

•**12–1.** A car starts from rest and with constant acceleration achieves a velocity of 15 m/s when it travels a distance of 200 m. Determine the acceleration of the car and the time required.

Kinematics:

$$v_0 = 0, v = 15 \text{ m/s}, s_0 = 0, \text{ and } s = 200 \text{ m.}$$

$$\left(\begin{array}{l} \rightarrow \\ \leftarrow \end{array} \right) \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$15^2 = 0^2 + 2a_c(200 - 0)$$

$$a_c = 0.5625 \text{ m/s}^2$$

Ans.

$$\left(\begin{array}{l} \rightarrow \\ \leftarrow \end{array} \right) \quad v = v_0 + a_c t$$

$$15 = 0 + 0.5625t$$

$$t = 26.7 \text{ s}$$

Ans.

12–2. A train starts from rest at a station and travels with a constant acceleration of 1 m/s². Determine the velocity of the train when $t = 30$ s and the distance traveled during this time.

Kinematics:

$$a_c = 1 \text{ m/s}^2, v_0 = 0, s_0 = 0, \text{ and } t = 30 \text{ s.}$$

$$\left(\begin{array}{l} \rightarrow \\ \leftarrow \end{array} \right) \quad v = v_0 + a_c t$$

$$= 0 + 1(30) = 30 \text{ m/s}$$

Ans.

$$\left(\begin{array}{l} \rightarrow \\ \leftarrow \end{array} \right) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$= 0 + 0 + \frac{1}{2}(1)(30^2)$$

$$= 450 \text{ m}$$

Ans.

12–3. An elevator descends from rest with an acceleration of 5 ft/s^2 until it achieves a velocity of 15 ft/s . Determine the time required and the distance traveled.

Kinematics:

$$a_c = 5 \text{ ft/s}^2, v_0 = 0, v = 15 \text{ ft/s, and } s_0 = 0.$$

$$(+\downarrow) \quad v = v_0 + a_c t$$

$$15 = 0 + 5t$$

$$t = 3 \text{ s}$$

Ans.

$$(+\downarrow) \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$15^2 = 0^2 + 2(5)(s - 0)$$

$$s = 22.5 \text{ ft}$$

Ans.

***12–4.** A car is traveling at 15 m/s , when the traffic light 50 m ahead turns yellow. Determine the required constant deceleration of the car and the time needed to stop the car at the light.

Kinematics:

$$v_0 = 0, s_0 = 0, s = 50 \text{ m and } v_0 = 15 \text{ m/s.}$$

$$(\pm) \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = 15^2 + 2a_c(50 - 0)$$

$$a_c = -2.25 \text{ m/s}^2 = 2.25 \text{ m/s}^2 \leftarrow$$

Ans.

$$(\pm) \quad v = v_0 + a_c t$$

$$0 = 15 + (-2.25)t$$

$$t = 6.67 \text{ s}$$

Ans.

•12–5. A particle is moving along a straight line with the acceleration $a = (12t - 3t^{1/2}) \text{ ft/s}^2$, where t is in seconds. Determine the velocity and the position of the particle as a function of time. When $t = 0$, $v = 0$ and $s = 15 \text{ ft}$.

Velocity:

(\pm)

$$dv = a \, dt$$

$$\int_0^v dv = \int_0^t (12t - 3t^{1/2}) dt$$

$$v \Big|_0^v = (6t^2 - 2t^{3/2}) \Big|_0^t$$

$$v = (6t^2 - 2t^{3/2}) \text{ ft/s}$$

Ans.

Position: Using this result and the initial condition $s = 15 \text{ ft}$ at $t = 0 \text{ s}$,

(\pm)

$$ds = v \, dt$$

$$\int_{15 \text{ ft}}^s ds = \int_0^t (6t^2 - 2t^{3/2}) dt$$

$$s \Big|_{15 \text{ ft}}^s = \left(2t^3 - \frac{4}{5}t^{5/2} \right) \Big|_0^t$$

$$s = \left(2t^3 - \frac{4}{5}t^{5/2} + 15 \right) \text{ ft}$$

Ans.

12–6. A ball is released from the bottom of an elevator which is traveling upward with a velocity of 6 ft/s . If the ball strikes the bottom of the elevator shaft in 3 s , determine the height of the elevator from the bottom of the shaft at the instant the ball is released. Also, find the velocity of the ball when it strikes the bottom of the shaft.

Kinematics: When the ball is released, its velocity will be the same as the elevator at the instant of release. Thus, $v_0 = 6 \text{ ft/s}$. Also, $t = 3 \text{ s}$, $s_0 = 0$, $s = -h$, and $a_c = -32.2 \text{ ft/s}^2$.

($+\uparrow$)

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-h = 0 + 6(3) + \frac{1}{2}(-32.2)(3^2)$$

$$h = 127 \text{ ft}$$

Ans.

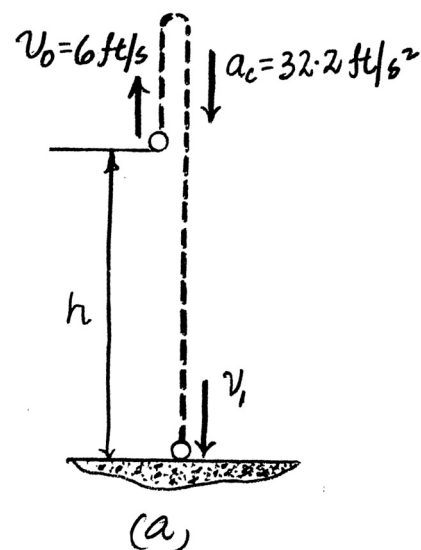
($+\uparrow$)

$$v = v_0 + a_c t$$

$$v = 6 + (-32.2)(3)$$

$$= -90.6 \text{ ft/s} = 90.6 \text{ ft/s} \downarrow$$

Ans.



12–7. A car has an initial speed of 25 m/s and a constant deceleration of 3 m/s². Determine the velocity of the car when $t = 4$ s. What is the displacement of the car during the 4-s time interval? How much time is needed to stop the car?

$$v = v_0 + a_c t$$

$$v = 25 + (-3)(4) = 13 \text{ m/s}$$

Ans.

$$\Delta s = s - s_0 = v_0 t + \frac{1}{2} a_c t^2$$

$$\Delta s = s - 0 = 25(4) + \frac{1}{2}(-3)(4)^2 = 76 \text{ m}$$

Ans.

$$v = v_0 + a_c t$$

$$0 = 25 + (-3)(t)$$

$$t = 8.33 \text{ s}$$

Ans.

***12–8.** If a particle has an initial velocity of $v_0 = 12$ ft/s to the right, at $s_0 = 0$, determine its position when $t = 10$ s, if $a = 2$ ft/s² to the left.

$$\begin{aligned} (\pm) \quad s &= s_0 + v_0 t + \frac{1}{2} a_c t^2 \\ &= 0 + 12(10) + \frac{1}{2}(-2)(10)^2 \\ &= 20 \text{ ft} \end{aligned}$$

Ans.

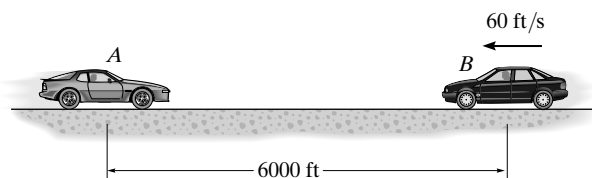
•12–9. The acceleration of a particle traveling along a straight line is $a = k/v$, where k is a constant. If $s = 0$, $v = v_0$ when $t = 0$, determine the velocity of the particle as a function of time t .

Velocity:

$$\begin{aligned} (\pm) \quad dt &= \frac{dv}{a} \\ \int_0^t dt &= \int_{v_0}^v \frac{dv}{k/v} \\ \int_0^t dt &= \int_{v_0}^v \frac{1}{k} v dv \\ t \Big|_0^t &= \frac{1}{2k} v^2 \Big|_{v_0}^v \\ t &= \frac{1}{2k} (v^2 - v_0^2) \\ v &= \sqrt{2kt + v_0^2} \end{aligned}$$

Ans.

12–10. Car *A* starts from rest at $t = 0$ and travels along a straight road with a constant acceleration of 6 ft/s^2 until it reaches a speed of 80 ft/s . Afterwards it maintains this speed. Also, when $t = 0$, car *B* located 6000 ft down the road is traveling towards *A* at a constant speed of 60 ft/s . Determine the distance traveled by car *A* when they pass each other.



Distance Traveled: Time for car *A* to achieve $v = 80 \text{ ft/s}$ can be obtained by applying Eq. 12–4.

$$\begin{aligned} \left(\begin{array}{l} \rightarrow \\ \leftarrow \end{array} \right) \quad v &= v_0 + a_c t \\ 80 &= 0 + 6t \\ t &= 13.33 \text{ s} \end{aligned}$$

The distance car *A* travels for this part of motion can be determined by applying Eq. 12–6.

$$\begin{aligned} \left(\begin{array}{l} \rightarrow \\ \leftarrow \end{array} \right) \quad v^2 &= v_0^2 + 2a_c (s - s_0) \\ 80^2 &= 0 + 2(6)(s_1 - 0) \\ s_1 &= 533.33 \text{ ft} \end{aligned}$$

For the second part of motion, car *A* travels with a constant velocity of $v = 80 \text{ ft/s}$ and the distance traveled in $t' = (t_1 - 13.33) \text{ s}$ (t_1 is the total time) is

$$\left(\begin{array}{l} \rightarrow \\ \leftarrow \end{array} \right) \quad s_2 = vt' = 80(t_1 - 13.33)$$

Car *B* travels in the opposite direction with a constant velocity of $v = 60 \text{ ft/s}$ and the distance traveled in t_1 is

$$\left(\begin{array}{l} \rightarrow \\ \leftarrow \end{array} \right) \quad s_3 = vt_1 = 60t_1$$

It is required that

$$\begin{aligned} s_1 + s_2 + s_3 &= 6000 \\ 533.33 + 80(t_1 - 13.33) + 60t_1 &= 6000 \\ t_1 &= 46.67 \text{ s} \end{aligned}$$

The distance traveled by car *A* is

$$s_A = s_1 + s_2 = 533.33 + 80(46.67 - 13.33) = 3200 \text{ ft} \quad \textbf{Ans.}$$

***12–12.** A sphere is fired downwards into a medium with an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t) \text{ m/s}^2$, where t is in seconds, determine the distance traveled before it stops.

Velocity: $v_0 = 27 \text{ m/s}$ at $t_0 = 0 \text{ s}$. Applying Eq. 12–2, we have

$$\begin{aligned} (+\downarrow) \quad dv &= a dt \\ \int_{27}^v dv &= \int_0^t -6t dt \\ v &= (27 - 3t^2) \text{ m/s} \end{aligned} \quad [1]$$

At $v = 0$, from Eq. [1]

$$0 = 27 - 3t^2 \quad t = 3.00 \text{ s}$$

Distance Traveled: $s_0 = 0 \text{ m}$ at $t_0 = 0 \text{ s}$. Using the result $v = 27 - 3t^2$ and applying Eq. 12–1, we have

$$\begin{aligned} (+\downarrow) \quad ds &= v dt \\ \int_0^s ds &= \int_0^t (27 - 3t^2) dt \\ s &= (27t - t^3) \text{ m} \end{aligned} \quad [2]$$

At $t = 3.00 \text{ s}$, from Eq. [2]

$$s = 27(3.00) - 3.00^3 = 54.0 \text{ m} \quad \text{Ans.}$$

•12–13. A particle travels along a straight line such that in 2 s it moves from an initial position $s_A = +0.5 \text{ m}$ to a position $s_B = -1.5 \text{ m}$. Then in another 4 s it moves from s_B to $s_C = +2.5 \text{ m}$. Determine the particle's average velocity and average speed during the 6-s time interval.

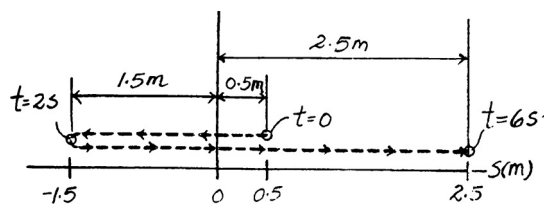
$$\Delta s = (s_C - s_A) = 2 \text{ m}$$

$$s_T = (0.5 + 1.5 + 1.5 + 2.5) = 6 \text{ m}$$

$$t = (2 + 4) = 6 \text{ s}$$

$$v_{avg} = \frac{\Delta s}{t} = \frac{2}{6} = 0.333 \text{ m/s} \quad \text{Ans.}$$

$$(v_{sp})_{avg} = \frac{s_T}{t} = \frac{6}{6} = 1 \text{ m/s} \quad \text{Ans.}$$



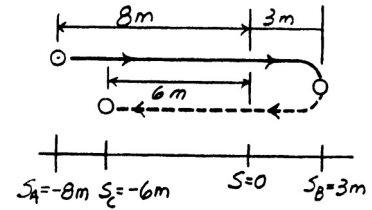
12–14. A particle travels along a straight-line path such that in 4 s it moves from an initial position $s_A = -8$ m to a position $s_B = +3$ m. Then in another 5 s it moves from s_B to $s_C = -6$ m. Determine the particle's average velocity and average speed during the 9-s time interval.

Average Velocity: The displacement from A to C is $\Delta s = s_C - s_A = -6 - (-8) = 2$ m.

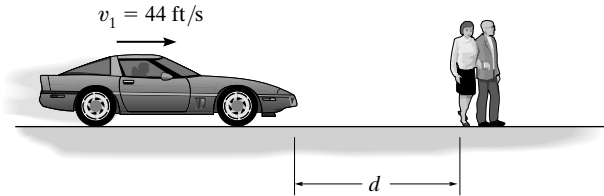
$$v_{\text{avg}} = \frac{\Delta s}{\Delta t} = \frac{2}{4 + 5} = 0.222 \text{ m/s} \quad \text{Ans.}$$

Average Speed: The distances traveled from A to B and B to C are $s_{A \rightarrow B} = 8 + 3 = 11.0$ m and $s_{B \rightarrow C} = 3 + 6 = 9.00$ m, respectively. Then, the total distance traveled is $s_{\text{Tot}} = s_{A \rightarrow B} + s_{B \rightarrow C} = 11.0 + 9.00 = 20.0$ m.

$$(v_{sp})_{\text{avg}} = \frac{s_{\text{Tot}}}{\Delta t} = \frac{20.0}{4 + 5} = 2.22 \text{ m/s} \quad \text{Ans.}$$



12–15. Tests reveal that a normal driver takes about 0.75 s before he or she can *react* to a situation to avoid a collision. It takes about 3 s for a driver having 0.1% alcohol in his system to do the same. If such drivers are traveling on a straight road at 30 mph (44 ft/s) and their cars can decelerate at 2 ft/s^2 , determine the shortest stopping distance d for each from the moment they see the pedestrians. *Moral:* If you must drink, please don't drive!



Stopping Distance: For normal driver, the car moves a distance of $d' = vt = 44(0.75) = 33.0$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 33.0$ ft and $v = 0$.

$$\begin{aligned} (\pm) \quad v^2 &= v_0^2 + 2a_c(s - s_0) \\ 0^2 &= 44^2 + 2(-2)(d - 33.0) \\ d &= 517 \text{ ft} \quad \text{Ans.} \end{aligned}$$

For a drunk driver, the car moves a distance of $d' = vt = 44(3) = 132$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 132$ ft and $v = 0$.

$$\begin{aligned} (\pm) \quad v^2 &= v_0^2 + 2a_c(s - s_0) \\ 0^2 &= 44^2 + 2(-2)(d - 132) \\ d &= 616 \text{ ft} \quad \text{Ans.} \end{aligned}$$

***12–16.** As a train accelerates uniformly it passes successive kilometer marks while traveling at velocities of 2 m/s and then 10 m/s. Determine the train's velocity when it passes the next kilometer mark and the time it takes to travel the 2-km distance.

Kinematics: For the first kilometer of the journey, $v_0 = 2$ m/s, $v = 10$ m/s, $s_0 = 0$, and $s = 1000$ m. Thus,

$$\begin{aligned} (\pm) \quad v^2 &= v_0^2 + 2a_c(s - s_0) \\ 10^2 &= 2^2 + 2a_c(1000 - 0) \\ a_c &= 0.048 \text{ m/s}^2 \end{aligned}$$

For the second kilometer, $v_0 = 10$ m/s, $s_0 = 1000$ m, $s = 2000$ m, and 0.048 m/s². Thus,

$$\begin{aligned} (\pm) \quad v^2 &= v_0^2 + 2a_c(s - s_0) \\ v^2 &= 10^2 + 2(0.048)(2000 - 1000) \\ v &= 14 \text{ m/s} \end{aligned}$$

Ans.

For the whole journey, $v_0 = 2$ m/s, $v = 14$ m/s, and 0.048 m/s². Thus,

$$\begin{aligned} (\pm) \quad v &= v_0 + a_c t \\ 14 &= 2 + 0.048t \\ t &= 250 \text{ s} \end{aligned}$$

Ans.

•12–17. A ball is thrown with an upward velocity of 5 m/s from the top of a 10-m high building. One second later another ball is thrown vertically from the ground with a velocity of 10 m/s. Determine the height from the ground where the two balls pass each other.

Kinematics: First, we will consider the motion of ball A with $(v_A)_0 = 5$ m/s, $(s_A)_0 = 0$, $s_A = (h - 10)$ m, $t_A = t'$, and $a_c = -9.81$ m/s². Thus,

$$\begin{aligned} (+\uparrow) \quad s_A &= (s_A)_0 + (v_A)_0 t_A + \frac{1}{2} a_c t_A^2 \\ h - 10 &= 0 + 5t' + \frac{1}{2}(-9.81)(t')^2 \\ h &= 5t' - 4.905(t')^2 + 10 \end{aligned} \quad (1)$$

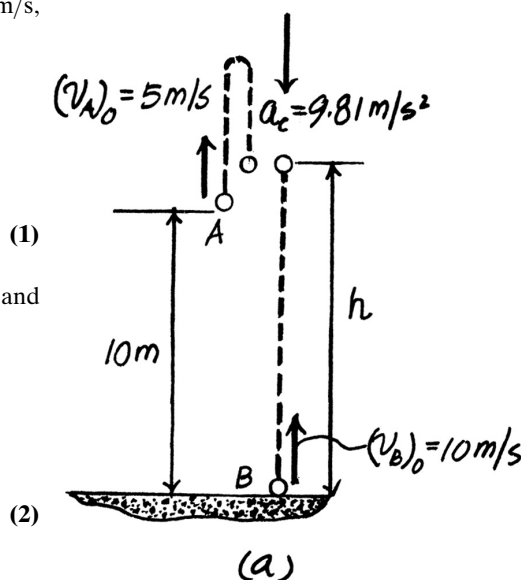
Motion of ball B is with $(v_B)_0 = 10$ m/s, $(s_B)_0 = 0$, $s_B = h$, $t_B = t' - 1$ and $a_c = -9.81$ m/s². Thus,

$$\begin{aligned} (+\uparrow) \quad s_B &= (s_B)_0 + (v_B)_0 t_B + \frac{1}{2} a_c t_B^2 \\ h &= 0 + 10(t' - 1) + \frac{1}{2}(-9.81)(t' - 1)^2 \\ h &= 19.81t' - 4.905(t')^2 - 14.905 \end{aligned} \quad (2)$$

Solving Eqs. (1) and (2) yields

$$\begin{aligned} h &= 4.54 \text{ m} \\ t' &= 1.68 \text{ s} \end{aligned}$$

Ans.



12–18. A car starts from rest and moves with a constant acceleration of 1.5 m/s^2 until it achieves a velocity of 25 m/s . It then travels with constant velocity for 60 seconds. Determine the average speed and the total distance traveled.

Kinematics: For stage (1) of the motion, $v_0 = 0$, $s_0 = 0$, $v = 25 \text{ m/s}$, and $a_c = 1.5 \text{ m/s}^2$.

$$\begin{aligned} \left(\pm \right) \quad v &= v_0 + a_c t \\ 25 &= 0 + 1.5 t_1 \\ t_1 &= 16.67 \text{ s} \end{aligned}$$

$$\begin{aligned} \left(\pm \right) \quad v^2 &= v_0^2 + 2a_c(s - s_0) \\ 25^2 &= 0 + 2(1.5)(s_1 - 0) \\ s_1 &= 208.33 \text{ m} \end{aligned}$$

For stage (2) of the motion, $s_0 = 108.22 \text{ ft}$, $v_0 = 25 \text{ ft/s}$, $t = 60 \text{ s}$, and $a_c = 0$. Thus,

$$\begin{aligned} \left(\pm \right) \quad s &= s_0 + v_0 t + \frac{1}{2} a_c t^2 \\ s &= 208.33 + 25(60) + 0 \\ &= 1708.33 \text{ ft} = 1708 \text{ m} \end{aligned}$$

Ans.

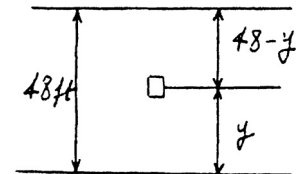
The average speed of the car is then

$$v_{\text{avg}} = \frac{s}{t_1 + t_2} = \frac{1708.33}{16.67 + 60} = 22.3 \text{ m/s}$$

Ans.

12–19. A car is to be hoisted by elevator to the fourth floor of a parking garage, which is 48 ft above the ground. If the elevator can accelerate at 0.6 ft/s^2 , decelerate at 0.3 ft/s^2 , and reach a maximum speed of 8 ft/s , determine the shortest time to make the lift, starting from rest and ending at rest.

$$\begin{aligned} +\uparrow \quad v^2 &= v_0^2 + 2 a_c (s - s_0) \\ v_{\text{max}}^2 &= 0 + 2(0.6)(y - 0) \\ 0 &= v_{\text{max}}^2 + 2(-0.3)(48 - y) \\ 0 &= 1.2 y - 0.6(48 - y) \\ y &= 16.0 \text{ ft}, \quad v_{\text{max}} = 4.382 \text{ ft/s} < 8 \text{ ft/s} \\ +\uparrow \quad v &= v_0 + a_c t \\ 4.382 &= 0 + 0.6 t_1 \\ t_1 &= 7.303 \text{ s} \\ 0 &= 4.382 - 0.3 t_2 \\ t_2 &= 14.61 \text{ s} \\ t &= t_1 + t_2 = 21.9 \text{ s} \end{aligned}$$



Ans.

***12–20.** A particle is moving along a straight line such that its speed is defined as $v = (-4s^2)$ m/s, where s is in meters. If $s = 2$ m when $t = 0$, determine the velocity and acceleration as functions of time.

$$v = -4s^2$$

$$\frac{ds}{dt} = -4s^2$$

$$\int_2^s s^{-2} ds = \int_0^t -4 dt$$

$$-s^{-1} \Big|_2^s = -4t \Big|_0^t$$

$$t = \frac{1}{4}(s^{-1} - 0.5)$$

$$s = \frac{2}{8t + 1}$$

$$v = -4 \left(\frac{2}{8t + 1} \right)^2 = \left(-\frac{16}{(8t + 1)^2} \right) \text{ m/s}$$

Ans.

$$a = \frac{dv}{dt} = \frac{16(2)(8t + 1)(8)}{(8t + 1)^4} = \left(\frac{256}{(8t + 1)^3} \right) \text{ m/s}^2$$

Ans.

•12–21. Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (6t - 3) \text{ ft/s}^2$ and $a_B = (12t^2 - 8) \text{ ft/s}^2$, where t is in seconds. Determine the distance between them when $t = 4 \text{ s}$ and the total distance each has traveled in $t = 4 \text{ s}$.

Velocity: The velocity of particles A and B can be determined using Eq. 12-2.

$$\begin{aligned} dv_A &= a_A dt \\ \int_0^{v_A} dv_A &= \int_0^t (6t - 3) dt \\ v_A &= 3t^2 - 3t \\ dv_B &= a_B dt \\ \int_0^{v_B} dv_B &= \int_0^t (12t^2 - 8) dt \\ v_B &= 4t^3 - 8t \end{aligned}$$

The times when particle A stops are

$$3t^2 - 3t = 0 \quad t = 0 \text{ s and } t = 1 \text{ s}$$

The times when particle B stops are

$$4t^3 - 8t = 0 \quad t = 0 \text{ s and } t = \sqrt{2} \text{ s}$$

Position: The position of particles A and B can be determined using Eq. 12-1.

$$\begin{aligned} ds_A &= v_A dt \\ \int_0^{s_A} ds_A &= \int_0^t (3t^2 - 3t) dt \\ s_A &= t^3 - \frac{3}{2}t^2 \\ ds_B &= v_B dt \\ \int_0^{s_B} ds_B &= \int_0^t (4t^3 - 8t) dt \\ s_B &= t^4 - 4t^2 \end{aligned}$$

The positions of particle A at $t = 1 \text{ s}$ and 4 s are

$$s_A|_{t=1 \text{ s}} = 1^3 - \frac{3}{2}(1^2) = -0.500 \text{ ft}$$

$$s_A|_{t=4 \text{ s}} = 4^3 - \frac{3}{2}(4^2) = 40.0 \text{ ft}$$

Particle A has traveled

$$d_A = 2(0.5) + 40.0 = 41.0 \text{ ft}$$

Ans.

The positions of particle B at $t = \sqrt{2} \text{ s}$ and 4 s are

$$s_B|_{t=\sqrt{2}} = (\sqrt{2})^4 - 4(\sqrt{2})^2 = -4 \text{ ft}$$

$$s_B|_{t=4} = (4)^4 - 4(4)^2 = 192 \text{ ft}$$

Particle B has traveled

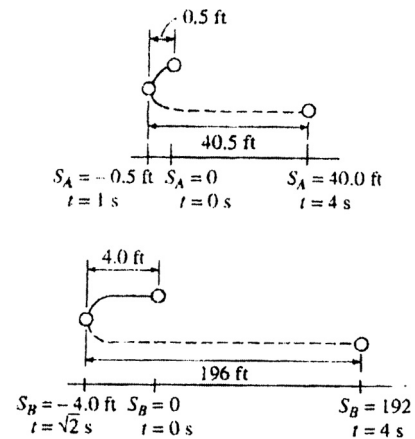
$$d_B = 2(4) + 192 = 200 \text{ ft}$$

Ans.

At $t = 4 \text{ s}$ the distance between A and B is

$$\Delta s_{AB} = 192 - 40 = 152 \text{ ft}$$

Ans.



12–22. A particle moving along a straight line is subjected to a deceleration $a = (-2v^3) \text{ m/s}^2$, where v is in m/s. If it has a velocity $v = 8 \text{ m/s}$ and a position $s = 10 \text{ m}$ when $t = 0$, determine its velocity and position when $t = 4 \text{ s}$.

Velocity: The velocity of the particle can be related to its position by applying Eq. 12–3.

$$ds = \frac{v dv}{a}$$

$$\int_{10\text{m}}^s ds = \int_{8\text{m/s}}^v -\frac{dv}{2v^2}$$

$$s - 10 = \frac{1}{2v} - \frac{1}{16}$$

$$v = \frac{8}{16s - 159} \quad [1]$$

Position: The position of the particle can be related to the time by applying Eq. 12–1.

$$dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_{10\text{m}}^s \frac{1}{8} (16s - 159) ds$$

$$8t = 8s^2 - 159s + 790$$

When $t = 4 \text{ s}$,

$$8(4) = 8s^2 - 159s + 790$$

$$8s^2 - 159s + 758 = 0$$

Choose the root greater than 10 m $s = 11.94 \text{ m} = 11.9 \text{ m}$

Ans.

Substitute $s = 11.94 \text{ m}$ into Eq. [1] yields

$$v = \frac{8}{16(11.94) - 159} = 0.250 \text{ m/s}$$

Ans.

12–23. A particle is moving along a straight line such that its acceleration is defined as $a = (-2v) \text{ m/s}^2$, where v is in meters per second. If $v = 20 \text{ m/s}$ when $s = 0$ and $t = 0$, determine the particle's position, velocity, and acceleration as functions of time.

$$a = -2v$$

$$\frac{dv}{dt} = -2v$$

$$\int_{20}^v \frac{dv}{v} = \int_0^t -2 dt$$

$$\ln \frac{v}{20} = -2t$$

$$v = (20e^{-2t}) \text{ m/s}$$

Ans.

$$a = \frac{dv}{dt} = (-40e^{-2t}) \text{ m/s}^2$$

Ans.

$$\int_0^s ds = \int_0^t v dt = \int_0^t (20e^{-2t}) dt$$

$$s = -10e^{-2t} \Big|_0^t = -10(e^{-2t} - 1)$$

$$s = 10(1 - e^{-2t}) \text{ m}$$

Ans.

***12–24.** A particle starts from rest and travels along a straight line with an acceleration $a = (30 - 0.2v) \text{ ft/s}^2$, where v is in ft/s. Determine the time when the velocity of the particle is $v = 30 \text{ ft/s}$.

Velocity:

$$(\rightarrow) \quad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{30 - 0.2v}$$

$$t|_0^t = -\frac{1}{0.2} \ln(30 - 0.2v) \Big|_0^v$$

$$t = 5 \ln \frac{30}{30 - 0.2v}$$

$$t = 5 \ln \frac{30}{30 - 0.2(50)} = 1.12 \text{ s}$$

Ans.

•12–25. When a particle is projected vertically upwards with an initial velocity of v_0 , it experiences an acceleration $a = -(g + kv^2)$, where g is the acceleration due to gravity, k is a constant and v is the velocity of the particle. Determine the maximum height reached by the particle.

Position:

$$(+\uparrow) \quad ds = \frac{v dv}{a}$$

$$\int_0^s ds = \int_{v_0}^v -\frac{v dv}{g + kv^2}$$

$$s|_0^s = -\left[\frac{1}{2k} \ln(g + kv^2) \right] \Big|_{v_0}^v$$

$$s = \frac{1}{2k} \ln \left(\frac{g + kv_0^2}{g + kv^2} \right)$$

The particle achieves its maximum height when $v = 0$. Thus,

$$h_{\max} = \frac{1}{2k} \ln \left(\frac{g + kv_0^2}{g} \right)$$

$$= \frac{1}{2k} \ln \left(1 + \frac{k}{g} v_0^2 \right)$$

Ans.

12–26. The acceleration of a particle traveling along a straight line is $a = (0.02e^t) \text{ m/s}^2$, where t is in seconds. If $v = 0$, $s = 0$ when $t = 0$, determine the velocity and acceleration of the particle at $s = 4 \text{ m}$.

Velocity: $a = 0.02e^{5.329} = 4.13 \text{ m/s}^2$

Ans.

$(\pm) \quad dv = a \, dt$

$$\int_0^v dv = \int_0^t 0.02e^t \, dt$$

$$v|_0^v = 0.02e^t \Big|_0^t$$

$$v = [0.02(e^t - 1)] \text{ m/s} \quad \textbf{(1)}$$

Position:

$(\pm) \quad ds = v \, dt$

$$\int_0^s ds = \int_0^t 0.02(e^t - 1) \, dt$$

$$s|_0^s = 0.02(e^t - t) \Big|_0^t$$

$$s = 0.02(e^t - t - 1) \text{ m}$$

When $s = 4 \text{ m}$,

$$4 = 0.02(e^t - t - 1)$$

$$e^t - t - 201 = 0$$

Solving the above equation by trial and error,

$$t = 5.329 \text{ s}$$

Thus, the velocity and acceleration when $s = 4 \text{ m}$ ($t = 5.329 \text{ s}$) are

$$v = 0.02(e^{5.329} - 1) = 4.11 \text{ m/s} \quad \textbf{Ans.}$$

$$a = 0.02e^{5.329} = 4.13 \text{ m/s}^2 \quad \textbf{Ans.}$$

12–27. A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2}) \text{ m/s}^2$, where s is in meters. Determine the particle's velocity when $s = 2 \text{ m}$, if it starts from rest when $s = 1 \text{ m}$. Use Simpson's rule to evaluate the integral.

$$a = \frac{5}{(3s^{1/3} + s^{5/2})}$$

$$a \, ds = v \, dv$$

$$\int_1^2 \frac{5 \, ds}{(3s^{1/3} + s^{5/2})} = \int_0^v v \, dv$$

$$0.8351 = \frac{1}{2} v^2$$

$$v = 1.29 \text{ m/s}$$

Ans.

***12–28.** If the effects of atmospheric resistance are accounted for, a falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})]$ m/s², where v is in m/s and the positive direction is downward. If the body is released from rest at a very *high altitude*, determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \rightarrow \infty$).

Velocity: The velocity of the particle can be related to the time by applying Eq. 12–2.

$$(+\downarrow) \quad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{9.81[1 - (0.01v)^2]}$$

$$t = \frac{1}{9.81} \left[\int_0^v \frac{dv}{2(1 + 0.01v)} + \int_0^v \frac{dv}{2(1 - 0.01v)} \right]$$

$$9.81t = 50 \ln \left(\frac{1 + 0.01v}{1 - 0.01v} \right)$$

$$v = \frac{100(e^{0.1962t} - 1)}{e^{0.1962t} + 1} \quad [1]$$

a) When $t = 5$ s, then, from Eq. [1]

$$v = \frac{100[e^{0.1962(5)} - 1]}{e^{0.1962(5)} + 1} = 45.5 \text{ m/s} \quad \text{Ans.}$$

b) If $t \rightarrow \infty$, $\frac{e^{0.1962t} - 1}{e^{0.1962t} + 1} \rightarrow 1$. Then, from Eq. [1]

$$v_{\max} = 100 \text{ m/s} \quad \text{Ans.}$$

•12–29. The position of a particle along a straight line is given by $s = (1.5t^3 - 13.5t^2 + 22.5t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint:* Plot the path to determine the total distance traveled.

Position: The position of the particle when $t = 6$ s is

$$s|_{t=6s} = 1.5(6^3) - 13.5(6^2) + 22.5(6) = -27.0 \text{ ft} \quad \text{Ans.}$$

Total Distance Traveled: The velocity of the particle can be determined by applying Eq. 12–1.

$$v = \frac{ds}{dt} = 4.50t^2 - 27.0t + 22.5$$

The times when the particle stops are

$$4.50t^2 - 27.0t + 22.5 = 0$$

$$t = 1 \text{ s} \quad \text{and} \quad t = 5 \text{ s}$$

The position of the particle at $t = 0$ s, 1 s and 5 s are

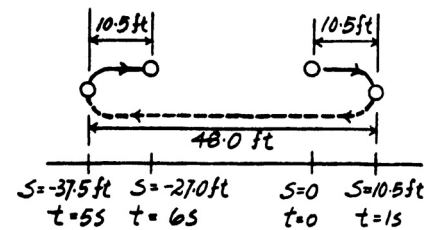
$$s|_{t=0s} = 1.5(0^3) - 13.5(0^2) + 22.5(0) = 0$$

$$s|_{t=1s} = 1.5(1^3) - 13.5(1^2) + 22.5(1) = 10.5 \text{ ft}$$

$$s|_{t=5s} = 1.5(5^3) - 13.5(5^2) + 22.5(5) = -37.5 \text{ ft}$$

From the particle's path, the total distance is

$$s_{\text{tot}} = 10.5 + 48.0 + 10.5 = 69.0 \text{ ft} \quad \text{Ans.}$$



12–30. The velocity of a particle traveling along a straight line is $v = v_0 - ks$, where k is constant. If $s = 0$ when $t = 0$, determine the position and acceleration of the particle as a function of time.

Position:

$$\left(\pm \right) \quad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{v_0 - ks}$$

$$t \Big|_0^t = -\frac{1}{k} \ln(v_0 - ks) \Big|_0^s$$

$$t = \frac{1}{k} \ln \left(\frac{v_0}{v_0 - ks} \right)$$

$$e^{kt} = \frac{v_0}{v_0 - ks}$$

$$s = \frac{v_0}{k} (1 - e^{-kt})$$

Ans.

Velocity:

$$v = \frac{ds}{dt} = \frac{d}{dt} \left[\frac{v_0}{k} (1 - e^{-kt}) \right]$$

$$v = v_0 e^{-kt}$$

Acceleration:

$$a = \frac{dv}{dt} = \frac{d}{dt} (v_0 e^{-kt})$$

$$a = -kv_0 e^{-kt}$$

Ans.

12–31. The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1) \text{ m/s}^2$, where t is in seconds. If $s = 1 \text{ m}$ and $v = 2 \text{ m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 6 \text{ s}$. Also, determine the total distance the particle travels during this time period.

$$\int_2^v dv = \int_0^t (2t - 1) dt$$

$$v = t^2 - t + 2$$

$$\int_1^s ds = \int_0^t (t^2 - t + 2) dt$$

$$s = \frac{1}{3} t^3 - \frac{1}{2} t^2 + 2t + 1$$

When $t = 6 \text{ s}$,

$$v = 32 \text{ m/s}$$

Ans.

$$s = 67 \text{ m}$$

Ans.

Since $v \neq 0$ then

$$d = 67 - 1 = 66 \text{ m}$$

Ans.

***12–32.** Ball A is thrown vertically upward from the top of a 30-m-high-building with an initial velocity of 5 m/s. At the same instant another ball B is thrown upward from the ground with an initial velocity of 20 m/s. Determine the height from the ground and the time at which they pass.

Origin at roof:

Ball A :

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-s = 0 + 5t - \frac{1}{2} (9.81)t^2$$

Ball B :

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-s = -30 + 20t - \frac{1}{2} (9.81)t^2$$

Solving,

$$t = 2 \text{ s}$$

$$s = 9.62 \text{ m}$$

Distance from ground,

$$d = (30 - 9.62) = 20.4 \text{ m}$$

Also, origin at ground,

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_A = 30 + 5t + \frac{1}{2} (-9.81)t^2$$

$$s_B = 0 + 20t + \frac{1}{2} (-9.81)t^2$$

Require

$$s_A = s_B$$

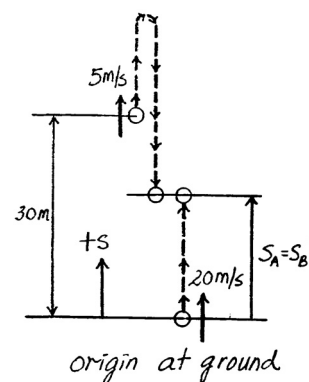
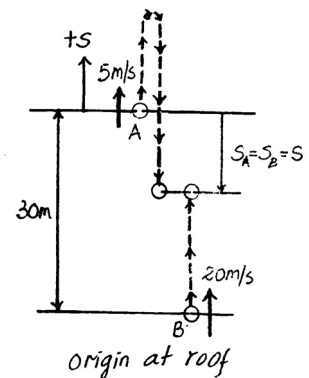
$$30 + 5t + \frac{1}{2} (-9.81)t^2 = 20t + \frac{1}{2} (-9.81)t^2$$

$$t = 2 \text{ s}$$

$$s_B = 20.4 \text{ m}$$

Ans.

Ans.



Ans.

Ans.

•12–33. A motorcycle starts from rest at $t = 0$ and travels along a straight road with a constant acceleration of 6 ft/s^2 until it reaches a speed of 50 ft/s . Afterwards it maintains this speed. Also, when $t = 0$, a car located 6000 ft down the road is traveling toward the motorcycle at a constant speed of 30 ft/s . Determine the time and the distance traveled by the motorcycle when they pass each other.

Motorcycle:

$$\left(\begin{array}{l} \rightarrow \end{array} \right) \quad v = v_0 + a_c t'$$

$$50 = 0 + 6t'$$

$$t' = 8.33 \text{ s}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$(50)^2 = 0 + 2(6)(s' - 0)$$

$$s' = 208.33 \text{ ft}$$

In $t' = 8.33 \text{ s}$ car travels

$$s'' = v_0 t' = 30(8.33) = 250 \text{ ft}$$

Distance between motorcycle and car:

$$6000 - 250 - 208.33 = 5541.67 \text{ ft}$$

When passing occurs for motorcycle,

$$s = v_0 t; \quad x = 50(t'')$$

For car:

$$s = v_0 t; \quad 5541.67 - x = 30(t'')$$

Solving,

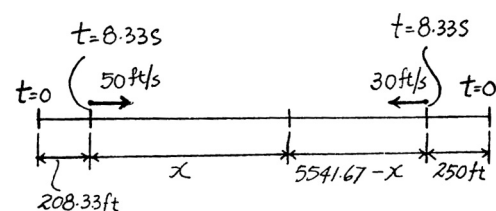
$$x = 3463.54 \text{ ft}$$

$$t'' = 69.27 \text{ s}$$

Thus, for the motorcycle,

$$t = 69.27 + 8.33 = 77.6 \text{ s}$$

$$s_m = 208.33 + 3463.54 = 3.67(10)^3 \text{ ft}$$



Ans.

Ans.

12–34. A particle moves along a straight line with a velocity $v = (200s)$ mm/s, where s is in millimeters. Determine the acceleration of the particle at $s = 2000$ mm. How long does the particle take to reach this position if $s = 500$ mm when $t = 0$?

Acceleration:

$$\left(\pm \right) \quad \frac{dv}{ds} = 200s$$

$$\text{Thus, } a = v \frac{dv}{ds} = (200s)(200) = 40(10^3)s \text{ mm/s}^2$$

When $s = 2000$ mm,

$$a = 40(10^3)(2000) = 80(10^6) \text{ mm/s}^2 = 80 \text{ km/s}^2 \quad \textbf{Ans.}$$

Position:

$$\begin{aligned} \left(\pm \right) \quad dt &= \frac{ds}{v} \\ \int_0^t dt &= \int_{500 \text{ mm}}^s \frac{ds}{200s} \\ t \Big|_0^t &= \frac{1}{200} \ln s \Big|_{500 \text{ mm}}^s \\ t &= \frac{1}{200} \ln \frac{s}{500} \end{aligned}$$

At $s = 2000$ mm,

$$t = \frac{1}{200} \ln \frac{2000}{500} = 6.93(10^{-3}) \text{ s} = 6.93 \text{ ms} \quad \textbf{Ans.}$$

■12–35. A particle has an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t) \text{ m/s}^2$, where t is in seconds, determine its velocity, after it has traveled 10 m. How much time does this take?

Velocity:

$$\begin{aligned} (\pm) \quad dv &= a \, dt \\ \int_{27}^v dv &= \int_0^t (-6t) \, dt \\ v \Big|_{27}^v &= (-3t^2) \Big|_0^t \\ v &= (27 - 3t^2) \text{ m/s} \end{aligned}$$

$$\begin{aligned} (\pm) \quad ds &= v \, dt \\ \int_0^s ds &= \int_0^t (27 - 3t^2) \, dt \\ s \Big|_0^s &= (27t - t^3) \Big|_0^t \\ s &= (27t - t^3) \text{ m/s} \end{aligned}$$

When $s = 100 \text{ m}$,

$$t = 0.372 \text{ s}$$

Ans.

$$v = 26.6 \text{ m/s}$$

Ans.

*12–36. The acceleration of a particle traveling along a straight line is $a = (8 - 2s) \text{ m/s}^2$, where s is in meters. If $v = 0$ at $s = 0$, determine the velocity of the particle at $s = 2 \text{ m}$, and the position of the particle when the velocity is maximum.

Velocity:

$$\begin{aligned} (\pm) \quad v \, dv &= a \, ds \\ \int_0^v v \, dv &= \int_0^s (8 - 2s) \, ds \\ \frac{v^2}{2} \Big|_0^v &= (8s - s^2) \Big|_0^s \\ v &= \sqrt{16s - 2s^2} \text{ m/s} \end{aligned}$$

At $s = 2 \text{ m}$,

$$v|_{s=2 \text{ m}} = \sqrt{16(2) - 2(2^2)} = \pm 4.90 \text{ m/s}$$

Ans.

When the velocity is maximum $\frac{dv}{ds} = 0$. Thus,

$$\begin{aligned} \frac{dv}{ds} &= \frac{16 - 4s}{2\sqrt{16s - 2s^2}} = 0 \\ 16 - 4s &= 0 \\ s &= 4 \text{ m} \end{aligned}$$

Ans.

•12–37. Ball A is thrown vertically upwards with a velocity of v_0 . Ball B is thrown upwards from the same point with the same velocity t seconds later. Determine the elapsed time $t < 2v_0/g$ from the instant ball A is thrown to when the balls pass each other, and find the velocity of each ball at this instant.

Kinematics: First, we will consider the motion of ball A with $(v_A)_0 = v_0$, $(s_A)_0 = 0$, $s_A = h$, $t_A = t'$, and $(a_c)_A = -g$.

$$\begin{aligned} (+\uparrow) \quad s_A &= (s_A)_0 + (v_A)_0 t_A + \frac{1}{2}(a_c)_A t_A^2 \\ h &= 0 + v_0 t' + \frac{1}{2}(-g)(t')^2 \\ h &= v_0 t' - \frac{g}{2} t'^2 \end{aligned} \quad (1)$$

$$\begin{aligned} (+\uparrow) \quad v_A &= (v_A)_0 + (a_c)_A t_A \\ v_A &= v_0 + (-g)(t') \\ v_A &= v_0 - g t' \end{aligned} \quad (2)$$

The motion of ball B requires $(v_B)_0 = v_0$, $(s_B)_0 = 0$, $s_B = h$, $t_B = t' - t$, and $(a_c)_B = -g$.

$$\begin{aligned} (+\uparrow) \quad s_B &= (s_B)_0 + (v_B)_0 t_B + \frac{1}{2}(a_c)_B t_B^2 \\ h &= 0 + v_0(t' - t) + \frac{1}{2}(-g)(t' - t)^2 \\ h &= v_0(t' - t) - \frac{g}{2}(t' - t)^2 \end{aligned} \quad (3)$$

$$\begin{aligned} (+\uparrow) \quad v_B &= (v_B)_0 + (a_c)_B t_B \\ v_B &= v_0 + (-g)(t' - t) \\ v_B &= v_0 - g(t' - t) \end{aligned} \quad (4)$$

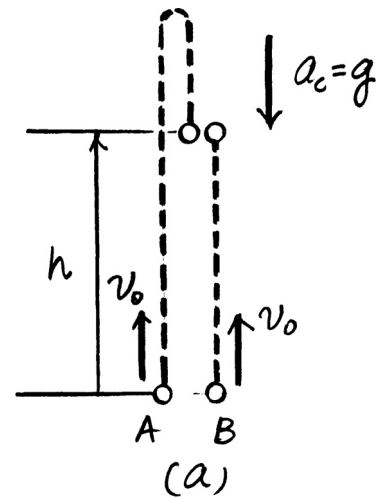
Solving Eqs. (1) and (3),

$$\begin{aligned} v_0 t' - \frac{g}{2} t'^2 &= v_0(t' - t) - \frac{g}{2}(t' - t)^2 \\ t' &= \frac{2v_0 + gt}{2g} \end{aligned} \quad \text{Ans.}$$

Substituting this result into Eqs. (2) and (4),

$$\begin{aligned} v_A &= v_0 - g\left(\frac{2v_0 + gt}{2g}\right) \\ &= -\frac{1}{2}gt = \frac{1}{2}gt \downarrow \end{aligned} \quad \text{Ans.}$$

$$\begin{aligned} v_B &= v_0 - g\left(\frac{2v_0 + gt}{2g} - t\right) \\ &= \frac{1}{2}gt \uparrow \end{aligned} \quad \text{Ans.}$$



12–38. As a body is projected to a high altitude above the earth's *surface*, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R + y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81 \text{ m/s}^2$ and $R = 6356 \text{ km}$, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \rightarrow \infty$.

$$v \, dv = a \, dy$$

$$\int_v^0 v \, dv = -g_0 R^2 \int_0^\infty \frac{dy}{(R + y)^2}$$

$$\frac{v^2}{2} \Big|_v^0 = \frac{g_0 R^2}{R + y} \Big|_0^\infty$$

$$v = \sqrt{2g_0 R}$$

$$= \sqrt{2(9.81)(6356)(10^3)}$$

$$= 11167 \text{ m/s} = 11.2 \text{ km/s}$$

Ans.

12–39. Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12–38), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500 \text{ km}$? Use the numerical data in Prob. 12–38.

From Prob. 12–38,

$$(+\uparrow) \quad a = -g_0 \frac{R^2}{(R + y)^2}$$

$$\text{Since } a \, dy = v \, dv$$

then

$$-g_0 R^2 \int_{y_0}^y \frac{dy}{(R + y)^2} = \int_0^v v \, dv$$

$$g_0 R^2 \left[\frac{1}{R + y} \right]_{y_0}^y = \frac{v^2}{2}$$

$$g_0 R^2 \left[\frac{1}{R + y} - \frac{1}{R + y_0} \right] = \frac{v^2}{2}$$

Thus

$$v = -R \sqrt{\frac{2g_0 (y_0 - y)}{(R + y)(R + y_0)}}$$

$$\text{When } y_0 = 500 \text{ km}, \quad y = 0,$$

$$v = -6356(10^3) \sqrt{\frac{2(9.81)(500)(10^3)}{6356(6356 + 500)(10^6)}}$$

$$v = -3016 \text{ m/s} = 3.02 \text{ km/s} \downarrow$$

Ans.

***12–40.** When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f . If this variation of the acceleration can be expressed as $a = (g/v_f^2)(v_f^2 - v^2)$, determine the time needed for the velocity to become $v = v_f/2$. Initially the particle falls from rest.

$$\frac{dv}{dt} = a = \left(\frac{g}{v_f^2}\right)(v_f^2 - v^2)$$

$$\int_0^v \frac{dv}{v_f^2 - v^2} = \frac{g}{v_f^2} \int_0^t dt$$

$$\frac{1}{2v_f} \ln \left(\frac{v_f + v}{v_f - v} \right) \Big|_0^v = \frac{g}{v_f^2} t$$

$$t = \frac{v_f}{2g} \ln \left(\frac{v_f + v}{v_f - v} \right)$$

$$t = \frac{v_f}{2g} \ln \left(\frac{v_f + v_f/2}{v_f - v_f/2} \right)$$

$$t = 0.549 \left(\frac{v_f}{g} \right)$$

Ans.

•12–41. A particle is moving along a straight line such that its position from a fixed point is $s = (12 - 15t^2 + 5t^3)$ m, where t is in seconds. Determine the total distance traveled by the particle from $t = 1$ s to $t = 3$ s. Also, find the average speed of the particle during this time interval.

Velocity:

$$\left(\pm \right) \quad v = \frac{ds}{dt} = \frac{d}{dt} (12 - 15t^2 + 5t^3)$$

$$v = -30t + 15t^2 \text{ m/s}$$

The velocity of the particle changes direction at the instant when it is momentarily brought to rest. Thus,

$$v = -30t + 15t^2 = 0$$

$$t(-30 + 15t) = 0$$

$$t = 0 \text{ and } 2 \text{ s}$$

Position: The positions of the particle at $t = 0$ s, 1 s, 2 s, and 3 s are

$$s|_{t=0 \text{ s}} = 12 - 15(0^2) + 5(0^3) = 12 \text{ m}$$

$$s|_{t=1 \text{ s}} = 12 - 15(1^2) + 5(1^3) = 2 \text{ m}$$

$$s|_{t=2 \text{ s}} = 12 - 15(2^2) + 5(2^3) = -8 \text{ m}$$

$$s|_{t=3 \text{ s}} = 12 - 15(3^2) + 5(3^3) = 12 \text{ m}$$

Using the above results, the path of the particle is shown in Fig. *a*. From this figure, the distance traveled by the particle during the time interval $t = 1$ s to $t = 3$ s is

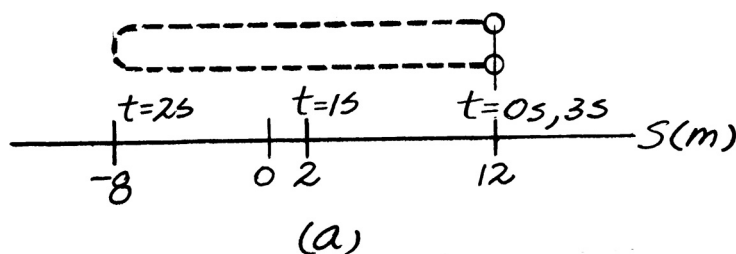
$$s_{\text{Tot}} = (2 + 8) + (8 + 12) = 30 \text{ m}$$

Ans.

The average speed of the particle during the same time interval is

$$v_{\text{avg}} = \frac{s_{\text{Tot}}}{\Delta t} = \frac{30}{3 - 1} = 15 \text{ m/s}$$

Ans.



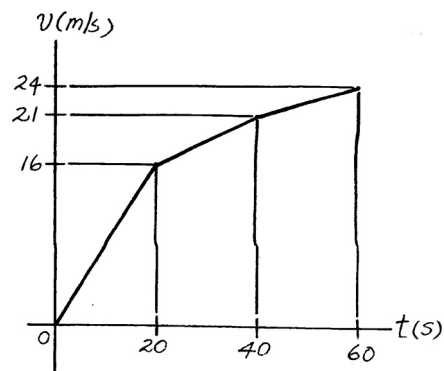
12–42. The speed of a train during the first minute has been recorded as follows:

t (s)	0	20	40	60
v (m/s)	0	16	21	24

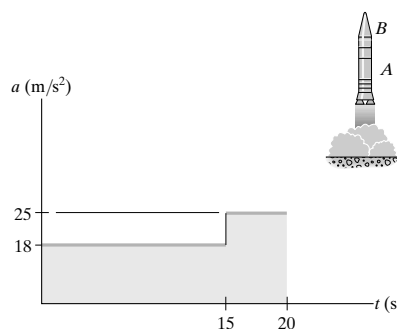
Plot the v – t graph, approximating the curve as straight-line segments between the given points. Determine the total distance traveled.

The total distance traveled is equal to the area under the graph.

$$s_T = \frac{1}{2}(20)(16) + \frac{1}{2}(40 - 20)(16 + 21) + \frac{1}{2}(60 - 40)(21 + 24) = 980 \text{ m} \quad \text{Ans.}$$



12–43. A two-stage missile is fired vertically from rest with the acceleration shown. In 15 s the first stage *A* burns out and the second stage *B* ignites. Plot the v - t and s - t graphs which describe the two-stage motion of the missile for $0 \leq t \leq 20$ s.



Since $v = \int a \, dt$, the constant lines of the a - t graph become sloping lines for the v - t graph.

The numerical values for each point are calculated from the total area under the a - t graph to the point.

$$\text{At } t = 15 \text{ s, } v = (18)(15) = 270 \text{ m/s}$$

$$\text{At } t = 20 \text{ s, } v = 270 + (25)(20 - 15) = 395 \text{ m/s}$$

Since $s = \int v \, dt$, the sloping lines of the v - t graph become parabolic curves for the s - t graph.

The numerical values for each point are calculated from the total area under the v - t graph to the point.

$$\text{At } t = 15 \text{ s, } s = \frac{1}{2}(15)(270) = 2025 \text{ m}$$

$$\text{At } t = 20 \text{ s, } s = 2025 + 270(20 - 15) + \frac{1}{2}(395 - 270)(20 - 15) = 3687.5 \text{ m} = 3.69 \text{ km}$$

Also:

$$0 \leq t \leq 15:$$

$$a = 18$$

$$v = v_0 + a_c t = 0 + 18t$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2 = 0 + 0 + 9t^2$$

$$\text{At } t = 15:$$

$$v = 18(15) = 270$$

$$s = 9(15)^2 = 2025$$

$$15 \leq t \leq 20:$$

$$a = 25$$

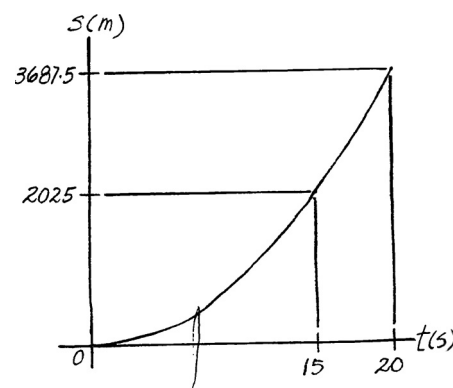
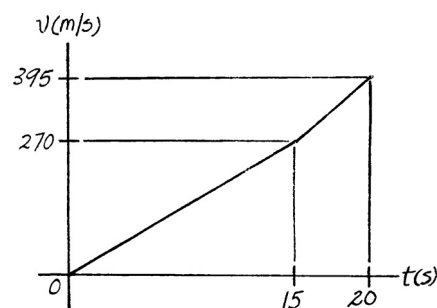
$$v = v_0 + a_c t = 270 + 25(t - 15)$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2 = 2025 + 270(t - 15) + \frac{1}{2}(25)(t - 15)^2$$

$$\text{When } t = 20:$$

$$v = 395 \text{ m/s}$$

$$s = 3687.5 \text{ m} = 3.69 \text{ km}$$



***12–44.** A freight train starts from rest and travels with a constant acceleration of 0.5 ft/s^2 . After a time t' it maintains a constant speed so that when $t = 160 \text{ s}$ it has traveled 2000 ft. Determine the time t' and draw the v – t graph for the motion.

Total Distance Traveled: The distance for part one of the motion can be related to time $t = t'$ by applying Eq. 12–5 with $s_0 = 0$ and $v_0 = 0$.

$$\left(\pm \right) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2} (0.5)(t')^2 = 0.25(t')^2$$

The velocity at time t can be obtained by applying Eq. 12–4 with $v_0 = 0$.

$$\left(\pm \right) \quad v = v_0 + a_c t = 0 + 0.5t = 0.5t \quad [1]$$

The time for the second stage of motion is $t_2 = 160 - t'$ and the train is traveling at a constant velocity of $v = 0.5t'$ (Eq. [1]). Thus, the distance for this part of motion is

$$\left(\pm \right) \quad s_2 = vt_2 = 0.5t'(160 - t') = 80t' - 0.5(t')^2$$

If the total distance traveled is $s_{\text{Tot}} = 2000$, then

$$s_{\text{Tot}} = s_1 + s_2$$

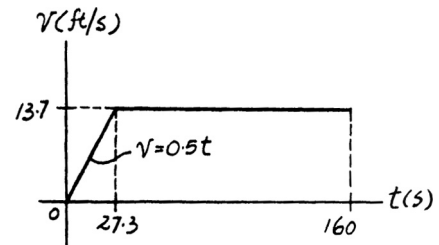
$$2000 = 0.25(t')^2 + 80t' - 0.5(t')^2$$

$$0.25(t')^2 - 80t' + 2000 = 0$$

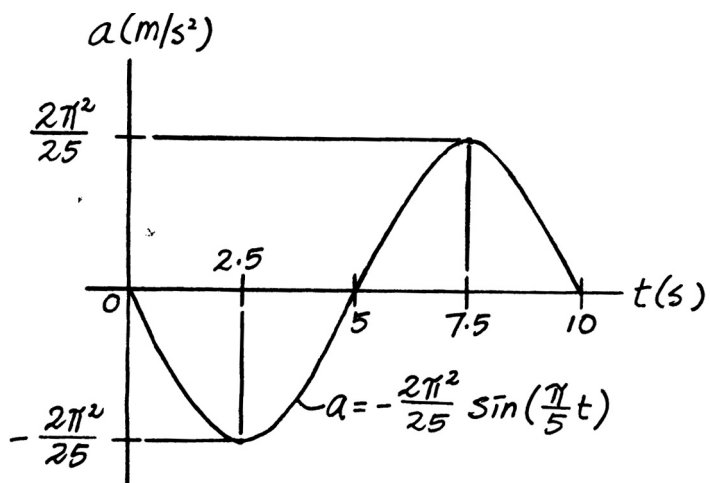
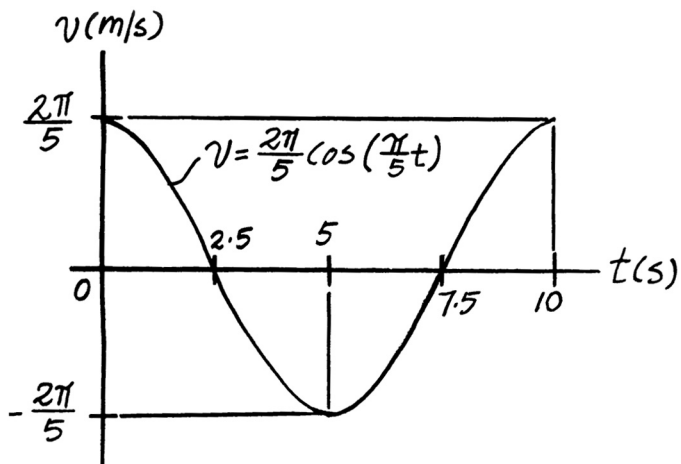
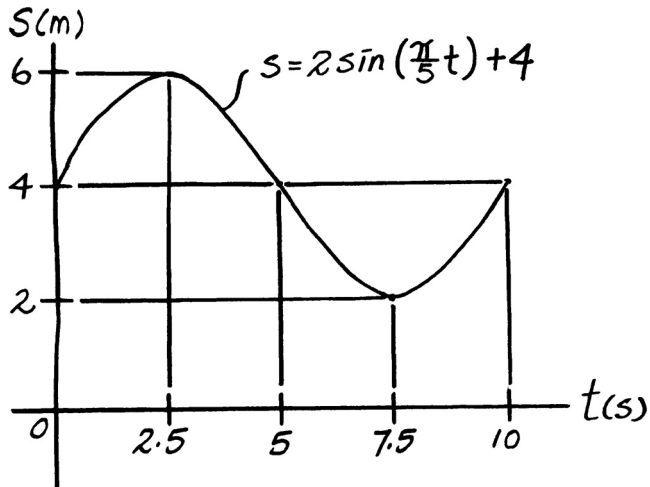
Choose a root that is less than 160 s, then

$$t' = 27.34 \text{ s} = 27.3 \text{ s} \quad \text{Ans.}$$

v – t Graph: The equation for the velocity is given by Eq. [1]. When $t = t' = 27.34 \text{ s}$, $v = 0.5(27.34) = 13.7 \text{ ft/s}$.



- 12–45. If the position of a particle is defined by $s = [2 \sin(\pi/5)t + 4]$ m, where t is in seconds, construct the s - t , v - t , and a - t graphs for $0 \leq t \leq 10$ s.



12–46. A train starts from station *A* and for the first kilometer, it travels with a uniform acceleration. Then, for the next two kilometers, it travels with a uniform speed. Finally, the train decelerates uniformly for another kilometer before coming to rest at station *B*. If the time for the whole journey is six minutes, draw the v - t graph and determine the maximum speed of the train.

For stage (1) motion,

$$\begin{aligned} (\pm) \quad v_1 &= v_0 + (a_c)_1 t \\ v_{\max} &= 0 + (a_c)_1 t_1 \\ v_{\max} &= (a_c)_1 t_1 \end{aligned} \quad (1)$$

$$\begin{aligned} (\pm) \quad v_1^2 &= v_0^2 + 2(a_c)_1(s_1 - s_0) \\ v_{\max}^2 &= 0 + 2(a_c)_1(1000 - 0) \\ (a_c)_1 &= \frac{v_{\max}^2}{2000} \end{aligned} \quad (2)$$

Eliminating $(a_c)_1$ from Eqs. (1) and (2), we have

$$t_1 = \frac{2000}{v_{\max}} \quad (3)$$

For stage (2) motion, the train travels with the constant velocity of v_{\max} for $t = (t_2 - t_1)$. Thus,

$$\begin{aligned} (\pm) \quad s_2 &= s_1 + v_1 t + \frac{1}{2}(a_c)_2 t^2 \\ 1000 + 2000 &= 1000 + v_{\max}(t_2 - t_1) + 0 \\ t_2 - t_1 &= \frac{2000}{v_{\max}} \end{aligned} \quad (4)$$

For stage (3) motion, the train travels for $t = 360 - t_2$. Thus,

$$\begin{aligned} (\pm) \quad v_3 &= v_2 + (a_c)_3 t \\ 0 &= v_{\max} - (a_c)_3(360 - t_2) \\ v_{\max} &= (a_c)_3(360 - t_2) \end{aligned} \quad (5)$$

$$\begin{aligned} (\pm) \quad v_3^2 &= v_2^2 + 2(a_c)_3(s_3 - s_2) \\ 0 &= v_{\max}^2 + 2[-(a_c)_3](4000 - 3000) \\ (a_c)_3 &= \frac{v_{\max}^2}{2000} \end{aligned} \quad (6)$$

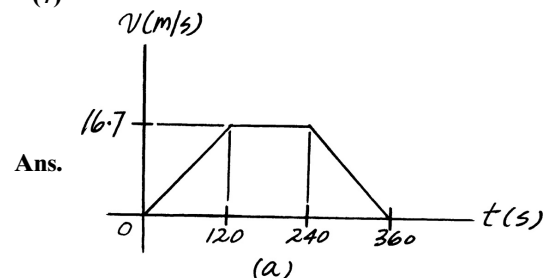
Eliminating $(a_c)_3$ from Eqs. (5) and (6) yields

$$360 - t_2 = \frac{2000}{v_{\max}} \quad (7)$$

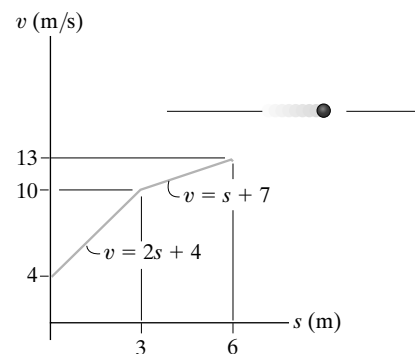
Solving Eqs. (3), (4), and (7), we have

$$\begin{aligned} t_1 &= 120 \text{ s} & t_2 &= 240 \text{ s} \\ v_{\max} &= 16.7 \text{ m/s} \end{aligned}$$

Based on the above results the v - t graph is shown in Fig. *a*.



12–47. The particle travels along a straight line with the velocity described by the graph. Construct the $a-s$ graph.



$a-s$ Graph: For $0 \leq s < 3$ m,

$$\left(\frac{\pm}{\rightarrow} \right) \quad a = v \frac{dv}{ds} = (2s + 4)(2) = (4s + 8) \text{ m/s}^2$$

At $s = 0$ m and 3 m,

$$a|_{s=0 \text{ m}} = 4(0) + 8 = 8 \text{ m/s}^2$$

$$a|_{s=3 \text{ m}} = 4(3) + 8 = 20 \text{ m/s}^2$$

For $3 \text{ m} < s \leq 6$ m,

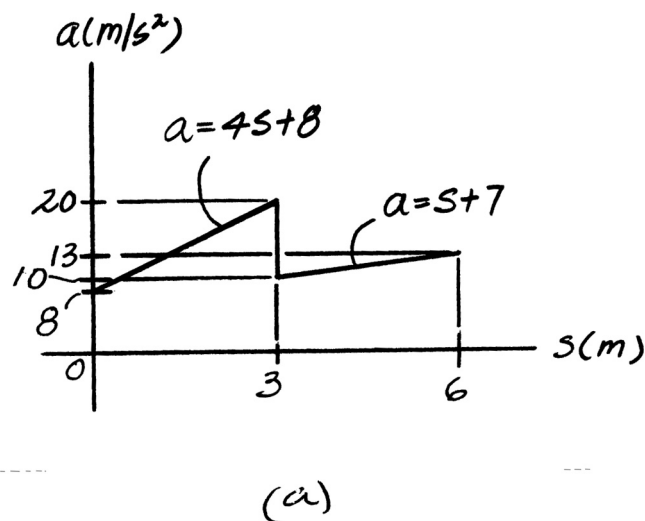
$$\left(\frac{\pm}{\rightarrow} \right) \quad a = v \frac{dv}{ds} = (s + 7)(1) = (s + 7) \text{ m/s}^2$$

At $s = 3$ m and 6 m,

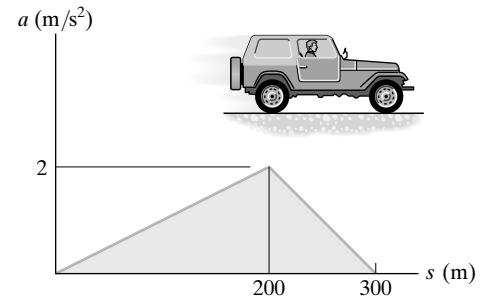
$$a|_{s=3 \text{ m}} = 3 + 7 = 10 \text{ m/s}^2$$

$$a|_{s=6 \text{ m}} = 6 + 7 = 13 \text{ m/s}^2$$

The $a-s$ graph is shown in Fig. a .



***12–48.** The a - s graph for a jeep traveling along a straight road is given for the first 300 m of its motion. Construct the v - s graph. At $s = 0$, $v = 0$.



a - s Graph: The function of acceleration a in terms of s for the interval $0 \text{ m} \leq s < 200 \text{ m}$ is

$$\frac{a - 0}{s - 0} = \frac{2 - 0}{200 - 0} \quad a = (0.01s) \text{ m/s}^2$$

For the interval $200 \text{ m} < s \leq 300 \text{ m}$,

$$\frac{a - 2}{s - 200} = \frac{0 - 2}{300 - 200} \quad a = (-0.02s + 6) \text{ m/s}^2$$

v - s Graph: The function of velocity v in terms of s can be obtained by applying $v dv = a ds$. For the interval $0 \text{ m} \leq s < 200 \text{ m}$,

$$v dv = ds$$

$$\int_0^v v dv = \int_0^s 0.01s ds$$

$$v = (0.1s) \text{ m/s}$$

At $s = 200 \text{ m}$, $v = 0.100(200) = 20.0 \text{ m/s}$

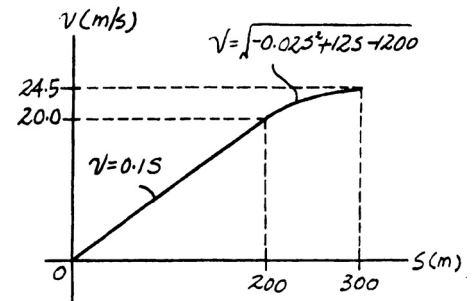
For the interval $200 \text{ m} < s \leq 300 \text{ m}$,

$$v dv = a ds$$

$$\int_{20.0 \text{ m/s}}^v v dv = \int_{200 \text{ m}}^s (-0.02s + 6) ds$$

$$v = \left(\sqrt{-0.02s^2 + 12s - 1200} \right) \text{ m/s}$$

At $s = 300 \text{ m}$, $v = \sqrt{-0.02(300^2) + 12(300) - 1200} = 24.5 \text{ m/s}$



•12-49. A particle travels along a curve defined by the equation $s = (t^3 - 3t^2 + 2t)$ m. where t is in seconds. Draw the $s - t$, $v - t$, and $a - t$ graphs for the particle for $0 \leq t \leq 3$ s.

$$s = t^3 - 3t^2 + 2t$$

$$v = \frac{ds}{dt} = 3t^2 - 6t + 2$$

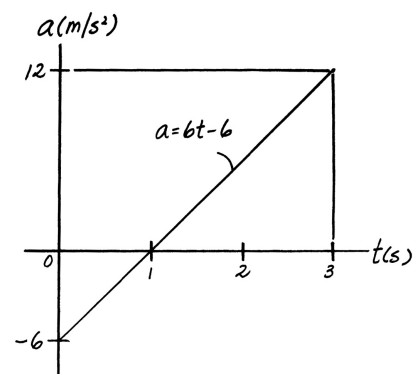
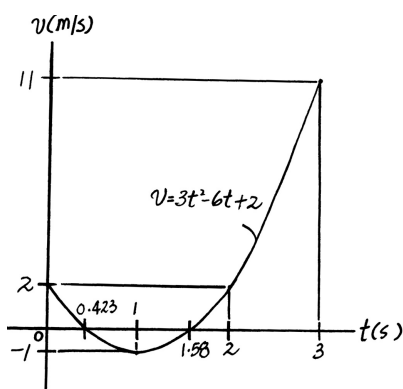
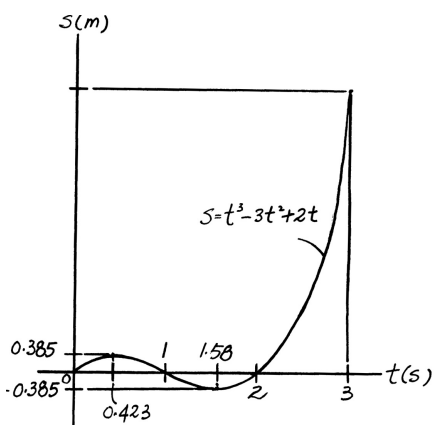
$$a = \frac{dv}{dt} = 6t - 6$$

$$v = 0 \text{ at } 0 = 3t^2 - 6t + 2$$

$$t = 1.577 \text{ s, and } t = 0.4226 \text{ s,}$$

$$s|_{t=1.577} = -0.386 \text{ m}$$

$$s|_{t=0.4226} = 0.385 \text{ m}$$



12-50. A truck is traveling along the straight line with a velocity described by the graph. Construct the $a - s$ graph for $0 \leq s \leq 1500$ ft.

$a - s$ Graph: For $0 \leq s < 625$ ft,

$$\left(\pm \right) \quad a = v \frac{dv}{ds} = (0.6s^{3/4}) \left[\frac{3}{4} (0.6)s^{-1/4} \right] = (0.27s^{1/2}) \text{ ft/s}^2$$

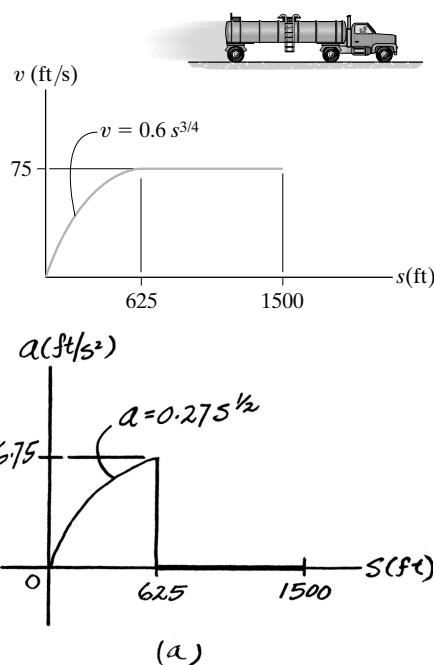
At $s = 625$ ft,

$$a|_{s=625 \text{ ft}} = 0.27(625^{1/2}) = 6.75 \text{ ft/s}^2$$

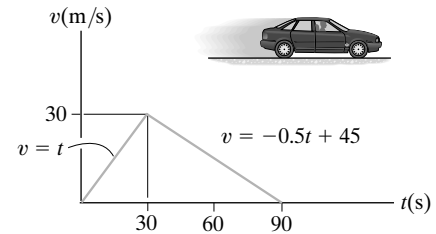
For $625 \text{ ft} < s < 1500$ ft,

$$\left(\pm \right) \quad a = v \frac{dv}{ds} = 75(0) = 0$$

The $a - s$ graph is shown in Fig. a.



12–51. A car starts from rest and travels along a straight road with a velocity described by the graph. Determine the total distance traveled until the car stops. Construct the s – t and a – t graphs.



s – t Graph: For the time interval $0 \leq t < 30$ s, the initial condition is $s = 0$ when $t = 0$ s.

$$\begin{aligned} (\pm) \quad ds &= v dt \\ \int_0^s ds &= \int_0^t t dt \\ s &= \left(\frac{t^2}{2} \right) \text{ m} \end{aligned}$$

When $t = 30$ s,

$$s = \frac{30^2}{2} = 450 \text{ m}$$

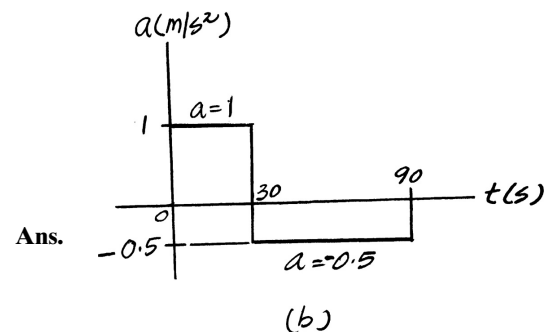
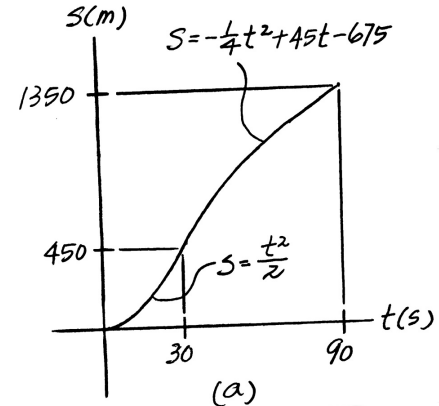
For the time interval $30 \text{ s} < t \leq 90$ s, the initial condition is $s = 450$ m when $t = 30$ s.

$$\begin{aligned} (\pm) \quad ds &= v dt \\ \int_{450 \text{ m}}^s ds &= \int_{30 \text{ s}}^t (-0.5t + 45) dt \\ s &= \left(-\frac{1}{4}t^2 + 45t - 675 \right) \text{ m} \end{aligned}$$

When $t = 90$ s,

$$s|_{t=90 \text{ s}} = -\frac{1}{4}(90^2) + 45(90) - 675 = 1350 \text{ m}$$

The s – t graph shown is in Fig. *a*.



a – t Graph: For the time interval $0 < t < 30$ s,

$$a = \frac{dv}{dt} = \frac{d}{dt}(t) = 1 \text{ m/s}^2$$

For the time interval $30 \text{ s} < t \leq 90$ s,

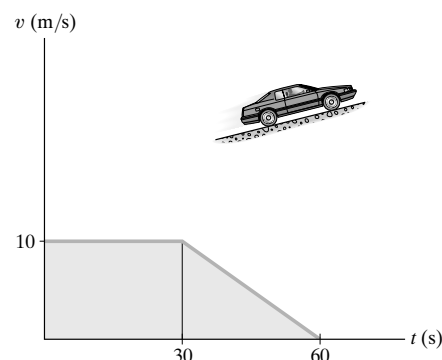
$$a = \frac{dv}{dt} = \frac{d}{dt}(-0.5t + 45) = -0.5 \text{ m/s}^2$$

The a – t graph is shown in Fig. *b*.

Note: Since the change in position of the car is equal to the area under the v – t graph, the total distance traveled by the car is

$$\begin{aligned} \Delta s &= \int v dt \\ s|_{t=90 \text{ s}} - 0 &= \frac{1}{2}(90)(30) \\ s|_{t=90 \text{ s}} &= 1350 \text{ m} \end{aligned}$$

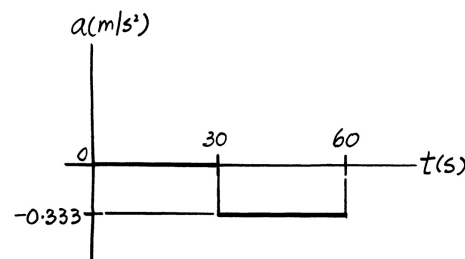
***12–52.** A car travels up a hill with the speed shown. Determine the total distance the car travels until it stops ($t = 60$ s). Plot the a – t graph.



Distance traveled is area under v – t graph.

$$s = (10)(30) + \frac{1}{2}(10)(30) = 450 \text{ m}$$

Ans.



•12–53. The snowmobile moves along a straight course according to the v – t graph. Construct the s – t and a – t graphs for the same 50-s time interval. When $t = 0$, $s = 0$.

s – t Graph: The position function in terms of time t can be obtained by applying $v = \frac{ds}{dt}$. For time interval $0 \leq t < 30$ s, $v = \frac{12}{30}t = \left(\frac{2}{5}t\right)$ m/s.

$$ds = v dt$$

$$\int_0^s ds = \int_0^t \frac{2}{5} t dt$$

$$s = \left(\frac{1}{5}t^2\right) \text{ m}$$

At $t = 30$ s, $s = \frac{1}{5}(30^2) = 180 \text{ m}$

For time interval $30 \text{ s} < t \leq 50 \text{ s}$,

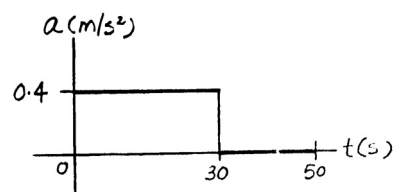
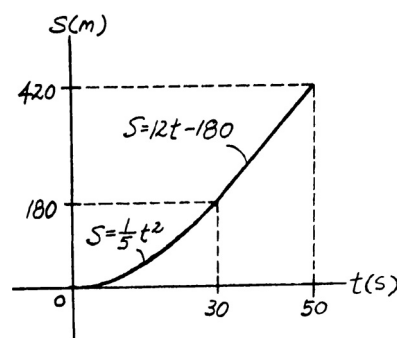
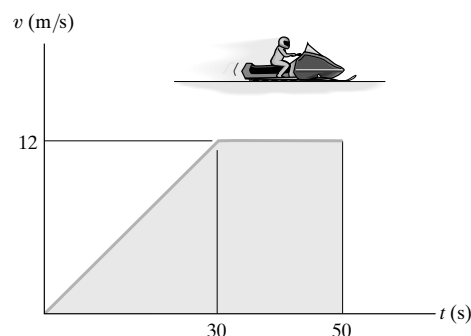
$$ds = v dt$$

$$\int_{180 \text{ m}}^s ds = \int_{30 \text{ s}}^t 12 dt$$

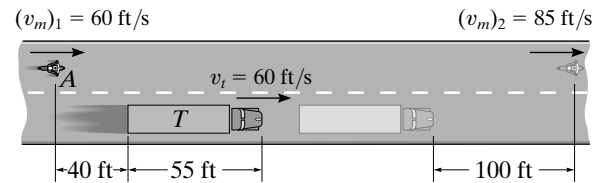
$$s = (12t - 180) \text{ m}$$

At $t = 50$ s, $s = 12(50) - 180 = 420 \text{ m}$

a – t Graph: The acceleration function in terms of time t can be obtained by applying $a = \frac{dv}{dt}$. For time interval $0 \text{ s} \leq t < 30 \text{ s}$ and $30 \text{ s} < t \leq 50 \text{ s}$, $a = \frac{dv}{dt} = \frac{2}{5} = 0.4 \text{ m/s}^2$ and $a = \frac{dv}{dt} = 0$, respectively.



12-54. A motorcyclist at A is traveling at 60 ft/s when he wishes to pass the truck T which is traveling at a constant speed of 60 ft/s. To do so the motorcyclist accelerates at 6 ft/s^2 until reaching a maximum speed of 85 ft/s. If he then maintains this speed, determine the time needed for him to reach a point located 100 ft in front of the truck. Draw the $v-t$ and $s-t$ graphs for the motorcycle during this time.



Motorcycle:

Time to reach 85 ft/s,

$$v = v_0 + a_c t$$

$$85 = 60 + 6t$$

$$t = 4.167 \text{ s}$$

$$v^2 = v_0^2 + 2a_c (s - s_0)$$

Distance traveled,

$$(85)^2 = (60)^2 + 2(6)(s_m - 0)$$

$$s_m = 302.08 \text{ ft}$$

In $t = 4.167 \text{ s}$, truck travels

$$s_t = 60(4.167) = 250 \text{ ft}$$

Further distance for motorcycle to travel: $40 + 55 + 250 + 100 - 302.08 = 142.92 \text{ ft}$

Motorcycle:

$$s = s_0 + v_0 t$$

$$(s + 142.92) = 0 + 85t'$$

Truck:

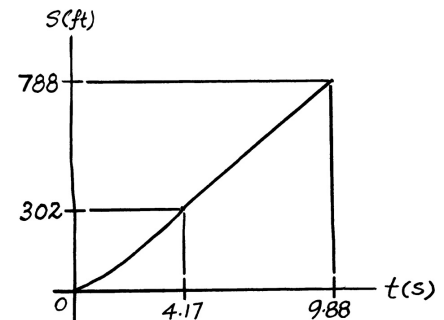
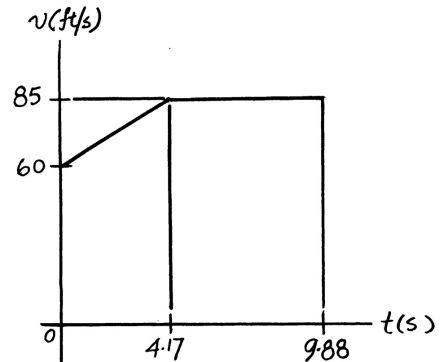
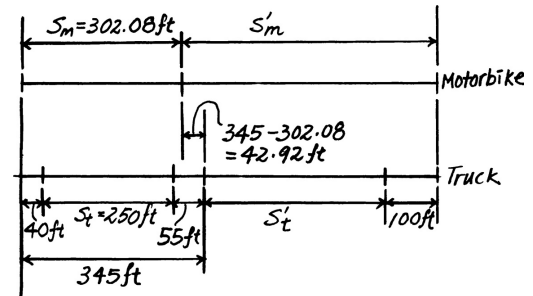
$$s = 0 + 60t'$$

Thus $t' = 5.717 \text{ s}$

$$t = 4.167 + 5.717 = 9.88 \text{ s}$$

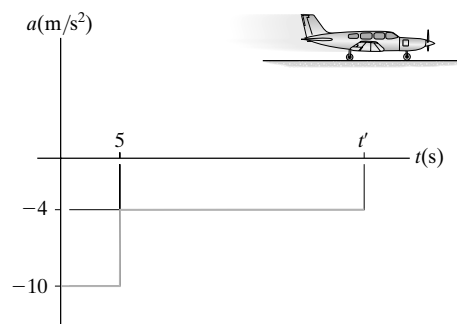
Total distance motorcycle travels

$$s_T = 302.08 + 85(5.717) = 788 \text{ ft}$$



Ans.

12-55. An airplane traveling at 70 m/s lands on a straight runway and has a deceleration described by the graph. Determine the time t' and the distance traveled for it to reach a speed of 5 m/s. Construct the $v-t$ and $s-t$ graphs for this time interval, $0 \leq t \leq t'$.



$v-t$ Graph: For the time interval $0 \leq t < 5$ s, the initial condition is $v = 70$ m/s when $t = 0$ s.

$$\begin{aligned} (\pm) \quad dv &= a dt \\ \int_{70 \text{ m/s}}^v dv &= \int_0^t -10 dt \\ v &= (-10t + 70) \text{ m/s} \end{aligned}$$

When $t = 5$ s,

$$v|_{t=5 \text{ s}} = -10(5) + 70 = 20 \text{ m/s}$$

For the time interval $5 \text{ s} < t \leq t'$, the initial condition is $v = 20$ m/s when $t = 5$ s.

$$\begin{aligned} (\pm) \quad dv &= a dt \\ \int_{20 \text{ m/s}}^v dv &= \int_5^t -4 dt \\ v &= (-4t + 40) \text{ m/s} \end{aligned}$$

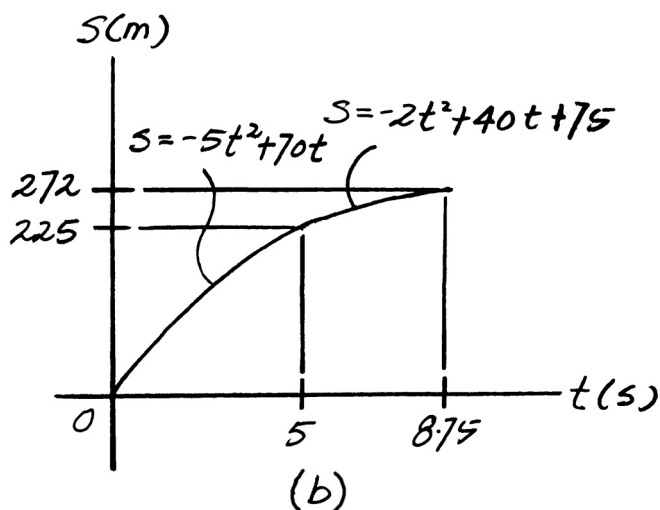
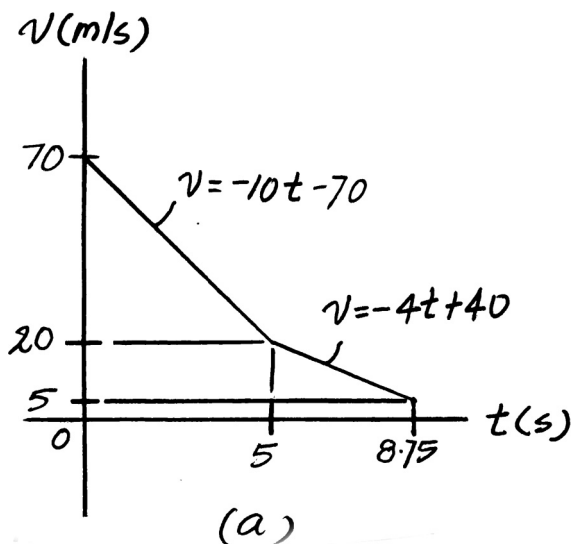
When $v = 5$ m/s,

$$5 = -4t' + 40 \quad t' = 8.75 \text{ s} \quad \text{Ans.}$$

Also, the change in velocity is equal to the area under the $a-t$ graph. Thus,

$$\begin{aligned} \Delta v &= \int a dt \\ 5 - 70 &= -[5(10) + 4(t' - 5)] \\ t' &= 8.75 \text{ s} \end{aligned}$$

The $v-t$ graph is shown in Fig. a.



12–55. Continued

$s-t$ Graph: For the time interval $0 \leq t < 5$ s, the initial condition is $s = 0$ when $t = 0$ s.

$$\begin{aligned} \left(\begin{array}{l} \pm \\ \rightarrow \end{array} \right) \quad ds &= v dt \\ \int_0^s ds &= \int_0^t (-10t + 70) dt \\ s &= (-5t^2 + 70t) \text{ m} \end{aligned}$$

When $t = 5$ s,

$$s|_{t=5 \text{ s}} = -5(5^2) + 70(5) = 225 \text{ m}$$

For the time interval $5 < t \leq t' = 8.75$ s the initial condition is $s = 225$ m when $t = 5$ s.

$$\begin{aligned} \left(\begin{array}{l} \pm \\ \rightarrow \end{array} \right) \quad ds &= v dt \\ \int_{225 \text{ m}}^s ds &= \int_5^{t'} (-4t + 40) dt \\ s &= (-2t^2 + 40t + 75) \text{ m} \end{aligned}$$

When $t = t' = 8.75$ s,

$$s|_{t=8.75 \text{ s}} = -2(8.75^2) + 40(8.75) + 75 = 271.875 \text{ m} = 272 \text{ m} \quad \textbf{Ans.}$$

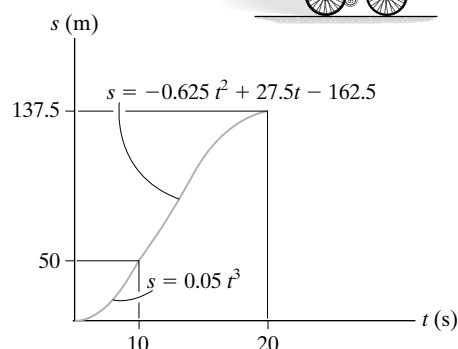
Also, the change in position is equal to the area under the $v-t$ graph. Referring to Fig. *a*, we have

$$\Delta s = \int v dt$$

$$s|_{t=8.75 \text{ s}} - 0 = \frac{1}{2} (70 + 20)(5) + \frac{1}{2} (20 + 5)(3.75) = 271.875 \text{ m} = 272 \text{ m} \quad \textbf{Ans.}$$

The $s-t$ graph is shown in Fig. *b*.

***12–56.** The position of a cyclist traveling along a straight road is described by the graph. Construct the v - t and a - t graphs.



v - t Graph: For the time interval $0 \leq t < 10$ s,

$$\left(\pm \right) \quad v = \frac{ds}{dt} = \frac{d}{dt}(0.05t^3) = (0.15t^2) \text{ m/s}$$

When $t = 0$ s and 10 s,

$$v|_{t=0} = 0.15(0^2) = 0 \quad v|_{t=10\text{ s}} = 0.15(10^2) = 15 \text{ m/s}$$

For the time interval $10 \text{ s} < t \leq 20$ s,

$$\left(\pm \right) \quad v = \frac{ds}{dt} = \frac{d}{dt}(-0.625t^2 + 27.5t - 162.5) = (-1.25t + 27.5) \text{ m/s}$$

When $t = 10$ s and 20 s,

$$v|_{t=10\text{ s}} = -1.25(10) + 27.5 = 15 \text{ m/s}$$

$$v|_{t=20\text{ s}} = -1.25(20) + 27.5 = 2.5 \text{ m/s}$$

The v - t graph is shown in Fig. *a*.

a - t Graph: For the time interval $0 \leq t < 10$ s,

$$\left(\pm \right) \quad a = \frac{dv}{dt} = \frac{d}{dt}(0.15t^2) = (0.3t) \text{ m/s}^2$$

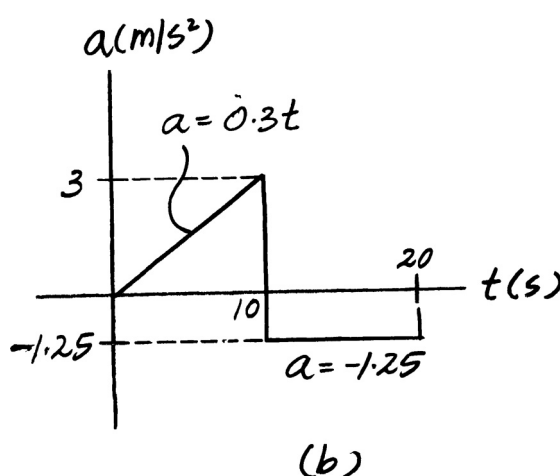
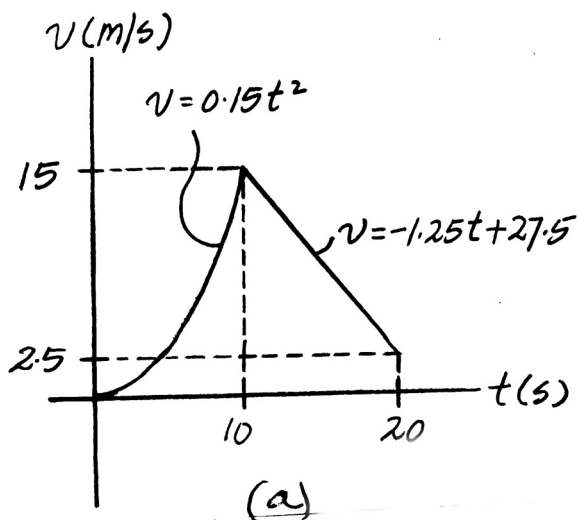
When $t = 0$ s and 10 s,

$$a|_{t=0\text{ s}} = 0.3(0) = 0 \quad a|_{t=10\text{ s}} = 0.3(10) = 3 \text{ m/s}^2$$

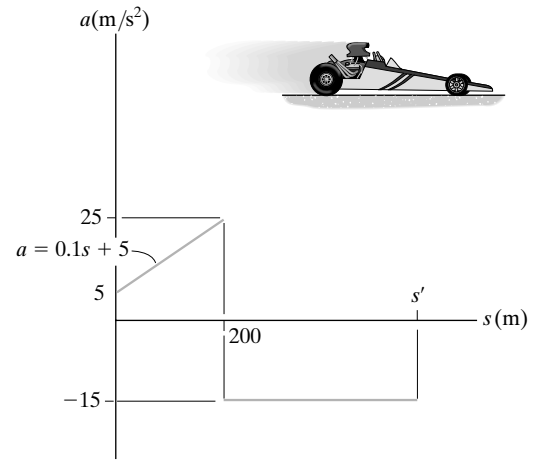
For the time interval $10 \text{ s} < t \leq 20$ s,

$$\left(\pm \right) \quad a = \frac{dv}{dt} = \frac{d}{dt}(-1.25t + 27.5) = -1.25 \text{ m/s}^2$$

the a - t graph is shown in Fig. *b*.



•12–57. The dragster starts from rest and travels along a straight track with an acceleration-deceleration described by the graph. Construct the v – s graph for $0 \leq s \leq s'$, and determine the distance s' traveled before the dragster again comes to rest.



v – s Graph: For $0 \leq s < 200$ m, the initial condition is $v = 0$ at $s = 0$.

$$\begin{aligned} (\pm) \quad v dv &= a ds \\ \int_0^v v dv &= \int_0^s (0.1s + 5) ds \\ \frac{v^2}{2} \Big|_0^v &= (0.05s^2 + 5s) \Big|_0^s \\ v &= \left(\sqrt{0.1s^2 + 10s} \right) \text{ m/s} \end{aligned}$$

At $s = 200$ m,

$$v|_{s=200 \text{ m}} = \sqrt{0.1(200^2) + 10(200)} = 77.46 \text{ m/s} = 77.5 \text{ m/s}$$

For $200 \text{ m} < s \leq s'$, the initial condition is $v = 77.46 \text{ m/s}$ at $s = 200$ m.

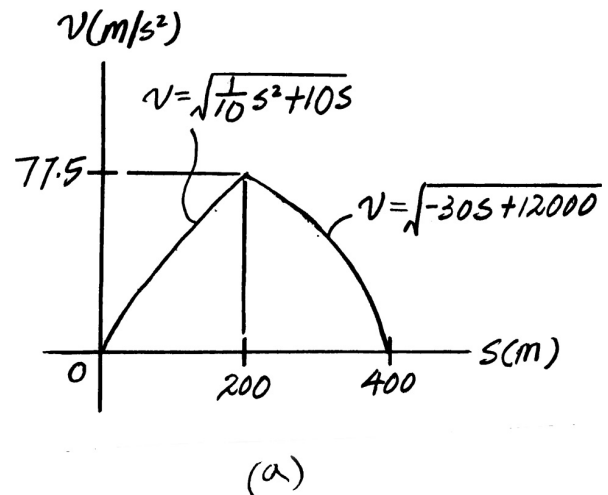
$$\begin{aligned} (\pm) \quad v dv &= a ds \\ \int_{77.46 \text{ m/s}}^v v dv &= \int_{200 \text{ m}}^s -15 ds \\ \frac{v^2}{2} \Big|_{77.46 \text{ m/s}}^v &= -15s \Big|_{200 \text{ m}}^s \\ v &= \left(\sqrt{-30s + 12000} \right) \text{ m/s} \end{aligned}$$

When $v = 0$,

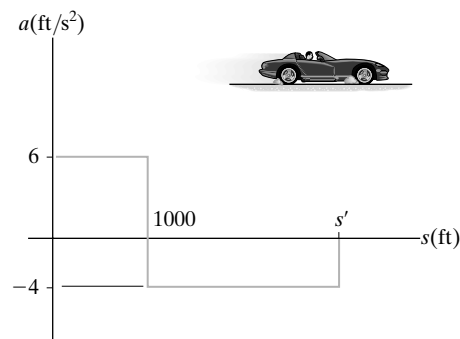
$$0 = \sqrt{-30s' + 12000} \quad s' = 400 \text{ m}$$

Ans.

The v – s graph is shown in Fig. *a*.



12–58. A sports car travels along a straight road with an acceleration-deceleration described by the graph. If the car starts from rest, determine the distance s' the car travels until it stops. Construct the $v-s$ graph for $0 \leq s \leq s'$.



$v-s$ Graph: For $0 \leq s < 1000$ ft, the initial condition is $v = 0$ at $s = 0$.

$$\begin{aligned} (\pm) \quad v dv &= a ds \\ \int_0^v v dv &= \int_0^s 6 ds \\ \frac{v^2}{2} &= 6s \\ v &= (\sqrt{12s^{1/2}}) \text{ ft/s} \end{aligned}$$

When $s = 1000$ ft,

$$v = \sqrt{12(1000)^{1/2}} = 109.54 \text{ ft/s} = 110 \text{ ft/s}$$

For $1000 \text{ ft} < s \leq s'$, the initial condition is $v = 109.54 \text{ ft/s}$ at $s = 1000$ ft.

$$\begin{aligned} (\pm) \quad v dv &= a ds \\ \int_{109.54 \text{ ft/s}}^v v dv &= \int_{1000 \text{ ft}}^s -4 ds \\ \frac{v^2}{2} \Big|_{109.54 \text{ ft/s}}^v &= -4s \Big|_{1000 \text{ ft}}^s \\ v &= (\sqrt{20\,000 - 8s}) \text{ ft/s} \end{aligned}$$

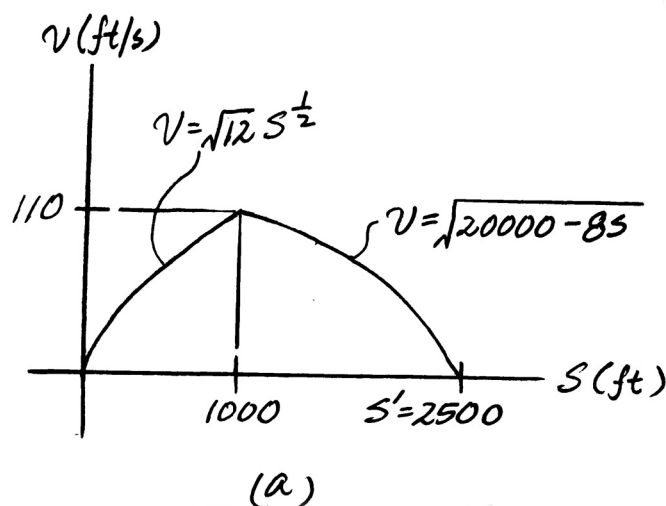
When $v = 0$,

$$0 = \sqrt{20\,000 - 8s'}$$

$$s' = 2500 \text{ ft}$$

Ans.

The $v-s$ graph is shown in Fig. a .



12–59. A missile starting from rest travels along a straight track and for 10 s has an acceleration as shown. Draw the v - t graph that describes the motion and find the distance traveled in 10 s.

For $t \leq 5$ s,

$$\begin{aligned} a &= 6t \\ dv &= a \, dt \\ \int_0^v dv &= \int_0^t 6t \, dt \\ v &= 3t^2 \end{aligned}$$

When $t = 5$ s,

$$v = 75 \text{ m/s}$$

For $5 < t < 10$ s,

$$\begin{aligned} a &= 2t + 20 \\ dv &= a \, dt \\ \int_{75}^v dv &= \int_5^t (2t + 20) \, dt \\ v - 75 &= t^2 + 20t - 125 \\ v &= t^2 + 20t - 50 \end{aligned}$$

When $t = 10$ s,

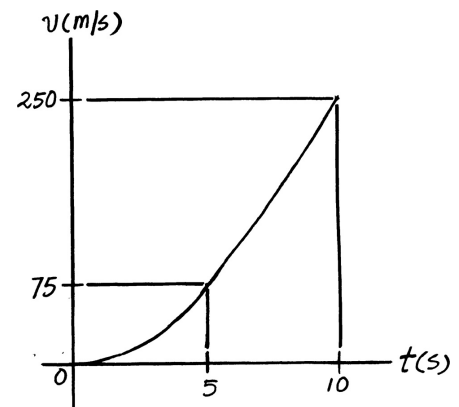
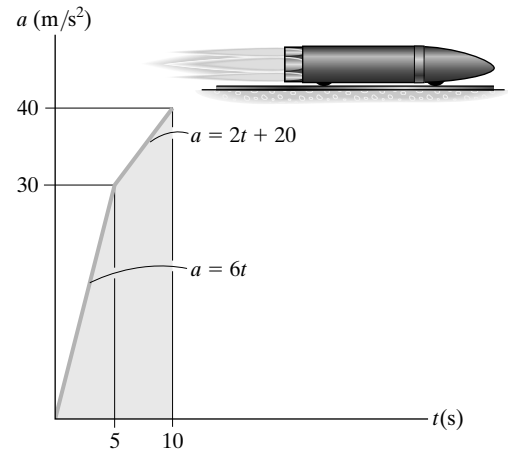
$$v = 250 \text{ m/s}$$

Distance at $t = 5$ s:

$$\begin{aligned} ds &= v \, dt \\ \int_0^s ds &= \int_0^5 3t^2 \, dt \\ s &= (5)^3 = 125 \text{ m} \end{aligned}$$

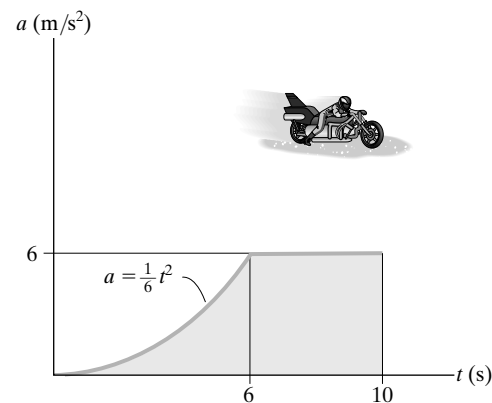
Distance at $t = 10$ s:

$$\begin{aligned} ds &= v \, dv \\ \int_{125}^s ds &= \int_5^{10} (t^2 + 20t - 50) \, dt \\ s - 125 &= \left[\frac{1}{3}t^3 + 10t^2 - 50t \right]_5^{10} \\ s &= 917 \text{ m} \end{aligned}$$



Ans.

***12–60.** A motorcyclist starting from rest travels along a straight road and for 10 s has an acceleration as shown. Draw the v – t graph that describes the motion and find the distance traveled in 10 s.



For $0 \leq t < 6$ $dv = a dt$

$$\int_0^v dv = \int_0^t \frac{1}{6} t^2 dt$$

$$v = \frac{1}{18} t^3$$

$$ds = v dt$$

$$\int_0^s ds = \int_0^t \frac{1}{18} t^3 dt$$

$$s = \frac{1}{72} t^4$$

When $t = 6$ s, $v = 12$ m/s $s = 18$ m

For $6 < t \leq 10$ $dv = a dt$

$$\int_{12}^v dv = \int_6^t 6 dt$$

$$v = 6t - 24$$

$$ds = v dt$$

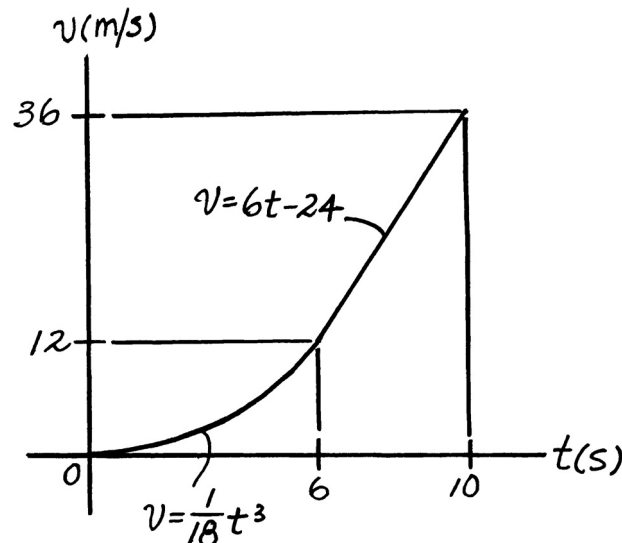
$$\int_{18}^s ds = \int_6^t (6t - 24) dt$$

$$s = 3t^2 - 24t + 54$$

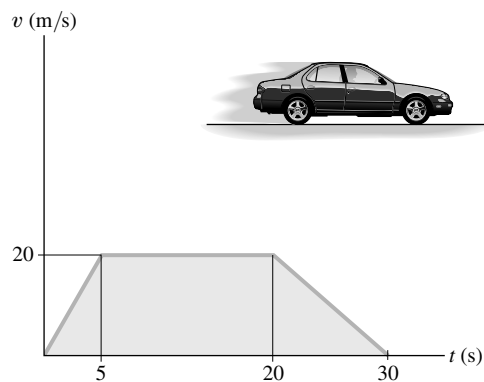
When $t = 10$ s, $v = 36$ m/s

$$s = 114 \text{ m}$$

Ans.



•12–61. The v – t graph of a car while traveling along a road is shown. Draw the s – t and a – t graphs for the motion.



$$0 \leq t \leq 5 \quad a = \frac{\Delta v}{\Delta t} = \frac{20}{5} = 4 \text{ m/s}^2$$

$$5 \leq t \leq 20 \quad a = \frac{\Delta v}{\Delta t} = \frac{20 - 20}{20 - 5} = 0 \text{ m/s}^2$$

$$20 \leq t \leq 30 \quad a = \frac{\Delta v}{\Delta t} = \frac{0 - 20}{30 - 20} = -2 \text{ m/s}^2$$

From the v – t graph at $t_1 = 5 \text{ s}$, $t_2 = 20 \text{ s}$, and $t_3 = 30 \text{ s}$,

$$s_1 = A_1 = \frac{1}{2}(5)(20) = 50 \text{ m}$$

$$s_2 = A_1 + A_2 = 50 + 20(20 - 5) = 350 \text{ m}$$

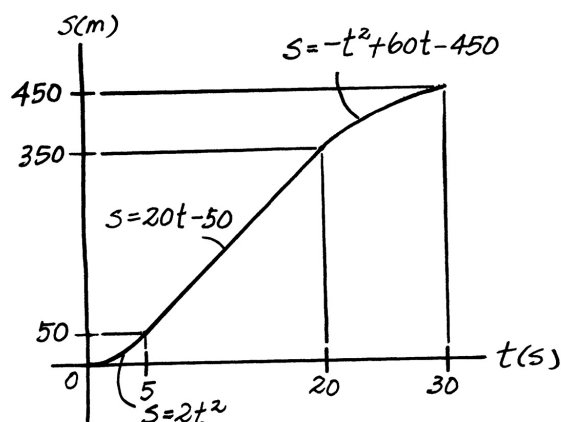
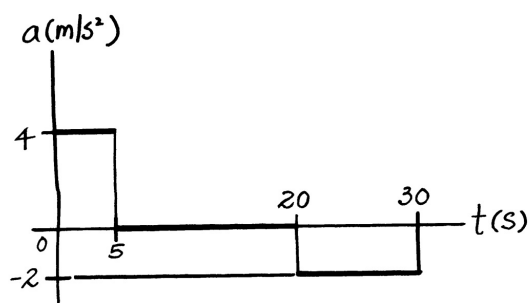
$$s_3 = A_1 + A_2 + A_3 = 350 + \frac{1}{2}(30 - 20)(20) = 450 \text{ m}$$

The equations defining the portions of the s – t graph are

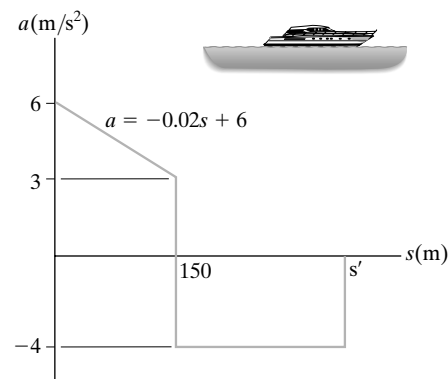
$$0 \leq t \leq 5 \text{ s} \quad v = 4t; \quad ds = v dt; \quad \int_0^s ds = \int_0^t 4t dt; \quad s = 2t^2$$

$$5 \leq t \leq 20 \text{ s} \quad v = 20; \quad ds = v dt; \quad \int_{50}^s ds = \int_5^t 20 dt; \quad s = 20t - 50$$

$$20 \leq t \leq 30 \text{ s} \quad v = 2(30 - t); \quad ds = v dt; \quad \int_{350}^s ds = \int_{20}^t 2(30 - t) dt; \quad s = -t^2 + 60t - 450$$



12–62. The boat travels in a straight line with the acceleration described by the $a-s$ graph. If it starts from rest, construct the $v-s$ graph and determine the boat's maximum speed. What distance s' does it travel before it stops?



$v-s$ Graph: For $0 \leq s < 150$ m, the initial condition is $v = 0$ at $s = 0$.

$$\begin{aligned} (\pm) \quad v dv &= a ds \\ \int_0^v v dv &= \int_0^s (-0.02s + 6) ds \\ \frac{v^2}{2} \Big|_0^v &= (-0.01s^2 + 6s) \Big|_0^s \\ v &= \left(\sqrt{-0.02s^2 + 12s} \right) \text{ m/s} \end{aligned}$$

The maximum velocity of the boat occurs at $s = 150$ m, where its acceleration changes sign. Thus,

$$v_{\max} = v|_{s=150 \text{ m}} = \sqrt{-0.02(150^2) + 12(150)} = 36.74 \text{ m/s} = 36.7 \text{ m/s} \quad \text{Ans.}$$

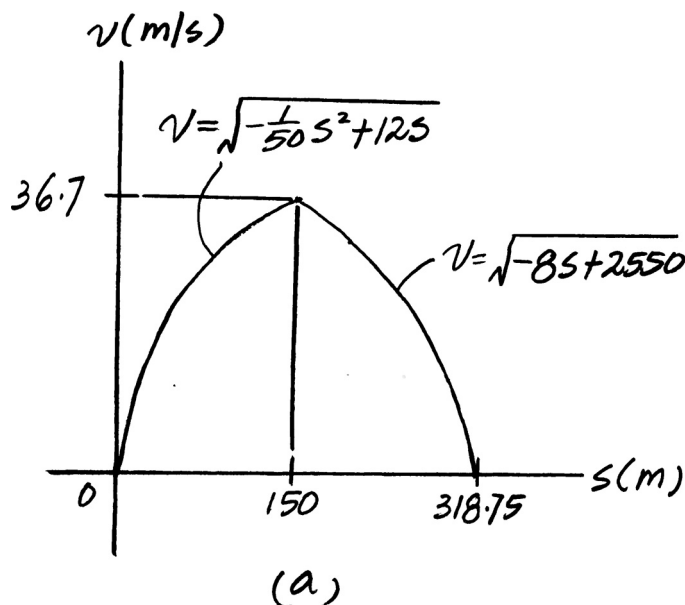
For $150 \text{ m} < s < s'$, the initial condition is $v = 36.74$ m/s at $s = 150$ m.

$$\begin{aligned} (\pm) \quad v dv &= a ds \\ \int_{36.74 \text{ m/s}}^v v dv &= \int_{150 \text{ m}}^s -4 ds \\ \frac{v^2}{2} \Big|_{36.74 \text{ m/s}}^v &= -4s \Big|_{150 \text{ m}}^s \\ v &= \sqrt{-8s + 2550} \text{ m/s} \end{aligned}$$

Thus, when $v = 0$,

$$0 = \sqrt{-8s' + 2550} \quad s' = 318.7 \text{ m} = 319 \text{ m} \quad \text{Ans.}$$

The $v-s$ graph is shown in Fig. a .



12–63. The rocket has an acceleration described by the graph. If it starts from rest, construct the v - t and s - t graphs for the motion for the time interval $0 \leq t \leq 14$ s.

v - t Graph: For the time interval $0 \leq t < 9$ s, the initial condition is $v = 0$ at $s = 0$.

$$\begin{aligned} (+\uparrow) \quad dv &= a dt \\ \int_0^v dv &= \int_0^t 6t^{1/2} dt \\ v &= (4t^{3/2}) \text{ m/s} \end{aligned}$$

When $t = 9$ s,

$$v|_{t=9 \text{ s}} = 4(9^{3/2}) = 108 \text{ m/s}$$

The initial condition is $v = 108$ m/s at $t = 9$ s.

$$\begin{aligned} (+\uparrow) \quad dv &= a dt \\ \int_{108 \text{ m/s}}^v dv &= \int_{9 \text{ s}}^t (4t - 18) dt \\ v &= (2t^2 - 18t + 108) \text{ m/s} \end{aligned}$$

When $t = 14$ s,

$$v|_{t=14 \text{ s}} = 2(14^2) - 18(14) + 108 = 248 \text{ m/s}$$

The v - t graph is shown in Fig. *a*.

s - t Graph: For the time interval $0 \leq t < 9$ s, the initial condition is $s = 0$ when $t = 0$.

$$\begin{aligned} (+\uparrow) \quad ds &= v dt \\ \int_0^s ds &= \int_0^t 4t^{3/2} dt \\ s &= \frac{8}{5} t^{5/2} \end{aligned}$$

When $t = 9$ s,

$$s|_{t=9 \text{ s}} = \frac{8}{5} (9^{5/2}) = 388.8 \text{ m}$$

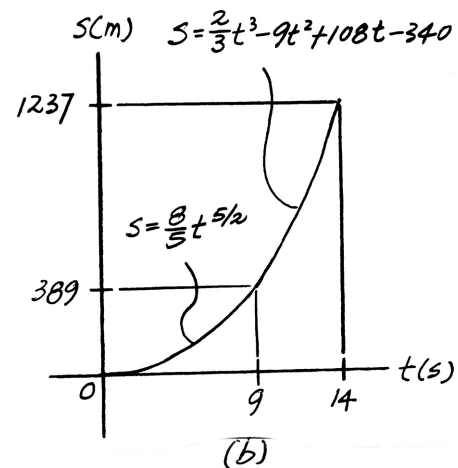
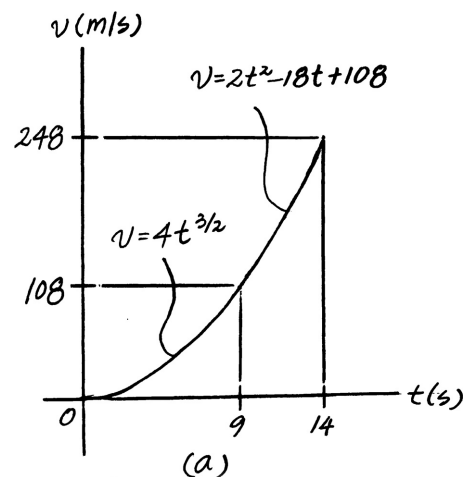
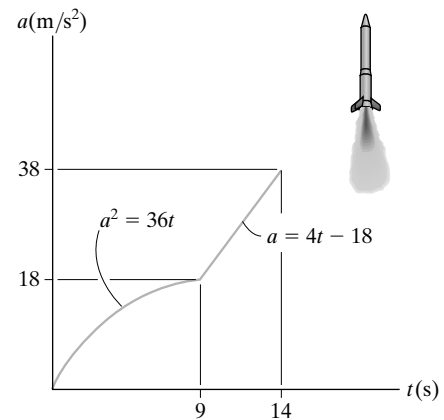
For the time interval $9 \text{ s} < t \leq 14$ s, the initial condition is $s = 388.8$ m when $t = 9$ s.

$$\begin{aligned} (+\uparrow) \quad ds &= v dt \\ \int_{388.8 \text{ m}}^s ds &= \int_{9 \text{ s}}^t (2t^2 - 18t + 108) dt \\ s &= \left(\frac{2}{3} t^3 - 9t^2 + 108t - 340.2 \right) \text{ m} \end{aligned}$$

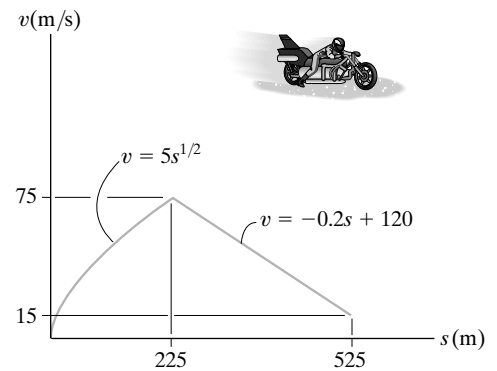
When $t = 14$ s,

$$s|_{t=14 \text{ s}} = \frac{2}{3} (14^3) - 9(14^2) + 108(14) - 340.2 = 1237 \text{ m}$$

The s - t graph is shown in Fig. *b*.



***12-64.** The jet bike is moving along a straight road with the speed described by the $v-s$ graph. Construct the $a-s$ graph.



$a-s$ Graph: For $0 \leq s < 225$ m,

$$\left(\pm \right) \quad a = v \frac{dv}{ds} = (5s^{1/2}) \left(\frac{5}{2} s^{-1/2} \right) = 12.5 \text{ m/s}^2$$

For $225 \text{ m} < s \leq 525$ m,

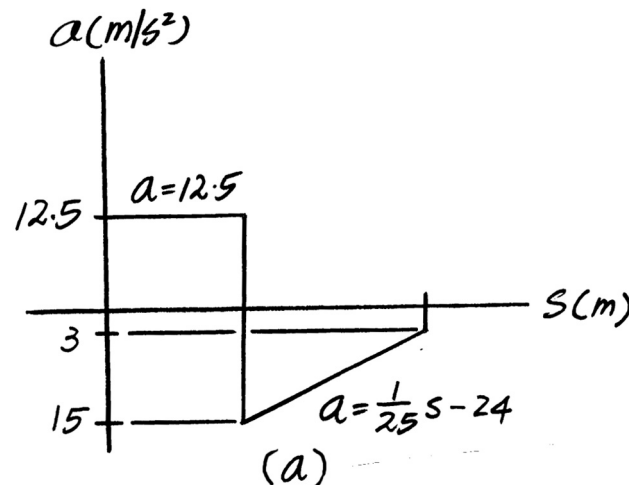
$$\left(\pm \right) \quad a = v \frac{dv}{ds} = (-0.2s + 120)(-0.2) = (0.04s - 24) \text{ m/s}^2$$

At $s = 225$ m and 525 m,

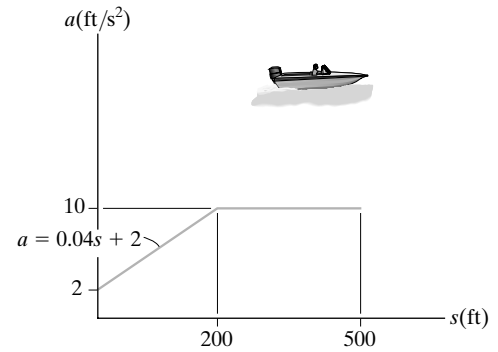
$$a|_{s=225 \text{ m}} = 0.04(225) - 24 = -15 \text{ m/s}^2$$

$$a|_{s=525 \text{ m}} = 0.04(525) - 24 = -3 \text{ m/s}^2$$

The $a-s$ graph is shown in Fig. a .



•12–65. The acceleration of the speed boat starting from rest is described by the graph. Construct the v – s graph.



v – s Graph: For $0 \leq s < 200$ ft, the initial condition is $v = 0$ at $s = 0$.

$$\begin{aligned} (\pm) \quad v dv &= a ds \\ \int_0^v v dv &= \int_0^s (0.04s + 2) ds \\ \frac{v^2}{2} \Big|_0^v &= 0.02s^2 + 2s \Big|_0^s \\ v &= \sqrt{0.04s^2 + 4s} \text{ ft/s} \end{aligned}$$

At $s = 200$ ft,

$$v|_{s=200 \text{ ft}} = \sqrt{0.04(200^2) + 4(200)} = 48.99 \text{ ft/s} = 49.0 \text{ ft/s}$$

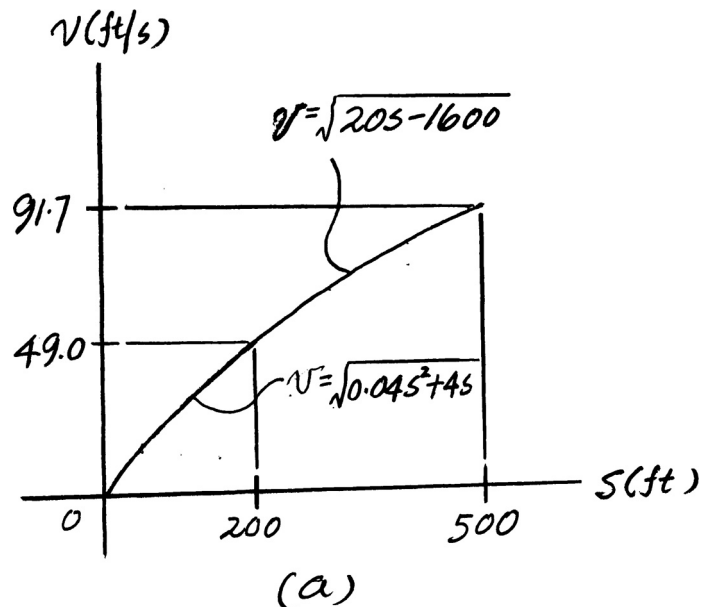
For $200 \text{ ft} < s \leq 500$ ft, the initial condition is $v = 48.99$ ft/s at $s = 200$ ft.

$$\begin{aligned} (\pm) \quad v dv &= a ds \\ \int_{48.99 \text{ ft/s}}^v v dv &= \int_{200 \text{ ft}}^s 10 ds \\ \frac{v^2}{2} \Big|_{48.99 \text{ ft/s}}^v &= 10s \Big|_{200 \text{ ft}}^s \\ v &= \sqrt{20s - 1600} \text{ ft/s} \end{aligned}$$

At $s = 500$ ft,

$$v|_{s=500 \text{ ft}} = \sqrt{20(500) - 1600} = 91.65 \text{ ft/s} = 91.7 \text{ ft/s}$$

The v – s graph is shown in Fig. a .



12–66. The boat travels along a straight line with the speed described by the graph. Construct the s – t and a – s graphs. Also, determine the time required for the boat to travel a distance $s = 400$ m if $s = 0$ when $t = 0$.

s – t Graph: For $0 \leq s < 100$ m, the initial condition is $s = 0$ when $t = 0$ s.

$$\begin{aligned} \left(\pm \right) \quad dt &= \frac{ds}{v} \\ \int_0^t dt &= \int_0^s \frac{ds}{2s^{1/2}} \\ t &= s^{1/2} \\ s &= (t^2) \text{ m} \end{aligned}$$

When $s = 100$ m,

$$100 = t^2 \quad t = 10 \text{ s}$$

For $100 < s \leq 400$ m, the initial condition is $s = 100$ m when $t = 10$ s.

$$\begin{aligned} \left(\pm \right) \quad dt &= \frac{ds}{v} \\ \int_{10 \text{ s}}^t dt &= \int_{100 \text{ m}}^s \frac{ds}{0.2s} \\ t - 10 &= 5 \ln \frac{s}{100} \\ \frac{t}{5} - 2 &= \ln \frac{s}{100} \\ e^{t/5-2} &= \frac{s}{100} \\ \frac{e^{t/5}}{e^2} &= \frac{s}{100} \\ s &= (13.53e^{t/5}) \text{ m} \end{aligned}$$

When $s = 400$ m,

$$\begin{aligned} 400 &= 13.53e^{t/5} \\ t &= 16.93 \text{ s} = 16.9 \text{ s} \end{aligned}$$

The s – t graph is shown in Fig. *a*.

a – s Graph: For $0 \text{ m} \leq s < 100$ m,

$$a = v \frac{dv}{ds} = (2s^{1/2})(s^{-1/2}) = 2 \text{ m/s}^2$$

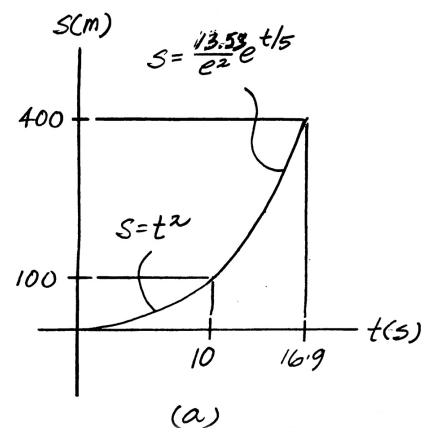
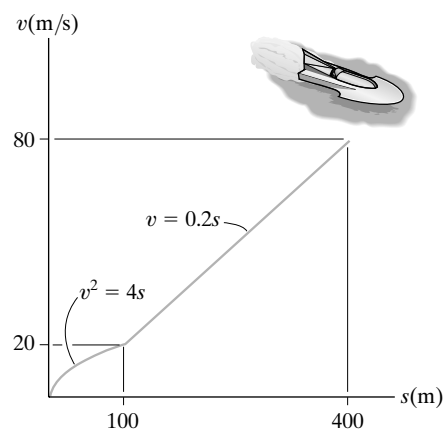
For $100 \text{ m} < s \leq 400$ m,

$$a = v \frac{dv}{ds} = (0.2s)(0.2) = 0.04s$$

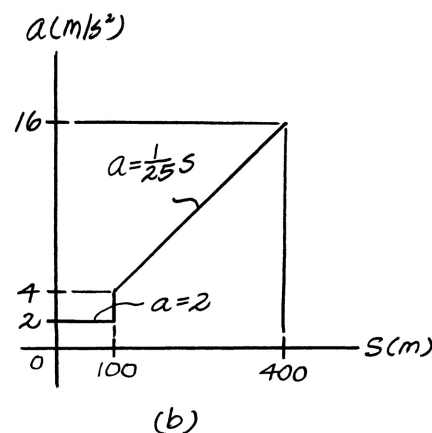
When $s = 100$ m and 400 m,

$$\begin{aligned} a|_{s=100 \text{ m}} &= 0.04(100) = 4 \text{ m/s}^2 \\ a|_{s=400 \text{ m}} &= 0.04(400) = 16 \text{ m/s}^2 \end{aligned}$$

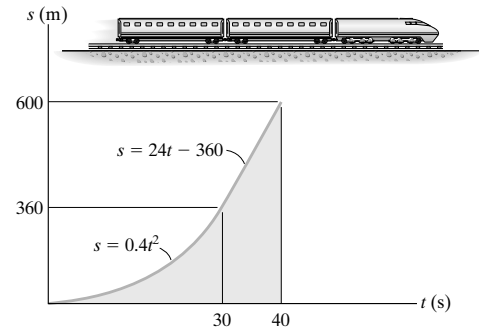
The a – s graph is shown in Fig. *b*.



Ans.



12-67. The $s-t$ graph for a train has been determined experimentally. From the data, construct the $v-t$ and $a-t$ graphs for the motion.



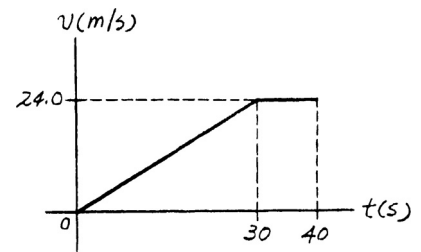
$v-t$ Graph: The velocity in terms of time t can be obtained by applying $v = \frac{ds}{dt}$.
For time interval $0 \text{ s} \leq t \leq 30 \text{ s}$,

$$v = \frac{ds}{dt} = 0.8t$$

When $t = 30 \text{ s}$, $v = 0.8(30) = 24.0 \text{ m/s}$

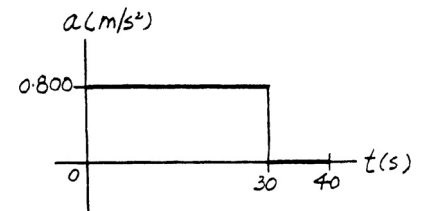
For time interval $30 \text{ s} < t \leq 40 \text{ s}$,

$$v = \frac{ds}{dt} = 24.0 \text{ m/s}$$

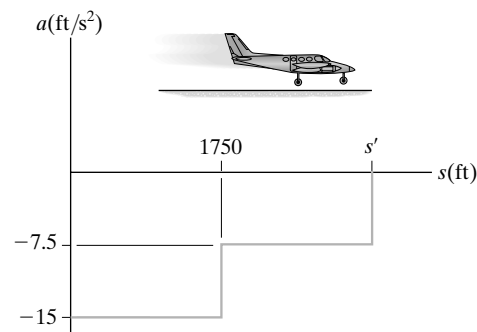


$a-t$ Graph: The acceleration in terms of time t can be obtained by applying $a = \frac{dv}{dt}$.

For time interval $0 \text{ s} \leq t < 30 \text{ s}$ and $30 \text{ s} < t \leq 40 \text{ s}$, $a = \frac{dv}{dt} = 0.800 \text{ m/s}^2$ and $a = \frac{dv}{dt} = 0$, respectively.



***12–68.** The airplane lands at 250 ft/s on a straight runway and has a deceleration described by the graph. Determine the distance s' traveled before its speed is decreased to 25 ft/s. Draw the s – t graph.



v – s Graph: For $0 \leq s < 1750$ ft, the initial condition is $v = 250$ ft/s at $s = 0$ s.

$$\begin{aligned} (\pm) \quad v dv &= a ds \\ \int_{250 \text{ ft/s}}^v v dv &= \int_0^s -15 ds \\ \frac{v^2}{2} \Big|_{250 \text{ ft/s}}^v &= -15s \Big|_0^s \\ v &= (\sqrt{62500 - 30s}) \text{ ft/s} \end{aligned}$$

At $s = 1750$ ft,

$$v|_{s=1750 \text{ ft}} = \sqrt{62500 - 30(1750)} = 100 \text{ ft/s}$$

For $1750 \text{ ft} < s < s'$, the initial condition is $v = 100$ ft/s at $s = 1750$ ft.

$$\begin{aligned} (\pm) \quad v dv &= a ds \\ \int_{100 \text{ ft/s}}^v v dv &= \int_{1750 \text{ ft}}^s -7.5 ds \\ \frac{v^2}{2} \Big|_{100 \text{ ft/s}}^v &= (-7.5s) \Big|_{1750 \text{ ft}}^s \\ v &= \sqrt{36250 - 15s} \end{aligned}$$

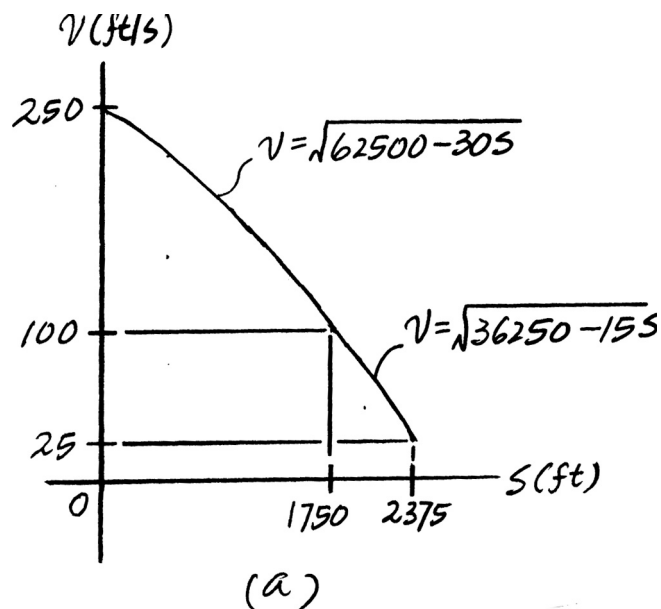
When $v = 25$ ft/s

$$25 = \sqrt{36250 - 15s}$$

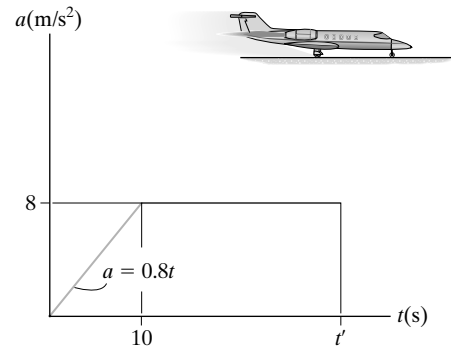
$$s' = 2375 \text{ ft}$$

Ans.

The v – s graph is shown in Fig. a .



•12–69. The airplane travels along a straight runway with an acceleration described by the graph. If it starts from rest and requires a velocity of 90 m/s to take off, determine the minimum length of runway required and the time t' for take off. Construct the $v-t$ and $s-t$ graphs.



$v-t$ graph: For the time interval $0 \leq t < 10$ s, the initial condition is $v = 0$ when $t = 0$ s.

$$\begin{aligned} (\pm) \quad dv &= a dt \\ \int_0^v dv &= \int_0^t 0.8t dt \\ v &= (0.4t^2) \text{ m/s} \end{aligned}$$

When $t = 10$ s,

$$v = 0.4(10^2) = 40 \text{ m/s}$$

For the time interval $10 \text{ s} < t \leq t'$, the initial condition is $v = 40$ m/s when $t = 10$ s.

$$\begin{aligned} (\pm) \quad dv &= a dt \\ \int_{40 \text{ m/s}}^v dv &= \int_{10 \text{ s}}^t 8 dt \\ v|_{40 \text{ m/s}}^v &= 8t|_{10 \text{ s}}^t \\ v &= (8t - 40) \text{ m/s} \end{aligned}$$

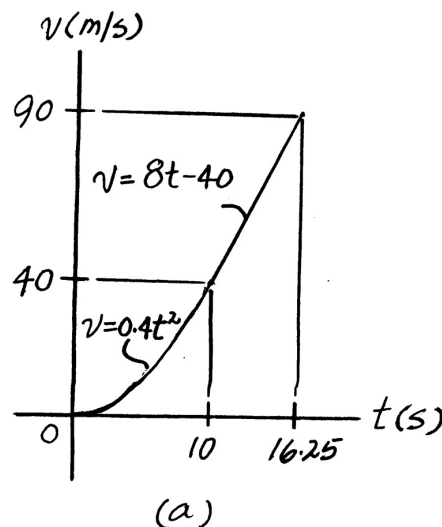
Thus, when $v = 90$ m/s,

$$90 = 8t' - 40 \qquad t' = 16.25 \text{ s} \qquad \text{Ans.}$$

Also, the change in velocity is equal to the area under the $a-t$ graph. Thus,

$$\begin{aligned} \Delta v &= \int a dt \\ 90 - 0 &= \frac{1}{2}(8)(10) + 8(t' - 10) \\ t' &= 16.25 \text{ s} \qquad \text{Ans.} \end{aligned}$$

The $v-t$ graph is shown in Fig. *a*.



•12–69. Continued

s – t Graph: For the time interval $0 \leq t < 10$ s, the initial condition is $s = 0$ when $t = 0$ s.

$$\begin{aligned} \left(\pm \right) \quad ds &= v dt \\ \int_0^s ds &= \int_0^t 0.4t^2 dt \\ s &= (0.1333t^3) \text{ m} \end{aligned}$$

When $t = 10$ s,

$$s|_{t=10 \text{ s}} = 0.1333(10^3) = 133.33 \text{ m}$$

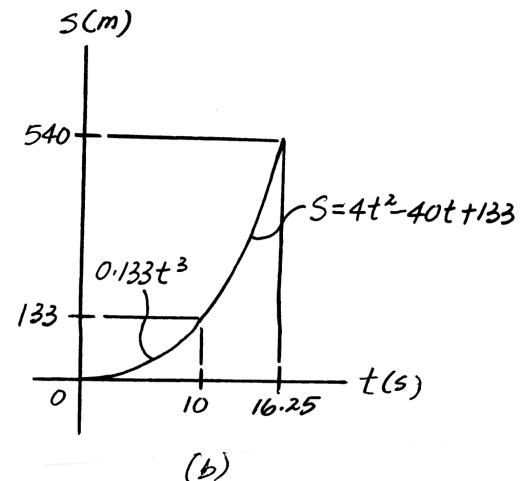
For the time interval $10 \text{ s} < t \leq t' = 16.25$ s, the initial condition is $s = 133.33$ m when $t = 10$ s.

$$\begin{aligned} \left(\pm \right) \quad ds &= v dt \\ \int_{133.33 \text{ m}}^s ds &= \int_{10 \text{ s}}^t (8t - 40) dt \\ s|_{133.33 \text{ m}}^s &= (4t^2 - 40t) \Big|_{10 \text{ s}}^t \\ s &= (4t^2 - 40t + 133.33) \text{ m} \end{aligned}$$

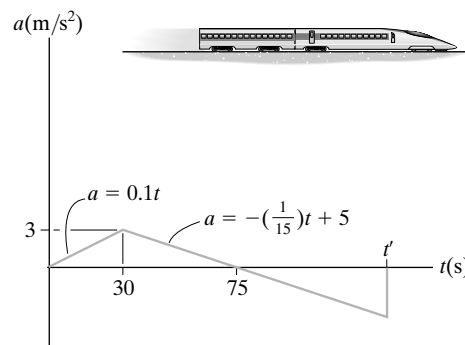
When $t = t' = 16.25$ s

$$s|_{t=16.25 \text{ s}} = 4(16.25)^2 - 40(16.25) + 133.33 = 539.58 \text{ m} = 540 \text{ m} \quad \mathbf{Ans.}$$

The s – t graph is shown in Fig. *b*.



12–70. The a – t graph of the bullet train is shown. If the train starts from rest, determine the elapsed time t' before it again comes to rest. What is the total distance traveled during this time interval? Construct the v – t and s – t graphs.



v – t Graph: For the time interval $0 \leq t < 30$ s, the initial condition is $v = 0$ when $t = 0$ s.

$$\begin{aligned} (\pm) \quad dv &= a dt \\ \int_0^v dv &= \int_0^t 0.1t dt \\ v &= (0.05t^2) \text{ m/s} \end{aligned}$$

When $t = 30$ s,

$$v|_{t=30 \text{ s}} = 0.05(30^2) = 45 \text{ m/s}$$

For the time interval $30 \text{ s} < t \leq t'$, the initial condition is $v = 45 \text{ m/s}$ at $t = 30$ s.

$$\begin{aligned} (\pm) \quad dv &= a dt \\ \int_{45 \text{ m/s}}^v dv &= \int_{30 \text{ s}}^t \left(-\frac{1}{15}t + 5 \right) dt \\ v &= \left(-\frac{1}{30}t^2 + 5t - 75 \right) \text{ m/s} \end{aligned}$$

Thus, when $v = 0$,

$$0 = -\frac{1}{30}t'^2 + 5t' - 75$$

Choosing the root $t' > 75$ s,

$$t' = 133.09 \text{ s} = 133 \text{ s}$$

Ans.

Also, the change in velocity is equal to the area under the a – t graph. Thus,

$$\begin{aligned} \Delta v &= \int a dt \\ 0 &= \frac{1}{2}(3)(75) + \frac{1}{2} \left[\left(-\frac{1}{15}t' + 5 \right) (t' - 75) \right] \\ 0 &= -\frac{1}{30}t'^2 + 5t' - 75 \end{aligned}$$

12-70. Continued

This equation is the same as the one obtained previously.

The slope of the v - t graph is zero when $t = 75$ s, which is the instant $a = \frac{dv}{dt} = 0$. Thus,

$$v|_{t=75\text{ s}} = -\frac{1}{30}(75^2) + 5(75) - 75 = 112.5 \text{ m/s}$$

The v - t graph is shown in Fig. *a*.

s - t Graph: Using the result of v , the equation of the s - t graph can be obtained by integrating the kinematic equation $ds = vdt$. For the time interval $0 \leq t < 30$ s, the initial condition $s = 0$ at $t = 0$ s will be used as the integration limit. Thus,

$$(\pm) \quad ds = vdt$$

$$\int_0^s ds = \int_0^t 0.05t^2 dt$$

$$s = \left(\frac{1}{60}t^3\right) \text{ m}$$

When $t = 30$ s,

$$s|_{t=30\text{ s}} = \frac{1}{60}(30^3) = 450 \text{ m}$$

For the time interval $30 \text{ s} < t \leq t' = 133.09$ s, the initial condition is $s = 450$ m when $t = 30$ s.

$$(\pm) \quad ds = vdt$$

$$\int_{450\text{ m}}^s ds = \int_{30\text{ s}}^t \left(-\frac{1}{30}t^2 + 5t - 75\right) dt$$

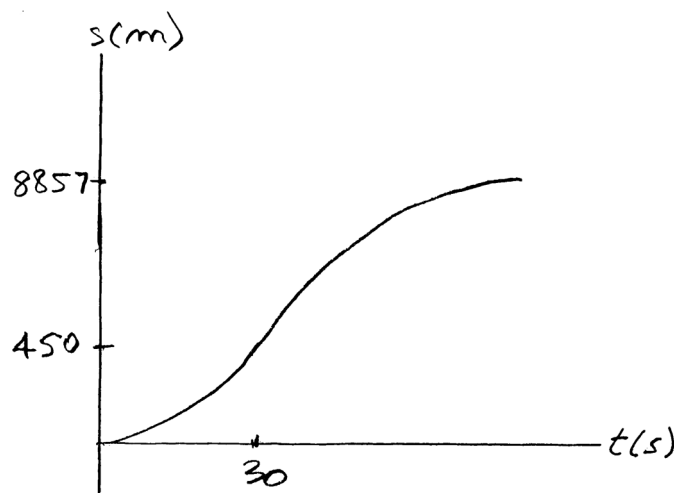
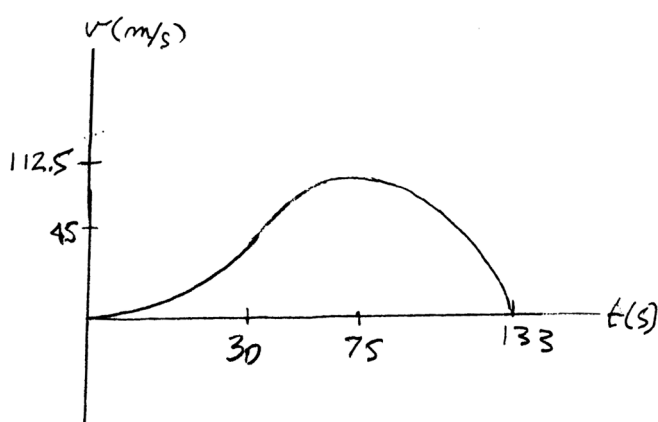
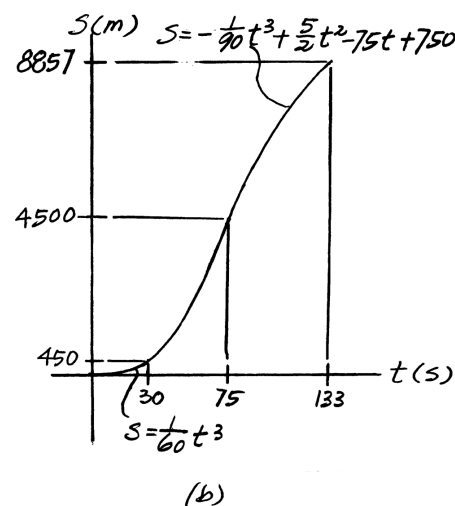
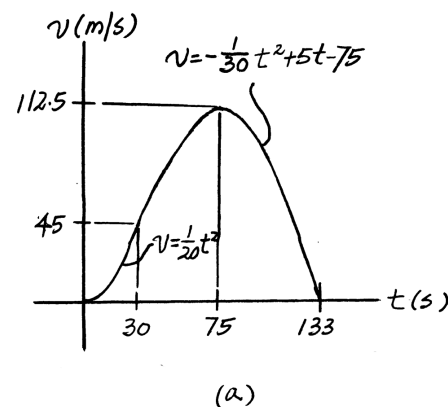
$$s = \left(-\frac{1}{90}t^3 + \frac{5}{2}t^2 - 75t + 750\right) \text{ m}$$

When $t = 75$ s and $t' = 133.09$ s,

$$s|_{t=75\text{ s}} = -\frac{1}{90}(75^3) + \frac{5}{2}(75^2) - 75(75) + 750 = 4500 \text{ m}$$

$$s|_{t=133.09\text{ s}} = -\frac{1}{90}(133.09^3) + \frac{5}{2}(133.09^2) - 75(133.09) + 750 = 8857 \text{ m} \quad \text{Ans.}$$

The s - t graph is shown in Fig. *b*.



12–71. The position of a particle is $\mathbf{r} = \{(3t^3 - 2t)\mathbf{i} - (4t^{1/2} + t)\mathbf{j} + (3t^2 - 2)\mathbf{k}\}$ m, where t is in seconds. Determine the magnitude of the particle's velocity and acceleration when $t = 2$ s.

Velocity:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt} \left[(3t^3 - 2t)\mathbf{i} - (4t^{1/2} + t)\mathbf{j} + (3t^2 - 2)\mathbf{k} \right] = \left[(9t^2 - 2)\mathbf{i} - (2t^{-1/2} + 1)\mathbf{j} + (6t)\mathbf{k} \right] \text{ m/s}$$

When $t = 2$ s,

$$\begin{aligned} \mathbf{v} &= \left[\left[9(2^2) - 2 \right] \mathbf{i} - \left[2(2^{-1/2}) + 1 \right] \mathbf{j} + 6(2)\mathbf{k} \right] \text{ m/s} \\ &= [34\mathbf{i} - 2.414\mathbf{j} + 12\mathbf{k}] \text{ m/s} \end{aligned}$$

Thus, the magnitude of the particle's velocity is

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{34^2 + (-2.414)^2 + 12^2} = 36.1 \text{ m/s} \quad \textbf{Ans.}$$

Acceleration:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d}{dt} \left[(9t^2 - 2)\mathbf{i} - (2t^{-1/2} + 1)\mathbf{j} + (6t)\mathbf{k} \right] \text{ m/s} = \left[(18t)\mathbf{i} + t^{-3/2}\mathbf{j} + 6\mathbf{k} \right] \text{ m/s}^2$$

When $t = 2$ s,

$$\mathbf{a} = [18(2)\mathbf{i} + 2^{-3/2}\mathbf{j} + 6\mathbf{k}] \text{ m/s}^2 = [36\mathbf{i} + 0.3536\mathbf{j} + 6\mathbf{k}] \text{ m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{36^2 + 0.3536^2 + 6^2} = 36.5 \text{ m/s}^2 \quad \textbf{Ans.}$$

***12–72.** The velocity of a particle is $\mathbf{v} = \{3\mathbf{i} + (6 - 2t)\mathbf{j}\}$ m/s, where t is in seconds. If $\mathbf{r} = \mathbf{0}$ when $t = 0$, determine the displacement of the particle during the time interval $t = 1$ s to $t = 3$ s.

Position: The position \mathbf{r} of the particle can be determined by integrating the kinematic equation $d\mathbf{r} = \mathbf{v}dt$ using the initial condition $\mathbf{r} = \mathbf{0}$ at $t = 0$ as the integration limit. Thus,

$$\begin{aligned} d\mathbf{r} &= \mathbf{v}dt \\ \int_0^{\mathbf{r}} d\mathbf{r} &= \int_0^t [3\mathbf{i} + (6 - 2t)\mathbf{j}] dt \\ \mathbf{r} &= \left[3t\mathbf{i} + (6t - t^2)\mathbf{j} \right] \text{ m} \end{aligned}$$

When $t = 1$ s and 3 s,

$$\begin{aligned} \mathbf{r}|_{t=1\text{ s}} &= 3(1)\mathbf{i} + [6(1) - 1^2]\mathbf{j} = [3\mathbf{i} + 5\mathbf{j}] \text{ m/s} \\ \mathbf{r}|_{t=3\text{ s}} &= 3(3)\mathbf{i} + [6(3) - 3^2]\mathbf{j} = [9\mathbf{i} + 9\mathbf{j}] \text{ m/s} \end{aligned}$$

Thus, the displacement of the particle is

$$\begin{aligned} \Delta\mathbf{r} &= \mathbf{r}|_{t=3\text{ s}} - \mathbf{r}|_{t=1\text{ s}} \\ &= (9\mathbf{i} + 9\mathbf{j}) - (3\mathbf{i} + 5\mathbf{j}) \\ &= [6\mathbf{i} + 4\mathbf{j}] \text{ m} \end{aligned}$$

Ans.

•12–73. A particle travels along the parabolic path $y = bx^2$. If its component of velocity along the y axis is $v_y = ct^2$, determine the x and y components of the particle's acceleration. Here b and c are constants.

Velocity:

$$\begin{aligned} dy &= v_y dt \\ \int_0^y dy &= \int_0^t ct^2 dt \\ y &= \frac{c}{3} t^3 \end{aligned}$$

Substituting the result of y into $y = bx^2$,

$$\begin{aligned} \frac{c}{3} t^3 &= bx^2 \\ x &= \sqrt{\frac{c}{3b}} t^{3/2} \end{aligned}$$

Thus, the x component of the particle's velocity can be determined by taking the time derivative of x .

$$v_x = \dot{x} = \frac{d}{dt} \left[\sqrt{\frac{c}{3b}} t^{3/2} \right] = \frac{3}{2} \sqrt{\frac{c}{3b}} t^{1/2}$$

Acceleration:

$$a_x = \dot{v}_x = \frac{d}{dt} \left(\frac{3}{2} \sqrt{\frac{c}{3b}} t^{1/2} \right) = \frac{3}{4} \sqrt{\frac{c}{3b}} t^{-1/2} = \frac{3}{4} \sqrt{\frac{c}{3b}} \frac{1}{\sqrt{t}} \quad \text{Ans.}$$

$$a_y = \dot{v}_y = \frac{d}{dt}(ct^2) = 2ct \quad \text{Ans.}$$

12–74. The velocity of a particle is given by $\mathbf{v} = \{16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\}$ m/s, where t is in seconds. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = 2$ s. Also, what is the x , y , z coordinate position of the particle at this instant?

Acceleration: The acceleration expressed in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{32t\mathbf{i} + 12t^2\mathbf{j} + 5\mathbf{k}\} \text{ m/s}^2$$

When $t = 2$ s, $\mathbf{a} = 32(2)\mathbf{i} + 12(2^2)\mathbf{j} + 5\mathbf{k} = \{64\mathbf{i} + 48\mathbf{j} + 5\mathbf{k}\} \text{ m/s}^2$. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{64^2 + 48^2 + 5^2} = 80.2 \text{ m/s}^2 \quad \textbf{Ans.}$$

Position: The position expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$d\mathbf{r} = \mathbf{v} dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t (16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}) dt$$

$$\mathbf{r} = \left[\frac{16}{3}t^3\mathbf{i} + t^4\mathbf{j} + \left(\frac{5}{2}t^2 + 2t \right)\mathbf{k} \right] \text{ m}$$

When $t = 2$ s,

$$\mathbf{r} = \frac{16}{3}(2^3)\mathbf{i} + (2^4)\mathbf{j} + \left[\frac{5}{2}(2^2) + 2(2) \right]\mathbf{k} = \{42.7\mathbf{i} + 16.0\mathbf{j} + 14.0\mathbf{k}\} \text{ m.}$$

Thus, the coordinate of the particle is

$$(42.7, 16.0, 14.0) \text{ m} \quad \textbf{Ans.}$$

12–75. A particle travels along the circular path $x^2 + y^2 = r^2$. If the y component of the particle's velocity is $v_y = 2r \cos 2t$, determine the x and y components of its acceleration at any instant.

Velocity:

$$dy = v_y dt$$

$$\int_0^y dy = \int_0^t 2r \cos 2t dt$$

$$y = r \sin 2t$$

Substituting this result into $x^2 + y^2 = r^2$, we obtain

$$x^2 + r^2 \sin^2 2t = r^2$$

$$x^2 = r^2(1 - \sin^2 2t)$$

$$x = \mp r \cos 2t$$

Thus,

$$v_x = \dot{x} = \frac{d}{dt}(\mp r \cos 2t) = \mp 2r \sin 2t$$

Acceleration:

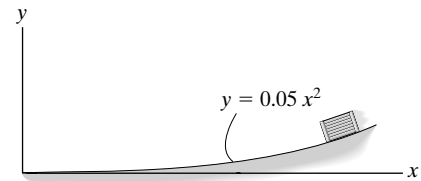
$$a_x = \dot{v}_x = \frac{d}{dt}(\mp 2r \sin 2t) = \mp 4r \cos 2t$$

Ans.

$$a_y = \dot{v}_y = \frac{d}{dt}(2r \cos 2t) = -4r \sin 2t$$

Ans.

***12–76.** The box slides down the slope described by the equation $y = (0.05x^2)$ m, where x is in meters. If the box has x components of velocity and acceleration of $v_x = -3$ m/s and $a_x = -1.5$ m/s² at $x = 5$ m, determine the y components of the velocity and the acceleration of the box at this instant.



Velocity: The x and y components of the box's velocity can be related by taking the first time derivative of the path's equation using the chain rule.

$$y = 0.05x^2$$

$$\dot{y} = 0.1x\dot{x}$$

or

$$v_y = 0.1xv_x$$

At $x = 5$ m, $v_x = -3$ m/s. Thus,

$$v_y = 0.1(5)(-3) = -1.5 \text{ m/s} = 1.5 \text{ m/s} \downarrow \quad \textbf{Ans.}$$

Acceleration: The x and y components of the box's acceleration can be obtained by taking the second time derivative of the path's equation using the chain rule.

$$\ddot{y} = 0.1[\dot{x}\dot{x} + x\ddot{x}] = 0.1(\dot{x}^2 + x\ddot{x})$$

or

$$a_y = 0.1(v_x^2 + xa_x)$$

At $x = 5$ m, $v_x = -3$ m/s and $a_x = -1.5$ m/s². Thus,

$$a_y = 0.1[(-3)^2 + 5(-1.5)] = 0.15 \text{ m/s}^2 \uparrow \quad \textbf{Ans.}$$

•12-77. The position of a particle is defined by $\mathbf{r} = \{5 \cos 2t \mathbf{i} + 4 \sin 2t \mathbf{j}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. Determine the magnitudes of the velocity and acceleration of the particle when $t = 1$ s. Also, prove that the path of the particle is elliptical.

Velocity: The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12-7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{-10 \sin 2t \mathbf{i} + 8 \cos 2t \mathbf{j}\} \text{ m/s}$$

When $t = 1$ s, $\mathbf{v} = -10 \sin 2(1) \mathbf{i} + 8 \cos 2(1) \mathbf{j} = \{-9.093 \mathbf{i} - 3.329 \mathbf{j}\}$ m/s. Thus, the magnitude of the velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-9.093)^2 + (-3.329)^2} = 9.68 \text{ m/s} \quad \text{Ans.}$$

Acceleration: The acceleration expressed in Cartesian vector form can be obtained by applying Eq. 12-9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{-20 \cos 2t \mathbf{i} - 16 \sin 2t \mathbf{j}\} \text{ m/s}^2$$

When $t = 1$ s, $\mathbf{a} = -20 \cos 2(1) \mathbf{i} - 16 \sin 2(1) \mathbf{j} = \{8.323 \mathbf{i} - 14.549 \mathbf{j}\}$ m/s². Thus, the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8.323^2 + (-14.549)^2} = 16.8 \text{ m/s}^2 \quad \text{Ans.}$$

Traveling Path: Here, $x = 5 \cos 2t$ and $y = 4 \sin 2t$. Then,

$$\frac{x^2}{25} = \cos^2 2t \quad [1]$$

$$\frac{y^2}{16} = \sin^2 2t \quad [2]$$

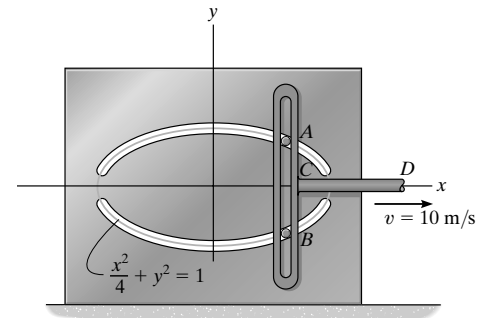
Adding Eqs [1] and [2] yields

$$\frac{x^2}{25} + \frac{y^2}{16} = \cos^2 2t + \sin^2 2t$$

However, $\cos^2 2t + \sin^2 2t = 1$. Thus,

$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \quad \text{(Equation of an Ellipse) (Q.E.D.)}$$

12–78. Pegs A and B are restricted to move in the elliptical slots due to the motion of the slotted link. If the link moves with a constant speed of 10 m/s, determine the magnitude of the velocity and acceleration of peg A when $x = 1$ m.



Velocity: The x and y components of the peg's velocity can be related by taking the first time derivative of the path's equation.

$$\begin{aligned}\frac{x^2}{4} + y^2 &= 1 \\ \frac{1}{4}(2x\dot{x}) + 2y\dot{y} &= 0 \\ \frac{1}{2}x\dot{x} + 2y\dot{y} &= 0\end{aligned}$$

or

$$\frac{1}{2}xv_x + 2yv_y = 0 \quad (1)$$

At $x = 1$ m,

$$\frac{(1)^2}{4} + y^2 = 1 \quad y = \frac{\sqrt{3}}{2} \text{ m}$$

Here, $v_x = 10$ m/s and $x = 1$. Substituting these values into Eq. (1),

$$\frac{1}{2}(1)(10) + 2\left(\frac{\sqrt{3}}{2}\right)v_y = 0 \quad v_y = -2.887 \text{ m/s} = 2.887 \text{ m/s} \downarrow$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{10^2 + 2.887^2} = 10.4 \text{ m/s} \quad \text{Ans.}$$

Acceleration: The x and y components of the peg's acceleration can be related by taking the second time derivative of the path's equation.

$$\begin{aligned}\frac{1}{2}(\dot{x}\dot{x} + x\ddot{x}) + 2(\dot{y}\dot{y} + y\ddot{y}) &= 0 \\ \frac{1}{2}(\dot{x}^2 + x\ddot{x}) + 2(\dot{y}^2 + y\ddot{y}) &= 0\end{aligned}$$

or

$$\frac{1}{2}(v_x^2 + xa_x) + 2(v_y^2 + ya_y) = 0 \quad (2)$$

Since v_x is constant, $a_x = 0$. When $x = 1$ m, $y = \frac{\sqrt{3}}{2}$ m, $v_x = 10$ m/s, and $v_y = -2.887$ m/s. Substituting these values into Eq. (2),

$$\begin{aligned}\frac{1}{2}(10^2 + 0) + 2\left[(-2.887)^2 + \frac{\sqrt{3}}{2}a_y\right] &= 0 \\ a_y &= -38.49 \text{ m/s}^2 = 38.49 \text{ m/s}^2 \downarrow\end{aligned}$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{0^2 + (-38.49)^2} = 38.5 \text{ m/s}^2 \quad \text{Ans.}$$

12–79. A particle travels along the path $y^2 = 4x$ with a constant speed of $v = 4 \text{ m/s}$. Determine the x and y components of the particle's velocity and acceleration when the particle is at $x = 4 \text{ m}$.

Velocity: The x and y components of the particle's velocity can be related by taking the first time derivative of the path's equation using the chain rule.

$$2y\dot{y} = 4\dot{x}$$

$$\dot{y} = \frac{2}{y} \dot{x}$$

or

$$v_y = \frac{2}{y} v_x \quad (1)$$

At $x = 4 \text{ m}$, $y = \sqrt{4(4)} = 4 \text{ m}$. Thus Eq. (1) becomes

$$v_y = \frac{1}{2} v_x \quad (2)$$

The magnitude of the particle's velocity is

$$v = \sqrt{v_x^2 + v_y^2} \quad (3)$$

Substituting $v = 4 \text{ m/s}$ and Eq. (2) into Eq. (3),

$$4 = \sqrt{v_x^2 + \left(\frac{1}{2}v_x\right)^2}$$

$$v_x = 3.578 \text{ m/s} = 3.58 \text{ m/s} \quad \text{Ans.}$$

Substituting the result of v_x into Eq. (2), we obtain

$$v_y = 1.789 \text{ m/s} = 1.79 \text{ m/s} \quad \text{Ans.}$$

Acceleration: The x and y components of the particle's acceleration can be related by taking the second time derivative of the path's equation using the chain rule.

$$2(\dot{y}\dot{y} + y\ddot{y}) = 4\ddot{x}$$

$$\dot{y}^2 + y\ddot{y} = 2\ddot{x}$$

or

$$v_y^2 + ya_y = 2a_x$$

When $x = 4 \text{ m}$, $y = 4 \text{ m}$, and $v_y = 1.789 \text{ m/s}$. Thus Eq. (4) becomes

$$1.789^2 + 4a_y = 2a_x$$

$$a_y = 0.5a_x - 0.8$$

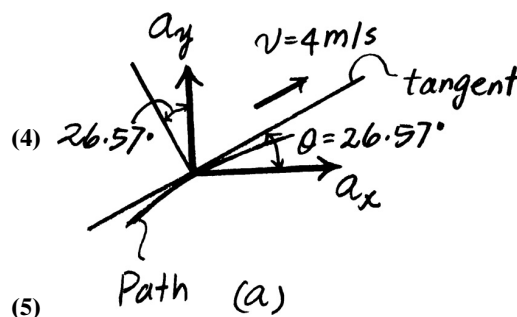
Since the particle travels with a constant speed along the path, its acceleration along the tangent of the path is equal to zero. Here, the angle that the tangent makes with the horizontal at $x = 4 \text{ m}$ is $\theta = \tan^{-1}\left(\frac{dy}{dx}\right)\bigg|_{x=4 \text{ m}} = \tan^{-1}\left(\frac{1}{x^{1/2}}\right)\bigg|_{x=4 \text{ m}} = \tan^{-1}(0.5) = 26.57^\circ$.

Thus, from the diagram shown in Fig. *a*,

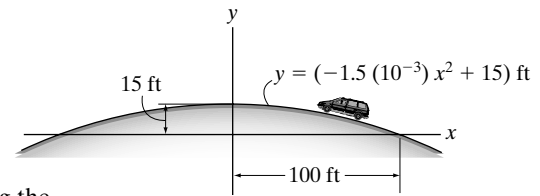
$$a_x \cos 26.57^\circ + a_y \sin 26.57^\circ = 0 \quad (6)$$

Solving Eqs. (5) and (6) yields

$$a_x = 0.32 \text{ m/s}^2 \quad a_y = -0.64 \text{ m/s}^2 = 0.64 \text{ m/s}^2 \downarrow \quad \text{Ans.}$$



***12–80.** The van travels over the hill described by $y = (-1.5(10^{-3})x^2 + 15)$ ft. If it has a constant speed of 75 ft/s, determine the x and y components of the van's velocity and acceleration when $x = 50$ ft.



Velocity: The x and y components of the van's velocity can be related by taking the first time derivative of the path's equation using the chain rule.

$$y = -1.5(10^{-3})x^2 + 15$$

$$\dot{y} = -3(10^{-3})x\dot{x}$$

or

$$v_y = -3(10^{-3})xv_x$$

When $x = 50$ ft,

$$v_y = -3(10^{-3})(50)v_x = -0.15v_x \quad (1)$$

The magnitude of the van's velocity is

$$v = \sqrt{v_x^2 + v_y^2} \quad (2)$$

Substituting $v = 75$ ft/s and Eq. (1) into Eq. (2),

$$75 = \sqrt{v_x^2 + (-0.15v_x)^2}$$

$$v_x = 74.2 \text{ ft/s} \leftarrow$$

Ans.

Substituting the result of v_x into Eq. (1), we obtain

$$v_y = -0.15(-74.17) = 11.12 \text{ ft/s} = 11.1 \text{ ft/s} \uparrow$$

Ans.

Acceleration: The x and y components of the van's acceleration can be related by taking the second time derivative of the path's equation using the chain rule.

$$\ddot{y} = -3(10^{-3})(\dot{x}\dot{x} + x\ddot{x})$$

or

$$a_y = -3(10^{-3})(v_x^2 + xa_x)$$

When $x = 50$ ft, $v_x = -74.17$ ft/s. Thus,

$$a_y = -3(10^{-3})\left[(-74.17)^2 + 50a_x\right]$$

$$a_y = -(16.504 + 0.15a_x) \quad (3)$$

Since the van travels with a constant speed along the path, its acceleration along the tangent of the path is equal to zero. Here, the angle that the tangent makes with the horizontal at

$$x = 50 \text{ ft is } \theta = \tan^{-1}\left(\frac{dy}{dx}\right)\bigg|_{x=50 \text{ ft}} = \tan^{-1}\left[-3(10^{-3})x\right]\bigg|_{x=50 \text{ ft}} = \tan^{-1}(-0.15) = -8.531^\circ.$$

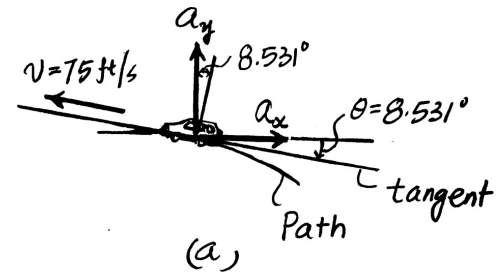
Thus, from the diagram shown in Fig. a ,

$$a_x \cos 8.531^\circ - a_y \sin 8.531^\circ = 0 \quad (4)$$

Solving Eqs. (3) and (4) yields

$$a_x = -2.42 \text{ ft/s}^2 = 2.42 \text{ ft/s}^2 \leftarrow \quad \text{Ans.}$$

$$a_y = -16.1 \text{ ft/s}^2 = 16.1 \text{ ft/s}^2 \downarrow \quad \text{Ans.}$$



•12–81. A particle travels along the circular path from A to B in 1 s. If it takes 3 s for it to go from A to C , determine its *average velocity* when it goes from B to C .

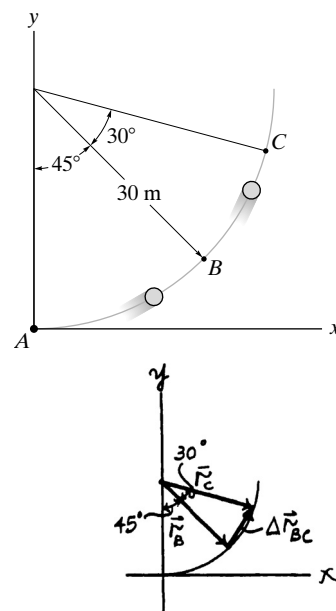
Position: The coordinates for points B and C are $[30 \sin 45^\circ, 30 - 30 \cos 45^\circ]$ and $[30 \sin 75^\circ, 30 - 30 \cos 75^\circ]$. Thus,

$$\begin{aligned}\mathbf{r}_B &= (30 \sin 45^\circ - 0)\mathbf{i} + [(30 - 30 \cos 45^\circ) - 30]\mathbf{j} \\ &= \{21.21\mathbf{i} - 21.21\mathbf{j}\} \text{ m}\end{aligned}$$

$$\begin{aligned}\mathbf{r}_C &= (30 \sin 75^\circ - 0)\mathbf{i} + [(30 - 30 \cos 75^\circ) - 30]\mathbf{j} \\ &= \{28.98\mathbf{i} - 7.765\mathbf{j}\} \text{ m}\end{aligned}$$

Average Velocity: The displacement from point B to C is $\Delta \mathbf{r}_{BC} = \mathbf{r}_C - \mathbf{r}_B$
 $= (28.98\mathbf{i} - 7.765\mathbf{j}) - (21.21\mathbf{i} - 21.21\mathbf{j}) = \{7.765\mathbf{i} + 13.45\mathbf{j}\} \text{ m}.$

$$(\mathbf{v}_{BC})_{\text{avg}} = \frac{\Delta \mathbf{r}_{BC}}{\Delta t} = \frac{7.765\mathbf{i} + 13.45\mathbf{j}}{3 - 1} = \{3.88\mathbf{i} + 6.72\mathbf{j}\} \text{ m/s} \quad \text{Ans.}$$



12–82. A car travels east 2 km for 5 minutes, then north 3 km for 8 minutes, and then west 4 km for 10 minutes. Determine the total distance traveled and the magnitude of displacement of the car. Also, what is the magnitude of the average velocity and the average speed?

Total Distance Traveled and Displacement: The total distance traveled is

$$s = 2 + 3 + 4 = 9 \text{ km} \quad \text{Ans.}$$

and the magnitude of the displacement is

$$\Delta r = \sqrt{(2 + 4)^2 + 3^2} = 6.708 \text{ km} = 6.71 \text{ km} \quad \text{Ans.}$$

Average Velocity and Speed: The total time is $\Delta t = 5 + 8 + 10 = 23 \text{ min} = 1380 \text{ s}.$
 The magnitude of average velocity is

$$v_{\text{avg}} = \frac{\Delta r}{\Delta t} = \frac{6.708(10^3)}{1380} = 4.86 \text{ m/s} \quad \text{Ans.}$$

and the average speed is

$$(v_{sp})_{\text{avg}} = \frac{s}{\Delta t} = \frac{9(10^3)}{1380} = 6.52 \text{ m/s} \quad \text{Ans.}$$

12–83. The roller coaster car travels down the helical path at constant speed such that the parametric equations that define its position are $x = c \sin kt$, $y = c \cos kt$, $z = h - bt$, where c , h , and b are constants. Determine the magnitudes of its velocity and acceleration.

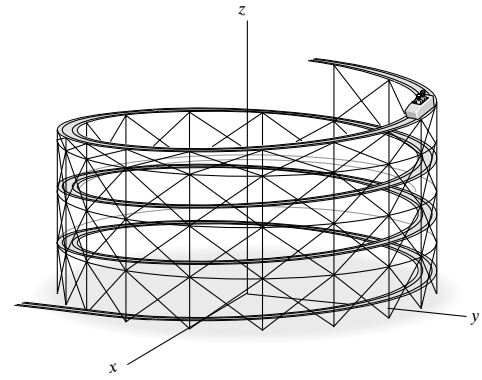
$$x = c \sin kt \quad \dot{x} = ck \cos kt \quad \ddot{x} = -ck^2 \sin kt$$

$$y = c \cos kt \quad \dot{y} = -ck \sin kt \quad \ddot{y} = -ck^2 \cos kt$$

$$z = h - bt \quad \dot{z} = -b \quad \ddot{z} = 0$$

$$v = \sqrt{(ck \cos kt)^2 + (-ck \sin kt)^2 + (-b)^2} = \sqrt{c^2 k^2 + b^2}$$

$$a = \sqrt{(-ck^2 \sin kt)^2 + (-ck^2 \cos kt)^2 + 0} = ck^2$$



Ans.

Ans.

***12–84.** The path of a particle is defined by $y^2 = 4kx$, and the component of velocity along the y axis is $v_y = ct$, where both k and c are constants. Determine the x and y components of acceleration when $y = y_0$.

$$y^2 = 4kx$$

$$2yv_y = 4kv_x$$

$$2v_y^2 + 2ya_y = 4ka_x$$

$$v_y = ct$$

$$a_y = c$$

$$2(ct)^2 + 2yc = 4ka_x$$

$$a_x = \frac{c}{2k} (y + ct^2)$$

Ans.

Ans.

•12–85. A particle moves along the curve $y = x - (x^2/400)$, where x and y are in ft. If the velocity component in the x direction is $v_x = 2$ ft/s and remains *constant*, determine the magnitudes of the velocity and acceleration when $x = 20$ ft.

Velocity: Taking the first derivative of the path $y = x - \frac{x^2}{400}$, we have

$$\begin{aligned}\dot{y} &= \dot{x} - \frac{1}{400}(2x\dot{x}) \\ \dot{y} &= \dot{x} - \frac{x}{200}\dot{x}\end{aligned}\quad [1]$$

However, $\dot{x} = v_x$ and $\dot{y} = v_y$. Thus, Eq. [1] becomes

$$v_y = v_x - \frac{x}{200}v_x \quad [2]$$

Here, $v_x = 2$ ft/s at $x = 20$ ft. Then, From Eq. [2]

$$v_y = 2 - \frac{20}{200}(2) = 1.80 \text{ ft/s}$$

Also,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{2^2 + 1.80^2} = 2.69 \text{ ft/s} \quad \text{Ans.}$$

Acceleration: Taking the second derivative of the path $y = x - \frac{x^2}{400}$, we have

$$\ddot{y} = \ddot{x} - \frac{1}{200}(\dot{x}^2 + x\ddot{x}) \quad [3]$$

However, $\ddot{x} = a_x$ and $\ddot{y} = a_y$. Thus, Eq. [3] becomes

$$a_y = a_x - \frac{1}{200}(v_x^2 + xa_x) \quad [4]$$

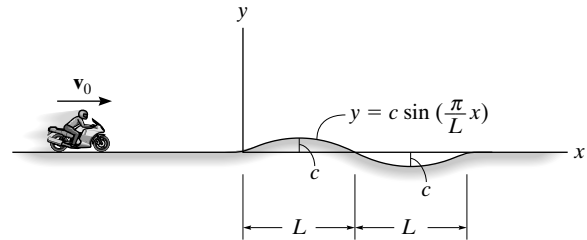
Since $v_x = 2$ ft/s is constant, hence $a_x = 0$ at $x = 20$ ft. Then, From Eq. [4]

$$a_y = 0 - \frac{1}{200}[2^2 + 20(0)] = -0.020 \text{ ft/s}^2$$

Also,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{0^2 + (-0.020)^2} = 0.0200 \text{ ft/s}^2 \quad \text{Ans.}$$

12–86. The motorcycle travels with constant speed v_0 along the path that, for a short distance, takes the form of a sine curve. Determine the x and y components of its velocity at any instant on the curve.



$$y = c \sin\left(\frac{\pi}{L}x\right)$$

$$y = \frac{\pi}{L}c \left(\cos\frac{\pi}{L}x\right)\dot{x}$$

$$v_y = \frac{\pi}{L}c v_x \left(\cos\frac{\pi}{L}x\right)$$

$$v_0^2 = v_y^2 + v_x^2$$

$$v_0^2 = v_x^2 \left[1 + \left(\frac{\pi}{L}c\right)^2 \cos^2\left(\frac{\pi}{L}x\right)\right]$$

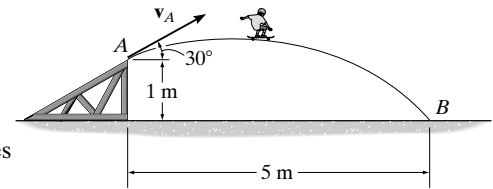
$$v_x = v_0 \left[1 + \left(\frac{\pi}{L}c\right)^2 \cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$

Ans.

$$v_y = \frac{v_0 \pi c}{L} \left(\cos\frac{\pi}{L}x\right) \left[1 + \left(\frac{\pi}{L}c\right)^2 \cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$

Ans.

12–87. The skateboard rider leaves the ramp at A with an initial velocity v_A at a 30° angle. If he strikes the ground at B , determine v_A and the time of flight.



Coordinate System: The x – y coordinate system will be set so that its origin coincides with point A .

x -Motion: Here, $(v_A)_x = v_A \cos 30^\circ$, $x_A = 0$ and $x_B = 5$ m. Thus,

$$(\rightarrow) \quad x_B = x_A + (v_A)_x t$$

$$5 = 0 + v_A \cos 30^\circ t$$

$$t = \frac{5}{v_A \cos 30^\circ}$$

(1)

y -Motion: Here, $(v_A)_y = v_A \sin 30^\circ$, $a_y = -g = -9.81 \text{ m/s}^2$, and $y_B = -1$ m. Thus,

$$(+\uparrow) \quad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-1 = 0 + v_A \sin 30^\circ t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - v_A \sin 30^\circ t - 1 = 0$$

(2)

Solving Eqs. (1) and (2) yields

$$v_A = 6.49 \text{ m/s} \quad t = 0.890 \text{ s}$$

Ans.

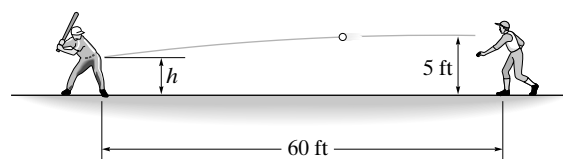
***12–88.** The pitcher throws the baseball horizontally with a speed of 140 ft/s from a height of 5 ft. If the batter is 60 ft away, determine the time for the ball to arrive at the batter and the height h at which it passes the batter.

$$(\rightarrow) \quad s = vt; \quad 60 = 140t$$

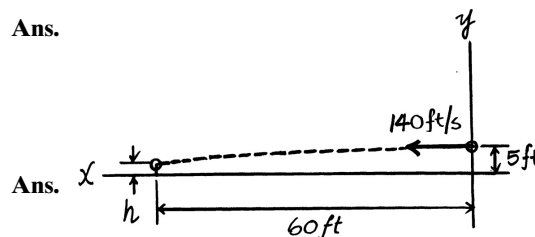
$$t = 0.4286 = 0.429 \text{ s}$$

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$h = 5 + 0 + \frac{1}{2} (-32.2)(0.4286)^2 = 2.04 \text{ ft}$$



Ans.



Ans.

•12–89. The ball is thrown off the top of the building. If it strikes the ground at B in 3 s, determine the initial velocity v_A and the inclination angle θ_A at which it was thrown. Also, find the magnitude of the ball's velocity when it strikes the ground.

Coordinate System: The x - y coordinate system will be set so that its origin coincides with point A .

x -Motion: Here, $(v_A)_x = v_A \cos \theta$, $x_A = 0$, and $x_B = 60 \text{ ft}$, and $t = 3 \text{ s}$. Thus,

$$(\rightarrow) \quad x_B = x_A + (v_A)_x t$$

$$60 = 0 + v_A \cos \theta (3)$$

$$v_A \cos \theta = 20$$

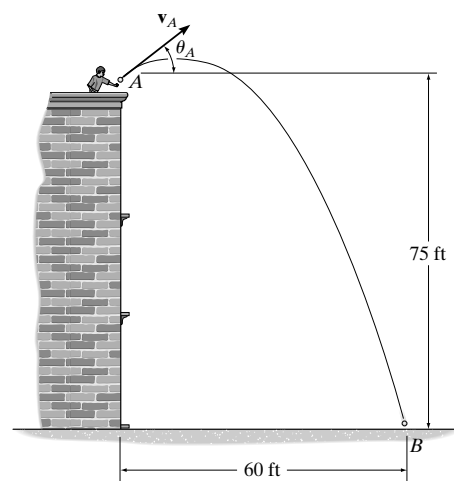
y -Motion: Here, $(v_A)_y = v_A \sin \theta$, $a_y = -g = -32.2 \text{ ft/s}^2$, $y_A = 0$, and $y_B = -75 \text{ ft}$, and $t = 3 \text{ s}$. Thus,

$$(+\uparrow) \quad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-75 = 0 + v_A \sin \theta (3) + \frac{1}{2} (-32.2)(3^2)$$

$$v_A \sin \theta = 23.3$$

(1)



(2)

Solving Eqs. (1) and (2) yields

$$\theta = 49.36^\circ = 49.4^\circ$$

$$v_A = 30.71 \text{ ft/s} = 30.7 \text{ ft/s}$$

Ans.

Using the result of θ and v_A , we obtain

$$(v_A)_x = 30.71 \cos 49.36^\circ = 20 \text{ ft/s}$$

$$(v_A)_y = 30.71 \sin 49.36^\circ = 23.3 \text{ ft/s}$$

Thus,

$$(+\uparrow) \quad (v_B)_y = (v_A)_y + a_y t$$

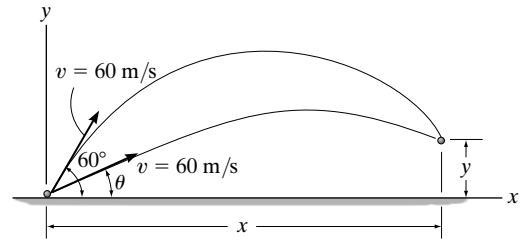
$$(v_B)_y = 23.3 + (-32.2)(3) = -73.3 \text{ ft/s} = 73.3 \text{ ft/s} \downarrow$$

Thus, the magnitude of the ball's velocity when it strikes the ground is

$$v_B = \sqrt{20^2 + 73.3^2} = 76.0 \text{ ft/s}$$

Ans.

12–90. A projectile is fired with a speed of $v = 60 \text{ m/s}$ at an angle of 60° . A second projectile is then fired with the same speed 0.5 s later. Determine the angle θ of the second projectile so that the two projectiles collide. At what position (x, y) will this happen?



x-Motion: For the motion of the first projectile, $v_x = 60 \cos 60^\circ = 30 \text{ m/s}$, $x_0 = 0$, and $t = t_1$. Thus,

$$\begin{aligned} (\rightarrow) \quad x &= x_0 + v_x t \\ x &= 0 + 30t_1 \end{aligned} \quad (1)$$

For the motion of the second projectile, $v_x = 60 \cos \theta$, $x_0 = 0$, and $t = t_1 - 0.5$. Thus,

$$\begin{aligned} (\rightarrow) \quad x &= x_0 + v_x t \\ x &= 0 + 60 \cos \theta (t_1 - 0.5) \end{aligned} \quad (2)$$

y-Motion: For the motion of the first projectile, $v_y = 60 \sin 60^\circ = 51.96 \text{ m/s}$, $y_0 = 0$, and $a_y = -g = -9.81 \text{ m/s}^2$. Thus,

$$\begin{aligned} (+\uparrow) \quad y &= y_0 + v_y t + \frac{1}{2} a_y t^2 \\ y &= 0 + 51.96t_1 + \frac{1}{2}(-9.81)t_1^2 \\ y &= 51.96t_1 - 4.905t_1^2 \end{aligned} \quad (3)$$

For the motion of the second projectile, $v_y = 60 \sin \theta$, $y_0 = 0$, and $a_y = -g = -9.81 \text{ m/s}^2$. Thus,

$$\begin{aligned} (+\uparrow) \quad y &= y_0 + v_y t + \frac{1}{2} a_y t^2 \\ y &= 0 + 60 \sin \theta (t_1 - 0.5) + \frac{1}{2}(-9.81)(t_1 - 0.5)^2 \\ y &= (60 \sin \theta)t_1 - 30 \sin \theta - 4.905t_1^2 + 4.905t_1 - 1.22625 \end{aligned} \quad (4)$$

Equating Eqs. (1) and (2),

$$\begin{aligned} 30t_1 &= 60 \cos \theta (t_1 - 0.5) \\ t_1 &= \frac{\cos \theta}{2 \cos \theta - 1} \end{aligned} \quad (5)$$

Equating Eqs. (3) and (4),

$$\begin{aligned} 51.96t_1 - 4.905t_1^2 &= (60 \sin \theta)t_1 - 30 \sin \theta - 4.905t_1^2 + 4.905t_1 - 1.22625 \\ (60 \sin \theta - 47.06)t_1 &= 30 \sin \theta + 1.22625 \\ t_1 &= \frac{30 \sin \theta + 1.22625}{60 \sin \theta - 47.06} \end{aligned} \quad (6)$$

12–90. Continued

Equating Eqs. (5) and (6) yields

$$\frac{\cos \theta}{2 \cos \theta - 1} = \frac{30 \sin \theta + 1.22625}{60 \sin \theta - 47.06}$$

$$49.51 \cos \theta - 30 \sin \theta = 1.22625$$

Solving by trial and error,

$$\theta = 57.57^\circ = 57.6^\circ \quad \textbf{Ans.}$$

Substituting this result into Eq. (5) (or Eq. (6)),

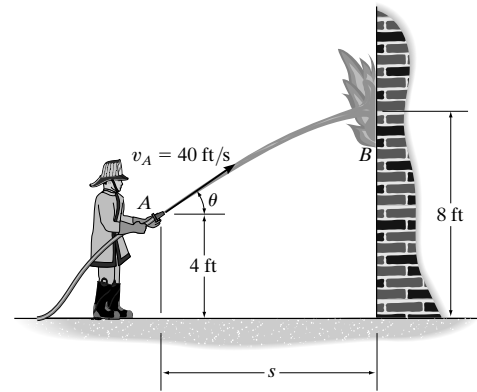
$$t_1 = \frac{\cos 57.57^\circ}{2 \cos 57.57^\circ - 1} = 7.3998 \text{ s}$$

Substituting this result into Eqs. (1) and (3),

$$x = 30(7.3998) = 222 \text{ m} \quad \textbf{Ans.}$$

$$y = 51.96(7.3998) - 4.905(7.3998^2) = 116 \text{ m} \quad \textbf{Ans.}$$

12–91. The fireman holds the hose at an angle $\theta = 30^\circ$ with horizontal, and the water is discharged from the hose at A with a speed of $v_A = 40 \text{ ft/s}$. If the water stream strikes the building at B , determine his two possible distances s from the building.



Coordinate System: The x - y coordinate system will be set so that its origin coincides with point A .

x -Motion: Here, $(v_A)_x = 40 \cos 30^\circ \text{ ft/s} = 34.64 \text{ ft/s}$, $x_A = 0$, and $x_B = s$. Thus,

$$\begin{aligned} (\rightarrow) \quad x_B &= x_A + (v_A)_x t \\ s &= 0 + 34.64t \\ s &= 34.64t \end{aligned} \quad (1)$$

y -Motion: Here, $(v_A)_y = 40 \sin 30^\circ \text{ ft/s} = 20 \text{ ft/s}$, $a_y = -g = -32.2 \text{ ft/s}^2$, $y_A = 0$, and $y_B = 8 - 4 = 4 \text{ ft}$. Thus,

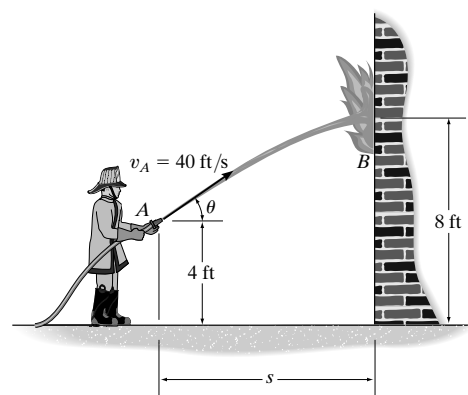
$$\begin{aligned} (+\uparrow) \quad y_B &= y_A + (v_A)_y t + \frac{1}{2} a_y t^2 \\ 4 &= 0 + 20t + \frac{1}{2} (-32.2)t^2 \\ 16.1t^2 - 20t + 4 &= 0 \\ t &= 0.2505 \text{ s and } 0.9917 \text{ s} \end{aligned}$$

Substituting these results into Eq. (1), the two possible distances are

$$s = 34.64(0.2505) = 8.68 \text{ ft} \quad \text{Ans.}$$

$$s = 34.64(0.9917) = 34.4 \text{ ft} \quad \text{Ans.}$$

***12–92.** Water is discharged from the hose with a speed of 40 ft/s. Determine the two possible angles θ the fireman can hold the hose so that the water strikes the building at B . Take $s = 20$ ft.



Coordinate System: The x - y coordinate system will be set so that its origin coincides with point A .

x -Motion: Here, $(v_A)_x = 40 \cos \theta$, $x_A = 0$, and $x_B = 20$ ft/s. Thus,

$$\begin{aligned} (\rightarrow) \quad x_B &= x_A + (v_A)_x t \\ 20 &= 0 + 40 \cos \theta t \\ t &= \frac{1}{2 \cos \theta} \end{aligned} \quad (1)$$

y -Motion: Here, $(v_A)_y = 40 \sin \theta$, $a_y = -g = -32.2 \text{ ft/s}^2$, $y_A = 0$, and $y_B = 8 - 4 = 4$ ft. Thus,

$$\begin{aligned} (+\uparrow) \quad y_B &= y_A + (v_A)_y t + \frac{1}{2} a_y t^2 \\ 4 &= 0 + 40 \sin \theta t + \frac{1}{2} (-32.2) t^2 \\ 16.1 t^2 - 40 \sin \theta t + 4 &= 0 \end{aligned} \quad (2)$$

Substituting Eq. (1) into Eq. (2) yields

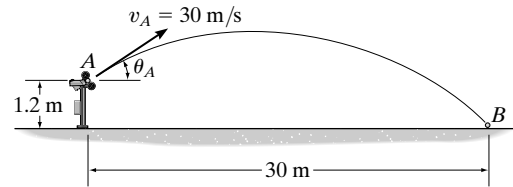
$$\begin{aligned} 16.1 \left(\frac{1}{2 \cos \theta} \right)^2 - 40 \sin \theta \left(\frac{1}{2 \cos \theta} \right) + 4 &= 0 \\ 20 \sin \theta \cos \theta - 4 \cos^2 \theta &= 4.025 \\ 10 \sin \theta \cos \theta - 2 \cos^2 \theta &= 2.0125 \\ 5 \sin 2\theta - (2 \cos^2 \theta - 1) - 1 &= 2.0125 \\ 5 \sin 2\theta - \cos 2\theta &= 3.0125 \end{aligned}$$

Solving by trial and error,

$$\theta = 23.8^\circ \text{ and } 77.5^\circ$$

Ans.

•12–93. The pitching machine is adjusted so that the baseball is launched with a speed of $v_A = 30 \text{ m/s}$. If the ball strikes the ground at B , determine the two possible angles θ_A at which it was launched.



Coordinate System: The x - y coordinate system will be set so that its origin coincides with point A .

x -Motion: Here, $(v_A)_x = 30 \cos \theta_A$, $x_A = 0$ and $x_B = 30 \text{ m}$. Thus,

$$\begin{aligned} (\rightarrow) \quad x_B &= x_A + (v_A)_x t \\ 30 &= 0 + 30 \cos \theta_A t \\ t &= \frac{1}{\cos \theta_A} \end{aligned} \quad (1)$$

y -Motion: Here, $(v_A)_y = 30 \sin \theta_A$, $a_y = -g = -9.81 \text{ m/s}^2$, and $y_B = -1.2 \text{ m}$. Thus,

$$\begin{aligned} (+\uparrow) \quad y_B &= y_A + (v_A)_y t + \frac{1}{2} a_y t^2 \\ -1.2 &= 0 + 30 \sin \theta_A t + \frac{1}{2} (-9.81) t^2 \\ 4.905 t^2 - 30 \sin \theta_A t - 1.2 &= 0 \end{aligned} \quad (2)$$

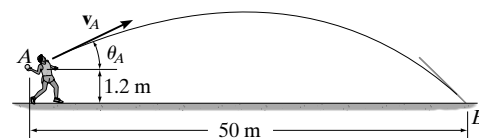
Substituting Eq. (1) into Eq. (2) yields

$$\begin{aligned} 4.905 \left(\frac{1}{\cos \theta_A} \right)^2 - 30 \sin \theta_A \left(\frac{1}{\cos \theta_A} \right) - 1.2 &= 0 \\ 1.2 \cos^2 \theta_A + 30 \sin \theta_A \cos \theta_A - 4.905 &= 0 \end{aligned}$$

Solving by trial and error,

$$\theta_A = 7.19^\circ \text{ and } 80.5^\circ \quad \text{Ans.}$$

12–94. It is observed that the time for the ball to strike the ground at B is 2.5 s. Determine the speed v_A and angle θ_A at which the ball was thrown.



Coordinate System: The x – y coordinate system will be set so that its origin coincides with point A .

x -Motion: Here, $(v_A)_x = v_A \cos \theta_A$, $x_A = 0$, $x_B = 50$ m, and $t = 2.5$ s. Thus,

$$\begin{aligned} (\rightarrow) \quad x_B &= x_A + (v_A)_x t \\ 50 &= 0 + v_A \cos \theta_A (2.5) \\ v_A \cos \theta_A &= 20 \end{aligned} \quad (1)$$

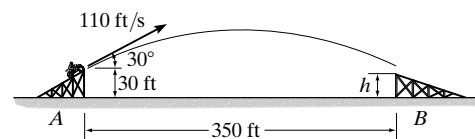
y -Motion: Here, $(v_A)_y = v_A \sin \theta_A$, $y_A = 0$, $y_B = -1.2$ m, and $a_y = -g = -9.81$ m/s². Thus,

$$\begin{aligned} (+\uparrow) \quad y_B &= y_A + (v_A)_y t + \frac{1}{2} a_y t^2 \\ -1.2 &= 0 + v_A \sin \theta_A (2.5) + \frac{1}{2} (-9.81) (2.5^2) \\ v_A \sin \theta_A &= 11.7825 \end{aligned} \quad (2)$$

Solving Eqs. (1) and (2) yields

$$\theta_A = 30.5^\circ \qquad v_A = 23.2 \text{ m/s} \qquad \text{Ans.}$$

12–95. If the motorcycle leaves the ramp traveling at 110 ft/s, determine the height h ramp B must have so that the motorcycle lands safely.



Coordinate System: The x – y coordinate system will be set so that its origin coincides with the take off point of the motorcycle at ramp A .

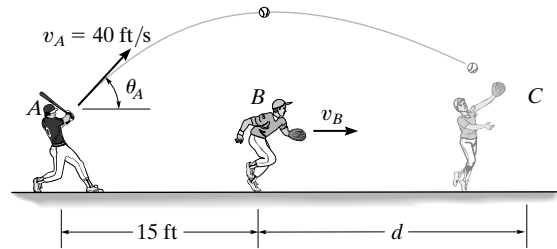
x -Motion: Here, $x_A = 0$, $x_B = 350$ ft, and $(v_A)_x = 110 \cos 30^\circ = 95.26$ ft/s. Thus,

$$\begin{aligned} (\rightarrow) \quad x_B &= x_A + (v_A)_x t \\ 350 &= 0 + 95.26 t \\ t &= 3.674 \text{ s} \end{aligned}$$

y -Motion: Here, $y_A = 0$, $y_B = h - 30$, $(v_A)_y = 110 \sin 30^\circ = 55$ ft/s, and $a_y = -g = -32.2$ ft/s². Thus, using the result of t , we have

$$\begin{aligned} (+\uparrow) \quad y_B &= y_A + (v_A)_y t + \frac{1}{2} a_y t^2 \\ h - 30 &= 0 + 55(3.674) + \frac{1}{2} (-32.2) (3.674^2) \\ h &= 14.7 \text{ ft} \end{aligned} \quad \text{Ans.}$$

***12–96.** The baseball player *A* hits the baseball with $v_A = 40 \text{ ft/s}$ and $\theta_A = 60^\circ$. When the ball is directly above of player *B* he begins to run under it. Determine the constant speed v_B and the distance d at which *B* must run in order to make the catch at the same elevation at which the ball was hit.



Vertical Motion: The vertical component of initial velocity for the football is $(v_0)_y = 40 \sin 60^\circ = 34.64 \text{ ft/s}$. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = 0$, respectively.

$$\begin{aligned}
 (+\uparrow) \quad s_y &= (s_0)_y + (v_0)_y t + \frac{1}{2} (a_c)_y t^2 \\
 0 &= 0 + 34.64t + \frac{1}{2} (-32.2)t^2 \\
 t &= 2.152 \text{ s}
 \end{aligned}$$

Horizontal Motion: The horizontal component of velocity for the baseball is $(v_0)_x = 40 \cos 60^\circ = 20.0 \text{ ft/s}$. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = R$, respectively.

$$\begin{aligned}
 (\pm) \quad s_x &= (s_0)_x + (v_0)_x t \\
 R &= 0 + 20.0(2.152) = 43.03 \text{ ft}
 \end{aligned}$$

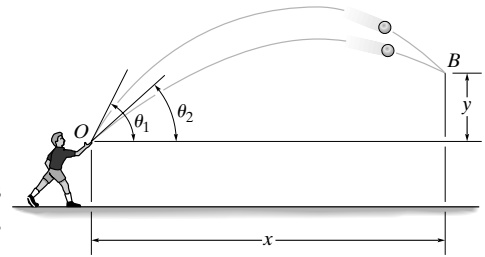
The distance for which player *B* must travel in order to catch the baseball is

$$d = R - 15 = 43.03 - 15 = 28.0 \text{ ft} \quad \textbf{Ans.}$$

Player *B* is required to run at a same speed as the horizontal component of velocity of the baseball in order to catch it.

$$v_B = 40 \cos 60^\circ = 20.0 \text{ ft/s} \quad \textbf{Ans.}$$

•12–97. A boy throws a ball at O in the air with a speed v_0 at an angle θ_1 . If he then throws another ball with the same speed v_0 at an angle $\theta_2 < \theta_1$, determine the time between the throws so that the balls collide in mid air at B .



Vertical Motion: For the first ball, the vertical component of initial velocity is $(v_0)_y = v_0 \sin \theta_1$ and the initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$\begin{aligned} (+\uparrow) \quad s_y &= (s_0)_y + (v_0)_y t + \frac{1}{2} (a_c)_y t^2 \\ y &= 0 + v_0 \sin \theta_1 t_1 + \frac{1}{2} (-g) t_1^2 \end{aligned} \quad [1]$$

For the second ball, the vertical component of initial velocity is $(v_0)_y = v_0 \sin \theta_2$ and the initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$\begin{aligned} (+\uparrow) \quad s_y &= (s_0)_y + (v_0)_y t + \frac{1}{2} (a_c)_y t^2 \\ y &= 0 + v_0 \sin \theta_2 t_2 + \frac{1}{2} (-g) t_2^2 \end{aligned} \quad [2]$$

Horizontal Motion: For the first ball, the horizontal component of initial velocity is $(v_0)_x = v_0 \cos \theta_1$ and the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$\begin{aligned} (\pm) \quad s_x &= (s_0)_x + (v_0)_x t \\ x &= 0 + v_0 \cos \theta_1 t_1 \end{aligned} \quad [3]$$

For the second ball, the horizontal component of initial velocity is $(v_0)_x = v_0 \cos \theta_2$ and the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$\begin{aligned} (\pm) \quad s_x &= (s_0)_x + (v_0)_x t \\ x &= 0 + v_0 \cos \theta_2 t_2 \end{aligned} \quad [4]$$

Equating Eqs. [3] and [4], we have

$$t_2 = \frac{\cos \theta_1}{\cos \theta_2} t_1 \quad [5]$$

Equating Eqs. [1] and [2], we have

$$v_0 t_1 \sin \theta_1 - v_0 t_2 \sin \theta_2 = \frac{1}{2} g (t_1^2 - t_2^2) \quad [6]$$

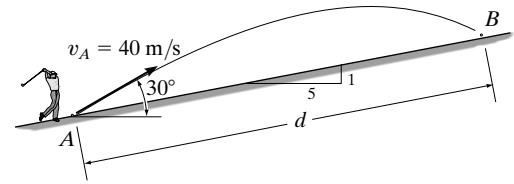
Solving Eq. [5] into [6] yields

$$\begin{aligned} t_1 &= \frac{2v_0 \cos \theta_2 \sin(\theta_1 - \theta_2)}{g(\cos^2 \theta_2 - \cos^2 \theta_1)} \\ t_2 &= \frac{2v_0 \cos \theta_1 \sin(\theta_1 - \theta_2)}{g(\cos^2 \theta_2 - \cos^2 \theta_1)} \end{aligned}$$

Thus, the time between the throws is

$$\begin{aligned} \Delta t = t_1 - t_2 &= \frac{2v_0 \sin(\theta_1 - \theta_2)(\cos \theta_2 - \cos \theta_1)}{g(\cos^2 \theta_2 - \cos^2 \theta_1)} \\ &= \frac{2v_0 \sin(\theta_1 - \theta_2)}{g(\cos \theta_2 + \cos \theta_1)} \end{aligned} \quad \text{Ans.}$$

12–98. The golf ball is hit at A with a speed of $v_A = 40 \text{ m/s}$ and directed at an angle of 30° with the horizontal as shown. Determine the distance d where the ball strikes the slope at B .



Coordinate System: The x - y coordinate system will be set so that its origin coincides with point A .

x -Motion: Here, $(v_A)_x = 40 \cos 30^\circ = 34.64 \text{ m/s}$, $x_A = 0$, and $x_B = d \left(\frac{5}{\sqrt{5^2 + 1}} \right) = 0.9806d$. Thus,

$$\begin{aligned} (+\rightarrow) \quad x_B &= x_A + (v_A)_x t \\ 0.9806d &= 0 + 34.64t \\ t &= 0.02831d \end{aligned} \quad (1)$$

y -Motion: Here, $(v_A)_y = 40 \sin 30^\circ = 20 \text{ m/s}$, $y_A = 0$, $y_B = d \left(\frac{1}{\sqrt{5^2 + 1}} \right) = 0.1961d$, and $a_y = -g = -9.81 \text{ m/s}^2$.

Thus,

$$\begin{aligned} (+\uparrow) \quad y_B &= y_A + (v_A)_y t + \frac{1}{2} a_y t^2 \\ 0.1961d &= 0 + 20t + \frac{1}{2} (-9.81)t^2 \\ 4.905t^2 - 20t + 0.1961d &= 0 \end{aligned} \quad (2)$$

Substituting Eq. (1) into Eq. (2) yields

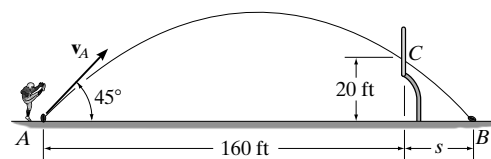
$$\begin{aligned} 4.905(0.02831d)^2 - 20(0.02831d) + 0.1961d &= 0 \\ 3.9303(10^{-3})d^2 - 0.37002d &= 0 \\ d[3.9303(10^{-3})d - 0.37002] &= 0 \end{aligned}$$

Since $d \neq 0$, then

$$\begin{aligned} 3.9303(10^{-3})d &= 0.37002 = 0 \\ d &= 94.1 \text{ m} \end{aligned}$$

Ans.

12–99. If the football is kicked at the 45° angle, determine its minimum initial speed v_A so that it passes over the goal post at C . At what distance s from the goal post will the football strike the ground at B ?



Coordinate System: The x - y coordinate system will be set so that its origin coincides with point A .

x -Motion: For the motion from A to C , $x_A = 0$, and $x_C = 160$ ft, $(v_A)_x = v_A \cos 45^\circ$, and $t = t_{AC}$. Thus,

$$\begin{aligned} (\rightarrow) \quad x_C &= x_A + (v_A)_x t \\ 160 &= 0 + v_A \cos 45^\circ t_{AC} \\ t_{AC} &= \frac{160}{v_A \cos 45^\circ} \end{aligned} \quad (1)$$

For the motion from A to B , $x_A = 0$, and $x_B = 160 + s$, $(v_A)_x = v_A \cos 45^\circ$, and $t = t_{AB}$. Thus,

$$\begin{aligned} (\rightarrow) \quad x_B &= x_A + (v_A)_x t \\ 160 + s &= 0 + v_A \cos 45^\circ t_{AB} \\ s &= v_A \cos 45^\circ (t_{AB}) - 160 \end{aligned} \quad (2)$$

y -Motion: For the motion from A to C , $y_A = 0$, and $y_C = 20$ ft, $(v_A)_y = v_A \sin 45^\circ$, and $a_y = -g = -32.2$ ft/s². Thus,

$$\begin{aligned} (+\uparrow) \quad y_C &= y_A + (v_A)_y t + \frac{1}{2} a_y t^2 \\ 20 &= 0 + v_A \sin 45^\circ t_{AC} + \frac{1}{2} (-32.2) t_{AC}^2 \\ 16.1 t_{AC}^2 - v_A \sin 45^\circ t_{AC} + 20 &= 0 \end{aligned} \quad (3)$$

For the motion from A to B , $y_A = y_B = 0$. Thus,

$$\begin{aligned} (+\uparrow) \quad y_B &= y_A + (v_A)_y t + \frac{1}{2} a_y t^2 \\ 0 &= 0 + v_A \sin 45^\circ (t_{AB}) + \frac{1}{2} (-32.2) t_{AB}^2 \\ t_{AB} (16.1 t_{AB} - v_A \sin 45^\circ) &= 0 \end{aligned}$$

Since $t_{AB} \neq 0$, then

$$16.1 t_{AB} - v_A \sin 45^\circ = 0 \quad (4)$$

Substituting Eq. (1) into Eq. (3) yields

$$\begin{aligned} 16.1 \left(\frac{160}{v_A \cos 45^\circ} \right)^2 - v_A \sin 45^\circ \left(\frac{160}{v_A \cos 45^\circ} \right) + 20 &= 0 \\ v_A &= 76.73 \text{ ft/s} = 76.7 \text{ ft/s} \end{aligned}$$

Ans.

Substituting this result into Eq. (4),

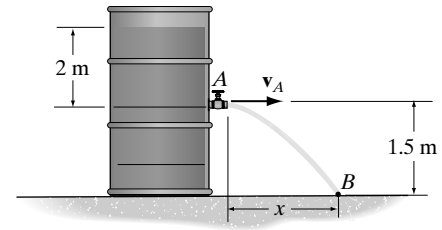
$$\begin{aligned} 16.1 t_{AB} - 76.73 \sin 45^\circ &= 0 \\ t_{AB} &= 3.370 \text{ s} \end{aligned}$$

Substituting the result of t_{AB} and v_A into Eq. (2),

$$\begin{aligned} s &= 76.73 \cos 45^\circ (3.370) - 160 \\ &= 22.9 \text{ ft} \end{aligned}$$

Ans.

***12–100.** The velocity of the water jet discharging from the orifice can be obtained from $v = \sqrt{2gh}$, where $h = 2$ m is the depth of the orifice from the free water surface. Determine the time for a particle of water leaving the orifice to reach point B and the horizontal distance x where it hits the surface.



Coordinate System: The x - y coordinate system will be set so that its origin coincides with point A . The speed of the water that the jet discharges from A is

$$v_A = \sqrt{2(9.81)(2)} = 6.264 \text{ m/s}$$

x-Motion: Here, $(v_A)_x = v_A = 6.264$ m/s, $x_A = 0$, $x_B = x$, and $t = t_A$. Thus,

$$\begin{aligned} (\rightarrow) \quad x_B &= x_A + (v_A)_x t \\ x &= 0 + 6.264 t_A \end{aligned} \quad (1)$$

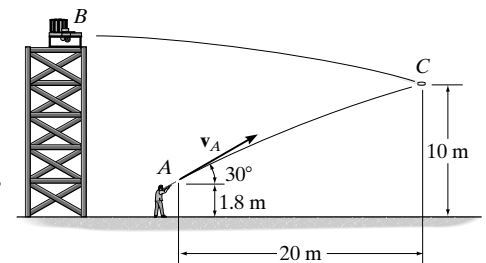
y-Motion: Here, $(v_A)_y = 0$, $a_y = -g = -9.81$ m/s², $y_A = 0$ m, $y_B = -1.5$ m, and $t = t_A$. Thus,

$$\begin{aligned} (+\uparrow) \quad y_B &= y_A + (v_A)_y t + \frac{1}{2} a_y t^2 \\ -1.5 &= 0 + 0 + \frac{1}{2} (-9.81) t_A^2 \\ t_A &= 0.553 \text{ s} \end{aligned}$$

Thus,

$$x = 0 + 6.264(0.553) = 3.46 \text{ m} \quad \text{Ans.}$$

•12–101. A projectile is fired from the platform at B . The shooter fires his gun from point A at an angle of 30° . Determine the muzzle speed of the bullet if it hits the projectile at C .



Coordinate System: The x - y coordinate system will be set so that its origin coincides with point A .

x-Motion: Here, $x_A = 0$ and $x_C = 20$ m. Thus,

$$\begin{aligned} (\rightarrow) \quad x_C &= x_A + (v_A)_x t \\ 20 &= 0 + v_A \cos 30^\circ t \end{aligned} \quad (1)$$

y-Motion: Here, $y_A = 1.8$, $(v_A)_y = v_A \sin 30^\circ$, and $a_y = -g = -9.81$ m/s². Thus,

$$\begin{aligned} (+\uparrow) \quad y_C &= y_A + (v_A)_y t + \frac{1}{2} a_y t^2 \\ 10 &= 1.8 + v_A \sin 30^\circ (t) + \frac{1}{2} (-9.81) (t)^2 \end{aligned}$$

Thus,

$$\begin{aligned} 10 - 1.8 &= \left(\frac{20 \sin 30^\circ}{\cos 30^\circ (t)} \right) (t) - 4.905 (t)^2 \\ t &= 0.8261 \text{ s} \end{aligned}$$

So that

$$v_A = \frac{20}{\cos 30^\circ (0.8261)} = 28.0 \text{ m/s} \quad \text{Ans.}$$

12–102. A golf ball is struck with a velocity of 80 ft/s as shown. Determine the distance d to where it will land.

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 80 \cos 55^\circ = 45.89$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = d \cos 10^\circ$, respectively.

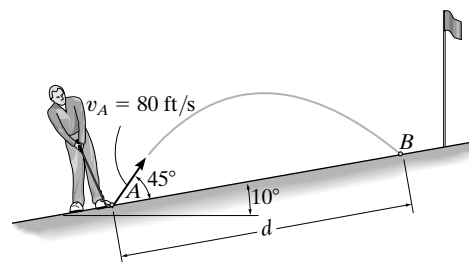
$$\begin{aligned} (\rightarrow) \quad s_x &= (s_0)_x + (v_0)_x t \\ d \cos 10^\circ &= 0 + 45.89t \end{aligned} \quad [1]$$

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 80 \sin 55^\circ = 65.53$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = d \sin 10^\circ$, respectively.

$$\begin{aligned} (+\uparrow) \quad s_y &= (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2 \\ d \sin 10^\circ &= 0 + 65.53t + \frac{1}{2}(-32.2)t^2 \end{aligned} \quad [2]$$

Solving Eqs. [1] and [2] yields

$$\begin{aligned} d &= 166 \text{ ft} \\ t &= 3.568 \text{ s} \end{aligned} \quad \text{Ans.}$$



12–103. The football is to be kicked over the goalpost, which is 15 ft high. If its initial speed is $v_A = 80$ ft/s, determine if it makes it over the goalpost, and if so, by how much, h .

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 80 \cos 60^\circ = 40.0$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = 25$ ft, respectively.

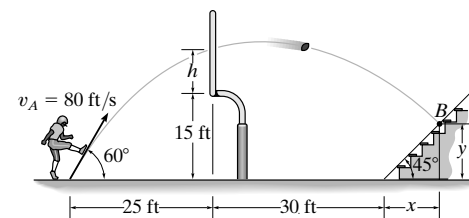
$$\begin{aligned} (\rightarrow) \quad s_x &= (s_0)_x + (v_0)_x t \\ 25 &= 0 + 40.0t \\ t &= 0.625 \text{ s} \end{aligned}$$

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 80 \sin 60^\circ = 69.28$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = H$, respectively.

$$\begin{aligned} (+\uparrow) \quad s_y &= (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2 \\ H &= 0 + 69.28(0.625) + \frac{1}{2}(-32.2)(0.625^2) \\ H &= 37.01 \text{ ft} \end{aligned}$$

Since $H > 15$ ft, the football is kicked over the goalpost.

$$h = H - 15 = 37.01 - 15 = 22.0 \text{ ft} \quad \text{Ans.}$$



***12–104.** The football is kicked over the goalpost with an initial velocity of $v_A = 80$ ft/s as shown. Determine the point $B(x, y)$ where it strikes the bleachers.

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 80 \cos 60^\circ = 40.0$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (55 + x)$, respectively.

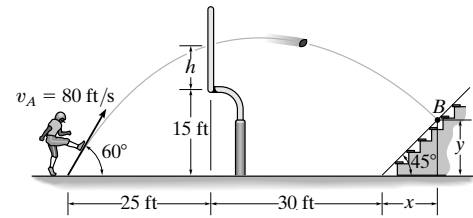
$$\begin{aligned} (\rightarrow) \quad s_x &= (s_0)_x + (v_0)_x t \\ 55 + x &= 0 + 40.0t \end{aligned} \quad [1]$$

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 80 \sin 60^\circ = 69.28$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y = x \tan 45^\circ = x$, respectively.

$$\begin{aligned} (+\uparrow) \quad s_y &= (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2 \\ x &= 0 + 69.28t + \frac{1}{2}(-32.2)t^2 \end{aligned} \quad [2]$$

Solving Eqs. [1] and [2] yields

$$\begin{aligned} t &= 2.969 \text{ s} \\ y = x &= 63.8 \text{ ft} \end{aligned} \quad \text{Ans.}$$



•12–105. The boy at A attempts to throw a ball over the roof of a barn with an initial speed of $v_A = 15$ m/s. Determine the angle θ_A at which the ball must be thrown so that it reaches its maximum height at C . Also, find the distance d where the boy should stand to make the throw.

Vertical Motion: The vertical component, of initial and final velocity are $(v_0)_y = (15 \sin \theta_A)$ m/s and $v_y = 0$, respectively. The initial vertical position is $(s_0)_y = 1$ m.

$$\begin{aligned} (+\uparrow) \quad v_y &= (v_0)_y + a_c t \\ 0 &= 15 \sin \theta_A + (-9.81)t \end{aligned} \quad [1]$$

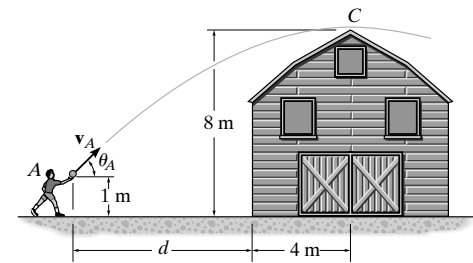
$$\begin{aligned} (+\uparrow) \quad s_y &= (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2 \\ 8 &= 1 + 15 \sin \theta_A t + \frac{1}{2}(-9.81)t^2 \end{aligned} \quad [2]$$

Solving Eqs. [1] and [2] yields

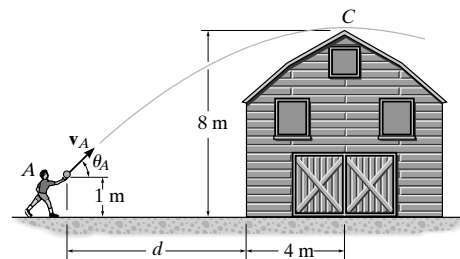
$$\begin{aligned} \theta_A &= 51.38^\circ = 51.4^\circ \\ t &= 1.195 \text{ s} \end{aligned} \quad \text{Ans.}$$

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = v_A \cos \theta_A = 15 \cos 51.38^\circ = 9.363$ m/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (d + 4)$ m, respectively.

$$\begin{aligned} (\rightarrow) \quad s_x &= (s_0)_x + (v_0)_x t \\ d + 4 &= 0 + 9.363(1.195) \\ d &= 7.18 \text{ m} \end{aligned} \quad \text{Ans.}$$



12–106. The boy at A attempts to throw a ball over the roof of a barn such that it is launched at an angle $\theta_A = 40^\circ$. Determine the minimum speed v_A at which he must throw the ball so that it reaches its maximum height at C . Also, find the distance d where the boy must stand so that he can make the throw.



Vertical Motion: The vertical components of initial and final velocity are $(v_0)_y = (v_A \sin 40^\circ) \text{ m/s}$ and $v_y = 0$, respectively. The initial vertical position is $(s_0)_y = 1 \text{ m}$.

$$\begin{aligned} (+\uparrow) \quad v_y &= (v_0)_y + a_c t \\ 0 &= v_A \sin 40^\circ + (-9.81) t \end{aligned} \quad [1]$$

$$\begin{aligned} (+\uparrow) \quad s_y &= (s_0)_y + (v_0)_y t + \frac{1}{2} (a_c)_y t^2 \\ 8 &= 1 + v_A \sin 40^\circ t + \frac{1}{2} (-9.81) t^2 \end{aligned} \quad [2]$$

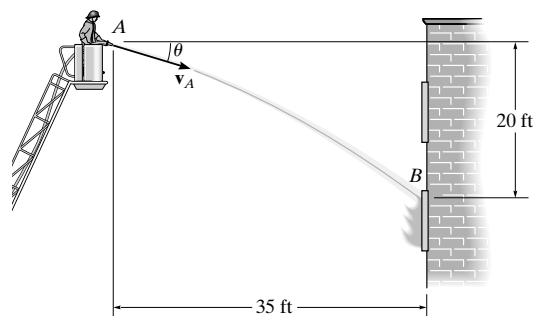
Solving Eqs. [1] and [2] yields

$$\begin{aligned} v_A &= 18.23 \text{ m/s} = 18.2 \text{ m/s} \\ t &= 1.195 \text{ s} \end{aligned} \quad \text{Ans.}$$

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = v_A \cos \theta_A = 18.23 \cos 40^\circ = 13.97 \text{ m/s}$. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (d + 4) \text{ m}$, respectively.

$$\begin{aligned} (\rightarrow) \quad s_x &= (s_0)_x + (v_0)_x t \\ d + 4 &= 0 + 13.97(1.195) \\ d &= 12.7 \text{ m} \end{aligned} \quad \text{Ans.}$$

12–107. The fireman wishes to direct the flow of water from his hose to the fire at B . Determine two possible angles θ_1 and θ_2 at which this can be done. Water flows from the hose at $v_A = 80 \text{ ft/s}$.



$$\begin{aligned} (\rightarrow) \quad s &= s_0 + v_0 t \\ 35 &= 0 + (80) \cos \theta \end{aligned}$$

$$\begin{aligned} (+\uparrow) \quad s &= s_0 + v_0 t + \frac{1}{2} a_c t^2 \\ -20 &= 0 - 80 \sin \theta t + \frac{1}{2} (-32.2) t^2 \end{aligned}$$

Thus,

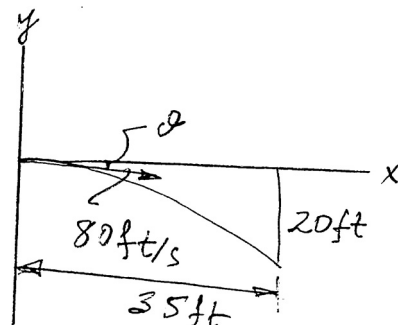
$$\begin{aligned} 20 &= 80 \sin \theta \frac{0.4375}{\cos \theta} t + 16.1 \left(\frac{0.1914}{\cos^2 \theta} \right) \\ 20 \cos^2 \theta &= 17.5 \sin 2\theta + 3.0816 \end{aligned}$$

Solving,

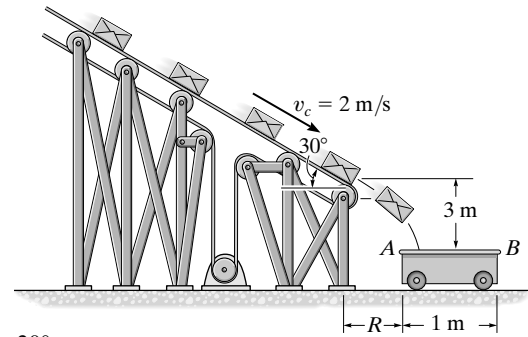
$$\begin{aligned} \theta_1 &= 25.0^\circ \quad (\text{below the horizontal}) \\ \theta_2 &= 85.2^\circ \quad (\text{above the horizontal}) \end{aligned}$$

Ans.

Ans.



***12–108.** Small packages traveling on the conveyor belt fall off into a 1-m-long loading car. If the conveyor is running at a constant speed of $v_c = 2 \text{ m/s}$, determine the smallest and largest distance R at which the end A of the car may be placed from the conveyor so that the packages enter the car.



Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 2 \sin 30^\circ = 1.00 \text{ m/s}$. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = 3 \text{ m}$, respectively.

$$\begin{aligned} (+\downarrow) \quad s_y &= (s_0)_y + (v_0)_y t + \frac{1}{2} (a_c)_y t^2 \\ 3 &= 0 + 1.00(t) + \frac{1}{2} (9.81)(t^2) \end{aligned}$$

Choose the positive root $t = 0.6867 \text{ s}$

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 2 \cos 30^\circ = 1.732 \text{ m/s}$ and the initial horizontal position is $(s_0)_x = 0$. If $s_x = R$, then

$$\begin{aligned} (\rightarrow) \quad s_x &= (s_0)_x + (v_0)_x t \\ R &= 0 + 1.732(0.6867) = 1.19 \text{ m} \end{aligned}$$

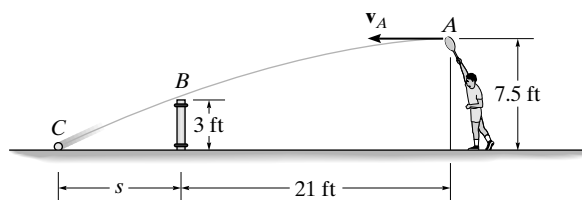
Ans.

If $s_x = R + 1$, then

$$\begin{aligned} (\rightarrow) \quad s_x &= (s_0)_x + (v_0)_x t \\ R + 1 &= 0 + 1.732(0.6867) \\ R &= 0.189 \text{ m} \end{aligned}$$

Ans.

•12–109. Determine the horizontal velocity v_A of a tennis ball at A so that it just clears the net at B . Also, find the distance s where the ball strikes the ground.



Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 0$. For the ball to travel from A to B , the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 3$ ft, respectively.

$$\begin{aligned} (+\uparrow) \quad s_y &= (s_0)_y + (v_0)_y t + \frac{1}{2} (a_c)_y t^2 \\ 3 &= 7.5 + 0 + \frac{1}{2} (-32.2) t_1^2 \\ t_1 &= 0.5287 \text{ s} \end{aligned}$$

For the ball to travel from A to C , the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 0$, respectively.

$$\begin{aligned} (+\uparrow) \quad s_y &= (s_0)_y + (v_0)_y t + \frac{1}{2} (a_c)_y t^2 \\ 0 &= 7.5 + 0 + \frac{1}{2} (-32.2) t_2^2 \\ t_2 &= 0.6825 \text{ s} \end{aligned}$$

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = v_A$. For the ball to travel from A to B , the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = 21$ ft, respectively. The time is $t = t_1 = 0.5287$ s.

$$\begin{aligned} (\leftarrow) \quad s_x &= (s_0)_x + (v_0)_x t \\ 21 &= 0 + v_A (0.5287) \\ v_A &= 39.72 \text{ ft/s} = 39.7 \text{ ft/s} \end{aligned} \quad \text{Ans.}$$

For the ball to travel from A to C , the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (21 + s)$ ft, respectively. The time is $t = t_2 = 0.6825$ s.

$$\begin{aligned} (\leftarrow) \quad s_x &= (s_0)_x + (v_0)_x t \\ 21 + s &= 0 + 39.72(0.6825) \\ s &= 6.11 \text{ ft} \end{aligned} \quad \text{Ans.}$$

12–110. It is observed that the skier leaves the ramp A at an angle $\theta_A = 25^\circ$ with the horizontal. If he strikes the ground at B , determine his initial speed v_A and the time of flight t_{AB} .

$$(\rightarrow) \quad s = v_0 t$$

$$100\left(\frac{4}{5}\right) = v_A \cos 25^\circ t_{AB}$$

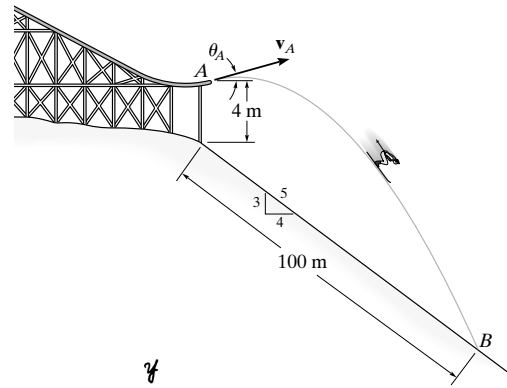
$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-4 - 100\left(\frac{3}{5}\right) = 0 + v_A \sin 25^\circ t_{AB} + \frac{1}{2}(-9.81)t_{AB}^2$$

Solving,

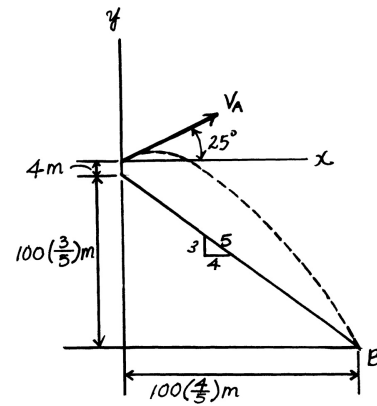
$$v_A = 19.4 \text{ m/s}$$

$$t_{AB} = 4.54 \text{ s}$$



Ans.

Ans.



12–111. When designing a highway curve it is required that cars traveling at a constant speed of 25 m/s must not have an acceleration that exceeds 3 m/s^2 . Determine the minimum radius of curvature of the curve.

Acceleration: Since the car is traveling with a constant speed, its tangential component of acceleration is zero, i.e., $a_t = 0$. Thus,

$$a = a_n = \frac{v^2}{\rho}$$

$$3 = \frac{25^2}{\rho}$$

$$\rho = 208 \text{ m}$$

Ans.

***12–112.** At a given instant, a car travels along a circular curved road with a speed of 20 m/s while decreasing its speed at the rate of 3 m/s^2 . If the magnitude of the car's acceleration is 5 m/s^2 , determine the radius of curvature of the road.

Acceleration: Here, the car's tangential component of acceleration of $a_t = -3 \text{ m/s}^2$. Thus,

$$a = \sqrt{a_t^2 + a_n^2}$$

$$5 = \sqrt{(-3)^2 + a_n^2}$$

$$a_n = 4 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho}$$

$$4 = \frac{20^2}{\rho}$$

$$\rho = 100 \text{ m}$$

Ans.

•**12–113.** Determine the maximum constant speed a race car can have if the acceleration of the car cannot exceed 7.5 m/s^2 while rounding a track having a radius of curvature of 200 m.

Acceleration: Since the speed of the race car is constant, its tangential component of acceleration is zero, i.e., $a_t = 0$. Thus,

$$a = a_n = \frac{v^2}{\rho}$$

$$7.5 = \frac{v^2}{200}$$

$$v = 38.7 \text{ m/s}$$

Ans.

12–114. An automobile is traveling on a horizontal circular curve having a radius of 800 ft. If the acceleration of the automobile is 5 ft/s^2 , determine the constant speed at which the automobile is traveling.

Acceleration: Since the automobile is traveling at a constant speed, $a_t = 0$.

Thus, $a_n = a = 5 \text{ ft/s}^2$. Applying Eq. 12–20, $a_n = \frac{v^2}{\rho}$, we have

$$v = \sqrt{\rho a_n} = \sqrt{800(5)} = 63.2 \text{ ft/s}$$

Ans.

12–115. A car travels along a horizontal circular curved road that has a radius of 600 m. If the speed is uniformly increased at a rate of 2000 km/h^2 , determine the magnitude of the acceleration at the instant the speed of the car is 60 km/h .

$$a_t = \left(\frac{2000 \text{ km}}{\text{h}^2} \right) \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right)^2 = 0.1543 \text{ m/s}^2$$

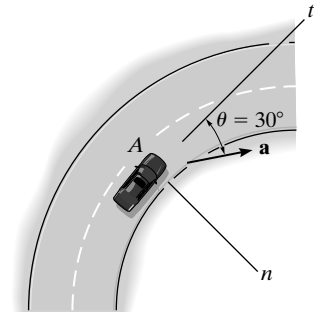
$$v = \left(\frac{60 \text{ km}}{\text{h}} \right) \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) = 16.67 \text{ m/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{16.67^2}{600} = 0.4630 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.1543^2 + 0.4630^2} = 0.488 \text{ m/s}^2$$

Ans.

***12–116.** The automobile has a speed of 80 ft/s at point *A* and an acceleration **a** having a magnitude of 10 ft/s², acting in the direction shown. Determine the radius of curvature of the path at point *A* and the tangential component of acceleration.



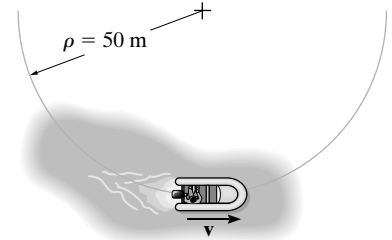
Acceleration: The tangential acceleration is

$$a_t = a \cos 30^\circ = 10 \cos 30^\circ = 8.66 \text{ ft/s}^2 \quad \text{Ans.}$$

and the normal acceleration is $a_n = a \sin 30^\circ = 10 \sin 30^\circ = 5.00 \text{ ft/s}^2$. Applying Eq. 12–20, $a_n = \frac{v^2}{\rho}$, we have

$$\rho = \frac{v^2}{a_n} = \frac{80^2}{5.00} = 1280 \text{ ft} \quad \text{Ans.}$$

•12–117. Starting from rest the motorboat travels around the circular path, $\rho = 50 \text{ m}$, at a speed $v = (0.8t) \text{ m/s}$, where t is in seconds. Determine the magnitudes of the boat's velocity and acceleration when it has traveled 20 m.



Velocity: The time for which the boat to travel 20 m must be determined first.

$$\begin{aligned} ds &= v dt \\ \int_0^{20 \text{ m}} ds &= \int_0^t 0.8t dt \\ t &= 7.071 \text{ s} \end{aligned}$$

The magnitude of the boat's velocity is

$$v = 0.8 (7.071) = 5.657 \text{ m/s} = 5.66 \text{ m/s} \quad \text{Ans.}$$

Acceleration: The tangential accelerations is

$$a_t = \dot{v} = 0.8 \text{ m/s}^2$$

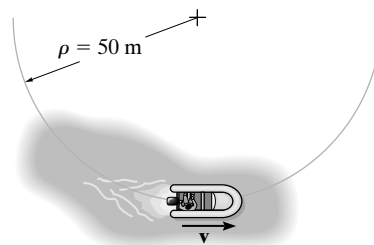
To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{5.657^2}{50} = 0.640 \text{ m/s}^2$$

Thus, the magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.8^2 + 0.640^2} = 1.02 \text{ m/s}^2 \quad \text{Ans.}$$

12–118. Starting from rest, the motorboat travels around the circular path, $\rho = 50$ m, at a speed $v = (0.2t^2)$ m/s, where t is in seconds. Determine the magnitudes of the boat's velocity and acceleration at the instant $t = 3$ s.



Velocity: When $t = 3$ s, the boat travels at a speed of

$$v = 0.2 (3^2) = 1.80 \text{ m/s} \quad \textbf{Ans.}$$

Acceleration: The tangential acceleration is $a_t = \dot{v} = (0.4t)$ m/s². When $t = 3$ s,

$$a_t = 0.4 (3) = 1.20 \text{ m/s}^2$$

To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{1.80^2}{50} = 0.0648 \text{ m/s}^2$$

Thus, the magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{1.20^2 + 0.0648^2} = 1.20 \text{ m/s}^2 \quad \textbf{Ans.}$$

12–119. A car moves along a circular track of radius 250 ft, and its speed for a short period of time $0 \leq t \leq 2$ s is $v = 3(t + t^2)$ ft/s, where t is in seconds. Determine the magnitude of the car's acceleration when $t = 2$ s. How far has it traveled in $t = 2$ s?

$$v = 3(t + t^2)$$

$$a_t = \frac{dv}{dt} = 3 + 6t$$

When $t = 2$ s,

$$a_t = 3 + 6(2) = 15 \text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{[3(2 + 2^2)]^2}{250} = 1.296 \text{ ft/s}^2$$

$$a = \sqrt{(15)^2 + (1.296)^2} = 15.1 \text{ ft/s}^2 \quad \textbf{Ans.}$$

$$ds = v dt$$

$$\int ds = \int_0^2 3(t + t^2) dt$$

$$\Delta s = \left[\frac{3}{2} t^2 + t^3 \right]_0^2$$

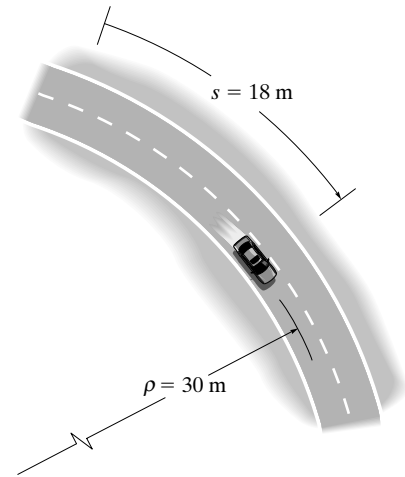
$$\Delta s = 14 \text{ ft} \quad \textbf{Ans.}$$

***12–120.** The car travels along the circular path such that its speed is increased by $a_t = (0.5e^t) \text{ m/s}^2$, where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 18 \text{ m}$ starting from rest. Neglect the size of the car.

$$\begin{aligned}\int_0^v dv &= \int_0^t 0.5e^t dt \\ v &= 0.5(e^t - 1) \\ \int_0^{18} ds &= 0.5 \int_0^t (e^t - 1) dt \\ 18 &= 0.5(e^t - t - 1)\end{aligned}$$

Solving,

$$\begin{aligned}t &= 3.7064 \text{ s} \\ v &= 0.5(e^{3.7064} - 1) = 19.85 \text{ m/s} = 19.9 \text{ m/s} \quad \text{Ans.} \\ a_t &= \dot{v} = 0.5e^t|_{t=3.7064 \text{ s}} = 20.35 \text{ m/s}^2 \\ a_n &= \frac{v^2}{\rho} = \frac{19.85^2}{30} = 13.14 \text{ m/s}^2 \\ a &= \sqrt{a_t^2 + a_n^2} = \sqrt{20.35^2 + 13.14^2} = 24.2 \text{ m/s}^2 \quad \text{Ans.}\end{aligned}$$



•12–121. The train passes point B with a speed of 20 m/s which is decreasing at $a_t = -0.5 \text{ m/s}^2$. Determine the magnitude of acceleration of the train at this point.

Radius of Curvature:

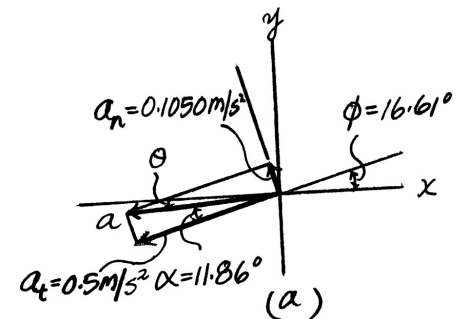
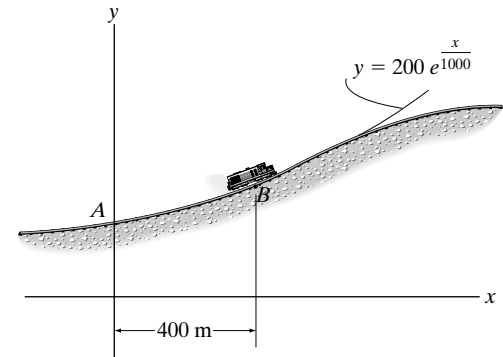
$$\begin{aligned}y &= 200e^{\frac{x}{1000}} \\ \frac{dy}{dx} &= 200\left(\frac{1}{1000}\right)e^{\frac{x}{1000}} = 0.2e^{\frac{x}{1000}} \\ \frac{d^2y}{dx^2} &= 0.2\left(\frac{1}{1000}\right)e^{\frac{x}{1000}} = 0.2(10^{-3})e^{\frac{x}{1000}} \\ \rho &= \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(0.2e^{\frac{x}{1000}}\right)^2\right]^{3/2}}{\left|0.2(10^{-3})e^{\frac{x}{1000}}\right|} \bigg|_{x=400 \text{ m}} = 3808.96 \text{ m}\end{aligned}$$

Acceleration:

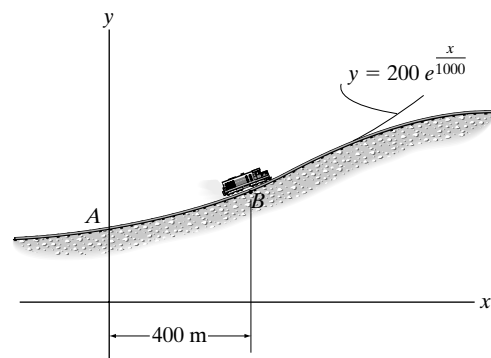
$$\begin{aligned}a_t &= \dot{v} = -0.5 \text{ m/s}^2 \\ a_n &= \frac{v^2}{\rho} = \frac{20^2}{3808.96} = 0.1050 \text{ m/s}^2\end{aligned}$$

The magnitude of the train's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-0.5)^2 + 0.1050^2} = 0.511 \text{ m/s}^2 \quad \text{Ans.}$$



12–122. The train passes point A with a speed of 30 m/s and begins to decrease its speed at a constant rate of $a_t = -0.25 \text{ m/s}^2$. Determine the magnitude of the acceleration of the train when it reaches point B , where $s_{AB} = 412 \text{ m}$.



Velocity: The speed of the train at B can be determined from

$$v_B^2 = v_A^2 + 2a_t(s_B - s_A)$$

$$v_B^2 = 30^2 + 2(-0.25)(412 - 0)$$

$$v_B = 26.34 \text{ m/s}$$

Radius of Curvature:

$$y = 200e^{\frac{x}{1000}}$$

$$\frac{dy}{dx} = 0.2e^{\frac{x}{1000}}$$

$$\frac{d^2y}{dx^2} = 0.2(10^{-3})e^{\frac{x}{1000}}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(0.2e^{\frac{x}{1000}}\right)^2\right]^{3/2}}{\left|0.2(10^{-3})e^{\frac{x}{1000}}\right|} \bigg|_{x=400 \text{ m}} = 3808.96 \text{ m}$$

Acceleration:

$$a_t = \dot{v} = -0.25 \text{ m/s}^2$$

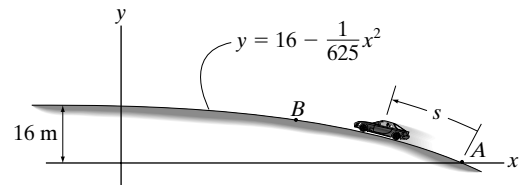
$$a_n = \frac{v^2}{\rho} = \frac{26.34^2}{3808.96} = 0.1822 \text{ m/s}^2$$

The magnitude of the train's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-0.25)^2 + 0.1822^2} = 0.309 \text{ m/s}^2$$

Ans.

12–123. The car passes point *A* with a speed of 25 m/s after which its speed is defined by $v = (25 - 0.15s)$ m/s. Determine the magnitude of the car's acceleration when it reaches point *B*, where $s = 51.5$ m.



Velocity: The speed of the car at *B* is

$$v_B = [25 - 0.15(51.5)] = 17.28 \text{ m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2(10^{-3})x$$

$$\frac{d^2y}{dx^2} = -3.2(10^{-3})$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(-3.2(10^{-3})x\right)^2\right]^{3/2}}{\left|-3.2(10^{-3})\right|} \bigg|_{x=50 \text{ m}} = 324.58 \text{ m}$$

Acceleration:

$$a_n = \frac{v_B^2}{\rho} = \frac{17.28^2}{324.58} = 0.9194 \text{ m/s}^2$$

$$a_t = v \frac{dv}{ds} = (25 - 0.15s)(-0.15) = (0.225s - 3.75) \text{ m/s}^2$$

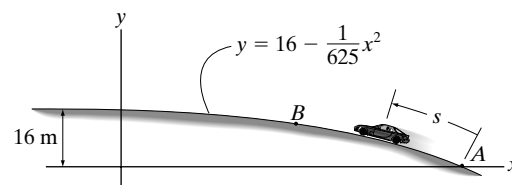
When the car is at *B* ($s = 51.5$ m)

$$a_t = [0.225(51.5) - 3.75] = -2.591 \text{ m/s}^2$$

Thus, the magnitude of the car's acceleration at *B* is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-2.591)^2 + 0.9194^2} = 2.75 \text{ m/s}^2 \quad \textbf{Ans.}$$

***12–124.** If the car passes point *A* with a speed of 20 m/s and begins to increase its speed at a constant rate of $a_t = 0.5 \text{ m/s}^2$, determine the magnitude of the car's acceleration when $s = 100 \text{ m}$.



Velocity: The speed of the car at *C* is

$$v_C^2 = v_A^2 + 2a_t(s_C - s_A)$$

$$v_C^2 = 20^2 + 2(0.5)(100 - 0)$$

$$v_C = 22.361 \text{ m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2(10^{-3})x$$

$$\frac{d^2y}{dx^2} = -3.2(10^{-3})$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(-3.2(10^{-3})x\right)^2\right]^{3/2}}{|-3.2(10^{-3})|} \bigg|_{x=0} = 312.5 \text{ m}$$

Acceleration:

$$a_t = \dot{v} = 0.5 \text{ m/s}^2$$

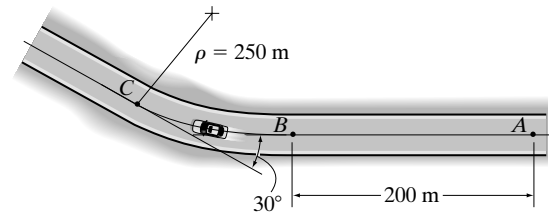
$$a_n = \frac{v_C^2}{\rho} = \frac{22.361^2}{312.5} = 1.60 \text{ m/s}^2$$

The magnitude of the car's acceleration at *C* is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 1.60^2} = 1.68 \text{ m/s}^2$$

Ans.

•12–125. When the car reaches point A it has a speed of 25 m/s. If the brakes are applied, its speed is reduced by $a_t = (-\frac{1}{4}t^{1/2})$ m/s². Determine the magnitude of acceleration of the car just before it reaches point C .



Velocity: Using the initial condition $v = 25$ m/s at $t = 0$ s,

$$\begin{aligned} dv &= a_t dt \\ \int_{25 \text{ m/s}}^v dv &= \int_0^t -\frac{1}{4}t^{1/2} dt \\ v &= \left(25 - \frac{1}{6}t^{3/2}\right) \text{ m/s} \end{aligned} \quad (1)$$

Position: Using the initial conditions $s = 0$ when $t = 0$ s,

$$\begin{aligned} \int ds &= \int v dt \\ \int_0^s ds &= \int_0^t \left(25 - \frac{1}{6}t^{3/2}\right) dt \\ s &= \left(25t - \frac{1}{15}t^{5/2}\right) \text{ m} \end{aligned}$$

Acceleration: When the car reaches C , $s_C = 200 + 250\left(\frac{\pi}{6}\right) = 330.90$ m. Thus,

$$330.90 = 25t - \frac{1}{15}t^{5/2}$$

Solving by trial and error,

$$t = 15.942 \text{ s}$$

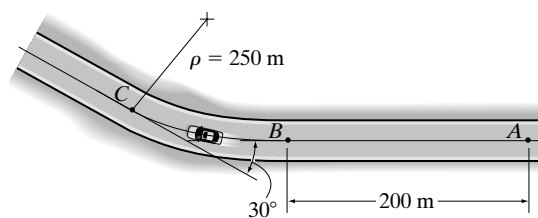
Thus, using Eq. (1).

$$\begin{aligned} v_C &= 25 - \frac{1}{6}(15.942)^{3/2} = 14.391 \text{ m/s} \\ (a_t)_C &= \dot{v} = -\frac{1}{4}(15.942^{1/2}) = -0.9982 \text{ m/s}^2 \\ (a_n)_C &= \frac{v_C^2}{\rho} = \frac{14.391^2}{250} = 0.8284 \text{ m/s}^2 \end{aligned}$$

The magnitude of the car's acceleration at C is

$$a = \sqrt{(a_t)_C^2 + (a_n)_C^2} = \sqrt{(-0.9982)^2 + 0.8284^2} = 1.30 \text{ m/s}^2 \quad \text{Ans.}$$

12–126. When the car reaches point *A*, it has a speed of 25 m/s. If the brakes are applied, its speed is reduced by $a_t = (0.001s - 1) \text{ m/s}^2$. Determine the magnitude of acceleration of the car just before it reaches point *C*.



Velocity: Using the initial condition $v = 25 \text{ m/s}$ at $t = 0 \text{ s}$,

$$\begin{aligned} dv &= ads \\ \int_{25 \text{ m/s}}^v v dv &= \int_0^s (0.001s - 1) ds \\ v &= \sqrt{0.001s^2 - 2s + 625} \end{aligned}$$

Acceleration: When the car is at point *C*, $s_C = 200 + 250\left(\frac{\pi}{6}\right) = 330.90 \text{ m}$. Thus, the speed of the car at *C* is

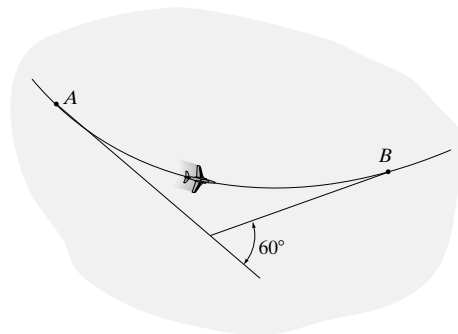
$$\begin{aligned} v_C &= \sqrt{0.001(330.90)^2 - 2(330.90) + 625} = 8.526 \text{ m/s} \\ (a_t)_C &= \dot{v} = [0.001(330.90) - 1] = -0.6691 \text{ m/s}^2 \\ (a_n)_C &= \frac{v_C^2}{\rho} = \frac{8.526^2}{250} = 0.2908 \text{ m/s}^2 \end{aligned}$$

The magnitude of the car's acceleration at *C* is

$$a = \sqrt{(a_t)_C^2 + (a_n)_C^2} = \sqrt{(-0.6691)^2 + 0.2908^2} = 0.730 \text{ m/s}^2$$

Ans.

12–127. Determine the magnitude of acceleration of the airplane during the turn. It flies along the horizontal circular path *AB* in 40 s, while maintaining a constant speed of 300 ft/s.



Acceleration: From the geometry in Fig. *a*, $2\phi + 60^\circ = 180^\circ$ or $\phi = 60^\circ$. Thus, $\frac{\theta}{2} = 90^\circ - 60^\circ$ or $\theta = 60^\circ = \frac{\pi}{3} \text{ rad}$.

$$s_{AB} = vt = 300(40) = 12\,000 \text{ ft}$$

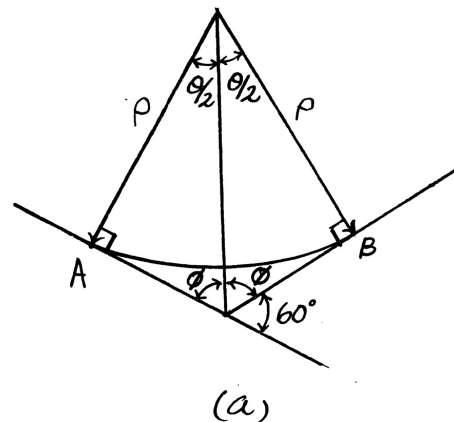
Thus,

$$\begin{aligned} \rho &= \frac{s_{AB}}{\theta} = \frac{12\,000}{\pi/3} = \frac{36\,000}{\pi} \text{ ft} \\ a_n &= \frac{v^2}{\rho} = \frac{300^2}{36\,000/\pi} = 7.854 \text{ ft/s}^2 \end{aligned}$$

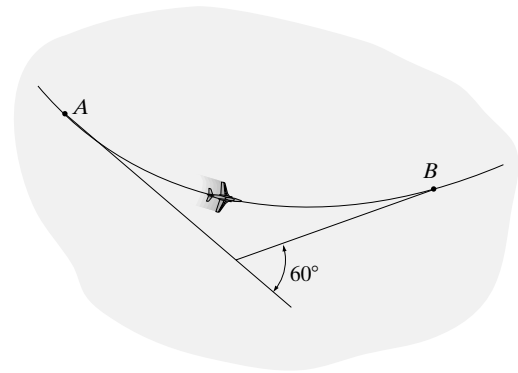
Since the airplane travels along the circular path with a constant speed, $a_t = 0$. Thus, the magnitude of the airplane's acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0^2 + 7.854^2} = 7.85 \text{ ft/s}^2$$

Ans.



***12–128.** The airplane flies along the horizontal circular path AB in 60 s. If its speed at point A is 400 ft/s, which decreases at a rate of $a_t = (-0.1t)$ ft/s², determine the magnitude of the plane's acceleration when it reaches point B .



Velocity: Using the initial condition $v = 400$ ft/s when $t = 0$ s,

$$\begin{aligned} dv &= a_t dt \\ \int_{400 \text{ ft/s}}^v dv &= \int_0^t -0.1t dt \\ v &= (400 - 0.05t^2) \text{ ft/s} \end{aligned}$$

Position: Using the initial condition $s = 0$ when $t = 0$ s,

$$\begin{aligned} \int ds &= \int v dt \\ \int_0^s ds &= \int_0^t (400 - 0.05t^2) dt \\ s &= (400t - 0.01667t^3) \text{ ft} \end{aligned}$$

Acceleration: From the geometry, $2\phi + 60^\circ = 180^\circ$ or $\phi = 60^\circ$. Thus, $\frac{\theta}{2} = 90^\circ - 60^\circ$ or $\theta = 60^\circ = \frac{\pi}{3}$ rad.

$$s_{AB} = 400(60) - 0.01667(60^3) = 20\,400 \text{ ft}$$

$$\rho = \frac{s_{AB}}{\theta} = \frac{20\,400}{\pi/3} = \frac{61\,200}{\pi} \text{ ft}$$

$$v_B = 400 - 0.05(60^2) = 220 \text{ ft/s}$$

$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{220^2}{61\,200/\pi} = 2.485 \text{ ft/s}^2$$

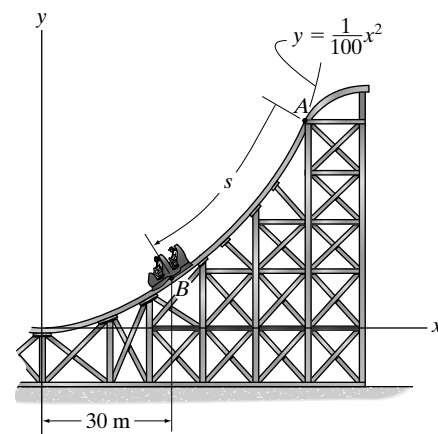
$$(a_t)_B = \dot{v} = -0.1(60) = -6 \text{ ft/s}^2$$

The magnitude of the airplane's acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-6)^2 + 2.485^2} = 6.49 \text{ ft/s}^2$$

Ans.

•12–129. When the roller coaster is at B , it has a speed of 25 m/s, which is increasing at $a_t = 3 \text{ m/s}^2$. Determine the magnitude of the acceleration of the roller coaster at this instant and the direction angle it makes with the x axis.



Radius of Curvature:

$$y = \frac{1}{100}x^2$$

$$\frac{dy}{dx} = \frac{1}{50}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{50}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(\frac{1}{50}x\right)^2\right]^{3/2}}{\left|\frac{1}{50}\right|} \bigg|_{x=30 \text{ m}} = 79.30 \text{ m}$$

Acceleration:

$$a_t = \dot{v} = 3 \text{ m/s}^2$$

$$a_n = \frac{v_B^2}{\rho} = \frac{25^2}{79.30} = 7.881 \text{ m/s}^2$$

The magnitude of the roller coaster's acceleration is

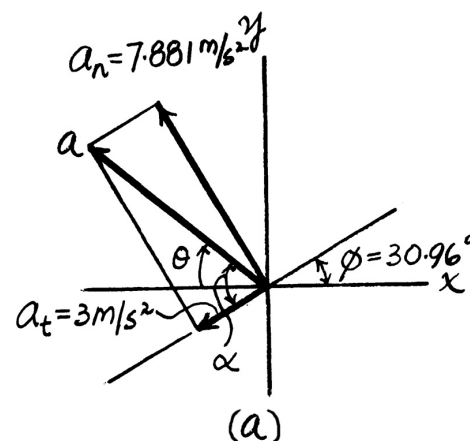
$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3^2 + 7.881^2} = 8.43 \text{ m/s}^2 \quad \text{Ans.}$$

The angle that the tangent at B makes with the x axis is $\phi = \tan^{-1}\left(\frac{dy}{dx}\bigg|_{x=30 \text{ m}}\right) = \tan^{-1}\left[\frac{1}{50}(30)\right] = 30.96^\circ$.

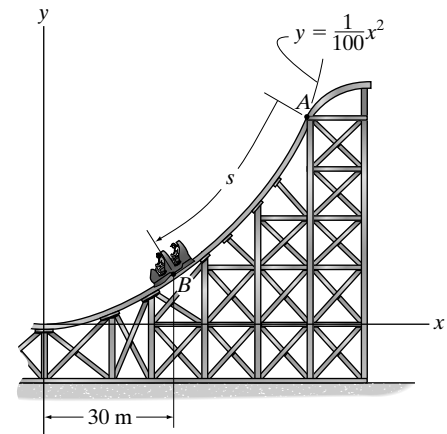
As shown in Fig. a , \mathbf{a}_n is always directed towards the center of curvature of the path. Here,

$\alpha = \tan^{-1}\left(\frac{a_n}{a_t}\right) = \tan^{-1}\left(\frac{7.881}{3}\right) = 69.16^\circ$. Thus, the angle θ that the roller coaster's acceleration makes with the x axis is

$$\theta = \alpha - \phi = 38.2^\circ \quad \text{Ans.}$$



12–130. If the roller coaster starts from rest at A and its speed increases at $a_t = (6 - 0.06s) \text{ m/s}^2$, determine the magnitude of its acceleration when it reaches B where $s_B = 40 \text{ m}$.



Velocity: Using the initial condition $v = 0$ at $s = 0$,

$$\begin{aligned} dv &= a_t dt \\ \int_0^v v dv &= \int_0^s (6 - 0.06s) ds \\ v &= \left(\sqrt{12s - 0.06s^2} \right) \text{ m/s} \end{aligned} \quad (1)$$

Thus,

$$v_B = \sqrt{12(40) - 0.06(40)^2} = 19.60 \text{ m/s}$$

Radius of Curvature:

$$\begin{aligned} y &= \frac{1}{100} x^2 \\ \frac{dy}{dx} &= \frac{1}{50} x \\ \frac{d^2y}{dx^2} &= \frac{1}{50} \\ \rho &= \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{\left| \frac{d^2y}{dx^2} \right|} = \frac{\left[1 + \left(\frac{1}{50} x \right)^2 \right]^{3/2}}{\left| \frac{1}{50} \right|} \bigg|_{x=30 \text{ m}} = 79.30 \text{ m} \end{aligned}$$

Acceleration:

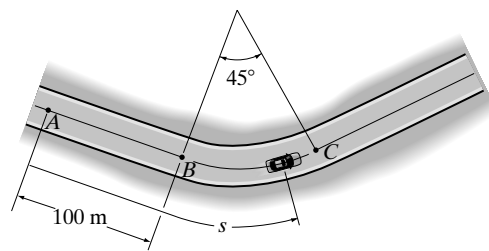
$$a_t = \dot{v} = 6 - 0.06(40) = 3.600 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{19.60^2}{79.30} = 4.842 \text{ m/s}^2$$

The magnitude of the roller coaster's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3.600^2 + 4.842^2} = 6.03 \text{ m/s}^2 \quad \text{Ans.}$$

12–131. The car is traveling at a constant speed of 30 m/s. The driver then applies the brakes at *A* and thereby reduces the car's speed at the rate of $a_t = (-0.08v)$ m/s², where v is in m/s. Determine the acceleration of the car just before it reaches point *C* on the circular curve. It takes 15 s for the car to travel from *A* to *C*.



Velocity: Using the initial condition $v = 30$ m/s when $t = 0$ s,

$$dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_{30 \text{ m/s}}^v \frac{dv}{-0.08v}$$

$$t = 12.5 \ln \frac{30}{v}$$

$$v = (30e^{-0.08t}) \text{ m/s} \quad (1)$$

Position: Using the initial condition $s = 0$ when $t = 0$ s,

$$ds = v dt$$

$$\int_0^s ds = \int_0^t 30e^{-0.08t} dt$$

$$s = [375(1 - e^{-0.08t})] \text{ m} \quad (2)$$

Acceleration:

$$s_C = 375(1 - e^{-0.08(15)}) = 262.05 \text{ m}$$

$$s_{BC} = s_C - s_B = 262.05 - 100 = 162.05 \text{ m}$$

$$\rho = \frac{s_{BC}}{\theta} = \frac{162.05}{\pi/4} = 206.33 \text{ m}$$

$$v_C = 30e^{-0.08(15)} = 9.036 \text{ m/s}$$

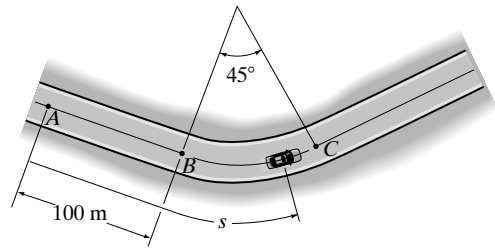
$$(a_n)_C = \frac{v_C^2}{\rho} = \frac{9.036^2}{206.33} = 0.3957 \text{ m/s}^2$$

$$(a_t)_C = \dot{v} = -0.08(9.036) = -0.7229 \text{ m/s}^2$$

The magnitude of the car's acceleration at point *C* is

$$a = \sqrt{(a_t)_C^2 + (a_n)_C^2} = \sqrt{(-0.7229)^2 + 0.3957^2} = 0.824 \text{ m/s}^2 \quad \text{Ans.}$$

***12–132.** The car is traveling at a speed of 30 m/s. The driver applies the brakes at *A* and thereby reduces the speed at the rate of $a_t = \left(-\frac{1}{8}t\right)$ m/s², where *t* is in seconds. Determine the acceleration of the car just before it reaches point *C* on the circular curve. It takes 15 s for the car to travel from *A* to *C*.



Velocity: Using the initial condition $v = 30$ m/s when $t = 0$ s,

$$dv = a_t dt$$

$$\int_{30 \text{ m/s}}^v dv = \int_0^t -\frac{1}{8} t dt$$

$$v = \left(30 - \frac{1}{16} t^2\right) \text{ m/s}$$

Position: Using the initial condition $s = 0$ when $t = 0$ s,

$$ds = v dt$$

$$\int_0^s ds = \int_0^t \left(30 - \frac{1}{16} t^2\right) dt$$

$$s = \left(30t - \frac{1}{48} t^3\right) \text{ m}$$

Acceleration:

$$s_C = 30(15) - \frac{1}{48} (15^3) = 379.6875 \text{ m}$$

$$s_{BC} = s_C - s_B = 379.6875 - 100 = 279.6875 \text{ m}$$

$$\rho = \frac{s_{BC}}{\theta} = \frac{279.6875}{\pi/4} = 356.11 \text{ m}$$

$$v_C = 30 - \frac{1}{16} (15^2) = 15.9375 \text{ m/s}$$

$$(a_t)_C = \dot{v} = -\frac{1}{8} (15) = -1.875 \text{ m/s}^2$$

$$(a_n)_C = \frac{v_C^2}{\rho} = \frac{15.9375^2}{356.11} = 0.7133 \text{ m/s}^2$$

The magnitude of the car's acceleration at point *C* is

$$a = \sqrt{(a_t)_C^2 + (a_n)_C^2} = \sqrt{(-1.875)^2 + 0.7133^2} = 2.01 \text{ m/s}^2$$

Ans.

•12–133. A particle is traveling along a circular curve having a radius of 20 m. If it has an initial speed of 20 m/s and then begins to decrease its speed at the rate of $a_t = (-0.25s)\text{m/s}^2$, determine the magnitude of the acceleration of the particle two seconds later.

Velocity: Using the initial condition $v = 20\text{ m/s}$ at $s = 0$.

$$v dv = a ds$$

$$\int_{20\text{ m/s}}^v v dv = \int_0^s -0.25 s ds$$

$$v = \left(\sqrt{400 - 0.25s^2} \right) \text{m/s}$$

Position: Using the initial condition $s = 0$ when $t = 0\text{ s}$.

$$dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{\sqrt{400 - 0.25s^2}} = 2 \int_0^s \frac{ds}{\sqrt{1600 - s^2}}$$

$$t = 2 \sin^{-1} \left(\frac{s}{40} \right)$$

$$s = (40 \sin(t/2)) \text{ m}$$

When $t = 2\text{ s}$,

$$s = 40 \sin(2/2) = 33.659\text{ m}$$

Acceleration:

$$a_t = \dot{v} = -0.25(33.659) = -8.415\text{ m/s}^2$$

$$v = \sqrt{400 - 0.25(33.659^2)} = 10.81\text{ m/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{10.81^2}{20} = 5.8385\text{ m/s}^2$$

The magnitude of the particle's acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-8.415)^2 + 5.8385^2} = 10.2\text{ m/s}^2$$

Ans.

12–134. A racing car travels with a constant speed of 240 km/h around the elliptical race track. Determine the acceleration experienced by the driver at A.

Radius of Curvature:

$$y^2 = 4\left(1 - \frac{x^2}{16}\right)$$

$$2y \frac{dy}{dx} = -\frac{x}{2} \quad (1)$$

$$\frac{dy}{dx} = -\frac{x}{4y} \quad (2)$$

Differentiating Eq. (1),

$$2\left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right) + 2y \frac{d^2y}{dx^2} = -\frac{1}{2}$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{1}{2} - 2\left(\frac{dy}{dx}\right)^2}{2y}$$

$$\frac{d^2y}{dx^2} = -\left[\frac{1 + 4\left(\frac{dy}{dx}\right)^2}{4y}\right] \quad (3)$$

Substituting Eq. (2) into Eq. (3) yields

$$\frac{d^2y}{dx^2} = -\left[\frac{4y^2 + x^2}{16y^3}\right]$$

Thus,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(-\frac{x}{4y}\right)^2\right]^{3/2}}{\left|\frac{4y^2 + x^2}{16y^3}\right|} = \frac{\left(1 + \frac{x^2}{16y^2}\right)^{3/2}}{\left(\frac{4y^2 + x^2}{16y^3}\right)} = \frac{(16y^2 + x^2)^{3/2}}{4(4y^2 + x^2)}$$

Acceleration: Since the race car travels with a constant speed along the track, $a_t = 0$.

At $x = 4$ km and $y = 0$,

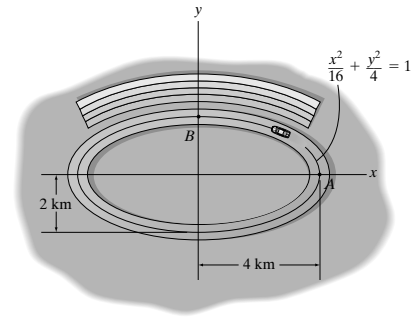
$$\rho_A = \frac{(16y^2 + x^2)^{3/2}}{4(4y^2 + x^2)} \bigg|_{\substack{x=4 \text{ km} \\ y=0}} = \frac{(0 + 4^2)^{3/2}}{4(0 + 4^2)} = 1 \text{ km} = 1000 \text{ m}$$

The speed of the race car is

$$v = \left(240 \frac{\text{km}}{\text{h}}\right) \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 66.67 \text{ m/s}$$

Thus,

$$a_A = \frac{v^2}{\rho_A} = \frac{66.67^2}{1000} = 4.44 \text{ m/s}^2 \quad \text{Ans.}$$



12–135. The racing car travels with a constant speed of 240 km/h around the elliptical race track. Determine the acceleration experienced by the driver at B .

Radius of Curvature:

$$y^2 = 4\left(1 - \frac{x^2}{16}\right)$$

$$2y \frac{dy}{dx} = -\frac{x}{2} \quad (1)$$

$$\frac{dy}{dx} = -\frac{x}{4y} \quad (2)$$

Differentiating Eq. (1),

$$2\left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right) + 2y \frac{d^2y}{dx^2} = -\frac{1}{2}$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{1}{2} - 2\left(\frac{dy}{dx}\right)^2}{2y}$$

$$\frac{d^2y}{dx^2} = -\left[\frac{1 + 4\left(\frac{dy}{dx}\right)^2}{4y}\right] \quad (3)$$

Substituting Eq. (2) into Eq. (3) yields

$$\frac{d^2y}{dx^2} = -\left[\frac{4y^2 + x^2}{16y^3}\right]$$

Thus,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(-\frac{x}{4y}\right)^2\right]^{3/2}}{\left|\frac{4y^2 + x^2}{16y^3}\right|} = \frac{\left(1 + \frac{x^2}{16y^2}\right)^{3/2}}{\left(\frac{4y^2 + x^2}{16y^3}\right)} = \frac{(16y^2 + x^2)^{3/2}}{4(4y^2 + x^2)}$$

Acceleration: Since the race car travels with a constant speed along the track, $a_t = 0$.

At $x = 0$ and $y = 2$ km

$$\rho_B = \frac{(16y^2 + x^2)^{3/2}}{4(4y^2 + x^2)} \bigg|_{\substack{x=0 \\ y=2 \text{ km}}} = \frac{[16(2^2) + 0]^{3/2}}{4[4(2^2) + 0]} = 8 \text{ km} = 8000 \text{ m}$$

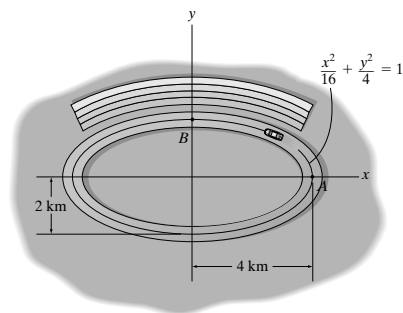
The speed of the car is

$$v = \left(240 \frac{\text{km}}{\text{h}}\right) \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 66.67 \text{ m/s}$$

Thus,

$$a_B = \frac{v^2}{\rho_B} = \frac{66.67^2}{8000} = 0.556 \text{ m/s}^2$$

Ans.



***12–136.** The position of a particle is defined by $\mathbf{r} = \{2 \sin(\frac{\pi}{4}t)\mathbf{i} + 2 \cos(\frac{\pi}{4}t)\mathbf{j} + 3t\mathbf{k}\}$ m, where t is in seconds. Determine the magnitudes of the velocity and acceleration at any instant.

Velocity:

$$\mathbf{r} = \left[2 \sin\left(\frac{\pi}{4}t\right)\mathbf{i} + 2 \cos\left(\frac{\pi}{4}t\right)\mathbf{j} + 3t\mathbf{k} \right] \text{ m}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \left[\frac{\pi}{2} \cos\left(\frac{\pi}{4}t\right)\mathbf{i} - \frac{\pi}{2} \sin\left(\frac{\pi}{4}t\right)\mathbf{j} + 3\mathbf{k} \right] \text{ m/s}$$

The magnitude of the velocity is

$$\begin{aligned} v &= \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{\left(\frac{\pi}{2} \cos \frac{\pi}{4} t\right)^2 + \left(-\frac{\pi}{2} \sin \frac{\pi}{4} t\right)^2 + 3^2} \\ &= \sqrt{\frac{\pi^2}{4} + 9} = 3.39 \text{ m/s} \end{aligned}$$

Ans.

Acceleration:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \left[-\frac{\pi^2}{8} \sin\left(\frac{\pi}{4}t\right)\mathbf{i} - \frac{\pi^2}{8} \cos\left(\frac{\pi}{4}t\right)\mathbf{j} \right] \text{ m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{\left(-\frac{\pi^2}{8} \sin \frac{\pi}{4} t\right)^2 + \left(-\frac{\pi^2}{8} \cos \frac{\pi}{4} t\right)^2} = \frac{\pi^2}{8} \text{ m/s}^2 = 1.23 \text{ m/s}^2 \quad \text{Ans.}$$

•12–137. The position of a particle is defined by $\mathbf{r} = \{t^3\mathbf{i} + 3t^2\mathbf{j} + 8t\mathbf{k}\}$ m, where t is in seconds. Determine the magnitude of the velocity and acceleration and the radius of curvature of the path when $t = 2$ s.

Velocity:

$$\mathbf{r} = [t^3\mathbf{i} + 3t^2\mathbf{j} + 8t\mathbf{k}] \text{ m}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = [3t^2\mathbf{i} + 6t\mathbf{j} + 8\mathbf{k}] \text{ m/s}$$

When $t = 2$ s,

$$\mathbf{v} = [3(2^2)\mathbf{i} + 6(2)\mathbf{j} + 8\mathbf{k}] = [12\mathbf{i} + 12\mathbf{j} + 8\mathbf{k}] \text{ m/s}$$

The magnitude of the velocity is

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{12^2 + 12^2 + 8^2} = 18.76 = 18.8 \text{ m/s} \quad \textbf{Ans.}$$

Acceleration:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = [6t\mathbf{i} + 6\mathbf{j}] \text{ m/s}^2$$

When $t = 2$ s,

$$\mathbf{a} = [6(2)\mathbf{i} + 6\mathbf{j}] = [12\mathbf{i} + 6\mathbf{j}] \text{ m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{12^2 + 6^2 + 0^2} = 13.42 = 13.4 \text{ m/s}^2 \quad \textbf{Ans.}$$

Since \mathbf{a}_t is parallel to \mathbf{v} , its magnitude can be obtained by the vector dot product

$a_t = \mathbf{a} \cdot \mathbf{u}_v$, where $\mathbf{u}_v = \frac{\mathbf{v}}{v} = 0.6396\mathbf{i} + 0.6396\mathbf{j} + 0.4264\mathbf{k}$. Thus,

$$a_t = (12\mathbf{i} + 6\mathbf{j}) \cdot (0.6396\mathbf{i} + 0.6396\mathbf{j} + 0.4264\mathbf{k}) = 11.51 \text{ m/s}^2$$

Thus,

$$a = \sqrt{a_t^2 + a_n^2}$$

$$13.42 = \sqrt{11.51^2 + a_n^2}$$

$$a_n = 6.889 \text{ m/s}^2$$

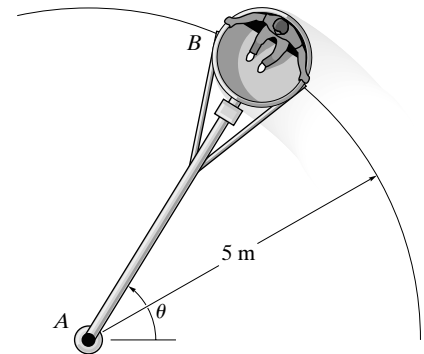
$$a_n = \frac{v^2}{\rho}$$

$$6.889 = \frac{18.76^2}{\rho}$$

$$\rho = 51.1 \text{ m}$$

Ans.

12–138. Car B turns such that its speed is increased by $(a_t)_B = (0.5e^t) \text{ m/s}^2$, where t is in seconds. If the car starts from rest when $\theta = 0^\circ$, determine the magnitudes of its velocity and acceleration when the arm AB rotates $\theta = 30^\circ$. Neglect the size of the car.



Velocity: The speed v in terms of time t can be obtained by applying $a = \frac{dv}{dt}$.

$$\begin{aligned} dv &= a dt \\ \int_0^v dv &= \int_0^t 0.5e^t dt \\ v &= 0.5(e^t - 1) \end{aligned} \quad [1]$$

When $\theta = 30^\circ$, the car has traveled a distance of $s = r\theta = 5\left(\frac{30^\circ}{180^\circ}\pi\right) = 2.618 \text{ m}$.

The time required for the car to travel this distance can be obtained by applying

$$v = \frac{ds}{dt}$$

$$\begin{aligned} ds &= v dt \\ \int_0^{2.618 \text{ m}} ds &= \int_0^t 0.5(e^t - 1) dt \\ 2.618 &= 0.5(e^t - t - 1) \end{aligned}$$

Solving by trial and error $t = 2.1234 \text{ s}$

Substituting $t = 2.1234 \text{ s}$ into Eq. [1] yields

$$v = 0.5(e^{2.1234} - 1) = 3.680 \text{ m/s} = 3.68 \text{ m/s} \quad \text{Ans.}$$

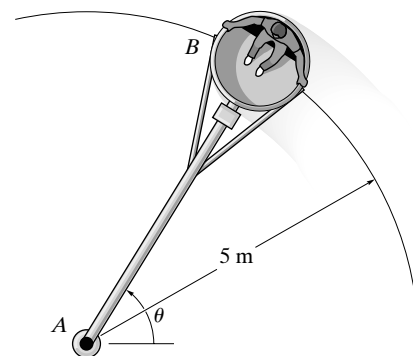
Acceleration: The tangential acceleration for the car at $t = 2.1234 \text{ s}$ is $a_t = 0.5e^{2.1234} = 4.180 \text{ m/s}^2$. To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{3.680^2}{5} = 2.708 \text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4.180^2 + 2.708^2} = 4.98 \text{ m/s}^2 \quad \text{Ans.}$$

12–139. Car B turns such that its speed is increased by $(a_t)_B = (0.5e^t) \text{ m/s}^2$, where t is in seconds. If the car starts from rest when $\theta = 0^\circ$, determine the magnitudes of its velocity and acceleration when $t = 2 \text{ s}$. Neglect the size of the car.



Velocity: The speed v in terms of time t can be obtained by applying $a = \frac{dv}{dt}$.

$$\begin{aligned} dv &= a dt \\ \int_0^v dv &= \int_0^t 0.5e^t dt \\ v &= 0.5(e^t - 1) \end{aligned}$$

When $t = 2 \text{ s}$, $v = 0.5(e^2 - 1) = 3.195 \text{ m/s} = 3.19 \text{ m/s}$ **Ans.**

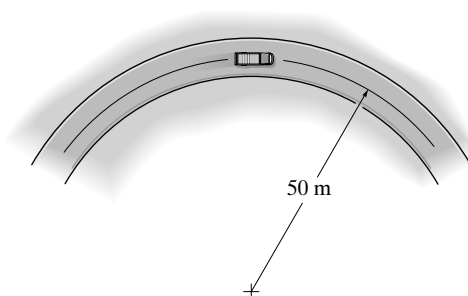
Acceleration: The tangential acceleration of the car at $t = 2 \text{ s}$ is $a_t = 0.5e^2 = 3.695 \text{ m/s}^2$. To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{3.195^2}{5} = 2.041 \text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3.695^2 + 2.041^2} = 4.22 \text{ m/s}^2 \quad \textbf{Ans.}$$

***12–140.** The truck travels at a speed of 4 m/s along a circular road that has a radius of 50 m . For a short distance from $s = 0$, its speed is then increased by $a_t = (0.05s) \text{ m/s}^2$, where s is in meters. Determine its speed and the magnitude of its acceleration when it has moved $s = 10 \text{ m}$.



Velocity: The speed v in terms of position s can be obtained by applying $v dv = a ds$.

$$\begin{aligned} v dv &= a ds \\ \int_{4 \text{ m/s}}^v v dv &= \int_0^s 0.05s ds \\ v &= (\sqrt{0.05s^2 + 16}) \text{ m/s} \end{aligned}$$

At $s = 10 \text{ m}$, $v = \sqrt{0.05(10^2) + 16} = 4.583 \text{ m/s} = 4.58 \text{ m/s}$ **Ans.**

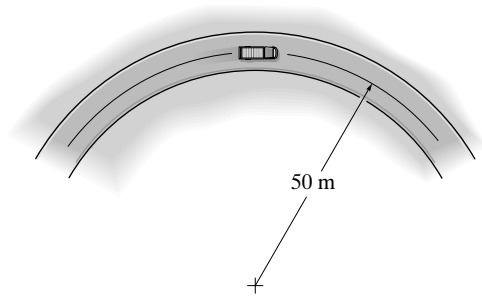
Acceleration: The tangential acceleration of the truck at $s = 10 \text{ m}$ is $a_t = 0.05(10) = 0.500 \text{ m/s}^2$. To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{4.583^2}{50} = 0.420 \text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.500^2 + 0.420^2} = 0.653 \text{ m/s}^2 \quad \textbf{Ans.}$$

•12–141. The truck travels along a circular road that has a radius of 50 m at a speed of 4 m/s. For a short distance when $t = 0$, its speed is then increased by $a_t = (0.4t) \text{ m/s}^2$, where t is in seconds. Determine the speed and the magnitude of the truck's acceleration when $t = 4 \text{ s}$.



Velocity: The speed v in terms of time t can be obtained by applying $a = \frac{dv}{dt}$.

$$\begin{aligned} dv &= a dt \\ \int_{4 \text{ m/s}}^v dv &= \int_0^t 0.4t dt \\ v &= (0.2t^2 + 4) \text{ m/s} \end{aligned}$$

When $t = 4 \text{ s}$, $v = 0.2(4^2) + 4 = 7.20 \text{ m/s}$ **Ans.**

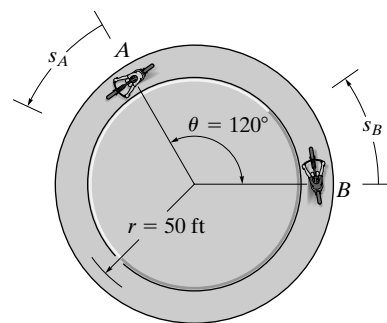
Acceleration: The tangential acceleration of the truck when $t = 4 \text{ s}$ is $a_t = 0.4(4) = 1.60 \text{ m/s}^2$. To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{7.20^2}{50} = 1.037 \text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{1.60^2 + 1.037^2} = 1.91 \text{ m/s}^2 \quad \textbf{Ans.}$$

12–142. Two cyclists, A and B , are traveling counterclockwise around a circular track at a constant speed of 8 ft/s at the instant shown. If the speed of A is increased at $(a_t)_A = (s_A) \text{ ft/s}^2$, where s_A is in feet, determine the distance measured counterclockwise along the track from B to A between the cyclists when $t = 1 \text{ s}$. What is the magnitude of the acceleration of each cyclist at this instant?



Distance Traveled: Initially, the distance between the cyclists is $d_0 = \rho\theta$
 $= 50\left(\frac{120^\circ}{180^\circ}\pi\right) = 104.72 \text{ ft}$. When $t = 1 \text{ s}$, cyclist B travels a distance of $s_B = 8(1)$
 $= 8 \text{ ft}$. The distance traveled by cyclist A can be obtained as follows

$$\begin{aligned} v_A dv_A &= a_A ds_A \\ \int_{8 \text{ ft/s}}^{v_A} v_A dv_A &= \int_0^{s_A} s_A ds_A \\ v_A &= \sqrt{s_A^2 + 64} \quad [1] \\ dt &= \frac{ds_A}{v_A} \\ \int_0^{1 \text{ s}} dt &= \int_0^{s_A} \frac{ds_A}{\sqrt{s_A^2 + 64}} \\ 1 &= \sinh^{-1}\left(\frac{s_A}{8}\right) \\ s_A &= 9.402 \text{ ft} \end{aligned}$$

Thus, the distance between the two cyclists after $t = 1 \text{ s}$ is

$$d = d_0 + s_A - s_B = 104.72 + 9.402 - 8 = 106 \text{ ft} \quad \text{Ans.}$$

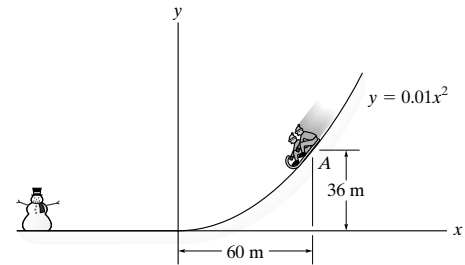
Acceleration: The tangential acceleration for cyclist A and B at $t = 1 \text{ s}$ is $(a_t)_A = s_A = 9.402 \text{ ft/s}^2$ and $(a_t)_B = 0$ (cyclist B travels at constant speed), respectively. At $t = 1 \text{ s}$, from Eq. [1], $v_A = \sqrt{9.402^2 + 64} = 12.34 \text{ ft/s}$. To determine normal acceleration, apply Eq. 12–20.

$$\begin{aligned} (a_n)_A &= \frac{v_A^2}{\rho} = \frac{12.34^2}{50} = 3.048 \text{ ft/s}^2 \\ (a_n)_B &= \frac{v_B^2}{\rho} = \frac{8^2}{50} = 1.28 \text{ ft/s}^2 \end{aligned}$$

The magnitude of the acceleration for cyclist A and B are

$$\begin{aligned} a_A &= \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{9.402^2 + 3.048^2} = 9.88 \text{ ft/s}^2 \quad \text{Ans.} \\ a_B &= \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{0^2 + 1.28^2} = 1.28 \text{ ft/s}^2 \quad \text{Ans.} \end{aligned}$$

12–143. A toboggan is traveling down along a curve which can be approximated by the parabola $y = 0.01x^2$. Determine the magnitude of its acceleration when it reaches point A , where its speed is $v_A = 10$ m/s, and it is increasing at the rate of $(a_t)_A = 3$ m/s².



Acceleration: The radius of curvature of the path at point A must be determined

first. Here, $\frac{dy}{dx} = 0.02x$ and $\frac{d^2y}{dx^2} = 0.02$, then

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \frac{[1 + (0.02x)^2]^{3/2}}{|0.02|} \Big|_{x=60 \text{ m}} = 190.57 \text{ m}$$

To determine the normal acceleration, apply Eq. 12–20.

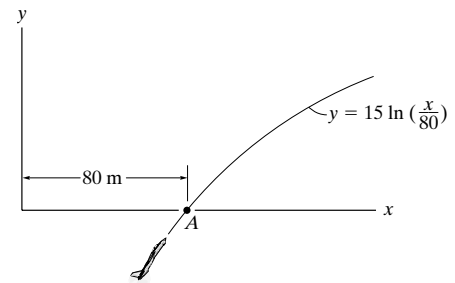
$$a_n = \frac{v^2}{\rho} = \frac{10^2}{190.57} = 0.5247 \text{ m/s}^2$$

Here, $a_t = \dot{v}_A = 3$ m/s. Thus, the magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3^2 + 0.5247^2} = 3.05 \text{ m/s}^2$$

Ans.

***12–144.** The jet plane is traveling with a speed of 120 m/s which is decreasing at 40 m/s² when it reaches point A . Determine the magnitude of its acceleration when it is at this point. Also, specify the direction of flight, measured from the x axis.



$$y = 15 \ln\left(\frac{x}{80}\right)$$

$$\frac{dy}{dx} = \frac{15}{x} \Big|_{x=80 \text{ m}} = 0.1875$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2} \Big|_{x=80 \text{ m}} = -0.002344$$

$$\rho \Big|_{x=80 \text{ m}} = \frac{[1 + (\frac{dy}{dx})^2]^{3/2}}{|\frac{d^2y}{dx^2}|} \Big|_{x=80 \text{ m}} = \frac{[1 + (0.1875)^2]^{3/2}}{|-0.002344|} = 449.4 \text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{(120)^2}{449.4} = 32.04 \text{ m/s}^2$$

$$a_t = -40 \text{ m/s}^2$$

$$a = \sqrt{(-40)^2 + (32.04)^2} = 51.3 \text{ m/s}^2$$

Ans.

Since

$$\frac{dy}{dx} = \tan \theta = 0.1875$$

$$\theta = 10.6^\circ$$

Ans.

•12–145. The jet plane is traveling with a constant speed of 110 m/s along the curved path. Determine the magnitude of the acceleration of the plane at the instant it reaches point A ($y = 0$).

$$y = 15 \ln\left(\frac{x}{80}\right)$$

$$\frac{dy}{dx} = \frac{15}{x} \bigg|_{x=80 \text{ m}} = 0.1875$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2} \bigg|_{x=80 \text{ m}} = -0.002344$$

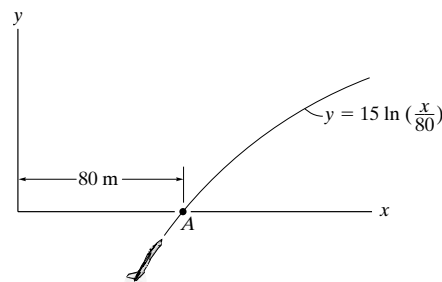
$$\rho \bigg|_{x=80 \text{ m}} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} \bigg|_{x=80 \text{ m}} = \frac{\left[1 + (0.1875)^2\right]^{3/2}}{|-0.002344|} = 449.4 \text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{(110)^2}{449.4} = 26.9 \text{ m/s}^2$$

Since the plane travels with a constant speed, $a_t = 0$. Hence

$$a = a_n = 26.9 \text{ m/s}^2$$

Ans.



12–146. The motorcyclist travels along the curve at a constant speed of 30 ft/s. Determine his acceleration when he is located at point A. Neglect the size of the motorcycle and rider for the calculation.

$$\frac{dy}{dx} = -\frac{500}{x^2} \bigg|_{x=100 \text{ ft}} = -0.05$$

$$\frac{d^2y}{dx^2} = \frac{1000}{x^3} \bigg|_{x=100 \text{ ft}} = 0.001$$

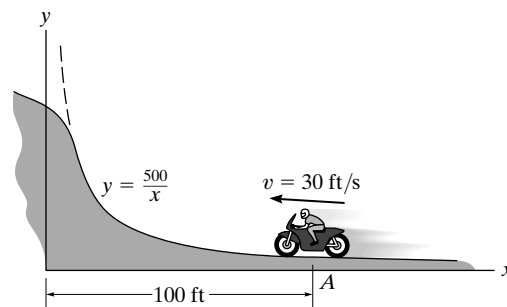
$$\rho \bigg|_{x=100 \text{ ft}} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} \bigg|_{x=100 \text{ ft}} = \frac{\left[1 + (-0.05)^2\right]^{3/2}}{|0.001|} = 1003.8 \text{ ft}$$

$$a_n = \frac{v^2}{\rho} = \frac{30^2}{1003.8} = 0.897 \text{ ft/s}^2$$

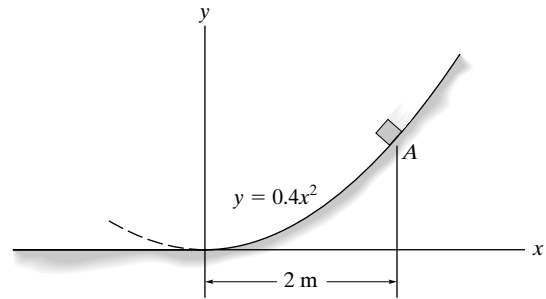
Since the motorcyclist travels with a constant speed, $a_t = 0$. Hence

$$a = a_n = 0.897 \text{ ft/s}^2$$

Ans.



12–147. The box of negligible size is sliding down along a curved path defined by the parabola $y = 0.4x^2$. When it is at A ($x_A = 2$ m, $y_A = 1.6$ m), the speed is $v_B = 8$ m/s and the increase in speed is $dv_B/dt = 4$ m/s². Determine the magnitude of the acceleration of the box at this instant.



$$y = 0.4x^2$$

$$\left. \frac{dy}{dx} \right|_{x=2 \text{ m}} = 0.8x \Big|_{x=2 \text{ m}} = 1.6$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=2 \text{ m}} = 0.8$$

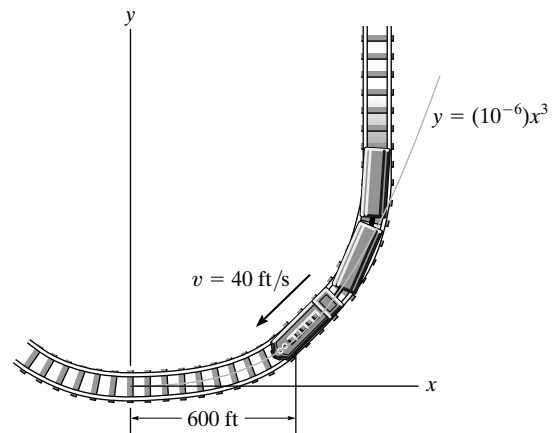
$$\rho = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{\left| \frac{d^2y}{dx^2} \right|} \Big|_{x=2 \text{ m}} = \frac{\left[1 + (1.6)^2 \right]^{3/2}}{|0.8|} = 8.396 \text{ m}$$

$$a_n = \frac{v_B^2}{\rho} = \frac{8^2}{8.396} = 7.622 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(4)^2 + (7.622)^2} = 8.61 \text{ m/s}^2$$

Ans.

***12–148.** A spiral transition curve is used on railroads to connect a straight portion of the track with a curved portion. If the spiral is defined by the equation $y = (10^{-6})x^3$, where x and y are in feet, determine the magnitude of the acceleration of a train engine moving with a constant speed of 40 ft/s when it is at point $x = 600$ ft.



$$y = (10^{-6})x^3$$

$$\left. \frac{dy}{dx} \right|_{x=600 \text{ ft}} = 3(10^{-6})x^2 \Big|_{x=600 \text{ ft}} = 1.08$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=600 \text{ ft}} = 6(10^{-6})x \Big|_{x=600 \text{ ft}} = 3.6(10)^{-3}$$

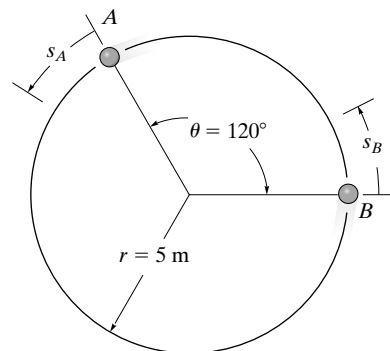
$$\rho \Big|_{x=600 \text{ ft}} = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{\left| \frac{d^2y}{dx^2} \right|} \Big|_{x=600 \text{ ft}} = \frac{\left[1 + (1.08)^2 \right]^{3/2}}{|3.6(10)^{-3}|} = 885.7 \text{ ft}$$

$$a_n = \frac{v^2}{\rho} = \frac{40^2}{885.7} = 1.81 \text{ ft/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0 + (1.81)^2} = 1.81 \text{ ft/s}^2$$

Ans.

•12–149. Particles A and B are traveling counterclockwise around a circular track at a constant speed of 8 m/s. If at the instant shown the speed of A begins to increase by $(a_t)_A = (0.4s_A) \text{ m/s}^2$, where s_A is in meters, determine the distance measured counterclockwise along the track from B to A when $t = 1 \text{ s}$. What is the magnitude of the acceleration of each particle at this instant?



Distance Traveled: Initially the distance between the particles is

$$d_0 = \rho d\theta = 5 \left(\frac{120^\circ}{180^\circ} \right) \pi = 10.47 \text{ m}$$

When $t = 1 \text{ s}$, B travels a distance of

$$d_B = 8(1) = 8 \text{ m}$$

The distance traveled by particle A is determined as follows:

$$\begin{aligned} v dv &= a ds \\ \int_{8 \text{ m/s}}^v v dv &= \int_0^s 0.4 s ds \\ v &= 0.6325 \sqrt{s^2 + 160} \\ dt &= \frac{ds}{v} \\ \int_0^t dt &= \int_0^s \frac{ds}{0.6325 \sqrt{s^2 + 160}} \\ 1 &= \frac{1}{0.6325} \left(\ln \left[\frac{\sqrt{s^2 + 160} + s}{\sqrt{160}} \right] \right) \\ s &= 8.544 \text{ m} \end{aligned} \quad (1)$$

Thus the distance between the two cyclists after $t = 1 \text{ s}$ is

$$d = 10.47 + 8.544 - 8 = 11.0 \text{ m} \quad \text{Ans.}$$

Acceleration:

For A , when $t = 1 \text{ s}$,

$$\begin{aligned} (a_t)_A &= \dot{v}_A = 0.4(8.544) = 3.4176 \text{ m/s}^2 \\ v_A &= 0.6325 \sqrt{8.544^2 + 160} = 9.655 \text{ m/s} \\ (a_n)_A &= \frac{v_A^2}{\rho} = \frac{9.655^2}{5} = 18.64 \text{ m/s}^2 \end{aligned}$$

The magnitude of the A 's acceleration is

$$a_A = \sqrt{3.4176^2 + 18.64^2} = 19.0 \text{ m/s}^2 \quad \text{Ans.}$$

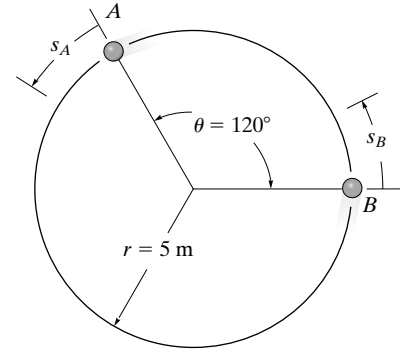
For B , when $t = 1 \text{ s}$,

$$\begin{aligned} (a_t)_B &= \dot{v}_B = 0 \\ (a_n)_B &= \frac{v_B^2}{\rho} = \frac{8^2}{5} = 12.80 \text{ m/s}^2 \end{aligned}$$

The magnitude of the B 's acceleration is

$$a_B = \sqrt{0^2 + 12.80^2} = 12.8 \text{ m/s}^2 \quad \text{Ans.}$$

12–150. Particles A and B are traveling around a circular track at a speed of 8 m/s at the instant shown. If the speed of B is increasing by $(a_t)_B = 4 \text{ m/s}^2$, and at the same instant A has an increase in speed of $(a_t)_A = 0.8t \text{ m/s}^2$, determine how long it takes for a collision to occur. What is the magnitude of the acceleration of each particle just before the collision occurs?



Distance Traveled: Initially the distance between the two particles is $d_0 = \rho\theta = 5\left(\frac{120^\circ}{180^\circ}\pi\right) = 10.47 \text{ m}$. Since particle B travels with a constant acceleration, distance can be obtained by applying equation

$$s_B = (s_0)_B + (v_0)_B t + \frac{1}{2} a_c t^2$$

$$s_B = 0 + 8t + \frac{1}{2} (4) t^2 = (8t + 2t^2) \text{ m} \quad [1]$$

The distance traveled by particle A can be obtained as follows.

$$dv_A = a_A dt$$

$$\int_{8 \text{ m/s}}^{v_A} dv_A = \int_0^t 0.8t dt$$

$$v_A = (0.4t^2 + 8) \text{ m/s} \quad [2]$$

$$ds_A = v_A dt$$

$$\int_0^{s_A} ds_A = \int_0^t (0.4t^2 + 8) dt$$

$$s_A = 0.1333t^3 + 8t$$

In order for the collision to occur

$$s_A + d_0 = s_B$$

$$0.1333t^3 + 8t + 10.47 = 8t + 2t^2$$

Solving by trial and error $t = 2.5074 \text{ s} = 2.51 \text{ s}$

Ans.

Note: If particle A strikes B then, $s_A = 5\left(\frac{240^\circ}{180^\circ}\pi\right) + s_B$. This equation will result in $t = 14.6 \text{ s} > 2.51 \text{ s}$.

Acceleration: The tangential acceleration for particle A and B when $t = 2.5074$ are $(a_t)_A = 0.8t = 0.8(2.5074) = 2.006 \text{ m/s}^2$ and $(a_t)_B = 4 \text{ m/s}^2$, respectively. When $t = 2.5074 \text{ s}$, from Eq. [1], $v_A = 0.4(2.5074^2) + 8 = 10.51 \text{ m/s}$ and $v_B = (v_0)_B + a_c t = 8 + 4(2.5074) = 18.03 \text{ m/s}$. To determine the normal acceleration, apply Eq. 12–20.

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{10.51^2}{5} = 22.11 \text{ m/s}^2$$

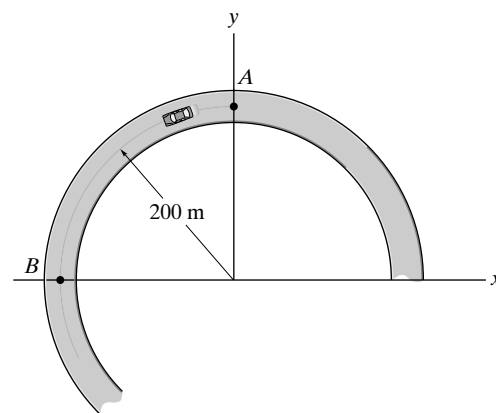
$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{18.03^2}{5} = 65.01 \text{ m/s}^2$$

The magnitude of the acceleration for particles A and B just before collision are

$$a_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{2.006^2 + 22.11^2} = 22.2 \text{ m/s}^2 \quad \text{Ans.}$$

$$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{4^2 + 65.01^2} = 65.1 \text{ m/s}^2 \quad \text{Ans.}$$

12–151. The race car travels around the circular track with a speed of 16 m/s. When it reaches point *A* it increases its speed at $a_t = (\frac{4}{3}v^{1/4})$ m/s², where v is in m/s. Determine the magnitudes of the velocity and acceleration of the car when it reaches point *B*. Also, how much time is required for it to travel from *A* to *B*?



$$a_t = \frac{4}{3} v^{\frac{1}{4}}$$

$$dv = a_t dt$$

$$dv = \frac{4}{3} v^{\frac{1}{4}} dt$$

$$\int_{16}^v 0.75 \frac{dv}{v^{\frac{1}{4}}} = \int_0^t dt$$

$$v^{\frac{3}{4}} \Big|_{16}^v = t$$

$$v^{\frac{3}{4}} - 8 = t$$

$$v = (t + 8)^{\frac{4}{3}}$$

$$ds = v dt$$

$$\int_0^s ds = \int_0^t (t + 8)^{\frac{4}{3}} dt$$

$$s = \frac{3}{7} (t + 8)^{\frac{7}{3}} \Big|_0^t$$

$$s = \frac{3}{7} (t + 8)^{\frac{7}{3}} - 54.86$$

$$\text{For } s = \frac{\pi}{2} (200) = 100\pi = \frac{3}{7} (t + 8)^{\frac{7}{3}} - 54.86$$

$$t = 10.108 \text{ s} = 10.1 \text{ s}$$

Ans.

$$v = (10.108 + 8)^{\frac{4}{3}} = 47.551 = 47.6 \text{ m/s}$$

Ans.

$$a_t = \frac{4}{3} (47.551)^{\frac{1}{4}} = 3.501 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(47.551)^2}{200} = 11.305 \text{ m/s}^2$$

$$a = \sqrt{(3.501)^2 + (11.305)^2} = 11.8 \text{ m/s}^2$$

Ans.

***12–152.** A particle travels along the path $y = a + bx + cx^2$, where a, b, c are constants. If the speed of the particle is constant, $v = v_0$, determine the x and y components of velocity and the normal component of acceleration when $x = 0$.

$$y = a + bx + cx^2$$

$$\dot{y} = b\dot{x} + 2cx\dot{x}$$

$$\ddot{y} = b\ddot{x} + 2c(\dot{x})^2 + 2cx\ddot{x}$$

$$\text{When } x = 0, \quad \dot{y} = b\dot{x}$$

$$v_0^2 = \dot{x}^2 + b^2\dot{x}^2$$

$$v_x = \dot{x} = \frac{v_0}{\sqrt{1 + b^2}}$$

Ans.

$$v_y = \frac{v_0 b}{\sqrt{1 + b^2}}$$

Ans.

$$a_n = \frac{v_0^2}{\rho}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|}$$

$$\frac{dy}{dx} = b + 2cx$$

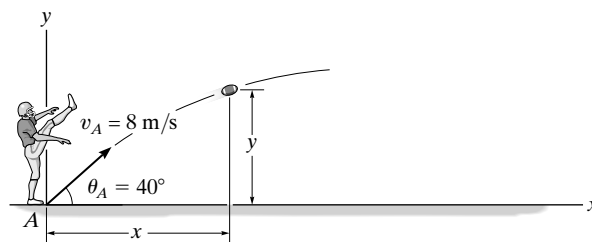
$$\frac{d^2y}{dx^2} = 2c$$

$$\text{At } x = 0, \quad \rho = \frac{(1 + b^2)^{3/2}}{2c}$$

$$a_n = \frac{2cv_0^2}{(1 + b^2)^{3/2}}$$

Ans.

•12–153. The ball is kicked with an initial speed $v_A = 8 \text{ m/s}$ at an angle $\theta_A = 40^\circ$ with the horizontal. Find the equation of the path, $y = f(x)$, and then determine the normal and tangential components of its acceleration when $t = 0.25 \text{ s}$.



Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 8 \cos 40^\circ = 6.128 \text{ m/s}$ and the initial horizontal and final positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$\begin{aligned} (\pm) \quad s_x &= (s_0)_x + (v_0)_x t \\ x &= 0 + 6.128t \end{aligned} \quad [1]$$

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 8 \sin 40^\circ = 5.143 \text{ m/s}$. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$\begin{aligned} (+\uparrow) \quad s_y &= (s_0)_y + (v_0)_y t + \frac{1}{2} (a_c)_y t^2 \\ y &= 0 + 5.143t + \frac{1}{2} (-9.81) (t^2) \end{aligned} \quad [2]$$

Eliminate t from Eqs [1] and [2], we have

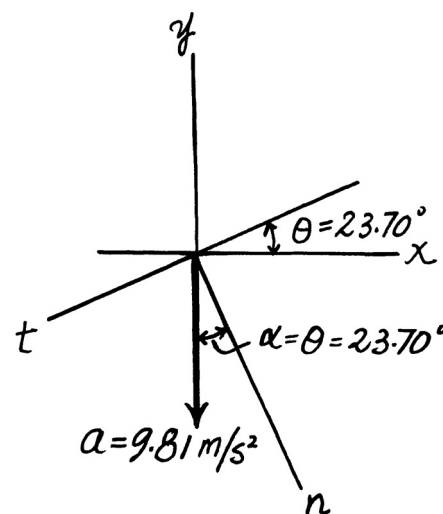
$$y = \{0.8391x - 0.1306x^2\} \text{ m} = \{0.839x - 0.131x^2\} \text{ m} \quad \text{Ans.}$$

The tangent of the path makes an angle $\theta = \tan^{-1} \frac{3.644}{4} = 42.33^\circ$ with the x axis.

Acceleration: When $t = 0.25 \text{ s}$, from Eq. [1], $x = 0 + 6.128(0.25) = 1.532 \text{ m}$. Here, $\frac{dy}{dx} = 0.8391 - 0.2612x$. At $x = 1.532 \text{ m}$, $\frac{dy}{dx} = 0.8391 - 0.2612(1.532) = 0.4389$ and the tangent of the path makes an angle $\theta = \tan^{-1} 0.4389 = 23.70^\circ$ with the x axis. The magnitude of the acceleration is $a = 9.81 \text{ m/s}^2$ and is directed downward. From the figure, $\alpha = 23.70^\circ$. Therefore,

$$a_t = a \sin \alpha = 9.81 \sin 23.70^\circ = 3.94 \text{ m/s}^2 \quad \text{Ans.}$$

$$a_n = a \cos \alpha = 9.81 \cos 23.70^\circ = 8.98 \text{ m/s}^2 \quad \text{Ans.}$$



12–154. The motion of a particle is defined by the equations $x = (2t + t^2)$ m and $y = (t^2)$ m, where t is in seconds. Determine the normal and tangential components of the particle's velocity and acceleration when $t = 2$ s.

Velocity: Here, $\mathbf{r} = \{(2t + t^2)\mathbf{i} + t^2\mathbf{j}\}$ m. To determine the velocity \mathbf{v} , apply Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{(2 + 2t)\mathbf{i} + 2t\mathbf{j}\} \text{ m/s}$$

When $t = 2$ s, $\mathbf{v} = [2 + 2(2)]\mathbf{i} + 2(2)\mathbf{j} = \{6\mathbf{i} + 4\mathbf{j}\}$ m/s. Then $v = \sqrt{6^2 + 4^2} = 7.21$ m/s. Since the velocity is always directed tangent to the path,

$$v_n = 0 \quad \text{and} \quad v_t = 7.21 \text{ m/s} \quad \textbf{Ans.}$$

The velocity \mathbf{v} makes an angle $\theta = \tan^{-1} \frac{4}{6} = 33.69^\circ$ with the x axis.

Acceleration: To determine the acceleration \mathbf{a} , apply Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{2\mathbf{i} + 2\mathbf{j}\} \text{ m/s}^2$$

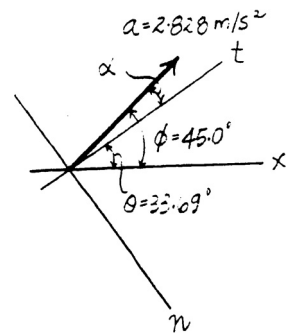
Then

$$a = \sqrt{2^2 + 2^2} = 2.828 \text{ m/s}^2$$

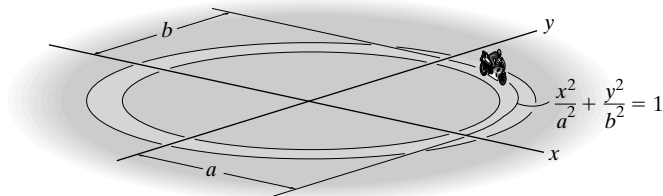
The acceleration \mathbf{a} makes an angle $\phi = \tan^{-1} \frac{2}{2} = 45.0^\circ$ with the x axis. From the figure, $\alpha = 45^\circ - 33.69^\circ = 11.31^\circ$. Therefore,

$$a_n = a \sin \alpha = 2.828 \sin 11.31^\circ = 0.555 \text{ m/s}^2 \quad \textbf{Ans.}$$

$$a_t = a \cos \alpha = 2.828 \cos 11.31^\circ = 2.77 \text{ m/s}^2 \quad \textbf{Ans.}$$



12–155. The motorcycle travels along the elliptical track at a constant speed v . Determine the greatest magnitude of the acceleration if $a > b$.



Acceleration: Differentiating twice the expression $y = \frac{b}{a}\sqrt{a^2 - x^2}$, we have

$$\frac{dy}{dx} = -\frac{bx}{a\sqrt{a^2 - x^2}}$$

$$\frac{d^2y}{dx^2} = -\frac{ab}{(a^2 - x^2)^{3/2}}$$

The radius of curvature of the path is

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(-\frac{bx}{a\sqrt{a^2 - x^2}}\right)^2\right]^{3/2}}{\left|-\frac{ab}{(a^2 - x^2)^{3/2}}\right|} = \frac{\left[1 + \frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2}}{\frac{ab}{(a^2 - x^2)^{3/2}}} \quad [1]$$

To have the maximum normal acceleration, the radius of curvature of the path must be a minimum. By observation, this happens when $y = 0$ and $x = a$. When $x \rightarrow a$,

$$\frac{b^2x^2}{a^2(a^2 - x^2)} \gg 1. \text{ Then, } \left[1 + \frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2} \rightarrow \left[\frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2} = \frac{b^3x^3}{a^3(a^2 - x^2)^{3/2}}.$$

Substituting this value into Eq. [1] yields $\rho = \frac{b^2}{a^4}x^3$. At $x = a$,

$$\rho = \frac{b^2}{a^4}(a^3) = \frac{b^2}{a}$$

To determine the normal acceleration, apply Eq. 12–20.

$$(a_n)_{\max} = \frac{v^2}{\rho} = \frac{v^2}{b^2/a} = \frac{a}{b^2}v^2$$

Since the motorcycle is traveling at a constant speed, $a_t = 0$. Thus,

$$a_{\max} = (a_n)_{\max} = \frac{a}{b^2}v^2 \quad \textbf{Ans.}$$

***12–156.** A particle moves along a circular path of radius 300 mm. If its angular velocity is $\theta = (2t^2)$ rad/s, where t is in seconds, determine the magnitude of the particle's acceleration when $t = 2$ s.

Time Derivatives:

$$\dot{r} = \ddot{r} = 0$$

$$\dot{\theta} = 2t^2|_{t=2\text{ s}} = 8 \text{ rad/s} \quad \ddot{\theta} = 4t|_{t=2\text{ s}} = 8 \text{ rad/s}^2$$

Velocity: The radial and transverse components of the particle's velocity are

$$v_r = \dot{r} = 0 \quad v_\theta = r\dot{\theta} = 0.3(8) = 2.4 \text{ m/s}$$

Thus, the magnitude of the particle's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0^2 + 2.4^2} = 2.4 \text{ m/s} \quad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.3(8^2) = -19.2 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.3(8) + 0 = 2.4 \text{ m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-19.2)^2 + 2.4^2} = 19.3 \text{ m/s}^2 \quad \textbf{Ans.}$$

•12–157. A particle moves along a circular path of radius 300 mm. If its angular velocity is $\dot{\theta} = (3t^2)$ rad/s, where t is in seconds, determine the magnitudes of the particle's velocity and acceleration when $\theta = 45^\circ$. The particle starts from rest when $\theta = 0^\circ$.

Time Derivatives: Using the initial condition $\theta = 0^\circ$ when $t = 0$ s,

$$d\theta = 3t^2 dt$$

$$\int_0^\theta d\theta = \int_0^t 3t^2 dt$$

$$\theta = (t^3) \text{ rad}$$

At $\theta = 45^\circ = \frac{\pi}{4}$ rad,

$$\frac{\pi}{4} = t^3 \quad t = 0.9226 \text{ s}$$

$$\dot{r} = \dot{r} = 0$$

$$\dot{\theta} = 3t^2|_{t=0.9226 \text{ s}} = 2.554 \text{ rad/s} \quad \ddot{\theta} = 6t|_{t=0.9226 \text{ s}} = 5.536 \text{ rad/s}^2$$

Velocity:

$$v_r = \dot{r} = 0 \quad v_\theta = r\dot{\theta} = 0.3(2.554) = 0.7661 \text{ m/s}$$

Thus, the magnitude of the particle's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0^2 + 0.7661^2} = 0.766 \text{ m/s}$$

Ans.

Acceleration:

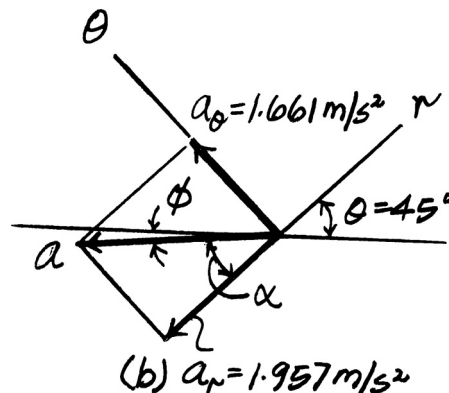
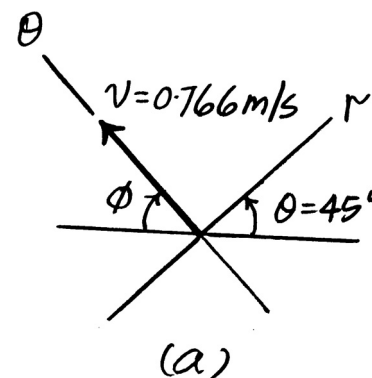
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.3(2.554^2) = -1.957 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.3(5.536) + 0 = 1.661 \text{ m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-1.957)^2 + 1.661^2} = 2.57 \text{ m/s}^2$$

Ans.



12–158. A particle moves along a circular path of radius 5 ft. If its position is $\theta = (e^{0.5t})$ rad, where t is in seconds, determine the magnitude of the particle's acceleration when $\theta = 90^\circ$.

Time Derivative: At $\theta = 90^\circ = \frac{\pi}{2}$ rad,

$$\frac{\pi}{2} = e^{0.5t} \quad t = 0.9032 \text{ s}$$

Using the result of t , the value of the first and second time derivative of r and θ are

$$\dot{r} = \ddot{r} = 0$$

$$\dot{\theta} = 0.5e^{0.5t} \Big|_{t=0.9032 \text{ s}} = 0.7854 \text{ rad/s}$$

$$\ddot{\theta} = 0.25e^{0.5t} \Big|_{t=0.9032 \text{ s}} = 0.3927 \text{ rad/s}^2$$

Acceleration:

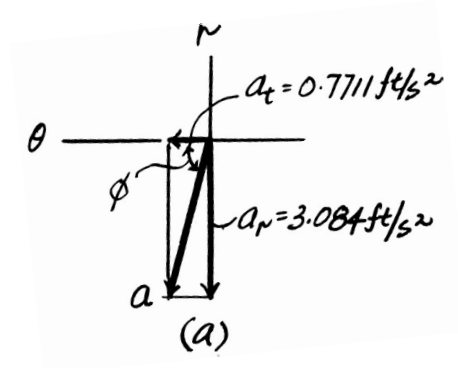
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5(0.7854^2) = -3.084 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 5(0.3927) + 0 = 1.963 \text{ ft/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-3.084)^2 + 1.963^2} = 3.66 \text{ ft/s}^2$$

Ans.



12–159. The position of a particle is described by $r = (t^3 + 4t - 4)$ mm and $\theta = (t^{3/2})$ rad, where t is in seconds. Determine the magnitudes of the particle's velocity and acceleration at the instant $t = 2$ s.

Time Derivatives: The first and second time derivative of r and θ when $t = 2$ s are

$$r = t^3 + 4t - 4 \Big|_{t=2\text{ s}} = 12 \text{ m}$$

$$\theta = t^{3/2}$$

$$\dot{r} = 3t^2 + 4 \Big|_{t=2\text{ s}} = 16 \text{ m/s}$$

$$\dot{\theta} = \frac{3}{2} t^{1/2} \Big|_{t=2\text{ s}} = 2.121 \text{ rad/s}$$

$$\ddot{r} = 6t \Big|_{t=2\text{ s}} = 12 \text{ m/s}^2$$

$$\ddot{\theta} = \frac{3}{4} t^{-1/2} \Big|_{t=2\text{ s}} = 0.5303 \text{ rad/s}^2$$

Velocity:

$$v_r = \dot{r} = 16 \text{ m/s}$$

$$v_\theta = r\dot{\theta} = 12(2.121) = 25.46 \text{ m/s}$$

Thus, the magnitude of the particle's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{16^2 + 25.46^2} = 30.1 \text{ m/s}$$

Ans.

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 12 - 12(2.121^2) = -42.0 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 12(0.5303) + 2(16)(2.121) = 74.25 \text{ m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-42.0)^2 + 74.25^2} = 85.3 \text{ m/s}^2$$

Ans.

***12–160.** The position of a particle is described by $r = (300e^{-0.5t})$ mm and $\theta = (0.3t^2)$ rad, where t is in seconds. Determine the magnitudes of the particle's velocity and acceleration at the instant $t = 1.5$ s.

Time Derivatives: The first and second time derivative of r and θ when $t = 1.5$ s are

$$\begin{aligned} r &= 300e^{-0.5t} \Big|_{t=1.5 \text{ s}} = 141.77 \text{ mm} & \theta &= 0.3t^2 \text{ rad} \\ \dot{r} &= -150e^{-0.5t} \Big|_{t=1.5 \text{ s}} = -70.85 \text{ mm/s} & \dot{\theta} &= 0.6t \Big|_{t=1.5 \text{ s}} = 0.9 \text{ rad/s} \\ \ddot{r} &= 75e^{-0.5t} \Big|_{t=1.5 \text{ s}} = 35.43 \text{ mm/s}^2 & \ddot{\theta} &= 0.6 \text{ rad/s}^2 \end{aligned}$$

Velocity:

$$v_r = \dot{r} = -70.85 \text{ mm/s} \quad v_\theta = r\dot{\theta} = 141.71(0.9) = 127.54 \text{ mm/s}$$

Thus, the magnitude of the particle's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-70.85)^2 + 127.54^2} = 146 \text{ mm/s} \quad \textbf{Ans.}$$

Acceleration:

$$\begin{aligned} a_r &= \ddot{r} - r\dot{\theta}^2 = 35.43 - 141.71(0.9^2) = -79.36 \text{ mm/s}^2 \\ a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} = 141.71(0.6) + 2(-70.85)(0.9) = -42.51 \text{ mm/s}^2 \end{aligned}$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-79.36)^2 + (-42.51)^2} = 90.0 \text{ mm/s}^2 \quad \textbf{Ans.}$$

•12–161. An airplane is flying in a straight line with a velocity of 200 mi/h and an acceleration of 3 mi/h². If the propeller has a diameter of 6 ft and is rotating at an angular rate of 120 rad/s, determine the magnitudes of velocity and acceleration of a particle located on the tip of the propeller.

$$v_{Pl} = \left(\frac{200 \text{ mi}}{\text{h}} \right) \left(\frac{5280 \text{ ft}}{1 \text{ mi}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) = 293.3 \text{ ft/s}$$

$$a_{Pl} = \left(\frac{3 \text{ mi}}{\text{h}^2} \right) \left(\frac{5280 \text{ ft}}{1 \text{ mi}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right)^2 = 0.00122 \text{ ft/s}^2$$

$$v_{Pr} = 120(3) = 360 \text{ ft/s}$$

$$v = \sqrt{v_{Pl}^2 + v_{Pr}^2} = \sqrt{(293.3)^2 + (360)^2} = 464 \text{ ft/s} \quad \textbf{Ans.}$$

$$a_{Pr} = \frac{v_{Pr}^2}{\rho} = \frac{(360)^2}{3} = 43200 \text{ ft/s}^2$$

$$a = \sqrt{a_{Pl}^2 + a_{Pr}^2} = \sqrt{(0.00122)^2 + (43200)^2} = 43.2(10^3) \text{ ft/s}^2 \quad \textbf{Ans.}$$

12–162. A particle moves along a circular path having a radius of 4 in. such that its position as a function of time is given by $\theta = (\cos 2t)$ rad, where t is in seconds. Determine the magnitude of the acceleration of the particle when $\theta = 30^\circ$.

$$\text{When } \theta = \frac{\pi}{6} \text{ rad,} \quad \frac{\pi}{6} = \cos 2t \quad t = 0.5099 \text{ s}$$

$$\dot{\theta} = \frac{d\theta}{dt} = -2 \sin 2t \bigg|_{t=0.5099 \text{ s}} = -1.7039 \text{ rad/s}$$

$$\ddot{\theta} = \frac{d^2\theta}{dt^2} = -4 \cos 2t \bigg|_{t=0.5099 \text{ s}} = -2.0944 \text{ rad/s}^2$$

$$r = 4 \quad \dot{r} = 0 \quad \ddot{r} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4(-1.7039)^2 = -11.6135 \text{ in./s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4(-2.0944) + 0 = -8.3776 \text{ in./s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-11.6135)^2 + (-8.3776)^2} = 14.3 \text{ in./s}^2$$

Ans.

12–163. A particle travels around a limaçon, defined by the equation $r = b - a \cos \theta$, where a and b are constants. Determine the particle's radial and transverse components of velocity and acceleration as a function of θ and its time derivatives.

$$r = b - a \cos \theta$$

$$\dot{r} = a \sin \theta \dot{\theta}$$

$$\dot{r} = a \cos \theta \dot{\theta}^2 + a \sin \theta \ddot{\theta}$$

$$v_r = \dot{r} = a \sin \theta \dot{\theta}$$

Ans.

$$v_\theta = r\dot{\theta} = (b - a \cos \theta)\dot{\theta}$$

Ans.

$$\begin{aligned} a_r = \ddot{r} - r\dot{\theta}^2 &= a \cos \theta \dot{\theta}^2 + a \sin \theta \ddot{\theta} - (b - a \cos \theta)\dot{\theta}^2 \\ &= (2a \cos \theta - b)\dot{\theta}^2 + a \sin \theta \ddot{\theta} \end{aligned}$$

Ans.

$$\begin{aligned} a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} &= (b - a \cos \theta)\ddot{\theta} + 2(a \sin \theta \dot{\theta})\dot{\theta} \\ &= (b - a \cos \theta)\ddot{\theta} + 2a\dot{\theta}^2 \sin \theta \end{aligned}$$

Ans.

***12–164.** A particle travels around a lituus, defined by the equation $r^2\theta = a^2$, where a is a constant. Determine the particle's radial and transverse components of velocity and acceleration as a function of θ and its time derivatives.

$$r^2\theta = a^2$$

$$r = a\theta^{-\frac{1}{2}}$$

$$r = a\left(-\frac{1}{2}\right)\theta^{-\frac{3}{2}}\dot{\theta}$$

$$\dot{r} = -\frac{1}{2}a\left(-\frac{3}{2}\theta^{-\frac{3}{2}}\dot{\theta}^2 + \theta^{-\frac{3}{2}}\ddot{\theta}\right)$$

$$v_r = \dot{r} = -\frac{1}{2}a\theta^{-\frac{3}{2}}\dot{\theta}$$

Ans.

$$v_\theta = r\dot{\theta} = a\theta^{-\frac{1}{2}}\dot{\theta}$$

Ans.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{1}{2}a\left(-\frac{3}{2}\theta^{-\frac{3}{2}}\dot{\theta}^2 + \theta^{-\frac{3}{2}}\ddot{\theta}\right) - a\theta^{-\frac{1}{2}}\dot{\theta}^2$$

$$= a\left[\left(\frac{3}{4}\theta^{-2} - 1\right)\theta^{-\frac{1}{2}}\dot{\theta}^2 - \frac{1}{2}\theta^{-\frac{3}{2}}\ddot{\theta}\right]$$

Ans.

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = a\theta^{-\frac{1}{2}}\ddot{\theta} + 2(a)\left(-\frac{1}{2}\right)\theta^{-\frac{3}{2}}\dot{\theta}\left(\dot{\theta}\right) = a\left[\ddot{\theta} - \frac{\dot{\theta}^2}{\theta}\right]\theta^{-\frac{1}{2}}$$

Ans.

•12–165. A car travels along the circular curve of radius $r = 300$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.4$ rad/s, which is increasing at the rate of $\ddot{\theta} = 0.2$ rad/s². Determine the magnitudes of the car's velocity and acceleration at this instant.

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = 0 \quad v_\theta = r\dot{\theta} = 300(0.4) = 120 \text{ ft/s}$$

Thus, the magnitude of the velocity of the car is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0^2 + 120^2} = 120 \text{ ft/s}$$

Ans.

Acceleration: Applying Eq. 12–29, we have

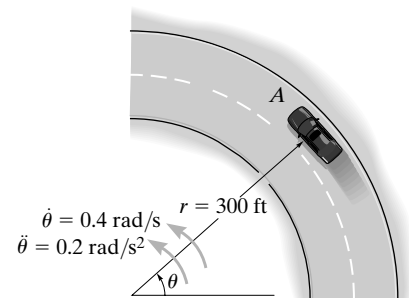
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 300(0.4^2) = -48.0 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 300(0.2) + 2(0)(0.4) = 60.0 \text{ ft/s}^2$$

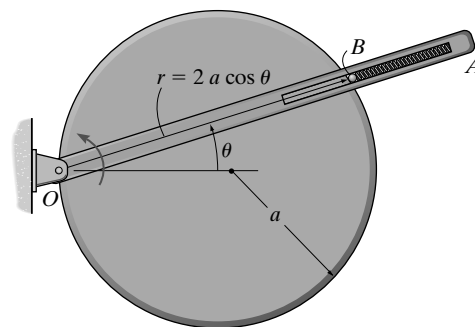
Thus, the magnitude of the acceleration of the car is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-48.0)^2 + 60.0^2} = 76.8 \text{ ft/s}^2$$

Ans.



12–166. The slotted arm OA rotates counterclockwise about O with a constant angular velocity of $\dot{\theta}$. The motion of pin B is constrained such that it moves on the fixed circular surface and along the slot in OA . Determine the magnitudes of the velocity and acceleration of pin B as a function of θ .



Time Derivatives:

$$r = 2a \cos \theta$$

$$\dot{r} = -2a \sin \theta \dot{\theta}$$

$$\ddot{r} = -2a [\cos \theta \ddot{\theta} + \sin \theta \dot{\theta}^2] = -2a [\cos \theta \ddot{\theta} + \sin \theta \dot{\theta}^2]$$

Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$. Thus,

$$\ddot{r} = -2a \cos \theta \dot{\theta}^2$$

Velocity:

$$v_r = \dot{r} = -2a \sin \theta \dot{\theta}$$

$$v_\theta = r\dot{\theta} = 2a \cos \theta \dot{\theta}$$

Thus, the magnitude of the pin's velocity is

$$\begin{aligned} v &= \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-2a \sin \theta \dot{\theta})^2 + (2a \cos \theta \dot{\theta})^2} \\ &= \sqrt{4a^2 \dot{\theta}^2 (\sin^2 \theta + \cos^2 \theta)} = 2a \dot{\theta} \end{aligned}$$

Ans.

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2a \cos \theta \dot{\theta}^2 - 2a \cos \theta \dot{\theta}^2 = -4a \cos \theta \dot{\theta}^2$$

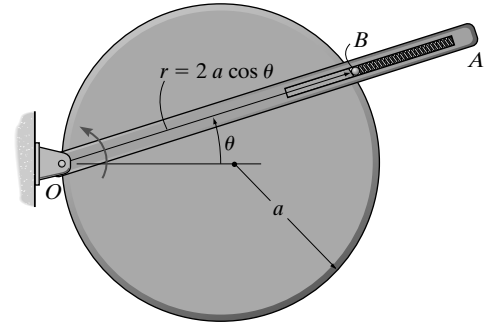
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-2a \sin \theta \dot{\theta})\dot{\theta} = -4a \sin \theta \dot{\theta}^2$$

Thus, the magnitude of the pin's acceleration is

$$\begin{aligned} a &= \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-4a \cos \theta \dot{\theta}^2)^2 + (-4a \sin \theta \dot{\theta}^2)^2} \\ &= \sqrt{16a^2 \dot{\theta}^4 (\cos^2 \theta + \sin^2 \theta)} = 4a \dot{\theta}^2 \end{aligned}$$

Ans.

12–167. The slotted arm OA rotates counterclockwise about O such that when $\theta = \pi/4$, arm OA is rotating with an angular velocity of $\dot{\theta}$ and an angular acceleration of $\ddot{\theta}$. Determine the magnitudes of the velocity and acceleration of pin B at this instant. The motion of pin B is constrained such that it moves on the fixed circular surface and along the slot in OA .



Time Derivatives:

$$r = 2a \cos \theta$$

$$\dot{r} = -2a \sin \theta \dot{\theta}$$

$$\ddot{r} = -2a[\cos \theta \ddot{\theta} + \sin \theta \dot{\theta}^2] = -2a[\cos \theta \ddot{\theta} + \sin \theta \dot{\theta}^2]$$

When $\theta = \frac{\pi}{4}$ rad,

$$r|_{\theta = \frac{\pi}{4}} = 2a \left(\frac{1}{\sqrt{2}} \right) = \sqrt{2}a$$

$$\dot{r}|_{\theta = \frac{\pi}{4}} = -2a \left(\frac{1}{\sqrt{2}} \right) \dot{\theta} = -\sqrt{2}a \dot{\theta}$$

$$\ddot{r}|_{\theta = \frac{\pi}{4}} = -2a \left(\frac{1}{\sqrt{2}} \ddot{\theta} + \frac{1}{\sqrt{2}} \dot{\theta}^2 \right) = -\sqrt{2}a(\ddot{\theta} + \dot{\theta}^2)$$

Velocity:

$$v_r = \dot{r} = -\sqrt{2}a \dot{\theta}$$

$$v_\theta = r \dot{\theta} = \sqrt{2}a \dot{\theta}$$

Thus, the magnitude of the pin's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-\sqrt{2}a \dot{\theta})^2 + (\sqrt{2}a \dot{\theta})^2} = 2a \dot{\theta}$$

Ans.

Acceleration:

$$a_r = \ddot{r} - r \dot{\theta}^2 = -\sqrt{2}a(\ddot{\theta} + \dot{\theta}^2) - \sqrt{2}a \dot{\theta}^2 = -\sqrt{2}a(2\dot{\theta}^2 + \ddot{\theta})$$

$$a_\theta = r \ddot{\theta} + 2\dot{r} \dot{\theta} = \sqrt{2}a \ddot{\theta} + 2(-\sqrt{2}a \dot{\theta})(\dot{\theta}) = \sqrt{2}a(\ddot{\theta} - 2\dot{\theta}^2)$$

Thus, the magnitude of the pin's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{[-\sqrt{2}a(2\dot{\theta}^2 + \ddot{\theta})]^2 + [\sqrt{2}a(\ddot{\theta} - 2\dot{\theta}^2)]^2} \\ = 2a \sqrt{4\dot{\theta}^4 + \ddot{\theta}^2}$$

Ans.

***12–168.** The car travels along the circular curve having a radius $r = 400$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.025$ rad/s, which is decreasing at the rate $\ddot{\theta} = -0.008$ rad/s². Determine the radial and transverse components of the car's velocity and acceleration at this instant and sketch these components on the curve.

$$r = 400 \quad \dot{r} = 0 \quad \ddot{r} = 0$$

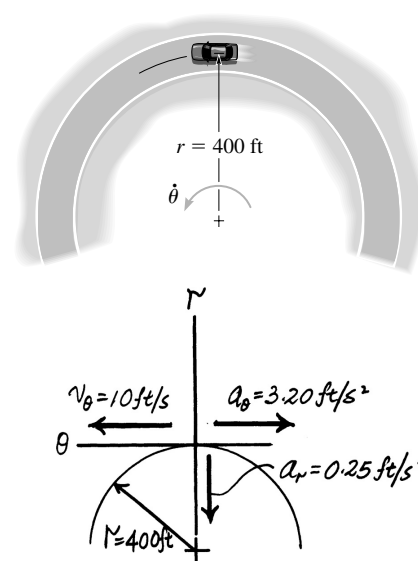
$$\dot{\theta} = 0.025 \quad \ddot{\theta} = -0.008$$

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 400(0.025) = 10 \text{ ft/s}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(0.025)^2 = -0.25 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 400(-0.008) + 0 = -3.20 \text{ ft/s}^2$$



Ans.

Ans.

Ans.

Ans.

•12–169. The car travels along the circular curve of radius $r = 400$ ft with a constant speed of $v = 30$ ft/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of the car's acceleration.

$$r = 400 \text{ ft} \quad \dot{r} = 0 \quad \ddot{r} = 0$$

$$v_r = \dot{r} = 0 \quad v_\theta = r\dot{\theta} = 400(\dot{\theta})$$

$$v = \sqrt{(0)^2 + (400\dot{\theta})^2} = 30$$

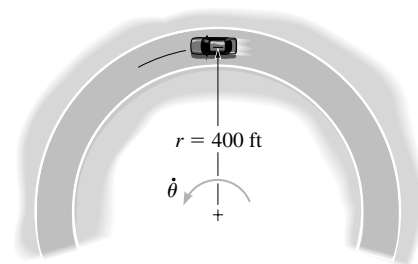
$$\dot{\theta} = 0.075 \text{ rad/s}$$

$$\ddot{\theta} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(0.075)^2 = -2.25 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 400(0) + 2(0)(0.075) = 0$$

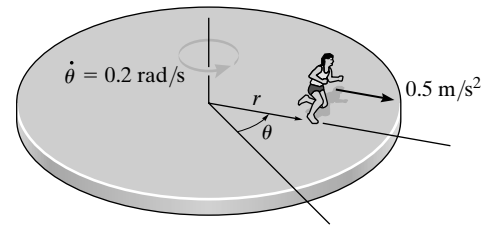
$$a = \sqrt{(-2.25)^2 + (0)^2} = 2.25 \text{ ft/s}^2$$



Ans.

Ans.

12–170. Starting from rest, the boy runs outward in the radial direction from the center of the platform with a constant acceleration of 0.5 m/s^2 . If the platform is rotating at a constant rate $\dot{\theta} = 0.2 \text{ rad/s}$, determine the radial and transverse components of the velocity and acceleration of the boy when $t = 3 \text{ s}$. Neglect his size.



Velocity: When $t = 3 \text{ s}$, the position of the boy is given by

$$s = (s_0)_r + (v_0)_r t + \frac{1}{2} (a_c)_r t^2$$

$$r = 0 + 0 + \frac{1}{2} (0.5)(3^2) = 2.25 \text{ m}$$

The boy's radial component of velocity is given by

$$\begin{aligned} v_r &= (v_0)_r + (a_c)_r t \\ &= 0 + 0.5(3) = 1.50 \text{ m/s} \end{aligned}$$

Ans.

The boy's transverse component of velocity is given by

$$v_\theta = r\dot{\theta} = 2.25(0.2) = 0.450 \text{ m/s}$$

Ans.

Acceleration: When $t = 3 \text{ s}$, $r = 2.25 \text{ m}$, $\dot{r} = v_r = 1.50 \text{ m/s}$, $\ddot{r} = 0.5 \text{ m/s}^2$, $\ddot{\theta} = 0$. Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0.5 - 2.25(0.2^2) = 0.410 \text{ m/s}^2$$

Ans.

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.25(0) + 2(1.50)(0.2) = 0.600 \text{ m/s}^2$$

Ans.

12–171. The small washer slides down the cord OA . When it is at the midpoint, its speed is 200 mm/s and its acceleration is 10 mm/s^2 . Express the velocity and acceleration of the washer at this point in terms of its cylindrical components.

$$OA = \sqrt{(400)^2 + (300)^2 + (700)^2} = 860.23 \text{ mm}$$

$$OB = \sqrt{(400)^2 + (300)^2} = 500 \text{ mm}$$

$$v_r = (200) \left(\frac{500}{860.23} \right) = 116 \text{ mm/s}$$

$$v_\theta = 0$$

$$v_z = (200) \left(\frac{700}{860.23} \right) = 163 \text{ mm/s}$$

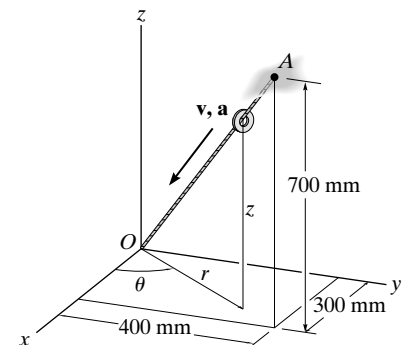
$$\text{Thus, } \mathbf{v} = \{-116\mathbf{u}_r - 163\mathbf{u}_z\} \text{ mm/s}$$

$$a_r = 10 \left(\frac{500}{860.23} \right) = 5.81$$

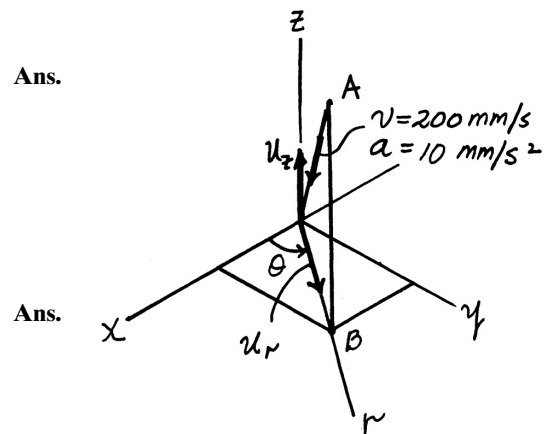
$$a_\theta = 0$$

$$a_z = 10 \left(\frac{700}{860.23} \right) = 8.14$$

$$\text{Thus, } \mathbf{a} = \{-5.81\mathbf{u}_r - 8.14\mathbf{u}_z\} \text{ mm/s}^2$$

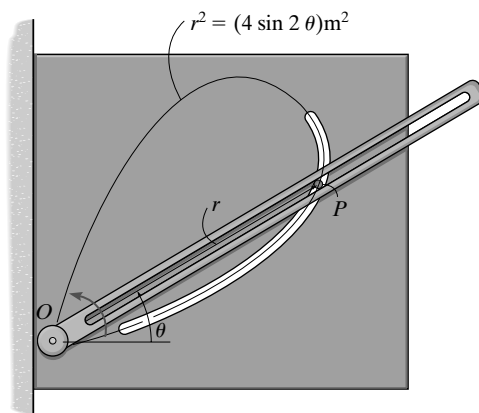


Ans.



Ans.

***12–172.** If arm OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 2 \text{ rad/s}$, determine the magnitudes of the velocity and acceleration of peg P at $\theta = 30^\circ$. The peg moves in the fixed groove defined by the lemniscate, and along the slot in the arm.



Time Derivatives:

$$r^2 = 4 \sin 2\theta$$

$$2r\dot{r} = 8 \cos 2\theta \dot{\theta}$$

$$\dot{r} = \left[\frac{4 \cos 2\theta \dot{\theta}}{r} \right] \text{ m/s} \quad \dot{\theta} = 2 \text{ rad/s}$$

$$2(r\ddot{r} + \dot{r}^2) = 8(-2 \sin 2\theta \dot{\theta}^2 + \cos 2\theta \ddot{\theta})$$

$$\ddot{r} = \left[\frac{4(\cos 2\theta \ddot{\theta} - 2 \sin 2\theta \dot{\theta}^2) - \dot{r}^2}{r} \right] \text{ m/s}^2 \quad \ddot{\theta} = 0$$

At $\theta = 30^\circ$,

$$r|_{\theta=30^\circ} = \sqrt{4 \sin 60^\circ} = 1.861 \text{ m}$$

$$\dot{r}|_{\theta=30^\circ} = \frac{(4 \cos 60^\circ)(2)}{1.861} = 2.149 \text{ m/s}$$

$$\ddot{r}|_{\theta=30^\circ} = \frac{4[0 - 2 \sin 60^\circ(2^2)] - (2.149)^2}{1.861} = -17.37 \text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = 2.149 \text{ m/s} \quad v_\theta = r\dot{\theta} = 1.861(2) = 3.722 \text{ m/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{2.149^2 + 3.722^2} = 4.30 \text{ m/s} \quad \text{Ans.}$$

Acceleration:

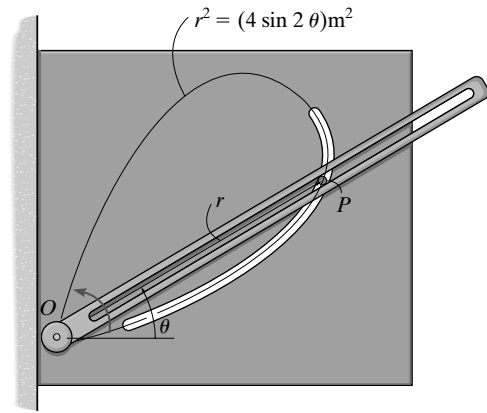
$$a_r = \ddot{r} - r\dot{\theta}^2 = -17.37 - 1.861(2^2) = -24.82 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(2.149)(2) = 8.597 \text{ m/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-24.82)^2 + 8.597^2} = 26.3 \text{ m/s}^2 \quad \text{Ans.}$$

•12–173. The peg moves in the curved slot defined by the lemniscate, and through the slot in the arm. At $\theta = 30^\circ$, the angular velocity is $\dot{\theta} = 2 \text{ rad/s}$, and the angular acceleration is $\ddot{\theta} = 1.5 \text{ rad/s}^2$. Determine the magnitudes of the velocity and acceleration of peg P at this instant.



Time Derivatives:

$$2r\dot{r} = 8 \cos 2\theta \dot{\theta}$$

$$\dot{r} = \left(\frac{4 \cos 2\theta \dot{\theta}}{r} \right) \text{ m/s}$$

$$\dot{\theta} = 2 \text{ rad/s}$$

$$2 \left(r\ddot{r} + \dot{r}^2 \right) = 8 \left(-2 \sin 2\theta \ddot{\theta} + \cos 2\theta \dot{\theta}^2 \right)$$

$$\ddot{r} = \left[\frac{4 \left(\cos 2\theta \ddot{\theta} - 2 \sin 2\theta \dot{\theta}^2 \right) - \dot{r}^2}{r} \right] \text{ m/s}^2$$

$$\ddot{\theta} = 1.5 \text{ rad/s}^2$$

At $\theta = 30^\circ$,

$$r|_{\theta=30^\circ} = \sqrt{4 \sin 60^\circ} = 1.861 \text{ m}$$

$$\dot{r}|_{\theta=30^\circ} = \frac{(4 \cos 60^\circ)(2)}{1.861} = 2.149 \text{ m/s}$$

$$\ddot{r}|_{\theta=30^\circ} = \frac{4 \left[\cos 60^\circ (1.5) - 2 \sin 60^\circ (2^2) \right] - (2.149)^2}{1.861} = -15.76 \text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = 2.149 \text{ m/s}$$

$$v_\theta = r\dot{\theta} = 1.861(2) = 3.722 \text{ m/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{a_r^2 + a_\theta^2} = \sqrt{2.149^2 + 3.722^2} = 4.30 \text{ m/s}$$

Ans.

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -15.76 - 1.861(2^2) = -23.20 \text{ m/s}^2$$

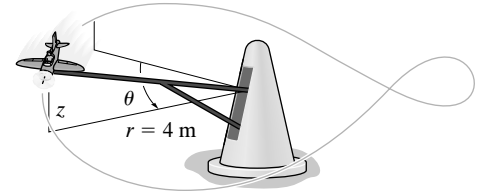
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.861(1.5) + 2(2.149)(2) = 11.39 \text{ m/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-23.20)^2 + 11.39^2} = 25.8 \text{ m/s}^2$$

Ans.

12–174. The airplane on the amusement park ride moves along a path defined by the equations $r = 4$ m, $\theta = (0.2t)$ rad, and $z = (0.5 \cos \theta)$ m, where t is in seconds. Determine the cylindrical components of the velocity and acceleration of the airplane when $t = 6$ s.



$$r = 4 \text{ m} \qquad \theta = 0.2t \Big|_{t=6 \text{ s}} = 1.2 \text{ rad}$$

$$\dot{r} = 0 \qquad \dot{\theta} = 0.2 \text{ rad/s}$$

$$\ddot{r} = 0 \qquad \ddot{\theta} = 0$$

$$z = 0.5 \cos \theta \qquad \dot{z} = -0.5 \sin \theta \dot{\theta} \Big|_{\theta=1.2 \text{ rad}} = -0.0932 \text{ m/s}$$

$$\ddot{z} = -0.5 [\cos \theta \ddot{\theta} + \sin \theta \dot{\theta}^2] \Big|_{\theta=1.2 \text{ rad}} = -0.007247 \text{ m/s}^2$$

$$v_r = \dot{r} = 0$$

Ans.

$$v_\theta = r\dot{\theta} = 4(0.2) = 0.8 \text{ m/s}$$

Ans.

$$v_z = \dot{z} = -0.0932 \text{ m/s}$$

Ans.

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4(0.2)^2 = -0.16 \text{ m/s}^2$$

Ans.

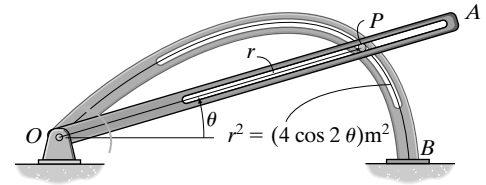
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4(0) + 2(0)(0.2) = 0$$

Ans.

$$a_z = \ddot{z} = -0.00725 \text{ m/s}^2$$

Ans.

12–175. The motion of peg P is constrained by the lemniscate curved slot in OB and by the slotted arm OA . If OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 3 \text{ rad/s}$, determine the magnitudes of the velocity and acceleration of peg P at $\theta = 30^\circ$.



Time Derivatives:

$$r^2 = 4 \cos 2\theta$$

$$2r\dot{r} = -8 \sin 2\theta \dot{\theta}$$

$$\dot{r} = \left(\frac{-4 \sin 2\theta \dot{\theta}}{r} \right) \text{ m/s} \quad \dot{\theta} = 3 \text{ rad/s}$$

$$2(r\ddot{r} + \dot{r}^2) = -8(\sin 2\theta \ddot{\theta} + 2 \cos 2\theta \dot{\theta}^2)$$

$$\ddot{r} = \left[\frac{-4(\sin 2\theta \ddot{\theta} + 2 \cos 2\theta \dot{\theta}^2) - \dot{r}^2}{r} \right] \text{ m/s}^2 \quad \ddot{\theta} = 0$$

$\dot{\theta} = 3 \text{ rad/s}$. Thus, when $\theta = 30^\circ$,

$$r|_{\theta=30^\circ} = \sqrt{4 \cos 60^\circ} = \sqrt{2} \text{ m}$$

$$\dot{r}|_{\theta=30^\circ} = \frac{-4 \sin 60^\circ (3)}{\sqrt{2}} = -7.348 \text{ m/s}$$

$$\ddot{r}|_{\theta=30^\circ} = \frac{-4[0 + 2 \cos 60^\circ (3)^2] - (-7.348)^2}{\sqrt{2}} = -63.64 \text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = -7.348 \text{ m/s} \quad v_\theta = r\dot{\theta} = \sqrt{2}(3) = 4.243 \text{ m/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{7.348^2 + (-4.243)^2} = 8.49 \text{ m/s} \quad \text{Ans.}$$

Acceleration:

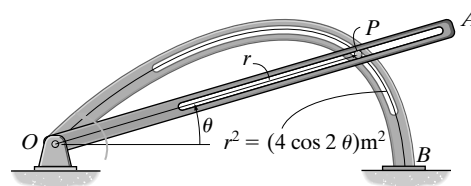
$$a_r = \ddot{r} - r\dot{\theta}^2 = -63.64 - \sqrt{2}(3)^2 = -76.37 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-7.348)(3) = -44.09 \text{ m/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-76.37)^2 + (-44.09)^2} = 88.2 \text{ m/s}^2 \quad \text{Ans.}$$

***12–176.** The motion of peg P is constrained by the lemniscate curved slot in OB and by the slotted arm OA . If OA rotates counterclockwise with an angular velocity of $\dot{\theta} = (3t^{3/2})$ rad/s, where t is in seconds, determine the magnitudes of the velocity and acceleration of peg P at $\theta = 30^\circ$. When $t = 0, \theta = 0^\circ$.



Time Derivatives:

$$r^2 = 4 \cos 2\theta$$

$$2r\dot{r} = -8 \sin 2\theta \dot{\theta}$$

$$\dot{r} = \left(\frac{-4 \sin 2\theta \dot{\theta}}{r} \right) \text{ m/s}$$

$$2(\dot{r}\dot{r} + r\ddot{r}) = -8(\sin 2\theta \ddot{\theta} + 2 \cos 2\theta \dot{\theta}^2)$$

$$\ddot{r} = \left[\frac{-4(\sin 2\theta \ddot{\theta} + 2 \cos 2\theta \dot{\theta}^2) - \dot{r}^2}{r} \right] \text{ m/s}^2$$

$$\frac{d\theta}{dt} = \dot{\theta} = 3t^{3/2}$$

$$\int_{0^\circ}^{\theta} d\theta = \int_0^t 3t^{3/2} dt$$

$$\theta = \left(\frac{6}{5} t^{5/2} \right) \text{ rad}$$

At $\theta = 30^\circ = \frac{\pi}{6} \text{ rad}$,

$$\frac{\pi}{6} = \frac{6}{5} t^{5/2} \quad t = 0.7177 \text{ s}$$

$$\dot{\theta} = 3t^{3/2} \Big|_{t=0.7177 \text{ s}} = 1.824 \text{ rad/s}$$

$$\ddot{\theta} = \frac{9}{2} t^{1/2} \Big|_{t=0.7177 \text{ s}} = 3.812 \text{ rad/s}^2$$

Thus,

$$r|_{\theta=30^\circ} = \sqrt{4 \cos 60^\circ} = \sqrt{2} \text{ m}$$

$$\dot{r}|_{\theta=30^\circ} = \frac{-4 \sin 60^\circ (1.824)}{\sqrt{2}} = -4.468 \text{ m/s}$$

$$\ddot{r}|_{\theta=30^\circ} = \frac{-4[\sin 60^\circ (3.812) + 2 \cos 60^\circ (1.824)^2] - (-4.468)^2}{\sqrt{2}} = -32.86 \text{ m/s}^2$$

12–176. Continued

Velocity:

$$v_r = \dot{r} = -4.468 \text{ m/s} \qquad v_\theta = r\dot{\theta} = \sqrt{2}(1.824) = 2.579 \text{ m/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-4.468)^2 + 2.579^2} = 5.16 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

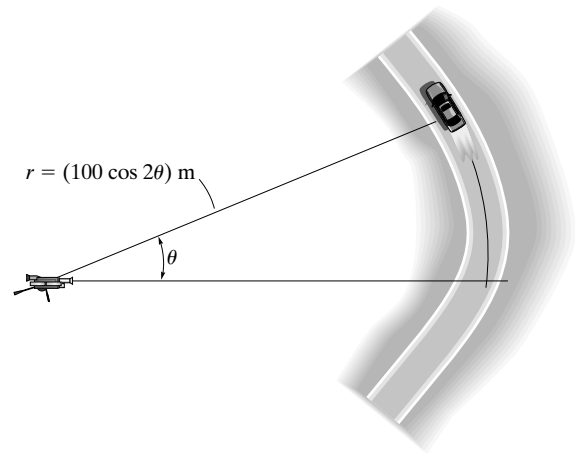
$$a_r = \ddot{r} - r\dot{\theta}^2 = -32.86 - \sqrt{2}(1.824)^2 = -37.57 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \sqrt{2}(3.812) + 2(-4.468)(1.824) = -10.91 \text{ m/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-37.57)^2 + (-10.91)^2} = 39.1 \text{ m/s}^2 \qquad \textbf{Ans.}$$

•12-177. The driver of the car maintains a constant speed of 40 m/s. Determine the angular velocity of the camera tracking the car when $\theta = 15^\circ$.



Time Derivatives:

$$r = 100 \cos 2\theta$$

$$\dot{r} = (-200 \sin 2\theta \dot{\theta}) \text{ m/s}$$

At $\theta = 15^\circ$,

$$r|_{\theta=15^\circ} = 100 \cos 30^\circ = 86.60 \text{ m}$$

$$\dot{r}|_{\theta=15^\circ} = -200 \sin 30^\circ \dot{\theta} = -100\dot{\theta} \text{ m/s}$$

Velocity: Referring to Fig. a, $v_r = -40 \cos \phi$ and $v_\theta = 40 \sin \phi$.

$$v_r = \dot{r}$$

$$-40 \cos \phi = -100\dot{\theta}$$

and

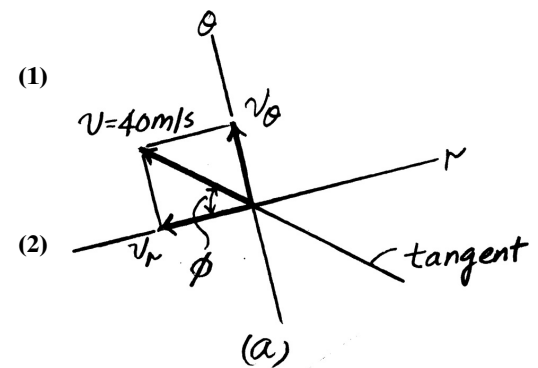
$$v_\theta = r\dot{\theta}$$

$$40 \sin \phi = 86.60\dot{\theta}$$

Solving Eqs. (1) and (2) yields

$$\phi = 40.89^\circ$$

$$\dot{\theta} = 0.3024 \text{ rad/s} = 0.302 \text{ rad/s}$$



Ans.

12–178. When $\theta = 15^\circ$, the car has a speed of 50 m/s which is increasing at 6 m/s^2 . Determine the angular velocity of the camera tracking the car at this instant.

Time Derivatives:

$$r = 100 \cos 2\theta$$

$$\dot{r} = (-200 \sin 2\theta \dot{\theta}) \text{ m/s}$$

$$\ddot{r} = -200[\sin 2\theta \ddot{\theta} + 2 \cos 2\theta \dot{\theta}^2] \text{ m/s}^2$$

At $\theta = 15^\circ$,

$$r|_{\theta=15^\circ} = 100 \cos 30^\circ = 86.60 \text{ m}$$

$$\dot{r}|_{\theta=15^\circ} = -200 \sin 30^\circ \dot{\theta} = -100\dot{\theta} \text{ m/s}$$

$$\ddot{r}|_{\theta=15^\circ} = -200[\sin 30^\circ \ddot{\theta} + 2 \cos 30^\circ \dot{\theta}^2] = (-100\ddot{\theta} - 346.41\dot{\theta}^2) \text{ m/s}^2$$

Velocity: Referring to Fig. *a*, $v_r = -50 \cos \phi$ and $v_\theta = 50 \sin \phi$. Thus,

$$v_r = \dot{r}$$

$$-50 \cos \phi = -100\dot{\theta}$$

and

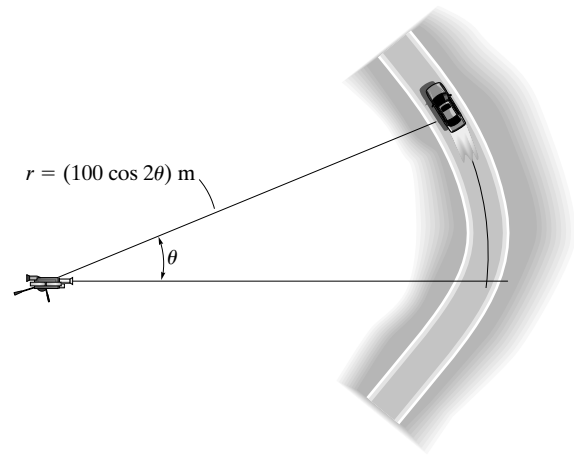
$$v_\theta = r\dot{\theta}$$

$$50 \sin \phi = 86.60\dot{\theta}$$

Solving Eqs. (1) and (2) yields

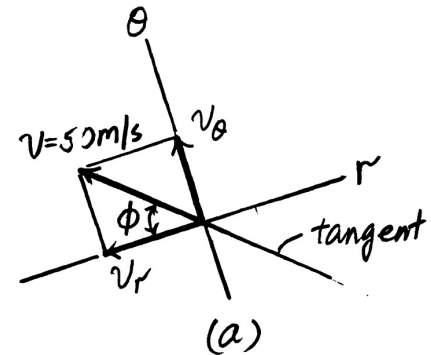
$$\phi = 40.89^\circ$$

$$\dot{\theta} = 0.378 \text{ rad/s}$$



(1)

(2)



Ans.

12–179. If the cam rotates clockwise with a constant angular velocity of $\dot{\theta} = 5 \text{ rad/s}$, determine the magnitudes of the velocity and acceleration of the follower rod *AB* at the instant $\theta = 30^\circ$. The surface of the cam has a shape of limaçon defined by $r = (200 + 100 \cos \theta) \text{ mm}$.

Time Derivatives:

$$r = (200 + 100 \cos \theta) \text{ mm}$$

$$\dot{r} = (-100 \sin \theta \dot{\theta}) \text{ mm/s}$$

$$\dot{\theta} = 5 \text{ rad/s}$$

$$\ddot{r} = -100[\sin \theta \ddot{\theta} + \cos \theta \dot{\theta}^2] \text{ mm/s}^2$$

$$\ddot{\theta} = 0$$

When $\theta = 30^\circ$,

$$r|_{\theta=30^\circ} = 200 + 100 \cos 30^\circ = 286.60 \text{ mm}$$

$$\dot{r}|_{\theta=30^\circ} = -100 \sin 30^\circ (5) = -250 \text{ mm/s}$$

$$\ddot{r}|_{\theta=30^\circ} = -100[0 + \cos 30^\circ (5^2)] = -2165.06 \text{ mm/s}^2$$

Velocity: The radial component gives the rod's velocity.

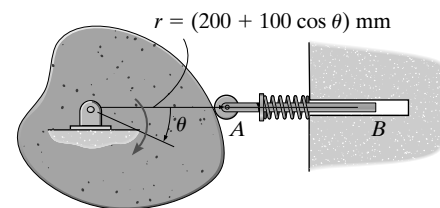
$$v_r = \dot{r} = -250 \text{ mm/s}$$

Ans.

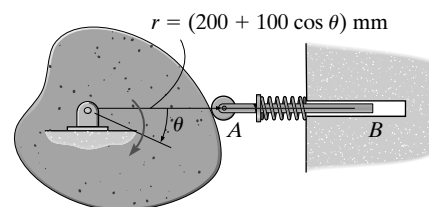
Acceleration: The radial component gives the rod's acceleration.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2165.06 - 286.60(5^2) = -9330 \text{ mm/s}^2$$

Ans.



***12–180.** At the instant $\theta = 30^\circ$, the cam rotates with a clockwise angular velocity of $\dot{\theta} = 5 \text{ rad/s}$ and angular acceleration of $\ddot{\theta} = 6 \text{ rad/s}^2$. Determine the magnitudes of the velocity and acceleration of the follower rod AB at this instant. The surface of the cam has a shape of a limaçon defined by $r = (200 + 100 \cos \theta) \text{ mm}$.



Time Derivatives:

$$r = (200 - 100 \cos \theta) \text{ mm}$$

$$\dot{r} = (-100 \sin \theta \dot{\theta}) \text{ mm/s}$$

$$\ddot{r} = -100[\sin \theta \ddot{\theta} + \cos \theta \dot{\theta}^2] \text{ mm/s}^2$$

When $\theta = 30^\circ$,

$$r|_{\theta=30^\circ} = 200 + 100 \cos 30^\circ = 286.60 \text{ mm}$$

$$\dot{r}|_{\theta=30^\circ} = -100 \sin 30^\circ (5) = -250 \text{ mm/s}$$

$$\ddot{r}|_{\theta=30^\circ} = -100[\sin 30^\circ (6) + \cos 30^\circ (5^2)] = -2465.06 \text{ mm/s}^2$$

Velocity: The radial component gives the rod's velocity.

$$v_r = \dot{r} = -250 \text{ mm/s}$$

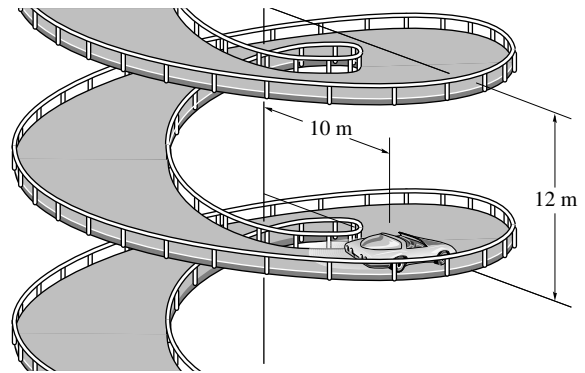
Ans.

Acceleration: The radial component gives the rod's acceleration.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2465.06 - 286.60(5^2) = -9630 \text{ mm/s}^2$$

Ans.

•12–181. The automobile travels from a parking deck down along a cylindrical spiral ramp at a constant speed of $v = 1.5$ m/s. If the ramp descends a distance of 12 m for every full revolution, $\theta = 2\pi$ rad, determine the magnitude of the car's acceleration as it moves along the ramp, $r = 10$ m. *Hint:* For part of the solution, note that the tangent to the ramp at any point is at an angle of $\phi = \tan^{-1}(12/[2\pi(10)]) = 10.81^\circ$ from the horizontal. Use this to determine the velocity components v_θ and v_z , which in turn are used to determine $\dot{\theta}$ and \dot{z} .



$$\phi = \tan^{-1}\left(\frac{12}{2\pi(10)}\right) = 10.81^\circ$$

$$v = 1.5 \text{ m/s}$$

$$v_r = 0$$

$$v_\theta = 1.5 \cos 10.81^\circ = 1.473 \text{ m/s}$$

$$v_z = -1.5 \sin 10.81^\circ = -0.2814 \text{ m/s}$$

Since

$$r = 10 \quad \dot{r} = 0 \quad \ddot{r} = 0$$

$$v_\theta = r \dot{\theta} = 1.473 \quad \dot{\theta} = \frac{1.473}{10} = 0.1473$$

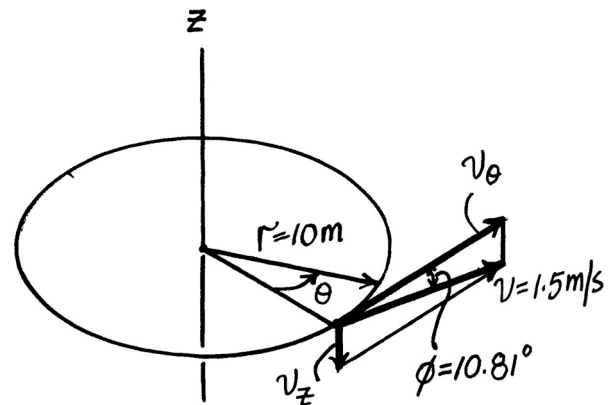
Since $\dot{\theta} = 0$

$$a_r = r - \dot{r}\theta^2 = 0 - 10(0.1473)^2 = -0.217$$

$$a_\theta = \dot{r}\ddot{\theta} + 2r\dot{\theta} = 10(0) + 2(0)(0.1473) = 0$$

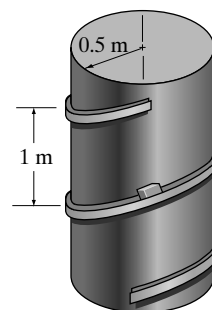
$$a_z = \ddot{z} = 0$$

$$a = \sqrt{(-0.217)^2 + (0)^2 + (0)^2} = 0.217 \text{ m/s}^2$$



Ans.

12–182. The box slides down the helical ramp with a constant speed of $v = 2$ m/s. Determine the magnitude of its acceleration. The ramp descends a vertical distance of 1 m for every full revolution. The mean radius of the ramp is $r = 0.5$ m.



Velocity: The inclination angle of the ramp is $\phi = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \left[\frac{1}{2\pi(0.5)} \right] = 17.66^\circ$.

Thus, from Fig. a , $v_\theta = 2 \cos 17.66^\circ = 1.906$ m/s and $v_z = 2 \sin 17.66^\circ = 0.6066$ m/s. Thus,

$$v_\theta = r\dot{\theta}$$

$$1.906 = 0.5\dot{\theta}$$

$$\dot{\theta} = 3.812 \text{ rad/s}$$

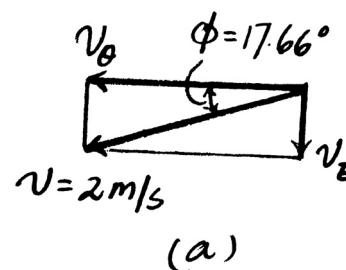
Acceleration: Since $r = 0.5$ m is constant, $\dot{r} = \ddot{r} = 0$. Also, $\dot{\theta}$ is constant, then $\ddot{\theta} = 0$. Using the above results,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.5(3.812)^2 = -7.264 \text{ m/s}^2$$

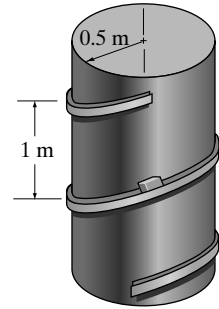
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(0) + 2(0)(3.812) = 0$$

Since \mathbf{v}_z is constant $a_z = 0$. Thus, the magnitude of the box's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2 + a_z^2} = \sqrt{(-7.264)^2 + 0^2 + 0^2} = 7.26 \text{ m/s}^2 \quad \text{Ans.}$$



12–183. The box slides down the helical ramp which is defined by $r = 0.5$ m, $\theta = (0.5t^3)$ rad, and $z = (2 - 0.2t^2)$ m, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the box at the instant $\theta = 2\pi$ rad.



Time Derivatives:

$$r = 0.5 \text{ m}$$

$$\dot{r} = \ddot{r} = 0$$

$$\dot{\theta} = (1.5t^2) \text{ rad/s} \quad \ddot{\theta} = 3(3t) \text{ rad/s}^2$$

$$z = 2 - 0.2t^2$$

$$\dot{z} = (-0.4t) \text{ m/s} \quad \ddot{z} = -0.4 \text{ m/s}^2$$

When $\theta = 2\pi$ rad,

$$2\pi = 0.5t^3 \quad t = 2.325 \text{ s}$$

Thus,

$$\dot{\theta}|_{t=2.325 \text{ s}} = 1.5(2.325)^2 = 8.108 \text{ rad/s}$$

$$\ddot{\theta}|_{t=2.325 \text{ s}} = 3(2.325) = 6.975 \text{ rad/s}^2$$

$$\dot{z}|_{t=2.325 \text{ s}} = -0.4(2.325) = -0.92996 \text{ m/s}$$

$$\ddot{z}|_{t=2.325 \text{ s}} = -0.4 \text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 0.5(8.108) = 4.05385 \text{ m/s}$$

$$v_z = \dot{z} = -0.92996 \text{ m/s}$$

Thus, the magnitude of the box's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2 + v_z^2} = \sqrt{0^2 + 4.05385^2 + (-0.92996)^2} = 4.16 \text{ m/s} \quad \text{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.5(8.108)^2 = -32.867 \text{ m/s}^2$$

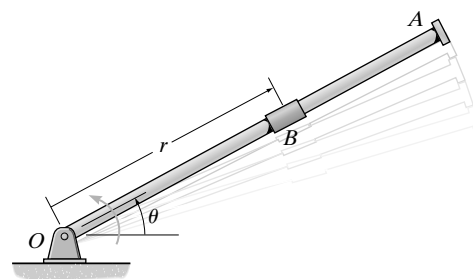
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(6.975) + 2(0)(8.108)^2 = 3.487 \text{ m/s}^2$$

$$a_z = \ddot{z} = -0.4 \text{ m/s}^2$$

Thus, the magnitude of the box's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2 + a_z^2} = \sqrt{(-32.867)^2 + 3.487^2 + (-0.4)^2} = 33.1 \text{ m/s}^2 \quad \text{Ans.}$$

***12–184.** Rod OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 6 \text{ rad/s}$. Through mechanical means collar B moves along the rod with a speed of $\dot{r} = (4t^2) \text{ m/s}$, where t is in seconds. If $r = 0$ when $t = 0$, determine the magnitudes of velocity and acceleration of the collar when $t = 0.75 \text{ s}$.



Time Derivatives: Using the initial condition $r = 0$ when $t = 0 \text{ s}$,

$$\frac{dr}{dt} = \dot{r} = 4t^2$$

$$\int_0^r dr = \int_0^t 4t^2 dt$$

$$r = \left[\frac{4}{3} t^3 \right] \text{ m}$$

$$r = \left. \frac{4}{3} t^3 \right|_{t=0.75 \text{ s}} = 0.5625 \text{ m}$$

$$\dot{r} = \left. 4t^2 \right|_{t=0.75 \text{ s}} = 2.25 \text{ m/s} \quad \dot{\theta} = 6 \text{ rad/s}$$

$$\ddot{r} = \left. 8t \right|_{t=0.75 \text{ s}} = 6 \text{ m/s}^2 \quad \ddot{\theta} = 0$$

Velocity:

$$v_r = \dot{r} = 2.25 \text{ m/s} \quad v_\theta = r\dot{\theta} = 0.5625(6) = 3.375 \text{ m/s}$$

Thus, the magnitude of the collar's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{2.25^2 + 3.375^2} = 4.06 \text{ m/s} \quad \textbf{Ans.}$$

Acceleration:

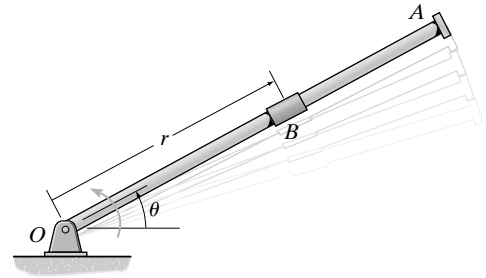
$$a_r = \ddot{r} - r\dot{\theta}^2 = 6 - 0.5625(6^2) = -14.25 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5625(0) + 2(2.25)(6) = 27 \text{ m/s}^2$$

Thus, the magnitude of the collar's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-14.25)^2 + (27)^2} = 30.5 \text{ m/s}^2 \quad \textbf{Ans.}$$

•12–185. Rod OA is rotating counterclockwise with an angular velocity of $\dot{\theta} = (2t^2)$ rad/s. Through mechanical means collar B moves along the rod with a speed of $\dot{r} = (4t^2)$ m/s. If $\theta = 0$ and $r = 0$ when $t = 0$, determine the magnitudes of velocity and acceleration of the collar at $\theta = 60^\circ$.



Position: Using the initial condition $\theta = 0$ when $t = 0$ s,

$$\frac{d\theta}{dt} = \dot{\theta} = 2t^2$$

$$\int_{0^\circ}^{\theta} d\theta = \int_0^t 2t^2 dt$$

$$\theta = \left[\frac{2}{3} t^3 \right] \text{ rad}$$

At $\theta = 60^\circ = \frac{\pi}{3}$ rad,

$$\frac{\pi}{3} = \frac{2}{3} t^3 \quad t = 1.162 \text{ s}$$

Using the initial condition $r = 0$ when $t = 0$ s,

$$\frac{dr}{dt} = \dot{r} = 4t^2$$

$$\int_0^r dr = \int_0^t 4t^2 dt$$

$$r = \left[\frac{4}{3} t^3 \right] \text{ m}$$

When $t = 1.162 \text{ s}$ $\left(\theta = \frac{\pi}{3} \text{ rad} \right)$,

$$r = \frac{4}{3} t^3 \bigg|_{t=1.162 \text{ s}} = 2.094 \text{ m}$$

Time Derivatives:

$$\dot{r} = 4t^2 \bigg|_{t=1.162 \text{ s}} = 5.405 \text{ m/s} \quad \dot{\theta} = 2t^2 \bigg|_{t=1.162 \text{ s}} = 2.703 \text{ rad/s}$$

$$\ddot{r} = 8t \bigg|_{t=1.162 \text{ s}} = 9.300 \text{ m/s}^2 \quad \ddot{\theta} = 4t \bigg|_{t=1.162 \text{ s}} = 4.650 \text{ rad/s}^2$$

Velocity:

$$v_r = \dot{r} = 5.405 \text{ m/s} \quad v_\theta = r\dot{\theta} = 2.094(2.703) = 5.660 \text{ m/s}$$

Thus, the magnitude of the collar's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{5.405^2 + 5.660^2} = 7.83 \text{ m/s} \quad \text{Ans.}$$

Acceleration:

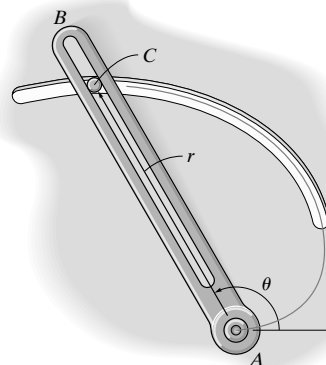
$$a_r = \ddot{r} - r\dot{\theta}^2 = 9.300 - 2.094(2.703^2) = -5.998 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.094(4.650) + 2(5.405)(2.703) = 38.95 \text{ m/s}^2$$

Thus, the magnitude of the collar's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-5.998)^2 + 38.95^2} = 39.4 \text{ m/s}^2 \quad \text{Ans.}$$

12–186. The slotted arm AB drives pin C through the spiral groove described by the equation $r = a\theta$. If the angular velocity is constant at $\dot{\theta}$, determine the radial and transverse components of velocity and acceleration of the pin.



Time Derivatives: Since $\dot{\theta}$ is constant, then $\ddot{\theta} = 0$.

$$r = a\theta \quad \dot{r} = a\dot{\theta} \quad \ddot{r} = a\ddot{\theta} = 0$$

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = a\dot{\theta}$$

Ans.

$$v_\theta = r\dot{\theta} = a\theta\dot{\theta}$$

Ans.

Acceleration: Applying Eq. 12–29, we have

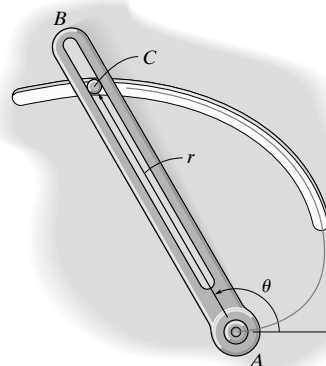
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - a\theta\dot{\theta}^2 = -a\theta\dot{\theta}^2$$

Ans.

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(a\dot{\theta})(\dot{\theta}) = 2a\dot{\theta}^2$$

Ans.

12–187. The slotted arm AB drives pin C through the spiral groove described by the equation $r = (1.5\theta)$ ft, where θ is in radians. If the arm starts from rest when $\theta = 60^\circ$ and is driven at an angular velocity of $\dot{\theta} = (4t)$ rad/s, where t is in seconds, determine the radial and transverse components of velocity and acceleration of the pin C when $t = 1$ s.



Time Derivatives: Here, $\dot{\theta} = 4t$ and $\ddot{\theta} = 4$ rad/s².

$$r = 1.5\theta \quad \dot{r} = 1.5\dot{\theta} = 1.5(4t) = 6t \quad \ddot{r} = 1.5\ddot{\theta} = 1.5(4) = 6 \text{ ft/s}^2$$

Velocity: Integrate the angular rate, $\int_{\frac{\pi}{3}}^{\theta} d\theta = \int_0^t 4t dt$, we have $\theta = \frac{1}{3}(6t^2 + \pi)$ rad.

Then, $r = \left\{ \frac{1}{2}(6t^2 + \pi) \right\}$ ft. At $t = 1$ s, $r = \frac{1}{2}[6(1^2) + \pi] = 4.571$ ft, $\dot{r} = 6(1) = 6.00$ ft/s

and $\dot{\theta} = 4(1) = 4$ rad/s. Applying Eq. 12–25, we have

$$v_r = \dot{r} = 6.00 \text{ ft/s}$$

Ans.

$$v_\theta = r\dot{\theta} = 4.571(4) = 18.3 \text{ ft/s}$$

Ans.

Acceleration: Applying Eq. 12–29, we have

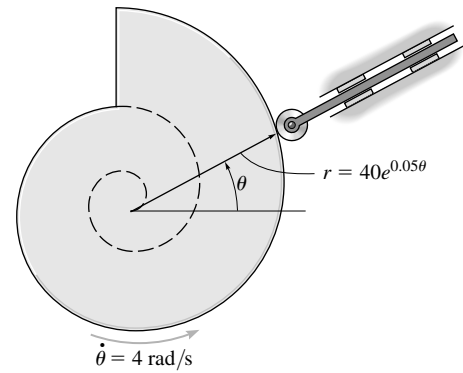
$$a_r = \ddot{r} - r\dot{\theta}^2 = 6 - 4.571(4^2) = -67.1 \text{ ft/s}^2$$

Ans.

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.571(4) + 2(6)(4) = 66.3 \text{ ft/s}^2$$

Ans.

***12–188.** The partial surface of the cam is that of a logarithmic spiral $r = (40e^{0.05\theta})$ mm, where θ is in radians. If the cam rotates at a constant angular velocity of $\dot{\theta} = 4$ rad/s, determine the magnitudes of the velocity and acceleration of the point on the cam that contacts the follower rod at the instant $\theta = 30^\circ$.



$$r = 40e^{0.05\theta}$$

$$\dot{r} = 2e^{0.05\theta} \dot{\theta}$$

$$\ddot{r} = 0.1e^{0.05\theta} (\dot{\theta})^2 + 2e^{0.05\theta} \ddot{\theta}$$

$$\theta = \frac{\pi}{6}$$

$$\dot{\theta} = 4$$

$$\ddot{\theta} = 0$$

$$r = 40e^{0.05(\frac{\pi}{6})} = 41.0610$$

$$\dot{r} = 2e^{0.05(\frac{\pi}{6})} (4) = 8.2122$$

$$\ddot{r} = 0.1e^{0.05(\frac{\pi}{6})} (4)^2 + 0 = 1.64244$$

$$v_r = \dot{r} = 8.2122$$

$$v_\theta = r\dot{\theta} = 41.0610(4) = 164.24$$

$$v = \sqrt{(8.2122)^2 + (164.24)^2} = 164 \text{ mm/s}$$

Ans.

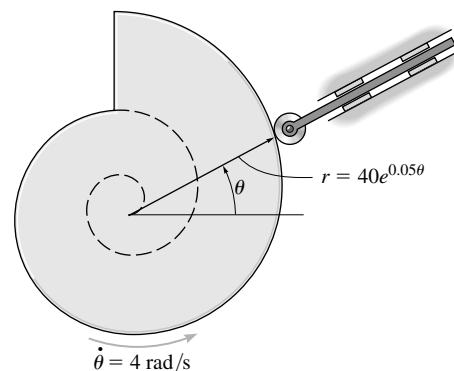
$$a_r = \ddot{r} - r\dot{\theta}^2 = 1.64244 - 41.0610(4)^2 = -655.33$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(8.2122)(4) = 65.6976$$

$$a = \sqrt{(-655.33)^2 + (65.6976)^2} = 659 \text{ mm/s}^2$$

Ans.

•12–189. Solve Prob. 12–188, if the cam has an angular acceleration of $\ddot{\theta} = 2 \text{ rad/s}^2$ when its angular velocity is $\dot{\theta} = 4 \text{ rad/s}$ at $\theta = 30^\circ$.



$$r = 40e^{0.05\theta}$$

$$r = 2e^{0.05\theta} \dot{\theta}$$

$$r = 0.1e^{0.05\theta} (\dot{\theta})^2 + 2e^{0.05\theta} \ddot{\theta}$$

$$\theta = \frac{\pi}{6}$$

$$\theta = 4$$

$$\theta = 2$$

$$r = 40e^{0.05(\frac{\pi}{6})} = 41.0610$$

$$r = 2e^{0.05(\frac{\pi}{6})} (4) = 8.2122$$

$$\ddot{r} = 0.1e^{0.05(\frac{\pi}{6})} (4)^2 + 2e^{0.05(\frac{\pi}{6})} (2) = 5.749$$

$$v_r = \dot{r} = 8.2122$$

$$v_\theta = r\dot{\theta} = 41.0610(4) = 164.24$$

$$v = \sqrt{(8.2122)^2 + (164.24)^2} = 164 \text{ mm/s}$$

Ans.

$$a_r = \ddot{r} - r\dot{\theta}^2 = 5.749 - 41.0610(4)^2 = -651.2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 41.0610(2) + 2(8.2122)(4) = 147.8197$$

$$a = \sqrt{(-651.2)^2 + (147.8197)^2} = 668 \text{ mm/s}^2$$

Ans.

12–190. A particle moves along an Archimedean spiral $r = (8\theta)$ ft, where θ is given in radians. If $\dot{\theta} = 4$ rad/s (constant), determine the radial and transverse components of the particle's velocity and acceleration at the instant $\theta = \pi/2$ rad. Sketch the curve and show the components on the curve.

Time Derivatives: Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$.

$$r = 8\theta = 8\left(\frac{\pi}{2}\right) = 4\pi \text{ ft} \quad \dot{r} = 8\dot{\theta} = 8(4) = 32.0 \text{ ft/s} \quad \ddot{r} = 8\ddot{\theta} = 0$$

Velocity: Applying Eq. 12–25, we have

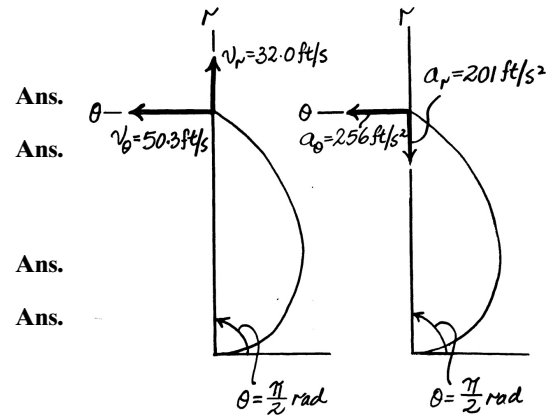
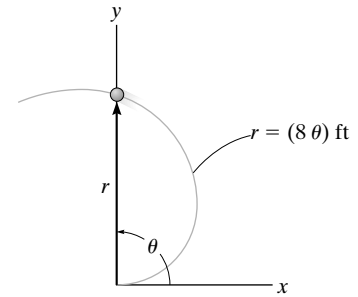
$$v_r = \dot{r} = 32.0 \text{ ft/s}$$

$$v_\theta = r\dot{\theta} = 4\pi(4) = 50.3 \text{ ft/s}$$

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4\pi(4^2) = -201 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(32.0)(4) = 256 \text{ ft/s}^2$$



12–191. Solve Prob. 12–190 if the particle has an angular acceleration $\ddot{\theta} = 5$ rad/s² when $\dot{\theta} = 4$ rad/s at $\theta = \pi/2$ rad.

Time Derivatives: Here,

$$r = 8\theta = 8\left(\frac{\pi}{2}\right) = 4\pi \text{ ft} \quad \dot{r} = 8\dot{\theta} = 8(4) = 32.0 \text{ ft/s}$$

$$\ddot{r} = 8\ddot{\theta} = 8(5) = 40 \text{ ft/s}^2$$

Velocity: Applying Eq. 12–25, we have

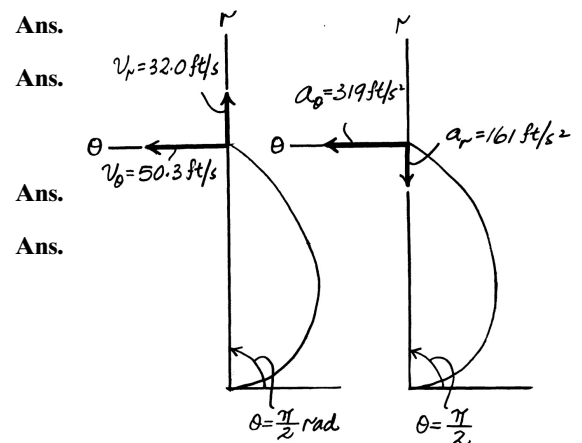
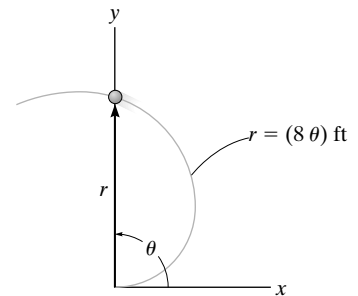
$$v_r = \dot{r} = 32.0 \text{ ft/s}$$

$$v_\theta = r\dot{\theta} = 4\pi(4) = 50.3 \text{ ft/s}$$

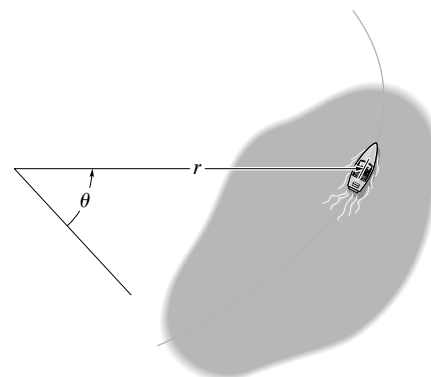
Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 40 - 4\pi(4^2) = -161 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4\pi(5) + 2(32.0)(4) = 319 \text{ ft/s}^2$$



***12–192.** The boat moves along a path defined by $r^2 = [10(10^3) \cos 2\theta] \text{ ft}^2$, where θ is in radians. If $\theta = (0.4t^2) \text{ rad}$, where t is in seconds, determine the radial and transverse components of the boat's velocity and acceleration at the instant $t = 1 \text{ s}$.



Time Derivatives: Here, $\theta = 0.4t^2$, $\dot{\theta} = 0.8t$ and $\ddot{\theta} = 0.8 \text{ rad/s}^2$. When $t = 1 \text{ s}$, $\theta = 0.4 (1^2) = 0.4 \text{ rad}$ and $\dot{\theta} = 0.8 (1) = 0.8 \text{ rad/s}$.

$$r^2 = 10(10^3) \cos 2\theta \quad r = 100\sqrt{\cos 2\theta}$$

$$2r\dot{r} = -20(10^3) \sin 2\theta \dot{\theta} \quad \dot{r} = -\frac{10(10^3) \sin 2\theta}{r} \dot{\theta}$$

$$2r\ddot{r} + 2\dot{r}^2 = -20(10^3) (2 \cos 2\theta \ddot{\theta} + \sin 2\theta \dot{\theta}^2)$$

$$\ddot{r} = \frac{-10(10^3) (2 \cos 2\theta \ddot{\theta} + \sin 2\theta \dot{\theta}^2) - \dot{r}^2}{r}$$

$$\text{At } \theta = 0.4 \text{ rad}, r = 100\sqrt{\cos 0.8} = 83.47 \text{ ft}, \dot{r} = -\frac{10(10^3) \sin 0.8}{83.47} (0.8) = -68.75 \text{ ft/s}$$

$$\text{and } \ddot{r} = \frac{-10(10^3) [2 \cos 0.8 (0.8^2) + \sin 0.8 (0.8)] - (-68.75)^2}{83.47} = -232.23 \text{ ft/s}^2.$$

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = -68.8 \text{ ft/s} \quad \text{Ans.}$$

$$v_\theta = r\dot{\theta} = 83.47(0.8) = 66.8 \text{ ft/s} \quad \text{Ans.}$$

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = -232.23 - 83.47(0.8^2) = -286 \text{ ft/s}^2 \quad \text{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 83.47(0.8) + 2(-68.75)(0.8) = -43.2 \text{ ft/s}^2 \quad \text{Ans.}$$

•12–193. A car travels along a road, which for a short distance is defined by $r = (200/\theta) \text{ ft}$, where θ is in radians. If it maintains a constant speed of $v = 35 \text{ ft/s}$, determine the radial and transverse components of its velocity when $\theta = \pi/3 \text{ rad}$.

$$r = \frac{200}{\theta} \bigg|_{\theta=\pi/3 \text{ rad}} = \frac{600}{\pi} \text{ ft}$$

$$\dot{r} = -\frac{200}{\theta^2} \dot{\theta} \bigg|_{\theta=\pi/3 \text{ rad}} = -\frac{1800}{\pi^2} \dot{\theta}$$

$$v_r = \dot{r} = -\frac{1800}{\pi^2} \dot{\theta} \quad v_\theta = r\dot{\theta} = \frac{600}{\pi} \dot{\theta}$$

$$v^2 = v_r^2 + v_\theta^2$$

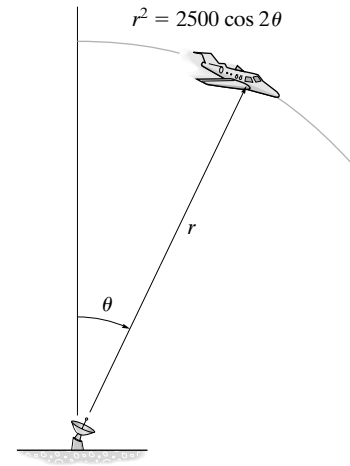
$$35^2 = \left(-\frac{1800}{\pi^2} \dot{\theta}\right)^2 + \left(\frac{600}{\pi} \dot{\theta}\right)^2$$

$$\dot{\theta} = 0.1325 \text{ rad/s}$$

$$v_r = -\frac{1800}{\pi^2} (0.1325) = -24.2 \text{ ft/s} \quad \text{Ans.}$$

$$v_\theta = \frac{600}{\pi} (0.1325) = 25.3 \text{ ft/s} \quad \text{Ans.}$$

12–194. For a short time the jet plane moves along a path in the shape of a lemniscate, $r^2 = (2500 \cos 2\theta) \text{ km}^2$. At the instant $\theta = 30^\circ$, the radar tracking device is rotating at $\dot{\theta} = 5(10^{-3}) \text{ rad/s}$ with $\ddot{\theta} = 2(10^{-3}) \text{ rad/s}^2$. Determine the radial and transverse components of velocity and acceleration of the plane at this instant.



Time Derivatives: Here, $\dot{\theta} = 5(10^{-3}) \text{ rad/s}$ and $\ddot{\theta} = 2(10^{-3}) \text{ rad/s}^2$.

$$\begin{aligned} r^2 &= 2500 \cos 2\theta & r &= 50\sqrt{\cos 2\theta} \\ 2r\dot{r} &= -5000 \sin 2\theta \dot{\theta} & \dot{r} &= -\frac{2500 \sin 2\theta}{r} \dot{\theta} \\ 2r\ddot{r} + 2\dot{r}^2 &= -5000(2 \cos 2\theta \ddot{\theta} + \sin 2\theta \dot{\theta}^2) \\ \ddot{r} &= \frac{-2500(2 \cos 2\theta \ddot{\theta} + \sin 2\theta \dot{\theta}^2) - \dot{r}^2}{r} \end{aligned}$$

$$\begin{aligned} \text{At } \theta = 30^\circ, r &= 50\sqrt{\cos 60^\circ} = 35.36 \text{ km}, \dot{r} = -\frac{2500 \sin 60^\circ}{35.36} [5(10^{-3})] = -0.3062 \text{ km/s} \\ \text{and } \ddot{r} &= \frac{-2500[2 \cos 60^\circ [5(10^{-3})]^2 + \sin 60^\circ [2(10^{-3})]] - (-0.3062)^2}{35.36} = -0.1269 \text{ km/s}^2. \end{aligned}$$

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = -0.3062 \text{ km/s} = 306 \text{ m/s} \quad \text{Ans.}$$

$$v_\theta = r\dot{\theta} = 35.36 [5(10^{-3})] = 0.1768 \text{ km/s} = 177 \text{ m/s} \quad \text{Ans.}$$

Acceleration: Applying Eq. 12–29, we have

$$\begin{aligned} a_r = \ddot{r} - r\dot{\theta}^2 &= -0.1269 - 35.36 [5(10^{-3})]^2 \\ &= -0.1278 \text{ km/s}^2 = -128 \text{ m/s}^2 \quad \text{Ans.} \end{aligned}$$

$$\begin{aligned} a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} &= 35.36 [2(10^{-3})] + 2(-0.3062) [5(10^{-3})] \\ &= 0.06765 \text{ km/s}^2 = 67.7 \text{ m/s}^2 \quad \text{Ans.} \end{aligned}$$

12–195. The mine car C is being pulled up the incline using the motor M and the rope-and-pulley arrangement shown. Determine the speed v_P at which a point P on the cable must be traveling toward the motor to move the car up the plane with a constant speed of $v = 2$ m/s.

$$2s_C + (s_C - s_P) = l$$

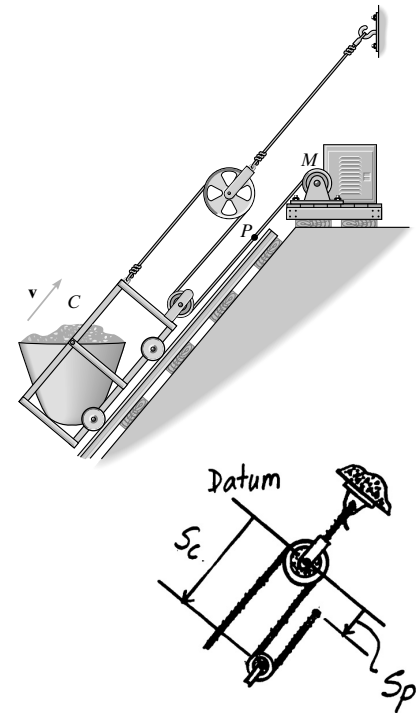
Thus,

$$3v_C - v_P = 0$$

Hence,

$$v_P = 3(-2) = -6 \text{ m/s} = 6 \text{ m/s} \nearrow$$

Ans.



***12–196.** Determine the displacement of the log if the truck at C pulls the cable 4 ft to the right.

$$2s_B + (s_B - s_C) = l$$

$$3s_B - s_C = l$$

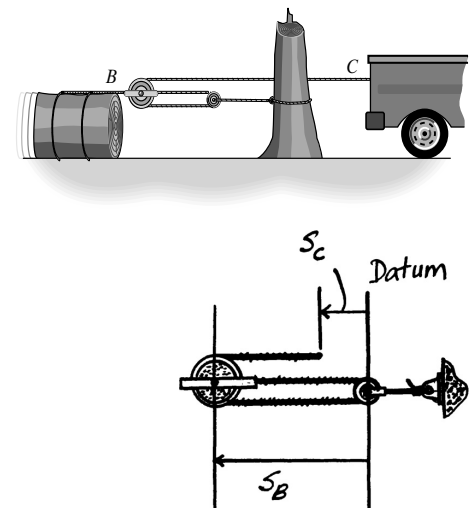
$$3\Delta s_B - \Delta s_C = 0$$

Since $\Delta s_C = -4$, then

$$3\Delta s_B = -4$$

$$\Delta s_B = -1.33 \text{ ft} = 1.33 \text{ ft} \rightarrow$$

Ans.



•12–197. If the hydraulic cylinder H draws in rod BC at 2 ft/s, determine the speed of slider A .

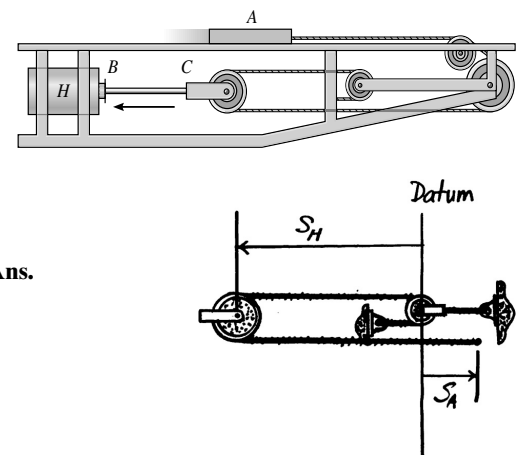
$$2s_H + s_A = l$$

$$2v_H = -v_A$$

$$2(2) = -v_A$$

$$v_A = -4 \text{ ft/s} = 4 \text{ ft/s} \leftarrow$$

Ans.



12–198. If end A of the rope moves downward with a speed of 5 m/s, determine the speed of cylinder B .

Position Coordinates: By referring to Fig. a , the length of the two ropes written in terms of the position coordinates s_A , s_B , and s_C are

$$\begin{aligned} s_B + 2a + 2s_C &= l_1 \\ s_B + 2s_C &= l_1 - 2a \end{aligned} \quad (1)$$

and

$$\begin{aligned} s_A + (s_A - s_C) &= l_2 \\ 2s_A - s_C &= l_2 \end{aligned} \quad (2)$$

Eliminating s_C from Eqs. (1) and (2) yields

$$s_B + 4s_A = l_1 - 2a + 2l_2$$

Time Derivative: Taking the time derivative of the above equation,

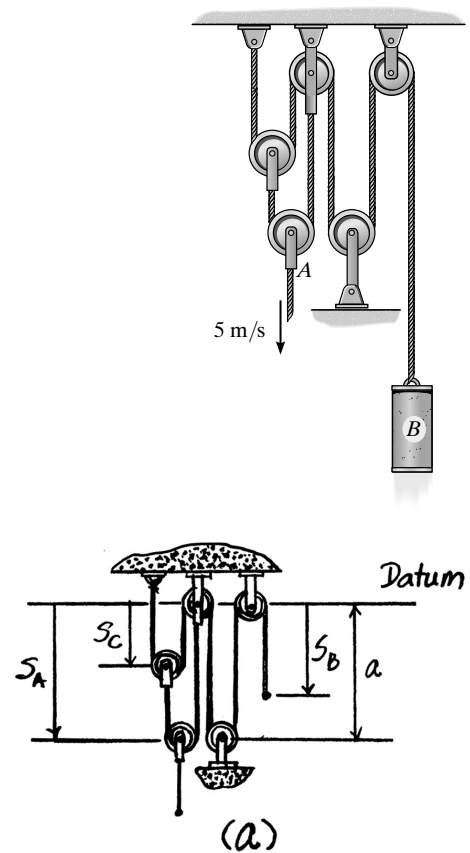
$$(+\downarrow) \quad v_B + 4v_A = 0$$

Here, $v_A = 5$ m/s. Thus,

$$v_B + 4(5) = 0$$

$$v_B = -20 \text{ m/s} = 20 \text{ m/s} \uparrow$$

Ans.



12–199. Determine the speed of the elevator if each motor draws in the cable with a constant speed of 5 m/s.

Position Coordinates: By referring to Fig. a , the length of the two cables written in terms of the position coordinates are

$$\begin{aligned} s_E + (s_E - s_A) + s_C &= l_1 \\ 2s_E - s_A + s_C &= l_1 \end{aligned} \quad (1)$$

and

$$\begin{aligned} (s_E - s_B) + 2(s_E - s_C) &= l_2 \\ 3s_E - s_B - 2s_C &= l_2 \end{aligned} \quad (2)$$

Eliminating s_C from Eqs. (1) and (2) yields

$$7s_E - 2s_A - s_B = 2l_1 + l_2$$

Time Derivative: Taking the time derivative of the above equation,

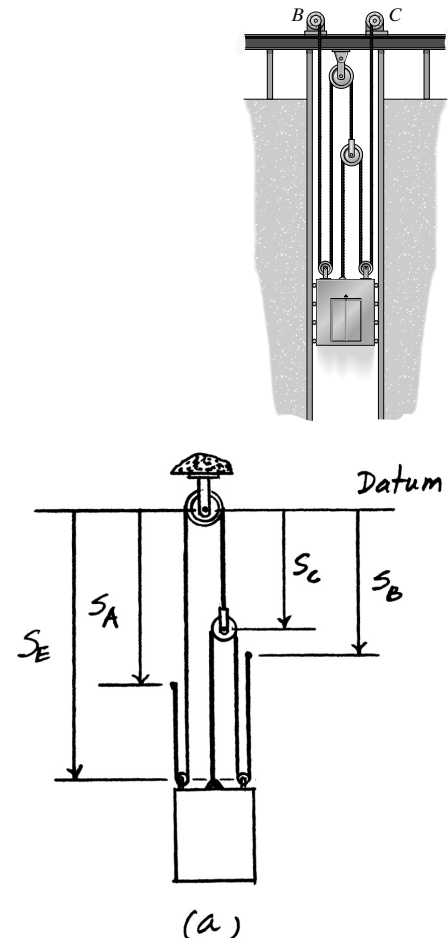
$$(+\downarrow) \quad 7v_E - 2v_A - v_B = 0$$

Here, $v_A = v_B = -5$ m/s. Thus,

$$7v_E - [2(-5)] - (-5) = 0$$

$$v_E = -2.14 \text{ m/s} = 2.14 \text{ m/s} \uparrow$$

Ans.



***12–200.** Determine the speed of cylinder A , if the rope is drawn towards the motor M at a constant rate of 10 m/s.

Position Coordinates: By referring to Fig. a , the length of the rope written in terms of the position coordinates s_A and s_M is

$$3s_A + s_M = l$$

Time Derivative: Taking the time derivative of the above equation,

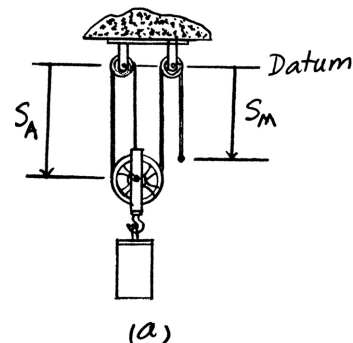
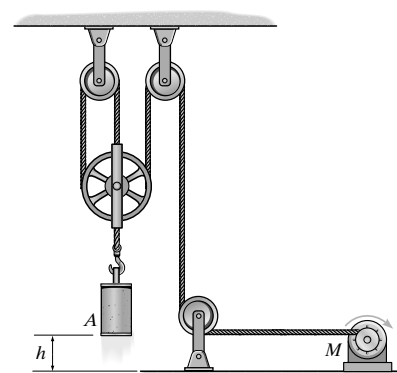
$$(+\downarrow) \quad 3v_A + v_M = 0$$

Here, $v_M = 10$ m/s. Thus,

$$3v_A + 10 = 0$$

$$v_A = -3.33 \text{ m/s} = 3.33 \text{ m/s} \uparrow$$

Ans.



•12–201. If the rope is drawn towards the motor M at a speed of $v_M = (5t^{3/2})$ m/s, where t is in seconds, determine the speed of cylinder A when $t = 1$ s.

Position Coordinates: By referring to Fig. a , the length of the rope written in terms of the position coordinates s_A and s_M is

$$3s_A + s_M = l$$

Time Derivative: Taking the time derivative of the above equation,

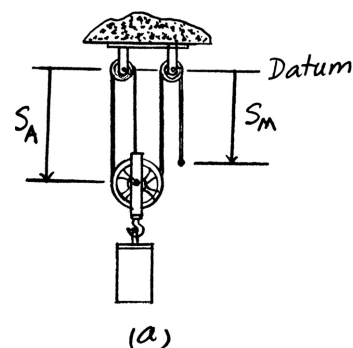
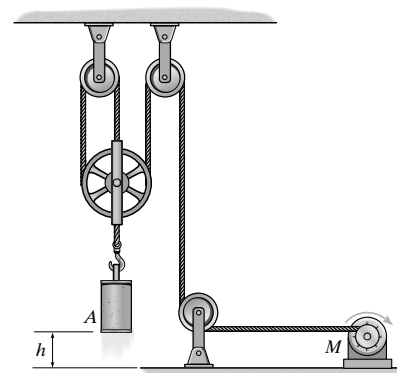
$$(+\downarrow) \quad 3v_A + v_M = 0$$

Here, $v_M = (5t^{3/2})$ m/s. Thus,

$$3v_A + 5t^{3/2} = 0$$

$$v_A = \left(-\frac{5}{3} t^{3/2} \right) \text{ m/s} = \left(\frac{5}{3} t^{3/2} \right) \text{ m/s} \Big|_{t=1 \text{ s}} = 1.67 \text{ m/s} \uparrow$$

Ans.



12–202. If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.

Position-Coordinate Equation: Datum is established at fixed pulley D . The position of point A , block B and pulley C with respect to datum are s_A , s_B and s_C , respectively. Since the system consists of two cords, two position-coordinate equations can be developed.

$$2s_C + s_A = l_1 \quad [1]$$

$$s_B + (s_B - s_C) = l_2 \quad [2]$$

Eliminating s_C from Eqs. [1] and [2] yields

$$s_A + 4s_B = l_1 + 2l_2$$

Time Derivative: Taking the time derivative of the above equation yields

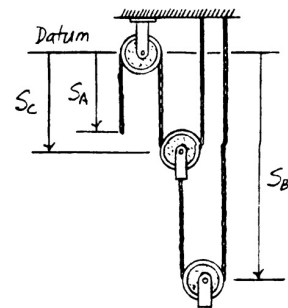
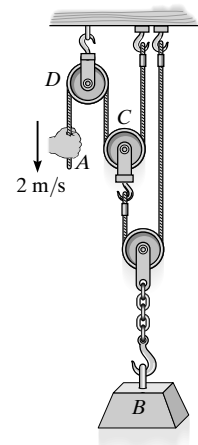
$$v_A + 4v_B = 0 \quad [3]$$

Since $v_A = 2$ m/s, from Eq. [3]

$$(+\downarrow) \quad 2 + 4v_B = 0$$

$$v_B = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow$$

Ans.



12–203. Determine the speed of B if A is moving downwards with a speed of $v_A = 4$ m/s at the instant shown.

Position Coordinates: By referring to Fig. a , the length of the two ropes written in terms of the position coordinates s_A , s_B , and s_C are

$$s_A + 2s_C = l_1 \quad (1)$$

and

$$s_B + (s_B - s_C) = l_2 \quad (2)$$

Eliminating s_C from Eqs. (1) and (2),

$$4s_B + s_A = 2l_2 + l_1 \quad (3)$$

Time Derivative: Taking the time derivative of Eq. (3),

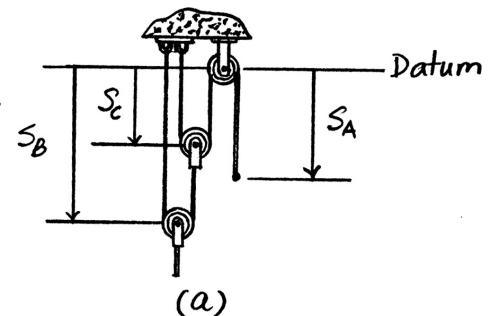
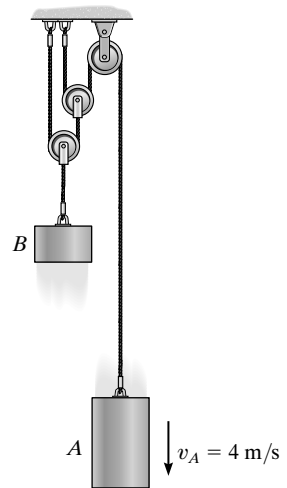
$$(+\downarrow) \quad 4v_B + v_A = 0$$

Here, $v_A = 4$ m/s. Thus,

$$4v_B + 4 = 0$$

$$v_B = -1 \text{ m/s} = 1 \text{ m/s} \uparrow$$

Ans.



***12–204.** The crane is used to hoist the load. If the motors at A and B are drawing in the cable at a speed of 2 ft/s and 4 ft/s, respectively, determine the speed of the load.

Position-Coordinate Equation: Datum is established as shown. The position of point A and B and load C with respect to datum are s_A , s_B and s_C , respectively.

$$4s_C + s_A + s_B + 2h = l$$

Time Derivative: Since h is a constant, taking the time derivative of the above equation yields

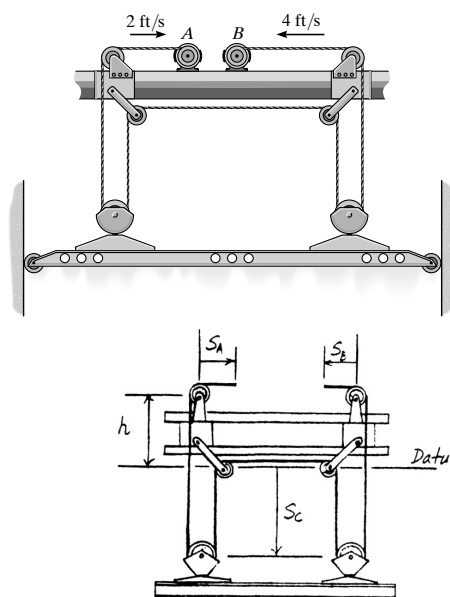
$$4v_C + v_A + v_B = 0 \quad [1]$$

Since $v_A = 2$ ft/s and $v_B = 4$ ft/s, from Eq. [1]

$$4v_C + 2 + 4 = 0$$

$$v_C = -1.50 \text{ ft/s} = 1.50 \text{ ft/s} \uparrow$$

Ans.



•12–205. The cable at B is pulled downwards at 4 ft/s, and the speed is decreasing at 2 ft/s². Determine the velocity and acceleration of block A at this instant.

$$2s_A + (h - s_C) = l$$

$$2v_A = v_C$$

$$s_C + (s_C - s_B) = l$$

$$2v_C = v_B$$

$$v_B = 4v_A$$

$$a_B = 4a_A$$

Thus,

$$-4 = 4v_A$$

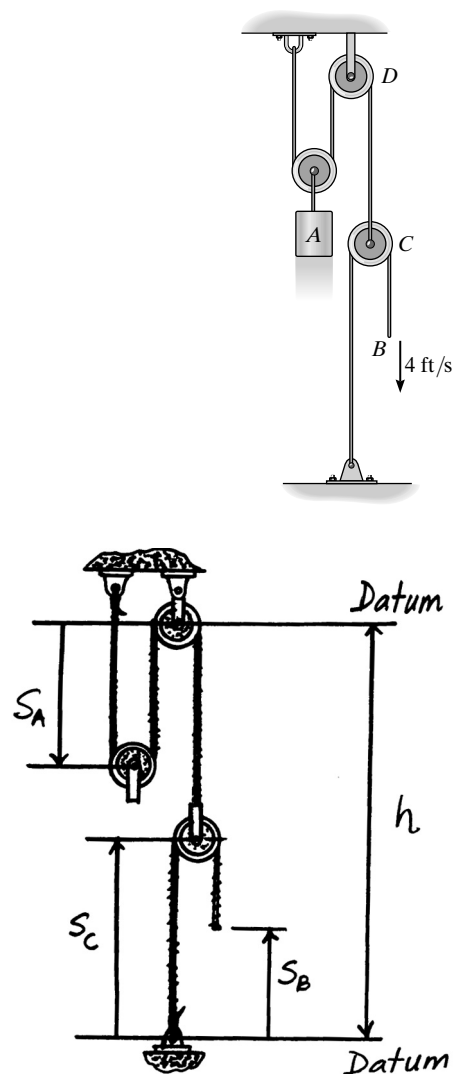
$$v_A = -1 \text{ ft/s} = 1 \text{ ft/s} \uparrow$$

$$2 = 4a_A$$

$$a_A = 0.5 \text{ ft/s}^2 = 0.5 \text{ ft/s}^2 \downarrow$$

Ans.

Ans.



12–206. If block A is moving downward with a speed of 4 ft/s while C is moving up at 2 ft/s, determine the speed of block B .

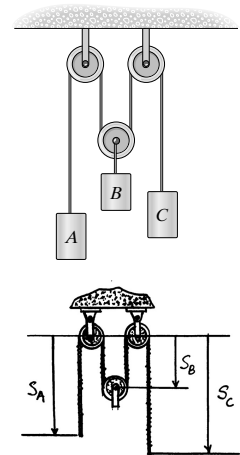
$$s_A + 2s_B + s_C = l$$

$$v_A + 2v_B + v_C = 0$$

$$4 + 2v_B - 2 = 0$$

$$v_B = -1 \text{ ft/s} = 1 \text{ ft/s} \uparrow$$

Ans.



12–207. If block A is moving downward at 6 ft/s while block C is moving down at 18 ft/s, determine the speed of block B .

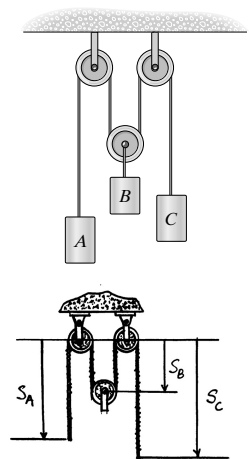
$$s_A = 2s_B + s_C = l$$

$$v_A = 2v_B + v_C = 0$$

$$6 + 2v_B + 18 = 0$$

$$v_B = -12 \text{ ft/s} = 12 \text{ ft/s} \uparrow$$

Ans.



***12–208.** If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block E rises.

Position-Coordinate Equation: Datum is established at fixed pulley. The position of point A , pulley B and C and block E with respect to datum are s_A , s_B , s_C and s_E , respectively. Since the system consists of three cords, three position-coordinate equations can be developed.

$$2s_B + s_A = l_1 \quad [1]$$

$$s_C + (s_C - s_B) = l_2 \quad [2]$$

$$s_E + (s_E - s_C) = l_3 \quad [3]$$

Eliminating s_C and s_B from Eqs. [1], [2] and [3], we have

$$s_A + 8s_E = l_1 + 2l_2 + 4l_3$$

Time Derivative: Taking the time derivative of the above equation yields

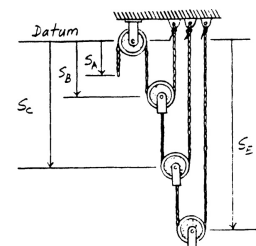
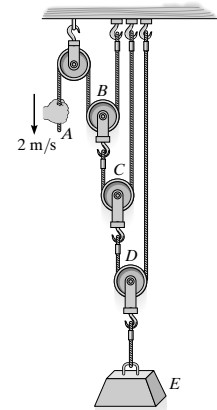
$$v_A + 8v_E = 0 \quad [4]$$

Since $v_A = 2 \text{ m/s}$, from Eq. [3]

$$(+\downarrow) \quad 2 + 8v_E = 0$$

$$v_E = -0.250 \text{ m/s} = 0.250 \text{ m/s} \uparrow$$

Ans.



•12–209. If motors at A and B draw in their attached cables with an acceleration of $a = (0.2t)$ m/s², where t is in seconds, determine the speed of the block when it reaches a height of $h = 4$ m, starting from rest at $h = 0$. Also, how much time does it take to reach this height?

$$s_A + 2s_D = l$$

$$(s_C - s_D) + s_C + s_B = l'$$

$$\Delta s_A = -2\Delta s_D \quad 2\Delta s_C - \Delta s_D + \Delta s_B = 0$$

If $\Delta s_C = -4$ and $\Delta s_A = \Delta s_B$, then,

$$\Delta s_A = -2\Delta s_D \quad 2(-4) - \Delta s_D + \Delta s_A = 0$$

$$\Delta s_D = -2.67 \text{ m} \quad \Delta s_A = \Delta s_B = 5.33 \text{ m}$$

Thus,

$$v_A = -2v_D$$

$$2v_C - v_D + v_B = 0$$

$$a = 0.2t$$

$$dv = a dt$$

$$\int_0^v dv = \int_0^t 0.2t dt$$

$$v = 0.1t^2$$

$$ds = v dt$$

$$\int_0^s ds = \int_0^t 0.1t^2 dt$$

$$s = \frac{0.1}{3} t^3 = 5.33$$

$$t = 5.428 \text{ s} = 5.43 \text{ s}$$

$$v = 0.1(5.428)^2 = 2.947 \text{ m/s}$$

$$v_A = v_B = 2.947 \text{ m/s}$$

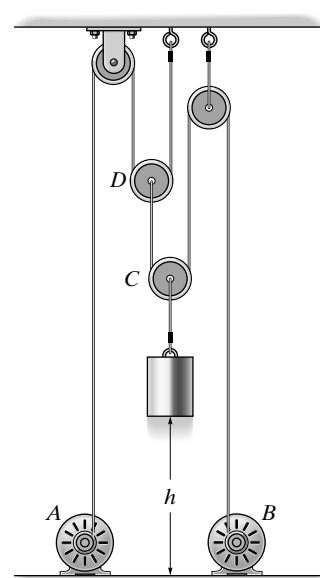
Thus, from Eqs. (1) and (2):

$$2.947 = -2v_D$$

$$v_D = -1.474$$

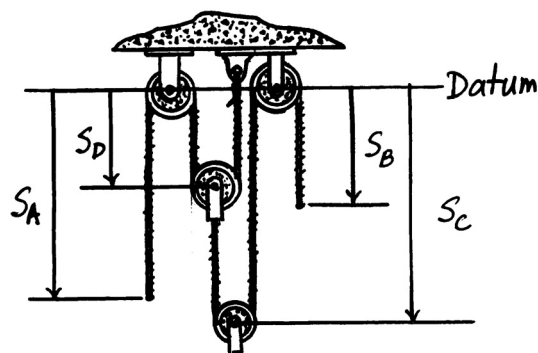
$$2v_C - (-1.474) + 2.947 = 0$$

$$v_C = -2.21 \text{ m/s} = 2.21 \text{ m/s} \uparrow$$



(1)

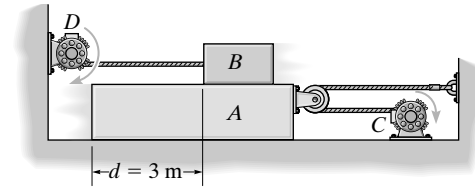
(2)



Ans.

Ans.

12–210. The motor at C pulls in the cable with an acceleration $a_C = (3t^2) \text{ m/s}^2$, where t is in seconds. The motor at D draws in its cable at $a_D = 5 \text{ m/s}^2$. If both motors start at the same instant from rest when $d = 3 \text{ m}$, determine (a) the time needed for $d = 0$, and (b) the velocities of blocks A and B when this occurs.



For A :

$$s_A + (s_A - s_C) = l$$

$$2v_A = v_C$$

$$2a_A = a_C = -3t^2$$

$$a_A = -1.5t^2 = 1.5t^2 \rightarrow$$

$$v_A = 0.5t^3 \rightarrow$$

$$s_A = 0.125t^4 \rightarrow$$

For B :

$$a_B = 5 \text{ m/s}^2 \leftarrow$$

$$v_B = 5t \leftarrow$$

$$s_B = 2.5t^2 \leftarrow$$

Require $s_A + s_B = d$

$$0.125t^4 + 2.5t^2 = 3$$

$$\text{Set } u = t^2 \quad 0.125u^2 + 2.5u = 3$$

The positive root is $u = 1.1355$. Thus,

$$t = 1.0656 = 1.07 \text{ s}$$

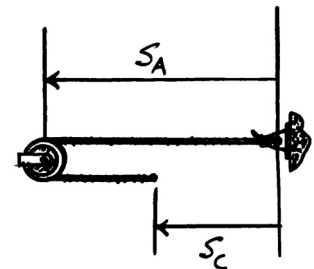
Ans.

$$v_A = .05(1.0656)^2 = 0.605 \text{ m/s}$$

Ans.

$$v_B = 5(1.0656) = 5.33 \text{ m/s}$$

Ans.



12–211. The motion of the collar at A is controlled by a motor at B such that when the collar is at $s_A = 3$ ft it is moving upwards at 2 ft/s and decreasing at 1 ft/s². Determine the velocity and acceleration of a point on the cable as it is drawn into the motor B at this instant.

$$\sqrt{s_A^2 + 4^2} + s_B = l$$

$$\frac{1}{2}(s_A^2 + 16)^{-\frac{1}{2}}(2s_A)\dot{s}_A + \dot{s}_B = 0$$

$$\dot{s}_B = -s_A\dot{s}_A(s_A^2 + 16)^{-\frac{1}{2}}$$

$$\dot{s}_B = -\left[\left(\dot{s}_A\right)^2(s_A^2 + 16)^{-\frac{1}{2}} + s_A\ddot{s}_A(s_A^2 + 16)^{-\frac{1}{2}} + s_A\dot{s}_A\left(-\frac{1}{2}\right)(s_A^2 + 16)^{-\frac{3}{2}}(2s_A\dot{s}_A)\right]$$

$$\dot{s}_B = \frac{(s_A\dot{s}_A)^2}{(s_A^2 + 16)^{\frac{3}{2}}} - \frac{(s_A)^2 + s_A\ddot{s}_A}{(s_A^2 + 16)^{\frac{1}{2}}}$$

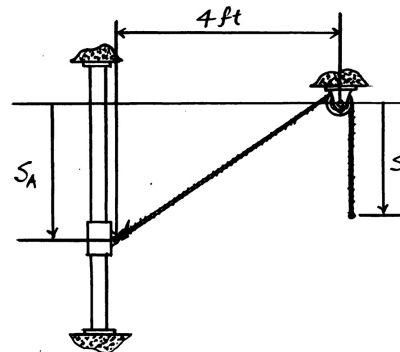
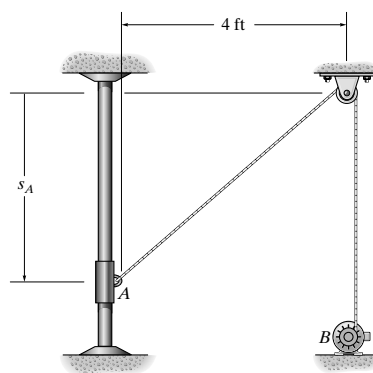
Evaluating these equations:

$$s_B = -3(-2)((3)^2 + 16)^{-\frac{1}{2}} = 1.20 \text{ ft/s } \downarrow$$

$$s_B = \frac{((3)(-2))^2}{((3)^2 + 16)^{\frac{3}{2}}} - \frac{(-2)^2 + 3(1)}{((3)^2 + 16)^{\frac{1}{2}}} = -1.11 \text{ ft/s}^2 = 1.11 \text{ ft/s}^2 \uparrow$$

Ans.

Ans.



***12–212.** The man pulls the boy up to the tree limb C by walking backward at a constant speed of 1.5 m/s. Determine the speed at which the boy is being lifted at the instant $x_A = 4$ m. Neglect the size of the limb. When $x_A = 0$, $y_B = 8$ m, so that A and B are coincident, i.e., the rope is 16 m long.

Position-Coordinate Equation: Using the Pythagorean theorem to determine l_{AC} , we have $l_{AC} = \sqrt{x_A^2 + 8^2}$. Thus,

$$l = l_{AC} + y_B$$

$$16 = \sqrt{x_A^2 + 8^2} + y_B$$

$$y_B = 16 - \sqrt{x_A^2 + 64}$$

[1]

Time Derivative: Taking the time derivative of Eq. [1] and realizing that $v_A = \frac{dx_A}{dt}$ and $v_B = \frac{dy_B}{dt}$, we have

$$v_B = \frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 64}} \frac{dx_A}{dt}$$

$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 64}} v_A$$

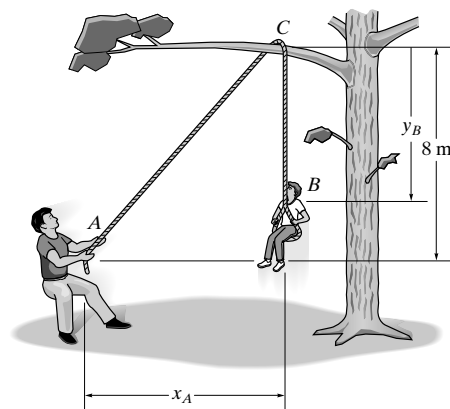
[2]

At the instant $x_A = 4$ m, from Eq. [2]

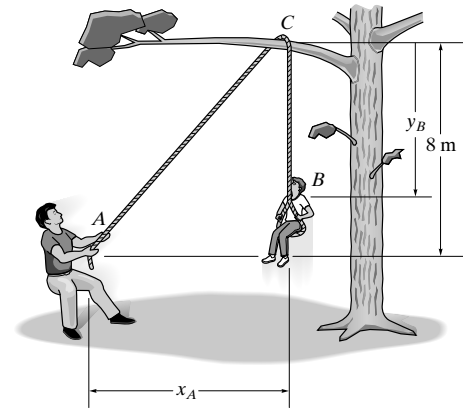
$$v_B = -\frac{4}{\sqrt{4^2 + 64}}(1.5) = -0.671 \text{ m/s} = 0.671 \text{ m/s } \uparrow$$

Ans.

Note: The negative sign indicates that velocity v_B is in the opposite direction to that of positive y_B .



•12–213. The man pulls the boy up to the tree limb C by walking backward. If he starts from rest when $x_A = 0$ and moves backward with a constant acceleration $a_A = 0.2 \text{ m/s}^2$, determine the speed of the boy at the instant $y_B = 4 \text{ m}$. Neglect the size of the limb. When $x_A = 0$, $y_B = 8 \text{ m}$, so that A and B are coincident, i.e., the rope is 16 m long.



Position-Coordinate Equation: Using the Pythagorean theorem to determine l_{AC} , we have $l_{AC} = \sqrt{x_A^2 + 8^2}$. Thus,

$$\begin{aligned} l &= l_{AC} + y_B \\ 16 &= \sqrt{x_A^2 + 8^2} + y_B \\ y_B &= 16 - \sqrt{x_A^2 + 64} \end{aligned} \quad [1]$$

Time Derivative: Taking the time derivative of Eq. [1] Where $v_A = \frac{dx_A}{dt}$ and $v_B = \frac{dy_B}{dt}$, we have

$$\begin{aligned} v_B &= \frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 64}} \frac{dx_A}{dt} \\ v_B &= -\frac{x_A}{\sqrt{x_A^2 + 64}} v_A \end{aligned} \quad [2]$$

At the instant $y_B = 4 \text{ m}$, from Eq. [1], $4 = 16 - \sqrt{x_A^2 + 64}$, $x_A = 8.944 \text{ m}$. The velocity of the man at that instant can be obtained.

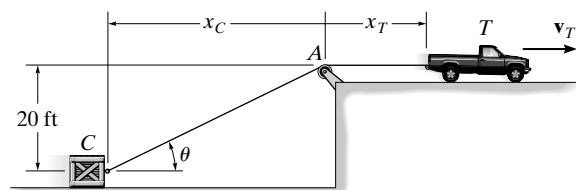
$$\begin{aligned} v_A^2 &= (v_0)_A^2 + 2(a_c)_A[s_A - (s_0)_A] \\ v_A^2 &= 0 + 2(0.2)(8.944 - 0) \\ v_A &= 1.891 \text{ m/s} \end{aligned}$$

Substitute the above results into Eq. [2] yields

$$v_B = -\frac{8.944}{\sqrt{8.944^2 + 64}} (1.891) = -1.41 \text{ m/s} = 1.41 \text{ m/s} \uparrow \quad \text{Ans.}$$

Note: The negative sign indicates that velocity v_B is in the opposite direction to that of positive y_B .

12–214. If the truck travels at a constant speed of $v_T = 6 \text{ ft/s}$, determine the speed of the crate for any angle θ of the rope. The rope has a length of 100 ft and passes over a pulley of negligible size at A. *Hint:* Relate the coordinates x_T and x_C to the length of the rope and take the time derivative. Then substitute the trigonometric relation between x_C and θ .



$$\sqrt{(20)^2 + x_C^2} + x_T = l = 100$$

$$\frac{1}{2} \left((20)^2 + (x_C)^2 \right)^{-\frac{1}{2}} (2x_C \dot{x}_C) + \dot{x}_T = 0$$

Since $\dot{x}_T = v_T = 6 \text{ ft/s}$, $v_C = \dot{x}_C$, and

$$x_C = 20 \cot \theta$$

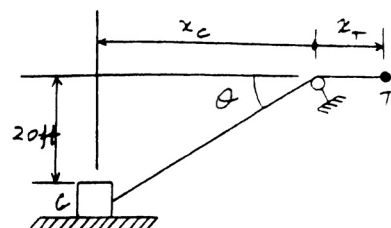
Then,

$$\frac{(20 \cot \theta) v_C}{(400 + 400 \cot^2 \theta)^{\frac{1}{2}}} = -6$$

Since $1 + \cot^2 \theta = \csc^2 \theta$,

$$\left(\frac{\cot \theta}{\csc \theta} \right) v_C = \cos \theta v_C = -6$$

$$v_C = -6 \sec \theta = (6 \sec \theta) \text{ ft/s} \rightarrow$$



Ans.

12–215. At the instant shown, car A travels along the straight portion of the road with a speed of 25 m/s. At this same instant car B travels along the circular portion of the road with a speed of 15 m/s. Determine the velocity of car B relative to car A.

Velocity: Referring to Fig. a, the velocity of cars A and B expressed in Cartesian vector form are

$$\mathbf{v}_A = [25 \cos 30^\circ \mathbf{i} - 25 \sin 30^\circ \mathbf{j}] \text{ m/s} = [21.65 \mathbf{i} - 12.5 \mathbf{j}] \text{ m/s}$$

$$\mathbf{v}_B = [15 \cos 15^\circ \mathbf{i} - 15 \sin 15^\circ \mathbf{j}] \text{ m/s} = [14.49 \mathbf{i} - 3.882 \mathbf{j}] \text{ m/s}$$

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$14.49 \mathbf{i} - 3.882 \mathbf{j} = 21.65 \mathbf{i} - 12.5 \mathbf{j} + \mathbf{v}_{B/A}$$

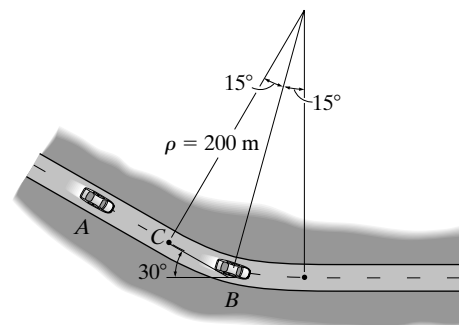
$$\mathbf{v}_{B/A} = [-7.162 \mathbf{i} + 8.618 \mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/A}$ is given by

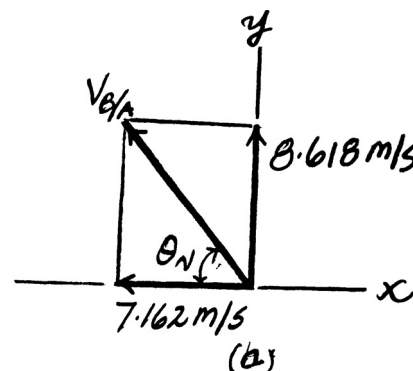
$$v_{B/A} = \sqrt{(-7.162)^2 + 8.618^2} = 11.2 \text{ m/s}$$

The direction angle θ_v of $\mathbf{v}_{B/A}$ measured from the x axis, Fig. a is

$$\theta_v = \tan^{-1} \left(\frac{8.618}{7.162} \right) = 50.3^\circ$$

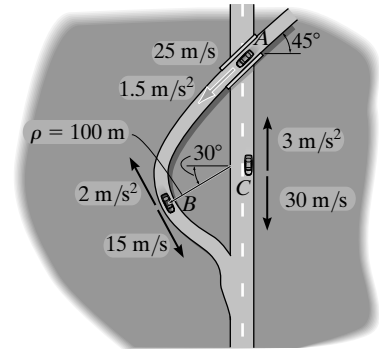


Ans.



Ans.

***12–216.** Car A travels along a straight road at a speed of 25 m/s while accelerating at 1.5 m/s^2 . At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2 . Determine the velocity and acceleration of car A relative to car C .



Velocity: The velocity of cars A and B expressed in Cartesian vector form are

$$\mathbf{v}_A = [-25 \cos 45^\circ \mathbf{i} - 25 \sin 45^\circ \mathbf{j}] \text{ m/s} = [-17.68 \mathbf{i} - 17.68 \mathbf{j}] \text{ m/s}$$

$$\mathbf{v}_C = [-30 \mathbf{j}] \text{ m/s}$$

Applying the relative velocity equation, we have

$$\mathbf{v}_A = \mathbf{v}_C + \mathbf{v}_{A/C}$$

$$-17.68 \mathbf{i} - 17.68 \mathbf{j} = -30 \mathbf{j} + \mathbf{v}_{A/C}$$

$$\mathbf{v}_{A/C} = [-17.68 \mathbf{i} + 12.32 \mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{A/C}$ is given by

$$v_{A/C} = \sqrt{(-17.68)^2 + 12.32^2} = 21.5 \text{ m/s} \quad \text{Ans.}$$

and the direction angle θ_v that $\mathbf{v}_{A/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{12.32}{17.68}\right) = 34.9^\circ \nearrow \quad \text{Ans.}$$

Acceleration: The acceleration of cars A and B expressed in Cartesian vector form are

$$\mathbf{a}_A = [-1.5 \cos 45^\circ \mathbf{i} - 1.5 \sin 45^\circ \mathbf{j}] \text{ m/s}^2 = [-1.061 \mathbf{i} - 1.061 \mathbf{j}] \text{ m/s}^2$$

$$\mathbf{a}_C = [3 \mathbf{j}] \text{ m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_C + \mathbf{a}_{A/C}$$

$$-1.061 \mathbf{i} - 1.061 \mathbf{j} = 3 \mathbf{j} + \mathbf{a}_{A/C}$$

$$\mathbf{a}_{A/C} = [-1.061 \mathbf{i} - 4.061 \mathbf{j}] \text{ m/s}^2$$

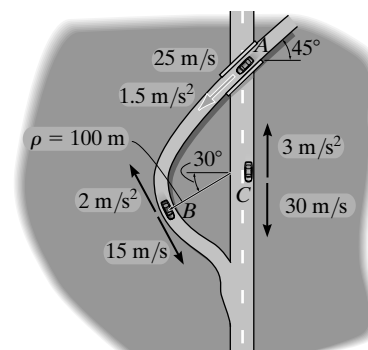
Thus, the magnitude of $\mathbf{a}_{A/C}$ is given by

$$a_{A/C} = \sqrt{(-1.061)^2 + (-4.061)^2} = 4.20 \text{ m/s}^2 \quad \text{Ans.}$$

and the direction angle θ_a that $\mathbf{a}_{A/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{4.061}{1.061}\right) = 75.4^\circ \searrow \quad \text{Ans.}$$

•12–217. Car B is traveling along the curved road with a speed of 15 m/s while decreasing its speed at 2 m/s^2 . At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2 . Determine the velocity and acceleration of car B relative to car C .



Velocity: The velocity of cars B and C expressed in Cartesian vector form are

$$\mathbf{v}_B = [15 \cos 60^\circ \mathbf{i} - 15 \sin 60^\circ \mathbf{j}] \text{ m/s} = [7.5\mathbf{i} - 12.99\mathbf{j}] \text{ m/s}$$

$$\mathbf{v}_C = [-30\mathbf{j}] \text{ m/s}$$

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$7.5\mathbf{i} - 12.99\mathbf{j} = -30\mathbf{j} + \mathbf{v}_{B/C}$$

$$\mathbf{v}_{B/C} = [7.5\mathbf{i} + 17.01\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/C}$ is given by

$$v_{B/C} = \sqrt{7.5^2 + 17.01^2} = 18.6 \text{ m/s} \quad \text{Ans.}$$

and the direction angle θ_v that $\mathbf{v}_{B/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{17.01}{7.5}\right) = 66.2^\circ \quad \text{Ans.}$$

Acceleration: The normal component of car B 's acceleration is $(a_B)_n = \frac{v_B^2}{\rho} = \frac{15^2}{100} = 2.25 \text{ m/s}^2$. Thus, the tangential and normal components of car B 's acceleration and the acceleration of car C expressed in Cartesian vector form are

$$(\mathbf{a}_B)_t = [-2 \cos 60^\circ \mathbf{i} + 2 \sin 60^\circ \mathbf{j}] = [-1\mathbf{i} + 1.732\mathbf{j}] \text{ m/s}^2$$

$$(\mathbf{a}_B)_n = [2.25 \cos 30^\circ \mathbf{i} + 2.25 \sin 30^\circ \mathbf{j}] = [1.9486\mathbf{i} + 1.125\mathbf{j}] \text{ m/s}^2$$

$$\mathbf{a}_C = [3\mathbf{j}] \text{ m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_C + \mathbf{a}_{B/C}$$

$$(-1\mathbf{i} + 1.732\mathbf{j}) + (1.9486\mathbf{i} + 1.125\mathbf{j}) = 3\mathbf{j} + \mathbf{a}_{B/C}$$

$$\mathbf{a}_{B/C} = [0.9486\mathbf{i} - 0.1429\mathbf{j}] \text{ m/s}^2$$

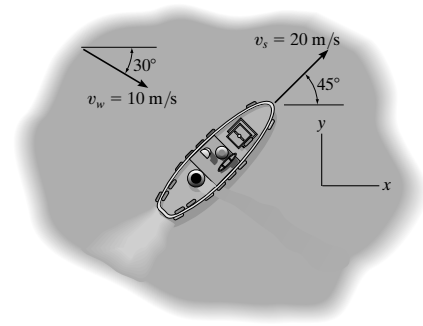
Thus, the magnitude of $\mathbf{a}_{B/C}$ is given by

$$a_{B/C} = \sqrt{0.9486^2 + (-0.1429)^2} = 0.959 \text{ m/s}^2 \quad \text{Ans.}$$

and the direction angle θ_a that $\mathbf{a}_{B/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{0.1429}{0.9486}\right) = 8.57^\circ \quad \text{Ans.}$$

12–218. The ship travels at a constant speed of $v_s = 20$ m/s and the wind is blowing at a speed of $v_w = 10$ m/s, as shown. Determine the magnitude and direction of the horizontal component of velocity of the smoke coming from the smoke stack as it appears to a passenger on the ship.



Solution I

Vector Analysis: The velocity of the smoke as observed from the ship is equal to the velocity of the wind relative to the ship. Here, the velocity of the ship and wind expressed in Cartesian vector form are $\mathbf{v}_s = [20 \cos 45^\circ \mathbf{i} + 20 \sin 45^\circ \mathbf{j}]$ m/s $= [14.14\mathbf{i} + 14.14\mathbf{j}]$ m/s and $\mathbf{v}_w = [10 \cos 30^\circ \mathbf{i} - 10 \sin 30^\circ \mathbf{j}] = [8.660\mathbf{i} - 5\mathbf{j}]$ m/s. Applying the relative velocity equation,

$$\mathbf{v}_w = \mathbf{v}_s + \mathbf{v}_{w/s}$$

$$8.660\mathbf{i} - 5\mathbf{j} = 14.14\mathbf{i} + 14.14\mathbf{j} + \mathbf{v}_{w/s}$$

$$\mathbf{v}_{w/s} = [-5.482\mathbf{i} - 19.14\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{w/s}$ is given by

$$v_w = \sqrt{(-5.482)^2 + (-19.14)^2} = 19.9 \text{ m/s} \quad \text{Ans.}$$

and the direction angle θ that $\mathbf{v}_{w/s}$ makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{19.14}{5.482}\right) = 74.0^\circ \quad \text{Ans.}$$

Solution II

Scalar Analysis: Applying the law of cosines by referring to the velocity diagram shown in Fig. *a*,

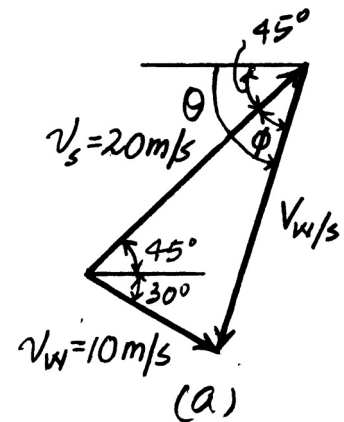
$$\begin{aligned} v_{w/s} &= \sqrt{20^2 + 10^2 - 2(20)(10) \cos 75^\circ} \\ &= 19.91 \text{ m/s} = 19.9 \text{ m/s} \end{aligned} \quad \text{Ans.}$$

Using the result of $v_{w/s}$ and applying the law of sines,

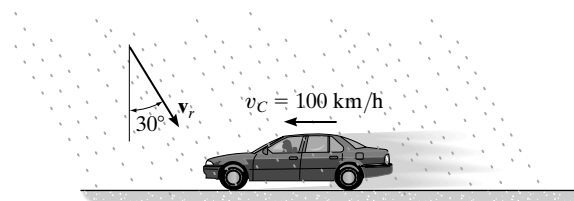
$$\frac{\sin \phi}{10} = \frac{\sin 75^\circ}{19.91} \quad \phi = 29.02^\circ$$

Thus,

$$\theta = 45^\circ + \phi = 74.0^\circ \quad \text{Ans.}$$



12–219. The car is traveling at a constant speed of 100 km/h. If the rain is falling at 6 m/s in the direction shown, determine the velocity of the rain as seen by the driver.



Solution I

Vector Analysis: The speed of the car is $v_c = \left(100 \frac{\text{km}}{\text{h}}\right) \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 27.78 \text{ m/s}$. The velocity of the car and the rain expressed in Cartesian vector form are $\mathbf{v}_c = [-27.78\mathbf{i}] \text{ m/s}$ and $\mathbf{v}_r = [6 \sin 30^\circ \mathbf{i} - 6 \cos 30^\circ \mathbf{j}] = [3\mathbf{i} - 5.196\mathbf{j}] \text{ m/s}$. Applying the relative velocity equation, we have

$$\mathbf{v}_r = \mathbf{v}_c + \mathbf{v}_{r/c}$$

$$3\mathbf{i} - 5.196\mathbf{j} = -27.78\mathbf{i} + \mathbf{v}_{r/c}$$

$$\mathbf{v}_{r/c} = [30.78\mathbf{i} - 5.196\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{r/c}$ is given by

$$v_{r/c} = \sqrt{30.78^2 + (-5.196)^2} = 31.2 \text{ m/s}$$

Ans.

and the angle θ makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{5.196}{30.78}\right) = 9.58^\circ$$

Ans.

Solution II

Scalar Analysis: Referring to the velocity diagram shown in Fig. *a* and applying the law of cosines,

$$\begin{aligned} v_{r/c} &= \sqrt{27.78^2 + 6^2 - 2(27.78)(6) \cos 120^\circ} \\ &= 19.91 \text{ m/s} = 19.9 \text{ m/s} \end{aligned}$$

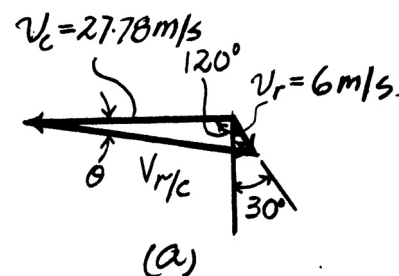
Ans.

Using the result of $v_{r/c}$ and applying the law of sines,

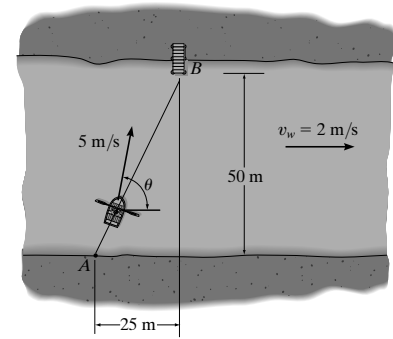
$$\frac{\sin \theta}{6} = \frac{\sin 120^\circ}{31.21}$$

$$\theta = 9.58^\circ$$

Ans.



***12–220.** The man can row the boat in still water with a speed of 5 m/s. If the river is flowing at 2 m/s, determine the speed of the boat and the angle θ he must direct the boat so that it travels from A to B .



Solution I

Vector Analysis: Here, the velocity \mathbf{v}_b of the boat is directed from A to B . Thus,

$\phi = \tan^{-1}\left(\frac{50}{25}\right) = 63.43^\circ$. The magnitude of the boat's velocity relative to the

flowing river is $v_{b/w} = 5$ m/s. Expressing \mathbf{v}_b , \mathbf{v}_w , and $\mathbf{v}_{b/w}$ in Cartesian vector form, we have $\mathbf{v}_b = v_b \cos 63.43^\circ \mathbf{i} + v_b \sin 63.43^\circ \mathbf{j} = 0.4472v_b \mathbf{i} + 0.8944v_b \mathbf{j}$, $\mathbf{v}_w = [2\mathbf{i}]$ m/s, and $\mathbf{v}_{b/w} = 5 \cos \theta \mathbf{i} + 5 \sin \theta \mathbf{j}$. Applying the relative velocity equation, we have

$$\mathbf{v}_b = \mathbf{v}_w + \mathbf{v}_{b/w}$$

$$0.4472v_b \mathbf{i} + 0.8944v_b \mathbf{j} = 2\mathbf{i} + 5 \cos \theta \mathbf{i} + 5 \sin \theta \mathbf{j}$$

$$0.4472v_b \mathbf{i} + 0.8944v_b \mathbf{j} = (2 + 5 \cos \theta)\mathbf{i} + 5 \sin \theta \mathbf{j}$$

Equating the \mathbf{i} and \mathbf{j} components, we have

$$0.4472v_b = 2 + 5 \cos \theta \quad (1)$$

$$0.8944v_b = 5 \sin \theta \quad (2)$$

Solving Eqs. (1) and (2) yields

$$v_b = 5.56 \text{ m/s} \quad \theta = 84.4^\circ \quad \text{Ans.}$$

Solution II

Scalar Analysis: Referring to the velocity diagram shown in Fig. *a* and applying the law of cosines,

$$5^2 = 2^2 + v_b^2 - 2(2)(v_b) \cos 63.43^\circ$$

$$v_b^2 - 1.789v_b - 21 = 0$$

$$v_b = \frac{-(-1.789) \pm \sqrt{(-1.789)^2 - 4(1)(-21)}}{2(1)}$$

Choosing the positive root,

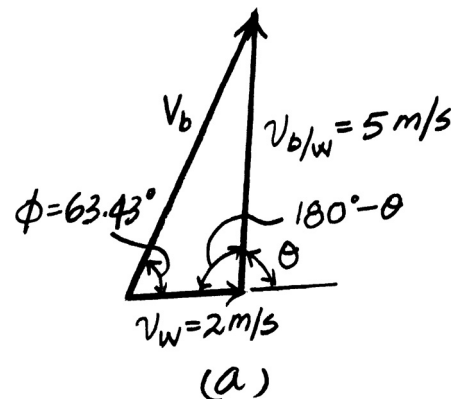
$$v_b = 5.563 \text{ m/s} = 5.56 \text{ m/s} \quad \text{Ans.}$$

Using the result of v_b and applying the law of sines,

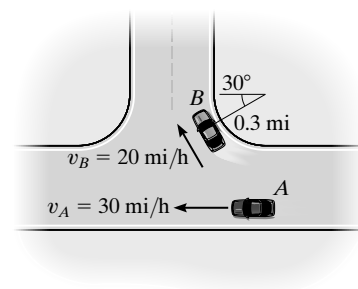
$$\frac{\sin 180^\circ - \theta}{5.563} = \frac{\sin 63.43^\circ}{5}$$

$$\theta = 84.4^\circ$$

Ans.



•12–221. At the instant shown, cars A and B travel at speeds of 30 mi/h and 20 mi/h, respectively. If B is increasing its speed by 1200 mi/h^2 , while A maintains a constant speed, determine the velocity and acceleration of B with respect to A .



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$\begin{array}{c} 20 \quad 30^\circ \\ \nearrow \\ \leftarrow 30 + (v_{B/A})_x + (v_{B/A})_y \\ \uparrow \end{array}$$

$$(\rightarrow) \quad -20 \sin 30^\circ = -30 + (v_{B/A})_x$$

$$(+\uparrow) \quad 20 \cos 30^\circ = (v_{B/A})_y$$

Solving

$$(v_{B/A})_x = 20 \rightarrow$$

$$(v_{B/A})_y = 17.32 \uparrow$$

$$v_{B/A} = \sqrt{(20)^2 + (17.32)^2} = 26.5 \text{ mi/h}$$

Ans.

$$\theta = \tan^{-1}\left(\frac{17.32}{20}\right) = 40.9^\circ \swarrow \theta$$

Ans.

$$(a_B)_n = \frac{(20)^2}{0.3} = 1333.3$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$\begin{array}{c} 1200 \quad 30^\circ \\ \nearrow \\ 1333.3 \\ \swarrow 30^\circ \\ 0 + (a_{B/A})_x + (a_{B/A})_y \\ \uparrow \end{array}$$

$$(\rightarrow) \quad -1200 \sin 30^\circ + 1333.3 \cos 30^\circ = (a_{B/A})_x$$

$$(+\uparrow) \quad 1200 \cos 30^\circ + 1333.3 \sin 30^\circ = (a_{B/A})_y$$

Solving

$$(a_{B/A})_x = 554.7 \rightarrow ; (a_{B/A})_y = 1705.9 \uparrow$$

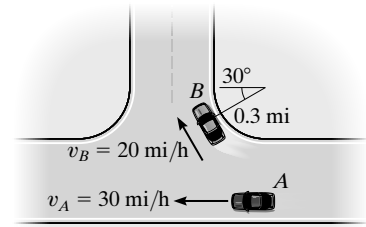
$$a_{B/A} = \sqrt{(554.7)^2 + (1705.9)^2} = 1.79(10^3) \text{ mi/h}^2$$

Ans.

$$\theta = \tan^{-1}\left(\frac{1705.9}{554.7}\right) = 72.0^\circ \swarrow \theta$$

Ans.

12–222. At the instant shown, cars A and B travel at speeds of 30 m/h and 20 mi/h, respectively. If A is increasing its speed at 400 mi/h² whereas the speed of B is decreasing at 800 mi/h², determine the velocity and acceleration of B with respect to A .



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$\begin{matrix} 20 & 30^\circ \\ \nearrow & \\ 30 & + (v_{B/A})_x + (v_{B/A})_y \\ \leftarrow & \rightarrow \quad \uparrow \end{matrix}$$

$$(\rightarrow) \quad -20 \sin 30^\circ = -30 + (v_{B/A})_x$$

$$(\uparrow) \quad 20 \cos 30^\circ = (v_{B/A})_y$$

Solving

$$(v_{B/A})_x = 20 \rightarrow$$

$$(v_{B/A})_y = 17.32 \uparrow$$

$$v_{B/A} = \sqrt{(20)^2 + (17.32)^2} = 26.5 \text{ mi/h}$$

Ans.

$$\theta = \tan^{-1}\left(\frac{17.32}{20}\right) = 40.9^\circ \nearrow \theta$$

Ans.

$$\mathbf{a}_B + \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$\begin{matrix} \frac{20^2}{0.3} = 1333.3 & + [800] & = [400] + [(a_{B/A})_x] + [(a_{B/A})_y] \\ \nearrow 30^\circ & \nearrow 30^\circ & \leftarrow \quad \rightarrow \quad \uparrow \end{matrix}$$

$$(\rightarrow) \quad 1333.3 \cos 30^\circ + 800 \sin 30^\circ = -400 + (a_{B/A})_x$$

$$(a_{B/A})_x = 1954.7 \rightarrow$$

$$(\uparrow) \quad 1333.3 \sin 30^\circ - 800 \cos 30^\circ = (a_{B/A})_y$$

$$(a_{B/A})_y = -26.154 = 26.154 \downarrow$$

$$(a_{B/A}) = \sqrt{(1954.7)^2 + (26.154)^2}$$

$$a_{B/A} = 1955 \text{ mi/h}^2$$

Ans.

$$\theta = \tan^{-1}\left(\frac{26.154}{1954.7}\right) = 0.767^\circ \searrow \theta$$

Ans.

12–223. Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 20$ ft/s and $v_B = 15$ ft/s, determine the velocity of boat A with respect to boat B . How long after leaving the shore will the boats be 800 ft apart?

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-20 \sin 30^\circ \mathbf{i} + 20 \cos 30^\circ \mathbf{j} = 15 \cos 45^\circ \mathbf{i} + 15 \sin 45^\circ \mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-20.61 \mathbf{i} + 6.714 \mathbf{j}\} \text{ ft/s}$$

$$v_{A/B} = \sqrt{(-20.61)^2 + (6.714)^2} = 21.7 \text{ ft/s}$$

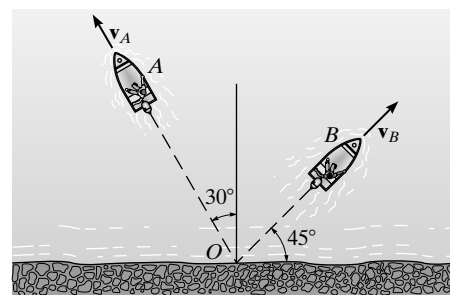
$$\theta = \tan^{-1} \left(\frac{6.714}{20.61} \right) = 18.0^\circ \text{ } \searrow$$

$$(800)^2 = (20t)^2 + (15t)^2 - 2(20t)(15t) \cos 75^\circ$$

$$t = 36.9 \text{ s}$$

Also

$$t = \frac{800}{v_{A/B}} = \frac{800}{21.68} = 36.9 \text{ s}$$

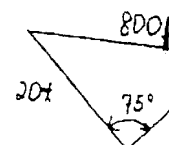


Ans.

Ans.

Ans.

Ans.



***12–224.** At the instant shown, cars A and B travel at speeds of 70 mi/h and 50 mi/h, respectively. If B is increasing its speed by 1100 mi/h^2 , while A maintains a constant speed, determine the velocity and acceleration of B with respect to A . Car B moves along a curve having a radius of curvature of 0.7 mi.

Relative Velocity:

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$50 \sin 30^\circ \mathbf{i} + 50 \cos 30^\circ \mathbf{j} = 70 \mathbf{j} + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = \{25.0 \mathbf{i} - 26.70 \mathbf{j}\} \text{ mi/h}$$

Thus, the magnitude of the relative velocity $\mathbf{v}_{B/A}$ is

$$v_{B/A} = \sqrt{25.0^2 + (-26.70)^2} = 36.6 \text{ mi/h}$$

Ans.

The direction of the relative velocity is the same as the direction of that for relative acceleration. Thus

$$\theta = \tan^{-1} \frac{26.70}{25.0} = 46.9^\circ \text{ } \swarrow$$

Ans.

Relative Acceleration: Since car B is traveling along a curve, its normal acceleration is $(a_B)_n = \frac{v_B^2}{\rho} = \frac{50^2}{0.7} = 3571.43 \text{ mi/h}^2$. Applying Eq. 12–35 gives

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$(1100 \sin 30^\circ + 3571.43 \cos 30^\circ) \mathbf{i} + (1100 \cos 30^\circ - 3571.43 \sin 30^\circ) \mathbf{j} = 0 + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{3642.95 \mathbf{i} - 833.09 \mathbf{j}\} \text{ mi/h}^2$$

Thus, the magnitude of the relative velocity $\mathbf{a}_{B/A}$ is

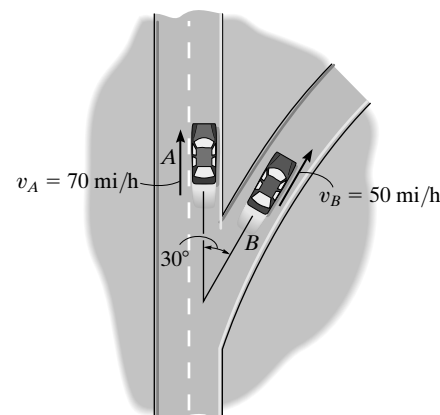
$$a_{B/A} = \sqrt{3642.95^2 + (-833.09)^2} = 3737 \text{ mi/h}^2$$

Ans.

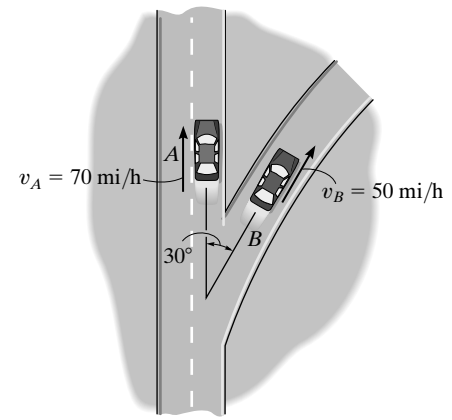
And its direction is

$$\phi = \tan^{-1} \frac{833.09}{3642.95} = 12.9^\circ \text{ } \swarrow$$

Ans.



•12–225. At the instant shown, cars A and B travel at speeds of 70 mi/h and 50 mi/h, respectively. If B is decreasing its speed at 1400 mi/h^2 while A is increasing its speed at 800 mi/h^2 , determine the acceleration of B with respect to A . Car B moves along a curve having a radius of curvature of 0.7 mi.



Relative Acceleration: Since car B is traveling along a curve, its normal acceleration

is $(a_B)_n = \frac{v_B^2}{\rho} = \frac{50^2}{0.7} = 3571.43 \text{ mi/h}^2$. Applying Eq. 12–35 gives

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$(3571.43 \cos 30^\circ - 1400 \sin 30^\circ)\mathbf{i} + (-1400 \cos 30^\circ - 3571.43 \sin 30^\circ)\mathbf{j} = 800\mathbf{j} + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{2392.95\mathbf{i} - 3798.15\mathbf{j}\} \text{ mi/h}^2$$

Thus, the magnitude of the relative acc. $\mathbf{a}_{B/A}$ is

$$a_{B/A} = \sqrt{2392.95^2 + (-3798.15)^2} = 4489 \text{ mi/h}^2$$

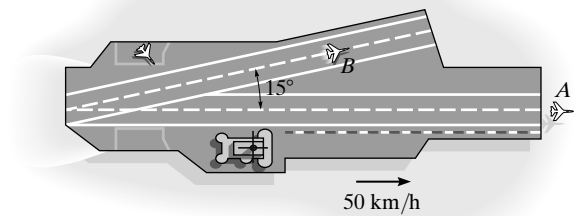
Ans.

And its direction is

$$\phi = \tan^{-1} \frac{3798.15}{2392.95} = 57.8^\circ \swarrow$$

Ans.

12–226. An aircraft carrier is traveling forward with a velocity of 50 km/h. At the instant shown, the plane at A has just taken off and has attained a forward horizontal air speed of 200 km/h, measured from still water. If the plane at B is traveling along the runway of the carrier at 175 km/h in the direction shown, determine the velocity of A with respect to B .



$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$\mathbf{v}_B = 50\mathbf{i} + 175 \cos 15^\circ \mathbf{i} + 175 \sin 15^\circ \mathbf{j} = 219.04\mathbf{i} + 45.293\mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$200\mathbf{i} = 219.04\mathbf{i} + 45.293\mathbf{j} + (v_{A/B})_x \mathbf{i} + (v_{A/B})_y \mathbf{j}$$

$$200 = 219.04 + (v_{A/B})_x$$

$$0 = 45.293 + (v_{A/B})_y$$

$$(v_{A/B})_x = -19.04$$

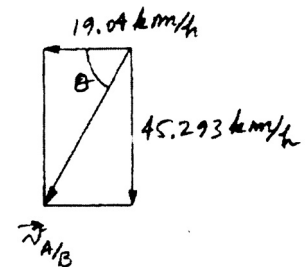
$$(v_{A/B})_y = -45.293$$

$$v_{A/B} = \sqrt{(-19.04)^2 + (-45.293)^2} = 49.1 \text{ km/h}$$

$$\theta = \tan^{-1} \left(\frac{45.293}{19.04} \right) = 67.2^\circ \searrow$$

Ans.

Ans.



12–227. A car is traveling north along a straight road at 50 km/h. An instrument in the car indicates that the wind is directed towards the east. If the car's speed is 80 km/h, the instrument indicates that the wind is directed towards the north-east. Determine the speed and direction of the wind.

Solution I

Vector Analysis: For the first case, the velocity of the car and the velocity of the wind relative to the car expressed in Cartesian vector form are $\mathbf{v}_c = [50\mathbf{j}]$ km/h and $\mathbf{v}_{w/c} = (v_{w/c})_1 \mathbf{i}$. Applying the relative velocity equation, we have

$$\begin{aligned}\mathbf{v}_c &= \mathbf{v}_w + \mathbf{v}_{w/c} \\ \mathbf{v}_w &= 50\mathbf{j} + (v_{w/c})_1 \mathbf{i} \\ \mathbf{v}_w &= (v_{w/c})_1 \mathbf{i} + 50\mathbf{j} \quad (1)\end{aligned}$$

For the second case, $\mathbf{v}_c = [80\mathbf{j}]$ km/h and $\mathbf{v}_{w/c} = (v_{w/c})_2 \cos 45^\circ \mathbf{i} + (v_{w/c})_2 \sin 45^\circ \mathbf{j}$. Applying the relative velocity equation, we have

$$\begin{aligned}\mathbf{v}_w &= \mathbf{v}_c + \mathbf{v}_{w/c} \\ \mathbf{v}_w &= 80\mathbf{j} + (v_{w/c})_2 \cos 45^\circ \mathbf{i} + (v_{w/c})_2 \sin 45^\circ \mathbf{j} \\ \mathbf{v}_w &= (v_{w/c})_2 \cos 45^\circ \mathbf{i} + [80 + (v_{w/c})_2 \sin 45^\circ] \mathbf{j} \quad (2)\end{aligned}$$

Equating Eqs. (1) and (2) and then the \mathbf{i} and \mathbf{j} components,

$$(v_{w/c})_1 = (v_{w/c})_2 \cos 45^\circ \quad (3)$$

$$50 = 80 + (v_{w/c})_2 \sin 45^\circ \quad (4)$$

Solving Eqs. (3) and (4) yields

$$(v_{w/c})_2 = -42.43 \text{ km/h} \quad (v_{w/c})_1 = -30 \text{ km/h}$$

Substituting the result of $(v_{w/c})_1$ into Eq. (1),

$$\mathbf{v}_w = [-30\mathbf{i} + 50\mathbf{j}] \text{ km/h}$$

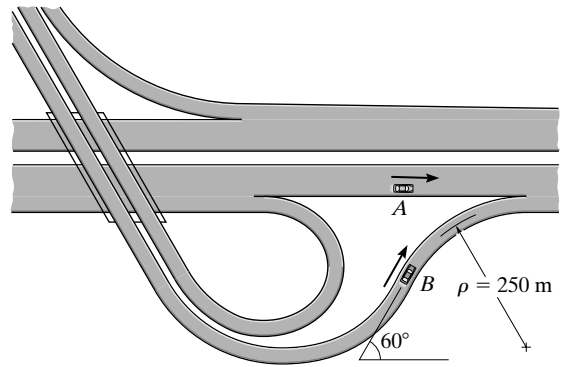
Thus, the magnitude of \mathbf{v}_w is

$$v_w = \sqrt{(-30)^2 + 50^2} = 58.3 \text{ km/h} \quad \text{Ans.}$$

and the directional angle θ that \mathbf{v}_w makes with the x axis is

$$\theta = \tan^{-1} \left(\frac{50}{-30} \right) = 59.0^\circ \searrow \quad \text{Ans.}$$

***12–228.** At the instant shown car A is traveling with a velocity of 30 m/s and has an acceleration of 2 m/s^2 along the highway. At the same instant B is traveling on the trumpet interchange curve with a speed of 15 m/s , which is decreasing at 0.8 m/s^2 . Determine the relative velocity and relative acceleration of B with respect to A at this instant.



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$15 \cos 60^\circ \mathbf{i} + 15 \sin 60^\circ \mathbf{j} = 30 \mathbf{i} + (v_{B/A})_x \mathbf{i} + (v_{B/A})_y \mathbf{j}$$

$$15 \cos 60^\circ = 30 + (v_{B/A})_x$$

$$15 \sin 60^\circ = 0 + (v_{B/A})_y$$

$$(v_{B/A})_x = -22.5 = 22.5 \text{ m/s} \leftarrow$$

$$(v_{B/A})_y = 12.99 \text{ m/s} \uparrow$$

$$v_{B/A} = \sqrt{(22.5)^2 + (12.99)^2} = 26.0 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{12.99}{22.5}\right) = 30^\circ \searrow$$

$$(a_B)_n = \frac{v^2}{\rho} = \frac{15^2}{250} = 0.9 \text{ m/s}^2$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$-0.8 \cos 60^\circ \mathbf{i} - 0.8 \sin 60^\circ \mathbf{j} + 0.9 \sin 60^\circ \mathbf{i} - 0.9 \cos 60^\circ \mathbf{j} = 2 \mathbf{i} + (a_{B/A})_x \mathbf{i} + (a_{B/A})_y \mathbf{j}$$

$$-0.8 \cos 60^\circ + 0.9 \sin 60^\circ = 2 + (a_{B/A})_x$$

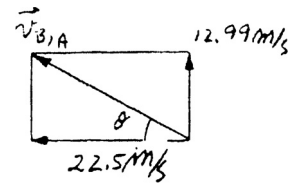
$$-0.8 \sin 60^\circ - 0.9 \cos 60^\circ = (a_{B/A})_y$$

$$(a_{B/A})_x = -1.6206 \text{ ft/s}^2 = 1.6206 \text{ m/s}^2 \leftarrow$$

$$(a_{B/A})_y = -1.1428 \text{ ft/s}^2 = 1.1428 \text{ m/s}^2 \downarrow$$

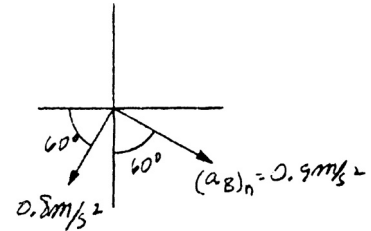
$$a_{B/A} = \sqrt{(1.6206)^2 + (1.1428)^2} = 1.98 \text{ m/s}^2$$

$$\phi = \tan^{-1}\left(\frac{1.1428}{1.6206}\right) = 35.2^\circ \nearrow$$



Ans.

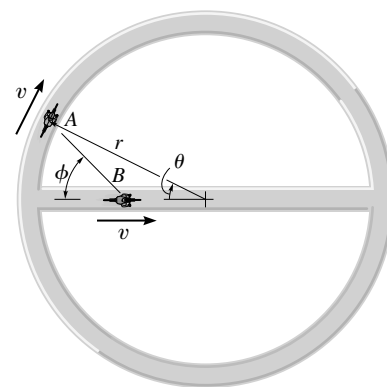
Ans.



Ans.

Ans.

•12–229. Two cyclists A and B travel at the same constant speed v . Determine the velocity of A with respect to B if A travels along the circular track, while B travels along the diameter of the circle.



$$\mathbf{v}_A = v \sin \theta \mathbf{i} + v \cos \theta \mathbf{j} \quad \mathbf{v}_B = v \mathbf{i}$$

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$= (v \sin \theta \mathbf{i} + v \cos \theta \mathbf{j}) - v \mathbf{i}$$

$$= (v \sin \theta - v) \mathbf{i} + v \cos \theta \mathbf{j}$$

$$v_{A/B} = \sqrt{(v \sin \theta - v)^2 + (v \cos \theta)^2}$$

$$= \sqrt{2v^2 - 2v^2 \sin \theta}$$

$$= v\sqrt{2(1 - \sin \theta)}$$

Ans.

12–230. A man walks at 5 km/h in the direction of a 20-km/h wind. If raindrops fall vertically at 7 km/h in *still air*, determine the direction in which the drops appear to fall with respect to the man. Assume the horizontal speed of the raindrops is equal to that of the wind.

Relative Velocity: The velocity of the rain must be determined first. Applying Eq. 12–34 gives

$$\mathbf{v}_r = \mathbf{v}_w + \mathbf{v}_{r/w} = 20\mathbf{i} + (-7\mathbf{j}) = \{20\mathbf{i} - 7\mathbf{j}\} \text{ km/h}$$

Thus, the relative velocity of the rain with respect to the man is

$$\mathbf{v}_r = \mathbf{v}_m + \mathbf{v}_{r/m}$$

$$20\mathbf{i} - 7\mathbf{j} = 5\mathbf{i} + \mathbf{v}_{r/m}$$

$$\mathbf{v}_{r/m} = \{15\mathbf{i} - 7\mathbf{j}\} \text{ km/h}$$

The magnitude of the relative velocity $\mathbf{v}_{r/m}$ is given by

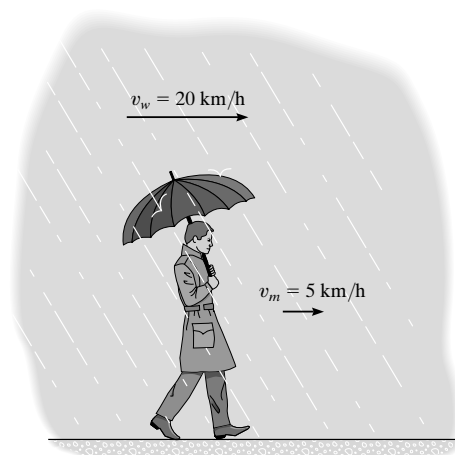
$$v_{r/m} = \sqrt{15^2 + (-7)^2} = 16.6 \text{ km/h}$$

Ans.

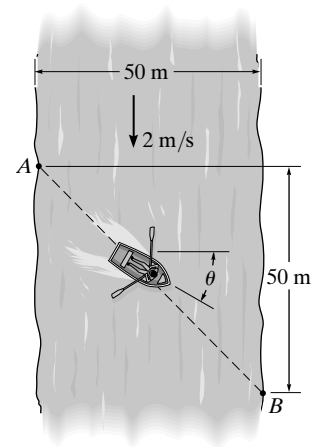
And its direction is given by

$$\theta = \tan^{-1} \frac{7}{15} = 25.0^\circ \swarrow$$

Ans.



12–231. A man can row a boat at 5 m/s in still water. He wishes to cross a 50-m-wide river to point *B*, 50 m downstream. If the river flows with a velocity of 2 m/s, determine the speed of the boat and the time needed to make the crossing.



Relative Velocity:

$$\mathbf{v}_b = \mathbf{v}_r + \mathbf{v}_{b/r}$$

$$v_b \sin 45^\circ \mathbf{i} - v_b \cos 45^\circ \mathbf{j} = -2\mathbf{j} + 5 \cos \theta \mathbf{i} - 5 \sin \theta \mathbf{j}$$

Equating **i** and **j** component, we have

$$v_b \sin 45^\circ = 5 \cos \theta \quad [1]$$

$$-v_b \cos 45^\circ = -2 - 5 \sin \theta \quad [2]$$

Solving Eqs. [1] and [2] yields

$$\theta = 28.57^\circ$$

$$v_b = 6.210 \text{ m/s} = 6.21 \text{ m/s} \quad \text{Ans.}$$

Thus, the time *t* required by the boat to travel from point *A* to *B* is

$$t = \frac{s_{AB}}{v_b} = \frac{\sqrt{50^2 + 50^2}}{6.210} = 11.4 \text{ s} \quad \text{Ans.}$$