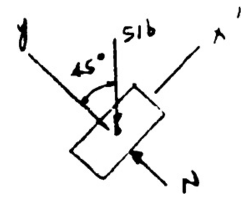
•15–1. A 5-lb block is given an initial velocity of 10 ft/s up a 45° smooth slope. Determine the time for it to travel up the slope before it stops.

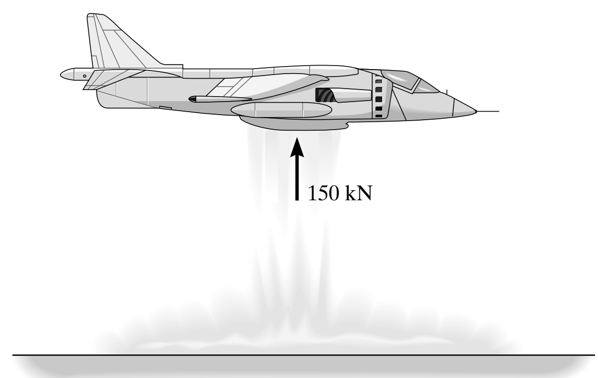
$$(\nearrow+) \qquad m(v_{x'})_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_{x'})_2$$

$$\frac{5}{32.2}(10) + (-5 \sin 45°)t = 0$$

$$t = 0.439 \text{ s}$$ **Ans.**



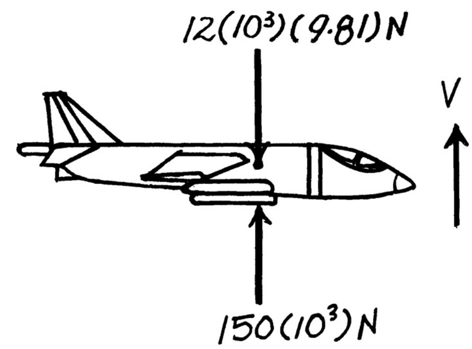
15–2. The 12-Mg "jump jet" is capable of taking off vertically from the deck of a ship. If its jets exert a constant vertical force of 150 kN on the plane, determine its velocity and how high it goes in $t = 6$ s, starting from rest. Neglect the loss of fuel during the lift.



$$(+\uparrow) \qquad m(v_y)_1 + \Sigma \int F_y\, dt = m(v_y)_2$$

$$0 + 150(10^3)(6) - 12(10^3)(9.81)(6) = 12(10^3)v$$

$$v = 16.14 \text{ m/s} = 16.1 \text{ m/s}$$ **Ans.**



$$(+\uparrow) \qquad v = v_0 + a_c t$$

$$16.14 = 0 + a(6)$$

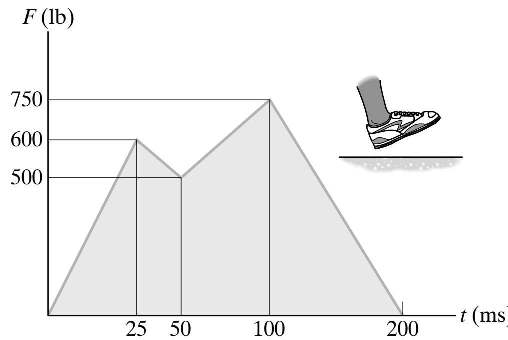
$$a = 2.690 \text{ m/s}^2$$

$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s = 0 + 0 + \frac{1}{2}(2.690)(6)^2$$

$$s = 48.4 \text{ m}$$ **Ans.**

15–3. The graph shows the vertical reactive force of the shoe-ground interaction as a function of time. The first peak acts on the heel, and the second peak acts on the forefoot. Determine the total impulse acting on the shoe during the interaction.



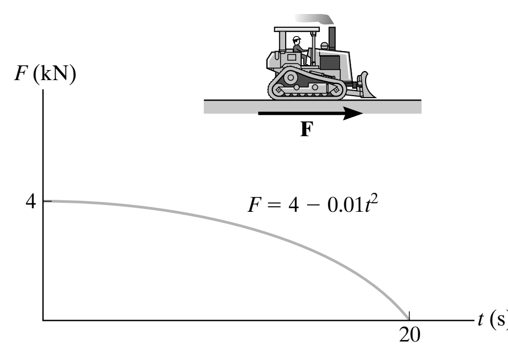
Impulse: The total impluse acting on the shoe can be obtained by evaluating the area under the $F-t$ graph.

$$I = \frac{1}{2}(600)\left[25\left(10^{-3}\right)\right] + \frac{1}{2}(500 + 600)(50 - 25)\left(10^{-3}\right)$$

$$+ \frac{1}{2}(500 + 750)(100 - 50)\left(10^{-3}\right) + \frac{1}{2}(750)\left[(200 - 100)\left(10^{-3}\right)\right]$$

$$= 90.0 \text{ lb} \cdot \text{s}$$ **Ans.**

***15–4.** The 28-Mg bulldozer is originally at rest. Determine its speed when $t = 4$ s if the horizontal traction **F** varies with time as shown in the graph.

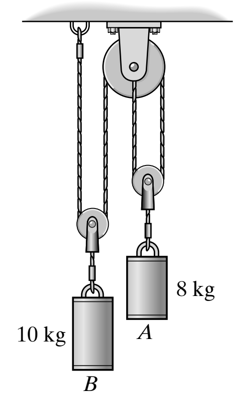


$$\left(\xrightarrow{+}\right) \qquad m(v_x)_1 + \Sigma\int_{t_1}^{-t_2} F_x\, dt = m(v_x)_2$$

$$0 + \int_0^4 (4 - 0.01t^2)(10^3)dt = 28(10^3)v$$

$$v = 0.564 \text{ m/s}$$ **Ans.**

•15–5. If cylinder A is given an initial downward speed of 2 m/s, determine the speed of each cylinder when $t = 3$ s. Neglect the mass of the pulleys.



Free-Body Diagram: The free-body diagram of blocks A and B are shown in Figs. b and c, respectively. Here, the final velocity of blocks A and B, $(\mathbf{v}_A)_2$ and $(\mathbf{v}_B)_2$ must be assumed to be directed downward so that they are consistent with the positive sense of s_A and s_B shown in Fig. a.

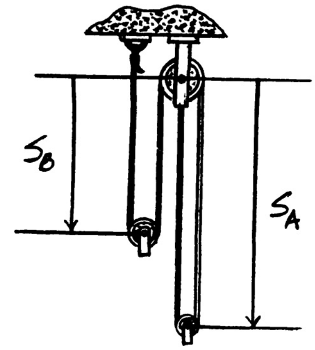
Kinematics: Expressing the length of the cable in terms of s_A and s_B by referring to Fig. a,

$$2s_A + 2s_B = l$$

$$s_A + s_B = l/2 \qquad \textbf{(1)}$$

Taking the time derivative of Eq. (1), we obtain

$$(+\downarrow) \qquad v_A + v_B = 0 \qquad \textbf{(2)}$$



(a)

Principle of Impulse and Momentum: Initially, the velocity of block A is directed downward. Thus, $(v_A)_1 = 2\text{ m/s}\downarrow$.

From Eq. (2),

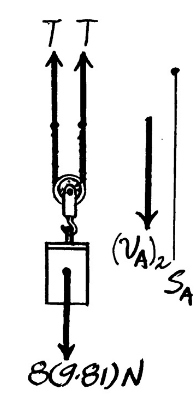
$$(+\downarrow) \qquad 2 + (v_B)_1 = 0 \qquad (v_B)_1 = -2\text{ m/s} = 2\text{ m/s}\uparrow$$

By referring to Fig. b,

$$(+\uparrow) \qquad m(v_A)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_A)_2$$

$$8(-2) + 2T(3) - 8(9.81)(3) = 8\left[-(v_A)_2\right]$$

$$6T = 251.44 - 8(v_A)_2 \qquad \textbf{(3)}$$



(b)

By referring Fig. c,

$$(+\uparrow) \qquad m(v_B)_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_B)_2$$

$$10(2) + 2T(3) - 10(9.81)(3) = 10\left[-(v_B)_2\right]$$

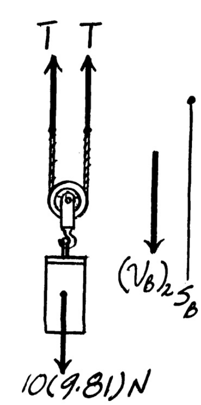
$$6T = 274.3 - 10(v_B)_2 \qquad \textbf{(4)}$$

Solving Eqs. (2), (3), and (4),

$$(v_A)_2 = -1.27\text{ m/s} = 1.27\text{ m/s}\uparrow \qquad \textbf{Ans.}$$

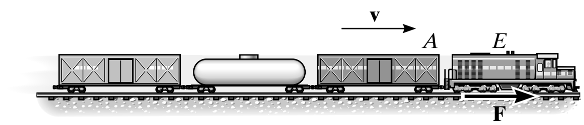
$$(v_B)_2 = 1.27\text{ m/s}\downarrow \qquad \textbf{Ans.}$$

$$T = 43.6\text{ N}$$



(c)

15–6. A train consists of a 50-Mg engine and three cars, each having a mass of 30 Mg. If it takes 80 s for the train to increase its speed uniformly to 40 km/h, starting from rest, determine the force T developed at the coupling between the engine E and the first car A. The wheels of the engine provide a resultant frictional tractive force $\mathbf{F}$ which gives the train forward motion, whereas the car wheels roll freely. Also, determine F acting on the engine wheels.



$(v_x)_2 = 40 \text{ km/h} = 11.11 \text{ m/s}$

Entire train:

$$\left(\xrightarrow{+}\right) \qquad m(v_x)_1 + \Sigma \int F_x \, dt = m(v_x)_2$$

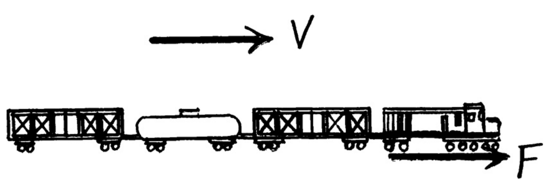
$$0 + F(80) = [50 + 3(30)]\left(10^3\right)(11.11)$$

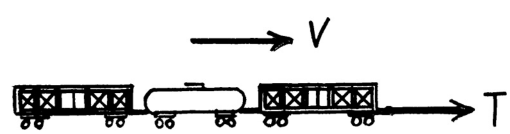
$$F = 19.4 \text{ kN} \qquad \textbf{Ans.}$$

Three cars:

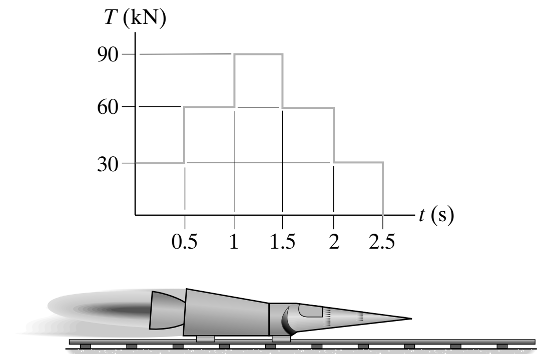
$$\left(\xrightarrow{+}\right) \qquad m(v_x)_1 + \Sigma \int F_x \, dt = m(v_x)_2$$

$$0 + T(80) = 3(30)\left(10^3\right)(11.11) \qquad T = 12.5 \text{ kN} \qquad \textbf{Ans.}$$





15–7. Determine the maximum speed attained by the 1.5-Mg rocket sled if the rockets provide the thrust shown in the graph. Initially, the sled is at rest. Neglect friction and the loss of mass due to fuel consumption.



Principle of Impulse and Momentum: The graph of thrust **T** vs. time t due to the successive ignition of the rocket is shown in Fig. *a*. The sled attains its maximum speed at the instant that all the rockets burn out their fuel, that is, at $t = 2.5$ s. The impulse generated by **T** during $0 \leq t \leq 2.5$ s is equal to the area under the T vs t graphs. Thus,

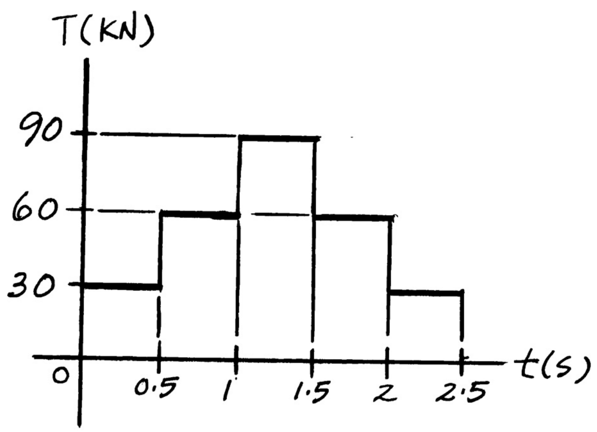
$$I = \int T\,dt = 30(10^3)(0.5 - 0) + 60(10^3)(1 - 0.5) + 90(10^3)(1.5 - 1)$$
$$+\ 60(10^3)(2 - 1.5) + 30(10^3)(25 - 2) = 135\,000 \text{ N} \cdot \text{s}$$

By referring to the free-body diagram of the sled shown in Fig. *a*,

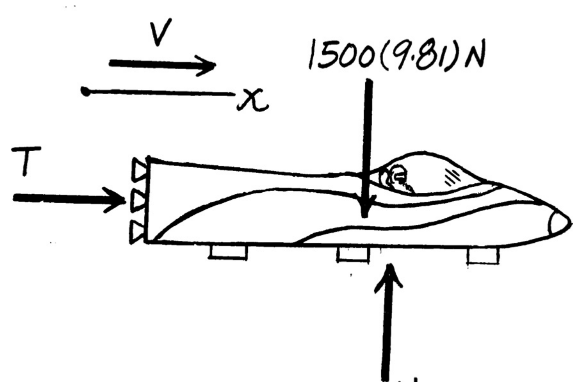
$$\left(\overset{+}{\rightarrow}\right) \qquad m(v_1)_x + \Sigma \int F_x dt = m(v_2)_x$$

$$1500(0) + 135000 = 1500 v_{\text{max}}$$

$$v_{\text{max}} = 90 \text{ m/s} \qquad \textbf{Ans.}$$

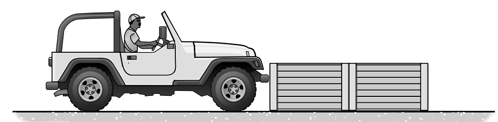


(a)

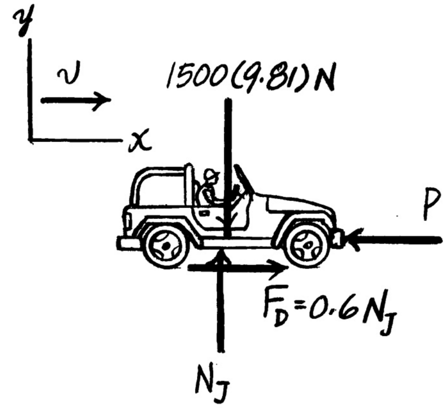


(b)

***15–8.** The 1.5-Mg four-wheel-drive jeep is used to push two identical crates, each having a mass of 500 kg. If the coefficient of static friction between the tires and the ground is $\mu_s = 0.6$, determine the maximum possible speed the jeep can achieve in 5 s without causing the tires to slip. The coefficient of kinetic friction between the crates and the ground is $\mu_k = 0.3$.



Free-Body Diagram: The free-body diagram of the jeep and crates are shown in Figs. *a* and *b*, respectively. Here, the maximum driving force for the jeep is equal to the maximum static friction between the tires and the ground, i.e., $F_D = \mu_s N_J = 0.6N_J$. The frictional force acting on the crate is $\left(F_f\right)_C = \mu_k N_C = 0.3N_C$.



(a)

Principle of Impulse and Momentum: By referring to Fig. *a*,

$$(+\uparrow) \qquad m\left(v_1\right)_y + \Sigma\int_{t_1}^{t_2} F_y dt = m\left(v_2\right)_y$$

$$1500(0) + N_J(5) - 1500(9.81)(5) = 1500(0)$$

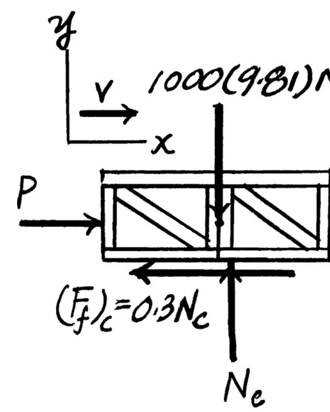
$$N_J = 14715 \text{ N}$$

$$(\xrightarrow{+}) \qquad m\left(v_1\right)_x + \Sigma\int_{t_1}^{t_2} F_x dt = m\left(v_2\right)_x$$

$$1500(0) + 0.6(14715)(5) - P(5) = 1500v$$

$$v = 29.43 - 3.333(10^{-3})P \qquad \textbf{(1)}$$

By considering Fig. *b*,



(b)

$$(+\uparrow) \qquad m\left(v_1\right)_y + \Sigma\int_{t_1}^{t_2} F_y dt = m\left(v_2\right)_y$$

$$1000(0) + N_C(5) - 1000(9.81)(5) = 1000(0)$$

$$N_C = 9810 \text{ N}$$

$$(\xrightarrow{+}) \qquad m\left(v_1\right)_x + \Sigma\int_{t_1}^{t_2} F_x dt = m\left(v_2\right)_x$$

$$1000(0) + P(5) - 0.3(9810)(5) = 1000v$$

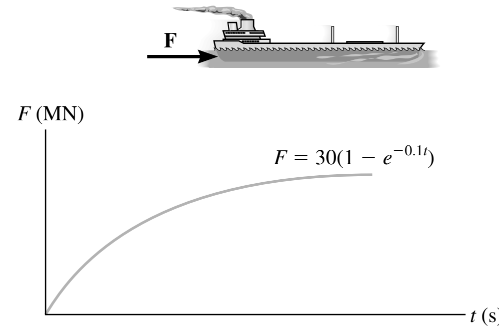
$$v = 0.005P - 14.715 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v = 11.772 \text{ m/s} = 11.8 \text{ m/s} \qquad \textbf{Ans.}$$

$$P = 5297.4 \text{ N}$$

•15–9. The tanker has a mass of 130 Gg. If it is originally at rest, determine its speed when $t = 10$ s. The horizontal thrust provided by its propeller varies with time as shown in the graph. Neglect the effect of water resistance.

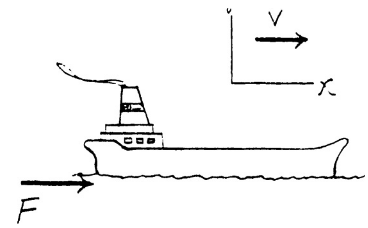


Principle of Linear Impulse and Momentum: Applying Eq. 15–4, we have

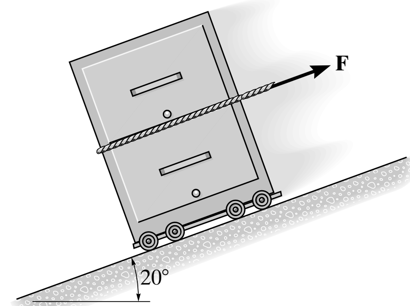
$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$(\overset{+}{\rightarrow}) \qquad 0 + \int_0^{10s} 30\left(10^6\right)\left(1 - e^{-0.1t}\right)dt = 0.130\left(10^9\right) v$$

$$v = 0.849 \text{ m/s} \qquad \textbf{Ans.}$$



15–10. The 20-lb cabinet is subjected to the force $F = (3 + 2t)$ lb, where t is in seconds. If the cabinet is initially moving down the plane with a speed of 6 ft/s, determine how long for the force to bring the cabinet to rest. **F** always acts parallel to the plane.



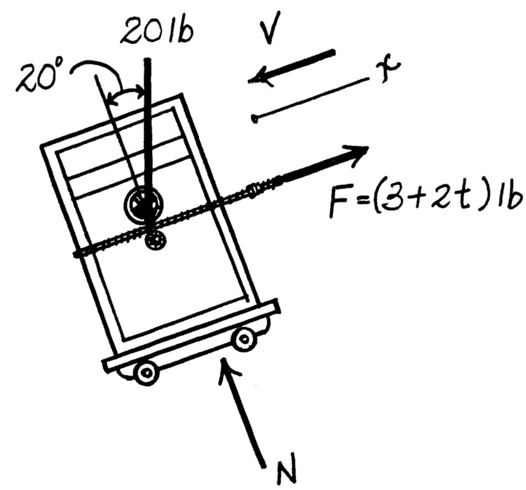
$$(+\swarrow) \qquad m(v_x)_1 + \Sigma \int F_x\,dt = m(v_x)_2$$

$$\left(\frac{20}{32.2}\right)(6) + 20(\sin 20°)t - \int_0^t (3 + 2t)\,dt = 0$$

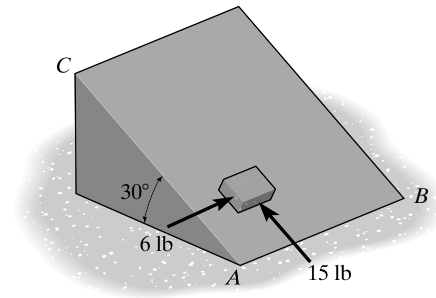
$$3.727 + 3.840t - t^2 = 0$$

Solving for the positive root,

$t = 4.64$ s **Ans.**



15–11. The small 20-lb block is placed on the inclined plane and subjected to 6-lb and 15-lb forces that act parallel with edges AB and AC, respectively. If the block is initially at rest, determine its speed when $t = 3$ s. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.2$.



Free-Body Diagram: Here, the x–y plane is set parallel with the inclined plane. Thus, the z axis is perpendicular to the inclined plane. The frictional force will act along but in the opposite sense to that of the motion, which makes an angle θ with the x axis. Its magnitude is $F_f = \mu_k N = 0.2N$.

Principle of Impulse and Momentum: By referring to Fig. a,

$$m(v_1)_z + \Sigma \int_{t_1}^{t_2} F_z\, dt = m(v_2)_z$$

$$\frac{20}{32.2}(0) + N(3) - 20\cos 30°(3) = \frac{20}{32.2}(0)$$

$$N = 17.32 \text{ lb}$$

and

$$m(v_1)_x + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_2)_x$$

$$\frac{20}{32.2}(0) + 6(3) - \left[0.2(17.32)\cos\theta\right](3) = \; = \frac{20}{32.2}(v\cos\theta)$$

$$\cos\theta(v + 16.73) = 28.98 \qquad \textbf{(1)}$$

and

$$m(v_1)_y + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_2)_y$$

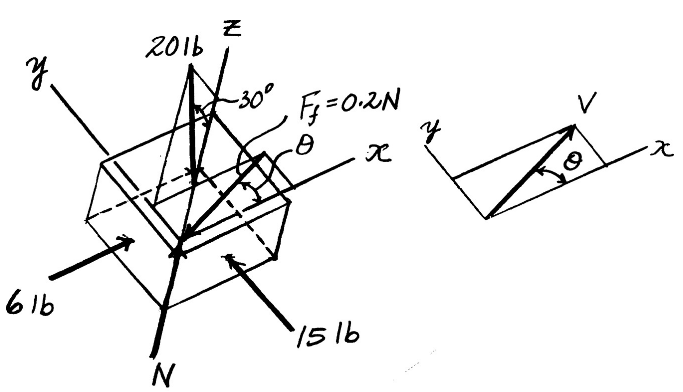
$$\frac{20}{32.2}(0) + 15(3) - (20\sin 30°)(3) - \left[0.2(17.32)\sin\theta\right](3) = \frac{20}{32.2}(v\sin\theta)$$

$$\sin\theta(v + 16.73) = 24.15 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

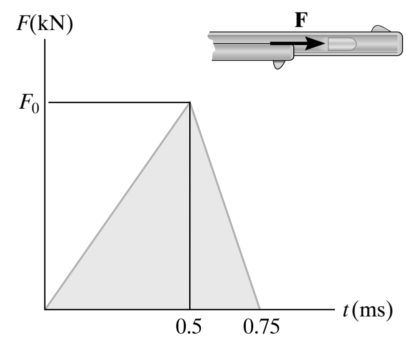
$$\theta = 39.80°$$

$$v = 20.99 \text{ ft/s} = 21.0 \text{ ft/s} \qquad \textbf{Ans.}$$

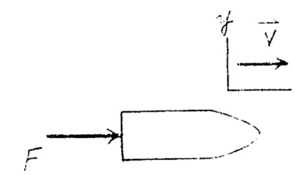


(a)

***15–12.** Assuming that the force acting on a 2-g bullet, as it passes horizontally through the barrel of a rifle, varies with time in the manner shown, determine the maximum net force F_0 applied to the bullet when it is fired. The muzzle velocity is 500 m/s when $t = 0.75$ ms. Neglect friction between the bullet and the rifle barrel.



Principle of Linear Impulse and Momentum: The total impluse acting on the bullet can be obtained by evaluating the area under the F–t graph. Thus, $I = \Sigma \int_{t_1}^{t_2} F_x\,dt = \frac{1}{2}(F_0)\left[0.5\left(10^{-3}\right)\right] + \frac{1}{2}(F_0)\left[(0.75 - 0.5)\left(10^{-3}\right)\right]$ $= 0.375\left(10^{-3}\right) F_0$. Applying Eq. 15–4, we have

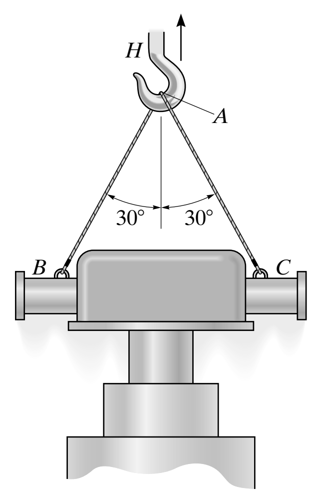


$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$(\overset{+}{\rightarrow}) \qquad 0 + 0.375\left(10^{-3}\right) F_0 = 2\left(10^{-3}\right)(500)$$

$$F_0 = 2666.67 \text{ N} = 2.67 \text{ kN} \qquad \textbf{Ans.}$$

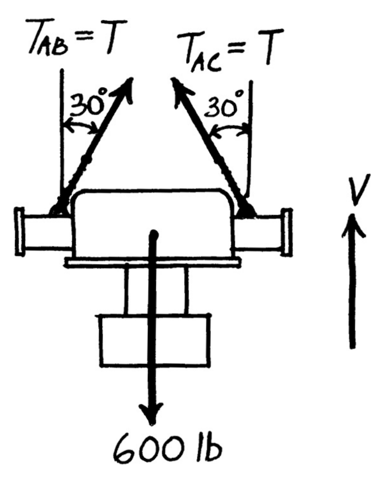
•15–13. The fuel-element assembly of a nuclear reactor has a weight of 600 lb. Suspended in a vertical position from H and initially at rest, it is given an upward speed of 5 ft/s in 0.3 s. Determine the average tension in cables AB and AC during this time interval.



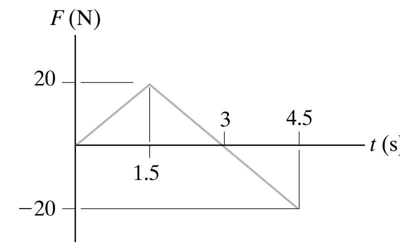
$$(+\uparrow) \qquad m(v_y)_1 + \Sigma \int F_y\,dt = m(v_y)_2$$

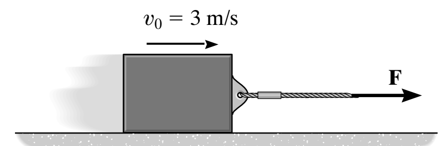
$$0 + 2(T\cos 30°)(0.3) - 600(0.3) = \left(\frac{600}{32.2}\right)(5)$$

$$T = 526 \text{ lb} \qquad \textbf{Ans.}$$



15–14. The 10-kg smooth block moves to the right with a velocity of $v_0 = 3$ m/s when force **F** is applied. If the force varies as shown in the graph, determine the velocity of the block when $t = 4.5$ s.



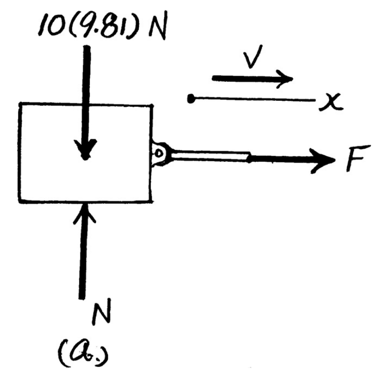


Principle of Impulse and Momentum: The impulse generated by force **F** during $0 \le t \le 4.5$ is equal to the area under the **F** vs. t graph, i.e., $I = \int F dt = \frac{1}{2}(20)(3-0) + \left[-\frac{1}{2}(20)(4.5-3)\right] = 15\ \text{N}\cdot\text{s}$. Referring to the free-body diagram of the block shown in Fig. a,

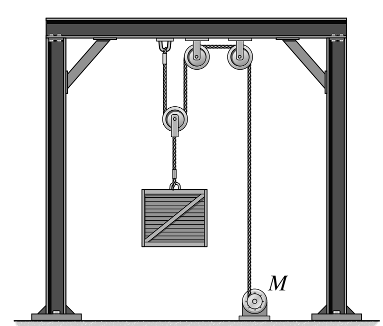
$$\left(\xrightarrow{+}\right) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x\, dt = m(v_2)_x$$

$$10(3) + 15 = 10v$$

$$v = 4.50 \text{ m/s} \qquad \textbf{Ans.}$$



15–15. The 100-kg crate is hoisted by the motor M. If the velocity of the crate increases uniformly from 1.5 m/s to 4.5 m/s in 5 s, determine the tension developed in the cable during the motion.

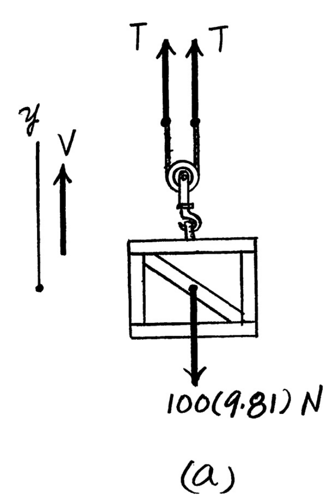


Principle of Impulse and Momentum: By referring to the free-body diagram of the crate shown in Fig. a,

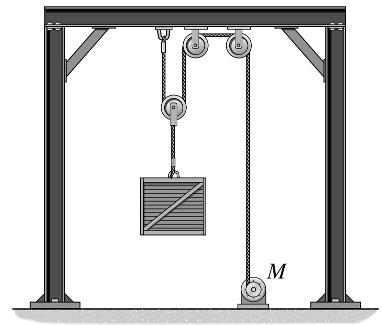
$$\left(+\uparrow\right) \qquad m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$100(1.5) + 2T(5) - 100(9.81)(5) = 100(4.5)$$

$$T = 520.5 \text{ N} \qquad \textbf{Ans.}$$



***15–16.** The 100-kg crate is hoisted by the motor M. The motor exerts a force on the cable of $T = (200t^{1/2} + 150)$ N, where t is in seconds. If the crate starts from rest at the ground, determine the speed of the crate when $t = 5$ s.



Free-Body Diagram: Here, force 2**T** must overcome the weight of the crate before it moves. By considering the equilibrium of the free-body diagram of the crate shown in Fig. *a*,

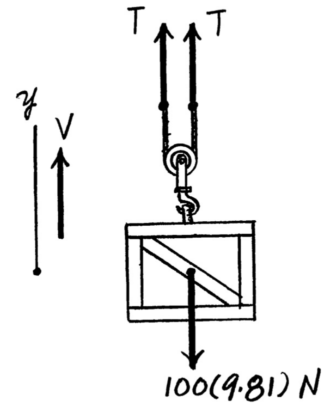
$$+\uparrow \Sigma F_y = 0; \qquad 2\left(200t^{1/2} + 150\right) - 100(9.81) = 0 \qquad t = 2.8985 \text{ s}$$

Principle of Impulse and Momentum: Here, only the impulse generated by force 2**T** after $t = 2.8186$ s contributes to the motion. Referring to Fig. *a*,

$$(+\uparrow) \qquad m\left(v_1\right)_y + \Sigma \int_{t_1}^{t_2} F_y\, dt = m\left(v_2\right)_y$$

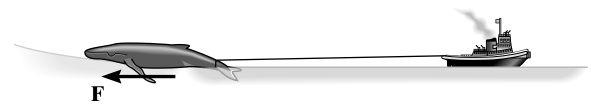
$$100(0) + 2\int_{2.898\text{ s}}^{5\text{s}} \left(200t^{1/2} + 150\right) dt - 100(9.81)(5 - 2.8985) = 100v$$

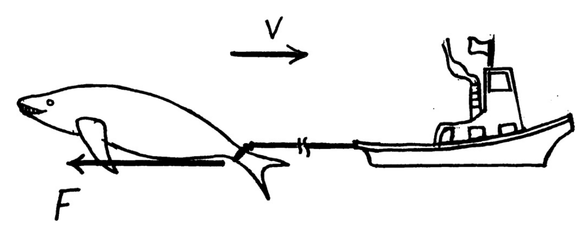
$$v = 2.34 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

•15–17. The 5.5-Mg humpback whale is stuck on the shore due to changes in the tide. In an effort to rescue the whale, a 12-Mg tugboat is used to pull it free using an inextensible rope tied to its tail. To overcome the frictional force of the sand on the whale, the tug backs up so that the rope becomes slack and then the tug proceeds forward at 3 m/s. If the tug then turns the engines off, determine the average frictional force **F** on the whale if sliding occurs for 1.5 s before the tug stops after the rope becomes taut. Also, what is the average force on the rope during the tow?

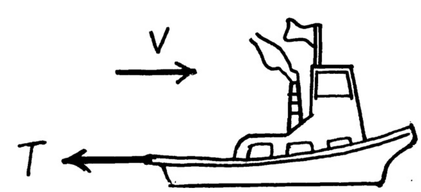




$$(\overset{+}{\rightarrow}) \qquad m_1 (v_x)_1 + \Sigma \int F_x\, dt = m_2(v_x)_2$$

$$0 + 12\left(10^3\right)(3) - F(1.5) = 0 + 0$$

$$F = 24 \text{ kN} \qquad \textbf{Ans.}$$



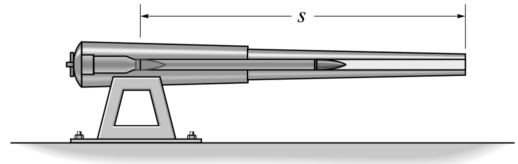
Tug:

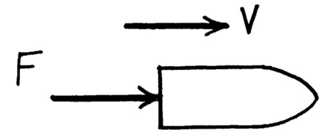
$$(\overset{+}{\rightarrow}) \qquad m\,(v_x)_1 + \Sigma \int F_x\, dt = m\,(v_x)_2$$

$$12\left(10^3\right)(3) - T(1.5) = 0$$

$$T = 24 \text{ kN} \qquad \textbf{Ans.}$$

15–18. The force acting on a projectile having a mass m as it passes horizontally through the barrel of the cannon is $F = C \sin(\pi t/t')$. Determine the projectile's velocity when $t = t'$. If the projectile reaches the end of the barrel at this instant, determine the length s.





$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \Sigma \int F_x\,dt = m(v_x)_2$$

$$0 + \int_0^t C \sin\left(\frac{\pi t}{t'}\right) = \mathrm{m}v$$

$$-C\left(\frac{t'}{\pi}\right)\cos\left(\frac{\pi t}{t'}\right)\Bigg|_0^{\mathrm{t}} = \mathrm{m}v$$

$$v = \frac{Ct'}{\pi m}\left(1 - \cos\left(\frac{\pi t}{t'}\right)\right)$$

When $t = t'$,

$$v_2 = \frac{2C\,t'}{\pi m} \qquad \text{`}\textbf{Ans.}$$

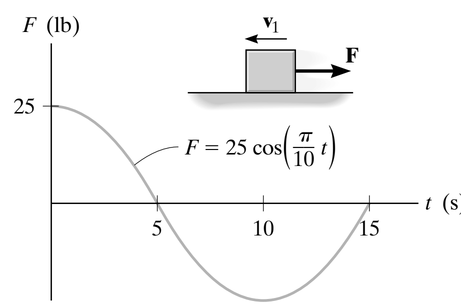
$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \left(\frac{C\,t'}{\pi\,m}\right)\left(1 - \cos\left(\frac{\pi t}{t'}\right)\right)\mathrm{dt}$$

$$s = \left(\frac{C\,t'}{\pi m}\right)\left[t - \frac{t'}{\pi}\sin\left(\frac{\pi t}{t'}\right)\right]_0^{t'}$$

$$s = \frac{Ct'^2}{\pi m} \qquad \textbf{Ans.}$$

15–19. A 30-lb block is initially moving along a smooth horizontal surface with a speed of $v_1 = 6$ ft/s to the left. If it is acted upon by a force **F**, which varies in the manner shown, determine the velocity of the block in 15 s.



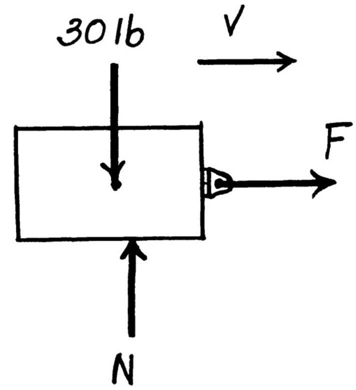
$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \Sigma \int F_x\,dt = m(v_x)_2$$

$$-\left(\frac{30}{32.2}\right)(6) + \int_0^{15} 25\cos\left(\frac{\pi}{10}t\right)dt = \left(\frac{30}{32.2}\right)(v_x)_2$$

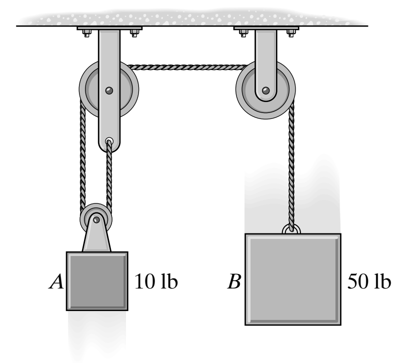
$$-5.59 + (25)\left[\sin\left(\frac{\pi}{10}t\right)\right]_0^{15}\left(\frac{10}{\pi}\right) = \left(\frac{30}{32.2}\right)(v_x)_2$$

$$-5.59 + (25)[-1]\left(\frac{10}{\pi}\right) = \left(\frac{30}{32.2}\right)(v_x)_2$$

$$(v_x)_2 = -91.4 = 91.4 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$



***15–20.** Determine the velocity of each block 2 s after the blocks are released from rest. Neglect the mass of the pulleys and cord.



Kinematics: The speed of block A and B can be related by using the position coordinate equation.

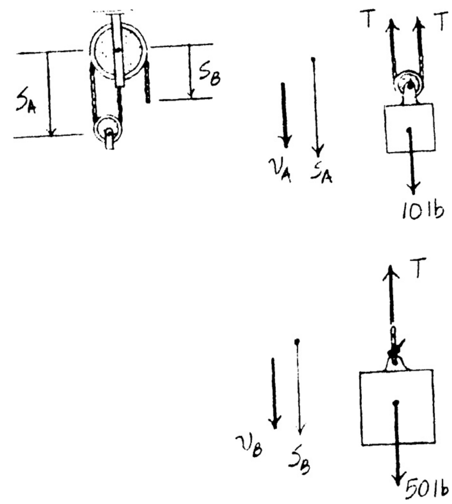
$$2s_A + s_B = l$$

$$2v_A + v_B = 0 \qquad \textbf{[1]}$$

Principle of Linear Impulse and Momentum: Applying Eq. 15–4 to block A, we have

$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$$(+\uparrow) \qquad -\left(\frac{10}{32.2}\right)(0) + 2T(2) - 10(2) = -\left(\frac{10}{32.2}\right)(v_A) \qquad \textbf{[2]}$$



Applying Eq. 15–4 to block B, we have

$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

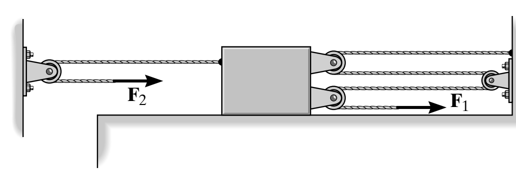
$$(+\uparrow) \qquad -\left(\frac{50}{32.2}\right)(0) + T(2) - 50(2) = -\left(\frac{50}{32.2}\right)(v_B) \qquad \textbf{[3]}$$

Solving Eqs. [1], [2] and [3] yields

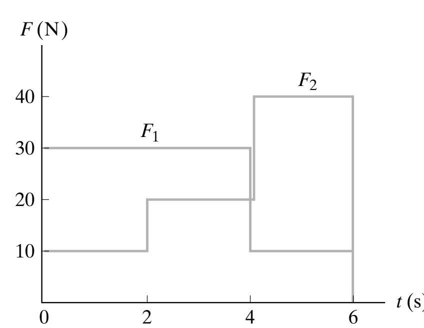
$$v_A = -27.6 \text{ ft/s} = 27.6 \text{ ft/s} \uparrow \qquad v_B = 55.2 \text{ ft/s} \downarrow \qquad \textbf{Ans.}$$

$$T = 7.143 \text{ lb}$$

•15–21. The 40-kg slider block is moving to the right with a speed of 1.5 m/s when it is acted upon by the forces $\mathbf{F}_1$ and $\mathbf{F}_2$. If these loadings vary in the manner shown on the graph, determine the speed of the block at $t = 6$ s. Neglect friction and the mass of the pulleys and cords.



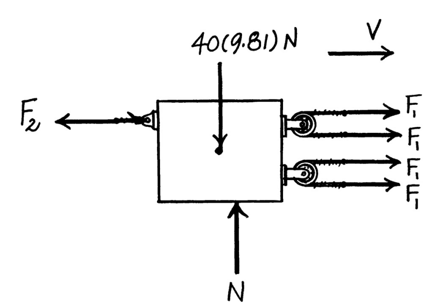
The impulses acting on the block are equal to the areas under the graph.



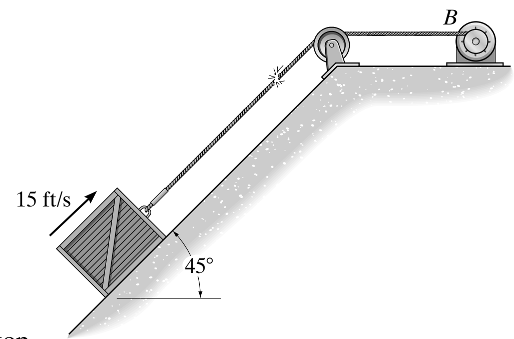
$$\left(\xrightarrow{+}\right) \qquad m(v_x)_1 + \Sigma \int F_x\,dt = m(v_x)_2$$

$$40(1.5) + 4[(30)4 + 10(6-4)] - [10(2) + 20(4-2) + 40(6-4)] = 40v_2$$

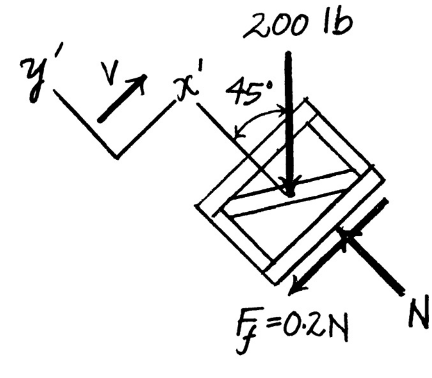
$$v_2 = 12.0 \text{ m/s} (\rightarrow) \qquad \textbf{Ans.}$$



15–22. At the instant the cable fails, the 200-lb crate is traveling up the plane with a speed of 15 ft/s. Determine the speed of the crate 2 s afterward. The coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.20$.



Free-Body Diagram: When the cable snaps, the crate will slide up the plane, stop, and then slide down the plane. The free-body diagram of the crate in both cases are shown in Figs. *a* and *b*. The frictional force acting on the crate in both cases can be computed from $F_f = \mu_k N = 0.2N$.



(a)

Principle of Impulse and Momentum: By referring to Fig. *a*,

$$+\nwarrow \quad m(v_1)_{y'} + \Sigma \int_{t_1}^{t_2} F_{y'}\, dt = m(v_2)_{y'}$$

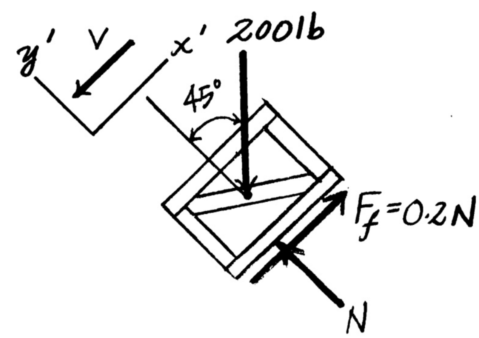
$$\frac{200}{32.2}(0) + N(t') - 200 \cos 45°(t') = \ = \frac{200}{32.2}(0)$$

$$N = 141.42 \text{ lb}$$

$$+\nearrow \quad m(v_1)_{x'} + \Sigma \int_{t_1}^{t_2} F_{x'}\, dt = m(v_2)_{x'}$$

$$\frac{200}{32.2}(15) - 200 \sin 45°(t') - 0.2(141.42)(t') = \frac{200}{32.2}(0)$$

$$t' = 0.5490 \text{ s}$$



(b)

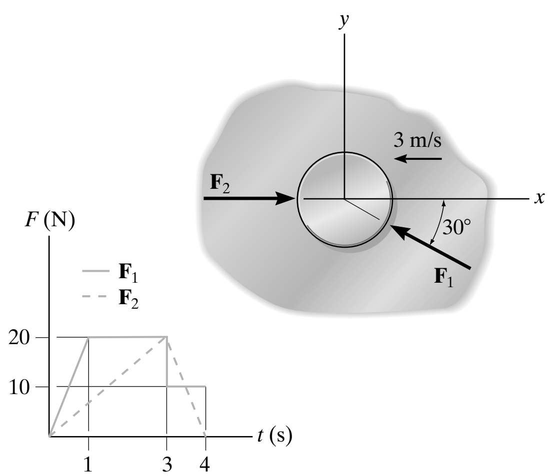
Thus, the time the crate takes to slide down the plane is $t'' = 2 - 0.5490 = 1.451$ s. Here, $N = 141.42$ for both cases. By referring to Fig. *b*,

$$+\nearrow \quad m(v_1)_{x'} + \Sigma \int_{t_1}^{t_2} F_{x'}\, dt = m(v_2)_{x'}$$

$$\frac{200}{32.2}(0) + 0.2(141.42)(1.451) - 200 \sin 45°(1.451) = \frac{200}{32.2}(-v)$$

$$v = 26.4 \text{ ft/s} \qquad \textbf{Ans.}$$

15–23. Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ vary as shown by the graph. The 5-kg smooth disk is traveling to the left with a speed of 3 m/s when $t = 0$. Determine the magnitude and direction of the disk's velocity when $t = 4$ s.



Principle of Impulse and Momentum: The impulse generated by $\mathbf{F}_1$ and $\mathbf{F}_2$ during the time period $0 \le t \le 4$ s is equal to the area under the F_1 vs t and F_2 vs t graphs, i.e., $I_1 = \frac{1}{2}(20)(1) + 20(3-1) + 10(4-3) = 60\text{N}\cdot\text{s}$ and $I_2 = \frac{1}{2}(20)(3-0) + \frac{1}{2}(20)(4-3) = 40\ \text{N}\cdot\text{s}$. By referring to the impulse and momentum diagram shown in Fig. *a*

$$\left(\xrightarrow{+}\right) \qquad m(v_1)_x + \Sigma \int_{t_1}^{t^2} F_x\,dt = m(v_2)_x$$

$$-5(3) + 40 - 60\cos 30^\circ = 5v_x$$

$$v_x = -5.392\text{ m/s} = 5.392\text{ m/s} \leftarrow$$

$$\left(+\uparrow\right) \qquad m(v_1)_y + \Sigma \int_{t_1}^{t_2} F_y\,dt = m(v_2)_y$$

$$0 + 60\sin 30^\circ = 5v_y$$

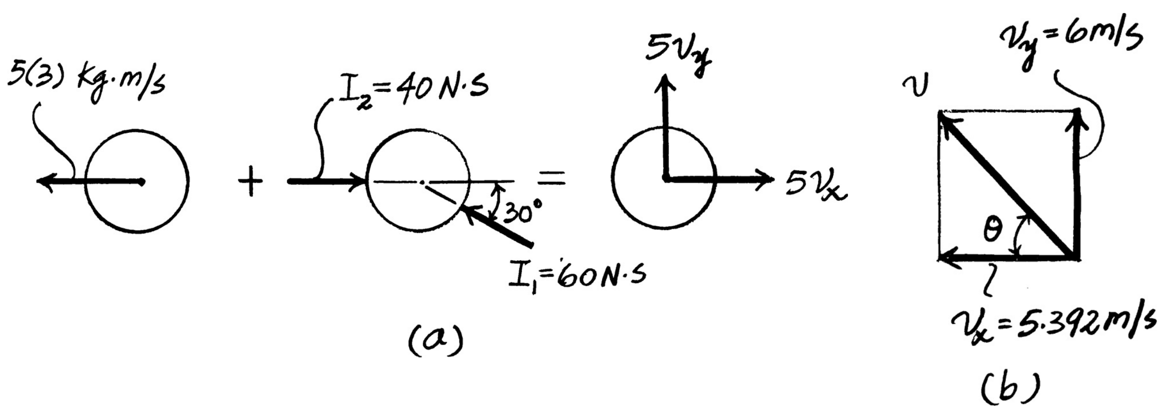
$$v_y = 6\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}$,

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{5.392^2 + 6^2} = 8.07\text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ makes with the horizontal is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{6}{5.392}\right) = 48.1^\circ \qquad \textbf{Ans.}$$



***15–24.** A 0.5-kg particle is acted upon by the force $\mathbf{F} = \{2t^2\mathbf{i} - (3t + 3)\mathbf{j} + (10 - t^2)\mathbf{k}\}$ N, where t is in seconds. If the particle has an initial velocity of $\mathbf{v}_0 = \{5\mathbf{i} + 10\mathbf{j} + 20\mathbf{k}\}$ m/s, determine the magnitude of the velocity of the particle when $t = 3$ s.

Principle of Impulse and Momentum:

$$m\mathbf{v}_1 + \Sigma \int_{t_1}^{t_2} \mathbf{F}dt = m\mathbf{v}_2$$

$$0.5(5\mathbf{i} + 10\mathbf{j} + 20\mathbf{k}) + \int_0^{3\text{ s}} \left[2t^2\mathbf{i} - \left(3t + 3\right)\mathbf{j} + \left(10 - t^2\right)\mathbf{k}\right] = 0.5\mathbf{v}_2$$

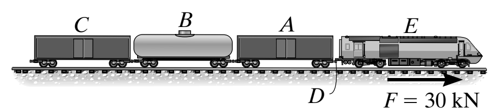
$$\mathbf{v}_2 = \left\{41\mathbf{i} - 35\mathbf{j} + 62\mathbf{k}\right\} \text{ m/s}$$

The magnitude of $\mathbf{v}_2$ is given by

$$v_2 = \sqrt{\left(v_2\right)_x{}^2 + \left(v_2\right)_y{}^2 + \left(v_2\right)_z{}^2} = \sqrt{\left(41\right)^2 + \left(-35\right)^2 + \left(62\right)^2}$$

$$= 82.2 \text{ m/s} \qquad \textbf{Ans.}$$

•15–25. The train consists of a 30-Mg engine E, and cars A, B, and C, which have a mass of 15 Mg, 10 Mg, and 8 Mg, respectively. If the tracks provide a traction force of $F = 30$ kN on the engine wheels, determine the speed of the train when $t = 30$ s, starting from rest. Also, find the horizontal coupling force at D between the engine E and car A. Neglect rolling resistance.

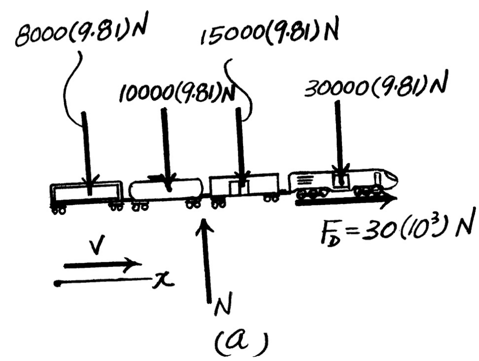


Principle of Impulse and Momentum: By referring to the free-body diagram of the entire train shown in Fig. a, we can write

$$\left(\overset{+}{\rightarrow}\right) \qquad m\left(v_1\right)_x + \Sigma \int_{t_1}^{t_2} F_x dt = m\left(v_2\right)_x$$

$$63\,000(0) + 30(10^3)(30) = 63\,000v$$

$$v = 14.29 \text{ m/s} \qquad \textbf{Ans.}$$

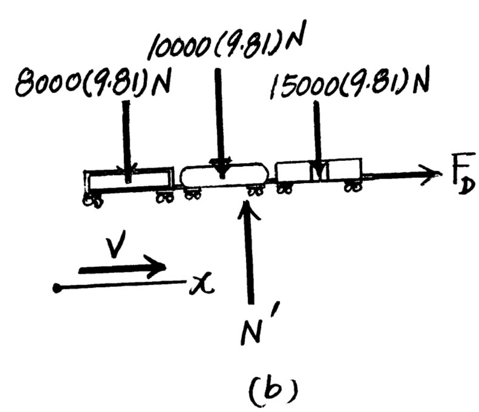


Using this result and referring to the free-body diagram of the train's car shown in Fig. b,

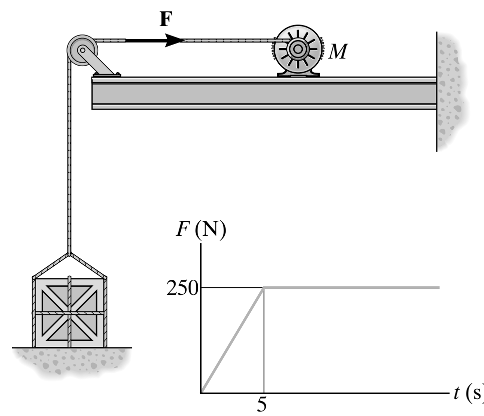
$$\left(\overset{+}{\rightarrow}\right) \qquad m\left(v_1\right)_x + \Sigma \int_{t_1}^{t_2} F_x\, dt = m\left(v_2\right)_x$$

$$33000(0) + F_D(30) = 33\,000\left(14.29\right)$$

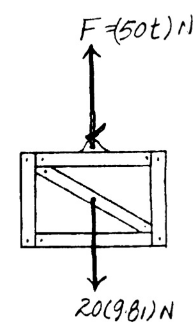
$$F_D = 15\,714.29 \text{ N} = 15.7 \text{ kN} \qquad \textbf{Ans.}$$



15–26. The motor M pulls on the cable with a force of **F**, which has a magnitude that varies as shown on the graph. If the 20-kg crate is originally resting on the floor such that the cable tension is zero at the instant the motor is turned on, determine the speed of the crate when $t = 6$ s. *Hint*: First determine the time needed to begin lifting the crate.



Equations of Equilibrium: For the period $0 \le t < 5$ s, $F = \dfrac{250}{5}t = (50t)$ N. The time needed for the motor to move the crate is given by



$$+\uparrow \Sigma F_y = 0; \qquad 50t - 20(9.81) = 0 \qquad t = 3.924 \text{ s} < 5 \text{ s}$$

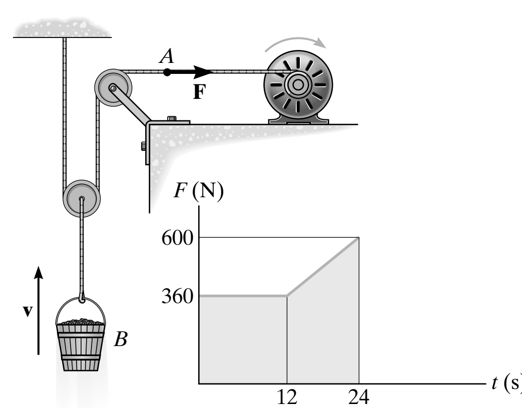
Principle of Linear Impulse and Momentum: The crate starts to move 3.924 s after the motor is turned on. Applying Eq. 15–4, we have

$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

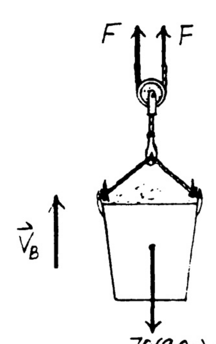
$$(+\uparrow) \qquad 20(0) + \int_{3.924 \text{ s}}^{5 \text{ s}} 50t\,dt + 250(6 - 5) - 20(9.81)(6 - 3.924) = 20v$$

$$v = 4.14 \text{ m/s} \qquad \textbf{Ans.}$$

15–27. The winch delivers a horizontal towing force **F** to its cable at A which varies as shown in the graph. Determine the speed of the 70-kg bucket when $t = 18$ s. Originally the bucket is moving upward at $v_1 = 3$ m/s.



Principle of Linear Impulse and Momentum: For the time period $12 \text{ s} \le t < 18$ s, $\dfrac{F - 360}{t - 12} = \dfrac{600 - 360}{24 - 12}$, $F = (20t + 120)$ N. Applying Eq. 15–4 to bucket B, we have

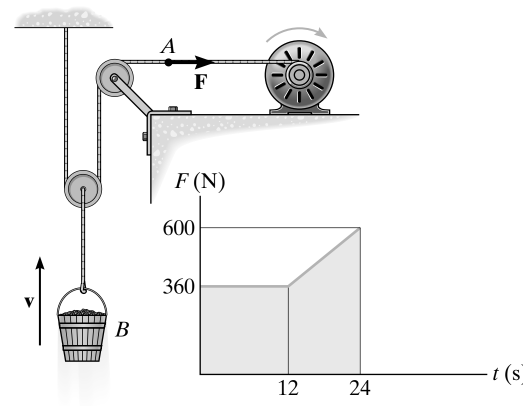


$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

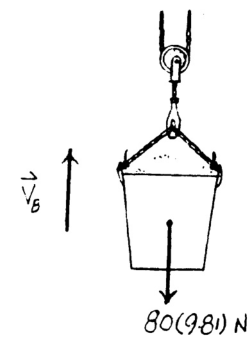
$$(+\uparrow) \qquad 70(3) + 2\left[360(12) + \int_{12 s}^{18 s} (20t + 120)dt\right] - 70(9.81)(18) = 70v_2$$

$$v_2 = 21.8 \text{ m/s} \qquad \textbf{Ans.}$$

***15–28.** The winch delivers a horizontal towing force **F** to its cable at A which varies as shown in the graph. Determine the speed of the 80-kg bucket when $t = 24$ s. Originally the bucket is released from rest.



Principle of Linear Impulse and Momentum: The total impluse exerted on bucket B can be obtained by evaluating the area under the F–t graph. Thus, $I = \Sigma \int_{t_1}^{t_2} F_y \, dt = 2\left[360(12) + \frac{1}{2}(360 + 600)(24 - 12)\right] = 20160 \text{ N} \cdot \text{s}$. Applying Eq. 15–4 to the bucket B, we have

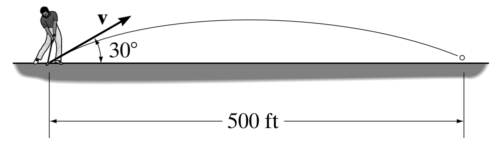


$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

$$(+\uparrow) \qquad 80(0) + 20160 - 80(9.81)(24) = 80v_2$$

$$v_2 = 16.6 \text{m/s}$$ **Ans.**

•15–29. The 0.1-lb golf ball is struck by the club and then travels along the trajectory shown. Determine the average impulsive force the club imparts on the ball if the club maintains contact with the ball for 0.5 ms.



Kinematics: By considering the x-motion of the golf ball, Fig. a,

$$\left(\overset{+}{\rightarrow}\right) \qquad s_x = \left(s_0\right) + \left(v_0\right)_x t$$

$$500 = 0 + v\cos 30°\, t$$

$$t = \frac{500}{v\cos 30°}$$

Subsequently, using the result of t and considering the y-motion of the golf ball,

$$\left(+\uparrow\right) \qquad s_y = \left(s_0\right)_y + \left(v_0\right)_y t + \frac{1}{2} a_y t^2$$

$$0 = 0 + v\sin 30°\left(\frac{500}{v\cos 30°}\right) + \frac{1}{2}(-32.2)\left(\frac{500}{v\cos 30°}\right)^2$$

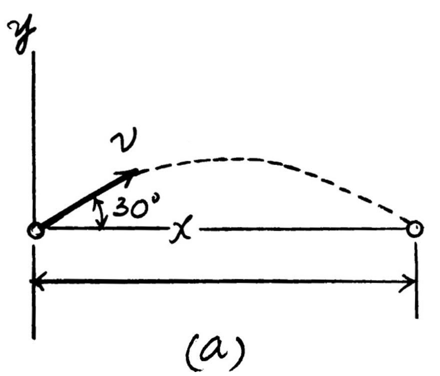
$$v = 136.35 \text{ ft/s}$$

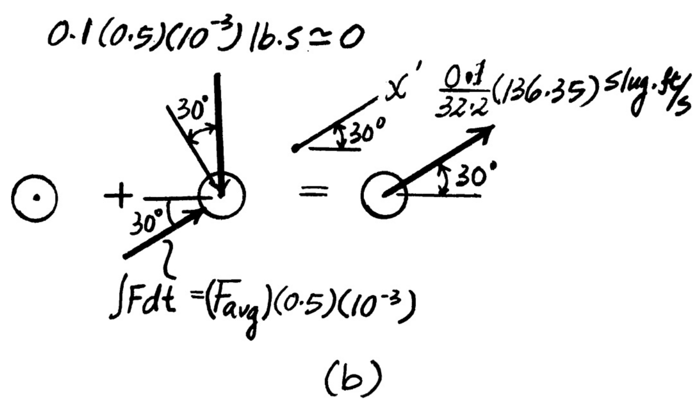
Principle of Impulse and Momentum: Here, the impulse generated by the weight of the golf ball is very small compared to that generated by the force of the impact. Hence, it can be neglected. By referring to the impulse and momentum diagram shown in Fig. b,

$$(\;) \qquad m\left(v_1\right)_{x'} + \Sigma\int_{t_1}^{t_2} F_{x'}\,dt = m\left(v_2\right)_{x'}$$

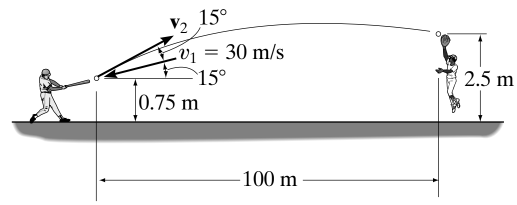
$$0 + F_{\text{avg}}\,(0.5)(10^{-3}) = \frac{0.1}{32.2}\left(136.35\right)$$

$$F_{\text{avg}} = 847 \text{ lb} \qquad \textbf{Ans.}$$





15–30. The 0.15-kg baseball has a speed of $v = 30$ m/s just before it is struck by the bat. It then travels along the trajectory shown before the outfielder catches it. Determine the magnitude of the average impulsive force imparted to the ball if it is in contact with the bat for 0.75 ms.



$(\xrightarrow{+})$
$$s_x = (s_0)_x + (v_0)_x t$$
$$100 = 0 + v\cos 30°\, t$$
$$t = \frac{100}{v\cos 30°}$$

Subsequently, using the result of t and considering the y-motion of the golf ball.

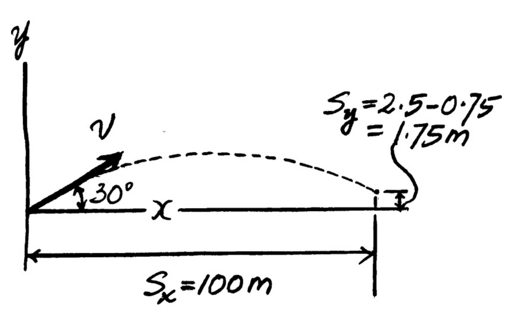
$(+\uparrow)$
$$x_y = (s_0)_y + (v_0)_y t + \frac{1}{2}a_y t^2$$
$$1.75 = 0 + v\sin 30°\left(\frac{100}{v\cos 30°}\right) + \frac{1}{2}(-9.81)\left(\frac{100}{v\cos 30°}\right)^2$$
$$v = 34.18 \text{ m/s}$$

Principle of Impulse and Momentum: Here, the impulse generated by the weight of the baseball is very small compared to that generated by the force of the impact. Hence, it can be neglected. By referring to the impulse and momentum diagram shown in Fig. *b*,

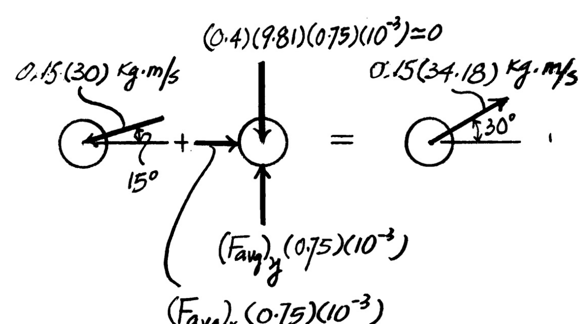
$(\xrightarrow{+})$
$$m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_2)_x$$
$$-0.15(30)\cos 15° + \left(F_{\text{avg}}\right)_x (0.75)\left(10^{-3}\right) = 0.15(34.18)\cos 30°$$
$$\left(F_{\text{avg}}\right)_x = 11\,715.7 \text{ N}$$

$(+\uparrow)$
$$m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_2)_y$$
$$-0.15(30)\sin 15° + \left(F_{\text{avg}}\right)_y (0.75)\left(10^{-3}\right) = 0.15(34.18)\sin 30°$$
$$\left(F_{\text{avg}}\right)_y = 4970.9 \text{ N}$$

Thus,
$$F_{\text{avg}} = \sqrt{\left(F_{\text{avg}}\right)_x^2 + \left(F_{\text{avg}}\right)_y^2} = \sqrt{11715.7^2 + 4970.9^2}$$
$$= 12.7 \text{ kN}$$ **Ans.**

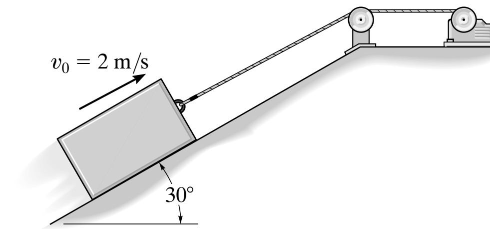


(a)



(b)

15–31. The 50-kg block is hoisted up the incline using the cable and motor arrangement shown. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.4$. If the block is initially moving up the plane at $v_0 = 2$ m/s, and at this instant ($t = 0$) the motor develops a tension in the cord of $T = (300 + 120\sqrt{t})$ N, where t is in seconds, determine the velocity of the block when $t = 2$ s.

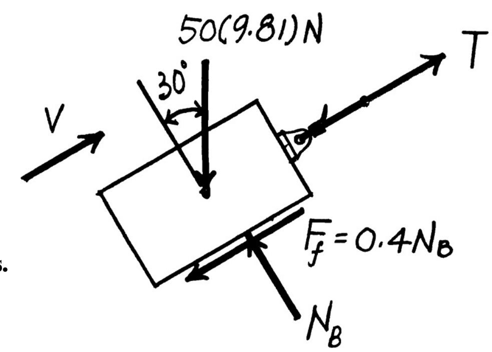


$+\nwarrow\Sigma F_x = 0; \qquad N_B - 50(9.81)\cos 30° = 0 \qquad N_B = 424.79$ N

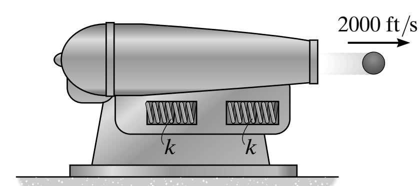
$$(+\nearrow) \qquad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$$

$$50(2) + \int_0^2 \left(300 + 120\sqrt{t}\right)dt - 0.4(424.79)(2)$$
$$- 50(9.81)\sin 30°(2) = 50v_2$$

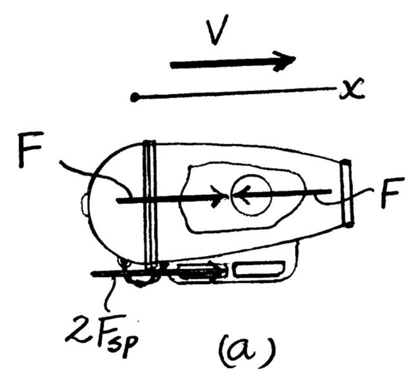
$$v_2 = 192 \text{ m/s}$$ **Ans.**



***15–32.** The 10-lb cannon ball is fired horizontally by a 500-lb cannon as shown. If the muzzle velocity of the ball is 2000 ft/s, measured relative to the ground, determine the recoil velocity of the cannon just after firing. If the cannon rests on a smooth support and is to be stopped after it has recoiled a distance of 6 in., determine the required stiffness k of the two identical springs, each of which is originally unstretched.



Free-Body Diagram: The free-body diagram of the cannon and ball system is shown in Fig. *a*. Here, the spring force $2\mathbf{F}_{sp}$ is nonimpulsive since the spring acts as a shock absorber. The pair of impulsive forces $\mathbf{F}$ resulting from the explosion cancel each other out since they are internal to the system



Conservation of Linear Momentum: Since the resultant of the impulsice force along the x axis is zero, the linear momentum of the system is conserved along the x axis.

$$\left(\overset{+}{\rightarrow}\right) \qquad m_C\left(v_C\right)_1 + m_b\left(v_b\right)_1 = m_C\left(v_C\right)_2 + m_b\left(v_b\right)_2$$

$$\frac{500}{32.2}(0) + \frac{10}{32.2}(0) = \frac{500}{32.2}\left(v_C\right)_2 + \frac{10}{32.2}(2000)$$

$$\left(v_C\right)_2 = -40 \text{ ft/s} = 40 \text{ ft/s} \leftarrow$$ **Ans.**

Conservation of Energy: The initial and final elastic potential energy in each spring are $\left(V_e\right)_i = \frac{1}{2}ks_i^{\,2} = 0$ and $\left(V_e\right)_f = \frac{1}{2}ks_f^{\,2} = \frac{1}{2}k(0.5^2) = 0.125k$. By referring to Fig. *a*,

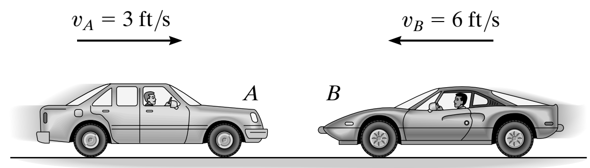
$$T_i + V_i = T_f + V_f$$

$$\frac{1}{2}m_C\left(v_C\right)_i^{\,2} + 2\left(V_e\right)_i = \frac{1}{2}m_C\left(v_C\right)_f^{\,2} + 2\left(V_e\right)_f$$

$$\frac{1}{2}\left(\frac{500}{32.2}\right)\left(40^2\right) + 2\left(0\right) = 0 + 2\left(0.125k\right)$$

$$k = 49\,689.44 \text{ lb/ft} = 49.7 \text{ kip/ft}$$ **Ans.**

15–33. A railroad car having a mass of 15 Mg is coasting at 1.5 m/s on a horizontal track. At the same time another car having a mass of 12 Mg is coasting at 0.75 m/s in the opposite direction. If the cars meet and couple together, determine the speed of both cars just after the coupling. Find the difference between the total kinetic energy before and after coupling has occurred, and explain qualitatively what happened to this energy.



$(\xrightarrow{+})\qquad \Sigma m v_1 = \Sigma m v_2$

$$15\,000(1.5) - 12\,000(0.75) = 27\,000(v_2)$$

$$v_2 = 0.5 \text{ m/s} \qquad \textbf{Ans.}$$

$$T_1 = \frac{1}{2}(15\,000)(1.5)^2 + \frac{1}{2}(12\,000)(0.75)^2 = 20.25 \text{ kJ}$$

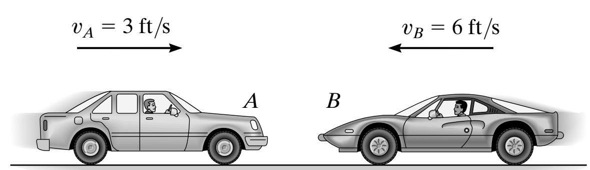
$$T_2 = \frac{1}{2}(27\,000)(0.5)^2 = 3.375 \text{ kJ}$$

$$\Delta T = T_1 - T_2$$

$$= 20.25 - 3.375 = 16.9 \text{ kJ} \qquad \textbf{Ans.}$$

This energy is dissipated as noise, shock, and heat during the coupling.

15–34. The car A has a weight of 4500 lb and is traveling to the right at 3 ft/s. Meanwhile a 3000-lb car B is traveling at 6 ft/s to the left. If the cars crash head-on and become entangled, determine their common velocity just after the collision. Assume that the brakes are not applied during collision.

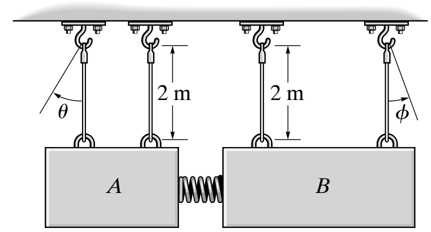


$(\xrightarrow{+})\qquad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$

$$\frac{4500}{32.2}(3) - \frac{3000}{32.2}(6) = \frac{7500}{32.2}v_2$$

$$v_2 = -0.600 \text{ ft/s} = 0.600 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

15–35. The two blocks A and B each have a mass of 5 kg and are suspended from parallel cords. A spring, having a stiffness of $k = 60$ N/m, is attached to B and is compressed 0.3 m against A as shown. Determine the maximum angles θ and ϕ of the cords when the blocks are released from rest and the spring becomes unstretched.



$$(\overset{+}{\rightarrow}) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$0 + 0 = -5v_A + 5v_B$$

$$v_A = v_B = v$$

Just before the blocks begin to rise:

$$T_1 + V_1 = T_2 + V_2$$

$$(0 + 0) + \frac{1}{2}(60)(0.3)^2 = \frac{1}{2}(5)(v)^2 + \frac{1}{2}(5)(v)^2 + 0$$

$$v = 0.7348 \text{ m/s}$$

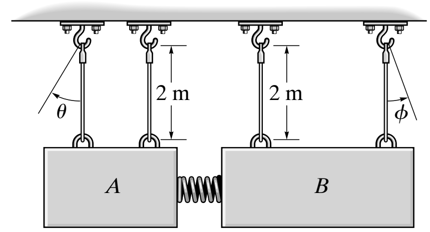
For A or B: Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(5)(0.7348)^2 + 0 = 0 + 5(9.81)(2)(1 - \cos\theta)$$

$$\theta = \phi = 9.52^\circ$$ **Ans.**

***15–36.** Block A has a mass of 4 kg and B has a mass of 6 kg. A spring, having a stiffness of $k = 40$ N/m, is attached to B and is compressed 0.3 m against A as shown. Determine the maximum angles θ and ϕ of the cords after the blocks are released from rest and the spring becomes unstretched.



$(\overset{+}{\rightarrow})\qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$0 + 0 = 6v_B - 4v_A$$

$$v_A = 1.5v_B$$

Just before the blocks begin to rise:

$$T_1 + V_1 = T_2 + V_2$$

$$(0 + 0) + \frac{1}{2}(40)(0.3)^2 = \frac{1}{2}(4)(v_A)^2 + \frac{1}{2}(6)(v_B)^2 + 0$$

$$3.6 = 4v_A^2 + 6v_B^2$$

$$3.6 = 4(1.5v_B)^2 + 6v_B^2$$

$$v_B = 0.4899 \text{ m/s} \qquad v_A = 0.7348 \text{ m/s}$$

For A:

Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(4)(0.7348)^2 + 0 = 0 + 4(9.81)(2)(1 - \cos\theta)$$

$\theta = 9.52°$ **Ans.**

For B:

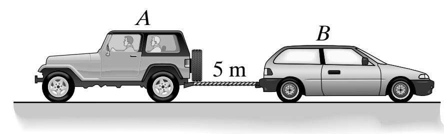
Datum at lowest point

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(6)(0.4899)^2 + 0 = 0 + 6(9.81)(2)(1 - \cos\phi)$$

$\phi = 6.34°$ **Ans.**

•15–37. The winch on the back of the Jeep A is turned on and pulls in the tow rope at 2 m/s measured relative to the Jeep. If both the 1.25-Mg car B and the 2.5-Mg Jeep A are free to roll, determine their velocities at the instant they meet. If the rope is 5 m long, how long will this take?



$$\left(\overset{+}{\rightarrow}\right) \qquad 0 + 0 = m_A v_A - m_B v_B \qquad \textbf{(1)}$$

$$0 = 2.5\left(10^3\right) v_A - 1.25\left(10^3\right) v_B$$

However, $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$$\left(\overset{+}{\rightarrow}\right) \qquad v_A = -v_B + 2 \qquad \textbf{(2)}$$

Substituting Eq. (2) into (1) yields:

$v_B = 1.33\text{m/s}$ **Ans.**

$v_A = 0.667 \text{ m/s}$ **Ans.**

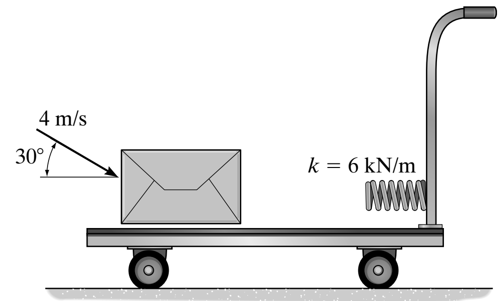
Kinematics:

$$\left(\overset{+}{\rightarrow}\right) \qquad s_{A/B} = v_{A/B}\, t$$

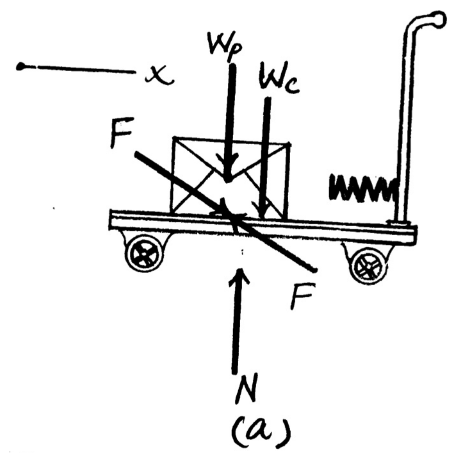
$$5 = 2t$$

$$t = 2.5 \text{ s} \qquad \textbf{Ans.}$$

15–38. The 40-kg package is thrown with a speed of 4 m/s onto the cart having a mass of 20 kg. If it slides on the smooth surface and strikes the spring, determine the velocity of the cart at the instant the package fully compresses the spring. What is the maximum compression of the spring? Neglect rolling resistance of the cart.



Conservation of Linear Momentum: By referring to the free-body diagram of the package and cart system shown in Fig. *a*, we notice the pair of impulsive forces **F** generated during the impact cancel each other since they are internal to the system. Thus, the resultant of the impulsive forces along the *x* axis is zero. As a result, the linear momentum of the system is conserved along the *x* axis. The cart does not move after the impact until the package strikes the spring. Thus,



$$(\xrightarrow{+}) \qquad m_p\left[\left(v_p\right)_1\right]_x + m_c(v_c)_1 = m_p\left(v_p\right)_2 + m_c(v_c)_2$$

$$40(4\cos 30^\circ) + 0 = 40\left(v_p\right)_2 + 0$$

$$\left(v_p\right)_2 = 3.464 \text{ m/s} \rightarrow$$

When the spring is fully compressed, the package momentarily stops sliding on the cart. At this instant, the package and the cart move with a common speed.

$$(\xrightarrow{+}) \qquad m_p\left(v_p\right)_2 + m_c(v_c)_2 = \left(m_p + m_c\right)v_3$$

$$40(3.464) + 0 = (40 + 20)v_3$$

$$v_3 = 2.309 \text{ m/s} = 2.31 \text{ m/s} \qquad \textbf{Ans.}$$

Conservation of Energy: We will consider the conservation of energy of the system. The initial and final elastic potential energies of the spring are $\left(V_e\right)_2 = \frac{1}{2}ks_2{}^2 = 0$ and $\left(V_e\right)_3 = \frac{1}{2}ks_3{}^2 = \frac{1}{2}(6000)s_{max}{}^2 = 3000s_{max}{}^2$.

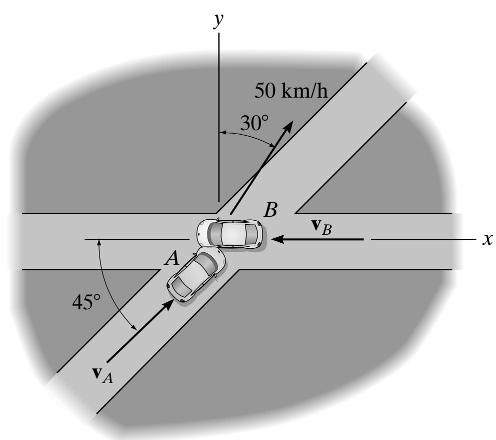
$$T_2 + V_2 = T_3 + V_3$$

$$\left[\frac{1}{2}m_p\left(v_p\right)^2 + \frac{1}{2}m_c(v_c)_2{}^2\right] + \left(V_e\right)_2 = \frac{1}{2}\left(m_p + m_p\right)v_3{}^2 + \left(V_e\right)_3$$

$$\left[\frac{1}{2}(40)(3.464^2) + 0\right] + 0 = \frac{1}{2}(40 + 20)\left(2.309^2\right) + 3000s_{max}{}^2$$

$$s_{max} = 0.1632 \text{ m} = 163 \text{ mm} \qquad \textbf{Ans.}$$

15–39. Two cars A and B have a mass of 2 Mg and 1.5 Mg, respectively. Determine the magnitudes of $\mathbf{v}_A$ and $\mathbf{v}_B$ if the cars collide and stick together while moving with a common speed of 50 km/h in the direction shown.



Conservation of Linear Momentum: Since the pair of impulsice forces $\mathbf{F}$ generated during the impact are internal to the system of cars A and B, they cancel each other out. Thus, the resultant impulsive force along the x and y axes are zero. Consequently, the linear momentum of the system is conserved along the x and y axes. The common speed of the system just after the impact is $v_2 = \left[50(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 13.89$ m/s. Thus, we can write

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_x + \left[-m_B(v_B)_x\right] = (m_A + m_B)(v_2)_x$$

$$2000v_A \cos 45° - 1500v_B = (2000 + 1500)(13.89 \sin 30°)$$

$$1414.21v_A - 1500v_B = 24305.56 \qquad \textbf{(1)}$$

and

$$(+\uparrow) \qquad m_A(v_A)_y + m_B(v_B)_y = (m_A + m_B)(v_2)_y$$

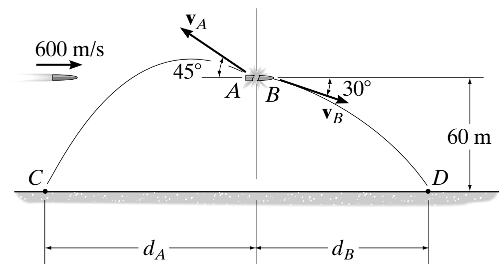
$$2000v_A \sin 45° + 0 = (2000 + 1500)(13.89 \cos 30°)$$

$$v_A = 29.77 \text{ m/s} = 29.8 \text{ m/s} \qquad \textbf{Ans.}$$

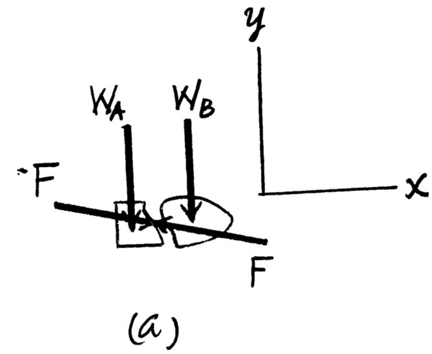
Substituting the result of $\mathbf{v}_A$ into Eq. (1),

$$v_B = 11.86 \text{ m/s} = 11.9 \text{ m/s} \qquad \textbf{Ans.}$$

***15–40.** A 4-kg projectile travels with a horizontal velocity of 600 m/s before it explodes and breaks into two fragments A and B of mass 1.5 kg and 2.5 kg, respectively. If the fragments travel along the parabolic trajectories shown, determine the magnitude of velocity of each fragment just after the explosion and the horizontal distance d_A where segment A strikes the ground at C.



Conservation of Linear Momentum: By referring to the free-body diagram of the projectile just after the explosion shown in Fig. *a*, we notice that the pair of impulsive forces **F** generated during the explosion cancel each other since they are internal to the system. Here, $\mathbf{W}_A$ and $\mathbf{W}_B$ are non-impulsive forces. Since the resultant impulsive force along the x and y axes is zero, the linear momentum of the system is conserved along these two axes.



$$\left(\xrightarrow{+}\right) \qquad mv_x = m_A(v_A)_x + m_B(v_B)_x$$

$$4(600) = -1.5v_A \cos 45° + 2.5v_B \cos 30°$$

$$2.165v_B - 1.061v_A = 2400 \qquad \textbf{(1)}$$

$$(+\uparrow) \qquad mv_y = m_A(v_A)_y + m_B(v_B)_y$$

$$0 = 1.5v_A \sin 45° - 2.5v_B \sin 30°$$

$$v_B = 0.8485v_A \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v_A = 3090.96 \text{ m/s} = 3.09(10^3) \text{ m/s} \qquad \textbf{Ans.}$$

$$v_B = 2622.77 \text{ m/s} = 2.62(10^3) \text{ m/s} \qquad \textbf{Ans.}$$

By considering the x and y motion of segment A,

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2} a_y t^2$$

$$-60 = 0 + 3090.96 \sin 45°\, t_{AC} + \frac{1}{2}(-9.81)t_{AC}{}^2$$

$$4.905t_{AC}{}^2 - 2185.64t_{AC} - 60 = 0$$

Solving for the positive root of this equation,

$$t_{AC} = 445.62 \text{ s}$$

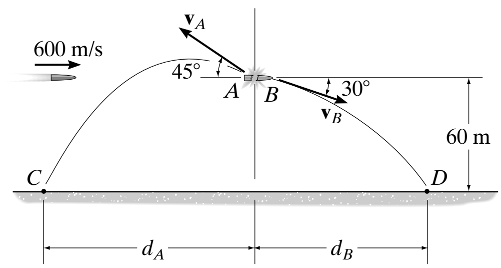
and

$$\left(\xleftarrow{+}\right) \qquad s_x = (s_0)_x + (v_0)_x t$$

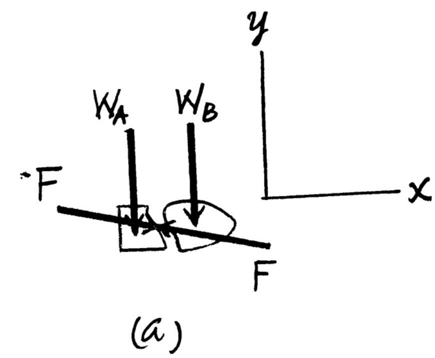
$$d_A = 0 + 3090.96 \cos 45°(445.62)$$

$$= 973.96\left(10^3\right) \text{ m} = 974 \text{ km} \qquad \textbf{Ans.}$$

•15–41. A 4-kg projectile travels with a horizontal velocity of 600 m/s before it explodes and breaks into two fragments A and B of mass 1.5 kg and 2.5 kg, respectively. If the fragments travel along the parabolic trajectories shown, determine the magnitude of velocity of each fragment just after the explosion and the horizontal distance d_B where segment B strikes the ground at D.



Conservation of Linear Momentum: By referring to the free-body diagram of the projectile just after the explosion shown in Fig. *a*, we notice that the pair of impulsive forces **F** generated during the explosion cancel each other since they are internal to the system. Here, $\mathbf{W}_A$ and $\mathbf{W}_B$ are non-impulsive forces. Since the resultant impulsive force along the x and y axes is zero, the linear momentum of the system is conserved along these two axes.



$(\xrightarrow{+})$ $$mv_x = m_A(v_A)_x + m_B(v_B)_x$$

$$4(600) = -1.5v_A \cos 45° + 2.5v_B \cos 30°$$

$$2.165v_B - 1.061v_A = 2400 \quad \textbf{(1)}$$

$(+\uparrow)$ $$mv_y = m_A(v_A)_y + m_B(v_B)_y$$

$$0 = 1.5v_A \sin 45° - 2.5v_B \sin 30°$$

$$v_B = 0.8485v_A \quad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v_A = 3090.96 \text{ m/s} = 3.09(10^3) \text{ m/s} \quad \textbf{Ans.}$$

$$v_B = 2622.77 \text{ m/s} = 2.62(10^3) \text{ m/s} \quad \textbf{Ans.}$$

By considering the x and y motion of segment B,

$(+\uparrow)$ $$s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}a_y t^2$$

$$-60 = 0 - 2622.77 \sin 30° \, t_{BD} + \frac{1}{2}(-9.81)t_{BD}^{\,2}$$

$$4.905t_{BD}^{\,2} + 1311.38t_{BD} - 60 = 0$$

Solving for the positive root of the above equation,

$$t_{BD} = 0.04574 \text{ s}$$

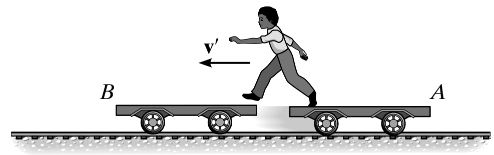
and

$(\xrightarrow{+})$ $$s_x = (s_0)_x + (v_0)_x t$$

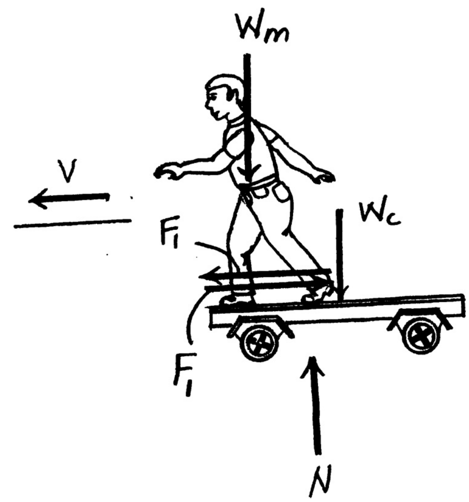
$$d_B = 0 + 2622.77 \cos 30°(0.04574)$$

$$= 103.91 \text{ m} = 104 \text{ m} \quad \textbf{Ans.}$$

15–42. The 75-kg boy leaps off cart A with a horizontal velocity of $v' = 3\text{ m/s}$ measured relative to the cart. Determine the velocity of cart A just after the jump. If he then lands on cart B with the same velocity that he left cart A, determine the velocity of cart B just after he lands on it. Carts A and B have the same mass of 50 kg and are originally at rest.



Free-Body Diagram: The free-body diagram of the man and cart system when the man leaps off and lands on the cart are shown in Figs. *a* and *b,* respectively. The pair of impulsive forces $\mathbf{F}_1$ and $\mathbf{F}_2$ generated during the leap and landing are internal to the system and thus cancel each other.



(a)

Kinematics: Applying the relative velocity equation, the relation between the velocity of the man and cart A just after leaping can be determined.

$$\mathbf{v}_m = \mathbf{v}_A + \mathbf{v}_{m/A}$$

$$(\overset{+}{\leftarrow}) \qquad (v_m)_2 = (v_A)_2 + 3 \qquad \textbf{(1)}$$

Conservation of Linear Momentum: Since the resultant of the impulse forces along the x axis is zero, the linear momentum of the system is conserved along the x axis for both cases. When the man leaps off cart A,

$$(\overset{+}{\leftarrow}) \qquad m_m(v_m)_1 + m_A(v_A)_1 = m_m(v_m)_2 + m_A(v_A)_2$$

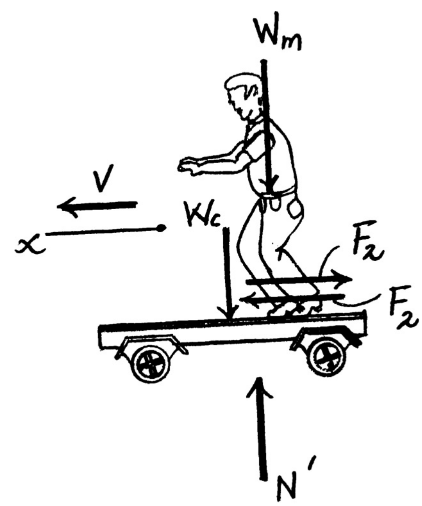
$$0 + 0 = 75(v_m)_2 + 50(v_A)_2$$

$$(v_m)_2 = -0.6667(v_A)_2$$

Solving Eqs. (1) and (2) yields

$$(v_A)_2 = -1.80\text{ m/s} = 1.80\text{ m/s} \rightarrow$$

$$(v_m)_2 = 1.20\text{ m/s} \leftarrow$$



(b)

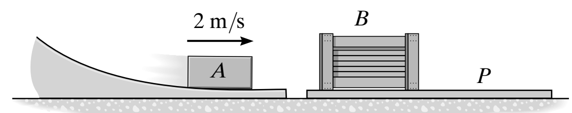
Using the result of $(v_m)_2$ and considering the man's landing on cart B,

$$(\overset{+}{\leftarrow}) \qquad m_m(v_m)_2 + m_B(v_B)_1 = (m_m + m_B)v$$

$$75(1.20) + 0 = (75 + 50)v$$

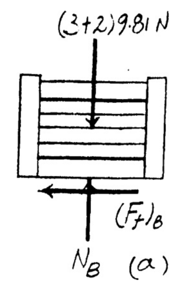
$$v = 0.720\text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

15–43. Block A has a mass of 2 kg and slides into an open ended box B with a velocity of 2 m/s. If the box B has a mass of 3 kg and rests on top of a plate P that has a mass of 3 kg, determine the distance the plate moves after it stops sliding on the floor. Also, how long is it after impact before all motion ceases? The coefficient of kinetic friction between the box and the plate is $\mu_k = 0.2$, and between the plate and the floor $\mu'_k = 0.4$. Also, the coefficient of static friction between the plate and the floor is $\mu'_s = 0.5$.



Equations of Equilibrium: From FBD(a).

$$+\uparrow \Sigma F_y = 0; \qquad N_B - (3+2)(9.81) = 0 \qquad N_B = 49.05 \text{ N}$$



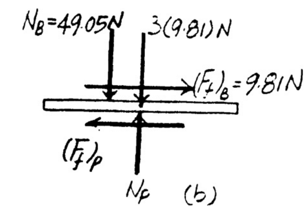
When box B slides on top of plate P, $(F_f)_B = \mu_k N_B = 0.2(49.05) = 9.81$ N. From FBD(b).

$$+\uparrow \Sigma F_y = 0; \qquad N_P - 49.05 - 3(9.81) = 0 \qquad N_P = 78.48 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 9.81 - (F_f)_P = 0 \qquad (F_f)_P = 9.81 \text{ N}$$

Since $(F_f)_P < \left[(F_f)_P\right]_{\max} = \mu_s' N_P = 0.5(78.48) = 39.24$ N, plate P does not move. Thus

$$s_P = 0 \qquad \textbf{Ans.}$$



Conservation of Linear Momentum: If we consider the block and the box as a system, then the impulsive force caused by the impact is *internal* to the system. Therefore, it will cancel out. As the result, linear momentum is conserved along the x axis.

$$m_A (v_A)_1 + m_R (v_R)_1 = (m_A + m_R) v_2$$

$$(\xrightarrow{+}) \qquad 2(2) + 0 = (2+3) v_2$$

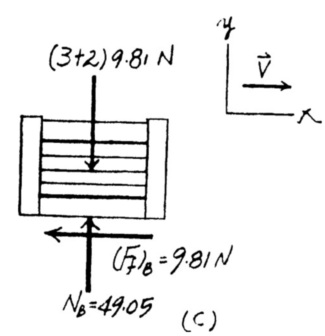
$$v_2 = 0.800 \text{ m/s} \rightarrow$$

Principle of Linear Impulse and Momentum: Applying Eq. 15–4, we have

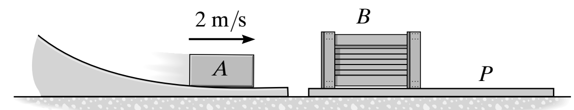
$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

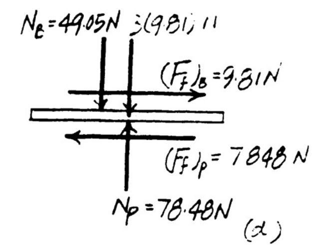
$$(\xrightarrow{+}) \qquad 5(0.8) + \left[-9.81(t)\right] = 5(0)$$

$$t = 0.408 \text{ s} \qquad \textbf{Ans.}$$



***15–44.** Block A has a mass of 2 kg and slides into an open ended box B with a velocity of 2 m/s. If the box B has a mass of 3 kg and rests on top of a plate P that has a mass of 3 kg, determine the distance the plate moves after it stops sliding on the floor. Also, how long is it after impact before all motion ceases? The coefficient of kinetic friction between the box and the plate is $\mu_k = 0.2$, and between the plate and the floor $\mu_k' = 0.1$. Also, the coefficient of static friction between the plate and the floor is $\mu_s' = 0.12$.





Equations of Equilibrium: From FBD(a),

$$+\uparrow \Sigma F_x = 0; \qquad N_B - (3+2)(9.81) = 0 \qquad N_B = 49.05 \text{ N}$$

When box B slides on top of plate P. $(F_f)_B = \mu_k N_B = 0.2(49.05) = 9.81$ N. From FBD(b).

$$+\uparrow \Sigma F_y = 0; \qquad N_P - 49.05 - 3(9.81) = 0 \qquad N_P = 78.48 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 9.81 - (F_f)_P = 0 \qquad (F_f)_P = 9.81 \text{ N}$$

Since $(F_f)_P > \left[(F_f)_P\right]_{\max} = \mu_s' N_P = 0.12(78.48) = 9.418\text{N}$, plate P slides. Thus, $(F_f)_P = \mu_k' N_P = 0.1(78.48) = 7.848$ N.

Conservation of Linear Momentum: If we consider the block and the box as a system, then the impulsive force caused by the impact is *internal* to the system. Therefore, it will cancel out. As the result, linear momentum is conserved along x axis.

$$m_A (v_A)_1 + m_R (v_R)_1 = (m_A + m_R)\, v_2$$

$$(\overset{+}{\rightarrow}) \qquad 2(2) + 0 = (2+3)\, v_2$$

$$v_2 = 0.800 \text{ m/s} \rightarrow$$

Principle of Linear Impulse and Momentum: In order for box B to stop sliding on plate P, both box B and plate P must have same speed v_3. Applying Eq. 15–4 to box B (FBD(c)], we have

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$(\overset{+}{\rightarrow}) \qquad 5(0.8) + \left[-9.81(t_1)\right] = 5v_3 \qquad \textbf{[1]}$$

Applying Eq. 15–4 to plate P[FBD(d)], we have

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$(\overset{+}{\rightarrow}) \qquad 3(0) + 9.81(t_1) - 7.848(t_1) = 3v_3 \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$t_1 = 0.3058 \text{ s} \qquad v_3 = 0.200 \text{ m/s}$$

Equation of Motion: From FBD(d), the acceleration of plate P when box B still slides on top of it is given by

$$\overset{+}{\rightarrow} \Sigma Fx = ma_x; \qquad 9.81 - 7.848 = 3(a_P)_1 \qquad (a_P)_1 = 0.654 \text{ m/s}^2$$

***15–44. Continued**

When box B stop slid ling on top of box B, $(F_f)_B = 0$. From this instant onward plate P and box B act as a unit and slide together. From FBD(d), the acceleration of plate P and box B is given by

$$\overset{+}{\rightarrow} \Sigma Fx = ma_x; \qquad -7.848 = 8(a_P)_2 \qquad (a_P)_2 = -0.981 \text{ m/s}^2$$

Kinematics: Plate P travels a distance s_1 before box B stop sliding.

$$\left(\overset{+}{\rightarrow}\right) \qquad s_1 = (v_0)_P\, t_1 + \frac{1}{2}(a_P)_1\, t_1^2$$

$$= 0 + \frac{1}{2}(0.654)\left(0.3058^2\right) = 0.03058 \text{ m}$$

The time t_2 for plate P to stop after box B stop slidding is given by

$$\left(\overset{+}{\rightarrow}\right) \qquad v_4 = v_3 + (a_P)_2\, t_2$$

$$0 = 0.200 + (-0.981)t_2 \qquad t_2 = 0.2039 \text{ s}$$

The distance s_2 traveled by plate P after box B stop sliding is given by

$$\left(\overset{+}{\rightarrow}\right) \qquad v_4^2 = v_3^2 + 2(a_P)_2\, s_2$$

$$0 = 0.200^2 + 2(-0.981)s_2 \qquad s_2 = 0.02039 \text{ m}$$

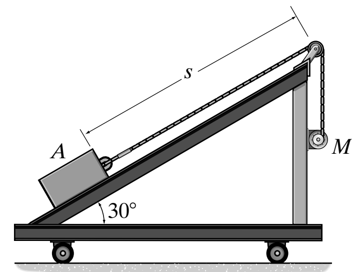
The total distance travel by plate P is

$$s_P = s_1 + s_2 = 0.03058 + 0.02039 = 0.05097 \text{ m} = 51.0 \text{ mm} \qquad \textbf{Ans.}$$

The total time taken to cease all the motion is

$$t_{\text{Tot}} = t_1 + t_2 = 0.3058 + 0.2039 = 0.510 \text{ s} \qquad \textbf{Ans.}$$

•15–45. The 20-kg block A is towed up the ramp of the 40-kg cart using the motor M mounted on the side of the cart. If the motor winds in the cable with a constant velocity of 5 m/s, measured relative to the cart, determine how far the cart will move when the block has traveled a distance $s = 2$ m up the ramp. Both the block and cart are at rest when $s = 0$. The coefficient of kinetic friction between the block and the ramp is $\mu_k = 0.2$. Neglect rolling resistance.



Conservation of Linear Momentum: The linear momentum of the block and cart system is conserved along the x axis since no impulsive forces act along the x axis.

$$(\xrightarrow{+}) \qquad m_B\left[(v_B)_x\right]_1 + m_C(v_C)_1 = m_B\left[(v_B)_x\right]_2 + m_C(v_C)_2$$

$$0 + 0 = 20(v_B)_x + 40v_C \qquad \textbf{(1)}$$

Kinematics: Here, the velocity of the block relative to the cart is directed up the ramp with a magnitude of $v_{B/C} = 5$ m/s. Applying the relative velocity equation and considering the motion of the block.

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$\begin{bmatrix} v_B \end{bmatrix} = \begin{bmatrix} v_C \\ \rightarrow \end{bmatrix} + \begin{bmatrix} 5 \end{bmatrix}$$

$$(\xrightarrow{+}) \qquad (v_B)_x = v_C + 5\cos 30° \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$v_C = -1.443 \text{ m/s} = 1.443 \text{ m/s} \leftarrow \qquad (v_B)_x = 2.887 \text{ m/s}$

The time required for the block to travel up the ramp a relative distance of $s_{B/C} = 2$ m is

$$( ) \qquad s_{B/C} = (s_{B/C})_0 + (v_{B/C})t$$

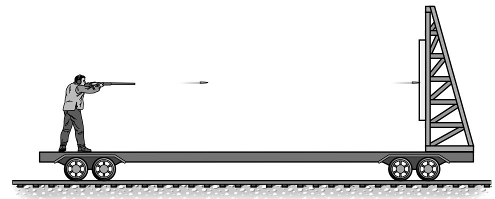
$$2 = 0 + 5t$$

$$t = 0.4 \text{ s}$$

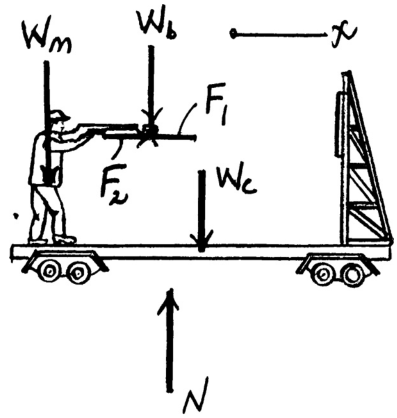
Thus, the distance traveled by the cart during time t is

$$(\xleftarrow{+}) \qquad s_C = v_C t = 1.443(0.4) = 0.577 \text{ m} \leftarrow \qquad \textbf{Ans.}$$

15–46. If the 150-lb man fires the 0.2-lb bullet with a horizontal muzzle velocity of 3000 ft/s, measured relative to the 600-lb cart, determine the velocity of the cart just after firing. What is the velocity of the cart when the bullet becomes embedded in the target? During the firing, the man remains at the same position on the cart. Neglect rolling resistance of the cart.



Free-Body Diagram: The free-body diagram of the bullet, man, and cart just after firing and at the instant the bullet hits the target are shown in Figs., *a* and *b*, respectively. The pairs of impulsive forces $\mathbf{F}_1$ and $\mathbf{F}_2$ generated during the firing and impact are internal to the system and thus cancel each other.



(a)

Kinematics: Applying the relative velocity equation, the relation between the velocity of the bullet and the cart just after firing can be determined

$$(\overset{+}{\rightarrow}) \qquad \mathbf{v}_b = \mathbf{v}_c + \mathbf{v}_{b/c}$$

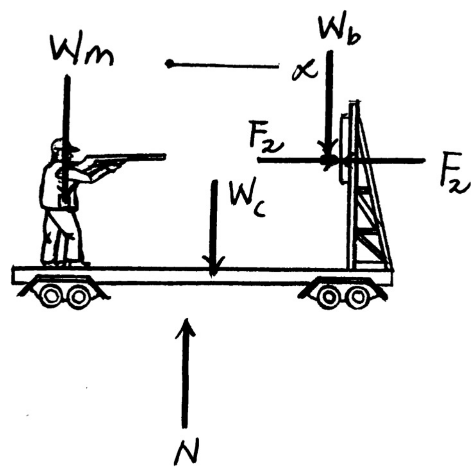
$$(v_b)_2 = (v_c)_2 + 3000 \qquad \textbf{(1)}$$

Conservation of Linear Momentum: Since the pair of resultant impulsive forces $\mathbf{F}_1$ and $\mathbf{F}_2$ generated during the firing and impact is zero along the x axis, the linear momentum of the system for both cases are conserved along the. x axis. For the case when the bullet is fired, momentum is conserved along the x' axis.

$$(\overset{+}{\rightarrow}) \qquad m_b(v_b)_1 + m_c(v_c)_1 = m_b(v_b)_2 + m_c(v_c)_2$$

$$0 + 0 = \left(\frac{0.2}{32.2}\right)(v_b)_2 + \left(\frac{150 + 600}{32.2}\right)(v_c)_2$$

$$(v_b)_2 = -3750(v_c)_2 \qquad \textbf{(2)}$$



(b)

Solving Eqs. (1) and (2) yields

$$(v_c)_2 = -0.7998 \text{ ft/s} = 0.800 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

$$(v_b)_2 = 2999.20 \text{ ft/s} \rightarrow$$

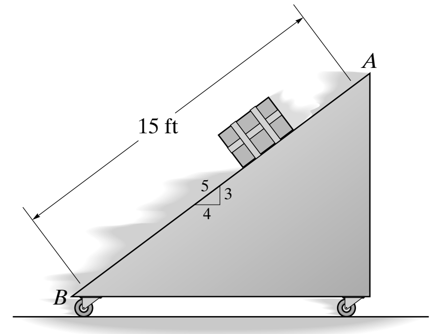
Using the results of $(v_c)_2$ and $(v_b)_2$ and considering the case when the bullet hits the target,

$$(\overset{+}{\rightarrow}) \qquad m_b(v_b)_2 + m_c(v_c)_2 = (m_b + m_c)v_3$$

$$\frac{0.2}{32.2}(2999.20) + \left[-\left(\frac{150 + 600}{32.2}\right)(0.7998)\right] = \left(\frac{150 + 600 + 0.2}{32.2}\right)v_3$$

$$v_3 = 0 \qquad \textbf{Ans.}$$

15–47. The free-rolling ramp has a weight of 120 lb. The crate whose weight is 80 lb slides from rest at A, 15 ft down the ramp to B. Determine the ramp's speed when the crate reaches B. Assume that the ramp is smooth, and neglect the mass of the wheels.



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 80\left(\frac{3}{5}\right)(15) = \frac{1}{2}\left(\frac{80}{32.2}\right)v_B^2 + \frac{1}{2}\left(\frac{120}{32.2}\right)v_r^2 \qquad \textbf{(1)}$$

$$(\overset{+}{\rightarrow})\Sigma\, m\, v_1 = \Sigma\, m\, v_2$$

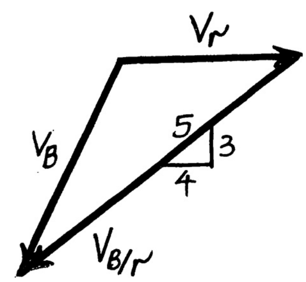
$$0 + 0 = \frac{120}{32.2}v_r - \frac{80}{32.2}(v_B)_x$$

$$(v_B)_x = 1.5v_r$$

$$\mathbf{v}_B = \mathbf{v}_r + \mathbf{v}_{B/r}$$

$$\left(\overset{+}{\rightarrow}\right) - (v_B)_x = vr - \frac{4}{5}v_{B/r} \qquad \textbf{(2)}$$

$$\left(+\uparrow\right)(-v_B)_y = 0 - \frac{3}{5}v_{B/r} \qquad \textbf{(3)}$$



Eliminating $(v_B)_{/r}$, from Eqs. (2) and (3) and substituting $(v_B)_y = 1.875\, v_r$, results in

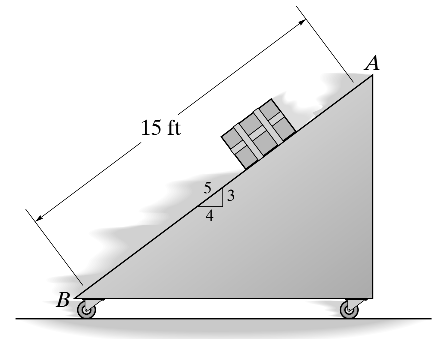
$$v_B^2 = (v_B)_x^2 + (v_B)_y^2 = (1.5v_r)^2 + (1.875v_r)^2 = 5.7656v_r^2 \qquad \textbf{(4)}$$

Substituting Eq. (4) into (1) yields:

$$80\left(\frac{3}{5}\right)(15) = \frac{1}{2}\left(\frac{80}{32.2}\right)\left(5.7656v_r^2\right) + \frac{1}{2}\left(\frac{120}{32.2}\right)v_r^2$$

$v_r = 8.93$ ft/s **Ans.**

***15–48.** The free-rolling ramp has a weight of 120 lb. If the 80-lb crate is released from rest at A, determine the distance the ramp moves when the crate slides 15 ft down the ramp to the bottom B.



$(\overset{+}{\rightarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0 = \frac{120}{32.2} v_r - \frac{80}{32.2} (v_B)_x$$

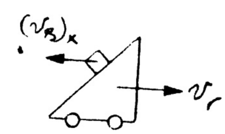
$$(v_B)_x = 1.5\, v_r$$

$$\mathbf{v}_B = \mathbf{v}_r + \mathbf{v}_{B/r}$$

$$-(v_B)_x = v_r - (v_{B/r})_x \left(\frac{4}{5}\right)$$

$$-1.5 v_r = v_r - (v_{B/r})_x \left(\frac{4}{5}\right)$$

$$2.5\, v_r = (v_{B/r})_x \left(\frac{4}{5}\right)$$



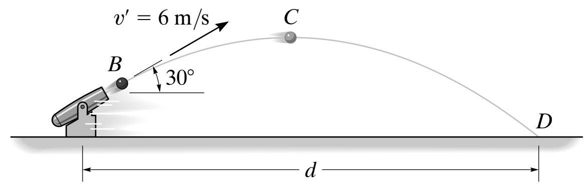
Integrate

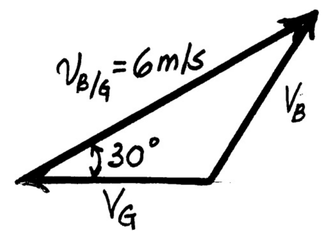
$$2.5\, s_r = (s_{B/r})_x \left(\frac{4}{5}\right)$$

$$2.5\, s_r = \left(\frac{4}{5}\right)(15)$$

$s_r = 4.8$ ft **Ans.**

•15–49. The 5-kg spring-loaded gun rests on the smooth surface. It fires a ball having a mass of 1 kg with a velocity of $v' = 6$ m/s relative to the gun in the direction shown. If the gun is originally at rest, determine the horizontal distance d the ball is from the initial position of the gun at the instant the ball strikes the ground at D. Neglect the size of the gun.





$(\overset{+}{\rightarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0 = 1(v_B)_x - 5v_G$$

$$(v_B)_x = 5v_G$$

$(\overset{+}{\rightarrow})$ $\quad v_B = v_G + v_{B/G}$

$$5v_G = -v_G + 6\cos 30°$$

$$v_G = 0.8660 \text{ m/s} \leftarrow$$

So that,

$(v_B)_x = 4.330 \text{ m/s} \rightarrow$

$(v_B)_y = 4.330 \tan 30° = 2.5 \text{ m/s} \uparrow$

Time of flight for the ball:

$(+\uparrow)$ $\quad v = v_0 + a_c t$

$$-2.5 = 2.5 - 9.81t$$

$$t = 0.5097 \text{ s}$$

Distance ball travels:

$(\overset{+}{\rightarrow})$ $\quad s = v_0 t$

$$s = 4.330(0.5097) = 2.207 \text{ m} \rightarrow$$

Distance gun travels:

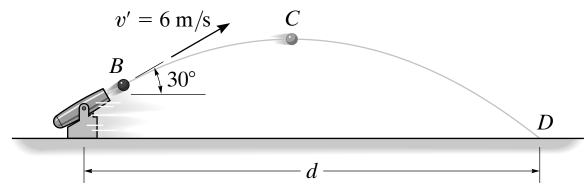
$(\overset{+}{\leftarrow})$ $\quad s = v_0 t$

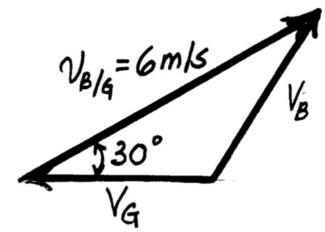
$$s' = 0.8660(0.5097) = 0.4414 \text{ m} \leftarrow$$

Thus,

$d = 2.207 + 0.4414 = 2.65$ m **Ans.**

15–50. The 5-kg spring-loaded gun rests on the smooth surface. It fires a ball having a mass of 1 kg with a velocity of $v' = 6$ m/s relative to the gun in the direction shown. If the gun is originally at rest, determine the distance the ball is from the initial position of the gun at the instant the ball reaches its highest elevation C. Neglect the size of the gun.





$(\overset{+}{\rightarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0 = 1(v_B)_x - 5v_G$$

$$(v_B)_x = 5v_G$$

$(\overset{+}{\rightarrow})$ $\quad v_B = v_G + v_{B/G}$

$$5v_G = -v_G + 6\cos 30^\circ$$

$$v_G = 0.8660 \text{ m/s} \leftarrow$$

So that,

$(v_B)_x = 4.330 \text{ m/s} \rightarrow$

$(v_B)_y = 4.330 \tan 30^\circ = 2.5 \text{ m/s} \uparrow$

Time of flight for the ball:

$(+\uparrow)$ $\quad v = v_0 + a_c t$

$$0 = 2.5 - 9.81t$$

$$t = 0.2548 \text{ s}$$

Height of ball:

$(+\uparrow)$ $\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$$0 = (2.5)^2 - 2(9.81)(h - 0)$$

$$h = 0.3186 \text{ m}$$

Distance ball travels:

$(\overset{+}{\rightarrow})$ $\quad s = v_0 t$

$$s = 4.330(0.2548) = 1.103 \text{ m} \rightarrow$$

Distance gun travels:

$(\overset{+}{\leftarrow})$ $\quad s = v_0 t$

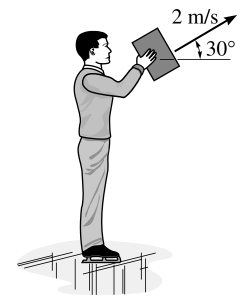
$$s' = 0.8660(0.2548) = 0.2207 \text{ m} \leftarrow$$

$1.103 + 0.2207 = 1.324$ m

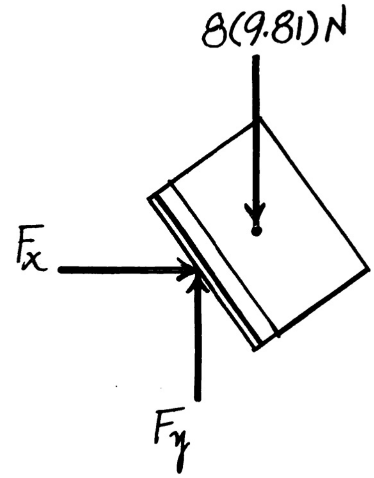
Distance from cannon to ball:

$\sqrt{(0.4587)^2 + (1.324)^2} = 1.36$ m **Ans.**

15–51. A man wearing ice skates throws an 8-kg block with an initial velocity of 2 m/s, measured relative to himself, in the direction shown. If he is originally at rest and completes the throw in 1.5 s while keeping his legs rigid, determine the horizontal velocity of the man just after releasing the block. What is the vertical reaction of both his skates on the ice during the throw? The man has a mass of 70 kg. Neglect friction and the motion of his arms.



$$\left(\xrightarrow{+}\right) \qquad 0 = -m_M\, v_M + m_B\, (v_B)_x \qquad \textbf{(1)}$$



However, $\mathbf{v}_B = \mathbf{v}_M + \mathbf{v}_{B/M}$

$$\left(\xrightarrow{+}\right) \qquad (v_B)_x = -v_M + 2\cos 30° \qquad \textbf{(2)}$$

$$(+\uparrow) \qquad (v_B)_y = 0 + 2\sin 30° = 1 \text{ m/s}$$

Substituting Eq. (2) into (1) yields:

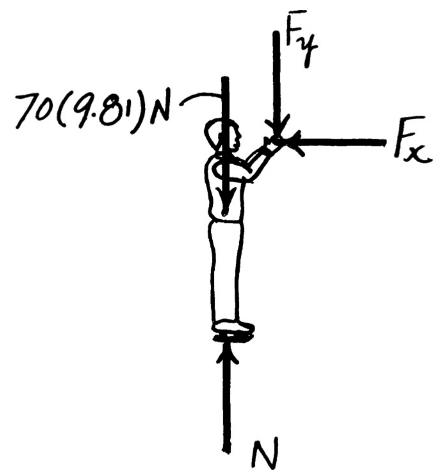
$$0 = -m_M\, v_M + m_B\,(-v_M + 2\cos 30°)$$

$$v_M = \frac{2m_B \cos 30°}{m_B + m_M} = \frac{2(8)\cos 30°}{8 + 70} = 0.178\text{m/s} \qquad \textbf{Ans.}$$

For the block:

$$(+\uparrow) \qquad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

$$0 + F_y(1.5) - 8(9.81)(1.5) = 8(2\sin 30°) \qquad F_y = 83.81 \text{ N}$$

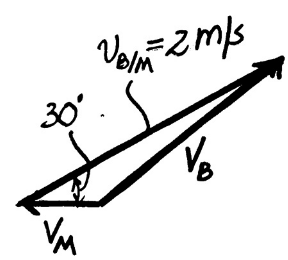


For the man:

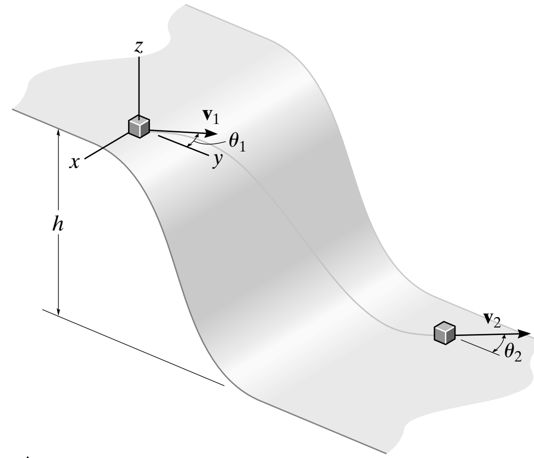
$$(+\uparrow) \qquad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

$$0 + N(1.5) - 70(9.81)(1.5) - 83.81(1.5) = 0$$

$$N = 771 \text{ N} \qquad \textbf{Ans.}$$



***15–52.** The block of mass m travels at v_1 in the direction θ_1 shown at the top of the smooth slope. Determine its speed v_2 and its direction θ_2 when it reaches the bottom.



There are no impulses in the : direction:

$$mv_1 \sin\theta_1 = mv_2 \sin\theta_2$$

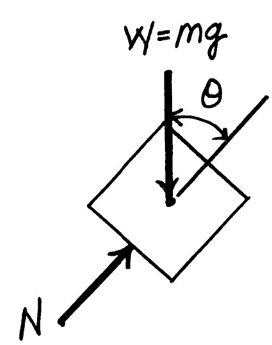
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1^2 + mgh = \frac{1}{2}mv_2^2 + 0$$

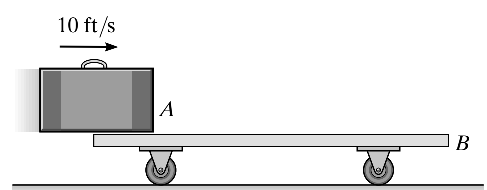
$$v_2 = \sqrt{v_1^2 + 2gh}$$ **Ans.**

$$\sin\theta_2 = \frac{v_t \sin\theta_1}{\sqrt{v_1^2 + 2gh}}$$

$$\theta_2 = \sin^{-1}\left(\frac{v_1 \sin\theta_1}{\sqrt{v_1^2 + 2gh}}\right)$$ **Ans.**



•15–53. The 20-lb cart B is supported on rollers of negligible size. If a 10-lb suitcase A is thrown horizontally onto the cart at 10 ft/s when it is at rest, determine the length of time that A slides relative to B, and the final velocity of A and B. The coefficient of kinetic friction between A and B is $\mu_k = 0.4$.



System

$(\overset{+}{\rightarrow})$ $\quad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$\left(\frac{10}{32.2}\right)(10) + 0 = \left(\frac{10 + 20}{32.2}\right)v$$

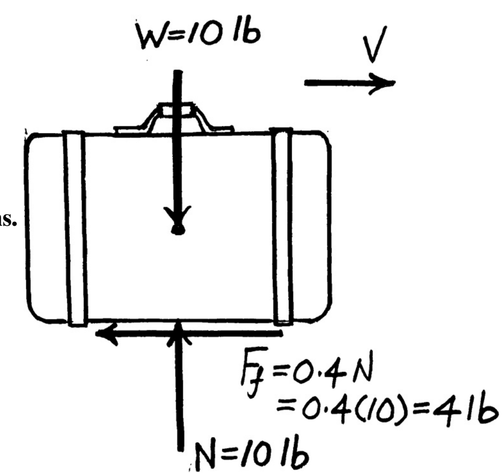
$$v = 3.33 \text{ ft/s}$$ **Ans.**

For A:

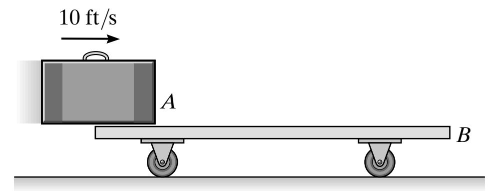
$$m\,v_1 + \Sigma\int F\,dt = m\,v_2$$

$$\left(\frac{10}{32.2}\right)(10) - 4t = \left(\frac{10}{32.2}\right)(3.33)$$

$$t = 0.5176 = 0.518 \text{ s}$$ **Ans.**



15–54. The 20-lb cart B is supported on rollers of negligible size. If a 10-lb suitcase A is thrown horizontally onto the cart at 10 ft/s when it is at rest, determine the time t and the distance B moves at the instant A stops relative to B. The coefficient of kinetic friction between A and B is $\mu_k = 0.4$.

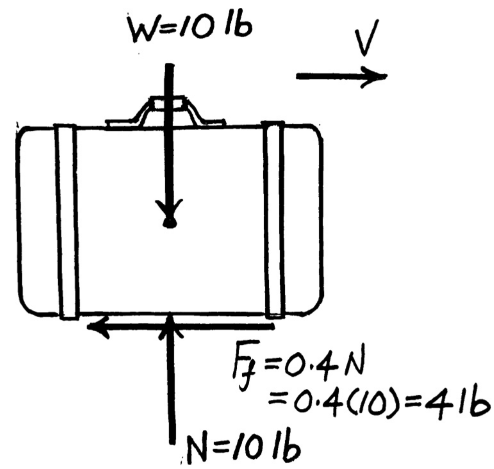


System:

$$\left(\xrightarrow{+}\right) \qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$$

$$\left(\frac{10}{32.2}\right)(10) + 0 = \left(\frac{10 + 20}{32.2}\right)v$$

$$v = 3.33 \text{ ft/s}$$



For A:

$$m v_1 + \Sigma \int F\, dt = m v_2$$

$$\left(\frac{10}{32.2}\right)(10) - 4t = \left(\frac{10}{32.2}\right)(3.33)$$

$$t = 0.5176 = 0.518 \text{ s}$$ **Ans.**

For B:

$$\left(\xrightarrow{+}\right) \qquad v = v_0 + a_c t$$

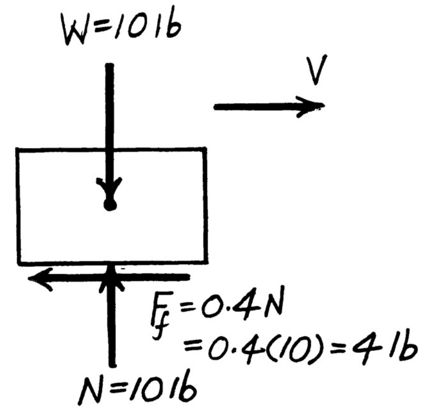
$$3.33 = 0 + a_c\,(0.5176)$$

$$a_c = 6.440 \text{ ft/s}^2$$

$$\left(\xrightarrow{+}\right) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s = 0 + 0 + \frac{1}{2}(6.440)(0.5176)^2 = 0.863 \text{ ft}$$ **Ans.**

15–55. A 1-lb ball A is traveling horizontally at 20 ft/s when it strikes a 10-lb block B that is at rest. If the coefficient of restitution between A and B is $e = 0.6$, and the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the time for the block B to stop sliding.



$$(\overset{+}{\rightarrow}) \qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$$

$$\left(\frac{1}{32.2}\right)(20) + 0 = \left(\frac{1}{32.2}\right)(v_A)_2 + \left(\frac{10}{32.2}\right)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 20$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.6 = \frac{(v_B)_2 - (v_A)_2}{20 - 0}$$

$$(v_B)_2 - (v_A)_2 = 12$$

Thus,

$(v_B)_2 = 2.909 \text{ ft/s} \rightarrow$

$(v_A)_2 = -9.091 \text{ ft/s} = 9.091 \text{ ft/s} \leftarrow$

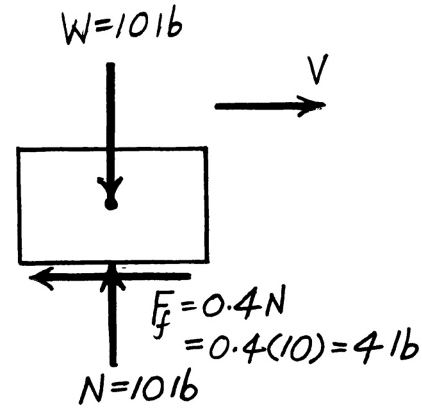
Block B:

$$(\overset{+}{\rightarrow}) \qquad m\, v_1 + \Sigma \int F\, dt = m\, v_2$$

$$\left(\frac{10}{32.2}\right)(2.909) - 4t = 0$$

$$t = 0.226 \text{ s} \qquad \textbf{Ans.}$$

***15–56.** A 1-lb ball A is traveling horizontally at 20 ft/s when it strikes a 10-lb block B that is at rest. If the coefficient of restitution between A and B is $e = 0.6$, and the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the distance block B slides on the plane before it stops sliding.



$$(\overset{+}{\rightarrow}) \qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$$

$$\left(\frac{1}{32.2}\right)(20) + 0 = \left(\frac{1}{32.2}\right)(v_A)_2 + \left(\frac{10}{32.2}\right)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 20$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.6 = \frac{(v_B)_2 - (v_A)_2}{20 - 0}$$

$$(v_B)_2 - (v_A)_2 = 12$$

Thus,

$(v_B)_2 = 2.909 \text{ ft/s} \rightarrow$

$(v_A)_2 = -9.091 \text{ ft/s} = 9.091 \text{ ft/s} \leftarrow$

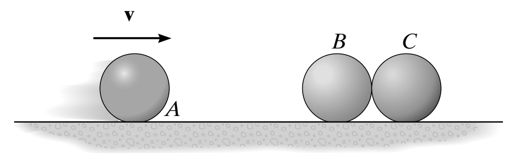
Block B:

$T_1 + \Sigma U_{1-2} = T_2$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(2.909)^2 - 4d = 0$$

$d = 0.329$ ft **Ans.**

•15–57. The three balls each have a mass m. If A has a speed v just before a direct collision with B, determine the speed of C after collision. The coefficient of restitution between each ball is e. Neglect the size of each ball.



Conservation of Momentum: When ball A strikes ball B, we have

$$m_A (v_A)_1 + m_B (v_B)_1 = m_A (v_A)_2 + m_B (v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad mv + 0 = m(v_A)_2 + m(v_B)_2 \qquad \textbf{[1]}$$

Coefficient of Restitution:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{v - 0} \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$(v_A)_2 = \frac{v(1 - e)}{2} \qquad (v_B)_2 = \frac{v(1 + e)}{2}$$

Conservation of Momentum: When ball B strikes ball C, we have

$$m_B (v_B)_2 + m_C (v_C)_1 = m_B (v_B)_3 + m_C (v_C)_2$$

$$(\overset{+}{\rightarrow}) \qquad m\left[\frac{v(1 + e)}{2}\right] + 0 = m(v_B)_3 + m(v_C)_2 \qquad \textbf{[3]}$$

Coefficient of Restitution:

$$e = \frac{(v_C)_2 - (v_B)_3}{(v_B)_2 - (v_C)_1}$$

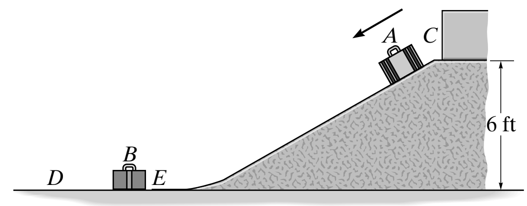
$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_C)_2 - (v_B)_3}{\frac{v(1 + e)}{2} - 0} \qquad \textbf{[4]}$$

Solving Eqs. [3] and [4] yields

$$(v_C)_2 = \frac{v(1 + e)^2}{4} \qquad \textbf{Ans.}$$

$$(v_B)_3 = \frac{v(1 - e^2)}{4}$$

15–58. The 15-lb suitcase A is released from rest at C. After it slides down the smooth ramp, it strikes the 10-lb suitcase B, which is originally at rest. If the coefficient of restitution between the suitcases is $e = 0.3$ and the coefficient of kinetic friction between the floor DE and each suitcase is $\mu_k = 0.4$, determine (a) the velocity of A just before impact, (b) the velocities of A and B just after impact, and (c) the distance B slides before coming to rest.



Conservation of Energy: The datum is set at lowest point E. When the suitcase A is at point C it is 6 ft *above* the datum. Its gravitational potential energy is $15(6) = 90.0$ ft · lb. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 90.0 = \frac{1}{2}\left(\frac{15}{32.2}\right)(v_A)_1^2 + 0$$

$$(v_A)_1 = 19.66 \text{ ft/s} = 19.7 \text{ ft/s} \qquad \textbf{Ans.}$$

Conservation of Momentum:

$$m_A\,(v_A)_1 + m_B\,(v_B)_1 = m_A\,(v_A)_2 + m_B\,(v_B)_2$$

$$(\overset{+}{\leftarrow}) \quad \left(\frac{15}{32.2}\right)(19.66) + 0 = \left(\frac{15}{32.2}\right)(v_A)_2 + \left(\frac{10}{32.2}\right)(v_B)_2 \qquad \textbf{[1]}$$

Coefficient of Restitution:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\leftarrow}) \quad 0.3 = \frac{(v_B)_2 - (v_A)_2}{19.66 - 0} \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$(v_A)_2 = 9.435 \text{ ft/s} = 9.44 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

$$(v_B)_2 = 15.33 \text{ ft/s} = 15.3 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

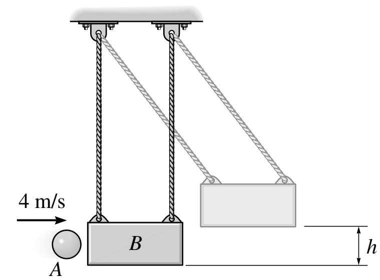
Principle of Work and Energy: $N_B = 10.0$ lb. Thus, the friction $F_f = \mu_k N_B = 0.4(10.0) = 4.00$ lb. The friction F_f which acts in the opposite direction to that of displacement does *negative* work. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)\left(15.33^2\right) + (-4.00 s_B) = 0$$

$$s_B = 9.13 \text{ ft} \qquad \textbf{Ans.}$$

15–59. The 2-kg ball is thrown at the suspended 20-kg block with a velocity of 4 m/s. If the coefficient of restitution between the ball and the block is $e = 0.8$, determine the maximum height h to which the block will swing before it momentarily stops.



System:

$$\left(\xrightarrow{+}\right) \qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$$

$$(2)(4) + 0 = (2)(v_A)_2 + (20)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 4$$

$$\left(\xrightarrow{+}\right) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.8 = \frac{(v_B)_2 - (v_A)_2}{4 - 0}$$

$$(v_B)_2 - (v_A)_2 = 3.2$$

Solving:

$(v_A)_2 = -2.545$ m/s

$(v_B)_2 = 0.6545$ m/s

Block:

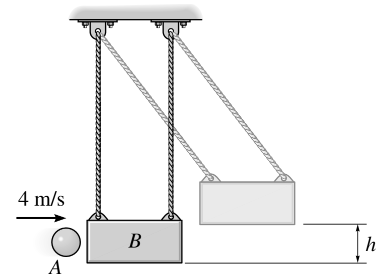
Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(20)(0.6545)^2 + 0 = 0 + 20(9.81)h$$

$h = 0.0218 \text{ m} = 21.8 \text{ mm}$ **Ans.**

***15–60.** The 2-kg ball is thrown at the suspended 20-kg block with a velocity of 4 m/s. If the time of impact between the ball and the block is 0.005 s, determine the average normal force exerted on the block during this time. Take $e = 0.8$.



System:

$$(\overset{+}{\rightarrow}) \qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$$

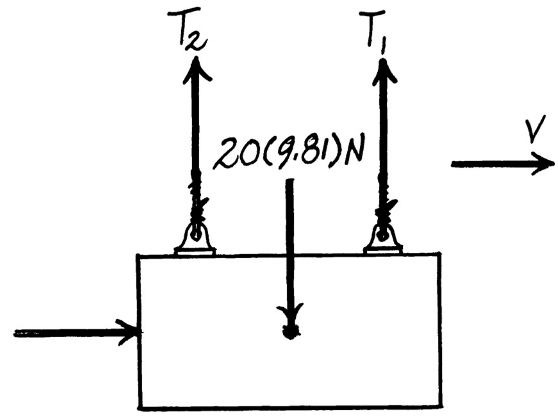
$$(2)(4) + 0 = (2)(v_A)_2 + (20)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 4$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.8 = \frac{(v_B)_2 + (v_A)_2}{4 - 0}$$

$$(v_B)_2 - (v_A)_2 = 3.2$$



Solving:

$(v_A)_2 = -2.545$ m/s

$(v_B)_2 = 0.6545$ m/s

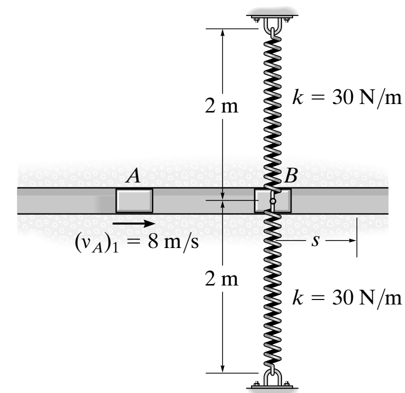
Block:

$$(\overset{+}{\rightarrow}) \qquad m v_1 + \Sigma \int F\, dt = m v_2$$

$$0 + F(0.005) = 20(0.6545)$$

$$F = 2618 \text{ N} = 2.62 \text{ kN} \qquad \textbf{Ans.}$$

•15–61. The slider block B is confined to move within the smooth slot. It is connected to two springs, each of which has a stiffness of $k = 30$ N/m. They are originally stretched 0.5 m when $s = 0$ as shown. Determine the maximum distance, s_{max}, block B moves after it is hit by block A which is originally traveling at $(v_A)_1 = 8$ m/s. Take $e = 0.4$ and the mass of each block to be 1.5 kg.



$(\xrightarrow{+})$ $\quad \Sigma m\, v_1 = \Sigma m\, v_2$

$$(1.5)(8) + 0 = (1.5)(v_A)_2 + (1.5)(v_B)_2$$

$(\xrightarrow{+})$ $\quad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$$0.4 = \frac{(v_B)_2 - (v_A)_2}{8 - 0}$$

Solving:

$(v_A)_2 = 2.40$ m/s

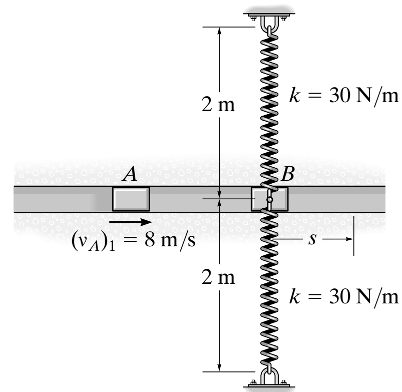
$(v_B)_2 = 5.60$ m/s

$T_1 + V_1 + T_2 + V_2$

$$\frac{1}{2}(1.5)(5.60)^2 + 2\left[\frac{1}{2}(30)(0.5)^2\right] = 0 + 2\left[\frac{1}{2}(3)\left(\sqrt{s_{max}^2 + 2^2} - 1.5\right)^2\right]$$

$s_{max} = 1.53$ m **Ans.**

15–62. In Prob. 15–61 determine the average net force between blocks A and B during impact if the impact occurs in 0.005 s.



$(\xrightarrow{+})$ $$\Sigma m\, v_1 = \Sigma m\, v_2$$

$$(1.5)(8) + 0 = (1.5)(v_A)_2 + (1.5)(v_B)_2$$

$(\xrightarrow{+})$ $$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

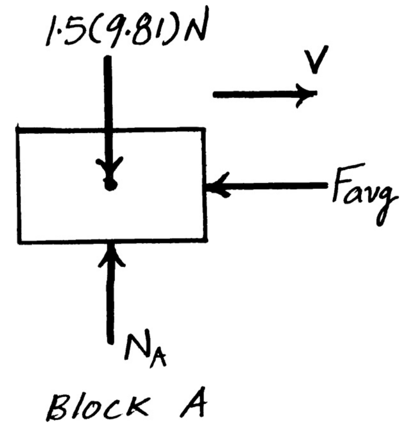
$$0.4 = \frac{(v_B)_2 - (v_A)_2}{8 - 0}$$

Solving:

$(v_A)_2 = 2.40$ m/s

$(v_B)_2 = 5.60$ m/s

Choosing block A:

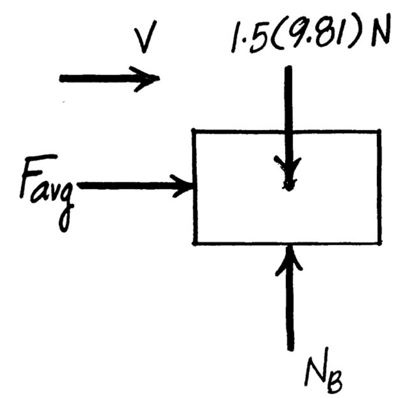


$(\xrightarrow{+})$ $$m v_1 + \Sigma \int F\, dt = m v_2$$

$$(1.5)(8) - F_{\text{avg}}(0.005) = 1.5(2.40)$$

$F_{\text{avg}} = 1.68$ kN **Ans.**

Choosing block B:

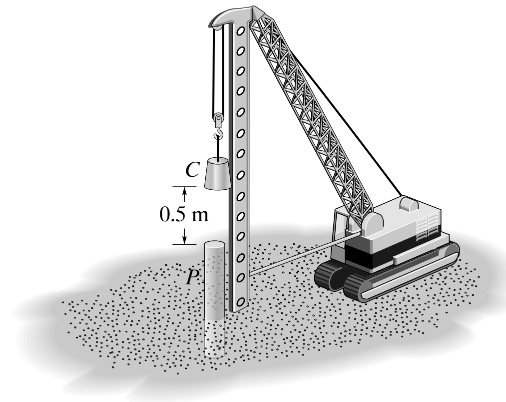


$(\xrightarrow{+})$ $$m v_1 + \Sigma \int F\, dt = m v_2$$

$$0 + F_{\text{avg}}(0.005) = 1.5(5.60)$$

$F_{\text{avg}} = 1.68$ kN **Ans.**

15–63. The pile P has a mass of 800 kg and is being driven into *loose sand* using the 300-kg hammer C which is dropped a distance of 0.5 m from the top of the pile. Determine the initial speed of the pile just after it is struck by the hammer. The coefficient of restitution between the hammer and the pile is $e = 0.1$. Neglect the impulses due to the weights of the pile and hammer and the impulse due to the sand during the impact.



The force of the sand on the pile can be considered nonimpulsive, along with the weights of each colliding body. Hence,

Counter weight: Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 300(9.81)(0.5) = \frac{1}{2}(300)(v)^2 + 0$$

$$v = 3.1321 \text{ m/s}$$

System:

$$(+\downarrow) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$300(3.1321) + 0 = 300(v_C)_2 + 800(v_P)_2$$

$$(v_C)_2 + 2.667(v_P)_2 = 3.1321$$

$$(+\downarrow) \qquad e = \frac{(v_P)_2 - (v_C)_2}{(v_C)_1 - (v_P)_1}$$

$$0.1 = \frac{(v_P)_2 - (v_C)_2}{3.1321 - 0}$$

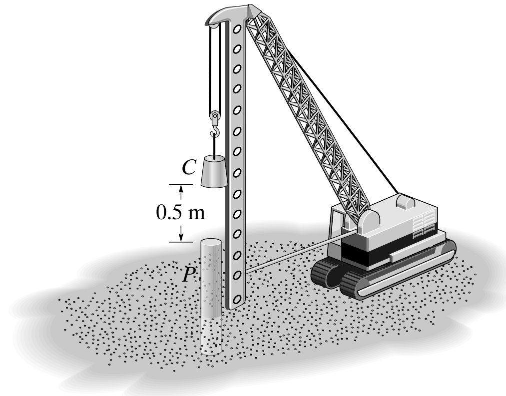
$$(v_P)_2 - (v_C)_2 = 0.31321$$

Solving:

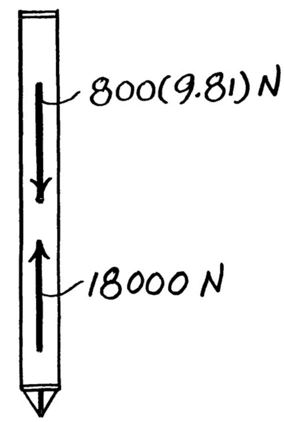
$(v_P)_2 = 0.940$ m/s **Ans.**

$(v_C)_2 = 0.626$ m/s

***15–64.** The pile P has a mass of 800 kg and is being driven into *loose sand* using the 300-kg hammer C which is dropped a distance of 0.5 m from the top of the pile. Determine the distance the pile is driven into the sand after one blow if the sand offers a frictional resistance against the pile of 18 kN. The coefficient of restitution between the hammer and the pile is $e = 0.1$. Neglect the impulses due to the weights of the pile and hammer and the impulse due to the sand during the impact.



The force of the sand on the pile can be considered nonimpulsive, along with the weights of each colliding body. Hence,



Counter weight: Datum at lowest point,

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 300(9.81)(0.5) = \frac{1}{2}(300)(v)^2 + 0$$

$$v = 3.1321 \text{ m/s}$$

System:

$$(+\downarrow) \qquad \Sigma m\, v_1 = \Sigma m\, v_2$$

$$300(3.1321) + 0 = 300(v_C)_2 + 800(v_P)_2$$

$$(v_C)_2 + 2.667(v_P)_2 = 3.1321$$

$$(+\downarrow) \qquad e = \frac{(v_P)_2 - (v_C)_2}{(v_C)_1 - (v_P)_1}$$

$$0.1 = \frac{(v_P)_2 - (v_C)_2}{3.1321 - 0}$$

$$(v_P)_2 - (v_C)_2 = 0.31321$$

Solving:

$$(v_P)_2 = 0.9396 \text{ m/s}$$

$$(v_C)_2 = 0.6264 \text{ m/s}$$

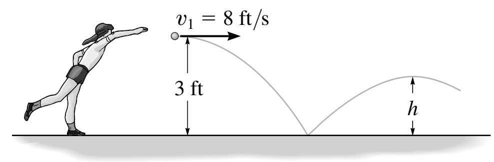
Pile:

$$T_2 + \Sigma U_{2-3} = T_3$$

$$\frac{1}{2}(800)(0.9396)^2 + 800(9.81)d - 18\,000d = 0$$

$d = 0.0348 \text{ m} = 34.8 \text{ mm}$ **Ans.**

•15–65. The girl throws the ball with a horizontal velocity of $v_1 = 8$ ft/s. If the coefficient of restitution between the ball and the ground is $e = 0.8$, determine (a) the velocity of the ball just after it rebounds from the ground and (b) the maximum height to which the ball rises after the first bounce.



Kinematics: By considering the vertical motion of the falling ball, we have

$$(+\downarrow) \qquad (v_1)_y^2 = (v_0)_y^2 + 2a_c\left[s_y - (s_0)_y\right]$$

$$(v_1)_y^2 = 0^2 + 2(32.2)(3 - 0)$$

$$(v_1)_y = 13.90 \text{ ft/s}$$

Coefficient of Restitution (*y*):

$$e = \frac{\left(v_g\right)_2 - (v_2)_y}{(v_1)_y - \left(v_g\right)_1}$$

$$(+\uparrow) \qquad 0.8 = \frac{0 - (v_2)_y}{-13.90 - 0}$$

$$(v_2)_y = 11.12 \text{ ft/s}$$

Conservation of "*x*" Momentum: The momentum is conserved along the *x* axis.

$$\left(\xrightarrow{+}\right) \qquad m(v_x)_1 = m(v_x)_2\,; \qquad (v_x)_2 = 8 \text{ ft/s} \rightarrow$$

The magnitude and the direction of the rebounding velocity for the ball is

$$v_2 = \sqrt{(v_x)_2^2 + \left(v_y\right)_2^2} = \sqrt{8^2 + 11.12^2} = 13.7 \text{ ft/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{11.12}{8}\right) = 54.3° \qquad \textbf{Ans.}$$

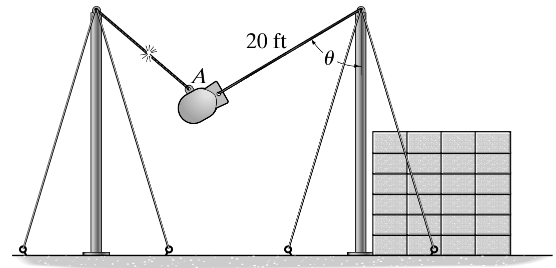
Kinematics: By considering the vertical motion of tfie ball after it rebounds from the ground, we have

$$(+\uparrow) \qquad (v)_y^2 = (v_2)_y^2 + 2a_c\left[s_y - (s_2)_y\right]$$

$$0 = 11.12^2 + 2(-32.2)(h - 0)$$

$$h = 1.92 \text{ ft} \qquad \textbf{Ans.}$$

15–66. During an impact test, the 2000-lb weight is released from rest when $\theta = 60°$. It swings downwards and strikes the concrete blocks, rebounds and swings back up to $\theta = 15°$ before it momentarily stops. Determine the coefficient of restitution between the weight and the blocks. Also, find the impulse transferred between the weight and blocks during impact. Assume that the blocks do not move after impact.



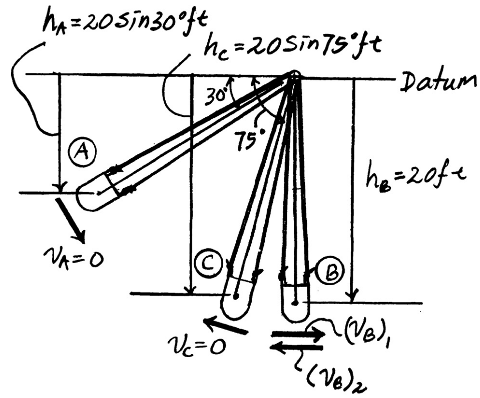
Conservation of Energy: First, consider the weight's fall from position A to position B as shown in Fig. a,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}m_A v_A{}^2 + \left(V_g\right)_A = \frac{1}{2}m(v_B)_1{}^2 + \left(V_g\right)_B$$

$$0 + [-2000(20\sin 30°)] = \frac{1}{2}\left(\frac{2000}{32.2}\right)(v_B)_1{}^2 + [-2000(20)]$$

$$(v_B)_1 = 25.38\,\text{ft/s} \rightarrow$$



(a)

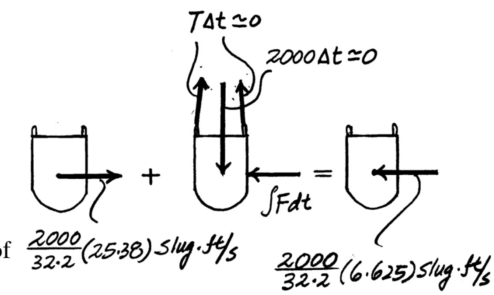
Subsequently, we will consider the weight rebounds from position B to position C.

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}m(v_B)_1{}^2 + \left(V_g\right)_B = \frac{1}{2}m v_C{}^2 + \left(V_g\right)_C$$

$$\frac{1}{2}\left(\frac{2000}{32.2}\right)(v_B)_1{}^2 + [-2000(20)] = 0 + [-2000(20\sin 75°)]$$

$$(v_B)_2 = 6.625\text{ ft/s} \leftarrow$$



(b)

Coefficient of Restitution: Since the concrete blocks do not move, the coefficient of restitution can be written as

$$(\overset{+}{\rightarrow}) \qquad e = -\frac{(v_B)_2}{(v_B)_1} = \frac{(-6.625)}{25.38} = 0.261 \qquad \textbf{Ans.}$$

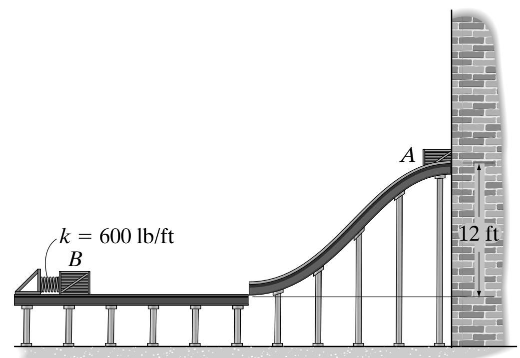
Principle of Impulse and Momentum: By referring to the Impulse and momentum diagrams shown in Fig. b,

$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma\int_{t_2}^{t_1} F_x dt = m(v_2)_x$$

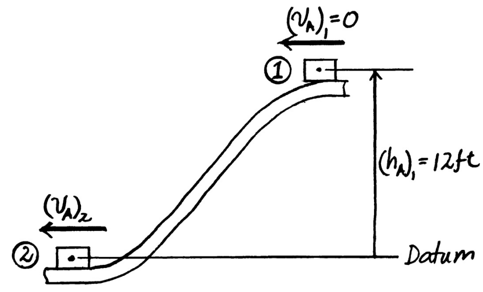
$$\frac{2000}{32.2}(25.38) - \int F dt = -\frac{2000}{32.2}(6.625)$$

$$\int F dt = 1987.70\text{ lb}\cdot\text{s} = 1.99\text{ kip}\cdot\text{s} \qquad \textbf{Ans.}$$

15–67. The 100-lb crate A is released from rest onto the smooth ramp. After it slides down the ramp it strikes the 200-lb crate B that rests against the spring of stiffness $k = 600$ lb/ft. If the coefficient of restitution between the crates is $e = 0.5$, determine their velocities just after impact. Also, what is the spring's maximum compression? The spring is originally unstretched.



Conservation of Energy: By considering crate A's fall from position (1) to position (2) as shown in Fig. a,



(a)

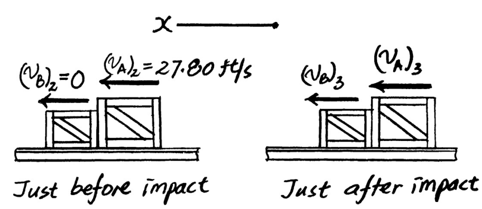
$$(T_A)_1 + (V_A)_1 = (T_A)_2 + (V_A)_2$$

$$\frac{1}{2}m_A(v_A)_1{}^2 + \left(V_g\right)_1 = \frac{1}{2}m_A(v_A)_2{}^2 + \left(V_g\right)_2$$

$$0 + 100(12) = \frac{1}{2}\left(\frac{100}{32.2}\right)(v_A)_2{}^2 + 0$$

$$(v_A)_2 = 27.80 \text{ ft/s}$$

Conservation of Linear Momentum: The linear momentum of the system is conserved along the x axis (line of impact). By referring to Fig. b,



(b)

$$(\overset{+}{\leftarrow}) \qquad m_A(v_A)_2 + m_B(v_B)_2 = m_A(v_A)_3 + m_B(v_B)_3$$

$$\left(\frac{100}{32.2}\right)(27.80) + 0 = \left(\frac{100}{32.2}\right)(v_A)_3 + \left(\frac{200}{32.2}\right)(v_B)_3$$

$$100(v_A)_3 + 200(v_B)_3 = 2779.93 \qquad \textbf{(1)}$$

Coefficient of Restitution:

$$(\overset{+}{\leftarrow}) \qquad e = \frac{(v_B)_3 - (v_A)_3}{(v_A)_2 - (v_B)_2}$$

$$0.5 = \frac{(v_B)_3 - (v_A)_3}{27.80 - 0}$$

$$(v_B)_3 - (v_A)_3 = 13.90 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$$(v_B)_3 = 13.90 \text{ ft/s} = 13.9 \text{ ft/s} \leftarrow \qquad (v_A)_3 = 0 \qquad \textbf{Ans.}$$

Conservation of Energy: The maximum compression of the spring occurs when crate B momentarily stops. By considering the conservation of energy of crate B,

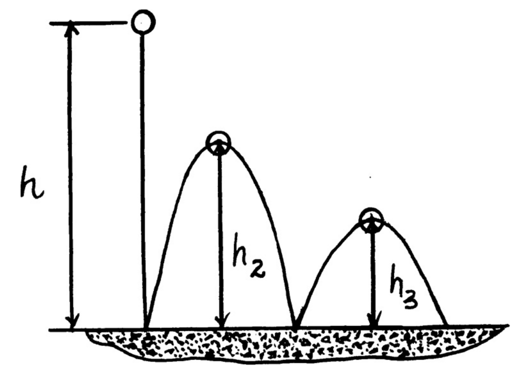
$$(T_B)_3 + (V_B)_3 = (T_B)_4 + (V_B)_4$$

$$\frac{1}{2}m_B(v_B)_3{}^2 + \frac{1}{2}ks_3{}^2 = \frac{1}{2}m_B(v_B)_4{}^2 + \frac{1}{2}ks_{\max}{}^2$$

$$\frac{1}{2}\left(\frac{200}{32.2}\right)(13.90^2) + 0 = 0 + \frac{1}{2}(600)s_{\max}{}^2$$

$$s_{\max} = 1.41 \text{ ft} \qquad \textbf{Ans.}$$

***15–68.** A ball has a mass m and is dropped onto a surface from a height h. If the coefficient of restitution is e between the ball and the surface, determine the time needed for the ball to stop bouncing.



Just before impact:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + mgh = \frac{1}{2}mv^2 + 0$$

$$v = \sqrt{2gh}$$

Time to fall:

$$(+\downarrow) \qquad v = v_0 + a_c t$$

$$v = v_0 + gt_1$$

$$\sqrt{2gh} = 0 + gt_1$$

$$t_1 = \sqrt{\frac{2h}{g}}$$

After impact:

$$(+\uparrow) \qquad e = \frac{v^2}{v}$$

$$v_2 = e\sqrt{2gh}$$

Height after first bounce: Datum at lowest point

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}m\left(e\sqrt{2gh}\right)^2 + 0 = 0 + mgh_2$$

$$h_2 = \frac{1}{2}e^2\left(\frac{2\,gh}{g}\right) = e^2h$$

Time to rise to h_2:

$$(+\uparrow) \qquad v = v_0 + a_c t$$

$$v_3 = v_2 - gt_2$$

$$0 = e\sqrt{2gh} - gt_2$$

$$t_2 = e\sqrt{\frac{2h}{g}}$$

***15–68. Continued**

Total time for first bounce

$$t_{1b} = t_1 + t_2 = \sqrt{\frac{2h}{g}} + e\sqrt{\frac{2h}{g}} = \sqrt{\frac{2h}{g}}(1 + e)$$

For the second bounce.

$$t_{2b} = \sqrt{\frac{2h_2}{g}}(1 + e) = \sqrt{\frac{2gh}{g}}(1 + e)e$$

For the third bounce.

$$h_3 = e^2 h_2 = e^2\left(e^2 h\right) = e^4 h$$

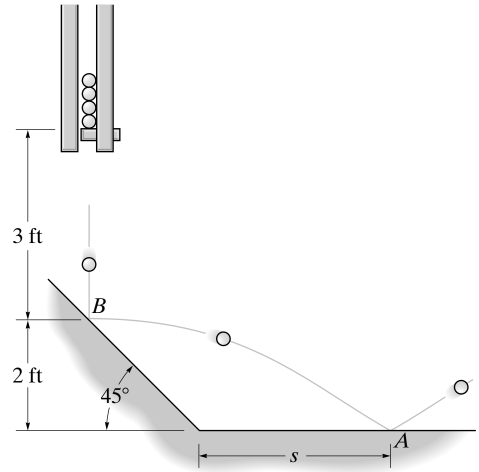
$$t_{3b} = \sqrt{\frac{2h_3}{g}}(1 + e) = \sqrt{\frac{2h}{g}}(1 + e)e^2$$

Thus the total time for an infinite number of bounces:

$$t_{tot} = \sqrt{\frac{2h}{g}}(1 + e)\left(1 + e + e^2 + e^3 + \dots\right)$$

$$t_{tot} = \sqrt{\frac{2h}{g}}\left(\frac{1 + e}{1 - e}\right)$$ **Ans.**

•15–69. To test the manufactured properties of 2-lb steel balls, each ball is released from rest as shown and strikes the 45° smooth inclined surface. If the coefficient of restitution is to be $e = 0.8$, determine the distance s to where the ball strikes the horizontal plane at A. At what speed does the ball strike point A?



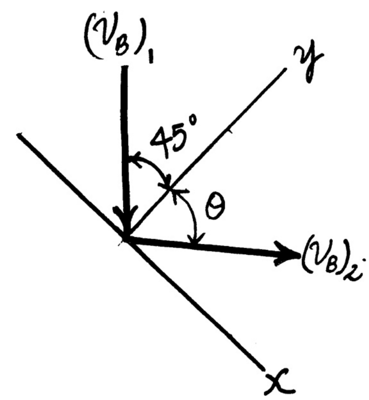
Just before impact Datum at lowest point

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (2)(3) = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_B)_1^2 + 0$$

$$(v_B)_1 = 13.900 \text{ ft/s}$$

At B:



$$(+\searrow) \qquad \Sigma m(v_B)_{x1} = \Sigma m(v_B)_{x2}$$

$$\left(\frac{2}{32.2}\right)(13.900) \sin 45° = \left(\frac{2}{32.2}\right)(v_B)_2 \sin \theta$$

$$(v_B)_2 \sin \theta = 9.829 \text{ ft/s} \qquad \textbf{(1)}$$

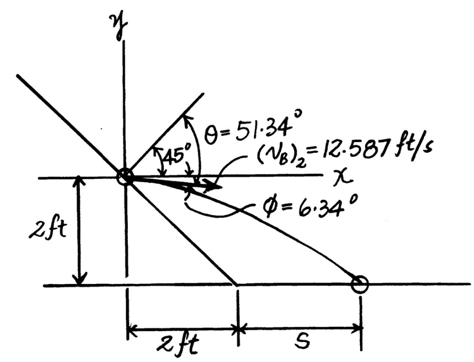
$$(+\nearrow) \qquad e = \frac{(v_B)_{y2} - 0}{0 - (v_B)_{y1}}$$

$$0.8 = \frac{(v_B)_2 \cos \theta - 0}{0 - (-13.900) \cos 45°}$$

$$(v_B)_2 \cos \theta = 7.863 \text{ ft/s} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2):

$$(v_B)_2 = 12.587 \text{ ft/s} \qquad \theta = 51.34°$$



$$\phi = 51.34° - 45° = 6.34°$$

$$(+\downarrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$\left(v_{A_r}\right)^2 = \left[12.587 \sin 6.34°\right]^2 + 2(32.2)(2 - 0)$$

$$v_{A_r} = 11.434 \text{ ft/s}$$

$$(+\downarrow) \qquad v = v_0 + a_c t$$

$$11.434 = 12.587 \sin 6.34° + 32.2t$$

$$t = 0.3119 \text{ s}$$

$$\left(\xrightarrow{+}\right) \qquad v_{Ax} = 12.587 \cos 6.34° = 12.510 \text{ ft/s}$$

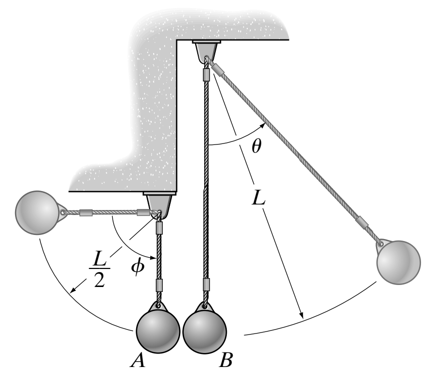
$$s_t = v_B t$$

$$s + \frac{2}{\tan 45°} = (12.51)(0.3119)$$

$$s = 1.90 \text{ ft} \qquad \textbf{Ans.}$$

$$v_A = \sqrt{(12.510)^2 + (11.434)^2} = 16.9 \text{ ft/s} \qquad \textbf{Ans.}$$

15–70. Two identical balls A and B of mass m are suspended from cords of length $L/2$ and L, respectively. Ball A is released from rest when $\phi = 90°$ and swings down to $\phi = 0°$, where it strikes B. Determine the speed of each ball just after impact and the maximum angle θ through which B will swing. The coefficient of restitution between the balls is e.



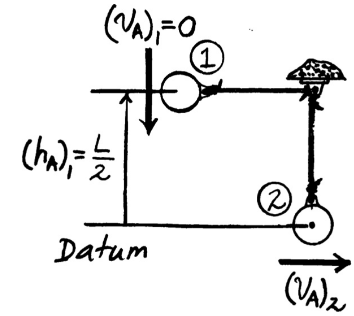
Conservation of Energy: First, we will consider bob A's swing from position (1) to position (2) as shown in Fig. a,

$$(T_A)_1 + (V_A)_1 = (T_A)_2 + (V_A)_2$$

$$\frac{1}{2}m_A(v_A)_1{}^2 + \left(V_g\right)_1 = \frac{1}{2}m_A(v_A)_2{}^2 + \left(V_g\right)_2$$

$$0 + mg\left(\frac{L}{2}\right) = \frac{1}{2}m(v_A)_2{}^2 + 0$$

$$(v_A)_2 = \sqrt{gL}$$



(a)

Conservation of Linear Momentum: The linear momentum of the system is conserved along the x axis (line of impact). By referring to Fig. b,

$$(\xrightarrow{+})\qquad m_A(v_A)_2 + m_B(v_B)_2 = m_A(v_A)_3 + m_B(v_B)_3$$

$$m\sqrt{gL} + 0 = m(v_A)_3 + m(v_B)_3$$

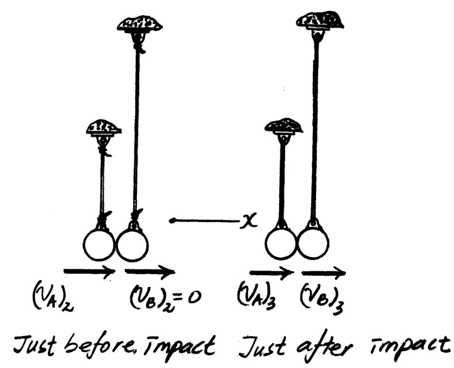
$$(v_A)_3 + (v_B)_3 = \sqrt{gL} \qquad \textbf{(1)}$$

Coefficient of Restitution: Applying Eq. 15–11 we have

$$(\xrightarrow{+})\qquad e = \frac{(v_B)_3 - (v_A)_3}{(v_A)_2 - (v_B)_2}$$

$$e = \frac{(v_B)_3 + (v_A)_3}{\sqrt{gL} - 0}$$

$$(v_B)_3 - (v_A)_3 = e\sqrt{gL} \qquad \textbf{(2)}$$



(b)

Solving Eqs. (1) and (2), yields

$$(v_A)_3 = \left(\frac{1-e}{2}\right)\sqrt{gL} \qquad \textbf{Ans.}$$

$$(v_B)_3 = \left(\frac{1+e}{2}\right)\sqrt{gL}$$

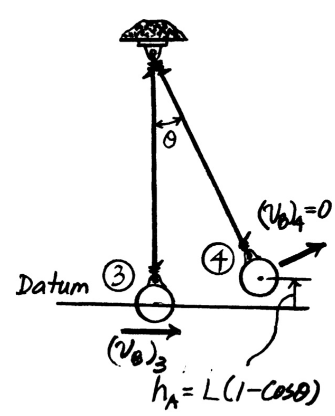
Conservation of Energy: We will now consider the swing of B from position (3) to position (4) as shown in Fig. c. Using the result of $(v_B)_3$,

$$(T_B)_3 + (V_B)_3 = (T_B)_4 + (V_B)_4$$

$$\frac{1}{2}m_B(v_B)_3{}^2 + \left(V_g\right)_3 = \frac{1}{2}m_B(v_B)_4{}^2 + \left(V_g\right)_4$$

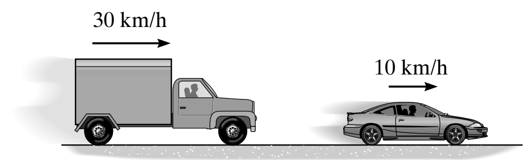
$$\frac{1}{2}m\left[\left(\frac{1+e}{2}\right)\sqrt{gL}\right]^2 + 0 = 0 + mg\left[L(1-\cos\theta)\right]$$

$$\theta = \cos^{-1}\left[1 - \frac{(1+e)^2}{8}\right] \qquad \textbf{Ans.}$$



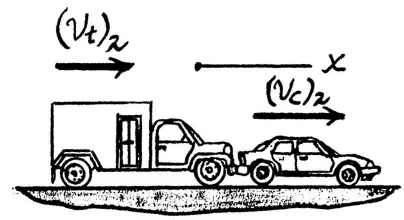
(C)

15–71. The 5-Mg truck and 2-Mg car are traveling with the free-rolling velocities shown just before they collide. After the collision, the car moves with a velocity of 15 km/h to the right *relative* to the truck. Determine the coefficient of restitution between the truck and car and the loss of energy due to the collision.



Conservation of Linear Momentum: The linear momentum of the system is conserved along the x axis (line of impact).

The initial speeds of the truck and car are $(v_t)_1 = \left[30\left(10^3\right)\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 8.333\text{ m/s}$ and $(v_c)_1 = \left[10\left(10^3\right)\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\,h}{3600\text{ s}}\right) = 2.778\text{ m/s}$.



Just after impact
(a)

By referring to Fig. *a*,

$$(\overset{+}{\rightarrow}) \qquad m_t\left(v_t\right)_1 + m_c\left(v_c\right)_1 = m_t\left(v_t\right)_2 + m_c\left(v_c\right)_2$$

$$5000(8.333) + 2000(2.778) = 5000\left(v_t\right)_2 + 2000\left(v_c\right)_2$$

$$5\left(v_t\right)_2 + 2\left(v_c\right)_2 = 47.22 \qquad \textbf{(1)}$$

Coefficient of Restitution: Here, $(v_{c/t}) = \left[15\left(10^3\right)\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 4.167\text{ m/s} \rightarrow$. Applying the relative velocity equation,

$$(\mathbf{v}_c)_2 = (\mathbf{v}_t)_2 + (\mathbf{v}_{c/t})_2$$

$$(\overset{+}{\rightarrow}) \qquad (v_c)_2 = (v_t)_2 + 4.167$$

$$(v_c)_2 - (v_t)_2 = 4.167 \qquad \textbf{(2)}$$

Applying the coefficient of restitution equation,

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_c)_2 - (v_t)_2}{(v_t)_1 - (v_c)_1}$$

$$e = \frac{(v_c)_2 - (v_t)_2}{8.333 - 2.778} \qquad \textbf{(3)}$$

15–71. Continued

Substituting Eq. (2) into Eq. (3),

$$e = \frac{4.167}{8.333 - 2.778} = 0.75$$ **Ans.**

Solving Eqs. (1) and (2) yields

$$(v_t)_2 = 5.556 \text{ m/s}$$

$$(v_c)_2 = 9.722 \text{ m/s}$$

Kinetic Energy: The kinetic energy of the system just before and just after the collision are

$$T_1 = \frac{1}{2} m_t (v_t)_1{}^2 + \frac{1}{2} m_c (v_c)_1{}^2$$

$$= \frac{1}{2}(5000)(8.333^2) + \frac{1}{2}(2000)(2.778^2)$$

$$= 181.33\left(10^3\right) \text{ J}$$

$$T_2 = \frac{1}{2} m_t (v_t)_2{}^2 + \frac{1}{2} m_c (v_c)_2{}^2$$

$$= \frac{1}{2}(5000)(5.556^2) + \frac{1}{2}(2000)(9.722^2)$$

$$= 171.68\left(10^3\right)\text{J}$$

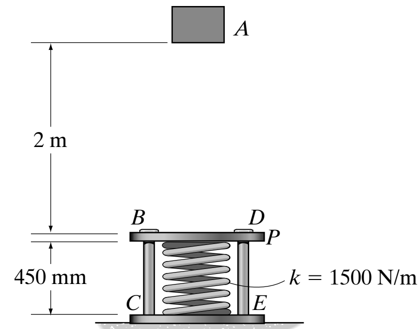
Thus,

$$\Delta E = T_1 - T_2 = 181.33\left(10^3\right) - 171.68\left(10^3\right)$$

$$= 9.645\left(10^3\right) \text{ J}$$

$$= 9.65\text{kJ}$$ **Ans.**

***15–72.** A 10-kg block A is released from rest 2 m above the 5-kg plate P, which can slide freely along the smooth vertical guides BC and DE. Determine the velocity of the block and plate just after impact. The coefficient of restitution between the block and the plate is $e = 0.75$. Also, find the maximum compression of the spring due to impact. The spring has an unstretched length of 600 mm.



Conservation of Energy: By considering block A's fall from position (1) to position (2) as shown in Fig. *a*,

$$(T_A)_1 + (V_A)_1 = (T_A)_2 + (V_A)_2$$

$$\frac{1}{2} m_A (v_A)_1{}^2 + \left(V_g\right)_1 = \frac{1}{2} m_A (v_A)_2{}^2 + \left(V_g\right)_2$$

$$0 + 10(9.81)(2) = \frac{1}{2}(10)\left(v_A\right)_2{}^2 + 0$$

$$(v_A)_2 = 6.264 \text{ m/s}$$

Conservation of Linear Momentum: Since the weight of block A and plate **P** and the force developed in the spring are nonimpulsive, the linear momentum of the system is conserved along the line of impact (y axis). By referring to Fig. *b*,

$$(+\downarrow) \qquad m_A(v_A)_2 + m_P(v_P)_2 = m_A(v_A)_3 + m_P(v_P)_2$$

$$10(6.262) + 0 = 10(v_A)_3 + 5(v_P)_2$$

$$(v_P)_2 + 2(v_A)_3 = 12.528 \qquad \textbf{(1)}$$

Coefficient of Restitution: Applying Eq. 15–11 we have

$$(+\downarrow) \qquad e = \frac{(v_P)_3 - (v_A)_3}{(v_A)_2 - (v_P)_2}$$

$$0.75 = \frac{(v_P)_3 - (v_A)_3}{6.264 - 0}$$

$$(v_P)_3 - (v_A)_3 = 4.698 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$(v_A)_3 = 2.610 \text{ m/s} \qquad (v_P)_3 = 7.308 \text{ m/s}$$

Conservation of Energy: The maximum compression of the spring occurs when plate P momentarily stops. If we consider the plate's fall fromposition (3) to position (4) as shown in Fig. *c*,

$$(T_P)_3 + (V_P)_3 = (T_P)_4 + (V_P)_4$$

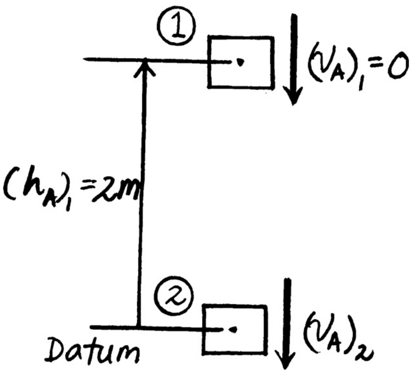
$$\frac{1}{2} m_P (v_P)_3{}^2 + \left[\left(V_g\right)_3 + (V_e)_3\right] = \frac{1}{2} m_P (v_P)_4{}^2 + \left[\left(V_g\right)_4 + (V_e)_4\right]$$

$$\frac{1}{2}(5)(7.308^2) + \left[5(9.81)s_{\max} + \frac{1}{2}(1500)(0.6 - 0.45)^2\right]$$

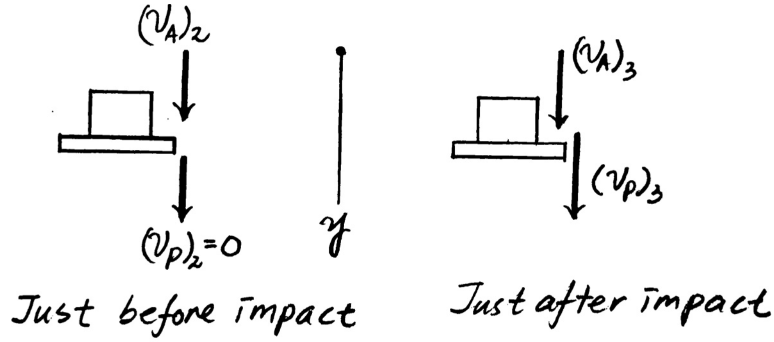
$$= 0 + \left[0 + \frac{1}{2}(1500)\left[s_{\max} + (0.6 - 0.45)\right]^2\right]$$

$$750s^2{}_{\max} + 175.95s_{\max} - 133.25 = 0$$

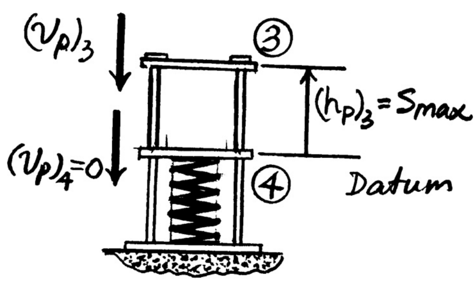
$$s_{\max} = 0.3202 \text{ m} = 320 \text{ mm} \qquad \textbf{Ans.}$$


①
$(v_A)_1 = 0$
$(h_A)_1 = 2m$
②
$(v_A)_2$
Datum

(a)

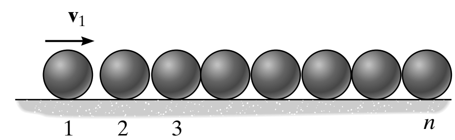

$(v_A)_2$
$(v_P)_2 = 0$
y
Just before impact
$(v_A)_3$
$(v_P)_3$
Just after impact

(b)

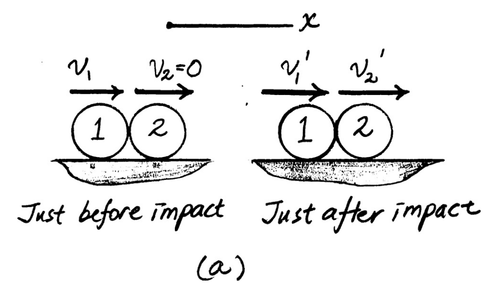

$(v_P)_3$
③
$(h_P)_3 = s_{max}$
$(v_P)_4 = 0$
④
Datum

(c)

•15–73. A row of n similar spheres, each of mass m, are placed next to each other as shown. If sphere 1 has a velocity of v_1, determine the velocity of the nth sphere just after being struck by the adjacent $(n-1)$th sphere. The coefficient of restitution between the spheres is e.



When sphere (1) strikes sphere (2), the linear momentum of the system is conserved along the x axis (line of impact). By referring to Fig. a,



$$(\overset{+}{\rightarrow}) \qquad mv_1 + 0 = mv_1' + mv_2'$$

$$v_1' + v_2' = v_1 \qquad \textbf{(1)}$$

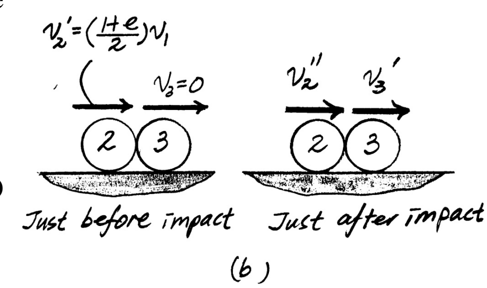
$$(\overset{+}{\rightarrow}) \qquad e = \frac{v_2' - v_1'}{v_1 - 0}$$

$$v_2' - v_1' = ev_1 \qquad \textbf{(2)}$$

Eliminating v_1' from Eqs. (1) and (2), we obtain

$$v_2' = \left(\frac{1+e}{2}\right)v_1$$

Subsequently, sphere (2) strikes sphere (3). By referring to Fig. b and using the result of v_2',



$$(\overset{+}{\rightarrow}) \qquad m\left(\frac{1+e}{2}\right)v_1 + 0 = mv_2'' + mv_3'$$

$$v_2'' + v_3' = \left(\frac{1+e}{2}\right)v_1 \qquad \textbf{(3)}$$

Applying Eq. 15–11 we have

$$(\overset{+}{\rightarrow}) \qquad e = \frac{v_3' - v_2''}{\left(\frac{1+e}{2}\right)v_1 - 0}$$

$$v_3' - v_2'' = \left[\frac{e(1+e)}{2}\right]v_1 \qquad \textbf{(4)}$$

Eliminating v_2'' from Eqs. (3) and (4), we obtain

$$v_3' = \left(\frac{1+e}{2}\right)^2 v_1$$

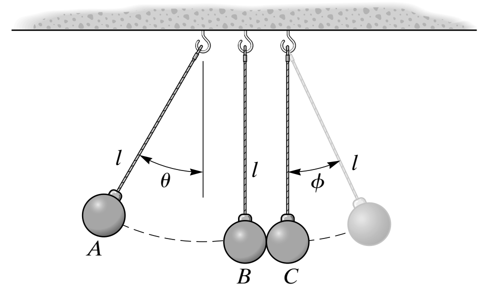
If we continue to use the above procedures to analyse the impact between spheres (3) and (4), the speed of sphere (4) after the impact.

$$v_4' = \left(\frac{1+e}{2}\right)^3 v_1$$

Thus, when sphere $(n-1)$ strikes sphere n, the speed of sphere n just after impact is

$$v_n' = \left(\frac{1+e}{2}\right)^{n-1} v_1 \qquad \textbf{Ans.}$$

15–74. The three balls each have a mass of m. If A is released from rest at θ, determine the angle ϕ to which C rises after collision. The coefficient of restitution between each ball is e.



Conservation of Energy: The datum is set at the initial position of ball B. When ball A is $l(1 - \cos\theta)$ *above* the datum its gravitational potential energy is $mg[l(l - \cos\theta)]$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + mg[l(1 - \cos\theta)] = \frac{1}{2} m(v_A)_1^2 + 0$$

$$(v_A)_1 = \sqrt{2gl(1 - \cos\theta)}$$

Conservation of Momentum: When ball A strikes ball B, we have

$$m_A (v_A)_1 + m_B (v_B)_1 = m_A (v_A)_2 + m_B (v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad m\sqrt{2gl(1 - \cos\theta)} + 0 = m(v_A)_2 + m(v_B)_2 \qquad \textbf{[1]}$$

Coefficient of Restitution:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{\sqrt{2gl(1 - \cos\theta)} - 0} \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$(v_A)_2 = \frac{(1 - e)\sqrt{2gl(1 - \cos\theta)}}{2}$$

$$(v_B)_2 = \frac{(1 + e)\sqrt{2gl(1 - \cos\theta)}}{2}$$

Conservation of Momentum: When ball B strikes ball C, we have

$$m_B (v_B)_2 + m_C (v_C)_1 = m_B (v_B)_3 + m_C (v_C)_2$$

$$(\overset{+}{\rightarrow}) \quad m\left[\frac{(1 + e)\sqrt{2gl(1 - \cos\theta)}}{2}\right] + 0 = m(v_B)_3 + m(v_C)_2 \qquad \textbf{[3]}$$

Coefficient of Restitution:

$$e = \frac{(v_C)_2 - (v_B)_3}{(v_B)_2 - (v_C)_1}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_C)_2 - (v_B)_3}{\dfrac{(1 + e)\sqrt{2gl(1 - \cos\theta)}}{2} - 0} \qquad \textbf{[4]}$$

Solving Eqs. [3] and [4] yields

$$(v_C)_2 = \frac{(1 + e)^2}{4}\sqrt{2gl(1 - \cos\theta)}$$

$$(v_B)_3 = \frac{(1 - e^2)}{4}\sqrt{2gl(1 - \cos\theta)}$$

15–74. Continued

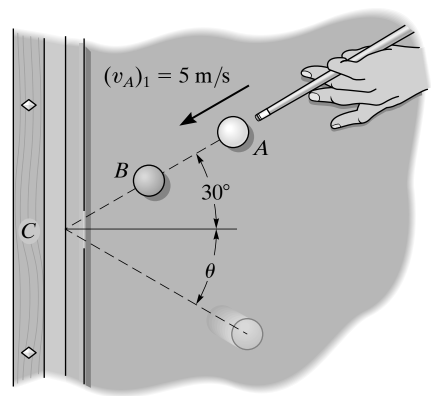
Conservation of Energy: The datum is set at the initial position of ball C. When ball C is $l(1 - \cos\phi)$ *above* the datum its gravitational potential energy is $mg[l(1 - \cos\phi)]$. Applying Eq. 14–21, we have

$$T_2 + V_2 = T_3 + V_3$$

$$0 + \frac{1}{2}m\left[\frac{(1+e)^2}{4}\sqrt{2gl(1-\cos\theta)}\right]^2 = 0 + mgl(1-\cos\phi)$$

$$\phi = \cos^{-1}\left[1 - \frac{(1+e)^4}{16}(1-\cos\theta)\right]$$ **Ans.**

15–75. The cue ball A is given an initial velocity $(v_A)_1 = 5$ m/s. If it makes a direct collision with ball B ($e = 0.8$), determine the velocity of B and the angle θ just after it rebounds from the cushion at C ($e' = 0.6$). Each ball has a mass of 0.4 kg. Neglect the size of each ball.



Conservation of Momentum: When ball A strikes ball B, we have

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

(+) $\quad 0.4(5) + 0 = 0.4(v_A)_2 + 0.4(v_B)_2$ **[1]**

Coefficient of Restitution:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$(\overset{+}{\leftarrow})$ $\quad 0.8 = \dfrac{(v_B)_2 - (v_A)_2}{5 - 0}$ **[2]**

Solving Eqs. [1] and [2] yields

$$(v_A)_2 = 0.500 \text{ m/s} \qquad (v_B)_2 = 4.50 \text{ m/s}$$

Conservation of "*y*" Momentum: When ball B strikes the cushion at C, we have

$$m_B\left(v_{B_y}\right)_2 = m_B\left(v_{B_y}\right)_3$$

$(+\downarrow)$ $\quad 0.4(4.50 \sin 30°) = 0.4(v_B)_3 \sin\theta$

$(v_B)_3 \sin\theta = 2.25$ **[3]**

Coefficient of Restitution (*x*):

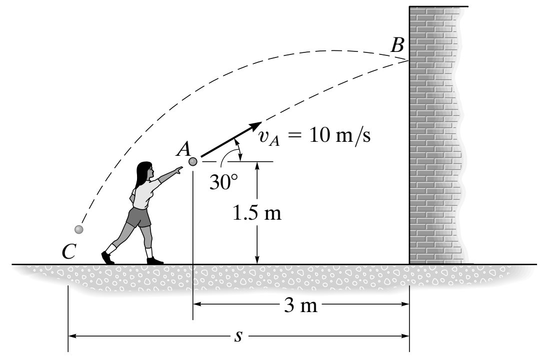
$$e = \frac{(v_C)_2 - \left(v_{B_x}\right)_3}{\left(v_{B_x}\right)_2 - (v_C)_1}$$

$(\overset{+}{\leftarrow})$ $\quad 0.6 = \dfrac{0 - \left[-(v_B)_3 \cos\theta\right]}{4.50 \cos 30° - 0}$ **[4]**

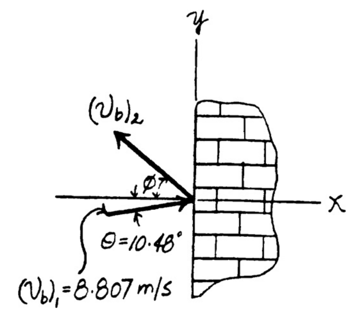
Solving Eqs. [1] and [2] yields

$$(v_B)_3 = 3.24 \text{ m/s} \qquad \theta = 43.9°$$ **Ans.**

***15–76.** The girl throws the 0.5-kg ball toward the wall with an initial velocity $v_A = 10$ m/s. Determine (a) the velocity at which it strikes the wall at B, (b) the velocity at which it rebounds from the wall if the coefficient of restitution $e = 0.5$, and (c) the distance s from the wall to where it strikes the ground at C.



Kinematics: By considering the horizontal motion of the ball before the impact, we have



$$(\xrightarrow{+}) \qquad s_x = (s_0)_x + v_x t$$

$$3 = 0 + 10\cos 30° t \qquad t = 0.3464 \text{ s}$$

By considering the vertical motion of the ball before the impact, we have

$$(+\uparrow) \qquad v_y = (v_0)_y + (a_c)_y t$$

$$= 10\sin 30° + (-9.81)(0.3464)$$

$$= 1.602 \text{ m/s}$$

The vertical position of point B above the ground is given by

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$(s_B)_y = 1.5 + 10\sin 30°(0.3464) + \frac{1}{2}(-9.81)\left(0.3464^2\right) = 2.643 \text{ m}$$

Thus, the magnitude of the velocity and its directional angle are

$$(v_b)_1 = \sqrt{(10\cos 30°)^2 + 1.602^2} = 8.807 \text{ m/s} = 8.81 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{1.602}{10\cos 30°} = 10.48° = 10.5° \qquad \textbf{Ans.}$$

Conservation of "y" Momentum: When the ball strikes the wall with a speed of $(v_b)_1 = 8.807$ m/s, it rebounds with a speed of $(v_b)_2$.

$$m_b\left(v_{b_y}\right)_1 = m_b\left(v_{b_y}\right)_2$$

$$(\xleftarrow{+}) \qquad m_b(1.602) = m_b\left[(v_b)_2\sin\phi\right]$$

$$(v_b)_2\sin\phi = 1.602 \qquad \textbf{[1]}$$

Coefficient of Restitution (x):

$$e = \frac{(v_w)_2 - \left(v_{b_x}\right)_2}{\left(v_{b_x}\right)_1 - (v_w)_1}$$

$$(\xrightarrow{+}) \qquad 0.5 = \frac{0 - \left[-(v_b)_2\cos\phi\right]}{10\cos 30° - 0} \qquad \textbf{[2]}$$

***15–76. Continued**

Solving Eqs. [1] and [2] yields

$$\phi = 20.30° = 20.3° \qquad (v_b)_2 = 4.617 \text{ m/s} = 4.62 \text{ m/s} \qquad \textbf{Ans.}$$

Kinematics: By considering the vertical motion of the ball after the impact, we have

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$-2.643 = 0 + 4.617 \sin 20.30° t_1 + \frac{1}{2}(-9.81)t_1^2$$

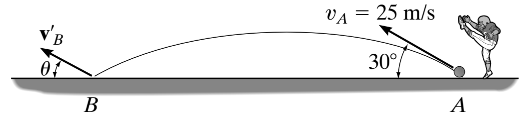
$$t_1 = 0.9153 \text{ s}$$

By considering the horizontal motion of the ball after the impact, we have

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + v_x t$$

$$s = 0 + 4.617 \cos 20.30°(0.9153) = 3.96 \text{ m} \qquad \textbf{Ans.}$$

•15–77. A 300-g ball is kicked with a velocity of $v_A = 25$ m/s at point A as shown. If the coefficient of restitution between the ball and the field is $e = 0.4$, determine the magnitude and direction θ of the velocity of the rebounding ball at B.



Kinematics: The parabolic trajectory of the football is shown in Fig. *a*. Due to the symmetrical properties of the trajectory, $v_B = v_A = 25$ m/s and $\phi = 30°$.

Conservation of Linear Momentum: Since no impulsive force acts on the football along the x axis, the linear momentum of the football is conserved along the x axis.

$$(\overset{+}{\leftarrow}) \qquad m(v_B)_x = m(v'_B)_x$$

$$0.3(25 \cos 30°) = 0.3(v'_B)_x$$

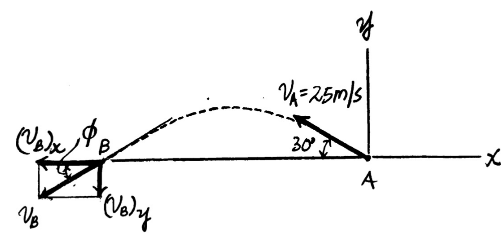
$$(v'_B)_x = 21.65 \text{ m/s} \leftarrow$$

Coefficient of Restitution: Since the ground does not move during the impact, the coefficient of restitution can be written as

$$(+\uparrow) \qquad e = \frac{0 - (v'_B)_y}{(v_B)_y - 0}$$

$$0.4 = \frac{-(v'_B)_y}{-25 \sin 30°}$$

$$(v'_B)_y = 5 \text{ m/s} \uparrow$$



(a)

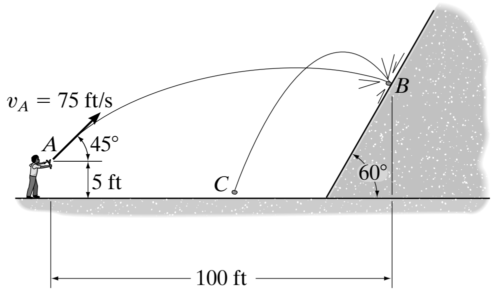
Thus, the magnitude of $\mathbf{v}'_B$ is

$$v'_B = \sqrt{(v'_B)_x + (v'_B)_y} = \sqrt{21.65^2 + 5^2} = 22.2 \text{ m/s} \qquad \textbf{Ans.}$$

and the angle of $\mathbf{v}'_B$ is

$$\theta = \tan^{-1}\left[\frac{(v'_B)_y}{(v'_B)_x}\right] = \tan^{-1}\left(\frac{5}{21.65}\right) = 13.0° \qquad \textbf{Ans.}$$

15–78. Using a slingshot, the boy fires the 0.2-lb marble at the concrete wall, striking it at B. If the coefficient of restitution between the marble and the wall is $e = 0.5$, determine the speed of the marble after it rebounds from the wall.



Kinematics: By considering the x and y motion of the marble from A to B, Fig. *a,*

$$\left(\xrightarrow{+}\right) \qquad (s_B)_x = (s_A)_x + (v_A)_x t$$

$$100 = 0 + 75 \cos 45^\circ t$$

$$t = 1.886 \text{ s}$$

and

$$(+\uparrow) \qquad (s_B)_y = (s_A)_y + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$(s_B)_y = 0 + 75 \sin 45^\circ(1.886) + \frac{1}{2}(-32.2)(1.886^2)$$

$$= 42.76 \text{ ft}$$

and

$$(+\uparrow) \qquad (v_B)_y = (v_A)_y + a_y t$$

$$(v_B)_y = 75 \sin 45^\circ + (-32.2)(1.886) = -7.684 \text{ ft/s} = 7.684 \text{ ft/s} \downarrow$$

Since $(v_B)_x = (v_A)_x = 75 \cos 45^\circ = 53.03$ ft/s, the magnitude of $\mathbf{v}_B$ is

$$v_B = \sqrt{(v_B)_x{}^2 + (v_B)_y{}^2} = \sqrt{53.03^2 + 7.684^2} = 53.59 \text{ ft/s}$$

and the direction angle of $\mathbf{v}_B$ is

$$\theta = \tan^{-1}\left[\frac{(v_B)_y}{(v_B)_x}\right] = \tan^{-1}\left(\frac{7.684}{53.03}\right) = 8.244^\circ$$

Conservation of Linear Momentum: Since no impulsive force acts on the marble along the inclined surface of the concrete wall (x' axis) during the impact, the linear momentum of the marble is conserved along the x' axis. Referring to Fig. *b,*

$$(+\nearrow) \qquad m_B(v'_B)_{x'} = m_B(v'_B)_{x'}$$

$$\frac{0.2}{32.2}\left(53.59 \sin 21.756^\circ\right) = \frac{0.2}{32.2}\left(v'_B \cos \phi\right)$$

$$v'_B \cos \phi = 19.862 \qquad \textbf{(1)}$$

15–78. Continued

Coefficient of Restitution: Since the concrete wall does not move during the impact, the coefficient of restitution can be written as

$$(+\nwarrow) \qquad e = \frac{0 - \left(v_B'\right)_{y'}}{\left(v_B\right)_{y'} - 0}$$

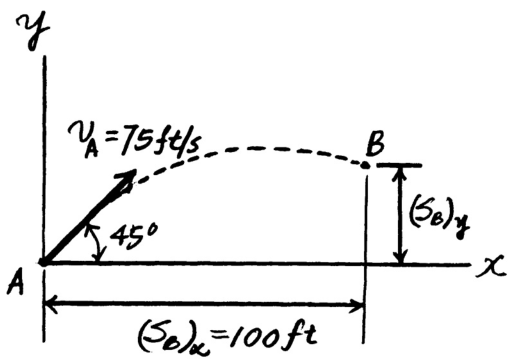
$$0.5 = \frac{-v_B' \sin \phi}{-53.59 \cos 21.756°}$$

$$v_B' \sin \phi = 24.885 \qquad \textbf{(2)}$$

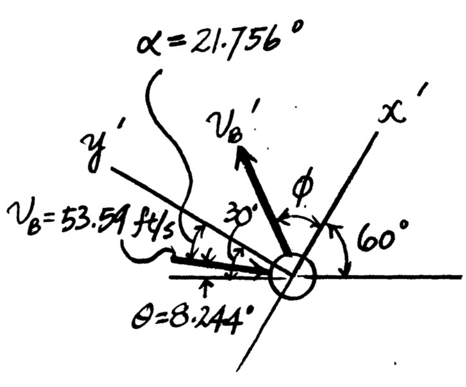
Solving Eqs. (1) and (2) yields

$$v_B' = 31.8 \text{ ft/s} \qquad \textbf{Ans.}$$

$$\phi = 51.40°$$

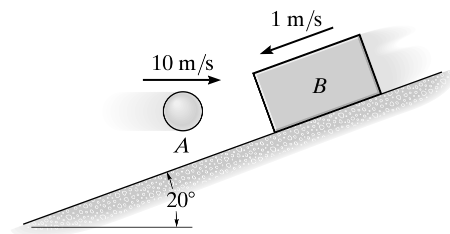


(a)



(b)

15–79. The 2-kg ball is thrown so that it travels horizontally at 10 m/s when it strikes the 6-kg block as it is traveling down the inclined plane at 1 m/s. If the coefficient of restitution between the ball and the block is $e = 0.6$, determine the speeds of the ball and the block just after the impact. Also, what distance does B slide up the plane before it momentarily stops? The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.4$.



System:

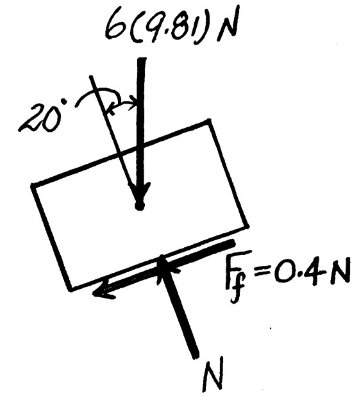
$(+\nearrow)$ $\qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$2(10\cos 20^\circ) - 6(1) = 2(v_{A_x})_2 + 6(v_{B_x})_2$$

$$(v_{A_x})_2 + 3(v_{B_x})_2 = 6.3969$$

$(+\nearrow)$ $\qquad e = \dfrac{(v_{B_x})_2 - (v_{A_x})_2}{(v_{A_x})_1 - v_{B_x})_1}; \qquad 0.6 = \dfrac{(v_{B_x})_2 - (v_{A_x})_2}{10\cos 20^\circ - (-1)}$

$$(v_{B_x})_2 - (v_{A_x})_2 = 6.23816$$



Solving:

$(v_{A_x})_2 = -3.0794$ m/s

$(v_{B_x})_2 = 3.1588$ m/s

Ball A:

$(+\nwarrow)$ $\qquad m_A\left(v_{A_y}\right)_1 = m_A\left(v_{A_y}\right)_2$

$$m_A(-10\sin 20^\circ) = m_A\left(v_{A_y}\right)_2$$

$$\left(v_{A_y}\right)_2 = -3.4202 \text{ m/s}$$

Thus,

$(v_A)_2 = \sqrt{(-3.0794)^2 + (-3.4202)^2} = 4.60$ m/s **Ans.**

$(v_B)_2 = 3.1588 = 3.16$ m/s **Ans.**

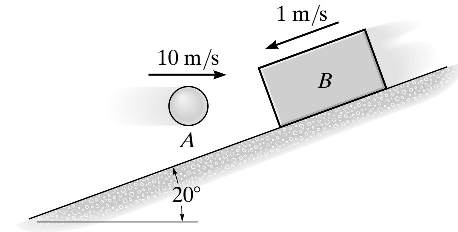
$+\nwarrow \Sigma F_y = 0; \qquad -6(9.81)\cos 20^\circ + N = 0 \qquad N = 55.31$ N

$$T_1 + \Sigma U_{1-2} = T_2$$

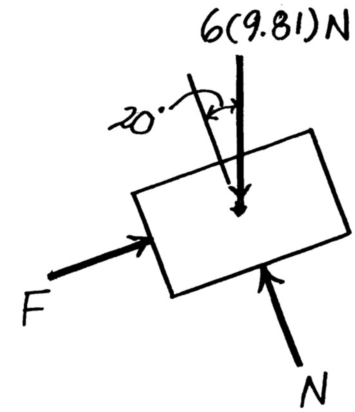
$$\frac{1}{2}(6)(3.1588)^2 - 6(9.81)\sin 20^\circ d - 0.4(55.31)d = 0$$

$d = 0.708$ m **Ans.**

***15–80.** The 2-kg ball is thrown so that it travels horizontally at 10 m/s when it strikes the 6-kg block as it travels down the smooth inclined plane at 1 m/s. If the coefficient of restitution between the ball and the block is $e = 0.6$, and the impact occurs in 0.006 s, determine the average impulsive force between the ball and block.



System:



$(+\nearrow)\qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$2(10\cos 20^\circ) - 6(1) = 2(v_{A_x})_2 + 6(v_{B_x})_2$$

$$(v_{A_x})_2 + 3(v_{B_x})_2 = 6.3969$$

$(+\nearrow)\qquad e = \dfrac{(v_{B_x}) - (v_{A_x})_2}{(v_{A_x})_1 - (v_{B_x})_1};\qquad 0.6 = \dfrac{(v_{B_x})_2 - (v_{A_x})_2}{10\cos 20^\circ - (-1)}$

$$(v_{B_x})_2 - (v_{A_x})_2 = 6.23816$$

Solving:

$(v_{A_x})_2 = -3.0794 \text{ m/s}$

$(v_{B_x})_2 = 3.1588 \text{ m/s}$

Block B.

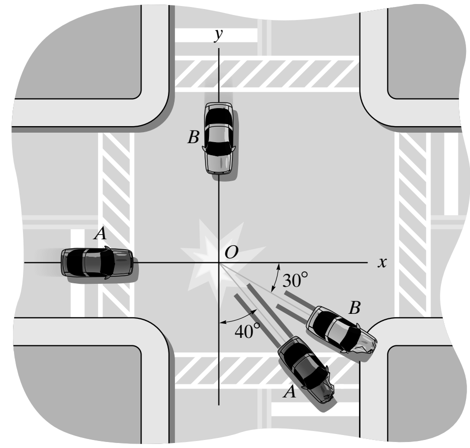
Neglect impulse of weight.

$(+\nearrow)\qquad mv_1 + \Sigma\displaystyle\int F\,dt = mv_2$

$$-6(1) + F(0.006) = 6(3.1588)$$

$F = 4.16 \text{ kN}$ **Ans.**

•15–81. Two cars A and B each have a weight of 4000 lb and collide on the icy pavement of an intersection. The direction of motion of each car after collision is measured from snow tracks as shown. If the driver in car A states that he was going 44 ft/s (30 mi/h) just before collision and that after collision he applied the brakes so that his car skidded 10 ft before stopping, determine the approximate speed of car B just before the collision. Assume that the coefficient of kinetic friction between the car wheels and the pavement is $\mu_k = 0.15$. *Note:* The line of impact has not been defined; however, this information is not needed for the solution.

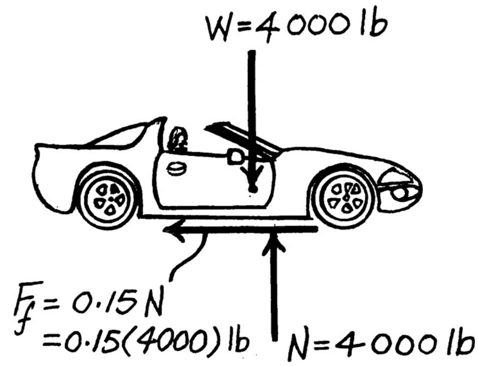


$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{4000}{32.2}\right)(v_A)_2^2 - (0.15)(4000)(10) = 0$$

$$(v_A)_2 = 9.829 \text{ ft/s}$$

$$(v_A)_1 = 44 \text{ ft/s}$$



$$\left(\xrightarrow{+}\right) \qquad \Sigma m_1 (v_x)_1 = \Sigma m_2 (v_x)_2$$

$$\left(\frac{4000}{32.2}\right)(44) + 0 = \left(\frac{4000}{32.2}\right)(9.829)\sin 40^\circ + \left(\frac{4000}{32.2}\right)(v_B)_2 \cos 30^\circ$$

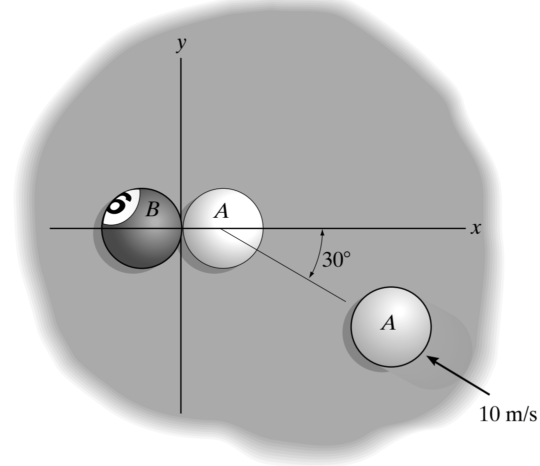
$$(v_B)_2 = 43.51 \text{ ft/s}$$

$$(+\uparrow) \qquad \Sigma m_1 (v_y)_1 = \Sigma m_2 (v_y)_2$$

$$0 - \left(\frac{4000}{32.2}\right)(v_B)_1 = -\left(\frac{4000}{32.2}\right)(9.829)\cos 40^\circ - \left(\frac{4000}{32.2}\right)(43.51)\sin 30^\circ$$

$$(v_B)_1 = 29.3 \text{ ft/s} \qquad \textbf{Ans.}$$

15–82. The pool ball A travels with a velocity of 10 m/s just before it strikes ball B, which is at rest. If the masses of A and B are each 200 g, and the coefficient of restitution between them is $e = 0.8$, determine the velocity of both balls just after impact.



Conservation of Linear Momentum: By referring to the impulse and momentum of the system of billiard balls shown in Fig. *a*, notice that the linear momentum of the system is conserved along the n axis (line of impact). Thus,

$$\left(\overset{+}{\leftarrow}\right) \qquad m_A\left(v_A\right)_n + m_B\left(v_B\right)_n = m_A\left(v'_A\right)_n + m_B\left(v'_B\right)_n$$

$$0.2(10)\cos 30° = 0.2v'_A \cos\theta_A + 0.2v'_B \cos\theta_B$$

$$v'_A \cos\theta_A + v'_B \cos\theta_B = 8.6603 \qquad \textbf{(1)}$$

Also, we notice that the linear momentum of each ball A and B is conserved along the t axis (tangent of plane impact). Thus,

$$\left(+\uparrow\right) \qquad m_A\left(v_A\right)_t = m_A\left(v'_A\right)_t$$

$$0.2(10)\sin 30° = 0.2v'_A \sin\theta_A$$

$$v'_A \sin\theta_A = 5 \qquad \textbf{(2)}$$

and

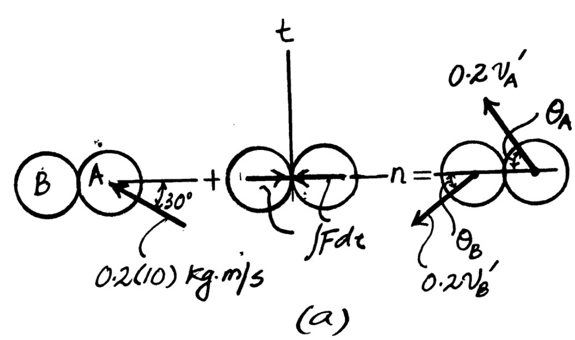
$$\left(+\uparrow\right) \qquad m_B\left(v\right)_t = m_B\left(v'_B\right)_t$$

$$0 = 0.2v'_B \sin\theta_B$$

$$v'_B \sin\theta_B = 0$$

Since $v'_B \neq 0$, then $\sin\theta_B = 0$. Thus

$\theta_B = 0$



(a)

Coefficient of Restitution: The coefficient of restitution equation written along the n axis (line of impact) gives

$$\left(\overset{+}{\leftarrow}\right) \qquad e = \frac{\left(v'_B\right)_n - \left(v'_A\right)_n}{\left(v_A\right)_n - \left(v_B\right)_n}$$

$$0.8 = \frac{v'_B \cos\theta_B - v'_A \cos\theta_A}{10\cos 30° - 0}$$

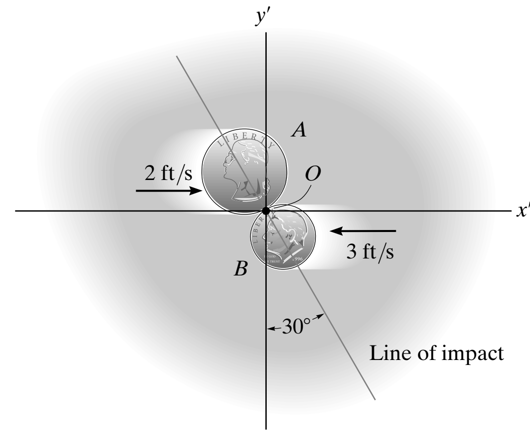
$$v'_B \cos\theta_B - v'_A \cos\theta_A = 6.928 \qquad \textbf{(3)}$$

Using the result of θ_B and solving Eqs. (1), (2), and (3),

$v'_A = 5.07\text{m/s} \qquad \theta_A = 80.2°$ **Ans.**

$v'_B = 7.79 \text{ m/s} \leftarrow$ **Ans.**

15–83. Two coins A and B have the initial velocities shown just before they collide at point O. If they have weights of $W_A = 13.2(10^{-3})$ lb and $W_B = 6.60(10^{-3})$ lb and the surface upon which they slide is smooth, determine their speeds just after impact. The coefficient of restitution is $e = 0.65$.



$$(+\searrow) \qquad m_A\left(v_{A_x}\right)_1 + m_B\left(v_{B_x}\right)_1 = m_A\left(v_{A_x}\right)_2 + m_B\left(v_{B_x}\right)_2$$

$$\left(\frac{13.2(10^{-3})}{32.2}\right)2\sin 30^\circ - \left(\frac{6.6(10^{-3})}{32.2}\right)3\sin 30^\circ$$

$$= \left(\frac{13.2(10^{-3})}{32.2}\right)(v_{A_x})_2 + \left(\frac{6.6(10^{-3})}{32.2}\right)(v_{B_x})_2$$

$$(+\searrow) \qquad e = \frac{(v_{B_x})_2 - (v_{A_x})_2}{(v_{A_x})_1 - (v_{B_x})_1} \qquad\qquad 0.65 = \frac{(v_{B_x})_2 - (v_{A_x})_2}{2\sin 30^\circ - (-3\sin 30^\circ)}$$

Solving:

$$(v_{A_x})_2 = -0.3750 \text{ ft/s}$$

$$(v_{B_x})_2 = 1.250 \text{ ft/s}$$

$$(+\nearrow) \qquad m_A\left(v_{A_y}\right)_2 = m_A\left(v_{A_y}\right)_2$$

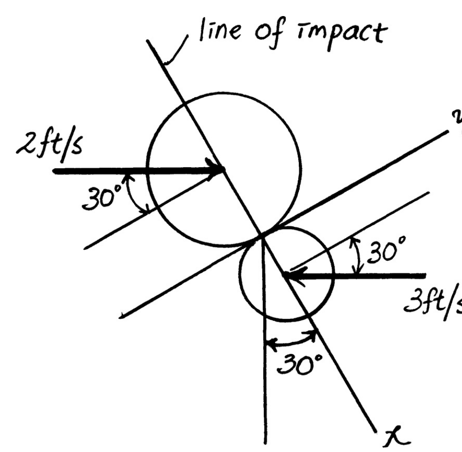
$$\left(\frac{13.2(10^{-3})}{32.2}\right)2\cos 30^\circ = \left(\frac{13.2(10^{-3})}{32.2}\right)(v_{A_y})_2$$

$$(v_{A_y})_2 = 1.732 \text{ ft/s}$$

$$(+\swarrow) \qquad m_B\left(v_{B_y}\right)_1 = m_B\left(v_{B_y}\right)_2$$

$$\left(\frac{6.6(10^{-3})}{32.2}\right)3\cos 30^\circ = \left(\frac{6.6(10^{-3})}{32.2}\right)(v_{B_y})_2$$

$$(v_{B_y})_2 = 2.598 \text{ ft/s}$$

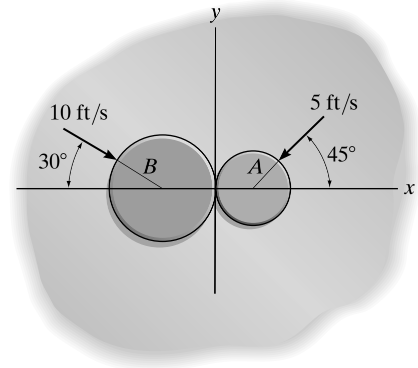


Thus:

$$(v_B)_2 = \sqrt{(1.250)^2 + (2.598)^2} = 2.88 \text{ ft/s} \qquad \textbf{Ans.}$$

$$(v_A)_2 = \sqrt{(-0.3750)^2 + (1.732)^2} = 1.77 \text{ ft/s} \qquad \textbf{Ans.}$$

***15–84.** Two disks *A* and *B* weigh 2 lb and 5 lb, respectively. If they are sliding on the smooth horizontal plane with the velocities shown, determine their velocities just after impact. The coefficient of restitution between the disks is $e = 0.6$.



Conservation of Linear Momentum: By referring to the impulse and momentum of the system of disks shown in Fig. *a,* notice that the linear momentum of the system is conserved along the *n* axis (line of impact). Thus,

$$\left(\overset{+}{\rightarrow}\right) \qquad m_A\left(v_A\right)_n + m_B\left(v_B\right)_n = m_A\left(v'_A\right)_n + m_B\left(v'_B\right)_n$$

$$-\frac{2}{32.2}(5)\cos 45° + \frac{5}{32.2}(10)\cos 30° = \frac{2}{32.2}v'_A\cos\theta_A + \frac{5}{32.2}v'_B\cos\theta_B$$

$$2v'_A\cos\theta_A + 5v'_B\cos\theta_B = 36.23 \qquad \textbf{(1)}$$

Also, we notice that the linear momentum a of disks *A* and *B* are conserved along the *t* axis (tangent to the plane of impact). Thus,

$$(+\downarrow) \qquad m_A\left(v_A\right)_t = m_A\left(v'_A\right)_t$$

$$\frac{2}{32.2}(5)\sin 45° = \frac{2}{32.2}v'_A\sin\theta_A$$

$$v'_A\sin\theta_A = 3.5355 \qquad \textbf{(2)}$$

and

$$(+\downarrow) \qquad m_B\left(v_B\right)_t = m_B\left(v'_B\right)_t$$

$$\frac{2}{32.2}(10)\sin 30° = = \frac{2}{32.2}v'_B\sin\theta_B$$

$$v'_B\sin\theta_B = 5 \qquad \textbf{(3)}$$

Coefficient of Restitution: The coefficient of restitution equation written along the *n* axis (line of impact) gives

$$\left(\overset{+}{\rightarrow}\right) \qquad e = \frac{\left(v'_A\right)_n - \left(v'_B\right)_n}{\left(v_B\right)_n - \left(v_A\right)_n}$$

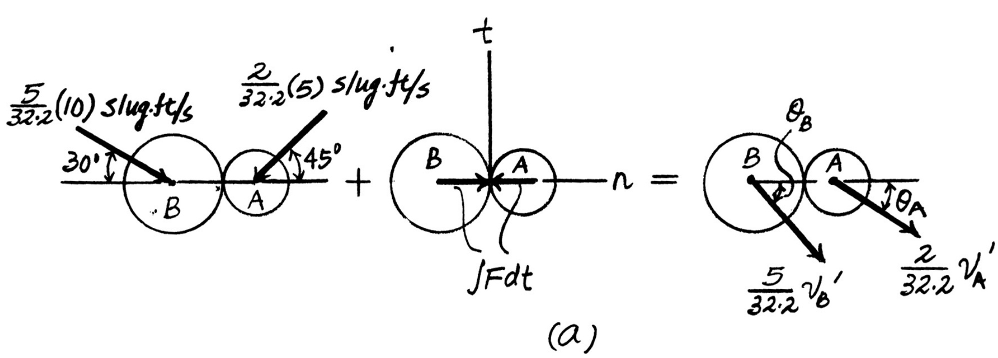
$$0.6 = \frac{v'_A\cos\theta_A - v'_B\cos\theta_B}{10\cos 30° - \left(-5\cos 45°\right)}$$

$$v'_A\cos\theta_A - v'_B\cos\theta_B = 7.317 \qquad \textbf{(4)}$$

Solving Eqs. (1), (2), (3), and (4), yields

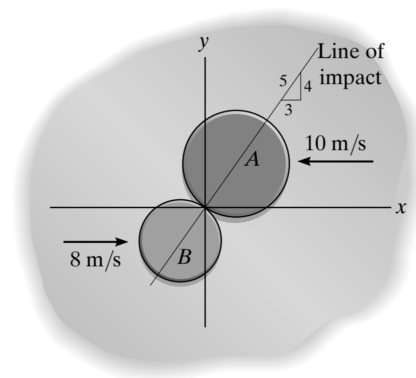
$v'_A = 11.0$ ft/s $\qquad \theta_A = 18.8°$ **Ans.**

$v'_B = 5.88$ ft/s $\qquad \theta_B = 58.3°$ **Ans.**



(a)

•15–85. Disks A and B have a mass of 15 kg and 10 kg, respectively. If they are sliding on a smooth horizontal plane with the velocities shown, determine their speeds just after impact. The coefficient of restitution between them is $e = 0.8$.



Conservation of Linear Momentum: By referring to the impulse and momentum of the system of disks shown in Fig. a, notice that the linear momentum of the system is conserved along the n axis (line of impact). Thus,

$$+\nearrow \quad m_A\left(v_A\right)_n + m_B\left(v_B\right)_n = m_A\left(v_A'\right)_n + m_B\left(v_B'\right)_n$$

$$15(10)\left(\frac{3}{5}\right) - 10(8)\left(\frac{3}{5}\right) = 15v_A' \cos\phi_A + 10v_B' \cos\phi_B$$

$$15v_A' \cos\phi_A + 10v_B' \cos\phi_B = 42 \qquad \textbf{(1)}$$

Also, we notice that the linear momentum of disks A and B are conserved along the t axis (tangent to? plane of impact). Thus,

$$+\nwarrow \quad m_A\left(v_A\right)_t = m_A\left(v_A'\right)_t$$

$$15(10)\left(\frac{4}{5}\right) = 15v_A' \sin\phi_A$$

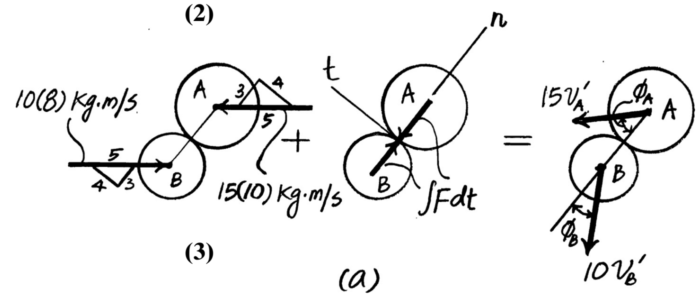
$$v_A' \sin\phi_A = 8 \qquad \textbf{(2)}$$

and

$$+\nwarrow \quad m_B\left(v_B\right)_t = m_B\left(v_B'\right)_t$$

$$10(8)\left(\frac{4}{5}\right) = 10\, v_B' \sin\phi_B$$

$$v_B' \sin\phi_B = 6.4 \qquad \textbf{(3)}$$



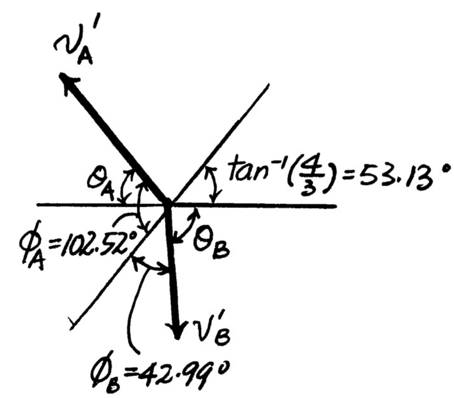
(a)

Coefficient of Restitution: The coefficient of restitution equation written along the n axis (line of impact) gives

$$+\nearrow \quad e = \frac{\left(v_B'\right)_n - \left(v_A'\right)_n}{\left(v_A\right)_n - \left(v_B\right)_n}$$

$$0.8 = \frac{v_B' \cos\phi_B - v_A' \cos\phi_A}{10\left(\frac{3}{5}\right) - \left[-8\left(\frac{3}{5}\right)\right]}$$

$$v_B' \cos\phi_B - v_A' \cos\phi_A = 8.64 \qquad \textbf{(4)}$$



(b)

Solving Eqs. (1), (2), (3), and (4), yeilds

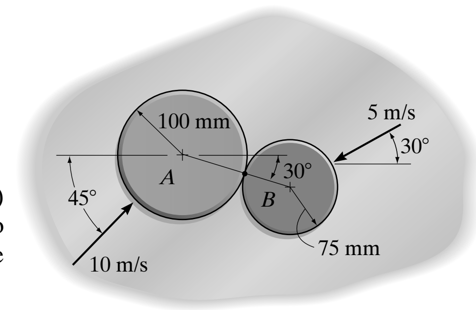
$$v_A' = 8.19 \text{ m/s} \qquad \textbf{Ans.}$$

$$\phi_A = 102.52°$$

$$v_B' = 9.38 \text{ m/s} \qquad \textbf{Ans.}$$

$$\phi_B = 42.99°$$

15–86. Disks A and B have a mass of 6 kg and 4 kg, respectively. If they are sliding on the smooth horizontal plane with the velocities shown, determine their speeds just after impact. The coefficient of restitution between the disks is $e = 0.6$.



Conservation of Linear Momentum: The orientation of the line of impact (n axis) and the tangent of the plane of contact (t axis) are shownn in Fig. *a*. By referring to the impulse and momentum of the system of disks shown in Fig. *b*, we notice that the linear momentum of the system is conserved along the n axis. Thus,

$$(\searrow +) \qquad m_A(v_A)_n + m_B(v_B)_n = m_A(v_A')_n + m_B(v_B')_n$$

$$-6(10)\cos 75^\circ + 4(5)\cos 60^\circ = 6(v_A'\cos\phi_A) - 4(v_B'\cos\phi_B)$$

$$4v_A'\cos\phi_A - 6v_B'\cos\phi_B = 5.529 \qquad \textbf{(1)}$$

Also, we notice that the linear momentum of disks A and B are conserved along the t axis. Thus,

$$(+\nearrow) \qquad m_A(v_A)_t = m_A(v_A')_t$$

$$6(10)\sin 75^\circ = 6\,v_A'\sin\phi_A$$

$$v_A'\sin\phi_A = 9.659 \qquad \textbf{(2)}$$

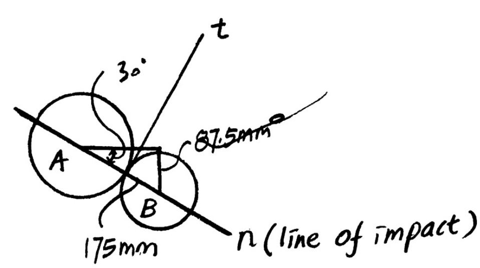
and

$$(+\nearrow) \qquad m_B(v_B)_t = m_B(v_B')_t$$

$$4(5)\sin 60^\circ = 4\,v_B'\sin\phi_B$$

$$v_B'\sin\phi_B = 4.330 \qquad \textbf{(3)}$$

Coefficient of Restitution: The coefficient of restitution equation written along the n axis (line of impact) gives



(a)

$$(\searrow +) \qquad e = \frac{(v_B')_n - (v_A')_n}{(v_A)_n - (v_B)_n}$$

$$0.6 = \frac{-v_B'\cos\phi_B - v_A'\cos\phi_A}{-10\cos 75^\circ - 5\cos 60^\circ}$$

$$v_B'\cos\phi_B + v_A'\cos\phi_A = 3.053 \qquad \textbf{(4)}$$

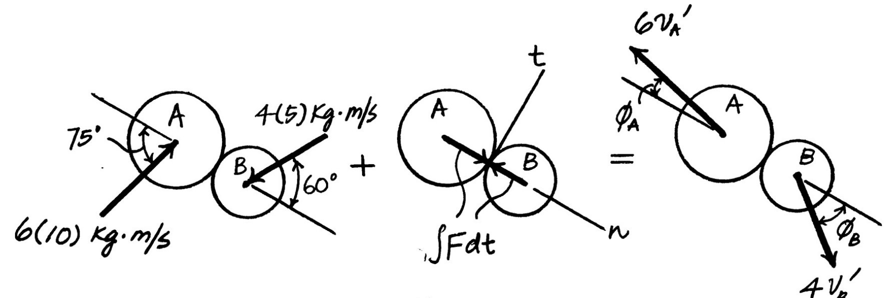
Solving Eqs. (1), (2), (3), and (4), yields

$$v_A' = 9.68 \text{ m/s} \qquad \textbf{Ans.}$$

$$\phi_A = 86.04^\circ$$

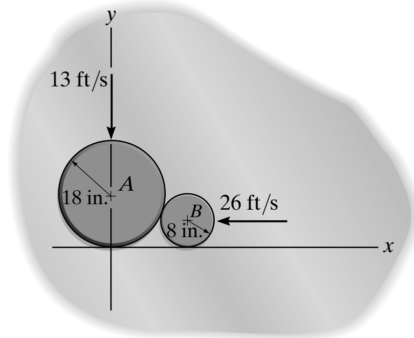
$$v_B' = 4.94 \text{ m/s} \qquad \textbf{Ans.}$$

$$\phi_B = 61.16^\circ$$



(b)

15–87. Disks A and B weigh 8 lb and 2 lb, respectively. If they are sliding on the smooth horizontal plane with the velocities shown, determine their speeds just after impact. The coefficient of restitution between them is $e = 0.5$.



Conservation of Linear Momentum: The orientation of the line of impact (n axis) and the tangent of the plane of contact, (t axis) are shown in Fig. a. By referring to the impulse and momentum of the system of disks shown in Fig. b, notice that the linear momentum of the system is conserved along the n axis. Thus,

$$m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$

$$-\frac{8}{32.2}(13)\left(\frac{5}{13}\right) + \frac{2}{32.2}(26)\left(\frac{12}{13}\right) = \frac{8}{32.2}(v'_A \cos\phi_A) - \frac{2}{32.2}(v'_B \cos\phi_B)$$

$$8v'_A \cos\phi_A - 2v'_B \cos\phi_B = 8 \tag{1}$$

Also, we notice that the linear momentum of disks A and B are conserved along the t axis. Thus,

$$m_A(v_A)_t = m_A(v'_A)_t$$

$$\frac{8}{32.2}(13)\left(\frac{12}{13}\right) = \frac{8}{32.2}v'_A \sin\phi_A$$

$$v'_B \sin\phi_A = 12 \tag{2}$$

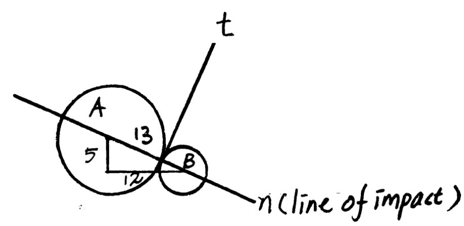
and

$$m_B(v_B)_t = m_B(v'_B)_t$$

$$\frac{2}{32.2}(26)\left(\frac{5}{13}\right) = \frac{2}{32.2}v'_B \sin\phi_B$$

$$v'_B \sin\phi_B = 10 \tag{3}$$

Coefficient of Restitution: The coefficient of restitution equation written along the n axis (line of impact) gives



(a)

$$e = \frac{(v'_B)_n - (v'_A)_n}{(v_A)_n - (v_A)_n}$$

$$0.5 = \frac{-v'_B \cos\phi_B - v'_A \cos\phi_A}{-13\left(\frac{5}{13}\right) - 26\left(\frac{12}{13}\right)}$$

$$v'_A \cos\phi_A + v'_B \cos\phi_B = 14.5 \tag{4}$$

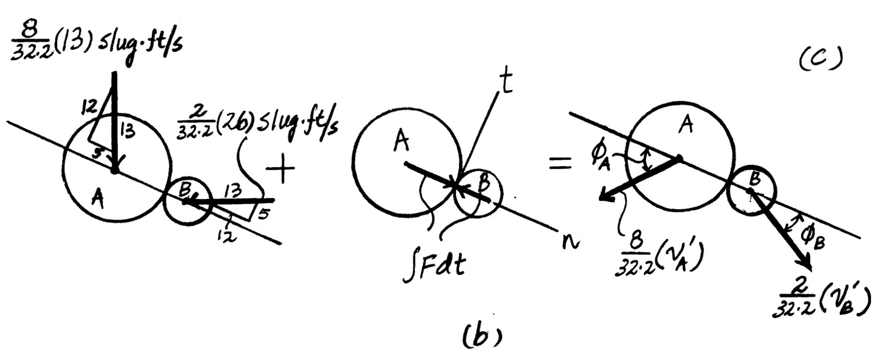
Solving Eqs. (1), (2), (3), and (4), yields

$v'_A = 12.6$ ft/s **Ans.**

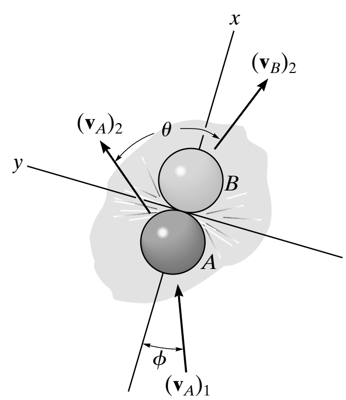
$\phi_A = 72.86°$

$v'_B = 14.7$ ft/s **Ans.**

$\phi_B = 42.80°$



***15–88.** Ball A strikes ball B with an initial velocity of $(\mathbf{v}_A)_1$ as shown. If both balls have the same mass and the collision is perfectly elastic, determine the angle θ after collision. Ball B is originally at rest. Neglect the size of each ball.



Velocity before impact:

$$(v_{Ax})_1 = (v_A)_1 \cos\phi \qquad (v_{Ay})_1 = (v_A)_1 \sin\phi$$

$$(v_{Bx})_1 = 0 \qquad (v_{By})_1 = 0$$

Velocity after impact

$$(v_{Ax})_2 = (v_A)_2 \cos\theta_1 \qquad (v_{Ay})_2 = (v_A)_2 \sin\theta_1$$

$$(v_{Bx})_2 = (v_B)_2 \cos\theta_2 \qquad (v_{By})_2 = -(v_B)_2 \sin\theta_2$$

Conservation of "y" momentum:

$$m_B\,(v_{By})_1 = m_B\,(v_{By})_2$$

$$0 = m\big[-(v_\theta)_2 \sin\theta_2\big] \qquad \theta_2 = 0°$$

Conservation of "x" momentum:

$$m_A\,(v_{Ax})_1 + m_B\,(v_{Bx})_1 = m_A\,(v_{Ax})_2 + m_B\,(v_{Bx})_2$$

$$m\,(v_A)_1 \cos\phi + 0 = m\,(v_A)_2 \cos\theta_1 + m\,(v_B)_2 \cos 0°$$

$$(v_A)_1 \cos\phi = (v_A)_2 \cos\theta_1 + (v_B)_2 \qquad \textbf{(1)}$$

Coefficient of Restitution (x direction):

$$e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}\,; \qquad 1 = \frac{(v_B)_2 \cos 0° - (v_A)_2 \cos\theta_1}{(v_A)_1 \cos\phi - 0}$$

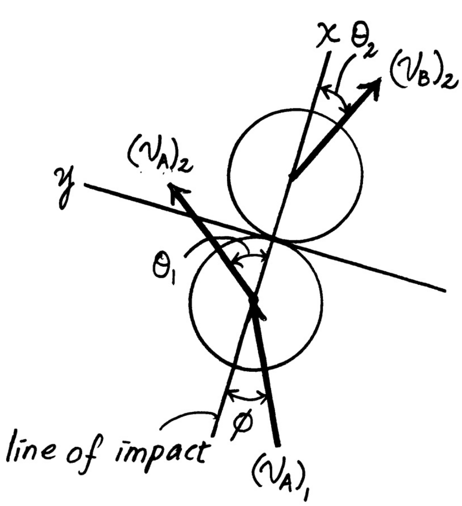
$$(v_A)_1 \cos\phi = -(v_A)_2 \cos\theta_1 + (v_B)_2 \qquad \textbf{(2)}$$

Subtracting Eq. (1) from Eq. (2) yields:

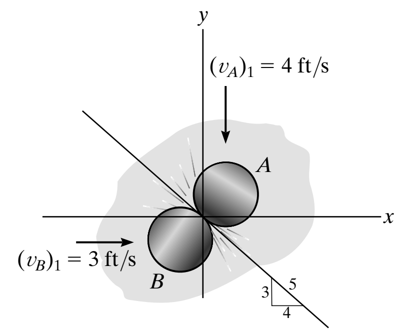
$$2\,(v_A)_2 \cos\theta_1 = 0 \qquad \text{Since } 2(v_A)_2 \neq 0$$

$$\cos\theta_1 = 0 \qquad \theta_1 = 90°$$

$$\theta = \theta_1 + \theta_2 = 90° + 0° = 90° \qquad \textbf{Ans.}$$



•15–89. Two disks A and B each have a weight of 2 lb and the initial velocities shown just before they collide. If the coefficient of restitution is $e = 0.5$, determine their speeds just after impact.



System:

$(+\nearrow)$ $\quad \Sigma\, mv_{1x} = \Sigma mv_{2x}$

$$\frac{2}{32.2}(3)\left(\frac{3}{5}\right) - \frac{2}{32.2}(4)\left(\frac{4}{5}\right) = \frac{2}{32.2}(v_B)_{2x} + \frac{2}{32.2}(v_A)_{2x}$$

$(+\nearrow)$ $\quad e = \dfrac{(v_{Ax})_2 - (v_{Bx})_2}{(v_{Bx})_1 - (v_{Ax})_1}\,; \qquad 0.5 = \dfrac{(v_{Ax})_2 - (v_{Bx})_2}{3\left(\frac{3}{5}\right) - \left[-4\left(\frac{4}{5}\right)\right]}$

Solving,

$(v_{Ax})_2 = 0.550$ ft/s

$(v_{Bx})_2 = -1.95$ ft/s $= 1.95$ ft/s $\leftarrow$

Ball A:

$(+\nwarrow)$ $\quad m_A\left(v_{Ay}\right)_1 = m_A\left(v_{Ay}\right)_2$

$$-\frac{2}{32.2}(4)\left(\frac{3}{5}\right) = \frac{2}{32.2}\left(v_{Ay}\right)_2$$

$$\left(v_{Ay}\right)_2 = -2.40 \text{ ft/s}$$

Ball B:

$(+\nwarrow)$ $\quad m_B\left(v_{By}\right)_1 = m_B\left(v_{By}\right)_2$

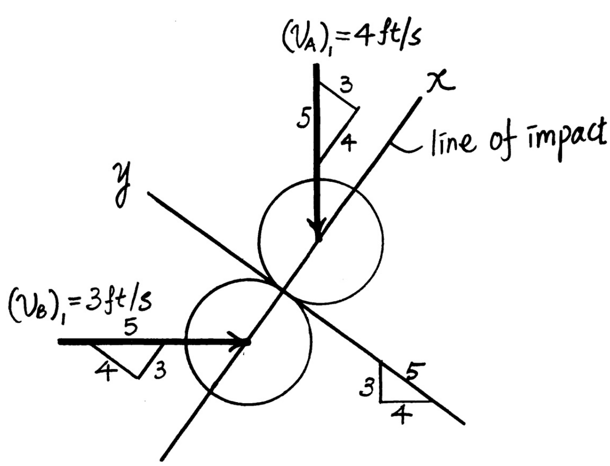
$$-\frac{2}{32.2}(3)\left(\frac{4}{5}\right) = \frac{2}{32.2}\left(v_{By}\right)_2$$

$$(v_{B_y})_2 = -2.40 \text{ ft/s}$$

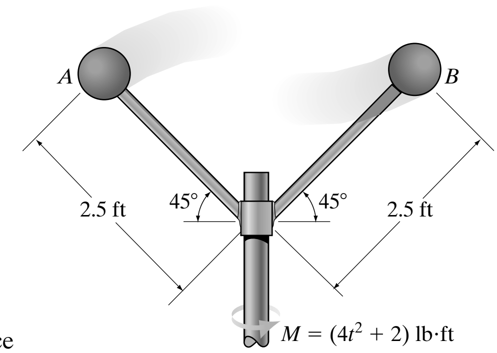
Thus,

$(v_A)_2 = \sqrt{(0.550)^2 + (2.40)^2} = 2.46$ ft/s **Ans.**

$(v_B)_2 = \sqrt{(1.95)^2 + (2.40)^2} = 3.09$ ft/s **Ans.**



15–90. The spheres A and B each weighing 4 lb, are welded to the light rods that are rigidly connected to a shaft as shown. If the shaft is subjected to a couple moment of $M = (4t^2 + 2)$ lb · ft, where t is in seconds, determine the speed of A and B when $t = 3$ s. The system starts from rest. Neglect the size of the spheres.



Free-Body Diagram: The free-body diagram of the system is shown in Fig. *a*. Since the moment reaction $\mathbf{M}_C$ has no component about the z axis, the force reaction $\mathbf{F}_C$ acts through the z axis, and the line of action of $\mathbf{W}_A$ and $\mathbf{W}_B$ are parallel to the z axis, they produce no angular impulse about the z axis.

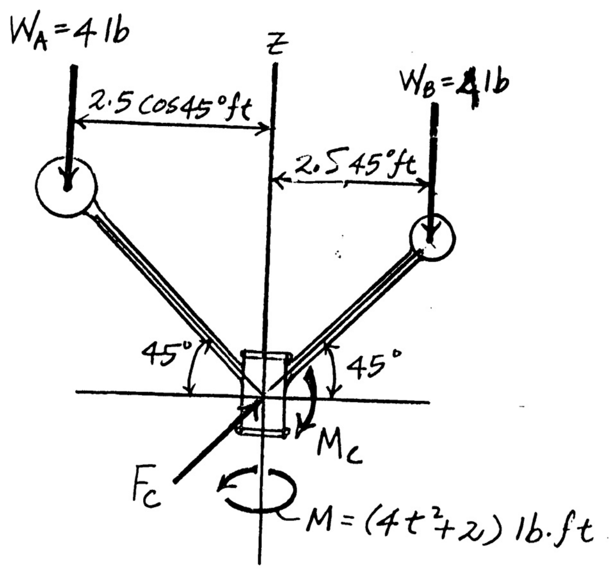
Principle of Angular Impulse and Momentum:

$$\left(H_1\right)_z + \Sigma \int_{t_2}^{t_1} M_z \, dt = \left(H_2\right)$$

$$0 + \int_0^{3\text{ s}} \left(4t^2 + 2\right)dt = 2\left(\frac{4}{32.2}(v)(2.5 \cos 45°)\right)$$

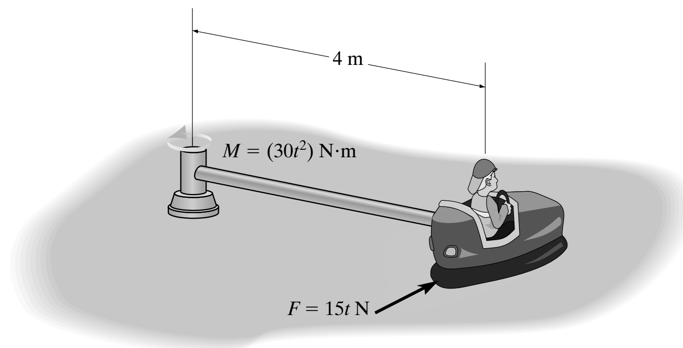
$$\frac{4t^3}{3} + 2t \Big|_0^{3\text{ s}} = 0.4392\, v$$

$v = 95.6$ ft/s **Ans.**



(a)

15–91. If the rod of negligible mass is subjected to a couple moment of $M = (30t^2)$ N · m and the engine of the car supplies a traction force of $F = (15t)$ N to the wheels, where t is in seconds, determine the speed of the car at the instant $t = 5$ s. The car starts from rest. The total mass of the car and rider is 150 kg. Neglect the size of the car.



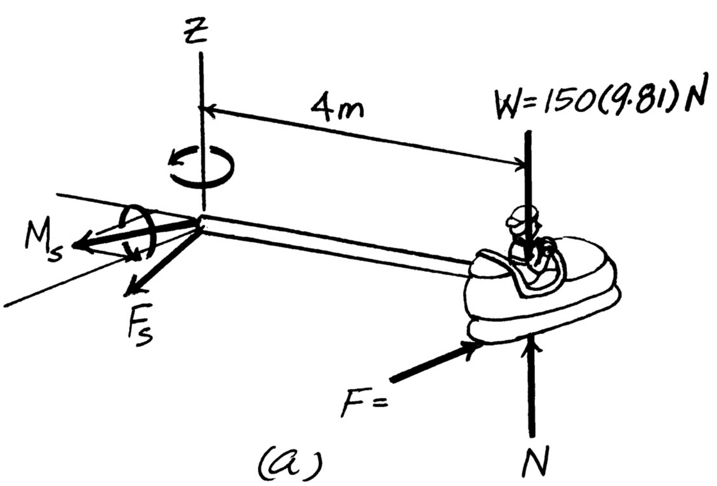
Free-Body Diagram: The free-body diagram of the system is shown in Fig. *a*. Since the moment reaction $\mathbf{M}_S$ has no component about the z axis, the force reaction $\mathbf{F}_S$ acts through the z axis, and the line of action of **W** and **N** are parallel to the z axis, they produce no angular impulse about the z axis.

Principle of Angular Impulse and Momentum:

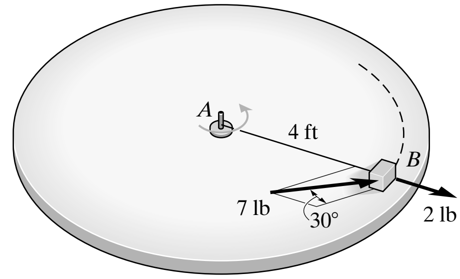
$$\left(H_1\right)_z + \Sigma \int_{t_2}^{t_1} M_z\, dt = \left(H_2\right)_z$$

$$0 + \int_0^{5\text{ s}} 30t^2\, dt + \int_0^{5\text{ s}} 15t(4)dt = 150v(4)$$

$v = 3.33$ m/s **Ans.**



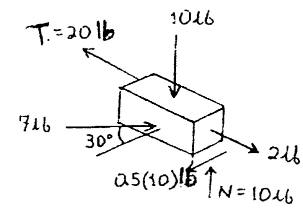
15–92. The 10-lb block rests on a surface for which $\mu_k = 0.5$. It is acted upon by a radial force of 2 lb and a horizontal force of 7 lb, always directed at 30° from the tangent to the path as shown. If the block is initially moving in a circular path with a speed $v_1 = 2$ ft/s at the instant the forces are applied, determine the time required before the tension in cord AB becomes 20 lb. Neglect the size of the block for the calculation.



$\Sigma F_n = ma_n;$

$$20 - 7\sin 30° - 2 = \frac{10}{32.2}\left(\frac{v^2}{4}\right)$$

$v = 13.67$ ft/s

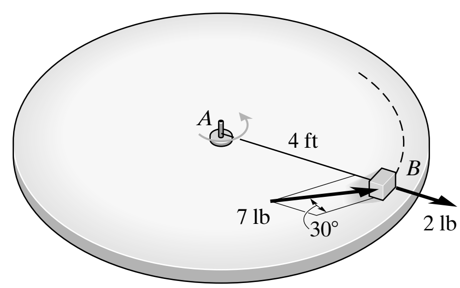


$$(H_A)_1 + \Sigma \int M_A\,dt = (H_A)_2$$

$$\left(\frac{10}{32.2}\right)(2)(4) + (7\cos 30°)(4)(t) - 0.5(10)(4)\,t = \frac{10}{32.2}(13.67)(4)$$

$t = 3.41$ s **Ans.**

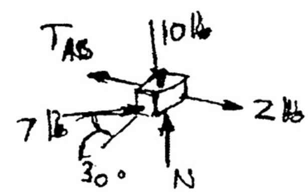
15–93. The 10-lb block is originally at rest on the smooth surface. It is acted upon by a radial force of 2 lb and a horizontal force of 7 lb, always directed at 30° from the tangent to the path as shown. Determine the time required to break the cord, which requires a tension $T = 30$ lb. What is the speed of the block when this occurs? Neglect the size of the block for the calculation.



$\Sigma F_n = ma_n;$

$$30 - 7\sin 30° - 2 = \frac{10}{32.2}\left(\frac{v^2}{4}\right)$$

$v = 17.764$ ft/s

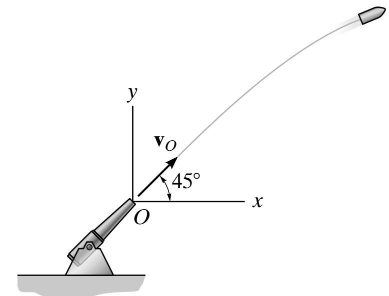


$$(H_A)_1 + \Sigma \int M_A\,dt = (H_A)_2$$

$$0 + (7\cos 30°)(4)(t) = \frac{10}{32.2}(17.764)(4)$$

$t = 0.910$ s **Ans.**

15–94. The projectile having a mass of 3 kg is fired from a cannon with a muzzle velocity of $v_0 = 500\,\text{m/s}$. Determine the projectile's angular momentum about point O at the instant it is at the maximum height of its trajectory.



At the maximum height, the projectile travels with a horizontal speed of $v = v_x = 500 \cos 45° = 353.6\ \text{m/s}^2$.

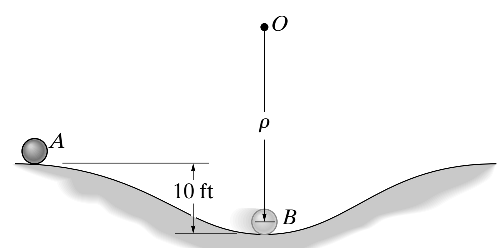
$$(+\uparrow) \qquad v_y^2 = (v_0)_y^2 + 2a_c\left[s_y - (s_0)_y\right]$$

$$0 = (500 \sin 45°)^2 + 2(-9.81)\left[(s_y)_{max} - 0\right]$$

$$(s_y)_{max} = 6371\ \text{m}$$

$$H_O = (d)(mv) = 6371(3)(353.6) = 6.76(10^6)\ \text{kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans.}$$

15–95. The 3-lb ball located at A is released from rest and travels down the curved path. If the ball exerts a normal force of 5 lb on the path when it reaches point B, determine the angular momentum of the ball about the center of curvature, point O. *Hint:* Neglect the size of the ball. The radius of curvature at point B must first be determined.

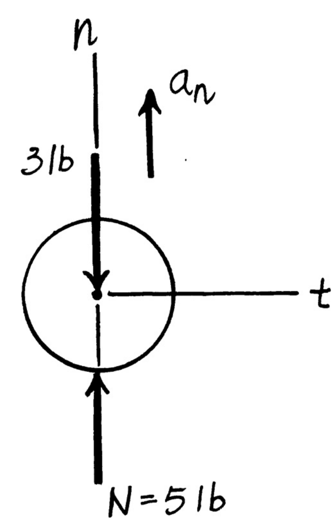


Datum at B:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 3(10) = \frac{1}{2}\left(\frac{3}{32.2}\right)(v_B)^2 + 0$$

$$v_B = 25.38\ \text{ft/s}$$

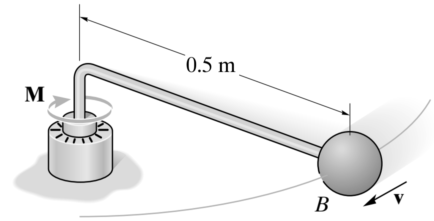


$$(+\uparrow)\Sigma F_n = ma_n; \qquad 5 - 3 = \left(\frac{3}{32.2}\right)\left(\frac{(25.38)^2}{\rho}\right)$$

$$\rho = 30\ \text{ft}$$

$$H_B = 30\left(\frac{3}{32.2}\right)(25.38) = 70.9\ \text{slug}\cdot\text{ft}^2/\text{s} \qquad \textbf{Ans.}$$

***15–96.** The ball B has a mass of 10 kg and is attached to the end of a rod whose mass can be neglected. If the shaft is subjected to a torque $M = (2t^2 + 4)\ \text{N} \cdot \text{m}$, where t is in seconds, determine the speed of the ball when $t = 2$ s. The ball has a speed $v = 2$ m/s when $t = 0$.

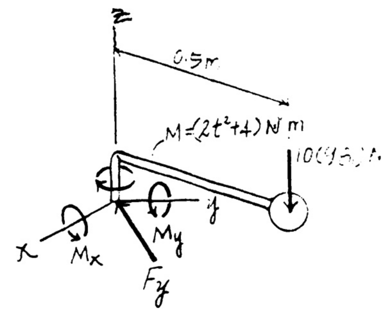


Principle of Angular Impluse and Momentum: Applying Eq. 15–22, we have

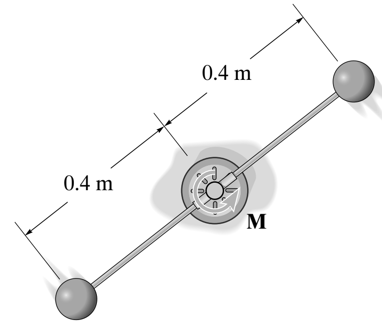
$$(H_z)_1 + \Sigma \int_{t_1}^{t_2} M_z\, dt = (H_z)_2$$

$$0.5(10)(2) + \int_0^{2\text{ s}} \left(2\,t^2 + 4\right) dt = 0.5(10)\, v$$

$$v = 4.67\text{ m/s} \qquad \textbf{Ans.}$$



•15–97. The two spheres each have a mass of 3 kg and are attached to the rod of negligible mass. If a torque $M = (6e^{0.2t})\ \text{N} \cdot \text{m}$, where t is in seconds, is applied to the rod as shown, determine the speed of each of the spheres in 2 s, starting from rest.

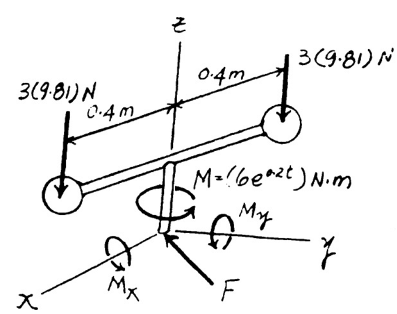


Principle of Angular Impluse and Momentum: Applying Eq. 15–22, we have

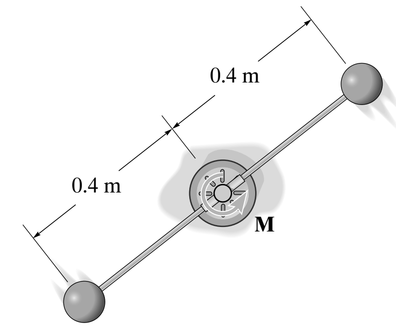
$$(H_z)_1 + \Sigma \int_{t_1}^{t_2} M_z\, dt = (H_z)_2$$

$$2[0.4\,(3)\,(0)] + \int_0^{2\text{ s}} \left(6e^{\,0.2\,t}\right)\, dt = 2\,[0.4\,(3)\,v]$$

$$v = 6.15\text{ m/s} \qquad \textbf{Ans.}$$



15–98. The two spheres each have a mass of 3 kg and are attached to the rod of negligible mass. Determine the time the torque $M = (8t)\ \text{N} \cdot \text{m}$, where t is in seconds, must be applied to the rod so that each sphere attains a speed of 3 m/s starting from rest.

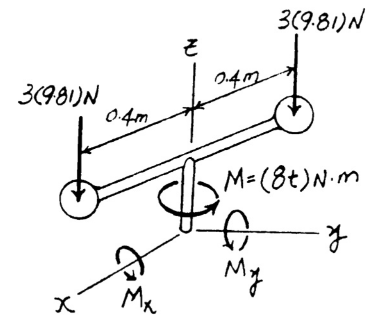


Principle of Angular Impluse and Momentum: Applying Eq. 15–22, we have

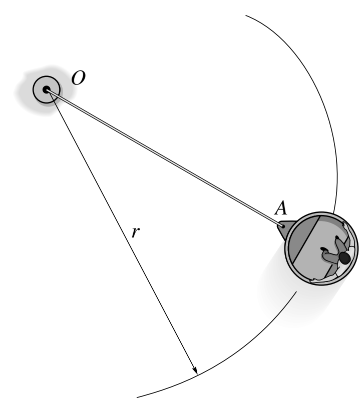
$$(H_z)_1 + \Sigma \int_{t_1}^{t_2} M_z\, dt = (H_z)_2$$

$$2[0.4\,(3)\,(0)] + \int_0^t (8t)\, dt = 2[0.4\,(3)\,(3)]$$

$$t = 1.34 \text{ s}$$ **Ans.**



15–99. An amusement park ride consists of a car which is attached to the cable OA. The car rotates in a horizontal circular path and is brought to a speed $v_1 = 4$ ft/s when $r = 12$ ft. The cable is then pulled in at the constant rate of 0.5 ft/s. Determine the speed of the car in 3 s.



Conservation of Angular Momentum: Cable OA is shorten by $\Delta r = 0.5(3) = 1.50$ ft. Thus, at this instant $r_2 = 12 - 1.50 = 10.5$ ft. Since no force acts on the car along the tangent of the moving path, the angular momentum is conserved about point O. Applying Eq. 15–23, we have

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$

$$r_1\, m v_1 = r_2\, m v'$$

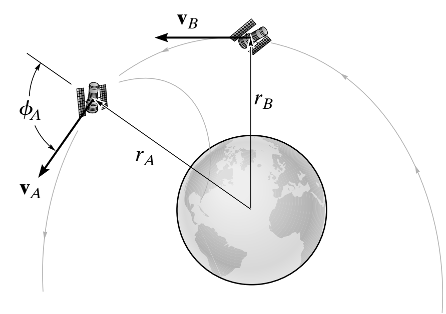
$$12(m)(4) = 10.5(m)\, v'$$

$$v' = 4.571 \text{ ft/s}$$

The speed of car after 3 s is

$$v_2 = \sqrt{0.5^2 + 4.571^2} = 4.60 \text{ ft/s}$$ **Ans.**

***15–100.** An earth satellite of mass 700 kg is launched into a free-flight trajectory about the earth with an initial speed of $v_A = 10$ km/s when the distance from the center of the earth is $r_A = 15$ Mm. If the launch angle at this position is $\phi_A = 70°$, determine the speed v_B of the satellite and its closest distance r_B from the center of the earth. The earth has a mass $M_e = 5.976(10^{24})$ kg. *Hint:* Under these conditions, the satellite is subjected only to the earth's gravitational force, $F = GM_e m_s/r^2$, Eq. 13–1. For part of the solution, use the conservation of energy.



$$(H_O)_1 = (H_O)_2$$

$$m_s\,(v_A \sin \phi_A) r_A = m_s\,(v_B) r_B$$

$$700[10(10^3)\sin 70°](15)(10^6) = 700(v_B)(r_B) \qquad \textbf{(1)}$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2} m_s\,(v_A)^2 - \frac{GM_e m_s}{r_A} = \frac{1}{2} m_s\,(v_B)^2 - \frac{GM_e m_s}{r_B}$$

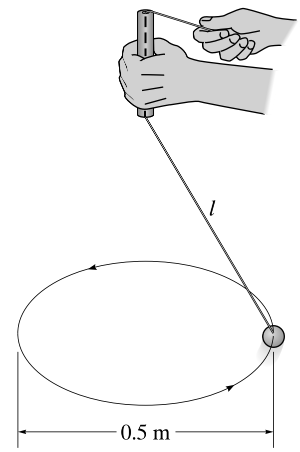
$$\frac{1}{2}(700)[10(10^3)]^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{[15(10^6)]} = \frac{1}{2}(700)(v_B)^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{r_B} \qquad \textbf{(2)}$$

Solving,

$v_\theta = 10.2$ km/s **Ans.**

$r_\theta = 13.8$ Mm **Ans.**

•15–101. The 2-kg ball rotates around a 0.5-m-diameter circular path with a constant speed. If the cord length is shortened from $l = 1$ m to $l' = 0.5$ m, by pulling the cord through the tube, determine the new diameter of the path d'. Also, what is the tension in the cord in each case?



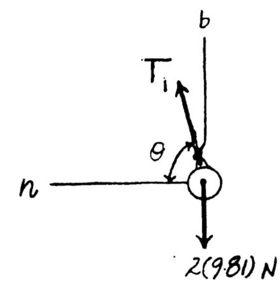
Equation of Motion: When the ball is travelling around the 0.5 m diameter circular path, $\cos\theta = \dfrac{0.25}{1} = 0.25$ and $\sin\theta = \dfrac{\sqrt{0.9375}}{1} = \sqrt{0.9375}$. Applying Eq. 13–8, we have

$$\Sigma F_b = 0; \qquad T_1\left(\sqrt{0.9375}\right) - 2(9.81) = 0$$

$$T_1 = 20.26 \text{ N} = 20.3 \text{ N} \qquad \textbf{Ans.}$$

$$\Sigma F_n = ma_n; \qquad 20.26(0.25) = 2\left(\frac{v_1^2}{0.25}\right)$$

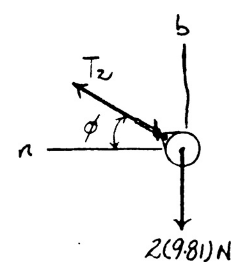
$$v_1 = 0.7958 \text{ m/s}$$



When the ball is travelling around d' the diameter circular path, $\cos\phi = \dfrac{d'/2}{0.5} = \text{d}'$ and $\sin\phi = \dfrac{\sqrt{0.25 - 0.25\,d'^2}}{0.5} = \sqrt{1 - d'^2}$. Applying Eq. 13–8, we have

$$\Sigma F_b = 0; \qquad T_2\left(\sqrt{1 - d^2}\right) - 2(9.81) = 0 \qquad \textbf{[1]}$$

$$\Sigma F_n = ma_n; \qquad T_2\,(d') = 2\left(\frac{v_2^2}{d'/2}\right) \qquad \textbf{[2]}$$



Conservation of Angular Momentum: Since no force acts on the ball along the tangent of the circular path, the angular momentum is conserved about z axis. Applying Eq. 15– 23, we have

$$(\mathbf{H}_z)_1 = (\mathbf{H}_z)_2$$

$$r_1\,mv_1 = r_2\,mv_2$$

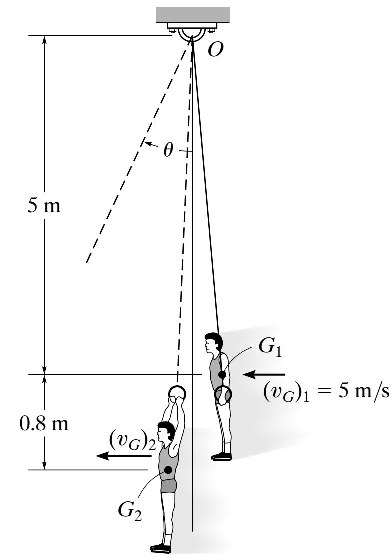
$$0.25\,(2)\,(0.7958) = \frac{d'}{2}\,(2)\,v_2 \qquad \textbf{[3]}$$

Solving Eqs. [1], [2] and [3] yields

$$d' = 0.41401 \text{ m} = 0.414 \text{ m} \qquad T_2 = 21.6 \text{ N} \qquad \textbf{Ans.}$$

$$v_2 = 0.9610 \text{ m/s}$$

15–102. A gymnast having a mass of 80 kg holds the two rings with his arms down in the position shown as he swings downward. His center of mass is located at point G_1. When he is at the lowest position of his swing, his velocity is $(v_G)_1 = 5$ m/s. At this position he *suddenly* lets his arms come up, shifting his center of mass to position G_2. Determine his new velocity in the upswing and the angle θ to which he swings before momentarily coming to rest. Treat his body as a particle.



$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$

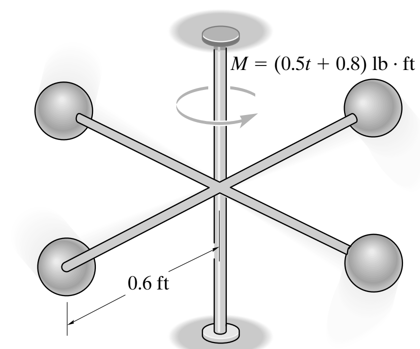
$$5\,(80)(5) = 5.8\,(80)\,v_2 \qquad v_2 = 4.310 \text{ m/s} = 4.31 \text{ m/s}$$ **Ans.**

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(80)(4.310)^2 + 0 = 0 + 80(9.81)\left[5.8\,(1 - \cos\theta)\right]$$

$$\theta = 33.2^\circ$$ **Ans.**

15–103. The four 5-lb spheres are rigidly attached to the crossbar frame having a negligible weight. If a couple moment $M = (0.5t + 0.8)$ lb·ft, where t is in seconds, is applied as shown, determine the speed of each of the spheres in 4 seconds starting from rest. Neglect the size of the spheres.

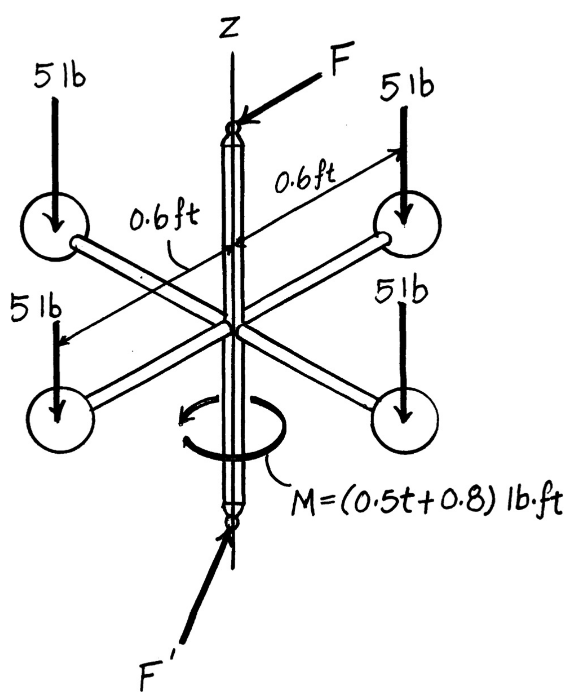


$$(H_z)_1 + \Sigma\int M_z\,dt = (H_z)_2$$

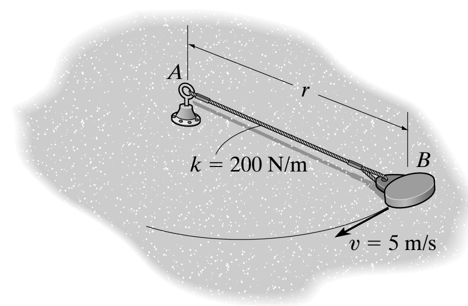
$$0 + \int_0^4 (0.5\,t + 0.8)\;dt = 4\left[\left(\frac{5}{32.2}\right)(0.6\,v_2)\right]$$

$$7.2 = 0.37267\,v_2$$

$$v_2 = 19.3 \text{ ft/s}$$ **Ans.**



***15–104.** At the instant $r = 1.5$ m, the 5-kg disk is given a speed of $v = 5$ m/s, perpendicular to the elastic cord. Determine the speed of the disk and the rate of shortening of the elastic cord at the instant $r = 1.2$ m. The disk slides on the smooth horizontal plane. Neglect its size. The cord has an unstretched length of 0.5 m.



Conservation of Energy: The intial and final stretch of the elastic cord is $s_1 = 1.5 - 0.5 = 1$ m and $s_2 = 1.2 - 0.5 = 0.7$ m. Thus,

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}ks_1^2 = \frac{1}{2}mv_2^2 = \frac{1}{2}ks_2^2$$

$$\frac{1}{2}5(5^2) + \frac{1}{2}(200)(1^2) = \frac{1}{2}(5)v_2^2 + \frac{1}{2}(200)(0.7^2)$$

$$v_2 = 6.738 \text{ m/s} \qquad \textbf{Ans.}$$

Conservation of Angular Momentum: Since no angular impulse acts on the disk about an axis perpendicular to the page passing through point O, its angular momentum of the system is conserved about this z axis. Thus,

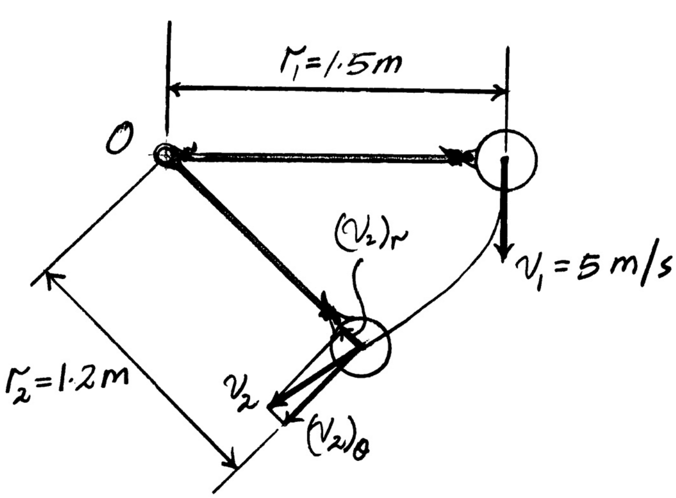
$$\left(H_O\right)_1 = \left(H_O\right)_2$$

$$r_1 m v_1 = r_2 m \left(v_2\right)_\theta$$

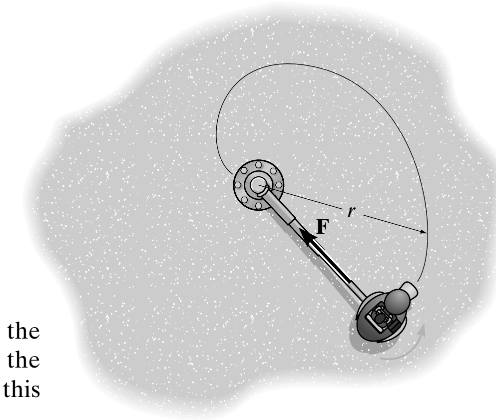
$$\left(v_2\right)_\theta = \frac{r_1 v_1}{r_2} = \frac{1.5(5)}{1.2} = 6.25 \text{ m/s}$$

Since $v_2^2 = \left(v_2\right)_\theta^2 + \left(v_2\right)_r^2$, then

$$\left(v_2\right)_r = \sqrt{v_2^2 - \left(v_2\right)_\theta^2} = \sqrt{6.738^2 - 6.25^2} = 2.52 \text{ m/s} \qquad \textbf{Ans.}$$



•15–105. The 150-lb car of an amusement park ride is connected to a rotating telescopic boom. When $r = 15$ ft, the car is moving on a horizontal circular path with a speed of 30 ft/s. If the boom is shortened at a rate of 3 ft/s, determine the speed of the car when $r = 10$ ft. Also, find the work done by the axial force **F** along the boom. Neglect the size of the car and the mass of the boom.



Conservation of Angular Momentum: By referring to Fig. *a*, we notice that the angular momentum of the car is conserved about an axis perpendicular to the page passing through point O, since no angular impulse acts on the car about this axis. Thus,

$$(H_O)_1 = (H_O)_2$$

$$r_1 m v_1 = r_2 m (v_2)_\theta$$

$$(v_2)_\theta = \frac{r_1 v_1}{r_2} = \frac{15(30)}{10} = 45 \text{ ft/s}$$

Thus, the magnitude of $\mathbf{v}_2$ is

$$v_2 = \sqrt{(v_2)_r{}^2 - (v_2)_\theta{}^2} = \sqrt{3^2 + 45^2} = 45.10 \text{ ft/s} = 45.1 \text{ ft/s} \qquad \textbf{Ans.}$$

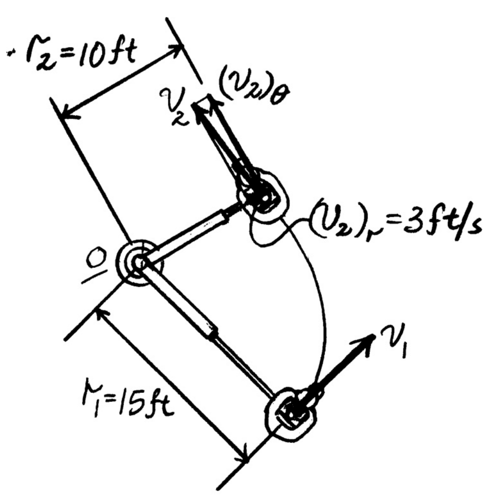
Principle of Work and Energy: Using the result of v_2,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2} m v_1{}^2 + U_F = \frac{1}{2} m v_2{}^2$$

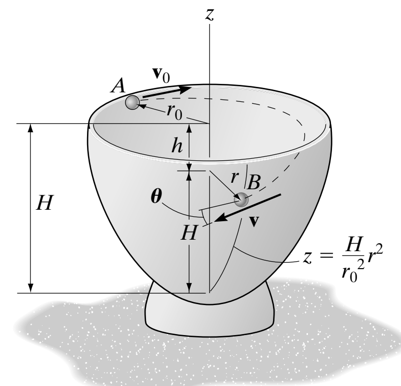
$$\frac{1}{2}\left(\frac{150}{32.2}\right)(30^2) + U_F = \frac{1}{2}\left(\frac{150}{32.2}\right)(45.10^2)$$

$$U_F = 2641 \text{ ft} \cdot \text{lb} \qquad \textbf{Ans.}$$



(a)

15–106. A small ball bearing of mass m is given a velocity of v_0 at A parallel to the horizontal rim of a smooth bowl. Determine the magnitude of the velocity $\mathbf{v}$ of the ball when it has fallen through a vertical distance h to reach point B. Angle θ is measured from $\mathbf{v}$ to the horizontal at point B.



Conservation of Angular Momentum: By observing the free-body diagram of the ball shown in Fig. *a*, notice that the weight $\mathbf{W}$ of the ball is parallel to the z axis and the line of action of the normal reaction $\mathbf{N}$ always intersect the z axis, and they produce no angular impulse about the z axis. Thus, the angular momentum of the ball is conserved about the z axis. At point B, $z = H - h$. Thus, $H - h = \frac{H}{r_0^2}r^2$ or $r = \sqrt{\frac{H-h}{H}}\,r_0$. Thus, we can write

$$(H_1)_z = (H_2)_z$$

$$r_0 m v_0 = r m v \cos\theta$$

$$r_0 v_0 = \left(\sqrt{\frac{H-h}{H}}\,r_0\right) v \cos\theta$$

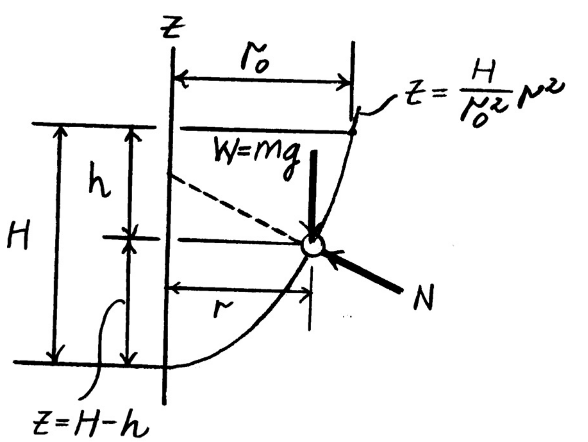
$$\cos\theta = \frac{v_0}{v}\sqrt{\frac{H}{Hh}} \qquad \textbf{(1)}$$

Conservation of Energy: By setting the datum at point B,

$$T_A + V_A = T_B + V_B$$

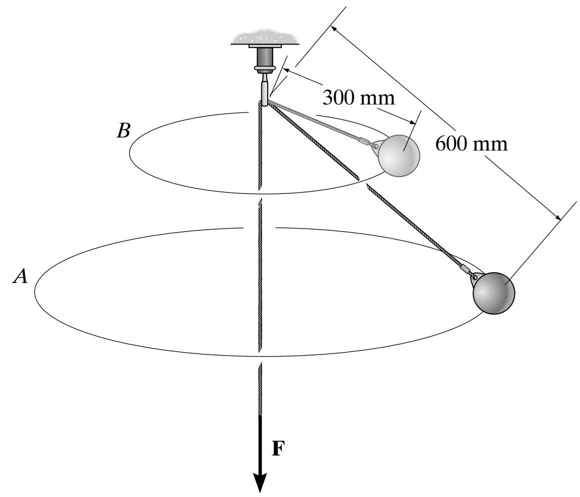
$$\frac{1}{2}mv_0^2 + mgh = \frac{1}{2}mv^2 + 0$$

$$v = \sqrt{v_0^2 + 2gh} \qquad \textbf{Ans.}$$



(a)

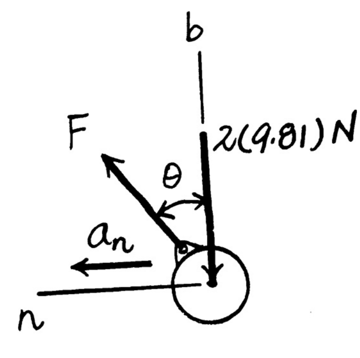
15–107. When the 2-kg bob is given a horizontal speed of 1.5 m/s, it begins to rotate around the horizontal circular path A. If the force **F** on the cord is increased, the bob rises and then rotates around the horizontal circular path B. Determine the speed of the bob around path B. Also, find the work done by force **F**.



Equations of Motion: By referring to the free-body diagram of the bob shown in Fig. a,

$$+\uparrow \Sigma F_b = 0; \qquad F\cos\theta - 2(9.81) = 0 \qquad \textbf{(1)}$$

$$\leftarrow \Sigma F_n = ma_n; \qquad F\sin\theta = 2\left(\frac{v^2}{l\sin\theta}\right) \qquad \textbf{(2)}$$



(a)

Eliminating F from Eqs. (1) and (2) yields

$$\frac{\sin^2\theta}{\cos\theta} = \frac{v^2}{9.81l}$$

$$\frac{1-\cos^2\theta}{\cos\theta} = \frac{v^2}{9.81l} \qquad \textbf{(3)}$$

When $l = 0.6$ m, $v = v_1 = 5$ m/s. Using Eq. (3), we obtain

$$\frac{1-\cos^2\theta_1}{\cos\theta_1} = \frac{1.5^2}{9.81(0.6)}$$

$$\cos^2\theta_1 + 0.3823\cos\theta_1 - 1 = 0$$

Solving for the root < 1, we obtain

$$\theta_1 = 34.21°$$

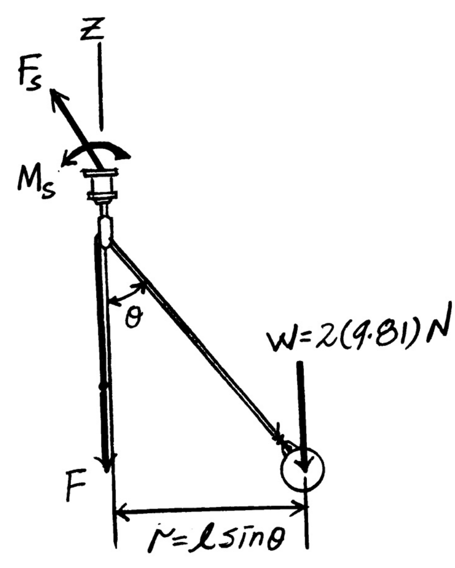
Conservation of Angular Momentum: By observing the free-body diagram of the system shown in Fig. b, notice that **W** and **F** are parallel to the z axis, $\mathbf{M}_S$ has no z component, and $\mathbf{F}_S$ acts through the z axis. Thus, they produce no angular impulse about the z axis. As a result, the angular momentum of the system is conserved about the z axis. When $\theta = \theta_1 = 34.21°$ and $\theta = \theta_2$, $r = r_1 = 0.6\sin 34.21° = 0.3373$ m and $r = r_2 = 0.3\sin\theta_2$. Thus,

$$(H_z)_1 = (H_z)_2$$

$$r_1 m v_1 = r_2 m v_2$$

$$0.3373(2)(1.5) = 0.3\sin\theta_2\,(2)v_2$$

$$v_2\sin\theta_2 = 1.6867 \qquad \textbf{(4)}$$



(b)

15–107. Continued

Substituting $l = 0.3$ and $\theta = \theta_2$ $v = v_2$ into Eq. (3) yields

$$\frac{1 - \cos^2\theta_2}{\cos\theta_2} = \frac{v^2}{9.81(0.3)}$$

$$\frac{1 - \cos^2\theta_2}{\cos\theta_2} = \frac{{v_2}^2}{2.943} \qquad \textbf{(5)}$$

Eliminating v_2 from Eqs. (4) and (5),

$$\sin^3\theta_2 \tan\theta_2 - 0.9667 = 0$$

Solving the above equation by trial and error, we obtain

$$\theta_2 = 57.866°$$

Substituting the result of θ_2 into Eq. (4), we obtain

$$v_2 = 1.992 \text{ m/s} = 1.99 \text{ m/s} \qquad \textbf{Ans.}$$

Principle of Work and Energy: When θ changes from θ_1 to θ_2, **W** displaces vertically upward $h = 0.6\cos 34.21° - 0.3\cos 57.866° = 0.3366$ m. Thus, **W** does negatives work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m{v_1}^2 + U_F + \left(-Wh\right) = \frac{1}{2}m{v_2}^2$$

$$\frac{1}{2}(2)(1.5^2) + U_F - 2(9.81)(0.3366) = \frac{1}{2}(2)(1.992)^2$$

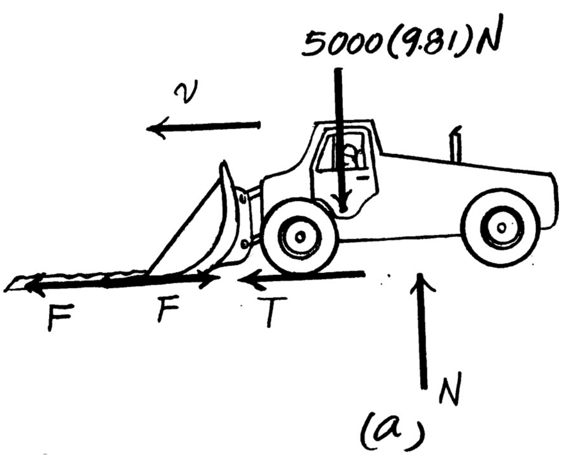
$$U_F = 8.32 \text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

***15–108.** A scoop in front of the tractor collects snow at a rate of 200 kg/s. Determine the resultant traction force **T** that must be developed on all the wheels as it moves forward on level ground at a constant speed of 5 km/h. The tractor has a mass of 5 Mg.

Here, the tractor moves with the constant speed of $v = \left[5\left(10^3\right)\frac{\text{m}}{\text{h}}\right]\left[\frac{1\text{ h}}{3600\text{ s}}\right]$ $= 1.389$ m/s. Thus, $v_{D/s} = v = 1.389$ m/s since the snow on the ground is at rest. The rate at which the tractor gains mass is $\frac{dm_s}{dt} = 200$ kg/s. Since the tractor is moving with a constant speeds $\frac{dv}{dt} = 0$. Referring to Fig. *a*,

$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} + v_{D/s}\frac{dm_s}{dt}; \qquad T = 0 + 1.389(200)$$

$$T = 278\text{ N} \qquad \textbf{Ans.}$$



•15–109. A four-engine commercial jumbo jet is cruising at a constant speed of 800 km/h in level flight when all four engines are in operation. Each of the engines is capable of discharging combustion gases with a velocity of 775 m/s relative to the plane. If during a test two of the engines, one on each side of the plane, are shut off, determine the new cruising speed of the jet. Assume that air resistance (drag) is proportional to the square of the speed, that is, $F_D = cv^2$, where c is a constant to be determined. Neglect the loss of mass due to fuel consumption.

Steady Flow Equation: Since the air is collected from a large source (the atmosphere), its entrance speed into the engine is negligible. The exit speed of the air from the engine is

$$\left(\xrightarrow{+}\right) \qquad v_e + v_p + v_{e/p}$$

When the four engines are in operation, the airplane has a constant speed of $v_p = \left[800(10^3)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 222.22$ m/s. Thus,

$$\left(\xrightarrow{+}\right) \qquad v_e = -222.22 + 775 = 552.78 \text{ m/s} \rightarrow$$

Referring to the free-body diagram of the airplane shown in Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right]; \qquad C(222.22^2) = 4\frac{dm}{dt}(552.78 - 0)$$

$$C = 0.44775\frac{dm}{dt}$$

When only two engines are in operation, the exit speed of the air is

$$\left(\xrightarrow{+}\right) \qquad v_e = -v_p + 775$$

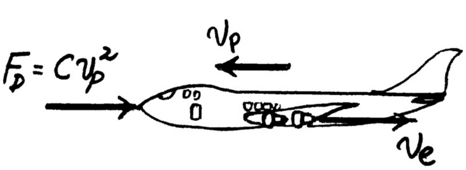
Using the result for C,

$$\xrightarrow{+} \Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right]; \quad \left(0.044775\frac{dm}{dt}\right)\left(v_p^{\,2}\right) = 2\frac{dm}{dt}\left[-v_p + 775\right) - 0\right]$$

$$0.044775v_p^{\,2} + 2v_p - 1550 = 0$$

Solving for the positive root,

$v_p = 165.06 \text{ m/s} = 594 \text{ km/h}$ **Ans.**



(a)

15–110. The jet dragster when empty has a mass of 1.25 Mg and carries 250 kg of solid propellent fuel. Its engine is capable of burning the fuel at a constant rate of 50 kg/s, while ejecting it at 1500 m/s relative to the dragster. Determine the maximum speed attained by the dragster starting from rest. Assume air resistance is $F_D = (10v^2)$ N, where v is the dragster's velocity in m/s. Neglect rolling resistance.

The free-body diagram of the dragster and exhaust system is shown in Fig. *a*, The pair of thrust **T** cancel each other since they are internal to the system. The mass of the dragster at any instant t is $m = (1250 + 250) - 50t = (1500 - 50t)$ kg.

$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \qquad -10v^2 = \left(1500 - 50t\right)\frac{dv}{dt} - 1500(50)$$

$$\frac{dt}{1500 - 50t} = \frac{dv}{75000 - 10v^2} \qquad \textbf{(1)}$$

The dragster acheives its maximum speed when all the fuel is consumed. The time it takes for this to occur is $t = \dfrac{250}{50} = 5$ s. Integratiing Eq. (1),

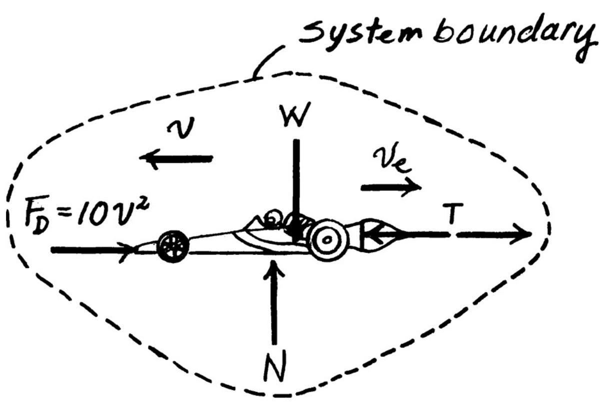
$$\int_0^{5\text{ s}} \frac{dt}{1500 - 50t} = \int_0^{v} \frac{dv}{75000 - 10v^2}$$

$$-\frac{1}{50}\,1\text{n}\left(1500 - 50t\right)\Big|_0^{5\text{ s}} = \frac{1}{2\sqrt{75000(10)}}\,1\text{n}\frac{\sqrt{75000} + \sqrt{10}v}{\sqrt{75000} - \sqrt{10}v}\Big|_0^{v}$$

$$1\text{n}\,\frac{\sqrt{75000} + \sqrt{10}v}{\sqrt{75000} - \sqrt{10}v} = 6.316$$

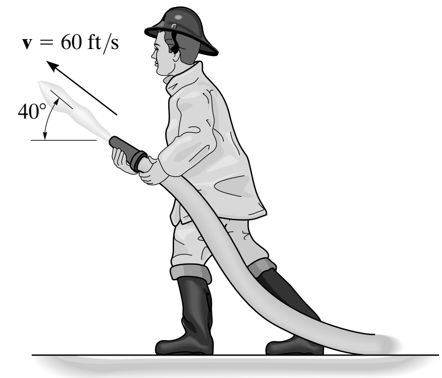
$$\frac{\sqrt{75000} + \sqrt{10}v}{\sqrt{75000} - \sqrt{10}v} = e^{6.316}$$

$$v = 86.3 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

15–111. The 150-lb fireman is holding a hose which has a nozzle diameter of 1 in. and hose diameter of 2 in. If the velocity of the water at discharge is 60 ft/s, determine the resultant normal and frictional force acting on the man's feet at the ground. Neglect the weight of the hose and the water within it. $\gamma_w = 62.4\ \text{lb/ft}^3$.



Originally, the water flow is horizontal. The fireman alters the direction of flow to 40° from the horizontal.

$$\frac{dm}{dt} = \rho v_B A_B = \frac{62.4}{32.2}(60)\left(\frac{\pi\left(\frac{1}{2}\right)^2}{(12)^2}\right) = 0.6342\ \text{slug/s}$$

Also, the velocity of the water through the hose is

$$\rho v_A A_A = \rho v_B A_B$$

$$\rho v_A\left(\frac{\pi(1)^2}{(12)^2}\right) = \rho\,(60)\left(\frac{\pi\left(\frac{1}{2}\right)^2}{(12)^2}\right)$$

$$v_A = 15\ \text{ft/s}$$

$$\overset{+}{\leftarrow}\ \Sigma F_x = \frac{dm}{dt}\left((v_B)_x - (v_A)_x\right)$$

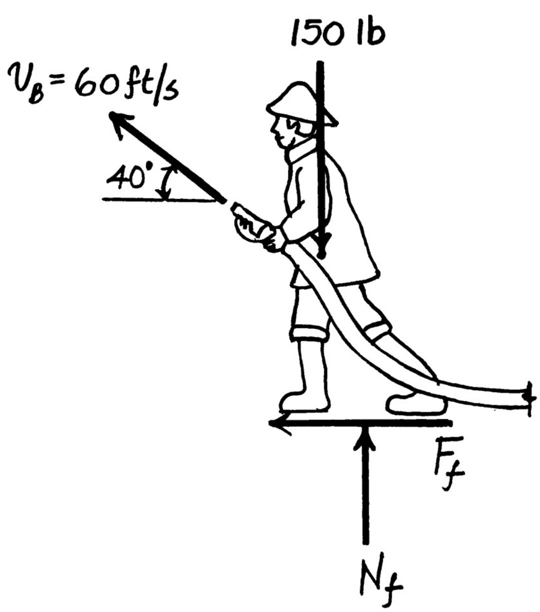
$$F_f = 0.6342\left[60\cos 40° - 15\right]$$

$F_f = 19.6$ lb **Ans.**

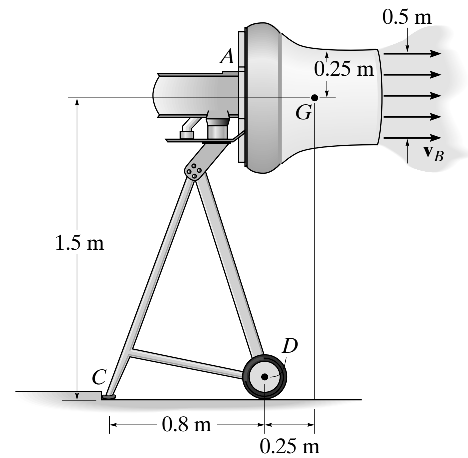
$$+\uparrow \Sigma F_y = \frac{dm}{dt}\left((v_B)_y - (v_A)_y\right)$$

$$N_f - 150 = 0.6342\left[60\sin 40° - 0\right]$$

$N_f = 174$ lb **Ans.**



***15–112.** When operating, the air-jet fan discharges air with a speed of $v_B = 20$ m/s into a slipstream having a diameter of 0.5 m. If air has a density of 1.22 kg/m^3, determine the horizontal and vertical components of reaction at C and the vertical reaction at each of the two wheels, D, when the fan is in operation. The fan and motor have a mass of 20 kg and a center of mass at G. Neglect the weight of the frame. Due to symmetry, both of the wheels support an equal load. Assume the air entering the fan at A is essentially at rest.



$$\frac{dm}{dt} = \rho v A = 1.22(20)(\pi)(0.25)^2 = 4.791 \text{ kg/s}$$

$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm}{dt}(v_{B_x} - v_{A_x})$$

$$C_x = 4.791(20 - 0)$$

$C_x = 95.8$ N **Ans.**

$$+\uparrow \Sigma F_y = 0; \qquad C_y + 2D_y - 20(9.81) = 0$$

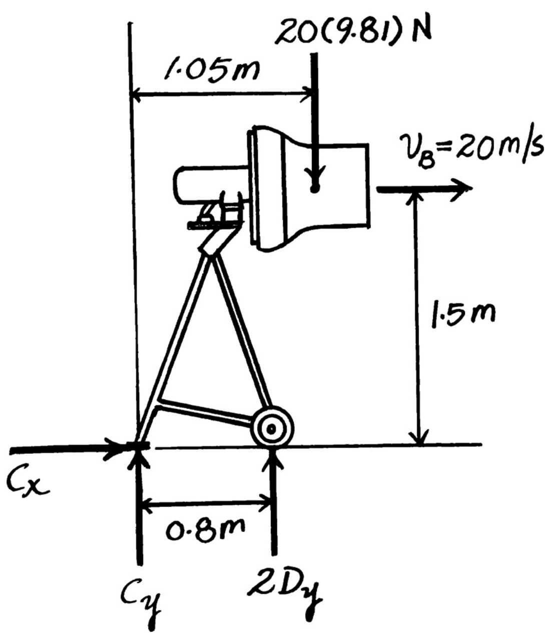
$$\zeta + \Sigma M_C = \frac{dm}{dt}(d_{CG} v_B - d_{CG} v_A)$$

$$2D_y(0.8) - 20(9.81)(1.05) = 4.791(-1.5(20) - 0)$$

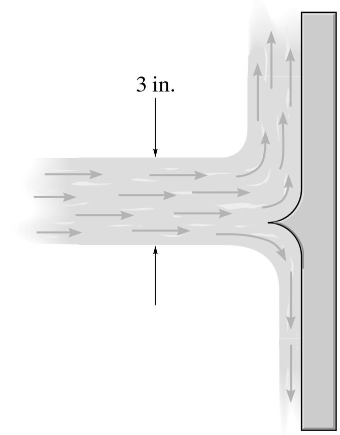
Solving:

$D_y = 38.9$ N **Ans.**

$C_y = 118$ N **Ans.**



•15–113. The blade divides the jet of water having a diameter of 3 in. If one-fourth of the water flows downward while the other three-fourths flows upwards, and the total flow is $Q = 0.5\ \text{ft}^3/\text{s}$, determine the horizontal and vertical components of force exerted on the blade by the jet, $\gamma_w = 62.4\ \text{lb/ft}^3$.



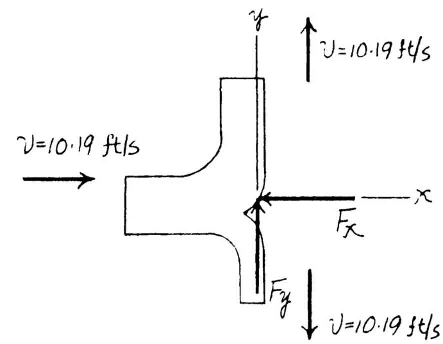
Equations of Steady Flow: Here, the flow rate $Q = 0.5\ \text{ft}^2/\text{s}$. Then, $v = \dfrac{Q}{A} = \dfrac{0.5}{\frac{\pi}{4}\left(\frac{3}{12}\right)^2} = 10.19\ \text{ft/s}$. Also, $\dfrac{dm}{dt} = \rho_w Q = \dfrac{62.4}{32.2}(0.5) = 0.9689\ \text{slug/s}$.

Applying Eq. 15–25 we have

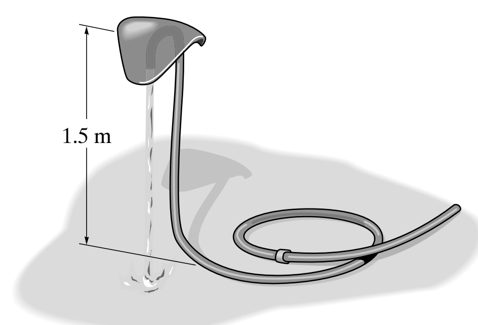
$$\Sigma F_x = \Sigma \frac{dm}{dt}\left(v_{\text{out}_s} - v_{\text{in}_s}\right);\ -F_x = 0 - 0.9689\,(10.19) \qquad F_x = 9.87\ \text{lb} \qquad \textbf{Ans.}$$

$$\Sigma F_y = \Sigma \frac{dm}{dt}\left(v_{\text{out}_y} - v_{\text{in}_y}\right);\ F_y = \frac{3}{4}(0.9689)(10.19) + \frac{1}{4}(0.9689)(-10.19)$$

$$F_y = 4.93\ \text{lb} \qquad \textbf{Ans.}$$



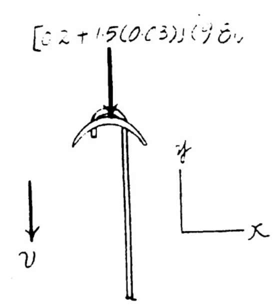
15–114. The toy sprinkler for children consists of a 0.2-kg cap and a hose that has a mass per length of 30 g/m. Determine the required rate of flow of water through the 5-mm-diameter tube so that the sprinkler will lift 1.5 m from the ground and hover from this position. Neglect the weight of the water in the tube. $\rho_w = 1\ \text{Mg/m}^3$.



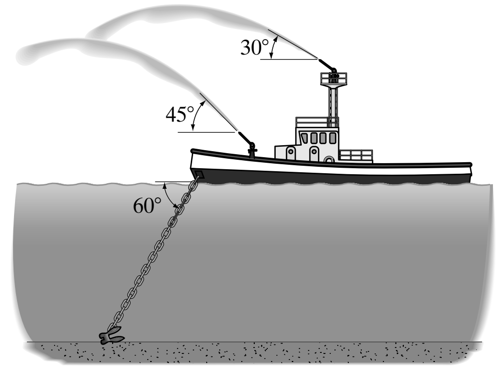
Equations of Steady Flow: Here, $v = \dfrac{Q}{\frac{\pi}{4}(0.005^2)} = \dfrac{Q}{6.25(10^{-6})\pi}$ and $\dfrac{dm}{dt} = \rho_w Q = 1000Q$. Applying Eq. 15–25, we have

$$\Sigma F_y = \frac{dm}{dt}\left(v_{B_y} - v_{A_y}\right);\ -[0.2 + 1.5\,(0.03)]\,(9.81) = 1000Q\left(-\frac{Q}{6.25\,(10^{-6})\ \pi} - 0\right)$$

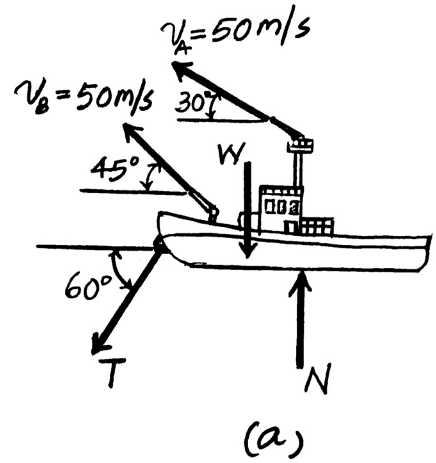
$$Q = 0.217\left(10^{-3}\right)\ \text{m}^3/\text{s} \qquad \textbf{Ans.}$$



15–115. The fire boat discharges two streams of seawater, each at a flow of $0.25\ \mathrm{m^3/s}$ and with a nozzle velocity of $50\ \mathrm{m/s}$. Determine the tension developed in the anchor chain, needed to secure the boat. The density of seawater is $\rho_{sw} = 1020\ \mathrm{kg/m^3}$.



Steady Flow Equation: Here, the mass flow rate of the sea water at nozzles A and B are $\dfrac{dm_A}{dt} = \dfrac{dm_B}{dt} = \rho_{sw}\,Q = 1020(0.25) = 225\ \mathrm{kg/s}$. Since the sea water is collected from the large reservoir (the sea), the velocity of the sea water entering the control volume can be considered zero. By referring to the free-body diagram of the control volume(the boat),

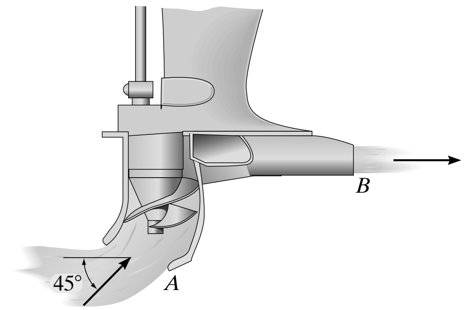


$$\overset{+}{\leftarrow} \Sigma F_x = \frac{dm_A}{dt}\left(v_A\right)_x + \frac{dm_B}{dt}\left(v_B\right)_x;$$

$$T\cos 60^\circ = 225\big(50\cos 30^\circ\big) + 225\big(50\cos 45^\circ\big)$$

$$T = 40\,114.87\ \mathrm{N} = 40.1\ \mathrm{kN}$$ **Ans.**

***15–116.** A speedboat is powered by the jet drive shown. Seawater is drawn into the pump housing at the rate of 20 ft^3/s through a 6-in.-diameter intake A. An impeller accelerates the water flow and forces it out horizontally through a 4-in.- diameter nozzle B. Determine the horizontal and vertical components of thrust exerted on the speedboat. The specific weight of seawater is $\gamma_{sw} = 64.3\ \text{lb/ft}^3$.



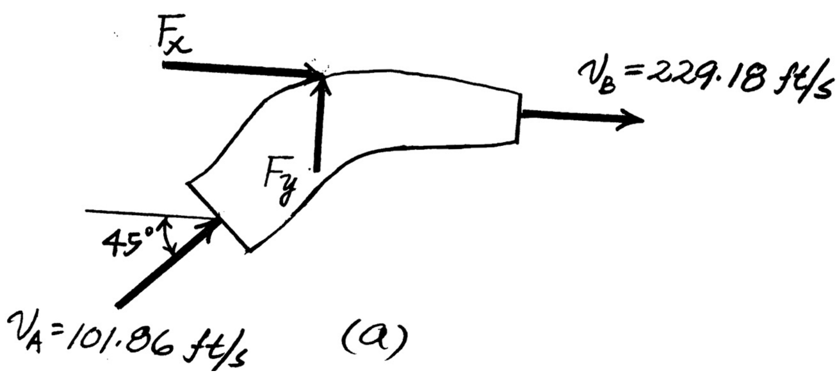
Steady Flow Equation: The speed of the sea water at the hull bottom A and B are $v_A = \dfrac{Q}{A_A} = \dfrac{20}{\frac{\pi}{4}\left(\frac{6}{12}\right)^2} = 101.86\ \text{ft/s}$ and $v_B = \dfrac{Q}{A_B} = \dfrac{20}{\frac{\pi}{4}\left(\frac{4}{12}\right)^2} = 229.18\ \text{ft/s}$ and the mass flow rate at the hull bottom A and nozle B are the same, i.e., $\dfrac{dm_A}{dt} = \dfrac{dm_B}{dt} = \dfrac{dm}{dt} = \rho_{sw}Q = \left(\dfrac{64.3}{32.2}\right)(20) = 39.94\ \text{slug/s}$. By referring to the free-body diagram of the control volume shown in Fig. a,

$$\left(\xrightarrow{+}\right)\Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right]; \qquad F_x = 39.94(229.18 - 101.86\cos 45^\circ)$$

$$= 6276.55\ \text{lb} = 6.28\ \text{kip} \qquad \textbf{Ans.}$$

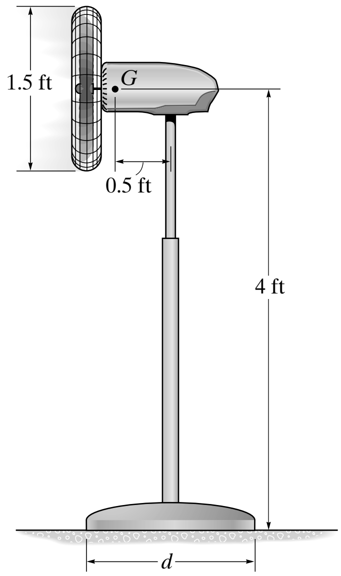
$$(+\uparrow)\Sigma F_y = \frac{dm}{dt}\left[(v_B)_y - (v_A)_y\right]; \qquad F_y = 39.94(101.86\sin 45^\circ - 0)$$

$$= 2876.53\ \text{lb} = 2.28\ \text{kip} \qquad \textbf{Ans.}$$



(a)

•15–117. The fan blows air at 6000 ft^3/min. If the fan has a weight of 30 lb and a center of gravity at G, determine the smallest diameter d of its base so that it will not tip over. The specific weight of air is $\gamma = 0.076\ \text{lb/ft}^3$.



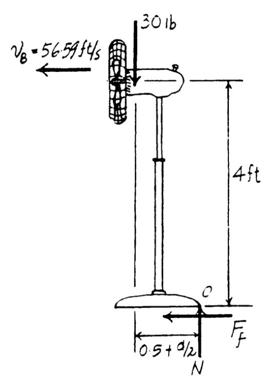
Equations of Steady Flow: Here $Q = \left(\dfrac{6000\ \text{ft}^3}{\text{min}}\right) \times \left(\dfrac{1\ \text{min}}{60\ \text{s}}\right) = 100\ \text{ft}^3/\text{s}$. Then,

$v = \dfrac{Q}{A} = \dfrac{100}{\frac{\pi}{4}(1.5^2)} = 56.59\ \text{ft/s}$. Also, $\dfrac{dm}{dt} = \rho_a Q = \dfrac{0.076}{32.2}(100) = 0.2360\ \text{slug/s}$.

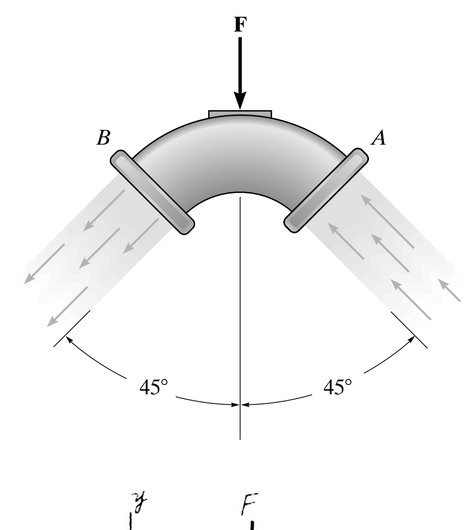
Applying Eq. 15–26 we have

$$\zeta + \Sigma M_O = \frac{dm}{dt}(d_{OB}\, v_B - d_{OA}\, v_A); \quad 30\left(0.5 + \frac{d}{2}\right) = 0.2360\,[4(56.59) - 0]$$

$$d = 2.56\ \text{ft} \qquad \textbf{Ans.}$$



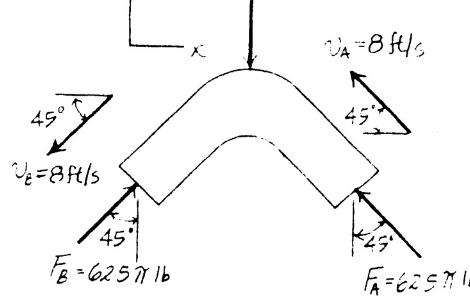
15–118. The elbow for a 5-in-diameter buried pipe is subjected to a static pressure of 10 lb/in^2. The speed of the water passing through it is $v = 8$ ft/s. Assuming the pipe connections at A and B do not offer any vertical force resistance on the elbow, determine the resultant vertical force **F** that the soil must then exert on the elbow in order to hold it in equilibrium. Neglect the weight of the elbow and the water within it. $\gamma_w = 62.4\ \text{lb/ft}^3$.



Equations of Steady Flow: Here. $Q = vA = 8\left[\dfrac{\pi}{4}\left(\dfrac{5}{12}\right)^2\right] = 1.091\ \text{ft}^3/\text{s}$. Then, the mass flow rate is $\dfrac{dm}{dt} = \rho_w Q = \dfrac{62.4}{32.2}(1.091) = 2.114\ \text{slug/s}$. Also, the force induced by the water pressure at A is $F = \rho A = 10\left[\dfrac{\pi}{4}(5^2)\right] = 62.5\pi$ lb. Applying Eq. 15–26, we have

$$\Sigma F_y = \frac{dm}{dt}\left(v_{B_y} - v_{A_y}\right); -F + 2(62.5\,\pi \cos 45°) = 2.114(-8 \sin 45° - 8 \sin 45°)$$

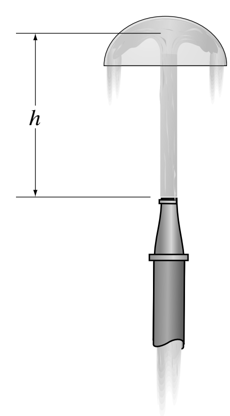
$$F = 302\ \text{lb} \qquad \textbf{Ans.}$$



$$\Sigma F_x = \frac{dm}{dt}\left(v_{B_x} - v_{A_x}\right); 62.5\,\pi \sin 45° - 62.5\,\pi \sin 45°$$
$$= 2.114[-8 \cos 45° - (-8 \cos 45°)]$$

$$0 = 0 \qquad \textbf{(Check!)}$$

15–119. The hemispherical bowl of mass m is held in equilibrium by the vertical jet of water discharged through a nozzle of diameter d. If the discharge of the water through the nozzle is Q, determine the height h at which the bowl is suspended. The water density is ρ_w. Neglect the weight of the water jet.



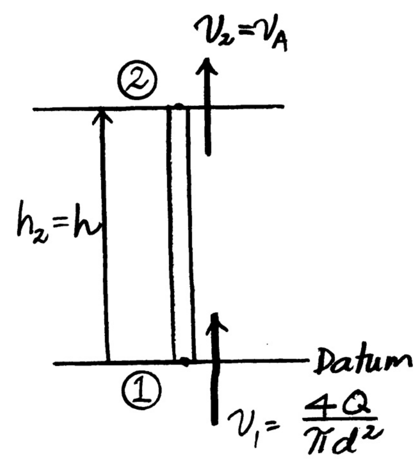
Conservation of Energy: The speed at which the water particle leaves the nozzle is $v_1 = \dfrac{Q}{A} = \dfrac{Q}{\frac{\pi}{4}d^2} = \dfrac{4Q}{\pi d^2}$. The speed of particle v_A when it comes in contact with the bowl can be determined using conservation of energy. With reference to the datum set in Fig. *a*,

$$T_1 + V_1 = T_2 + V_2 0$$

$$\frac{1}{2}mv_1{}^2 + \left(V_g\right)_1 = \frac{1}{2}mv_2{}^2 + \left(V_g\right)_2$$

$$\frac{1}{2}m\left(\frac{4Q}{\pi d^2}\right)^2 + 0 = \frac{1}{2}mv_A{}^2 + mgh$$

$$v_A = \sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}$$



(a)

Steady Flow Equation: The mass flow rate of the water jet that enters the control volume at A is $\dfrac{dm_A}{dt} = \rho_w Q$, and exits from the control volume at B is $\dfrac{dm_B}{dt} = \dfrac{dm_A}{dt} = \rho_w Q$. Thus, $v_B = v_A = \sqrt{\dfrac{16Q^2}{\pi^2 d^4} - 2gh}$. Here, the vertical force acting on the control volume is equal to the weight of the bowl. By referring to the free - body diagram of the control volume, Fig. *b*,

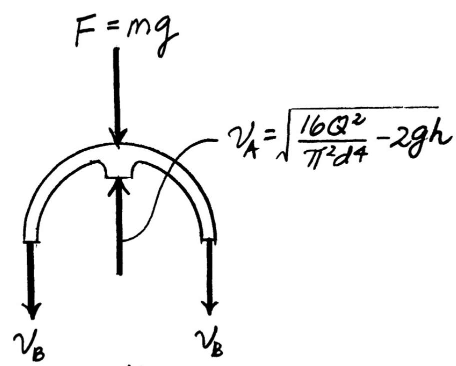
$$+\uparrow \Sigma F_y = 2\frac{dm_B}{dt}v_B - \frac{dm_A}{dt}v_A;$$

$$-mg = -(\rho_w Q)\left(\sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}\right) - \rho_w Q\left(\sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}\right)$$

$$mg = 2\rho_w Q\left(\sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}\right)$$

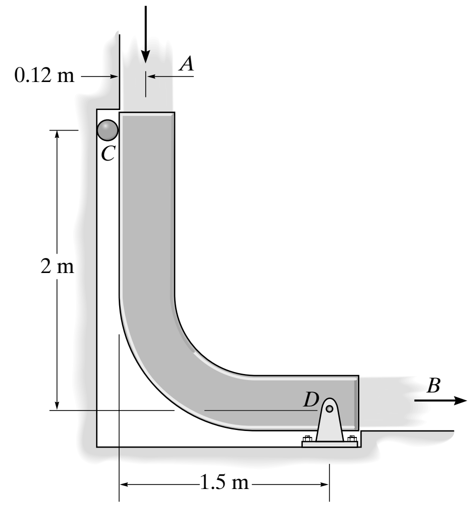
$$m^2 g^2 = 4\rho_w{}^2 Q^2\left(\frac{16Q^2}{\pi^2 d^4} - 2gh\right)$$

$$h = \frac{8Q^2}{\pi^2 d^4 g} - \frac{m^2 g}{8\rho_w{}^2 Q^2} \qquad \textbf{Ans.}$$



(b)

***15–120.** The chute is used to divert the flow of water, $Q = 0.6\ \text{m}^3/\text{s}$. If the water has a cross-sectional area of $0.05\ \text{m}^2$, determine the force components at the pin D and roller C necessary for equilibrium. Neglect the weight of the chute and weight of the water on the chute. $\rho_w = 1\ \text{Mg/m}^3$.



Equations of Steady Flow: Here, the flow rate $Q = 0.6\ \text{m}^2/\text{s}$. Then, $v = \dfrac{Q}{A} = \dfrac{0.6}{0.05} = 12.0\ \text{m/s}$. Also, $\dfrac{dm}{dt} = \rho_w Q = 1000\,(0.6) = 600\ \text{kg/s}$. Applying Eqs. 15–26 and 15–28, we have

$$\circlearrowleft + \Sigma M_A = \frac{dm}{dt}\left(d_{DB}\, v_B - d_{DA}\, v_A\right);$$

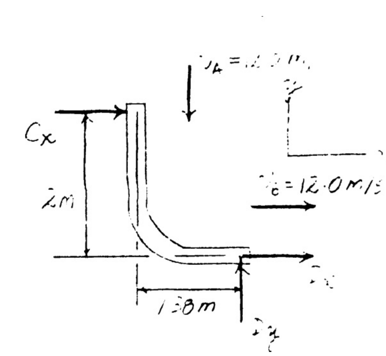
$$-C_x\,(2) = 600\,[0 - 1.38(12.0)] \qquad C_x = 4968\ \text{N} = 4.97\ \text{kN} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = \frac{dm}{dt}\left(v_{B_x} - v_{A_x}\right);$$

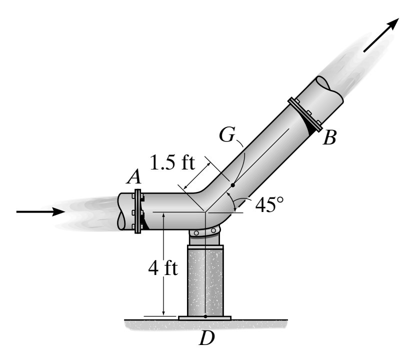
$$D_x + 4968 = 600\,(12.0 - 0) \qquad D_x = 2232\text{N} = 2.23\ \text{kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = \Sigma\frac{dm}{dt}\left(v_{\text{out}_y} - v_{\text{in}_y}\right);$$

$$D_y = 600[0 - (-12.0)] \qquad D_y = 7200\ \text{N} = 7.20\ \text{kN} \qquad \textbf{Ans.}$$



•15–121. The bend is connected to the pipe at flanges A and B as shown. If the diameter of the pipe is 1 ft and it carries a discharge of $50\ \text{ft}^3/\text{s}$, determine the horizontal and vertical components of force reaction and the moment reaction exerted at the fixed base D of the support. The total weight of the bend and the water within it is 500 lb, with a mass center at point G. The gauge pressure of the water at the flanges at A and B are 15 psi and 12 psi, respectively. Assume that no force is transferred to the flanges at A and B. The specific weight of water is $\gamma_w = 62.4\ \text{lb/ft}^3$.



Free-Body Diagram: The free-body of the control volume is shown in Fig. *a*. The force exerted on sections A and B due to the water pressure is $F_A = P_A A_A = 15\left[\dfrac{\pi}{4}\left(12^2\right)\right] = 1696.46\ \text{lb}$ and $F_B = P_B A_B = 12\left[\dfrac{\pi}{4}\left(12^2\right)\right] = 1357.17\ \text{lb}$. The speed of the water at, sections A and B are $v_A = v_B = \dfrac{Q}{A} = \dfrac{50}{\frac{\pi}{4}\left(1^2\right)} = 63.66\ \text{ft/s}$. Also, the mass flow rate at these two sections are $\dfrac{dm_A}{dt} = \dfrac{dm_B}{dt} = \rho_W Q = \left(\dfrac{62.4}{32.2}\right)(50) = 96.894\ \text{slug/s}$.

Steady Flow Equation: The moment steady flow equation will be written about point D to eliminate D_x and D_y.

$$\circlearrowleft +\Sigma M_D = \frac{dm_B}{dt} dv_B - \frac{dm_A}{dt} dv_A;$$

$$M_D + 1357.17 \cos 45°(4) - 500\left(1.5 \cos 45°\right) - 1696.46(4)$$

$$= -96.894(4)\left(63.66 \cos 45°\right) - [-96.894(4)(63.66)]$$

$$M_D = 10\,704.35 \text{ lb} \cdot \text{ft} = 10.7 \text{ kip} \cdot \text{ft}$$ **Ans.**

Writing the force steady flow equation along the x and y axes,

$$(+\uparrow)\Sigma F_y = \frac{dm}{dt}\left[(v_B)_y - (v_A)_y\right];$$

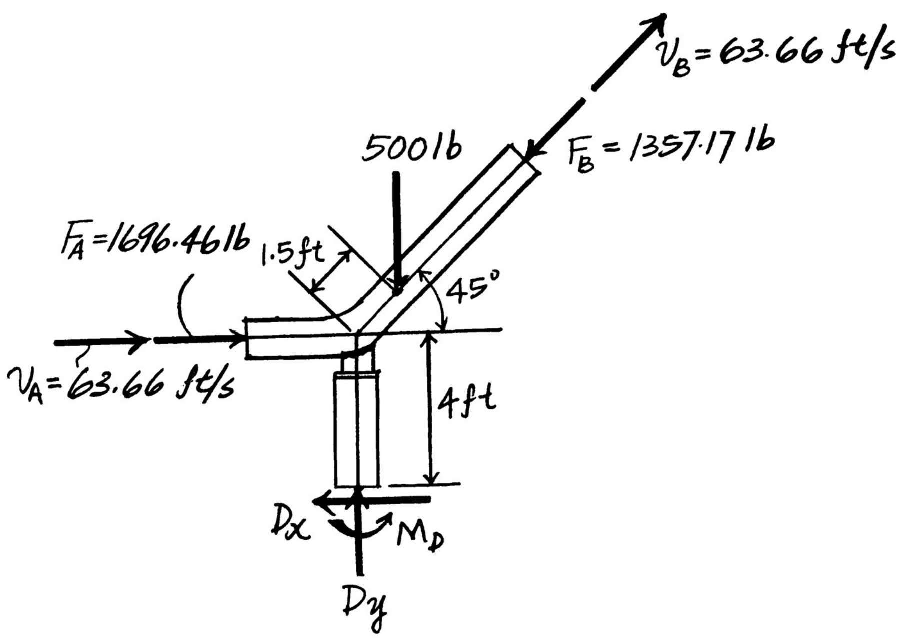
$$D_y - 500 - 1357.17 \sin 45° = 96.894(63.66 \sin 45° - 0)$$

$$D_y = 5821.44 \text{ lb} = 5.82 \text{ kip}$$ **Ans.**

$$\left(\xrightarrow{+}\right)\Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right];$$

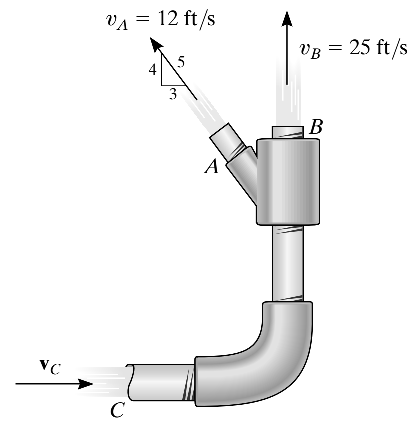
$$1696.46 - 1357.17 \cos 45° - D_x = 96.894[63.66 \cos 45° - 63.66]$$

$$D_x = 2543.51 \text{ lb} = 2.54 \text{ kip}$$ **Ans.**



(a)

15–122. The gauge pressure of water at C is 40 lb/in^2. If water flows out of the pipe at A and B with velocities $v_A = 12$ ft/s and $v_B = 25$ ft/s, determine the horizontal and vertical components of force exerted on the elbow necessary to hold the pipe assembly in equilibrium. Neglect the weight of water within the pipe and the weight of the pipe. The pipe has a diameter of 0.75 in. at C, and at A and B the diameter is 0.5 in. $\gamma_w = 62.4$ lb/ft^3.



$$\frac{dm_A}{dt} = \frac{63.4}{32.2}(25)(\pi)\left(\frac{025}{15}\right)^2 = 0.03171 \text{ slug/s}$$

$$\frac{dm_B}{dt} = \frac{62.4}{32.2}(25)(\pi)\left(\frac{0.25}{12}\right)^2 = 0.06606 \text{ slug/s}$$

$$\frac{dm_C}{dt} = 0.03171 + 0.06606 = 0.09777 \text{ slug/s}$$

$$v_C A_C = v_A A_A + v_B A_B$$

$$v_C(\pi)\left(\frac{0.375}{12}\right)^2 = 12(\pi)\left(\frac{0.25}{12}\right)^2 + 25(\pi)\left(\frac{0.25}{12}\right)^2$$

$$v_C = 16.44 \text{ ft/s}$$

$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm_B}{dt} v_{B_s} + \frac{dm_A}{dt} v_{A_s} - \frac{dm_C}{dt} v_{C_s}$$

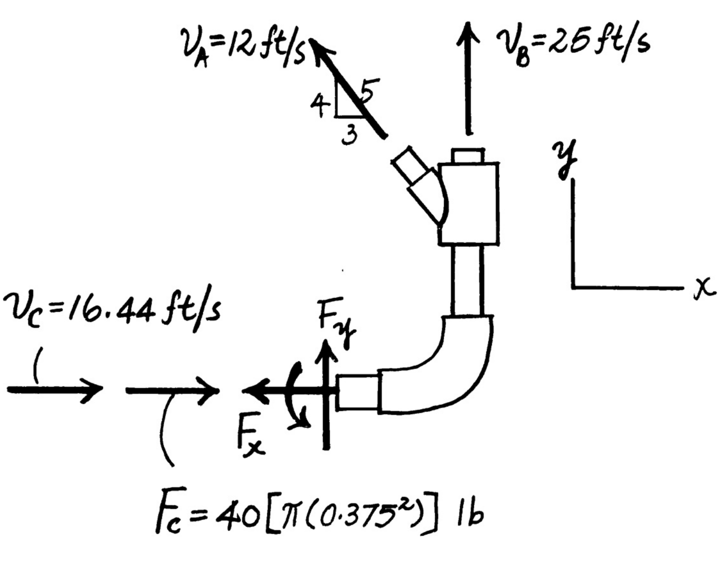
$$40(\pi)(0.375)^2 - F_x = 0 - 0.03171(12)\left(\frac{3}{5}\right) - 0.09777(16.44)$$

$F_x = 19.5$ lb **Ans.**

$$+\uparrow \Sigma F_y = \frac{dm_B}{dt} v_{B_y} + \frac{dm_A}{dt} v_{A_y} - \frac{dm_C}{dt} v_{C_y}$$

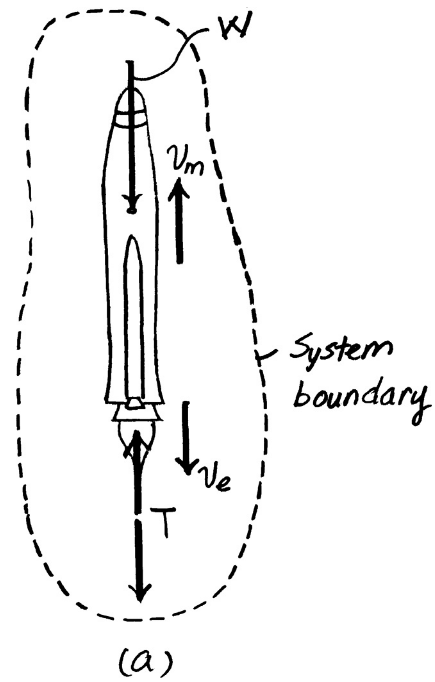
$$F_y = 0.06606(25) + 0.03171\left(\frac{4}{5}\right)(12) - 0$$

$F_y = 1.9559 = 1.96$ lb **Ans.**



15–123. A missile has a mass of 1.5 Mg (without fuel). If it consumes 500 kg of solid fuel at a rate of 20 kg/s and ejects it with a velocity of 2000 m/s relative to the missile, determine the velocity and acceleration of the missile at the instant all the fuel has been consumed. Neglect air resistance and the variation of its weight with altitude. The missile is launched vertically starting from rest.

By referring to the free-body diagram of the missile system in Fig. *a*, notice that the pair of thrust **T** cancel each other since they are internal to the system. The mass of the missile at any instant t after lauch is given by $m = (1500 + 500) - 20t = (2000 - 20t)$kg. Thus, the weight at the same instant is $W = (2000 - 20t)(9.81)$.



$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \qquad -(200-20t)(9.81) = (2000-20t)\frac{dv}{dt} - 2000(20)$$

$$a = \frac{dv}{dt} = \frac{2000}{100-t} - 9.81 \qquad \textbf{(1)}$$

The time taken for all the fuel to the consumed is $t = \frac{500}{20} = 25$ s. Substituting the result of t into Eq. (1),

$$a = \frac{2000}{100-25} - 9.81 = 16.9 \text{ m/s}^2\uparrow \qquad \textbf{Ans.}$$

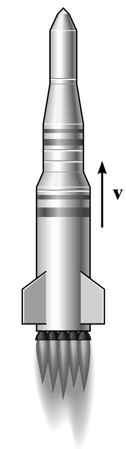
Integrating Eq. (1),

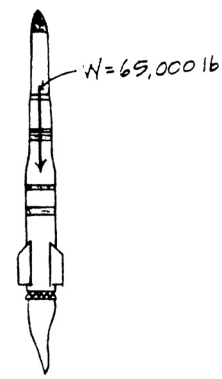
$$\int_0^v dv = \int_0^{25\,s}\left(\frac{2000}{100-t} - 9.81\right)dt$$

$$v = \left(-2000\ln(100 - t) - 9.81t\right)\Big|_0^{25\text{ s}}$$

$$= 330 \text{ m/s} \qquad \textbf{Ans.}$$

***15–124.** The rocket has a weight of 65 000 lb including the solid fuel. Determine the constant rate at which the fuel must be burned so that its thrust gives the rocket a speed of 200 ft/sin 10 s starting from rest. The fuel is expelled from the rocket at a relative speed of 3000 ft/s relative to the rocket. Neglect the effects of air resistance and assume that g is constant.





A System That Loses Mass: Here, $W = \left(m_0 - \frac{dm_r}{dt}t\right)g$. Applying Eq. 15–28, we have

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt};$$

$$-\left(m_0 - \frac{dm_e}{dt}t\right)g = \left(m_0 - \frac{dm_e}{dt}t\right)\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

$$\frac{dv}{dt} = \frac{v_{D/e}\frac{dm_e}{dt}}{m_0 - \frac{dm_e}{dt}t} - g$$

$$\int_0^v dv = \int_0^t \left(\frac{v_{D/e}\frac{dm_e}{dt}}{m_0 - \frac{dm_e}{dt}t} - g\right)dt$$

$$v = \left[-v_{D/e}1\text{n}\left(m_0 - \frac{dm_e}{dt}\text{t}\right) - \text{gt}\right]\Bigg|_0^{\text{t}}$$

$$v = v_{D/e}1\text{n}\left(\frac{m_0}{m_0 - \frac{dm_e}{dt}t}\right) - \text{gt} \qquad \textbf{[1]}$$

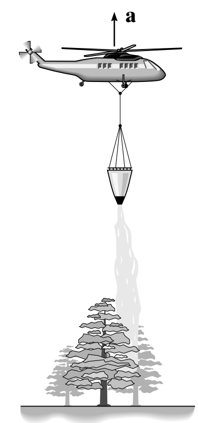
Substitute Eq. [1] with $m_0 = \frac{65\,000}{32.2} = 2018.63$ slug, $v_{D/e} = 3000$ ft/s, $v = 200$ ft/s and $t = 10\ s$, we have

$$200 = 3000\ 1\text{n}\left[\frac{2018.63}{2018.63 - \frac{dm_e}{dt}(10)}\right] - 32.2(10)$$

$$e^{0\ 174} = \frac{2018.63}{2018.63 - \frac{dm_e}{dt}(10)}$$

$$\frac{dm_e}{dt} = 32.2 \text{ slug/s} \qquad \textbf{Ans.}$$

•15–125. The 10-Mg helicopter carries a bucket containing 500 kg of water, which is used to fight fires. If it hovers over the land in a fixed position and then releases 50 kg/s of water at 10 m/s, measured relative to the helicopter, determine the initial upward acceleration the helicopter experiences as the water is being released.



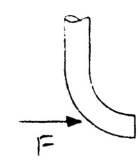
$$+\uparrow \Sigma F_t = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

Initially, the bucket is full of water, hence $m = 10(10^3) + 0.5(10^3) = 10.5(10^3)$ kg

$0 = 10.5(10^3)a - (10)(50)$

$a = 0.0476 \text{ m/s}^2$ **Ans.**

15–126. A plow located on the front of a locomotive scoops up snow at the rate of 10 ft^3/s and stores it in the train. If the locomotive is traveling at a constant speed of 12 ft/s, determine the resistance to motion caused by the shoveling. The specific weight of snow is $\gamma_s = 6$ lb/ft^3.

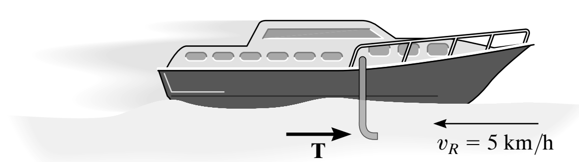


$$\Sigma F_x = m\frac{dv}{dt} + v_{D/t}\frac{dm_t}{dt}$$

$$F = 0 + (12 - 0)\left(\frac{10(6)}{32.2}\right)$$

$F = 22.4$ lb **Ans.**

15–127. The boat has a mass of 180 kg and is traveling forward on a river with a constant velocity of 70 km/h, measured *relative* to the river. The river is flowing in the opposite direction at 5 km/h. If a tube is placed in the water, as shown, and it collects 40 kg of water in the boat in 80 s, determine the horizontal thrust T on the tube that is required to overcome the resistance due to the water collection and yet maintain the constant speed of the boat. $\rho_w = 1$ Mg/m^3.



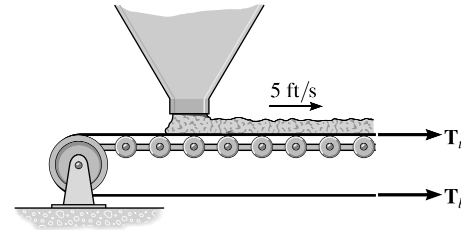
$$\frac{dm}{dt} = \frac{40}{80} = 0.5 \text{ kg/s}$$

$$v_{D/t} = (70)\left(\frac{1000}{3600}\right) = 19.444 \text{ m/s}$$

$$\Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$T = 0 + 19.444(0.5) = 9.72$ N **Ans.**

***15–128.** The bin deposits gravel onto the conveyor belt at the rate of 1500 lb/min. If the speed of the belt is 5 ft/s, determine how much greater the tension in the top portion of the belt must be than that in the bottom portion in order to pull the belt forward.

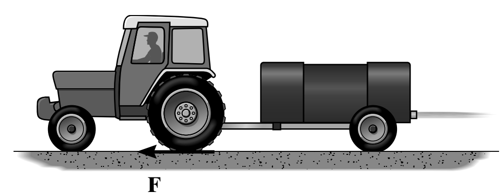


A System That Gains Mass: Here, $v_{D/t} = 5$ ft/s, $\dfrac{dv}{dt} = 0$ and $\dfrac{dm_t}{dt} = \left(\dfrac{1500 \text{ lb}}{\text{min}}\right) \times \left(\dfrac{1 \text{ slug}}{32.2 \text{ lb}}\right) \times \left(\dfrac{1 \text{ sin}}{60 \text{ s}}\right) = 0.7764$ slug/s. Applying Eq. 15–29, we have

$$\xrightarrow{+} \Sigma F_s = m\frac{dv}{dt} + v_{D/t}\frac{dm_s}{dt}; \qquad T_t - T_b = 0 + 5(0.7764)$$

$$\Delta T = 3.88 \text{ lb} \qquad \textbf{Ans.}$$

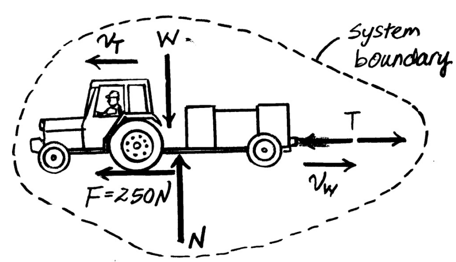
•15–129. The tractor together with the empty tank has a total mass of 4 Mg. The tank is filled with 2 Mg of water. The water is discharged at a constant rate of 50 kg/s with a constant velocity of 5 m/s, measured relative to the tractor. If the tractor starts from rest, and the rear wheels provide a resultant traction force of 250 N, determine the velocity and acceleration of the tractor at the instant the tank becomes empty.



The free-body diagram of the tractor and water jet is shown in Fig. *a*. The pair of thrust **T** cancel each other since they are internal to the system. The mass of the tractor and the tank at any instant t is given by $m = (4000 + 2000) - 50t = (6000 - 50t)$kg.

$$\xleftarrow{+} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \qquad 250 = (6000 - 50t)\frac{dv}{dt} - 5(50)$$

$$a = \frac{dv}{dt} = \frac{10}{120 - t} \qquad \textbf{(1)}$$



(a)

The time taken to empty the tank is $t = \dfrac{2000}{50} = 40$ s. Substituting the result of t into Eq. (1),

$$a = \frac{10}{120 - 40} = 0.125\text{m/s}^2 \qquad \textbf{Ans.}$$

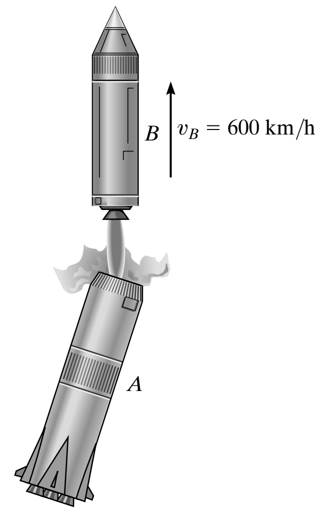
Integrating Eq. (1),

$$\int_0^v dv = \int_0^{40 \text{ s}} \frac{10}{120 - t}dt$$

$$v = -10\ln(120 - t)\Big|_0^{40 \text{ s}}$$

$$= 4.05 \text{ m/s} \qquad \textbf{Ans.}$$

15–130. The second stage B of the two-stage rocket has a mass of 5 Mg (empty) and is launched from the first stage A with an initial velocity of 600 km/h. The fuel in the second stage has a mass of 0.7 Mg and is consumed at the rate of 4 kg/s. If it is ejected from the rocket at the rate of 3 km/s, measured relative to B, determine the acceleration of B at the instant the engine is fired and just before all the fuel is consumed. Neglect the effects of gravitation and air resistance.



A System That Loses Mass: At the instant when stage B of rocket is launched, the total mass of the rocket is $m = 5000 + 5700$ kg. Applying Eq. 15–29, we have

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt};$$

$$0 = (5700)\frac{dv}{dt} - 3\left(10^3\right)(4) \qquad a = \frac{dv}{dt} = 2.11 \text{ m/s}^2$$ **Ans.**

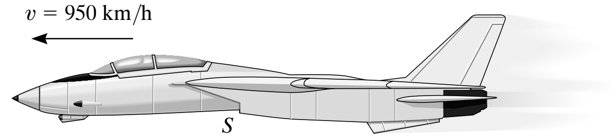
At the instant just before all the fuel being consumed, the mass of the rocket is $m = 5000$ kg. Applying Eq. 15–29, we have

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt};$$

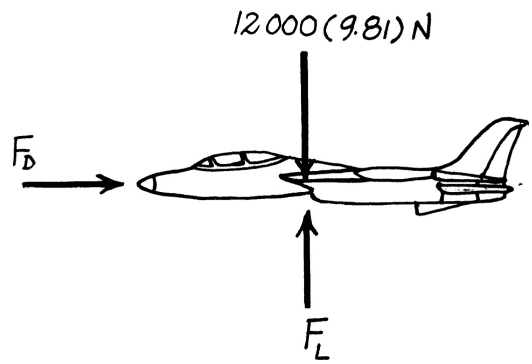
$$0 = (5000)\frac{dv}{dt} - 3\left(10^2\right)(4) \qquad a = \frac{dv}{dt} = 2.40 \text{ m/s}^2$$ **Ans.**

15–131. The 12-Mg jet airplane has a constant speed of 950 km/h when it is flying along a horizontal straight line. Air enters the intake scoops S at the rate of 50 m^3/s. If the engine burns fuel at the rate of 0.4 kg/s and the gas (air and fuel) is exhausted relative to the plane with a speed of 450 m/s, determine the resultant drag force exerted on the plane by air resistance. Assume that air has a constant density of 1.22 kg/m^3. *Hint:* Since mass both enters and exits the plane, Eqs. 15–28 and 15–29 must be combined to yield.

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}.$$



$$\Sigma F_s = m\frac{dv}{dt} - \frac{dm_e}{dt}(v_{D/E}) + \frac{dm_i}{dt}(v_{D/i}) \qquad \textbf{(1)}$$



$$v = 950 \text{ km/h} = 0.2639 \text{ km/s}, \qquad \frac{dv}{dt} = 0$$

$$v_{D/E} = 0.45 \text{ km/s}$$

$$v_{D/t} = 0.2639 \text{ km/s}$$

$$\frac{dm_t}{dt} = 50(1.22) = 61.0 \text{ kg/s}$$

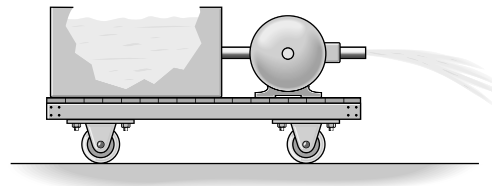
$$\frac{dm_e}{dt} = 0.4 + 61.0 = 61.4 \text{ kg/s}$$

Forces T and R are incorporated into Eq. (1) as the last two terms in the equation.

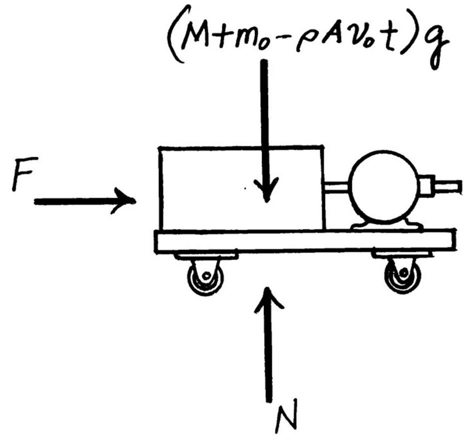
$$(\overset{+}{\leftarrow}) \quad -F_D = 0 - (0.45)(61.4) + (0.2639)(61)$$

$$F_D = 11.5 \text{ kN} \qquad \textbf{Ans.}$$

***15–132.** The cart has a mass M and is filled with water that has a mass m_0. If a pump ejects the water through a nozzle having a cross-sectional area A at a constant rate of v_0 relative to the cart, determine the velocity of the cart as a function of time. What is the maximum speed of the cart assuming all the water can be pumped out? The frictional resistance to forward motion is F. The density of the water is ρ.



$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\left(\frac{dm_e}{dt}\right)$$



$$\frac{dm_e}{dt} = \rho A v_0$$

$$-F = (M + m_0 - \rho A v_0 t)\frac{dv}{dt} - v_0(\rho A v_0)$$

$$\int_0^t \frac{dt}{M + m_0 - \rho A v_0 t} = \int_0^v \frac{dv}{\rho A v_0^2 - F}$$

$$-\left(\frac{1}{\rho A v_0}\right)\ln(M + m_0) = \rho A v_0 t) + \left(\frac{1}{\rho A v_0}\right)\ln(M + m_0) = \frac{v}{\rho A v_0^2 - F}$$

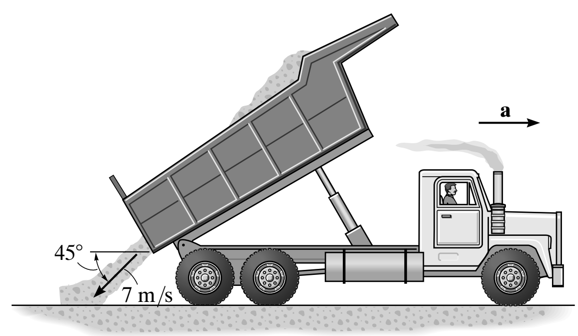
$$\left(\frac{1}{\rho A v_0}\right)\ln\left(\frac{M + m_0}{M + m_0 - \rho A v_0 t}\right) = \frac{v}{\rho A v_0^2 - F}$$

$$v = \left(\frac{\rho A v_0^2 - F}{\rho A v_0}\right)\ln\left(\frac{M + m_0}{M + m_0 - \rho A v_0 t}\right)$$ **Ans.**

v_{max} occurs when $t = \dfrac{m_0}{\rho A v_0}$, or,

$$v_{max} = \left(\frac{\rho A v_0^2 - F}{\rho A v_0}\right)\ln\left(\frac{M + m_0}{M}\right)$$ **Ans.**

•15–133. The truck has a mass of 50 Mg when empty. When it is unloading 5 m^3 of sand at a constant rate of 0.8 m^3/s, the sand flows out the back at a speed of 7 m/s, measured relative to the truck, in the direction shown. If the truck is free to roll, determine its initial acceleration just as the sand begins to fall out. Neglect the mass of the wheels and any frictional resistance to motion. The density of sand is $\rho_s = 1520$ kg/m^3.

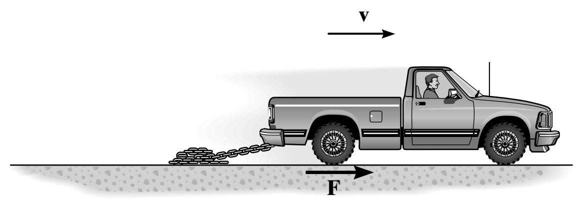


A System That Loses Mass: Initially, the total mass of the truck is $m = 50\left(10^3\right) + 5(1520) = 57.6\left(10^3\right)$ kg and $\dfrac{dm_e}{dt} = 0.8(1520) = 1216$ kg/s. Applying Eq. 15–29, we have

$$\overset{+}{\rightarrow} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \quad 0 = 57.6\left(10^3\right)a - (0.8\cos 45°)(1216)$$

$$a = 0.104 \text{ m/s}^2$$ **Ans.**

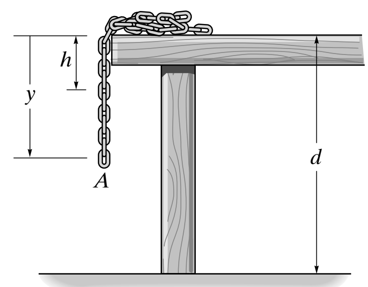
15–134. The truck has a mass m_0 and is used to tow the smooth chain having a total length l and a mass per unit of length m'. If the chain is originally piled up, determine the tractive force $\mathbf{F}$ that must be supplied by the rear wheels of the truck necessary to maintain a constant speed v while the chain is being drawn out.



A System That Loses Mass: Here, $v_{D/t} = v$, $\dfrac{dv}{dt} = 0$ and $\dfrac{dm_t}{dt} = m'v$. Applying Eq. 15–29, we have

$$\overset{+}{\rightarrow}\Sigma F_S = m\frac{dv}{dt} + v_{D/t}\frac{dm_t}{dt}; \qquad F = 0 + v(m'v) = m'v^2 \qquad \textbf{Ans.}$$

15–135. The chain has a total length $L < d$ and a mass per unit length of m'. If a portion h of the chain is suspended over the table and released, determine the velocity of its end A as a function of its position y. Neglect friction.



$$\Sigma F_s = m\frac{dv}{dt} + v_{D/e}\frac{dm_e}{dt}$$

$$m'gy = m'y\frac{dv}{dt} + v(m'v)$$

$$m'gy = m'\left(y\frac{dv}{dt} + v^2\right)$$

Since $dt = \dfrac{dy}{v}$, we have

$$gy = vy\frac{dv}{dy} + v^2$$

Multiply by $2y$ and integrate:

$$\int 2gy^2\,dy = \int\left(2vy^2\frac{dv}{dy} + 2yv^2\right)dy$$

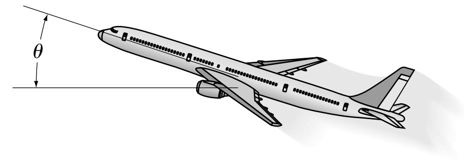
$$\frac{2}{3}g^3y^3 + C = v^2y^2$$

when $v = 0$, $y = h$, so that $C = -\dfrac{2}{3}gh^3$

Thus, $v^2 = \dfrac{2}{3}g\left(\dfrac{y^3 - h^3}{y^2}\right)$

$$v = \sqrt{\frac{2}{3}g\left(\frac{y^3 - h^3}{y^2}\right)} \qquad \textbf{Ans.}$$

***15–136.** A commercial jet aircraft has a mass of 150 Mg and is cruising at a constant speed of 850 km/h in level flight ($\theta = 0°$). If each of the two engines draws in air at a rate of 1000 kg/s and ejects it with a velocity of 900 m/s, relative to the aircraft, determine the maximum angle of inclination θ at which the aircraft can fly with a constant speed of 750 km/h. Assume that air resistance (drag) is proportional to the square of the speed, that is, $F_D = cv^2$, where c is a constant to be determined. The engines are operating with the same power in both cases. Neglect the amount of fuel consumed.

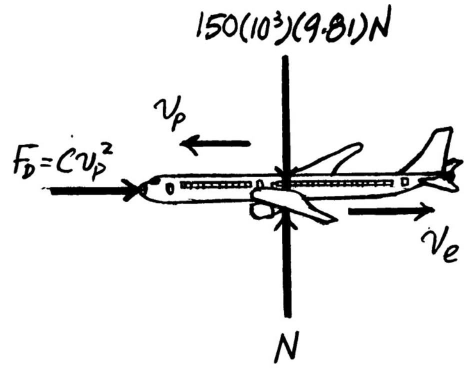


Steady Flow Equation: Since the air is collected from a large source (the atmosphere), its entrance speed into the engine is negligible. The exit speed of the air from the engine is given by

$$v_e = v_p + v_{e/p}$$

When the airplane is in level flight, it has a constant speed of $v_p = \left[850(10^3)\,\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 236.11\text{m/s}$. Thus,

$$\left(\overset{+}{\rightarrow}\right) \qquad v_e = -236.11 + 900 = 663.89\text{ m/s} \rightarrow$$



(a)

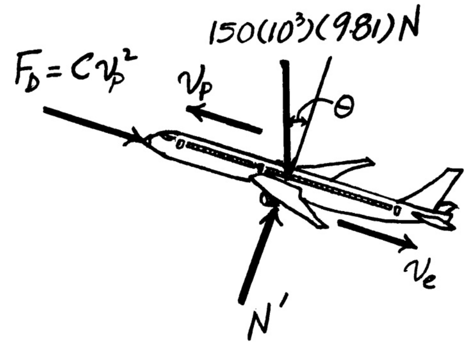
By referring to the free-body diagram of the airplane shown in Fig. *a*,

$$\left(\overset{+}{\rightarrow}\right)\Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right]; \qquad C(236.11^2) = 2(1000)(663.89 - 0)$$

$$C = 23.817\text{ kg}\cdot\text{s/m}$$

When the airplane is in the inclined position, it has a constant speed of $v_p = \left[750\left(10^3\right)\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 208.33\text{ m/s}$. Thus,

$$v_e = -208.33 + 900 = 691.67\text{ m/s}$$



(b)

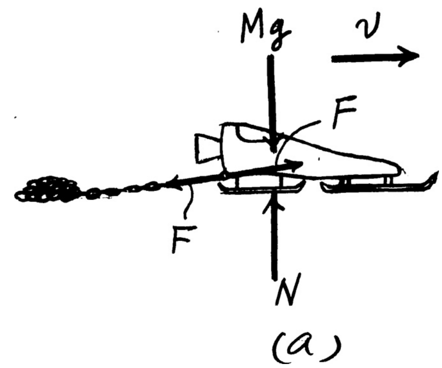
By referring to the free-body diagram of the airplane shown in Fig. *b* and using the result of C, we can write

$$\nwarrow + \Sigma F_{x'} = \frac{dm}{dt}\left[(v_B)_{x'} - (v_A)_{x'}\right]; \qquad 23.817\left(208.33^2\right) + 150\left(10^3\right)(9.81)\sin\theta = 2(1000)(691.67 - 0)$$

$$\theta = 13.7° \qquad \textbf{Ans.}$$

•15–137. A coil of heavy open chain is used to reduce the stopping distance of a sled that has a mass M and travels at a speed of v_0. Determine the required mass per unit length of the chain needed to slow down the sled to $(1/2)v_0$ within a distance $x = s$ if the sled is hooked to the chain at $x = 0$. Neglect friction between the chain and the ground.

Observing the free-body diagram of the system shown in Fig. *a*, notice that the pair of forces **F**, which develop due to the change in momentum of the chain, cancel each other since they are internal to the system. Here, $v_{D/s} = v$ since the chain on the ground is at rest. The rate at which the system gains mass is $\frac{dm_s}{dt} = m'v$ and the mass of the system is $m = m'x + M$. Referring to Fig. *a*,



$$\left(\xrightarrow{+}\right)\Sigma F_s = m\frac{dv}{dt} + v_{D/s}\frac{dm_s}{dt}; \qquad 0 = \left(m'x + M\right)\frac{dv}{dt} + v\left(m'v\right)$$

$$0 = \left(m'x + M\right)\frac{dv}{dt} + m'v^2 \qquad \textbf{(1)}$$

Since $\frac{dx}{dt} = v$ or $dt = \frac{dx}{v}$,

$$\left(m'x + M\right)v\frac{dv}{dx} + m'v^2 = 0$$

$$\frac{dv}{v} = -\left(\frac{m'}{m'x + M}\right)dx \qquad \textbf{(2)}$$

Integrating using the limit $v = v_0$ at $x = 0$ and $v = \frac{1}{2}v_0$ at $x = s$,

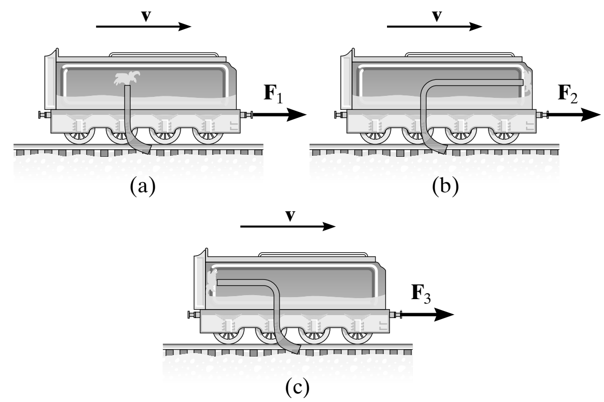
$$\int_{v_0}^{\frac{1}{2}v_0}\frac{dv}{v} = -\int_0^s\left(\frac{m'}{m'x + M}\right)dx$$

$$\ln v\Big|_{v_0}^{\frac{1}{2}v_0} = -\ln\left(m'x + M\right)\Big|_0^s$$

$$\frac{1}{2} = \frac{M}{m's + M}$$

$$m' = \frac{M}{s} \qquad \textbf{Ans.}$$

15–138. The car is used to scoop up water that is lying in a trough at the tracks. Determine the force needed to pull the car forward at constant velocity $\mathbf{v}$ for each of the three cases. The scoop has a cross-sectional area A and the density of water is ρ_w.



The system consists of the car and the scoop. In all cases

$$\Sigma F_t = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

$$F = 0 - v(\rho)(A)\,v$$

$$F = v^2\,\rho\,A$$ **Ans.**

15–139. A rocket has an empty weight of 500 lb and carries 300 lb of fuel. If the fuel is burned at the rate of 1.5 lb/s and ejected with a velocity of 4400 ft/s relative to the rocket, determine the maximum speed attained by the rocket starting from rest. Neglect the effect of gravitation on the rocket.

$$+\uparrow \Sigma F_s = \frac{dv}{dt} - v_{\,D/e}\frac{dm_e}{dt}$$

At a time t, $m = m_0 - ct$, where $c = \dfrac{dm_e}{dt}$. In space the weight of the rocket is zero.

$$0 = (m_0 - ct)\frac{dv}{dt} - v_{D/e}\,c$$

$$\int_0^v dv = \int_0^t \left(\frac{c v_{D/e}}{m_0 - ct}\right)dt$$

$$v = v_{D/e}\,1\text{n}\left(\frac{m_0}{m_0 - ct}\right) \qquad [1]$$

The maximum speed occurs when all the fuel is consumed, that is, when $t = \frac{300}{15} = 20$ s.

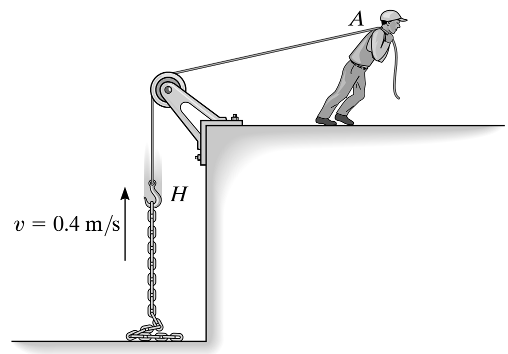
Here, $m_0 = \frac{500\,+\,300}{32.2} = 24.8447$ slug, $c = \frac{15}{32.2} = 0.4658$ slug/s, $v_{D/e} = 4400$ ft/s.

Substitute the numerical into Eq. [1]:

$$v_{\max} = 4400\,1\text{n}\left(\frac{24.8447}{24.8447 - (0.4658(20))}\right)$$

$$v_{\max} = 2068 \text{ ft/s}$$ **Ans.**

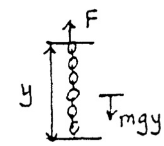
***15–140.** Determine the magnitude of force **F** as a function of time, which must be applied to the end of the cord at A to raise the hook H with a constant speed $v = 0.4$ m/s. Initially the chain is at rest on the ground. Neglect the mass of the cord and the hook. The chain has a mass of 2 kg/m.



$$\frac{dv}{dt} = 0, \qquad y = vt$$

$$m_i = my = mvt$$

$$\frac{dm_i}{dt} = mv$$



$$+\uparrow \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\left(\frac{dm_i}{dt}\right)$$

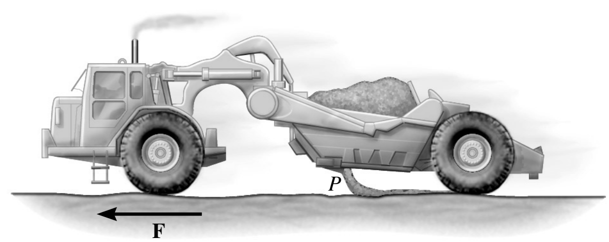
$$F - mgvt = 0 + v(mv)$$

$$F = m(gvt + v^2)$$

$$= 2[9.81(0.4)t + (0.4)^2]$$

$$F = (7.85t + 0.320)\text{ N}$$ **Ans.**

•15–141. The earthmover initially carries 10 m^3 of sand having a density of 1520 kg/m^3. The sand is unloaded horizontally through a 2.5-m^2 dumping port P at a rate of 900 kg/s measured relative to the port. If the earthmover maintains a constant resultant tractive force $F = 4$ kN at its front wheels to provide forward motion, determine its acceleration when half the sand is dumped. When empty, the earthmover has a mass of 30 Mg. Neglect any resistance to forward motion and the mass of the wheels. The rear wheels are free to roll.



When half the sand remains,

$$m = 30\,000 + \frac{1}{2}(10)(1520) = 37\,600\text{ kg}$$

$$\frac{dm}{dt} = 900\text{ kg/s} = \rho\, v_{D/e} A$$

$$900 = 1520(v_{D/e})(2.5)$$

$$v_{D/e} = 0.237\text{ m/s}$$

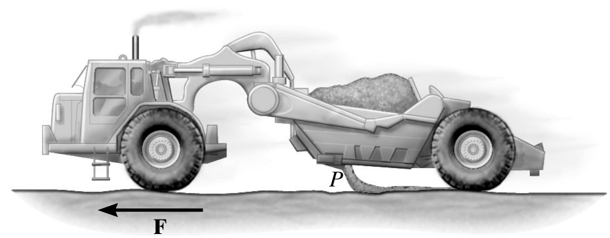
$$a = \frac{dv}{dt} = 0.1$$

$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} - \frac{dm}{dt}v$$

$$F = 37\,600(0.1) - 900(0.237)$$

$$F = 3.55\text{ kN}$$ **Ans.**

15–142. The earthmover initially carries 10 m^3 of sand having a density of 1520 kg/m^3. The sand is unloaded horizontally through a 2.5-m^2 dumping port P at a rate of 900 kg/s measured relative to the port. Determine the resultant tractive force **F** at its front wheels if the acceleration of the earthmover is 0.1 m/s^2 when half the sand is dumped. When empty, the earthmover has a mass of 30 Mg. Neglect any resistance to forward motion and the mass of the wheels. The rear wheels are free to roll.



When half the sand remains,

$$m = 30\,000 + \frac{1}{2}(10)(1520) = 37\,600 \text{ kg}$$

$$\frac{dm}{dt} = 900 \text{ kg/s} = \rho\, v_{D/e}\, A$$

$$900 = 1520(v_{D/e})(2.5)$$

$$v_{D/e} = 0.237 \text{ m/s}$$

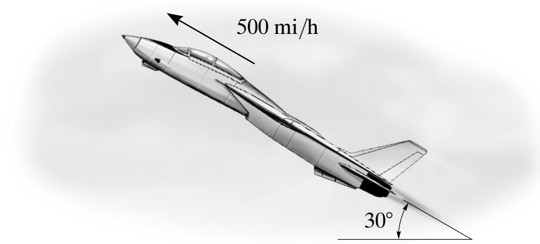
$$a = \frac{dv}{dt} = 0.1$$

$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} - \frac{dm}{dt}\text{v}$$

$$F = 37\,600(0.1) - 900(0.237)$$

$F = 3.55$ kN **Ans.**

15–143. The jet is traveling at a speed of 500 mi/h, 30° with the horizontal. If the fuel is being spent at 3 lb/s, and the engine takes in air at 400 lb/s, whereas the exhaust gas (air and fuel) has a relative speed of 32 800 ft/s, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = (0.7v^2)$ lb, where the speed is measured in ft/s. The jet has a weight of 15 000 lb. *Hint:* See Prob. 15–131.

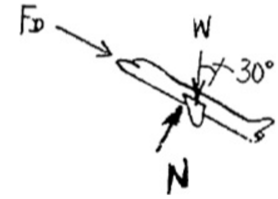


$$\frac{dm_i}{dt} = \frac{400}{32.2} = 12.42 \text{ slug/s}$$

$$\frac{dm_e}{dt} = \frac{403}{32.2} = 12.52 \text{ slug/s}$$

$$v = v_{D/i} = 500 \text{ mi/h} = 733.3 \text{ ft/s}$$

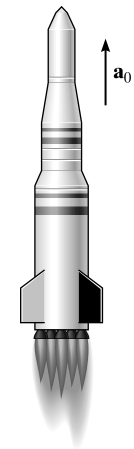
$$\nwarrow + \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}$$



$$-(15\,000)\sin 30° - 0.7(733.3)^2 = \frac{15\,000}{32.2}\frac{dv}{dt} - 32\,800(12.52) + 733.3(12.42)$$

$$a = \frac{dv}{dt} = 37.5 \text{ ft/s}^2$$ **Ans.**

***15–144.** The rocket has an initial mass m_0, including the fuel. For practical reasons desired for the crew, it is required that it maintain a constant upward acceleration a_0. If the fuel is expelled from the rocket at a relative speed $v_{e/r}$ determine the rate at which the fuel should be consumed to maintain the motion. Neglect air resistance, and assume that the gravitational acceleration is constant.



$$a_0 = \frac{dv}{dt}$$

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - \mathrm{v}_{D/e}\frac{dm_e}{dt}$$

$$-mg = ma_0 - v_{e/r}\frac{dm}{dt}$$

$$v_{e/r}\frac{dm}{m} = (a_0 + g)\,dt \qquad \textbf{(1)}$$

Since $v_{c/r}$ is constant, integrating, with $t = 0$ when $m = m_0$ yields

$$v_{e/r}\ln\left(\frac{m}{m_0}\right) = (a_0 + g)\,t$$

$$\frac{m}{m_0} = e^{[(a_0+g)/v_{e/r}]t}$$

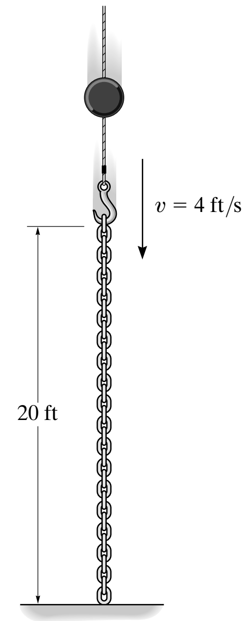
The time rate of fuel consumption is determined from Eq. (1).

$$\frac{dm}{dt} = m\left(\frac{(a_0 + g)}{v_{e/r}}\right)$$

$$\frac{dm}{dt} = m_0\left(\frac{(a_0 + g)}{v_{e/r}}\right)e^{[(a_0+g)v_{e/r}]t} \qquad \textbf{Ans.}$$

Note: $v_{c/r}$ must be considered a negative quantity.

•15–145. If the chain is lowered at a constant speed, determine the normal reaction exerted on the floor as a function of time. The chain has a weight of 5 lb/ft and a total length of 20 ft.



At time t, the weight of the chain on the floor is $W = mg(vt)$

$$\frac{dv}{dt} = 0, \qquad m_i = m(vt)$$

$$\frac{dm_i}{dt} = mv$$

$$\Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$$R - mg(vt) = 0 + v(mv)$$

$$R = m(gvt + v^2)$$

$$R = \frac{5}{32.2}(32.2(4)(t) + (4)^2)$$

$$R = (20t + 2.48)\text{ lb} \qquad \textbf{Ans.}$$