•14–1. A 1500-lb crate is pulled along the ground with a constant speed for a distance of 25 ft, using a cable that makes an angle of 15° with the horizontal. Determine the tension in the cable and the work done by this force. The coefficient of kinetic friction between the ground and the crate is $\mu_k = 0.55$.

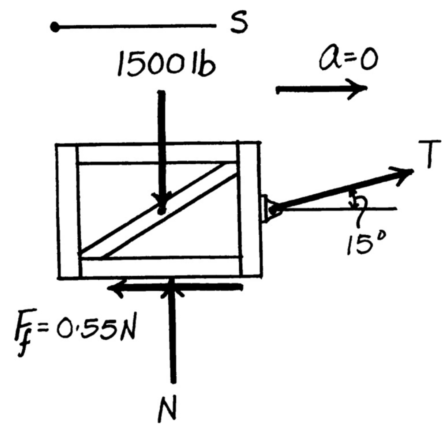
$\xrightarrow{+} \Sigma F_x = 0; \qquad T\cos 15° - 0.55N = 0$

$+\uparrow \Sigma F_y = 0; \qquad N + T\sin 15° - 1500 = 0$

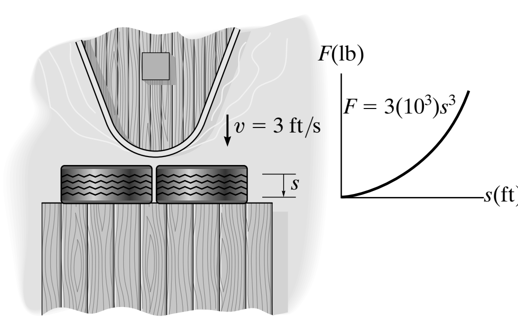
$N = 1307$ lb

$T = 744.4 \text{ lb} = 744 \text{ lb}$ **Ans.**

$U_T = (744.4 \cos 15°)(25) = 18.0(10^3) \text{ ft} \cdot \text{lb}$ **Ans.**



14–2. The motion of a 6500-lb boat is arrested using a bumper which provides a resistance as shown in the graph. Determine the maximum distance the boat dents the bumper if its approaching speed is 3 ft/s.



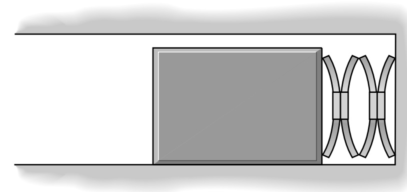
Principle of Work and Energy: Here, the bumper resisting force F does *negative* work since it acts in the opposite direction to that of displacement. Since the boat is required to stop, $T_2 = 0$. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{6500}{32.2}\right)(3^2) + \left[-\int_0^s 3(10^3)\, s^3 ds\right] = 0$$

$$s = 1.05 \text{ ft}$$ **Ans.**

14–3. The smooth plug has a weight of 20 lb and is pushed against a series of Belleville spring washers so that the compression in the spring is $s = 0.05$ ft. If the force of the spring on the plug is $F = (3s^{1/3})$ lb, where s is given in feet, determine the speed of the plug after it moves away from the spring. Neglect friction.

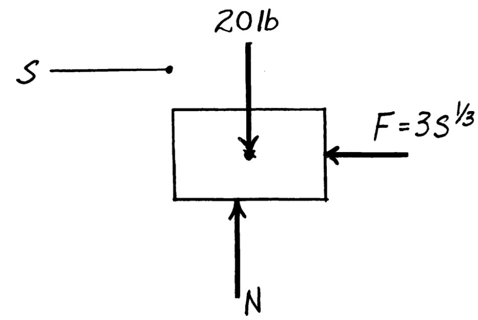


$$T_1 + \Sigma U_{1-2} = T_2$$

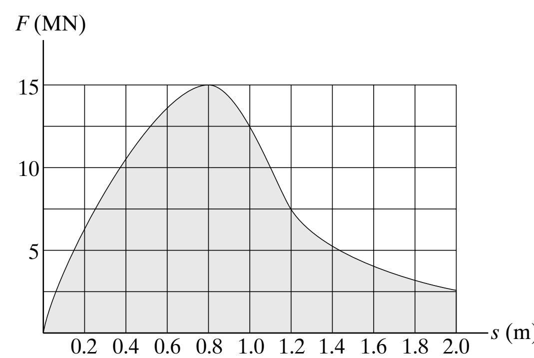
$$0 + \int_0^{0.05} 3s^{\frac{1}{3}}\,ds = \frac{1}{2}\left(\frac{20}{32.2}\right)v^2$$

$$3\left(\frac{3}{4}\right)(0.05)^{\frac{4}{3}} = \frac{1}{2}\left(\frac{20}{32.2}\right)v^2$$

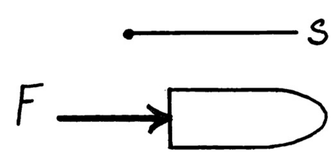
$v = 0.365$ ft/s **Ans.**



***14–4.** When a 7-kg projectile is fired from a cannon barrel that has a length of 2 m, the explosive force exerted on the projectile, while it is in the barrel, varies in the manner shown. Determine the approximate muzzle velocity of the projectile at the instant it leaves the barrel. Neglect the effects of friction inside the barrel and assume the barrel is horizontal.



The work done is measured as the area under the force–displacement curve. This area is approximately 31.5 squares. Since each square has an area of $2.5\left(10^6\right)(0.2)$,

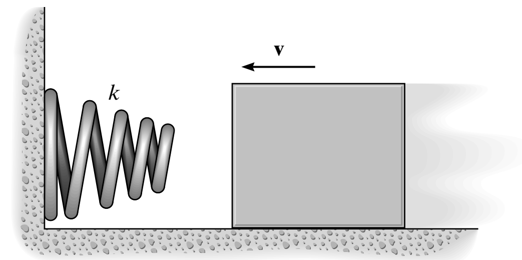


$$T_1 + \Sigma U_{1-2} = T_2$$

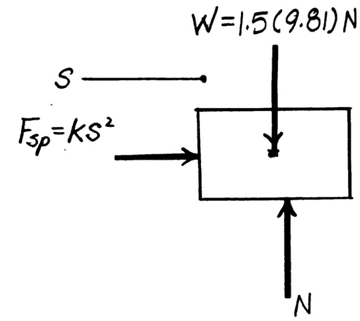
$$0 + \left[(31.5)(2.5)\left(10^6\right)(0.2)\right] = \frac{1}{2}(7)(v_2)^2$$

$v_2 = 2121$ m/s $= 2.12$ km/s (approx.) **Ans.**

•14–5. The 1.5-kg block slides along a smooth plane and strikes a *nonlinear spring* with a speed of $v = 4$ m/s. The spring is termed "nonlinear" because it has a resistance of $F_s = ks^2$, where $k = 900$ N/m^2. Determine the speed of the block after it has compressed the spring $s = 0.2$ m.



Principle of Work and Energy: The spring force F_{sp} which acts in the opposite direction to that of displacement does *negative* work. The normal reaction N and the weight of the block do not displace hence do no work. Applying Eq. 14–7, we have



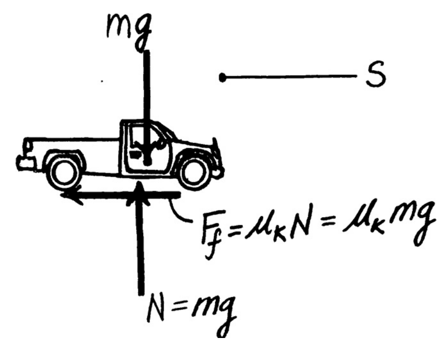
$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(1.5)\left(4^2\right) + \left[-\int_0^{0.2\text{ m}} 900s^2\,ds\right] = \frac{1}{2}(1.5)\,v^2$$

$$v = 3.58 \text{ m/s} \qquad \textbf{Ans.}$$

14–6. When the driver applies the brakes of a light truck traveling 10 km/h, it skids 3 m before stopping. How far will the truck skid if it is traveling 80 km/h when the brakes are applied?

$$10 \text{ km/h} = \frac{10\left(10^3\right)}{3600} = 2.778 \text{ m/s} \qquad 80 \text{ km/h} = 22.22 \text{ m/s}$$



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m(2.778)^2 - \mu_k mg(3) = 0$$

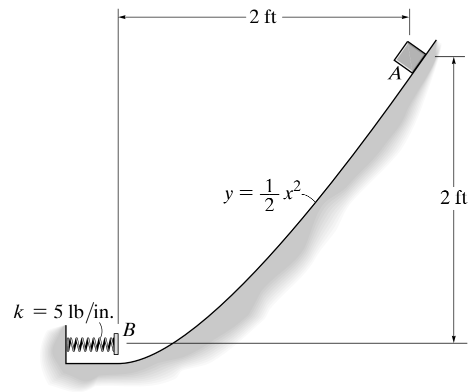
$$\mu_k g = 1.286$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m(22.22)^2 - (1.286)m(d) = 0$$

$$d = 192 \text{ m} \qquad \textbf{Ans.}$$

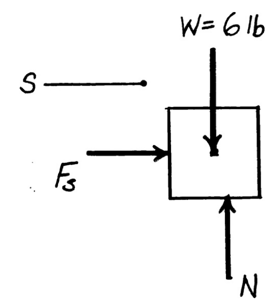
14–7. The 6-lb block is released from rest at A and slides down the smooth parabolic surface. Determine the maximum compression of the spring.



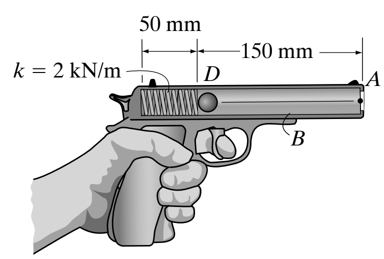
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 2(6) - \frac{1}{2}(5)(12)s^2 = 0$$

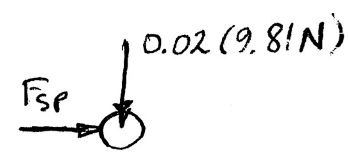
$s = 0.632$ ft $= 7.59$ in. **Ans.**



***14–8.** The spring in the toy gun has an unstretched length of 100 mm. It is compressed and locked in the position shown. When the trigger is pulled, the spring unstretches 12.5 mm, and the 20-g ball moves along the barrel. Determine the speed of the ball when it leaves the gun. Neglect friction.



Principle of Work and Energy: Referring to the free-body diagram of the ball bearing shown in Fig. a, notice that $\mathbf{F}_{sp}$ does positive work. The spring has an initial and final compression of $s_1 = 0.1 - 0.05 = 0.05$ m and $s_2 = 0.1 - (0.05 + 0.0125) = 0.0375$ m.



(a)

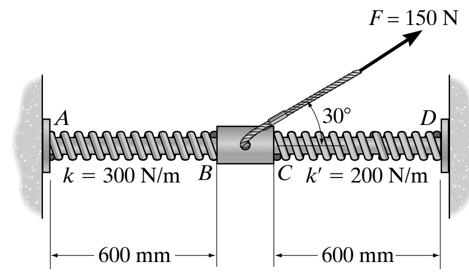
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left[\frac{1}{2}ks_1{}^2 - \frac{1}{2}ks_2{}^2\right] = \frac{1}{2}mv_A{}^2$$

$$0 + \left[\frac{1}{2}(2000)(0.05)^2 - \frac{1}{2}(2000)(0.0375^2)\right] = \frac{1}{2}(0.02)v_A{}^2$$

$v_A = 10.5$ m/s **Ans.**

•14–9. Springs AB and CD have a stiffness of $k = 300$ N/m and $k' = 200$ N/m, respectively, and both springs have an unstretched length of 600 mm. If the 2-kg smooth collar starts from rest when the springs are unstretched, determine the speed of the collar when it has moved 200 mm.

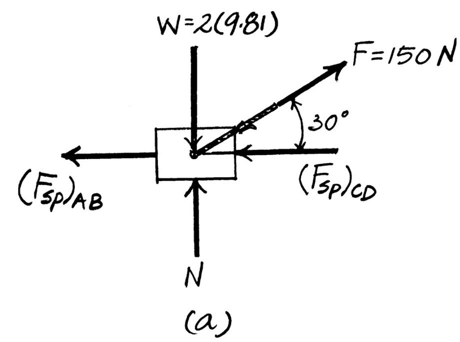


Principle of Work and Energy: By referring to the free-body diagram of the collar, notice that $\mathbf{W}$, $\mathbf{N}$, and $F_y = 150 \sin 30°$ do no work. However, $F_x = 150 \cos 30°$ N does positive work and $(\mathbf{F}_{sp})_{AB}$ and $(\mathbf{F}_{sp})_{CD}$ do negative work.

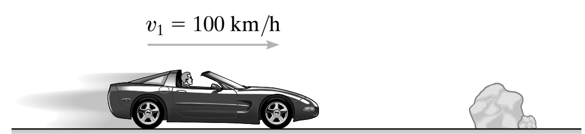
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 150 \cos 30°(0.2) + \left[-\frac{1}{2}(300)(0.2^2)\right] + \left[-\frac{1}{2}(200)(0.2^2)\right] = \frac{1}{2}(2)v^2$$

$$v = 4.00 \text{ m/s} \qquad \textbf{Ans.}$$



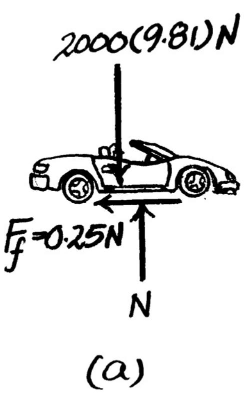
14–10. The 2-Mg car has a velocity of $v_1 = 100$ km/h when the driver sees an obstacle in front of the car. If it takes 0.75 s for him to react and lock the brakes, causing the car to skid, determine the distance the car travels before it stops. The coefficient of kinetic friction between the tires and the road is $\mu_k = 0.25$.



Free-Body Diagram: The normal reaction $\mathbf{N}$ on the car can be determined by writing the equation of motion along the y axis. By referring to the free-body diagram of the car, Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 2000(9.81) = 2000(0) \qquad N = 19\,620 \text{ N}$$

Since the car skids, the frictional force acting on the car is $F_f = \mu_k N = 0.25(19620) = 4905N$.



Principle of Work and Energy: By referring to Fig. a, notice that only $\mathbf{F}_f$ does work, which is negative. The initial speed of the car is $v_1 = \left[100(10^3)\frac{\text{m}}{\text{h}}\right]\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) =$ 27.78 m/s. Here, the skidding distance of the car is denoted as s'.

$$T_1 + \Sigma U_{1-2} = T_2$$

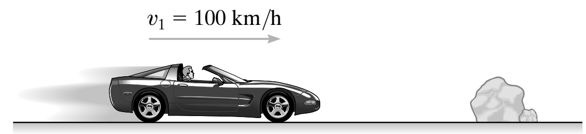
$$\frac{1}{2}(2000)(27.78^2) + (-4905s') = 0$$

$$s' = 157.31 \text{ m}$$

The distance traveled by the car during the reaction time is $s'' = v_1 t = 27.78(0.75) = 20.83$ m. Thus, the total distance traveled by the car before it stops is

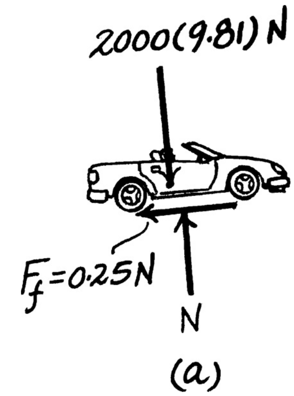
$$s = s' + s'' = 157.31 + 20.83 = 178.14 \text{ m} = 178 \text{ m} \qquad \textbf{Ans.}$$

14–11. The 2-Mg car has a velocity of $v_1 = 100\,\text{km/h}$ when the driver sees an obstacle in front of the car. It takes 0.75 s for him to react and lock the brakes, causing the car to skid. If the car stops when it has traveled a distance of 175 m, determine the coefficient of kinetic friction between the tires and the road.



Free-Body Diagram: The normal reaction **N** on the car can be determined by writing the equation of motion along the y axis and referring to the free-body diagram of the car, Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 2000(9.81) = 2000(0) \qquad N = 19\,620\text{ N}$$



Since the car skids, the frictional force acting on the car can be computed from $F_f = \mu_k N = \mu_k(19\,620)$.

Principle of Work and Energy: By referring to Fig. a, notice that only $\mathbf{F}_f$ does work, which is negative. The initial speed of the car is $v_1 = \left[100(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) =$ 27.78 m/s. Here, the skidding distance of the car is s'.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(2000)(27.78^2) + \left[-\mu_k(19\,620)s'\right] = 0$$

$$s' = \frac{39.327}{\mu_k}$$

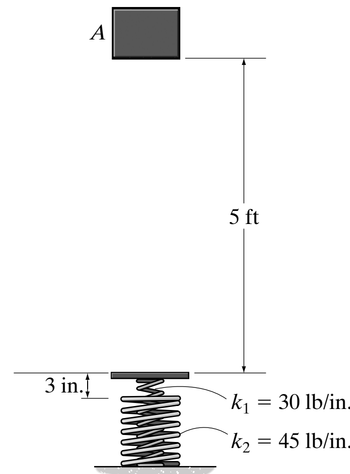
The distance traveled by the car during the reaction time is $s'' = v_1 t = 27.78(0.75) = 20.83$ m. Thus, the total distance traveled by the car before it stops is

$$s = s' + s''$$

$$175 = \frac{39.327}{\mu_k} + 20.83$$

$$\mu_k = 0.255 \qquad \textbf{Ans.}$$

***14–12.** The 10-lb block is released from rest at A. Determine the compression of each of the springs after the block strikes the platform and is brought momentarily to rest. Initially both springs are unstretched. Assume the platform has a negligible mass.



Free-Body Diagram: The free-body diagram of the block in contact with both springs is shown in Fig. a. When the block is brought momentarily to rest, springs (1) and (2) are compressed by $s_1 = y$ and $s_2 = (y - 3)$, respectively.

Principle of Work and Energy: When the block is momentarily at rest, **W** which displaces downward $h = [5(12) + y]$in. $= (60 + y)$ in., does positive work, whereas $(\mathbf{F}_{sp})_1$ and $(\mathbf{F}_{sp})_2$ both do negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 10(60 + y) + \left[-\frac{1}{2}(30)y^2\right] + \left[-\frac{1}{2}(45)(y - 3)^2\right] = 0$$

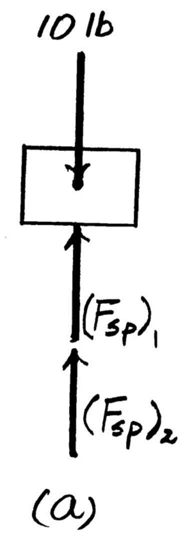
$$37.5y^2 - 145y - 397.5 = 0$$

Solving for the positive root of the above equation,

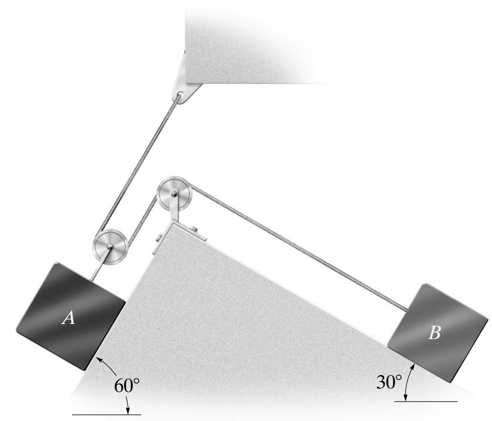
$$y = 5.720 \text{ in.}$$

Thus,

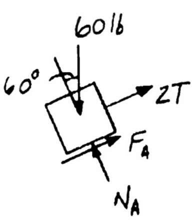
$s_1 = 5.72$ in. $\qquad s_2 = 5.720 - 3 = 2.72$ in. $\qquad$ **Ans.**



14–13. Determine the velocity of the 60-lb block A if the two blocks are released from rest and the 40-lb block B moves 2 ft up the incline. The coefficient of kinetic friction between both blocks and the inclined planes is $\mu_k = 0.10$.



Block A:



$+\nwarrow\Sigma F_y = ma_y;\qquad N_A - 60\cos 60° = 0$

$N_A = 30 \text{ lb}$

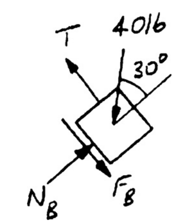
$F_A = 0.1(30) = 3 \text{ lb}$

Block B:

$+\nearrow\Sigma F_y = ma_y;\qquad N_B - 40\cos 30° = 0$

$N_B = 34.64 \text{ lb}$

$F_B = 0.1(34.64) = 3.464 \text{ lb}$



Use the system of both blocks. N_A, N_B, T, and R do no work.

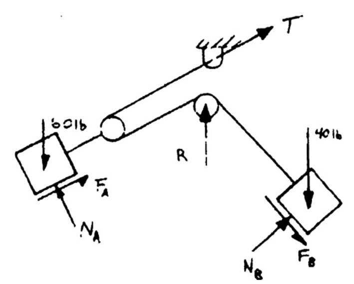
$T_1 + \Sigma U_{1-2} = T_2$

$$(0 + 0) + 60\sin 60°|\Delta s_A| - 40\sin 30°|\Delta s_B| - 3|\Delta s_A| - 3.464|\Delta s_B| = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{40}{32.2}\right)v_B^2$$

$2s_A + s_B = l$

$2\Delta s_A = -\Delta s_B$

When $|\Delta s_B| = 2 \text{ ft}$, $|\Delta s_A| = 1 \text{ ft}$



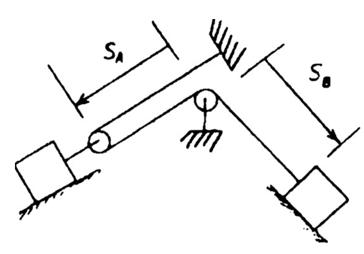
Also,

$2v_A = -v_B$

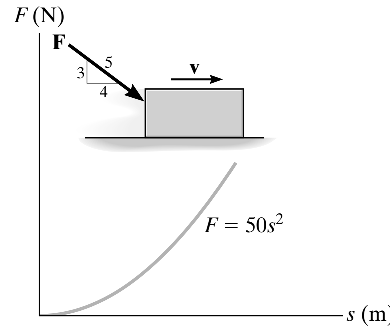
Substituting and solving,

$v_A = 0.771 \text{ ft/s}$ **Ans.**

$v_B = -1.54 \text{ ft/s}$

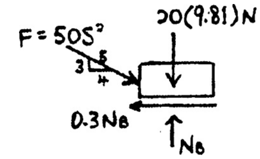


14–14. The force **F**, acting in a constant direction on the 20-kg block, has a magnitude which varies with the position s of the block. Determine how far the block slides before its velocity becomes 5 m/s. When $s = 0$ the block is moving to the right at 2 m/s. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.



$+\uparrow \Sigma F_y = 0;\qquad N_B - 20(9.81) - \frac{3}{5}(50\, s^2) = 0$

$$N_B = 196.2 + 30\, s^2$$



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(20)(2)^2 + \frac{4}{5}\int_0^s 50\, s^2\, ds - 0.3(196.2)(s) - 0.3\int_0^s 30\, s^2\, ds = \frac{1}{2}(20)(5)^2$$

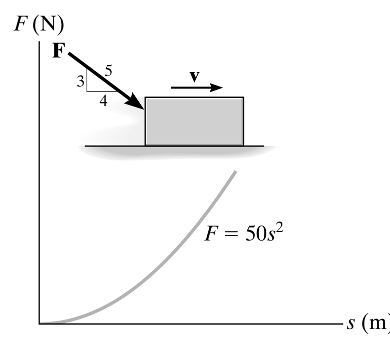
$$40 + 13.33\, s^3 - 58.86\, s - 3\, s^3 = 250$$

$$s^3 - 5.6961\, s - 20.323 = 0$$

Solving for the real root yields

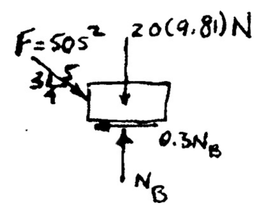
$s = 3.41$ m **Ans.**

14–15. The force **F**, acting in a constant direction on the 20-kg block, has a magnitude which varies with position s of the block. Determine the speed of the block after it slides 3 m. When $s = 0$ the block is moving to the right at 2 m/s. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.



$+\uparrow \Sigma F_y = 0;\qquad N_B - 20(9.81) - \frac{3}{5}(50\, s^2) = 0$

$$N_B = 196.2 + 30\, s^2$$



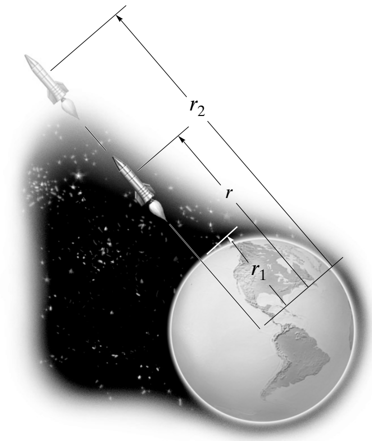
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(20)(2)^2 + \frac{4}{5}\int_0^3 50\, s^2\, ds - 0.3(196.2)(3) - 0.3\int_0^3 30\, s^2\, ds = \frac{1}{2}(20)(v)^2$$

$$40 + 360 - 176.58 - 81 = 10\, v^2$$

$v = 3.77$ m/s **Ans.**

14–16. A rocket of mass m is fired vertically from the surface of the earth, i.e., at $r = r_1$. Assuming no mass is lost as it travels upward, determine the work it must do against gravity to reach a distance r_2. The force of gravity is $F = GM_e m/r^2$ (Eq. 13–1), where M_e is the mass of the earth and r the distance between the rocket and the center of the earth.

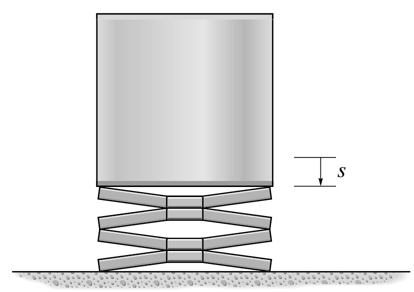


$$F = G\frac{M_e m}{r^2}$$

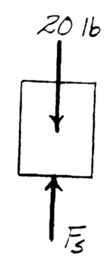
$$F_{1-2} = \int F\,dr = GM_e m \int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$= GM_e m\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad \textbf{Ans.}$$

•**14–17.** The cylinder has a weight of 20 lb and is pushed against a series of Belleville spring washers so that the compression in the spring is $s = 0.05$ ft. If the force of the spring on the cylinder is $F = (100s^{1/3})$ lb, where s is given in feet, determine the speed of the cylinder just after it moves away from the spring, i.e., at $s = 0$.



Principle of Work and Energy: The spring force which acts in the direction of displacement does *positive* work, whereas the weight of the block does *negative* work since it acts in the opposite direction to that of displacement. Since the block is initially at rest, $T_1 = 0$. Applying Eq. 14–7, we have

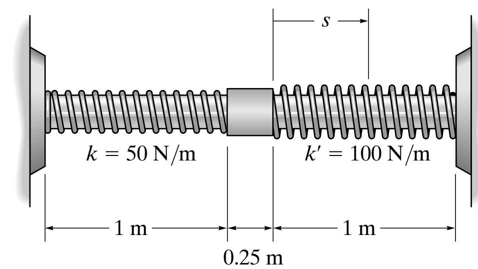


$$T_1 + \sum U_{1-2} = T_2$$

$$0 + \int_0^{0.05\text{ ft}} 100s^{1/3}\,ds - 20(0.05) = \frac{1}{2}\left(\frac{20}{32.2}\right)v^2$$

$$v = 1.11 \text{ ft/s} \qquad \textbf{Ans.}$$

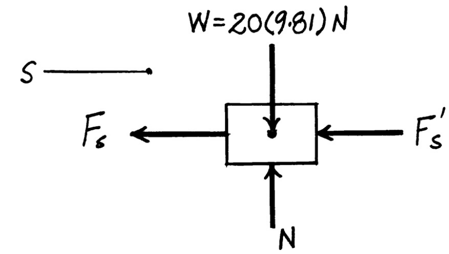
14–18. The collar has a mass of 20 kg and rests on the smooth rod. Two springs are attached to it and the ends of the rod as shown. Each spring has an uncompressed length of 1 m. If the collar is displaced $s = 0.5$ m and released from rest, determine its velocity at the instant it returns to the point $s = 0$.



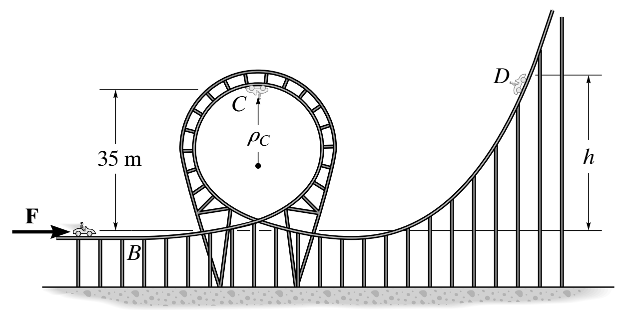
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \frac{1}{2}(50)(0.5)^2 + \frac{1}{2}(100)(0.5)^2 = \frac{1}{2}(20)v_C^2$$

$v_C = 1.37$ m/s **Ans.**

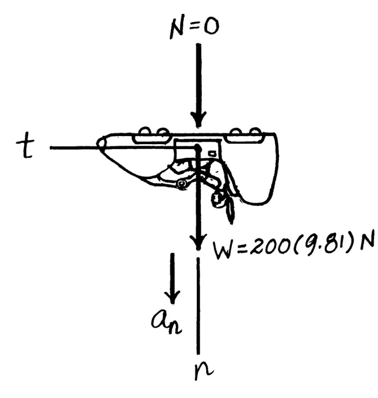


14–19. Determine the height h of the incline D to which the 200-kg roller coaster car will reach, if it is launched at B with a speed just sufficient for it to round the top of the loop at C without leaving the track. The radius of curvature at C is $\rho_c = 25$ m.



Equations of Motion: Here, it is required that $N = 0$. Applying Eq. 13–8 to FBD(a), we have

$$\Sigma F_n = ma_n; \qquad 200(9.81) = 200\left(\frac{v_C^2}{25}\right) \qquad v_C^2 = 245.25 \text{ m}^2/\text{s}^2$$



(a)

Principle of Work and Energy: The weight of the roller coaster car and passengers do *negative* work since they act in the opposite direction to that of displacement. When the roller coaster car travels from B to C, applying Eq. 14–7, we have

$$T_B + \sum U_{B-C} = T_C$$

$$\frac{1}{2}(200)\,v_B^2 - 200(9.81)\,(35) = \frac{1}{2}(200)(245.25)$$

$$v_B = 30.53 \text{ m/s}$$

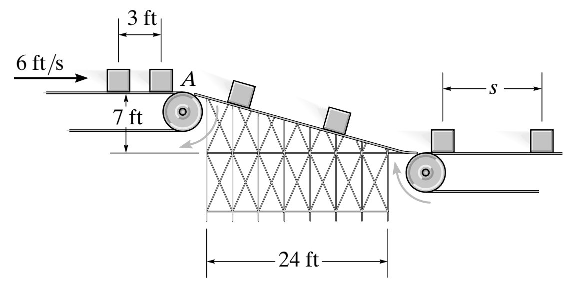
When the roller coaster car travels from B to D, it is required that the car stops at D, hence $T_D = 0$.

$$T_B + \sum U_{B-D} = T_D$$

$$\frac{1}{2}(200)\left(30.53^2\right) - 200(9.81)(h) = 0$$

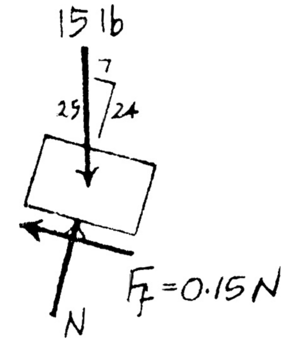
$h = 47.5$ m **Ans.**

***14–20.** Packages having a weight of 15 lb are transferred horizontally from one conveyor to the next using a ramp for which $\mu_k = 0.15$. The top conveyor is moving at 6 ft/s and the packages are spaced 3 ft apart. Determine the required speed of the bottom conveyor so no sliding occurs when the packages come horizontally in contact with it. What is the spacing s between the packages on the bottom conveyor?



Equations of Motion:

$$+\Sigma F_{y'} = ma_{y'}; \qquad N - 15\left(\frac{24}{25}\right) = \frac{15}{32.2}(0) \qquad N = 14.4 \text{ lb}$$



Principle of Work and Energy: Only force components parallel to the inclined plane which are in the direction of displacement [15(7/25) lb and $F_f = \mu_k N = 0.15(14.4) = 2.16$ lb] do work, whereas the force components perpendicular to the inclined plane [15(24/25) lb and normal reaction N] do no work since no displacement occurs in this direction. Here, the 15(7/25) lb force does *positive* work and $F_f = 2.16$ lb does *negative* work. Slipping at the contact surface between the package and the belt will not occur if the speed of belt is the same as the speed of the package at B. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

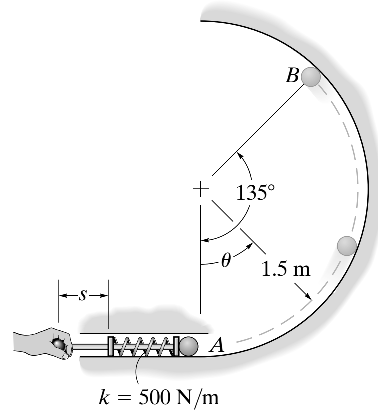
$$\frac{1}{2}\left(\frac{15}{32.2}\right)(6^2) + 15\left(\frac{7}{25}\right)(25) - 2.16(25) = \frac{1}{2}\left(\frac{15}{32.2}\right)v^2$$

$$v = 15.97 \text{ ft/s} = 16.0 \text{ ft/s} \qquad \textbf{Ans.}$$

The time between two succesive packages to reach point B is $t = \frac{3}{6} = 0.5$ s. Hence, the distance between two succesive packages on the lower belt is

$$s = vt = 15.97(0.5) = 7.98 \text{ ft} \qquad \textbf{Ans.}$$

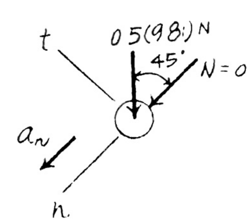
•14–21. The 0.5-kg ball of negligible size is fired up the smooth vertical circular track using the spring plunger. The plunger keeps the spring compressed 0.08 m when $s = 0$. Determine how far s it must be pulled back and released so that the ball will begin to leave the track when $\theta = 135°$.



Equations of Motion:

$$\Sigma F_n = ma_n; \qquad 0.5(9.81)\cos 45° = 0.5\left(\frac{v_B^2}{1.5}\right) \qquad v_B^2 = 10.41 \text{ m}^2/\text{s}^2$$

Principle of Work and Energy: Here, the weight of the ball is being displaced vertically by $s = 1.5 + 1.5\sin 45° = 2.561$ m and so it does *negative* work. The spring force, given by $F_{sp} = 500(s + 0.08)$, does positive work. Since the ball is at rest initially, $T_1 = 0$. Applying Eq. 14–7, we have

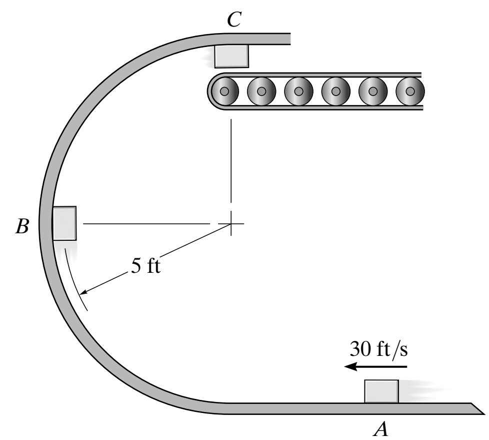


$$T_A + \sum U_{A-B} = T_B$$

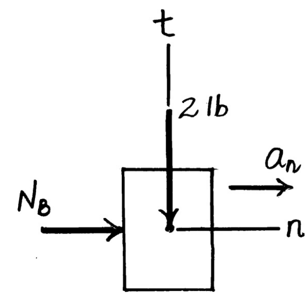
$$0 + \int_0^s 500(s + 0.08)\,ds - 0.5(9.81)(2.561) = \frac{1}{2}(0.5)(10.41)$$

$$s = 0.1789 \text{ m} = 179 \text{ mm} \qquad \textbf{Ans.}$$

14–22. The 2-lb box slides on the smooth circular ramp. If the box has a velocity of 30 ft/s at A, determine the velocity of the box and normal force acting on the ramp when the box is located at B and C. Assume the radius of curvature of the path at C is still 5 ft.



Point B:



$$T_1 + \Sigma U_{1-2} = T_2$$

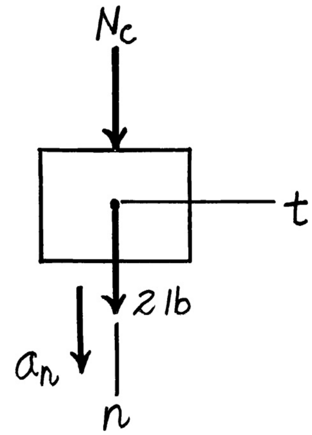
$$\frac{1}{2}\left(\frac{2}{32.2}\right)(30)^2 - 2(5) = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_B)^2$$

$v_B = 24.0$ ft/s **Ans.**

$\rightarrow \Sigma F_n = ma_n;\qquad N_B = \left(\frac{2}{32.2}\right)\left(\frac{(24.0)^2}{5}\right)$

$N_B = 7.18$ lb **Ans.**

Point C:



$$T_1 + \Sigma U_{1-2} = T_2$$

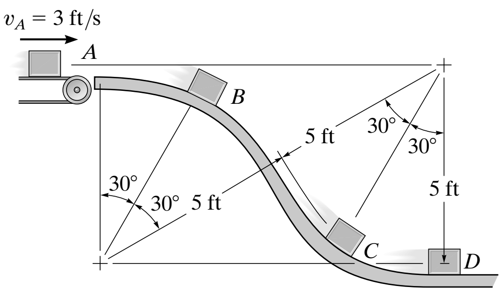
$$\frac{1}{2}\left(\frac{2}{32.2}\right)(30)^2 - 2(10) = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_C)^2$$

$v_C = 16.0$ ft/s **Ans.**

$+\downarrow \Sigma F_n = ma_n;\qquad N_C + 2 = \left(\frac{2}{32.2}\right)\left(\frac{(16.0)^2}{5}\right)$

$N_C = 1.18$ lb **Ans.**

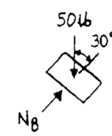
14–23. Packages having a weight of 50 lb are delivered to the chute at $v_A = 3$ ft/s using a conveyor belt. Determine their speeds when they reach points B, C, and D. Also calculate the normal force of the chute on the packages at B and C. Neglect friction and the size of the packages.



$$T_A + \Sigma U_{A-B} = T_B$$

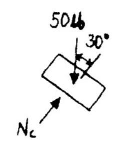
$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 50(5)(1 - \cos 30°) = \frac{1}{2}\left(\frac{50}{32.2}\right)v_B^2$$

$v_B = 7.221 = 7.22$ ft/s **Ans.**



$$+\swarrow\Sigma F_n = ma_n; \qquad -N_B + 50\cos 30° = \left(\frac{50}{32.2}\right)\left[\frac{(7.221)^2}{5}\right]$$

$N_B = 27.1$ lb **Ans.**



$$T_A + \Sigma U_{A-C} = T_C$$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 50(5\cos 30°) = \frac{1}{2}\left(\frac{50}{32.2}\right)v_C^2$$

$v_C = 16.97 = 17.0$ ft/s **Ans.**

$$+\nearrow\Sigma F_n = ma_n; \qquad N_C - 50\cos 30° = \left(\frac{50}{32.2}\right)\left[\frac{(16.97)^2}{5}\right]$$

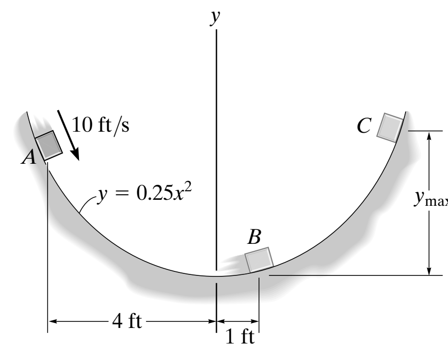
$N_C = 133$ lb **Ans.**

$$T_A + \Sigma U_{A-D} = T_D$$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3)^2 + 50(5) = \frac{1}{2}\left(\frac{50}{32.2}\right)v_D^2$$

$v_D = 18.2$ ft/s **Ans.**

***14–24.** The 2-lb block slides down the smooth parabolic surface, such that when it is at A it has a speed of 10 ft/s. Determine the magnitude of the block's velocity and acceleration when it reaches point B, and the maximum height y_{max} reached by the block.

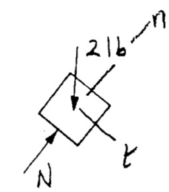


$$y = 0.25x^2$$

$$y_A = 0.25(-4)^2 = 4 \text{ ft}$$

$$y_B = 0.25(1)^2 = 0.25 \text{ ft}$$

$$T_A + \Sigma U_{A-B} = T_B$$

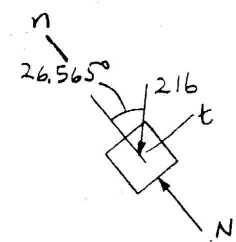


$$\frac{1}{2}\left(\frac{2}{32.2}\right)(10)^2 + 2(4-0.25) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2$$

$$v_B = 18.48 \text{ ft/s} = 18.5 \text{ ft/s}$$ **Ans.**

$$\frac{dy}{dx} = \tan\theta = 0.5x\Big|_{x=1} = 0.5 \qquad \theta = 26.565^\circ$$

$$\frac{d^2y}{dx^2} = 0.5$$



$$+\nearrow\Sigma F_t = ma_t; \qquad -2\sin 26.565^\circ = \left(\frac{2}{32.2}\right)a_t$$

$$a_t = -14.4 \text{ ft/s}^2$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (0.5)^2\right]^{\frac{3}{2}}}{|0.5|} = 2.795 \text{ ft}$$

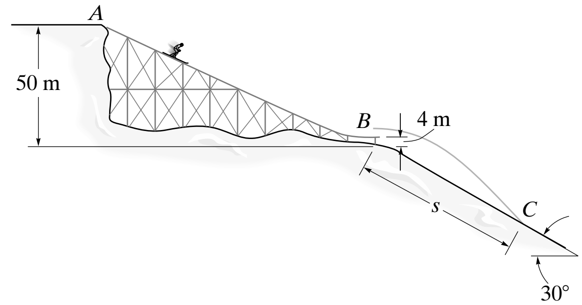
$$a_n = \frac{v_B^2}{\rho} = \frac{(18.48)^2}{2.795} = 122.2 \text{ ft/s}^2$$

$$a_B = \sqrt{(-14.4)^2 + (122.2)^2} = 123 \text{ ft/s}^2$$ **Ans.**

$$T_A + \Sigma U_{A-C} = T_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(10)^2 - 2(v_{max} - 4) = 0 \qquad y_{max} = 5.55 \text{ ft}$$ **Ans.**

•14–25. The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, find the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass of 70 kg.



$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 70(9.81)(46) = \frac{1}{2}(70)(v_B)^2$$

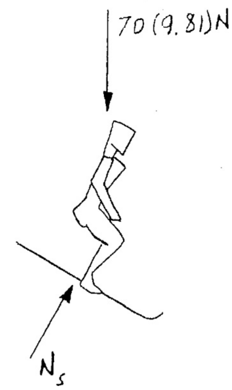
$v_B = 30.04 \text{ m/s} = 30.0 \text{ m/s}$ **Ans.**

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$s \cos 30^\circ = 0 + 30.04t$$

$(+\downarrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$s \sin 30^\circ + 4 = 0 + 0 + \frac{1}{2}(9.81)t^2$$



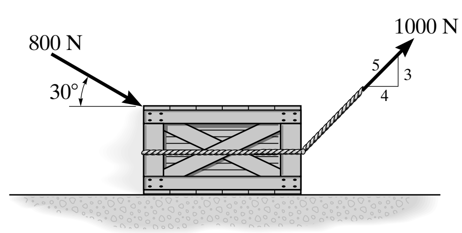
Eliminating t,

$$s^2 - 122.67s - 981.33 = 0$$

Solving for the positive root

$s = 130 \text{ m}$ **Ans.**

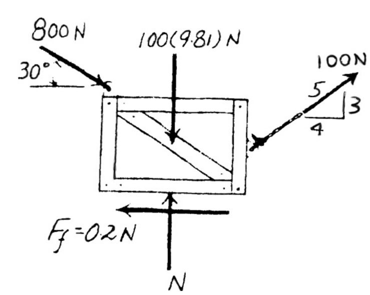
14–26. The crate, which has a mass of 100 kg, is subjected to the action of the two forces. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



Equations of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.2N$. Applying Eq. 13–7, we have

$$+\uparrow \Sigma F_y = ma_y; \qquad N + 1000\left(\frac{3}{5}\right) - 800 \sin 30^\circ - 100(9.81) = 100(0)$$

$$N = 781 \text{ N}$$



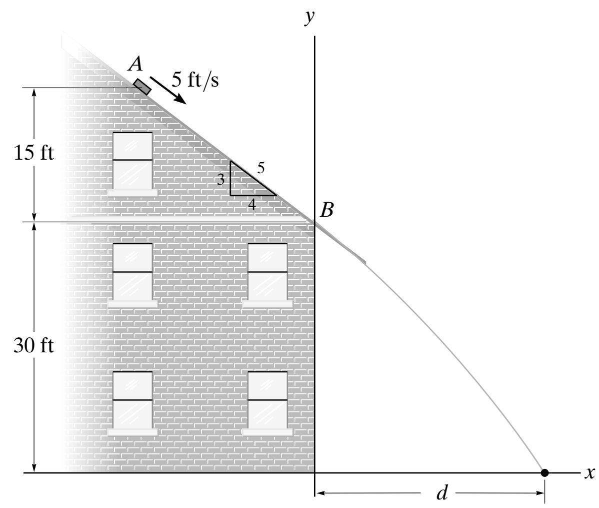
Principle of Work and Energy: The horizontal components of force 800 N and 1000 N which act in the direction of displacement do *positive* work, whereas the friction force $F_f = 0.2(781) = 156.2$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of 800 N and 1000 N force and the weight of the crate do not displace, hence they do no work. Since the crate is originally at rest, $T_1 = 0$. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 800 \cos 30^\circ(s) + 1000\left(\frac{4}{5}\right)s - 156.2s = \frac{1}{2}(100)\left(6^2\right)$$

$s = 1.35\text{m}$ **Ans.**

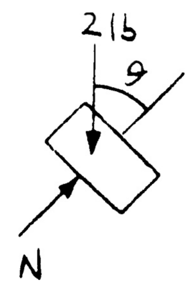
14–27. The 2-lb brick slides down a smooth roof, such that when it is at A it has a velocity of 5 ft/s. Determine the speed of the brick just before it leaves the surface at B, the distance d from the wall to where it strikes the ground, and the speed at which it hits the ground.



$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(15) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2$$

$v_B = 31.48 \text{ ft/s} = 31.5 \text{ ft/s}$ **Ans.**



$$\left(\xrightarrow{+}\right) \qquad s = s_0 + v_0 t$$

$$d = 0 + 31.48\left(\frac{4}{5}\right)t$$

$$(+\downarrow) \qquad s = s_0 + v_0 t - \frac{1}{2}a_c t^2$$

$$30 = 0 + 31.48\left(\frac{3}{5}\right)t + \frac{1}{2}(32.2)t^2$$

$$16.1t^2 + 18.888t - 30 = 0$$

Solving for the positive root,

$$t = 0.89916 \text{ s}$$

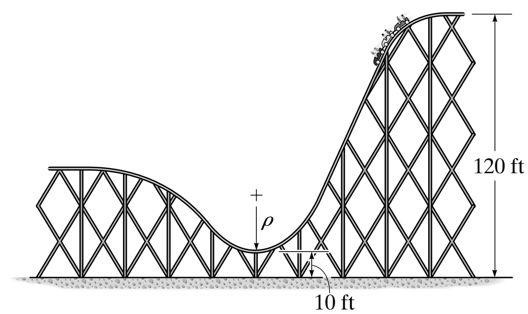
$d = 31.48\left(\frac{4}{5}\right)(0.89916) = 22.6 \text{ ft}$ **Ans.**

$$T_A + \Sigma U_{A-C} = T_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(45) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_C^2$$

$v_C = 54.1 \text{ ft/s}$ **Ans.**

***14–28.** Roller coasters are designed so that riders will not experience a normal force that is more than 3.5 times their weight against the seat of the car. Determine the smallest radius of curvature ρ of the track at its lowest point if the car has a speed of 5 ft/s at the crest of the drop. Neglect friction.



Principle of Work and Energy: Here, the rider is being displaced vertically (downward) by $s = 120 - 10 = 110$ ft and does *positive* work. Applying Eq. 14–7 we have

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{W}{32.2}\right)(5^2) + W(110) = \frac{1}{2}\left(\frac{W}{32.2}\right)v^2$$

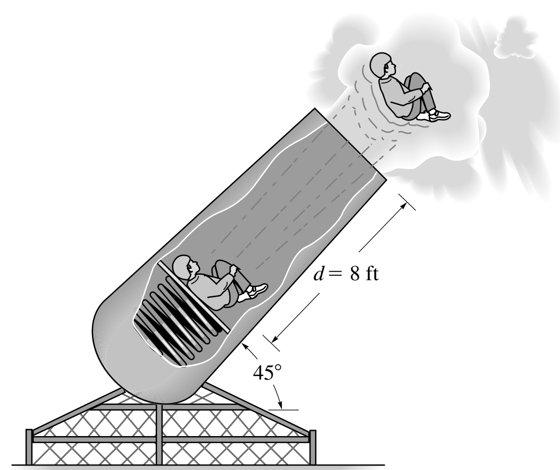
$$v^2 = 7109\ \text{ft}^2/\text{s}^2$$

Equations of Motion: It is required that $N = 3.5W$. Applying Eq. 13–7, we have

$$\Sigma F_n = ma_n; \qquad 3.5W - W = \left(\frac{W}{32.2}\right)\left(\frac{7109}{\rho}\right)$$

$$\rho = 88.3\ \text{ft} \qquad \textbf{Ans.}$$

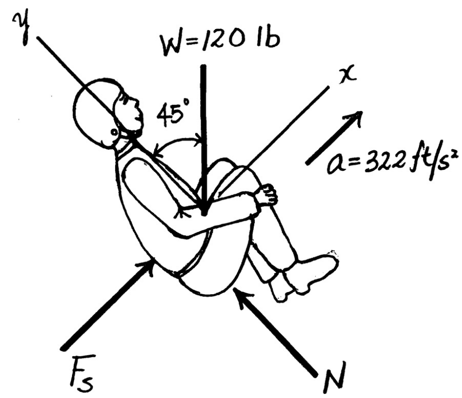
•14–29. The 120-lb man acts as a human cannonball by being "fired" from the spring-loaded cannon shown. If the greatest acceleration he can experience is $a = 10g = 322\ \text{ft/s}^2$, determine the required stiffness of the spring which is compressed 2 ft at the moment of firing. With what velocity will he exit the cannon barrel, $d = 8$ ft, when the cannon is fired? When the spring is compressed $s = 2$ ft then $d = 8$ ft. Neglect friction and assume the man holds himself in a rigid position throughout the motion.



Initial acceleration is $10g = 322\ \text{ft/s}^2$

$$+\nearrow\Sigma F_x = ma_x; \qquad F_s - 120\sin 45^\circ = \left(\frac{120}{32.2}\right)(322), \qquad F_s = 1284.85\ \text{lb}$$

For $s = 2$ ft; $\quad 1284\ 85 = k(2) \qquad k = 642.4 = 642\ \text{lb/ft}$ **Ans.**

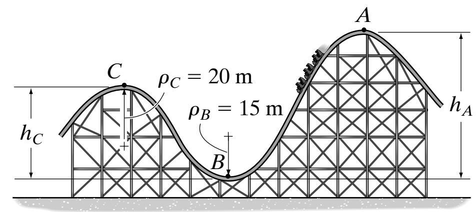


$$T_1 + \Sigma U_{1-2} = T_2$$

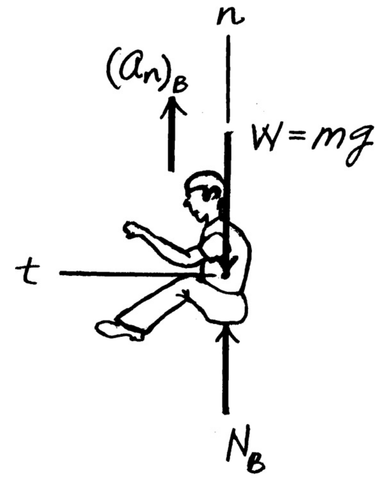
$$0 + \left[\frac{1}{2}(642.2)(2)^2 - 120(8)\sin 45^\circ\right] = \frac{1}{2}\left(\frac{120}{32.2}\right)v_2^2$$

$$v_2 = 18.0\ \text{ft/s} \qquad \textbf{Ans.}$$

14–30. If the track is to be designed so that the passengers of the roller coaster do not experience a normal force equal to zero or more than 4 times their weight, determine the limiting heights h_A and h_C so that this does not occur. The roller coaster starts from rest at position A. Neglect friction.



Free-Body Diagram: The free-body diagram of the passenger at positions B and C are shown in Figs. a and b, respectively.



(a)

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho}$. The requirement at position B is that $N_B = 4mg$. By referring to Fig. a,

$$+\uparrow \Sigma F_n = ma_n; \qquad 4mg - mg = m\left(\frac{v_B{}^2}{15}\right)$$

$$v_B{}^2 = 45g$$

At position C, N_C is required to be zero. By referring to Fig. b,

$$+\downarrow \Sigma F_n = ma_n; \qquad mg - 0 = m\left(\frac{v_C{}^2}{20}\right)$$

$$v_C{}^2 = 20g$$

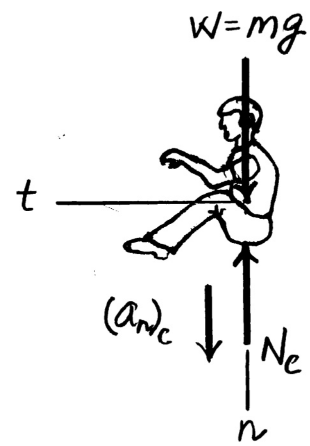
Principle of Work and Energy: The normal reaction **N** does no work since it always acts perpendicular to the motion. When the rollercoaster moves from position A to B, **W** displaces vertically downward $h = h_A$ and does positive work.

We have

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + mgh_A = \frac{1}{2}m(45g)$$

$$h_A = 22.5 \text{ m} \qquad \textbf{Ans.}$$



(b)

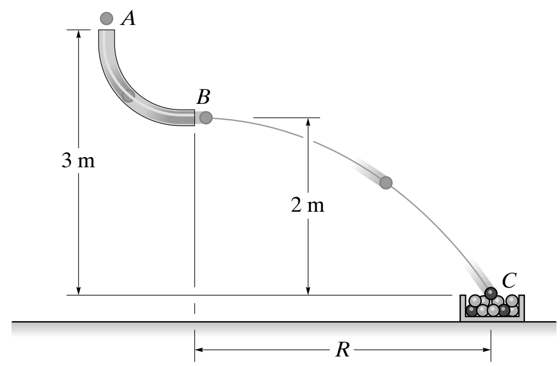
When the rollercoaster moves from position A to C, **W** displaces vertically downward $h = h_A - h_C = (22.5 - h_C)$ m.

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + mg(22.5 - h_C) = \frac{1}{2}m(20g)$$

$$h_C = 12.5 \text{ m} \qquad \textbf{Ans.}$$

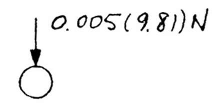
14–31. Marbles having a mass of 5 g fall from rest at A through the glass tube and accumulate in the can at C. Determine the placement R of the can from the end of the tube and the speed at which the marbles fall into the can. Neglect the size of the can.



$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + [0.005(9.81)(3 - 2)] = \frac{1}{2}(0.005)v_B^2$$

$$v_B = 4.429 \text{ m/s}$$



$$(+\downarrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$2 = 0 + 0 = \frac{1}{2}(9.81)t^2$$

$$t = 0.6386 \text{ s}$$

$$\left(\overset{+}{\rightarrow}\right) \qquad s = s_0 + v_0 t$$

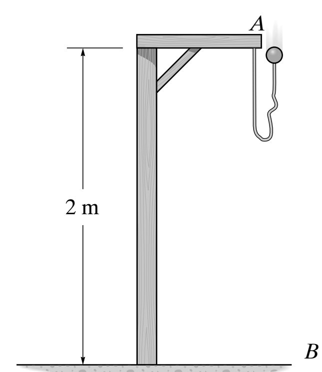
$$R = 0 + 4.429(0.6386) = 2.83 \text{ m}$$ **Ans.**

$$T_A + \Sigma U_{A-C} = T_1$$

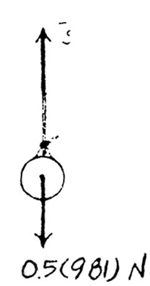
$$0 + [0.005(9.81)(3) = \frac{1}{2}(0.005)v_C^2$$

$$v_C = 7.67 \text{ m/s}$$ **Ans.**

***14–32.** The ball has a mass of 0.5 kg and is suspended from a rubber band having an unstretched length of 1 m and a stiffness $k = 50$ N/m. If the support at A to which the rubber band is attached is 2 m from the floor, determine the greatest speed the ball can have at A so that it does not touch the floor when it reaches its lowest point B. Neglect the size of the ball and the mass of the rubber band.



Principle of Work and Energy: The weight of the ball, which acts in the direction of displacement, does *positive* work, whereas the force in the rubber band does *negative* work since it acts in the opposite direction to that of displacement. Here it is required that the ball displace 2 m downward and stop, hence $T_2 = 0$. Applying Eq. 14–7, we have

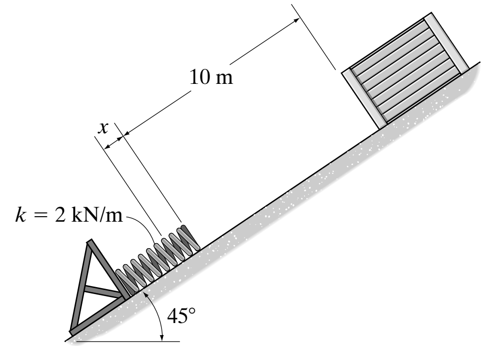


$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(0.5)v^2 + 0.5(9.81)(2) - \frac{1}{2}(50)(2 - 1)^2 = 0$$

$$v = 7.79 \text{ m/s}$$ **Ans.**

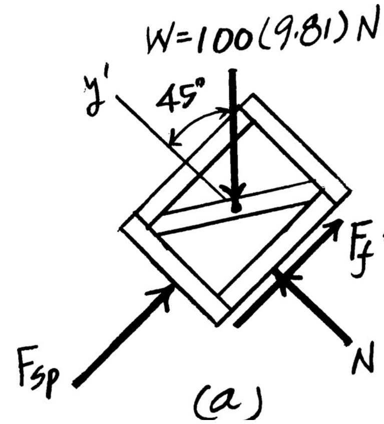
•**14–33.** If the coefficient of kinetic friction between the 100-kg crate and the plane is $\mu_k = 0.25$, determine the compression x of the spring required to bring the crate momentarily to rest. Initially the spring is unstretched and the crate is at rest.



Free-Body Diagram: The normal reaction **N** on the crate can be determined by writing the equation of motion along the y' axis and referring to the free-body diagram of the crate when it is in contact with the spring, Fig. *a*.

$$\nwarrow + F_{y'} = ma_{y'}; \qquad N - 100(9.81)\cos 45° = 100(0) \qquad N = 693.67 \text{ N}$$

Thus, the frictional force acting on the crate is $F_f = \mu_k N = 0.25(693.67)\text{ N} = 173.42 \text{ N}$.



Principle of Work and Energy: By referring to Fig. *a*, we notice that **N** does no work. Here, **W** which displaces downward through a distance of $h = (10 + x)\sin 45°$ does positive work, whereas $\mathbf{F}_f$ and $\mathbf{F}_{sp}$ do negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

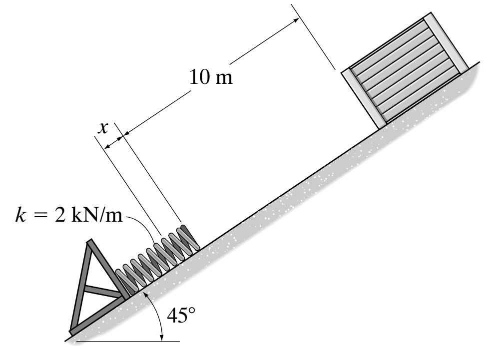
$$0 + 100(9.81)\left[(10 + x)\sin 45°\right] + \left[-173.42(10 + x)\right] + \left[-\frac{1}{2}(2000)x^2\right] = 0$$

$$1000x^2 - 520.25x - 5202.54 = 0$$

Solving for the positive root

$$x = 2.556 \text{ m} = 2.57 \text{ m} \qquad \textbf{Ans.}$$

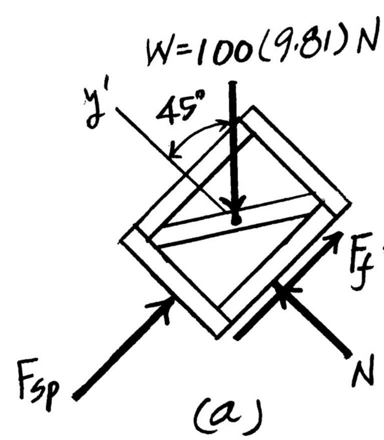
14–34. If the coefficient of kinetic friction between the 100-kg crate and the plane is $\mu_k = 0.25$, determine the speed of the crate at the instant the compression of the spring is $x = 1.5$ m. Initially the spring is unstretched and the crate is at rest.



Free-Body Diagram: The normal reaction **N** on the crate can be determined by writing the equation of motion along the y' axis and referring to the free-body diagram of the crate when it is in contact with the spring, Fig. *a*.

$$\nwarrow + F_{y'} = ma_{y'}; \qquad N - 100(9.81)\cos 45° = 100(0) \qquad N = 693.67 \text{ N}$$

Thus, the frictional force acting on the crate is $F_f = \mu_k N = 0.25(693.67)\text{ N} = 173.42$ N. The force developed in the spring is $F_{sp} = kx = 2000x$.



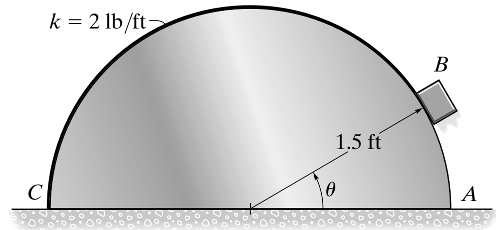
Principle of Work and Energy: By referring to Fig. *a*, notice that **N** does no work. Here, **W** which displaces downward through a distance of $h = (10 + 1.5)\sin 45° = 8.132$ m does positive work, whereas $\mathbf{F}_f$ and $\mathbf{F}_{sp}$ do negative work.

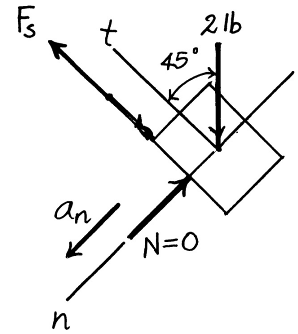
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 100(9.81)(8.132) + \left[-173.42(10 + 1.5)\right] + \left[-\frac{1}{2}(2000)(1.5^2)\right] = \frac{1}{2}(100)v^2$$

$$v = 8.64 \text{m/s} \qquad \textbf{Ans.}$$

14–35. A 2-lb block rests on the smooth semicylindrical surface. An elastic cord having a stiffness $k = 2$ lb/ft is attached to the block at B and to the base of the semicylinder at point C. If the block is released from rest at $A(\theta = 0°)$, determine the unstretched length of the cord so that the block begins to leave the semicylinder at the instant $\theta = 45°$. Neglect the size of the block.





$$+\swarrow \Sigma F_n = ma_n; \qquad 2 \sin 45° = \frac{2}{32.2}\left(\frac{v^2}{1.5}\right)$$

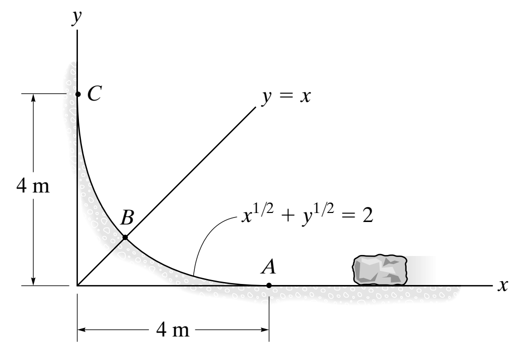
$$v = 5.844 \text{ ft/s}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

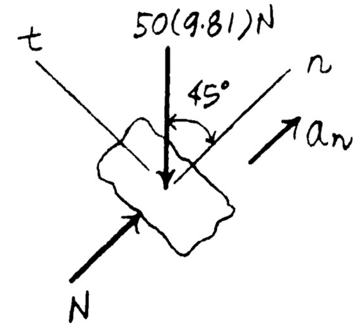
$$0 + \frac{1}{2}(2)\left[\pi(1.5) - l_0\right]^2 - \frac{1}{2}(2)\left[\frac{3\pi}{4}(1.5) - l_0\right]^2 - 2(1.5 \sin 45°) = \frac{1}{2}\left(\frac{2}{32.2}\right)(5.844)^2$$

$l_0 = 2.77$ ft **Ans.**

***14–36.** The 50-kg stone has a speed of $v_A = 8$ m/s when it reaches point A. Determine the normal force it exerts on the incline when it reaches point B. Neglect friction and the stone's size.



Geometry: Here, $x^{1/2} + y^{1/2} = 2$. At point B, $y = x$, hence $2x^{1/2} = 2$ and $x = y = 1$ m.



$$y^{-1/2}\frac{dy}{dx} = -x^{-1/2} \qquad \frac{dy}{dx} = \left.\frac{x^{-1/2}}{y^{-1/2}}\right|_{x=1\text{ m},\, y=1\text{ m}} = -1$$

$$y^{-1/2}\frac{d^2y}{dx^2} + \left(-\frac{1}{2}\right)y^{-3/2}\left(\frac{dy}{dx}\right)^2 = -\left(-\frac{1}{2}x^{-3/2}\right)$$

$$\frac{d^2y}{dx^2} = y^{1/2}\left.\left[\frac{1}{2y^{3/2}}\left(\frac{dy}{dx}\right)^2 + \frac{1}{2x^{3/2}}\right]\right|_{x=1\text{ m},\, y=1\text{ m}} = 1$$

The slope angle θ at point B is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=1\text{ m},\, y=1\text{ m}} = -1 \qquad \theta = -45.0^\circ$$

and the radius of curvature at point B is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + (-1)^2\right]^{3/2}}{|1|}\right| = 2.828\text{ m}$$

Principle of Work and Energy: The weight of the block which acts in the opposite direction to that of the vertical displacement does *negative* work when the block displaces 1 m vertically. Applying Eq. 14–7, we have

$$T_A + \sum U_{A-B} = T_B$$

$$\frac{1}{2}(50)\left(8^2\right) - 50(9.81)(1) = \frac{1}{2}(50)\,v_B^2$$

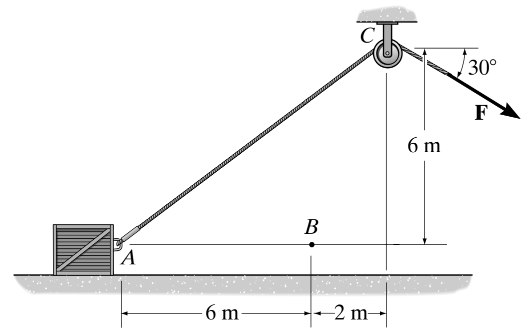
$$v_B^2 = 44.38\text{ m}^2/\text{s}^2$$

Equations of Motion: Applying Eq. 13–8 with $\theta = 45.0^\circ$, $v_B^2 = 44.38\text{ m}^2/\text{s}^2$ and $\rho = 2.828$ m, we have

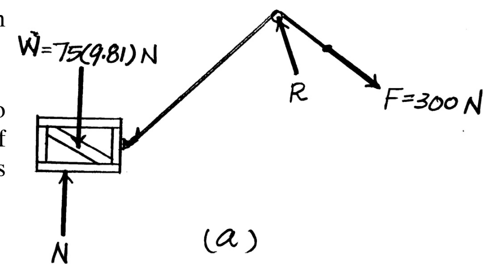
$$+\nearrow\Sigma F_n = ma_n; \qquad N - 50(9.81)\cos 45^\circ = 50\left(\frac{44.38}{2.828}\right)$$

$$N = 1131.37\text{ N} = 1.13\text{ kN} \qquad \textbf{Ans.}$$

•14–37. If the 75-kg crate starts from rest at A, determine its speed when it reaches point B. The cable is subjected to a constant force of $F = 300$ N. Neglect friction and the size of the pulley.



Free-Body Diagram: The free-body diagram of the crate and cable system at an arbitrary position is shown in Fig. *a*.



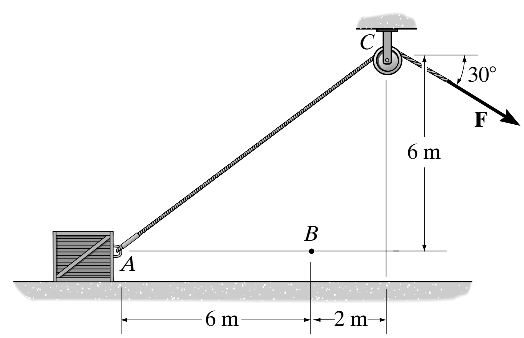
Principle of Work and Energy: By referring to Fig. *a*, notice that **N**, **W**, and **R** do no work. When the crate moves from A to B, force **F** displaces through a distance of $s = AC - BC = \sqrt{8^2 + 6^2} - \sqrt{2^2 + 6^2} = 3.675$ m. Here, the work of **F** is positive.

$$T_1 + \Sigma U_{1-2} = T_2$$

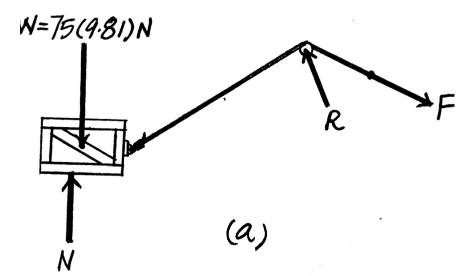
$$0 + 300(3.675) = \frac{1}{2}(75)v_B{}^2$$

$$v_B = 5.42 \text{ m/s}$$ **Ans.**

14–38. If the 75-kg crate starts from rest at A, and its speed is 6 m/s when it passes point B, determine the constant force **F** exerted on the cable. Neglect friction and the size of the pulley.



Free-Body Diagram: The free-body diagram of the crate and cable system at an arbitrary position is shown in Fig. *a*.



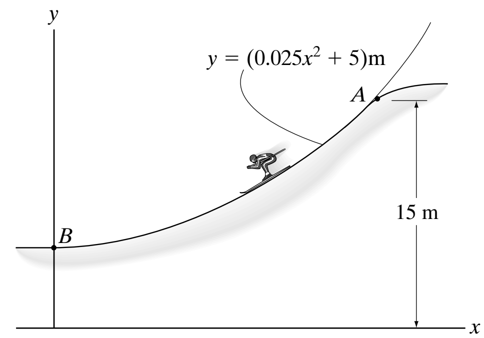
Principle of Work and Energy: By referring to Fig. *a*, notice that **N**, **W**, and **R** do no work. When the crate moves from A to B, force **F** displaces through a distance of $s = AC - BC = \sqrt{8^2 + 6^2} - \sqrt{2^2 + 6^2} = 3.675$ m. Here, the work of **F** is positive.

$$T_1 + \Sigma U_{1-2} = T_2$$

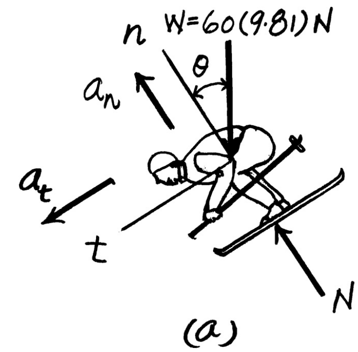
$$0 + F(3.675) = \frac{1}{2}(75)(6^2)$$

$$F = 367 \text{ N}$$ **Ans.**

14–39. If the 60-kg skier passes point A with a speed of $5\,\text{m/s}$, determine his speed when he reaches point B. Also find the normal force exerted on him by the slope at this point. Neglect friction.



Free-Body Diagram: The free-body diagram of the skier at an arbitrary position is shown in Fig. *a*.



Principle of Work and Energy: By referring to Fig. *a*, we notice that **N** does no work since it always acts perpendicular to the motion. When the skier slides down the track from A to B, **W** displaces vertically downward $h = y_A - y_B = 15 - \left[0.025\left(0^2\right) + 5\right] = 10\text{ m}$ and does positive work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}(60)(5^2) + \left[60(9.81)(10)\right] = \frac{1}{2}(60)v_B{}^2$$

$$v_B = 14.87\text{ m/s} = 14.9\text{ m/s} \qquad \textbf{Ans.}$$

$$dy/dx = 0.05x$$

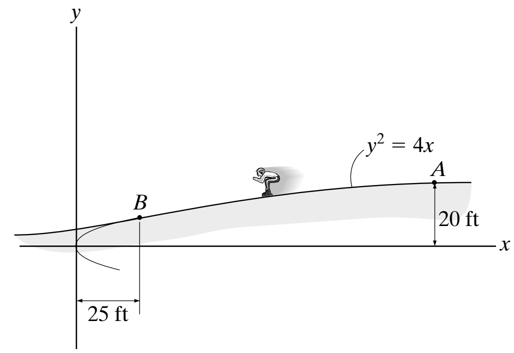
$$d^2y/dx^2 = 0.05$$

$$\rho = \frac{[1 + 0]^{3/2}}{0.5} = 20\text{ m}$$

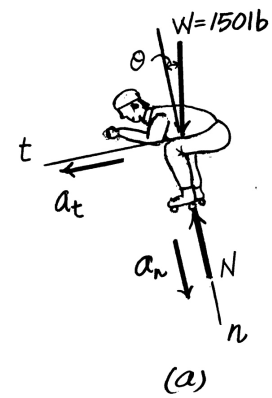
$$+\uparrow \Sigma F_n = ma_n; \qquad N - 60(9.81) = 60\left(\frac{(14.87)^2}{20}\right)$$

$$N = 1.25\text{ kN} \qquad \textbf{Ans.}$$

***14–40.** The 150-lb skater passes point A with a speed of 6 ft/s. Determine his speed when he reaches point B and the normal force exerted on him by the track at this point. Neglect friction.



Free-Body Diagram: The free-body diagram of the skater at an arbitrary position is shown in Fig. *a*.



Principle of Work and Energy: By referring to Fig. *a*, notice that **N** does no work since it always acts perpendicular to the motion. When the skier slides down the track from A to B, **W** displaces vertically downward $h = y_A - y_B = 20 - \left[2(25)^{1/2}\right] = 10$ ft and does positive work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{150}{32.2}\right)\left(6^2\right) + \left[150(10)\right] = \frac{1}{2}\left(\frac{150}{32.2}\right)v_B{}^2$$

$$v_B = 26.08 \text{ ft/s} = 26.1 \text{ ft/s} \qquad \textbf{Ans.}$$

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho}$. By referring to Fig. *a*,

$$\searrow + \Sigma F_n = ma_n; \qquad 150 \cos\theta - N = \frac{150}{32.2}\left(\frac{v^2}{\rho}\right)$$

$$N = 150 \cos\theta - \frac{150}{32.2}\left(\frac{v^2}{\rho}\right) \qquad \textbf{(1)}$$

Geometry: Here, $y = 2x^{1/2}$, $\dfrac{dy}{dx} = \dfrac{1}{x^{1/2}}$, and $\dfrac{d^2y}{dx^2} = -\dfrac{1}{2x^{3/2}}$. The slope that the track at position B makes with the horizontal is $\theta_B = \tan^{-1}\left(\dfrac{dx}{dy}\right)\Bigg|_{x=25\text{ ft}}$ $= \tan\left(\dfrac{1}{x^{1/2}}\right)\Bigg|_{x=25\text{ ft}} = 11.31°$. The radius of curvature of the track at position B is given by

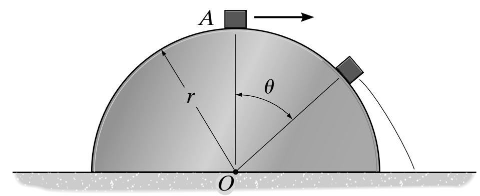
$$\rho_B = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(\dfrac{1}{x^{1/2}}\right)^2\right]^{3/2}}{\left|-\dfrac{1}{2x^{3/2}}\right|}\Bigg|_{x=25\text{ ft}} = 265.15 \text{ ft}$$

Substituting $\theta = \theta_B = 11.31°$, $v = v_B = 26.08$ ft/s, and $\rho = \rho_B = 265.15$ ft into Eq. (1),

$$N_B = 150 \cos 11.31° - \frac{150}{32.2}\left(\frac{26.08^2}{265.15}\right)$$

$$= 135 \text{ lb} \qquad \textbf{Ans.}$$

•**14–41.** A small box of mass m is given a speed of $v = \sqrt{\frac{1}{4}gr}$ at the top of the smooth half cylinder. Determine the angle θ at which the box leaves the cylinder.

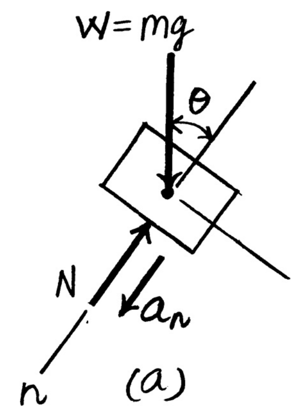


Principle of Work and Energy: By referring to the free-body diagram of the block, Fig. a, notice that **N** does no work, while **W** does positive work since it displaces downward though a distance of $h = r - r\cos\theta$.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m\left(\frac{1}{4}gr\right) + mg(r - r\cos\theta) = \frac{1}{2}mv^2$$

$$v^2 = gr\left(\frac{9}{4} - 2\cos\theta\right) \qquad (1)$$



Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{gr\left(\frac{9}{4} - 2\cos\theta\right)}{r} = g\left(\dfrac{9}{4} - 2\cos\theta\right)$. By referring to Fig. a,

$$\Sigma F_n = ma_n; \qquad mg\cos\theta - N = m\left[g\left(\frac{9}{4} - 2\cos\theta\right)\right]$$

$$N = mg\left(3\cos\theta - \frac{9}{4}\right)$$

It is required that the block leave the track. Thus, $N = 0$.

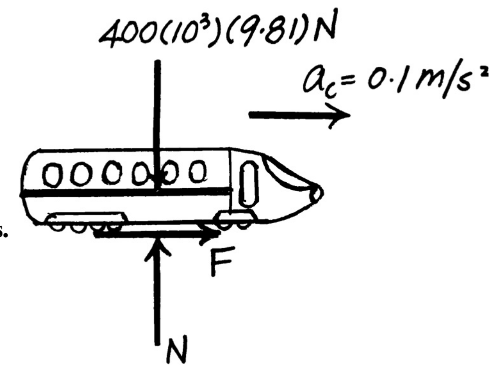
$$0 = mg\left(3\cos\theta - \frac{9}{4}\right)$$

Since $mg \neq 0$,

$$3\cos\theta - \frac{9}{4} = 0$$

$$\theta = 41.41° = 41.4° \qquad \textbf{Ans.}$$

14–42. The diesel engine of a 400-Mg train increases the train's speed uniformly from rest to 10 m/s in 100 s along a horizontal track. Determine the average power developed.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + U_{1-2} = \frac{1}{2}(400)\left(10^3\right)(10^2)$$

$$U_{1-2} = 20\left(10^6\right)\text{ J}$$

$$P_{\text{avg}} = \frac{U_{1-2}}{t} = \frac{20(10^6)}{100} = 200\text{ kW}$$ **Ans.**

Also,

$$v = v_0 + a_c t$$

$$10 = 0 + a_c(100)$$

$$a_c = 0.1\text{ m/s}^2$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad F = 400\left(10^3\right)(0.1) = 40\left(10^3\right)\text{ N}$$

$$P_{\text{avg}} = \mathbf{F} \cdot \mathbf{v}_{\text{avg}} = 40\left(10^3\right)\left(\frac{10}{2}\right) = 200\text{ kW}$$ **Ans.**

14–43. Determine the power input for a motor necessary to lift 300 lb at a constant rate of 5 ft/s. The efficiency of the motor is $\epsilon = 0.65$.

Power: The power output can be obtained using Eq. 14–10.

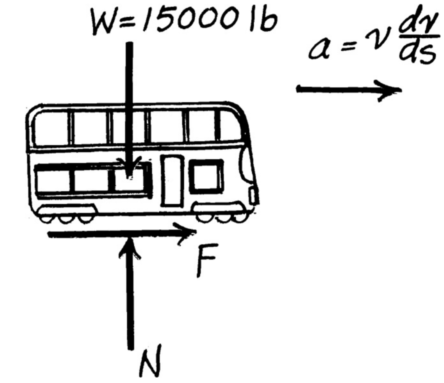
$$P = \mathbf{F} \cdot \mathbf{v} = 300(5) = 1500\text{ ft}\cdot\text{lb/s}$$

Using Eq. 14–11, the required power input for the motor to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\epsilon}$$

$$= \frac{1500}{0.65} = 2307.7\text{ ft}\cdot\text{lb/s} = 4.20\text{ hp}$$ **Ans.**

***14–44.** An electric streetcar has a weight of 15 000 lb and accelerates along a horizontal straight road from rest so that the power is always 100 hp. Determine how far it must travel to reach a speed of 40 ft/s.



$$F = ma = \frac{W}{g}\left(\frac{v\,dv}{ds}\right)$$

$$P = Fv = \left[\left(\frac{W}{g}\right)\left(\frac{v\,dv}{ds}\right)\right]v$$

$$\int_0^s P\,ds = \int_0^v \frac{W}{g}v^2\,dv$$

P = constant

$$Ps = \frac{W}{g}\left(\frac{1}{3}\right)v^3 \qquad s = \frac{W}{3gP}v^3$$

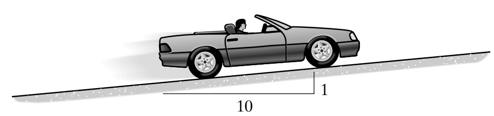
$$s = \frac{(15\,000)(40)^3}{3(32.2)(100)(550)} = 181 \text{ ft}$$ **Ans.**

•14–45. The Milkin Aircraft Co. manufactures a turbojet engine that is placed in a plane having a weight of 13000 lb. If the engine develops a constant thrust of 5200 lb, determine the power output of the plane when it is just ready to take off with a speed of 600 mi/h.

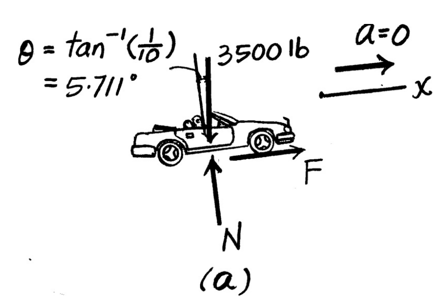
At 600 ms/h.

$$P = 5200(600)\left(\frac{88 \text{ ft/s}}{60 \text{ m/h}}\right)\frac{1}{550} = 8.32\ (10^3) \text{ hp}$$ **Ans.**

14–46. The engine of the 3500-lb car is generating a constant power of 50 hp while the car is traveling up the slope with a constant speed. If the engine is operating with an efficiency of $\epsilon = 0.8$, determine the speed of the car. Neglect drag and rolling resistance.



Equations of Motion: By referring to the free-body diagram of the car shown in Fig. *a*,



$$+\nearrow\Sigma F_{x'} = ma_{x'}; \qquad F - 3500 \sin 5.711° = \frac{3500}{32.2}(0) \qquad F = 348.26 \text{ lb}$$

Power: The power input of the car is $P_{in} = (50 \text{ hp})\left(\frac{550 \text{ ft}\cdot\text{lb/s}}{1 \text{ hp}}\right) = 27\,500 \text{ ft}\cdot\text{lb/s}$. Thus, the power output is given by $P_{out} = \varepsilon P_{in} = 0.8(27\,500) = 22\,000 \text{ ft}\cdot\text{lb/s}$.

$$P_{out} = \mathbf{F}\cdot\mathbf{v}$$

$$22\,000 = 348.26v$$

$$v = 63.2 \text{ ft/s}$$ **Ans.**

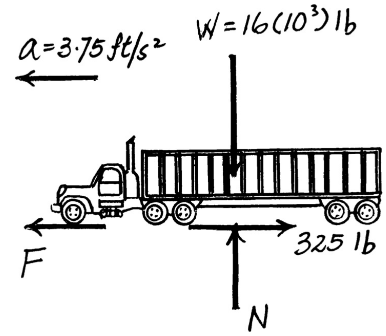
14–47. A loaded truck weighs $16(10^3)$ lb and accelerates uniformly on a level road from 15 ft/s to 30 ft/s during 4 s. If the frictional resistance to motion is 325 lb, determine the maximum power that must be delivered to the wheels.

$$a = \frac{\Delta v}{\Delta t} = \frac{30 - 15}{4} = 3.75 \text{ ft/s}^2$$

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad F - 325 = \left(\frac{16(10^3)}{32.2}\right)(3.75)$$

$$F = 2188.35 \text{ lb}$$

$$P_{\max} = \mathbf{F} \cdot \mathbf{v}_{\max} = \frac{2188.35(30)}{550} = 119 \text{ hp}$$ **Ans.**



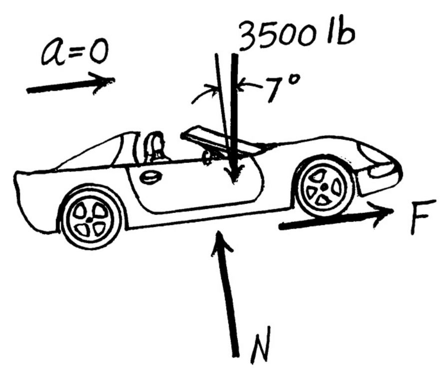
***14–48.** An automobile having a weight of 3500 lb travels up a 7° slope at a constant speed of $v = 40$ ft/s. If friction and wind resistance are neglected, determine the power developed by the engine if the automobile has a mechanical efficiency of $\epsilon = 0.65$.

$$s = vt = 40(1) = 40 \text{ ft}$$

$$U_{1-2} = (3500)(40 \sin 7°) = 17.062(10^3) \text{ ft} \cdot \text{lb}$$

$$P_{in} = \frac{P_o}{e} = \frac{17.062(10^3) \text{ ft} \cdot \text{lb/s}}{0.65} = 26.249(10^3) \text{ ft} \cdot \text{lb/s} = 47.7 \text{ hp}$$

$$P_{out} = \frac{U_{1-2}}{t} = \frac{17.602(10^3)}{1} = 17.062\,(10^3) \text{ ft} \cdot \text{lb/s}$$ **Ans.**



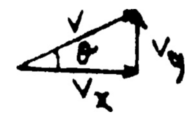
Also,

$$F = 3500 \sin 7° = 426.543 \text{ lb}$$

$$P_{out} = \mathbf{F} \cdot \mathbf{v} = 426.543\,(40) = 17.062\,(10^3) \text{ ft} \cdot \text{lb/s}$$

$$P_{in} = \frac{P_o}{e} = \frac{17.062\,(10^3) \text{ ft} \cdot \text{lb/s}}{0.65} = 26.249(10^3) \text{ ft} \cdot \text{lb/s} = 47.7 \text{ hp}$$ **Ans.**

•14–49. An escalator step moves with a constant speed of 0.6 m/s. If the steps are 125 mm high and 250 mm in length, determine the power of a motor needed to lift an average mass of 150 kg per step. There are 32 steps.



Step height: 0.125 m

The number of steps: $\dfrac{4}{0.125} = 32$

Total load: $32(150)(9.81) = 47\ 088$ N

If load is placed at the center height, $h = \dfrac{4}{2} = 2$ m, then

$$U = 47\ 088\left(\frac{4}{2}\right) = 94.18 \text{ kJ}$$

$$v_s = v \sin\theta = 0.6\left(\frac{4}{\sqrt{(32(0.25))^2 + 4^2}}\right) = 0.2683 \text{ m/s}$$

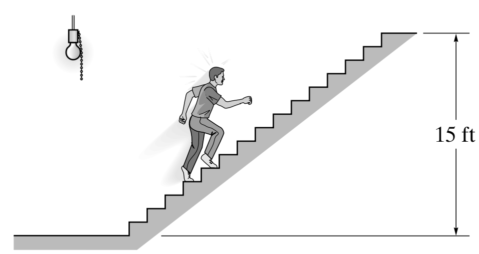
$$t = \frac{h}{v_y} = \frac{2}{0.2683} = 7.454 \text{ s}$$

$$P = \frac{U}{t} = \frac{94.18}{7.454} = 12.6 \text{ kW}$$ **Ans.**

Also,

$$P = \mathbf{F} \cdot \mathbf{v} = 47\ 088(0.2683) = 12.6 \text{ kW}$$ **Ans.**

14–50. The man having the weight of 150 lb is able to run up a 15-ft-high flight of stairs in 4 s. Determine the power generated. How long would a 100-W light bulb have to burn to expend the same amount of energy? *Conclusion:* Please turn off the lights when they are not in use!



Power: The work done by the man is

$$U = Wh = 150(15) = 2250 \text{ ft} \cdot \text{lb}$$

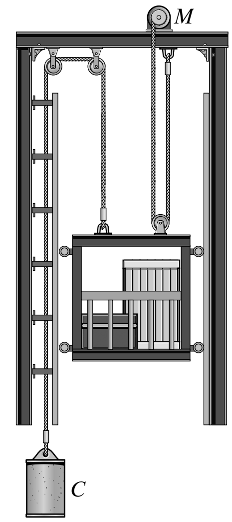
Thus, the power generated by the man is given by

$$P_{\max} = \frac{U}{t} = \frac{2250}{4} = 562.5 \text{ ft} \cdot \text{lb/s} = 1.02 \text{ hp}$$ **Ans.**

The power of the bulb is $P_{\text{bulb}} = 100\ W \times \left(\dfrac{1 \text{ hp}}{746 \text{ W}}\right) \times \left(\dfrac{550 \text{ ft} \cdot \text{lb/s}}{1 \text{ hp}}\right) =$ 73.73 ft · lb/s. Thus,

$$t = \frac{U}{P_{\text{bulb}}} = \frac{2250}{73.73} = 30.5 \text{ s}$$ **Ans.**

14–51. The material hoist and the load have a total mass of 800 kg and the counterweight C has a mass of 150 kg. At a given instant, the hoist has an upward velocity of 2 m/s and an acceleration of 1.5 m/s^2. Determine the power generated by the motor M at this instant if it operates with an efficiency of $\epsilon = 0.8$.



Equations of Motion: Here, $a = 1.5$ m/s^2. By referring to the free-body diagram of the hoist and counterweight shown in Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad 2T + T' - 800(9.81) = 800(1.5) \qquad \textbf{(1)}$$

$$+\downarrow \Sigma F_y = ma_y; \qquad 150(9.81) - T' = 150(1.5)$$

Solving,

$$T' = 1246.5 \text{ N}$$

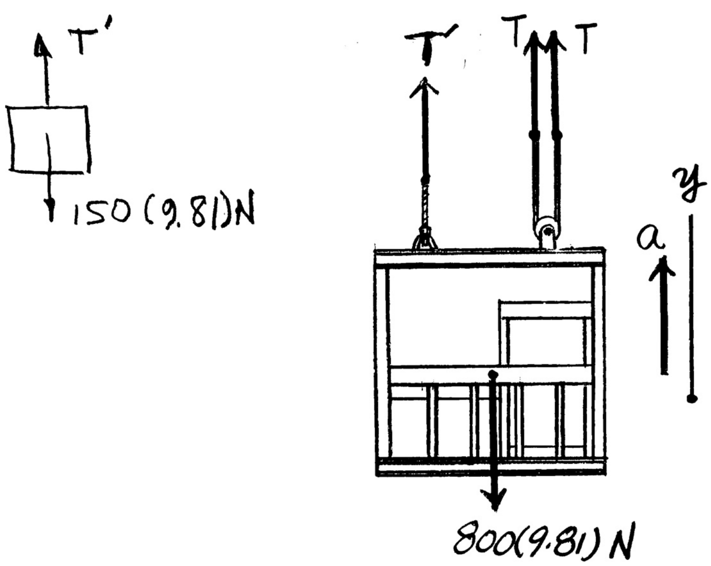
$$T = 3900.75 \text{ N}$$

Power:

$$P_{\text{out}} = 2\mathbf{T}\cdot\mathbf{v} = 2(3900.75)(2) = 15\,603 \text{ W}$$

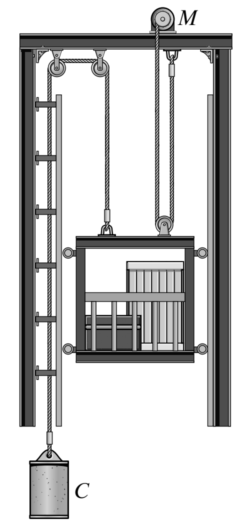
Thus,

$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{15603}{0.8} = 19.5(10^3) \text{ W} = 19.5 \text{ kW} \qquad \textbf{Ans.}$$



(a)

***14–52.** The material hoist and the load have a total mass of 800 kg and the counterweight C has a mass of 150 kg. If the upward speed of the hoist increases uniformly from 0.5 m/s to 1.5 m/s in 1.5 s, determine the average power generated by the motor M during this time. The motor operates with an efficiency of $\epsilon = 0.8$.



Kinematics: The acceleration of the hoist can be determined from

$$(+\uparrow) \qquad v = v_0 + a_c t$$

$$1.5 = 0.5 + a(1.5)$$

$$a = 0.6667 \text{ m/s}^2$$

Equations of Motion: Using the result of **a** and referring to the free-body diagram of the hoist and block shown in Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad 2T + T' - 800(9.81) = 800(0.6667)$$

$$+\downarrow \Sigma F_y = ma_y; \qquad 150(9.81) - T' = 150(0.6667)$$

Solving,

$$T' = 1371.5 \text{ N}$$

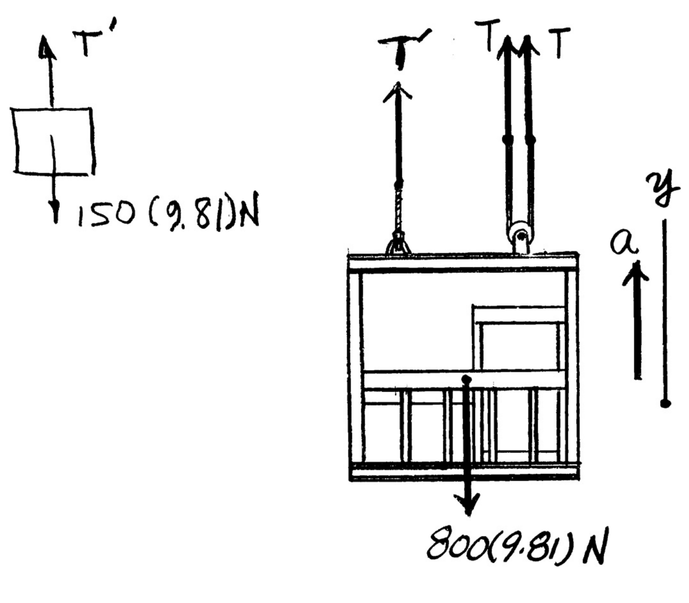
$$T = 3504.92 \text{ N}$$

Power:

$$(P_{\text{out}})_{\text{avg}} = 2\mathbf{T} \cdot \mathbf{v}_{\text{avg}} = 2(3504.92)\left(\frac{1.5 + 0.5}{2}\right) = 7009.8 \text{ W}$$

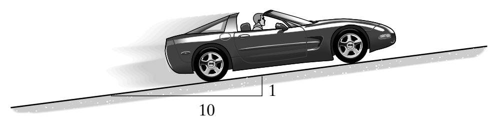
Thus,

$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{7009.8}{0.8} = 8762.3 \quad \text{W} = 8.76 \text{ kW} \qquad \textbf{Ans.}$$



(a)

•**14–53.** The 2-Mg car increases its speed uniformly from rest to 25 m/s in 30 s up the inclined road. Determine the maximum power that must be supplied by the engine, which operates with an efficiency of $\epsilon = 0.8$. Also, find the average power supplied by the engine.

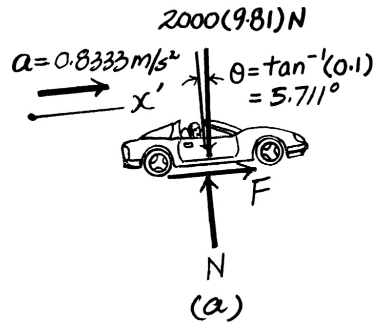


Kinematics: The constant acceleration of the car can be determined from

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$25 = 0 + a_c(30)$$

$$a_c = 0.8333 \text{ m/s}^2$$



Equations of Motion: By referring to the free-body diagram of the car shown in Fig. *a*,

$$\Sigma F_{x'} = ma_{x'}; \qquad F - 2000(9.81)\sin 5.711° = 2000(0.8333)$$

$$F = 3618.93\text{N}$$

Power: The maximum power output of the motor can be determined from

$$(P_{\text{out}})_{\max} = \mathbf{F} \cdot \mathbf{v}_{\max} = 3618.93(25) = 90\,473.24 \text{ W}$$

Thus, the maximum power input is given by

$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{90473.24}{0.8} = 113\,091.55 \text{ W} = 113 \text{ kW} \qquad \textbf{Ans.}$$

The average power output can be determined from

$$(P_{\text{out}})_{\text{avg}} = \mathbf{F} \cdot \mathbf{v}_{\text{avg}} = 3618.93\left(\frac{25}{2}\right) = 45\,236.62 \text{ W}$$

Thus,

$$(P_{\text{in}})_{\text{avg}} = \frac{(P_{\text{out}})_{\text{avg}}}{\varepsilon} = \frac{45236.62}{0.8} = 56\,545.78 \text{ W} = 56.5 \text{ kW} \qquad \textbf{Ans.}$$

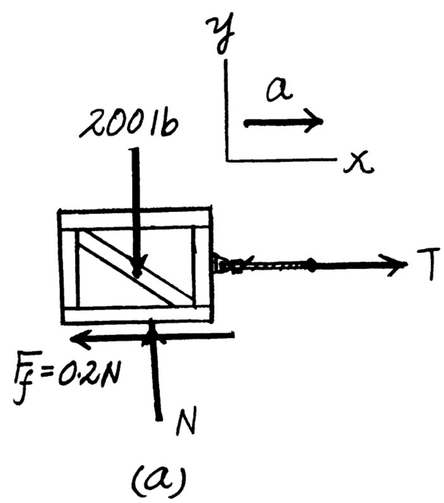
14–54. Determine the velocity of the 200-lb crate in 15 s if the motor operates with an efficiency of $\epsilon = 0.8$. The power input to the motor is 2.5 hp. The coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.2$.

Equations of Motion: By referring to the free-body diagram of the crate shown in Fig. *a*,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 200 = \frac{200}{32.2}(0) \qquad N = 200 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad T - 0.2(200) = \frac{200}{32.2}(a)$$

$$T = \left(\frac{200}{32.2}a + 40\right) \text{lb} \qquad \textbf{(1)}$$



Power: Here, the power input is $P_{in} = (2.5 \text{ hp})\left(\frac{550 \text{ ft} \cdot \text{lb/s}}{1 \text{ hp}}\right) = 1375 \text{ ft} \cdot \text{lb/s}$. Thus,

$P_{out} = \varepsilon P_{in} = 0.8(1375) = 1100 \text{ ft} \cdot \text{lb/s}$.

$$P_{out} = \mathbf{T} \cdot \mathbf{v}$$

$$1100 = Tv \qquad \textbf{(2)}$$

Kinematics: The speed of the crate is

$$\left(\xrightarrow{+}\right) \qquad v = v_0 + a_c t$$

$$v = 0 + a(15)$$

$$v = 15a \qquad \textbf{(3)}$$

Substituting Eq. (3) into Eq. (2) yields

$$T = \frac{73.33}{a} \qquad \textbf{(4)}$$

Substituting Eq. (4) into Eq. (1) yields

$$\frac{73.33}{a} = \frac{200}{32.2}a + 40$$

Solving for the positive root,

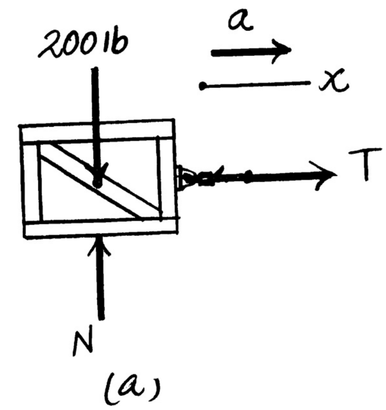
$$a = 1.489 \text{ ft/s}^2$$

Substituting the result into Eq. (3),

$$v = 15(1.489) = 22.3 \text{ ft/s} \qquad \textbf{Ans.}$$

14–55. A constant power of 1.5 hp is supplied to the motor while it operates with an efficiency of $\epsilon = 0.8$. Determine the velocity of the 200-lb crate in 15 seconds, starting from rest. Neglect friction.

Equations of Motion: Here, $a = \dfrac{dv}{dt}$. By referring to the free-body diagram of the crate shown in Fig. a,



$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad T = \frac{200}{32.2}\left(\frac{dv}{dt}\right) \qquad \textbf{(1)}$$

Power: Here, the power input is $P_{\text{in}} = (1.5\text{ hp})\left(\dfrac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = 825\text{ ft}\cdot\text{lb/s}$. Thus,

$P_{\text{out}} = \varepsilon P_{\text{in}} = 0.8(825) = 660\text{ ft}\cdot\text{lb/s}$.

$$P_{\text{out}} = \mathbf{T}\cdot\mathbf{v}$$

$$660 = \frac{200}{32.2}\left(\frac{dv}{dt}\right)v$$

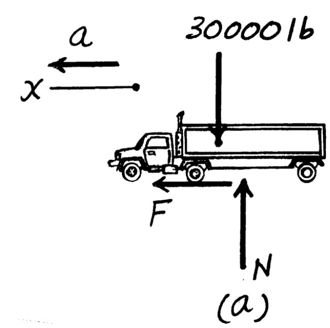
$$\int_0^v v\,dv = \int_0^{15\text{ s}} 106.26\,dt$$

$$\left.\frac{v^2}{2}\right|_0^v = 106.26\,dt\Big|_0^{15\text{ s}}$$

$$v = 56.5\text{ ft/s} \qquad \textbf{Ans.}$$

***14–56.** The fluid transmission of a 30 000-lb truck allows the engine to deliver constant power to the rear wheels. Determine the distance required for the truck traveling on a level road to increase its speed from 35 ft/s to 60 ft/s if 90 hp is delivered to the rear wheels. Neglect drag and rolling resistance.

Equations of Motion: Here, $a = v\dfrac{dv}{ds}$. By referring to the fre -body diagram of the truck shown in Fig. a,



$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad F = \left(\frac{30000}{32.2}\right)\left(v\frac{dv}{ds}\right) \qquad \textbf{(1)}$$

Power: Here, the power output is $P_{\text{out}} = (90\text{ hp})\left(\dfrac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = 49500\text{ ft}\cdot\text{lb/s}$. Using Eq. (1),

$$P_{\text{out}} = \mathbf{F}\cdot\mathbf{v}$$

$$49500 = \left(\frac{30000}{32.2}\right)\left(v\frac{dv}{ds}\right)v$$

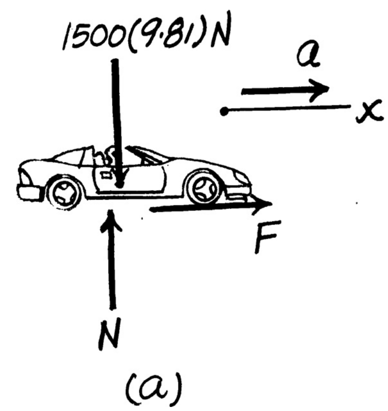
$$\int_0^s 53.13\,ds = \int_{35\text{ ft/s}}^{60\text{ ft/s}} v^2\,dv$$

$$53.13s\Big|_0^s = \left.\frac{v^3}{3}\right|_{35\text{ ft/s}}^{60\text{ ft/s}}$$

$$s = 1086\text{ ft} \qquad \textbf{Ans.}$$

•14–57. If the engine of a 1.5-Mg car generates a constant power of 15 kW, determine the speed of the car after it has traveled a distance of 200 m on a level road starting from rest. Neglect friction.

Equations of Motion: Here, $a = v\dfrac{dv}{ds}$. By referring to the free-body diagram of the car shown in Fig. *a*,



$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F = 1500\left(v\frac{dv}{ds}\right)$$

Power:

$$P_{\text{out}} = \mathbf{F} \cdot \mathbf{v}$$

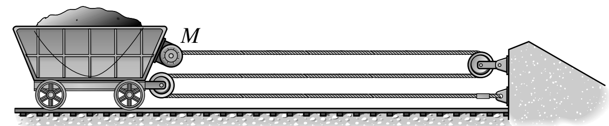
$$15(10^3) = 1500\left(v\frac{dv}{ds}\right)v$$

$$\int_0^{200\text{ m}} 10ds = \int_0^v v^2\,dv$$

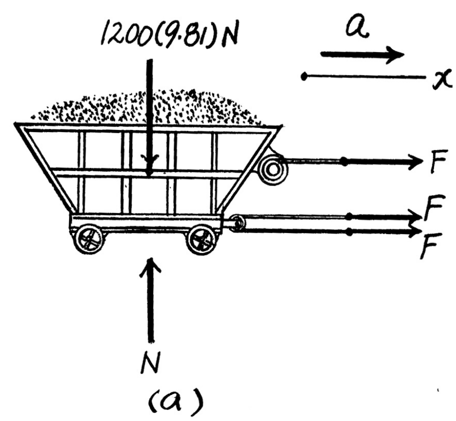
$$10s\Big|_0^{200\text{ m}} = \frac{v^3}{3}\Big|_0^v$$

$$v = 18.7 \text{ m/s} \qquad \textbf{Ans.}$$

14–58. The 1.2-Mg mine car is being pulled by the winch *M* mounted on the car. If the winch exerts a force of $F = (150t^{3/2})$ N on the cable, where t is in seconds, determine the power output of the winch when $t = 5$ s, starting from rest.



Equations of Motion: By referring to the free-body diagram of the mine car shown in Fig. *a*,



$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 3\left(150t^{3/2}\right) = 1200a$$

$$a = \left(0.375t^{3/2}\right) \text{m/s}^2$$

Kinematics: The speed of the mine car at $t = 5$ s can be determined by integrating the kinematic equation $dv = adt$ and using the result of **a**.

$$\left(\xrightarrow{+}\right) \qquad \int_0^v dv = \int_0^{5\text{ s}} 0.375t^{3/2}\,dt$$

$$v = 0.15t^{5/2}\Big|_0^{5\text{ s}} = 8.385 \text{ m/s}$$

Power: At $t = 5$ s, $F = 150(5^{3/2}) = 1677.05$ N.

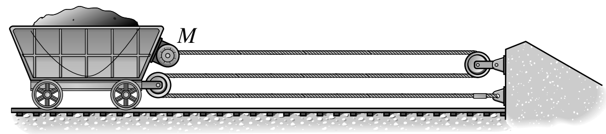
$$P_{\text{out}} = 3\mathbf{F} \cdot \mathbf{v}$$

$$= 3(1677.05)(8.385)$$

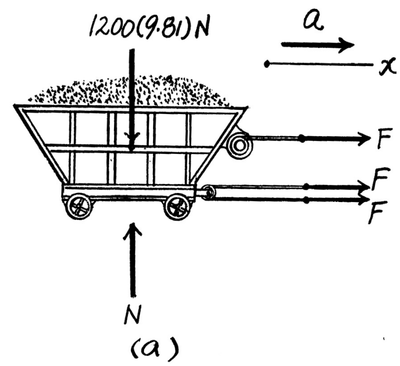
$$= 42.1875(10^3) \text{ W}$$

$$= 42.2 \text{ kW} \qquad \textbf{Ans.}$$

14–59. The 1.2-Mg mine car is being pulled by the winch M mounted on the car. If the winch generates a constant power output of 30 kW, determine the speed of the car at the instant it has traveled a distance of 30 m, starting from rest.



Equations of Motion: Here, $a = v\dfrac{dv}{ds}$. By referring to the free-body diagram of the mine car shown in Fig. a,



$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 3F = 1200\left(v\frac{dv}{ds}\right) \qquad \textbf{(1)}$$

Power:

$$P_{\text{out}} = 3\mathbf{F}\cdot\mathbf{v}$$

$$30(10^3) = 3Fv \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

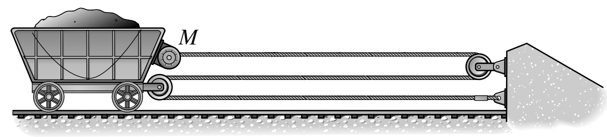
$$30(10^3) = 1200\left(v\frac{dv}{ds}\right)v$$

$$\int_0^v v^2 dv = \int_0^{30\text{ m}} 25 ds$$

$$\left.\frac{v^3}{3}\right|_0^v = 25s\Big|_0^{30\text{ m}}$$

$$v = 13.1 \text{ m/s} \qquad \textbf{Ans.}$$

***14–60.** The 1.2-Mg mine car is being pulled by winch M mounted on the car. If the winch generates a constant power output of 30 kW, and the car starts from rest, determine the speed of the car when $t = 5$ s.



Equations of Motion: Here, $a = v\dfrac{dv}{ds}$. By referring to the free-body diagram of the mine car shown in Fig. a,

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 3F = 1200\left(v\frac{dv}{ds}\right) \qquad \textbf{(1)}$$

Power:

$$P_{\text{out}} = 3\mathbf{F}\cdot\mathbf{v}$$

$$30(10^3) = 3Fv \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

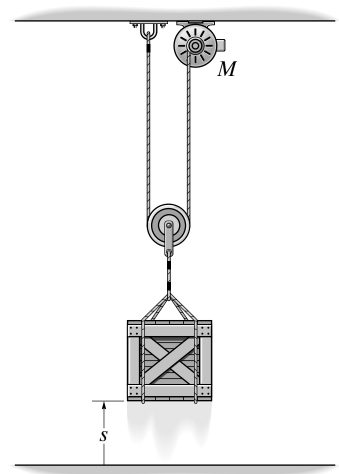
$$30(10^3) = \left(1200\frac{dv}{ds}\right)v$$

$$\int_0^v vdv = \int_0^{5\text{ s}} 25dt$$

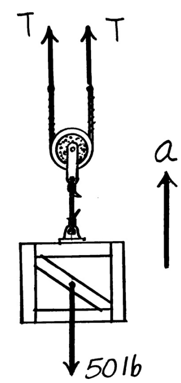
$$\left.\frac{v^2}{2}\right|_0^v = 25t\Big|_0^{5\text{ s}}$$

$$v = 15.8 \text{ m/s} \qquad \textbf{Ans.}$$

•14–61. The 50-lb crate is hoisted by the motor M. If the crate starts from rest and by constant acceleration attains a speed of 12 ft/s after rising $s = 10$ ft, determine the power that must be supplied to the motor at the instant $s = 10$ ft. The motor has an efficiency $\epsilon = 0.65$. Neglect the mass of the pulley and cable.



$+\uparrow \Sigma F_y = ma_y; \qquad 2T - 50 = \frac{50}{32.2} a$



$(+\uparrow)\ v^2 = v_0^2 + 2\,a_c\,(s - s_0)$

$(12)^2 = 0 + 2(a)(10 - 0)$

$a = 7.20\ \text{ft/s}^2$

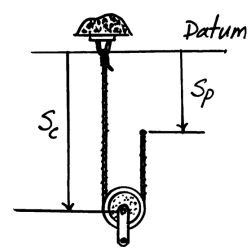
Thus, $T = 30.59$ lb

$s_C + (s_C - s_P) = l$

$2\,v_C = v_P$

$2(12) = v_P = 24$ ft/s

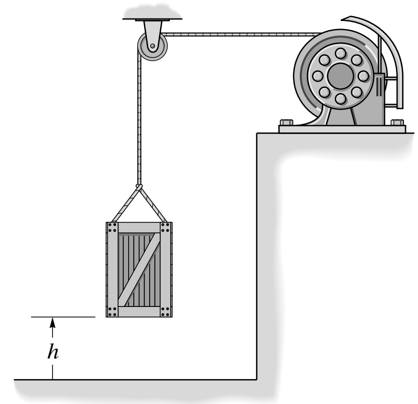
$P_o = 30.59(24) = 734.16$



$P_i = \frac{734.16}{0.65} = 1129.5\ \text{ft} \cdot \text{lb/s}$

$P_i = 2.05$ hp **Ans.**

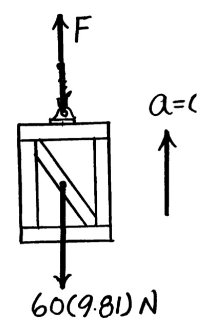
14–62. A motor hoists a 60-kg crate at a constant velocity to a height of $h = 5$ m in 2 s. If the indicated power of the motor is 3.2 kW, determine the motor's efficiency.



Equations of Motion:

$$+\uparrow \Sigma F_y = ma_y; \qquad F - 60(9.81) = 60(0) \qquad F = 588.6 \text{ N}$$

Power: The crate travels at a constant speed of $v = \dfrac{5}{2} = 2.50$ m/s. The power output can be obtained using Eq. 14–10.

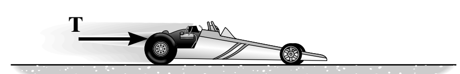


$$P = \mathbf{F} \cdot \mathbf{v} = 588.6\,(2.50) = 1471.5 \text{ W}$$

Thus, from Eq. 14–11, the efficiency of the motor is given by

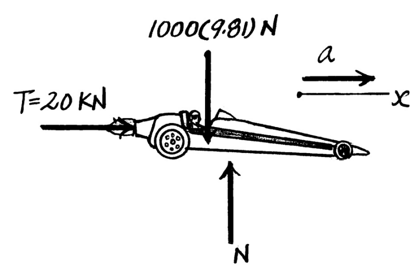
$$\varepsilon = \frac{\text{power output}}{\text{power input}} = \frac{1471.5}{3200} = 0.460 \qquad \textbf{Ans.}$$

14–63. If the jet on the dragster supplies a constant thrust of $T = 20$ kN, determine the power generated by the jet as a function of time. Neglect drag and rolling resistance, and the loss of fuel. The dragster has a mass of 1 Mg and starts from rest.



Equations of Motion: By referring to the free-body diagram of the dragster shown in Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 20(10^3) = 1000(a) \qquad a = 20 \text{ m/s}^2$$



(a)

Kinematics: The velocity of the dragster can be determined from

$$\left(\xrightarrow{+}\right) \qquad v = v_0 + a_c t$$

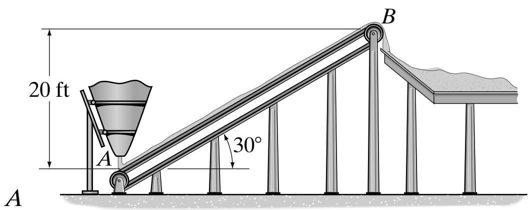
$$v = 0 + 20t = (20t) \text{ m/s}$$

Power:

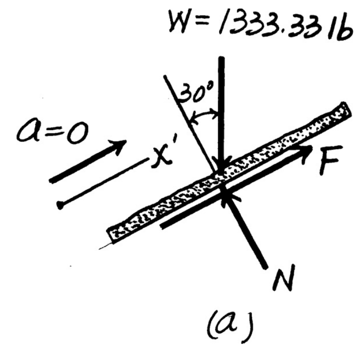
$$P = \mathbf{F} \cdot \mathbf{v} = 20(10^3)(20t)$$

$$= \left[400(10^3)t\right] \text{ W} \qquad \textbf{Ans.}$$

***14–64.** Sand is being discharged from the silo at A to the conveyor and transported to the storage deck at the rate of 360 000 lb/h. An electric motor is attached to the conveyor to maintain the speed of the belt at 3 ft/s. Determine the average power generated by the motor.



Equations of Motion: The time required for the conveyor to move from point A point B is $t_{AB} = \dfrac{s_{AB}}{v} = \dfrac{20/\sin 30°}{3} = 13.33$ s. Thus, the weight of the sand on the conveyor at any given instant is $W = (360\,000 \text{ lb/h})\left(\dfrac{1 \text{ h}}{3600 \text{ s}}\right)(13.33 \text{ s}) = 1333.33$ lb. By referring to the free-body diagram of the sand shown in Fig. a,



$$+\nearrow\Sigma F_{x'} = ma_{x'}; \qquad F - 1333.3 \sin 30° = \frac{1333.33}{32.2}(0)$$

$$F = 666.67 \text{ lb}$$

Power: Using the result of **F**,

$$P = \mathbf{F} \cdot \mathbf{v} = 666.67(3) = 2000 \text{ ft} \cdot \text{lb/s}$$

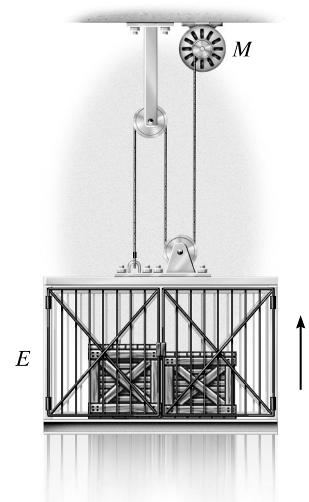
Thus,

$$P = (2000 \text{ ft} \cdot \text{lb/s})\left(\frac{1 \text{ hp}}{550 \text{ ft} \cdot \text{lb/s}}\right) = 3.64 \text{ hp} \qquad \textbf{Ans.}$$

Note that P can also be determined in a more direct manner using

$$P_{\text{out}} = \frac{dW}{dt}(h) = \left(360\,000 \frac{\text{lb}}{\text{h}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)(20 \text{ ft}) = 2000 \text{ ft} \cdot \text{lb/s}$$

14–65. The 500-kg elevator starts from rest and travels upward with a constant acceleration $a_c = 2 \text{ m/s}^2$. Determine the power output of the motor M when $t = 3$ s. Neglect the mass of the pulleys and cable.



$$+\uparrow\Sigma F_y = m\,a_y; \qquad 3T - 500(9.81) = 500(2)$$

$$T = 1968.33 \text{ N}$$

$$3s_E - s_P = l$$

$$3\,v_E = v_P$$

When $t = 3$ s,

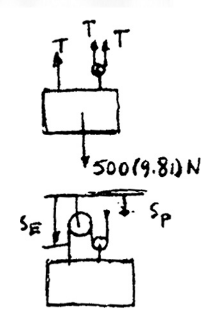
$$(+\uparrow) \quad v_0 + a_c\,t$$

$$v_E = 0 + 2(3) = 6 \text{ m/s}$$

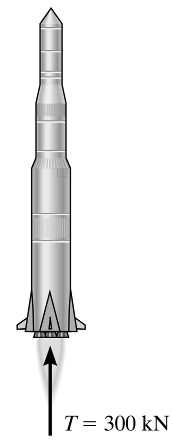
$$v_P = 3(6) = 18 \text{ m/s}$$

$$P_O = 1968.33(18)$$

$$P_O = 35.4 \text{ kW} \qquad \textbf{Ans.}$$



14–66. A rocket having a total mass of 8 Mg is fired vertically from rest. If the engines provide a constant thrust of $T = 300$ kN, determine the power output of the engines as a function of time. Neglect the effect of drag resistance and the loss of fuel mass and weight.

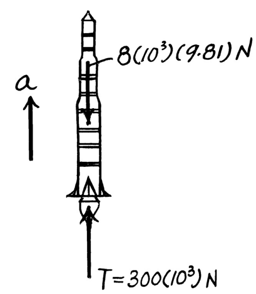


$+\uparrow \Sigma F_y = ma_y; \qquad 300(10^3) - 8(10^3)(9.81) = 8(10^3)a \qquad a = 27.69 \text{ m/s}^2$

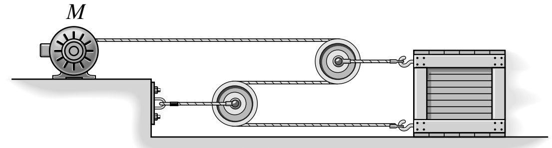
$(+\uparrow) \qquad v = v_0 + a_c t$

$\qquad = 0 + 27.69t = 27.69t$

$P = \mathbf{T} \cdot \mathbf{v} = 300\,(10^3)\,(27.69t) = 8.31\,t \text{ MW}$ **Ans.**



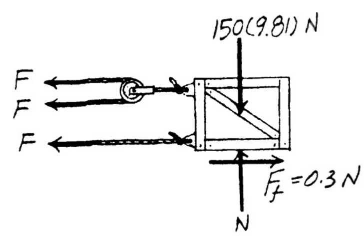
14–67. The crate has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively. If the motor M supplies a cable force of $F = (8t^2 + 20)$ N, where t is in seconds, determine the power output developed by the motor when $t = 5$ s.



Equations of Equilibrium: If the crate is on the verge of slipping, $F_f = \mu_s N = 0.3N$. From FBD(a),

$+\uparrow \Sigma F_y = 0; \qquad N - 150(9.81) = 0 \qquad N = 1471.5 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 0.3(1471.5) - 3\left(8t^2 + 20\right) = 0 \qquad t = 3.9867 \text{ s}$



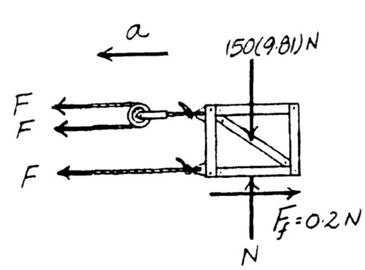
(a)

Equations of Motion: Since the crate moves 3.9867 s later, $F_f = \mu_k N = 0.2N$. From FBD(b),

$+\uparrow \Sigma F_y = ma_y; \qquad N - 150(9.81) = 150\,(0) \qquad N = 1471.5 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 0.2\,(1471.5) - 3\left(8t^2 + 20\right) = 150\,(-a)$

$$a = \left(0.160t^2 - 1.562\right) \text{ m/s}^2$$



Kinematics: Applying $dv = a\,dt$, we have

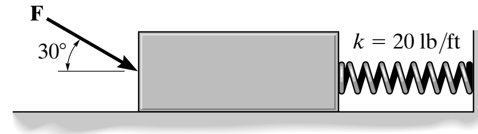
$$\int_0^v dv = \int_{3.9867\text{ s}}^{5} \left(0.160t^2 - 1.562\right) dt$$

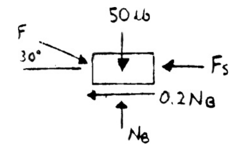
$$v = 1.7045 \text{ m/s}$$

Power: At $t = 5$ s, $F = 8\left(5^2\right) + 20 = 220$ N. The power can be obtained using Eq. 14–10.

$P = \mathbf{F} \cdot \mathbf{v} = 3\,(220)\,(1.7045) = 1124.97 \text{ W} = 1.12 \text{ kW}$ **Ans.**

***14–68.** The 50-lb block rests on the rough surface for which the coefficient of kinetic friction is $\mu_k = 0.2$. A force $F = (40 + s^2)$ lb, where s is in ft, acts on the block in the direction shown. If the spring is originally unstretched ($s = 0$) and the block is at rest, determine the power developed by the force the instant the block has moved $s = 1.5$ ft.





$$+\uparrow \Sigma F_y = 0; \qquad N_B - \left(40 + s^2\right)\sin 30° - 50 = 0$$

$$N_B = 70 + 0.5s^2$$

$$T_1 + \Sigma U_{1-2} + T_2$$

$$0 + \int_0^{1.5} \left(40 + s^2\right)\cos 30°\, ds - \frac{1}{2}(20)(1.5)^2 - 0.2\int_0^{1.5}\left(70 + 0.5s^2\right) ds = \frac{1}{2}\left(\frac{50}{32.2}\right)v_2^2$$

$$0 + 52.936 - 22.5 - 21.1125 = 0.7764\, v_2^2$$

$$v_2 = 3.465 \text{ ft/s}$$

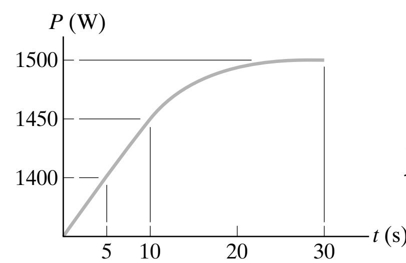
When $s = 1.5$ ft,

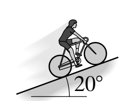
$$F = 40 + (1.5)^2 = 42.25 \text{ lb}$$

$$P = \mathbf{F}\cdot\mathbf{v} = (42.25\cos 30°)(3.465)$$

$$P = 126.79 \text{ ft}\cdot\text{lb/s} = 0.231 \text{ hp}$$ **Ans.**

•14–69. Using the biomechanical power curve shown, determine the maximum speed attained by the rider and his bicycle, which have a total mass of 92 kg, as the rider ascends the 20° slope starting from rest.

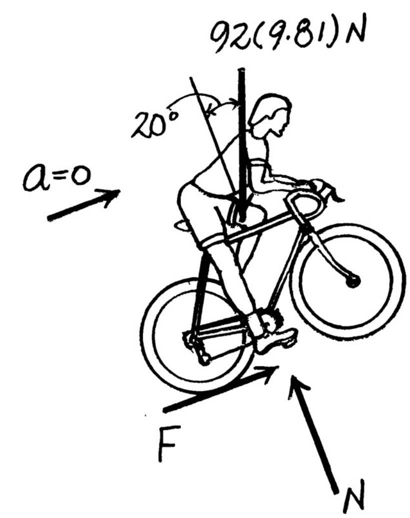




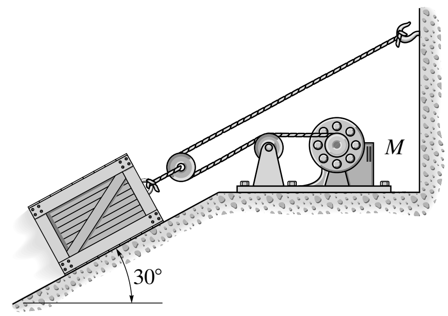
$$F = 92(9.81)\sin 20° = 308.68 \text{ N}$$

$$P = \mathbf{F}\cdot\mathbf{v}; \qquad 1500 = 308.68\,v$$

$$v = 4.86 \text{ m/s}$$ **Ans.**

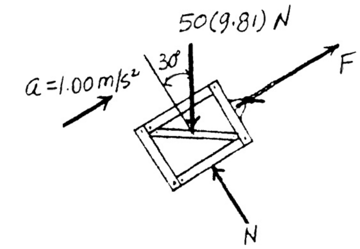


14–70. The 50-kg crate is hoisted up the 30° incline by the pulley system and motor M. If the crate starts from rest and, by constant acceleration, attains a speed of 4 m/s after traveling 8 m along the plane, determine the power that must be supplied to the motor at the instant the crate has moved 8 m. Neglect friction along the plane. The motor has an efficiency of $\epsilon = 0.74$.



Kinematics: Applying equation $v^2 = v_0^2 + 2a_c(s - s_0)$, we have

$$4^2 = 0^2 + 2a(8 - 0) \qquad a = 1.00 \text{ m/s}^2$$



Equations of Motion:

$$+\Sigma F_{x'} = ma_{x'}; \qquad F - 50(9.81)\sin 30° = 50(1.00) \qquad F = 295.25 \text{ N}$$

Power: The power output at the instant when $v = 4$ m/s can be obtained using Eq. 14–10.

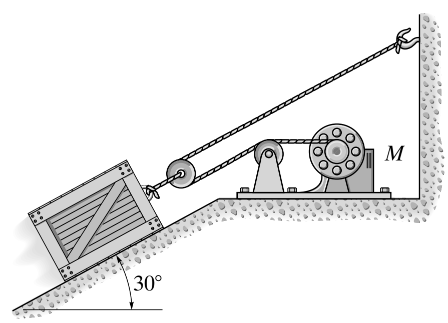
$$P = \mathbf{F} \cdot \mathbf{v} = 295.25\,(4) = 1181 \text{ W} = 1.181 \text{ kW}$$

Using Eq. 14–11, the required power input to the motor in order to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

$$= \frac{1.181}{0.74} = 1.60 \text{ kW} \qquad \textbf{Ans.}$$

14–71. Solve Prob. 14–70 if the coefficient of kinetic friction between the plane and the crate is $\mu_k = 0.3$.



Kinematics: Applying equation $v^2 = v_0^2 + 2a_c(s - s_0)$, we have

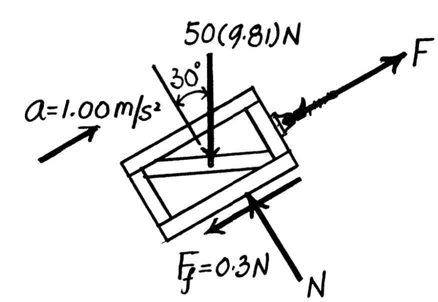
$$4^2 = 0^2 + 2a(8 - 0) \qquad a = 1.00 \text{ m/s}^2$$

Equations of Motion:

$$+\Sigma F_{y'} = ma_{y'}; \qquad N - 50(9.81)\cos 30° = 50(0) \qquad N = 424.79 \text{ N}$$

$$+\Sigma F_{x'} = ma_{x'}; \qquad F - 0.3\,(424.79) - 50(9.81)\sin 30° = 50(1.00)$$

$$F = 422.69 \text{ N}$$



Power: The power output at the instant when $v = 4$ m/s can be obtained using Eq. 14–10.

$$P = \mathbf{F} \cdot \mathbf{v} = 422.69\,(4) = 1690.74 \text{ W} = 1.691 \text{ kW}$$

Using Eq. 14–11, the required power input to the motor to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

$$= \frac{1.691}{0.74} = 2.28 \text{ kW} \qquad \textbf{Ans.}$$

***14–72.** Solve Prob. 14–12 using the conservation of energy equation.

Put Datum at center of block at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 10(60 + y) = 0 + \left[\frac{1}{2}(30)y^2\right] + \left[\frac{1}{2}(45)(y - 3)^2\right] = 0$$

$$37.5y^2 - 145y - 397.5 = 0$$

Solving for the positive root of the above equation,

$$y = 5.720 \text{ in.}$$

Thus,

$s_1 = 5.72$ in. $\quad s_2 = 5.720 - 3 = 2.72$ in. **Ans.**

•14–73. Solve Prob. 14–7 using the conservation of energy equation.

Datum at B:

$$T_A + V_A = T_B + V_B$$

$$0 + 6(2) = 0 + \frac{1}{2}(5)(12)(x)^2$$

$x = 0.6325$ ft $= 7.59$ in. **Ans.**

14–74. Solve Prob. 14–8 using the conservation of energy equation.

The spring has an initial and final compression of $s_1 = 0.1 - 0.05 = 0.05$ m and $s_2 = 0.1 - (0.05 + 0.0125) = 0.0375$ m.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \left[\frac{1}{2}k{s_1}^2\right] + \left[-Wh\right] = \frac{1}{2}m{v_A}^2 + \frac{1}{2}k{s_2}^2 + 0$$

$$0 + \left[\frac{1}{2}(2000)(0.05)^2\right] = \frac{1}{2}(0.02){v_A}^2 + \frac{1}{2}(2000)(0.0375^2)$$

$v_A = 10.5$ m/s **Ans.**

14–75. Solve Prob. 14–18 using the conservation of energy equation.

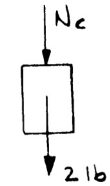
$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(100)(0.5)^2 + \frac{1}{2}(50)(0.5)^2 = \frac{1}{2}(20)v^2 + 0$$

$v = 1.37$ m/s **Ans.**

***14–76.** Solve Prob. 14–22 using the conservation of energy equation.

Datum at A



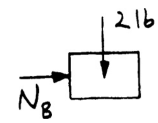
$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(30)^2 + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2 + 2(5)$$

$v_B = 24.042 = 24.0$ ft/s **Ans.**

$$\overset{+}{\rightarrow} \Sigma F_n = ma_n; \qquad N_B = \left(\frac{2}{32.2}\right)\left(\frac{(24.042)^2}{5}\right)$$

$N_B = 7.18$ lb **Ans.**



$$T_A + V_A = T_C + V_C$$

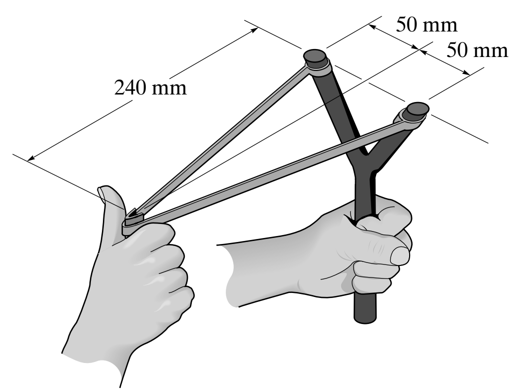
$$\frac{1}{2}\left(\frac{2}{32.2}\right)(30)^2 + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_C^2 + 2(10)$$

$v_C = 16.0$ ft/s **Ans.**

$$+\downarrow \Sigma F_n = ma_n; \qquad N_C + 2 = \left(\frac{2}{32.2}\right)\left(\frac{(16.0)^2}{5}\right)$$

$N_C = 1.18$ lb **Ans.**

•14–77. Each of the two elastic rubber bands of the slingshot has an unstretched length of 200 mm. If they are pulled back to the position shown and released from rest, determine the speed of the 25-g pellet just after the rubber bands become unstretched. Neglect the mass of the rubber bands. Each rubber band has a stiffness of $k = 50$ N/m.

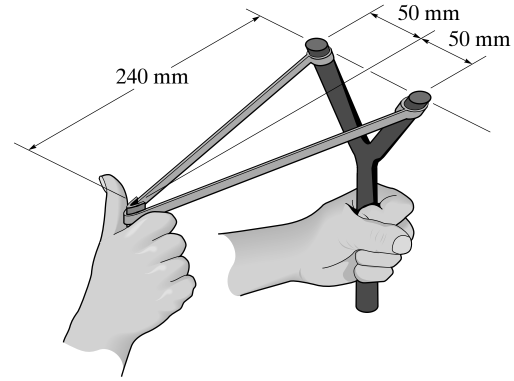


$$T_1 + V_1 = T_2 + V_2$$

$$0 + (2)\left(\frac{1}{2}\right)(50)[\sqrt{(0.05)^2 + (0.240)^2} - 0.2]^2 = \frac{1}{2}(0.025)v^2$$

$v = 2.86$ m/s **Ans.**

14–78. Each of the two elastic rubber bands of the slingshot has an unstretched length of 200 mm. If they are pulled back to the position shown and released from rest, determine the maximum height the 25-g pellet will reach if it is fired vertically upward. Neglect the mass of the rubber bands and the change in elevation of the pellet while it is constrained by the rubber bands. Each rubber band has a stiffness $k = 50$ N/m.

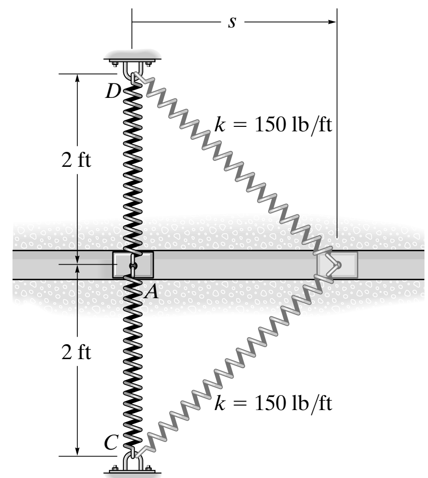


$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left(\frac{1}{2}\right)(50)[\sqrt{(0.05)^2 + (0.240)^2} - 0.2]^2 = 0 + 0.025(9.81)h$$

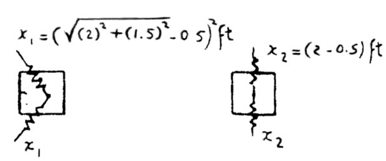
$h = 0.416 \text{ m} = 416 \text{ mm}$ **Ans.**

14–79. Block A has a weight of 1.5 lb and slides in the smooth horizontal slot. If the block is drawn back to $s = 1.5$ ft and released from rest, determine its speed at the instant $s = 0$. Each of the two springs has a stiffness of $k = 150$ lb/ft and an unstretched length of 0.5 ft.



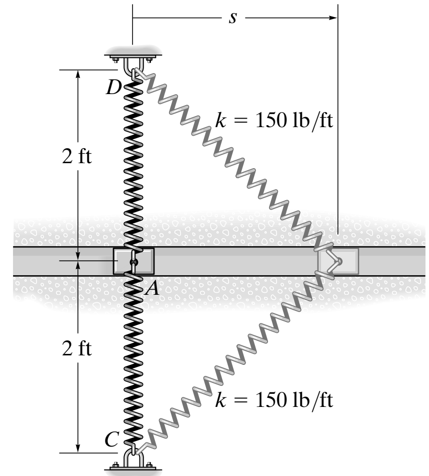
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}(150)\left(\sqrt{(2)^2 + (1.5)^2} - 0.5\right)^2\right] = \frac{1}{2}\left(\frac{1.5}{32.2}\right)(v_2)^2 + 2\left[\frac{1}{2}(150)(2 - 0.5)^2\right]$$



$v_2 = 106$ ft/s **Ans.**

***14–80.** The 2-lb block A slides in the smooth horizontal slot. When $s = 0$ the block is given an initial velocity of 60 ft/s to the right. Determine the maximum horizontal displacement s of the block. Each of the two springs has a stiffness of $k = 150$ lb/ft and an unstretched length of 0.5 ft.



$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(60)^2 + 2\left[\frac{1}{2}(150)(2 - 0.5)^2\right] = 0 + 2\left[\frac{1}{2}(150)\left(\sqrt{(2)^2 + s^2} - 0.5\right)^2\right]$$

Set $d = \sqrt{(2)^2 + s^2}$ then

$$d^2 - d - 2.745 = 0$$

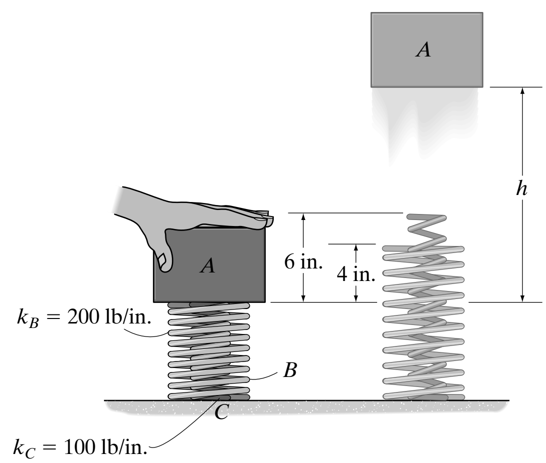
Solving for the positive root,

$$d = 2.231$$

$$(2.231)^2 = (2)^2 + s^2$$

$s = 0.988$ ft **Ans.**

•14–81. The 30-lb block A is placed on top of two nested springs B and C and then pushed down to the position shown. If it is then released, determine the maximum height h to which it will rise.



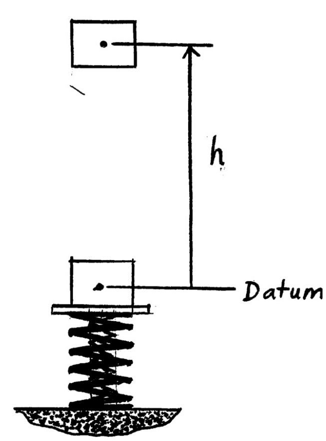
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}mv_2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

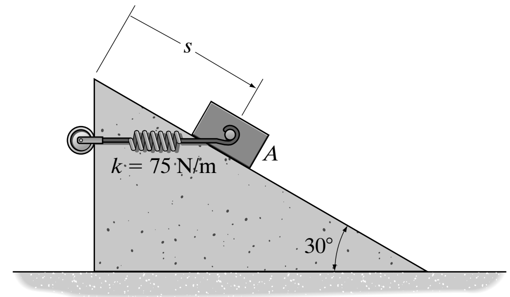
$$0 + 0 + \frac{1}{2}(200)(4)^2 + \frac{1}{2}(100)(6)^2 = 0 + h(30) + 0$$

$h = 113$ in. **Ans.**



(a)

14–82. The spring is unstretched when $s = 1\text{ m}$ and the 15-kg block is released from rest at this position. Determine the speed of the block when $s = 3\text{ m}$. The spring remains horizontal during the motion, and the contact surfaces between the block and the inclined plane are smooth.



Potential Energy: With reference to the datum set in Fig. *a,* the gravitational potential energy of the block at positions (1) and (2) are $\left(V_g\right)_1 = mgh_1 = 15(9.81)(0) = 0$ and $\left(V_g\right)_2 = mgh_2 = 15(9.81)\left[-2\sin 30°\right] = -147.15\text{ J}$. When the block is at position (1) the spring is unstretched. Thus, the elastic potential energy of the spring at this instant is $\left(V_e\right)_1 = \frac{1}{2}ks_1{}^2 = 0$. The spring is stretched $s_2 = 2\cos 30°$ m when the block is at position (2). Thus, $\left(V_e\right)_2 = \frac{1}{2}ks_2{}^2 = \frac{1}{2}(75)(2\cos 30°)^2 = 112.5\text{ J}$ since it is being stretched $s_2 = x$.

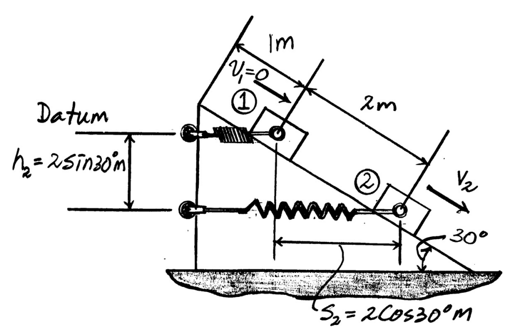
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1{}^2 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}mv_2{}^2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

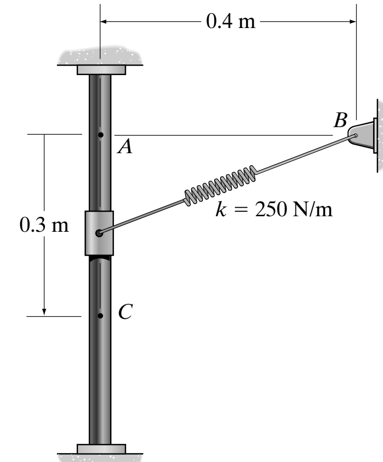
$$0 + \left(0 + 0\right) = \frac{1}{2}(15)v_2{}^2 + \left[-147.15 + 112.5\right]$$

$$v_2 = 2.15\text{ m/s}$$ **Ans.**



(a)

14–83. The vertical guide is smooth and the 5-kg collar is released from rest at A. Determine the speed of the collar when it is at position C. The spring has an unstretched length of 300 mm.



Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the collar at positions A and C are $(V_g)_A = mgh_A = 5(9.81)(0) = 0$ and $(V_g)_C = mgh_C = 5(9.81)(-0.3) = -14.715$ J. When the collar is at positions A and C, the spring stretches $s_A = 0.4 - 0.3 = 0.1$ m and $s_C = \sqrt{0.4^2 + 0.3^2} - 0.3 = 0.2$ m. The elastic potential energy of the spring when the collar is at these two positions are $(V_e)_A = \frac{1}{2}ks_A{}^2 = \frac{1}{2}(250)(0.1^2) = 1.25$ J and $(V_e)_C = \frac{1}{2}ks_C{}^2 = \frac{1}{2}(250)(0.2^2) = 5$ J.

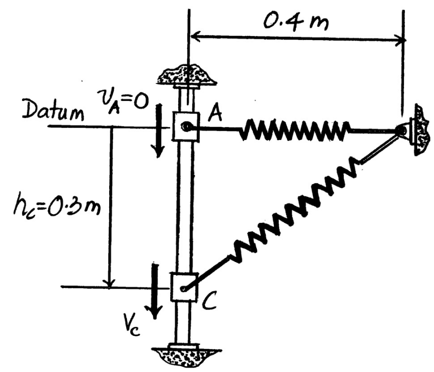
Conservation of Energy:

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}mv_A{}^2 + \left[(V_g)_A + (V_e)_A\right] = \frac{1}{2}mv_C{}^2 + \left[(V_g)_C + (V_e)_C\right]$$

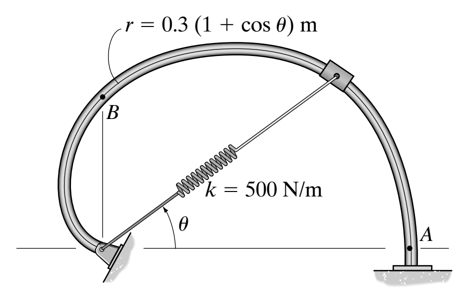
$$0 + (0 + 1.25) = \frac{1}{2}(5)v_C{}^2 + (-14.715 + 5)$$

$$v_C = 2.09 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

***14–84.** The 5-kg collar slides along the smooth vertical rod. If the collar is nudged from rest at A, determine its speed when it passes point B. The spring has an unstretched length of 200 mm.



Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the collar at positions A and B are $(V_g)_A = mgh_A = 5(9.81)(0) = 0$ and $(V_g)_B = mgh_B = 5(9.81)(0.3) = 14.715$ J. The spring stretches $s_A = 0.6 - 0.2 = 0.4$ m and $s_B = 0.3 - 0.2 = 0.1$ m when the collar is at positions A and B, respectively. Thus, the elastic potential energy of the spring when the collar is at these two positions are $(V_e)_A = \frac{1}{2}ks_A{}^2 = \frac{1}{2}(500)(0.4^2) = 40$ J and $(V_e)_B = \frac{1}{2}ks_B{}^2 = \frac{1}{2}(500)(0.1^2) = 2.5$ J.

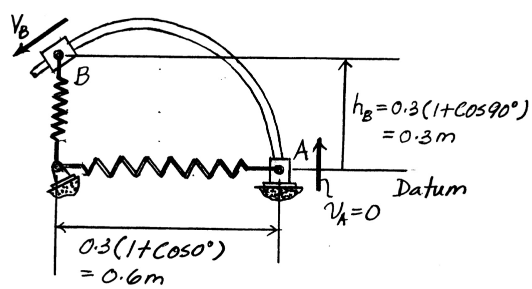
Conservation of Energy:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A{}^2 + \left[\left(V_g\right)_A + (V_e)_A\right] = \frac{1}{2}mv_B{}^2 + \left[\left(V_g\right)_B + (V_e)_B\right]$$

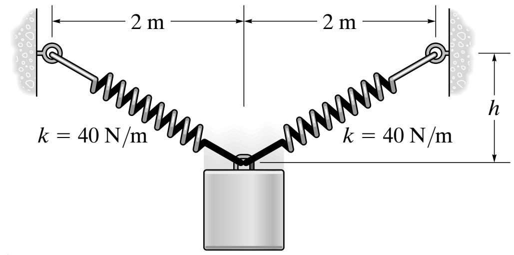
$$0 + (0 + 40) = \frac{1}{2}(5)v_B{}^2 + (14.715 + 2.5)$$

$$v_B = 3.02 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

•14–85. The cylinder has a mass of 20 kg and is released from rest when $h = 0$. Determine its speed when $h = 3$ m. The springs each have an unstretched length of 2 m.



Potential Energy: Datum is set at the cylinder position when $h = 0$. When the cylinder moves to a position $h = 3$ m *below* the datum, its gravitational potential energy at this position is $20(9.81)(-3) = -588.6$ J. The initial and final elastic potential energy are $2\left[\frac{1}{2}(40)(2-2)^2\right] = 0$ and $2\left[\frac{1}{2}(40)\left(\sqrt{2^2+3^2}-2\right)^2\right] = 103.11$ J, respectively.

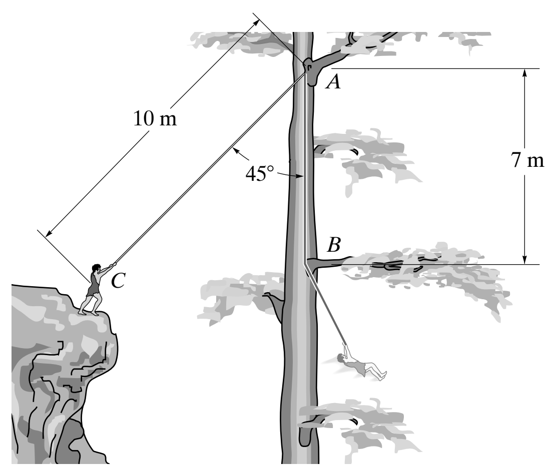
Conservation of Energy:

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$0 + 0 = \frac{1}{2}(20)\,v^2 + 103.11 + (-588.6)$$

$$v = 6.97 \text{ m/s} \qquad \textbf{Ans.}$$

14–86. Tarzan has a mass of 100 kg and from rest swings from the cliff by rigidly holding on to the tree vine, which is 10 m measured from the supporting limb A to his center of mass. Determine his speed just after the vine strikes the lower limb at B. Also, with what force must he hold on to the vine just before and just after the vine contacts the limb at B?

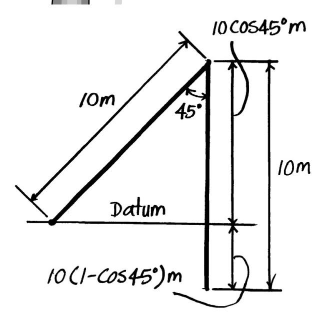


Datum at C

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}(100)(v_C)^2 - 100(9.81)(10)(1 - \cos 45°)$$

$$v_C = 7.581 = 7.58 \text{ m/s} \qquad \textbf{Ans.}$$



Just before striking B, $\rho = 10$ m:

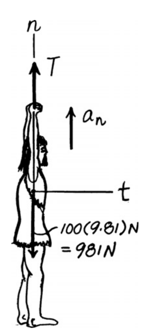
$$+\uparrow \Sigma F_n = m a_n; \qquad T - 981 = 100\left(\frac{(7.581)^2}{10}\right)$$

$$T = 1.56 \text{ kN} \qquad \textbf{Ans.}$$

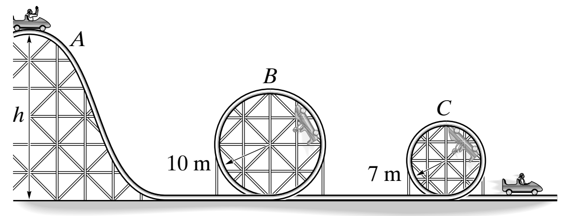
Just after striking B, $\rho = 3$ m:

$$+\uparrow \Sigma F_n = m a_n; \qquad T - 981 = 100\left(\frac{(7.581)^2}{3}\right)$$

$$T = 2.90 \text{ kN} \qquad \textbf{Ans.}$$

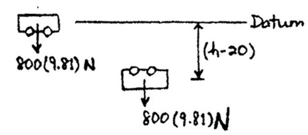


14–87. The roller-coaster car has a mass of 800 kg, including its passenger, and starts from the top of the hill A with a speed $v_A = 3$ m/s. Determine the minimum height h of the hill so that the car travels around both inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and at C?



$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(800)(3)^2 + 0 = \frac{1}{2}(800)(v_B^2) - 800(9.81)(h - 20)$$

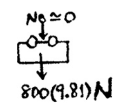


$$+\downarrow \Sigma F_n = m\, a_n; \qquad 800(9.81) = 800\left(\frac{v_B^2}{10}\right)$$

Thus,

$$v_B = 9.90 \text{ m/s}$$

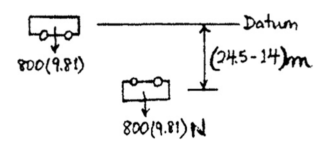
$$h = 24.5 \text{ m} \qquad \textbf{Ans.}$$



At B: $\qquad N_B = 0 \qquad$ (For h to be minimum.) **Ans.**

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}(800)(3)^2 + 0 = \frac{1}{2}(800)(v_C)^2 - 800(9.81)(24.5 - 14)$$

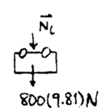


$$v_C = 14.69 \text{ m/s}$$

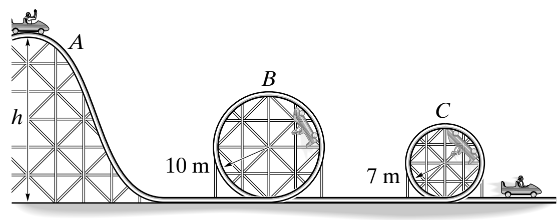
$$a_n = \frac{14.69^2}{7}$$

$$+\downarrow \Sigma F_n = m\, a_n; \qquad N_C + 800(9.81) = 800\left(\frac{14.69^2}{7}\right)$$

$$N_C = 16.8 \text{ kN} \qquad \textbf{Ans.}$$



***14–88.** The roller-coaster car has a mass of 800 kg, including its passenger. If it is released from rest at the top of the hill A, determine the minimum height h of the hill so that the car travels around both inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and at C?

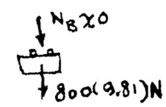


Since friction is neglected, the car will travel around the 7-m loop provided it first travels around the 10-m loop.

$$T_A + V_A = T_B + V_B$$

$$0 + 0 = \frac{1}{2}(800)(v_B^2) - 800(9.81)(h - 20)$$

$$+\downarrow \Sigma F_n = m\,a_n; \qquad 800(9.81) = 800\left(\frac{v_B^2}{10}\right)$$



Thus,

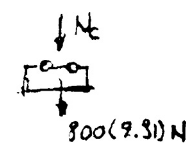
$$v_B = 9.90 \text{ m/s}$$

$$h = 25.0 \text{ m} \qquad \textbf{Ans.}$$

At B: $\qquad N_B = 0 \qquad$ (For h to be minimum.) **Ans.**

$$T_A + V_A = T_C + V_C$$

$$0 + 0 = \frac{1}{2}(800)(v_C)^2 - 800(9.81)(25 - 14)$$

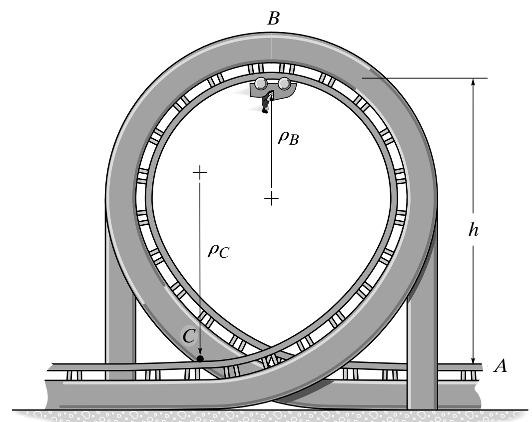


$$v_C = 14.69 \text{ m/s}$$

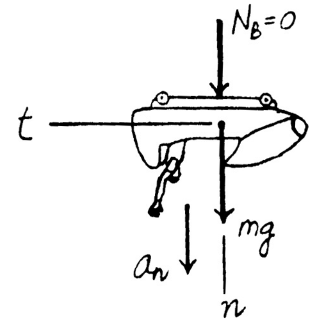
$$+\downarrow \Sigma F_n = m\,a_n; \qquad N_C + 800(9.81) = 800\left(\frac{(14.69)^2}{7}\right)$$

$$N_C = 16.8 \text{ kN} \qquad \textbf{Ans.}$$

•14–89. The roller coaster and its passenger have a total mass m. Determine the smallest velocity it must have when it enters the loop at A so that it can complete the loop and not leave the track. Also, determine the normal force the tracks exert on the car when it comes around to the bottom at C. The radius of curvature of the tracks at B is ρ_B, and at C it is ρ_C. Neglect the size of the car. Points A and C are at the same elevation.



Equations of Motion: In order for the roller coaster to just pass point B without falling off the track, it is required that $N_B = 0$. Applying Eq. 13–8, we have



$$\Sigma F_n = ma_n; \qquad mg = m\left(\frac{v_B^2}{\rho_B}\right) \qquad v_B^2 = \rho_B\, g$$

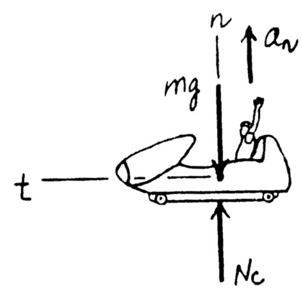
Potential Energy: Datum is set at lowest point A. When the roller coaster is at point B, its position is h *above* the datum. Thus, the gravitational potential energy at this point is mgh.

Conservation of Energy: When the roller coaster travels from A to B, we have

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^2 + 0 = \frac{1}{2}m(\rho_B\, g) + mgh$$

$$v_A = \sqrt{\rho_B\, g + 2gh} \qquad \textbf{Ans.}$$



When the roller coaster travels from A to C, we have

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}m(\rho_B\, g + 2gh) + 0 = \frac{1}{2}mv_C^2 + 0$$

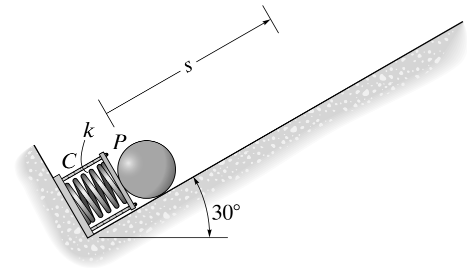
$$v_C^2 = \rho_B\, g + 2gh$$

Equations of Motion:

$$\Sigma F_n = ma_n; \qquad N_C - mg = m\left(\frac{\rho_B\, g + 2gh}{\rho_C}\right)$$

$$N_C = \frac{mg}{\rho_C}(\rho_B + \rho_C + 2h) \qquad \textbf{Ans.}$$

14–90. The 0.5-lb ball is shot from the spring device. The spring has a stiffness $k = 10$ lb/in. and the four cords C and plate P keep the spring compressed 2 in. when no load is on the plate. The plate is pushed back 3 in. from its initial position. If it is then released from rest, determine the speed of the ball when it reaches a position $s = 30$ in. on the smooth inclined plane.



Potential Energy: The datum is set at the lowest point (compressed position). Finally, the ball is $\frac{30}{12}\sin 30° = 1.25$ ft *above* the datum and its gravitational potential energy is $0.5(1.25) = 0.625$ ft·lb. The initial and final elastic potential energy are $\frac{1}{2}(120)\left(\frac{2+3}{12}\right)^2 = 10.42$ ft·lb and $\frac{1}{2}(120)\left(\frac{2}{12}\right)^2 = 1.667$ ft·lb, respectively.

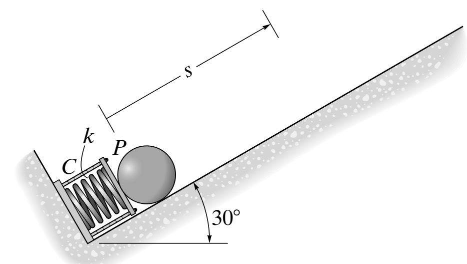
Conservation of Energy:

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$0 + 10.42 = \frac{1}{2}\left(\frac{0.5}{32.2}\right)v^2 + 0.625 + 1.667$$

$$v = 32.3 \text{ ft/s} \qquad \textbf{Ans.}$$

14–91. The 0.5-lb ball is shot from the spring device shown. Determine the smallest stiffness k which is required to shoot the ball a maximum distance $s = 30$ in. up the plane after the spring is pushed back 3 in. and the ball is released from rest. The four cords C and plate P keep the spring compressed 2 in. when no load is on the plate.



Potential Energy: The datum is set at the lowest point (compressed position). Finally, the ball is $\frac{30}{12}\sin 30° = 1.25$ ft *above* the datum and its gravitational potential energy is $0.5(1.25) = 0.625$ ft·lb. The initial and final elastic potential energy are $\frac{1}{2}(k)\left(\frac{2+3}{12}\right)^2 = 0.08681k$ and $\frac{1}{2}(k)\left(\frac{2}{12}\right)^2 = 0.01389k$, respectively.

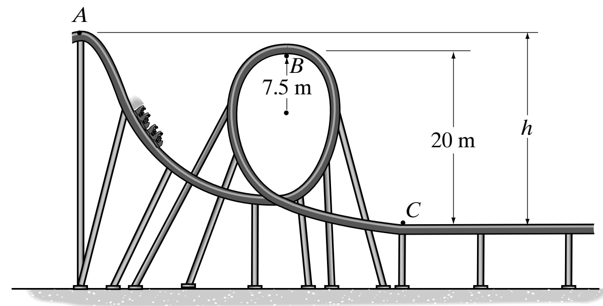
Conservation of Energy:

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$0 + 0.08681k = 0 + 0.625 + 0.01389k$$

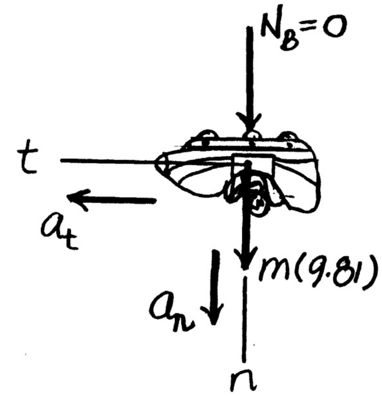
$$k = 8.57 \text{ lb/ft} \qquad \textbf{Ans.}$$

***14–92.** The roller coaster car having a mass m is released from rest at point A. If the track is to be designed so that the car does not leave it at B, determine the required height h. Also, find the speed of the car when it reaches point C. Neglect friction.



Equation of Motion: Since it is required that the roller coaster car is about to leave the track at B, $N_B = 0$. Here, $a_n = \dfrac{v_B^2}{\rho_B} = \dfrac{v_B^2}{7.5}$. By referring to the free-body diagram of the roller coaster car shown in Fig. a,

$$\Sigma F_n = ma_n; \qquad m(9.81) = m\left(\frac{v_B^2}{7.5}\right) \qquad v_B^2 = 73.575 \text{ m}^2/\text{s}^2$$



(a)

Potential Energy: With reference to the datum set in Fig. b, the gravitational potential energy of the rollercoaster car at positions A, B, and C are $(V_g)_A = mgh_A = m(9.81)h = 9.81mh$, $(V_g)_B = mgh_B = m(9.81)(20) = 196.2$ m, and $(V_g)_C = mgh_C = m(9.81)(0) = 0$.

Conservation of Energy: Using the result of v_B^2 and considering the motion of the car from position A to B,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^2 + (V_g)_A = \frac{1}{2}mv_B^2 + (V_g)_B$$

$$0 + 9.81mh = \frac{1}{2}m(73.575) + 196.2m$$

$$h = 23.75 \text{ m}$$ **Ans.**

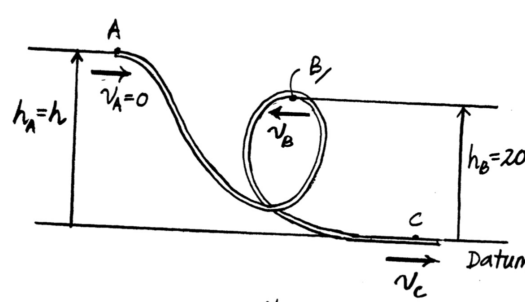
Also, considering the motion of the car from position B to C,

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}mv_B^2 + (V_g)_B = \frac{1}{2}mv_C^2 + (V_g)_C$$

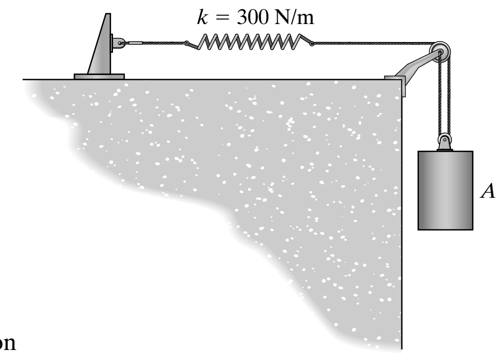
$$\frac{1}{2}m(73.575) + 196.2m = \frac{1}{2}mv_C^2 + 0$$

$$v_C = 21.6 \text{ m/s}$$ **Ans.**



(b)

•14–93. When the 50-kg cylinder is released from rest, the spring is subjected to a tension of 60 N. Determine the speed of the cylinder after it has fallen 200 mm. How far has it fallen when it momentarily stops?

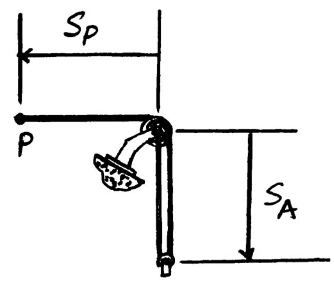


Kinematics: We can express the length of the cord in terms of the position coordinates s_A and s_P. By referring to Fig. *a*,

$$s_P + 2s_A = l$$

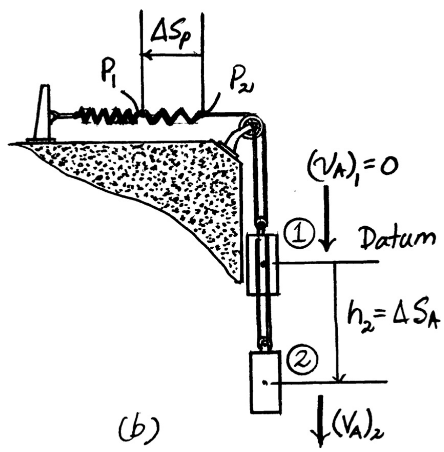
Thus,

$$\Delta s_P + 2\Delta s_A = 0 \qquad \textbf{(1)}$$



(a)

Potential Energy: By referring to the datum set in Fig. *b*, the gravitational potential energy of the cylinder at positions (1) and (2) are $(V_g)_1 = mgh_1 = 50(9.81)(0) = 0$ and $(V_g)_2 = mgh_2 = 50(9.81)(-\Delta s_A) = -490.5\Delta s_A$. When the cylinder is at positions (1) and (2), the stretch of the springs are $s_1 = \dfrac{F}{k} = \dfrac{60}{300} = 0.2$ m and $s_2 = s_1 + \Delta s_P = (0.2 + \Delta s_P)$ m. Thus, the elastic potential energy of the spring at these two instances are $(V_e)_1 = \dfrac{1}{2}ks_1^{\,2} = \dfrac{1}{2}(300)(0.2^2) = 6$ J and $(V_e)_2 = \dfrac{1}{2}ks_2^{\,2} = \dfrac{1}{2}(300)(0.2 + \Delta s_P)^2 = 150(0.2 + \Delta s_P)^2$.



Conservation of Energy: For the case when $\Delta s_A = 0.2$ m, from Eq. (1), we obtain $\Delta s_P + 2(0.2) = 0$ or $\Delta s_P = -0.4 \text{ m} = 0.4 \text{ m} \rightarrow$. We have

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m_A(v_A)_1^{\,2} + \left[\left(V_g\right)_1 + (V_e)_1\right] = \frac{1}{2}m_A(v_A)_2^{\,2} + \left[\left(V_g\right)_2 + (V_e)_2\right]$$

$$0 + \left(0 + 6\right) = \frac{1}{2}(50)(v_A)_2^{\,2} + \left[-490.5(0.2) + 150\left(0.2 + 0.4\right)^2\right]$$

$$(v_A)_2 = 1.42 \text{ m/s} \qquad \textbf{Ans.}$$

For the case when the cylinder momentarily stops at position (2), from Eq. (1), $\Delta s_P = |-2\Delta s_A|$ Also, $(v_A)_2 = 0$.

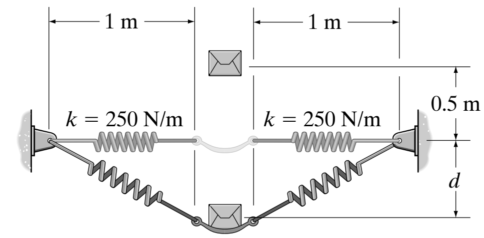
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m_A(v_A)_1^{\,2} + \left[\left(V_g\right)_1 + (V_e)_1\right] = \frac{1}{2}m_A(v_A)_2^{\,2} + \left[\left(V_g\right)_2 + (V_e)_2\right]$$

$$0 + \left(0 + 6\right) = 0 + \left[(-490.5\Delta s_A) + 150\left(0.2 + 2\Delta s_A\right)^2\right]$$

$$\Delta s_A = 0.6175 \text{ m} = 617.5 \text{ mm} \qquad \textbf{Ans.}$$

14–94. A pan of negligible mass is attached to two identical springs of stiffness $k = 250$ N/m. If a 10-kg box is dropped from a height of 0.5 m above the pan, determine the maximum vertical displacement d. Initially each spring has a tension of 50 N.



Potential Energy: With reference to the datum set in Fig. a, the gravitational potential energy of the box at positions (1) and (2) are $(V_g)_1 = mgh_1 = 10(9.81)(0) = 0$ and $(V_g)_2 = mgh_2 = 10(9.81)[-(0.5 + d)] = -98.1(0.5 + d)$. Initially, the spring stretches $s_1 = \frac{50}{250} = 0.2$ m. Thus, the unstretched length of the spring is $l_0 = 1 - 0.2 = 0.8$ m and the initial elastic potential of each spring is $(V_e)_1 = (2)\frac{1}{2}ks_1{}^2 = 2(250/2)(0.2^2) = 10$ J. When the box is at position (2), the spring stretches $s_2 = \left(\sqrt{d^2 + 1^2} - 0.8\right)$ m. The elastic potential energy of the springs when the box is at this position is

$$(V_e)_2 = (2)\frac{1}{2}ks_2{}^2 = 2(250/2)\left[\sqrt{d^2+1} - 0.8\right]^2 = 250\left(d^2 - 1.6\sqrt{d^2+1} + 1.64\right).$$

Conservation of Energy:

$$T_1 + V_1 + T_2 + V_2$$

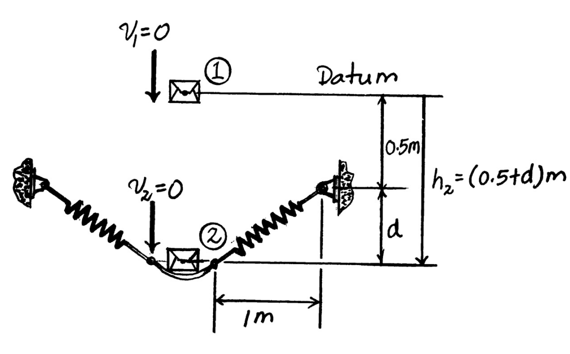
$$\frac{1}{2}mv_1{}^2 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}mv_2{}^2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + (0 + 10) = 0 + \left[-98.1(0.5 + d) + 250\left(d^2 - 1.6\sqrt{d^2+1} + 1.64\right)\right]$$

$$250d^2 - 98.1d - 400\sqrt{d^2+1} + 350.95 = 0$$

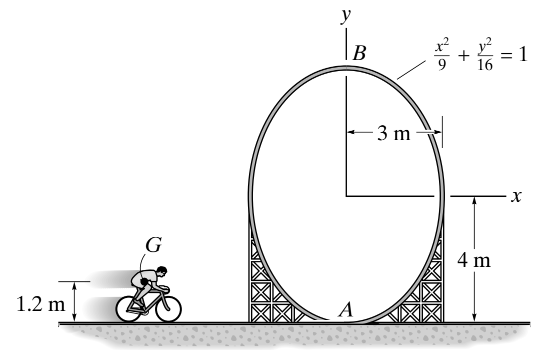
Solving the above equation by trial and error,

$$d = 1.34 \text{ m}$$ **Ans.**



(a)

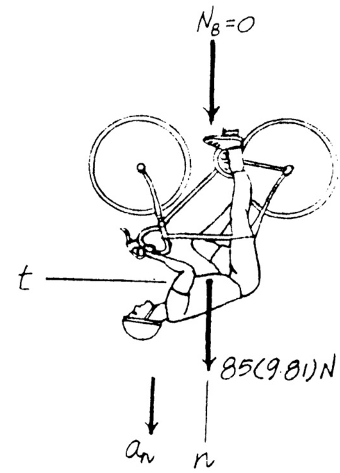
14–95. The man on the bicycle attempts to coast around the ellipsoidal loop without falling off the track. Determine the speed he must maintain at A just before entering the loop in order to perform the stunt. The bicycle and man have a total mass of 85 kg and a center of mass at G. Neglect the mass of the wheels.



Geometry: Here, $y = \frac{4}{3}\sqrt{9 - x^2}$.

$$\frac{dy}{dx} = -\frac{4}{3}x\left(9 - x^2\right)^{-1/2}\Big|_{x=0} = 0$$

$$\frac{d^2y}{dx^2} = -\frac{4}{3}\left[x^2\left(9 - x^2\right)^{-3/2} + (9 - x)^{-1/2}\right]\Bigg|_{x=0} = -0.4444$$



The slope angle θ at point B is given by

$$\tan\theta = \frac{dy}{dx}\Big|_{x=0} = 0 \qquad \theta = 0°$$

and the radius of curvature at point B is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \frac{\left[1 + 0^2\right]^{3/2}}{|-0.4444|}\Bigg| = 2.25 \text{ m}$$

Since the center of mass for the cyclist is 1.2 m off the track, the radius of curvature for the cyclist is

$$\rho' = \rho - 1.2 = 1.05 \text{ m}$$

Equations of Motion: In order for the cyclist to just pass point B without falling off the track, it is required that $N_B = 0$. Applying Eq. 13–8 with $\theta = 0°$ and $\rho = 1.05$ m, we have

$$\Sigma F_n = ma_n; \qquad 85(9.81) = 85\left(\frac{v_B^2}{1.05}\right) \qquad v_B^2 = 10.30 \text{ m}^2/\text{s}^2$$

Potential Energy: Datum is set at the center of mass of the cyclist before he enters the track. When the cyclist is at point B, his position is $(8 - 1.2 - 1.2) = 5.6$ m *above* the datum. Thus, his gravitational potential energy at this point is $85(9.81)(5.6) = 4669.56$ J.

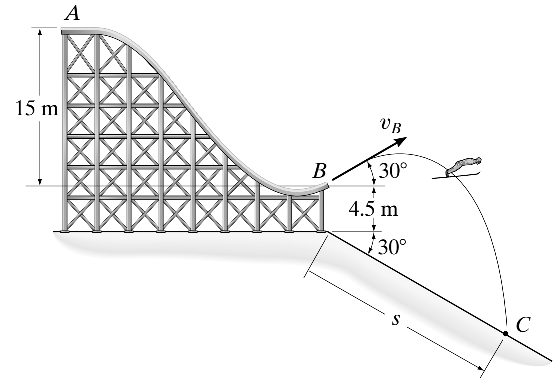
Conservation of Energy:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(85)v_A^2 + 0 = \frac{1}{2}(85)(10.30) + 4669.56$$

$$v_A = 11.0 \text{ m/s} \qquad \textbf{Ans.}$$

***14–96.** The 65-kg skier starts from rest at A. Determine his speed at B and the distance s where he lands at C. Neglect friction.



Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the skier at positions A and B are $\left(V_g\right)_A = mgh_A = 65(9.81)(0) = 0$ and $\left(V_g\right)_B = mgh_B = 65(9.81)(-15) = -9564.75$ J.

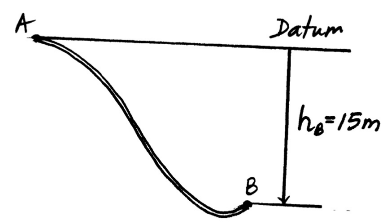
Conservation of Energy:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A{}^2 + \left(V_g\right)_A = \frac{1}{2}mv_B{}^2 + \left(V_g\right)_B$$

$$0 + 0 = \frac{1}{2}(65)v_B{}^2 + \left(-9564.75\right)$$

$$v_B = 17.16 \text{ m/s} = 17.2 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

Kinematics: By considering the x-motion of the skier, Fig. *b*,

$$\left(\xrightarrow{+}\right) \qquad s_x = \left(s_B\right)_x + \left(v_B\right)_x t$$

$$s \cos 30^\circ = 0 + 17.16 \cos 30^\circ(t)$$

$$s = 17.16t \qquad \textbf{(1)}$$

By considering the y-motion of the skier, Fig. *a*,

$$\left(+\uparrow\right) \qquad s_y = \left(s_B\right)_y + \left(v_B\right)_y t + \frac{1}{2}a_y t^2$$

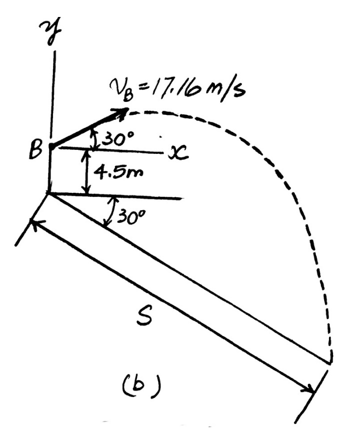
$$-\left(4.5 + s \sin 30^\circ\right) = 0 + 17.16 \sin 30^\circ\, t + \frac{1}{2}\left(-9.81\right)t^2$$

$$0 = 4.5 + 0.5s + 8.5776t - 4.905t^2 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

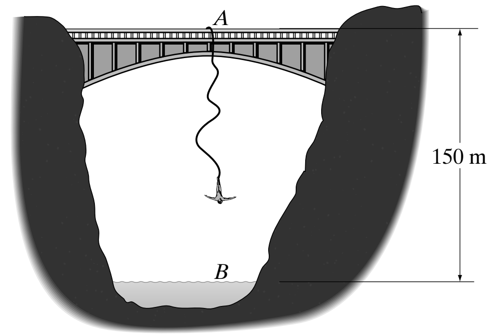
$$s = 64.2 \text{ m} \qquad \textbf{Ans.}$$

$$t = 3.743 \text{ s}$$



(b)

•14–97. The 75-kg man bungee jumps off the bridge at A with an initial downward speed of 1.5 m/s. Determine the required unstretched length of the elastic cord to which he is attached in order that he stops momentarily just above the surface of the water. The stiffness of the elastic cord is $k = 3$ kN/m. Neglect the size of the man.



Potential Energy: With reference to the datum set at the surface of the water, the gravitational potential energy of the man at positions A and B are $\left(V_g\right)_A = mgh_A = 75(9.81)(150) = 110362.5$ J and $\left(V_g\right)_B = mgh_B = 75(9.81)(0) = 0$. When the man is at position A, the elastic cord is unstretched ($s_A = 0$), whereas the elastic cord stretches $s_B = \left(150 - l_0\right)$ m, where l_0 is the unstretched length of the cord. Thus, the elastic potential energy of the elastic cord when the man is at these two positions are $\left(V_e\right)_A = \frac{1}{2}ks_A{}^2 = 0$ and $\left(V_e\right)_B = \frac{1}{2}ks_B{}^2 = \frac{1}{2}(3000)(150 - l_0)^2 = 1500(150 - l_0)^2$.

Conservation of Energy:

$$T_A + V_A = T_B + V_B$$

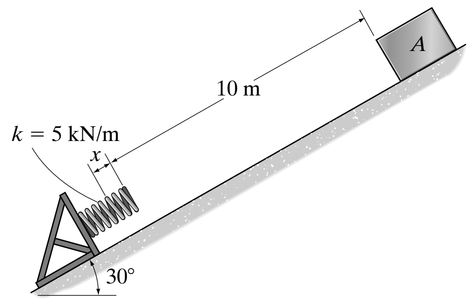
$$\frac{1}{2}mv_A{}^2 + \left[\left(V_g\right)_A + \left(V_e\right)_A\right] = \frac{1}{2}mv_B{}^2 + \left[\left(V_g\right)_B + \left(V_e\right)_B\right]$$

$$\frac{1}{2}(75)(1.5^2) + \left(110362.5 + 0\right) = 0 + \left[0 + 1500(150 - l_0)^2\right]$$

Ans.

$$l_0 = 141 \text{ m}$$

14–98. The 10-kg block A is released from rest and slides down the smooth plane. Determine the compression x of the spring when the block momentarily stops.



Potential Energy: With reference to the datum set in Fig. a, the gravitational potential energy of the block at positions (1) and (2) are $(V_g)_1 = mgh_1 = 10(9.81)(0) = 0$ and $(V_g)_2 = mgh_2 = 10(9.81)\left[-(10+x)\sin 30°\right] = -49.05(10+x)$, respectively. The spring is unstretched initially, thus the initial elastic potential energy of the spring is $(V_e)_1 = 0$. The final elastic energy of the spring is $(V_e)_2 = \frac{1}{2}ks_2{}^2 = \frac{1}{2}(5)(10^3)x^2$ since it is being compressed $s_2 = x$.

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

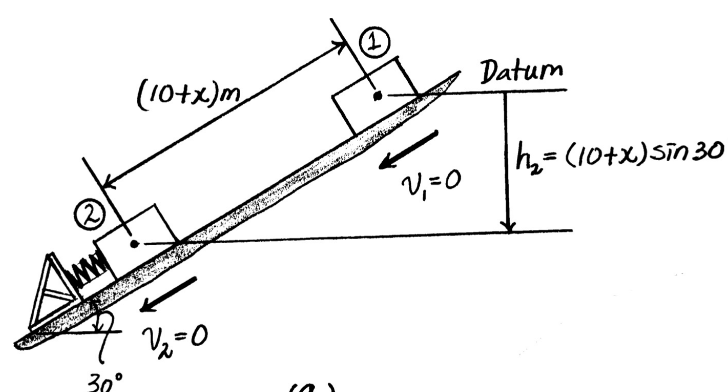
$$\frac{1}{2}mv_1{}^2 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}mv_2{}^2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + (0+0) = 0 + \left[-49.05(10+x)\right] + \frac{1}{2}(5)(10^3)x^2$$

$$2500x^2 - 49.05x - 490.5 = 0$$

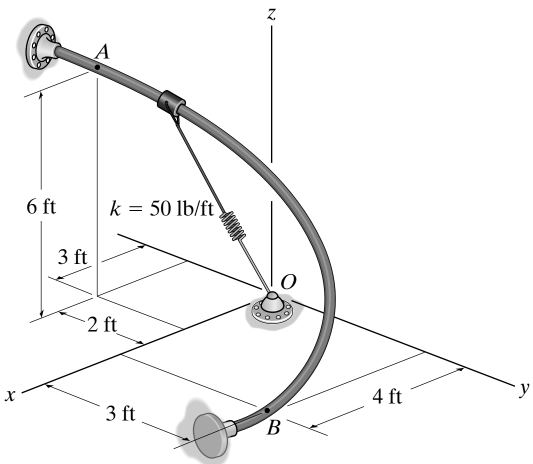
Solving for the positive root of the above equation,

$$x = 0.4529 \text{ m} = 453 \text{ mm} \qquad \textbf{Ans.}$$



(a)

14–99. The 20-lb smooth collar is attached to the spring that has an unstretched length of 4 ft. If it is released from rest at position A, determine its speed when it reaches point B.



Potential Energy: With reference to the datum set at the x–y plane, the gravitational potential energy of the collar at positions A and B are $(V_g)_A = Wh_A = 20(6) = 120 \text{ ft}\cdot\text{lb}$ and $(V_g)_B = Wh_B = 20(0) = 0$. The stretch of the spring when the collar is at positions A and B are $s_A = OA - l_0 = \sqrt{(3-0)^2 + (-2-0)^2 + (6-0)^2} - 4 = 3 \text{ ft}$ and $s_B = OB - l_0 = \sqrt{(4-0)^2 + (3-0)^2} - 4 = 1 \text{ ft}$. Thus, the elastic potential energy of the spring when the collar is at these two positions are $(V_e)_A = \frac{1}{2}ks_A{}^2 = \frac{1}{2}(50)(3^2) = 225 \text{ ft}\cdot\text{lb}$ and $(V_e)_B = \frac{1}{2}ks B^2 = \frac{1}{2}(50)(1^2) = 25 \text{ ft}\cdot\text{lb}$.

Conservation of Energy:

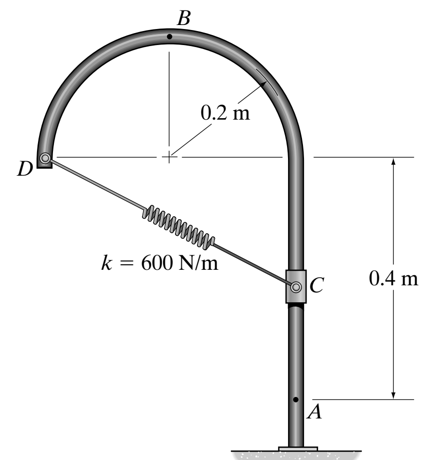
$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A{}^2 + \left[(V_g)_A + (V_e)_A\right] = \frac{1}{2}mv_B{}^2 + \left[(V_g)_B + (V_e)_B\right]$$

$$0 + (120 + 225) = \frac{1}{2}\left(\frac{20}{32.2}\right)v_B{}^2 + (0 + 25)$$

$$v_B = 32.1 \text{ ft/s} \qquad \textbf{Ans.}$$

***14–100.** The 2-kg collar is released from rest at A and travels along the smooth vertical guide. Determine the speed of the collar when it reaches position B. Also, find the normal force exerted on the collar at this position. The spring has an unstretched length of 200 mm.

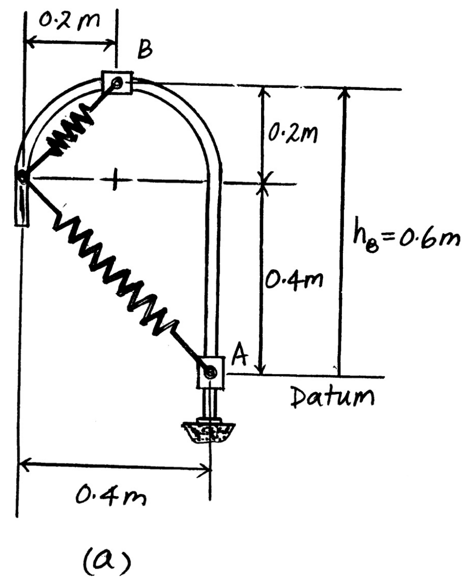


Potential Energy: With reference to the datum set in Fig. a, the gravitational potential energy of the collar at positions A and B are $(V_g)_A = mgh_A = 2(9.81)(0) = 0$ and $(V_g)_B = mgh_B = 2(9.81)(0.6) = 11.772$ J. When the collar is at positions A and B, the spring stretches $s_A = \sqrt{0.4^2 + 0.4^2} - 0.2 = 0.3657$ m and $s_B = \sqrt{0.2^2 + 0.2^2} - 0.2 = 0.08284$ m. Thus, the elastic potential energy of the spring when the collar is at these two positions are

$$(V_e)_A = \frac{1}{2}ks_A{}^2 = \frac{1}{2}(600)(0.3657^2) = 40.118 \text{ J}$$

and

$$(V_e)_B = \frac{1}{2}ks_B{}^2 = \frac{1}{2}(600)(0.08284^2) = 2.0589 \text{ J.}$$



Conservation of Energy:

$$T_A + V_A = T_B + V_B$$

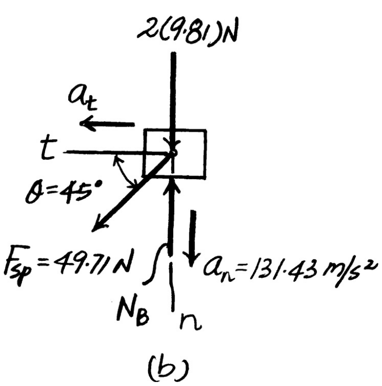
$$\frac{1}{2}mv_A{}^2 + \left[(V_g)_A + (V_e)_A\right] = \frac{1}{2}mv_B{}^2 + \left[(V_g)_B + (V_e)_B\right]$$

$$0 + (0 + 40.118) = \frac{1}{2}(2)v_B{}^2 + (11.772 + 2.0589)$$

$$v_B = 5.127 \text{ m/s} = 5.13 \text{ m/s} \qquad \textbf{Ans.}$$

Equation of Motion: When the collar is at position B, $\theta = \tan^{-1}\left(\frac{0.2}{0.2}\right) = 45°$ and $F_{sp} = ks_B = 600(0.08284) = 49.71$ N. Here,

$$a_n = \frac{v^2}{\rho} = \frac{v_B{}^2}{0.2} = \frac{(5.127)^2}{0.2} = 131.43 \text{ m/s}^2.$$



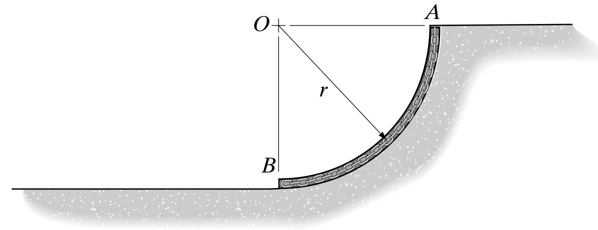
By referring to the free-body diagram of the collar shown in Fig. b,

$$\Sigma F_n = ma_n; \qquad 2(9.81) + 49.71 \sin 45° - N_B = 2(131.43)$$

$$N_B = -208.09 \text{ N} = 208 \text{ N}\downarrow \qquad \textbf{Ans.}$$

Note: The negative sign indicates that $\mathbf{N}_B$ acts in the opposite sense to that shown on the free-body diagram.

•14–101. A quarter-circular tube AB of mean radius r contains a smooth chain that has a mass per unit length of m_0. If the chain is released from rest from the position shown, determine its speed when it emerges completely from the tube.



Potential Energy: The location of the center of gravity G of the chain at positions (1) and (2) are shown in Fig. a. The mass of the chain is $m = m_0\left(\frac{\pi}{2}r\right) = \frac{\pi}{2}m_0 r$. Thus, the center of mass is at $h_1 = r - \frac{2r}{\pi} = \left(\frac{\pi - 2}{\pi}\right)r$. With reference to the datum set in Fig. a the gravitational potential energy of the chain at positions (1) and (2) are

$$\left(V_g\right)_1 = mgh_1 = \left(\frac{\pi}{2}m_0 rg\right)\left(\frac{\pi - 2}{\pi}\right)r = \left(\frac{\pi - 2}{2}\right)m_0 r^2 g$$

and

$$\left(V_g\right)_2 = mgh_2 = 0$$

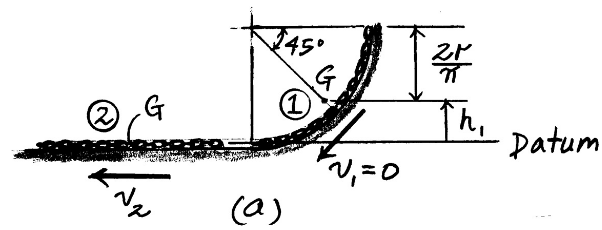
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1^{\,2} + \left(V_g\right)_1 = \frac{1}{2}mv_2^{\,2} + \left(V_g\right)_2$$

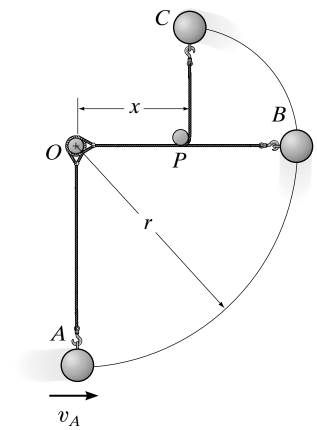
$$0 + \left(\frac{\pi - 2}{2}\right)m_0 r^2 g = \frac{1}{2}\left(\frac{\pi}{2}m_0 r\right)v_2^{\,2} + 0$$

$$v_2 = \sqrt{\frac{2}{\pi}(\pi - 2)gr} \qquad \textbf{Ans.}$$

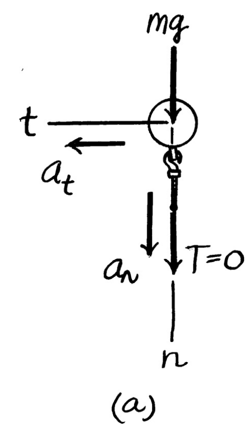


(a)

14–102. The ball of mass m is given a speed of $v_A = \sqrt{3gr}$ at position A. When it reaches B, the cord hits the small peg P, after which the ball describes a smaller circular path. Determine the position x of P so that the ball will just be able to reach point C.



Equation of Motion: If the ball is just about to complete the small circular path, the cord will become slack at position C, i.e., $T = 0$. Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{v_C^{\,2}}{r - x}$. By referring to the free-body diagram of the ball shown in Fig. a,



$$\Sigma F_n = ma_n; \qquad mg = m\left(\frac{v_C^{\,2}}{r - x}\right) \qquad v_C^{\,2} = g(r - x) \qquad \textbf{(1)}$$

Potential Energy: With reference to the datum set in Fig. b, the gravitational potential energy of the ball at positions A and C are $(V_g)_A = mgh_A = mg(0) = 0$ and $(V_g)_C = mgh_C = mg(2r - x)$.

Conservation of Energy:

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}mv_A^{\,2} + (V_g)_A = \frac{1}{2}mv_C^{\,2} + (V_g)_C$$

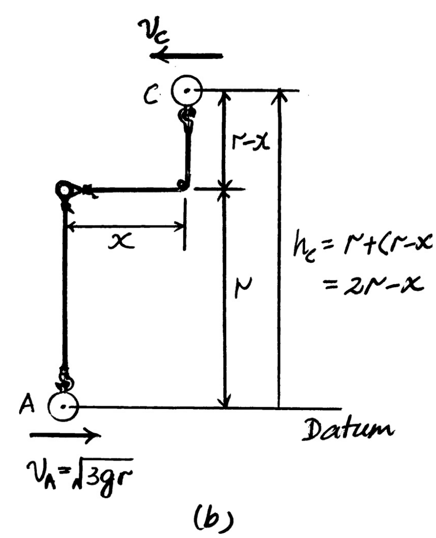
$$\frac{1}{2}m(3gr) + 0 = \frac{1}{2}mv_C^{\,2} + mg(2r - x)$$

$$v_C^{\,2} = g(2x - r) \qquad \textbf{(2)}$$

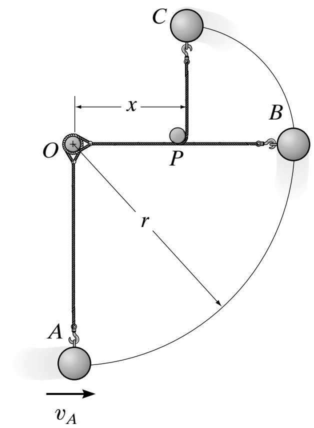
Solving Eqs. (1) and (2) yields

$$x = \frac{2}{3}r \qquad \textbf{Ans.}$$

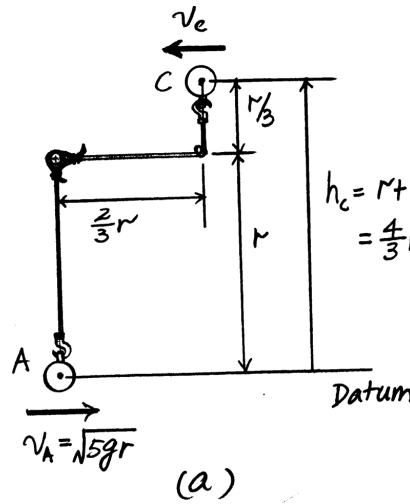
$$v_C = \sqrt{\frac{1}{2}gr}$$



14–103. The ball of mass m is given a speed of $v_A = \sqrt{5gr}$ at position A. When it reaches B, the cord hits the peg P, after which the ball describes a smaller circular path. If $x = \frac{2}{3}r$, determine the speed of the ball and the tension in the cord when it is at the highest point C.



Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the ball at positions A and C are $\left(V_g\right)_A = mgh_A = mg(0) = 0$ and $\left(V_g\right)_C = mgh_C = mg\left(\frac{4}{3}r\right) = \frac{4}{3}mgr$.



Conservation of Energy:

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}mv_A{}^2 + \left(V_g\right)_A = \frac{1}{2}mv_C{}^2 + \left(V_g\right)_C$$

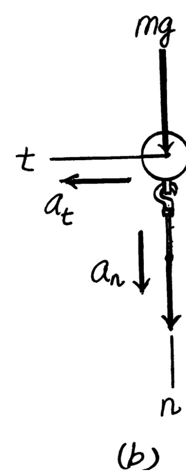
$$\frac{1}{2}m(5gr) + 0 = \frac{1}{2}mv_C{}^2 + \frac{4}{3}mgr$$

$$v_C = \sqrt{\frac{7}{3}gr} \qquad \textbf{Ans.}$$

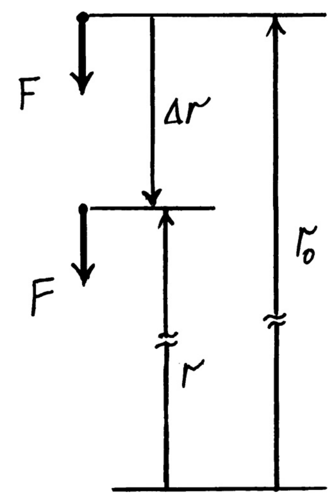
Equations of Motion: Here, $a_n = \dfrac{v_C{}^2}{\rho} = \dfrac{\frac{7}{3}gr}{r/3}$. By referring to the free-body diagram of the ball shown in Fig. *b*,

$$\Sigma F_n = ma_n; \qquad T + mg = m(7g)$$

$$T = 6mg \qquad \textbf{Ans.}$$



***14–104.** If the mass of the earth is M_e, show that the gravitational potential energy of a body of mass m located a distance r from the center of the earth is $V_g = -GM_e m/r$. Recall that the gravitational force acting between the earth and the body is $F = G(M_e m/r^2)$, Eq. 13–1. For the calculation, locate the datum an "infinite" distance from the earth. Also, prove that **F** is a conservative force.



The work is computed by moving F from position r_1 to a farther position r_2.

$$V_g = -U = -\int F\,dr$$

$$= -G\,M_e\,m\int_{r1}^{r2}\frac{dr}{r^2}$$

$$= -G\,M_e\,m\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

As $r_1 \rightarrow \infty$, let $r_2 = r_1, F_2 = F_1$, then

$$V_g \rightarrow \frac{-G\,M_e\,m}{r}$$

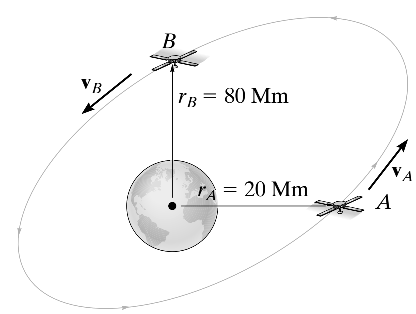
To be conservative, require

$$F = -\nabla V_g = -\frac{\partial}{\partial r}\left(-\frac{G\,M_e\,m}{r}\right)$$

$$= \frac{-G\,M_e\,m}{r^2}$$

Q.E.D.

•14–105. A 60-kg satellite travels in free flight along an elliptical orbit such that at A, where $r_A = 20$ Mm, it has a speed $v_A = 40$ Mm/h. What is the speed of the satellite when it reaches point B, where $r_B = 80$ Mm? *Hint:* See Prob. 14–104, where $M_e = 5.976(10^{24})$ kg and $G = 66.73(10^{-12})\ \mathrm{m^3/(kg\cdot s^2)}$.



$v_A = 40\ \mathrm{Mm/h} = 11\,111.1\ \mathrm{m/s}$

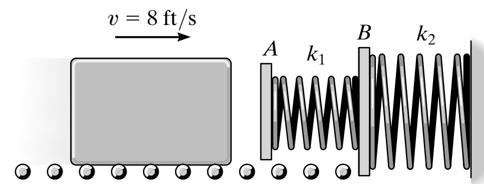
Since $V = -\dfrac{GM_e\,m}{r}$

$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}(60)(11\,111.1)^2 - \frac{66.73(10)^{-12}(5.976)(10)^{23}(60)}{20(10)^6} = \frac{1}{2}(60)v_B^2 - \frac{66.73(10)^{-12}(5.976)(10)^{24}(60)}{80(10)^6}$$

$v_B = 9672\ \mathrm{m/s} = 34.8\ \mathrm{Mm/h}$ **Ans.**

14–106. The double-spring bumper is used to stop the 1500-lb steel billet in the rolling mill. Determine the maximum displacement of the plate A if the billet strikes the plate with a speed of 8 ft/s. Neglect the mass of the springs, rollers and the plates A and B. Take $k_1 = 3000$ lb/ft, $k_2 = 45000$ lb/ft.



$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}\left(\frac{1500}{32.2}\right)(8)^2 + 0 = 0 + \frac{1}{2}(3000)s_L^2 + \frac{1}{2}(4500)s\frac{2}{2} \qquad \textbf{(1)}$$

$$F_s = 3000s_1 = 4500s_2; \qquad \textbf{(2)}$$

$s_1 = 1.5s_2$

Solving Eqs. (1) and (2) yields:

$s_2 = 0.5148$ ft $\qquad s_1 = 0.7722$ ft

$s_A = s_1 + s_2 = 0.7722 + 0.5148 = 1.29$ ft **Ans.**