•16–1. A disk having a radius of 0.5 ft rotates with an initial angular velocity of 2 rad/s and has a constant angular acceleration of 1 rad/s^2. Determine the magnitudes of the velocity and acceleration of a point on the rim of the disk when $t = 2$ s.

$\omega = \omega_0 + \alpha_c t;$

$\omega = 2 + 1(2) = 4$ rad/s

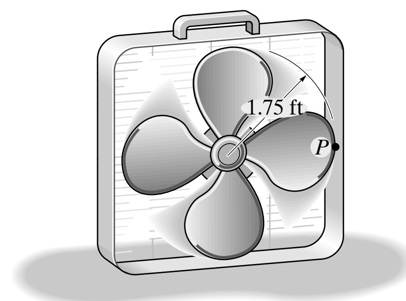
$v = r\omega; \qquad v = 0.5(4) = 2$ ft/s **Ans.**

$a_t = r\alpha; \qquad a_t = 0.5(1) = 0.5$ ft/s^2

$a_n = \omega^2 r; \qquad a_n = (4)^2(0.5) = 8$ ft/s^2

$a = \sqrt{8^2 + (0.5)^2} = 8.02$ ft/s^2 **Ans.**

16–2. Just after the fan is turned on, the motor gives the blade an angular acceleration $\alpha = (20e^{-0.6t})$ rad/s^2, where t is in seconds. Determine the speed of the tip P of one of the blades when $t = 3$ s. How many revolutions has the blade turned in 3 s? When $t = 0$ the blade is at rest.



$d\omega = \alpha\, dt$

$$\int_0^\omega d\omega = \int_0^t 20e^{-0.6t}\, dt$$

$$\omega = -\frac{20}{0.6} e^{-0.6t}\Big|_0^t = 33.3\left(1 - e^{-0.6t}\right)$$

$\omega = 27.82$ rad/s when $t = 3$s

$v_p = \omega r = 27.82(1.75) = 48.7$ ft/s **Ans.**

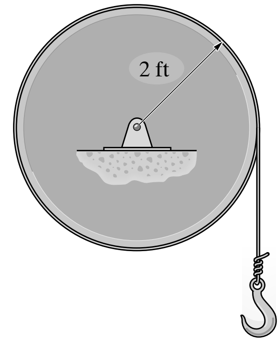
$d\theta = \omega\, dt$

$$\int_0^\theta d\theta = \int_0^t 33.3\left(1 - e^{-0.6t}\right) dt$$

$$\theta = 33.3\left(t + \left(\frac{1}{0.6}\right)e^{-0.6t}\right)\Big|_0^3 = 33.3\left[3 + \left(\frac{1}{0.6}\right)\left(e^{-0.6(3)} - 1\right)\right]$$

$\theta = 53.63$ rad $= 8.54$ rev **Ans.**

16–3. The hook is attached to a cord which is wound around the drum. If it moves from rest with an acceleration of 20 ft/s^2, determine the angular acceleration of the drum and its angular velocity after the drum has completed 10 rev. How many more revolutions will the drum turn after it has first completed 10 rev and the hook continues to move downward for 4 s?



Angular Motion: The angular acceleration of the drum can be determine by applying Eq. 16–11.

$$a_t = \alpha r; \qquad 20 = \alpha(2) \qquad \alpha = 10.0 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Applying Eq. 16–7 with $\alpha_c = \alpha = 10.0 \text{ rad/s}^2$ and $\theta = (10 \text{ rev}) \times \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 20\pi$ rad, we have

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$\omega^2 = 0 + 2(10.0)(20\pi - 0)$$

$$\omega = 35.45 \text{ rad/s} = 35.4 \text{ rad/s} \qquad \textbf{Ans.}$$

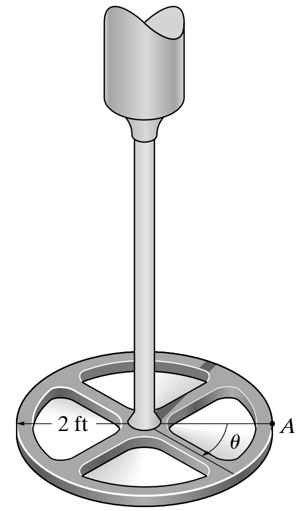
The angular displacement of the drum 4 s after it has completed 10 revolutions can be determined by applying Eq. 16–6 with $\omega_0 = 35.45$ rad/s.

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$$

$$= 0 + 35.45(4) + \frac{1}{2}(10.0)\left(4^2\right)$$

$$= (221.79 \text{ rad}) \times \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 35.3\text{rev} \qquad \textbf{Ans.}$$

***16–4.** The torsional pendulum (wheel) undergoes oscillations in the horizontal plane, such that the angle of rotation, measured from the equilibrium position, is given by $\theta = (0.5 \sin 3t)$ rad, where t is in seconds. Determine the maximum velocity of point A located at the periphery of the wheel while the pendulum is oscillating. What is the acceleration of point A in terms of t?



Angular Velocity: Here, $\theta = (0.5 \sin 3t)$ rad/s. Applying Eq. 16–1, we have

$$\omega = \frac{d\theta}{dt} = (1.5 \cos 3t) \text{ rad/s}$$

By observing the above equation, the angular velocity is maximum if $\cos 3t = 1$. Thus, the maximum angular velocity is $\omega_{max} = 1.50$ rad/s. The maximum speed of point A can be obtained by applying Eq. 16–8.

$$(v_A)_{max} = \omega_{max}\, r = 1.50(2) = 3.00 \text{ ft/s} \qquad \textbf{Ans.}$$

Angular Acceleration: Applying Eq. 16–2, we have

$$\alpha = \frac{d\omega}{dt} = (-4.5 \sin 3t) \text{ rad/s}^2$$

The tangential and normal components of the acceleration of point A can be determined using Eqs. 16–11 and 16–12, respectively.

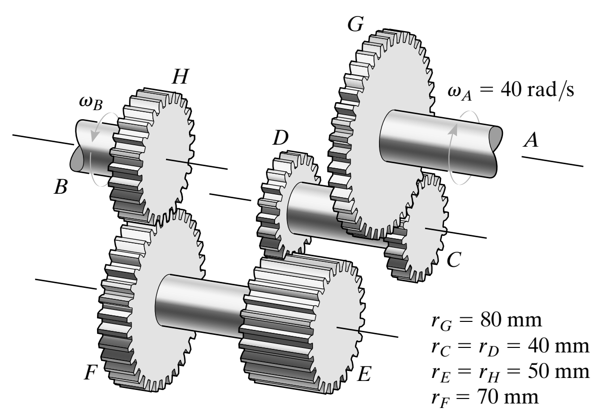
$$a_t = \alpha r = (-4.5 \sin 3t)(2) = (-9 \sin 3t) \text{ ft/s}^2$$

$$a_n = \omega^2 r = (1.5 \cos 3t)^2 (2) = \left(4.5 \cos^2 3t\right) \text{ ft/s}^2$$

Thus,

$$\mathbf{a}_A = \left(-9 \sin 3t\mathbf{u}_t + 4.5 \cos^2 3t\mathbf{u}_n\right) \text{ ft/s}^2 \qquad \textbf{Ans.}$$

•16–5. The operation of reverse gear in an automotive transmission is shown. If the engine turns shaft A at $\omega_A = 40$ rad/s, determine the angular velocity of the drive shaft, ω_B. The radius of each gear is listed in the figure.



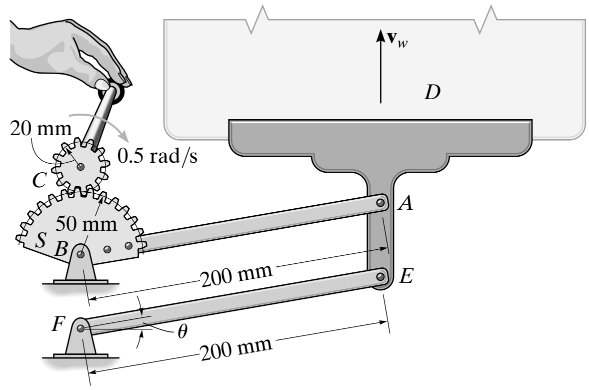
$r_A \omega_A = r_C \omega_C$: $\quad 80(40) = 40\omega_C \quad \omega_C = \omega_D = 80$ rad/s

$\omega_E r_E = \omega_D r_D$: $\quad \omega_E(50) = 80(40) \quad \omega_E = \omega_F = 64$ rad/s

$\omega_F r_F = \omega_B r_B$: $\quad 64(70) = \omega_B(50) \quad \omega_B = 89.6$ rad/s

$\omega_B = 89.6$ rad/s **Ans.**

16–6. The mechanism for a car window winder is shown in the figure. Here the handle turns the small cog C, which rotates the spur gear S, thereby rotating the fixed-connected lever AB which raises track D in which the window rests. The window is free to slide on the track. If the handle is wound at 0.5 rad/s, determine the speed of points A and E and the speed v_w of the window at the instant $\theta = 30°$.



$$v_C = \omega_C\, r_C = 0.5(0.02) = 0.01 \text{ m/s}$$

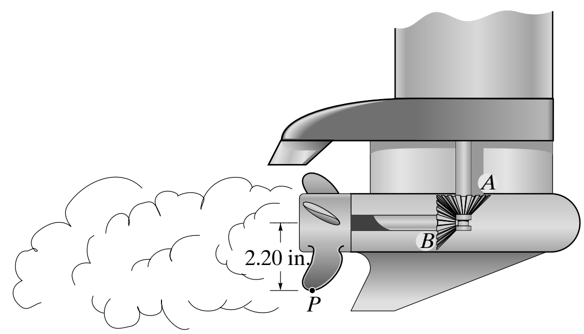
$$\omega_S = \frac{v_C}{r_S} = \frac{0.01}{0.05} = 0.2 \text{ rad/s}$$

$$v_A = v_E = \omega_S\, r_A = 0.2(0.2) = 0.04 \text{ m/s} = 40 \text{ mm/s} \qquad \textbf{Ans.}$$

Points A and E move along circular paths. The vertical component closes the window.

$$v_w = 40 \cos 30° = 34.6 \text{ mm/s} \qquad \textbf{Ans.}$$

16–7. The gear A on the drive shaft of the outboard motor has a radius $r_A = 0.5$ in. and the meshed pinion gear B on the propeller shaft has a radius $r_B = 1.2$ in. Determine the angular velocity of the propeller in $t = 1.5$ s, if the drive shaft rotates with an angular acceleration $\alpha = (400t^3)$ rad/s^2, where t is in seconds. The propeller is originally at rest and the motor frame does not move.



Angular Motion: The angular velocity of gear A at $t = 1.5$ s must be determined first. Applying Eq. 16–2, we have

$$d\omega = \alpha dt$$

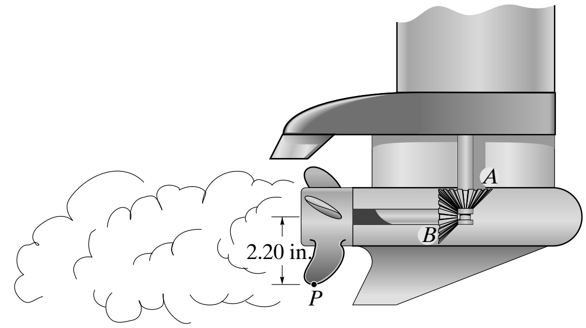
$$\int_0^{\omega_A} d\omega = \int_0^{1.5\,s} 400t^3\, dt$$

$$\omega_A = 100t^4\big|_0^{1.5\,s} = 506.25 \text{ rad/s}$$

However, $\omega_A\, r_A = \omega_B\, r_B$ where ω_B is the angular velocity of propeller. Then,

$$\omega_B = \frac{r_A}{r_B}\,\omega_A = \left(\frac{0.5}{1.2}\right)(506.25) = 211 \text{ rad/s} \qquad \textbf{Ans.}$$

***16–8.** For the outboard motor in Prob. 16–7, determine the magnitude of the velocity and acceleration of point P located on the tip of the propeller at the instant $t = 0.75$ s.



Angular Motion: The angular velocity of gear A at $t = 0.75$ s must be determined first. Applying Eq. 16–2, we have

$$d\omega = \alpha dt$$

$$\int_0^{\omega_A} d\omega = \int_0^{0.75\,s} 400t^3\, dt$$

$$\omega_A = 100t^4 \Big|_0^{0.75\,s} = 31.64 \text{ rad/s}$$

The angular acceleration of gear A at $t = 0.75$ s is given by

$$\alpha_A = 400\left(0.75^3\right) = 168.75 \text{ rad/s}^2$$

However, $\omega_A r_A = \omega_B r_B$ and $\alpha_A r_A = \alpha_B r_B$ where ω_B and α_B are the angular velocity and acceleration of propeller. Then,

$$\omega_B = \frac{r_A}{r_B}\omega_A = \left(\frac{0.5}{1.2}\right)(31.64) = 13.18 \text{ rad/s}$$

$$\alpha_B = \frac{r_A}{r_B}\alpha_A = \left(\frac{0.5}{1.2}\right)(168.75) = 70.31 \text{ rad/s}^2$$

Motion of *P*: The magnitude of the velocity of point P can be determined using Eq. 16–8.

$$v_P = \omega_B r_P = 13.18\left(\frac{2.20}{12}\right) = 2.42 \text{ ft/s} \qquad \textbf{Ans.}$$

The tangential and normal components of the acceleration of point P can be determined using Eqs. 16–11 and 16–12, respectively.

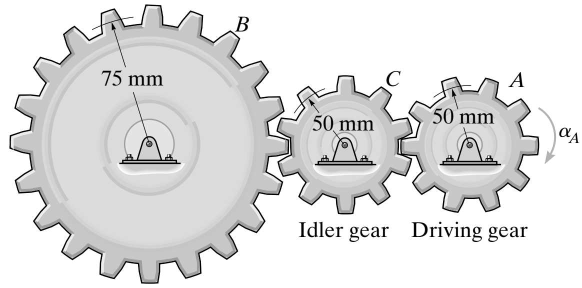
$$a_r = \alpha_B r_P = 70.31\left(\frac{2.20}{12}\right) = 12.89 \text{ ft/s}^2$$

$$a_n = \omega_B^2 r_P = \left(13.18^2\right)\left(\frac{2.20}{12}\right) = 31.86 \text{ ft/s}^2$$

The magnitude of the acceleration of point P is

$$a_P = \sqrt{a_r^2 + a_n^2} = \sqrt{12.89^2 + 31.86^2} = 34.4 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

•16–9. When only two gears are in mesh, the driving gear A and the driven gear B will always turn in opposite directions. In order to get them to turn in the *same direction* an idler gear C is used. In the case shown, determine the angular velocity of gear B when $t = 5$ s, if gear A starts from rest and has an angular acceleration of $\alpha_A = (3t + 2)$ rad/s^2, where t is in seconds.



$d\omega = \alpha\, dt$

$$\int_0^{\omega_A} d\omega_A = \int_0^t (3t + 2)\, dt$$

$\omega_A = 1.5t^2 + 2t|_{t=5} = 47.5$ rad/s

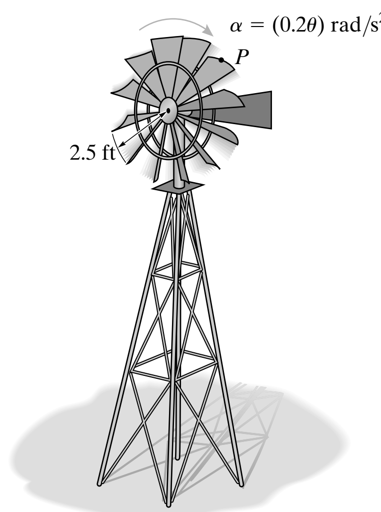
$(47.5)(50) = \omega_C\,(50)$

$\omega_C = 47.5$ rad/s

$\omega_B\,(75) = 47.5(50)$

$\omega_B = 31.7$ rad/s **Ans.**

16–10. During a gust of wind, the blades of the windmill are given an angular acceleration of $\alpha = (0.2\theta)$ rad/s^2, where θ is in radians. If initially the blades have an angular velocity of 5 rad/s, determine the speed of point P, located at the tip of one of the blades, just after the blade has turned two revolutions.



Angular Motion: The angular velocity of the blade can be obtained by applying Eq. 16–4.

$$\omega d\omega = \alpha d\theta$$

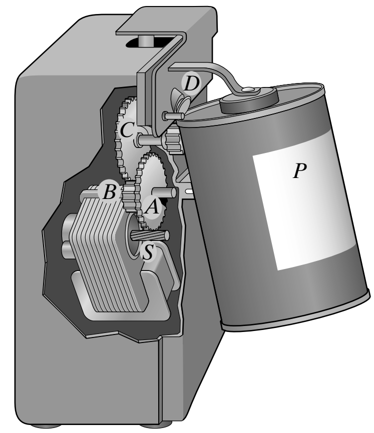
$$\int_{5\text{ rad/s}}^{\omega} \omega d\omega = \int_0^{4\pi} 0.2\theta d\theta$$

$$\omega = 7.522 \text{ rad/s}$$

Motion of *P*: The speed of point P can be determined using Eq. 16–8.

$v_P = \omega r_P = 7.522(2.5) = 18.8$ ft/s **Ans.**

16–11. The can opener operates such that the can is driven by the drive wheel D. If the armature shaft S on the motor turns with a constant angular velocity of 40 rad/s, determine the angular velocity of the can. The radii of S, can P, drive wheel D, gears A, B, and C, are $r_S = 5$ mm, $r_P = 40$ mm, $r_D = 7.5$ mm, $r_A = 20$ mm, $r_B = 10$ mm, and $r_C = 25$ mm, respectively.



Gears A and B will have the same angular velocity since they are mounted on the same axle. Thus,

$$\omega_A r_A = \omega_s r_s$$

$$\omega_B = \omega_A = \left(\frac{r_s}{r_A}\right)\omega_s = \left(\frac{5}{20}\right)(40) = 10 \text{ rad/s}$$

Wheel D is mounted on the same axle as gear C, which in turn is in mesh with gear B.

$$\omega_C r_C = \omega_B r_B$$

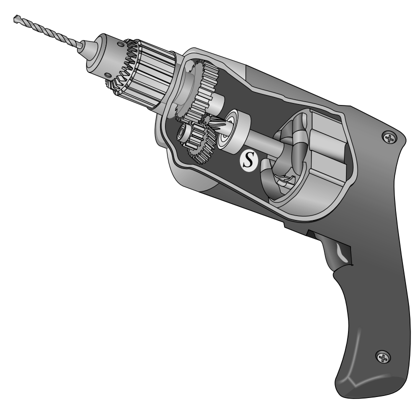
$$\omega_D = \omega_C = \left(\frac{r_B}{r_C}\right)\omega_B = \left(\frac{10}{25}\right)(10) = 4 \text{ rads/s}$$

Finally, the rim of can P is in mesh with wheel D.

$$\omega_P r_P = \omega_D r_D$$

$$\omega_P = \left(\frac{r_D}{r_P}\right)\omega_D = \left(\frac{7.5}{40}\right)(4) = 0.75 \text{ rad/s} \qquad \textbf{Ans.}$$

***16–12.** If the motor of the electric drill turns the armature shaft S with a constant angular acceleration of $\alpha_S = 30 \text{ rad/s}^2$, determine the angular velocity of the shaft after it has turned 200 rev, starting from rest.



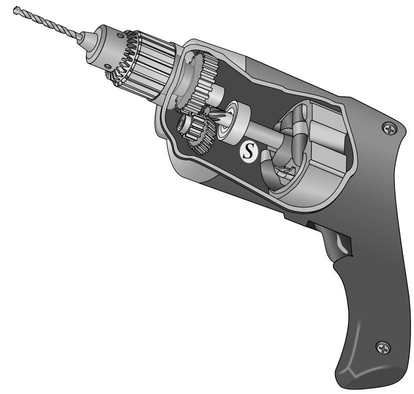
Motion of Pulley A: Here, $\theta_s = (200 \text{ rev})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 400\pi$ rad. Since the angular acceleration of shaft s is constant, its angular velocity can be determined from

$$\omega_s{}^2 = (\omega_s)_0{}^2 + 2\alpha_C\left[\theta_s - (\theta_s)_0\right]$$

$$\omega_s{}^2 = 0^2 + 2(30)(400\pi - 0)$$

$$\omega_s = 274.6 \text{ rad/s} \qquad \textbf{Ans.}$$

•16–13. If the motor of the electric drill turns the armature shaft S with an angular velocity of $\omega_S = (100t^{1/2})$ rad/s, determine the angular velocity and angular acceleration of the shaft at the instant it has turned 200 rev, starting from rest.



Motion of Armature Shaft *S*: Here, $\theta_s = (200 \text{ rev})\left(\dfrac{2\pi \text{rad}}{1 \text{ rev}}\right) = 400\pi$. The angular velocity of A can be determined from

$$\int d\theta_s = \int \omega_s dt$$

$$\int_0^{\theta_s} \theta_s = \int_0^t 100t^{1/2} dt$$

$$\theta_s\Big|_0^{\theta_s} = 66.67t^{3/2}\Big|_0^t$$

$$\theta_s = \left(66.67t^{3/2}\right)\text{rad}$$

When $\theta_s = 400\pi$ rad,

$$400\pi = 66.67t^{3/2}$$

$$t = 7.083 \text{ s}$$

Thus, the angular velocity of the shaft after it turns 200 rev ($t = 7.083$ s) is

$$\omega_s = 100(7.083)^{1/2} = 266 \text{ rad/s} \qquad \textbf{Ans.}$$

The angular acceleration of the shaft is

$$\alpha_s = \frac{d\omega_s}{dt} = 100\left(\frac{1}{2}t^{-1/2}\right) = \left(\frac{50}{t^{1/2}}\right) \text{rad/s}^2$$

When $t = 7.083$ s,

$$\alpha_s = \frac{50}{7.083^{1/2}} = 18.8 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

16–14. A disk having a radius of 6 in. rotates about a fixed axis with an angular velocity of $\omega = (2t + 3)$ rad/s, where t is in seconds. Determine the tangential and normal components of acceleration of a point located on the rim of the disk at the instant the angular displacement is $\theta = 40$ rad.

Motion of the Disk: We have

$$\int d\theta = \int \omega dt$$

$$\int_0^\theta d\theta = \int_0^t (2t + 3)dt$$

$$\theta\Big|_0^\theta = \left(t^2 + 3t\right)\Big|_0^t$$

$$\theta = \left(t^2 + 3t\right) \text{ rad}$$

When $\theta = 40$ rad,

$$40 = t^2 + 3t$$

$$t^2 + 3t - 40 = 0$$

Solving for the positive root,

$$t = 5 \text{ s}$$

Also,

$$\alpha = \frac{d\omega}{dt} = 2 \text{ rad/s}^2$$

When $t = 5$ s($\theta = 40$ rad),

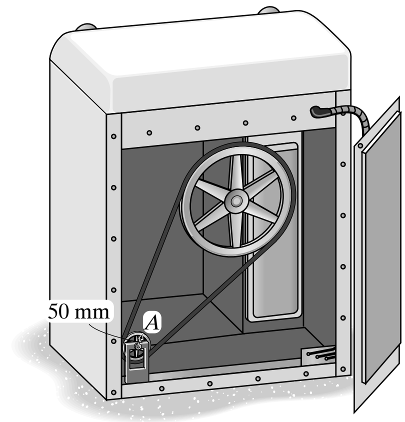
$$\omega = 2(5) + 3 = 13 \text{ rad/s}$$

Motion of Point *P*: Using the result for ω and α, the tangential and normal components of the acceleration of point P are

$$a_t = \alpha r_p = 2\left(\frac{6}{12}\right) = 1 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$a_n = \omega^2 r_p = (13)^2\left(\frac{6}{12}\right) = 84.5 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

16–15. The 50-mm-radius pulley A of the clothes dryer rotates with an angular acceleration of $\alpha_A = (27\theta_A^{1/2})$ rad/s^2, where θ_A is in radians. Determine its angular acceleration when $t = 1$ s, starting from rest.



Motion of Pulley A: The angular velocity of pulley A can be determined from

$$\int \omega_A \, d\omega_A = \int \alpha_A \, d\theta_A$$

$$\int_0^{\omega_A} \omega_A d\omega_A = \int_0^{\theta_A} 27\theta_A{}^{1/2} d\theta_A$$

$$\left.\frac{\omega_A{}^2}{2}\right|_0^{\omega_A} = 18\theta_A{}^{3/2}\Big|_0^{\theta_A}$$

$$\omega_A = \left(6\theta_A{}^{3/4}\right) \text{ rad/s}$$

Using this result, the angular displacement of A as a function of t can be determined from

$$\int dt = \int \frac{d\theta_A}{\omega_A}$$

$$\int_0^t dt = \int_0^{\theta_A} \frac{d\theta_A}{6\theta_A{}^{3/4}}$$

$$t\Big|_0^t = \left.\frac{2}{3}\theta_A{}^{1/4}\right|_0^{\theta_A}$$

$$t = \left(\frac{2}{3}\theta_A{}^{1/4}\right) \text{s}$$

$$\theta_A = \left(\frac{3}{2}t\right)^4 \text{ rad}$$

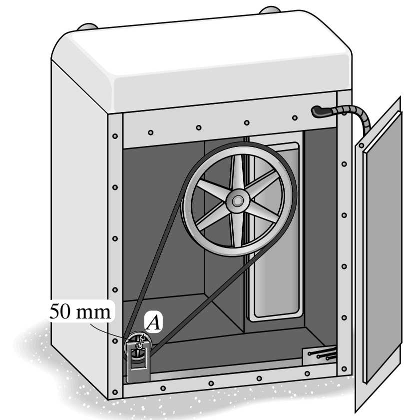
When $t = 1$ s

$$\theta_A = \left[\frac{3}{2}(1)\right]^4 = 5.0625 \text{ rad}$$

Thus, when $t = 1$ s, α_A is

$$\alpha_A = 27\left(5.0625^{1/2}\right) = 60.8 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

***16–16.** If the 50-mm-radius motor pulley A of the clothes dryer rotates with an angular acceleration of $\alpha_A = (10 + 50t)$ rad/s^2, where t is in seconds, determine its angular velocity when $t = 3$ s, starting from rest.



Motion of Pulley A: The angular velocity of pulley A can be determined from

$$\int d\omega_A = \int \alpha_A dt$$

$$\int_0^{\omega_A} d\omega_A = \int_0^t (10 + 50t)dt$$

$$\omega_A\Big|_0^{\omega_A} = \left(10t + 25t^2\right)\Big|_0^t$$

$$\omega_A = \left(10t + 25t^2\right)\text{rad/s}$$

When $t = 3$ s

$$\omega_A = 10(3) + 25\left(3^2\right) = 225 \text{ rad/s} \qquad \textbf{Ans.}$$

•16–17. The vacuum cleaner's armature shaft S rotates with an angular acceleration of $\alpha = 4\omega^{3/4}$ rad/s^2, where ω is in rad/s. Determine the brush's angular velocity when $t = 4$ s, starting from rest. The radii of the shaft and the brush are 0.25 in. and 1 in., respectively. Neglect the thickness of the drive belt.

Motion of the Shaft: The angular velocity of the shaft can be determined from

$$\int dt = \int \frac{d\omega_S}{\alpha_S}$$

$$\int_0^t dt = \int_0^{\omega_s} \frac{d\omega_S}{4\omega_S{}^{3/4}}$$

$$t\Big|_0^t = \omega_S{}^{1/4}\Big|_0^{\omega_s}$$

$$t = \omega_S{}^{1/4}$$

$$\omega_S = \left(t^4\right) \text{ rad/s}$$

When $t = 4$ s

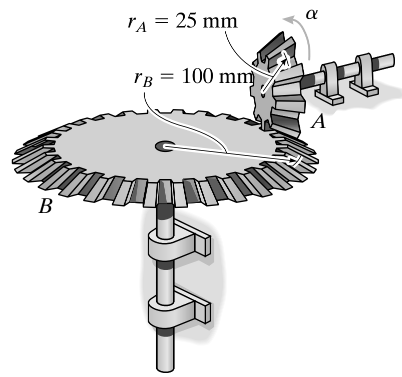
$$\omega_s = 4^4 = 256 \text{ rad/s}$$

Motion of the Beater Brush: Since the brush is connected to the shaft by a non-slip belt, then

$$\omega_B\, r_B = \omega_s\, r_s$$

$$\omega_B = \left(\frac{r_s}{r_B}\right)\omega_s = \left(\frac{0.25}{1}\right)(256) = 64 \text{ rad/s} \qquad \textbf{Ans.}$$

16–18. Gear A is in mesh with gear B as shown. If A starts from rest and has a constant angular acceleration of $\alpha_A = 2\ \text{rad/s}^2$, determine the time needed for B to attain an angular velocity of $\omega_B = 50\ \text{rad/s}$.



Angular Motion: The angular acceleration of gear B must be determined first. Here, $\alpha_A r_A = \alpha_B r_B$. Then,

$$\alpha_B = \frac{r_A}{r_B}\alpha_A = \left(\frac{25}{100}\right)(2) = 0.5\ \text{rad/s}^2$$

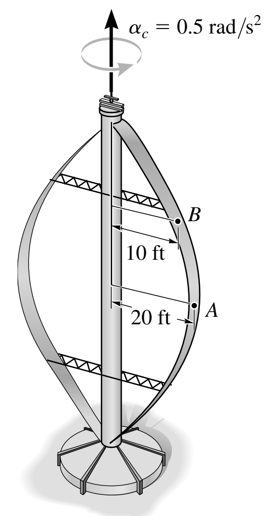
The time for gear B to attain an angular velocity of $\omega_B = 50\ \text{rad/s}$ can be obtained by applying Eq. 16–5.

$$\omega_B = (\omega_0)_B + \alpha_B t$$

$$50 = 0 + 0.5t$$

$$t = 100\ \text{s} \qquad \textbf{Ans.}$$

16–19. The vertical-axis windmill consists of two blades that have a parabolic shape. If the blades are originally at rest and begin to turn with a constant angular acceleration of $\alpha_c = 0.5\ \text{rad/s}^2$, determine the magnitude of the velocity and acceleration of points A and B on the blade after the blade has rotated through two revolutions.



Angular Motion: The angular velocity of the blade after the blade has rotated $2(2\pi) = 4\pi$ rad can be obtained by applying Eq. 16–7.

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$\omega^2 = 0^2 + 2(0.5)(4\pi - 0)$$

$$\omega = 3.545\ \text{rad/s}$$

Motion of A and B: The magnitude of the velocity of point A and B on the blade can be determined using Eq. 16–8.

$$v_A = \omega r_A = 3.545(20) = 70.9\ \text{ft/s} \qquad \textbf{Ans.}$$

$$\omega_B = \omega r_B = 3.545(10) = 35.4\ \text{ft/s} \qquad \textbf{Ans.}$$

The tangential and normal components of the acceleration of point A and B can be determined using Eqs. 16–11 and 16–12 respectively.

$$(a_t)_A = \alpha r_A = 0.5(20) = 10.0\ \text{ft/s}^2$$

$$(a_n)_A = \omega^2 r_A = \left(3.545^2\right)(20) = 251.33\ \text{ft/s}^2$$

$$(a_t)_B = \alpha r_B = 0.5(10) = 5.00\ \text{ft/s}^2$$

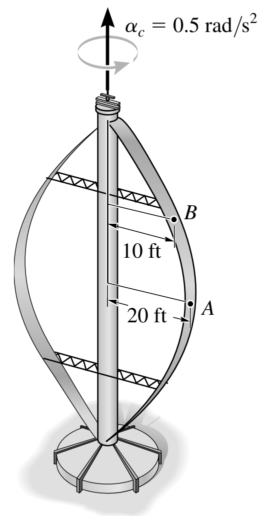
$$(a_n)_B = \omega^2 r_B = \left(3.545^2\right)(10) = 125.66\ \text{ft/s}^2$$

The magnitude of the acceleration of points A and B are

$$(a)_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{10.0^2 + 251.33^2} = 252\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$(a)_B = \ = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{5.00^2 + 125.66^2} = 126\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

***16–20.** The vertical-axis windmill consists of two blades that have a parabolic shape. If the blades are originally at rest and begin to turn with a constant angular acceleration of $\alpha_c = 0.5\ \text{rad/s}^2$, determine the magnitude of the velocity and acceleration of points A and B on the blade when $t = 4$ s.



Angular Motion: The angular velocity of the blade at $t = 4$ s can be obtained by applying Eq. 16–5.

$$\omega = \omega_0 + \alpha_c t = 0 + 0.5(4) = 2.00\ \text{rad/s}$$

Motion of *A* and *B*: The magnitude of the velocity of points A and B on the blade can be determined using Eq. 16–8.

$$v_A = \omega r_A = 2.00(20) = 40.0\ \text{ft/s} \qquad \textbf{Ans.}$$

$$v_B = \omega r_B = 2.00(10) = 20.0\ \text{ft/s} \qquad \textbf{Ans.}$$

The tangential and normal components of the acceleration of points A and B can be determined using Eqs. 16–11 and 16–12 respectively.

$$(a_t)_A = \alpha r_A = 0.5(20) = 10.0\ \text{ft/s}^2$$

$$(a_n)_A = \omega^2 r_A = \left(2.00^2\right)(20) = 80.0\ \text{ft/s}^2$$

$$(a_t)_B = \alpha r_B = 0.5(10) = 5.00\ \text{ft/s}^2$$

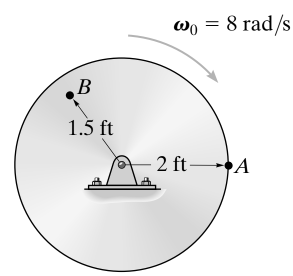
$$(a_n)_B = \omega^2 r_B = \left(2.00^2\right)(10) = 40.0\ \text{ft/s}^2$$

The magnitude of the acceleration of points A and B are

$$(a)_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{10.0^2 + 80.0^2} = 80.6\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$(a)_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{5.00^2 + 40.0^2} = 40.3\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

16.21. The disk is originally rotating at $\omega_0 = 8$ rad/s. If it is subjected to a constant angular acceleration of $\alpha = 6\ \text{rad/s}^2$, determine the magnitudes of the velocity and the n and t components of acceleration of point A at the instant $t = 0.5$ s.



$$\omega = \omega_0 + \alpha_c t$$

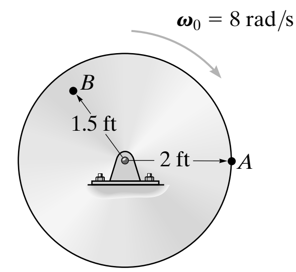
$$\omega = 8 + 6(0.5) = 11\ \text{rad/s}$$

$$v = r\omega; \qquad v_A = 2(11) = 22\ \text{ft/s} \qquad \textbf{Ans.}$$

$$a_t = r\alpha; \qquad (a_A)_t = 2(6) = 12.0\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$a_n = \omega^2 r; \qquad (a_A)_n = (11)^2(2) = 242\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

16–22. The disk is originally rotating at $\omega_0 = 8$ rad/s. If it is subjected to a constant angular acceleration of $\alpha = 6$ rad/s^2, determine the magnitudes of the velocity and the n and t components of acceleration of point B just after the wheel undergoes 2 revolutions.



$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$\omega^2 = (8)^2 + 2(6)[2(2\pi) - 0]$$

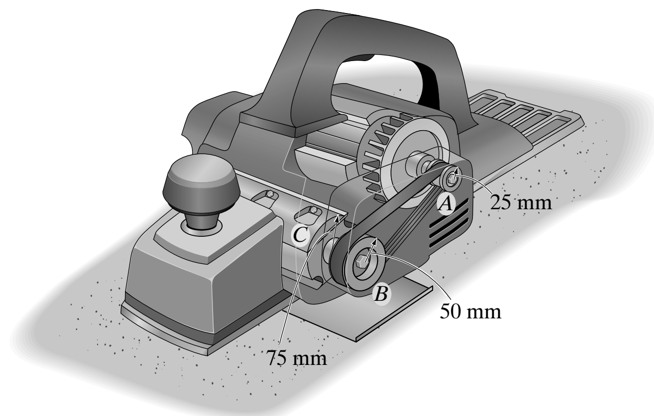
$$\omega = 14.66 \text{ rad/s}$$

$$v_B = \omega r = 14.66(1.5) = 22.0 \text{ ft/s}$$ **Ans.**

$$(a_B)_t = \alpha r = 6(1.5) = 9.00 \text{ ft/s}^2$$ **Ans.**

$$(a_B)_n = \omega^2 r = (14.66)^2(1.5) = 322 \text{ ft/s}^2$$ **Ans.**

16–23. The blade C of the power plane is driven by pulley A mounted on the armature shaft of the motor. If the constant angular acceleration of pulley A is $\alpha_A = 40$ rad/s^2, determine the angular velocity of the blade at the instant A has turned 400 rev, starting from rest.



Motion of Pulley A: Here, $\theta_A = (400 \text{ rev})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 800\pi$ rad. Since the angular velocity can be determined from

$$\omega_A{}^2 = (\omega_A)_0{}^2 + 2\alpha_C\left[\theta_A - (\theta_A)_0\right]$$

$$\omega_A{}^2 = 0^2 + 2(40)(800\pi - 0)$$

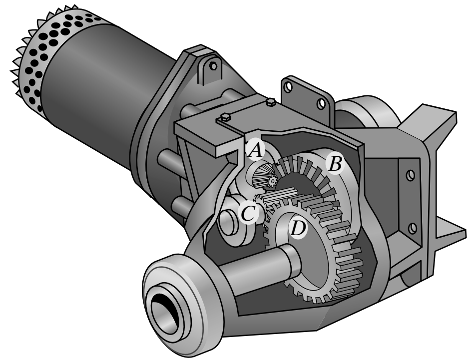
$$\omega_A = 448.39 \text{ rad/s}$$

Motion of Pulley B: Since blade C and pulley B are on the same axle, both will have the same angular velocity. Pulley B is connected to pulley A by a nonslip belt. Thus,

$$\omega_B r_B = \omega_A r_A$$

$$\omega_C = \omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{25}{50}\right)(448.39) = 224 \text{ rad/s}$$ **Ans.**

***16–24.** For a short time the motor turns gear A with an angular acceleration of $\alpha_A = (30t^{1/2})$ rad/s^2, where t is in seconds. Determine the angular velocity of gear D when $t = 5$ s, starting from rest. Gear A is initially at rest. The radii of gears A, B, C, and D are $r_A = 25$ mm, $r_B = 100$ mm, $r_C = 40$ mm, and $r_D = 100$ mm, respectively.



Motion of the Gear *A*: The angular velocity of gear A can be determined from

$$\int d\omega_A = \int \alpha dt$$

$$\int_0^{\omega_A} d\omega_A = \int_0^t 30t^{1/2} dt$$

$$\omega_A \Big|_0^{\omega_A} = 20t^{3/2} \Big|_0^t$$

$$\omega_A = \left(20t^{3/2}\right) \text{ rad/s}$$

When $t = 5$ s

$$\omega_A = 20\left(5^{3/2}\right) = 223.61 \text{ rad/s}$$

Motion of Gears *B*, *C*, and *D*: Gears B and C which are mounted on the same axle will have the same angular velocity. Since gear B is in mesh with gear A, then

$$\omega_B r_B = \omega_A r_A$$

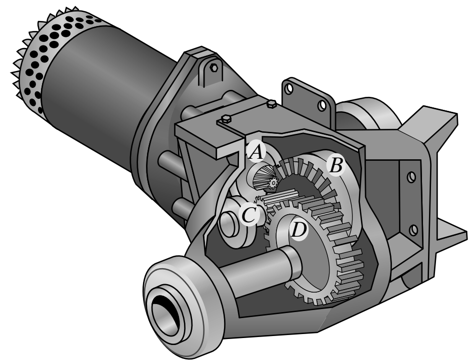
$$\omega_C = \omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{25}{100}\right)(223.61) = 55.90 \text{ rad/s}$$

Also, gear D is in mesh with gear C. Then

$$\omega_D r_D = \omega_C r_C$$

$$\omega_D = \left(\frac{r_C}{r_D}\right)\omega_C = \left(\frac{40}{100}\right)(55.90) = 22.4 \text{ rad/s} \qquad \textbf{Ans.}$$

•16–25. The motor turns gear A so that its angular velocity increases uniformly from zero to 3000 rev/min after the shaft turns 200 rev. Determine the angular velocity of gear D when $t = 3$ s. The radii of gears A, B, C, and D are $r_A = 25$ mm, $r_B = 100$ mm, $r_C = 40$ mm, and $r_D = 100$ mm, respectively.



Motion of Wheel *A*: Here, $\omega_A = \left(3000\,\dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{rev}}\right) = 100\pi$ rad/s

when $\theta_A = (200\text{ rev})\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right) = 400\pi$ rad. Since the angular acceleration of gear A is constant, it can be determined from

$$\omega_A{}^2 = (\omega_A)_0{}^2 + 2\alpha_A\left[\theta_A - (\theta_A)_0\right]$$

$$(100\pi)^2 = 0^2 + 2\alpha_A(400\pi - 0)$$

$$\alpha_A = 39.27\text{ rad/s}^2$$

Thus, the angular velocity of gear A when $t = 3$ s is

$$\omega_A = (\omega_A)_0 + \alpha_A t$$

$$= 0 + 39.27(3)$$

$$= 117.81\text{ rad/s}$$

Motion of Gears *B*, *C*, and *D*: Gears B and C which are mounted on the same axle will have the same angular velocity. Since gear B is in mesh with gear A, then

$$\omega_B r_B = \omega_B r_A$$

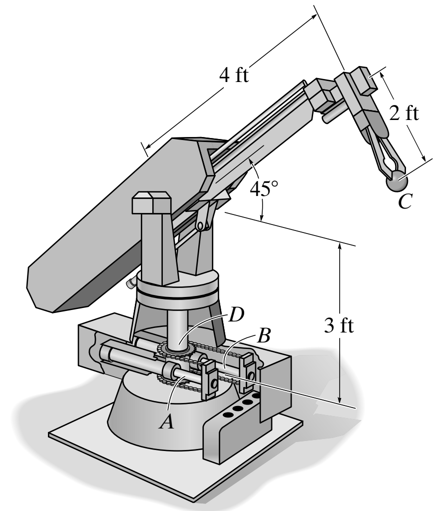
$$\omega_C = \omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{25}{100}\right)(117.81) = 29.45\text{ rad/s}$$

Also, gear D is in mesh with gear C. Then

$$\omega_D r_D = \omega_C r_C$$

$$\omega_D = \left(\frac{r_C}{r_D}\right)\omega_C = \left(\frac{40}{100}\right)(29.45) = 11.8\text{ rad/s}$$ **Ans.**

16–26. Rotation of the robotic arm occurs due to linear movement of the hydraulic cylinders A and B. If this motion causes the gear at D to rotate clockwise at 5 rad/s, determine the magnitude of velocity and acceleration of the part C held by the grips of the arm.



Motion of Part *C*: Since the shaft that turns the robot's arm is attached to gear D, then the angular velocity of the robot's arm $\omega_R = \omega_D = 5.00$ rad/s. The distance of part C from the rotating shaft is $r_C = 4\cos 45° + 2\sin 45° = 4.243$ ft. The magnitude of the velocity of part C can be determined using Eq. 16–8.

$$v_C = \omega_R\, r_C = 5.00(4.243) = 21.2 \text{ ft/s} \qquad \textbf{Ans.}$$

The tangential and normal components of the acceleration of part C can be determined using Eqs. 16–11 and 16–12 respectively.

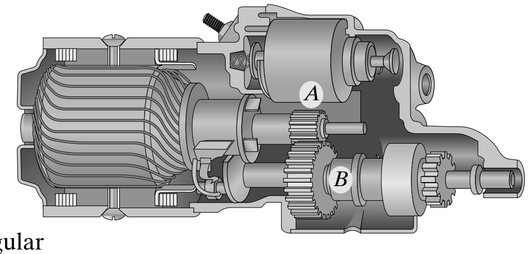
$$a_t = \alpha r_C = 0$$

$$a_n = \omega_R^2\, r_C = \left(5.00^2\right)(4.243) = 106.07 \text{ ft/s}^2$$

The magnitude of the acceleration of point C is

$$a_C = \sqrt{a_t^2 + a_n^2} = \sqrt{0^2 + 106.07^2} = 106 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

16–27. For a short time, gear A of the automobile starter rotates with an angular acceleration of $\alpha_A = (450t^2 + 60)\ \text{rad/s}^2$, where t is in seconds. Determine the angular velocity and angular displacement of gear B when $t = 2$ s, starting from rest. The radii of gears A and B are 10 mm and 25 mm, respectively.



Motion of Gear A: Applying the kinematic equation of variable angular acceleration,

$$\int d\omega_A \int \alpha_A dt$$

$$\int_0^{\omega_A} d\omega_A = \int_0^t \left(450t^2 + 60\right)dt$$

$$\omega_A\Big|_0^{\omega_A} = 150t^3 + 60t\Big|_0^t$$

$$\omega_A = \left(150t^3 + 60t\right)\ \text{rad/s}$$

When $t = 2$ s,

$$\omega_A = 150(2)^3 + 60(2) = 1320\ \text{rad/s}$$

$$\int d\theta_A = \int \omega_A\, dt$$

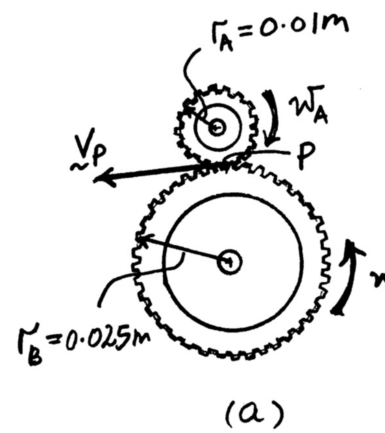
$$\int_0^{\theta_A} d\theta_A = \int_0^t \left(150t^3 + 60t\right)dt$$

$$\theta_A\Big|_0^{\theta_A} = 37.5t^4 + 30t^2\Big|_0^t$$

$$\theta_A = \left(37.5t^4 + 30t^2\right)\ \text{rad}$$

When $t = 2$ s

$$\theta_A = 37.5(2)^4 + 30(2)^2 = 720\ \text{rad}$$



(a)

Motion of Gear B: Since gear B is meshed with gear A, Fig. a, then

$$v_p = \omega_A r_A = \omega_B r_B$$

$$\omega_B = \omega_A\left(\frac{r_A}{r_B}\right)$$

$$= (1320)\left(\frac{0.01}{0.025}\right)$$

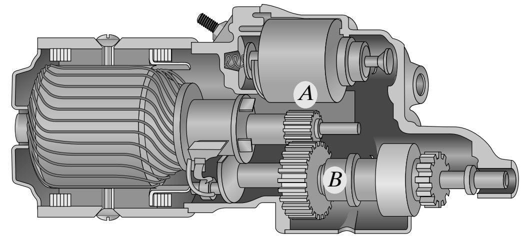
$$= 528\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\theta_B = \theta_A\left(\frac{r_A}{r_B}\right)$$

$$= 720\left(\frac{0.01}{0.025}\right)$$

$$= 288\ \text{rad} \qquad \textbf{Ans.}$$

***16–28.** For a short time, gear A of the automobile starter rotates with an angular acceleration of $\alpha_A = (50\omega^{1/2})$ rad/s^2, where ω is in rad/s. Determine the angular velocity of gear B after gear A has rotated 50 rev, starting from rest. The radii of gears A and B are 10 mm and 25 mm, respectively.



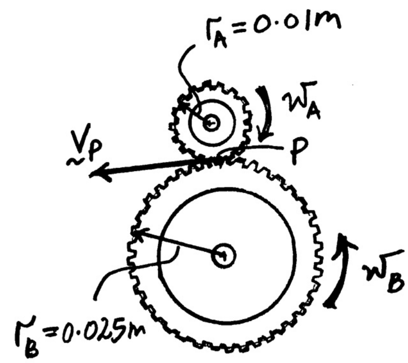
Motion of Gear *A*: We have

$$\int dt = \int \frac{d\omega_A}{\alpha_A}$$

$$\int_0^t dt = \int_0^{\omega_A} \frac{d\omega_A}{50{\omega_A}^{1/2}}$$

$$t\Big|_0^t = \frac{1}{25}{\omega_A}^{1/2}\Big|_0^{\omega_A}$$

$$\omega_A = \left(625t^2\right) \text{ rad/s}$$



(a)

The angular displacement of gear A can be determined using this result.

$$\int d\theta_A = \int \omega_A\, dt$$

$$\int_0^{\theta_A} d\theta_A = \int_0^t \left(625t^2\right)dt$$

$$\theta_A\Big|_0^{\theta_A} = 208.33t^3\Big|_0^t$$

$$\theta_A = \left(208.33t^3\right) \text{ rad}$$

When $\theta_A = 50 \text{ rev}\left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 100\pi$ rad,

$$100\pi = 208.33t^3$$

$$t = 1.147 \text{ s}$$

Thus, the angular velocity of gear A at $t = 1.147$ s$(\theta_A = 100\pi$ rad) is

$$\omega_A = 625(1.147^2) = 821.88 \text{ rad/s}$$

Motion of Gear *B*: Since gear B is meshed with gear A, Fig. *a*, then

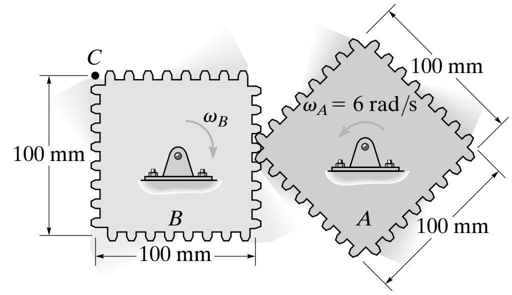
$$v_p = \omega_A r_A = \omega_B r_B$$

$$\omega_B = \omega_A\left(\frac{r_A}{r_B}\right)$$

$$= 821.88\left(\frac{0.01}{0.025}\right)$$

$$= 329 \text{ rad/s} \qquad \textbf{Ans.}$$

•16–29. Gear A rotates with a constant angular velocity of $\omega_A = 6$ rad/s. Determine the largest angular velocity of gear B and the speed of point C.



$(r_B)_{\max} = (r_A)_{\max} = 50\sqrt{2}$ mm

$(r_B)_{\min} = (r_A)_{\min} = 50$ mm

When r_A is max., r_B is min.

$\omega_B\,(r_B) = \omega_A\, r_A$

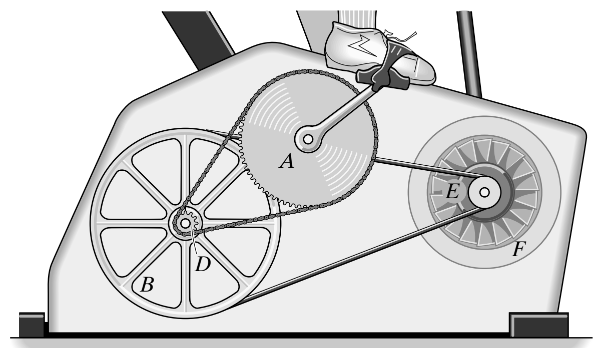
$$(\omega_B)_{\max} = 6\left(\frac{r_A}{r_B}\right) = 6\left(\frac{50\sqrt{2}}{50}\right)$$

$(\omega_B)_{\max} = 8.49$ rad/s **Ans.**

$v_C = (\omega_B)_{\max}\, r_C = 8.49\left(0.05\sqrt{2}\right)$

$v_C = 0.6$ m/s **Ans.**

16–30. If the operator initially drives the pedals at 20 rev/min, and then begins an angular acceleration of 30 rev/min^2, determine the angular velocity of the flywheel F when $t = 3$ s. Note that the pedal arm is fixed connected to the chain wheel A, which in turn drives the sheave B using the fixed connected clutch gear D. The belt wraps around the sheave then drives the pulley E and fixed-connected flywheel.



$r_A = 125$ mm $\quad r_B = 175$ mm
$r_D = 20$ mm $\quad r_E = 30$ mm

$\omega = \omega_0 + \alpha_c\, t$

$$\omega_A = 20 + 30\left(\frac{3}{60}\right) = 21.5 \text{ rev/min}$$

$\omega_A\, r_A = \omega_D\, r_D$

$21.5(125) = \omega_D\,(20)$

$\omega_D = \omega_B = 134.375$

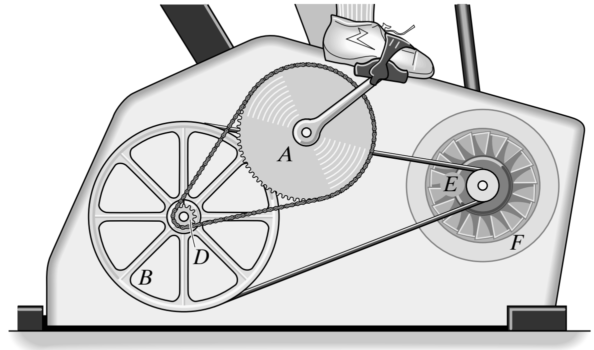
$\omega_B\, r_B = \omega_E\, r_E$

$134.375(175) = \omega_E(30)$

$\omega_E = 783.9$ rev/min

$\omega_F = 784$ rev/min **Ans.**

16–31. If the operator initially drives the pedals at 12 rev/min, and then begins an angular acceleration of 8 rev/min^2, determine the angular velocity of the flywheel F after the pedal arm has rotated 2 revolutions. Note that the pedal arm is fixed connected to the chain wheel A, which in turn drives the sheave B using the fixed-connected clutch gear D. The belt wraps around the sheave then drives the pulley E and fixed-connected flywheel.



$r_A = 125$ mm $\quad r_B = 175$ mm
$r_D = 20$ mm $\quad r_E = 30$ mm

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$\omega^2 = (12)^2 + 2(8)(2 - 0)$$

$$\omega = 13\,266 \text{ rev/min}$$

$$\omega_A r_A = \omega_D r_D$$

$$13\,266(125) = \omega_D(20)$$

$$\omega_D = \omega_D = 82.916$$

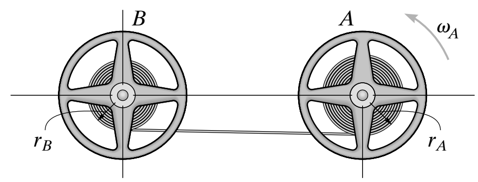
$$\omega_B r_B = \omega_E r_E$$

$$82.916(175) = \omega_E(30)$$

$$\omega_E = 483.67$$

$$\omega_F = 484 \text{ rev/min}$$ **Ans.**

***16–32.** The drive wheel A has a constant angular velocity of ω_A. At a particular instant, the radius of rope wound on each wheel is as shown. If the rope has a thickness T, determine the angular acceleration of wheel B.



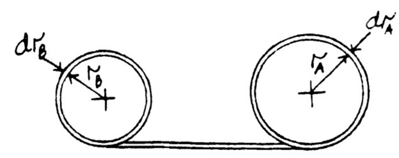
Angular Motion: The angular velocity between wheels A and B can be related by

$$\omega_A r_A = \omega_B r_B \text{ or } \omega_B = \frac{r_A}{r_B}\omega_A$$

During time dt, the volume of the tape exchange between the wheel is

$$-2\pi r_B\, dr_B = 2\pi r_A\, dr_A$$

$$dr_B = -\left(\frac{r_A}{r_B}\right)dr_A \qquad \textbf{[1]}$$

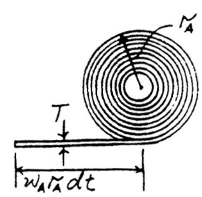


Applying Eq. 16–2 with $\omega_B = \frac{r_A}{r_B}\omega_A$, we have

$$\alpha_B = \frac{d\omega_B}{dt} = \frac{d}{dt}\left[\frac{r_A}{r_B}\omega_A\right] = \omega_A\left(\frac{1}{r_B}\frac{dr_A}{dt} - \frac{r_A}{r_B^2}\frac{dr_B}{dt}\right) \qquad \textbf{[2]}$$

Substituting Eq.[1] into [2] yields

$$\alpha_B = \omega_A\left(\frac{r_A^2 + r_B^2}{r_B^3}\right)\frac{dr_A}{dt} \qquad \textbf{[3]}$$



The volume of tape coming out from wheel A in time dt is

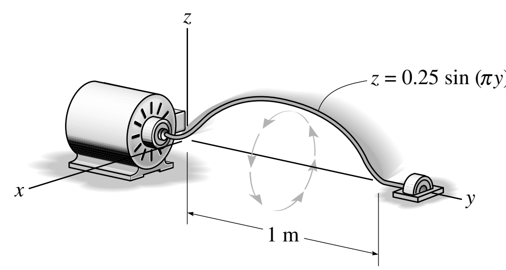
$$2\pi r_A\, dr_A = (\omega_A r_A\, dt)\, T$$

$$\frac{dr_A}{dt} = \frac{\omega_A T}{2\pi} \qquad \textbf{[4]}$$

Substitute Eq.[4] into [3] gives

$$\alpha_B = \frac{\omega_A^2 T}{2\pi r_B^3}\left(r_A^2 + r_B^2\right) \qquad \textbf{Ans.}$$

•16–33. If the rod starts from rest in the position shown and a motor drives it for a short time with an angular acceleration of $\alpha = (1.5e^t)$ rad/s^2, where t is in seconds, determine the magnitude of the angular velocity and the angular displacement of the rod when $t = 3$ s. Locate the point on the rod which has the greatest velocity and acceleration, and compute the magnitudes of the velocity and acceleration of this point when $t = 3$ s. The rod is defined by $z = 0.25 \sin(\pi y)$ m, where the argument for the sine is given in radians and y is in meters.



$$d\omega = \alpha\, dt$$

$$\int_0^\omega d\omega = \int_0^t 1.5e^t\, dt$$

$$\omega = 1.5e^t\big|_0^t = 1.5\left[e^t - 1\right]$$

$$d\theta = \omega\, dt$$

$$\int_0^\theta d\theta = 1.5\int_0^t \left[e^t - 1\right] dt$$

$$\theta = 1.5\left[e^t - t\right]_0^t = 1.5\left[e^t - t - 1\right]$$

When $t = 3$ s

$$\omega = 1.5\left[e^3 - 1\right] = 28.63 = 28.6 \text{ rad/s} \qquad \textbf{Ans.}$$

$$\theta = 1.5\left[e^3 - 3 - 1\right] = 24.1 \text{ rad} \qquad \textbf{Ans.}$$

The point having the greatest velocity and acceleration is located furthest from the axis of rotation. This is at $y = 0.5$ m, where $z = 0.25 \sin(\pi 0.5) = 0.25$ m.

Hence,

$$v_P = \omega(z) = 28.63(0.25) = 7.16 \text{ m/s} \qquad \textbf{Ans.}$$

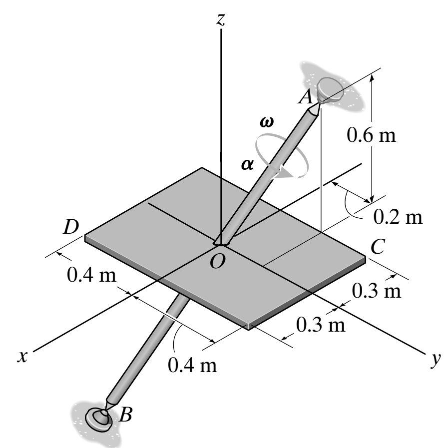
$$(a_t)_P = \alpha(z) = \left(1.5e^3\right)(0.25) = 7.532 \text{ m/s}^2$$

$$(a_n)_P = \omega^2(z) = (28.63)^2(0.25) = 204.89 \text{ m/s}^2$$

$$a_P = \sqrt{(a_t)_P^2 + (a_n)_P^2} = \sqrt{(7.532)^2 + (204.89)^2}$$

$$a_P = 205 \text{ m/s}^2 \qquad \textbf{Ans.}$$

16–34. If the shaft and plate rotate with a constant angular velocity of $\omega = 14\ \text{rad/s}$, determine the velocity and acceleration of point C located on the corner of the plate at the instant shown. Express the result in Cartesian vector form.



We will first express the angular velocity ω of the plate in Cartesian vector form. The unit vector that defines the direction of ω is

$$\mathbf{u}_{OA} = \frac{-0.3\mathbf{i} + 0.2\mathbf{j} + 0.6\mathbf{k}}{\sqrt{(-0.3)^2 + 0.2^2 + 0.6^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

Thus,

$$\omega = \omega\mathbf{u}_{OA} = 14\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}]\ \text{rad/s}$$

Since ω is constant

$$\alpha = 0$$

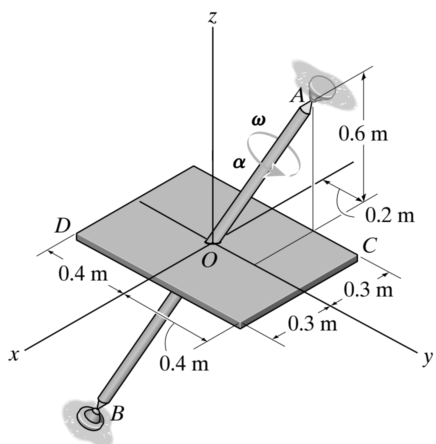
For convenience, $\mathbf{r}_C = [-0.3\mathbf{i} + 0.4\mathbf{j}]$ m is chosen. The velocity and acceleration of point C can be determined from

$$\mathbf{v}_C = \omega \times \mathbf{r}_C$$
$$= (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})$$
$$= [-4.8\mathbf{i} - 3.6\mathbf{j} - 1.2\mathbf{k}]\ \text{m/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_C = \alpha \times \mathbf{r}_C$$
$$= 0 + (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times [(-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})]$$
$$= [38.4\mathbf{i} - 64.8\mathbf{j} + 40.8\mathbf{k}]\text{m/s}^2 \qquad \textbf{Ans.}$$

16–35. At the instant shown, the shaft and plate rotates with an angular velocity of $\omega = 14$ rad/s and angular acceleration of $\alpha = 7$ rad/s^2. Determine the velocity and acceleration of point D located on the corner of the plate at this instant. Express the result in Cartesian vector form.



We will first express the angular velocity ω of the plate in Cartesian vector form. The unit vector that defines the direction of ω and α is

$$\mathbf{u}_{OA} = \frac{-0.3\mathbf{i} + 0.2\mathbf{j} + 0.6\mathbf{k}}{\sqrt{(-0.3)^2 + 0.2^2 + 0.6^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

Thus,

$$\omega = \omega\mathbf{u}_{OA} = 14\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}] \text{ rad/s}$$

$$\alpha = \alpha\mathbf{u}_{OA} = 7\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}] \text{ rad/s}$$

For convenience, $\mathbf{r}_D = [-0.3\mathbf{i} + 0.4\mathbf{j}]$ m is chosen. The velocity and acceleration of point D can be determined from

$$\mathbf{v}_D = \omega \times r_D$$

$$= (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})$$

$$= [4.8\mathbf{i} + 3.6\mathbf{j} + 1.2\mathbf{k}]\text{m/s} \qquad \textbf{Ans.}$$

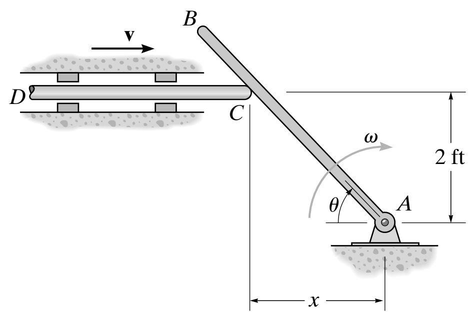
and

$$\mathbf{a}_D = \alpha \times \mathbf{r}_D - \omega^2\,\mathbf{r}_D$$

$$= (-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j}) + (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times [(-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})]$$

$$= [-36.0\mathbf{i} + 66.6\mathbf{j} + 40.2\mathbf{k}]\text{m/s}^2 \qquad \textbf{Ans.}$$

***16–36.** Rod CD presses against AB, giving it an angular velocity. If the angular velocity of AB is maintained at $\omega = 5$ rad/s, determine the required magnitude of the velocity **v** of CD as a function of the angle θ of rod AB.

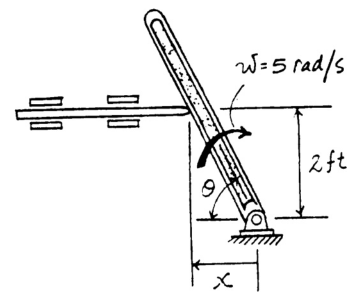


Position Coordinate Equation: From the geometry,

$$x = \frac{2}{\tan\theta} = 2\cot\theta \qquad \textbf{[1]}$$

Time Derivatives: Taking the time derivative of Eq. [1], we have

$$\frac{dx}{dt} = -2\csc^2\theta\frac{d\theta}{dt} \qquad \textbf{[2]}$$

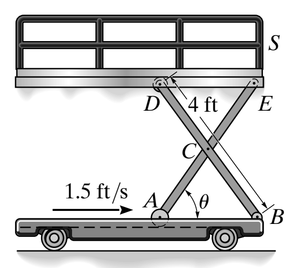


However, $\dfrac{dx}{dt} = v$ and $\dfrac{d\theta}{dt} = \omega = 5$ rad/s, then from Eq. [2]

$$v = -2\csc^2\theta(5) = \left(-10\csc^2\theta\right) \qquad \textbf{Ans.}$$

Note: Negative sign indicates that v is directed in the opposite direction to that of positive x.

•16–37. The scaffold S is raised by moving the roller at A toward the pin at B. If A is approaching B with a speed of 1.5 ft/s, determine the speed at which the platform rises as a function of θ. The 4-ft links are pin connected at their midpoint.



Position Coordinate Equation:

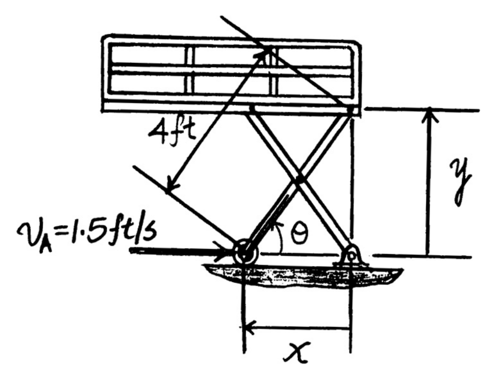
$$x = 4\cos\theta \qquad y = 4\sin\theta$$

Time Derivatives:

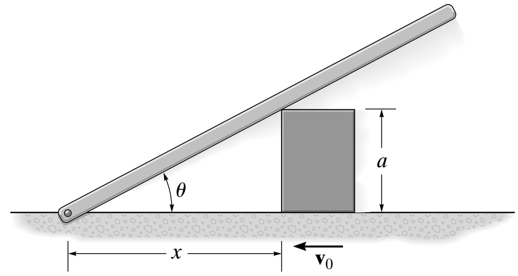
$$\dot{x} = -4\sin\theta\dot{\theta} \qquad \text{However,} \quad \dot{x} = -v_A = -1.5 \text{ ft/s}$$

$$-1.5 = -4\sin\theta\dot{\theta} \qquad \dot{\theta} = \frac{0.375}{\sin\theta}$$

$$\dot{y} = v_y = 4\cos\theta\dot{\theta} = 4\cos\theta\left(\frac{0.375}{\sin\theta}\right) = 1.5\cot\theta \qquad \textbf{Ans.}$$



16–38. The block moves to the left with a constant velocity $\mathbf{v}_0$. Determine the angular velocity and angular acceleration of the bar as a function of θ.



Position Coordinate Equation: From the geometry,

$$x = \frac{a}{\tan\theta} = a\cot\theta \qquad [1]$$

Time Derivatives: Taking the time derivative of Eq. [1], we have

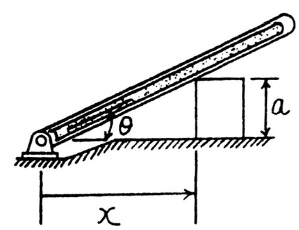
$$\frac{dx}{dt} = -a\csc^2\theta\frac{d\theta}{dt} \qquad [2]$$

Since v_0 is directed toward negative x, then $\frac{dx}{dt} = -v_0$. Also, $\frac{d\theta}{dt} = \omega$.

From Eq.[2],

$$-v_0 = -a\csc^2\theta(\omega)$$

$$\omega = \frac{v_0}{a\csc^2\theta} = \frac{v_0}{a}\sin^2\theta \qquad \textbf{Ans.}$$



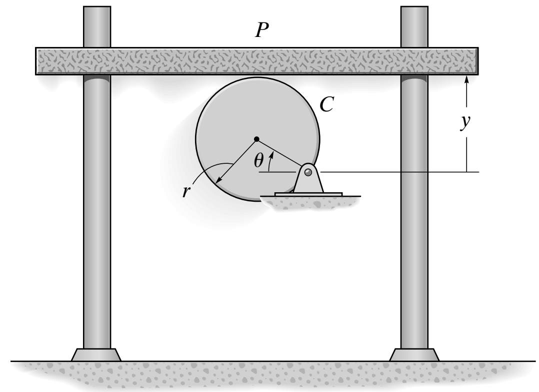
Here, $\alpha = \frac{d\omega}{dt}$. Then from the above expression

$$\alpha = \frac{v_0}{a}(2\sin\theta\cos\theta)\frac{d\theta}{dt} \qquad [3]$$

However, $2\sin\theta\cos\theta = \sin 2\theta$ and $\omega = \frac{d\theta}{dt} = \frac{v_0}{a}\sin^2\theta$. Substitute these values into Eq.[3] yields

$$\alpha = \frac{v_0}{a}\sin 2\theta\left(\frac{v_0}{a}\sin^2\theta\right) = \left(\frac{v_0}{a}\right)^2\sin 2\theta\sin^2\theta \qquad \textbf{Ans.}$$

16–39. Determine the velocity and acceleration of platform P as a function of the angle θ of cam C if the cam rotates with a constant angular velocity $\boldsymbol{\omega}$. The pin connection does not cause interference with the motion of P on C. The platform is constrained to move vertically by the smooth vertical guides.



Position Coordinate Equation: From the geometry.

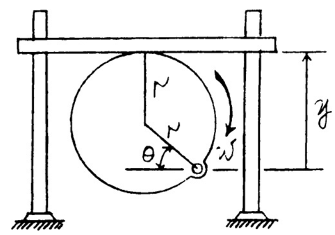
$$y = r\sin\theta + r \qquad \textbf{[1]}$$

Time Derivatives: Taking the time derivative of Eq. [1], we have

$$\frac{dy}{dt} = r\cos\theta\,\frac{d\theta}{dt} \qquad \textbf{[2]}$$

However $v = \dfrac{dy}{dt}$ and $\omega = \dfrac{d\theta}{dt}$. From Eq.[2],

$$v = \omega r\cos\theta \qquad \textbf{Ans.}$$



Taking the time derivative of the above expression, we have

$$\frac{dv}{dt} = r\left[\omega(-\sin\theta)\,\frac{d\theta}{dt} + \cos\theta\frac{d\omega}{dt}\right]$$

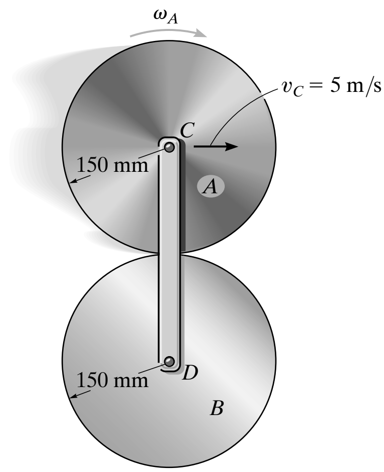
$$= r\left(\cos\theta\,\frac{d\omega}{dt} - \omega^2\sin\theta\right) \qquad \textbf{[4]}$$

However $a = \dfrac{dv}{dt}$ and $\alpha = \dfrac{d\omega}{dt} = 0$. From Eq.[4],

$$a = -\omega^2 r\sin\theta \qquad \textbf{Ans.}$$

Note: Negative sign indicates that a is directed in the opposite direction to that of positive y.

***16–40.** Disk A rolls without slipping over the surface of the *fixed* cylinder B. Determine the angular velocity of A if its center C has a speed $v_C = 5$ m/s. How many revolutions will A rotate about its center just after link DC completes one revolution?



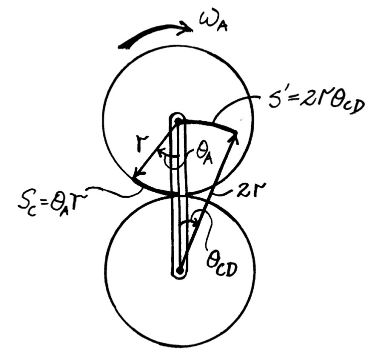
As shown by the construction, as A rolls through the arc $s = \theta_A r$, the center of the disk moves through the same distance $s' = s$. Hence,

$$s = \theta_A r$$

$$\dot{s} = \dot{\theta}_A r$$

$$5 = \omega_A (0.15)$$

$$\omega_A = 33.3 \text{ rad/s} \qquad \textbf{Ans.}$$



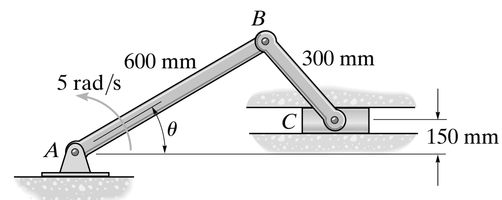
Link

$$s' = 2r\theta_{CD} = s = \theta_A r$$

$$2\theta_{CD} = \theta_A$$

Thus, A makes 2 revolutions for each revolution of CD. **Ans.**

•**16–41.** Crank AB rotates with a constant angular velocity of 5 rad/s. Determine the velocity of block C and the angular velocity of link BC at the instant $\theta = 30°$.



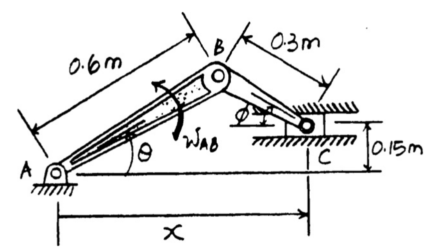
Position Coordinate Equation: From the geometry,

$$x = 0.6\cos\theta + 0.3\cos\phi \qquad \textbf{[1]}$$

$$0.6\sin\theta = 0.15 + 0.3\sin\phi \qquad \textbf{[2]}$$

Eliminate ϕ from Eqs. [1] and [2] yields

$$x = 0.6\cos\theta + 0.3\sqrt{2\sin\theta - 4\sin^2\theta + 0.75} \qquad \textbf{[3]}$$



Time Derivatives: Taking the time derivative of Eq. [3], we have

$$\frac{dx}{dt} = \left[-0.6\sin\theta + \frac{0.15(2\cos\theta - 4\sin 2\theta)}{\sqrt{2\sin\theta - 4\sin^2\theta + 0.75}}\right]\frac{d\theta}{dt} \qquad \textbf{[4]}$$

However, $\dfrac{dx}{dt} = v_C$ and $\dfrac{d\theta}{dt} = \omega_{AB}$, then from Eq.[4]

$$v_C = \left[-0.6\sin\theta + \frac{0.15(2\cos\theta - 4\sin 2\theta)}{\sqrt{2\sin\theta - 4\sin^2\theta + 0.75}}\right]\omega_{AB} \qquad \textbf{[5]}$$

At the instant $\theta = 30°$, $\omega_{AB} = 5$ rad/s. Substitute into Eq.[5] yields

$$v_C = \left[-0.6\sin 30° + \frac{0.15(2\cos 30° - 4\sin 60°)}{\sqrt{2\sin 30° - 4\sin^2 30° + 0.75}}\right](5) = -3.00\text{ m/s} \qquad \textbf{Ans.}$$

Taking the time derivative of Eq. [2], we have

$$0.6\cos\theta\frac{d\theta}{dt} = 0.3\cos\phi\frac{d\phi}{dt} \qquad \textbf{[6]}$$

However, $\dfrac{d\phi}{dt} = \omega_{BC}$ and $\dfrac{d\theta}{dt} = \omega_{AB}$, then from Eq.[6]

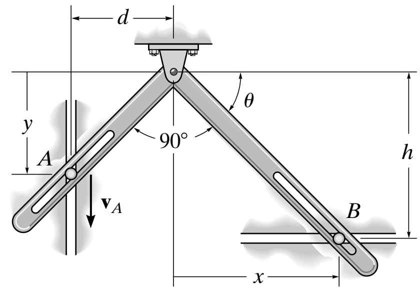
$$\omega_{BC} = \left(\frac{2\cos\theta}{\cos\phi}\right)\omega_{AB} \qquad \textbf{[7]}$$

At the instant $\theta = 30°$, from Eq.[2], $\phi = 30.0°$. From Eq.[7]

$$\omega_{BC} = \left(\frac{2\cos 30°}{\cos 30.0°}\right)(5) = 10.0\text{ rad/s} \qquad \textbf{Ans.}$$

Note: Negative sign indicates that v_C is directed in the opposite direction to that of positive x.

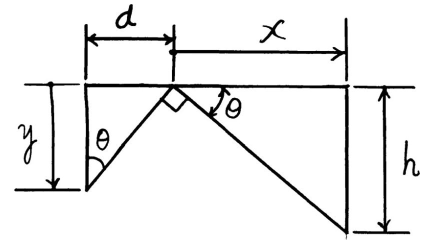
16–42. The pins at A and B are constrained to move in the vertical and horizontal tracks. If the slotted arm is causing A to move downward at $\mathbf{v}_A$, determine the velocity of B as a function of θ.



Position Coordinate Equation:

$$\tan\theta = \frac{h}{x} = \frac{d}{y}$$

$$x = \left(\frac{h}{d}\right)y$$

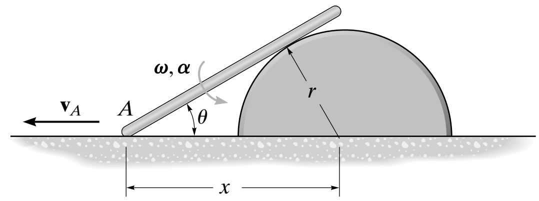


Time Derivatives:

$$\dot{x} = \left(\frac{h}{d}\right)\dot{y}$$

$$v_B = \left(\frac{h}{d}\right)v_A \qquad \textbf{Ans.}$$

16–43. End A of the bar moves to the left with a constant velocity $\mathbf{v}_A$. Determine the angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$ of the bar as a function of its position x.

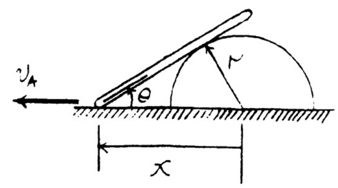


Position Coordinate Equation: From the geometry.

$$x = \frac{r}{\sin\theta} \qquad \textbf{[1]}$$

Time Derivatives: Taking the time derivative of Eq.[1], we have

$$\frac{dx}{dt} = -\frac{r\cos\theta\, d\theta}{r\sin^2\theta\, dt} \qquad \textbf{[2]}$$



Since v_0 is directed toward positive x, then $\frac{dx}{dt} = v_A$. Also, $\frac{d\theta}{dt} = \omega$. From the geometry, $\sin\theta = \frac{r}{x}$ and $\cos\theta = \frac{\sqrt{x^2 - r^2}}{x}$. Substitute these values into Eq.[2], we have

$$v_A = -\left(\frac{r\left(\sqrt{x^2 - r^2}/x\right)}{(r/x)^2}\right)\omega$$

$$\omega = -\left(\frac{r}{x\sqrt{x^2 - r^2}}\right)v_A \qquad \textbf{Ans.}$$

Taking the time derivative of Eq. [2], we have

$$\frac{d^2x}{dt^2} = \frac{r}{\sin^2\theta} = \left[\left(\frac{1 + \cos^2\theta}{\sin\theta}\right)\left(\frac{d\theta}{dt}\right)^2 - \cos\theta\frac{d^2\theta}{dt^2}\right] \qquad \textbf{[3]}$$

Here, $\frac{d^2x}{dt^2} = a = 0$ and $\frac{d^2\theta}{dt^2} = \alpha$. Substitute into Eq.[3], we have

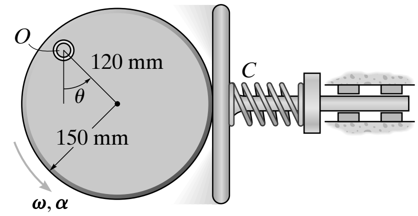
$$0 = \frac{r}{\sin^2\theta}\left[\left(\frac{1 + \cos^2\theta}{\sin\theta}\right)\omega^2 - \alpha\cos\theta\right]$$

$$\alpha = \left(\frac{1 + \cos^2\theta}{\sin\theta\cos\theta}\right)\omega^2 \qquad \textbf{[4]}$$

However, $\sin\theta = \frac{r}{x}$, $\cos\theta = \frac{\sqrt{x^2 - r^2}}{x}$ and $\omega = -\left(\frac{r}{x\sqrt{x^2 - r^2}}\right)v_A$. Substitute these values into Eq.[4] yields

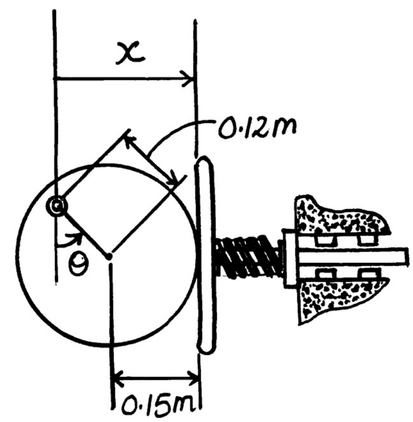
$$\alpha = \left[\frac{r(2x^2 - r^2)}{x^2(x^2 - r^2)^{3/2}}\right]v_A^2 \qquad \textbf{Ans.}$$

***16–44.** Determine the velocity and acceleration of the plate at the instant $\theta = 30°$, if at this instant the circular cam is rotating about the fixed point O with an angular velocity $\omega = 4$ rad/s and an angular acceleration $\alpha = 2$ rad/s^2.



Position Coordinate Equation: From the geometry,

$$x = 0.12 \sin\theta + 0.15 \qquad \textbf{[1]}$$



Time Derivatives: Taking the time derivative of Eq. [1], we have

$$\frac{dx}{dt} = 0.12 \cos\theta \frac{d\theta}{dt} \qquad \textbf{[2]}$$

However $v = \dfrac{dx}{dt}$ and $\omega = \dfrac{d\theta}{dt}$. From Eq.[2],

$$v = 0.12\omega \cos\theta \qquad \textbf{[3]}$$

At the instant $\theta = 30°$, $\omega = 4$ rad/s, then substitute these values into Eq.[3] yields

$$v = 0.12(4)\cos 30° = 0.416 \text{ m/s} \qquad \textbf{Ans.}$$

Taking the time derivative of Eq. [3], we have

$$\frac{dv}{dt} = 0.12\left[\omega(-\sin\theta)\frac{d\theta}{dt} + \cos\theta\frac{d\omega}{dt}\right]$$

$$= 0.12\left(\cos\theta\frac{d\omega}{dt} - \omega^2 \sin\theta\right) \qquad \textbf{[4]}$$

However $a = \dfrac{dv}{dt}$ and $\alpha = \dfrac{d\omega}{dt}$. From Eq.[4],

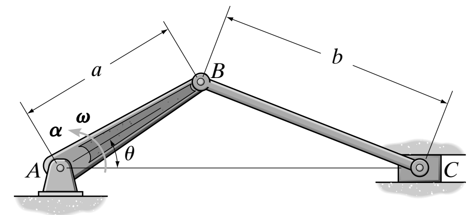
$$a = 0.12\left(\alpha\cos\theta - \omega^2\sin\theta\right) \qquad \textbf{[5]}$$

At the instant $\theta = 30°$, $\omega = 4$ rad/s and $\alpha = 2$ rad/s^2, then substitute these values into Eq.[5] yields

$$a = 0.12\left(2\cos 30° - 4^2 \sin 30°\right) = -0.752 \text{ m/s}^2 \qquad \textbf{Ans.}$$

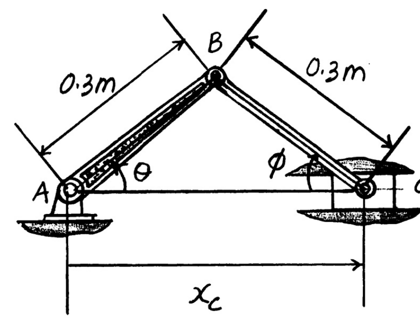
Note: Negative sign indicates that a is directed in the opposite direction to that of positive x.

•16–45. At the instant $\theta = 30°$, crank AB rotates with an angular velocity and angular acceleration of $\omega = 10$ rad/s and $\alpha = 2$ rad/s^2, respectively. Determine the velocity and acceleration of the slider block C at this instant. Take $a = b = 0.3$ m.



Position Coordinates: Due to symmetry, $\phi = \theta$. Thus, from the geometry shown in Fig. a,

$$x_C = 2[0.3\cos\theta]\text{m} = 0.6\cos\theta \text{ m}$$



(a)

Time Derivative: Taking the time derivative,

$$v_C = \dot{x}_C = \left(-0.6\sin\theta\dot{\theta}\right)\text{m/s} \qquad \textbf{(1)}$$

When $\theta = 30°$, $\dot{\theta} = \omega = 10$ rad/s Thus,

$$v_C = -0.6\sin 30°(10) = -3 \text{ m/s} = 3 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

The time derivative of Eq. (1) gives

$$a_C = \ddot{x}_C = -0.6\left(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2\right) \text{ m/s}^2$$

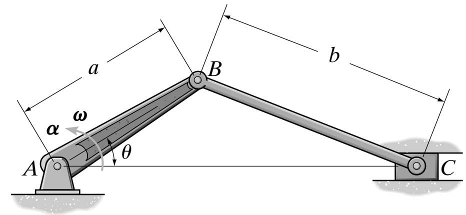
When $\theta = 30°$, $\ddot{\theta} = \alpha = 2$ rad/s^2, and $\dot{\theta} = 10$ rad/s. Thus,

$$a_C = -0.6\left[\sin 30°(2) + \cos 30°(10^2)\right]$$

$$= -52.6 \text{ m/s}^2 = 52.6 \text{ m/s}^2 \leftarrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{v}_C$ and $\mathbf{a}_C$ are in the negative sense of x_C.

16–46. At the instant $\theta = 30°$, crank AB rotates with an angular velocity and angular acceleration of $\omega = 10$ rad/s and $\alpha = 2$ rad/s^2, respectively. Determine the angular velocity and angular acceleration of the connecting rod BC at this instant. Take $a = 0.3$ m and $b = 0.5$ m.



Position Coordinates: The angles θ and ϕ can be related using the law of sines and referring to the geometry shown in Fig. *a*.

$$\frac{\sin\phi}{0.3} = \frac{\sin\theta}{0.5}$$

$$\sin\phi = 0.6\sin\theta \qquad \textbf{(1)}$$

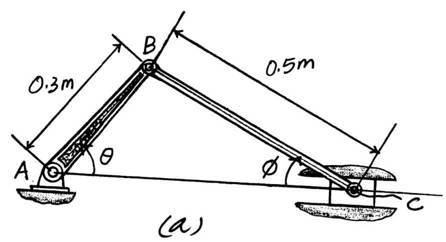
When $\theta = 30°$,

$$\phi = \sin^{-1}(0.6\sin 30°) = 17.46°$$

Time Derivative: Taking the time derivative of Eq. (1),

$$\cos\phi\dot{\phi} = 0.6\cos\theta\dot{\theta} \qquad \textbf{(2)}$$

$$\omega_{BC} = \dot{\phi} = \frac{0.6\cos\theta}{\cos\phi}\dot{\theta}$$



When $\theta = 30°$, $\phi = 17.46°$ and $\dot{\theta} = 10$ rad/s,

$$\omega_{BC} = \dot{\phi} = \frac{0.6\cos 30°}{\cos 17.46°}(10) = 5.447 \text{ rad/s} = 5.45 \text{ rad/s} \qquad \textbf{Ans.}$$

The time derivative of Eq. (2) gives

$$\cos\phi\ddot{\phi} - \sin\phi\dot{\phi}^2 = 0.6\left(\cos\theta\ddot{\theta} - \sin\theta\dot{\theta}^2\right)$$

$$\alpha_{BC} = \ddot{\phi} = \frac{0.6\left(\cos\theta\ddot{\theta} - \sin\theta\dot{\theta}^2\right) + \sin\phi\dot{\phi}^2}{\cos\phi}$$

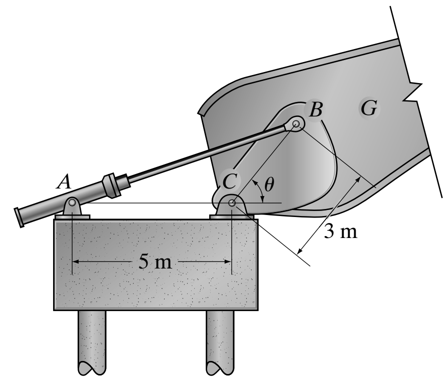
When $\theta = 30°$, $\phi = 17.46°$, $\dot{\theta} = 10$ rad/s, $\dot{\phi} = 5.447$ rad/s and $\ddot{\theta} = \alpha = 2$ rad/s^2,

$$\alpha_{BC} = \frac{0.6\left[\cos 30°(2) - \sin 30°(10^2)\right] + \sin 17.46°(5.447^2)}{\cos 17.46°}$$

$$= -21.01 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

The negative sign indicates that α_{BC} acts counterclockwise.

16–47. The bridge girder G of a bascule bridge is raised and lowered using the drive mechanism shown. If the hydraulic cylinder AB shortens at a constant rate of 0.15 m/s, determine the angular velocity of the bridge girder at the instant $\theta = 60°$.



Position Coordinates: Applying the law of cosines to the geometry shown in Fig. a,

$$s^2 = 3^2 + 5^2 - 2(3)(5)\cos\left(180°-\theta\right)$$

$$s^2 = 34 - 30\cos\left(180°-\theta\right)$$

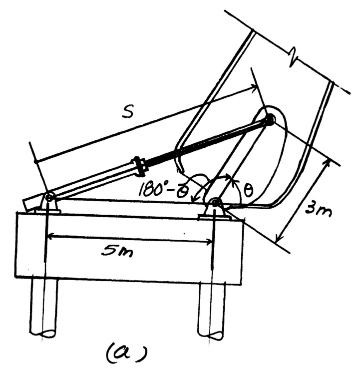
However, $\cos\left(180°-\theta\right) = -\cos\theta$. Thus,

$$s^2 = 34 + 30\cos\theta$$

Time Derivatives: Taking the time derivative,

$$2s\dot{s} = 0 + 30\left(-\sin\theta\dot{\theta}\right)$$

$$s\dot{s} = -15\sin\theta\dot{\theta} \qquad \textbf{(1)}$$

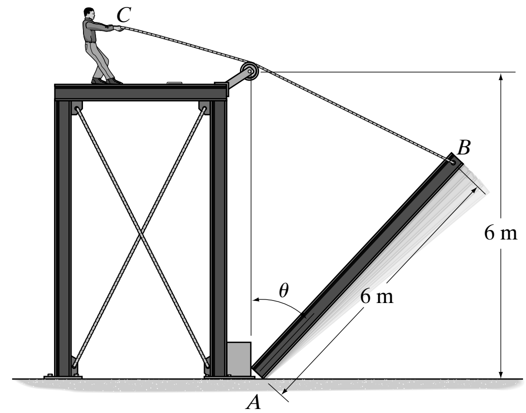


When $\theta = 60°$, $s = \sqrt{34+30\cos 60°} = 7$ m. Also, $\dot{s} = -0.15$ m/s since $\dot{s}$ is directed towards the negative sense of s. Thus, Eq. (1) gives

$$7\left(-0.15\right) = -15\sin 60°\dot{\theta}$$

$$\omega = \dot{\theta} = 0.0808 \text{ rad/s} \qquad \textbf{Ans.}$$

***16–48.** The man pulls on the rope at a constant rate of 0.5 m/s. Determine the angular velocity and angular acceleration of beam AB when $\theta = 60°$. The beam rotates about A. Neglect the thickness of the beam and the size of the pulley.



Position Coordinates: Applying the law of cosines to the geometry,

$$s^2 = 6^2 + 6^2 - 2(6)(6)\cos\theta$$

$$s^2 = \left(72 - 72\cos\theta\right)\mathrm{m}^2$$

Time Derivatives: Taking the time derivative,

$$2s\dot{s} = 0 - 72\left(-\sin\theta\dot{\theta}\right)$$

$$s\dot{s} = 36\sin\theta\dot{\theta} \qquad \textbf{(1)}$$

Here, $\dot{s} = -0.5\ \mathrm{m/s}$ since $\dot{s}$ acts in the negative sense of s. When $\theta = 60°$, $s = \sqrt{72 - 72\cos 60°} = 6$ m. Thus, Eq. (1) gives

$$6\left(-0.5\right) = 36\sin 60°\dot{\theta}$$

$$\omega = \dot{\theta} = -0.09623\ \mathrm{rad/s} - 0.0962\ \mathrm{rad/s} \qquad \textbf{Ans.}$$

The negative sign indicates that ω acts in the negative rotational sense of θ. The time derivative of Eq.(1) gives

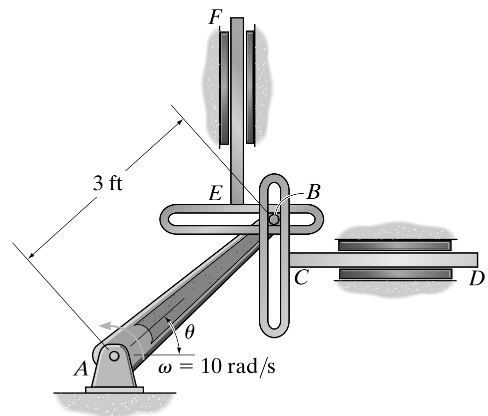
$$s\ddot{s} + \dot{s}^2 = 36\left(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2\right) \qquad \textbf{(2)}$$

Since $\dot{s}$ is constant, $\ddot{s} = 0$. When $\theta = 60°$.

$$6(0) + (-0.5)^2 = 36\left[\sin 60°\,\ddot{\theta} + \cos 60°\left(-0.09623\right)^2\right]$$

$$\alpha = \ddot{\theta} = 0.00267\ \mathrm{rad/s^2} \qquad \textbf{Ans.}$$

•16–49. Peg B attached to the crank AB slides in the slots mounted on follower rods, which move along the vertical and horizontal guides. If the crank rotates with a constant angular velocity of $\omega = 10$ rad/s, determine the velocity and acceleration of rod CD at the instant $\theta = 30°$.



Position Coordinates: From the geometry shown in Fig.*a*,

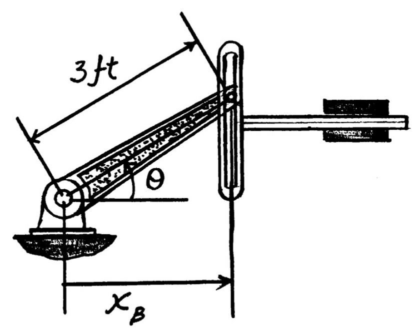
$$x_B = 3\cos\theta \text{ ft}$$

Time Derivative: Taking the time derivative,

$$v_{CD} = \dot{x}_B = -3\sin\theta\dot{\theta} \text{ ft/s} \qquad \textbf{(1)}$$

Here, $\dot{\theta} = \omega = 10$ rad/s since ω acts in the positive rotational sense of θ. When $\theta = 30°$,

$$v_{CD} = -3\sin 30°\,(10) = -15 \text{ ft/s} = 15 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$



(a)

Taking the time derivative of Eq.(1) gives

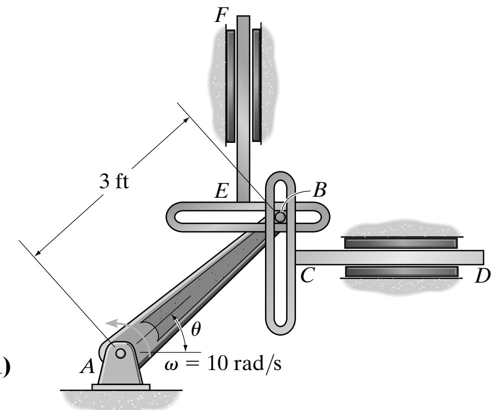
$$a_{CD} = \ddot{x}_B = -3\left(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2\right)$$

Since ω is constant, $\ddot{\theta} = \alpha = 0$. When $\theta = 30°$,

$$a_{CD} = -3\left[\sin 30°(0) + \cos 30°\,(10^2)\right]$$
$$= -259.80 \text{ ft/s} = 260 \text{ ft/s}^2 \leftarrow \qquad \textbf{Ans.}$$

The negative signs indicates that $\mathbf{v}_{CD}$ and $\mathbf{a}_{CD}$ act towards the negative sense of x_B.

16–50. Peg B attached to the crank AB slides in the slots mounted on follower rods, which move along the vertical and horizontal guides. If the crank rotates with a constant angular velocity of $\omega = 10$ rad/s, determine the velocity and acceleration of rod EF at the instant $\theta = 30°$.



Position Coordinates: From the geometry shown in Fig.a,

$$y_B = 3 \sin \theta \text{ ft}$$

Time Derivatives: Taking the time derivative,

$$v_{EF} = \dot{y}_B = 3 \cos \theta \dot{\theta} \text{ ft/s} \qquad \textbf{(1)}$$

Here, $\dot{\theta} = \omega = 10$ rad/s since ω acts in the positive rotational sense of θ. When $\theta = 30°$,

$$v_{EF} = 3 \cos 30°(10) = 25.98 \text{ ft/s} = 26 \text{ ft/s}\uparrow \qquad \textbf{Ans.}$$

The time derivative of Eq.(1) gives

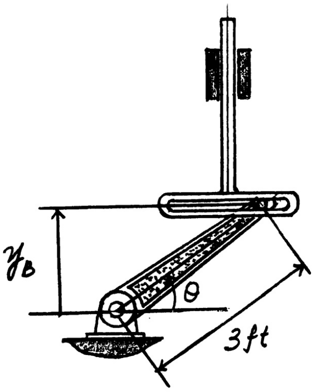
$$a_{EF} = \ddot{y}_B = 3\left[\cos \theta \ddot{\theta} - \sin \theta \dot{\theta}^2\right] \text{ft/s}^2$$

Since ω is constant, $\ddot{\theta} = \alpha = 0$. When $\theta = 30°$,

$$a_{EF} = 3\left[\cos 30°(0) - \sin 30°(10^2)\right]$$

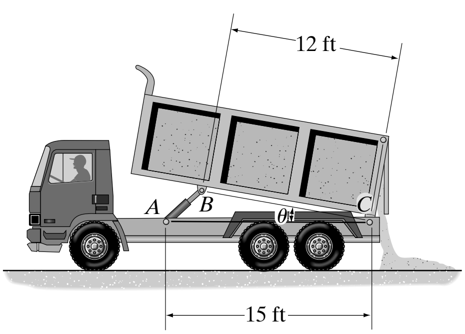
$$= -150 \text{ ft/s}^2 = 150 \text{ ft/s}^2\downarrow \qquad \textbf{Ans.}$$

The negative signs indicates that $\mathbf{a}_{EF}$ acts towards the negative sense of y_B.



(a)

16–51. If the hydraulic cylinder AB is extending at a constant rate of 1 ft/s, determine the dumpster's angular velocity at the instant $\theta = 30°$.



Position Coordinates: Applying the law of cosines to the geometry shown in Fig. a,

$$s^2 = 15^2 + 12^2 - 2(15)(12) \cos \theta$$

$$s^2 = (369 - 360 \cos \theta) \text{ ft}^2 \qquad \textbf{(1)}$$

Time Derivatives: Taking the time derivative,

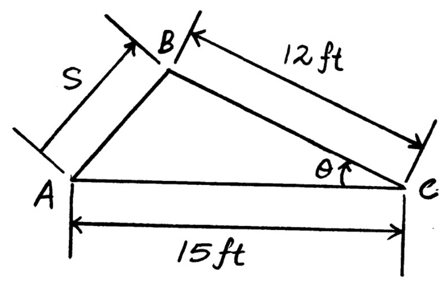
$$2s\dot{s} = 360 \sin \theta \dot{\theta}$$

$$s\dot{s} = 180 \sin \theta \dot{\theta} \qquad \textbf{(2)}$$

$\dot{s} = +1$ ft/s since the hydraulic cylinder is extending towards the positive sense of s. When $\theta = 30°$, from Eq. (1), $s = \sqrt{369 - 360 \cos 30°} = 7.565$ ft. Thus, Eq.(2) gives

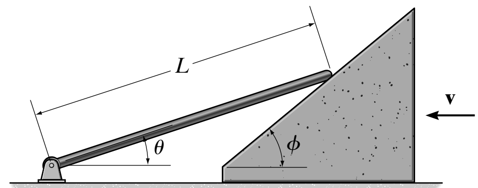
$$7.565(1) = 180 \sin 30° \dot{\theta}$$

$$\dot{\theta} = 0.0841 \text{ rad/s} \qquad \textbf{Ans.}$$



(a)

***16–52.** If the wedge moves to the left with a constant velocity **v**, determine the angular velocity of the rod as a function of θ.



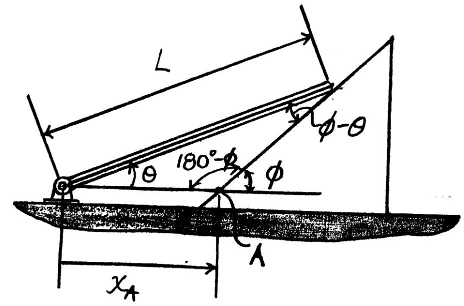
Position Coordinates: Applying the law of sines to the geometry shown in Fig. *a*,

$$\frac{x_A}{\sin(\phi - \theta)} = \frac{L}{\sin\left(180^\circ - \phi\right)}$$

$$x_A = \frac{L\sin(\phi - \theta)}{\sin\left(180^\circ - \phi\right)}$$

However, $\sin\left(180^\circ - \phi\right) = \sin\phi$. Therefore,

$$x_A = \frac{L\sin(\phi - \theta)}{\sin\phi}$$



(a)

Time Derivative: Taking the time derivative,

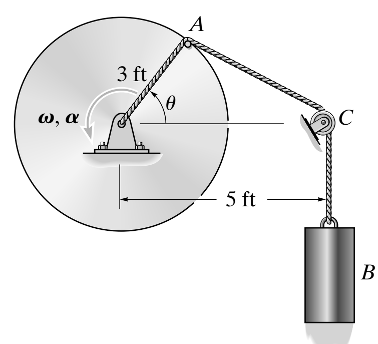
$$\dot{x}_A = \frac{L\cos(\phi - \theta)(-\dot{\theta})}{\sin\phi}$$

$$v_A = \dot{x}_A = -\frac{L\cos(\phi - \theta)\dot{\theta}}{\sin\phi} \qquad \textbf{(1)}$$

Since point A is on the wedge, its velocity is $v_A = -v$. The negative sign indicates that $\mathbf{v}_A$ is directed towards the negative sense of x_A. Thus, Eq. (1) gives

$$\dot{\theta} = \frac{v\sin\phi}{L\cos(\phi - \theta)} \qquad \textbf{Ans.}$$

•16–53. At the instant shown, the disk is rotating with an angular velocity of $\boldsymbol{\omega}$ and has an angular acceleration of $\boldsymbol{\alpha}$. Determine the velocity and acceleration of cylinder B at this instant. Neglect the size of the pulley at C.



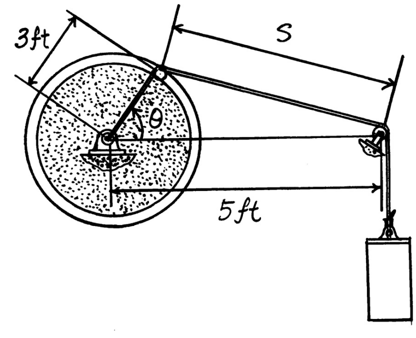
$$s = \sqrt{3^2 + 5^2 - 2(3)(5)\cos\theta}$$

$$v_B = \dot{s} = \frac{1}{2}(34 - 30\cos\theta)^{-\frac{1}{2}}(30\sin\theta)\dot{\theta}$$

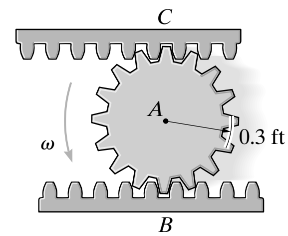
$$v_B = \frac{15\,\omega\sin\theta}{(34 - 30\cos\theta)^{\frac{1}{2}}} \qquad \textbf{Ans.}$$

$$a_B = \dot{s} = \frac{15\,\omega\cos\theta\dot{\theta} + 15\dot{\omega}\sin\theta}{\sqrt{34 - 30\cos\theta}} + \frac{\left(-\frac{1}{2}\right)(15\omega\sin\theta)\left(30\sin\theta\dot{\theta}\right)}{(34 - 30\cos\theta)^{\frac{3}{2}}}$$

$$= \frac{15\,(\omega^2\cos\theta + \alpha\sin\theta)}{(34 - 30\cos\theta)^{\frac{1}{2}}} - \frac{225\,\omega^2\sin^2\theta}{(34 - 30\cos\theta)^{\frac{3}{2}}} \qquad \textbf{Ans.}$$



16–54. Pinion gear A rolls on the fixed gear rack B with an angular velocity $\omega = 4$ rad/s. Determine the velocity of the gear rack C.

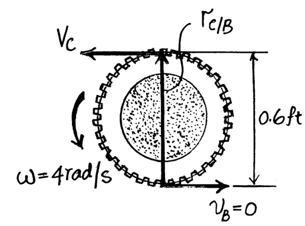


$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$(\overset{+}{\leftarrow})$ $\quad v_C = 0 + 4(0.6)$

$v_C = 2.40$ ft/s **Ans.**

Also:

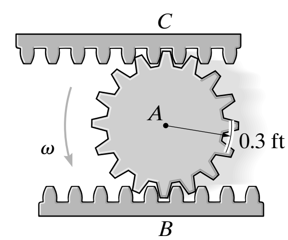


$$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$$

$$-v_C\,\mathbf{i} = 0 + (4\mathbf{k}) \times (0.6\mathbf{j})$$

$v_C = 2.40$ ft/s **Ans.**

16–55. Pinion gear A rolls on the gear racks B and C. If B is moving to the right at 8 ft/s and C is moving to the left at 4 ft/s, determine the angular velocity of the pinion gear and the velocity of its center A.



$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$(\overset{+}{\rightarrow})$ $\quad -4 = 8 - 0.6(\omega)$

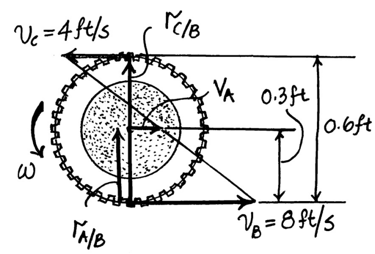
$\omega = 20$ rad/s **Ans.**

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$(\overset{+}{\rightarrow})$ $\quad v_A = 8 - 20(0.3)$

$v_A = 2$ ft/s $\rightarrow$ **Ans.**

Also,



$$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$$

$$-4\mathbf{i} = 8\mathbf{i} + (\omega\mathbf{k}) \times (0.6\mathbf{j})$$

$$-4 = 8 - 0.6\omega$$

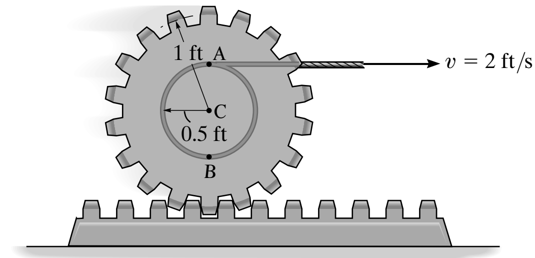
$\omega = 20$ rad/s **Ans.**

$$\mathbf{v}_A = \mathbf{v}_B + \omega \times \mathbf{r}_{A/B}$$

$$v_A\,\mathbf{i} = 8\mathbf{i} + 20\mathbf{k} \times (0.3\mathbf{j})$$

$v_A = 2$ ft/s $\rightarrow$ **Ans.**

***16–56.** The gear rests in a fixed horizontal rack. A cord is wrapped around the inner core of the gear so that it remains horizontally tangent to the inner core at A. If the cord is pulled to the right with a constant speed of 2 ft/s, determine the velocity of the center of the gear, C.



$$\mathbf{v}_A = \mathbf{v}_D + \mathbf{v}_{A/D}$$

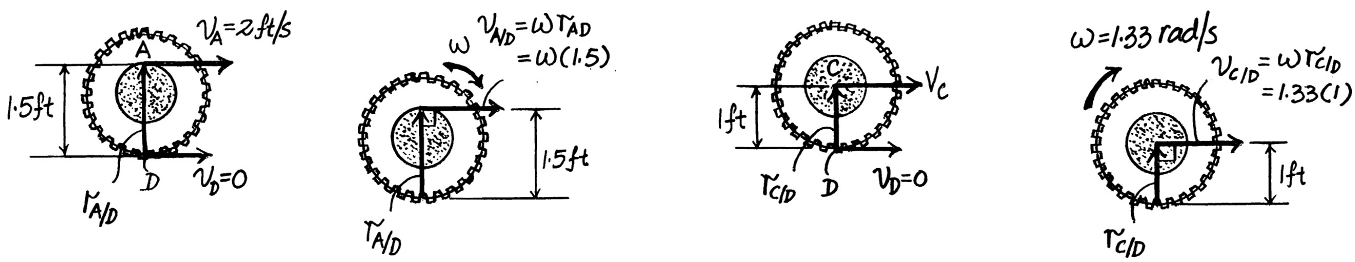
$$\left[\underset{\rightarrow}{2}\right] = 0 + \left[\underset{\rightarrow}{\omega(1.5)}\right]$$

$(\overset{+}{\rightarrow})$ $\quad 2 = 1.5\omega \qquad \omega = 1.33 \text{ rad/s}$

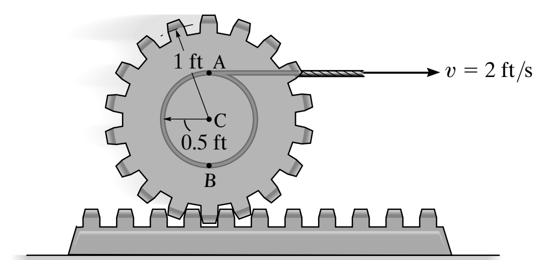
$$\mathbf{v}_C = \mathbf{v}_D + \mathbf{v}_{C/D}$$

$$\left[\underset{\rightarrow}{v_C}\right] = 0 + \left[\underset{\rightarrow}{1.33\,(1)}\right]$$

$(\overset{+}{\rightarrow})$ $\quad v_C = 1.33 \text{ ft/s} \rightarrow$ **Ans.**



•16–57. Solve Prob. 16–56 assuming that the cord is wrapped around the gear in the opposite sense, so that the end of the cord remains horizontally tangent to the inner core at B and is pulled to the right at 2 ft/s.



$$\mathbf{v}_B = \mathbf{v}_D + \mathbf{v}_{B/D}$$

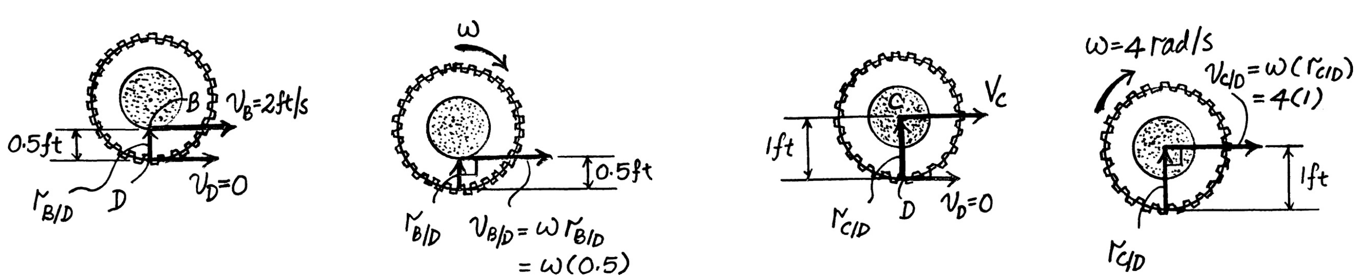
$$\left[\underset{\rightarrow}{2}\right] = 0 + \left[\underset{\rightarrow}{\omega(0.5)}\right]$$

$(\overset{+}{\rightarrow})$ $\quad 2 = 0.5\omega \qquad \omega = 4 \text{ rad/s}$

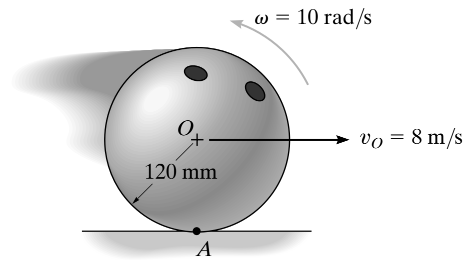
$$\mathbf{v}_C = \mathbf{v}_D + \mathbf{v}_{C/D}$$

$$\left[\underset{\rightarrow}{v_C}\right] = 0 + \left[\underset{\rightarrow}{4\,(1)}\right]$$

$(\overset{+}{\rightarrow})$ $\quad v_C = 4 \text{ ft/s} \rightarrow$ **Ans.**



16–58. A bowling ball is cast on the "alley" with a backspin of $\omega = 10$ rad/s while its center O has a forward velocity of $v_O = 8$ m/s. Determine the velocity of the contact point A in contact with the alley.

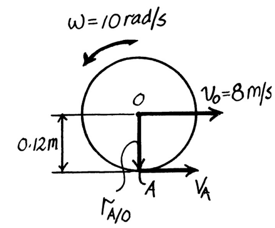


$$\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A/O}$$

$$(\overset{+}{\rightarrow}) \qquad v_A = 8 + 10(0.12)$$

$$v_A = 9.20 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$

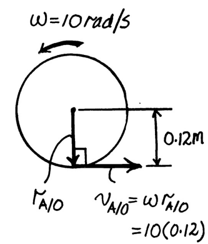
Also,



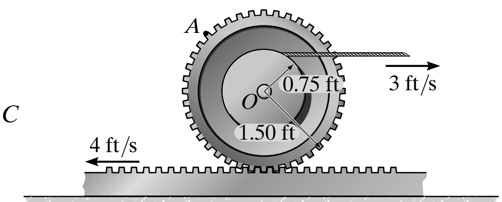
$$\mathbf{v}_A = \mathbf{v}_O + \omega \times \mathbf{r}_{A/O}$$

$$v_A\,\mathbf{i} = 8\mathbf{i} + (10\mathbf{k}) \times (-0.12\mathbf{j})$$

$$(\overset{+}{\rightarrow}) \qquad v_A = 9.20 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$



16–59. Determine the angular velocity of the gear and the velocity of its center O at the instant shown.



General Plane Motion: Applying the relative velocity equation to points B and C and referring to the kinematic diagram of the gear shown in Fig. a,

$$\mathbf{v}_B = \mathbf{v}_C + \omega \times \mathbf{r}_{B/C}$$

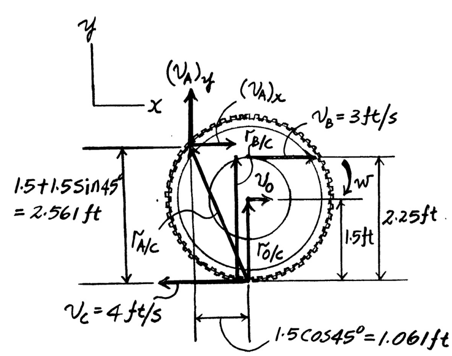
$$3\mathbf{i} = -4\mathbf{i} + \left(-\omega\mathbf{k}\right) \times \left(2.25\mathbf{j}\right)$$

$$3\mathbf{i} = \left(2.25\omega - 4\right)\mathbf{i}$$

Equating the **i** components yields

$$3 = 2.25\omega - 4 \qquad \textbf{(1)}$$

$$\omega = 3.111 \text{ rad/s} \qquad \textbf{Ans. (2)}$$



(a)

For points O and C,

$$\mathbf{v}_O = \mathbf{v}_C + \omega \times \mathbf{r}_{O/C}$$

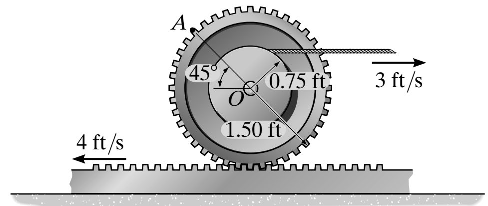
$$= -4\mathbf{i} + \left(-3.111\mathbf{k}\right) \times \left(1.5\mathbf{j}\right)$$

$$= [0.6667\mathbf{i}] \text{ ft/s}$$

Thus,

$$v_O = 0.667 \text{ ft/s} \rightarrow \qquad \textbf{Ans.}$$

***16–60.** Determine the velocity of point A on the rim of the gear at the instant shown.



General Plane Motion: Applying the relative velocity equation to points B and C and referring to the kinematic diagram of the gear shown in Fig. a,

$$\mathbf{v}_B = \mathbf{v}_C + \omega \times \mathbf{r}_{B/C}$$

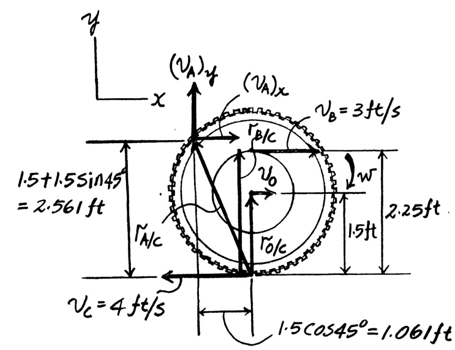
$$3\mathbf{i} = -4\mathbf{i} + \left(-\omega\mathbf{k}\right) \times \left(2.25\mathbf{j}\right)$$

$$3\mathbf{i} = \left(2.25\omega - 4\right)\mathbf{i}$$

Equating the $\mathbf{i}$ components yields

$$3 = 2.25\omega - 4 \qquad \textbf{(1)}$$

$$\omega = 3.111 \text{ rad/s} \qquad \textbf{(2)}$$



(a)

For points A and C,

$$\mathbf{v}_A = \mathbf{v}_C + \omega \times \mathbf{r}_{A/C}$$

$$\left(v_A\right)_x \mathbf{i} + \left(v_A\right)_y \mathbf{j} = -4\mathbf{i} + \left(-3.111\mathbf{k}\right) \times \left(-1.061\mathbf{i} + 2.561\mathbf{j}\right)$$

$$\left(v_A\right)_x \mathbf{i} + \left(v_A\right)_y \mathbf{j} = 3.9665\mathbf{i} + 3.2998\mathbf{j}$$

Equating the $\mathbf{i}$ and $\mathbf{j}$ components yields

$$\left(v_A\right)_x = 3.9665 \text{ ft/s} \qquad \left(v_A\right)_y = 3.2998 \text{ ft/s}$$

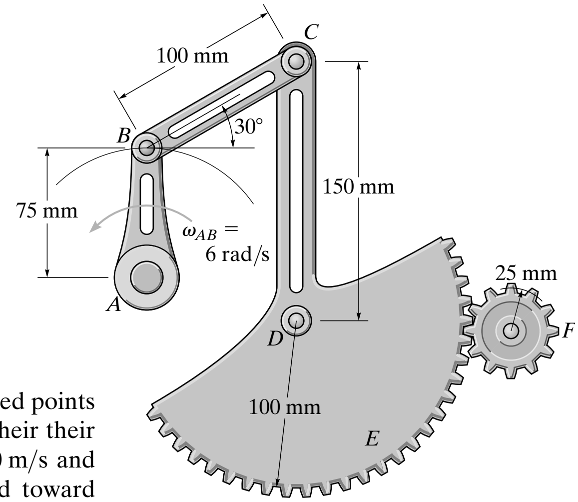
Thus, the magnitude of v_A is

$$v_A = \sqrt{\left(v_A\right)_x^{\,2} + \left(v_A\right)_y^{\,2}} = \sqrt{3.9665^2 + 3.2998^2} = 5.16 \text{ ft/s} \qquad \textbf{Ans.}$$

and its direction is

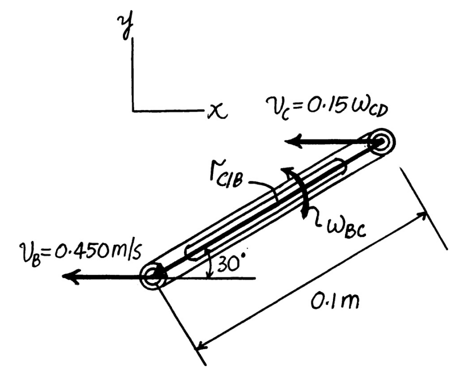
$$\theta = \tan^{-1}\left[\frac{\left(v_A\right)_y}{\left(v_A\right)_x}\right] = \tan^{-1}\left(\frac{3.2998}{3.9665}\right) = 39.8° \qquad \textbf{Ans.}$$

•16–61. The rotation of link AB creates an oscillating movement of gear F. If AB has an angular velocity of $\omega_{AB} = 6$ rad/s, determine the angular velocity of gear F at the instant shown. Gear E is rigidly attached to arm CD and pinned at D to a fixed point.



Kinematic Diagram: Since link AB and arm CD are rotating about the fixed points A and D respectively, then $\mathbf{v}_B$ and $\mathbf{v}_C$ are always directed perpendicular their their respective arms with the magnitude of $v_B = \omega_{AB}\, r_{AB} = 6(0.075) = 0.450$ m/s and $v_C = \omega_{CD}\, r_{CD} = 0.15\omega_{CD}$. At the instant shown, $\mathbf{v}_B$ and $\mathbf{v}_C$ are directed toward negative x axis.

Velocity Equation: Here, $\mathbf{r}_{B/C} = \{-0.1 \cos 30°\mathbf{i} + 0.1 \sin 30°\mathbf{j}\}$ m $= \{-0.08660\mathbf{i} + 0.05\mathbf{j}\}$ m. Applying Eq. 16–16, we have



$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$$

$$-0.450\mathbf{i} = -0.15\omega_{CD}\,\mathbf{i} + (\omega_{BC}\mathbf{k}) \times (0.08660\mathbf{i} + 0.05\mathbf{j})$$

$$-0.450\mathbf{i} = -(0.05\omega_{BC} + 0.15\omega_{CD})\mathbf{i} + 0.08660\omega_{BC}\mathbf{j}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components gives

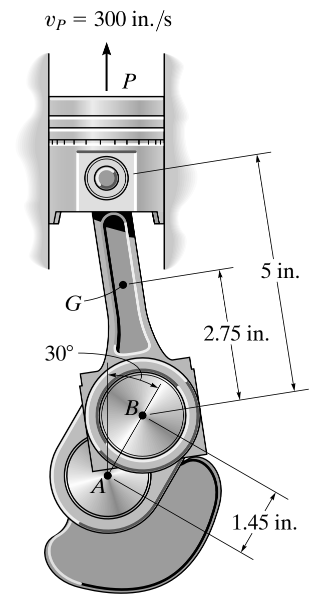
$$0 = 0.08660\omega_{BC} \qquad \omega_{BC} = 0$$

$$-0.450 = -[0.05(0) + 0.15\omega_{CD}] \qquad \omega_{CD} = 3.00 \text{ rad/s}$$

Angular Motion About a Fixed Point: The angular velocity of gear E is the same with arm CD since they are attached together. Then, $\omega_E = \omega_{CD} = 3.00$ rad/s. Here, $\omega_E\, r_E = \omega_F\, r_F$ where ω_F is the angular velocity of gear F.

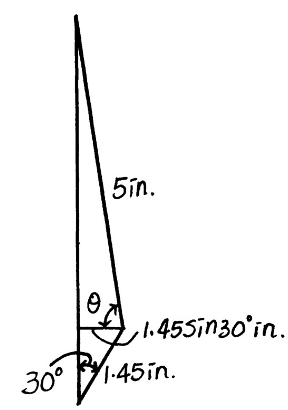
$$\omega_F = \frac{r_E}{r_F}\,\omega_E = \left(\frac{100}{25}\right)(3.00) = 12.0 \text{ rad/s} \qquad \textbf{Ans.}$$

16–62. Piston P moves upward with a velocity of 300 in./s at the instant shown. Determine the angular velocity of the crankshaft AB at this instant.



From the geometry:

$$\cos\theta = \frac{1.45\sin 30^\circ}{5} \qquad \theta = 81.66^\circ$$



For link BP

$\mathbf{v}_P = \{300\mathbf{j}\}$ in/s $\qquad \mathbf{v}_B = -v_B\cos 30^\circ\mathbf{i} + v_B\sin 30^\circ\mathbf{j} \qquad \omega = -\omega_{BP}\mathbf{k}$

$\mathbf{r}_{P/B} = \{-5\cos 81.66^\circ\mathbf{i} + 5\sin 81.66^\circ\mathbf{j}\}$ in.

$\mathbf{v}_P = \mathbf{v}_B + \omega \times \mathbf{r}_{P/B}$

$300\mathbf{j} = (-v_B\cos 30^\circ\mathbf{i} + v_B\sin 30^\circ\mathbf{j}) + (-\omega_{BP}\mathbf{k}) \times (-5\cos 81.66^\circ\mathbf{i} + 5\sin 81.66^\circ\mathbf{j})$

$300\mathbf{j} = (-v_B\cos 30^\circ\mathbf{i} + 5\sin 81.66^\circ\omega_{BP})\mathbf{i} + (v_B\sin 30^\circ + 5\cos 81.66^\circ\,\omega_{BP})\mathbf{j}$

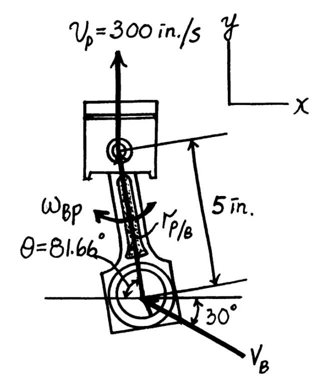
Equating the **i** and **j** components yields:

$$0 = -v_B\cos 30^\circ + 5\sin 81.66^\circ\,\omega_{BP} \qquad \textbf{(1)}$$

$$300 = v_B\sin 30^\circ + 5\cos 81.66^\circ\,\omega_{BP} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$\omega_{BP} = 83.77\text{ rad/s} \qquad v_B = 478.53\text{ in./s}$$

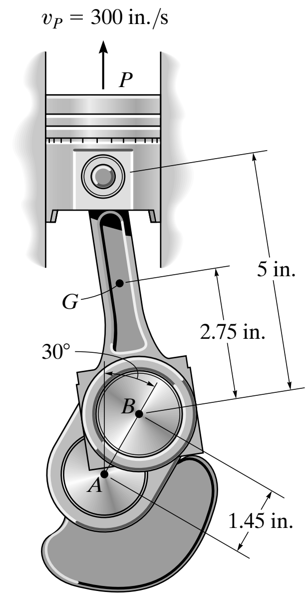


For crankshaft AB: Crankshaft AB rotates about the fixed point A. Hence

$$v_B = \omega_{AB}\,r_{AB}$$

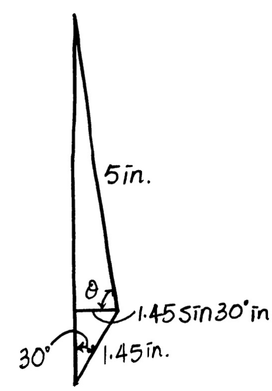
$$478.53 = \omega_{AB}(1.45) \qquad \omega_{AB} = 330\text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

16–63. Determine the velocity of the center of gravity G of the connecting rod at the instant shown. Piston P is moving upward with a velocity of 300 in./s.



From the geometry:

$$\cos\theta = \frac{1.45 \sin 30^\circ}{5} \qquad \theta = 81.66^\circ$$



For link BP

$\mathbf{v}_P = \{300\mathbf{j}\}$ in/s $\qquad \mathbf{v}_B = -v_B \cos 30^\circ \mathbf{i} + v_B \sin 30^\circ \mathbf{j} \qquad \omega = -\omega_{BP}\mathbf{k}$

$\mathbf{r}_{P/B} = \{-5 \cos 81.66^\circ \mathbf{i} + 5 \sin 81.66^\circ \mathbf{j}\}$ in.

$\mathbf{v}_P = \mathbf{v}_B + \omega \times \mathbf{r}_{P/B}$

$300\mathbf{j} = (-v_B \cos 30^\circ \mathbf{i} + v_B \sin 30^\circ \mathbf{j}) + (-\omega_{BP}\mathbf{k}) \times (-5 \cos 81.66^\circ \mathbf{i} + 5 \sin 81.66^\circ \mathbf{j})$

$300\mathbf{j} = (-v_B \cos 30^\circ + 5 \sin 81.66^\circ\, \omega_{BP})\mathbf{i} + (v_B \sin 30^\circ + 5 \cos 81.66^\circ\, \omega_{BP})\mathbf{j}$

Equating the **i** and **j** components yields:

$$0 = -v_B \cos 30^\circ + 5 \sin 81.66^\circ\, \omega_{BP} \qquad \textbf{(1)}$$

$$300 = v_B \sin 30^\circ + 5 \cos 81.66^\circ\, \omega_{BP} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$\omega_{BP} = 83.77$ rad/s $\qquad v_B = 478.53$ in./s

$\mathbf{v}_P = \{300\mathbf{j}\}$ in/s $\qquad \omega = \{-83.77\mathbf{k}\}$ rad/s

$\mathbf{r}_{G/P} = \{2.25 \cos 81.66^\circ \mathbf{i} - 2.25 \sin 81.66^\circ \mathbf{j}\}$ in.

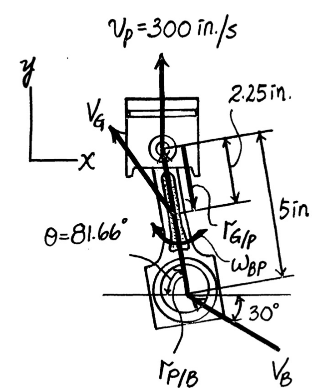
$\mathbf{v}_G = \mathbf{v}_P + \omega \times \mathbf{r}_{G/P}$

$= 300\mathbf{j} + (-83.77\mathbf{k}) \times (2.25 \cos 81.66^\circ \mathbf{i} - 2.25 \sin 81.66^\circ \mathbf{j})$

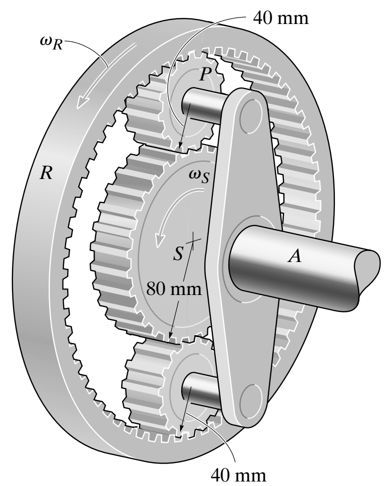
$= \{-186.49\mathbf{i} + 272.67\mathbf{j}\}$ in./s

$$v_G = \sqrt{(-186.49)^2 + 272.67^2} = 330 \text{ in./s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{272.67}{186.49}\right) = 55.6^\circ \qquad \textbf{Ans.}$$



***16–64.** The planetary gear system is used in an automatic transmission for an automobile. By locking or releasing certain gears, it has the advantage of operating the car at different speeds. Consider the case where the ring gear R is held fixed, $\omega_R = 0$, and the sun gear S is rotating at $\omega_S = 5$ rad/s. Determine the angular velocity of each of the planet gears P and shaft A.



$v_A = 5(80) = 400$ mm/s $\leftarrow$

$v_B = 0$

$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$

$0 = -400\mathbf{i} + (\omega_p\,\mathbf{k}) \times (80\mathbf{j})$

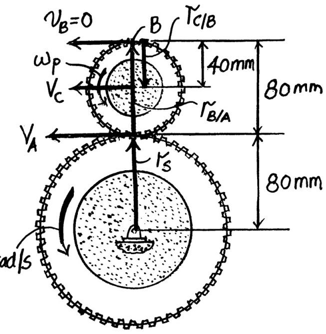
$0 = -400\mathbf{i} - 80\omega_p\,\mathbf{i}$

$\omega_P = -5 \text{ rad/s} = 5 \text{ rad/s}$ **Ans.**

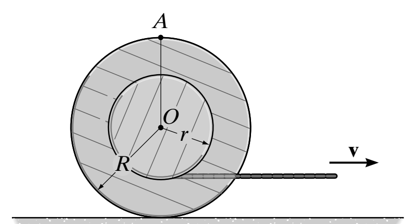
$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$

$\mathbf{v}_C = 0 + (-5\mathbf{k}) \times (-40\mathbf{j}) = -200\mathbf{i}$

$\omega_A = \dfrac{200}{120} = 1.67 \text{ rad/s}$ **Ans.**



•16–65. Determine the velocity of the center O of the spool when the cable is pulled to the right with a velocity of $\mathbf{v}$. The spool rolls without slipping.



Kinematic Diagram: Since the spool rolls without slipping, the velocity of the contact point P is zero. The kinematic diagram of the spool is shown in Fig. a.

General Plane Motion: Applying the relative velocity equation and referring to Fig. a,

$$\mathbf{v}_B = \mathbf{v}_P + \omega \times \mathbf{r}_{B/D}$$

$$v\mathbf{i} = \mathbf{0} + (-\omega\mathbf{k}) \times \left[(R - r)\mathbf{j}\right]$$

$$v\mathbf{i} = \omega(R - r)\mathbf{i}$$

Equating the $\mathbf{i}$ components, yields

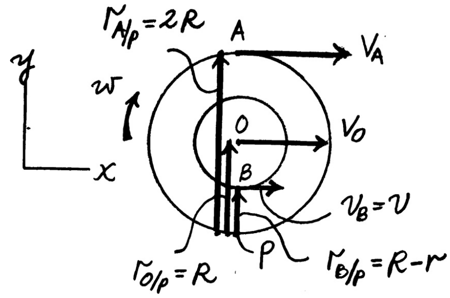
$$v = \omega(R - r) \qquad \omega = \frac{v}{R - r}$$

Using this result,

$$\mathbf{v}_O = \mathbf{v}_P + \omega \times \mathbf{r}_{O/P}$$

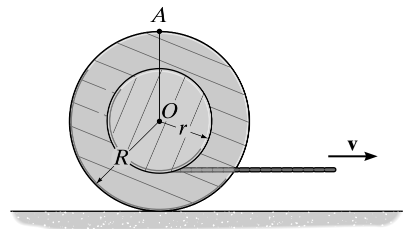
$$= \mathbf{0} + \left(-\frac{v}{R - r}\mathbf{k}\right) \times R\mathbf{j}$$

$$\mathbf{v}_O = \left(\frac{R}{R - r}\right)v \rightarrow$$ **Ans.**



(a)

16–66. Determine the velocity of point A on the outer rim of the spool at the instant shown when the cable is pulled to the right with a velocity of $\mathbf{v}$. The spool rolls without slipping.



Kinematic Diagram: Since the spool rolls without slipping, the velocity of the contact point P is zero. The kinematic diagram of the spool is shown in Fig. a.

General Plane Motion: Applying the relative velocity equation and referring to Fig. a,

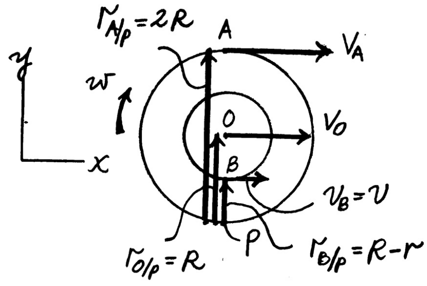
$$\mathbf{v}_B = \mathbf{v}_P + \omega \times \mathbf{r}_{B/D}$$

$$v\mathbf{i} = \mathbf{0} + (-\omega\mathbf{k}) \times \left[(R - r)\mathbf{j}\right]$$

$$v\mathbf{i} = \omega(R - r)\mathbf{i}$$

Equating the $\mathbf{i}$ components, yields

$$v = \omega(R - r) \qquad \omega = \frac{v}{R - r}$$



(a)

Using this result,

$$\mathbf{v}_A = \mathbf{v}_P + \omega \times \mathbf{r}_{A/P}$$

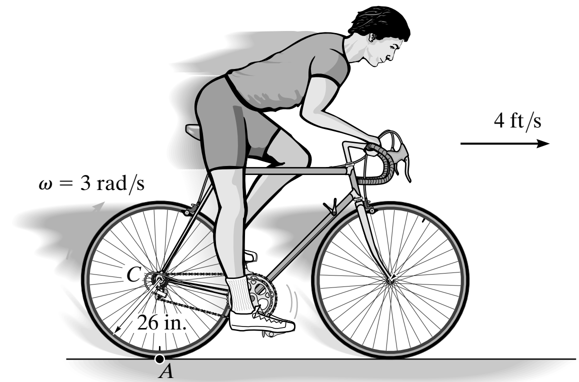
$$= \mathbf{0} + \left(-\frac{v}{R - r}\mathbf{k}\right) \times 2R\mathbf{j}$$

$$= \left[\left(\frac{2R}{R - r}\right)v\right]\mathbf{i}$$

Thus,

$$v_A = \left(\frac{2R}{R - r}\right)v \rightarrow \qquad \textbf{Ans.}$$

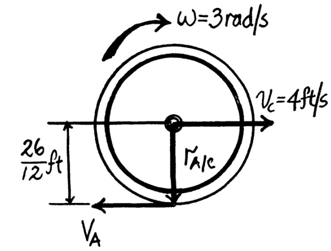
16–67. The bicycle has a velocity $v = 4$ ft/s, and at the same instant the rear wheel has a clockwise angular velocity $\omega = 3$ rad/s, which causes it to slip at its contact point A. Determine the velocity of point A.



$$\mathbf{v}_A = \mathbf{v}_C + \mathbf{v}_{A/C}$$

$$\begin{bmatrix} v_A \\ \leftarrow \end{bmatrix} = \begin{bmatrix} 4 \\ \rightarrow \end{bmatrix} + \begin{bmatrix} \left(\dfrac{26}{12}\right)(3) \\ \leftarrow \end{bmatrix}$$

$v_A = 2.5$ ft/s $\leftarrow$ **Ans.**



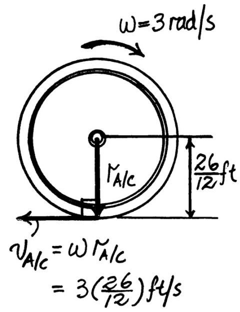
Also,

$$\mathbf{v}_A = \mathbf{v}_C + \omega \times \mathbf{r}_{A/C}$$

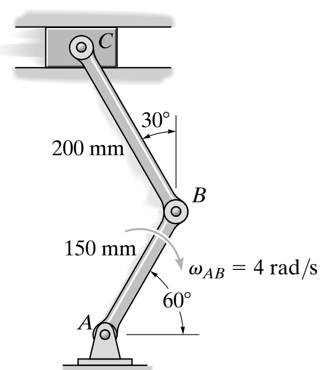
$$\mathbf{v}_A = 4\mathbf{i} + (-3\mathbf{k}) \times \left(-\frac{26}{12}\mathbf{j}\right)$$

$$\mathbf{v}_A = 4\mathbf{i} - 6.5\mathbf{i} = -2.5\mathbf{i}$$

$v_A = 2.5$ ft/s $\leftarrow$ **Ans.**



***16–68.** If bar AB has an angular velocity $\omega_{AB} = 4$ rad/s, determine the velocity of the slider block C at the instant shown.



For link AB: Link AB rotates about a fixed point A. Hence

$$v_B = \omega_{AB}\, r_{AB} = 4(0.15) = 0.6 \text{ m/s}$$

For link BC

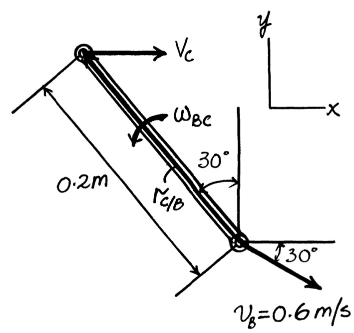
$$\mathbf{v}_B = \{0.6\cos 30°\mathbf{i} - 0.6\sin 30°\mathbf{j}\}\text{m/s} \qquad \mathbf{v}_C = v_C\mathbf{i} \qquad \omega = \omega_{BC}\,\mathbf{k}$$

$$\mathbf{r}_{C/B} = \{-0.2\sin 30°\mathbf{i} + 0.2\cos 30°\mathbf{j}\}\text{ m}$$

$$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$$

$$v_C\,\mathbf{i} = (0.6\cos 30°\mathbf{i} - 0.6\sin 30°\mathbf{j}) + (\omega_{BC}\,\mathbf{k}) \times (-0.2\sin 30°\mathbf{i} + 0.2\cos 30°\mathbf{j})$$

$$v_C\mathbf{i} = (0.5196 - 0.1732\omega_{BC})\mathbf{i} - (0.3 + 0.1\omega_{BC})\mathbf{j}$$

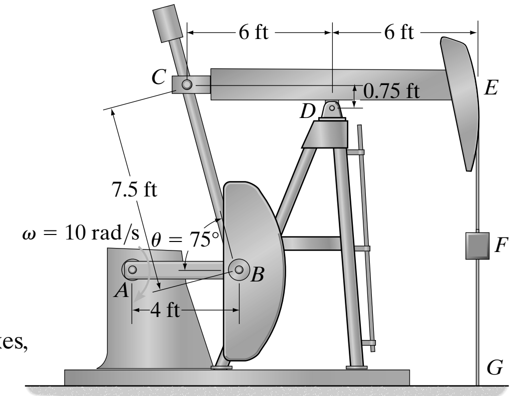


Equating the $\mathbf{i}$ and $\mathbf{j}$ components yields:

$$0 = 0.3 + 0.1\omega_{BC} \qquad \omega_{BC} = -3 \text{ rad/s}$$

$v_C = 0.5196 - 0.1732(-3) = 1.04$ m/s $\rightarrow$ **Ans.**

•16–69. The pumping unit consists of the crank pitman AB, connecting rod BC, walking beam CDE and pull rod F. If the crank is rotating with an angular velocity of $\omega = 10$ rad/s, determine the angular velocity of the walking beam and the velocity of the pull rod EFG at the instant shown.

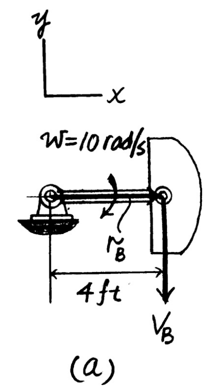


Rotation About a Fixed Axis: The crank and walking beam rotate about fixed axes, Figs. *a* and *b*. Thus, the velocity of points *B, C,* and *E* can be determined from

$$v_B = \omega \times r_B = \left(-10\mathbf{k}\right) \times \left(4\mathbf{i}\right) = \left[-40\mathbf{j}\right]\text{ft/s}$$

$$v_C = \omega_{CDE} \times r_{DC} = \left(\omega_{CDE}\mathbf{k}\right) \times \left(-6\mathbf{i} + 0.75\mathbf{j}\right) = -0.75\omega_{CDE}\mathbf{i} - 6\omega_{CDE}\mathbf{j}$$

$$v_E = \omega_{CDE} \times r_{DE} = \left(\omega_{CDE}\mathbf{k}\right) \times \left(6\mathbf{i}\right) = 6\omega_{CDE}\mathbf{j} \quad \textbf{(1)}$$



General Plane Motion: Applying the relative velocity equation and referring to the kinematic diagram of link BC shown in Fig. *c*,

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$$

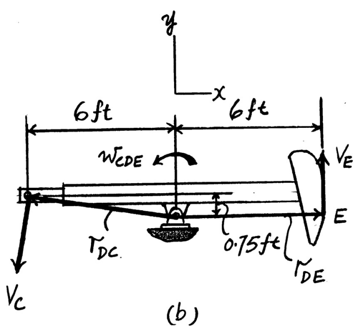
$$-0.75\omega_{CDE}\mathbf{i} - 6\omega_{CDE}\mathbf{j} = -40\mathbf{j} + (\omega_{BC}\mathbf{k}) \times (-7.5\cos 75^\circ\,\mathbf{i} + 7.5\sin 75^\circ\,\mathbf{j})$$

$$-0.75\omega_{CDE}\mathbf{i} - 6\omega_{CDE}\mathbf{j} = -7.244\omega_{BC}\,\mathbf{i} - (1.9411\omega_{BC} + 40)\mathbf{j}$$

Equating the **i** and **j** components

$$-0.75\omega_{CDE} = -7.244\omega_{BC} \quad \textbf{(2)}$$

$$-6\omega_{CDE} = -(1.9411\omega_{BC} + 40) \quad \textbf{(3)}$$



Solving Eqs. (1) and (2) yields

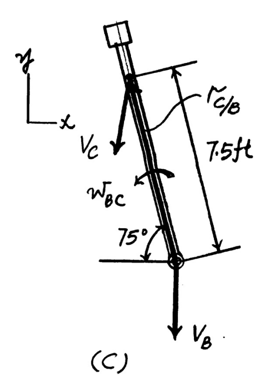
$\omega_{BC} = 0.714$ rad/s $\qquad \omega_{CDE} = 6.898$ rad/s $= 6.90$ rad/s **Ans.**

Substituting the result for ω_{CDE} into Eq. (1),

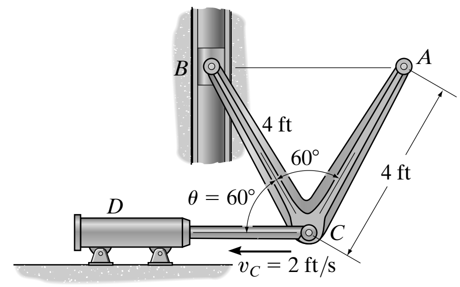
$$\mathbf{v}_E = \theta(6.898) = [41.39\mathbf{j}] \text{ ft/s}$$

Thus,

$$v_E = 41.4 \text{ ft/s} \uparrow \quad \textbf{Ans.}$$



16–70. If the hydraulic cylinder shortens at a constant rate of $v_C = 2$ ft/s, determine the angular velocity of link ACB and the velocity of block B at the instant shown.



General Plane Motion: Applying the relative velocity equation to points B and C and referring to the kinematic diagram of link ABC shown in Fig. a,

$$\mathbf{v}_B = \mathbf{v}_C + \omega \times \mathbf{r}_{B/C}$$

$$v_B\mathbf{j} = -2\mathbf{i} + (-\omega\mathbf{k}) \times (-4\cos 60°\mathbf{i} + 4\sin 60°\,\mathbf{j})$$

$$v_B\mathbf{j} = (3.464\omega - 2)\mathbf{i} + 2\omega\mathbf{j}$$

Equating the **i** and **j** components yields

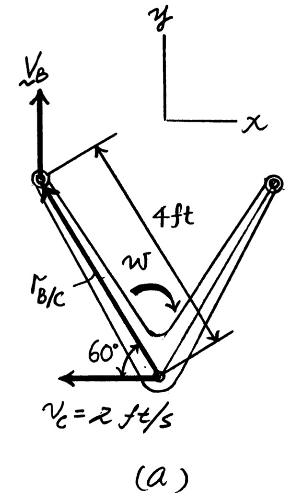
$$0 = 3.464\omega - 2$$

$$v_B = 2\omega$$

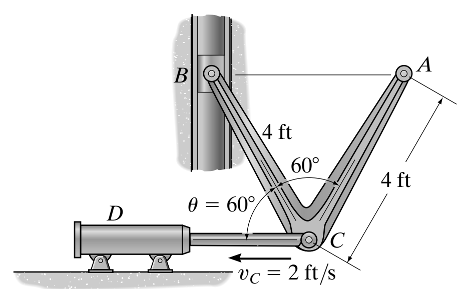
Solving,

$$\omega = 0.577 \text{ rad/s} \qquad \textbf{Ans.}$$

$$v_B = 1.15\text{ft/s} \uparrow \qquad \textbf{Ans.}$$



16–71. If the hydraulic cylinder shortens at a constant rate of $v_C = 2$ ft/s, determine the velocity of end A of link ACB at the instant shown.



General Plane Motion: First, applying the relative velocity equation to points B and C and referring to the kinematic diagram of link ABC shown in Fig. a,

$$\mathbf{v}_B = \mathbf{v}_C + \omega \times \mathbf{r}_{B/C}$$

$$v_B\mathbf{j} + -2\mathbf{i} + (-\omega\mathbf{k}) \times (-4\cos 60°\,\mathbf{i} + 4\sin 60°\,\mathbf{j})$$

$$v_B\,\mathbf{j} = (3.464\omega - 2)\mathbf{i} + 2\omega\mathbf{j}$$

Equating the **i** components yields

$$0 = 3.464\omega - 2 \qquad \omega = 0.5774 \text{ rad/s}$$

Then, for points A and C using the result of ω,

$$\mathbf{v}_A = \mathbf{v}_C + \omega \times \mathbf{r}_{A/C}$$

$$(v_A)_x\,\mathbf{i} + (v_A)_y\,\mathbf{j} = -2\mathbf{i} + (-0.5774\mathbf{k}) \times (4\cos 60°\,\mathbf{i} + 4\sin 60°\,\mathbf{j})$$

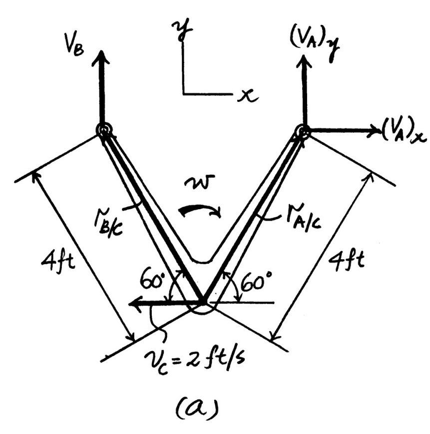
$$(v_A)_x\,\mathbf{i} + (v_A)_y\,\mathbf{j} = -1.1547\mathbf{j}$$

Equating the **i** and **j** components yields

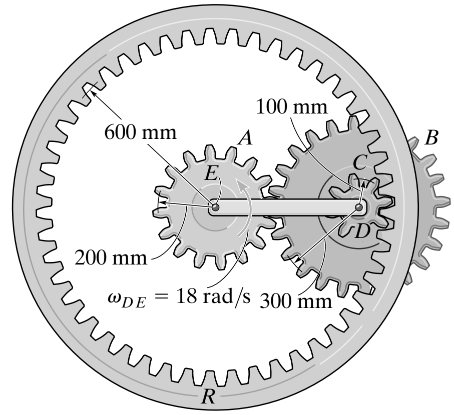
$$(v_A)_x = 0 \qquad (v_A)_y = -1.1547 \text{ ft/s} = 1.1547 \text{ ft/s} \downarrow$$

Thus,

$$v_A = (v_A)_y = 1.15 \text{ ft/s}\downarrow \qquad \textbf{Ans.}$$



***16–72.** The epicyclic gear train consists of the sun gear A which is in mesh with the planet gear B. This gear has an inner hub C which is fixed to B and in mesh with the fixed ring gear R. If the connecting link DE pinned to B and C is rotating at $\omega_{DE} = 18$ rad/s about the pin at E, determine the angular velocities of the planet and sun gears.



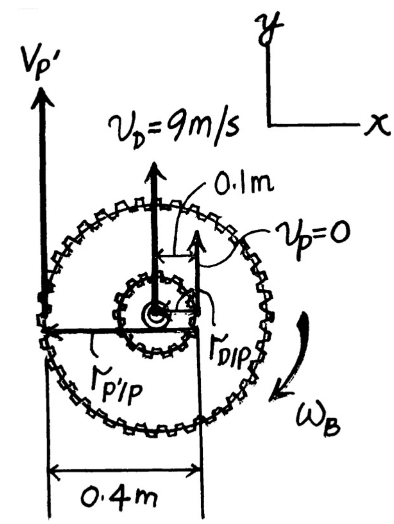
$v_D = r_{DE}\,\omega_{DE} = (0.5)(18) = 9$ m/s ↑

The velocity of the contact point P with the ring is zero.

$\mathbf{v}_D = \mathbf{v}_P + \omega \times \mathbf{r}_{D/P}$

$9\mathbf{j} = 0 + (-\omega_B\,\mathbf{k}) \times (-0.1\mathbf{i})$

$\omega_B = 90$ rad/s ↻ **Ans.**



Let P' be the contact point between A and B.

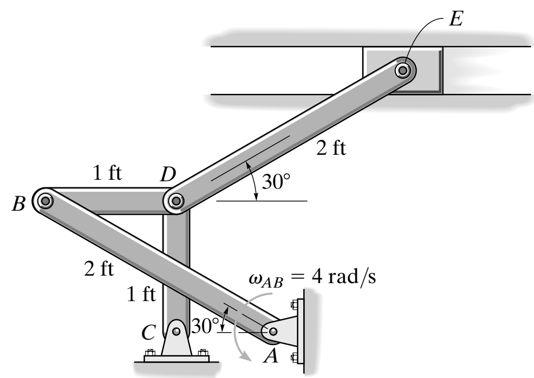
$\mathbf{v}_{P'} = \mathbf{v}_P + \omega \times \mathbf{r}_{P'/P}$

$v_{P'}\,\mathbf{j} = \mathbf{0} + (-90\mathbf{k}) \times (-0.4\mathbf{i})$

$v_{P'} = 36$ m/s ↑

$\omega_A = \dfrac{v_{P'}}{r_A} = \dfrac{36}{0.2} = 180$ rad/s ↺ **Ans.**

•**16–73.** If link AB has an angular velocity of $\omega_{AB} = 4$ rad/s at the instant shown, determine the velocity of the slider block E at this instant. Also, identify the type of motion of each of the four links.



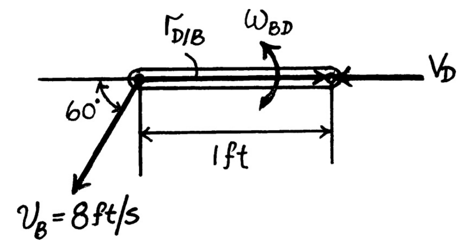
Link AB rotates about the fixed point A. Hence

$$v_B = \omega_{AB}\, r_{AB} = 4(2) = 8 \text{ ft/s}$$

For link BD

$$\mathbf{v}_B = \{-8\cos 60°\mathbf{i} - 8\sin 60°\,\mathbf{j}\} \text{ ft/s} \qquad \mathbf{v}_D = -v_D\mathbf{i} \qquad \boldsymbol{\omega}_{BD} = \omega_{BD}\,\mathbf{k}$$

$$\mathbf{r}_{D/B} = \{1\mathbf{i}\} \text{ ft}$$



$$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B}$$

$$-v_D\mathbf{i} = (-8\cos 60°\mathbf{i} - 8\sin 60°\mathbf{j}) + (\omega_{BD}\mathbf{k}) \times (1\mathbf{i})$$

$$-v_D\,\mathbf{i} = -8\cos 60°\mathbf{i} + (\omega_{BD} - 8\sin 60°)\mathbf{j}$$

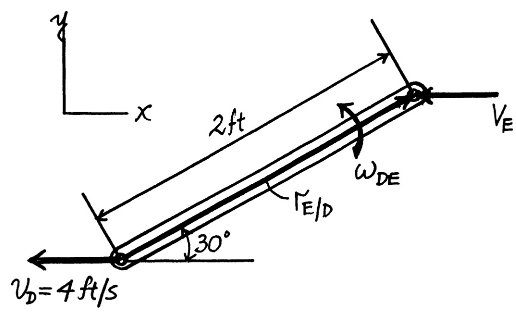
$(\overset{+}{\rightarrow})$ $\quad -v_D = -8\cos 60° \qquad v_D = 4$ ft/s

$(+\uparrow)$ $\quad 0 = \omega_{BD} - 8\sin 60° \qquad \omega_{BD} = 6.928$ rad/s

For Link DE

$$\mathbf{v}_D = \{-4\mathbf{i}\} \text{ ft/s} \qquad \boldsymbol{\omega}_{DE} = \omega_{DE}\,\mathbf{k} \qquad \mathbf{v}_E = -v_E\mathbf{i}$$

$$\mathbf{r}_{E/D} = \{2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j}\} \text{ ft}$$



$$\mathbf{v}_E = \mathbf{v}_D + \boldsymbol{\omega}_{DE} \times \mathbf{r}_{E/D}$$

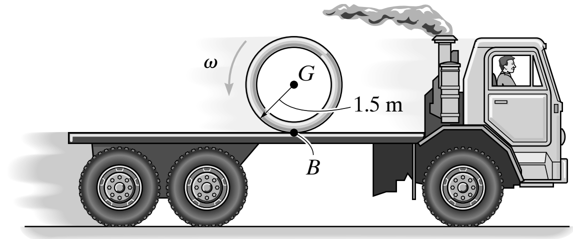
$$-v_E\mathbf{i} = -4\mathbf{i} + (\omega_{DE}\mathbf{k}) \times (2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j})$$

$$-v_E\mathbf{i} = (-4 - 2\sin 30°\,\omega_{DE})\mathbf{i} + 2\cos 30°\omega_{DE}\mathbf{j}$$

$(\overset{+}{\rightarrow})$ $\quad 0 = 2\cos 30°\,\omega_{DE} \qquad \omega_{DE} = 0$

$(+\uparrow)$ $\quad -v_E = -4 - 2\sin 30°(0) \qquad v_E = 4\text{ft/s} \quad \leftarrow$ **Ans.**

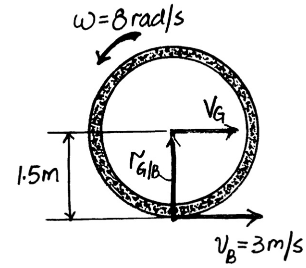
16–74. At the instant shown, the truck travels to the right at 3 m/s, while the pipe rolls counterclockwise at $\omega = 8$ rad/s without slipping at B. Determine the velocity of the pipe's center G.



$$\mathbf{v}_G = \mathbf{v}_B + \mathbf{v}_{G/B}$$

$$\left[\underset{\rightarrow}{\mathbf{v}_G}\right] = \left[\underset{\rightarrow}{3}\right] + \left[\underset{\leftarrow}{1.5(8)}\right]$$

$v_G = 9 \text{ m/s} \leftarrow$ **Ans.**



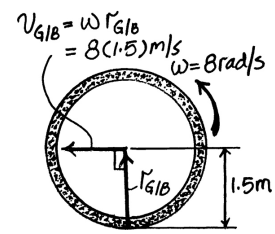
Also:

$$\mathbf{v}_G = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{G/B}$$

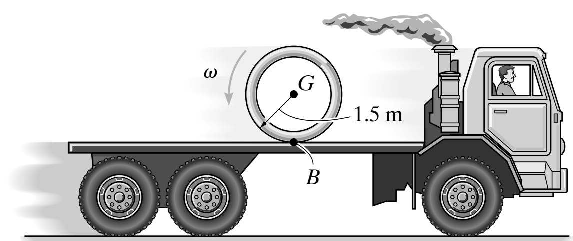
$$v_G\mathbf{i} = 3\mathbf{i} + (8\mathbf{k}) \times (1.5\mathbf{j})$$

$$v_G = 3 - 12$$

$v_G = -9 \text{ m/s} = 9 \text{ m/s} \leftarrow$ **Ans.**



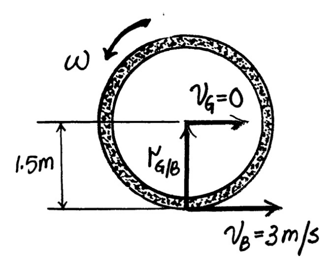
16–75. At the instant shown, the truck travels to the right at 8 m/s. If the pipe does not slip at B, determine its angular velocity if its mass center G appears to remain stationary to an observer on the ground.



$$\mathbf{v}_G = \mathbf{v}_B + \mathbf{v}_{G/B}$$

$$0 = \left[\underset{\rightarrow}{8}\right] + \left[\underset{\leftarrow}{1.5\omega}\right]$$

$\omega = \dfrac{8}{1.5} = 5.33 \text{ rad/s} \circlearrowleft$ **Ans.**



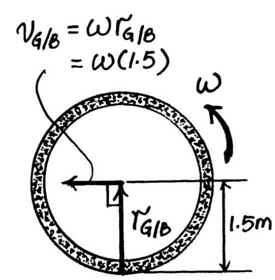
Also:

$$\mathbf{v}_G = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{G/B}$$

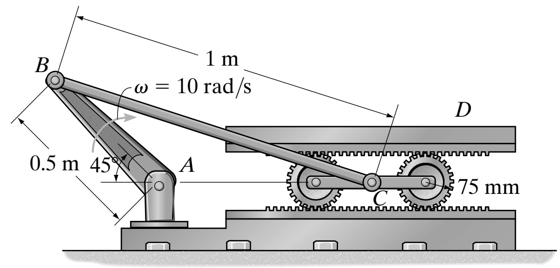
$$0\mathbf{i} = 8\mathbf{i} + (\omega\mathbf{k}) \times (1.5\mathbf{j})$$

$$0 = 8 - 1.5\omega$$

$\omega = \dfrac{8}{1.5} = 5.33 \text{ rad/s} \circlearrowleft$ **Ans.**



***16–76.** The mechanism of a reciprocating printing table is driven by the crank AB. If the crank rotates with an angular velocity of $\omega = 10$ rad/s, determine the velocity of point C at the instant shown.

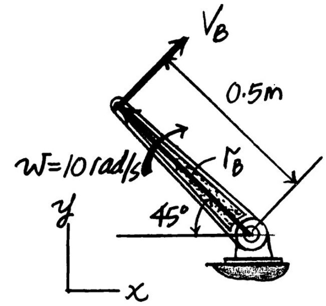


Rotation About a Fixed Axis: Referring to Fig. *a*,

$$v_B = \omega \times \mathbf{r}_B$$

$$= (-10\mathbf{k}) \times (-0.5\cos 45°\,\mathbf{i} + 0.5\sin 45°\mathbf{j})$$

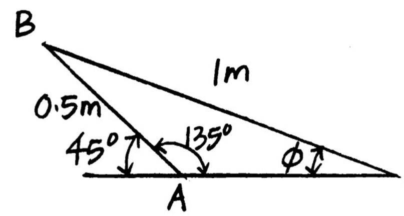
$$= [3.536\mathbf{i} + 3.536\mathbf{j}]\text{ m}$$



(a)

General Plane Motion: Applying the law of sines to the geometry shown in Fig. *b*,

$$\frac{\sin\phi}{0.5} = \frac{\sin 135°}{1} \qquad \phi = 20.70°$$



(b)

Applying the relative velocity equation to the kinematic diagram of link BC shown in Fig. *c*,

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$3.536\mathbf{i} + 3.536\mathbf{j} = v_C\,\mathbf{i} + (-\omega_{BC}\,\mathbf{k}) \times (-1\cos 20.70°\,\mathbf{i} + 1\sin 20.70°\,\mathbf{j})$$

$$3.536\mathbf{i} + 3.536\mathbf{j} = (v_C + 0.3536\omega_{BC})\mathbf{i} + 0.9354\omega_{BC}\,\mathbf{j}$$

Equating the **i** and **j** components yields,

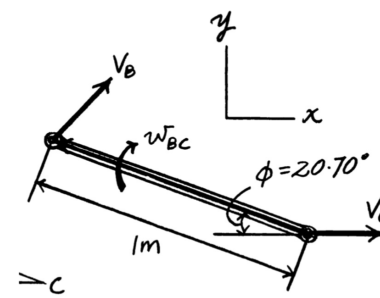
$$3.536 = v_C + 0.3536\omega_{BC}$$

$$3.536 = 0.9354\omega_{BC}$$

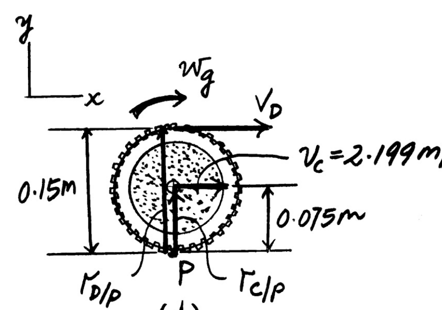
Solving,

$$\omega_{BC} = 3.780\text{ rad/s} \qquad \textbf{Ans.}$$

$$v_C = 2.199\text{ m/s} \qquad \textbf{Ans.}$$

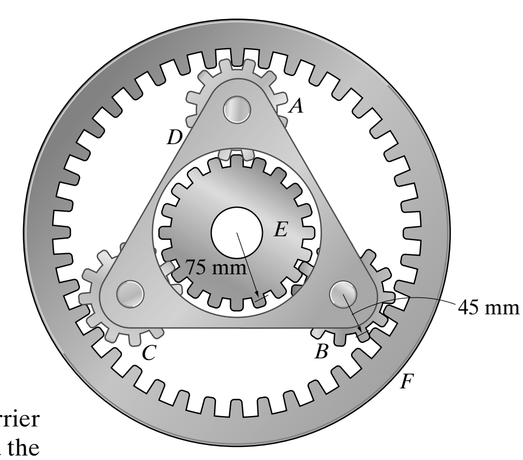


(c)



(d)

•16–77. The planetary gear set of an automatic transmission consists of three planet gears A, B, and C, mounted on carrier D, and meshed with the sun gear E and ring gear F. By controlling which gear of the planetary set rotates and which gear receives the engine's power, the automatic transmission can alter a car's speed and direction. If the carrier is rotating with a counterclockwise angular velocity of $\omega_D = 20$ rad/s while the ring gear is rotating with a clockwise angular velocity of $\omega_F = 10$ rad/s, determine the angular velocity of the planet gears and the sun gear. The radii of the planet gears and the sun gear are 45 mm and 75 mm, respectively.



Rotation About a Fixed Axis: Here, the ring gear, the sun gear, and the carrier rotate about a fixed axis. Thus, the velocity of the center O of the planet gear and the contact points P' and P with the ring and sun gear can be determined from

$$v_O = \omega_D r_O = 20(0.045 + 0.075) = 2.4 \text{ m/s} \leftarrow$$

$$v_{P'} = \omega_F r_F = 10(0.045 + 0.045 + 0.075) = 1.65 \text{ m/s} \rightarrow$$

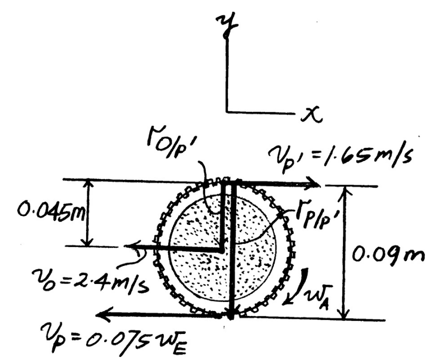
$$v_P = \omega_E r_E = \omega_E(0.075) = 0.075\omega_E$$

General Plane Motion: First, applying the relative velocity equation for O and P' and referring to the kinematic diagram of planet gear A shown in Fig. *a*,

$$\mathbf{v}_O = \mathbf{v}_{P'} + \boldsymbol{\omega}_A \times \mathbf{r}_{O/P'}$$

$$-2.4\mathbf{i} = 1.65\mathbf{i} + (-\omega_A \mathbf{k}) \times (-0.045\mathbf{j})$$

$$-2.4\mathbf{i} = (1.65 - 0.045\omega_A)\mathbf{i}$$



(a)

Thus,

$$-2.4 = 1.65 - 0.045\omega_A$$

$$\omega_A = 90 \text{ rad/s} \qquad \textbf{Ans.}$$

Using this result to apply the relative velocity equation for P' and P,

$$\mathbf{v}_P = \mathbf{v}_{P'} + \boldsymbol{\omega}_A \times \mathbf{r}_{P/P'}$$

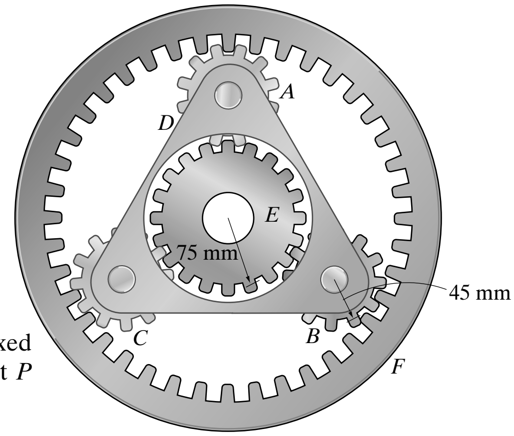
$$-0.075\omega_E\mathbf{i} = 1.65\mathbf{i} + (-90\mathbf{j}) \times (-0.09\mathbf{j})$$

$$-0.075\omega_E\mathbf{i} = -6.45\mathbf{i}$$

Thus,

$$-0.075\omega_E = -6.45 \qquad \omega_E = 86 \text{ rad/s} \qquad \textbf{Ans.}$$

16–78. The planetary gear set of an automatic transmission consists of three planet gears A, B, and C, mounted on carrier D, and meshed with sun gear E and ring gear F. By controlling which gear of the planetary set rotates and which gear receives the engine's power, the automatic transmission can alter a car's speed and direction. If the ring gear is held stationary and the carrier is rotating with a clockwise angular velocity of $\omega_D = 20$ rad/s, determine the angular velocity of the planet gears and the sun gear. The radii of the planet gears and the sun gear are 45 mm and 75 mm, respectively.

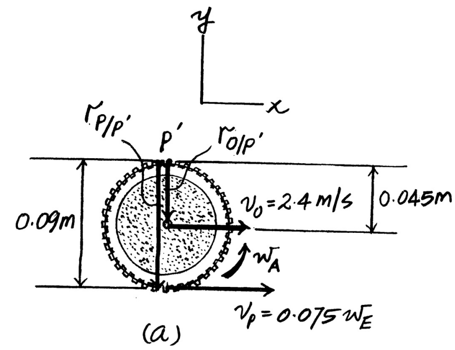


Rotation About a Fixed Axis: Here, the carrier and the sun gear rotate about a fixed axis. Thus, the velocity of the center O of the planet gear and the contact point P with the sun gear can be determined from

$$v_O = \omega_D r_D = 20(0.045 + 0.075) = 2.4 \text{ m/s}$$

$$v_P = \omega_E r_E = \omega_E (0.075) = 0.075\omega_E$$

General Plane Motion: Since the ring gear is held stationary, the velocity of the contact point P' with the planet gear A is zero. Applying the relative velocity equation for O and P' and referring to the kinematic diagram of planet gear A shown in Fig. a,



$$\mathbf{v}_O = \mathbf{v}_{P'} + \boldsymbol{\omega}_A \times \mathbf{r}_{O/P'}$$

$$2.4\mathbf{i} = \mathbf{0} + (\omega_A\mathbf{k}) \times (-0.045\mathbf{j})$$

$$2.4\mathbf{i} = 0.045\omega_A\,\mathbf{i}$$

Thus,

$$2.4 = 0.045\omega_A$$

$$\omega_A = 53.33 \text{ rad/s} = 53.3 \text{ rad/s} \qquad \textbf{Ans.}$$

Using this result to apply the relative velocity equation for points P' and P,

$$\mathbf{v}_P = \mathbf{v}_{P'} + \boldsymbol{\omega}_A \times \mathbf{r}_{P/P'}$$

$$0.075\omega_E\,\mathbf{i} = \mathbf{0} + (53.33\mathbf{k}) \times (-0.09\mathbf{j})$$

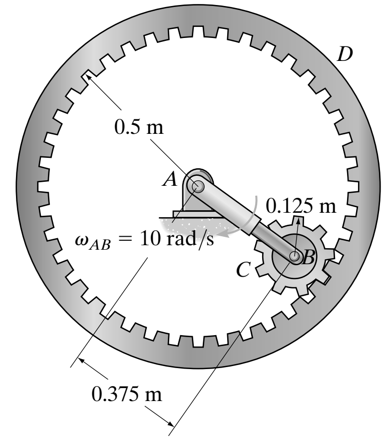
$$0.075\omega_E\,\mathbf{i} = 4.8\mathbf{i}$$

Thus,

$$0.075\omega_E = 4.8 \qquad \textbf{Ans.}$$

$$\omega_E = 64 \text{ rad/s} \qquad \textbf{Ans.}$$

16–79. If the ring gear D is held fixed and link AB rotates with an angular velocity of $\omega_{AB} = 10$ rad/s, determine the angular velocity of gear C.



Rotation About a Fixed Axis: Since link AB rotates about a fixed axis, Fig. a, the velocity of the center B of gear C is

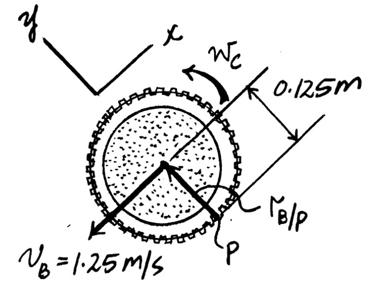
$$v_B = \omega_{AB}\, r_{AB} = 10(0.375) = 3.75 \text{ m/s}$$

General Plane Motion: Since gear D is fixed, the velocity of the contact point P between the gears is zero. Applying the relative velocity equation and referring to the kinematic diagram of gear C shown in Fig. b,

$$\mathbf{v}_B = \mathbf{v}_P + \omega_C \times \mathbf{r}_{B/P}$$

$$-3.75\mathbf{i} = \mathbf{0} + (\omega_C\,\mathbf{k}) \times (0.125\mathbf{j})$$

$$-3.75\mathbf{i} = -0.125\omega_C\mathbf{i}$$



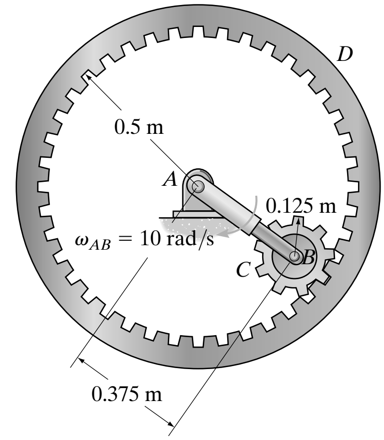
(a)

Thus,

$$-3.75 = -0.125\omega_C$$

$$\omega_C = 30 \text{ rad/s} \qquad \textbf{Ans.}$$

***16–80.** If the ring gear D rotates counterclockwise with an angular velocity of $\omega_D = 5$ rad/s while link AB rotates clockwise with an angular velocity of $\omega_{AB} = 10$ rad/s, determine the angular velocity of gear C.



Rotation About a Fixed Axis: Since link AB and gear D rotate about a fixed axis, Fig. a, the velocity of the center B and the contact point of gears D and C is

$$v_B = \omega_{AB}\, r_B = 10(0.375) = 3.75 \text{ m/s}$$

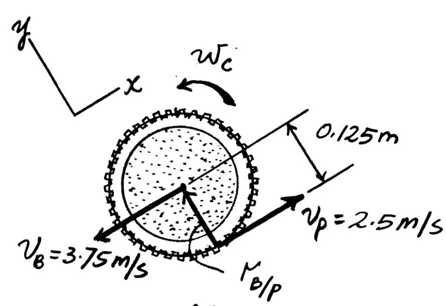
$$v_P = \omega_D\, r_P = 5(0.5) = 2.5 \text{ m/s}$$

General Plane Motion: Applying the relative velocity equation and referring to the kinematic diagram of gear C shown in Fig. b,

$$\mathbf{v}_B = \mathbf{v}_P + \omega_C \times r_{B/P}$$

$$-3.75\mathbf{i} = 2.5\mathbf{i} + (\omega_C\,\mathbf{k}) \times (0.125\mathbf{j})$$

$$-3.75\mathbf{i} = (2.5 - 0.125\omega_C)\mathbf{i}$$



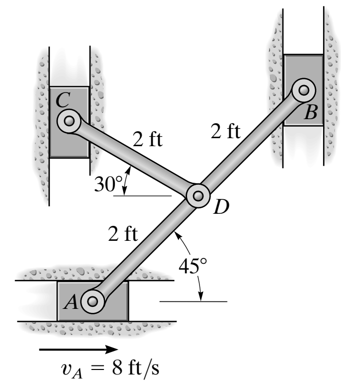
(a)

Thus,

$$-3.75 = 2.5 - 0.125\omega_C$$

$$\omega_C = 50 \text{ rad/s} \qquad \textbf{Ans.}$$

•16–81. If the slider block A is moving to the right at $v_A = 8$ ft/s, determine the velocity of blocks B and C at the instant shown. Member CD is pin connected to member ADB.



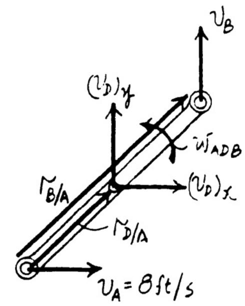
Kinematic Diagram: Block B and C are moving along the guide and directed towards the *positive* y axis and *negative* y axis, respectively. Then, $\mathbf{v}_B = v_B\,\mathbf{j}$ and $\mathbf{v}_C = -v_C\,\mathbf{j}$. Since the direction of the velocity of point D is unknown, we can assume that its x and y components are directed in the *positive direction* of their respective axis.

Velocity Equation: Here, $\mathbf{r}_{B/A} = \{4\cos 45°\mathbf{i} + 4\sin 45°\mathbf{j}\}$ ft $= \{2.828\mathbf{i} + 2.828\mathbf{j}\}$ ft and $\mathbf{r}_{D/A} = \{2\cos 45°\mathbf{i} + 2\sin 45°\mathbf{j}\}$ ft $= \{1.414\mathbf{i} + 1.414\mathbf{j}\}$ ft. Applying Eq. 16–16 to link ADB, we have

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{ADB} \times \mathbf{r}_{B/A}$$

$$v_B\,\mathbf{j} = 8\mathbf{i} + (\omega_{ADB}\,\mathbf{k}) \times (2.828\mathbf{i} + 2.828\mathbf{j})$$

$$v_B\,\mathbf{j} = (8 - 2.828\omega_{ADB})\,\mathbf{i} + 2.828\omega_{ADB}\,\mathbf{j}$$



Equating **i** and **j** components gives

$$0 = 8 - 2.828\omega_{ADB} \qquad \textbf{[1]}$$

$$v_B = 2.828\omega_{ADB} \qquad \textbf{[2]}$$

Solving Eqs.[1] and [2] yields

$$\omega_{ADB} = 2.828 \text{ rad/s}$$

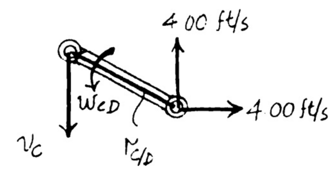
$$v_B = 8.00 \text{ ft/s} \uparrow \qquad \textbf{Ans.}$$

The x and y component of velocity of v_D are given by

$$\mathbf{v}_D = \mathbf{v}_A + \omega_{ADB} \times \mathbf{r}_{D/A}$$

$$(v_D)_x\,\mathbf{i} + (v_D)_y\mathbf{j} = 8\mathbf{i} + (2.828\mathbf{k}) \times (1.414\mathbf{i} + 1.414\mathbf{j})$$

$$(v_D)_x\,\mathbf{i} + (v_D)_y\,\mathbf{j} = 4.00\mathbf{i} + 4.00\mathbf{j}$$



Equating **i** and **j** components gives

$$(v_D)_x = 4.00 \text{ ft/s} \qquad (v_D)_y = 4.00 \text{ ft/s}$$

Here, $r_{C/D} = \{-2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j}\}$ ft $= \{-1.732\mathbf{i} + 1\mathbf{j}\}$ ft. Applying Eq. 16–16 to link CD, we have

$$\mathbf{v}_C = \mathbf{v}_D + \omega_{CD} \times \mathbf{r}_{C/D}$$

$$-v_C\mathbf{j} = 4.00\mathbf{i} + 4.00\mathbf{j} + (\omega_{CD}\mathbf{k}) \times (-1.732\mathbf{i} + 1\mathbf{j})$$

$$-v_C\,\mathbf{j} = (4.00 - \omega_{CD})\,\mathbf{i} + (4 - 1.732\omega_{CD})\,\mathbf{j}$$

Equating **i** and **j** components gives

$$0 = 4.00 - \omega_{CD} \qquad \textbf{[3]}$$

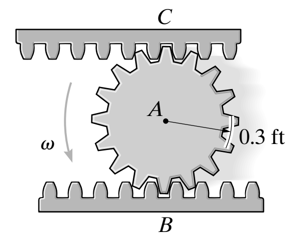
$$-v_C = 4 - 1.732\omega_{CD} \qquad \textbf{[4]}$$

Solving Eqs. [3] and [4] yields

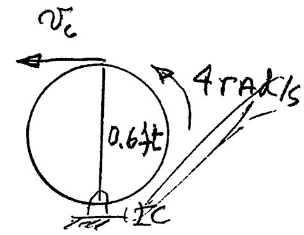
$$\omega_{CD} = 4.00 \text{ rad/s}$$

$$v_C = 2.93 \text{ ft/s} \downarrow \qquad \textbf{Ans.}$$

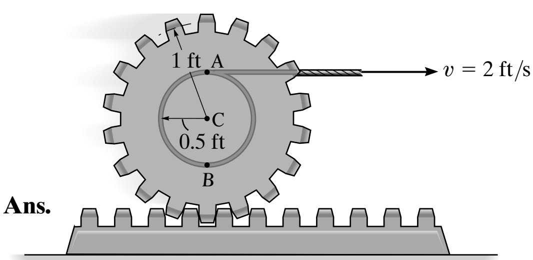
16–82. Solve Prob. 16–54 using the method of instantaneous center of zero velocity.



$$v_C = 4\ \text{rad/s}(0.6\ \text{ft}) = 2.40\ \text{ft/s}$$ **Ans.**

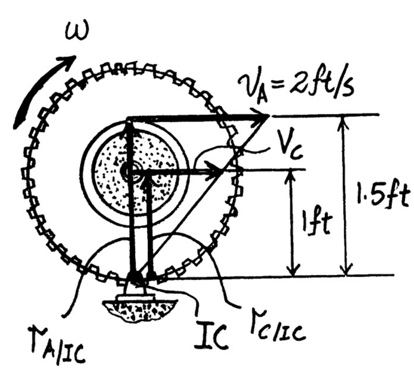


16–83. Solve Prob. 16–56 using the method of instantaneous center of zero velocity.

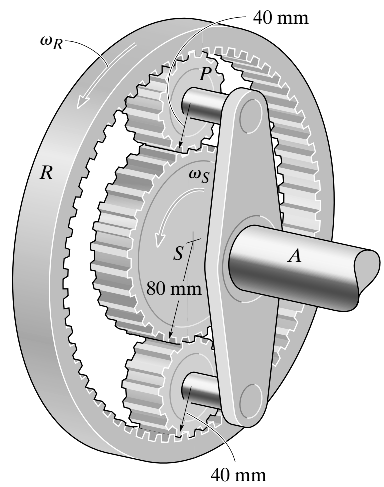


$$\omega = \frac{2}{1.5} = 1.33\ \text{rad/s}$$

$$v_C = 1(1.33) = 1.33\ \text{ft/s} \rightarrow$$ **Ans.**



***16–84.** Solve Prob. 16–64 using the method of instantaneous center of zero velocity.

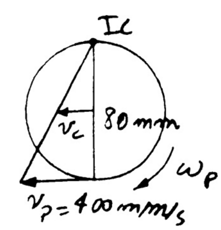


$$v_P = (80)(5) = 400 \text{ mm/s}$$

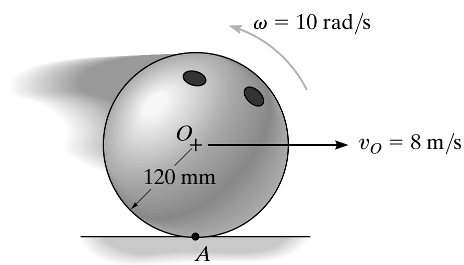
$$\omega_P = \frac{400}{80} = 5 \text{ rad/s} \quad \textbf{Ans.}$$

$$v_C = (5)(40) = 200 \text{ mm/s}$$

$$\omega_A = \frac{200}{(80 + 40)} = 1.67 \text{ rad/s} \quad \textbf{Ans.}$$

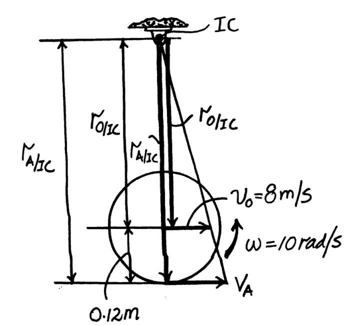


•16–85. Solve Prob. 16–58 using the method of instantaneous center of zero velocity.

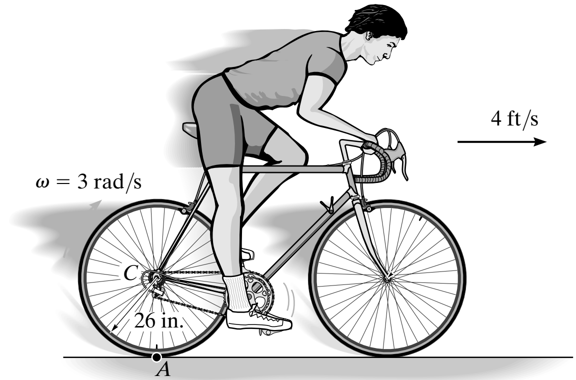


$$r_{O/IC} = \frac{8}{10} = 0.8 \text{ m}$$

$$v_A = 10(0.8 + 0.120) = 9.20 \text{ m/s} \quad \textbf{Ans.}$$



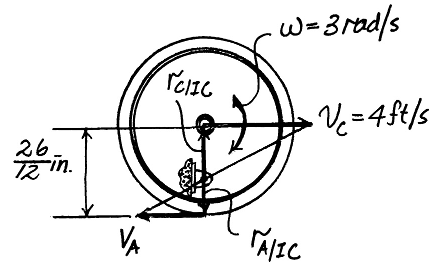
16–86. Solve Prob. 16–67 using the method of instantaneous center of zero velocity.



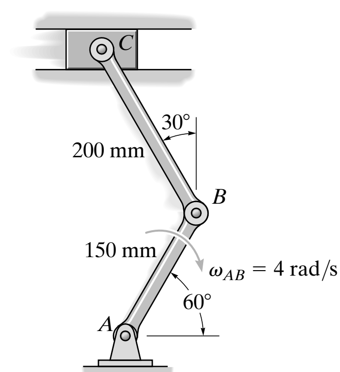
$$r_{C/IC} = \ = \frac{4}{3} = 1.33 \text{ ft}$$

$$r_{A/IC} = \ = \frac{26}{12} - 1.33 \text{ ft} = 0.833 \text{ ft}$$

$$v_A = 3(0.833) = 2.5 \text{ ft/s} \quad \leftarrow$$ **Ans.**



16–87. Solve Prob. 16–68 using the method of instantaneous center of zero velocity.



$$v_B = 4(0.150) = 0.6 \text{ m/s}$$

$$\frac{r_{C/IC}}{\sin 120^\circ} = \frac{0.2}{\sin 30^\circ}$$

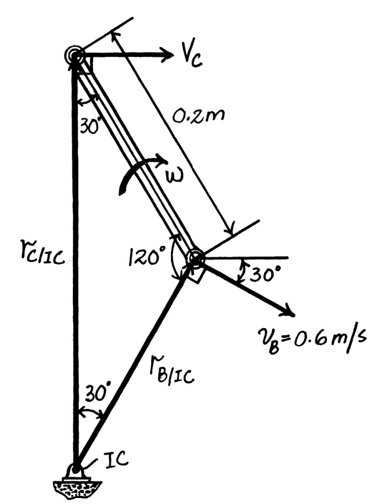
$$r_{C/IC} = 0.34641 \text{ m}$$

$$\frac{r_{B/IC}}{\sin 30^\circ} = \frac{0.2}{\sin 30^\circ}$$

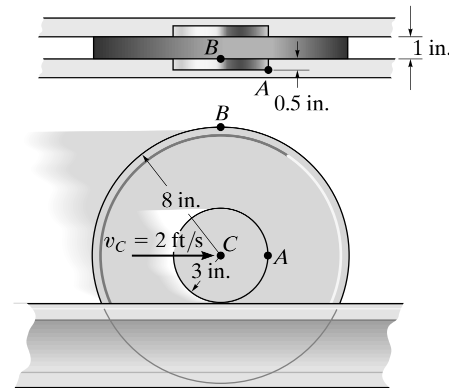
$$r_{B/IC} = 0.2 \text{ m}$$

$$\omega = \frac{0.6}{0.2} = 3 \text{ rad/s}$$

$$v_C = 0.34641(3) = 1.04 \text{ m/s} \quad \rightarrow$$ **Ans.**



***16–88.** The wheel rolls on its hub without slipping on the horizontal surface. If the velocity of the center of the wheel is $v_C = 2$ ft/s to the right, determine the velocities of points A and B at the instant shown.



$$v_C = \omega r_{C/IC}$$

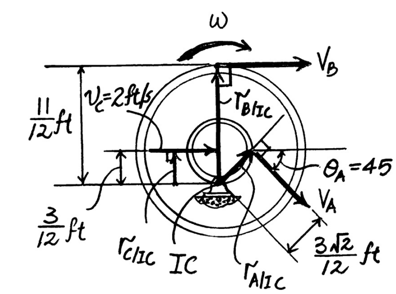
$$2 = \omega\left(\frac{3}{12}\right)$$

$$\omega = 8 \text{ rad/s}$$

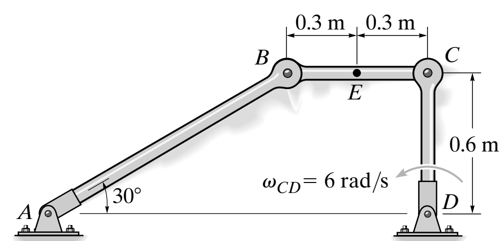
$$v_B = \omega r_{B/IC} = 8\left(\frac{11}{12}\right) = 7.33 \text{ ft/s} \quad \rightarrow$$ **Ans.**

$$v_A = \omega r_{A/IC} = 8\left(\frac{3\sqrt{2}}{12}\right) = 2.83 \text{ ft/s}$$ **Ans.**

$$\theta_A = \tan^{-1}\left(\frac{3}{3}\right) = 45°$$ **Ans.**



•16–89. If link CD has an angular velocity of $\omega_{CD} = 6$ rad/s, determine the velocity of point E on link BC and the angular velocity of link AB at the instant shown.



$$v_C = \omega_{CD}\,(r_{CD}) = (6)(0.6) = 3.60 \text{ m/s}$$

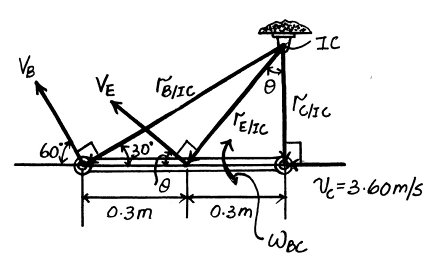
$$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{3.60}{0.6 \tan 30°} = 10.39 \text{ rad/s}$$

$$v_B = \omega_{BC}\, r_{B/IC} = (10.39)\left(\frac{0.6}{\cos 30°}\right) = 7.20 \text{ m/s}$$

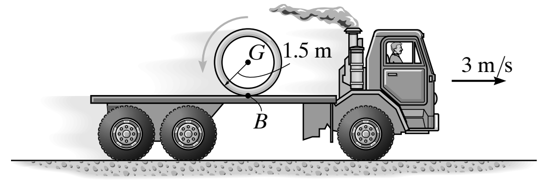
$$\omega_{AB} = \frac{v_B}{r_{AB}} = \frac{7.20}{\left(\dfrac{0.6}{\sin 30°}\right)} = 6 \text{ rad/s} \circlearrowleft$$ **Ans.**

$$v_E = \omega_{BC}\, r_{E/IC} = 10.39\sqrt{(0.6 \tan 30°)^2 + (0.3)^2} = 4.76 \text{ m/s}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{0.3}{0.6 \tan 30°}\right) = 40.9°$$ **Ans.**

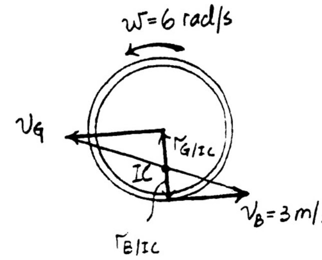


16–90. At the instant shown, the truck travels to the right at 3 m/s, while the pipe rolls counterclockwise at $\omega = 6$ rad/s without slipping at B. Determine the velocity of the pipe's center G.



Kinematic Diagram: Since the pipe rolls without slipping, then the velocity of point B must be the same as that of the truck, i.e; $v_B = 3$ m/s.

Instantaneous Center: $r_{B/IC}$ must be determined first in order to locate the the instantaneous center of zero velocity of the pipe.



$$v_B = \omega r_{B/IC}$$

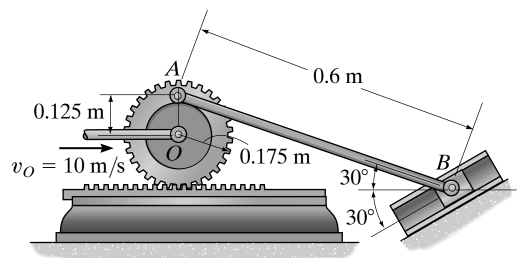
$$3 = 6(r_{B/IC})$$

$$r_{B/IC} = 0.5 \text{ m}$$

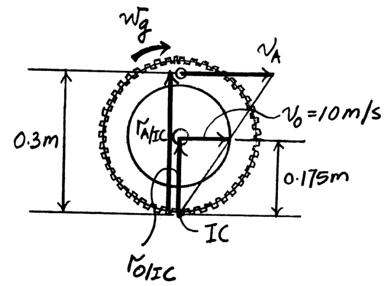
Thus, $r_{G/IC} = 1.5 - r_{B/IC} = 1.5 - 0.5 = 1.00$ m. Then

$$v_G = \omega r_{G/IC} = 6(1.00) = 6.00 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

16–91. If the center O of the gear is given a velocity of $v_O = 10$ m/s, determine the velocity of the slider block B at the instant shown.



General Plane Motion: Since the gear rack is stationary, the IC of the gear is located at the contact point between the gear and the rack, Fig. a. Here, $r_{O/IC} = 0.175$ m and $r_{A/IC} = 0.6$ m. Thus, the velocity of point A can be determined using the similar triangles shown in Fig. a,



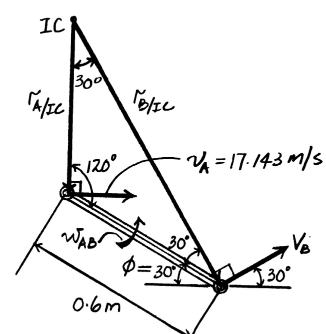
(a)

$$\omega_g = \frac{v_A}{r_{A/IC}} = \frac{v_O}{r_{O/IC}}$$

$$\frac{v_A}{0.3} = \frac{10}{0.175}$$

$$v_A = 17.143 \text{ m/s} \rightarrow$$

The location of the IC for rod AB is indicated in Fig. b. From the geometry shown in Fig. b,



(b)

$$r_{A/IC} = 0.6 \text{ m}$$

$$r_{B/IC} = 2(0.6 \cos 30°) = 1.039 \text{ m}$$

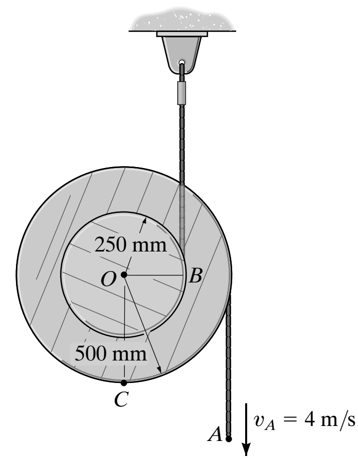
Thus, the angular velocity of the gear can be determined from

$$\omega_{AB} = \frac{v_A}{r_{A/IC}} = \frac{17.143}{0.6} = 28.57 \text{ rad/s}$$

Then,

$$v_B = \omega_{AB}\, r_{B/IC} = 28.57(1.039) = 29.7 \text{ m/s} \qquad \textbf{Ans.}$$

***16–92.** If end A of the cord is pulled down with a velocity of $v_A = 4$ m/s, determine the angular velocity of the spool and the velocity of point C located on the outer rim of the spool.



General Plane Motion: Since the contact point B between the rope and the spool is at rest, the IC is located at point B, Fig. a. From the geometry of Fig. a,

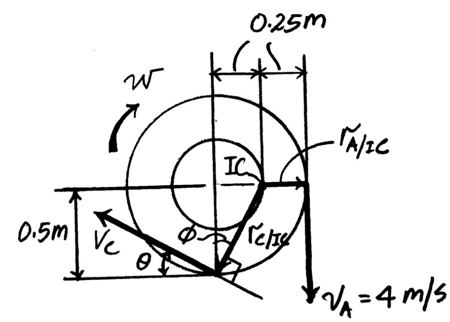
$$r_{A/IC} = 0.25 \text{ m}$$

$$r_{C/IC} = \sqrt{0.25^2 + 0.5^2} = 0.5590 \text{ m}$$

$$\phi = \tan^{-1}\left(\frac{0.25}{0.5}\right) = 26.57°$$

Thus, the angular velocity of the spool can be determined from

$$\omega = \frac{v_A}{r_{A/IC}} = \frac{4}{0.25} = 16 \text{rad/s} \qquad \textbf{Ans.}$$



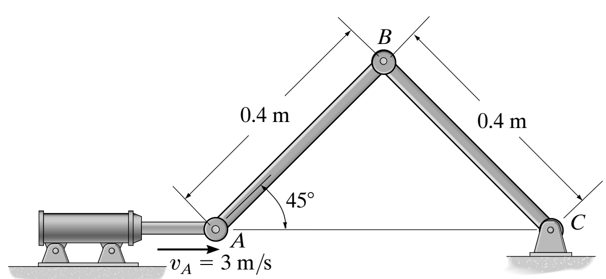
Then,

$$v_C = \omega r_{C/IC} = 16(0.5590) = 8.94 \text{m/s} \qquad \textbf{Ans.}$$

and its direction is

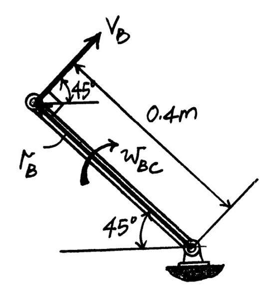
$$\theta = \phi = 26.6° \qquad \textbf{Ans.}$$

•16–93. If end A of the hydraulic cylinder is moving with a velocity of $v_A = 3$ m/s, determine the angular velocity of rod BC at the instant shown.



Rotation About a Fixed Axis: Referring to Fig. a,

$$v_B = \omega_{BC}\, r_B = \omega_{BC}\,(0.4)$$



(a)

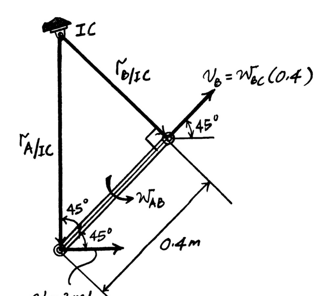
General Plane Motion: The location of the IC for rod AB is indicated in Fig. b. From the geometry shown in this figure, we obtain

$$r_{A/IC} = \frac{0.4}{\cos 45°} \qquad\qquad r_{A/IC} = 0.5657 \text{ m}$$

$$r_{B/IC} = 0.4 \tan 45° = 0.4 \text{ m}$$

Thus, the angular velocity of rod AB can be determined from

$$\omega_{AB} = \frac{v_A}{r_{A/IC}} = \frac{3}{0.5657} = 5.303 \text{ rad/s}$$



(b)

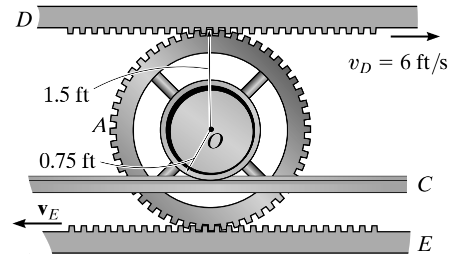
Then,

$$v_B = \omega_{AB}\, r_{B/IC}$$

$$\omega_{BC}\,(0.4) = 5.303(0.4)$$

$$\omega_{BC} = 5.30 \text{ rad/s} \qquad \textbf{Ans.}$$

16–94. The wheel is rigidly attached to gear A, which is in mesh with gear racks D and E. If D has a velocity of $v_D = 6$ ft/s to the right and wheel rolls on track C without slipping, determine the velocity of gear rack E.



General Plane Motion: Since the wheel rolls without slipping on track C, the IC is located there, Fig. a. Here,

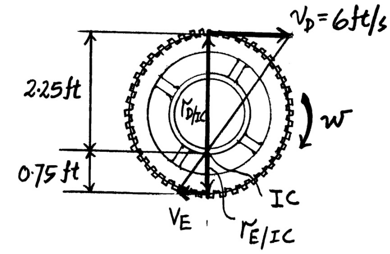
$$r_{D/IC} = 2.25 \text{ ft} \qquad r_{E/IC} = 0.75 \text{ ft}$$

Thus, the angular velocity of the gear can be determined from

$$\omega = \frac{v_D}{r_{D/IC}} = \frac{6}{2.25} = 2.667 \text{ rad/s}$$

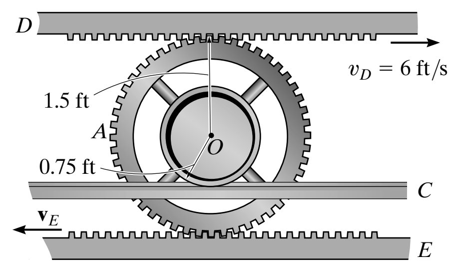
Then,

$$v_E = \omega r_{E/IC} = 2.667(0.75) = 2 \text{ ft/s} \leftarrow$$ **Ans.**



(a)

16–95. The wheel is rigidly attached to gear A, which is in mesh with gear racks D and E. If the racks have a velocity of $v_D = 6$ ft/s and $v_E = 10$ ft/s, show that it is necessary for the wheel to slip on the fixed track C. Also find the angular velocity of the gear and the velocity of its center O.



General Plane Motion: The location of the IC can be found using the similar triangles shown in Fig. a,

$$\frac{r_{D/IC}}{6} = \frac{3 - r_{D/IC}}{10} \qquad r_{D/IC} = 1.125 \text{ ft}$$

Thus,

$$r_{O/IC} = 1.5 - r_{D/IC} = 1.5 - 1.125 = 0.375\text{ft}$$

$$r_{F/IC} = 2.25 - r_{D/IC} = 2.25 - 1.125 = 1.125\text{ft}$$

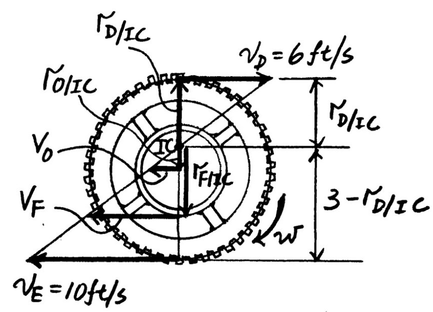
Thus, the angular velocity of the gear is

$$\omega = \frac{v_D}{r_{D/IC}} = \frac{6}{1.125} = 5.333 \text{ rad/s} = 5.33 \text{ rad/s}$$ **Ans.**

The velocity of the contact point F between the wheel and the track is

$$v_F = \omega r_{F/IC} = 5.333(1.125) = 6 \text{ ft/s} \leftarrow$$

Since $v_F \neq 0$, **the wheel slips on the track** **(Q.E.D.)**

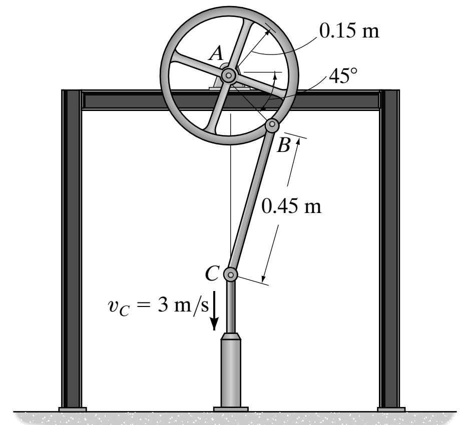


(a)

The velocity of center O of the gear is

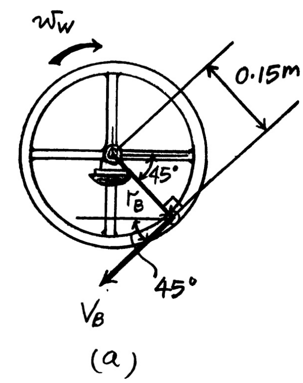
$$v_O = \omega r_{O/IC} = 5.333(0.375) = 2\text{ft/s} \leftarrow$$ **Ans.**

***16–96.** If C has a velocity of $v_C = 3$ m/s, determine the angular velocity of the wheel at the instant shown.



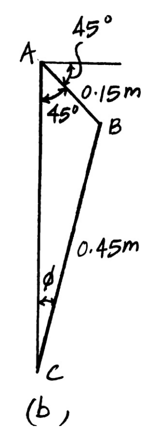
Rotation About a Fixed Axis: Referring to Fig. *a*,

$$v_B = \omega_W r_B = \omega_W(0.15)$$



General Plane Motion: Applying the law of sines to the geometry shown in Fig. *b*,

$$\frac{\sin\phi}{0.15} = \frac{\sin 45^\circ}{0.45} \qquad \phi = 13.63^\circ$$



The location of the IC for rod BC is indicated in Fig. *c*. Applying the law of sines to the geometry of Fig. *c*,

$$\frac{r_{C/IC}}{\sin 58.63^\circ} = \frac{0.45}{\sin 45^\circ} \qquad r_{C/IC} = 0.5434\text{ m}$$

$$\frac{r_{B/IC}}{\sin 76.37^\circ} = \frac{0.45}{\sin 45^\circ} \qquad r_{B/IC} = 0.6185\text{ m}$$

Thus, the angular velocity of rod BC is

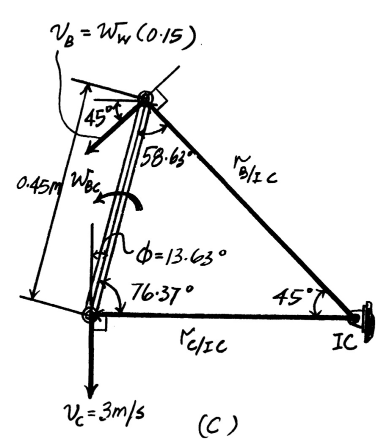
$$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{3}{0.5434} = 5.521\text{ rad/s}$$

and

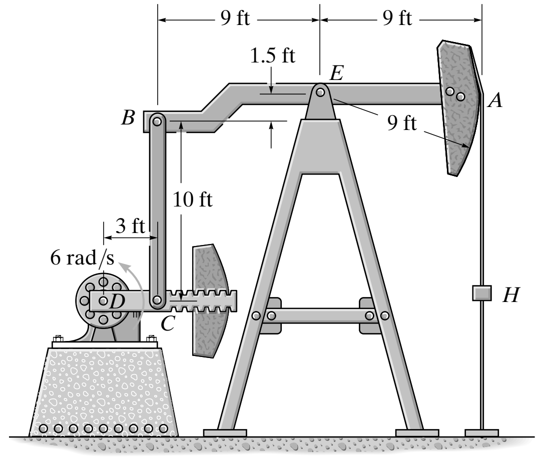
$$v_B = \omega_{BC}\, r_{B/IC}$$

$$\omega_W(0.15) = 5.521(0.6185)$$

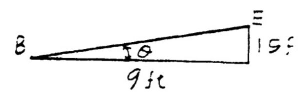
$$\omega_W = 22.8\text{ rad/s} \qquad \textbf{Ans.}$$



•16–97. The oil pumping unit consists of a walking beam AB, connecting rod BC, and crank CD. If the crank rotates at a constant rate of 6 rad/s, determine the speed of the rod hanger H at the instant shown. *Hint:* Point B follows a circular path about point E and therefore the velocity of B is *not* vertical.



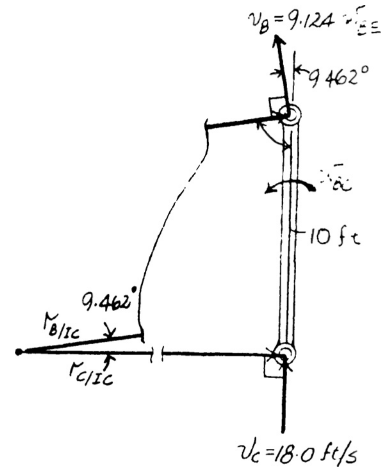
Kinematic Diagram: From the geometry, $\theta = \tan^{-1}\left(\frac{1.5}{9}\right) = 9.462°$ and $r_{BE} = \sqrt{9^2 + 1.5^2} = 9.124$ ft. Since crank CD and beam BE are rotating about fixed points D and E, then $\mathbf{v}_C$ and $\mathbf{v}_B$ are always directed perpendicular to crank CD and beam BE, respectively. The magnitude of $\mathbf{v}_C$ and $\mathbf{v}_B$ are $v_C = \omega_{CD} r_{CD} = 6(3) = 18.0$ ft/s and $v_B = \omega_{BE} r_{BE} = 9.124\omega_{BE}$. At the instant shown, $\mathbf{v}_C$ is directed vertically while $\mathbf{v}_B$ is directed with an angle 9.462° with the vertical.



Instantaneous Center: The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. From the geometry

$$r_{B/IC} = \frac{10}{\sin 9.462°} = 60.83 \text{ ft}$$

$$r_{C/IC} = \frac{10}{\tan 9.462°} = 60.0 \text{ ft}$$



The angular velocity of link BC is given by

$$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{18.0}{60.0} = 0.300 \text{ rad/s}$$

Thus, the angular velocity of beam BE is given by

$$v_B = \omega_{BC} r_{B/IC}$$

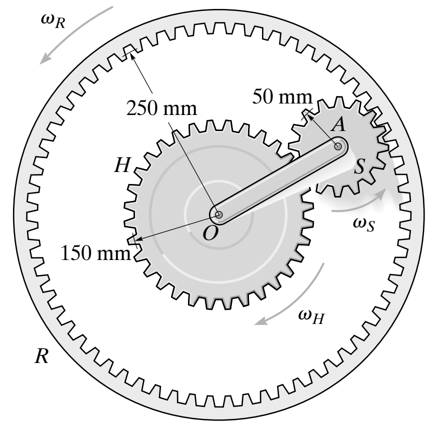
$$9.124\omega_{BE} = 0.300(60.83)$$

$$\omega_{BE} = 2.00 \text{ rad/s}$$

The speed of rod hanger H is given by

$$v_H = \omega_{BE} r_{EA} = 2.00(9) = 18.0 \text{ ft/s}$$ **Ans.**

16–98. If the hub gear H and ring gear R have angular velocities $\omega_H = 5$ rad/s and $\omega_R = 20$ rad/s, respectively, determine the angular velocity ω_S of the spur gear S and the angular velocity of arm OA.



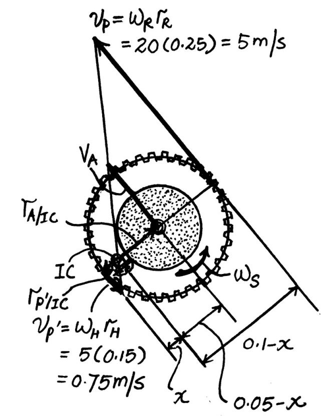
$$\frac{5}{0.1 - x} = \frac{0.75}{x}$$

$$x = 0.01304 \text{ m}$$

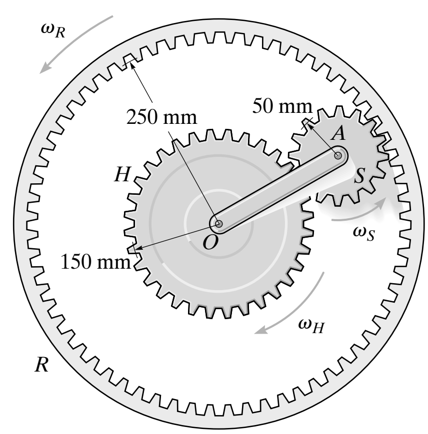
$$\omega_s = \frac{0.75}{0.01304} = 57.5 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

$$v_A = 57.5(0.05 - 0.01304) = 2.125 \text{ m/s}$$

$$\omega_{OA} = \frac{2.125}{0.2} = 10.6 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$



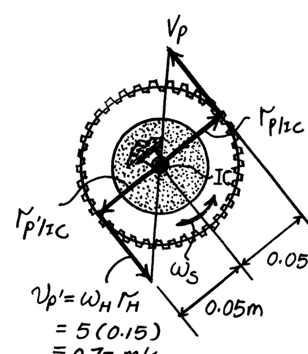
16–99. If the hub gear H has an angular velocity $\omega_H = 5$ rad/s, determine the angular velocity of the ring gear R so that the arm OA which is pinned to the spur gear S remains stationary ($\omega_{OA} = 0$). What is the angular velocity of the spur gear?



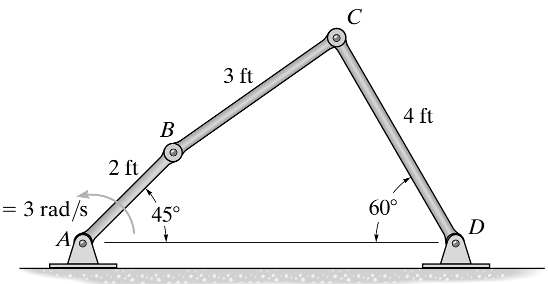
The IC is at A.

$$\omega_S = \frac{0.75}{0.05} = 15.0 \text{ rad/s} \qquad \textbf{Ans.}$$

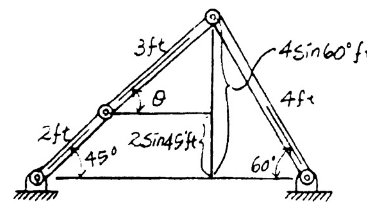
$$\omega_R = \frac{0.75}{0.250} = 3.00 \text{ rad/s} \qquad \textbf{Ans.}$$



***16–100.** If rod AB is rotating with an angular velocity $\omega_{AB} = 3$ rad/s, determine the angular velocity of rod BC at the instant shown.



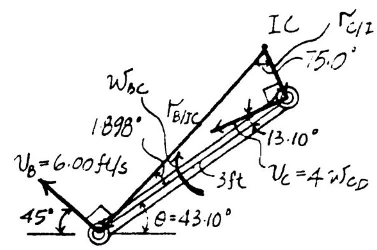
Kinematic Diagram: From the geometry, $\theta = \sin^{-1}\left(\dfrac{4\sin 60° - 2\sin 45°}{3}\right) = 43.10°$. Since links AB and CD is rotating about fixed points A and D, then $\mathbf{v}_B$ and $\mathbf{v}_C$ are always directed perpendicular to links AB and CD, respectively. The magnitude of $\mathbf{v}_B$ and $\mathbf{v}_C$ are $v_B = \omega_{AB} r_{AB} = 3(2) = 6.00$ ft/s and $v_C = \omega_{CD} r_{CD} = 4\omega_{CD}$. At the instant shown, $\mathbf{v}_B$ is directed at an angle of 45° while $\mathbf{v}_C$ is directed at 30°



Instantaneous Center: The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sines, we have

$$\frac{r_{B/IC}}{\sin 103.1°} = \frac{3}{\sin 75°} \qquad r_{B/IC} = 3.025 \text{ ft}$$

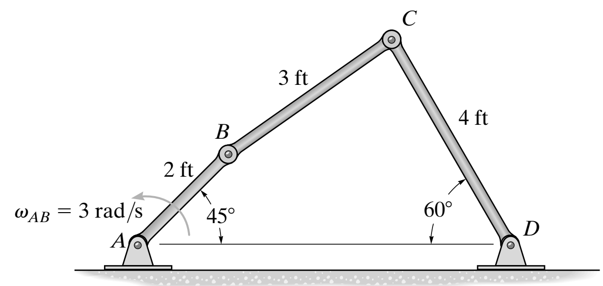
$$\frac{r_{C/IC}}{\sin 1.898°} = \frac{3}{\sin 75°} \qquad r_{C/IC} = 0.1029 \text{ ft}$$



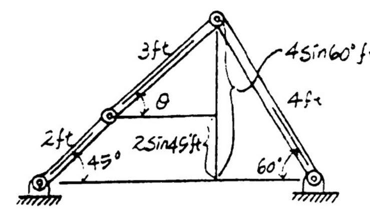
The angular velocity of link BC is given by

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{6.00}{3.025} = 1.983 \text{ rad/s} = 1.98 \text{ rad/s} \qquad \textbf{Ans.}$$

•16–101. If rod AB is rotating with an angular velocity $\omega_{AB} = 3$ rad/s, determine the angular velocity of rod CD at the instant shown.



Kinematic Diagram: From the geometry. $\theta = \sin^{-1}\left(\dfrac{4\sin 60° - 2\sin 45°}{3}\right) = 43.10°$. Since links AB and CD is rotating about fixed points A and D, then $\mathbf{v}_B$ and $\mathbf{v}_C$ are always directed perpendicular to links AB and CD, respectively. The magnitude of $\mathbf{v}_B$ and $\mathbf{v}_C$ are $v_B = \omega_{AB} r_{AB} = 3(2) = 6.00$ ft/s and $v_C = \omega_{CD} r_{CD} = 4\omega_{CD}$. At the instant shown, $\mathbf{v}_B$ is directed at an angle of 45° while $\mathbf{v}_C$ is directed at 30°.



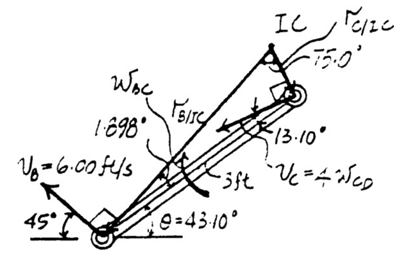
Instantaneous Center: The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sines, we have

$$\frac{r_{B/IC}}{\sin 103.1°} = \frac{3}{\sin 75°} \qquad r_{B/IC} = 3.025 \text{ ft}$$

$$\frac{r_{C/IC}}{\sin 1.898°} = \frac{3}{\sin 75°} \qquad r_{C/IC} = 0.1029 \text{ ft}$$

The angular velocity of link BC is given by

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{6.00}{3.025} = 1.983 \text{ rad/s}$$



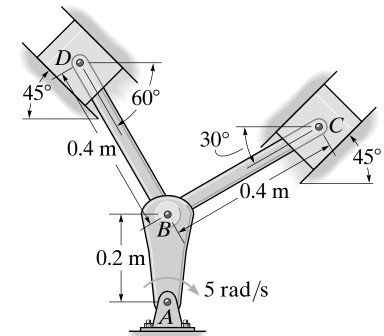
Thus, the angular velocity of link CD is given by

$$v_C = \omega_{BC} r_{C/IC}$$

$$4\omega_{CD} = 1.983(0.1029)$$

$$\omega_{CD} = 0.0510 \text{ rad/s} \qquad \textbf{Ans.}$$

16–102. The mechanism used in a marine engine consists of a crank AB and two connecting rods BC and BD. Determine the velocity of the piston at C the instant the crank is in the position shown and has an angular velocity of 5 rad/s.



$v_B = 0.2(5) = 1 \text{ m/s} \rightarrow$

Member BC:

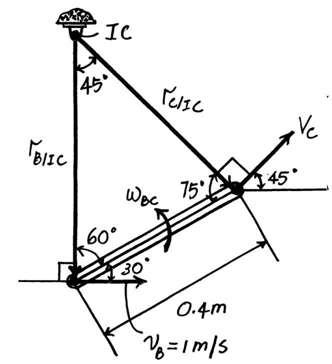
$$\frac{r_{C/IC}}{\sin 60^\circ} = \frac{0.4}{\sin 45^\circ}$$

$r_{C/IC} = 0.4899$ m

$$\frac{r_{B/IC}}{\sin 75^\circ} = \frac{0.4}{\sin 45^\circ}$$

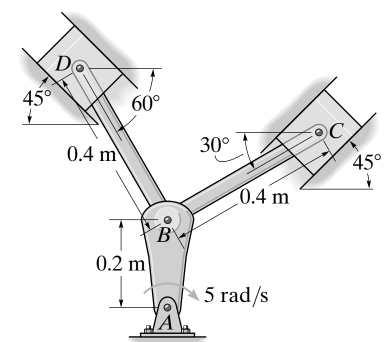
$r_{B/IC} = 0.5464$ m

$$\omega_{BC} = \frac{1}{0.5464} = 1.830 \text{ rad/s}$$



$v_C = 0.4899(1.830) = 0.897 \text{ m/s} \nearrow$ **Ans.**

16–103. The mechanism used in a marine engine consists of a crank AB and two connecting rods BC and BD. Determine the velocity of the piston at D the instant the crank is in the position shown and has an angular velocity of 5 rad/s.



$v_B = 0.2(5) = 1 \text{ m/s} \rightarrow$

Member BD:

$$\frac{r_{B/IC}}{\sin 105^\circ} = \frac{0.4}{\sin 45^\circ}$$

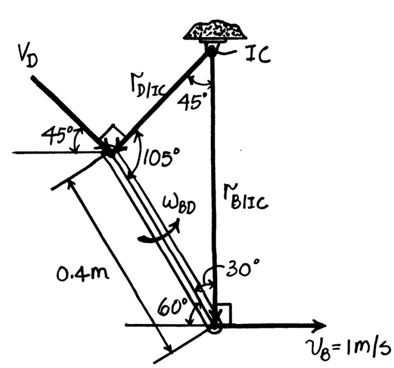
$r_{B/IC} = 0.54641$ m

$$\frac{r_{D/IC}}{\sin 30^\circ} = \frac{0.4}{\sin 45^\circ}$$

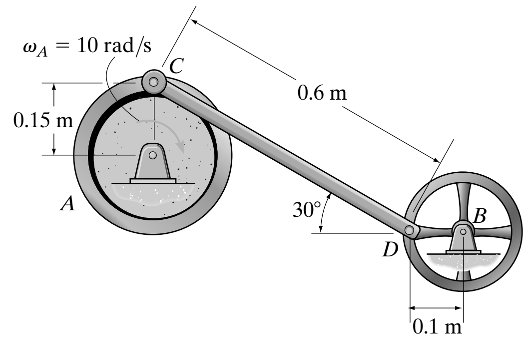
$r_{D/IC} = 0.28284$ m

$$\omega_{BD} = \frac{1}{0.54641} = 1.830 \text{ rad/s}$$

$v_D = 1.830(0.28284) = 0.518 \text{ m/s} \searrow$ **Ans.**



***16–104.** If flywheel A is rotating with an angular velocity of $\omega_A = 10$ rad/s, determine the angular velocity of wheel B at the instant shown.



Rotation About a Fixed Axis: Referring to Figs. *a* and *b*,

$$v_C = \omega_A r_C = 10(0.15) = 1.5 \text{ m/s} \rightarrow$$

$$v_D = \omega_B r_D = \omega_B(0.1) \downarrow$$

General Plane Motion: The location of the *IC* for rod *CD* is indicated in Fig. *c*. From the geometry of this figure, we obtain

$$r_{C/IC} = 0.6 \sin 30° = 0.3 \text{ m}$$

$$r_{D/IC} = 0.6 \cos 30° = 0.5196 \text{ m}$$

Thus, the angular velocity of rod *CD* can be determined from

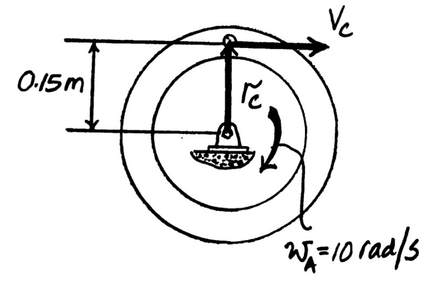
$$\omega_{CD} = \frac{v_D}{r_{C/IC}} = \frac{1.5}{0.3} = 5 \text{ rad/s}$$

Then,

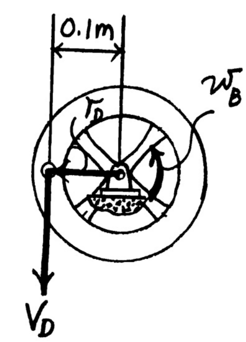
$$v_D = \omega_{CD}\, r_{D/IC}$$

$$\omega_B(0.1) = 5(0.5196)$$

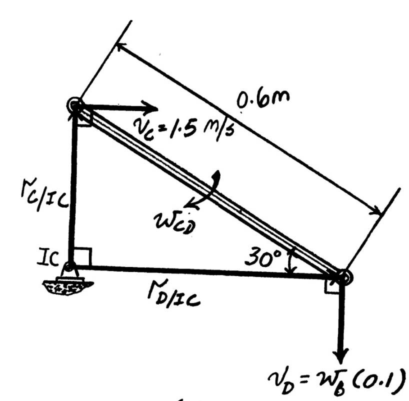
$$\omega_B = 26.0 \text{ rad/s} \qquad \textbf{Ans.}$$



(a)

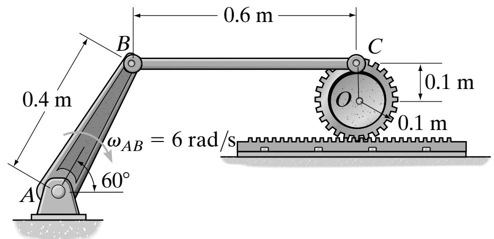


(b)



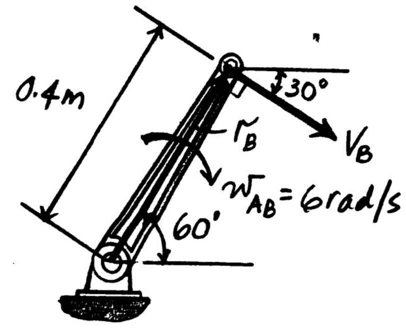
(c)

•16–105. If crank AB is rotating with an angular velocity of $\omega_{AB} = 6$ rad/s, determine the velocity of the center O of the gear at the instant shown.



Rotation About a Fixed Axis: Referring to Fig. a,

$$v_B = \omega_{AB}\, r_B = 6(0.4) = 2.4 \text{ m/s}$$



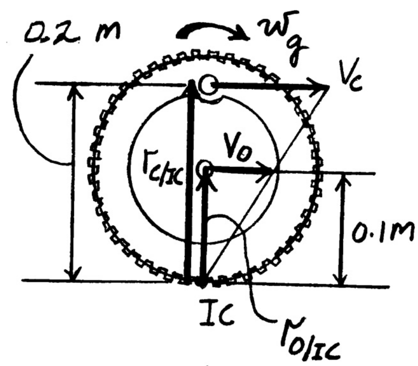
(a)

General Plane Motion: Since the gear rack is stationary, the IC of the gear is located at the contact point between the gear and the rack, Fig. b. Thus, $\mathbf{v}_O$ and $\mathbf{v}_C$ can be related using the similar triangles shown in Fig. b,

$$\omega_g = \frac{v_C}{r_{C/IC}} = \frac{v_O}{r_{O/IC}}$$

$$\frac{v_C}{0.2} = \frac{v_O}{0.1}$$

$$v_C = 2v_O$$



(b)

The location of the IC for rod BC is indicated in Fig. c. From the geometry shown,

$$r_{B/IC} = \frac{0.6}{\cos 60^\circ} = 1.2 \text{ m}$$

$$r_{C/IC} = 0.6 \tan 60^\circ = 1.039 \text{ m}$$

Thus, the angular velocity of rod BC can be determined from

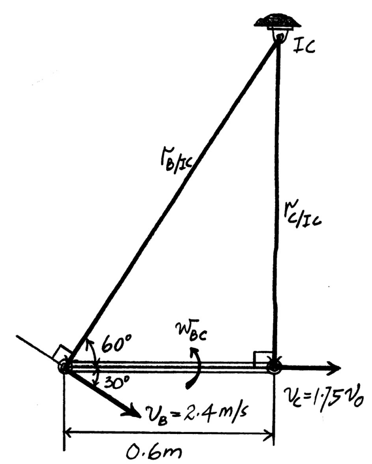
$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{2.4}{1.2} = 2 \text{ rad/s}$$

Then,

$$v_C = \omega_{BC}\, r_{C/IC}$$

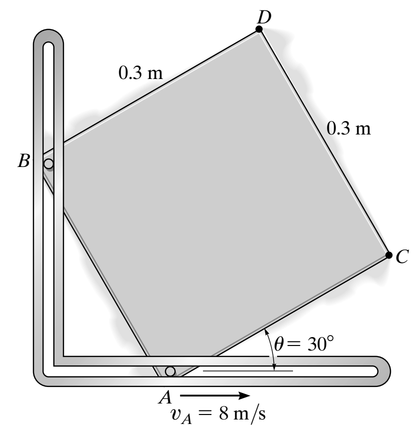
$$2v_O = 2(1.039)$$

$$v_O = 1.04 \text{ m/s} \rightarrow$$ **Ans.**



(c)

16–106. The square plate is constrained within the slots at A and B. When $\theta = 30°$, point A is moving at $v_A = 8$ m/s. Determine the velocity of point C at this instant.



$$r_{A/IC} = 0.3 \cos 30° = 0.2598 \text{ m}$$

$$\omega = \frac{8}{0.2598} = 30.792 \text{ rad/s}$$

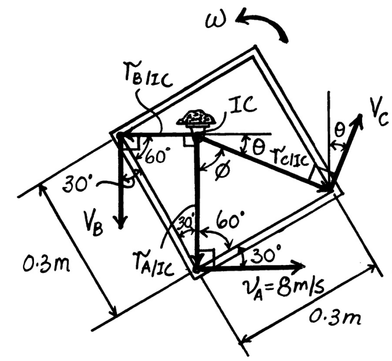
$$r_{C/IC} = \sqrt{(0.2598)^2 + (0.3)^2 - 2(0.2598)(0.3)\cos 60°} = 0.2821 \text{ m}$$

$$v_C = (0.2821)(30.792) = 8.69 \text{ m/s}$$ **Ans.**

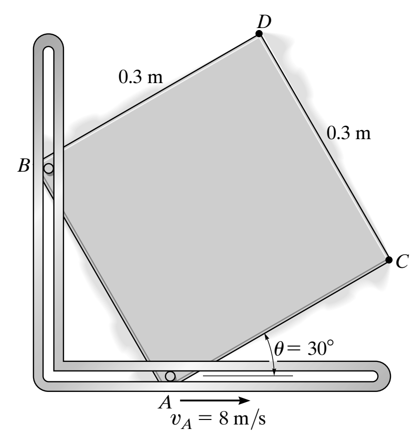
$$\frac{\sin \phi}{0.3} = \frac{\sin 60°}{0.2821}$$

$$\phi = 67.09°$$

$$\theta = 90° - 67.09° = 22.9°$$ **Ans.**



16–107. The square plate is constrained within the slots at A and B. When $\theta = 30°$, point A is moving at $v_A = 8$ m/s. Determine the velocity of point D at this instant.



$$r_{A/IC} = 0.3 \cos 30° = 0.2598 \text{ m}$$

$$\omega = \frac{8}{0.2598} = 30.792 \text{ rad/s}$$

$$r_{B/IC} = 0.3 \sin 30° = 0.15 \text{ m}$$

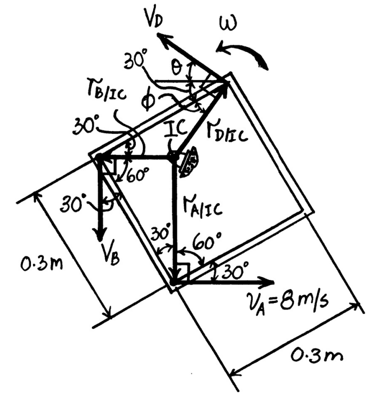
$$r_{D/IC} = \sqrt{(0.3)^2 + (0.15)^2 - 2(0.3)(0.15)\cos 30°} = 0.1859 \text{ m}$$

$$v_D = (30.792)(0.1859) = 5.72 \text{ m/s}$$ **Ans.**

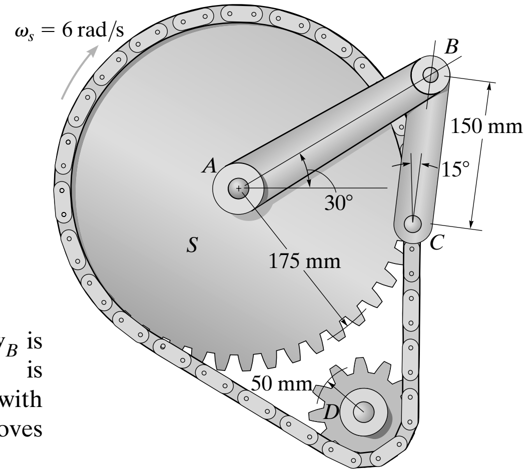
$$\frac{\sin \phi}{0.15} = \frac{\sin 30°}{0.1859}$$

$$\phi = 23.794°$$

$$\theta = 90° - 30° - 23.794° = 36.2°$$ **Ans.**



***16–108.** The mechanism produces intermittent motion of link AB. If the sprocket S is turning with an angular velocity of $\omega_S = 6$ rad/s, determine the angular velocity of link BC at this instant. The sprocket S is mounted on a shaft which is separate from a collinear shaft attached to AB at A. The pin at C is attached to one of the chain links.

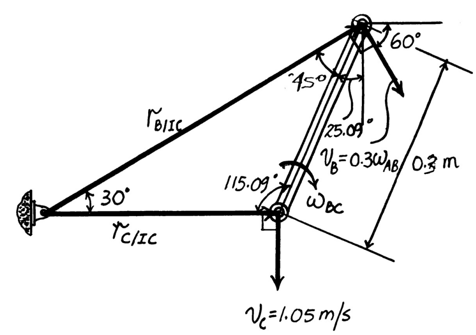


Kinematic Diagram: Since link AB is rotating about the fixed point A, then $\mathbf{v}_B$ is always directed perpendicular to link AB and its magnitude is $v_B = \omega_{AB}\, r_{AB} = 0.2\omega_{AB}$. At the instant shown, $\mathbf{v}_B$ is directed at an angle 60° with the horizontal. Since point C is attached to the chain, at the instant shown, it moves vertically with a speed of $v_C = \omega_S\, r_S = 6(0.175) = 1.05$ m/s.

Instantaneous Center: The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sines, we have

$$\frac{r_{B/IC}}{\sin 105^\circ} = \frac{0.15}{\sin 30^\circ} \qquad r_{B/IC} = 0.2898 \text{ m}$$

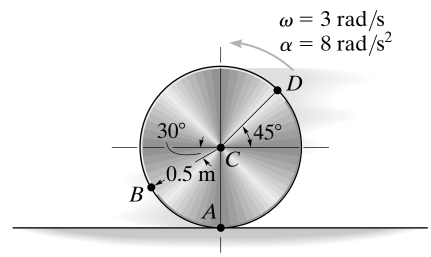
$$\frac{r_{C/IC}}{\sin 45^\circ} = \frac{0.15}{\sin 30^\circ} \qquad r_{C/IC} = 0.2121 \text{ m}$$



The angular velocity of bar BC is given by

$$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{1.05}{0.2121} = 4.950 \text{ rad/s}$$ **Ans.**

•16–109. The disk is moving to the left such that it has an angular acceleration $\alpha = 8\ \text{rad/s}^2$ and angular velocity $\omega = 3\ \text{rad/s}$ at the instant shown. If it does not slip at A, determine the acceleration of point B.



$$a_C = 0.5(8) = 4\ \text{m/s}^2$$

$$\mathbf{a}_B = \mathbf{a}_C + \mathbf{a}_{B/C}$$

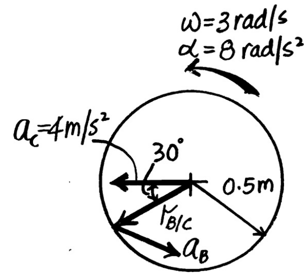
$$\mathbf{a}_B = \left[\underset{\leftarrow}{4}\right] + \left[\underset{\nearrow 30^\circ}{(3)^2(0.5)}\right] + \left[\underset{\searrow 30^\circ}{(0.5)(8)}\right]$$

$(\overset{+}{\rightarrow})$ $\quad (a_B)_x = -4 + 4.5\cos 30^\circ + 4\sin 30^\circ = 1.897\ \text{m/s}^2$

$(+\uparrow)$ $\quad (a_B)_y = 0 + 4.5\sin 30^\circ - 4\cos 30^\circ = -1.214\ \text{m/s}^2$

$$a_B = \sqrt{(1.897)^2 + (-1.214)^2} = 2.25\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{1.214}{1.897}\right) = 32.6^\circ \ \searrow \qquad \textbf{Ans.}$$



Also,

$$\mathbf{a}_B = \mathbf{a}_C + \alpha \times \mathbf{r}_{B/C} - \omega^2\,\mathbf{r}_{B/C}$$

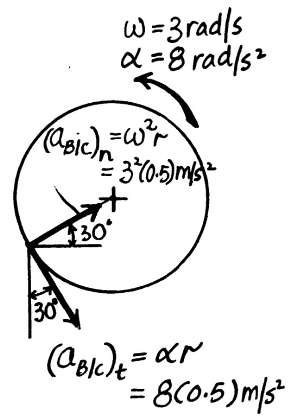
$$(a_B)_x\,\mathbf{i} + (a_B)_y\,\mathbf{j} = -4\mathbf{i} + (8\mathbf{k}) \times (-0.5\cos 30^\circ\mathbf{i} - 0.5\sin 30^\circ\mathbf{j}) - (3)^2\,(-0.5\cos 30^\circ\mathbf{i} - 0.5\sin 30^\circ\mathbf{j})$$

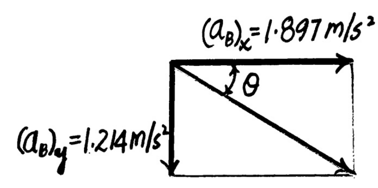
$(\overset{+}{\rightarrow})$ $\quad (a_B)_x = -4 + 8(0.5\sin 30^\circ) + (3)^2(0.5\cos 30^\circ) = 1.897\ \text{m/s}^2$

$(+\uparrow)$ $\quad (a_B)_y = 0 - 8(0.5\cos 30^\circ) + (3)^2\,(0.5\sin 30^\circ) = -1.214\ \text{m/s}^2$

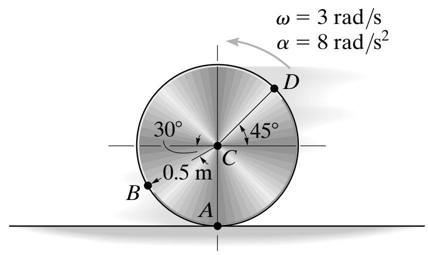
$$\theta = \tan^{-1}\left(\frac{1.214}{1.897}\right) = 32.6^\circ \ \searrow \qquad \textbf{Ans.}$$

$$a_B = \sqrt{(1.897)^2 + (-1.214)^2} = 2.25\ \text{m/s}^2 \qquad \textbf{Ans.}$$





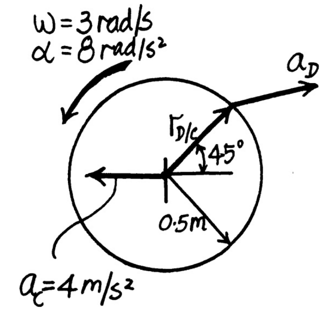
16–110. The disk is moving to the left such that it has an angular acceleration $\alpha = 8\ \text{rad/s}^2$ and angular velocity $\omega = 3\ \text{rad/s}$ at the instant shown. If it does not slip at A, determine the acceleration of point D.



$$a_C = 0.5(8) = 4\ \text{m/s}^2$$

$$\mathbf{a}_D = \mathbf{a}_C + \mathbf{a}_{D/C}$$

$$\mathbf{a}_D = \begin{bmatrix} 4 \\ \leftarrow \end{bmatrix} + \begin{bmatrix} (3)^2(0.5) \\ \measuredangle\ 45° \end{bmatrix} + \begin{bmatrix} 8\,(0.5) \\ 45° \searrow \end{bmatrix}$$



$(\xrightarrow{+})$ $\quad (a_D)_x = -4 - 4.5 \sin 45° - 4 \cos 45° = -10.01\ \text{m/s}^2$

$(+\uparrow)$ $\quad (a_D)_y = 0 - 4.5 \cos 45° + 4 \sin 45° = -0.3536\ \text{m/s}^2$

$$\theta = \tan^{-1}\left(\frac{0.3536}{10.01}\right) = 2.02° \qquad \textbf{Ans.}$$

$$a_D = \sqrt{(-10.01)^2 + (-0.3536)^2} = 10.0\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Also,

$$\mathbf{a}_D = \mathbf{a}_C + \alpha \times \mathbf{r}_{D/C} - \omega^2\, \mathbf{r}_{D/C}$$

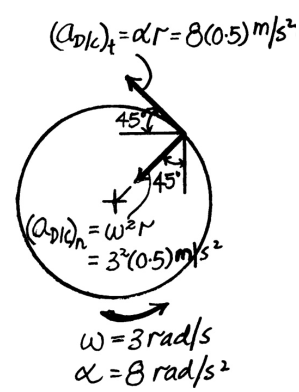
$$(a_D)_x\,\mathbf{i} + (a_D)_y\,\mathbf{j} = -4\mathbf{i} + (8\mathbf{k}) \times (0.5 \cos 45°\mathbf{i} + 0.5 \sin 45°\mathbf{j}) - (3)^2\,(0.5 \cos 45°\mathbf{i} + 0.5 \sin 45°\mathbf{j})$$

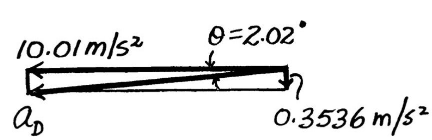
$(\xrightarrow{+})$ $\quad (a_D)_x = -4 - 8(0.5 \sin 45°) - (3)^2(0.5 \cos 45°) = -10.01\ \text{m/s}^2$

$(+\uparrow)$ $\quad (a_D)_y = +8(0.5 \cos 45°) - (3)^2\,(0.5 \sin 45°) = -0.3536\ \text{m/s}^2$

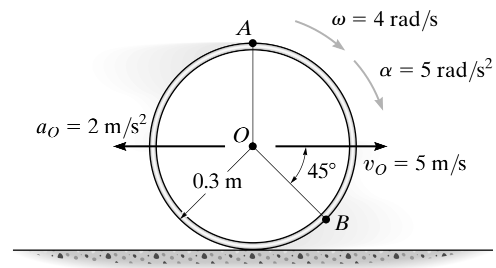
$$\theta = \tan^{-1}\left(\frac{0.3536}{10.01}\right) = 2.02° \qquad \textbf{Ans.}$$

$$a_D = \sqrt{(-10.01)^2 + (-0.3536)^2} = 10.0\ \text{m/s}^2 \qquad \textbf{Ans.}$$





16–111. The hoop is cast on the rough surface such that it has an angular velocity $\omega = 4$ rad/s and an angular acceleration $\alpha = 5$ rad/s^2. Also, its center has a velocity $v_O = 5$ m/s and a deceleration $a_O = 2$ m/s^2. Determine the acceleration of point A at this instant.



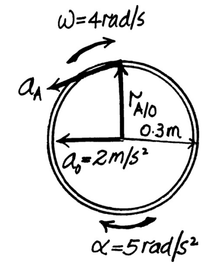
$$\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}_{A/O}$$

$$\mathbf{a}_A = \left[\underset{\leftarrow}{2}\right] + \left[\underset{\downarrow}{(4)^2(0.3)}\right] + \left[\underset{\rightarrow}{5(0.3)}\right]$$

$$\mathbf{a}_A = \left[\underset{\leftarrow}{0.5}\right] + \left[\underset{\downarrow}{4.8}\right]$$

$$a_A = 4.83 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{4.8}{0.5}\right) = 84.1° \qquad \textbf{Ans.}$$



Also,

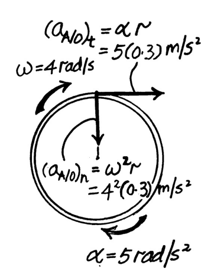
$$\mathbf{a}_A = \mathbf{a}_O - \omega^2 \mathbf{r}_{A/O} + \alpha \times \mathbf{r}_{A/O}$$

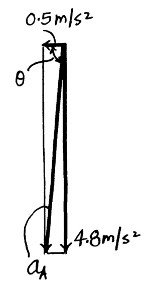
$$\mathbf{a}_A = -2\mathbf{i} - (4)^2(0.3\mathbf{j}) + (-5\mathbf{k}) \times (0.3\mathbf{j})$$

$$\mathbf{a}_A = \{-0.5\mathbf{i} - 4.8\mathbf{j}\} \text{ m/s}^2$$

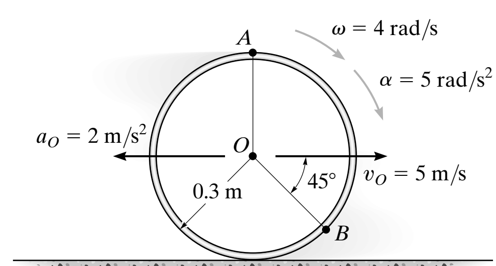
$$a_A = 4.83 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{4.8}{0.5}\right) = 84.1° \qquad \textbf{Ans.}$$





***16–112.** The hoop is cast on the rough surface such that it has an angular velocity $\omega = 4$ rad/s and an angular acceleration $\alpha = 5$ rad/s^2. Also, its center has a velocity of $v_O = 5$ m/s and a deceleration $a_O = 2$ m/s^2. Determine the acceleration of point B at this instant.



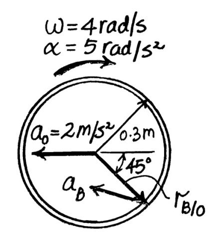
$$\mathbf{a}_B = \mathbf{a}_O + \mathbf{a}_{B/O}$$

$$\mathbf{a}_B = \left[\underset{\leftarrow}{2}\right] + \left[5(0.3)\right] + \left[(4)^2(0.3)\right]$$

$$\mathbf{a}_B = \left[\underset{\leftarrow}{6.4548}\right] + \left[\underset{\uparrow}{2.333}\right]$$

$$a_B = 6.86 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{2.333}{6.4548}\right) = 19.9°$$



Also:

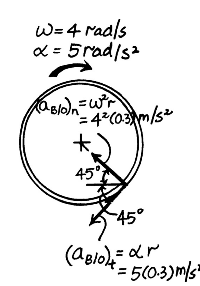
$$\mathbf{a}_B = \mathbf{a}_O + \alpha \times \mathbf{r}_{B/O} - \omega^2 \mathbf{r}_{B/O}$$

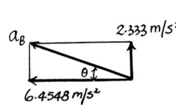
$$\mathbf{a}_B = -2\mathbf{i} + (-5\mathbf{k}) \times (0.3 \cos 45°\mathbf{i} - 0.3 \sin 45°\mathbf{j}) - (4)^2(0.3 \cos 45°\mathbf{i} - 0.3 \sin 45°\mathbf{j})$$

$$\mathbf{a}_B = \{-6.4548\mathbf{i} + 2.333\mathbf{j}\} \text{ m/s}^2$$

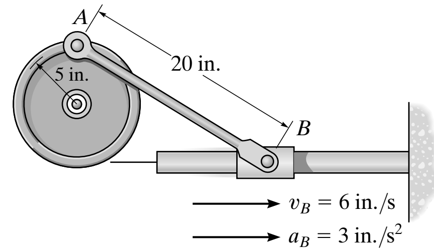
$$a_B = 6.86 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{2.333}{6.4548}\right) = 19.9°$$

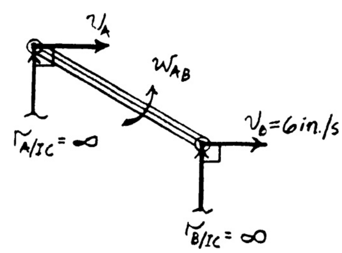




•16–113. At the instant shown, the slider block B is traveling to the right with the velocity and acceleration shown. Determine the angular acceleration of the wheel at this instant.



Velocity Analysis: The angular velocity of link AB can be obtained by using the method of instantaneous center of zero velocity. Since $\mathbf{v}_A$ and $\mathbf{v}_B$ are parallel, $r_{A/IC} = r_{B/IC} = \infty$. Thus, $\omega_{AB} = 0$. Since $\omega_{AB} = 0$, $v_A = v_B = 6$ in./s. Thus, the angular velocity of the wheel is $\omega_W = \dfrac{v_A}{r_{OA}} = \dfrac{6}{5} = 1.20$ rad/s.



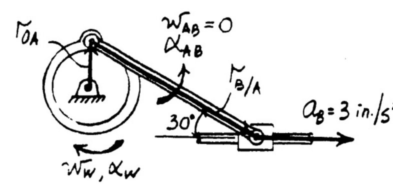
Acceleration Equation: The acceleration of point A can be obtained by analyzing the angular motion of link OA about point O. Here, $\mathbf{r}_{OA} = \{5\mathbf{j}\}$ in..

$$\mathbf{a}_A = \alpha_W \times \mathbf{r}_{OA} - \omega_W^2\,\mathbf{r}_{OA}$$

$$= (-\alpha_W\,\mathbf{k}) \times (5\mathbf{j}) - 1.20^2\,(5\mathbf{j})$$

$$= \{5\alpha_W\,\mathbf{i} - 7.20\mathbf{j}\}\ \text{in./s}^2$$

Link AB is subjected to general plane motion. Applying Eq. 16–18 with $\mathbf{r}_{B/A} = \{20\cos 30°\mathbf{i} - 20\sin 30°\mathbf{j}\}$ in. $= \{17.32\mathbf{i} - 10.0\mathbf{j}\}$ in., we have



$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\,\mathbf{r}_{B/A}$$

$$3\mathbf{i} = 5\alpha_W\,\mathbf{i} - 7.20\mathbf{j} + \alpha_{AB}\,\mathbf{k} \times (17.32\mathbf{i} - 10.0\mathbf{j}) - \mathbf{0}$$

$$3\mathbf{i} = (10.0\alpha_{AB} + 5\alpha_W)\,\mathbf{i} + (17.32\alpha_{AB} - 7.20)\,\mathbf{j}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components, we have

$$3 = 10.0\alpha_{AB} + 5\alpha_W \qquad \textbf{[1]}$$

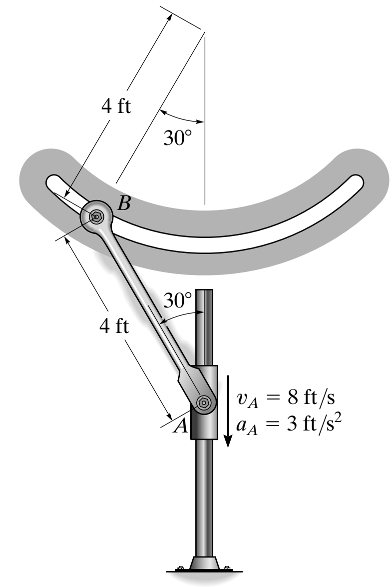
$$0 = 17.32\alpha_{AB} - 7.20 \qquad \textbf{[2]}$$

Solving Eqs.[1] and [2] yields

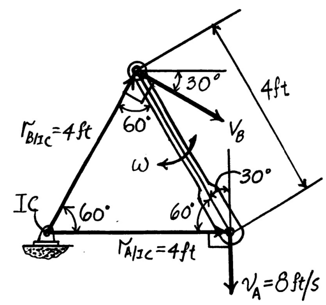
$$\alpha_{AB} = 0.4157\ \text{rad/s}^2$$

$$\alpha_W = -0.2314\ \text{rad/s}^2 = 0.231\ \text{rad/s}^2 \;\circlearrowright \qquad \textbf{Ans.}$$

16–114. The ends of bar AB are confined to move along the paths shown. At a given instant, A has a velocity of 8 ft/s and an acceleration of 3 ft/s². Determine the angular velocity and angular acceleration of AB at this instant.



$$\omega = \frac{8}{4} = 2 \text{ rad/s} \quad \circlearrowright$$ **Ans.**



$$v_B = 4(2) = 8 \text{ ft/s}$$

$$(a_B)_n = \frac{(8)^2}{4} = 16 \text{ ft/s}^2$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$\begin{bmatrix} 16 \\ \nearrow 30^\circ \end{bmatrix} + \begin{bmatrix} (a_B)_t \\ \searrow 30^\circ \end{bmatrix} = \begin{bmatrix} 3 \\ \downarrow \end{bmatrix} + \begin{bmatrix} \alpha(4) \\ \nearrow 60^\circ \end{bmatrix} + \begin{bmatrix} (2)^2(4) \\ \searrow 60^\circ \end{bmatrix}$$

$(\overset{+}{\rightarrow})$ $\quad 16 \sin 30^\circ + (a_B)_t \cos 30^\circ = 0 + \alpha(4) \sin 60^\circ + 16 \cos 60^\circ$

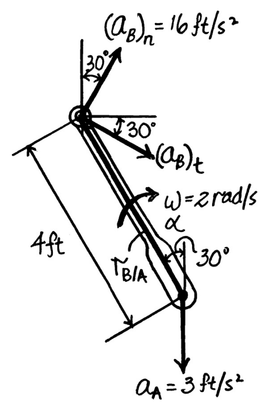
$(+\uparrow)$ $\quad 16 \cos 30^\circ - (a_B)_t \sin 30^\circ = -3 + \alpha(4) \cos 60^\circ - 16 \sin 60^\circ$

$$\alpha = 7.68 \text{ rad/s}^2 \quad \circlearrowright$$ **Ans.**

$$(a_B)_t = 30.7 \text{ ft/s}^2$$

Also,

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$



$$(a_B)_t \cos 30^\circ\mathbf{i} - (a_B)_t \sin 30^\circ\mathbf{j} + \left(\frac{(8)^2}{4}\right) \sin 30^\circ\mathbf{i} + \left(\frac{(8)^2}{4}\right) \cos 30^\circ\mathbf{j} = -3\mathbf{j}$$

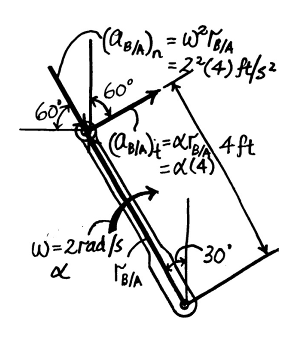
$$-(\alpha\mathbf{k}) \times (-4 \sin 30^\circ\mathbf{i} + 4 \cos 30^\circ\mathbf{j}) - (2)^2(-4 \sin 30^\circ\mathbf{i} + 4 \cos 30^\circ\mathbf{j})$$

$(\overset{+}{\rightarrow})$ $\quad (a_B)_t \cos 30^\circ + 8 = -3.464\alpha + 8$

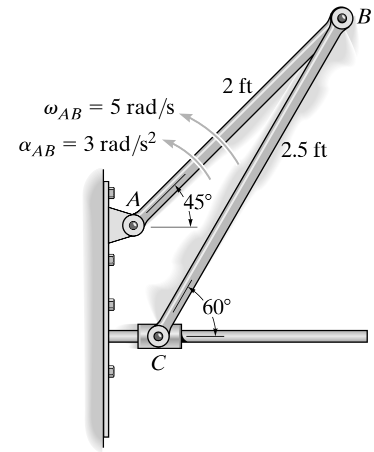
$(+\uparrow)$ $\quad -(a_B)_t \sin 30^\circ + 13.8564 = -3 + 2\alpha - 13.8564$

$$\alpha = 7.68 \text{ rad/s}^2 \quad \circlearrowright$$ **Ans.**

$$(a_B)_t = 30.7 \text{ ft/s}^2$$



16–115. Rod AB has the angular motion shown. Determine the acceleration of the collar C at this instant.



$$\frac{r_{B/IC}}{\sin 30^\circ} = \frac{2.5}{\sin 135^\circ}$$

$$r_{B/IC} = 1.7678 \text{ ft}$$

$$\omega = \frac{10}{1.7678} = 5.66 \text{ rad/s} \quad ↺$$

$$(a_B)_n = 25(2) = 50 \text{ ft/s}^2$$

$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C/B}$$

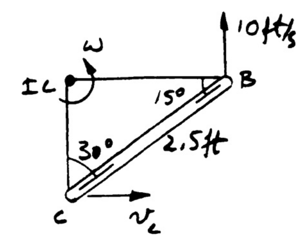
$$\left[\underset{\rightarrow}{a_C}\right] = \left[\underset{45^\circ \searrow}{6}\right] + \left[\underset{45^\circ \swarrow}{50}\right] + \left[\underset{\measuredangle 60^\circ}{(5.66)^2(2.5)}\right] + \left[\underset{\searrow 30^\circ}{\alpha(2.5)}\right]$$

$$(\overset{+}{\rightarrow}) \qquad a_C = -6\cos 45^\circ - 50\cos 45^\circ + 80\cos 60^\circ + \alpha(2.5)\cos 30^\circ$$

$$(+\uparrow) \qquad 0 = 6\sin 45^\circ - 50\sin 45^\circ + 80\sin 60^\circ - \alpha(2.5)\sin 30^\circ$$

$$\alpha = 30.5 \text{ rad/s}^2 \quad ↺$$

$$a_C = 66.5 \text{ ft/s}^2 \rightarrow$$ **Ans.**



Also,

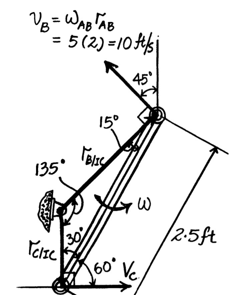
$$v_B = 5(2) = 10 \text{ ft/s}$$

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$$-v_C\mathbf{i} = -10\cos 45^\circ\mathbf{i} + 10\sin 45^\circ\mathbf{j} + \omega\mathbf{k} \times (-2.5\sin 30^\circ\mathbf{i} - 2.5\cos 30^\circ\mathbf{j})$$

$$(+\uparrow) \qquad 0 = 10\sin 45^\circ - 2.5\,\omega\sin 30^\circ$$

$$\omega = 5.66 \text{ rad/s}$$



$$\mathbf{a}_C = \mathbf{a}_B + \alpha \times \mathbf{r}_{C/B} - \omega^2\mathbf{r}_{C/B}$$

$$a_C\,\mathbf{i} = -\frac{(10)^2}{2}\cos 45^\circ\mathbf{i} - \frac{(10)^2}{2}\sin 45^\circ\mathbf{j} - 6\cos 45^\circ\mathbf{i} + 6\sin 45^\circ\mathbf{j}$$

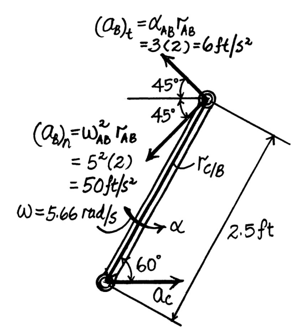
$$+ (\alpha\mathbf{k}) \times (-2.5\cos 60^\circ\mathbf{i} - 2.5\sin 60^\circ\mathbf{j}) - (5.66)^2(-2.5\cos 60^\circ\mathbf{i} - 2.5\sin 60^\circ\mathbf{j})$$

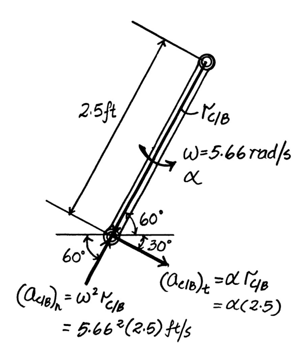
$$(\overset{+}{\rightarrow}) \qquad a_C = -35.355 - 4.243 + 2.165\alpha + 40$$

$$(+\uparrow) \qquad 0 - -35.355 + 4.243 - 1.25\alpha + 69.282$$

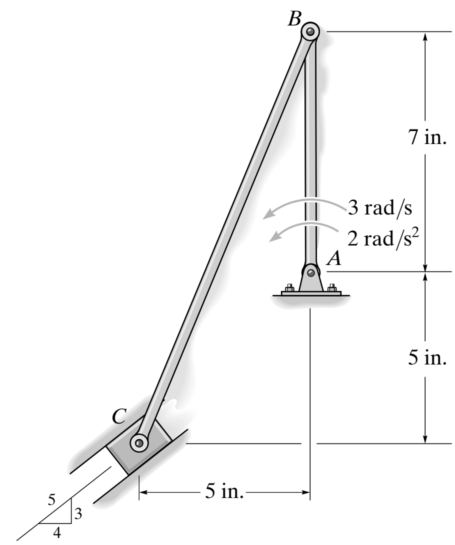
$$\alpha = 30.5 \text{ rad/s}^2 \quad ↺$$

$$a_C = 66.5 \text{ ft/s}^2 \quad \rightarrow$$ **Ans.**





***16–116.** At the given instant member AB has the angular motions shown. Determine the velocity and acceleration of the slider block C at this instant.



$v_B = 3(7) = 21 \text{ in./s} \leftarrow$

$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$

$-v_C\left(\frac{4}{5}\right)\mathbf{i} - v_C\left(\frac{3}{5}\right)\mathbf{j} = -21\mathbf{i} + \omega\mathbf{k} \times (-5\mathbf{i} - 12\mathbf{j})$

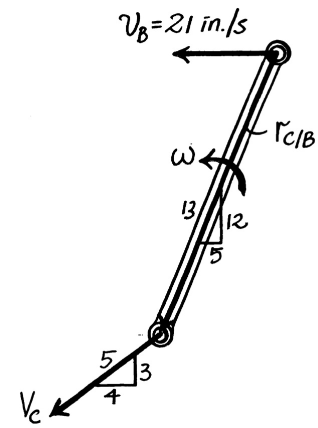
$(\overset{+}{\rightarrow}) \qquad -0.8v_C = -21 + 12\omega$

$(+\uparrow) \qquad -0.6v_C = -5\omega$

Solving:

$\omega = 1.125 \text{ rad/s}$

$v_C = 9.375 \text{ in./s} = 9.38 \text{ in./s}$ **Ans.**



$(a_B)_n = (3)^2(7) = 63 \text{ in./s}^2 \downarrow$

$(a_B)_t = (2)(7) = 14 \text{ in./s}^2 \leftarrow$

$\mathbf{a}_C = a_B + \alpha \times \mathbf{r}_{C/B} - \omega^2\,\mathbf{r}_{C/B}$

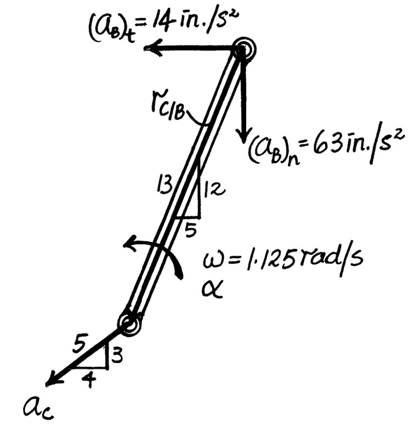
$-a_C\left(\frac{4}{5}\right)\mathbf{i} - a_C\left(\frac{3}{5}\right)\mathbf{j} = -14\mathbf{i} - 63\mathbf{j} + (\alpha\mathbf{k}) \times (-5\mathbf{i} - 12\mathbf{j}) - (1.125)^2(-5\mathbf{i} - 12\mathbf{j})$

$(\overset{+}{\rightarrow}) \qquad -0.8a_C = -14 + 12\alpha + 6.328$

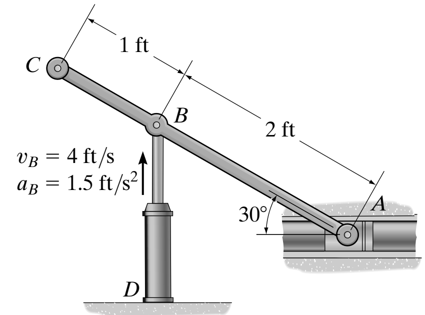
$(+\uparrow) \qquad -0.6a_C = -63 - 5\alpha + 15.1875$

$a_C = 54.7 \text{ in./s}^2$ **Ans.**

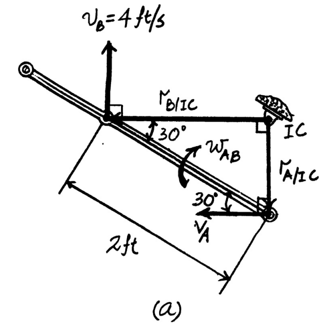
$\alpha = -3.00 \text{ rad/s}^2$



•16–117. The hydraulic cylinder D extends with a velocity of $v_B = 4$ ft/s and an acceleration of $a_B = 1.5$ ft/s^2. Determine the acceleration of A at the instant shown.



Angular Velocity: The location of the IC for rod AB is indicated in Fig. a. From the geometry of this figure,



$$r_{B/IC} = 2\cos 30° = 1.732 \text{ ft}$$

Thus,

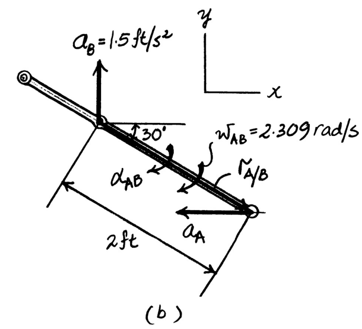
$$\omega_{AB} = \frac{v_B}{r_{B/IC}} = \frac{4}{1.732} = 2.309 \text{ rad/s}$$

Acceleration and Angular Acceleration: Here, $\mathbf{r}_{A/B} = 2\cos 30°\mathbf{i} - 2\sin 30°\,\mathbf{j} = [1.732\mathbf{i} - 1\mathbf{j}]$ ft. Applying the relative acceleration equation and referring to Fig. b,

$$\mathbf{a}_A = \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}{}^2\,\mathbf{r}_{A/B}$$

$$-a_A\mathbf{i} = 1.5\mathbf{j} + (-\alpha_{AB}\mathbf{k})\times(1.732\mathbf{i} - 1\mathbf{j}) - 2.309^2(1.732\mathbf{i} - 1\mathbf{j})$$

$$-a_A\mathbf{i} = -(\alpha_{AB} + 9.238)\mathbf{i} + (6.833 - 1.732\alpha_{AB})\mathbf{j}$$



Equating the $\mathbf{i}$ and $\mathbf{j}$ components,

$$-a_A = -(\alpha_{AB} + 9.238) \quad \textbf{(1)}$$

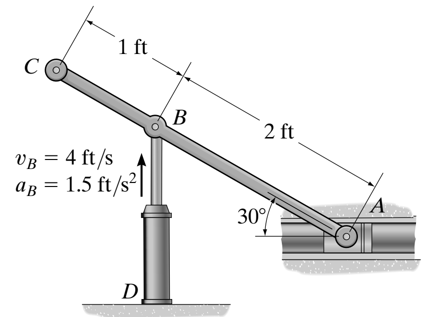
$$0 = 6.833 - 1.732\alpha_{AB} \quad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\alpha_{AB} = 3.945 \text{ rad/s}^2$$

$$a_A = 13.2 \text{ft/s}^2 \leftarrow \qquad \textbf{Ans.}$$

16–118. The hydraulic cylinder D extends with a velocity of $v_B = 4$ ft/s and an acceleration of $a_B = 1.5$ ft/s^2. Determine the acceleration of C at the instant shown.



Angular Velocity: The location of the IC for rod AB is indicated in Fig. a. From the geometry of this figure, $r_{B/IC} = 2\cos 30° = 1.732$ ft

Thus,

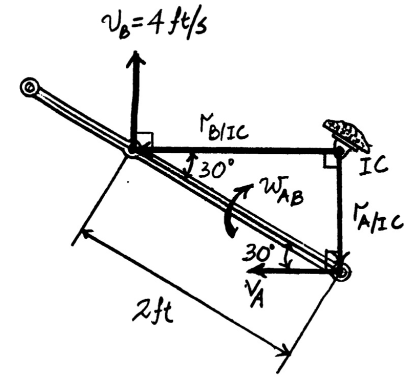
$$\omega_{AB} = \frac{v_B}{r_{B/IC}} = \frac{4}{1.732} = 2.309 \text{ rad/s}$$

Acceleration and Angular Acceleration: Here, $\mathbf{r}_{A/B} = 2\cos 30°\,\mathbf{i} - 2\sin 30°\,\mathbf{j}$ $= [1.732\mathbf{i} - 1\mathbf{j}]$ ft. Applying the relative acceleration equation to points A and B and referring to Fig. b,

$$\mathbf{a}_A = \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}{}^2\,\mathbf{r}_{A/B}$$

$$-a_A\mathbf{i} = 1.5\mathbf{j} + (-\alpha_{AB}\mathbf{k}) \times (1.732\mathbf{i} - 1\mathbf{j}) - 2.309^2(1.732\mathbf{i} - 1\mathbf{j})$$

$$-a_A\,\mathbf{i} = -(\alpha_{AB} + 9.2376)\mathbf{i} + (6.833 - 1.732\alpha_{AB})\mathbf{j}$$



(a)

Equating the $\mathbf{i}$ and $\mathbf{j}$ components, we obtain

$$0 = 6.833 - 1.732\alpha_{AB} \qquad\qquad \alpha_{AB} = 3.945 \text{ rad/s}^2$$

Using this result and $\mathbf{r}_{C/B} = -1\cos 30°\,\mathbf{i} + 1\sin 30°\mathbf{j} = [-0.8660\mathbf{i} + 0.5\mathbf{j}]$ ft, the relative acceleration equation is applied at points B and C, Fig. b, which gives

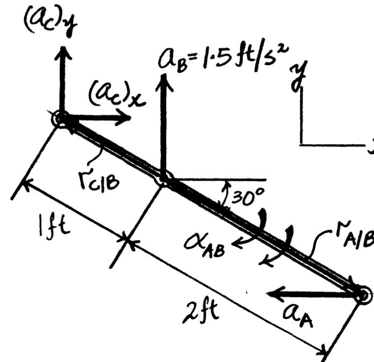
$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{C/B} - \omega_{AB}{}^2\mathbf{r}_{C/B}$$

$$(a_C)_x\,\mathbf{i} + (a_C)_y\mathbf{j} = 1.5\mathbf{j} + (-3.945\mathbf{k}) \times (-0.8660\mathbf{i} + 0.5\mathbf{j}) - (2.309)^2(-0.8660\mathbf{i} + 0.5\mathbf{j})$$

$$(a_C)_x\,\mathbf{i} + (a_C)_y\,\mathbf{j} = 6.591\mathbf{i} + 2.25\mathbf{j}$$

Equating the $\mathbf{i}$ and $\mathbf{j}$ components,

$$(a_C)_x = 6.591 \text{ ft/s}^2 \rightarrow \qquad\qquad (a_C)_y = 2.25 \text{ ft/s}^2 \uparrow$$



(b)

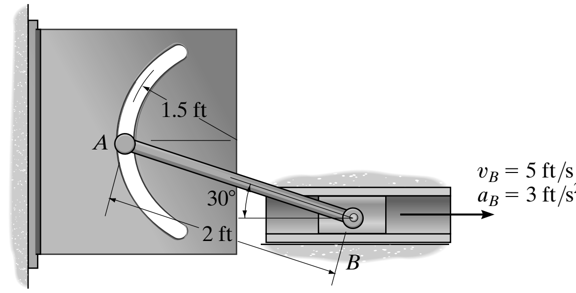
Thus, the magnitude of a_C is

$$a_C = \sqrt{(a_C)_x{}^2 + (a_C)_y{}^2} = \sqrt{6.591^2 + 2.25^2} = 6.96 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

and its direction is

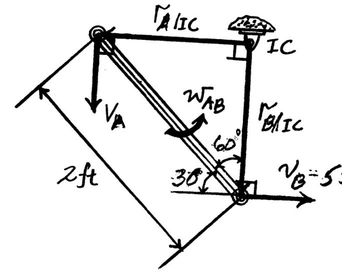
$$\theta = \tan^{-1}\left[\frac{(a_C)_y}{(a_C)_x}\right] = \tan^{-1}\left(\frac{2.25}{6.591}\right) = 18.8° \;\measuredangle \qquad \textbf{Ans.}$$

16–119. The slider block moves with a velocity of $v_B = 5$ ft/s and an acceleration of $a_B = 3$ ft/s^2. Determine the angular acceleration of rod AB at the instant shown.



Angular Velocity: The velocity of point A is directed along the tangent of the circular slot. Thus, the location of the IC for rod AB is indicated in Fig. a. From the geometry of this figure,

$$r_{B/IC} = 2\sin 30° = 1 \text{ ft} \qquad r_{A/IC} = 2\cos 30° = 1.732 \text{ ft}$$



(a)

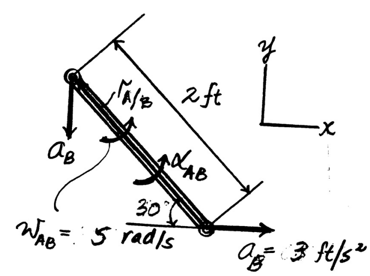
Thus,

$$\omega_{AB} = \frac{v_B}{r_{B/IC}} = \frac{5}{1} = 5 \text{ rad/s}$$

Then

$$v_A = \omega_{AB}\, r_{A/IC} = 5(1.732) = 8.660 \text{ ft/s}$$

Acceleration and Angular Acceleration: Since point A travels along the circular slot, the normal component of its acceleration has a magnitude of $(a_A)_n = \dfrac{v_A{}^2}{\rho} = \dfrac{8.660^2}{1.5} = 50$ ft/s^2 and is directed towards the center of the circular slot. The tangential component is directed along the tangent of the slot. Applying the relative acceleration equation and referring to Fig. b,



(b)

$$\mathbf{a}_A = \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}{}^2\,\mathbf{r}_{A/B}$$

$$50\mathbf{i} - (a_A)_t\,\mathbf{j} = 3\mathbf{i} + (\alpha_{AB}\,\mathbf{k}) \times (-2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j}) - 5^2(-2\cos 30°\,\mathbf{i} + 2\sin 30°\mathbf{j})$$

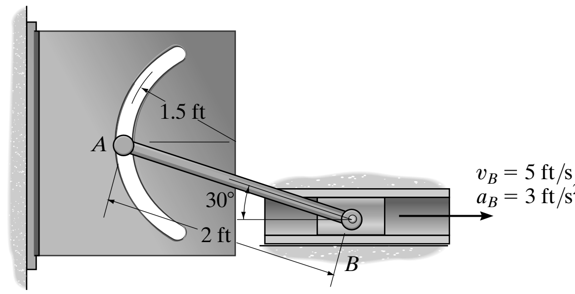
$$50\mathbf{i} - (a_A)_t\,\mathbf{j} = (46.30 - \alpha_{AB})\mathbf{i} + (1.732\alpha_{AB} + 25)\mathbf{j}$$

Equating the **i** components,

$$50 = 46.30 - \alpha_{AB}$$

$$\alpha_{AB} = -3.70 \text{ rad/s}^2 = 3.70 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

***16–120.** The slider block moves with a velocity of $v_B = 5$ ft/s and an acceleration of $a_B = 3$ ft/s^2. Determine the acceleration of A at the instant shown.



Angualr Velocity: The velocity of point A is directed along the tangent of the circular slot. Thus, the location of the IC for rod AB is indicated in Fig. a. From the geometry of this figure,

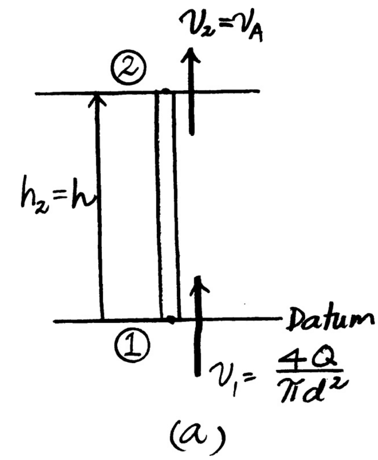
$$r_{B/IC} = 2\sin 30° = 1\text{ ft} \qquad r_{A/IC} = 2\cos 30° = 1.732\text{ ft}$$

Thus,

$$\omega_{AB} = \frac{v_B}{r_{B/IC}} = \frac{5}{1} = 5\text{ rad/s}$$

Then

$$v_A = \omega_{AB}\, r_{A/IC} = 5(1.732) = 8.660\text{ ft/s}$$



Acceleration and Angular Acceleration: Since point A travels along the circular slot, the normal component of its acceleration has a magnitude of $(a_A)_n = \frac{v_A{}^2}{\rho} = \frac{8.660^2}{1.5} = 50$ ft/s^2 and is directed towards the center of the circular slot. The tangential component is directed along the tangent of the slot. Applying the relative acceleration equation and referring to Fig. b,

$$\mathbf{a}_A = \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}{}^2\,\mathbf{r}_{A/B}$$

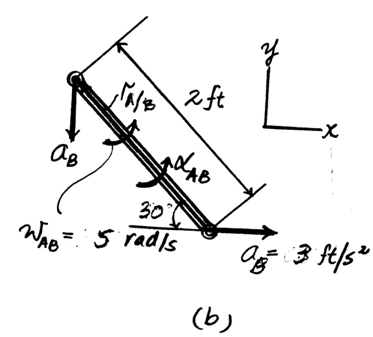
$$50\mathbf{i} - (a_A)_t\,\mathbf{j} = 3\mathbf{i} + (\alpha_{AB}\,\mathbf{k}) \times (-2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j}) - 5^2(-2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j})$$

$$50\mathbf{i} - (a_A)_t\,\mathbf{j} = (46.30 - \alpha_{AB})\mathbf{i} - (1.732\alpha_{AB} + 25)\mathbf{j}$$

Equating the **i** and **j** components,

$$50 = 46.30 - \alpha_{AB}$$

$$-(a_A)_t = -(1.732\alpha_{AB} + 25)$$



Solving,

$$\alpha_{AB} = -3.70\text{ rad/s}^2$$

$$(a_A)_t = 18.59\text{ ft/s}^2 \downarrow$$

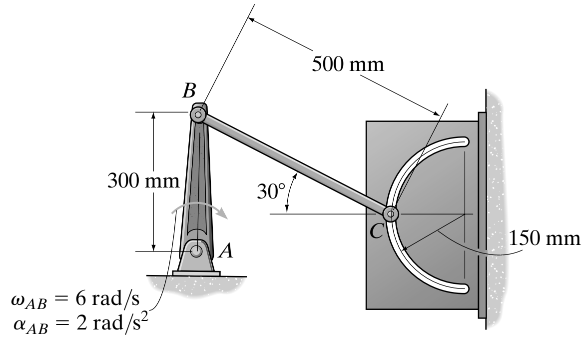
Thus, the magnitude of $\mathbf{a}_A$ is

$$a_A = \sqrt{(a_A)_t{}^2 + (a_A)_n{}^2} = \sqrt{18.59^2 + 50^2} = 53.3\text{ft/s}^2 \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \tan^{-1}\left[\frac{(a_A)_t}{(a_A)_n}\right] = \tan^{-1}\left(\frac{18.59}{50}\right) = 20.4° \qquad \textbf{Ans.}$$

•16–121. Crank AB rotates with an angular velocity of $\omega_{AB} = 6\text{ rad/s}$ and an angular acceleration of $\alpha_{AB} = 2\text{ rad/s}^2$. Determine the acceleration of C and the angular acceleration of BC at the instant shown.



Angular Velocity: Since crank AB rotates about a fixed axis, then

$$v_B = \omega_{AB}\, r_B = 6(0.3) = 1.8\text{ m/s} \rightarrow$$

The location of the IC for rod BC is

$r_{B/IC} = 0.5 \sin 30° = 0.25\text{ m}$ $\qquad$ $r_{C/IC} = 0.5 \cos 30° = 0.4330\text{ m}$

Then,

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{1.8}{0.25} = 7.2\text{ rad/s}$$

and

$$v_C = \omega_{BC}\, r_{C/IC} = 7.2(0.4330) = 3.118\text{ ft/s}$$

Acceleration and Angular Acceleration: Since crank AB rotates about a fixed axis, then

$$\begin{aligned}\mathbf{a}_B &= \alpha'_{AB} \times \mathbf{r}_B - \omega_{AB}{}^2\mathbf{r}_B \\ &= (-2\mathbf{k}) \times (0.3\mathbf{j}) - 6^2(0.3\mathbf{j}) \\ &= \{0.6\mathbf{i} - 10.8\mathbf{j}\}\text{ m/s}^2\end{aligned}$$

Since point C travels along a circular slot, the normal component of its acceleration has a magnitude of $(a_C)_n = \dfrac{v_C{}^2}{\rho} = \dfrac{3.118^2}{0.15} = 64.8\text{ m/s}^2$ and is directed towards the center of the circular slot. The tangential component is directed along the tangent of the slot. Applying the relative acceleration equation,

$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}{}^2\mathbf{r}_{C/B}$$

$$64.8\mathbf{i} - (a_C)_t\,\mathbf{j} = (0.6\mathbf{i} - 10.8\mathbf{j}) + (\alpha_{BC}\mathbf{k}) \times (0.5\cos 30°\mathbf{i} - 0.5\sin 30°\,\mathbf{j}) - 7.2^2(0.5\cos 30°\mathbf{i} - 0.5\sin 30°\,\mathbf{j})$$

$$64.8\mathbf{i} - (a_C)_t\,\mathbf{j} = -(0.25\alpha_{BC} - 21.85)\mathbf{i} + (2.16 + 0.4330\alpha_{BC})\mathbf{j}$$

Equating the **i** and **j** components,

$$64.8 = -(0.25\alpha_{BC} - 21.85)$$

$$-(a_C)_t = 2.16 + 0.4330\alpha_{BC}$$

Solving,

$$\alpha_{BC} = -346.59\text{ rad/s}^2 = 347\text{rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

$$(a_C)_t = -152.24\text{ m/s}^2 = 152.24\text{ m/s}^2 \uparrow$$

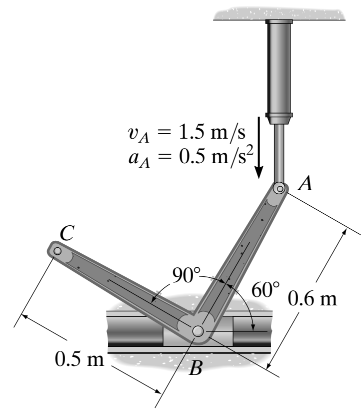
Thus, the magnitude of $\mathbf{a}_C$ is

$$a_C = \sqrt{(a_C)_t{}^2 + (a_C)_n{}^2} = \sqrt{152.24^2 + 64.7^2} = 165\text{m/s}^2 \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \tan^{-1}\left[\frac{(a_C)_t}{(a_C)_n}\right] = \tan^{-1}\left(\frac{152.24}{64.8}\right) = 66.9° \qquad \textbf{Ans.}$$

16–122. The hydraulic cylinder extends with a velocity of $v_A = 1.5\text{ m/s}$ and an acceleration of $a_A = 0.5\text{ m/s}^2$. Determine the angular acceleration of link ABC and the acceleration of end C at the instant shown. Point B is pin connected to the slider block.



Angular Velocity: The location of the IC for link ABC is indicated in Fig. a. From the geometry of this figure,

$$r_{A/IC} = 0.6 \cos 60° = 0.3\text{ m}$$

Then

$$\omega_{ABC} = \frac{v_A}{r_{A/IC}} = \frac{1.5}{0.3} = 5\text{ rad/s}$$

Acceleration and Angular Acceleration: Applying the relative acceleration equation to points A and B,

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{ABC} \times \mathbf{r}_{B/A} - \omega_{ABC}{}^2 r_{B/A}$$

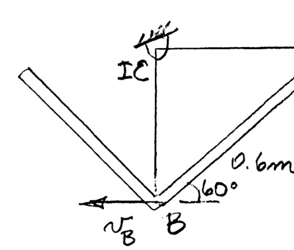
$$-a_B\,\mathbf{i} = -0.5\mathbf{j} + (-\alpha_{ABC}\,\mathbf{k}) \times (-0.6 \cos 60°\mathbf{i} - 0.6 \sin 60°\,\mathbf{j}) - 5^2(-0.6 \cos 60°\mathbf{i} - 0.6 \sin 60°\mathbf{j})$$

$$-a_B\,\mathbf{i} = (7.5 - 0.5196\alpha_{ABC})\mathbf{i} + (0.3\alpha_{ABC} + 12.490)\mathbf{j}$$

Equating the $\mathbf{i}$ and $\mathbf{j}$ components,

$$-a_B = 7.5 - 0.5196\alpha_{ABC} \qquad \textbf{(1)}$$

$$0 = 0.3\alpha_{ABC} + 12.490 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2),

$$\alpha_{ABC} = -41.63\text{ rad/s}^2 = 41.6\text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$a_B = -29.13\text{ m/s}^2$$

From points B and C,

$$a_C = \mathbf{a}_B + \alpha_{ABC} \times \mathbf{r}_{C/B} - \omega_{ABC}{}^2 r_{C/B}$$

$$(a_C)_x\,\mathbf{i} + (a_C)_y\,\mathbf{j} = [-(-29.13)\mathbf{i}] + [-(-41.63)\mathbf{k}] \times (-0.5 \cos 30°\,\mathbf{i} + 0.5 \sin 30°\mathbf{j}) - 5^2(-0.5 \cos 30°\mathbf{i} + 0.5 \sin 30°\mathbf{j})$$

$$(a_C)_x\,\mathbf{i} + (a_C)_y\,\mathbf{j} = 29.55\mathbf{i} - 24.28\,\mathbf{j}$$

Equating the $\mathbf{i}$ and $\mathbf{j}$ components,

$$(a_C)_x = 29.55\text{ m/s}^2 \qquad (a_C)_y = -24.28\text{ m/s}^2 = 24.28\text{ m/s}^2 \downarrow$$

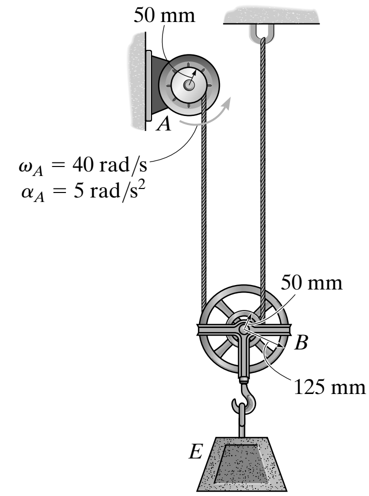
Thus, the magnitude of a_C is

$$a_C = \sqrt{(a_C)_x{}^2 + (a_C)_y{}^2} = \sqrt{29.55^2 + 24.28^2} = 38.2\text{ m/s}^2 \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \tan^{-1}\left[\frac{(a_C)_y}{(a_C)_x}\right] = \tan^{-1}\left(\frac{24.28}{29.55}\right) = 39.4° \qquad \textbf{Ans.}$$

16–123. Pulley A rotates with the angular velocity and angular acceleration shown. Determine the angular acceleration of pulley B at the instant shown.

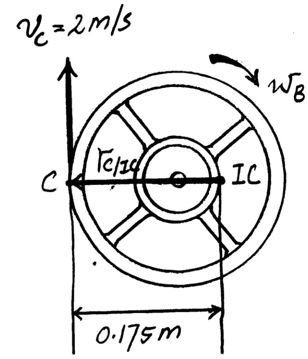


Angular Velocity: Since pulley A rotates about a fixed axis,

$$v_C = \omega_A r_A = 40(0.05) = 2 \text{ m/s} \uparrow$$

The location of the IC is indicated in Fig. a. Thus,

$$\omega_B = \frac{v_C}{r_{C/IC}} = \frac{2}{0.175} = 11.43 \text{ rad/s}$$



(a)

Acceleration and Angular Acceleration: For pulley A,

$$(a_C)_t = \alpha_A r_A = 5(0.05) = 0.25 \text{ m/s}^2 \uparrow$$

Using this result and applying the relative acceleration equation to points C and D by referring to Fig. b,

$$\mathbf{a}_D = \mathbf{a}_C + \alpha_B \times \mathbf{r}_{D/C} - \omega_B{}^2 r_{D/C}$$

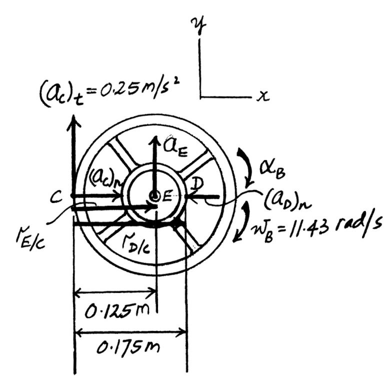
$$(a_D)_n\,\mathbf{i} = (a_C)_n\,\mathbf{i} + 0.25\mathbf{j} + (-\alpha_B\,\mathbf{k}) \times (0.175\mathbf{i}) - 11.43^2(0.175\mathbf{i})$$

$$(a_D)_n\,\mathbf{i} = [(a_C)_n - 22.86]\mathbf{i} + (0.25 - 0.175\alpha_B)\mathbf{j}$$

Equating the $\mathbf{j}$ components,

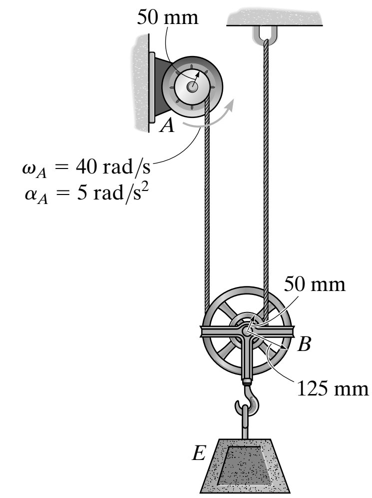
$$0 = 0.25 - 0.175\alpha_B$$

$$\alpha_B = 1.43 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



(b)

***16–124.** Pulley A rotates with the angular velocity and angular acceleration shown. Determine the acceleration of block E at the instant shown.

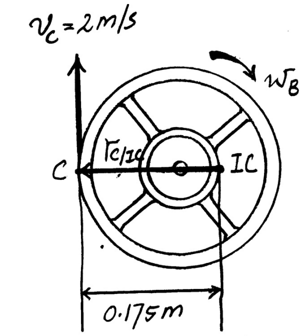


Angular Velocity: Since pulley A rotates about a fixed axis,

$$v_C = \omega_A r_A = 40(0.05) = 2 \text{ m/s} \uparrow$$

The location of the IC is indicated in Fig. a. Thus,

$$\omega_B = \frac{v_C}{r_{C/IC}} = \frac{2}{0.175} = 11.43 \text{ rad/s}$$



(a)

Acceleration and Angular Acceleration: For pulley A,

$$(a_C)_t = \alpha_A r_A = 5(0.05) = 0.25 \text{ m/s}^2 \uparrow$$

Using this result and applying the relative acceleration equation to points C and D by referring to Fig. b,

$$\mathbf{a}_D = \mathbf{a}_C + \alpha_B \times \mathbf{r}_{D/C} - \omega_B{}^2 r_{D/C}$$

$$(a_D)_n \mathbf{i} = (a_C)_n \mathbf{i} + 0.25\mathbf{j} + (-\alpha_B \mathbf{k}) \times (0.175\mathbf{i}) - 11.43^2(0.175\mathbf{i})$$

$$(a_D)_n \mathbf{i} = [(a_C)_n - 22.86]\mathbf{i} + (0.25 - 0.175\alpha_B)\mathbf{j}$$

Equating the $\mathbf{j}$ components,

$$0 = 0.25 - 0.175\alpha_B$$

$$\alpha_B = 1.429 \text{ rad/s} = 1.43 \text{ rad/s}^2$$

Using this result, the relative acceleration equation applied to points C and E, Fig. b, gives

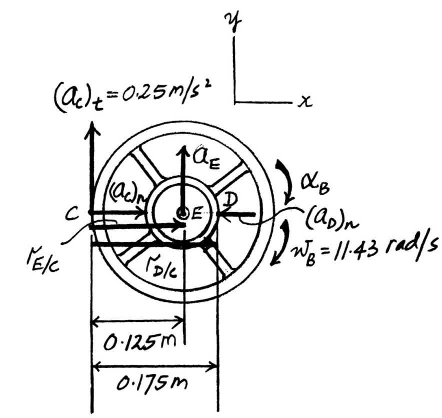
$$\mathbf{a}_E = \mathbf{a}_C + \alpha_B \times \mathbf{r}_{E/C} - \omega_B{}^2 r_{E/C}$$

$$a_E \mathbf{j} = [(a_C)_n \mathbf{i} + 0.25\mathbf{j}] + (-1.429\mathbf{k}) \times (0.125\mathbf{i}) - 11.43^2(0.125\mathbf{i})$$

$$a_E \mathbf{j} = [(a_C)_n - 16.33]\mathbf{i} + 0.0714\mathbf{j}$$

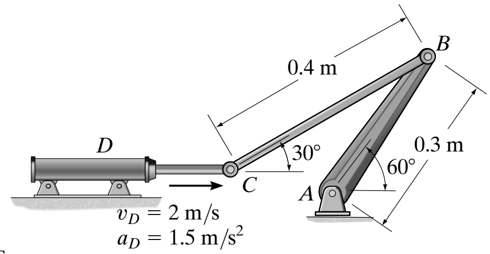
Equating the $\mathbf{j}$ components,

$$a_E = 0.0714 \text{ m/s}^2 \uparrow$$ **Ans.**



(b)

•16–125. The hydraulic cylinder is extending with the velocity and acceleration shown. Determine the angular acceleration of crank AB and link BC at the instant shown.

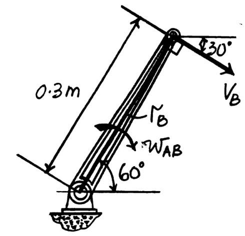


Angular Velocity: Crank AB rotates about a fixed axis. Thus,

$$v_B = \omega_{AB}\, r_B = \omega_{AB}\,(0.3)$$

The location of the IC for link BC is indicated in Fig. b. From the geometry of this figure,

$$r_{C/IC} = 0.4 \text{ m} \qquad\qquad r_{B/IC} = 2(0.4 \cos 30°) = 0.6928 \text{ m}$$



(a)

Then

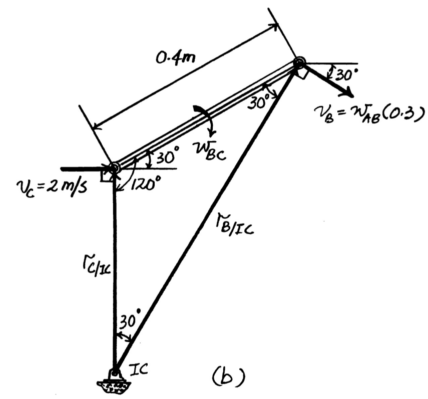
$$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{2}{0.4} = 5 \text{ rad/s}$$

and

$$v_B = \omega_{BC}\, r_{B/IC}$$

$$\omega_{AB}(0.3) = 5(0.6928)$$

$$\omega_{AB} = 11.55 \text{ rad/s}$$



(b)

Acceleration and Angular Acceleration: Since crank AB rotates about a fixed axis, Fig. c,

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_B - \omega_{AB}{}^2 \mathbf{r}_B$$

$$= (-\alpha_{AB}\mathbf{k}) \times (0.3 \cos 60°\mathbf{i} + 0.3 \sin 60°\mathbf{j}) - 11.55^2(0.3 \cos 60°\mathbf{i} + 0.3 \sin 60°\mathbf{j})$$

$$= (0.2598\alpha_{AB})\mathbf{i} - (0.15\alpha_{AB} + 34.64)\mathbf{j}$$

Using these results and applying the relative acceleration equation to points B and C of link BC, Fig. d,

$$\mathbf{a}_B = \mathbf{a}_C + \alpha_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}{}^2 r_{B/C}$$

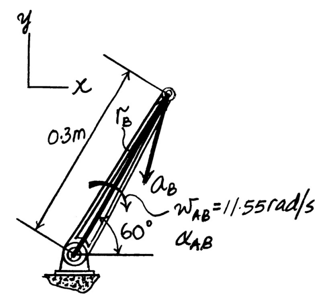
$$(0.2598\alpha_{AB} - 20)\mathbf{i} - (0.15\alpha_{AB} + 34.64)\mathbf{j} = 1.5\mathbf{i} + (\alpha_{BC}\mathbf{k}) \times (0.4 \cos 30°\mathbf{i} + 0.4 \sin 30°\mathbf{j}) - 5^2(0.4 \cos 30°\mathbf{i} + 0.4 \sin 30°\mathbf{j})$$

$$(0.2598\alpha_{AB} - 20)\mathbf{i} - (0.15\alpha_{AB} + 34.64)\mathbf{j} = -(0.2\alpha_{BC} + 7.160)\mathbf{i} + (0.3464\alpha_{BC} - 5)\mathbf{j}$$

Equating the **i** and **j** components,

$$0.2598\alpha_{AB} - 20 = -(0.2\alpha_{BC} + 7.160)$$

$$-(0.15\alpha_{AB} + 34.64) = 0.3464\alpha_{BC} - 5$$

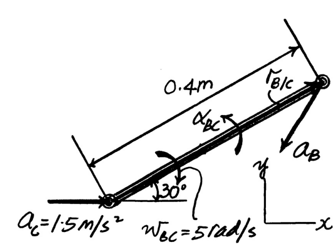


(c)

Solving,

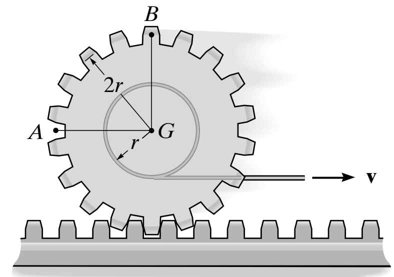
$$\alpha_{BC} = -160.44 \text{ rad/s}^2 = 160 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$\alpha_{AB} = 172.93 \text{ rad/s}^2 = 173 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



(d)

16–126. A cord is wrapped around the inner spool of the gear. If it is pulled with a constant velocity **v**, determine the velocities and accelerations of points A and B. The gear rolls on the fixed gear rack.

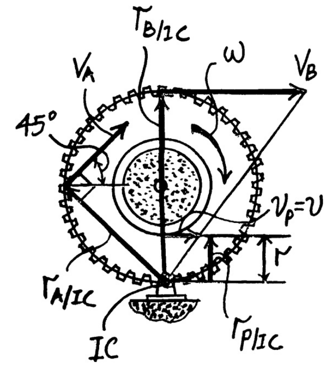


Velocity Analysis:

$$\omega = \frac{v}{r}$$

$$v_B = \omega r_{B/IC} = \frac{v}{r}(4r) = 4v \rightarrow \qquad \textbf{Ans.}$$

$$v_A = \omega r_{A/IC} = \frac{v}{r}\left(\sqrt{(2r)^2 + (2r)^2}\right) = 2\sqrt{2}v \quad \measuredangle 45^\circ \qquad \textbf{Ans.}$$



Acceleration Equation: From Example 16–3, Since $a_G = 0, \alpha = 0$

$$\mathbf{r}_{B/G} = 2r\,\mathbf{j} \qquad \mathbf{r}_{A/G} = -2r\,\mathbf{i}$$

$$\mathbf{a}_B = \mathbf{a}_G + \alpha \times \mathbf{r}_{B/G} - \omega^2 \mathbf{r}_{B/G}$$

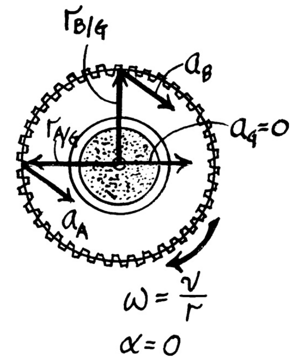
$$= \mathbf{0} + \mathbf{0} - \left(\frac{v}{r}\right)^2 (2r\,\mathbf{j}) = -\frac{2v^2}{r}\mathbf{j}$$

$$a_B = \frac{2v^2}{r} \downarrow \qquad \textbf{Ans.}$$

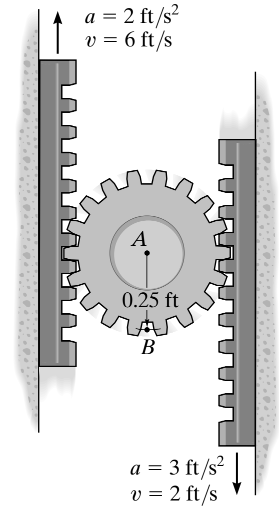
$$\mathbf{a}_A = \mathbf{a}_G + \alpha \times \mathbf{r}_{A/G} - \omega^2 \mathbf{r}_{A/G}$$

$$= 0 + 0 - \left(\frac{v}{r}\right)^2 (-2r\mathbf{i}) = \frac{2v^2}{r}\mathbf{i}$$

$$a_A = \frac{2v^2}{r} \rightarrow \qquad \textbf{Ans.}$$



16–127. At a given instant, the gear racks have the velocities and accelerations shown. Determine the acceleration of points *A* and *B*.



Velocity Analysis: The angular velocity of the gear can be obtained by using the method of instantaneous center of zero velocity. From similar triangles,

$$\omega = \frac{v_D}{r_{D/IC}} = \frac{v_C}{r_{C/IC}}$$

$$\frac{6}{r_{D/IC}} = \frac{2}{r_{C/IC}} \qquad \textbf{[1]}$$

Where

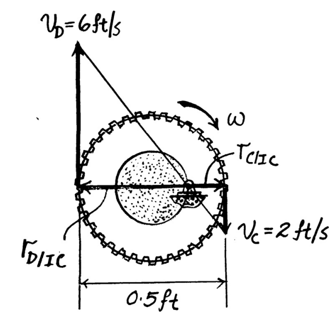
$$r_{D/IC} + r_{C/IC} = 0.5 \qquad \textbf{[2]}$$

Solving Eqs.[1] and [2] yields

$$r_{D/IC} = 0.375 \text{ ft} \qquad r_{C/IC} = 0.125 \text{ ft}$$

Thus,

$$\omega = \frac{v_D}{r_{D/IC}} = \frac{6}{0.375} = 16.0 \text{ rad/s}$$



Acceleration Equation: The angular acceleration of the gear can be obtained by analyzing the angular motion point *C* and *D*. Applying Eq. 16–18 with $\mathbf{r}_{D/C} = \{-0.5\mathbf{i}\}$ ft, we have

$$\mathbf{a}_D = \mathbf{a}_C + \alpha \times \mathbf{r}_{D/C} - \omega^2 \mathbf{r}_{D/C}$$

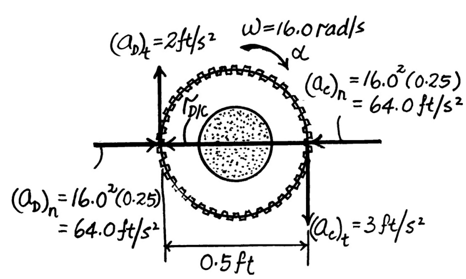
$$64.0\mathbf{i} + 2\mathbf{j} = -64.0\mathbf{i} - 3\mathbf{j} + (-\alpha\mathbf{k}) \times (-0.5\mathbf{i}) - 16.0^2(-0.5\mathbf{i})$$

$$64.0\mathbf{i} + 2\mathbf{j} = 64.0\mathbf{i} + (0.5\alpha - 3)\mathbf{j}$$

Equating **i** and **j** components, we have

$$64.0 = 64.0 \ (\textbf{\textit{Check!}})$$

$$2 = 0.5\,\alpha - 3 \qquad \alpha = 10.0 \text{ rad/s}^2$$



The acceleration of point *A* can be obtained by analyzing the angular motion point *A* and *C*. Applying Eq. 16–18 with $\mathbf{r}_{A/C} = \{-0.25\mathbf{i}\}$ ft, we have

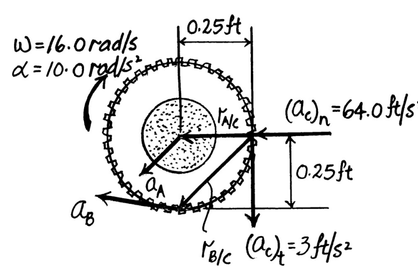
$$\mathbf{a}_A = \mathbf{a}_C + \alpha \times \mathbf{r}_{A/C} - \omega^2 \mathbf{r}_{A/C}$$

$$= -64.0\mathbf{i} - 3\mathbf{j} + (-10.0\mathbf{k}) \times (-0.25\mathbf{i}) - 16.0^2(-0.25\mathbf{i})$$

$$= \{0.500\mathbf{j}\} \text{ ft/s}^2$$

Thus,

$$a_A = 0.500 \text{ ft/s}^2 \downarrow \qquad \textbf{Ans.}$$



The acceleration of point *B* can be obtained by analyzing the angular motion point *B* and *C*. Applying Eq. 16–18 with $\mathbf{r}_{B/C} = \{-0.25\mathbf{i} - 0.25\mathbf{j}\}$ ft, we have

$$\mathbf{a}_B = \mathbf{a}_C + \alpha \times \mathbf{r}_{B/C} - \omega^2 \mathbf{r}_{B/C}$$

$$= -64.0\mathbf{i} - 3\mathbf{j} + (-10.0\mathbf{k}) \times (-0.25\mathbf{i} - 0.25\mathbf{j}) - 16.0^2(-0.25\mathbf{i} - 0.25\mathbf{j})$$

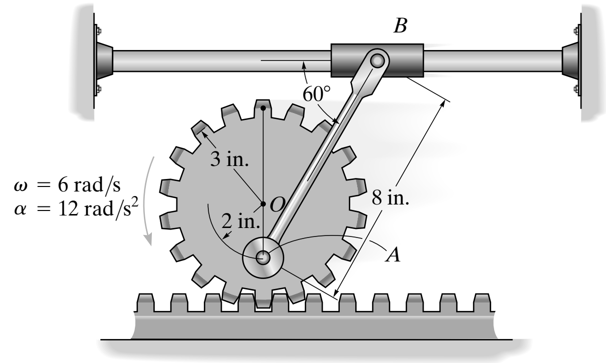
$$= \{-2.50\mathbf{i} + 63.5\mathbf{j}\} \text{ ft/s}^2$$

The magnitude and direction of the acceleration of point *B* are given by

$$a_C = \sqrt{(-2.50)^2 + 63.5^2} = 63.5 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{63.5}{2.50} = 87.7^\circ \qquad \textbf{Ans.}$$

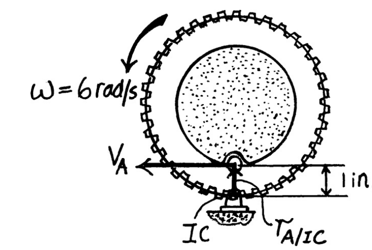
***16–128.** At a given instant, the gear has the angular motion shown. Determine the accelerations of points A and B on the link and the link's angular acceleration at this instant.



For the gear

$v_A = \omega r_{A/IC} = 6(1) = 6 \text{ in./s}$

$\mathbf{a}_O = -12(3)\mathbf{i} = \{-36\mathbf{i}\} \text{ in./s}^2 \qquad \mathbf{r}_{A/O} = \{-2\mathbf{j}\} \text{ in.} \qquad \alpha = \{12\mathbf{k}\} \text{ rad/s}^2$



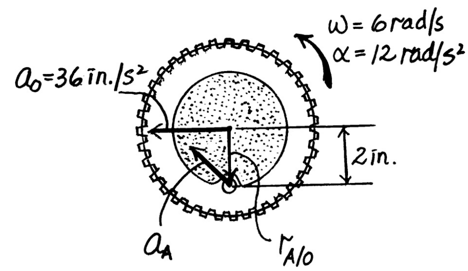
$\mathbf{a}_A = \mathbf{a}_0 + \alpha \times \mathbf{r}_{A/O} - \omega^2\mathbf{r}_{A/O}$

$= -36\mathbf{i} + (12\mathbf{k}) \times (-2\mathbf{j}) - (6)^2(-2\mathbf{j})$

$= \{-12\mathbf{i} + 72\mathbf{j}\} \text{ in./s}^2$

$a_A = \sqrt{(-12)^2 + 72^2} = 73.0 \text{ in./s}^2$ **Ans.**

$\theta = \tan^{-1}\left(\frac{72}{12}\right) = 80.5°$ **Ans.**



For link AB

The IC is at ∞, so $\omega_{AB} = 0$, i.e.,

$\omega_{AB} = \frac{v_A}{r_{A/IC}} = \frac{6}{\infty} = 0$

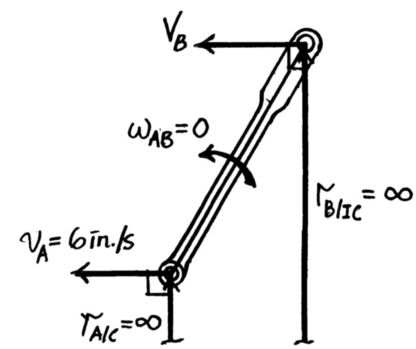
$\mathbf{a}_B = a_B\mathbf{i} \qquad \alpha_{AB} = -\alpha_{AB}\,\mathbf{k} \qquad \mathbf{r}_{B/A} = \{8 \cos 60°\mathbf{i} + 8 \sin 60°\mathbf{j}\} \text{ in.}$

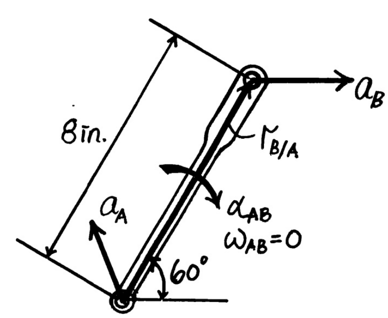
$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} - \omega^2\,\mathbf{r}_{B/A}$

$a_B\,\mathbf{i} = (-12\mathbf{i} + 72\mathbf{j}) + (-\alpha_{AB}\,\mathbf{k}) \times (8 \cos 60°\mathbf{i} + 8 \sin 60°\mathbf{j}) - \mathbf{0}$

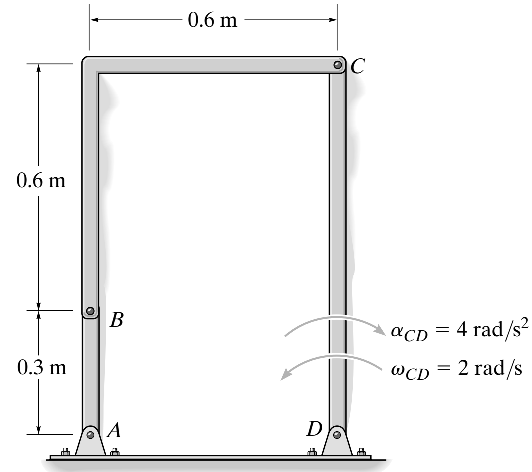
$(\overset{+}{\rightarrow}) \qquad a_B = -12 + 8 \sin 60°(18) = 113 \text{ in./s}^2 \rightarrow$ **Ans.**

$(+\uparrow) \qquad 0 = 72 - 8 \cos 60°\alpha_{AB} \qquad \alpha_{AB} = 18 \text{ rad/s}^2$ **Ans.**





•16–129. Determine the angular acceleration of link AB if link CD has the angular velocity and angular deceleration shown.



IC is at ∞, thus

$\omega_{BC} = 0$

$v_B = v_C = (0.9)(2) = 1.8\ \text{m/s}$

$(a_C)_n = (2)^2(0.9) = 3.6\ \text{m/s}^2 \downarrow$

$(a_C)_t = 4(0.9) = 3.6\ \text{m/s}^2 \rightarrow$

$(a_B)_n = \dfrac{(1.8)^2}{0.3} = 10.8\ \text{m/s}^2 \downarrow$

$\mathbf{a}_B = \mathbf{a}_C + \alpha_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2\,\mathbf{r}_{B/C}$

$(a_B)_t\,\mathbf{i} - 10.8\mathbf{j} = 3.6\mathbf{i} - 3.6\mathbf{j} + (\alpha_{BC}\mathbf{k}) \times (-0.6\mathbf{i} - 0.6\mathbf{j}) - \mathbf{0}$

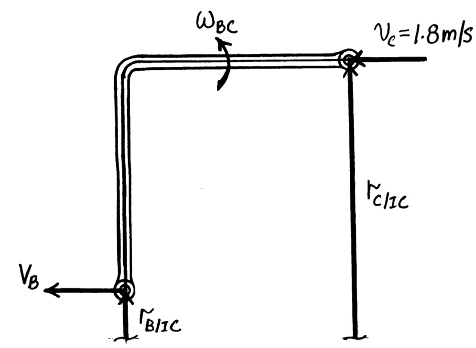
$(\xrightarrow{+})\qquad (a_B)_t = 3.6 + 0.6\,\alpha_{BC}$

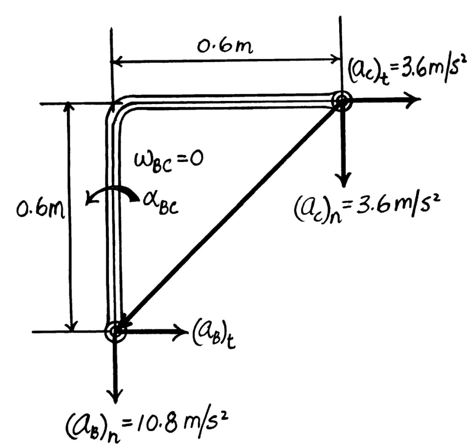
$(+\uparrow)\qquad -10.8 = -3.6 - 0.6\,\alpha_{BC}$

$\alpha_{BC} = 12\ \text{rad/s}^2$

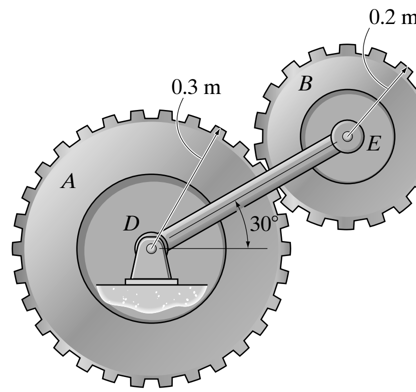
$(a_B)_t = 10.8\ \text{m/s}^2$

$\alpha_{AB} = \dfrac{10.8}{0.3} = 36\ \text{rad/s}^2 \circlearrowright$ **Ans.**





16–130. Gear A is held fixed, and arm DE rotates clockwise with an angular velocity of $\omega_{DE} = 6$ rad/s and an angular acceleration of $\alpha_{DE} = 3\text{ rad/s}^2$. Determine the angular acceleration of gear B at the instant shown.

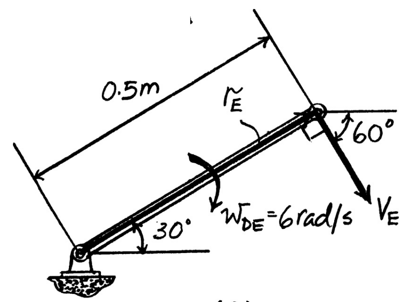


Angular Velocity: Arm DE rotates about a fixed axis, Fig. a. Thus,

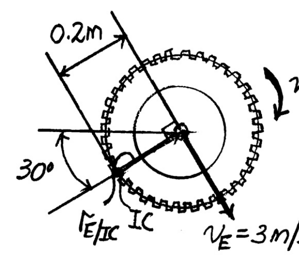
$$v_E = \omega_{DE}\, r_E = 6(0.5) = 3\text{ m/s}$$

The IC for gear B is located at the point where gears A and B are meshed, Fig. b. Thus,

$$\omega_B = \frac{v_E}{r_{E/IC}} = \frac{3}{0.2} = 15\text{ rad/s}$$



(a)



(b)

Acceleration and Angular Acceleration: Since arm DE rotates about a fixed axis, Fig. c,

$$\mathbf{a}_E = \alpha_{DE} \times \mathbf{r}_E - \omega_{DE}{}^2\,\mathbf{r}_E$$

$$= (-3\mathbf{k}) \times (0.5\cos 30°\mathbf{i} + 0.5\sin 30°\,\mathbf{j}) - 6^2\,(0.5\cos 30°\,\mathbf{i} + 0.5\sin 30°\,\mathbf{j})$$

$$= [-14.84\mathbf{i} - 10.30\mathbf{j}]\text{ m/s}^2$$

Using these results to apply the relative acceleration equation to points E and F of gear B, Fig. d, we have

$$\mathbf{a}_F = \mathbf{a}_E + \alpha_B \times \mathbf{r}_{F/E} - \omega_B{}^2 r_{F/E}$$

$$a_F\cos 30°\mathbf{i} + a_F\sin 30°\mathbf{j} = (-14.84\mathbf{i} - 10.30\mathbf{j}) + (-\alpha_B\,\mathbf{k}) \times$$

$$(-0.2\cos 30°\,\mathbf{i} - 0.2\sin 30°\mathbf{j}) - 15^2(-0.2\cos 30°\mathbf{i} - 0.2\sin 30°\mathbf{j})$$

$$a_F\cos 30°\,\mathbf{i} + a_F\sin 30°\mathbf{j} = (24.13 - 0.1\alpha_B)\mathbf{i} + (0.1732\alpha_B + 12.20)\mathbf{j}$$

Equating the **i** and **j** components yields

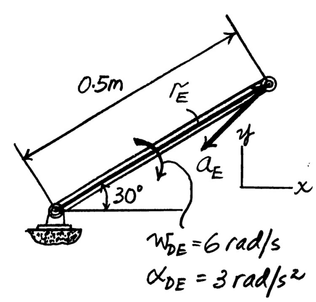
$$0.8660a_F = 24.13 - 0.1\alpha_B$$

$$0.5a_F = 0.1732\alpha_B + 12.20$$

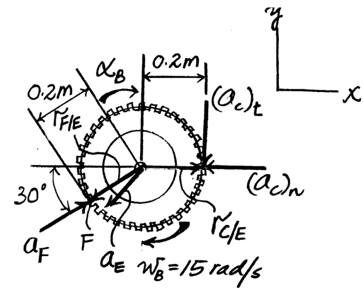
Solving,

$$a_F = 27\text{ m/s}^2$$

$$\alpha_B = 7.5\text{ rad/s}^2 \qquad \textbf{Ans.}$$

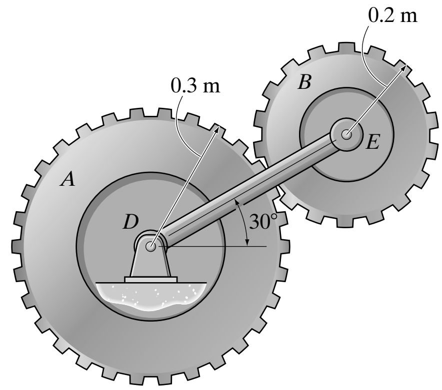


(c)



(d)

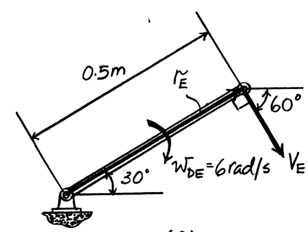
16–131. Gear A rotates counterclockwise with a constant angular velocity of $\omega_A = 10$ rad/s, while arm DE rotates clockwise with an angular velocity of $\omega_{DE} = 6$ rad/s and an angular acceleration of $\alpha_{DE} = 3$ rad/s^2. Determine the angular acceleration of gear B at the instant shown.



Angular Velocity: Arm DE and gear A rotate about a fixed axis, Figs. a and b. Thus,

$$v_E = \omega_{DE}\, r_E = 6(0.5) = 3 \text{ m/s}$$

$$v_F = \omega_A\, r_F = 10(0.3) = 3 \text{ m/s}$$



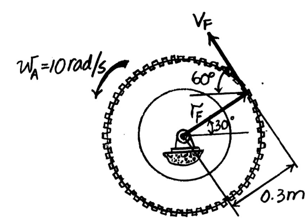
(a)

The location of the IC for gear B is indicated in Fig. c. Thus,

$$r_{E/IC} = r_{F/IC} = 0.1 \text{ m}$$

Then,

$$\omega_B = \frac{v_E}{r_{E/IC}} = \frac{3}{0.1} = 30 \text{ rad/s}$$



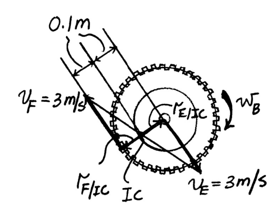
(b)

Acceleration and Angular Acceleration: Since arm DE rotates about a fixed axis, Fig. c, then

$$\mathbf{a}_E = \alpha_{DE} \times \mathbf{r}_E - \omega_{DE}^{\,2}\, \mathbf{r}_E$$

$$= (-3\mathbf{k}) \times (0.5 \cos 30°\mathbf{i} + 0.5 \sin 30°\,\mathbf{j}) - 6^2\,(0.5 \cos 30°\,\mathbf{i} + 0.5 \sin 30°\,\mathbf{j})$$

$$= [-14.84\mathbf{i} - 10.30\mathbf{j}] \text{ m/s}^2$$



(c)

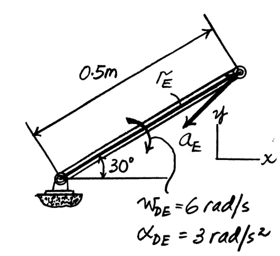
Using these results and applying the acceleration equation to points E and F of gear B, Fig. e,

$$\mathbf{a}_F = \mathbf{a}_E + \alpha_B \times \mathbf{r}_{F/E} - \omega_B^{\,2}\mathbf{r}_{F/E}$$

$$a_F \cos 30°\mathbf{i} + a_F \sin 30°\mathbf{j} = (-14.84\mathbf{i} - 10.30\mathbf{j}) + (-\alpha_B\,\mathbf{k}) \times$$

$$(-0.2 \cos 30°\,\mathbf{i} - 0.2 \sin 30°\mathbf{j}) - 30^2(-0.2 \cos 30°\mathbf{i} - 0.2 \sin 30°\mathbf{j})$$

$$0.8660a_F\,\mathbf{i} + 0.5a_F\,\mathbf{j} = (141.05 - 0.1\alpha_B)\mathbf{i} + (79.70 + 0.1732\alpha_B)\mathbf{j}$$



(d)

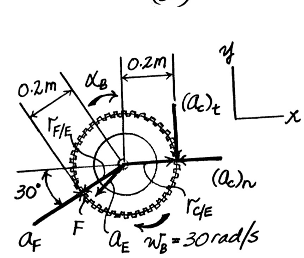
Equating the $\mathbf{i}$ and $\mathbf{j}$ components yields

$$0.8660a_F = 141.05 - 0.1\alpha_B$$

$$0.5a_F = 79.70 + 0.1732\alpha_B$$

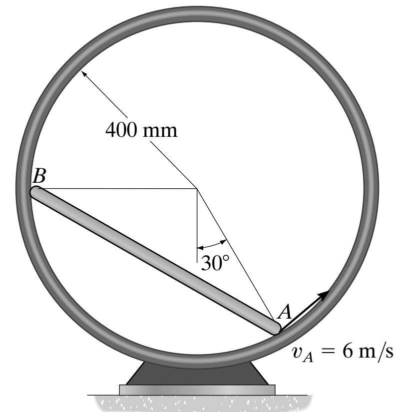
$$a_F = 162 \text{ m/s}^2$$

$$\alpha_B = 7.5 \text{ rad/s}^2$$ **Ans.**



(e)

***16–132.** If end A of the rod moves with a constant velocity of $v_A = 6$ m/s, determine the angular velocity and angular acceleration of the rod and the acceleration of end B at the instant shown.



Angular Velocity: The location of the IC is indicated in Fig. a. Thus,

$$r_{A/IC} = r_{B/IC} = 0.4 \text{ m}$$

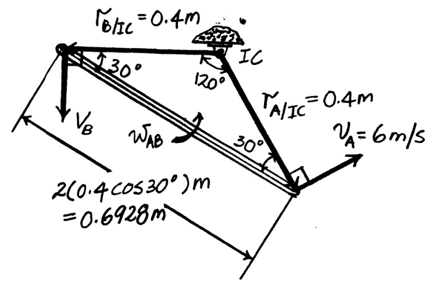
Then,

$$\omega_{AB} = \frac{v_A}{r_{A/IC}} = \frac{6}{0.4} = 15 \text{ rad/s}$$

and

$$v_B = \omega_{AB}\, r_{B/IC} = 15(0.4) = 6 \text{ m/s}$$

Acceleration and Angular Acceleration: The magnitude of the normal component of its acceleration of points A and B are $(a_A)_n = \frac{v_A{}^2}{\rho} = \frac{6^2}{0.4} = 90 \text{ m/s}^2$ and $(a_B)_n = \frac{v_B{}^2}{\rho} = \frac{6^2}{0.4} = 90 \text{ m/s}^2$ and both are directed towards the center of the circular track. Since $\mathbf{v}_A$ is constant, $(a_A)_t = 0$. Thus, $a_A = 90 \text{ m/s}^2$. Applying the relative acceleration equation to points A and B, Fig. b,



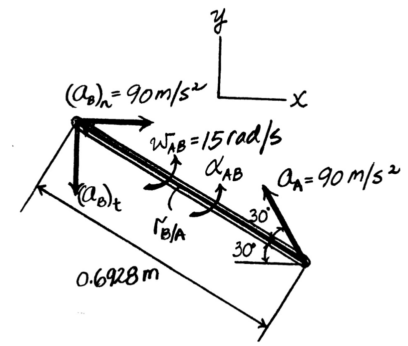
(a)

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}{}^2\mathbf{r}_{B/A}$$

$$90\mathbf{i} - (a_B)_t\,\mathbf{j} = (-90 \cos 60°\mathbf{i} + 90 \sin 60°\mathbf{j}) + (\alpha_{AB}\,\mathbf{k}) \times$$

$$(-0.6928 \cos 30°\mathbf{i} + 0.6928 \sin 30°\mathbf{j}) - 15^2(-0.6928 \cos 30°\mathbf{i} + 0.6928 \sin 30°\mathbf{j})$$

$$90\mathbf{i} - (a_B)_t\,\mathbf{j} = (-0.3464\alpha_{AB} + 90)\mathbf{i} - (0.6\alpha_{AB})\mathbf{j}$$



(b)

Equating the **i** and **j** components yields

$$90 = -0.3464\alpha_{AB} + 90$$

$$-(a_B) = -0.6\alpha_{AB}$$

$$\alpha_{AB} = 0 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$(a_B)_t = 0 \text{ m/s}^2$$

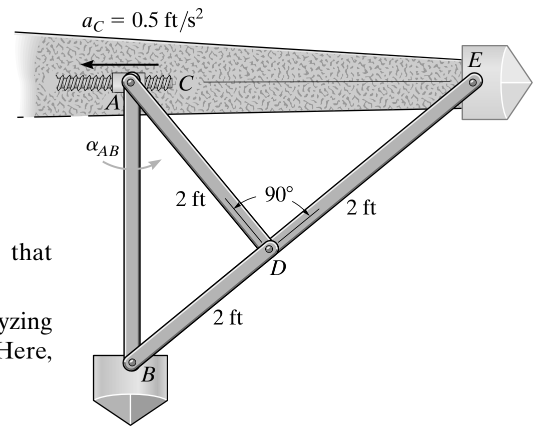
Thus, the magnitude of a_B is

$$a_B = \sqrt{(a_B)_t{}^2 + (a_B)_n{}^2} = \sqrt{0^2 + 90^2} = 90 \text{ m/s}^2 \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \tan^{-1}\left[\frac{(a_B)_t}{(a_B)_n}\right] = \tan^{-1}\left(\frac{0}{90}\right) = 0° \rightarrow \qquad \textbf{Ans.}$$

•16–133. The retractable wing-tip float is used on an airplane able to land on water. Determine the angular accelerations α_{CD}, α_{BD}, and α_{AB} at the instant shown if the trunnion C travels along the horizontal rotating screw with an acceleration of $a_C = 0.5\ \text{ft/s}^2$. In the position shown, $v_C = 0$. Also, points A and E are pin connected to the wing and points A and C are coincident at the instant shown.



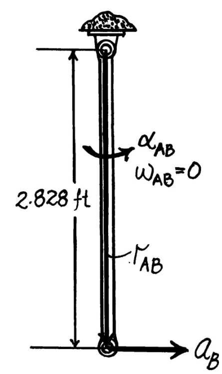
Velocity Analysis: Since $v_C = 0$, then $\omega_{CD} = 0$. Also, one can then show that $\omega_{BD} = \omega_{AB} = \omega_{ED} = 0$.

Acceleration Equation: The acceleration of point D can be obtained by analyzing the angular motion of link ED about point E. Here, $\mathbf{r}_{ED} = \{-2\cos 45°\mathbf{i} - 2\sin 45°\mathbf{j}\}\ \text{ft} = \{-1.414\mathbf{i} - 1.414\mathbf{j}\}\ \text{ft}$.

$$\mathbf{a}_D = \alpha_{ED} \times \mathbf{r}_{ED} - \omega_{ED}^2\,\mathbf{r}_{ED}$$

$$= (\alpha_{ED}\,\mathbf{k}) \times (-1.414\mathbf{i} - 1.414\mathbf{j}) - \mathbf{0}$$

$$= \{1.414\,\alpha_{ED}\,\mathbf{i} - 1.414\,\alpha_{ED}\,\mathbf{j}\}\ \text{ft/s}^2$$



The acceleration of point B can be obtained by analyzing the angular motion of links AB about point A. Here, $\mathbf{r}_{AB} = \{-2.828\mathbf{j}\}$ ft

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\,\mathbf{r}_{B/A}$$

$$= (\alpha_{AB}\,\mathbf{k}) \times (-2.828\mathbf{j}) - \mathbf{0}$$

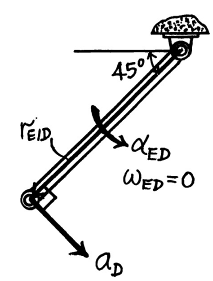
$$= \{2.828\,\alpha_{AB}\,\mathbf{i}\}\ \text{ft/s}^2$$

Link CD is subjected to general plane motion. Applying Eq. 16–18 with $\mathbf{r}_{D/C} = \{2\cos 44°\mathbf{i} - 2\sin 45°\mathbf{j}\}\ \text{ft} = \{1.414\mathbf{i} - 1.414\mathbf{j}\}$ ft, we have

$$\mathbf{a}_D = \mathbf{a}_C + \alpha_{CD} \times \mathbf{r}_{D/C} - \omega_{CD}^2\mathbf{r}_{D/C}$$

$$1.414\,\alpha_{ED}\,\mathbf{i} - 1.414\,\alpha_{ED}\,\mathbf{j} = -0.5\mathbf{i} + \alpha_{CD}\mathbf{k} \times (1.414\mathbf{i} - 1.414\mathbf{j}) - \mathbf{0}$$

$$1.414\,\alpha_{ED}\,\mathbf{i} - 1.414\,\alpha_{ED}\,\mathbf{j} = (1.414\,\alpha_{CD} - 0.5)\,\mathbf{i} + 1.414\,\alpha_{CD}\mathbf{j}$$



Equating $\mathbf{i}$ and $\mathbf{j}$ components, we have

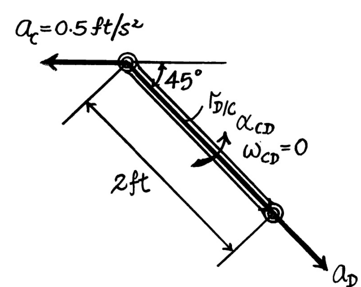
$$1.414\,\alpha_{ED} = 1.414\alpha_{CD} - 0.5 \qquad [1]$$

$$-1.414\alpha_{ED} = 1.414\alpha_{CD} \qquad [2]$$

Solving Eqs.[1] and [2] yields

$$\alpha_{ED} = -0.1768\ \text{rad/s}^2$$

$$\alpha_{CD} = 0.177\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

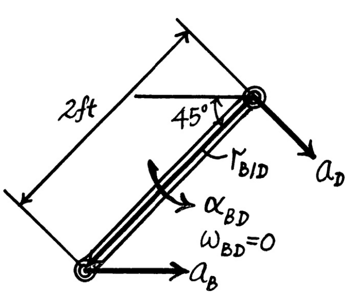


Link BD is subjected to general plane motion. Applying Eq. 16–18 with $\mathbf{r}_{B/D} = \{-2\cos 45°\mathbf{i} - 2\sin 45°\mathbf{j}\}\ \text{ft} = \{-1.414\mathbf{i} - 1.414\mathbf{j}\}$ ft and $\mathbf{a}_D = [1.414\,(-0.1768)\,\mathbf{i} - 1.414\,(-0.1768)\mathbf{j} = \{-0.25\mathbf{i} + 0.25\mathbf{j}\}\ \text{rad/s}^2$, we have

$$\mathbf{a}_B = \mathbf{a}_D + \alpha_{BD} \times \mathbf{r}_{B/D} - \omega_{BD}^2\mathbf{r}_{B/D}$$

$$2.828\,\alpha_{AB}\,\mathbf{i} = -0.25\mathbf{i} + 0.25\mathbf{j} + \alpha_{BD}\mathbf{k} \times (-1.414\mathbf{i} - 1.414\mathbf{j}) - \mathbf{0}$$

$$2.828\,\alpha_{AB}\,\mathbf{i} = (1.414\,\alpha_{BD} - 0.25)\,\mathbf{i} + (0.25 - 1.414\,\alpha_{BD})\,\mathbf{j}$$



Equating $\mathbf{i}$ and $\mathbf{j}$ components, we have

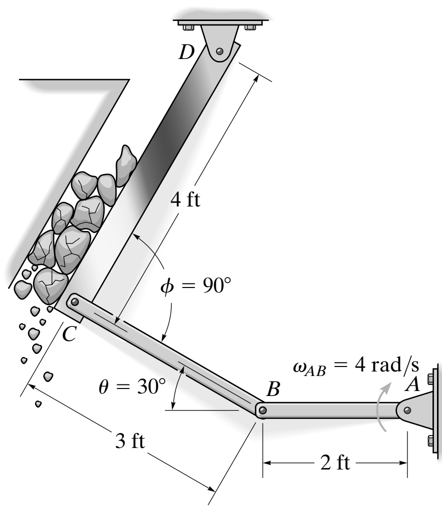
$$2.828\,\alpha_{AB} = 1.414\,\alpha_{BD} - 0.25 \qquad [3]$$

$$0 = 0.25 - 1.414\,\alpha_{BD} \qquad [4]$$

Solving Eqs. [3] and [4] yields

$$\alpha_{BD} = 0.177\ \text{rad/s}^2 \qquad \alpha_{AB} = 0 \qquad \textbf{Ans.}$$

16–134. Determine the angular velocity and the angular acceleration of the plate CD of the stone-crushing mechanism at the instant AB is horizontal. At this instant $\theta = 30^\circ$ and $\phi = 90^\circ$. Driving link AB is turning with a constant angular velocity of $\omega_{AB} = 4$ rad/s.

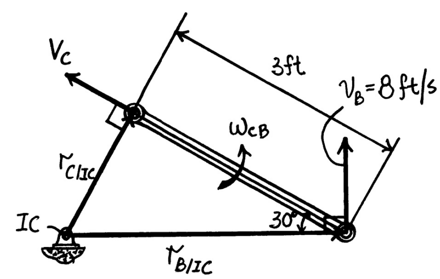


$$v_B = \omega_{AB}\, r_{BA} = (4)(2) = 8 \text{ ft/s} \uparrow$$

$$\omega_{CB} = \frac{v_B}{r_{B/IC}} = \frac{8}{3/\cos 30^\circ} = 2.309 \text{ rad/s}$$

$$v_C = \omega_{BC}\, r_{C/IC} = (2.309)(3 \tan 30^\circ) = 4 \text{ ft/s}$$

$$\omega_{CD} = \frac{v_C}{r_{CD}} = \frac{4}{4} = 1 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$



$$a_B = (a_B)_n = (4)^2(2) = 32 \text{ ft/s}^2 \rightarrow$$

$$(\mathbf{a}_C)_t + (\mathbf{a}_C)_n = \mathbf{a}_B + \alpha_{CB} \times \mathbf{r}_{C/B} - \omega^2 \mathbf{r}_{C/B}$$

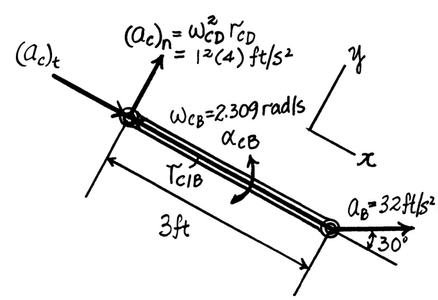
$$(a_C)_t\,\mathbf{i} + (1)^2(4)\mathbf{j} = 32 \cos 30^\circ \mathbf{i} + 32 \sin 30^\circ \mathbf{j} + (\alpha_{CB}\,\mathbf{k}) \times (-3\mathbf{i}) - (2.309)^2(-3\mathbf{i})$$

$$(a_C)_t = 32 \cos 30^\circ - (2.309)^2(-3) = 43.71 \text{ ft/s}^2$$

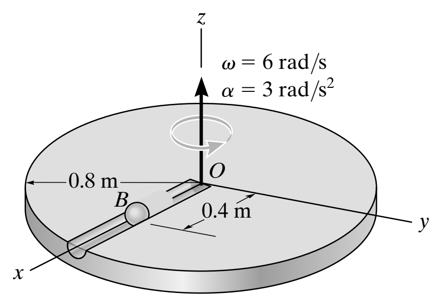
$$4 = 32 \sin 30^\circ - \alpha_{CB}(3)$$

$$\alpha_{CB} = 4 \text{ rad/s}^2 \circlearrowleft$$

$$\alpha_{CD} = \frac{43.71}{4} = 10.9 \text{ rad/s}^2 \circlearrowleft \qquad \textbf{Ans.}$$



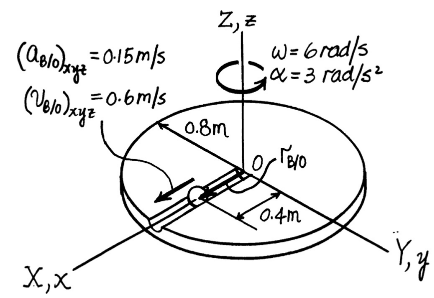
16–135. At the instant shown, ball B is rolling along the slot in the disk with a velocity of 600 mm/s and an acceleration of 150 mm/s^2, both measured relative to the disk and directed away from O. If at the same instant the disk has the angular velocity and angular acceleration shown, determine the velocity and acceleration of the ball at this instant.



Kinematic Equations:

$$\mathbf{v}_B = \mathbf{v}_O + \Omega \times \mathbf{r}_{B/O} + (\mathbf{v}_{B/O})_{xyz} \quad \textbf{(1)}$$

$$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{B/O} + \Omega \times (\Omega \times \mathbf{r}_{B/O}) + 2\Omega \times (\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz} \quad \textbf{(2)}$$



$\mathbf{v}_O = \mathbf{0}$

$\mathbf{a}_O = \mathbf{0}$

$\Omega = \{6\mathbf{k}\}$ rad/s

$\Omega = \{3\mathbf{k}\}$ rad/s^2

$\mathbf{r}_{B/O} = \{0.4\,\mathbf{i}\,\}$ m

$(\mathbf{v}_{B/O})_{xyz} = \{0.6\mathbf{i}\}$ m/s

$(\mathbf{a}_{B/O})_{xyz} = \{0.15\mathbf{i}\}$ m/s^2

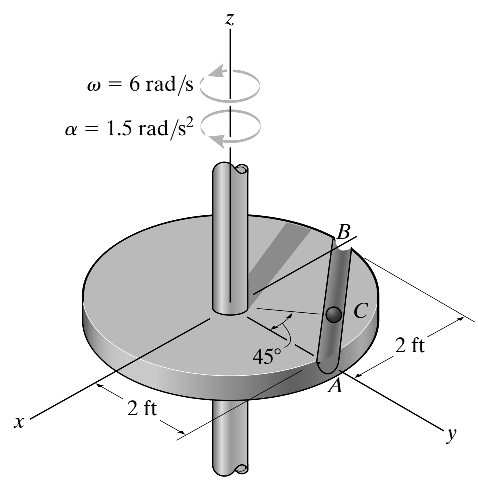
Substitute the data into Eqs.(1) and (2) yields:

$$\mathbf{v}_B = \mathbf{0} + (6\mathbf{k}) \times (0.4\mathbf{i}) + (0.6\mathbf{i}) = \{0.6\mathbf{i} + 2.4\mathbf{j}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_B = \mathbf{0} + (3\mathbf{k}) \times (0.4\mathbf{i}) + (6\mathbf{k}) \times [(6\mathbf{k}) \times (0.4\mathbf{i})\,] + 2\,(6\mathbf{k}) \times (0.6\mathbf{i}) + (0.15\mathbf{i})$$

$$= \{-14.2\mathbf{i} + 8.40\mathbf{j}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

***16–136.** Ball C moves along the slot from A to B with a speed of 3 ft/s, which is increasing at 1.5 ft/s^2, both measured relative to the circular plate. At this same instant the plate rotates with the angular velocity and angular deceleration shown. Determine the velocity and acceleration of the ball at this instant.



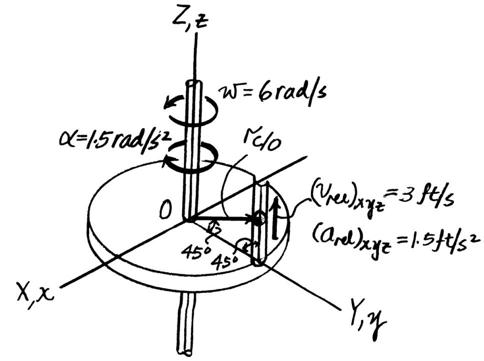
Reference Frames: The xyz rotating reference frame is attached to the plate and coincides with the fixed reference frame XYZ at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is

$$\mathbf{v}_O = \mathbf{a}_O = \mathbf{0} \qquad \omega = [6\mathbf{k}]\ \text{rad/s} \qquad \dot{\omega} = \alpha = [-1.5\mathbf{k}]\ \text{rad/s}^2$$

For the motion of ball C with respect to the xyz frame,

$$(\mathbf{v}_{\text{rel}})_{xyz} = (-3\sin 45°\mathbf{i} - 3\cos 45°\mathbf{j})\ \text{ft/s} = [-2.121\mathbf{i} - 2.121\mathbf{j}]\ \text{ft/s}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = (-1.5\sin 45°\mathbf{i} - 1.5\cos 45°\mathbf{j})\ \text{ft/s}^2 = [-1.061\mathbf{i} - 1.061\mathbf{j}]\ \text{ft/s}^2$$



(a)

From the geometry shown in Fig. b, $r_{C/O} = 2\cos 45° = 1.414$ ft. Thus,

$$\mathbf{r}_{C/O} = (-1.414\sin 45°\mathbf{i} + 1.414\cos 45°\mathbf{j})\text{ft} = [-1\mathbf{i} + 1\mathbf{j}]\ \text{ft}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_C = \mathbf{v}_O + \omega \times \mathbf{r}_{C/O} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (6\mathbf{k}) \times (-1\mathbf{i} + 1\mathbf{j}) + (-2.121\mathbf{i} - 2.121\mathbf{j})$$

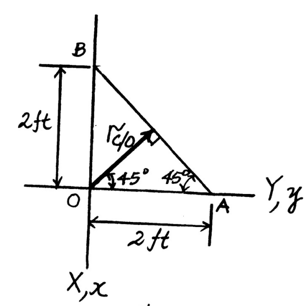
$$= [-8.12\mathbf{i} - 8.12\mathbf{j}]\ \text{ft/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation, we have

$$\mathbf{a}_C = \mathbf{a}_O + \dot{\omega} \times \mathbf{r}_{C/O} + \omega \times (\omega \times \mathbf{r}_{C/O}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (a_{\text{rel}})_{xyz}$$

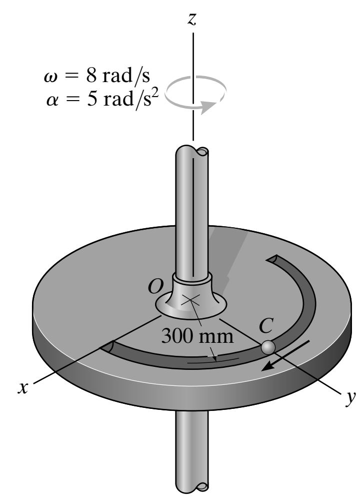
$$= 0 + (1.5\mathbf{k}) \times (-1\mathbf{i} + 1\mathbf{j}) + (6\mathbf{k}) \times [(6\mathbf{k}) \times (-1\mathbf{i} + 1\mathbf{j})] + 2(6\mathbf{k}) \times (-2.121\mathbf{i} - 2.121\mathbf{j}) + (-1.061\mathbf{i} - 1.061\mathbf{j})$$

$$= [61.9\mathbf{i} - 61.0\mathbf{j}]\text{ft/s}^2 \qquad \textbf{Ans.}$$



(b)

•16–137. Ball C moves with a speed of 3 m/s, which is increasing at a constant rate of 1.5 m/s², both measured relative to the circular plate and directed as shown. At the same instant the plate rotates with the angular velocity and angular acceleration shown. Determine the velocity and acceleration of the ball at this instant.



Reference Frames: The xyz rotating reference frame is attached to the plate and coincides with the fixed reference frame XYZ at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is

$$\mathbf{v}_O = \mathbf{a}_O = \mathbf{0} \qquad \omega = [8\mathbf{k}] \text{ rad/s} \qquad \dot{\omega} = \alpha = [5\mathbf{k}] \text{ rad/s}^2$$

For the motion of ball C with respect to the xyz frame, we have

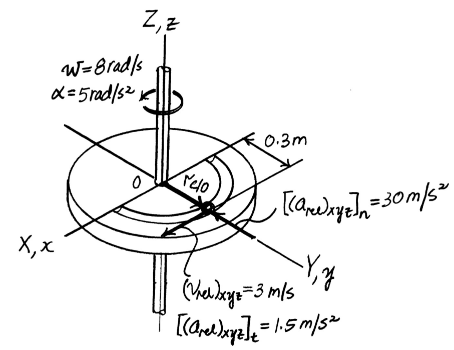
$$\mathbf{r}_{C/O} = [0.3\mathbf{j}] \text{ m}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = [3\mathbf{i}] \text{ m/s}$$

The normal component of $\left(\mathbf{a}_{\text{rel}}\right)_{xyz}$ is $\left[\left(a_{\text{rel}}\right)_{xyz}\right]_n = \dfrac{\left(v_{\text{rel}}\right)_{xyz}{}^2}{\rho} = \dfrac{3^2}{0.3} = 30 \text{ m/s}^2$.

Thus,

$$\left(\mathbf{a}_{\text{rel}}\right)_{xyz} = [1.5\mathbf{i} - 30\mathbf{j}] \text{ m/s}$$



(a)

Velocity: Applying the relative velocity equation,

$$\begin{aligned}\mathbf{v}_C &= \mathbf{v}_O + \omega \times \mathbf{r}_{C/O} + (\mathbf{v}_{\text{rel}})_{xyz} \\ &= \mathbf{0} + (8\mathbf{k}) \times (0.3\mathbf{j}) + (3\mathbf{i}) \\ &= [0.6\mathbf{i}] \text{ m/s}\end{aligned}$$

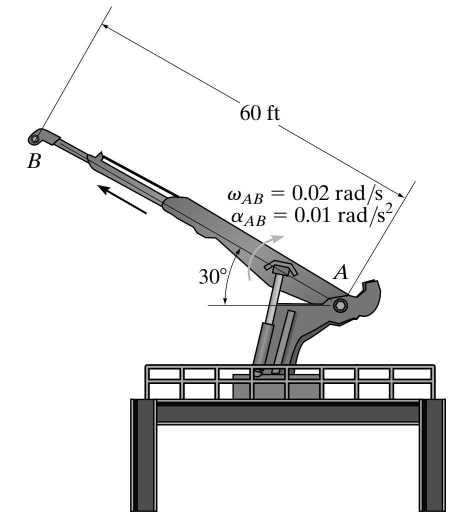
Ans.

Acceleration: Applying the relative acceleration equation.

$$\begin{aligned}\mathbf{a}_C &= \mathbf{a}_O + \dot{\omega} \times \mathbf{r}_{C/O} + \omega \times (\omega \times \mathbf{r}_{C/O}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz} \\ &= \mathbf{0} + (5\mathbf{k}) \times (0.3\mathbf{j}) + (8\mathbf{k}) \times [(8\mathbf{k}) \times (0.3\mathbf{j})] + \mathbf{2}(8\mathbf{k}) \times (3\mathbf{i}) + (1.5\mathbf{i} - 30\mathbf{j}) \\ &= [-1.2\mathbf{j}] \text{ m/s}^2\end{aligned}$$

Ans.

16–138. The crane's telescopic boom rotates with the angular velocity and angular acceleration shown. At the same instant, the boom is extending with a constant speed of 0.5 ft/s, measured relative to the boom. Determine the magnitudes of the velocity and acceleration of point B at this instant.



Reference Frames: The xyz rotating reference frame is attached to boom AB and coincides with the XY fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xy frame with respect to the XY frame is

$\mathbf{v}_A = \mathbf{a}_A = \mathbf{0}$ $\qquad \omega_{AB} = [-0.02\mathbf{k}]$ rad/s $\qquad \dot{\omega}_{AB} = \alpha = [-0.01\mathbf{k}]$ rad/s^2

For the motion of point B with respect to the xyz frame, we have

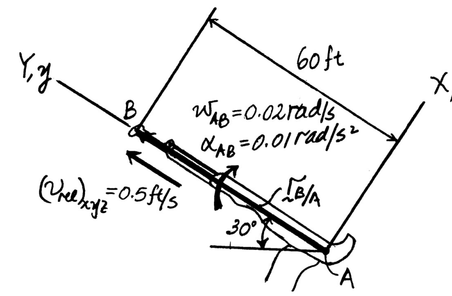
$\mathbf{r}_{B/A} = [60\mathbf{j}]$ ft $\qquad (\mathbf{v}_{\text{rel}})_{xyz} = [0.5\mathbf{j}]$ ft/s $\qquad (\mathbf{a}_{\text{rel}})_{xyz} = \mathbf{0}$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (-0.02\mathbf{k}) \times (60\mathbf{j}) + 0.5\mathbf{j}$$

$$= [1.2\mathbf{i} + 0.5\mathbf{j}] \text{ ft / s}$$



(a)

Thus, the magnitude of $\mathbf{v}_B$, Fig. b, is

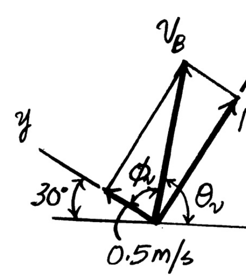
$$v_B = \sqrt{1.2^2 + 0.5^2} = 1.30 \text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\omega}_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A}) + 2\omega_{AB} \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (-0.01\mathbf{k}) \times (60\mathbf{j}) + (-0.02\mathbf{k}) \times [(-0.02\mathbf{k}) \times (60\mathbf{j})] + 2(-0.02\mathbf{k}) \times (0.5\mathbf{j}) + \mathbf{0}$$

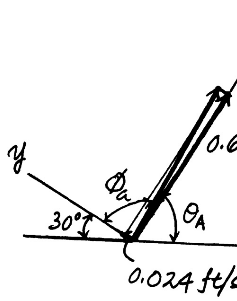
$$= [0.62\mathbf{i} - 0.024\,\mathbf{j}] \text{ ft/s}^2$$



(b)

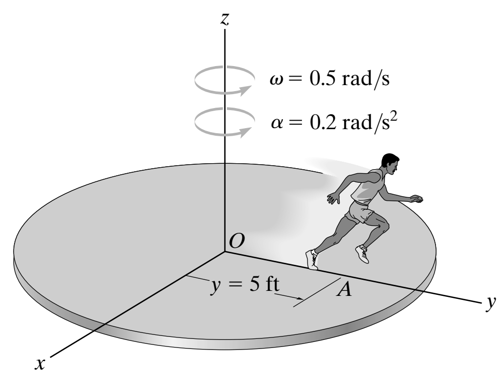
Thus, the magnitude of a_B, Fig. c, is

$$a_B = \sqrt{0.62^2 + (-0.024)^2} = 0.6204 \text{ ft/s}^2 \qquad \textbf{Ans.}$$



(c)

16–139. The man stands on the platform at O and runs out toward the edge such that when he is at A, $y = 5$ ft, his mass center has a velocity of 2 ft/s and an acceleration of 3 ft/s^2, both measured relative to the platform and directed along the positive y axis. If the platform has the angular motions shown, determine the velocity and acceleration of his mass center at this instant.



$$\mathbf{v}_A = \mathbf{v}_O + \Omega \times \mathbf{r}_{A/O} + (\mathbf{v}_{A/O})_{xyz}$$

$$\mathbf{v}_A = \mathbf{0} + (0.5\mathbf{k}) \times (5\mathbf{j}) + 2\mathbf{j}$$

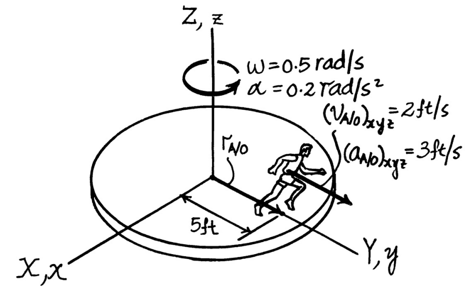
$$\mathbf{v}_A = \{-2.50\mathbf{i} + 2.00\mathbf{j}\} \text{ ft/s}$$ **Ans.**

$$\mathbf{a}_A = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{A/O} + \Omega \times (\Omega \times \mathbf{r}_{A/O}) + 2\Omega \times (\mathbf{v}_{A/O})_{xyz} + (\mathbf{a}_{A/O})_{xyz}$$

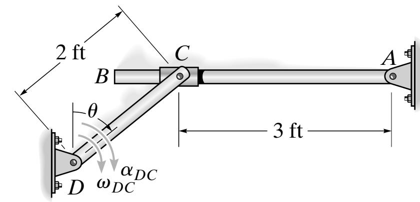
$$\mathbf{a}_A = \mathbf{0} + (0.2\mathbf{k}) \times (5\mathbf{j}) + (0.5\mathbf{k}) \times (0.5\mathbf{k} \times 5\mathbf{j}) + 2(0.5\mathbf{k}) \times (2\mathbf{j}) + 3\mathbf{j}$$

$$\mathbf{a}_A = -1\mathbf{i} - 1.25\mathbf{j} - 2\mathbf{i} + 3\mathbf{j}$$

$$\mathbf{a}_A = \{-3.00\mathbf{i} + 1.75\mathbf{j}\} \text{ ft/s}^2$$ **Ans.**



***16–140.** At the instant $\theta = 45°$, link DC has an angular velocity of $\omega_{DC} = 4$ rad/s and an angular acceleration of $\alpha_{DC} = 2$ rad/s^2. Determine the angular velocity and angular acceleration of rod AB at this instant. The collar at C is pin connected to DC and slides freely along AB.



$\mathbf{v}_A = 0$

$\mathbf{a}_A = \mathbf{0}$

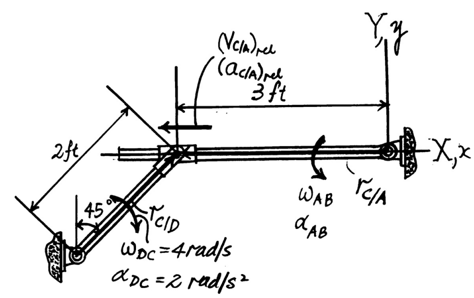
$\Omega = \omega_{AB}\,\mathbf{k}$

$\dot{\Omega} = \alpha_{AB}\,\mathbf{k}$

$\mathbf{r}_{C/A} = \{-3\mathbf{i}\}$ ft

$(\mathbf{v}_{C/A})_{xyz} = (v_{C/A})_{rel}\,\mathbf{i}$

$(\mathbf{a}_{C/A})_{xyz} = (a_{C/A})_{rel}\,\mathbf{i}$



$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D} = (-4\mathbf{k}) \times (2\sin 45°\mathbf{i} + 2\cos 45°\mathbf{j}) = \{5.6569\mathbf{i} - 5.6569\mathbf{j}\}$ ft/s

$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\,\mathbf{r}_{C/D}$

$= (-2\mathbf{k}) \times (2\sin 45°\mathbf{i} + 2\cos 45°\mathbf{j}) - (4)^2\,(2\sin 45°\mathbf{i} + 2\cos 45°\mathbf{j})$

$= \{-19.7990\mathbf{i} - 25.4558\mathbf{j}\}$ ft/s^2

$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$

$5.6569\mathbf{i} - 5.6569\mathbf{j} = \mathbf{0} + (\omega_{AB}\mathbf{k}) \times (-3\mathbf{i}) + (v_{C/A})_{xyz}\,\mathbf{i}$

$5.6569\mathbf{i} - 5.6569\mathbf{j} = (v_{C/A})_{xyz}\,\mathbf{i} - 3\omega_{AB}\,\mathbf{j}$

Solving:

$(v_{C/A})_{xyz} = 5.6569$ ft/s

$\omega_{AB} = 1.89$ rad/s ↺ **Ans.**

$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$

$-19.7990\mathbf{i} - 25.4558\mathbf{j} = \mathbf{0} + (\alpha_{AB}\,\mathbf{k}) \times (-3\mathbf{i}) + (1.89\mathbf{k}) \times [(1.89\mathbf{k}) \times (-3\mathbf{i})] + 2(1.89\mathbf{k}) \times (5.6569\mathbf{i}) + (a_{C/A})_{xyz}\,\mathbf{i}$

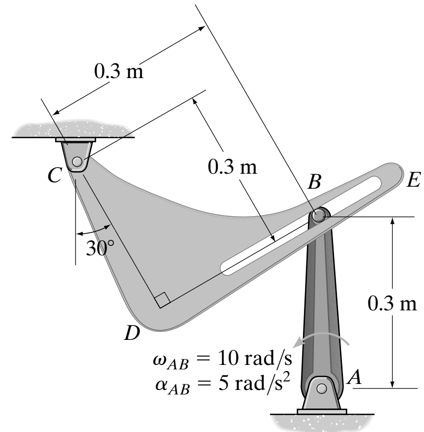
$-19.7990\mathbf{i} - 25.4558\mathbf{j} = [10.6667 + (a_{C/A})_{xyz}]\mathbf{i} + (21.334 - 3\alpha_{AB})\mathbf{j}$

Solving:

$(a_{C/A})_{xyz} = -30.47$ ft/s^2

$\alpha_{AB} = 15.6$ rad/s^2 ↺ **Ans.**

•16–141. Peg B fixed to crank AB slides freely along the slot in member CDE. If AB rotates with the motion shown, determine the angular velocity of CDE at the instant shown.



Reference Frame: The xyz rotating reference frame is attached to member CDE and coincides with the XYZ fixed reference frame at the instant considered. Thus, the motion of the xyz reference frame with respect to the XYZ frame is

$$\mathbf{v}_C = \mathbf{0} \qquad \omega_{CDE} = \omega_{CDE}\mathbf{k}$$

For the motion of point B with respect to the xyz frame,

$$\mathbf{r}_{B/C} = \left[\sqrt{0.3^2 + 0.3^2}\mathbf{i}\right] \text{m} = 0.3\sqrt{2}\mathbf{i}$$

$$\left(\mathbf{v}_{\text{rel}}\right)_{xyz} = \left(v_{\text{rel}}\right)_{xyz} \cos 45°\mathbf{i} + \left(v_{\text{rel}}\right)_{xyz} \sin 45°\mathbf{j} = 0.7071\left(v_{\text{rel}}\right)_{xyz}\mathbf{i} + 0.7071\left(v_{\text{rel}}\right)_{xyz}\mathbf{j}$$

Since crank AB rotates about a fixed axis, $\mathbf{v}_B$ and $\mathbf{a}_B$ with respect to the XYZ reference frame can be determined from

$$\begin{aligned}\mathbf{v}_B &= \omega_{AB} \times \mathbf{r}_B \\ &= (10\mathbf{k}) \times (-0.3 \cos 75°\mathbf{i} + 0.3 \sin 75°\mathbf{j}) \\ &= [-2.898\mathbf{i} - 0.7765\mathbf{j}] \text{ m/s}\end{aligned}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{CDE} \times \mathbf{r}_{B/C} + \left(\mathbf{v}_{\text{rel}}\right)_{xyz}$$

$$(-2.898\mathbf{i} - 0.7765\mathbf{j}) = \mathbf{0} + (\omega_{CDE}\mathbf{k}) \times \left(0.3\sqrt{2}\mathbf{i}\right) + 0.7071\left(v_{\text{rel}}\right)_{xyz}\mathbf{i} + 0.7071\left(v_{rel}\right)_{xyz}\mathbf{j}$$

$$-2.898\mathbf{i} - 0.7765\mathbf{j} = 0.7071\left(v_{\text{rel}}\right)_{xyz}\mathbf{i} + \left[0.3\sqrt{2}\omega_{CDE} + 0.7071\left(v_{\text{rel}}\right)_{xyz}\right]\mathbf{j}$$

Equating the $\mathbf{i}$ and $\mathbf{j}$ components yields

$$-2.898 = 0.7071(v_{\text{rel}})_{xyz} \qquad \textbf{(1)}$$

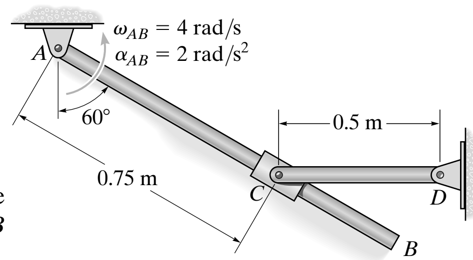
$$-0.7765 = 0.3\sqrt{2}\omega_{CDE} + 0.7071(v_{\text{rel}})_{xyz} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$(v_{\text{rel}})_{xyz} = -4.098 \text{ m/s}$$

$$\omega_{CDE} = 5 \text{ rad/s} \qquad \textbf{Ans.}$$

16–142. At the instant shown rod AB has an angular velocity $\omega_{AB} = 4$ rad/s and an angular acceleration $\alpha_{AB} = 2$ rad/s^2. Determine the angular velocity and angular acceleration of rod CD at this instant. The collar at C is pin connected to CD and slides freely along AB.



Coordinate Axes: The origin of both the fixed and moving frames of reference are located at point A. The x, y, z moving frame is attached to and rotate with rod AB since collar C slides along rod AB.

Kinematic Equation: Applying Eqs. 16–24 and 16–27, we have

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz} \quad \textbf{[1]}$$

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz} \quad \textbf{[2]}$$

Motion of moving reference

$\mathbf{v}_A = \mathbf{0}$

$\mathbf{a}_A = \mathbf{0}$

$\Omega = 4\mathbf{k}$ rad/s

$\dot{\Omega} = 2\mathbf{k}$ rad/s^2

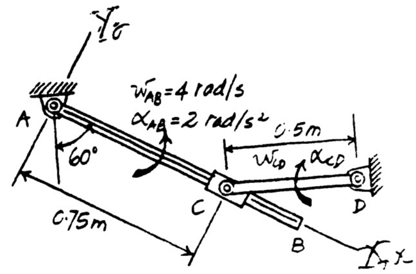
Motion of C with respect to moving reference

$r_{C/A} = \{0.75\mathbf{i}\}$m

$(\mathbf{v}_{C/A})_{xyz} = (v_{C/A})_{xyz}\,\mathbf{i}$

$(\mathbf{a}_{C/A})_{xyz} = (a_{C/A})_{xyz}\,\mathbf{i}$

The velocity and acceleration of collar C can be determined using Eqs. 16–9 and 16–14 with $\mathbf{r}_{C/D} = \{-0.5\cos 30°\mathbf{i} - 0.5\sin 30°\mathbf{j}\}\text{m} = \{-0.4330\mathbf{i} - 0.250\mathbf{j}\}$ m.



$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D} = -\omega_{CD}\mathbf{k} \times (-0.4330\mathbf{i} - 0.250\mathbf{j})$$

$$= -0.250\omega_{CD}\mathbf{i} + 0.4330\omega_{CD}\mathbf{j}$$

$$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\mathbf{r}_{C/D}$$

$$= -\alpha_{CD}\,\mathbf{k} \times (-0.4330\mathbf{i} - 0.250\mathbf{j}) - \omega_{CD}^2(-0.4330\mathbf{i} - 0.250\mathbf{j})$$

$$= \left(0.4330\omega_{CD}^2 - 0.250\,\alpha_{CD}\right)\mathbf{i} + \left(0.4330\alpha_{CD} + 0.250\omega_{CD}^2\right)\mathbf{j}$$

Substitute the above data into Eq.[1] yields

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$

$$-0.250\,\omega_{CD}\,\mathbf{i} + 0.4330\omega_{CD}\mathbf{j} = \mathbf{0} + 4\mathbf{k} \times 0.75\mathbf{i} + (v_{C/A})_{xyz}\,\mathbf{i}$$

$$-0.250\omega_{CD}\mathbf{i} + 0.4330\omega_{CD}\mathbf{j} = (v_{C/A})_{xyz}\,\mathbf{i} + 3.00\mathbf{j}$$

Equating **i** and **j** components and solve, we have

$$(v_{C/A})_{xyz} = -1.732 \text{ m/s}$$

$$\omega_{CD} = 6.928 \text{ rad/s} = 6.93 \text{ rad/s} \qquad \textbf{Ans.}$$

Substitute the above data into Eq.[2] yields

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (v_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$

$$\left[0.4330\left(6.928^2\right) - 0.250\,\alpha_{CD}\right]\mathbf{i} + \left[0.4330\alpha_{CD} + 0.250\left(6.928^2\right)\right]\mathbf{j}$$

$$= \mathbf{0} + 2\mathbf{k} \times 0.75\mathbf{i} + 4\mathbf{k} \times (4\mathbf{k} \times 0.75\mathbf{i}) + \mathbf{2}\,(4\mathbf{k}) \times (-1.732\mathbf{i}) + (a_{C/A})_{xyz}\,\mathbf{i}$$

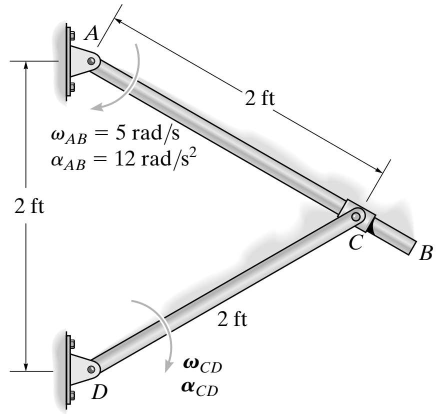
$$(20.78 - 0.250\alpha_{CD})\mathbf{i} + (0.4330\,\alpha_{CD} + 12)\mathbf{j} = \left[(a_{C/A})_{xyz} - 12.0\right]\mathbf{i} - 12.36\mathbf{j}$$

Equating **i** and **j** components, we have

$$(a_{C/A})_{xyz} = 46.85 \text{ m/s}^2$$

$$\alpha_{CD} = -56.2 \text{ rad/s}^2 = 56.2 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

16–143. At a given instant, rod AB has the angular motions shown. Determine the angular velocity and angular acceleration of rod CD at this instant. There is a collar at C.



$\mathbf{v}_A = \mathbf{0}$

$\mathbf{a}_A = \mathbf{0}$

$\Omega = \{-5\mathbf{k}\}$ rad/s

$\dot{\Omega} = \{-12\mathbf{k}\}$ rad/s^2

$\mathbf{r}_{C/A} = \{2\mathbf{i}\}$ ft

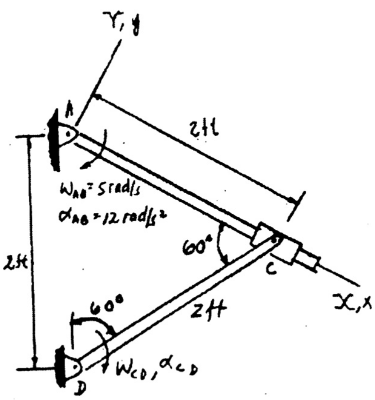
$(\mathbf{v}_{C/A})_{xyz} = (v_{C/A})_{xyz}\,\mathbf{i}$

$(\mathbf{a}_{C/A})_{xyz} = (a_{C/A})_{xyz}\,\mathbf{i}$

$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$

$\mathbf{v}_C = \mathbf{0} + (-5\mathbf{k}) \times (2\mathbf{i}) + (v_{C/A})_{xyz}\mathbf{i}$

$\quad = (v_{C/A})_{xyz}\,\mathbf{i} - 10\mathbf{j}$



$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{CD}$$

$(v_{C/A})_{xyz}\,\mathbf{i} - 10\mathbf{j} = (-\omega_{CD}\,\mathbf{k}) \times (2\cos 60°\mathbf{i} + 2\sin 60°\mathbf{j})$

$(v_{C/A})_{xyz}\,\mathbf{i} - 10\mathbf{j} = 1.732\omega_{CD}\,\mathbf{i} - \omega_{CD}\mathbf{j}$

Solving:

$$\omega_{CD} = 10 \text{ rad/s} \quad \circlearrowright \qquad \textbf{Ans.}$$

$(v_{C/A})_{xyz} = 1.732(10) = 17.32$ ft/s

$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$

$\mathbf{a}_C = \mathbf{0} + (-12\mathbf{k}) \times (2\mathbf{i}) + (-5\mathbf{k}) \times [(-5\mathbf{k}) \times (2\mathbf{i})] + 2(-5\mathbf{k}) \times [(v_{C/A})_{xyz}\mathbf{i}] + (a_{C/A})_{xyz}\,\mathbf{i}$

$\quad = [(a_{C/A})_{xyz} - 50]\mathbf{i} - [10(v_{C/A})_{xyz} + 24]\mathbf{j}$

$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\,\mathbf{r}_{CD}$

$[(a_{C/A})_{xyz} - 50]\mathbf{i} - [10(17.32) + 24]\mathbf{j} = (-\alpha_{CD}\,\mathbf{k}) \times (2\cos 60°\mathbf{i} + 2\sin 60°\mathbf{j}) - (10)^2(2\cos 60°\mathbf{i} + 2\sin 60°\mathbf{j})$

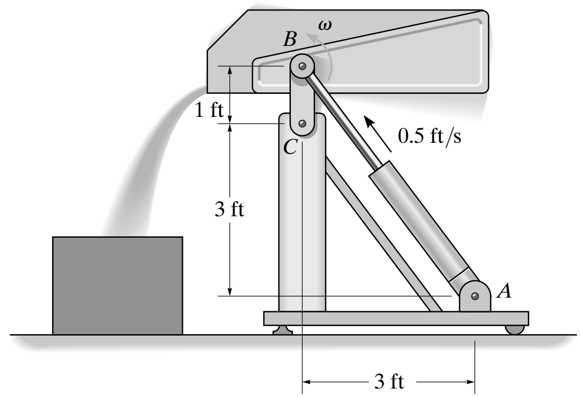
$[(a_{C/A})_{xyz} - 50]\mathbf{i} - (10(17.32) + 24]\mathbf{j} = (1.732\,\alpha_{CD} - 100)\mathbf{i} - (\alpha_{CD} + 173.2)\mathbf{j}$

Solving:

$-[10(17.32) + 24] = -(\alpha_{CD} + 173.2) \qquad \alpha_{CD} = 24 \text{ rad/s}^2 \quad \circlearrowright \qquad$ **Ans.**

$(a_{C/A})_{xyz} - 50 = 1.732(24) - 100 \qquad (a_{C/A})_{xyz} = -8.43 \text{ ft/s}^2$

***16–144.** The dumpster pivots about C and is operated by the hydraulic cylinder AB. If the cylinder is extending at a constant rate of 0.5 ft/s, determine the angular velocity $\boldsymbol{\omega}$ of the container at the instant it is in the horizontal position shown.



$$\mathbf{r}_{B/A} = 5\,\mathbf{j}$$

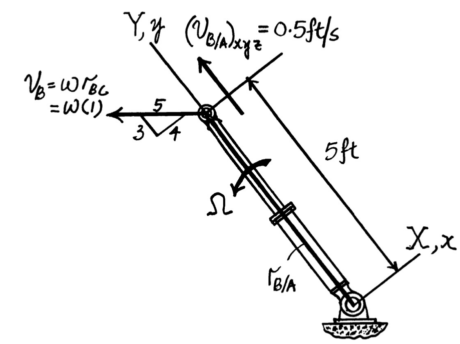
$$\mathbf{v}_{B/A} = 0.5\mathbf{j}$$

$$\mathbf{v}_B = -\frac{4}{5}\,\omega(1)\mathbf{i} + \frac{3}{5}\,\omega(1)\mathbf{j}$$

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$-\frac{4}{5}\,\omega(1)\mathbf{i} + \frac{3}{5}\,\omega(1)\mathbf{j} = \mathbf{0} + (\Omega\mathbf{k}) \times (5\mathbf{j}) + 0.5\mathbf{j}$$

$$-\frac{4}{5}\,\omega(1)\mathbf{i} + \frac{3}{5}\omega(1)\mathbf{j} = -\Omega(5)\mathbf{i} + 0.5\mathbf{j}$$

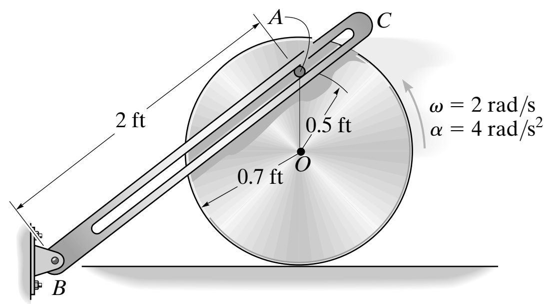


Thus,

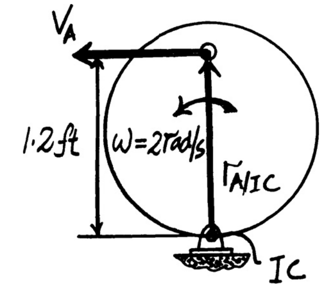
$$\omega = 0.833 \text{ rad/s} \qquad \textbf{Ans.}$$

$$\Omega = 0.133 \text{ rad/s}$$

•16–145. The disk rolls without slipping and at a given instant has the angular motion shown. Determine the angular velocity and angular acceleration of the slotted link BC at this instant. The peg at A is fixed to the disk.



$$\mathbf{v}_A = -(1.2)(2)\mathbf{i} = -2.4\mathbf{i} \text{ ft/s}$$



$$\mathbf{a}_A = \mathbf{a}_O + \alpha \times \mathbf{r}_{A/O} - \omega^2 \mathbf{r}_{A/O}$$

$$\mathbf{a}_A = -4(0.7)\mathbf{i} + (4\mathbf{k}) \times (0.5\mathbf{j}) - (2)^2(0.5\mathbf{j})$$

$$\mathbf{a}_A = -4.8\,\mathbf{i} - 2\mathbf{j}$$

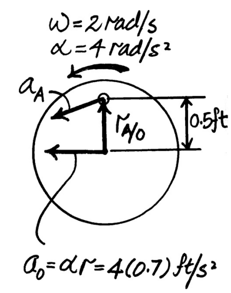
$$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz}$$

$$-2.4\mathbf{i} = \mathbf{0} + (\omega_{BC}\mathbf{k}) \times (1.6\mathbf{i} + 1.2\mathbf{j}) + v_{A/B}\left(\frac{4}{5}\right)\mathbf{i} + v_{A/B}\left(\frac{3}{5}\right)\mathbf{j}$$

$$-2.4\mathbf{i} = 1.6\,\omega_{BC}\,\mathbf{j} - 1.2\,\omega_{BC}\,\mathbf{i} + 0.8v_{A/B}\mathbf{i} + 0.6v_{A/B}\,\mathbf{j}$$

$$-2.4 = -1.2\,\omega_{BC} + 0.8\,v_{A/B}$$

$$0 = 1.6\omega_{BC} + 0.6v_{A/B}$$



Solving,

$$\omega_{BC} = 0.720 \text{ rad/s} \quad \circlearrowleft$$ **Ans.**

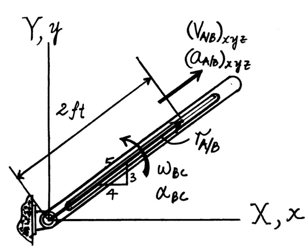
$$v_{A/B} = -1.92 \text{ ft/s}$$

$$\mathbf{a}_A = \mathbf{a}_B + \Omega \times \mathbf{r}_{A/B} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$$

$$-4.8\mathbf{i} - 2\mathbf{j} = \mathbf{0} + (\alpha_{BC}\,\mathbf{k}) \times (1.6\mathbf{i} + 1.2\mathbf{j}) + (0.72\mathbf{k}) \times (0.72\mathbf{k} \times (1.6\mathbf{i} + 1.2\mathbf{j}))$$

$$+2(0.72\mathbf{k}) \times \left[-(0.8)(1.92)\mathbf{i} - 0.6(1.92)\mathbf{j}\right] + 0.8\,a_{B/A}\mathbf{i} + 0.6a_{B/A}\,\mathbf{j}$$

$$-4.8\mathbf{i} - 2\mathbf{j} = 1.6\alpha_{BC}\mathbf{j} - 1.2\alpha_{BC}\,\mathbf{i} - 0.8294\mathbf{i} - 0.6221\mathbf{j} - 2.2118\mathbf{j} + 1.6589\mathbf{i} + 0.8a_{B/A}\,\mathbf{i} + 0.6a_{B/A}\,\mathbf{j}$$



$$-4.8 = -1.2\alpha_{BC} - 0.8294 + 1.6589 + 0.8a_{B/A}$$

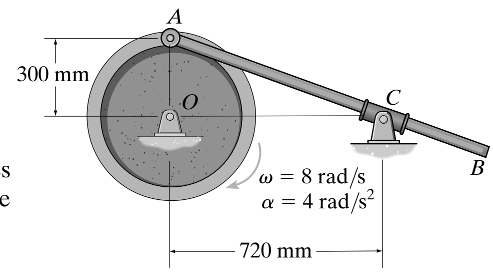
$$-2 = 1.6\alpha_{BC} - 0.6221 - 2.2118 + 0.6a_{B/A}$$

Solving,

$$\alpha_{BC} = 2.02 \text{ rad/s}^2 \quad \circlearrowleft$$ **Ans.**

$$a_{B/A} = -4.00 \text{ ft/s}^2$$

16–146. The wheel is rotating with the angular velocity and angular acceleration at the instant shown. Determine the angular velocity and angular acceleration of the rod at this instant. The rod slides freely through the smooth collar.



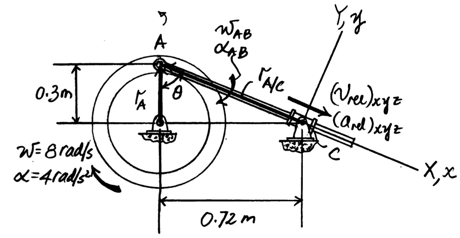
Reference Frame: The xyz rotating reference frame is attached to C and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xyz reference frame with respect to the XYZ frame is

$$\mathbf{v}_C = \mathbf{a}_C = 0 \qquad \omega_{AB} = -\omega_{AB}\mathbf{k} \qquad \dot{\omega}_{AB} = -\alpha_{AB}\,\mathbf{k}$$

From the geometry shown in Fig.a,

$$r_{A/C} = \sqrt{0.3^2 + 0.72^2} = 0.78 \text{ m}$$

$$\theta = \tan^{-1}\left(\frac{0.72}{0.3}\right) = 67.38°$$



(a)

For the motion of point A with respect to the xyz frame,

$$\mathbf{r}_{A/C} = [-0.78\mathbf{i}] \text{ m} \qquad (\mathbf{v}_{\text{rel}})_{xyz} = (v_{\text{rel}})_{xyz}\,\mathbf{i} \qquad (\mathbf{a}_{\text{rel}})_{xyz} = (a_{\text{rel}})_{xyz}\,\mathbf{i}$$

Since the wheel A rotates about a fixed axis, $\mathbf{v}_A$ and $\mathbf{a}_A$ with respect to the XYZ reference frame can be determined from

$$\mathbf{v}_A = \omega \times \mathbf{r}_A$$
$$= (-8\mathbf{k}) \times (-0.3 \cos 67.38°\mathbf{i} + 0.3 \sin 67.38°\mathbf{j})$$
$$= [2.215\mathbf{i} + 0.9231\mathbf{j}] \text{ m/s}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A - \omega^2\mathbf{r}_A$$
$$= (-4\mathbf{k}) \times (-0.3 \cos 67.38°\mathbf{i} + 0.3 \sin 67.38°\,\mathbf{j}) - 8^2(-0.3 \cos 67.38°\mathbf{i} + 0.3 \sin 67.38°\mathbf{j})$$
$$= [8.492\mathbf{i} - 17.262\mathbf{j}] \text{ m/s}^2$$

Velocity: Applying the relative velocity equation, we have

$$\mathbf{v}_A = \mathbf{v}_C + \omega_{AB} \times \mathbf{r}_{A/C} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$2.215\mathbf{i} + 0.9231\mathbf{j} = \mathbf{0} + (-\omega_{AB}\mathbf{k}) \times (-0.78\mathbf{i}) + (v_{\text{ rel}})_{xyz}\,\mathbf{i}$$

$$2.215\mathbf{i} + 0.9231\mathbf{j} = (v_{\text{rel}})_{xyz}\,\mathbf{i} + 0.78\omega_{AB}\,\mathbf{j}$$

Equating the **i** and **j** components yields

$$(v_{\text{rel}})_{xyz} = 2.215 \text{ m/s}$$

$$0.78\omega_{AB} = 0.9231 \qquad \omega_{AB} = 1.183 \text{ rad/s} = 1.18 \text{ rad/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation.

$$\mathbf{a}_A = \mathbf{a}_C + \dot{\omega}_{AB} \times \mathbf{r}_{A/C} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{A/C}) + 2\omega_{AB} \times (\mathbf{v}_{\text{rel}})_{xyx} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$8.492\mathbf{i} - 17.262\mathbf{j} = \mathbf{0} + (-\alpha_{AB}\mathbf{k}) \times (-0.78\mathbf{i}) + (-1.183\mathbf{k}) \times [(-1.183\mathbf{k}) \times (-0.78\mathbf{i})] + 2(-1.183\mathbf{k}) \times (2.215\mathbf{i}) + (a_{\text{ rel}})_{xyz}\mathbf{i}$$

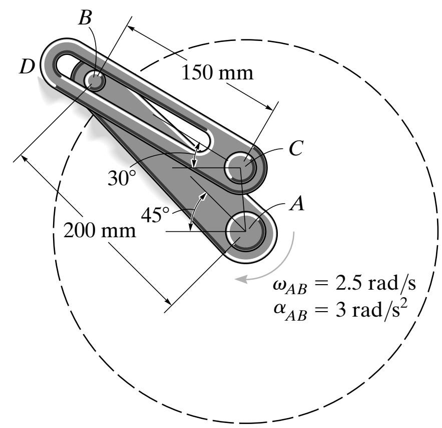
$$8.492\mathbf{i} - 17.262\mathbf{j} = \left[\left(a_{\text{rel}}\right)_{xyz} + 1.092\right]\mathbf{i} + \left(0.78\alpha_{AB} - 5.244\right)\mathbf{j}$$

Equating the **j** components yields

$$-17.262 = 0.78\alpha_{AB} - 5.244$$

$$\alpha_{AB} = -15.41 \text{ rad/s}^2 = 15.4 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

16–147. The two-link mechanism serves to amplify angular motion. Link AB has a pin at B which is confined to move within the slot of link CD. If at the instant shown, AB (input) has an angular velocity of $\omega_{AB} = 2.5$ rad/s and an angular acceleration of $\alpha_{AB} = 3\ \text{rad/s}^2$, determine the angular velocity and angular acceleration of CD (output) at this instant.



$\mathbf{v}_C = \mathbf{0}$

$\mathbf{a}_C = \mathbf{0}$

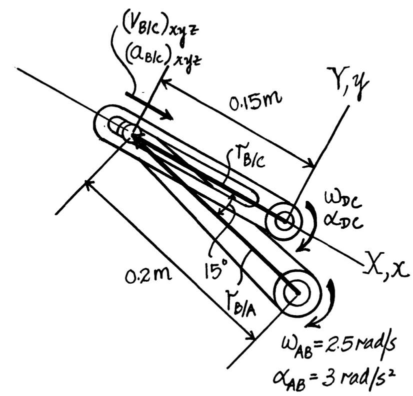
$\Omega = -\omega_{DC}\,\mathbf{k}$

$\dot{\Omega} = -\alpha_{DC}\,\mathbf{k}$

$\mathbf{r}_{B/C} = \{-0.15\,\mathbf{i}\}$ m

$(\mathbf{v}_{B/C})_{xyz} = (v_{B/C})_{xyz}\,\mathbf{i}$

$(\mathbf{a}_{B/C})_{xyz} = (a_{B/C})_{xyz}\,\mathbf{i}$



$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = (-2.5\mathbf{k}) \times (-0.2\cos 15°\mathbf{i} + 0.2\sin 15°\mathbf{j})$

$\quad = \{0.1294\mathbf{i} + 0.4830\mathbf{j}\}$ m/s

$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\,\mathbf{r}_{B/A}$

$\quad = (-3\mathbf{k}) \times (-0.2\cos 15°\mathbf{i} + 0.2\sin 15°\mathbf{j}) - (2.5)^2(-0.2\cos 15°\mathbf{i} + 0.2\sin 15°\mathbf{j})$

$\quad = \{1.3627\mathbf{i} + 0.2560\mathbf{j}\}\ \text{m/s}^2$

$\mathbf{v}_B = \mathbf{v}_C + \Omega \times \mathbf{r}_{B/C} + (\mathbf{v}_{B/C})_{xyz}$

$0.1294\mathbf{i} + 0.4830\mathbf{j} = \mathbf{0} + (-\omega_{DC}\mathbf{k}) \times (-0.15\mathbf{i}) + (v_{B/C})_{xyz}\,\mathbf{i}$

$0.1294\mathbf{i} + 0.4830\mathbf{j} = (v_{B/C})_{xyz}\,\mathbf{i} + 0.15\omega_{DC}\,\mathbf{j}$

Solving:

$$(v_{B/C})_{xyz} = 0.1294 \text{ m/s}$$

$$\omega_{DC} = 3.22 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$

$\mathbf{a}_B = \mathbf{a}_C + \dot{\Omega} \times \mathbf{r}_{B/C} + \Omega \times (\Omega \times \mathbf{r}_{B/C}) + 2\Omega \times (\mathbf{v}_{B/C})_{xyz} + (\mathbf{a}_{B/C})_{xyz}$

$1.3627\mathbf{i} + 0.2560\mathbf{j} = \mathbf{0} + (-\alpha_{DC}\mathbf{k}) \times (-0.15\mathbf{i}) + (-3.22\mathbf{k}) \times \left[(-3.22\mathbf{k}) \times (-0.15\mathbf{i})\right] + 2(-3.22\mathbf{k}) \times (0.1294\mathbf{i}) + (a_{B/C})_{xyz}\,\mathbf{i}$

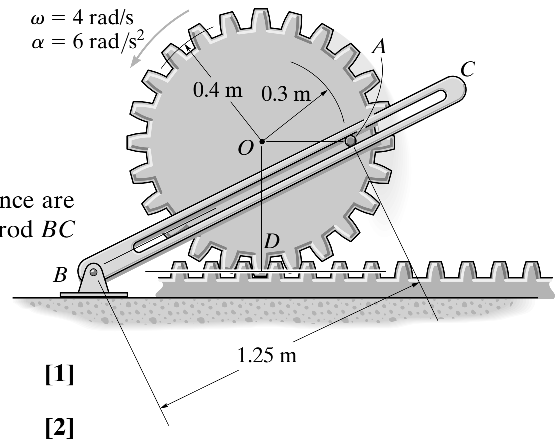
$1.3627\mathbf{i} + 0.2560\mathbf{j} = \left[1.5550 + (a_{B/C})_{xyz}\right]\mathbf{i} + (0.15\,\alpha_{DC} - 0.8333)\mathbf{j}$

Solving:

$$(a_{B/C})_{xyz} = -0.1923\ \text{m/s}^2$$

$$\alpha_{DC} = 7.26\ \text{rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

***16–148.** The gear has the angular motion shown. Determine the angular velocity and angular acceleration of the slotted link BC at this instant. The peg A is fixed to the gear.



Coordinate Axes: The origin of both the fixed and moving frames of reference are located at point B. The x, y, z moving frame is attached to and rotates with rod BC since peg A slides along slot in member BC.

Kinematic Equation: Applying Eqs. 16–24 and 16–27, we have

$$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz} \qquad [1]$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{A/B} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz} \qquad [2]$$

Motion of moving reference

$\mathbf{v}_B = \mathbf{0}$

$\mathbf{a}_B = \mathbf{0}$

$\Omega = \omega_{BC}\,\mathbf{k}$

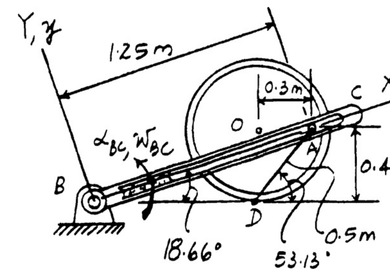
$\dot{\Omega} = \alpha_{BC}\,\mathbf{k}$

Motion of C with respect to moving reference

$\mathbf{r}_{A/B} = \{1.25\mathbf{i}\}$ m

$(\mathbf{v}_{A/B})_{xyz} = (v_{A/B})_{xyz}\,\mathbf{i}$

$(\mathbf{a}_{A/B})_{xyz} = (a_{A/B})_{xyz}\,\mathbf{i}$



The velocity and acceleration of peg A can be determined using Eqs. 16–16 and 16–18 with $\mathbf{r}_{A/D} = \{0.5\cos 34.47°\mathbf{i} + 0.5\sin 34.47°\mathbf{j}\}$ m $= \{0.4122\mathbf{i} + 0.2830\mathbf{j}\}$ m.

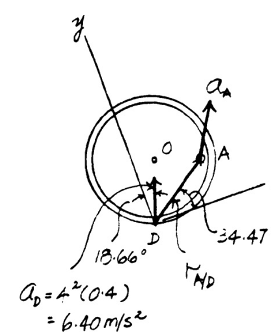
$$\mathbf{v}_A = \mathbf{v}_D + \omega \times \mathbf{r}_{A/D} = \mathbf{0} + 4\mathbf{k} \times (0.4122\mathbf{i} + 0.2830\mathbf{j})$$

$$= \{-1.1319\mathbf{i} + 1.6489\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_A = \mathbf{a}_D + \alpha \times \mathbf{r}_{A/D} - \omega^2\mathbf{r}_{A/D}$$

$$= 6.40\sin 18.66°\mathbf{i} + 6.40\cos 18.66°\mathbf{j} + 6\mathbf{k} \times (0.4122\mathbf{i} + 0.2830\mathbf{j}) - 4^2(0.4122\mathbf{i} + 0.2830\mathbf{j})$$

$$= \{-6.2454\mathbf{i} + 4.0094\mathbf{j}\} \text{ m/s}^2$$



Substitute the above data into Eq.[1] yields

$$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz}$$

$$-1.1319\mathbf{i} + 1.6489\mathbf{j} = \mathbf{0} + \omega_{BC}\mathbf{k} \times 1.25\mathbf{i} + (v_{A/B})_{xyz}\,\mathbf{i}$$

$$-1.1319\mathbf{i} + 1.6489\mathbf{j} = (v_{A/B})_{xyz}\,\mathbf{i} + 1.25\,\omega_{BC}\mathbf{j}$$

Equating **i** and **j** components and solving, we have

$$(v_{A/B})_{xyz} = -1.1319 \text{ m/s}$$

$$\omega_{BC} = 1.3191 \text{ rad/s} = 1.32 \text{ rad/s} \qquad \textbf{Ans.}$$

Substitute the above data into Eq.[2] yields

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{A/B} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$$

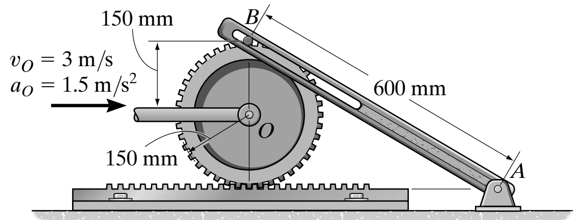
$$-6.2454\mathbf{i} + 4.0094\mathbf{j} = \mathbf{0} + \alpha_{BC}\,\mathbf{k} \times 1.25\mathbf{i} + 1.3191\mathbf{k} \times (1.3191\mathbf{k} \times 1.25\mathbf{i}) + 2(1.3191\mathbf{k}) \times (-1.1319\mathbf{i}) + (a_{A/B})_{xyz}\,\mathbf{i}$$

$$-6.2454\mathbf{i} + 4.0094\mathbf{j} = \left[(a_{A/B})_{xyz} - 2.1751\right]\mathbf{i} + (1.25\alpha_{BC} - 2.9861)\mathbf{j}$$

Equating **i** and **j** components, we have

$$(a_{A/B})_{xyz} = -4.070 \text{ m/s}^2$$

$$\alpha_{BC} = 5.60 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



•16–149. Peg B on the gear slides freely along the slot in link AB. If the gear's center O moves with the velocity and acceleration shown, determine the angular velocity and angular acceleration of the link at this instant.

Gear Motion: The IC of the gear is located at the point where the gear and the gear rack mesh, Fig. a. Thus,

$$\omega = \frac{v_O}{r_{O/IC}} = \frac{3}{0.15} = 20 \text{ rad/s}$$

Then,

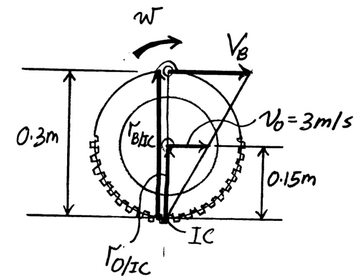
$$v_B = \omega r_{B/IC} = 20(0.3) = 6 \text{ m/s} \rightarrow$$

Since the gear rolls on the gear rack, $\alpha = \dfrac{a_O}{r} = \dfrac{1.5}{0.15} = 10$ rad/s. By referring to Fig. b,

$$\mathbf{a}_B = \mathbf{a}_O + \alpha \times \mathbf{r}_{B/O} - \omega^2 \mathbf{r}_{B/O}$$

$$(a_B)_t \mathbf{i} - (a_B)_n \mathbf{j} = 1.5\mathbf{i} + (-10\mathbf{k}) \times 0.15\mathbf{j} - 20^2(0.15\mathbf{j})$$

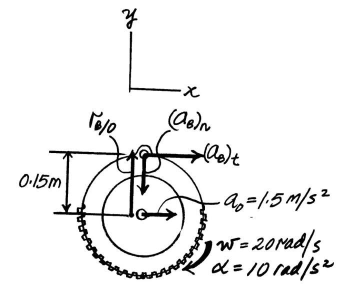
$$(a_B)_t \mathbf{i} - (a_B)_n \mathbf{j} = 3\mathbf{i} - 60\mathbf{j}$$



(a)

Thus,

$$(a_B)_t = 3 \text{ m/s}^2 \qquad (a_B)_n = 60 \text{ m/s}^2$$



(b)

Reference Frame: The $x'y'z'$ rotating reference frame is attached to link AB and coincides with the XYZ fixed reference frame, Figs. c and d. Thus, $\mathbf{v}_B$ and $\mathbf{a}_B$ with respect to the XYZ frame is

$$\mathbf{v}_B = [6 \sin 30°\mathbf{i} - 6 \cos 30°\,\mathbf{j}] = [3\mathbf{i} - 5.196\mathbf{j}] \text{ m/s}$$

$$\mathbf{a}_B = (3 \sin 30° - 60 \cos 30°)\mathbf{i} + (-3 \cos 30° - 60 \sin 30°)\mathbf{j}$$

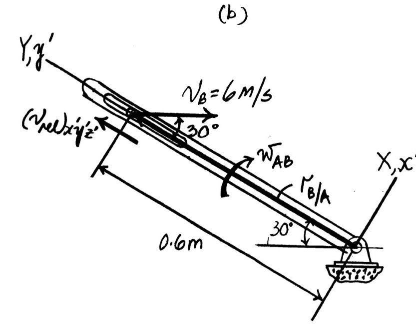
$$= [-50.46\mathbf{i} - 32.60\mathbf{j}] \text{ m/s}^2$$

For motion of the $x'y'z'$ frame with reference to the XYZ reference frame,

$$\mathbf{v}_A = \mathbf{a}_A = \mathbf{0} \qquad \omega_{AB} = -\omega_{AB}\mathbf{k} \qquad \dot{\omega}_{AB} = -\alpha_{AB}\,\mathbf{k}$$

For the motion of point B with respect to the $x'y'z'$ frame is

$$\mathbf{r}_{B/A} = [0.6\mathbf{j}]\text{m} \qquad (\mathbf{v}_{\text{rel}})_{x'y'z'} = (v_{\text{rel}})_{x'y'z'}\mathbf{j} \qquad (\mathbf{a}_{\text{rel}})_{x'y'z'} = (a_{\text{rel}})_{x'y'z'}\,\mathbf{j}$$



(C)

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A} + (\mathbf{v}_{\text{rel}})_{x'y'z'}$$

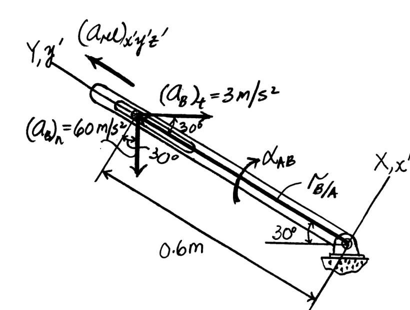
$$3\mathbf{i} - 5.196\mathbf{j} = \mathbf{0} + (-\omega_{AB}\mathbf{k}) \times (0.6\mathbf{j}) + (v_{\text{rel}})_{x'y'z'}\,\mathbf{j}$$

$$3\mathbf{i} - 5.196\mathbf{j} = 0.6\omega_{AB}\,\mathbf{i} + (v_{\text{rel}})_{x'y'z'}\mathbf{j}$$

Equating the **i** and **j** components yields

$$3 = 0.6\omega_{AB} \qquad \omega_{AB} = 5 \text{ rad/s} \qquad \textbf{Ans.}$$

$$(v_{\text{rel}})_{x'y'z'} = -5.196 \text{ m/s}$$



(d)

Acceleration: Applying the relative acceleration equation.

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\omega}_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A}) + 2\omega_{AB} \times (\mathbf{v}_{\text{rel}})_{x'y'z'} + (\mathbf{a}_{\text{rel}})_{x'y'z'}$$

$$-50.46\mathbf{i} - 32.60\mathbf{j} = \mathbf{0} + (-\alpha_{AB}\mathbf{k}) \times (0.6\mathbf{j}) + (-5\mathbf{k}) \times [(-5\mathbf{k}) \times (0.6\mathbf{j})] + 2(-5\mathbf{k}) \times (-5.196\mathbf{j}) + (a_{\text{rel}})_{x'y'z'}\mathbf{j}$$

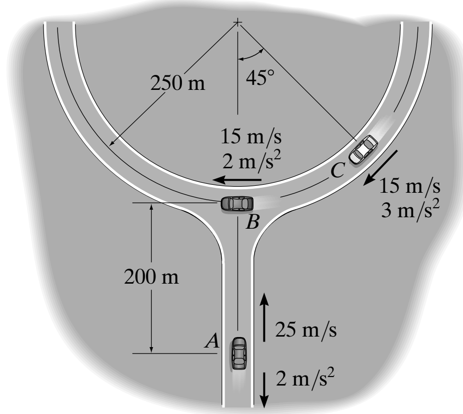
$$-50.46\mathbf{i} - 32.60\mathbf{j} = (0.6\alpha_{AB} - 51.96)\mathbf{i} + \left[(a_{\text{rel}})_{x'y'z'} - 15\right]\mathbf{j}$$

Equating the **i** components,

$$-50.46 = 0.6\alpha_{AB} - 51.96$$

$$\alpha_{AB} = 2.5 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

16–150. At the instant shown, car A travels with a speed of 25 m/s, which is decreasing at a constant rate of 2 m/s^2, while car B travels with a speed of 15 m/s, which is increasing at a constant rate of 2 m/s^2. Determine the velocity and acceleration of car A with respect to car B.



Reference Frames: The xyz rotating reference frame is attached to car B and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Since car B moves along the circular road, its normal component of acceleration is $(a_B)_n = \dfrac{v_B{}^2}{\rho} = \dfrac{15^2}{250} = 0.9$ m/s^2. Thus, the motion of car B with respect to the XYZ frame is

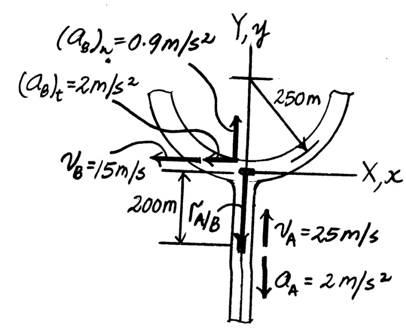
$$\mathbf{v}_B = [-15\mathbf{i}] \text{ m/s}$$

$$\mathbf{a}_B = [-2\mathbf{i} + 0.9\mathbf{j}] \text{ m/s}^2$$

Also, the angular velocity and angular acceleration of the xyz frame with respect to the XYZ frame is

$$\omega = \frac{v_B}{\rho} = \frac{15}{250} = 0.06 \text{ rad/s} \qquad \omega = [-0.06\mathbf{k}] \text{ rad/s}$$

$$\dot{\omega} = \frac{(a_B)_t}{\rho} = \frac{2}{250} = 0.008 \text{ rad/s}^2 \qquad \dot{\omega} = [-0.008\mathbf{k}] \text{ rad/s}^2$$



(a)

The velocity of car A with respect to the XYZ reference frame is

$$\mathbf{v}_A = [25\mathbf{j}] \text{ m/s} \qquad \mathbf{a}_A = [-2\mathbf{j}] \text{ m/s}^2$$

From the geometry shown in Fig. a,

$$\mathbf{r}_{A/B} = [-200\mathbf{j}] \text{ m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_A = \mathbf{v}_B + \omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$25\mathbf{j} = -15\mathbf{i} + (-0.06\mathbf{k}) \times (-200\mathbf{j}) + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$25\mathbf{j} = -27\mathbf{i} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = [27\mathbf{i} + 25\mathbf{j}] \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

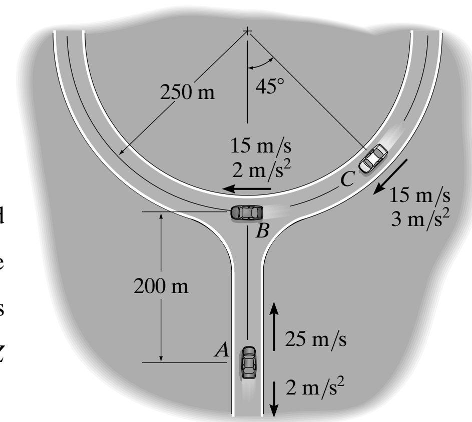
$$\mathbf{a}_A = \mathbf{a}_B + \dot{\omega} \times \mathbf{r}_{A/B} + \omega \times (\omega \times \mathbf{r}_{A/B}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{j} = (-2\mathbf{i} + 0.9\mathbf{j}) + (-0.008\mathbf{k}) \times (-200\mathbf{j}) + (-0.06\mathbf{k}) \times [(-0.06\mathbf{k}) \times (-200\mathbf{j})] + 2(-0.06\mathbf{k}) \times (27\mathbf{i} + 25\mathbf{j}) + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{j} = -0.6\mathbf{i} - 1.62\mathbf{j} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = [0.6\mathbf{i} - 0.38\mathbf{j}] \text{ m/s}^2 \qquad \textbf{Ans.}$$

16–151. At the instant shown, car A travels with a speed of 25 m/s, which is decreasing at a constant rate of 2 m/s^2, while car C travels with a speed of 15 m/s, which is increasing at a constant rate of 3 m/s^2. Determine the velocity and acceleration of car A with respect to car C.



Reference Frame: The xyz rotating reference frame is attached to car C and coincides with the XYZ fixed reference frame at the instant considered, Fig. *a*. Since car C moves along the circular road, its normal component of acceleration is $(a_C)_n = \dfrac{v_C{}^2}{\rho} = \dfrac{15^2}{250} = 0.9\text{ m/s}^2$. Thus, the motion of car C with respect to the XYZ frame is

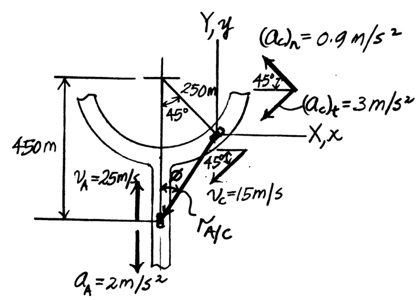
$$\mathbf{v}_C = -15\cos 45°\mathbf{i} - 15\sin 45°\mathbf{j} = [-10.607\mathbf{i} - 10.607\mathbf{j}]\text{ m/s}$$

$$\mathbf{a}_C = (-0.9\cos 45° - 3\cos 45°)\mathbf{i} + (0.9\sin 45° - 3\sin 45°)\mathbf{j} = [-2.758\mathbf{i} - 1.485\mathbf{j}]\text{ m/s}^2$$

Also, the angular velocity and angular acceleration of the xyz reference frame is

$$\omega = \frac{v_C}{\rho} = \frac{15}{250} = 0.06\text{ rad/s} \qquad \omega = [-0.06\mathbf{k}]\text{ rad/s}$$

$$\dot{\omega} = \frac{(a_C)_t}{\rho} = \frac{3}{250} = 0.012\text{ rad/s}^2 \qquad \dot{\omega} = [-0.012\mathbf{k}]\text{ rad/s}^2$$



(a)

The velocity and accdeleration of car A with respect to the XYZ frame is

$$\mathbf{v}_A = [25\mathbf{j}]\text{ m/s} \qquad \mathbf{a}_A = [-2\mathbf{j}]\text{ m/s}^2$$

From the geometry shown in Fig. *a*,

$$\mathbf{r}_{A/C} = -250\sin 45°\mathbf{i} - (450 - 250\cos 45°)\mathbf{j} = [-176.78\mathbf{i} - 273.22\mathbf{j}]\text{ m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_A = \mathbf{v}_C + \omega \times \mathbf{r}_{A/C} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$25\mathbf{j} = (-10.607\mathbf{i} - 10.607\mathbf{j}) + (-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 273.22\mathbf{j}) + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$25\mathbf{j} = -27\mathbf{i} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = [27\mathbf{i} + 25\mathbf{j}]\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

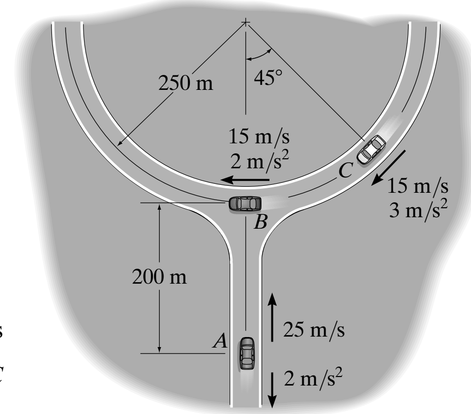
$$\mathbf{a}_A = \mathbf{a}_C + \dot{\omega} \times r_{A/C} + \omega \times (\omega \times \mathbf{r}_{A/C}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{j} = (-2.758\mathbf{i} - 1.485\mathbf{j}) + (-0.012\mathbf{k}) \times (-176.78\mathbf{i} - 273.22\mathbf{j})$$
$$+ (-0.06\mathbf{k}) \times [(-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 273.22\mathbf{j})] + 2(-0.06\mathbf{k}) \times (27\mathbf{i} + 25\mathbf{j}) + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{j} = -2.4\mathbf{i} - 1.62\mathbf{j} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = [2.4\mathbf{i} - 0.38\mathbf{j}]\text{ m/s}^2 \qquad \textbf{Ans.}$$

***16–152.** At the instant shown, car B travels with a speed of 15 m/s, which is increasing at a constant rate of $2\,\text{m/s}^2$, while car C travels with a speed of 15 m/s, which is increasing at a constant rate of $3\,\text{m/s}^2$. Determine the velocity and acceleration of car B with respect to car C.



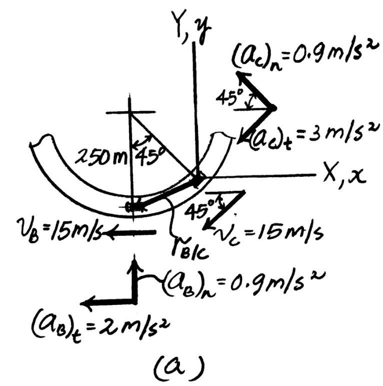
Reference Frame: The xyz rotating reference frame is attached to C and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Since B and C move along the circular road, their normal components of acceleration are $(a_B)_n = \dfrac{{v_B}^2}{\rho} = \dfrac{15^2}{250} = 0.9\ \text{m/s}^2$ and $(a_C)_n = \dfrac{{v_C}^2}{\rho} = \dfrac{15^2}{250} = 0.9\ \text{m/s}^2$. Thus, the motion of cars B and C with respect to the XYZ frame are

$$\mathbf{v}_B = [-15\mathbf{i}]\ \text{m/s}$$

$$\mathbf{v}_C = [-15\cos 45°\mathbf{i} - 15\sin 45°\mathbf{j}] = [-10.607\mathbf{i} - 10.607\mathbf{j}]\ \text{m/s}$$

$$\mathbf{a}_B = [-2\mathbf{i} + 0.9\mathbf{j}]\ \text{m/s}^2$$

$$\mathbf{a}_C = (-0.9\cos 45° - 3\cos 45°)\mathbf{i} + (0.9\sin 45° - 3\sin 45°)\mathbf{j} = [-2.758\mathbf{i} - 1.485\,\mathbf{j}]\ \text{m/s}^2$$



(a)

Also, the angular velocity and angular acceleration of the xyz reference frame with respect to the XYZ reference frame are

$$\omega = \frac{v_C}{\rho} = \frac{15}{250} = 0.06\ \text{rad/s} \qquad \omega = [-0.06\mathbf{k}]\ \text{rad/s}$$

$$\dot{\omega} = \frac{(a_C)_t}{\rho} = \frac{3}{250} = 0.012\ \text{rad/s}^2 \qquad \dot{\omega} = [-0.012\mathbf{k}]\ \text{rad/s}^2$$

From the geometry shown in Fig. a,

$$\mathbf{r}_{B/C} = -250\sin 45°\mathbf{i} - (250 - 250\cos 45°)\mathbf{j} = [-176.78\mathbf{i} - 73.22\,\mathbf{j}]\ \text{m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \omega \times r_{B/C} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$-15\mathbf{i} = (-10.607\mathbf{i} - 10.607\mathbf{j}) + (-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 73.22\mathbf{j}) + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$-15\mathbf{i} = -15\mathbf{i} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = \mathbf{0} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

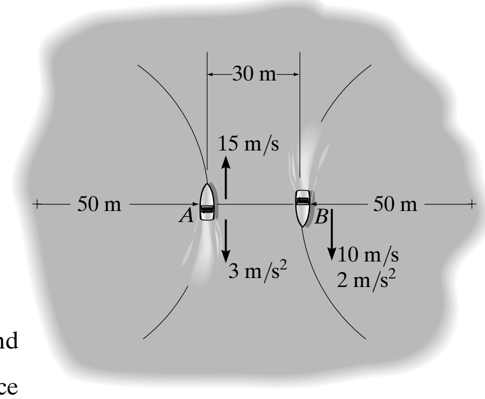
$$\mathbf{a}_B = \mathbf{a}_C + \dot{\omega} \times \mathbf{r}_{B/C} + \omega \times (\omega \times \mathbf{r}_{B/C}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{i} + 0.9\mathbf{j} = (-2.758\mathbf{i} - 1.485\mathbf{j}) + (-0.012\mathbf{k}) \times (-176.78\mathbf{i} - 73.22\mathbf{j})$$
$$+(-0.06\mathbf{k}) \times [(-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 73.22\mathbf{j})] + 2(-0.06\mathbf{k}) \times \mathbf{0} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{i} + 0.9\mathbf{j} = -3\mathbf{i} + 0.9\mathbf{j} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(a_{\text{rel}})_{xyz} = [1\mathbf{i}]\ \text{m/s}^2 \qquad \textbf{Ans.}$$

•16–153. At the instant shown, boat A travels with a speed of 15 m/s, which is decreasing at 3 m/s^2, while boat B travels with a speed of 10 m/s, which is increasing at 2 m/s^2. Determine the velocity and acceleration of boat A with respect to boat B at this instant.



Reference Frame: The xyz rotating reference frame is attached to boat B and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Since boats A and B move along the circular paths, their normal components of acceleration are $(a_A)_n = \dfrac{v_A{}^2}{\rho} = \dfrac{15^2}{50} = 4.5\text{ m/s}^2$ and $(a_B)_n = \dfrac{v_B{}^2}{\rho} = \dfrac{10^2}{50} = 2\text{ m/s}^2$. Thus, the motion of boats A and B with respect to the XYZ frame are

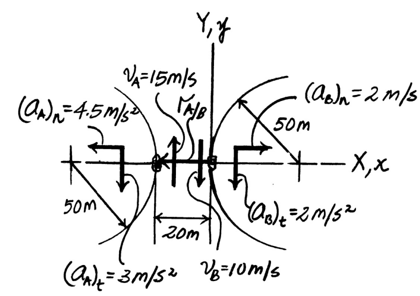
$$\mathbf{v}_A = [15\mathbf{j}]\text{ m/s} \qquad\qquad \mathbf{v}_B = [-10\mathbf{j}]\text{ m/s}$$

$$\mathbf{a}_A = [-4.5\mathbf{i} - 3\mathbf{j}]\text{ m/s}^2 \qquad\qquad \mathbf{a}_B = [2\mathbf{i} - 2\mathbf{j}]\text{ m/s}^2$$

Also, the angular velocity and angular acceleration of the xyz reference frame with respect to the XYZ reference frame are

$$\omega = \frac{v_B}{\rho} = \frac{10}{50} = 0.2\text{ rad/s} \qquad\qquad \omega = [0.2\mathbf{k}]\text{ rad/s}$$

$$\dot{\omega} = \frac{(a_B)_t}{\rho} = \frac{2}{50} = 0.04\text{ rad/s}^2 \qquad\qquad \dot{\omega} = [0.04\mathbf{k}]\text{ rad/s}^2$$



(a)

And the position of boat A with respect to B is

$$\mathbf{r}_{A/B} = [-20\mathbf{i}]\text{ m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_A = \mathbf{v}_B + \omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$15\mathbf{j} = -10\mathbf{j} + (0.2\mathbf{k}) \times (-20\mathbf{i}) + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$15\mathbf{j} = -14\mathbf{j} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = [29\mathbf{j}]\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

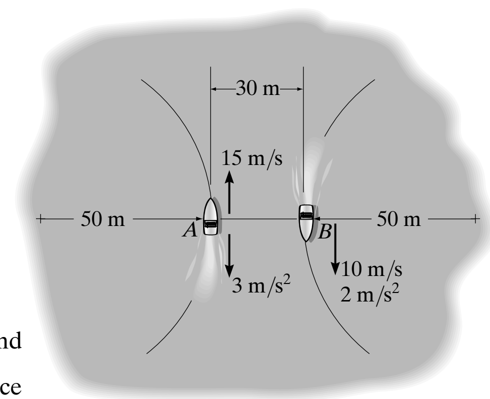
$$\mathbf{a}_A = \mathbf{a}_B + \dot{\omega} \times \mathbf{r}_{A/B} + \omega \times (\omega \times \mathbf{r}_{A/B}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(-4.5\mathbf{i} - 3\mathbf{j}) = (2\mathbf{i} - 2\mathbf{j}) + (0.04\mathbf{k}) \times (-20\mathbf{i}) + (0.2\mathbf{k}) \times \left[(0.2\mathbf{k}) \times (-20\mathbf{i})\right] + 2(0.2\mathbf{k}) \times (29\mathbf{j}) + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-4.5\mathbf{i} - 3\mathbf{j} = -8.8\mathbf{i} - 2.8\mathbf{j} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = [4.3\mathbf{i} - 0.2\mathbf{j}]\text{ m/s}^2 \qquad \textbf{Ans.}$$

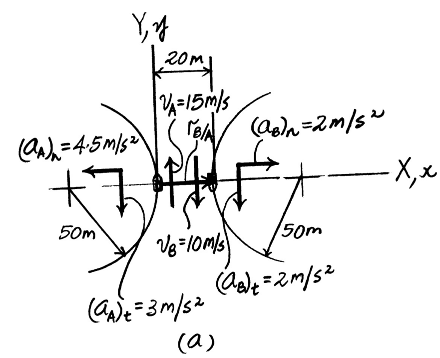
16–154. At the instant shown, boat A travels with a speed of 15 m/s, which is decreasing at 3 m/s^2, while boat B travels with a speed of 10 m/s, which is increasing at 2 m/s^2. Determine the velocity and acceleration of boat B with respect to boat A at this instant.



Reference Frame: The xyz rotating reference frame is attached to boat A and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Since boats A and B move along the circular paths, their normal components of acceleration are $(a_A)_n = \dfrac{{v_A}^2}{\rho} = \dfrac{15^2}{50} = 4.5 \text{ m/s}^2$ and $(a_B)_n = \dfrac{{v_B}^2}{\rho} = \dfrac{10^2}{50} = 2 \text{ m/s}^2$. Thus, the motion of boats A and B with respect to the XYZ frame are

$$\mathbf{v}_A = [15\mathbf{j}] \text{ m/s} \qquad \mathbf{v}_B = [-10\mathbf{j}] \text{ m/s}$$

$$\mathbf{a}_A = [-4.5\mathbf{i} - 3\mathbf{j}] \text{ m/s}^2 \qquad \mathbf{a}_B = [2\mathbf{i} - 2\mathbf{j}] \text{ m/s}^2$$



Also, the angular velocity and angular acceleration of the xyz reference frame with respect to the XYZ reference frame are

$$\omega = \frac{v_A}{\rho} = \frac{15}{50} = 0.3 \text{ rad/s} \qquad \omega = [0.3\mathbf{k}] \text{ rad/s}$$

$$\dot{\omega} = \frac{(a_A)_t}{\rho} = \frac{3}{50} = 0.06 \text{ rad/s}^2 \qquad \dot{\omega} = [-0.06\mathbf{k}] \text{ rad/s}^2$$

And the position of boat B with respect to boat A is

$$\mathbf{r}_{B/A} = [20\mathbf{i}] \text{ m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$-10\mathbf{j} = 15\mathbf{j} + (0.3\mathbf{k}) \times (20\mathbf{i}) + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$-10\mathbf{j} = 21\mathbf{j} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = [-31\mathbf{j}] \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

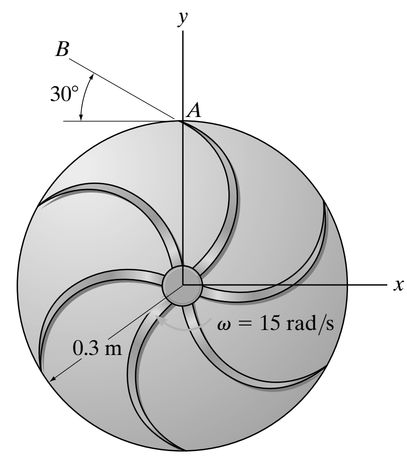
$$\mathbf{a}_B = \mathbf{a}_A + \dot{\omega} \times \mathbf{r}_{B/A} + \omega(\omega \times \mathbf{r}_{B/A}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(2\mathbf{i} - 2\mathbf{j}) = (-4.5\mathbf{i} - 3\mathbf{j}) + (-0.06\mathbf{k}) \times (20\mathbf{i}) + (0.3\mathbf{k}) \times \left[(0.3\mathbf{k}) \times (20\mathbf{i})\right] + 2(0.3\mathbf{k}) \times (-31\mathbf{j}) + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$2\mathbf{i} - 2\mathbf{j} = 12.3\mathbf{i} - 4.2\mathbf{j} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = [-10.3\mathbf{i} + 2.2\mathbf{j}] \text{ m/s}^2 \qquad \textbf{Ans.}$$

16–155. Water leaves the impeller of the centrifugal pump with a velocity of 25 m/s and acceleration of 30 m/s², both measured relative to the impeller along the blade line AB. Determine the velocity and acceleration of a water particle at A as it leaves the impeller at the instant shown. The impeller rotates with a constant angular velocity of $\omega = 15$ rad/s.



Reference Frame: The xyz rotating reference frame is attached to the impeller and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is

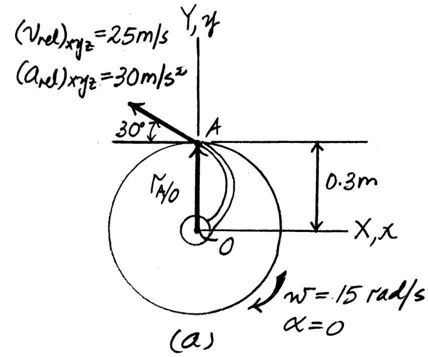
$$\mathbf{v}_O = \mathbf{a}_O = \mathbf{0} \qquad \omega = [-15\mathbf{k}] \text{ rad/s} \qquad \dot{\omega} = \mathbf{0}$$

The motion of point A with respect to the xyz frame is

$$\mathbf{r}_{A/O} = [0.3\mathbf{j}] \text{ m}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = (-25\cos 30°\,\mathbf{i} + 25\sin 30°\,\mathbf{j}) = [-21.65\mathbf{i} + 12.5\mathbf{j}] \text{ m/s}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = (-30\cos 30°\,\mathbf{i} + 30\sin 30°\,\mathbf{j}) = [-25.98\mathbf{i} + 15\mathbf{j}] \text{ m/s}^2$$



Velocity: Applying the relative velocity equation.

$$\mathbf{v}_A = \mathbf{v}_O + \omega \times \mathbf{r}_{A/O} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (-15\mathbf{k}) \times (0.3\mathbf{j}) + (-21.65\mathbf{i} + 12.5\mathbf{j})$$

$$= [-17.2\mathbf{i} + 12.5\mathbf{j}] \text{ m/s}$$ **Ans.**

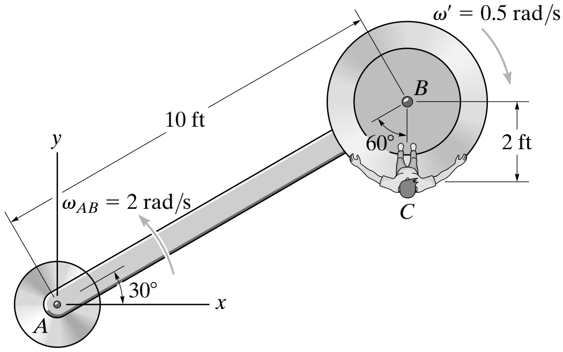
Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_O + \dot{\omega} \times \mathbf{r}_{A/O} + \omega \times (\omega \times \mathbf{r}_{A/O}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (-15\mathbf{k}) \times [(-15\mathbf{k}) \times (0.3\mathbf{j})] + 2(-15\mathbf{k}) \times (-21.65\mathbf{i} + 12.5\mathbf{j}) + (-25.98\mathbf{i} + 15\mathbf{j})$$

$$= [349\mathbf{i} + 597\mathbf{j}] \text{ m/s}^2$$ **Ans.**

***16–156.** A ride in an amusement park consists of a rotating arm AB having a constant angular velocity $\omega_{AB} = 2$ rad/s about point A and a car mounted at the end of the arm which has a constant angular velocity $\boldsymbol{\omega}' = \{-0.5\mathbf{k}\}$ rad/s, measured relative to the arm. At the instant shown, determine the velocity and acceleration of the passenger at C.



$$\mathbf{r}_{B/A} = (10\cos 30°\,\mathbf{i} + 10\sin 30°\,\mathbf{j}) = \{8.66\mathbf{i} + 5\mathbf{j}\}\text{ ft}$$

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = 2\mathbf{k} \times (8.66\mathbf{i} + 5\mathbf{j}) = \{-10.0\mathbf{i} + 17.32\mathbf{j}\}\text{ ft/s}$$

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\,\mathbf{r}_{B/A}$$

$$= 0 - (2)^2(8.66\mathbf{i} + 5\mathbf{j}) = \{-34.64\mathbf{i} - 20\mathbf{j}\}\text{ ft/s}^2$$

$$\Omega = (2 - 0.5)\mathbf{k} = 1.5\mathbf{k}$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= -10.0\mathbf{i} + 17.32\mathbf{j} + 1.5\mathbf{k} \times (-2\mathbf{j}) + 0$$

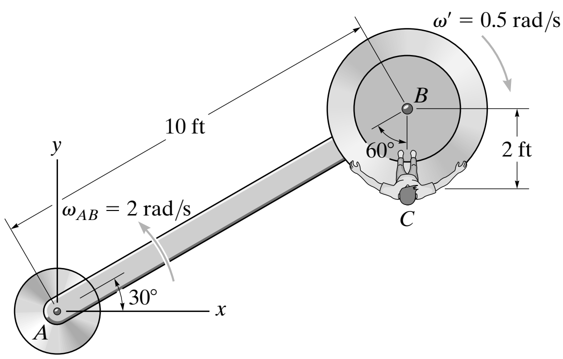
$$= \{-7.00\mathbf{i} + 17.3\mathbf{j}\}\text{ ft/s}$$ **Ans.**

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= -34.64\mathbf{i} - 20\mathbf{j} + 0 + (1.5\mathbf{k}) \times (1.5\mathbf{k}) \times (-2\mathbf{j}) + 0 + 0$$

$$= \{-34.6\mathbf{i} - 15.5\mathbf{j}\}\text{ ft/s}^2$$ **Ans.**

•16–157. A ride in an amusement park consists of a rotating arm AB that has an angular acceleration of $\alpha_{AB} = 1\text{ rad/s}^2$ when $\omega_{AB} = 2$ rad/s at the instant shown. Also at this instant the car mounted at the end of the arm has an angular acceleration of $\boldsymbol{\alpha}' = \{-0.6\mathbf{k}\}\text{ rad/s}^2$ and angular velocity of $\boldsymbol{\omega}' = \{-0.5\mathbf{k}\}$ rad/s, measured relative to the arm. Determine the velocity and acceleration of the passenger C at this instant.



$$\mathbf{r}_{B/A} = (10\cos 30°\,\mathbf{i} + 10\sin 30°\,\mathbf{j}) = \{8.66\mathbf{i} + 5\mathbf{j}\}\text{ ft}$$

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = 2\mathbf{k} \times (8.66\mathbf{i} + 5\mathbf{j}) = \{-10.0\mathbf{i} + 17.32\mathbf{j}\}\text{ ft/s}$$

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\,\mathbf{r}_{B/A}$$

$$= (1\mathbf{k}) \times (8.66\mathbf{i} + 5\mathbf{j}) - (2)^2(8.66\mathbf{i} + 5\mathbf{j}) = \{-39.64\mathbf{i} - 11.34\mathbf{j}\}\text{ ft/s}^2$$

$$\Omega = (2-0.5)\mathbf{k} = 1.5\mathbf{k}$$

$$\dot{\Omega} = (1 - 0.6)\mathbf{k} = 0.4\mathbf{k}$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= -10.0\mathbf{i} + 17.32\mathbf{j} + 1.5\mathbf{k} \times (-2\mathbf{j}) + 0$$

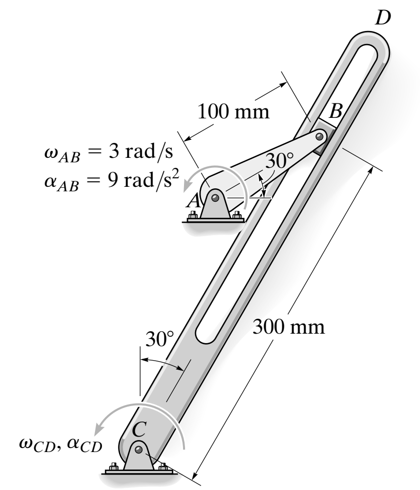
$$= \{-7.00\mathbf{i} + 17.3\mathbf{j}\}\text{ ft/s}$$ **Ans.**

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= -39.64\mathbf{i} - 11.34\mathbf{j} + (0.4\mathbf{k}) \times (-2\mathbf{j}) + (1.5\mathbf{k}) \times (1.5\mathbf{k}) \times (-2\mathbf{j}) + 0 + 0$$

$$= \{-38.8\mathbf{i} - 6.84\mathbf{j}\}\text{ ft/s}^2$$ **Ans.**

16–158. The "quick-return" mechanism consists of a crank AB, slider block B, and slotted link CD. If the crank has the angular motion shown, determine the angular motion of the slotted link at this instant.



$v_B = 3(0.1) = 0.3$ m/s

$(a_B)_t = 9(0.1) = 0.9$ m/s^2

$(a_B)_n = (3)^2\,(0.1) = 0.9$ m/s^2

$\mathbf{v}_B = \mathbf{v}_C + \Omega \times \mathbf{r}_{B/C} + (\mathbf{v}_{B/C})_{xyz}$

$0.3 \cos 60°\mathbf{i} + 0.3 \sin 60°\mathbf{j} = \mathbf{0} + (\omega_{CD}\mathbf{k}) \times (0.3\mathbf{i}) + v_{B/C}\,\mathbf{i}$

$v_{B/C} = 0.15$ m/s

$\omega_{CD} = 0.866$ rad/s ↺ **Ans.**

$\mathbf{a}_B = \mathbf{a}_C + \dot{\Omega} \times \mathbf{r}_{B/C} + \Omega \times (\Omega \times \mathbf{r}_{B/C}) + 2\Omega \times (\mathbf{v}_{B/C})_{xyz} + (\mathbf{a}_{B/C})_{xyz}$

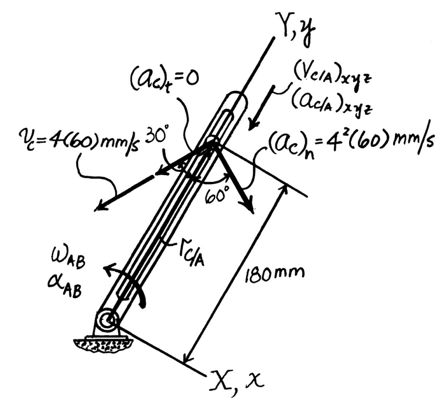
$0.9 \cos 60°\mathbf{i} - 0.9 \cos 30°\mathbf{i} + 0.9 \sin 60°\mathbf{j} + 0.9 \sin 30°\mathbf{j} = \mathbf{0} + (\alpha_{CD}\,\mathbf{k}) \times (0.3\mathbf{i})$

$+(0.866\mathbf{k}) \times (0.866\mathbf{k} \times 0.3\mathbf{i}) + 2(0.866\mathbf{k} \times 0.15\mathbf{i}) + a_{B/C}\,\mathbf{i}$

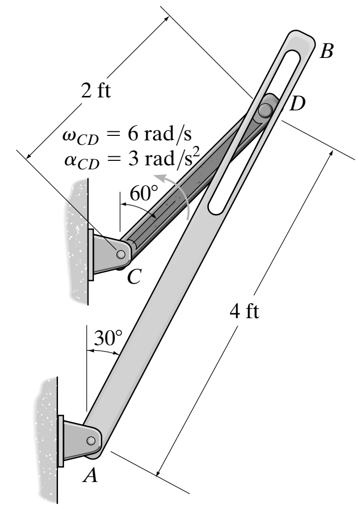
$-0.3294\mathbf{i} + 1.2294\mathbf{j} = 0.3\alpha_{CD}\,\mathbf{j} - 0.225\mathbf{i} + 0.2598\mathbf{j} + a_{B/C}\,\mathbf{i}$

$a_{B/C} = -0.104$ m/s^2

$\alpha_{CD} = 3.23$ rad/s^2 ↺ **Ans.**



16–159. The quick return mechanism consists of the crank CD and the slotted arm AB. If the crank rotates with the angular velocity and angular acceleration at the instant shown, determine the angular velocity and angular acceleration of AB at this instant.



Reference Frame: The xyz rotating reference frame is attached to slotted arm AB and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xyz reference frame with respect to the XYZ frame is

$$\mathbf{v}_A = a_A = \mathbf{0} \qquad \omega_{AB} = \omega_{AB}\mathbf{k} \qquad \dot{\omega}_{AB} = \alpha_{AB}\,\mathbf{k}$$

For the motion of point D with respect to the xyz frame, we have

$$\mathbf{r}_{D/A} = [4\mathbf{i}]\text{ ft} \qquad (\mathbf{v}_{\text{rel}})_{xyz} = (v_{\text{rel}})_{xyz}\mathbf{i} \qquad (\mathbf{a}_{\text{rel}})_{xyz} = (a_{\text{rel}})_{xyz}\,\mathbf{i}$$

Since the crank CD rotates about a fixed axis, $\mathbf{v}_D$ and $\mathbf{a}_D$ with respect to the XYZ reference frame can be determined from

$$\mathbf{v}_D = \omega_{CD} \times \mathbf{r}_D$$

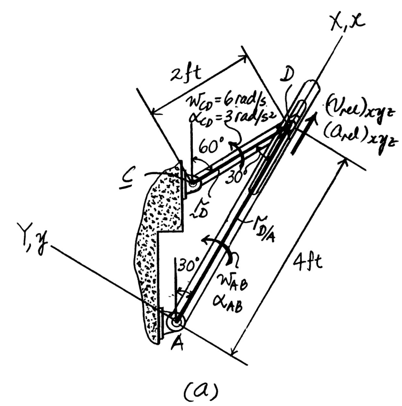
$$= (6\mathbf{k}) \times (2\cos 30°\,\mathbf{i} - 2\sin 30°\,\mathbf{j})$$

$$= [6\mathbf{i} + 10.39\mathbf{j}]\text{ ft/s}$$

$$\mathbf{a}_D = \alpha_{CD} \times \mathbf{r}_D - \omega_{CD}{}^2\mathbf{r}_D$$

$$= (3\mathbf{k}) \times (2\cos 30°\,\mathbf{i} - 2\sin 30°\,\mathbf{j}) - 6^2(2\cos 30°\,\mathbf{i} - 2\sin 30°\,\mathbf{j})$$

$$= [-59.35\mathbf{i} + 41.20\mathbf{j}]\text{ ft/s}^2$$



(a)

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_D = \mathbf{v}_A + \omega_{AB} \times r_{D/A} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$6\mathbf{i} + 10.39\,\mathbf{j} = \mathbf{0} + (\omega_{AB}\mathbf{k}) \times (4\mathbf{i}) + (v_{\text{rel}})_{xyz}\,\mathbf{i}$$

$$6\mathbf{i} + 10.39\mathbf{j} = (v_{\text{rel}})_{xyz}\,\mathbf{i} + 4\omega_{AB}\,\mathbf{j}$$

Equating the **i** and **j** components yields

$$(v_{\text{rel}})_{xyz} = 6\text{ ft/s}$$

$$10.39 = 4\omega_{AB} \qquad \omega_{AB} = 2.598\text{ rad/s} = 2.60\text{ rad/s} \quad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_D = \mathbf{a}_A + \dot{\omega}_{AB} \times \mathbf{r}_{D/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{AB}) + 2\omega_{AB} \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-59.35\mathbf{i} + 41.20\,\mathbf{j} = \mathbf{0} + (\alpha_{AB}\mathbf{k}) \times 4\mathbf{i} + 2.598\mathbf{k} \times [(2.598\mathbf{k}) \times (4\mathbf{i})] + 2(2.598\mathbf{k}) \times (6\mathbf{i}) + (\mathbf{a}_{\text{rel}})_{xyz}\,\mathbf{i}$$

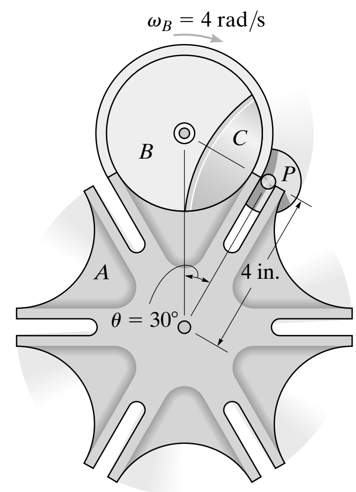
$$-59.35\mathbf{i} + 41.20\,\mathbf{j} = \left[(a_{\text{rel}})_{xyz} - 27\right]\mathbf{i} + (4\alpha_{AB} + 31.18)\mathbf{j}$$

Equating the **i** and **j** components yields

$$41.20 = 4\alpha_{AB} + 31.18$$

$$\alpha_{AB} = 2.50\text{ rad/s}^2 \qquad \textbf{Ans.}$$

***16–160.** The Geneva mechanism is used in a packaging system to convert constant angular motion into intermittent angular motion. The star wheel A makes one sixth of a revolution for each full revolution of the driving wheel B and the attached guide C. To do this, pin P, which is attached to B, slides into one of the radial slots of A, thereby turning wheel A, and then exits the slot. If B has a constant angular velocity of $\omega_B = 4$ rad/s, determine $\boldsymbol{\omega}_A$ and $\boldsymbol{\alpha}_A$ of wheel A at the instant shown.



The circular path of motion of P has a radius of

$$r_P = 4 \tan 30° = 2.309 \text{ in.}$$

Thus,

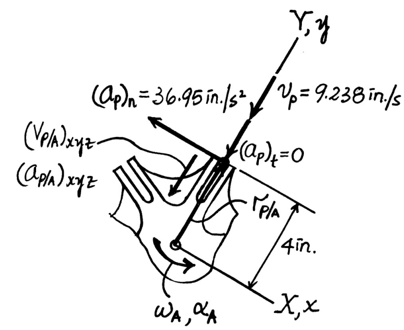
$$\mathbf{v}_P = -4(2.309)\mathbf{j} = -9.238\mathbf{j}$$

$$\mathbf{a}_P = -(4)^2(2.309)\mathbf{i} = -36.95\mathbf{i}$$

Thus,

$$\mathbf{v}_P = \mathbf{v}_A + \Omega \times \mathbf{r}_{P/A} + (\mathbf{v}_{P/A})_{xyz}$$

$$-9.238\mathbf{j} = \mathbf{0} + (\omega_A\,\mathbf{k}) \times (4\mathbf{j}) - v_{P/A}\,\mathbf{j}$$



Solving,

$$\omega_A = 0 \qquad \textbf{Ans.}$$

$$v_{P/A} = 9.238 \text{ in./s}$$

$$\mathbf{a}_P = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{P/A} + \Omega \times (\Omega \times \mathbf{r}_{P/A}) + 2\Omega \times (\mathbf{v}_{P/A})_{xyz} + (\mathbf{a}_{P/A})_{xyz}$$

$$-36.95\mathbf{i} = \mathbf{0} + (\alpha_A\mathbf{k}) \times (4\mathbf{j}) + \mathbf{0} + \mathbf{0} - a_{P/A}\,\mathbf{j}$$

Solving,

$$-36.95 = -4\alpha_A$$

$$\alpha_A = 9.24 \text{ rad/s}^2 \circlearrowleft \qquad \textbf{Ans.}$$

$$a_{P/A} = 0$$