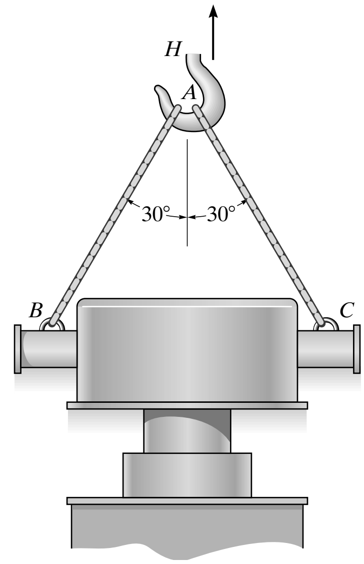
•13–1. The casting has a mass of 3 Mg. Suspended in a vertical position and initially at rest, it is given an upward speed of 200 mm/s in 0.3 s using a crane hook H. Determine the tension in cables AC and AB during this time interval if the acceleration is constant.



Kinematics: Applying the equation $v = v_0 + a_c t$, we have

$(+\uparrow) \qquad 0.2 = 0 + a(0.3) \qquad a = 0.6667 \text{ m/s}^2$

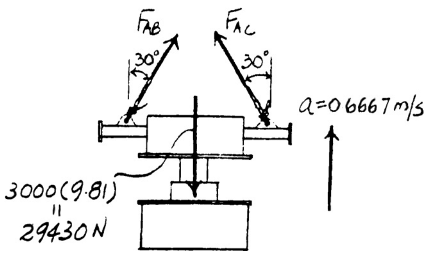
Equations of Motion:

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_{AB} \sin 30 - F_{AC} \sin 30° = 0$

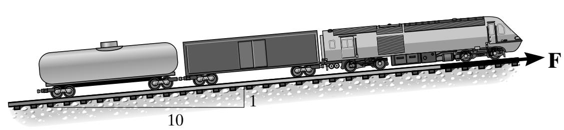
$$F_{AB} = F_{AC} = F$$

$+\uparrow \Sigma F_y = ma_y; \qquad 2F\cos 30° - 29430 = 3000(0.6667)$

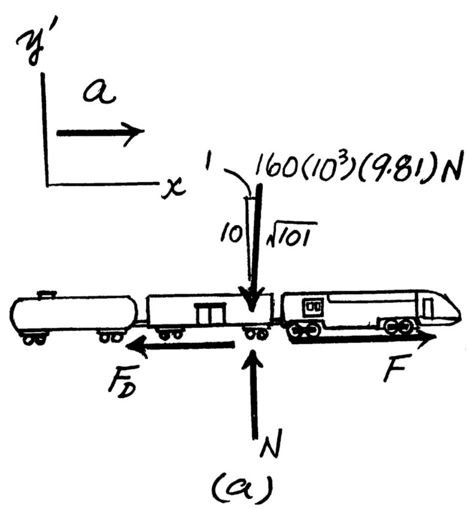
$$F_{AB} = F_{AC} = F = 18146.1 \text{ N} = 18.1 \text{ kN} \qquad \textbf{Ans.}$$



13–2. The 160-Mg train travels with a speed of 80 km/h when it starts to climb the slope. If the engine exerts a traction force **F** of 1/20 of the weight of the train and the rolling resistance $\mathbf{F}_D$ is equal to 1/500 of the weight of the train, determine the deceleration of the train.



Free-Body Diagram: The tractive force and rolling resistance indicated on the free-body diagram of the train, Fig. (a), are $F = \left(\frac{1}{20}\right)(160)(10^3)(9.81) \text{ N} = 78\,480 \text{ N}$ and $F_D = \left(\frac{1}{500}\right)(160)(10^3)(9.81)\text{N} = 3139.2 \text{ N}$, respectively.

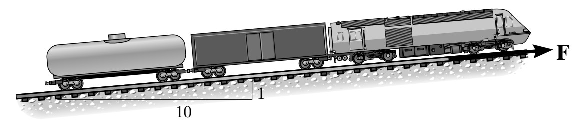


Equations of Motion: Here, the acceleration **a** of the train will be assumed to be directed up the slope. By referring to Fig. (a),

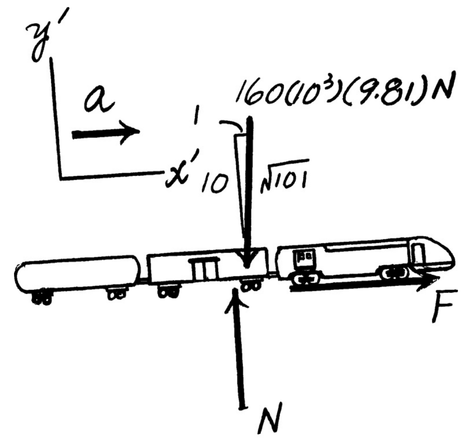
$+\nearrow \Sigma F_{x'} = ma_{x'}; \qquad 78\,480 - 3139.2 - 160(10^3)(9.81)\left(\frac{1}{\sqrt{101}}\right) = 160(10^3)a$

$$a = -0.5057 \text{ m/s}^2 \qquad \textbf{Ans.}$$

13–3. The 160-Mg train starts from rest and begins to climb the slope as shown. If the engine exerts a traction force **F** of 1/8 of the weight of the train, determine the speed of the train when it has traveled up the slope a distance of 1 km. Neglect rolling resistance.



Free-Body Diagram: Here, the tractive force indicated on the free-body diagram of the train, Fig. (a), is $F = \frac{1}{8}(160)(10^3)(9.81)\text{ N} = 196.2(10^3)\text{ N}$.



(a)

Equations of Motion: Here, the acceleration **a** of the train will be assumed directed up the slope. By referring to Fig. (a),

$$+\nearrow \Sigma F_{x'} = ma_{x'}; \qquad 196.2(10^3) - 160(10^3)(9.81)\left(\frac{1}{\sqrt{101}}\right) = 160(10^3)a$$

$$a = 0.2501\text{ m/s}^2$$

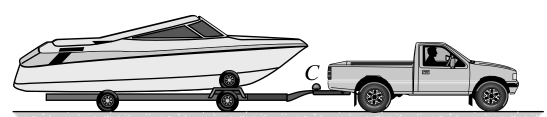
Kinematics: Using the result of **a**,

$$(+\nearrow) \qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$v^2 = 0 + 2(0.2501)(1000 - 0)$$

$$v = 22.4\text{ m/s} \qquad \textbf{Ans.}$$

***13–4.** The 2-Mg truck is traveling at 15 m/s when the brakes on all its wheels are applied, causing it to skid for a distance of 10 m before coming to rest. Determine the constant horizontal force developed in the coupling *C*, and the frictional force developed between the tires of the truck and the road during this time. The total mass of the boat and trailer is 1 Mg.

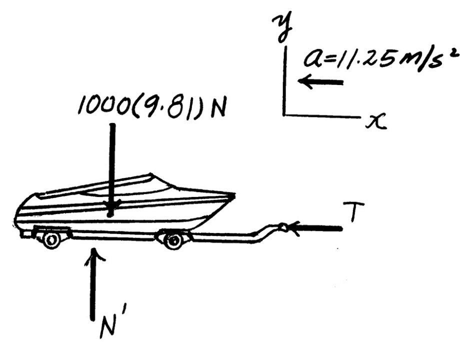


Kinematics: Since the motion of the truck and trailer is known, their common acceleration **a** will be determined first.

$$(\xrightarrow{+}) \qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$0 = 15^2 + 2a(10 - 0)$$

$$a = -11.25\text{ m/s}^2 = 11.25\text{ m/s}^2 \leftarrow$$



(a)

Free-Body Diagram: The free-body diagram of the truck and trailer are shown in Figs. (a) and (b), respectively. Here, **F** representes the frictional force developed when the truck skids, while the force developed in coupling *C* is represented by **T**.

Equations of Motion: Using the result of **a** and referrning to Fig. (a),

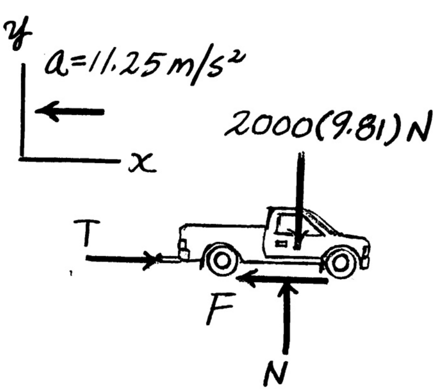
$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -T = 1000(-11.25)$$

$$T = 11\,250\text{ N} = 11.25\text{ kN} \qquad \textbf{Ans.}$$

Using the results of **a** and **T** and referring to Fig. (b),

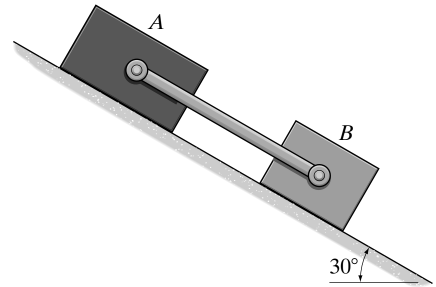
$$+\uparrow \Sigma F_x = ma_x; \qquad 11\,250 - F = 2000(-11.25)$$

$$F = 33\,750\text{ N} = 33.75\text{ kN} \qquad \textbf{Ans.}$$

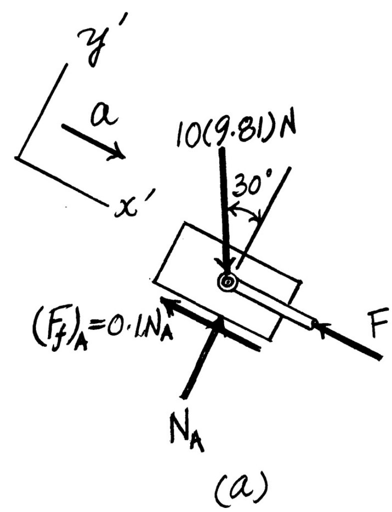


(b)

•13–5. If blocks A and B of mass 10 kg and 6 kg, respectively, are placed on the inclined plane and released, determine the force developed in the link. The coefficients of kinetic friction between the blocks and the inclined plane are $\mu_A = 0.1$ and $\mu_B = 0.3$. Neglect the mass of the link.



Free-Body Diagram: Here, the kinetic friction $(F_f)_A = \mu_A N_A = 0.1N_A$ and $(F_f)_B = \mu_B N_B = 0.3N_B$ are required to act up the plane to oppose the motion of the blocks which are down the plane. Since the blocks are connected, they have a common acceleration **a**.



Equations of Motion: By referring to Figs. (a) and (b),

$+\nearrow \Sigma F_{y'} = ma_{y'}$; $\quad N_A - 10(9.81)\cos 30° = 10(0)$

$N_A = 84.96 \text{ N}$

$\searrow + \Sigma F_{x'} = ma_{x'}$; $\quad 10(9.81)\sin 30° - 0.1(84.96) - F = 10a$

$$40.55 - F = 10a \qquad \textbf{(1)}$$

and

$+\nearrow \Sigma F_{y'} = ma_{y'}$; $\quad N_B - 6(9.81)\cos 30° = 6(0)$

$N_B = 50.97 \text{ N}$

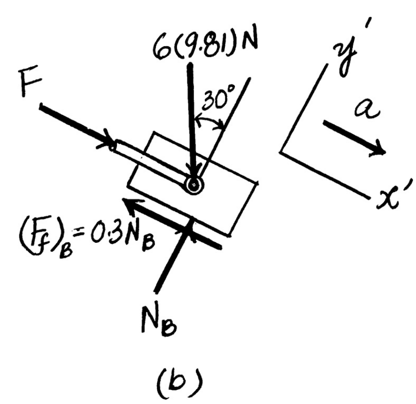
$\searrow + \Sigma F_{x'} = ma_{x'}$; $\quad F + 6(9.81)\sin 30° - 0.3(50.97) = 6a$

$$F + 14.14 = 6a \qquad \textbf{(2)}$$

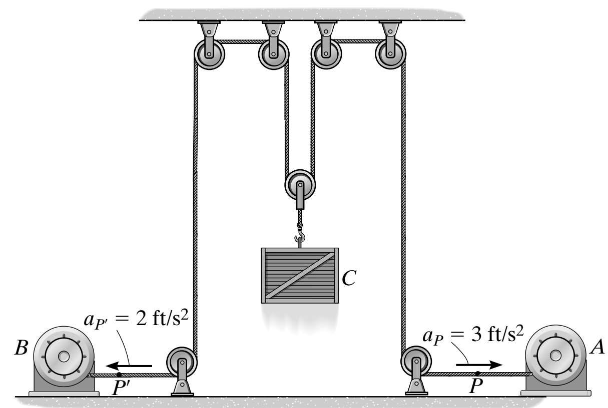
Solving Eqs. (1) and (2) yields

$a = 3.42 \text{ m/s}^2$

$F = 6.37 \text{ N}$ **Ans.**



13–6. Motors A and B draw in the cable with the accelerations shown. Determine the acceleration of the 300-lb crate C and the tension developed in the cable. Neglect the mass of all the pulleys.



Kinematics: We can express the length of the cable in terms of s_P, $s_{P'}$, and s_C by referring to Fig. (a).

$$s_P + s_{P'} + 2s_C = l$$

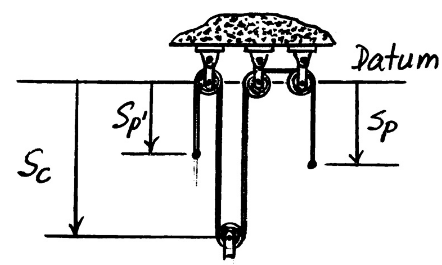
The second time derivative of the above equation gives

$$(+\downarrow) \qquad a_P + a_{P'} + 2a_C = 0 \qquad \textbf{(1)}$$

Here, $a_P = 3\text{ ft/s}^2$ and $a_{P'} = 2\text{ ft/s}^2$. Substituting these values into Eq. (1),

$$3 + 2 + 2a_C = 0$$

$$a_C = -2.5\text{ ft/s}^2 = 2.5\text{ ft/s}^2 \uparrow \qquad \textbf{Ans.}$$



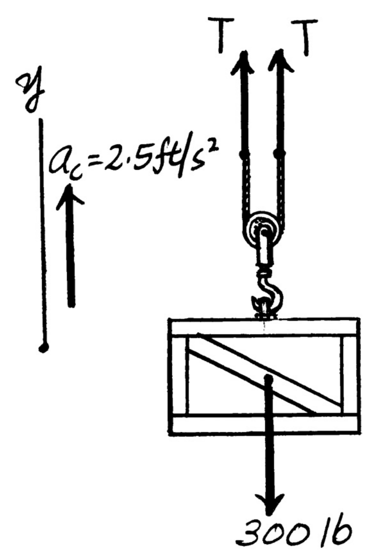
(a)

Free-Body Diagram: The free-body diagram of the crate is shown in Fig. (b).

Equations of Motion: Using the result of $\mathbf{a}_C$ and referring to Fig. (b),

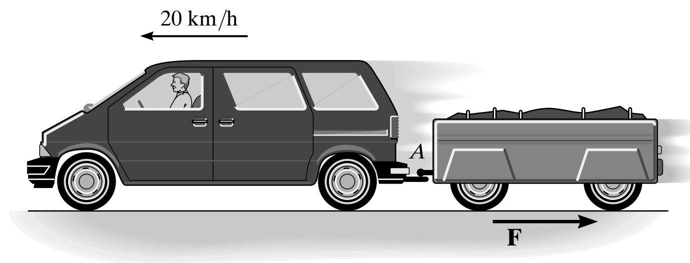
$$+\uparrow \Sigma F_y = ma_y; \qquad 2T - 300 = \frac{300}{32.2}(2.5)$$

$$T = 162\text{ lb} \qquad \textbf{Ans.}$$



(b)

13–7. The van is traveling at 20 km/h when the coupling of the trailer at A fails. If the trailer has a mass of 250 kg and coasts 45 m before coming to rest, determine the constant horizontal force F created by rolling friction which causes the trailer to stop.



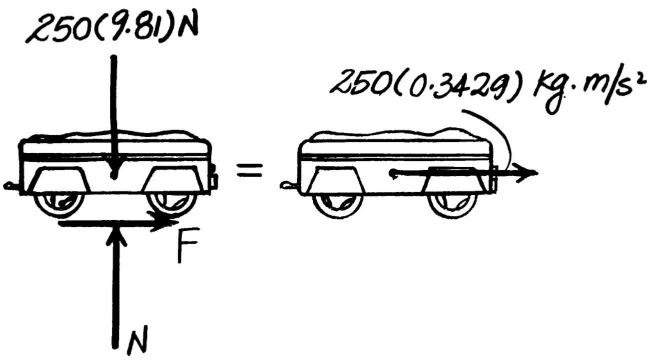
$$20 \text{ km/h} = \frac{20(10^3)}{3600} = 5.556 \text{ m/s}$$

$$(\overset{+}{\leftarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

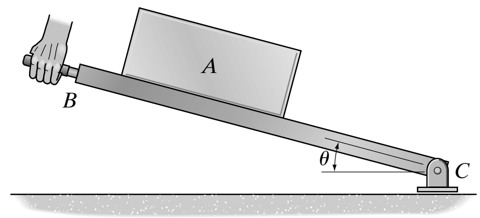
$$0 = 5.556^2 + 2(a)(45 - 0)$$

$$a = -0.3429 \text{ m/s}^2 = 0.3429 \text{ m/s}^2 \rightarrow$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad F = 250(0.3429) = 85.7 \text{ N} \qquad \textbf{Ans.}$$



***13–8.** If the 10-lb block A slides down the plane with a constant velocity when $\theta = 30°$, determine the acceleration of the block when $\theta = 45°$.



Free-Body Diagram: The free-body diagrams of the block when $\theta = 30°$ and $\theta = 45°$ are shown in Figs. (a) and (b), respectively. Here, the kinetic friction $F_f = \mu_k N$ and $F_{f'} = \mu_k N'$ are required to act up the plane to oppose the motion of the block which is directed down the plane for both cases.

Equations of Motion: Since the block has constant velocity when $\theta = 30°$, $a_{x'} = a = 0$. Also, $a_{y'} = 0$. By referring to Fig. (a), we can write

$$+\nearrow\Sigma F_{y'} = ma_{y'}; \qquad N - 10\cos 30° = \frac{10}{32.2}(0)$$

$$N = 8.660 \text{ lb}$$

$$\searrow+\Sigma F_{x'} = ma_{x'}; \qquad 10\sin 30° - \mu_k(8.660) = \frac{10}{32.2}(0)$$

$$\mu_k = 0.5774$$

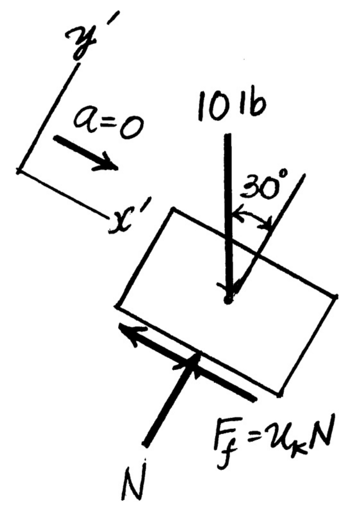
Using the results of μ_k and referring to Fig. (b),

$$+\nearrow\Sigma F_{y'} = ma_{y'}; \qquad N' - 10\cos 45° = \frac{10}{32.2}(0)$$

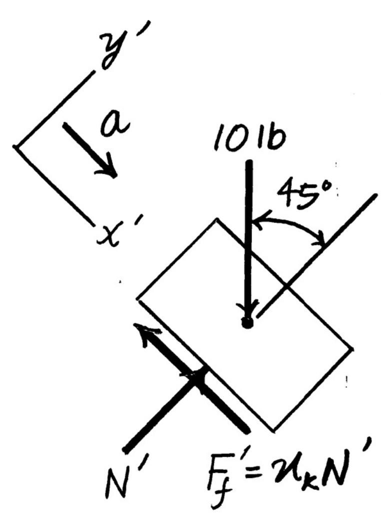
$$N' = 7.071 \text{ lb}$$

$$\searrow+\Sigma F_{x'} = ma_{x'}; \qquad 10\sin 45° - 0.5774(7.071) = \frac{10}{32.2}a$$

$$a = 9.62 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

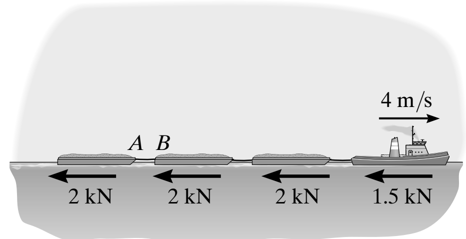


(a)



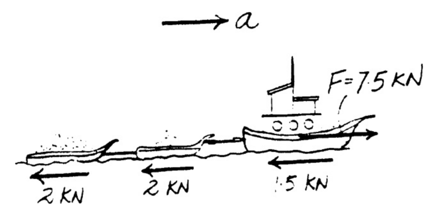
(b)

•13–9. Each of the three barges has a mass of 30 Mg, whereas the tugboat has a mass of 12 Mg. As the barges are being pulled forward with a constant velocity of 4 m/s, the tugboat must overcome the frictional resistance of the water, which is 2 kN for each barge and 1.5 kN for the tugboat. If the cable between A and B breaks, determine the acceleration of the tugboat.



Equations of Motion: When the tugboat and barges are travelling at a constant velocity, the driving force F can be determined by applying Eq. 13–7.

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F - 1.5 - 2 - 2 = 0 \qquad F = 7.50 \text{ kN}$$

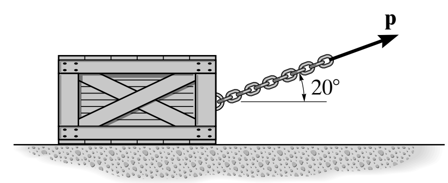


If the cable between barge A and B breaks and the driving force F remains the same, the acceleration of the tugboat and barge is given by

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad (7.50 - 1.5 - 2 - 2)\left(10^3\right) = (12\,000 + 30\,000 + 3000)a$$

$$a = 0.0278 \text{ m/s}^2 \qquad \textbf{Ans.}$$

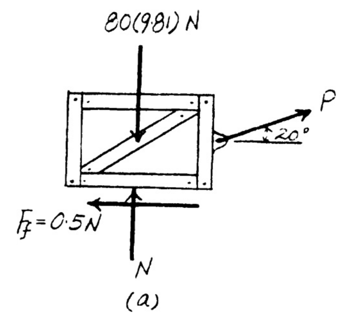
13–10. The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. If the magnitude of **P** is increased until the crate begins to slide, determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.5$ and the coefficient of kinetic friction is $\mu_k = 0.3$.



Equations of Equilibrium: If the crate is on the verge of slipping, $F_f = \mu_s N = 0.5N$. From FBD(a),

$$+\uparrow \Sigma F_y = 0; \qquad N + P \sin 20° - 80(9.81) = 0 \qquad \textbf{(1)}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P \cos 20° - 0.5N = 0 \qquad \textbf{(2)}$$



Solving Eqs.(1) and (2) yields

$$P = 353.29 \text{ N} \qquad N = 663.97 \text{ N}$$

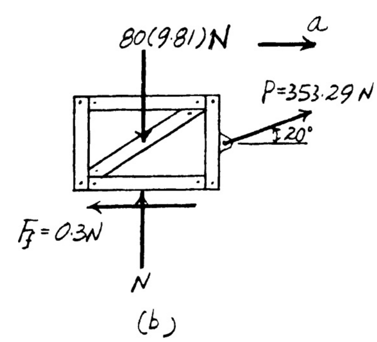
Equations of Motion: The friction force developed between the crate and its contacting surface is $F_f = \mu_k N = 0.3N$ since the crate is moving. From FBD(b),

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 80(9.81) + 353.29 \sin 20° = 80(0)$$

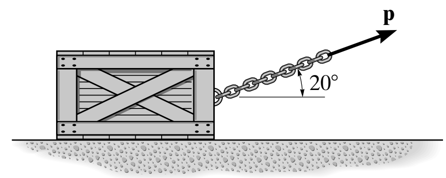
$$N = 663.97 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 353.29 \cos 20° - 0.3(663.97) = 80a$$

$$a = 1.66 \text{ m/s}^2 \qquad \textbf{Ans.}$$



13–11. The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. Determine the crate's acceleration in $t = 2$ s if the coefficient of static friction is $\mu_s = 0.4$, the coefficient of kinetic friction is $\mu_k = 0.3$, and the towing force is $P = (90t^2)$ N, where t is in seconds.



Equations of Equilibrium: At $t = 2$ s, $P = 90\left(2^2\right) = 360$ N. From FBD(a)

$$+\uparrow \Sigma F_y = 0; \qquad N + 360 \sin 20° - 80(9.81) = 0 \qquad N = 661.67 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 360 \cos 20° - F_f = 0 \qquad F_f = 338.29\text{N}$$

Since $F_f > (F_f)_{\max} = \mu_s N = 0.4(661.67) = 264.67$ N, the crate accelerates.

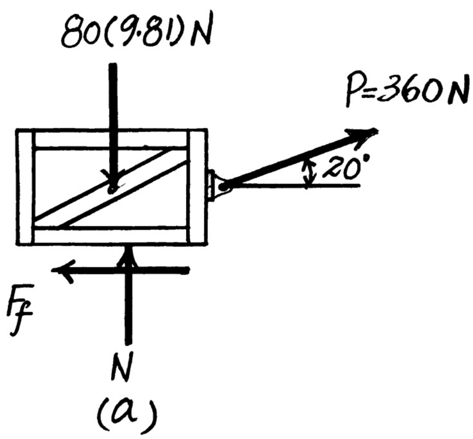
Equations of Motion: The friction force developed between the crate and its contacting surface is $F_f = \mu_k N = 0.3N$ since the crate is moving. From FBD(b),

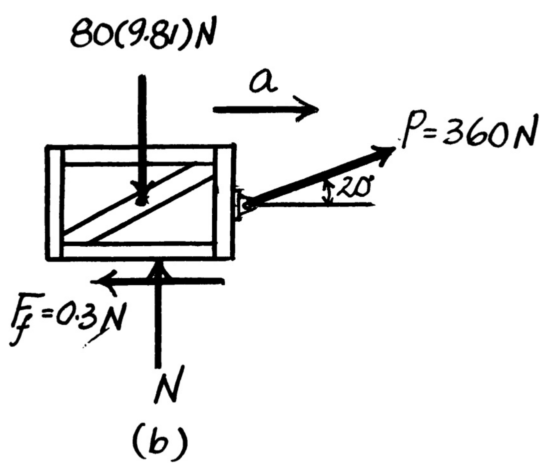
$$+\uparrow \Sigma F_y = ma_y; \qquad N - 80(9.81) + 360 \sin 20° = 80(0)$$

$$N = 661.67 \text{ N}$$

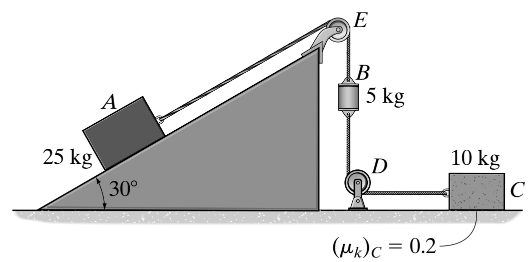
$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 360 \cos 20° - 0.3(661.67) = 80a$$

$$a = 1.75 \text{ m/s}^2 \qquad \textbf{Ans.}$$





***13–12.** Determine the acceleration of the system and the tension in each cable. The inclined plane is smooth, and the coefficient of kinetic friction between the horizontal surface and block C is $(\mu_k)_C = 0.2$.



Free-Body Diagram: The free-body diagram of block A, cylinder B, and block C are shown in Figs. (a), (b), and (c), respectively. The frictional force $(F_f)_C = (\mu_k)_C N_C = 0.2N_C$ must act to the right to oppose the motion of block C which is to the left.

Equations of Motion: Since block A, cylinder B, and block C move together as a single unit, they share a common acceleration **a**. By referring to Figs. (a), (b), and (c),

$$\Sigma F_{x'} = ma_{x'}; \qquad T_1 - 25(9.81)\sin 30° = 25(-a) \qquad \textbf{(1)}$$

and

$$+\uparrow \Sigma F_y = ma_y; \qquad T_1 - T_2 - 5(9.81) = 5(a) \qquad \textbf{(2)}$$

and

$$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 10(9.81) = 10(0)$$

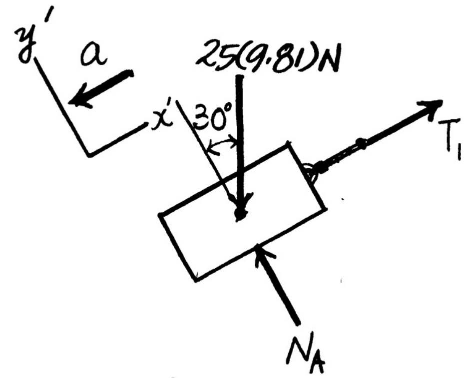
$$N_C = 98.1 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -T_2 + 0.2(98.1) = 10(-a) \qquad \textbf{(3)}$$

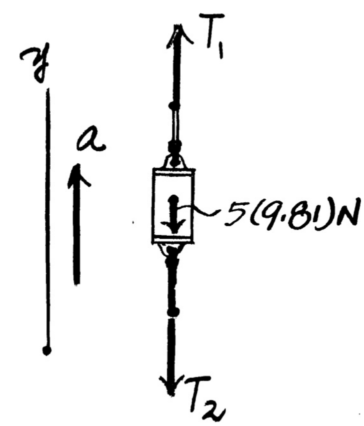
Solving Eqs. (1), (2), and (3), yields

$$a = 1.349 \text{ m/s}^2 \qquad T_1 = 88.90 \text{ N} = 88.9\text{N} \qquad \textbf{Ans.}$$

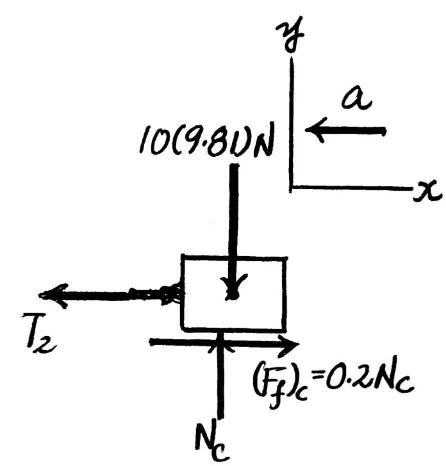
$$T_2 = 33.11 \text{ N} = 33.1 \text{ N} \qquad \textbf{Ans.}$$



(a)

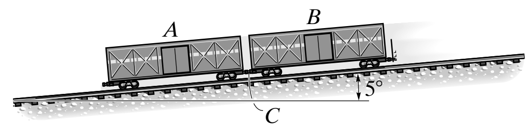


(b)



(c)

•13–13. The two boxcars A and B have a weight of 20 000 lb and 30 000 lb, respectively. If they coast freely down the incline when the brakes are applied to all the wheels of car A causing it to skid, determine the force in the coupling C between the two cars. The coefficient of kinetic friction between the wheels of A and the tracks is $\mu_k = 0.5$. The wheels of car B are free to roll. Neglect their mass in the calculation. *Suggestion:* Solve the problem by representing single resultant normal forces acting on A and B, respectively.



Car A:

$+\nwarrow\Sigma F_y = 0; \qquad N_A - 20\,000\cos 5° = 0 \qquad N_A = 19\,923.89\text{ lb}$

$+\nearrow\Sigma F_x = ma_x; \qquad 0.5(19\,923.89) - T - 20\,000\sin 5° = \left(\dfrac{20\,000}{32.2}\right)a$ **(1)**

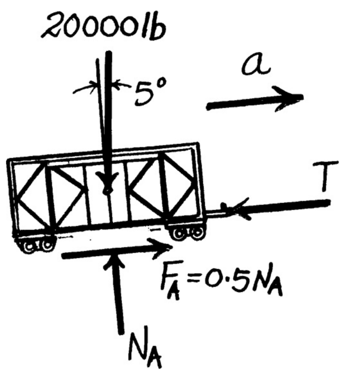
Both cars:

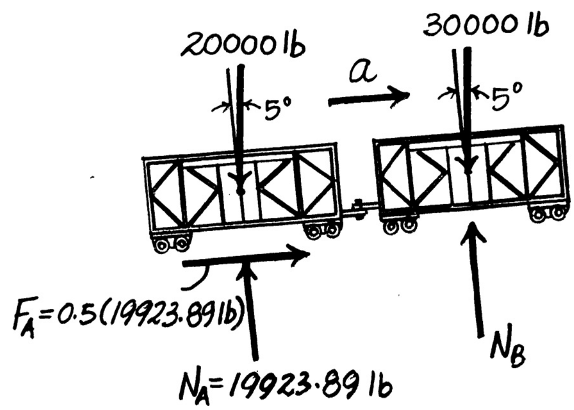
$+\nearrow\Sigma F_x = ma_x; \qquad 0.5(19\,923.89) - 50\,000\sin 5° = \left(\dfrac{50\,000}{32.2}\right)a$

Solving,

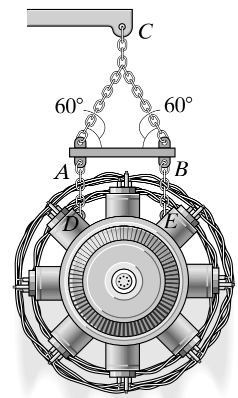
$a = 3.61\text{ ft/s}^2$

$T = 5.98\text{ kip}$ **Ans.**





13–14. The 3.5-Mg engine is suspended from a spreader beam AB having a negligible mass and is hoisted by a crane which gives it an acceleration of 4 m/s^2 when it has a velocity of 2 m/s. Determine the force in chains CA and CB during the lift.



System:

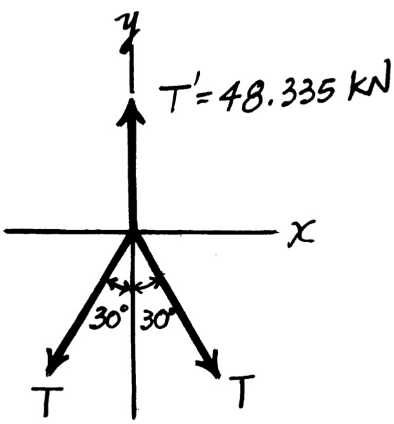
$$+\uparrow \Sigma F_y = ma_y; \qquad T' - 3.5\left(10^3\right)(9.81) = 3.5\left(10^3\right)(4)$$

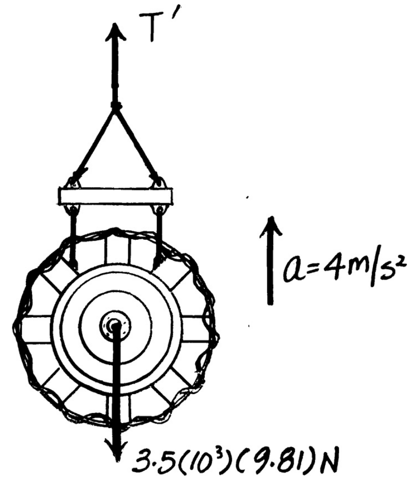
$$T' = 48.335 \text{ kN}$$

Joint C:

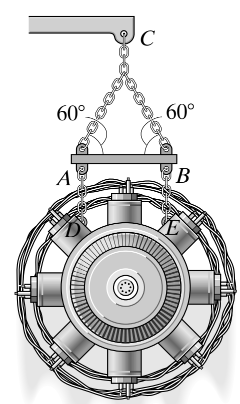
$$+\uparrow \Sigma F_y = ma_y; \qquad 48.335 - 2\,T\cos 30° = 0$$

$$T = T_{CA} = T_{CB} = 27.9 \text{ kN} \qquad \textbf{Ans.}$$





13–15. The 3.5-Mg engine is suspended from a spreader beam having a negligible mass and is hoisted by a crane which exerts a force of 40 kN on the hoisting cable. Determine the distance the engine is hoisted in 4 s, starting from rest.



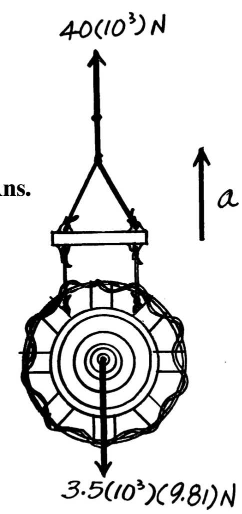
System:

$$+\uparrow \Sigma F_y = ma_y; \qquad 40\left(10^3\right) - 3.5\left(10^3\right)(9.81) = 3.5\left(10^3\right)a$$

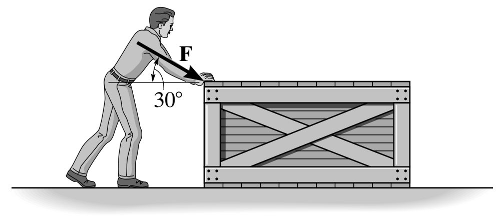
$$a = 1.619 \text{ m/s}^2$$

$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s = 0 + 0 + \frac{1}{2}(1.619)(4)^2 = 12.9 \text{ m} \qquad \textbf{Ans.}$$



***13–16.** The man pushes on the 60-lb crate with a force **F**. The force is always directed down at 30° from the horizontal as shown, and its magnitude is increased until the crate begins to slide. Determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.6$ and the coefficient of kinetic friction is $\mu_k = 0.3$.



Force to produce motion:

$\xrightarrow{+} \Sigma F_x = 0; \qquad F\cos 30° - 0.6N = 0$

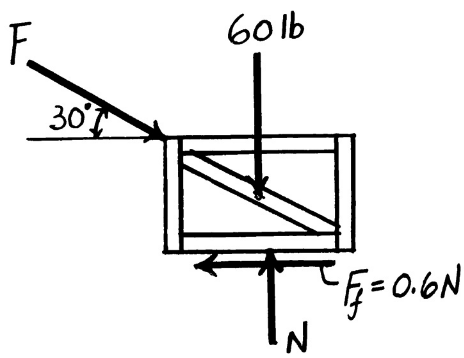
$+\uparrow \Sigma F_y = 0; \qquad N - 60 - F\sin 30° = 0$

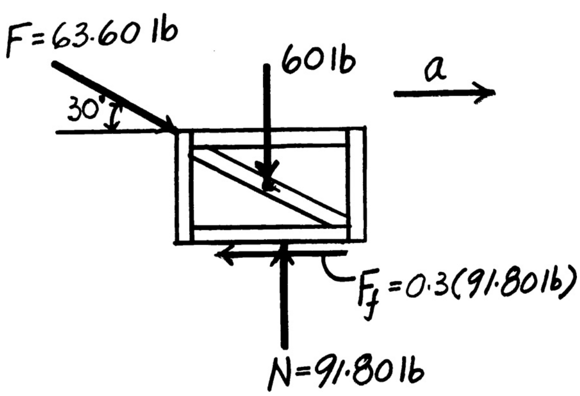
$N = 91.80 \text{ lb} \qquad F = 63.60 \text{ lb}$

Since $N = 91.80$ lb,

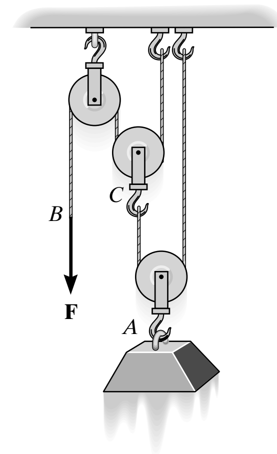
$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 63.60\cos 30° - 0.3(91.80) = \left(\frac{60}{32.2}\right)a$

$a = 14.8 \text{ ft/s}^2$ **Ans.**





•13–17. A force of $F = 15$ lb is applied to the cord. Determine how high the 30-lb block A rises in 2 s starting from rest. Neglect the weight of the pulleys and cord.



Block:

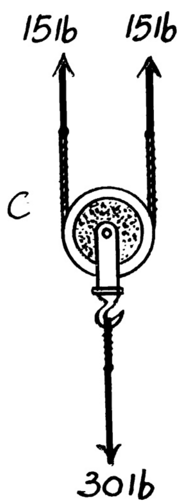
$$+\uparrow \Sigma F_y = ma_y; \qquad -30 + 60 = \left(\frac{30}{32.2}\right)a_A$$

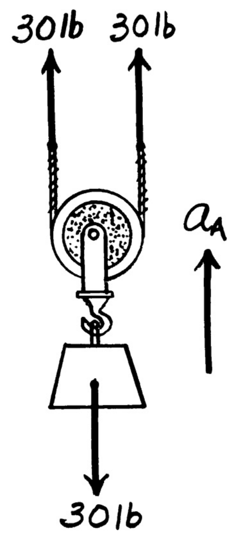
$$a_A = 32.2 \text{ ft/s}^2$$

$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

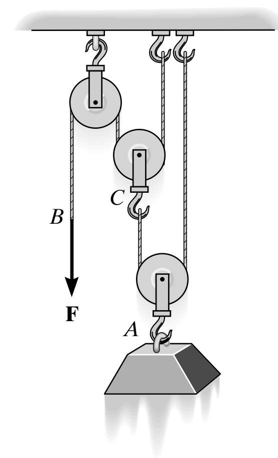
$$s = 0 + 0 + \frac{1}{2}(32.2)(2)^2$$

$$s = 64.4 \text{ ft}$$ **Ans.**





13–18. Determine the constant force **F** which must be applied to the cord in order to cause the 30-lb block A to have a speed of 12 ft/s when it has been displaced 3 ft upward starting from rest. Neglect the weight of the pulleys and cord.



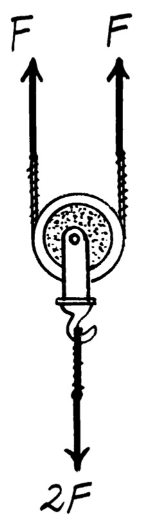
$(+\uparrow)$ $\quad v^2 = v_0^2 + 2a_c(s - s_0)$

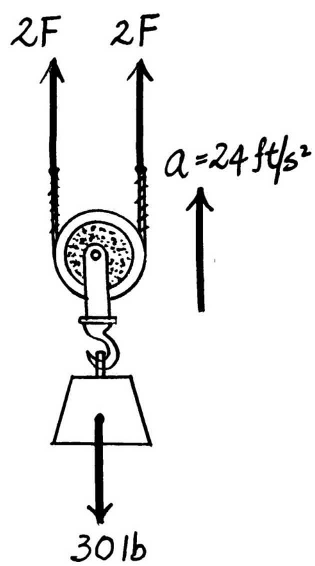
$(12)^2 = 0 + 2(a)(3)$

$a = 24\text{ ft/s}^2$

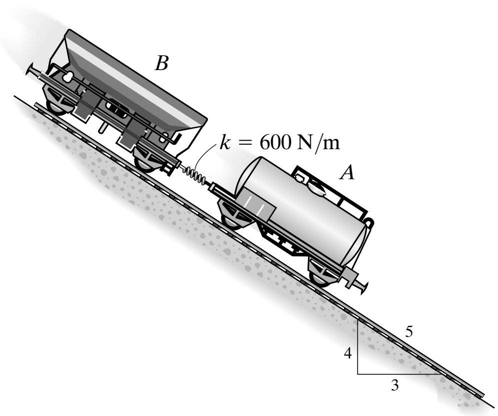
$+\uparrow\Sigma F_y = ma_y;\qquad -30 + 4F = \left(\dfrac{30}{32.2}\right)(24)$

$F = 13.1\text{ lb}$ **Ans.**





13–19. The 800-kg car at B is connected to the 350-kg car at A by a spring coupling. Determine the stretch in the spring if (a) the wheels of both cars are free to roll and (b) the brakes are applied to all four wheels of car B, causing the wheels to skid. Take $(\mu_k)_B = 0.4$. Neglect the mass of the wheels.



a) Equations of Motion: Applying Eq. 13–7 to FBD(a), we have

$\searrow + \Sigma F_{x'} = ma_{x'}$; $\qquad (800 + 350)(9.81)\sin 53.13^\circ = (800 + 350)a$

$$a = 7.848 \text{ m/s}^2$$

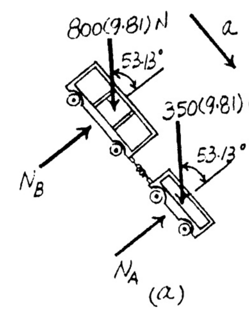
For FBD(b),

$\searrow + \Sigma F_{x'} = ma_{x'}$; $\qquad 350(9.81)\sin 53.13^\circ + F_{sp} = 350(7.848)$

$$F_{sp} = 0$$

The stretch of spring is given by

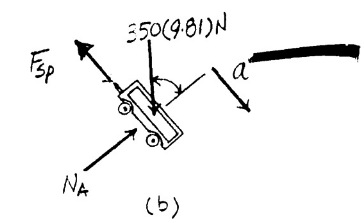
$$x = \frac{F_{sp}}{k} = 0 \qquad \textbf{Ans.}$$



b) Equations of Motion: The friction force developed between the wheels of car B and the inclined plane is $(F_f)_B = (\mu_k)_B N_B = 0.4N_B$. For car B only [FBD(c)],

$+\nearrow \Sigma F_{y'} = ma_{y'}$; $\qquad N_B - 800(9.81)\cos 53.13^\circ = 800(0)$

$$N_B = 4708.8 \text{ N}$$



For the whole system (FBD(c)],

$\searrow + \Sigma F_{x'} = ma_{x'}$; $\qquad (800 + 350)(9.81)\sin 53.13^\circ - 0.4(4708.8) = (800 + 350)a$

$$a = 6.210 \text{ m/s}^2$$

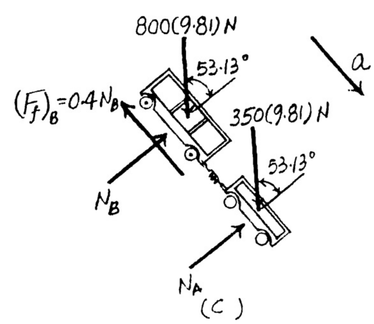
For FBD(b),

$\searrow + \Sigma F_{x'} = ma_{x'}$; $\qquad 350(9.81)\sin 53.13^\circ - F_{sp} = 350\,(6.210)$

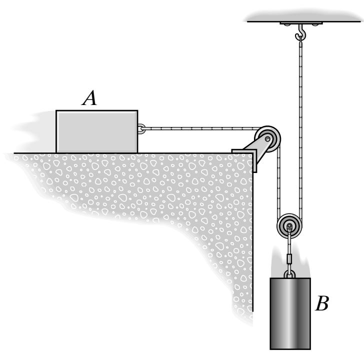
$$F_{sp} = 573.25 \text{ N}$$

The stretch of spring is given by

$$x = \frac{F_{sp}}{k} = \frac{573.25}{600} = 0.955 \text{ m} \qquad \textbf{Ans.}$$



***13–20.** The 10-lb block A travels to the right at $v_A = 2$ ft/s at the instant shown. If the coefficient of kinetic friction is $\mu_k = 0.2$ between the surface and A, determine the velocity of A when it has moved 4 ft. Block B has a weight of 20 lb.



Block A:

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad -T + 2 = \left(\frac{10}{32.2}\right)a_A \tag{1}$$

Weight B:

$$+\downarrow \Sigma F_y = ma_y; \qquad 20 - 2T = \left(\frac{20}{32.2}\right)a_B \tag{2}$$

Kinematics:

$$s_A + 2s_B = l$$

$$a_A = -2a_B \tag{3}$$

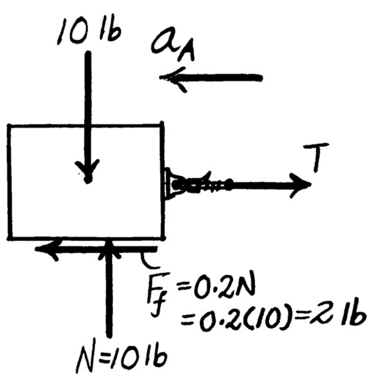
Solving Eqs. (1)–(3):

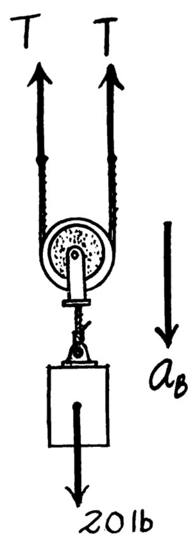
$a_A = -17.173 \text{ ft/s}^2 \qquad a_B = 8.587 \text{ ft/s}^2 \qquad T = 7.33 \text{ lb}$

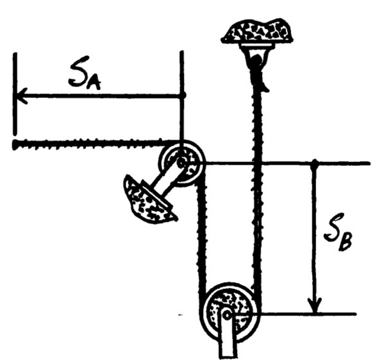
$v^2 = v_0^2 + 2a_c(s - s_0)$

$v^2 = (2)^2 + 2(17.173)(4 - 0)$

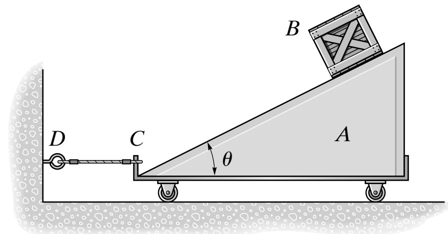
$$v = 11.9 \text{ ft/s}$$ **Ans.**







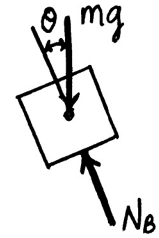
•13–21. Block B has a mass m and is released from rest when it is on top of cart A, which has a mass of $3m$. Determine the tension in cord CD needed to hold the cart from moving while B slides down A. Neglect friction.



Block B:

$+\nwarrow \Sigma F_y = ma_y;\qquad N_B - mg\cos\theta = 0$

$N_B = mg\cos\theta$

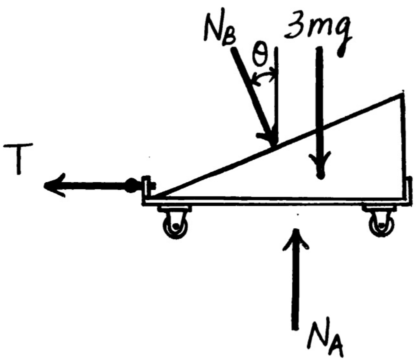


Cart:

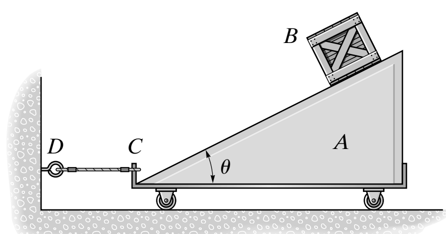
$\xrightarrow{+} \Sigma F_x = ma_x;\qquad -T + N_B\sin\theta = 0$

$T = mg\sin\theta\cos\theta$

$T = \left(\dfrac{mg}{2}\right)\sin 2\theta$ **Ans.**



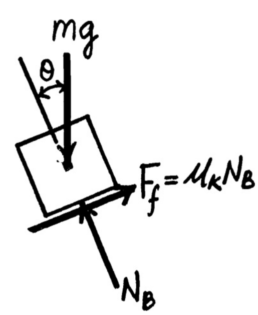
13–22. Block B has a mass m and is released from rest when it is on top of cart A, which has a mass of $3m$. Determine the tension in cord CD needed to hold the cart from moving while B slides down A. The coefficient of kinetic friction between A and B is μ_k.



Block B:

$+\nwarrow \Sigma F_y = ma_y;\qquad N_B - mg\cos\theta = 0$

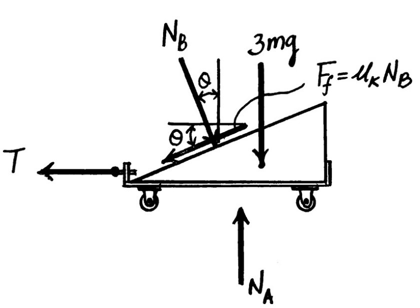
$N_B = mg\cos\theta$



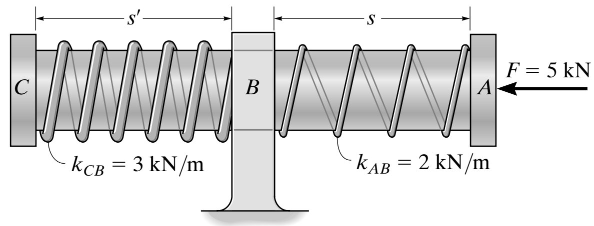
Cart:

$\xrightarrow{+} \Sigma F_x = ma_x;\qquad -T + N_B\sin\theta - \mu_k N_B\cos\theta = 0$

$T = mg\cos\theta(\sin\theta - \mu_k\cos\theta)$ **Ans.**



13–23. The 2-kg shaft CA passes through a smooth journal bearing at B. Initially, the springs, which are coiled loosely around the shaft, are unstretched when no force is applied to the shaft. In this position $s = s' = 250$ mm and the shaft is at rest. If a horizontal force of $F = 5$ kN is applied, determine the speed of the shaft at the instant $s = 50$ mm, $s' = 450$ mm. The ends of the springs are attached to the bearing at B and the caps at C and A.



$F_{CB} = k_{CB}x = 3000x \qquad F_{AB} = k_{AB}x = 2000x$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad 5000 - 3000x - 2000x = 2a$

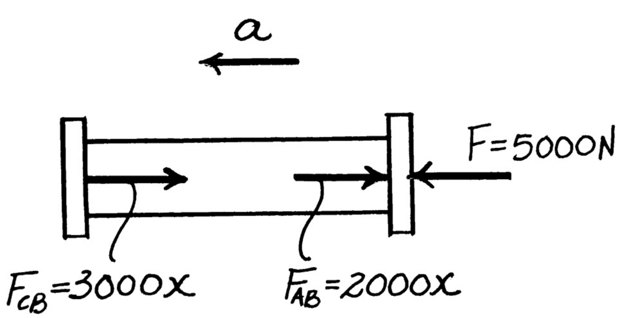
$$2500 - 2500x = a$$

$a\,dx - v\,dv$

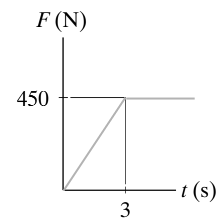
$$\int_0^{0.2} (2500 - 2500x)\,dx = \int_0^v v\,dv$$

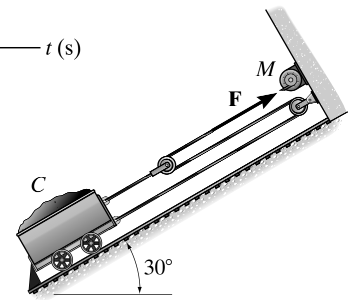
$$2500(0.2) - \left(\frac{2500(0.2)^2}{2}\right) = \frac{v^2}{2}$$

$v = 30$ m/s **Ans.**



***13–24.** If the force of the motor M on the cable is shown in the graph, determine the velocity of the cart when $t = 3$ s. The load and cart have a mass of 200 kg and the car starts from rest.





Free-Body Diagram: The free-body diagram of the rail car is shown in Fig. (a).

Equations of Motion: For $0 \leq t < 3$ s, $F = \frac{450}{3}t = (150t)$ N. By referring to Fig. (a), we can write

$$+\nearrow\Sigma F_{x'} = ma_{x'}; \qquad 3(150t) - 200(9.81)\sin 30° = 200a$$

$$a = (2.25t - 4.905)\ \text{m/s}^2$$

For $t > 3$ s, $F = 450$ N. Thus,

$$+\nearrow\Sigma F_{x'} = ma_{x'}; \qquad 3(450) - 200(9.81)\sin 30° = 200a$$

$$a = 1.845\ \text{m/s}^2$$

Equilibrium: For the rail car to move, force **3F** must overcome the weight component of the rail crate. Thus, the time required to move the rail car is given by

$$\Sigma F_{x'} = 0; \quad 3(150t) - 200(9.81)\sin 30° = 0 \qquad t = 2.18\ \text{s}$$

Kinematics: The velocity of the rail car can be obtained by integrating the kinematic equation, $dv = adt$. For $2.18\ \text{s} \leq t < 3$ s, $v = 0$ at $t = 2.18$ s will be used as the integration limit. Thus,

$$(+\uparrow) \qquad \int dv = \int adt$$

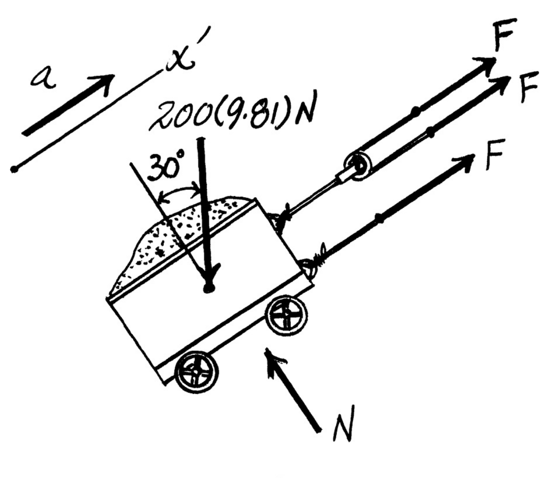
$$\int_0^v dv = \int_{2.18\ \text{s}}^t (2.25t - 4.905)dt$$

$$v = \left(1.125t^2 - 4.905t\right)\Big|_{2.18\ \text{s}}^t$$

$$= \left(1.125t^2 - 4.905t + 5.34645\right)\text{m/s}$$

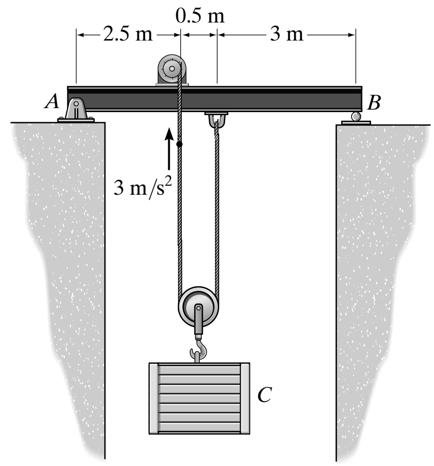
When $t = 3$ s,

$$v = 1.125(3)^2 - 4.905(3) + 5.34645 = 0.756\text{m/s} \qquad \textbf{Ans.}$$



(a)

•13–25. If the motor draws in the cable with an acceleration of $3\,\text{m/s}^2$, determine the reactions at the supports A and B. The beam has a uniform mass of 30 kg/m, and the crate has a mass of 200 kg. Neglect the mass of the motor and pulleys.



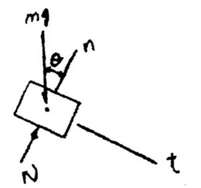
$+\searrow\Sigma F_t = ma_t; \qquad mg \sin\theta = ma_t \qquad a_t = g\sin\theta$

$v\,dv = a_t\,ds = g\sin\theta\,ds \qquad \text{However } dy = ds\sin\theta$

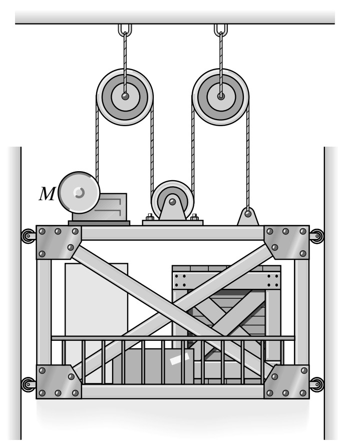
$$\int_0^v v\,dv = \int_0^h g\,dy$$

$$\frac{v^2}{2} = gh$$

$$v = \sqrt{2gh}$$ **Q.E.D.**



13–26. A freight elevator, including its load, has a mass of 500 kg. It is prevented from rotating by the track and wheels mounted along its sides. When $t = 2$ s, the motor M draws in the cable with a speed of 6 m/s, *measured relative to the elevator.* If it starts from rest, determine the constant acceleration of the elevator and the tension in the cable. Neglect the mass of the pulleys, motor, and cables.



$3s_E + s_P = l$

$3v_E = -v_P$

$(+\downarrow) \qquad v_P = v_E + v_{P/E}$

$-3v_E = v_E + 6$

$$v_E = -\frac{6}{4} = -1.5\text{ m/s} = 1.5\text{ m/s}\uparrow$$

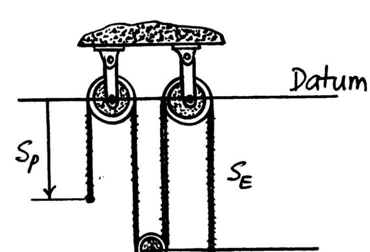
$(+\uparrow) \qquad v = v_0 + a_c t$

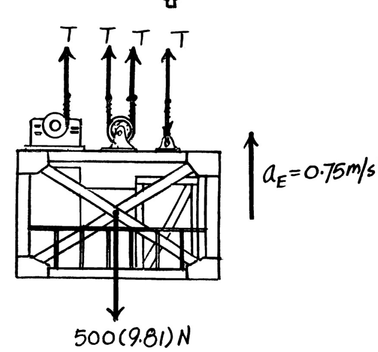
$1.5 = 0 + a_E(2)$

$a_E = 0.75\text{ m/s}^2\uparrow$ **Ans.**

$+\uparrow\Sigma F_y = ma_y; \qquad 4T - 500(9.81) = 500(0.75)$

$T = 1320\text{ N} = 1.32\text{ kN}$ **Ans.**



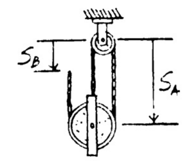


13–27. Determine the required mass of block A so that when it is released from rest it moves the 5-kg block B a distance of 0.75 m up along the smooth inclined plane in $t = 2$ s. Neglect the mass of the pulleys and cords.

Kinematic: Applying equation $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$, we have

$$(\nwarrow +) \qquad 0.75 = 0 + 0 + \frac{1}{2} a_B \left(2^2\right) \qquad a_B = 0.375 \text{ m/s}^2$$

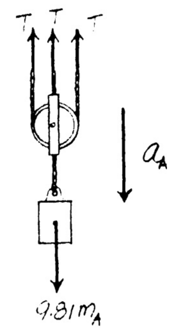
Establishing the position - coordinate equation, we have



$$2s_A + (s_A - s_B) = l \qquad 3s_A - s_B = l$$

Taking time derivative twice yields

$$3a_A - a_B = 0 \tag{1}$$



(a)

From Eq.(1),

$$3a_A - 0.375 = 0 \qquad a_A = 0.125 \text{ m/s}^2$$

Equation of Motion: The tension T developed in the cord is the same throughout the entire cord since the cord passes over the smooth pulleys. From FBD(b),

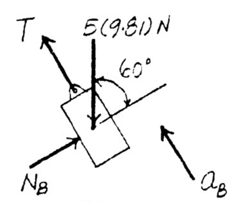
$$\nwarrow + \Sigma F_{y'} = ma_{y'}; \qquad T - 5(9.81)\sin 60^\circ = 5(0.375)$$

$$T = 44.35 \text{ N}$$

From FBD(a),

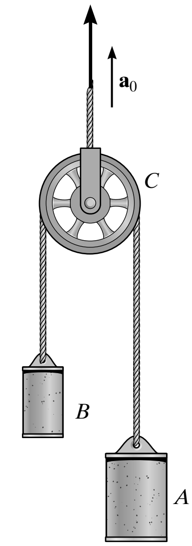
$$+\uparrow \Sigma F_y = ma_y; \qquad 3(44.35) - 9.81m_A = m_A(-0.125)$$

$$m_A = 13.7 \text{ kg} \qquad \textbf{Ans.}$$



(b)

***13–28.** Blocks A and B have a mass of m_A and m_B, where $m_A > m_B$. If pulley C is given an acceleration of $\mathbf{a}_0$, determine the acceleration of the blocks. Neglect the mass of the pulley.



Free-Body Diagram: The free-body diagram of blocks A and B are shown in Figs, (a) and (b), respectively. Here, $\mathbf{a}_A$ and $\mathbf{a}_B$ are assumed to be directed upwards. Since pulley C is smooth, the tension in the cord remains constant for the entire cord.

Equations of Motion: By referring to Figs. (a) and (b),

$$+\uparrow \Sigma F_y = ma_y; \qquad T - m_A g = m_A a_A \qquad \textbf{(1)}$$

and

$$+\uparrow \Sigma F_y = ma_y; \qquad T - m_B g = m_B a_B \qquad \textbf{(2)}$$

Eliminating **T** from Eqs. (1) and (2) yields

$$(m_A - m_B)g = m_B a_B - m_A a_A \qquad \textbf{(3)}$$

Kinematics: The acceleration of blocks A and B relative to pulley C will be of the same magnitude, i.e., $a_{A/C} = a_{B/C} = a_{\text{rel}}$. If we assume that $\mathbf{a}_{A/C}$ is directed downwards, $\mathbf{a}_{B/C}$ must also be directed downwards to be consistent. Applying the relative acceleration equation,

$$(+\uparrow) \qquad \mathbf{a}_A = \mathbf{a}_C + \mathbf{a}_{A/C}$$

$$a_A = a_O - a_{\text{rel}} \qquad \textbf{(4)}$$

and

$$(+\uparrow) \qquad \mathbf{a}_B = \mathbf{a}_C + \mathbf{a}_{B/C}$$

$$a_B = a_O - a_{\text{rel}} \qquad \textbf{(5)}$$

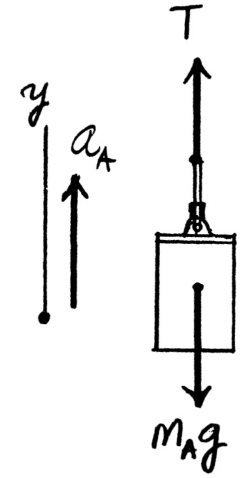
Eliminating a_{rel} from Eqs.(4) and (5),

$$a_A + a_B = 2a_O \qquad \textbf{(6)}$$

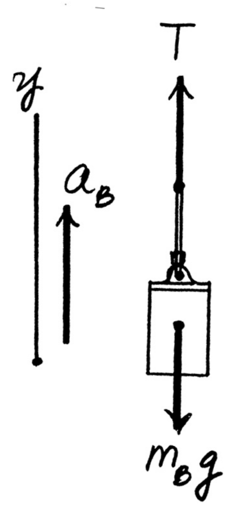
Solving Eqs. (3) and (6), yields

$$a_A = \frac{2mg\, a_O - (m_A - m_B)g}{m_A + m_B} \uparrow \qquad \textbf{Ans.}$$

$$a_B = \frac{2m_A a_O + (m_A - m_B)g}{m_A + m_B} \uparrow \qquad \textbf{Ans.}$$

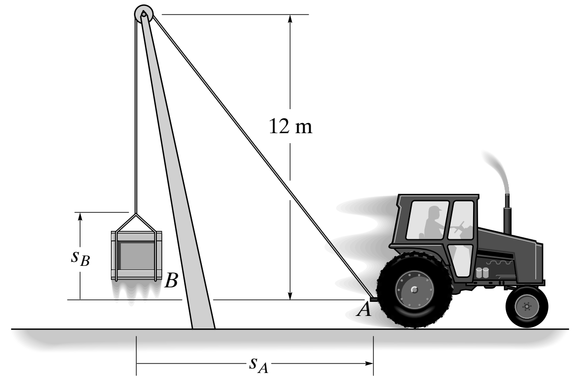


(a)



(b)

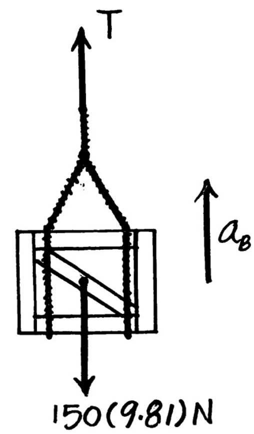
•13–29. The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right at a constant speed of 4 m/s, determine the tension in the rope when $s_A = 5$ m. When $s_A = 0$, $s_B = 0$.



$$12 - s_B + \sqrt{s_A^2 + (12)^2} = 24$$

$$-\dot{s}_B + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A \dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \ddot{s}_A\right) = 0$$



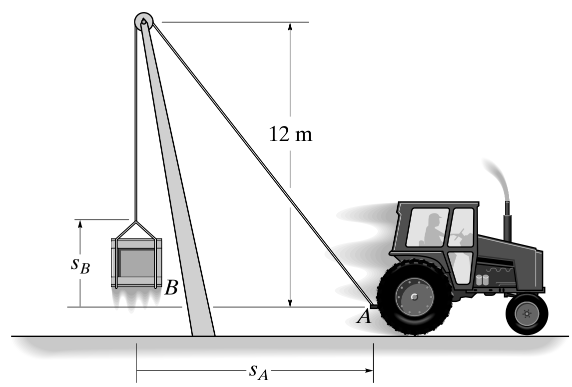
$$\ddot{s}_B = -\left[\frac{s_A^2 \dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A \ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + 0}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 1.0487 \text{ m/s}^2$$

$+\uparrow \Sigma F_y = ma_y;\qquad T - 150(9.81) = 150(1.0487)$

$T = 1.63$ kN **Ans.**

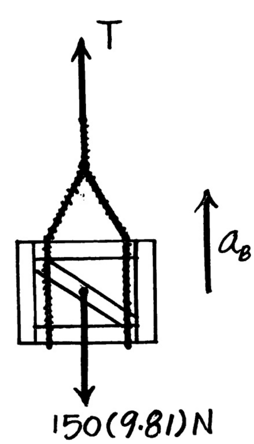
13–30. The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right with an acceleration of 3 m/s^2 and has a velocity of 4 m/s at the instant $s_A = 5$ m, determine the tension in the rope at this instant. When $s_A = 0$, $s_B = 0$.



$$12 = s_B + \sqrt{s_A^2 + (12)^2} = 24$$

$$-\dot{s}_B + \frac{1}{2}\left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(2 s_A \dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A \dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \ddot{s}_A\right) = 0$$



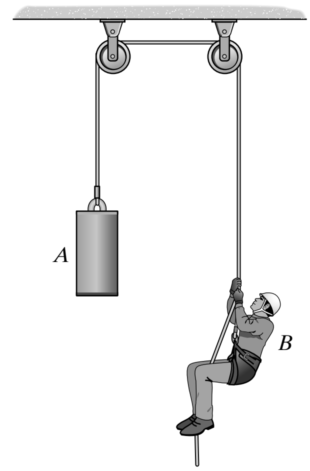
$$\ddot{s}_B = -\left[\frac{s_A^2 \dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A \ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + (5)(3)}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 2.2025 \text{ m/s}^2$$

$+\uparrow \Sigma F_y = ma_y;\qquad T - 150(9.81) = 150(2.2025)$

$T = 1.80$ kN **Ans.**

13–31. The 75-kg man climbs up the rope with an acceleration of $0.25\,\text{m/s}^2$, measured relative to the rope. Determine the tension in the rope and the acceleration of the 80-kg block.



Free-Body Diagram: The free-body diagram of the man and block A are shown in Figs. (a) and (b), respectively. Here, the acceleration of the man a_m and the block a_A are assumed to be directed upwards.

Equations of Motion: By referring to Figs. (a) and (b),

$$+\uparrow \Sigma F_y = ma_y; \qquad T - 75(9.81) = 75a_m \qquad \textbf{(1)}$$

and

$$+\uparrow \Sigma F_y = ma_y; \qquad T - 80(9.81) = 80a_A \qquad \textbf{(2)}$$

Kinematics: Here, the rope has an acceleration with a magnitude equal to that of block A, i.e., $a_r = a_A$ and is directed downward. Applying the relative acceleration equation,

$$(+\uparrow) \qquad \mathbf{a}_m = \mathbf{a}_r + \mathbf{a}_{m/r}$$

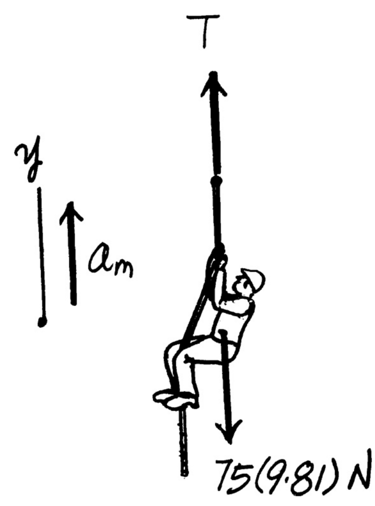
$$a_m = -a_A + 0.25 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3) yields

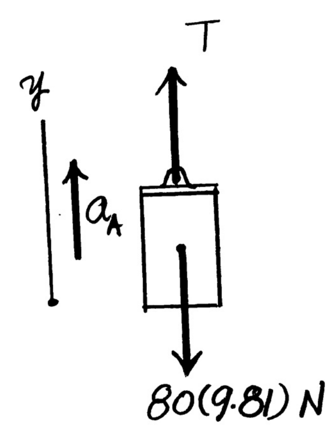
$$a_A = -0.19548\text{ m/s}^2 = 0.195\text{ m/s}^2\downarrow \qquad \textbf{Ans.}$$

$$T = 769.16\text{ N} = 769\text{ N} \qquad \textbf{Ans.}$$

$$a_m = 0.4455\text{ m/s}^2$$

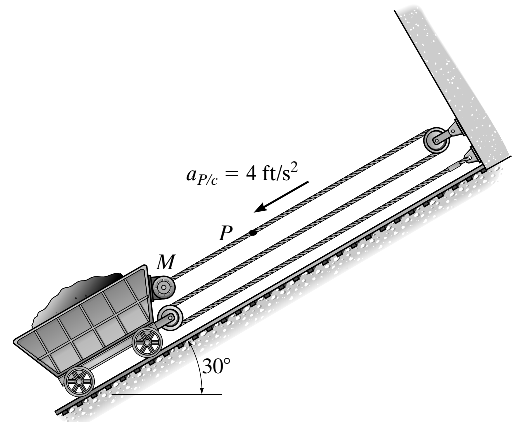


(a)



(b)

***13–32.** Motor M draws in the cable with an acceleration of $4\,\text{ft/s}^2$, measured relative to the 200-lb mine car. Determine the acceleration of the car and the tension in the cable. Neglect the mass of the pulleys.



Free-Body Diagram: The free-body diagram of the mine car is shown in Fig. (a). Here, its acceleration $\mathbf{a}_C$ is assumed to be directed down the inclined plane so that it is consistent with the position coordinate s_C of the mine car as indicated on Fig. (b).

Equations of Motion: By referring to Fig. (a),

$$+\nearrow\Sigma F_{x'} = ma_{x'}; \qquad 3T - 200\sin 30^\circ = \frac{200}{32.2}(-a_C) \tag{1}$$

Kinematics: We can express the length of the cable in terms of s_P and s_C by referring to Fig. (b).

$$s_P + 2s_C = 0$$

The second derivative of the above equation gives

$$a_P + 2a_C = 0 \tag{2}$$

Applying the relative acceleration equation,

$$\mathbf{a}_P = \mathbf{a}_C + \mathbf{a}_{P/C}$$

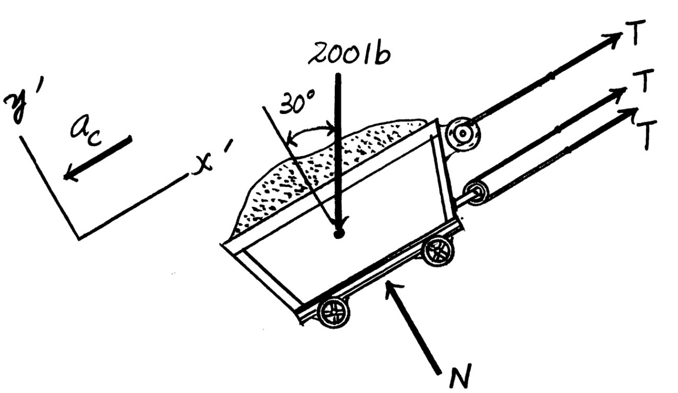
$$a_P = a_C + 4 \tag{3}$$

Solving Eqs. (1), (2), and (3) yields

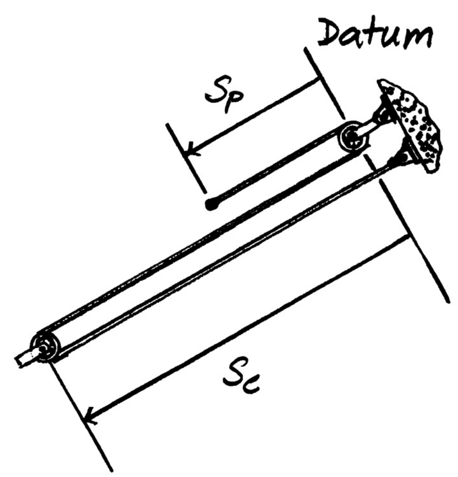
$$a_C = -1.333\ \text{ft/s}^2 = 1.33\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$T = 36.1\ \text{lb} \qquad \textbf{Ans.}$$

$$a_P = 2.667\ \text{ft/s}^2$$

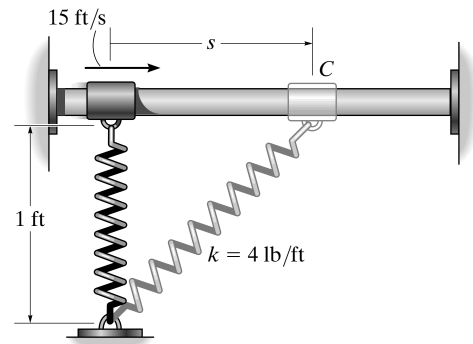


(a)



(b)

•13–33. The 2-lb collar C fits loosely on the smooth shaft. If the spring is unstretched when $s = 0$ and the collar is given a velocity of 15 ft/s, determine the velocity of the collar when $s = 1$ ft.



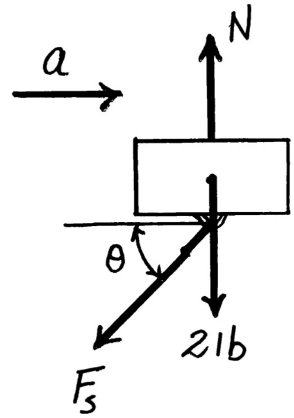
$F_s = kx; \qquad F_s = 4\left(\sqrt{1 + s^2} - 1\right)$

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -4\left(\sqrt{1 + s^2} - 1\right)\left(\dfrac{s}{\sqrt{1 + s^2}}\right) = \left(\dfrac{2}{32.2}\right)\left(v\dfrac{dv}{ds}\right)$

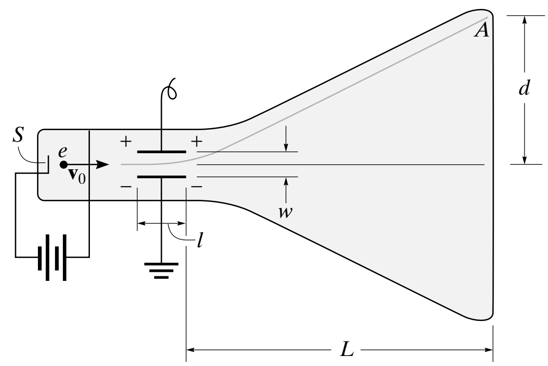
$-\displaystyle\int_0^1 \left(4s\,ds - \dfrac{4s\,ds}{\sqrt{1 + s^2}}\right) = \int_{15}^{v} \left(\dfrac{2}{32.2}\right) v\,dv$

$-\left[2s^2 - 4\sqrt{1 + s^2}\right]_0^1 = \dfrac{1}{32.2}\left(v^2 - 15^2\right)$

$v = 14.6$ ft/s **Ans.**



13–34. In the cathode-ray tube, electrons having a mass m are emitted from a source point S and begin to travel horizontally with an initial velocity $\mathbf{v}_0$. While passing between the grid plates a distance l, they are subjected to a vertical force having a magnitude eV/w, where e is the charge of an electron, V the applied voltage acting across the plates, and w the distance between the plates. After passing clear of the plates, the electrons then travel in straight lines and strike the screen at A. Determine the deflection d of the electrons in terms of the dimensions of the voltage plate and tube. Neglect gravity which causes a slight vertical deflection when the electron travels from S to the screen, and the slight deflection between the plates.



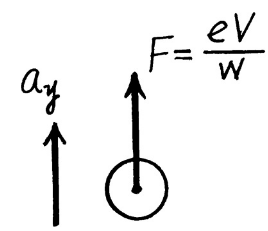
$$v_x = v_0$$

$$t_1 = \frac{l}{v_0}$$

t_1 is the time between plates.

$$t_1 = \frac{L}{v_0}$$

t_2 is the tune to reach screen.



$$+\uparrow \Sigma F_y = ma_y; \qquad \frac{eV}{w} = ma_y$$

$$a_y = \frac{eV}{mw}$$

During t_1 constant acceleration,

$$(+\uparrow) \qquad v = v_0 + a_c t$$

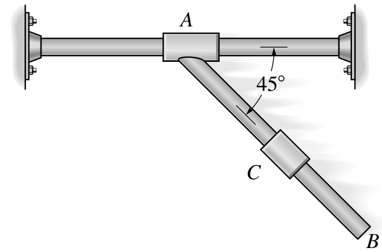
$$v_y = a_y t_1 = \left(\frac{eV}{mw}\right)\left(\frac{l}{v_0}\right)$$

During time t_2, $a_y = 0$

$$d = v_y t_2 = \left(\frac{eVl}{mwv_0}\right)\left(\frac{L}{v_0}\right)$$

$$d = \frac{eVLl}{v_0^2 wm} \qquad \textbf{Ans.}$$

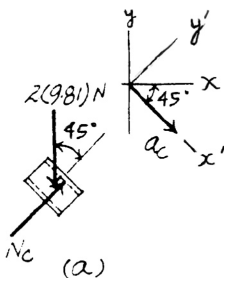
13–35. The 2-kg collar C is free to slide along the smooth shaft AB. Determine the acceleration of collar C if (a) the shaft is fixed from moving, (b) collar A, which is fixed to shaft AB, moves to the left at constant velocity along the horizontal guide, and (c) collar A is subjected to an acceleration of $2\ \text{m/s}^2$ to the left. In all cases, the motion occurs in the vertical plane.



a, b) Equation of Motion: Applying Eq. 13–7 to FBD(a), we have

$\searrow + \Sigma F_{x'} = ma_{x'}$; $2(9.81)\sin 45° = 2a_C$

$$a_C = 6.94\ \text{m/s}^2 \searrow \qquad \textbf{Ans.}$$



c) Equation of Motion: Applying Eq. 13–7 to FBD(b), we have

$\searrow + \Sigma F_{x'} = ma_{x'}$; $2(9.81)\sin 45° = 2a_{C/A} + 2(-2\cos 45°)$

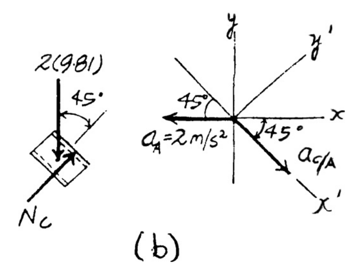
$$a_{C/A} = 8.351\ \text{m/s}^2$$

Relative Acceleration:

$\overset{+}{\rightarrow}$ $\mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{C/A}$

$$= -2\mathbf{i} + 8.351\cos 45°\mathbf{i} - 8.351\sin 45°\mathbf{j}$$

$$= \{3.905\mathbf{i} - 5.905\mathbf{j}\}\ \text{m/s}^2$$



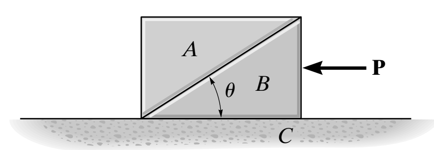
Thus, the magnitude of the acceleration $\mathbf{a}_c$ is

$$a_C = \sqrt{3.905^2 + (-5.905)^2} = 7.08\ \text{m/s}^2 \qquad \textbf{Ans.}$$

and its directional angle is

$$\theta = \tan^{-1}\left(\frac{5.905}{3.905}\right) = 56.5° \searrow \qquad \textbf{Ans.}$$

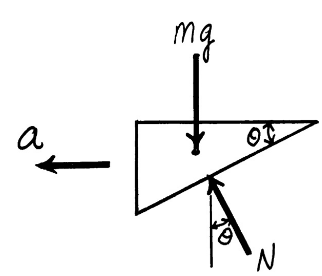
***13–36.** Blocks A and B each have a mass m. Determine the largest horizontal force $\mathbf{P}$ which can be applied to B so that A will not move relative to B. All surfaces are smooth.



Require

$a_A = a_B = a$

Block A:

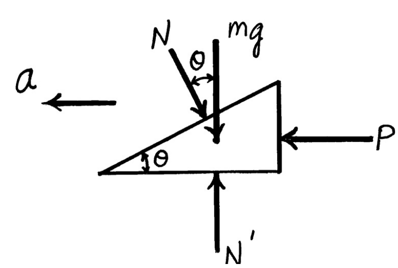


$+\uparrow \Sigma F_y = 0$; $N\cos\theta - mg = 0$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x$; $N\sin\theta = ma$

$$a = g\tan\theta$$

Block B:



$\overset{+}{\leftarrow} \Sigma F_x = ma_x$; $P - N\sin\theta = ma$

$$P - mg\tan\theta = mg\tan\theta$$

$$P = 2mg\tan\theta \qquad \textbf{Ans.}$$

•13–37. Blocks A and B each have a mass m. Determine the largest horizontal force $\mathbf{P}$ which can be applied to B so that A will not slip on B. The coefficient of static friction between A and B is μ_s. Neglect any friction between B and C.

Require

$$a_A = a_B = a$$

Block A:

$$+\uparrow \Sigma F_y = 0; \qquad N\cos\theta - \mu_s N\sin\theta - mg = 0$$

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad N\sin\theta + \mu_s N\cos\theta = ma$$

$$N = \frac{mg}{\cos\theta - \mu_s\sin\theta}$$

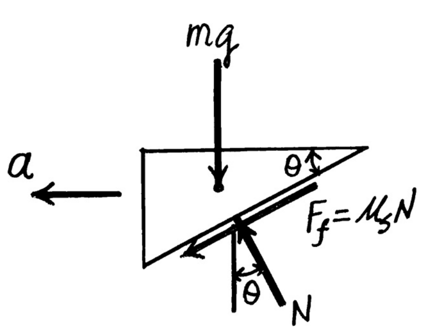
$$a = g\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

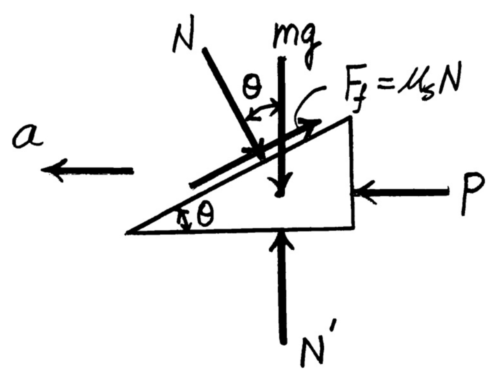
Block B:

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad P - \mu_s N\cos\theta - N\sin\theta = ma$$

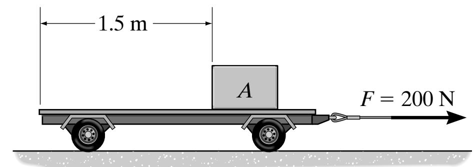
$$P - mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) = mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

$$P = 2mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) \qquad \textbf{Ans.}$$

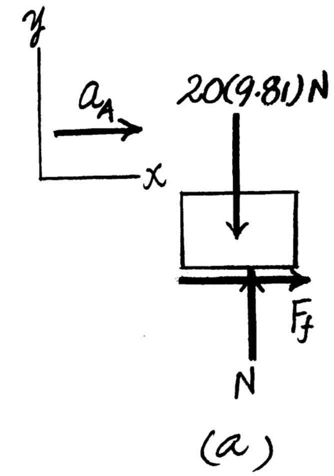




13–38. If a force $F = 200$ N is applied to the 30-kg cart, show that the 20-kg block A will slide on the cart. Also determine the time for block A to move on the cart 1.5 m. The coefficients of static and kinetic friction between the block and the cart are $\mu_s = 0.3$ and $\mu_k = 0.25$. Both the cart and the block start from rest.



Free-Body Diagram: The free-body diagram of block A and the cart are shown in Figs. (a) and (b), respectively.



Equations of Motion: If block A does not slip, it will move together with the cart with a common acceleration, i.e., $a_A = a_C = a$. By referring to Figs. (a) and (b),

$+\uparrow \Sigma F_y = ma_y; \qquad N - 20(9.81) = 2(0)$

$N = 196.2 \text{ N}$

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_f = 20a \qquad \textbf{(1)}$

and

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 200 - F_f = 30a \qquad \textbf{(2)}$

Solving Eqs. (1) and (2) yields

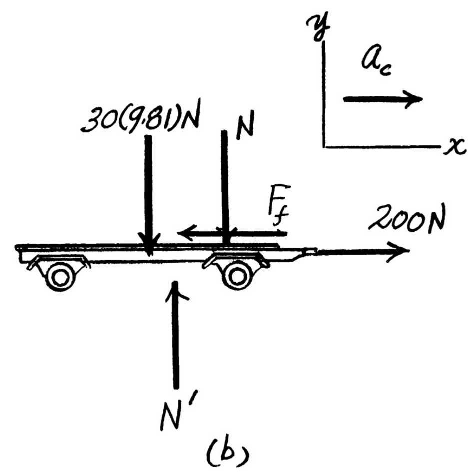
$a = 4 \text{ m/s}^2 \qquad F_f = 80 \text{ N}$

Since $F_f > (F_f)_{\max} = \mu_S N = 0.3(196.2) = 58.86$ N, the *block A will slide on the cart*. As such $F_f = \mu_k N = 0.25(196.2) = 49.05$ N. Again, by referring to Figs. (a) and (b),

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 49.05 = 20a_A \qquad a_A = 2.4525 \text{m/s}^2$

and

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 200 - 49.05 = 30a_C \qquad a_C = 5.0317 \text{ m/s}^2$



Kinematics: The acceleration of block A relative to the cart can be determined by applying the relative acceleration equation

$$\mathbf{a}_A = \mathbf{a}_C + \mathbf{a}_{A/C}$$

$$\left(\xrightarrow{+}\right) \qquad 2.4525 = 5.0317 + a_{A/C}$$

$$a_{A/C} = -2.5792 \text{ m/s}^2 = 2.5792 \text{ m/s}^2 \leftarrow$$

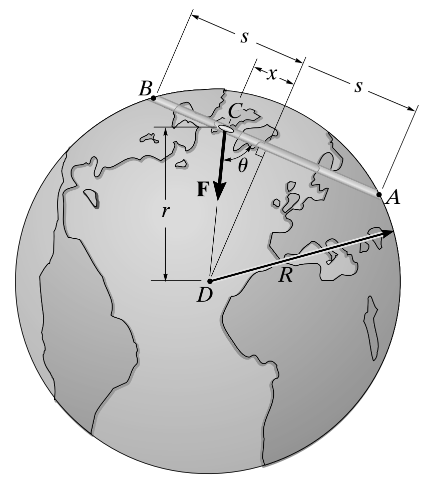
Here, $s_{A/C} = 1.5 \text{ m} \leftarrow$. Thus,

$$s_{A/C} = \left(s_{A/C}\right)_O + \left(v_{A/C}\right)_O t + \frac{1}{2} a_{A/C} t^2$$

$$\left(\xrightarrow{+}\right) \qquad 1.5 = 0 + 0 + \frac{1}{2}(2.5792)t^2$$

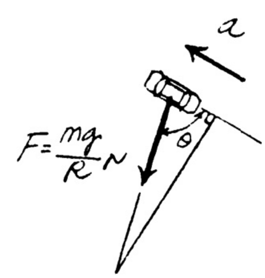
$$t = 1.08 \text{ s} \qquad \textbf{Ans.}$$

13–39. Suppose it is possible to dig a smooth tunnel through the earth from a city at A to a city at B as shown. By the theory of gravitation, any vehicle C of mass m placed within the tunnel would be subjected to a gravitational force which is always directed toward the center of the earth D. This force **F** has a magnitude that is directly proportional to its distance r from the earth's center. Hence, if the vehicle has a weight of $W = mg$ when it is located on the earth's surface, then at an arbitrary location r the magnitude of force **F** is $F = (mg/R)r$, where $R = 6328$ km, the radius of the earth. If the vehicle is released from rest when it is at B, $x = s = 2$ Mm, determine the time needed for it to reach A, and the maximum velocity it attains. Neglect the effect of the earth's rotation in the calculation and assume the earth has a constant density. *Hint:* Write the equation of motion in the x direction, noting that $r \cos \theta = x$. Integrate, using the kinematic relation $v\,dv = a\,dx$, then integrate the result using $v = dx/dt$.



Equation of Motion: Applying Eq. 13–7, we have

$$\nwarrow + \Sigma F_{x'} = ma_{x'}; \qquad -\frac{mg}{R} r \cos\theta = ma \qquad a = -\frac{g}{R} r \cos\theta = -\frac{g}{R} x$$



Kinematics: Applying equation $v\,dv = a dx$, we have

$$(\nwarrow +) \qquad \int_0^v v dv = -\frac{g}{R}\int_s^x x\,dx$$

$$\frac{v^2}{2} = \frac{g}{2R}\left(s^2 - x^2\right)$$

$$v = -\sqrt{\frac{g}{R}\left(s^2 - x^2\right)} \qquad \textbf{(1)}$$

Note: The negative sign indicates that the velocity is in the opposite direction to that of positive x.

Applying equation $dt = dx/v$, we have

$$(\nwarrow +) \qquad \int_0^t dt = -\sqrt{\frac{R}{g}}\int_s^x \frac{dx}{\sqrt{s^2 - x^2}}$$

$$t = \sqrt{\frac{R}{g}}\left(\frac{\pi}{2} - \sin^{-1}\frac{x}{s}\right)$$

At $x = -s$,
$$t = \sqrt{\frac{R}{g}}\left(\frac{\pi}{2} - \sin^{-1}\frac{-s}{s}\right) = \pi\sqrt{\frac{R}{g}} \qquad \textbf{(2)}$$

Substituting $R = 6328\left(10^3\right)$ m and $g = 9.81$ m/s^2 into Eq.(2) yields

$$t = \pi\sqrt{\frac{6328(10^3)}{9.81}} = 2523.2 \text{ s} = 42.1 \text{ min} \qquad \textbf{Ans.}$$

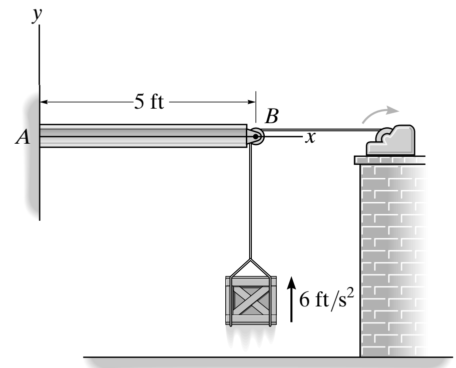
The maximum velocity occurs at $x = 0$. From Eq.(1)

$$v_{\max} = -\sqrt{\frac{g}{R}\left(s^2 - 0^2\right)} = -\sqrt{\frac{g}{R}}s \qquad \textbf{(3)}$$

Substituting $R = 6328\left(10^3\right)$ m, $s = 2\left(10^6\right)$ m, and $g = 9.81$ m/s^2 into Eq.(3) yields

$$v_{\max} = -\left(\sqrt{\frac{9.81}{6328\left(10^3\right)}}\right)\left[2\left(10^6\right)\right] = -2490.18 \text{ m/s} = 2.49 \text{ km/s} \qquad \textbf{Ans.}$$

***13–40.** The 30-lb crate is being hoisted upward with a constant acceleration of 6 ft/s^2. If the uniform beam AB has a weight of 200 lb, determine the components of reaction at the fixed support A. Neglect the size and mass of the pulley at B. *Hint:* First find the tension in the cable, then analyze the forces in the beam using statics.



Crate:

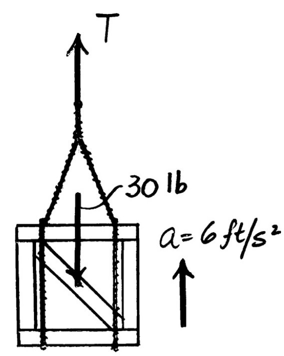
$$+\uparrow \Sigma F_y = ma_y; \qquad T - 30 = \left(\frac{30}{32.2}\right)(6) \qquad T = 35.59 \text{ lb}$$

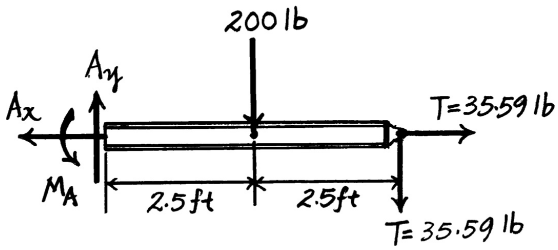
Beam:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -A_x + 35.59 = 0 \qquad A_x = 35.6 \text{ lb} \qquad \textbf{Ans.}$$

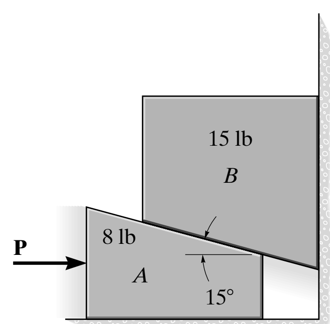
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 200 - 35.59 = 0 \qquad A_y = 236 \text{ lb} \qquad \textbf{Ans.}$$

$$+\Sigma M_A = 0; \qquad M_A - 200(2.5) - (35.59)(5) = 0 \qquad M_A = 678 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$





•13–41. If a horizontal force of $P = 10$ lb is applied to block A, determine the acceleration of block B. Neglect friction. *Hint:* Show that $a_B = a_A \tan 15°$.

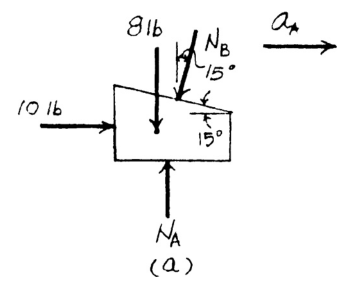


Equations of Motion: Applying Eq. 13–7 to FBD(a), we have

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 10 - N_B \sin 15° = \left(\frac{8}{32.2}\right)a_A \qquad \textbf{(1)}$$

Applying Eq. 13–7 to FBD(b), we have

$$+\uparrow \Sigma F_y = ma_y; \qquad N_B \cos 15° - 15 = \left(\frac{15}{32.2}\right)a_B \qquad \textbf{(2)}$$



Kinematics: From the geometry of Fig. (c),

$$s_B = s_A \tan 15°$$

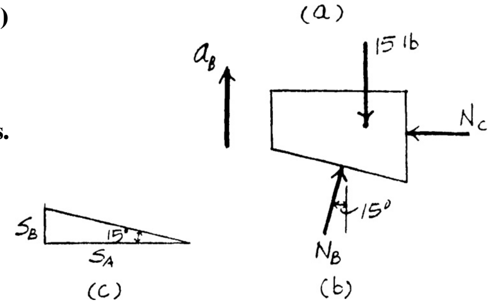
Taking the time derivative twice to the above expression yields

$$a_B = a_A \tan 15° \text{ (Q.E.D.)} \qquad \textbf{(3)}$$

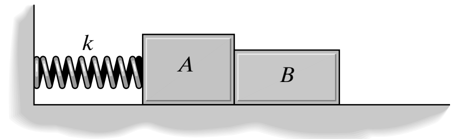
Solving Eqs.(1), (2) and (3) yields

$$a_B = 5.68 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$a_A = 21.22 \text{ ft/s}^2 \qquad N_B = 18.27 \text{ lb}$$



13–42. Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, determine the distance both blocks slide on the smooth surface before they begin to separate. What is their velocity at this instant?



Block A:

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -k(x - d) - N = m_A a_A$$

Block B:

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad N = m_B a_B$$

Since $a_A = a_B = a$,

$$-k(x - d) - m_B a = m_A a$$

$$a = \frac{k(d - x)}{(m_A + m_B)} \qquad N = \frac{km_B (d - x)}{(m_A + m_B)}$$

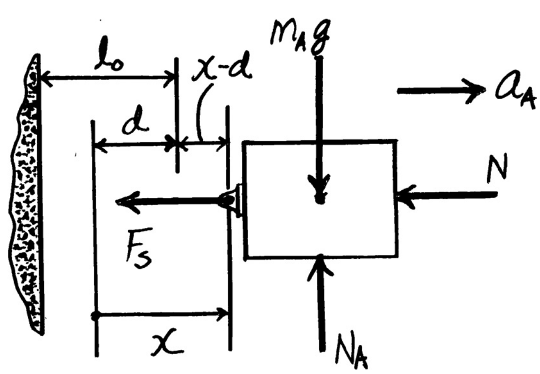
$N = 0$ when $\quad d - x = 0, \quad$ or $\quad x = d$ **Ans.**

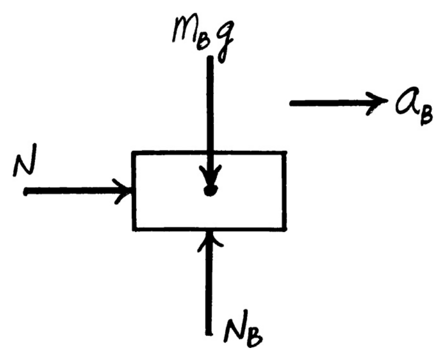
$$v\,dv = a\,dx$$

$$\int_0^v v\,dv = \int_0^d \frac{k(d - x)}{(m_A + m_B)}\,dx$$

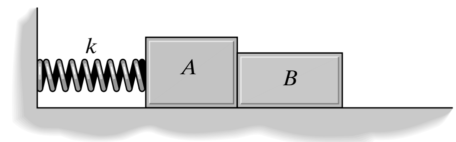
$$\frac{1}{2}v^2 = \frac{k}{(m_A + m_B)}\left[(d)x - \frac{1}{2}x^2\right]_0^d = \frac{1}{2}\frac{kd^2}{(m_A + m_B)}$$

$$v = \sqrt{\frac{kd^2}{(m_A + m_B)}}$$ **Ans.**





13–43. Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, show that for separation to occur it is necessary that $d > 2\mu_k g(m_A + m_B)/k$, where μ_k is the coefficient of kinetic friction between the blocks and the ground. Also, what is the distance the blocks slide on the surface before they separate?



Block A:

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -k(x - d) - N - \mu_k m_A g = m_A a_A$

Block B:

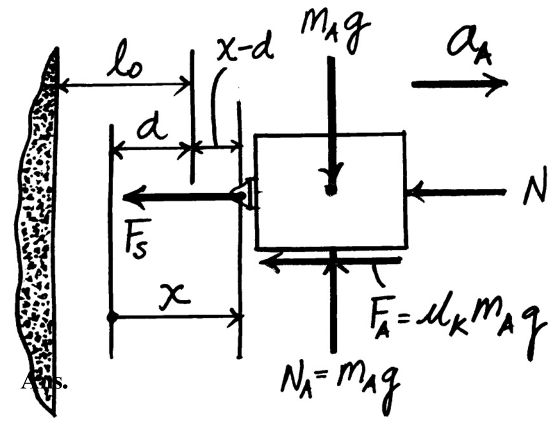
$\xrightarrow{+} \Sigma F_x = ma_x; \qquad N - \mu_k m_B g = m_B a_B$

Since $a_A = a_B = a$,

$$a = \frac{k(d - x) - \mu_k g(m_A + m_B)}{(m_A + m_B)} = \frac{k(d - x)}{(m_A + m_B)} - \mu_k g$$

$$N = \frac{km_B(d - x)}{(m_A + m_B)}$$

$N = 0$, then $x = d$ for separation. **Ans.**



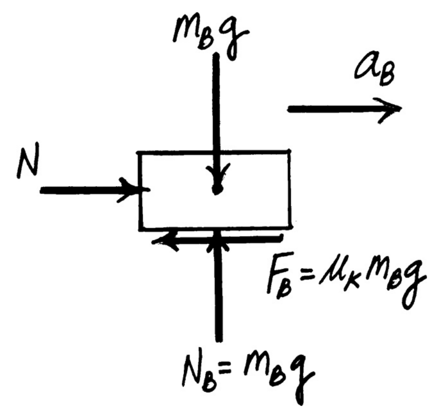
At the moment of separation:

$v\,dv = a\,dx$

$$\int_0^v v\,dv = \int_0^d \left[\frac{k(d - x)}{(m_A + m_B)} - \mu_k g\right] dx$$

$$\frac{1}{2}v^2 = \frac{k}{(m_A + m_B)}\left[(d)x - \frac{1}{2}x^2 - \mu_k g\,x\right]_0^d$$

$$v = \sqrt{\frac{kd^2 - 2\mu_k g(m_A + m_B)d}{(m_A + m_B)}}$$



Require $v > 0$, so that

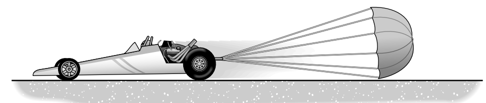
$kd^2 - 2\mu_k g(m_A + m_B)d > 0$

Thus,

$kd > 2\mu_k g(m_A + m_B)$

$$d > \frac{2\mu_k g}{k}(m_A + m_B) \qquad \textbf{Q.E.D.}$$

***13–44.** The 600-kg dragster is traveling with a velocity of 125 m/s when the engine is shut off and the braking parachute is deployed. If air resistance imposed on the dragster due to the parachute is $F_D = (6000 + 0.9v^2)$ N, where v is in m/s, determine the time required for the dragster to come to rest.



Free-Body Diagram: The free-body diagram of the dragster is shown in Fig. (a).

Equations of Motion: By referring to Fig. (a),

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 6000 + 0.9v^2 = 600(-a)$$

$$a = -\left[10 + 1.5(10^{-3})v^2\right] \text{ m/s}^2$$

$$= -1.5(10^{-3})\left[6666.67 + v^2\right] \text{ m/s}^2$$

Kinematics: Using the result of **a**, the time the dragster takes to stop can be obtained by integrating.

$$(\xleftarrow{+}) \qquad \int dt = \int \frac{dv}{a}$$

$$\int_0^t dt = \int_{125 \text{ m/s}}^{v} \frac{dv}{-1.5(10^{-3})(6666.67 + v^2)}$$

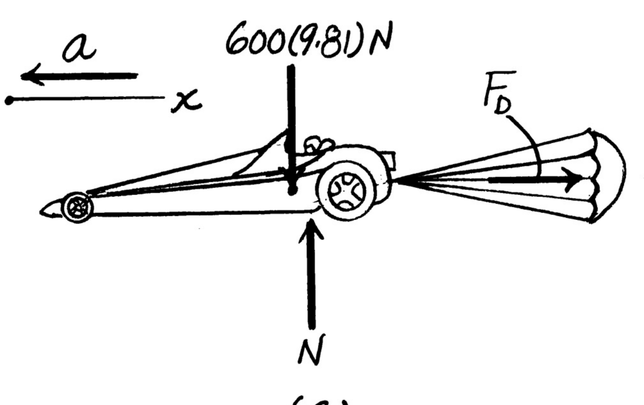
$$t = -666.67 \int_{125 \text{ m/s}}^{v} \frac{dv}{6666.67 + v^2}$$

$$= -666.67\left[\frac{1}{\sqrt{6666.67(1)}} \tan^{-1}\left(\frac{v}{\sqrt{6666.7}}\right)\right]\Bigg|_{125 \text{ m/s}}^{v}$$

$$= 8.165\left[0.9922 - \tan^{-1}(0.01225v)\right]$$

When $v = 0$,

$$t = 8.165\left[0.9922 - \tan^{-1}(0)\right] = 8.10 \text{ s} \qquad \textbf{Ans.}$$



(a)

•13–45. The buoyancy force on the 500-kg balloon is $F = 6\,\text{kN}$, and the air resistance is $F_D = (100v)\,\text{N}$, where v is in m/s. Determine the terminal or maximum velocity of the balloon if it starts from rest.

Free-Body Diagram: The free-body diagram of the balloon is shown in Fig. (a).

Equations of Motion: By referring to Fig. (a),

$$+\uparrow \Sigma F_y = ma_y; \qquad 6000 - 500(9.81)100v = 500a$$

$$a = (2.19 - 0.2v)\ \text{m/s}^2$$

Kinematics: Using the result of **a**, the velocity of the balloon as a function of t can be determined by integrating the kinematic equation, $dt = \dfrac{dv}{a}$. Here, the initial condition $v = 0$ at $t = 0$ will be used as the integration limit. Thus,

$$(+\uparrow) \qquad \int dt = \int \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{2.19 - 0.2v}$$

$$t = -\frac{1}{0.2}\ln(2.19 - 0.2v)\Big|_0^v$$

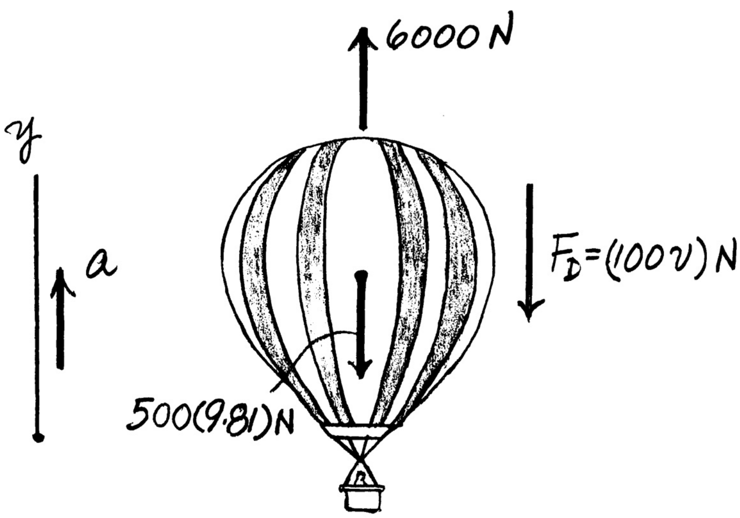
$$t = 5\ln\left(\frac{2.19}{2.19 - 0.2v}\right)$$

$$e^{t/5} = \frac{2.19}{2.19 - 0.2v}$$

$$v = 10.95(1 - e^{-t/5})$$

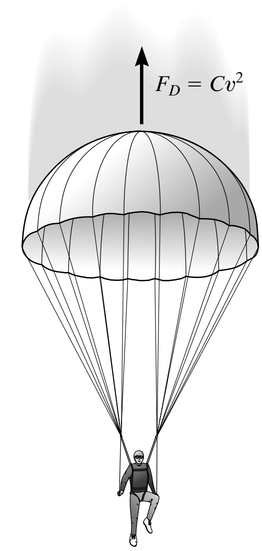
When $t \to \infty$, the balloon achieves its terminal velocity. Since $e^{-t/5} \to 0$ when $t \to \infty$,

$$v_{\max} = 10.95\ \text{m/s} \qquad \textbf{Ans.}$$



(a)

13–46. The parachutist of mass m is falling with a velocity of v_0 at the instant he opens the parachute. If air resistance is $F_D = Cv^2$, determine her maximum velocity (terminal velocity) during the descent.



Free-Body Diagram: The free-body diagram of the parachutist is shown in Fig. (a).

Equations of Motion: By referring to Fig. (a),

$$+\downarrow \Sigma F_y = ma_y; \qquad mg - cv^2 = ma$$

$$a = \frac{mg - cv^2}{m} = g - \frac{c}{m}v^2 \downarrow$$

Kinematics: Using the result of **a**, the velocity of the parachutist as a function of t can be determined by integrating the kinematic equation, $dt = \frac{dv}{a}$. Here, the initial condition $v = v_0$ at $t = 0$ will be used as the integration limit. Thus,

$$(+\downarrow) \qquad \int dt = \int \frac{dv}{a}$$

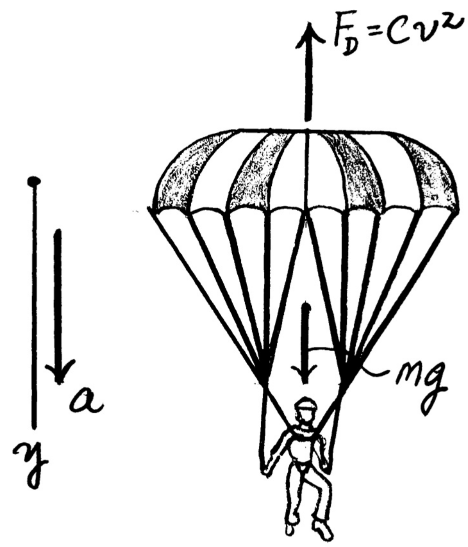
$$\int_0^t dt = \int_{v_0}^{v} \frac{dv}{g - \frac{c}{m}v^2}$$

$$t = \frac{1}{2\sqrt{\frac{gc}{m}}} \ln\left(\frac{\sqrt{g} + \sqrt{\frac{c}{m}}v}{\sqrt{g} - \sqrt{\frac{c}{m}}v}\right)\Bigg|_{v_0}^{v}$$

$$t = \frac{1}{2}\sqrt{\frac{m}{gc}} \ln\left[\frac{\sqrt{\frac{mg}{c}} + v}{\sqrt{\frac{mg}{c}} - v}\right]\Bigg|_{v_0}^{v}$$

$$2\sqrt{\frac{m}{gc}}t = \ln\left[\frac{\left(\sqrt{\frac{mg}{c}} + v\right)\left(\sqrt{\frac{mg}{c}} - v\right)}{\left(\sqrt{\frac{mg}{c}} - v\right)\left(\sqrt{\frac{mg}{c}} + v_0\right)}\right]$$

$$e^{2\sqrt{\frac{gc}{m}}t} = \left[\frac{\left(\sqrt{\frac{mg}{c}} + v\right)\left(\sqrt{\frac{mg}{c}} - v_0\right)}{\left(\sqrt{\frac{mg}{c}} - v\right)\left(\sqrt{\frac{mg}{c}} + v_0\right)}\right]$$



(a)

13–46. Continued

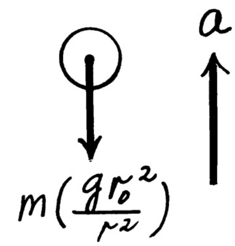
$$v = \frac{\sqrt{\frac{mg}{c}}\left[\left(\frac{\sqrt{\frac{m}{gc}} + v_0}{\sqrt{\frac{m}{gc}} - v_0}\right)e^{2\sqrt{\frac{gc}{m}}t} - 1\right]}{1 + \left(\frac{\sqrt{\frac{m}{gc}} + v_0}{\sqrt{\frac{m}{gc}} - v_0}\right)e^{2\sqrt{\frac{gc}{m}}t}}$$

When $t \rightarrow \infty$, $\left(\frac{\sqrt{\frac{m}{gc}} + v_0}{\sqrt{\frac{m}{gc}} - v_0}\right)e^{2\sqrt{\frac{gc}{m}}t} - 1 \cong 1 + \left(\frac{\sqrt{\frac{m}{gc}} + v_0}{\sqrt{\frac{m}{gc}} - v_0}\right)e^{2\sqrt{\frac{gc}{m}}t}$. Thus, the terminal velocity of the parachutist is

$$v_{max} = \sqrt{\frac{mg}{c}} \qquad \textbf{Ans.}$$

Note: The terminal velocity of the parachutist is independent of the initial velocity v_0.

13–47. The weight of a particle varies with altitude such that $W = m(gr_0^2)/r^2$, where r_0 is the radius of the earth and r is the distance from the particle to the earth's center. If the particle is fired vertically with a velocity v_0 from the earth's surface, determine its velocity as a function of position r. What is the smallest velocity v_0 required to escape the earth's gravitational field, what is r_{max}, and what is the time required to reach this altitude?



$$+\uparrow \Sigma F_y = ma_y; \qquad -m\left(\frac{gr_0^2}{r^2}\right) = ma$$

$$a = \frac{gr_0^2}{r^2}$$

$$v\,dv = a\,dr$$

$$\int_{v_0}^{v} v\,dv = \int_{r_0}^{r} -gr_0^2 \frac{dr}{r^2}$$

$$\frac{1}{2}(v^2 - v_0^2) = -gr_0^2\left[-\frac{1}{r}\right]_0^r = gr_0^2\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

$$v = \sqrt{v_0^2 - 2gr_0\left(1 - \frac{r_0}{r}\right)}$$ **Ans. (1)**

For minimum escape, require $v = 0$,

$$v_0^2 - 2gr_0\left(1 - \frac{r_0}{r}\right) = 0$$

$$r_{max} = \frac{2gr_0^2}{2gr_0 - v_0^2}$$ **Ans.**

$r_{max} \rightarrow \infty$ when $v_0^2 \rightarrow 2gr_0$

Escape velocity is

$$v_{esc} = \sqrt{2gr_0}$$ **Ans. (2)**

From Eq. (1), using the value for v from Eq. (2),

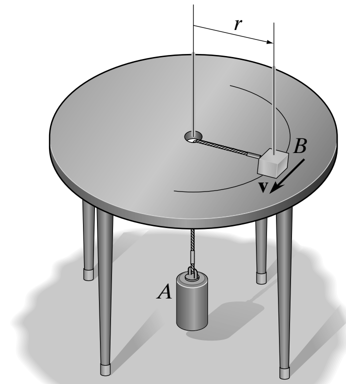
$$v = \frac{dr}{dt} = \sqrt{\frac{2gr_0^2}{r}}$$

$$\int_{r_0}^{r} \frac{dr}{\sqrt{\frac{2gr_0^2}{r}}} = \int_0^t dt$$

$$\frac{1}{\sqrt{2gr_0^2}}\left[\frac{2}{3}r^{\frac{3}{2}}\right]_{r_0}^{r_{max}} = t$$

$$t = \frac{2}{3r_0\sqrt{2g}}\left(r_{max}^{\frac{3}{2}} - r_0^{\frac{3}{2}}\right)$$ **Ans.**

***13–48.** The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block is given a speed of $v = 10\,\text{m/s}$, determine the radius r of the circular path along which it travels.

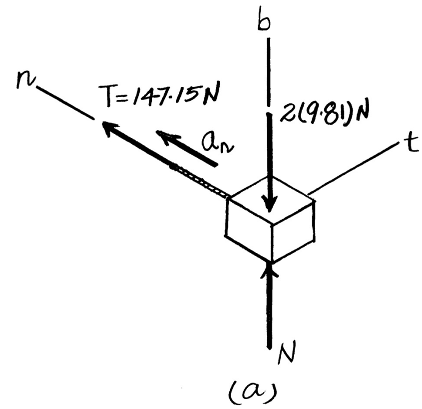


Free-Body Diagram: The free-body diagram of block B is shown in Fig. (a). The tension in the cord is equal to the weight of cylinder A, i.e., $T = 15(9.81)\text{N} = 147.15\text{ N}$. Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

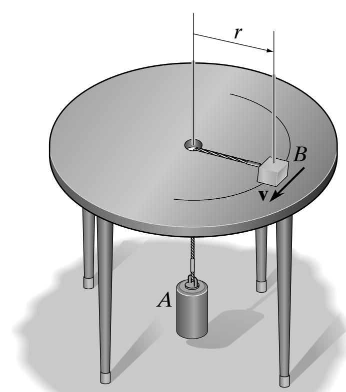
Equations of Motion: Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{10^2}{r}$ and referring to Fig. (a),

$$\Sigma F_n = ma_n; \qquad 147.15 = 2\left(\frac{10^2}{r}\right)$$

$$r = 1.36\text{ m} \qquad \textbf{Ans.}$$



•13–49. The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block travels along a circular path of radius $r = 1.5\,\text{m}$, determine the speed of the block.

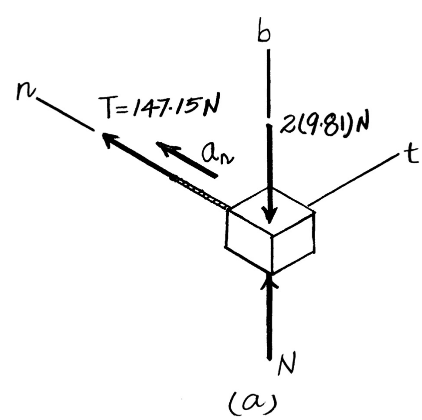


Free-Body Diagram: The free-body diagram of block B is shown in Fig. (a). The tension in the cord is equal to the weight of cylinder A, i.e., $T = 15(9.81)\text{N} = 147.15\text{N}$. Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

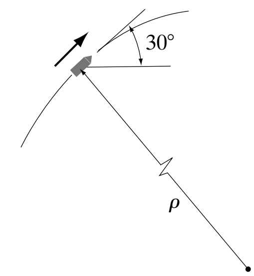
Equations of Motion: Realizing that $a_n = \dfrac{v^2}{r} = \dfrac{v^2}{1.5}$ and referring to Fig. (a),

$$\Sigma F_n = ma_n; \qquad 147.15 = 2\left(\frac{v^2}{1.5}\right)$$

$$v = 10.5\text{ m/s} \qquad \textbf{Ans.}$$



13–50. At the instant shown, the 50-kg projectile travels in the vertical plane with a speed of $v = 40\,\text{m/s}$. Determine the tangential component of its acceleration and the radius of curvature ρ of its trajectory at this instant.



Free-Body Diagram: The free-body diagram of the projectile is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature of the trajectory (positive n axis).

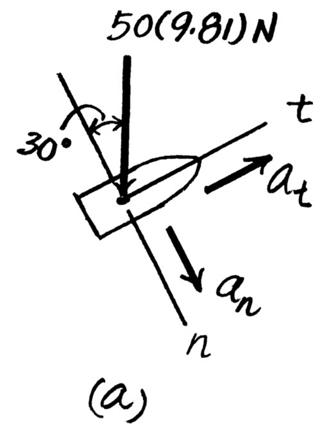
Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{40^2}{\rho}$. By referring to Fig. (a),

$+\nearrow\Sigma F_t = ma_t;$ $\qquad -50(9.81)\sin 30° = 50a_t$

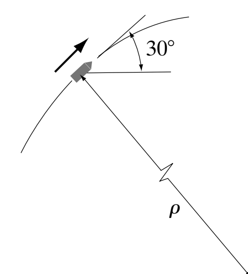
$$a_t = -4.905\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$+\searrow\Sigma F_n = ma_n;$ $\qquad 50(9.81)\cos 30° = 50\left(\dfrac{40^2}{\rho}\right)$

$$\rho = 188\ \text{m} \qquad \textbf{Ans.}$$



13–51. At the instant shown, the radius of curvature of the vertical trajectory of the 50-kg projectile is $\rho = 200\,\text{m}$. Determine the speed of the projectile at this instant.

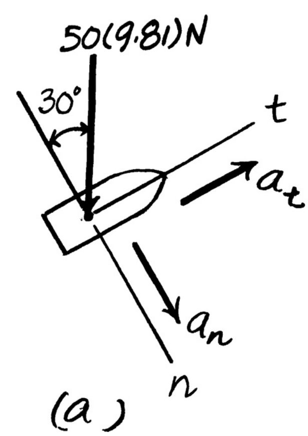


Free-Body Diagram: The free-body diagram of the projectile is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature of the trajectory (positive n axis).

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{v^2}{200}$. By referring to Fig. (a),

$\searrow+\Sigma F_n = ma_n;$ $\qquad 50(9.81)\cos 30° = 50\left(\dfrac{v^2}{200}\right)$

$$v = 41.2\ \text{m/s} \qquad \textbf{Ans.}$$

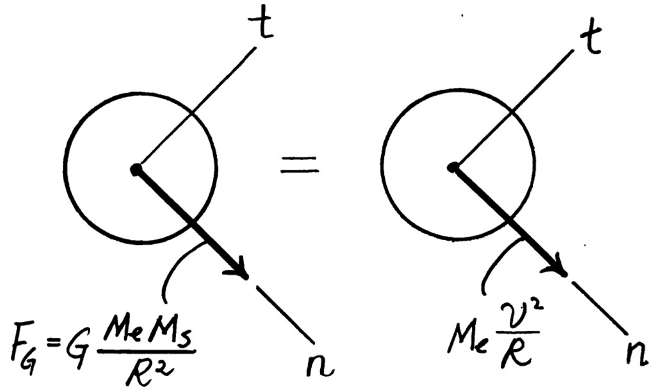


***13–52.** Determine the mass of the sun, knowing that the distance from the earth to the sun is $149.6(10^6)$ km. *Hint:* Use Eq. 13–1 to represent the force of gravity acting on the earth.

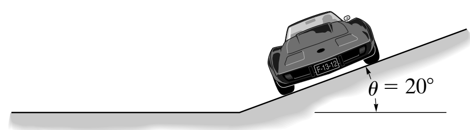
$$\searrow + \Sigma F_n = ma_n; \qquad G\frac{M_e M_s}{R^2} = M_e \frac{v^2}{R} \qquad M_s = \frac{v^2 R}{G}$$

$$v = \frac{s}{t} = \frac{2\pi(149.6)(10^9)}{365(24)(3600)} = 29.81\left(10^3\right)\text{ m/s}$$

$$M_s = \frac{\left[(29.81)(10^3)\right]^2(149.6)(10^9)}{66.73(10^{-12})} = 1.99\left(10^{30}\right)\text{ kg} \qquad \textbf{Ans.}$$



•13–53. The sports car, having a mass of 1700 kg, travels horizontally along a 20° banked track which is circular and has a radius of curvature of $\rho = 100$ m. If the coefficient of static friction between the tires and the road is $\mu_s = 0.2$, determine the *maximum constant speed* at which the car can travel without sliding up the slope. Neglect the size of the car.

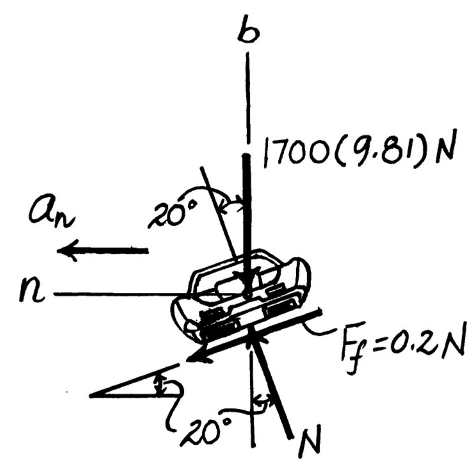


$$+\uparrow \Sigma F_b = 0; \qquad N\cos 20° - 0.2N\sin 20° - 1700(9.81) = 0$$

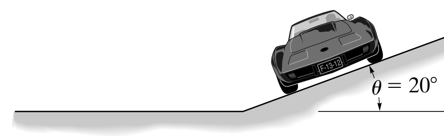
$$N = 19\,140.6\text{ N}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad 19\,140.6\sin 20° + 0.2(19\,140.6)\cos 20° = 1700\left(\frac{v_{max}^2}{100}\right)$$

$$v_{max} = 24.4\text{ m/s} \qquad \textbf{Ans.}$$



13–54. Using the data in Prob. 13–53, determine the *minimum speed* at which the car can travel around the track without sliding down the slope.

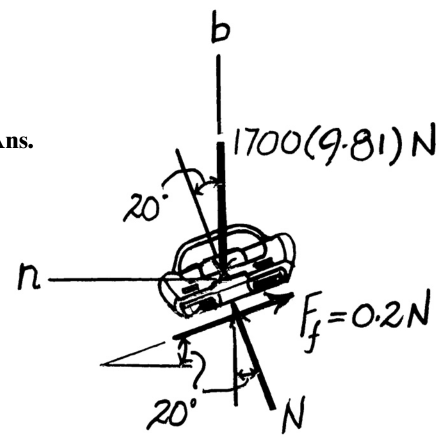


$$+\uparrow \Sigma F_b = 0; \qquad N \cos 20° + 0.2N \sin 20° - 1700(9.81) = 0$$

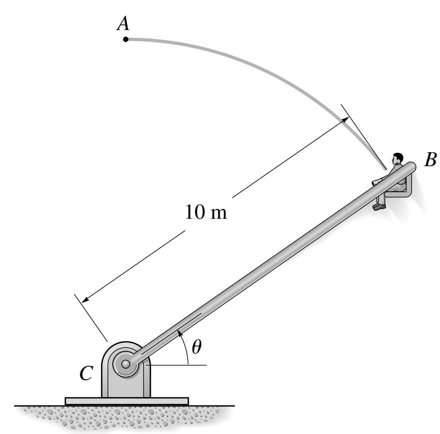
$$N = 16543.1 \text{ N}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad 16543.1 \sin 20° - 0.2(16543.1) \cos 20° = 1700\left(\frac{v_{min}^2}{100}\right)$$

$$v_{min} = 12.2 \text{ m/s} \qquad \textbf{Ans.}$$



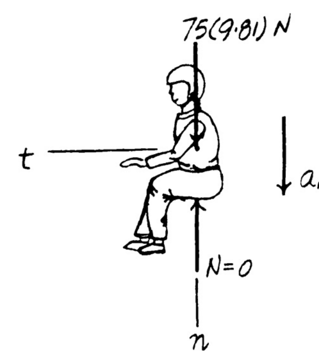
13–55. The device shown is used to produce the experience of weightlessness in a passenger when he reaches point A, $\theta = 90°$, along the path. If the passenger has a mass of 75 kg, determine the minimum speed he should have when he reaches A so that he does not exert a normal reaction on the seat. The chair is pin-connected to the frame BC so that he is always seated in an upright position. During the motion his speed remains constant.



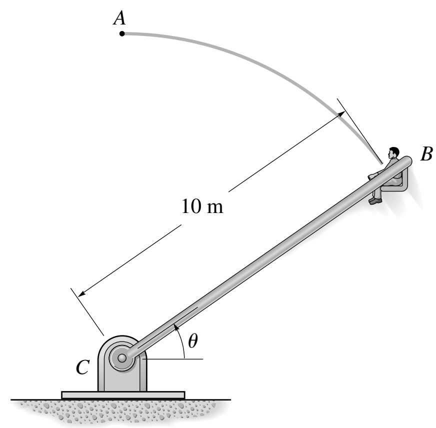
Equation of Motion: If the man is about to fly off from the seat, the normal reaction $N = 0$. Applying Eq. 13–8, we have

$$+\downarrow \Sigma F_n = ma_n; \qquad 75(9.81) = 75\left(\frac{v^2}{10}\right)$$

$$v = 9.90 \text{ m/s} \qquad \textbf{Ans.}$$



***13–56.** A man having the mass of 75 kg sits in the chair which is pin-connected to the frame BC. If the man is always seated in an upright position, determine the horizontal and vertical reactions of the chair on the man at the instant $\theta = 45°$. At this instant he has a speed of 6 m/s, which is increasing at 0.5 m/s².



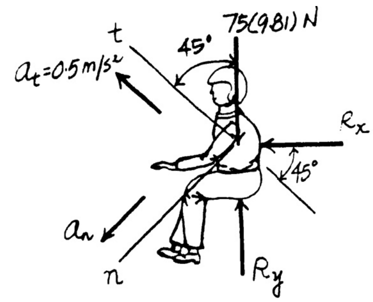
Equation of Motion: Applying Eq. 13–8, we have

$+\nwarrow\Sigma F_t = ma_t;$ $\quad R_x \cos 45° + R_y \cos 45° - 75(9.81)\cos 45° = 75(0.5)$ **(1)**

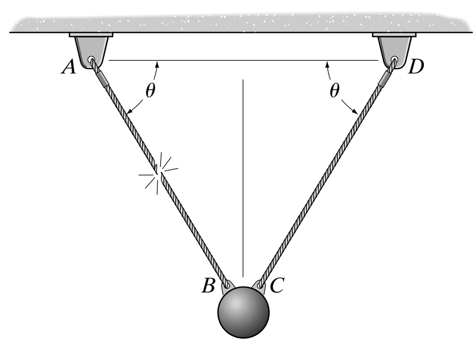
$+\swarrow\Sigma F_n = ma_n;$ $\quad R_x \sin 45° - R_y \sin 45° + 75(9.81)\sin 45° = 75\left(\frac{6^2}{10}\right)$ **(2)**

Solving Eqs.(1) and (2) yields

$R_x = 217 \text{ N} \qquad R_y = 571 \text{ N}$ **Ans.**



•13–57. Determine the tension in wire CD just after wire AB is cut. The small bob has a mass m.

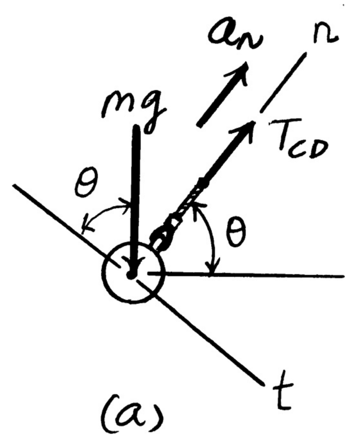


Free-Body Diagram: The free-body diagram of the bob is shown in Fig. (a).

Equations of Motion: Since the speed of the bob is zero just after the wire AB is cut, its normal component of acceleration is $a_n = \dfrac{v^2}{\rho} = 0$. By referring to Fig. (a),

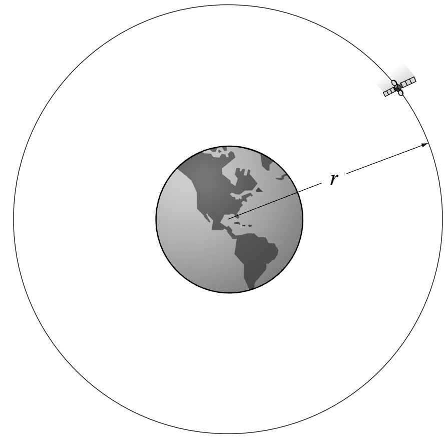
$+\nearrow\Sigma F_n = ma_n;$ $\quad T_{CD} - mg \sin\theta = m(0)$

$T_{CD} = mg \sin\theta$ **Ans.**



(a)

13–58. Determine the time for the satellite to complete its orbit around the earth. The orbit has a radius r measured from the center of the earth. The masses of the satellite and the earth are m_s and M_e, respectively.



Free-Body Diagram: The free-body diagram of the satellite is shown in Fig. (a). The force **F** which is directed towards the center of the orbit (positive n axis) is given by $F = \dfrac{GM_e m_s}{r^2}$ (Eq. 12–1). Also, $\mathbf{a}_n$ must be directed towards the positive n axis.

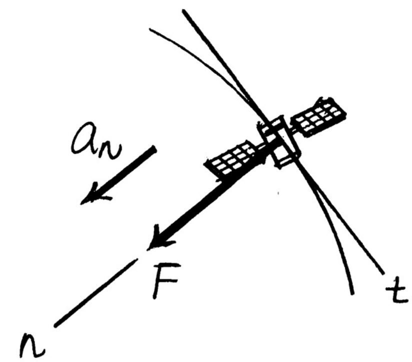
Equations of Motion: Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{{v_s}^2}{r}$ and referring to Fig. (a),

$$+\swarrow \Sigma F_n = ma_n; \qquad \frac{GM_e m_s}{r^2} = m_s\left(\frac{{v_s}^2}{r}\right)$$

$$v_s = \sqrt{\frac{GM_e}{r}}$$

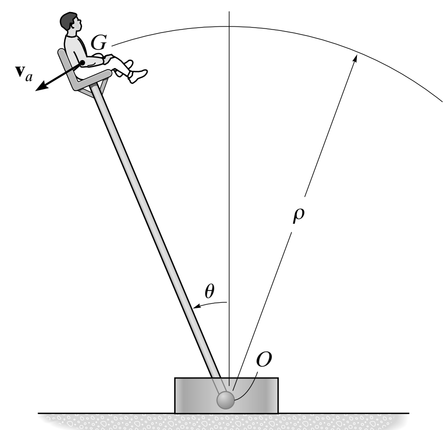
The period is the time required for the satellite to complete one revolution around the orbit. Thus,

$$T = \frac{2\pi r}{v_s} = \frac{2\pi r}{\sqrt{\dfrac{GM_e}{r}}} = 2\pi\sqrt{\frac{r^3}{GM_e}} \qquad \textbf{Ans.}$$



(a)

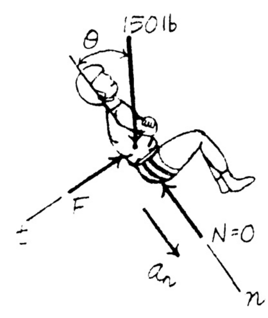
13–59. An acrobat has a weight of 150 lb and is sitting on a chair which is perched on top of a pole as shown. If by a mechanical drive the pole rotates downward at a constant rate from $\theta = 0°$, such that the acrobat's center of mass G maintains a *constant speed* of $v_a = 10$ ft/s, determine the angle θ at which he begins to "fly" out of the chair. Neglect friction and assume that the distance from the pivot O to G is $\rho = 15$ ft.



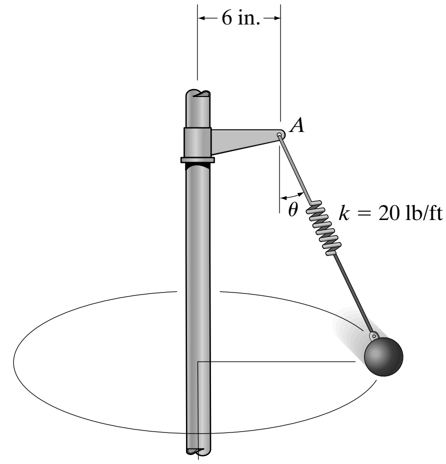
Equations of Motion: If the acrobat is about to fly off the chair, the normal reaction $N = 0$. Applying Eq. 13–8, we have

$$+\searrow \Sigma F_n = ma_n; \qquad 150\cos\theta = \frac{150}{32.2}\left(\frac{10^2}{15}\right)$$

$$\theta = 78.1° \qquad \textbf{Ans.}$$



***13–60.** A spring, having an unstretched length of 2 ft, has one end attached to the 10-lb ball. Determine the angle θ of the spring if the ball has a speed of 6 ft/s tangent to the horizontal circular path.

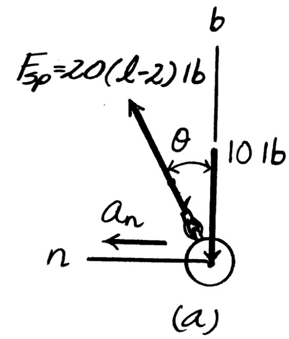


Free-Body Diagram: The free-body diagram of the bob is shown in Fig. (a). If we denote the stretched length of the spring as l, then using the springforce formula, $F_{sp} = ks = 20(l - 2)$ lb. Here, $\mathbf{a}_n$ must be directed towards the center of the horizontal circular path (positive n axis).

Equations of Motion: The radius of the horizontal circular path is $r = 0.5 + l \sin\theta$. Since $a_n = \dfrac{v^2}{r} = \dfrac{6^2}{0.5 + l\sin\theta}$, by referring to Fig. (a),

$$+\uparrow \Sigma F_b = 0; \qquad 20(l - 2)\cos\theta - 10 = 0 \qquad \textbf{(1)}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad 20(l - 2)\sin\theta = \frac{10}{32.2}\left(\frac{6^2}{0.5 + l\sin\theta}\right) \qquad \textbf{(2)}$$

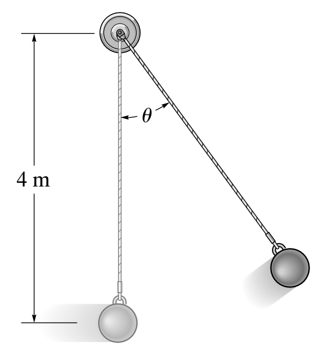


Solving Eqs. (1) and (2) yields

$$\theta = 31.26° = 31.3° \qquad \textbf{Ans.}$$

$$l = 2.585 \text{ ft} \qquad \textbf{Ans.}$$

•13–61. If the ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$, determine the tension in the cord at this instant. Also, determine the angle θ to which the ball swings and momentarily stops. Neglect the size of the ball.



$$+\uparrow \Sigma F_n = ma_n; \qquad T - 30(9.81) = 30\left(\frac{(4)^2}{4}\right)$$

$$T = 414 \text{ N} \qquad \textbf{Ans.}$$

$$+\nearrow \Sigma F_t = ma_t; \qquad -30(9.81)\sin\theta = 30a_t$$

$$a_t = -9.81\sin\theta$$

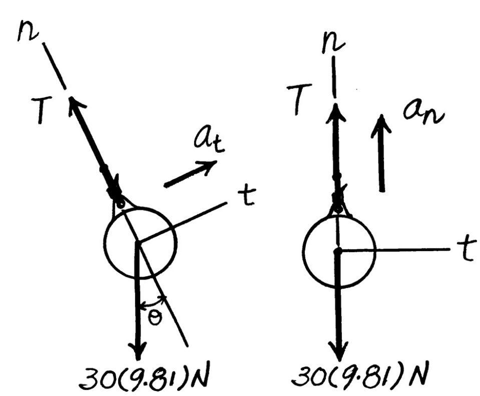
$a_t\,ds = v\,dv$ Since $ds = 4\,d\theta$, then

$$-9.81\int_0^\theta \sin\theta(4\,d\theta) = \int_4^0 v\,dv$$

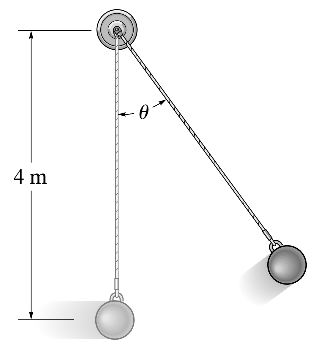
$$\left[9.81(4)\cos\theta\right]_0^\theta = -\frac{1}{2}(4)^2$$

$$39.24(\cos\theta - 1) = -8$$

$$\theta = 37.2° \qquad \textbf{Ans.}$$



13–62. The ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$. Determine the tension in the cord and the rate at which the ball's speed is decreasing at the instant $\theta = 20°$. Neglect the size of the ball.



$$+\nwarrow \Sigma F_n = ma_n; \qquad T - 30(9.81)\cos\theta = 30\left(\frac{v^2}{4}\right)$$

$$+\nearrow \Sigma F_t = ma_t; \qquad -30(9.81)\sin\theta = 30a_t$$

$$a_t = -9.81\sin\theta$$

$a_t\,ds = v\,dv$ Since $ds = 4\,d\theta$, then

$$-9.81\int_0^\theta \sin\theta\,(4\,d\theta) = \int_4^v v\,dv$$

$$9.81(4)\cos\theta\Big|_0^\theta = \frac{1}{2}(v)^2 - \frac{1}{2}(4)^2$$

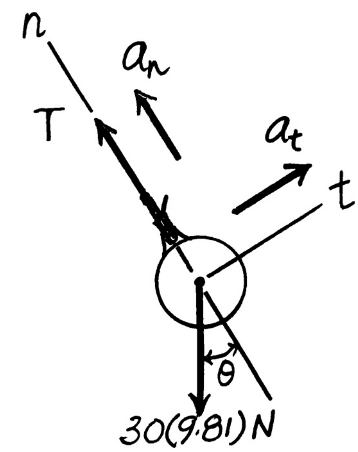
$$39.24(\cos\theta - 1) + 8 = \frac{1}{2}v^2$$

At $\theta = 20°$

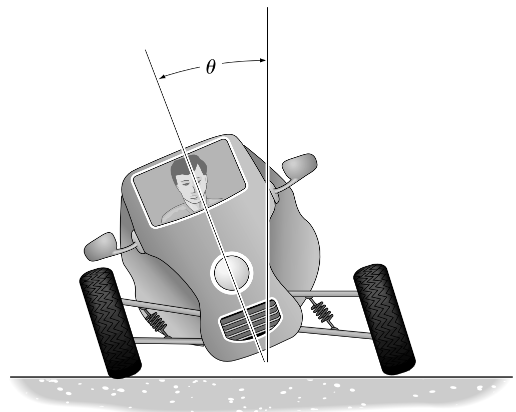
$v = 3.357$ m/s

$a_t = -3.36\text{ m/s}^2 = 3.36\text{ m/s}^2 \swarrow$ **Ans.**

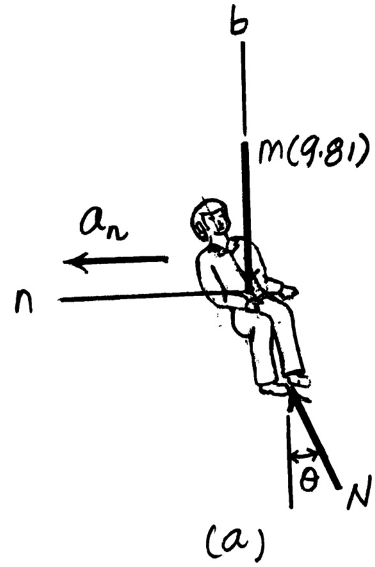
$T = 361$ N **Ans.**



13–63. The vehicle is designed to combine the feel of a motorcycle with the comfort and safety of an automobile. If the vehicle is traveling at a constant speed of 80 km/h along a circular curved road of radius 100 m, determine the tilt angle θ of the vehicle so that only a normal force from the seat acts on the driver. Neglect the size of the driver.



Free-Body Diagram: The free-body diagram of the passenger is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).



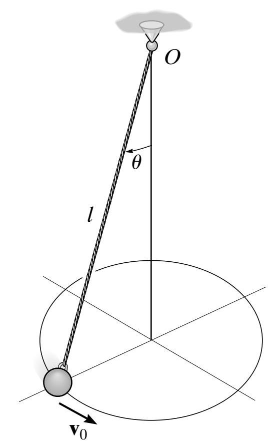
Equations of Motion: The speed of the passenger is $v = \left(80\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$ $= 22.22$ m/s. Thus, the normal component of the passenger's acceleration is given by $a_n = \frac{v^2}{\rho} = \frac{22.22^2}{100} = 4.938\text{ m/s}^2$. By referring to Fig. (a),

$$+\uparrow \Sigma F_b = 0; \qquad N\cos\theta - m(9.81) = 0 \qquad N = \frac{9.81m}{\cos\theta}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad \frac{9.81m}{\cos\theta}\sin\theta = m(4.938)$$

$$\theta = 26.7^\circ \qquad \textbf{Ans.}$$

***13–64.** The ball has a mass m and is attached to the cord of length l. The cord is tied at the top to a swivel and the ball is given a velocity $\mathbf{v}_0$. Show that the angle θ which the cord makes with the vertical as the ball travels around the circular path must satisfy the equation $\tan\theta\sin\theta = v_0^2/gl$. Neglect air resistance and the size of the ball.



$\xrightarrow{+} \Sigma F_n = ma_n; \qquad T\sin\theta = m\left(\frac{v_0^2}{r}\right)$

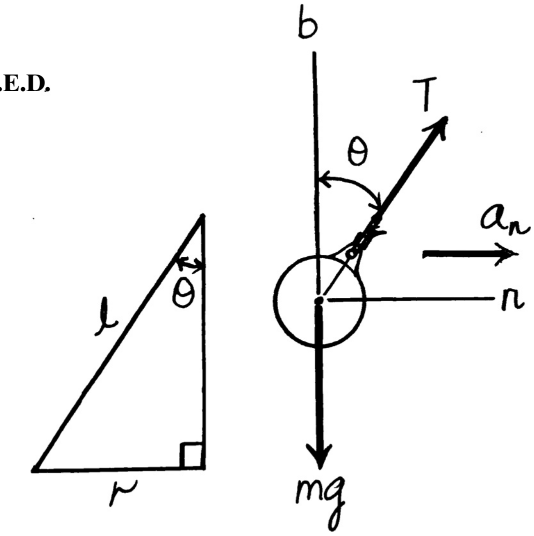
$+\uparrow \Sigma F_b = 0; \qquad T\cos\theta - mg = 0$

Since $r = l\sin\theta \qquad T = \dfrac{mv_0^2}{l\sin^2\theta}$

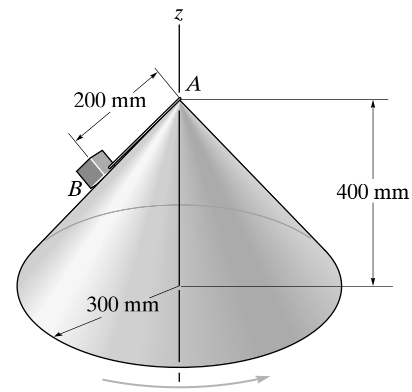
$$\left(\frac{mv_0^2}{l}\right)\left(\frac{\cos\theta}{\sin^2\theta}\right) = mg$$

$$\tan\theta\sin\theta = \frac{v_0^2}{gl}$$

Q.E.D.



•**13–65.** The smooth block B, having a mass of 0.2 kg, is attached to the vertex A of the right circular cone using a light cord. If the block has a speed of 0.5 m/s around the cone, determine the tension in the cord and the reaction which the cone exerts on the block. Neglect the size of the block.



$$\frac{\rho}{200} = \frac{300}{500}; \qquad \rho = 120\text{ mm} = 0.120\text{ m}$$

$$+\nearrow \Sigma F_y = ma_y; \qquad T - 0.2(9.81)\left(\frac{4}{5}\right) = \left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{3}{5}\right)$$

$$T = 1.82\text{ N} \qquad \textbf{Ans.}$$

$$+\nwarrow \Sigma F_x = ma_x; \qquad N_B - 0.2(9.81)\left(\frac{3}{5}\right) = -\left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{4}{5}\right)$$

$$N_B = 0.844\text{ N} \qquad \textbf{Ans.}$$

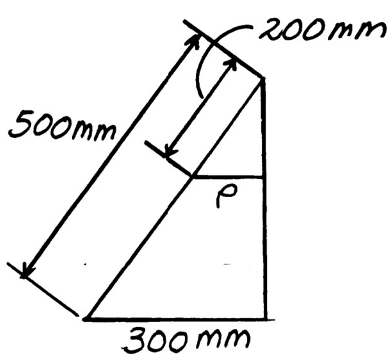
Also,

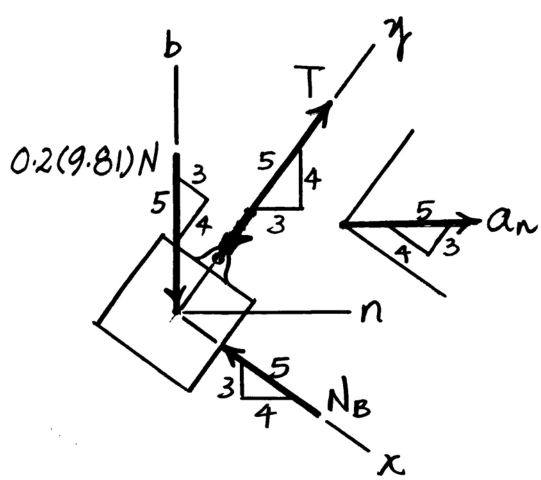
$$\xrightarrow{+} \Sigma F_n = ma_n; \qquad T\left(\frac{3}{5}\right) - N_B\left(\frac{4}{5}\right) = 0.2\left(\frac{(0.5)^2}{0.120}\right)$$

$$+\uparrow \Sigma F_b = 0; \qquad T\left(\frac{4}{5}\right) + N_B\left(\frac{3}{5}\right) - 0.2(9.81) = 0$$

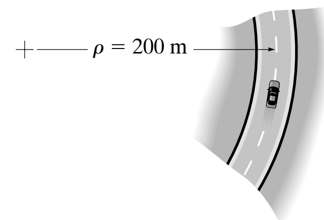
$$T = 1.82\text{ N} \qquad \textbf{Ans.}$$

$$N_B = 0.844\text{ N} \qquad \textbf{Ans.}$$

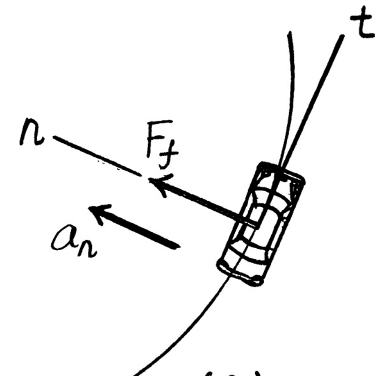




13–66. Determine the minimum coefficient of static friction between the tires and the road surface so that the 1.5-Mg car does not slide as it travels at 80 km/h on the curved road. Neglect the size of the car.



Free-Body Diagram: The frictional force $\mathbf{F}_f$ developed between the tires and the road surface and $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis) as indicated on the free-body diagram of the car, Fig. (a).



(a)

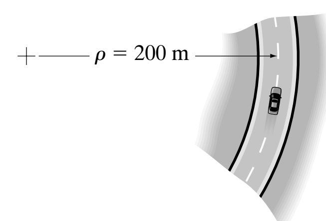
Equations of Motion: Here, the speed of the car is $v = \left(80\,\dfrac{\text{km}}{\text{h}}\right)\left(\dfrac{1000\text{ m}}{1\text{ km}}\right)\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 22.22\text{ m/s}$. Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{22.22^2}{20} = 2.469\text{ m/s}^2$ and referring to Fig. (a),

$$+\nwarrow\Sigma F_n = ma_n; \qquad F_f = 1500(2.469) = 3703.70\text{ N}$$

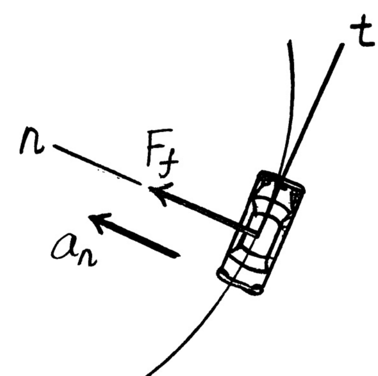
The normal reaction acting on the car is equal to the weight of the car, i.e., $N = 1500(9.81) = 14\,715$ N. Thus, the required minimum μ_s is given by

$$\mu_s = \frac{F_f}{N} = \frac{370.70}{14\,715} = 0.252 \qquad \textbf{Ans.}$$

13–67. If the coefficient of static friction between the tires and the road surface is $\mu_s = 0.25$, determine the maximum speed of the 1.5-Mg car without causing it to slide when it travels on the curve. Neglect the size of the car.



Free-Body Diagram: The frictional force $\mathbf{F}_f$ developed between the tires and the road surface and $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis) as indicated on the free-body diagram of the car, Fig. (a).



(a)

Equations of Motion: Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{v^2}{200}$ and referring to Fig. (a),

$$+\nwarrow\Sigma F_n = ma_n; \qquad F_f = 1500\left(\frac{v^2}{200}\right) = 7.5v^2$$

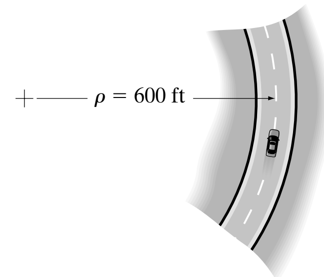
The normal reaction acting on the car is equal to the weight of the car, i.e., $N = 1500(9.81) = 14\,715$ N. When the car is on the verge of sliding,

$$F_f = \mu_s N$$

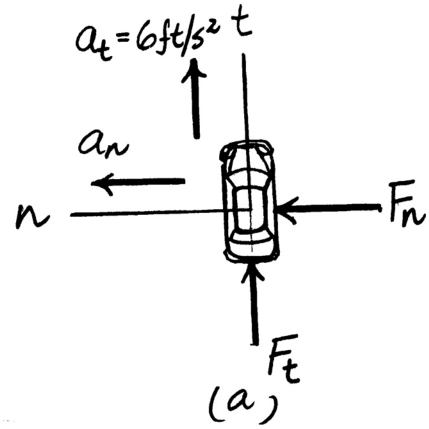
$$7.5\,v^2 = 0.25(14\,715)$$

$$v = 22.1\text{ m/s} \qquad \textbf{Ans.}$$

***13–68.** At the instant shown, the 3000-lb car is traveling with a speed of 75 ft/s, which is increasing at a rate of 6 ft/s^2. Determine the magnitude of the resultant frictional force the road exerts on the tires of the car. Neglect the size of the car.



Free-Body Diagram: Here, the force acting on the tires will be resolved into its n and t components $\mathbf{F}_n$ and $\mathbf{F}_t$ as indicated on the free-body diagram of the car, Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature of the road (positive n axis).



(a)

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{75^2}{600} = 9.375$ ft/s^2. By referring to Fig. (a),

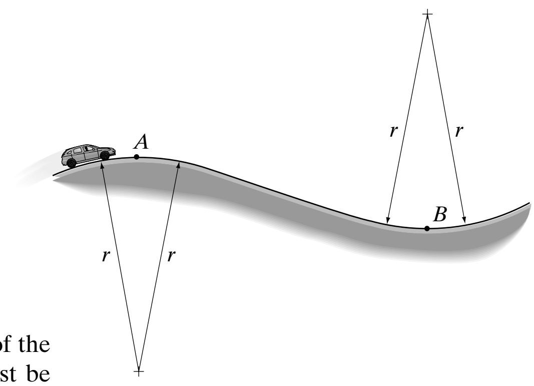
$$+\uparrow \Sigma F_t = ma_t; \qquad F_t = \frac{3000}{32.2}(6) = 559.01 \text{ lb}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad F_n = \frac{3000}{32.2}(9.375) = 873.45 \text{ lb}$$

Thus, the magnitude of force **F** acting on the tires is

$$F = \sqrt{F_t^{\,2} + F_n^{\,2}} = \sqrt{559.01^2 + 873.45^2} = 1037 \text{ lb} \qquad \textbf{Ans.}$$

•13–69. Determine the maximum speed at which the car with mass m can pass over the top point A of the vertical curved road and still maintain contact with the road. If the car maintains this speed, what is the normal reaction the road exerts on the car when it passes the lowest point B on the road?



Free-Body Diagram: The free-body diagram of the car at the top and bottom of the vertical curved road are shown in Figs. (a) and (b), respectively. Here, $\mathbf{a}_n$ must be directed towards the center of curvature of the vertical curved road (positive n axis).

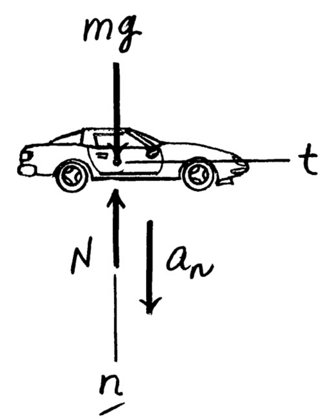
Equations of Motion: When the car is on top of the vertical curved road, it is required that its tires are about to lose contact with the road surface. Thus, $N = 0$. Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{v^2}{r}$ and referring to Fig. (a),

$$+\downarrow \Sigma F_n = ma_n; \qquad mg = m\left(\frac{v^2}{r}\right) \qquad v = \sqrt{gr} \qquad \textbf{Ans.}$$

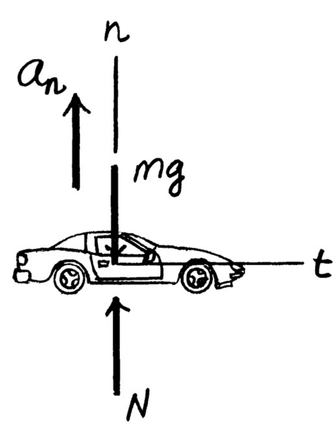
Using the result of v, the normal component of car acceleration is $a_n = \dfrac{v^2}{\rho} = \dfrac{gr}{r} = g$ when it is at the lowest point on the road. By referring to Fig. (b),

$$+\uparrow \Sigma F_n = ma_n; \qquad N - mg = mg$$

$$N = 2mg \qquad \textbf{Ans.}$$

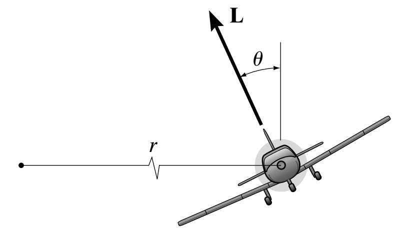


(a)



(b)

13–70. A 5-Mg airplane is flying at a constant speed of 350 km/h along a horizontal circular path of radius $r = 3000\,\text{m}$. Determine the uplift force **L** acting on the airplane and the banking angle θ. Neglect the size of the airplane.



Free-Body Diagram: The free-body diagram of the airplane is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).

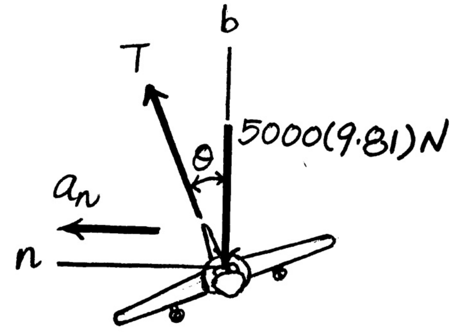
Equations of Motion: The speed of the airplane is $v = \left(350\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 97.22\text{ m/s}$. Realizing that $a_n = \frac{v^2}{\rho} = \frac{97.22^2}{3000} = 3.151\text{ m/s}^2$ and referring to Fig. (a),

$$+\uparrow \Sigma F_b = 0; \qquad T\cos\theta - 5000(9.81) = 0 \qquad \textbf{(1)}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad T\sin\theta = 5000(3.151) \qquad \textbf{(2)}$$

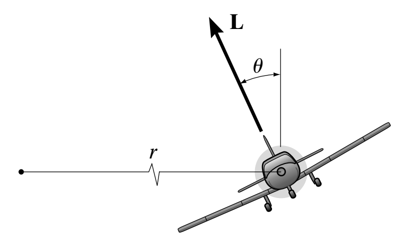
Solving Eqs. (1) and (2) yields

$\theta = 17.8^\circ \qquad T = 51517.75 = 51.5\text{ kN}$ **Ans.**



(a)

13–71. A 5-Mg airplane is flying at a constant speed of 350 km/h along a horizontal circular path. If the banking angle $\theta = 15^\circ$, determine the uplift force **L** acting on the airplane and the radius r of the circular path. Neglect the size of the airplane.



Free-Body Diagram: The free-body diagram of the airplane is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).

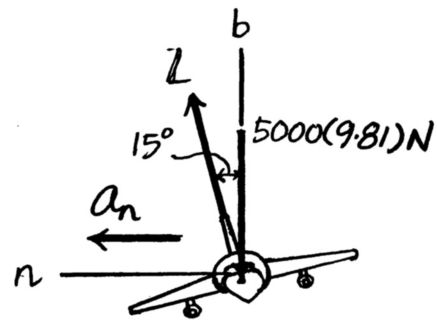
Equations of Motion: The speed of the airplane is $v = \left(350\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 97.22\text{ m/s}$. Realizing that $a_n = \frac{v^2}{\rho} = \frac{97.22^2}{r}$ and referring to Fig. (a),

$$+\uparrow \Sigma F_b = 0; \qquad L\cos 15^\circ - 5000(9.81) = 0$$

$$L = 50780.30\text{ N} = 50.8\text{ kN} \qquad \textbf{Ans.}$$

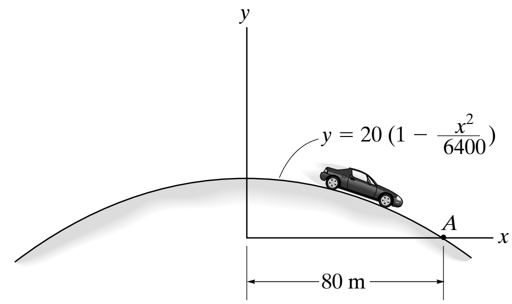
$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad 50780.30\sin 15^\circ = 5000\left(\frac{97.22^2}{r}\right)$$

$$r = 3595.92\text{ m} = 3.60\text{ km} \qquad \textbf{Ans.}$$



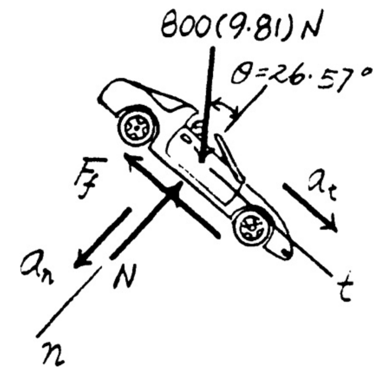
(a)

***13–72.** The 0.8-Mg car travels over the hill having the shape of a parabola. If the driver maintains a constant speed of 9 m/s, determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at the instant it reaches point A. Neglect the size of the car.



Geometry: Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57^\circ$$



and the radius of curvature at point A is

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1 + (-0.00625x)^2]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61\text{ m}$$

Equations of Motion: Here, $a_t = 0$. Applying Eq. 13–8 with $\theta = 26.57^\circ$ and $\rho = 223.61$ m, we have

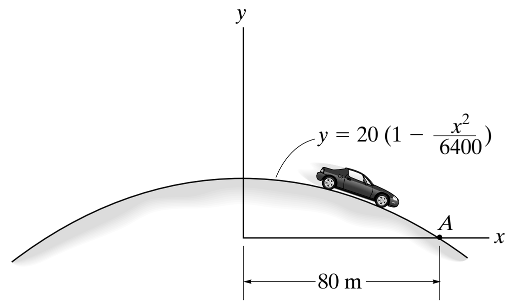
$$\Sigma F_t = ma_t; \qquad 800(9.81)\sin 26.57^\circ - F_f = 800(0)$$

$$F_f = 3509.73\text{ N} = 3.51\text{ kN} \qquad \textbf{Ans.}$$

$$\Sigma F_n = ma_n; \qquad 800(9.81)\cos 26.57^\circ - N = 800\left(\frac{9^2}{223.61}\right)$$

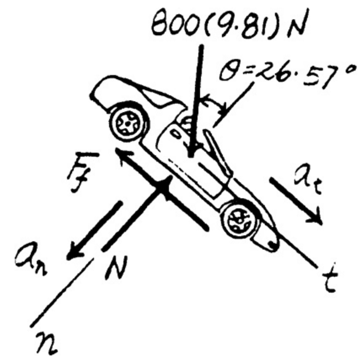
$$N = 6729.67\text{ N} = 6.73\text{ kN} \qquad \textbf{Ans.}$$

•13–73. The 0.8-Mg car travels over the hill having the shape of a parabola. When the car is at point A, it is traveling at 9 m/s and increasing its speed at 3 m/s^2. Determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at this instant. Neglect the size of the car.



Geometry: Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57°$$



and the radius of curvature at point A is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + (-0.00625x)^2\right]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61\text{ m}$$

Equation of Motion: Applying Eq. 13–8 with $\theta = 26.57°$ and $\rho = 223.61$ m, we have

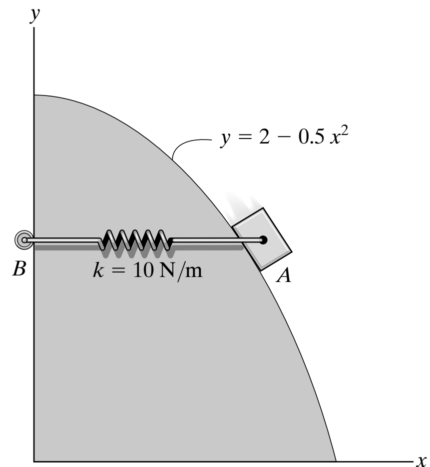
$$\Sigma F_t = ma_t; \qquad 800(9.81)\sin 26.57° - F_f = 800(3)$$

$$F_f = 1109.73\text{ N} = 1.11\text{ kN} \qquad \textbf{Ans.}$$

$$\Sigma F_n = ma_n; \qquad 800(9.81)\cos 26.57° - N = 800\left(\frac{9^2}{223.61}\right)$$

$$N = 6729.67\text{ N} = 6.73\text{ kN} \qquad \textbf{Ans.}$$

13–74. The 6-kg block is confined to move along the smooth parabolic path. The attached spring restricts the motion and, due to the roller guide, always remains horizontal as the block descends. If the spring has a stiffness of $k = 10$ N/m, and unstretched length of 0.5 m, determine the normal force of the path on the block at the instant $x = 1$ m when the block has a speed of 4 m/s. Also, what is the rate of increase in speed of the block at this point? Neglect the mass of the roller and the spring.

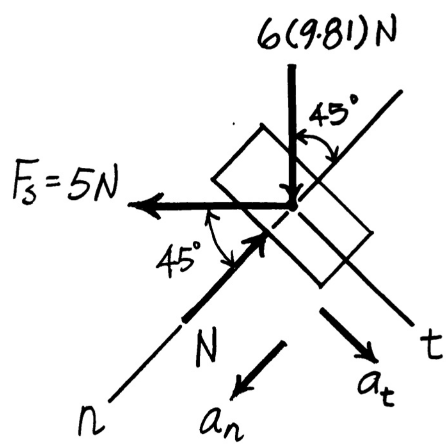


$$v = 2 - 0.5x^2$$

$$\frac{dv}{dx} = \tan\theta = -x\bigg|_{x=1} = -1 \qquad \theta = -45^\circ$$

$$\frac{d^2y}{dx^2} = -1$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (-1)^2\right]^{\frac{3}{2}}}{|-1|} = 2.8284 \text{ m}$$



$$F_s = kx = 10(1 - 0.5) = 5 \text{ N}$$

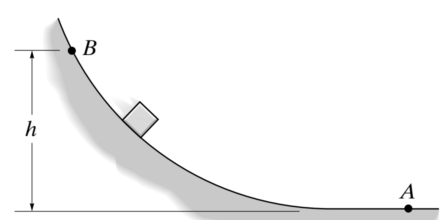
$$+\swarrow\Sigma F_n = ma_n; \qquad 6(9.81)\cos 45^\circ - N + 5\cos 45^\circ = 6\left(\frac{(4)^2}{2.8284}\right)$$

$$N = 11.2 \text{ N} \qquad \textbf{Ans.}$$

$$+\searrow\Sigma F_t = ma_t; \qquad 6(9.81)\sin 45^\circ - 5\sin 45^\circ = 6a_t$$

$$a_t = 6.35 \text{ m/s}^2 \qquad \textbf{Ans.}$$

13–75. Prove that if the block is released from rest at point B of a smooth path of *arbitrary shape*, the speed it attains when it reaches point A is equal to the speed it attains when it falls freely through a distance h; i.e., $v = \sqrt{2gh}$.



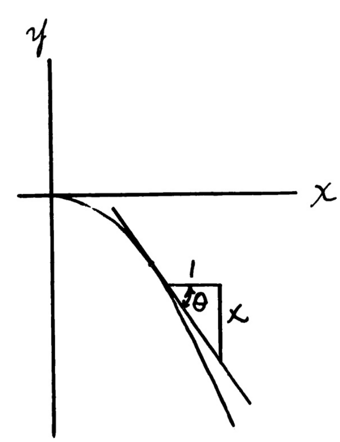
$$+\searrow\Sigma F_t = ma_t; \qquad mg\sin\theta = ma_t \qquad a_t = g\sin\theta$$

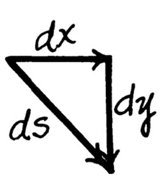
$$v\,dv = a_t\,ds = g\sin\theta\,ds \qquad \text{However} \quad dy = ds\sin\theta$$

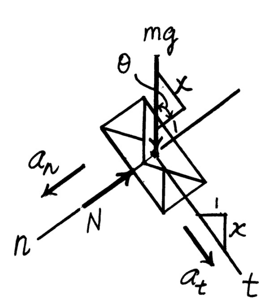
$$\int_0^v v\,dv = \int_0^h g\,dy$$

$$\frac{v^2}{2} = gh$$

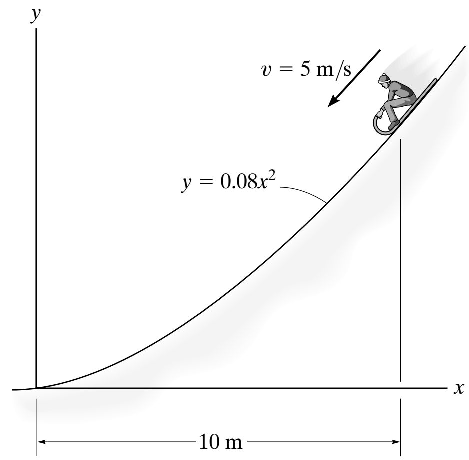
$$v = \sqrt{2gh} \qquad \textbf{Q.E.D.}$$







***13–76.** A toboggan and rider of total mass 90 kg travel down along the (smooth) slope defined by the equation $y = 0.08x^2$. At the instant $x = 10$ m, the toboggan's speed is 5 m/s. At this point, determine the rate of increase in speed and the normal force which the slope exerts on the toboggan. Neglect the size of the toboggan and rider for the calculation.



Geometry: Here, $\dfrac{dy}{dx} = 0.16x$ and $\dfrac{d^2y}{dx^2} = 0.16$. The slope angle θ at $x = 10$ m is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=10\text{ m}} = 0.16(10) \qquad \theta = 57.99^\circ$$

and the radius of curvature at $x = 10$ m is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{\left|d^2y/dx^2\right|} = \left.\frac{\left[1 + (0.16x)^2\right]^{3/2}}{|0.16|}\right|_{x=10\text{ m}} = 41.98\text{ m}$$

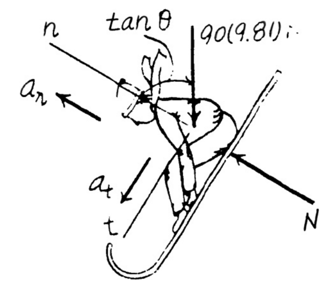
Equations of Motion: Applying Eq. 13–8 with $\theta = 57.99^\circ$ and $\rho = 41.98$ m, we have

$$\Sigma F_t = ma_t; \qquad 90(9.81)\sin 57.99^\circ = 90a_t$$

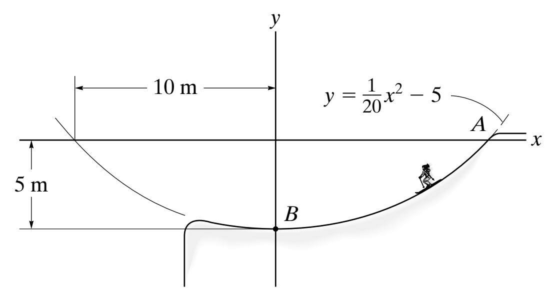
$$a_t = 8.32\text{ m/s}^2 \qquad \textbf{Ans.}$$

$$\Sigma F_n = ma_n; \qquad -90(9.81)\cos 57.99^\circ + N = 90\left(\frac{5^2}{41.98}\right)$$

$$N = 522\text{ N} \qquad \textbf{Ans.}$$

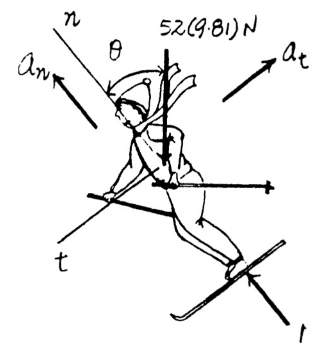


•**13–77.** The skier starts from rest at $A(10\text{ m}, 0)$ and descends the smooth slope, which may be approximated by a parabola. If she has a mass of 52 kg, determine the normal force the ground exerts on the skier at the instant she arrives at point B. Neglect the size of the skier. *Hint:* Use the result of Prob. 13–75.



Geometry: Here, $\dfrac{dy}{dx} = \dfrac{1}{10}x$ and $\dfrac{d^2y}{dx^2} = \dfrac{1}{10}$. The slope angle θ at point B is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=0\text{ m}} = 0 \qquad \theta = 0°$$



and the radius of curvature at point B is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + \left(\frac{1}{10}x\right)^2\right]^{3/2}}{|1/10|}\right|_{x=0\text{ m}} = 10.0\text{ m}$$

Equations of Motion:

$+\swarrow \Sigma F_t = ma_t; \qquad 52(9.81)\sin\theta = -52a_t \qquad a_t = -9.81\sin\theta$

$+\nwarrow \Sigma F_n = ma_n; \qquad N - 52(9.81)\cos\theta = m\left(\dfrac{v^2}{\rho}\right) \qquad$ **(1)**

Kinematics: The speed of the skier can be determined using $v\,dv = a_t\,ds$. Here, a_t must be in the direction of positive ds. Also, $ds = \sqrt{1 + (dy/dx)^2}dx = \sqrt{1 + \frac{1}{100}x^2}dx$

Here, $\tan\theta = \dfrac{1}{10}x$. Then, $\sin\theta = \dfrac{x}{10\sqrt{1 + \frac{1}{100}x^2}}$.

$$(+\ ) \qquad \int_0^v v\,dv = -9.81\int_{10\text{ m}}^{0}\left(\frac{x}{10\sqrt{1 + \frac{1}{100}x^2}}\right)\left(\sqrt{1 + \frac{1}{100}x^2}dx\right)$$

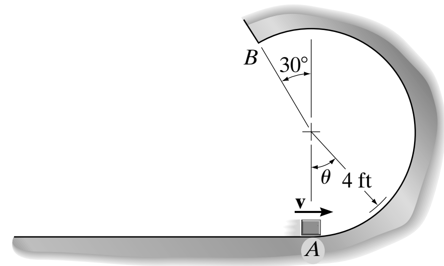
$$v^2 = 9.81\text{ m}^2/\text{s}^2$$

Substituting $v^2 = 98.1\text{ m}^2/\text{s}^2$, $\theta = 0°$, and $\rho = 10.0$ m into Eq.(1) yields

$$N - 52(9.81)\cos 0° = 52\left(\frac{98.1}{10.0}\right)$$

$$N = 1020.24\text{ N} = 1.02\text{ kN} \qquad \textbf{Ans.}$$

13–78. The 5-lb box is projected with a speed of 20 ft/s at A up the vertical circular smooth track. Determine the angle θ when the box leaves the track.



Free-Body Diagram: The free-body diagram of the box at an arbitrary position θ is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of the vertical circular path (positive n axis), while $\mathbf{a}_t$ is assumed to be directed toward the positive t axis.

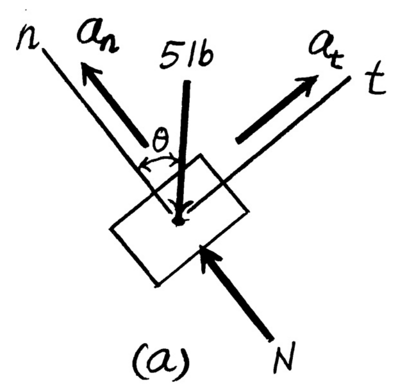
Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{v^2}{4}$. Also, the box is required to leave the track, so that $N = 0$. By referring to Fig. (a),

$$+\nearrow \Sigma F_t = ma_t; \qquad -5 \sin \theta = \frac{5}{32.2} a_t$$

$$a_t = -(32.2 \sin \theta) \text{ ft/s}^2$$

$$+\nwarrow \Sigma F_n = ma_n; \qquad -5 \cos \theta = \frac{5}{32.2}\left(\frac{v^2}{4}\right)$$

$$v^2 = -128.8 \cos \theta \qquad \textbf{(1)}$$



Kinematics: Using the result of $\mathbf{a}_t$, the speed of the box can be determined by integrating the kinematic equation $v\,dv = a_t\,ds$, where $ds = r\,d\theta = 4\,d\theta$. Using the initial condition $v = 20$ ft/s at $\theta = 0°$ as the integration limit,

$$\int_{20\text{ft/s}}^{v} v\,dv = \int_{0°}^{\theta} -32.2 \sin \theta (4\,d\theta)$$

$$\left.\frac{v^2}{2}\right|_{20 \text{ ft/s}}^{v} = 128.8 \cos \theta \Big|_{0°}^{\theta}$$

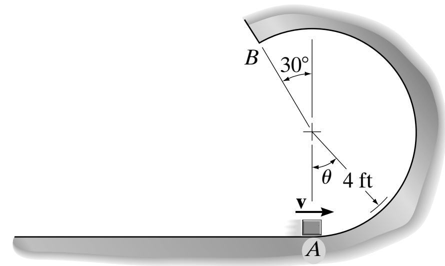
$$v^2 = 257.6 \cos \theta + 142.4 \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2),

$$386.4 \cos \theta + 142.4 = 0$$

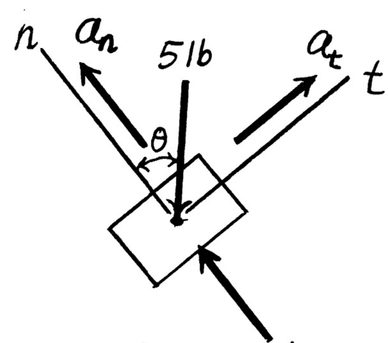
$$\theta = 111.62° = 112° \qquad \textbf{Ans.}$$

13–79. Determine the minimum speed that must be given to the 5-lb box at A in order for it to remain in contact with the circular path. Also, determine the speed of the box when it reaches point B.



Free-Body Diagram: The free-body diagram of the box at an arbitrary position θ is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of the vertical circular path (positive n axis), while $\mathbf{a}_t$ is assumed to be directed toward the positive t axis.

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{v^2}{4}$. Also, the box is required to leave the track, so that $N = 0$. By referring to Fig. (a),



(a)

$$+\nearrow\Sigma F_t = ma_t; \qquad -5\sin\theta = \frac{5}{32.2}a_t$$

$$a_t = -(32.2\sin\theta)\ \text{ft/s}^2$$

$$+\nwarrow\Sigma F_n = ma_n; \qquad N - 5\cos\theta = \frac{5}{32.2}\left(\frac{v^2}{4}\right)$$

$$N = 0.03882v^2 + 5\cos\theta \qquad \textbf{(1)}$$

Kinematics: Using the result of $\mathbf{a}_t$, the speed of the box can be determined by intergrating the kinematic equation $v\,dv = a_t\,ds$, where $ds = r\,d\theta = 4\,d\theta$. Using the initial condition $v = v_0$ at $\theta = 0°$ as the integration limit,

$$\int_{v_0}^{v} v\,dv = \int_{0°}^{\theta} -32.2\sin\theta(4d\theta)$$

$$\left.\frac{v^2}{2}\right|_{v_0}^{v} = 128.8\cos\theta\Big|_{0°}^{\theta}$$

$$v^2 = 257.6\cos\theta - 257.6 + {v_0}^2 \qquad \textbf{(2)}$$

Provided the box does not leave the vertical circular path at $\theta = 180°$, then it will remain in contact with the path. Thus, it is required that the box is just about to leave the path at $\theta = 180°$, Thus, $N = 0$. Substituting these two values into Eq. (1),

$$0 = 00.03882v^2 + 5\cos 180°$$

$$v = 11.35\ \text{ft/s}$$

Substituting the result of v and $v_0 = v_{\min}$ into Eq. (2),

$$11.35^2 = 257.6\cos 180° - 257.6 + {v_{\min}}^2$$

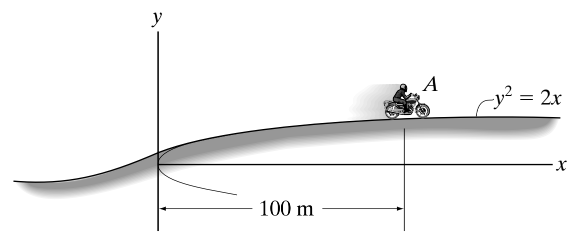
$$v_{\min} = 25.38\ \text{ft/s} = 25.4\ \text{ft/s} \qquad \textbf{Ans.}$$

At point B, $\theta = 210°$. Substituting this value and $v_0 = v_{\min} = 25.38$ ft/s into Eq. (2),

$$v_B{}^2 = 257.6\cos 210° - 257.6 + 25.38^2$$

$$v_B = 12.8\ \text{ft/s} \qquad \textbf{Ans.}$$

***13–80.** The 800-kg motorbike travels with a constant speed of 80 km/h up the hill. Determine the normal force the surface exerts on its wheels when it reaches point A. Neglect its size.

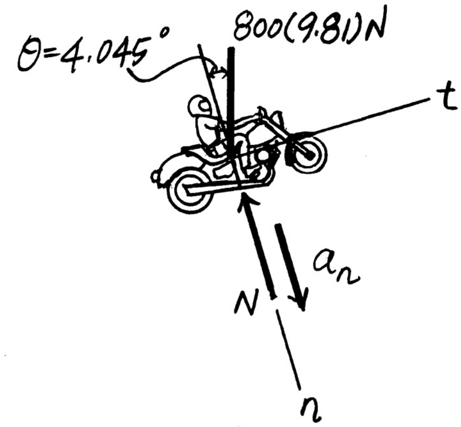


Geometry: Here, $y = \sqrt{2}x^{1/2}$. Thus, $\frac{dy}{dx} = \frac{\sqrt{2}}{2x^{1/2}}$ and $\frac{d^2y}{dx^2} = -\frac{\sqrt{2}}{4x^{3/2}}$. The angle that the hill slope at A makes with the horizontal is

$$\theta = \tan^{-1}\left(\frac{dy}{dx}\right)\bigg|_{x=100\text{ m}} = \tan^{-1}\left(\frac{\sqrt{2}}{2x^{1/2}}\right)\bigg|_{x=100\text{ m}} = 4.045°$$

The radius of curvature of the hill at A is given by

$$\rho_A = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\Bigg|_{x=100\text{ m}} = \frac{\left[1+\left(\frac{\sqrt{2}}{2(100^{1/2})}\right)^2\right]^{3/2}}{\left|-\frac{\sqrt{2}}{4(100^{3/2})}\right|} = 2849.67\text{ m}$$



(a)

Free-Body Diagram: The free-body diagram of the motorcycle is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).

Equations of Motion: The speed of the motorcycle is

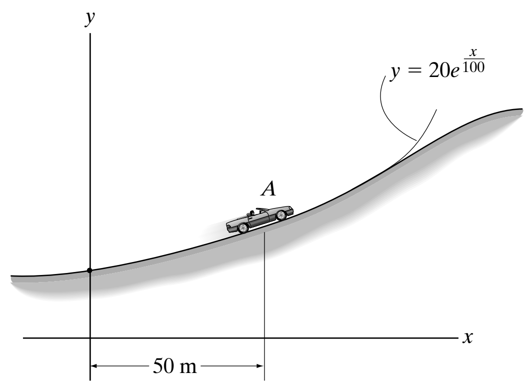
$$v = \left(80\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 22.22\text{ m/s}$$

Thus, $a_n = \frac{v^2}{\rho_A} = \frac{22.22^2}{2849.67} = 0.1733\text{ m/s}^2$. By referring to Fig. (a),

$$\searrow + \Sigma F_n = ma_n; \qquad 800(9.81)\cos 4.045° - N = 800(0.1733)$$

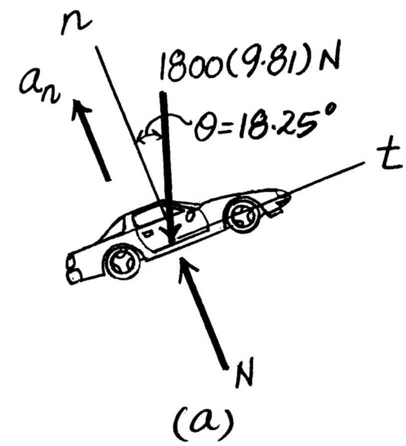
$$N = 7689.82\text{ N} = 7.69\text{ kN} \qquad \textbf{Ans.}$$

•13–81. The 1.8-Mg car travels up the incline at a constant speed of 80 km/h. Determine the normal reaction of the road on the car when it reaches point A. Neglect its size.



Geometry: Here, $\dfrac{dy}{dx} = 20\left(\dfrac{1}{100}\right)e^{x/100} = 0.2e^{x/100}$ and $\dfrac{d^2y}{dx^2} = 0.2\left(\dfrac{1}{100}\right)e^{x/100} = 0.002e^{x/100}$. The angle that the slope of the road at A makes with the horizontal is $\theta = \tan^{-1}\left(\dfrac{dy}{dx}\right)\Big|_{x=50\text{ m}} = \tan^{-1}\left(0.2e^{50/100}\right) = 18.25°$. The radius of curvature of the road at A is given by

$$\rho_A = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|}\Bigg|_{x=50\text{ m}} = \frac{\left[1 + \left(0.2e^{50/100}\right)^2\right]^{3/2}}{\left|0.002e^{50/100}\right|} = 354.05\text{ m}$$



Free-Body Diagram: The free-body diagram of the car is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).

Equations of Motion: The speed of the car is

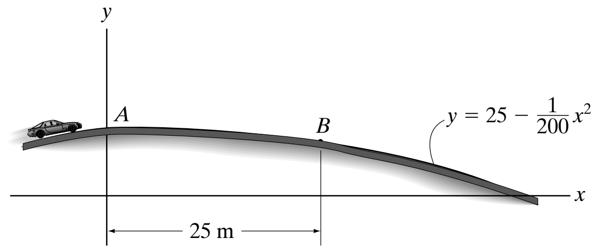
$$v = \left(80\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 22.22\text{ m/s}^2$$

Thus, $a_n = \dfrac{v^2}{\rho_A} = \dfrac{22.22^2}{354.05} = 1.395\text{ m/s}^2$. By referring to Fig. (a),

$$+\nwarrow\Sigma F_n = ma_n; \qquad N - 1800(9.81)\cos 18.25° = 1800(1.395)$$

$$N = 19280.46\text{ N} = 19.3\text{ kN}$$ **Ans.**

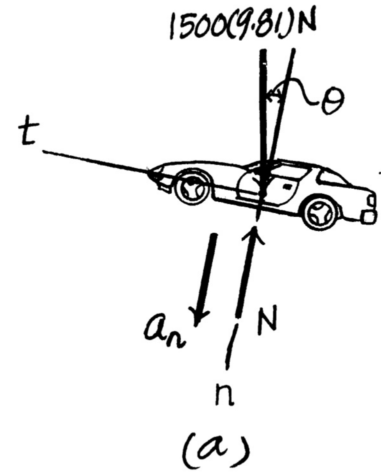
13–82. Determine the maximum speed the 1.5-Mg car can have and still remain in contact with the road when it passes point A. If the car maintains this speed, what is the normal reaction of the road on it when it passes point B? Neglect the size of the car.



Geometry: Here, $\dfrac{dx}{dy} = -0.01x$ and $\dfrac{d^2y}{dx^2} = -0.01$. The angle that the slope of the road makes with the horizontal at A and B are $\theta_A = \tan^{-1}\left(\dfrac{dy}{dx}\right)\Bigg|_{x=0\text{ m}}$ $= tan^{-1}(0) = 0°$ and $\theta_B = \tan^{-1}\left(\dfrac{dy}{dx}\right)\Bigg|_{x=25\text{ m}} = \tan^{-1}\big(-0.01(25)\big) = -14.04°$. The radius of curvature of the road at A and B are

$$\rho_A = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|}\Bigg|_{x=0\text{ m}} = \frac{\left[1 + (0)^2\right]^{3/2}}{0.01} = 100\text{ m}$$

$$\rho_B = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|}\Bigg|_{x=25\text{ m}} = \frac{\left[1 + (0.25)^2\right]^{3/2}}{0.01} = 109.52\text{ m}$$



Free-Body Diagram: The free-body diagram of the car at an arbitrary position x is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature of the road (positive n axis).

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho}$. By referring to Fig. (a),

$$\Sigma F_n = ma_n; \qquad 1500(9.81)\cos\theta - N = 1500\left(\frac{v^2}{\rho}\right)$$

$$N = 14\,715\cos\theta - \frac{1500v^2}{\rho} \qquad \textbf{(1)}$$

Since the car is required to just about lose contact with the road at A, then $N = N_A = 0$, $\theta = \theta_A = 0$ and $\rho = \rho_A = 100$ m. Substituting these values into Eq. (1),

$$0 = 14\,715\cos 0° - \frac{1500v^2}{100}$$

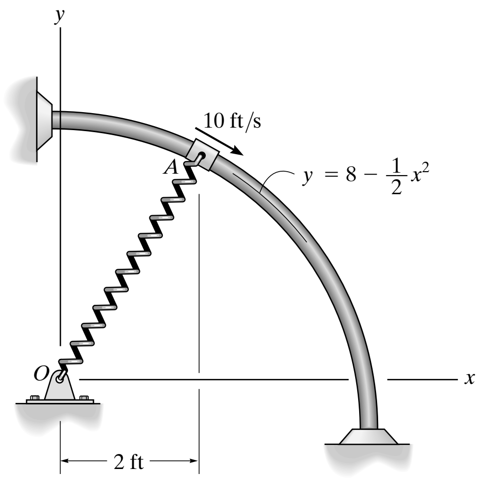
$$v = 31.32\text{ m/s} = 31.3\text{ m/s} \qquad \textbf{Ans.}$$

When the car is at B, $\theta = \theta_B = 14.04°$ and $\rho = \rho_B = 109.52$ m. Substituting these values into Eq. (1), we obtain

$$N_B = 14\,715\cos 14.04° - \frac{1500\big(31.32^2\big)}{109.52}$$

$$= 839.74\text{N} = 840\text{ N} \qquad \textbf{Ans.}$$

13–83. The 5-lb collar slides on the smooth rod, so that when it is at A it has a speed of 10 ft/s. If the spring to which it is attached has an unstretched length of 3 ft and a stiffness of $k = 10$ lb/ft, determine the normal force on the collar and the acceleration of the collar at this instant.



$$y = 8 - \frac{1}{2}x^2$$

$$\frac{dy}{dx} = \tan\theta = -x\bigg|_{x=2} = -2 \qquad \theta = -63.435^\circ$$

$$\frac{d^2y}{dx^2} = -1$$

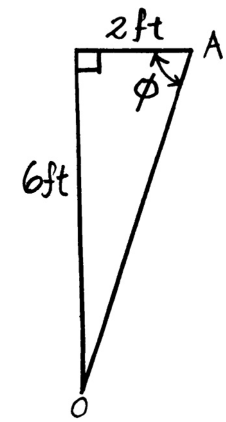
$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{(1 + (-2)^2)^{\frac{3}{2}}}{|-1|} = 11.18\text{ ft}$$

$$y = 8 - \frac{1}{2}(2)^2 = 6$$

$$OA = \sqrt{(2)^2 + (6)^2} = 6.3246$$

$$F_s = kx = 10(6.3246 - 3) = 33.246\text{ lb}$$

$$\tan\phi = \frac{6}{2}\ \phi 1.565^\circ$$



$+\swarrow\Sigma F_n = ma_n;$ $\quad 5\cos 63.435^\circ - N + 33.246\cos 8.1301^\circ = \left(\frac{5}{32.2}\right)\left(\frac{(10)^2}{11.18}\right)$

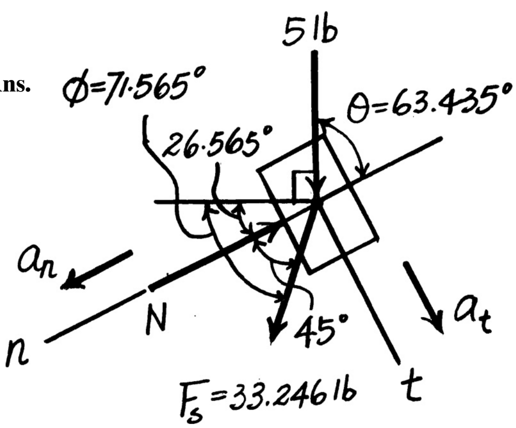
$N = 33.8\text{ lb}$ **Ans.**

$+\searrow\Sigma F_t = ma_t;$ $\quad 5\sin 63.435^\circ + 33.246\sin 8.1301^\circ = \left(\frac{5}{32.2}\right)a_t$

$$a_t = 59.08\text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(10)^2}{11.18} = 8.9443\text{ ft/s}^2$$

$a = \sqrt{(59.08)^2 + (8.9443)^2} = 59.8\text{ ft/s}^2$ **Ans.**



***13–84.** The path of motion of a 5-lb particle in the horizontal plane is described in terms of polar coordinates as $r = (2t + 1)$ ft and $\theta = (0.5t^2 - t)$ rad, where t is in seconds. Determine the magnitude of the resultant force acting on the particle when $t = 2$ s.

$r = 2t + 1|_{t=2\,s} = 5\text{ ft} \qquad \dot{r} = 2\text{ ft/s} \qquad \ddot{r} = 0$

$\theta = 0.5t^2 - t|_{t=2\,s} = 0\text{ rad} \qquad \dot{\theta} = t - 1|_{t=2\,s} = 1\text{ rad/s} \qquad \ddot{\theta} = 1\text{ rad/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5(1)^2 = -5\text{ ft/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 5(1) + 2(2)(1) = 9\text{ ft/s}^2$

$\Sigma F_r = ma_r; \qquad F_r = \dfrac{5}{32.2}(-5) = -0.7764\text{ lb}$

$\Sigma F_\theta = ma_\theta; \qquad F_\theta = \dfrac{5}{32.2}(9) = 1.398\text{ lb}$

$F = \sqrt{F_r^2 + F_\theta^2} = \sqrt{(-0.7764)^2 + (1.398)^2} = 1.60\text{ lb}$ **Ans.**

•13–85. Determine the magnitude of the resultant force acting on a 5-kg particle at the instant $t = 2$ s, if the particle is moving along a horizontal path defined by the equations $r = (2t + 10)$ m and $\theta = (1.5t^2 - 6t)$ rad, where t is in seconds.

$r = 2t + 10|_{t=2\,s} = 14$

$\dot{r} = 2$

$\ddot{r} = 0$

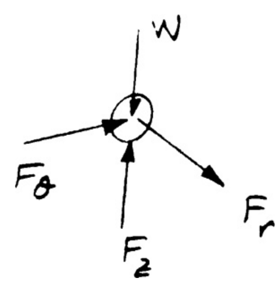
$\theta = 1.5t^2 - 6t$

$\dot{\theta} = 3t - 6\Big|_{t=2\,s} = 0$

$\ddot{\theta} = 3$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0 = 0$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 14(3) + 0 = 42$



Hence,

$\Sigma F_r = ma_r; \qquad F_r = 5(0) = 0$

$\Sigma F_\theta = ma_\theta; \qquad F_\theta = 5(42) = 210\text{ N}$

$F = \sqrt{(F_r)^2 + (F_\theta)^2} = 210\text{ N}$ **Ans.**

13–86. A 2-kg particle travels along a horizontal smooth path defined by

$$r = \left(\frac{1}{4}t^3 + 2\right)\text{m}, \ \theta = \left(\frac{t^2}{4}\right)\text{rad},$$

where t is in seconds. Determine the radial and transverse components of force exerted on the particle when $t = 2\,\text{s}$.

Kinematics: Since the motion of the particle is known, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ will be determined first. The values of $\mathbf{r}$ and the time derivative of r and θ evaluated at $t = 2$ s are

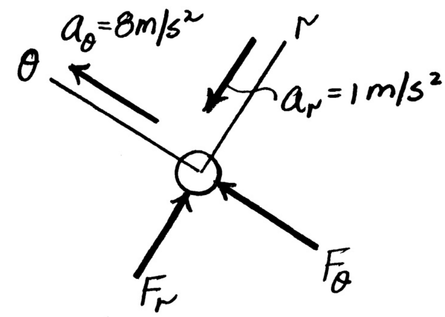
$$r|_{t=2\,\text{s}} = \frac{1}{4}t^3 + 2\bigg|_{t=2\,\text{s}} = 4\text{ m} \qquad \dot{r}|_{t=2\,\text{s}} = \frac{3}{4}t^2\bigg|_{t=2\,\text{s}} = 3\text{ m/s} \qquad \ddot{r}|_{t=2\,\text{s}} = \frac{3}{2}t\bigg|_{t=2\,\text{s}} = 3\text{ m/s}^2$$

$$\dot{\theta} = \frac{t}{2}\bigg|_{t=2\,\text{s}} = 1\text{ rad/s} \qquad \ddot{\theta}|_{t=2\,\text{s}} = 0.5\text{ rad/s}^2$$

Using the above time derivative,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 3 - 4(1^2) = -1\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4(0.5) + 2(3)(1) = 8\text{ m/s}^2$$



(a)

Equations of Motion: By referring to the free-body diagram of the particle in Fig. (a),

$\Sigma F_r = ma_r;$ $\qquad F_r = 2(-1) = -2\text{ N}$ **Ans.**

$\Sigma F_\theta = ma_\theta;$ $\qquad F_\theta = 2(8) = 16\text{ N}$ **Ans.**

Note: The negative sign indicates that $\mathbf{F}_r$ acts in the opposite sense to that shown on the free-body diagram.

13–87. A 2-kg particle travels along a path defined by

$$r = (3 + 2t^2)\,\text{m},\ \theta = \left(\frac{1}{3}t^3 + 2\right)\text{rad}$$

and $z = (5 - 2t^2)$ m, where t is in seconds. Determine the r, θ, z components of force that the path exerts on the particle at the instant $t = 1$ s.

Kinematics: Since the motion of the particle is known, $\mathbf{a}_r$, $\mathbf{a}_\theta$, and $\mathbf{a}_z$ will be determined first. The values of r and the time derivative of r, θ, and z evaluated at $t = 1$ s are

$$r|_{t=1\text{ s}} = 3 + 2t^2\Big|_{t=1\text{ s}} = 5\text{ m} \qquad \dot{r}|_{t=1\text{ s}} = 4t|_{t=1\text{ s}} = 4\text{ m/s} \qquad \ddot{r}|_{t=1\text{ s}} = 4\text{ m/s}^2$$

$$\dot{\theta}|_{t=1\text{ s}} = t^2\Big|_{t=1\text{ s}} = 1\text{ rad/s} \qquad \ddot{\theta}|_{t=1\text{ s}} = 2t|_{t=1\text{ s}} = 2\text{ rad/s}^2$$

$$\dot{z} = -4t \qquad \ddot{z} = -4\text{ m/s}^2$$

Using the above time derivative,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4 - 5(1^2) = -1\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 5(2) + 2(4)(1) = 18\text{ m/s}^2$$

$$a_z = \ddot{z} = -4\text{ m/s}^2$$

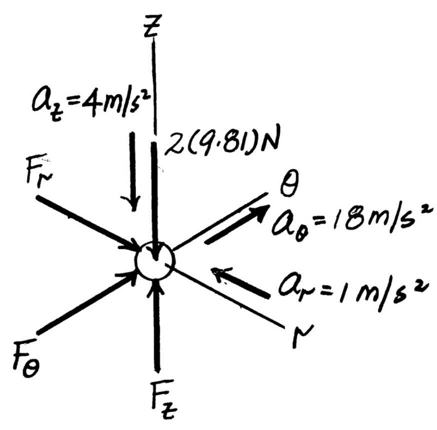
Equations of Motion: By referring to the free-body diagram of the particle in Fig. (a),

$\Sigma F_r = ma_r;$ $\quad F_r = 2(-1) = -2\text{ N}$ **Ans.**

$\Sigma F_\theta = ma_\theta;$ $\quad F_\theta = 2(18) = 36\text{ N}$ **Ans.**

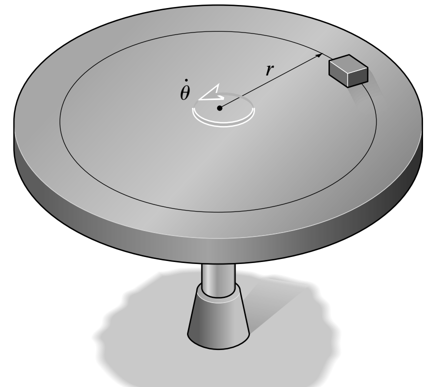
$\Sigma Fz = ma_z;$ $\quad F_z - 2(9.81) = 2(-4);$ $\quad F_z = 11.6\text{ N}$ **Ans.**

Note: The negative sign indicates that $\mathbf{F}_r$ acts in the opposite sense to that shown on the free-body diagram.



(a)

***13–88.** If the coefficient of static friction between the block of mass m and the turntable is μ_s, determine the maximum constant angular velocity of the platform without causing the block to slip.



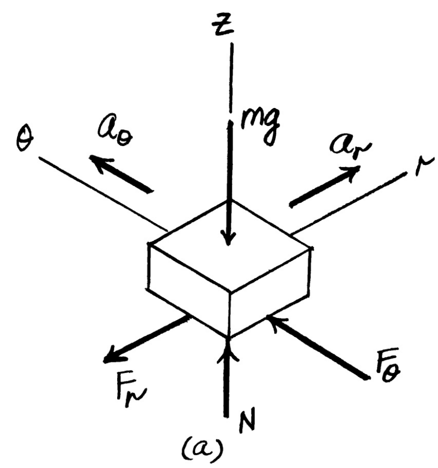
Free-Body Diagram: The free-body diagram of the block is shown in Fig. (a). Here, the frictional force developed is resolved into its radial and transversal components $\mathbf{F}_r$, $\mathbf{F}_\theta$, $\mathbf{a}_r$, and $\mathbf{a}_\theta$ are assumed to be directed towards their positive axes.

Equations of Motion: By referring to Fig. (a),

$$\Sigma F_r = ma_r; \qquad -F_r = ma_r \qquad \textbf{(1)}$$

$$\Sigma F_\theta = ma_\theta; \qquad F_\theta = ma_\theta \qquad \textbf{(2)}$$

$$\Sigma F_z = ma_z; \qquad N - mg = m(0) \qquad N = mg$$



Kinematics: Since r and $\dot{\theta}$ are constant, $\dot{r} = \ddot{r} = 0$ and $\ddot{\theta} = 0$.

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - r\dot{\theta}^2 = -r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$$

Substituting the results of $\mathbf{a}_r$ and $\mathbf{a}_\theta$ into Eqs. (1) and (2),

$$F_r = mr\,\dot{\theta}^2 \qquad F_\theta = 0$$

Thus, the magnitude of the frictional force is given by

$$F = \sqrt{F_r^{\,2} + F_\theta^{\,2}} = \sqrt{(mr\dot{\theta})^2 + 0} = mr\dot{\theta}^2$$

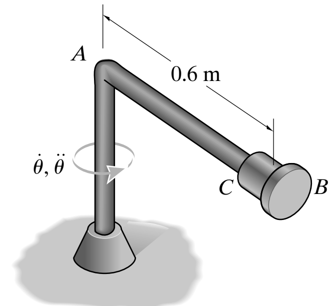
Since the block is required to be on the verge of slipping,

$$F = \mu_s N$$

$$mr\dot{\theta}^2 = \mu_s mg$$

$$\dot{\theta} = \sqrt{\frac{\mu_s g}{r}} \qquad \textbf{Ans.}$$

•13–89. The 0.5-kg collar C can slide freely along the smooth rod AB. At a given instant, rod AB is rotating with an angular velocity of $\dot{\theta} = 2$ rad/s and has an angular acceleration of $\ddot{\theta} = 2$ rad/s^2. Determine the normal force of rod AB and the radial reaction of the end plate B on the collar at this instant. Neglect the mass of the rod and the size of the collar.



Free-Body Diagram: The free-body diagram of the collar is shown in Fig. (a). Here, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ are assumed to be directed towards the positive of their respective axes.

Equations of Motion: By referring to Fig. (a),

$$\Sigma F_r = ma_r; \qquad -N_B = 0.5a_r \qquad \textbf{(1)}$$

$$\Sigma F_\theta = ma_\theta; \qquad F_{AB} = 0.5a_\theta \qquad \textbf{(2)}$$

Kinematics: Since $r = 0.6$ m is constant, $\dot{r} = \ddot{r} = 0$.

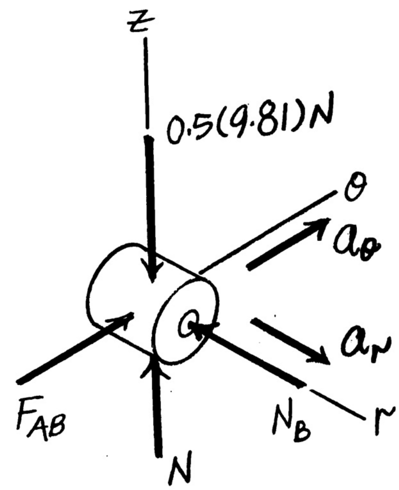
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - (0.6)(2^2) = -2.4 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6(2) + 0 = 1.2 \text{ m/s}^2$$

Substituting the results of $\mathbf{a}_r$ and $\mathbf{a}_\theta$ into Eqs. (1) and (2) yields

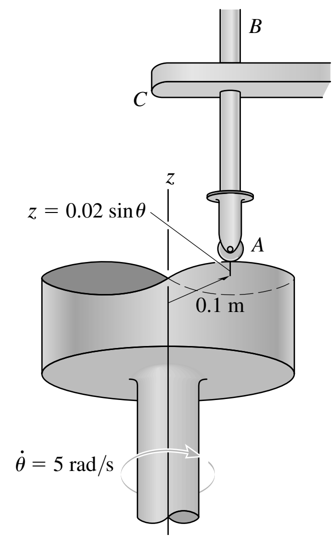
$$N_B = 1.20 \text{ N} \qquad \textbf{Ans.}$$

$$F_{AB} = 0.6 \text{ N} \qquad \textbf{Ans.}$$



(a)

13–90. The 2-kg rod AB moves up and down as its end slides on the smooth contoured surface of the cam, where $r = 0.1$ m and $z = (0.02 \sin \theta)$ m. If the cam is rotating with a constant angular velocity of 5 rad/s, determine the force on the roller A when $\theta = 90°$. Neglect friction at the bearing C and the mass of the roller.



Kinematics: Taking the required time derivatives, we have

$$\dot{\theta} = 5 \text{ rad/s} \qquad \ddot{\theta} = 0$$

$$z = 0.02 \sin \theta \qquad \dot{z} = 0.02 \cos \theta \dot{\theta} \qquad \ddot{z} = 0.02\left(\cos \theta \ddot{\theta} - \sin \theta \dot{\theta}^2\right)$$

Thus,

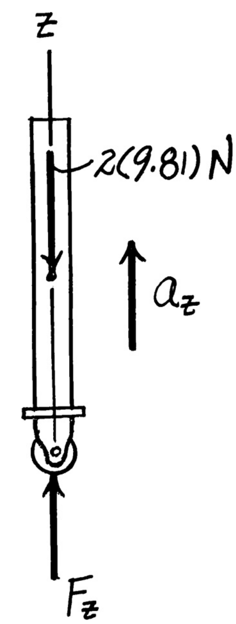
$$a_z = \ddot{z} = 0.02\left[\cos \theta (0) - \sin \theta \left(5^2\right)\right] = -0.5 \sin \theta$$

At $\theta = 90°$, $\qquad a_z = -0.5 \sin 90° = -0.500 \text{ m/s}^2$

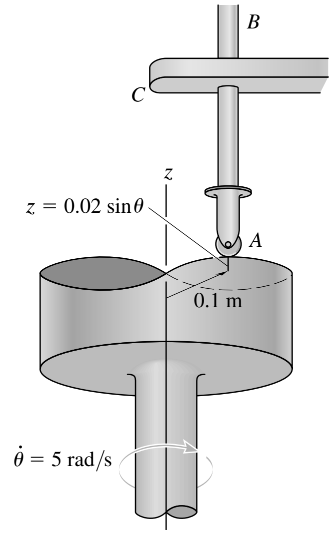
Equations of Motion:

$\Sigma F_z = ma_z;$ $\qquad F_z - 2(9.81) = 2(-0.500)$

$\qquad F_z = 18.6 \text{ N}$ **Ans.**



13–91. The 2-kg rod AB moves up and down as its end slides on the smooth contoured surface of the cam, where $r = 0.1$ m and $z = (0.02 \sin \theta)$ m. If the cam is rotating at a constant angular velocity of 5 rad/s, determine the maximum and minimum force the cam exerts on the roller at A. Neglect friction at the bearing C and the mass of the roller.



Kinematics: Taking the required time derivatives, we have

$$\dot{\theta} = 5 \text{ rad/s} \qquad \ddot{\theta} = 0$$

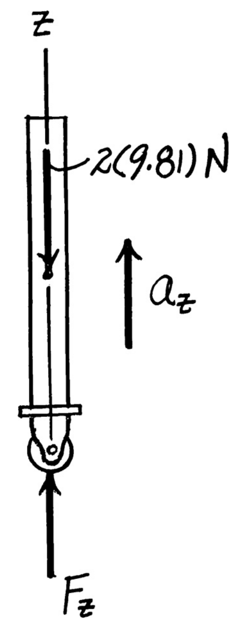
$$z = 0.02 \sin \theta \qquad \dot{z} = 0.02 \cos \theta\dot{\theta} \qquad \ddot{z} = 0.02\left(\cos \theta\ddot{\theta} - \sin \theta\dot{\theta}^2\right)$$

Thus,

$$a_z = \ddot{z} = 0.02\left[\cos \theta(0) - \sin \theta\left(5^2\right)\right] = -0.5 \sin \theta$$

At $\theta = 90°$, $\qquad a_z = -0.5 \sin 90° = -0.500 \text{ m/s}^2$

At $\theta = -90°$, $\qquad a_z = -0.5 \sin (-90°) = 0.500 \text{ m/s}^2$



Equations of Motion: At $\theta = 90°$, applying Eq. 13–9, we have

$$\Sigma F_z = ma_z; \qquad (F_z)_{\min} - 2(9.81) = 2(-0.500)$$

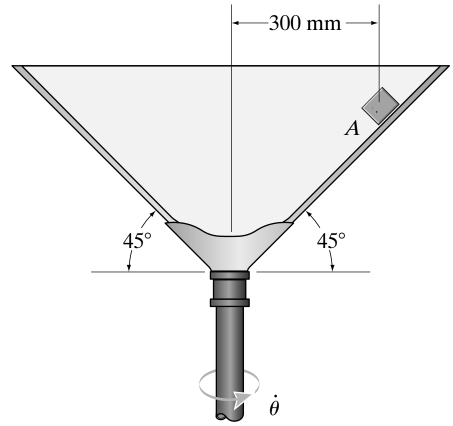
$$(F_z)_{\min} = 18.6 \text{ N} \qquad \textbf{Ans.}$$

At $\theta = -90°$, we have

$$\Sigma F_z = ma_z; \qquad (F_z)_{\max} - 2(9.81) = 2(0.500)$$

$$(F_z)_{\max} = 20.6 \text{ N} \qquad \textbf{Ans.}$$

***13–92.** If the coefficient of static friction between the conical surface and the block of mass m is $\mu_s = 0.2$, determine the minimum constant angular velocity $\dot{\theta}$ so that the block does not slide downwards.



Free-Body Diagram: The free-body diagram of the block is shown in Fig. (a). Since the block is required to be on the verge of sliding down the conical surface, $F_f = \mu_k N = 0.2N$ must be directed up the conical surface. Here, $\mathbf{a}_r$ is assumed to be directed towards the positive r axis.

Equations of Motion: By referring to Fig. (a),

$$+\uparrow \Sigma F_z = ma_z; \quad N \cos 45° + 0.2N \sin 45° - m(9.81) = m(0) \qquad N = 11.56m$$

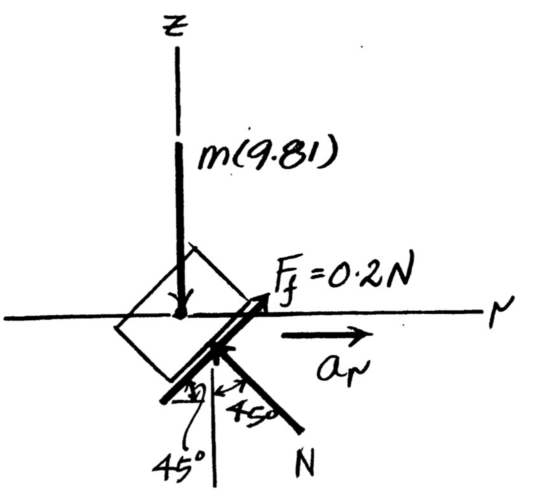
$$\overset{+}{\rightarrow} \Sigma F_r = ma_r; \quad 0.2(11.56m) \cos 45° - (11.56m) \sin 45° = ma_r \quad a_r = -6.54 \text{ m/s}^2$$

Kinematics: Since $r = 0.3$ m is constant, $\dot{r} = \ddot{r} = 0$.

$$a_r = \ddot{r} - r\dot{\theta}^2$$

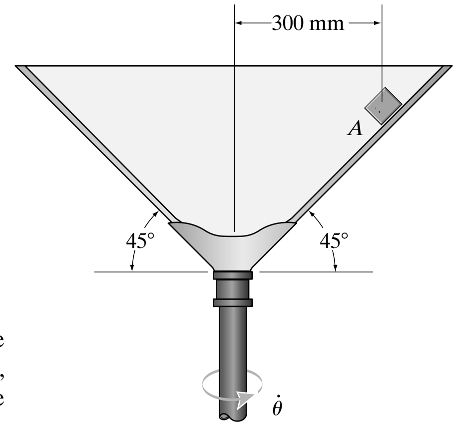
$$-6.54 = 0 - 0.3\dot{\theta}^2$$

$$\dot{\theta} = 4.67 \text{ rad/s} \qquad \textbf{Ans.}$$



(a)

•13–93. If the coefficient of static friction between the conical surface and the block is $\mu_s = 0.2$, determine the maximum constant angular velocity $\dot{\theta}$ without causing the block to slide upwards.



Free-Body Diagram: The free-body diagram of the block is shown in Fig. (a). Since the block is required to be on the verge of sliding up the conical surface, $F_f = \mu_k N = 0.2N$ must be directed down the conical surface. Here, $\mathbf{a}_r$ is assumed to be directed towards the positive r axis.

Equations of Motion: By referring to Fig. (a),

$$+\uparrow \Sigma F_z = ma_z; \quad N\cos 45° - 0.2N\sin 45° - m(9.81) = m(0) \qquad N = 17.34m$$

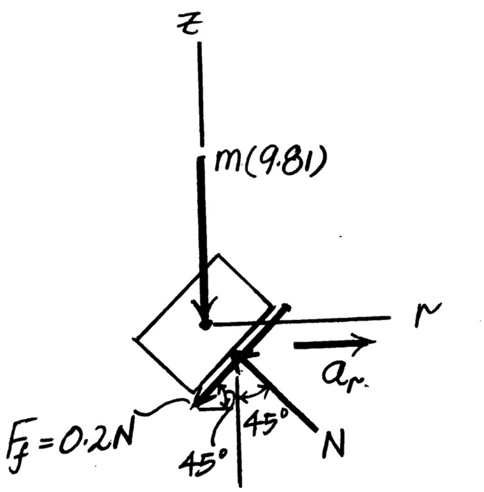
$$\xrightarrow{+} \Sigma F_r = ma_r; \quad -17.34m\sin 45° - 0.2(17.34m)\cos 45° = ma_r \quad a_r = -14.715 \text{ m/s}^2$$

Kinematics: Since $r = 0.3$ m is constant, $\dot{r} = \ddot{r} = 0$.

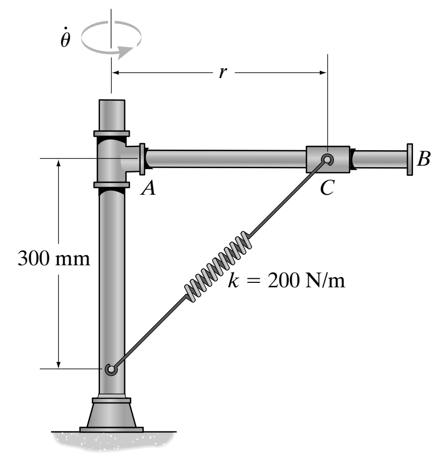
$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$-14.715 = 0 - 0.3\dot{\theta}^2$$

$$\dot{\theta} = 7.00 \text{ rad/s} \qquad \textbf{Ans.}$$



13–94. If the position of the 3-kg collar C on the smooth rod AB is held at $r = 720$ mm, determine the constant angular velocity $\dot{\theta}$ at which the mechanism is rotating about the vertical axis. The spring has an unstretched length of 400 mm. Neglect the mass of the rod and the size of the collar.



Free-Body Diagram: The free-body diagram of the collar is shown in Fig. (a). The force in the spring is given by $F_{sp} = ks = 200\left(\sqrt{0.72^2 + 0.3^2} - 0.4\right) = 76$ N. Here, $\mathbf{a}_r$ is assumed to be directed towards the positive r axis.

Equations of Motion: By referring to Fig. (a),

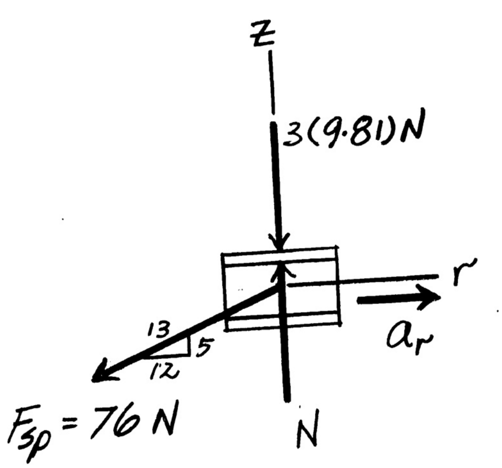
$$\overset{+}{\rightarrow} \Sigma F_r = ma_r; \qquad -76\left(\frac{12}{13}\right) = 3a_r \qquad a_r = -23.38 \text{ m/s}^2$$

Kinematics: Since $r = 0.72$ m is constant, $\dot{r} = \ddot{r} = 0$.

$$a_r = \ddot{r} - r\dot{\theta}^2$$

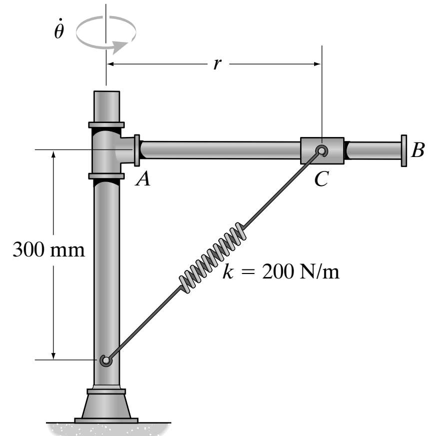
$$-23.38 = 0 - 0.72\dot{\theta}^2$$

$$\dot{\theta} = 5.70 \text{ rad/s}$$ **Ans.**



(a)

13–95. The mechanism is rotating about the vertical axis with a constant angular velocity of $\dot{\theta} = 6\,\text{rad/s}$. If rod AB is smooth, determine the constant position r of the 3-kg collar C. The spring has an unstretched length of 400 mm. Neglect the mass of the rod and the size of the collar.



Free-Body Diagram: The free-body diagram of the collar is shown in Fig. (a). The force in the spring is given by $F_{sp} = ks = 200\left(\sqrt{r^2 + 0.3^2} - 0.4\right)$. Here, $\mathbf{a}_r$ is assumed to be directed towards the positive r axis.

Equations of Motion: By referring to Fig. (a),

$$\xrightarrow{+} \Sigma F_r = ma_r; \qquad -200\left(\sqrt{r^2 + 0.3^2} - 0.4\right)\cos\alpha = 3a_r \qquad \textbf{(1)}$$

However, from the geometry shown in Fig. (b),

$$\cos\alpha = \frac{r}{\sqrt{r^2 + 0.3^2}}$$

Thus, Eq. (1) can be rewritten as

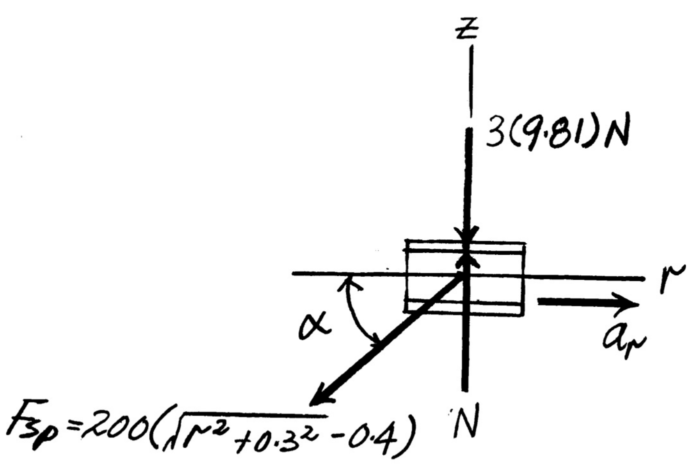
$$-200\left(r - \frac{0.4r}{\sqrt{r^2 + 0.3^2}}\right) = 3a_r \qquad \textbf{(2)}$$

Kinematics: Since r is constant, $\dot{r} = \ddot{r} = 0$.

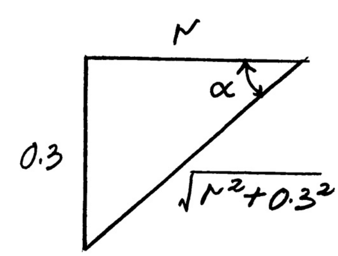
$$a_r = \ddot{r} - r\dot{\theta}^2 = -r(6^2) \qquad \textbf{(3)}$$

Substituting Eq. (3) into Eq. (2) and solving,

$$r = 0.8162\text{ m} = 816\text{ mm} \qquad \textbf{Ans.}$$

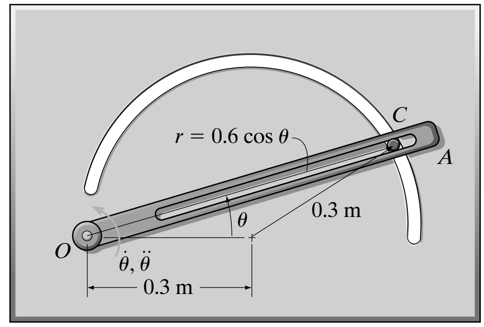


(a)



(b)

***13–96.** Due to the constraint, the 0.5-kg cylinder C travels along the path described by $r = (0.6\cos\theta)$ m. If arm OA rotates counterclockwise with an angular velocity of $\dot{\theta} = 2$ rad/s and an angular acceleration of $\ddot{\theta} = 0.8$ rad/s^2 at the instant $\theta = 30°$, determine the force exerted by the arm on the cylinder at this instant. The cylinder is in contact with only one edge of the smooth slot, and the motion occurs in the horizontal plane.



Kinematics: Since the motion of cylinder C is known, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ will be determined first. The values of r and the time derivatives at the instant $\theta = 30°$ are evaluated below.

$$r = 0.6\cos\theta\big|_{\theta=30°} = 0.6\cos 30° = 0.5196 \text{ m}$$

$$\dot{r} = -0.6\sin\theta\dot{\theta}\big|_{\theta=30°} = -0.6\sin 30°(2) = -0.6 \text{ m/s}$$

$$\ddot{r} = -0.6(\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta})\big|_{\theta=30°} = -0.6\left[\cos 30°\left(2^2\right) + \sin 30°(0.8)\right] = -2.318 \text{ m/s}^2$$

Using the above time derivatives, we obtain

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2.318 - 0.5196\left(2^2\right) = -4.397 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5196(0.8) + 2(-0.6)(2) = -1.984 \text{ m/s}^2$$

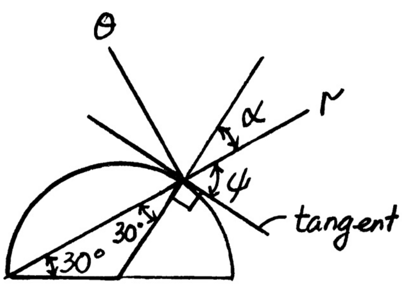
Free-Body Diagram: From the geometry shown in Fig. (a), we notice that $\alpha = 30°$. The free-body diagram of the cylinder C is shown in Fig. (b).

Equations of Motion: By referring to Fig. (b),

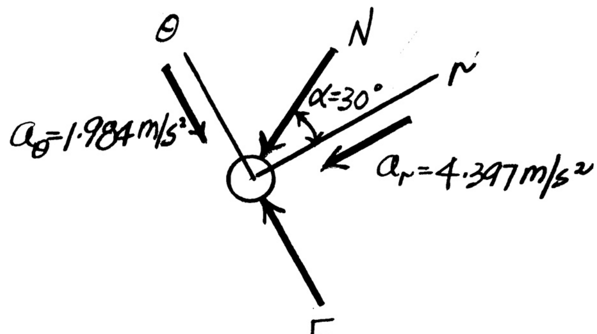
$+\nearrow\Sigma F_r = ma_r;\quad -N\cos 30° = 0.5(-4.397)\qquad N = 2.539\text{ N}$

$+\nwarrow\Sigma F_\theta = ma_\theta;\quad F_{OA} - 2.539\sin 30° = 0.5(-1.984)\qquad F_{OA} = 0.277\text{ N}$ **Ans.**

Kinematics: The values of r and the time derivatives at the instant $\theta = 30°$ are evaluated below.

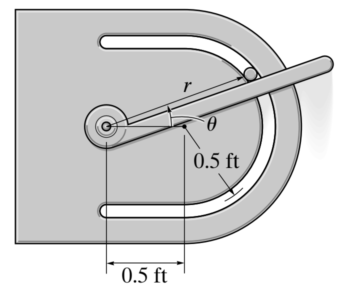


(a)



(b)

•13–97. The 0.75-lb smooth can is guided along the circular path using the arm guide. If the arm has an angular velocity $\dot{\theta} = 2$ rad/s and an angular acceleration $\ddot{\theta} = 0.4$ rad/s^2 at the instant $\theta = 30°$, determine the force of the guide on the can. Motion occurs in the *horizontal plane.*



$$r = \cos\theta\big|_{\theta=30°} = 0.8660 \text{ ft}$$

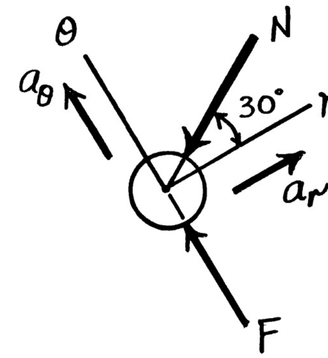
$$\dot{r} = -\sin\theta\dot{\theta}\big|_{\theta=30°} = -1.00 \text{ ft/s}$$

$$\ddot{r} = -(\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta})\big|_{\theta=30°} = -3.664 \text{ ft/s}^2$$

Using the above time derivative, we obtain

$$a_r = \ddot{r} - r\dot{\theta}^2 = -3.664 - 0.8660\left(2^2\right) = -7.128 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.8660(4) + 2(-1)(2) = -0.5359 \text{ ft/s}^2$$

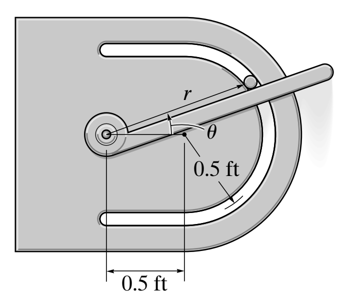


Equations of Motion: By referring to Fig. (a),

$\Sigma F_r = ma_r$; $\quad -N\cos 30° = \dfrac{0.75}{32.2}(-7.128) \quad N = 0.1917$ lb

$\Sigma F_\theta = ma_\theta$; $\quad F - 0.1917\sin 30° = \dfrac{0.75}{32.2}(-0.5359) \quad F = 0.0835$ lb **Ans.**

13–98. Solve Prob. 13–97 if motion occurs in the *vertical plane*.



Kinematics: The values of r and the time derivatives at the instant $\theta = 30°$ are evaluated below.

$$r = \cos\theta\big|_{\theta=30°} = 0.8660 \text{ ft}$$

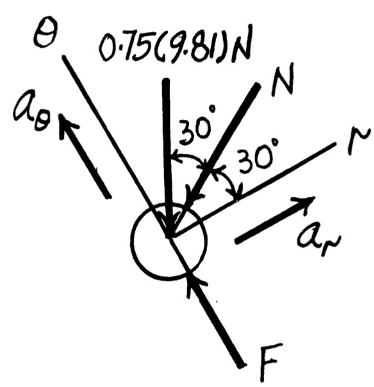
$$\dot{r} = -\sin\theta\dot{\theta}\big|_{\theta=30°} = -1.00 \text{ ft/s}$$

$$\ddot{r} = -(\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta})\big|_{\theta=30°} = -3.664 \text{ ft/s}^2$$

Using the above time derivative, we obtain

$$a_r = \ddot{r} - r\dot{\theta}^2 = -3.664 - 0.8660\left(2^2\right) = -7.128 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.8660(4) + 2(-1)(2) = -0.5359 \text{ ft/s}^2$$



Equations of Motion: By referring to Fig. (a),

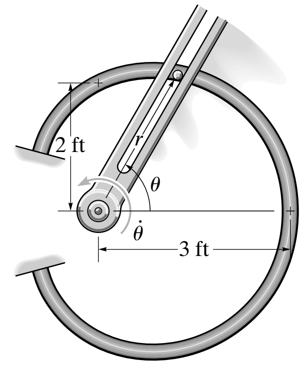
$\Sigma F_r = ma_r$; $\quad -N\cos 30° - 0.75\cos 60° = \dfrac{0.75}{32.2}(-7.128)$

$N = -0.2413$ lb

$\Sigma F_\theta = ma_\theta$; $\quad F + 0.2413\sin 30° - 0.75\sin 60° = \dfrac{0.75}{32.2}(-0.5359)$

$F = 0.516$ lb **Ans.**

13–99. The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If at all times $\dot{\theta} = 0.5$ rad/s, determine the force which the rod exerts on the particle at the instant $\theta = 90°$. The fork and path contact the particle on only one side.



$r = 2 + \cos\theta$

$\dot{r} = -\sin\theta\dot{\theta}$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta}$

At $\theta = 90°$, $\dot{\theta} = 0.5$ rad/s, and $\ddot{\theta} = 0$

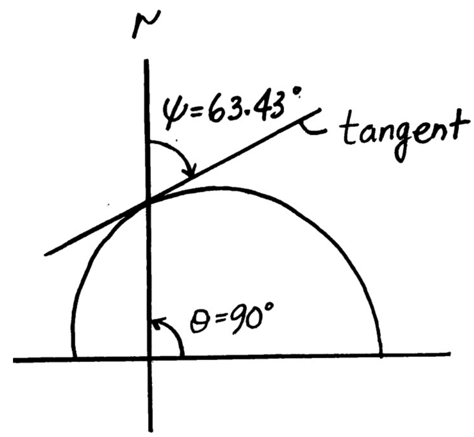
$r = 2 + \cos 90° = 2$ ft

$\dot{r} = -\sin 90°(0.5) = -0.5$ ft/s

$\ddot{r} = -\cos 90°(0.5)^2 - \sin 90°(0) = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 2(0.5)^2 = -0.5 \text{ ft/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2(0) + 2(-0.5)(0.5) = -0.5 \text{ ft/s}^2$

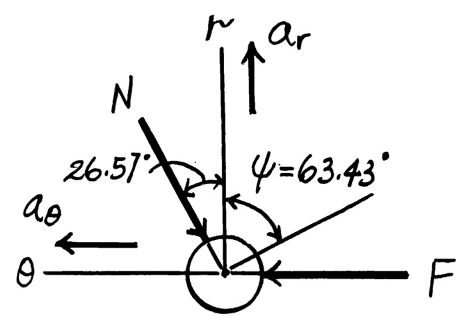


$$\tan\psi = \frac{r}{dr/d\theta} = \frac{2 + \cos\theta}{-\sin\theta}\bigg|_{\theta=90°} = -2 \qquad \psi = -63.43°$$

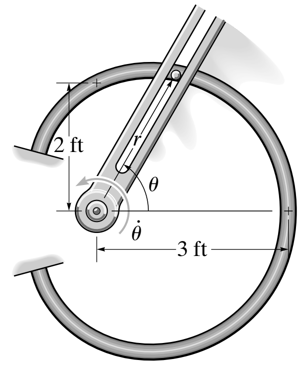
$$+\uparrow\Sigma F_r = ma_r; \qquad -N\cos 26.57° = \frac{2}{32.2}(-0.5) \qquad N = 0.03472 \text{ lb}$$

$$\overset{+}{\leftarrow}\Sigma F_\theta = ma_\theta; \qquad F - 0.03472\ \sin 26.57° = \frac{2}{32.2}(-0.5)$$

$$F = -0.0155 \text{ lb} \qquad \textbf{Ans.}$$



***13–100.** Solve Prob. 13–99 at the instant $\theta = 60^\circ$.



$r = 2 + \cos\theta$

$\dot{r} = -\sin\theta\dot{\theta}$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta}$

At $\theta = 60^\circ$, $\dot{\theta} = 0.5$ rad/s, and $\ddot{\theta} = 0$

$r = 2 + \cos 60^\circ = 2.5$ ft

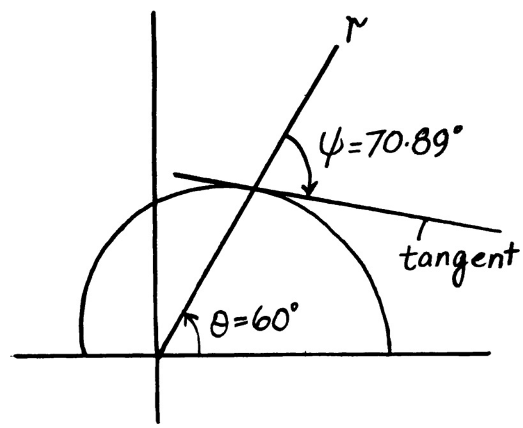
$\dot{r} = -\sin 60^\circ(0.5) = -0.4330$ ft/s

$\ddot{r} = -\cos 60^\circ(0.5)^2 - \sin 60^\circ(0) = -0.125$ ft/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.125 - 2.5(0.5)^2 = -0.75$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.5(0) + 2(-0.4330)(0.5) = -0.4330$ ft/s^2

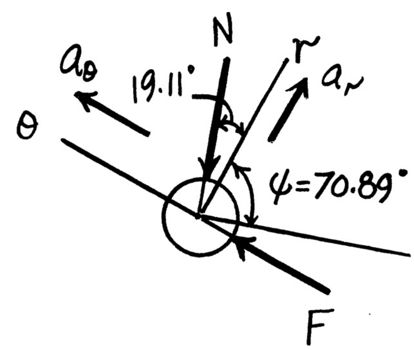
$$\tan\psi = \frac{r}{dr/d\theta} = \frac{2 + \cos\theta}{-\sin\theta}\bigg|_{\theta=60^\circ} = -2.887 \qquad \psi = -70.89^\circ$$



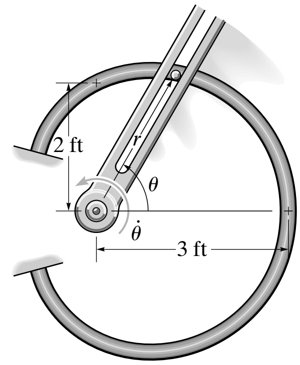
$+\nearrow\Sigma F_r = ma_r;\qquad -N\cos 19.11^\circ = \dfrac{2}{32.2}(-0.75)\qquad N = 0.04930$ lb

$+\nwarrow\Sigma F_\theta = ma_\theta;\qquad F - 0.04930\sin 19.11^\circ = \dfrac{2}{32.2}(-0.4330)$

$F = -0.0108$ lb **Ans.**



•13–101. The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If $\theta = (0.5t^2)$ rad, where t is in seconds, determine the force which the rod exerts on the particle at the instant $t = 1$ s. The fork and path contact the particle on only one side.



$r = 2 + \cos\theta \qquad \theta = 0.5t^2$

$\dot{r} = -\sin\theta\theta \qquad \dot{\theta} = t$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta} \qquad \ddot{\theta} = 1\ \text{rad/s}^2$

At $t = 1$ s, $\theta = 0.5$ rad, $\theta = 1$ rad/s, and $\ddot{\theta} = 1\ \text{rad/s}^2$

$r = 2 + \cos 0.5 = 2.8776$ ft

$\dot{r} = -\sin 0.5(1) = -0.4974\ \text{ft/s}^2$

$\ddot{r} = -\cos 0.5(1)^2 - \sin 0.5(1) = -1.357\ \text{ft/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -1.375 - 2.8776(1)^2 = -4.2346\ \text{ft/s}^2$

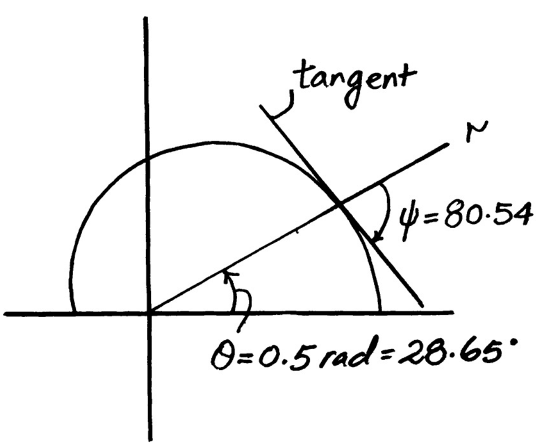
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.8776(1) + 2(-0.4794)(1) = 1.9187\ \text{ft/s}^2$

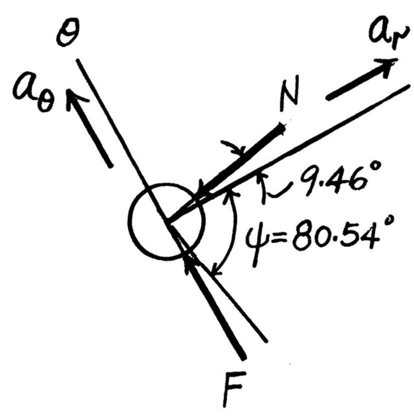
$$\tan\psi = \frac{r}{dr/d\theta} = \left.\frac{2 + \cos\theta}{-\sin\theta}\right|_{\theta = 0.5\ \text{rad}} = -6.002 \qquad \psi = -80.54^\circ$$

$$+\nearrow\Sigma F_r = ma_r; \qquad -N\cos 9.46^\circ = \frac{2}{32.2}(-4.2346) \qquad N = 0.2666\ \text{lb}$$

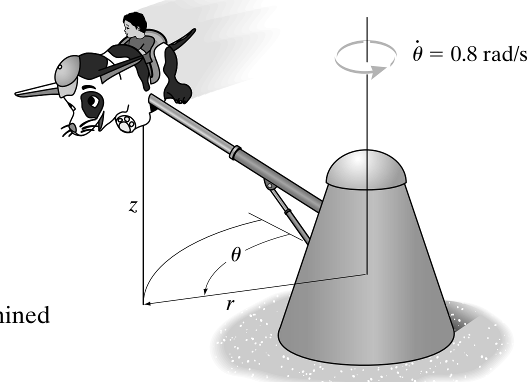
$$+\nwarrow\Sigma F_\theta = ma_\theta; \qquad F - 0.2666\sin 9.46^\circ = \frac{2}{32.2}(1.9187)$$

$$F = 0.163\ \text{lb}$$ **Ans.**





13–102. The amusement park ride rotates with a constant angular velocity of $\dot{\theta} = 0.8\,\text{rad/s}$. If the path of the ride is defined by $r = (3\sin\theta + 5)\,\text{m}$ and $z = (3\cos\theta)\,\text{m}$, determine the r, θ, and z components of force exerted by the seat on the 20-kg boy when $\theta = 120°$.



Kinematics: Since the motion of the boy is known, $\mathbf{a}_r$, $\mathbf{a}_\theta$, and $\mathbf{a}_z$ will be determined first. The value of r and its time derivatives at the instant $\theta = 120°$ are

$$r = (3\sin\theta + 5)\big|_{\theta=120°} = 3\sin 120° + 5 = 7.598\text{ m}$$

$$\dot{r} = 3\cos\theta\dot{\theta}\big|_{\theta=120°} = 3\cos 120°(0.8) = -1.2\text{ m/s}$$

$$\ddot{r} = 3(\cos\theta\ddot{\theta} - \sin\theta\dot{\theta}^2)\big|_{\theta=120°}$$

$$= 3\left[\cos 120°(0) - \sin 120°(0.8^2)\right] = -1.663\text{ m/s}^2$$

Using the above time derivatives, we obtain

$$a_r = \ddot{r} - r\dot{\theta}^2 = -1.663 - 7.598(0.8^2) = -6.526\text{ m/s}^2$$

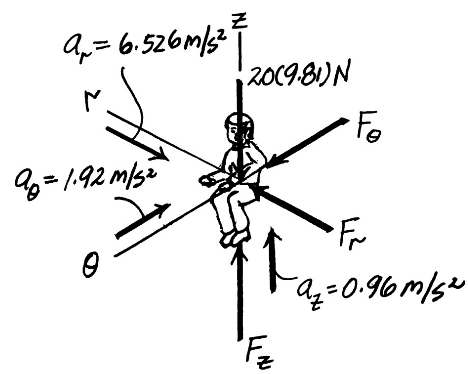
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 7.598(0) + 2(-1.2)(0.8) = -1.92\text{ m/s}^2$$

Also,

$$z = 3\cos\theta\,\text{m} \qquad \dot{z} = -3\sin\theta\dot{\theta}\text{ m/s}$$

$$a_z = \ddot{z} = -3(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2)\Big|_{\theta=120°} = -3\left[\sin 120°(0) + \cos 120°(0.8^2)\right]$$

$$= 0.96\text{ m/s}^2$$



(a)

Equations of Motion: By referring to the free-body diagram of the boy shown in Fig. (a),

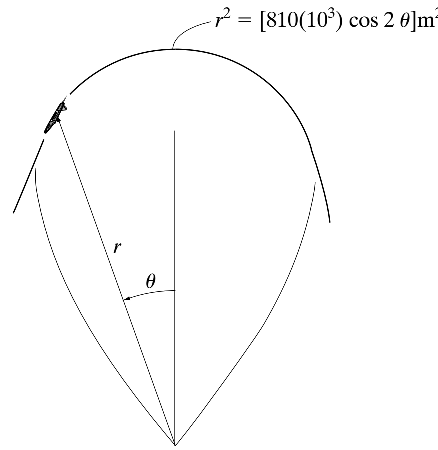
$\Sigma F_r = ma_r;$ $\qquad F_r = 20(-6.526) = -131\text{ N}$ **Ans.**

$\Sigma F_\theta = ma_\theta;$ $\qquad F_\theta = 20(-1.92) = -38.4\text{ N}$ **Ans.**

$\Sigma F_z = ma_z;$ $\qquad F_z - 20(9.81) = 20(0.96) \qquad F_z = 215\text{ N}$ **Ans.**

Note: The negative signs indicate that $\mathbf{F}_r$ and $\mathbf{F}_\theta$ act in the opposite sense to those shown on the free-body diagram.

13–103. The airplane executes the vertical loop defined by $r^2 = [810(10^3)\cos 2\theta]\,\text{m}^2$. If the pilot maintains a constant speed $v = 120\,\text{m/s}$ along the path, determine the normal force the seat exerts on him at the instant $\theta = 0°$. The pilot has a mass of 75 kg.



Kinematics: Since the motion of the airplane is known, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ will be determined first. The value of r and θ at $\theta = 0°$ are

$$r^2 = 810(10^3)\cos 2\theta\Big|_{\theta=0°} = 810(10^3)\cos 0°$$

$$r = 900\text{ m}$$

and

$$2r\dot{r} = -810(10^3)\sin 2\theta(2\dot{\theta})$$

$$\dot{r} = \frac{-810(10^3)\sin 2\theta\dot{\theta}}{r}\Bigg|_{\theta=0°} = \frac{-810(10^3)\sin 0°\theta}{900} = 0$$

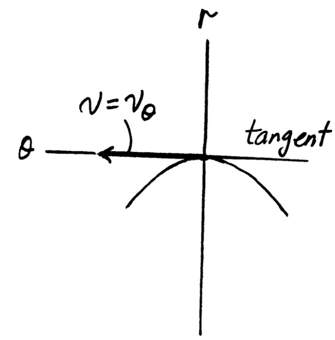
and

$$r\ddot{r} + \dot{r}^2 = -810(10^3)\left[\sin 2\theta\ddot{\theta} + 2\cos 2\theta\dot{\theta}^2\right]$$

$$\ddot{r} = \frac{-810(10^3)\left[\sin 2\theta\ddot{\theta} + 2\cos 2\theta\dot{\theta}^2\right] - \dot{r}^2}{r}\Bigg|_{\theta=0°}$$

$$= \frac{-810(10^3)\left[\sin 0°\ddot{\theta} + 2\cos 0°\dot{\theta}^2\right] - 0}{900}$$

$$= -1800\dot{\theta}^2$$



(a)

The radial and transversal components of the airplane's velocity are given by

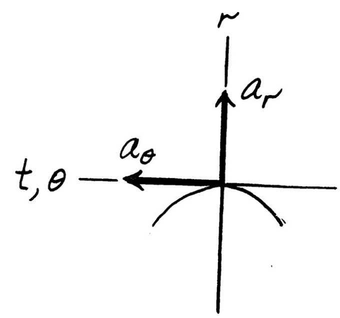
$$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 900\dot{\theta}$$

Thus,

$$v = v_\theta$$

$$120 = 900\dot{\theta}$$

$$\dot{\theta} = 0.1333\text{ rad/s}^2$$



(b)

Substituting the result of $\dot{\theta}$ into $\ddot{r}$, we obtain

$$\ddot{r} = -1800(0.1333^2) = -32\text{ m/s}^2$$

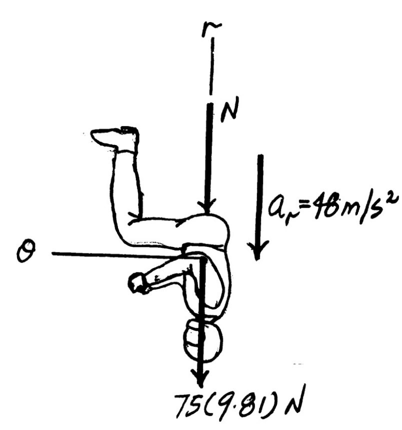
Since $v = v_\theta$ and v are always directed along the tangent, then the tangent of the path at $\theta = 0°$ coincide with the θ axis, Fig. (a). As a result $a_\theta = a_t = 0$, Fig. (b), because v is constant. Using the results of $\ddot{r}$ and $\dot{\theta}$, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = -32 - 900(1.1333^2) = -48\text{ m/s}^2$$

Equations of Motion: By referring to the free-body diagram of the pilot shown in Fig. (c),

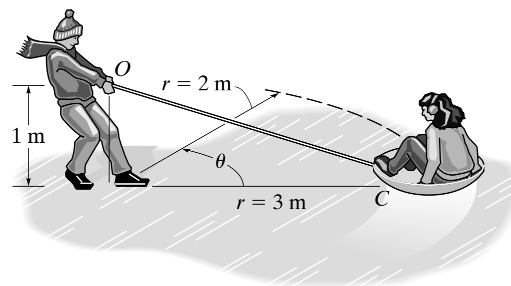
$$+\uparrow\Sigma F_r = ma_r; \qquad -N - 75(9.81) = 75(-48)$$

$$N = 2864.25\text{ N} = 2.86\text{ kN} \qquad \textbf{Ans.}$$



(c)

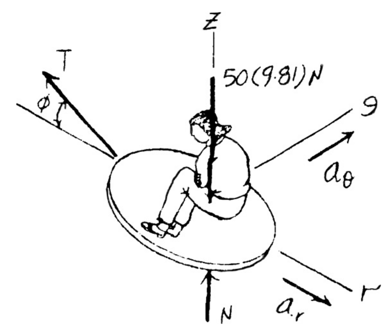
***13–104.** A boy standing firmly spins the girl sitting on a circular "dish" or sled in a circular path of radius $r_0 = 3$ m such that her angular velocity is $\dot{\theta}_0 = 0.1$ rad/s. If the attached cable OC is drawn inward such that the radial coordinate r changes with a constant speed of $\dot{r} = -0.5$ m/s, determine the tension it exerts on the sled at the instant $r = 2$ m. The sled and girl have a total mass of 50 kg. Neglect the size of the girl and sled and the effects of friction between the sled and ice. *Hint:* First show that the equation of motion in the θ direction yields $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1/r)\,d/dt(r^2\dot{\theta}) = 0$. When integrated, $r^2\dot{\theta} = C$, where the constant C is determined from the problem data.



Equations of Motion: Applying Eq.13–9, we have

$$\Sigma F_\theta = ma_\theta; \qquad 0 = 50\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$

$$\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right) = \frac{1}{r}\frac{d\left(r^2\dot{\theta}\right)}{dt} = 0 \qquad \textbf{Q.E.D.}$$



Thus, $\displaystyle\int \frac{d\left(r^2\dot{\theta}\right)}{dt} = C$. Then, $r^2\,\dot{\theta} = C$. At $r = r_0 = 3$ m, $\dot{\theta} = \dot{\theta}_0 = 0.1$ rad/s.

Hence, $r = 2$ m,

$$\left(2^2\right)\dot{\theta} = \left(3^2\right)(0.1) \qquad \dot{\theta} = 0.225 \text{ rad/s}$$

Here, $\dot{r} = -0.5$ m/s and $\ddot{r} = 0$. Applying Eqs. 12–29, we have

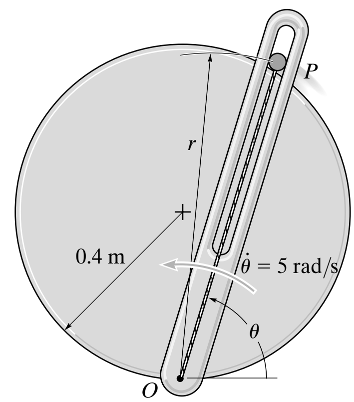
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 2\left(0.225^2\right) = -0.10125 \text{ m/s}^2$$

At $r = 2$ m, $\phi = \tan^{-1}\left(\dfrac{1}{2}\right) = 26.57°$. Then

$$\Sigma F_r = ma_r; \qquad -T\cos 26.57° = 50(-0.10125)$$

$$T = 5.66 \text{ N} \qquad \textbf{Ans.}$$

13–105. The smooth particle has a mass of 80 g. It is attached to an elastic cord extending from O to P and due to the slotted arm guide moves along the *horizontal* circular path $r = (0.8 \sin \theta)$ m. If the cord has a stiffness $k = 30$ N/m and an unstretched length of 0.25 m, determine the force of the guide on the particle when $\theta = 60°$. The guide has a constant angular velocity $\dot{\theta} = 5$ rad/s.



$r = 0.8 \sin \theta$

$\dot{r} = 0.8 \cos \theta \, \dot{\theta}$

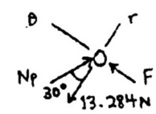
$\ddot{r} = -0.8 \sin \theta \, (\dot{\theta})^2 + 0.8 \cos \theta \ddot{\theta}$

$\dot{\theta} = 5, \qquad \ddot{\theta} = 0$

At $\theta = 60°$, $\quad r = 0.6928$

$\dot{r} = 2$

$\ddot{r} = -17.321$



$a_r = \ddot{r} - r(\dot{\theta})^2 = -17.321 - 0.6928(5)^2 = -34.641$

$a_\theta = r\ddot{\theta} + 2\,\dot{r}\dot{\theta} = 0 + 2(2)(5) = 20$

$F_s = ks; \qquad F_s = 30(0.6928 - 0.25) = 13.284$ N

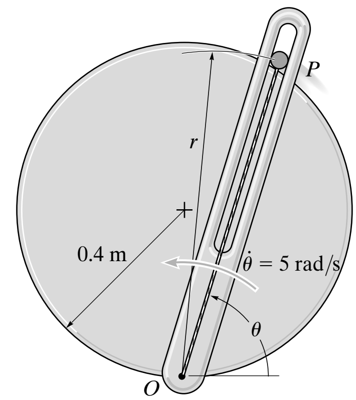
$\nearrow + \Sigma F_r = m a_r; \qquad -13.284 + N_P \cos 30° = 0.08(-34.641)$

$\nwarrow + \Sigma F_\theta = m a_\theta; \qquad F - N_P \sin 30° = 0.08(20)$

$F = 7.67$ N **Ans.**

$N_P = 12.1$ N

13–106. Solve Prob. 13–105 if $\ddot{\theta} = 2\ \text{rad/s}^2$ when $\dot{\theta} = 5\ \text{rad/s}$ and $\theta = 60°$.



$r = 0.8 \sin\theta$

$\dot{r} = 0.8 \cos\theta\,\dot{\theta}$

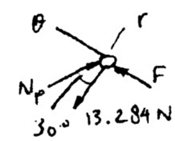
$\ddot{r} = -0.8 \sin\theta\,(\dot{\theta})^2 + 0.8 \cos\theta\ddot{\theta}$

$\dot{\theta} = 5, \qquad \ddot{\theta} = 2$

At $\theta = 60°$, $\quad r = 0.6928$

$\dot{r} = 2$

$\ddot{r} = -16.521$



$a_r = \ddot{r} - r(\dot{\theta})^2 = -16.521 - 0.6928(5)^2 = -33.841$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6925(2) + 2(2)(5) = 21.386$

$F_s = ks; \qquad F_s = 30(0.6928 - 0.25) = 13.284\ \text{N}$

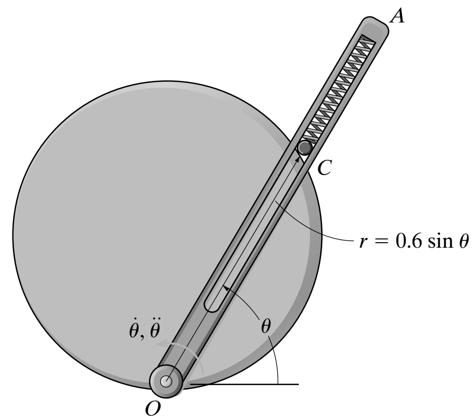
$\nearrow + \Sigma F_r = m\,a_r; \qquad -13.284 + N_P \cos 30° = 0.08(-33.841)$

$+\nwarrow \Sigma F_\theta = m a_\theta; \qquad F - N_P \sin 30° = 0.08(21.386)$

$F = 7.82\ \text{N}$ **Ans.**

$N_P = 12.2\ \text{N}$

13–107. The 1.5-kg cylinder C travels along the path described by $r = (0.6 \sin\theta)$ m. If arm OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 3$ rad/s, determine the force exerted by the smooth slot in arm OA on the cylinder at the instant $\theta = 60°$. The spring has a stiffness of 100 N/m and is unstretched when $\theta = 30°$. The cylinder is in contact with only one edge of the slotted arm. Neglect the size of the cylinder. Motion occurs in the horizontal plane.



Kinematics: Since the motion of cylinder C is known, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ will be determined first. The values of r and its time derivatives at the instant $\theta = 60°$ are evaluated below.

$$r = 0.6 \sin\theta\big|_{\theta=60°} = 0.6 \sin 60° = 0.5196 \text{ m}$$

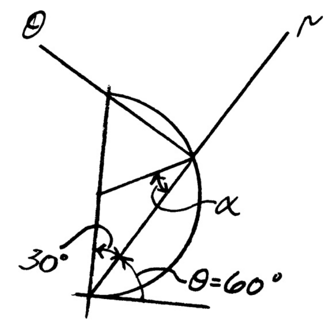
$$\dot{r} = 0.6 \cos\theta\dot{\theta}\big|_{\theta=60°} = 0.6 \cos 60°(3) = 0.9 \text{ m/s}$$

$$\ddot{r} = 0.6(\cos\theta\ddot{\theta} - \sin\theta(\dot{\theta})^2)\Big|_{\theta=60°} = 0.6\left[\cos 60°(0) - \sin 60°(3^2)\right] = -4.677 \text{ m/s}^2$$

Using the above time derivatives,

$$a_r = \ddot{r} - r\dot{\theta}^2 = -4.677 - 0.5196(3^2) = -9.353 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5196(0) + 2(0.9)(3) = 5.4 \text{ m/s}^2$$



(a)

Free-Body Diagram: From the geometry shown in Fig. (a), we notice that $\alpha = 30°$. The force developed in the spring is given by $F_{sp} = ks = 100(0.6 \sin 60° - 0.6 \sin 30°) = 21.96$ N. The free-body diagram of the cylinder C is shown in Fig. (b).

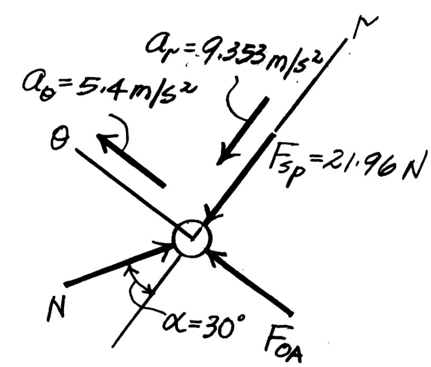
Equations of Motion: By referring to Fig. (a),

$+\nearrow \Sigma F_r = ma_r;$ $\qquad N \cos 30° - 21.96 = 1.5(-9.353)$

$\qquad N = 9.159 \text{ N}$

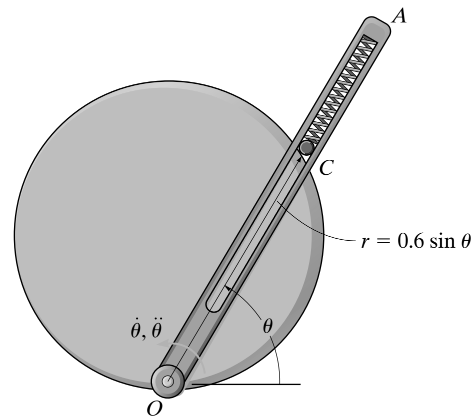
$\nwarrow + \Sigma F_\theta = ma_\theta;$ $\qquad F_{OA} - 9.159 \sin 30° = 1.5(5.4)$

$\qquad F_{OA} = 12.68 \text{ N} = 12.7 \text{ N}$ **Ans.**



(b)

***13–108.** The 1.5-kg cylinder C travels along the path described by $r = (0.6 \sin \theta)$ m. If arm OA is rotating counterclockwise with an angular velocity of $\dot{\theta} = 3$ rad/s, determine the force exerted by the smooth slot in arm OA on the cylinder at the instant $\theta = 60°$. The spring has a stiffness of 100 N/m and is unstretched when $\theta = 30°$. The cylinder is in contact with only one edge of the slotted arm. Neglect the size of the cylinder. Motion occurs in the vertical plane.

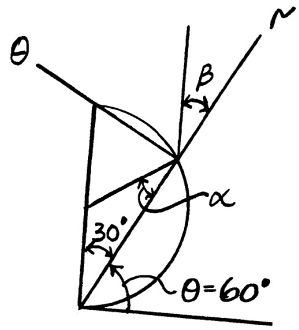


Kinematics: Since the motion of cylinder C is known, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ will be determined first. The values of r and its time derivatives at the instant $0 = 60°$ are evaluated below.

$$r = 0.6 \sin \theta\big|_{\theta=60°} = 0.6 \sin 60° = 0.5196 \text{ m}$$

$$\dot{r} = 0.6 \cos \theta\dot{\theta}\Big|_{\theta=60°} = 0.6 \cos 60°(3) = 0.9 \text{ m/s}$$

$$\ddot{r} = 0.6(\cos \theta\ddot{\theta} - \sin \theta(\dot{\theta})^2)\Big|_{\theta=60°} = 0.6\left[\cos 60°(0) - \sin 60°(3^2)\right] = -4.677 \text{ m/s}^2$$



(a)

Using the above time derivatives,

$$a_r = \ddot{r} - r\dot{\theta}^2 = -4.677 - 0.5196(3^2) = -9.353 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5196(0) + 2(0.9)(3) = 5.4 \text{ m/s}^2$$

Free-Body Diagram: From the geometry shown in Fig. (a), we notice that $\alpha = 30°$ and $\beta = 30°$. The force developed in the spring is given by $F_{sp} = ks = 100(0.6 \sin 60° - 0.6 \sin 30°) = 21.96$ N. The free-body diagram of the cylinder C is shown in Fig. (b).

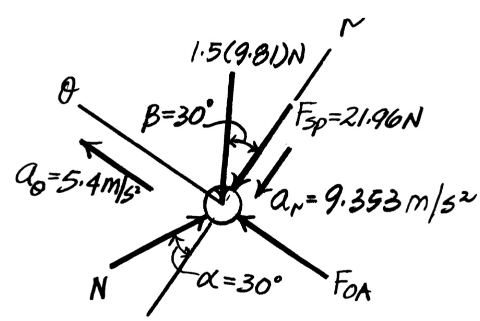
Equations of Motion: By referring to Fig. (a),

$$+\nearrow\Sigma F_r = ma_r; \qquad N \cos 30° - 21.96 - 1.5(9.81) \cos 30° = 1.5(-9.353)$$

$$N = 23.87 \text{ N}$$

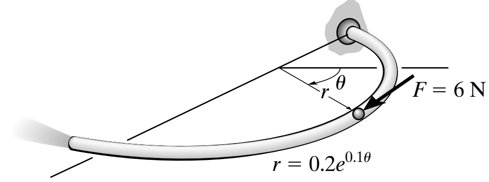
$$\nwarrow+\Sigma F_\theta = ma_\theta; \qquad F_{OA} - 1.5(9.81) \sin 30° - 23.87 \sin 30° = 1.5(5.4)$$

$$F_{OA} = 27.4 \text{ N} \qquad \textbf{Ans.}$$



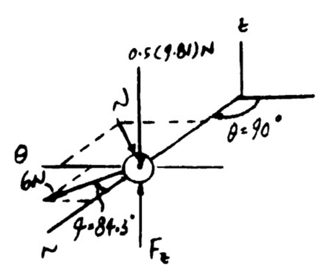
(b)

•13–109. Using air pressure, the 0.5-kg ball is forced to move through the tube lying in the horizontal plane and having the shape of a logarithmic spiral. If the tangential force exerted on the ball due to air pressure is 6 N, determine the rate of increase in the ball's speed at the instant $\theta = \pi/2$. Also, what is the angle ψ from the extended radial coordinate r to the line of action of the 6-N force?

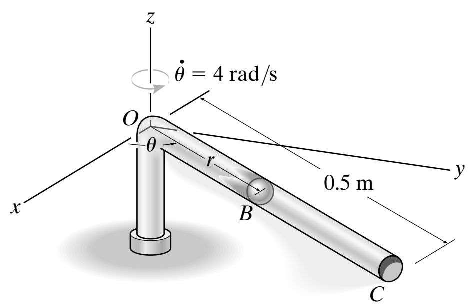


$$\tan \psi = \frac{r}{dr/d\theta} = \frac{0.2e^{0.1\,\theta}}{0.02e^{0.1\theta}} = 10 \qquad \psi = 84.3° \qquad \textbf{Ans.}$$

$$\Sigma F_t = ma_t; \qquad 6 = 0.5a_t \qquad a_t = 12 \text{ m/s}^2 \qquad \textbf{Ans.}$$



13–110. The tube rotates in the horizontal plane at a constant rate of $\dot{\theta} = 4$ rad/s. If a 0.2-kg ball B starts at the origin O with an initial radial velocity of $\dot{r} = 1.5$ m/s and moves outward through the tube, determine the radial and transverse components of the ball's velocity at the instant it leaves the outer end at C, $r = 0.5$ m. *Hint:* Show that the equation of motion in the r direction is $\ddot{r} - 16r = 0$. The solution is of the form $r = Ae^{-4t} + Be^{4t}$. Evaluate the integration constants A and B, and determine the time t when $r = 0.5$ m. Proceed to obtain v_r and v_θ.

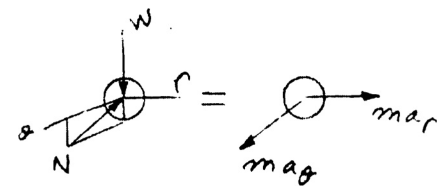


$$\dot{\theta} = 4 \qquad \ddot{\theta} = 0$$

$$\Sigma F_r = ma_r; \qquad 0 = 0.2\left[\ddot{r} - r(4)^2\right]$$

$$\ddot{r} - 16r = 0$$

Solving this second-order differential equation,



$$r = Ae^{-4t} + Be^{4t} \qquad \textbf{(1)}$$

$$\dot{r} = -4Ae^{-4t} + 4Be^{4t} \qquad \textbf{(2)}$$

At $t = 0, r = 0, \dot{r} = 1.5$:

$$0 = A + B \qquad \frac{1.5}{4} = -A + B$$

$$A = -0.1875 \qquad B = 0.1875$$

From Eq. (1) at $r = 0.5$ m,

$$0.5 = 0.1875\left(-e^{-4t} + e^{4t}\right)$$

$$\frac{2.667}{2} = \frac{(-e^{-4t} + e^{4t})}{2}$$

$$1.333 = \sin\text{h}(4t)$$

$$t = \frac{1}{4}\sin\text{h}^{-1}(1.333) \qquad t = 0.275\text{ s}$$

Using Eq. (2),

$$\dot{r} = 4(0.1875)\left(e^{-4t} + e^{4t}\right)$$

$$\dot{r} = 8(0.1875)\left(\frac{e^{-4t} + e^{4t}}{2}\right) = 8(0.1875)(\cos\text{h}(4t))$$

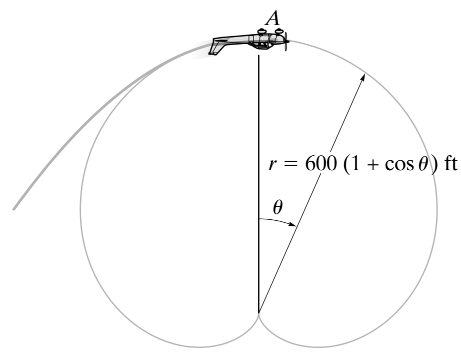
At $t = 0.275$ s:

$$\dot{r} = 1.5\cos\text{h}[4(0.275)]$$

$$v_t = r = 2.50\text{ m/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 0.5(4) = 2\text{ m/s} \qquad \textbf{Ans.}$$

13–111. The pilot of an airplane executes a vertical loop which in part follows the path of a cardioid, $r = 600(1 + \cos\theta)$ ft. If his speed at A ($\theta = 0°$) is a constant $v_P = 80$ ft/s, determine the vertical force the seat belt must exert on him to hold him to his seat when the plane is upside down at A. He weighs 150 lb.



$$r = 600(1 + \cos\theta)\big|_{\theta=0°} = 1200 \text{ ft}$$

$$\dot{r} = -600 \sin\theta\dot{\theta}\big|_{\theta=0°} = 0$$

$$\ddot{r} = -600 \sin\theta\ddot{\theta} - 600\cos\theta\dot{\theta}^2\big|_{\theta=0°} = -600\dot{\theta}^2$$

$$v_p^2 = \dot{r}^2 + \left(r\dot{\theta}\right)^2$$

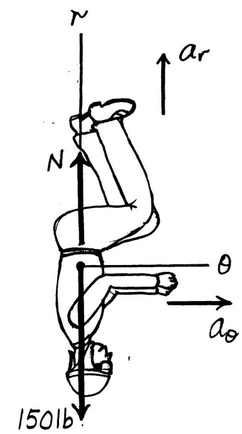
$$(80)^2 = 0 + \left(1200\dot{\theta}\right)^2 \qquad \dot{\theta} = 0.06667$$

$$2v_p v_p = 2r\ddot{r} + 2\left(r\dot{\theta}\right)\left(\dot{r}\theta + r\ddot{\theta}\right)$$

$$0 = 0 + 0 + 2r^2\theta\ddot{\theta} \qquad \ddot{\theta} = 0$$

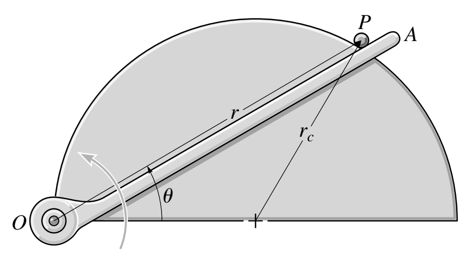
$$a_r = \ddot{r} - r\dot{\theta}^2 = -600(0.06667)^2 - 1200(0.06667)^2 = -8 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 0 = 0$$



$+\uparrow\Sigma F_r = ma_r;\qquad N - 150 = \left(\dfrac{150}{32.2}\right)(-8) \qquad N = 113 \text{ lb}$ **Ans.**

*__13–112.__ The 0.5-lb ball is guided along the vertical circular path $r = 2r_c\cos\theta$ using the arm OA. If the arm has an angular velocity $\dot{\theta} = 0.4$ rad/s and an angular acceleration $\ddot{\theta} = 0.8$ rad/s^2 at the instant $\theta = 30°$, determine the force of the arm on the ball. Neglect friction and the size of the ball. Set $r_c = 0.4$ ft.



$$r = 2(0.4)\cos\theta = 0.8\cos\theta$$

$$\dot{r} = -0.8\sin\theta\dot{\theta}$$

$$\ddot{r} = -0.8\cos\theta\dot{\theta}^2 - 0.8\sin\theta\ddot{\theta}$$

At $\theta = 30°$, $\dot{\theta} = 0.4$ rad/s, and $\ddot{\theta} = 0.8$ rad/s^2

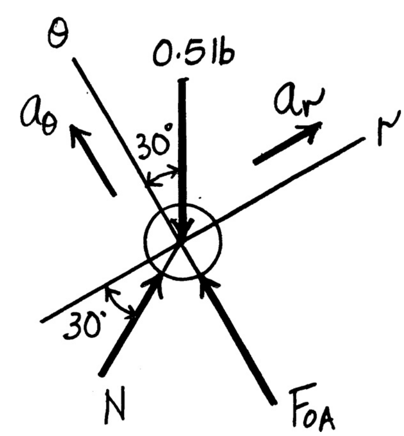
$$r = 0.8\cos 30° = 0.6928 \text{ ft}$$

$$\dot{r} = -0.8\sin 30°(0.4) = -0.16 \text{ ft/s}$$

$$\ddot{r} = -0.8\cos 30°(0.4)^2 - 0.8\sin 30°(0.8) = -0.4309 \text{ ft/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -0.4309 - 0.6928(0.4)^2 = -0.5417 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6928(0.8) + 2(-0.16)(0.4) = 0.4263 \text{ ft/s}^2$$

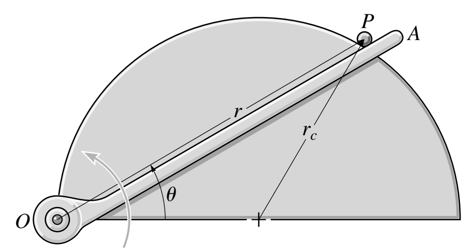


$+\nearrow\Sigma F_r = ma_r;\qquad N\cos 30° - 0.5\sin 30° = \dfrac{0.5}{32.2}(-0.5417) \qquad N = 0.2790 \text{ lb}$

$\nwarrow+\Sigma F_\theta = ma_\theta;\qquad F_{OA} + 0.2790\sin 30° - 0.5\cos 30° = \dfrac{0.5}{32.2}(0.4263)$

$F_{OA} = 0.300 \text{ lb}$ **Ans.**

•13–113. The ball of mass m is guided along the vertical circular path $r = 2r_c \cos\theta$ using the arm OA. If the arm has a constant angular velocity $\dot{\theta}_0$, determine the angle $\theta \leq 45°$ at which the ball starts to leave the surface of the semicylinder. Neglect friction and the size of the ball.



$r = 2r_c \cos\theta$

$\dot{r} = -2r_c \sin\theta\dot{\theta}$

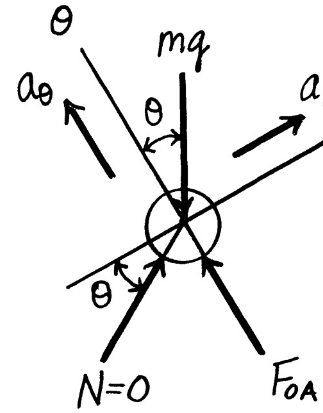
$\ddot{r} = -2r_c \cos\theta\dot{\theta}^2 - 2r_c \sin\theta\ddot{\theta}$

Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$.

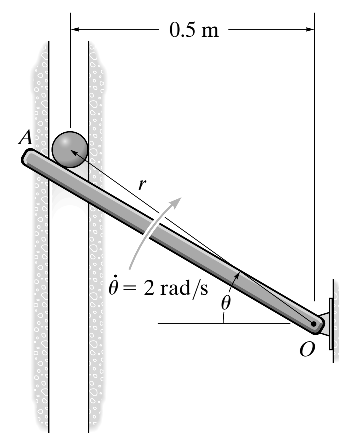
$$a_r = \ddot{r} - r\dot{\theta}^2 = -2r_c \cos\theta\dot{\theta}_0^2 - 2r_c \cos\theta\dot{\theta}_0^2 = -4r_c \cos\theta\dot{\theta}_0^2$$

$+\nearrow\Sigma F_r = ma_r;\qquad -mg\sin\theta = m(-4r_c\cos\theta\dot{\theta}_0^2)$

$$\tan\theta = \frac{4r_c\,\dot{\theta}_0^2}{g}\qquad \theta = \tan^{-1}\left(\frac{4r_c\,\dot{\theta}_0^2}{g}\right)$$ **Ans.**



13–114. The ball has a mass of 1 kg and is confined to move along the smooth vertical slot due to the rotation of the smooth arm OA. Determine the force of the rod on the ball and the normal force of the slot on the ball when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 3$ rad/s. Assume the ball contacts only one side of the slot at any instant.



Kinematics: Here, $\dot{\theta} = 3$ rad/s and $\ddot{\theta} = 0$. Taking the required time derivatives at $\theta = 30°$, we have

$$r = \frac{0.5}{\cos\theta}\bigg|_{\theta=30°} = 0.5774\text{ m}$$

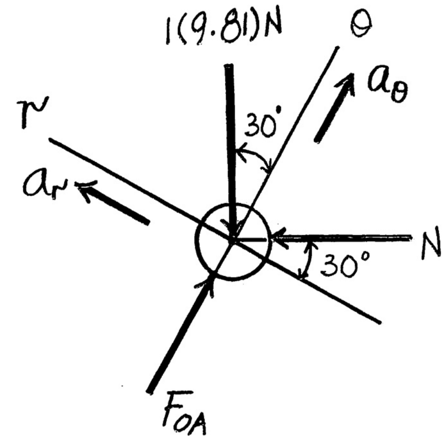
$$\dot{r} = \frac{0.5\sin\theta}{\cos^2\theta}\dot{\theta} = 0.5\tan\theta\sec\theta\dot{\theta}\bigg|_{\theta=30°}\ 1.00\text{ m/s}$$

$$\ddot{r} = 0.5\left[\tan\theta\sec\theta\ddot{\theta} + \left(\sec^3\theta + \tan^2\theta\sec\theta\right)\dot{\theta}^2\right]\bigg|_{\theta=30°} = 8.660\text{ m/s}^2$$

Applying Eqs. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 8.660 - 0.5774\left(3^2\right) = 3.464\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(0) + 2(1.00)(3) = 6.00\text{m/s}^2$$



Equations of Motion:

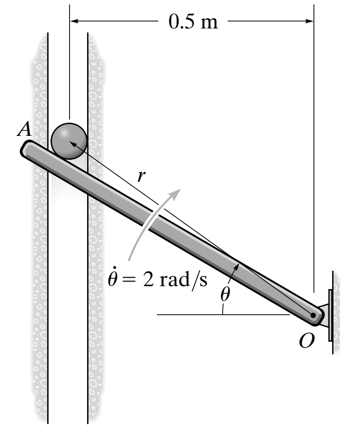
$\nwarrow+\Sigma F_r = ma_r;\qquad N\cos 30° - 1(9.81)\sin 30° = 1(3.464)$

$N = 9.664\text{ N} = 9.66\text{ N}$ **Ans.**

$+\nearrow\Sigma F_\theta = ma_\theta;\qquad F_{OA} - 1(9.81)\cos 30° - 9.664\sin 30° = 1(6.00)$

$F_{OA} = 19.3\text{ N}$ **Ans.**

13–115. Solve Prob. 13–114 if the arm has an angular acceleration of $\ddot{\theta} = 2\ \text{rad/s}^2$ when $\dot{\theta} = 3\ \text{rad/s}$ at $\theta = 30°$.



Kinematics: Here, $\dot{\theta} = 3\ \text{rad/s}$ and $\ddot{\theta} = 2\ \text{rad/s}^2$. Taking the required time derivatives at $\theta = 30°$, we have

$$r = \frac{0.5}{\cos\theta}\bigg|_{\theta=30°} = 0.5774\ \text{m}$$

$$\dot{r} = \frac{0.5\sin\theta}{\cos^2\theta}\dot{\theta} = 0.5\tan\theta\sec\theta\dot{\theta}\bigg|_{\theta=30°} = 1.00\ \text{m/s}$$

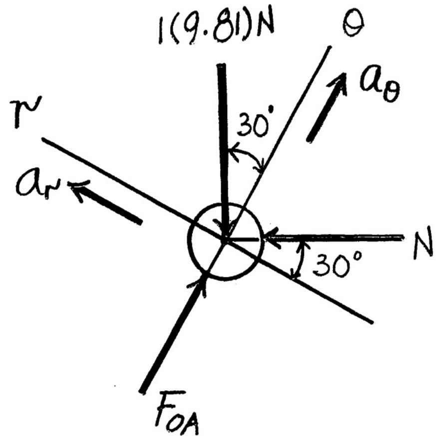
$$\ddot{r} = 0.5\left[\tan\theta\sec\theta\ddot{\theta} + \left(\sec^2\theta + \tan^2\theta\sec\theta\right)\dot{\theta}^2\right]\bigg|_{\theta=30°} = 9.327\ \text{m/s}^2$$

Applying Eqs. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 9.327 - 0.5774\left(3^2\right) = 4.131\ \text{m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(2) + 2(1.00)(3) = 7.155\ \text{m/s}^2$$

Equations of Motion:



$\Sigma F_r = ma_r;\qquad N\cos 30° - 1(9.81)\sin 30° = 1(4.131)$

$N = 10.43\ \text{N} = 10.4\ \text{N}$ **Ans.**

$\Sigma F_\theta = ma_\theta;\qquad F_{OA} - 1(9.81)\cos 30° - 10.43\sin 30° = 1(7.155)$

$F_{OA} = 20.9\ \text{N}$ **Ans.**

***13–116.** Prove Kepler's third law of motion. *Hint:* Use Eqs. 13–19, 13–28, 13–29, and 13–31.

From Eq. 13–19,

$$\frac{1}{r} = C\cos\theta + \frac{GM_s}{h^2}$$

For $\theta = 0°$ and $\theta = 180°$,

$$\frac{1}{r_p} = C + \frac{GM_s}{h^2}$$

$$\frac{1}{r_a} = -C + \frac{GM_s}{h^2}$$

Eliminating C, from Eqs. 13–28 and 13–29,

$$\frac{2a}{b^2} = \frac{2GM_s}{h^2}$$

From Eq. 13–31,

$$T = \frac{\pi}{h}(2a)(b)$$

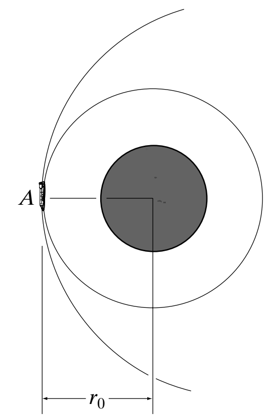
Thus,

$$b^2 = \frac{T^2h^2}{4\pi^2a^2}$$

$$\frac{4\pi^2a^3}{T^2h^2} = \frac{GM_s}{h^2}$$

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)a^3 \qquad \textbf{Q.E.D.}$$

•13–117. The Viking explorer approaches the planet Mars on a parabolic trajectory as shown. When it reaches point A its velocity is 10 Mm/h. Determine r_0 and the required velocity at A so that it can then maintain a circular orbit as shown. The mass of Mars is 0.1074 times the mass of the earth.



When the Viking explorer approaches point A on a parabolic trajectory, its velocity at point A is given by

$$v_A = \sqrt{\frac{2GM_M}{r_0}}$$

$$\left[10(10^6)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = \sqrt{\frac{2(66.73)(10^{-12})\left[0.1074(5.976)(10^{24})\right]}{r_0}}$$

$$r_0 = 11.101(10^6)\text{ m} = 11.1\text{ Mm} \qquad \textbf{Ans.}$$

When the explorer travels along a circular orbit of $r_0 = 11.101(10^6)$ m, its velocity is

$$v_{A'} = \sqrt{\frac{GM_r}{r_0}} = \sqrt{\frac{66.73(10^{-12})\left[0.1074(5.976)(10^{24})\right]}{11.101(10^6)}}$$

$$= 1964.19\text{ m/s}$$

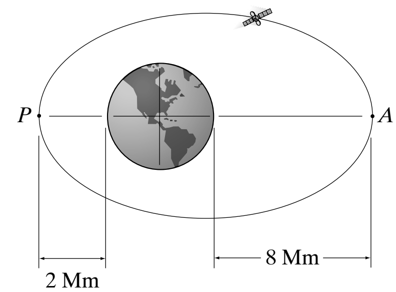
Thus, the required sudden decrease in the explorer's velocity is

$$\Delta v_A = v_A - v_{A'}$$

$$= 10(10^6)\left(\frac{1}{3600}\right) - 1964.19$$

$$= 814\text{ m/s} \qquad \textbf{Ans.}$$

13–118. The satellite is in an elliptical orbit around the earth as shown. Determine its velocity at perigee P and apogee A, and the period of the satellite.



Here,

$$r_0 = r_p = 2(10^6) + 6378(10^3) = 8.378(10^6)\text{ m}$$

and

$$r_a = 8(10^6) + 6378(10^3)$$

$$= 14.378(10^6)\text{ m}$$

$$r_a = \frac{r_0}{\dfrac{2GM_e}{r_0 v_0^2} - 1}$$

$$14.378(10^6) = \frac{8.378(10^6)}{\dfrac{2(66.73)(10^{-12})(5.976)(10^{24})}{8.378(10^6)v_0^{\,2}} - 1}$$

$$v_p = v_0 = 7755.53\text{ m/s} = 7.76\text{ km/s} \qquad \textbf{Ans.}$$

Using the result of v_p, we have

$$h = r_p v_p = r_a v_a$$

$$h = 8.378(10^6)(7755.53\text{ m/s}) = 14.378(10^6)v_a$$

$$v_A = v_a = 4519.12\text{ m/s} = 4.52\text{ km/s} \qquad \textbf{Ans.}$$

$$h = r_p v_p = 8.378(10^6)(7755.53)$$

$$= 64.976(10^9)\text{ m}^2/\text{s}$$

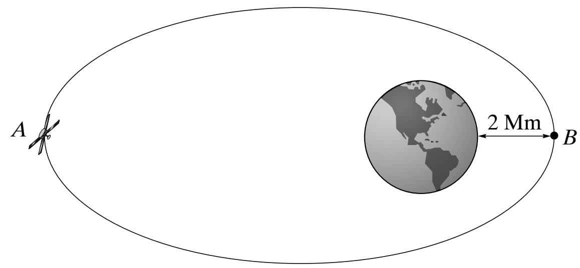
Thus,

$$T = \frac{\pi}{h}\left(r_p + r_a\right)\sqrt{r_p r_a}$$

$$= \frac{\pi}{64.976(10^9)}\left[8.378(10^6) + 14.378(10^6)\right]\sqrt{8.378(10^6)(14.378)(10^6)}$$

$$= 12\,075.71\text{ s} = 3.35\text{ hr} \qquad \textbf{Ans.}$$

13–119. The satellite is moving in an elliptical orbit with an eccentricity $e = 0.25$. Determine its speed when it is at its maximum distance A and minimum distance B from the earth.



$$e = \frac{Ch^2}{GM_e}$$

where $C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)$ and $h = r_0\, v_0$.

$$e = \frac{1}{GM_e\, r_0}\left(1 - \frac{GM_e}{r_0\, v_0^2}\right)(r_0\, v_0)^2$$

$$e = \left(\frac{r_0\, v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0\, v_0^2}{GM_e} = e + 1 \qquad v_0 = \sqrt{\frac{GM_e\,(e+1)}{r_0}}$$

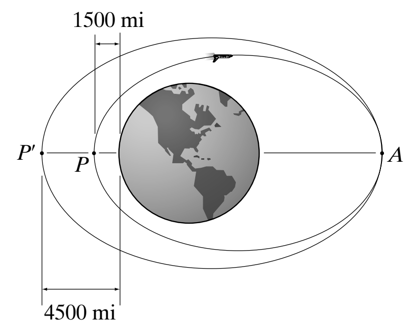
where $r_0 = r_p = 2\left(10^6\right) + 6378\left(10^3\right) = 8.378\left(10^6\right)$ m.

$$v_B = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.25+1)}{8.378(10^6)}} = 7713 \text{ m/s} = 7.71 \text{ km/s} \qquad \textbf{Ans.}$$

$$r_a = \frac{r_0}{\frac{2GM_e}{r_0\, v_0} - 1} = \frac{8.378(10^6)}{\frac{2(66.73)(10^{-12})(5.976)(10^{24})}{8.378(10^6)(7713)^2} - 1} = 13.96\left(10^6\right) \text{ m}$$

$$v_A = \frac{r_p}{r_a} v_B = \frac{8.378(10^6)}{13.96(10^6)}(7713) = 4628 \text{ m/s} = 4.63 \text{ km/s} \qquad \textbf{Ans.}$$

***13–120.** The space shuttle is launched with a velocity of 17 500 mi/h parallel to the tangent of the earth's surface at point P and then travels around the elliptical orbit. When it reaches point A, its engines are turned on and its velocity is suddenly increased. Determine the required increase in velocity so that it enters the second elliptical orbit. Take $G = 34.4(10^{-9})\,\text{ft}^4/\text{lb}\cdot\text{s}^4$, $M_e = 409(10^{21})$ slug, and $r_e = 3960\,\text{mi}$, where 5280 ft = mi.



For the first elliptical orbit,

$$r_P = 1500 + 3960 = (5460\text{ mi})\left(\frac{5280\text{ ft}}{1\text{ mi}}\right) = 28.8288(10^6)\text{ ft}$$

and

$$v_P = \left(17500\,\frac{\text{mi}}{\text{h}}\right)\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)\left(\frac{1\text{h}}{3600\text{ s}}\right) = 25\,666.67\text{ ft/s}$$

Using the results of r_p and v_p,

$$r_a = \frac{r_P}{\dfrac{2GM_e}{r_P\,{v_P}^2} - 1} = \frac{28.8288(10^6)}{\dfrac{2(34.4)(10^{-9})(409)(10^{21})}{28.8288(10^6)(25666.67^2)} - 1}$$

$$= 59.854(10^6)\text{ ft/s}$$

Since $h = r_P\,v_P = 28.8288(10^6)(25666.67) = 739.94(10^9)\text{ ft}^2/\text{s}$ is constant,

$$r_a\,v_a = h$$

$$59.854(10^6)v_a = 739.94(10^9)$$

$$v_a = 12\,362.40\text{ ft/s}$$

When the shuttle enters the second elliptical orbit, $r_P' = 4500 + 3960 = 8460\text{ mi}\left(\dfrac{5280\text{ ft}}{1\text{ mi}}\right) = 44.6688(10^6)\text{ ft}$ and $r_a' = r_a = 59.854(10^6)$ ft.

$$r_a' = \frac{r_P'}{\dfrac{2GM_e}{r_P'\left(v_P'\right)^2} - 1}$$

$$59.854(10^6) = \frac{44.6688(10^6)}{\dfrac{2(34.4)(10^{-9})(409)(10^{21})}{44.6688(10^6)\left(v_P'\right)^2} - 1}$$

$$v_P' = 18\,993.05\text{ ft/}s$$

Since $h' = r_P'\,v_P' = 44.6688(10^6)(18\,993.05) = 848.40(10^9)\text{ ft}^2/\text{s}$ is constant,

$$r_a'\,v_a' = h'$$

$$59.854(10^6)v_a' = 848.40(10^9)$$

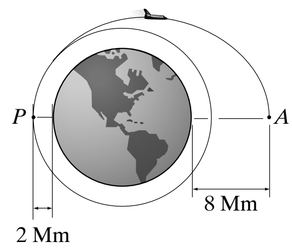
$$v_a' = 14\,174.44\text{ ft/s}$$

Thus, the required increase in the shuttle's speed at point A is

$$\Delta v_A = v_{A'} - v_A = 14\,174.44 - 12\,362.40$$

$$= 1812.03\text{ ft/s} = 1812\text{ ft/s} \qquad \textbf{Ans.}$$

•13–121. Determine the increase in velocity of the space shuttle at point P so that it travels from a circular orbit to an elliptical orbit that passes through point A. Also, compute the speed of the shuttle at A.



When the shuttle is travelling around the circular orbit of radius $r_o = 2(10^6) + 6378(10^3) = 8.378(10^6)$ m, its speed is

$$v_o = \sqrt{\frac{GM_e}{r_o}} = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{8.378(10^6)}} = 6899.15 \text{ m/s}$$

When the shuttle enters the elliptical orbit, $r_p = r_o = 8.378(10^6)$ m and $r_a = 8(10^6) + 6378(10^3) = 14.378(10^6)$ m.

$$r_a = \frac{r_p}{\dfrac{2GM_e}{r_p v_p^2} - 1}$$

$$14.378(10^6) = \frac{8.378(10^6)}{\dfrac{2(66.73)(10^{-12})(5.976)(10^{24})}{8.378(10^6)v_p^2} - 1}$$

$$v_p = 7755.54 \text{ m/s}$$

Thus, the required increase in speed for the shuttle at point P is

$$\Delta v_p = v_p - v_o = 7755.54 - 6899.15 = 856.39 \text{ m/s} = 856 \text{ m/s} \qquad \textbf{Ans.}$$

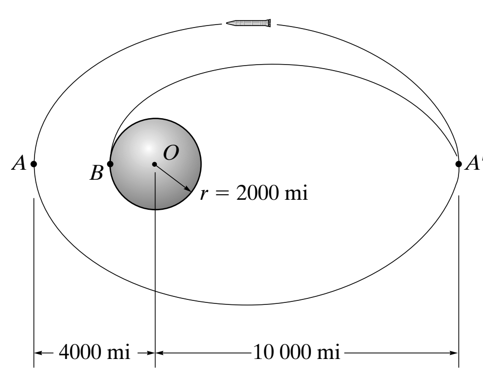
Since $h = r_p v_p = 8.378(10^6)(7755.54) = 64.976(10^9)\ \text{m}^2/\text{s}$ is constant,

$$r_a v_a = h$$

$$14.378(10^6)v_a = 64.976(10^9)$$

$$v_A = 4519.11 \text{ m/s} = 4.52 \text{ km/s} \qquad \textbf{Ans.}$$

13–122. The rocket is in free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.60 times that of the earth. If the orbit has the apoapsis and periapsis shown, determine the rocket's velocity when it is at point A. Take $G = 34.4(10^{-9})(\text{lb}\cdot\text{ft}^2)/\text{slug}^2$, $M_e = 409(10^{21})$ slug, 1 mi = 5280 ft.



$$r_0 = OA = (4000)(5280) = 21.12\left(10^6\right) \text{ ft}$$

$$OA' = (10\,000)(5280) = 52.80\left(10^6\right)\text{ft}$$

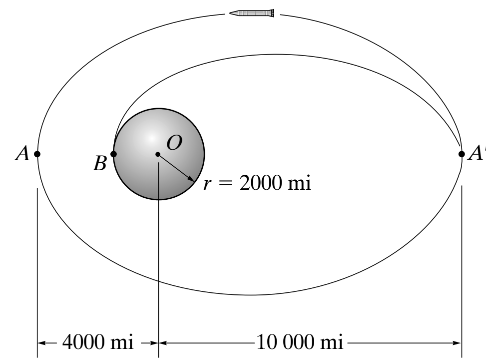
$$M_P = \left(409\left(10^{21}\right)\right)(0.6) = 245.4\left(10^{21}\right) \text{ slug}$$

$$OA' = \frac{OA}{\left(\dfrac{2GM_p}{OAv_0^2} - 1\right)}$$

$$v_0 = \sqrt{\frac{2GM_P}{OA\left(\dfrac{OA}{OA'} + 1\right)}} = \sqrt{\frac{2(34.4)(10^{-9})(245.4)(10^{21})}{21.12(10^6)\left(\dfrac{21.12}{52.80} + 1\right)}}$$

$$v_0 = 23.9\left(10^3\right) \text{ ft/s} \qquad \textbf{Ans.}$$

13–123. If the rocket is to land on the surface of the planet, determine the required free-flight speed it must have at A' so that the landing occurs at B. How long does it take for the rocket to land, in going from A' to B? The planet has no atmosphere, and its mass is 0.6 times that of the earth. Take $G = 34.4(10^{-9})(\text{lb}\cdot\text{ft}^2)/\text{slug}^2$, $M_e = 409(10^{21})$ slug, 1 mi = 5280 ft.



$$M_P = 409\left(10^{21}\right)(0.6) = 245.4\left(10^{21}\right)\text{ slug}$$

$$OA' = (10\,000)(5280) = 52.80\left(10^{6}\right)\text{ ft} \qquad OB = (2000)(5280) = 10.56\left(10^{6}\right)\text{ ft}$$

$$OA' = \frac{OB}{\left(\dfrac{2GM_P}{OBv_0^2} - 1\right)}$$

$$v_0 = \sqrt{\frac{2GM_P}{OB\left(\dfrac{OB}{OA'} + 1\right)}} = \sqrt{\frac{2(34.4(10^{-9}))245.4(10^{21})}{10.56(10^6)\left(\dfrac{10.56}{52.80} + 1\right)}}$$

$$v_0 = 36.50\left(10^{3}\right)\text{ ft/s} \qquad \text{(speed at } B\text{)}$$

$$v_{A'} = \frac{OBv_0}{OA'}$$

$$v_{A'} = \frac{10.56(10^6)36.50(10^3)}{52.80(10^6)}$$

$$v_{A'} = 7.30\left(10^{3}\right)\text{ ft/s}$$ **Ans.**

$$T = \frac{\pi}{h}(OB + OA')\sqrt{(OB)(OA')}$$

$$h = (OB)(v_0) = 10.56\left(10^{6}\right)36.50\left(10^{3}\right) = 385.5\left(10^{9}\right)$$

Thus,

$$T = \frac{\pi(10.56 + 52.80)(10^6)}{385.5(10^9)}\left(\sqrt{(10.56)(52.80)}\right)\left(10^{6}\right)$$

$$T = 12.20\left(10^{3}\right)\text{ s}$$

$$t = \frac{T}{2} = 6.10\left(10^{3}\right)\text{ s} = 1.69\text{ h}$$ **Ans.**

***13–124.** A communications satellite is to be placed into an equatorial circular orbit around the earth so that it always remains directly over a point on the earth's surface. If this requires the period to be 24 hours (approximately), determine the radius of the orbit and the satellite's velocity.

$$\frac{GM_eM_s}{r^2} = \frac{M_s v^2}{r}$$

$$\frac{GM_e}{r} = v^2$$

$$\frac{GM_e}{r} = \left[\frac{2\pi r}{24(3600)}\right]^2$$

$$\frac{66.73(10^{-12})(5.976)(10^{24})}{\left[\frac{2\pi}{24(3600)}\right]^2} = r^3$$

$$r = 42.25(10^6)\text{ m} = 42.2\text{ Mm} \qquad \textbf{Ans.}$$

$$v = \frac{2\pi(42.25)(10^6)}{24(3600)} = 3.07\text{ km/s} \qquad \textbf{Ans.}$$

•13–125. The speed of a satellite launched into a circular orbit about the earth is given by Eq. 13–25. Determine the speed of a satellite launched parallel to the surface of the earth so that it travels in a circular orbit 800 km from the earth's surface.

For a 800-km orbit

$$v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{(800 + 6378)(10^3)}}$$

$$= 7453.6\text{ m/s} = 7.45\text{ km/s} \qquad \textbf{Ans.}$$

13–126. The earth has an orbit with eccentricity $e = 0.0821$ around the sun. Knowing that the earth's minimum distance from the sun is $151.3(10^6)$ km, find the speed at which a rocket travels when it is at this distance. Determine the equation in polar coordinates which describes the earth's orbit about the sun.

$$e = \frac{Ch^2}{GM_S} \qquad \text{where } C = \frac{1}{r_0}\left(1 - \frac{GM_S}{r_0 v_0^2}\right) \text{ and } h = r_0 v_0$$

$$e = \frac{1}{GM_S r_0}\left(1 - \frac{GM_S}{r_0 v_0^2}\right)(r_0 v_0)^2 \qquad e = \left(\frac{r_0 v_0^2}{GM_S} - 1\right) \qquad \frac{r_0 v_0^2}{GM_S} = e + 1$$

$$v_0 = \sqrt{\frac{GM_S(e+1)}{r_0}}$$

$$= \sqrt{\frac{66.73(10^{-12})(1.99)(10^{30})(0.0821 + 1)}{151.3(10^9)}} = 30818 \text{ m/s} = 30.8 \text{ km/s} \qquad \textbf{Ans.}$$

$$\frac{1}{r} = \frac{1}{r_0}\left(1 - \frac{GM_S}{r_0 v_0^2}\right)\cos\theta + \frac{GM_S}{r_0^2 v_0^2}$$

$$\frac{1}{r} = \frac{1}{151.3(10^9)}\left(1 - \frac{66.73(10^{-12})(1.99)(10^{30})}{151.3(10^9)(30818)^2}\right)\cos\theta + \frac{66.73(10^{-12})(1.99)(10^{30})}{\left[151.3(10^9)\right]^2(30818)^2}$$

$$\frac{1}{r} = 0.502(10^{-12})\cos\theta + 6.11(10^{-12}) \qquad \textbf{Ans.}$$

13–127. A rocket is in a free-flight elliptical orbit about the earth such that the eccentricity of its orbit is e and its perigee is r_0. Determine the minimum increment of speed it should have in order to escape the earth's gravitational field when it is at this point along its orbit.

To escape the earth's gravitational field, the rocket has to make a parabolic trajectory.

Parabolic Trajectory:

$$v_e = \sqrt{\frac{2GM_e}{r_0}}$$

Elliptical Orbit:

$$e = \frac{Ch^2}{GM_e} \qquad \text{where } C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right) \text{ and } h = r_0 v_0$$

$$e = \frac{1}{GM_e r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)(r_0 v_0)^2$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1 \qquad v_0 = \sqrt{\frac{GM_e(e+1)}{r_0}}$$

$$\Delta v = \sqrt{\frac{2GM_e}{r_0}} - \sqrt{\frac{GM_e(e+1)}{r_0}} = \sqrt{\frac{GM_e}{r_0}}\left(\sqrt{2} - \sqrt{1+e}\right) \qquad \textbf{Ans.}$$

***13–128.** A rocket is in circular orbit about the earth at an altitude of $h = 4$ Mm. Determine the minimum increment in speed it must have in order to escape the earth's gravitational field.

Circular Orbit:

$$v_C = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})5.976(10^{24})}{4000(10^3) + 6378(10^3)}} = 6198.8 \text{ m/s}$$

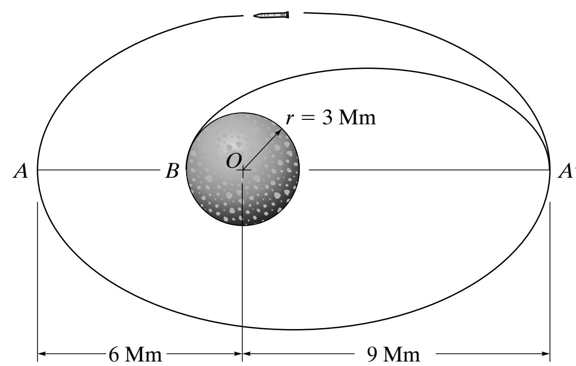
Parabolic Orbit:

$$v_e = \sqrt{\frac{2GM_e}{r_0}} = \sqrt{\frac{2(66.73)(10^{-12})5.976(10^{24})}{4000(10^3) + 6378(10^3)}} = 8766.4 \text{ m/s}$$

$$\Delta v = v_e - v_C = 8766.4 - 6198.8 = 2567.6 \text{ m/s}$$

$$\Delta v = 2.57 \text{ km/s}$$ **Ans.**

•13–129. The rocket is in free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.70 times that of the earth. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point A.

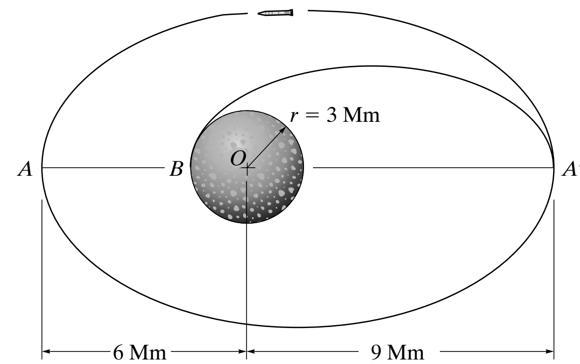


Central-Force Motion: Use $r_a = \dfrac{r_0}{\left(2\,GM/r_0\,v_0^2\right) - 1}$, with $r_0 = r_p = 6\left(10^6\right)$ m and $M = 0.70M_e$, we have

$$9\left(10^6\right) = \frac{6(10)^6}{\left(\dfrac{2(66.73)\,(10^{-12})\,(0.7)\,[5.976(10^{24})]}{6(10^6)v_P^2}\right) - 1}$$

$$v_P = 7471.89 \text{ m/s} = 7.47 \text{ km/s}$$ **Ans.**

13–130. If the rocket is to land on the surface of the planet, determine the required free-flight speed it must have at A' so that it strikes the planet at B. How long does it take for the rocket to land, going from A' to B along an elliptical path? The planet has no atmosphere, and its mass is 0.70 times that of the earth.



Central-Force Motion: Use $r_A = \dfrac{r_0}{\left(2GM/r_0 v_0^2\right) - 1}$, with $r_A = 9\left(10^6\right)$ m, $r_0 = r_P = 3\left(10^6\right)$ m, and $M = 0.70M_e$. We have

$$9\left(10^6\right) = \frac{3(10^6)}{\left(\dfrac{2(66.73)(10^{-12})\,(0.7)[5.976(10^{24})]}{3(10^6)v_P^2}\right) - 1}$$

$$v_P = 11814.08 \text{ m/s}$$

Applying Eq. 13–20, we have

$$v_A = \left(\frac{r_P}{r_A}\right)v_P = \left[\frac{3(10^6)}{9(10^6)}\right](11814.08) = 3938.03 \text{ m/s} = 3.94 \text{ km/s} \qquad \textbf{Ans.}$$

Eq. 13–20 gives $h = r_p\, v_p = 3\left(10^6\right)(11814.08) = 35.442\left(10^9\right)$ m^2/s. Thus, applying Eq.13–31, we have

$$T = \frac{\pi}{6}\left(r_P + r_A\right)\sqrt{r_P\, r_A}$$

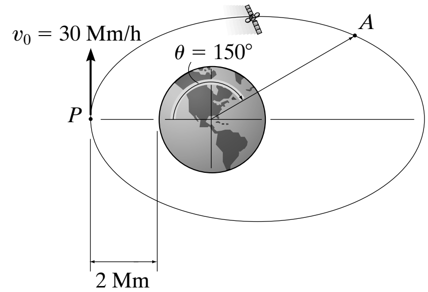
$$= \frac{\pi}{35.442(10^9)}\left[(9 + 3)\left(10^6\right)\right]\sqrt{3(10^6)\,9\,(10^6)}$$

$$= 5527.03 \text{ s}$$

The time required for the rocket to go from A' to B (half the orbit) is given by

$$t = \frac{T}{2} = 2763.51 \text{ s} = 46.1 \text{ min} \qquad \textbf{Ans.}$$

13–131. The satellite is launched parallel to the tangent of the earth's surface with a velocity of $v_0 = 30$ Mm/h from an altitude of 2 Mm above the earth as shown. Show that the orbit is elliptical, and determine the satellite's velocity when it reaches point A.



Here,

$$r_0 = 2(10^6) + 6378(10^3) = 8.378(10^6)\text{ m}$$

and

$$v_0 = \left[30(10^6)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 8333.33\text{m/s}$$

$$h = r_0 v_0 = 8.378(10^6)(8333.33) = 69.817(10^9)\text{ m}^2/\text{s}$$

and

$$C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0{}^2}\right)$$

$$= \frac{1}{8.378(10^6)}\left[1 - \frac{66.73(10^{-12})(5.976)(10^{24})}{8.378(10^6)(8333.33^2)}\right]$$

$$= 37.549(10^{-9})\text{ m}^{-1}$$

The eccentricity of the satellite orbit is given by

$$e = \frac{Ch^2}{GM_e} = \frac{37.549(10^{-9})\left[69.817(10^9)\right]^2}{66.73(10^{-12})(5.976)(10^{24})} = 0.459$$

Since $e < 1$, the satellite orbit is *elliptical* (Q.E.D.). $r = r_A$ at $\theta = 150°$, we obtain

$$\frac{1}{r} = C\cos\theta + \frac{GM_e}{h^2}$$

$$\frac{1}{r_A} = 37.549(10^{-9})\cos 150° + \frac{66.73(10^{-12})(5.976)(10^{24})}{\left[69.817(10^9)\right]^2}$$

$$r_A = 20.287(10^6)\text{ m}$$

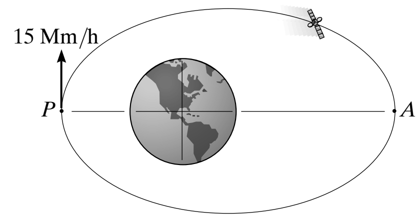
Since h is constant,

$$r_A v_A = h$$

$$20.287(10^6)v_A = 69.817(10^9)$$

$$v_A = 3441.48\text{ m/s} = 3.44\text{ km/s} \qquad \textbf{Ans.}$$

***13–132.** The satellite is in an elliptical orbit having an eccentricity of $e = 0.15$. If its velocity at perigee is $v_P = 15\,\text{Mm/h}$, determine its velocity at apogee A and the period of the satellite.



Here, $v_P = \left[15(10^6)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 4166.67\text{ m/s}.$

$$h = r_P v_P$$

$$h = r_P\,(4166.67) = 4166.67 r_p \qquad \textbf{(1)}$$

and

$$C = \frac{1}{r_P}\left(1 - \frac{GM_e}{r_P\,{v_P}^2}\right)$$

$$C = \frac{1}{r_p}\left[1 - \frac{66.73(10^{-12})(5.976)(10^{24})}{r_p(4166.67^2)}\right]$$

$$C = \frac{1}{r_P}\left[1 - \frac{22.97(10^6)}{r_P}\right] \qquad \textbf{(2)}$$

$$e = \frac{Ch^2}{GM_e}$$

$$0.15 = \frac{\frac{1}{r_P}\left[1 - \frac{22.97(10^6)}{r_P}\right](4166.67\,r_P)^2}{66.73(10^{-12})(5.976)(10^{24})}$$

$$r_P = 26.415(10^6)\text{ m}$$

Using the result of r_p

$$r_A = \frac{r_P}{\frac{2GM_e}{r_P\,{v_P}^2} - 1}$$

$$= \frac{26.415(10^6)}{\frac{2(66.73)(10^{-12})(5.976)(10^{24})}{26.415(10^6)(4166.67^2)} - 1}$$

$$= 35.738(10^6)\text{ m}$$

Since $h = r_P\,v_P = 26.415(10^6)(4166.67^2) = 110.06(10^9)\text{ m}^2\text{/s}$ is constant,

$$r_A v_A = h$$

$$35.738(10^6)v_A = 110.06(10^9)$$

$$v_A = 3079.71\text{ m/s} = 3.08\text{ km/s} \qquad \textbf{Ans.}$$

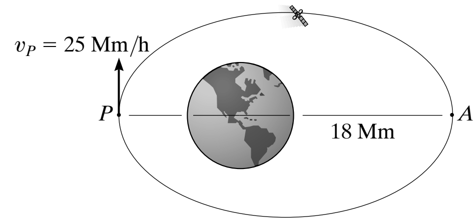
Using the results of h, r_A, and r_P,

$$T = \frac{\pi}{6}\left(r_P + r_A\right)\sqrt{r_P\,r_A}$$

$$= \frac{\pi}{110.06(10^9)}\left[26.415(10^6) + 35.738(10^6)\right]\sqrt{26.415(10^6)(35.738)(10^6)}$$

$$= 54\,508.43\text{ s} = 15.1\text{ hr} \qquad \textbf{Ans.}$$

•13–133. The satellite is in an elliptical orbit. When it is at perigee P, its velocity is $v_P = 25\,\text{Mm/h}$, and when it reaches point A, its velocity is $v_A = 15\,\text{Mm/h}$ and its altitude above the earth's surface is 18 Mm. Determine the period of the satellite.



Here,

$$v_A = \left[15\left(10^6\right)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 4166.67\text{ m/s}$$

and

$$r_A = 18\left(10^6\right) + 6378\left(10^3\right) = 24.378\left(10^6\right)\text{ m}$$

$$h = r_A v_A\left[24.378\left(10^6\right)\right](4166.67) = 101.575\left(10^9\right)\text{ m}^2\text{/s}$$

Since h is constant and $v_P = \left[25\left(10^6\right)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 6944.44\text{ m/s}$,

$$r_P v_P = h$$

$$r_P\,(6944.44) = 101.575(10^9)$$

$$r_P = 14.6268(10^6)\text{ m}$$

Using the results of h, r_A, and r_P,

$$T = \frac{\pi}{6}\left(r_P + r_A\right)\sqrt{r_P r_A}$$

$$= \frac{\pi}{101.575\left(10^9\right)}\left[14.6268\left(10^6\right) + 24.378\left(10^6\right)\right]\sqrt{14.6268\left(10^6\right)(24.378)\left(10^6\right)}$$

$$= 430158.48\text{ s} = 119\text{ h} \qquad \textbf{Ans.}$$

13–134. A satellite is launched with an initial velocity $v_0 = 4000$ km/h parallel to the surface of the earth. Determine the required altitude (or range of altitudes) above the earth's surface for launching if the free-flight trajectory is to be (a) circular, (b) parabolic, (c) elliptical, and (d) hyperbolic.

$$v_0 = \frac{4000(10^3)}{3600} = 1111\text{ m/s}$$

(a) For circular trajectory, $e = 0$

$$v_0 = \sqrt{\frac{GM_e}{r_0}} \qquad r_0 = \frac{GM_e}{v_0^2} = \frac{(66.73)(10^{-12})(5.976)(10^{24})}{(1111)^2} = 323(10^3)\text{ km}$$

$$r = r_0 - 6378\text{ km} = 317\left(10^3\right)\text{ km} = 317\text{ Mm} \qquad \textbf{Ans.}$$

(b) For parabolic trajectory, $e = 1$

$$v_0 = \sqrt{\frac{2GM_e}{r_0}} \qquad r_0 = \frac{2GM_e}{v_0^2} = \frac{2(66.73)(10^{-12})(5.976)(10^{24})}{1111^2} = 646\left(10^3\right)\text{ km}$$

$$r = r_0 - 6378\text{ km} = 640\left(10^3\right)\text{ km} = 640\text{ Mm} \qquad \textbf{Ans.}$$

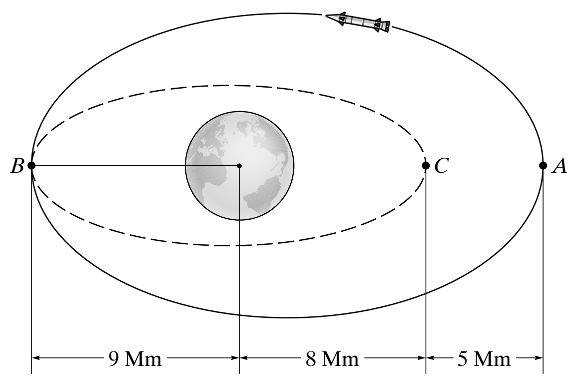
(c) For elliptical trajectory, $e < 1$

$$317\text{ Mm} < r < 640\text{ Mm} \qquad \textbf{Ans.}$$

(d) For hyperbolic trajectory, $e > 1$

$$r > 640\text{ Mm} \qquad \textbf{Ans.}$$

13–135. The rocket is in a free-flight elliptical orbit about the earth such that $e = 0.76$ as shown. Determine its speed when it is at point A. Also determine the sudden change in speed the rocket must experience at B in order to travel in free flight along the orbit indicated by the dashed path.



$$e = \frac{Ch^2}{GM_e} \qquad \text{where} \quad C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right) \text{ and } h = r_0 v_0$$

$$e = \frac{1}{GM_e r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)(r_0 v_0)^2$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1 \qquad \text{or} \qquad \frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1} \qquad \textbf{(1)}$$

$$r_a = \frac{r_0}{\frac{2GM_e}{r_0 v_0^2} - 1} \qquad \textbf{(2)}$$

Substituting Eq.(1) into (2) yields:

$$r_a = \frac{r_0}{2\left(\frac{1}{e+1}\right) - 1} = \frac{r_0(e+1)}{1-e} \qquad \textbf{(3)}$$

From Eq.(1),

$$\frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1} \qquad v_0 = \sqrt{\frac{GM_e(e+1)}{r_0}}$$

$$v_B = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.76+1)}{9(10^6)}} = 8831 \text{ m/s}$$

$$v_A = \frac{r_p}{r_a} v_B = \frac{9(10^6)}{13(10^6)}(8831) = 6113 \text{ m/s} = 6.11 \text{ km/s} \qquad \textbf{Ans.}$$

From Eq.(3),

$$r_a = \frac{r_0(e+1)}{1-e}$$

$$9(10)^6 = \frac{8(10^6)(e+1)}{1-e} \qquad e = 0.05882$$

From Eq. (1),

$$\frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1} \qquad v_0 = \sqrt{\frac{GM_e(e+1)}{r_0}}$$

$$v_C = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.05882+1)}{8(10^6)}} = 7265 \text{ m/s}$$

$$v_B = \frac{r_p}{r_a} v_C = \frac{8(10^6)}{9(10^6)}(7265) = 6458 \text{ m/s}$$

$$\Delta v_B = 6458 - 8831 = -2374 \text{ m/s} = -2.37 \text{ km/s} \qquad \textbf{Ans.}$$

***13–136.** A communications satellite is in a circular orbit above the earth such that it always remains directly over a point on the earth's surface. As a result, the period of the satellite must equal the rotation of the earth, which is approximately 24 hours. Determine the satellite's altitude h above the earth's surface and its orbital speed.

The period of the satellite around the circular orbit of radius $r_0 = h + r_e = \left[h + 6.378(10^6)\right]$ m is given by

$$T = \frac{2\pi r_0}{v_s}$$

$$24(3600) = \frac{2\pi\left[h + 6.378(10^6)\right]}{v_s}$$

$$v_s = \frac{2\pi\left[h + 6.378(10^6)\right.}{86.4(10^3)} \qquad \textbf{(1)}$$

The velocity of the satellite orbiting around the circular orbit of radius $r_0 = h + r_e = \left[h + 6.378(10^6)\right]$ m is given by

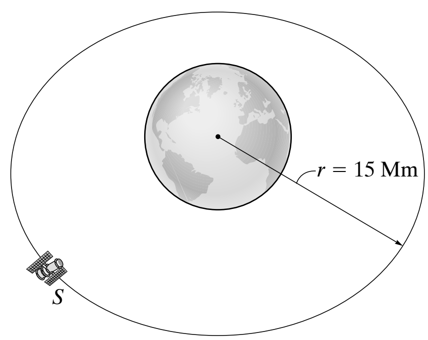
$$v_S = \sqrt{\frac{GM_e}{r_0}}$$

$$v_S = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{h + 6.378(10^6)}} \qquad \textbf{(2)}$$

Solving Eqs.(1) and (2),

$h = 35.87(10^6)$ m $= 35.9$ Mm $\qquad v_S = 3072.32$ m/s $= 3.07$ km/s **Ans.**

•13–137. Determine the constant speed of satellite S so that it circles the earth with an orbit of radius $r = 15$ Mm. *Hint:* Use Eq. 13–1.



$$F = G\frac{m_s m_e}{r^2} \qquad \text{Also} \qquad F = m_s\left(\frac{v_s^2}{r}\right) \qquad \text{Hence}$$

$$m_s\left(\frac{v_0^2}{r}\right) = G\frac{m_s m_e}{r^2}$$

$$v = \sqrt{G\frac{m_e}{r}} = \sqrt{66.73(10^{-12})\left(\frac{5.976(10^{24})}{15(10^6)}\right)} = 5156 \text{ m/s} = 5.16 \text{ km/s} \qquad \textbf{Ans.}$$