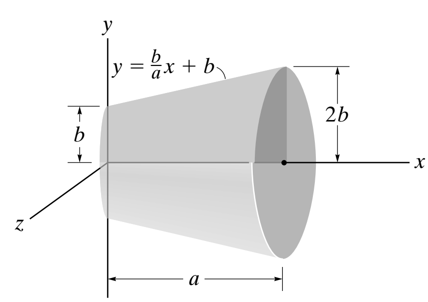
$$m = \int dm = \int_0^{200} \pi\,\rho\,(50x)\,dx$$

$$= \rho\,\pi\,(50)\left[\frac{1}{2}x^2\right]_0^{200}$$

$$= \rho\,\pi\left(\frac{50}{2}\right)(200)^2$$

$$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{50}{3}(200)} = 57.7\ \text{mm}$$ **Ans.**

***17–4.** The frustum is formed by rotating the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the frustum. The frustum has a constant density ρ.



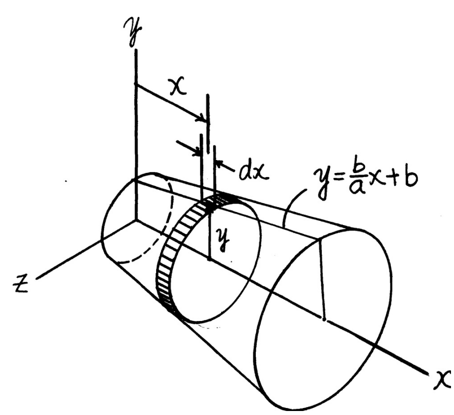
$$dm = \rho\,dV = \rho\pi y^2\,dx = \rho\pi\left(\frac{b^2}{a^2}x^2 + \frac{2b^2}{a}x + b^2\right)dx$$

$$dI_x = \frac{1}{2}dmy^2 = \frac{1}{2}\rho\pi y^4\,dx$$

$$dI_x = \frac{1}{2}\rho\pi\left(\frac{b^4}{a^4}x^4 + \frac{4\,b^4}{a^3}x^3 + \frac{6\,b^4}{a^2}x^2 + \frac{4b^4}{a}x + b^4\right)dx$$

$$I_x = \int dI_x = \frac{1}{2}\rho\pi\int_0^a \left(\frac{b^4}{a^4}x^4 + \frac{4b^4}{a^3}x^3 + \frac{6\,b^4}{a^2}x^2 + \frac{4\,b^4}{a}x + b^4\right)dx$$

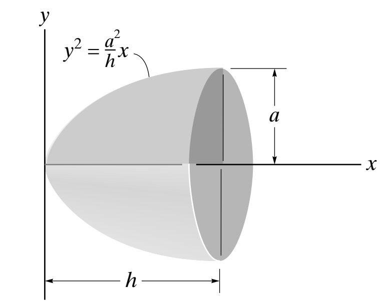
$$= \frac{31}{10}\rho\pi ab^4$$



$$m = \int_m dm = \rho\pi\int_0^a \left(\frac{b^2}{a^2}x^2 + \frac{2b^2}{a}x + b^2\right)dx = \frac{7}{3}\rho\pi ab^2$$

$$I_x = \frac{93}{70}mb^2$$ **Ans.**

•17–5. The paraboloid is formed by revolving the shaded area around the x axis. Determine the moment of inertia about the x axis and express the result in terms of the total mass m of the paraboloid. The material has a constant density ρ.



$$dm = \rho\, dV = \rho\,(\pi\, y^2\, dx)$$

$$d\,I_x = \frac{1}{2}\,dm\, y^2 = \frac{1}{2}\rho\,\pi\, y^4\, dx$$

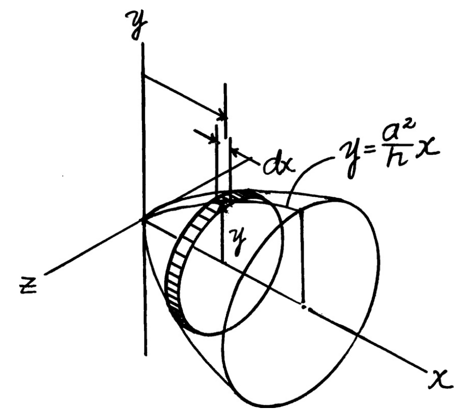
$$I_x = \int_0^h \frac{1}{2}\rho\,\pi\left(\frac{a^4}{h^2}\right)x^2\, dx$$

$$= \frac{1}{6}\,\pi\,\rho a^4\, h$$

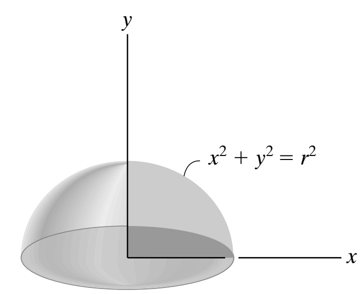
$$m = \int_0^h \frac{1}{2}\rho\,\pi\left(\frac{a^2}{h}\right)x\, dx$$

$$= \frac{1}{2}\,\rho\,\pi\, a^2\, h$$

$$I_x = \frac{1}{3}\,ma^2$$ **Ans.**



17–6. The hemisphere is formed by rotating the shaded area around the y axis. Determine the moment of inertia I_y and express the result in terms of the total mass m of the hemisphere. The material has a constant density ρ.



$$m = \int_V \rho\, dV = \rho\int_0^r \pi\, x^2\, dy = \rho\pi\int_0^r (r^2 - y^2)dy$$

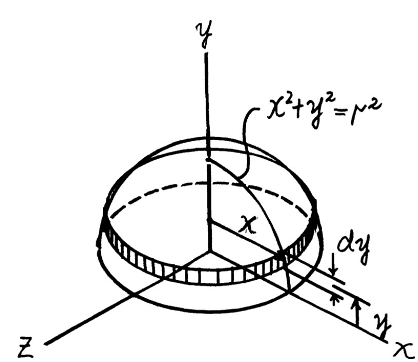
$$= \rho\pi\left[r^2\, y - \frac{1}{3}\,y^3\right]_0^r = \frac{2}{3}\rho\pi\, r^3$$

$$I_y = \int_m \frac{1}{2}\,(dm)\, x^2 = \frac{\rho}{2}\int_0^r \pi x^4\, dy = \frac{\rho\pi}{2}\int_0^r (r^2 - y^2)^2\, dy$$

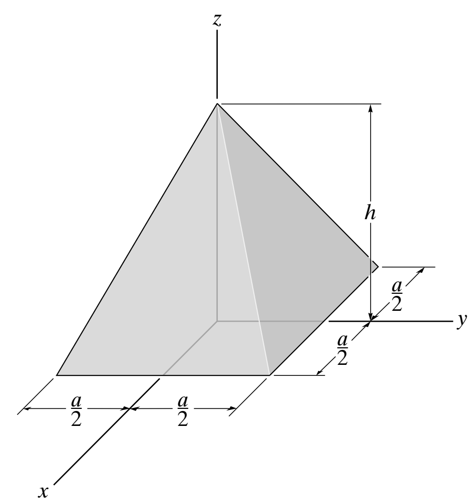
$$= \frac{\rho\pi}{2}\left[r^4 y - \frac{2}{3}\,r^2\, y^3 + \frac{y^5}{5}\right]_0^r = \frac{4\rho\pi}{15}\,r^5$$

Thus,

$$I_y = \frac{2}{5}\,m\, r^2$$ **Ans.**



17–7. Determine the moment of inertia of the homogeneous pyramid of mass m about the z axis. The density of the material is ρ. *Suggestion:* Use a rectangular plate element having a volume of $dV = (2x)(2y)dz$.



$$dI_z = \frac{dm}{12}\left[(2y)^2 + (2y)^2\right] = \frac{2}{3}y^2\,dm$$

$$dm = 4\rho y^2\,dz$$

$$dI_z = \frac{8}{3}\rho y^4\,dz = \frac{8}{3}\rho(h-z)^4\left(\frac{a^4}{16h^4}\right)dz$$

$$I_z = \frac{\rho}{6}\left(\frac{a^4}{h^4}\right)\int_0^h (h^4 - 4h^3z + 6h^2z^2 - 4hz^3 + z^4)dz = \frac{\rho}{6}\left(\frac{a^4}{h^4}\right)\left[h^5 - 2h^5 + 2h^5 - h^5 + \frac{1}{5}h^5\right]$$

$$= \frac{\rho a^4 h}{30}$$

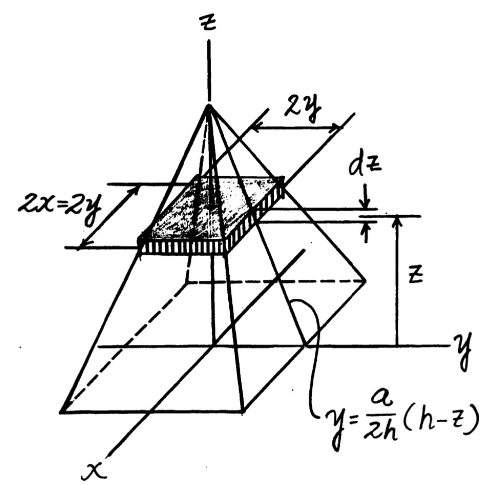
$$m = \int_0^h 4\rho(h-z)^2\left(\frac{a^2}{4h^2}\right)dz = \frac{\rho a^2}{h^2}\int_0^h (h^2 - 2hz + z^2)dz$$

$$= \frac{\rho a^2}{h^2}\left[h^3 - h^3 + \frac{1}{3}h^3\right]$$

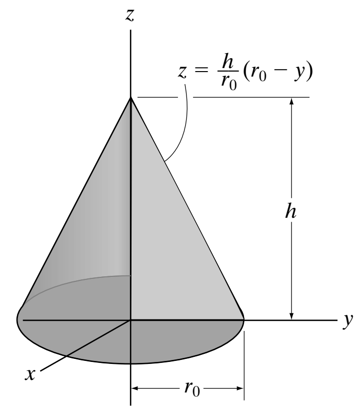
$$= \frac{\rho a^2 h}{3}$$

Thus,

$$I_z = \frac{m}{10}a^2$$ **Ans.**

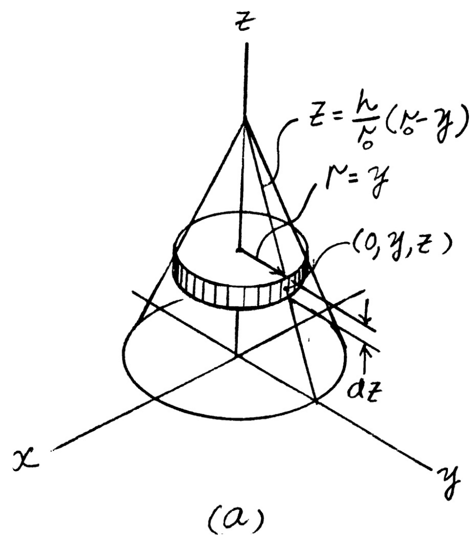


***17–8.** Determine the mass moment of inertia I_z of the cone formed by revolving the shaded area around the z axis. The density of the material is ρ. Express the result in terms of the mass m of the cone.



Differential Element: The mass of the disk element shown shaded in Fig. a is $dm = \rho\, dV = \rho\pi r^2 dz$. Here, $r = y = r_o - \frac{r_o}{h}z$. Thus, $dm = \rho\pi\left(r_o - \frac{r_o}{h}z\right)^2 dz$. The mass moment of inertia of this element about the z axis is

$$dI_z = \frac{1}{2}dmr^2 = \frac{1}{2}(\rho\pi r^2 dz)r^2 = \frac{1}{2}\rho\pi r^4 dz = \frac{1}{2}\rho\pi\left(r_o - \frac{r_o}{h}z\right)^4 dz$$



Mass: The mass of the cone can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^h \rho\pi\left(r_o - \frac{r_o}{h}z\right)^2 dz$$

$$= \rho\pi\left[\frac{1}{3}\left(r_o - \frac{r_o}{h}z\right)^3\left(-\frac{h}{r_o}\right)\right]\Bigg|_0^h = \frac{1}{3}\rho\pi r_o^{\,2}h$$

Mass Moment of Inertia: Integrating dI_z, we obtain

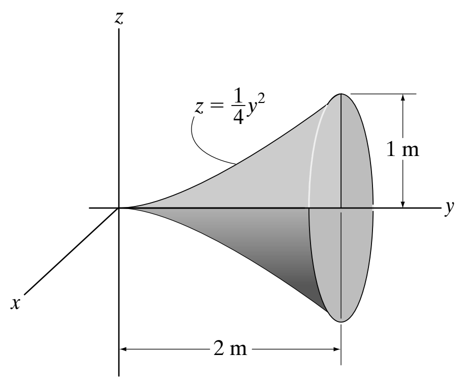
$$I_z = \int dI_z = \int_0^h \frac{1}{2}\rho\pi\left(r_o - \frac{r_o}{h}z\right)^4 dz$$

$$= \frac{1}{2}\rho\pi\left[\frac{1}{5}\left(r_o - \frac{r_o}{h}z\right)^3\left(-\frac{h}{r_o}\right)\right]\Bigg|_0^h = \frac{1}{10}\rho\pi r_o^{\,4}h$$

From the result of the mass, we obtain $\rho\pi r_o^{\,2}h = 3m$. Thus, I_z can be written as

$$I_z = \frac{1}{10}\left(\rho\pi r_o^{\,2}h\right)r_o^{\,2} = \frac{1}{10}(3m)r_o^{\,2} = \frac{3}{10}mr_o^{\,2}$$ **Ans.**

•17–9. Determine the mass moment of inertia I_y of the solid formed by revolving the shaded area around the y axis. The density of the material is ρ. Express the result in terms of the mass m of the solid.



Differential Element: The mass of the disk element shown shaded in Fig. a is $dm = \rho\, dV = \rho\pi r^2 dy$. Here, $r = z = \frac{1}{4}y^2$. Thus, $dm = \rho\pi\left(\frac{1}{4}y^2\right)^2 dy = \frac{\rho\pi}{16}y^4\, dy$.

The mass moment of inertia of this element about the y axis is

$$dI_y = \frac{1}{2}dmr^2 = \frac{1}{2}(\rho\pi r^2 dy)r^2 = \frac{1}{2}\rho\pi r^4 dy = \frac{1}{2}\rho\pi\left(\frac{1}{4}y^2\right)^4 dy = \frac{\rho\pi}{512}y^8 dy$$

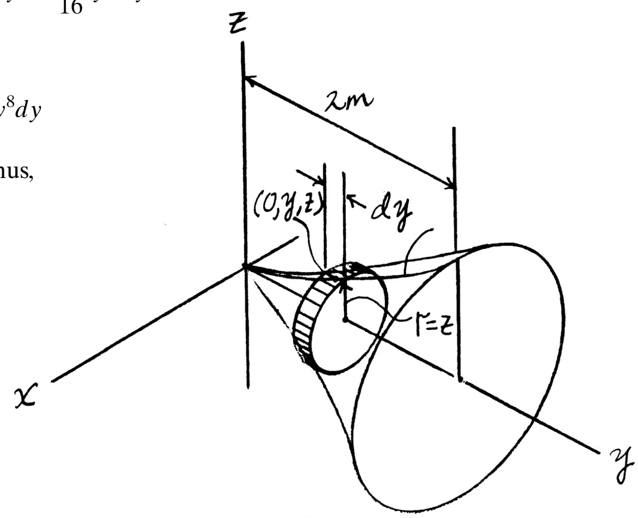
Mass: The mass of the solid can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^{2\text{ m}} \frac{\rho\pi}{16}y^4 dy = \frac{\rho\pi}{16}\left(\frac{y^5}{5}\right)\Bigg|_0^{2\text{ m}} = \frac{2}{5}\rho\pi$$

Mass Moment of Inertia: Integrating dI_y, we obtain

$$I_y = \int dI_y = \int_0^{2\text{ m}} \frac{\rho\pi}{572}y^8 dy$$

$$= \frac{\rho\pi}{512}\left(\frac{y^9}{9}\right)\Bigg|_0^{2\text{ m}} = \frac{\pi\rho}{9}$$

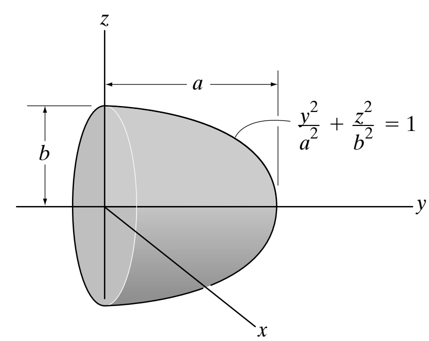


(a)

From the result of the mass, we obtain $\pi\rho = \frac{5m}{2}$. Thus, I_y can be written as

$$I_y = \frac{1}{9}\left(\frac{5m}{2}\right) = \frac{5}{18}m \qquad \textbf{Ans.}$$

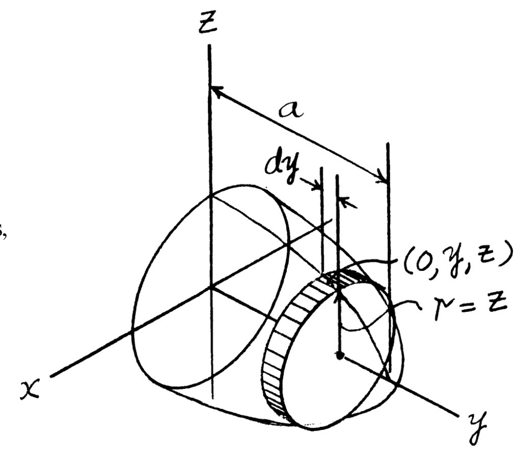
17–10. Determine the mass moment of inertia I_y of the solid formed by revolving the shaded area around the y axis. The density of the material is ρ. Express the result in terms of the mass m of the semi-ellipsoid.



Differential Element: The mass of the disk element shown shaded in Fig. a is $dm = \rho\, dV = \rho\pi r^2 dy$. Here, $r = z = b\sqrt{1 - \frac{y^2}{a^2}}$. Thus, $dm = \rho\pi\left(b\sqrt{1 - \frac{y^2}{a^2}}\right)^2 dz$ $= \rho\pi b^2\left(1 - \frac{y^2}{a^2}\right)dy$. The mass moment of inertia of this element about the y axis is

$$dI_y = \frac{1}{2}dmr^2 = \frac{1}{2}(\rho\pi r^2 dy)r^2 = \frac{1}{2}\rho\pi r^4 dy = \frac{1}{2}\rho\pi\left(b\sqrt{1 - \frac{y^2}{a^2}}\right)^4 dy$$

$$= \frac{1}{2}\rho\pi b^4\left(1 - \frac{y^2}{a^2}\right)^2 dy = \frac{1}{2}\rho\pi b^4\left(1 + \frac{y^4}{a^4} - \frac{2y^2}{a^2}\right)dy$$



(a)

Mass: The mass of the semi-ellipsoid can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^a \rho\pi b^2\left(1 - \frac{y^2}{a^2}\right)dy = \rho\pi b^2\left(y - \frac{y^3}{3a^2}\right)\Bigg|_0^a = \frac{2}{3}\rho\,\pi\, ab^2$$

Mass Moment of Inertia: Integrating dI_y, we obtain

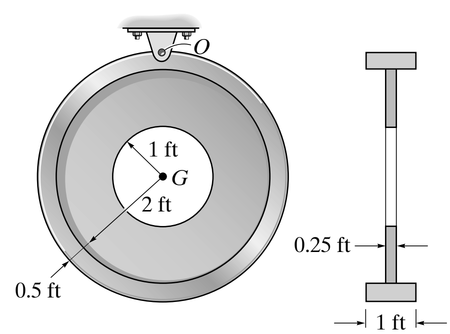
$$I_y = \int dI_y = \int_0^a \frac{1}{2}\rho\pi b^4\left(H\frac{y^4}{a^4} - \frac{2y^2}{a^2}\right)dy$$

$$= \frac{1}{2}\rho\pi b^4\left(y + \frac{y^5}{5a^4} - \frac{2y^3}{3a^2}\right)\Bigg|_0^a = \frac{4}{15}\rho\,\pi\, ab^4$$

From the result of the mass, we obtain $\rho\pi ab^2 = \frac{3m}{2}$. Thus, I_y can be written as

$$I_y = \frac{4}{15}(\rho\pi ab^2)b^2 = \frac{4}{15}\left(\frac{3m}{2}\right)b^2 = \frac{2}{5}mb^2$$ **Ans.**

17–11. Determine the moment of inertia of the assembly about an axis that is perpendicular to the page and passes through the center of mass G. The material has a specific weight of $\gamma = 90\ \text{lb/ft}^3$.

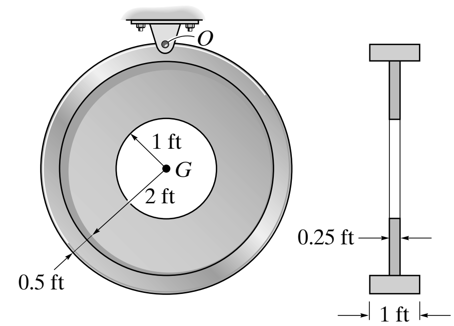


$$I_G = \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2.5)^2(1)\right](2.5)^2 - \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2)^2(1)\right](2)^2$$

$$+ \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2)^2(0.25)\right](2)^2 - \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(1)^2(0.25)\right](1)^2$$

$$= 118\ \text{slug}\cdot\text{ft}^2$$ **Ans.**

***17–12.** Determine the moment of inertia of the assembly about an axis that is perpendicular to the page and passes through point O. The material has a specific weight of $\gamma = 90\ \text{lb/ft}^3$.



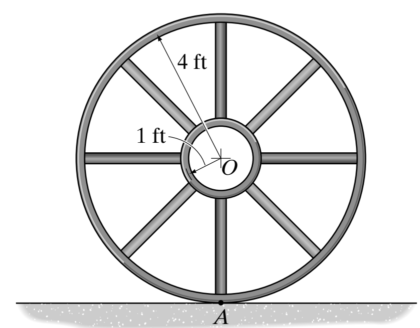
$$I_G = \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2.5)^2(1)\right](2.5)^2 - \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2)^2(1)\right](2)^2$$
$$+ \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(2)^2(0.25)\right](2)^2 - \frac{1}{2}\left[\left(\frac{90}{32.2}\right)\pi(1)^2(0.25)\right](1)^2$$
$$= 117.72\ \text{slug}\cdot\text{ft}^2$$

$$I_O = I_G + md^2$$

$$m = \left(\frac{90}{32.2}\right)\pi(2^2 - 1^2)(0.25) + \left(\frac{90}{32.2}\right)\pi(2.5^2 - 2^2)(1) = 26.343\ \text{slug}$$

$$I_O = 117.72 + 26.343(2.5)^2 = 282\ \text{slug}\cdot\text{ft}^2$$ **Ans.**

•17–13. If the large ring, small ring and each of the spokes weigh 100 lb, 15 lb, and 20 lb, respectively, determine the mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point A.



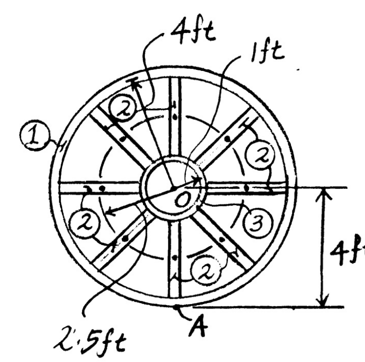
Composite Parts: The wheel can be subdivided into the segments shown in Fig. a. The spokes which have a length of $(4 - 1) = 3$ ft and a center of mass located at a distance of $\left(1 + \frac{3}{2}\right)$ ft $= 2.5$ ft from point O can be grouped as segment (2).

Mass Moment of Inertia: First, we will compute the mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point O.

$$I_O = \left(\frac{100}{32.2}\right)(4^2) + 8\left[\frac{1}{12}\left(\frac{20}{32.2}\right)(3^2) + \left(\frac{20}{32.2}\right)(2.5^2)\right] + \left(\frac{15}{32.2}\right)(1^2)$$
$$= 84.94\ \text{slug}\cdot\text{ft}^2$$

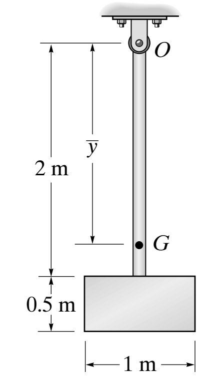
The mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point A can be found using the parallel-axis theorem $I_A = I_O + md^2$, where $m = \frac{100}{32.2} + 8\left(\frac{20}{32.2}\right) + \frac{15}{32.2} = 8.5404$ slug and $d = 4$ ft. Thus,

$$I_A = 84.94 + 8.5404(4^2) = 221.58\ \text{slug}\cdot\text{ft}^2 = 222\ \text{slug}\cdot\text{ft}^2$$ **Ans.**



(a)

17–14. The pendulum consists of the 3-kg slender rod and the 5-kg thin plate. Determine the location $\bar{y}$ of the center of mass G of the pendulum; then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.



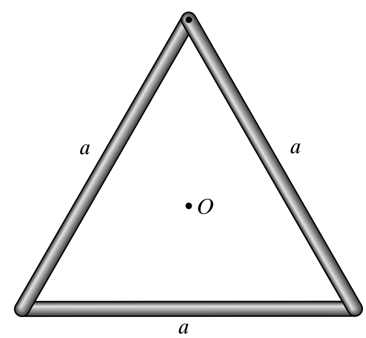
$$\bar{y} = \frac{\Sigma\, \bar{y} m}{\Sigma\, m} = \frac{1(3) + 2.25(5)}{3 + 5} = 1.781 \text{ m} = 1.78 \text{ m}$$ **Ans.**

$$I_G = \Sigma \bar{I}_G + md^2$$

$$= \frac{1}{12}(3)(2)^2 + 3(1.781 - 1)^2 + \frac{1}{12}(5)(0.5^2 + 1^2) + 5(2.25 - 1.781)^2$$

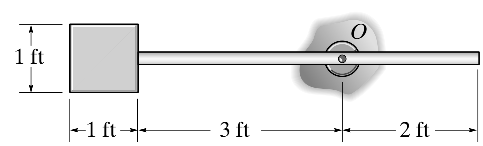
$$= 4.45 \text{ kg}\cdot\text{m}^2$$ **Ans.**

17–15. Each of the three slender rods has a mass m. Determine the moment of inertia of the assembly about an axis that is perpendicular to the page and passes through the center point O.



$$I_O = 3\left[\frac{1}{12} ma^2 + m\left(\frac{a \sin 60^\circ}{3}\right)^2\right] = \frac{1}{2} ma^2$$ **Ans.**

***17–16.** The pendulum consists of a plate having a weight of 12 lb and a slender rod having a weight of 4 lb. Determine the radius of gyration of the pendulum about an axis perpendicular to the page and passing through point O.



$$I_O = \Sigma I_G + md^2$$

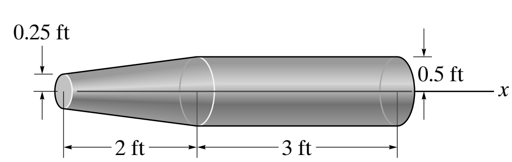
$$= \frac{1}{12}\left(\frac{4}{32.2}\right)(5)^2 + \left(\frac{4}{32.2}\right)(0.5)^2 + \frac{1}{12}\left(\frac{12}{32.2}\right)(1^2 + 1^2) + \left(\frac{12}{32.2}\right)(3.5)^2$$

$$= 4.917 \text{ slug}\cdot\text{ft}^2$$

$$m = \left(\frac{4}{32.2}\right) + \left(\frac{12}{32.2}\right) = 0.4969 \text{ slug}$$

$$k_O = \sqrt{\frac{I_O}{m}} = \sqrt{\frac{4.917}{0.4969}} = 3.15 \text{ ft}$$ **Ans.**

•**17–17.** Determine the moment of inertia of the solid steel assembly about the x axis. Steel has a specific weight of $\gamma_{st} = 490 \text{ lb/ft}^3$.

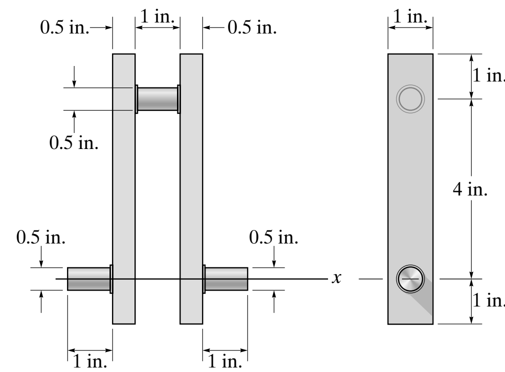


$$I_x = \frac{1}{2} m_1 (0.5)^2 + \frac{3}{10} m_2 (0.5)^2 - \frac{3}{10} m_3 (0.25)^2$$

$$= \left[\frac{1}{2}\pi(0.5)^2(3)(0.5)^2 + \frac{3}{10}\left(\frac{1}{3}\right)\pi(0.5)^2(4)(0.5)^2 - \frac{3}{10}\left(\frac{1}{2}\right)\pi(0.25)^2(2)(0.25)^2\right]\left(\frac{490}{32.2}\right)$$

$$= 5.64 \text{ slug}\cdot\text{ft}^2$$ **Ans.**

17–18. Determine the moment of inertia of the center crank about the x axis. The material is steel having a specific weight of $\gamma_{st} = 490\ \text{lb/ft}^3$.

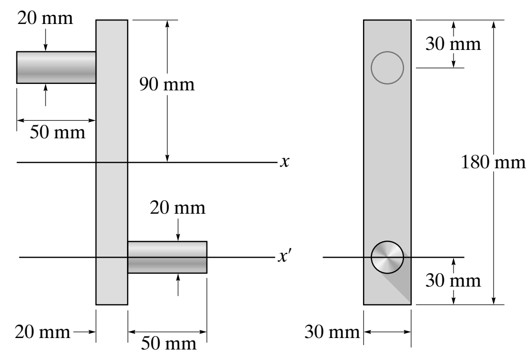


$$m_s = \frac{490}{32.2}\left(\frac{\pi\,(0.25)^2(1)}{(12)^3}\right) = 0.0017291\ \text{slug}$$

$$m_p = \frac{490}{32.2}\left(\frac{(6)(1)(0.5)}{(12)^3}\right) = 0.02642\ \text{slug}$$

$$I_x = 2\left[\frac{1}{12}(0.02642)\big((1)^2 + (6)^2\big) + (0.02642)(2)^2\right]$$
$$+ 2\left[\frac{1}{2}(0.0017291)(0.25)^2\right] + \frac{1}{2}(0.0017291)(0.25)^2 + (0.0017291)(4)^2\Big]$$
$$= 0.402\ \text{slug}\cdot\text{in}^2$$ **Ans.**

17–19. Determine the moment of inertia of the overhung crank about the x axis. The material is steel for which the density is $\rho = 7.85\ \text{Mg/m}^3$.

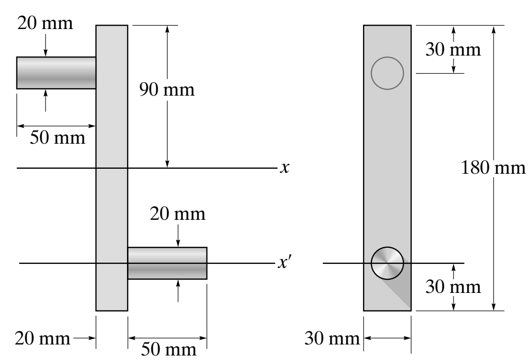


$$m_c = 7.85\big(10^3\big)\big((0.05)\pi(0.01)^2\big) = 0.1233\ \text{kg}$$

$$m_p = 7.85\big(10^3\big)((0.03)(0.180)(0.02)) = 0.8478\ \text{kg}$$

$$I_x = 2\left[\frac{1}{2}(0.1233)(0.01)^2 + (0.1233)(0.06)^2\right]$$
$$+ \left[\frac{1}{12}(0.8478)\big((0.03)^2 + (0.180)^2\big)\right]$$
$$= 0.00325\ \text{kg}\cdot\text{m}^2 = 3.25\ \text{g}\cdot\text{m}^2$$ **Ans.**

***17–20.** Determine the moment of inertia of the overhung crank about the x' axis. The material is steel for which the density is $\rho = 7.85\ \text{Mg/m}^3$.

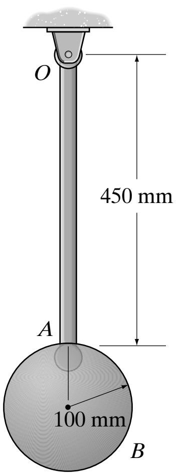


$$m_c = 7.85\big(10^3\big)\big((0.05)\pi(0.01)^2\big) = 0.1233\ \text{kg}$$

$$m_p = 7.85\big(10^3\big)((0.03)(0.180)(0.02)) = 0.8478\ \text{kg}$$

$$I_x = \left[\frac{1}{2}(0.1233)(0.01)^2\right] + \left[\frac{1}{2}(0.1233)(0.02)^2 + (0.1233)(0.120)^2\right]$$
$$+ \left[\frac{1}{12}(0.8478)\big((0.03)^2 + (0.180)^2\big) + (0.8478)(0.06)^2\right]$$
$$= 0.00719\ \text{kg}\cdot\text{m}^2 = 7.19\ \text{g}\cdot\text{m}^2$$ **Ans.**

•17–21. Determine the mass moment of inertia of the pendulum about an axis perpendicular to the page and passing through point O. The slender rod has a mass of 10 kg and the sphere has a mass of 15 kg.



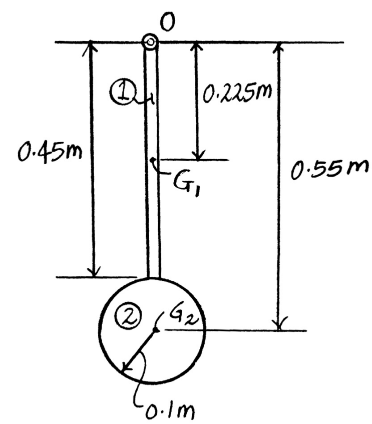
Composite Parts: The pendulum can be subdivided into two segments as shown in Fig. *a*. The perpendicular distances measured from the center of mass of each segment to the point O are also indicated.

Moment of Inertia: The moment of inertia of the slender rod segment (1) and the sphere segment (2) about the axis passing through their center of mass can be computed from $(I_G)_1 = \frac{1}{12}ml^2$ and $(I_G)_2 = \frac{2}{5}mr^2$. The mass moment of inertia of each segment about an axis passing through point O can be determined using the parallel-axis theorem.

$$I_O = \Sigma I_G + md^2$$

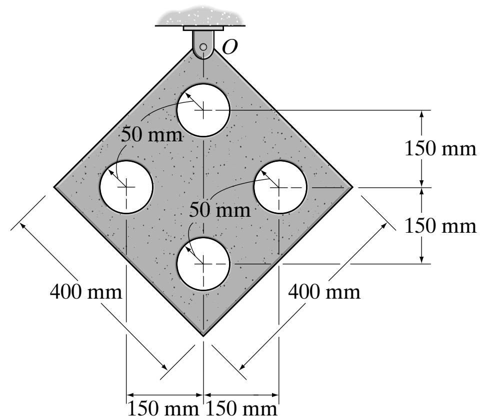
$$= \left[\frac{1}{12}(10)(0.45^2) + 10(0.225^2)\right] + \left[\frac{2}{5}(15)(0.1^2) + 15(0.55^2)\right]$$

$$= 5.27 \text{ kg} \cdot \text{m}^2$$ **Ans.**



(a)

17–22. Determine the mass moment of inertia of the thin plate about an axis perpendicular to the page and passing through point O. The material has a mass per unit area of $20\ \text{kg/m}^2$.



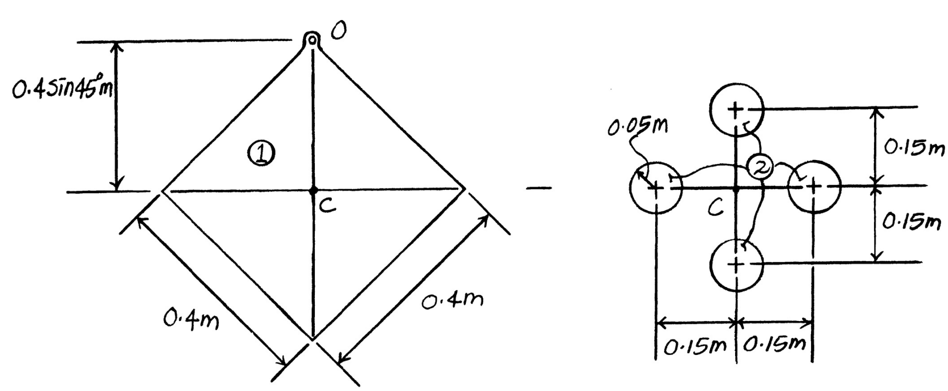
Composite Parts: The plate can be subdivided into the segments shown in Fig. *a*. Here, the four similar holes of which the perpendicular distances measured from their centers of mass to point C are the same and can be grouped as segment (2). This segment should be considered as a negative part.

Mass Moment of Inertia: The mass of segments (1) and (2) are $m_1 = (0.4)(0.4)(20) = 3.2$ kg and $m_2 = \pi(0.05^2)(20) = 0.05\pi$ kg, respectively. The mass moment of inertia of the plate about an axis perpendicular to the page and passing through point C is

$$I_C = \frac{1}{12}(3.2)(0.4^2 + 0.4^2) - 4\left[\frac{1}{2}(0.05\pi)(0.05^2) + 0.05\pi(0.15^2)\right]$$
$$= 0.07041\ \text{kg}\cdot\text{m}^2$$

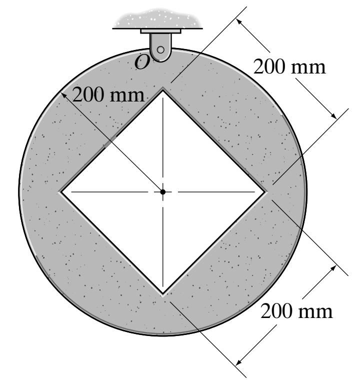
The mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point O can be determined using the parallel-axis theorem $I_O = I_C + md^2$, where $m = m_1 - m_2 = 3.2 - 4(0.05\pi) = 2.5717$ kg and $d = 0.4 \sin 45° m$. Thus,

$$I_O = 0.07041 + 2.5717(0.4 \sin 45°)^2 = 0.276\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$



(a)

17–23. Determine the mass moment of inertia of the thin plate about an axis perpendicular to the page and passing through point O. The material has a mass per unit area of $20\ \mathrm{kg/m^2}$.



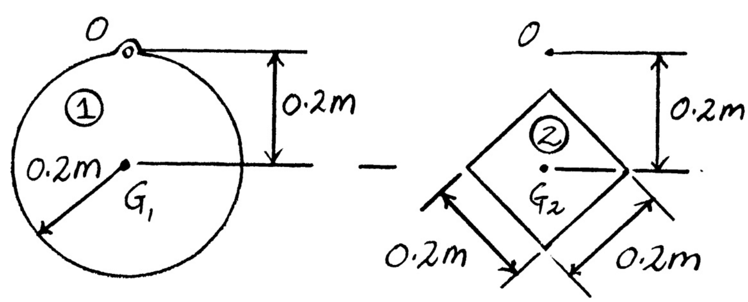
Composite Parts: The plate can be subdivided into two segments as shown in Fig. *a*. Since segment (2) is a hole, it should be considered as a negative part. The perpendicular distances measured from the center of mass of each segment to the point O are also indicated.

Mass Moment of Inertia: The moment of inertia of segments (1) and (2) are computed as $m_1 = \pi(0.2^2)(20) = 0.8\pi$ kg and $m_2 = (0.2)(0.2)(20) = 0.8$ kg. The moment of inertia of the plate about an axis perpendicular to the page and passing through point O for each segment can be determined using the parallel-axis theorem.

$$I_O = \Sigma I_G + md^2$$

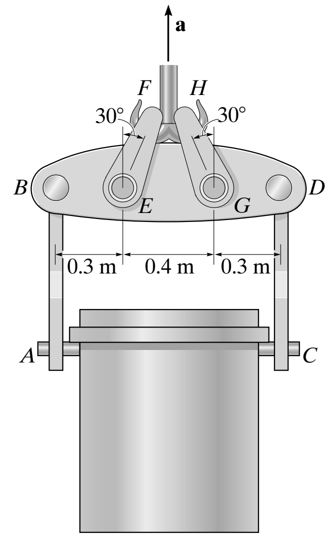
$$= \left[\frac{1}{2}(0.8\pi)(0.2^2) + 0.8\pi(0.2^2)\right] - \left[\frac{1}{12}(0.8)(0.2^2 + 0.2^2) + 0.8(0.2^2)\right]$$

$$= 0.113\ \mathrm{kg \cdot m^2}$$ **Ans.**



(a)

***17–24.** The 4-Mg uniform canister contains nuclear waste material encased in concrete. If the mass of the spreader beam BD is 50 kg, determine the force in each of the links AB, CD, EF, and GH when the system is lifted with an acceleration of $a = 2\ \text{m/s}^2$ for a short period of time.



Canister:

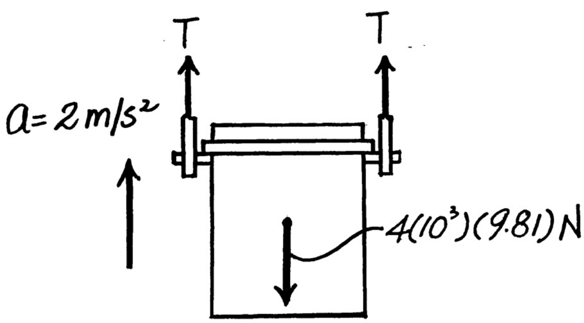
$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2T - 4(10^3)(9.81) = 4(10^3)(2)$

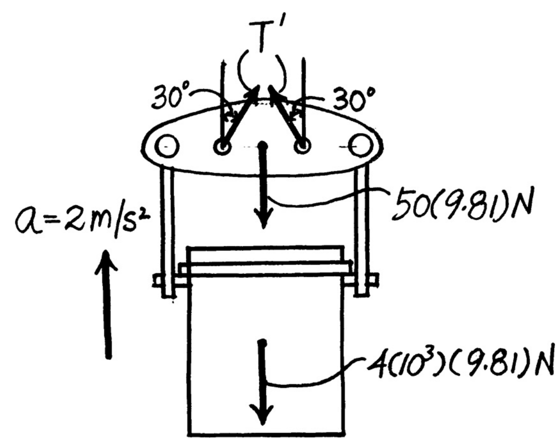
$T_{AB} = T_{CD} = T = 23.6\ \text{kN}$ **Ans.**

System:

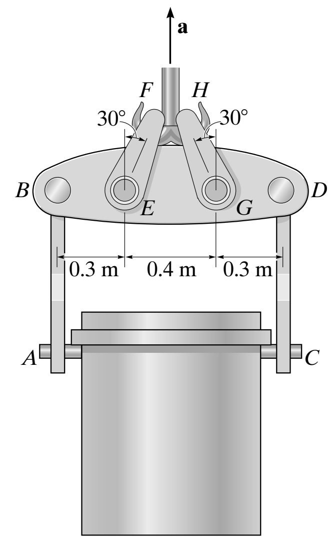
$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2T' \cos 30° - 4050(9.81) = 4050(2)$

$T_{EF} = T_{GH} = T' = 27.6\ \text{kN}$ **Ans.**





•17–25. The 4-Mg uniform canister contains nuclear waste material encased in concrete. If the mass of the spreader beam BD is 50 kg, determine the largest vertical acceleration $\mathbf{a}$ of the system so that each of the links AB and CD are not subjected to a force greater than 30 kN and links EF and GH are not subjected to a force greater than 34 kN.



Canister:

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad 2(30)\left(10^3\right) - 4\left(10^3\right)(9.81) = 4\left(10^3\right)a$

$a = 5.19 \text{ m/s}^2$

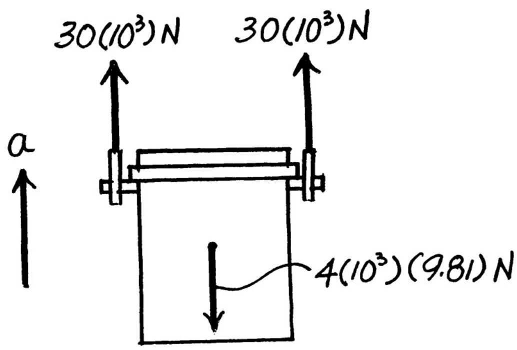
System:

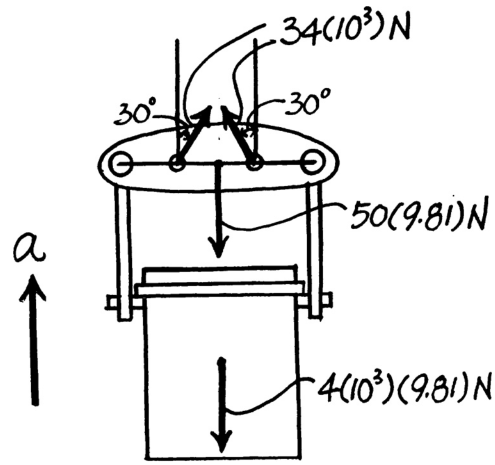
$+\uparrow \Sigma F_y = m(a_G)_y;\qquad 2\left[34\left(10^3\right)\cos 30°\right] - 4050(9.81) = 4050a$

$a = 4.73 \text{ m/s}^2$

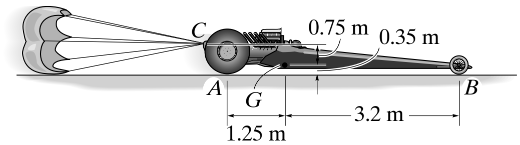
Thus,

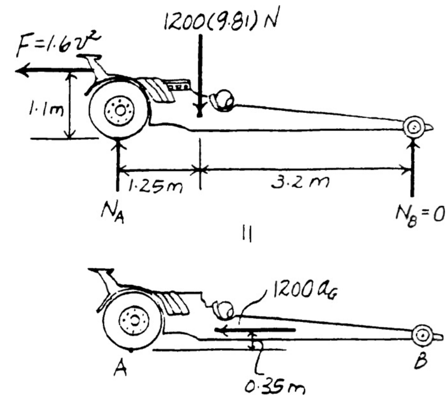
$a_{\max} = 4.73 \text{ m/s}^2$ **Ans.**





17–26. The dragster has a mass of 1200 kg and a center of mass at G. If a braking parachute is attached at C and provides a horizontal braking force of $F = (1.6v^2)$ N, where v is in meters per second, determine the critical speed the dragster can have upon releasing the parachute, such that the wheels at B are on the verge of leaving the ground; i.e., the normal reaction at B is zero. If such a condition occurs, determine the dragster's initial deceleration. Neglect the mass of the wheels and assume the engine is disengaged so that the wheels are free to roll.





If the front wheels are on the verge of lifting off the ground, then $N_B = 0$.

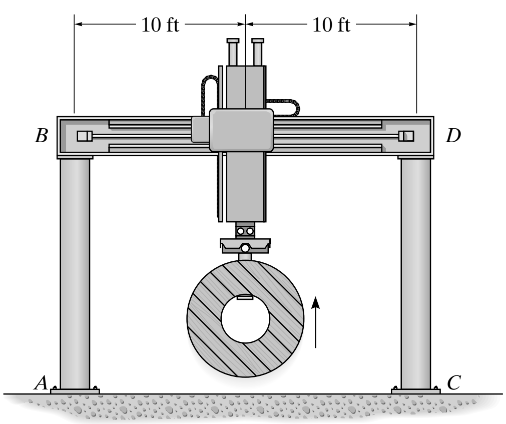
$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad 1.6\,v^2\,(1.1) - 1200(9.81)(1.25) = 1200a_G(0.35)$ **(1)**

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad 1.6v^2 = 1200a_G$ **(2)**

Solving Eqs. (1) and (2) yields

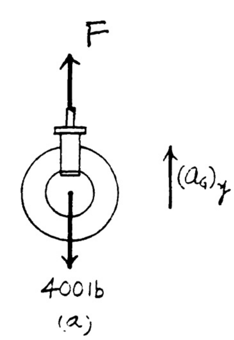
$a_G = 16.35 \text{ m/s}^2 \qquad v = 111 \text{ m/s}$ **Ans.**

17–27. When the lifting mechanism is operating, the 400-lb load is given an upward acceleration of 5 ft/s^2. Determine the compressive force the load creates in each of the columns, AB and CD. What is the compressive force in each of these columns if the load is moving upward at a constant velocity of 3 ft/s? Assume the columns only support an axial load.



Equations of Motion: Applying Eq. 17–12 to FBD(a), we have

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad F - 400 = \left(\dfrac{400}{32.2}\right)(a_G)_y$ **(1)**



Equation of Equilibrium: Due to symmetry $F_{CD} = F_{AB}$. From FBD(b).

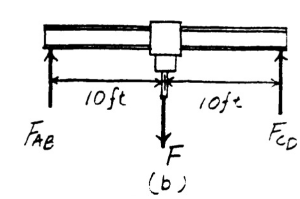
$+\uparrow \Sigma F_y = 0; \qquad 2F_{AB} - F = 0$ **(2)**

If $(a_G)_y = 5 \text{ ft/s}^2$, from Eq. (1), $F = 462.11$ lb. Substitute into Eq. (2) yields

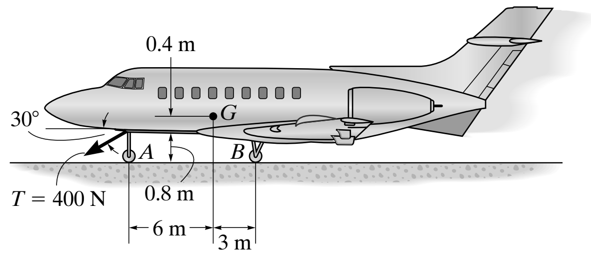
$F_{AB} = F_{CD} = 231 \text{ lb}$ **Ans.**

If the load travels with a constant speed, $(a_G)_y = 0$. From Eq. (1), $F = 400$ lb. Substitute into Eq. (2) yields

$F_{AB} = F_{CD} = 200 \text{ lb}$ **Ans.**



***17–28.** The jet aircraft has a mass of 22 Mg and a center of mass at G. If a towing cable is attached to the upper portion of the nose wheel and exerts a force of $T = 400$ N as shown, determine the acceleration of the plane and the normal reactions on the nose wheel and each of the two wing wheels located at B. Neglect the lifting force of the wings and the mass of the wheels.



$\leftarrow \Sigma F_x = m(a_G)_x;$ $\quad 400 \cos 30° = 22(10^3)\, a_G$

$a_G = 0.01575 \text{ m/s}^2 = 0.0157 \text{ m/s}^2$ **Ans.**

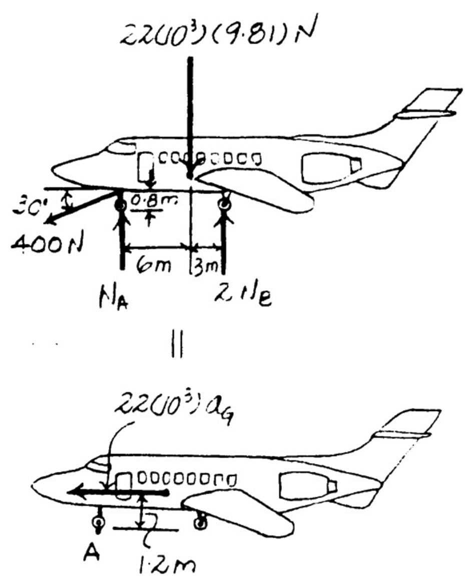
$\zeta + \Sigma M_A = \Sigma (M_k)_A;$ $\quad 400 \cos 30° \,(0.8) + 2N_B\,(9) - 22(10^3)\,(9.81)(6)$

$= 22(10^3)(0.01575)(1.2)$

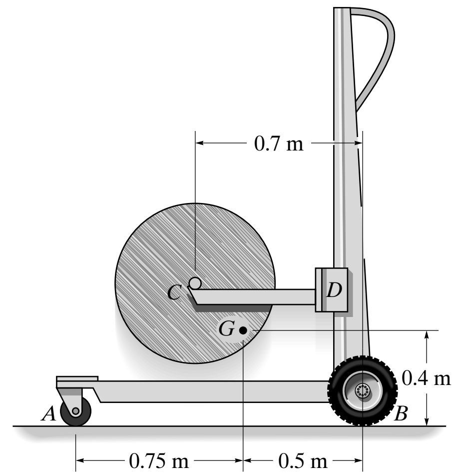
$N_B = 71\,947.70 \text{ N} = 71.9 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad N_A + 2(71\,947.70) - 22(10^3)(9.81) - 400 \sin 30° = 0$

$N_A = 72\,124.60 \text{ N} = 72.1 \text{ kN}$ **Ans.**



•17–29. The lift truck has a mass of 70 kg and mass center at G. If it lifts the 120-kg spool with an acceleration of $3\ \text{m/s}^2$, determine the reactions on each of the four wheels. The loading is symmetric. Neglect the mass of the movable arm CD.



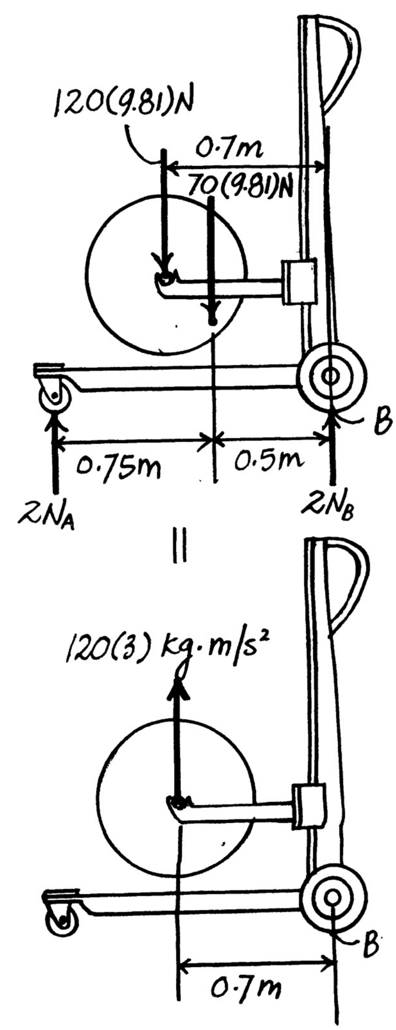
$\zeta + \Sigma M_B = \Sigma(M_k)_B; \qquad 70(9.81)(0.5) + 120(9.81)(0.7) - 2N_A(1.25)$

$= -120(3)(0.7)$

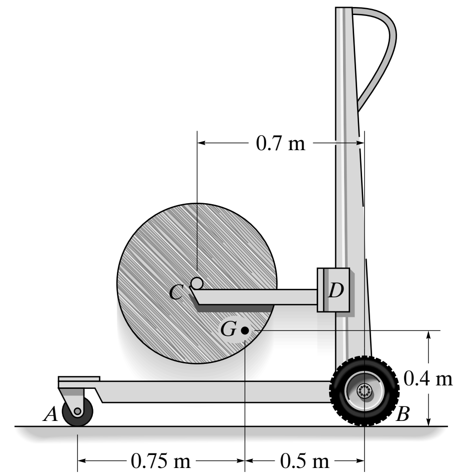
$N_A = 567.76\ \text{N} = 568\ \text{N}$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2(567.76) + 2N_B - 120(9.81) - 70(9.81) = 120(3)$

$N_B = 544\ \text{N}$ **Ans.**



17–30. The lift truck has a mass of 70 kg and mass center at G. Determine the largest upward acceleration of the 120-kg spool so that no reaction on the wheels exceeds 600 N.



Assume $N_A = 600$ N.

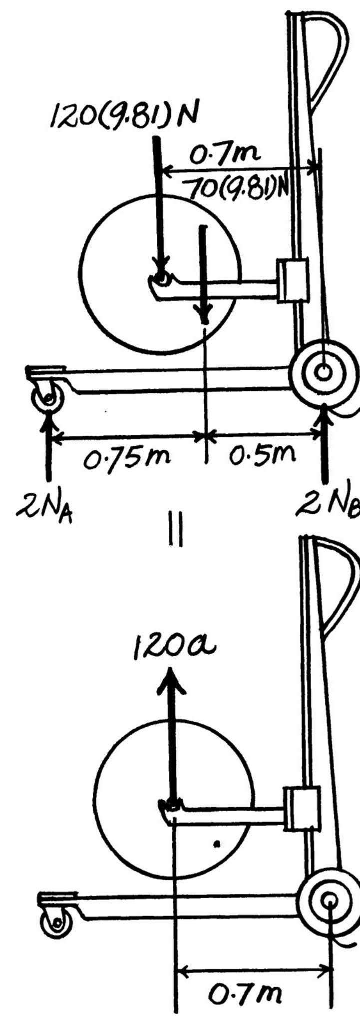
$\zeta + \Sigma M_B = \Sigma (M_k)_B;\qquad 70(9.81)(0.5) + 120(9.81)(0.7) - 2(600)(1.25) = -120a(0.7)$

$$a = 3.960 \text{ m/s}^2$$

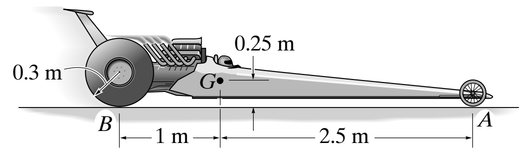
$+\uparrow \Sigma F_y = m(a_G)_y;\qquad 2(600) + 2N_B - 120(9.81) - 70(9.81) = 120(3.960)$

$N_B = 570 \text{ N} < 600 \text{ N}$ **OK**

Thus $a = 3.96 \text{ m/s}^2$ **Ans.**



17–31. The dragster has a mass of 1500 kg and a center of mass at G. If the coefficient of kinetic friction between the rear wheels and the pavement is $\mu_k = 0.6$, determine if it is possible for the driver to lift the front wheels, A, off the ground while the rear drive wheels are slipping. Neglect the mass of the wheels and assume that the front wheels are free to roll.



If the front wheels A lift off the ground, then $N_A = 0$.

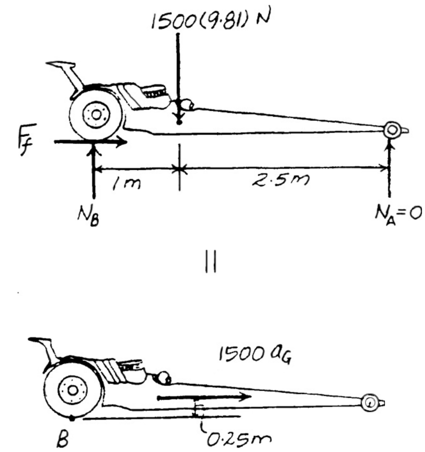
$\zeta + \Sigma M_B = \Sigma(M_k)_B$; $\quad -1500(9.81)(1) = -1500a_G(0.25)$

$$a_G = 39.24 \text{ m/s}^2$$

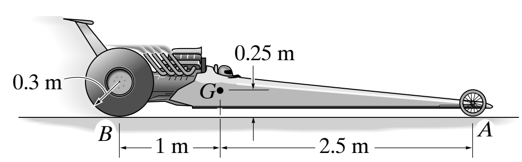
$\xrightarrow{+} \Sigma F_x = m(a_G)_x$; $\quad F_f = 1500(39.24) = 58860 \text{ N}$

$+\uparrow \Sigma F_y = m(a_G)_y$; $\quad N_B - 1500(9.81) = 0 \quad N_B = 14715 \text{ N}$

Since the required friction $F_f > (F_f)_{max} = \mu_k N_B = 0.6(14715) = 8829$ N, **it is not possible to lift the front wheels off the ground**. **Ans.**



***17–32.** The dragster has a mass of 1500 kg and a center of mass at G. If no slipping occurs, determine the frictional force $\mathbf{F}_B$ which must be developed at each of the rear drive wheels B in order to create an acceleration of $a = 6 \text{ m/s}^2$. What are the normal reactions of each wheel on the ground? Neglect the mass of the wheels and assume that the front wheels are free to roll.



$\zeta + \Sigma M_B = \Sigma(M_k)_B$; $\quad 2N_A(3.5) - 1500(9.81)(1) = -1500(6)(0.25)$

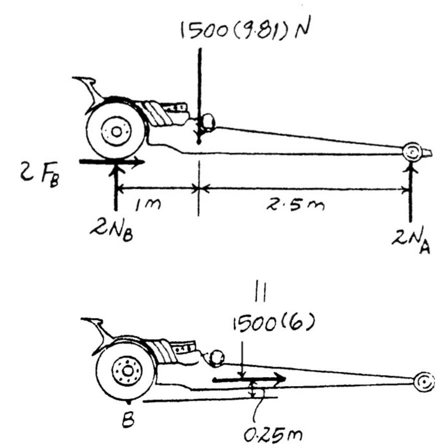
$$N_A = 1780.71 \text{ N} = 1.78 \text{ kN}$$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y$; $\quad 2N_B + 2(1780.71) - 1500(9.81) = 0$

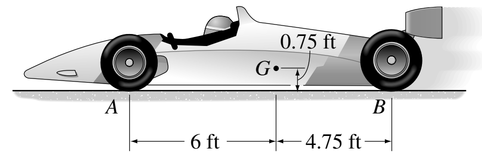
$$N_B = 5576.79 \text{ N} = 5.58 \text{ kN}$$ **Ans.**

$\xrightarrow{+} \Sigma F_x = m(a_G)_x$; $\quad 2F_B = 1500(6)$

$$F_B = 4500 \text{ N} = 4.50 \text{ kN}$$ **Ans.**



•17–33. At the start of a race, the rear drive wheels B of the 1550-lb car slip on the track. Determine the car's acceleration and the normal reaction the track exerts on the front pair of wheels A and rear pair of wheels B. The coefficient of kinetic friction is $\mu_k = 0.7$, and the mass center of the car is at G. The front wheels are free to roll. Neglect the mass of all the wheels.



Equations of Motion: Since the rear wheels B are required to slip, the frictional force developed is $F_B = \mu_s N_B = 0.7N_B$.

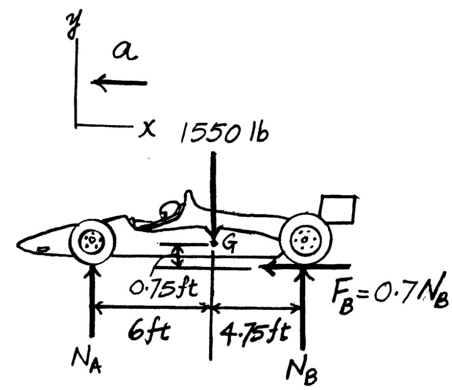
$$\xleftarrow{+} \Sigma F_x = m(a_G)_x; \qquad 0.7N_B = \frac{1550}{32.2}a \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + N_B - 1550 = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_G = 0; \qquad N_B(4.75) - 0.7N_B(0.75) - N_A(6) = 0 \qquad \textbf{(3)}$$

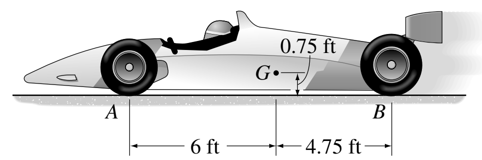
Solving Eqs. (1), (2), and (3) yields

$N_A = 640.46 \text{ lb} = 640 \text{ lb}$ $\qquad N_B = 909.54 \text{ lb} = 910 \text{ lb}$ $\qquad a = 13.2 \text{ ft/s}^2$ **Ans.**



(a)

17–34. Determine the maximum acceleration that can be achieved by the car without having the front wheels A leave the track or the rear drive wheels B slip on the track. The coefficient of static friction is $\mu_s = 0.9$. The car's mass center is at G, and the front wheels are free to roll. Neglect the mass of all the wheels.



Equations of Motion:

$$\xleftarrow{+} \Sigma F_x = m(a_G)_x; \qquad F_B = \frac{1550}{32.2}a \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + N_B - 1550 = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_G = 0; \qquad N_B(4.75) - F_B(0.75) - N_A(6) = 0 \qquad \textbf{(3)}$$

If we assume that the front wheels are about to leave the track, $N_A = 0$. Substituting this value into Eqs. (2) and (3) and solving Eqs. (1), (2), (3),

$N_B = 1550 \text{ lb}$ $\qquad F_B = 9816.67 \text{ lb}$ $\qquad a = 203.93 \text{ ft/s}^2$

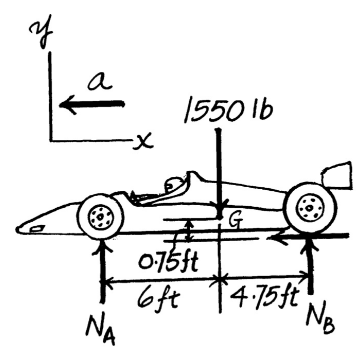
Since $F_B > (F_B)_{max} = \mu_s N_B = 0.9(1550) \text{ lb} = 1395 \text{ lb}$, the rear wheels will slip. Thus, the solution must be reworked so that the rear wheels are about to slip.

$$F_B = \mu_s N_B = 0.9N_B \qquad \textbf{(4)}$$

Solving Eqs. (1), (2), (3), and (4) yields

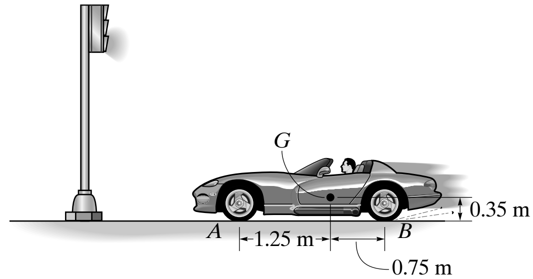
$N_A = 626.92 \text{ lb}$ $\qquad N_B = 923.08 \text{ lb}$

$a = 17.26 \text{ ft/s}^2 = 17.3 \text{ ft/s}^2$ **Ans.**



(a)

17–35. The sports car has a mass of 1.5 Mg and a center of mass at G. Determine the shortest time it takes for it to reach a speed of 80 km/h, starting from rest, if the engine only drives the rear wheels, whereas the front wheels are free rolling. The coefficient of static friction between the wheels and the road is $\mu_s = 0.2$. Neglect the mass of the wheels for the calculation. If driving power could be supplied to all four wheels, what would be the shortest time for the car to reach a speed of 80 km/h?



$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\qquad 0.2N_A + 0.2N_B = 1500a_G$ **(1)**

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A + N_B - 1500(9.81) = 0$ **(2)**

$\circlearrowleft + \Sigma M_G = 0;\qquad -N_A(1.25) + N_B(0.75) - (0.2N_A + 0.2N_B)(0.35) = 0$ **(3)**

For Rear-Wheel Drive:

Set the friction force $0.2N_A = 0$ in Eqs. (1) and (3).

Solving yields:

$N_A = 5.18\text{ kN} > 0$ **(OK)** $\qquad N_B = 9.53\text{ kN} \qquad a_G = 1.271\text{m/s}^2$

Since $v = 80\text{ km/h} = 22.22\text{ m/s}$, then

$$\left(\overset{+}{\leftarrow}\right)\qquad v = v_0 + a_G t$$

$$22.22 = 0 + 1.271t$$

$$t = 17.5\text{ s}$$ **Ans.**

For 4-Wheel Drive:

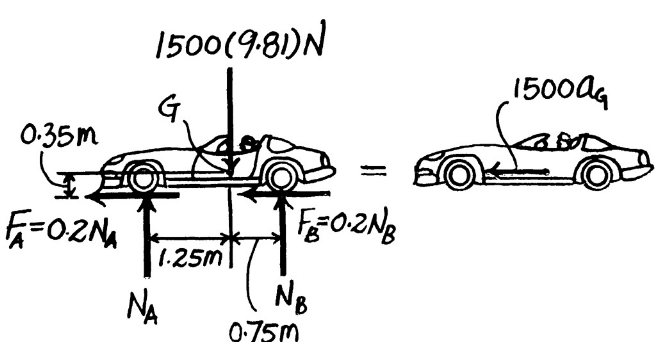
$N_A = 5.00\text{ kN} > 0$ **(OK)** $\qquad N_B = 9.71\text{ kN} \qquad a_G = 1.962\text{m/s}^2$

Since $v_2 = 80\text{ km/h} = 22.22\text{ m/s}$, then

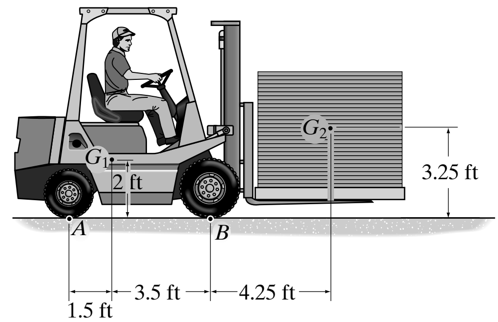
$$v_2 = v_1 + a_G t$$

$$22.22 = 0 + 1.962t$$

$$t = 11.3\text{ s}$$ **Ans.**



***17–36.** The forklift travels forward with a constant speed of 9 ft/s. Determine the shortest stopping distance without causing any of the wheels to leave the ground. The forklift has a weight of 2000 lb with center of gravity at G_1, and the load weighs 900 lb with center of gravity at G_2. Neglect the weight of the wheels.



Equations of Motion: Since it is required that the rear wheels are about to leave the ground, $N_A = 0$. Applying the moment equation of motion of about point B,

$$\circlearrowleft + \Sigma M_B = (M_k)_B; \qquad 2000(3.5) - 900(4.25) = \left(\frac{2000}{32.2}a\right)(2) + \left(\frac{900}{32.2}a\right)(3.25)$$

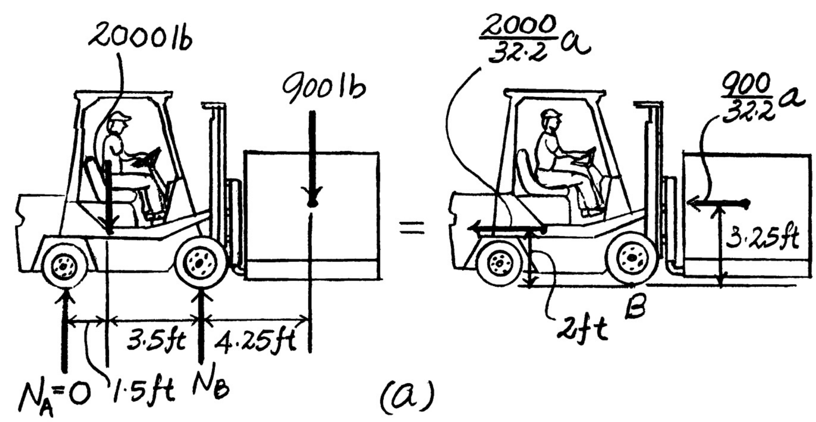
$$a = 14.76 \text{ ft/s}^2 \leftarrow$$

Kinematics: Since the acceleration of the forklift is constant,

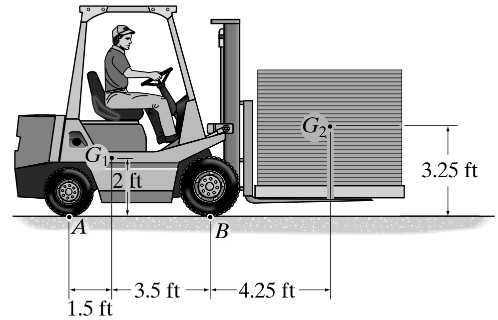
$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = 9^2 + 2(-14.76)(s - 0)$$

$$s = 2.743 \text{ ft} = 2.74 \text{ ft} \qquad \textbf{Ans.}$$



•17–37. If the forklift's rear wheels supply a combined traction force of $F_A = 300$ lb, determine its acceleration and the normal reactions on the pairs of rear wheels and front wheels. The forklift has a weight of 2000 lb, with center of gravity at G_1, and the load weighs 900 lb, with center of gravity at G_2. The front wheels are free to roll. Neglect the weight of the wheels.



Equations of Motion: The acceleration of the forklift can be obtained directly by writing the force equation of motion along the x axis.

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 300 = \frac{2000}{32.2}a + \frac{900}{32.2}a$$

$$a = 3.331 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Using this result and writing the moment equation of motion about point A,

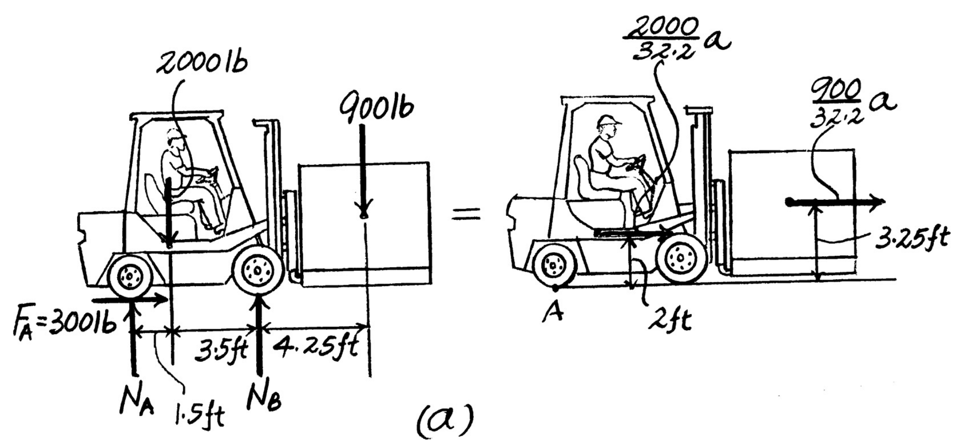
$$\circlearrowleft + \Sigma M_A = (M_k)_A; \qquad N_B(5) - 2000(1.5) - 900(9.25) = -\left(\frac{2000}{32.2}\right)(3.331)(2) - \left(\frac{900}{32.2}\right)(3.331)(3.25)$$

$$N_B = 2121.72 \text{ lb} = 2122 \text{ lb} \qquad \textbf{Ans.}$$

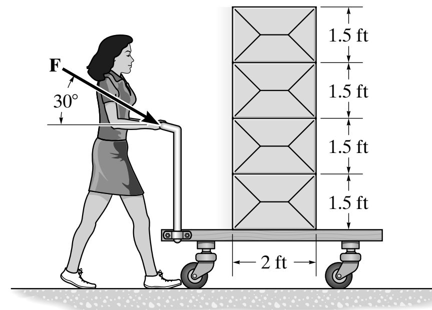
Finally, writing the force equation of motion along the y axis and using this result,

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + 2121.72 - 2000 - 900 = 0$$

$$N_A = 778.28 \text{ lb} = 778 \text{ lb} \qquad \textbf{Ans.}$$



17–38. Each uniform box on the stack of four boxes has a weight of 8 lb. The stack is being transported on the dolly, which has a weight of 30 lb. Determine the maximum force **F** which the woman can exert on the handle in the direction shown so that no box on the stack will tip or slip. The coefficient of the static friction at all points of contact is $\mu_s = 0.5$. The dolly wheels are free to roll. Neglect their mass.

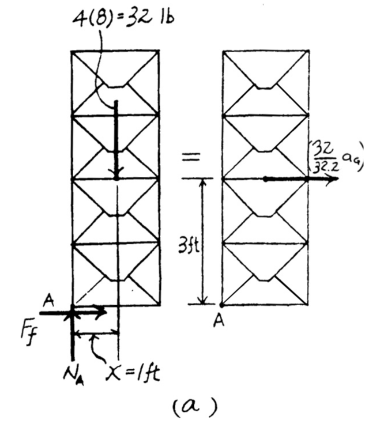


Assume that the boxes up, then $x = 1$ ft. Applying Eq. 17–12 to FBD(a). we have

$$\zeta + \Sigma M_A = \Sigma(M_k)_A; \qquad -32(1) = -\left[\left(\frac{32}{32.2}\right)a_G\right](3) \qquad a_G = 10.73\ \text{ft/s}^2$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A - 32 = 0 \qquad N_A = 32.0\ \text{lb}$$

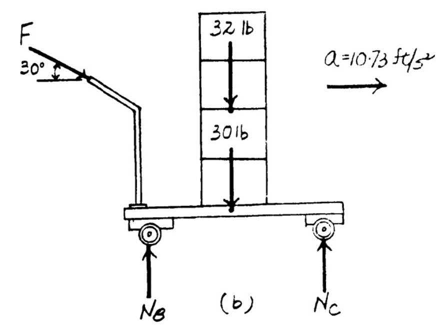
$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad F_f = \left(\frac{32}{32.2}\right)(10.73) = 10.67\ \text{lb}$$



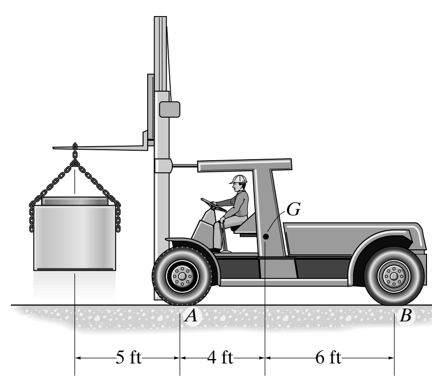
Since $F_f < (F_f)_{\max} = \mu_s N_A = 0.5(32.0) = 16.0$ lb. slipping will not occur. Hence, the boxes and the dolly moves as a unit. From FBD(b),

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad F\cos 30° = \left(\frac{32 + 30}{32.2}\right)(10.73)$$

$$F = 23.9\ \text{lb}$$ **Ans.**



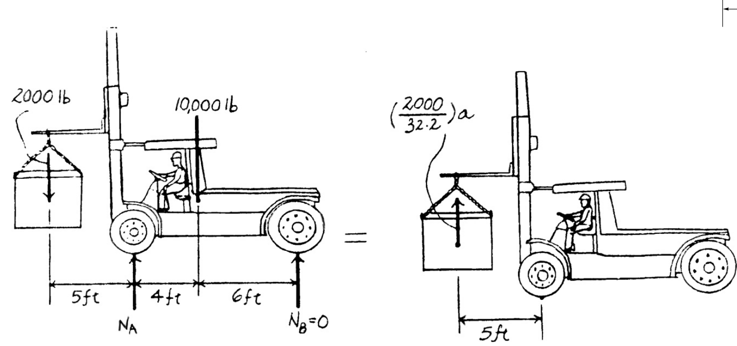
17–39. The forklift and operator have a combined weight of 10 000 lb and center of mass at G. If the forklift is used to lift the 2000-lb concrete pipe, determine the maximum vertical acceleration it can give to the pipe so that it does not tip forward on its front wheels.



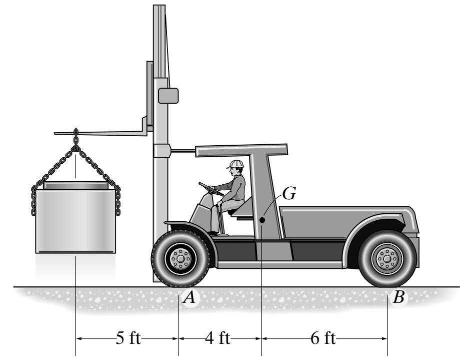
It is required that $N_B = 0$.

$$\zeta + \Sigma M_A = \Sigma(M_k)_A; \qquad 2000(5) - 10000(4) = -\left[\left(\frac{2000}{32.2}\right)a\right](5)$$

$$a = 96.6\ \text{ft/s}^2$$ **Ans.**



***17–40.** The forklift and operator have a combined weight of 10 000 lb and center of mass at G. If the forklift is used to lift the 2000-lb concrete pipe, determine the normal reactions on each of its four wheels if the pipe is given an upward acceleration of 4 ft/s^2.

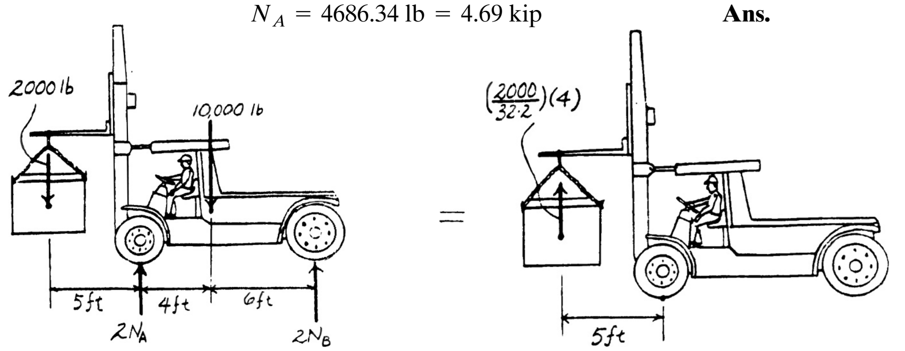


$$\zeta + \Sigma M_A = \Sigma(M_k)_A; \qquad 2000(5) + 2N_B(10) - 10000(4) = -\left[\left(\frac{2000}{32.2}\right)(4)\right](5)$$

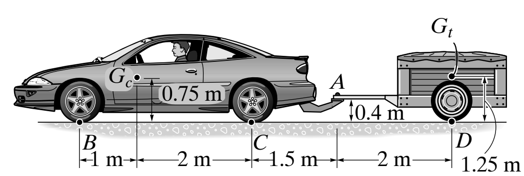
$$N_B = 1437.89 \text{ lb} = 1.44 \text{ kip} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2N_A + 2(1437.89) - 2000 - 10000 = \left(\frac{2000}{32.2}\right)(4)$$

$$N_A = 4686.34 \text{ lb} = 4.69 \text{ kip} \qquad \textbf{Ans.}$$



•17–41. The car, having a mass of 1.40 Mg and mass center at G_c, pulls a loaded trailer having a mass of 0.8 Mg and mass center at G_t. Determine the normal reactions on both the car's front and rear wheels and the trailer's wheels if the driver applies the car's rear brakes C and causes the car to skid. Take $\mu_C = 0.4$ and assume the hitch at A is a pin or ball-and-socket joint. The wheels at B and D are free to roll. Neglect their mass and the mass of the driver.

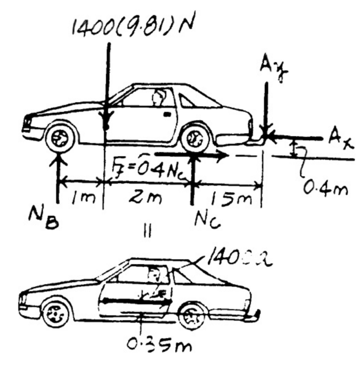


Equations of Motion: Since the car skids, then $F_f = \mu_C N_C = 0.4N_C$. Applying Eq. 17–12 to FBD(a), we have

$$\zeta + \Sigma M_A = \Sigma(M_k)_A; \qquad 1400(9.81)(3.5) + 0.4N_C(0.4) - N_B(4.5) - N_C(1.5) = -1400a(0.35) \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_B + N_C - 1400(9.81) - A_y = 0 \qquad \textbf{(2)}$$

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 0.4N_C - A_x = 1400a \qquad \textbf{(3)}$$



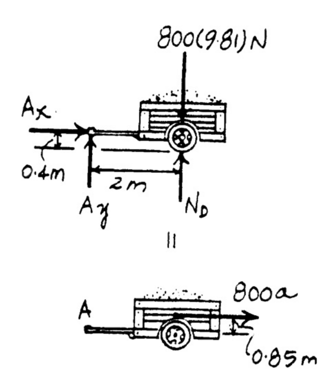
(a)

From FBD(b),

$$\zeta + \Sigma M_A = \Sigma(M_k)_A; \qquad N_D(2) - 800(9.81)(2) = -800a(0.85) \qquad \textbf{(4)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_D + A_y - 800(9.81) = 0 \qquad \textbf{(5)}$$

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad A_x = 800a \qquad \textbf{(6)}$$



(b)

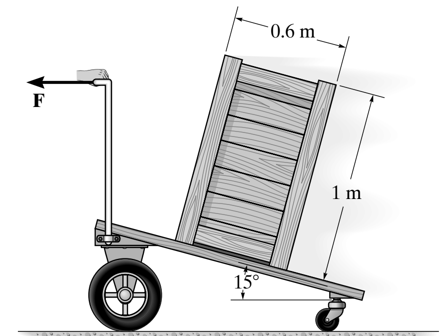
Solving Eqs. (1), (2), (3), (4), (5), and (6) yields

$$N_B = 9396.95 \text{ N} = 9.40 \text{ kN} \qquad N_C = 4622.83 \text{ N} = 4.62 \text{ kN} \qquad \textbf{Ans.}$$

$$N_D = 7562.23 \text{ N} = 7.56 \text{ kN} \qquad \textbf{Ans.}$$

$$a = 0.8405 \text{ m/s}^2 \qquad A_x = 672.41 \text{ N} \qquad A_y = 285.77 \text{ N}$$

17–42. The uniform crate has a mass of 50 kg and rests on the cart having an inclined surface. Determine the smallest acceleration that will cause the crate either to tip or slip relative to the cart. What is the magnitude of this acceleration? The coefficient of static friction between the crate and the cart is $\mu_s = 0.5$.



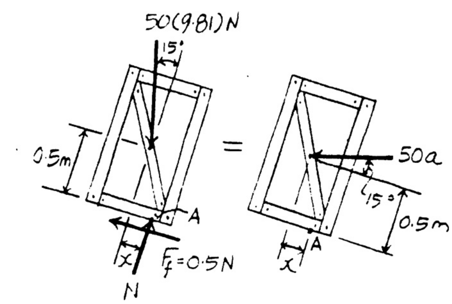
Equations of Motion: Assume that the crate slips, then $F_f = \mu_s N = 0.5N$.

$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \qquad 50(9.81)\cos 15°(x) - 50(9.81)\sin 15°(0.5)$$

$$= 50a\cos 15°(0.5) + 50a\sin 15°(x) \qquad \textbf{(1)}$$

$$+\nearrow \Sigma F_{y'} = m(a_G)_{y'}; \qquad N - 50(9.81)\cos 15° = -50a\sin 15° \qquad \textbf{(2)}$$

$$\searrow + \Sigma F_{x'} = m(a_G)_{x'}; \qquad 50(9.81)\sin 15° - 0.5N = -50a\cos 15° \qquad \textbf{(3)}$$



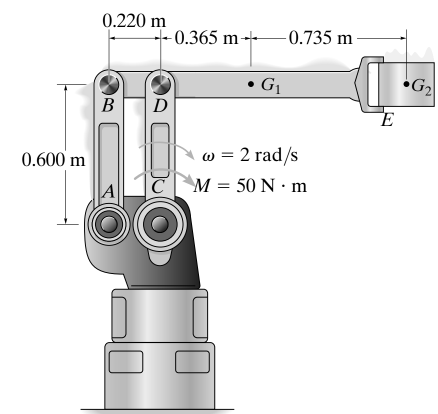
Solving Eqs. (1), (2), and (3) yields

$$N = 447.81 \text{ N} \qquad x = 0.250 \text{ m}$$

$$a = 2.01 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Since $x < 0.3$ m, then crate will not tip. Thus, **the crate slips**. **Ans.**

17–43. Arm BDE of the industrial robot is activated by applying the torque of $M = 50$ N·m to link CD. Determine the reactions at pins B and D when the links are in the position shown and have an angular velocity of 2 rad/s. Arm BDE has a mass of 10 kg with center of mass at G_1. The container held in its grip at E has a mass of 12 kg with center of mass at G_2. Neglect the mass of links AB and CD.



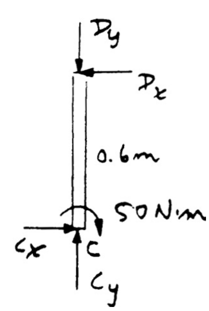
Curvilinear translation:

$$(a_D)_n = (a_G)_n = (2)^2(0.6) = 2.4 \text{ m/s}^2$$

Member DC:

$$\circlearrowleft + \Sigma M_C = 0; \qquad -D_x(0.6) + 50 = 0$$

$$D_x = 83.33 \text{ N} = 83.3 \text{ N} \qquad \textbf{Ans.}$$



Member BDE:

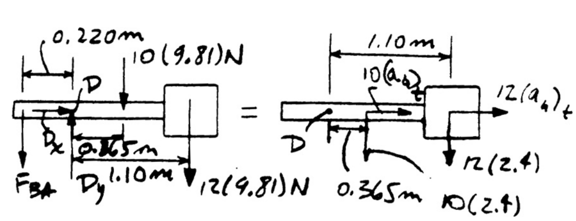
$$\circlearrowleft + \Sigma M_D = \Sigma (M_k)_D; \qquad -F_{BA}(0.220) + 10(9.81)(0.365) + 12(9.81)(1.10)$$

$$= 10(2.4)(0.365) + 12(2.4)(1.10)$$

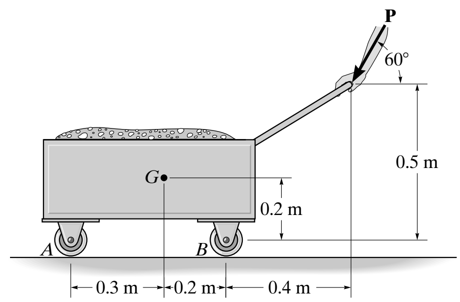
$$F_{BA} = 567.54 \text{ N} = 568 \text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad -567.54 + D_y - 10(9.81) - 12(9.81) = -10(2.4) - 12(2.4)$$

$$D_y = 731 \text{ N} \qquad \textbf{Ans.}$$



***17–44.** The handcart has a mass of 200 kg and center of mass at G. Determine the normal reactions at each of the two wheels at A and at B if a force of $P = 50$ N is applied to the handle. Neglect the mass of the wheels.

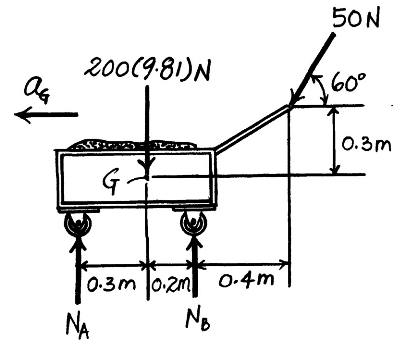


$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x\,; \qquad 50 \cos 60° = 200 a_G$

$+\uparrow \Sigma F_y = m(a_G)_y\,; \qquad N_A + N_B - 200(9.81) - 50 \sin 60° = 0$

$\circlearrowleft + \Sigma M_G = 0; \qquad -N_A(0.3) + N_B(0.2) + 50 \cos 60°(0.3) - 50 \sin 60°(0.6) = 0$

$a_G = 0.125 \text{ m/s}^2 \qquad N_A = 765.2 \text{ N} \qquad N_B = 1240 \text{ N}$

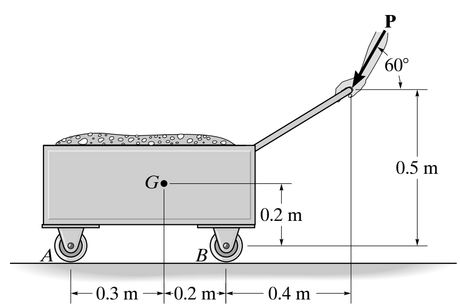


At each wheel,

$$N_A' = \frac{N_A}{2} = 383 \text{ N} \qquad \textbf{Ans.}$$

$$N_B' = \frac{N_B}{2} = 620 \text{ N} \qquad \textbf{Ans.}$$

•17–45. The handcart has a mass of 200 kg and center of mass at G. Determine the largest magnitude of force **P** that can be applied to the handle so that the wheels at A or B continue to maintain contact with the ground. Neglect the mass of the wheels.



$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x\,; \qquad P \cos 60° = 200 a_G$

$+\uparrow \Sigma F_y = m(a_G)_y\,; \qquad N_A + N_B - 200(9.81) - P \sin 60° = 0$

$\circlearrowleft + \Sigma M_G = 0; \qquad -N_A(0.3) + N_B(0.2) + P \cos 60°(0.3) - P \sin 60°(0.6) = 0$

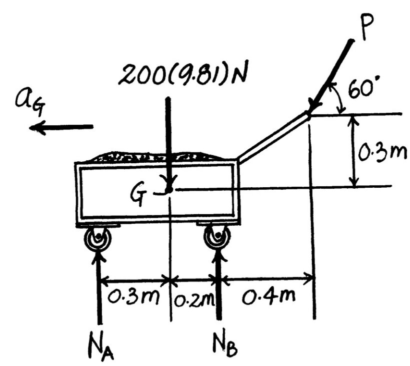
For P_{max}, require

$$N_A = 0$$

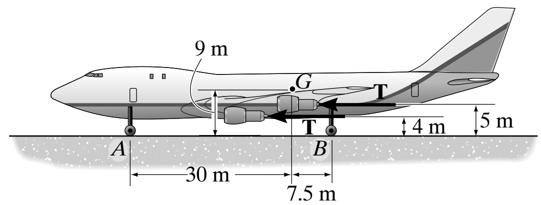
$$P = 1998 \text{ N} = 2.00 \text{ kN} \qquad \textbf{Ans.}$$

$$N_B = 3692 \text{ N}$$

$$a_G = 4.99 \text{ m/s}^2$$



17–46. The jet aircraft is propelled by four engines to increase its speed uniformly from rest to 100 m/s in a distance of 500 m. Determine the thrust **T** developed by each engine and the normal reaction on the nose wheel A. The aircraft's total mass is 150 Mg and the mass center is at point G. Neglect air and rolling resistance and the effect of lift.



Kinematics: The acceleration of the aircraft can be determined from

$$v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$100^2 = 0^2 + 2a(500 - 0)$$

$$a = 10 \text{ m/s}^2$$

Equations of Motion: The thrust **T** can be determined directly by writing the force equation of motion along the x axis.

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 4T = 150(10^3)(10)$$

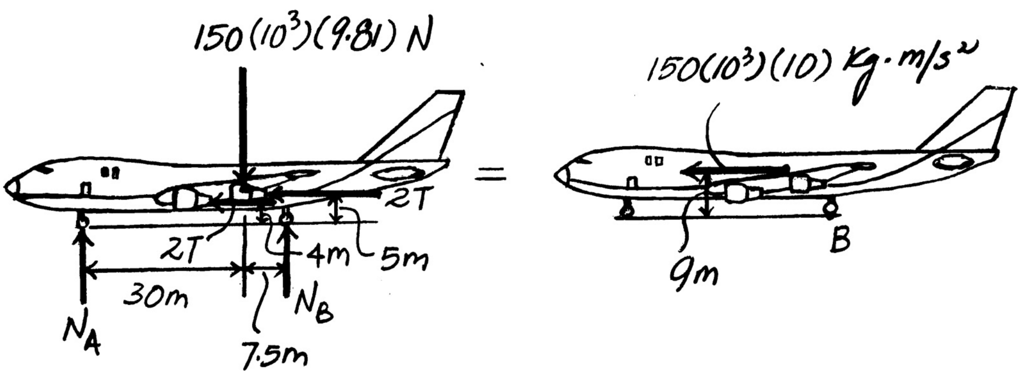
$$T = 375(10^3)\text{N} = 375 \text{ kN} \qquad \textbf{Ans.}$$

Writing the moment equation of equilibrium about point B and using the result of **T**,

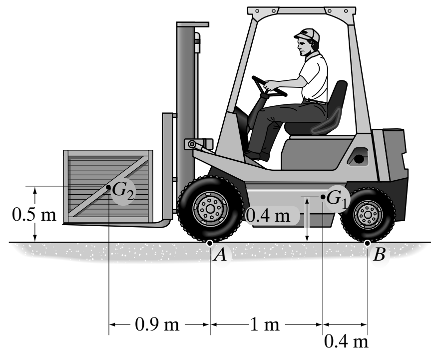
$$\circlearrowleft + \Sigma M_B = (M_k)_B; \qquad 150(10^3)(9.81)(7.5) + 2\left[375(10^3)\right](5) + 2\left[375(10^3)\right](4)$$

$$- N_A(37.5) = 150(10^3)(10)(9)$$

$$N_A = 114.3(10^3)\text{N} = 114 \text{ kN} \qquad \textbf{Ans.}$$



17–47. The 1-Mg forklift is used to raise the 750-kg crate with a constant acceleration of $2\,\text{m/s}^2$. Determine the reaction exerted by the ground on the pairs of wheels at A and at B. The centers of mass for the forklift and the crate are located at G_1 and G_2, respectively.



Equations of Motion: $\mathbf{N}_B$ can be obtained directly by writing the moment equation of motion about point A.

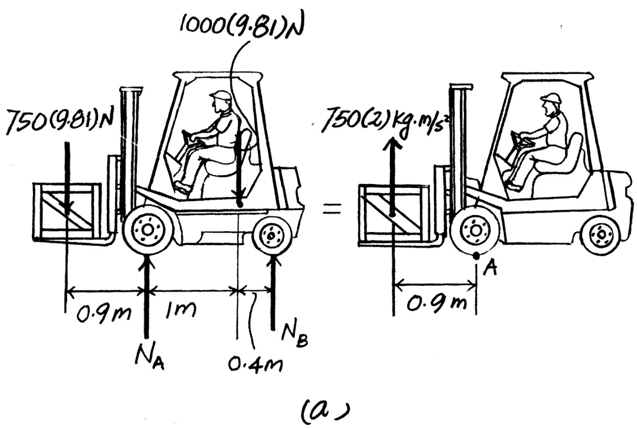
$$\circlearrowleft + \Sigma M_A = (M_k)_A; \quad N_B(1.4) + 750(9.81)(0.9) - 1000(9.81)(1) = -750(2)(0.9)$$

$$N_B = 1313.03\text{ N} = 1.31\text{ kN} \qquad \textbf{Ans.}$$

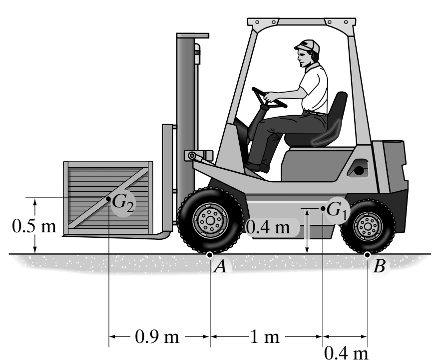
Using this result to write the force equation of motion along the y axis,

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + 1313.03 - 750(9.81) - 1000(9.81) = 750(2)$$

$$N_A = 17354.46\text{ N} = 17.4\text{ kN} \qquad \textbf{Ans.}$$



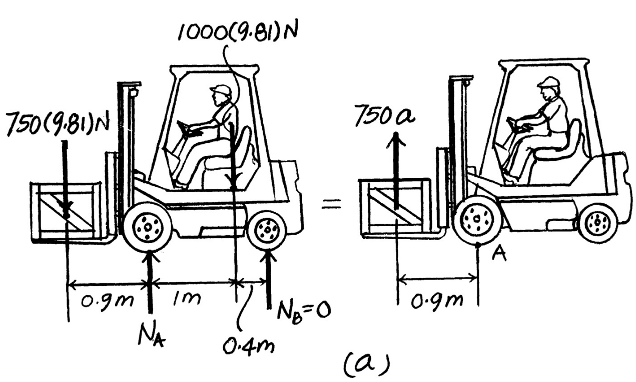
***17–48.** Determine the greatest acceleration with which the 1-Mg forklift can raise the 750-kg crate, without causing the wheels at B to leave the ground. The centers of mass for the forklift and the crate are located at G_1 and G_2, respectively.



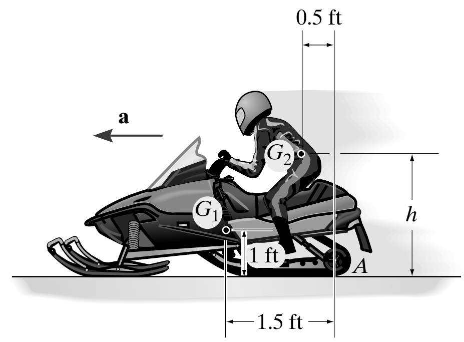
Equations of Motion: Since the wheels at B are required to just lose contact with the ground, $N_B = 0$. The direct solution for $\mathbf{a}$ can be obtained by writing the moment equation of motion about point A.

$$\circlearrowleft + \Sigma M_A = (M_k)_A; \quad 750(9.81)(0.9) - 1000(9.81)(1) = -750a(0.9)$$

$$a = 4.72\text{ m/s}^2 \qquad \textbf{Ans.}$$



•17–49. The snowmobile has a weight of 250 lb, centered at G_1, while the rider has a weight of 150 lb, centered at G_2. If the acceleration is $a = 20 \text{ ft/s}^2$, determine the maximum height h of G_2 of the rider so that the snowmobile's front skid does not lift off the ground. Also, what are the traction (horizontal) force and normal reaction under the rear tracks at A?



Equations of Motion: Since the front skid is required to be on the verge of lift off, $N_B = 0$. Writing the moment equation about point A and referring to Fig. a,

$$\circlearrowleft +\Sigma M_A = (M_k)_A; \qquad 250(1.5) + 150(0.5) = \frac{150}{32.2}(20)(h_{\max}) + \frac{250}{32.2}(20)(1)$$

$$h_{\max} = 3.163 \text{ ft} = 3.16 \text{ ft} \qquad \textbf{Ans.}$$

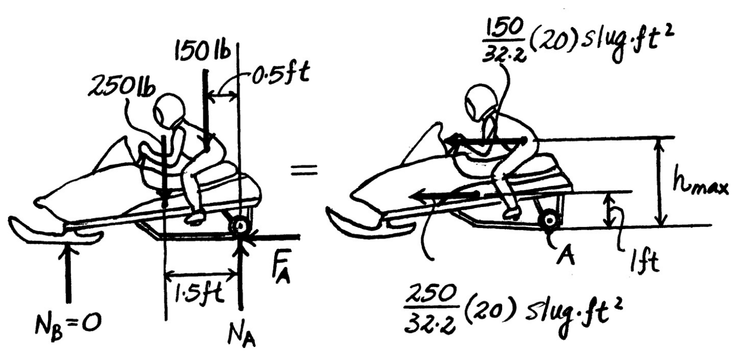
Writing the force equations of motion along the x and y axes,

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_A = \frac{150}{32.2}(20) + \frac{250}{32.2}(20)$$

$$F_A = 248.45 \text{ lb} = 248 \text{ lb} \qquad \textbf{Ans.}$$

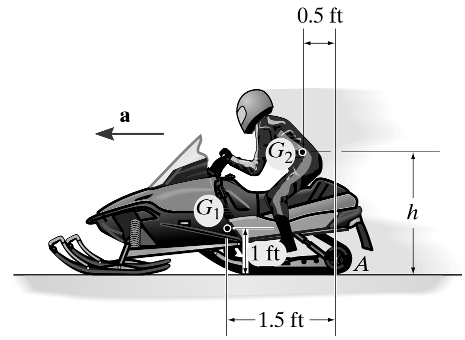
$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 250 - 150 = 0$$

$$N_A = 400 \text{ lb} \qquad \textbf{Ans.}$$



(a)

17–50. The snowmobile has a weight of 250 lb, centered at G_1, while the rider has a weight of 150 lb, centered at G_2. If $h = 3$ ft, determine the snowmobile's maximum permissible acceleration **a** so that its front skid does not lift off the ground. Also, find the traction (horizontal) force and the normal reaction under the rear tracks at A.



Equations of Motion: Since the front skid is required to be on the verge of lift off, $N_B = 0$. Writing the moment equation about point A and referring to Fig. a,

$$\circlearrowleft + \Sigma M_A = (M_k)_A; \qquad 250(1.5) + 150(0.5) = \left(\frac{150}{32.2} a_{\max}\right)(3) + \left(\frac{250}{32.2} a_{\max}\right)(1)$$

$$a_{\max} = 20.7 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

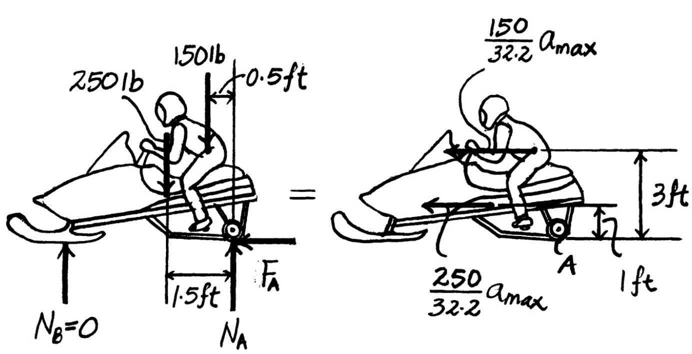
Writing the force equations of motion along the x and y axes and using this result, we have

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_A = \frac{150}{32.2}(20.7) + \frac{250}{32.2}(20.7)$$

$$F_A = 257.14 \text{ lb} = 257 \text{ lb} \qquad \textbf{Ans.}$$

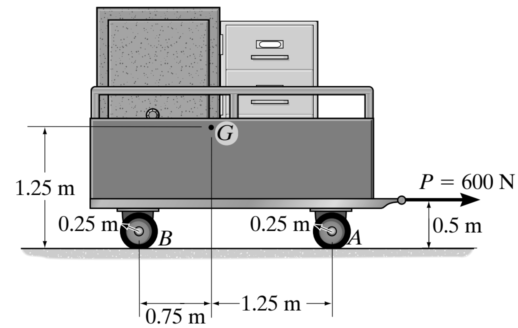
$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 150 - 250 = 0$$

$$N_A = 400 \text{ lb} \qquad \textbf{Ans.}$$



(a)

17–51. The trailer with its load has a mass of 150 kg and a center of mass at G. If it is subjected to a horizontal force of $P = 600$ N, determine the trailer's acceleration and the normal force on the pair of wheels at A and at B. The wheels are free to roll and have negligible mass.



Equations of Motion: Writing the force equation of motion along the x axis,

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\quad 600 = 150a \qquad a = 4 \text{ m/s}^2 \rightarrow$ **Ans.**

Using this result to write the moment equation about point A,

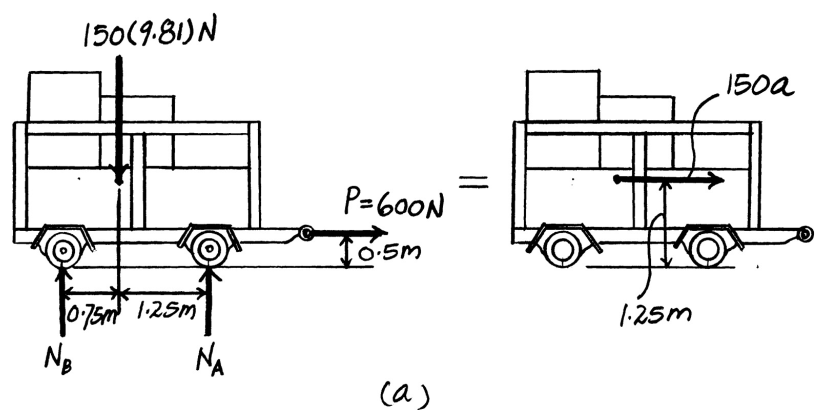
$\circlearrowleft + \Sigma M_A = (M_k)_A;\quad 150(9.81)(1.25) - 600(0.5) - N_B(2) = -150(4)(1.25)$

$N_B = 1144.69 \text{ N} = 1.14 \text{ kN}$ **Ans.**

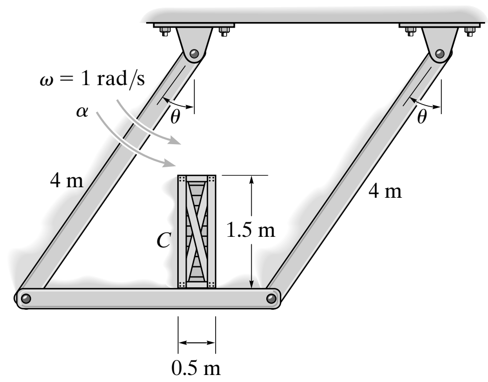
Using this result to write the force equation of motion along the y axis,

$+\uparrow \Sigma F_y = m(a_G)_y;\quad N_A + 1144.69 - 150(9.81) = 150(0)$

$N_A = 326.81 \text{ N} = 327 \text{ N}$ **Ans.**



***17–52.** The 50-kg uniform crate rests on the platform for which the coefficient of static friction is $\mu_s = 0.5$. If the supporting links have an angular velocity $\omega = 1$ rad/s, determine the greatest angular acceleration α they can have so that the crate does not slip or tip at the instant $\theta = 30°$.



Curvilinear Translation:

$(a_G)_n = (1)^2(4) = 4\text{ m/s}^2$

$(a_G)_t = \alpha(4)\text{ m/s}^2$

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad F_C = 50(4)\sin 30° + 50(\alpha)(4)\cos 30°$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_C - 50(9.81) = 50(4)\cos 30° - 50(\alpha)(4)\sin 30°$

$\circlearrowleft + \Sigma M_G = \Sigma(M_k)_G;\qquad N_C(x) - F_C(0.75) = 0$

Assume crate is about to slip. $F_C = 0.5N_C$

Thus,

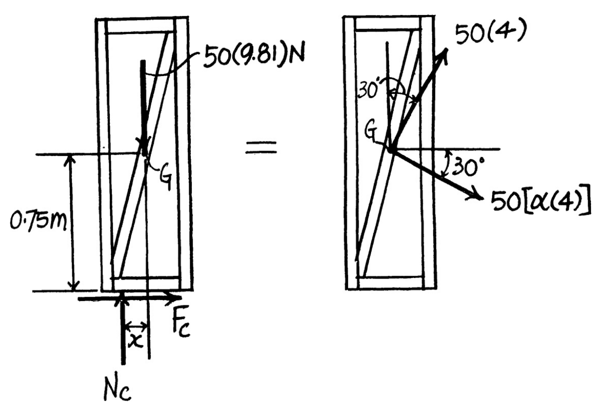
$$x = 0.375\text{ m} > 0.25\text{ m}$$

Crate must tip. Set $x = 0.25$ m.

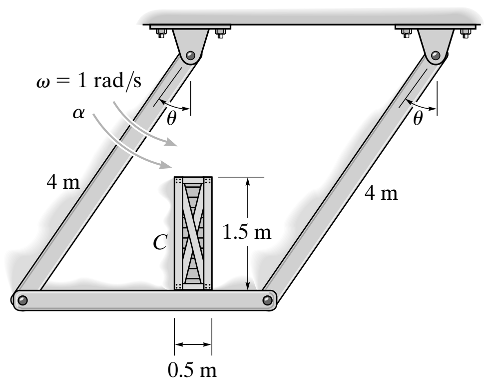
$$N_C = 605\text{ N}\qquad F_C = 202\text{ N}$$

$$\alpha = 0.587\text{ rad/s}^2$$ **Ans.**

Note: $(F_C)_{max} = 0.5(605) = 303\text{ N} > 202\text{ N}$ **O.K.**



•17–53. The 50-kg uniform crate rests on the platform for which the coefficient of static friction is $\mu_s = 0.5$. If at the instant $\theta = 30°$ the supporting links have an angular velocity $\omega = 1$ rad/s and angular acceleration $\alpha = 0.5$ rad/s^2, determine the frictional force on the crate.



Curvilinear Translation:

$(a_G)_n = (1)^2(4) = 4 \text{ m/s}^2$

$(a_G)_t = 0.5(4) \text{ m/s}^2 = 2 \text{ m/s}^2$

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad F_C = 50(4) \sin 30° + 50(2) \cos 30°$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_C - 50(9.81) = 50(4) \cos 30° - 50(2) \sin 30°$

Solving,

$F_C = 186.6 \text{ N}$

$N_C = 613.7 \text{ N}$

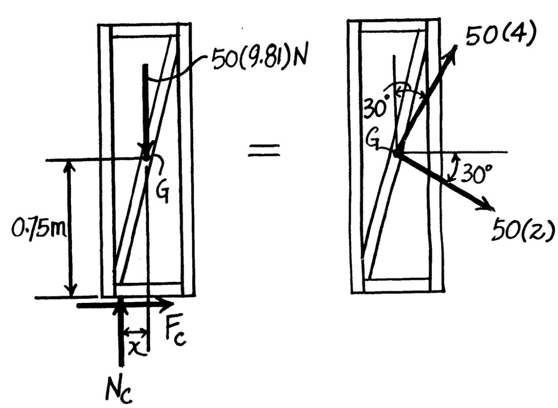
$(F_C)_{\max} = 0.5(613.7) = 306.9 \text{ N} > 186.6 \text{ N}$ **OK**

$\circlearrowleft + \Sigma M_G = \Sigma(M_k)_G; \qquad N_C(x) - F_C(0.75) = 0$

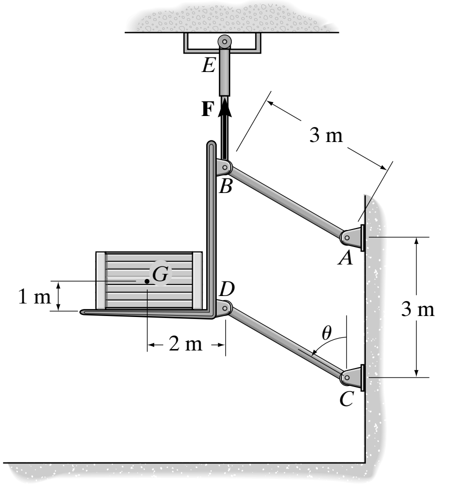
$613.7(x) - 186.6(0.75) = 0$

$x = 0.228 \text{ m} < 0.25 \text{ m}$ **OK**

Thus, $F_C = 187$ N **Ans.**



17–54. If the hydraulic cylinder BE exerts a vertical force of $F = 1.5$ kN on the platform, determine the force developed in links AB and CD at the instant $\theta = 90°$. The platform is at rest when $\theta = 45°$. Neglect the mass of the links and the platform. The 200-kg crate does not slip on the platform.



Equations of Motion: The free-body diagram of the crate and platform at the general position is shown in Fig. *a*. Here, $(a_G)_t = \alpha r = \alpha(3)$ and $(a_G)_n = \omega^2 r = \omega^2(3)$, where ω and α are the angular velocity and acceleration of the links. Writing the force equation of motion along the t axis by referring to Fig. *a*, we have

$$+\nearrow\Sigma F_t = m(a_G)_t; \qquad 200(9.81)\sin\theta - 1500\sin\theta = 200\left[\alpha(3)\right]$$

$$\alpha = 0.77\sin\theta$$

Kinematics: Using this result, the angular velocity of the links can be obtained by integrating

$$\int \omega\,d\omega = \int \alpha\,d\theta$$

$$\int_0^{\omega} \omega\,d\omega = \int_{45°}^{\theta} 0.77\sin\theta\,d\theta$$

$$\omega = \sqrt{1.54(0.7071 - \cos\theta)}$$

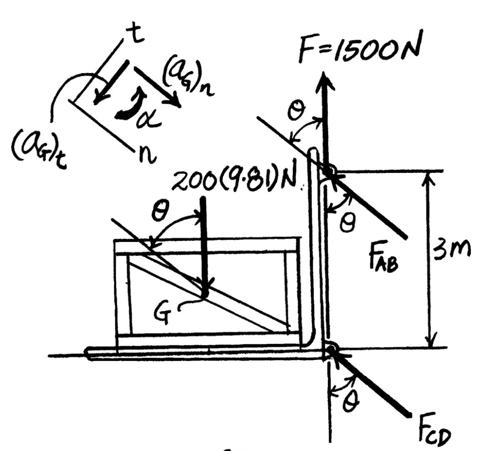
When $\theta = 90°$, $\omega = 1.044$ rad/s. Referring to the free-body diagram of the crate and platform when $\theta = 90°$, Fig. *b*,

$$\overset{+}{\rightarrow}\Sigma F_n = m(a_G)_n; \qquad F_{AB} - F_{CD} = 200\left[1.044^2(3)\right] \qquad \textbf{(1)}$$

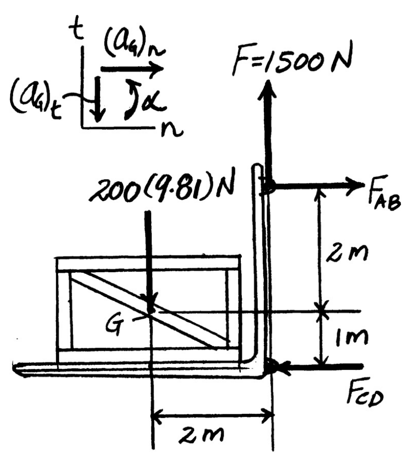
$$\circlearrowleft + \Sigma M_G = 0; \qquad 1500(2) - F_{AB}(2) - F_{CD}(1) = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$F_{AB} = 1217.79\text{ N} = 1.22\text{ kN} \qquad F_{CD} = 564.42\text{ N} = 564\text{N} \qquad \textbf{Ans.}$$

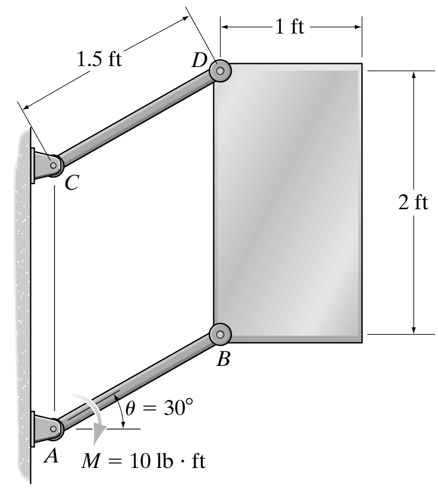


(a)



(b)

17–55. A uniform plate has a weight of 50 lb. Link AB is subjected to a couple moment of $M = 10\ \text{lb}\cdot\text{ft}$ and has a clockwise angular velocity of $2\ \text{rad/s}$ at the instant $\theta = 30°$. Determine the force developed in link CD and the tangential component of the acceleration of the plate's mass center at this instant. Neglect the mass of links AB and CD.



Equations of Motion: Since the plate undergoes the cantilever translation, $(a_G)_n = (2^2)(1.5) = 6\ \text{ft/s}^2$. Referring to the free-body diagram of the plate shown in Fig. *a*,

$$\Sigma F_n = m(a_G)_n; \qquad -F_{CD} - B_x \cos 30° - B_y \sin 30° + 50 \sin 30° = \left(\frac{50}{32.2}\right)(6) \quad \textbf{(1)}$$

$$\Sigma F_t = m(a_G)_t; \qquad B_x \sin 30° - B_y \cos 30° + 50 \cos 30° = \frac{50}{32.2}(a_G)_t \quad \textbf{(2)}$$

$$\circlearrowleft +\Sigma M_G = 0; \qquad B_x(1) - B_y(0.5) - F_{CD} \cos 30°(1) - F_{CD} \sin 30°(0.5) = 0 \quad \textbf{(3)}$$

Since the mass of link AB can be neglected, we can apply the moment equation of equilibrium to link AB. Referring to its free-body diagram, Fig. *b*,

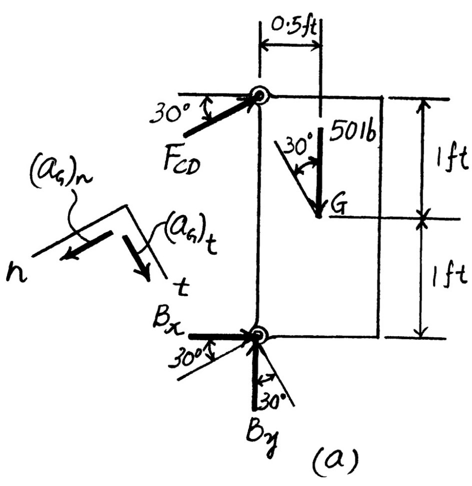
$$\circlearrowleft +\Sigma M_A = 0; \qquad B_x(1.5 \sin 30°) - B_y(1.5 \cos 30°) - 10 = 0 \quad \textbf{(4)}$$

Solving Eqs. (1) through (4) yields

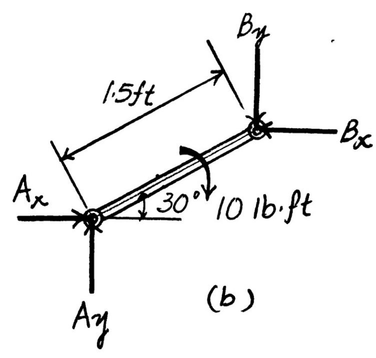
$B_x = 8.975\ \text{lb}$ $\qquad$ $B_y = -2.516\ \text{lb}$

$F_{CD} = 9.169\ \text{lb} = 9.17\ \text{lb}$ **Ans.**

$(a_G)_t = 32.18\ \text{ft/s}^2 = 32.2\ \text{ft/s}^2$ **Ans.**

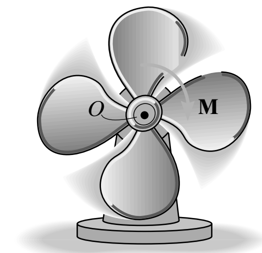


(a)



(b)

***17–56.** The four fan blades have a total mass of 2 kg and moment of inertia $I_O = 0.18\ \text{kg}\cdot\text{m}^2$ about an axis passing through the fan's center O. If the fan is subjected to a moment of $M = 3(1 - e^{-0.2t})\ \text{N}\cdot\text{m}$, where t is in seconds, determine its angular velocity when $t = 4$ s starting from rest.



$$\circlearrowleft + \Sigma M_O = I_O\alpha; \qquad 3\left(1 - e^{-0.2t}\right) = 0.18\alpha$$

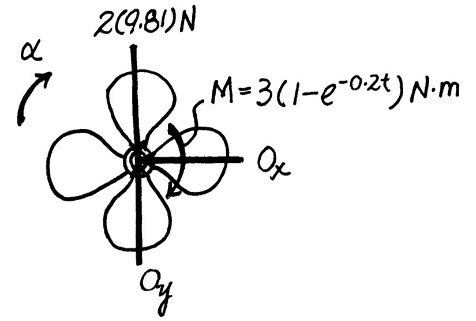
$$\alpha = 16.67\left(1 - e^{-0.2t}\right)$$

$$d\omega = \alpha\, dt$$

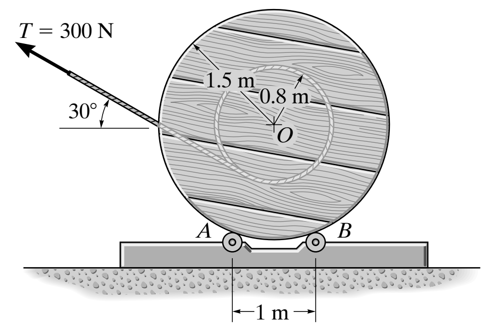
$$\int_0^\omega d\omega = \int_0^4 16.67\left(1 - e^{-0.2t}\right) dt$$

$$\omega = 16.67\left[t + \frac{1}{0.2}e^{-0.2t}\right]_0^4$$

$$\omega = 20.8 \text{ rad/s}$$ **Ans.**



•17–57. Cable is unwound from a spool supported on small rollers at A and B by exerting a force of $T = 300$ N on the cable in the direction shown. Compute the time needed to unravel 5 m of cable from the spool if the spool and cable have a total mass of 600 kg and a centroidal radius of gyration of $k_O = 1.2$ m. For the calculation, neglect the mass of the cable being unwound and the mass of the rollers at A and B. The rollers turn with no friction.



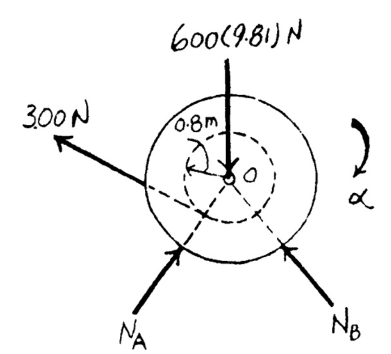
Equations of Motion: The mass moment of inertia of the spool about point O is given by $I_O = mk_O^2 = 600\left(1.2^2\right) = 864\ \text{kg}\cdot\text{m}^2$. Applying Eq. 17–16, we have

$$\circlearrowright + \Sigma M_O = I_O\,\alpha; \qquad -300(0.8) = -864\alpha \qquad \alpha = 0.2778 \text{ rad/s}^2$$

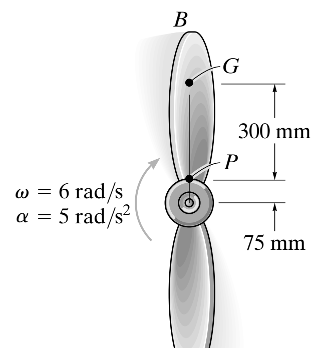
Kinematics: Here, the angular displacement $\theta = \dfrac{s}{r} = \dfrac{5}{0.8} = 6.25$ rad. Applying equation $\theta = \theta_0 + \omega_0 t + \dfrac{1}{2}\alpha t^2$, we have

$$(\circlearrowleft +) \qquad 6.25 = 0 + 0 + \frac{1}{2}(0.2778)t^2$$

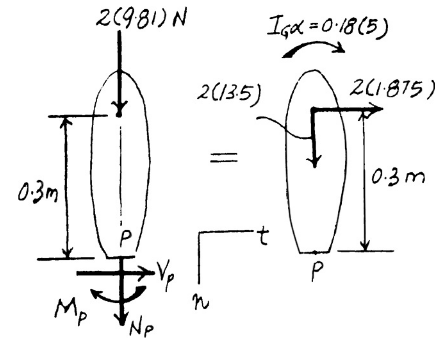
$$t = 6.71 \text{ s}$$ **Ans.**



17–58. The single blade PB of the fan has a mass of 2 kg and a moment of inertia $I_G = 0.18\ \text{kg}\cdot\text{m}^2$ about an axis passing through its center of mass G. If the blade is subjected to an angular acceleration $\alpha = 5\ \text{rad/s}^2$, and has an angular velocity $\omega = 6$ rad/s when it is in the vertical position shown, determine the internal normal force N, shear force V, and bending moment M, which the hub exerts on the blade at point P.



Equations of Motion: Here, $(a_G)_t = \alpha r_G = 5(0.375) = 1.875\ \text{m/s}^2$ and $(a_G)_n = \omega^2 r_G = 6^2(0.375) = 13.5\ \text{m/s}^2$.



$\circlearrowleft + \Sigma M_P = \Sigma (M_k)_P;\qquad -M_P = -0.18(5) - 2(1.875)(0.3)$

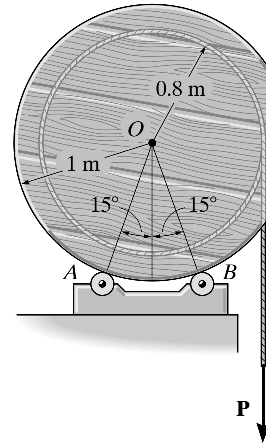
$M_P = 2.025\ \text{N}\cdot\text{m}$ **Ans.**

$\Sigma F_n = m(a_G)_n;\qquad N_P + 2(9.81) = 2(13.5)$

$N_P = 7.38\ \text{N}$ **Ans.**

$\Sigma F_t = m(a_G)_t;\qquad V_P = 2(1.875) = 3.75\ \text{N}$ **Ans.**

17–59. The uniform spool is supported on small rollers at A and B. Determine the constant force $\mathbf{P}$ that must be applied to the cable in order to unwind 8 m of cable in 4 s starting from rest. Also calculate the normal forces on the spool at A and B during this time. The spool has a mass of 60 kg and a radius of gyration about O of $k_O = 0.65$ m. For the calculation neglect the mass of the cable and the mass of the rollers at A and B.



$(\downarrow +)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$8 = 0 + 0 + \frac{1}{2} a_c (4)^2$

$a_c = 1\ \text{m/s}^2$

$\alpha = \frac{1}{0.8} = 1.25\ \text{rad/s}^2$

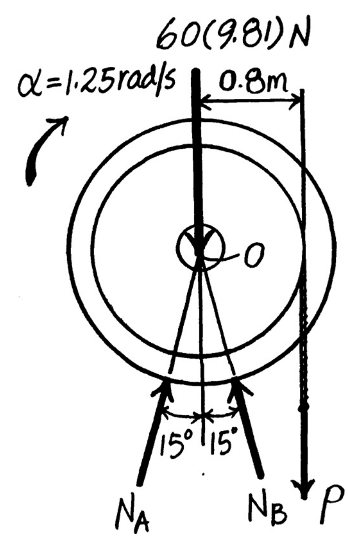
$\circlearrowright + \Sigma M_O = I_O \alpha;\qquad P(0.8) = 60(0.65)^2(1.25)$

$P = 39.6\ \text{N}$ **Ans.**

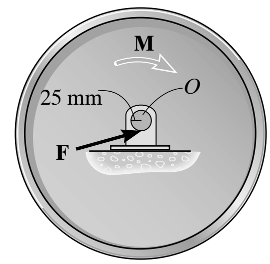
$\xrightarrow{+} \Sigma F_x = ma_x;\qquad N_A \sin 15° - N_B \sin 15° = 0$

$+\uparrow \Sigma F_y = ma_y;\qquad N_A \cos 15° + N_B \cos 15° - 39.6 - 588.6 = 0$

$N_A = N_B = 325\ \text{N}$ **Ans.**



***17–60.** A motor supplies a constant torque $M = 2\ \text{N} \cdot \text{m}$ to a 50-mm-diameter shaft O connected to the center of the 30-kg flywheel. The resultant bearing friction **F**, which the bearing exerts on the shaft, acts tangent to the shaft and has a magnitude of 50 N. Determine how long the torque must be applied to the shaft to increase the flywheel's angular velocity from 4 rad/s to 15 rad/s. The flywheel has a radius of gyration $k_O = 0.15$ m about its center O.



$\circlearrowright + \Sigma M_O = I_O\,\alpha;\qquad 2 - 50(0.025) = 30(0.15)^2\alpha$

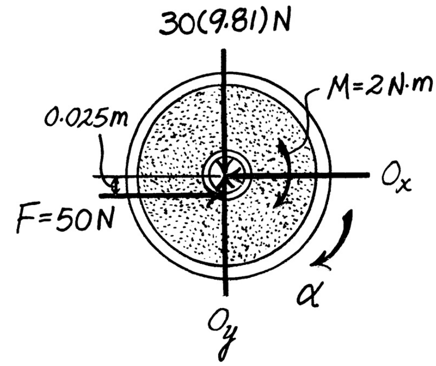
$$\alpha = 1.11\ \text{rad/s}^2$$

$\circlearrowright +$

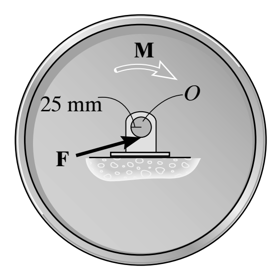
$$\omega = \omega_0 + \alpha_c\,t$$

$$15 = 4 + (1.11)t$$

$$t = 9.90\ \text{s}$$ **Ans.**



•17–61. If the motor in Prob. 17–60 is disengaged from the shaft once the flywheel is rotating at 15 rad/s, so that $M = 0$, determine how long it will take before the resultant bearing frictional force $F = 50$ N stops the flywheel from rotating.



$\circlearrowright + \Sigma M_O = I_O\,\alpha;\qquad 50(0.025) = 30(0.15)^2\alpha$

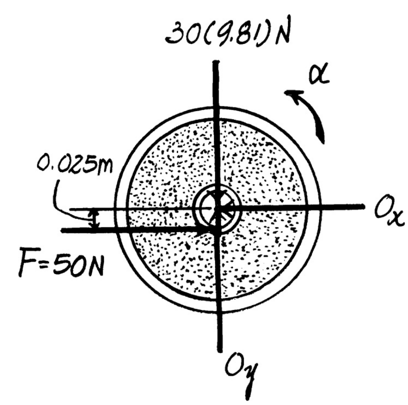
$$\alpha = 1.852\ \text{rad/s}^2$$

$\circlearrowright +$

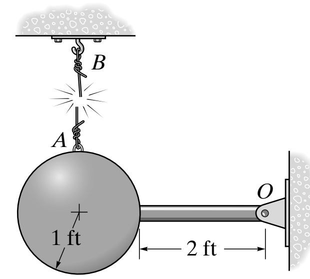
$$\omega = \omega_0 + \alpha_c\,t$$

$$0 = -15 + (1.852)t$$

$$t = 8.10\ \text{s}$$ **Ans.**



17–62. The pendulum consists of a 30-lb sphere and a 10-lb slender rod. Compute the reaction at the pin O just after the cord AB is cut.

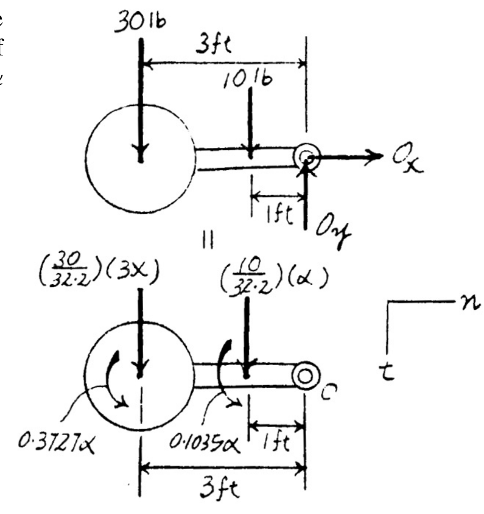


Mass Moment Inertia: From the inside back cover of the text.

$$(I_G)_S = \frac{2}{5}mr^2 = \frac{2}{5}\left(\frac{30}{32.2}\right)(1^2) = 0.3727 \text{ slug}\cdot\text{ft}^2$$

$$(I_G)_R = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{10}{32.2}\right)(2^2) = 0.1035 \text{ slug}\cdot\text{ft}^2$$

Equations of Motion: At the instant shown, the normal component of acceleration of the mass center for the sphere and the rod are $\left[(a_G)_n\right]_S = \left[(a_G)_n\right]_R = 0$ since the angular velocity of the pendulum $\omega = 0$ at that instant. The tangential component of acceleration of the mass center for the sphere and the rod are $\left[(a_G)_t\right]_S = \alpha r_S = 3\alpha$ and $\left[(a_G)_t\right]_R = \alpha r_R = \alpha$.



$$\circlearrowleft + \Sigma M_O = \Sigma(M_k)_O; \qquad 30(3) + 10(1) = 0.3727\alpha + 0.1035\alpha + \left(\frac{30}{32.2}\right)(3\alpha)(3) + \left(\frac{10}{32.2}\right)(\alpha)(1)$$

$$\alpha = 10.90 \text{ rad/s}^2$$

$$\Sigma F_n = m(a_G)_n; \qquad O_x = 0$$

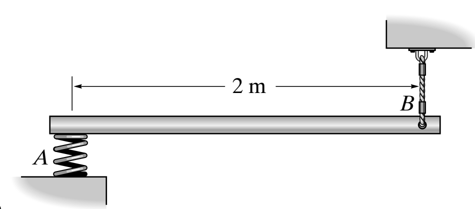
$$\Sigma F_t = m(a_G)_t; \qquad 30 + 10 - O_y = \left(\frac{30}{32.2}\right)[3(10.90)] + \left(\frac{10}{32.2}\right)(10.90)$$

$$O_y = 6.140 \text{ lb}$$

Thus,

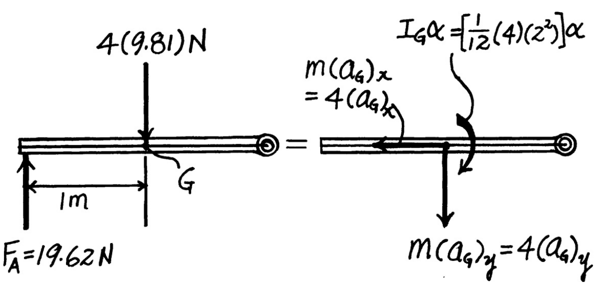
$$F_O = \sqrt{O_x^2 + O_y^2} = \sqrt{0^2 + 6.140^2} = 6.14 \text{ lb} \qquad \textbf{Ans.}$$

17–63. The 4-kg slender rod is supported horizontally by a spring at A and a cord at B. Determine the angular acceleration of the rod and the acceleration of the rod's mass center at the instant the cord at B is cut. *Hint:* The stiffness of the spring is not needed for the calculation.



Since the deflection of the spring is unchanged at the instant the cord is cut, the reaction at A is

$$F_A = \frac{4}{2}(9.81) = 19.62 \text{ N}$$



$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 0 = 4(a_G)_x$$

$$+\downarrow \Sigma F_y = m(a_G)_y; \qquad 4(9.81) - 19.62 = 4(a_G)_y$$

$$\circlearrowright + \Sigma M_G = I_G\alpha; \qquad (19.62)(1) = \left[\frac{1}{12}(4)(2)^2\right]\alpha$$

Solving:

$$(a_G)_x = 0$$

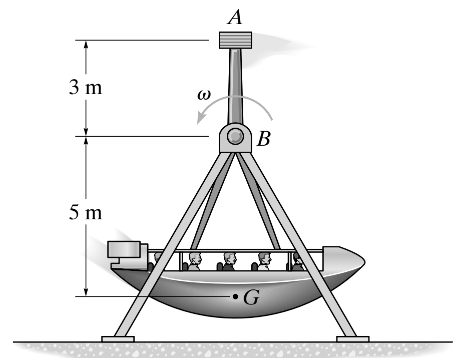
$$(a_G)_y = 4.905 \text{ m/s}^2$$

$$\alpha = 14.7 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

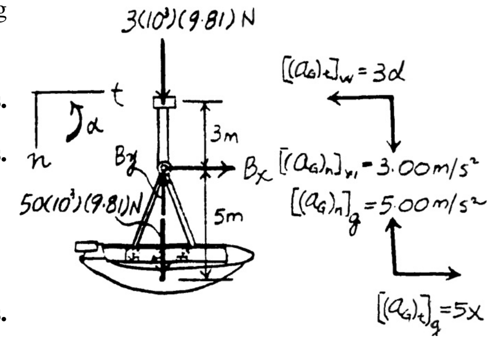
Thus,

$$(a_G) = 4.90 \text{ m/s}^2 \qquad \textbf{Ans.}$$

***17–64.** The passengers, the gondola, and its swing frame have a total mass of 50 Mg, a mass center at G, and a radius of gyration $k_B = 3.5$ m. Additionally, the 3-Mg steel block at A can be considered as a point of concentrated mass. Determine the horizontal and vertical components of reaction at pin B if the gondola swings freely at $\omega = 1$ rad/s when it reaches its lowest point as shown. Also, what is the gondola's angular acceleration at this instant?



Equations of Motion: The mass moment of inertia of the gondola and the counter weight about point B is given by $I_B = m_g k_B^2 + m_W r_W^2 = 50(10^3)(3.5^2) + 3(10^3)(3^2) = 639.5(10^3)$ kg·m². At the instant shown, the normal component of acceleration of the mass center for the gondola and the counter weight are $[(a_G)_n]_g = \omega^2 r_g = 1^2(5) = 5.00$ m/s² and $[(a_G)_n]_W = \omega^2 r_W = 1^2(3) = 3.00$ m/s². The tangential component of acceleration of the mass center for the gondola and the counter weight are $[(a_G)_t]_g = \alpha r_g = 5\alpha$ and $[(a_G)_t]_W = \alpha r_W = 3\alpha$. Applying Eq. 17–16, we have



$\circlearrowleft + \Sigma M_B = I_B\,\alpha; \qquad 0 = 639.5(10^3)\alpha \qquad \alpha = 0$ **Ans.**

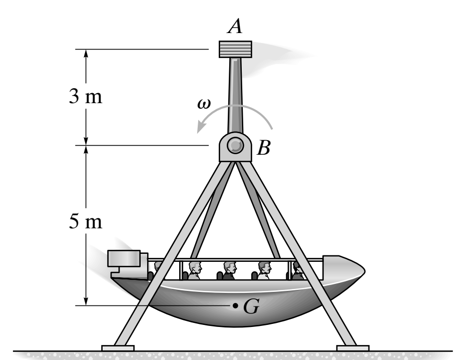
$\Sigma F_t = m(a_G)_t; \qquad B_x = 0$ **Ans.**

$\Sigma F_n = m(a_G)_n; \qquad 3(10^3)(9.81) + 50(10^3)(9.81) - B_y$

$$= 3(10^3)(3.00) - 50(10^3)(5.00)$$

$B_y = 760.93(10^3)\text{ N} = 761\text{ kN}$ **Ans.**

•17–65. The passengers, the gondola, and its swing frame have a total mass of 50 Mg, a mass center at G, and a radius of gyration $k_B = 3.5$ m. Additionally, the 3-Mg steel block at A can be considered as a point of concentrated mass. Determine the angle θ to which the gondola will swing before it stops momentarily, if it has an angular velocity of $\omega = 1$ rad/s at its lowest point.

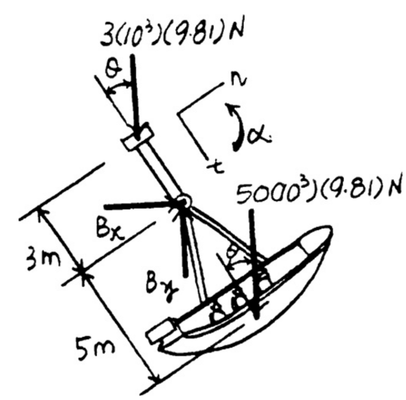


Equations of Motion: The mass moment of inertia of the gondola and the counter weight about point B is given by $I_B = m_g k_B^2 + m_W r_W^2 = 50(10^3)(3.5^2) + 3(10^3)(3^2) = 639.5(10^3)$ kg·m². Applying Eq. 17–16, we have

$\circlearrowleft + \Sigma M_B = I_B\,\alpha; \qquad 3(10^3)(9.81)\sin\theta(3)$

$$-50(10^3)(9.81)\sin\theta(5) = 639.5(10^3)\,\alpha$$

$$\alpha = -3.6970\sin\theta$$

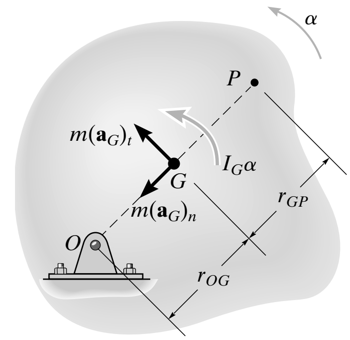


Kinematics: Applying equation $\omega\,d\omega = \alpha\,d\theta$, we have

$$\int_{1\text{ rad/s}}^{0} \omega\,d\omega = \int_{0°}^{\theta} -3.6970\sin\theta\,d\theta$$

$\theta = 30.1°$ **Ans.**

17–66. The kinetic diagram representing the general rotational motion of a rigid body about a fixed axis passing through O is shown in the figure. Show that $I_G\boldsymbol{\alpha}$ may be eliminated by moving the vectors $m(\mathbf{a}_G)_t$ and $m(\mathbf{a}_G)_n$ to point P, located a distance $r_{GP} = k_G^2/r_{OG}$ from the center of mass G of the body. Here k_G represents the radius of gyration of the body about an axis passing through G. The point P is called the *center of percussion* of the body.



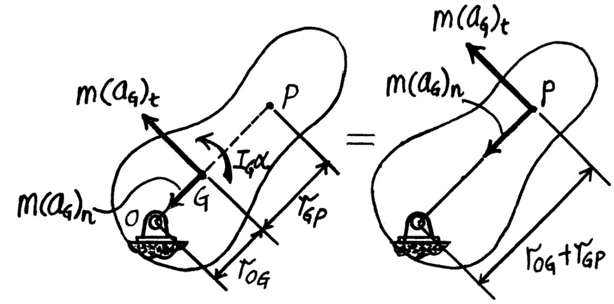
$$m(a_G)_t\, r_{OG} + I_G\,\alpha = m(a_G)_t\, r_{OG} + \left(mk_G^2\right)\alpha$$

However,

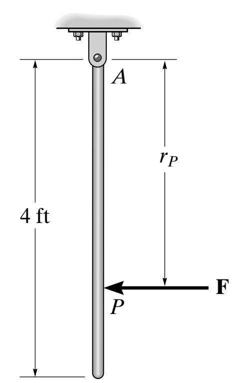
$$k_G^2 = r_{OG}\, r_{GP} \text{ and } \alpha = \frac{(a_G)_t}{r_{OG}}$$

$$m(a_G)_t\, r_{OG} + I_G\,\alpha = m(a_G)_t\, r_{OG} + (mr_{OG}\, r_{GP})\left[\frac{(a_G)_t}{r_{OG}}\right]$$

$$= m(a_G)_t(r_{OG} + r_{GP}) \qquad \textbf{Q.E.D.}$$



17–67. Determine the position r_P of the center of percussion P of the 10-lb slender bar. (See Prob. 17–66.) What is the horizontal component of force that the pin at A exerts on the bar when it is struck at P with a force of $F = 20$ lb?



Using the result of Prob 17–66,

$$r_{GP} = \frac{k_G^2}{r_{AG}} = \frac{\left[\sqrt{\dfrac{1}{12}\left(\dfrac{ml^2}{m}\right)}\right]^2}{\dfrac{l}{2}} = \frac{1}{6}l$$

Thus,

$$r_P = \frac{1}{6}l + \frac{1}{2}l = \frac{2}{3}l = \frac{2}{3}(4) = 2.67 \text{ ft} \qquad \textbf{Ans.}$$

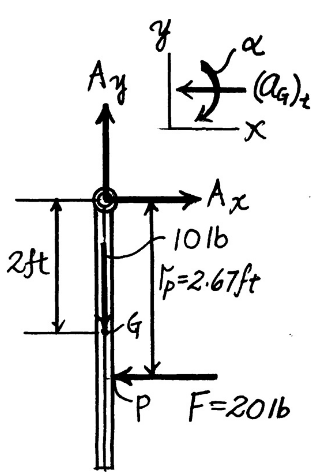
$$\circlearrowleft +\Sigma M_A = I_A\,\alpha; \qquad 20(2.667) = \left[\frac{1}{3}\left(\frac{10}{32.2}\right)(4)^2\right]\alpha$$

$$\alpha = 32.2 \text{ rad/s}^2$$

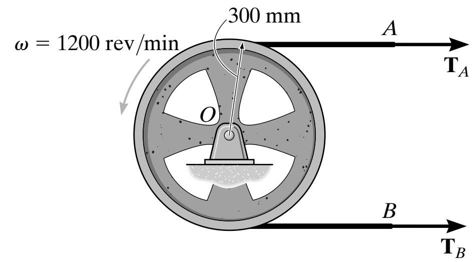
$$(a_G)_t = 2(32.2) = 64.4 \text{ ft/s}^2$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad -A_x + 20 = \left(\frac{10}{32.2}\right)(64.4)$$

$$A_x = 0 \qquad \textbf{Ans.}$$

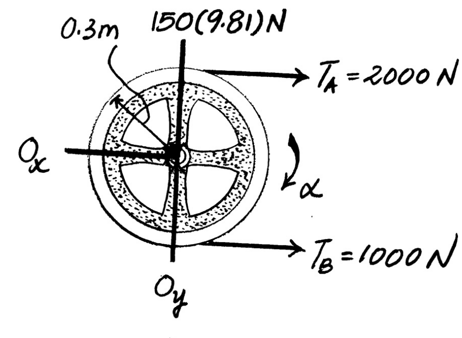


***17–68.** The 150-kg wheel has a radius of gyration about its center of mass O of $k_O = 250$ mm. If it rotates counterclockwise with an angular velocity of $\omega = 1200$ rev/min at the instant the tensile forces $T_A = 2000$ N and $T_B = 1000$ N are applied to the brake band at A and B, determine the time needed to stop the wheel.



Equations of Motion: Here, the mass moment of inertia of the flywheel about its mass center O is $I_O = mk_O{}^2 = 150\left(0.25^2\right) = 9.375\ \text{kg}\cdot\text{m}^2$. Referring to the free-body diagram of the flywheel in Fig. b, we have

$$\circlearrowleft + \Sigma M_O = I_O\alpha; \qquad 1000(0.3) - 2000(0.3) = -9.375\alpha \qquad \alpha = 32\ \text{rad/s}^2$$



(a)

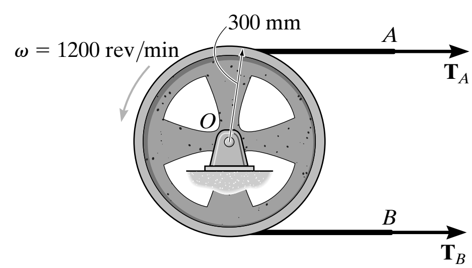
Kinematics: Here, $\omega_0 = \left(1200\,\dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right) = 40\pi$ rad. Since the angular acceleration is constant, we can apply

$$\circlearrowleft + \qquad \omega = \omega_0 + \alpha_c t$$

$$0 = 40\pi + (-32)t$$

$$t = 3.93\ \text{s} \qquad \textbf{Ans.}$$

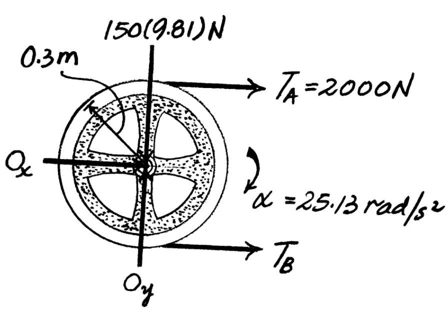
•17–69. The 150-kg wheel has a radius of gyration about its center of mass O of $k_O = 250$ mm. If it rotates counterclockwise with an angular velocity of $\omega = 1200$ rev/min and the tensile force applied to the brake band at A is $T_A = 2000$ N, determine the tensile force $\mathbf{T}_B$ in the band at B so that the wheel stops in 50 revolutions after $\mathbf{T}_A$ and $\mathbf{T}_B$ are applied.



Kinematics: Here, $\omega_0 = \left(1200\,\dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right) = 40\pi$ rad and

$$\theta = (50\ \text{rev})$$

$$\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 100\pi\text{rad}$$



(a)

Since the angular acceleration is constant,

$$\circlearrowleft + \qquad \omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$$

$$0 = (40\pi)^2 + 2\alpha(100\pi - 0)$$

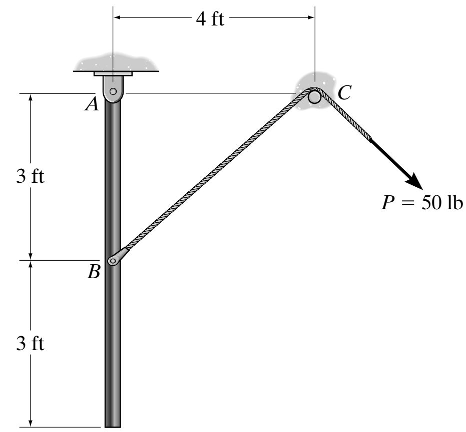
$$\alpha = -25.13\ \text{rad/s}^2 = 25.13\ \text{rad/s}^2$$

Equations of Motion: Here, the mass moment of inertia of the flywheel about its mass center O is $I_O = mk_O{}^2 = 150\left(0.25^2\right) = 9.375\ \text{kg}\cdot\text{m}^2$. Referring to the free-body diagram of the flywheel,

$$\circlearrowleft + \Sigma M_O = I_O\alpha; \qquad T_B(0.3) - 2000(0.3) = -9.375(25.13)$$

$$T_B = 1214.60\ \text{N} = 1.21\ \text{kN} \qquad \textbf{Ans.}$$

17–70. The 100-lb uniform rod is at rest in a vertical position when the cord attached to it at B is subjected to a force of $P = 50$ lb. Determine the rod's initial angular acceleration and the magnitude of the reactive force that pin A exerts on the rod. Neglect the size of the smooth peg at C.



Equations of Motion: Since the rod rotates about a fixed axis passing through point A, $(a_G)_t = \alpha r_G = \alpha(3)$ and $(a_G)_n = \omega^2 r_G = 0$. The mass moment of inertia of the rod about G is $I_G = \frac{1}{12}\left(\frac{100}{32.2}\right)(6^2) = 9.317\ \text{slug}\cdot\text{ft}^2$. Writing the moment equation of motion about point A,

$$\circlearrowleft + \Sigma M_A = (M_k)_A; \qquad 50\left(\frac{4}{5}\right)(3) = \frac{100}{32.2}\left[\alpha(3)\right](3) + 9.317\alpha$$

$$\alpha = 3.220\ \text{rad/s}^2 = 3.22\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

This result can also be obtained by applying $\Sigma M_A = I_A\alpha$, where

$$I_A = 9.317 + \left(\frac{100}{32.2}\right)(3^2) = 37.267\ \text{slug}\cdot\text{ft}^2$$

Thus,

$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad 50\left(\frac{4}{5}\right)(3) = 37.267\alpha$$

$$\alpha = 3.220\ \text{rad/s}^2 = 3.22\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

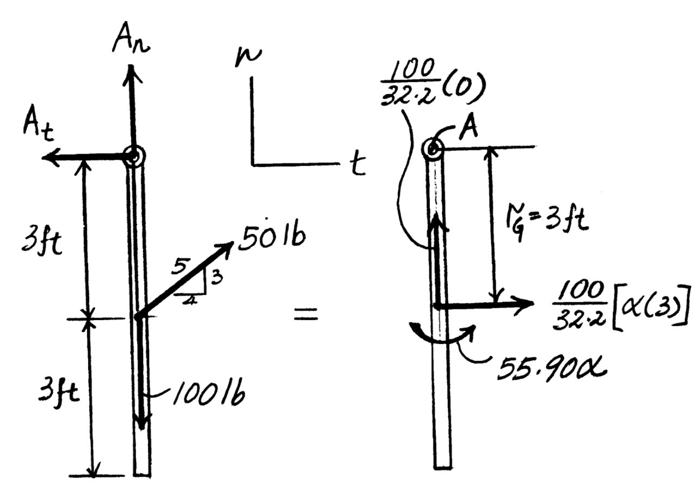
Using this result to write the force equation of motion along the n and t axes,

$$+\uparrow \Sigma F_n = m(a_G)_n; \qquad A_n + 50\left(\frac{3}{5}\right) - 100 = \frac{100}{32.2}(0) \qquad A_n = 70\ \text{lb}$$

$$\overset{+}{\rightarrow} \Sigma F_t = m(a_G)_t; \qquad 50\left(\frac{4}{5}\right) - A_t = \frac{100}{32.2}\left[3.220(3)\right] \qquad A_t = 10.0\ \text{lb}$$

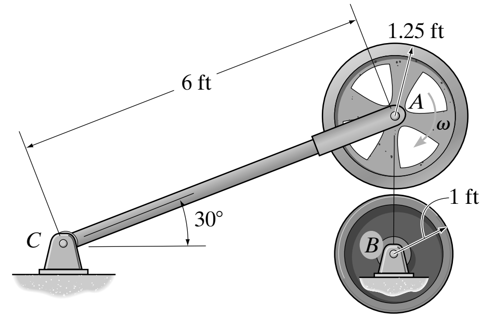
Thus,

$$F_A = \sqrt{A_t^{\,2} + A_n^{\,2}} = \sqrt{10^2 + 70^2} = 70.7\ \text{lb} \qquad \textbf{Ans.}$$

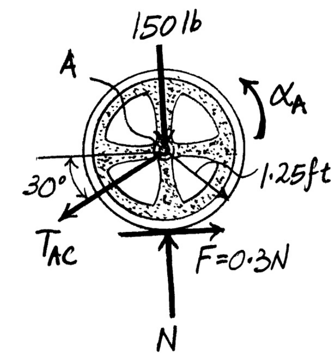


(a)

17–71. Wheels A and B have weights of 150 lb and 100 lb, respectively. Initially, wheel A rotates clockwise with a constant angular velocity of $\omega = 100$ rad/s and wheel B is at rest. If A is brought into contact with B, determine the time required for both wheels to attain the same angular velocity. The coefficient of kinetic friction between the two wheels is $\mu_k = 0.3$ and the radii of gyration of A and B about their respective centers of mass are $k_A = 1$ ft and $k_B = 0.75$ ft. Neglect the weight of link AC.



Equations of Motion: Wheel A will slip on wheel B until both wheels attain the same angular velocity. The frictional force developed at the contact point is $F = \mu_k N = 0.3N$. The mass moment of inertia of wheel A about its mass center is $I_A = m_A k_A{}^2 = \dfrac{150}{32.2}\left(1^2\right)$ slug·ft². Referring to the free-body diagram of wheel A shown in Fig. *a*.



(a)

$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \qquad 0.3N - T_{AC}\cos 30° = 0$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \quad N - T_{AC}\sin 30° - 150 = 0$$

$$\circlearrowleft+\Sigma M_A = I_A\,\alpha_A; \qquad 0.3N(1.25) = \left[\frac{150}{32.2}\left(1^2\right)\right]\alpha_A$$

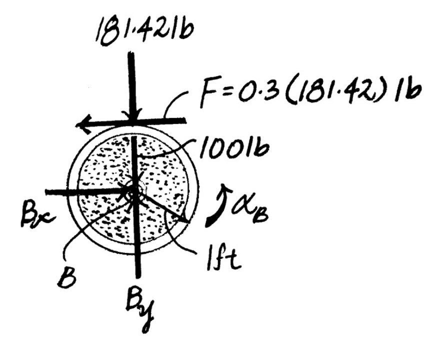
Solving,

$$N = 181.42 \text{ lb} \qquad T_{AC} = 62.85 \text{ lb} \qquad \alpha_A = 14.60 \text{ rad/s}$$

The mass moment of inertia of wheel B about its mass center is

$$I_B = m_B\,k_B{}^2 = \frac{100}{32.2}\left(0.75^2\right) \text{slug}\cdot\text{ft}^2$$

Writing the moment equation of motion about point B using the free-body diagram of wheel B shown in Fig. *b*,



(b)

$$+\Sigma M_B = I_B\,\alpha_B; \qquad 0.3(181.42)(1) = \frac{100}{32.2}\left(0.75^2\right)\alpha_B$$

$$\alpha_B = 31.16 \text{ rad/s}^2$$

Kinematics: Since the angular acceleration of both wheels is constant,

$$\circlearrowleft+ \qquad \omega_A = (\omega_A)_0 + \alpha_A\,t$$

$$\omega_A = 100 + (-14.60)t$$

and

$$\circlearrowleft+ \qquad \omega_B = (\omega_B)_0 + \alpha_B\,t$$

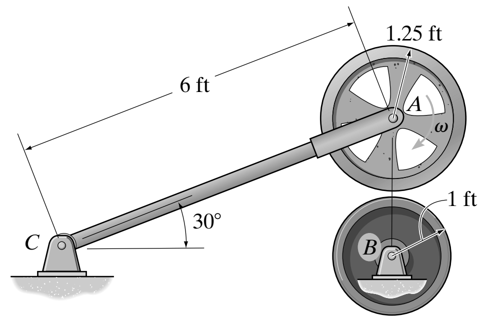
$$\omega_B = 0 + 31.16t$$

Since ω_A is required to be equal to ω_B, we obtain

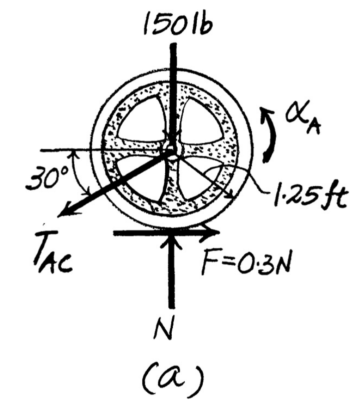
$$100 + (-14.60)t = 31.16t$$

$$t = 2.185 \text{ s} = 2.19 \text{ s} \qquad \textbf{Ans.}$$

***17–72.** Initially, wheel A rotates clockwise with a constant angular velocity of $\omega = 100$ rad/s. If A is brought into contact with B, which is held fixed, determine the number of revolutions before wheel A is brought to a stop. The coefficient of kinetic friction between the two wheels is $\mu_k = 0.3$, and the radius of gyration of A about its mass center is $k_A = 1$ ft. Neglect the weight of link AC.



Equations of Motion: Since wheel B is fixed, wheel A will slip on wheel B. The frictional force developed at the contact point is $F = \mu_k N = 0.3N$. The mass moment of inertia of wheel A about its mass center is $I_A = m_A k_A^2 = \frac{150}{32.2}(1^2)$ slug · ft². Referring to the free-body diagram of wheel A shown in Fig. a,



$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \quad 0.3N - T_{AC}\cos 30° = 0$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \quad N - T_{AC}\sin 30° - 150 = 0$$

$$\circlearrowleft +\Sigma M_A = I_A\alpha_A; \quad 0.3N(1.25) = \left[\frac{150}{32.2}(1^2)\right]\alpha_A$$

Solving,

$$N = 181.42 \text{ lb} \qquad T_{AC} = 62.85 \text{ lb} \qquad \alpha_A = 14.60 \text{ rad/s}$$

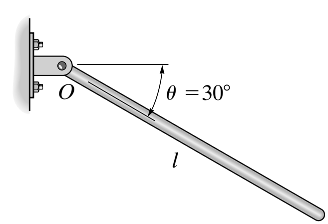
Kinematics: Since the angular acceleration is constant,

$$\circlearrowleft + \qquad \omega_A{}^2 = (\omega_A)^2{}_0 + 2\alpha_A(\theta - \theta_0)$$

$$0^2 = 100^2 + 2(-14.60)(\theta - 0)$$

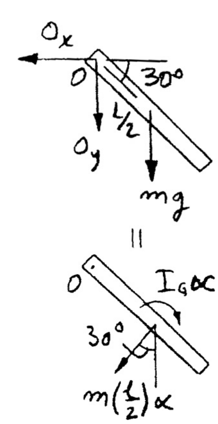
$$\theta = 342.36 \text{ rad}\left(\frac{1 \text{ rad}}{2\pi \text{ rad}}\right) = 54.49 \text{ rev} = 54.5 \text{ rev} \qquad \textbf{Ans.}$$

•17–73. The bar has a mass m and length l. If it is released from rest from the position $\theta = 30°$, determine its angular acceleration and the horizontal and vertical components of reaction at the pin O.



$$\circlearrowleft +\Sigma M_O = I_O\alpha; \quad (mg)\left(\frac{l}{2}\right)\cos 30° = \frac{1}{3}ml^2\alpha$$

$$\alpha = \frac{1.299g}{l} = \frac{1.30g}{l} \qquad \textbf{Ans.}$$



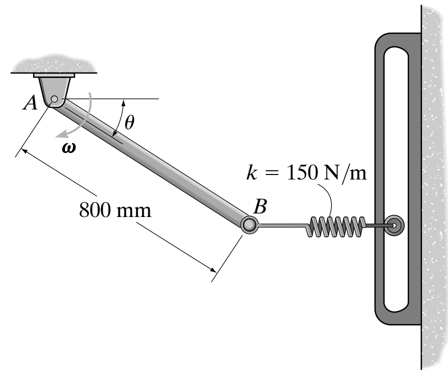
$$\overset{+}{\leftarrow}\Sigma F_x = m(a_G)_x; \quad O_x = m\left(\frac{l}{2}\right)\left(\frac{1.299g}{l}\right)\sin 30°$$

$$O_x = 0.325mg \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \quad O_y - mg = -m\left(\frac{l}{2}\right)\left(\frac{1.299g}{l}\right)\cos 30°$$

$$O_y = 0.438mg \qquad \textbf{Ans.}$$

17–74. The uniform slender rod has a mass of 9 kg. If the spring is unstretched when $\theta = 0°$, determine the magnitude of the reactive force exerted on the rod by pin A when $\theta = 45°$, if at this instant $\omega = 6$ rad/s. The spring has a stiffness of $k = 150$ N/m and always remains in the horizontal position.



Equations of Motion: The stretch of the spring when $\theta = 45°$ is $s = 0.8 - 0.8 \cos 45° = 0.2343$ m. Thus, $F_{sp} = ks = 150(0.2343) = 35.15$ N. Since the rod rotates about a fixed axis passing through point A, $(a_G)_t = \alpha r_G = \alpha(0.4)$ and $(a_G)_n = \omega^2 r_G = 6^2(0.4) = 14.4\ \text{m/s}^2$. The mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12} ml^2 = \frac{1}{12}(9)(0.8^2) = 0.48\ \text{kg} \cdot \text{m}^2$. Writing the moment equation of motion about point A, Fig. a,

$$\zeta + \Sigma M_A = \Sigma(M_k)_A; \qquad 35.15 \cos 45°(0.8) - 9(9.81) \cos 45°(0.4)$$

$$= -9[\alpha(0.4)](0.4) - 0.48\alpha$$

$$\alpha = 2.651\ \text{rad/s}^2$$

The above result can also be obtained by applying $\Sigma M_A = I_A\,\alpha$, where

$$I_A = I_G + md^2 = \frac{1}{12}(9)(0.8^2) + 9(0.4^2) = 1.92\ \text{kg} \cdot \text{m}^2$$

Thus,

$$\zeta + \Sigma M_A = I_A\,\alpha; \qquad 35.15 \cos 45°(0.8) - 9(9.81) \cos 45°(0.4) = -1.92\alpha$$

$$\alpha = 2.651\ \text{rad/s}^2$$

Using this result and writing the force equation of motion along the n and t axes,

$$+\swarrow \Sigma F_t = m(a_G)_t; \qquad 9(9.81) \cos 45° - 35.15 \cos 45° - A_t = 9[2.651(0.4)]$$

$$A_t = 28.03\ \text{N}$$

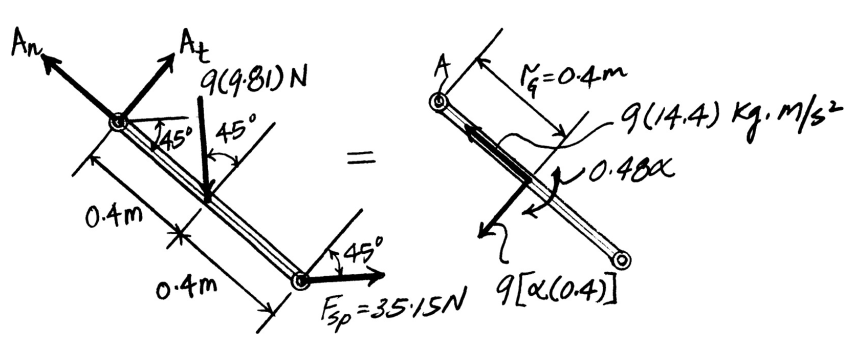
$$+\nwarrow \Sigma F_n = m(a_G)_n; \qquad A_n - 9(9.81) \sin 45° - 35.15 \sin 45° = 9(14.4)$$

$$A_n = 216.88\ \text{N}$$

Thus,

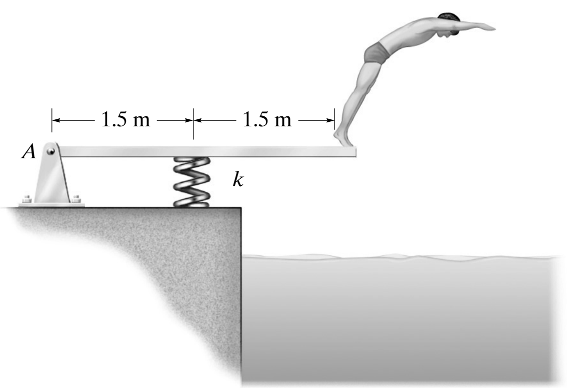
$$F_A = \sqrt{A_t^{\,2} + A_n^{\,2}} = \sqrt{28.03^2 + 216.88^2}$$

$$= 218.69\ \text{N} = 219\ \text{N} \qquad \textbf{Ans.}$$



(a)

17–75. Determine the angular acceleration of the 25-kg diving board and the horizontal and vertical components of reaction at the pin A the instant the man jumps off. Assume that the board is uniform and rigid, and that at the instant he jumps off the spring is compressed a maximum amount of 200 mm, $\omega = 0$, and the board is horizontal. Take $k = 7$ kN/m.



$$\zeta + \sum M_A = I_A\alpha; \qquad 1.5(1400 - 245.25) = \left[\frac{1}{3}(25)(3)^2\right]\alpha$$

$$+\uparrow \sum F_t = m(a_G)_t; \qquad 1400 - 245.25 - A_y = 25(1.5\alpha)$$

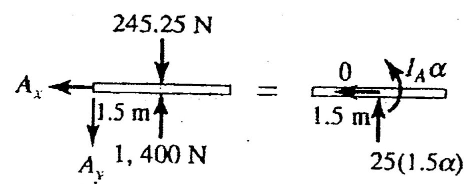
$$\overset{+}{\leftarrow} \sum F_n = m(a_G)_n; \qquad A_x = 0$$

Solving,

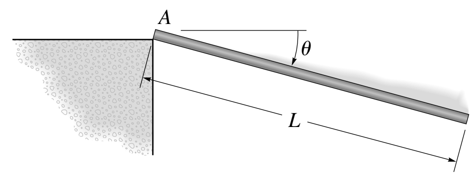
$A_x = 0$ **Ans.**

$A_y = 289$ N **Ans.**

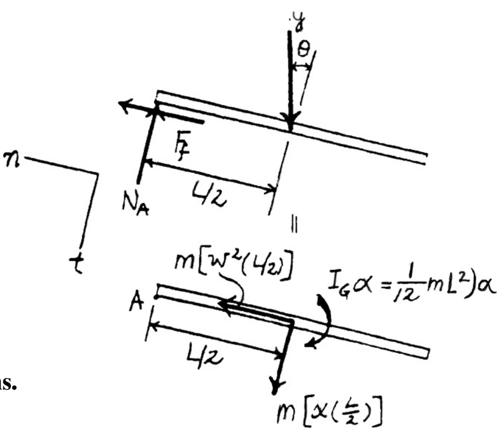
$\alpha = 23.1$ rad/s^2 **Ans.**



***17–76.** The slender rod of length L and mass m is released from rest when $\theta = 0°$. Determine as a function of θ the normal and the frictional forces which are exerted by the ledge on the rod at A as it falls downward. At what angle θ does the rod begin to slip if the coefficient of static friction at A is μ?



Equations of Motion: The mass moment inertia of the rod about its mass center is given by $I_G = \frac{1}{12}mL^2$. At the instant shown, the normal component of acceleration of the mass center for the rod is $(a_G)_n = \omega^2 r_G = \omega^2\left(\frac{L}{2}\right)$. The tangential component of acceleration of the mass center for the rod is $(a_G)_t = \alpha r_s = \alpha\left(\frac{L}{2}\right)$.



$$\zeta + \Sigma M_A = \Sigma(M_k)_O; \qquad -mg\cos\theta\left(\frac{L}{2}\right) = -\left(\frac{1}{12}mL^2\right)\alpha - m\left[\alpha\left(\frac{L}{2}\right)\right]\left(\frac{L}{2}\right)$$

$$\alpha = \frac{3g}{2L}\cos\theta$$

$$+\swarrow \Sigma F_t = m(a_G)_t; \qquad mg\cos\theta - N_A = m\left[\frac{3g}{2L}\cos\theta\left(\frac{L}{2}\right)\right]$$

$$N_A = \frac{mg}{4}\cos\theta \qquad \textbf{Ans.}$$

$$\nwarrow + \Sigma F_n = m(a_G)_n; \qquad F_f - mg\sin\theta = m\left[\omega^2\left(\frac{L}{2}\right)\right] \qquad \textbf{(1)}$$

Kinematics: Applying equation $\omega\, d\omega = a\, d\theta$, we have

$$\int_0^\omega \omega\, d\omega = \int_{0°}^{\theta} \frac{3g}{2L}\cos\theta\, d\theta$$

$$\omega^2 = \frac{3g}{L}\sin\theta$$

Substitute $\omega^2 = \frac{3g}{L}\sin\theta$ into Eq. (1) gives

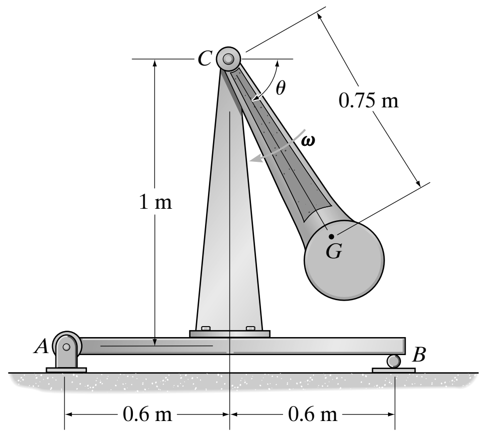
$$F_f = \frac{5mg}{2}\sin\theta \qquad \textbf{Ans.}$$

If the rod is on the verge of slipping at A, $F_f = \mu N_A$. Substitute the data obtained above, we have

$$\frac{5mg}{2}\sin\theta = \mu\left(\frac{mg}{4}\cos\theta\right)$$

$$\theta = \tan^{-1}\left(\frac{\mu}{10}\right) \qquad \textbf{Ans.}$$

•17–77. The 100-kg pendulum has a center of mass at G and a radius of gyration about G of $k_G = 250$ mm. Determine the horizontal and vertical components of reaction on the beam by the pin A and the normal reaction of the roller B at the instant $\theta = 90°$ when the pendulum is rotating at $\omega = 8$ rad/s. Neglect the weight of the beam and the support.



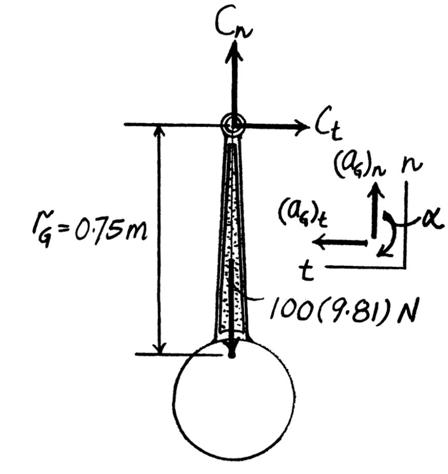
Equations of Motion: Since the pendulum rotates about the fixed axis passing through point C, $(a_G)_t = \alpha r_G = \alpha(0.75)$ and $(a_G)_n = \omega^2 r_G = 8^2(0.75) = 48\ \text{m/s}^2$. Here, the mass moment of inertia of the pendulum about this axis is $I_C = 100(0.25)^2 + 100(0.75^2) = 62.5\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about point C and referring to the free-body diagram of the pendulum, Fig. a, we have

$$\circlearrowleft +\Sigma M_C = I_C\alpha; \qquad 0 = 62.5\alpha \qquad \alpha = 0$$

Using this result to write the force equations of motion along the n and t axes,

$$\overset{+}{\leftarrow} \Sigma F_t = m(a_G)_t; \qquad -C_t = 100[0(0.75)] \qquad C_t = 0$$

$$+\uparrow \Sigma F_n = m(a_G)_n; \qquad C_n - 100(9.81) = 100(48) \qquad C_n = 5781\ \text{N}$$



(a)

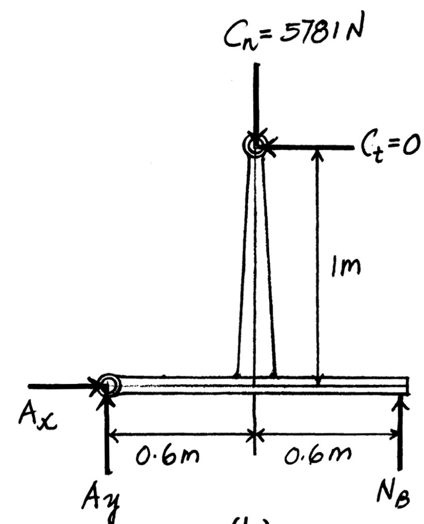
Equilibrium: Writing the moment equation of equilibrium about point A and using the free-body diagram of the beam in Fig. b, we have

$$+\Sigma M_A = 0; \qquad N_B(1.2) - 5781(0.6) = 0 \qquad N_B = 2890.5\ \text{N} = 2.89\ \text{kN} \qquad \textbf{Ans.}$$

Using this result to write the force equations of equilibrium along the x and y axes, we have

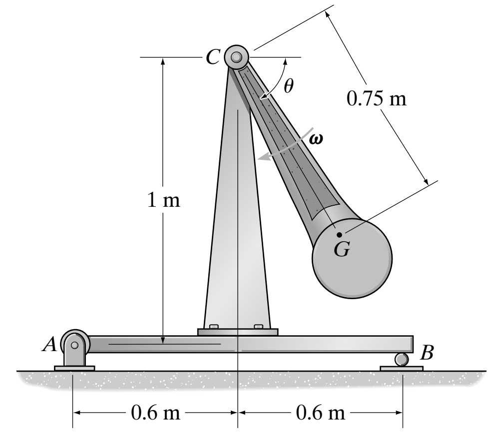
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + 2890.5 - 5781 = 0 \qquad A_y = 2890.5\ \text{N} = 2.89\ \text{kN} \qquad \textbf{Ans.}$$



(b)

17–78. The 100-kg pendulum has a center of mass at G and a radius of gyration about G of $k_G = 250$ mm. Determine the horizontal and vertical components of reaction on the beam by the pin A and the normal reaction of the roller B at the instant $\theta = 0°$ when the pendulum is rotating at $\omega = 4$ rad/s. Neglect the weight of the beam and the support.



Equations of Motion: Since the pendulum rotates about the fixed axis passing through point C, $(a_G)_t = \alpha r_G = \alpha(0.75)$ and $(a_G)_n = \omega^2 r_G = 4^2(0.75) = 12\ \text{m/s}^2$. Here, the mass moment of inertia of the pendulum about this axis is $I_C = 100(0.25^2) + 100(0.75)^2 = 62.5\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about point C and referring to the free-body diagram shown in Fig. a,

$$\circlearrowleft +\Sigma M_C = I_C\alpha; \qquad -100(9.81)(0.75) = -62.5\alpha \qquad \alpha = 11.772\ \text{rad/s}^2$$

Using this result to write the force equations of motion along the n and t axes, we have

$$+\uparrow \Sigma F_t = m(a_G)_t; \qquad C_t - 100(9.81) = -100[11.772(0.75)] \qquad C_t = 98.1\ \text{N}$$

$$\overset{+}{\leftarrow} \Sigma F_n = m(a_G)_n; \qquad C_n = 100(12) \qquad C_n = 1200\ \text{N}$$

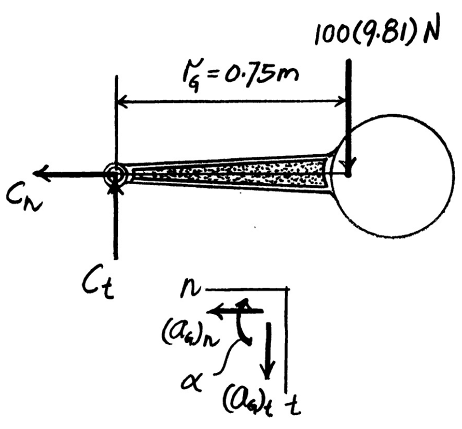
Equilibrium: Writing the moment equation of equilibrium about point A and using the free-body diagram of the beam in Fig. b,

$$+\Sigma M_A = 0; \qquad N_B(1.2) - 98.1(0.6) - 1200(1) = 0 \qquad N_B = 1049.05\ \text{N} = 1.05\ \text{kN} \quad \textbf{Ans.}$$

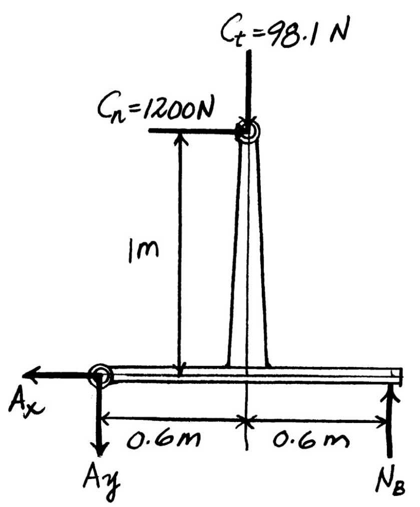
Using this result to write the force equations of equilibrium along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 1200 - A_x = 0 \qquad A_x = 1200\ \text{N} = 1.20\ \text{kN} \quad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 1049.05 - 98.1 - A_y = 0 \qquad A_y = 950.95\ \text{N} = 951\ \text{N} \quad \textbf{Ans.}$$

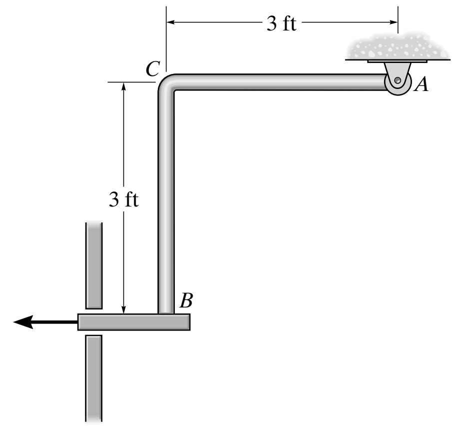


(a)

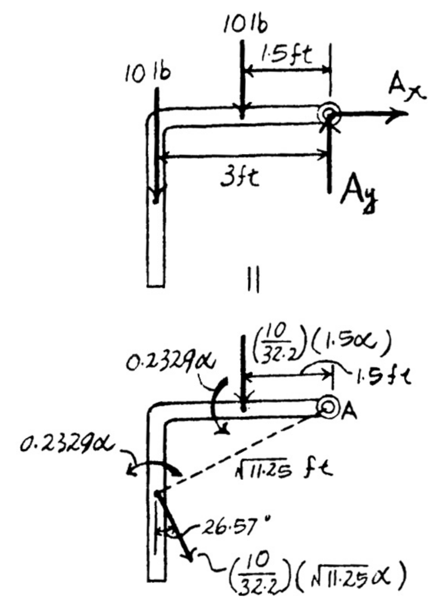


(b)

17–79. If the support at B is suddenly removed, determine the initial horizontal and vertical components of reaction that the pin A exerts on the rod ACB. Segments AC and CB each have a weight of 10 lb.



Equations of Motion: The mass moment inertia of the rod segment AC and BC about their respective mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{10}{32.2}\right)(3^2)$ $= 0.2329 \text{ slug} \cdot \text{ft}^2$. At the instant shown, the normal component of acceleration of the mass center for rod segment AB and BC are $[(a_G)_n]_{AB} = [(a_G)_n]_{BC} = 0$ since the angular velocity of the assembly $\omega = 0$ at that instant. The tangential component of acceleration of the mass center for rod segment AC and BC are $[(a_G)_t]_{AB} = 1.5\,\alpha$ and $[(a_G)_t]_{BC} = \sqrt{11.25}\alpha$.



$$\zeta + \Sigma M_A = \Sigma(M_k)_A; \qquad 10(1.5) + 10(3) = 0.2329\alpha + \left(\frac{10}{32.2}\right)(1.5\alpha)(1.5)$$

$$+ 0.2329\,\alpha + \left(\frac{10}{32.2}\right)(\sqrt{11.25}\alpha)\,(\sqrt{11.25})$$

$$\alpha = 9.660 \text{ rad/s}^2$$

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad A_x = \left(\frac{10}{32.2}\right)[\sqrt{11.25}\,(9.660)]\sin 26.57^\circ$$

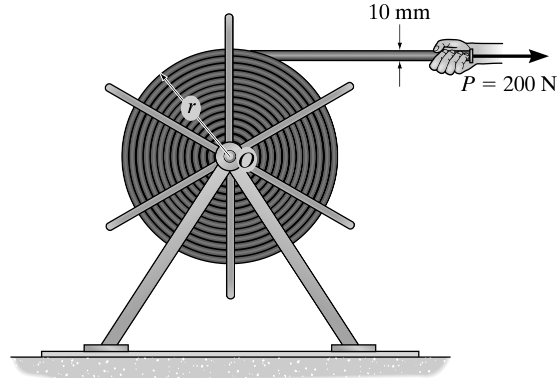
$$A_x = 4.50 \text{ lb}$$ **Ans.**

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad A_y - 20 = -\left(\frac{10}{32.2}\right)[1.5(9.660)]$$

$$-\left(\frac{10}{32.2}\right)[\sqrt{11.25}\,(9.660)]\cos 26.57^\circ$$

$$A_y = 6.50 \text{ lb}$$ **Ans.**

***17–80.** The hose is wrapped in a spiral on the reel and is pulled off the reel by a horizontal force of $P = 200$ N. Determine the angular acceleration of the reel after it has turned 2 revolutions. Initially, the radius is $r = 500$ mm. The hose is 15 m long and has a mass per unit length of 10 kg/m. Treat the wound-up hose as a disk.



Equations of Motion: The mass of the hose on the reel when it rotates through an angle θ is $m = 15(10) - r\theta(10) = (150 - 10r\theta)$ kg. Then, the mass moment of inertia of the reel about point O at any instant is $I_O = \frac{1}{2}mr^2 = \frac{1}{2}(150 - 10r\theta)r^2$. Also, the acceleration of the unwound hose is $a = \alpha r$. Writing the moment equation of motion about point O,

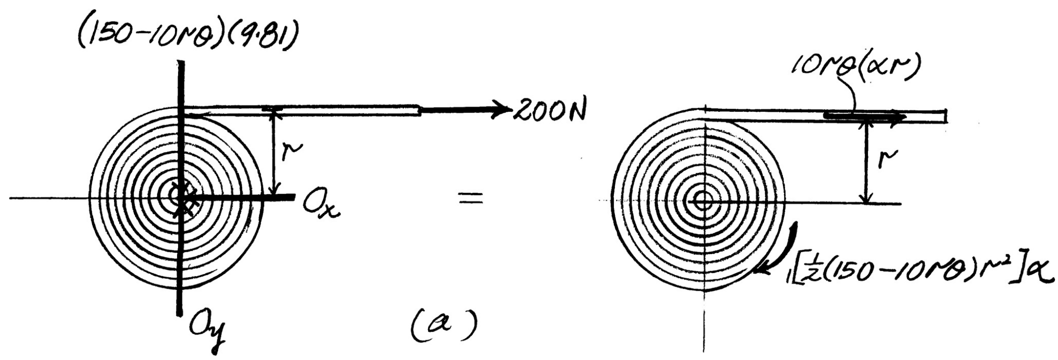
$$\circlearrowleft + \Sigma M_O = \Sigma(M_k)_O; \qquad -200(r) = -\left[\frac{1}{2}(150 - 10r\theta)r^2\right]\alpha - 10r\theta(\alpha r)r$$

$$\alpha = \frac{200}{75r + 5r^2\theta}$$

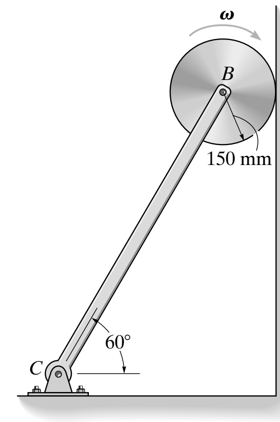
However, $r = 0.5 - \frac{\theta}{2\pi}(0.01) = 0.5 - \frac{0.005}{\pi}\theta$. Thus, when $\theta = 2\text{ rev}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 4\pi$ rad, $r = 0.48$ m. Then

$$\alpha = \frac{200}{75(0.48) + 5(0.48^2)(4\pi)}$$

$$= 3.96\text{ rad/s}^2 \qquad \textbf{Ans.}$$



•17–81. The disk has a mass of 20 kg and is originally spinning at the end of the strut with an angular velocity of $\omega = 60$ rad/s. If it is then placed against the wall, where the coefficient of kinetic friction is $\mu_k = 0.3$, determine the time required for the motion to stop. What is the force in strut BC during this time?



$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad F_{CB}\sin 30° - N_A = 0$

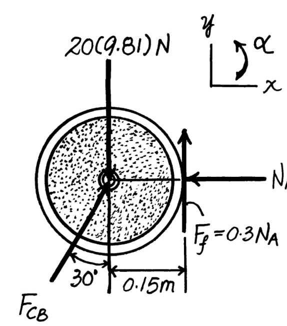
$+\uparrow \Sigma F_y = m(a_G)_y; \qquad F_{CB}\cos 30° - 20(9.81) + 0.3N_A = 0$

$\circlearrowleft + \Sigma M_B = I_B\,\alpha; \qquad 0.3N_A(0.15) = \left[\frac{1}{2}(20)(0.15)^2\right]\alpha$

$$N_A = 96.6 \text{ N}$$

$$F_{CB} = 193 \text{ N} \qquad \textbf{Ans.}$$

$$\alpha = 19.3 \text{ rad/s}^2$$

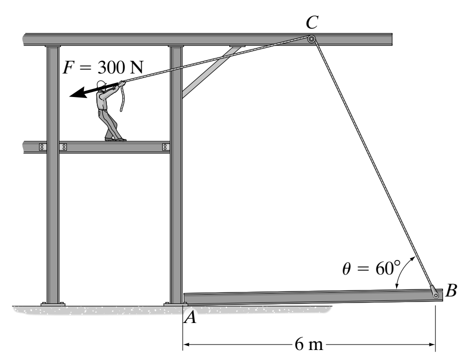


$\circlearrowleft +$ $$\omega = \omega_0 + \alpha_c t$$

$$0 = 60 + (-19.3)\,t$$

$$t = 3.11 \text{ s} \qquad \textbf{Ans.}$$

17–82. The 50-kg uniform beam (slender rod) is lying on the floor when the man exerts a force of $F = 300$ N on the rope, which passes over a small smooth peg at C. Determine the initial angular acceleration of the beam. Also find the horizontal and vertical reactions on the beam at A (considered to be a pin) at this instant.



Equations of Motion: Since the beam rotates about a fixed axis passing through point A, $(a_G)_t = \alpha r_G = \alpha(3)$ and $(a_G)_n = \omega^2 r_G = \omega^2(3)$. However, the beam is initially at rest, so $\omega = 0$. Thus, $(a_G)_n = 0$. Here, the mass moment of inertia of the beam about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(50)(6^2) = 150 \text{ kg}\cdot\text{m}^2$. Writing the moment equation of motion about point A, Fig. a,

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad 300\sin 60°(6) - 50(9.81)(3) = 50[\alpha(3)](3) + 150\alpha$

$$\alpha = 0.1456 \text{ rad/s}^2 = 0.146 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

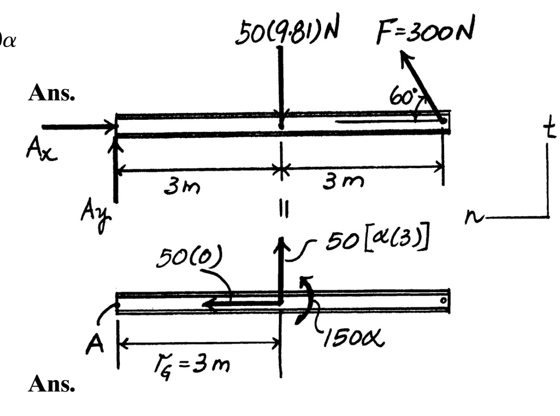
This result can also be obtained by applying $\Sigma M_A = I_A\,\alpha$, where

$$I_A = \frac{1}{12}(50)(6^2) + 50(3^2) = 600 \text{ kg}\cdot\text{m}^2$$

Thus,

$\circlearrowleft + \Sigma M_A = \Sigma(\mu_k)_A; \qquad 300\sin 60°(6) - 50(9.81)(3) = 600\alpha$

$$\alpha = 0.1456 \text{ rad/s}^2 = 0.146 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



(a)

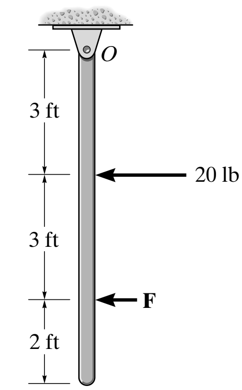
Using this result to write the force equations of motion along the n and t axes,

$\xleftarrow{+} \Sigma F_t = m(a_G)_t; \qquad 300\cos 60° - A_x = 50(0) \qquad A_x = 150 \text{ N} \qquad \textbf{Ans.}$

$+\uparrow \Sigma F_n = m(a_G)_n; \quad A_y + 300\sin 60° - 50(9.81) = 50[0.1456(3)]$

$$A_y = 252.53 \text{ N} = 253 \text{ N} \qquad \textbf{Ans.}$$

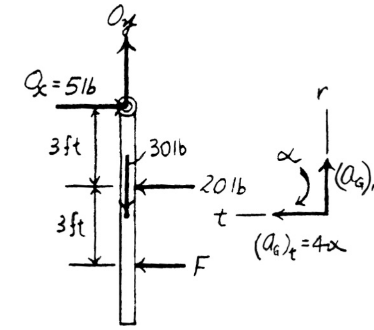
17–83. At the instant shown, two forces act on the 30-lb slender rod which is pinned at O. Determine the magnitude of force $\mathbf{F}$ and the initial angular acceleration of the rod so that the horizontal reaction which the *pin exerts on the rod* is 5 lb directed to the right.



Equations of Motion: The mass moment of inertia of the rod about point O is given by $I_O = I_G = mr_G^2 = \frac{1}{12}\left(\frac{30}{32.2}\right)(8^2) + \left(\frac{30}{32.2}\right)(4^2) = 19.88 \text{ slug}\cdot\text{ft}^2$. At the instant shown, the tangential component of acceleration of the mass center for the rod is $(a_G)_t = \alpha r_g = 4\alpha$. Applying Eq. 17–16, we have

$$\circlearrowleft + \Sigma M_O = I_O\,\alpha; \qquad -20(3) - F(6) = -19.88\alpha \qquad \textbf{(1)}$$

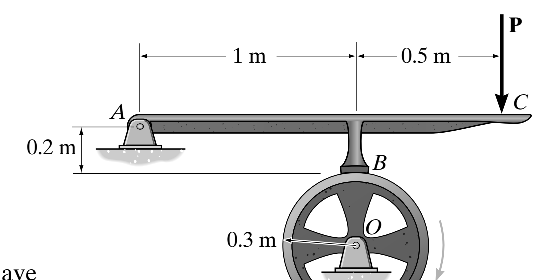
$$\Sigma F_t = m(a_G)_t\,; \qquad 20 + F - 5 = \left(\frac{30}{32.2}\right)(4\alpha) \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields:

$$\alpha = 12.1 \text{ rad/s}^2 \qquad F = 30.0 \text{ lb} \qquad \textbf{Ans.}$$

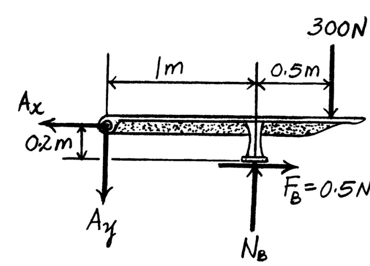
***17–84.** The 50-kg flywheel has a radius of gyration about its center of mass of $k_O = 250$ mm. It rotates with a constant angular velocity of 1200 rev/min before the brake is applied. If the coefficient of kinetic friction between the brake pad B and the wheel's rim is $\mu_k = 0.5$, and a force of $P = 300$ N is applied to the braking mechanism's handle, determine the time required to stop the wheel.



Equilibrium: Writing the moment equation of equilibrium about point A, we have

$$\circlearrowleft + \Sigma M_A = 0; \qquad N_B(1) + 0.5N_B(0.2) - 300(1.5) = 0$$

$$N_B = 409.09 \text{ N}$$

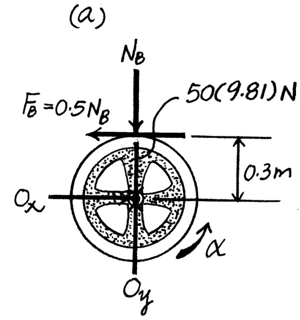


(a)

Equations of Motion: The mass moment of inertia of the flywheel about its center is $I_O = 50(0.25^2) = 3.125 \text{ kg}\cdot\text{m}^2$. Referring to the free-body diagram of the flywheel shown in Fig. *b*, we have

$$+\Sigma M_O = I_O\,\alpha; \qquad 0.5(409.09)(0.3) = 3.125\alpha$$

$$\alpha = 19.64 \text{ rad/s}^2$$



(b)

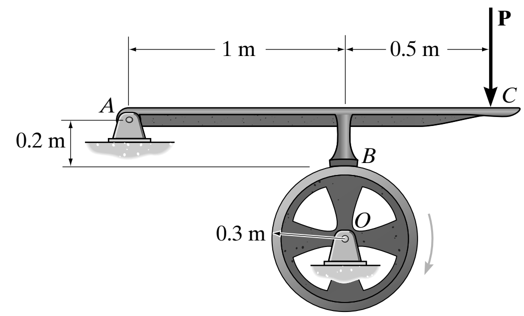
Kinematics: Here, $\omega_0 = \left(1200\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 40\pi$ rad/s. Since the angular acceleration is constant,

$$\circlearrowleft + \qquad \omega = \omega_0 + \alpha t$$

$$0 = 40\pi + (-19.64)t$$

$$t = 6.40 \text{ s} \qquad \textbf{Ans.}$$

•17–85. The 50-kg flywheel has a radius of gyration about its center of mass of $k_O = 250$ mm. It rotates with a constant angular velocity of 1200 rev/min before the brake is applied. If the coefficient of kinetic friction between the brake pad B and the wheel's rim is $\mu_k = 0.5$, determine the constant force **P** that must be applied to the braking mechanism's handle in order to stop the wheel in 100 revolutions.



Kinematics: Here,

$$\omega_0 = \left(1200\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 40\pi\text{ rad/s}$$

and

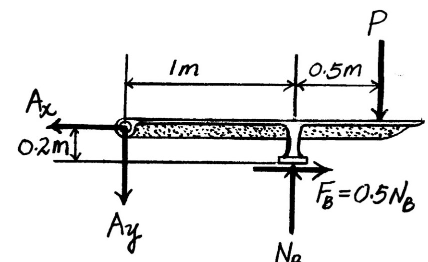
$$\theta = (100\text{ rev})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 200\pi\text{ rad}$$

Since the angular acceleration is constant,

$$\circlearrowleft + \quad \omega^2 = \omega_0{}^2 + \alpha(\theta - \theta_0)$$

$$0^2 = (40\pi)^2 + 2\alpha(200\pi - 0)$$

$$\alpha = -12.57\text{ rad/s}^2 = 12.57\text{ rad/s}^2$$

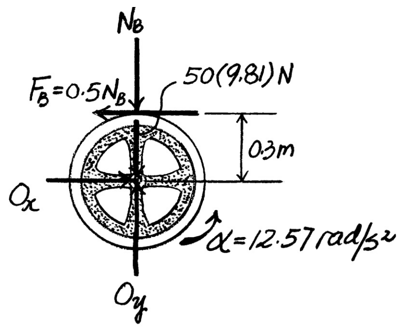


(a)

Equilibrium: Writing the moment equation of equilibrium about point A using the free-body diagram of the brake shown in Fig. a,

$$\circlearrowleft + \Sigma M_A = 0; \qquad N_B\,(1) + 0.5N_B\,(0.2) - P(1.5) = 0$$

$$N_B = 1.3636P$$



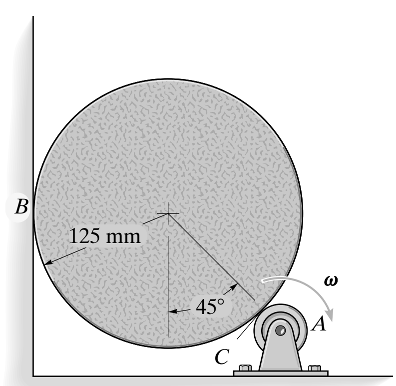
(b)

Equations of Motion: The mass moment of inertia of the flywheel about its center is $I_O = mk_O{}^2 = 50(0.25^2) = 3.125\text{ kg}\cdot\text{m}^2$. Referring to the free-body diagram of the flywheel shown in Fig. b,

$$+\Sigma M_O = I_O\alpha; \qquad 0.5(1.3636\,P)(0.3) = 3.125(12.57)$$

$$P = 191.98\text{ N} = 192\text{ N} \qquad \textbf{Ans.}$$

17–86. The 5-kg cylinder is initially at rest when it is placed in contact with the wall B and the rotor at A. If the rotor always maintains a constant clockwise angular velocity $\omega = 6$ rad/s, determine the initial angular acceleration of the cylinder. The coefficient of kinetic friction at the contacting surfaces B and C is $\mu_k = 0.2$.

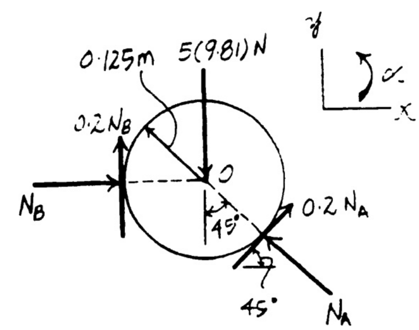


Equations of Motion: The mass moment of inertia of the cylinder about point O is given by $I_O = \dfrac{1}{2}mr^2 = \dfrac{1}{2}(5)(0.125^2) = 0.0390625\text{ kg}\cdot\text{m}^2$. Applying Eq. 17–16, we have

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x\,; \qquad N_B + 0.2N_A\cos 45° - N_A\sin 45° = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y\,; \qquad 0.2N_B + 0.2N_A\sin 45° + N_A\cos 45° - 5(9.81) = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_O = I_O\,\alpha; \qquad 0.2N_A\,(0.125) - 0.2N_B\,(0.125) = 0.0390625\alpha \qquad \textbf{(3)}$$

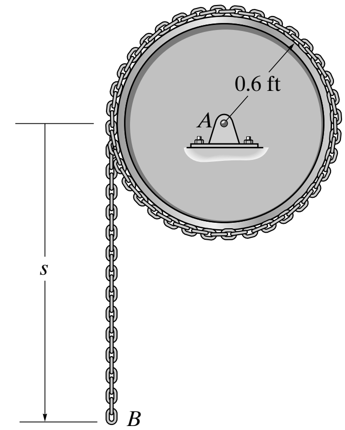


Solving Eqs. (1), (2), and (3) yields;

$$N_A = 51.01\text{ N} \qquad N_B = 28.85\text{ N}$$

$$\alpha = 14.2\text{ rad/s}^2 \qquad \textbf{Ans.}$$

17–87. The drum has a weight of 50 lb and a radius of gyration $k_A = 0.4$ ft. A 35-ft-long chain having a weight of 2 lb/ft is wrapped around the outer surface of the drum so that a chain length of $s = 3$ ft is suspended as shown. If the drum is originally at rest, determine its angular velocity after the end B has descended $s = 13$ ft. Neglect the thickness of the chain.



$$\zeta + \Sigma M_A = \Sigma(M_k)_A; \quad 2s(0.6) = \left(\frac{2s}{32.2}\right)[(\alpha)(0.6)](0.6) + \left[\left(\frac{50}{32.2}\right)(0.4)^2 + \frac{2(35-s)}{32.2}(0.6)^2\right]\alpha$$

$$1.2s = 0.02236s\alpha + (0.24845 + 0.7826 - 0.02236s)\alpha$$

$$1.164s = \alpha$$

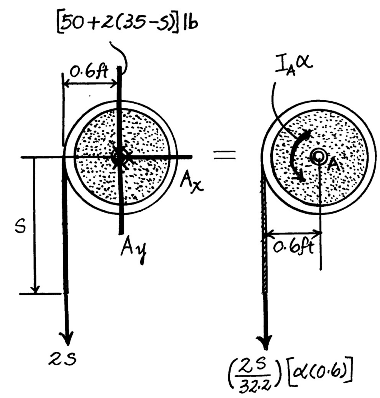
$$\alpha\, d\theta = \alpha\left(\frac{ds}{0.6}\right) = \omega\, d\omega$$

$$1.164s\left(\frac{ds}{0.6}\right) = \omega\, d\omega$$

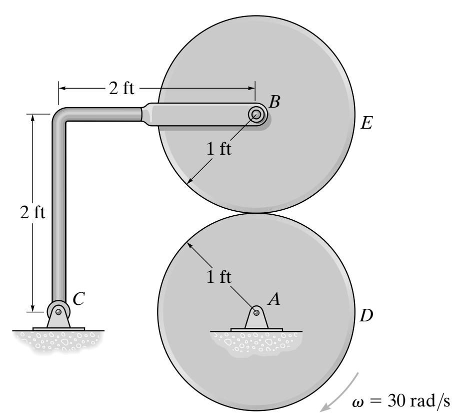
$$1.9398\int_3^{13} s\, ds = \int_0^{\omega} \omega\, d\omega$$

$$1.9398\left[\frac{(13)^2}{2} - \frac{(3)^2}{2}\right] = \frac{1}{2}\omega^2$$

$$\omega = 17.6 \text{ rad/s}$$ **Ans.**



***17–88.** Disk D turns with a constant clockwise angular velocity of 30 rad/s. Disk E has a weight of 60 lb and is initially at rest when it is brought into contact with D. Determine the time required for disk E to attain the same angular velocity as disk D. The coefficient of kinetic friction between the two disks is $\mu_k = 0.3$. Neglect the weight of bar BC.



Equations of Motion: The mass moment of inertia of disk E about point B is given by $I_B = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{60}{32.2}\right)(1^2) = 0.9317 \text{ slug}\cdot\text{ft}^2$. Applying Eq. 17–16, we have

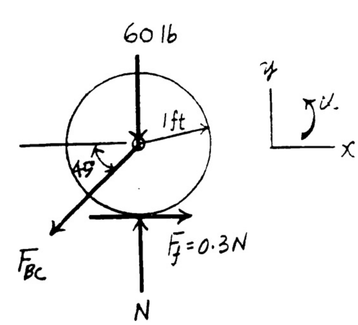
$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \qquad 0.3N - F_{BC}\cos 45° = 0 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \qquad N - F_{BC}\sin 45° - 60 = 0 \qquad \textbf{(2)}$$

$$\zeta + \Sigma M_O = I_O\,\alpha; \qquad 0.3N(1) = 0.9317\alpha \qquad \textbf{(3)}$$

Solving Eqs. (1), (2) and (3) yields:

$$F_{BC} = 36.37 \text{ lb} \qquad N = 85.71 \text{ lb} \qquad \alpha = 27.60 \text{ rad/s}^2$$

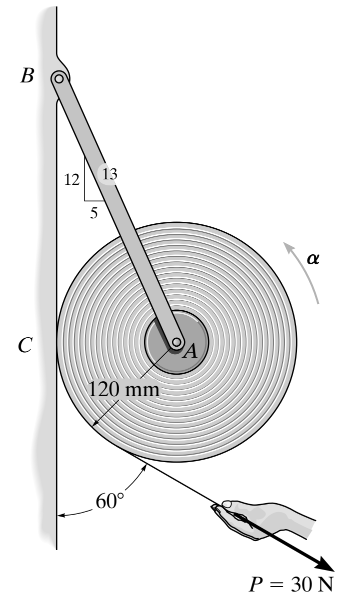


Kinematics: Applying equation $\omega = \omega_0 + \alpha_t$, we have

$$(\zeta+) \qquad 30 = 0 + 27.60t$$

$$t = 1.09 \text{ s}$$ **Ans.**

•17–89. A 17-kg roll of paper, originally at rest, is supported by bracket AB. If the roll rests against a wall where the coefficient of kinetic friction is $\mu_C = 0.3$, and a constant force of 30 N is applied to the end of the sheet, determine the tension in the bracket as the paper unwraps, and the angular acceleration of the roll. For the calculation, treat the roll as a cylinder.



Equations of Motion: The mass moment of inertia of the paper roll about point A is given by $I_A = \frac{1}{2}mr^2 = \frac{1}{2}(17)\left(0.12^2\right) = 0.1224\ \text{kg}\cdot\text{m}^2$. Applying Eq. 17–16, we have

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad N_C - F_{AB}\left(\frac{5}{13}\right) + 30\sin 60° = 0 \qquad \textbf{(1)}$$

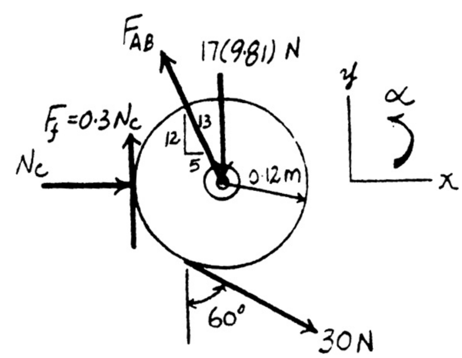
$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 0.3N_C + F_{AB}\left(\frac{12}{13}\right) - 30\cos 60° - 17(9.81) = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad 30(0.12) - 0.3N_C(0.12) = 0.1224\alpha \qquad \textbf{(3)}$$

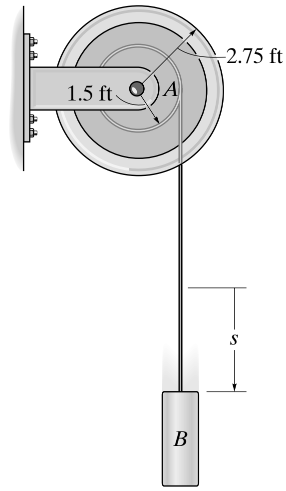
Solving Eqs. (1), (2), and (3) yields:

$$F_{AB} = 183\ \text{N} \qquad \alpha = 16.4\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

$$N_C = 44.23\ \text{N}$$



17–90. The cord is wrapped around the inner core of the spool. If a 5-lb block B is suspended from the cord and released from rest, determine the spool's angular velocity when $t = 3$ s. Neglect the mass of the cord. The spool has a weight of 180 lb and the radius of gyration about the axle A is $k_A = 1.25$ ft. Solve the problem in two ways, first by considering the "system" consisting of the block and spool, and then by considering the block and spool separately.



System:

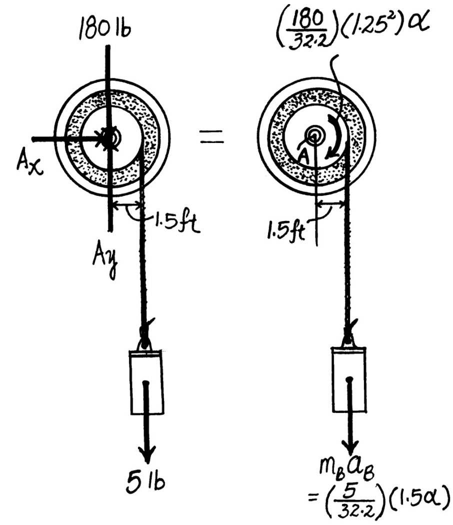
$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \qquad 5(1.5) = \left(\frac{180}{32.2}\right)(1.25)^2\alpha + \left(\frac{5}{32.2}\right)(1.5\alpha)(1.5)$$

$$\alpha = 0.8256 \text{ rad/s}^2$$

$$(\circlearrowleft +) \qquad \omega = \omega_0 + a_c t$$

$$\omega = 0 + (0.8256)(3)$$

$$\omega = 2.48 \text{ rad/s} \qquad \textbf{Ans.}$$



Also,

Spool:

$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad T(1.5) = \left(\frac{180}{32.2}\right)(1.25)^2\,\alpha$$

Weight:

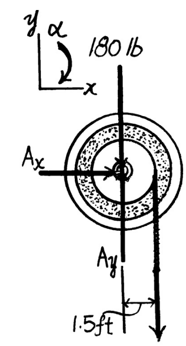
$$+\downarrow \Sigma F_y = m(a_G)_y\,; \qquad 5 - T = \left(\frac{5}{32.2}\right)(1.5\alpha)$$

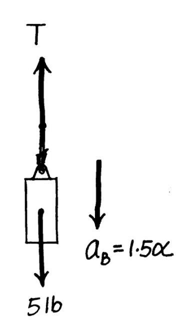
$$\alpha = 0.8256 \text{ rad/s}^2$$

$$(\circlearrowleft +) \qquad \omega = \omega_0 + a_c t$$

$$\omega = 0 + (0.8256)(3)$$

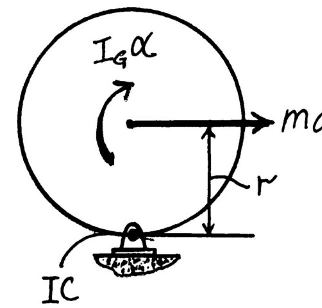
$$\omega = 2.48 \text{ rad/s} \qquad \textbf{Ans.}$$





17–91. If a disk *rolls without slipping* on a horizontal surface, show that when moments are summed about the instantaneous center of zero velocity, IC, it is possible to use the moment equation $\Sigma M_{IC} = I_{IC}\alpha$, where I_{IC} represents the moment of inertia of the disk calculated about the instantaneous axis of zero velocity.

$\zeta + \Sigma M_{IC} = \Sigma\,(M_k)_{IC}\,; \qquad \Sigma M_{IC} = I_G\,\alpha + (ma_G)r$



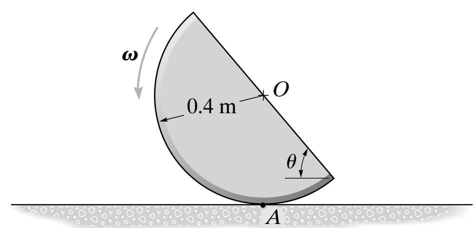
Since there is no slipping, $a_G = \alpha r$.

Thus, $\Sigma M_{IC} = (I_G + mr^2)\,\alpha$.

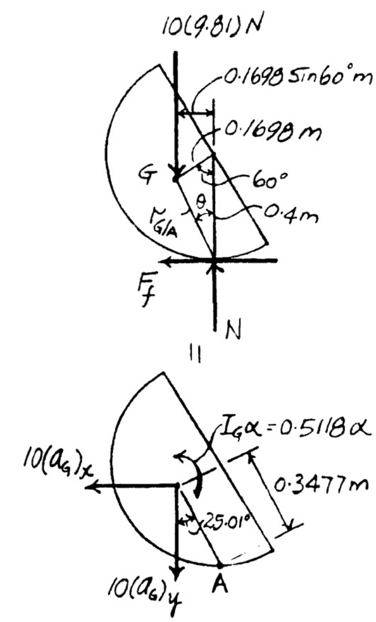
By the parallel-axis theorem, the term in parenthesis represents I_{IC}. Thus,

$$\Sigma M_{IC} = I_{IC}\,\alpha \qquad \textbf{Q.E.D.}$$

***17–92.** The 10-kg semicircular disk is rotating at $\omega = 4$ rad/s at the instant $\theta = 60°$. Determine the normal and frictional forces it exerts on the ground at A at this instant. Assume the disk does not slip as it rolls.



Equations of Motion: The mass moment of inertia of the semicircular disk about its center of mass is given by $I_G = \frac{1}{2}(10)\left(0.4^2\right) - 10\left(0.1698^2\right) = 0.5118\ \text{kg}\cdot\text{m}^2$. From the geometry, $r_{G/A} = \sqrt{0.1698^2 + 0.4^2 - 2(0.1698)(0.4)\cos 60°} = 0.3477$ m. Also, using the law of sines, $\dfrac{\sin\theta}{0.1698} = \dfrac{\sin 60°}{0.3477}$, $\theta = 25.01°$. Applying Eq. 17–16, we have



$\zeta + \Sigma M_A = \Sigma(M_k)_A\,; \qquad 10(9.81)(0.1698\sin 60°) = 0.5118\alpha$

$$+\,10(a_G)_x\cos 25.01°\,(0.3477)$$

$$+\,10(a_G)_y\sin 25.01°(0.3477) \qquad \textbf{(1)}$$

$\overset{+}{\leftarrow}\Sigma F_x = m\,(a_G)_x; \qquad F_f = 10(a_G)_x \qquad \textbf{(2)}$

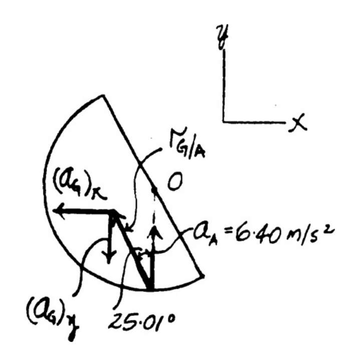
$+\uparrow F_y = m(a_G)_y\,; \qquad N - 10(9.81) = -10(a_G)_y \qquad \textbf{(3)}$

Kinematics: Since the semicircular disk does not slip at A, then $(a_A)_x = 0$. Here, $\mathbf{r}_{G/A} = \{-0.3477\sin 25.01°\mathbf{i} + 0.3477\cos 25.01°\mathbf{j}\}$ m $= \{-0.1470\mathbf{i} + 0.3151\mathbf{j}\}$ m. Applying Eq. 16–18, we have

$$\mathbf{a}_G = \mathbf{a}_A + \boldsymbol{\alpha}\times\mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$-(a_G)_x\,\mathbf{i} - (a_G)_y\mathbf{j} = 6.40\mathbf{j} + \alpha\mathbf{k}\times(-0.1470\mathbf{i} + 0.3151\mathbf{j}) - 4^2(-0.1470\mathbf{i} + 0.3151\mathbf{j})$$

$$-(a_G)_x\,\mathbf{i} - (a_G)_y\,\mathbf{j} = (2.3523 - 0.3151\alpha)\,\mathbf{i} + (1.3581 - 0.1470\alpha)\mathbf{j}$$



Equating **i** and **j** components, we have

$$(a_G)_x = 0.3151\alpha - 2.3523 \qquad \textbf{(4)}$$

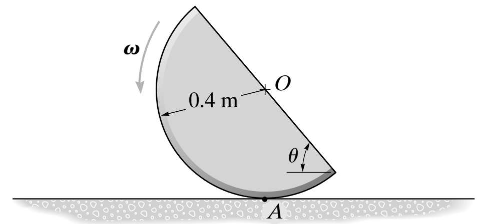
$$(a_G)_y = 0.1470\alpha - 1.3581 \qquad \textbf{(5)}$$

Solving Eqs. (1), (2), (3), (4), and (5) yields:

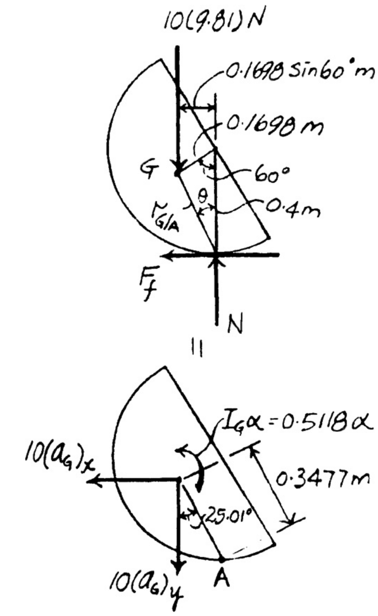
$\alpha = 13.85\ \text{rad/s}^2 \qquad (a_G)_x = 2.012\ \text{m/s}^2 \qquad (a_G)_y = 0.6779\ \text{m/s}^2$

$F_f = 20.1$ N $\qquad N = 91.3$ N $\qquad$ **Ans.**

•17–93. The semicircular disk having a mass of 10 kg is rotating at $\omega = 4$ rad/s at the instant $\theta = 60°$. If the coefficient of static friction at A is $\mu_s = 0.5$, determine if the disk slips at this instant.



Equations of Motion: The mass moment of inertia of the semicircular disk about its center of mass is given by $I_G = \frac{1}{2}(10)\left(0.4^2\right) - 10\,(0.1698^2) = 0.5118\ \text{kg}\cdot\text{m}^2$. From the geometry, $r_{G/A} = \sqrt{0.1698^2 + 0.4^2 - 2(0.1698)\,(0.4)\cos 60°} = 0.3477$ m Also, using law of sines, $\dfrac{\sin\theta}{0.1698} = \dfrac{\sin 60°}{0.3477}$, $\theta = 25.01°$. Applying Eq. 17–16, we have



$$\zeta + \Sigma M_A = \Sigma(M_k)_A; \qquad 10(9.81)(0.1698\sin 60°) = 0.5118\alpha + 10(a_G)_x\cos 25.01°(0.3477) + 10(a_G)_y\sin 25.01°(0.3477) \qquad \textbf{(1)}$$

$$\overset{+}{\leftarrow}\Sigma F_x = m(a_G)_x; \qquad F_f = 10(a_G)_x \qquad \textbf{(2)}$$

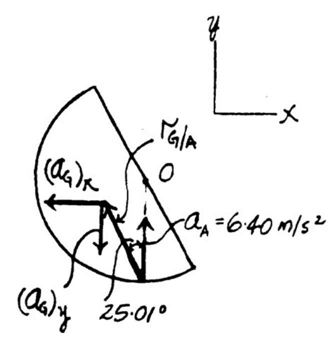
$$+\uparrow F_y = m(a_G)_y; \qquad N - 10(9.81) = -10(a_G)_y \qquad \textbf{(3)}$$

Kinematics: Assume that the semicircular disk does not slip at A, then $(a_A)_x = 0$. Here, $\mathbf{r}_{G/A} = \{-0.3477\sin 25.01°\mathbf{i} + 0.3477\cos 25.01°\mathbf{j}\}$ m $= \{-0.1470\mathbf{i} + 0.3151\mathbf{j}\}$ m. Applying Eq. 16–18, we have

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$-(a_G)_x\,\mathbf{i} - (a_G)_y\mathbf{j} = 6.40\mathbf{j} + \alpha\mathbf{k} \times (-0.1470\mathbf{i} + 0.3151\mathbf{j}) - 4^2(-0.1470\mathbf{i} + 0.3151\mathbf{j})$$

$$-(a_G)_x\,\mathbf{i} - (a_G)_y\,\mathbf{j} = (2.3523 - 0.3151\,\alpha)\,\mathbf{i} + (1.3581 - 0.1470\alpha)\mathbf{j}$$



Equating **i** and **j** components, we have

$$(a_G)_x = 0.3151\alpha - 2.3523 \qquad \textbf{(4)}$$

$$(a_G)_y = 0.1470\alpha - 1.3581 \qquad \textbf{(5)}$$

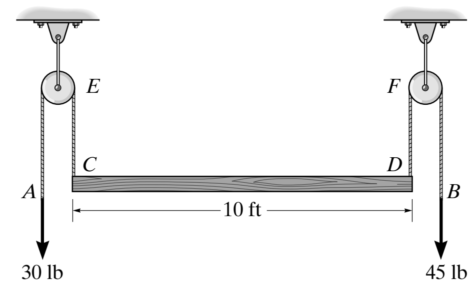
Solving Eqs. (1), (2), (3), (4), and (5) yields:

$$\alpha = 13.85\ \text{rad/s}^2 \qquad (a_G)_x = 2.012\ \text{m/s}^2 \qquad (a_G)_y = 0.6779\ \text{m/s}^2$$

$$F_f = 20.12\ \text{N} \qquad N = 91.32\ \text{N}$$

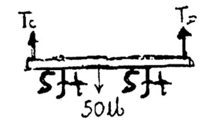
Since $F_f < (F_f)_{max} = \mu_s N = 0.5(91.32) = 45.66$ N, then the semicircular **disk does not slip**. **Ans.**

17–94. The uniform 50-lb board is suspended from cords at C and D. If these cords are subjected to constant forces of 30 lb and 45 lb, respectively, determine the initial acceleration of the board's center and the board's angular acceleration. Assume the board is a thin plate. Neglect the mass of the pulleys at E and F.



$+\uparrow \Sigma F_y = m(a_G)_y;\qquad 45 + 30 - 50 = \left(\dfrac{50}{32.2}\right) a_G$

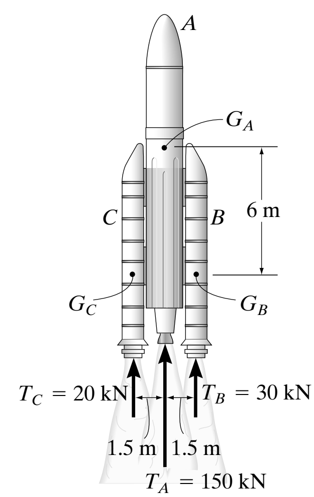
$a_G = 16.1\ \text{ft/s}^2$ **Ans.**



$\circlearrowleft + \Sigma M_G = I_G\,\alpha;\qquad -30(5) + 45(5) = \left[\dfrac{1}{12}\left(\dfrac{50}{32.2}\right)(10)^2\right]\alpha$

$\alpha = 5.80\ \text{rad/s}^2$ **Ans.**

17–95. The rocket consists of the main section A having a mass of 10 Mg and a center of mass at G_A. The two identical booster rockets B and C each have a mass of 2 Mg with centers of mass at G_B and G_C, respectively. At the instant shown, the rocket is traveling vertically and is at an altitude where the acceleration due to gravity is $g = 8.75\ \text{m/s}^2$. If the booster rockets B and C suddenly supply a thrust of $T_B = 30\ \text{kN}$ and $T_C = 20\ \text{kN}$, respectively, determine the angular acceleration of the rocket. The radius of gyration of A about G_A is $k_A = 2\ \text{m}$ and the radii of gyration of B and C about G_B and G_C are $k_B = k_C = 0.75\ \text{m}$.



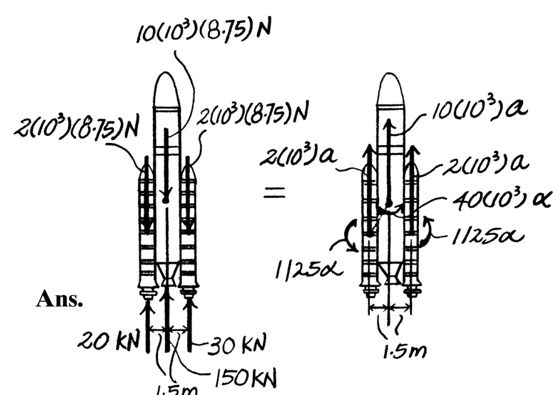
Equations of Motion: The mass moment of inertia of the main section and booster rockets about G is

$$\left(I_G\right)_A = 10\left(10^3\right)\left(2^2\right) + 2\left(2\left(10^3\right)\left(0.75^2\right) + 2\left(10^3\right)\left(1.5^2 + 6^2\right)\right)$$

$$= 195.25\left(10^3\right)\ \text{kg}\cdot\text{m}^2$$

$+\uparrow \Sigma F_y = m\left(a_G\right)_y;$

$$150\left(10^3\right) + 20\left(10^3\right) + 30\left(10^3\right) - \left[2\left(10^3\right) + 2\left(10^3\right) + 10\left(10^3\right)\right](8.75) = \left[2\left(10^3\right) + 2\left(10^3\right) + 10\left(10^3\right)\right]a$$



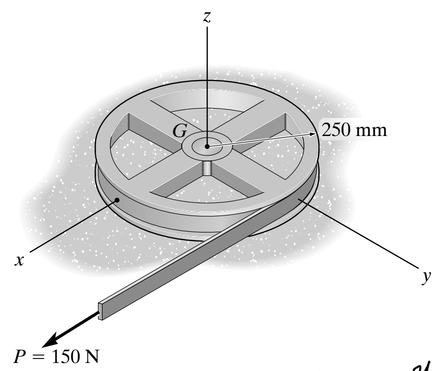
(a)

$a = 5.536\ \text{m/s}^2 = 5.54\ \text{m/s}^2\uparrow$ **Ans.**

$+\Sigma\left(M_G\right)_A = \Sigma\left(I_G\right)_A\,\alpha;\qquad 30\left(10^3\right)(1.5) - 20\left(10^3\right)(1.5) = 195.25\left(10^3\right)\alpha$

$\alpha = 0.0768\ \text{rad/s}^2$ **Ans.**

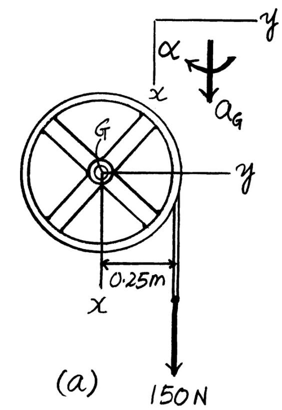
***17–96.** The 75-kg wheel has a radius of gyration about the z axis of $k_z = 150$ mm. If the belt of negligible mass is subjected to a force of $P = 150$ N, determine the acceleration of the mass center and the angular acceleration of the wheel. The surface is smooth and the wheel is free to slide.



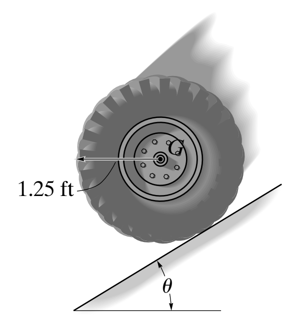
Equations of Motion: The mass moment of inertia of the wheel about the z axis is $(I_G)_z = mk_z^2 = 75(0.15^2) = 1.6875\ \text{kg}\cdot\text{m}^2$. Referring to the free-body diagram of the wheel shown in Fig. a, we have

$+\downarrow \Sigma F_x = m(a_G)_x;\qquad 150 = 75a_G \qquad a_G = 2\text{m/s}^2$ **Ans.**

$\circlearrowleft + \Sigma M_G = I_G\alpha;\qquad -150(0.25) = -1.6875\alpha \qquad \alpha = 22.22\ \text{rad/s}^2$ **Ans.**



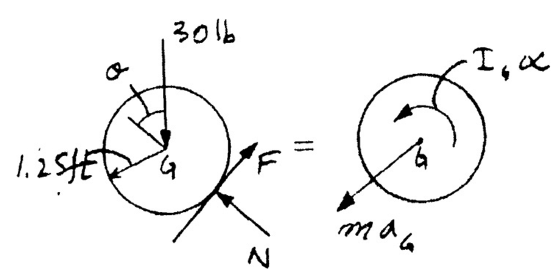
•17–97. The wheel has a weight of 30 lb and a radius of gyration of $k_G = 0.6$ ft. If the coefficients of static and kinetic friction between the wheel and the plane are $\mu_s = 0.2$ and $\mu_k = 0.15$, determine the wheel's angular acceleration as it rolls down the incline. Set $\theta = 12°$.



$+\swarrow \Sigma F_x = m(a_G)_x;\qquad 30\sin 12° - F = \left(\frac{30}{32.2}\right)a_G$

$+\nwarrow \Sigma F_y = m(a_G)_y;\qquad N - 30\cos 12° = 0$

$\circlearrowleft + \Sigma M_G = I_G\,\alpha;\qquad F(1.25) = \left[\left(\frac{30}{32.2}\right)(0.6)^2\right]\alpha$



Assume the wheel does not slip.

$$a_G = (1.25)\alpha$$

Solving:

$F = 1.17$ lb

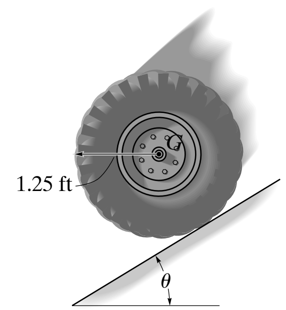
$N = 29.34$ lb

$a_G = 5.44\ \text{ft/s}^2$

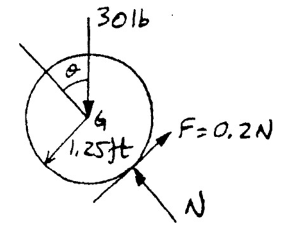
$\alpha = 4.35\ \text{rad/s}^2$ **Ans.**

$F_{max} = 0.2(29.34) = 5.87\ \text{lb} > 1.17\ \text{lb}$ **OK**

17–98. The wheel has a weight of 30 lb and a radius of gyration of $k_G = 0.6$ ft. If the coefficients of static and kinetic friction between the wheel and the plane are $\mu_s = 0.2$ and $\mu_k = 0.15$, determine the maximum angle θ of the inclined plane so that the wheel rolls without slipping.



Since wheel is on the verge of slipping:



$$+\swarrow\Sigma F_x = m(a_G)_x; \qquad 30\sin\theta - 0.2N = \left(\frac{30}{32.2}\right)(1.25\alpha) \qquad \textbf{(1)}$$

$$+\nwarrow\Sigma F_y = m(a_G)_y; \qquad N - 30\cos\theta = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft+\Sigma M_C = I_G\,\alpha; \qquad 0.2N(1.25) = \left[\left(\frac{30}{32.2}\right)(0.6)^2\right]\alpha \qquad \textbf{(3)}$$

Substituting Eqs.(2) and (3) into Eq. (1),

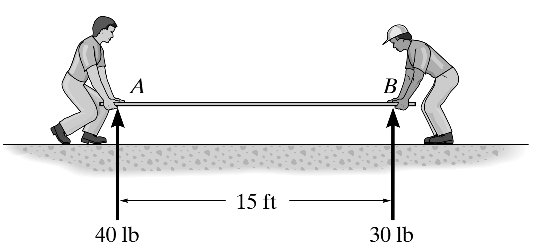
$$30\sin\theta - 6\cos\theta = 26.042\cos\theta$$

$$30\sin\theta = 32.042\cos\theta$$

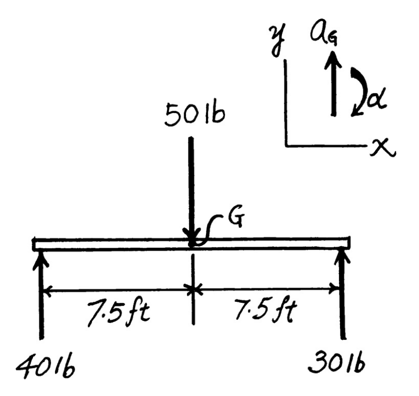
$$\tan\theta = 1.068$$

$$\theta = 46.9^\circ \qquad \textbf{Ans.}$$

17–99. Two men exert constant vertical forces of 40 lb and 30 lb at ends A and B of a uniform plank which has a weight of 50 lb. If the plank is originally at rest in the horizontal position, determine the initial acceleration of its center and its angular acceleration. Assume the plank to be a slender rod.



Equations of Motion: The mass moment of inertia of the plank about its mass center is given by $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{50}{32.2}\right)\left(15^2\right) = 29.115\ \text{slug}\cdot\text{ft}^2$ Applying Eq. 17–14, we have



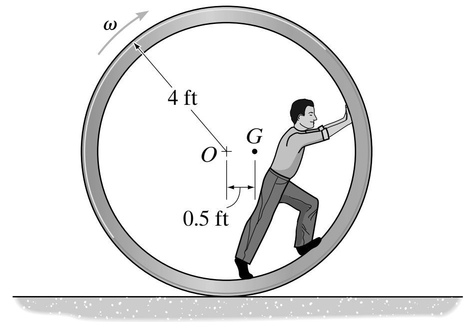
$$+\uparrow\Sigma F_y = m(a_G)_y; \qquad 40 + 30 - 50 = \left(\frac{50}{32.2}\right)a_G$$

$$a_G = 12.9\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$\circlearrowleft+\Sigma M_G = I_G\,\alpha; \qquad 30(7.5) - 40(7.5) = -29.115\,\alpha$$

$$\alpha = 2.58\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

***17–100.** The circular concrete culvert rolls with an angular velocity of $\omega = 0.5$ rad/s when the man is at the position shown. At this instant the center of gravity of the culvert and the man is located at point G, and the radius of gyration about G is $k_G = 3.5$ ft. Determine the angular acceleration of the culvert. The combined weight of the culvert and the man is 500 lb. Assume that the culvert rolls without slipping, and the man does not move within the culvert.



Equations of Motion: The mass moment of inertia of the system about its mass center is $I_G = mk_G{}^2 = \frac{500}{32.2}\left(3.5^2\right) = 190.22\ \text{slug}\cdot\text{ft}^2$. Writing the moment equation of motion about point A, Fig. a,

$$+\Sigma M_A = \Sigma(M_k)_A;\quad -500(0.5) = -\frac{500}{32.2}(a_G)_x(4) - \frac{500}{32.2}(a_G)_y(0.5) - 190.22\alpha \qquad \textbf{(1)}$$

Kinematics: Since the culvert rolls without slipping,

$$a_0 = \alpha r = \alpha(4) \rightarrow$$

Applying the relative acceleration equation and referring to Fig. b,

$$a_G = a_O + \alpha \times r_{G/O} - \omega^2\, \mathbf{r}_{G/A}$$

$$(a_G)_x\,\mathbf{i} - (a_G)_y\mathbf{j} = 4\alpha\mathbf{i} + (-\alpha\mathbf{k}) \times (0.5\mathbf{i}) - 0.5^2(0.5\mathbf{i})$$

$$(a_G)_x\,\mathbf{i} - (a_G)_y\,\mathbf{j} = (4\alpha - 0.125)\mathbf{i} - 0.5\alpha\mathbf{j}$$

Equating the **i** and **j** components,

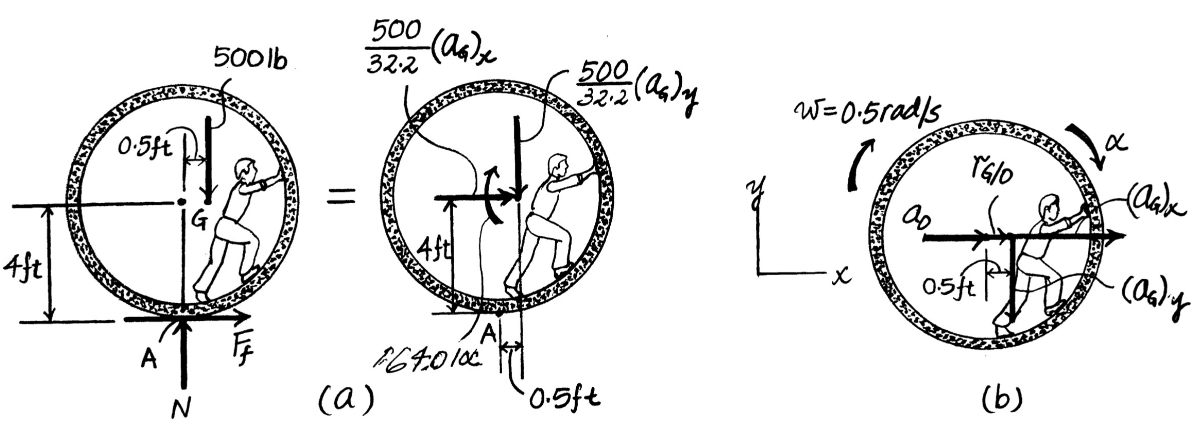
$$(a_G)_x = 4\alpha - 0.125 \qquad \textbf{(2)}$$

$$(a_G)_y = 0.5\alpha \qquad \textbf{(3)}$$

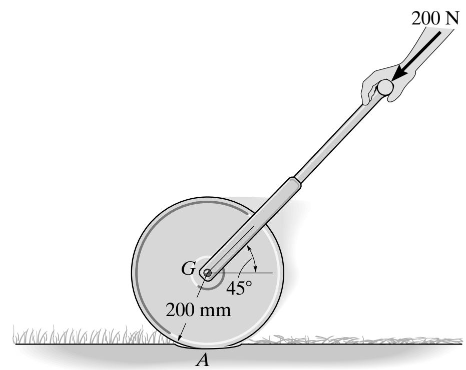
Substituting Eqs. (2) and (3) into Eq. (1),

$$-500(0.5) = -\frac{500}{32.2}(4\alpha - 0.125)(4) - \frac{500}{32.2}(0.5\alpha)(0.5) - 190.22\alpha$$

$$\alpha = 0.582\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



•17–101. The lawn roller has a mass of 80 kg and a radius of gyration $k_G = 0.175$ m. If it is pushed forward with a force of 200 N when the handle is at 45°, determine its angular acceleration. The coefficients of static and kinetic friction between the ground and the roller are $\mu_s = 0.12$ and $\mu_k = 0.1$, respectively.



$\stackrel{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 200 \cos 45° - F_A = 80a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A - 80(9.81) - 200 \sin 45° = 0$

$\circlearrowleft + \Sigma M_G = I_G\, \alpha; \qquad F_A(0.2) = 80(0.175)^2\, \alpha$

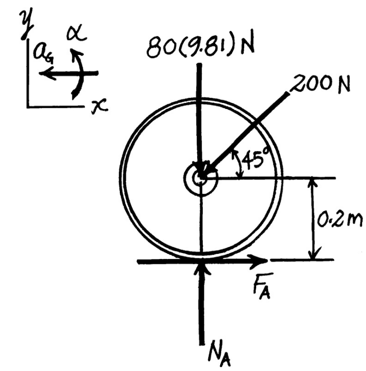
Assume no slipping: $a_G = 0.2\alpha$

$F_A = 61.32$ N

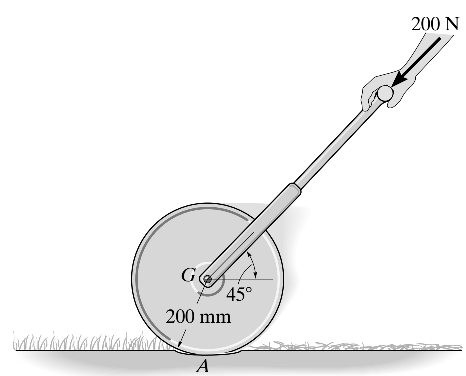
$N_A = 926.2$ N

$\alpha = 5.01 \text{ rad/s}^2$ **Ans.**

$(F_A)_{\max} = \mu_s N_A = 0.12(926.2) = 111.1 \text{ N} > 61.32 \text{ N}$ **OK**



17–102. Solve Prob. 17–101 if $\mu_s = 0.6$ and $\mu_k = 0.45$.



$\stackrel{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 200 \cos 45° - F_A = 80a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A - 80(9.81) - 200 \sin 45° = 0$

$\circlearrowleft + \Sigma M_G = I_G\, \alpha; \qquad F_A(0.2) = 80(0.175)^2\, \alpha$

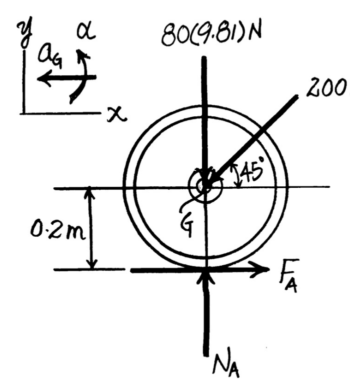
Assume no slipping: $a_G = 0.2\, \alpha$

$F_A = 61.32$ N

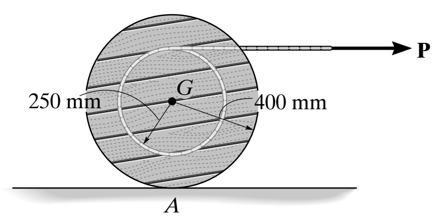
$N_A = 926.2$ N

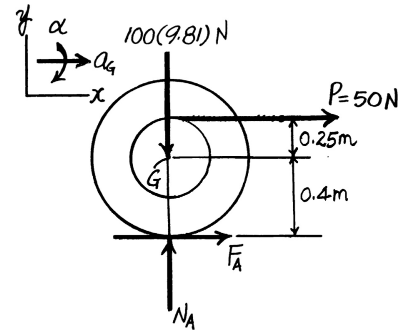
$\alpha = 5.01 \text{ rad/s}^2$ **Ans.**

$(F_A)_{\max} = \mu_s N_A = 0.6(926.2 \text{ N}) = 555.7 \text{ N} > 61.32 \text{ N}$ **OK**



17–103. The spool has a mass of 100 kg and a radius of gyration of $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 50$ N.





$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad 50 + F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A - 100(9.81) = 0$

$\circlearrowleft + \Sigma M_G = I_G\alpha;\qquad 50(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

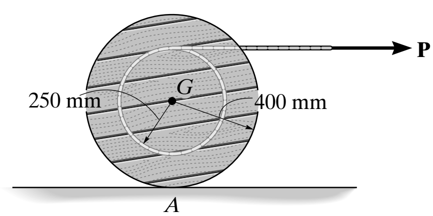
Assume no slipping: $a_G = 0.4\alpha$

$\alpha = 1.30\ \text{rad/s}^2$ **Ans.**

$a_G = 0.520\ \text{m/s}^2 \qquad N_A = 981\ \text{N} \qquad F_A = 2.00\ \text{N}$

Since $(F_A)_{\max} = 0.2(981) = 196.2\ \text{N} > 2.00\ \text{N}$ **OK**

***17–104.** Solve Prob. 17–103 if the cord and force $P = 50$ N are directed vertically upwards.

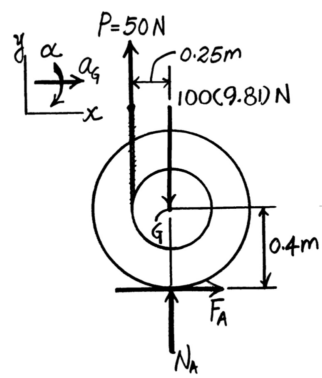


$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A + 50 - 100(9.81) = 0$

$\circlearrowleft + \Sigma M_G = I_G\,\alpha;\qquad 50(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping: $a_G = 0.4\,\alpha$

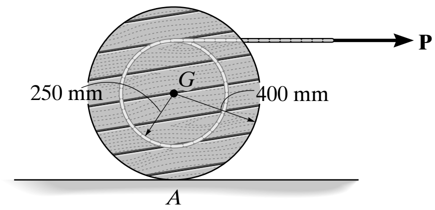


$\alpha = 0.500\ \text{rad/s}^2$ **Ans.**

$a_G = 0.2\ \text{m/s}^2 \qquad N_A = 931\ \text{N} \qquad F_A = 20\ \text{N}$

Since $(F_A)_{\max} = 0.2(931) = 186.2\ \text{N} > 20\ \text{N}$ **OK**

•17–105. The spool has a mass of 100 kg and a radius of gyration $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 600$ N.



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad 600 + F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A - 100(9.81) = 0$

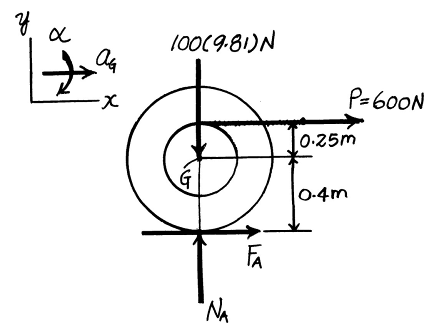
$\circlearrowleft + \Sigma M_G = I_G\,\alpha;\qquad 600(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping: $a_G = 0.4\alpha$

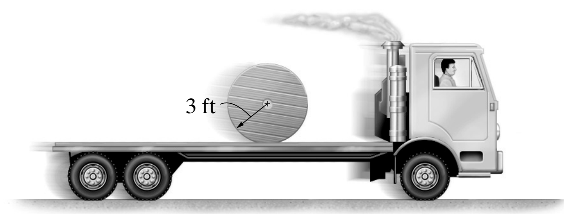
$$\alpha = 15.6 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$a_G = 6.24 \text{ m/s}^2 \qquad N_A = 981 \text{ N} \qquad F_A = 24.0 \text{ N}$$

Since $(F_A)_{max} = 0.2(981) = 196.2 \text{ N} > 24.0 \text{ N}$ **OK**

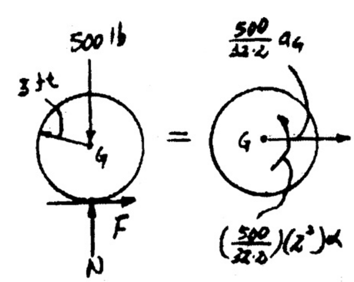


17–106. The truck carries the spool which has a weight of 500 lb and a radius of gyration of $k_G = 2$ ft. Determine the angular acceleration of the spool if it is not tied down on the truck and the truck begins to accelerate at 3 ft/s². Assume the spool does not slip on the bed of the truck.



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad F = \left(\dfrac{500}{32.2}\right)a_G \qquad \textbf{(1)}$

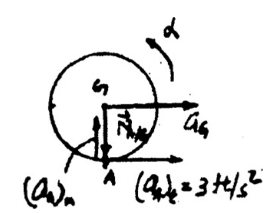
$\circlearrowleft + \ \Sigma M_G = I_G\,\alpha;\qquad F(3) = \left(\dfrac{500}{32.2}\right)(2)^2\,\alpha \qquad \textbf{(2)}$



$$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$$

$$\left[\underset{\rightarrow}{(a_A)_t}\right] + \left[\underset{\uparrow}{(a_A)_n}\right] = \left[\underset{\rightarrow}{a_G}\right] + \left[\underset{\rightarrow}{3\alpha}\right] + \left[\underset{\uparrow}{(a_{A/G})_n}\right]$$

$\left(\xrightarrow{+}\right)\quad 3 = a_G + 3\alpha \qquad \textbf{(3)}$

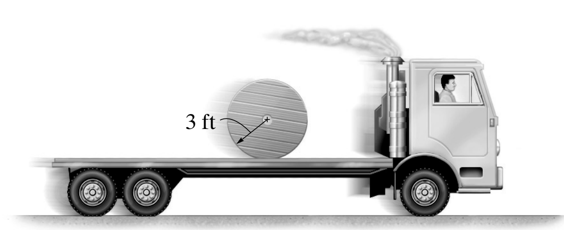


Solving Eqs. (1), (2), and (3) yields:

$$F = 14.33 \text{ lb} \qquad a_G = 0.923 \text{ ft/s}^2$$

$$\alpha = 0.692 \text{ rad/s}^2. \qquad \textbf{Ans.}$$

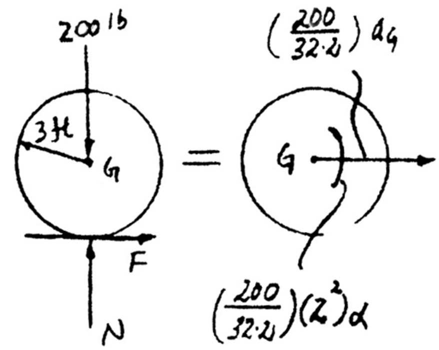
17–107. The truck carries the spool which has a weight of 200 lb and a radius of gyration of $k_G = 2$ ft. Determine the angular acceleration of the spool if it is not tied down on the truck and the truck begins to accelerate at 5 ft/s^2. The coefficients of static and kinetic friction between the spool and the truck bed are $\mu_s = 0.15$ and $\mu_k = 0.1$, respectively.



$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N - 200 = 0 \qquad N = 200 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;\qquad F = \left(\dfrac{200}{32.2}\right)a_G \qquad$ **(1)**

$\circlearrowleft + \Sigma M_G = I_G\alpha;\qquad F(3) = \left(\dfrac{200}{32.2}\right)(2)^2\alpha \qquad$ **(2)**

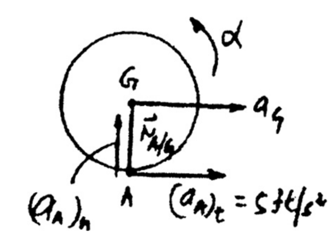


Assume no slipping occurs at the point of contact. Hence, $(a_A)_t = 5$ ft/s^2.

$$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$$

$$\begin{bmatrix}(a_A)_t\\ \rightarrow\end{bmatrix} + \begin{bmatrix}(a_A)_n\\ \uparrow\end{bmatrix} = \begin{bmatrix}a_G\\ \rightarrow\end{bmatrix} + \begin{bmatrix}3\alpha\\ \rightarrow\end{bmatrix} + \begin{bmatrix}(a_{A/G})_n\\ \uparrow\end{bmatrix}$$

$(\overset{+}{\rightarrow})\qquad 5 = a_G + 3\alpha \qquad$ **(3)**



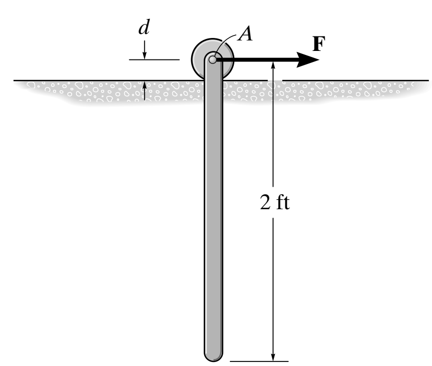
Solving Eqs. (1), (2), and (3) yields:

$$F = 9.556 \text{ lb} \qquad a_G = 1.538 \text{ ft/s}^2$$

$$\alpha = 1.15 \text{ rad/s}^2$$ **Ans.**

Since $F_{max} = (200 \text{ lb})(0.15) = 30 \text{ lb} > 9.556$ lb **OK**

***17–108.** A uniform rod having a weight of 10 lb is pin supported at A from a roller which rides on a horizontal track. If the rod is originally at rest, and a horizontal force of $F = 15$ lb is applied to the roller, determine the acceleration of the roller. Neglect the mass of the roller and its size d in the computations.

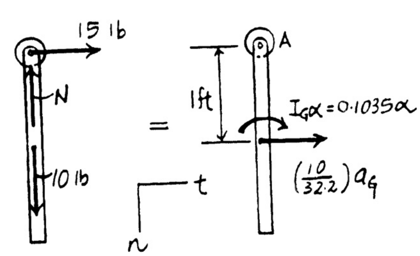


Equations of Motion: The mass moment of inertia of the rod about its mass center is given by $I_G = \dfrac{1}{12}ml^2 = \dfrac{1}{12}\left(\dfrac{10}{32.2}\right)(2^2) = 0.1035 \text{ slug}\cdot\text{ft}^2$. At the instant force **F** is applied, the angular velocity of the rod $\omega = 0$. Thus, the normal component of acceleration of the mass center for the rod $(a_G)_n = 0$. Applying Eq. 17–16, we have

$\Sigma F_t = m(a_G)_t;\qquad 15 = \left(\dfrac{10}{32.2}\right)a_G \quad a_G = 48.3 \text{ ft/s}^2$

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A;\qquad 0 = \left(\dfrac{10}{32.2}\right)(48.3)(1) - 0.1035\,\alpha$

$$\alpha = 144.9 \text{ rad/s}^2$$



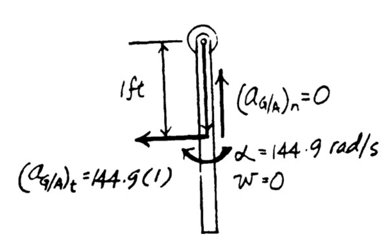
Kinematics: Since $\omega = 0$, $(a_{G/A})_n = 0$. The acceleration of roller A can be obtain by analyzing the motion of points A and G. Applying Eq. 16–17, we have

$$\mathbf{a}_G = \mathbf{a}_A + (\mathbf{a}_{G/A})_t + (\mathbf{a}_{G/A})_n$$

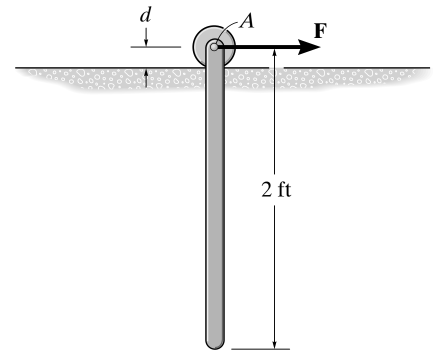
$$\begin{bmatrix}48.3\\ \rightarrow\end{bmatrix} = \begin{bmatrix}a_A\\ \rightarrow\end{bmatrix} + \begin{bmatrix}144.9(1)\\ \leftarrow\end{bmatrix} + \begin{bmatrix}0\end{bmatrix}$$

$(\overset{+}{\rightarrow})\qquad 48.3 = a_A - 144.9$

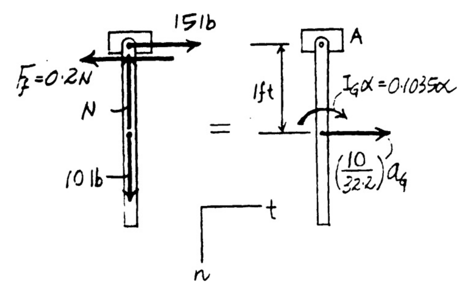
$$a_A = 193 \text{ ft/s}^2$$ **Ans.**



•17–109. Solve Prob. 17–108 assuming that the roller at A is replaced by a slider block having a negligible mass. The coefficient of kinetic friction between the block and the track is $\mu_k = 0.2$. Neglect the dimension d and the size of the block in the computations.



Equations of Motion: The mass moment of inertia of the rod about its mass center is given by $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{10}{32.2}\right)(2^2) = 0.1035 \text{ slug}\cdot\text{ft}^2$. At the instant force **F** is applied, the angular velocity of the rod $\omega = 0$. Thus, the normal component of acceleration of the mass center for the rod $(a_G)_n = 0$. Applying Eq. 17–16, we have



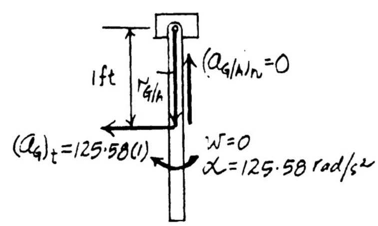
$$\Sigma F_n = m(a_G)_n; \qquad 10 - N = 0 \qquad N = 10.0 \text{ lb}$$

$$\Sigma F_t = m(a_G)_t; \qquad 15 - 0.2(10.0) = \left(\frac{10}{32.2}\right)a_G \qquad a_G = 41.86 \text{ ft/s}^2$$

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad 0 = \left(\frac{10}{32.2}\right)(41.86)(1) - 0.1035\alpha$$

$$\alpha = 125.58 \text{ rad/s}^2$$

Kinematics: Since $\omega = 0$, $(a_{G/A})_n = 0$. The acceleration of block A can be obtain by analyzing the motion of points A and G. Applying Eq. 16–17, we have



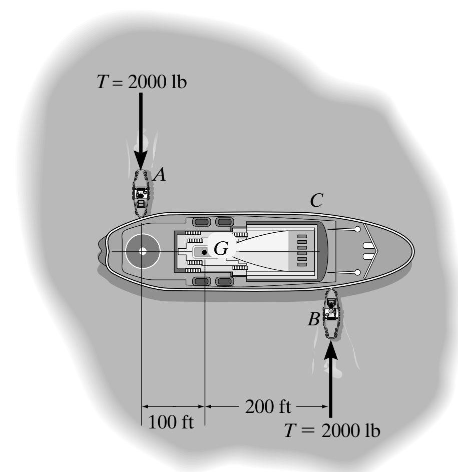
$$\mathbf{a}_G = \mathbf{a}_A + (\mathbf{a}_{G/A})_t + (\mathbf{a}_{G/A})_n$$

$$\left[\underset{\rightarrow}{41.86}\right] = \left[\underset{\rightarrow}{a_A}\right] + \left[\underset{\leftarrow}{125.58(1)}\right] + [0]$$

$$(\overset{+}{\rightarrow}) \qquad 41.86 = a_A - 125.58$$

$$a_A = 167 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

17–110. The ship has a weight of $4(10^6)$ lb and center of gravity at G. Two tugboats of negligible weight are used to turn it. If each tugboat pushes on it with a force of $T = 2000$ lb, determine the initial acceleration of its center of gravity G and its angular acceleration. Its radius of gyration about its center of gravity is $k_G = 125$ ft. Neglect water resistance.



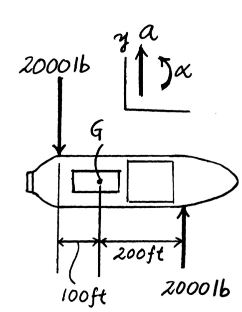
Equations of Motion: Here, the mass moment of inertia of the ship about its mass center is $I_G = mk_G{}^2 = \frac{4(10^6)}{32.2}(125^2) = 1.941(10^9) \text{ slug}\cdot\text{ft}^2$. Referring to the free-body diagrams of the ship shown in Fig. a,

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2000 - 2000 = \frac{4(10^6)}{32.2}a$$

$$a = 0 \qquad \textbf{Ans.}$$

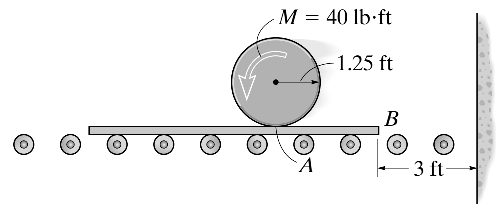
$$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad 2000(100) + 2000(200) = 1.941(10^9)\alpha$$

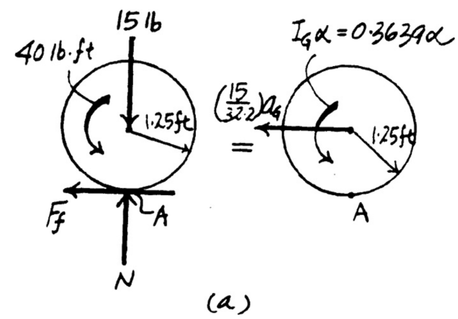
$$\alpha = 0.30912(10^{-3}) \text{ rad/s}^2 = 0.309(10^{-3}) \text{ rad/s}^2 \qquad \textbf{Ans.}$$



(a)

17–111. The 15-lb cylinder is initially at rest on a 5-lb plate. If a couple moment $M = 40 \text{ lb} \cdot \text{ft}$ is applied to the cylinder, determine the angular acceleration of the cylinder and the time needed for the end B of the plate to travel 3 ft to the right and strike the wall. Assume the cylinder does not slip on the plate, and neglect the mass of the rollers under the plate.



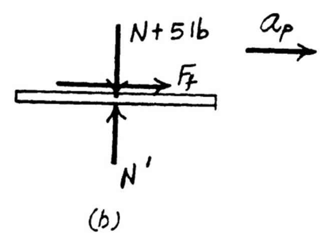


Equation of Motions: The mass moment of inertia of the cylinder about its mass center is given by $I_G = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{15}{32.2}\right)(1.25^2) = 0.3639 \text{ slug} \cdot \text{ft}^2$. Applying Eq. 17–16 to the cylinder [FBD(a)], we have

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad -40 = -\left(\frac{15}{32.2}\right)a_G(1.25) - 0.3639\alpha \qquad \textbf{(1)}$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_f = \left(\frac{15}{32.2}\right)a_G \qquad \textbf{(2)}$$

Applying the equation of motion to the place [FBD(b)], we have



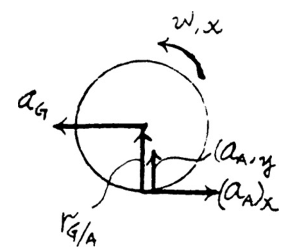
$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad F_f = \left(\frac{5}{32.2}\right)a_P \qquad \textbf{(3)}$$

Kinematics: Analyzing the motion of points G and A by applying Eq. 16–18 with $\mathbf{r}_{G/A} = \{1.25\mathbf{j}\}$ ft, we have

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$-a_G\mathbf{i} = (a_A)_x\,\mathbf{i} + (a_A)_y\,\mathbf{j} + \alpha\mathbf{k} \times (1.25\mathbf{j}) - \omega^2(1.25\mathbf{j})$$

$$-a_G\mathbf{i} = \left[(a_A)_x - 1.25\alpha\right]\mathbf{i} + \left[(a_A)_y - 1.25\,\omega^2\right]\mathbf{j}$$



Equating **i** components, we have

$$a_G = 1.25\alpha - (a_A)_x \qquad \textbf{(4)}$$

Since the cylinder rolls without slipping on the plate, then $a_P = (a_A)_x$. Substitute into Eq. (4) yields

$$a_G = 1.25\,\alpha - a_P \qquad \textbf{(5)}$$

Solving Eqs. (1), (2), (3), and (5) yields:

$$\alpha = 73.27 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$a_G = 22.90 \text{ ft/s}^2 \qquad a_P = 68.69 \text{ ft/s}^2 \qquad F_f = 10.67 \text{ lb}$$

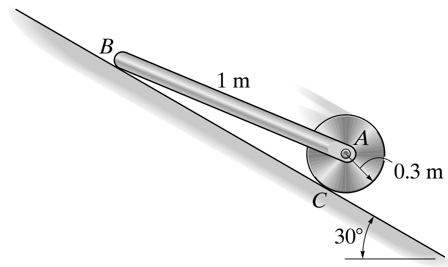
The time required for the plate to travel 3 ft is given by

$$s = s_o + v_o\,t + \frac{1}{2}a_P\,t^2$$

$$3 = 0 + 0 + \frac{1}{2}(68.69)t^2$$

$$t = 0.296 \text{ s} \qquad \textbf{Ans.}$$

***17–112.** The assembly consists of an 8-kg disk and a 10-kg bar which is pin connected to the disk. If the system is released from rest, determine the angular acceleration of the disk. The coefficients of static and kinetic friction between the disk and the inclined plane are $\mu_s = 0.6$ and $\mu_k = 0.4$, respectively. Neglect friction at B.



Equation of Motions:

Disk:

$+\searrow\Sigma F_x = m(a_G)_x;\qquad A_x - F_C + 8(9.81)\sin 30° = 8a_G$ **(1)**

$+\nearrow\Sigma F_y = m(a_G)_y;\qquad N_C - A_y - 8(9.81)\cos 30° = 0$ **(2)**

$\circlearrowleft +\Sigma M_A = I_A\alpha;\qquad F_C(0.3) = \left[\frac{1}{2}(8)(0.3)^2\right]\alpha$ **(3)**

Bar:

$+\searrow\Sigma F_x = m(a_G)_x;\qquad 10(9.81)\sin 30° - A_x = 10a_G$ **(4)**

$+\nearrow\Sigma F_y = m(a_G)_y;\qquad N_B + A_y - 10(9.81)\cos 30° = 0$ **(5)**

$\circlearrowleft +\Sigma M_G = I_G\alpha;\qquad -N_B(0.5\cos 17.46°) + A_x(0.5\sin 17.46°)$

$+ A_y(0.5\cos 17.46°) = 0$ **(6)**

Kinematics: Assume no slipping of the disk:

$$a_G = 0.3\alpha$$ **(7)**

Solving Eqs. (1) through (7):

$A_x = 8.92\text{ N}\qquad A_y = 41.1\text{ N}\qquad N_B = 43.9\text{ N}$

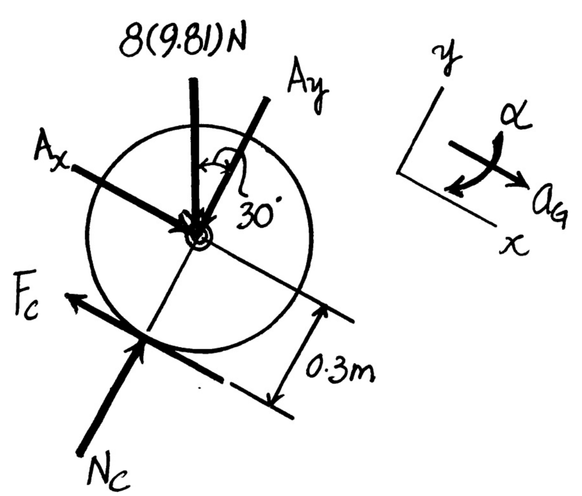
$a_G = 4.01\text{ m/s}^2$

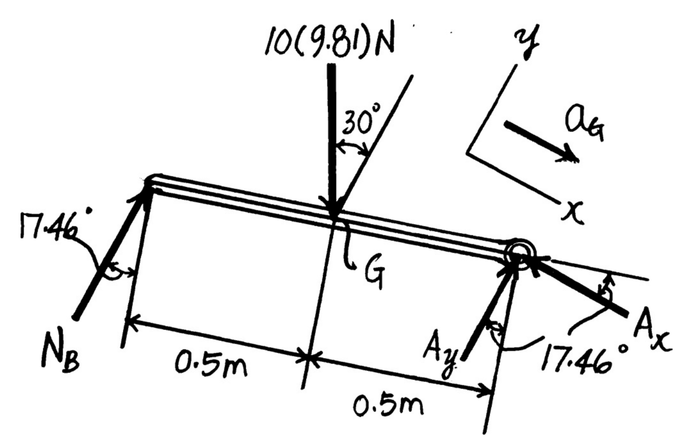
$\alpha = 13.4\text{ rad/s}^2$ **Ans.**

$N_C = 109\text{ N}$

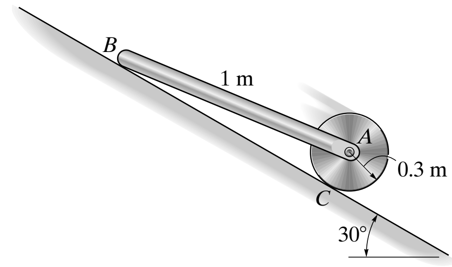
$F_C = 16.1\text{ N}$

$(F_C)_{max} = 0.6(109) = 65.4\text{ N} > 16.1\text{ N}$ **OK**





•17–113. Solve Prob. 17–112 if the bar is removed. The coefficients of static and kinetic friction between the disk and inclined plane are $\mu_s = 0.15$ and $\mu_k = 0.1$, respectively.



Equation of Motions:

$+\searrow\Sigma F_x = m(a_G)_x;\quad 8(9.81)\sin 30° - F_C = 8a_G$ **(1)**

$+\nearrow\Sigma F_y = m(a_G)_y;\quad -8(9.81)\cos 30° + N_C = 0$ **(2)**

$\circlearrowright +\Sigma M_G = I_G\alpha;\quad F_C\,(0.3) = \left[\frac{1}{2}(8)(0.3)^2\right]\alpha$ **(3)**

Kinematics: Assume no slipping: $a_G = 0.3\alpha$

Solving Eqs. (1)–(3):

$$N_C = 67.97\text{ N}$$

$$a_G = 3.27\text{ m/s}^2$$

$$\alpha = 10.9\text{ rad/s}^2$$

$$F_C = 13.08\text{ N}$$

$$(F_C)_{\max} = 0.15(67.97) = 10.2\text{ N} < 13.08\text{ N}$$ **NG**

Slipping occurs:

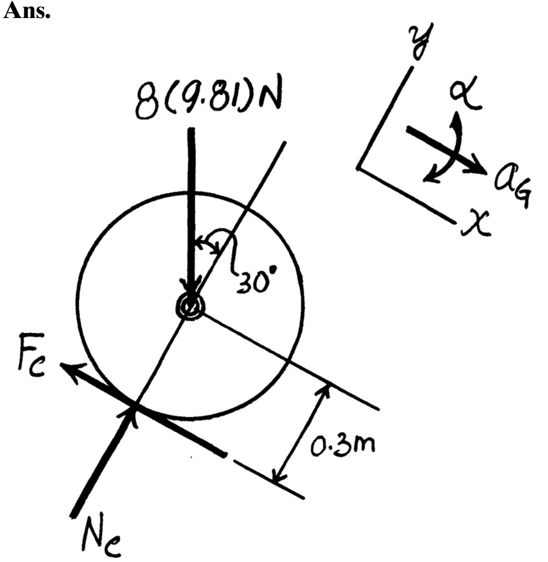
$$F_C = 0.1N_C$$

Solving Eqs. (1) through (3):

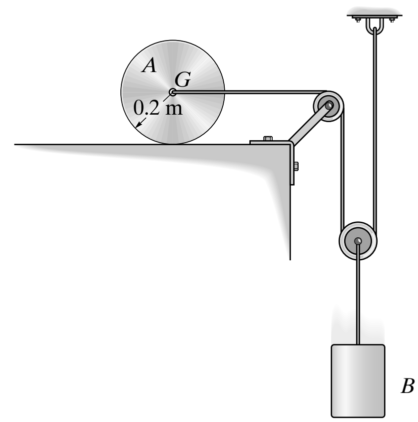
$$N_C = 67.97\text{ N}$$

$$\alpha = 5.66\text{ rad/s}^2$$ **Ans.**

$$a_G = 4.06\text{ m/s}^2$$



17–114. The 20-kg disk A is attached to the 10-kg block B using the cable and pulley system shown. If the disk rolls without slipping, determine its angular acceleration and the acceleration of the block when they are released. Also, what is the tension in the cable? Neglect the mass of the pulleys.



Equation of Motions:

Disk:

$$\circlearrowleft +\Sigma M_{IC} = \Sigma(M_k)_{IC}; \qquad T(0.2) = -\left[\frac{1}{2}(20)(0.2)^2 + 20(0.2)^2\right]\alpha \qquad \textbf{(1)}$$

Block:

$$+\downarrow \Sigma F_y = m(a_G)_y; \qquad 10(9.81) - 2T = 10a_B \qquad \textbf{(2)}$$

Kinematics:

$$2s_B + s_A = l$$

$$2a_B = -a_A$$

Also,

$$a_A = 0.2\alpha$$

Thus,

$$a_B = -0.1\alpha \qquad \textbf{(3)}$$

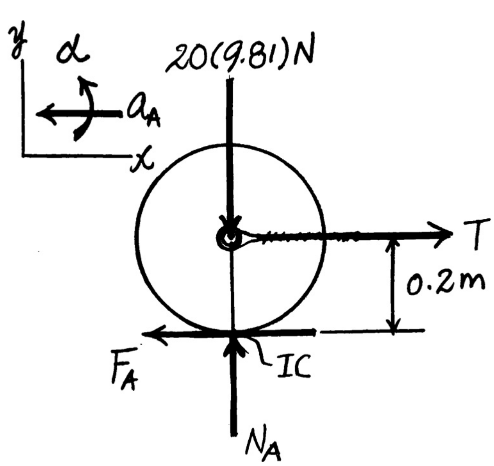
Note the direction for α and a_B are the same for all equations.

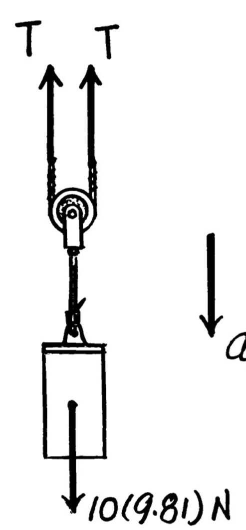
Solving Eqs. (1) through (3):

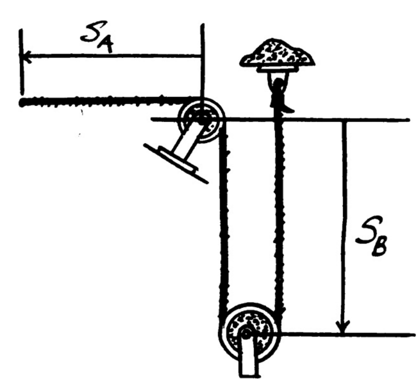
$$a_B = 0.755 \text{ m/s}^2 = 0.755 \text{ m/s}^2 \downarrow \qquad \textbf{Ans.}$$

$$\alpha = -7.55 \text{ rad/s}^2 = 7.55 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

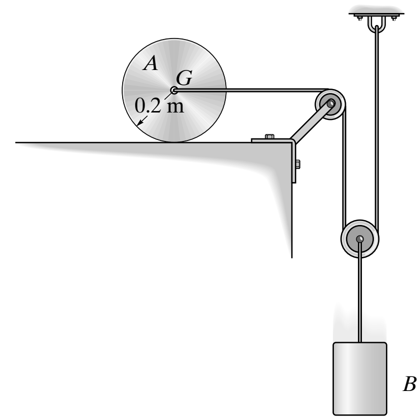
$$T = 45.3 \text{ N} \qquad \textbf{Ans.}$$







17–115. Determine the minimum coefficient of static friction between the disk and the surface in Prob. 17–114 so that the disk will roll without slipping. Neglect the mass of the pulleys.



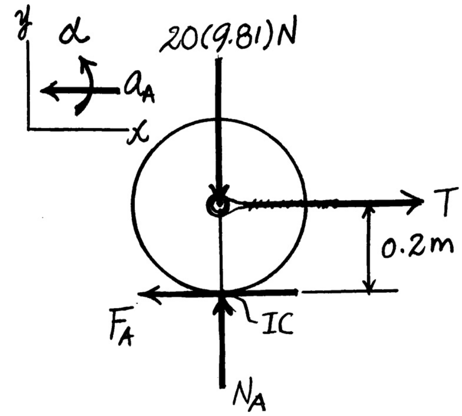
Equation of Motions:

Disk:

$$\circlearrowleft + \Sigma M_{IC} = \Sigma (M_k)_{IC}; \quad T(0.2) = -\left[\frac{1}{2}(20)(0.2)^2 + 20(0.2)^2\right]\alpha \qquad \textbf{(1)}$$

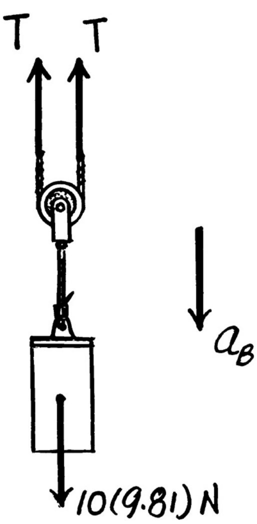
$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad -T + F_A = 20a_A$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A - 20(9.81) = 0$$



Block:

$$+\downarrow \Sigma F_y = m(a_G)_y; \qquad 10(9.81) - 2T = 10a_B \qquad \textbf{(2)}$$



Kinematics:

$$2s_B + s_A = l$$

$$2a_B = -a_A$$

Also,

$$a_A = 0.2\alpha$$

Thus,

$$a_B = -0.1\alpha \qquad \textbf{(3)}$$

Note the direction for α and a_B are the same for all equations. Solving Eqs. (1) through (3):

$$a_B = 0.755 \text{ m/s}^2$$

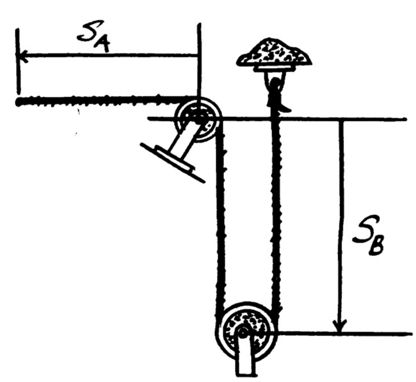
$$\alpha = -7.55 \text{ rad/s}^2$$

$$T = 45.3 \text{ N}$$

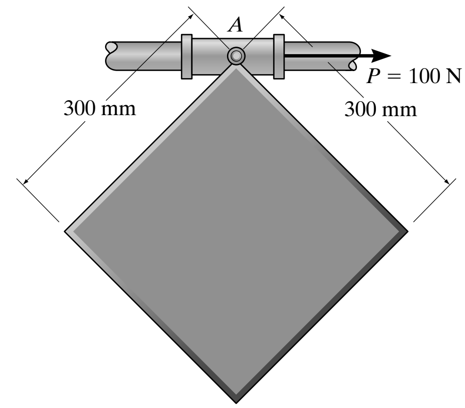
Also,

$$a_A = 0.2(-7.55) = -1.509 \text{ m/s}^2, \qquad N_A = 196.2 \text{ N}, \qquad F_A = 15.09 \text{ N}$$

$$\mu_{\min} = \frac{15.09}{196.2} = 0.0769 \qquad \textbf{Ans.}$$



***17–116.** The 20-kg square plate is pinned to the 5-kg smooth collar. Determine the initial angular acceleration of the plate when $P = 100$ N is applied to the collar. The plate is originally at rest.



Equations of Motion: The mass moment of inertia of the plate about its mass center is $I_G = \frac{1}{12}m\left(a^2 + b^2\right) = \frac{1}{12}(20)\left(0.3^2 + 0.3^2\right) = 0.3 \text{ kg} \cdot \text{m}^2$.

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad 100 = 5a_A + 20(a_G)_x \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = \Sigma(\mu_k)_A; \qquad 0 = 20(a_G)_x(0.3 \sin 45°) - 0.3\alpha \qquad \textbf{(2)}$$

Kinematics: Applying the relative acceleration equation and referring to Fig. b,

$$a_G = a_A + \alpha \times \mathbf{r}_{G/A} - \omega^2 \mathbf{r}_{G/A}$$

$$(a_G)_x \mathbf{i} + (a_G)_y \mathbf{j} = a_A\mathbf{i} + (-\alpha\mathbf{k}) \times (-0.3 \sin 45° \,\mathbf{j}) - 0$$

$$(a_G)_x \mathbf{i} + (a_G)_y \mathbf{j} = (a_A - 0.2121\alpha)\mathbf{i}$$

Equating the **i** and **j** components,

$$(a_G)_x = a_A - 0.2121\alpha \qquad \textbf{(3)}$$

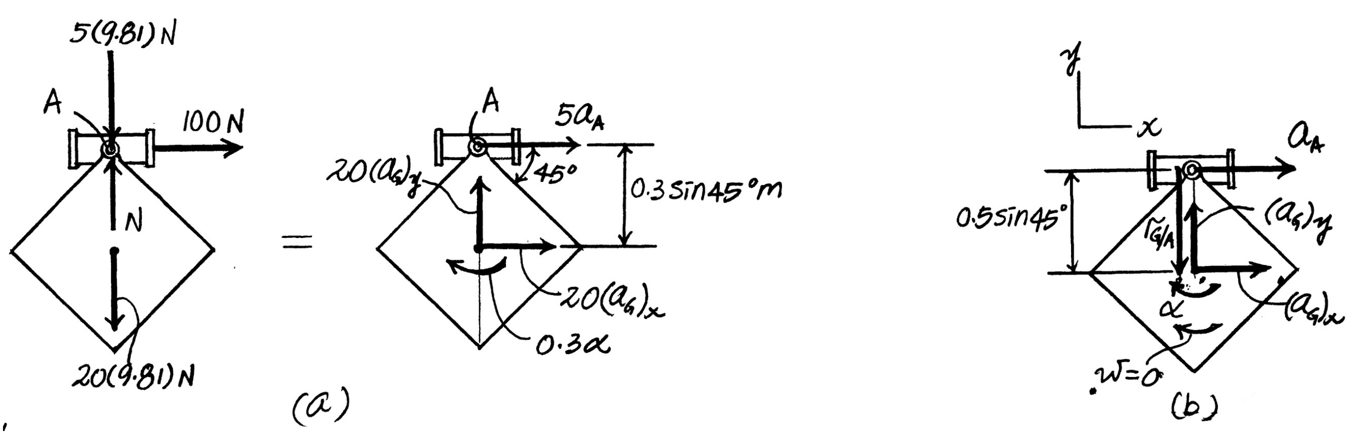
$$(a_G)_y = 0$$

Solving Eqs. (1) through (3) yields:

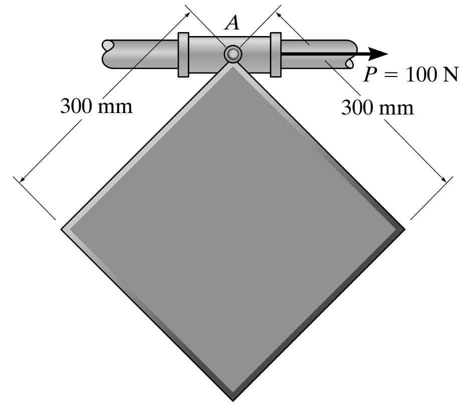
$$a_A = 10 \text{ m/s}^2 \rightarrow$$

$$(a_G)_x = 2.5 \text{ m/s}^2 \rightarrow$$

$$\alpha = 35.4 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



•17–117. The 20-kg square plate is pinned to the 5-kg smooth collar. Determine the initial acceleration of the collar when $P = 100$ N is applied to the collar. The plate is originally at rest.



Equations of Motion: The mass moment of inertia of the plate about its mass center is $I_G = \frac{1}{12}m\left(a^2 + b^2\right) = \frac{1}{12}(20)\left(0.3^2 + 0.3^2\right) = 0.3\ \text{kg}\cdot\text{m}^2$.

$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x;\qquad 100 = 5a_x + 20(a_G)_x$ **(1)**

$\circlearrowleft +\Sigma M_A = \Sigma(\mu_k)_A;\qquad 0 = 20(a_G)_x(0.3\sin 45°) - 0.3\alpha$ **(2)**

Kinematics: Applying the relative acceleration equation and referring to Fig. *b*,

$$a_G = a_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$(a_G)_x\,\mathbf{i} + (a_G)_y\,\mathbf{j} = a_A\mathbf{i} + (-\alpha\mathbf{k}) \times (-0.3\sin 45°\mathbf{j}) - 0$$

$$(a_G)_x\,\mathbf{i} + (a_G)_y\,\mathbf{j} = (a_A - 0.2121\alpha)\mathbf{i}$$

Equating the **i** and **j** components,

$$(a_G)_x = a_A - 0.2121\alpha \qquad \textbf{(3)}$$

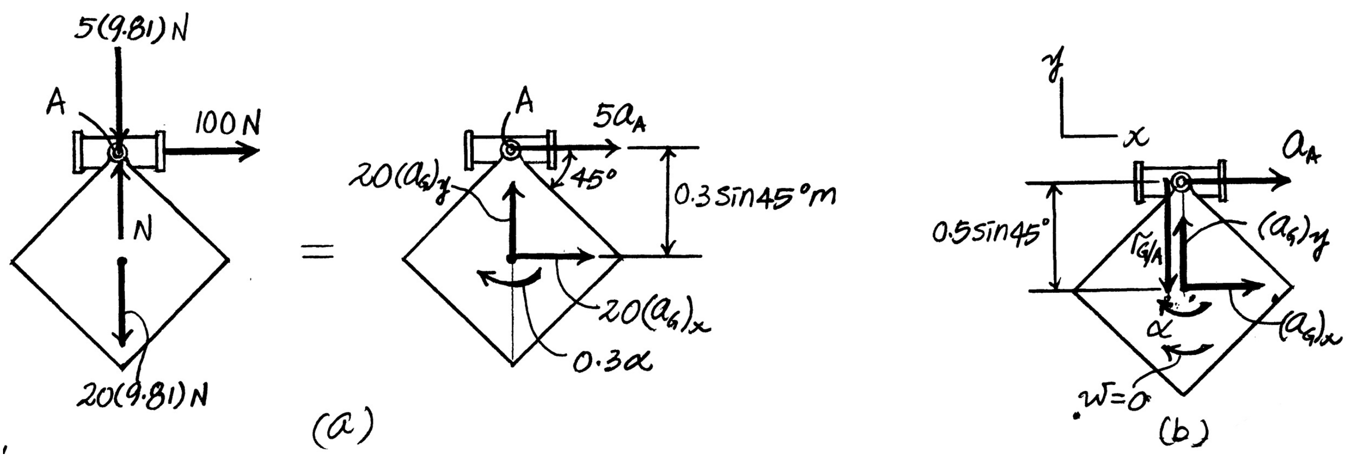
$$(a_G)_y = 0$$

Solving Eqs. (1) through (3) yields

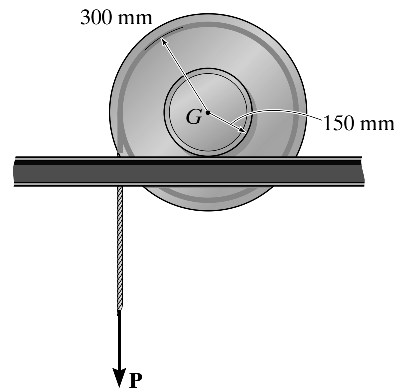
$$a_A = 10\ \text{m/s}^2 \rightarrow \qquad \textbf{Ans.}$$

$$(a_G)_x = 2.5\ \text{m/s}^2 \rightarrow$$

$$\alpha = 35.4\ \text{rad/s}^2$$



17–118. The spool has a mass of 100 kg and a radius of gyration of $k_G = 200$ mm about its center of mass G. If a vertical force of $P = 200$ N is applied to the cable, determine the acceleration of G and the angular acceleration of the spool. The coefficients of static and kinetic friction between the rail and the spool are $\mu_s = 0.3$ and $\mu_k = 0.25$, respectively.



Equations of Motion: The mass moment of inertia of the spool about its mass center is $I_G = mk_G{}^2 = 100\left(0.2^2\right) = 4\ \text{kg}\cdot\text{m}^2$.

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_f = 100a_G \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - 100(9.81) - 200 = 0 \qquad N = 1181\ \text{N}$$

$$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad 200(0.3) - F_f(0.15) = 4\alpha \qquad \textbf{(2)}$$

Kinematics: Assuming that the spool rolls without slipping on the rail,

$$a_G = \alpha r_G = \alpha(0.15) \qquad \textbf{(3)}$$

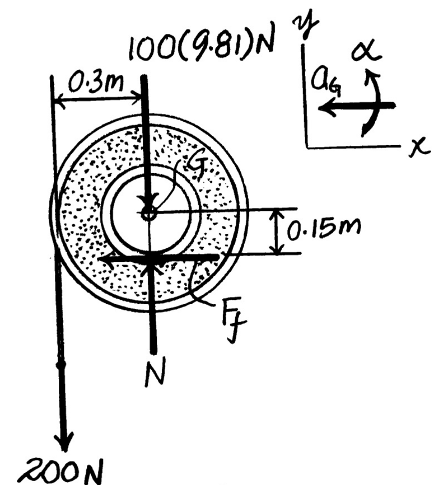
Solving Eqs. (1) through (3) yields:

$$\alpha = 9.60\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

$$a_G = 1.44\ \text{m/s}^2 \leftarrow \qquad \textbf{Ans.}$$

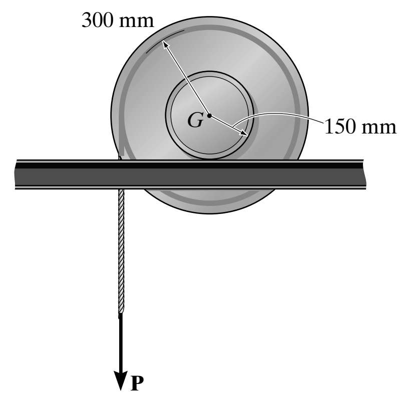
$$F_f = 144\ \text{N}$$

Since $F_f < \mu_k N = 0.3(1181) = 354.3$ N, the spool does not slip as assumed.



(a)

17–119. The spool has a mass of 100 kg and a radius of gyration of $k_G = 200$ mm about its center of mass G. If a vertical force of $P = 500$ N is applied to the cable, determine the acceleration of G and the angular acceleration of the spool. The coefficients of static and kinetic friction between the rail and the spool are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively.



Equations of Motion: The mass moment of inertia of the spool about its mass center is $I_G = mk_G{}^2 = 100\left(0.2^2\right) = 4 \text{ kg}\cdot\text{m}^2$.

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_f = 100\,a_G \qquad \textbf{(1)}$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - 100(9.81) - 500 = 0 \qquad N = 1481 \text{ N}$

$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad 500(0.3) - F_f(0.15) = 4\alpha \qquad \textbf{(2)}$

Kinematics: Assuming that the spool rolls without slipping on the rail,

$$a_G = \alpha r_G = \alpha(0.15) \qquad \textbf{(3)}$$

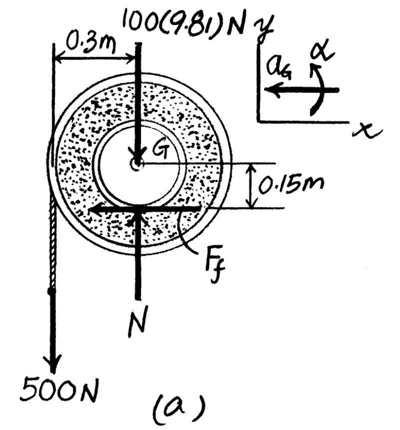
Solving Eqs. (1) through (3) yields:

$\alpha = 24 \text{ rad/s}^2 \qquad a_G = 3.6 \text{ m/s}^2 \qquad F_f = 360 \text{ N}$

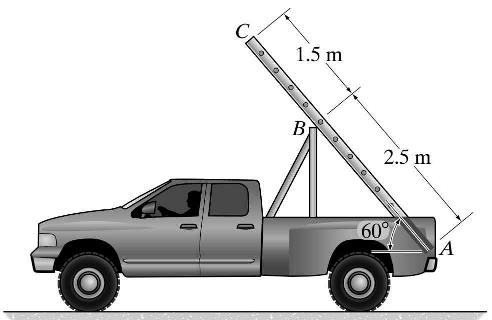
Since $F_f > \mu_k N = 0.2(1481) = 296.2$ N, the spool slips. Thus, the solution must be reworked using $F_f = \mu_k N = 0.15(1481) = 222.15$ N. Substituting this result into Eqs. (1) and (2),

$222.15 = 100a_G \qquad a_G = 2.22 \text{ m/s}^2 \leftarrow$ **Ans.**

$500(0.3) - 222.15(0.15) = 4\alpha \qquad \alpha = 29.17 \text{ rad/s}^2 = 29.2 \text{ rad/s}^2$ **Ans.**



***17–120.** If the truck accelerates at a constant rate of $6\ \text{m/s}^2$, starting from rest, determine the initial angular acceleration of the 20-kg ladder. The ladder can be considered as a uniform slender rod. The support at B is smooth.



Equations of Motion: We must first show that the ladder will rotate when the acceleration of the truck is $6\ \text{m/s}^2$. This can be done by determining the minimum acceleration of the truck that will cause the ladder to lose contact at B, $N_B = 0$. Writing the moment equation of motion about point A using Fig. *a*,

$$\circlearrowleft +\Sigma M_A = \Sigma(M_k)_A; \qquad 20(9.81)\cos 60°(2) = 20a_{\text{min}}\,(2\sin 60°)$$

$$a_{\text{min}} = 5.664\ \text{m/s}^2$$

Since $a_{\text{min}} < 6\ \text{m/s}^2$, the ladder will in the fact rotate. The mass moment of inertia about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(20)\left(4^2\right) = 26.67\ \text{kg}\cdot\text{m}^2$. Referring to Fig. *b*,

$$\circlearrowleft +\Sigma M_A = \Sigma(M_k)_A; \qquad 20(9.81)\cos 60°(2) = -20(a_G)_x(2\sin 60°)$$

$$-\ 20(a_G)_y(2\cos 60°) - 26.67\alpha \qquad \textbf{(1)}$$

Kinematics: The acceleration of A is equal to that of the truck. Thus, $a_A = 6\ \text{m/s}^2 \leftarrow$. Applying the relative acceleration equation and referring to Fig. *c*,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -6\mathbf{i} + (-\alpha\mathbf{k}) \times (-2\cos 60°\,\mathbf{i} + 2\sin 60°\,\mathbf{j}) - \mathbf{0}$$

$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = (2\sin 60°\,\alpha - 6)\mathbf{i} + \alpha\mathbf{j}$$

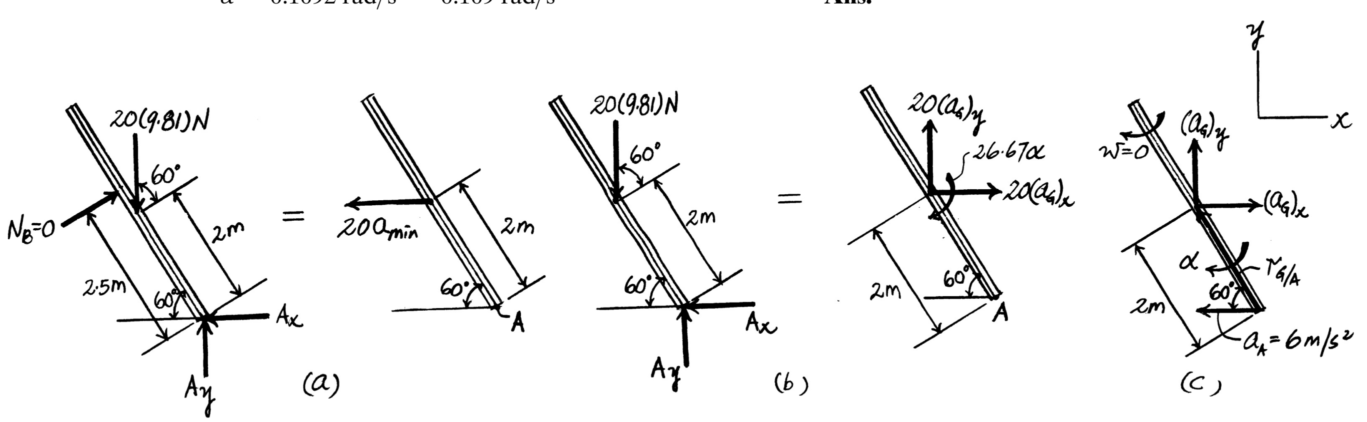
Equating the **i** and **j** components,

$$(a_G)_x = 2\sin 60°\,\alpha - 6 \qquad \textbf{(2)}$$

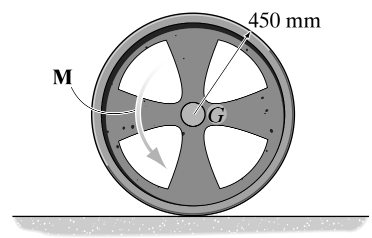
$$(a_G)_y = \alpha \qquad \textbf{(3)}$$

Substituting Eqs. (2) and (3) into Eq. (1),

$$\alpha = 0.1092\ \text{rad/s}^2 = 0.109\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



•**17–121.** The 75-kg wheel has a radius of gyration about its mass center of $k_G = 375$ mm. If it is subjected to a torque of $M = 100\ \text{N}\cdot\text{m}$, determine its angular acceleration. The coefficients of static and kinetic friction between the wheel and the ground are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively.



Equations of Motion: The mass moment of inertia of the wheel about its mass center is $I_G = mk_G{}^2 = 75\left(0.375^2\right) = 10.55\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about point A,

$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \quad 100 = 75a_G(0.45) + 10.55\alpha \qquad \textbf{(1)}$$

Assuming that the wheel rolls without slipping.

$$a_G = \alpha r_G = \alpha(0.45) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$\alpha = 3.886\ \text{rad/s}^2 = 3.89\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

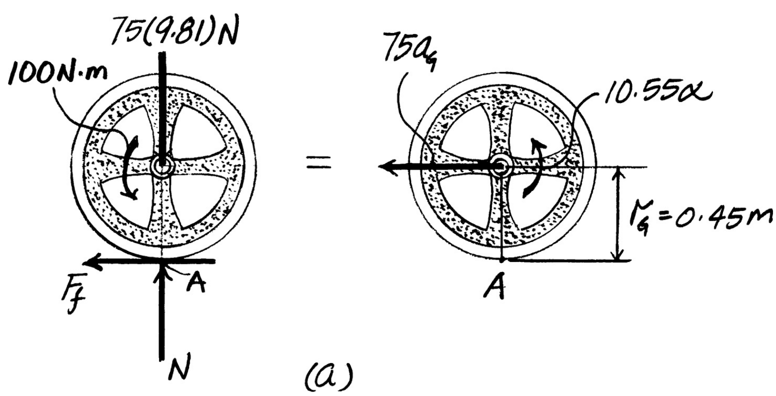
$$a_G = 1.749\ \text{m/s}^2$$

Writing the force equation of motion along the x and y axes,

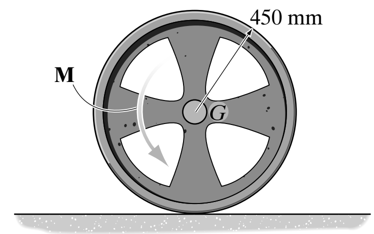
$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N - 75(9.81) = 0 \qquad N = 735.75\ \text{N}$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad F_f = 75(1.749) = 131.15\ \text{N}$$

Since $F_f < \mu_k N = 0.2(735.75) = 147.15\ \text{N}$, the wheel does not slip as assumed.



17–122. The 75-kg wheel has a radius of gyration about its mass center of $k_G = 375$ mm. If it is subjected to a torque of $M = 150\ \text{N}\cdot\text{m}$, determine its angular acceleration. The coefficients of static and kinetic friction between the wheel and the ground are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively.



Equations of Motion: The mass moment of inertia of the wheel about its mass center is $I_G = mk_G{}^2 = 75\left(0.375^2\right) = 10.55\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about point A, we have

$$\circlearrowleft +\Sigma M_A = \Sigma(\mu_k)_A; \quad 150 = 75a_G(0.45) + 10.55\alpha \qquad \textbf{(1)}$$

Assuming that the wheel rolls without slipping,

$$a_G = \alpha r_G = \alpha(0.45) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$a_G = 2.623\ \text{m/s}^2$$

$$\alpha = 5.829\ \text{rad/s}^2$$

Writing the force equations of motion along the x and y axes,

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - 75(9.81) = 0 \qquad N = 735.75\ \text{N}$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_f = 75(2.623) = 196.72\ \text{N}$$

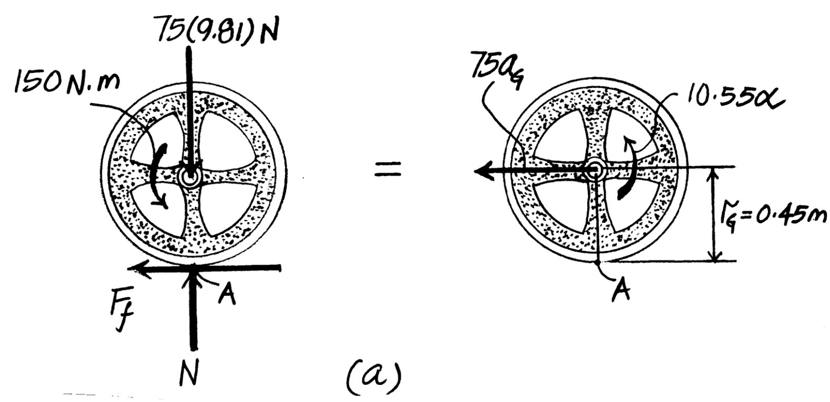
Since $F_f > \mu_k N = 0.2(735.75) = 147.15$ N, the wheel slips. The solution must be reworked using $F_f = \mu_k N = 0.15(735.75) = 110.36$ N. Thus,

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 110.36 = 75a_G \qquad a_G = 1.4715\ \text{m/s}^2$$

Substituting this result into Eq. (1), we obtain

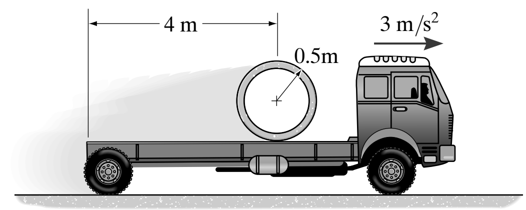
$$150 = 75(1.4715)(0.45) + 10.55\alpha$$

$$\alpha = 9.513\ \text{rad/s}^2 = 9.51\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



(a)

17–123. The 500-kg concrete culvert has a mean radius of 0.5 m. If the truck has an acceleration of $3\,\mathrm{m/s^2}$, determine the culvert's angular acceleration. Assume that the culvert does not slip on the truck bed, and neglect its thickness.



Equations of Motion: The mass moment of inertia of the culvert about its mass center is $I_G = mr^2 = 500\left(0.5^2\right) = 125\ \mathrm{kg \cdot m^2}$. Writing the moment equation of motion about point A using Fig. a,

$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \quad 0 = 125\alpha - 500a_G(0.5) \qquad \textbf{(1)}$$

Kinematics: Since the culvert does not slip at A, $(a_A)_t = 3\ \mathrm{m/s^2}$. Applying the relative acceleration equation and referring to Fig. b,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2 r_{G/A}$$

$$a_G\mathbf{i} - 3\mathbf{i} + (a_A)_n\,\mathbf{j} + (\alpha\mathbf{k} \times 0.5\mathbf{j}) - \omega^2(0.5\mathbf{j})$$

$$a_G\mathbf{i} = (3 - 0.5\alpha)\mathbf{i} + \left[(a_A)_n - 0.5\omega^2\right]\mathbf{j}$$

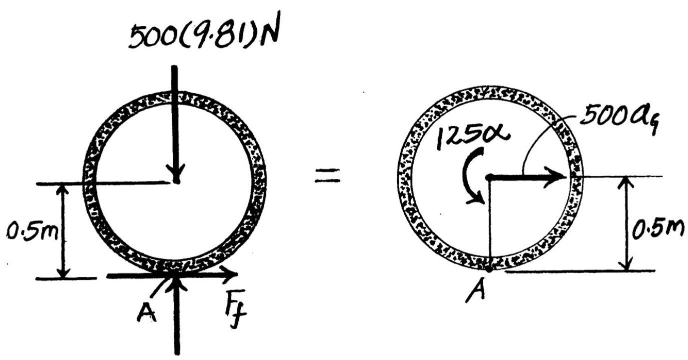
Equating the **i** components,

$$a_G = 3 - 0.5\alpha \qquad \textbf{(2)}$$

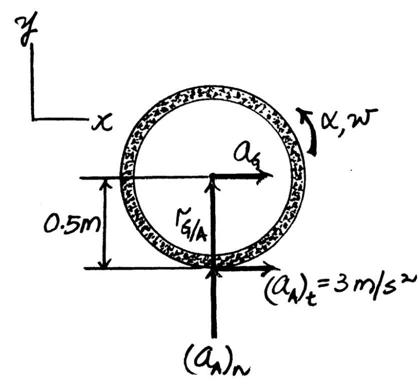
Solving Eqs. (1) and (2) yields

$$a_G = 1.5\ \mathrm{m/s^2} \rightarrow$$

$$\alpha = 3\ \mathrm{rad/s^2} \qquad \textbf{Ans.}$$



(a)



(b)